Bézier Curves

CPSC 453 – Fall 2016
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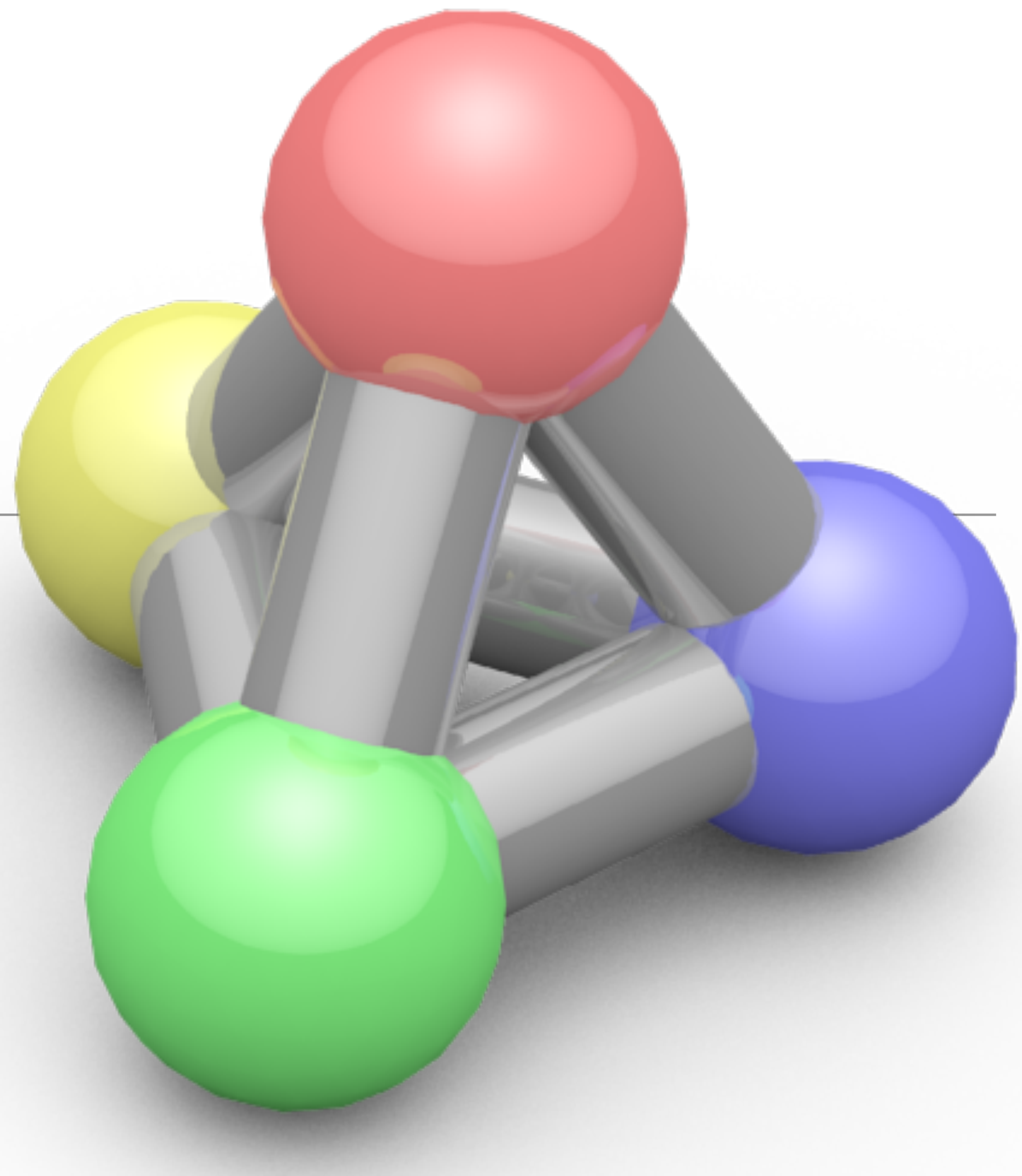


Today's Outline

- Motivation
- Quadratic Bézier curves
  - de Casteljau formulation
  - Bernstein polynomial form

How might we represent a

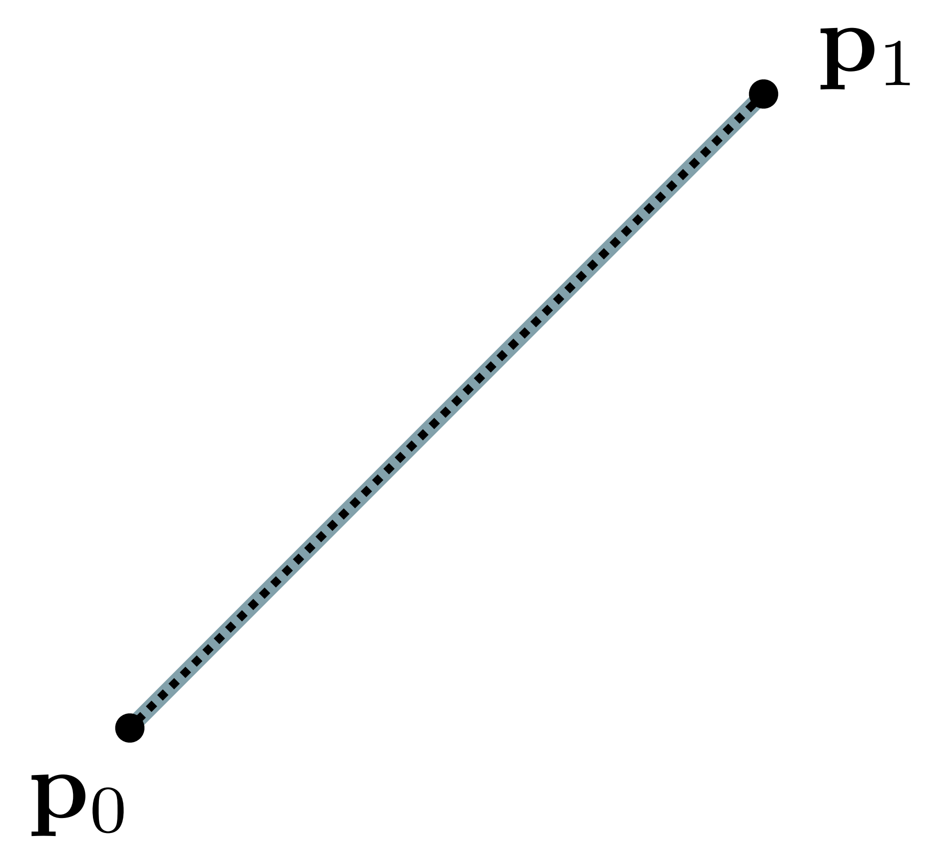
freeform shape?

The Goal:

Create a system that provides an accurate, complete, and indisputable definition of freeform shapes.

Parametric Segment by Linear Interpolation

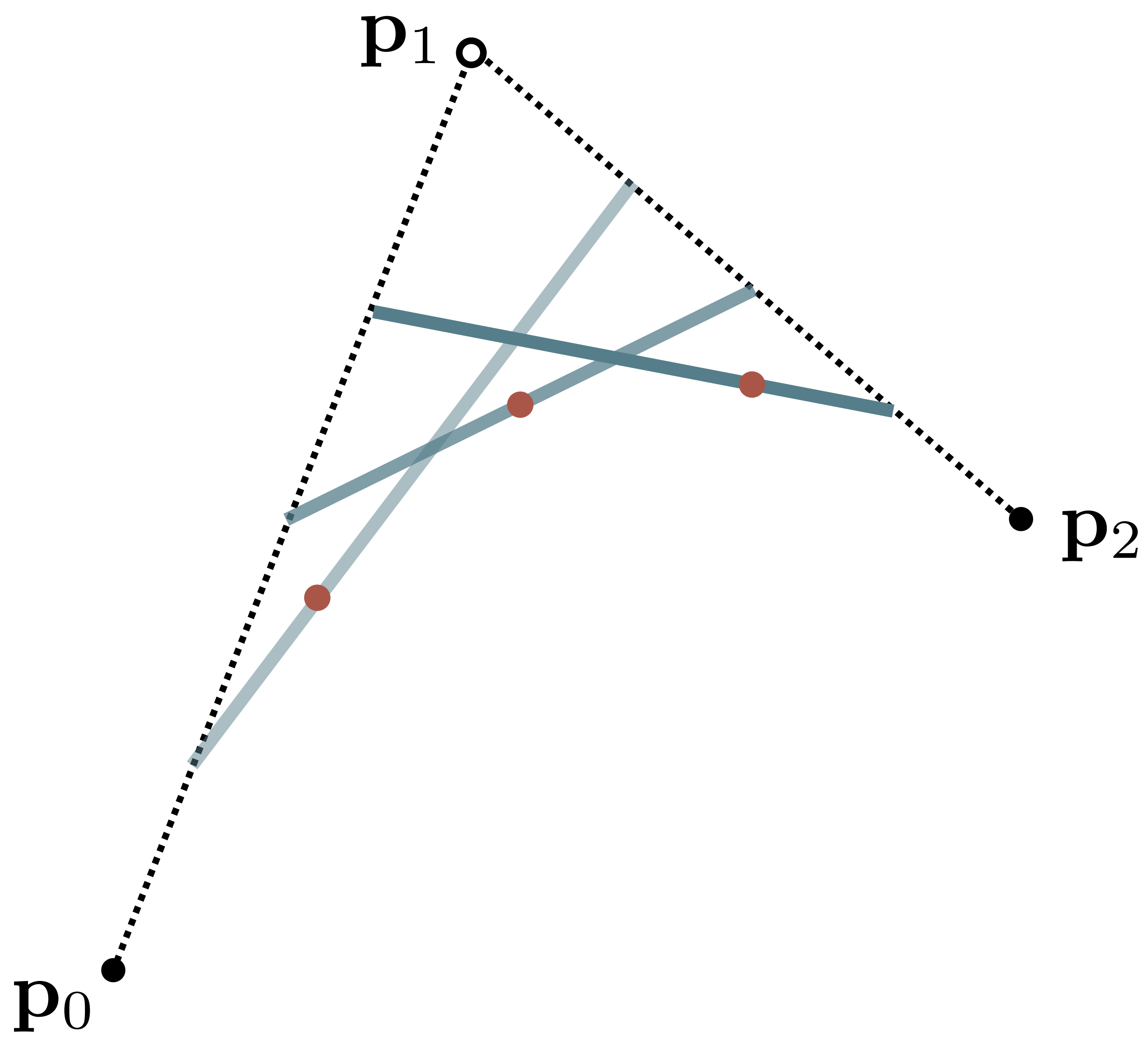
$$\begin{aligned}\mathbf{p}(u) &= \mathrm{lerp}(\mathbf{p}_0, \mathbf{p}_1, u) \\ &= (1-u)\mathbf{p}_0 + u\mathbf{p}_1\end{aligned}$$



de Casteljau's Algorithm (quadratic)

$$\mathbf{p}_0^1 = \mathrm{lerp}(\mathbf{p}_0, \mathbf{p}_1, u)$$
$$\mathbf{p}_1^1 = \mathrm{lerp}(\mathbf{p}_1, \mathbf{p}_2, u)$$

$$\mathbf{p}(u) = \mathrm{lerp}(\mathbf{p}_0^1, \mathbf{p}_1^1, u)$$

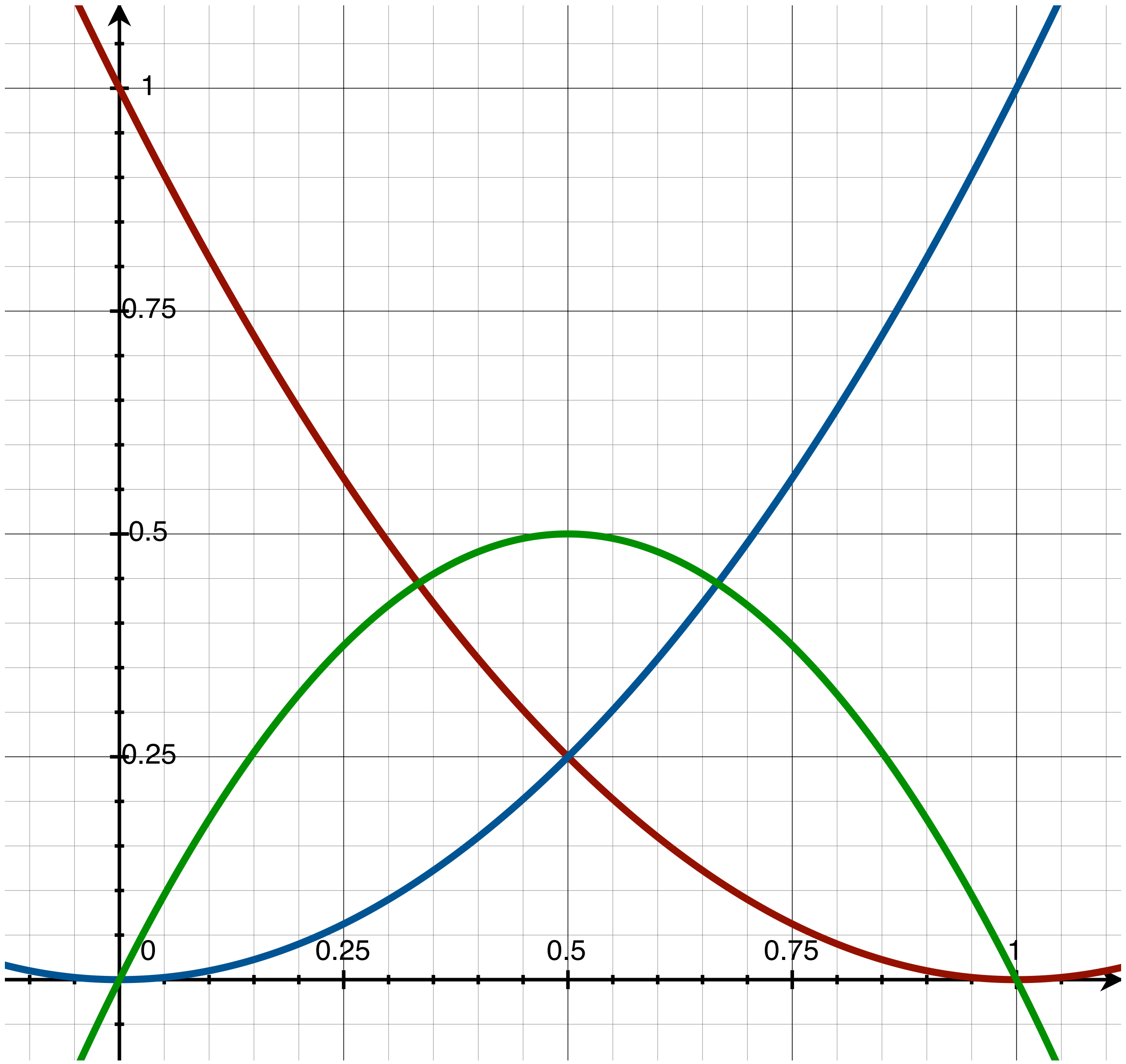


Bernstein Polynomials

$$b_{0,2}(u) = (1-u)^2$$

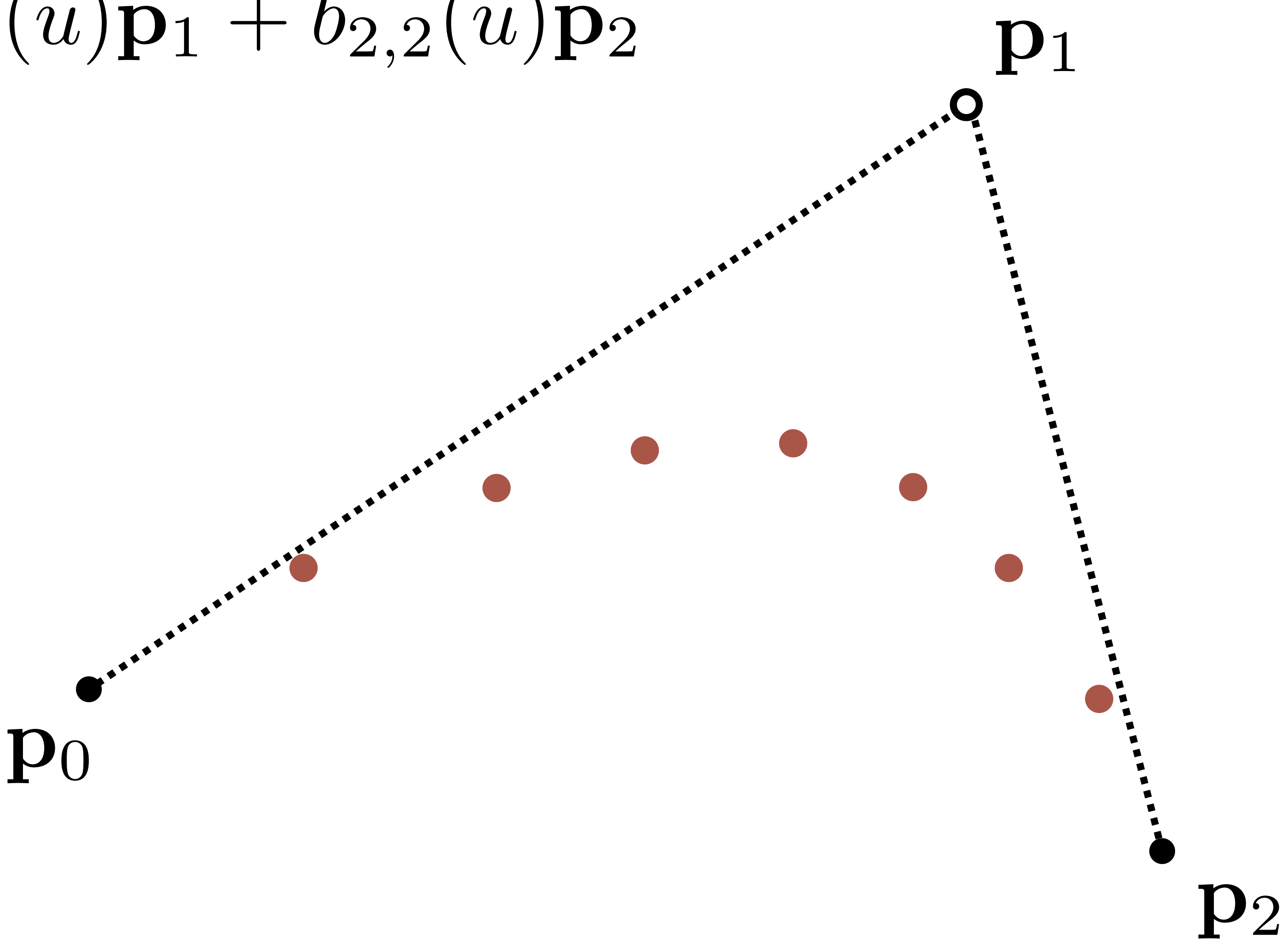
$$b_{1,2}(u) = 2u(1-u)$$

$$b_{2,2}(u) = u^2$$



Bernstein Form of a Quadratic Bézier

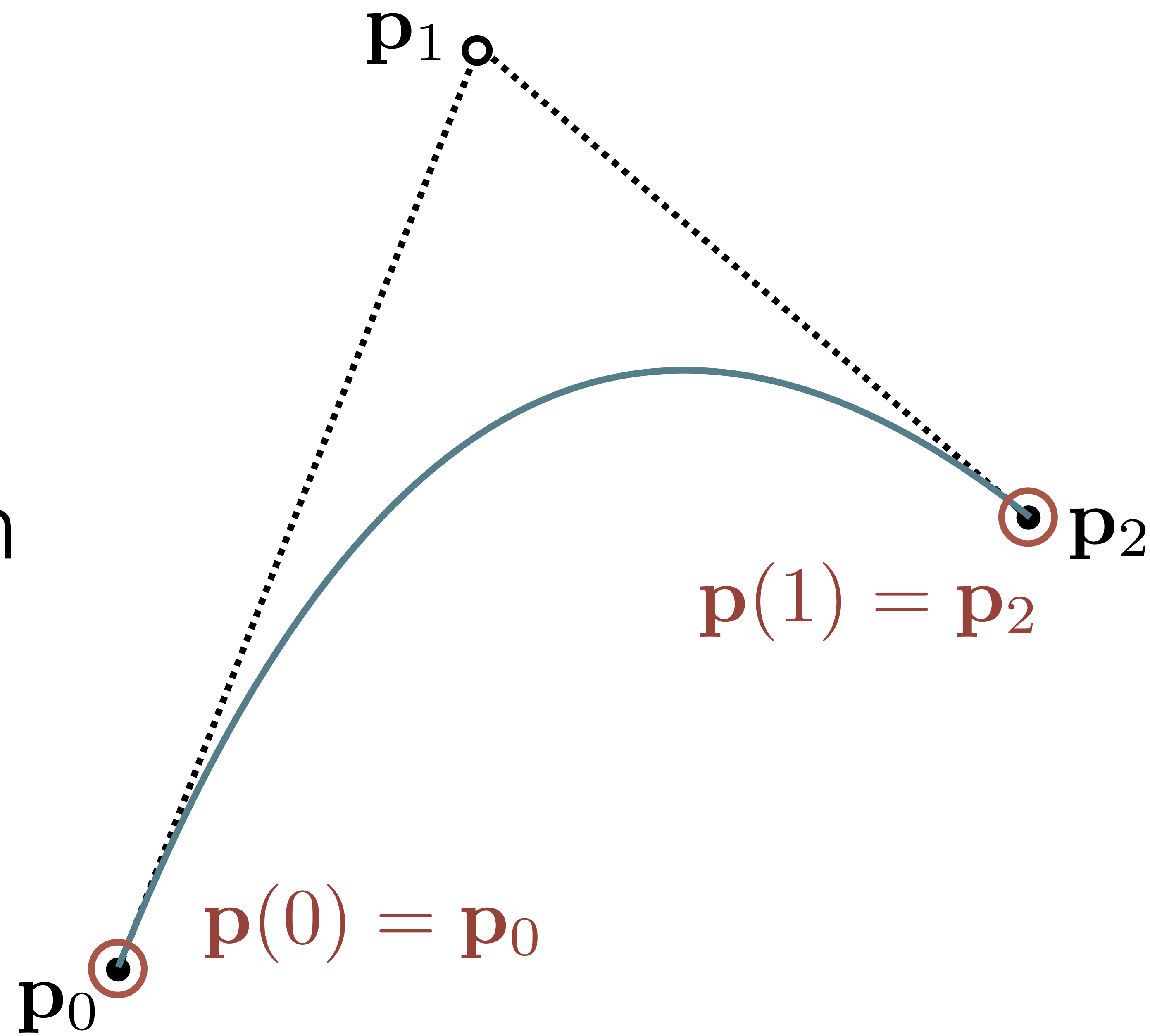
$$\begin{aligned}\mathbf{p}(u) &= (1-u)^2\mathbf{p}_0 + 2u(1-u)\mathbf{p}_1 + u^2\mathbf{p}_2 \\ &= b_{0,2}(u)\mathbf{p}_0 + b_{1,2}(u)\mathbf{p}_1 + b_{2,2}(u)\mathbf{p}_2 \\ &= \sum_{i=0}^{2} b_{i,2}(u)\mathbf{p}_i\end{aligned}$$



Property #1:

endpoint interpolation

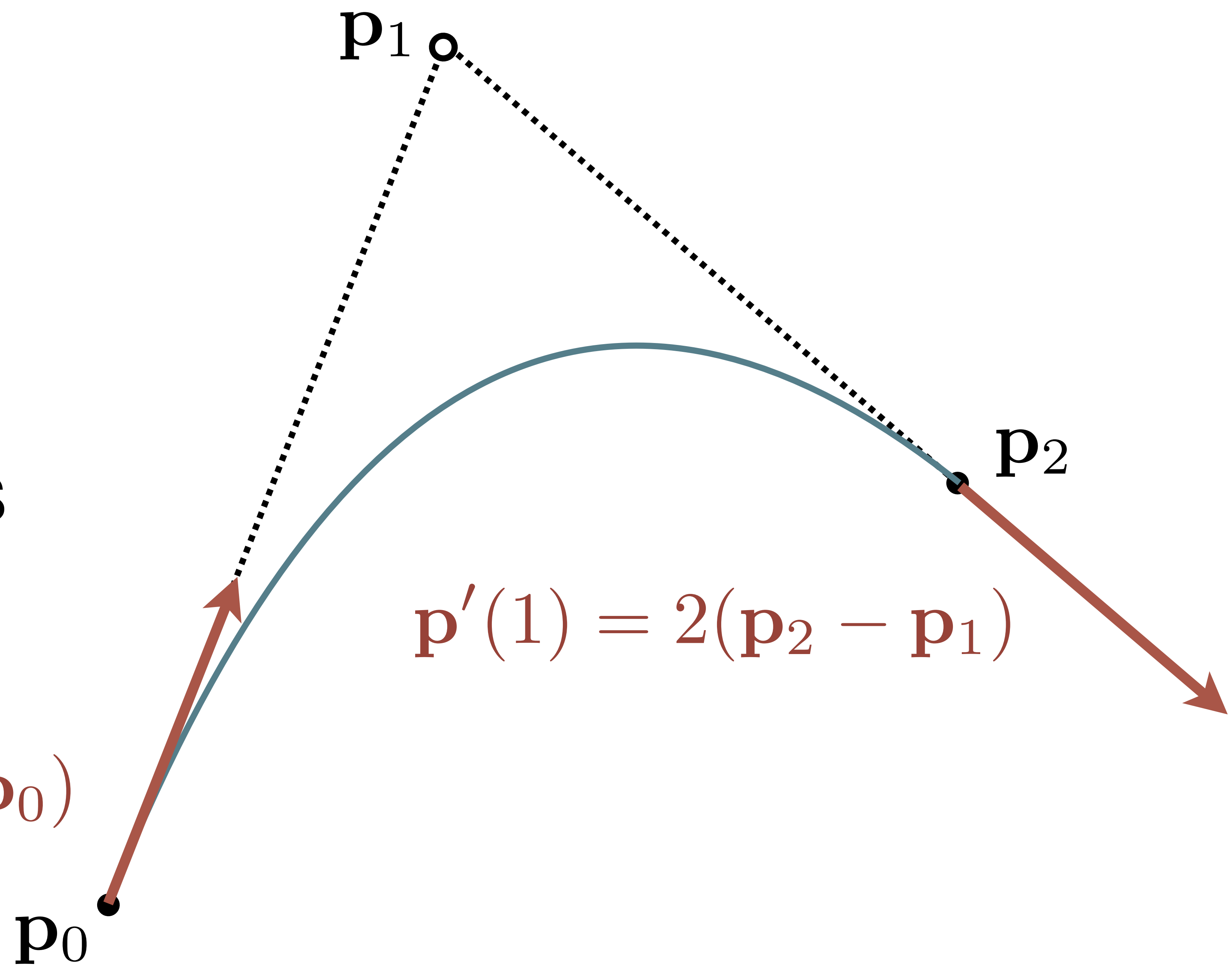
Endpoint Interpolation



Property #2:

endpoint tangents

Endpoint Tangents

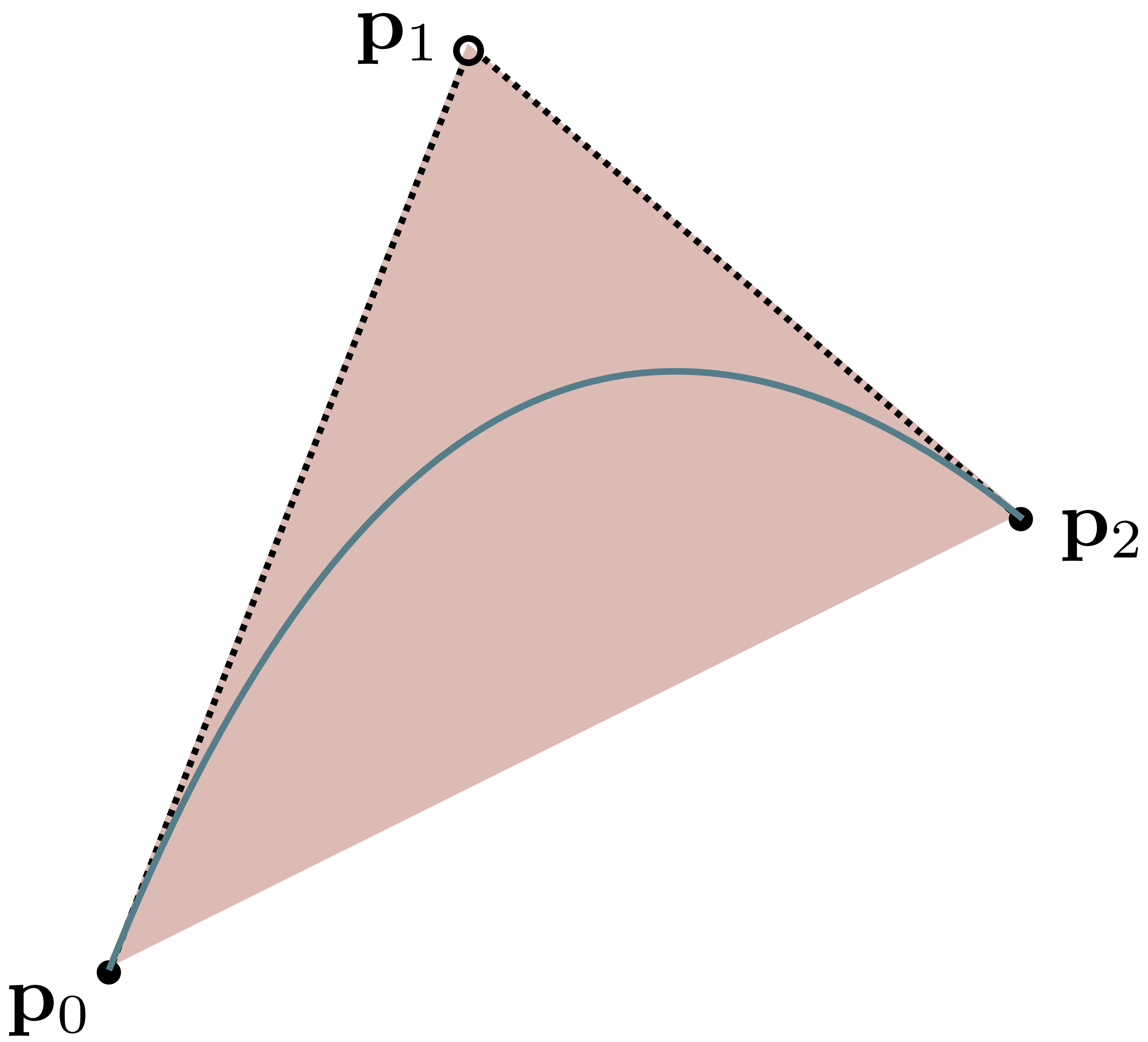


$$\mathbf{p}'(0) = 2(\mathbf{p}_1 - \mathbf{p}_0)$$

Property #3:

convex hull

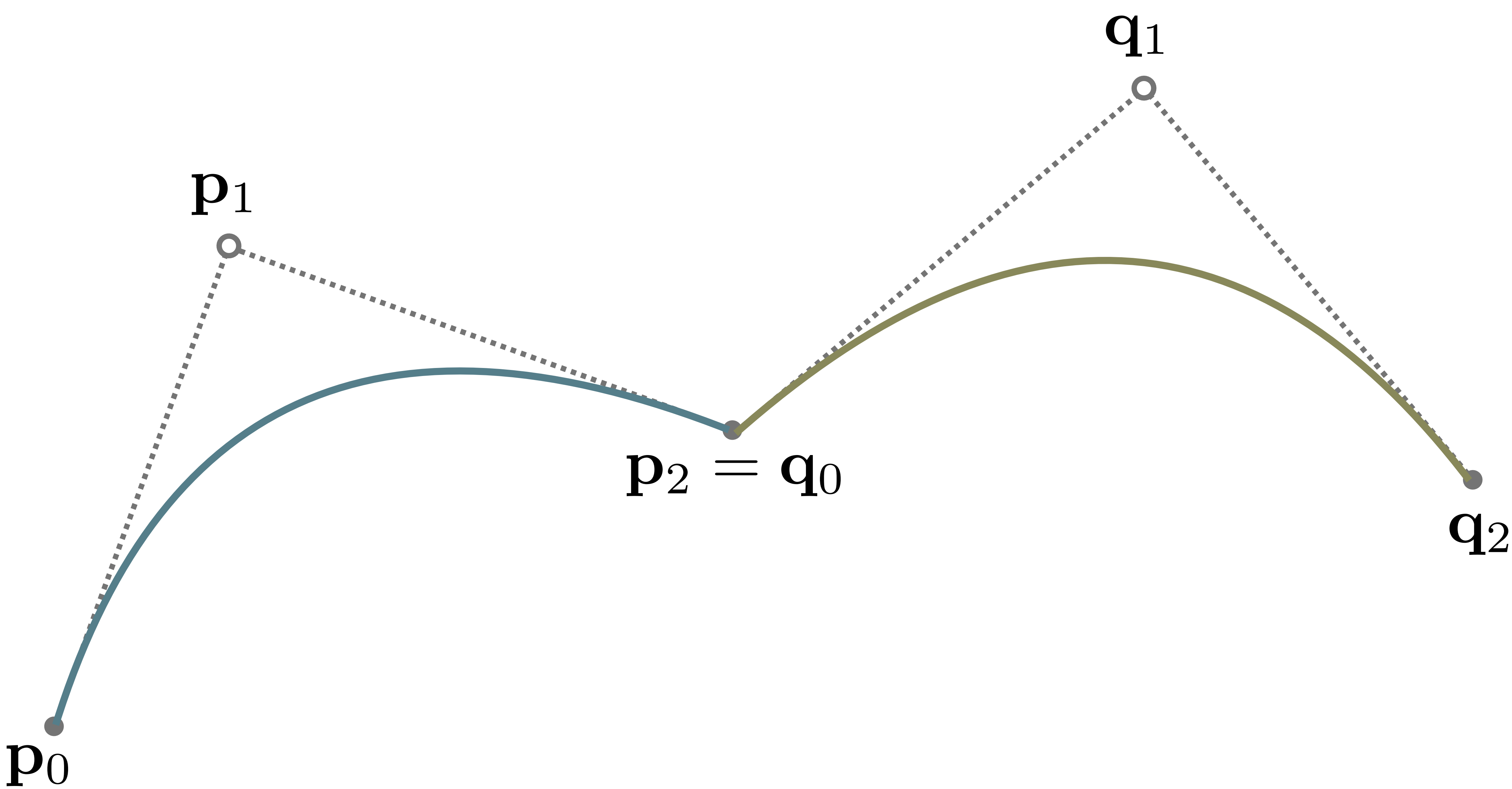
Convex Hull



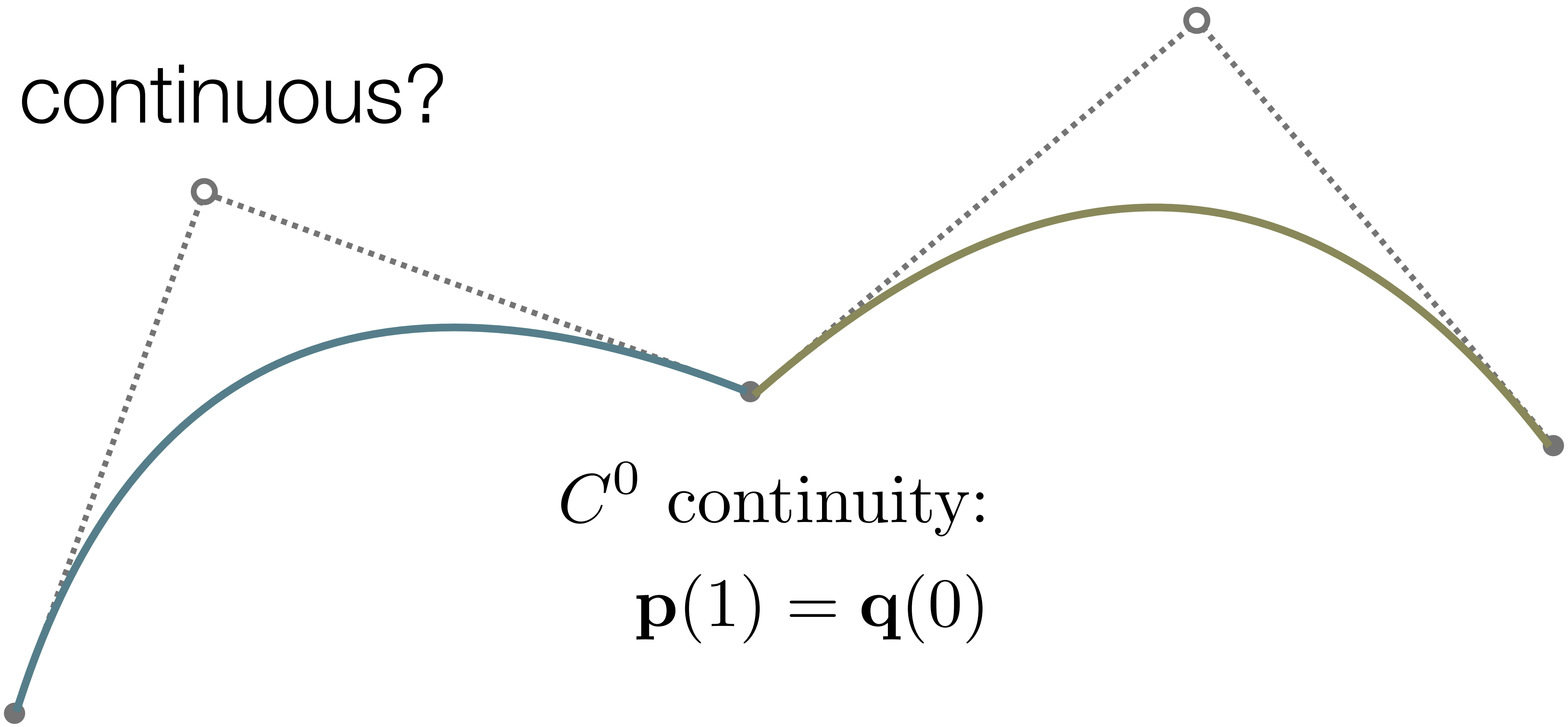
Bézier Splines

of the 2nd degree

Splines are just curve segments joined together:



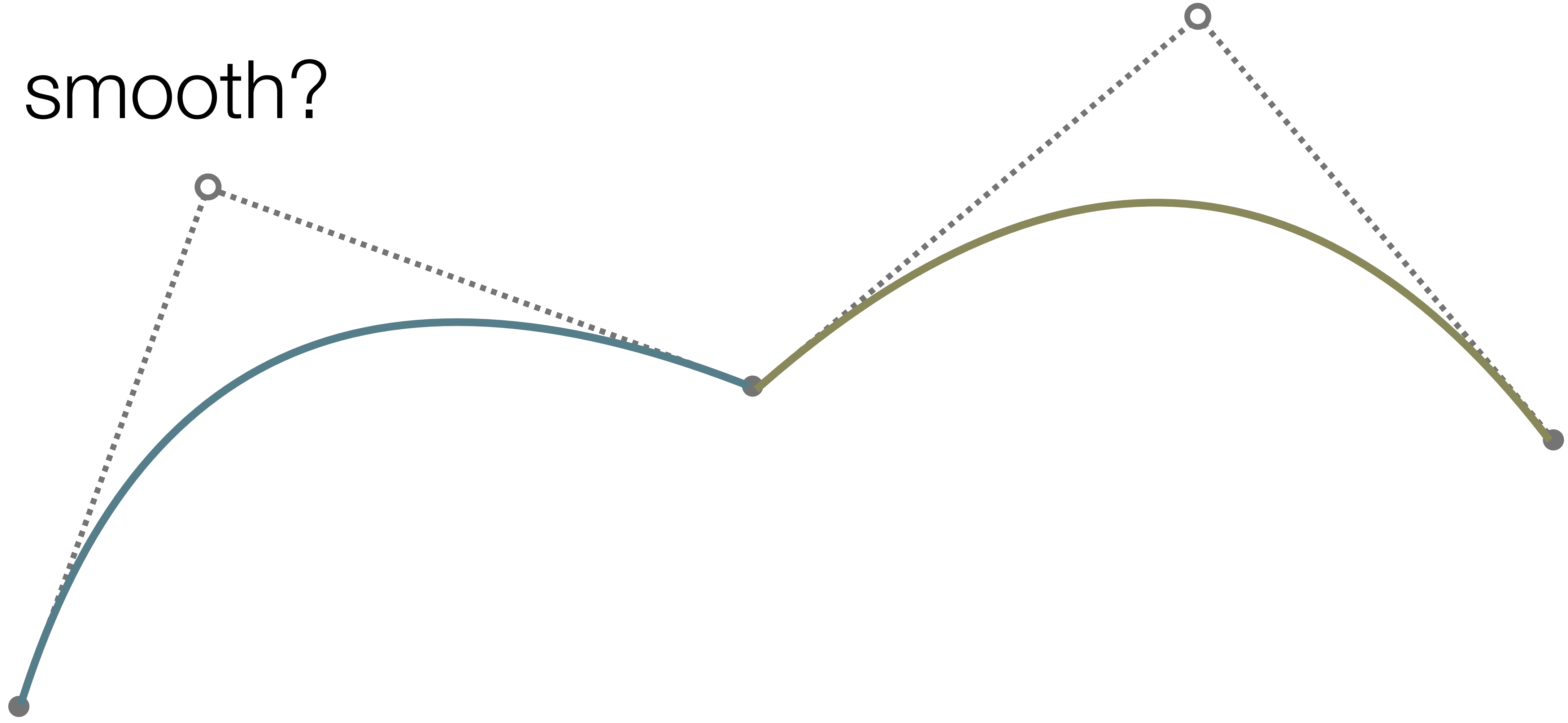
Is it continuous?



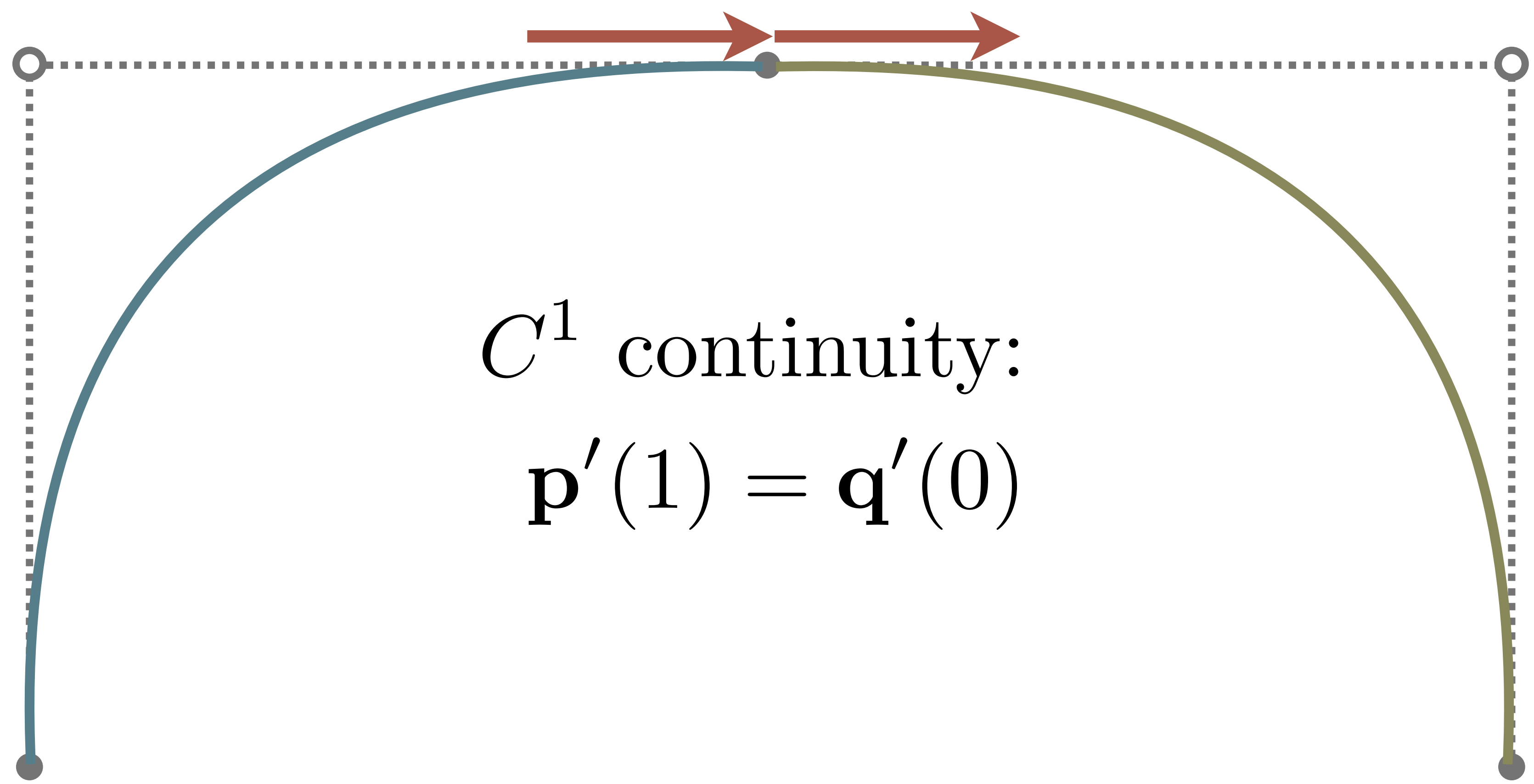
C^0 continuity:

$$\mathbf{p}(1) = \mathbf{q}(0)$$

Is it smooth?



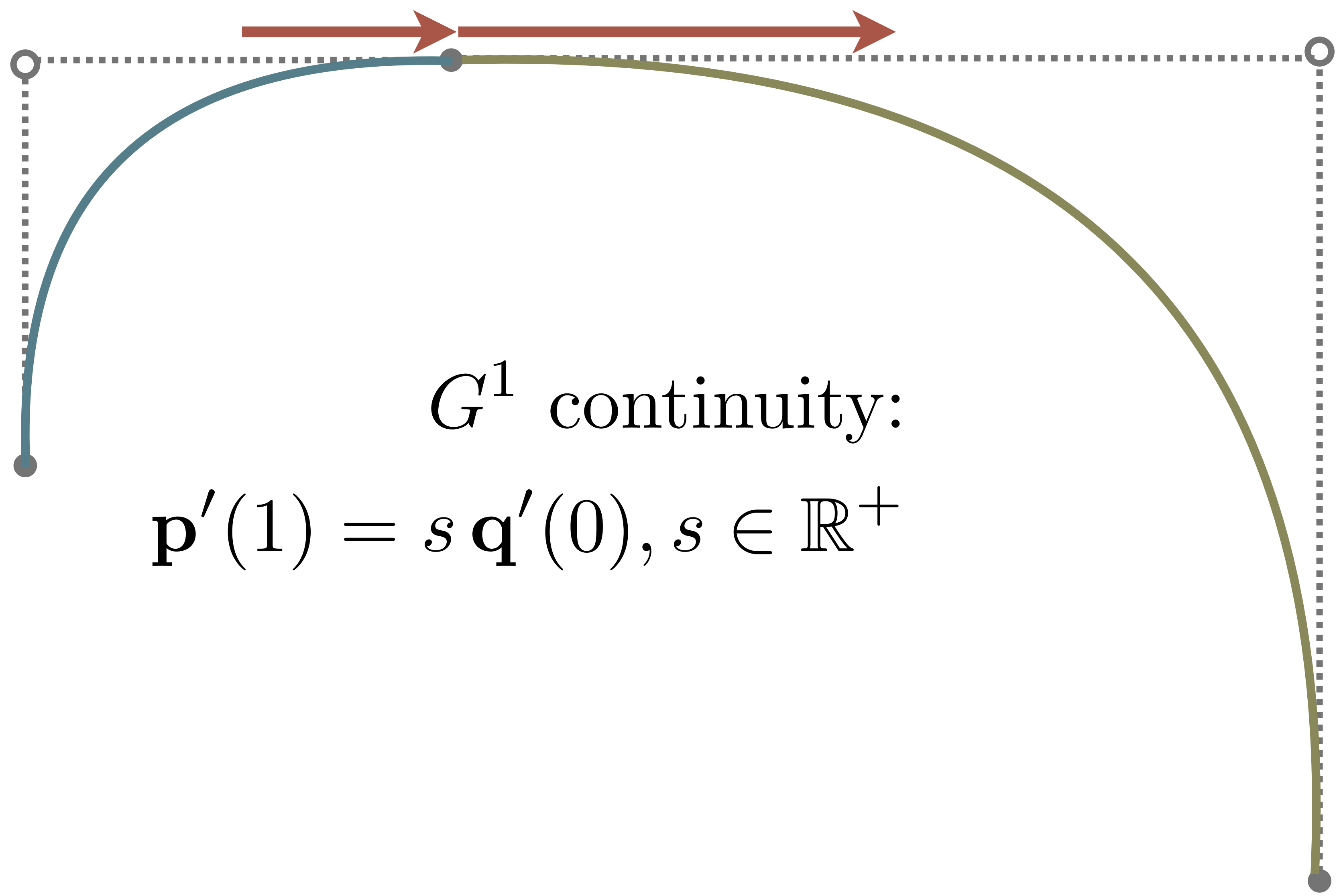
Is it smooth?



C^1 continuity:

$$\mathbf{p}'(1) = \mathbf{q}'(0)$$

Is it smooth?



G^1 continuity:

$$\mathbf{p}'(1) = s\,\mathbf{q}'(0), s \in \mathbb{R}^+$$

To be continued…