## ENGG 201 - WINTER 2017 - MIDTERM EXAM #2

#### Wednesday March 22, 2017 - 19h00-21h00 (120 minutes)

- a) Attempt all four (4) questions
- b) Show all work for full marks.
- c) Closed Text, Closed Notes, Schulich Calculators (only) are allowed.
- d) Print your name and lecture number below, and put your ID number on each additional page in the blanks provided.

Surname Section L0_	SOLUTION		
	SOLUTION	Section	L0_

L01 (330pm TR): Dr. Sumon // L02 (1230pm TR): Dr. Kallos

#### STUDENT IDENTIFICATION

Each candidate must sign the Seating List confirming presence at the examination. All candidates for final examinations are required to place their University of Calgary student I.D. cards on their desks for the duration of the examination. (Students writing mid-term tests can also be asked to provide identity proof.) Students without an I.D. card who can produce an acceptable alternative I.D., e.g., one with a printed name and photograph, are allowed to write the examination.

A student without acceptable I.D. will be required to complete an Identification Form. The form indicates that there is no guarantee that the examination paper will be graded if any discrepancies in identification are discovered after verification with the student's file. A Student who refuses to produce identification or who refuses to complete and sign the Identification Form is not permitted to write the examination.

#### **EXAMINATION RULES**

- (1) Students late in arriving will not normally be admitted after one-half hour of the examination time has passed:
- (2) No candidate will be permitted to leave the examination room until one-half hour has elapsed after the opening of the examination, nor during the last 15 minutes of the examination. All candidates remaining during the last 15 minutes of the examination period must remain at their desks until their papers have been collected by an invigilator.
- (3) All enquiries and requests must be addressed to supervisors only.
- (4) Candidates are strictly cautioned against:
  - (a) speaking to other candidates or communicating with them under any circumstances whatsoever;
  - (b) bringing into the examination room any textbook, notebook or memoranda not authorized by the examiner;
  - (c) making use of calculators and/or portable computing machines not authorized by the instructor;
  - (d) leaving answer papers exposed to view;
  - (e) attempting to read other student's examination papers.
  - The penalty for violation of these rules is suspension or expulsion or such other penalty as may be determined.
- (5) Candidates are requested to write on both sides of the pages unless the examiner has asked that the left hand page be reserved for rough drafts or calculations.
- (6) Discarded matter is to be struck out and not removed by mutilation of the examination answer book.
- (7) Candidates are cautioned against writing in their answer book any matter extraneous to the actual answering of the question set
- (8) The candidate is to write his/her name on each answer book as directed and is to number each book.
- (9) A candidate must report to a supervisor before leaving the examination room.
- (10) Answer books must be handed to the supervisor-in-charge promptly when the signal is given. Failure to comply with this regulation will be cause for rejection of an answer paper.
- (11) If during the course of an examination a student becomes ill or receives word of domestic affliction, the student should report at once to the supervisor, hand in the unfinished paper and request that it be cancelled. If physical and/or emotional ill health is the cause, the student must report at once to a physician/counselor so that subsequent application for a deferred examination is supported by a completed Physician/Counselor Statement form. Students can consult professionals at University Health Services or University Counseling Services during normal working hours or consult their physician/counselor in the community.
  - Should a student write an examination, hand in the paper for marking, and later report extenuating circumstances to support a request for cancellation of the paper and for another examination, such a request will be denied.
- (12) Smoking during examinations is strictly prohibited.

Question Number I (25 Marks ~ 30 minutes)

Assume ideal gas behavior at all conditions in Question 1.

## PART A - SPHERICAL VESSELS 1, 2, and 3 (/12)

Use the data in the table below to answer Question 1 Part A. Vessels 1,2 and 3 are spherical.

Vessel Number	Component	Molar Mass	Vessel Volume	Temperature	Pressure	Mass
		(kg/kmol)	(m³)	(°C)	(kPa)	(kg)
1	CH <sub>4</sub>	16	-	0	800	5
2	N <sub>2</sub>	28		27	100	a=

a) Vessel 1 contains 5 kg of methane gas at 0°C and 800 kPa. Calculate the internal volume of vessel 1. (/3)

$$N = m/M = 5/16 = 0.3125 \text{ km}\text{s}$$

$$V = \frac{NRT}{P} = \frac{(0.3125)(8.314)(273.15)}{800} = \boxed{0.8871 \text{ m}^3}$$

b) Vessel 2 contains  $N_2$  at 27°C and 100 kPa. What is the density (kg/m³) of  $N_2$  in vessel 2? (/3)

$$P = NRT = \frac{m}{m} RT$$

$$P = \frac{m}{v} = \frac{PM}{RT} = \frac{(100 \text{ k/k})(28 \frac{\text{kg}}{\text{kmod}})}{(8.314 \frac{\text{k/k} \cdot \text{ms}}{\text{kmod} \cdot \text{k}})(300.15 \text{ k})}$$

$$= 1.123 \frac{\text{kg/m}^3}{\text{kmod} \cdot \text{k}}$$

c) The contents of Vessel 1 and Vessel 2 are transferred to a new Vessel 3(volume = 0.8314 m³), and the resulting mixture in Vessel 3 is brought to 27°C and 6.003 MPa. Determine the number of moles of gas mixture in Vessel 3. (/3)

$$\begin{array}{lll}
\mathcal{V}_{3} &=& 0.8314 \text{ m}^{3} \\
T_{3} &=& 300.15 \text{ K} \\
P_{3} &=& 6.003 \times 10^{3} \text{ k/h}
\end{array}$$

$$\begin{array}{lll}
N &=& \frac{Pv}{RT} = \frac{(6.003 \times 10^{3} \text{ k/h})(.8314 \text{ m}^{3})}{(8.314 \text{ k/h} \cdot \text{m}^{3})} \\
&=& \frac{(8.314 \text{ k/h} \cdot \text{m}^{3})}{(8.314 \text{ k/h} \cdot \text{m}^{3})} \\
&=& \frac{12 \text{ kmef}}{(8.314 \text{ k/h} \cdot \text{m}^{3})}$$

d) Determine the amount of N<sub>2</sub> (kg) that was in vessel 2. (/3)

$$n_{tot} = n_{eH_1} + n_{N_2}$$
  
 $\Rightarrow 2 = 0.3125 + n_{N_2}$   
 $\Rightarrow n_{N_2} = 1.6875 \text{ kmel} = 1.6875 \times 28 = 47.25 \text{ kg}$ 

## PART B - CUBE-SHAPED CONTAINER (/13)

A cube container (same length on each side) is used to store a mixture of 3 kmol of nitrogen and 5 kmol of methane at 27°C and 6.003 MPa.

a) Determine the partial pressures of nitrogen and methane in the cube container.

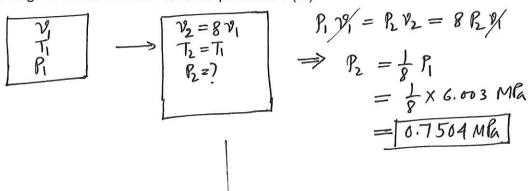
$$\begin{array}{c} (12) \\ N_{N_2} = 3 \text{ kmrf} \\ N_{eHq} = 5 \text{ kmrf} \\ T = 300.15 \text{ k} \\ P = 6.003 \times 10^3 \text{ kfa} \\ \end{array} \begin{array}{c} \lambda_{N_2} = 0.625 \\ \hline P_{N_1} = .625 \times 6.503 \text{ Mfa} \\ \hline P_{N_1} = .625 \times 6.503 \text{ Mfa} \\ \hline P_{N_2} = \frac{3}{3.752 \text{ Mfa}} \\ \hline P_{N_2} = P - P_{cHq} = \frac{3}{3.752 \text{ Mfa}} \\ \hline P_{N_2} =$$

b) Calculate the number density of the gas mixture in the cube container. (/3)

$$P_{N} = \frac{P}{KT} = \frac{6.003 \times 10^{6} \text{ fa}}{1.3805 \times 10^{-23} \text{ fa} \cdot \text{m}^{3}} \times 300.15 \text{ K}$$

$$= 1.45 \times 10^{27} \text{ molecules/m}^{3}$$

c) The content of the cube container is transferred isothermally to a NEW larger cube container. The length of each side of the NEW larger cube is twice the length of the original cube. What is the new pressure? (/3)



d) The content of the NEW larger cube is now heated to a temperature of 100°C. What is the new pressure? (/3) ↓

e) What is the average molecular weight of gas mixture in the NEW larger cube? (/2)

$$\overline{M} = \Sigma /; M; = 0.325 \times 28 + 0.625 \times 16$$
  
=  $20.5 \text{ kg/kmel}$ 

Question Number II (25 Marks ~ 30 minutes)

Nitrogen (M=14) has a critical temperature of -146.9°C, critical pressure of 3.369 MPa, and collision diameter of 3.64 Å.

a) Calculate the average speed of N<sub>2</sub> molecules at 500°C and 3 atm. (/3)

b) How far would an average  $N_2$  molecule travel between collisions at 500°C and 3 atm? (/3)

$$\lambda = \frac{1}{\sqrt{2} \pi \sigma^2 \rho_N} = \frac{kT}{\sqrt{2} \pi \sigma^2 \rho}$$

$$\lambda = \frac{1.3805 \times 10^{-23} \times 773.15}{\sqrt{2} \pi (3.64 \times 10^{-10})^2 (3 \times 101325)}$$

$$\lambda = \frac{5.965 \times 10^{-8} \text{ M}}{\sqrt{2} \pi (3.64 \times 10^{-8})^2 (3 \times 101325)}$$

c) How much energy would a single nitrogen molecule gain if the temperature is raised from 100°C to 200°C at 1atm? (/3)

sed from 100°C to 200°C at 1atm? (/3)
$$\Delta E_{k} = E_{k2} - E_{k4} \qquad T_{1} = 100°C \\
= \frac{1}{2}M(C^{2}z - C^{2}_{1}) \\
= \frac{1}{2}\frac{28}{6.023\times10^{26}}\left(\frac{3R}{M}(T_{2} - T_{1})\right) \qquad C^{2} = \frac{3RT}{M}$$

$$= 2.324\times10^{-26}\left(\frac{39.08}{10^{-24}}\right)$$

$$\Delta E_{k} = 2.07\times10^{-24} \text{ FJ}$$

d) How many nitrogen molecules would fit in a room that is 5m wide, 5 m deep and 4 m high at 25°C and 1 atm? Assume that air is 78% N<sub>2</sub> on a molar basis. (/4)

$$PN = \frac{N}{V} = \frac{P}{KT} = \frac{101325 Pa}{1.38 \times 10^{-23} \times 298.15}$$

$$PN = 2.4617 \times 10^{25} \text{ (all air)}$$

$$PN(Nz) = 0.78 \times P = 1.92 \times 10^{25} \text{ molec/m}^3$$

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$$Molec = PN V = (1.92 \times 10^{25}) (5 \times 5 \times 4) = \boxed{1.92 \times 10^{27} \text{ molecodes}}$$

$$(1.00 \text{ m}^3)$$

e) The Schmidt number is defined as the ratio of the momentum diffusivity (viscosity) to mass diffusivity and can be calculated from the following formula:

$$Sc = \frac{\mu}{\rho D}$$

Where  $\mu$  = viscosity,  $\rho$  = density and D = diffusivity.

Calculate the Sc for N<sub>2</sub> at 25°C and 1 atm assuming ideal gas behaviour. (/4)

f) How would the Sc change if the temperature were raised to 100°C and the pressure to 4 atm? (/3)

g) Nitrogen is trapped between two window panes separated by 4 mm. Outside the window is -30°C on a cold winter day and the house inside is at 25°C. Calculate how much heat would be lost through a 1m by 1m window in a 24h period. You can assume no temperature gradient through the glass on either side of the nitrogen. (/5)

$$\frac{Q}{A} = -k \frac{dT}{dz} \longrightarrow \Delta q = -A \Delta t k \frac{dT}{dz}$$

Question Number III (25 Marks ~ 30 minutes)

### PART A /14 marks

A fixed amount (1 kmol) of ammonia is at 10 MPa and 415.6 K in a rigid container. The gas follows van der Waals (vdW) Equation of State, and the vdW parameters are given below.

Molar Mass	а	b
17 kg/kmol	4.2 atm (m³/kmal)2	0.0374 (m <sup>3</sup> /kmol)
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a) Determine the molar volume of the gas at 10 MPa and 415.6 K using the ideal gas law. (/3)

$$PV_{m}^{G} = RT$$

$$V_{m}^{G} = \frac{RT}{P} = \frac{8.314 \frac{kPa \cdot m^{3}}{k mol \cdot K} \times 415.6 \, K}{10 \times 16^{3} \, kPa} = \frac{6.3455 \frac{m^{3}}{k mol}}{10 \times 16^{3} \, kPa}$$

b) Determine the molar volume of ammonia at 10 MPa and 415.6 K using the vdW Equation of State. Show two iterations. (/7)

Coefficients: 
$$\frac{P}{p} = b + V_{m}^{T6} = b + V_{m}^{T6} = 0.0374 + 0.3455 = 0.38293 \frac{m^{3}}{kmf}$$

$$\frac{A}{p} = \frac{4.2 \times 101.325 \text{ kfa.} (m^{3}/\text{kmf})^{2}}{10 \times 10^{3} \text{ kfa}} = 0.04257 (\frac{m^{3}}{kmf})^{2}$$

$$\frac{Ab}{p} = b * \frac{A}{p} = 0.0374 \times 0.04276 = 0.001592 (\frac{m^{3}}{kmf})^{2}$$

$$\frac{VdW^{\circ}}{p} = 0.38293 - \frac{0.04257}{V_{m}^{(1)}} + \frac{0.001592}{V_{m}^{(1)}}$$

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$$\frac{VdW^{\circ}}{p} = 0.38293 - \frac{0.04257}{V_{m}^{(1)}} + \frac{0.001592}{V_{m}^{(1)}}$$

$$\frac{VdW^{\circ}}{p} = 0.3455 \text{ m}^{3}/\text{kmf}$$

$$\frac{1}{p} = 0.3455 \text{ m}^{3}/\text{kmf}$$

$$\frac{1}{p} = 0.3455 \text{ m}^{3}/\text{kmf}$$

$$\frac{1}{p} = 0.245 \text{ m}^{3}/\text{kmf}$$

... b) continued

c) Determine the compressibility factor of methane based on the molar volumes obtained in parts (a) and (b) (/2)

$$Z = \frac{V_{\rm m}}{V_{\rm m}} = \frac{0.245}{0.3455} = \boxed{0.709}$$

d) Explain whether the following statement is true or false and explain why (no calculation needed): "If the temperature of the gas in the rigid container is doubled to 831.2 K, the pressure will double to 20 MPa". (/2)

False.

For ideal gas, it will be double (
$$P \times T$$
). But,

Ammonia is not ideal gas ( $Z \neq 1$  on  $a \neq 0$ ,  $b \neq \delta$ ).

 $P_1 \times P_2 = P_1 \times P_2 \times P_2 \times P_2 \times P_3 \times P_4 = P_1 \times P_2 \times P_4 \times P_4 \times P_5 = P_4 \times P_4 \times P_5 \times P_5 = P_4 \times P_5 \times P_6 \times P_6$ 

### PART B /11 marks

The pressure exerted by 1 mol of a real gas in a 0.5 litre vessel at 298 K is 44.6 atm.

Gas	Molar Mass	а	b
Methane (CH₄)	M=16 kg/kmol	2.25 atm.(L/mol) <sup>2</sup>	?

 $[L/mol = m^3/kmol]$ 

a) Determine the van der Waals parameter b for this gas. (/4)

$$P = \frac{RT}{V_{m} - b} - \frac{a}{V_{n}^{2}}$$

$$\Rightarrow P + \frac{a}{V_{m}^{2}} = \frac{RT}{V_{m} - b}$$

$$\Rightarrow V_{m} - b = \frac{RT}{P + \frac{a}{V_{n}^{2}}} = \frac{0.08206 \frac{L \cdot atm}{mol \cdot K} \times 298K}{44.6 atm + \frac{2.25 atm \cdot (L/mol)^{2}}{0.5^{2}(L/md)^{2}}}$$

$$= 0.496 \frac{L}{mol}$$

$$= 0.496 \frac{L}{mol}$$

$$= 0.496 \frac{L}{mol}$$

$$= 0.0437 \frac{L}{mol}$$

b) Determine the critical temperature and pressure of this gas. (/4)

c) Determine the compressibility factor of this gas using the generalized compressibility chart. (ONLY if you are unable to get critical properties for this gas, use  $T_c$ =400K and  $P_c$ =4 atm) (/3)

$$T_{R} = T/T_{c} = 298/185.5 = 1.61$$
 $P_{R} = P/P_{c} = 44.6/43.44 = 1.63$ 
 $T \approx 6.93$ 

# Question Number IV (25 Marks ~ 30 minutes)

A storage tank contains air (78% nitrogen, 21% oxygen, 1% argon on a molar basis) at 40 atm and 21.57°C. Properties of the components are shown below:

6	M (kg/kmol)	T <sub>c</sub> (K)	P <sub>c</sub> (atm)	ω
Nitrogen	14	126.2	33.5	0.04
Oxygen	16	154.6	49.8	0.021
Argon	40	150.8	48	-0.004

a) Use the ideal gas law to calculate the molar volume of the gas. (/3)

b) Use the Law of Corresponding States and the Pitzer Curl tables to calculate the molar volume of the gas. (/8)

$$T_{PL} = 0.78(126.2) + 0.21(154.6) + 0.01(150.8) = 132.41 \text{ K}$$
 $P_{PL} = 0.78(33.5) + 0.21(49.8) + 0.01(48) = 37.068 \text{ orbin}$ 
 $T_{PL} = 0.78(33.5) + 0.21(49.8) + 0.01(48) = 37.068 \text{ orbin}$ 
 $T_{PL} = \frac{T}{T_{PL}} = \frac{294.72}{132.41} = 2.226$ 
 $P_{PL} = \frac{P}{P_{PL}} = \frac{40}{37.068} = 1.08$ 
 $T_{PL} = \frac{T}{T_{PL}} = \frac{294.72}{132.41} = (2.23)$ 
 $T_{PL} = \frac{P}{T_{PL}} = \frac{40}{37.068} = 1.08$ 

$$W = \frac{2}{5}y_{1}W_{1} = 0.03557$$

OPTION #1  $Z = 0.9765$   $V_{M} = \frac{2}{7}RT = \frac{0.5903 \, \text{m}^{3}/\text{kmd}}{\rho}$ 

OPTION #2  $Z = 0.9940$   $V_{M} = \frac{2}{7}RT = \frac{0.6009 \, \text{m}^{3}/\text{kmd}}{\rho}$ 

See Next pages

OPTION #1 - Use Tr = 2.0 (max on charts) Tr = 2.0 Tr = 2.0

 $Z = Z^{\circ} + \bar{\omega} Z'$ Z = 0.9734 + 0.03557(0.088) = 0.9765 = Z

```
OPTION #Z - Extrapolate using Tr=1.9 and 2.0
            1.0 1.08 1.20
 70
              1.9 0.968 01
                                     0.962
              2.0 0.975 @ 2
                                      0.971
              2.23
  - Step 1 - Interpolate @ Tr=1.9 to get Pr=1.08

• 1 = 0.9656

- Interpolate @ Tr=2.0 to get Pr=1.08
                  · Z = 0.9734
  - Step Z - Make equation @ Pr=1,08 to see
                   effect of Tr
                                                    Tr @ Pr=1.00
                                                    and Pr=7,20
                                                     first and
                                                     then extrapolate
                                  2.23
                    M = \frac{0.58}{0.000} = 0.9734 - 0.9656 = 0.078
      y = mx + b
     0.9656 = 0.078 (1.9) + 6
        6 = 0.8174
        y=0.078x+0.8174
       70=0.078 (Tr) +0.8174
     Tr = 2.23 -0 20 = 0.99134
```

$$Z' = 0.06 T_r + 0.208$$
  
 $Z' = -0.06 (2.23) + 0.208$   
 $Z' = 0.0742$ 

$$Z = Z^{\circ} + \omega Z'$$
  
 $Z = 0.99134 + 0.03557(0.0742)$   
 $Z = 0.9940$ 

c) Compare your answer in b) to that obtained from the Generalized Compressibility chart. (/3)

d) How much gas (kg) would be held in a cylindrical tank 2 m diameter and 4 m high? You can use your answer from b) or c). (/5)

e) What error (%) in your answer to d) would you have obtained if you had used the

ideal gas law? (/4)
$$\frac{9}{6} Error = \frac{Real - Ideal}{Real} 100$$

$$\frac{1}{608.14 - 602} = \frac{1.018}{608.14}$$
The dother in you had used the ideal gas law? (/4)
$$1 = \frac{12.566}{0.6045} = 20.787$$
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The ideal gas law? (/4)

f) What non-ideal factors do the van der Waals a and b parameters represent? (/2)