QUESTION 1 (*11 marks*)

PART A: The Rayleigh Number (Ra) is a dimensionless number used to predict the onset of natural convection in a fluid. It is being proposed that Ra for fluid travelling in a cylindrical pipe can be calculated using the equation to the right:

$$\underbrace{Ra}_{\text{TERM A}} = \underbrace{\frac{g\beta(T_s - T_{air})D^3}{\left(\frac{\mu}{\rho}\right)^2}Pr}_{\text{TERM B}} + \underbrace{A}_{\text{TERM C}}$$

where: β = Coefficient of volumetric expansion
T_s = Temperature of the pipe surface, K
Pr = Prandtl Number (dimensionless)
μ = Viscosity of fluid, kg/(m·s)
g = gravitational acceleration
T_{air} = Temperature of air, K
ρ = Density of fluid
D = Pipe Diameter

(a) Determine the dimensions of A.

1 Assuming the equation is dimensionally homogenous (i.e. each term has the same dimensions), A would be dimensionless

(b) Use dimensional analysis to provide the dimensions of the coefficient of volumetric expansion, β. Also, provide typical SI units for β.

3 [TERM B] = [TERM A]

$$\frac{\frac{[L]}{[t]^2}[\beta][T][L]^3}{\left(\frac{[M]/[L][t]}{[M]/[L]^3}\right)^2} = 1$$

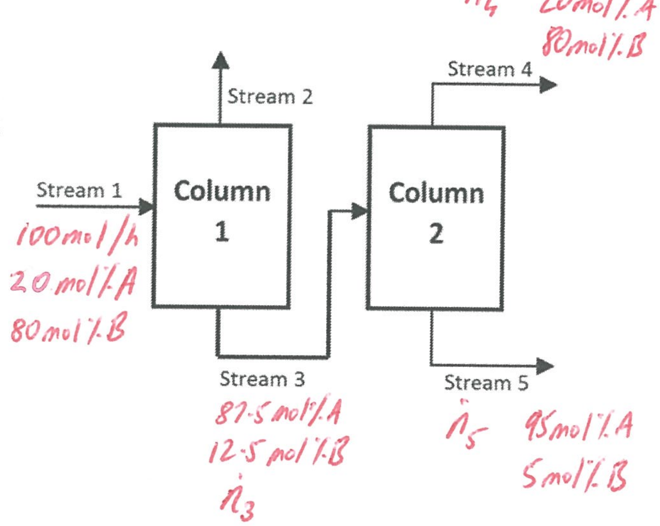
$$\frac{[L]^4[\beta][T]}{[t]^2} = \frac{[M]^2}{[L]^2[t]^2}\frac{[L]^6}{[M]^2}$$

$$\boxed{[\beta] = \frac{1}{[T]}}$$

$\boxed{\text{typical SI units: } \frac{1}{K}}$

PART B: A mixture containing 20 mol% Component A and 80 mol% Component B enters Distillation Column 1 at a rate of 100 mol/h in Stream 1. Streams 2 and 3, which are leaving Column 1, have identical flow rates of A. Stream 3 contains 12.5 mol% B. Stream 3 enters Distillation Column 2, and separates into two product streams. Stream 4 leaves Column 2 at a rate of $\dot{n}_4$ and contains 20 mol% A. Stream 5 leaves Column 2 at a rate of $\dot{n}_5$ and contains 95 mol% A. The entire process operates at steady state.

↳ no accumulation



QUESTION 1 ***Contd.***

(c) Determine the flow rate (in mol/h) of Component A in Stream 2.

2

Flow rate of A in stream 1 = (0.2)(100 mol/h) = 20 mol/h.

This flow rate of A is split between stream 2 and stream 3

∴ Flow rate of Component A in Stream 2 is $\frac{20\,mol}{h}/2 = \boxed{10\,mol/h}$

(d) Calculate the flow rate (in mol/h) of Component B in Stream 3

Flow rate of A in stream 3 = Flow rate of A in stream 2 = 10 mol/h.

2

This represents 87.5 mol% of the total flow rate ($\dot{n}_3$)

$$\therefore \quad \dot{n}_3 = \frac{10\,mol/h}{0.875} = 11.428\ mol/h.$$

∴ Flow rate of Component B in stream 3 = 11.428 mol/h − 10 mol/h = $\boxed{1.428\ \frac{mol}{h}}$

(e) What is the flow rate (in mol/h) of Stream 5?

3

We can perform a balance on each component for column 2

Balance on A: $10\,mol/h = (0.2)\dot{n}_4 + (0.95)\dot{n}_5$

$\therefore \quad \dot{n}_4 = 50\,mol/h - 4.75\,\dot{n}_5$ EQN (1)

Balance on B: $1.428\,mol/h = (0.8)\dot{n}_4 + (0.05)\dot{n}_5$

$1.786\,mol/h = \dot{n}_4 + 0.0625\,\dot{n}_5$ EQN (2)

Sub EQN (1) into (2)

$$1.786\,mol/h = (50\,mol/h - 4.75\,\dot{n}_5) + 0.0625\,\dot{n}_5$$

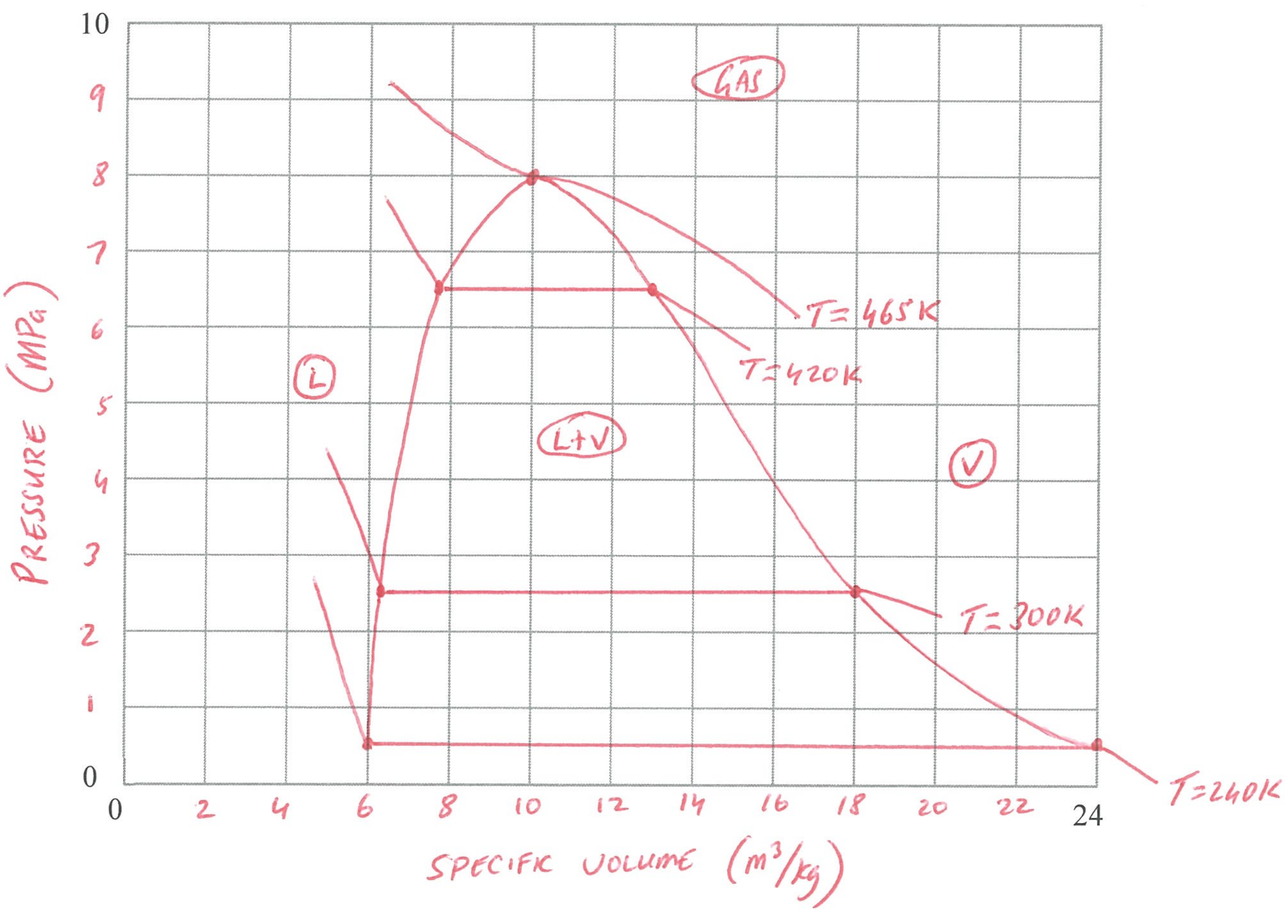
$$\boxed{\dot{n}_5 = 10.28\,mol/h}$$

Question 2 (*17 Marks*)

The table below contains vapour liquid equilibrium data for Substance Q. Use this data to answer the questions below.

Vapour Pressure (MPa)	T (K)	V_L (m^3/kg)	V_V (m^3/kg)
0.5	240	6	24
2.5	300	6.5	18
6.5	420	7.5	13
8	465	10	10

3 (a) Sketch a P-V phase diagram using the given information. Be sure to include the isotherms, and label the liquid, vapour, gas and two-phase regions.



Question 2 *Contd.*

(b) A mass of 100 g of Substance Q is placed in a 0.6 m^3 vessel at a temperature of 300 K. Determine the mass (in g) of each phase present at equilibrium.

1.5

$V = \frac{v}{m} = \frac{0.6 m^3}{0.1 kg} = 6 m^3/kg \longrightarrow \boxed{100 g \text{ of LIQUID}}$

(c) Approximate the volume (in m^3) that 300 g of Substance Q would occupy at a pressure of 0.505 MPa and a temperature of 240 K.

At this point, you would be just to the left of the bubble point specific volume at 240K.

1.5 $V_L \approx 6 m^3/kg \longrightarrow v = (6 m^3/kg)(0.3 kg) = 1.8 m^3$

Technically, since you are to the left of $V_L = 6 m^3/kg$, the volume would be slightly lower than $1.8 m^3$

(d) Calculate the degrees of freedom remaining at the critical point (*show your work*).

1 $F = C - P + 2 = 1 - 2 + 2 = 1$

However, the one degree of freedom is used up because $\rho_L = \rho_V$

$\therefore$ F remaining $= 0$

(e) 200 g of Substance Q is held at 420 K in a 4.5 m^3 container. Will the pressure under these conditions be less than, equal to, or greater than the vapour pressure at 420 K?

1 $V = \frac{v}{m} = \frac{4.5 m^3}{0.2 kg} = 22.5 m^3/kg \longrightarrow$ this is to the right of the dew point curve at T = 420K.

$\therefore$ P would be less than the vapour pressure.

(f) A mass of 50 g of Substance Q is placed in a rigid 500 L vessel at a temperature of 240 K.

(i) Calculate the mass (g) of each phase present at equilibrium.

3 $v = 500 L = 0.5 m^3 \longrightarrow V = \frac{v}{m} = \frac{0.5 m^3}{0.05 kg} = 10 m^3/kg$

L ——————— V

$V_L = 6 m^3/kg$ $V_{mix} = 10 m^3/kg$ $V_V = 24 m^3/kg$

$$\frac{M_L}{M_{mix}} = \frac{V_V - V_{mix}}{V_V - V_L}$$

$$\frac{M_L}{50 g} = \frac{24 m^3/kg - 10 m^3/kg}{24 m^3/kg - 6 m^3/kg}$$

$M_L = 38.9 g$ $\longrightarrow \therefore M_V = 50 g - 38.9 g = \boxed{11.1 g}$

Question 2 *Contd.*

(ii) How much Substance Q (in g) would you have to isothermally remove from the vessel to ensure that the mass of each phase present is equal at equilibrium?

2

When the mass of the liquid and vapor are identical, the value of V_{mix} must be right in the middle between V_v and V_L.

$$\therefore V_{mix} = 15\ m^3/kg \longrightarrow m = \frac{V}{V_{mix}} = \frac{0.5\ m^3}{15\ m^3/kg} = 0.0333\ kg$$

$\hookrightarrow 33.3\ g$

$\therefore$ mass removed $= 50g - 33.3g = \boxed{16.7g}$

(g) 200 g of Substance Q are held at 420 K in a variable volume container. You vary the size of the container so that each phase occupies an equal volume. Determine the new volume (in m^3) of the container at equilibrium.

4

volume of the liquid phase $= (V_L)(m_L) = (7.5\ m^3/kg)(m_L)$

volume of the vapor phase $= (V_v)(m_v) = (13\ m^3/kg)(m_v)$

In this case, we are told the two volumes are equal

$$\therefore (7.5\ m^3/kg)m_L = (13\ m^3/kg)m_v$$

$$m_L = 1.7\bar{3}\ m_v \quad \text{EQN (1)}$$

but we also know that $m_L + m_v = 0.2\ kg$

$$\therefore m_L = 0.2kg - m_v \quad \text{EQN (2)}$$

sub EQN (2) into (1)

$$(0.2kg - m_v) = 1.7\bar{3}\ m_v$$

$$m_v = 0.0732\ kg$$

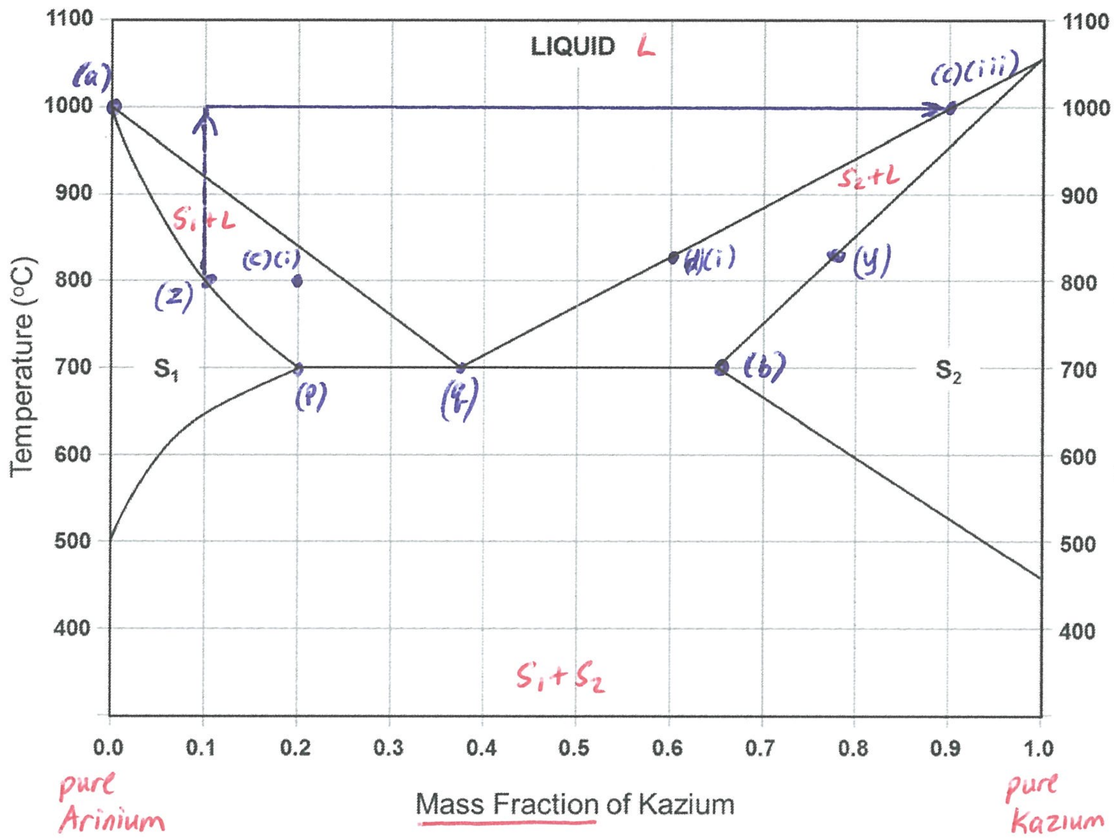
$\therefore$ volume of the vapor phase $= (13\ m^3/kg)(0.0732\ kg) = 0.951\ m^3$

This is also equal to the volume of the liquid phase.

$\therefore$ volume of container $= 0.951\ m^3 + 0.951\ m^3 = \boxed{1.902\ m^3}$

Question 3 (*19 Marks*)

Shown below is a ***Solid-Liquid*** phase diagram describing a binary mixture containing the components Kazium and Arinium at a constant pressure of 2 atm. The molar mass of Kazium is $M_K = 100$ kg/kmol. The molar mass of Arinium is $M_A = 44.44$ kg/kmol.



Use the phase diagram above to answer the following questions (*note: in order to receive full marks, you must show all of your work*).

(a) What is the melting point temperature (in °C) of pure Arinium?

1 1000 °C (see point (a) on graph)

(b) What is the maximum solubility (in mass %) of Arinium in Kazium?

1 35 mass % Arinium in Kazium (see point (b) on graph)

Question 3 *Contd.*

(c) 250 kg of a mixture with 0.8 mass fraction Arinium is maintained at a temperature of 800°C.

(i) Calculate the mass (in kg) of each phase present under these conditions?

3 0.8 mass fraction Arinium $\rightarrow$ 0.2 mass fraction Kazium
see point (c)(i) on graph
Since we are being asked how much of each phase we have, we can use the lever rule.

S_1 ——— L

$X_{S_1} = 0.1$ $X_{mix} = 0.2$ $X_L = 0.25$

$$\frac{M_{S_1}}{M_{mix}} = \frac{X_L - X_{mix}}{X_L - X_{S_1}}$$

$$\frac{M_{S_1}}{250\,kg} = \frac{0.25 - 0.2}{0.25 - 0.1}$$

$$\boxed{M_{S_1} = 83.33\,kg} \rightarrow \therefore M_L = 250\,kg - 83.33\,kg = \boxed{166.67\,kg}$$

(ii) Determine the mole fraction of Arinium in the mixture.

2 Since mol fraction is an intensive variable, we can answer this by assuming any convenient amount. Let A = Arinium, K = Kazium

Assume we have 100 kg

Mass of A $= (0.8)(100\,kg) = 80\,kg$
mass of K $= (0.2)(100\,kg) = 20\,kg$.

moles of A $= \left(80\,kg / 44.44\,kg/kmol\right) = 1.8\,kmol$

moles of K $= \left(20\,kg / 100\,kg/kmol\right) = 0.2\,kmol$

$\therefore$ mol fraction of Arinium $= \dfrac{1.8\,kmol}{1.8\,kmol + 0.2\,kmol} = \boxed{0.9}$

Question 3 *Contd.*

(iii) While maintaining a temperature of 800°C, the liquid phase is completely removed from the container and discarded. The remaining solid phase is then heated to 1000°C, and pure Kazium is added to the mixture. Determine the minimum mass (in kg) of Kazium that needs to be added to the mixture for solid S_2 to appear.

3

After discarding the liquid, you would be at point (z)
Point (c)(iii) is where the solid S_2 would appear (follow the arrows from (z) to (c)(iii))

In the initial solid : mass of Kazium = (0.1)(83.33 kg) = 8.333 kg
mass of Arinium = (0.9)(83.33 kg) = 75 kg.

Assume you added m_p kilograms of Kazium.
Final mixture : mass of Kazium = 8.333 kg + m_p
mass of arinium = 75 kg.

$$\frac{8.333\,kg + m_p}{8.333\,kg + m_p + 75\,kg} = 0.9 \longrightarrow m_p = \boxed{666.67\,kg}$$

(d) You start with a mixture containing 0.6 mass fraction Kazium at 1000°C.

→ at constant composition, solid will only appear if mixture is cooled.

2

(i) At what temperature (in °C) will the first solid appear? Determine the composition (in mass %) of this solid phase.

825 °C (see point (d)(i) on graph)
Composition : 78 mass % Kazium, 22 mass % Arinium (see point (y) on graph)

3

(ii) If the mixture described in (d)(i) is maintained at equilibrium at the eutectic temperature, list the phases present, and provide the composition of each listed phase.

Solid S_1 : 20 mass % KAZIUM (see point (p) on graph)
LIQUID : 37 mass % KAZIUM (see point (q) on graph)
SOLID S_2 : 65 mass % KAZIUM (see point (b) on graph)

1

(iii) Calculate the degrees of freedom remaining on the three phase line (*show all your work*)

$F = C - P + 2 = 2 - 3 + 2 = 1$

but this degree of freedom was used up by setting P to 2 atm.

$\therefore$ F remaining = $\boxed{0}$

<u>Question 3</u> ***Contd.***

(e) An equal mass of solid Kazium and solid Arinium are placed into a vessel and heated.

(i) While heating these solids, what is the minimum temperature (in °C) required to ensure that only a liquid exists within the vessel?

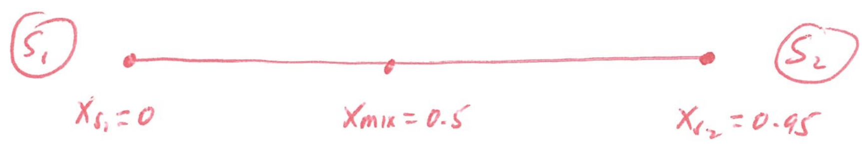
1 The temperature would have to be at the melting point of whichever pure component has the higher melting point $\boxed{1055\,°C}$

(ii) If the contents of the vessel are heated to 1100°C and then cooled to 490°C, what is the solubility (in mass%) of Kazium in Arinium?

1 0 mass % Kazium in Arinium

(iii) Following from (e)(ii) above, determine the mass fraction of S_1 in the container at equilibrium.

1



$$\text{mass fraction of } S_1 = \frac{X_{S_2} - X_{mix}}{X_{S_2} - X_{S_1}} = \frac{0.95 - 0.5}{0.95 - 0} = \boxed{0.474}$$

Question 4 *(13 Marks)*

A variable volume vessel (Container 1) is filled with 347 mol of air. Air can be assumed to be composed of 21 mol% oxygen (O_2) with the remainder being nitrogen (N_2). A second vessel (Container 2) has a fixed volume of 10 m^3, and is filled with helium (He) at a pressure of 3 atm and temperature of 200°C. The two containers are connected to one another via a valve. Answer the questions below assuming that the gases act in an ideal manner. The atomic mass of He = 4 kg/kmol, O = 16 kg/kmol, N = 14 kg/kmol.

(a) Find the initial amount (in mol) of He in Container 2.

2

$$PV = nRT \rightarrow n = \frac{PV}{RT}$$

$$= \frac{(3\,atm)(10\,m^3)}{\left(0.08205\,\frac{m^3\,atm}{kmol\,K}\right)(200+273.15)K} = 0.773\,kmol \rightarrow \boxed{773\,mol}$$

(b) The valve between the Containers is opened to allow helium to flow from Container 2 into Container 1. The valve is closed once the pressure in Container 2 drops to 2 atm. No air moves from Container 1 to Container 2 during this process. Assume the process is isothermal for Container 2

(i) Determine the amount (in mol) of Helium added to Container 1. $V_1 = V_2 \quad T_1 = T_2$

2

$$\frac{P_1 \cancel{V_1}}{n_1 \cancel{RT_1}} = \frac{P_2 \cancel{V_2}}{n_2 \cancel{RT_2}}$$

$$\therefore n_2 = \frac{P_2 n_1}{P_1} = \frac{(2\,atm)(773\,mol)}{3\,atm} = 515.3\,mol \rightarrow$$

$\therefore$ amt of He added to Container 1 $= 773\,mol - 515.3\,mol = 257.7\,mol$

(ii) Determine the composition (in mol% of O_2, N_2 and He) of the gas mixture in Container 1 after the helium has been added.

3

moles of $N_2 = (0.79)(347\,mol) = 274.1$

moles of $O_2 = (0.21)(347\,mol) = 72.9$

moles of He $= 257.7\,mol$

total moles $= 604.7\,mol$

$$\therefore mol\%\,N_2 = \frac{274.1\,mol}{604.7\,mol} \times 100\% = \boxed{45.3\%}$$

$$mol\%\,O_2 = \frac{72.9\,mol}{604.7\,mol} \times 100\% = \boxed{12.1\%}$$

$$mol\%\,He = \frac{257.7\,mol}{604.7\,mol} \times 100\% = \boxed{42.6\%}$$

Question 4 ***Contd.***

(iii) Determine the average molar mass (in kg/kmol) of the mixture in Container 1 after the helium has been added.

1 $\bar{M} = y_{N_2} M_{N_2} + y_{O_2} M_{O_2} + y_{He} M_{He}$

$= (0.453)(2)(14\ kg/kmol) + (0.121)(2)(16\ kg/kmol) + (0.426)(4\ kg/kmol)$

$= \boxed{18.26\ kg/kmol}$

(iv) The internal volume of Container 1 is set to 10 m^3. If the partial pressure of He in Container 1 is 1 atm, determine the temperature (in K) of the gas mixture in Container 1.

2 $P_{He} = y_{He} P \rightarrow P = \frac{P_{He}}{y_{He}} = \frac{1\,atm}{0.426} = 2.347\ atm.$

$$T = \frac{PV}{nR} = \frac{(2.347\,atm)(10\,m^3)}{(0.6047\,kmol)\left(0.08205\ \frac{atm\, m^3}{kmol\, K}\right)}$$

$T = \boxed{473\ K}$

(v) Container 1 develops a leak, and the gas mixture leaks out into the atmosphere until the pressure in the vessel is 1 atm. If the volume of the vessel remains unchanged at 10 m^3, determine the specific volume (in m^3/kg) of gas remaining in Container 1 at a temperature of 20°C.

3 $$\text{moles of gas remaining} = \frac{PV}{RT} = \frac{(1\,atm)(10\,m^3)}{\left(0.08205\ \frac{atm\, m^3}{kmol\, K}\right)(273.15+20)\,K} = 0.416\ kmol$$

$$\text{mass of remaining gas} = (0.416\ kmol)(18.26\ kg/kmol) = 7.59\ kg$$

$$\therefore\ V = \frac{v}{m} = \frac{10\,m^3}{7.59\,kg} = \boxed{1.32\ m^3/kg}$$