# Regular languages

For some of the problems below, we have a regular language $L$ and we want to prove that a language $L'$ obtained from $L$ by some operation is also regular. One approach is to take a DFA for $L$ and to show that there exists an NFA which recognizes $L'$. We do not need to construct explicitly the new NFA – it is enough to show that it exists.

**1.** Let $L \subseteq \Sigma^*$ be an arbitrary regular language. Prove that the following languages are regular.

a) $\text{prefmax}(L) = \{x \in L | xy \in L \iff y = \epsilon\}$.

b) $\text{sufmin}(L) = \{xy \in L | y \in L \iff x = \epsilon\}$.

c) $\text{left}(L) = \{x \in \Sigma^* | xy \in L \text{ for some } y \in \Sigma^* \text{ where } |x| = |y|\}$.

d) $\text{right}(L) = \{y \in \Sigma^* | xy \in L \text{ for some } x \in \Sigma^* \text{ where } |x| = |y|\}$.

e) $\text{everyother}(L) = \{\text{everyother}(w) | w \in L\}$, where $\text{everyother}(w)$ is the subsequence of $w$ containing every other symbol. For example, $\text{everyother}(\text{EVERYOTHER}) = \text{VROHR}$.

f) $\text{cycle}(L) = \{xy | x, y \in \Sigma^* \text{ and } yx \in L\}$.

**2.** Let $A/B = \{w \in B | wx \in A \text{ for some } x \in B\}$. Show that if $A$ is regular and $B$ is any language, then $A/B$ is regular.

**3.** Let $B$ and $C$ be languages over $\Sigma = \{0, 1\}$. Define

$$B \stackrel{1}{\leftarrow} C = \{w \in B | \text{for some } y \in C, \text{ strings } w \text{ and } y \text{ contain equal numbers of 1s}\}.$$

Prove that the class of regular languages is closed under the $\stackrel{1}{\leftarrow}$ operation.