CPSC 313 Spring 2016

University of Calgary
Faculty of Science
Midterm

8 June 2016

Time available: 150 minutes.

No books or calculators are permitted (two pages of notes are permitted).

Write answers in this booklet only.

Do not open the exam until you are told to do so.

Name: ______________________________

1) **Don't panic!** Partial credit is available.

2) There are 31 points in total and 3 bonus points. Full answer is 31 points.

3) Please read the entire exam before writing anything. Please ask for clarification if any question is unclear.

4) Please note that points may be deducted for unintelligible handwriting.

5) You may use the blank pages at the end of the exam if you need more space for your answers. Please indicate clearly when your answers are continued on these pages.

6) For your convenience, the last page contains a collection of information.

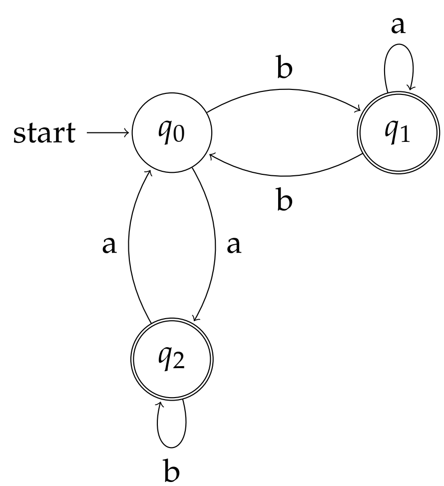
7) **Good luck!**

Problem	Possible score	Score
1	16	
2	6	
3	5	
4	4	
5	3 bonus	

Problem 1 Short answers (16 points, 2 points each)

1) Give a regular expression for the language $L = \{w \in \{a, b, c\}^* |$ the number of a's in w is a multiple of $3\}$. For instance $\epsilon \in L$, $abacba \in L$, $aaaaaba \in L$, but $aba \notin L$.

2) Give a regular expression for the language accepted by the following NFA.



3) Give an NFA having only three states accepting the language $L = \{w \in \{a,b\}^* | w \text{ does not contain the substring } aba\}$.

4) Give a context-free grammar for the language $L = \{a^n b^n c^k |\ n \geq 0,\ k \geq 3\}$.

5) Give a regular expression for the language consisting of all the bitstrings over $\{0,1\}$ that have both 00 and 11 as substrings.

6) Give a DFA accepting the language $L = \{w \in \{0,1\}^* | \ |w| \text{ is divisible by 2 but not by 3}\}$. Here $|w|$ denotes the length of the string w. For example, $|\epsilon| = 0$ and $|01| = 2$.

7) Give an NFA accepting the language of all strings over $\{0,1\}^*$ that contain at least one occurrence of the substring 000, but no occurrences of the substring 001.

8) Give a context-free grammar for the language consisting of all the strings in $\{0, 1\}^*$ that are not palindromes. Briefly justify.

(Blank space for continuing your answers to problem 1).

Problem 2 True/False questions (6 points, 1 point each)

Decide whether each the following statements is true or false. Give a short proof for each statement you believe is true, and give a counter-example to each statement you believe is false.

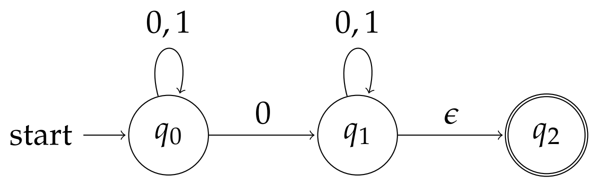
1) If L is any non-regular language over an alphabet Σ, then the complement of L is also non-regular.

2) The class of regular languages is closed under infinite union.

3) The number of regular languages over the alphabet $\Sigma = \{0, 1\}$ is infinite.

True/False questions

4) The language of the NFA below is $L = \{w \in \{0,1\}^* \mid w$ has odd length and its middle character is $0\}$



5) If L is a regular language over the alphabet $\{0,1\}$, then the language $\text{Even}(L) = \{w \in L \mid |w| \text{ is even }\}$ is regular. Here $|w|$ denotes the length of the string w. For example, $|\epsilon| = 0$ and $|01| = 2$.

6) The regular expression $0(120)^*12$ generates the same language as the regular expression $01(201)^*2$.

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Problem 3 DFA and regex (5 points: 3 + 2) We say that two bitstrings have the same 1-parity if and only if both strings contain an odd number of 1s or both strings contain an even number of 1s. We define over the alphabet $\Sigma = \{0, 1, \#\}$ the following language

$$1\text{PAR} = \{u\#v | u, v \in \{0,1\}^* \text{ and } u \text{ and } v \text{ have the same 1-parity}\}$$

For example # $\in$ 1PAR, 01#111 $\in$ 1PAR, and 0011#11001010 $\in$ 1PAR. However 1#0 $\notin$ 1PAR, ## $\notin$ 1PAR, and $\epsilon \notin$ 1PAR.

a) Describe a DFA for 1PAR. A formal description or a drawing without an English explanation will receive no credit, even if it is correct.

b) Write a regular expression for 1PAR.

(Blank page for continuing your answers to problem 3).

Problem 4 Constructive proof (4 points: 1 + 3).

If $x = a_1a_2 \ldots a_n$ and $y = b_1b_2 \ldots b_n$ are two strings of the same length n, define $\text{alt}(x, y)$ to be the string in which the symbols of x and y alternate, starting with the first symbol of x, that is,

$$\text{alt}(x, y) = a_1b_1a_2b_2 \ldots a_nb_n.$$

If L and M are languages, define $\text{alt}(L, M)$ to be the language of all strings of the form $\text{alt}(x, y)$, where x is any string in L and y is any string in M of the same length.

a) Write a regular expression for $\text{alt}(0^*1, 01^*)$.

b) If L and M are regular languages, prove that $\text{alt}(L, M)$ is regular.

(Blank page for continuing your answers to problem 4).

Problem 5 CFG design (3 bonus points).

This problem is intended to be a challenge.

Let $D = \{xy|\ x, y \in \{0,1\}^* \text{ and } |x| = |y| \text{ but } x \neq y\}$. Give a context-free grammar for D.

(Blank page for continuing your answers to any problem).

Theorem 1. *Let L_1 and L_2 be regular languages over the alphabet Σ. Then the following languages are regular.*

1. $\overline{L_1} = \{w \in \Sigma^* \mid w \notin L_1\}$,
2. $L_1 \cup L_2$,
3. $L_1 \cap L_2$,
4. L_1^*,
5. $L_1 L_2$,
6. $L_1^R = \{w \in \Sigma^* \mid w^R \in L_1\}$.

Theorem 2. *Any finite language is regular.*

Theorem 3. *Let $L_1 = \{w \in \{0,1\}^* \mid w = w^R\}$ and $L_2 = \{0^n 1^n \mid n \geq 0\}$. Then L_1 and L_2 are not regular.*

(Blank space for continuing your answers to any problem).