8.5 Turbulent Flow in Pipes

- Most engineering applications of pipe flow involve turbulent flow.
- Turbulent flow is more difficult to analyze theoretically, hence some empirical consideration are used.
- Actual velocity at any point in the pipe, even at steady-state conditions, is not constant but fluctuates (Fig 8-8a).
- However, if you consider the time average over sufficient length of time, it would remain constant (Fig 8-8b).
- Hence, in turbulent flow it is necessary to deal with time averaged velocity values.
- Figure 8-8(b) shows that the velocity profile is more flat than in laminar flow.

One approximate equation (empirical) is

$$\frac{\bar{u}}{u_{\max}} = \left[1 - \frac{r}{r_w}\right]^{1/7}$$

This equation is called the "one seventh power law".

8.5.1 The friction factor

Friction factor is a very useful parameter in calculations involving turbulent flow in pipes.

<u>Definition:</u> $f = \dfrac{2t_w}{r\,\bar{u}^2}$ (Fanning friction factor)

Another popular friction factor is Blasius or Darcy friction factor which is four times higher (f_D = 4f)

Assuming that the equation relating pressure drop to wall shear stress is valid for turbulent flow also,

From the definition of f, $t_w = \dfrac{fr\,\bar{u}^2}{2}$ and $r_w = \dfrac{1}{2}D$

Therefore,

$$-\left[\frac{\Delta P}{L} + r g \frac{\Delta h}{L}\right] = \frac{2 f \bar{u}^2 r}{D}$$

For laminar case the right hand side was $\dfrac{32 m \bar{u}}{D^2}$, therefore,

$$\frac{2 f \bar{u}^2 r}{D} = \frac{32 m \bar{u}^2}{D^2}$$

or

$$f = \frac{16}{D \bar{u} r / m} = \frac{16}{R_e}$$

Thus for laminar flow, the Fanning friction factor is equal to 16 divided by R_e.

For Turbulent Flow:

The relationship between f and Re is not so simple. Another parameter that affects the friction factor is the roughness of the pipe surface.

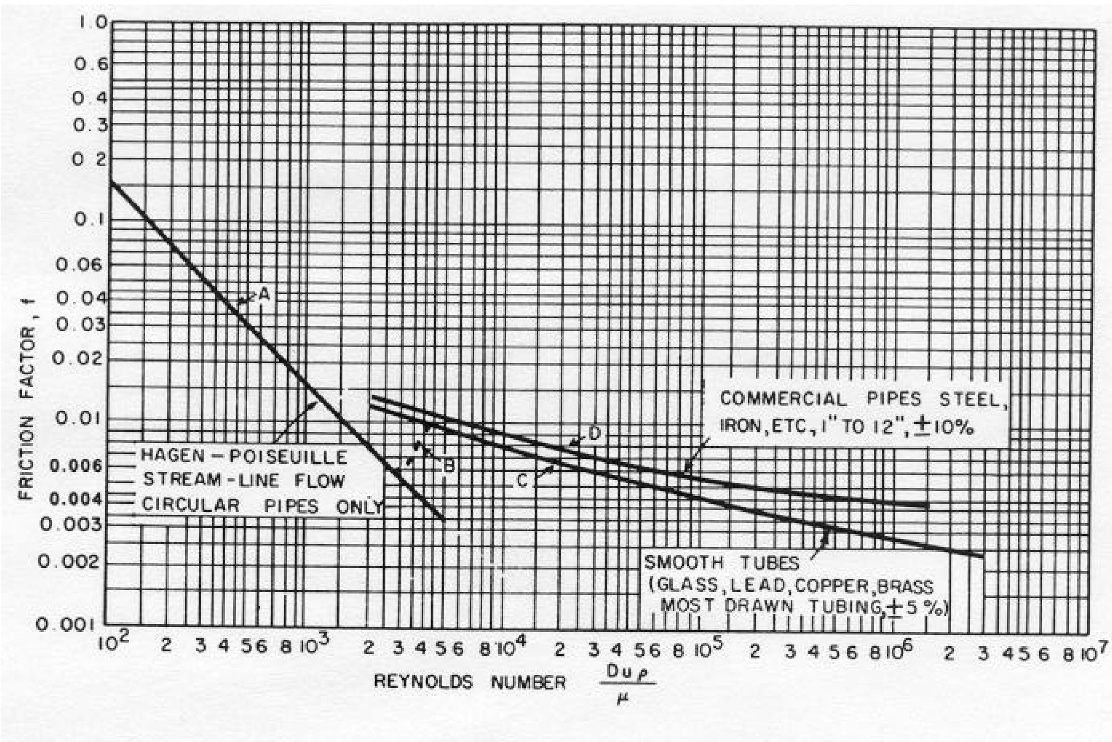

Figure 8-9 Friction Factor for Newtonian Fluids in Circular Pipes

In practice we use "Friction Factor Chart." For smooth pipes, the following approximation can be used:

$$f = 0.079\left(\frac{1}{R_e}\right)^{1/4}$$

Other, better correlations are available for estimating f as a function of Re and the pipe roughness.

Use of Friction Factor Chart

Quite straight forward in most calculations:

- Calculate Re
- Determine the pipe roughness
- Read the value of f at appropriate Re and pipe roughness
- Use

$-\left[\frac{\Delta P}{L} + rg\frac{\Delta h}{L}\right] = \frac{2f\bar{u}^2 r}{D}$ to calculate pressure drop.

The use is somewhat more involved when the pressure drop is given and you have to calculate the flow rate or average velocity.

In this case a trial and error procedure is needed. You guess the velocity and calculate the pressure drop using the procedure outlined above and see if the calculated pressure drop matches the given pressure drop.

8.5.3 Power Consumption

- This is a quantity of great practical interest in engineering applications of pipe flow calculations.
- In order to transport a fluid from one place to another, through a pipeline, you need to supply energy to overcome the frictional losses of mechanical energy (energy dissipation).
- The calculation of energy requirement or power needed is important for designing pumping stations and for estimating their operating cost. You need to know how big a pump to use and how much it will cost in terms of daily energy consumption.

Power = Q.DP

So if you know Q, and the pipe size, you can calculate DP and then calculate the power needed by the above relationship.

Example Problem 8-4

Crude oil (viscosity 0.1 Pa s; density 930 kg/m^3) is pumped through a 0.75 m inside diameter "smooth" steel pipe at the rate of 100,000 m^3 per day. The length of the pipe is 180 km.

(a) Calculate the Reynolds number of the flow.

(b) Estimate the pressure drop in the pipe. Assume the pipe to be horizontal and straight.

Solution

(a) The Reynolds number is $D\bar{u}\rho/\mu$; $\bar{u}$ for this quantity must be found from $Q = \bar{u}\,(\pi D^2/4)$. Now,

$$Q = 100{,}000\frac{m^3}{\text{day}} \times \frac{1\text{ day}}{24\text{ hours}} \times \frac{1\text{ hour}}{3600\text{ s}}$$

$$= 1.157\ m^3/s.$$

Therefore,

$$\bar{u} = (1.157)(4)/\pi(0.75)^2 = 2.62\ m/s.$$

Hence,

$$Re = (0.75)(2.62)(930)/0.1 = 1.83 \times 10^4$$

The flow of the oil is obviously turbulent.

(b) We can use Equation 8.60, with $\Delta h = 0$. So

$$-\Delta P = \frac{2f\bar{u}^2\rho L}{D}.$$

The friction factor (f) is found from Figure 8-9 to be 0.0065. (The Blasius equation gives $f = 0.0068$). So,

$$-\Delta P = P_1 - P_2 = 2(0.0065)(2.62)^2(930)(180 \times 10^3)/0.75$$

$$= 19.92\ MPa.$$

The pressure at the pipe inlet would have to be greater than the pressure at the pipe exit by 19.92 MPa to overcome the friction due to shear forces in the oil. To avoid having to compress the crude oil at the inlet to such high pressures two or more pumping stations may be installed along the length of the pipe.

Example Problem 8-5

What power is required to pump the crude oil in the Example Problem 8-4?

Solution

$$\text{Power} = (1.157)\,\frac{\text{m}^3}{\text{s}} \times 19.92\ \text{MPa}$$

$$= 23.05\ \text{MW}$$

This is the *minimum* power required by the compressors for delivery of the oil.

END OF CHAPTER 8 COVERAGE

NOTE THAT WE SKIPPED SECTIONS 8.5.3 AND 8.6