CPSC 313 (Fall 2011) L01

University of Calgary
Faculty of Science
Midterm

November 3, 2011.
Time available: 120 minutes.
No books or calculators are permitted (two 8.5 × 11 sheets of notes are permitted).

Write answers in this booklet only.
Do not open this exam until you are told to do so.

Name: ______________________________

Lab section:

☐	☐	☐
T02 Mon 17:00 Jibran	T01 Tues 10:00 Zahra	T03 Wed 12:00 Fatemeh

There are 6 (six) problems in total. There are 34 points (31 points and 3 bonus points) in total. Full answer is 29 points.

For your convenience, the last page (page 14) contains a collection of information.

You may use the blank pages at the end of the exam if you need more space for your answers. Please indicate clearly when your answers are continued on these pages.

Please note that points may be deducted for untidy handwriting.

Problem	Possible score	Score
1	12	
2	3	
3	8	
4	5	
5	3	
6	3 bonus	
Total	31 + 3 bonus	

1. Short answers (12 points)

1. Give two distinct non-regular languages L_1 and L_2 such that $L_1 \cup L_2$ is non-regular.

2. Give a regular expression for the language $L = \{w \in \Sigma^* \mid 1 \leq |w|_a \leq 3\}$. Here $\Sigma = \{a, b\}$.

3. Draw a **DFA** M accepting the language L^* where $L = \{a, aaaa, abba, b\}$.

4. The following claim is **false**. Explain why it is false and make a (small) modification to the claim so that it becomes true.

Claim 1 *Let $N = (Q, \Sigma, \delta, q_0, F)$ be an NFA with a unique final state, $F = \{q_f\}$. Suppose we* add *the production $\delta(q_f, \lambda) = q_0$ to δ. Then the resulting NFA accepts the language $L(N)^*$.*

5. Give a contextfree grammar for the language $L = \{a^i b^j c^j d^i \mid i, j \geq 0\}$.

6. Give a non-regular language L such that $L \cup L^R$ is regular.

2. DFA Construction (3 points)

Draw a **DFA** accepting the language $L = \{w \in \Sigma^* \mid bba \text{ is } \textbf{not} \text{ a substring of } w\}$. Here $\Sigma = \{a, b\}$. Briefly explain why your FA is deterministic and why it accepts L.

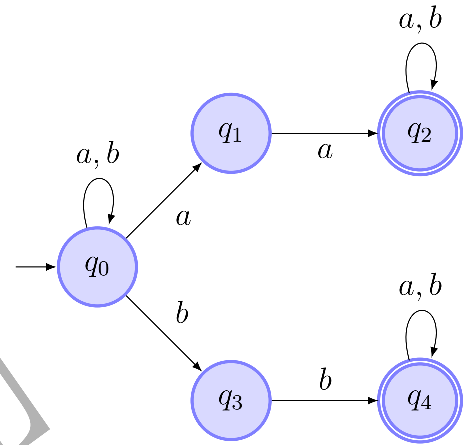
3. True or False (8 points)

Answer True or False. No justification required. All languages (except in question 9) are over the alphabet $\Sigma = \{a, b\}$. Here $|w|_a$ denotes the number of a symbols in the string w.

	Question	True	False
1	$\{w \in \Sigma^* \mid abba$ occurs as a substring of $w\}$ is regular		
2	$\{w \in \Sigma^* \mid w$ is *not* a palindrome$\}$ is regular		
3	If L is regular, so is $L \cdot L$		
4	$(L^*)^+ = L^*$		
5	$(L^+)^+ = L \cdot L^*$		
6	$L^+ = L^* \setminus \{\lambda\}$		
7	$(L^+)^* = L^*$		
8	$\{\lambda, a\}^* = \{a^i \mid i = 2^n$ for some integer $n\}$		
9	The grammar $S \to aA; A \to Bc; B \to bS$ generates the language $L = \{a^i b^i c^i \mid i \geq 0\}$		
10	If L is a non-regular then so is $\overline{L}$		
11	Every finite language is context-free		
12	There exists a right-linear grammar that generates the language $L((\mathbf{a} \cup \mathbf{b}^*)\mathbf{a}^+)$		
13	$L = \{w \mid$ for all prefixes u of w, $-1 \leq \lvert u\rvert_a - \lvert u\rvert_b \leq 1\}$ is regular		
14	$L = \{w \in \Sigma^* \mid \lvert w\rvert_a$ is divisible by $7\}$ is regular		
15	Every infinite language is non-regular		
16	If L is regular, so is the language $\mathsf{suffix}(L) = \{v \mid \exists u$ such that $uv \in L\}$		

4. Complementation of an NFA (5 points)

Let $L = L(N)$ be the language accepted by the following NFA $N = (Q, \Sigma, \delta, q_0, F)$, having final states $F = \{q_2, q_4\}$.



1. Give a regular expression $\mathbf{r}$ such that $L(\mathbf{r}) = L$.

2. Let $\overline{F} = Q \setminus F = \{q_0, q_1, q_3\}$. Explain why the NFA $(Q, \Sigma, \delta, q_0, \overline{F})$ does *not* accept $\overline{L}$.

3. Give an NFA K that accepts $\overline{L}$. Explain why your NFA K accepts $\overline{L}$.

5. Closure properties (3 points)

Show that the following language is non-regular. You may, without proof, use the Pumping Lemma and the Closure Properties stated on the last page of this midterm (page 14). You may also use, without proof, that the following four languages are non-regular.

1. $L_1 = \{ww \mid w \in \{a,b\}^*\}$
2. $L_2 = \{ww^R \mid w \in \{a,b\}^*\}$
3. $L_3 = \{a^i b^i \mid i > 0\}$
4. $L_4 = \{a^i b^j \mid i \neq j\}$

Show that $L = \{b^r c^i abd^j ba^k g^l d \mid r,i,j,k,l \geq 0; j + l = i\}$ is non-regular.

6. (Hard) Regular Expression to DFA (3 bonus points)

Let

$$\mathbf{r} = \big((\mathbf{a} \cup \mathbf{ba}^+)^*(\mathbf{a} \cup \mathbf{b} \cup \boldsymbol{\lambda})\big) \cap \big((\mathbf{b} \cup \mathbf{ab}^+)^*(\mathbf{b} \cup \mathbf{a} \cup \boldsymbol{\lambda})\big).$$

Give a minimal DFA accepting the language $L(\mathbf{r})$.

(Full points to correct solutions. No points to partial solutions. No justification required.)

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Theorem 2 (Closure Properties) *Let L_1 and L_2 be regular languages over the same alphabet Σ. Then the following eight languages are regular as well.*

1. $\overline{L_1} = \{w \in \Sigma^* \mid w \notin L_1\}$
2. $L_1 \cup L_2$
3. $L_1 \cap L_2$
4. L_1^*
5. $L_1 L_2$
6. $L_1^{\mathrm{R}} = \{w \in \Sigma^* \mid w^R \in L_1\}$
7. $\mathrm{prefix}(L_1) = \{u \in \Sigma^* \mid \exists v \in \Sigma^* \text{ such that } uv \in L_1\}$
8. $h(L_1)$, where $h : \Sigma \to \Gamma^*$ is a homomorphism and Γ is an alphabet.

Theorem 3 (Pumping Lemma for Regular Languages) *Let L be a regular language. Then there*

exists integer $m \in \mathbb{Z}$ such that for all $w \in L$ with $|w| \geq m$,

there exist strings x, y, and z, such that $w = x \cdot y \cdot z$ and

1. $|x \cdot y| \leq m$,
2. $|y| \geq 1$, *and*
3. *for all* $i \geq 0$, $x \cdot y^i \cdot z \in L$.
