Assignment 1
Due no later than May 18 at 13:00

For full credit it is enough to accumulate 7 points.

Exercise 1 (2 points).

1. Consider the alphabet $\Sigma = \{a, b\}$. Let $L_2 = \{a^i b^j | \ |i - j| \leq 2\}$, $L_a = \{aa\}$ and $L_b = \{bbbb\}$. Compute $L = L_a(L_2 L_b \cap L_2)$.

2. Prove that, for any languages A and B, we have

$$(A \cup B)^* = A^*(BA^*)^*.$$

Exercise 2 (1 point). You are given a 3×9 grid of points, and you color each point in the grid either red or black. Prove that there exist four points of the same color that form the corners of a rectangle. (*Hint: pigeonhole principle*).

Exercise 3 (2 points). Consider the following series: $1, 2, 3, 4, 5, 10, 20, 40, \ldots$, which starts as an arithmetic series, but after the first five terms becomes a geometric series. Prove that any positive integer can be written as a sum of distinct numbers from the series.

Exercise 4 (3 points). Give a regular expression for the following languages over the alphabet $\Sigma = \{a, b\}$. Briefly explain why your regular expression captures the language.

1. The language L_1 consisting of strings of length at least two, where the first letter is the same as the last letter, and the second letter is the same as the second to last letter. For example $a \notin L_1$, $aa \in L_1$, $aaa \in L$, $aba \in L_1$, $bbabba \notin L_1$.

2. The language $L_2 = \{w \in \Sigma^* | \ |w|$ is even or w starts with $a\}$.

3. The language $L_3 = \{w \in \Sigma^* | \ $ every fourth a is surrounded by b's $\}$.

Submission

You may turn in your assignment using the drop boxes on the second floor in the math science building, or by giving your assignment in person to one of the TAs. You must submit your assignment on or before Wednesday, May 18, 2016, 13:00. No late submissions will be accepted. The deadline is **firm**. For extenuating circumstances please contact the instructor.

Use the last page of this assignment as the front page of your assignment. Assignments submitted without the front page will be deducted 1 point.

Collaboration and plagiarism

You are welcome to work and discuss the assignment with other students enrolled in this course (i.e., CPSC 313 Spring 2016). You must clearly state whom your collaborators are, if any, for each problem on the assignment.

Verbal collaboration is allowed. Written collaboration is strictly forbidden. For instance, notes, papers, emails, messages, texting, twitter, chats, blogs, discussion boards, whiteboards, blackboards, and photos used as communication devices are strictly forbidden. All written work that you submit must be your own sole work. Anything else will be considered plagiarism. When you are discussing this assignment with others, do not use any form of writing.

The use of published literature is allowed. If you use any published literature (texts, articles, websites, etc) to complete your assignment, you must quote your sources. I suggest that you develop your own solutions however, without the use of any published materials. You will be asked to answer similar questions on the exams for this course and during the exams no such sources will be available.

You may read about the regulations on plagiarism in the calendar here: `http://www.ucalgary.ca/pubs/calendar/current/k-2.html`. If you have any doubt whether a collaboration is allowed or not, ask the lecturer before entering the collaboration.

CPSC 313 Assignment 1 Spring 2016

Name: _____

My sources and my collaborators, if any, on this assignment were:

Exercise 1: _____

Exercise 2: _____

Exercise 3: _____

Exercise 4: _____