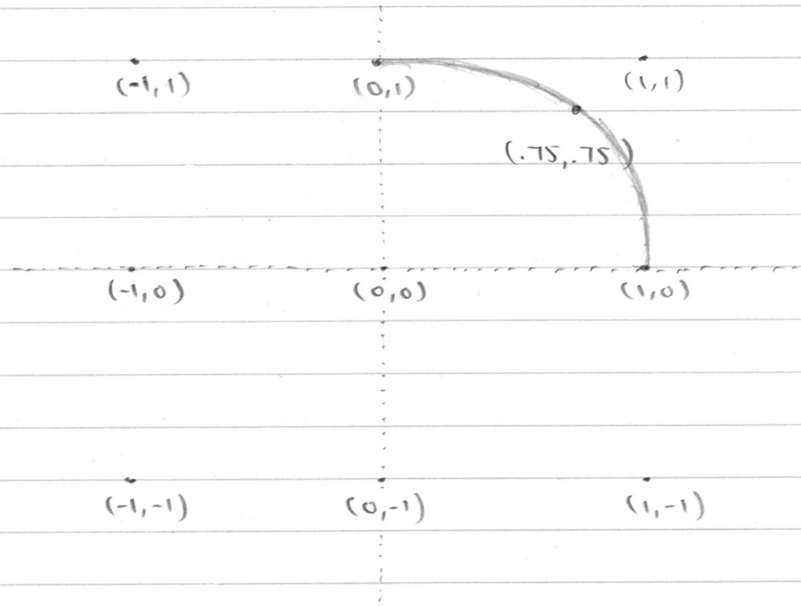
1\. A. To find the position of point in middle of curve, we employ the Bernstein form of a Quadratic Bezier with control points at (1,0), (1,1), and (0,1). By evaluating at $u = 0.5$, we get

$$
\begin{aligned}
p(u) &= (1-u)^2 p_0 + 2u(1-u)p_1 + u^2 p_2 \\
&= (1-.5)^2(1,0) + 2(.5)(1-.5)(1,1) + (.5)^2(0,1) \\
&= .25(1,0) + .5(1,1) + .25(0,1) \\
&= (.25, 0) + (.5, .5) + (0, .25) \\
&= (.75, .75)
\end{aligned}
$$

Thus, the position of point in middle of curve is (0.75, 0.75).

B. To determine the shape of this curve, we consider the points given as four mirrored copies of segment from 1A.



The shape formed would therefore look like a circle. However, we note that at $u = 0.5$ from 1A, the point is $(.75, .75)$, so it does not lie on unit circle since $\sqrt{.75^2 + .75^2} > 1$. Thus, it is more akin to a circle in the process of shifting towards a square (or vice versa)

c. Since the first and last control points are always the endpoints of a curve, we know that $(0,1)$ and $(1,0)$ are two of the four control points. To make use of information regarding tangents, we note that the Bernstein form of a cubic Bezier is

$$p(u) = (1-u)^3 p_0 + 3u(1-u)^2 p_1 + 3u^2(1-u) p_2 + u^3 p_3$$

Taking the derivative of this, we get

$$p'(u) = 3(1-u)^2(p_1 - p_0) + 6u(1-u)(p_2 - p_1) + 3u^2(p_3 - p_2)$$

At $u = 0$, we have

$$p'(0) = 3(1-0)^2(p_1 - p_0)$$
$$= 3(p_1 - (1,0)).$$

However, from problem, the tangent at $(1,0)$ is parallel to vector $(0,1)$. So, we have

$$c_1(0,1) = 3((x_1, y_1) - (1,0))$$
$$= 3(x_1 - 1, y_1)$$
$$(0, c_1) = (3x_1 - 3, 3y_1)$$

Thus $3x_1 - 3 = 0 \rightarrow 3x_1 = 3 \rightarrow x_1 = 1$

$3y_1 = c_1 \rightarrow y_1 = c_1/3$

At $u = 1$, we have

$$p'(1) = 3(1)^2(p_3 - p_2)$$
$$= 3((0,1) - p_2)$$

However, from problem, the tangent at $(0,1)$ is parallel to vector $(1,0)$, so we have

$$c_2(1,0) = 3((0,1) - (x_2, y_2))$$
$$= 3(-x_2, 1-y_2)$$
$$(c_2, 0) = (-3x_2, 3-3y_2)$$

Thus $-3x_2 = c_2 \rightarrow x_2 = -c_2/3$

$3-3y_2 = 0 \rightarrow 3y_2 = 3 \rightarrow y_2 = 1$

Knowing this, we simply need to determine c_1 and c_2 to find y_1 and x_2. Since the midpoint is at $(\sqrt{2}/2, \sqrt{2}/2)$, we substitute $u = 0.5$ and $p(u) = (\sqrt{2}/2, \sqrt{2}/2)$ to find

$$p(u) = (1-u)^3 p_0 + 3u(1-u)^2 p_1 + 3u^2(1-u)p_2 + u^3 p_3$$
$$= (1-.5)^3 p_0 + 3(.5)(1-.5)^2 p_1 + 3(.5)^2(1-.5)p_2 + (.5)^3 p_3$$
$$= .125(1,0) + .375(1, c_1/3) + .375(-c_2/3, 1) + .125(0,1)$$
$$= (.125, 0) + (.375, c_1/8) + (-c_2/8, .375) + (0, .125)$$
$$= (.5 - c_2/8, .5 + c_1/8)$$
$$(\sqrt{2}/2, \sqrt{2}/2) = (.5 - c_2/8, .5 + c_1/8)$$

Solving this, we get

$$\sqrt{2}/2 = 1/2 - c_2/8 \rightarrow c_2/8 = \frac{1-\sqrt{2}}{2} \rightarrow c_2 = 4 - 4\sqrt{2}$$
$$\sqrt{2}/2 = 1/2 + c_1/8 \rightarrow c_1/8 = \frac{\sqrt{2}-1}{2} \rightarrow c_1 = 4\sqrt{2} - 4$$

Thus, $x_2 = -c_2/3 = (4\sqrt{2}-4)/3$, $y_1 = c_1/3 = (4\sqrt{2}-4)/3$

The four control points are therefore $(1,0)$, $(1, (4\sqrt{2}-4)/3)$, $((4\sqrt{2}-4)/3, 1)$, $(0,1)$

D. Yes, it is likely that the curve created in 1C forms part of a circular arc. We know this is the case because the endpoints denote the start and end of the curve. Furthermore, we know that the midpoint of curve is at $(\sqrt{2}/2, \sqrt{2}/2)$, which lies on the unit circle. Thus, we have all three points lying on the unit circle, with symmetry between P_0, P_3 and P_1, P_2 along the

Hilroy

line $y=x$, so it is quite likely that the curve from 1C is part of a circular arc. Testing at other points such as $u=0.25$ confirms that other points also lie on unit circle.

E. We note that we simply apply formula from 1C, but now taking into account midpoint $= (.75, .75)$. The first condition of 1C is the same for 1A. Furthermore, we also note that the second and third conditions of 1C also match 1A, since the tangents at $(1,0)$ and $(0,1)$ are both vertical and horizontal respectively. That is, in determining the four control points of a cubic Bezier that forms a curve identical to 1A, we simply find c_1 and c_2 when $u=0.5$, $p(u) = (.75, .75)$.

$$p(u) = (1-u)^3 p_0 + 3u(1-u)^2 p_1 + 3u^2(1-u)p_2 + u^3 p_3$$
$$= (.5 - c_2/8, \ .5 + c_1/8) \longleftarrow \text{From 1C.}$$
$$(.75, .75) = (.5 - c_2/8, \ .5 + c_1/8)$$

Solving this, we get

$$3/4 = 2/4 - c_2/8 \rightarrow c_2/8 = -1/4 \rightarrow c_2 = -2$$
$$3/4 = 2/4 + c_1/8 \rightarrow c_1/8 = 1/4 \rightarrow c_1 = 2$$

Thus, $x_2 = -c_2/3 = 2/3$, $y_1 = c_1/3 = 2/3$

The four control points are therefore $(1,0), (1, 2/3), (2/3, 1), (0,1)$.

2. A. We note that the class C^0 consists of all continuous functions. Therefore, the knots on the spline that are C^0 continuous are

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14 since the curves are joined ($p(1) = q(0)$)

B. We note that the class G^1 require the curves share a common tangent direction at the join point. Therefore, the knots on the spline that are G^1 continuous are

3, 4, 5, 6, 7, 10, 11, 12, 13, 14 since ($p'(1) = sq'(0)$ for $s \in \mathbb{R}^{+}$)

C. We note that the class C^1 requires that aside from touching at the join point, the curves must also share a common tangent direction at the join point. Therefore, the knots on the spline that are C^1 continuous are

4, 5, 6, 11, 12, 13 since the first derivatives are continuous ($p'(1) = q'(0)$)
we determine these on diagram since off curve control points on either side of knot are same length.

3. A. Since we scale the text so that an average of 10 letters fit across a 1000×1000 pixel window, this means each letter takes up around 100 pixels on average horizontally. Since we want to scroll the text at around 4 characters per second, this means that around $4 \times 100 = 400$ pixels are traversed in a second. Since we have 60 frames per second, we would need $400/60 = 20/3 \approx 6.67$ pixels are moved at every frame. That is, the text is moved at around 6.67 pixels per frame.

B. Since we now want to determine the amount moved at every frame for 24 frames per second instead of 60 frames per second, we substitute 24 for 60 in above to get $400/24 = 50/3 \approx 16.67$ pixels moved per frame. The text would have to be moved at around 16.67 pixels per frame to maintain the same scroll rate.

To account for this difference, we would have to consider the change in the number of pixels moved each frame for the refresh rate of 60 frames per second compared with 24 frames per second if we are to maintain the scroll rate. Since we need to scroll text at comfortable speed and provide a means to interactively adjust text scroll rate in program, we need consider this and provide correct calculations to ensure text scrolls across screen at desired rate of choice. We would need to adjust the distance moved per frame to account for this difference.

Hilroy

If our goal is to scroll the text at 4 characters per second (fixed rate) regardless of what computer we run our program on, we would have to make sure that our program performs the calculations shown after determining the frame rate of the computer system. In this case, it would perform 400/f, where f denotes the frames per second of the particular system. It would then use this number to move the text by this many number of pixels per frame.