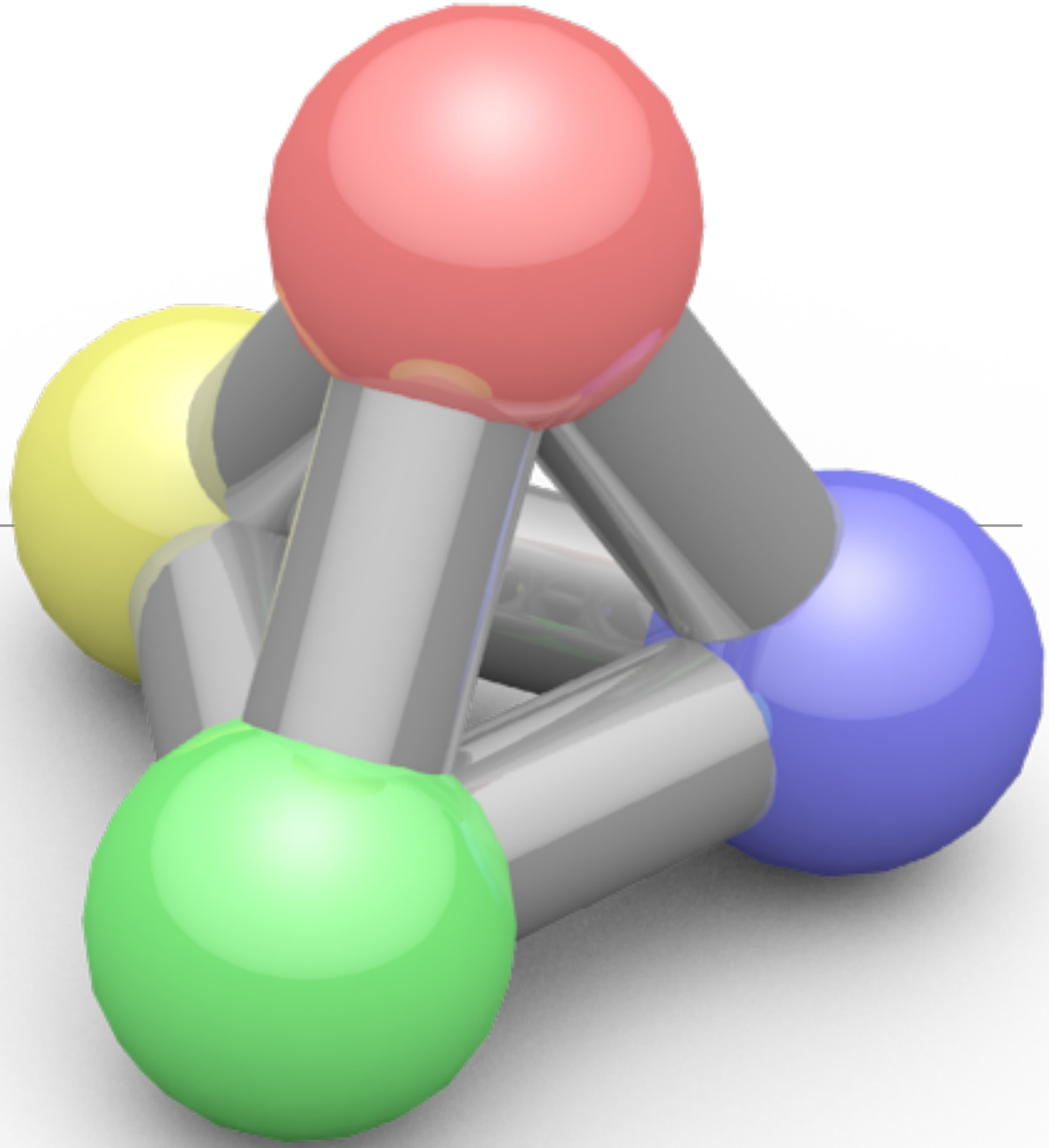
Some Leftovers

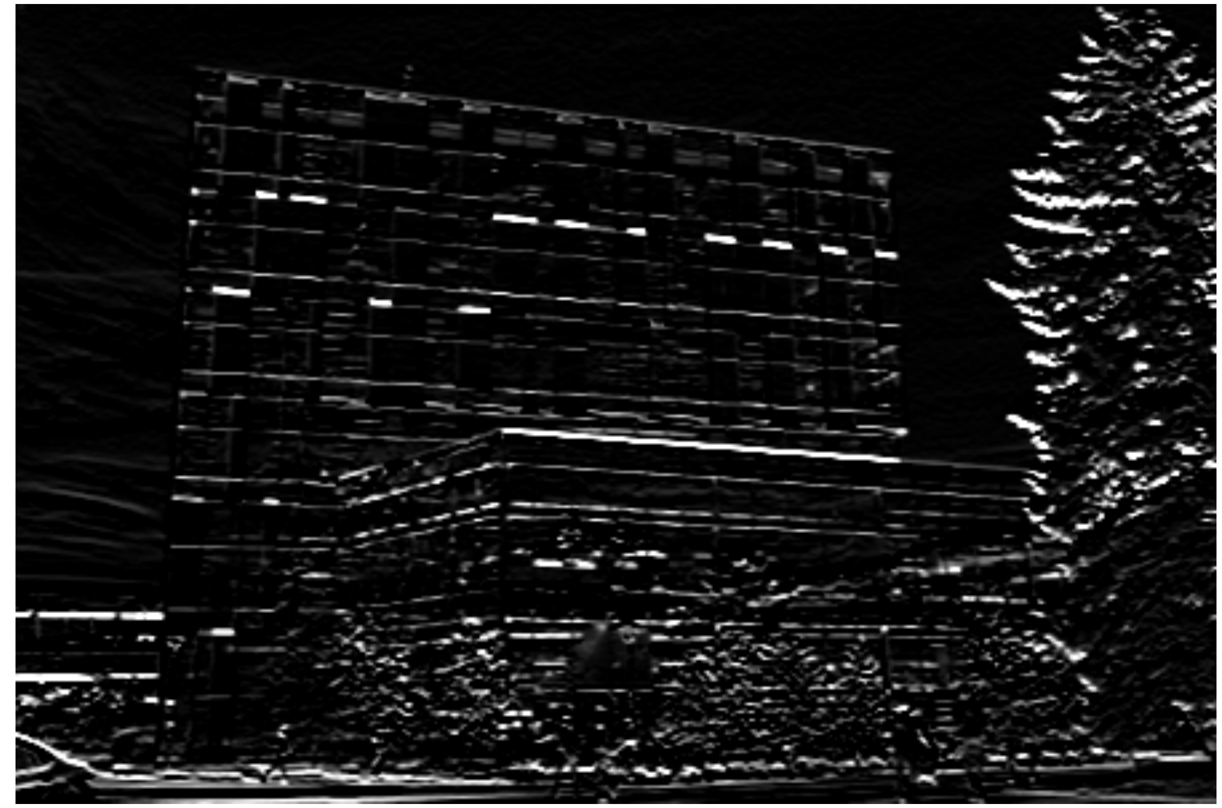


CPSC 453 – Fall 2016
Sonny Chan

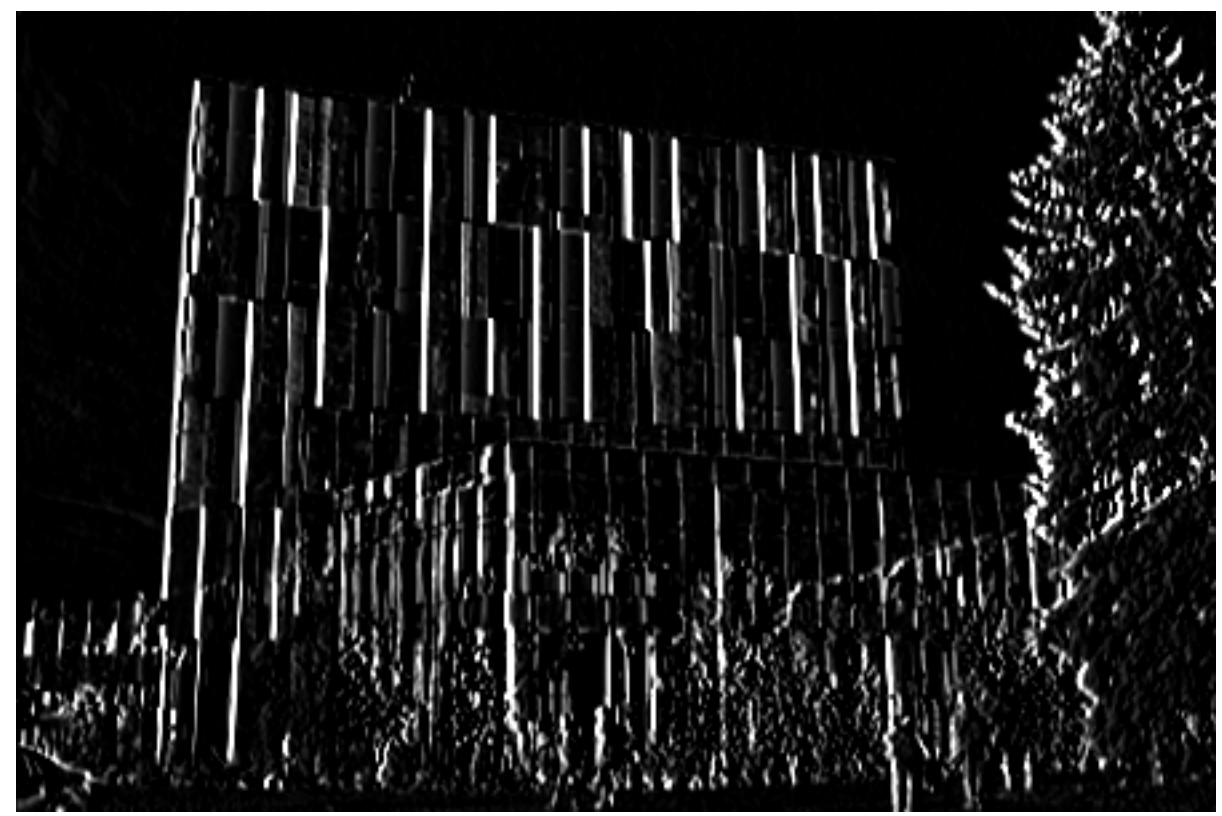
Today’s Outline

- Convolution Filters on Images
    - edge detection
    - smoothing
- Signal Reconstruction
    - interpolation, and how it do it perfectly
- Colour Conversion

original



horizontal edges

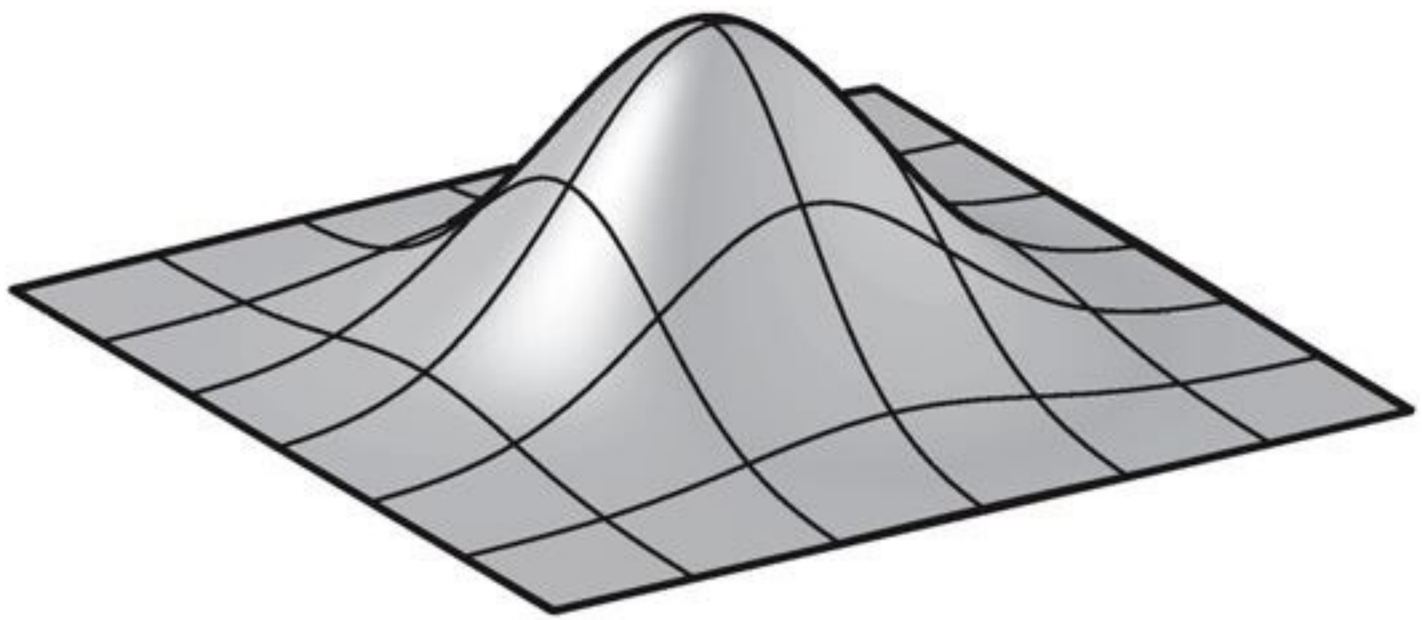


vertical edges

What about

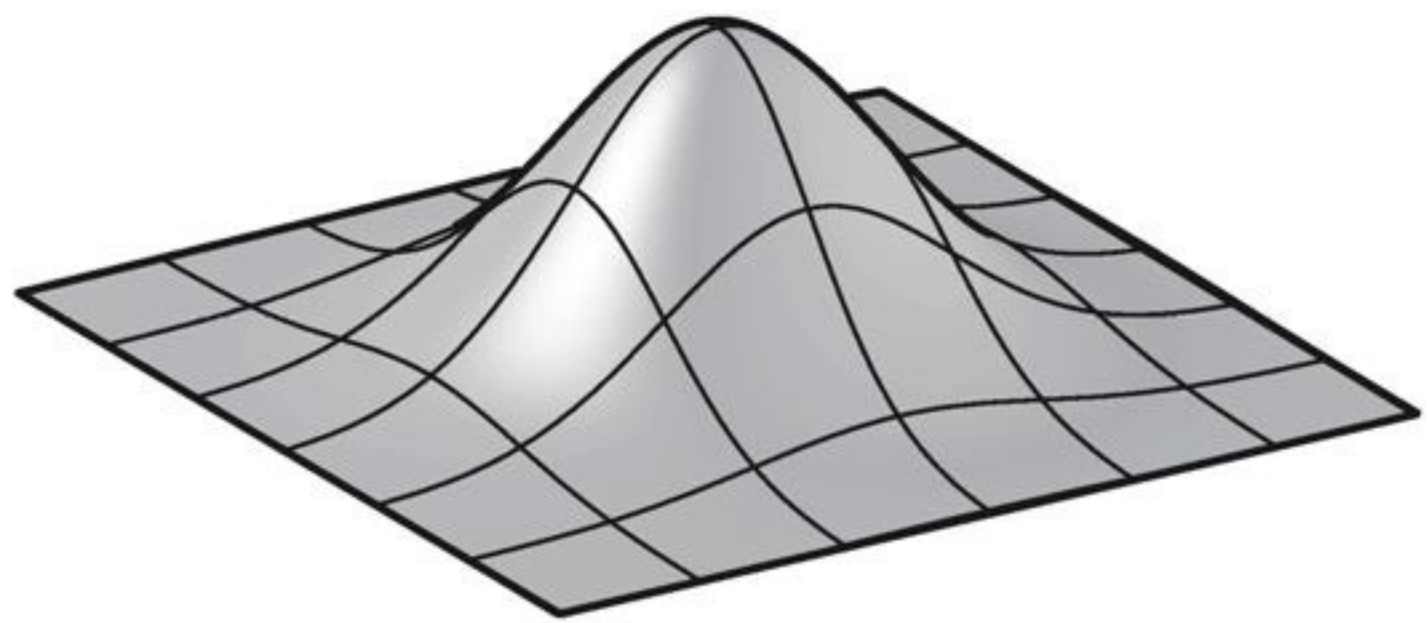
blurring?

The 2D Gaussian Function



$$G(x, y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

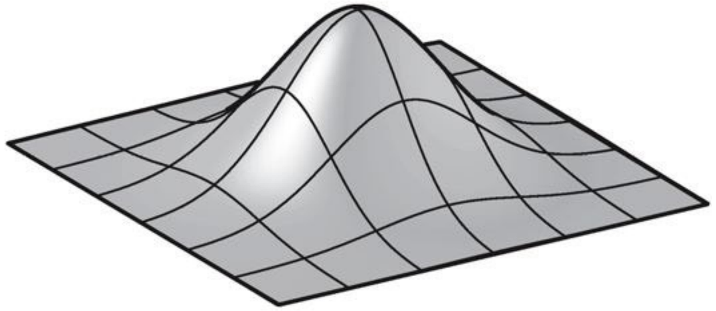
A Discrete 2D Gaussian



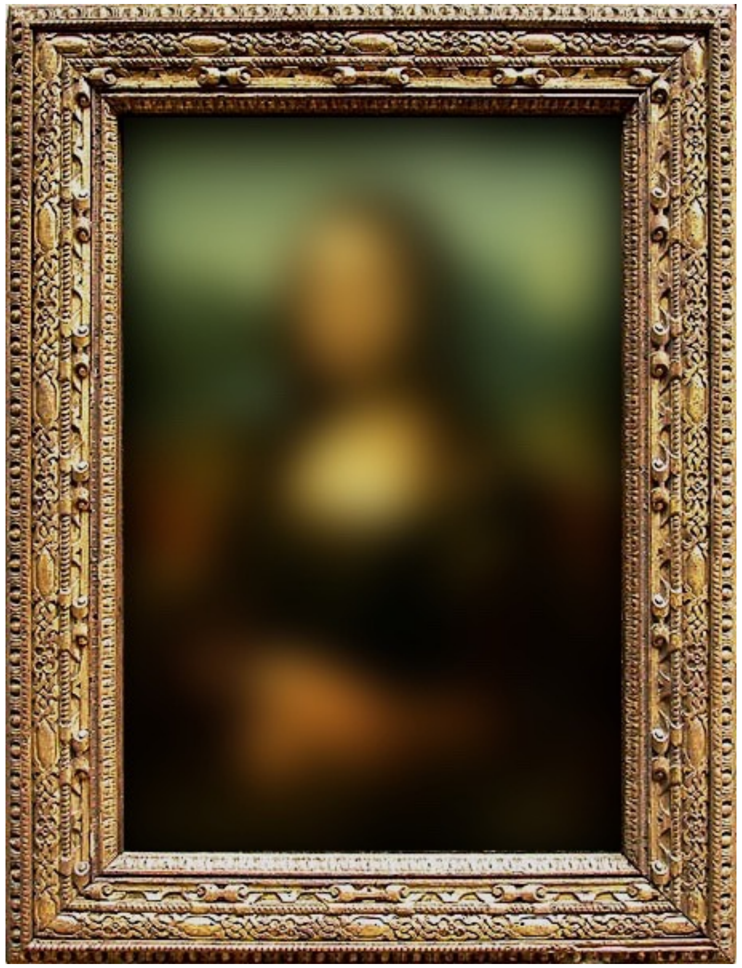
$$G[i,j] = \begin{bmatrix} 0.04 & 0.12 & 0.04 \\ 0.12 & 0.36 & 0.12 \\ 0.04 & 0.12 & 0.04 \end{bmatrix}$$

Convolution with a 2D Gaussian

$\star$



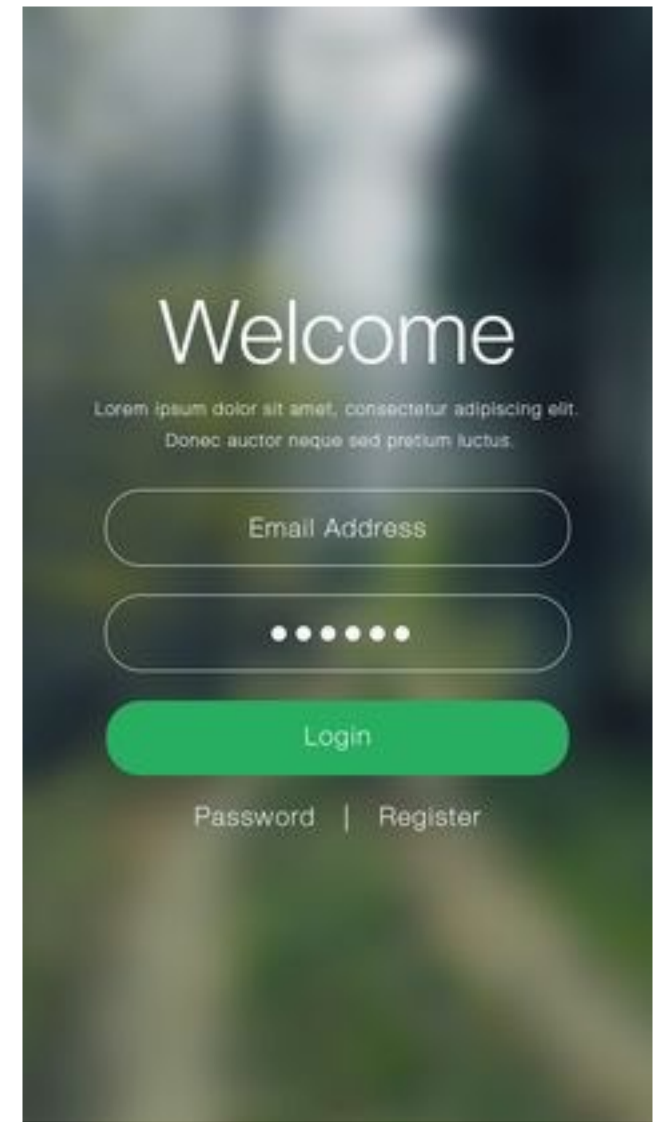
$=$

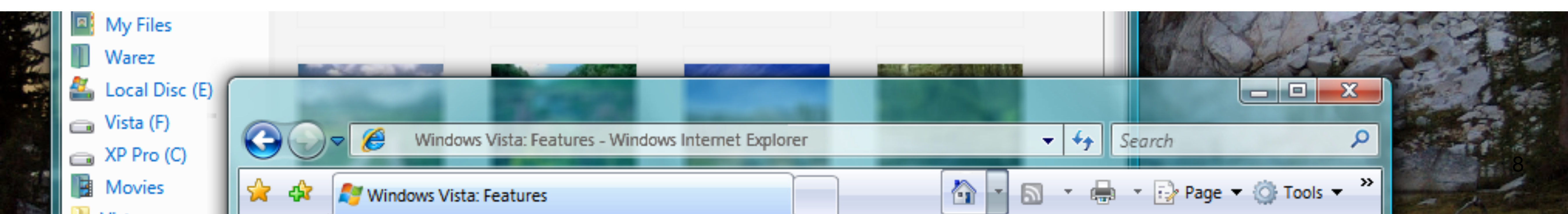


Why would you ever want to do this???

Because it

looks cool!





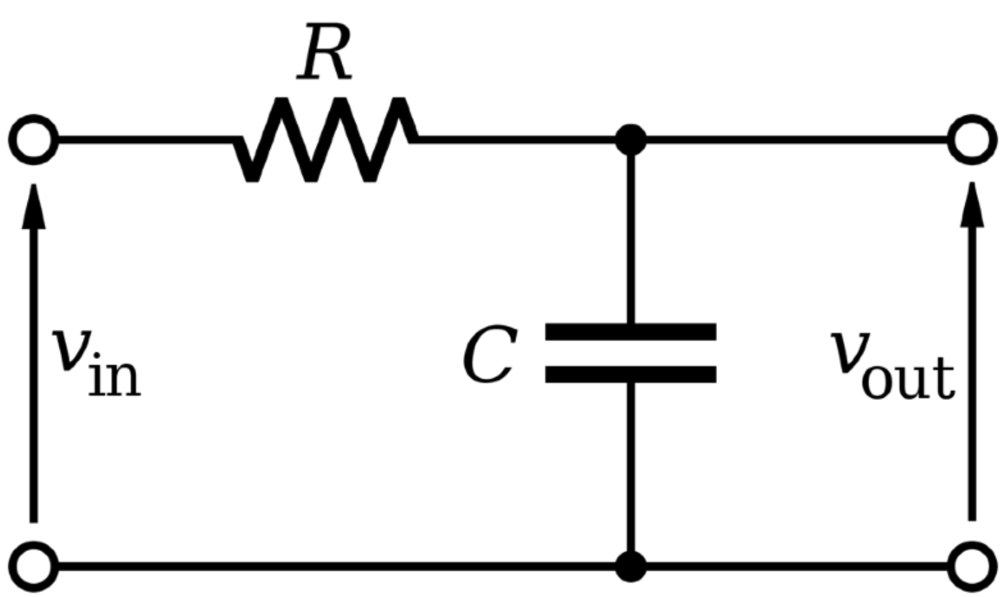
Anti-Aliasing in 2D

How does it work?

No jaggies!

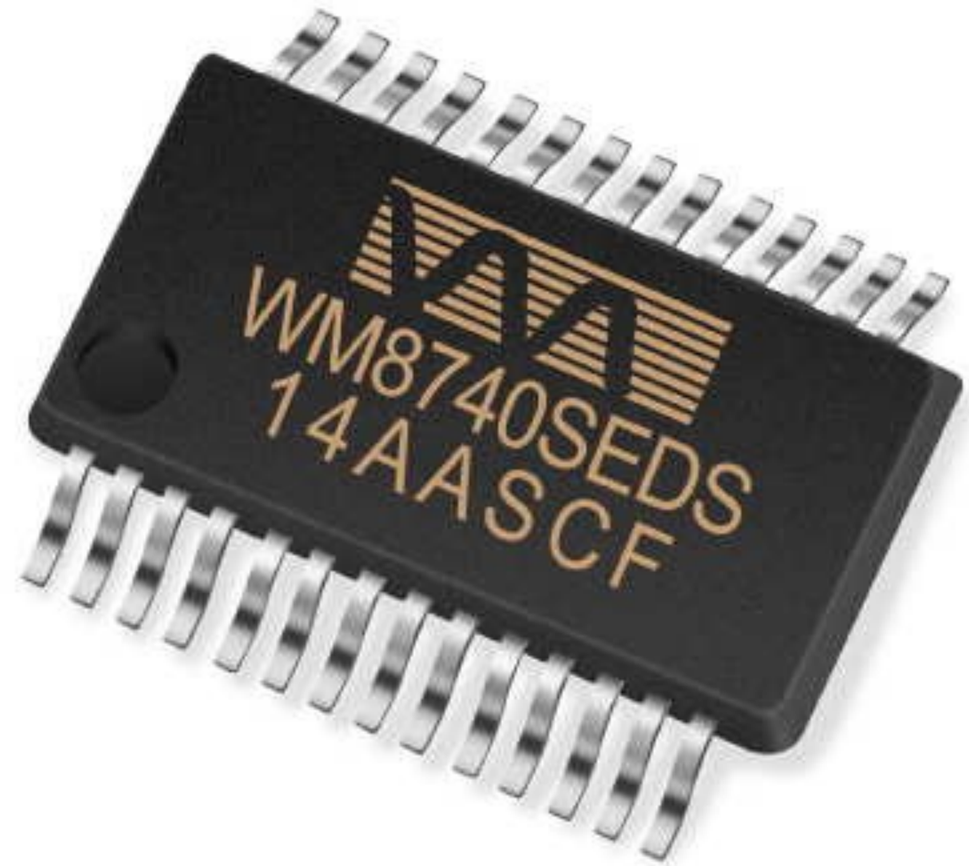
Anti-Aliasing in 2D

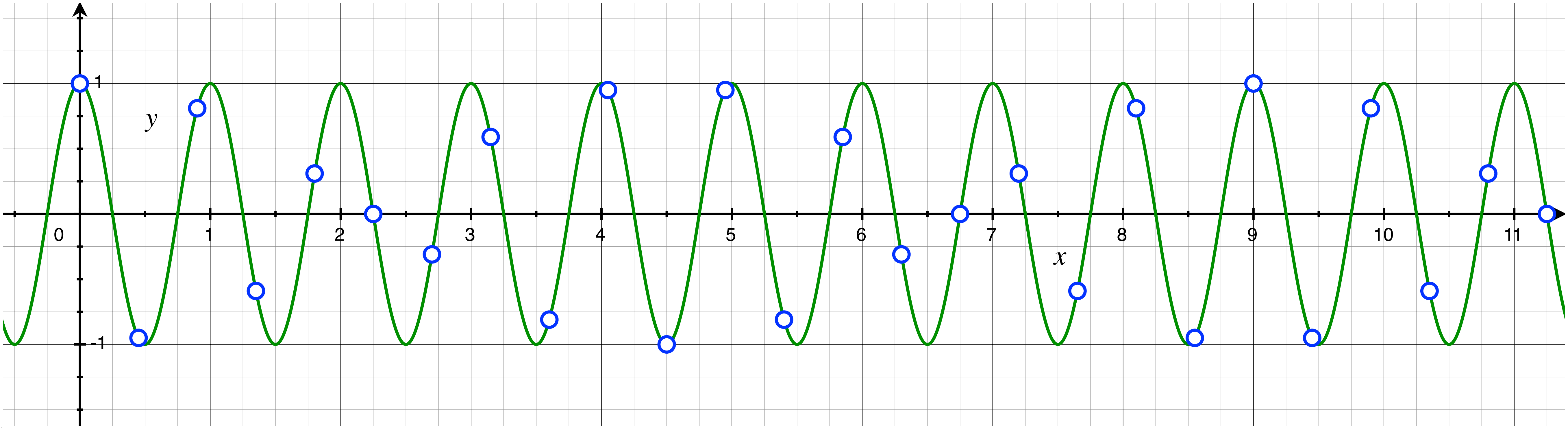
How does it work?



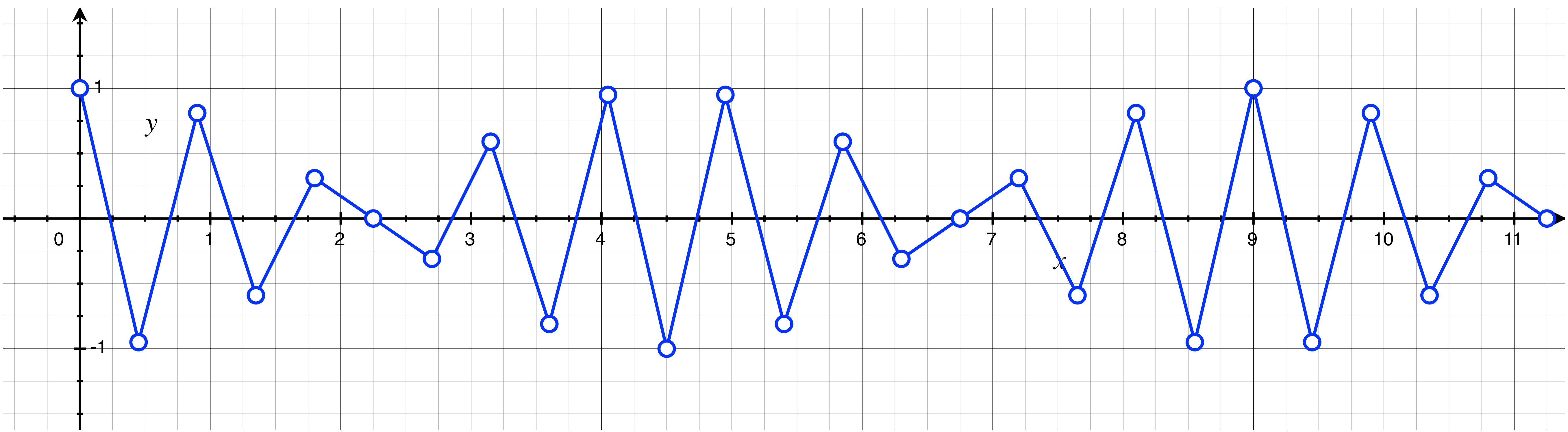
No jaggies!

Signal Reconstruction





Let’s see if we can shed some light on this problem…



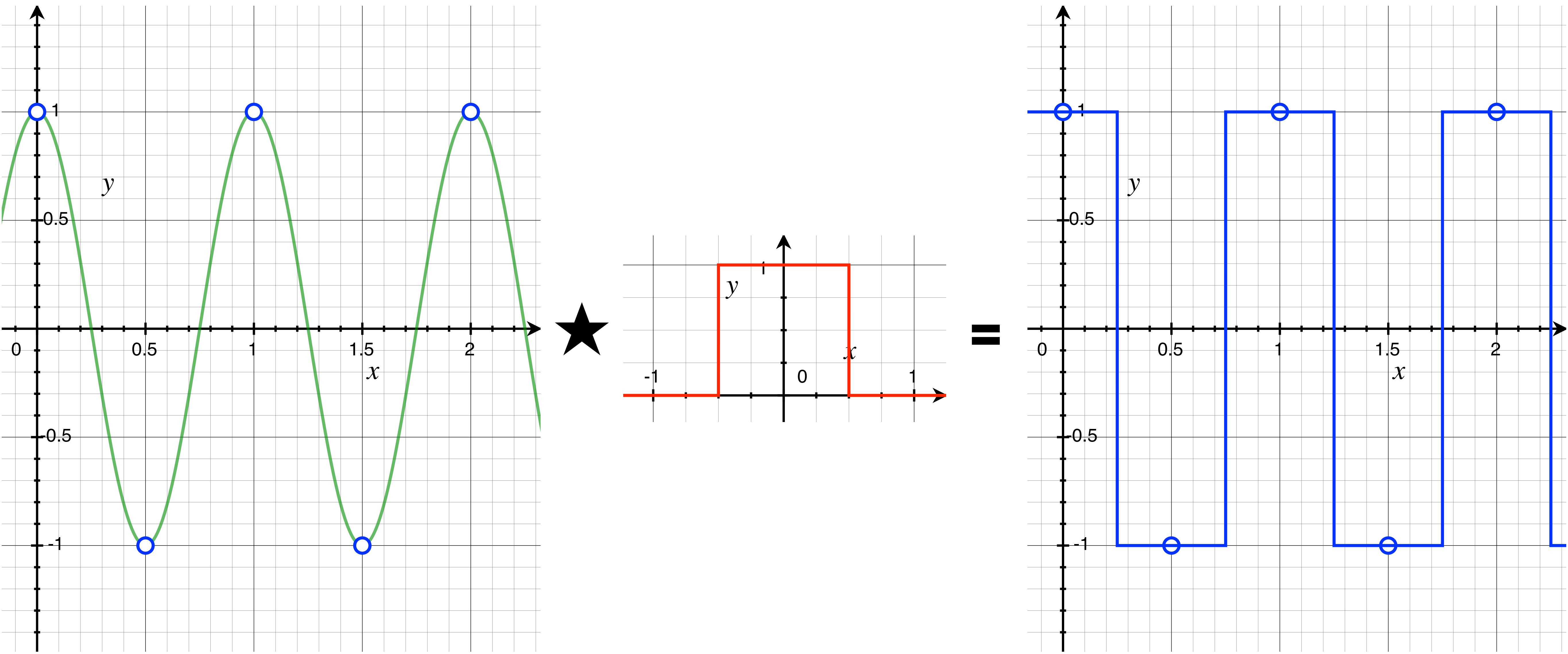
To **faithfully** reconstruct a band-limited signal from its samples, the sampling rate must be at least twice that of its highest frequency component.

–Shannon Sampling Theorem, or *Nyquist Frequency*

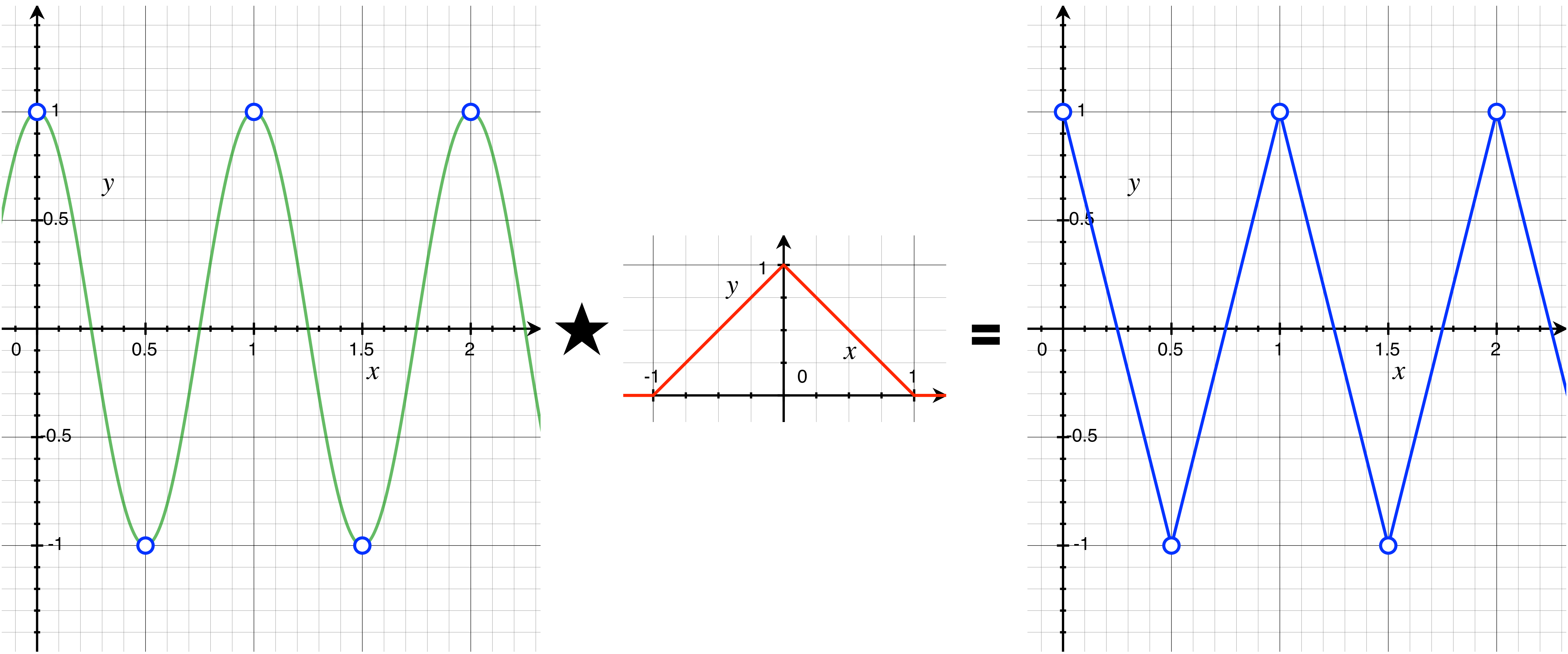
Interpolation

is

Convolution

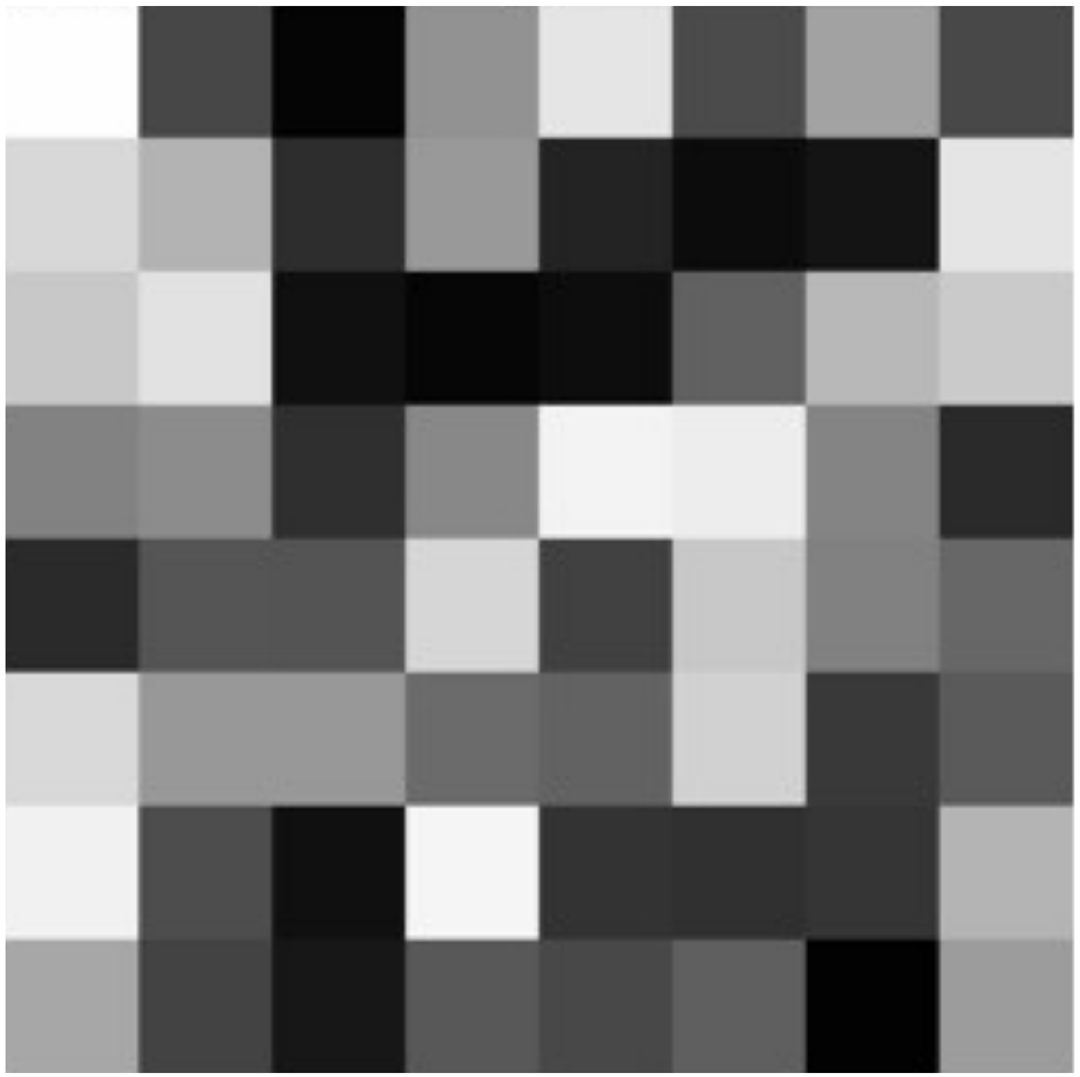


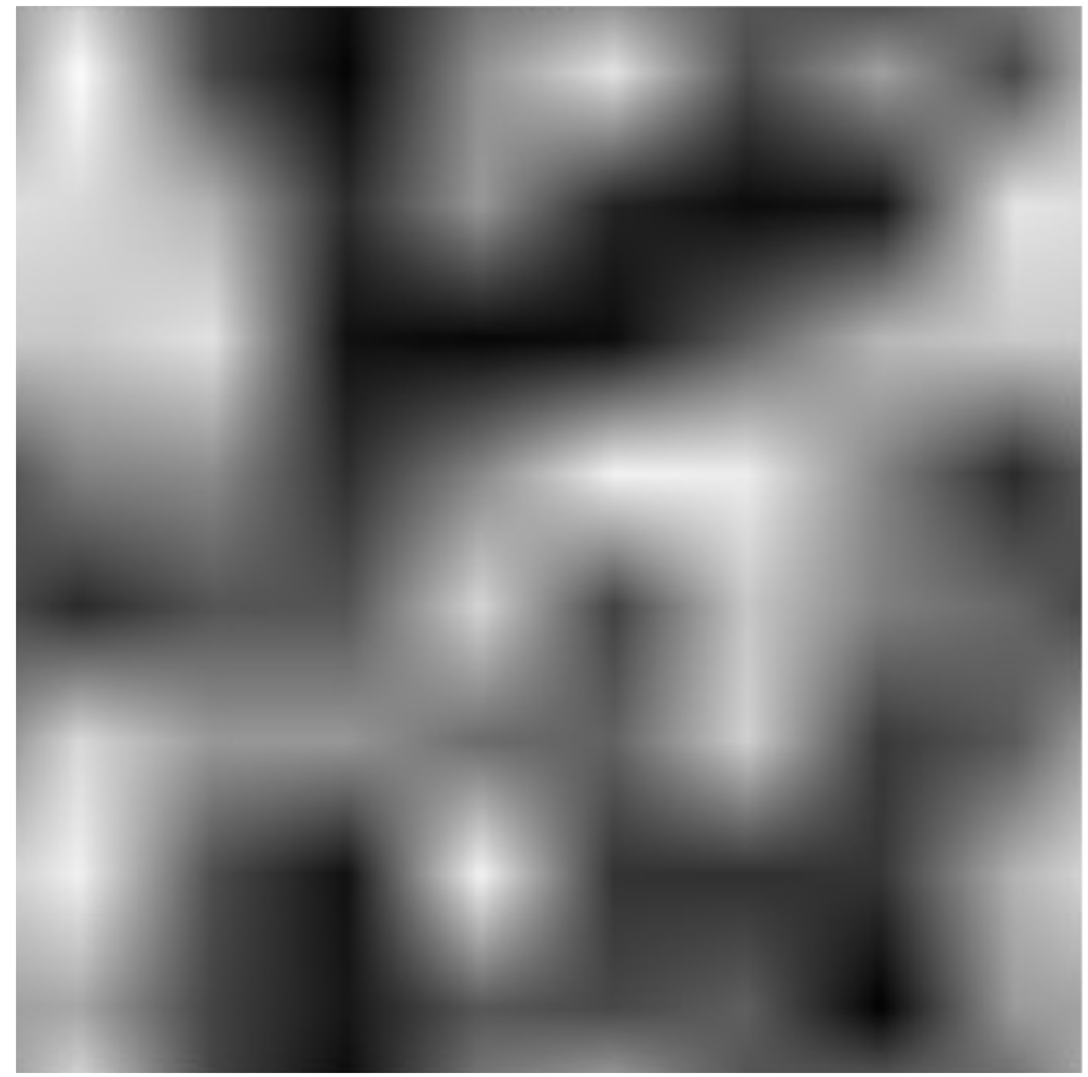
“nearest neighbour” interpolation

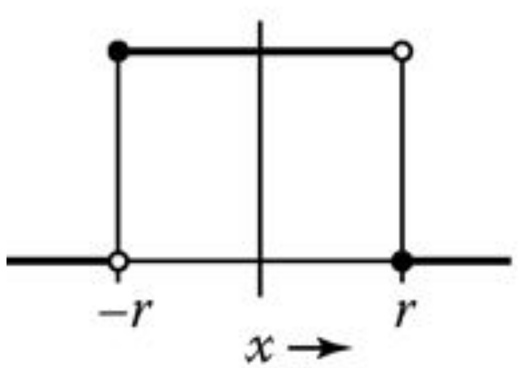


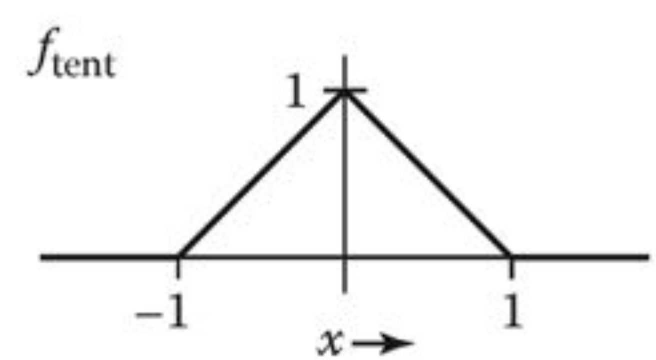
linear interpolation

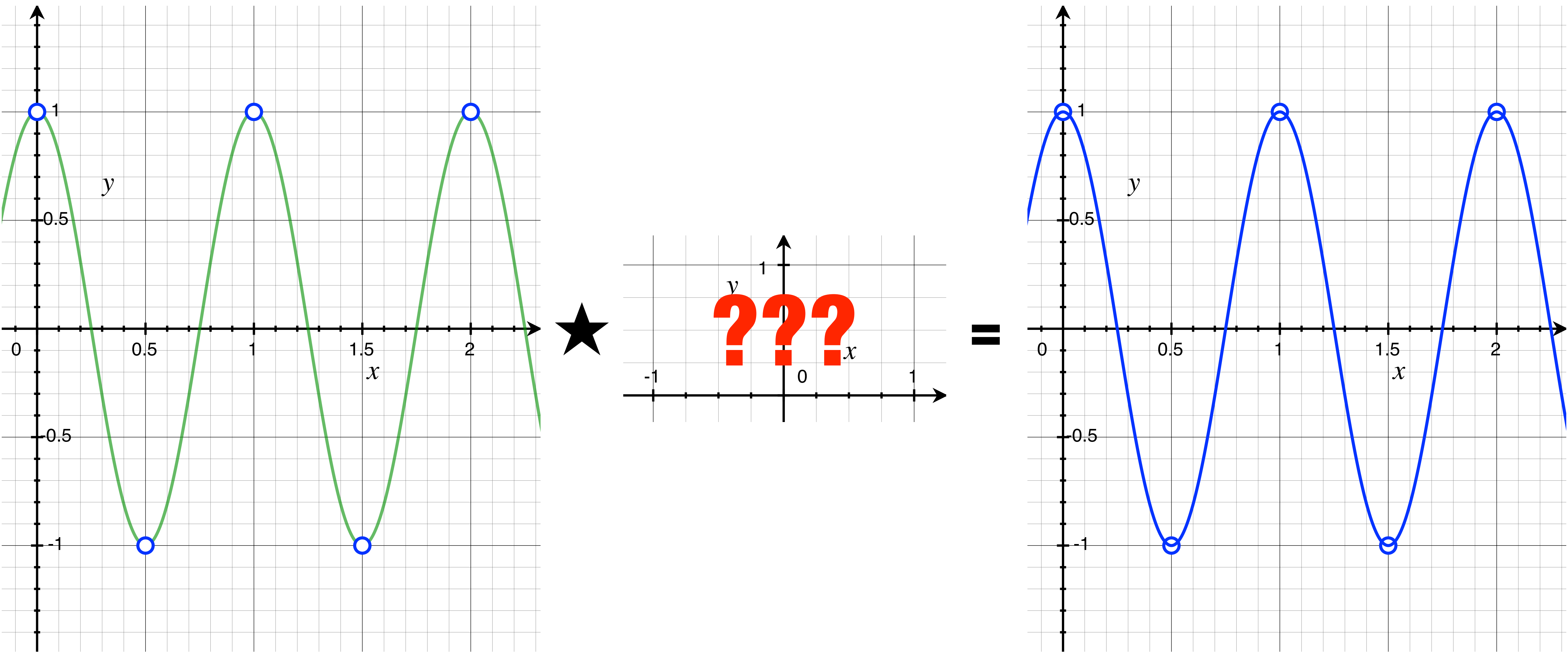
In Two Dimensions





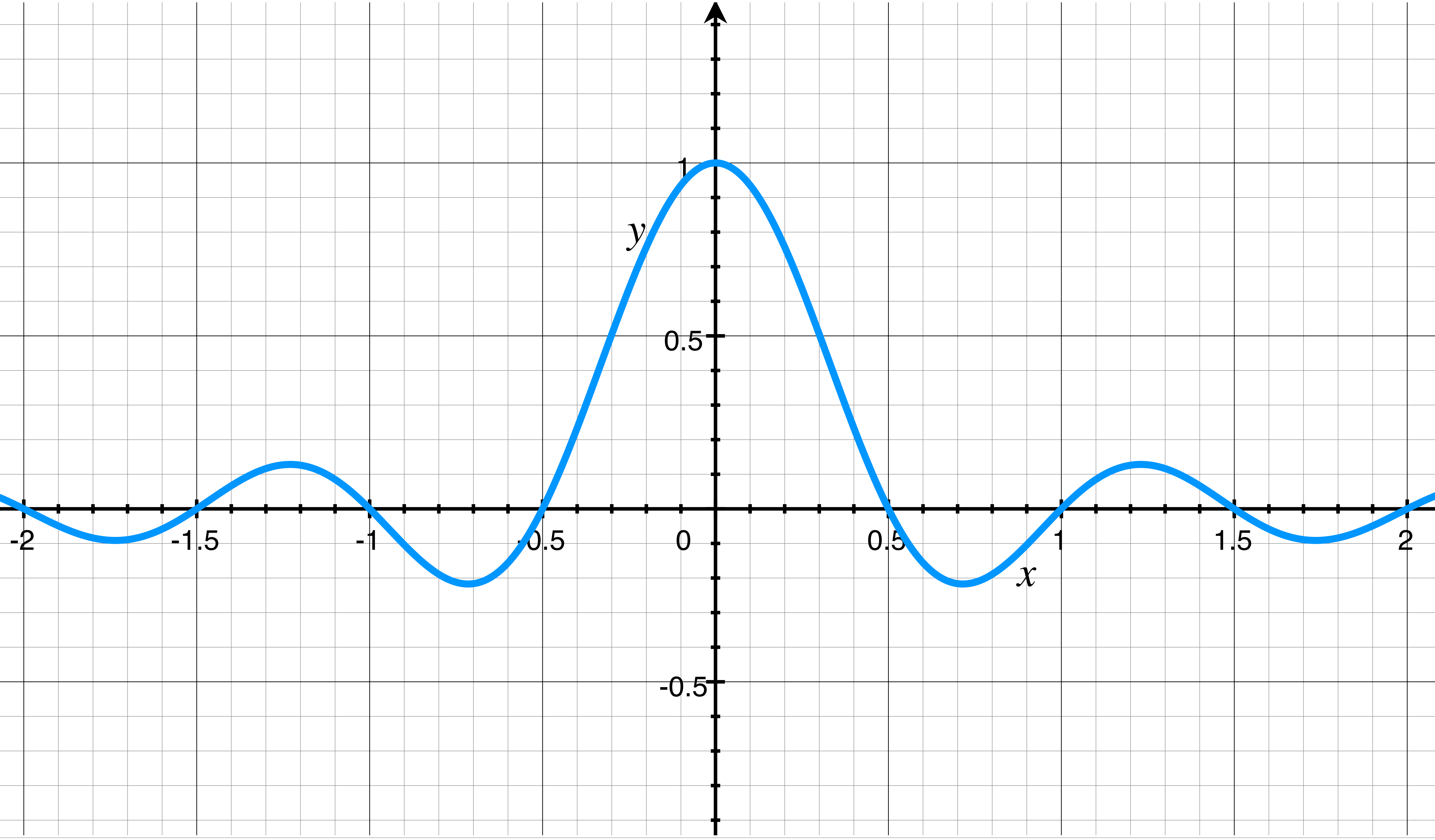






How do we get a “faithful” reconstruction?


ELECTRONICS
Fry's
EXIT
ENTRANCE

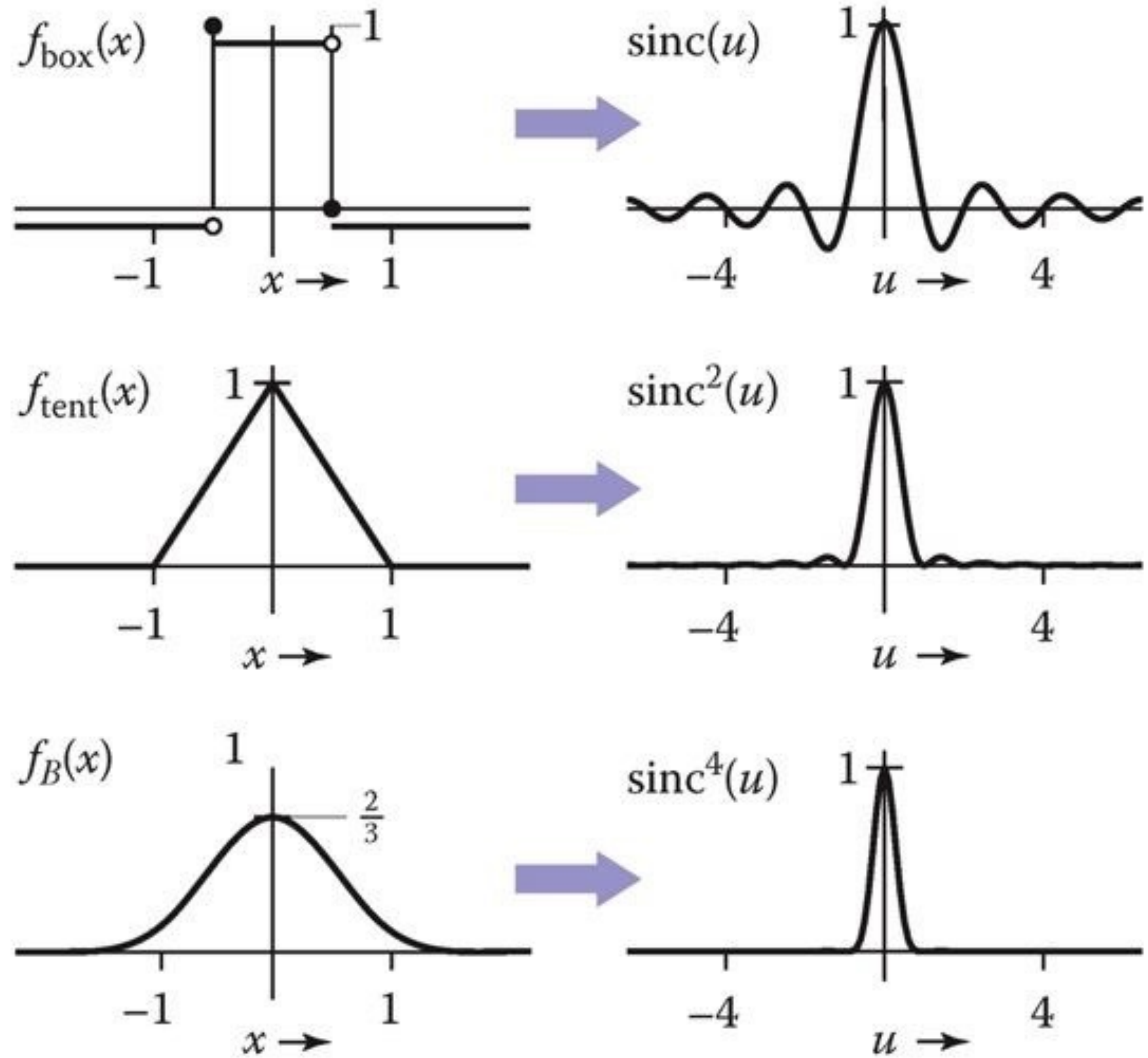


The sinc function

$$\mathrm{sinc}(x) = \frac{\sin x}{x}$$

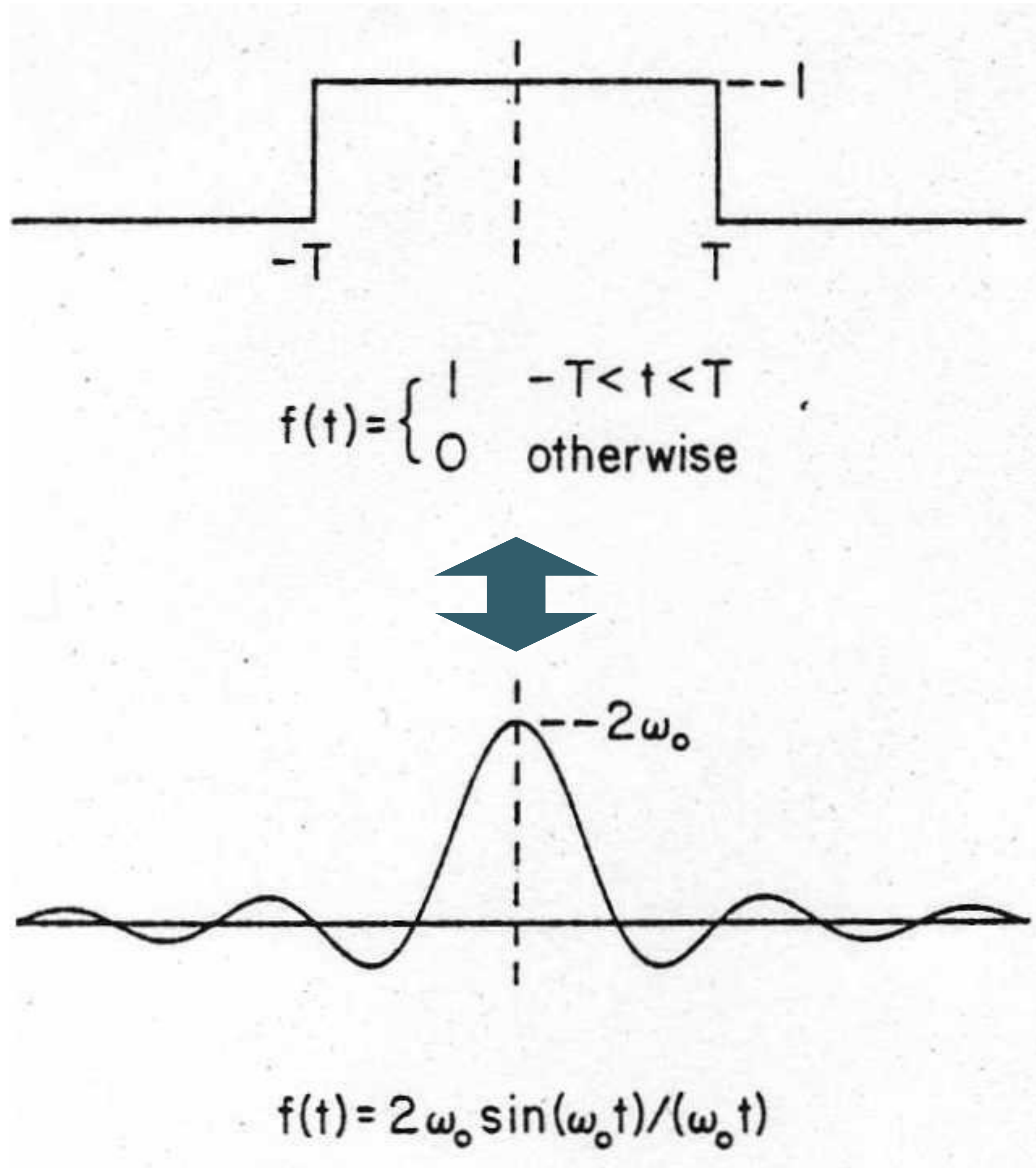
Reconstruction Filters

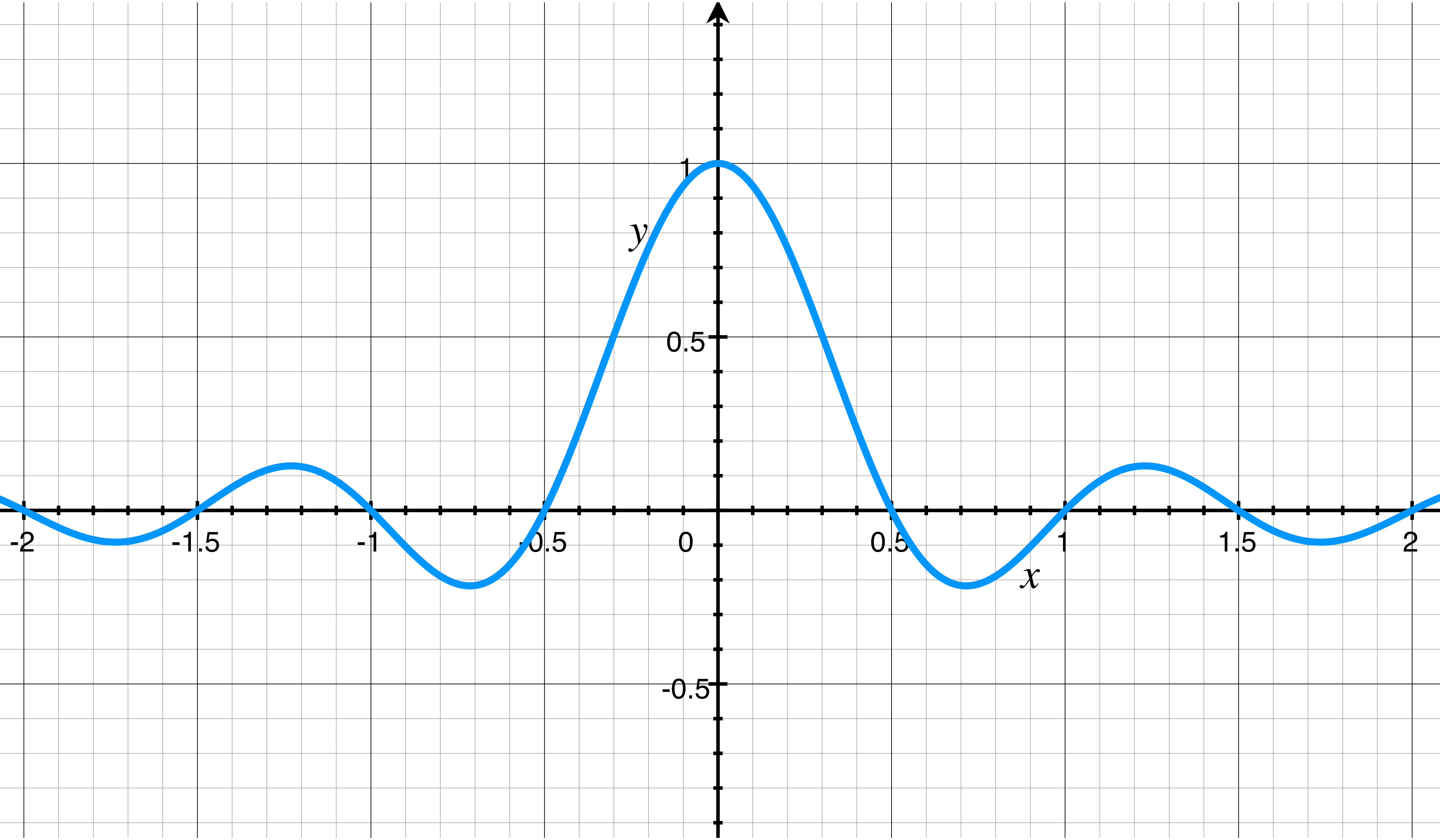
Frequency Content



Frequency of sinc

The Fourier Transform and its inverse are symmetric!



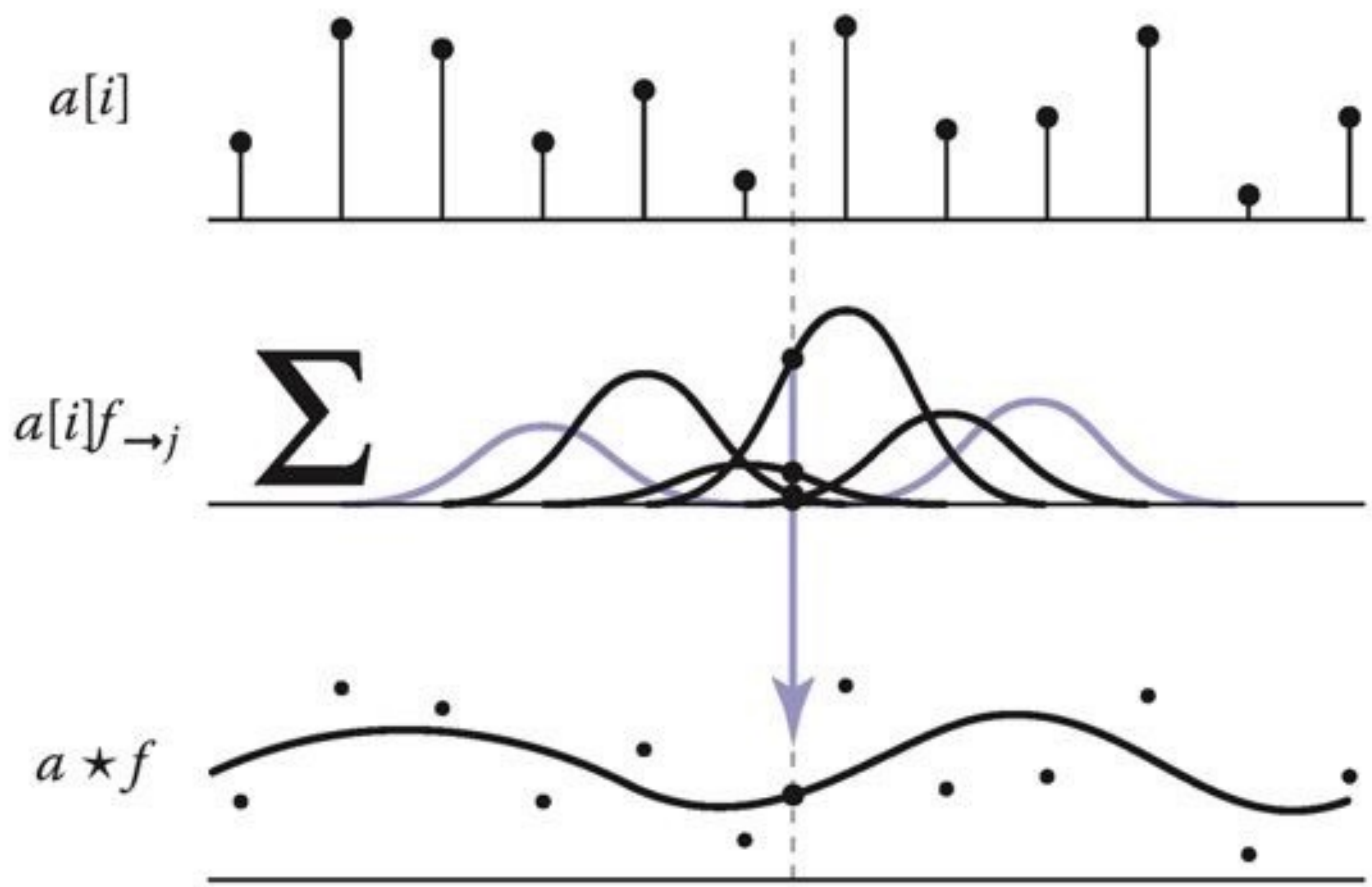


Perfect Reconstruction

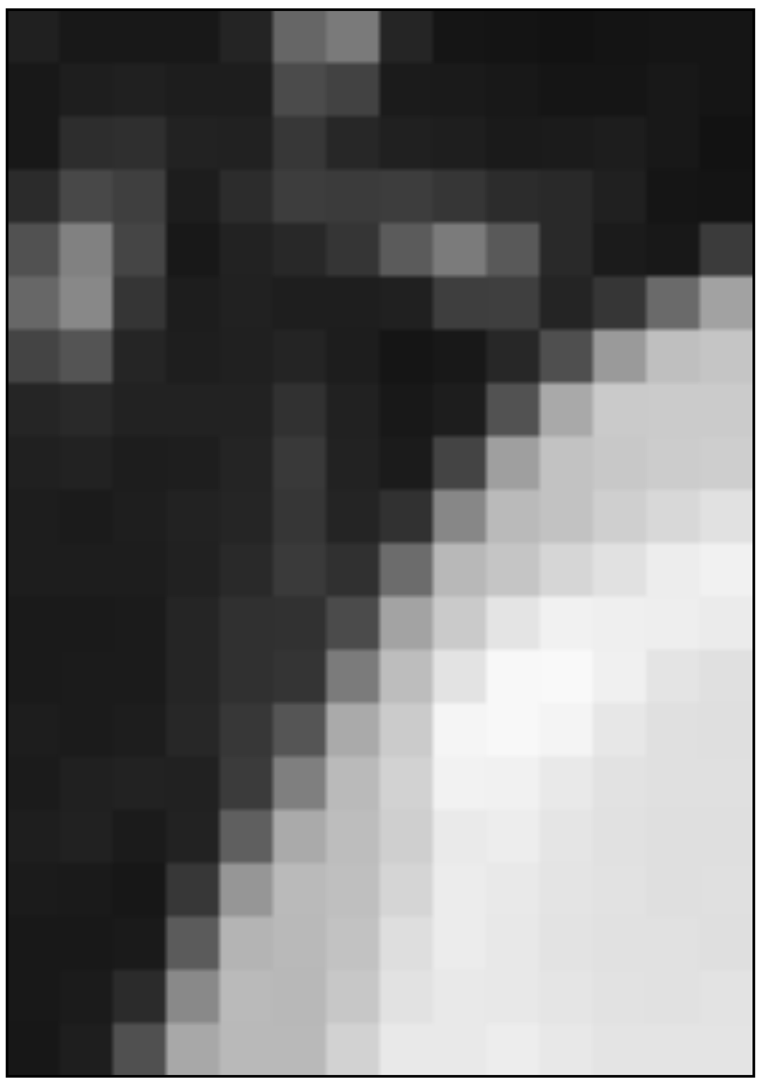
$$a[i] \star \mathrm{sinc}(x)$$

Signal Reconstruction

by convolution with reconstruction kernel (e.g. cubic spline)



Perfect Reconstruction?

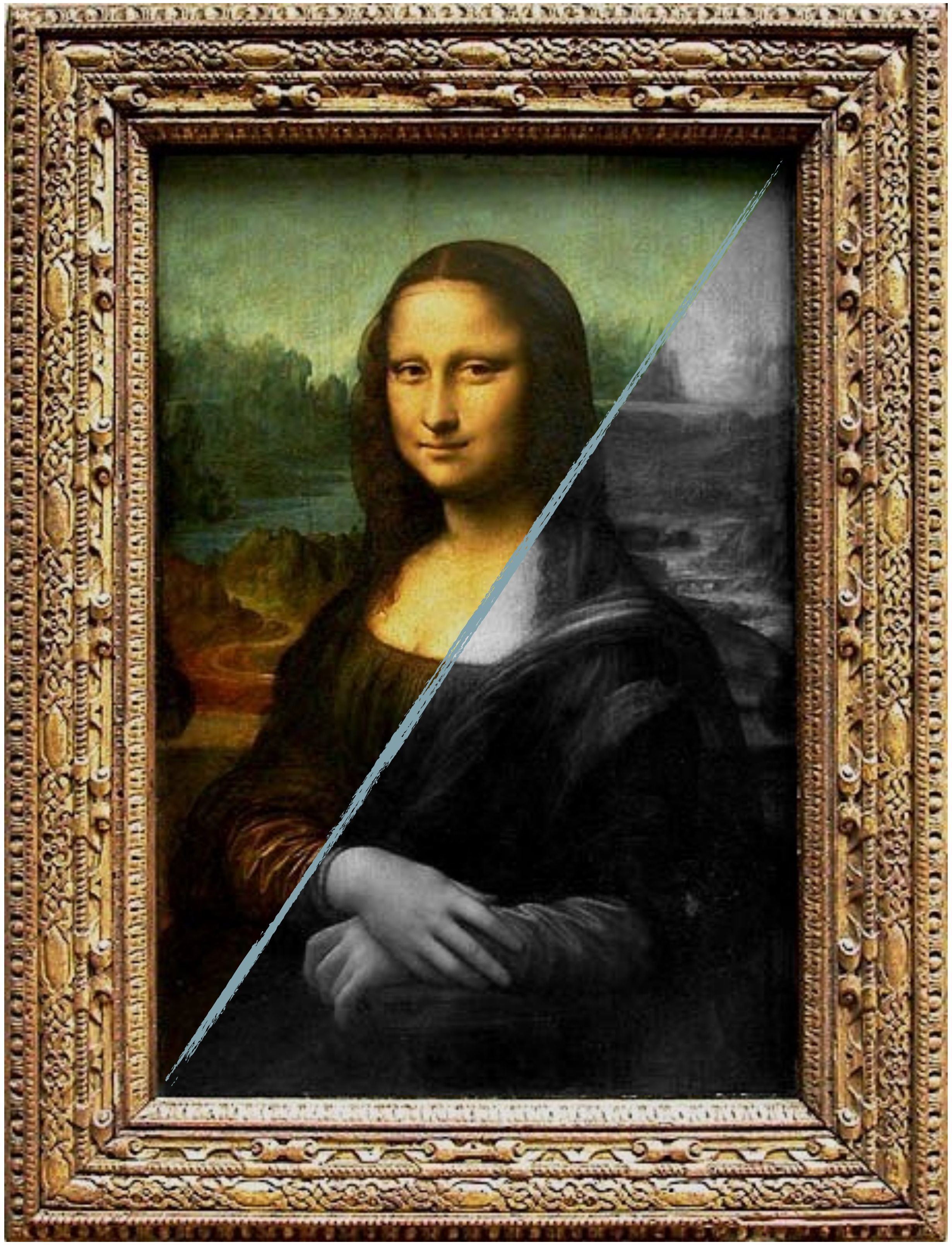


nearest

linear

sinc

Colour Conversion



Colour Images

- A sampled colour image maps integer pairs to three colour channels:

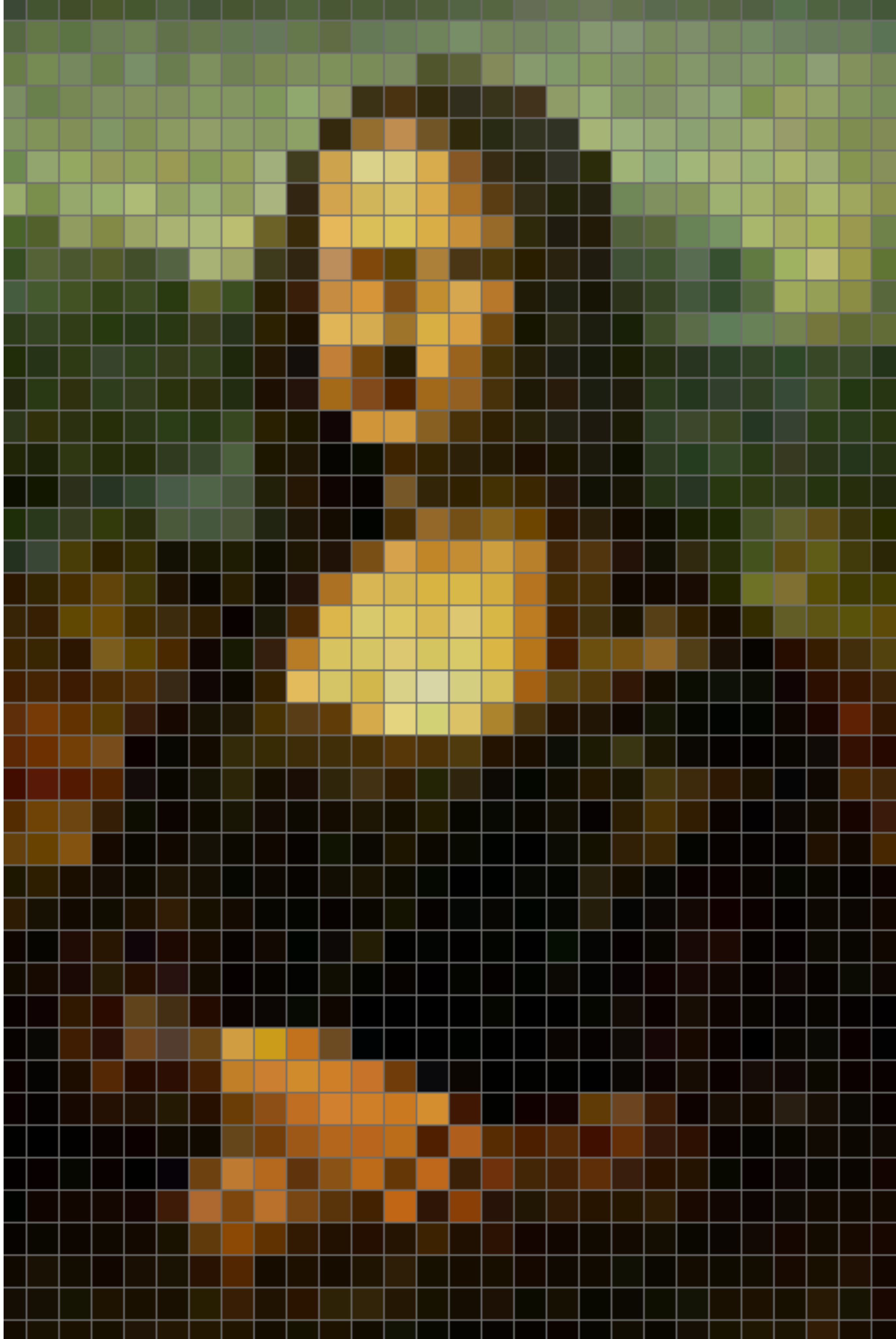
$$I[i, j] : \mathbb{Z}^2 \mapsto \mathbb{R}^3$$

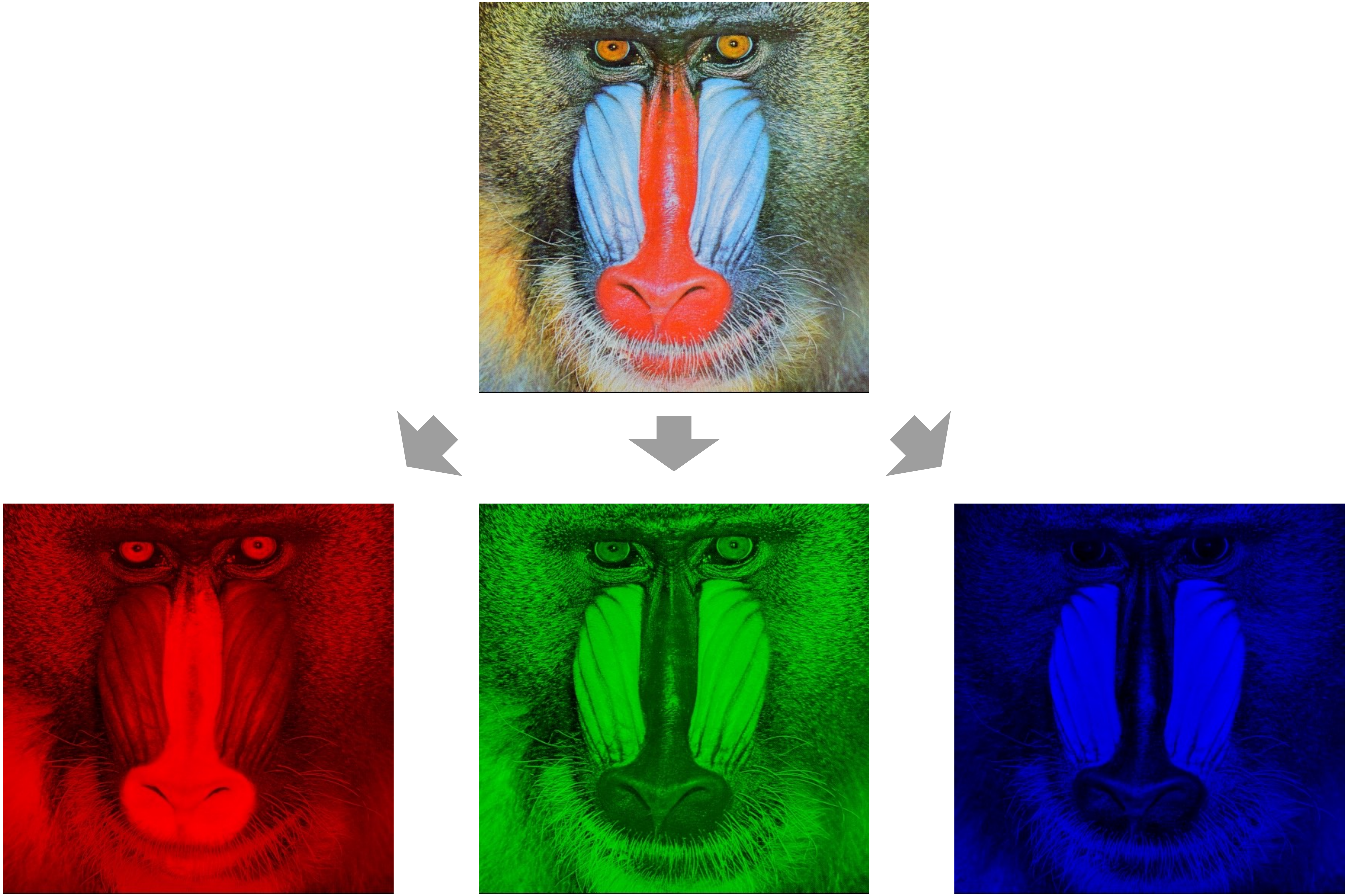
- We can consider each colour as a separate function to apply filters:

$$R[i, j] : \mathbb{Z}^2 \mapsto \mathbb{R}$$

$$G[i, j] : \mathbb{Z}^2 \mapsto \mathbb{R}$$

$$B[i, j] : \mathbb{Z}^2 \mapsto \mathbb{R}$$



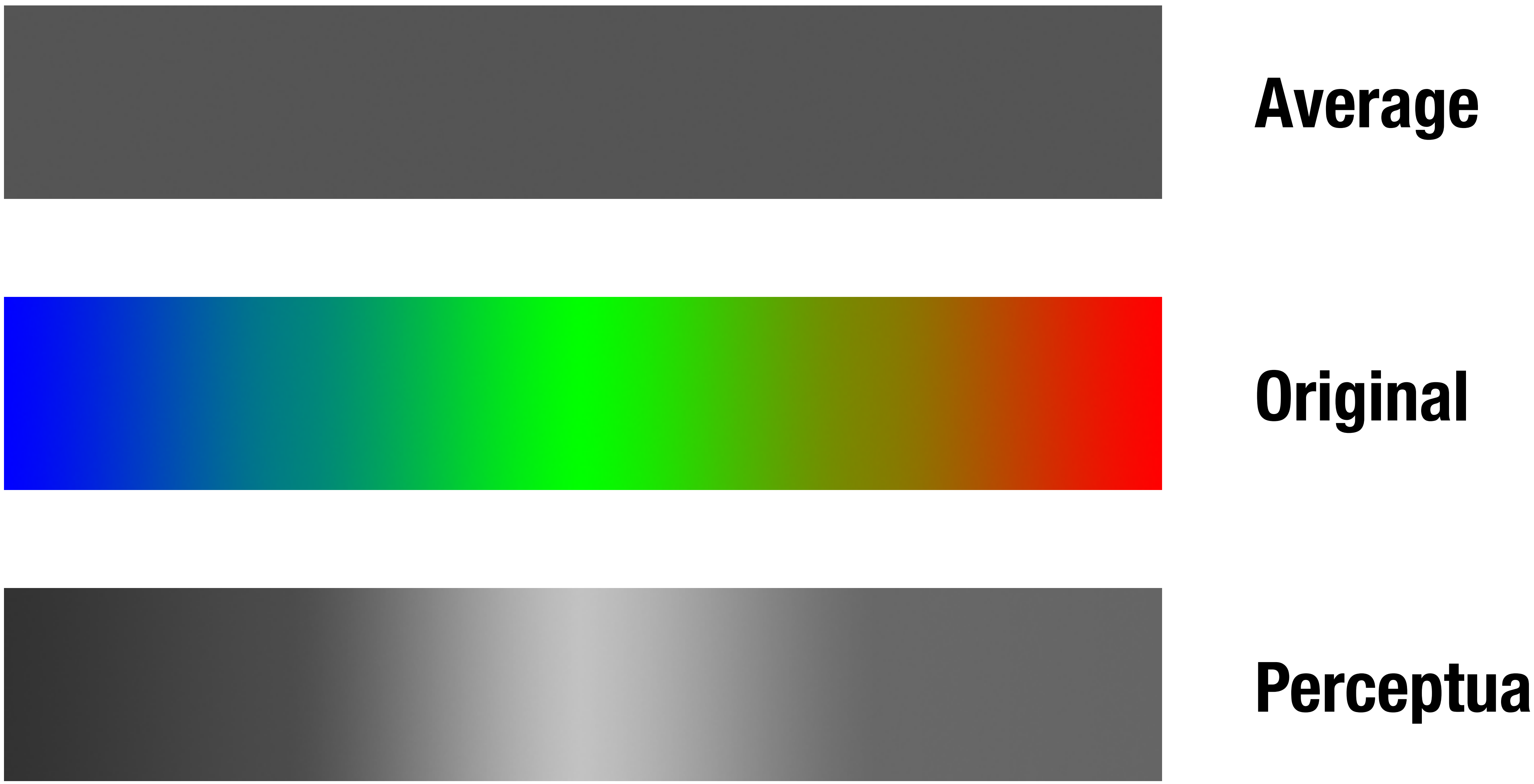


$R[i, j]$ $G[i, j]$ $B[i, j]$

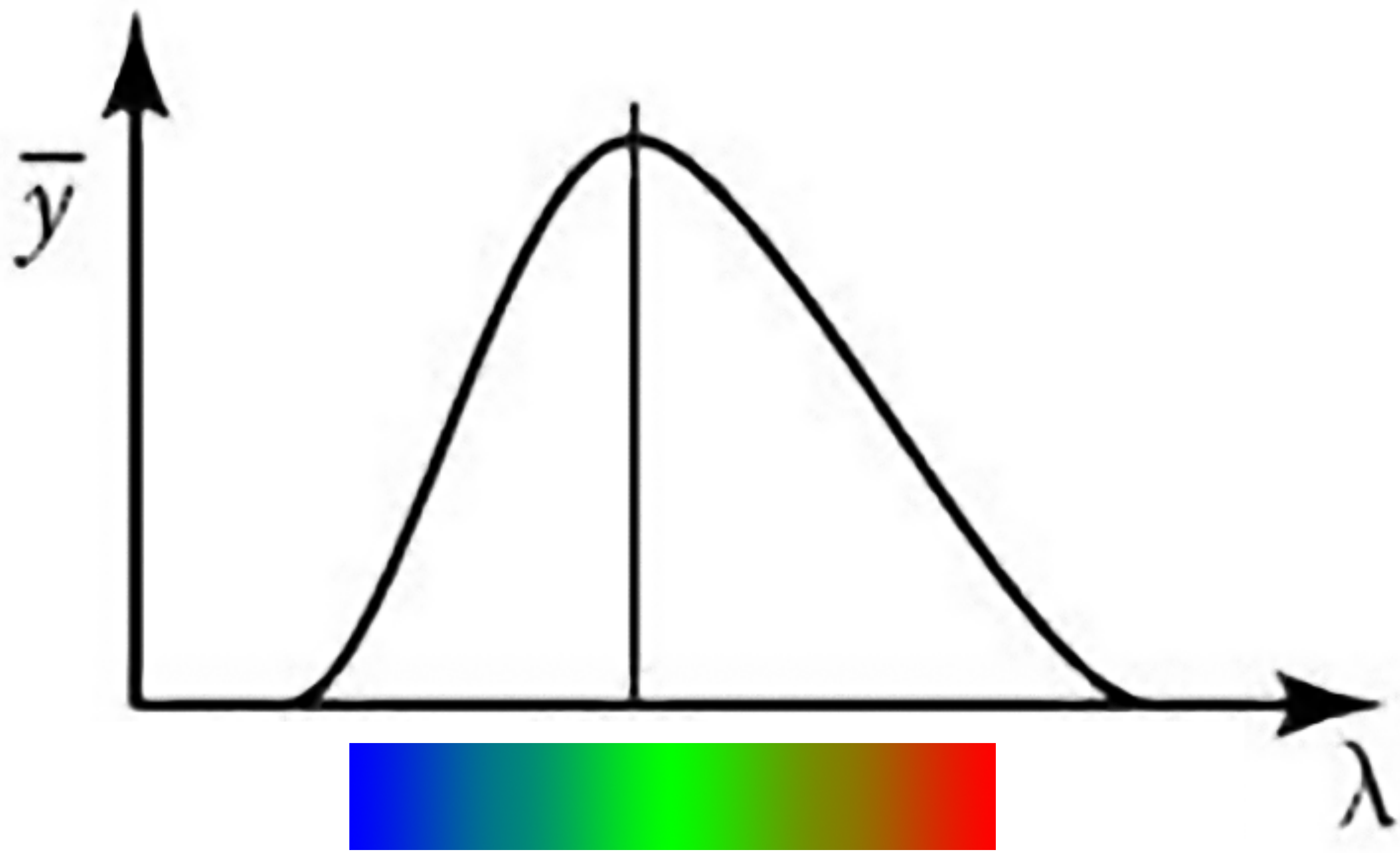
Or maybe we can average the channels?

Which one is “correct”?

Which greyscale do you see?



Recall Luminous Efficiency



Things to Remember

- Convolution is used for many important things:
    - dazzling image effects
    - low-pass filter for anti-aliasing
- Interpolation is convolution
    - perfect signal reconstruction is convolution with sinc
- Don’t simply average colours to get a greyscale image!

Let’s talk a bit about

vectors!