* Let $L \subseteq \Sigma^*$ be an arbitrary regular language. Prove that the language

$$\mathrm{left}(L) = \{x \in \Sigma^* \mid xy \in L \text{ for some } y \in \Sigma^* \text{ where } |x| = |y|\}$$

is also regular.

S. Let $M = (Q, \Sigma, \delta, q_0, F)$ be a DFA which accepts L. We construct an NFA which simulates two copies of M in parallel: one copy moves forward reading x, the other moves backward from a final state of M by guessing y. If the two copies arrive at the same state, we accept. Formally, the NFA which accepts left(L) is $N = (Q', \Sigma, \delta', Q_0', F)$, where

$Q' = Q \times Q$,

$Q_0' = \{(q_0, q_f) \mid q_f \in F\}$,

$\delta'((q_1, q_2), a) = \{(\delta(q_1, a), q_3) \mid$ there exists $b \in \Sigma$ with $\delta(q_3, b) = q_2\}$,

$F = \{(q, q)\}$.

* Let $A/B = \{w \mid wx \in A$ for some $x \in B\}$. Show that if A is regular and B is any language, then A/B is regular.

S. Let Σ be the union of the alphabets for A and B. Let $M = (Q, \Sigma, \delta, q_0, F)$ be a DFA which accepts A. We construct $M' = (Q, \Sigma, \delta, q_0, F')$, where

$F' = \{ q \in Q \mid \exists x \in B$ such that M goes from q to some final state while reading $x\}$.

Then M' is a DFA for A/B.