UNIVERSITY OF CALGARY

MO SHUILE TOGAM SUAS

FINAL EXAMINATION
(Fall, 2001-2002)

ENGG 201 - Behaviour of Liquids, Gases and Solids

December 13, 2001 Time Allowed: 180 minutes.

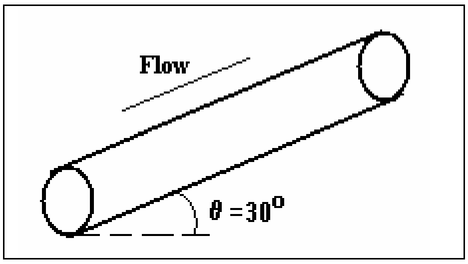
1. Attempt all six (6) questions. Weighting as noted. Total marks = 100.
2. The textbook and electronic calculators are permitted. Notes are not allowed.
3. Print your name clearly in the space provided on the cover page. Write your ID No. on the last page and all other pages in the provided space.

Surname: ___**SOLUTION**_______ Given Name(s):____________________ Section* ______

*__L01__ for MWF Lectures (Dr. Maini);
__L02__ for T R Lectures (Dr. Azaiez).

Question No.	Maximum Mark	Mark Obtained
I	15	
II	15	
III	20	
IV	15	
V	20	
VI	15	
Total (out of 100)		

Problem I: (15 points)
The inclined pipe shown in the figure is used to carry oil (ρ=900kg/m^3, μ=9x10^{-3}Pa.s). The pipe is made of steel, has a diameter D=20cm and a length L=300m.



(a) Determine the pressure drop if the average velocity in the pipe is equal to 3m/s. (**10 marks**)

$Re = \rho uD/\mu = 900*3*0.2/0.009 = 6x10^4$

➔ *The flow is turbulent, and $f \approx 0.006$ (Chart)*

$-(\Delta P+\rho g\Delta h) = 2f\, u^2\, \rho L/D$

$\Delta h = L.\sin(\theta)$

$-\Delta P = 2f\, u^2\, \rho L/D + \rho g\, L.\sin(\theta)$

$= 2*0.006*(3)^2*900*300/0.2 + 900*9.81*300*\sin(30)$

$= \underline{1.47\ MPa}$

Pressure Drop=1.47 MPa

(b) What would be the pressure drop for the same average velocity if the oil is treated as an ideal (inviscid) fluid. (**5 marks**)

If the flow was considered as inviscid, then the viscous losses are zero and,

$-\Delta P = \rho g\Delta h = \rho gL.\sin(\theta) =$

$= 900*9.81*300*\sin(30)$

$= \underline{1.32\ MPa}$

N.B. *This result can be also obtained from Bernoulli equation:*

$P/\rho + u^2/2 + gh = c^{st}$

Pressure Drop=1.32 MPa

Problem II: (15 points)
A $250 m^3$ tank contains 2300kg of a mixture of CO_2 (M=44kg/kmol) and CH_4 (M=16kg/kmol). The pressure and temperature in the tank are 1MPa and 300K respectively.

(a) Calculate the mole fraction of CH4 in the mixture using the ideal gas assumption. (**6 marks**)

*n = PV/RT = 1,000,000 * 250/(8.314 * 300) = 100233 mol = 100.233 kmol*
Average molar mass = 2300 kg/100.233 kmol = 22.95 kg/kmol
Let x be the mole fraction of CH4 in the mixture. Then the average molar mass is given by:

$$16 * x + 44 * (1-x) = 22.95, \text{ or}$$
$$28 x = 21.05 \text{ or } x = 0.752$$

Mole fraction of CH_4 in the mixture is 0.752 *Mole fraction of CO_2 in the mixture is 0.248*

(b) Given that the tank now contains 1,600kg of methane and 4,400kg of CO_2 at 300K, calculate the pressure in the tank using Kay's rule and the compressibility factor chart. (**9 marks**)

Component	Mass, kg	kmol	Mole Fract.	T_c, K	P_c, atm	$y_i.T_c$	$y_i.P_c$
CH_4	1600	100	0.5	190.6	45.4	95.3	22.7
CO_2	4400	100	0.5	304.2	72.8	152.1	36.4
SUM						247.4	59.1

$T_r = 300/247.4 = 1.21$

P_r is to be calculated, but we need P_r to look up the value of Z. This will require a trial and error procedure.

To start with assume Z = 1.0 and calculate P and P_r. Use this P_r and the previously calculated T_r to obtain a value of Z from the chart.
Recalculate P using this value of Z and continue the iteration until convergence is achieved. These calculations are summarized below.

$$Z=PVm/RT=Rv/(nRT)=5.01\times10^{-7} P \Rightarrow P=1995360.Z$$
$$P_r=P/P_{pc}=P/(59.1\times101325)=1.67\times10^{-7} P$$

Z	P (Pa)	Pr	Revised Z
1	1995360	0.333	0.93
0.93	1855684.8	0.310	0.92
0.92	1835731.2	0.307	0.92

P=1835.731kPa

Problem III: (20 points)

The following information is available on pure component properties of ethylene and propane.

Component	Normal boiling point, (K)	Latent heat of vaporization, (kJ/kmol)
Propane	231	19600
Ethylene	169	13500

(a) Using this information, determine the relationship between the temperature and vapour pressures for propane and ethylene, i.e. calculate the values of A and B in the equation $\ln(P_V) = \frac{-A}{T} + B$, for both components (Use units of kPa for P_V). (**10 marks**)

For Propane: *$A = \Delta H_V/R$ = 19600 (kJ.kmol)/8.314 (kJ/kmol.K) = 2357.47 K*
At normal boiling point P_V = 101.325 kPa, therefore
ln(101.325) = -2357.47/231 + B ➔ B = 14.82

For Ethylene: *$A = \Delta H_V/R$ = 13500 (kJ.kmol)/8.314 (kJ/kmol.K) = 1623.77 K*
At normal boiling point P_V = 101.325 kPa, therefore
ln(101.325) = -1623.77/169 + B ➔ B = 14.23

(b) A two-phase vapour-liquid mixture of ethylene and propane is kept in a tank at a temperature of 250K. Given that the vapor phase equilibrium contains 40mol% propane, use use Raoult's law to calculate the pressure of the mixture and mole percent of propane in the liquid phase. (**10 marks**)

Using the above correlations we have:

For Propane (1): ln(Pv1)=-2357.47/250 +14.82 ➔ Pv1=219.23 kPa
For Ethylene (2): ln(Pv2)=-1623.77/250 +14.23 ➔ Pv2=2286.83 kPa

α_{21}= Pv2/Pv1=10.43

We know that: $y_1 = x_1/[x_1 + \alpha_{21}x_2] = x_1/[x_1(1-\alpha_{21}) + \alpha_{21}]$

➔ $x_1 = \alpha_{21} y_1/[1 + (\alpha_{21}-1)y_1]$ =0.874

$P = x_1 Pv1 + x_2 Pv2$ =479.75kPa

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<u>Problem IV</u>: (15 points)
Two insulated steel pipes are carrying 200°C steam. The internal diameter of both pipes is 2.0cm and both are insulated with 2.5cm thick layer of asbestos on the outside. The wall thickness of the first pipe is 0.5cm while the wall of the other pipe is 2.0cm thick. The temperature at the outside surface of the insulation is kept constant at 25°C. Thermal conductivities of steel and asbestos are 50 W/mK and 0.15 W/mK respectively.

(a) Calculate the rate of heat loss per unit length for both pipes. (**6 marks**)

<u>For thin walled pipe:</u>

r_1 = 1.0 cm, r_2 = 1.5 cm, r_3 = 4 cm, κ_1 = 50 W/mK and κ_2 = 0.15 W/mK

$$\frac{Q}{L} = \frac{-2\pi\Delta T}{\frac{\ln(r_2/r_1)}{\kappa_1} + \frac{\ln(r_3/r_2)}{\kappa_2}} = \frac{2\pi * (200-25)}{\frac{\ln(1.5/1)}{50} + \frac{\ln(4/1.5)}{0.15}} = \text{167.9 W/m}$$

$$\boxed{\text{Q/L=167.95 W/m}}$$

<u>For thick walled pipe:</u>

r_1 = 1.0 cm, r_2 = 3.0 cm, r_3 = 5.5 cm, κ_1 = 50 W/mK and κ_2 = 0.15 W/mK

$$\frac{Q}{L} = \frac{-2\pi\Delta T}{\frac{\ln(r_2/r_1)}{\kappa_1} + \frac{\ln(r_3/r_2)}{\kappa_2}} = \frac{2\pi * (200-25)}{\frac{\ln(3/1)}{50} + \frac{\ln(5.5/3)}{0.15}} = \text{270.9 W/m}$$

$$\boxed{\text{Q/L=270.63 W/m}}$$

(b) Calculate the temperature at the interface between the steel and the insulation in case of the thicker pipe. (**6 marks**)

Considering the heat transfer only in the steel for the case of thick pipe,

$$-\Delta T = \frac{Q}{L}\frac{\ln(r_2/r_1)}{2\pi\kappa_1} = \frac{270.9\ln(3/1)}{2\pi * 50} = 0.95\ ^oC$$

Temperature at the interface = 200 – 0.95 = 199.05 ^oC

$$\boxed{T=199.05\ ^oC}$$

(c) Explain why the thick walled pipe looses more heat. (**3 marks**)

In this situation the steel offers very little resistance to heat loss. The temperature drop in the steel is less than 1 ^oC, even for the thick pipe. In the case of the thick walled pipe a much larger area of the outer steel surface is maintained at high temperature. This larger area results in larger heat loss, even though the temperature difference across the insulation layer would be slightly smaller in the case of thicker pipe.

<u>Problem V:</u> (20 points)

(a) The mean free path of a gas at a temperature T_1 and a pressure P_1 is $7*10^{-5}$cm. At these temperature and pressure, there are $2*10^{18}$ molecules/cm^3.

Use results from kinetic theory to determine the collision diameter, σ. (**5 marks**)

$$\lambda = \frac{1}{\sqrt{2}\pi\ \rho_N\ \sigma^2} \Rightarrow \sigma^2 = \frac{1}{\sqrt{2}\pi\ \rho_N\ \lambda} = \frac{1}{\sqrt{2}\pi\ 2x10^{24}.7x10^{-7}} = 16.077x10^{-20}\ m^2$$

$$\Rightarrow \sigma = 4.01\,\overset{o}{A}$$

$\boxed{\sigma = 4.010 \text{ Angstrom}}$

(b) The pressure of the gas is doubled such that $P_2=2*P_1$, while keeping its temperature fixed. What is the new value of the mean free path? (**3 marks**)

$$\rho_N = \frac{P}{kT} \Rightarrow \quad \lambda = \frac{kT}{\sqrt{2}\pi\ P\ \sigma^2}$$

$$\lambda_2 = \frac{kT_2}{\sqrt{2}\pi\ P_2\ \sigma^2} = \frac{kT_1}{\sqrt{2}\pi\,(2P_1)\,\sigma^2} = \frac{\lambda_1}{2} = 3.5*10^{-5}\,cm$$

$\boxed{\lambda = 3.5*10^{-5}\text{cm}}$

(c) The gas is now held in a cubic container of side a. The temperature T of the container is kept constant. One of the walls of the container has a small circular hole of radius r, from which the gas escapes. If we call X_n the percentage of molecules of the gas that escape through the small hole after a time Δt, show that X_n and Δt are given by the following relationship:

$$\Delta t = \frac{4X_n a^3}{r^2}\sqrt{\frac{M}{8\pi RT}}$$

(Assume that the pressure remains constant in the container). (**10 marks**)

The collision rate per unit area: $Z_m = \dfrac{\rho_N\,\bar{c}}{4} = \dfrac{\Delta n}{A\Delta\ t}$

➔ $\Delta\ t = \dfrac{\Delta\ n}{A.Z_m} = \dfrac{4.\Delta\ n}{A.\rho_N\ \bar{c}}$

The percent of molecules lost is $X_n=\Delta n/n_1$ ➔ $\Delta n= X_n.n_1$

but $n_1=\rho_n.v=\rho_n.a^3$, *therefore:* $\Delta n= X_n.\rho_n.a^3$

➔ $\Delta\ t = \dfrac{4.X_n.\,\rho_N\ a^3}{(\pi.r^2).\rho_N\,\bar{c}} = \dfrac{4.X_n.\,a^3}{(\pi.r^2).\bar{c}} = \dfrac{4.X_n.\,a^3}{\pi.r^2}\sqrt{\dfrac{\pi\ M}{8RT}} = \dfrac{4.X_n.\,a^3}{r^2}\sqrt{\dfrac{M}{8\ \pi\ RT}}$

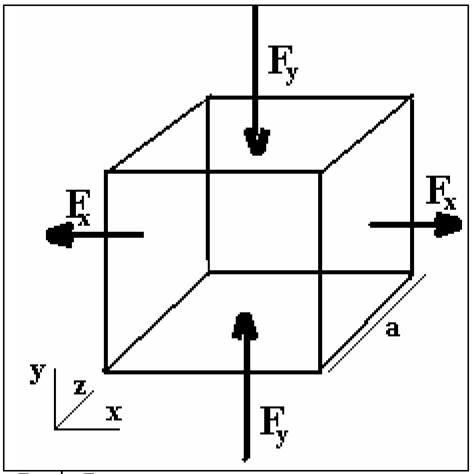
(d) Determine the time Δt for the container to lose 10% of the gas, if M=40kg/kmol, T=300K, a=20cm and r=0.1mm. (**2 marks**)

$$\Delta\ t = \frac{4x0.1x(0.2)^3}{(10^{-4})^2}\sqrt{\frac{40}{8\ \pi x8314x300}} = 255.62s \quad or \quad 4.26\,min$$

$\boxed{\Delta t = 255s = 4.26min}$

Problem VI: (15 points)

Consider a metal that has a cubic shape and which is subject to forces in the x and y directions as shown in the figure. Note that there are no forces applied in the z-direction.



(a) Show that if the tensile force in the x-direction and the compressive force in the y-direction have the same magnitude, then there is no change in the volume of the cube, i.e. $\Delta V=0$. (**6 marks**)

For the system of forces shown in the figure we have:

$$\varepsilon_x = \frac{\sigma_x}{E} - \upsilon\frac{\sigma_y}{E},$$

$$\varepsilon_y = \frac{\sigma_y}{E} - \upsilon\frac{\sigma_x}{E}, \qquad \Rightarrow \frac{\Delta V}{V} \approx \varepsilon_x + \varepsilon_y + \varepsilon_z = (1-2\upsilon)\frac{\sigma_x + \sigma_y}{E}$$

$$\varepsilon_z = -\upsilon\frac{\sigma_x + \sigma_y}{E},$$

But $\sigma_x = F_x/A$ and $\sigma_y = -F_y/A$. Therefore if $|F_x| = |F_y|$, then $\sigma_x + \sigma_y = 0$ or $\mathbf{\Delta V=0}$.

(b) It is found that if the magnitudes of the forces in the x and y directions are equal to 200N and 100N respectively, the relative change in the volume of the cube $\Delta V/V$, is equal to 10^{-5}. We give a=1cm, and the Poisson ratio $\nu=0.4$. Determine:

(ι) The value of the Young modulus of the metal. (**3 marks**)

$\sigma_x = F_x/A = F_x/a^2 = 200/(0.01)^2 = 2*10^6 Pa.$

$\sigma_y = -F_y/A = -F_y/a^2 = -100/(0.01)^2 = -10^6 Pa.$

Therefore:

$$\boldsymbol{E} = \frac{\Delta V}{V} \approx \varepsilon_x + \varepsilon_y + \varepsilon_z = (1-2\upsilon)\frac{\sigma_x + \sigma_y}{\left(\frac{\Delta V}{V}\right)} = (1-2\times 0.4)\frac{10^6}{10^{-5}} = 20\boldsymbol{GPa}.$$

$\boxed{E = 20.0GPa}$

(ii) The value of the Shear modulus of the metal. (**3 marks**)

$$\boldsymbol{G} = \frac{\boldsymbol{E}}{2(1+\upsilon)} = \frac{20\boldsymbol{GPa}}{2(1+0.4)} = 7.14\boldsymbol{GPa}.$$

$\boxed{G = 7.14GPa}$

(iii) The value of the strain in the x-direction, ε_x. (**3 marks**)

$$\varepsilon_x = \frac{\sigma_x}{E} - \upsilon\frac{\sigma_y}{E} = \frac{2.10^6}{2.10^{10}} - 0.4\frac{(-10^6)}{2.10^{10}} = 1.2\times 10^{-4}$$

$\boxed{\varepsilon_x = 1.2x10^{-4}}$

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