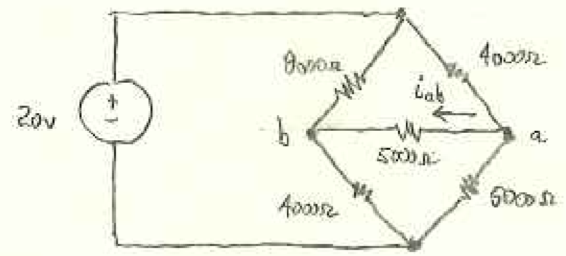
Question 1



At node b: $\frac{V_b}{4000} + \frac{V_b - 20}{8000} + \frac{V_b - V_a}{5000} = 0$

(x40K) $10V_b + 5V_b - 100 + 8V_b - 8V_a = 0$

$$23V_b - 8V_a = 100 \qquad (1)$$

At node a: $\frac{V_a}{5000} + \frac{V_a - 20}{4000} + \frac{V_a - V_b}{5000} = 0$

(x 20K) $4V_a + 5V_a - 100 + 4V_a - 4V_b = 0$

$$13V_a - 4V_b = 100 \qquad (2)$$

Solving these equations, we get, from (2)

$$V_b = \frac{13V_a - 100}{4} \qquad (3)$$

Substituting into (1) gives $\frac{23(13V_a - 100)}{4} - 8V_a = 100$

$74.75V_a - 575 - 8V_a = 100$

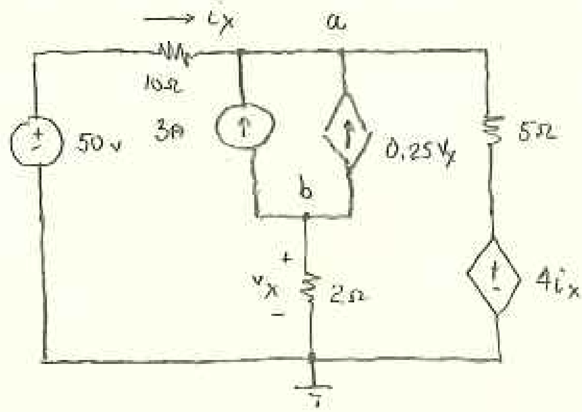
$66.75V_a = 675$

Therefore, $V_a = 10.11236$ V

Substituting back into (3) gives $V_b = 7.86517$ V

Finally, $i_{ab} = \frac{V_a - V_b}{5000} = \boxed{0.4494 \text{ mA}}$

Question 2



At node b: $\frac{V_b}{2} + 3 + 0.25\,V_x = 0$

And we observe that $V_b = V_x$, so

$$\frac{V_x}{2} + 3 + 0.25\,V_x = 0$$

$$(\times 2) \quad V_x + 6 + 0.5\,V_x = 0$$

$$1.5V_x = -6$$

$$\text{Giving} \quad V_x = -4\,\text{V} = V_b$$

Then, at node a: $\frac{V_a - 50}{10} - 3 - 0.25\,V_x + \frac{V_1 - 4i_x}{5} = 0$

$$(\times 10) \quad V_a - 50 - 30 - 2.5(-4) + 2V_a - 8i_x = 0$$

$$3V_a - 8i_x = 70 \qquad (1)$$

We see that $i_x = \frac{50 - V_a}{10}$, so substituting into (1) gives

$$3V_a - 8\left(\frac{50 - V_a}{10}\right) = 70$$

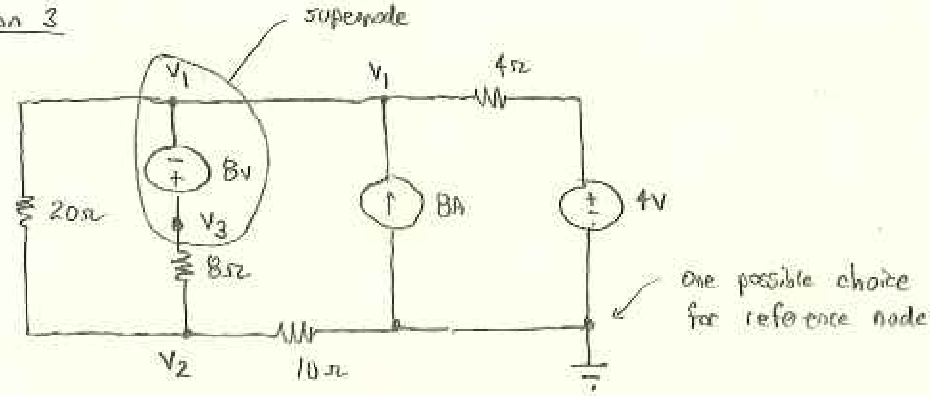
$$(\times 10) \quad 30V_a - 400 + 8V_a = 700$$

$$38\,V_a = 1100$$

$$\text{so} \quad V_a = 28.9474\ \text{V}$$

And that $i_x = \frac{50 - 28.9474}{10} = \boxed{2.105\ \text{A}}$

<u>Question 3</u>



With the supernode defined as above, the equations are

Nodes V_1, V_3:
$$-8 + \frac{V_1 - 4}{4} + \frac{V_1 - V_2}{20} + \frac{V_3 - V_2}{8} = 0$$

$$(\times 40) \quad -320 + 10V_1 - 40 + 2V_1 - 2V_2 + 5V_3 - 5V_2 = 0$$
$$-360 + 12V_1 - 7V_2 + 5V_3 = 0 \qquad (1)$$

Supernode dependence: $V_3 = V_1 + 8 \qquad (2)$

Substituting (2) into (1) gives:

$$-360 + 12V_1 - 7V_2 + 5(V_1 + 8) = 0$$
$$-360 + 12V_1 - 7V_2 + 5V_1 + 40 = 0$$
$$17V_1 - 7V_2 = 320 \qquad (3)$$

Node V_2:
$$\frac{V_2 - V_3}{8} + \frac{V_2 - V_1}{20} + \frac{V_2}{10} = 0$$

$$(\times 40) \quad 5V_2 - 5V_3 + 2V_2 - 2V_1 + 4V_2 = 0$$
$$-2V_1 + 11V_2 - 5V_3 = 0 \qquad (4)$$

Substituting (2) into (4)

$$-2V_1 + 11V_2 - 5(V_1 + 8) = 0$$
$$-2V_1 + 11V_2 - 5V_1 - 40 = 0$$
$$-7V_1 + 11V_2 = 40 \qquad (5)$$

Equation (3) gives $V_1 = \dfrac{7V_2 + 320}{17}$

Substituting into equation (5) gives

$$-7\left(\frac{7V_2 + 320}{17}\right) + 11V_2 = 40$$

$$-2.88235\,V_2 + 11V_2 = 40 + \left(\frac{320 \times 7}{17}\right)$$

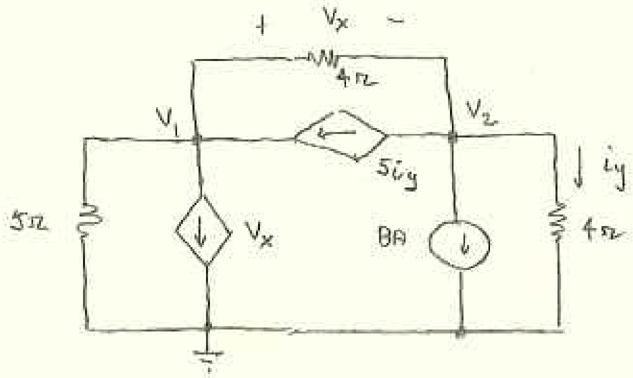
$$8.1176V_2 = 152.941$$

$$V_2 = 21.159\ \text{v}$$

And $V_1 = 27.563\ \text{v}$

Finally, $V_x = V_1 - V_2 = \boxed{6.377\ \text{v}}$

Question 4



Node v_1 : $$\frac{V_1}{5} + V_x - 5i_y + \frac{V_1 - V_2}{4} = 0 \qquad (1)$$

where we also observe that $V_x = V_1 - V_2$
$i_y = V_2/4$

Substituting into (1) : $$\frac{V_1}{5} + V_1 - V_2 - 5\frac{V_2}{4} + \frac{V_1 - V_2}{4} = 0$$

(x 20) $$4V_1 + 20V_1 - 20V_2 - 25V_2 + 5V_1 - 5V_2 = 0$$
$$29V_1 - 50V_2 = 0 \qquad (2)$$

Node V_2 : $$\frac{V_2 - V_1}{4} + 5i_y + 8 + \frac{V_2}{4} = 0$$

$$\frac{V_2 - V_1}{4} + \frac{5V_2}{4} + 8 + \frac{V_2}{4} = 0$$

(x4) $$V_2 - V_1 + 5V_2 + 32 + V_2 = 0$$
$$-V_1 + 7V_2 = -32 \qquad (3)$$

or $V_1 = 7V_2 + 32$

Substituting into equation (2),

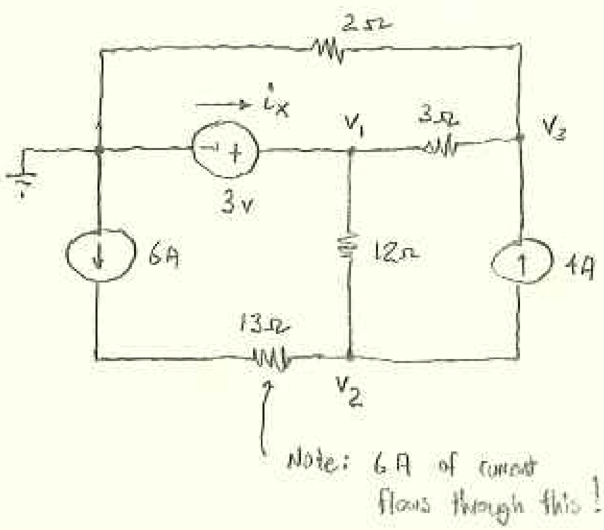
$$29(7V_2 + 32) - 50V_2 = 0$$
$$153V_2 = -928$$

Giving $V_2 = -6.06536$ v

and $V_1 = -10.4575$ v

Finally, $V_x = V_1 - V_2 = \boxed{-4.392\text{ v}}$

Question 5



where we see that $V_1 = 3$ v.

Node 2: $-6 + \frac{V_2 - 3}{12} + 4 = 0$

$$\frac{V_2 - 3}{12} = 2$$
$$V_2 - 3 = 24$$

so $V_2 = 27$ v

Node 3: $-4 + \frac{V_3 - 3}{3} + \frac{V_3}{2} = 0$

$$(\times 6) \quad -24 + 2V_3 - 6 + 3V_3 = 0$$
$$5V_3 = 30$$

so $V_3 = 6$

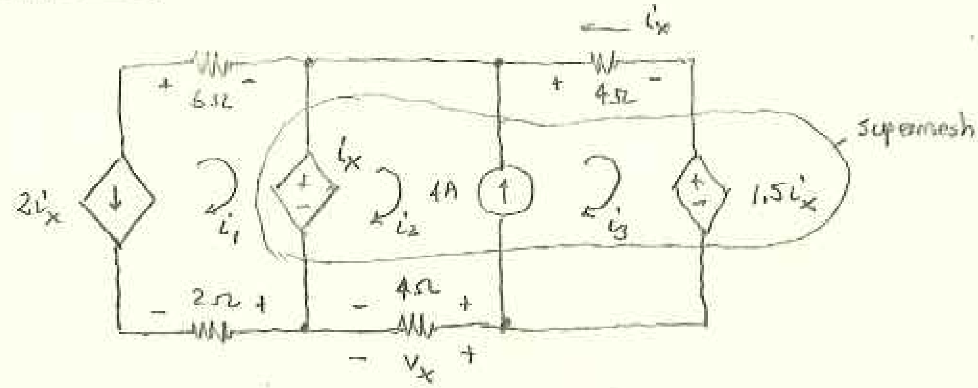
At node v_1, we have $\frac{v_1 - v_2}{12} + \frac{v_1 - v_3}{3} - i_x = 0$

$$(\times 12) \quad v_1 - v_2 + 4v_1 - 4v_3 = 12 i_x$$
$$5v_1 - v_2 - 4v_3 = 12 i_x$$
$$5(3) - 27 - 4(6) = 12 i_x$$

$$12 i_x = -36$$

$$\boxed{i_x = -3A}$$

Question 6



From this circuit, we see that $i_1 = -2i_x$
$i_3 = -i_x$

Supermesh i_2, i_3: $\quad -i_x + 4i_3 + 1.5 i_x + 4i_2 = 0$
$$0.5 i_x + 4i_3 + 4i_2 = 0$$

Now substitute for $i_3 = -i_x$

$$-(-i_3) + 4i_3 + 1.5(-i_3) + 4i_2 = 0$$
$$i_3 + 4i_3 - 1.5 i_3 + 4i_2 = 0$$

$$3.5 i_3 + 4 i_2 = 0 \qquad (1)$$

Supermesh dependence: $\quad i_3 - i_2 = 4$
or $\quad i_3 = 4 + i_2 \qquad (2)$

Substitute (2) into (1) giving

$$3.5(4 + i_2) + 4i_2 = 0$$
$$7.5 i_2 = -14$$

$i_2 = -1.8667$ A, so $v_x = 4 i_2 = \boxed{-7.467 \text{ V}}$