Monday, March 14, 2016

STEADY-STATE SINUSOIDAL ANALYSIS

So far, we have considered circuits in which sources are DC. We now investigate circuits where sources deliver sinusoidal (AC) currents and voltages

- methods of analysis are identical
- arithmetic changes from real to complex

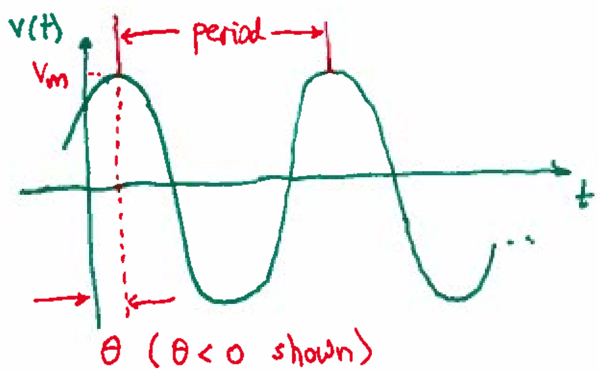
Sinusoidal currents and voltages

Let $v(t) = V_m \cos(\omega t + \theta)$

peak value (V_m)

θ, phase angle (radians)

ω, angular frequency (radians/sec)



The sinusoid is periodic with period T. We have one complete period when the angle increases by 2π.

$$\omega t \Big|_{t=T} = 2\pi, \quad \text{so} \quad \omega T = 2\pi$$

$$\text{and} \quad T = \frac{2\pi}{\omega}$$

Frequency is defined as the number of complete periods (cycles) per second

$$f = \frac{1}{T}, \quad f = \text{frequency in Hertz (Hz)}$$

We also have

$$\omega = \frac{2\pi}{T}, \quad \text{so} \quad \omega = 2\pi f \text{ rads/sec.}$$

By convention, we use cosine and not sine. They are related by

$$\sin(\omega t) = \cos(\omega t - \pi/2)$$
$$= \cos(\omega t - 90^\circ)$$

We say $\sin(\omega t)$ has a phase angle of -90°.

Root-mean-square values

We often express voltages and currents in terms of their peak values, V_m, I_m, but also in terms of their root-mean-square (rms) values).

Consider power in a resistor over one period of the waveforms. Instantaneous power is

$$p(t) = v(t)\,i(t) = v(t)\frac{v(t)}{R} = \frac{v^2(t)}{R}$$

The energy over one period is $E_T = \int_0^T p(t)\,dt$

An important measure is average power over one period.

$$P_{avg} = \frac{E_T}{T} = \frac{1}{T}\int_0^T p(t)\,dt$$

$$\text{or} \quad P_{avg} = \frac{1}{T}\int_0^T \frac{v^2(t)}{R}\,dt$$

which can be expressed as

$$P_{avg} = \frac{\left[\sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt}\right]^2}{R} = \frac{V_{rms}^2}{R}$$

Thus, we define rms voltage

$$V_{rms} = \underset{\text{root}}{\sqrt{\vphantom{\int}}}\;\underbrace{\frac{1}{T}\int_0^T}_{\text{mean}} \overset{\text{square}}{v^2(t)}\,dt$$

Rms values are sometimes called effective values. In the real world,

- AC voltages are specified in rms, not peak (e.g., household voltages are 120 v)
- Power is <u>average power</u>, not instantaneous power (e.g., a 100 W light bulb uses 100 W of average power)

<u>Relating to DC circuits</u>

$$v(t) = V_m \cos(\omega t + \theta), \text{ where } \omega = 0, \ \theta = 0$$

$$\text{so } \quad v(t) = V_m$$

and also

$$v(t) = V_{rms} = V_m$$
$$i(t) = I_{rms} = I_m$$
$$p(t) = P_{avg}$$

For sinusoidal voltages and currents, peak and rms values are not equal.

It can be shown that

$$\boxed{V_{rms} = \frac{V_m}{\sqrt{2}}}$$

SINUSOIDAL RMS VALUE

The voltage in your home is $V_{rms} = 120$ v, so

$$v(t) = 120\sqrt{2}\cos(\omega t + \theta), \text{ where } \omega = 2\pi f$$
$$= 2\pi \times 60 \text{ Hz}$$

$$v(t) = 169.7 \cos(120\pi t + \theta)$$

<u>Example</u>: Let $v(t) = 10 \sin(1000\pi t + 30°)$

Express as a cosine, give angular frequency, frequency in Hz, the rms voltage, and average power in a $10\,\Omega$ resistor.

We have

$$v(t) = 10 \sin(1000\pi t + 30°)$$
$$= 10 \cos(\underbrace{1000\pi t}_{\text{angular frequency}} + 30° - 90°)$$

angular frequency

$\omega = 1000\pi$ rad/sec

$f = \omega / 2\pi = 500$ Hz.