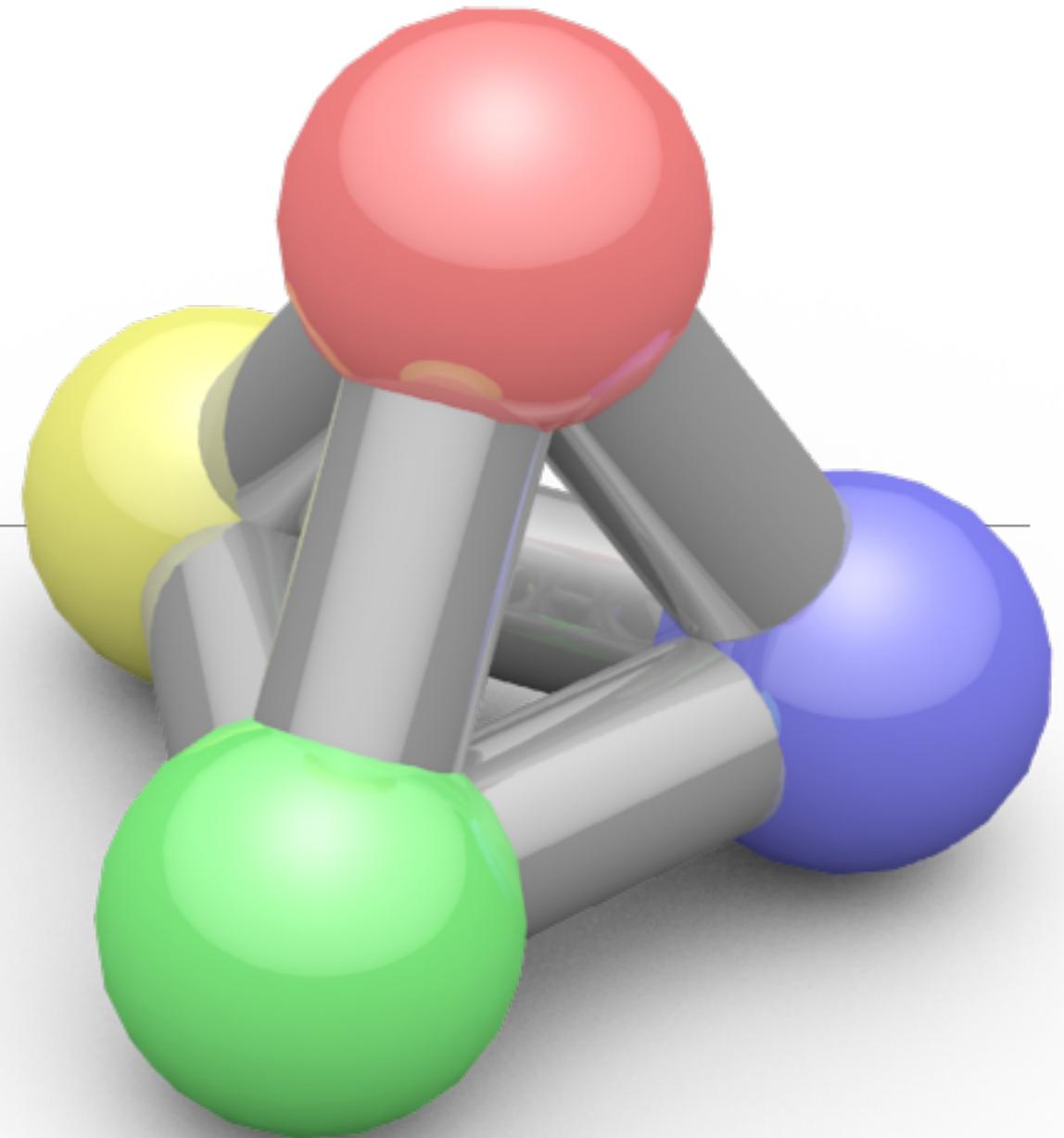


Sampling

CPSC 453 – Fall 2016

Sonny Chan



Today's Outline

- Sampling: what it is and how to do it properly!
- Aliasing in one and two dimensions
- The nuances of reconstructing a signal



Sampling | and aliases

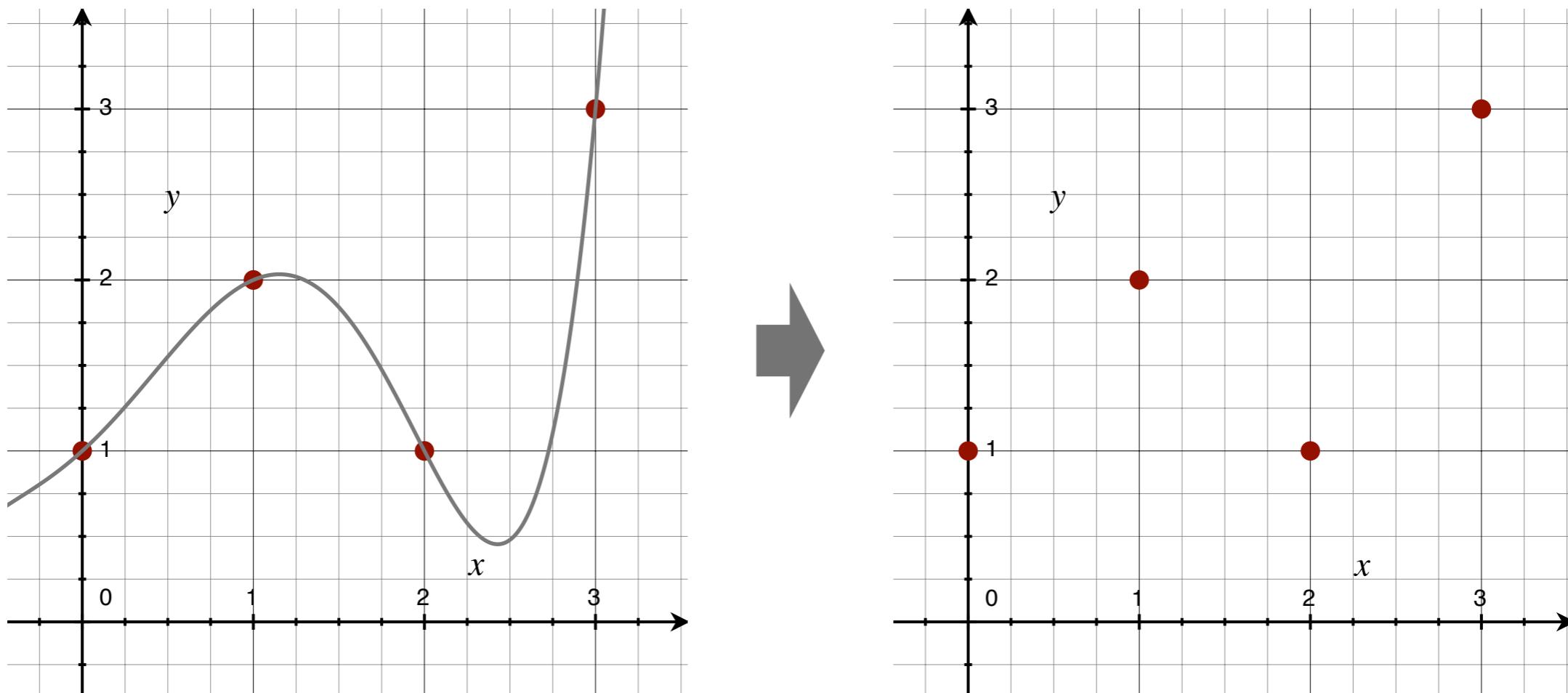
What do we mean by
sampling?

Sampling is recording a continuous signal at discrete points.

-(again, don't quote me on this)

Sampling in One Dimension

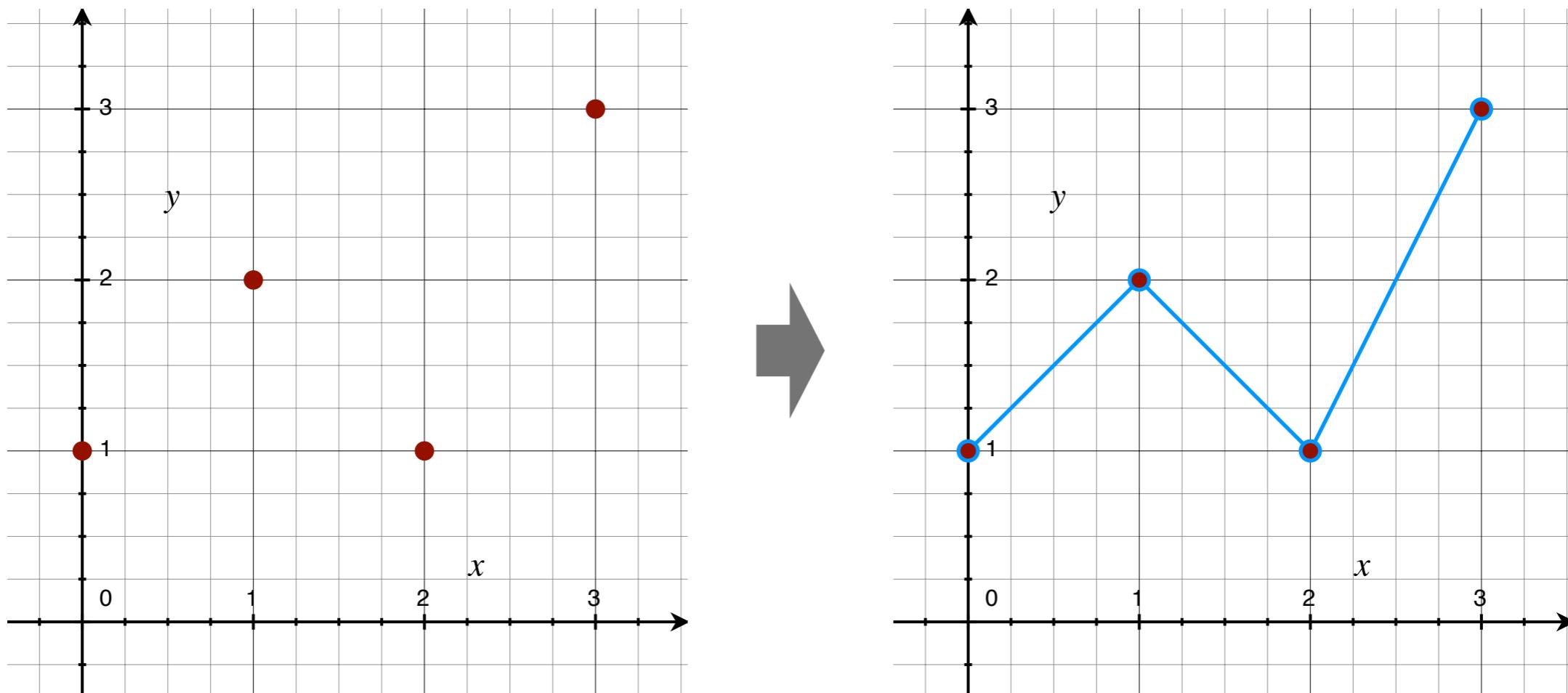
- Converting a continuous function to a discrete one:



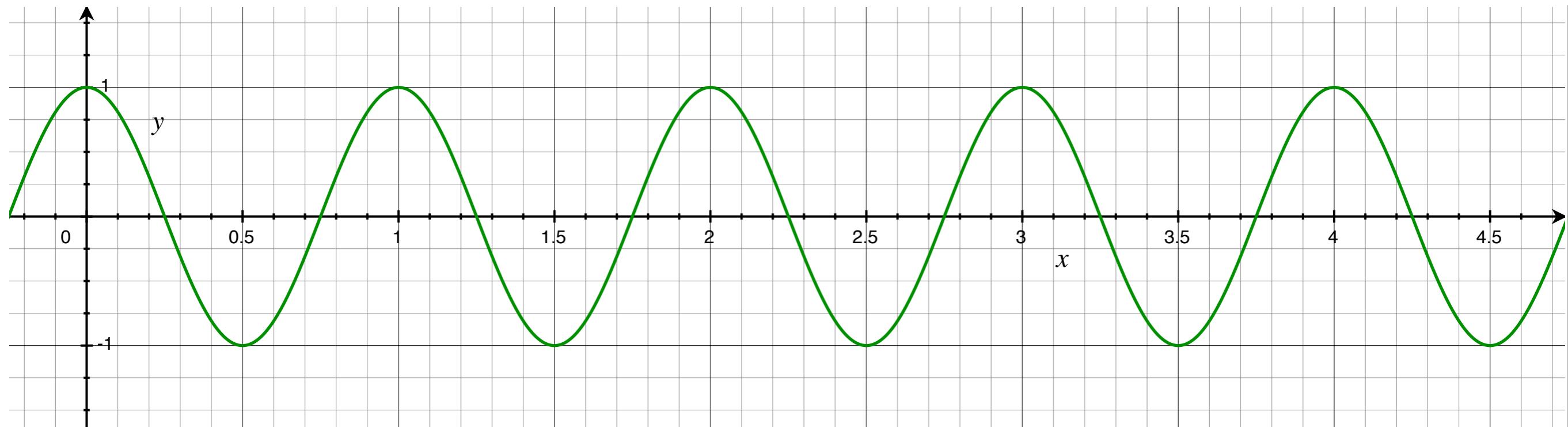
Can we get the original (continuous) function back?

Signal Reconstruction

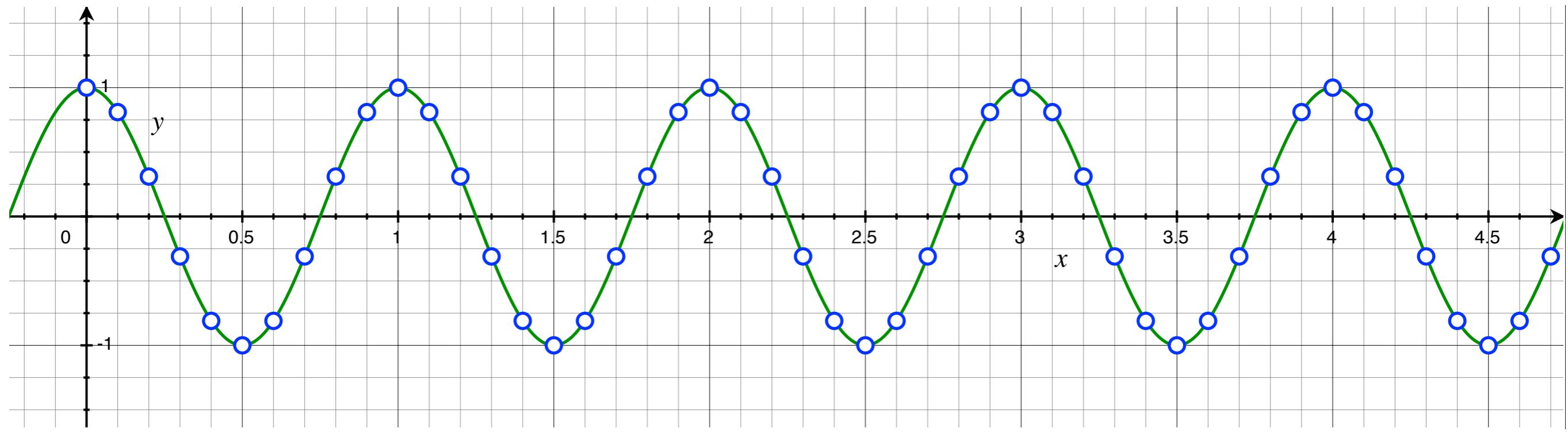
- That's just interpolation!



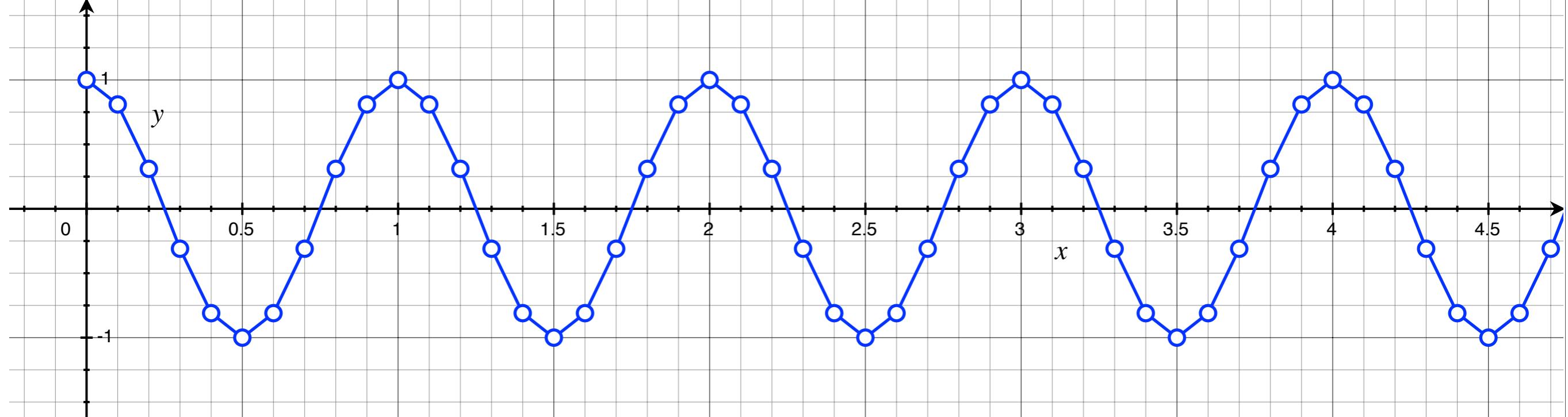
But wait a minute... that looks kind of different!



How many samples | do we need?



Looks pretty good!

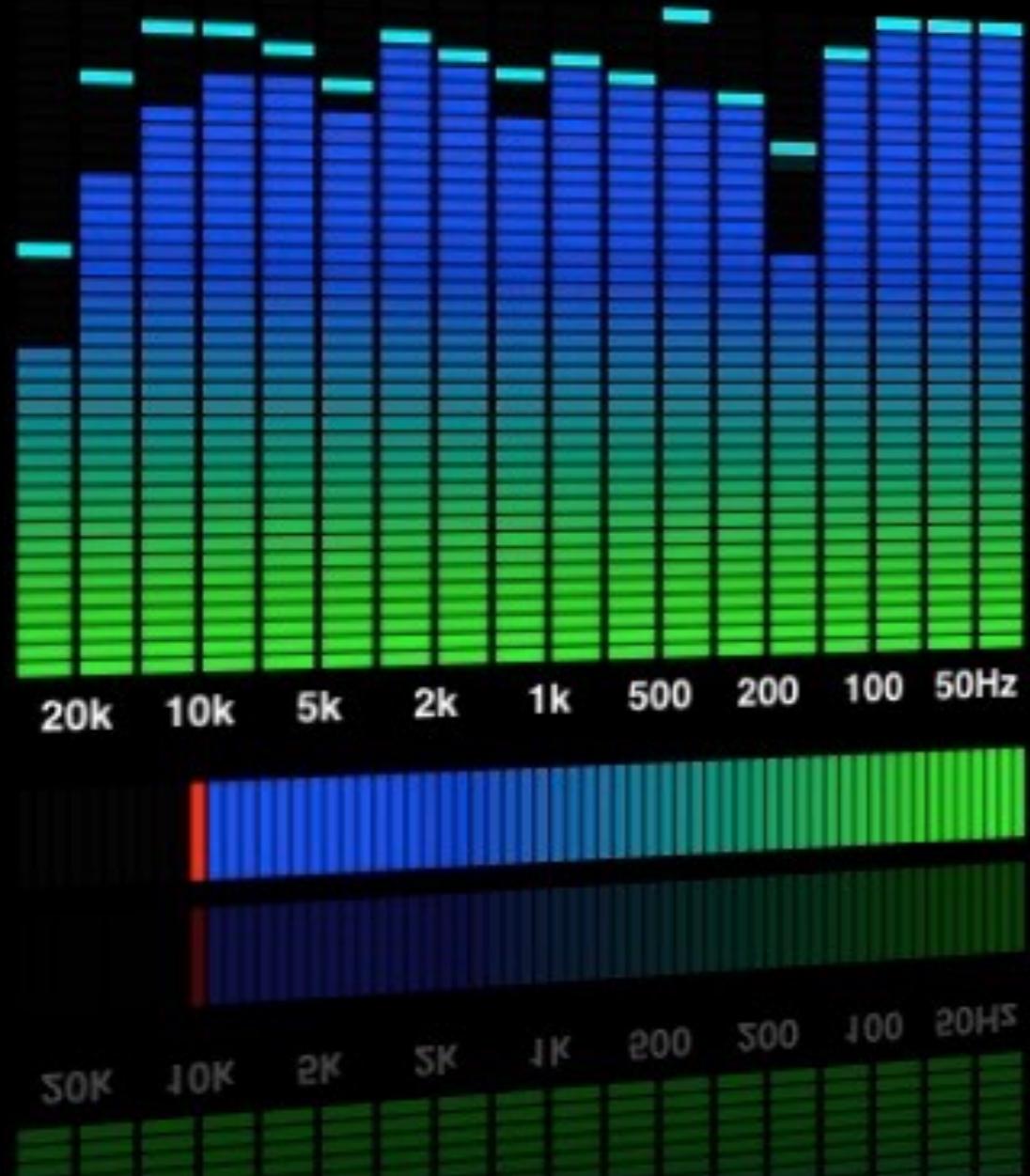


To faithfully reconstruct a band-limited signal from its samples, the sampling rate must be at least twice that of its highest frequency component.

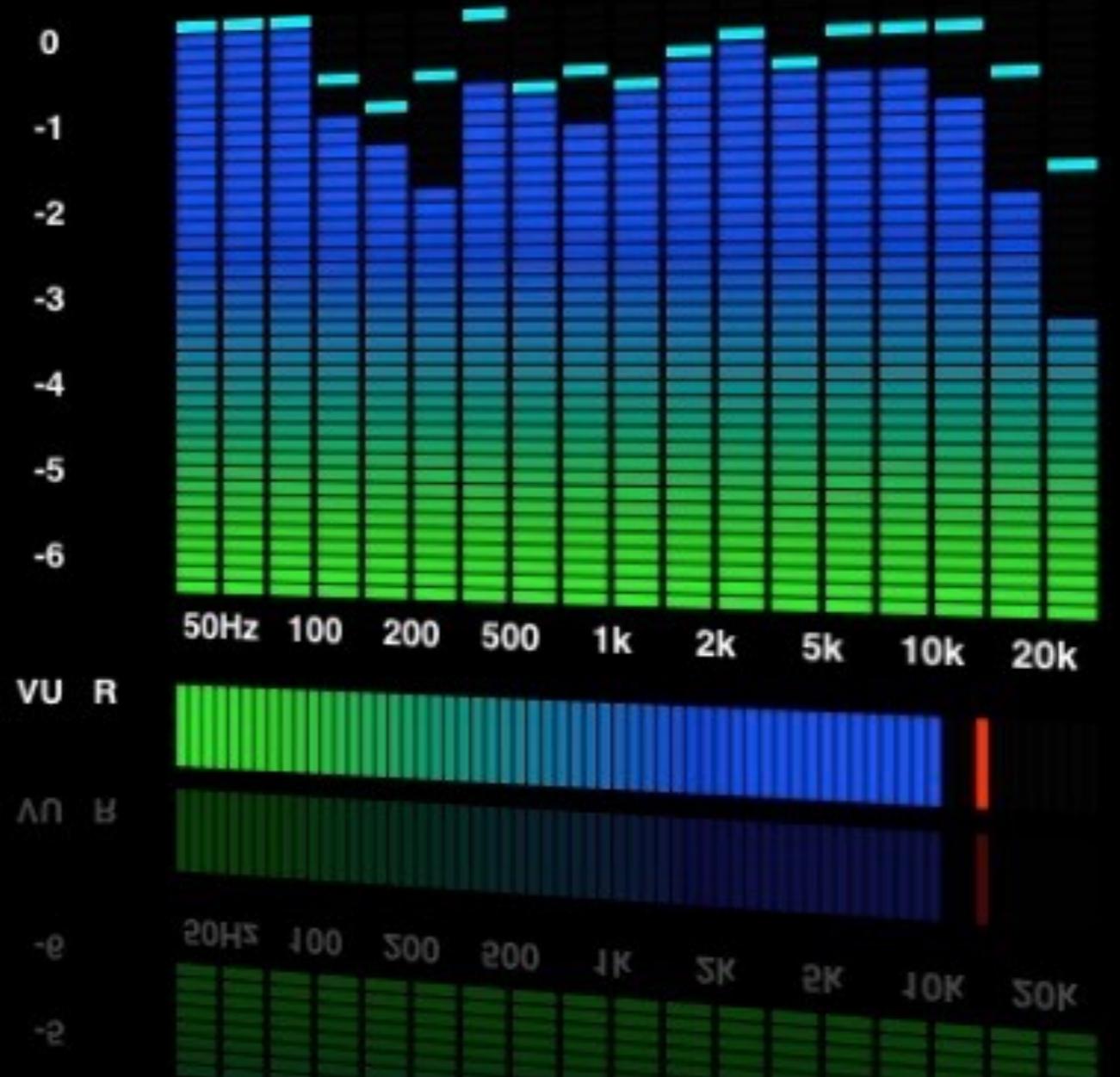
–Shannon Sampling Theorem, or *Nyquist Frequency*

Every continuous and periodic function may be expressed as the sum of a series of sine or cosine functions, each with specific amplitude and phase.

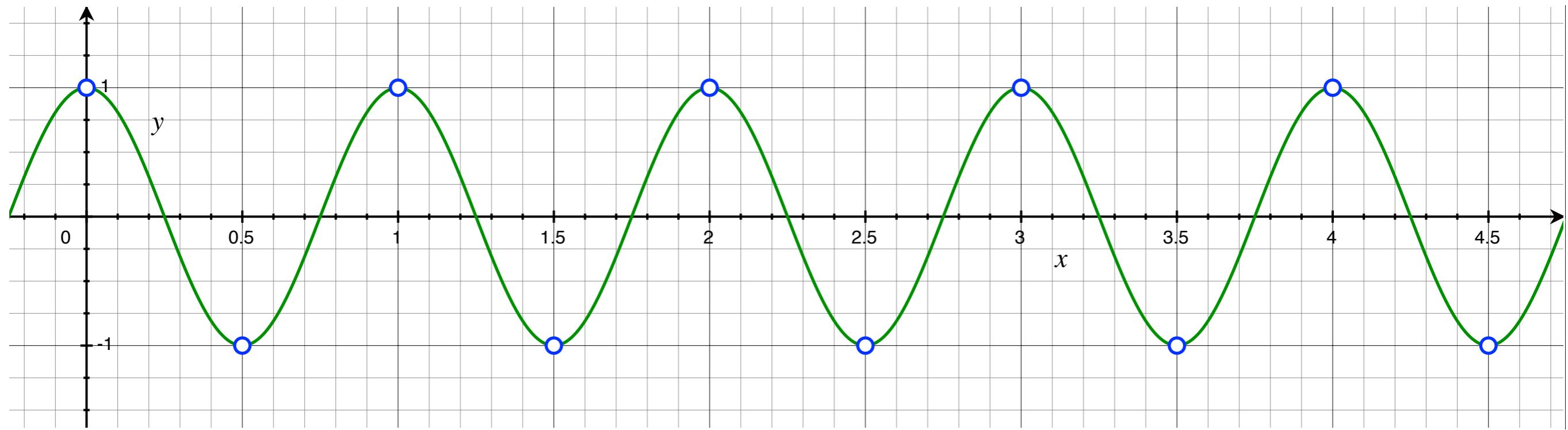
–Jean-Baptiste Fourier



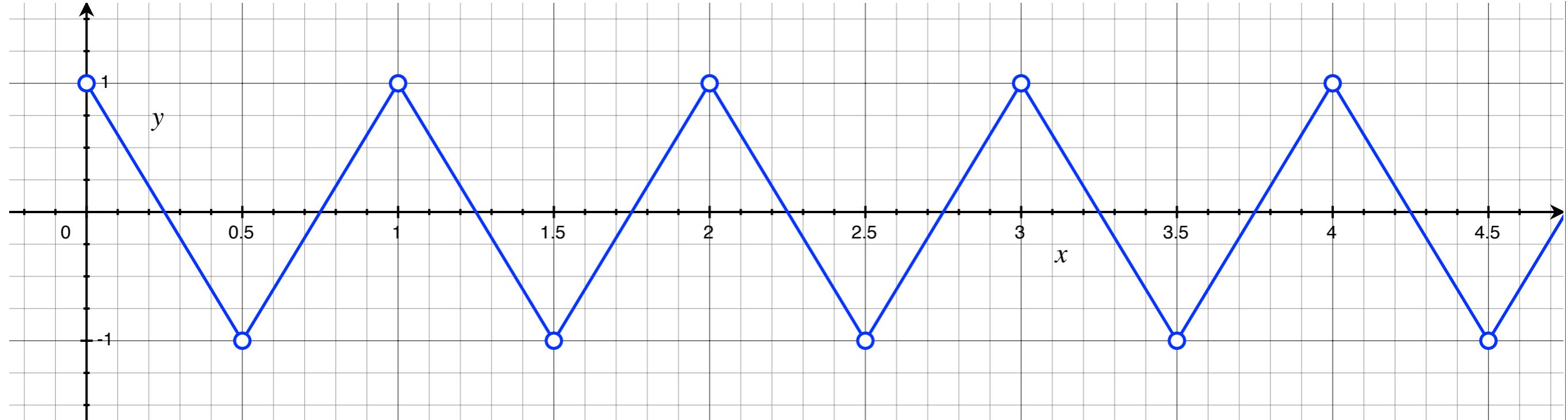
Fourier Transform

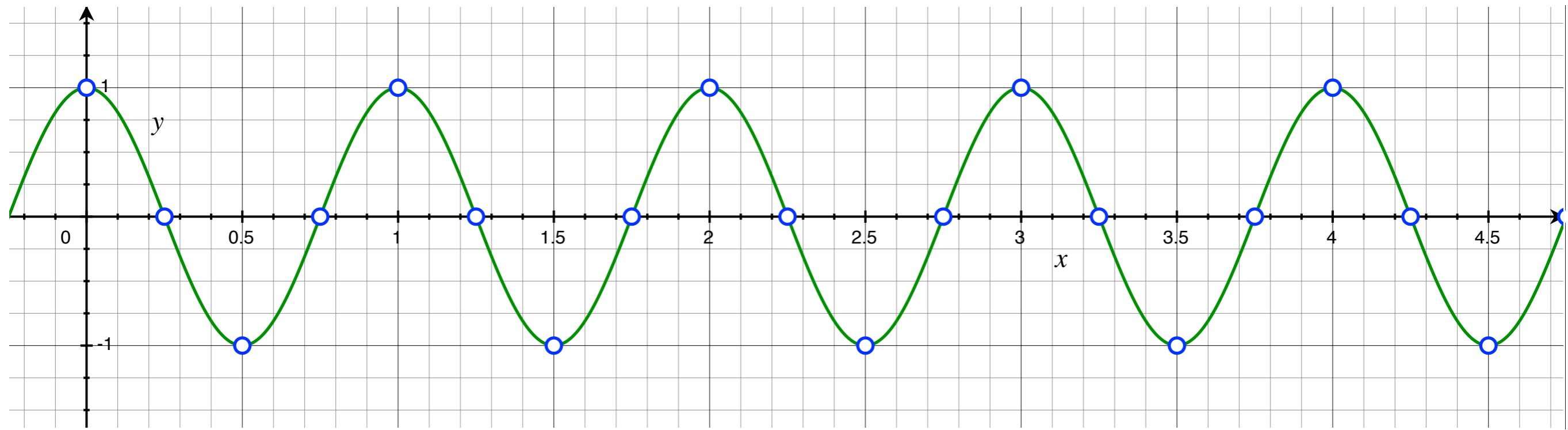


Spectrum Analyzer for iTunes

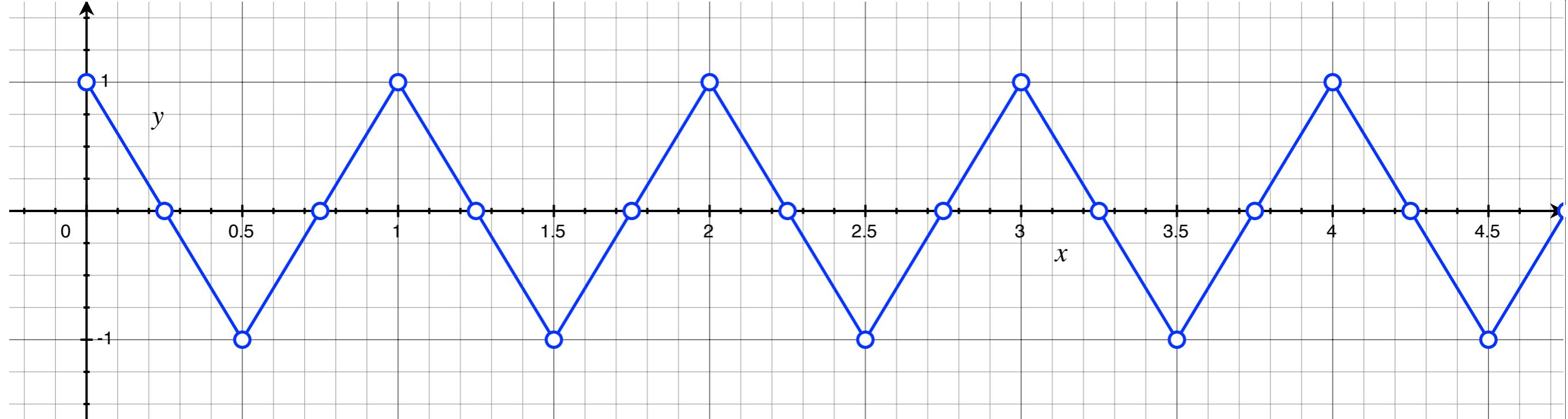


Hmmm... looks a little off, don't you think?

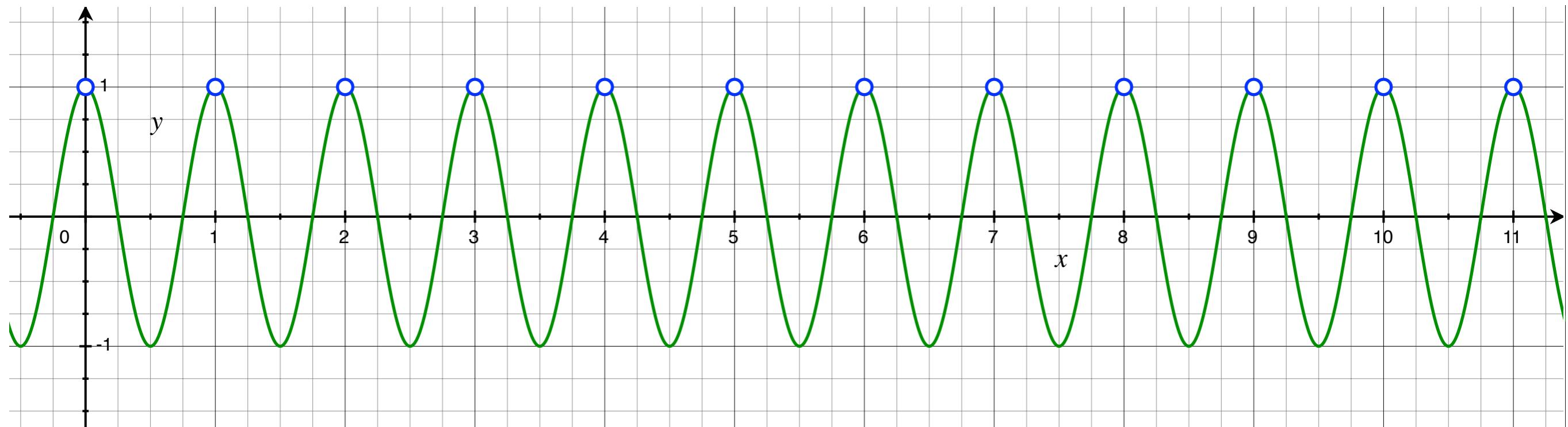




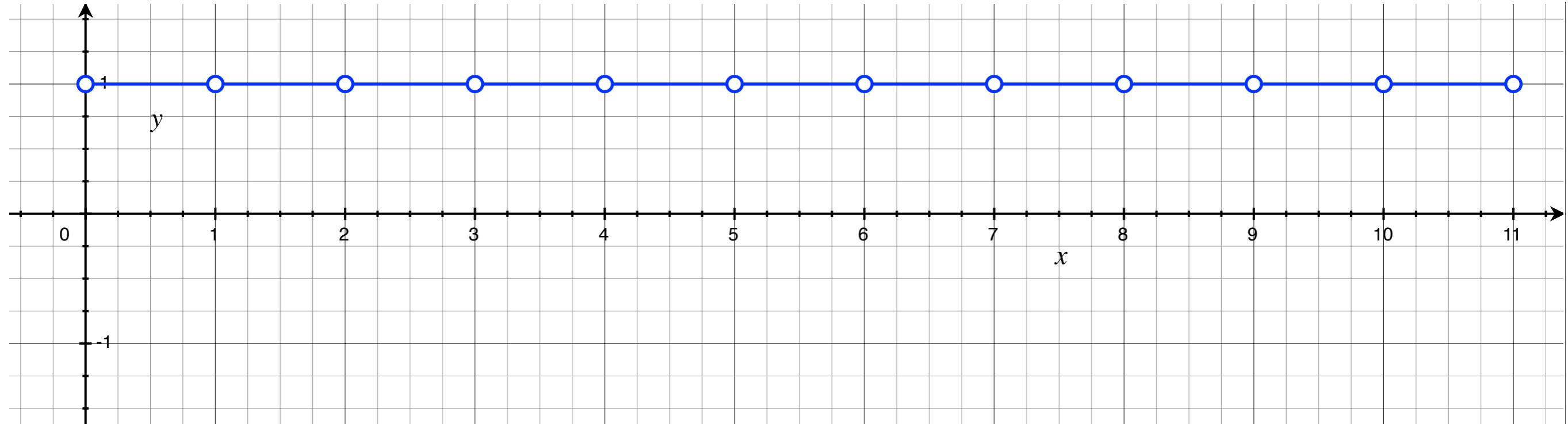
Huh. That still didn't seem to help.

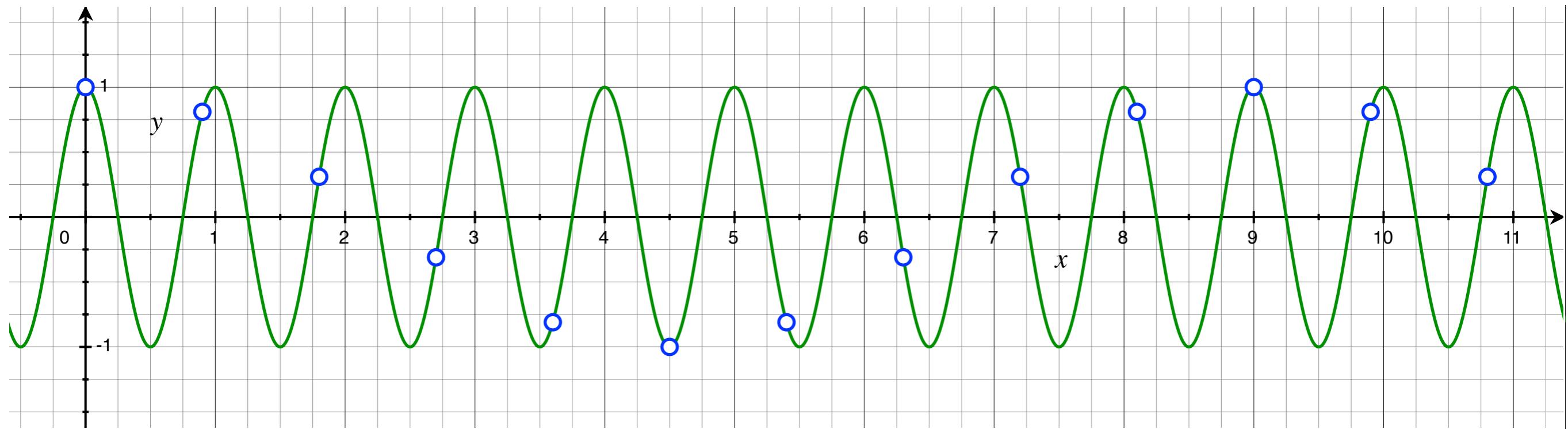


So what happens if we
under-sample?

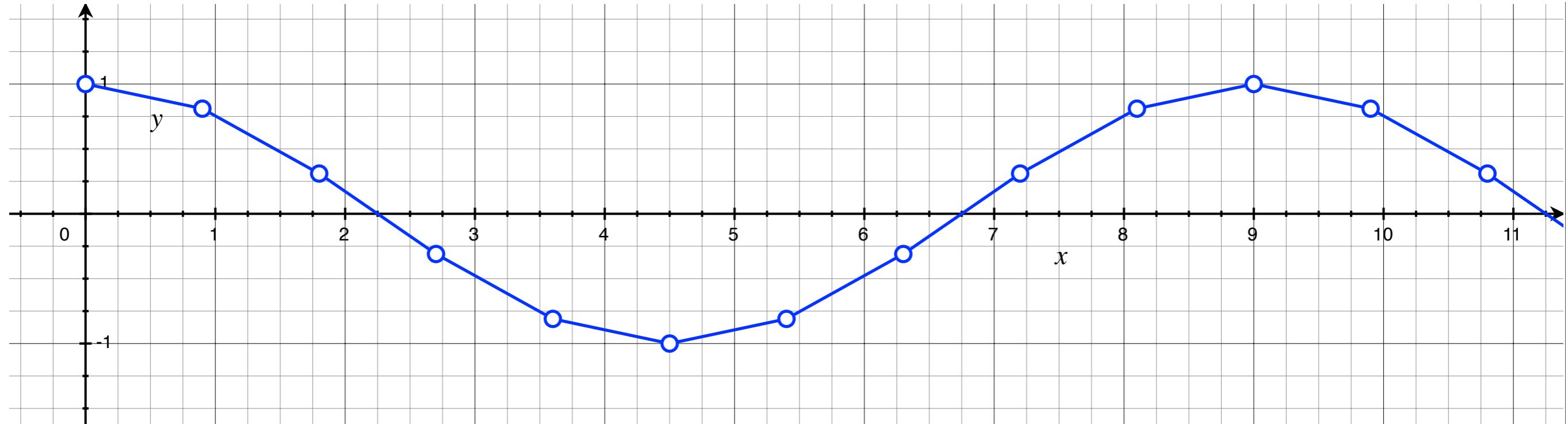


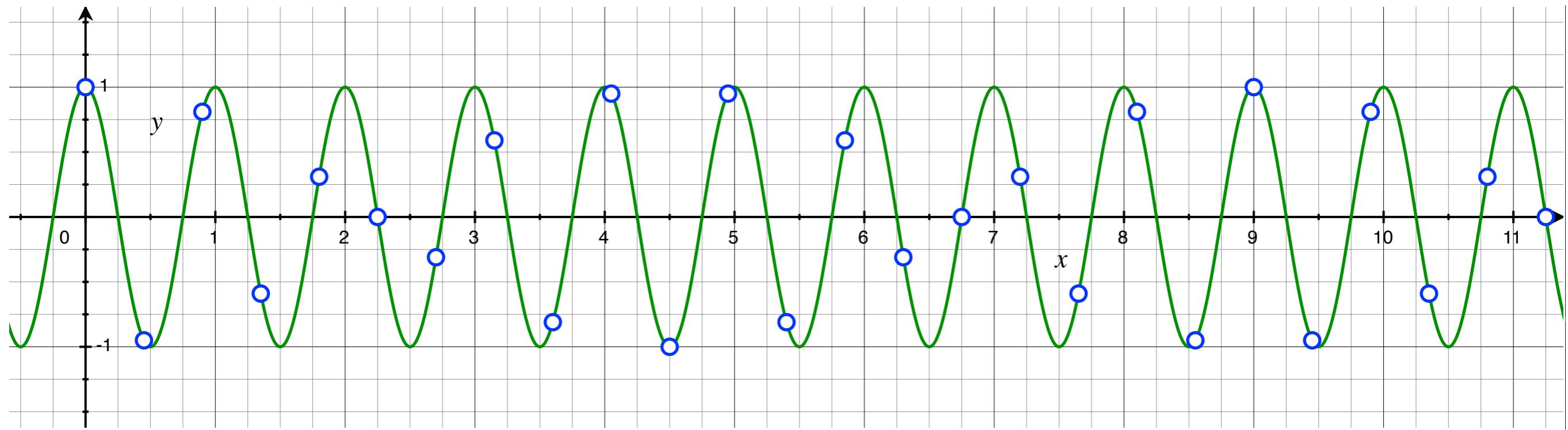
Well, that kinda sucked.



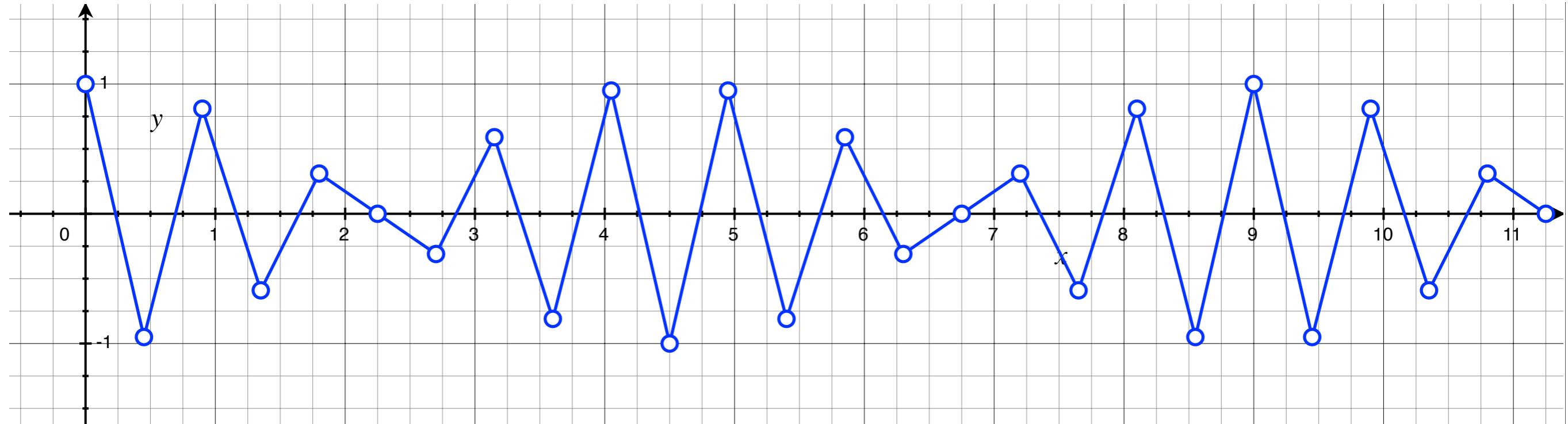


Aliasing!!!



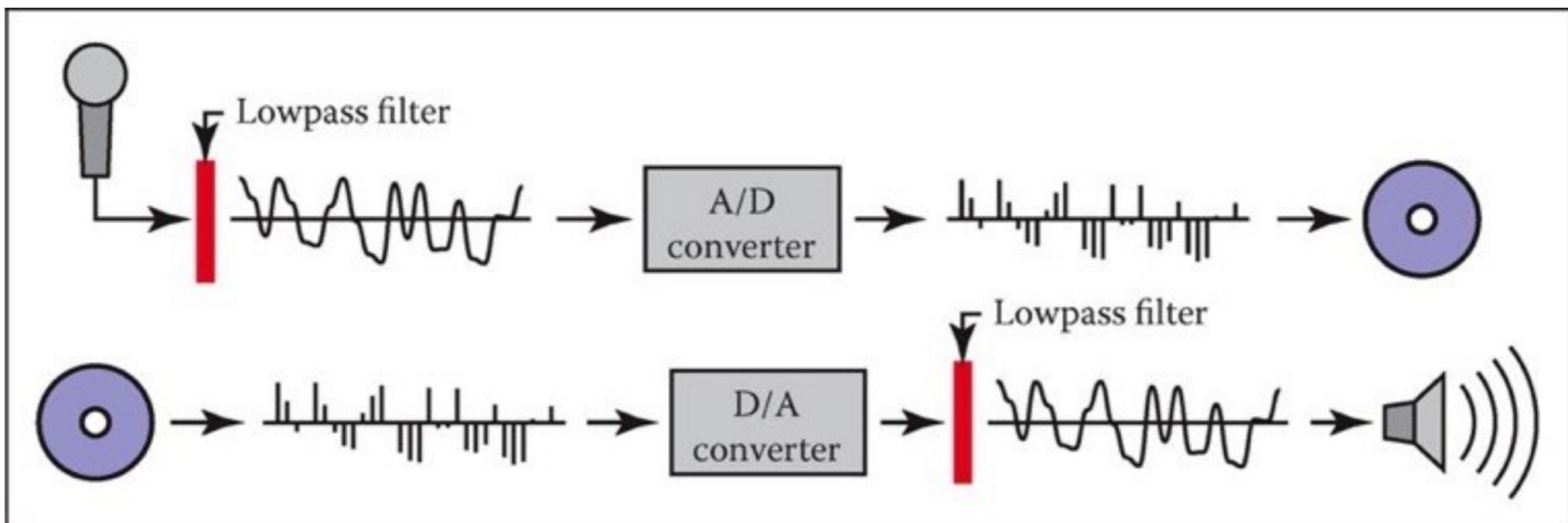


Wow, that's really weird. Let's come back to that...



Digital Audio & Music

- One of the first and most prevalent uses of digital sampling and signal reconstruction



Sampling Audio

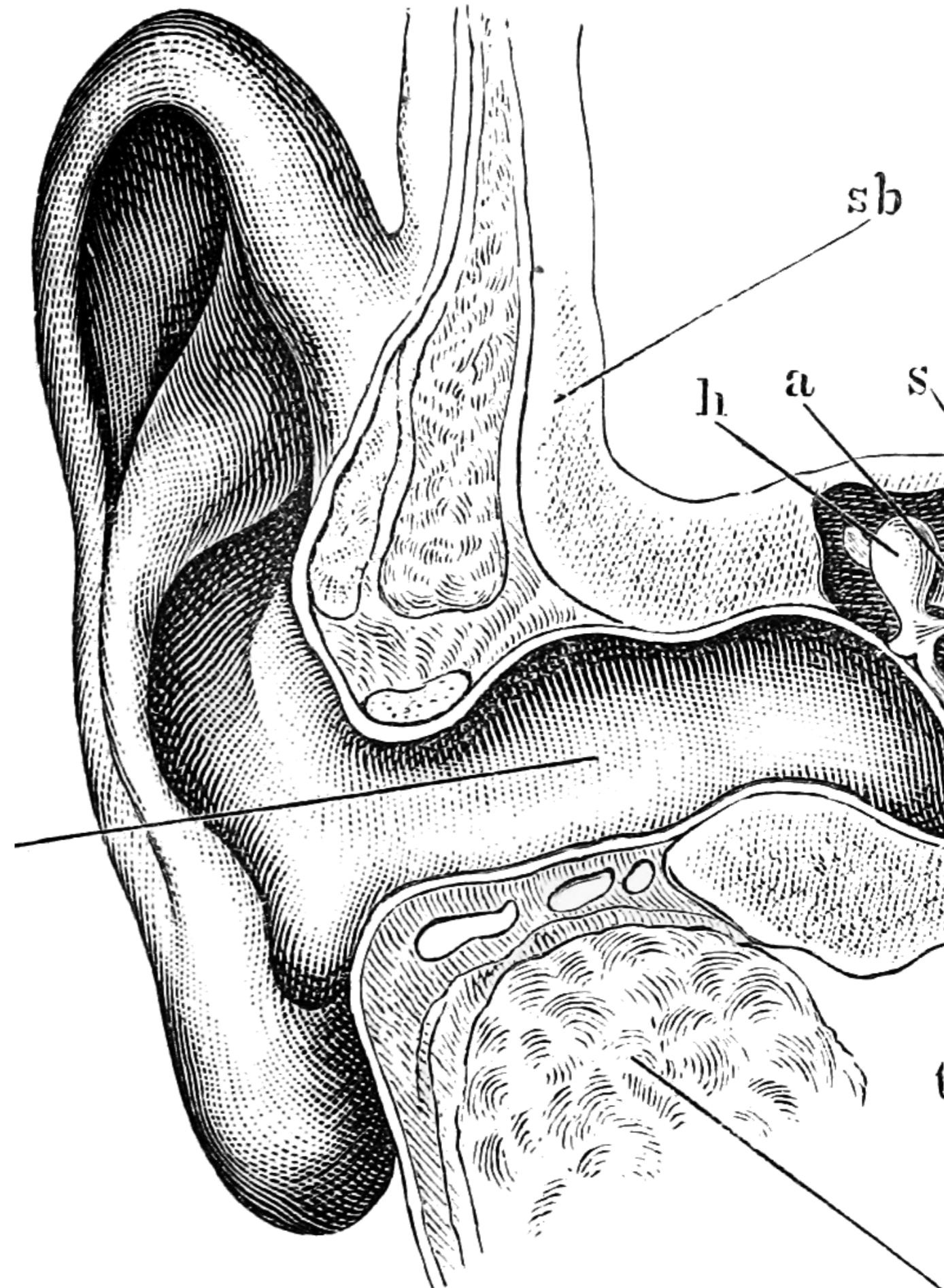
- What's the typical range of audible frequencies?

20 Hz – 20 kHz

- What are typical sampling rates for digital audio recordings?

44.1 kHz – 96 kHz

- What happens if we under-sample?

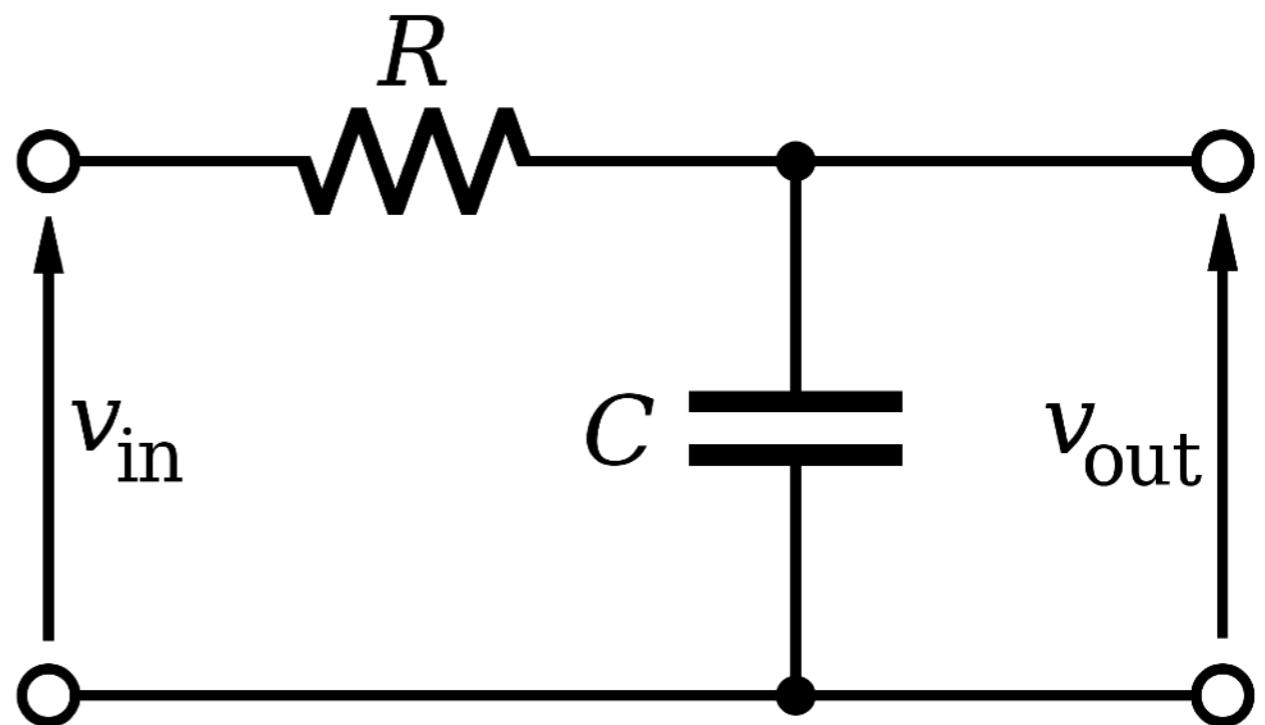


George Gershwin

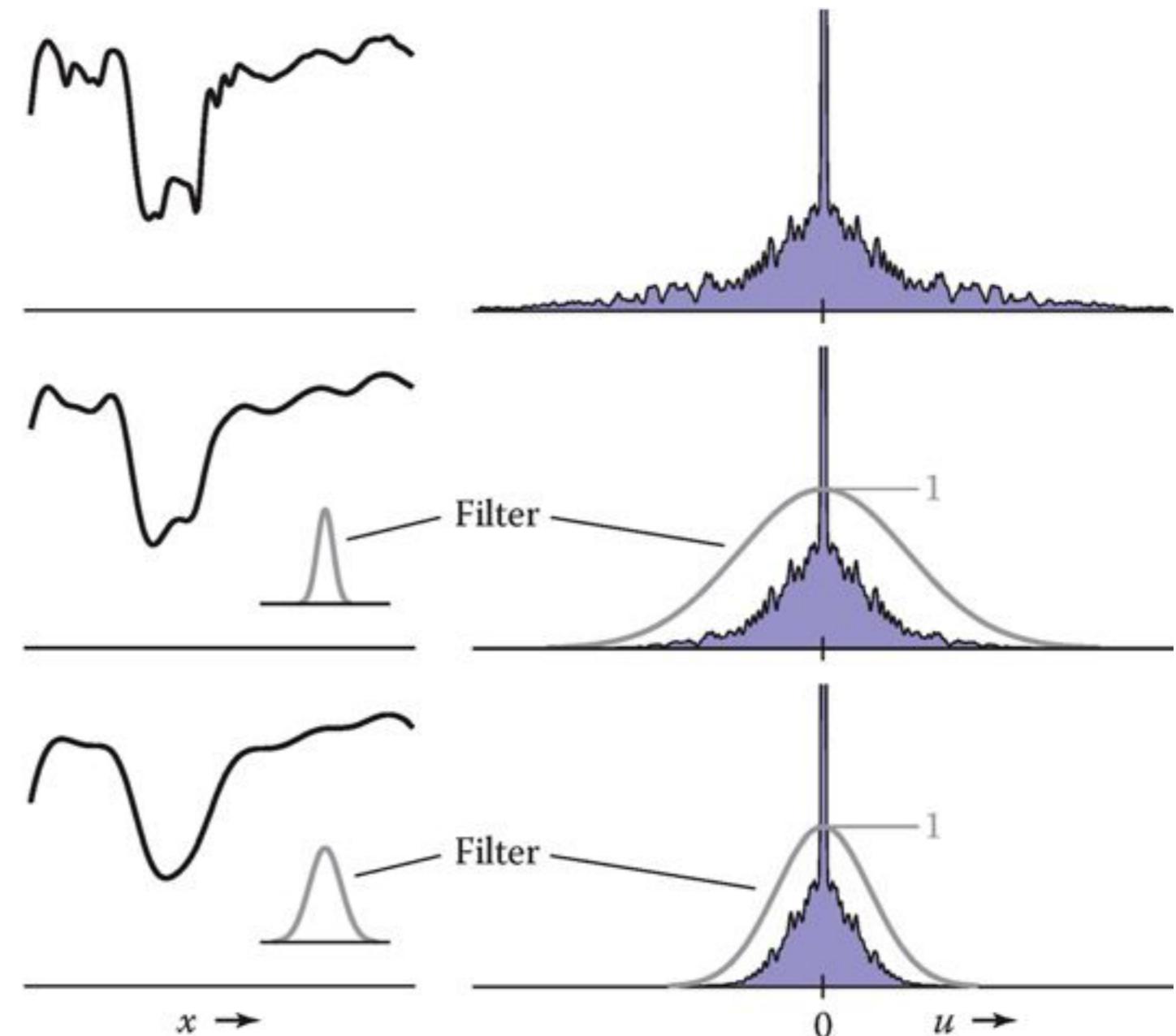
Rhapsody in Blue

Anti-Aliasing

Now that we know what aliasing is,
how do we get rid of it?

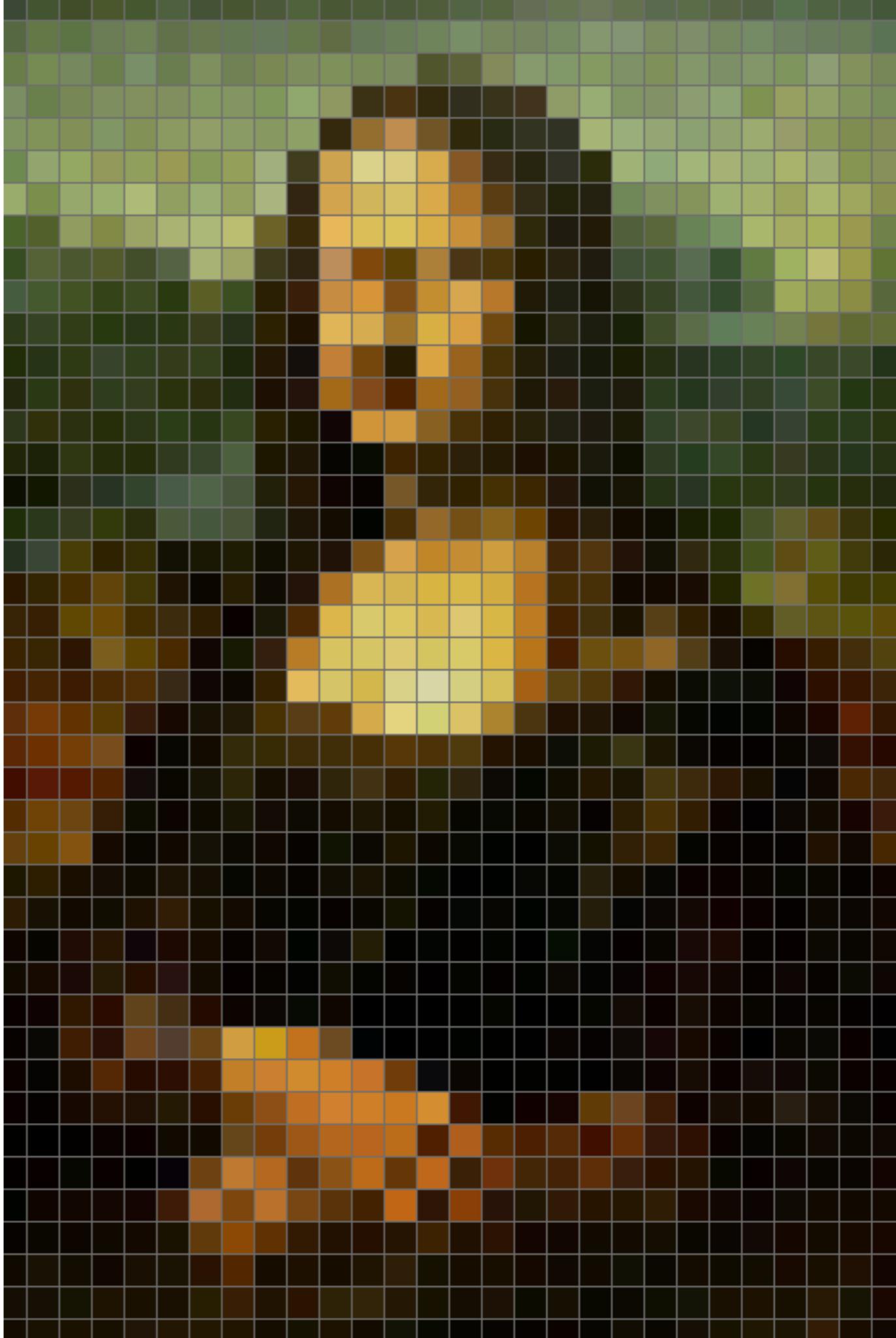


Low-Pass Filter



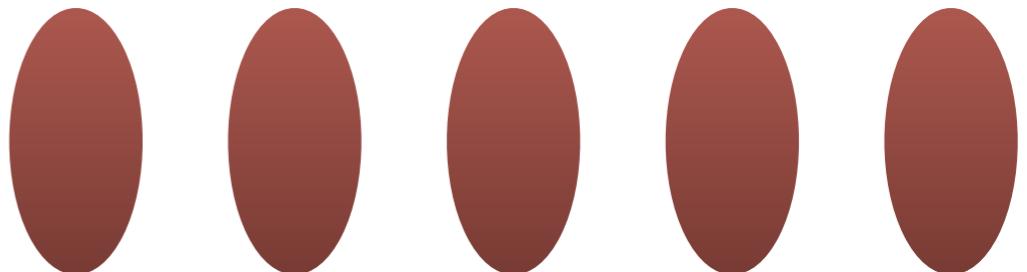
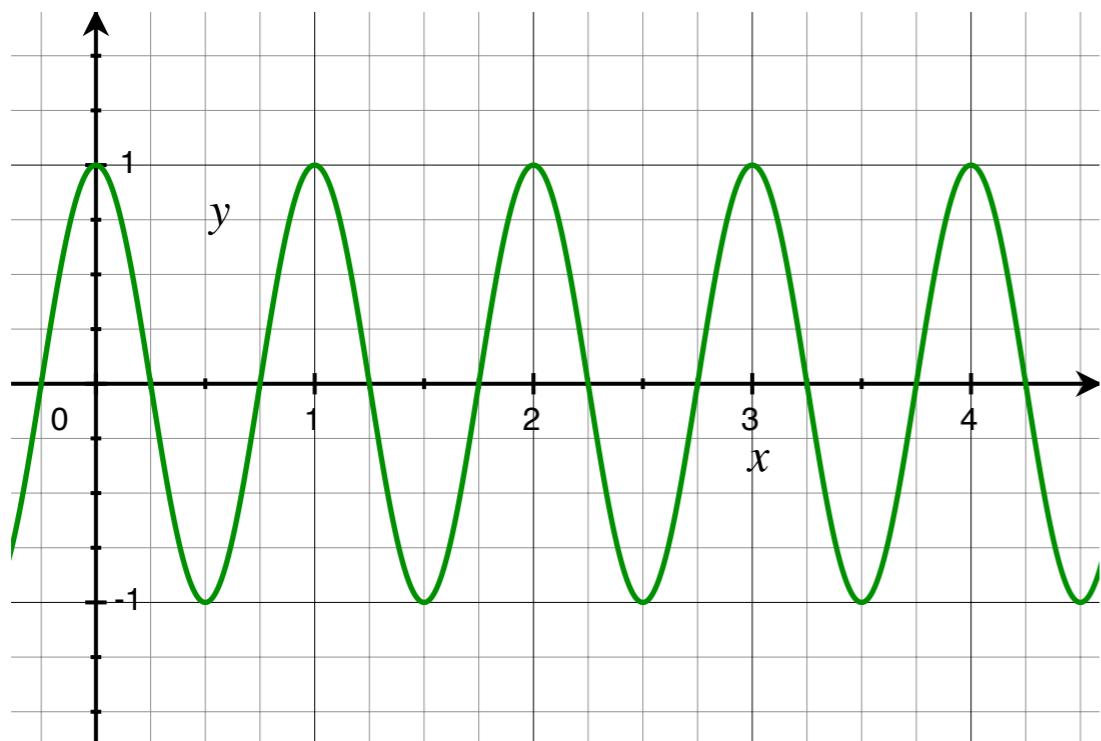
So, what does all this have to do with
computer graphics?

Digital Images



Spatial Frequency

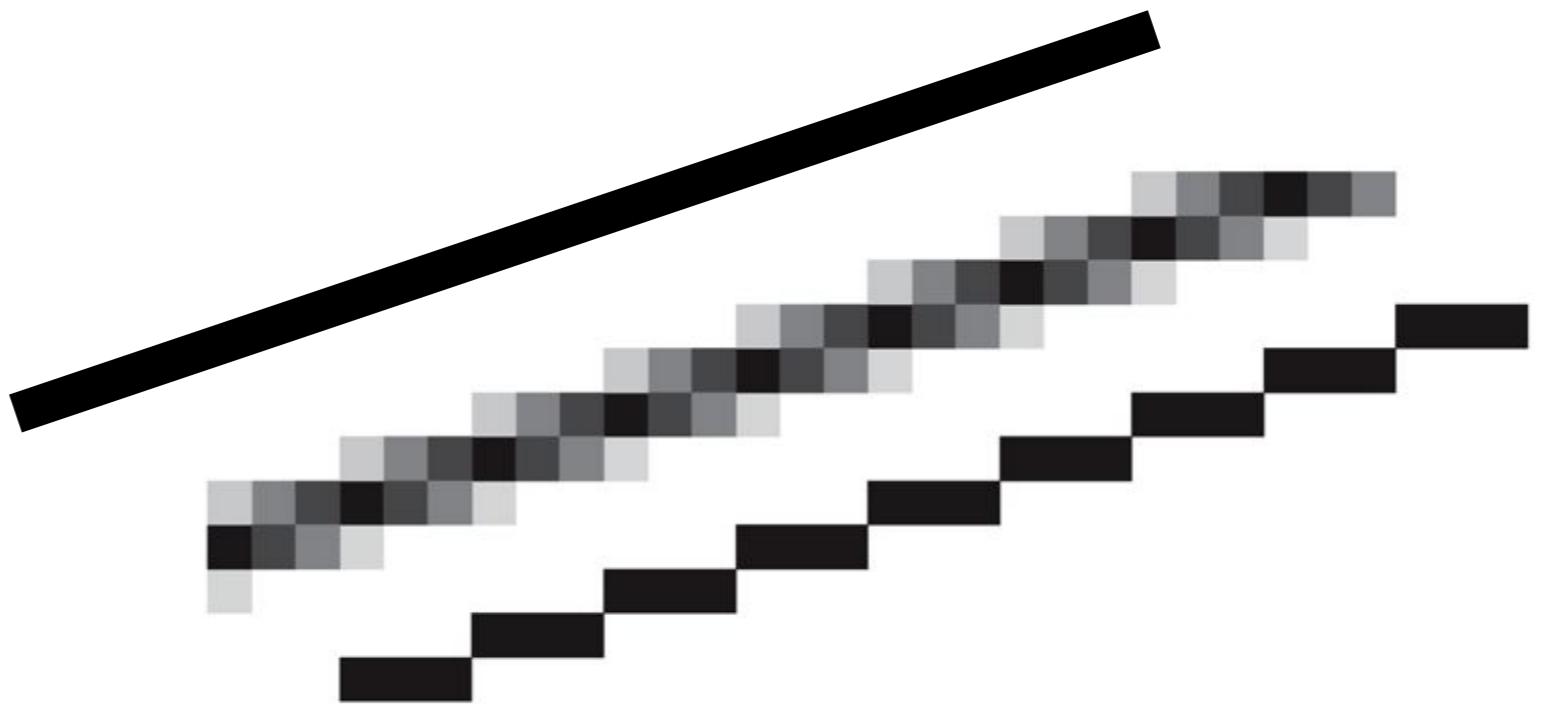
- How is **temporal** frequency related to **spatial** frequency?

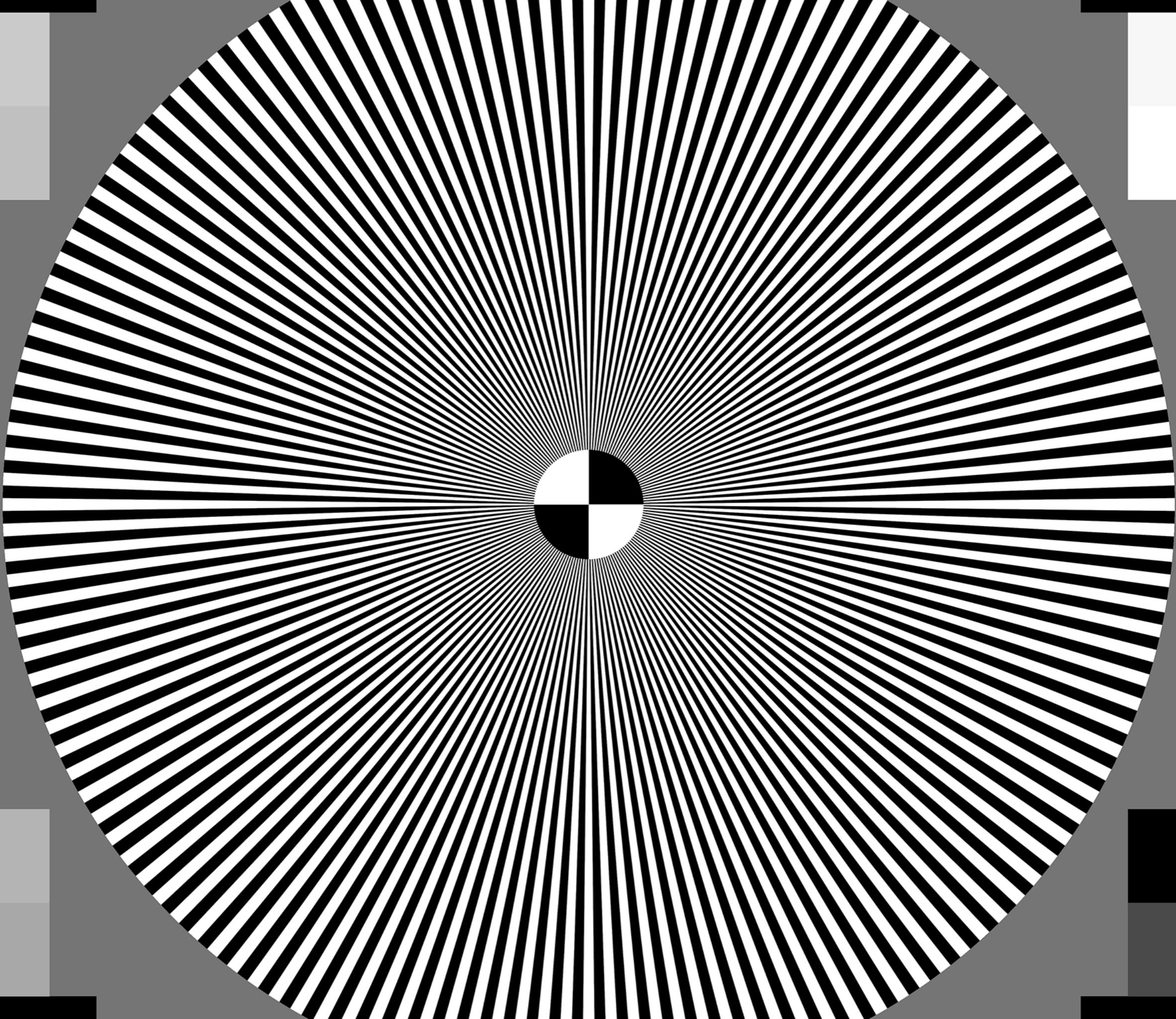


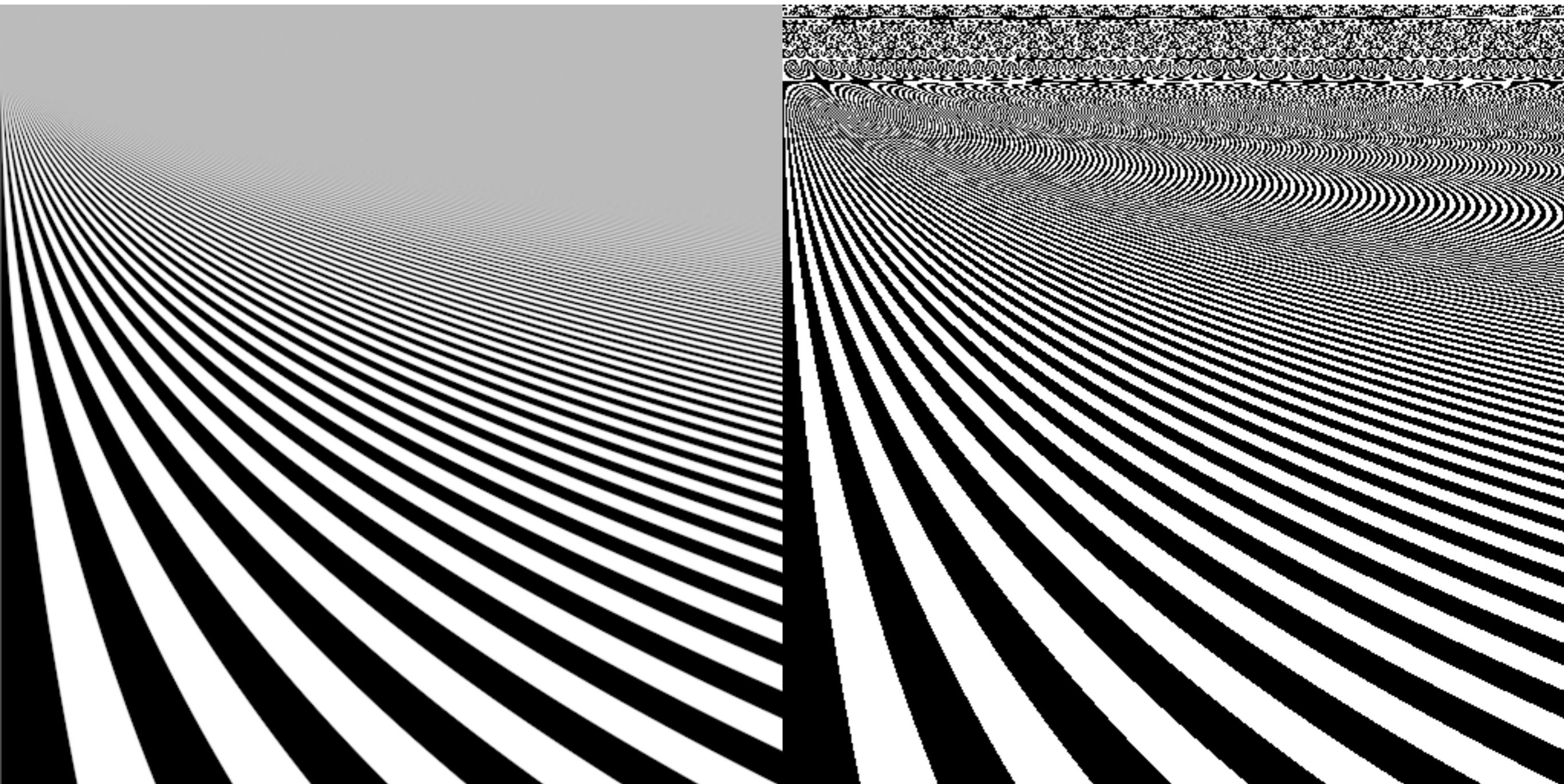
- What **units** are each measured in?

What exactly is spatial
aliasing in 2D?

Aliasing in 2D?







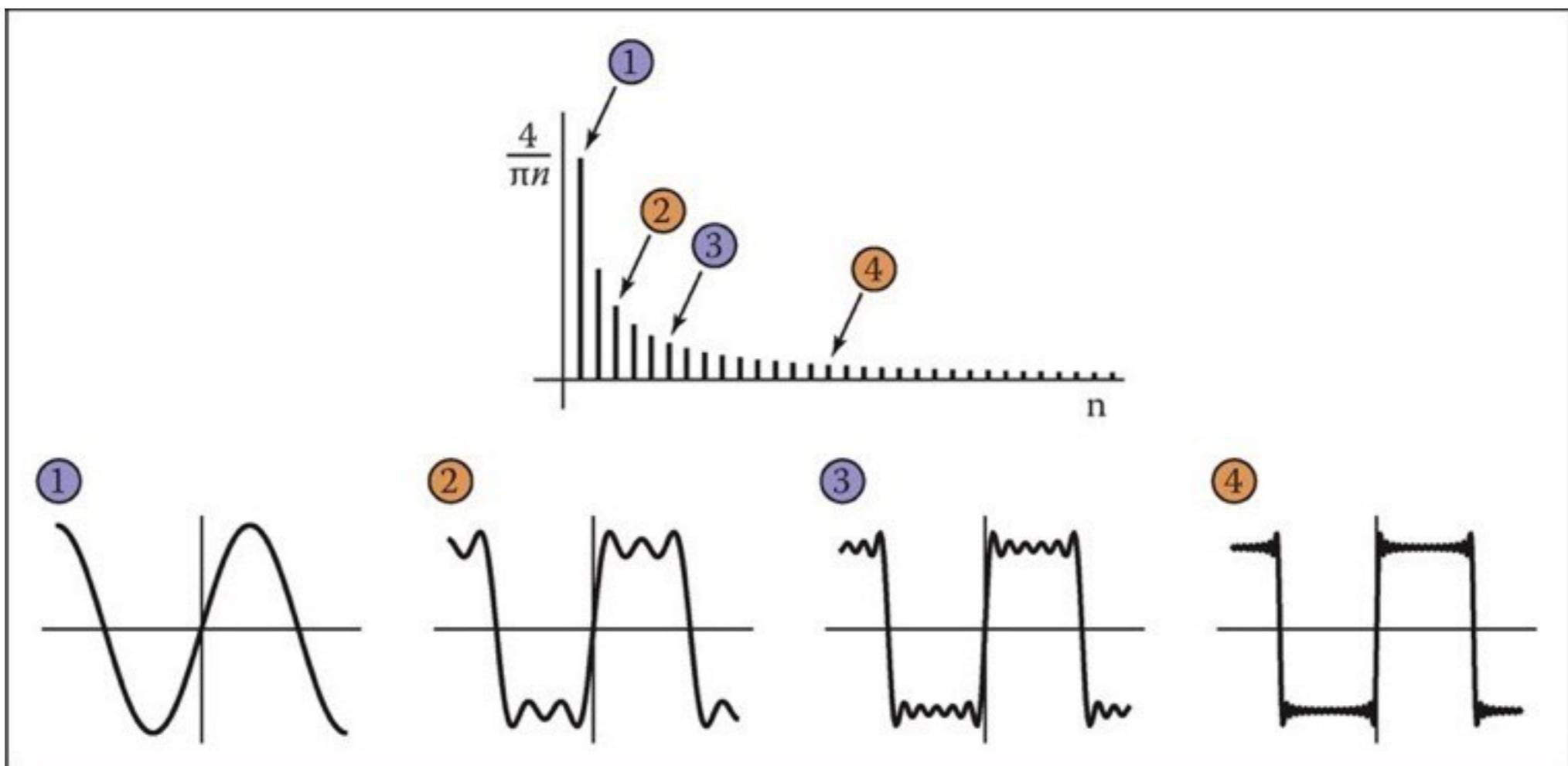
Spatial Aliasing

in two dimensions

**What is the frequency content of a
shape or polygon?**

Spectrum of a Step Function

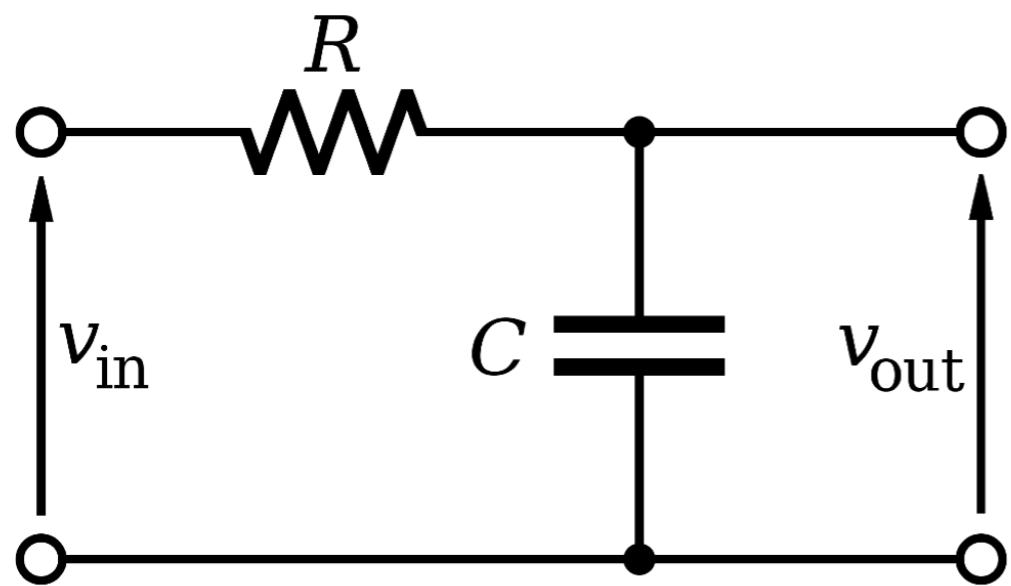
- Fourier said we can decompose it!



... but it has an infinite spectrum.

Anti-Aliasing in 2D

How does it work?



No jaggies!
(I'll have to punt this one)

Things to Remember

- **Interpolation** is the most important function in graphics
 - Converts a discrete function into a continuous function
 - You can interpolate almost anything you can add!
- **Sampling** goes the other way: continuous → discrete
 - Nyquist rate: need to sample at twice the highest frequency
- **Aliasing** happens when we under-sample a signal
 - This is generally not very appealing to our senses!