1. <u>Equilibrium</u>

We have arrived at the central concept of this course on Statics, namely, the notion of *equilibrium*. Since forces and couples are roughly understood to be the causes for the change of motion, and since we have already determined that every system of forces and/or couples is ultimately reducible to a single force-couple system (producing equivalent effects), it is not unreasonable to establish as a matter of definition that *a system of forces is in equilibrium whenever the equivalent force-couple system vanishes*. In other words, equilibrium implies that *the vector sum of all the forces of the system as well as the vector sum of the moments of all forces (and couples) with respect to any point vanish*. For a two-dimensional system (in the x, y plane, say) the equilibrium condition boils down to the three scalar equations

$$\Sigma F_x = 0 \qquad \Sigma F_y = 0 \qquad \Sigma M_A = 0$$

where *A* is an arbitrary point in the plane.

As definitions go, this is clear enough. There are, however, a few caveats that we would like to elucidate at this point.

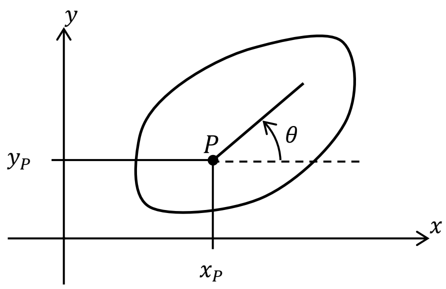
(i) As engineers we are not just interested in an axiomatic formulation, but we want to know how the concept of equilibrium can be related to Physics in general and, more particularly, to Dynamics. For this reason we must recall that Newton's first and second laws apply only for the so-called *inertial observers*. It is only such observers who will verify that a material particle free of all external forces moves necessarily in a straight line at a uniform speed. Moreover, it is only an inertial observer who will verify that an applied force results in the development of an acceleration vector proportional to the force. So, if we would like to regard equilibrium not just as the absence of forces and couples but also as something to do with the absence of motion, these factors should be taken into consideration. In an age of space exploration these issues have acquired a more poignant meaning than before.

(ii) Let us, therefore, assume that we have ascertained that we are, exactly or approximately, inertial observers and let us assume that we are looking at a rigid object of finite extent and referring our measurements to a Cartesian frame with coordinates x, y, z. If there are no external forces or couples acting on this object, is it necessarily at rest with respect to our frame? According to Newton's first law, the absence of a force resultant implies that the object can at most be moving in a straight line at a constant speed. We could, therefore, claim that all we have to do to take care of this problem is to move along with the object. In doing this, we still have an inertial observer and the object appears to be at rest with respect to our frame. But is it, really? Not quite. All we can ascertain is that the *centre of mass* of the object is at rest. But what about a rotational motion? We may think that the vanishing of the couple ensures that there is no rotation. Unfortunately, this is not so. A rigid body acted upon by no forces or couples will in general undergo a rotation with constant angular velocity about a particular axis passing through its centre of mass! If we decide to rotate with the object, we become no longer inertial observers.

(iii) From the engineering point of view, therefore, particularly when working on earth-bound structures, such as buildings, we prefer to regard equilibrium as the absence of motion (over a given time period of interest). We are then naturally led to the concept of *structural supports*. These are devices that suppress all possible rigid body motions. In doing so, the supports apply forces on the structure that ensure that the equations of equilibrium alluded to above (namely the vanishing of the sum of forces and of the sum of moments) are satisfied. To make these reactions manifest and to calculate their values, we detach and isolate the object under consideration from its supports. This is, in a nutshell, the art of the structural engineer.

2. Counting and suppressing degrees of freedom

How many degrees of freedom does a rigid object have? At the outset, let us recall that an object is said to be *rigid* if the distance between every pair of points in the object remains constant. As a consequence, rigid objects cannot experience any change in shape or in size. This is obviously an idealization. Real objects do undergo such changes, but when the material is very stiff these changes can be neglected in a first analysis, such as in this course. If you consider a rubber ball and subject it to a compression with your fingers, the deformation may become visible. Similarly, if we inflate a balloon, large changes in shape and size are noticeable. As a consequence, the forces may change in direction and their points of application evolve, so that the calculation of sum of forces and sum of moments depends on the very process of deformation. These difficult topics will be discussed in other courses. For now, therefore, we concentrate our attention on objects that can be considered to be ideally rigid.

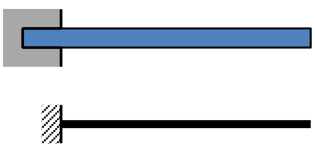


To simplify matters, let us work in a two-dimensional context. We look, therefore, at a rigid object, such as a disk or a bar, confined to live in the plane x, y. The lines of action of all forces of interest belong to this plane and, moreover, the couples have a line of action perpendicular to this plane (namely, the curved arrows representing these couples can be drawn in this plane). We ask: how many *degrees of freedom* does such an object have? Put differently, how many parameters do we have to specify to precisely locate this two-dimensional body within the plane? To answer this question, we can choose any point P within the object and specify its two coordinates (x_P, y_P). In order to pin down the other points, all we need to add is the angle θ that some material line of the body passing through P forms with the x-axis, as shown in the figure. We conclude that the number of parameters (degrees of freedom) of a rigid body in the plane is 3.

If we want to fix this object in the plane, we need to suppress exactly 3 independent degrees of freedom. Any less, would leave the body with some ability to move. A *support* is a device that suppresses a degree of freedom.

3. Common supports for plane structures

The most severe support that can be placed at a point of a structure is the *fixed support* or *built-in support*, also called a *clamp*. When applied at a point of a plane structure it prevents all displacements (say, in the x, y directions) and all rotations. A common occurrence of fixed supports in everyday life can be found in city lampposts. A *cantilever beam* is a bar which has been built-in at one of its ends. All three degrees of freedom have been suppressed and the bar will be in equilibrium under the action of arbitrary



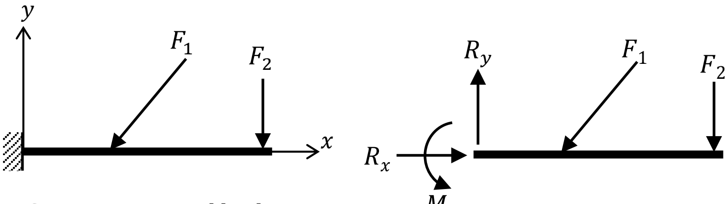
Cantilever beams

forces and couples (unless the support breaks). Common ways to indicate a built-in end are shown in the figure.

How does a fixed support prevent displacements and rotations? The answer is: by applying on the structure a force-couple system. To make these *reactions* manifest, we detach the structure from the rest of the universe and obtain an isolated system whose schematic drawing is known as a *free-body diagram*, or FBD for short. Let us show an example.

<u>Example</u>: A cantilever beam is subjected to two external forces, as shown. Draw the FBD of the beam, clearly indicating the support reactions. <u>Solution</u>: We reason as follows: Since the fixed support prevents horizontal and vertical displacements, it must do so by applying, respectively, a horizontal and a vertical force (any other two directions would also do). We indicate these forces as R_x and R_y and regard them as components of the reaction force $\boldsymbol{R}$ applied by the eliminated support on the structure. By Newton's third law, we know that the structure applies on the support an opposite force, but this is not our concern at this point. The fixed end, by its nature, not only prevents the appearance of displacements but also precludes the rotation. To do so, it applies on the structure a couple M, which we will indicate with a curved arrow. The FBD is shown on the right of the figure.

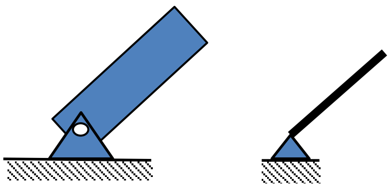
<u>Note</u>: The three reactions R_x, R_y, M are *external forces* acting on the free-body diagram. Conditions of equilibrium will be eventually imposed on the collection of all external forces, both those applied by the loading and those arising from the reactions of the supports. It would be *a grave conceptual error* to draw the reactions on the original structure. The reactions only appear when the supports have been removed (as in the FBD).



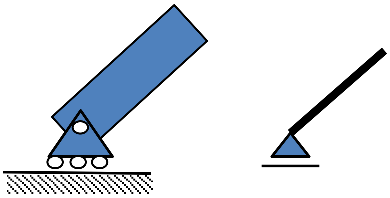
Given structure and loading

Free-body diagram

The second kind of support is known as a *hinge*, or a *pin connection*. This support, fixed to the ground, is connected to the structure by means of a frictionless pin about which rotations are permitted. Thus, the pinned support only prevents the development of displacements in two directions, but does not impose any restriction on rotations. It is usually indicated by a triangular shape, as shown. To prevent the two components of displacement, the hinge applies to the structure the corresponding components of reactive force. But, since it does not prevent the rotation, it cannot apply a reactive moment on the structure. This is a *duality principle* that goes very deep into the nature of the physical world.

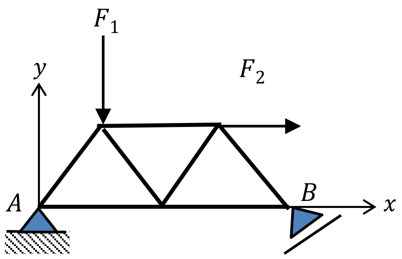

Pin supports

A hinge, in and of itself, is not enough to prevent a rigid object from moving. Indeed, we concluded that the number of independent degrees of freedom of a rigid object in the plane is 3. We must, therefore, restrict one more degree of freedom. This can be achieved, for example, by means of the third kind of support known as a *roller*. It is similar to a hinge but, instead of being completely fixed to the ground, it is mounted on rollers that permit the support to displace itself in the direction parallel to the ground surface (usually, but not necessarily, horizontal). It can only apply a reaction force perpendicular to the plane of rolling.

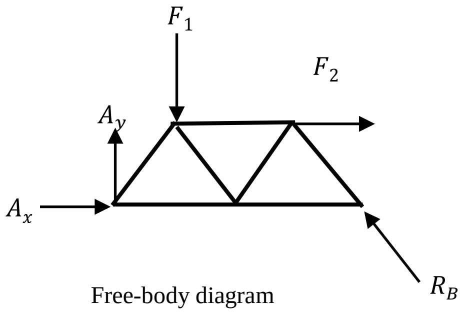


Rollers

Example: A bridge is supported at one end A by a pin support and at the other end B by a roller. Draw the corresponding FBD. Solution: Detaching the bridge from its supports, we apply the corresponding reaction forces, as shown.



Given structure and loading



Free-body diagram

4. Degrees of freedom and statical determinacy

We have established that for a structure to be in equilibrium (not to move) under the action of arbitrary loadings its degrees of freedom must be suppressed by means of supports. We have also learned that, since a rigid object in the plane has exactly three degrees of freedom, we must apply supports that prevent three (independent) motions. In so doing, the number of support reaction components is exactly 3. But we ask ourselves: What happens if we apply even more supports? Certainly, this seems to be beneficial to the overall stability of the structure. Since we have established that to have equilibrium we need to satisfy the vanishing of the sum of forces and of the sum of moments with respect to some point, and since in the plane this amounts to exactly 3 equations, we arrive at the following three case for the equilibrium f a rigid object:

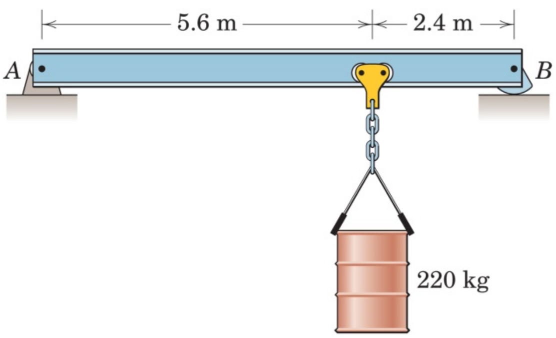
(a) The number of support conditions is less than 3 (for example, if our bridge were to be supported only at the left end by means of pin). In this case, the structure has a remaining ability to move (in the bridge example, it could still rotate about point A). We have 3 equations of equilibrium, but only 2 unknown quantities (in the bridge example, A_x and A_y). The equations cannot in general be satisfied and equilibrium cannot be established (except, perhaps, for special loadings. (Can you guess which, in the bridge example?) Such systems are called *statically under-determined* or *hypo-static*. They are effectively *mechanisms*.

(b) The number of supports conditions is exactly 3. In this case, the structure has no ability to move. We have exactly 3 independent equations of equilibrium and 3 unknown reaction components (if we consider the support of the bridge at *B* being present, then we have the three components A_x, A_y, R_B, or, in the case of the cantilever, we have R_x, R_y, M). Such systems are called *statically determinate* or *iso-static*.

(c) If we have extra support conditions, beyond the 3 needed to prevent the 3 degrees of freedom of a rigid object, we still have only 3 equations of Statics but more than 3 unknowns. Mathematically, the system of equations has either no solution at all (if there is a contradiction in the data) or, it has an infinite number of solutions satisfying the equations. We have then a case of a system that is *statically indeterminate* or *hyper-static*.

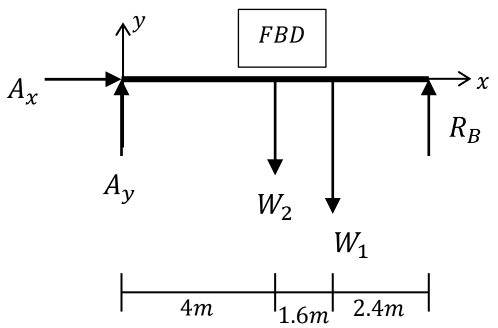
In this course *we will only consider statically determinate (iso-static) systems*. In other words, our systems will always be *minimally supported* to ensure their stability, but not beyond that.

5. Example



The 450 kg uniform beam supports the load shown. Determine the reactions at the supports.

Solution: The support at *A* is a pin. At *B* we have roller on a horizontal plane. Although we have represented the roller support in its most common fashion, there are other equivalent supports that have the same effect as a roller. What we have here is sometimes called a *rocker*. Clearly, it does not prevent small side displacements or rotations. It only prevents a displacement in the direction perpendicular to the plane of the rocker. We have to get used to these various engineering realizations of a support. Our first step towards the solution is to introduce a coordinate system and to draw a free-body diagram. Recall that the FBD (as indicated by the first initial) is free in the sense that it is detached from any connection to the rest of the world. It only interacts with it via forces and/or couples. A schematic FBD is shown below. The reactions are clearly indicated. We indicate the expected sense of the reaction components and let the solution of the equilibrium equations tell us whether we were right (if the corresponding answer is positive) or wrong (if it is negative, in which case we just reverse the assumed direction). The hanging mass is replaced by its weight W_1, namely, the action of the earth gravity on that part of the structure. Since the beam is described as 'uniform', the total weight W_2 of the beam results in a force at its mid-point. Notice that it is a good policy to repeat the relevant dimensions in the FBD.



We write the equilibrium equations as

$$\sum_{\rightarrow} F_x = A_x = 0 \qquad \sum_{\uparrow} F_y = A_y - W_2 - W_1 + R_B = 0 \qquad \sum_{CCW} M_A = -W_2(4m) - W_1(5.6m) + R_B(8m) = 0$$

We have chosen *A* as the point of reference for taking moments. Any other point would have been appropriate. It is usually not a bad idea to take moments with respect to a point that is common to as many unknown forces as possible, so as to de-couple the system of equations as much as possible. In this case, we can solve the third equation directly to obtain

$$R_B = \frac{(450kg)\left(9.81\frac{m}{s^2}\right)(4m) + (220kg)\left(9.81\frac{m}{s^2}\right)(5.6m)}{8m} = 3718N$$

The fact that the result is positive means that our assumption in the FBD was correct (the force points upwards). Similarly, from the second equation, we obtain

$$A_y = -3718N + (450kg)\left(9.81\frac{m}{s^2}\right) + (220kg)\left(9.81\frac{m}{s^2}\right) = 2855N$$

Again, the positive result means that the force is as assumed (to the right). Finally the first equation provides the trivial result $A_x = 0$.

6. Statics and Linear Algebra

There are important parallels between some of the results that you encountered in Linear Algebra and some of the physical facts that arise from considerations of equilibrium in Statics. Understanding these parallels enhances the comprehension of both disciplines. If you are not interested in this kind of things, just skip to the next section.

In Linear Algebra you learned that in a system of linear equations it is permitted, without in any way altering the solution of the system, to replace any equation by itself plus an arbitrary linear combination of the remaining equations of the system. For example, the solution of the system

$$\begin{cases} x + y = 1 \\ 2x + 3y = 4 \end{cases}$$

is the same as the solution of the system

$$\begin{cases} x + y = 1 \\ 4x + 5y = 6 \end{cases}$$

obtained by replacing the second equation by itself plus twice the first. What is the meaning of this kind of substitution in Statics? It is roughly the following:

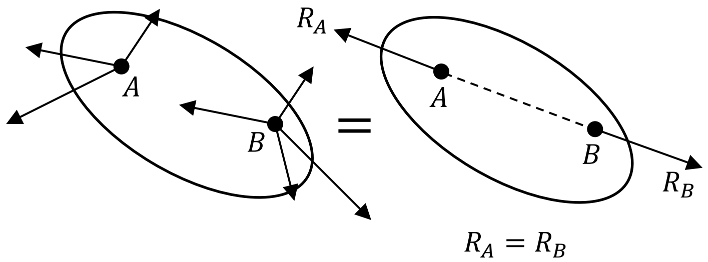
(i) If we combine two equations of sums of forces with respect to two directions, we obtain another equation proportional to the sum of forces in a new direction.

(ii) If we combine an equation of sum of moments with one (or both) of the equations of sum of forces, we obtain an equation of sum of moment with respect to a new point.

(iii) Conversely, any equation of sum of force components (in the direction of some line) or of sum of moments (about some point) can be obtained as a linear combination of a given set of independent equations of statics.

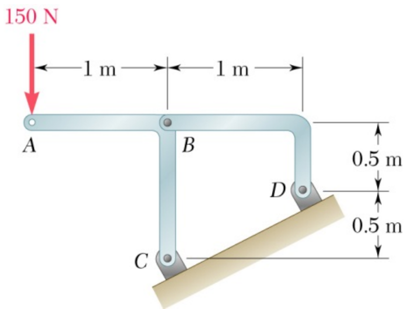
The physical meaning of this result is that not only can we choose our two axes of projection and our point of moments arbitrarily, but *we can replace one or both of the equations of projection with an* *equation of moments!* This conclusion is sometimes of great help in formulating the equations of equilibrium. There are some restrictions, however. If we use two equations of moment (about points *A* and *B*, say) and one equation of forces (in a direction *m*), then we must make sure that the direction *m* is not perpendicular to the line *AB*. Moreover, if we replace both equations of force by two equations of moment, so that we end up with equations of moment with respect to 3 points *A*,*B*,*C*, we must make sure that the three points are not collinear (that is, they form a triangle). The proof of these facts can be carried out either by physical reasoning (from Statics) or by invoking the theory of determinants (from Linear Algebra). <u>Exercise</u>: Prove the above propositions.

7. <u>Two-force members</u>

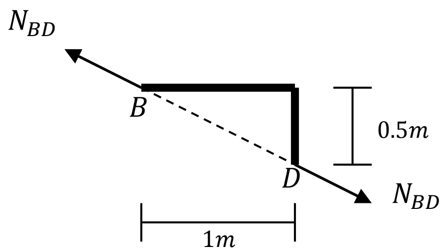


Consider a rigid object acted upon by forces (not couples) applied at 2 points only, such as *A*,*B* in the figure. We call this a *two-force member*. We now reason as follows: If this member is in equilibrium, then the sum of moments with respect to point *A* must vanish and, therefore, *the resultant of all the forces acting at point B must necessarily pass through point A*. Similarly, the resultant of all the forces acting at *A* must pass through point *B*. In other words, both resultants act along the line *AB*. But, taking sum of forces along this line, we conclude that these two resultants must be equal in magnitude and opposite in direction. The realization that a member of a structure is a two-force member can greatly help in the solution of a problem. It is important to emphasize a terminological inconsistency. The member itself, as a physical object, does not have any special property. What is important in determining whether or not an object acts as a two-force member is to ascertain that the applied forces necessarily concur at two points and that there are no couples.

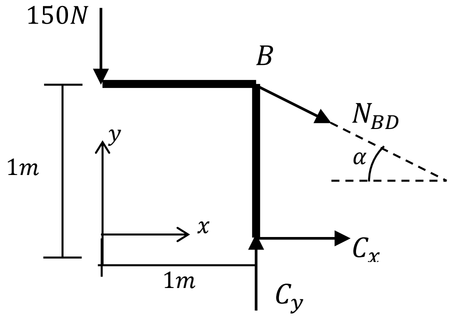
<u>Example</u>: Determine the support reactions at *C* and *D*.



<u>Solution</u>: Here we have a system of two rigid bodies (ABC and BD) interconnected by means of a frictionless hinge at *B*. This seems to go beyond what we have been doing so far, namely, considering a single rigid body. We observe, however, that the member BD is unloaded, except for the pin forces at *B* and *D*. This means that if we draw the FBD of this member the only two forces will be applied at these points. Since the pins are frictionless, they cannot transmit any couples at the point of attachment. The FBD of this bracket is shown in the figure. We have chosen one of the two possible senses. Notice that these two forces point in two different directions. We don't have yet any basis for calculating their magnitude, but one thing we do know and that is that, when we draw the FBD of the bracket ABC, the force at *B* must be reversed. Why? Because it is the *reaction* (which, according to Newton's third law) the body BD applies to the body ABC at point *B*.



Turning now to the FBD of ABC, shown in the figure, we obtain the basis for our equations of equilibrium. Notice that the force at B is known in direction, if not in magnitude. The direction is given by $\alpha = \tan^{-1} 0.5 = 26.565°$. We write the equations of equilibrium as



$$\Sigma F_x = C_x + N_{BD} \cos\alpha = 0$$

$$\Sigma F_y = -150N + C_y - N_{BD} \sin\alpha = 0$$

$$\Sigma M_B = (150N)(1m) + (C_x)(1m) = 0$$

Solving the system, we obtain

$$C_x = -150\ N\ \ (opposite\ to\ shown)$$

$$N_{BD} = \frac{-(-150N)}{\cos\alpha} = 167.7\ N\ (all\ the\ N_{BD}\ arrows\ are\ as\ shown)$$

$$C_y = 150N + (167.7N)(\sin\alpha) = 225.0\ N\ (as\ shown)$$

The magnitude of the reaction at C is $R_C = \sqrt{C_x^2 + C_y^2} = 270.4\ N$

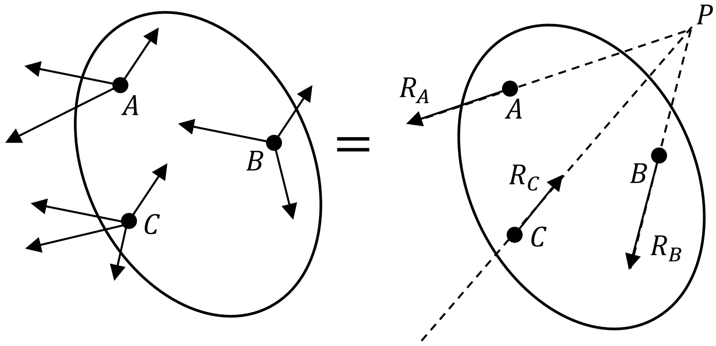
Digression No. 1: When we look at this problem we may ask: How is it possible to have 4 conditions of support (2 for each pin support) rather than 3? Didn't we say that to minimally support a rigid object we need to suppress just 3 dofs (degrees of freedom)? The answer is that we have here not just one but rather 2 rigid objects (namely, the bracket ABC and the bracket BD). If we *dismember* our system (as we have done, in fact, in our solution above) we obtain two rigid bodies with a total of 3+3=6 dofs. Since these two objects are interconnected by a pin at *B*, however, the number of dofs is reduced by 2 (this pin prevents the relative displacements between the two objects but does not impose any restriction on their relative rotation). Thus we are left with 6-2=4 dofs (for example, the 3 dofs of the left bracket plus an additional rotation of the second bracket about point *B*). To suppress the remaining 4 dofs we need 4 conditions of support, supplied in this case by the two pin supports at *C* and *D*.

8. Three-force members

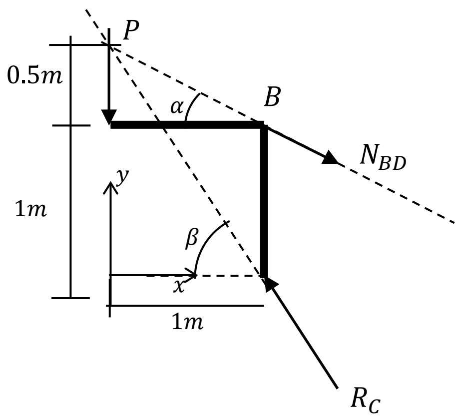
It should be clear that, useful as two-force members may be to simplify the analysis of certain structures, they are not, strictly speaking, a necessary part of the analysis. Considering, for instance, that our computers can perform billions of operations per second it seems that small improvements of the type provided by the recognition of a single two-force member are not worth our while. On the other hand, two-force members help us in understanding the overall functioning of a structure and predicting qualitatively some of the results. Three-force members are somewhat less useful, but still are worthy of some attention.

A three-force member is a rigid object acted upon by no couples and only by forces whose lines of action converge at most at 3 different points, as shown in the figure. Consider two of the resultant forces, say R_A and R_B. If their lines of action are not parallel, then they must meet at some point P in the plane. But, if we demand the object to be in equilibrium, taking moments with respect to point P

we must conclude that the force R_C must also pass through P! If, on the other hand, the forces R_A and R_B act on parallel lines of action, then the force at C must also be parallel (otherwise, we would have a contradiction). Thus, in a 3-force object the forces must be either concurrent or parallel. [Note: you may say that parallel forces concur at a point in the horizon, if you so prefer].



Examples: an example at hand is, in fact, provided by the previous question. Indeed, the bracket ABC is, quite obviously, a three-force body, since the forces are applied only at A, B or C. Moreover, after having determined the line of action of the force at B (via an argument pertaining to the two-force member BD), we know this line and also the line of action of the applied load at A, which happens to be vertical. We can immediately determine the intersection P of these two lines of action. We conclude, therefore, that the line of action of the reaction force at C must pass through P, as shown in the figure below. From simple geometry, we obtain the angle β as $\beta = \tan^{-1}\frac{!+0.5}{1} = 56.31°$. We can now either proceed graphically (by drawing the polygon of forces) or, more safely, by implementing the vanishing of the sum of forces. [We no longer need some of moments. Why?] We obtain



$$\sum_{\rightarrow} F_x = -R_C \cos\beta + N_{BD} \cos\alpha = 0$$

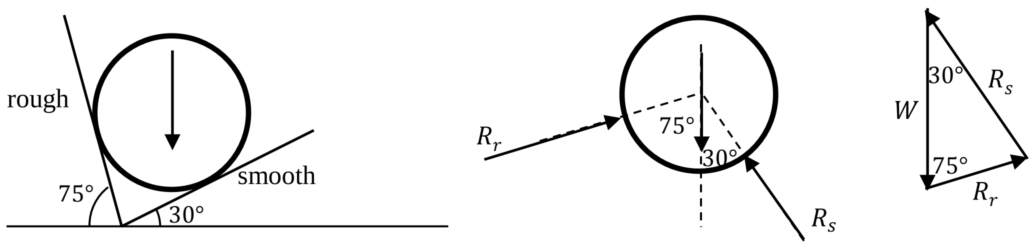
$$\sum_{\uparrow} F_y = -150N + R_C \sin\beta - N_{BD} \sin\alpha = 0$$

We solve this linear system and obtain

$$R_C = \frac{(150N)\cos\alpha}{\sin(\beta - \alpha)} = 270.49\ N$$

$$N_{BD} = \frac{(150N)\cos\beta}{\sin(\beta - \alpha)} = 167.75N$$

Another, less clever, example is provided by the following question. A bowling ball rests against an ice incline at 30° from the horizontal and against a rough wall at 75°, as shown in the figure. Assuming the ball to be uniform, find the reactions at the points of contact with the two surfaces. Denoting by W the weight of the ball and, by uniformity, placing it at the centre of the ball, and taking into consideration that the reaction exerted by the ice is necessarily perpendicular to the plane (no friction) and, therefore, passes through the centre, we conclude that the reaction at the rough wall must also pass through the centre. Had the ball not been uniform (so that its weight would not pass through the centre, the method would still apply, but the point of intersection would be eccentric. Notice that the other wall must be rough, otherwise we would still have a degree of freedom of rotation left, in contradiction with the tenets of our course (on Statics).



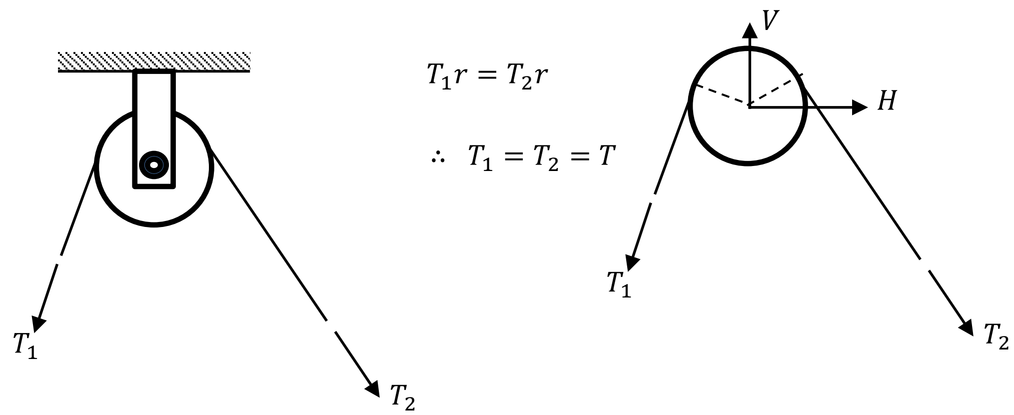
We can now proceed analytically (by two equations of sum of forces) or graphically. For the sake of variety, let us solve this graphically. If we draw the polygon of forces arising from the FBD above, we must obtain, for equilibrium, a (closed) triangle. In this particular case, rather than using the cosine law, we notice that this triangle happens to be isosceles. We obtain immediately

$$R_s = W$$

$$R_r = 2W \cos 75° = 0.52\,W$$

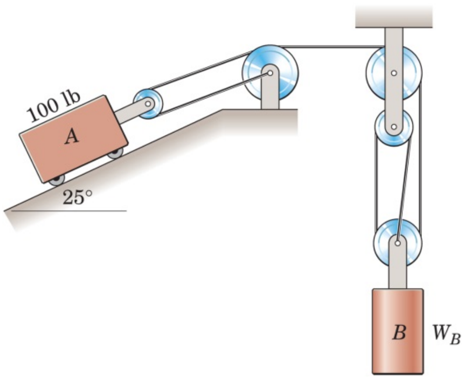
9. Pulleys

A *pulley* is a mechanism whose main function is to change the line of action of a force in a rope without affecting its magnitude. It consists of a wheel of radius r attached at its centre to a frictionless axle. The rope does not slide with respect to the wheel. In the figure, we have a pulley suspended from a roof whose rope is subjected to two tensions T_1 and T_2, which could in principle be different in magnitude. An equation of moments with respect to the centre of the wheel, however, reveals immediately that (under the assumption that the attachment to the axle is attained by a frictionless pin so that the wheel is free to rotate) these two tensions must be equal. Suspending pulleys from other pulleys, it is possible to lift heavy weights by means of forces much smaller than the weights themselves. Pulleys were widely used in antiquity and are still in wide use today. Archimedes (287-

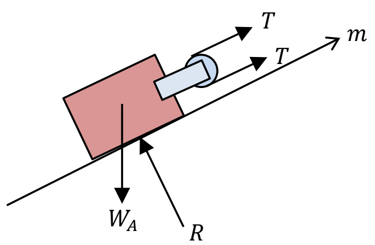


212) is credited with having developed ingenious pulley mechanisms by means of which warships could be displaced. Since nothing is gained without some cost, he used to declare that ‘what you gain in force you lose in displacement’, an intuitive formulation of the *principle of virtual work*. Indeed, the force may be smaller, but the amount of rope to be pulled increases with the diminution of the force.

Consider the following example (3.15 in the text). We are asked to find the weight W_B corresponding to equilibrium of the system. We assume the contact with the incline to be frictionless and the pins of all pulleys to be frictionless (so that they cannot apply couples to the axles!).

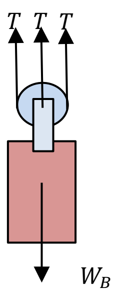


Solution: As in all problems involving more than a single rigid object, a good policy is to *dismember* or separate the system into its rigid components and draw separate FBDs for each of the parts. We note that, since we have a single continuous rope wound around various pulleys, according to the theory the tension T in all parts of the rope is constant. For this reason, we don't really need to invoke the FBDs of the separate pulleys themselves. We first draw the FBD of the body A by severing all its contacts with the rest of the world. In so doing, we will keep one of the pulleys and cut through the rope twice. The force applied by the incline is perpendicular to its plane, since the connection is by means of a rolling mechanism. Summing force components in the m direction yields the equation



$$-W_A \sin 25° + 2T = 0$$

The FBD of the body B, including the attached pulley and cutting three times through the rope, provides us with an equation of equilibrium in the vertical (y) direction as follows



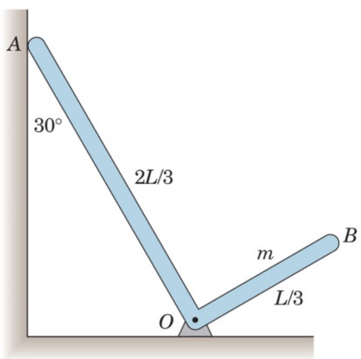
$$-W_B + 3T = 0$$

The solution of this system is

$$W_B = \frac{3}{2} W_A \sin 25° = 63.4\ lb$$

10. Additional example

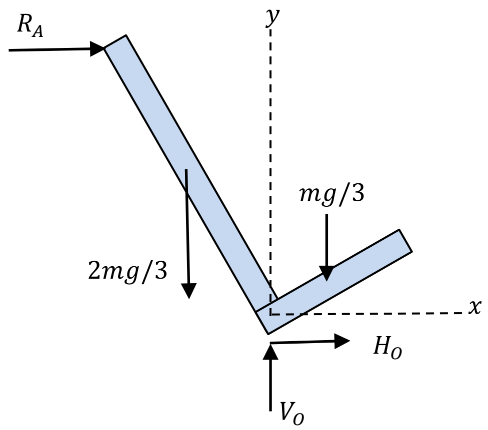
This is Problem 3.30 from the text. We are asked to find the reactions knowing that the wall is smooth and the pin is frictionless. The bar is assumed to be uniform, so that the mass m is distributed uniformly along the bracket.



Solution: Since the vertical wall is smooth (no friction), it can only apply a horizontal reaction. Since the pin at O is frictionless, it can only apply forces (but no couples). From the FBD, taking moments with respect to point O, we obtain

$$\sum\nolimits_{ccw} M_O = -R_A \frac{2L}{3}\cos 30° + \frac{2mg}{3}\left(\frac{L}{3}\sin 30°\right) - \frac{mg}{3}\left(\frac{L}{6}\cos 30°\right) = 0$$

Solving for R_A we get



$$R_A = \frac{mg}{2}\left(\frac{2}{3}\tan 30° - \frac{1}{6}\right) = 0.109\,mg$$

By sum of forces in the x direction

$$R_A + H_O = 0$$

Hence

$$H_O = -0.109\,mg.$$

The negative sign indicates 'opposite to shown in FBD'. Finally, in the y direction,

$$-mg + V_O = 0$$

Hence

$$V_O = mg$$

The positive sign indicates 'as shown in FBD'. The magnitude of the reaction at O can be obtained as

$$R_O = \sqrt{H_O^2 + V_O^2} = \sqrt{(0.109)^2 + 1^2}\,mg = 1.006\,mg$$

Note: This problem could have been solved as a three-force body problem, but the line of action of the total weight would have to be found first, thus unnecessarily adding to the computational effort.