Question 1

From the current waveform, we first must determine the voltage waveform. We have

$$V_C(t) = \frac{1}{C}\int_{t_1}^{t_2} i_C(t)dt + V_C(t_1)$$

- Interval $0 < t \leq 1$ ms

$$i_C(t) = 0, \quad \text{so} \quad V_C(t) = 10\text{ v} \quad \text{(unchanged from initial value)}$$

- Interval $1 < t \leq 3$ ms

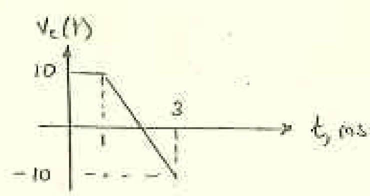
$$V_C(t) = \frac{1}{10^{-6}}\int_{1\text{ ms}}^{t}(-10\times10^{-3})\,dt + V_C(1\text{ ms})$$

$$= -10^{6}\times 10\times10^{-3}\, t\Big|_{1\text{ ms}}^{t} + 10$$

$$= -10^{4}\left[t - 0.001\right] + 10$$

$$= -10^{4}t + 10 + 10$$
$$= -10^{4}t + 20$$

At time $t = 0.003$, $\quad V_C(t) = -30 + 20 = -10$



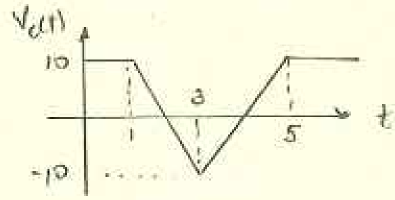
- Interval $3 < t \leq 5$ ms

$$V_C(t) = \frac{1}{C}\int_{3\text{ ms}}^{t} i_c(t) + V_C(3\text{ ms})$$

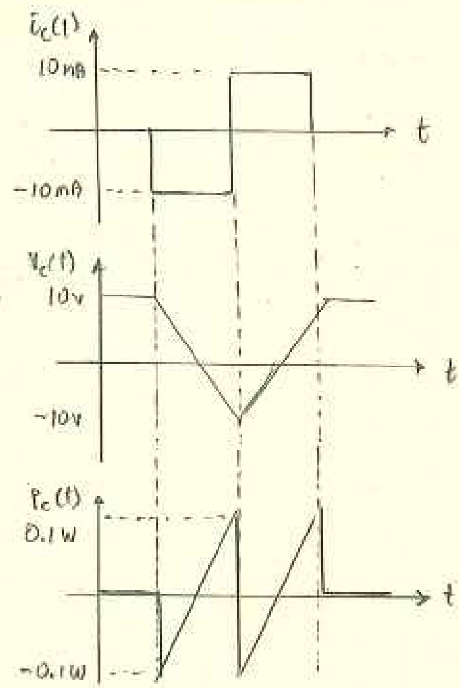
$$= +10^{4}\left[t - 0.003\right] - 10$$
$$= 10^{4}t - 30 - 10$$

$$= 10^{4}t - 40$$

At $t = 0.005$ s, $\quad V_C(t) = 50 - 40$
$$= 10\text{ v}$$



Multiplying this curve with $i_c(t)$ gives the power curve



Question 2

There is no initial current in the inductor, since the switch is open

$$i_L(t) = \frac{1}{L}\int_0^t V_L(t)\,dt + 0$$

$$= \frac{1}{4}\int_0^t (8t+6)\,dt$$

$$= \frac{1}{4}\left[4t^2 + 6t\right]\Big|_0^t = t^2 + 1.5t$$

At $t = 2s$, $i_L(2) = 4 + 3 = 7\ A$

And power $= V_L(2)\,i_L(2) = (8\times 2 + 6)\times 7$

$= 22V \times 7A$

$$\boxed{P_L(2) = 154\,W}$$

Question 3

In question 2, we determined $i_L(2) = 7$ A. The energy is therefore

$$w_L(2) = \frac{1}{2} L\, i_L^2(2)$$
$$= \frac{1}{4} \times 4 \times 7^2$$

$$\boxed{w_L(2) = 98\ J}$$

Question 4

We have $v_c(t) = 10 \sin(10^4 t) + 5\cos(10^4 t)$, and

$$i_c(t) = C\frac{dv_c(t)}{dt}$$
$$= 20\times10^{-6}\left[10^5 \cos(10^4 t) - 5\times10^4 \sin(10^4 t)\right]\Big|_{t=0.5ms}$$
$$= 20\times10^{-1}\left[\cos(5) - 0.5\sin(5)\right]$$
$$= 2\cos(5) - \sin(5)$$
$$= 1.5262\ A$$

And the voltage at $t = 0.5ms$ $\quad v_c(t) = 10\sin(5) + 5\cos(5)$
$$= -8.1709$$

The power, therefore, is $\quad P_c(t) = v_c(t)\, i_c(t)$
$$= -8.1709 \times 1.5262$$

$$\boxed{P_c(t) = -12.471\ W}$$

Question 5

For the time between $t = 0$ and $0.5s$, we have

$$v_c(t) = 40t$$

and $\quad i_c(t) = C\frac{dv_c(t)}{dt} = 40\times10^{-6}\times 40$
$$= 1.6\times10^{-3}\ A$$

And $v_c(t) = 40\times(0.4) = 16\ V$

Power: $\quad P_c(t) = v_c(t)\, i_c(t) = 16 \times 1.6\times10^{-3} = \boxed{25.6\ mW}$

Question 6

To calculate $w_L(t)$, we must first obtain the current waveform $i_L(t)$

$$i_L(t) = \frac{1}{L}\int_{t_1}^{t_2} v_L(t)\,dt + v_L(t_1)$$

- Interval $0 < t \leq 3\,s$

$$i_L(t) = \frac{1}{2}\int_0^t 10\,dt + i_L(0)$$

$$= \frac{1}{2}\left[10t\right]_0^t + 2$$

$$= 5t + 2$$

When $t = 3$, $i_L(3) = 15 \times 3 + 2 = 17\,A$

- Interval $3 < t \leq 6\,s$

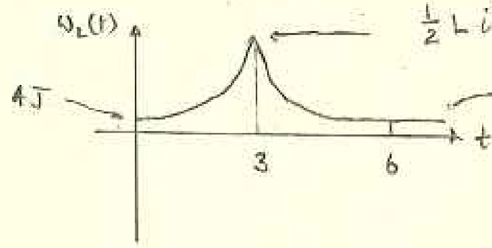
$$i_L(t) = \frac{1}{2}\left[-10t\right]_3^t + 17$$

$$= -5(t-3) + 17$$

$$= -5t + 32$$

When $t = 5$, $i_L(5) = -5 \times 6 + 32 = 2\,A$

The energy waveform is determined from $w_L(t) = \frac{1}{2} L\, i_L^2(t)$



$\frac{1}{2} L i^2 = \frac{1}{2} \times 2 \times 17^2 = 289\,J$

$\frac{1}{2} \times 2 \times 2^2 = 4\,J$

Question 7

The voltage on the inductor is $v_L(t) = L\dfrac{di_L(t)}{dt}$

- Interval $0 < t \leq 1$, slope of $i_L(t) = 0$, so

$$v_L(t) = 0$$

- Interval $1 < t \leq 3$ s

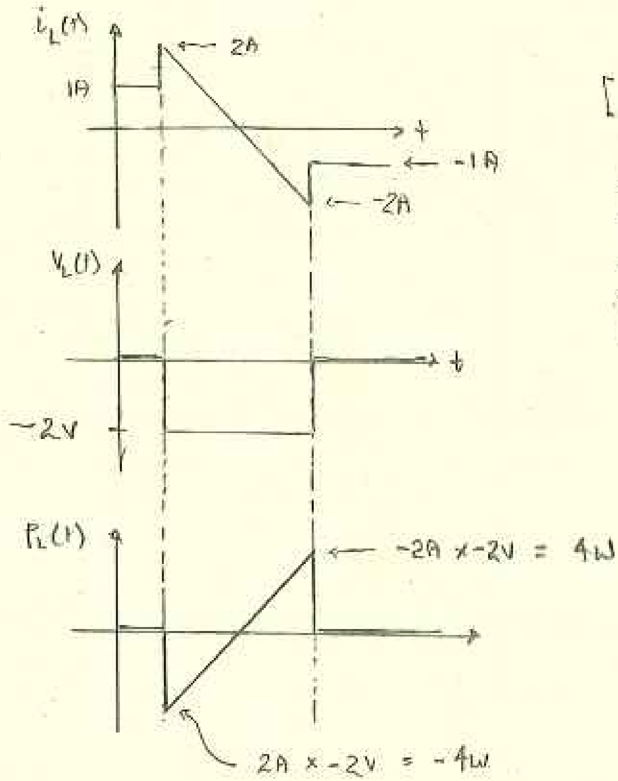
$$\text{slope} = -\frac{4}{2} = -2$$

so $V_L(t) = 1 \times -2 = -2V$

- Interval $t > 3$, slope $= 0$, so

$$V_L(t) = 0$$

The power waveform $P_L(t) = V_L(t)\, i_L(t)$



[Note that current changes instantaneously in this question, which should cause an infinite voltage ; acceptable answers here are the one shown, and "none of the above"]