ENGG 201
Handout – Linear Interpolation

Case #1 – Two-Variable Interpolation

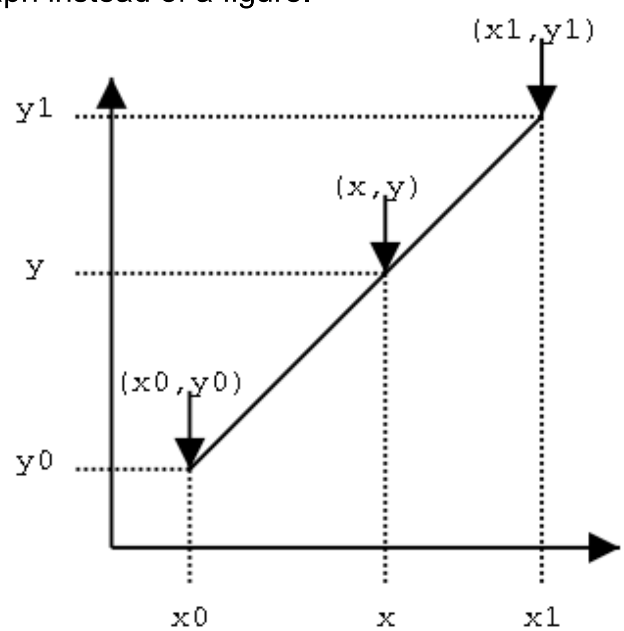
Interpolation – a mathematical prediction that is made <u>between</u> known values of data.

You interpolate when you have a figure or table with discrete data points and you know the coordinates (x_0,y_0) and (x_1,y_1), and want to find points in between by knowing either an x or y in the interval (x_0...x_1,y_0...y_1). You assume that a line joining the two points is a straight line.

For example you may have a table as shown below (given data in bold), and want to find the value of y that corresponds to a value of x between x0 and x1.

X	Y
x_0	**y_0**
x	y=?
x_1	**y_1**

It may be easier to visualize this as a graph instead of a figure.



By inspecting the figure we see that:

$$\frac{y - y_0}{y_1 - y_0} = \frac{x - x_0}{x_1 - x_0}$$

By manipulating this algebraically, we get:

$$y = y_0 + \frac{y_1 - y_0}{x_1 - x_0}(x - x_0)$$

This is similar to the familiar equation for a straight line:

$$y = mx + b$$

where: $m = \frac{y_1 - y_0}{x_1 - x_0}$ and $b = y_0 - x_0 \frac{y_1 - y_0}{x_1 - x_0}$

The same formula can easily be derived for x when y is known.

Example #1

The following table gives pizza price (the Y value) as a function of the number of slices of pepperoni added (the X value) for a pizza with 2.0 lbs of cheese.

Number of Slices of Pepperoni (#)	Pizza Price ($)
10	**$12.00**
20	**$18.00**

We want to find the price of a pepperoni pizza with 17 slices of pepperoni and 2 lbs of cheese, as shown in the modified table below:

Number of Slices of Pepperoni (#)	Pizza Price ($)
10	**$12.00**
17	Price = ??
20	**$18.00**

We expect the price to be between $12.00 and $18.00 (check at the end), and we assume a linear relationship between the number of slices and the price.

Using the formulas given above, we have:

$$y = y_0 + \frac{y_1 - y_0}{x_1 - x_0}(x - x_0)$$

where y = pizza price, and x = slices of pepperoni

Substituting numbers in from our original table, we get a general equation to interpolate the price between 10 and 20 pepperoni slices:

$$y = \$12.00 + \frac{\$18.00 - \$12.00}{20 - 10}(x - 10)$$

Substituting 17 in for x, we can solve for the price of our pizza:

$$y = \$12.00 + \frac{\$18.00 - \$12.00}{20 - 10}(17 - 10) = \$16.20$$

The price of **$16.20** is in the range we expected!

Case #2 – Three-Variable Interpolation

Use this when you have a figure or table and you know the third variable (z) at the coordinates (x_0,y_0), (x_0,y_1), (x_1,y_0), and (x_1,y_1), (this means you know z_{00}, z_{10}, z_{01}, and z_{11}). You want to find points in between (z_{xy}) by knowing both an x or y in the interval (x_0...x_1,y_0...y_1).

For example you may have a table as shown below (given data in bold), and want to find the value of z that corresponds to a value of x between x_0 and x_1, and a value of y between between y_0 and y_1,

z = f(x,y)		Y		
		y_0	y	**y_1**
X	**x_0**	**z_{00}**		**z_{01}**
	x	z_{x0}	z_{xy}=?	z_{x1}
	x_1	**z_{10}**		**z_{11}**

The procedure is simple – you need to perform 2 Two-Variable interpolations (like above in Case #1) to get z_{x0} and z_{x1}, and then a final Two-Variable interpolation between those values to get z_{xy}.

Example #2

The following table gives pizza price (the Z value) as a function of the number of slices of pepperoni added (the X value), and the pounds of cheese (the Y value).

Pizza Price (as a function of pepperoni and cheese)		Cheese (lb)	
		1.00	**2.00**
Pepperoni (slices)	**10**	**$9.00**	**$12.00**
	20	**$15.00**	**$18.00**

We want to find the price of a pepperoni pizza with 17 slices of pepperoni and 1.4 lbs of cheese, and as shown in the modified table below:

Pizza Price (as a function of pepperoni and cheese)		Cheese (lb)		
		1.00	1.40	**2.00**
Pepperoni (slices)	**10**	**$9.00**		**$12.00**
	17	w	Price = ??	$16.20
	20	**$15.00**		**$18.00**

Notice that we already have calculated (in Example #1) the price of a pizza with 17 slices of pepperoni <u>and 2.0 lbs of cheese</u>.

Now, we need to find the value of w so that we can interpolate between w and $16.20 (i.e. between 1.0 and 2.0 lbs of cheese on a 17-pepperoni pizza).

Substituting numbers in from our table, we get a general equation to interpolate the price between 10 and 20 pepperoni slices <u>at 1.00 lbs of cheese</u>:

$$y = \$9.00 + \frac{\$15.00 - \$9.00}{20 - 10}(x - 10)$$

Substituting 17 in for x, we can solve for w:

$$y = \$9.00 + \frac{\$15.00 - \$9.00}{20 - 10}(17 - 10) = \$13.20$$

Our table now looks like this:

Pizza Price (as a function of pepperoni and cheese)		Cheese (lb)		
		1.00	1.40	**2.00**
Pepperoni (slices)	**10**	**$9.00**		**$12.00**
	17	$13.20	Price = ??	$16.20
	20	**$15.00**		**$18.00**

Now we just need to interpolate between $13.20 and $16.20 (the prices at 1.0 lb and 2.0 lb of cheese for a 17-pepperoni pizza).

$$y = \$13.20 + \frac{\$16.20 - \$13.20}{2.00 - 1.00}(x - 1.00)$$

Substituting 1.40 lb in for x, we can solve for the price of our pizza:

$$y = \$13.20 + \frac{\$16.20 - \$13.20}{2.00 - 1.00}(1.40 - 1.00) = \$14.40$$

So, the price of a pizza with 1.40 lb of cheese and 17 slices of pepperoni is <u>**$14.40!**</u>