EXAMPLE — RATE OF HEAT CONDUCTION IN PIPES

CH9

(FALL 1993 FE)

A copper tube, 5 cm inside diameter and 5 mm in wall thickness, is carrying a refrigerant at -30°C. The pipe is covered with two layers of insulating material. The thickness of the inner and outer layers are 1 cm and 2 cm, respectively. The thermal conductivity of the inner layer is 0.3 W/mK. The material for the outer layer is polyurethane foam. The temperature at the outside surface of the foam layer is 10°C.

a) Calculate the rate of heat flow per unit length of pipe without any insulation, assuming that the outside surface of the uninsulated pipe is at 0°C.

b) Calculate the rate of heat flow per unit length of the insulated pipe

c) Calculate the amount of heat loss in one day from a 100 m long insulated pipe.

d) Calculate the temperature at the interface between the two insulation layers.

FORMULA SHEET (continued)

Thermal Expansion of Solids

$$\alpha_L = \frac{1}{L}\frac{dL}{dT}$$

Heat Conduction

Fourier's Law

$$Q = -\kappa A \frac{dT}{dx} = -\kappa A \frac{\Delta T}{\Delta x}$$

Composite Planar Wall

$$Q = \frac{-A\ \Delta T}{\left[\frac{\Delta x_1}{\kappa_1} + \frac{\Delta x_2}{\kappa_2} + \frac{\Delta x_3}{\kappa_3} + \ldots\right]}$$

Simple Cylinder

$$Q = \frac{-2\pi\kappa L\,\Delta T}{\ln\left(\frac{r_2}{r_1}\right)}$$

Composite Cylinder (Pipe)

$$Q = \frac{-2\pi L \Delta T}{\frac{\ln(r_2/r_1)}{\kappa_1} + \frac{\ln(r_3/r_2)}{\kappa_2} + \frac{\ln(r_4/r_3)}{\kappa_3} + \ldots\ldots}$$

Stress and Strain in Solids

Normal Stress

$$\varepsilon = \frac{1}{E}\sigma \qquad \upsilon = -\frac{\varepsilon_x}{\varepsilon_y}$$

$$\varepsilon_x = \frac{1}{E}\sigma_x - \frac{\upsilon}{E}(\sigma_y + \sigma_z) \qquad \varepsilon_y = \frac{1}{E}\sigma_y - \frac{\upsilon}{E}(\sigma_x + \sigma_z) \qquad \varepsilon_z = \frac{1}{E}\sigma_z - \frac{\upsilon}{E}(\sigma_y + \sigma_x)$$

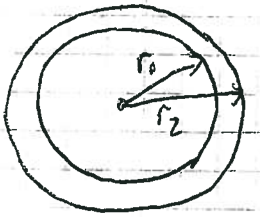
Bulk Modulus for Volume Change

$$\frac{\Delta V}{V} = -\frac{P}{K} \qquad K = \frac{E}{3(1-2\upsilon)}$$

Shear Stress

$$\tau = G\gamma \qquad G = \frac{E}{2(1+\upsilon)}$$

Ai: a) Heat flow/unit length uninsulated pipe



inside diameter = 5 cm $\rightarrow$ $r_1 = 2.5\times10^{-2}\,m$

thickness = 5 mm $\equiv 5\times10^{-3}\,m \rightarrow r_2 = r_1 + t$

$r_2 = 2.5\times10^{-2}\,m + 5\times10^{-3}\,m$

$T_1 = 273.15 - 30 = 243.15\,K$ $\qquad r_2 = 3.0\times10^{-2}\,m$ $\qquad T_2 = 273.15\,K$

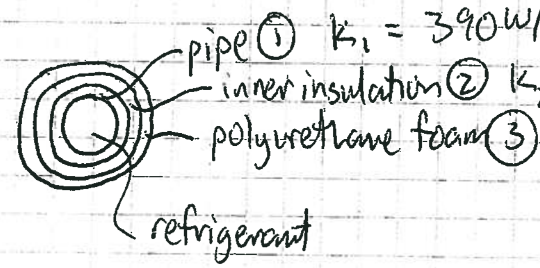
Copper tube thermal conductivity (TABLE 9-3) $k = 390\,W/mK$

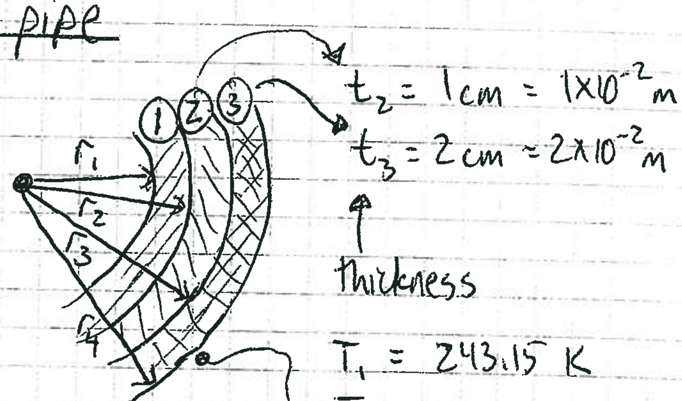
$$Q = \frac{2\pi L k (T_1 - T_2)}{\ln(r_2/r_1)} \rightarrow \frac{Q}{L} = \frac{2\pi k (T_1 - T_2)}{\ln(r_2/r_1)}$$

$$\frac{Q}{L} = \frac{2\pi \times 390\,W/mK\,(243.15 - 273.15\,K)}{\ln(3.0\times10^{-2}\,m / 2.5\times10^{-2}\,m)} = -403206\,\frac{W}{m}$$

$\boxed{\frac{Q}{L} = -403\,\frac{kW}{m}}$ $\Longrightarrow$ negative means heat loss

b) Heat flow/unit length insulated pipe





$T_1 = 243.15\,K$
$T_4 = 283.15\,K$ (10°C)

$r_1 = 2.5\times10^{-2}\,m$ (2.5 cm)
$r_2 = 3.0\times10^{-2}\,m$ (3.0 cm)
$r_3 = r_2 + t_2 = 3.0\times10^{-2}\,m + 1\times10^{-2}\,m = 4\times10^{-2}\,m$ (4 cm)
$r_4 = r_3 + t_3 = 4\times10^{-2}\,m + 2\times10^{-2}\,m = 6\times10^{-2}\,m$ (6 cm)

Polyurethane foam thermal conductivity (TABLE 9-3) $k_3 = 0.05\,W/mK$

$$Q = \frac{-2\pi L \Delta T}{\frac{\ln(r_2/r_1)}{k_1} + \frac{\ln(r_3/r_2)}{k_2} + \frac{\ln(r_4/r_3)}{k_3}}$$

$\Delta T = T_4 - T_1$
$\Delta T = 283.15 - 243.15$
$\Delta T = 40\,K$

$$\frac{Q}{L} = \frac{-2\pi(40K)}{\frac{\ln\left(\frac{3.0\times10^{-2}m}{2.5\times10^{-2}m}\right)}{390\,W/mK} + \frac{\ln\left(\frac{4\times10^{-2}m}{3\times10^{-2}m}\right)}{0.3\,W/mK} + \frac{\ln\left(\frac{6\times10^{-2}m}{4\times10^{-2}m}\right)}{0.05\,W/mK}}$$

$$\boxed{\frac{Q}{L} = -27.71\ \frac{W}{m}}$$ ← Much less than the uninsulated pipe!

c) <u>Heat loss from 100 m of insulated pipe in one day</u>

$$\frac{Q}{L} = -27.71\ \frac{W}{m} \rightarrow Q = (-27.71\ \tfrac{W}{m})(100m) = -2771\ W$$

$Q = -2771\ \frac{J}{s}$ one day $= 24h \times 60\frac{min}{h} \times \frac{60s}{min} = 86400s$

Heat loss $= -2771\ \frac{J}{s} \times 86400s = \boxed{-239\ MJ}$ $(-239\times10^6\ J)$

d) <u>Calculate temperature at interface between insulation layers</u>

(see diagram for b))

Overall Rate of heat loss Q = Heat loss rate through each layer

$Q = Q_1 = Q_2 = Q_3$ Need T_3 between layer 2 and layer 3

$$\frac{Q_3}{L} = \frac{Q}{L} = \frac{2\pi k_3(T_3 - T_4)}{\ln(r_4/r_3)} = -27.71\ \frac{W}{m} = \frac{2\pi(0.05\frac{W}{mK})(T_3 - 283.15K)}{\ln(6\times10^{-2}m/4\times10^{-2}m)}$$

$$\boxed{T_3 = 247.4\ K \quad (-25.8\ °C)}$$

$T_4 = 10°C$ $T_1 = -30°C$ ← Large temperature gradient in outer polyurethane foam insulation. (good insulator / poor conductor)