CPSC 313 (Fall 2012) L01

University of Calgary
Faculty of Science
Midterm

November 1, 2012.
Time available: 120 minutes.
No books or calculators are permitted (two 8.5×11 sheets of notes are permitted).

Write answers in this booklet only.
Do not open this exam until you are told to do so.

Name: ____

Lab section:

☐	☐	☐
T02	T01	T03
Mon 17:00	Tues 10:00	Wed 12:00
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There are 6 (six) problems in total. There are 34 points (31 points and 3 bonus points) in total. Full answer is 29 points.

For your convenience, the last page (page 16) contains a collection of information.

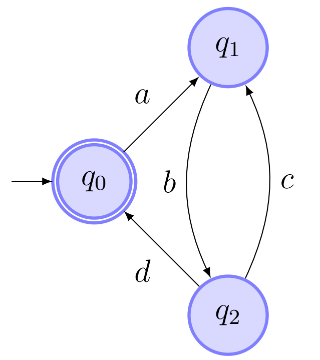
You may use the blank pages at the end of the exam if you need more space for your answers. Please indicate clearly when your answers are continued on these pages.

Please note that points may be deducted for untidy handwriting.

Problem	Possible score	Score
1	2	
2	16	
3	3	
4	4	
5	6	
6	3 bonus	
Total	31 + 3 bonus	

1. Regular expressions (2 points)

Give a regular expression for the language accepted by the following NFA.



2. Short answers (16 points)

1. Give a regular expression for the language $L = \{w \in \Sigma^\star \mid |w| \text{ is even or } w \text{ starts with an } a\}$. Here $\Sigma = \{a, b\}$.

2. Give an NFA accepting the regular language $L = \{a^i b^j \mid i, j \geq 0 \text{ and } |i - j| \text{ is odd}\}$.

3. Give a non-regular language L such that $L \subseteq L(\mathbf{b}^\star)$.

4. The following claim is **false**. Explain why it is false and make a (small) modification to the claim so that it becomes true.

Claim 1 *Let $N = (Q, \Sigma, \delta, q_0, F)$ be an NFA with $m = |Q|$ states. Suppose we* ***add*** *a new state q_{new} to N which we make the new start state, and that we* ***add*** *m productions $\delta(q_{\text{new}}, \lambda) = q_i$ to δ, one for every $q_i \in Q$. Then the resulting NFA accepts the language* $\mathsf{suffix}(L)$*.*

5. Give a **succinct** description of the language $L = L(G)$ generated by the grammar G containing the following seven productions $S \to aSb \mid A \mid B;\ A \to aA \mid a;\ B \to bB \mid b$.

6. Give a minimal DFA that has exactly two final states.

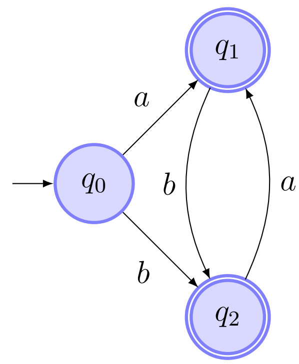
7. Give an infinite regular language L_{reg} such that

$$L_{\text{reg}} \subseteq \{w \in \{a,b\}^\star \mid |w|_a = |w|_b\}.$$

8. Give a grammar in Chomsky Normal Form for the language $L = \{a^i b a^i \mid i \geq 0\}$.

3. Finite automata (3 points)

Let $L = L(N)$ be the language accepted by the following NFA $N = (Q, \Sigma, \delta, q_0, F)$, having final states $F = \{q_1, q_2\}$ and alphabet $\Sigma = \{a, b\}$.



1. Give a regular expression **r** such that $L(\mathbf{r}) = \overline{L}$ is the complement of the language L.

2. Give a DFA that accepts L^+. Briefly explain why your FA is deterministic and why it accepts L^+.

4. Closure properties (4 points)

Show that the following language is non-regular. You may, without proof, use the Pumping Lemma and the Closure Properties stated on the last page of this midterm (page 16). You may also use, without proof, that the following four languages are non-regular.

1. $L_1 = \{ww \mid w \in \{a,b\}^*\}$
2. $L_2 = \{ww^R \mid w \in \{a,b\}^*\}$
3. $L_3 = \{a^i b^i \mid i > 0\}$
4. $L_4 = \{a^i b^j \mid i \neq j\}$

Show that $L = \{uvu \mid u, v \in \{a,b\}^* \text{ and } |u| = |v|\}$ is non-regular.

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5. True or False (6 points)
Answer True or False. No justification required. All languages (except in questions 3, 8, and 11) are over the alphabet $\Sigma = \{a, b\}$.

	Question	True	False
1	If $L^* = \Sigma^*$ then L is regular		
2	Every context-free language is non-regular		
3	The grammar $S \to abca$; $ca \to ba$ generates the language $\{abba\}$		
4	$L^* \setminus L^+ = \{\lambda\}$		
5	$L^* = L^+ \cup \{\lambda\}$		
6	The grammar $S \to aSb \mid bSa \mid \lambda$ generates a context-free language		
7	There exists a context-free language that is finite		
8	The language $\{\lambda, a\}^*$ is regular		
9	The grammar $S \to aSaS \mid bSbS \mid \lambda$ generates the language $L = \{ww \mid w \in \Sigma^*\}$		
10	The grammar $S \to SaS \mid b$ generates a non-regular language		
11	The grammar $S \to SS \mid a$ generates the language $\{a^{2^i} \mid i \geq 0\}$		
12	If $L \cap L(\mathbf{a}^\star\mathbf{b}^\star)$ is non-regular, so is L		

6. (Hard) Minimal NFAs (3 bonus points)
Give a minimal NFA (non-deterministic finite automata) accepting the finite language

$$L = \{w \in \{a,b\}^\star \mid |w|_a = 4 \text{ and } |w|_b = 1\}.$$

Full points to correct solutions. No points to partial solutions. No justification required.

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Theorem 2 (Closure Properties) *Let L_1 and L_2 be regular languages over the same alphabet Σ. Then the following nine languages are regular as well.*

1. $\overline{L_1} = \{w \in \Sigma^* \mid w \notin L_1\}$
2. $L_1 \cup L_2$
3. $L_1 \cap L_2$
4. L_1^*
5. $L_1 L_2$
6. $L_1^{\mathrm{R}} = \{w \in \Sigma^* \mid w^R \in L_1\}$
7. $\mathsf{prefix}(L_1) = \{u \in \Sigma^* \mid \exists v \in \Sigma^* \text{ such that } uv \in L_1\}$
8. $\mathsf{suffix}(L_1) = \{v \in \Sigma^* \mid \exists u \in \Sigma^* \text{ such that } uv \in L_1\}$
9. $h(L_1),$ where $h : \Sigma \to \Gamma^*$ is a homomorphism and Γ is an alphabet.

Theorem 3 (Pumping Lemma for Regular Languages) *Let L be a regular language. Then there*

exists integer $m \in \mathbb{Z}$ such that for all $w \in L$ with $|w| \geq m$,

there exist strings x, y, and z, such that $w = x \cdot y \cdot z$ and

1. $|x \cdot y| \leq m$,
2. $|y| \geq 1$, *and*
3. *for all* $i \geq 0$, $x \cdot y^i \cdot z \in L$.
