CHAPTER 6 QUANTUM THEORY AND ATOMIC STRUCTURE

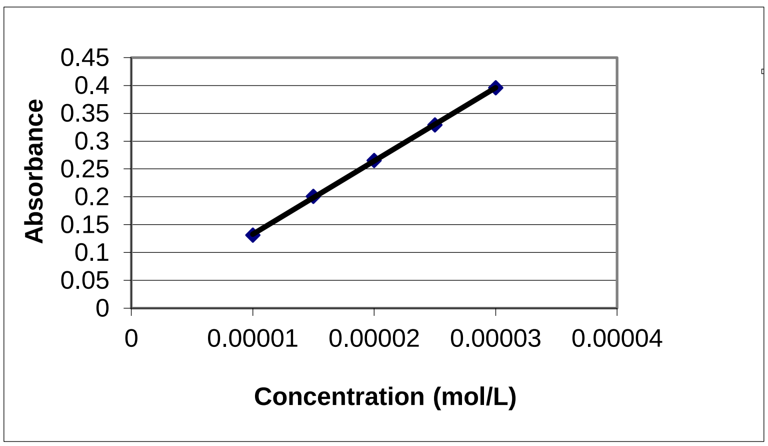
The value for the speed of light will be 3.00×10^8 m/s except when more significant figures are necessary, in which cases, 2.9979×10^8 m/s will be used.

TOOLS OF THE LABORATORY BOXED READING PROBLEMS

B6.1 Plan: Plot absorbance on the *y*-axis and concentration on the *x*-axis. Since this is a linear plot, the graph is of the type $y = mx + b$, with m = slope and b = intercept. Any two points may be used to find the slope, and the slope is used to find the intercept. Once the equation for the line is known, the absorbance of the solution in part b) is used to find the concentration of the diluted solution, after which the dilution equation is used to find the concentration (mol/L) of the original solution.

Solution:

a) Absorbance vs. Concentration:



This is a linear plot, thus, using the first and last points given:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(0.396 - 0.131)}{(3.0\text{x}10^{-5} - 1.0\text{x}10^{-5})\text{mol / L}} = 13{,}250 \text{ /mol/L} = \mathbf{1.3x10^4/mol/L}$$

Using the slope just calculated and any of the data points, the value of the intercept may be found.

$b = y - mx = 0.396 - (13{,}250\text{/mol/L})(3.0\text{x}10^{-5}\text{ mol/L}) = -0.0015 = \mathbf{0.00}$ (absorbance has no units)

b) Use the equation just determined: $y = (1.3\text{x}10^4\text{/mol/L})\, x + 0.00$.

$x = (y - 0.00)/(1.3\text{x}10^4\text{/mol/L}) = (0.236/1.3\text{x}10^4)\text{ mol/L} = 1.81538\text{x}10^{-5}\text{ mol/L} = \mathbf{1.8x10^{-5}\ mol/L}$

This value is c_f in a dilution problem $(c_iV_i) = (c_fV_f)$ with V_i = 20.0 mL and V_f = 150. mL.

$$M_i = \frac{(c_f)(V_f)}{(V_i)} = \frac{(1.81538\text{x}10^{-5}\text{ mol / L})(150.\text{ mL})}{(20.0\text{ mL})} = 1.361538\text{x}10^{-4}\text{ mol/L} = \mathbf{1.4x10^{-4}\ mol/L}$$

END–OF–CHAPTER PROBLEMS

6.2 Plan: Recall that the shorter the wavelength, the higher the frequency and the greater the energy. Figure 6.3 describes the electromagnetic spectrum by wavelength and frequency.

Solution:

a) Wavelength increases from left (10^{-2} nm) to right (10^{12} nm) in Figure 6.3. The trend in increasing wavelength is: **x-ray < ultraviolet < visible < infrared < microwave < radio wave**.

b) Frequency is inversely proportional to wavelength according to the equation $c = \lambda\nu$, so frequency has the opposite trend: **radio wave < microwave < infrared < visible < ultraviolet < x-ray**.
c) Energy is directly proportional to frequency according to the equation $E = h\nu$. Therefore, the trend in increasing energy matches the trend in increasing frequency: **radio wave < microwave < infrared < visible < ultraviolet < x-ray**.

6.7 Plan: Wavelength is related to frequency through the equation $c = \lambda\nu$. Recall that a Hz is a reciprocal second, or $1/\text{s} = \text{s}^{-1}$. Assume that the number "950" has three significant figures.
Solution:
$c = \lambda\nu$

$$\lambda\ (\text{m}) = \frac{c}{\nu} = \frac{3.00\text{x}10^{8}\ \text{m/s}}{(950.\ \text{kHz})\left(\frac{10^{3}\ \text{Hz}}{1\ \text{kHz}}\right)\left(\frac{\text{s}^{-1}}{\text{Hz}}\right)} = 315.789\ \text{m} = \mathbf{316\ m}$$

$$\lambda\ (\text{nm}) = \frac{c}{\nu} = (315.789\ \text{m})\left(\frac{1\ \text{nm}}{10^{-9}\ \text{m}}\right) = 3.15789\text{x}10^{11}\ \text{m} = \mathbf{3.16x10^{11}\ nm}$$

6.9 Plan: Frequency is related to energy through the equation $E = h\nu$. Note that $1\ \text{Hz} = 1\ \text{s}^{-1}$.
Solution:
$E = h\nu$
$E = (6.626\text{x}10^{-34}\ \text{J•s})(3.8\text{x}10^{10}\ \text{s}^{-1}) = 2.51788\text{x}10^{-23}\ \text{J} = \mathbf{2.5x10^{-23}\ J}$

6.11 Plan: Energy is inversely proportional to wavelength ($E = \frac{hc}{\lambda}$). As wavelength decreases, energy increases.
Solution:
In terms of increasing energy the order is **red < yellow < blue**.

6.13 Plan: Wavelength is related to frequency through the equation $c = \lambda\nu$. Recall that a Hz is a reciprocal second, or $1/\text{s} = \text{s}^{-1}$.
Solution:

$$\nu = (\text{s}^{-1}) = (22.235\ \text{GHz})\left(\frac{10^{9}\ \text{Hz}}{1\ \text{GHz}}\right)\left(\frac{\text{s}^{-1}}{\text{Hz}}\right) = 2.2235\text{x}10^{10}\ \text{s}^{-1}$$

$$\lambda\ (\text{nm}) = \frac{c}{\nu} = \frac{2.9979\text{x}10^{8}\ \text{m/s}}{2.2235\text{x}10^{10}\ \text{s}^{-1}}\left(\frac{1\ \text{nm}}{10^{-9}\ \text{m}}\right) = 1.3482797\text{x}10^{7}\ \text{nm} = \mathbf{1.3483x10^{7}\ nm}$$

6.16 Plan: The least energetic photon in part a) has the longest wavelength (242 nm). The most energetic photon in part b) has the shortest wavelength (220 nm). Use the relationship $c = \lambda\nu$ to find the frequency of the photons and relationship $E = \frac{hc}{\lambda}$ to find the energy.
Solution:
a) $c = \lambda\nu$

$$\nu = \frac{c}{\lambda} = \frac{3.00\text{x}10^{8}\ \text{m/s}}{242\ \text{nm}}\left(\frac{1\ \text{nm}}{10^{-9}\ \text{m}}\right) = 1.239669\text{x}10^{15}\ \text{s}^{-1} = \mathbf{1.24x10^{15}\ s^{-1}}$$

$$E = \frac{hc}{\lambda} = \frac{(6.626\text{x}10^{-34}\ \text{J•s})(3.00\text{x}10^{8}\ \text{m/s})}{242\ \text{nm}}\left(\frac{1\ \text{nm}}{10^{-9}\ \text{m}}\right) = 8.2140\text{x}10^{-19}\ \text{J} = \mathbf{8.21x10^{-19}\ J}$$

b) $\nu = \dfrac{c}{\lambda} = \dfrac{3.00\text{x}10^8\ \text{m/s}}{220\ \text{nm}}\left(\dfrac{1\ \text{nm}}{10^{-9}\ \text{m}}\right)$ = $1.3636\text{x}10^{15}\ \text{s}^{-1}$ = $\mathbf{1.4x10^{15}\ s^{-1}}$

$E = \dfrac{hc}{\lambda} = \dfrac{\left(6.626\text{x}10^{-34}\ \text{J•s}\right)\left(3.00\text{x}10^8\ \text{m/s}\right)}{220\ \text{nm}}\left(\dfrac{1\ \text{nm}}{10^{-9}\ \text{m}}\right)$ = $9.03545\text{x}10^{-19}$ J= $\mathbf{9.0x10^{-19}\ J}$

6.18 Bohr's key assumption was that the electron in an atom does not radiate energy while in a stationary state, and the electron can move to a different orbit by absorbing or emitting a photon whose energy is equal to the difference in energy between two states. These differences in energy correspond to the wavelengths in the known spectra for the hydrogen atoms. A Solar System model does not allow for the movement of electrons between levels.

6.20 Plan: The quantum number *n* is related to the energy level of the electron. An electron *absorbs* energy to change from lower energy (lower *n*) to higher energy (higher *n*), giving an absorption spectrum. An electron *emits* energy as it drops from a higher energy level (higher *n*) to a lower one (lower *n*), giving an emission spectrum.
Solution:
a) The electron is moving from a lower value of *n* (2) to a higher value of *n* (4): **absorption**
b) The electron is moving from a higher value of *n* (3) to a lower value of *n* (1): **emission**
c) The electron is moving from a higher value of *n* (5) to a lower value of *n* (2):**emission**
d) The electron is moving from a lower value of *n* (3) to a higher value of *n* (4): **absorption**

6.22 The Bohr model has successfully predicted the line spectra for the H atom and Be^{3+} ion since both are one-electron species. The energies could be predicted from $E_n = \dfrac{-\left(Z^2\right)\left(2.18\text{x}10^{-18}\ \text{J}\right)}{n^2}$ where *Z* is the atomic number for the atom or ion. The line spectra for H would not match the line spectra for Be^{3+} since the H nucleus contains one proton while the Be^{3+} nucleus contains 4 protons (the *Z* values in the equation do not match); the force of attraction of the nucleus for the electron would be greater in the beryllium ion than in the hydrogen atom. This means that the pattern of lines would be similar, but at different wavelengths.

6.23 Plan: Calculate wavelength by substituting the given values into Equation 7.3, where $n_1 = 2$ and $n_2 = 5$ because $n_2 > n_1$. Although more significant figures could be used, five significant figures are adequate for this calculation.
Solution:

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \qquad R = 1.096776\text{x}10^7\ \text{m}^{-1}$$

$n_1 = 2 \quad n_2 = 5$

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) = \left(1.096776\text{x}10^7\ \text{m}^{-1}\right)\left(\frac{1}{2^2} - \frac{1}{5^2}\right) = 2{,}303{,}229.6\ \text{m}^{-1}$$

$$\lambda\ (\text{nm}) = \left(\frac{1}{2{,}303{,}229.6\ \text{m}^{-1}}\right)\left(\frac{1\ \text{nm}}{10^{-9}\ \text{m}}\right) = 434.1729544\ \text{nm} = \mathbf{434.17\ nm}$$

6.25 Plan: The Rydberg equation is needed. For the infrared series of the H atom, n_1 equals 3. The least energetic spectral line in this series would represent an electron moving from the next highest energy level, $n_2 = 4$. Although more significant figures could be used, five significant figures are adequate for this calculation.
Solution:

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) = \left(1.096776\text{x}10^7\ \text{m}^{-1}\right)\left(\frac{1}{3^2} - \frac{1}{4^2}\right) = 533{,}155\ \text{m}^{-1}$$

$$\lambda\ (\text{nm}) = \left(\frac{1}{533{,}155\ \text{m}^{-1}}\right)\left(\frac{1\ \text{nm}}{10^{-9}\ \text{m}}\right) = 1875.627\ \text{nm} = \mathbf{1875.6\ nm}$$

6.27 Plan: To find the transition energy, use the equation for the energy of an electron transition and multiply by Avogadro's number to convert to energy per mole.
Solution:

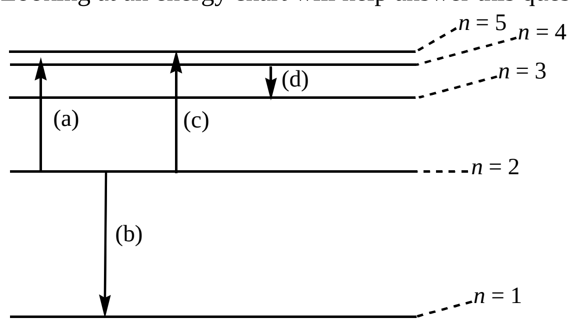
$$\Delta E = \left(-2.18\text{x}10^{-18}\text{ J}\right)\left(\frac{1}{n_{\text{final}}^2} - \frac{1}{n_{\text{initial}}^2}\right)$$

$$\Delta E = \left(-2.18\text{x}10^{-18}\text{ J}\right)\left(\frac{1}{2^2} - \frac{1}{5^2}\right) = -4.578\text{x}10^{-19}\text{ J/photon}$$

$$\Delta E = \left(\frac{-4.578\text{x}10^{-19}\text{ J}}{\text{photon}}\right)\left(\frac{6.022\text{x}10^{23}\text{ photons}}{1\text{ mol}}\right) = -2.75687\text{x}10^5\text{ J/mol} = \mathbf{-2.76x10^5\ J/mol}$$

The energy has a negative value since this electron transition to a lower n value is an emission of energy.

6.29 Plan: Determine the relative energy of the electron transitions. Remember that energy is directly proportional to frequency ($E = h\nu$).
Solution:
Looking at an energy chart will help answer this question.



Frequency is proportional to energy so the smallest frequency will be d) $n = 4$ to $n = 3$; levels 3 and 4 have a smaller ΔE than the levels in the other transitions. The largest frequency is b) $n = 2$ to $n = 1$ since levels 1 and 2 have a larger ΔE than the levels in the other transitions. Transition a) $n = 2$ to $n = 4$ will be smaller than transition c) $n = 2$ to $n = 5$ since level 5 is a higher energy than level 4. In order of increasing frequency the transitions are **d < a < c < b**.

6.31 Plan: Use the Rydberg equation. Since the electron is in the ground state (lowest energy level), $n_1 = 1$. Convert the wavelength from nm to units of meters.
Solution:

$$\lambda = \left(97.20\text{ nm}\right)\left(\frac{10^{-9}\text{ m}}{1\text{ nm}}\right) = 9.720\text{x}10^{-8}\text{ m}$$

ground state: $n_1 = 1$; $n_2 = ?$

$$\frac{1}{\lambda} = \left(1.096776\text{x}10^{7}\text{ m}^{-1}\right)\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\frac{1}{9.72\text{x}10^{-8}\text{ m}} = \left(1.096776\text{x}10^{7}\text{ m}^{-1}\right)\left(\frac{1}{1^2} - \frac{1}{n_2^2}\right)$$

$$0.93803 = \left(\frac{1}{1^2} - \frac{1}{n_2^2}\right)$$

$$\frac{1}{n_2^2} = 1 - 0.93803 = 0.06197$$

$$n_2^2 = 16.14$$

$n_2 = \mathbf{4}$

6.37 Macroscopic objects have significant mass. A large m in the denominator of $\lambda = h/mu$ will result in a very small wavelength. Macroscopic objects do exhibit a wavelike motion, but the wavelength is too small for humans to see it.

6.39 Plan: Use the de Broglie equation. Velocity in km/h must be converted to m/s because a joule is equivalent to kg•m²/s².

Solution:

a)

$$\text{Velocity (m/s)} = \left(\frac{19.8\text{ km}}{\text{h}}\right)\left(\frac{10^3\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 5.50\text{ m/s}$$

$$\lambda = \frac{h}{mu} = \frac{\left(6.626\text{x}10^{-34}\text{ J•s}\right)}{(105\text{ kg})\left(5.50\frac{\text{m}}{\text{s}}\right)}\left(\frac{\text{kg•m}^2/\text{s}^2}{\text{J}}\right) = 1.147359\text{x}10^{-36}\text{ m} = \mathbf{1.15x10^{-36}\ m}$$

$$\text{b) Uncertainty in velocity (m/s)} = \left(\frac{0.1\text{ km}}{\text{h}}\right)\left(\frac{10^3\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 0.02778\text{ m/s}$$

$$\Delta x \bullet m\Delta v \geq \frac{h}{4\pi}$$

$$\Delta x \geq \frac{h}{4\pi m\Delta v} \geq \frac{\left(6.626\text{x}10^{-34}\text{ J•s}\right)}{4\pi(105\text{ kg})\left(\frac{0.02778\text{ m}}{\text{s}}\right)}\left(\frac{\text{kg•m}^2/\text{s}^2}{\text{J}}\right) \geq 1.807674\text{x}10^{\cancel{-33}\,\underline{-35}}\text{ m} \geq \mathbf{2x10^{\cancel{-33}\,\underline{-35}}\ m}$$

6.41 Plan: Use the de Broglie equation. Mass in g must be converted to kg and wavelength in nm must be converted to m because a joule is equivalent to kg•m²/s².

Solution:

$$\text{Mass (kg)} = (56.5\text{ g})\left(\frac{1\text{ kg}}{10^3\text{ g}}\right) = 0.0565\text{ kg}$$

$$\text{Wavelength (m)} = (5400\text{ nm})\left(\frac{10^{-9}\text{ m}}{1\text{ nm}}\right) = 5.4\text{x}10^{-7}\text{ m}$$

$$\lambda = \frac{h}{mu}$$

$$u = \frac{h}{m\lambda} = \frac{\left(6.626\text{x}10^{-34}\text{ J•s}\right)}{(0.0565\text{ kg})\left(5.4\text{x}10^{-7}\text{ m}\right)}\left(\frac{\text{kg•m}^2/\text{s}^2}{\text{J}}\right) = 2.1717\text{x}10^{-26}\text{ m/s} = \mathbf{2.2x10^{-26}\ m/s}$$

6.43 Plan: The de Broglie wavelength equation will give the mass equivalent of a photon with known wavelength and velocity. The term "mass equivalent" is used instead of "mass of photon" because photons are quanta of electromagnetic energy that have no mass. A light photon's velocity is the speed of light, $3.00\text{x}10^8$ m/s. Wavelength in nm must be converted to m.

Solution:

$$\text{Wavelength (m)} = (589\text{ nm})\left(\frac{10^{-9}\text{ m}}{1\text{ nm}}\right) = 5.89\text{x}10^{-7}\text{ m}$$

$$\lambda = \frac{h}{mu}$$

$$m = \frac{h}{\lambda u} = \frac{\left(6.626\text{x}10^{-34}\text{ J•s}\right)}{\left(5.89\text{x}10^{-7}\text{ m}\right)\left(3.00\text{x}10^8\text{ m/s}\right)}\left(\frac{\text{kg•m}^2/\text{s}^2}{\text{J}}\right) = 3.7499\text{x}10^{-36}\text{ kg/photon} = \mathbf{3.75x10^{-36}\ kg/photon}$$

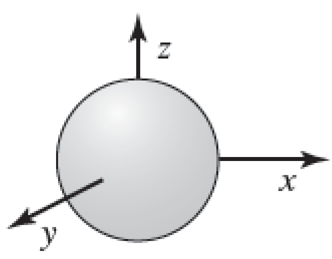
6.47 A peak in the radial probability distribution at a certain distance means that the total probability of finding the electron is greatest within a thin spherical volume having a radius very close to that distance. Since principal quantum number (n) correlates with distance from the nucleus, the peak for $n = 2$ would occur at a greater distance from the nucleus than 52.9 pm. Thus, the probability of finding an electron at 52.9 pm is much greater for the 1*s* orbital than for the 2*s*.

6.48 a) Principal quantum number, n, relates to the size of the orbital. More specifically, it relates to the distance from the nucleus at which the probability of finding an electron is greatest. This distance is determined by the energy of the electron.
b) Angular momentum quantum number, l, relates to the shape of the orbital. It is also called the azimuthal quantum number.
c) Magnetic quantum number, m_l, relates to the orientation of the orbital in space in three-dimensional space.

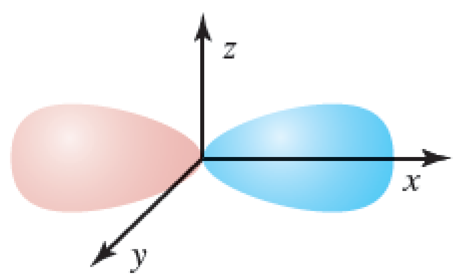
6.49 Plan: The following letter designations correlate with the following l quantum numbers:
$l = 0 = s$ orbital; $l = 1 = p$ orbital; $l = 2 = d$ orbital; $l = 3 = f$ orbital. Remember that allowed m_l values are $-l$ to $+l$. The number of orbitals of a particular type is given by the number of possible m_l values.
Solution:
a) There is only a single *s* orbital in any shell. $l = 1$ and $m_l = 0$: one value of m_l = **one** *s* orbital.
b) There are five *d* orbitals in any shell. $l = 2$ and $m_l = -2, -1, 0, +1, +2$. Five values of m_l = **five** *d* orbitals.
c) There are three *p* orbitals in any shell. $l = 1$ and $m_l = -1, 0, +1$. Three values of m_l = **three** *p* orbitals.
d) If $n = 3$, $l = 0(s)$, $1(p)$, and $2(d)$. There is a 3*s* (1 orbital), a 3*p* set (3 orbitals), and a 3*d* set (5 orbitals) for a total of **nine** orbitals (1 + 3 + 5 = 9).

6.51 Plan: Magnetic quantum numbers (m_l) can have integer values from $-l$ to $+l$. The l quantum number can have integer values from 0 to $n - 1$.
Solution:
a) $l = 2$ so m_l = **–2, –1, 0, +1, +2**
b) $n = 1$ so $l = 1 - 1 = 0$ and m_l = **0**
c) $l = 3$ so m_l = **–3, –2, –1, 0, +1, +2, +3**

6.53 Plan: The *s* orbital is spherical; *p* orbitals have two lobes; the subscript *x* indicates that this orbital lies along the *x*-axis.
Solution:
a) *s*: spherical



b) p_x: 2 lobes along the *x*-axis



The variations in colouring of the *p* orbital are a consequence of the quantum mechanical derivation of atomic orbitals that are beyond the scope of this course.

6.55 Plan: The following letter designations for the various subshell (orbitals) correlate with the following l quantum numbers: $l = 0 = s$ orbital; $l = 1 = p$ orbital; $l = 2 = d$ orbital; $l = 3 = f$ orbital. Remember that allowed m_l values are $-l$ to $+l$. The number of orbitals of a particular type is given by the number of possible m_l values.
Solution:

subshell	allowable m_l	# of possible orbitals
a) d ($l = 2$)	–2, –1, 0, +1, +2	5
b) p ($l = 1$)	–1, 0, +1	3
c) f ($l = 3$)	–3, –2, –1, 0, +1, +2, +3	7

6.57 Plan: The integer in front of the letter represents the n value. The letter designates the l value: $l = 0 = s$ orbital; $l = 1 = p$ orbital; $l = 2 = d$ orbital; $l = 3 = f$ orbital. Remember that allowed m_l values are $-l$ to $+l$.
Solution:
a) For the 5*s* subshell, $\boldsymbol{n = 5}$ and $\boldsymbol{l = 0}$. Since $m_l = 0$, there is **one** orbital.
b) For the 3*p* subshell, $\boldsymbol{n = 3}$ and $\boldsymbol{l = 1}$. Since $m_l = -1, 0, +1$, there are **three** orbitals.
c) For the 4*f* subshell, $\boldsymbol{n = 4}$ and $\boldsymbol{l = 3}$. Since $m_l = -3, -2, -1, 0, +1, +2, +3$, there are **seven** orbitals.

6.59 Plan: Allowed values of quantum numbers: n = positive integers; l = integers from 0 to $n - 1$; m_l = integers from $-l$ through 0 to $+l$.
Solution:
a) $n = 2$; $l = 0$; $m_l = -1$: With $n = 2$, l can be 0 or 1; with $l = 0$, the only allowable m_l value is 0. This combination is not allowed. To correct, either change the l or m_l value.
Correct: $n = 2$; $l = 1$; $m_l = -1$ or $n = 2$; $l = 0$; $m_l = 0$.
b) $n = 4$; $l = 3$; $m_l = -1$: With $n = 4$, l can be 0, 1, 2, or 3; with $l = 3$, the allowable m_l values are $-3, -2, -1, 0, +1, +2, +3$. Combination is allowed.
c) $n = 3$; $l = 1$; $m_l = 0$: With $n = 3$, l can be 0, 1, or 2; with $l = 1$, the allowable m_l values are $-1, 0, +1$. Combination is allowed.
d) $n = 5$; $l = 2$; $m_l = +3$: With $n = 5$, l can be 0, 1, 2, 3, or 4; with $l = 2$, the allowable m_l values are $-2, -1, 0, +1, +2$. +3 is not an allowable m_l value. To correct, either change l or m_l value.
Correct: $n = 5$; $l = 3$; $m_l = +3$ or $n = 5$; $l = 2$; $m_l = 0$.

6.62 Plan: For Part a, use the values of the constants h, π, m_e, and a_0 to find the overall constant in the equation. Use the resulting equation to calculate ΔE in part b). Use the relationship $E = \frac{hc}{\lambda}$ to calculate the wavelength in part c). Remember that a joule is equivalent to kg•m^2/s^2.
Solution:
a) $h = 6.626\text{x}10^{-34}$ J•s; $m_e = 9.1094\text{x}10^{-31}$ kg; $a_0 = 52.92\text{x}10^{-12}$ m

$$E = -\frac{h^2}{8\pi^2 m_e a_0^2 n^2} = -\frac{h^2}{8\pi^2 m_e a_0^2}\left(\frac{1}{n^2}\right)$$

$$E = -\frac{\left(6.626\text{x}10^{-34}\ \text{J•s}\right)^2}{8\pi^2\left(9.1094\text{x}10^{-31}\ \text{kg}\right)\left(52.92\text{x}10^{-12}\ \text{m}\right)^2}\left(\frac{\text{kg•m}^2/\text{s}^2}{\text{J}}\right)\left(\frac{1}{n^2}\right)$$

$$= -(2.17963\text{x}10^{-18}\ \text{J})\left(\frac{1}{n^2}\right) = -(2.180\text{x}10^{-18}\ \text{J})\left(\frac{1}{n^2}\right)$$

This is identical with the result from Bohr's theory. For the H atom, $Z = 1$ and Bohr's constant = $-2.18\text{x}10^{-18}$ J. For the hydrogen atom, derivation using classical principles or quantum-mechanical principles yields the same constant.
b) The $n = 3$ energy level is higher in energy than the $n = 2$ level. Because the zero point of the atom's energy is defined as an electron's infinite distance from the nucleus, a larger negative number describes a lower energy level. Although this may be confusing, it makes sense that an energy *change* would be a positive number.

$$\Delta E = -(2.180\text{x}10^{-18}\ \text{J})\left(\frac{1}{3^2} - \frac{1}{2^2}\right) = 3.027778\text{x}10^{-19}\ \text{J} = \mathbf{3.028x10^{-19}\ J}$$

c) $E = \frac{hc}{\lambda}$

$$\lambda\ (\text{m}) = \frac{hc}{E} = \frac{\left(6.626\text{x}10^{-34}\ \text{J•s}\right)\left(2.9979\text{x}10^{8}\ \text{m/s}\right)}{\left(3.027778\text{x}10^{-19}\ \text{J}\right)} = 6.56061\text{x}10^{-7}\ \text{m} = \mathbf{6.561x10^{-7}\ m}$$

$$\lambda\ (\text{nm}) = \left(6.56061\text{x}10^{-7}\ \text{m}\right)\left(\frac{1\ \text{nm}}{10^{-9}\ \text{m}}\right) = 656.061\ \text{nm} = \mathbf{656.1\ nm}$$

This is the wavelength for the observed red line in the hydrogen spectrum.

6.63 Plan: When light of sufficient frequency (energy) shines on metal, electrons in the metal break free and a current flows.

Solution:

a) The lines do not begin at the origin because an electron must absorb a minimum amount of energy before it has enough energy to overcome the attraction of the nucleus and leave the atom. This minimum energy is the energy of photons of light at the threshold frequency.

b) The lines for K and Ag do not begin at the same point. The amount of energy that an electron must absorb to leave the K atom is less than the amount of energy that an electron must absorb to leave the Ag atom, where the attraction between the nucleus and outer electron is stronger than in a K atom.

c) Wavelength is inversely proportional to energy. Thus, the metal that requires a larger amount of energy to be absorbed before electrons are emitted will require a shorter wavelength of light. Electrons in Ag atoms require more energy to leave, so Ag requires a shorter wavelength of light than K to eject an electron.

d) The slopes of the line show an increase in kinetic energy as the frequency (or energy) of light is increased. Since the slopes are the same, this means that for an increase of one unit of frequency (or energy) of light, the increase in kinetic energy of an electron ejected from K is the same as the increase in the kinetic energy of an electron ejected from Ag. After an electron is ejected, the energy that it absorbs above the threshold energy becomes the kinetic energy of the electron. For the same increase in energy above the threshold energy, for either K or Ag, the kinetic energy of the ejected electron will be the same.

6.66 Plan: The Bohr model has been successfully applied to predict the spectral lines for one-electron species other than H. Common one-electron species are small cations with all but one electron removed. Since the problem specifies a metal ion, assume that the possible choices are Li^{2+} or Be^{3+}. Use the relationship $E = h\nu$ to convert the frequency to energy and then solve Bohr's equation $E = \left(2.18\text{x}10^{-18}\text{ J}\right)\left(\frac{Z^2}{n^2}\right)$ to verify if a whole number for Z can be calculated. Recall that the negative sign is a convention based on the zero point of the atom's energy; it is deleted in this calculation to avoid taking the square root of a negative number.

Solution:

The highest energy line corresponds to the transition from $n = 1$ to $n = \infty$.

$E = h\nu = (6.626\text{x}10^{-34}\text{ J•s})\ (2.961\text{x}10^{16}\text{ Hz})\ (\text{s}^{-1}/\text{Hz}) = 1.9619586\text{x}10^{-17}\text{ J}$

$E = \left(2.18\text{x}10^{-18}\text{ J}\right)\left(\frac{Z^2}{n^2}\right)$ Z = charge of the nucleus

$$Z^2 = \frac{En^2}{2.18\text{x}10^{-18}\text{J}} = \frac{1.9619586\text{x}10^{-17}(1^2)}{2.18\text{x}10^{-18}\text{ J}} = 8.99998$$

Then $Z^2 = 9$ and $Z = 3$.

Therefore, the ion is $\mathbf{Li^{2+}}$ with an atomic number of 3.

6.68 Plan: The electromagnetic spectrum shows that the visible region goes from 400 to 750 nm. Thus, wavelengths b, c, and d are for the three transitions in the visible series with $n_{\text{final}} = 2$. Wavelength a is in the ultraviolet region of the spectrum and the ultraviolet series has $n_{\text{final}} = 1$. Wavelength e is in the infrared region of the spectrum and the infrared series has $n_{\text{final}} = 3$. Use the Rydberg equation to find the n_{initial} for each line. Convert the wavelengths from nm to units of m.

Solution:

$n = ? \rightarrow n = 1$; $\lambda = 121$ nm (shortest λ corresponds to the largest ΔE)

$$\lambda\ (\text{m}) = \left(121\text{ nm}\right)\left(\frac{10^{-9}\text{ m}}{1\text{ nm}}\right) = 1.21\text{x}10^{-7}\text{ m}$$

$$\frac{1}{\lambda} = \left(1.096776\text{x}10^{7}\text{ m}^{-1}\right)\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\left(\frac{1}{1.21\text{x}10^{-7}\text{ m}}\right) = \left(1.096776\text{x}10^{7}\text{ m}^{-1}\right)\left(\frac{1}{1^2} - \frac{1}{n_2^2}\right)$$

$$0.7535233 = \left(\frac{1}{1^2} - \frac{1}{n_2^2}\right)$$

$$\left(\frac{1}{n_2^2}\right) = 1 - 0.7535233$$

$$\left(\frac{1}{n_2^2}\right) = 0.2464767$$

$\boldsymbol{n_2 = 2}$ for line (a) ($n = 2 \rightarrow n = 1$)
$n = ? \rightarrow n = 3$; $\lambda = 1094$ nm (longest λ corresponds to the smallest ΔE)

$$\lambda\text{ (m)} = (1094\text{ nm})\left(\frac{10^{-9}\text{ m}}{1\text{ nm}}\right) = 1.094\text{x}10^{-6}\text{ m}$$

$$\frac{1}{\lambda} = (1.096776\text{x}10^{7}\text{ m}^{-1})\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\left(\frac{1}{1.094\text{x}10^{-6}\text{ m}}\right) = (1.096776\text{x}10^{7}\text{ m}^{-1})\left(\frac{1}{3^2} - \frac{1}{n_2^2}\right)$$

$$0.083342158 = \left(\frac{1}{3^2} - \frac{1}{n_2^2}\right)$$

$$0.083342158 = 0.111111111 - \frac{1}{n_2^2}$$

$$\left(\frac{1}{n_2^2}\right) = 0.11111111 - 0.083342158$$

$$\left(\frac{1}{n_2^2}\right) = 0.0277689535$$

$$n_2^2 = 36.01143$$

$\boldsymbol{n_2 = 6}$ for line (e) ($n = 6 \rightarrow n = 3$)
For the other three lines, $n_1 = 2$.
For line (d), $\boldsymbol{n_2 = 3}$ (largest $\lambda \rightarrow$ smallest ΔE).
For line (b), $\boldsymbol{n_2 = 5}$ (smallest $\lambda \rightarrow$ largest ΔE).
For line (c), $\boldsymbol{n_2 = 4}$.

6.72 Plan: Allowed values of quantum numbers: n = positive integers; l = integers from 0 to $n - 1$; m_l = integers from $- l$ through 0 to $+ l$.

Solution:

a) The l value must be at least 1 for m_l to be – 1, but cannot be greater than $n - 1 = 3 - 1 = 2$. Increase the l value to 1 or 2 to create an allowable combination.
b) The l value must be at least 1 for m_l to be +1, but cannot be greater than $n - 1 = 3 - 1 = 2$. Decrease the l value to 1 or 2 to create an allowable combination.
c) The l value must be at least 3 for m_l to be +3, but cannot be greater than $n - 1 = 7 - 1 = 6$. Increase the l value to 3, 4, 5, or 6 to create an allowable combination.
d) The l value must be at least 2 for m_l to be –2, but cannot be greater than $n - 1 = 4 - 1 = 3$. Increase the l value to 2 or 3 to create an allowable combination.

6.74 Plan: Ionization occurs when the electron is completely removed from the atom, or when $n_{final} = \infty$. We can use the equation for the energy of an electron transition to find the quantity of energy needed to remove completely the electron, called the ionization energy (IE). To obtain the ionization energy per mole of species, multiply by Avogadro's number. The charge on the nucleus must affect the IE because a larger nucleus would exert a greater pull on the escaping electron. The Bohr equation applies to H and other one-electron species. Use the expression to determine the ionization energy of B^{4+} and to find the energies of the transitions listed. Use $E = \frac{hc}{\lambda}$ to convert energy to wavelength.

Solution:

a) $E = \left(-2.18\text{x}10^{-18}\text{ J}\right)\left(\frac{Z^2}{n^2}\right)$ Z = atomic number

$$\Delta E = \left(-2.18\text{x}10^{-18}\text{ J}\right)\left(\frac{1}{n_{\text{final}}^2} - \frac{1}{n_{\text{initial}}^2}\right)Z^2$$

$$\Delta E = \left(-2.18\text{x}10^{-18}\text{ J}\right)\left(\frac{1}{\infty^2} - \frac{1}{n_{\text{initial}}^2}\right)Z^2\left(\frac{6.022\text{x}10^{23}}{1\text{ mol}}\right)$$

$= (1.312796\text{x}10^6)\ Z^2$ for $n = 1$

b) In the ground state $n = 1$, the initial energy level for the single electron in B^{4+}. Once ionized, $n = \infty$ is the final energy level.

$Z = 5$ for B^{4+}.

ΔE = IE = $(1.312796\text{x}10^6)\ Z^2 = (1.312796\text{x}10^6\text{ J/mol})(5^2) = 3.28199\text{x}10^7$ J/mol= **$3.28\text{x}10^7$ J/mol**

c) $n_{final} = \infty$, $n_{initial} = 3$, and $Z = 2$ for He^+.

$$\Delta E = \left(-2.18\text{x}10^{-18}\text{J}\right)\left(\frac{1}{n_{\text{final}}^2} - \frac{1}{n_{\text{initial}}^2}\right)Z^2 = \left(-2.18\text{x}10^{-18}\text{J}\right)\left(\frac{1}{\infty^2} - \frac{1}{3^2}\right)2^2 = 9.68889\text{x}10^{-19}\text{ J}$$

$$E = \frac{hc}{\lambda}$$

$$\lambda\text{ (m)} = \frac{hc}{E} = \frac{\left(6.626\text{x}10^{-34}\text{ J•s}\right)\left(3.00\text{x}10^8\text{ m/s}\right)}{9.68889\text{x}10^{-19}\text{ J}} = 2.051628\text{x}10^{-7}\text{ m}$$

$$\lambda\text{ (nm)} = \left(2.051628\text{x}10^{-7}\text{ m}\right)\left(\frac{1\text{ nm}}{10^{-9}\text{ m}}\right) = 205.1628\text{ nm} = \mathbf{205\text{ nm}}$$

d) $n_{final} = \infty$, $n_{initial} = 2$, and $Z = 4$ for Be^{3+}.

$$\Delta E = \left(-2.18\text{x}10^{-18}\text{ J}\right)\left(\frac{1}{\infty^2} - \frac{1}{n_{\text{initial}}^2}\right)Z^2 = \left(-2.18\text{x}10^{-18}\text{J}\right)\left(\frac{1}{\infty^2} - \frac{1}{2^2}\right)4^2 = 8.72\text{x}10^{-18}\text{ J}$$

$$\lambda\text{ (m)} = \frac{hc}{E} = \frac{\left(6.626\text{x}10^{-34}\text{ J•s}\right)\left(3.00\text{x}10^8\text{ m/s}\right)}{8.72\text{x}10^{-18}\text{ J}} = 2.279587\text{x}10^{-8}\text{ m}$$

$$\lambda\text{ (nm)} = \left(2.279587\text{x}10^{-8}\text{ m}\right)\left(\frac{1\text{ nm}}{10^{-9}\text{ m}}\right) = 22.79587\text{ nm} = \mathbf{22.8\text{ nm}}$$

6.76 Plan: Use the values and the equation given in the problem to calculate the appropriate values.

Solution:

a) $r_n = \dfrac{n^2 h^2 \varepsilon_0}{\pi m_e e^2}$

$$r_1 = \frac{1^2\left(6.626\text{x}10^{-34}\ \text{J•s}\right)^2\left(8.854\text{x}10^{-12}\ \dfrac{\text{C}^2}{\text{J•m}}\right)}{\pi\left(9.109\text{x}10^{-31}\ \text{kg}\right)\left(1.602\text{x}10^{-19}\ \text{C}\right)^2}\left(\frac{\text{kg•m}^2/\text{s}^2}{\text{J}}\right) = 5.2929377\text{x}10^{-11}\ \text{m} = \mathbf{5.293x10^{-11}\ m}$$

b) $$r_{10} = \frac{10^2\left(6.626\text{x}10^{-34}\ \text{J•s}\right)^2\left(8.854\text{x}10^{-12}\ \dfrac{\text{C}^2}{\text{J•m}}\right)}{\pi\left(9.109\text{x}10^{-31}\ \text{kg}\right)\left(1.602\text{x}10^{-19}\ \text{C}\right)^2}\left(\frac{\text{kg•m}^2/\text{s}^2}{\text{J}}\right) = 5.2929377\text{x}10^{-9}\ \text{m} = \mathbf{5.293x10^{-9}\ m}$$

6.78 Plan: Refer to Chapter 5 for the calculation of the amount of heat energy absorbed by a substance from its specific heat capacity and temperature change ($q = c$ x mass x ΔT). Using this equation, calculate the energy absorbed by the water. This energy equals the energy from the microwave photons. The energy of each photon can be calculated from its wavelength: $E = hc/\lambda$. Dividing the total energy by the energy of each photon gives the number of photons absorbed by the water.

Solution:

$q = c$ x mass x ΔT

q = (4.184 J/g°C)(252 g)(98 – 20.)°C = 8.22407x10^4 J

$$E = \frac{hc}{\lambda} = \frac{\left(6.626\text{x}10^{-34}\ \text{J•s}\right)\left(3.00\text{x}10^{8}\ \text{m/s}\right)}{1.55\text{x}10^{-2}\ \text{m}} = 1.28245\text{x}10^{-23}\ \text{J/photon}$$

Number of photons = $\left(8.22407\text{x}10^4\ \text{J}\right)\left(\dfrac{1\ \text{photon}}{1.28245\text{x}10^{-23}\ \text{J}}\right)$ = 6.41278x10^{27} photons= $\mathbf{6.4x10^{27}}$ **photons**

6.80 Plan: In general, to test for overlap of the two series, compare the longest wavelength in the "n" series with the shortest wavelength in the "n+1" series. The longest wavelength in any series corresponds to the transition between the n_1 level and the next level above it; the shortest wavelength corresponds to the transition between the n_1 level and the $n = \infty$ level. Use the relationship $\dfrac{1}{\lambda} = R\left(\dfrac{1}{n_1^2} - \dfrac{1}{n_2^2}\right)$ to calculate the wavelengths.

Solution:

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) = \left(1.096776\text{x}10^7\ \text{m}^{-1}\right)\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

a) The overlap between the n_1 = 1 series and the n_1 = 2 series would occur between the longest wavelengths for n_1 = 1 and the shortest wavelengths for n_1 = 2.

Longest wavelength in n_1 = 1 series has n_2 equal to 2.

$$\frac{1}{\lambda} = \left(1.096776\text{x}10^7\ \text{m}^{-1}\right)\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = 8{,}225{,}820\ \text{m}^{-1}$$

$$\lambda = \frac{1}{8{,}225{,}820\ \text{m}^{-1}} = 1.215684272\text{x}10^{-7}\ \text{m} = \mathbf{1.215684x10^{-7}\ m}$$

Shortest wavelength in the n_1 = 2 series:

$$\frac{1}{\lambda} = \left(1.096776\text{x}10^7\ \text{m}^{-1}\right)\left(\frac{1}{2^2} - \frac{1}{\infty^2}\right) = 2{,}741{,}940\ \text{m}^{-1}$$

$$\lambda = \frac{1}{2{,}741{,}940\ \text{m}^{-1}} = 3.647052817\text{x}10^{-7}\ \text{m} = \mathbf{3.647053x10^{-7}\ m}$$

Since the longest wavelength for $n_1 = 1$ series is shorter than shortest wavelength for $n_1 = 2$ series, there is **no overlap** between the two series.

b) The overlap between the $n_1 = 3$ series and the $n_1 = 4$ series would occur between the longest wavelengths for $n_1 = 3$ and the shortest wavelengths for $n_1 = 4$.

Longest wavelength in $n_1 = 3$ series has n_2 equal to 4.

$$\frac{1}{\lambda} = \left(1.096776\text{x}10^7 \text{ m}^{-1}\right)\left(\frac{1}{3^2} - \frac{1}{4^2}\right) = 533{,}155 \text{ m}^{-1}$$

$$\lambda = \frac{1}{533{,}155 \text{ m}^{-1}} = 1.875627163\text{x}10^{-6} \text{ m} = \mathbf{1.875627x10^{-6}\ m}$$

Shortest wavelength in $n_1 = 4$ series has $n_2 = \infty$.

$$\frac{1}{\lambda} = \left(1.096776\text{x}10^7 \text{ m}^{-1}\right)\left(\frac{1}{4^2} - \frac{1}{\infty^2}\right) = 685{,}485 \text{ m}^{-1}$$

$$\lambda = \frac{1}{685{,}485 \text{ m}^{-1}} = 1.458821127\text{x}10^{-6} \text{ m} = \mathbf{1.458821x10^{-6}\ m}$$

Since the $n_1 = 4$ series shortest wavelength is shorter than the $n_1 = 3$ series longest wavelength, the **series do overlap**.

c) Shortest wavelength in $n_1 = 5$ series has $n_2 = \infty$.

$$\frac{1}{\lambda} = \left(1.096776\text{x}10^7 \text{ m}^{-1}\right)\left(\frac{1}{5^2} - \frac{1}{\infty^2}\right) = 438{,}710.4 \text{ m}^{-1}$$

$$\lambda = \frac{1}{438{,}710.4 \text{ m}^{-1}} = 2.27940801\text{x}10^{-6} \text{ m} = \mathbf{2.279408x10^{-6}\ m}$$

Calculate the first few longest lines in the $n_1 = 4$ series to determine if any overlap with the shortest wavelength in the $n_1 = 5$ series:

For $n_1 = 4$, $n_2 = 5$:

$$\frac{1}{\lambda} = \left(1.096776\text{x}10^7 \text{ m}^{-1}\right)\left(\frac{1}{4^2} - \frac{1}{5^2}\right) = 246{,}774.6 \text{ m}^{-1}$$

$$\lambda = \frac{1}{246{,}774.6 \text{ m}^{-1}} = \mathbf{4.052281x10^{-6}\ m}$$

For $n_1 = 4$, $n_2 = 6$:

$$\frac{1}{\lambda} = \left(1.096776\text{x}10^7 \text{ m}^{-1}\right)\left(\frac{1}{4^2} - \frac{1}{6^2}\right) = 380{,}825 \text{ m}^{-1}$$

$$\lambda = \frac{1}{380{,}825 \text{ m}^{-1}} = = \mathbf{2.625878x10^{-6}\ m}$$

For $n_1 = 4$, $n_2 = 7$:

$$\frac{1}{\lambda} = \left(1.096776\text{x}10^7 \text{ m}^{-1}\right)\left(\frac{1}{4^2} - \frac{1}{7^2}\right) = 461{,}653.2 \text{ m}^{-1}$$

$$\lambda = \frac{1}{461{,}653.2 \text{ m}^{-1}} = \mathbf{2.166128x10^{-6}\ m}$$

The wavelengths of the first **two lines** of the $n_1 = 4$ series are longer than the shortest wavelength in the $n_1 = 5$ series. Therefore, only the first **two lines** of the $n_1 = 4$ series overlap the $n_1 = 5$ series.

d) At longer wavelengths (i.e., lower energies), there is increasing overlap between the lines from different series (i.e., with different n_1 values). The hydrogen spectrum becomes more complex, since the lines begin to merge into a more-or-less continuous band, and much more care is needed to interpret the information.

6.82 Plan: The energy differences sought may be determined by looking at the energy changes in steps. The wavelength is calculated from the relationship $\lambda = \frac{hc}{E}$.

Solution:

a) The difference between levels 3 and 2 (E_{32}) may be found by taking the difference in the energies for the 3 → 1 transition (E_{31}) and the 2 → 1 transition (E_{21}).

$E_{32} = E_{31} - E_{21} = (4.854\text{x}10^{-17}\text{ J}) - (4.098\text{x}10^{-17}\text{ J}) = \mathbf{7.56x10^{-18}\ J}$

$$\lambda = \frac{hc}{E} = \frac{\left(6.626\text{x}10^{-34}\text{ J}\bullet\text{s}\right)\left(3.00\text{x}10^{8}\text{ m/s}\right)}{\left(7.56\text{x}10^{-18}\text{ J}\right)} = 2.629365\text{x}10^{-8}\text{ m} = \mathbf{2.63x10^{-8}\ m}$$

b) The difference between levels 4 and 1 (E_{41}) may be found by adding the energies for the 4 → 2 transition (E_{42}) and the 2 → 1 transition (E_{21}).

$E_{41} = E_{42} + E_{21} = (1.024\text{x}10^{-17}\text{ J}) + (4.098\text{x}10^{-17}\text{ J}) = \mathbf{5.122x10^{-17}\ J}$

$$\lambda = \frac{hc}{E} = \frac{\left(6.626\text{x}10^{-34}\text{ J}\bullet\text{s}\right)\left(3.00\text{x}10^{8}\text{ m/s}\right)}{\left(5.122\text{x}10^{-17}\text{ J}\right)} = 3.88091\text{x}10^{-9}\text{ m} = \mathbf{3.881x10^{-9}\ m}$$

c) The difference between levels 5 and 4 (E_{54}) may be found by taking the difference in the energies for the 5 → 1 transition (E_{51}) and the 4 → 1 transition (see part b)).

$E_{54} = E_{51} - E_{41} = (5.242\text{x}10^{-17}\text{ J}) - (5.122\text{x}10^{-17}\text{ J}) = \mathbf{1.2x10^{-18}\ J}$

$$\lambda = \frac{hc}{E} = \frac{\left(6.626\text{x}10^{-34}\text{ J}\bullet\text{s}\right)\left(3.00\text{x}10^{8}\text{ m/s}\right)}{\left(1.2\text{x}10^{-18}\text{ J}\right)} = 1.6565\text{x}10^{-7}\text{ m} = \mathbf{1.66x10^{-7}\ m}$$

6.84 Plan: For part a), use the equation for kinetic energy, $E_k = ½mu^2$. For part b), use the relationship $E = hc/\lambda$ to find the energy of the photon absorbed. From that energy subtract the kinetic energy of the dislodged electron to obtain the work function.

Solution:

a) The energy of the electron is a function of its speed leaving the surface of the metal. The mass of the electron is $9.109\text{x}10^{-31}$ kg.

$$E_k = \frac{1}{2}mu^2 = \frac{1}{2}\left(9.109\text{x}10^{-31}\text{ kg}\right)\left(6.40\text{x}10^{5}\text{ m/s}\right)^2\left(\frac{\text{J}}{\text{kg}\bullet\text{m}^2/\text{s}^2}\right) = 1.86552\text{x}10^{-19}\text{ J} = \mathbf{1.87x10^{-19}\ J}$$

b) The minimum energy required to dislodge the electron (ϕ) is a function of the incident light. In this example, the incident light is higher than the threshold frequency, so the kinetic energy of the electron, E_k, must be subtracted from the total energy of the incident light, $h\nu$, to yield the work function, ϕ. (The number of significant figures given in the wavelength requires more significant figures in the speed of light.)

$$\lambda\text{ (m)} = \left(358.1\text{ nm}\right)\left(\frac{10^{-9}\text{ m}}{1\text{nm}}\right) = 3.581\text{x}10^{-7}\text{ m}$$

$$E = hc/\lambda = \frac{\left(6.626\text{x}10^{-34}\text{ J}\bullet\text{s}\right)\left(2.9979\text{x}10^{8}\text{ m/s}\right)}{\left(3.581\text{x}10^{-7}\text{ m}\right)} = 5.447078\text{x}10^{-19}\text{ J}$$

$\Phi = h\nu - E_k = (5.447078\text{x}10^{-19}\text{ J}) - (1.86552\text{x}10^{-19}\text{ J}) = 3.581558\text{x}10^{-19}\text{ J} = \mathbf{3.58x10^{-19}\ J}$

6.86 Plan: Examine Figure 7.3 and match the given wavelengths to their colours. For each salt, convert the mass of salt to moles and multiply by Avogadro's number to find the number of photons emitted by that amount of salt (assuming that each atom undergoes one-electron transition). Use the relationship $E = \frac{hc}{\lambda}$ to find the energy of one photon and multiply by the total number of photons for the total energy of emission.

Solution:
a) Figure 7.3 indicates that the 641 nm wavelength of Sr falls in the **red** region and the 493 nm wavelength of Ba falls in the **green** region.
b) $SrCl_2$

$$\text{Number of photons} = \left(5.00\text{ g SrCl}_2\right)\left(\frac{1\text{ mol SrCl}_2}{158.52\text{ g SrCl}_2}\right)\left(\frac{6.022\text{x}10^{23}\text{ photons}}{1\text{ mol SrCl}_2}\right) = 1.8994449\text{x}10^{22}\text{ photons}$$

$$\lambda\text{ (m)} = \left(641\text{ nm}\right)\left(\frac{10^{-9}\text{ m}}{1\text{nm}}\right) = 6.41\text{x}10^{-7}\text{ m}$$

$$E_{\text{photon}} = \frac{hc}{\lambda} = \frac{\left(6.626\text{x}10^{-34}\text{ J•s}\right)\left(3.00\text{x}10^{8}\text{ m/s}\right)}{6.41\text{x}10^{-7}\text{ m}}\left(\frac{1\text{ kJ}}{10^{3}\text{ J}}\right) = 3.10109\text{x}10^{-22}\text{ kJ/photon}$$

$$E_{\text{total}} = \left(1.8994449\text{x}10^{22}\text{ photons}\right)\left(\frac{3.10109\text{x}10^{-22}\text{ kJ}}{1\text{ photon}}\right) = 5.89035\text{ kJ} = \mathbf{5.89\text{ kJ}}$$

$BaCl_2$

$$\text{Number of photons} = \left(5.00\text{ g BaCl}_2\right)\left(\frac{1\text{ mol BaCl}_2}{208.2\text{ g BaCl}_2}\right)\left(\frac{6.022\text{x}10^{23}\text{ photons}}{1\text{ mol BaCl}_2}\right) = 1.44620557\text{x}10^{22}\text{ photons}$$

$$\lambda\text{ (m)} = \left(493\text{ nm}\right)\left(\frac{10^{-9}\text{ m}}{1\text{nm}}\right) = 4.93\text{x}10^{-7}\text{ m}$$

$$E_{\text{photon}} = \frac{hc}{\lambda} = \frac{\left(6.626\text{x}10^{-34}\text{ J•s}\right)\left(3.00\text{x}10^{8}\text{m/s}\right)}{4.93\text{x}10^{-7}\text{ m}}\left(\frac{1\text{ kJ}}{10^{3}\text{ J}}\right) = 4.0320487\text{x}10^{-22}\text{ kJ/photon}$$

$$E_{\text{total}} = \left(1.44620557\text{x}10^{22}\text{ photons}\right)\left(\frac{4.0320487\text{x}10^{-22}\text{ kJ}}{1\text{ photon}}\right) = 5.83117\text{ kJ} = \mathbf{5.83\text{ kJ}}$$

6.88 Plan: Examine Figure 7.3 to find the region of the electromagnetic spectrum in which the wavelength lies. Compare the absorbance of the given concentration of Vitamin A to the absorbance of the given amount of fish-liver oil to find the concentration of Vitamin A in the oil.
Solution:
a) At this wavelength the sensitivity to absorbance of light by Vitamin A is maximized while minimizing interference due to the absorbance of light by other substances in the fish-liver oil.
b) The wavelength 329 nm lies in the **ultraviolet region** of the electromagnetic spectrum.
c) A known quantity of vitamin A ($1.67\text{x}10^{-3}$ g) is dissolved in a known volume of solvent (250. mL) to give a *standard* concentration with a known response (1.018 units). This can be used to find the unknown quantity of Vitamin A that gives a response of 0.724 units. An equality can be made between the two concentration-to-absorbance ratios.

$$\text{Concentration } (C_1\text{, g/mL) of Vitamin A} = \left(\frac{1.67\text{x}10^{-3}\text{ g}}{250.\text{ mL}}\right) = 6.68\text{x}10^{-6}\text{ g/mL Vitamin A}$$

Absorbance (A_1) of Vitamin A = 1.018 units.
Absorbance (A_2) of fish-liver oil = 0.724 units
Concentration (g/mL) of Vitamin A in fish-liver oil sample = C_2

$$\frac{A_1}{C_1} = \frac{A_2}{C_2}$$

$$C_2 = \frac{A_2C_1}{A_1} = \frac{\left(0.724\right)\left(6.68\text{x}10^{-6}\text{ g/mL}\right)}{\left(1.018\right)} = 4.7508\text{x}10^{-6}\text{ g/mL Vitamin A}$$

Mass (g) of Vitamin A in oil sample = $(500.\ \text{mL oil})\left(\dfrac{4.7508\text{x}10^{-6}\ \text{g Vitamin A}}{1\ \text{mL oil}}\right)$ = $2.3754\text{x}10^{-3}$ g Vitamin A

Concentration of Vitamin A in oil sample = $\dfrac{(2.3754\text{x}10^{-3}\ \text{g})}{(0.1232\ \text{g Oil})}$ = $1.92808\text{x}10^{-2}$ g= **$1.93\text{x}10^{-2}$ g Vitamin A/g oil**

6.92 Plan: First find the energy in joules from the light that shines on the text. Each watt is one joule/s for a total of 75 J; take 5% of that amount of joules and then 10% of that amount. Use $E = \dfrac{hc}{\lambda}$ to find the energy of one photon of light with a wavelength of 550 nm. Divide the energy that shines on the text by the energy of one photon to obtain the number of photons.

Solution:

The amount of energy is calculated from the wavelength of light:

λ (m) = $(550\ \text{nm})\left(\dfrac{10^{-9}\ \text{m}}{1\ \text{nm}}\right)$ = $5.50\text{x}10^{-7}$ m

$E = \dfrac{hc}{\lambda} = \dfrac{(6.626\text{x}10^{-34}\ \text{J•s})(3.00\text{x}10^{8}\ \text{m/s})}{5.50\text{x}10^{-7}\ \text{m}}$ = $3.614182\text{x}10^{-19}$ J/photon

Amount of power from the bulb = $(75\ \text{W})\left(\dfrac{1\ \text{J/s}}{1\ \text{W}}\right)$ = 75 J/s

Amount of power converted to light = $(75\ \text{J/s})\left(\dfrac{5\%}{100\%}\right)$ = 3.75 Js

Amount of light shining on book = $(3.75\ \text{J/s})\left(\dfrac{10\%}{100\%}\right)$ = 0.375 J/s

Number of photons: $\left(\dfrac{0.375\ \text{J}}{\text{s}}\right)\left(\dfrac{1\ \text{photon}}{3.614182\text{x}10^{-19}\ \text{J}}\right)$ = $1.0376\text{x}10^{18}$ photons/s= **$1.0\text{x}10^{18}$ photons/s**

6.95 Plan: In the visible series with $n_{\text{final}} = 2$, the transitions will end in either the 2*s* or 2*p* orbitals since those are the only two types of orbitals in the second main energy level. With the restriction that the angular momentum quantum number can change by only ±1, the allowable transitions are from a *p* orbital to 2*s* ($l = 1$ to $l = 0$), from an *s* orbital to 2*p* ($l = 0$ to $l = 1$), and from a *d* orbital to 2*p* ($l = 2$ to $l = 1$). The problem specifies a change in *energy level*, so n_{init} must be 3, 4, 5, etc. (Although a change from 2*p* to 2*s* would result in a +1 change in *l*, this is not a change in energy level.)

Solution:

The first four transitions are as follows:

3*s* → 2*p*

3*d* → 2*p*

4*s* → 2*p*

3*p* → 2*s*