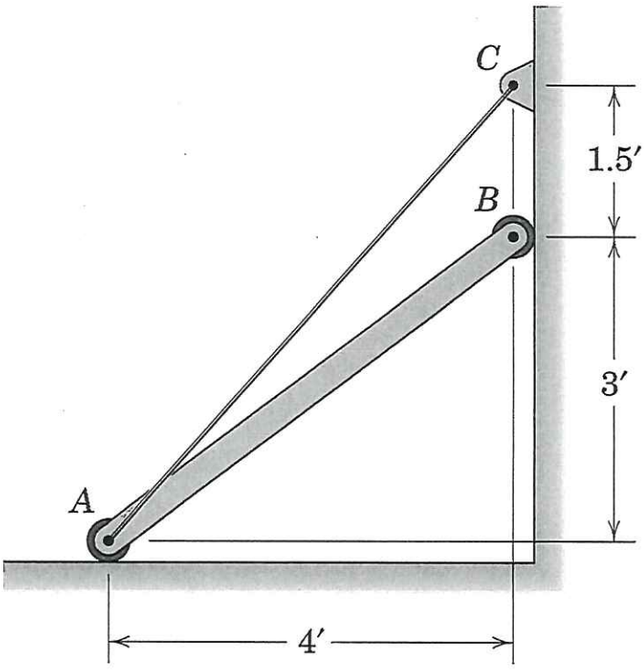
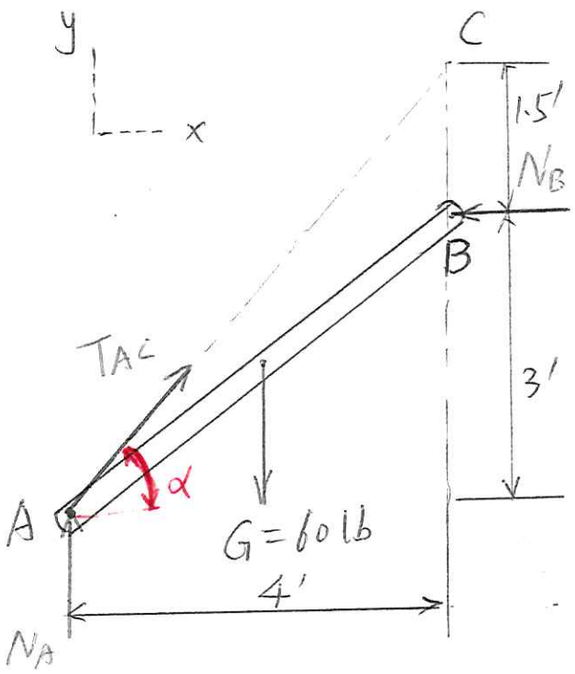
Q1.

The uniform bar with end rollers weighs 60 lb and is in equilibrium supported by the smooth horizontal and vertical surfaces and by the cable AC. Calculate the tension in the cable and the reactions against the rollers at A and B.





Solution

$+\circlearrowleft \Sigma M_A = 0$ $\quad N_B(3) - G\left(\frac{4}{2}\right) = 0$

$$N_B = \frac{(60)(2)}{3} = 40 \text{ lb}$$

$\pm\rightarrow \Sigma F_x = 0$ $\quad T_{AC}\left(\frac{4}{\sqrt{4^2+(3+1.5)^2}}\right) - N_B = 0$ *

$$T_{AC} = 60.21 \text{ lb}$$

$+\uparrow \Sigma F_y = 0$ $\quad N_A + T_{AC}\left(\frac{1.5+3}{\sqrt{4^2+(1.5+3)^2}}\right) - G = 0$ *

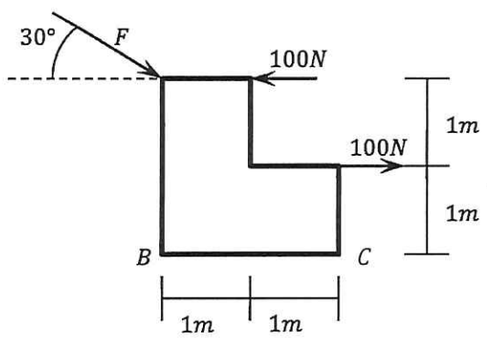
$$N_A = 15.00 \text{ lb}$$

* OR USE ANGLE $\alpha = \tan^{-1}\frac{4.5'}{4'} = 48.37°$

Points: /7.5

QUESTION 2

A plate is subjected to the 3 forces shown (a) If $F = 100N$ determine the equivalent force-couple system at B. (b) Find the point at which the resultant intersects the line AB. (c) Determine the value of F so that the resultant of the system would pass through point C.



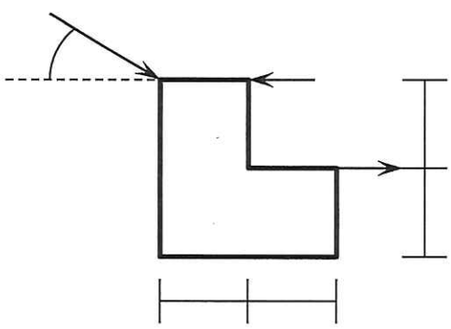
Solution: The 3 parts of the question can be answered (if so wished) independently of each other.

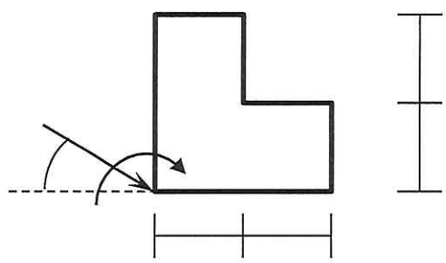
(a) The moment of F about B is found (by Varignon's lemma) as

$$M_B^F = -F\cos 30°\,(2m) = -(100\ N)\cos 30°(2m) = -173.2\ Nm$$

Therefore, the total couple at B is

$$C_B = (-173.2 + 100)Nm = -73.2\ Nm$$



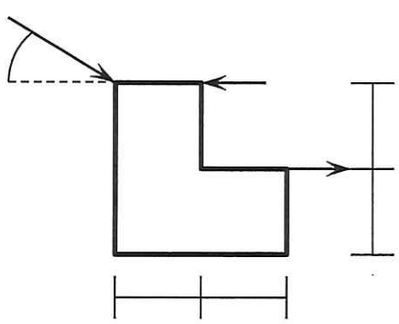


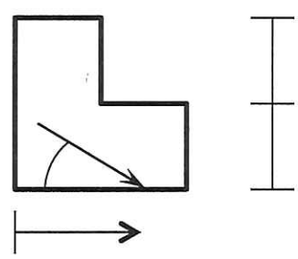
(b) Let D denote the desired point on line AB, and let x_D denote its abscissa. The moment of the system with respect to this point must vanish. We obtain

$$-(100\ N)\cos 30°\ (2m) + (100\ N)\sin 30°(x_D) + 100\ Nm = 0$$

Solving for x_D we obtain

$$x_D = \frac{73.2\ Nm}{(100\ N)\sin 30°} = 1.46\ m$$





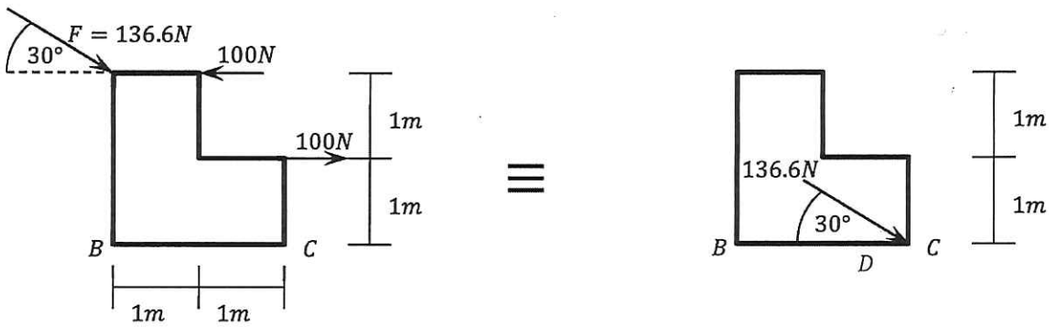
Clearly, the same solution is obtained if starting from the answer to part (a).

(c) We now let F be unknown, but make the moment with respect to C vanish. Thus

$$-(F)\cos 30°\ (2m) + (F)\sin 30°(2m) + 100\ Nm = 0$$

Solving for F we obtain

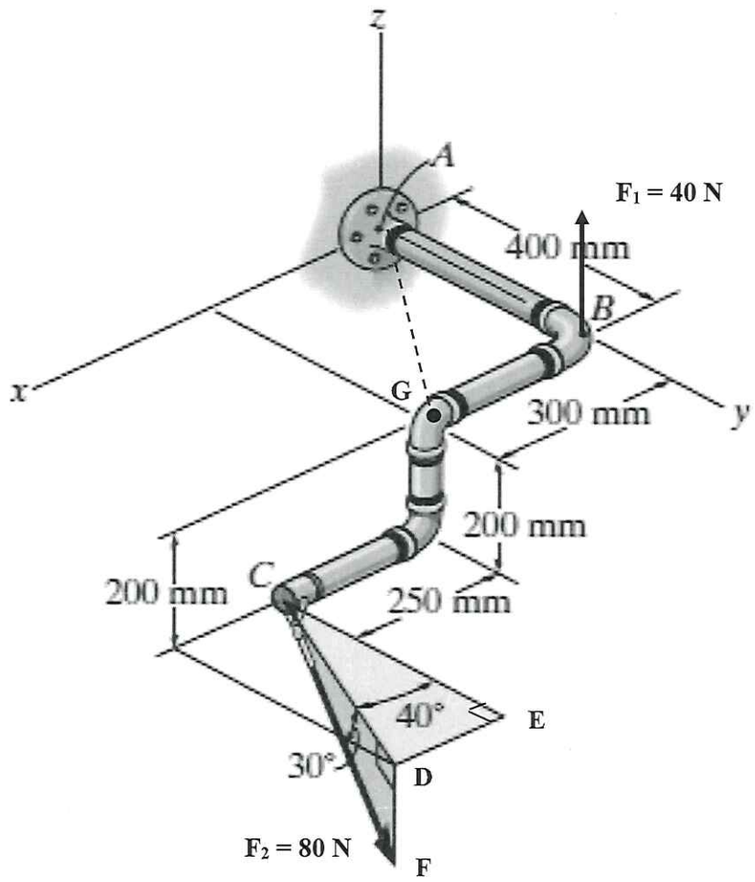
$$F = \frac{100\ Nm}{\cos 30°\ (2m) - \sin 30°\ (2m)} = 136.6\ N$$



Q3.

Determine the resultant moment (in terms of vector form) produced by forces, $\mathbf{F}_1$ and $\mathbf{F}_2$ about the point A and the axis AG, respectively.

$\mathbf{F}_1$ is applied at point *B* situated on y-axis and parallel to the *z*-axis; while $\mathbf{F}_2$ is applied at point C. The magnitudes of $\mathbf{F}_1$ and $\mathbf{F}_2$ are 40 N and 80 N, respectively. Note that the plane containing CDE is parallel to x-y plane and the plane containing CDF is perpendicular to x-y plane. The angle between lines CD and CF is 30°, and the angle between lines CD and CE is 40°. The point A is the origin of the given coordinate system, and the point G is on the x-y plane. The line CE is parallel to the y-axis.



A (0, 0, 0)

B (0, 0.4, 0)

C (0.55, 0.4, −0.2)

G (0.3, 0.4, 0)

Solution

(a) moment about point A

moment of F_1 about point A

$$\vec{M}_{A-F_1} = \vec{r}_{AB} \times \vec{F}_1$$

$$\vec{r}_{AB} = 0.4\vec{j}\ m \qquad \vec{F}_1 = 40\vec{k}\ N$$

$$\vec{M}_{A-F_1} = (0.4\vec{j}) \times (40\vec{k}) = 16\vec{i} \quad N\text{-}m$$

Points: /9

moment of F_2 about point A

$$\vec{M}_{A\text{-}F_2} = \vec{r}_{AC} \times \vec{F}_2$$

$$\vec{r}_{AC} = 0.55\vec{i} + 0.4\vec{j} - 0.2\vec{k} \quad m$$

$$\vec{F}_2 = (80)(\cos 30^\circ)(\sin 40^\circ)\vec{i} + (80)(\cos 30^\circ)(\cos 40^\circ)\vec{j} - (80)(\sin 30^\circ)\vec{k}$$

$$= 44.53\vec{i} + 53.07\vec{j} - 40\vec{k} \quad N$$

$$\vec{M}_{A\text{-}F_2} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0.55 & 0.4 & -0.2 \\ 44.53 & 53.07 & -40 \end{vmatrix}$$

$$= \begin{vmatrix} 0.4 & -0.2 \\ 53.07 & -40 \end{vmatrix}\vec{i} - \begin{vmatrix} 0.55 & -0.2 \\ 44.53 & -40 \end{vmatrix}\vec{j} + \begin{vmatrix} 0.55 & 0.4 \\ 44.53 & 53.07 \end{vmatrix}\vec{k}$$

$$= -5.39\vec{i} + 13.09\vec{j} + 11.38\vec{k} \quad N\text{-}m$$

The resultant moment about point A

$$\vec{M}_A = \vec{M}_{A\text{-}F_1} + \vec{M}_{A\text{-}F_2} = 16\vec{i} + (-5.39\vec{i} + 13.09\vec{j} + 11.38\vec{k})$$

$$= 10.61\vec{i} + 13.09\vec{j} + 11.38\vec{k} \quad N\text{-}m$$

(b) moment about axis AG

$$M_{AG} = \vec{M}_A \cdot \vec{n}_{AG}$$

$$\vec{r}_{AG} = 0.3\vec{i} + 0.4\vec{j} \quad m$$

$$\vec{n}_{AG} = \frac{\vec{r}_{AG}}{r_{AG}} = \frac{0.3\vec{i} + 0.4\vec{j}}{\sqrt{0.3^2 + 0.4^2}} = 0.6\vec{i} + 0.8\vec{j}$$

$$M_{AG} = (10.61\vec{i} + 13.09\vec{j} + 11.38\vec{k}) \cdot (0.6\vec{i} + 0.8\vec{j})$$

$$= (10.61)(0.6) + (13.09)(0.8)$$

$$= 16.84 \quad N$$

$$\vec{M}_{AG} = M_{AG}\vec{n}_{AG} = 16.84(0.6\vec{i} + 0.8\vec{j})$$

$$= 10.10\vec{i} + 13.47\vec{j} \quad N\text{-}m$$

VERSION 1

Q4.

Circle the correct answer.

Q4.1 Which statement is correct?

a) Two non intersecting lines in space are necessary parallel
b) Three non-vanishing parallel forces cannot form a couple
c) If a force intersects an axis, the moment of the force about the axis vanishes (is 0)
d) The cross product of two vectors is independent of the order of the factors

Q4.2 A system of forces and couples in space

a) can always be replaced by a single force without a couple
b) can never be replaced by a single force without a couple
c) can be replaced by a single force without a couple under special conditions
d) none of the above

Q4.3 A two-force member is

a) An object subjected to only one couple
b) An object subjected to two forces acting at the same point
c) An object subjected to one force and one couple
d) An object subjected only to forces acting at two points on the object itself

Q4.4 The moment of a force about an axis

a) Has the same magnitude as the moment of the force about a point on the axis
b) Is the component along the axis of a moment about a point lying on the axis itself
c) Is parallel to the force itself
d) Is perpendicular to the axis

Q4.5 Given two spatial forces forming a couple

a) The couple moment about any point in the plane of the forces vanishes
b) The couple moment is a vector perpendicular to the plane of the forces
c) The couple moment is a vector lying in the plane of the forces
d) None of the above

Q4.6 Given two spatial forces of magnitude $F > 0$ forming a couple of magnitude Fd

a) They can be replaced by any other couple as long as the product Fd is preserved
b) They amount to a zero force and can be eliminated entirely
c) They can be replaced by a single force at an appropriately determined location
d) None of the above

Points: /6

VERSION 2

Q4.

Circle the correct answer.

Q4.1 A system of forces and couples in space

a) can always be replaced by a single force without a couple
b) can never be replaced by a single force without a couple
c) can be replaced by a single force without a couple under special conditions
d) none of the above

Q4.2 Given two spatial forces forming a couple

a) The couple moment about any point in the plane of the forces vanishes
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c) They can be replaced by a single force at an appropriately determined location
d) None of the above

Q4.4 Which statement is correct?

a) Two non intersecting lines in space are necessary parallel
b) Three non-vanishing parallel forces cannot form a couple
c) If a force intersects an axis, the moment of the force about the axis vanishes (is 0)
d) The cross product of two vectors is independent of the order of the factors

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Q4.6 The moment of a force about an axis

a) Has the same magnitude as the moment of the force about a point on the axis
b) Is the component along the axis of a moment about a point lying on the axis itself
c) Is parallel to the force itself
d) Is perpendicular to the axis

Points: /6

VERSION 3

Q4.

Circle the correct answer.

Q4.1 Given two spatial forces forming a couple

a) The couple moment about any point in the plane of the forces vanishes
b) The couple moment is a vector perpendicular to the plane of the forces
c) The couple moment is a vector lying in the plane of the forces
d) None of the above

Q4.2 A system of forces and couples in space

a) can always be replaced by a single force without a couple
b) can never be replaced by a single force without a couple
c) can be replaced by a single force without a couple under special conditions
d) none of the above

Q4.3 Given two spatial forces of magnitude $F > 0$ forming a couple of magnitude Fd

a) They can be replaced by any other couple as long as the product Fd is preserved
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b) Three non-vanishing parallel forces cannot form a couple
c) If a force intersects an axis, the moment of the force about the axis vanishes (is 0)
d) The cross product of two vectors is independent of the order of the factors

Points: /6