Lecture #1

Topic: **Units and Dimensions**

Learning Objectives:

1. The difference between units and dimensions
2. Seven basic SI units and their dimensions
3. How to determine dimensions of various quantities
4. How to check dimensional consistency of equations

Consider the following list of terms:

miles, mm, g, tons, kg, feet, inches, km, lb, ounces, cm, metres, mg, light-years.

You are asked to sort this list into two groups and I give you one member of each group.

Group A	miles
Group B	lb

Go ahead and use the next minute or so to sort the list. Let us now see what we have. I need a volunteer to come to the blackboard and complete the sorting.

Group A	miles, mm, feet, inches, km, cm. metres, light-years
Group B	lb, g, tons, kg, ounces, mg

What types of terms are we dealing with in this list? (Answer: All of these terms are units)

What are units? Units are measures that are used to describe the magnitude of a physical attribute.

What is special about the terms in group A that makes them belong to the same group? (Answer: All terms in Group A are units of distance or length)

What about group B? (Answer: All terms in Group B are units of mass)

We see that different units can be used to describe the same attribute. Length can be expressed in inches, feet, metres, km or light-years. Mass can be expressed in lb, g, kg or tons. The generic scientific name for the type of quantity that a specific unit represents is "**Dimension**"

This list involves two basic dimensions: **length and mass.**

Another basic dimension that you should be familiar with is **time.** The concept of time is used to describe how slowly or quickly things change or happen. The most natural measure of time, which all humans experience, is in days. A day represents the time for earth to complete one revolution around its axis. The earth goes around the sun, in its orbit, at the rate of one full cycle per year. The basic unit of time in the SI system is second.

Remember that dimensions describe the basic nature of what a physical quantity represents. You can describe the magnitude of something in many different units but the basic nature of what is being described remains unchanged, i.e. the dimensions of the quantity remain the same. The three basic dimensions are represented by these symbols,

[M] for mass,
[L] for length, and
[t] for time.

Seven basic SI units

The combinations of the three fundamental dimensions ([M], [L] & [t]) are sufficient to express many of the physical properties in engineering. However, to define certain other properties, such as temperature, electrical current and intensity of illumination, it becomes necessary to define a few other basic dimensions.

The seven basic units in the SI system are:

Quantity	Unit	Symbol	Dimension
Length	metre	m	[L]
Mass	kilogram	kg	[M]
Time	second	s	[t]
Electric current	ampere	A	[A]
Thermodynamic temperature	Kelvin	K	[T]
Amount of substance	mole	mol	[mol]
Luminous intensity	candela	cd	[cd]

2.2 Derived units and basic dimensions

Many of the complex units that you will encounter in engineering are nothing more than combination of these three basic dimensions. For example the speed of a moving object is expressed in terms of distance travelled per unit time. It represents the ratio of the distance travelled and the time elapsed. In terms of fundamental dimensions, it can be represented as

$$[u] = [L]/[t] \text{ which can be written as } [L]\,[t]^{-1}$$

Acceleration is the change in velocity per unit time. In terms of the fundamental dimensions, it can be written as,

$$[a] = [u]/[t] = [L]\,[t]^{-1}/[t] = [L]\,[t]^{-2}.$$

We know that force is equal to mass time acceleration. In terms of the fundamental dimensions, it means,

$$[F] = [M]\,[a] = [M]\,[L]\,[t]^{-2}.$$

Work done or energy is equal to force multiplied by distance. Therefore,

$$[W] = [F]\,[L] = [M]\,[L]^2\,[t]^{-2}.$$

Power is work done per unit time. Hence

$$[P] = [W]/[t] = [M]\,[L]^2\,[t]^{-3}.$$

Thus the dimensions of any physical quantity can be expressed as

$$[A] = [M]^a\,[L]^b\,[t]^c.$$

A physical descriptor is called dimensionless if and only if $a = 0$, $b = 0$ and $c = 0$. If any of the three exponents (a, b, c) is non-zero, the quantity is dimensional. Dimensionless quantities are very important in engineering for two reasons. Firstly, their magnitude does not depend on the system of units being used. For example, density of a substance is mass per unit volume, which is clearly a dimensional quantity with dimensions of $[M]\,[L]^{-3}$. The value of density of same substance expressed in different units will be different. In g/cc, the density of water is 1.0 while in units of kg/m^3 it is 1000. In units of pounds/ft^3, the density of water is about 62.4. The specific gravity of a substance is defined as

(Density of the substance)/(density of water at standard conditions).

Since density appears in both numerator and denominator, the dimensions cancel out and the specific gravity is clearly a dimensionless quantity. Its value for a given substance would be the same, no matter what system of units is being used.

The other advantage of dimensionless numbers in engineering comes in analyzing complex situations involving a large number of independent variables. By combining several variables into dimensionless groups, it becomes easier to evaluate the combined effect of several variables simultaneously.

Determining Dimensions of a quantity.
There are two ways for determining the dimensions of something. You can use the basic definition of what that quantity represents or very often you can determine the dimensions from its SI units. We will see how this is done.

Let us take specific heat.

<u>Definition</u>

c = Energy required to increase the temperature of a unit mass of the material by 1 ^{o}C.

c = (energy)/(mass x temperature increase)

$$[c] = \frac{[energy]}{[M][T]} = \frac{[M][L]^2[t]^{-2}}{[M]\{T\}} = [L]^2[t]^{-2}[T]^{-1}$$

<u>Using SI units to determine dimensions</u>

The dimensions of a quantity can sometimes be inferred from its SI units. For example, let us consider surface tension. Its SI units are N/m or kg/s^2. You know that kg represents mass and s is the unit for time. Therefore you can write

[Surface tension] = $[M][t]^{-2}$.

Checking dimensional consistency of an equation

When you are dealing with an equation of the type

$A + B = C + D$

Where A, B, C , D represent different physical quantities, the equation is dimensionally consistent if and only if

$[A] = [B] = [C] = [D]$

That is all terms must have the same dimensions.

Example Problem

The flow of a fluid in a circular tube is described by the following equation:

$$\rho \frac{\partial u}{\partial t} = \frac{P_o - P_L}{L} + \mu \frac{\partial^2 u}{\partial r^2} + \frac{\mu}{r}\frac{\partial u}{\partial r}$$

where:
- ρ: the fluid density,
- μ: the fluid viscosity (S.I. units: $kg.m^{-1}.s^{-1}$),
- L: length of the tube,
- u: velocity of the fluid,
- t: time,
- r: radial position in the tube (length),
- P_o, P_L: the pressures at the entrance and exit of the tube.

Show that the above equation is dimensionally homogeneous (use dimensions and not units).

Solution:

$[P_o]=[P_L] = [Force/Area]== [M\,L\,t^{-2}]/[L^2]== [M]\,[L]^{-1}\,[t]^{-2}$

$[\mu] = [\,M]\,[L]^{-1}[\,t]^{-1}$

$[\rho] = [mass/volume] ==[M][\,L]^{-3}$

$[u] = [L][t]^{-1}$

$[(P_o-P_L)/L] ==[\,P_o-P_L][L]^{-1} ==[M]\,[L]^{-1}\,[t]^{-2} *[L]^{-1} ==[M]\,[L\,]^{-2}[t]^{-2}$

$[\mu\ (\delta^2u/\delta r^2)\,] ==[\mu][(u/L^2)] ==[M][L]^{-1}\,[t]^{-1}*\,[L][t]^{-1}\,[L]^{-2} ==[M]\,[L\,]^{-2}[t]^{-2}$

$[\mu\,/r(\,\delta u/\delta r)\,] ==[\mu][L]^{-1}\,[(u\,/L)\,] ==[M][L]^{-1}[t]^{-1}*\,[L]^{-1}\,*[L][t]^{-1}\,*[L]^{-1} ==[M]\,[L\,]^{-2}[t]^{-2}$

All terms appearing on the right hand side (RHS) have a dimension of $[M][L]^{-2}[t]^{-2}$, therefore it is possible to add them and the dimension of the right hand side term is:

$$[RHS] == [M]\,[L]^{-2}[t]^{-2}$$

The left hand side term (LHS) has dimension:

$[L.H.S] \quad == [\rho][u][t]^{-1} \quad == [M][L]^{-3} * [L][\,t]^{-1} * [t]^{-1} \qquad == [M]\,[L\,]^{-2}[t]^{-2}$

As a conclusion, the LHS and RHS terms have the same dimension. Therefore the equation is dimensionally homogeneous.