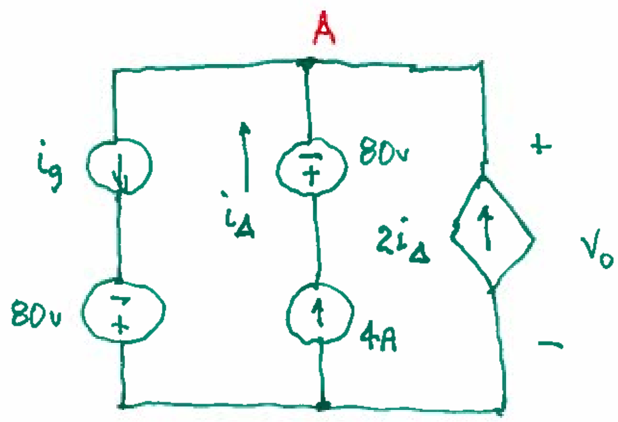
Friday, January 22, 2016

Example 2: For the following circuit, set $V_o = 100$ v, and find the total power in the circuit using KVL, KCL.



Solution: We need all voltages and currents

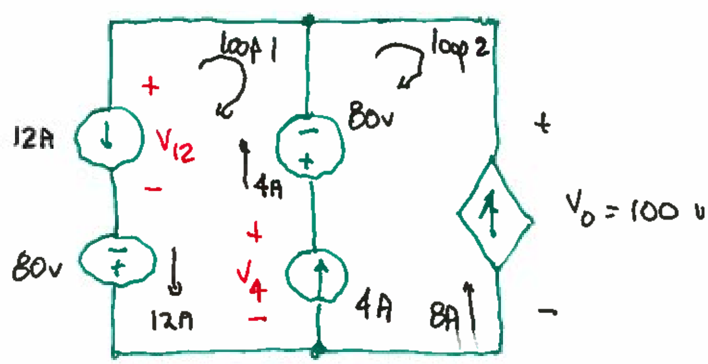
Apply KCL at node A: $i_\Delta + 2i_\Delta - i_g = 0$

We know $i_\Delta = 4$ A

so $4 + 2 \times 4 - i_g = 0$

$\therefore i_g = 12$ A

Redraw and add our own labels for loops and unknown voltages



We know all the currents; have two unknown voltages V_{12}, V_4.

KVL around loop 2: $-V_4 + 80 + 100 = 0$

$\therefore V_4 = 180$ v

KVL around loop 1: $80 - V_{12} - 80 + V_4 = 0$

$\therefore V_{12} = V_4 = 180$ v.

Now find power (using passive reference convention)

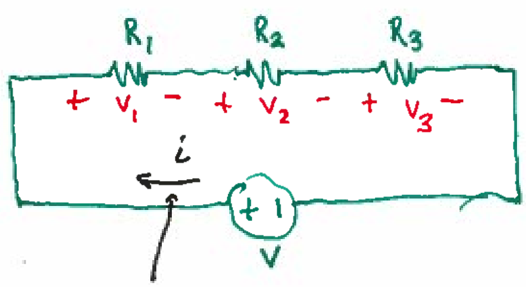
element	power
12 A	$p = vi = v_{12} \times 12 = 2160$ W
left 80 v	$p = -vi = -80 \times 12 = -960$ W
middle 80 v	$p = vi = 80 \times 4 = 320$ W
4A	$p = -vi = -v_4 \times 4 = -720$ W
dependent	$p = -vi = -100 \times 8 = -800$ W
	0 W (energy balance!)

RESISTIVE CIRCUITS

KVL, KCL, and Ohm's Law give us all the tools we need to begin circuit analysis.

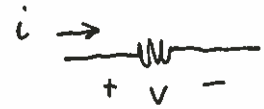
Resistances in series and parallel

Series resistances:



common current i.

Ohm's Law



$v = iR$

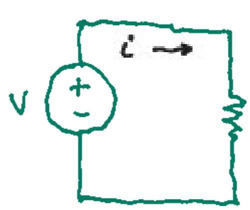
By KVL, we have

$$-V + v_1 + v_2 + v_3 = 0$$

$$-V + iR_1 + iR_2 + iR_3 = 0$$

$$\text{so} \quad V = i(R_1 + R_2 + R_3)$$

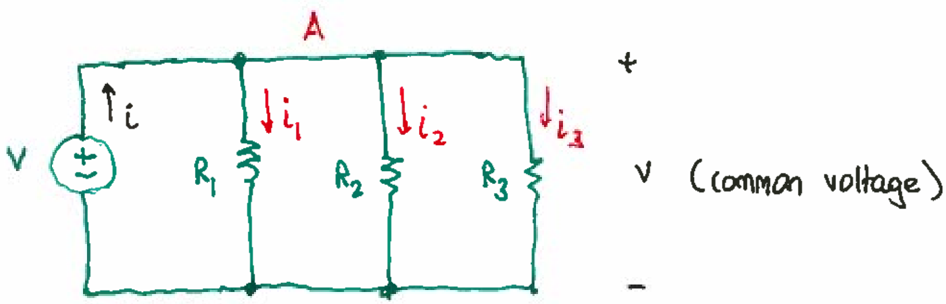
Can replace with a single equivalent resistance R_{eq}



$R_{eq} = R_1 + R_2 + R_3$

→ $\boxed{\text{Resistances in series add}}$

Parallel resistances:



By KCL at node A, $i - i_1 - i_2 - i_3 = 0$

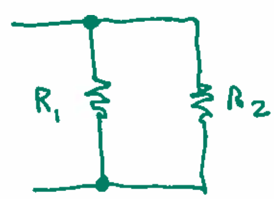
and Ohm's Law, $i - \frac{v}{R_1} - \frac{v}{R_2} - \frac{v}{R_3} = 0$

so $i = v\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) = v\left(G_1 + G_2 + G_3\right)$

→ $\boxed{\text{Conductances in parallel add}}$

or $v = i\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)^{-1}$

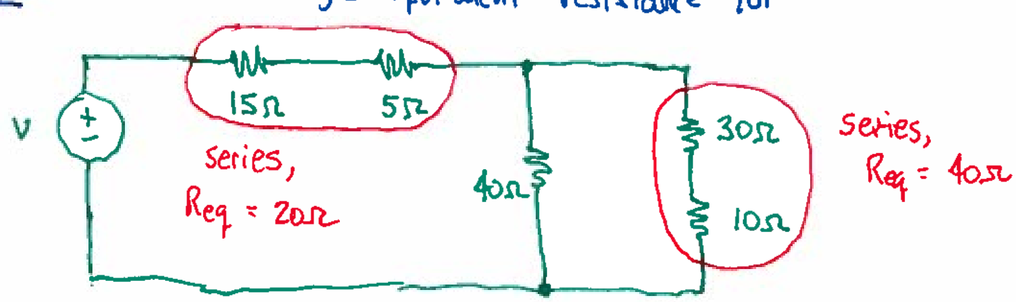
Very common resistor configuration:



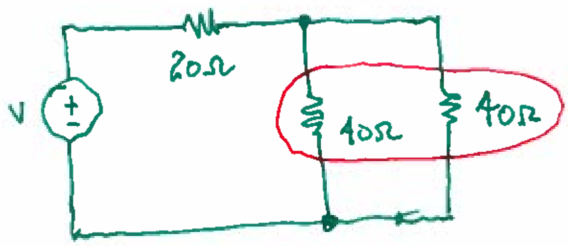
$$R_{eq} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$$

$$= \frac{R_1 R_2}{R_1 + R_2}$$

Example: Find a single equivalent resistance for



$\Downarrow$



parallel, $R_{eq} = \dfrac{40 \times 40}{40 + 40}$

$= 20\Omega$

$\Downarrow$

series

Final $R_{eq} = 20 + 20 = 40\Omega$