



**FINAL EXAMINATION
(Fall, 2001-2002)**

ENGG 201 - Behaviour of Liquids, Gases and Solids

December 13, 2001

Time Allowed: 180 minutes.

1. Attempt all six (6) questions. Weighting as noted. Total marks = 100.
2. The textbook and electronic calculators are permitted. Notes are not allowed.
3. Print your name clearly in the space provided on the cover page. Write your ID No. on the last page and all other pages in the provided space.

Surname: <u> SOLUTION </u> Given Name(s): _____ Section* _____

***L01** for MWF Lectures (Dr. Maini);
L02 for T R Lectures (Dr. Azaiez).

STUDENT IDENTIFICATION

Each candidate must sign the Seating List confirming presence at the examination. All candidates for final examinations are required to place their University of Calgary student I.D. cards on their desks for the duration of the examination. (Students writing mid-term tests can also be asked to provide identity proof.) Students without an I.D. card who can produce an **acceptable** alternative I.D. e.g. one with a printed name and photograph, are allowed to write the examination.

A student without acceptable I.D. will be required to complete an Identification Form. The form indicates that there is no guarantee that the examination paper will be graded if any discrepancies in identification are discovered after verification with the student's file. A **Student who refuses to produce identification or who refuses to complete and sign the Identification Form is not permitted to write the examination.**

EXAMINATION RULES

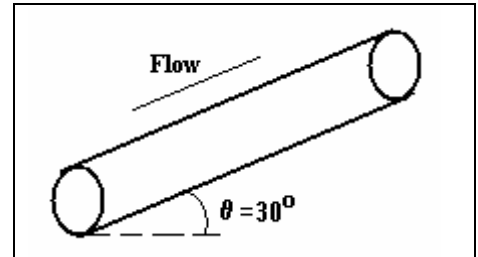
- (1) Students late in arriving will not normally be admitted after one-half hour of the examination time has passed.
- (2) No candidate will be permitted to leave the examination room until one-half hour has elapsed after the opening of the examination, nor during the last 15 minutes of the examination. All candidates remaining during the last 15 minutes of the examination period must remain at their desks until their papers have been collected by an invigilator.
- (3) All inquiries and requests must be addressed to supervisors only.
- (4) **Candidates are strictly cautioned against:**
 - (a) speaking to other candidates or communicating with them under any circumstances whatsoever;
 - (b) bringing into the examination room any textbook, notebook or memoranda not authorized by the examiner;
 - (c) making use of calculators and/or portable computing machines not authorized by the instructor;
 - (d) leaving answer papers exposed to view;
 - (e) attempting to read other student's examination papers.

The penalty for violation of these rules is suspension or expulsion or such other penalty as may be determined.
- (5) Candidates are requested to write on both sides of the page, unless the examiner has asked that the left hand page be reserved for rough drafts or calculations.
- (6) Discarded matter is to be struck out and not removed by mutilation of the examination answer book.
- (7) Candidates are cautioned against writing in their answer book any matter extraneous to the actual answering of the question set.
- (8) The candidate is to write his/her name on each answer book as directed and is to number each book.
- (9) A candidate must report to a supervisor before leaving the examination room.
- (10) Answer books must be handed to the supervisor-in-charge promptly when the signal is given. Failure to comply with this regulation will be cause for rejection of an answer paper.
- (11) If during the course of an examination a student becomes ill or receives word of domestic affliction, the student should report at once to the supervisor, hand in the unfinished paper and request that it be cancelled. If physical and/or emotional ill health is the cause, the student must report at once to a physician/counsellor so that subsequent application for a deferred examination is supported by a completed Physician/Counsellor Statement form. Students can consult professionals at University Health Services or university Counselling

Question No.	Maximum Mark	Mark Obtained
I	15	
II	15	
III	20	
IV	15	
V	20	
VI	15	
Total (out of 100)		

Problem I: (15 points)

The inclined pipe shown in the figure is used to carry oil ($\rho=900\text{kg/m}^3$, $\mu=9\times 10^{-3}\text{Pa.s}$). The pipe is made of steel, has a diameter $D=20\text{cm}$ and a length $L=300\text{m}$.



- (a) Determine the pressure drop if the average velocity in the pipe is equal to 3m/s . (10 marks)

$$Re = \rho u D / \mu = 900 * 3 * 0.2 / 0.009 = 6 \times 10^4$$

→ The flow is turbulent, and $f \approx 0.006$ (Chart)

$$-(\Delta P + \rho g \Delta h) = 2f u^2 \rho L / D$$

$$\Delta h = L \sin(\theta)$$

$$\begin{aligned} -\Delta P &= 2f u^2 \rho L / D + \rho g L \sin(\theta) \\ &= 2 * 0.006 * (3)^2 * 900 * 300 / 0.2 + 900 * 9.81 * 300 * \sin(30) \\ &= \underline{1.47 \text{ MPa}} \end{aligned}$$

Pressure Drop = 1.47 MPa

- (b) What would be the pressure drop for the same average velocity if the oil is treated as an ideal (inviscid) fluid. (5 marks)

If the flow was considered as inviscid, then the viscous losses are zero and,

$$\begin{aligned} -\Delta P &= \rho g \Delta h = \rho g L \sin(\theta) = \\ &= 900 * 9.81 * 300 * \sin(30) \\ &= \underline{1.32 \text{ MPa}} \end{aligned}$$

N.B. This result can be also obtained from Bernoulli equation:

$$P/\rho + u^2/2 + gh = c^{\text{st}}$$

Pressure Drop = 1.32 MPa

Problem II: (15 points)

A 250m³ tank contains 2300kg of a mixture of CO₂ (M=44kg/kmol) and CH₄ (M=16kg/kmol). The pressure and temperature in the tank are 1MPa and 300K respectively.

- (a) Calculate the mole fraction of CH₄ in the mixture using the ideal gas assumption. **(6 marks)**

$$n = PV/RT = 1,000,000 * 250 / (8.314 * 300) = 100233 \text{ mol} = 100.233 \text{ kmol}$$

$$\text{Average molar mass} = 2300 \text{ kg} / 100.233 \text{ kmol} = 22.95 \text{ kg/kmol}$$

Let x be the mole fraction of CH₄ in the mixture. Then the average molar mass is given by:

$$16 * x + 44 * (1-x) = 22.95, \text{ or}$$

$$28x = 21.05 \text{ or } x = 0.752$$

Mole fraction of CH₄ in the mixture is 0.752
Mole fraction of CO₂ in the mixture is 0.248

- (b) Given that the tank now contains 1,600kg of methane and 4,400kg of CO₂ at 300K, calculate the pressure in the tank using Kay's rule and the compressibility factor chart. **(9 marks)**

Component	Mass, kg	kmol	Mole Fract.	T_c, K	$P_c, \text{ atm}$	$y_i T_c$	$y_i P_c$
CH ₄	1600	100	0.5	190.6	45.4	95.3	22.7
CO ₂	4400	100	0.5	304.2	72.8	152.1	36.4
SUM						247.4	59.1

$$T_r = 300/247.4 = 1.21$$

P_r is to be calculated, but we need P_r to look up the value of Z . This will require a trial and error procedure.

To start with assume $Z = 1.0$ and calculate P and P_r . Use this P_r and the previously calculated T_r to obtain a value of Z from the chart.

Recalculate P using this value of Z and continue the iteration until convergence is achieved. These calculations are summarized below.

$$Z = PV_m/RT = Rv/(nRT) = 5.01 \times 10^{-7} P \rightarrow P = 1995360.Z$$

$$P_r = P/P_{pc} = P/(59.1 \times 101325) = 1.67 \times 10^{-7} P$$

Z	$P \text{ (Pa)}$	P_r	Revised Z
1	1995360	0.333	0.93
0.93	1855684.8	0.310	0.92
0.92	1835731.2	0.307	0.92

$$P = 1835.731 \text{ kPa}$$

Problem III: (20 points)

The following information is available on pure component properties of ethylene and propane.

Component	Normal boiling point, (K)	Latent heat of vaporization, (kJ/kmol)
Propane	231	19600
Ethylene	169	13500

- (a) Using this information, determine the relationship between the temperature and vapour pressures for propane and ethylene, i.e. calculate the values of A and B in the equation

$$\ln(P_v) = \frac{-A}{T} + B, \text{ for both components (Use units of kPa for } P_v\text{). (10 marks)}$$

For Propane: $A = \Delta H_v/R = 19600 \text{ (kJ.kmol)}/8.314 \text{ (kJ/kmol.K)} = \underline{2357.47 \text{ K}}$
 At normal boiling point $P_v = 101.325 \text{ kPa}$, therefore
 $\ln(101.325) = -2357.47/231 + B \rightarrow \underline{B = 14.82}$

For Ethylene: $A = \Delta H_v/R = 13500 \text{ (kJ.kmol)}/8.314 \text{ (kJ/kmol.K)} = \underline{1623.77 \text{ K}}$
 At normal boiling point $P_v = 101.325 \text{ kPa}$, therefore
 $\ln(101.325) = -1623.77/169 + B \rightarrow \underline{B = 14.23}$

- (b) A two-phase vapour-liquid mixture of ethylene and propane is kept in a tank at a temperature of 250K. Given that the vapor phase equilibrium contains 40mol% propane, use Raoult's law to calculate the pressure of the mixture and mole percent of propane in the liquid phase. (10 marks)

Using the above correlations we have:

For Propane (1): $\ln(P_{v1}) = -2357.47/250 + 14.82 \rightarrow P_{v1} = 219.23 \text{ kPa}$
For Ethylene (2): $\ln(P_{v2}) = -1623.77/250 + 14.23 \rightarrow P_{v2} = 2286.83 \text{ kPa}$

$$\alpha_{21} = P_{v2}/P_{v1} = 10.43$$

We know that: $y_1 = x_1/[x_1 + \alpha_{21}x_2] = x_1/[x_1(1-\alpha_{21}) + \alpha_{21}]$
 $\rightarrow x_1 = \alpha_{21} y_1/[1 + (\alpha_{21}-1)y_1] = \underline{0.874}$

$$P = x_1 P_{v1} + x_2 P_{v2} = \underline{479.75 \text{ kPa}}$$

Problem IV: (15 points)

Two insulated steel pipes are carrying 200°C steam. The internal diameter of both pipes is 2.0cm and both are insulated with 2.5cm thick layer of asbestos on the outside. The wall thickness of the first pipe is 0.5cm while the wall of the other pipe is 2.0cm thick. The temperature at the outside surface of the insulation is kept constant at 25°C. Thermal conductivities of steel and asbestos are 50 W/mK and 0.15 W/mK respectively.

(a) Calculate the rate of heat loss per unit length for both pipes. (6 marks)

For thin walled pipe:

$r_1 = 1.0 \text{ cm}, r_2 = 1.5 \text{ cm}, r_3 = 4 \text{ cm}, \kappa_1 = 50 \text{ W/mK and } \kappa_2 = 0.15 \text{ W/mK}$

$$\frac{Q}{L} = \frac{-2\pi\Delta T}{\frac{\ln(r_2/r_1)}{\kappa_1} + \frac{\ln(r_3/r_2)}{\kappa_2}} = \frac{2\pi * (200 - 25)}{\frac{\ln(1.5/1)}{50} + \frac{\ln(4/1.5)}{0.15}} = 167.9 \text{ W/m}$$

$Q/L = 167.95 \text{ W/m}$

For thick walled pipe:

$r_1 = 1.0 \text{ cm}, r_2 = 3.0 \text{ cm}, r_3 = 5.5 \text{ cm}, \kappa_1 = 50 \text{ W/mK and } \kappa_2 = 0.15 \text{ W/mK}$

$$\frac{Q}{L} = \frac{-2\pi\Delta T}{\frac{\ln(r_2/r_1)}{\kappa_1} + \frac{\ln(r_3/r_2)}{\kappa_2}} = \frac{2\pi * (200 - 25)}{\frac{\ln(3/1)}{50} + \frac{\ln(5.5/3)}{0.15}} = 270.9 \text{ W/m}$$

$Q/L = 270.63 \text{ W/m}$

(b) Calculate the temperature at the interface between the steel and the insulation in case of the thicker pipe. (6 marks)

Considering the heat transfer only in the steel for the case of thick pipe,

$$-\Delta T = \frac{Q}{L} \frac{\ln(r_2/r_1)}{2\pi\kappa_1} = \frac{270.9 \ln(3/1)}{2\pi * 50} = 0.95 \text{ } ^\circ\text{C}$$

$$\text{Temperature at the interface} = 200 - 0.95 = 199.05 \text{ } ^\circ\text{C}$$

$T = 199.05 \text{ } ^\circ\text{C}$

(c) Explain why the thick walled pipe loses more heat. (3 marks)

In this situation the steel offers very little resistance to heat loss. The temperature drop in the steel is less than 1 °C, even for the thick pipe. In the case of the thick walled pipe a much larger area of the outer steel surface is maintained at high temperature. This larger area results in larger heat loss, even though the temperature difference across the insulation layer would be slightly smaller in the case of thicker pipe.

Problem V: (20 points)

(a) The mean free path of a gas at a temperature T_1 and a pressure P_1 is 7×10^{-5} cm. At these temperature and pressure, there are 2×10^{18} molecules/cm³.

Use results from kinetic theory to determine the collision diameter, σ . (5 marks)

$$\lambda = \frac{l}{\sqrt{2} \pi \rho_N \sigma^2} \Rightarrow \sigma^2 = \frac{l}{\sqrt{2} \pi \rho_N \lambda} = \frac{l}{\sqrt{2} \pi 2 \times 10^{24} \cdot 7 \times 10^{-7}} = 16.077 \times 10^{-20} \text{ m}^2$$

$$\Rightarrow \sigma = 4.01 \text{ \AA}$$

$\sigma = 4.010 \text{ Angstrom}$

(b) The pressure of the gas is doubled such that $P_2 = 2P_1$, while keeping its temperature fixed. What is the new value of the mean free path? (3 marks)

$$\rho_N = \frac{P}{kT} \Rightarrow \lambda = \frac{kT}{\sqrt{2} \pi P \sigma^2}$$

$$\lambda_2 = \frac{kT_2}{\sqrt{2} \pi P_2 \sigma^2} = \frac{kT_1}{\sqrt{2} \pi (2P_1) \sigma^2} = \frac{\lambda_1}{2} = 3.5 \times 10^{-5} \text{ cm}$$

$\lambda = 3.5 \times 10^{-5} \text{ cm}$

(c) The gas is now held in a cubic container of side a . The temperature T of the container is kept constant. One of the walls of the container has a small circular hole of radius r , from which the gas escapes. If we call X_n the percentage of molecules of the gas that escape through the small hole after a time Δt , show that X_n and Δt are given by the following relationship:

$$\Delta t = \frac{4 X_n a^3}{r^2} \sqrt{\frac{M}{8 \pi R T}}$$

(Assume that the pressure remains constant in the container). (10 marks)

The collision rate per unit area: $Z_m = \frac{\rho_N \bar{c}}{4} = \frac{\Delta n}{A \Delta t}$

$$\Rightarrow \Delta t = \frac{\Delta n}{A Z_m} = \frac{4 \Delta n}{A \rho_N \bar{c}}$$

The percent of molecules lost is $X_n = \Delta n / n_1 \Rightarrow \Delta n = X_n \cdot n_1$
but $n_1 = \rho_N \cdot V = \rho_N \cdot a^3$, therefore: $\Delta n = X_n \cdot \rho_N \cdot a^3$

$$\Rightarrow \Delta t = \frac{4 X_n \cdot \rho_N a^3}{(\pi \cdot r^2) \cdot \rho_N \bar{c}} = \frac{4 X_n \cdot a^3}{(\pi \cdot r^2) \cdot \bar{c}} = \frac{4 X_n \cdot a^3}{\pi \cdot r^2} \sqrt{\frac{\pi M}{8 R T}} = \frac{4 X_n \cdot a^3}{r^2} \sqrt{\frac{M}{8 \pi R T}}$$

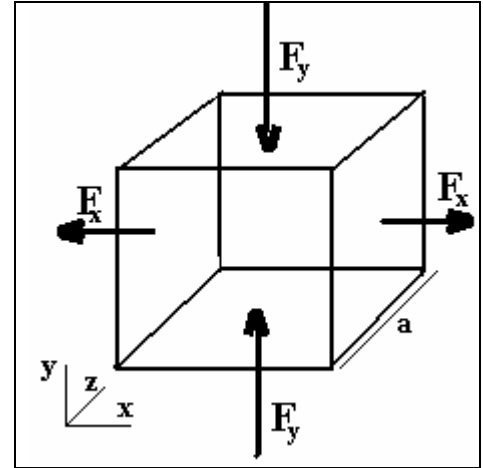
(d) Determine the time Δt for the container to lose 10% of the gas, if $M = 40$ kg/kmol, $T = 300$ K, $a = 20$ cm and $r = 0.1$ mm. (2 marks)

$$\Delta t = \frac{4 \times 0.1 \times (0.2)^3}{(10^{-4})^2} \sqrt{\frac{40}{8 \pi \times 8314 \times 300}} = 255.62 \text{ s} \quad \text{or} \quad 4.26 \text{ min}$$

$\Delta t = 255 \text{ s} = 4.26 \text{ min}$

Problem VI: (15 points)

Consider a metal that has a cubic shape and which is subject to forces in the x and y directions as shown in the figure. Note that there are no forces applied in the z-direction.



(a) Show that if the tensile force in the x-direction and the compressive force in the y-direction have the same magnitude, then there is no change in the volume of the cube, i.e. $\Delta V=0$. (6 marks)

For the system of forces shown in the figure we have:

$$\epsilon_x = \frac{\sigma_x}{E} - \nu \frac{\sigma_y}{E},$$

$$\epsilon_y = \frac{\sigma_y}{E} - \nu \frac{\sigma_x}{E}, \quad \Rightarrow \frac{\Delta V}{V} \approx \epsilon_x + \epsilon_y + \epsilon_z = (1 - 2\nu) \frac{\sigma_x + \sigma_y}{E}$$

$$\epsilon_z = -\nu \frac{\sigma_x + \sigma_y}{E},$$

But $\sigma_x = F_x/A$ and $\sigma_y = -F_y/A$. Therefore if $|F_x| = |F_y|$, then $\sigma_x + \sigma_y = 0$ or $\Delta V = 0$.

(b) It is found that if the magnitudes of the forces in the x and y directions are equal to 200N and 100N respectively, the relative change in the volume of the cube $\Delta V/V$, is equal to 10^{-5} . We give $a=1\text{cm}$, and the Poisson ratio $\nu=0.4$. Determine:

(i) The value of the Young modulus of the metal. (3 marks)

$$\sigma_x = F_x/A = F_x/a^2 = 200/(0.01)^2 = 2 \times 10^6 \text{ Pa.}$$

$$\sigma_y = -F_y/A = -F_y/a^2 = -100/(0.01)^2 = -10^6 \text{ Pa.}$$

Therefore:

$$E = \frac{\Delta V}{V} \approx \epsilon_x + \epsilon_y + \epsilon_z = (1 - 2\nu) \frac{\sigma_x + \sigma_y}{\left(\frac{\Delta V}{V}\right)} = (1 - 2 \times 0.4) \frac{10^6}{10^{-5}} = 20 \text{ GPa.}$$

$E = 20.0 \text{ GPa}$

(ii) The value of the Shear modulus of the metal. (3 marks)

$$G = \frac{E}{2(1 + \nu)} = \frac{20 \text{ GPa}}{2(1 + 0.4)} = 7.14 \text{ GPa.}$$

$G = 7.14 \text{ GPa}$

(iii) The value of the strain in the x-direction, ϵ_x . (3 marks)

$$\epsilon_x = \frac{\sigma_x}{E} - \nu \frac{\sigma_y}{E} = \frac{2 \cdot 10^6}{2 \cdot 10^{10}} - 0.4 \frac{(-10^6)}{2 \cdot 10^{10}} = 1.2 \times 10^{-4}$$

$\epsilon_x = 1.2 \times 10^{-4}$
