Question Number II *(27 Marks ~ 32 minutes)*

Answer the following questions about ethane (C_2H_6) (M=30.07 kg/kmol) molecules using the kinetic theory of gases and assuming ideal gas behaviour. The collision diameter of C_2H_6 can be assumed to be 4.44 Å.

a) Calculate the kinetic energy of a single ethane molecule at 250°C and 3 atm. (/4)

$$E_k = \frac{1}{2} m \overline{c^2}$$

$$m = \frac{M}{N_A} = \frac{30.07 \text{ kg/kmol}}{6.023\times10^{26}/\text{kmol}}$$

$$E_k = \frac{1}{2} m \frac{3RT}{M}$$

$$m = 4.9867\times10^{-26} \text{ kg}$$

$$\overline{c^2} = \frac{3RT}{M}$$

$$E_k = \frac{1}{2}(4.9867\times10^{-26}) \times \frac{3\times8.314\times(250+273.15)}{30.07}$$

$$\boxed{E_k = 1.082\times10^{-23} \text{ kJ}}$$

b) Calculate the effective separation distance between ethane molecules (Å) at 250°C and 3 atm. (/3)

$$\delta = \left[\frac{kT}{P}\right]^{1/3}$$

$$\delta = \left[\frac{1.3805\times10^{-23} \text{ J/K} \times (250+273.15)}{3\times101325 \text{ Pa}}\right]^{1/3}$$

$$\delta = 6.193\times10^{-9} \text{ m}$$

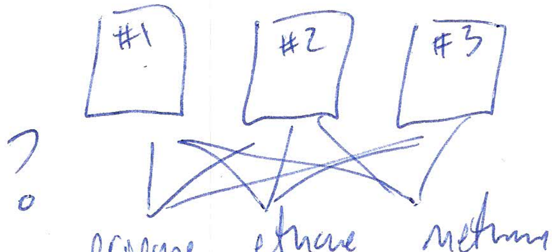
$$\boxed{\delta = 61.9 \text{ Å}}$$

c) What would be the average speed of the ethane molecules at 250°C and 3 atm? (/3)

$$\bar{c} = \sqrt{\frac{8RT}{\pi M}} = \sqrt{\frac{8 \times 8314 \frac{Pa\,m^3}{kmol\,K} \times (250+273.15)}{\pi \times 30.07}}$$

$$\boxed{\bar{c} = 606.9 \text{ m/s}}$$

d) Three identical containers (#1, #2 and #3) each contain a different single ideal gas, each at 200°C and 2 atm. It is known that each tank contains either propane (44.1 kg/kmol), ethane (30.07 kg/kmol), or methane (16 kg/kmol). The ratio of the density of Tank#2 to Tank #1 is 2.756 and the ratio of the rms speeds of Tank #3 to Tank #2 is 1.211. Identify which gas is in which container. (/5)



$$\frac{\rho_2}{\rho_1} = 2.756$$

$$\frac{c_{rms3}}{c_{rms2}} = 1.211$$

$$\rho_i = \frac{PM}{RT}$$

$$\frac{\rho_2}{\rho_1} = \frac{P_2 M_2}{RT_2} \cdot \frac{RT_1}{P_1 M_1} = \frac{M_2}{M_1} = 2.756$$

Try $M_1 = 16$
then $M_2 = 44.1$ ✓

$$\boxed{\begin{array}{l} (1)\ \text{methane} \\ (2)\ \text{propane} \\ (3)\ \text{ethane} \end{array}}$$

OR use $c_{rms} = \sqrt{\frac{3RT}{M}}$

$$\frac{c_{rms3}}{c_{rms2}} = \frac{\sqrt{M_2}}{\sqrt{M_3}} = \frac{\sqrt{44.1}}{\sqrt{30.07}} = 1.21 \checkmark$$

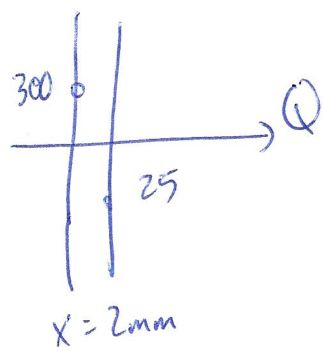
e) Calculate the thermal conductivity of ethane gas at 250°C and 3 atm. (/3)

$$k = \frac{C_v}{N_A \pi \sigma^2} \sqrt{\frac{RT}{\pi M}} \qquad C_v = \frac{3}{2} R$$

$$k = \frac{\frac{3}{2}(8.314)}{6.023 \times 10^{26} \pi (4.44 \times 10^{-10})^2} \sqrt{\frac{8314 \times (250 + 273.15)}{\pi \times 30.07}}$$

$$\boxed{k = 7.174 \times 10^{-6} \frac{kW}{mK}}$$

f) Ethane gas is trapped in a 2 mm wide gap within the hollow wall of a reactor. A small portion of the wall is uninsulated (2m x 2m) which means heat can escape the reactor. On each side of the gap is a thin metal wall. The inner wall, near the reactor contents, is at 300°C, while the outer wall is at 25°C. The pressure in the gap is 2 atm. Calculate the amount of heat lost in one day from the reactor to the outside through this uninsulated section, assuming no resistance through the actual metal wall, and assuming steady state. Also assuming that no gas or heat escape the edges of the gap. (/7)



$$\frac{Q}{A} = -k \frac{dT}{dx}$$

need k @ $\left(\frac{300+25}{2}\right) = 162.5°C \rightarrow 435.65 K$

$$\frac{k_1}{k_2} = \frac{\sqrt{T_1}}{\sqrt{T_2}} \rightarrow k_2 = \frac{\sqrt{T_2}}{\sqrt{T_1}} k_1 = \frac{\sqrt{435.65}}{\sqrt{523.15}} \times 7.17 \times 10^{-6} = 6.55 \times 10^{-6} \frac{kW}{mK}$$

$$\frac{Q}{A} = -6.55 \times 10^{-6} \frac{kW}{mK} \times \frac{(25 - 300)}{2 \times 10^{-3} m} = 0.900 \frac{kW}{m^2}$$

$$\frac{0.900 kJ}{s\, m^2} \times \underbrace{(2m^2 \times)}_{A} \underbrace{\left(\frac{24h}{d} \times \frac{60 min}{h} \times \frac{60 s}{min}\right)}_{t} = \boxed{155628 kJ}$$