Strings, sets and languages

1.

a) Prove that $\{\epsilon\} \bullet L = L \bullet \{\epsilon\} = L$ for any language L.

b) Prove that $(A \bullet B) \bullet C = A \bullet (B \bullet C)$, for all languages A, B, and C.

c) Prove that $|A \bullet B| = |A| \cdot |B|$, for all languages A and B. (The second $\cdot$ is multiplication).

d) Prove that L^* is finite if and only if $L = \varnothing$ or $L = \{\epsilon\}$.

e) Prove that $AB = BC$ implies $A^*B = BC^* = A^*BC^*$, for all languages A, B, and C.

2. Let L be a language. Prove that $L^+ = L^*$ if and only if $\epsilon \in L$.

3. The reversal L^R of any language L is the set of reversals of all strings in L.

a) Prove that $(AB)^R = B^R A^R$ for all languages A and B.

b) Prove that $(L^R)^R = L$ for every language L.

c) Prove that $(L^*)^R = (L^R)^*$ for every language L.

d) Prove or disprove the following claim: for any languages A and B it holds that $(A \cup B)^R = A^R \cup B^R$.

4. Let L be a language. Prove that $L = L^+$ if and only if $LL \subseteq L$.

5. Consider the following recursively-defined sets of strings of left brackets (and right brackets):

- A string x is balanced if it satisfies one of the following conditions:
  - x is the empty string, or
  - $x = (y)z$, where y and z are balanced strings.
- A string x is erasable if it satisfies one of two conditions
  - x is the empty string, or

- $x = y()z$, where yz is an erasable string.

Prove the following statements.

a) For any erasable strings x and y, the string xy is also erasable.

b) Every balanced string is erasable.