Question Number IV *(25 Marks ~ 20 minutes)*

a) An ideal gas mixture is held at a temperature of 100°C, and a pressure of 180kPa with a molar composition of 30% hydrogen (H_2, M=2 kg/kmol), 50% methane (CH_4, M=16 kg/kmol), and 20% pentane (C_5H_{12}, M=72kg/kmol)

i. What is the average molar mass of the mixture? *(/4)*

$$\bar{M} = \sum y_i M_i = (0.30)(2) + (0.50)(16) + (0.20)(72) = \boxed{23.0 \text{ kg/kmol}}$$

ii. What are the mass fractions of the three components? *(/5)*

Assume 1 kmol

0.30 kmol H_2 × 2 kg/kmol = 0.6 kg
0.50 kmol CH_4 × 16 kg/kmol = 8.0 kg
0.20 kmol C_5H_{12} × 72 kg/kmol = 14.4 kg / 23.0 kg

$0.6/23 = 0.026 = w_{H_2}$
$8.0/23 = 0.347 = w_{CH_4}$
$14.4/23 = 0.626 = w_{C_5H_{12}}$
$\Sigma = 1.0$ ✓

iii. What is the density of the mixture? *(/3)* $(Pv = nRT)$

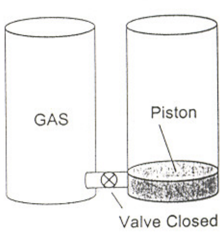
$$\rho = \frac{m}{V} = \frac{\bar{M}n}{V} = \frac{\bar{M}P}{\bar{R}T} = \frac{(23.0 \text{ kg/kmol})(180 \text{ kPa})}{(8.314 \frac{\text{kPa m}^3}{\text{kmol K}})(373 \text{ K})} = \boxed{1.335 \text{ kg/m}^3}$$

iv. What are the partial pressures of each component? *(/3)*

$\bar{p}_i = y_i P$

$\bar{p}_{H_2} = 0.30(180 \text{ kPa}) = \boxed{54 \text{ kPa}} = \bar{p}_{H_2}$
$\bar{p}_{CH_4} = 0.50(180 \text{ kPa}) = \boxed{90 \text{ kPa}} = \bar{p}_{CH_4}$
$\bar{p}_{C_5H_{12}} = 0.20(180 \text{ kPa}) = \boxed{36 \text{ kPa}} = \bar{p}_{C_5H_{12}}$
$\Sigma = 180$ ✓

b) Two identical rigid cylinders 2 m high with a diameter of 0.5 m are connected by a short length of tubing (ignore the tube volume) with a valve. The top of the second cylinder has been removed and replaced by a 20kg piston which can freely move up and down. Atmospheric pressure can be taken as 100kPa.



i. What mass of methane gas would be stored in the first cylinder at a pressure of 0.16 MPa and a temperature of 300K if the valve were closed? *(/5)*

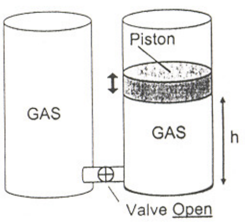
$V = \pi r^2 h$
$V = \pi \left(\frac{0.5m}{2}\right)^2 (2m)$
$V = 0.3925 \, m^3$
$Pv = nRT$

$$n = \frac{PV}{\bar{R}T} = \frac{(0.16 \times 10^3 \text{ kPa})(0.3925 m^3)}{(8.314 \frac{\text{kPa m}^3}{\text{kmol K}})(300 \text{ K})}$$

$n = 2.518 \times 10^{-2}$ kmol

$m = Mn = (2.518 \times 10^{-2} \text{ kmol})(16 \text{ kg/kmol})$

$\boxed{m = 0.4028 \text{ kg}}$ or (402.8 g)



ii. The valve between the cylinders is opened (see diagram), and the system is allowed to come to equilibrium at the same temperature (300K). What is the new height of the piston (h)? *(/5)*

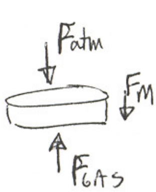
new h → need new V → $(Pv = nRT)$ → need new P

$\boxed{F_{GAS} = F_{atm} + F_m}$

$P_{GAS} A_{piston} = P_{atm} A_{piston} + M_{piston} g$

$A_{piston} = \pi r^2$
$= \pi \left(\frac{0.5m}{2}\right)^2$
$A = 0.1963 \, m^2$

$P_{GAS}(0.1963 m^2) = (100 \times 10^3 Pa)(0.1963 m^2) + (20 kg)(9.81 m/s^2)$



$P_{GAS} = 100\,999$ Pa
$P_{GAS} = 100.999$ kPa
$Pv = nRT$

$$V = \frac{nRT}{P} = \frac{(2.518 \times 10^{-2} \text{ kmol})(8.314)(300)}{100.999 \text{ kPa}} = 0.6218 \, m^3$$

$V = V_1 + V_2$
$0.6218 \, m^3 = 0.3925 \, m^3 + V_2$
$V_2 = 0.2293 \, m^3$
$V_2 = \pi r^2 h_2$
$0.2293 m^3 = \pi \left(\frac{0.5}{2}\right)^2 h_2$ → $h_2 = \boxed{1.168 m}$

Question Number V *(25 Marks ~ 20 minutes)*

At low pressures and high temperatures, nitrogen (M_{N2}=28 kg/kmol) can be assumed to behave like an ideal gas.

a) What is the velocity that the highest number of nitrogen molecules would be expected to be traveling at if the conditions are 250°C and 2 bars? *(/4)*

$$C_{mp} = \sqrt{\frac{2RT}{M}} = \sqrt{\frac{2\left(8.314\,\frac{kPa\,m^3}{kmol\,K}\right)(250+273)\times 1000\,\frac{Pa}{kPa}}{28\,kg/kmol}} = \underline{557.3\,m/s}$$

b) What is the root mean square velocity of nitrogen molecules at 250°C and 2 bars? *(/4)*

$$\sqrt{\overline{c^2}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3\left(8.314\,\frac{kPa\,m^3}{kmol\,K}\right)(523\,K)\times 1000\,\frac{Pa}{kPa}}{28\,kg/kmol}} = \underline{682.6\,m/s}$$

c) What is the mean separation distance between nitrogen molecules at 250°C and 2 bars? *(/4)*

$$k = \frac{R}{N_A} = \frac{8.314\,\frac{kPa\,m^3}{kmol\,K}}{6.023\times10^{26}/kmol} = 1.38\times10^{-26}\,kJ/K$$

$$\delta = \left[\frac{kT}{P}\right]^{1/3}$$

$$= \left[\frac{1.38\times10^{-26}\,kJ/K \times 523\,K}{200\,kPa}\right]^{1/3} = \underline{3.3\times10^{-9}\,m} \text{ or } \underline{33\,\text{Å}}$$

d) If the viscosity of nitrogen is 5x10^{-5} Pa.s at 250°C and 2 bars, what would the viscosity be at 500°C and 4 bars? *(/4)*

$$\mu_1 = \frac{M}{N_A\pi\sigma^2}\sqrt{\frac{RT_1}{\pi M}} \qquad \frac{\mu_2}{\mu_1} = \frac{\sqrt{T_2}}{\sqrt{T_1}} \qquad \mu_2 = \frac{\sqrt{773\,K}}{\sqrt{523\,K}}\,5\times10^{-5}\,Pa.s$$

$$\mu_2 = \frac{M}{N_A\pi\sigma^2}\sqrt{\frac{RT_2}{\pi M}} \qquad \mu_2 = \frac{\sqrt{T_2}}{\sqrt{T_1}}\times\mu_1 \qquad \boxed{\mu_2 = 6.08\times10^{-5}\,Pa.s}$$

e) Using the information in (d), what is the collision diameter of a nitrogen molecule? *(/4)* – You can use condition ① or ② → same answer

$$\sigma = \sqrt{\frac{M}{N_A\pi\mu_1}\sqrt{\frac{RT}{\pi M}}}$$

$$\sigma = \sqrt{\frac{28\,kg/kmol}{6.023\times10^{26}/kmol\;\pi\;(5\times10^{-5}\,Pa.s)}\sqrt{\frac{8.314\,\frac{kPa\,m^3}{kmol\,K}\cdot 523\,K\times 1000\,\frac{Pa}{kPa}}{\pi\times 28\,kg/kmol}}}$$

$$\sigma = \boxed{2.565\times10^{-10}\,m} \text{ or } \underline{2.565\,\text{Å}}$$

f) What would be the force pushing on the inside surface of a spherical 1 m diameter balloon filled with nitrogen at 500°C and 4 bars? *(/5)*

$$F = PA \qquad A = SA = 4\pi r^2$$

$$= 4(\pi)\left(\frac{1\,m}{2}\right)^2$$

$$F = (400\,kPa)(3.14159\,m^2) \qquad = 3.14159\,m^2$$

$$\boxed{F = 1256\,kN} \qquad P = 4\times100\,kPa = 400\,kPa$$

FORMULA SHEET

Constants / Conversions

$$R = 8.314\frac{kPa.m^3}{kmol.K} = 8.314\frac{J}{mol.K} \qquad N_A = 6.023x10^{26}\,\frac{molecules}{kmol} \qquad g = 9.81 m/s^2$$

$101.325\ kPa = 1\ atm$ $\qquad$ $1\ bar = 100\ kPa$ $\qquad$ $1\ L = 1000\ cm^3$

Geometric Shapes

$$V_{sphere} = \frac{4}{3}\pi r^3 \qquad SA_{sphere} = 4\pi r^2 \qquad V_{cylinder} = \pi r^2 h$$

Ideal Gas

$$Pv = nRT$$

Kinetic Theory of Gases

$$c_{mp} = \sqrt{\frac{2RT}{M}} \qquad \sqrt{\overline{c}^2} = \sqrt{\frac{3RT}{M}} \qquad \overline{c} = \sqrt{\frac{8RT}{\pi M}}$$

$$P = \frac{N_A m\overline{c}^2}{3V_m} \qquad E_k = \frac{1}{2}m\overline{c}^2 \qquad k = \frac{R}{N_A}$$

$$\lambda = \frac{1}{\sqrt{2}\pi\sigma^2\rho_N} \qquad \delta = \left[\frac{kT}{P}\right]^{1/3} \qquad \rho_N = \frac{N_A}{V_m} = \cancel{\frac{kT}{P}}\ \frac{P}{kT}$$

(TYPO)

Kinetic Theory of Gases - Transport Properties

$$\mu = \frac{\rho_N \overline{c}\lambda m}{2} \qquad \mu = \frac{M}{N_A\pi\sigma^2}\sqrt{\frac{RT}{\pi M}}$$

$$\kappa = \frac{\lambda\rho_N\overline{c}}{2}\frac{C_v}{N_A} \qquad \kappa = \frac{C_v}{N_A\pi\sigma^2}\sqrt{\frac{RT}{\pi M}}$$