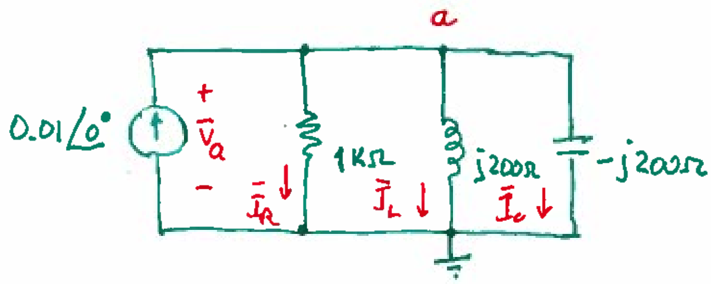
Thursday, March 24, 2016



Write a node equation at a.

Node a: $$-0.01 + \frac{\bar{V}}{1000} + \frac{\bar{V}}{j200} + \frac{\bar{V}}{-j200} = 0$$

(x1000) $$-10 + \bar{V} + \frac{5\bar{V}}{j} + \frac{5\bar{V}}{-j} = 0$$

$$-10 + \bar{V} - j5\bar{V} + j5\bar{V} = 0$$

so $\bar{V} = 10$ V.

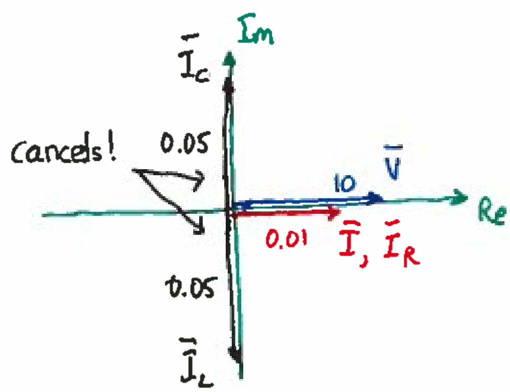
Then, $$\bar{I}_R = \frac{10}{1000} = 0.01 \text{ A}$$

$$\bar{I}_L = \frac{10}{j200} = -j0.05 \text{ A}$$

$$I_c = \frac{10}{-j200} = j0.05 \text{ A}$$

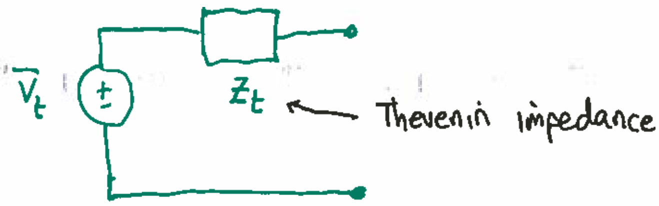
and $\bar{I} = \bar{I}_R + \bar{I}_L + \bar{I}_c = 0.01$

Phasor diagram:



Thevenin equivalent AC circuits

As for DC circuits, we can reduce an AC circuit to a Thevenin or Norton equivalent.

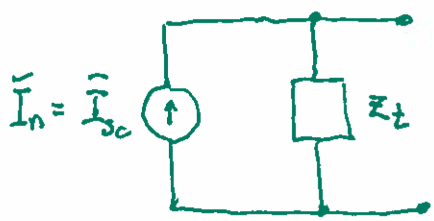


In the same way as DC circuits:

$$\bar{V}_t = \bar{V}_{oc}$$

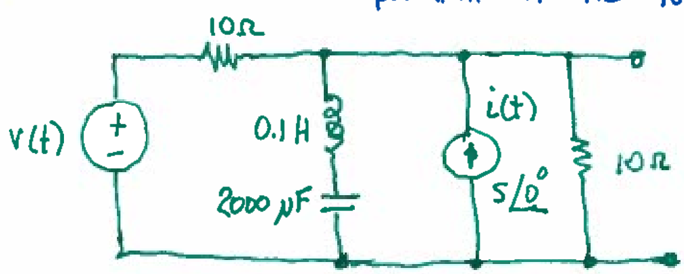
$$Z_t = \frac{\bar{V}_{oc}}{\bar{I}_{sc}}$$

The Norton equivalent



$$\bar{V}_{oc} = \bar{I}_n Z_t$$

Example: Find the Thevenin equivalent of the following:



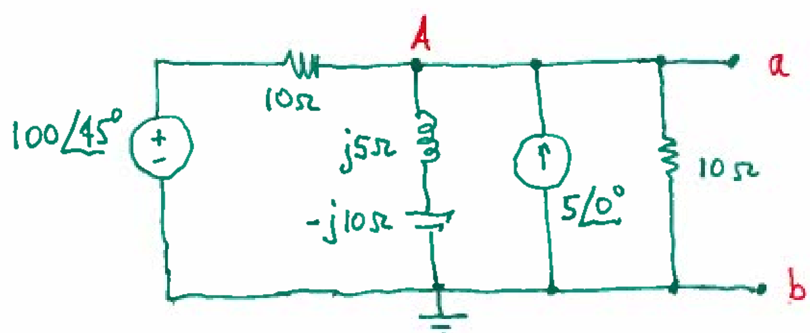
$v(t) = 100\cos(50t + 45°)$
$i(t) = 5\cos(50t)$

In terms of phasors and complex impedances

$$Z_L = j\omega L = j \times 50 \times 0.1 = j5\Omega$$

$$Z_C = \frac{1}{j\omega C} = \frac{1}{j \times 50 \times 2000 \times 10^{-6}} = -j10\Omega$$

Find V_t.



Single node-voltage equation needed at A.

$$\frac{\bar{V}_A - 100\angle 45°}{10} + \frac{\bar{V}_A}{j5 - j10} - 5 + \frac{\bar{V}_A}{10} = 0$$

$$\frac{\bar{V}_A - 100\left[\cos(45^\circ) + j\sin(45^\circ)\right]}{10} + \frac{\bar{V}_A}{-j5} - 5 + \frac{\bar{V}_A}{10} = 0$$

$$(\times 10) \quad \bar{V}_A - 70.1 - j70.1 + 2j\bar{V}_A - 50 + \bar{V}_A = 0$$

$$\bar{V}_A(2 + j2) = 120.7 + j70.1$$

$$\bar{V}_A = \frac{120.7 + j70.1}{2 + j2}$$

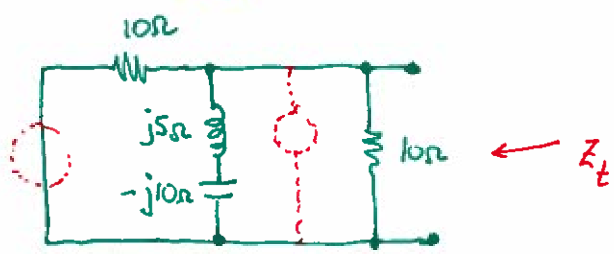
Let's switch to polar to make division easier

$$\bar{V}_A = \frac{139.9\angle 30.6^\circ}{2.828\angle 45^\circ} = 49.4\angle -14.4^\circ$$

$$\text{so} \quad \bar{V}_t = \bar{V}_A = 49.4\angle -14.4^\circ$$

$$\text{and} \quad v(t) = 49.4\cos(50t - 14.4^\circ)$$

Now find Z_t. Good news for this example – no dependent sources! OK to zero the independent ones.



Series-parallel impedance combinations

$$Z_t = 10\Omega \,//\, (j5\Omega - j10\Omega) \,//\, 10\Omega$$

$$= 10\Omega \,//\, -j5\Omega \,//\, 10\Omega$$

$$Z_t = \left[\frac{1}{10} - \frac{1}{j5} + \frac{1}{10}\right]^{-1}$$

$$= \left[\frac{1}{5} - \frac{1}{j5}\right]^{-1} = \left[\frac{j5 - 5}{j25}\right]^{-1}$$

$$\text{so} \quad Z_t = \frac{j25}{-5 + j5} \times \frac{-5 - j5}{-5 - j5} = \frac{-j125 - j^2 125}{25 + j25 - j25 - j^2 25}$$

$$= \frac{125 - j125}{50} = \frac{5}{2} - j\frac{5}{2}\ \Omega$$

This is a resistor and capacitor in series

negative indicates capacitance.

Frequency-dependent circuits

The frequency-dependent nature of inductors and capacitors opens the way to a wide number of applications of AC circuits.