NFAs

1. For each of the following languages in $\{0,1\}^*$, describe a nondeterministic finite automaton with the specified number of states that accepts that language.

a) Strings that end with 00 with three states.

b) Strings that contain the substring 0101 with five states.

c) The language $0^*1^*0^+$ with three states.

d) The language $1^*(001^+)^*$ with three states.

e) The language $(01 + 011 + 0111)^*$ with four states.

2. Prove that all finite languages are regular.

3. Find an NFA without ϵ-transitions and with a single final state which accepts the language $L = \{0\} \cup \{1^n | n \geq 1\}$. The alphabet is $\{0,1\}^*$.

4. Prove that, for every NFA with an arbitrary number of final states, there is an equivalent NFA with only one final state.

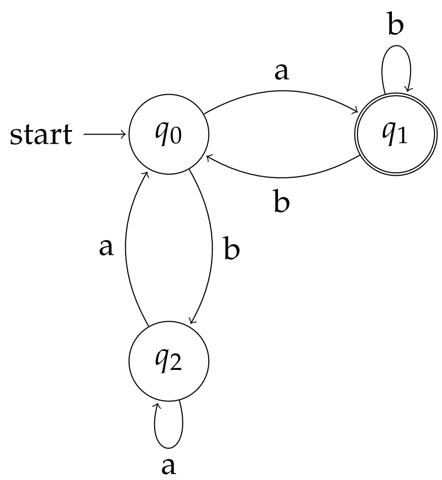
5. Find an NFA that accepts $\{a\}^*$ such that, if we remove a single edge in its transition graph (without any other changes), the resulting automaton accepts $\{a\}$.

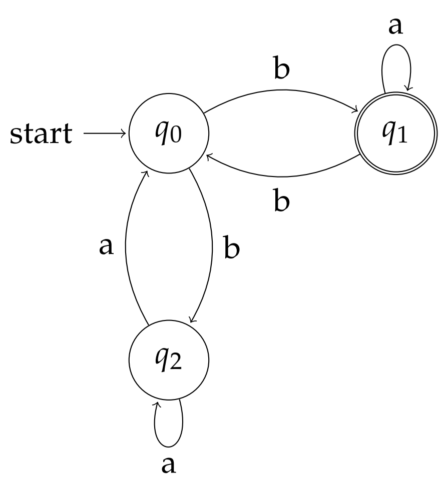
6. Let L be the language generated by the regular expression $aa^* + bb^*$ over the alphabet $\{a,b\}$.

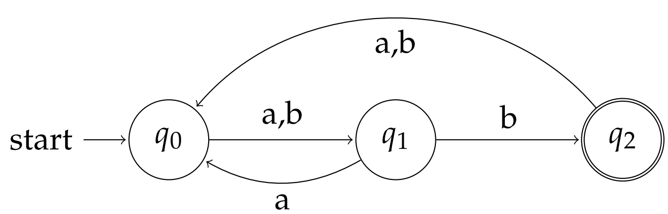
a) Prove that any DFA accepting L must have at least two final states.

b) Give an NFA with a single final state that accepts L.

7. Give regular expressions for the language accepted by the following NFAs. Briefly justify.

a) 

b) 

c) 

8. Recall that the complement of a language L over a finite alphabet Σ is the set $\overline{L} = \Sigma^* - L$.

a) Show that, if M is a DFA that recognizes the language B, swapping the accept and nonaccept states in M yields a new DFA recognizing the complement of B. Conclude that the class of regular languages is closed under complement.

b) Show by giving an example that, if M is an NFA that recognizes the language C, swapping the accept and nonaccept states in M does not necessarily yield a new NFA that recognizes the complement of C. Is the class of languages recognized by NFAs closed under complement? Explain your answer.