Question 1: Geometry & OpenGL (10 points) 10

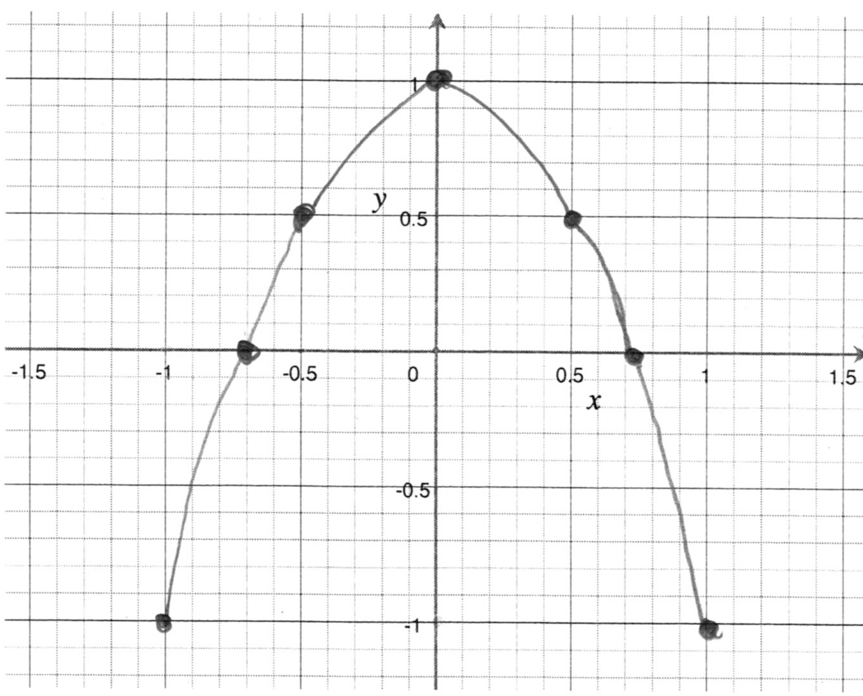
A. Consider the following parametric equation that expresses the (x, y) coordinates of points on a curve as a function of the parameter u:

$$x = \sin u \qquad\qquad u \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$
$$y = \cos 2u$$

(i) Fill in the table below by evaluating the parametric equations at the given values of u.

u	x	y
$-\frac{\pi}{2}$ $(-90°)$	-1	-1
$-\frac{\pi}{4}$ $(-45°)$	-0.707 , or $-\frac{\sqrt{2}}{2}$	0
$-\frac{\pi}{6}$ $(-30°)$	-0.5	0.5
0	0	1

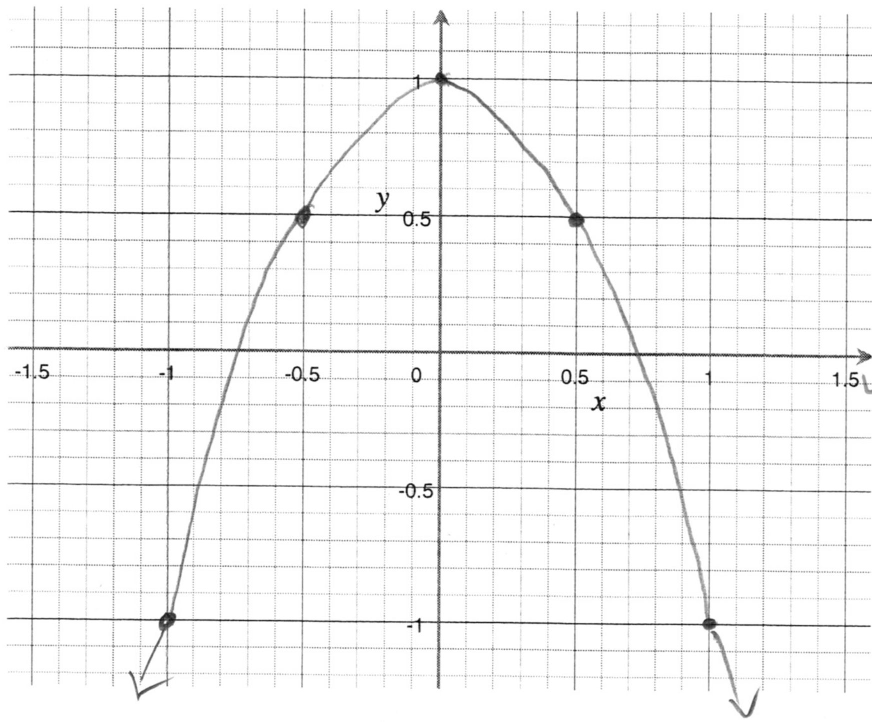
(ii) Draw the curve over its full parametric range, $u \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, on the grid below. Be as precise as you can so that we may see points that the curve passes through.



B. Now consider the curve formed by the following implicit equation:

$$2x^2 + y - 1 = 0 \quad \text{(or in explicit form, } y = -2x^2 + 1\text{)}$$

Draw the part of this curve that fits within the grid below.



C. Is the curve in 1A identical to the curve in 1B within the domain of the unit square (*i.e.* -1 < x < 1 and -1 < y < 1)? Provide justification for your answer.

Yes. at points where $x = -1, -0.5, 0, 0.5$ and 1, points match with other curve. x intercepts also match. This provides evidence that curves are same.

we can substitute $x = \sin(u)$ and $y = \cos(2u)$ to get

$2(\sin(u))^2 + \cos(2u) - 1 = 0$

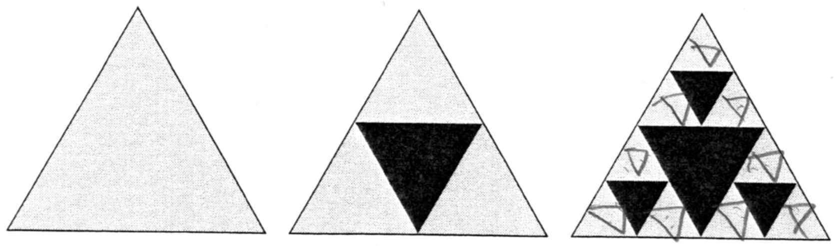
$2\sin^2(u) + \cos(2u) - 1 = 0$

$2\sin^2(u) + (1 - 2\sin^2(u)) - 1 = 0$

$0 = 0$

Excellent!

Recall the Sierpinski triangle in your first CPSC 453 assignment. It is also possible to draw the *inverse* of the Sierpinski triangle, shown below. The original equilateral triangle and its iterative divisions are shown in grey, and the inverse triangles, which fill the spaces in between, are shown in black. As you can see, no triangles are drawn for the inverse Sierpinski triangle at the base level, one triangle is drawn after 1 iteration, and four triangles are drawn after 2 iterations.



Base Triangle | 1 Iteration | 2 Iterations | 3 iterate

0 | 1 | 4 | 13

D. Imagine you are drawing the inverse Sierpinski triangle with `GL_TRIANGLES` primitives on a graphics system that can accommodate a maximum vertex buffer size of only 1000 *vertices*. What is the highest fractal iteration of the inverse Sierpinski that you can draw with a single call to `glDrawArrays`?

relation to current appears to be 3 times previous iteration + 1.

For each inverse triangle, we need 3 vertices.
So dividing 1000 by 3 gives # of triangles that vertex buffer can accomodate. 1000/3 = 333.3 ...

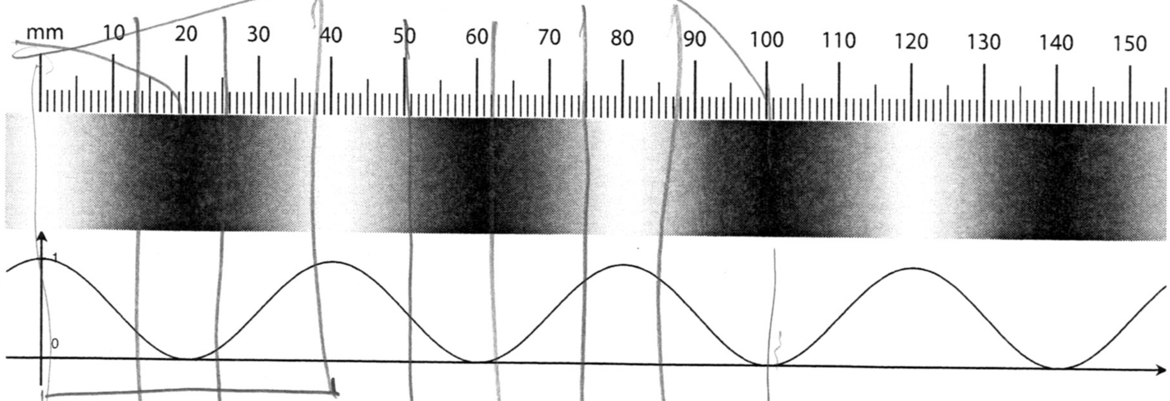
we make a table

Iteration	1	2	3	4	5	6	7	8	9	10
vertices	1	4	13	40	121	364	-	-	-	-

Thus, highest iteration is **5th** iteration.

Question 2: Digital Images & Effects (10 points) 10

A. Examine the horizontal, one-dimensional pattern depicted below. A ruler is drawn above the pattern and a graph of the pattern's intensity variation is drawn below it for your reference. You may assume that this pattern is periodic.



(i) What is the highest spatial frequency present in the pattern? Be sure to specify the units of measure.

wavelength = 40 mm

frequency = $1/\lambda$ = $1/40$ mm = 0.025 1/mm

Highest frequency is $1/40$ 1/mm

(ii) If you wanted to encode the above pattern as a digital image accurately (*i.e.* such that it can in principle be reproduced exactly, or very nearly so, and without aliasing), what is the minimum frequency you would need to sample it at?

According to Nyquist Frequency, we need to sample at twice highest frequency = 2 × 0.025 = 0.05 1/mm

Minimum frequency needed to sample is $1/20$ 1/mm.

B. Create a monochromatic, one-dimensional digital image of the first half of the pattern from 2A by sampling at an interval (period) of 1 cm. Take your first (leftmost) sample at 0 mm. Write your pixel values in the boxes below.

100/7 =

1	0.5	0	0.5	1	0.5	0	0.5

C. Convolve your 1D image from 2B with the discrete, three-point kernel shown on the right.

1	0	-1

1D Kernel

(i) Write the result of your convolution in the boxes below. You may handle the edges in any reasonable manner you'd like.

Treat as 0

Flip kernel to -1 0 1

0.5	-1	0	1	0	-1	0	0

(ii) Briefly describe the effect this convolution has on your image.

This kernel has the effect of enhancing wave and making values 1 or -1 depending on whether the slope is ±1. when slope decreasing, it gives -1, when increasing gives 1.

D. Let's do this one more time!

(i) Write a 1D convolution kernel (discrete function) that, when applied to an image, has the same effect as convolving *twice* with the kernel from 2C above.

convolve with itself

0 1 0 -1 0 Image.

-1 0 1 kernel

Treat edge as 0

...	0	1	0	-2	0	1	0	...
		-2	-1	i=0	+1	+2		

(ii) Write the result of convolving your image from 2B with the kernel from 2C twice, in the boxes below. Again, you may handle the edges in any reasonable manner.

Apply -1 0 1 again

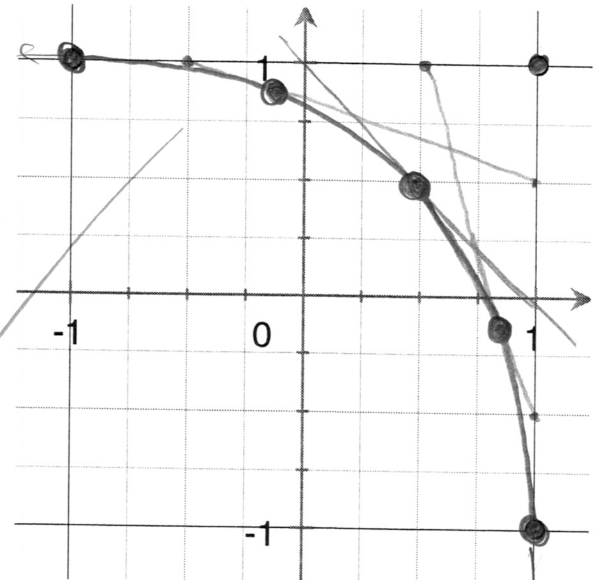
-1	-0.5	2	0	2	0	1	0

(you're missing a -, but ok)

8

Question 3: Bézier Curves & Splines (10 points)

A. Draw the *quadratic* Bézier curve defined by the sequence of three control points, (-1, 1), (1, 1), and (1, -1), shown as circles, on the grid to the right. Be as precise as possible so that we can clearly see any key points the curve may pass through.

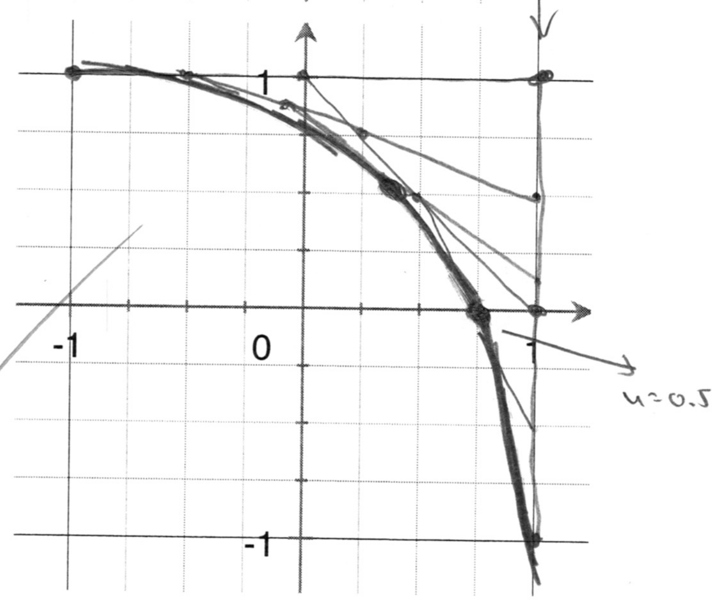


B. Draw the *cubic* Bézier curve defined by the sequence of four control points, (-1, 1), (1, 1), (1, -1) and (1, -1). Again, try to be precise and at minimum show where the (parametric) midpoint of the curve lies.

Apply formulas

Evaluate at u=0.5

.125(-1

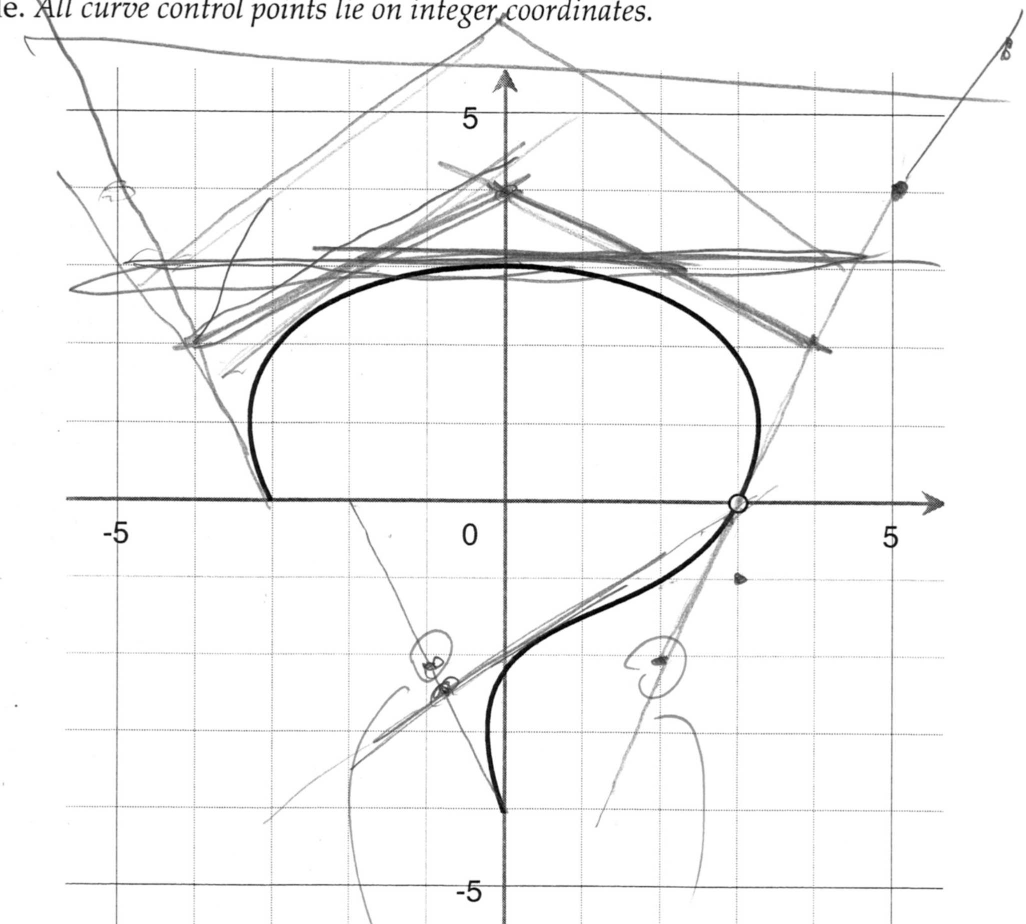


C. Is the Bézier curve in 3B identical to the curve in 3A? Briefly justify your answer.

It is similar, but it is not identical.

we can see visually that at u=0.5, different locations.

Examine the cubic Bézier spline formed from two curve segments shown on the grid below. This spline is G^1 continuous at the single knot that connects the two segments, shown by the open circle. *All curve control points lie on integer coordinates.*



D. Write the (x, y) coordinate pairs for the control points of the two curve segments that form this spline. Justify your answer visually, mathematically, with a brief written explanation, or any combination thereof.

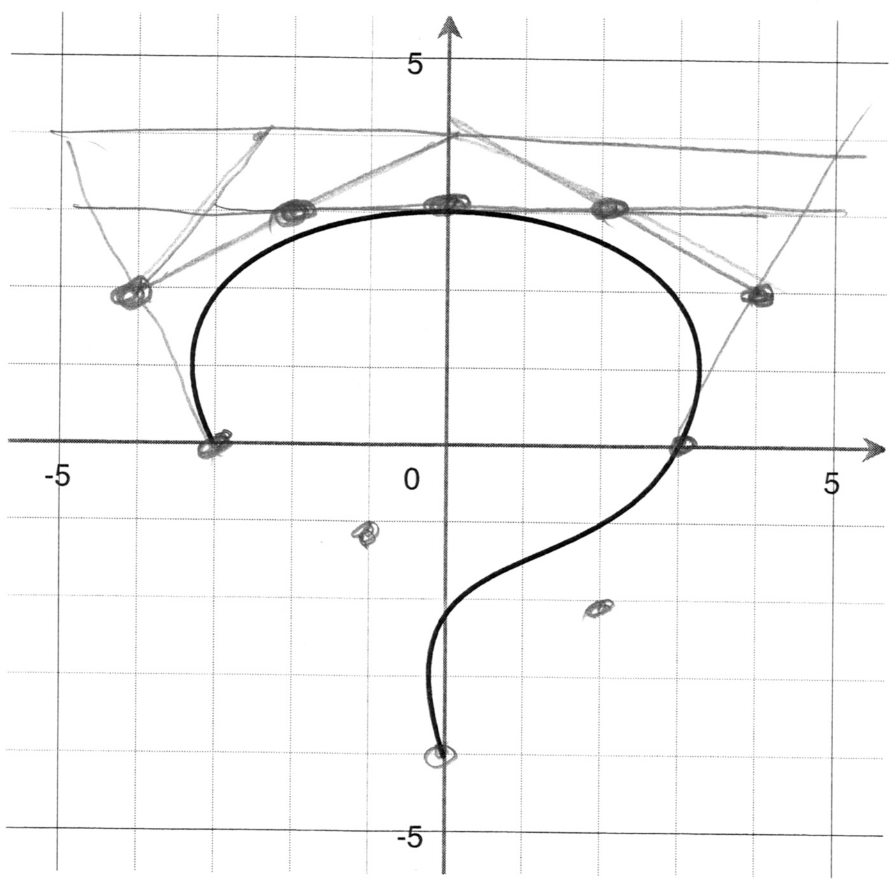
(0, −4), (3, 0)

(3, 0), (5, 4), (−5, 4), (−3, 0)

E. The same curve can be written as a C^1 continuous spline by using *three* cubic Bézier segments instead of two. Write the (x, y) coordinate pairs for the control points of three curve segments that form an identical, but C^1 continuous spline. The curve is shown again below for you to use as scratch space to draw on if you'd like.

2

(-3,0) (



Bonus Question (4 points)

A. Evaluate, simplify, or otherwise communicate the result of the following expression:

$$f(x) = \int_{-\infty}^{\infty} \frac{e^{-2\pi ixt} \sin \pi t}{\pi t}\, dt$$

$e^{\pi i}(e^{-2xt})$

Fourier of sinc functn

B. What category of typeface was used to typeset the main body text of this exam?

oldstyle. 2

C. What category of typeface was used to typeset the question headings of this exam?

x2

Sans serif

Total: 10+10+8+2 = 30/30

Well done!