CHAPTER 10

STRESS-STRAIN RELATIONSHIP FOR SOLIDS

More emphasis on sections 10.1 and 10.2

In this chapter we will study
- Stress-strain relationship for solids
- Why some solids fail and others do not
- Phenomena such as elastic behaviour, plasticity, creep and brittleness

10.1 Stress and Strain in Solids

Stress is simply force per unit area. Two types of stresses are considered: (1) normal [see Figure 10-1] and (2) shear [see Figure 10-2].

10.1.1 Normal and Shear Stresses

Normal stress (denoted by σ)

$s = \frac{F}{A}$ with F perpendicular to the surface. For solids the normal stress can have three components:

s_x, s_y, s_z. These components can have different values, unlike fluids where the pressure (which is normal stress) is same in all directions.

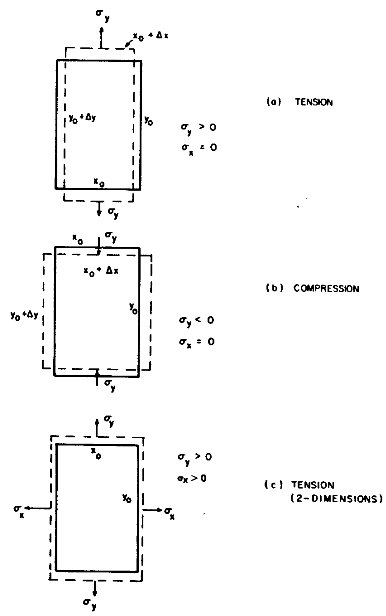


Figure 10-1 **Normal (Hydrostatic) Stress and Strain**
(a) Tension
(b) Compression
(c) Tension in Two Dimensions

Normal stress in solids can be away from body (tension) or toward the body (compression). By convension tension is considered positive and compression negative.

Shear Stress (denoted by τ)

τ = F/A, with force parallel to the surface. Note that we can have both types of stresses present at the same time.

Figure 10 -2 shows examples of shear stresses in solids.

Fig. 10-2 (a): cubic (rectangular solid)
Fig. 10-2(b): cylindrical solid (tube).

The area A is easy to determine for a rectangular solid. For the cylindrical tube,

$A = \pi(r_o^2 - r_i^2) = \pi(r_o + r_i)(r_o - r_i) = 2\pi r_o(r_o - r_i)$ since r_o and r_i are of the same order.

10.1.2 Normal and Shear Strains

Strain is a measure of fractional deformation of a solid that occurs due to the applied stress.

If we apply normal stress, the strain will also be normal. Similarly, shear stress will produce shear strain. Let us look at the normal strain first.

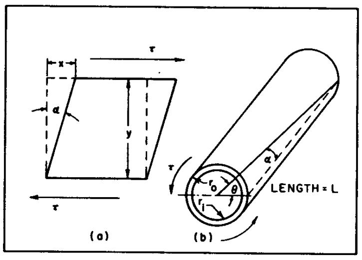


Figure 10-2 Shear Stress and Strain
(a) In Rectangular Co-ordinates $\gamma = x/y$
(b) In Cylindrical Co-ordinates $\gamma = \theta r_o/L$

Normal Strain (denoted by ε)
Normal strain is fractional change in linear dimensions of the solid. Even a one-dimensional stress can produce dimensional changes in all three directions. So we have to consider ε_x, ε_y, ε_z as the normal strains in x, y, and z directions. Let us look at a two dimensional object, [Fig 10 (c)].

The initial dimensions are x_o and y_o. Under stress, the new dimensions are (x_o + Δx) and (y_o +Δy). The normal strains in x and y direction are defined as:

$$e_x = \frac{\Delta x}{x_o}$$

$$e_y = \frac{\Delta y}{y_o}$$

When stress is positive, strain is also positive. So the sign of strain depends on the sign of stress.

Shear Strain (denoted by γ)

Shear strain is the fractional deformation due to a shear stress. It is a measure of the distortion in shape of the solid. Refer to Figure 10-2.

In rectangular systems,

$$g = \tan a = \frac{x}{y}$$

In cylindrical systems

$$g = \frac{q\, r_o}{L}$$

For thick walled tubes

$$g = \frac{q}{L}\left[\frac{r_o - r_i}{\ln(r_o / r_i)}\right]$$

The quantity in the square brackets is called log-mean radius.

10.1.3 Typical Stress-Strain Behaviour

Figure 10-3 shows behaviour for 3 types of solids:

(a) Steel (crystalline)
(b) Plaster of Paris
(c) Soft rubber

At low values of stress, stress and strain are linearly related, i.e. the relative elongation is directly proportional to the applied stress. This behaviour lasts only up to a certain value of stress, called proportional limit.

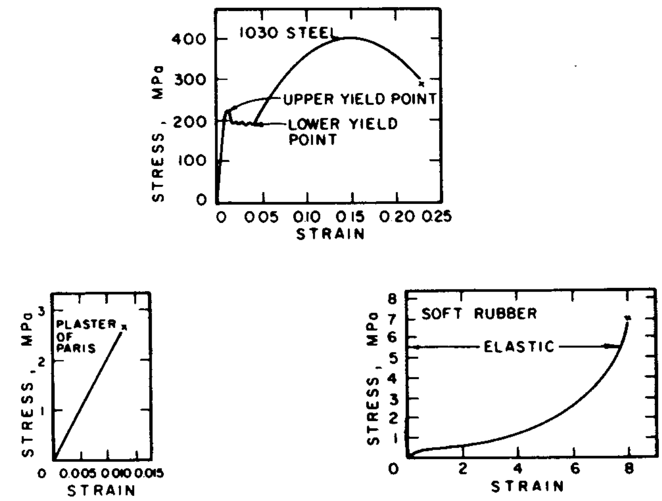


Figure 10-3 Engineering Stress - Strain Curves for:
(a) 1030 Steel
(b) Plaster of Paris
(c) Soft Rubber

The stress-strain behaviour of rubber (elastomers) is highly non-linear; but these substances are highly elastic.

Also note that stresses in steel are much higher.

Figure 10-4 (p348) shows typical stress-strain behaviour. Such figures are based on tests on material specimens, involving application of gradually increasing tensile stress until the material fails.

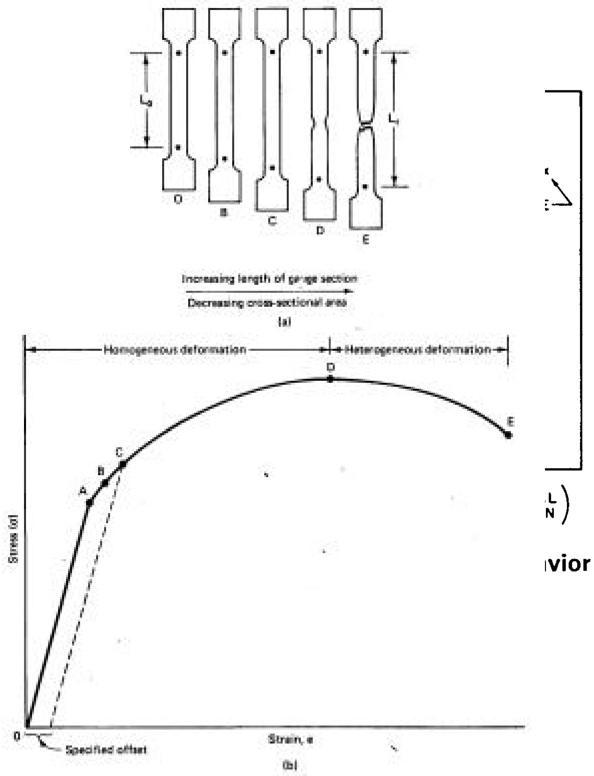


Figure 8-4 (a) Appearance of tensile specimen at various stages in the tension test; (b) typical stress–strain curve for a ductile metal: A, proportional limit; B, elastic limit; C, yield strength; D, ultimate tensile strength; E, fracture strength.

<u>Proportional limit:</u> The proportional limit is the highest value of stress at which the stress-strain relationship is linear.

<u>Elastic Limit:</u> The highest stress imposed on the material such that there is no permanent deformation remaining when the stress is removed.

<u>Yield Strength:</u> The yield strength corresponds to the stress required to produce a small specific amount of permanent deformation.

<u>Ultimate Tensile Strength:</u> The ultimate tensile strength is a measure of the maximum tensile stress that a material can withstand under conditions of uniaxial loading.

The effect of compression on materials is somewhat similar to that of tension. However, some materials fracture rather easily under tension whereas they can withstand large compression, e.g. glass, cement marble etc.

10.2 Elastic Deformation in Solids

Hooke's Law -- Linear elasticity of materials

Stress/strain = constant. (up to about 1% elongation in metals)

10.2.1 Young's Modulus for Linear Deformation

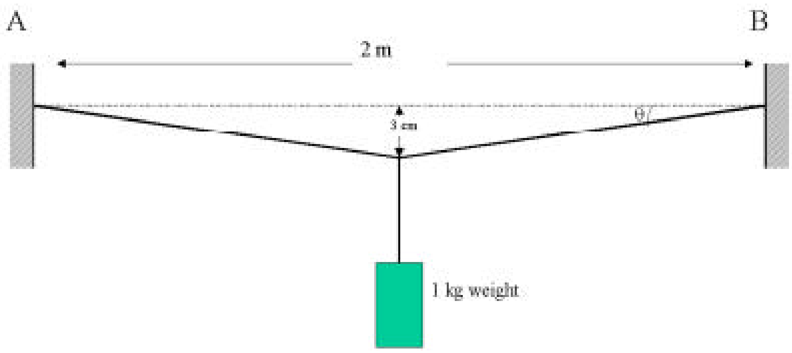
For one-directional normal stress

$$s_y = Ee_y$$

E is called Young's modulus or the modulus of elasticity. Its units are Pa.

Table 10-1 lists values of E for several materials. E is generally higher for higher melting point materials.

<u>Example Problem (Fall 1999 Final)</u>



A wire of 2mm diameter is stretched between two immobile anchor points, A and B, that are 2m apart. The wire is initially straight (totally horizontal) and has a negligible tensile stress. Subsequently, when a 1-kg weight is suspended from the midpoint, the vertical movement of the midpoint (at mechanical equilibrium) is 3cm.
Note: At equilibrium the vertical force exerted by the suspended weight is balanced by the vertical components of tension in the two arms of the wire.

(a) What is the total length of the wire after the weight has been suspended? (2 point)

Length of the wire after suspending the weight = $2\sqrt{1+(0.03)^2} = 2.0009$

(b) What is the axial strain in the wire? (2 point)

Strain e_y ***= (2.0009 - 2.0)/2 = 0.00045***

(c) What is the value of tensile stress in the wire? (5 pts)

$Sinq = \frac{0.03}{\sqrt{1+0.03^2}} = 0.02999$ ***Therfore,*** q ***= 1.72°***

Force balance in the vertical direction gives
2TxSin q ***= 1.0 x 9.81***
or, T = 9.81/(2 x 0.02999) = 163.6 N

Original cross-sectional area = p***D²/4 = 3.1416 x(0.002)²/4 = 3.14 x 10⁻⁶ m²***

Tensile stress in the wire = F/A = 163.6/3.14 x10⁻⁶ = 52 MPa

(a) Calculate the Young's modulus of the wire material. (3 pts)

Stress, s_y ***= 52 MPa***

Strain e_y ***= (1.00045 -1.0)/1 = 0.00045***

Young's modulus, E = 52 MPa/0.00045 = <u>115.6 GPa</u>

(e) Given that the Poisson ratio of the wire material is 0.33, calculate the diameter of the wire after the weight has been suspended. (4 pts)

Total axial stress in the wire = 52 MPa

$$e_x = \frac{-n\ s_y}{E} = \frac{-0.33x52\,MPa}{115.6\,GPa} = -0.00015$$

Diameter of the wire under axial stress of 52 MPa = 2 mm x (1 - 0.00015) = <u>1.9997 mm</u>