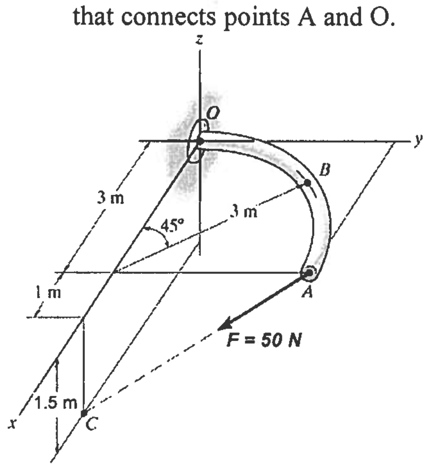
Q1. The curved rod lies in the xy plane and has a radius of 3 m. The force **F**, acts at its end on point A as shown. Determine the:

a) angle between the lines AC and AO.
b) magnitude of the component of the force that acts parallel to the line that connects points A and O.



$\overline{u_{AC}} = \frac{1\hat{i} - 3\hat{j} - 1.5\hat{k}}{3.5} = 0.286\hat{i} - 0.857\hat{j} - 0.429\hat{k}$

$\overline{u_{AO}} = \frac{-3\hat{i} - 3\hat{j}}{4.24} = -0.707\hat{i} - 0.707\hat{j}$

$\cos\theta = \overline{u_{AC}} \circ \overline{u_{AO}} = 0.4041$

$\theta = 66.17°$

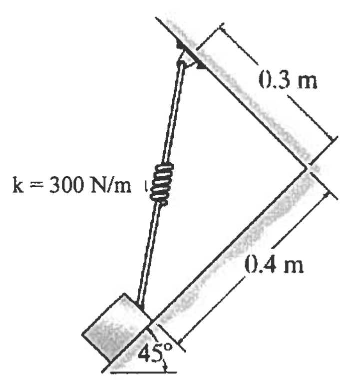
$F_{IIOA} = 50\cos 66.17° = 20.20N$
(or dot product)

ANSWER: (a) angle $\theta_{AO\text{-}AC}$ = 66.17° ° /1.5 marks

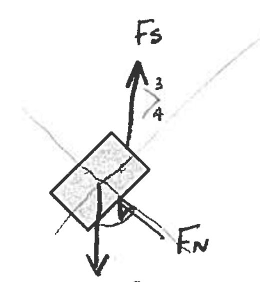
(b) F_{IIOA} = 20.20 N /1 mark

Q2. The block has a mass of 10 kg and rests on the smooth plane.

a) Draw the free body diagram of the block in the space given.
b) Determine the unstretched length of the spring in mm.



FBD of Block



$\Sigma F_x' = 0$

$\frac{4}{5}F_S - 98.1\cos 45 = 0$

$F_S = 86.7N$

ANSWER: (a) FBD of block (use box above) /1.5 marks

(b) $L_{unstretched}$ = 211.0 mm /1.5 marks

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$F_S = K\Delta L$

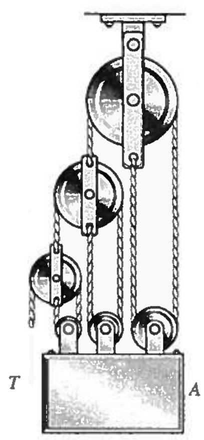
$\Delta L = \frac{86.7}{300} = 0.289m.$

$\Delta L = l - l_o$

$l_o = l - \Delta L = 0.5 - 0.289$

$l_o = 211.0mm.$

Q3. The mass of the suspended object A is m_A and the masses of the pulleys are negligible. Determine the force T necessary for the system to be in equilibrium.



T_1 $\quad$ T_2 $\quad$ F_1

A $\quad$ B $\quad$ C

T T T $\quad$ T_1 T_1 T_1 $\quad$ T_2 T_2 T_2

$T_1 = 3T$ $\quad$ $T_2 = 3T_1$

$= 3(3T) = 9T$

T T T_1 T_1 T_2 T_2

$M_A g$

$\Sigma F_y = 0$

$2T + 2T_1 + 2T_2 - M_A g = 0$

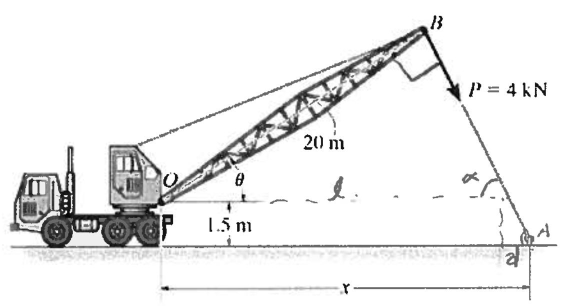
$2T + 2(3T) + 2(9T) - M_A g = 0$

$T = \dfrac{M_A g}{26}$

ANSWER: T = $M_A g / 26$ N $\qquad$ /2 marks

Q4. The towline exerts a force **P** = 4 kN at the end of a 20 m long crane boom. If θ = 20°, determine:

a) the placement x of the hook at A so that this force creates a maximum moment about point O.
b) the magnitude and direction of the moment about O created by the force P.



$\cos 20 = \dfrac{20}{l}$

$l = 21.283\,m$

$d = \dfrac{1.5}{\tan 70} = 0.546$

$x = l + d = 21.83\,m$

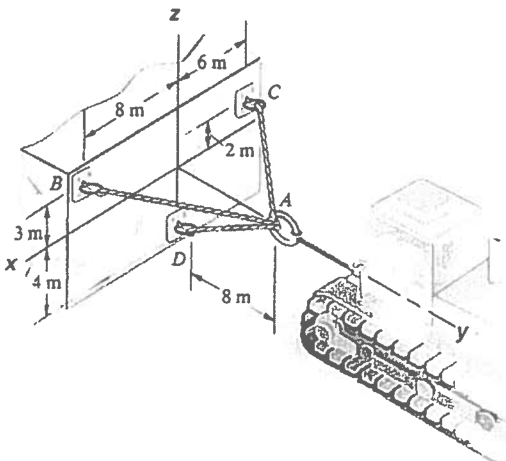
$M_o = 4\,KN \cdot 20\,m = 80\,KN{\cdot}m$

ANSWER: (a) x = 21.83 m $\qquad$ /1.5 marks

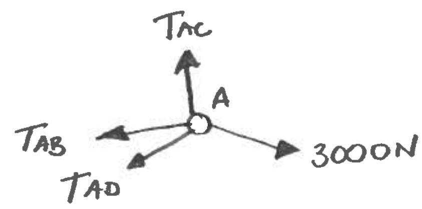
(b) M_o = 80 cw kNm $\qquad$ /1 mark

Q5. The bulldozer exerts a force **F** = 3000 N at A, directed along the positive y-axis. If the system is in equilibrium, what are the magnitudes of the tensions in cables AB, AC, and AD?

/10 marks



FBD of A



$$\overline{T_{AB}} = T_{AB}\left(\frac{8\hat{i} - 8\hat{j} + 3\hat{k}}{11.70}\right) = 0.6835T_{AB}\hat{i} - 0.6835T_{AB}\hat{j} + 0.2563T_{AB}\hat{k}$$

$$\overline{T_{AC}} = T_{AC}\left(\frac{-6\hat{i} - 8\hat{j} + 2\hat{k}}{10.20}\right) = -0.5883T_{AC}\hat{i} - 0.7845T_{AC}\hat{j} + 0.1961T_{AC}\hat{k}$$

$$\overline{T_{AD}} = T_{AD}\left(\frac{0\hat{i} - 8\hat{j} - 4\hat{k}}{8.94}\right) = -0.8944T_{AD}\hat{j} - 0.4472T_{AD}\hat{k}$$

$$\overline{F} = 3000\hat{j}$$

$\Sigma F_x = 0$; $0.6835T_{AB} - 0.5883T_{AC} = 0$

$T_{AB} = 0.8608T_{AC}$

$\Sigma F_z = 0$; $0.2563T_{AB} + 0.1961T_{AC} - 0.4472T_{AD} = 0$

$0.2563(0.8608T_{AC}) + 0.1961T_{AC} - 0.4472T_{AD} = 0$

$T_{AD} = 0.9319T_{AC}$

$\Sigma F_y = 0$; $-0.6835T_{AB} - 0.7845T_{AC} - 0.8944T_{AD} + 3000 = 0$

$-0.6835(0.8608T_{AC}) - 0.7845T_{AC} - 0.8944(0.9319T_{AC}) + 3000 = 0$

$2.206T_{AC} = 3000$

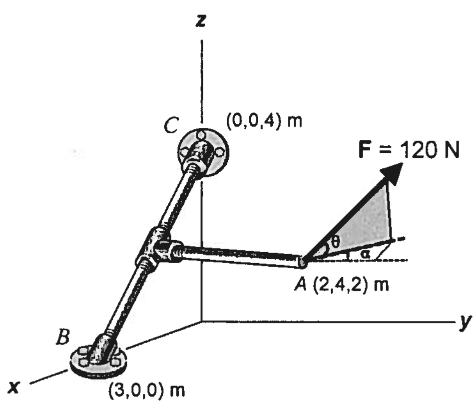
$T_{AC} = 1359.7\text{ N}$

$T_{AB} = 1170.47\text{ N}$

$T_{AD} = 1267.1\text{ N}$

Q6. The bar CB is located in the *xz* plane. An applied force **F**, located at point A, has a magnitude of F = 120N. The angle from F to the *xy* plane is $\theta = 65°$, and the angle $\alpha = 20°$ is to a line parallel to the *y* axis.
Determine the magnitude and direction (state your answer with magnitude and directional angles θ_x, θ_y, θ_z) of the moment created by **F** around the axis CB.

/10 marks



$$\bar{F} = 120\,(-\cos 65 \sin 20\,\hat{\imath} + \cos 65 \cos 20\,\hat{\jmath} + \sin 65\,\hat{k})$$

$$= -17.35\,\hat{\imath} + 47.66\,\hat{\jmath} + 108.76\,\hat{k} \;\; N$$

$$\bar{u}_{CB} = \left(\frac{3\hat{\imath} + 0\hat{\jmath} - 4\hat{k}}{5}\right) = 0.6\,\hat{\imath} - 0.8\,\hat{k}$$

$$\bar{r}_{BA} = (-1\hat{\imath} + 4\hat{\jmath} + 2\hat{k})\,m \quad \text{or} \quad \bar{r}_{CA} = (2\hat{\imath} + 4\hat{\jmath} - 2\hat{k})\,m.$$

$$\text{or} \quad \bar{r}_{DA} = (0.5\hat{\imath} + 0.4\hat{\jmath})\,m$$

$$M_{CB} = \bar{u}_{CB} \cdot (\bar{r}_{BA} \times \bar{F}) = \begin{vmatrix} 0.6 & 0 & -0.8 \\ -1 & 4 & 2 \\ -17.35 & 47.66 & 108.76 \end{vmatrix}$$

$$= 208.8 + 0 - 17.38$$

$$\underline{M_{CB} = 186.45 \;\; N \cdot m}$$

6

$$\bar{M}_{CB} = 186.45\,(0.6\,\hat{\imath} - 0.8\,\hat{k}) = (111.87\,\hat{\imath} - 149.16\,\hat{k})\; N \cdot m$$

$$\cos\theta_x = \frac{111.87}{186.45} \;\; (\text{or just } 0.6) \qquad \theta_x = 53.1°$$

$$\cos\theta_y = 0 \qquad \theta_y = 90°$$

$$\cos\theta_z = \frac{-149.16}{186.45} \;\; (\text{or just } -0.8) \qquad \theta_z = 143.13°$$

$$\underline{M_{CB} = 186.5 \;\; N \cdot m \qquad \theta_x = 53.1°,\; \theta_y = 90°,\; \theta_z = 143.1°}$$