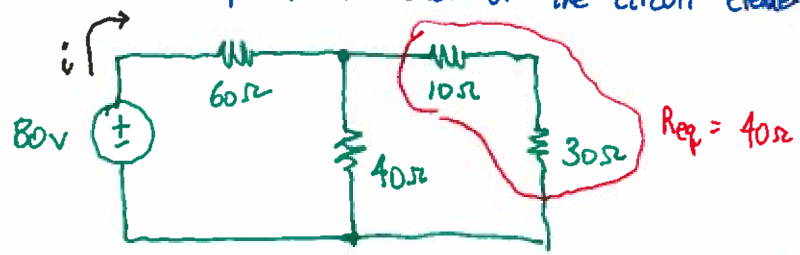
Monday, January 25, 2016

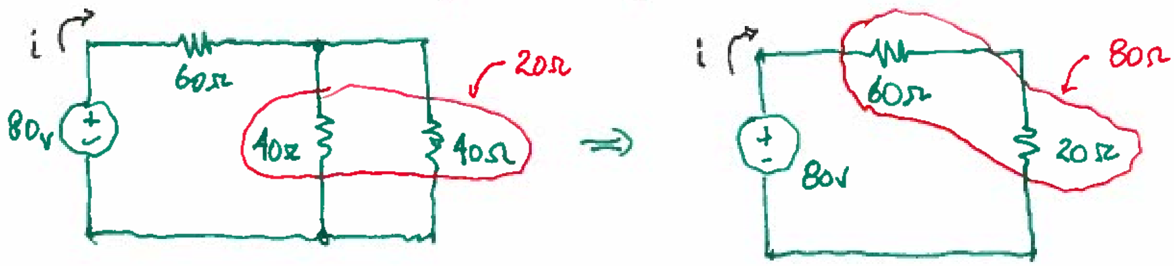
Circuit Analysis using series-parallel equivalents

Circuit analysis: this is a procedure for determining all voltages and currents in every circuit element.

We may employ the above simple resistor equivalents to analyze a circuit.

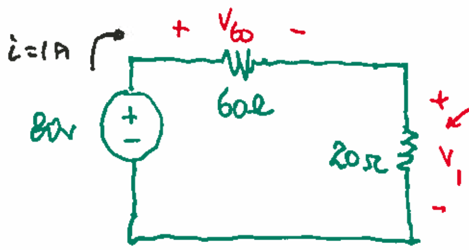
Example: Find the power in each of the circuit elements





We may now determine $i = \dfrac{80\,V}{80\,\Omega} = 1\,A$

Now reconstruct the original circuit, step-by-step. Let's define V_{60} and V_1



remember this is a combo' of 3 resistors

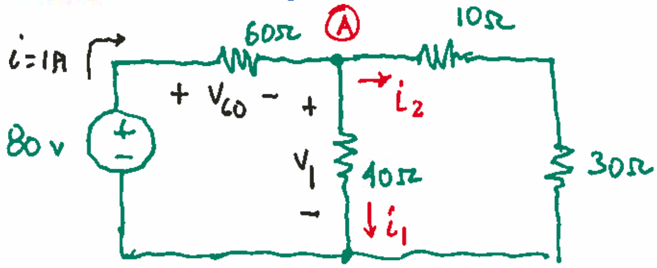
We have $V_{60} = i \cdot 60 = 60\,V$

and $V_1 = i \cdot 20 = 20\,V$

KVL check: $-80 + V_{60} + V_1 = 0$

$-80 + 60 + 20 = 0$ ✓

Now reconstruct the rest



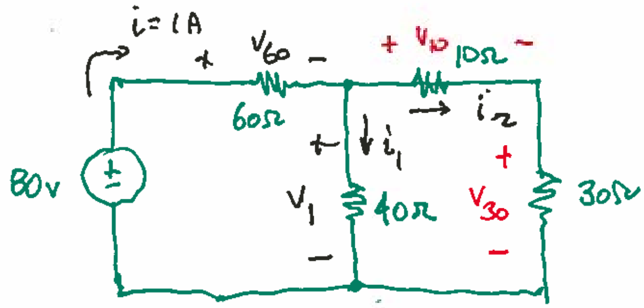
We have $i_1 = \frac{V_1}{40} = 0.5$

and $i_2 = \frac{V_1}{30+10} = 0.5$

KCL check at node A: $i - i_1 - i_2 = 0$

$1 - 0.5 - 0.5 = 0$ ✓

Finally,



Finally,

$V_{10} = i_2 \cdot 10 = 5v$

$V_{30} = i_2 \cdot 30 = 15v$

Now power:

element	power
80v	$P_{80} = -vi = -80 \times 1 = -80\,W$
60Ω	$P_{60} = i^2(60) = 60\,W$
40Ω	$P_{40} = i_1^2(40) = 10\,W$
10Ω	$P_{10} = i_2^2(10) = 2.5\,W$
30Ω	$P_{30} = i_2^2(30) = 7.5\,W$

Energy balance: $-80 + 60 + 10 + 2.5 + 7.5 = 0$

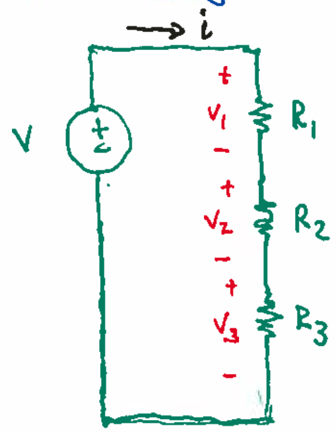
Later, we will use well-established systemic methods to do analysis:

- node-voltage method
- mesh-current method
- Thevenin equivalents
- superposition

Other simple resistor circuits – voltage and current dividers

Voltage dividers:

In a series connection of resistors, the total applied voltage divides among them.



We have

$$R_{eq} = R_1 + R_2 + R_3$$

so $i = \frac{V}{R_{eq}} = \frac{V}{R_1 + R_2 + R_3}$

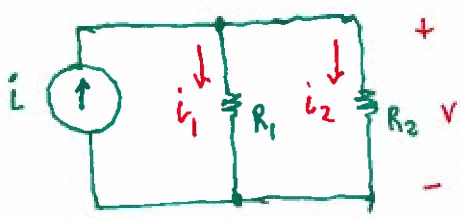
Individual voltages $v_1 = iR_1$
$v_2 = iR_2$
$v_3 = iR_3$

so, for R_1: $v_1 = \left(\frac{R_1}{R_1 + R_2 + R_3}\right)V$ ← R_1's portion of total resistance is same as v_1's portion of total voltage.

Similarly for R_2, R_3

Current dividers:

In a parallel connection of resistors, total applied current divides among them.



We have

$$R_{eq} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1}$$

$$= \frac{R_1 R_2}{R_1 + R_2}$$

so $v = i\left(\frac{R_1 R_2}{R_1 + R_2}\right)$