

## Assignment 2

Due: Friday, June 10 at 4:00pm

For any solutions to the problems below that include the specification of a CFG (whether it is to prove that a language is context-free or because you were asked explicitly for a CFG), you do not need to give a formal proof of correctness for the CFG. However, if it is too difficult to verify the correctness of your CFGs, then you may lose points—so please aim to make them as simple and clear as possible, and feel free to include short explanations if you believe it will be helpful.

1. [6 points] For each of the following languages, give a CFG that generates the language:

- (a)  $\{w \in \{0,1\}^* : |w|_0 = 2|w|_1\}$ .
- (b)  $\overline{\text{PAL}}$ , where PAL is the language of palindromes (as defined in Lecture 7).
- (c)  $\overline{\text{BAL}}$ , where BAL is the language of balanced parentheses (also as defined in Lecture 7).

(For part (a) and for question 4 below,  $|w|_\sigma$  is defined as the number of times the symbol  $\sigma$  appears in  $w$ .)

2. [6 points] Let  $\Sigma = \{0,1\}$  and let  $A \subseteq \Sigma^*$  be a context-free language. Prove that the following languages are also context-free:

- (a)  $B = \{uv : u, v \in \Sigma^* \text{ and there exists } \sigma \in \Sigma \text{ such that } u\sigma v \in A\}$ .
- (b)  $C = \{u\sigma v : u, v \in \Sigma^*, \sigma \in \Sigma, \text{ and } uv \in A\}$ .

In other words,  $B$  is the language of all strings you can obtain by choosing a string from  $A$  and removing exactly one symbol from that string, while  $C$  is the language of all strings you can obtain by choosing a string from  $A$  and inserting exactly one additional symbol from  $\Sigma$  anywhere into that string.

3. [6 points] Let  $\Sigma = \{0,1\}$  and let  $A \subseteq \Sigma^*$  be a regular language. Prove that the following languages are context-free:

- (a)  $B = \{ww^R : w \in A\}$ .
- (b)  $C = \{uv^R : u, v \in A, |u| = |v|\}$ .
- (c)  $D = \{u1v : u, v \in \Sigma^*, |u| = |v|, uv \in A\}$ .

4. [6 points] Prove that the following language is not context-free:

$$A = \{w \in \{0,1,2\}^* : |w|_0 \leq |w|_1 \leq |w|_2\}.$$

5. [1 point] For each of the questions above, list the full name of each of your 360 classmates with whom you worked on that question. (If you didn't work with anyone, that is fine: just indicate that you worked alone.)