

Prove that if a language L is regular then so is the language $\text{prefix}(L)$.

Example Proof:

(The tools and assumptions we need)

Assume L is regular. Then there exists a DFA $\mathbf{M}=(Q,\Sigma,\delta,q_0,F)$ such that L is accepted by \mathbf{M} . Also u is a prefix of $uv \in L$ if and only if there is a path in \mathbf{M} from q_0 (start state) to $\delta(q_0,u)$ (state after processing the prefix u) and there is a path in \mathbf{M} from $\delta(q_0,u)$ to a state in F . (an accepting state)

(Using the tools and assumption we established)

Let T be the set containing all states q such that there is a path in \mathbf{M} from q_0 to q and there is a path in \mathbf{M} from q to a state in F . The language $\text{prefix}(L)$ is accepted by the DFA $\mathbf{M}' = ((Q,\Sigma,\delta,q_0,F \cup T)$ by the definition of prefix above and therefore $\text{prefix}(L)$ is regular.