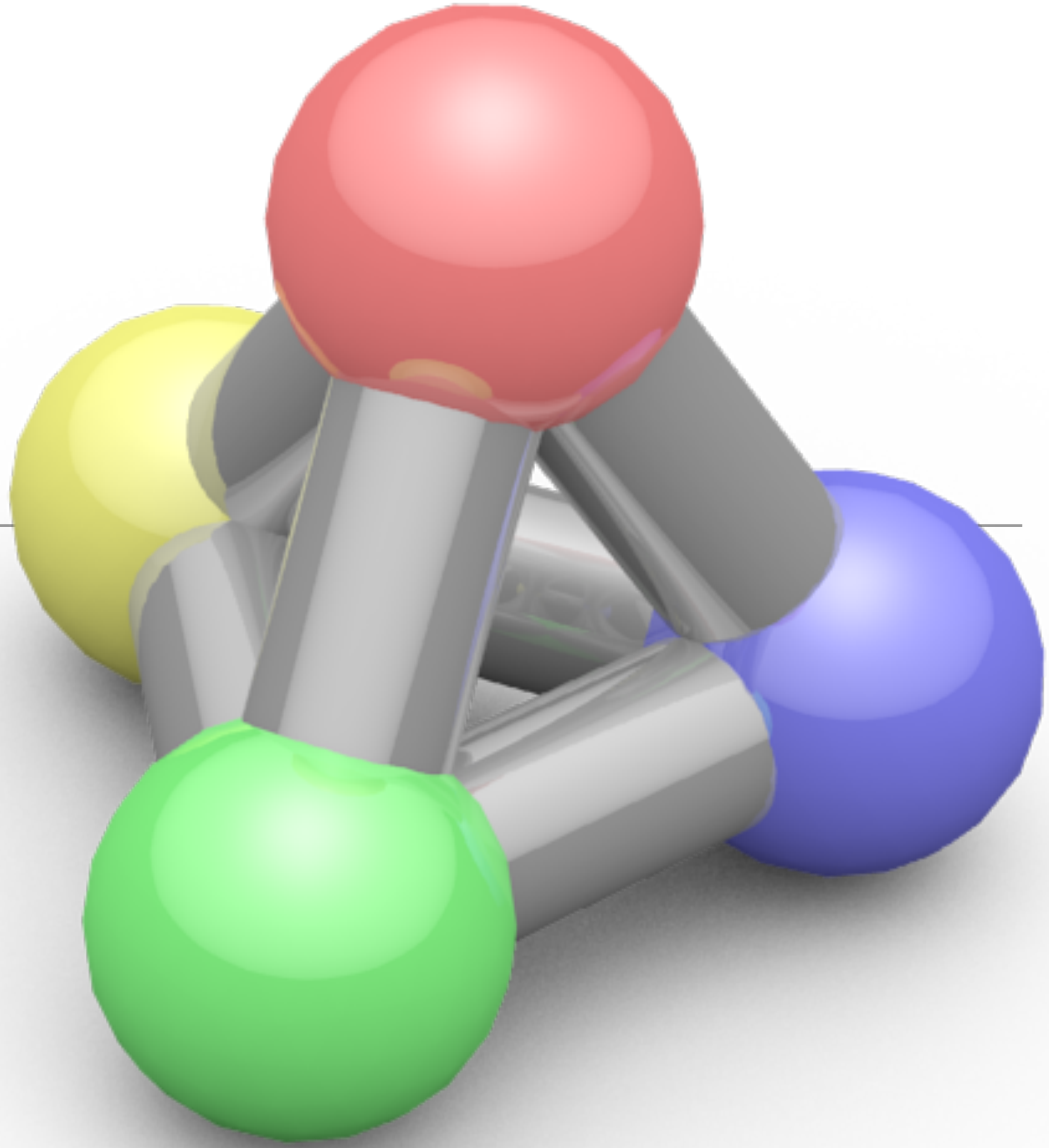
Ray Tracing

CPSC 453 – Fall 2016
Sonny Chan

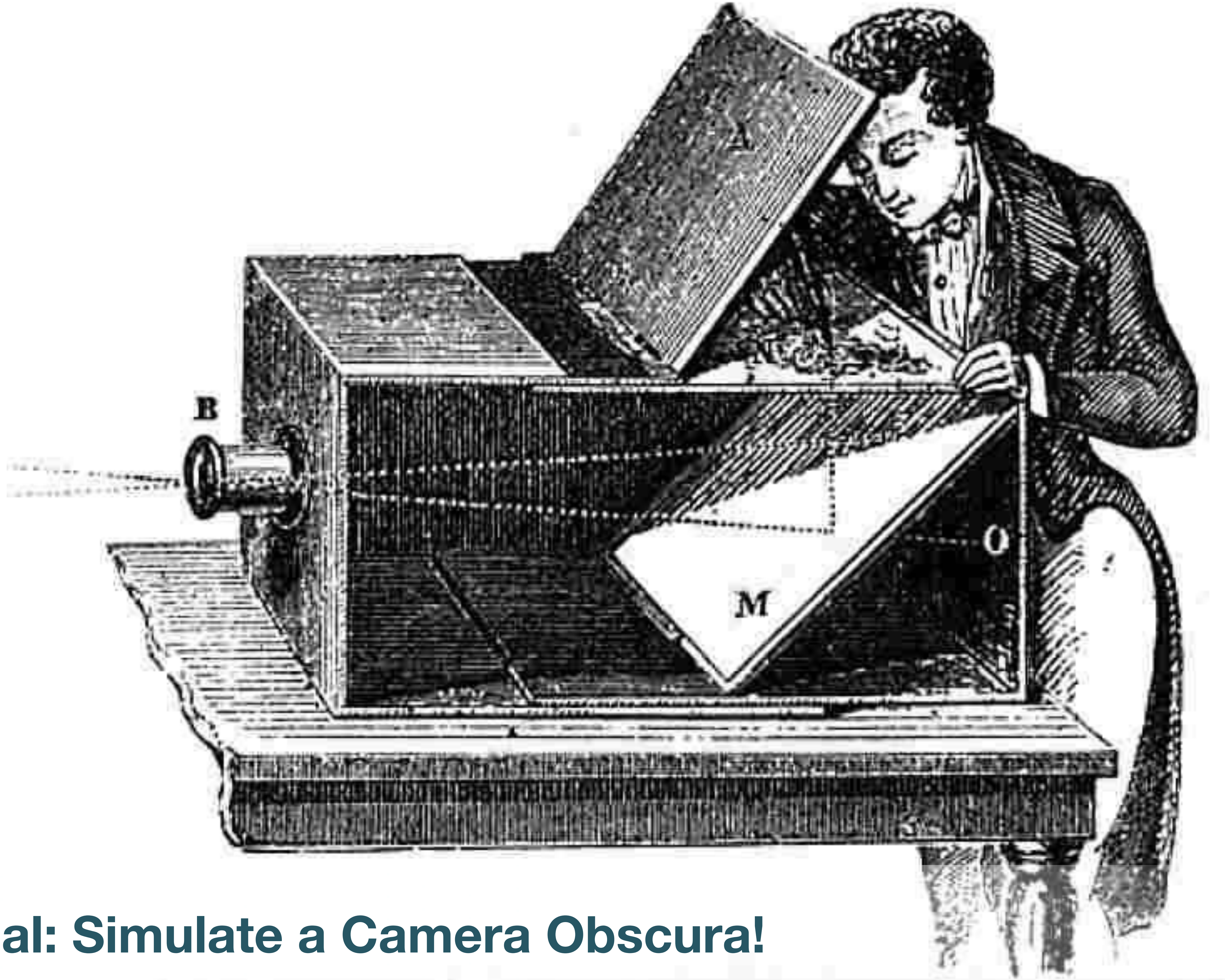


Ray Tracing

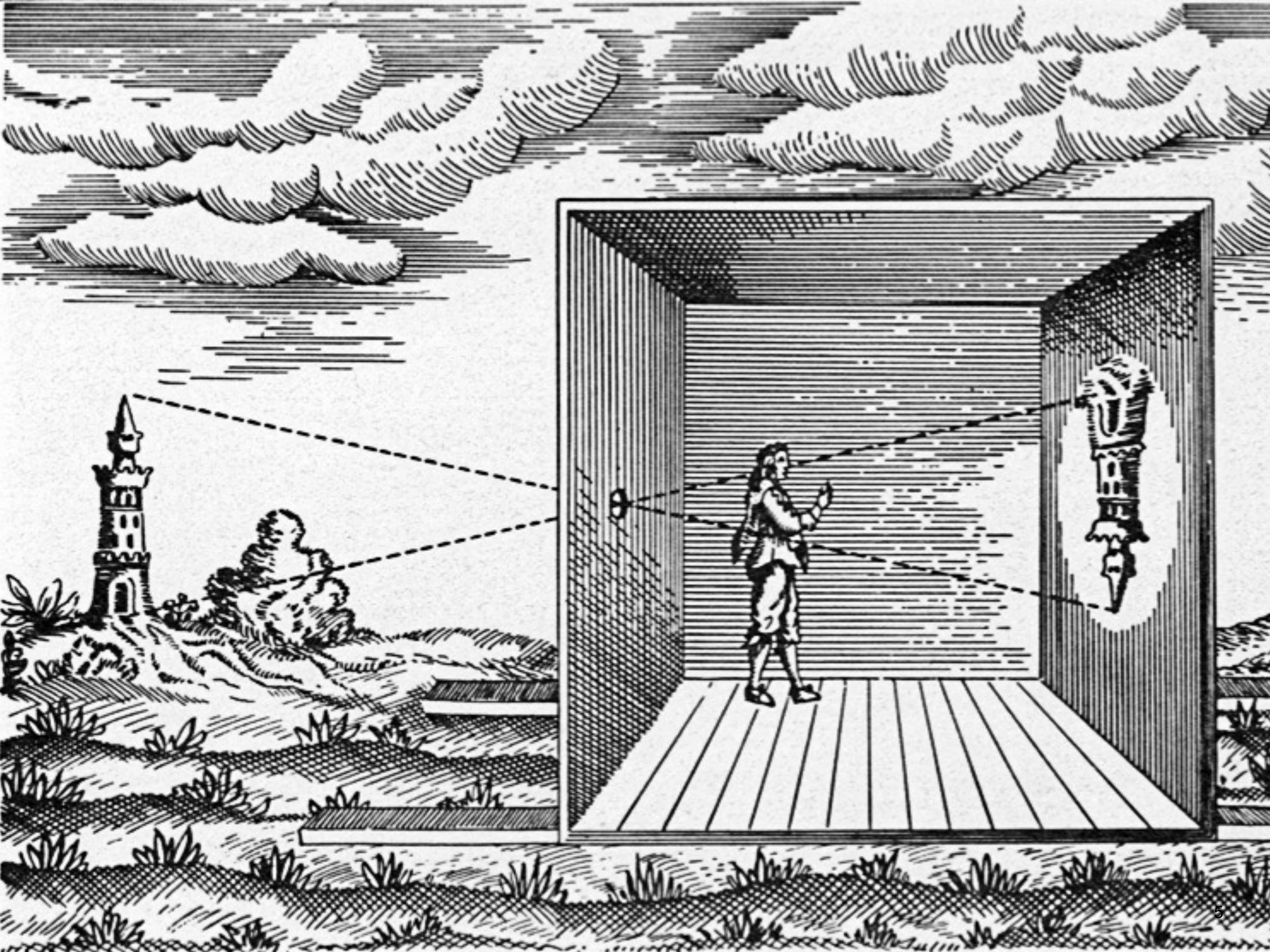
A method for synthesizing images of virtual 3D scenes.

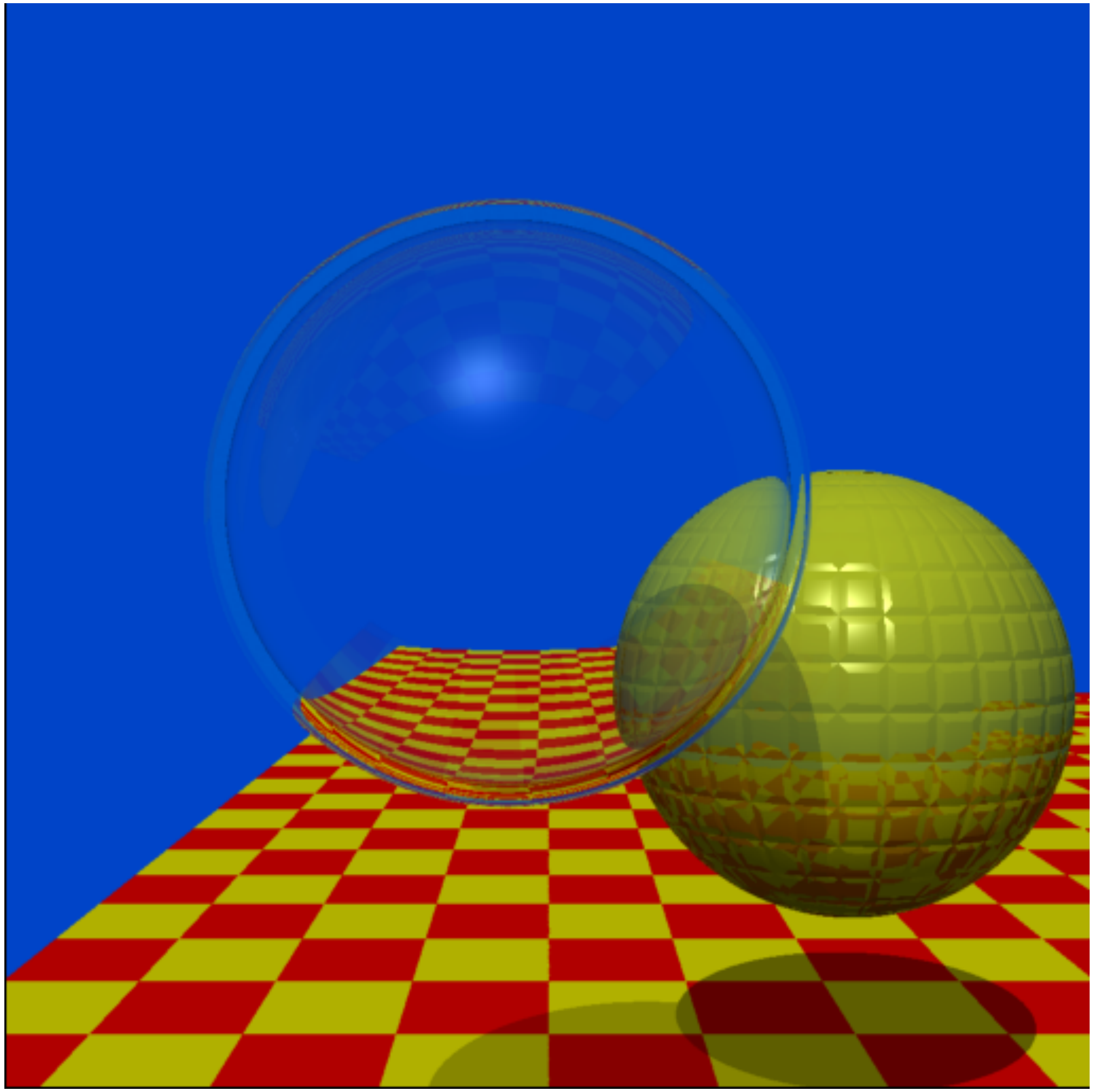
Image Capture Devices

Which one shall we use?



Goal: Simulate a Camera Obscura!



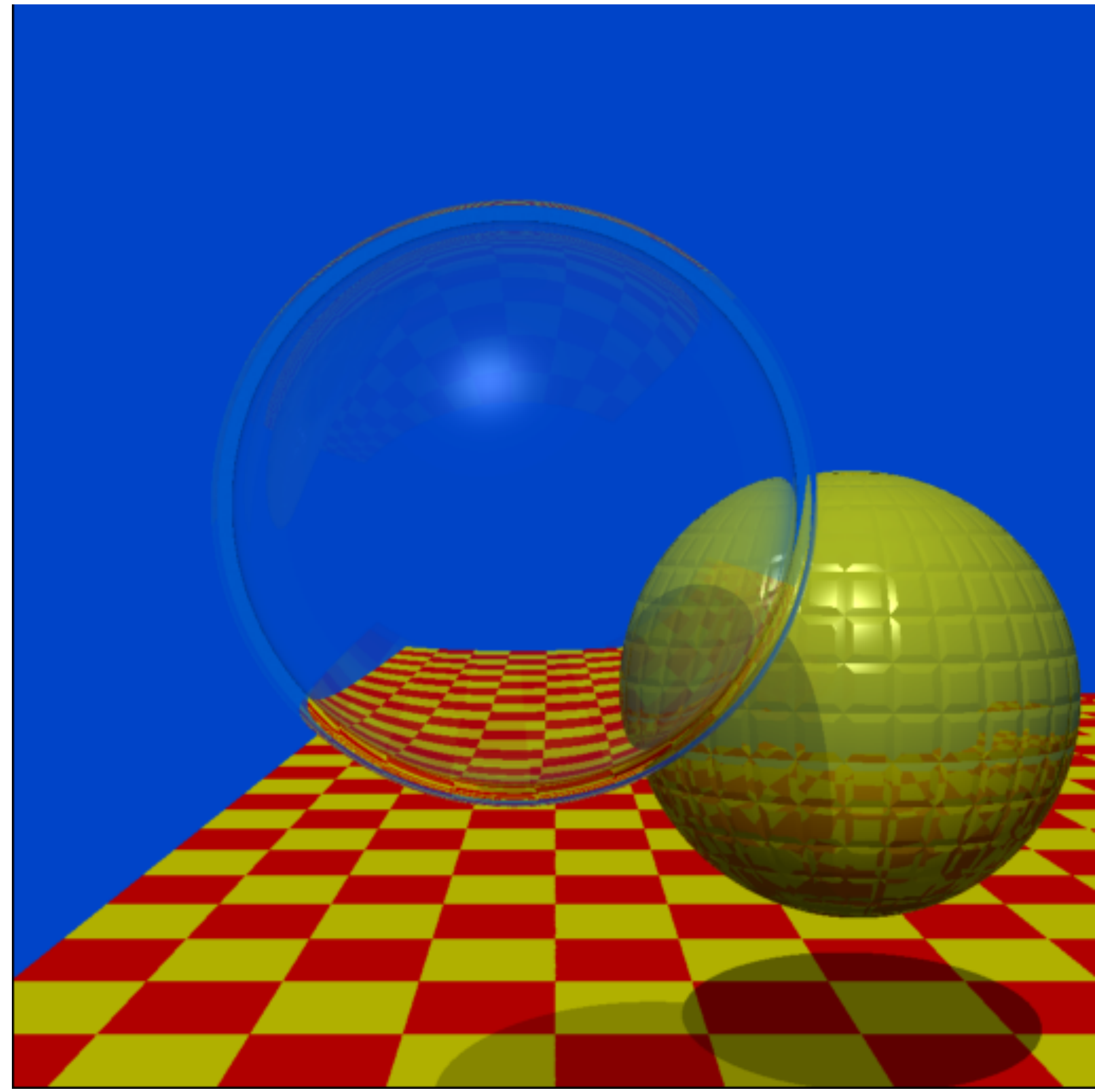


“Spheres & Checkerboard”

Turner Whitted, 1979

Whitted Ray Tracing

- “An improved illumination model for shaded display”
  - T. Whitted, *SIGGRAPH* 1979
- 512 × 512 image
- Rendered on VAX 11/780
- 74 minutes compute time
- How fast would your computer render this today?



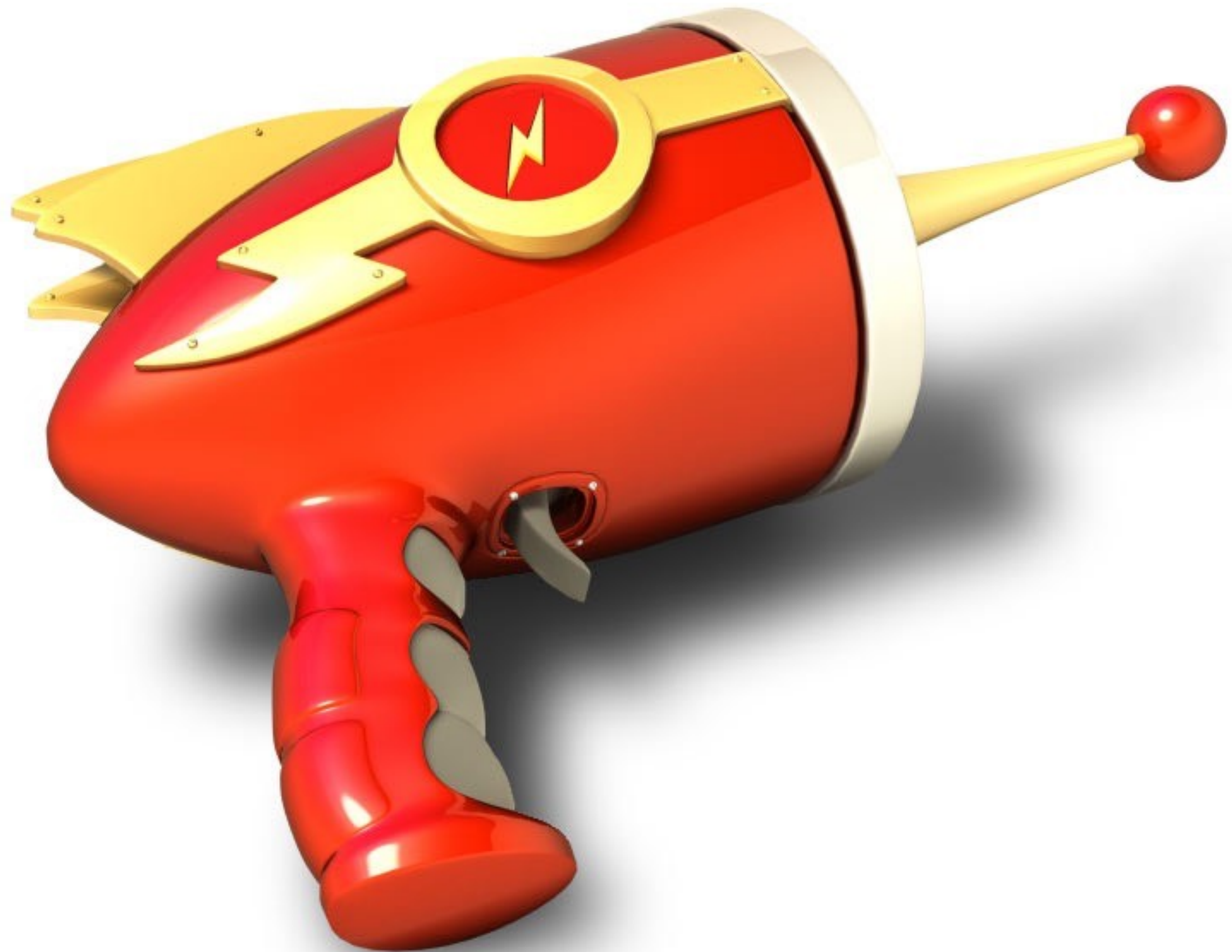
[image courtesy of P. Hanrahan, Stanford University]

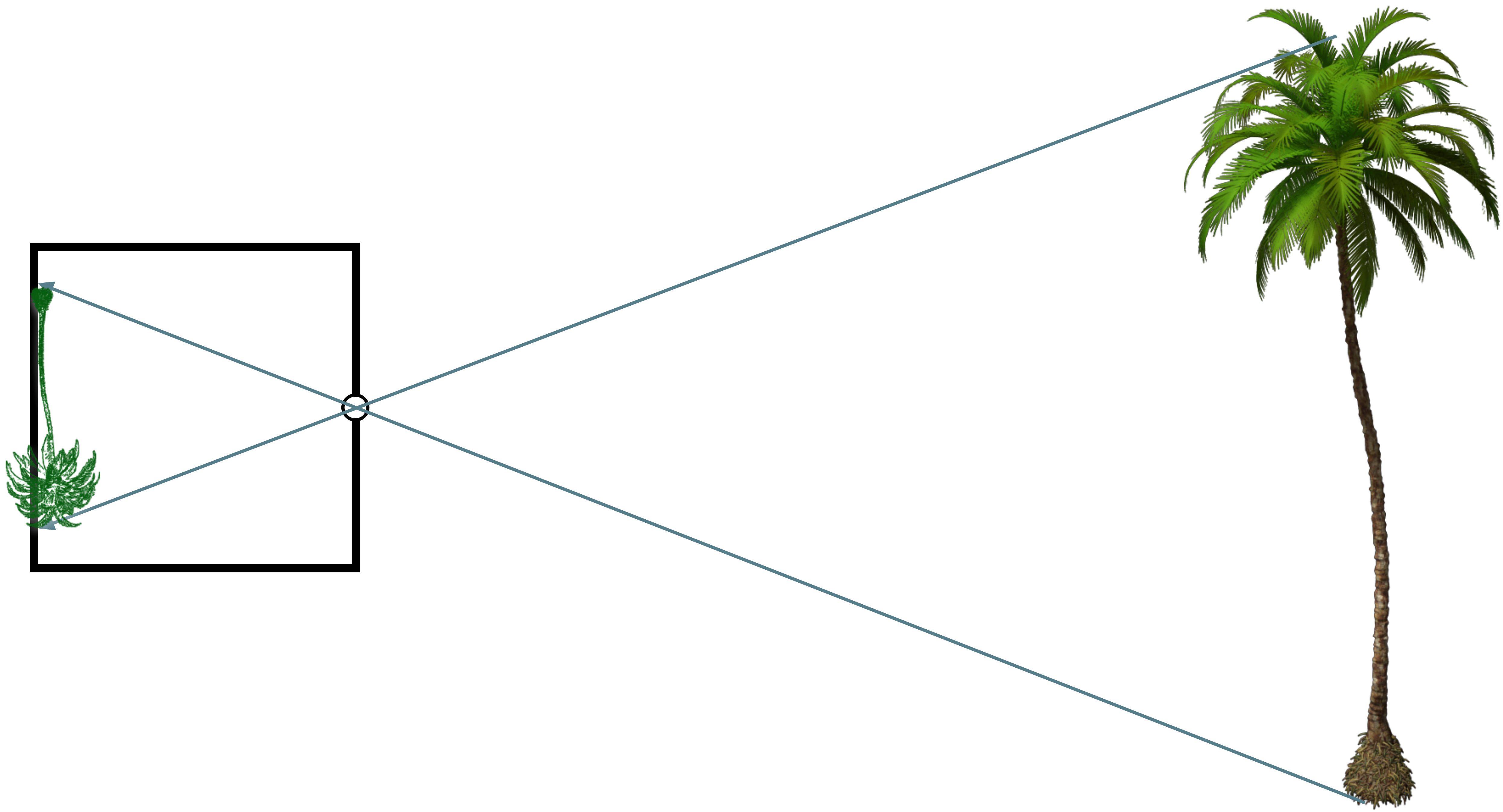

happy girl
kitchen co
certified organ
LIMITED PRODUCTIONS
orange marmalade
REFRIGERATE AFTER OPENING · NET WT 8 OZ
ALICE WATERS
HEZ PANISSE FRUIT
ALICE WATERS

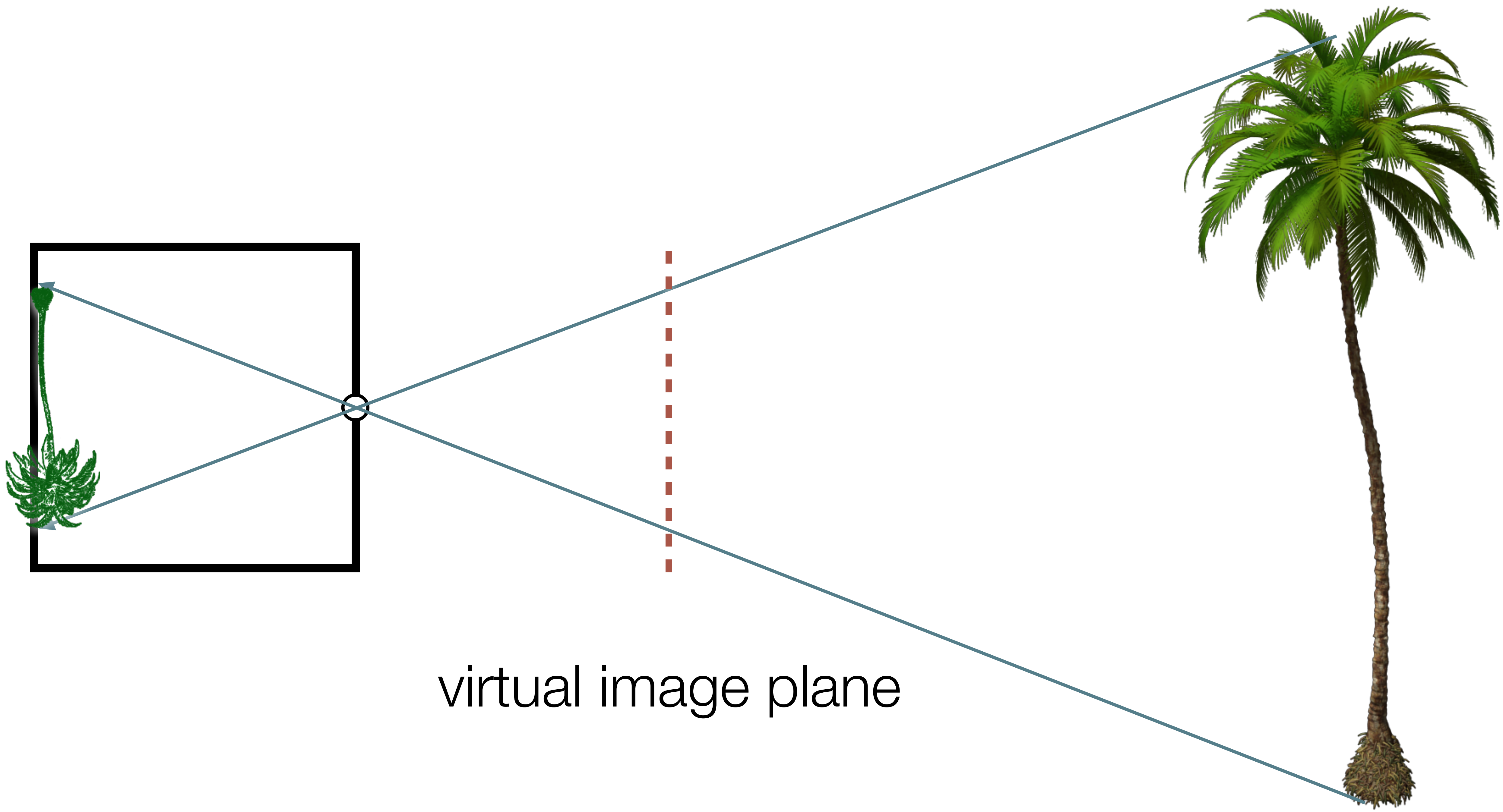
The Ray Tracing Algorithm

- For every pixel or position on your image, do
    1. **ray generation**: where am I looking?
    2. **ray intersection**: do I see something?
    3. **shading**: what colour is it? (next week's topic)

Ray Generation





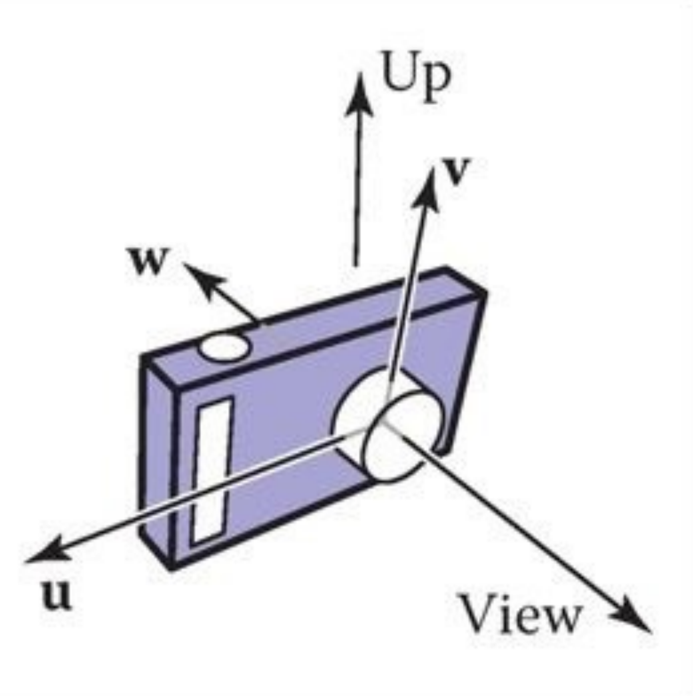


virtual image plane

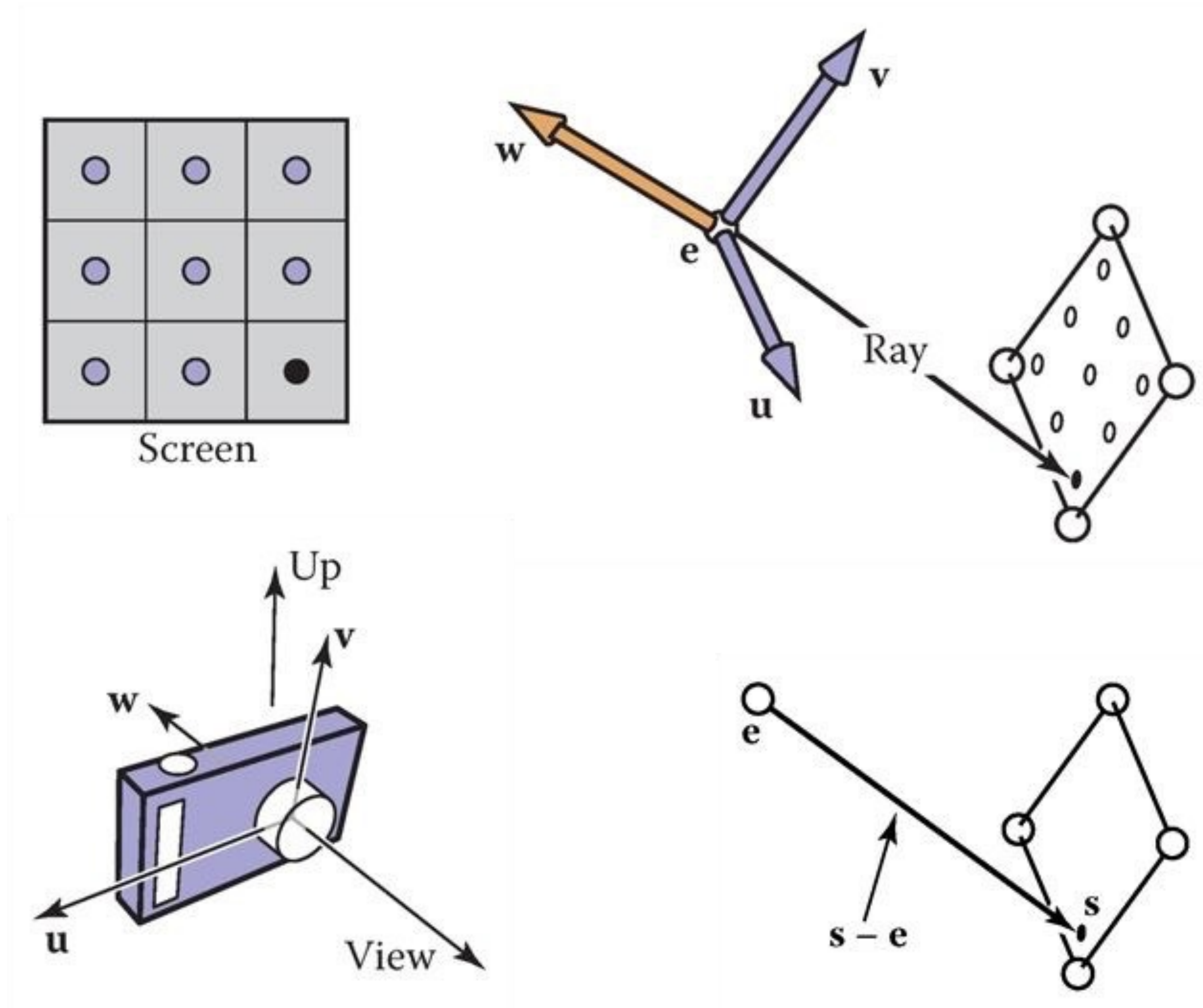
[from photojojo.com]

Let’s see how we might

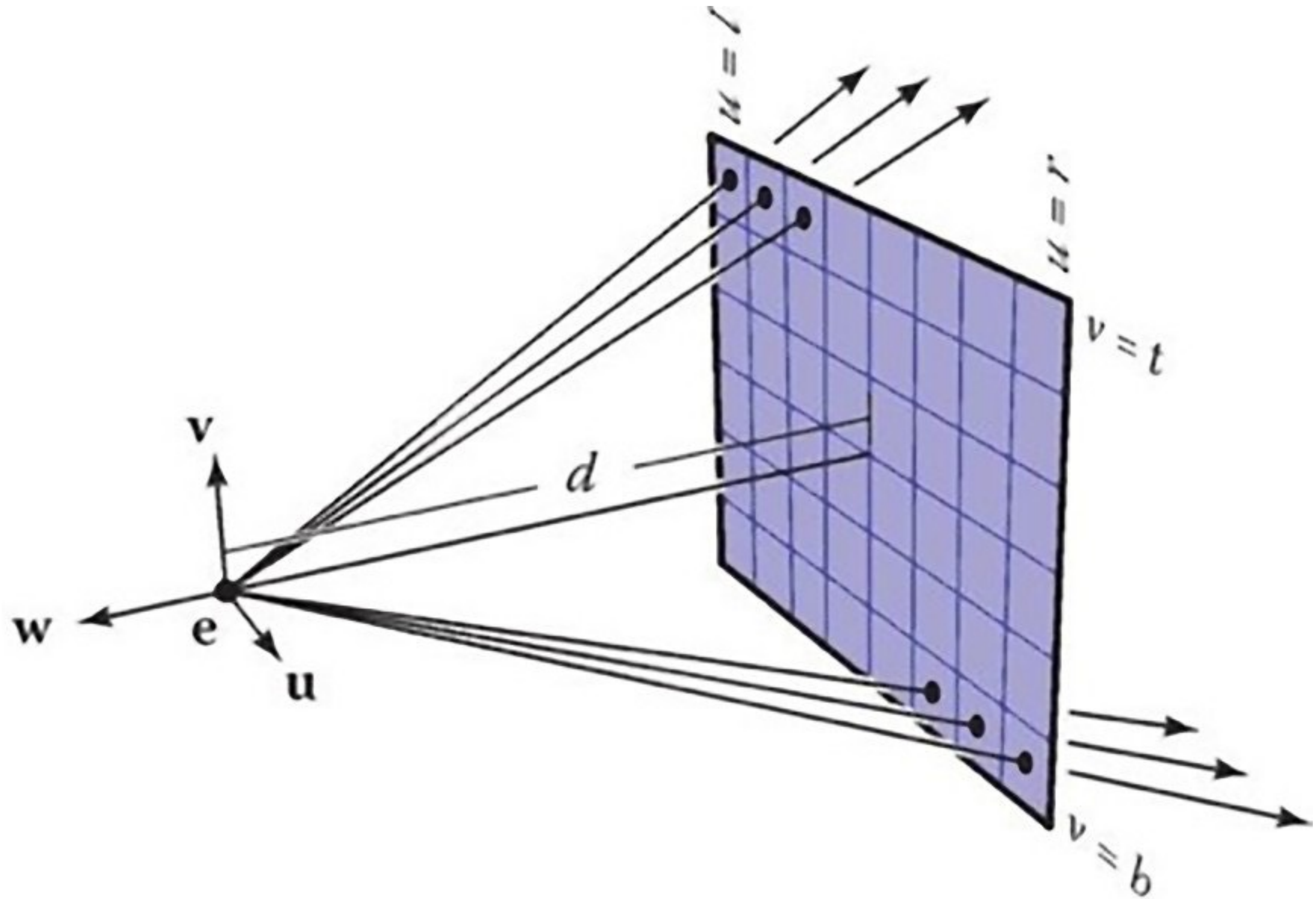
generate those rays



Ray Generation



$$\mathbf{r}(t) = \mathbf{e} + t(\mathbf{s} - \mathbf{e})$$



u = l
u = r
v = t
v = b
v
d
w
e
u

Ray Intersection

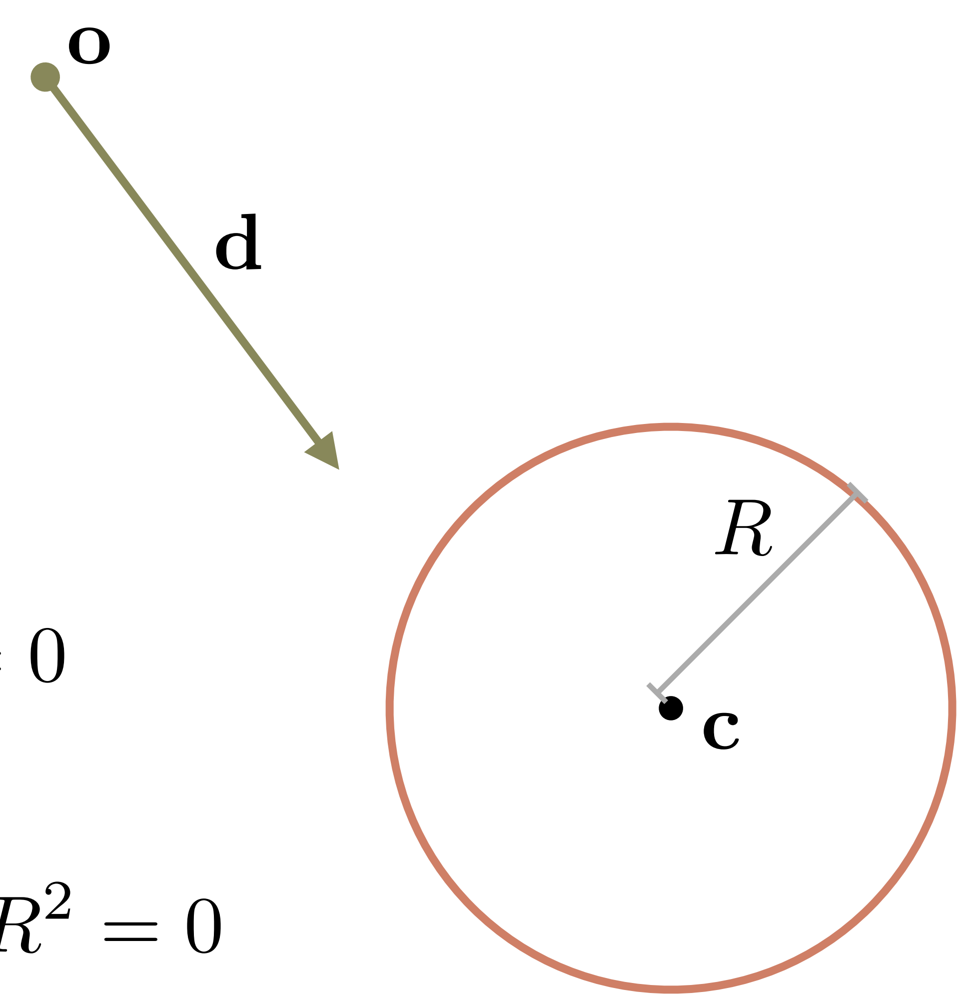
What do we see?

Ray Intersection

- For every object we have in our scene, we need a way to determine whether or not we can see it
- Usually amounts to solving an equation of one variable
- We will examine three key intersection tests today:
    - spheres
    - planes
    - triangles


spheres

Ray-Sphere Intersection



A ray: $\mathbf{r}(t) = \mathbf{o} + t\mathbf{d}$

A sphere: $||\mathbf{p} - \mathbf{c}|| = R$

$$(\mathbf{p} - \mathbf{c}) \cdot (\mathbf{p} - \mathbf{c}) - R^2 = 0$$

Intersection: $(\mathbf{r}(t) - \mathbf{c}) \cdot (\mathbf{r}(t) - \mathbf{c}) - R^2 = 0$

$$(\mathbf{o} + t\mathbf{d} - \mathbf{c}) \cdot (\mathbf{o} + t\mathbf{d} - \mathbf{c}) - R^2 = 0$$

Solve for t…

planes

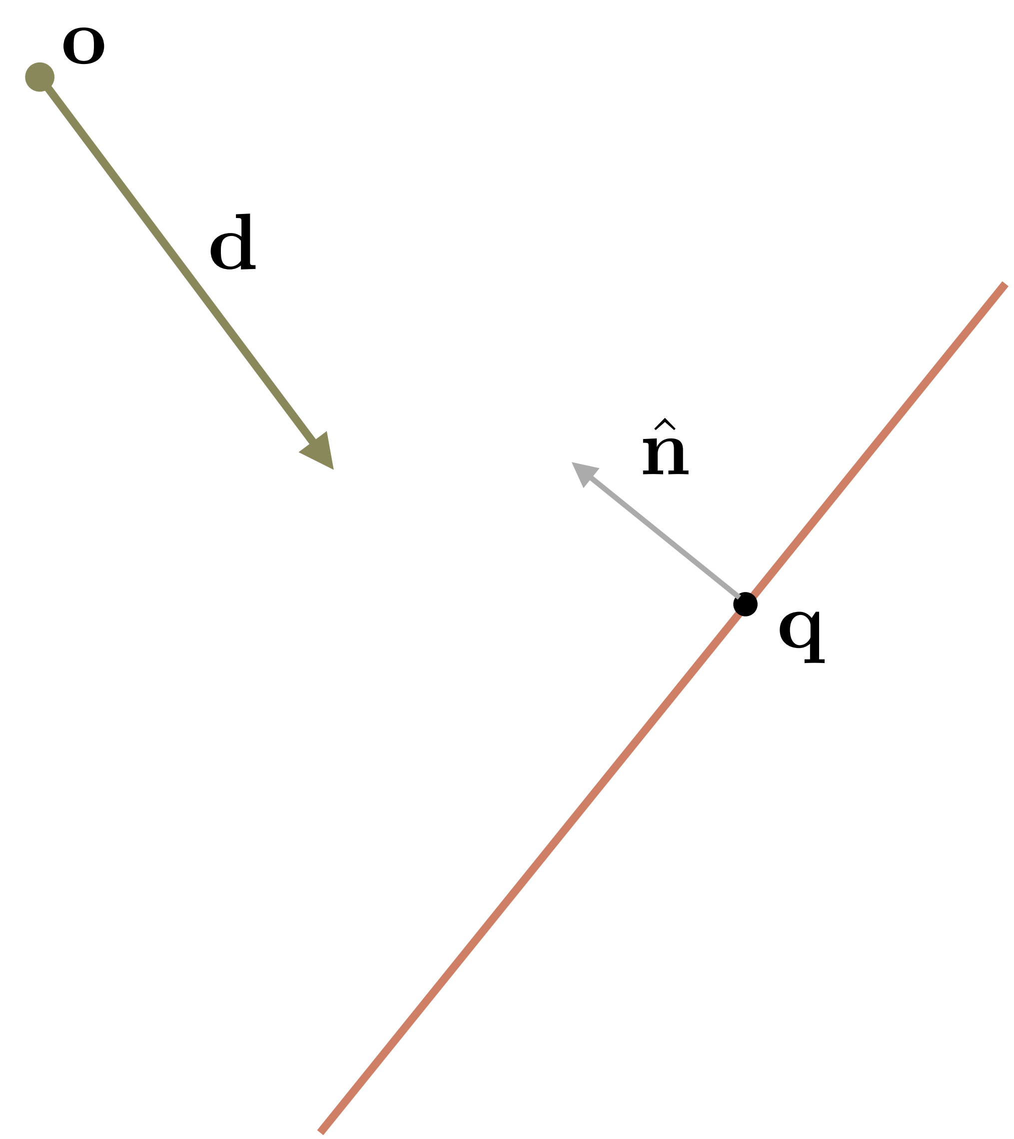
Ray-Plane Intersection

A ray: $\mathbf{r}(t) = \mathbf{o} + t\mathbf{d}$

A plane: $(\mathbf{p} - \mathbf{q}) \cdot \hat{\mathbf{n}} = 0$

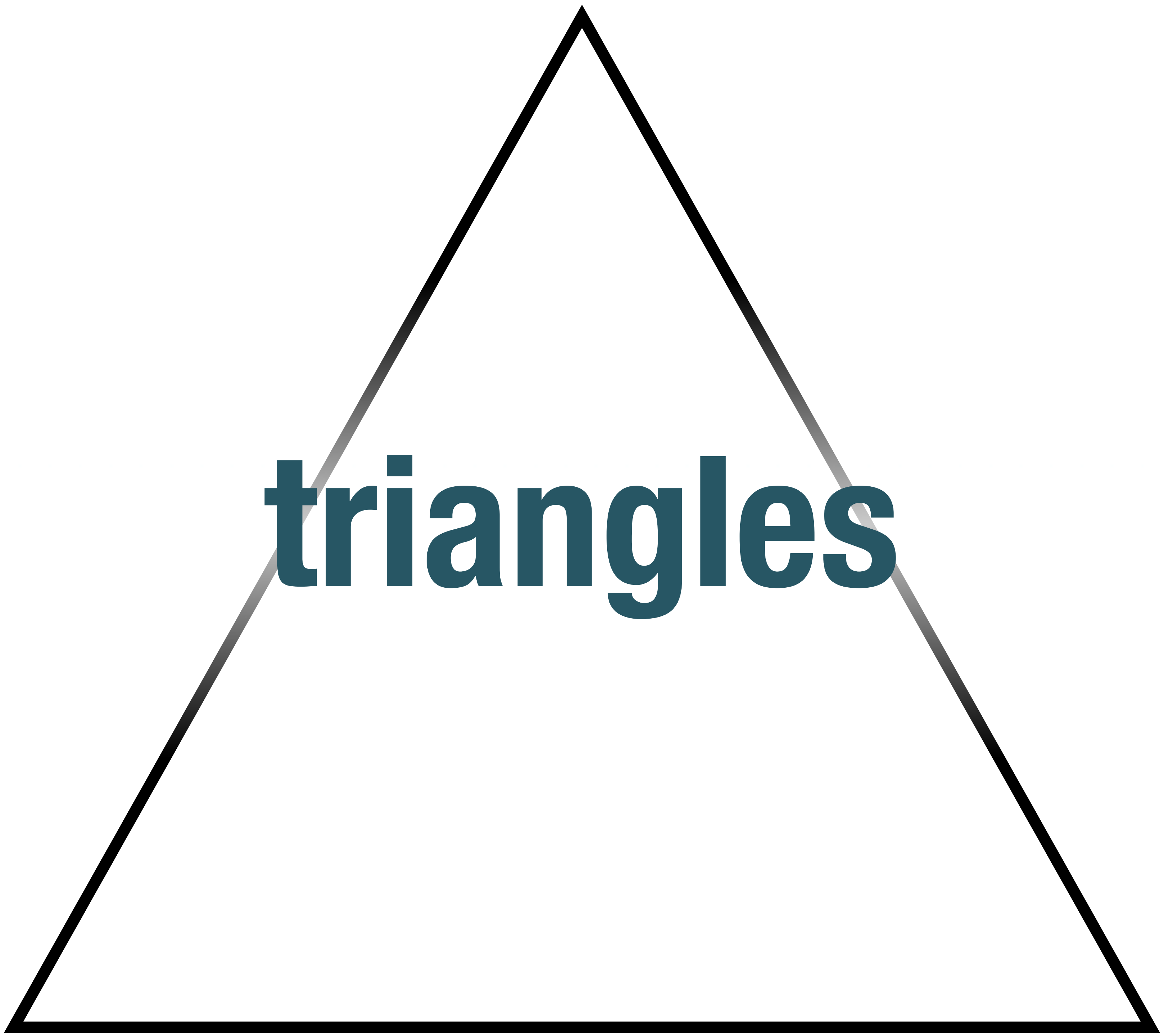
Intersection: $(\mathbf{r}(t) - \mathbf{q}) \cdot \hat{\mathbf{n}} = 0$

$(\mathbf{o} + t\mathbf{d} - \mathbf{q}) \cdot \hat{\mathbf{n}} = 0$



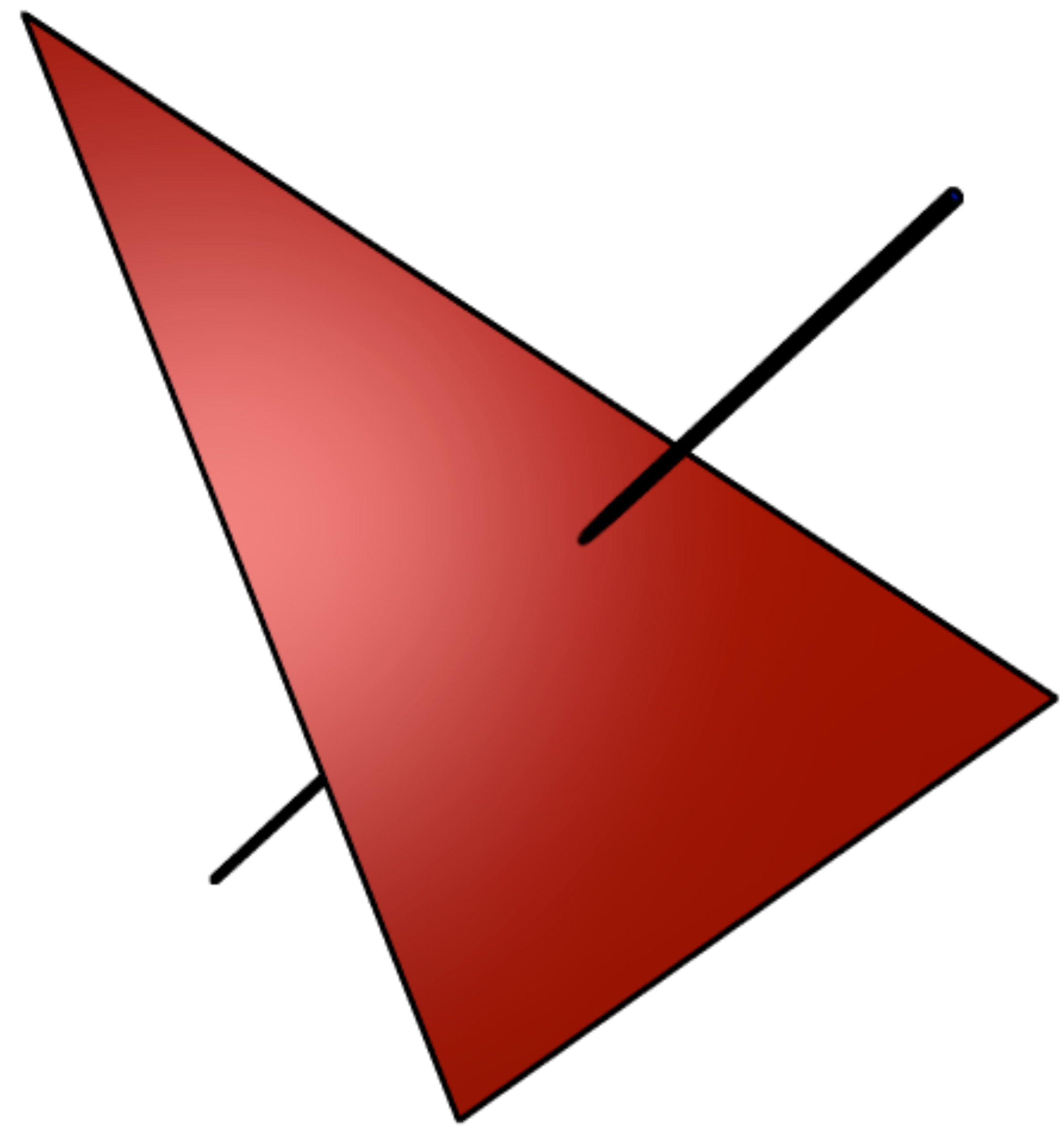
Solve for t…

triangles

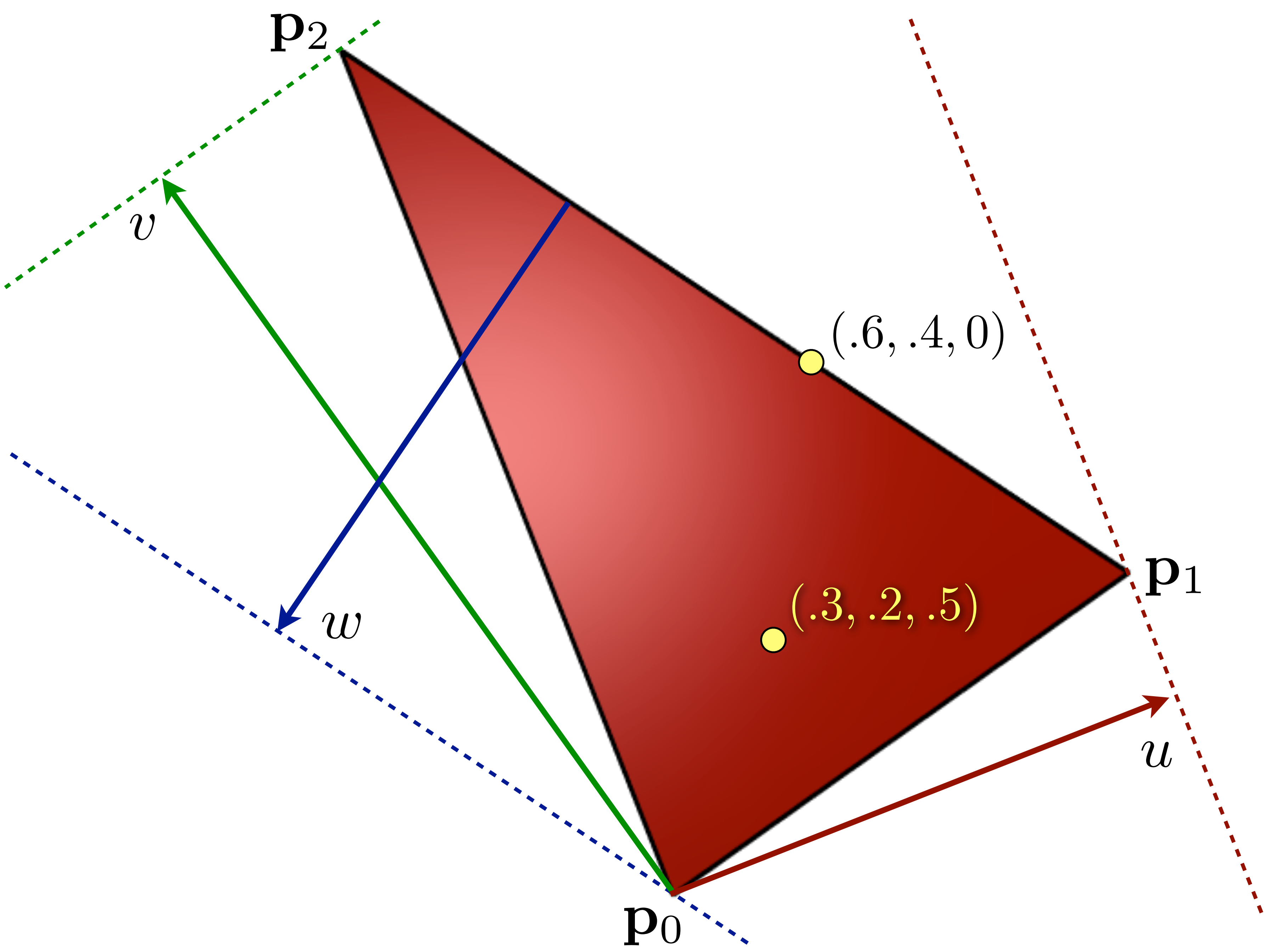


How might we test

intersection of a ray and a triangle?



Barycentric Coordinates



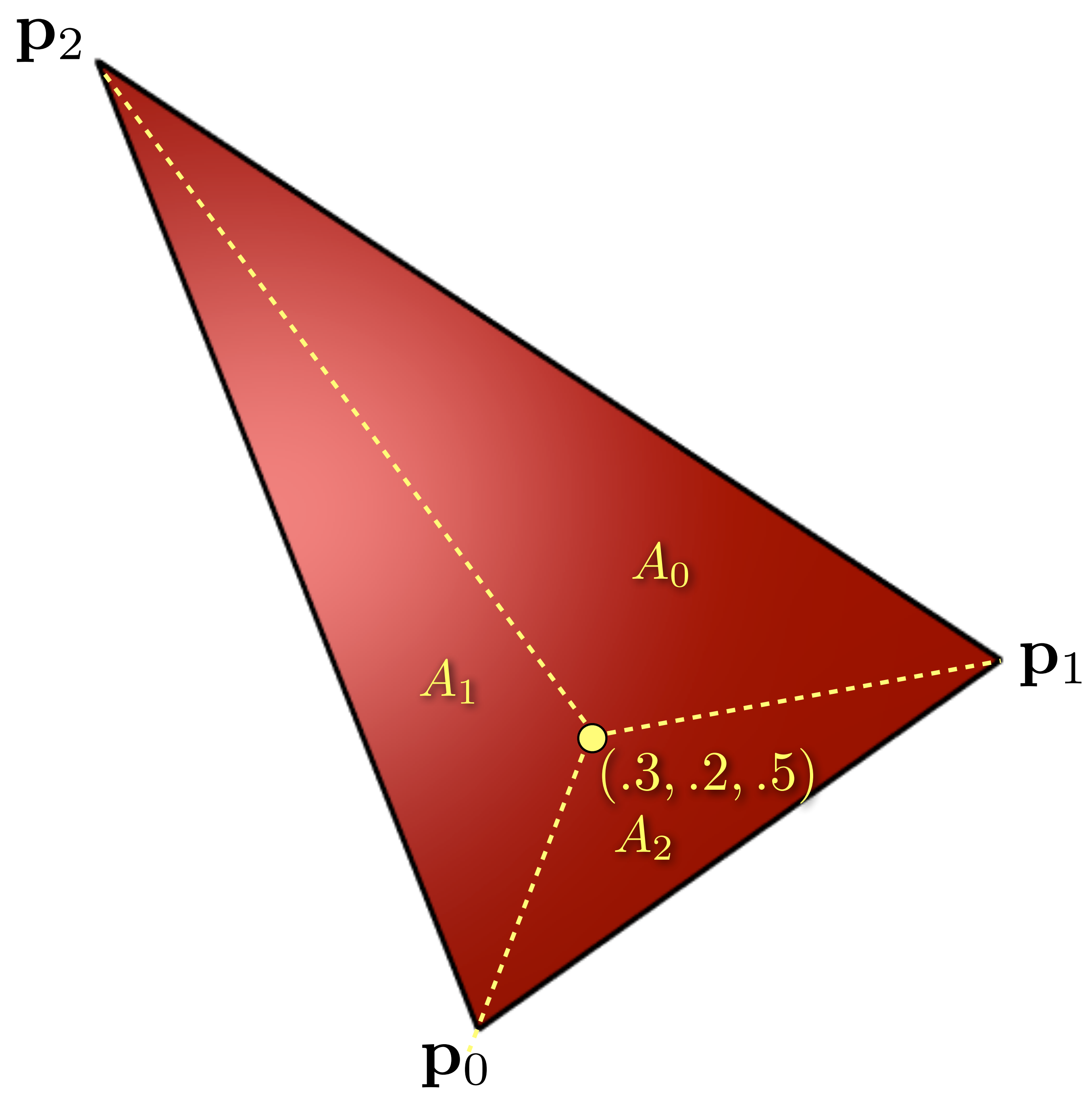
$$\mathbf{f}(u, v) = (1 - u - v)\mathbf{p}_0 + u\mathbf{p}_1 + v\mathbf{p}_2$$

Another Interpretation

Ratio of (signed) areas of triangles:

$$A = \tfrac{1}{2}(\mathbf{p}_1 - \mathbf{p}_0) \times (\mathbf{p}_2 - \mathbf{p}_0)$$

$$u = \frac{A_1}{A} \qquad v = \frac{A_2}{A}$$



$$\mathbf{f}(u, v) = (1 - u - v)\mathbf{p}_0 + u\mathbf{p}_1 + v\mathbf{p}_2$$

A Direct Approach for Intersection

A ray: $\mathbf{r}(t) = \mathbf{o} + t\mathbf{d}$

A triangle: $\mathbf{f}(u, v) = (1 - u - v)\mathbf{p}_0 + u\mathbf{p}_1 + v\mathbf{p}_2$

Ray-triangle intersect: $\mathbf{o} + t\mathbf{d} = (1 - u - v)\mathbf{p}_0 + u\mathbf{p}_1 + v\mathbf{p}_2$

Rearrange terms: $$\begin{pmatrix} -\mathbf{d} & \mathbf{p}_1 - \mathbf{p}_0 & \mathbf{p}_2 - \mathbf{p}_0 \end{pmatrix} \begin{pmatrix} t \\ u \\ v \end{pmatrix} = \mathbf{o} - \mathbf{p}_0$$

Solve for t, u, and v…

Cramer's Rule

- Given a set of linear equations in matrix form

$$\begin{pmatrix} \mathbf{a} & \mathbf{b} & \mathbf{c} \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \mathbf{d}$$

- Write the determinant of the matrix

$$\det(\mathbf{a}, \mathbf{b}, \mathbf{c}) = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

- Then the solutions are

$$x = \frac{\det(\mathbf{d}, \mathbf{b}, \mathbf{c})}{\det(\mathbf{a}, \mathbf{b}, \mathbf{c})} \qquad y = \frac{\det(\mathbf{a}, \mathbf{d}, \mathbf{c})}{\det(\mathbf{a}, \mathbf{b}, \mathbf{c})} \qquad z = \frac{\det(\mathbf{a}, \mathbf{b}, \mathbf{d})}{\det(\mathbf{a}, \mathbf{b}, \mathbf{c})}$$

A Direct Approach for Intersection

Our equation:

$$\begin{pmatrix} -\mathbf{d} & \mathbf{p}_1 - \mathbf{p}_0 & \mathbf{p}_2 - \mathbf{p}_0 \end{pmatrix} \begin{pmatrix} t \\ u \\ v \end{pmatrix} = \mathbf{o} - \mathbf{p}_0$$

Applying Cramer's rule:

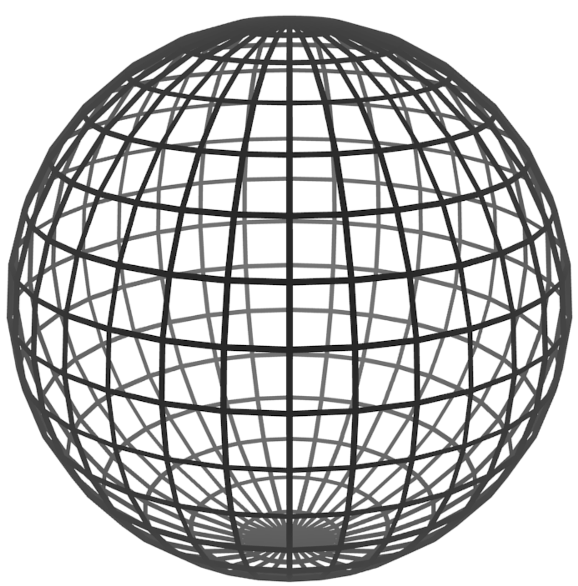
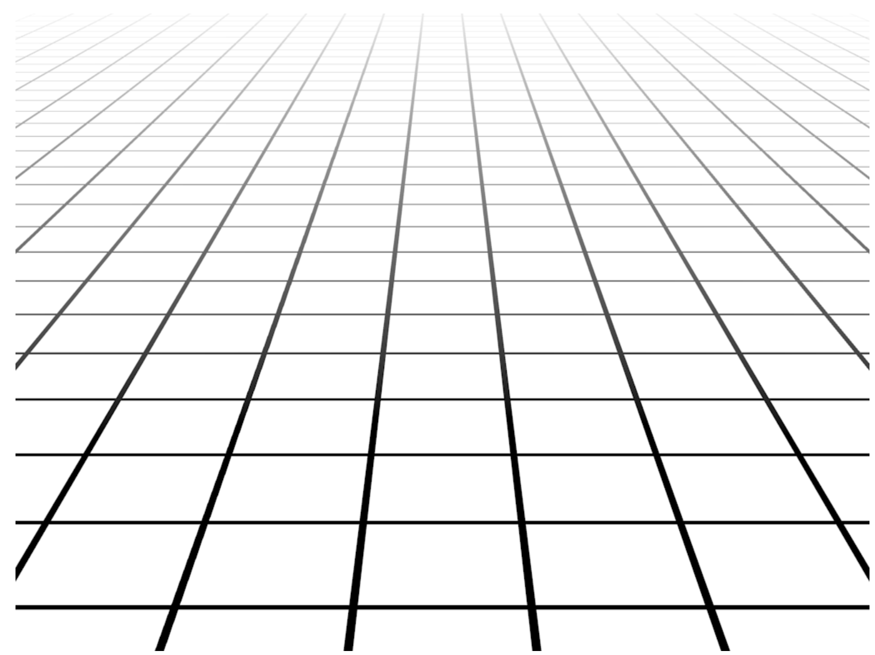
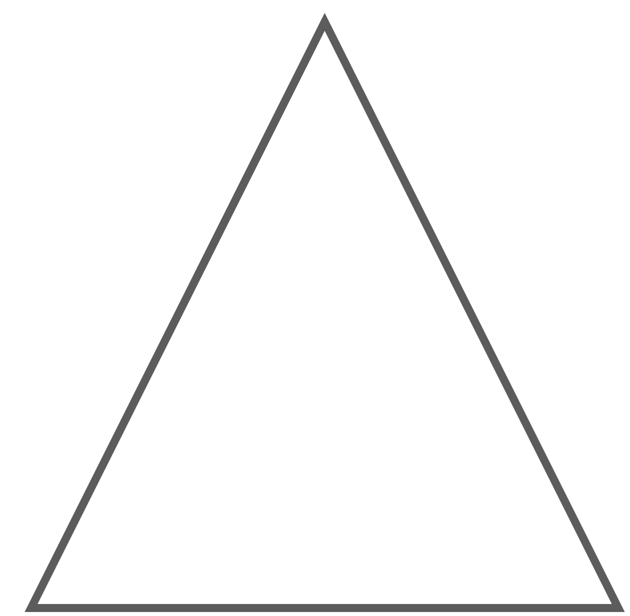
$$\begin{pmatrix} t \\ u \\ v \end{pmatrix} = \frac{1}{\det(-\mathbf{d}, \mathbf{e}_1, \mathbf{e}_2)} \begin{pmatrix} \det(\mathbf{s}, \mathbf{e}_1, \mathbf{e}_2) \\ \det(-\mathbf{d}, \mathbf{s}, \mathbf{e}_2) \\ \det(-\mathbf{d}, \mathbf{e}_1, \mathbf{s}) \end{pmatrix}$$

where $\mathbf{e}_1 = \mathbf{p}_1 - \mathbf{p}_0, \ \mathbf{e}_2 = \mathbf{p}_2 - \mathbf{p}_0, \ \mathbf{s} = \mathbf{o} - \mathbf{p}_0$

Check that t > 0 and u, v, u+v are within [0,1] interval!

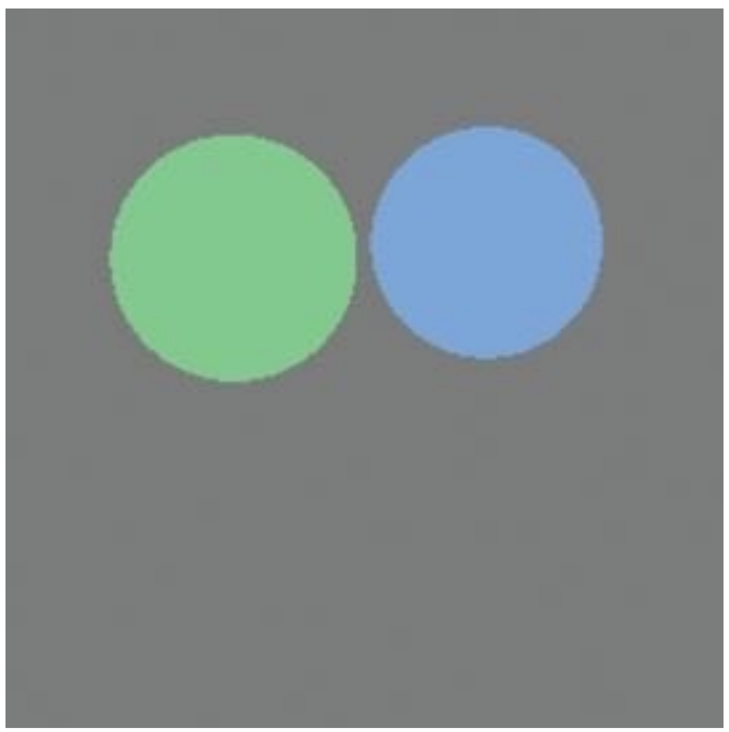
Ray-Object Intersection Tests

- Now we know how to draw spheres, planes, and triangles in perspective with a ray tracer!

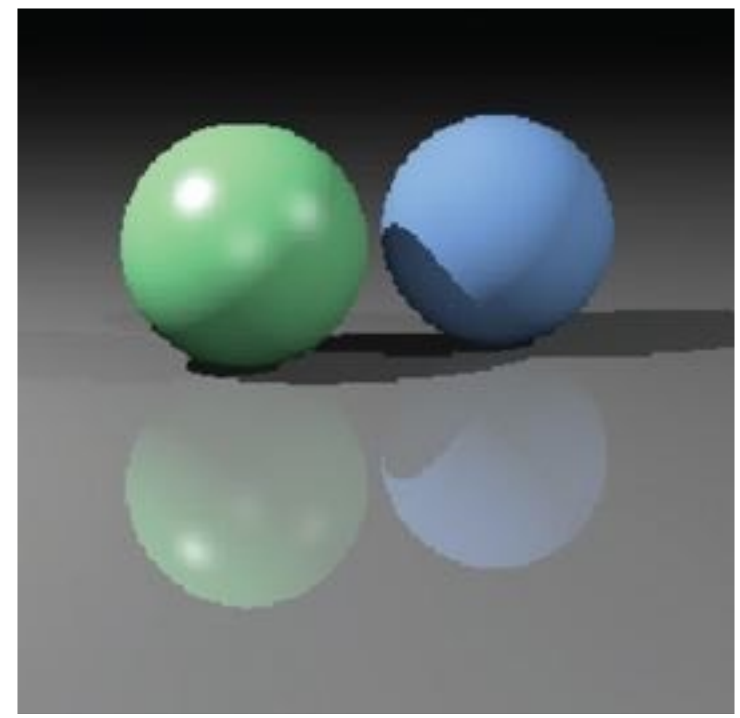




- What else can we do?

I think we need some shading…

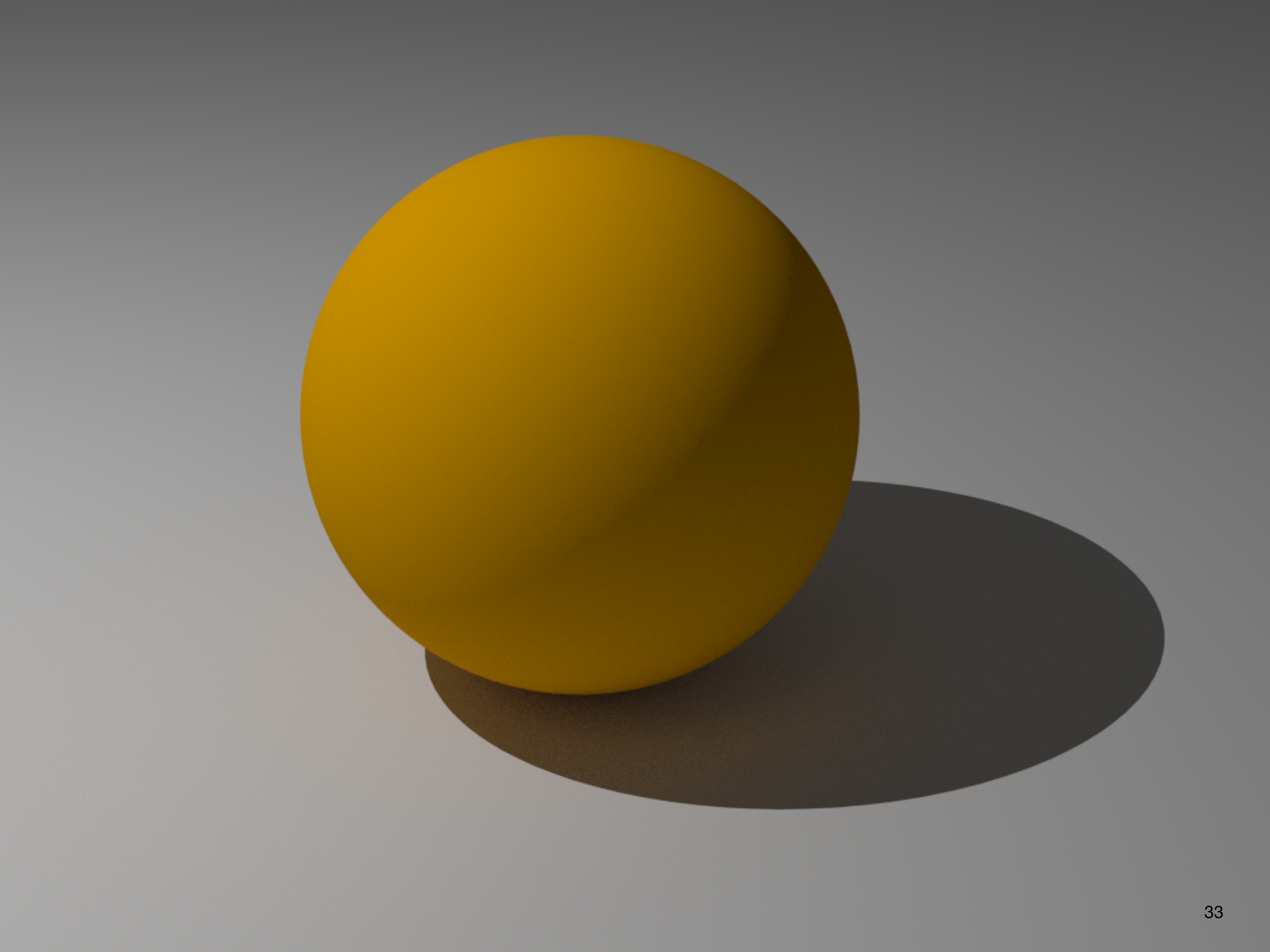


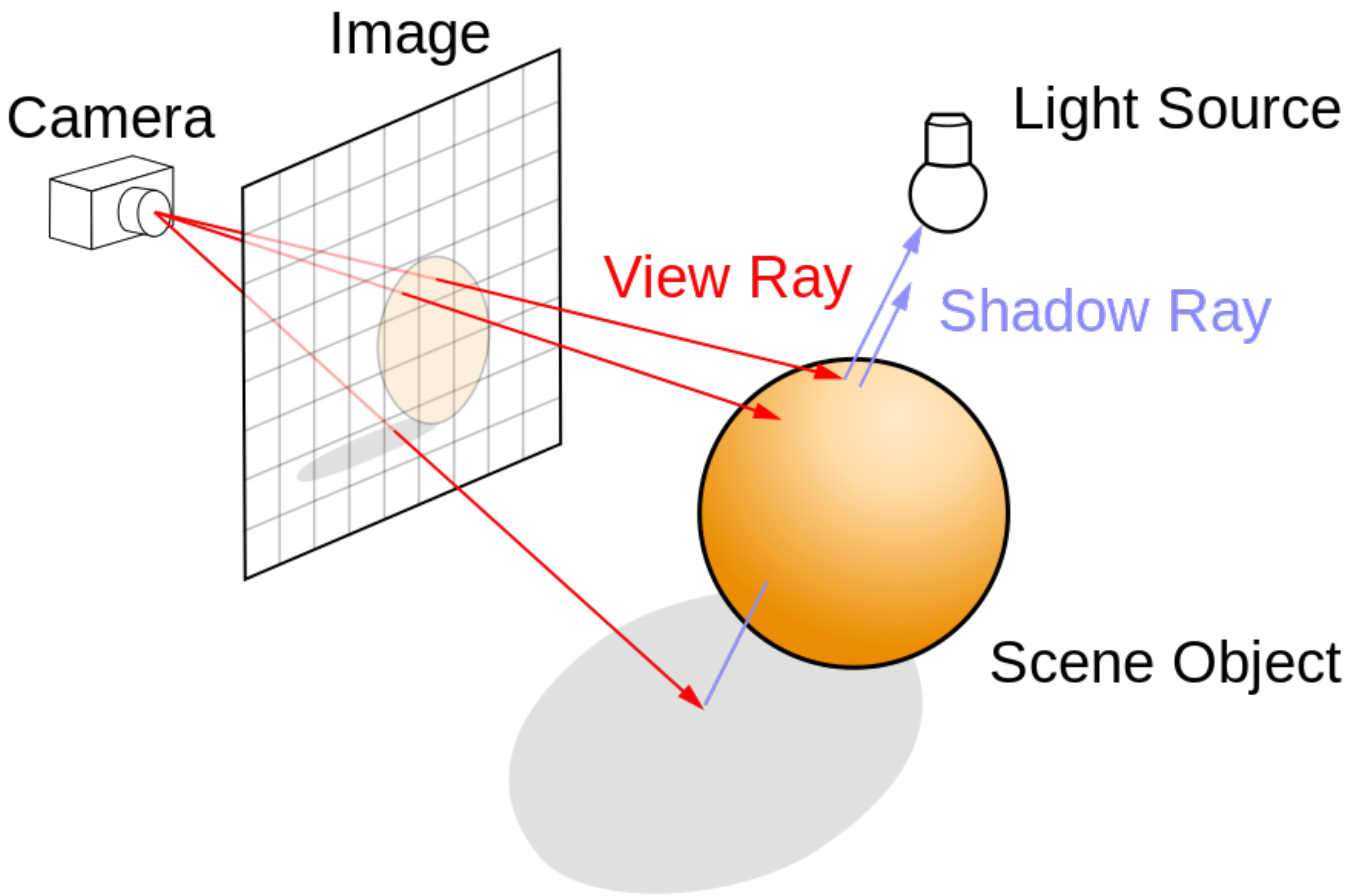
What we've got now…



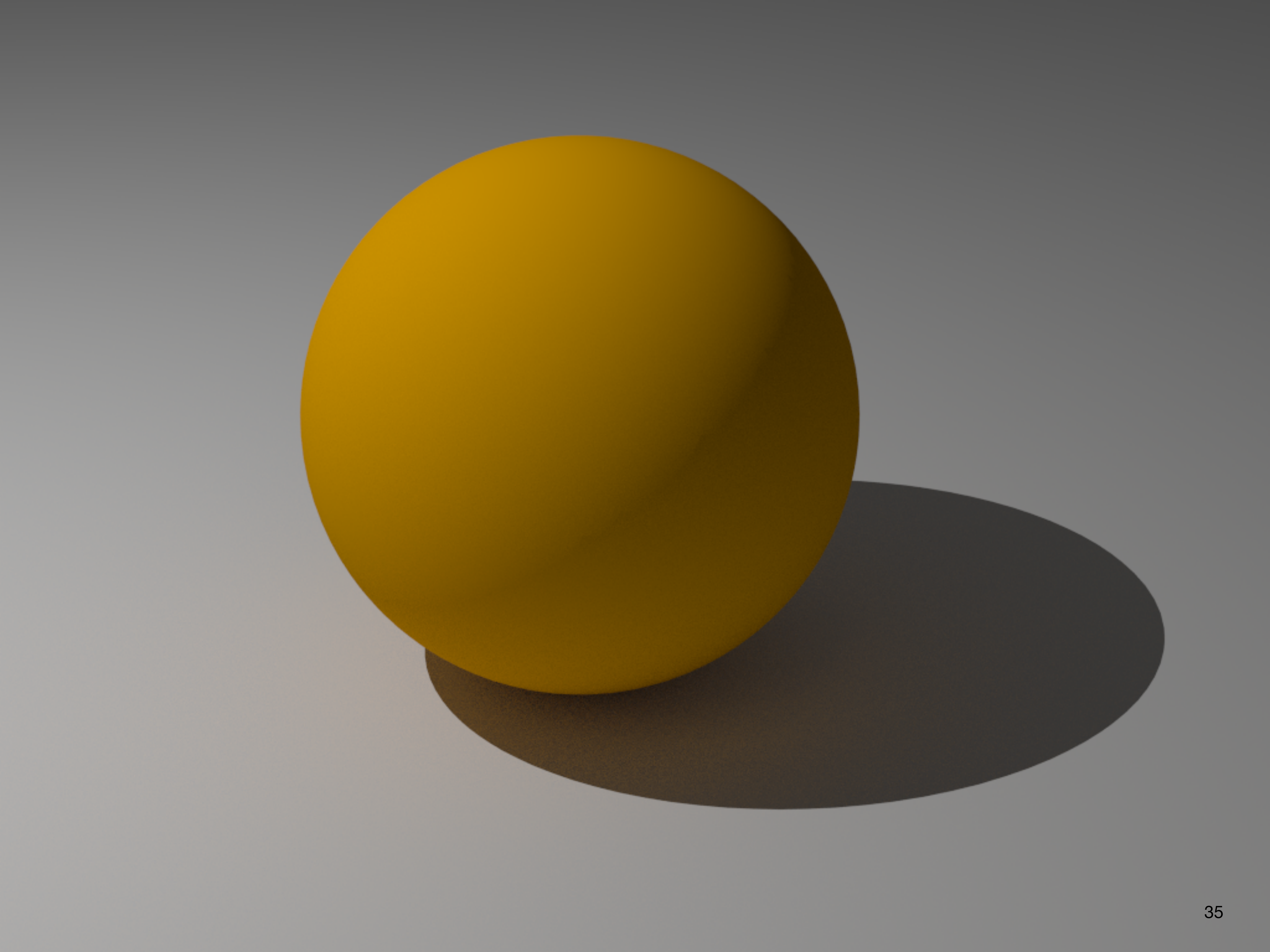
and what we really want!

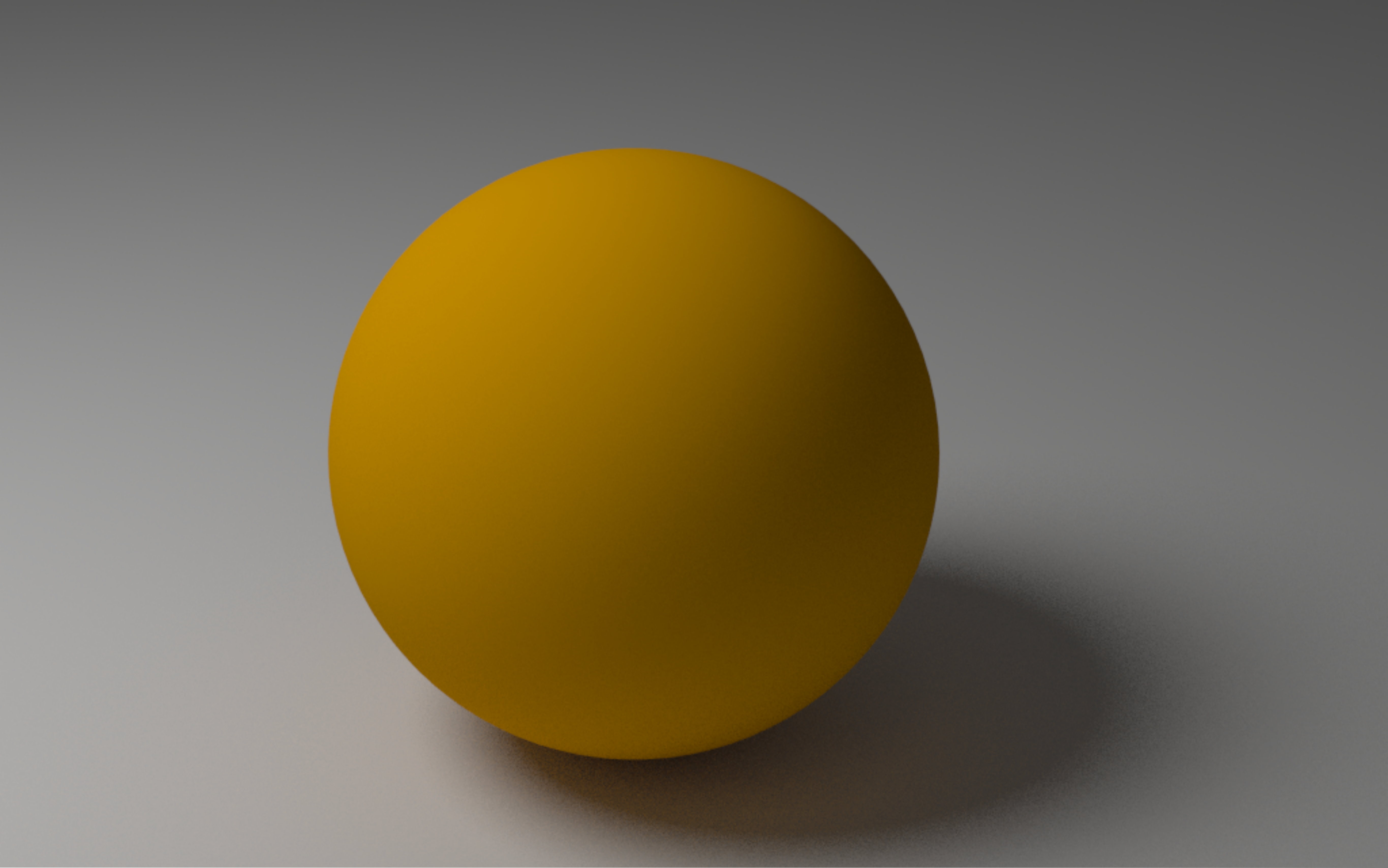
Shadows & More





What happens if we have more than one light?

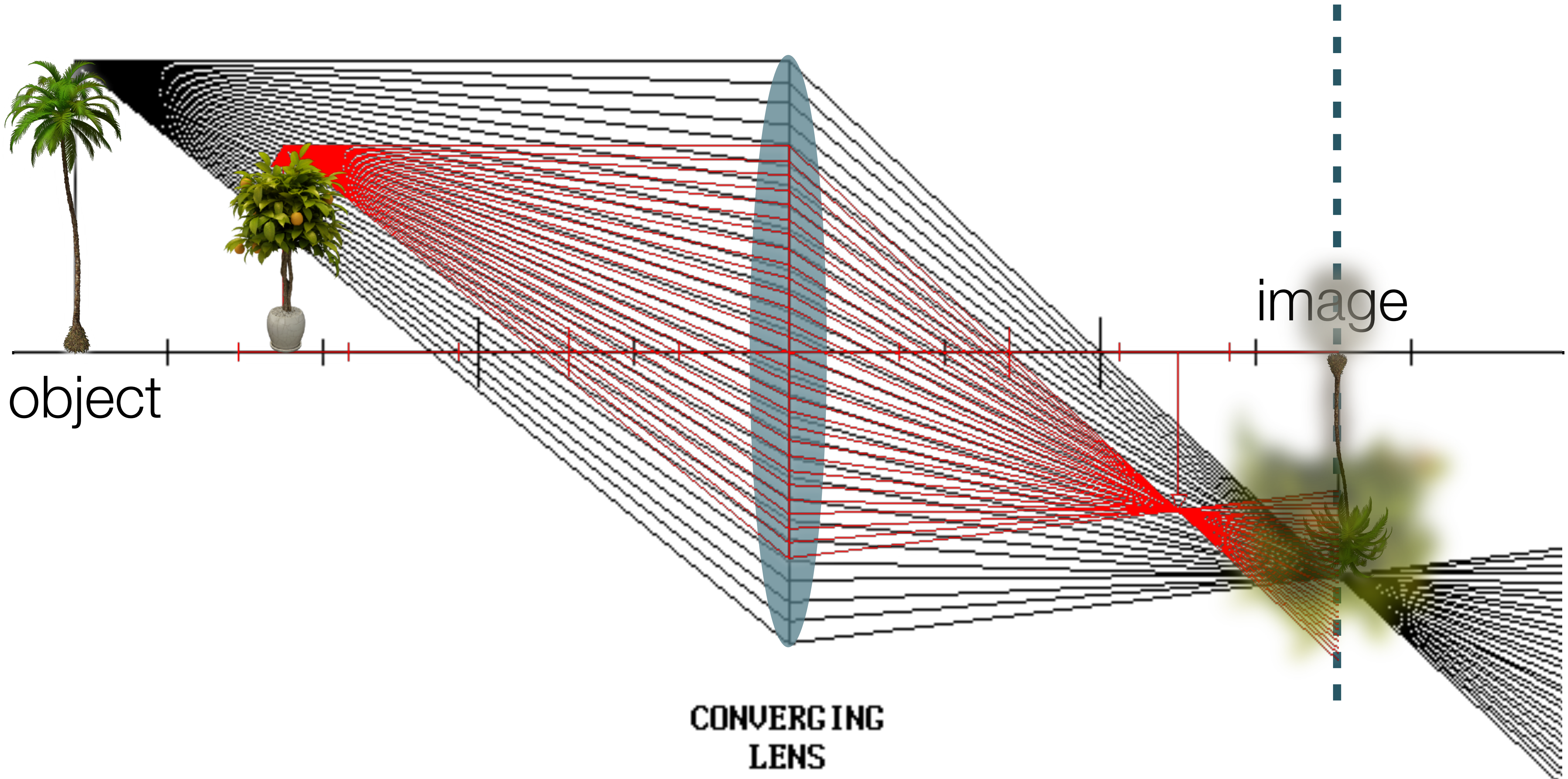




How do we get these nice, soft shadows?

Depth of Field

Can we ray trace it?



object
image
CONVERGING
LENS

Things to Remember

- Ray tracing is a relatively simple way for us to synthesize images of a 3D scene in perspective
    - simulates the optics of a pinhole camera
- Each image pixel generates a parametric viewing ray
- A ray-object intersection test is needed to render each kind of object geometry in the scene
- Some effects like shadows are relatively easy to compute