Binary T-x Phase Diagrams (Tutorial)

Winter 1997 (Mid-Term)

Use the temperature-composition diagram for vapour-liquid mixtures of A and B at 1 atm to answer the following questions.

(a) (i) What is the boiling point of A?
(ii) What is the maximum solubility of B in A?
(iii) At what temperature do all vapour and liquid phases co-exist?
(iii) Identify all co-existing phases at $T = 80°C$ and $w_A = 0.2$.

(b) A sample, prepared by mixing 7 kg of A and 3 kg of B, is held at 90°C.
(i) Give the composition of phase(s) that will be present in the sample.
(ii) Calculate the mass of each phase present.

(c) In an experiment, pure B is added slowly to the sample described in Part (b) while maintaining the temperature constant at 90°C. Determine:
(i) The minimum mass of pure B that must be added, to the sample in Part (b), in order to make the entire sample exist as a homogenous mixture.
(ii) Specify the composition of the phase(s) that will be present in the mixture after 25 kg of pure B has been added to the sample in Part (b).

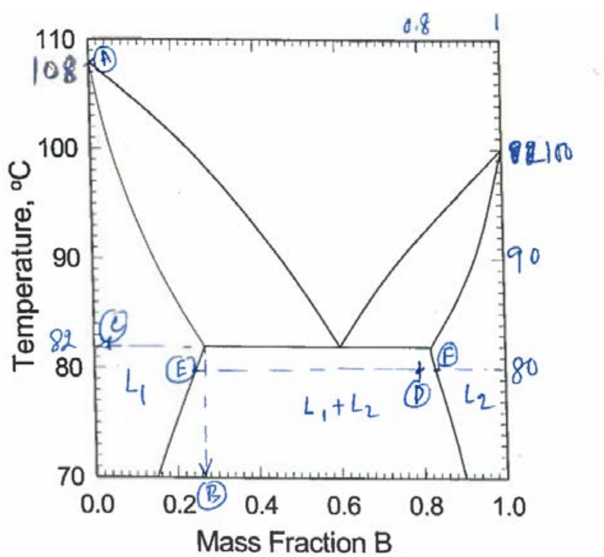
Solution: Part (a)

a(i) 108°C [point A]

a(ii) 26 % [point B]

a(iii) 82°C [point C]

a(iv) $W_B = 1 - W_A = 0.8$, T = 80°C. The system is at point D. It is in two-phase region. The coexisting phases are L_1 and L_2. (or, we can call them L-alpha and L-beta).



Explanatory figure for part (a)

Part (b)

<u>b(i)</u> $W_B = m_A/(m_A + m_B) = 0.3$, T = 90°C. The system is at point G. It is in two-phase region, the coexisting phases are L_1 and V.

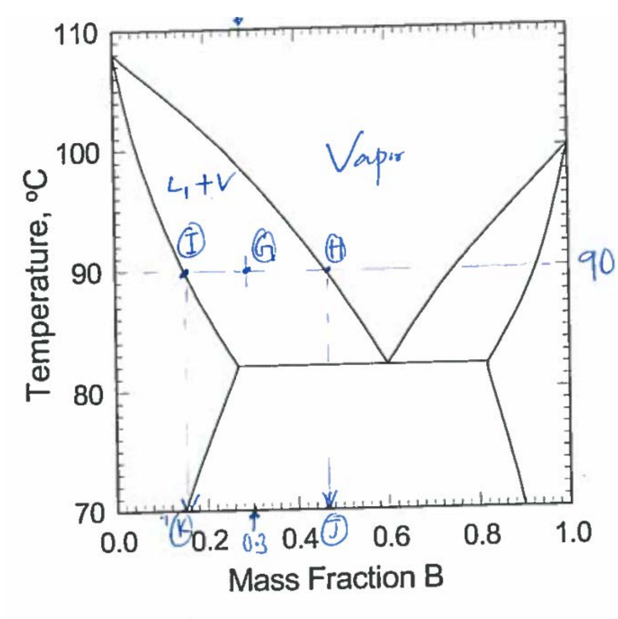
Compositions of L1 = 15 mass percent B, Compositions of phase V = 46 mass percent B.

[<u>*Note: Steps to determine compositions when two phases exist:*</u> The system is at point G. Therefore, the material inside the container exists as two separate phases, L_1 and V. To find composition, first, draw a tie-line through G. It intersects the phase boundaries at points I and H. Next, draw perpendicular from points I and H. Then read compositions of phase L1 at point K and compositions of V (vapor) at point J. The phase L1 is composed of 15 mass percent B and phase V is composed of 46 mass percent B.]

b(ii) Applying lever rule,

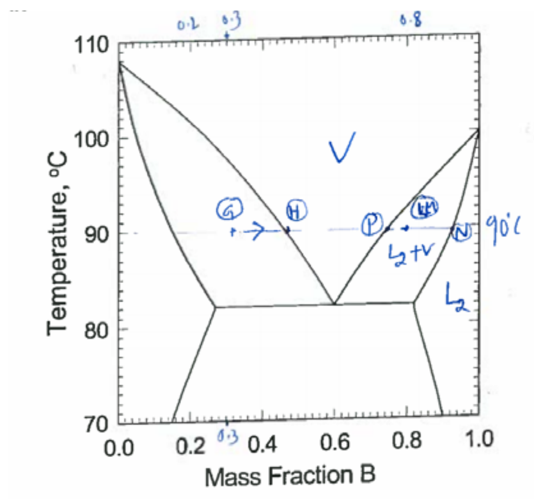
mass of L1/ total mass = $(W_{vapor} - W_{overall})/(W_{vapor} - W_{liquid})$= (46-30)/(46-15)=0.516

Then, mass of L1 = 0.516* 10 kg = 5.16 kg, mass of vapor = 10-5.16 = 4.84 kg.



Explanatory figure for part (b)

Part (c)



Explanatory figure for part (c)

c(i) We start from point **G**. When we add B at constant temperature, we change the overall compositions, that is we move to the right horizontally. When we reach **H**, the whole system is essentially made of vapor. Then, the entire system exists a homogeneous mixture – the vapor phase. Since the entire system is vapor, then vapor compositions = overall compositions. We draw a perpendicular from H, and read the composition, $W_B = 0.46$. Let the amount of B added is x kg. At point H, total B = 3+x kg, total A = 7 kg, total mass = 3+x+7 = 10+x. Mass fraction of B = (3+x)/(10+x) = 0.46. Solving for x, we obtain, **x = 2.46 kg.**

[Note: At G, we have two homogeneous mixtures, the vapor and L_1, therefore, overall, the entire system is not homogeneous, part of it is vapor and part of it is liquid. However, at H, all the liquid is converted to vapor, so we have uniformity everywhere]

c(ii) After addition of 25 kg of B, total B = 25+3 = 28 kg, and total A = 7 kg, total or overall mass = 28+7= 35 kg. Mass fraction of B, W_B = 28/35 =0.8. T = 90°C. The system is at point M. It is two phase region. The material inside splits into two phases, L_2 and vapor V. Following the previously mentioned procedure, we draw a tie-line PN, draw perpendiculars from P and N, and read the compositions

liquid L_2 = 95 mass percent B, and

vapor=75 mass percent B.

Question:

Part (a) The constant-pressure (1 atm) T-x phase diagram for the mixture of lead (Pb) and tin (Sn) is shown in Figure 1 below. For a mixture of lead and tin (30 wt% Sn), determine the phases present, their ccompositions and their amount (in percent of total mass) at 300, 200, 184, 182, and 0°C.

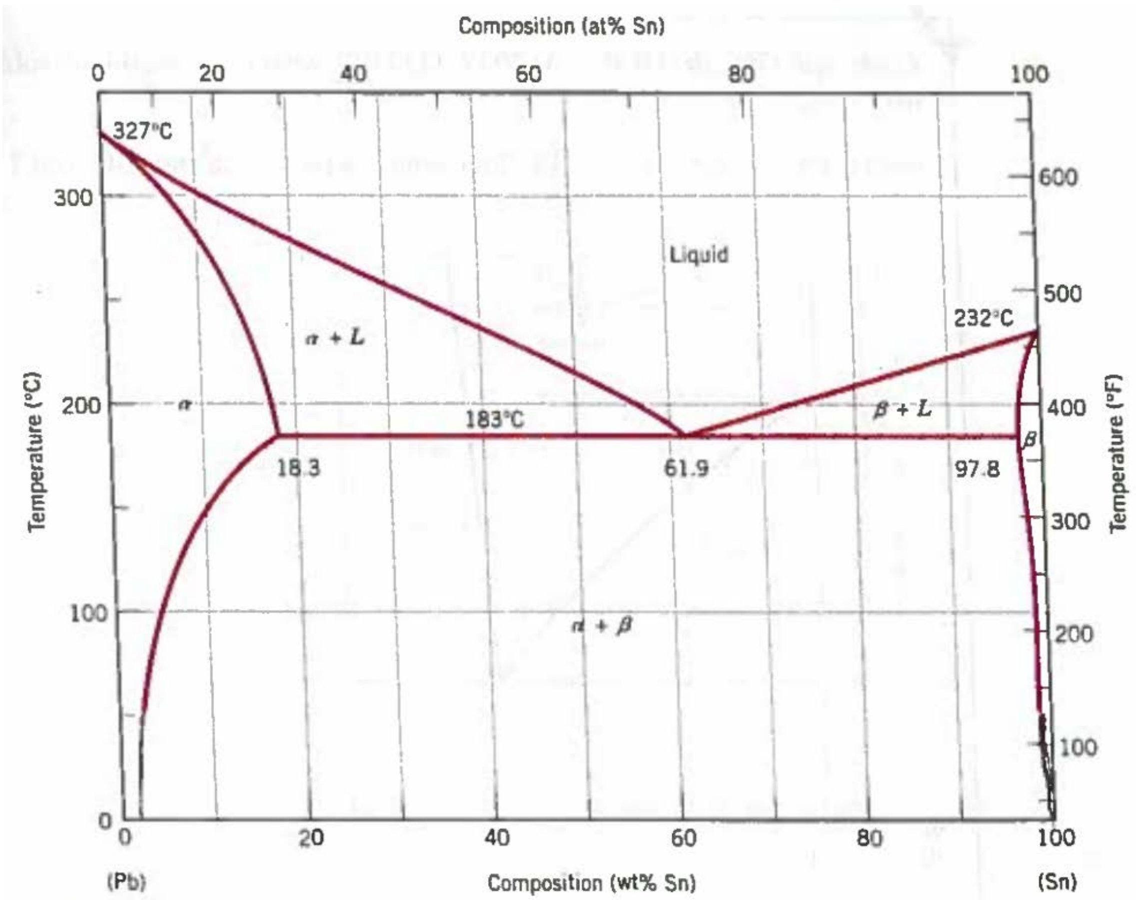


Figure 1: Pb-Sn phsae diaram (Ref: Fundamentals fo Materials Science and Engineering, W. D. Callister, Jr., and D. G. Rethwisch)

Ans:

3(a)

Temp (°C)	Phases	Composition (wt%)	Relative Amounts
300	L	L: 30% Sn	L = 100% of total
200	α+L	L: 56% Sn α: 17.5%	$\frac{m_L}{m_{tot}} = \frac{30-17.5}{56-17.5} = 32.5\%$ α Overall L 17.5 30 56 $\frac{m_\alpha}{m_{tot}} = (100-32.5) = 67.5\%$
184	α+L	L: 61.9% Sn α: 18.3% Sn (approx)	α Ov L 18.3 30 61.9 $\frac{m_L}{m_{tot}} = \frac{30-18.3}{61.9-18.3}$ $= 26.8\%$ $\frac{m_\alpha}{m_{tot}} = 73.2\%$
182	α+β	α: 18.3% Sn β: 97.8% Sn	α Ov β 18.3 30 97.8 $\frac{m_\alpha}{m_{tot}} = \frac{97.8-\cancel{18.3}\,30}{97.8-18.3}$ $= 85.3\%$ $\frac{m_\beta}{m_{tot}} = 14.7\%$
0	α+β	α: 2% Sn β: 100% Sn	$\frac{m_\alpha}{m_{tot}} = \frac{100-30}{100-2} = 71\%$ $\frac{m_\beta}{m_{tot}} = \frac{30-2}{100-2} = 29\%$

Part (b) The constant-pressure T-x phase diagram for the mixture of lead (Pb) and tin (Sn) is shown in Figure 2 below. Consider a number of different binary mixtures of Pb-Sn at different points (A, B, C, D, etc.) in the diagram. Which of the following statements are true ?

i) The compositions of solid alpha for the two mixtures at points **A** and **B** are same.

Ans: False. The mixture at B has 10 wt% Sn, the mixture at A has about 5 wt% Sn.

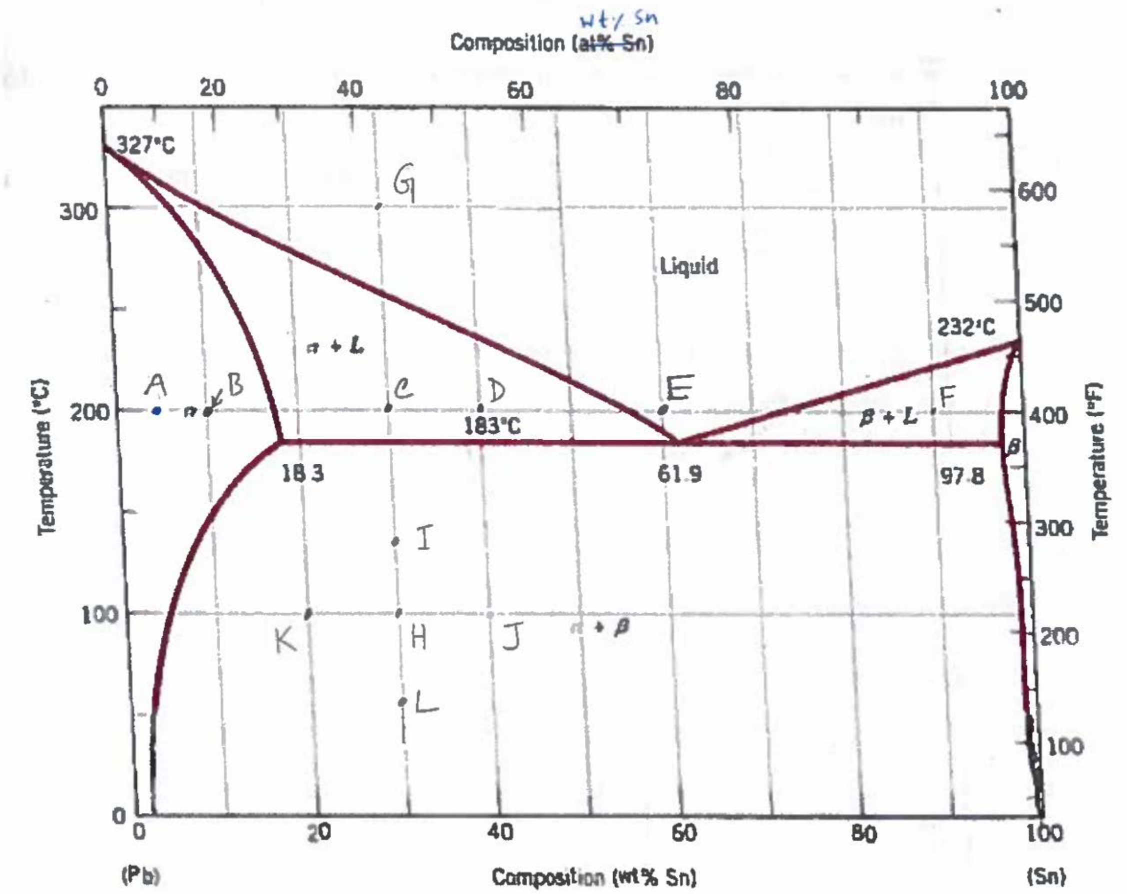


Figure 2. Pb-Sn phase diagram. Imagine different mixtures at poitns A, B, C..etc.

ii) The compositions of solid alpha for the two mixtures at points **C** and **D** are same.

Ans: True. The mixture at point C splits into solid alpha (17.5 wt% Sn) and liquid (56 wt% Sn). The mixture at point D also splits into solid alpha (17.5 wt% Sn) and liquid (56 wt% Sn). Hoever, the mass of solid alpha in mixture at C and the mass of solid alpha in mixture at D are different.

iii) The compositions of solid alpha for the two mixtures at points **I** and **L** are same.

Ans: False. The mixture at point I splits into solid alpha (about 9 wt% Sn) and liquid (about 98 wt% Sn). The mixture at point L also splits into solid alpha (about 3 wt% Sn) and liquid (about 99 wt% Sn).

Part (c) (i) 30 kg of Sn and 270 kg of Pb are mixed at 1 atm and at 200°C. At equilibrium how many phases are present?

Ans: One (point B), solid alpha phase

(ii) 30 kg of Sn and 70 kg of Pb are mixed at 1 atm and at 200°C. At equilibrium how many phases are present?

Ans: Two (point C), solid alpha and liquid phase.

(iii) 30 kg of Sn and 20 kg of Pb are mixed at 1 atm and at 200°C. At equilibrium how many phases are present?

Ans: One (point E), liquid phase.

(iv) 30 kg of Sn and 70 kg of Pb are mixed at 1 atm and at 100°C. At equilibrium how many phases are present?

Ans: Two (point H), solid alpha and solid beta

(v) Consider the mixture in part (iv): 30 kg of Sn and 70 kg of Pb, at 1 atm and at 100°C which is at point **H**. How can you bring this system at point **K**?

Ans: We keep the pressure and temperature same. Just add more Pb, so that the overall mass percent of Sn reduces to 20%. Let's say we add x kg of Pb. Then, 30/(100+x) = 0.2 or 30=20+0.2x, then, x =50 kg. Then, after addition, at K, we have total 150 kg mixture of which 30 kg is Sn, that is Sn is 20 wt%.

(vi) Consider the mixture in part (iv): 30 kg of Sn and 70 kg of Pb, at 1 atm and at 100°C which is at point **H**. How can you bring this system at point **J**?

Ans: We keep the pressure and temperature same. Just add more Sn, so that the overall mass percent of Sn increaes to 40%. Let's say we add x kg of Sn. Then, (30+x)/(100+x) = 0.4 or 30+x=40+0.4x, solving, x =16.67 kg. Then at J, we have total 116.67 kg mixture of which 46.67 kg is Sn, that is Sn is 40 wt%.

[**Note:** one could also write the equation uisng Pb mole fraction, at J, 70/(100+x) =0.6, then, x=16.67 kg]