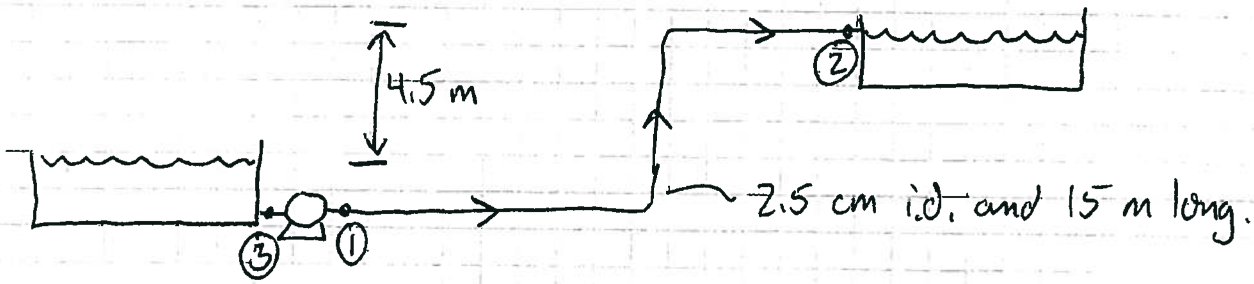
EXAMPLE – PUMPING OF LIQUIDS, PRESSURE DROP IN PIPES | Ch 8

Q: The following pipeline system is available for pumping a liquid from a ground-level reservoir to an over-head tank located at a height of 4.5 m. Both tanks are open to the atmosphere at 100 kPa. The pump is capable of developing a pressure of 200 kPa. The pipeline is 2.5 cm in diameter and 15 m long. Ignore losses due to bends and fittings.



Calculate the average velocity and pumping rate if

(a) the liquid is water at 10°C
  (i) the pipe is smooth
  (ii) the pipe is rough (commercial grade)

(b) the liquid is an oil with $\mu = 100\ mPa.s$ and $\rho = 900\ kg/m^3$

For all cases, calculate the power supplied by the pump.

A: a) water at 10°C $\mu = 1.3\ mPas = 1.3 \times 10^{-3}\ Pas$
$\rho = 1000\ kg/m^3$

$$-\left[\frac{\Delta P}{L} + \rho g \frac{\Delta h}{L}\right] = \frac{2f\bar{v}^2\rho}{D}$$

$\Delta P = P_2 - P_1$
$\Delta h = h_2 - h_1$
$L = \Delta L = L_2 - L_1$

Choose station ① as the pump discharge
② as the overhead tank inlet

$$-\left[\frac{100\times10^3\,Pa - 200\times10^3\,Pa}{15\,m} + \frac{1000\frac{kg}{m^3}\times 9.81\frac{m}{s^2}\times(4.5-0m)}{15\,m}\right] = \frac{2f\bar{v}^2\,1000\,kg/m^3}{0.025\,m}$$

$f = \frac{0.0465}{\bar{v}^2}$ can't solve directly

(i) smooth pipe
- Use the lower curve in Fig 8-9 (f vs. Re)
- ~~Don't know whether laminar or turbulent~~
→ Trial and Error
→ Assume: turbulent (ie $0.002 < f < 0.01$)

Try $f = 0.004$ (Guess #1) → $\bar{v}^2 = 0.0465/f = 11.6$ → $\bar{v} = 3.41\ m/s$

check: $Re = \frac{D\bar{v}\rho}{\mu} = \frac{0.025\times3.41\times1000}{1.3\times10^{-3}} = 66000$ (Turbulent!)

Read f from Fig 8-9 @ Re = 66000 → f = 0.005 (not good)

Try $f = 0.005$ (Guess #2) → $\bar{v}^2 = 9.3$ → $\bar{v} = 3.05\ m/s$

Check: $Re = \frac{D\bar{v}\rho}{\mu} = 59000$ $\xrightarrow[\text{@ Re = 59000}]{\text{Read f from chart}}$ $f = 0.005$ good!

Answer $\boxed{\bar{v} = 3.05\ m/s}$

Pumping Rate (Q) $Q = \frac{\pi}{4}D^2\bar{v} = \frac{\pi}{4}(0.025)^2(3.05) = 0.0015\ m^3/s$

or $\boxed{Q = 1.5\ L/s}$

(ii) Rough Pipe

- Use upper curve in Fig 8-9 (f vs. Re)

Try $f = 0.006 \rightarrow \bar{u}^2 = 7.75 \rightarrow \bar{u} = 2.78$ m/s

Check Re $= 54{,}000 \rightarrow f = 0.0061$ (greater than 0.006)

Try $f = 0.0061 \rightarrow \bar{u}^2 = 7.64 \rightarrow \bar{u} = 2.76$ m/s

Check Re $= 53{,}600 \rightarrow f = 0.0061$ good!

Answer $\boxed{\bar{u} = 2.76 \text{ m/s}}$

$\therefore Q = 0.0014 \text{ m}^3/\text{s} \rightarrow \boxed{1.4 \text{ L/s}}$ - less than (i) - smooth pipe

← rough pipe

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b) Oil $\mu = 0.1$ Pa.s ; $\rho = 900$ kg/m^3

$$-\left[\frac{-100\times10^3}{15} + \frac{900\times9.81\times4.5}{15}\right] = \frac{2f\bar{u}^2 900}{0.025}$$

or $f = 0.0558/\bar{u}^2$

Try $f = 0.005 \rightarrow \bar{u}^2 = 11.16 \rightarrow \bar{u} = 3.34$ m/s

Check $Re = \frac{0.025\times3.34\times900}{0.1} = 752$ Laminar! $\left(f = \frac{16}{752} = 0.21\right)$

For laminar flow, don't need trial + error

$$f = \frac{16}{Re} = \frac{16}{D\bar{u}\rho/\mu} = \frac{0.0558}{\bar{u}^2} \rightarrow \bar{u} = \frac{0.0558\times0.025\times900}{16\times0.1} = 0.785 \text{ m/s}$$

Re = 176.6 , $f = 0.0906$

← less than 2100, laminar

Answer $\boxed{\bar{u} = 0.785 \text{ m/s}}$

$Q = \frac{\pi}{4}(0.025)^2(0.785) = 0.000385 \text{ m}^3/\text{s} \sim \rightarrow \boxed{0.4 \text{ L/s}}$

(only 26% of smooth pipe water flow)

Note: For laminar flow, smooth or rough pipe does not matter.

(C) Power

Power = $Q(\Delta P)$ (across the pump)

Stations (3) to (1) $P_3 = 100 \times 10^3$ Pa

$P_1 = 200 \times 10^3$ Pa $> \Delta P = 100 \times 10^3$ Pa

(a) (i) Power $= 0.0015 \times 100 \times 10^3 = \underline{150\,W}$

(ii) Power $= 0.0014 \times 100 \times 10^3 = \underline{140\,W}$

(b) Power $= 0.000385 \times 100 \times 10^3 = \underline{39\,W}$