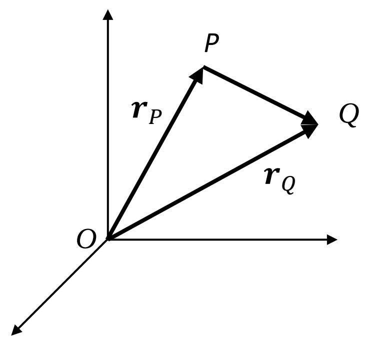
1. Given a point P in space and a (Cartesian) coordinate system x, y, z, we define the <u>position vector</u> $\boldsymbol{r}_P$ of P as the vector issuing from the origin O with its tip at P. It is given by

$$\boldsymbol{r}_P = x_P\boldsymbol{i} + y_P\boldsymbol{j} + z_P\boldsymbol{k}$$



Therefore, given two points, P and Q, the oriented segment PQ is measured by the vector difference

$$\overrightarrow{PQ} = \boldsymbol{r}_Q - \boldsymbol{r}_P$$

In other words,

$$\overrightarrow{PQ} = (x_Q - x_P)\boldsymbol{i} + (y_Q - y_P)\boldsymbol{j} + (z_Q - z_P)\boldsymbol{k}$$

In this particular question we are given two points, A and B, in the plane x, y, and a force $\boldsymbol{F}$ of magnitude F= 1250 lb acting along the line AB, as shown. We are asked to find the unit vector $\boldsymbol{n}_{AB}$ along this line and use it to determine the Cartesian components of the force. Clearly, the force (or any vector, for that matter) can be expressed as

$$\boldsymbol{F} = F\,\boldsymbol{n}_{AB}$$

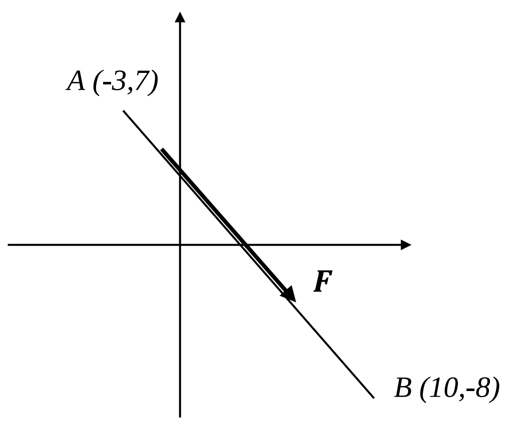
To obtain the unit vector, we divide the segment by its length, namely,

$$\boldsymbol{n}_{AB} = \frac{\overrightarrow{AB}}{AB} = \frac{\big(10-(-3)\big)\boldsymbol{i} + (-8-7)\boldsymbol{j}}{AB} = \frac{13\boldsymbol{i} - 15\boldsymbol{j}}{\sqrt{13^2 + 15^2}} = 0.655\boldsymbol{i} + 0.756\boldsymbol{j}$$

Notice that the components of the unit vector are precisely the director cosines of the direction under consideration. The sum of their squares is, of course, equal to 1. We now write

$$\boldsymbol{F} = F\,\boldsymbol{n}_{AB} = (1250\ lb)(0.655\boldsymbol{i} + 0.756\boldsymbol{j}) = (818.75\boldsymbol{i} - 945.00\boldsymbol{j})lb$$

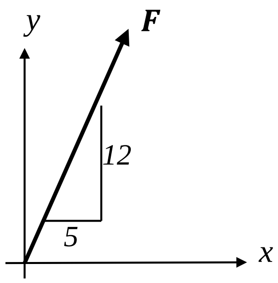
The answer is $F_x = 818.75\ lb,\ \ F_y = 945.00\ lb$



2. In this question (ignoring the artistic drawing) we are given a force $\boldsymbol{F}$ acting along a line with a slope specified by its rise and its run. We know that the y component of this force is $F_y = 70\ lb$. We are asked to find the magnitude F of the force and its x component.

The angle α between the line of action of the force and the x axis is obtained as $\alpha = \tan^{-1}\left(\frac{12}{5}\right) = 67.38°$. Since $F_y = Fsin\ \alpha$, we obtain

$$F = \frac{F_y}{\sin\alpha} = \frac{70\ lb}{\sin 67.38°} = 75.83\ lb$$



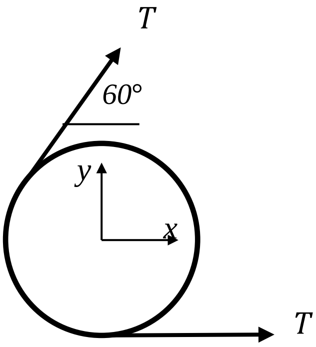
Alternatively, if we realize that the hypotenuse of the triangle is of length 13, we could have obtained the sine of the angle directly as 12/13. Finally, we obtain

$$F_x = F\cos\alpha = (75.83\ lb)\cos 67.38° = 29.17\ lb$$

Alternatively (to avoid propagation of a possible error in the first result) we could have stated

$$F_x = \frac{F_y}{\tan\alpha} = \frac{70\ lb}{12/5} = 29.17\ lb$$

3.



The two forces shown are equal in magnitude (400 N), but have different lines of action (in a physical context, they represent the tensions at both ends of a rope wound around a pulley). We are asked to find the resultant (vector sum) **R** of these two forces.

We can write

$$\boldsymbol{R} = (400\ N)\boldsymbol{i} + \big((400N)\cos 60°\ \boldsymbol{i} + (400N)\sin 60°\ \boldsymbol{j}\big) = (600\boldsymbol{i} + 346.41\boldsymbol{j})\ N$$

The magnitude R of $\boldsymbol{R}$ is

$$R = \sqrt{600^2 + 346.41^2}\ N = 692.82\ N$$

The angle α of this force with the x axis is obtained as

$$\alpha = \tan^{-1}\left(\frac{346.41}{600}\right) = 30°$$

Remark: Notice that in this case the graphical solution happens to be much faster. Since the two forces are equal in magnitude and form an angle of 60 degrees, they are nothing but the sides of an equilateral triangle. Completing the parallelogram we get a rhombus. The diagonal, therefore, exactly bisects the angle, hence the 30 degrees.