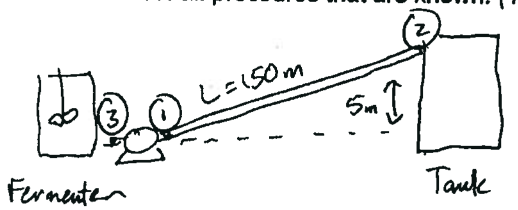
Question Number V (20 Marks ~ 36 min)

1. You have been hired to design a pipeline at Huge Stone Brewery that will be used to transport Cricket Beverage from a fermenter through 150 m of pipeline to a storage tank. The pressure at the inlet of the pipe is 100 kPa (at the fermenter), and pressure at the outlet of the pipeline (at the storage tank) is atmospheric pressure, and the outlet is at a height 5 m above the inlet. The density of the liquid is 1100 kg/m^3, and the viscosity is 0.105 Pa.s. You have two choices of pipe available – a 15 cm diameter smooth plastic pipe, and a 3 cm diameter rough steel pipe. You will also have to install a pump at the beginning of the pipe (i.e. just after the fermenter).

   a. Draw a diagram showing the fermenter, pipe, pump and storage tank. Be sure to label all pressures that are known. *(1 mark)*



$P_2 = 101.325$ kPa
$P_3 = 100$ kPa
$P_1 = ?$

   b. Calculate what the velocity of the fluid (m/s) would be for each of the two pipe choices to maintain a volumetric flow rate of 1000 L/min through the pipeline. *(3 marks)*

Pipe #1 – Smooth Plastic

$D = 15\text{cm} = 0.15\text{m}$

$Q = 1000\frac{L}{min} \times \frac{1 min}{60 s} \times \frac{1 m^3}{1000 L} = 0.0167\ m^3/s$

$A = \frac{\pi}{4}D^2 = \frac{\pi}{4}(0.15m)^2 = 0.01767 m^2$

$Q = \bar{v}A \rightarrow \boxed{\bar{v} = Q/A = 0.943 m/s}$

Pipe #2 – Rough Steel

$D = 3cm = 0.03m$

$Q = 0.0167\ m^3/s$

$A = \frac{\pi}{4}D^2 = \frac{\pi}{4}(0.03m)^2 = 0.00071 m^2$

$\boxed{\bar{v} = \frac{Q}{A} = 23.58 m/s}$

   c. Calculate the power required for the pump if the smooth plastic pipe is installed, and the volumetric flow rate is 1000 L/min. *(4 marks)* *

smooth plastic

$Re = \frac{D\bar{v}\rho}{\mu} = \frac{(0.15m)(0.943m/s)(1100 kg/m^3)}{0.105 Pa\,s} = 1482$ LAMINAR

$-\left[\frac{\Delta P}{L} + \rho g\frac{\Delta h}{L}\right] = \frac{32\mu\bar{v}}{D^2}$

$\Delta P = ?$ $L = 150m$
$\rho = 1100 kg/m^3$ $\Delta h = h_2 - h_1 = 5m$
$g = 9.81 m/s^2$ $\mu = 0.105 Pa\,s$
$\bar{v} = 0.943 m/s$ $D = 0.15m$

solving

$\Delta P = -75{,}078$ Pa

$\Delta P = P_2 - P_1 = 101325 Pa - P_1$

$P_1 = 176{,}403$ Pa

Pump Power $= Q\Delta P' = Q(P_1 - P_3)$

Power $= (0.0167 \frac{m^3}{s})(176{,}403 Pa - 100{,}000 Pa)$

$\boxed{\text{Power} = 1275.9 W}$

Continued...

Question Number V (20 Marks ~ 36 min) *(Continued)*

d. Calculate the power required for the pump if the rough steel pipe is installed and the volumetric flow rate is 1000 L/min. *(4 marks)*

$$Re = \frac{D\bar{U}\rho}{\mu} = \frac{(0.03\,m)(23.58\,m/s)(1100\,kg/m^3)}{0.105\,Pa\,s} = 7410 \quad \text{TURBULENT}$$

$$-\left[\frac{\Delta P}{L} + \rho g \frac{\Delta h}{L}\right] = \frac{2f\bar{U}^2\rho}{D}$$

$\Delta P = ?$ $L = 150\,m$ $\rho = 1100\,kg/m^3$

$\Delta h = 5\,m$ $g = 9.81\,m/s^2$

$\bar{U} = 23.58\,m/s$ $D = 0.03\,m$

$f = 0.0095$

solving

$\Delta P = -441{,}313$ Pa

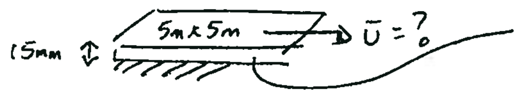
$\Delta P = P_2 - P_1 = 101325 - P_1$

$P_1 = 542{,}638$ Pa

$\text{Power} = Q\Delta P' = Q(P_1 - P_3)$

$= (0.0167\,m^3/s)(542{,}638 - 100{,}000) = \boxed{7392\text{ W} = \text{Power}}$

2. A thin 15 mm layer of oil has spilt on the floor of an auto body shop. This oil is a power law fluid with parameters K=0.58 Pa.s$^{0.4}$ and n=0.4. A piece of sheet metal 5 m by 5 m has fallen on top of the layer of oil. One of the mechanics tries to use the sheet of metal as a surf-board. Assuming he initially launches his entire body weight (100 kg) along the surface of the metal at 2 m/s^2, what is the velocity of the piece of sheet metal as it flies along the oil? *(4 Marks)*

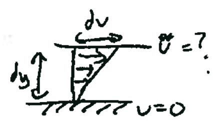


$K = 0.58\,Pa\,s^{0.4}$ $n = 0.4$

$\tau = K\left(\frac{du}{dy}\right)^n$

$$\tau = \frac{F}{A} = \frac{ma}{A} = \frac{(100\,kg)(2\,m/s^2)}{(5\,m)^2} = 8\,Pa$$

$8\,Pa = 0.58\left(\frac{du}{dy}\right)^{0.4}$



$\frac{du}{dy} = 706.6\,s^{-1}$ $dy = 0.015\,m$

$\therefore du = 10.5985\,m/s \rightarrow \boxed{U = 10.6\,m/s}$

3. What are the main differences between an ideal fluid, a Newtonian fluid, and a non-Newtonian fluid? *(3 marks)*

Ideal $\rightarrow \mu = 0$

Newtonian $\rightarrow \mu = \text{const} \neq f\left(\frac{du}{dy}\right)$

Non-Newtonian $\rightarrow \mu = f\left(\frac{du}{dy}\right) \neq \text{constant}$

4. Draw a velocity profile for laminar flow in a pipe (i.e. draw how the velocity changes as you move from the center of the pipe to the pipe wall). *(1 mark)*