ENGG 202
Jan 30 Week 4

Problems

2/5 SYSTEM ISOLATION AND THE FREE-BODY DIAGRAM

Equilibrium is the condition in which the resultant of all forces and moments acting on a body is zero.

Necessary and sufficient conditions for complete equilibrium in two dimensions:

$$\boxed{\mathbf{R} = \Sigma\mathbf{F} = \mathbf{0} \qquad \mathbf{M} = \Sigma\mathbf{M} = \mathbf{0}} \qquad \mathbf{(3/1)}$$

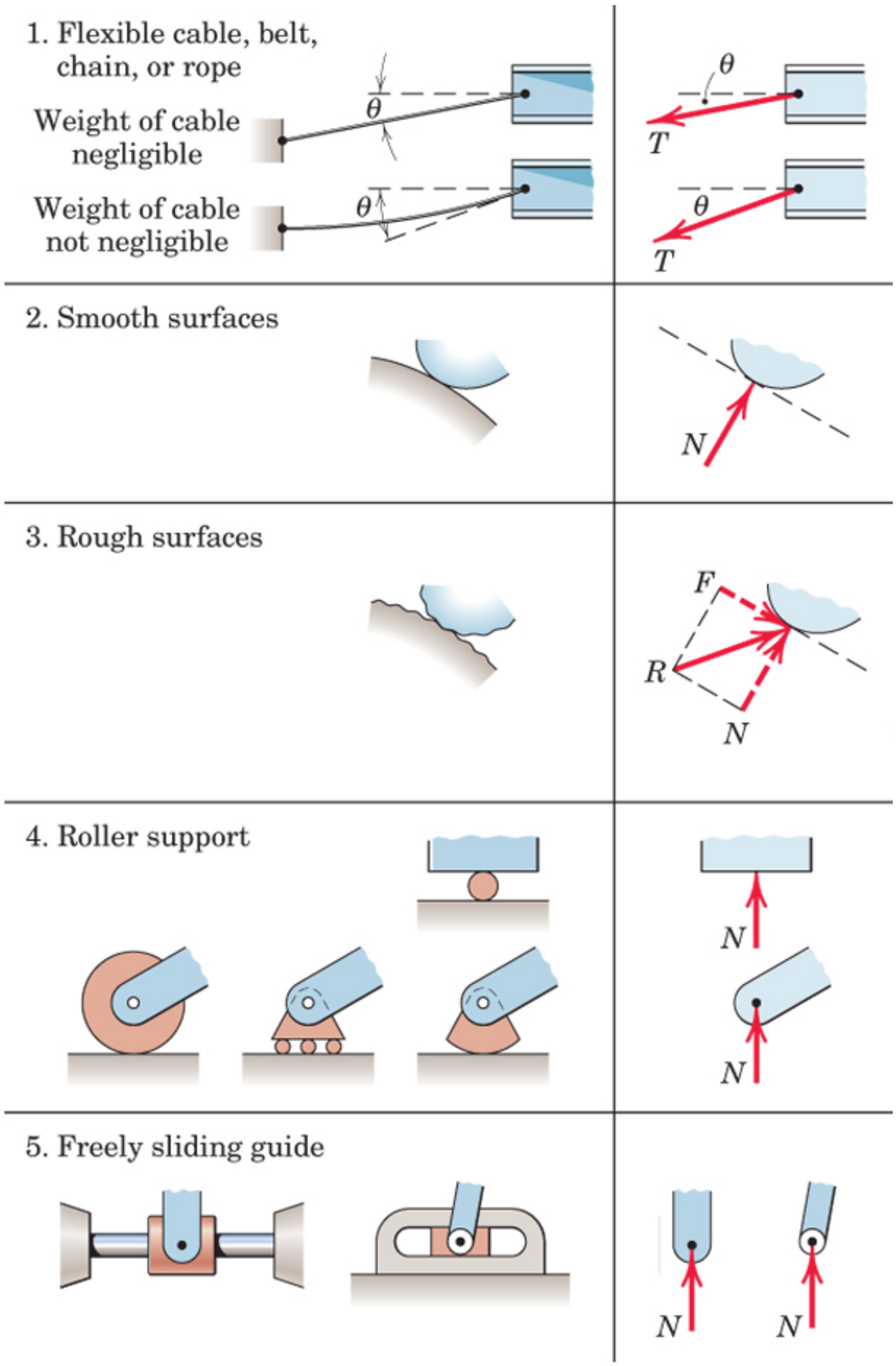
Before we apply Eqs. 3/1, we must define unambiguously the particular body or mechanical system to be analyzed and represent clearly and completely all forces acting on the body.

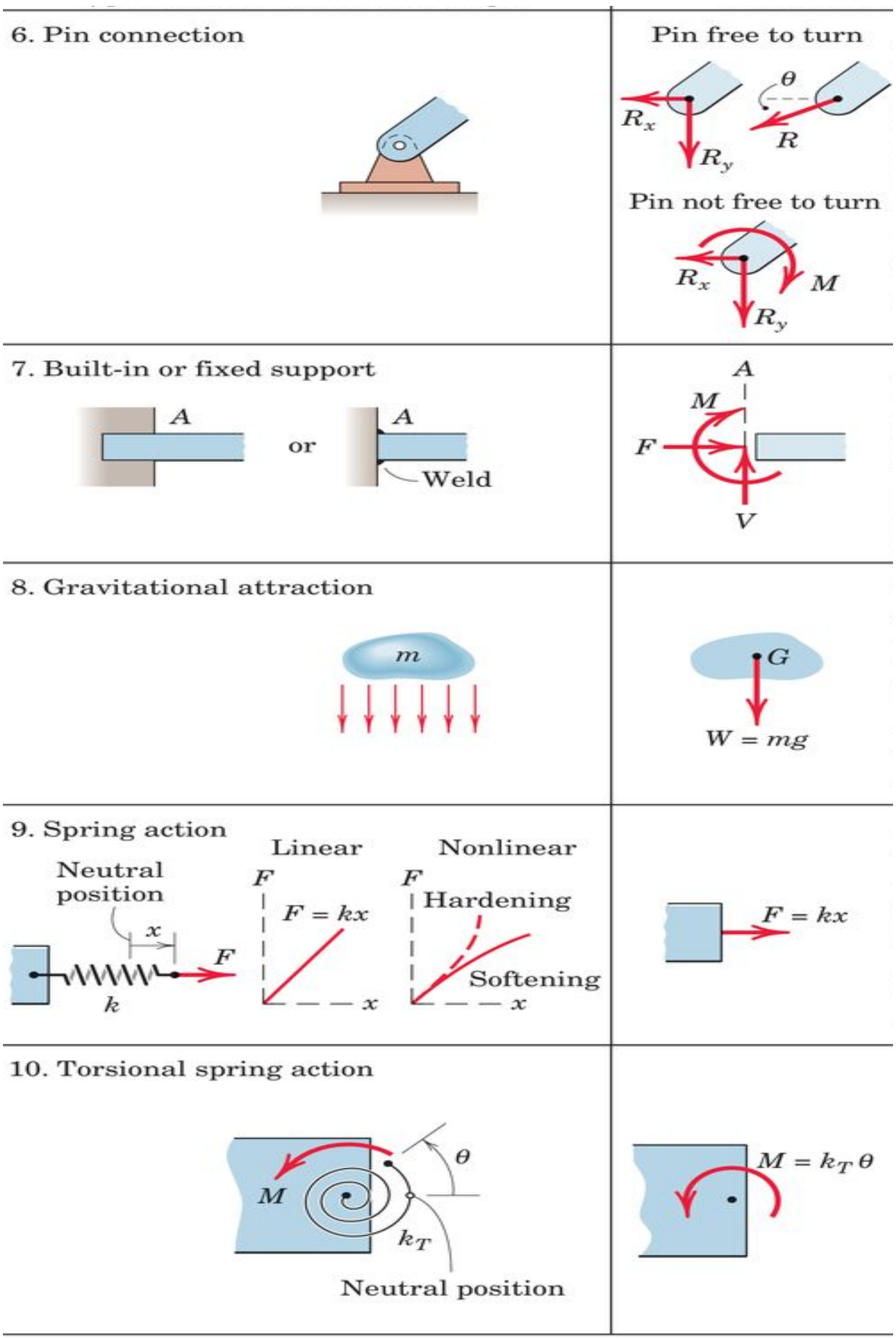
FREE-BODY DIAGRAM

MUST INCLUDE:
a) all external forces
b) magnitudes and directions of all known external forces
c) relevant dimensions
d) unknown external forces (usually the “support reactions”)

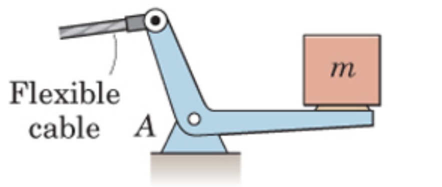
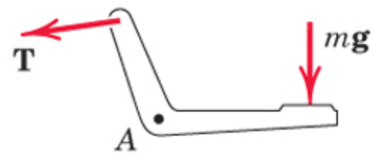
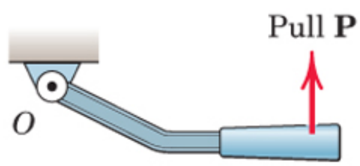
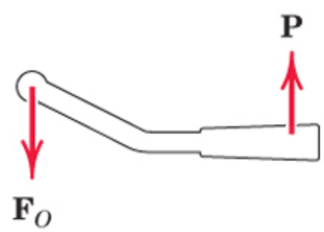
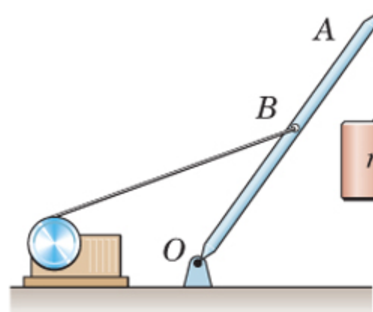
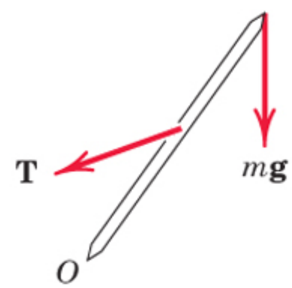
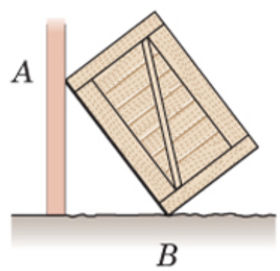
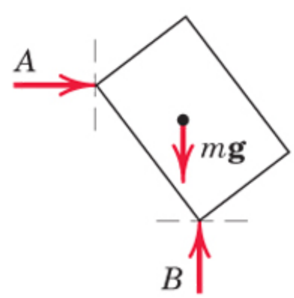
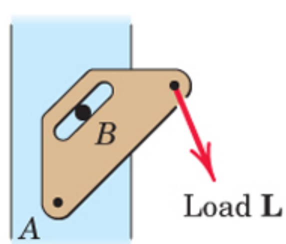
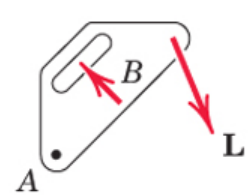
Modeling the action of forces in 2-D analysis

The FIGURE below shows the common types of force application on mechanical systems for analysis in two dimensions.. The force exerted on the body in question by a contacting or supporting member is <u>always in the sense to oppose the movement of the isolated body, which would occur if the contacting or supporting body were removed.</u>

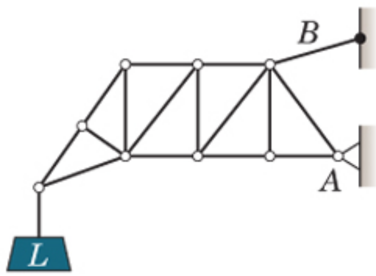



6. Pin connection
Pin free to turn
R_x
R_y
θ
R
Pin not free to turn
R_x
R_y
M
7. Built-in or fixed support
A
or
A
Weld
A
M
F
V
8. Gravitational attraction
m
G
$W = mg$
9. Spring action
Neutral position
x
F
k
Linear
F
$F = kx$
x
Nonlinear
F
Hardening
Softening
x
$F = kx$
10. Torsional spring action
M
θ
k_T
Neutral position
$M = k_T\,\theta$

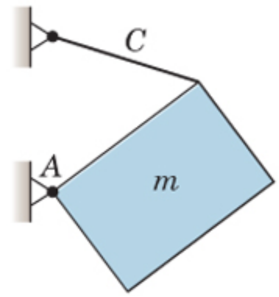
EXAMPLES FREE-BODY DIAGRAM

	Body	Incomplete FBD
1. Bell crank supporting mass m with pin support at A.		
2. Control lever applying torque to shaft at O.		
3. Boom OA, of negligible mass compared with mass m. Boom hinged at O and supported by hoisting cable at B.		
4. Uniform crate of mass m leaning against smooth vertical wall and supported on a rough horizontal surface.		
5. Loaded bracket supported by pin connection at A and fixed pin in smooth slot at B.		

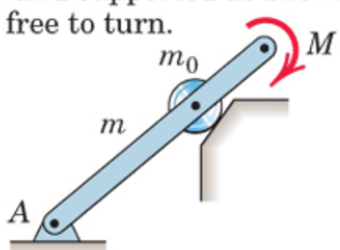
3. Loaded truss supported by pin joint at A and by cable at B.



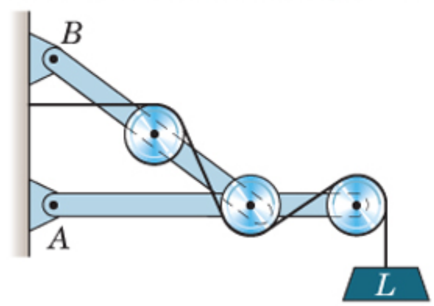
7. Uniform heavy plate of mass m supported in vertical plane by cable C and hinge A.



4. Uniform bar of mass m and roller of mass m_0 taken together. Subjected to couple M and supported as shown. Roller is free to turn.

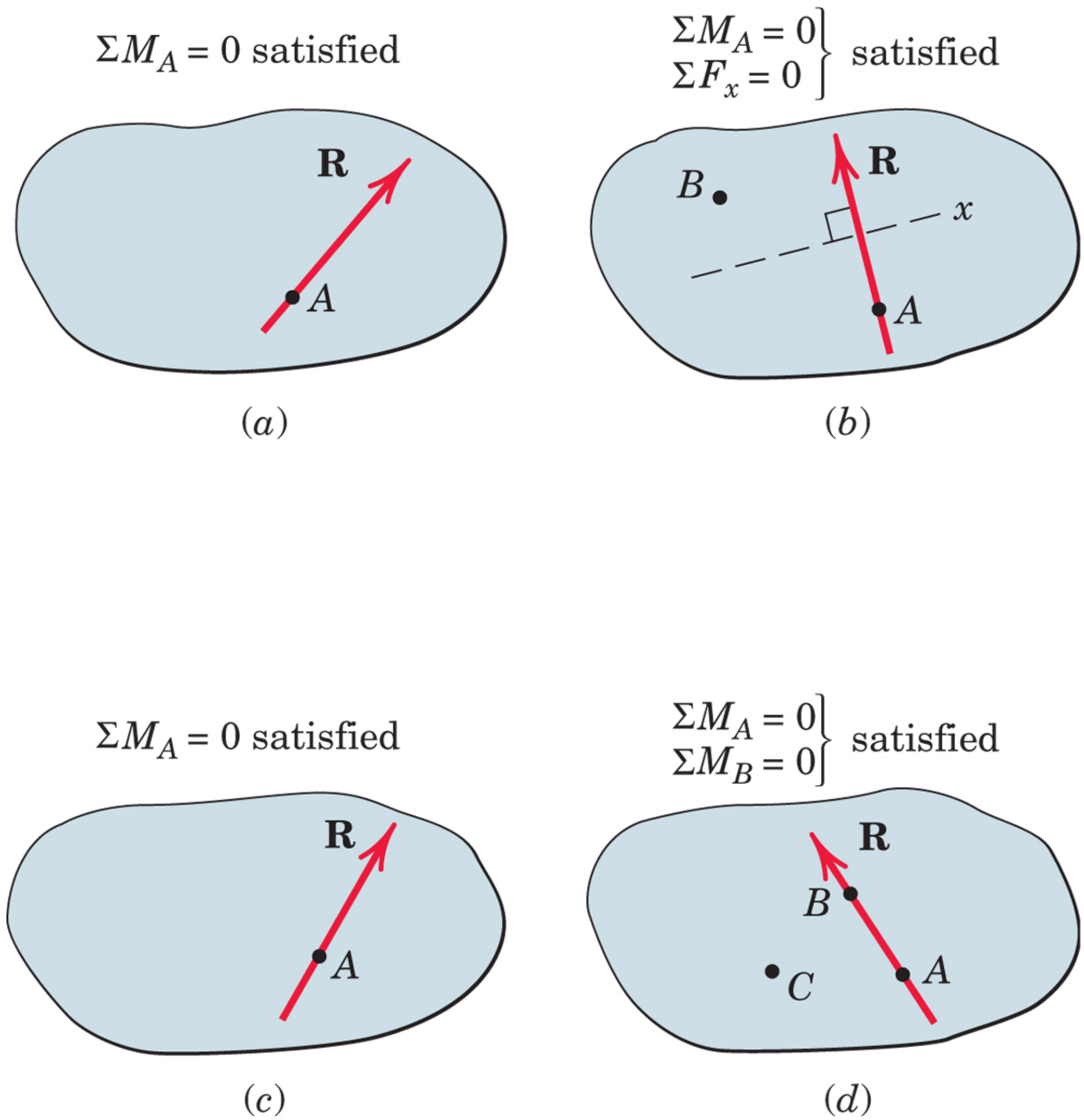


8. Entire frame, pulleys, and contacting cable to be isolated as a single unit.

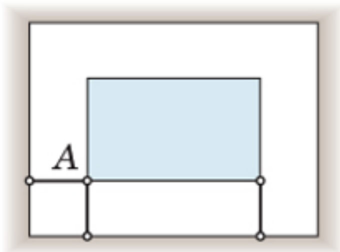


Equilibrium Conditions in 2-D

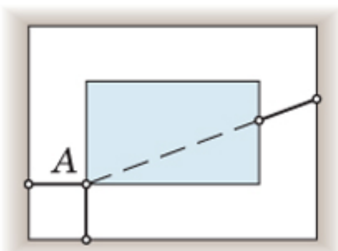
$$\boxed{\Sigma F_x = 0 \qquad \Sigma F_y = 0 \qquad \Sigma M_O = 0} \tag{3/2}$$



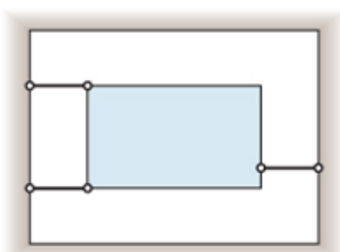
Constraints and Statical Determinacy



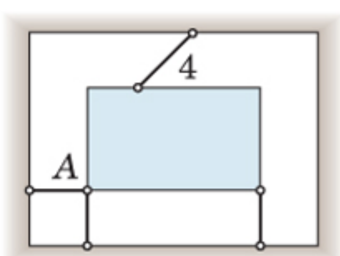
(*a*) Complete fixity
Adequate constraints



(*b*) Incomplete fixity
Partial constraints



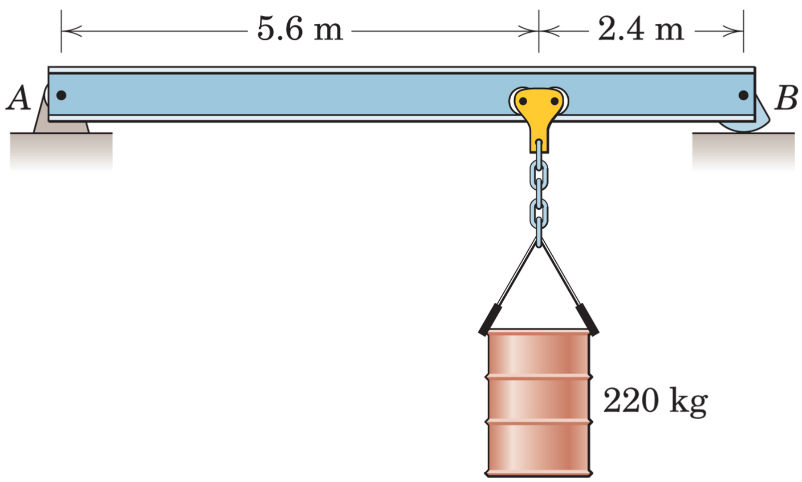
(*c*) Incomplete fixity
Partial constraints

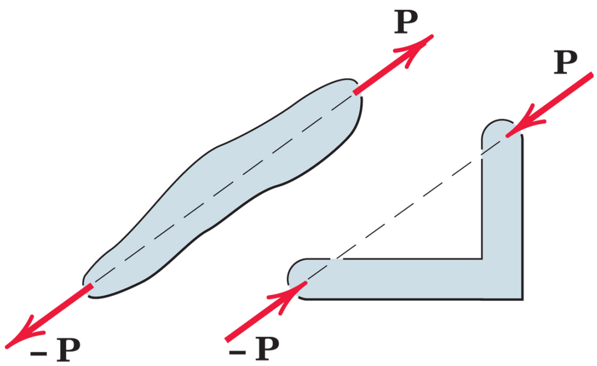


(*d*) Excessive fixity
Redundant constraint

Problem 3/4

The 450-kg uniform I-beam supports the load shown. Determine the reactions at the supports.

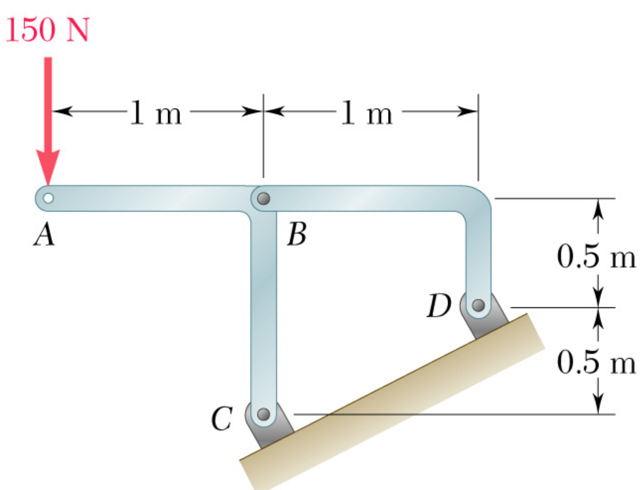


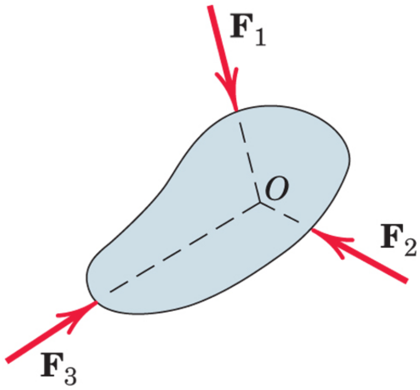


Two-force members

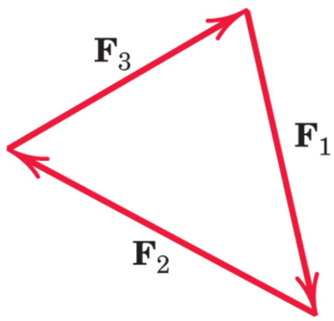
Example Problem

Determine the suppport reactions at C and D





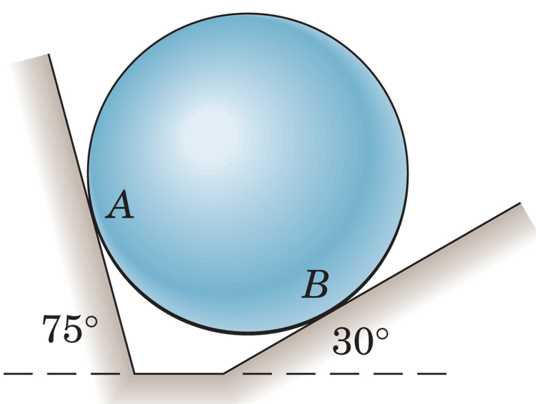
(a) Three-force member



(b) Closed polygon satisfies $\Sigma \mathbf{F} = \mathbf{0}$

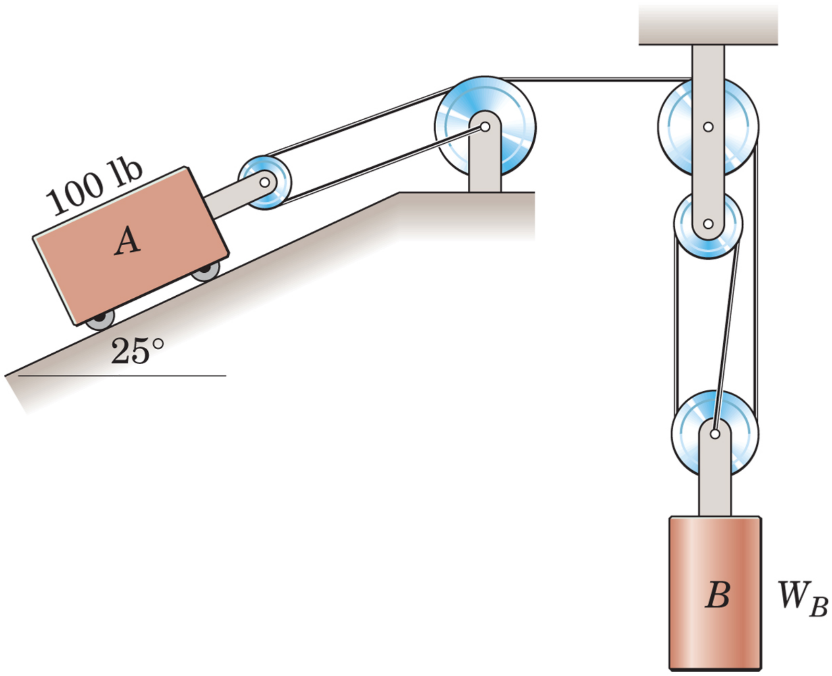
Problem 3/6

The 20-kg homogeneous smooth sphere rests on the two inclines as shown. Determine the contact forces at A and B.



Problem 3/15

What weight W_B will cause the system to be in equilibrium? Neglect all friction, and state any other assumptions.



IMPORTANT NOTE:
In the absence of friction at the axle and assuming no sliding between the rope and the wheel, the moment equilibrium at centre of the wheel requires that the two tensions in the rope be equal.
The tension is equal all along the string's length

Problem 3/30

The right-angle uniform slender bar AOB has mass m. If friction at the pivot O is neglected, determine the magnitude of the normal force at A and the magnitude of the pin reaction at O.