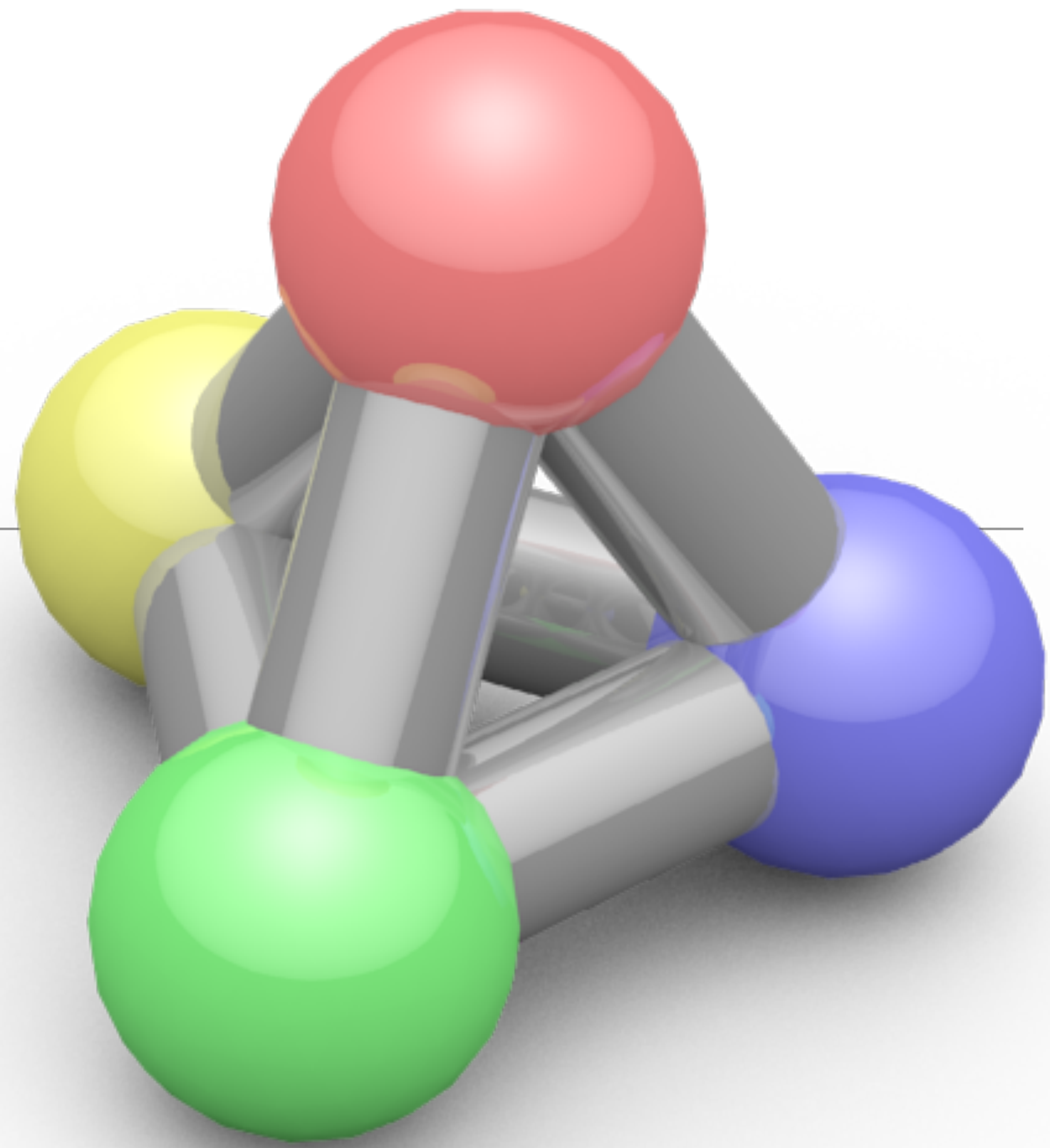
Bézier Splines

CPSC 453 – Fall 2016
Sonny Chan

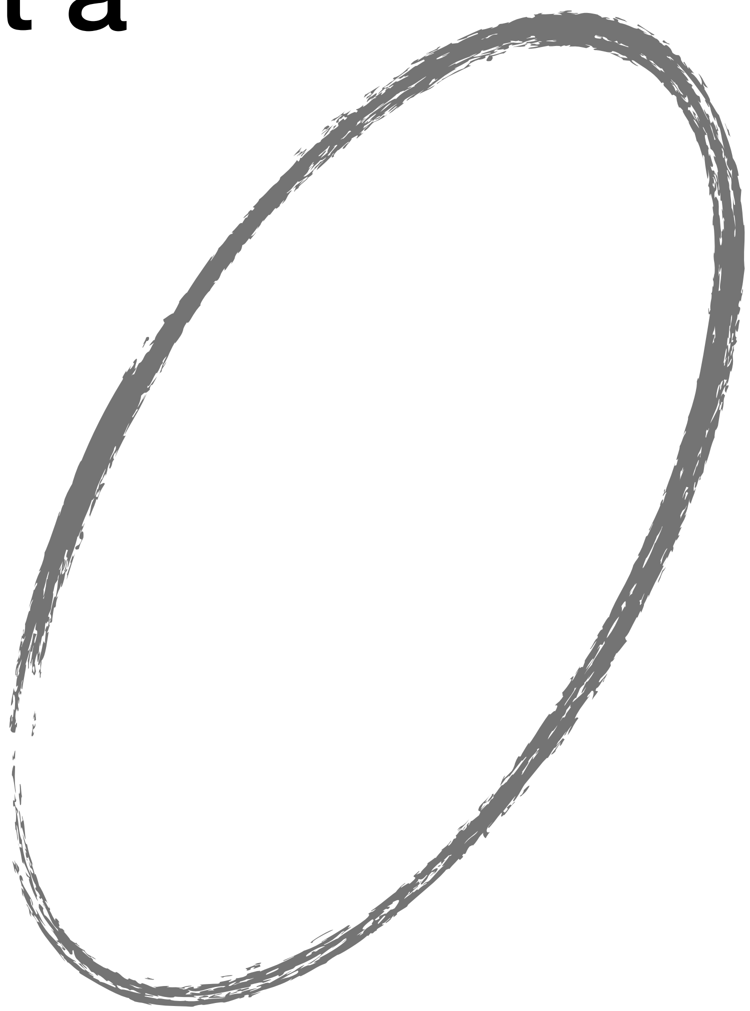


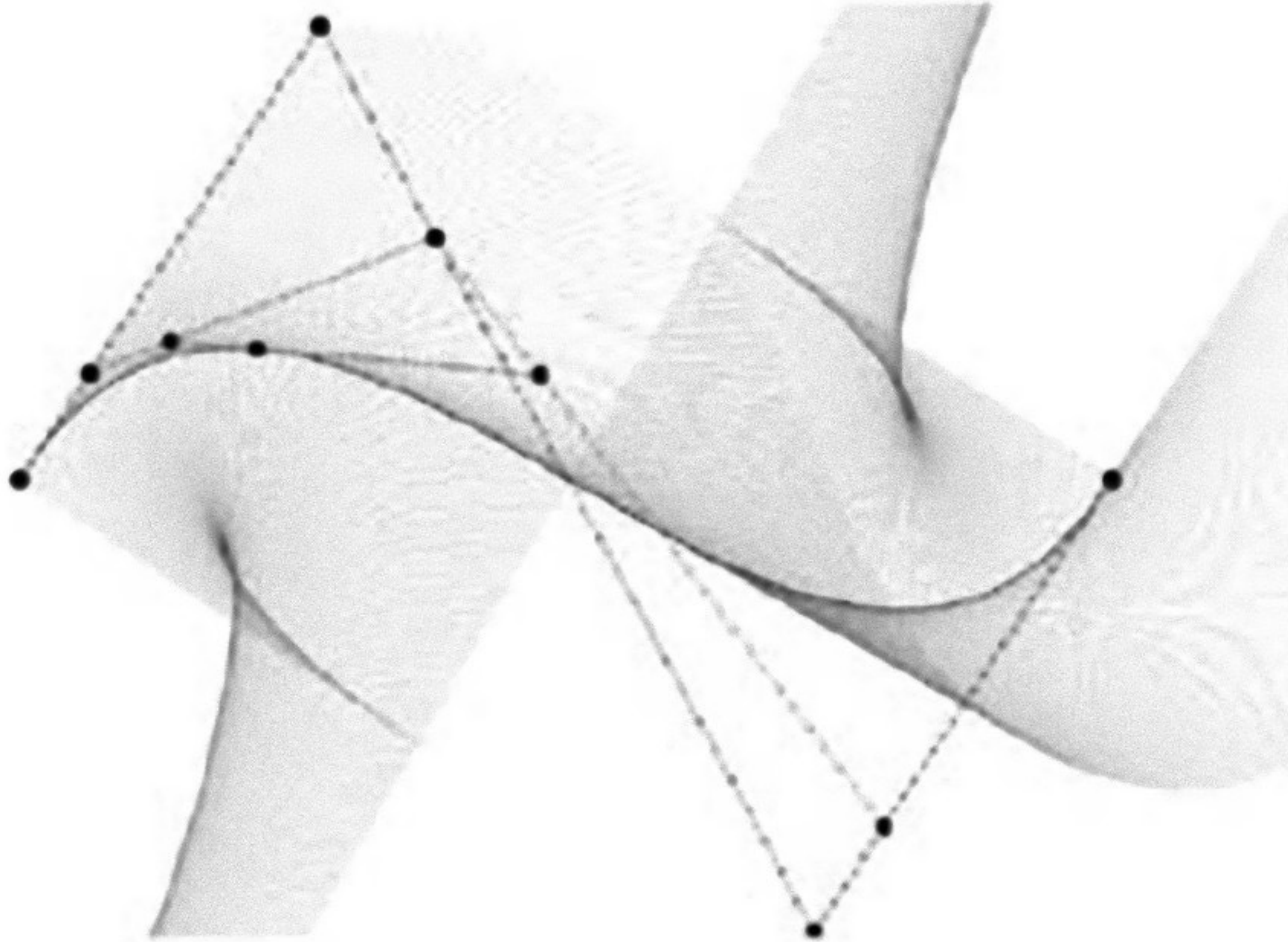
Today’s Outline

- Bézier splines
- Cubic Bézier curves
- Drawing Bézier curves

Using a Bézier spline to represent a

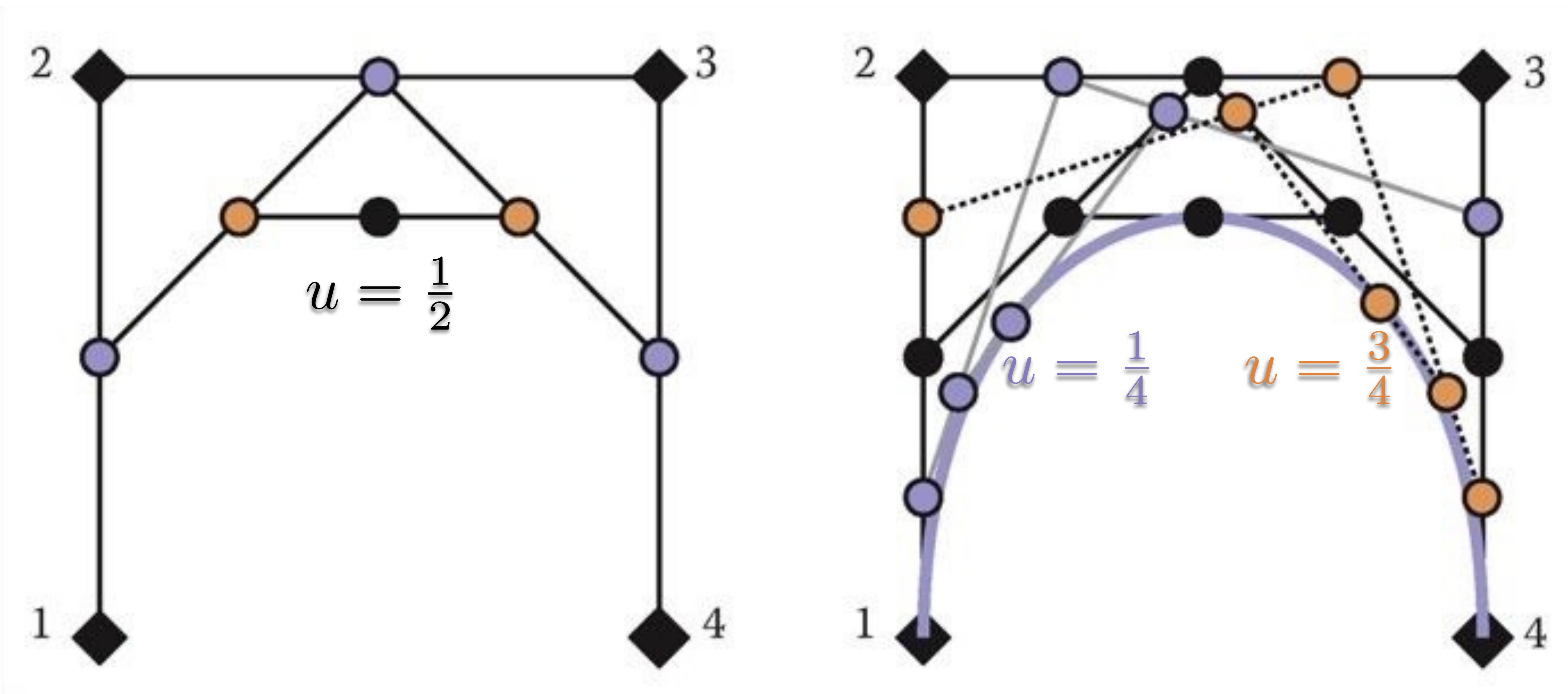
freeform shape





Bézier Curves
of the 3rd degree

de Casteljau Construction



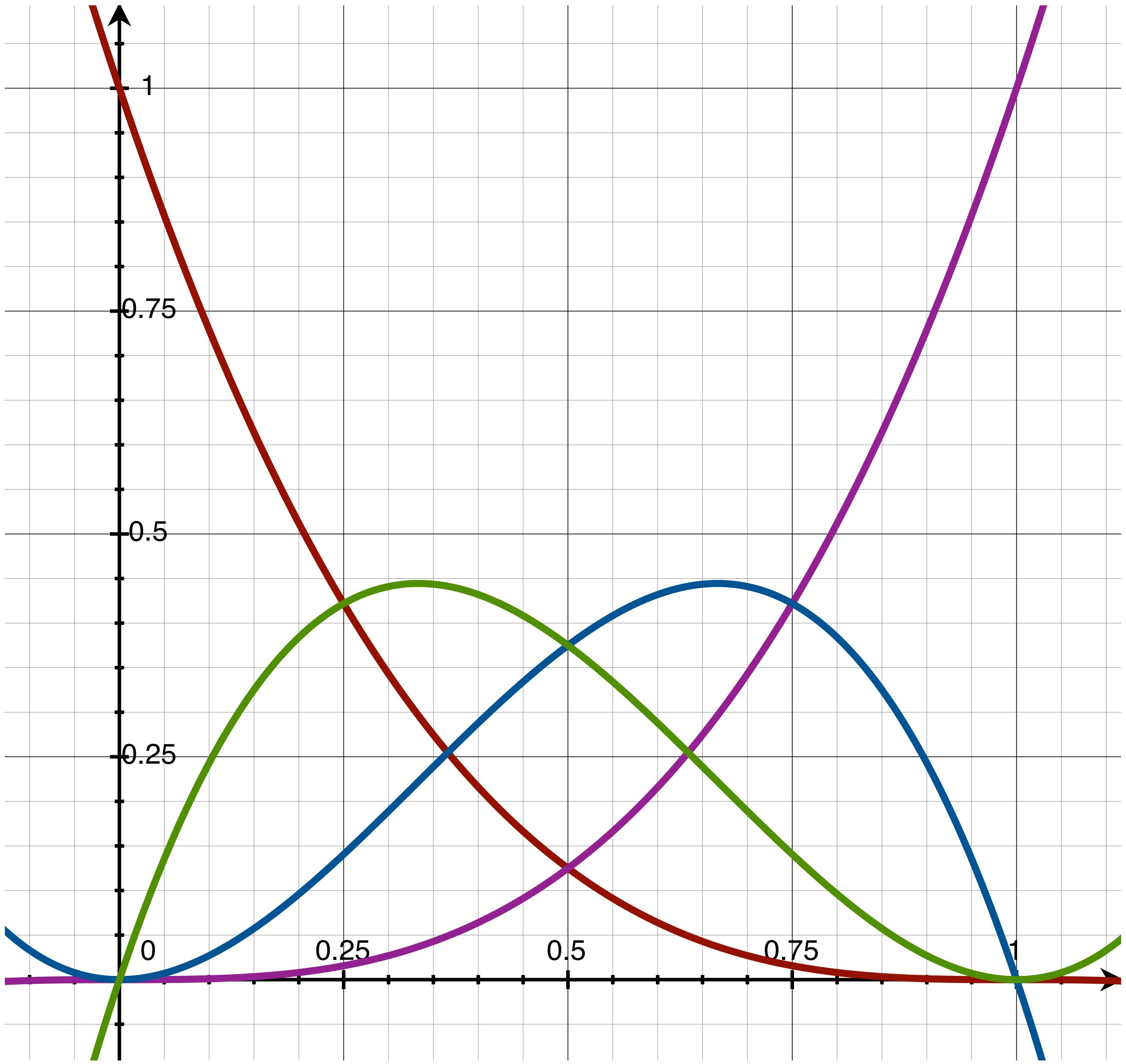
Third Degree Bernstein Polynomials

$$b_{0,3}(u) = (1-u)^3$$

$$b_{1,3}(u) = 3u(1-u)^2$$

$$b_{2,3}(u) = 3u^2(1-u)$$

$$b_{3,3}(u) = u^3$$

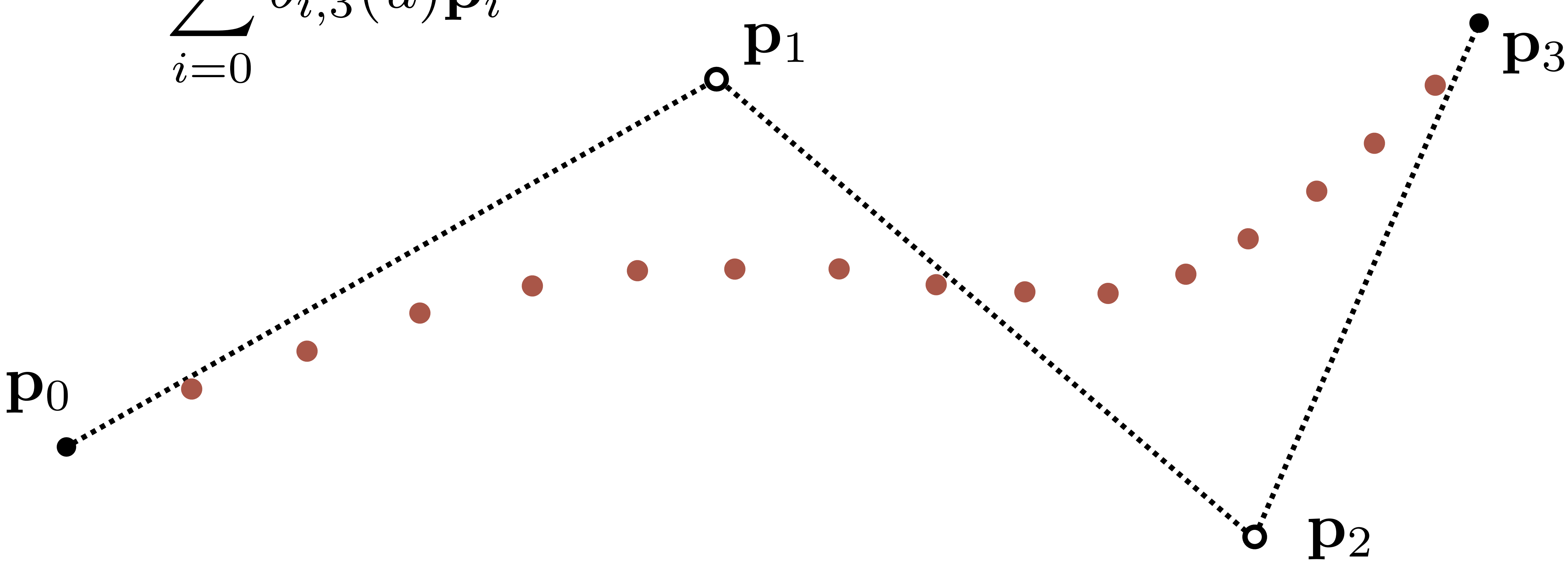


Bernstein form of a Cubic Bézier

$$\mathbf{p}(u) = (1-u)^3\mathbf{p}_0 + 3u(1-u)^2\mathbf{p}_1 + 3u^2(1-u)\mathbf{p}_2 + u^3\mathbf{p}_3$$

$$= b_{0,3}(u)\mathbf{p}_0 + b_{1,3}(u)\mathbf{p}_1 + b_{2,3}(u)\mathbf{p}_2 + b_{3,3}(u)\mathbf{p}_3$$

$$= \sum_{i=0}^{3} b_{i,3}(u)\mathbf{p}_i$$



Do our properties still apply?

✓ **Endpoints**

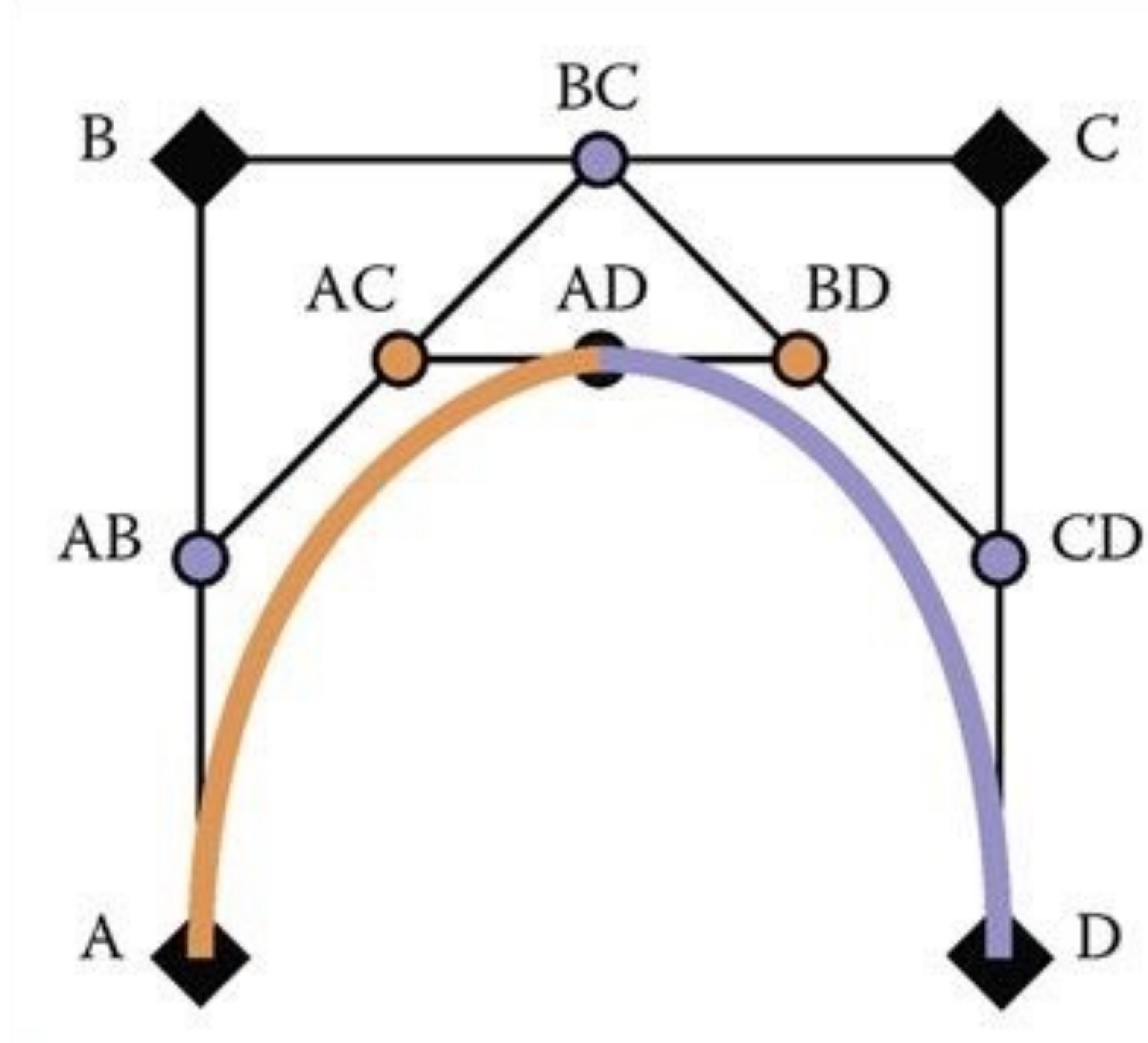
✓ **Tangents**

✓ **Convex Hull**

Property #4:

curve subdivision

Curve Subdivision

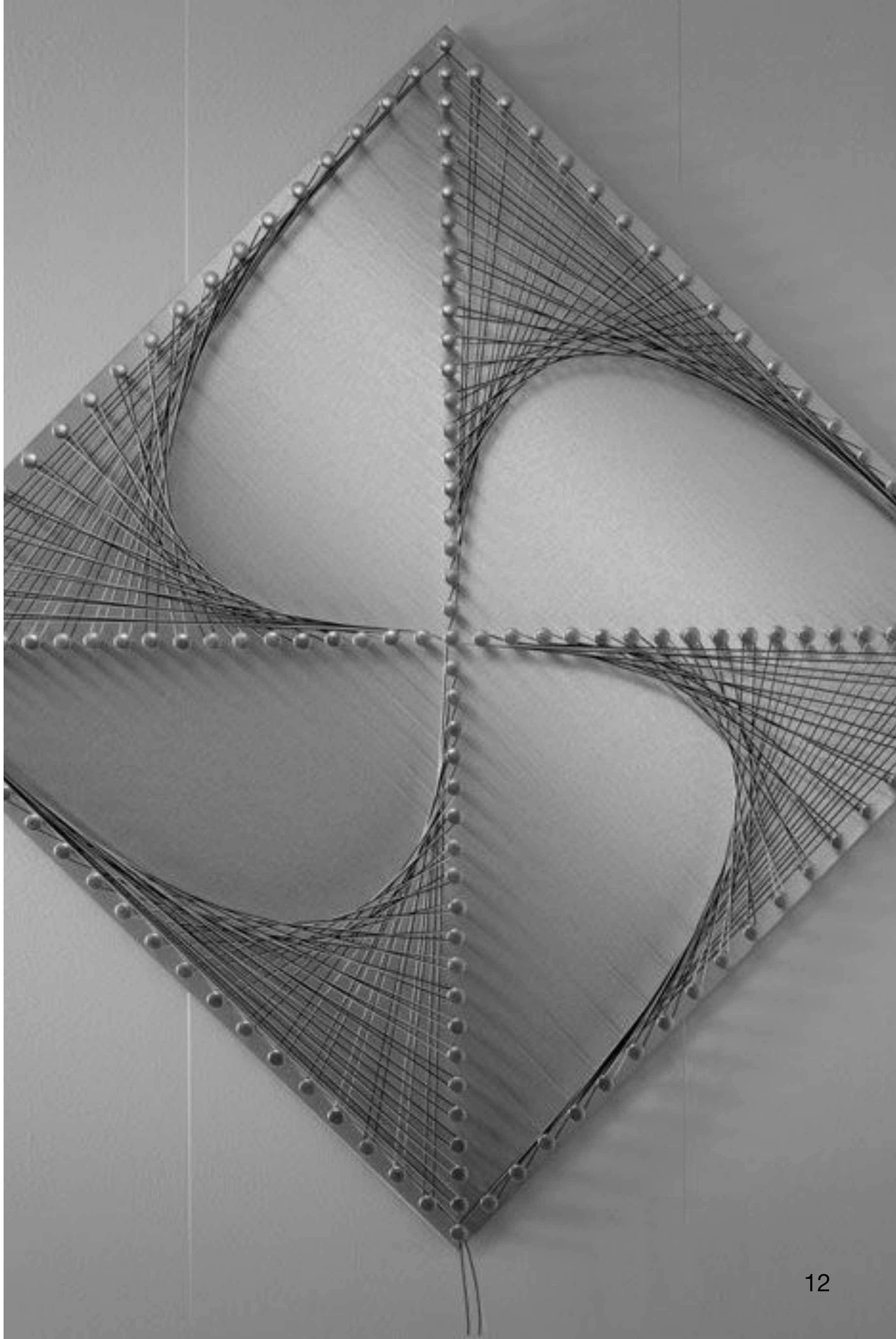


Cubic Bézier Splines

Let me show you how they work…

Drawing Curves

Or how to approximate them with straight line segments.



Two methods for evaluating your curve

$$
\begin{tikzcd}
\mathbf{p}_0 \arrow[dr] & & \mathbf{p}_1 \arrow[dl] \arrow[dr] & & \mathbf{p}_2 \arrow[dl] \arrow[dr] & & \mathbf{p}_3 \arrow[dl] \\
& \mathbf{p}_0^1 \arrow[dr, "1-u"'] & & \mathbf{p}_1^1 \arrow[dl] \arrow[dr] & & \mathbf{p}_2^1 \arrow[dl, "u"] & \\
& & \mathbf{p}_0^2 \arrow[dr] & & \mathbf{p}_1^2 \arrow[dl] & & \\
& & & \mathbf{p}(u) & & &
\end{tikzcd}
$$

de Casteljau

Bernstein

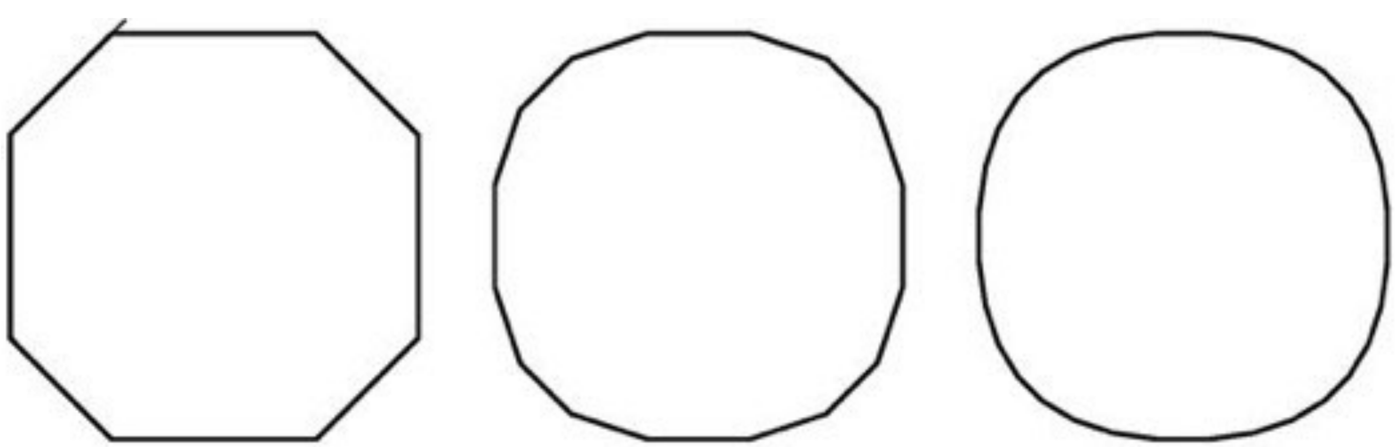
$$\mathbf{p}(u) = \sum_{i=0}^{n} b_{i,n}(u)\mathbf{p}_i$$

$$b_{i,n}(u) = \binom{n}{i} u^i (1-u)^{n-i}$$

Which one do you think is more efficient?

The key question:

How many line segments do you need?



Things to Remember

- Bézier curves and splines can provide an accurate, complete, and indisputable definition of freeform shapes
  - defined by de Casteljau construction or Bernstein polynomials
  - quadratic (3 points), cubic (4 points), or higher order
- Splines are just curve segments joined together
  - can have various degrees of parametric/geometric continuity
- Draw splines by approximating them with line segments, just like parametric curves! (same considerations apply)