

* CFG for $L = \{0^m 1^m \mid 2m \leq m \leq 3m\}$.

For each 0, we need to generate either 11 or 111.

$$S \rightarrow 0S11 \mid 0S111 \mid \epsilon.$$

* CFG for $L = \{0^i 1^j 2^{i+j} \mid i, j \geq 0\}$.

Any string in the language can be written in the form $0^i 1^j 2^j 2^i$.

$$S \rightarrow 0S2 \mid T \mid \epsilon$$

$$T \rightarrow 1T2 \mid \epsilon.$$

S generates the outer strings, of the form $0^i T 2^i$, and T generates the inner strings, of the form $1^j 2^j$.

* $L = \{0^m w w^R 0^m \mid m \geq 0, w \in \{0,1\}^*\}$.

$$S \rightarrow 0S0 \mid T$$

$$T \rightarrow 0T0 \mid 1T1 \mid \epsilon$$

S generates the outer part, T generates the inner part (palindromes).

* $L = \{0^m 1^m \mid m \leq m+3\}$

$$S \rightarrow 000T$$

$$T \rightarrow 0T1 \mid W$$

$$W \rightarrow W1 \mid \epsilon.$$

* CFG for $L = \{0^m 1^m \mid m \geq 0\}$

$$S \rightarrow E_0 T \mid T E_1$$

$$E_0 \rightarrow E_0 0 \mid 0$$

$$E_1 \rightarrow E_1 1 \mid 1$$

$$T \rightarrow 0 T 1 \mid \epsilon.$$

E_0 and E_1 generate extra 0's or 1's, and T generates the rest of the string, with an equal number of 0's and 1's.

* CFG for the set of strings over $\{0,1\}$ with more 0's than 1's.

We generate first a string with two 0's and a single 1. On each possible position, we use recursion.

$$S \rightarrow S0S0S1S \mid S0S1S0S \mid S1S0S0S \mid \epsilon.$$

* $L = \{x \# y \mid x, y \in \{0,1\}^* \text{ and } |x| = |y|\}$.

$$S \rightarrow 0S1 \mid 0S0 \mid 1S0 \mid 1S1 \mid \#.$$

* CFG for $L = \{0^i 1^j 2^k \mid i=j \text{ or } j=k\}$

$$S \rightarrow E_{01} T \mid W E_{12}$$

$$E_{01} \rightarrow 0 E_{01} 1 \mid \epsilon$$

$$E_{12} \rightarrow 1 E_{12} 2 \mid \epsilon$$

$$T \rightarrow 2 T \mid \epsilon$$

$$W \rightarrow 0 W \mid \epsilon$$

E_{01} generates the strings with the same number of 0's and 1's; E_{12} generates the strings with the same number of 1's and 2's.