UNIVERSITY OF CALGARY

MID-TERM EXAMINATION
(FALL, 2002-2003)

ENGG 201 - Behaviour of Liquids, Gases and Solids

October 15, 2002 Time Allowed: 90 minutes

1. Attempt all **four (4)** questions. Weighting as noted. Total marks =100.
2. Electronic calculators are permitted. Books and Notes are not allowed.
3. Print your name clearly in the space provided on the cover page. Write your ID No. on the last page and all other pages in the provided space.

Surname:___SOLUTION_________**Given Name(s):**___________________ **Section**_____

STUDENT IDENTIFICATION

Each candidate must sign the Seating List confirming presence at the examination. All candidates for final examinations are required to place their University of Calgary student I.D. cards on their desks for the duration of the examination. (Students writing mid-term tests can also be asked to provide identity proof.) Students without an I.D. card who can produce an **acceptable** alternative I.D. e.g. one with a printed name and photograph, are allowed to write the examination.

A student without acceptable I.D. will be required to complete an Identification Form. The form indicates that there is no guarantee that the examination paper will be graded if any discrepancies in identification are discovered after verification with the student's file. A **Student who refuses to produce identification or who refuses to complete and sign the Identification Form is not permitted to write the examination.**

EXAMINATION RULES

(1) Students late in arriving will not normally be admitted after one-half hour of the examination time has passed.

(2) No candidate will be permitted to leave the examination room until one-half hour has elapsed after the opening of the examination, nor during the last 15 minutes of the examination. All candidates remaining during the last 15 minutes of the examination period must remain at their desks until their papers have been collected by an invigilator.

(3) All inquiries and requests must be addressed to supervisors only.

(4) **Candidates are strictly cautioned against:**

(a) Speaking to other candidates or communicating with them under any circumstances whatsoever: (b) bringing into the examination room any textbook, notebook or memoranda not authorized by the examiner;

(c) Making use of calculators and/or portable computing machines not authorized by the instructor: (d) leaving answer papers exposed to view;

(e) Attempting to read other student's examination papers.

The penalty for violation of these rules is suspension or expulsion or such other penalty as may be determined.

(5) Candidates are requested to write on both sides of the page, unless the examiner has asked that the left hand page be reserved for rough drafts or calculations.

(6) Discarded matter is to be struck out and not removed by mutilation of the examination answer book.

(7) Candidates are cautioned against writing in their answer book any matter extraneous to the actual answering of the question set.

(8) The candidate is to write his/her name on each answer book as directed and is to number each book.

(9) A candidate must report to a supervisor before leaving the examination room.

(10) Answer books must be handed to the supervisor-in-charge promptly when the signal is given. Failure to comply with this regulation will be cause for rejection of an answer paper.

(11) If during the course of an examination a student becomes ill or receives word of domestic affliction, the student should report at once to the supervisor, hand in the unfinished paper and request that it be cancelled. If physical and/or emotional ill health is the cause, the student must report at once to a physician/counsellor so that subsequent application for a deferred examination is supported by a completed Physician/Counsellor Statement form. Students can consult professionals at University Health Services or university Counselling Services during normal working hours or consult their physician/counsellor in the community.

Should a student write an examination, hand in the paper for marking, and later report extenuating circumstances to support a request for cancellation of the paper and for another examination, such a request will be denied.

(12) Smoking during examinations is strictly prohibited.

RO 93-02

PROBLEM I (20 points)

$$\rho.v.\frac{\partial u}{\partial y} = -\frac{\partial P}{\partial x} + [\rho - \rho.\alpha.(T - T_o)].g + \mu.\frac{\partial^2 u}{\partial y^2}$$

The above equation uses the Boussinesq approximation to describe a fluid flow in which the density (ρ) varies with the temperature (T). In the equation, u and v stand for the components of the velocity, x and y have dimension of length, g is gravitational acceleration=9.8m/s^2, P is pressure, T_o is a reference temperature and μ the viscosity (Pa.s)..

Use the fact that the above equation is dimensionally homogeneous to determine the dimensions of the coefficient of volume expansion α, and of the viscosity μ.
N.B. The derivative operator ∂ is dimensionless.

Answer

For the equation to be dimensionally homogeneous, the two terms subtracted in the expression [$\rho-\rho.\alpha.(T-T_o)$] must have the same dimensions, ,i.e.

$$[\rho] \equiv [\rho.\alpha.(T-T_o)] \Rightarrow [\rho] \equiv [\rho].[\alpha].[T] \Rightarrow \underline{[\alpha] \equiv [T]^{-1}}$$

For the dimension of the viscosity, we may write that:

$$[\mu].[\partial^2 u/\partial y^2] \equiv [\rho.v.\partial u/\partial y] \Rightarrow [\mu].[u]/[y^2] \equiv [\rho][v][u]/[y]$$
$$\Rightarrow [\mu].[Lt^{-1}]/[L^2] \equiv [ML^{-3}].[Lt^{-1}][Lt^{-1}]/[L]$$
$$\Rightarrow [\mu].[L^{-1}][t^{-1}] \equiv [M][L^{-2}][t^{-2}]$$
$$\Rightarrow \underline{[\mu] \equiv [M][L]^{-1}[t]^{-1}}$$

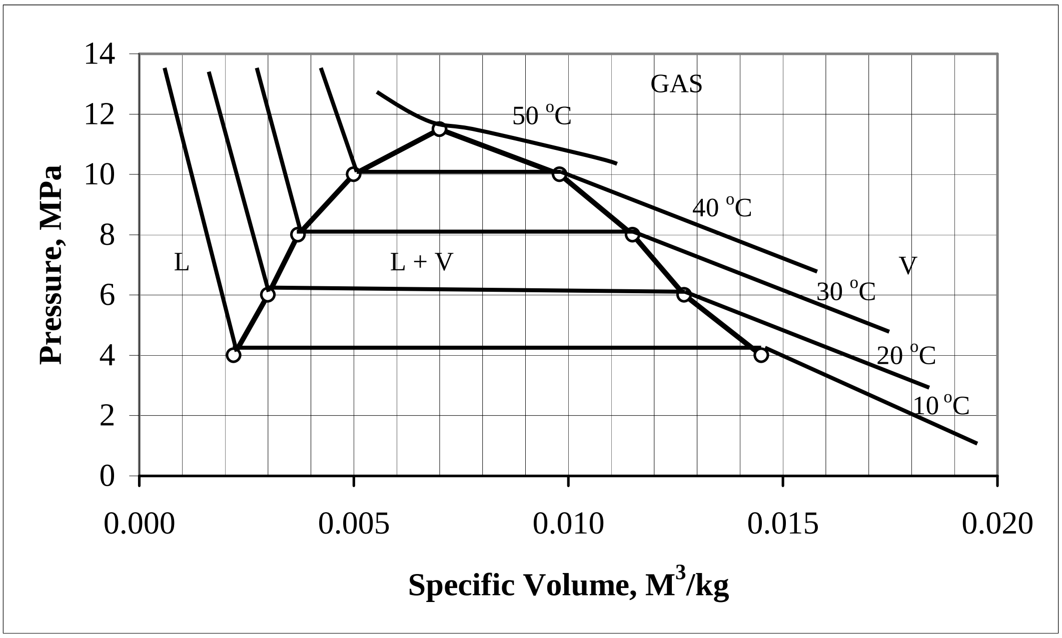
N.B. The same answer can be obtained by imposing the dimension of the term [$\mu.\partial^2 u/\partial y^2$] to be the same as that of any of the other three terms in the equation.

<u>PROBLEM II</u> (25 points)

The following table presents the vapour-liquid equilibrium data for a single component industrial material of 49 kg/kmol molar mass.

Temperature (^oC)	Pressure (MPa)	Specific volume of liquid (m^3/kg)	Specific volume of vapour (m^3/kg)
10	4.0	0.0022	0.0145
20	6.0	0.0030	0.0127
30	8.0	0.0037	0.0115
40	10.0	0.0050	0.0098
50	11.5	0.0070	0.0070

i. Plot a P-v diagram for this material showing the two-phase envelope and the approximate isotherms for the five listed temperatures. Label all phase regions. (10 points)



ii. What are the critical properties of this material (Pressure, Temperature and Specific volume)?(3 Marks)

<u>**T_c = 50 ^oC**</u>
<u>**P_c = 11.5 MPa**</u>
<u>**V_c = 0.0070 m^3/kg**</u>

iii. A 1.0 kg sample of this material is kept at 30 ^oC in a vessel of 6 litre volume. Answer the following questions concerning this sample.

a. What phases are present in the vessel? (2 points)

Specific volume = 6.0 l/kg = 0.006 m^3/kg, T=30^oC

➔ Phases present are <u>Liquid (L) and Vapour (V)</u>

b. What is the pressure in the vessel? (2 points)

The pressure , read from the isotherm in 2-phase zone, is <u>8.0 MPa</u>

c. What is the ratio of mass of liquid to mass of vapour in the vessel? (4 points)

$$\frac{\textbf{Mass of liquid}}{\textbf{Mass of vapour}} = \frac{\mathbf{0.0115 - 0.006}}{\mathbf{0.006 - 0.0037}} = \mathbf{2.39}$$

d. What is density of the liquid present in the vessel?(2 points)

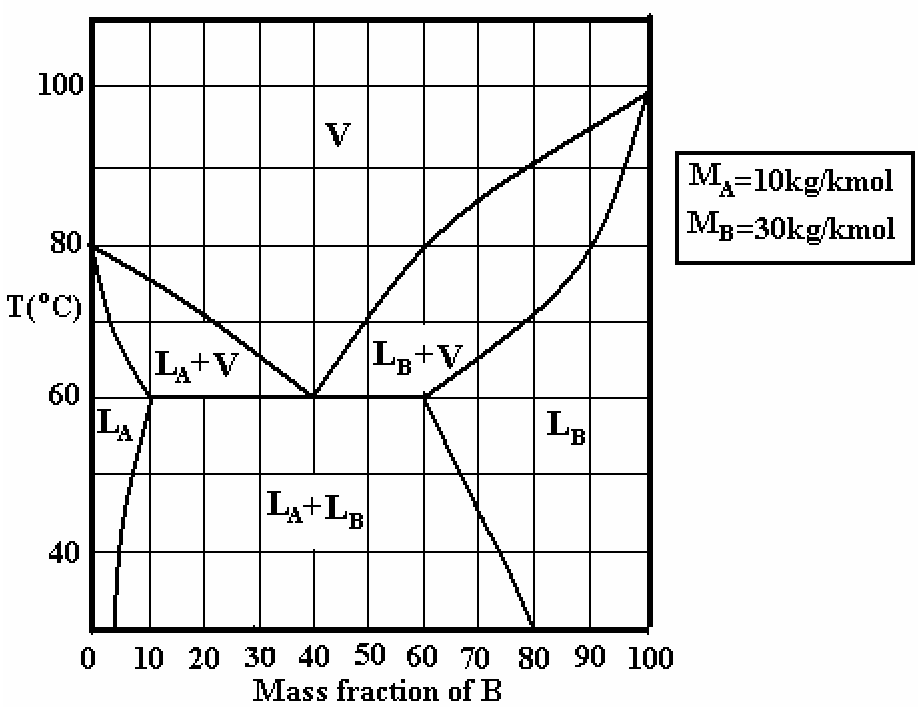
Specific volume of the liquid: 0.0037 kg/m^3

➔ Density of liquid = 1/0.0037 kg/m^3 = <u>270.3 kg/m^3</u>

iv. The temperature of the sample in Part (iii) is increased to 55 ^{o}C. What are the phases present at this temperature? (2 points)

Since the temperature is higher than the critical temperature, the phase present is <u>GAS</u>

PROBLEM III (30 points)



Use the above figure for a Liquid-Vapour phase diagram at P=1atm for a mixture of two components A and B, to answer the following questions:

a. What is the normal boiling point of pure A? (2 points)

T_A= 80 °C

b. Are A and B, miscible, partially miscible or immiscible? (2 points)

Partially miscible

c. A mixture consisting of 3kg of A and 12 kg of B is held at a temperature T=100°C. Name the phases present and their corresponding masses. (2 points)

The mass composition of the mixture is $m_B/(m_A+m_B)= m_B /m_t$=12/15=0.8 or 80wt%B. Placing this point at T=100°C on the diagram, shows that <u>there is one phase: vapour of a mass equal to m_t=15kg.</u>

d. The mixture in question (c) is brought to a temperature T=80°C.

(i) Name the phases present. (2 points)

At T=80°C, <u>there are two phases: L_B and V</u>

(ii) State the mass composition of all phases. (2 points)

<u>L_B: 90wt% B</u>

<u>V: 60wt%B</u>

(iii) Determine <u>the molar composition of the liquid phase</u>. (4 points)

To convert the mass composition of the liquid phase to a molar composition: Consider 1kg of mixture which is 90wt%B,

➔ the masses of B and A in this mixture are: $\boxed{\begin{array}{l} m_B = 0.9\text{kg B} \\ m_A = 0.1\text{kg A} \end{array}}$

➔ The number of moles B and A in this mixture are:

$$\boxed{\begin{array}{l} n_B = m_B / M_B = 0.9/30 = 0.03\text{kmol} \\ n_A = m_A / M_A = 0.1/10 = 0.01\text{kmol} \end{array}}$$

➔ Therefore the molar composition of the liquid phase is:

$x_B = n_B / (n_B + n_A) = n_B / n_t = 0.03/0.04 = 0.75$ **or** <u>**75mol% B**</u>

(iv) Determine <u>the mass of the liquid phase</u>. (5 points)

Use the lever rule to determine the mass of the liquid phase:

$m_L/m_t = (80-60)/(90-60) = 2/3$

➔ $m_L = 2^* m_t /3 = 2^*15/3 = \underline{10\text{kg.}}$

(v) Determine <u>the mass of B in the liquid phase</u>. (5 points)

The liquid phase has a mass of 10kg.

The liquid phase is 90wt%B

➔ the mass of B in the liquid phase is 0.9*10=<u>9kg.</u>

e. An amount of A is slowly added to the mixture in question (d). Determine the minimum amount of A to add to have the whole mixture in a vapour form. (6 points)

After adding A, the mixture will be all vapour when its composition is 60wt%B.

➔ $0.6 = m_B/m_t$, **since only A has been added, ,the mass of B is not changed, i.e.** $m_B = 12\text{kg}$.

➔ $m_t = m_B/0.6 = 12/0.6 = 20\text{kg}$

➔ $m_t = m_{t(initial)} + m_{A(added)} = 15\text{kg} + m_{A(added)}$

➔ $\underline{m_{A(added)} = 5\text{kg.}}$

PROBLEM IV **(25 points)**
A gas tank of 1.00 m^3 internal volume that is kept at a temperature of 25 ^oC contains a mixture of three ideal gases. The following information is given about the components of this mixture.

Component	Molar Mass (kg/kmol)	Mass of the component in the tank (kg)	kmol of the component in the tank	Partial pressure of the component in the tank (kPa)
CH_4	16	4.00	**0.250**	**620**
CO_2	44	**5.5**	0.125	**310**
N_2	28	**9.04**	**0.323**	800

i. Fill in the blanks in the above table by calculating the missing information. (12 points)
ii. Calculate the mass fraction of each component in the mixture. (3 points)
iii. Calculate the mole fraction of each component in the mixture. (3 points)
iv. Calculate the average molar mass of the gas mixture. (3 points)
v. If the tank is heated to 100 ^oC, what will be the new total pressure in the tank. (4 points)

Answer

(i) No. of Moles of CH_4 = 4.00/16.00 = 0.25 kmol = 250 mol

Partial pressure of CH_4 =

$$\frac{nRT}{V} = \frac{250(mol)\times 8.314(m^3Pa/mol\,K)\times(273.15+25)(K)}{1.00(m^3)} = 619{,}705\ Pa \approx \underline{620kPa}$$

Mass of CO_2 = kmol of CO_2 x Molar mass = 0.125 kmol x 44 kg/kmol = 5.50 kg

Partial pressure of CO_2 =

$$\frac{nRT}{V} = \frac{125(mol)\times 8.314(m^3Pa/mol\,K)\times(273.15+25)(K)}{1.00(m^3)} = 309853\ Pa \approx \underline{310kPa}$$

Moles of N_2 $= \frac{PV}{RT} = \frac{800{,}000\times 1}{8.314\times 298.15} = 322.7 = \underline{\mathbf{0.323\ kmol}}$

Mass of N2 = 28 kg/kmol x 0.323 kmol = 9.04 kg

(ii) and (iii)

Component	Mass (kg)	kmol	Mass Fraction	Mole fraction
CH_4	**4.00**	**0.250**	**0.216**	**0.358**
CO_2	**5.5**	**0.125**	**0.297**	**0.179**
N_2	**9.04**	**0.323**	**0.487**	**0.463**
Total	**18.54**	**0.698**	**1.00**	**1.00**

(iv) Average molar mass = 18.54 kg/0.698 kmol = 26.56 kg/kmol

(v) The total pressure in the tank at T_1=298.15K is 620+310+800=1730kPa.

$$p_2 = p_1 \frac{T_2}{T_1} = 1730(\text{kPa}) \frac{373.15\text{K}}{298.15\text{K}} = 2165 \text{ kPa} = \underline{\mathbf{2165kPa}}$$