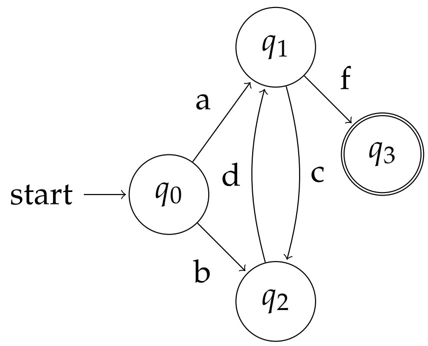
Practice Midterm Problems
CPSC 313 Spring 2016

1) The midterm will cover the material discussed in the lectures before June 6.
2) The length of the midterm: 150 minutes.
3) There will be no problems about induction or pigeonhole principle, but these concepts may be needed to solve problems related to DFAs, NFAs, regular languages, CFGs.
4) All the theory for the exam was covered in class.
5) Two letter-sized pages of notes are permitted.
6) The best source of problems for the midterm are the examples discussed in class, the assignments, and the exercises which accompany the lectures. Many other problems can be found in the textbook and in the resources listed on the webpage.
7) All problems on the midterm will require some justification. For instance, if you are asked to design a DFA, you will need to describe the DFA in English.
8) Below you can find some more practice problems.

Problem 1 Give a regular expression for the language $L = \{w \in \{0,1,2\}^* | \#(0,w) = 1$ and $\#(1,w) = 1\}$. Briefly justify. Here $\#(0,w)$ denotes the number of 0s in the bitstring w and $\#(1,w)$ denotes the number of 1s in w.

Problem 2 Give a regular expression for the language accepted by the following NFA. Briefly justify.



Problem 3 Give an NFA that accepts the language $\{0\}^*$ such that, if in its transition graph a single edge is removed (without other changes), the resulting automata accepts $\{0\}$.

Problem 4 Give a context-free grammar for the language $L = \{w \in \{a\}^* |\ |w| \not\equiv 1 \pmod 3\}$. Briefly justify. Here $|w|$ denotes the length of the string w.

Problem 5 Give a context-free grammar for the language $L = \{0^i 1^j 2^k |\ i = j \text{ or } j = k\}$. Briefly justify.

Problem 6 Let L be the language corresponding to the regular expression 0^*1^*. Prove that any DFA for L must have at least two final states. The alphabet is $\Sigma = \{0,1\}$.

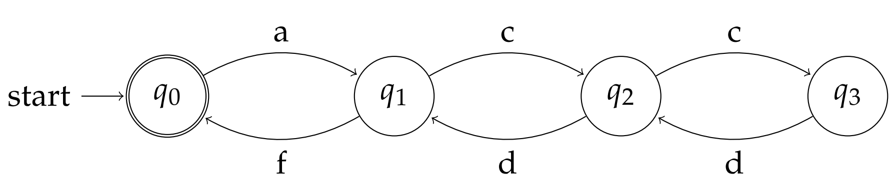
Problem 7 Let $L \subseteq \Sigma^*$ be an arbitrary regular language. We define the language

$$\text{left}(L) = \{x \in \Sigma^* | xy \in L \text{ for some } y \in \Sigma^*, \text{ where } |x| = |y|\}.$$

Prove that left(L) is regular.

Problem 8 Give a DFA accepting the language of all strings over $\{0,1,2\}$ that end with 100 or 200.

Problem 9 Give a regular expression for the language accepted by the following NFA. Briefly justify.



Problem 10 Decide whether each the following statements is true or false. Give a short proof for each statement you believe is true, and give a counter-example to each statement you believe is false. The alphabet is $\Sigma = \{0, 1\}$.

1) The language of an NFA with only one state must be finite.

2) The intersection of a regular language with a nonregular language is not regular.

3) Let A and B two non-regular languages. Then $A \cap B$ is non-regular.

4) If the language A^* is regular, then A is also regular.

5) The grammar $S \to 0S1 \mid 0S \mid S1 \mid \epsilon$ generates a regular language.

6) If A, B are two languages such that $A \subseteq B$, and B is regular, then A is regular.

Problem 11 Recall that the complement of a language L over a finite alphabet Σ is the set $\bar{L} = \Sigma^* - L$.

a) Show that, if M is a DFA that recognizes the language B, swapping the accept and nonaccept states in M yields a new DFA recognizing the complement of B. Conclude that the class of regular languages is closed under complement.

b) Show by giving an example that, if M is an NFA that recognizes the language C, swapping the accept and nonaccept states in M does not necessarily yield a new NFA that recognizes the complement of C. Is the class of languages recognized by NFAs closed under complement? Explain your answer.

Problem 12 Give a regular expression for the language consisting of all the bitstrings over $\{0, 1\}$ in which every pair of adjacent 0's appears before any pair of adjacent 1's.

Problem 13 Give a context-free grammar for the set of bitstrings containing twice as many 0's as 1's.

Problem 14 Give a language L_1 such that $L = L_1 L_2$ is regular, where $L_2 = \{a^i b^j \mid i \leq j\}$.

Problem 15 Give a regular expression for the language consisting of all the bitstrings over $\{0, 1\}$ that do not end in 101.

Problem 16 For the alphabet $\Sigma = \{a, b, c, d, e, f\}$, construct an NFA for the language $L = \{w \in \Sigma^* \mid \text{the last character of } w \text{ appears nowhere else in the string and } |w| \geq 1\}$.

Problem 17 Let L be a language. Prove that $L = L^+$ if and only if $LL \subseteq L$.

Problem 18 Give a context-free grammar for the language $L = \{0^i 1^j 2^k \mid i > j \text{ or } i > k\}$. Briefly justify.

Problem 19 Let Σ be any alphabet, and let L be a regular language over Σ. We define the languages

$$\text{Prefix}(L) = \{x \in \Sigma^* \mid \text{there exists } v \in \Sigma^* \text{ such that } xv \in L\}$$

and

$$\text{Suffix}(L) = \{x \in \Sigma^* \mid \text{there exists } u \in \Sigma^* \text{ such that } ux \in L\}.$$

Prove that $\text{Prefix}(L)$ and $\text{Suffix}(L)$ are also regular.