Computer Science 331
Stacks

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Lecture #13

Outline

Definition of a Stack ADT

A *stack* is a collection of objects that can be accessed in "last-in, first-out" order: The only visible element is the (remaining) one that was most recently added.

It is easy to implement such a simple data structure extremely efficiently — and it can be used to several several interesting problems.

Indeed, a *stack* is used to execute recursive programs — making this one of the more widely used data structures (even though you generally don't notice it!)

Stack ADT

Stack Interface:

```
public interface Stack<T> {
  public push(T x);
  public T peek();
  public T pop();
  public boolean isEmpty();
}
```

Stack Invariant:

- The object that is visible at the top of the stack is the object that has most recently been pushed onto it (and not yet removed)

A Stack Interface: Methods

1. `void push(T obj)`:
   - *Precondition:* Interface invariant
   - *Postcondition:*
     a) The input object has been pushed onto the stack (which is otherwise unchanged)
2. `T peek()` (called `top` in the textbook):
   - *Precondition:*
     a) Interface Invariant
     b) The stack is not empty
   - *Postcondition:*
     a) Value returned is the object on the top of the stack
     b) The stack has not been changed
   - *Exception:* An `EmptyStackException` is thrown if the stack is empty when this method is called

A Stack Interface: Methods

3. `T pop()`:
   - *Precondition:* Same as for `peek`
   - *Postcondition:*
     a) Value returned is the object on the top of the stack
     b) This top element has been removed from the stack
   - *Exception:* An `EmptyStackException` is thrown if the stack is empty when this method is called
4. `boolean isEmpty()`:
   - *Precondition:* Interface Invariant
   - *Postcondition:*
     a) The stack has not been changed.
     b) Value returned is `true` if the stack is empty and `false` otherwise

Example

Initial stack

S:	
	5 ← top
	10
S:	15

1) `S.peek()`

	5 ← top
	10
S:	15

Output: 5

2) `S.pop()`

	10 ← top
S:	15

Output: 5

3) `S.push(3)`

	3 ← top
	10
S:	15

Output: no output

4) `S.push(4)`

	4 ← top
	3
	10
S:	15

Output: no output

5) `S.peek()`

	4 ← top
	3
	10
S:	15

Output: 4

Problem: Parenthesis Matching

Consider an expression, given as a string of text, that might include various kinds of brackets.

How can we confirm that the brackets in the expression are properly matched? Eg. $[(3 \times 4) + (2 - (3 + 6))]$

Solution using a Stack (provable by induction on the length of the expression):

- Begin with an empty bounded stack (whose capacity is greater than or equal to the length of the given expression)
- Sweep over the expression, moving from left to right
- Push a left bracket onto the stack whenever one is found
- Try to pop a left bracket off the stack every time a right bracket is seen, checking that these two brackets have the same type
- Ignore non-bracket symbols

Solution Using a Stack (continued)

Then parentheses are matched if and only if:

- Stack is never empty when we want to pop a left bracket off it, and
- Compared left and right brackets always *do* have the same type, and
- The stack is empty after the last symbol in the expression has been processed.

Exercise: trace execution of this algorithm on the preceding example.

Two possibilities

Dynamic array implementation:

- stack's contents stored in cells $0, \ldots, top - 1$; top element in $top - 1$
- can use a static array if size of stack is bounded

Linked implementation:

- identify top of stack with the head of a singly-linked list
- works well because stack operations only require access to the top of the stack, and linked list operations with the head are especially efficient

Implementation Using an Array

Initial Stack

	5	← top
	10	
S:	15	

	0	1	2	3	4	5
S:	15	10	5	?	?	?

`top = 2`

Effect of `S.pop()`

	10	← top
S:	15	

	0	1	2	3	4	5
S:	15	10				

`top = 1`

Implementation Using an Array

Effect of `S.push(3)`

	3	← top
	10	
S:	15	

	0	1	2	3	4	5
S:	15	10	3			

`top = 2`

Effect of `S.push(4)`

	4	← top
	3	
	10	
S:	15	

	0	1	2	3	4	5
S:	15	10	3	4		

`top = 3`

Implementation of Stack Operations

```
public class ArrayStack<T> implements Stack<T> {
  private T[] stack;
  private int top;

  public ArrayStack()  { top = -1; stack = (T[]) new Object[6]; }
  public boolean isEmpty()  { return (top == -1); }
  public int size()  { return top+1; }
  public void push(T x)  { ++top; stack[top] = x; }
  public T peek() {
    if (isEmpty())  throw new EmptyStackException();
    return stack[top];
  }
  public T pop() {
    if (isEmpty())  throw new EmptyStackException();
    T e = stack[top]; stack[top] = null; --top; return e;
  }
}
```


Cost of Operations

All operations cost $\Theta(1)$ (constant time, independent of stack size)

Problem: What should we do if the stack size exceeds the array size?

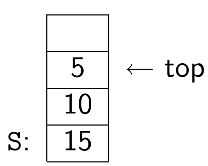
- modify push to reallocate a larger stack (or use a dynamic array)

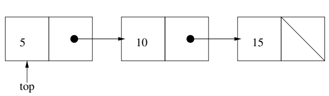
```
public void push(T x) {
  ++top;
  if (top == stack.length) {
    T [] stackNew = (T[]) new Object[2*stack.length];
    System.arraycopy(stackNew,0,stack,0,stack.length);
    stack = stackNew;
  }
  stack[top] = x;
}
```

Revised cost: $\Theta(n)$ in the worst case, $\Theta(1)$ amortized cost

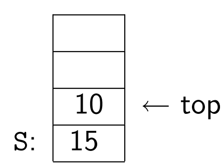
Implementation Using a Linked List

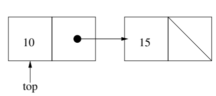
Initial Stack





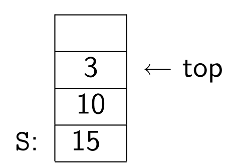
Effect of `S.pop()`

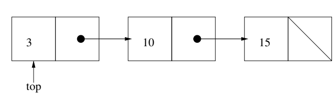




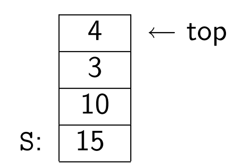
Implementation Using a Linked List

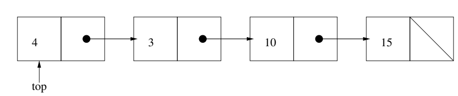
Effect of `S.push(3)`





Effect of `S.push(4)`





Implementation of Stack Operations

```
public class LinkedListStack<T> implements Stack<T> {
  private class StackNode<T> {
    private T value;
    private StackNode<T> next;

    private StackNode(T x, StackNode<T> n)
      { value = x; next = n; }
  }

  private StackNode<T> top;
  private int size;

  public LinkedListStack()
    { size = 0; top = (StackNode<T>) null; }
  public boolean isEmpty()  { return (size == 0); }
  public int size()  { return size; }
```


Implementation of Stack Operations (cont.)

```
public void push(T x)  {
  ++size; top = new StackNode<T>(x,top);
}

public T peek() {
  if (isEmpty())  throw new EmptyStackException();
  return top.value;
}

public T pop() {
  if (isEmpty())  throw new EmptyStackException();
  T x = top.value; top = top.next; --size; return x;
}
```

Cost of stack operations: $\Theta(1)$ (independent of stack size)

Variation: Bounded Stacks

Size-Bounded Stacks — Similar to stacks (as defined above) with the following exception:

- Stacks are created to have a maximum `capacity` (possibly user-defined — so that two constructors are needed)
- If the `capacity` would be exceeded when a new element is added to the top of the stack then `push` throws a `StackOverflowException` and leaves the stack unchanged
- A *static array* whose `length` is the stack's `capacity` can be used to implement a size-bounded stack, extremely simply and efficiently

Most "hardware" and physical stacks are bounded stacks.

Stacks in Java and the Textbook

Implementation in Java 8:

- Java 8 includes a `Stack` class as an extension of the `Vector` class (a dynamic array).
  Unfortunately, this implementation is somewhat problematic (Stack inheirit's Vector's methods, too!)

Introduction to Algorithms

- by Cormen, Lieserson, Rivest, and Stein
- Section 10.1

Data Structures: Abstraction and Design Using Java

- by Elliot B. Koffman and Paul A. T. Wolfgang
- Chapter 3