Monday, February 8, 2016

Similar to the node-voltage method, we can see an important pattern arise when we rearrange this equation:

$$\underbrace{(1200+300)}_{\substack{\text{total resistance}\\ \text{around mesh a}}} i_a - \underbrace{1200}_{\substack{\text{resistance}\\ \text{shared with}\\ \text{mesh b}}} i_b - \underbrace{300}_{\text{with } i_c} i_c - \underbrace{15}_{\text{total voltage.}} = 0 \qquad (1)$$

Mesh b: $250(i_b - i_c) + 1200(i_b - i_a) + 1000\, i_b = 0$

so $-1200 i_a + 2450 i_b - 250 i_c = 0 \qquad (2)$

Mesh c: $300(i_c - i_a) + 250(i_c - i_b) + 200\, i_c = 0$

so $-300 i_a - 250 i_b + 750 i_c = 0 \qquad (3)$

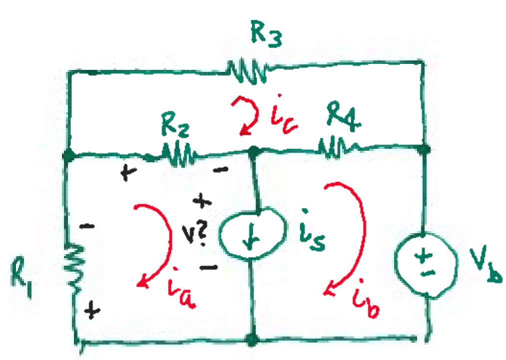
Three equations, 3 unknowns. For power in the 250Ω resistor,

$$p = i^2 (250)$$
$$= (i_c - i_b)^2 \times 250$$

Mesh-current method – special case

As in the node-voltage method, there is one special case we must handle. It is when there is a current source shared by meshes.

Here's one situation:



- i_s shared by meshes a and b.

In mesh a, we have an unknown voltage V.

$$i_a R_1 + (i_a - i_c) R_2 + V = 0 \qquad (1)$$

another unknown

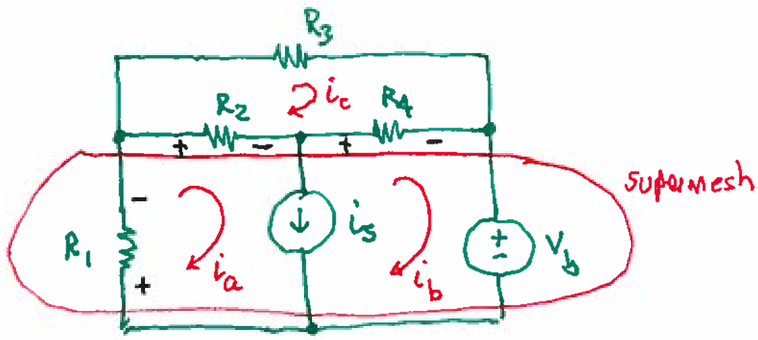
From mesh (b), in ~~the~~ terms of the same unknown voltage V.

$$-V + (i_b - i_c) R_4 + V_b = 0 \qquad (2)$$

Adding (1) and (2) eliminates unknown V.

$$i_a R_1 + (i_a - i_c) R_2 + V - V + (i_b - i_c) R_4 + V_b = 0$$

We may obtain this equation directly by considering a and b as a supermesh.

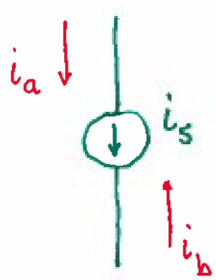


We may write a supermesh equation in one step.

$$\boxed{\underbrace{i_a R_1 + (i_a - i_c) R_2}_{\text{side of supermesh in mesh a}} + \underbrace{(i_b - i_c) R_4 + V_b}_{\text{side of supermesh in mesh b}} = 0}$$

SUPERMESH EQUATION.

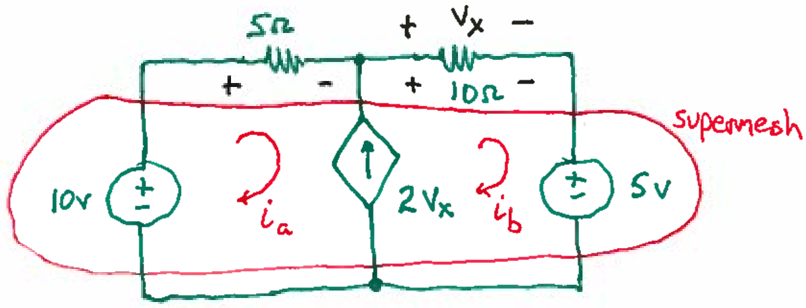
We also have a dependence equation for the two meshes within the supermesh.



$$\boxed{i_a - i_b = i_s}$$

DEPENDENCE EQUATION.

Example 1: Solve for mesh currents.



We'll need one supermesh equation and a dependence equation.

Supermesh: $\underbrace{-10 + 5i_a}_{\text{from mesh a}} + \underbrace{10i_b + 5}_{\text{from mesh b.}} = 0$

$$5i_a + 10i_b = 5 \qquad (1)$$

Dependence: $i_b - i_a = 2V_x.$ (2)

And the dependent current source,

$$V_x = 10i_b \qquad (3)$$

Combining (2) and (3)

$$i_b - i_a = 20i_b$$

$$\text{so } i_a = -19i_b$$

Solving (1) and (4) $\quad i_b = -0.0588\text{ A}$

$$i_a = 1.118\text{ A}.$$