

## ENGG 201 Winter 2013 – Volumetric Behaviour of Liquids Problem Set

### Problem #1 – Winter 2012 Midterm 2

Use the data at the end of this exam (Table 7-1 and Table 7-2) to answer the following questions about liquid carbon tetrachloride ( $M=153.8$  kg/kmol,  $T_c=556.4$ K,  $P_c=4493$  kPa).

- Calculate the volume occupied by 5 kg of carbon tetrachloride at 35°C and 1 atm using the data in Table 7-2. (/6)
- Estimate the average isobaric coefficient of volume expansion for carbon tetrachloride between 35°C and 55°C at 1 atm. (/5)
- Using Tait's law, calculate the volume occupied by 5 kg of carbon tetrachloride at 35°C and 200 atm. (/7) (use c and d from solution)
- Use the Generalized Compressibility chart to estimate the molar volume of carbon tetrachloride at 35°C and 1347 kPa. (/4)

Answers: a)  $3.2 \times 10^{-3} \text{ m}^3$ ; b)  $1.2925 \times 10^{-3} \text{ K}^{-1}$ ; c)  $c=0.10384$ ,  $d=856.6 \text{ atm}$ ,  $3.125 \times 10^{-3} \text{ m}^3$ ; d)  $0.095 \text{ m}^3/\text{kmol}$

### Problem #2 – Winter 2011 Quiz 4

The critical pressure of hexane is 3020 kPa and the critical temperature is 234.5°C. Use this information and the generalized compressibility chart to estimate the volume of 2 kmol of liquid hexane at 1420 kPa and 183.75°C. (/2)

Answer:  $0.4815 \text{ m}^3$

### Problem #3 – Winter 2008 Quiz 4

Calculate the specific volume of carbon tetrachloride at 25°C and 400 atm using the information in Table 7-1. The specific volume of carbon tetrachloride at 25°C and 1 atm is  $6.312 \times 10^{-4} \text{ m}^3/\text{kg}$ . (/3)

Answer:  $c=0.1066$ ,  $d=985.14 \text{ atm}$ ,  $6.0827 \times 10^{-4} \text{ m}^3/\text{kg}$

### Problem #4 – Winter 2005 Quiz 4

The isothermal compressibility of carbon tetrachloride ( $M=153.8$  kg/kmol) at 35°C and 1 atm is  $11.95 \times 10^{-10} \text{ m}^2/\text{N}$ , and at 35°C and 1000 atm is  $5.52 \times 10^{-10} \text{ m}^2/\text{N}$ . Use the data to calculate the specific volume of liquid carbon tetrachloride at 700 atm and 35°C, given that the specific volume of liquid carbon tetrachloride at 1 atm and 35°C is  $6.39 \times 10^{-4} \text{ m}^3/\text{kg}$ . (3 Marks).

Answer:  $c=0.1039$ ,  $d=857.7 \text{ atm}$ ,  $5.994 \times 10^{-4} \text{ m}^3/\text{kg}$

## Volumetric Behaviour of Liquids

$$\beta_T = -\frac{1}{V} \left( \frac{\partial V}{\partial P} \right)_T$$

$$\alpha_P = \frac{1}{V} \left( \frac{\partial V}{\partial T} \right)_P$$

$$\gamma_V = \frac{1}{P} \left( \frac{\partial P}{\partial T} \right)_V$$

$$V_T = V_{T_0} (1 + A\theta + B\theta^2 + C\theta^3)$$

### Tait's Equation

$$\beta_T = \frac{c}{P + d}$$

$$\frac{V_o - V}{V_o} = c \ln \left[ \frac{P + d}{d} \right]$$

**Table 7-1 Isothermal Compressibilities of Selected Liquids**

Liquid	Temperature °C	$\beta \times 10^{10}, \text{m}^2/\text{N}$	
		1 atm	1000 atm
Benzene	25	9.67	5.07
	35	10.43	5.28
	45	11.32	5.50
	55	12.29	5.73
	65	13.39	5.98
Carbon-tetrachloride	25	10.67	5.30
	35	11.95	5.52
	45	12.54	5.75
	55	13.63	5.97
	65	14.87	6.22
n Hexane	0	13.04	5.92
	25	16.06	6.51
	40	18.31	6.89
	60	21.93	8.87
Mercury	20	0.40	0.39
Water	25	4.57	3.46
	35	4.48	3.42
	45	4.44	3.40
	55	4.44	3.42
	65	4.48	3.47

**Table 7-2 Coefficients of Cubical Expansion of Liquids  
at 1 atm,  $T_0 = 0^\circ\text{C}$**

	$A \times 10^3$	$B \times 10^6$	$C \times 10^8$	$V_{T_0} \times 10^3$ $\text{m}^3/\text{kg}$
Acetone	1.324	3.809	-0.87983	1.230
Benzene	1.17626	1.27755	0.80648	1.1109
Carbon-tetrachloride	1.18384	0.89881	1.35135	0.6126
Mercury	0.18169	0.00295	0.01146	0.07356
Water	-0.05325	7.6153	-4.3722	1.00013
n - Pentane	1.50697	3.435	0.975	1.549

**Question Number IV (22 Marks ~ 26 minutes)**

Use the data at the end of this exam to answer the following questions about liquid carbon tetrachloride ( $M=153.8 \text{ kg/kmol}$ ,  $T_c=556.4\text{K}$ ,  $P_c=4493 \text{ kPa}$ ).

- a) Calculate the volume occupied by 5 kg of carbon tetrachloride at  $35^\circ\text{C}$  and 1 atm using the data in Table 7-2. (/6)

$$V_T = V_{T0} (1 + A\theta + B\theta^2 + C\theta^3)$$

$$\theta = 35 - 0$$

$$V_T = 0.6126 \times 10^{-3} (1 + 1.18384 \times 10^{-3} (35) + 0.89881 \times 10^{-6} (35)^2 + 1.35135 \times 10^{-8} (35)^3)$$

$$V_{T0} = 0.6126 \times 10^{-3} \text{ m}^3/\text{kg}$$

$$V_T = 6.39 \times 10^{-4} \text{ m}^3/\text{kg}$$

$$V = m V_T = 5 \text{ kg} \times 6.39 \times 10^{-4} \text{ m}^3/\text{kg} = \boxed{3.20 \times 10^{-3} \text{ m}^3}$$

- b) Estimate the average isobaric coefficient of volume expansion for carbon tetrachloride between  $35^\circ\text{C}$  and  $55^\circ\text{C}$  at 1 atm. (/5)

$$V_{35} = 6.39 \times 10^{-4} \text{ m}^3/\text{kg}$$

$$V_{55} = 0.6126 \times 10^{-3} (1 + 1.18384 \times 10^{-3} (65) + 0.89881 \times 10^{-6} (65)^2 + 1.35135 \times 10^{-8} (65)^3)$$

$$V_{55} = 6.56 \times 10^{-4} \text{ m}^3/\text{kg}$$

$$\alpha_p = \left( \frac{\Delta V/V}{\Delta T} \right)_p = \frac{(V_{55} - V_{35})/V_{35}}{\Delta T}$$

$$\alpha_p = \frac{(6.56 - 6.39) \times 10^{-4} \text{ m}^3/\text{kg}}{20} \div 6.39 \times 10^{-4} \text{ m}^3/\text{kg}$$

$$\boxed{\alpha_p = 1.2925 \times 10^{-3} \text{ K}^{-1}}$$

- c) Using Tait's law, calculate the volume occupied by 5 kg of carbon tetrachloride at 35°C and 200 atm. (17)

$$\frac{V_0 - V}{V_0} = c \ln \left[ \frac{P+d}{d} \right] \quad @ \quad 35^\circ\text{C}$$

$$P_1 = 1 \text{ atm} \quad \beta_1 = 11.95 \times 10^{-10} \text{ m}^2/\text{N} \times 101325 \text{ Pa/atm} = 1.21 \times 10^{-4} \text{ atm}^{-1}$$

$$P_2 = 1000 \text{ atm} \quad \beta_2 = 5.52 \times 10^{-10} \text{ m}^2/\text{N} \times 101325 = 5.593 \times 10^{-5} \text{ atm}^{-1}$$

$$\beta_1 = \frac{c}{P_1 + d} \quad \beta_2 = \frac{c}{P_2 + d} \quad \rightarrow \quad c = \beta_1 (P_1 + d) = \beta_2 (P_2 + d)$$

$$d = \frac{\beta_1 P_1 - \beta_2 P_2}{\beta_2 - \beta_1} = \frac{(1.21 \times 10^{-4})(1) - (5.593 \times 10^{-5})(1000)}{5.593 \times 10^{-5} - 1.21 \times 10^{-4}} = 856.617 \text{ atm}$$

$$c = \beta_1 (P_1 + d) = 1.21 \times 10^{-4} (1 + 856.617) = 0.10384$$

$$\frac{V_0 - V}{V_0} = c \ln \left[ \frac{P+d}{d} \right] \quad \rightarrow \quad V = V_0 \left( 1 - c \ln \frac{P+d}{d} \right)$$

$$V_0 = 6.39 \times 10^{-4} \text{ m}^3/\text{kg} \quad @ \quad 1 \text{ atm}, 35^\circ\text{C}$$

$$V = 6.39 \times 10^{-4} \left( 1 - 0.10384 \ln \left( \frac{200 + 856.617}{856.617} \right) \right)$$

$$V = 6.25 \times 10^{-4} \text{ m}^3/\text{kg}$$

$$V = \frac{v}{M} \quad \rightarrow \quad v = V_M = 5 \text{ kg} \times 6.25 \times 10^{-4} \text{ m}^3/\text{kg}$$

$$v = 3.125 \times 10^{-3} \text{ m}^3$$

- d) Use the Generalized Compressibility chart to estimate the molar volume of carbon tetrachloride at 35°C and 1347 kPa. (14)

$$T_r = T/T_c = 308.15/556.4 = 0.55$$

$$P_r = P/P_c = 1347/4493 = 0.30$$

$$z = 0.05 \quad (\text{chart})$$

$$V_m = \frac{zRT}{P} = \frac{(0.05)(8.314)(308.15)}{1347} = 0.095 \text{ m}^3/\text{kmol}$$

$$\left( \text{EXTRA} \rightarrow V = \frac{0.095}{153.8} = 6.18 \times 10^{-4} \text{ m}^3/\text{kg} \right)$$

2. The critical pressure of hexane is 3020 kPa and the critical temperature is 234.5°C. Use this information and the generalized compressibility chart to estimate the volume of 2 kmol of liquid hexane at 1420 kPa and 183.75°C. (/2)

$$P_c = 3020 \text{ kPa}$$

$$P = 1420 \text{ kPa}$$

$$T_c = 234.5^\circ\text{C} = 507.65$$

$$T = 183.75^\circ\text{C} = 456.9 \text{ K}$$

$$T_r = \frac{T}{T_c} = \frac{456.9}{507.65} = \underline{0.90}$$

$$P_r = \frac{P}{P_c} = \frac{1420}{3020} = \underline{0.47}$$

$$Z_c = 0.09$$

$$PV_m = ZRT$$

$$V_m = \frac{ZRT}{P} = \frac{0.09 \left( 8.314 \frac{\text{kPa} \cdot \text{m}^3}{\text{kmol} \cdot \text{K}} \right) (456.9 \text{ K})}{1420 \text{ kPa}}$$

$$V_m = 0.24076 \text{ m}^3/\text{kmol}$$

$$V_m = \frac{V}{n} \rightarrow V = V_m n$$

$$= 0.24076 \times 2 \text{ kmol}$$

$$V = 0.4815 \text{ m}^3$$

4. Calculate the specific volume of carbon tetrachloride at 25°C and 400 atm using the information in the Table below. The specific volume of carbon tetrachloride at 25°C and 1 atm is  $6.312 \times 10^{-4} \text{ m}^3/\text{kg}$ . (3)

Table 7-1 Isothermal Compressibilities of Selected Liquids

Liquid	Temperature °C	$\beta \times 10^{10}, \text{m}^2/\text{N}$	
		1 atm	1000 atm
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	55	12.29	5.73
	65	13.39	5.98
Carbon-tetrachloride	25	10.67	5.30
	35	11.95	5.52
	45	12.54	5.75
	55	13.63	5.97
	65	14.87	6.22

$$V = ? @ 25^\circ\text{C}, 400 \text{ atm}$$

$$\frac{V_0 - V}{V_0} = c \ln \left[ \frac{P+d}{J} \right]$$

need  $c, d$  @ 25°C

$$\beta_1 = \frac{c}{P_1 + d} \quad \beta_2 = \frac{c}{P_2 + d}$$

$$c = \beta_1 (P_1 + d) \quad c = \beta_2 (P_2 + d)$$

$$\beta_1 P_1 + d \beta_1 = \beta_2 P_2 + d \beta_2$$

$$d = \frac{\beta_2 P_2 - \beta_1 P_1}{\beta_1 - \beta_2}$$

$$d = \frac{(5.3 \times 10^{-5})(1000) - (1.081 \times 10^{-4})(1)}{1.081 \times 10^{-4} - 5.37 \times 10^{-5}}$$

$$\triangleright d = 985.14 \text{ atm}$$

$$c = \beta_1 (P_1 + d) = 1.081 \times 10^{-4} (1 + 985.14)$$

$$\triangleright c = 0.1066$$

$$1 - \frac{V}{V_0} = c \ln \left[ \frac{P+d}{J} \right] \rightarrow V_0 \left[ 1 - c \ln \left[ \frac{P+d}{J} \right] \right] = V$$

$$V = 6.312 \times 10^{-4} \frac{\text{m}^3}{\text{kg}} \left[ 1 - 0.1066 \ln \left[ \frac{400 + 985.14}{985.14} \right] \right]$$

$$\boxed{V = 6.0827 \times 10^{-4} \frac{\text{m}^3}{\text{kg}}}$$

$$P_1 = 1 \text{ atm}$$

$$\beta_1 = 10.67 \times 10^{-10} \frac{\text{m}^2}{\text{N}} \times \frac{1}{\text{Pa}}$$

$$\beta_1 = 1.081 \times 10^{-4} \text{ atm}^{-1} @ P_1 = 1 \text{ atm}$$

$$\beta_2 = 5.3 \times 10^{-10} \frac{\text{m}^2}{\text{N}}$$

$$\beta_2 = 5.37 \times 10^{-5} \text{ atm}^{-1} @ P_2 = 1000 \text{ atm}$$

2. The isothermal compressibility of carbon tetrachloride ( $M=153.8 \text{ kg/kmol}$ ) at  $35^\circ\text{C}$  and 1 atm is  $11.95 \times 10^{-10} \text{ m}^2/\text{N}$ , and at  $35^\circ\text{C}$  and 1000 atm is  $5.52 \times 10^{-10} \text{ m}^2/\text{N}$ . Use the data to calculate the specific volume of liquid carbon tetrachloride at 700 atm and  $35^\circ\text{C}$ , given that the specific volume of liquid carbon tetrachloride at 1 atm and  $35^\circ\text{C}$  is  $6.39 \times 10^{-4} \text{ m}^3/\text{kg}$ . (3 Marks).

$$V = ? \text{ @ } 35^\circ\text{C}, 700 \text{ atm}$$

$$\frac{V_0 - V}{V_0} = c \ln \left[ \frac{P+d}{d} \right]$$

need  $c, d$  @  $35^\circ\text{C}$

$$\beta_1 = \frac{c}{P_1 + d}$$

$$\beta_2 = \frac{c}{P_2 + d}$$

$$c = \beta_1 (P_1 + d) \quad c = \beta_2 (P_2 + d)$$

$$\beta_1 P_1 + d\beta_1 = \beta_2 P_2 + d\beta_2$$

$$d = \frac{\beta_2 P_2 - \beta_1 P_1}{\beta_1 - \beta_2}$$

Solve for  $c=0.1039$ ,  $d=857.7 \text{ atm}$  given data in problem

$$1 - \frac{V}{V_0} = c \ln \left[ \frac{P+d}{d} \right] \rightarrow V_0 \left[ 1 - c \ln \left[ \frac{P+d}{d} \right] \right] = V$$

$$V = 6.39 \times 10^{-4} \frac{\text{m}^3}{\text{kg}} \left[ 1 - 0.1039 \ln \left[ \frac{700 + 857.7}{857.7} \right] \right]$$

$$V = 5.994 \times 10^{-4} \frac{\text{m}^3}{\text{kg}}$$