THE UNIVERSITY OF CALGARY
Schulich School of Engineering

ENGG 202 – Engineering Statics
Second Midterm Exam
March 28, 2017

1. The examination is closed textbook
2. There are 2 comprehensive questions and 4 multiple-choice questions. Answer all questions directly on the question sheets.
3. Only the SSE sanctioned, non-programmable, scientific calculators are permitted.
4. **Free body diagrams are required** on all long-answer **equilibrium** questions to obtain full marks. Diagrams must be separate from the given figure.

DO NOT OPEN THE EXAM BOOKLET UNTIL INSTRUCTED TO DO SO

Student's Last name: ______________________________

Student's First name: ______________________________

Lecture Section (Circle One):

L01	TuTh	8:00	Di Martino
L02	MWF	9:00	Epstein
L03	MWF	8:00	Epstein
L04	MWF	15:00	He

Student ID#: ________________

TRIGONOMETRIC FORMULAE:

Sine Law: $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$

Cosine Law: $c^2 = a^2 + b^2 - 2ab\cos C$

Question	Maximum mark	Mark
Q1	11	
Q2	11	
Q3	8	
Total	30	

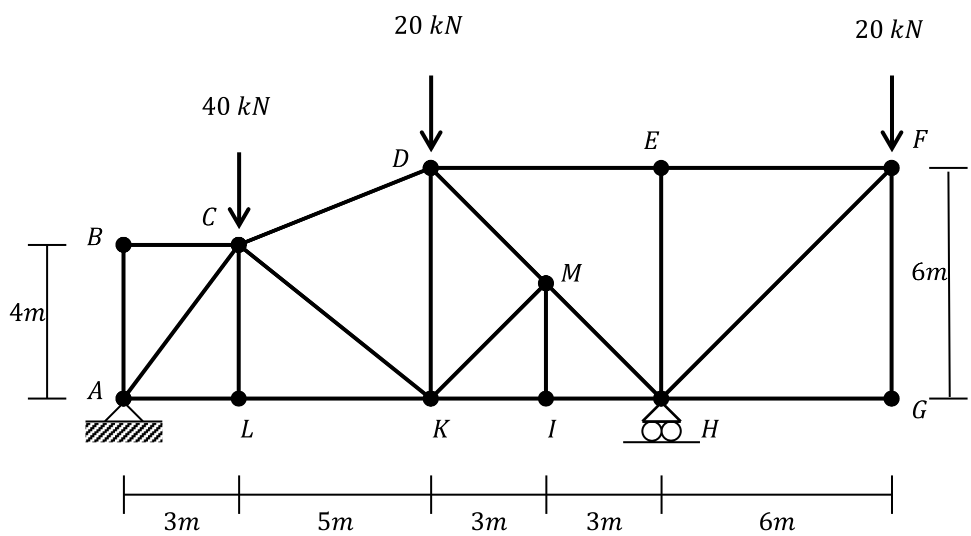
Q1.

For the truss and loading shown:

(a) Identify all the zero-force members;

(b) Using the method of joints, determine the internal forces in all the bars;

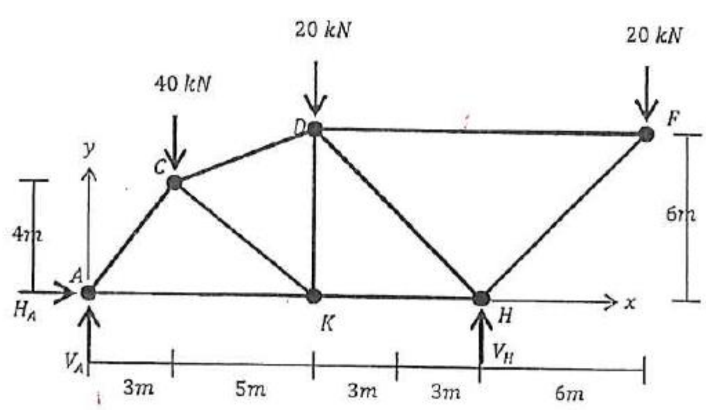
(c) Using the method of sections, verify the results obtained for members CK and HM.



<u>Solution:</u>

(a) Zero-force members: AB, BC, CL, IM, KM, EH, GH, EF, GF.

(b) This part should be considered as independent from part (a), whether the student implements or doesn't implement the zero-force members. Below is the FBD of the truss devoid of the zero-force members, but a long solution with all the original nodes is also acceptable.



<u>Find reactions at supports</u>

[Note: in theory, the reactions are not needed for this part, but I don't think there will be a single student that would think they aren't. Moreover, the reactions are indeed needed for part (c)]

Equilibrium equations are:

$$\Sigma F_x = H_A = 0$$
$$\Sigma F_y = V_A + V_H - 40\ kN - 20\ kN - 20\ kN = 0$$
$$\Sigma M_A = -(40kN)(3m) - (20kN)(8m) - (20kN)(20m) + V_H(14m) = 0$$

Solving, we obtain

$H_A = 0 \qquad V_A = 31.43\ kN\ \ (as\ shown) \qquad V_H = 48.57\ kN\ \ (as\ shown)$

<u>FBDs of individual nodes</u>

NODE F

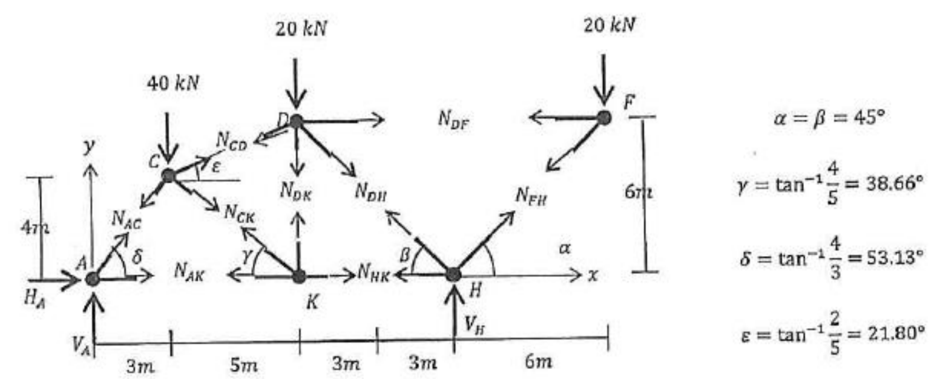
$\Sigma F_y = -20kN - N_{FH}\sin 45° = 0 \quad \therefore \quad N_{FH} = -28.28\ kN\ \ (compr.)$

$\Sigma F_x = -N_{DF} - N_{FH}\cos 45° = 0 \quad \therefore \quad N_{DF} = 20.00\ kN\ \ (tension)$

NODE H

$\Sigma F_y = N_{FH}\sin 45° + N_{DH}\sin 45° + V_H = 0 \quad \therefore \quad N_{DH} = -40.41\ kN\ \ (compr.)$

$\Sigma F_x = -N_{HK} - N_{DH}\cos 45° + N_{FH}\cos 45° = 0 \quad \therefore \quad N_{HK} = 8.58\ kN\ \ (tension)$



NODE A

$\Sigma F_y = N_{AC}\sin\delta + V_A = 0 \quad \therefore \quad N_{AC} = -39.29\ kN\ \ (compr.)$

$\Sigma F_x = H_A + N_{AK} + N_{AC}\cos\delta = 0 \quad \therefore \quad N_{AK} = 23.57\ kN\ \ (tension)$

NODE K

$\Sigma F_x = -N_{AK} + N_{HK} - N_{CK}\cos\gamma = 0 \quad \therefore \quad N_{CK} = -19.19\ kN\ \ (compr.)$

$\Sigma F_y = N_{CK}\sin\gamma + N_{DK} = 0 \quad \therefore \quad N_{DK} = 12.00\ kN\ \ (tension)$

NODE D

$\Sigma F_x = N_{DF} + N_{DH}\cos 45° - N_{CD}\cos\varepsilon = 0 \quad \therefore \quad N_{CD} = -9.23\ kN\ \ (compr.)$

We have now the forces in all bars. Let us verify the satisfaction of the 3 extra equilibrium equations:

At Node D we have

$$\Sigma F_y = -20\ kN - N_{DK} - N_{DH} \sin 45° - N_{CD} \sin \varepsilon = 0.002\ kN$$

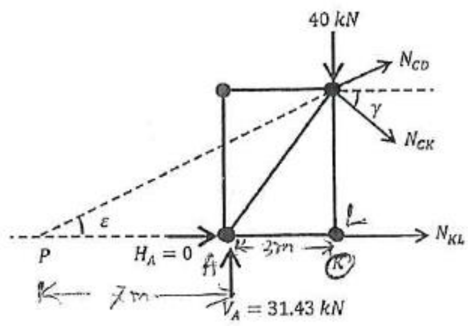
At node C:

$$\Sigma F_x = N_{CD} \cos \varepsilon + N_{CK} \cos \gamma - N_{AC} \cos \delta = 0.019\ kN$$

$$\Sigma F_y = -40\ kN - N_{AC} \sin \delta - N_{CK} \sin \gamma + N_{CD} \sin \varepsilon = -0.008\ kN$$

(c) <u>Method of sections</u>

BAR CK



$$PK = \frac{4m}{\tan \varepsilon} = 10.00m$$

$$\Sigma M_P = -(40kN)(10m) + (31.43kN)(7m) - N_{CK} \sin \gamma\ (10m) - N_{CK} \cos \gamma (4m) = 0$$

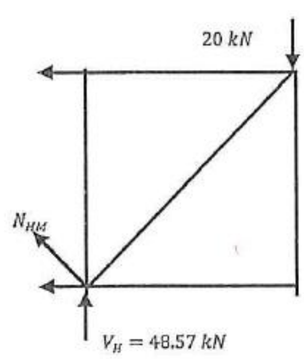
Solving, we get

$$N_{CK} = -19.21\ kN \quad (compr.)$$

BAR HM

$$\Sigma F_y = -20kN + 48.57 + N_{HM} \sin 45° = 0$$

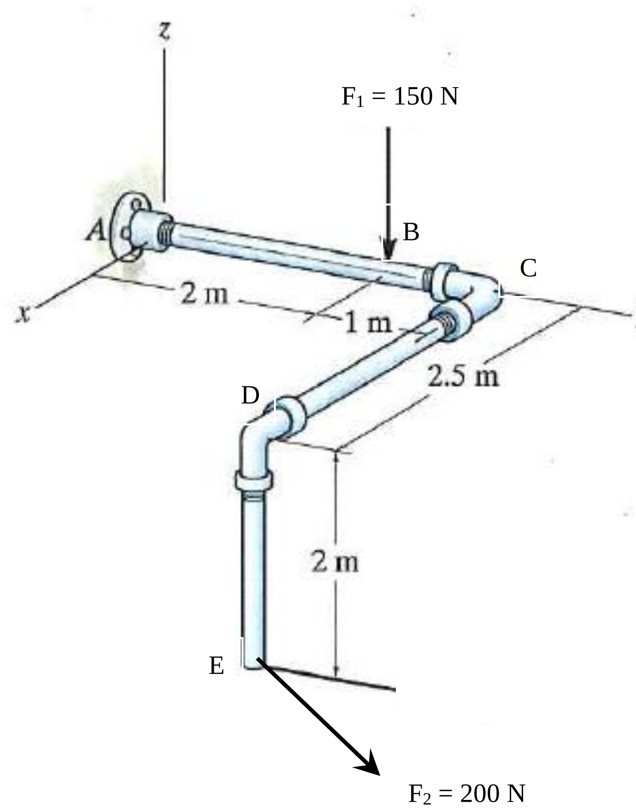
Thus, $N_{HM} = -40.40\ kN\ (compr.)$



Notice that this force is the same as N_{DH}

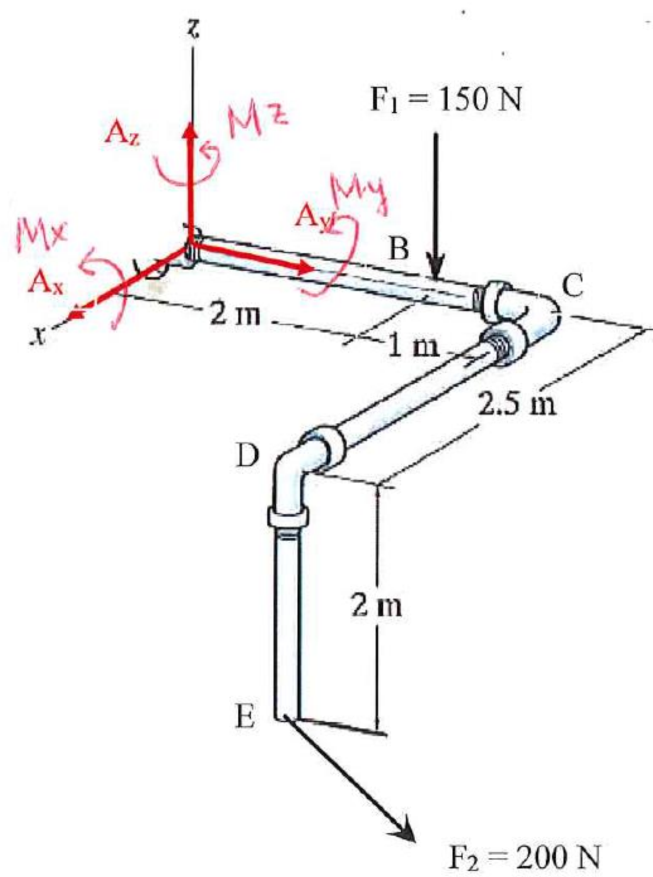
Points: /11

Q2.
Determine the support reactions at the fixed support at point A. The force F_1 of a magnitude of 150 N acting at point B is parallel to the z-axis. The 200 N force, F_2, acting at point E is in the direction of the unit vector of 0.6 **i** + 0.6 **j** - 0.53 **k**.



Solution:

Free body diagram



- Calculation of resultant force

$$\vec{F_1} = -150\,\vec{k} \quad N$$

$$\vec{F_2} = F_2\,\vec{n} = 200\,(0.6\,\vec{i} + 0.6\,\vec{j} - 0.53\,\vec{k})$$

$$= 120\,\vec{i} + 120\,\vec{j} - 106\,\vec{k} \quad N$$

$$\vec{F_A} = A_x\,\vec{i} + A_y\,\vec{j} + A_z\,\vec{k} \quad N$$

$\Sigma F_x = 0:$ $\quad 120 + A_x = 0 \Rightarrow A_x = -120\ N$

$\Sigma F_y = 0:$ $\quad 120 + A_y = 0 \Rightarrow A_y = -120\ N$

$\Sigma F_z = 0:$ $\quad -150 - 106 + A_z = 0 \Rightarrow A_z = 256\ N$

- calculation of resultant moment (about point A)

moment of F_1 about point A

$$\vec{M_1} = -(150)(2)\,\vec{i} = -300\,\vec{i} \quad N\text{-}m$$

moment of F_2 about point A

$A(0,0,0) \qquad E(2.5, 3, -2)$

$$\vec{r}_{AE} = (2.5-0)\,\vec{i} + (3-0)\,\vec{j} + (-2-0)\,\vec{k}$$

$$= 2.5\,\vec{i} + 3\,\vec{j} - 2\,\vec{k} \quad m$$

$$\vec{M_2} = \vec{r}_{AE} \times \vec{F_2} = (2.5\,\vec{i} + 3\,\vec{j} - 2\,\vec{k}) \times (120\,\vec{i} + 120\,\vec{j} - 106\,\vec{k})$$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2.5 & 3 & -2 \\ 120 & 120 & -106 \end{vmatrix}$$

$$= \begin{vmatrix} 3 & -2 \\ 120 & -106 \end{vmatrix} \vec{i} - \begin{vmatrix} 2.5 & -2 \\ 120 & -106 \end{vmatrix} \vec{j} + \begin{vmatrix} 2.5 & 3 \\ 120 & 120 \end{vmatrix} \vec{k}$$

$$= \Big((3)(-106) - (-2)(120)\Big)\vec{i} - \Big((2.5)(-106) - (-2)(120)\Big)\vec{j}$$

$$+ \Big((2.5)(120) - (3)(120)\Big)\vec{k}$$

$$= -78\vec{i} + 25\vec{j} - 60\vec{k} \quad N\text{-}m$$

moment at fixed support

$$\vec{M}_A = M_{Ax}\vec{i} + M_{Ay}\vec{j} + M_{Az}\vec{k}$$

$\Sigma M_x = 0: \ -300 - 78 + M_{Ax} = 0 \Rightarrow M_{Ax} = 378 \ N\text{-}m$

$\Sigma M_y = 0: \quad 25 + M_{Ay} = 0 \Rightarrow M_{Ay} = -25 \ N\text{-}m$

$\Sigma M_z = 0: \quad -60 + M_{Az} = 0 \Rightarrow M_{Az} = 60 \ N\text{-}m$

The reaction force at A

$$\vec{F}_A = A_x\vec{i} + A_y\vec{j} + A_z\vec{k} = -120\vec{i} - 120\vec{j} + 256\vec{k} \ N$$

$$F_A = \sqrt{(-120)^2 + (-120)^2 + 256^2} = 307.14 \ N$$

The reaction couple at A

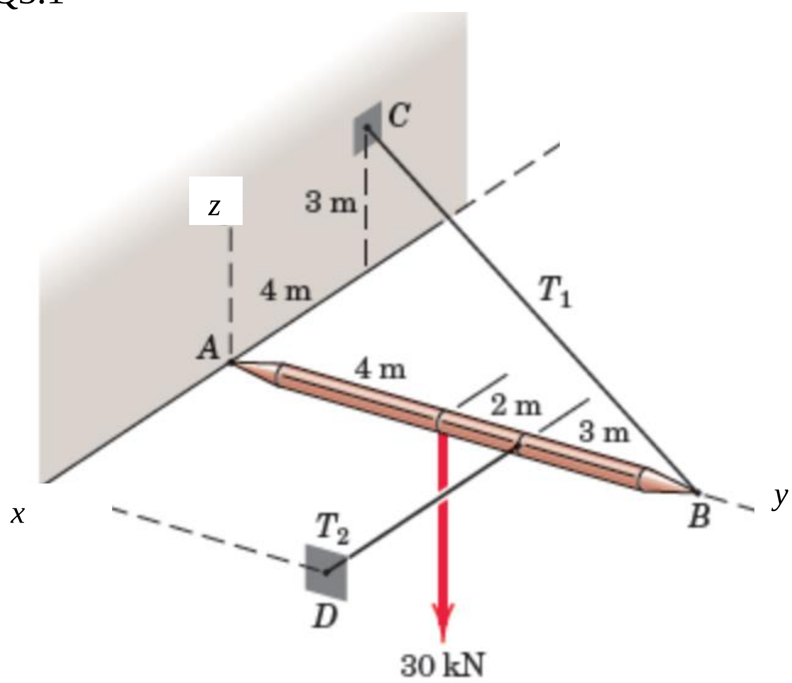
$$\vec{M}_A = M_{Ax}\vec{i} + M_{Ay}\vec{j} + M_{Az}\vec{k} = 378\vec{i} - 25\vec{j} + 60\vec{k} \ N\text{-}m$$

$$M_A = \sqrt{378^2 + (-25)^2 + 60^2} = 383.55 \ N\text{-}m$$

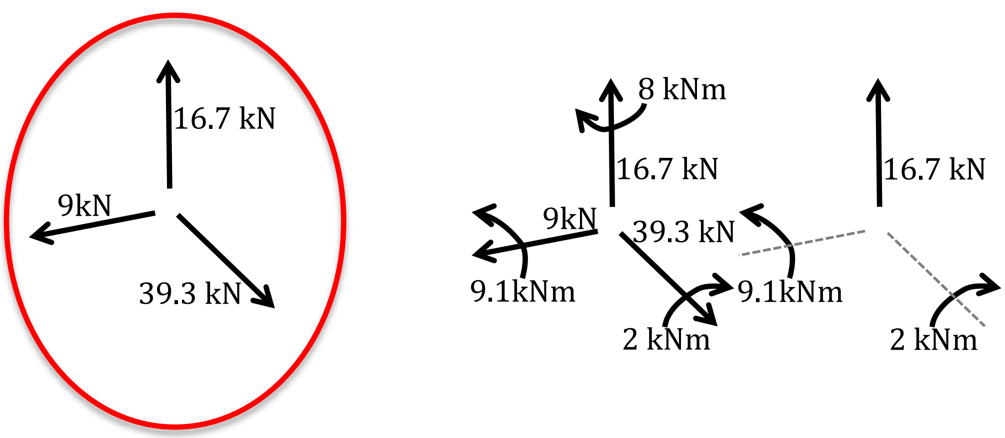
Points: /11

Q3.

Q3.1



For the ball-and-socket support shown at point A in the picture above what are the correct support reactions? (Circle the correct one)

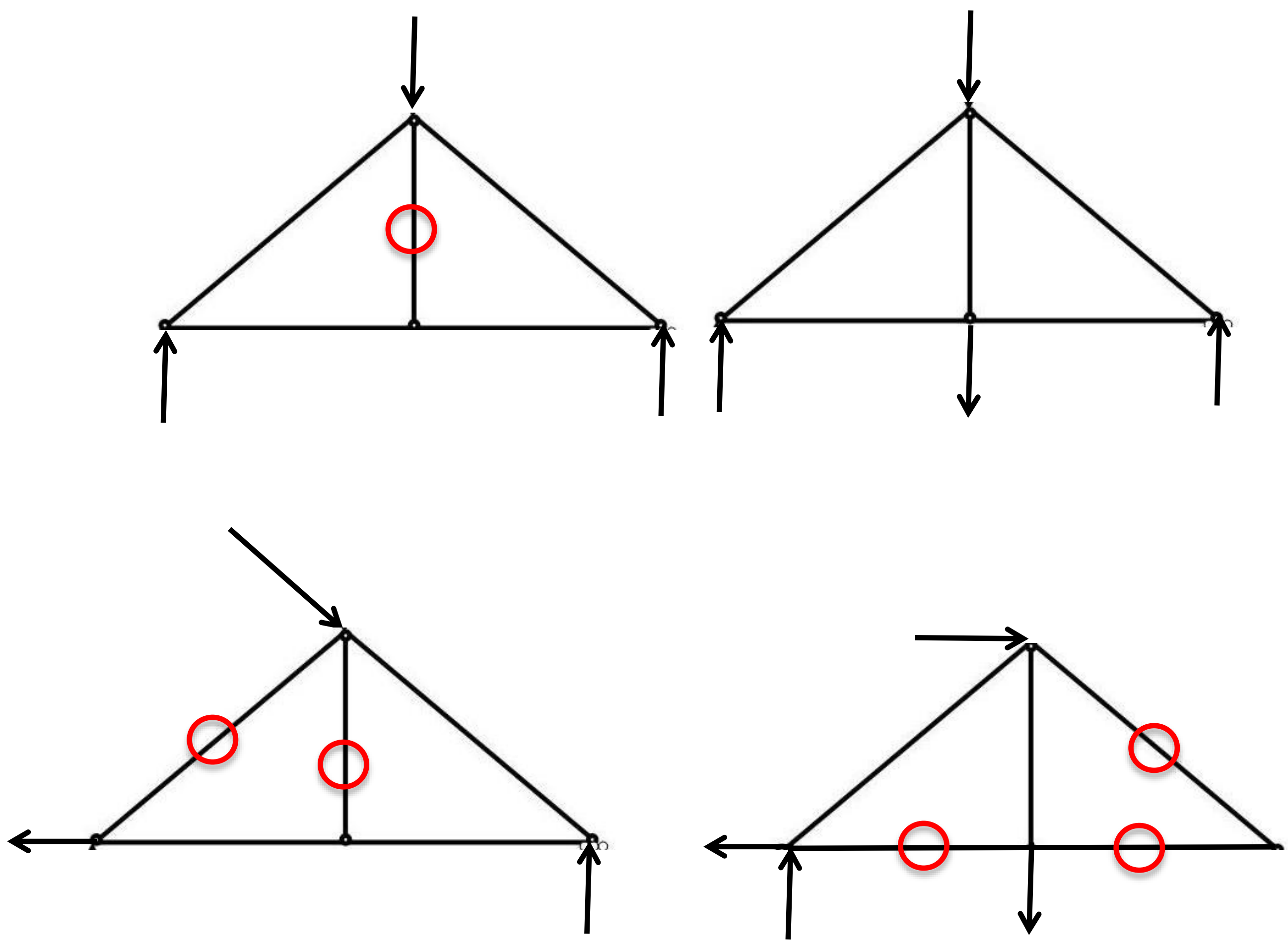


Q3.2

Which statement is true (choose only one):

a) A two-force member is a member loaded by a force and a couple
b) A two-force member is subjected to forces and couples at two points
c) A two-force member is a member with no internal force
d) None of the above

Q3.3
Identify the zero-force members in the following trusses (circle all zero-force members):



Q3.4
Which statement is true (choose only one):

a) A roller in 3D provides two support reactions
b) A roller in 3D provides one support reaction
c) A roller is equivalent to a ball-and-socket joint
d) None of the above

Note: In Q3.4, both a) and b) are correct.

Points: /8