Apparent power (continued)

Friday, April 1, 2016.

$$P_{app} = V_{rms} I_{rms}$$

Units of apparent power

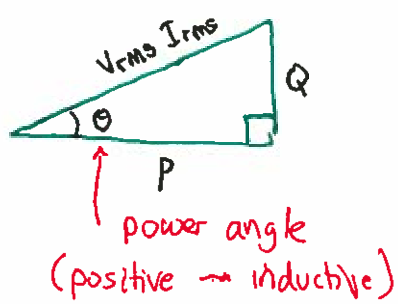
VOLT-AMPS : VAs

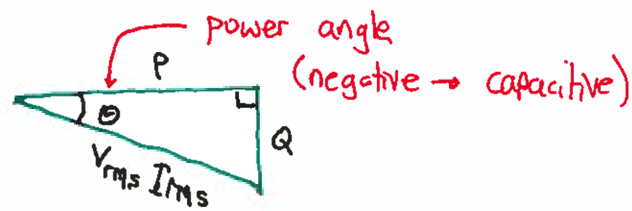
Sample specifications :

units	meaning
5 KW load	$P = 5000$ W
10 KVA load	$V_{rms} I_{rms} = 10{,}000$ VAs
15 KVAR load	$Q = 15{,}000$ VARs

The power triangle

Each of P, Q, and apparent power can be represented in a triangle.





Additional power relationships

It is easy to calculate P, Q, and apparent power directly for an impedance.

We have $Z = |Z|\angle\theta = R + jX$

and $\cos(\theta) = \dfrac{R}{|Z|}$, $\sin(\theta) = \dfrac{X}{|Z|}$

We also have

$$P = \frac{V_m I_m}{2}\cos(\theta) = \frac{V_m I_m}{2} \times \frac{R}{|Z|}$$

and $I_m = \dfrac{V_m}{|Z|}$

so $P = \frac{I_m^2 R}{2}$

and $\boxed{P = I_{rms}^2 R}$ AVERAGE POWER IN Z.

Similarly, $\boxed{Q = I_{rms}^2 X}$ REACTIVE POWER IN Z.

and $\boxed{P_{app} = \sqrt{P^2 + Q^2}}$ APPARENT POWER IN Z.

Finally, complex power is defined as

$$\bar{S} = \frac{1}{2} \bar{V} \bar{I}^*$$

We have

$$\bar{S} = \frac{1}{2} (V_m \angle \theta_V) \times (I_m \angle -\theta_I)$$

$$= \frac{V_m I_m}{2} \angle \theta_V - \theta_I$$

where $\theta = \theta_V - \theta_I$ (power angle)

Expanding $\bar{S}$ in rectangular form

$$\bar{S} = \frac{V_m I_m}{2} \cos(\theta) + j \frac{V_m I_m}{2} \sin(\theta)$$

which we may write

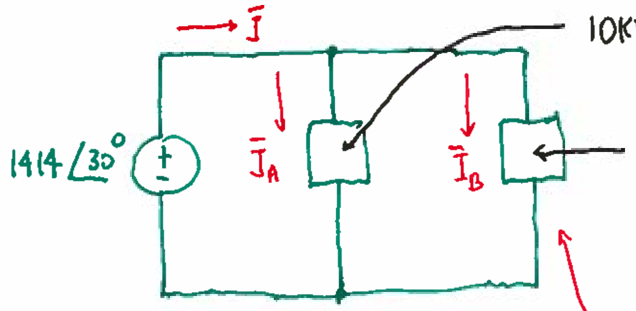
$$\bar{S} = P + jQ$$

The apparent power P_{app}

$$P_{app} = |\bar{S}| = \sqrt{P^2 + Q^2}$$

Example : using power triangles

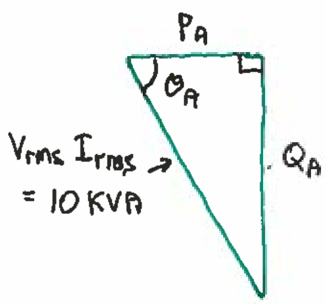
Find $\bar{I}$ in the circuit below



Note this load is specified in terms of apparent power (P_{app}, in KVA)

this load specified in terms of average power P (P, in Watts)

For branch A, we have



Power factor $\cos(\theta_A) = 0.5$ leading

Remember a leading PF means current leads the voltage.

$\theta_I > \theta_V$, so $\theta_A = \theta_V - \theta_I$ (negative angle)

Tip:

PF	θ_V, θ_I	power angle θ	
leading	$\theta_I > \theta_V$	$\theta = \theta_V - \theta_I$	NEGATIVE
lagging	$\theta_I < \theta_V$	$\theta = \theta_V - \theta_I$	POSITIVE

The power angle is therefore $\theta_A = -\left[\cos^{-1}(0.5)\right]$

$= -60^\circ$

For branch A, $P_A = V_{rms} I_{rms} \cos(\theta_A)$

$= 10{,}000 \times 0.5$

$= 5{,}000$ W.

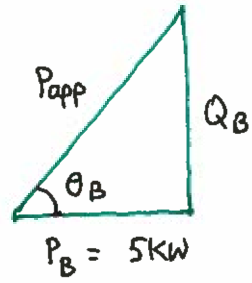
$Q_A = V_{rms} I_{rms} \sin(\theta_A)$

$= -10{,}000 \times 0.866$

$= -8.660$ KVAR

For branch B,

We know $\theta_B = \cos^{-1}(0.7)$
$= 45.57^\circ$



We have P_B, and need Q_B, where

$$\tan(\theta_B) = \frac{Q_B}{P_B} \; ; \quad \text{so} \quad Q_B = P_B \tan(\theta_B)$$

$$Q_B = 5000 \times \tan(45.57^\circ)$$
$$= 5.101 \text{ KVAR}$$

The total amount of power in both loads

$$P = P_A + P_B = 5\text{ KW} + 5\text{ KW} = 10\text{ KW}$$

$$Q = Q_A + Q_B = -8.660\text{ KVAR} + 5.101\text{ KVAR} = -3.559\text{ KVAR}$$

$$\bar{S} = P + jQ = 10{,}000 - j\,3559 \text{ VA}.$$