Turing machines

1. Describe Turing machines that decide each of the following languages:

a) Palindromes over the alphabet $\{0, 1\}$.

b) The complement of the language in part a).

c) $\{ww \mid w \in \{0, 1\}^*\}$.

d) $\{1^p \mid p \text{ is prime }\}$.

2. Show that there exists a Turing machine that on input 01^i01^j0 outputs $01^{i \cdot j}0$ for all positive integers i, j.

3. Prove that any standard Turing machine can be simulated by a Turing machine with only three states.

4. Let M be a one-tape Turing machine. Let w be an input of length n such that, when processing w, the machine does not move its head to the left in the first $n + q + 1$ steps. Prove that M never moves its head left on this input.

5. Show that the collection of decidable languages is closed under union, intersection, concatenation, star, and complementation.

6. Show that the collection of Turing-recognizable languages is closed under union, intersection, concatenation, and star.

7. A Turing machine with stay put instead of left is similar to an ordinary Turing machine, but the transition function has the form $\delta : Q \times \Gamma \to Q \times \Gamma \times \{R, S\}$. At each point, the machine can move its head right or let it stay in the same position. Show that the language recognized by such a machine is a regular language.