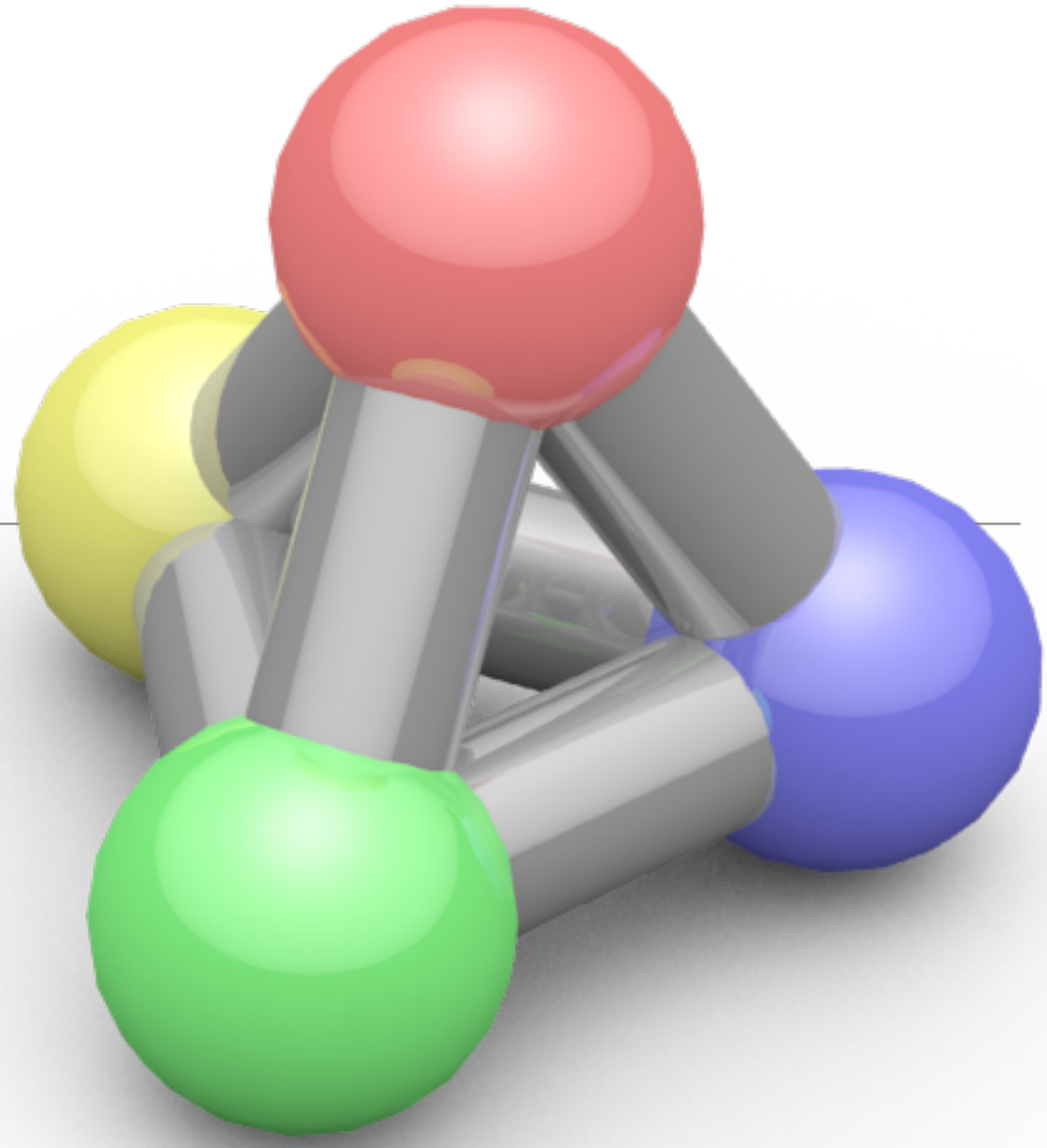
Interpolation



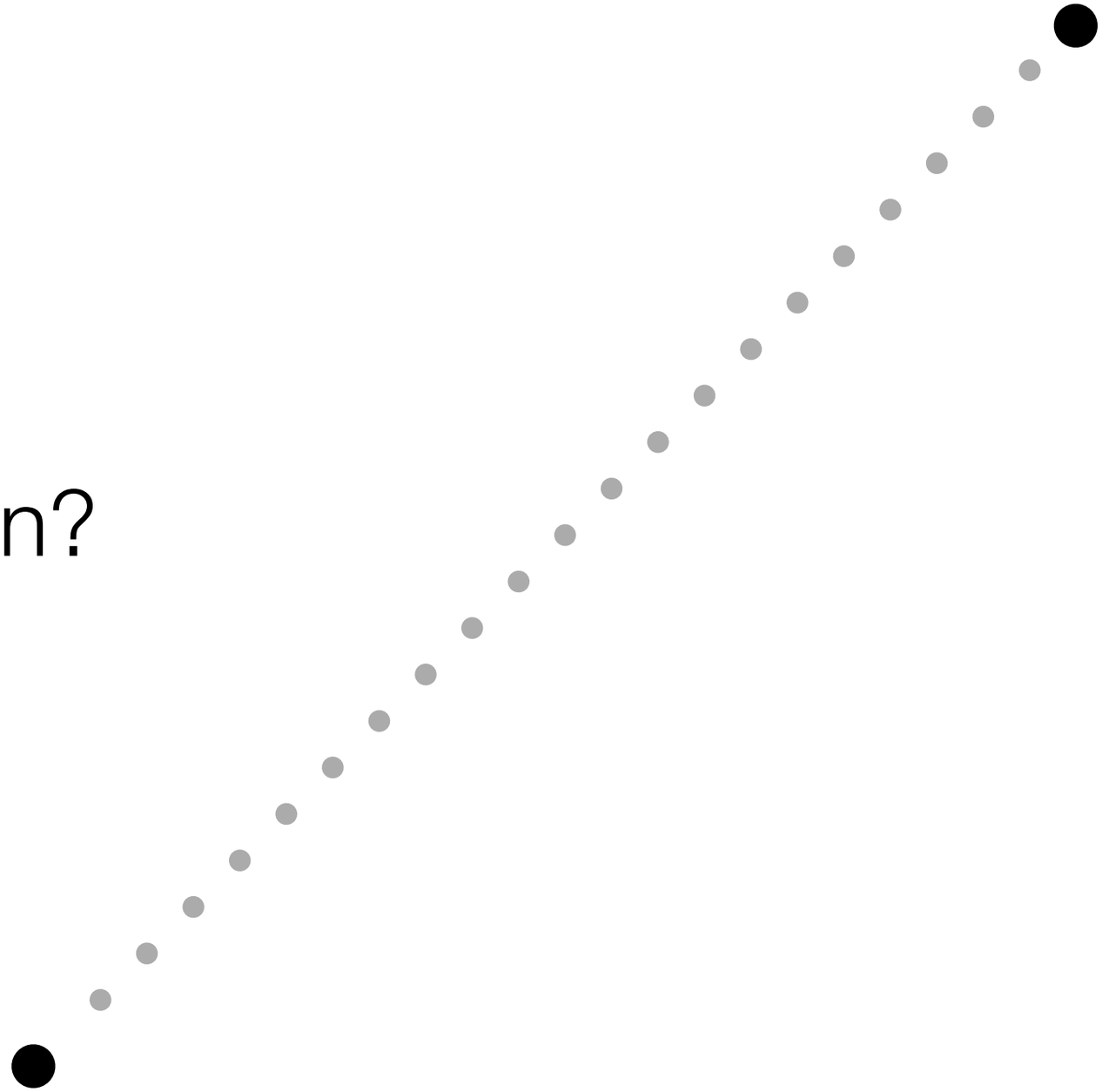
CPSC 453 – Fall 2016
Sonny Chan

Today's Outline

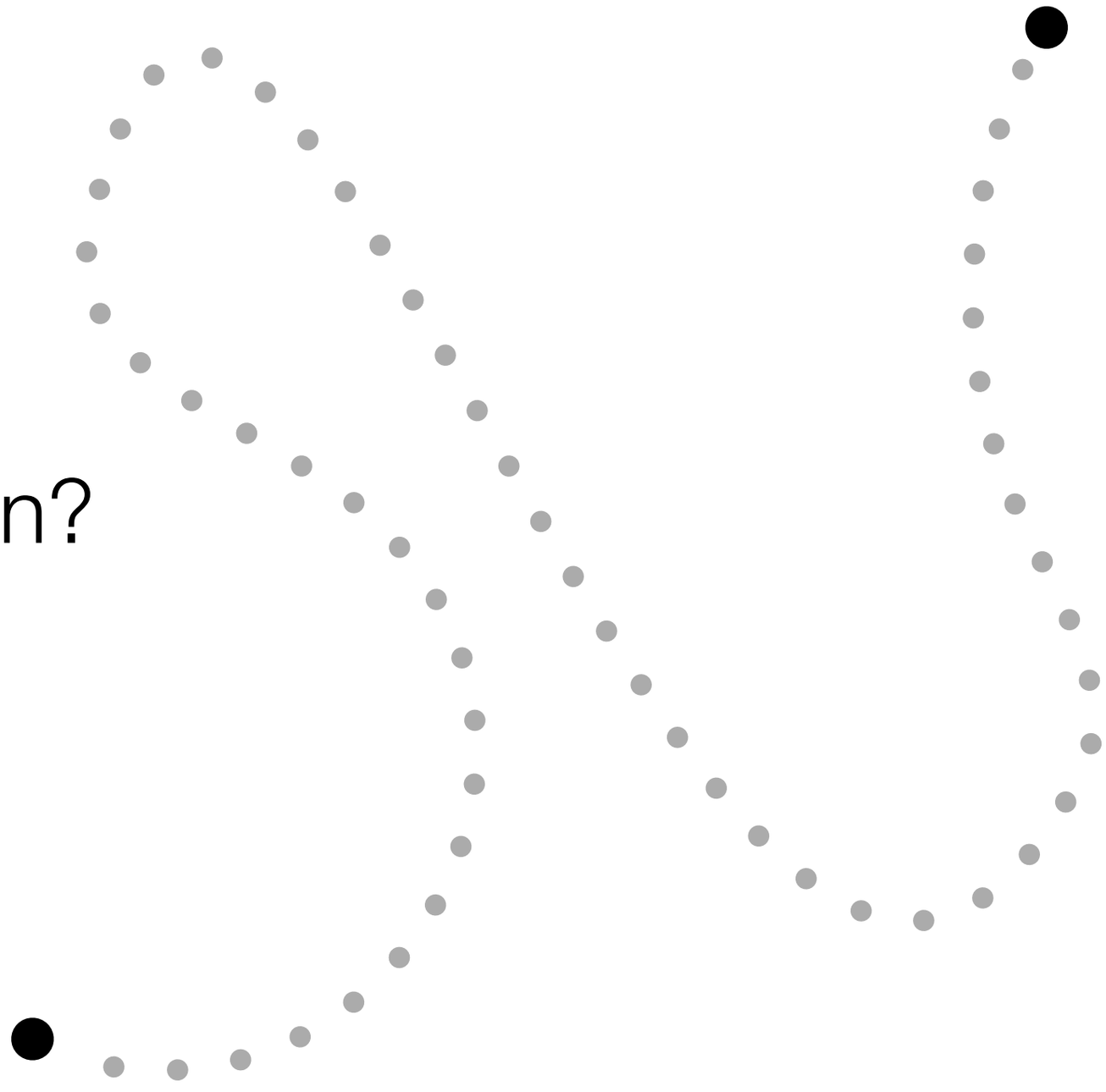
- One-dimensional linear interpolation
- Examples of interpolation
- Bi-linear interpolation

What exactly is interpolation?

Is this interpolation?



Is this interpolation?



Interpolation is finding “in-between” values.

–(don’t quote me on this one)

Linear Interpolation

- Basic form:

$$R = (1-t)A + tB,\ t \in [0,1]$$

- In code:

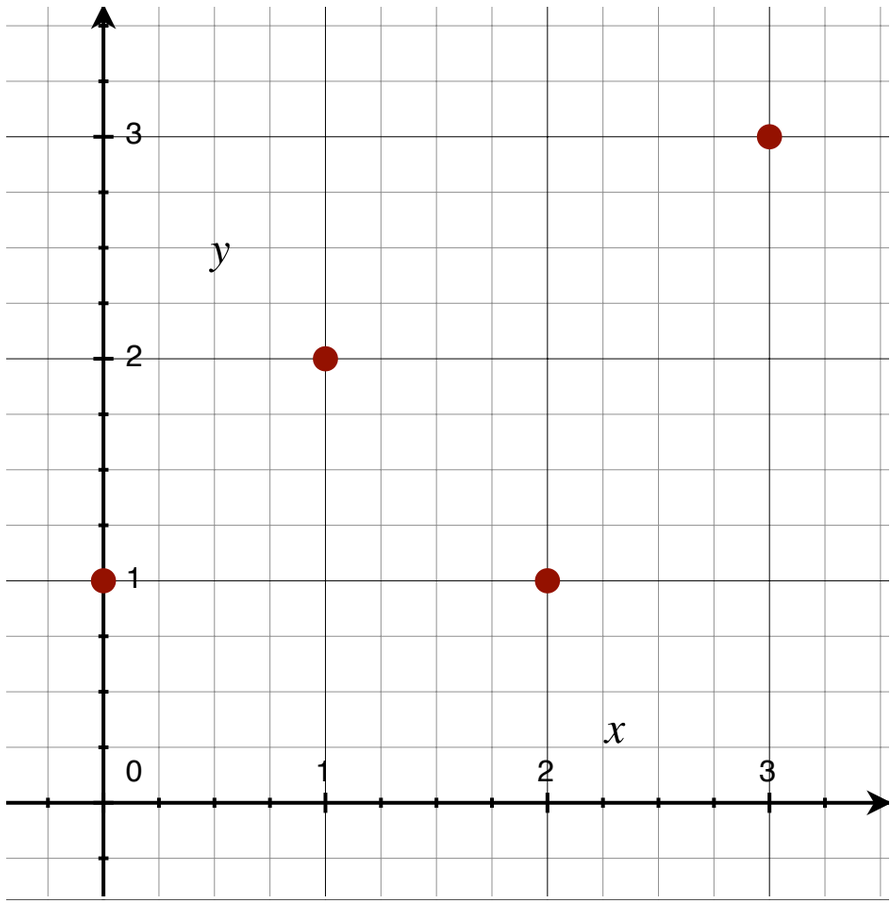
```
lerp(A, B, t) { return (1-t)*A + t*B; }  // C++
mix(x, y, a)  { return x*(1-a) + y*a; }  // glsl
```

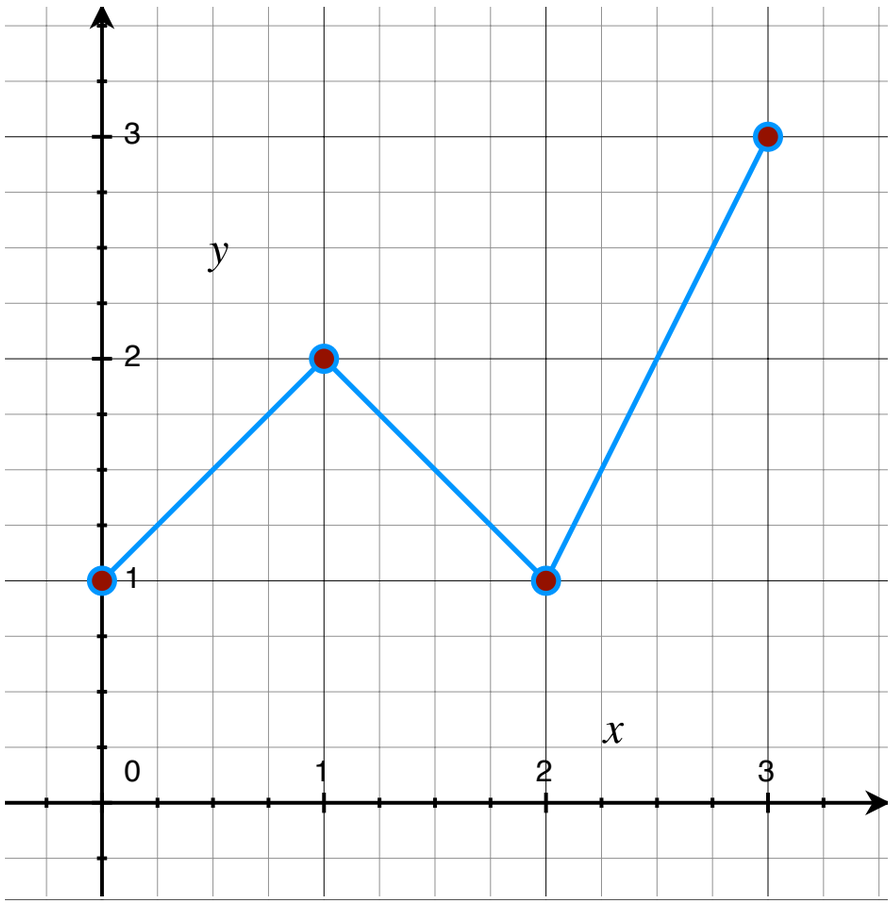
- Possibly the most used function in computer graphics!

Basic Example

- Convert a discrete function to a continuous one:

$$f(x) : \mathbb{Z} \mapsto \mathbb{R} \quad \Rightarrow \quad g(x) : \mathbb{R} \mapsto \mathbb{R}$$

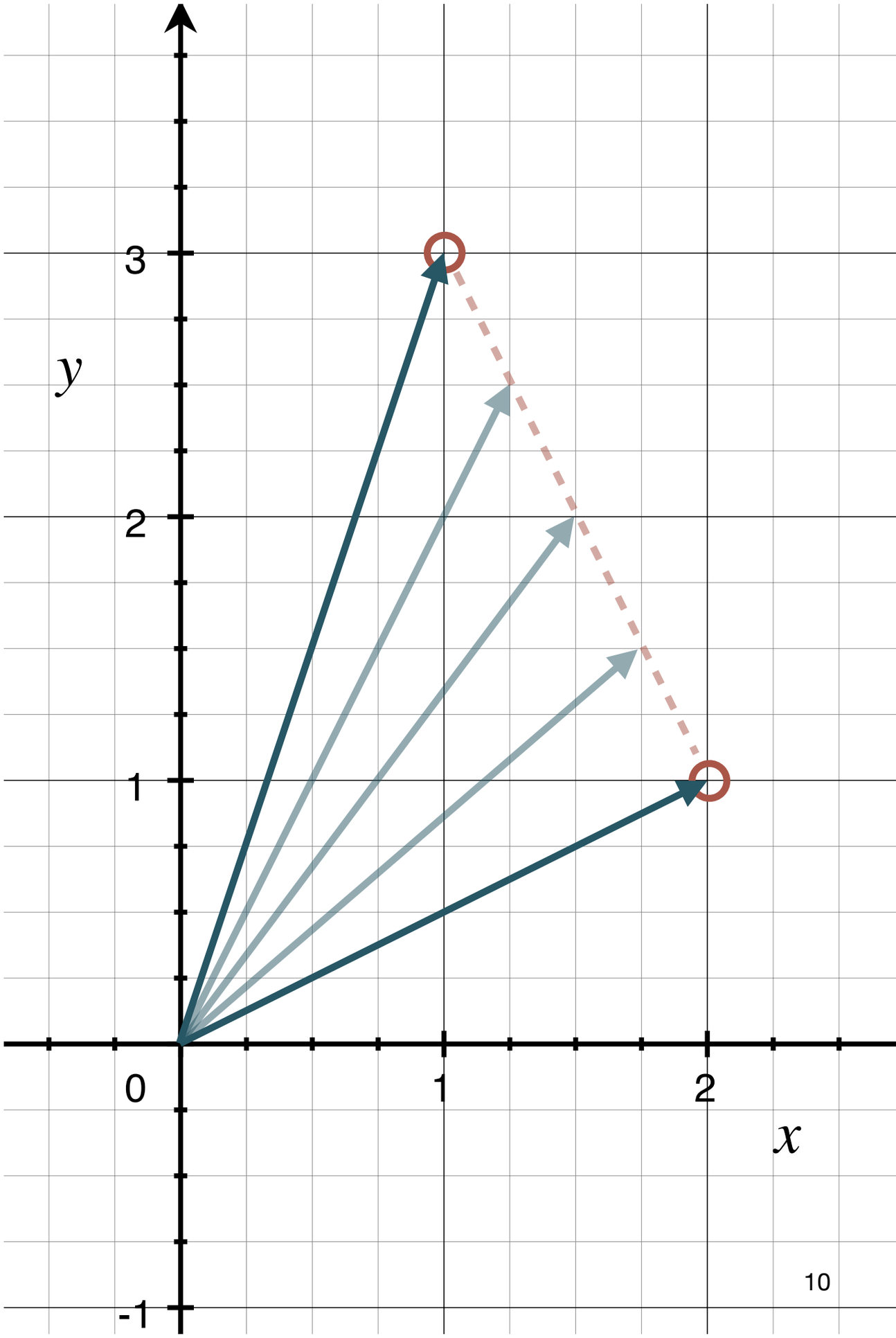




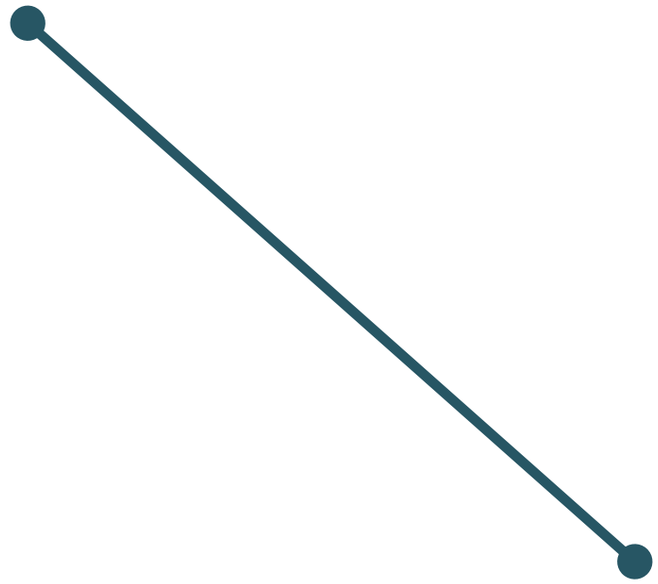
What other kinds of things can we interpolate?

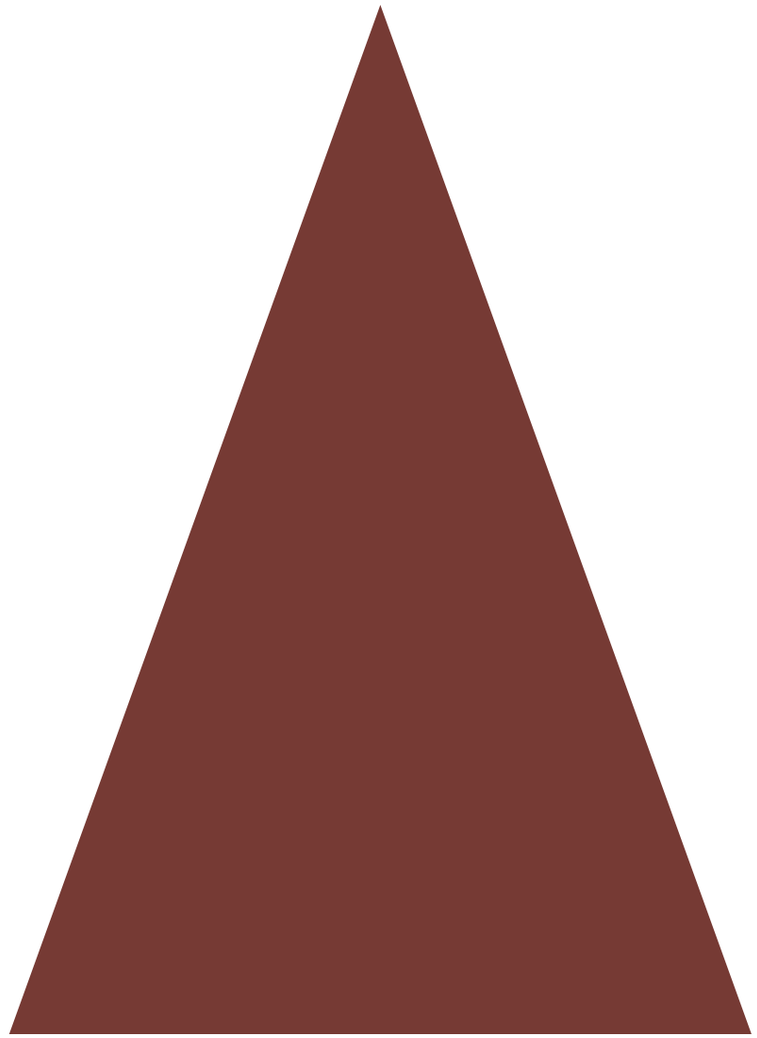
Vectors & Points

(though we technically shouldn't be adding points)

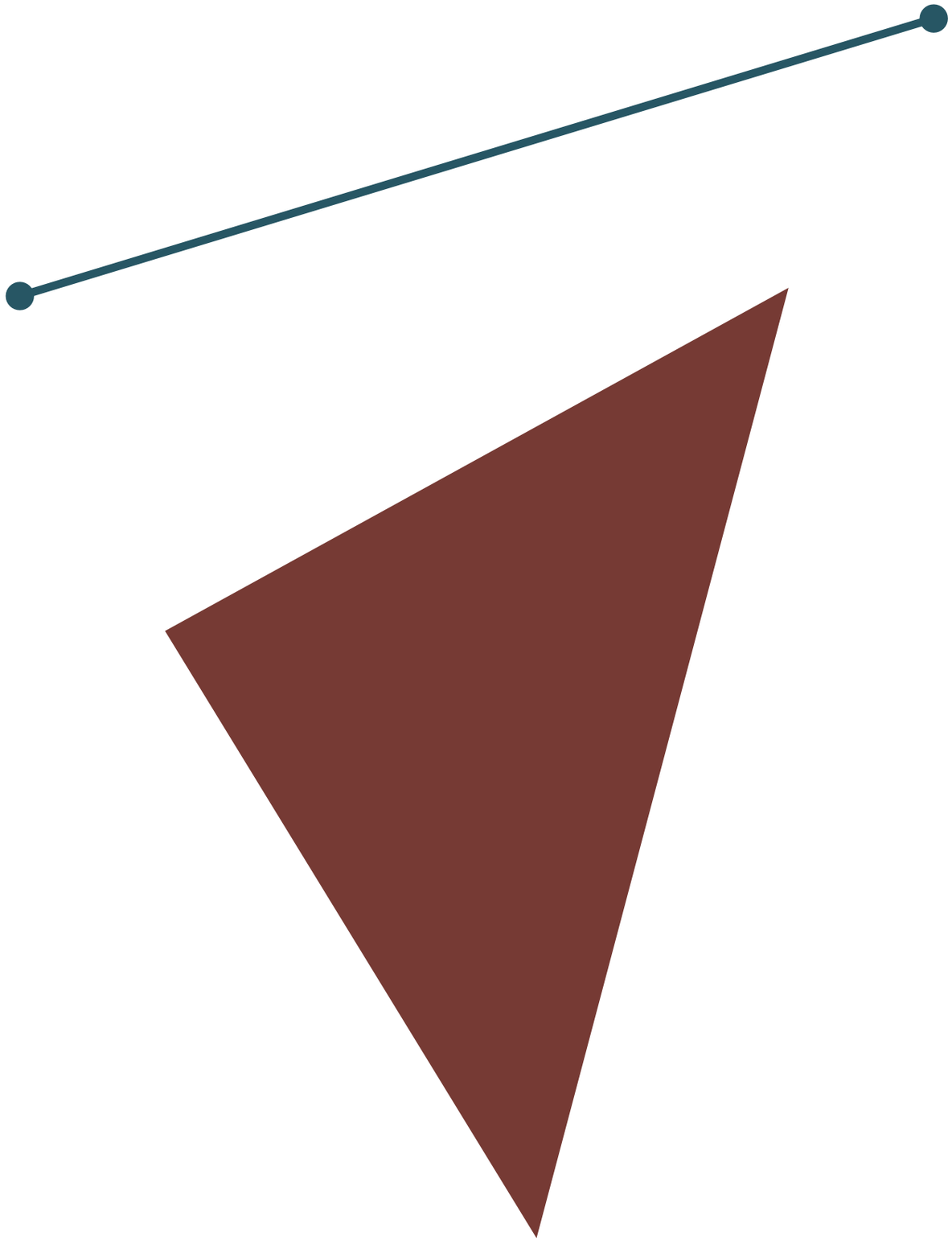


Lines & Triangles?

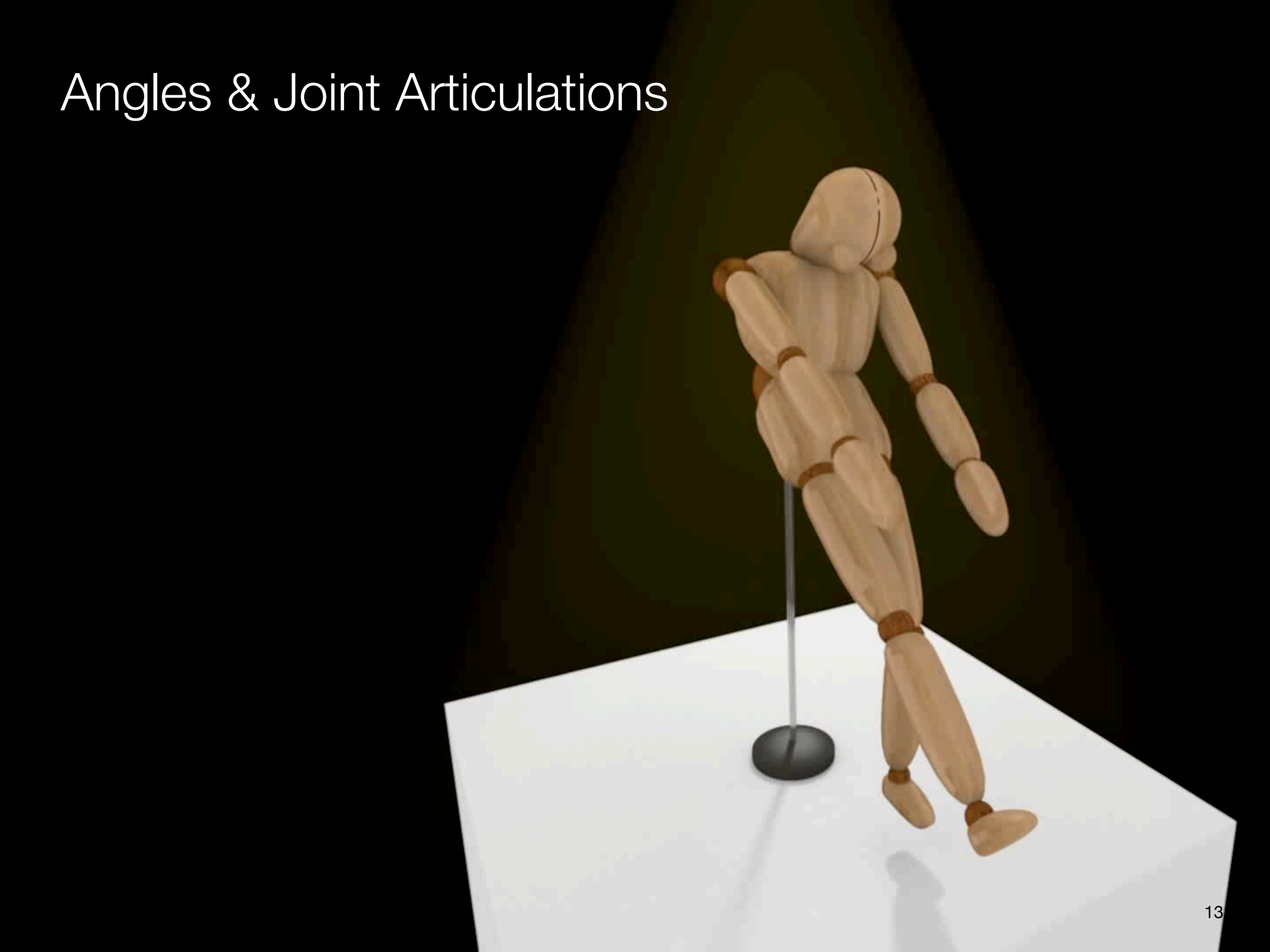




Lines & Triangles?



Angles & Joint Articulations

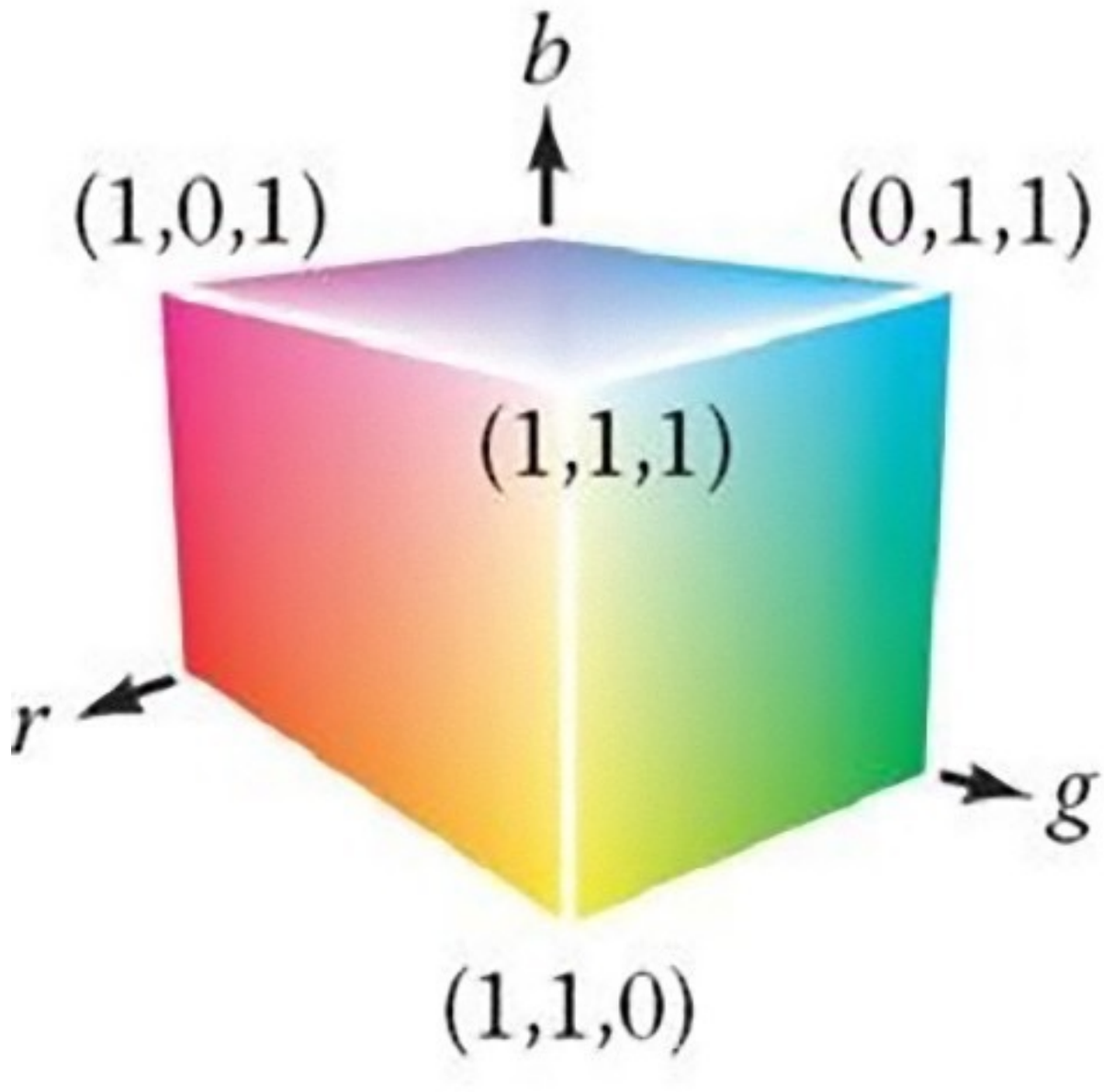


Orientations

and camera angles

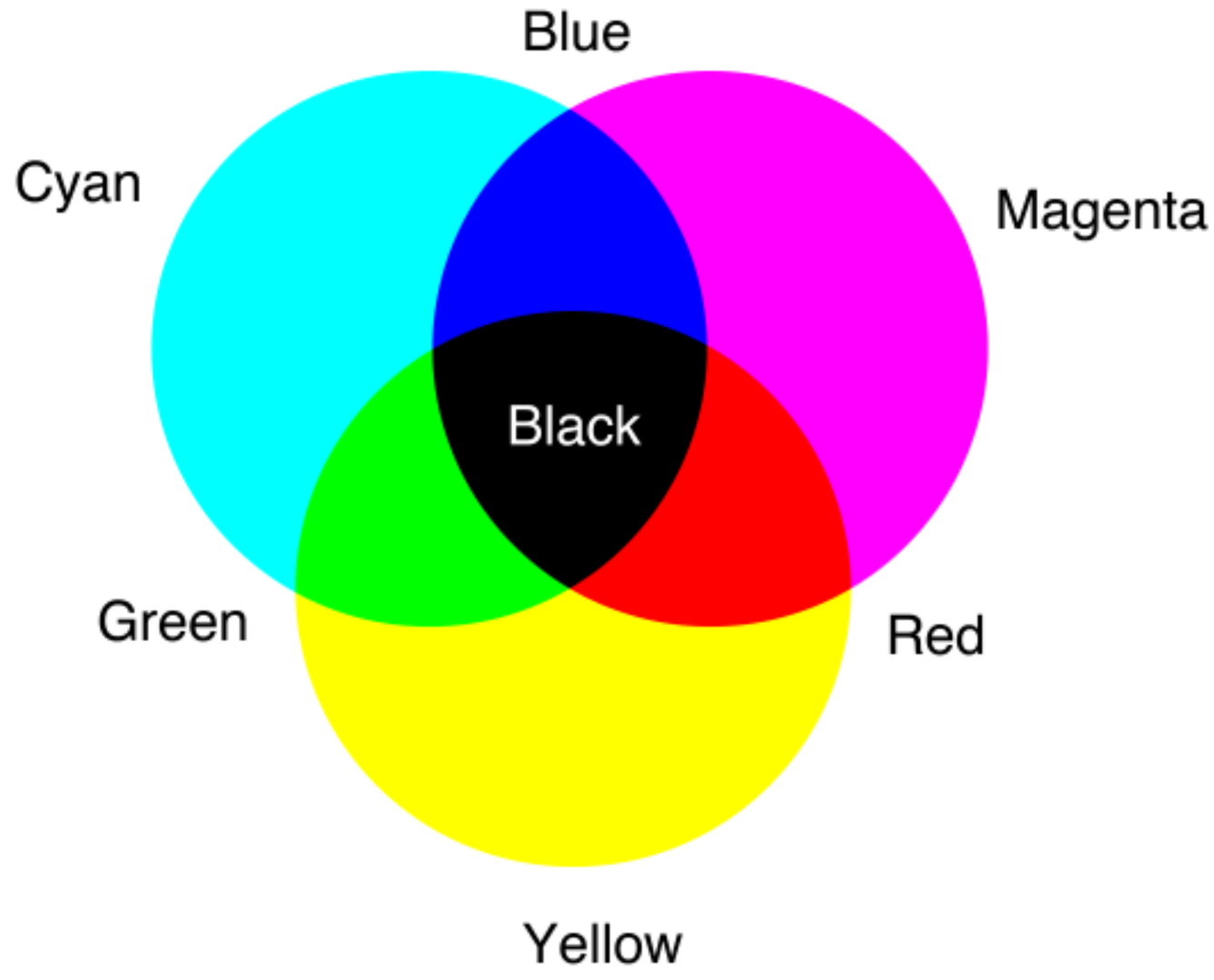
Colours

- What does it mean to add two colours?
- Does component-wise interpolation make sense?
  - $\mathbf{R}_C = (1-t)\,\mathbf{R}_A + t\,\mathbf{R}_B$
  - $\mathbf{G}_C = (1-t)\,\mathbf{G}_A + t\,\mathbf{G}_B$
  - $\mathbf{B}_C = (1-t)\,\mathbf{B}_A + t\,\mathbf{B}_B$



Not so fast!

What about CMYK?



What other kinds of things can we interpolate?

What interpolations did you observe?

features / lines

displacement vectors

colours

image pixels

time progression

Feature-Based Image Metamorphosis

[from T. Beier & S. Neely, *ACM SIGGRAPH*, 1992]

Feature-Based Image Metamorphosis

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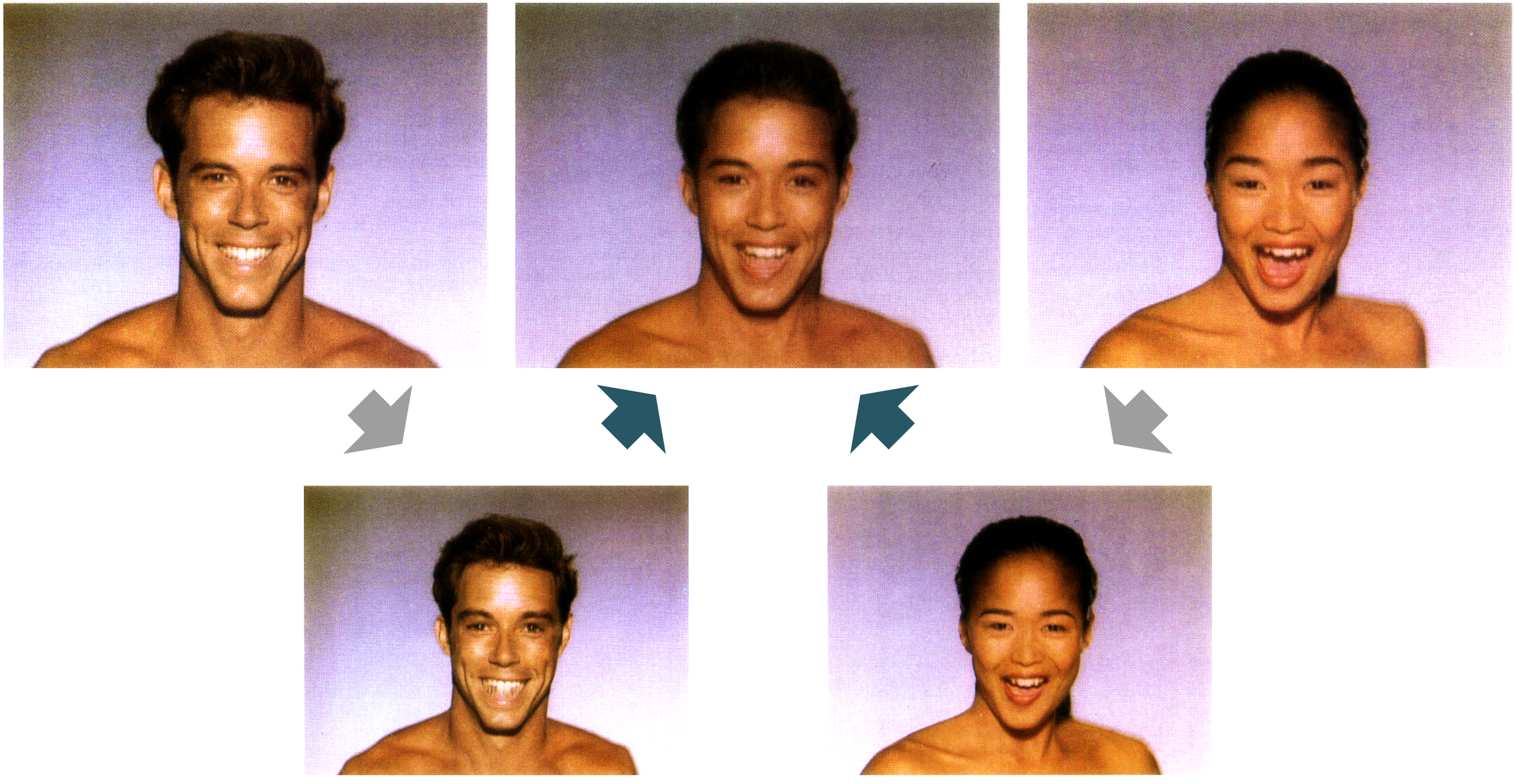
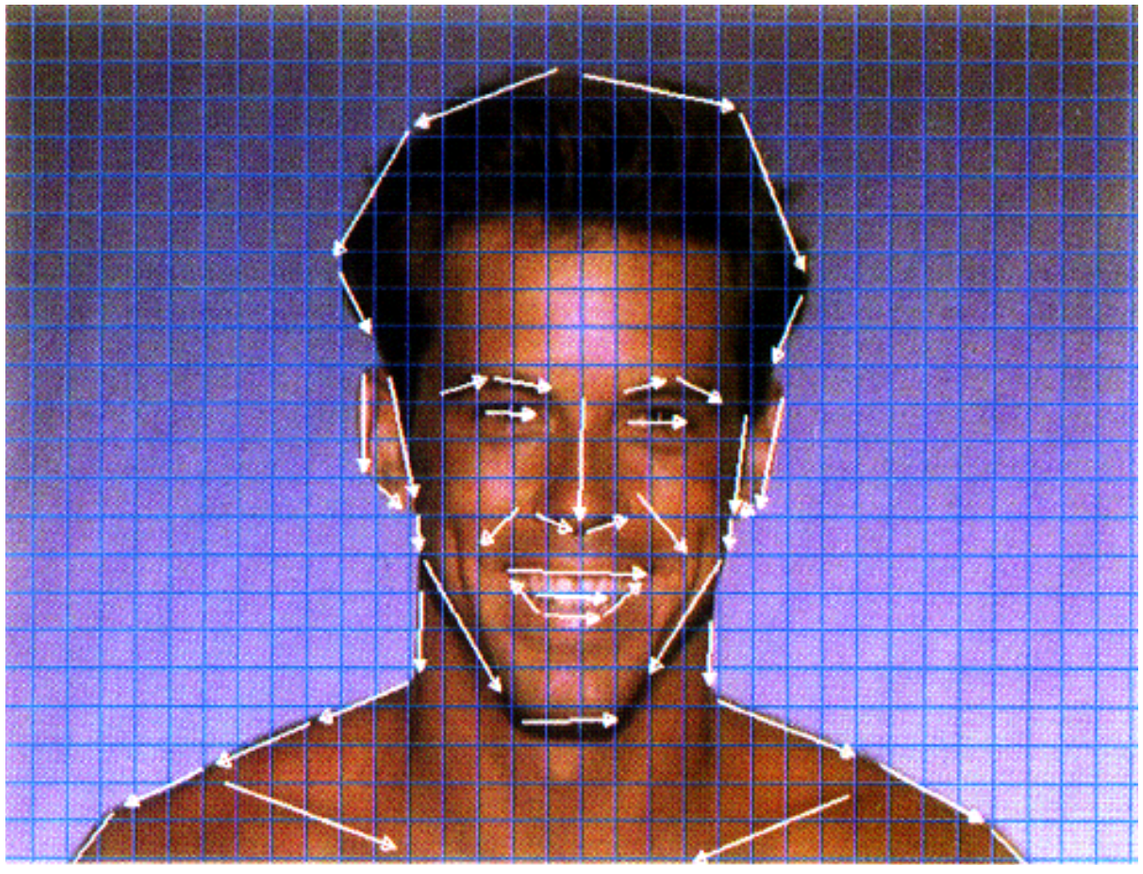
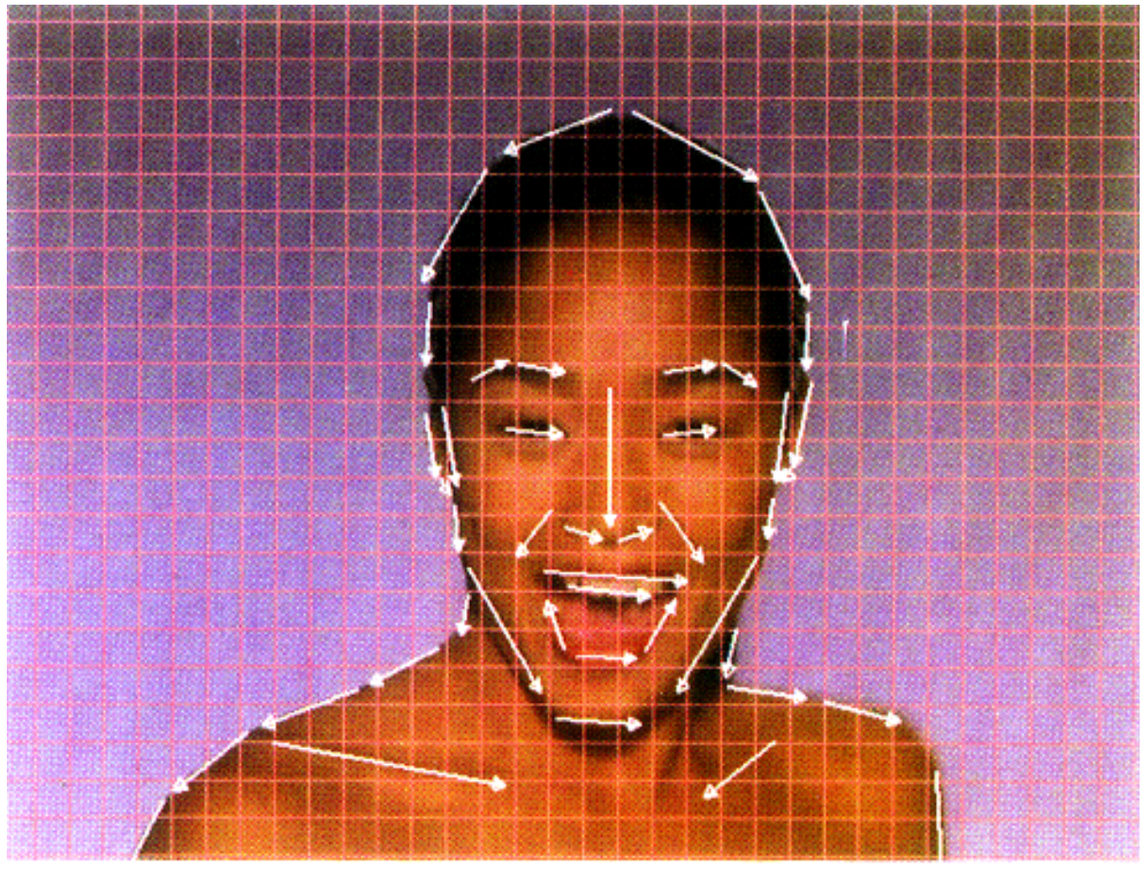
Feature-Based Image Metamorphosis



[from T. Beier & S. Neely, *ACM SIGGRAPH*, 1992]

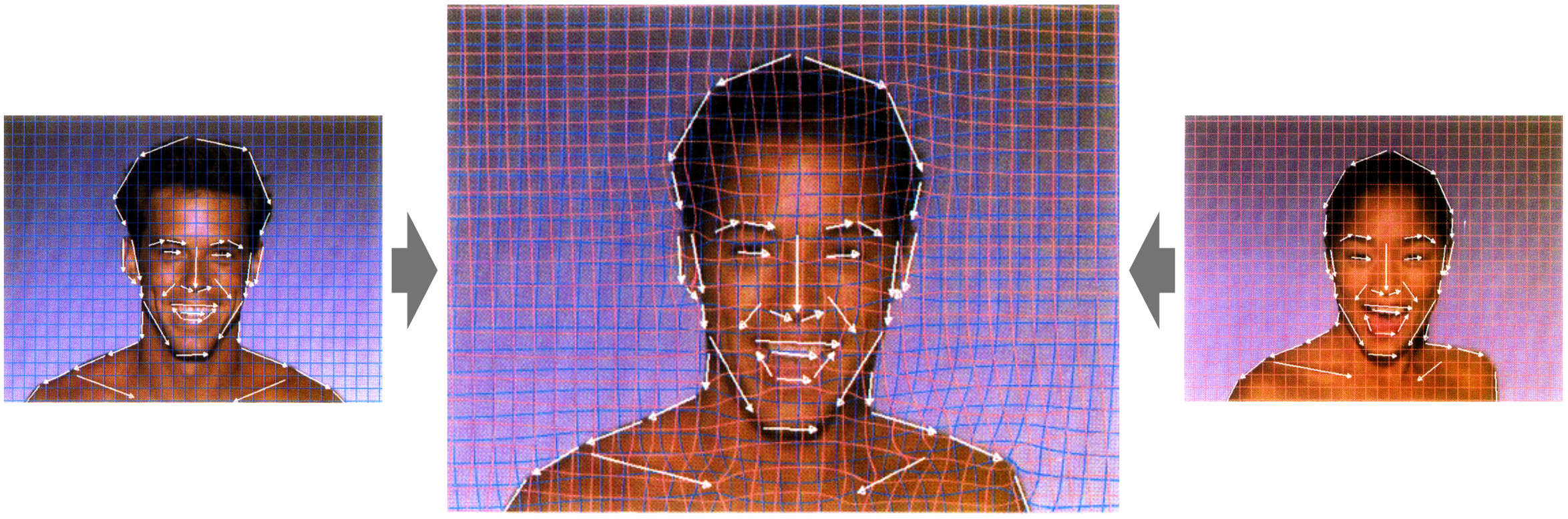
Feature-Based Image Metamorphosis





[from T. Beier & S. Neely, *ACM SIGGRAPH*, 1992]

Feature-Based Image Metamorphosis



[from T. Beier & S. Neely, *ACM SIGGRAPH*, 1992]

Two-Dimensional Interpolation

- Convert a discrete 2D function to a continuous one:

$$f(x, y) : \mathbb{Z}^2 \mapsto \mathbb{R}^2 \quad \Rightarrow \quad g(x, y) : \mathbb{R}^2 \mapsto \mathbb{R}^2$$

- When would you want to do this?

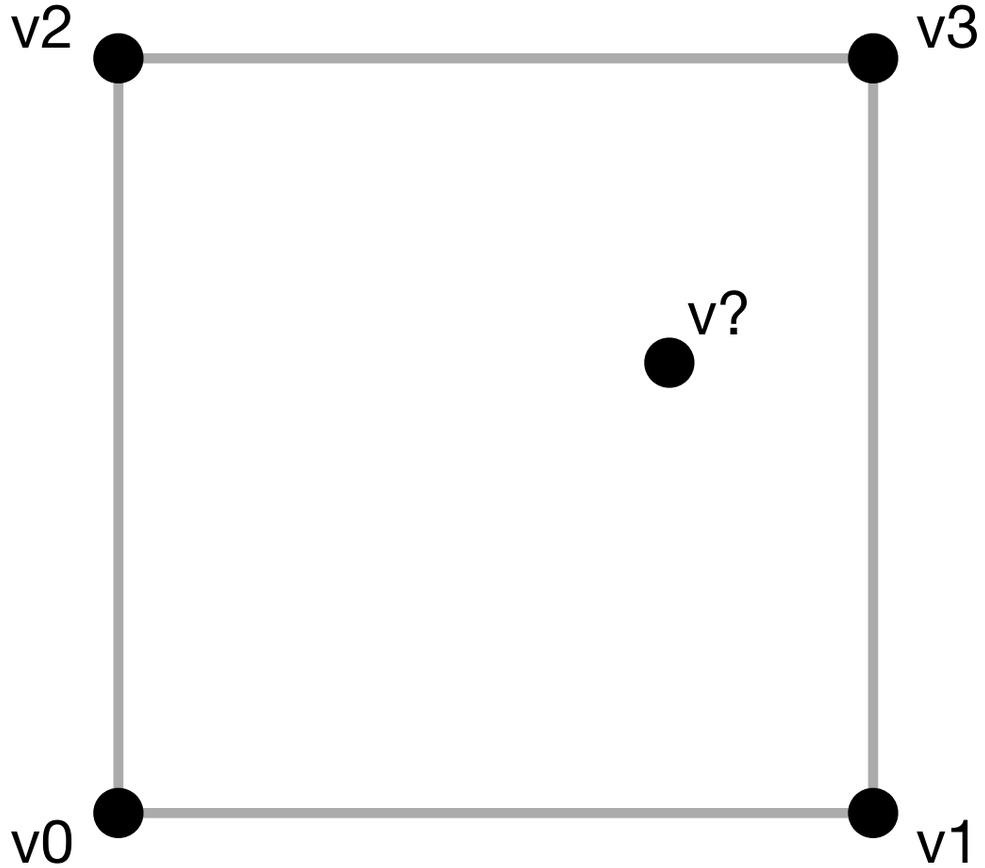
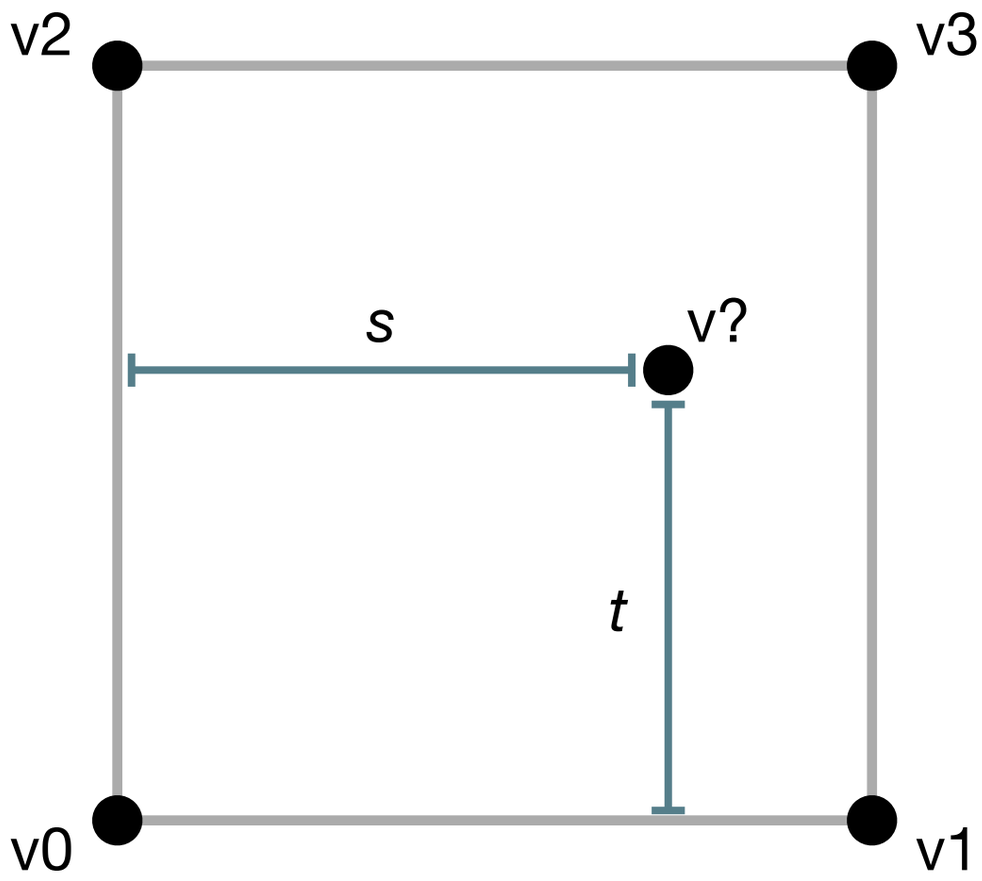
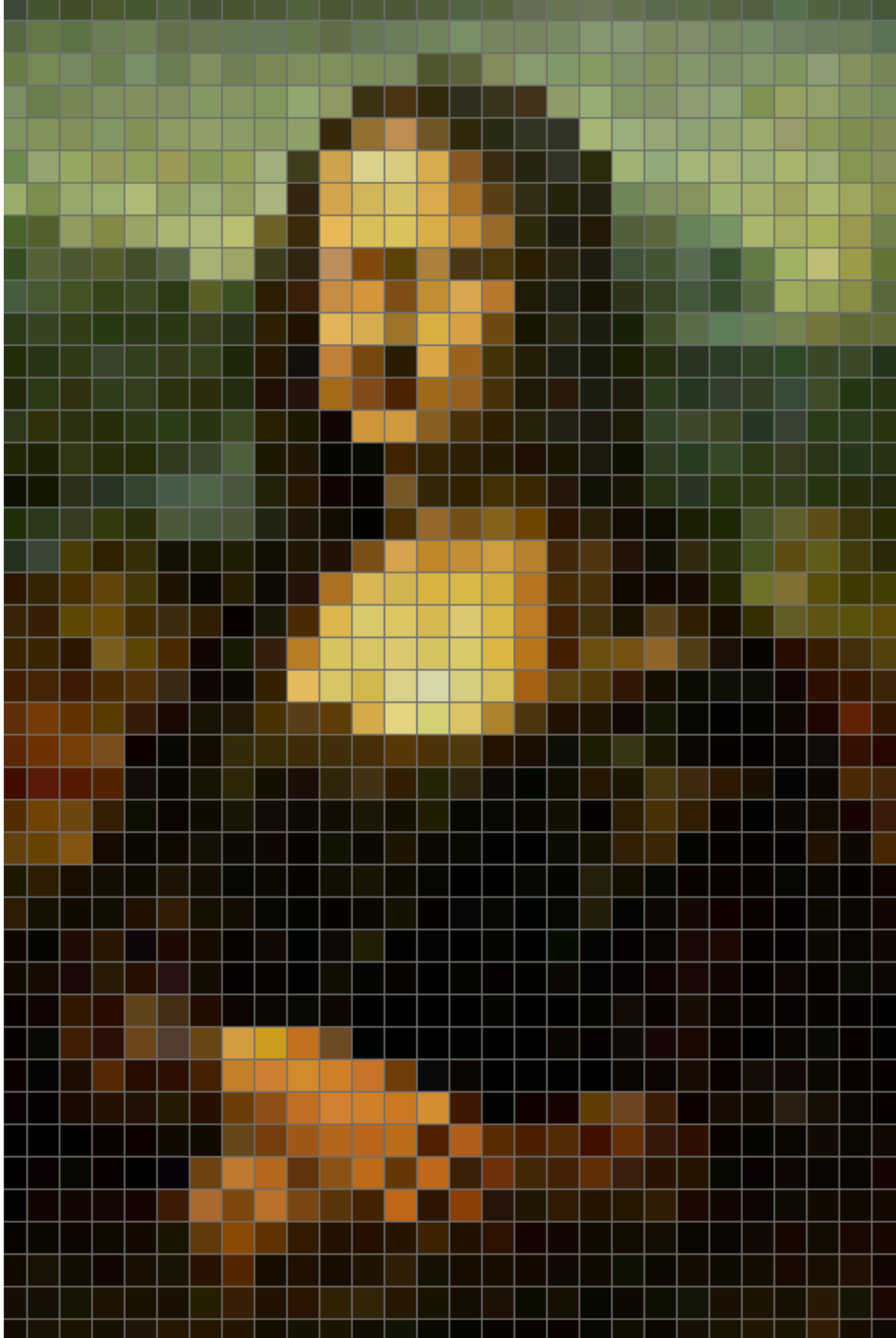


Image Manipulation!

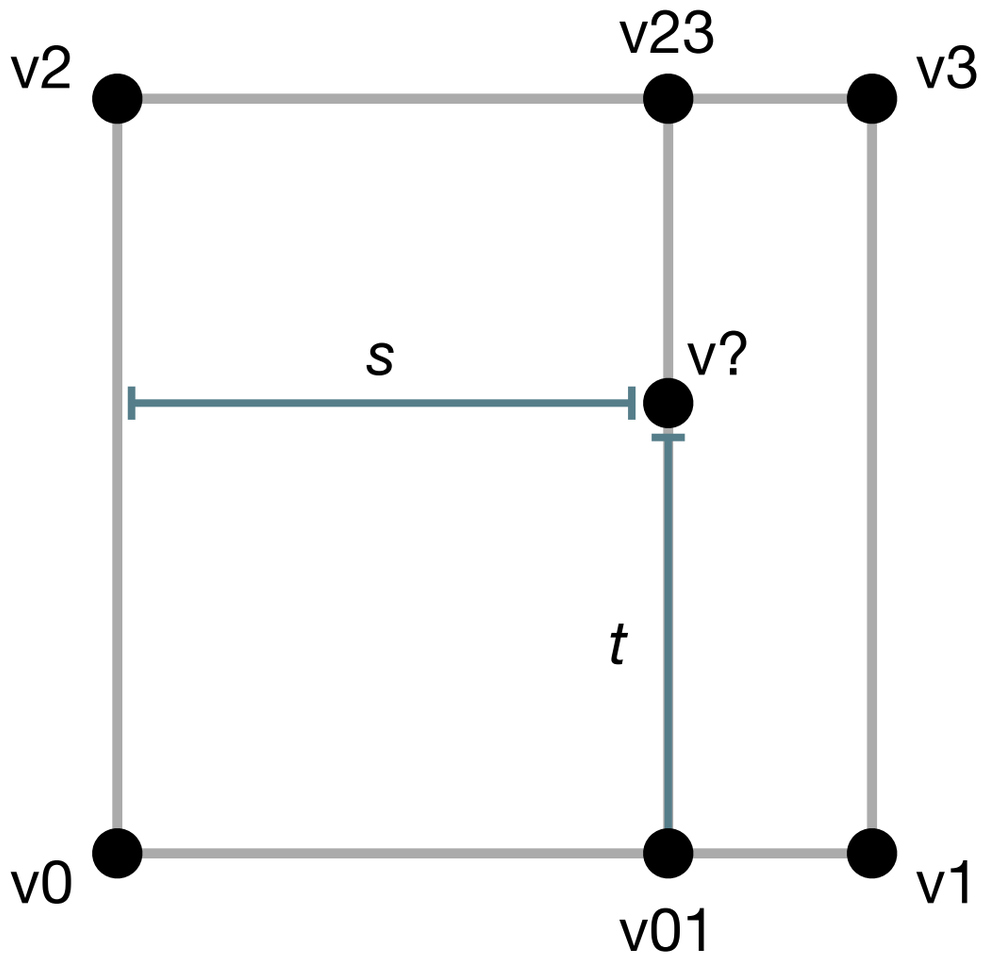
- What's the value at the interpolated point?





Bilinear Interpolation

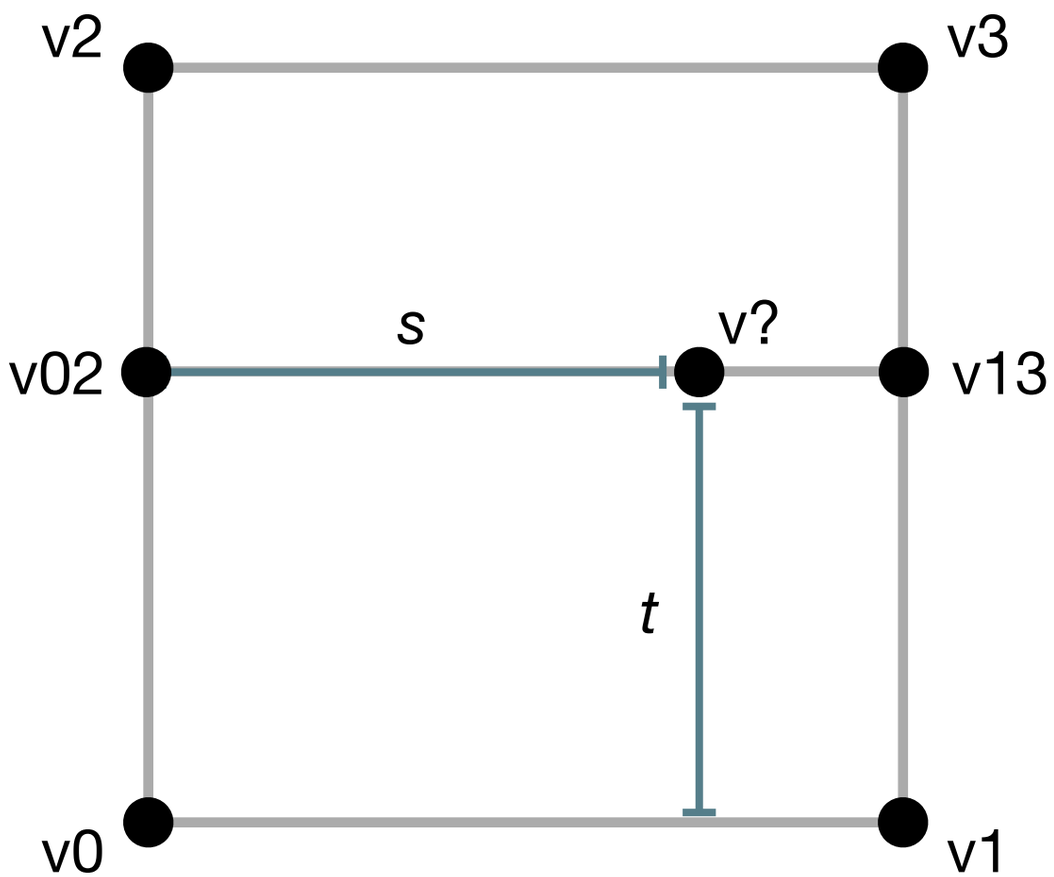
- Can be computed as three distinct linear interpolations




```
bilerp(v0, v1, v2, v3, s, t)
{
    v01 = lerp(v0, v1, s);
    v23 = lerp(v2, v3, s);
    v = lerp(v01, v23, t);
    return v;
}
```

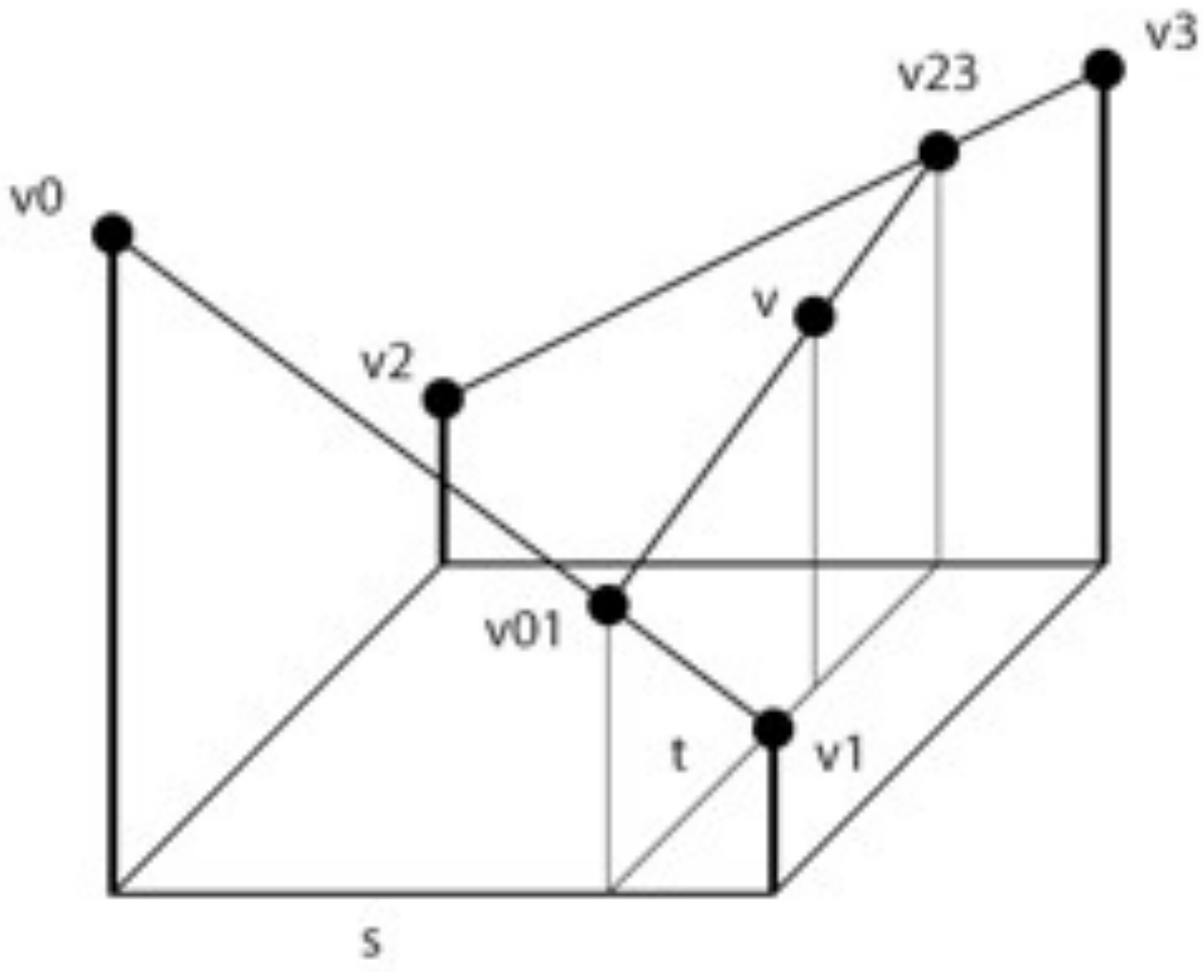
Bilinear Interpolation

- What if we do it this way? Are they equivalent?




```
bilerp(v0, v1, v2, v3, s, t)
{
    v02 = lerp(v0, v2, t);
    v13 = lerp(v1, v3, t);
    v = lerp(v02, v13, s);
    return v;
}
```

Bilinear Interpolation as a Height Function



What kind of surface does the interpolated continuous function form?

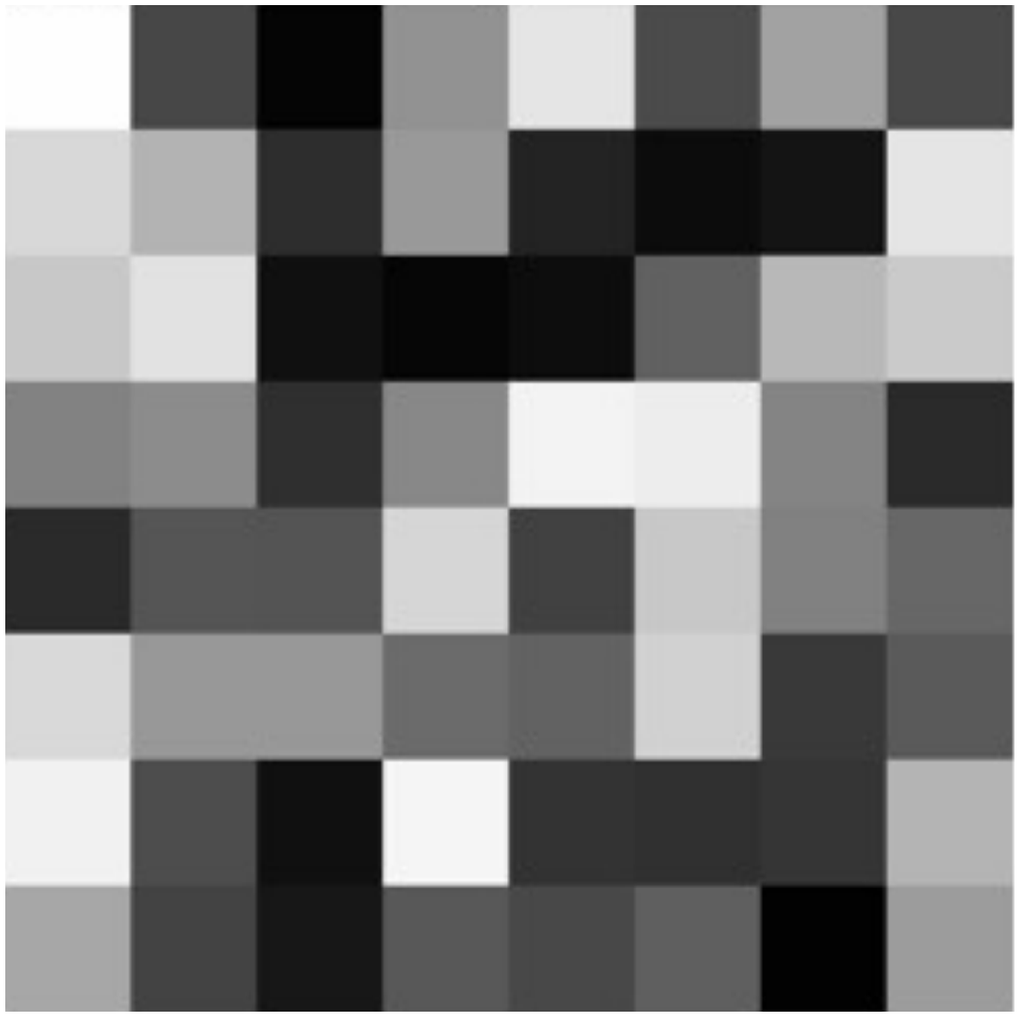
[courtesy of P. Hanrahan, Stanford University]

Ruled Surface

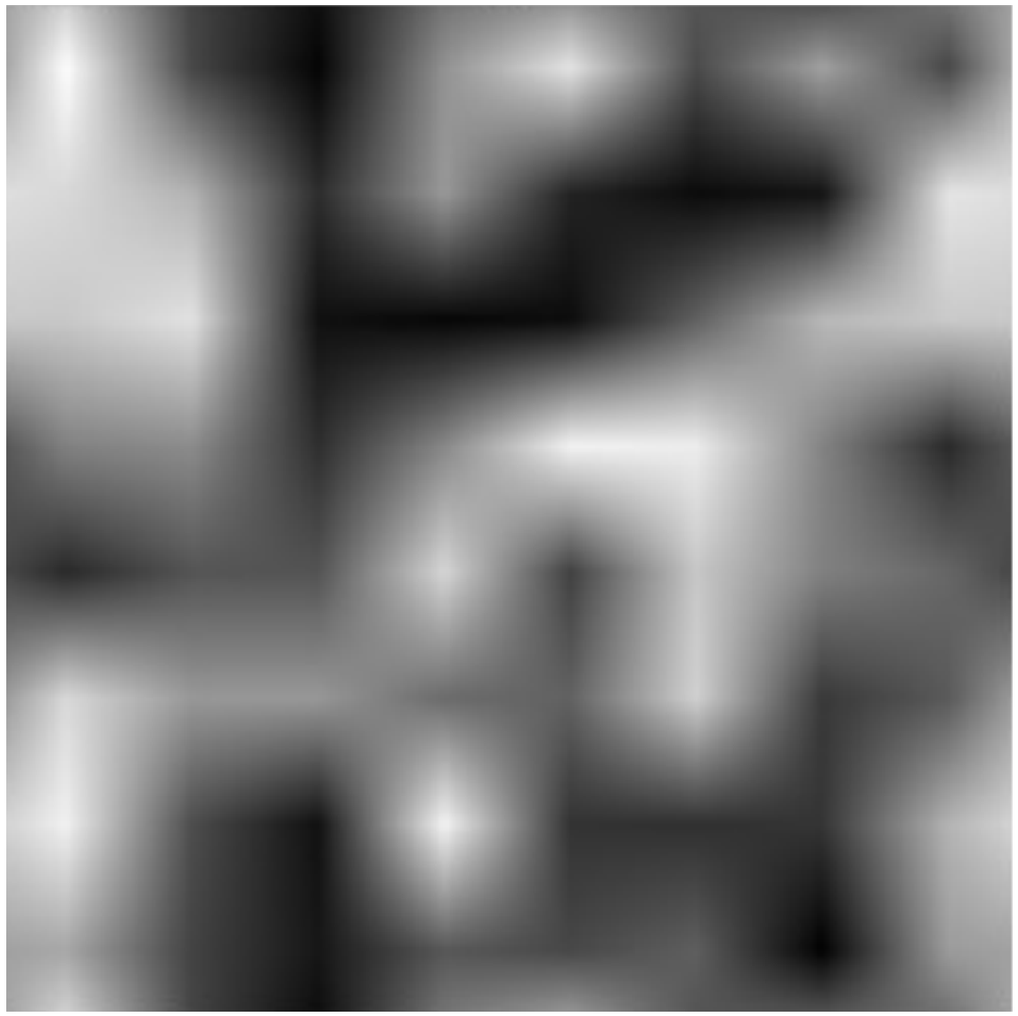
It’s a hyperboloid

[from math.arizona.edu]

Nearest Neighbour vs Bilinear Interpolation



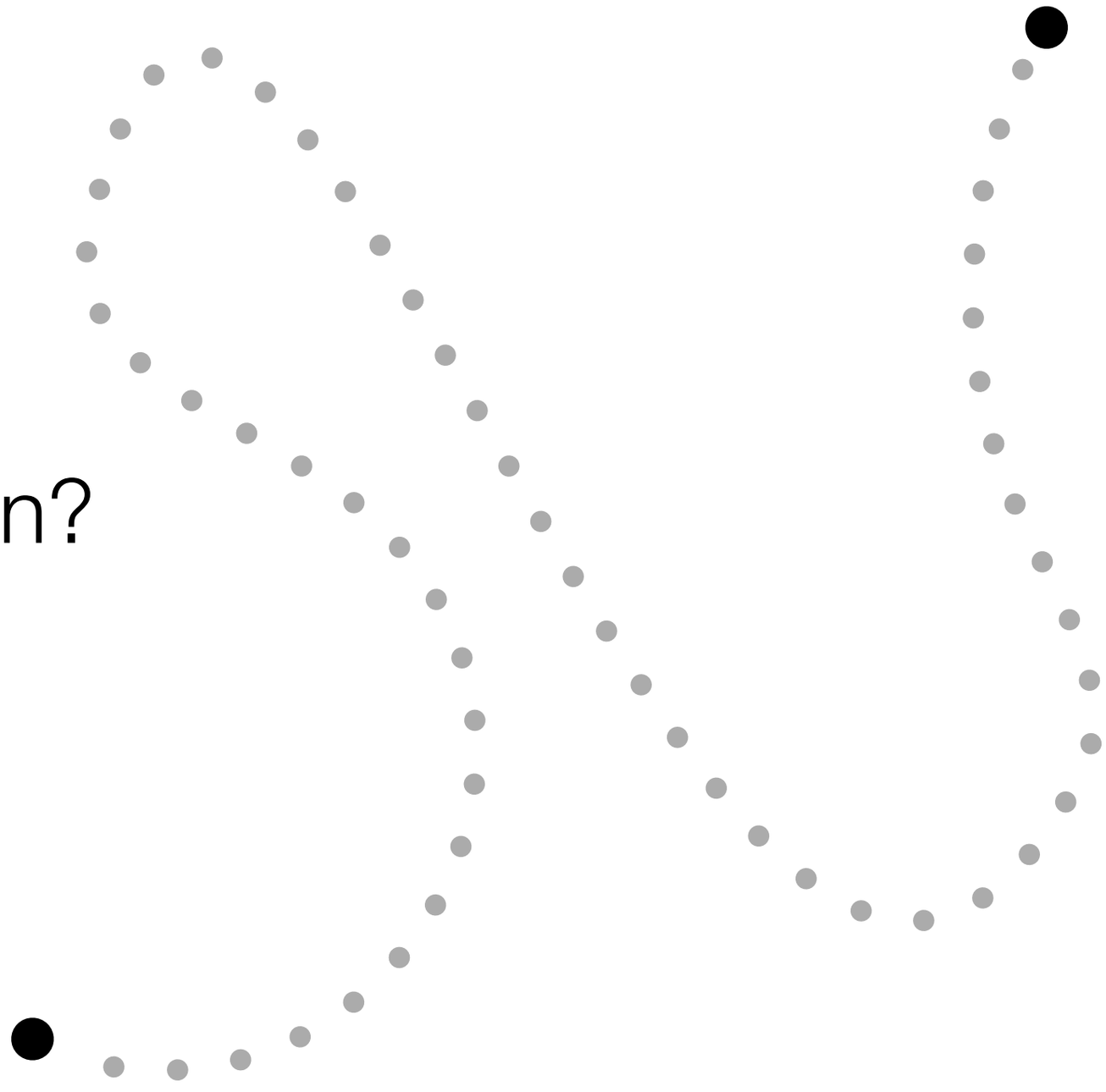
nearest pixel



bilinear

[courtesy of P. Hanrahan, Stanford University]

Is this interpolation?



Things to Remember

- Interpolation is the probably the most important function in computer graphics

- Converts a discrete function into a continuous function

- You can interpolate almost anything you can add!