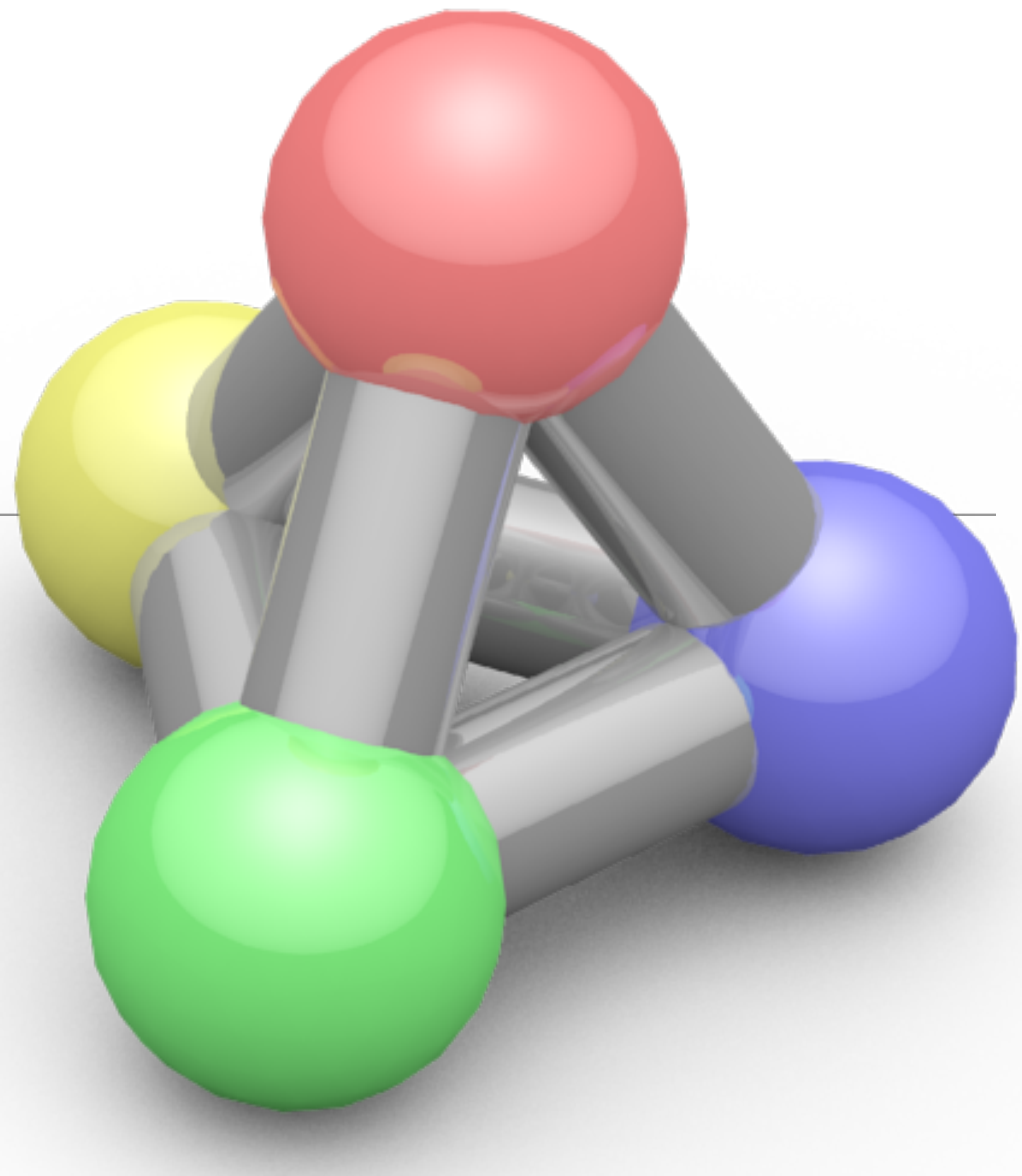
Digital Image Effects

CPSC 453 – Fall 2016

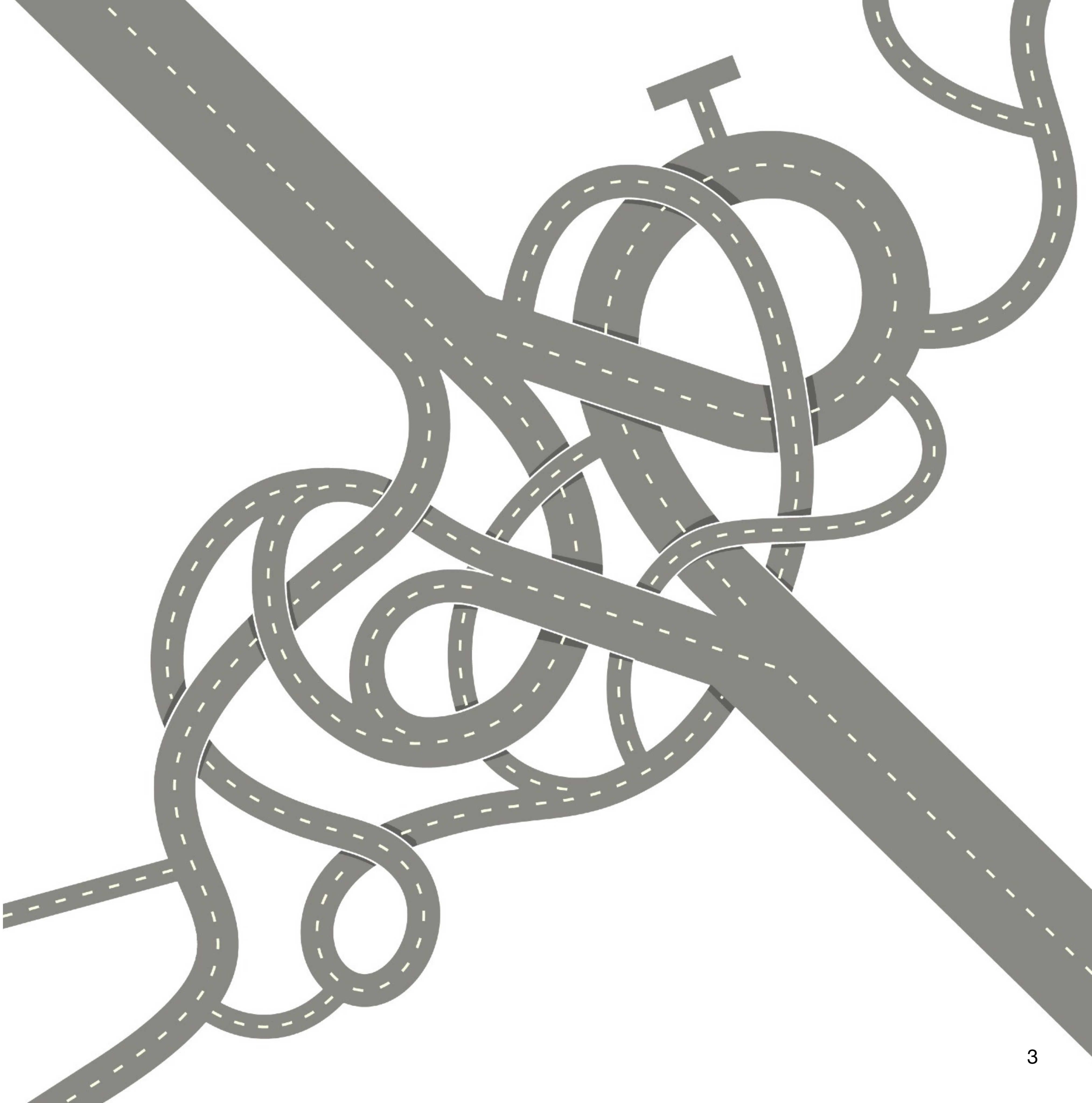
Sonny Chan



Today's Outline

- Convolution
  - discrete and continuous
  - one- and two-dimensional
- Convolution Filters on Images
  - edge detection
  - smoothing
- Colour Conversion

Convolution



Convolution is an operation on two functions.

$$f(x) \star g(x) \Rightarrow (f \star g)(x)$$

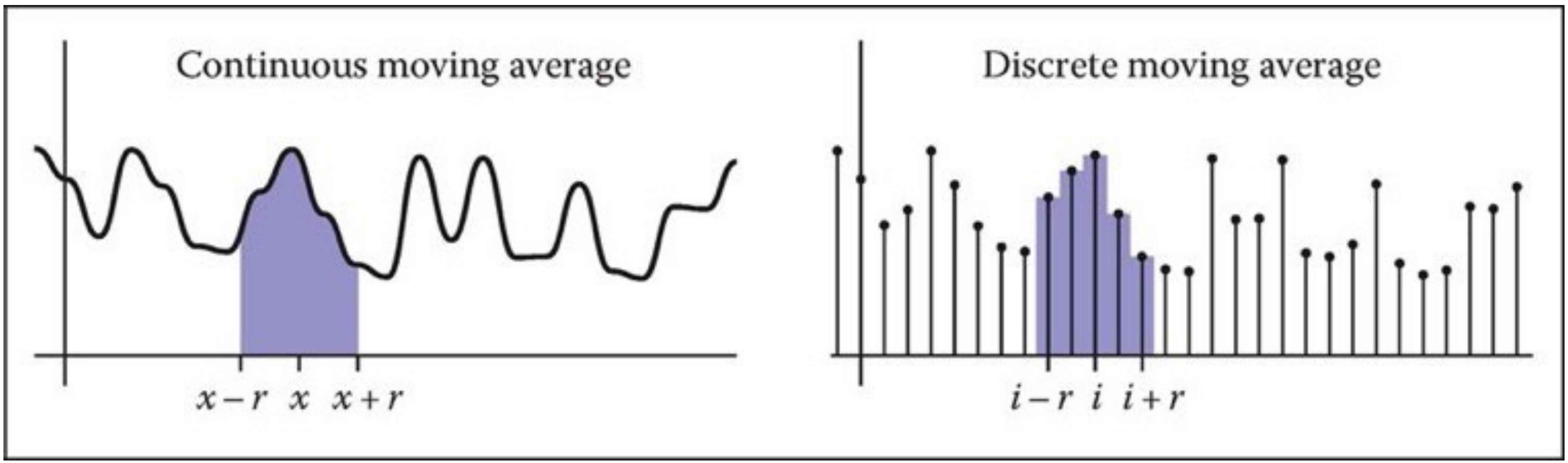
Example: Moving Average

continuous function

$$f(x),\ x \in \mathbb{R}$$

discrete function

$$a[i],\ i \in \mathbb{Z}$$



$$h(x) = \frac{1}{2r} \int_{x-r}^{x+r} f(t)dt$$

$$c[i] = \frac{1}{2r+1} \sum_{j=i-r}^{i+r} a[j]$$

Convolution Defined

- Moving averages:

$$h(x) = \frac{1}{2r} \int_{x-r}^{x+r} f(t)dt \qquad c[i] = \frac{1}{2r+1} \sum_{j=i-r}^{i+r} a[j]$$

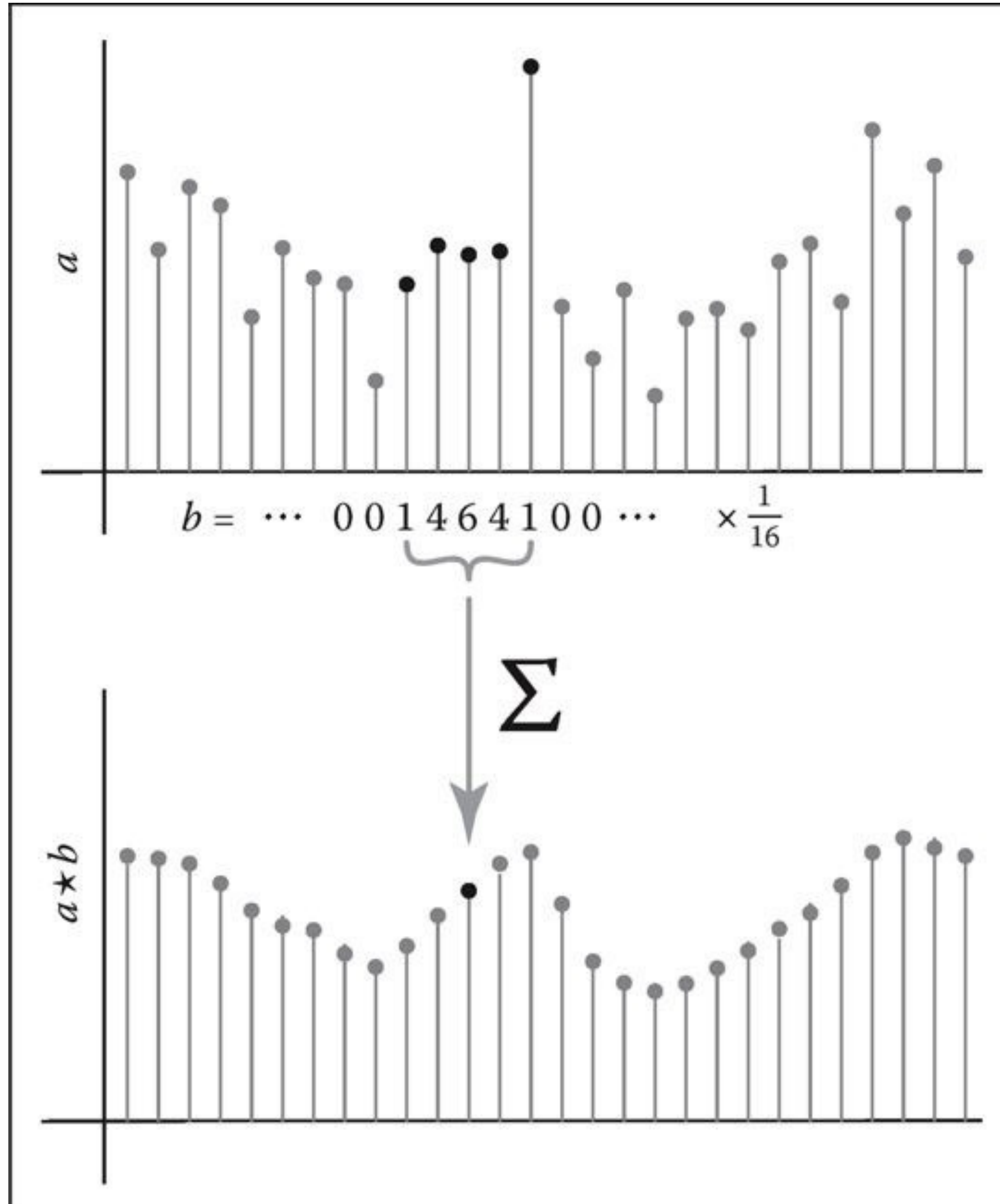
- Convolution is a weighted moving average:

$$(f \star g)(x) = \int_{-\infty}^{\infty} f(t)g(x-t)dt \qquad (a \star b)[i] = \sum_j a[j]b[i-j]$$

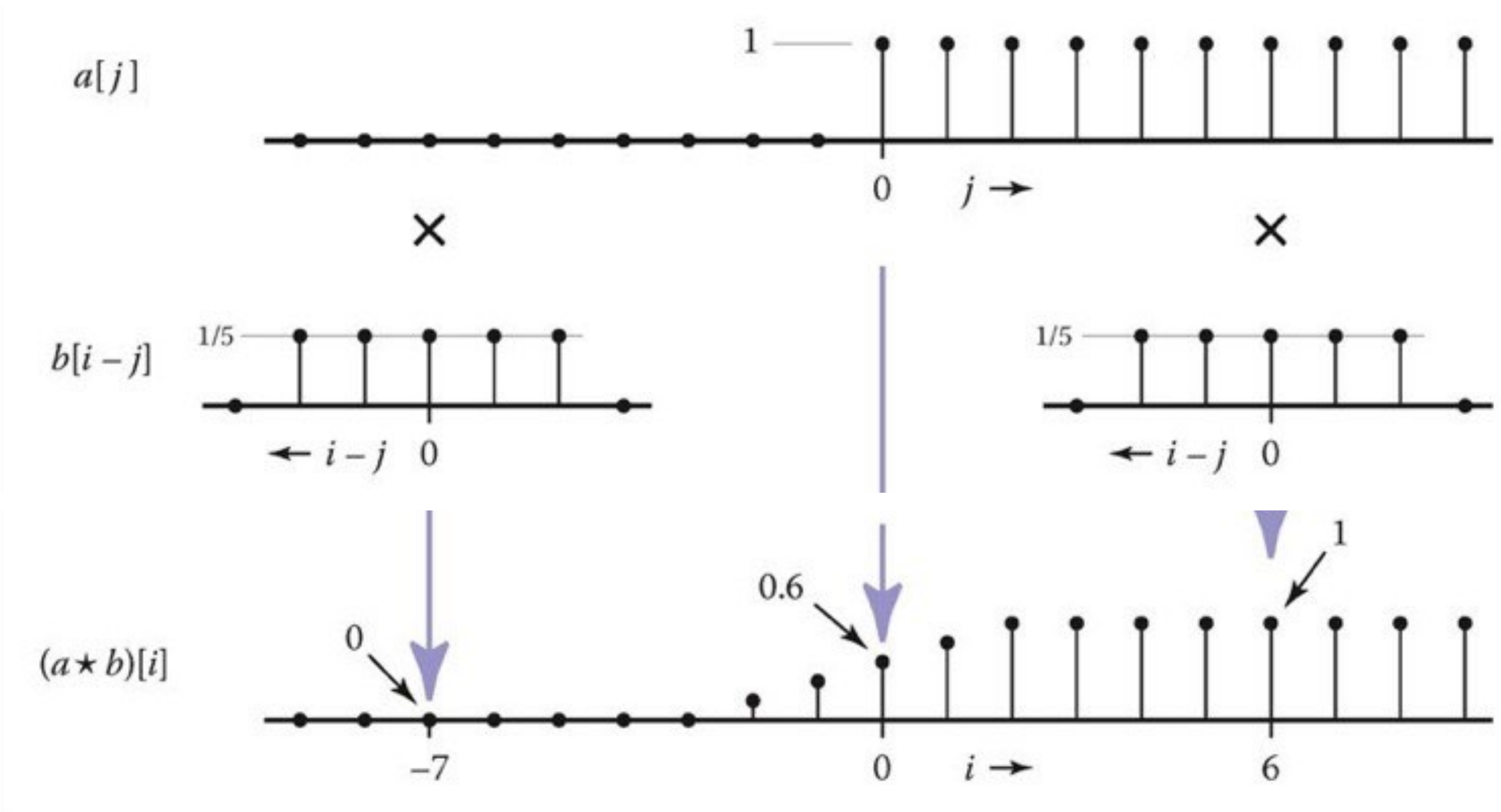
Finite Support

- Most convolution *kernels* we use in the real world have *finite support*
- Second function is zero outside of a certain radius

$$(a \star b)[i] = \sum_{j=i-r}^{i+r} a[j]b[i-j]$$



Example: Discrete Convolution with Box Filter



Is convolution

commutative?

$$f \star g = g \star f$$

Is convolution

associative?

$$(f \star g) \star h = f \star (g \star h)$$

Is convolution

distributive?

$$(f + g) \star h =$$
$$(f \star h) + (g \star h)$$

What is convolution's

identity?

$$(f \star \delta)(x) = f(x)$$

Does convolution have an

inverse?

$$(f \star f^{-1})(x) = \delta(x)$$

Summary of Convolution Properties

- Commutative $$f \star g = g \star f$$
- Associative $$(f \star g) \star h = f \star (g \star h)$$
- Distributive (over addition) $$(f + g) \star h = (f \star h) + (g \star h)$$
- Identity $$(f \star \delta)(x) = f(x)$$

Discrete-Continuous Convolution

- If we have one function of each type:

$$a[i] : \mathbb{Z} \mapsto \mathbb{R}$$
$$f(x) : \mathbb{R} \mapsto \mathbb{R}$$

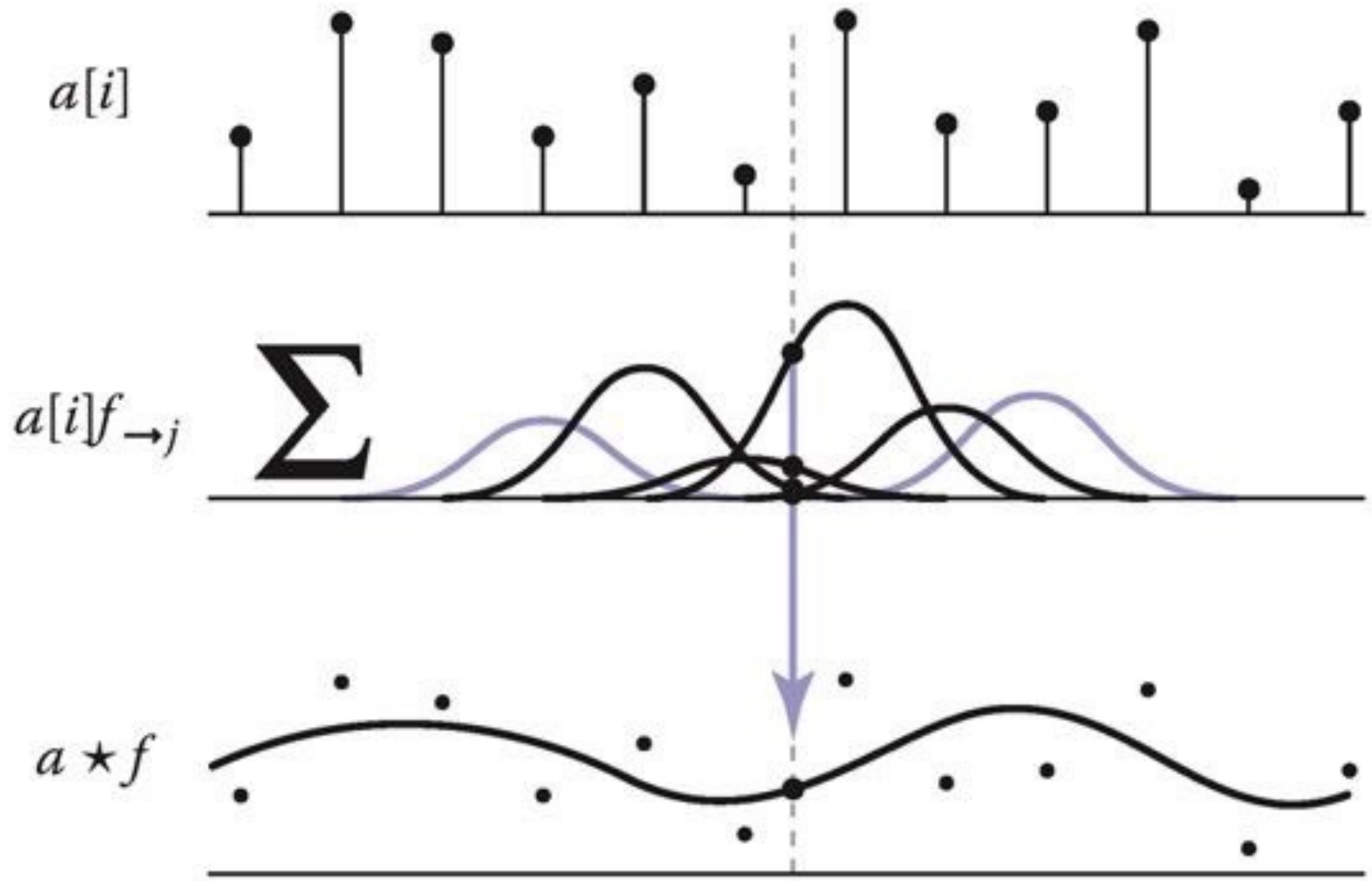
- We can define:

$$(a \star f)(x) = \sum_i a[i] f(x - i)$$

- When might we want to use this?

Signal Reconstruction

by convolution with reconstruction kernel (e.g. cubic spline)



Convolution in 2D

- If we have functions of two (or more) variables:

$$f(x, y), g(x, y) : \mathbb{R}^2 \mapsto \mathbb{R}$$
$$a[i, j], b[i, j] : \mathbb{Z}^2 \mapsto \mathbb{R}$$

- We can also define 2D convolution:

$$(a \star b)[i, j] = \sum_{i'} \sum_{j'} a[i', j']b[i - i', j - j']$$

- Where might we apply this?

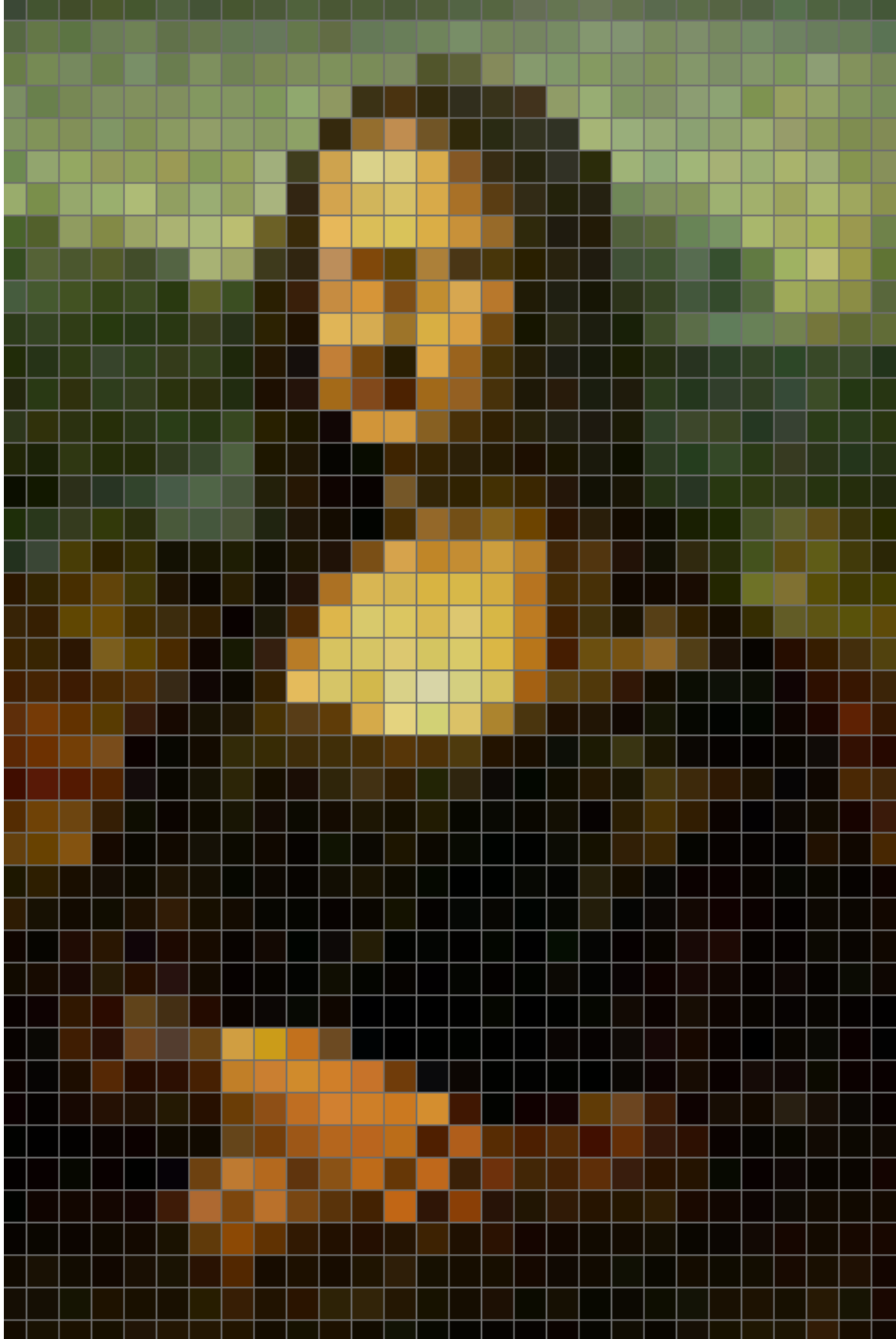
Images!

- A sampled digital image is a discrete function of two variables:

$$I[i,j] : \mathbb{Z}^2 \mapsto \mathbb{R}$$

- Though, technically, for colour images it's like this:

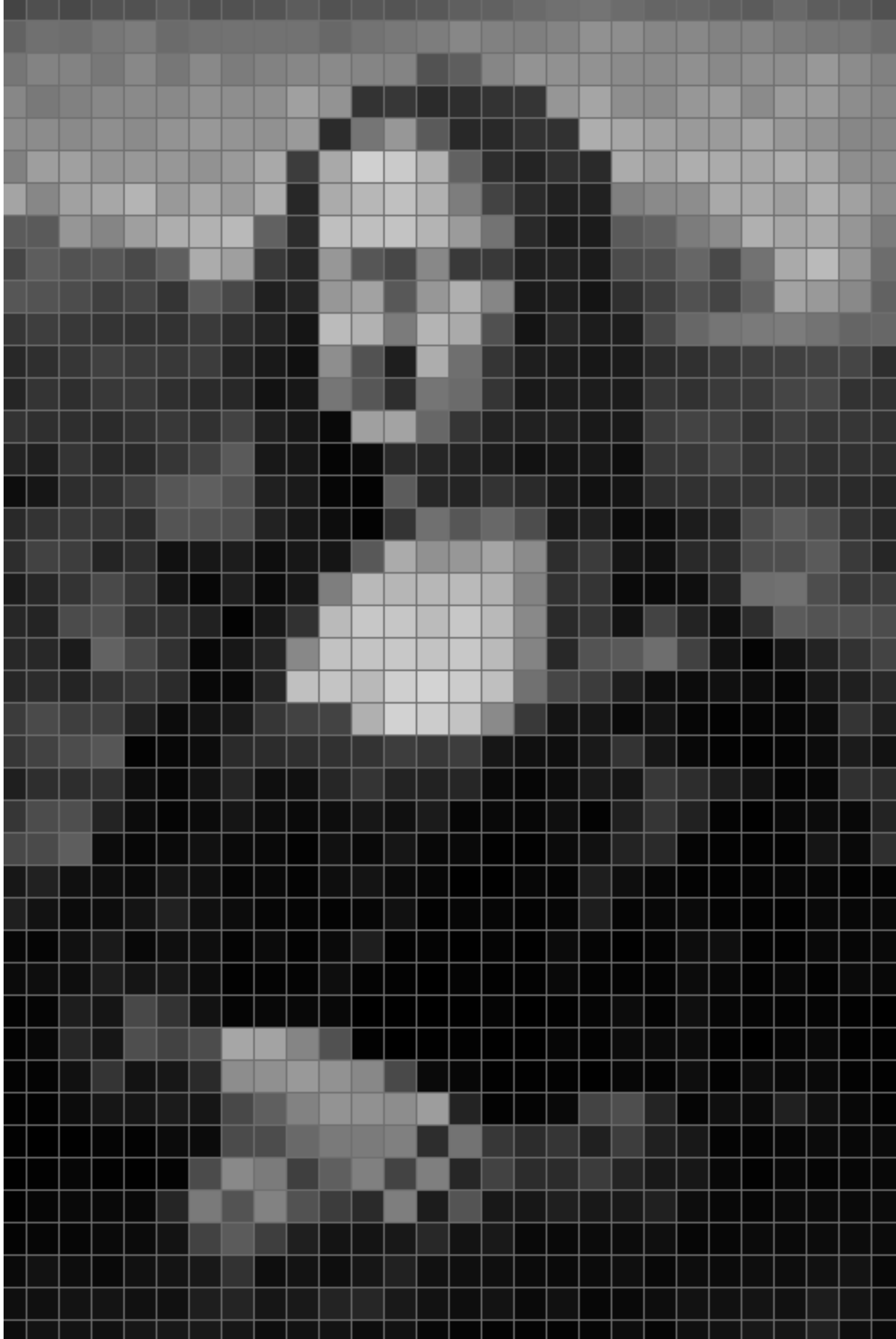
$$I[i,j] : \mathbb{Z}^2 \mapsto \mathbb{R}^3$$



Images!

- A sampled digital image is a discrete function of two variables:

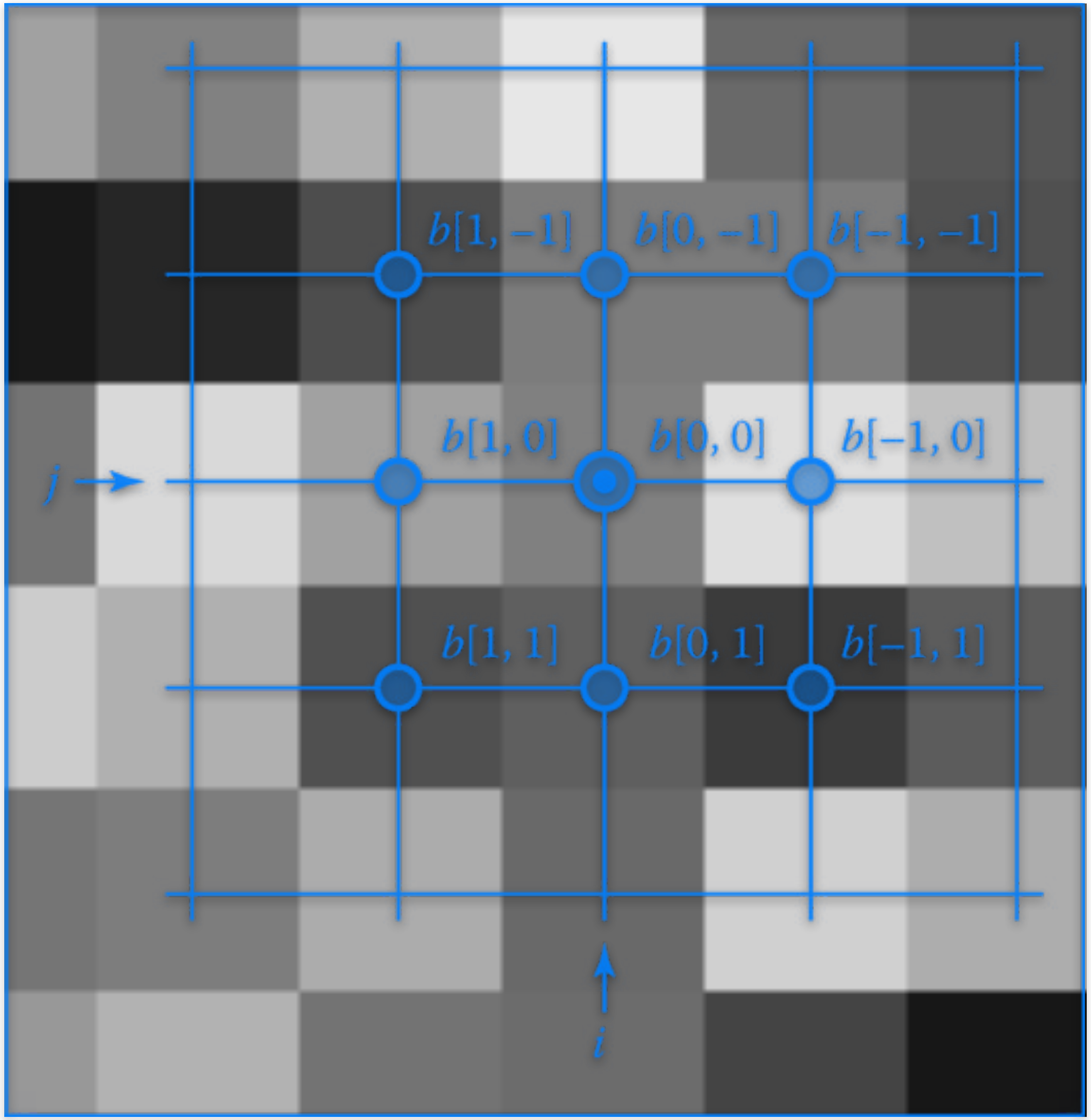
$$I[i,j] : \mathbb{Z}^2 \mapsto \mathbb{R}$$



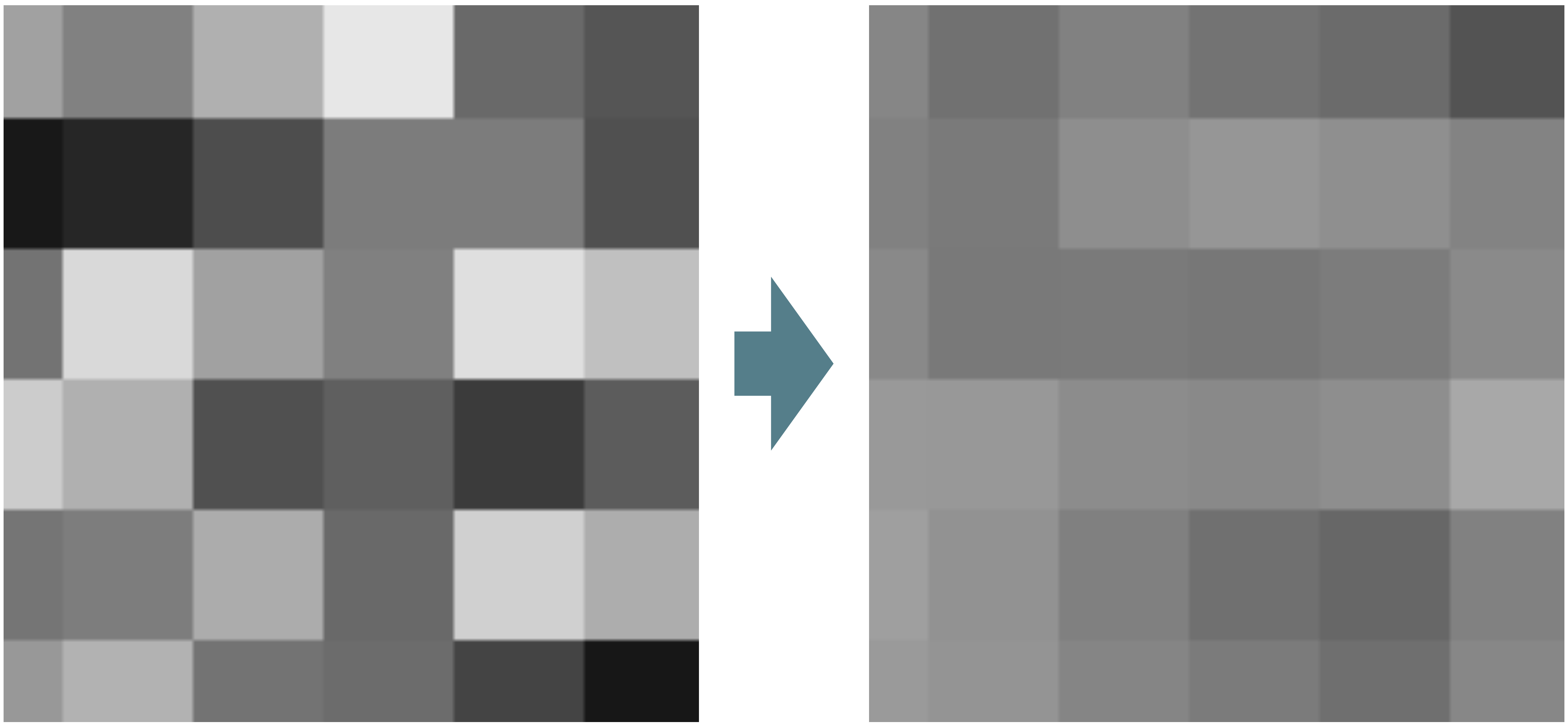
Example: Convolution with 2D Box Filter

$$(a \star b)[i,j] = \sum_{i'} \sum_{j'} a[i',j'] b[i-i', j-j']$$

$$b[i,j] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & 0 \\ 0 & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & 0 \\ 0 & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$



Convolution with 2D Box Filter

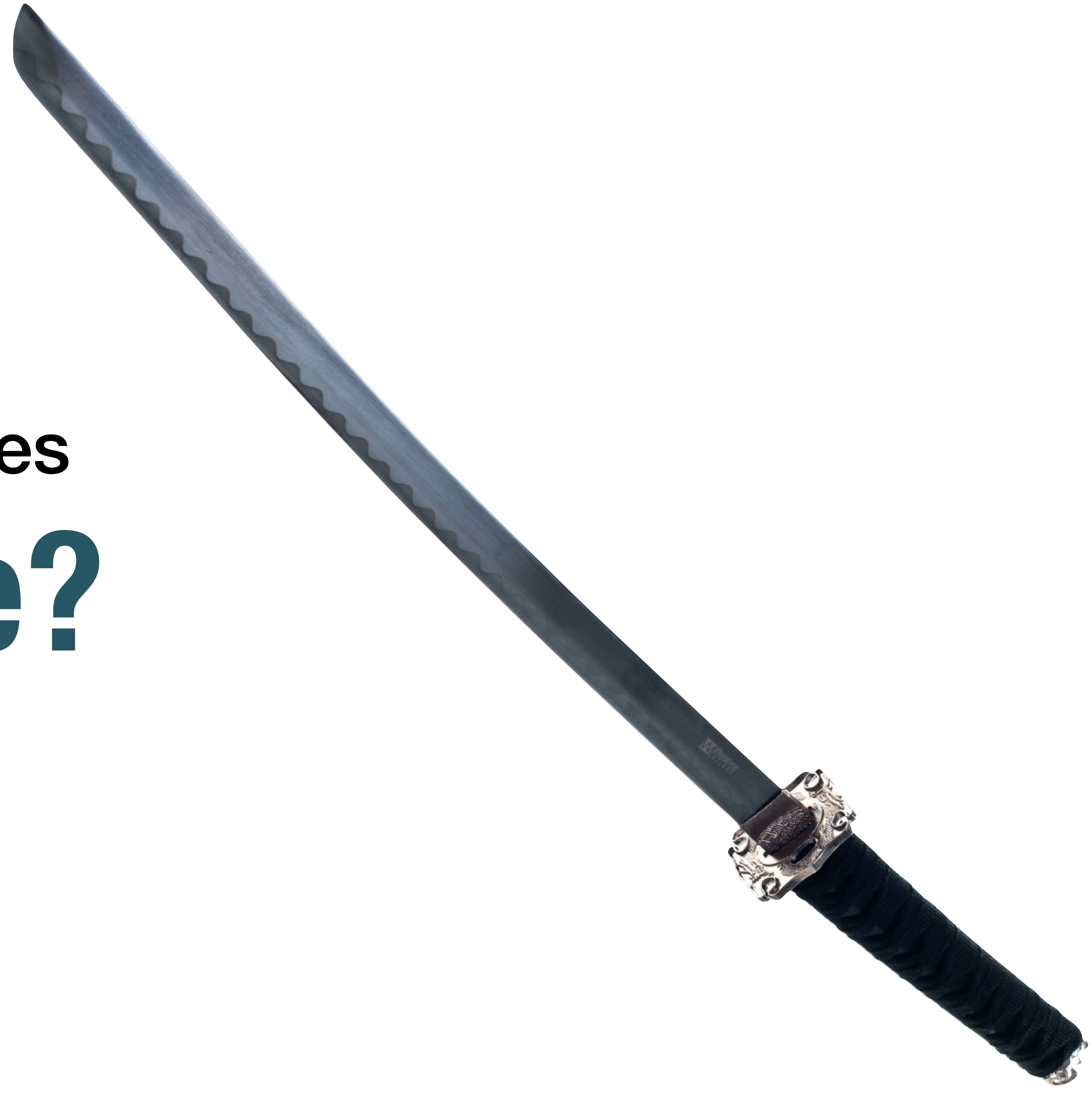


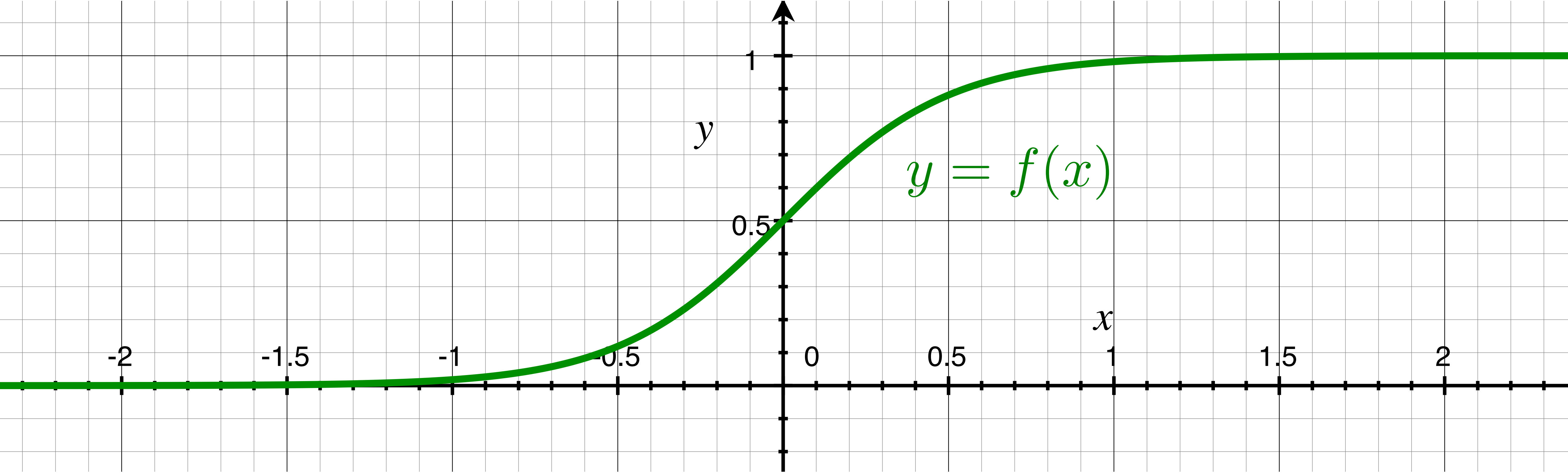
Can we achieve any

other effects?

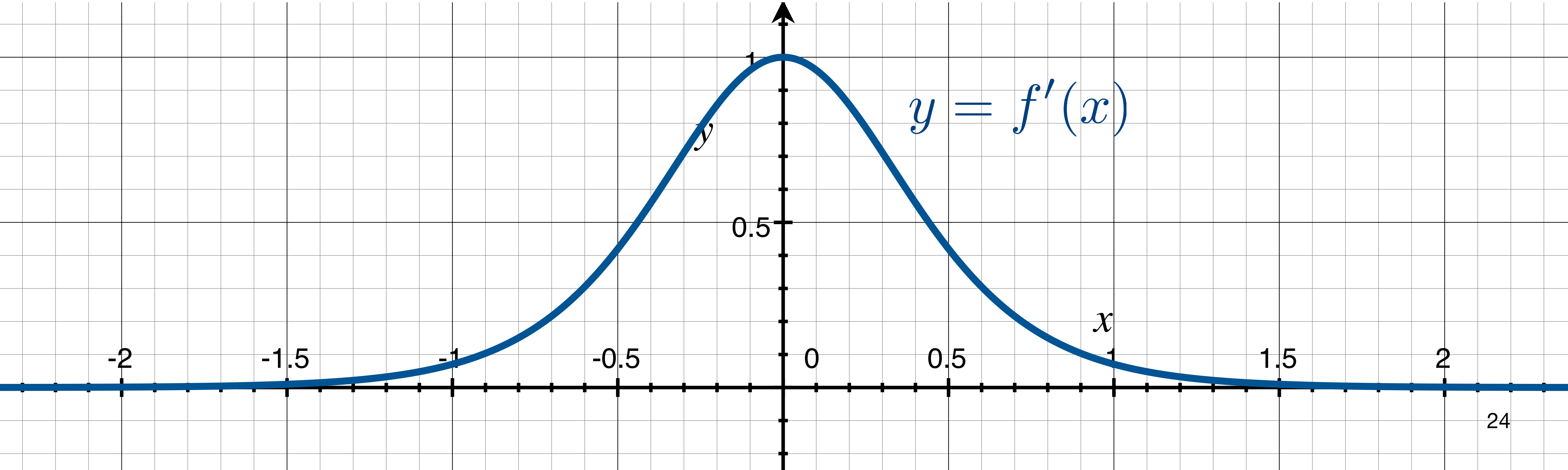
What constitutes

an edge?





How might we "detect" this edge?



An Edge Detector

- Derivative $$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$
- Forward difference $$f'(x) \approx \frac{f(x+h) - f(x)}{h}$$
- Central difference $$f'(x) \approx \frac{f(x+\frac{h}{2}) - f(x-\frac{h}{2})}{h}$$
- Discrete convolution $$b[i] = \begin{bmatrix} \ldots 0 & +1 & 0 & -1 & 0 \ldots \end{bmatrix}$$

Edge Detection in 2D: Sobel Filters

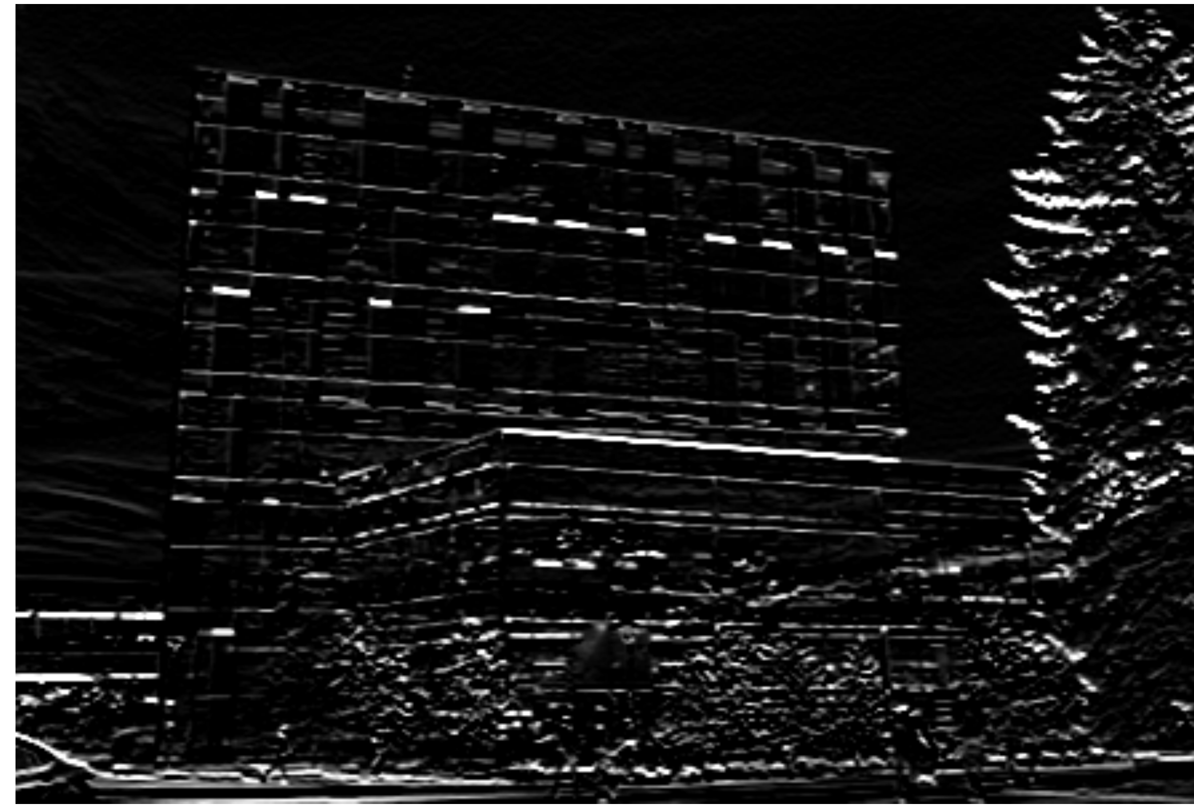
$$G_x[i,j] = \begin{bmatrix} +1 & 0 & -1 \\ +2 & 0 & -2 \\ +1 & 0 & -1 \end{bmatrix}$$

vertical edges

$$G_y[i,j] = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ +1 & +2 & +1 \end{bmatrix}$$

horizontal edges

original



horizontal edges

vertical edges

Things to Remember

- Convolution is an operation on two functions
  - can be applied on continuous or discrete functions
  - can be done in 1D, 2D, or more dimensions
  - continuous ★ discrete = continuous output function
- Convolution is used for many important things:
  - signal reconstruction (interpolation)
  - dazzling image effects, and more…