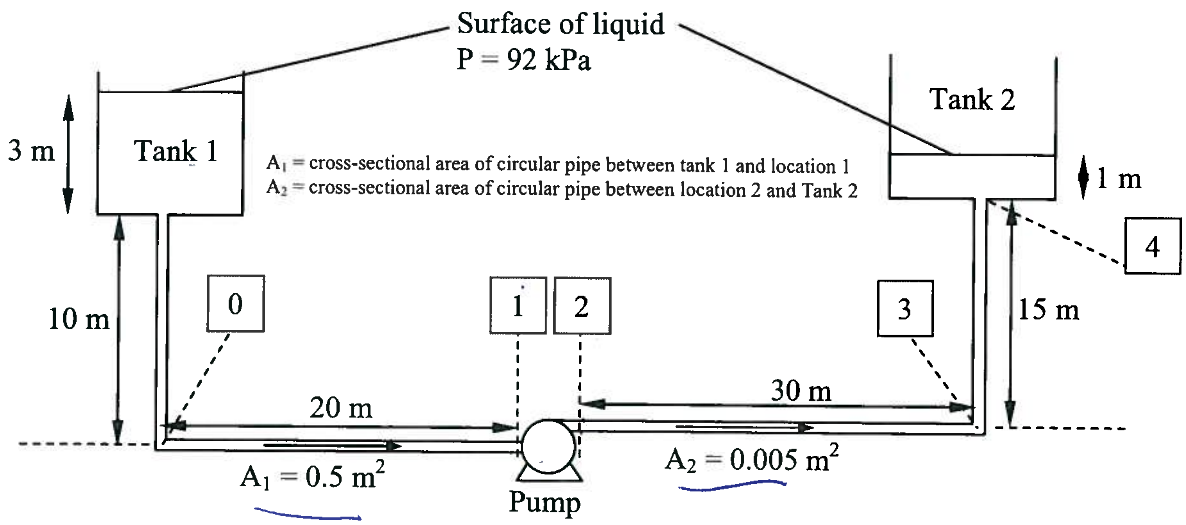
Sohrabi

Question Number IV *(22 Marks ~ 40 minutes)*

PART A

A pumping station is displayed in the following Figure.



The mass flow rate through the system is 1320 kg/h. The density and viscosity of the liquid are 980 kg/m^3 and 1.2 mPa s, respectively. The pipes can be treated as rough commercial steel.

$A = \frac{\pi}{4} D^2 \rightarrow D = \sqrt{\frac{4A}{\pi}} = 0.7979\,m$

a) Is the flow laminar or turbulent at location 1? **(/2)**

$\dot{m} = \rho Q \rightarrow Q = \frac{\dot{m}}{\rho} = \frac{1320\,kg/h}{980\,kg/m^3} = 1.347\,\frac{m^3}{h} \times \frac{1h}{3600\,s} = 3.74\times10^{-4}\,\frac{m^3}{s}$

$Q = vA \rightarrow \bar{v} = \frac{Q}{A} = \frac{3.74\times10^{-4}\,m^3/s}{0.5\,m^2} = 7.48\times10^{-4}\,m/s$ $\quad Re = \frac{D\bar{v}\rho}{\mu} = \frac{(0.7979\,m)(7.48\times10^{-4}\,\frac{m}{s})980}{1.2\times10^{-3}}$

$\boxed{Re = 487.4 \text{ Laminar}}$

b) At location 1, what is the maximum velocity in the circular pipe? **(/1)**

$\frac{U_{max}}{2} = \bar{U} \rightarrow U_{max} = \bar{U}\,2$ (laminar)

$U_{max} = 2\times7.48\times10^{-4}\,m/s = \boxed{1.496\times10^{-3}\,m/s = U_{max}}$

c) Is the flow laminar or turbulent at location 2? **(/2)**

$\rightarrow$ location 2 $\rightarrow Q = Q_{pipe\,1} = 3.74\times10^{-4}\,m^3/s$

$\bar{V}_2 = \frac{Q}{A} = \frac{3.74\times10^{-4}\,m^3/s}{0.005\,m^2} = 0.0748\,m/s$ $\quad A = \frac{\pi}{4}D^2 \rightarrow D = \sqrt{\frac{4A}{\pi}} = 0.07979\,m$

$Re = \frac{D\bar{v}\rho}{\mu} = \frac{0.07979\times0.0748\times980}{1.2\times10^{-3}} = \boxed{4874 \quad \text{Turbulent}}$

continued…

Question Number IV *(Continued)*

d) What is the pressure at location 1? *(/3)*

section #1 → pressure @ bottom of Tank 1 = 92000 Pa + (980)(3)(9.81)

LAMINAR FLOW (bottom of Tank 1 to $\boxed{1}$) = 120841 Pa

$$-\left[\frac{\Delta P}{L} + \rho g \frac{\Delta h}{L}\right] = \frac{32\mu\bar{v}}{D^2}$$

$\Delta h = 0 - 10\,m$

$\Delta P = P_1 - 120841\ Pa$

$$-\left[\frac{\Delta P}{30} + 980 \times 9.81 \frac{(-10)}{30}\right] = \frac{32 \times 1.2\times10^{-3} \times 7.48\times10^{-4}}{(0.7979\,m)^2} = 4.51166\times10^{-5}$$

$$\Delta P + 980 \times 9.81 \times (-10) = -1.35\times10^{-3}$$

96138

$$\Delta P = 96137.99865 = P_1 - 120841$$

$$\boxed{P_1 = 216978.9986\ Pa} \quad (216.98\ kPa)$$

e) What is the pressure at location 2? *(/4)*

section #2 → pressure @ bottom of Tank 2 = 92000 Pa + 980×1×9.81

TURBULENT FLOW (bottom of Tank 2 to $\boxed{2}$) = 101613.8 Pa = P_4

$$-\left[\frac{\Delta P}{L} + \rho g \frac{\Delta h}{L}\right] = \frac{2f\bar{v}^2\rho}{D}$$

$\Delta h = 15 - 0$

$\Delta P = P_4 - P_2 = 101613.8 - P_2$

$Re = 4.874\times10^3 \rightarrow f = 0.012$ (accept close to this)

$$-\left[\frac{\Delta P}{45} + 980 \times 9.81 \frac{15}{45}\right] = \frac{2 \times 0.012 \times (0.0748\,m/s)^2 \times 980}{0.07979} = 1.649$$

$$\Delta P + 980 \times 9.81 \times 15 = -74.217$$

$$\Delta P = -144132\ Pa = 101613.8 - P_2$$

$$\boxed{P_2 = 245746.6\ Pa}$$

f) What is the power of the pump? *(/2)*

Power = $Q\Delta P$

$\Delta P = P_2 - P_1 = 28767.58\ Pa$

Power = $(3.74\times10^{-4})(28767)$

$Q = 3.74\times10^{-4}\ m^3/s$

$$\boxed{Power = 10.76\ W}$$

continued...

Question Number IV *(Continued)*

PART B - In a bearing, the gap between the moving surface and a stationary surface is 0.05 mm. The lubrication liquid is at 80°C and has density equal to 870 kg/m^3 and viscosity equal to 140 cP. The moving surface slides past the stationary one at 0.778 m/s. The contact area between the surfaces is 0.78 cm^2.

a) If the flow is laminar between the two surfaces, what is the shear stress exerted by the steel part on the liquid in the gap? ***(/2)***

$$\frac{du}{dy} = \frac{\Delta v}{\Delta y} = \frac{0.778\ m/s}{0.05\times10^{-3} m} = 15560\ s^{-1}$$

$$\mu = \tau/(du/dy) \rightarrow \tau = \mu\frac{du}{dy} = (140\times10^{-3} Pa\,s)(15560\ s^{-1}) = \boxed{2178.4\ Pa}$$

b) What is the force applied to the steel part to keep it in motion? ***(/2)***

$$\tau = \frac{F}{A} \qquad A = 0.78\ cm^2 \times \left(\frac{1\ m}{100\ cm}\right)^2 = 7.8\times10^{-5} m^2$$

$$F = \tau A = (2178.4\ Pa)(7.8\times10^{-5} m^2) = \boxed{0.1699\ N}$$

c) The lubricant is mixed with another liquid. Viscometer tests of the liquid mixture yield the following data:

τ Shear Stress, Pa	du/dy Shear Rate, s^{-1}	μ_{APP}
510.29	10.9	46.82 Pa s
1370.23	38.7	35.41 Pa s

i. Does this liquid exhibit power-law behaviour? ***(/1)***

$\mu_{APP} = \tau/(du/dy) \rightarrow$ Non-newtonian

$\boxed{\text{yes}}$

ii. What is the apparent viscosity at a shear rate of 20 s^{-1}? ***(/3)***

$$\tau = K\left(\frac{du}{dy}\right)^n \rightarrow \log\tau = \log K + n\log du/dy$$

$$\log(510.29) = \log K + n\log(10.9)$$
$$\log(1370.23) = \log K + n\log(38.7)$$

$$\log\left(\frac{510.29}{1370.23}\right) = n\log(10.9/38.7)$$

$$-0.4289764 \qquad -0.55028$$

$$\boxed{n = 0.7796}$$

$$510.29 = K(10.9)^{0.77956}$$

$$\boxed{K = 79.26\ Pa\,s^{0.77956}}$$

$$\mu_{APP} = \frac{\tau}{du/dy} = K\left(\frac{du}{dy}\right)^{n-1}$$

$$\mu_{APP} = 79.26(20)^{0.77956-1} = \boxed{42.966\ Pa\,s = \mu_{APP}}$$