Decidability and reductions

1. Show that $\leq_m$ is a transitive relation.

2. Let $T = \{\langle M \rangle \mid M$ is a Turing machine that accepts w^R whenever it accepts $w\}$. Show that T is undecidable.

3. Consider the problem of determining whether a two-tape Turing machine M ever writes a non-blank symbol on its second tape when it is run on some input w. Formulate this problem as a language and show that it is undecidable.

4. Consider the problem of determining whether a two-tape Turing machine M ever writes a non-blank symbol on its second tape during the course of its computation on any input string. Formulate this problem as a language and show that it is undecidable.

5. Consider the problem of testing whether a Turing machine M on an input w ever attempts to move its head left when its head is on the left-most tape cell. Formulate this problem as a language and show that it is undecidable.