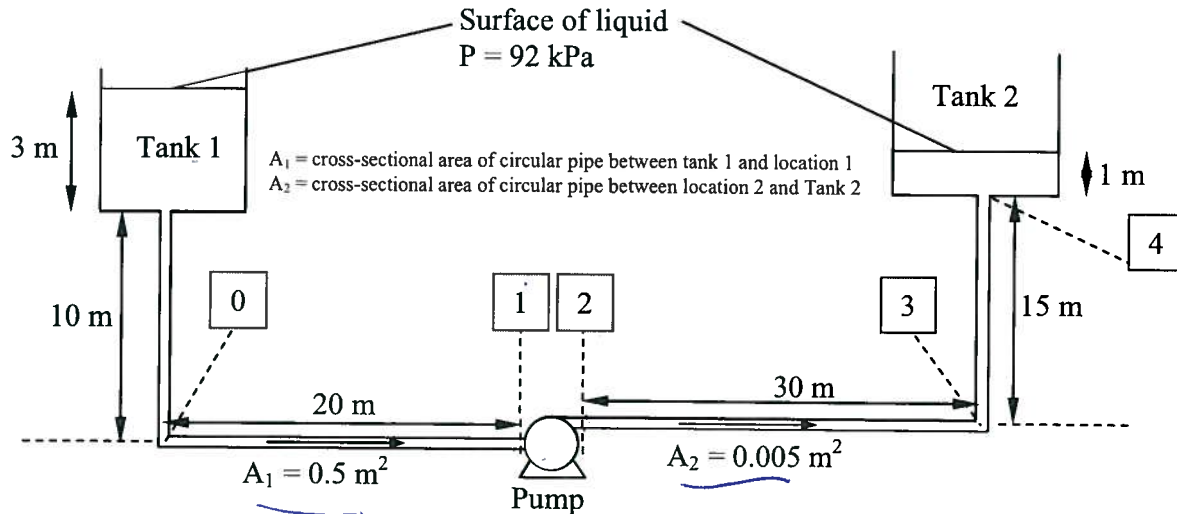


**Question Number IV (22 Marks ~ 40 minutes)****PART A**

A pumping station is displayed in the following Figure.



The mass flow rate through the system is 1320 kg/h. The density and viscosity of the liquid are 980 kg/m³ and 1.2 mPa s, respectively. The pipes can be treated as rough commercial steel.

a) Is the flow laminar or turbulent at location 1? (2)

$$\dot{m} = \rho Q \rightarrow Q = \frac{\dot{m}}{\rho} = \frac{1320 \text{ kg/h}}{980 \text{ kg/m}^3} = 1.347 \frac{\text{m}^3}{\text{h}} \times \frac{1 \text{ h}}{3600 \text{ s}} = 3.74 \times 10^{-4} \frac{\text{m}^3}{\text{s}}$$

$$Q = \bar{v} A \rightarrow \bar{v} = \frac{Q}{A} = \frac{3.74 \times 10^{-4} \text{ m}^3/\text{s}}{0.5 \text{ m}^2} = 7.48 \times 10^{-4} \text{ m/s}$$

b) At location 1, what is the maximum velocity in the circular pipe? (1)

$$\frac{v_{\max}}{2} = \bar{v} \rightarrow v_{\max} = 2\bar{v} \text{ (laminar)}$$

$$v_{\max} = 2 \times 7.48 \times 10^{-4} \text{ m/s} = 1.496 \times 10^{-3} \text{ m/s} = v_{\max}$$

$$Re = \frac{D \bar{v} \rho}{\mu} = \frac{(0.7979 \text{ m})(7.48 \times 10^{-4} \text{ m/s})(980 \text{ kg/m}^3)}{1.2 \times 10^{-3} \text{ Pa}\cdot\text{s}} = 487.4$$

Laminar

c) Is the flow laminar or turbulent at location 2? (2)

$$\rightarrow \text{location 2} \rightarrow Q = Q_{\text{pipe 1}} = 3.74 \times 10^{-4} \text{ m}^3/\text{s}$$

$$\bar{v}_2 = \frac{Q}{A} = \frac{3.74 \times 10^{-4} \text{ m}^3/\text{s}}{0.005 \text{ m}^2} = 0.0748 \text{ m/s}$$

$$A = \frac{\pi}{4} D^2 \rightarrow D = \sqrt{\frac{4A}{\pi}} = 0.07979 \text{ m}$$

$$Re = \frac{D \bar{v} \rho}{\mu} = \frac{0.07979 \times 0.0748 \times 980}{1.2 \times 10^{-3}} = 4874 \text{ Turbulent}$$

continued...

## Question Number IV (Continued)

d) What is the pressure at location 1? (/3)

$$\text{section \#1} \rightarrow \text{pressure @ bottom of Tank 1} = 92000 \text{ Pa} + (980)(3)(9.81)$$

$$\text{LAMINAR FLOW (bottom of Tank 1 to [1])} = 120841 \text{ Pa}$$

$$-\left[\frac{\Delta P}{L} + \rho g \frac{\Delta h}{L}\right] = \frac{32 \mu \bar{v}}{D^2} \quad \Delta h = 0 - 10 \text{ m}$$

$$\Delta P = P_1 - 120841 \text{ Pa}$$

$$-\left[\frac{\Delta P}{30} + 980 \times 9.81 \left(\frac{-10}{30}\right)\right] = \frac{32 \times 1.2 \times 10^{-3} \times 7.48 \times 10^{-4}}{(0.7979 \text{ m})^2} = 4.51166 \times 10^{-5}$$

$$\Delta P + 980 \times 9.81 \times (-10) = -1.35 \times 10^{-3}$$

$$96138$$

$$\Delta P = 96137.99865 = P_1 - 120841$$

$$\boxed{P_1 = 216978.9986 \text{ Pa}} \quad (216.98 \text{ kPa})$$

e) What is the pressure at location 2? (/4)

$$\text{section \#2} \rightarrow \text{pressure @ bottom of Tank 2} = 92000 \text{ Pa} + 980 \times 1 \times 9.81$$

$$\text{TURBULENT FLOW (bottom of Tank 2 to [2])} = 101613.8 \text{ Pa} = P_4$$

$$-\left[\frac{\Delta P}{L} + \rho g \frac{\Delta h}{L}\right] = \frac{2 f \bar{v}^2 \rho}{D} \quad \Delta h = 15 - 0$$

$$\Delta P = P_4 - P_2 = 101613.8 - P_2$$

$$Re = 4.874 \times 10^3 \rightarrow f = 0.012 \quad (\text{accept close to this})$$

$$-\left[\frac{\Delta P}{45} + 980 \times 9.81 \left(\frac{15}{45}\right)\right] = \frac{2 \times 0.012 \times (0.0748 \text{ m/s})^2 \times 980}{0.07979} = 1.649$$

$$\Delta P + 980 \times 9.81 \times 15 = -74,217$$

$$\Delta P = -144132 \text{ Pa} = 101613.8 - P_2$$

$$\boxed{P_2 = 245746.6 \text{ Pa}}$$

f) What is the power of the pump? (/2)

$$\text{Power} = Q \Delta P$$

$$\Delta P = P_2 - P_1 = 28767.58 \text{ Pa}$$

$$\text{Power} = (3.74 \times 10^{-4}) (28767)$$

$$Q = 3.74 \times 10^{-4} \text{ m}^3/\text{s}$$


$$\boxed{\text{Power} = 10.76 \text{ W}}$$

continued...

**Question Number IV (Continued)**

**PART B** - In a bearing, the gap between the moving surface and a stationary surface is 0.05 mm. The lubrication liquid is at 80°C and has density equal to 870 kg/m<sup>3</sup> and viscosity equal to 140 cP. The moving surface slides past the stationary one at 0.778 m/s. The contact area between the surfaces is 0.78 cm<sup>2</sup>.

- a) If the flow is laminar between the two surfaces, what is the shear stress exerted by the steel part on the liquid in the gap? (/2)

0.05 mm  $\uparrow$    $\frac{dv}{dy} = \frac{\Delta v}{\Delta y} = \frac{0.778 \text{ m/s}}{0.05 \times 10^{-3} \text{ m}} = 15560 \text{ s}^{-1}$

$\mu = \tau / (dv/dy) \rightarrow \tau = \mu \frac{dv}{dy} = (140 \times 10^{-3} \text{ Pa}\cdot\text{s})(15560 \text{ s}^{-1}) = \boxed{2178.4 \text{ Pa}}$

- b) What is the force applied to the steel part to keep it in motion? (/2)

$\tau = \frac{F}{A}$   $A = 0.78 \text{ cm}^2 \times \left(\frac{1 \text{ m}}{100 \text{ cm}}\right)^2 = 7.8 \times 10^{-5} \text{ m}^2$

$F = \tau A = (2178.4 \text{ Pa})(7.8 \times 10^{-5} \text{ m}^2) = \boxed{0.1699 \text{ N}}$

- c) The lubricant is mixed with another liquid. Viscometer tests of the liquid mixture yield the following data:

Shear Stress, Pa	Shear Rate, s <sup>-1</sup>
510.29	10.9
1370.23	38.7

$\mu_{\text{APP}}$   
46.82 Pa·s  
35.41 Pa·s

- i. Does this liquid exhibit power-law behaviour? (/1)

$\mu_{\text{APP}} = \tau / (dv/dy) \rightarrow \text{non-newtonian}$   
 $\boxed{\text{yes}}$

- ii. What is the apparent viscosity at a shear rate of 20 s<sup>-1</sup>? (/3)

$\tau = K \left(\frac{dv}{dy}\right)^n \rightarrow \log \tau = \log K + n \log \frac{dv}{dy}$   
 $\log(510.29) = \log K + n \log(10.9)$   
 $\log(1370.23) = \log K + n \log(38.7)$   
 $\log\left(\frac{510.29}{1370.23}\right) = n \log\left(\frac{10.9}{38.7}\right)$   
 $-0.4289764 = -0.55028 n$   
 $\bar{n} = 0.7796$   
 $510.29 = K(10.9)^{0.77956}$   
 $K = 79.26 \text{ Pa}\cdot\text{s}^{0.77956}$

$\mu_{\text{APP}} = \frac{\tau}{dv/dy} = K \left(\frac{dv}{dy}\right)^{n-1}$

$\mu_{\text{APP}} = 79.26 (20)^{0.77956-1} = \boxed{42.966 \text{ Pa}\cdot\text{s} = \mu_{\text{APP}}}$