Chapter 8: The Motion of Fluids

Sections to be omitted from this chapter: 8.5.3 and 8.6

- This chapter is a brief introduction to mechanics of fluids.
- Here we will examine the importance and application of fluid properties in their movement/transportation through pipes or tubes.
- Viscosity and density are two of the important properties in the motion of fluids.

8.1.1 Stress and Strain in Fluids

Mechanical equilibrium requires that the net force must be equal to zero. When a fluid is at rest, the force exerted on it by surroundings must be equal and opposite to the force exerted by the fluid on its surroundings.

In the presence of an externally applied force, the fluid is said to be under **stress**. We will consider two types of stress: normal and shear.

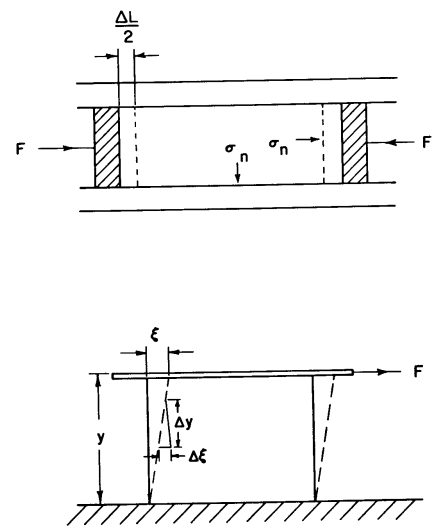

Figure 8-1 Stress in Fluids
(a) Normal Stress
(b) Shear Stress

Normal Stress (σ)

This type of stress is distributed uniformly throughout the fluid and it acts in outward direction, normal to all solid surfaces that the fluid is in contact with.

σ_n = -P =[F/A]

The application of pressure results in some small contraction of the fluid volume. Fractional change in volume = $\Delta V/V$

For the cylinder and piston, if A is constant, then

$$\frac{\Delta V}{V} = \frac{(\pi/4)D^2\Delta L}{(\pi/4)D^2 L_o} = \frac{\Delta L}{L_0}$$

= Fractional change in volume, called "normal strain."

Shear Stress

This type of stress is exerted on the surface of the fluid and acts parallel to the surface.

The fluid offers resistance to deformation, which is different from the simple compression. Shear stress is transferred to (imaginary) layers of the fluid.

Shear Stress = τ

Shear Strain = $\dfrac{\Delta\xi}{\Delta y}\, or \,\dfrac{d\xi}{dy}$ (ξ is Greek letter xi)

For an elastic material, $\tau \propto \dfrac{d\xi}{dy}$, which means the deformation is proportional to the applied shear stress. The material deforms under shear to an extent that depends on the magnitude of the applied shear stress. Also, in elastic materials, the deformation disappears when the applied stress is removed.

In case of fluids, the presence of shear stress results in deformation that continues to increase as long as the stress is there. Generally, the shear stress is proportional to rate of strain, i.e.

$$\tau = f\left(\frac{d}{dt}\left(\frac{d\xi}{dy}\right)\right) = f\left(\frac{d}{dy}\left(\frac{d\xi}{dt}\right)\right) = f\left(\frac{du}{dy}\right),$$

where, u is the velocity in one direction and y is the direction normal to the direction of velocity.

If $\tau = f\left(\dfrac{du}{dy}\right)$, then, the simplest functional form is

$$\tau = C\frac{du}{dy}.$$

Where, C is the constant of proportionality. Depending on the nature of C, fluids can be divided into three categories.

A. Ideal Fluids

An ideal fluid is an imaginary fluid for which C = 0. That is the shear stress resulting from a shear strain is zero. It is equivalent to assuming that the viscosity of the fluid is zero. Ideal fluids are also called:

- Frictionless
- Perfect
- Inviscid.

The resulting flow pattern is called "potential" or "ideal."

B. Newtonian Fluids

The fluids that display a linear relationship between shear stress and the rate of shear strain, i.e. for which C = constant (not equal to zero) are called Newtonian fluids. And for such fluids,

$\tau = -\mu \dfrac{du}{dy}$, where μ = viscosity or coefficient of dynamic viscosity.

Kinematic Viscosity

The ratio μ/ρ (dynamic viscosity/fluid density) is called kinematic viscosity (γ). Its units are m^2/s or mm^2/s or cSt.

C. Non-Newtonian Fluids

For a Newtonian Fluid, by definition,

$$\mu = \frac{\tau}{-\dfrac{du}{dy}} = \text{constant, i.e.} \neq f\left(\frac{du}{dy}\right)$$

Therefore, the viscosity is a function of temperature and pressure only. It does not depend on the flow conditions or flow history.

For non-Newtonian fluids, shear stress (τ) is not directly proportional to shear rate (du/dy). Generally the shear stress is a function of shear rate and time.

<u>Apparent viscosity for non-Newtonian Fluids</u>

The Newton's law of viscosity (which is the basis for defining viscosity) does not apply to non-Newtonian fluid, because the shear stress is not proportional to shear rate. However, for practical calculations, we can define "apparent viscosity" as the ratio of shear stress and shear rate.

$$\mu_{app} = \frac{\tau}{-du/dy} = f\left(\frac{du}{dy}\right)$$

The apparent viscosity for non-Newtonian fluids is a function of the shear rate. It can also be a function of shear history of the fluid.

<u>Common types of non-Newtonian Fluids</u>

(a) <u>Thixotropic Fluids</u> --> apparent viscosity decreases with time under shear to reach an ultimate value. Examples are slurries - e.g. Bentonite slurry.

Some kind of structure is formed at rest and broken as a result of shear.

(b) <u>Rheopectic Fluids</u> --> apparent viscosity increases with time under shear to reach a limiting value. This is opposite of thixotropic behaviour. Often emulsions and foams behave in this manner. The underlying mechanism is a change in size distribution of the dispersed phase. Some slurries can also be rheopectic.

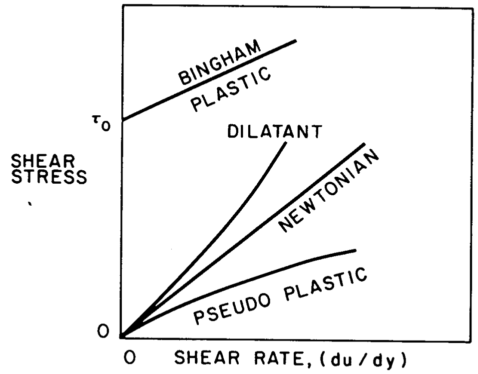

Figure 8-2 Steady Flow Fluid Behavior

(c) <u>Viscoelastic Fluids</u> --> these exhibit "rubber-like" properties, i.e. stretching under shear and partial elastic recovery when shear is removed. Synthetic fibres during their manufacture often show viscoelastic behaviour.

(d) Power-Law Fluids --> for these fluids, shear stress is proportional to shear rate raised to a power n. The Ostwald de Waele model

$$\tau = -K\left|\frac{du}{dy}\right|^{n} = -K\left|\frac{du}{dy}\right|^{n-1}\frac{du}{dy}$$

where, K = fluid consistency index and
n = fluid behaviour index.

Note that when n = 1, the fluid is Newtonian with $K \equiv \mu$, else

$$\mu_{app} = \frac{\tau}{du/dy} = K\left|\frac{du}{dy}\right|^{n-1}$$

The apparent viscosity of these fluids is only a function of shear rate, i.e. independent of the shear history.

For n > 1.0, the apparent viscosity increases with increasing shear rate and the fluid is called dilatant.

For n < 1.0, the apparent viscosity decreases with increasing shear rate and the fluid is called pseudo plastic.

By taking log of both sides of the equation $\tau = K\left|\dfrac{du}{dy}\right|^n$,

$$\log\tau = \log K + n\log\left(\frac{du}{dy}\right)$$

Therefore, a plot of log(τ) against log(du/dy) will be a straight line with slope = n, and the y intercept = log(K).

<u>(e)</u> <u>Bingham Plastic Fluids</u>

These fluids require a threshold value of shear stress before any flow can occur. The relationship between shear stress and shear rate can be modelled as:

$$\tau = \tau_o - \mu_o\left(\frac{du}{dy}\right) \text{ for } \tau \geq \tau_o$$

and du/dy = 0 for $\tau < \tau_o$

or $(\tau - \tau_o) = -\mu_o \dfrac{du}{dy}$

8.1.6 The Measurement of Viscosity

The actual or apparent viscosity of a fluid represents the ratio of shear stress to the shear rate corresponding to that shear stress. It means that the value of viscosity can be determined by simultaneously measuring shear stress and shear rate in a simple flow situation. The instruments that accomplish this are called viscometers. There are hundreds of possible variations of viscometer designs. Some of the more popular types are discussed below.

<u>**For Low Viscosities:**</u>

<u>Capillary Viscometer</u> --> Pressure drop across a capillary tube is measured at a constant flow rate.

$$\mu = \frac{\pi(-\Delta P)D^4}{128QL}$$

<u>Saybolt Viscometer (Cannon-Fenske)</u> --> A fixed volume of fluid is allowed to drain through a narrow tube under the force of gravity. The time for drainage is related to the fluid viscosity and density.

$$\nu \equiv \frac{\mu}{\rho} = At - \frac{B}{t}$$

For High Viscosities:

Falling Ball Viscometer --> A spherical ball (usually steel) is allowed to fall through a column of stationary fluid. Its terminal velocity is measured by measuring the time it takes to travel a fixed distance after reaching the terminal velocity. The terminal velocity provides a measure of fluid viscosity.

$\mu = \frac{2R^2(\rho_s - \rho_l)g}{9u_t}$, where

ρ_s = density of solid, and

ρ_l = density of liquid.

Co-axial Cylinder Viscometer (e.g. Fann Viscometer) --> one cylinder is rotated at fixed RPM, while the other is kept stationary by application of a counteracting torque. The torque needed to keep the cylinder stationary is related to the speed of rotation and the viscosity of the fluid by

$\mu = \frac{\Gamma}{k_o \Omega}$, where

Γ = Torque,

Ω = Angular velocity, and

k_o = apparatus constant

Cone-and-Plate Viscometer --> similar in principal to the concentric cylinder viscometer. Advantageous when only a small volume sample is available.