Chapter 5: Ideal gas, kinetic theory, transport properties

Midterm 2, Winter 2016

Assume ideal gas behavior. Data - Molar masses (kg/kmol): H=1, C=12, O=16. Collision diameter for dioxide (CO_2) = 342 pm.

a) Calculate the rms speed of CO_2 molecules at 300°C and 2 atm.

b) Determine the average time between collisions for molecules of CO_2 at 300°C and 2 atm.

c) How many molecules of CO_2 at 300°C and 2 atm would occupy a volume of 1 mm^3?

d) Calculate the total kinetic energy of all of the CO_2 gas in a 1 mm^3 volume at 300°C and 2 atm.

a) $$C_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3 \times 8314 \frac{J}{kmol \cdot K} \times 573.15\,K}{44 \frac{kg}{kmol}}}$$

$$= 570 \sqrt{J/kg}$$

$$= 570 \text{ m/s} \qquad \left[\text{Note: } \frac{J}{kg} = \frac{N \cdot m}{kg} = \frac{m^2}{s^2}\right]$$

b) $$t_{avg} \simeq \frac{\lambda}{\bar{c}}$$

$$\lambda = \frac{1}{\sqrt{2}\pi\sigma^2 \rho_N} = \frac{kT}{\sqrt{2}\pi\sigma^2 P}$$

$\sigma = 342 \text{ pm} = 3.42 \times 10^{-10}$ m

$\sqrt{2}\pi\sigma^2 = 51.965736 \times 10^{-20}$ m^2

$$\lambda = \frac{(1.3805 \times 10^{-23} \frac{Pa \cdot m^3}{K})(573.15\,K)}{(51.965736 \times 10^{-20} m^2)(2 \times 101325\,Pa)}$$

$$= 7.5134 \times 10^{-8} \text{ m}$$

$$\bar{c} = \sqrt{\frac{8RT}{\pi M}} = \sqrt{\frac{8 \times 8314 \frac{J}{kmol \cdot K} \times 573.15\,K}{\pi \times 44 \frac{kg}{kmol}}}$$

$$= 525.15 \sqrt{J/kg} = 525.15 \frac{m}{s}$$

c) $$n = \frac{PV}{RT} = \frac{(2 \times 101.325\,kPa)(10^{-9} m^3)}{8.314 \frac{kPa \cdot m^3}{kmol \cdot K} \times 573.15\,K}$$

$1 \text{ mm} = 10^{-3}$ m

$\Rightarrow 1 \text{ mm}^3 = 10^{-9}$ m^3

$$= 42.527347 \times 10^{-9} \text{ mol}$$

Number of molecule $N = nN_A = n \times 6.023 \times 10^{23} \frac{\text{molecules}}{\text{mol}}$

$= 2.5614 \times 10^{16}$ molecules.

d) $$\bar{E}_{K,tot} = N \times (\tfrac{3}{2}kT)$$
$$= N \times (1.5 \times 1.3805 \times 10^{-23} \tfrac{J}{K} \times 573.15\,K) = 3.04 \times 10^{-4}\,J.$$

Midterm Exam, Winter 2003

At low pressures and high temperatures, nitrogen (Molar mass = 28 kg/kmol) can be assumed to behave like an ideal gas.

a) What is the velocity that the highest number of nitrogen molecules would be expected to be travelling at if the conditions are 250°C and 2 bar (1 bar = 100 kPa)?

b) What is the root mean square velocity of nitrogen molecules at 250°C and 2 bar ?

c) What is the mean separation distance between nitrogen molecules at 250°C and 2 bar ?

d) If the viscosity of nitrogen is 5x10^{-5} Pa.s at 250°C and 2 bar, what would the viscosity be at 500°C and 4 bar?

e) Using the information in (d), what is the collision diameter of a nitrogen molecule?

f) What would be the force pushing on the inside surface of a spherical 1 m diameter balloon filled with nitrogen at 500°C and 4 bar?

a) Most probable velocity = the velocity of most of the molecules.

$$C_{mp} = \sqrt{\frac{2RT}{M}} = \sqrt{\frac{2 \times 8314 \frac{J}{kmol \cdot K} \times 523.15\ K}{28 \frac{kg}{kmol}}} = 557.3 \frac{m}{s}$$

b) $$C_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3 \times 8314 \frac{J}{kmol \cdot K} \times 523.15\ K}{28 \frac{kg}{kmol}}} = 682.6 \frac{m}{s}$$

c) $$\delta = \left(\frac{KT}{P}\right)^{1/3} = \left(\frac{1.3805 \times 10^{-23}\ \cancel{Pa} \cdot m^3/K \times 523.15 K}{200 \times 1000\ \cancel{Pa}}\right)^{1/3} = 33 \mathring{A}$$

d) $$\mu_1 = \frac{M}{N_A \pi \sigma^2}\sqrt{\frac{RT_1}{\pi M}}$$
$$\mu_2 = \frac{M}{N_A \pi \sigma^2}\sqrt{\frac{RT_2}{\pi M}}$$
$$\Rightarrow \frac{\mu_1}{\mu_2} = \sqrt{\frac{T_1}{T_2}}$$
$$\Rightarrow \frac{5 \times 10^{-5}\ Pa.s}{\mu_2} = \sqrt{\frac{523.15}{773.15}} = 0.822586$$
$$\Rightarrow \mu_2 = 6.078 \times 10^{-5}\ Pa.s.$$

e) $$\mu_1 = \frac{M}{N_A \pi \sigma^2}\sqrt{\frac{RT_1}{\pi M}}$$
$$\Rightarrow \sigma^2 = \frac{M}{N_A \pi \mu_1}\sqrt{\frac{RT_1}{\pi M}} = \left[\frac{28 \frac{kg}{kmol}}{(6.023 \times 10^{26} / kmol) \times 3.1416 \times 5 \times 10^{-5} \frac{kg}{m \cdot s}}\right] \times \sqrt{\frac{8314 \frac{J}{kmol \cdot K} \times 523.15 K}{3.1416 \times 28\ kg/kmol}}$$
$$= \left(\frac{0.029595\ m \cdot s}{10^{20}}\right)\left(222.36 \frac{m}{s}\right) = \frac{6.58}{10^{20}}\ m^2$$
$$\Rightarrow \sigma = 2.565 \mathring{A}$$

f) Area $= 4\pi r^2 = 3.1416\ m^2$
$P = 400$ kPa, $F = P \times A = 1256$ kN.