ENGG 201 – Chapter 6 Example – PVT Calculations for a Real Gas Mixture

Problem:

Consider the transportation of a natural gas, at an annual rate of 1 trillion standard cubic feet (measured at 0ºC and 1 atm), through 4 parallel pipelines, each 1 meter in diameter.

The composition of the gas (mole basis) is: 85% CH_4, 10% C_2H_6 and 5% CO_2. The actual pressure and temperature are 100 atm and 10ºC, respectively. The molar masses of the components are: CH_4=16 kg/kmol, C_2H_6=30 kg/kmol and 5% CO_2=44 kg/kmol.

Calculate the actual yearly volumetric rate and average velocity by

a) the ideal gas law
b) Kay's pseudocritical method
c) The van der Waals EOS (and mixing rules)
d) The Pitzer-Curl tables

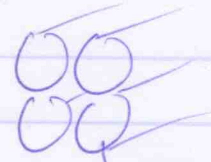
Notes: 1) 1 ft^3 = 0.02832 m^3
2) 1 std. cu. ft. = volume of an ideal gas that is 1 ft^3 when measured at 0ºC, 1 atm

Answers

a) 2.940×10^8 m^3/yr, 2.96 m/s
b) 2.205×10^8 m^3/yr, 2.22 m/s
c) 2.040×10^8 m^3/yr, 2.06 m/s
d) 2.16×10^8 m^3/yr, 2.18 m/s

CHAPTER 6 – REAL GAS MIXTURE EXAMPLE

SOLUTION



each 1 m in diameter

Rate = 1×10^{12} ft^3(standard) /yr

$\hookrightarrow T_0 = 0°C$

$P_0 = 1$ atm

Actual $T = 10°C$
$P = 100$ atm

$1 \text{ year} = 365 \times 24 \times 3600 = 3.1536\times10^7 s$

$$\frac{1\times10^{12} ft^3}{yr} \times \frac{\cancel{1ft^3}}{\cancel{0.0283}} \frac{0.02832 m^3}{1 ft^3} = 2.832\times10^{10} m^3/yr = v_0$$

$$n = \frac{P_0 v_0}{R T_0} = \frac{(101.325 kPa)(2.832\times10^{10} m^3)}{(8.314 \frac{kPa\, m^3}{kmol\, K})(273.15 K)} = 1.264\times10^9 \frac{kmol}{yr}$$

(a) ~~a~~ IDEAL GAS

$$Pv = nRT \rightarrow v = \frac{nRT}{P} = \frac{(1.264\times10^9 kmol)(8.314)(283.15)}{100\times101.325}$$

$$\boxed{v = 2.94\times10^8 m^3/yr}$$

$$2.94\times10^8 \frac{m^3}{yr} \times \frac{1 yr}{3.1536\times10^7 s} = 9.309 m^3/s$$

Pipe Area = $4\times\pi r^2 = 4\pi(0.5m)^2 = 3.14 m^2$

$$\bar{v} = \frac{Q}{A} = \frac{9.309 m^3/s}{3.14 m/s} = \boxed{2.963 m/s}$$

(b) KAY's Pseudocritical Method

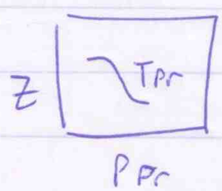
$T_{PC} = \sum y_i T_{ci} \qquad P_{PC} = \sum y_i P_{ci}$

GAS	y_i	T_{ci} (K)	P_{ci} (atm)	$y_i T_{ci}$	$y_i P_{ci}$
CH_4	0.85	190.6	45.4	162.01	38.59
C_2H_6	0.10	305.4	48.2	30.54	4.82
CO_2	0.05	304.2	72.8	15.21	3.64
				T_{PC} = 207.76 K	P_{PC} = 47.05 atm

Mixture pseudo-reduced properties

$T_{Pr} = T/T_{PC} = 283.15/207.76 = 1.36$

$P_{Pr} = P/P_{PC} = 100/47.05 = 2.13$



$Z_m = 0.75$ (chart)

$PV = n Z_m RT$

$$V = \frac{n Z_m RT}{P} = \frac{(1.264 \times 10^9 \text{ kmol})(0.75)(8.314)(283.15)}{100 \times 101.325} = \boxed{2.203 \times 10^8 \text{ m}^3/\text{yr}}$$

$2.203 \times 10^9 / 3.1536 \times 10^7 = 6.98 \text{ m}^3/\text{s}$

$$\bar{U} = \frac{6.98 \text{ m}^3/\text{s}}{3.14 \text{ m}^2} = \boxed{2.22 \text{ m/s}}$$

(c) van der Waals EOS (mixing rules)

$$a = \frac{27}{64}\frac{R^2 T_c^2}{P_c} \qquad b = \frac{R T_c}{8 P_c}$$

$$\bar{a} = \left[\sum y_i \sqrt{a_i}\right]^2 \qquad \bar{b} = \sum y_i b_i$$

GAS	y_i	P_{c_i}	T_{c_i}	a_i	b_i	$y_i\sqrt{a_i}$	$y_i b_i$
CH_4	0.85	45.4	190.6	2.275	0.04306	1.282	0.03660
C_2H_6	0.10	48.2	305.4	5.497	0.06504	0.234	0.006504
CO_2	0.05	72.8	304.2	3.612	0.04284	0.095	0.002142
				atm$(m^3/kmol)^2$	$\frac{m^3}{kmol}$	1.611	$\boxed{\bar{b} = 0.04525 \ m^3/kmol}$

1.611 ↓ square

$$\boxed{\bar{a} = 2.595 \ atm(m^3/kmol)^2}$$

$$V_m^3 - \left[b + \frac{RT}{P}\right]V_m^2 + \frac{a}{P}V_m - \frac{ab}{P} = 0$$

$$V_m^3 - \left[0.04525 + \frac{0.08205 \times 283.15}{100}\right]V_m^2 + \frac{2.595}{100}V_m - \frac{2.595 \times 0.04525}{100} = 0$$

$$V_m^3 - 0.27745\,V_m^2 + 0.02595\,V_m - 0.00117 = 0$$

$$V_m - 0.27745 + 0.02595/V_m - 0.00117/V_m^2 = 0$$

$$V_m = 0.27745 - 0.02595/V_m + 0.00117/V_m^2$$

$$V_m = f(V_m)$$

First guess = ideal gas V_m

$V_m = 2.94 \times 10^8 \, m^3 / 1.264 \times 10^9 \, kmol = 0.232 \, m^3/kmol = V_{m1}$

$V_{m1} = 0.232 \quad f(V_{m1}) = 0.27745 - \frac{0.02595}{0.232} + \frac{0.00117}{(0.232)^2} = 0.1873$

$V_{m2} = 0.1873 \quad f(V_{m2}) = 0.1722$

$V_{m3} = 0.1722 \quad f(V_{m3}) = 0.1662$

$\vdots$

$V_m = 0.1617 \, m^3/kmol$

$V = 0.1617 \, m^3/kmol \times 1.264 \times 10^9 \, kmol = \boxed{2.04 \times 10^8 \, m^3/yr}$

$\frac{2.04 \times 10^8}{3.1536 \times 10^7} = 6.48 \, m^3/s$

$\bar{u} = 6.48 / 3.14 = \boxed{2.06 \, m/s}$

(d) Pitzer-Curl

$T_{pr} = 1.36 \quad P_{pr} = 2.13$ (from b)

$CH_4 \quad \omega = 0.008 \quad C_2H_6 \quad \omega = 0.098 \quad CO_2 \quad \omega = 0.225$

$\bar{\omega} = \Sigma y_i \omega_i = 0.02785$

$Z^{(0)}$

		P_r	
	2.0	2.13	2.20
T_r 1.30	0.691		0.671
1.36	0.7426	$\boxed{0.73025}$	0.7236
1.40	0.777		0.759

$Z^{(1)}$

	2.0	2.13	2.20
T_r 1.30	0.20		0.20
1.36	0.194	0.1979	0.20
1.40	0.19		0.20

$$Z = Z^{(0)} + \omega Z^{(1)}$$

$$Z = 0.73025 + 0.02785\,(0.1979)$$

$$Z = 0.73576$$

$$V = V_{ideal}\, Z = \boxed{2.16 \times 10^{8}\ m^3/yr}$$

$$2.16 \times 10^{8} / 3.1536 \times 10^{7} = 6.86\ m^3/s$$

$$6.86 / 3.14\ m^2 = \boxed{2.18\ m/s}$$