8.2 Potential Flow

Potential flow is an idealised flow that represents the situation in which the viscosity of the fluid can be assumed to be zero. Zero viscosity means there is no friction and therefore no conversion of the mechanical energy to heat. It is an approximation of the real flow under conditions in which the viscous forces are very small compared to inertial effects.

All fluid flows are governed by the conservation principles:

- Conservation of mass
- Conservation of momentum
- Conservation of energy

Some of the important terms in evaluating flow behaviour are:

- Potential energy
- Kinetic energy
- Pressure
- Work done by/on the fluid

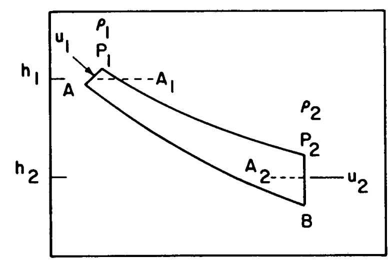


Figure 8-3 Streamlines of Flow of a Fluid in a Gravitational Field

8.2.1 Bernoulli's Equation

The path followed by the movement of a particle or pocket of fluid in potential flow is called "streamline" or "pathline". Streamlines do not intersect each other and no particle can flow across one streamline to another.

The Bernoulli’s equation relates to the conservation of energy in frictionless flow. The energy balance for a unit volume of fluid gives:

$$P + \frac{1}{2}\rho u^2 + \rho h g = \text{constant, or}$$

$$\frac{\mathrm{P}}{\rho} + \frac{1}{2}u^2 + hg = \text{constant (for } \rho = \text{constant)}$$

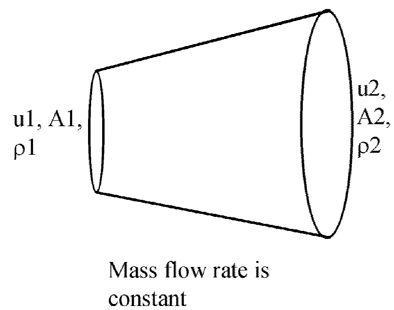
This expression is known as the Bernoulli's Equation. For the situation shown in Figure 8-3,

$$\frac{P_1}{\rho}+\frac{1}{2}u_1^2+h_1g=\frac{P_2}{\rho}+\frac{1}{2}u_2^2+h_2g$$

Bernoulli's equation is a very important equation in hydrodynamics and aerodynamics. There are several special cases are of particular interest.

(i) Flow through variable area duct.

The continuity equation (mass conservation equation) is



$$u_1A_1\rho_1=u_2A_2\rho_2$$

or

$$u_2=\frac{u_1A_1\rho_1}{A_2\rho_2}=\frac{u_1A_1}{A_2}\quad(\text{when}\,\rho_1=\rho_2)$$

Bernoulli's Equation is

$$\frac{P_1}{\rho}+\frac{1}{2}u_1^2+h_1g=\frac{P_2}{\rho}+\frac{1}{2}u_2^2+h_2g$$

If the duct is horizontal then $h_1 = h_2$, and

$$\frac{P_1}{\rho}+\frac{1}{2}u_1^2=\frac{P_2}{\rho}+\frac{1}{2}u_2^2=\frac{P_2}{\rho}+\frac{1}{2}u_1^2\left(\frac{A_1\rho_1}{A_2\rho_2}\right)^2, \text{ or}$$

$$\frac{P_1}{\rho}-\frac{P_2}{\rho}=\frac{1}{2}u_1^2\left(\frac{A_1\rho_1}{A_2\rho_2}\right)^2-\frac{1}{2}u_1^2=\frac{1}{2}u_1^2\left[\left(\frac{A_1\rho_1}{A_2\rho_2}\right)^2-1\right]$$

If P_1 and P_2 are of the same order of magnitude then the change in density with pressure for liquids can be neglected. Therefore:

$$\frac{P_1}{\rho}-\frac{P_2}{\rho}=\frac{1}{2}u_1^2\left[\left(\frac{A_1}{A_2}\right)^2-1\right]=\frac{1}{2}u_2^2\left[1-\frac{A_2^2}{A_1^2}\right]$$

<u>(ii) Flow through a constriction or orifice</u>

In this case there is a sudden and drastic reduction in flow area, as the fluid passes through an orifice. In this case the area of the orifice is much smaller than the area of the conduit, i.e. $A_2 << A_1$ or $\frac{A_2}{A_1} \approx 0$.

Then

$$\frac{P_1 - P_2}{\rho} = \frac{u_2^2}{2}$$ or

$$u_2 = \sqrt{\frac{\Delta P}{2\rho}}$$

However, for real fluids, the frictional loss can not be neglected. When the frictional loss is taken into account the equation becomes

$$u_2 = C_0\sqrt{\frac{\Delta P}{\rho}}$$

C_o is determined experimentally for the fluid involved.

(i) <u>Hydrostatic pressure of a column of liquid</u>

For a constant diameter pipe A1 = A2, and if the density is constant then

$$\frac{P_1}{\rho} + \frac{1}{2}u_1^2 + h_1 g = \frac{P_2}{\rho} + \frac{1}{2}u_2^2 + h_2 g$$ and $u_1 = u_2$

Therefore,

$$\frac{P_2 - P_1}{\rho} = (h_1 - h_2)g$$

Or,

P_2-P_1 = $\rho g(h_1$-$h_2)$

Can be used when $u_1 = u_2$ and $A_1 = A_2$ (moving situation) or whenever $u_1 = u_2 = 0$.

Example Problem 8-2

The 10 cm long nozzle at the tip of a water hose is a truncated cone with a diameter of 7.5 cm at the end connected to the hose, and a diameter of 2.5 cm at the open end. If the water flow rate is 10.0 kg/s and its density is 1000 kg/m^3, calculate the pressure at the larger end if the water discharges into ambient air at 1 atm.

Solution

The height difference between the nozzle ends is negligible and water is essentially incompressible at the prevailing conditions. Equation 8.25 can be used in this problem,

$$\frac{P_2 - P_1}{\rho} + \frac{u_2^2 - u_1^2}{2} = 0$$

where subscript 1 denotes the larger nozzle end. First, one can calculate the velocities:

$$u_1 = \frac{\text{mass flow rate}}{\text{density x area}} = \frac{10 \times 4}{1000(\pi)(0.075)^2} \text{ m/s}$$

$$= 2.264 \text{ m/s.}$$

The second velocity is evaluated from

$$u_1 A_1 = u_2 A_2$$

$$\text{or,} \quad u_2 = \frac{A_1}{A_2} u_1$$

but $A_1 = 9A_2$.

$$\text{so,} \quad u_2 = (9)(2.264) = 20.37 \text{ m/s}$$

$$\text{and} \quad P_2 - P_1 = -\rho \left[\frac{u_2^2 - u_1^2}{2}\right]$$

$$= -1000 \; \frac{(20.37)^2 - (2.26)^2}{2}$$

$$= -204.91 \text{ kPa} \; .$$

The exit pressure, P_2, is 1 atm. or 101.33 kPa. Therefore, the pressure at the nozzle inlet, P_1 must be 101.33 − (−204.91) or 306.24 kPa. This is equivalent to a pressure of 3.02 atm at the inlet of the nozzle.

8.3 Flow of Viscous (Real) Fluids

Real fluids experience a certain resistance to flow. Overcoming this resistance requires work. It means that a portion of the total mechanical energy (pressure + gravitational potential + kinetic energy) is dissipated (converted to heat) during the flow.

For low flow rates, the heat generated is too small to make any significant difference in temperature. At high flow velocities in well-insulated systems, the temperature rise due to energy dissipation can be quite large.

When the velocity is low, fluid particles move in smooth streamlines.

When the flow velocity is high, eddies or circulatory motion of fluids takes place. This increases the rate of energy dissipation.

Thus there are two distinct patterns of flow: Laminar (no eddies) and Turbulent (eddies)

The transition from laminar flow turbulent flow occurs at threshold set of conditions that are predictable.

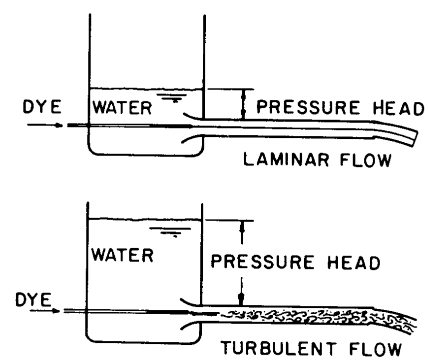


Figure 8-4 An Apparatus for Reynolds' Experiment

Reynolds Number

A dimensionless ratio of parameters

$$R_e = \frac{D\bar{u}\,\rho}{\mu}, \text{ Where,}$$

D = pipe diameter, m

$\bar{u}$ = average velocity, m/s

ρ= density, kg/m^3

μ= viscosity, Pa.s or kg/m.s

Laminar flow in pipes when Re < 2,100

Turbulent flow in pipes when Re > 4,000

For 2100 < Re < 4000, Intermediate or transition region. It means some turbulence has started but it is not fully developed.