ENGG 225

Fundamentals of Electrical Circuits and Machines

Laboratory #4
Steady-State AC Circuits

1 Introduction

Earlier in ENGG 225, we studied DC circuits along with some common and important circuit configurations, such as the Wheatstone bridge. We further used DC circuits to present, and demonstrate in detail, the systematic high-level methods of circuit analysis, such as the node-voltage and mesh-current methods, Thévenin's theorem, and the principle of superposition.

AC circuits also clearly make up an important and large class of electric circuits, and in this lab, you will apply the same circuit analysis methods to analyze them. There are many important practical applications of AC circuits, and you will explore one such application here, that of *signal conditioning,* simply called "filtering."

1.1 Learning Outcomes

At the end of this laboratory session, you will be able to analyze, design, and test AC circuits, and to design and experiment with practical filter circuits. Specifically, you will learn:

- to verify Thévenin's theorem for an AC circuit;
- to design and build a complex voltage divider that acts as a simple lowpass filter circuit with a prescribed "cutoff frequency;"
- how a simple series-resonant circuit can be used to reject a sinusoidal voltage having a particular frequency;

- a practical application of a simple lowpass filter circuit for reducing contaminating noise interference in an electrocardiogram (ECG) measurement of a heartbeat.

1.2 UCEE Acknoweldgment

The course instructors for ENGG 225 continue to greatly acknowledge the University of Calgary Engineering Endowment (UCEE) Fund for substantial funding toward the purchase of important high-quality laboratory test equipment and supplies needed to enhance the learning value of the laboratories in the course.

2 Background Review

2.1 Thévenin Equivalent Circuits

Thévenin's theorem states for DC circuits that:

> *Any two-terminal linear DC circuit can be replaced with an equivalent circuit consisting of a DC voltage source and a series resistance.*

It is the same for AC circuits, except that in AC circuits, the series resistance is in general replaced by an impedance, and the DC source by an AC source. You may wish to review Thévenin equivalent circuits in Sections 2.6 (DC circuits) and and 5.6 (AC circuits) in the course textbook.

2.2 Frequency-Dependent Circuits – Filters

Consider the simple AC circuit in Fig. 1, which consists of sinusoidal voltage source $v_{in}(t)$ in series with a resistor and capacitor. This is nothing more than a voltage divider circuit of the type considered in Lab #2, but now with sinusoidal voltages and currents. Although this is clearly a simple circuit, it has a very useful practical application called "filtering."

Let the frequency of the sinusoidal voltage source $v_{in}(t)$ be $\omega = 2\pi f$. The frequency of the sinusoidal voltages and currents throughout the circuit are all the same as $v_{in}(t)$. However, depending on the value of ω, the amplitude and phase of the voltages and currents elsewhere in the circuit can differ significantly from that of $v_{in}(t)$. The introduction of one or more *frequency-dependent* circuit elements, such as the capacitor in Fig. 1, brings about this behaviour. The impedance of a resistor is always a real-valued constant $Z_R = R$, independent of ω, while the impedance of the capacitor is the *frequency-dependent* complex value given by

$$Z_C = \frac{1}{j\omega C}. \tag{1}$$

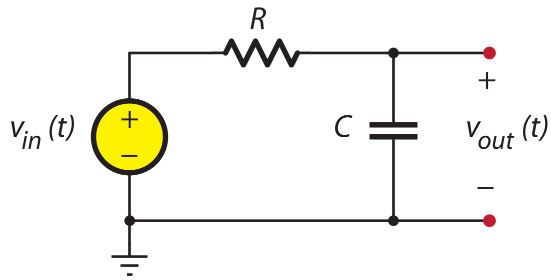

Fig. 1. A simple lowpass filter circuit

2.2.1 Operation as a "lowpass" filter

Consider that $v_{in}(t)$ is the "input" to the filter and $v_{out}(t)$ is the "output" of the filter. A very important characteristic of filter circuits such as this is its *frequency response*. The frequency response quantifies how the amplitude and phase of the output sinusoid $v_{out}(t)$ vary with the frequency ω of the input sinusoid $v_{in}(t)$. Such a circuit may be used to reject, or "filter out," sinusoidal signals on the basis of their frequency ω.

The filter circuit in Fig. 1 is called a *lowpass filter*. By equation (1), the impedance of the capacitor decreases as $\omega = 2\pi f$ increases, and when viewed as a simple voltage divider, the voltage across the capacitor is proportional to its impedance. Therefore, sinusoids with higher frequencies are "rejected" and lower frequencies are "passed" by the filter.

Let the input and output phasor voltages for the circuit in Fig. 1 be given by $\mathbf{V}_{in}$ and $\mathbf{V}_{out}$, respectively. Furthermore, let us assume that the input voltage is simply $v_{in}(t) = \cos(\omega t)$, so that $\mathbf{V}_{in} = 1\angle 0$. Using the principle of voltage division,

$$\mathbf{V}_{out} = \frac{Z_C}{R + Z_C} \times 1\angle 0. \tag{2}$$

Using the capacitor impedance Z_C given in equation (1), this becomes

$$\mathbf{V}_{out} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{1}{1 + j\omega RC}. \tag{3}$$

The magnitude of this complex function is called the *magnitude frequency response* function, and is given by

$$|\mathbf{V}_{out}| = \frac{1}{|1 + j\omega RC|} = \frac{1}{\sqrt{1^2 + (\omega RC)^2}}. \tag{4}$$

2.2.2 The "Cutoff Frequency"

The frequency at which the lowpass filter distinguishes between a "low" frequency and a "high frequency" is called the *cutoff frequency*, denoted as ω_c radians per second. The cutoff frequency in Hz is expressed as $f_c = \omega_c/2\pi$. The cutoff frequency is usually defined as the frequency at which the amplitude of the output sinusoid is $\frac{1}{\sqrt{2}}$ of the input sinusoid.

From equation (4), it should be easy to see that $|\mathbf{V}_{out}| = \frac{1}{\sqrt{2}}$ when the ωRC term in the denominator becomes 1. Therefore, the cutoff frequency ω_c occurs when

$$\omega = \omega_c = \frac{1}{RC}. \tag{5}$$

2.2.3 A Simple Series-Resonant Circuit

Consider the circuit in Fig. 2, which now places a series inductor between the resistor and capacitor in the circuit of Fig. 1.

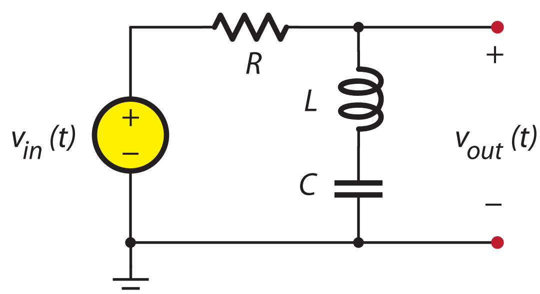

Fig. 2. A simple series-resonant circuit

Following the phasor analysis above for a complex voltage divider, we may write

$$\mathbf{V}_{out} = \frac{Z_C + Z_L}{R + Z_C + R_L} \times 1\angle 0. \tag{6}$$

Using the capacitor impedance Z_C given in equation (1), and $Z_L = j\omega L$ for an inductor, this becomes

$$\mathbf{V}_{out} = \frac{\frac{1}{j\omega C} + j\omega L}{R + \frac{1}{j\omega C} + j\omega L} = \frac{j\left(\omega L - \frac{1}{\omega C}\right)}{R + j\left(\omega L - \frac{1}{\omega C}\right)}. \tag{7}$$

Note that, at a certain frequency $\omega = \omega_0$, the numerator in equation (7) becomes *exactly zero*. This is the frequency at which the inductive impedance exactly cancels the capacitive impedance, turning the combined series impedance of the inductor and capacitor into a short circuit. When this happens, the output voltage $v_{out}(t) = 0$. This is called the *resonant frequency*, $f_0 = \omega_0/2\pi$, of the circuit. From equation (7),

$$\omega_0 = \sqrt{\frac{1}{LC}} \,. \tag{8}$$

Filters of this type are often called *notch filters* because they reject sinusoidal input voltages having a very narrow range of frequencies near the resonant frequency, and f_0 is often called the *notch frequency*. There are many applications for notch filters, such as filtering out 60 Hz electrical noise in electronic signals.

3 Pre-Lab Exercises

The pre-lab work must be completed prior to the laboratory session and will be checked prior to the beginning of the experimental exercises.

3.1 Thévenin Equivalent Circuits

Consider the resistor circuit shown in Fig. 3, and calculate the Thévenin equivalent circuit between the terminals **X** and **Y** using the input sinusoid given by

$$v_{in}(t) = \cos(\omega t)\ V \tag{9}$$

where $\omega = 2\pi f$ and $f = 1000$ Hz. Sketch the Thévenin equivalent circuit, indicating the value of R_t and the time-domain expression for $v_t(t)$.

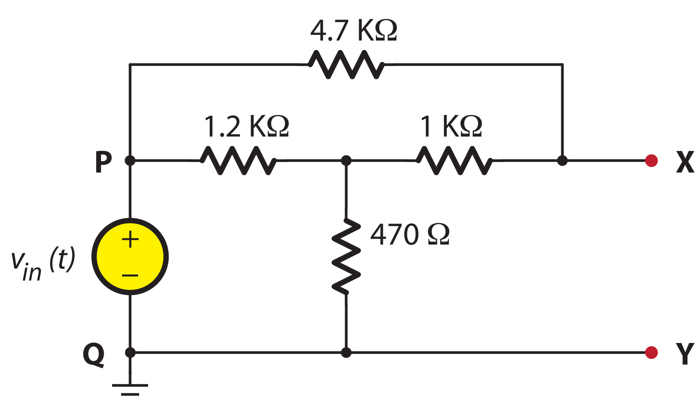

Fig. 3. Find the Thévenin equivalent circuit

3.2 The Notch Filter Circuit

For the circuits in Fig. 1 and Fig. 2, let the input and output sinusoids be given by

$$v_{in}(t) = \cos(\omega t) \tag{10}$$
$$v_{out}(t) = V_{out}\cos(\omega t + \theta_{out}) \tag{11}$$

where ω is the independent frequency variable, and V_{out} and θ_{out} are the unknowns.

Let $C = 0.047\mu$F, $L = 100$ mH, and $r = 820\Omega$ in the circuit of Fig. 2.

1. Using equation (8) above, determine the notch frequency f_0, in Hz.
2. At the notch frequency, calculate the voltage across the capacitor, and express it in the form $v_C(t) = V_C\cos(2\pi f_0 t + \theta_C)$. Is $v_C(t)$ *leading* or *lagging* $v(t)$?

3.3 The Lowpass Filter Circuit

Let $C = 0.047\mu$F in the circuit of Fig. 1. Using equation (5) above, determine what value of R is required for the filter to have a cutoff frequency of $f_c = 340$ Hz ($\omega_c = 2135$ rads/sec). For $v_{in}(t)$ given in equation (9), f_c is the frequency at which $V_{out} = \frac{1}{\sqrt{2}}$.

4 Procedure

One report per group is required, and must be completed during the lab session.

In the instructions below, you may wish to refer to the printed "Lab Instrument Reference" document attached to each lab station. You will use the oscilloscope as both an AC voltage source and to make measurements on your circuits.

4.1 Verifying the Thévenin Equivalent Circuit

Here, you will experimentally determine the Thévenin equivalent circuit corresponding to Fig. 3, and compare it with your pre-lab calculations.

1. Construct the circuit shown in Fig. 3 *(leave this circuit intact for Part 4.2, below).* Connect the oscilloscope's function generator output to the input of your circuit using a BNC cable to the breadboard and a wire to your circuit.
2. Locate and press the ""Wave Gen" button to enable the oscilloscope's function generator. The button should turn bright blue. Using the soft keys at the bottom of the display and the "Nav Knob" (by the illuminated circular arrow), set the frequency to 1000 Hz and 2 Volts Peak-to-Peak V_{PP}. (See the back page of the Lab Instrument Reference.)

3. Using an oscilloscope probe connected to Channel 1, arrange to view $v_{in}(t)$ produced by the function generator. Use the "Meas" button on the oscilloscope to report the actual peak-to-peak voltage. If this reading is different from the function generator setting, adjust the function generator voltage so that the measured value of v_{in} is 2 V_{PP}.

4. Make the following measurements to determine the Thévenin equivalent circuit.

   (a) Connect the probe to node **X** in your circuit and measure the the peak-to-peak Thévenin voltage. Record this value. V_t is the *amplitude* of this waveform, which is half its peak-to-peak value.

   (b) Disconnect the sinusoidal voltage source from the circuit, and place a short circuit between the nodes **P** and **Q**. Now use the DMM to measure resistance, and measure the resistance between nodes **X** and **Y**. This is R_t. Record this value.

> **IMPORTANT**: *Always make sure that the power sources are disconnected from a circuit on which you are making resistance measurements, otherwise the DMM may be damaged.*

5. Compare the measured values of R_t and V_t with those obtained in the pre-lab exercise.

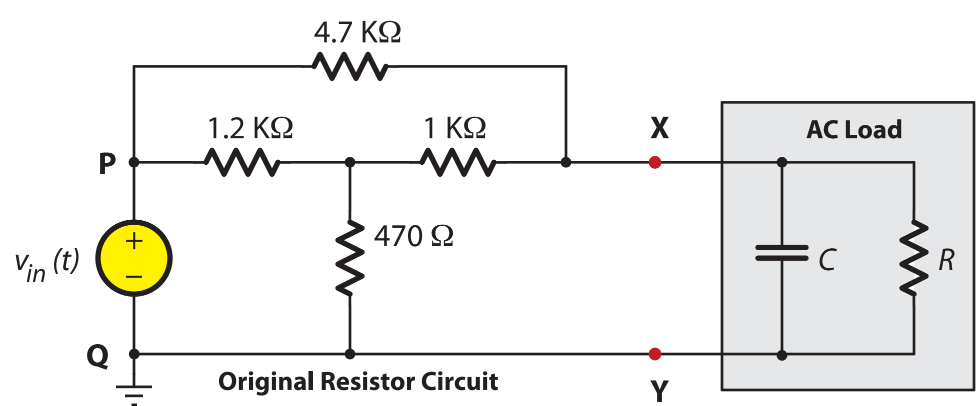

Fig. 4. Circuit of Fig. 3 with AC load, $R = 1K\Omega$ and $C = 0.047\mu F$.

4.2 Application of Thévenin Equivalent Circuit

In this part, you will test that your Thévenin equivalent circuit behaves in the same way as the original circuit in Fig. 3 when an electrical load is connected to its terminals.

4.2.1 Applying an AC load to the original resistor circuit

1. Disconnect the DMM, remove the short circuit between nodes **P** and **Q**. and reconnect $v_{in}(t)$. Next connect a resistor and capacitor to the terminals **X** and **Y**, as illustrated in Fig. 4. Use $R = 1K\Omega$ and $C = 0.047\mu$F.

2. Make sure that the amplitude of $v_{in}(t)$ remains at 2 V_{PP}, adjusting as necessary. Measure and record the new peak-to-peak voltage at node **X**. Change the frequency to 4000 Hz, and re-measure.

4.2.2 Applying the same AC load to the Thévenin equivalant

3. Construct the Thévenin equivalent circuit shown in Fig. 5 from your pre-lab analysis of Fig. 3. Set the function generator to produce your value of V_t and 1000 Hz. Attach the same AC load as in Fig. 4.

4. Measure and record the peak-to-peak voltage at **X** for input frequencies of $f = 1000$ Hz and 4000 Hz. Compare these measurements with those in Step 2, above.

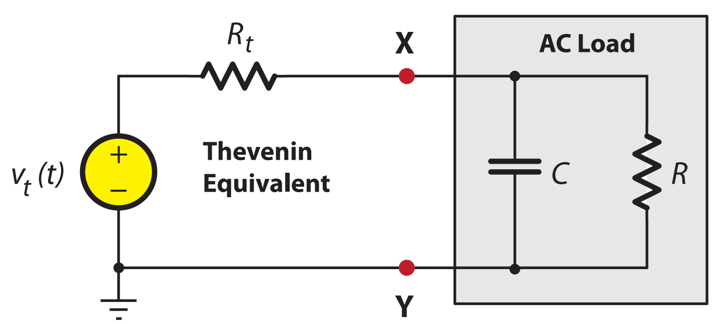

Fig. 5. Thévenin equivalent circuit with same AC load.

4.3 Measurements on the Notch Filter Circuit

Here you will experimentally confirm the operation of the notch frequency that you analyzed in your pre-lab calculations.

1. Construct the notch filter circuit in Fig. 2 on the breadboard with $C = 0.047\mu$F, L =100 mH, and $R = 820\ \Omega$. Use an oscilloscope probe connected to Channel 1 to measure $v_{in}(t)$ and a second oscilloscope probe to Channel 2 to measure $v_{out}(t)$.

2. Make sure that the function generator voltage remains at 2 V_{PP}.

3. Now adjust the frequency on the function generator until $v_{out}(t)$ is the smallest that you can make it (you might have to reduce the VOLTS/DIV setting on oscilloscope Channel 2 to get a good look).

4. Record the minimum peak-to-peak value of $v_{out}(t)$ that you find. Is this exactly zero as expected? If not, why?

5. Record the value of the frequency, and compare this value with f_0 from your pre-lab calculation. Comment on any difference.

6. Move the oscilloscope probe for Channel 2 to observe the capacitor voltage $v_C(t)$. Record the peak-to-peak value, and compare its peak amplitude (i.e., half the peak-to-peak value) to that which was calculated in your pre-lab work for V_C. Confirm your pre-lab calculation of whether the phase angle θ_C is *leading* or *lagging* $v_{in}(t)$.

4.4 Measurements on the Lowpass Filter Circuit

Here, you will experimentally verify the cutoff frequency of the circuit you designed in your pre-lab calculations.

1. Construct the lowpass filter circuit in Fig. 1 on the breadboard with $C = 0.047\mu$F and the value of R that you determined to provide a cutoff frequency of $f_c = 340$ Hz. Use an oscilloscope probe connected to Channel 1 to measure $v_{in}(t)$ and a second oscilloscope probe to Channel 2 to measure $v_{out}(t)$.

2. Set the function generator frequency to 340 Hz and make sure that the voltage remains at 2 V_{PP}.

3. Now adjust the frequency on the function generator until $V_{out} = 2/\sqrt{2} = 1.414$ V_{PP}. Record this frequency and compare with your pre-lab value for f_c. Comment on any difference.

4. Now decrease the frequency below f_c, and note what happens to the amplitude V_{out}; similarly, increase the frequency above f_c, and again note what happens. Is this behaving like a lowpass filter? Briefly explain.

4.5 Lowpass Filtering an ECG Waveform

In this part, you will explore a filtering application of AC circuits. An electrical filter is a circuit used to "condition" electronic signals in useful ways, most often to improve the quality of the information content in a signal by reducing interference signals such as "noise." In this exercise, we provide an actual ECG recording of the heartbeat of a rat, recorded in the presence of simulated electrical noise. This noise can be modeled as the summation of numerous high-frequency sinusoids, giving a distorted "thick-looking" waveform. You will use the lowpass filter circuit just investigated to try to reduce this noise.

1. Use your lowpass filter circuit designed for f_c = 340 Hz.
2. Disconnect the function generator from your circuit. The new input $v_{in}(t)$ instead will be from the Dual Audio Buffer module (shown at right) used in Lab #3. Following the procedure in Lab #3, set this up as follows.
   (a) Supply power (+15V, GND, -15V) to the buffer module. Connect the audio patch cord to the "Right Input" and "Left Input" on the buffer module.
   (b) Choose either the "Right Output" or "Left Output" on the module to connect to your circuit to supply $v_{in}(t)$ in place of the function generator.
   (c) On the PC at your lab station, navigate in Windows 7 to the network drive **N:** and locate the folder `ENGG`, subfolder `225`, and `Lab4`. There will be an audio file there called `RatECG-noisy.wav`. Double-click on this file to play it through the PC's audio output. As before, make sure the audio levels are at maximum.
3. Use Channel 1 of the oscilloscope again to measure $v_{in}(t)$ of your circuit, and use Channel 2 again to measure the output of your filter circuit. Adjust the time and voltage settings on the oscilloscope display appropriately. Make an approximate sketch of the input and output waveforms on graph paper in your notebook.
   - Briefly describe what the filter has done to the input signal.