

Reference node:

- This is our zero-volt reference; all node voltages are relative to the reference node.
- Always makes the job easier by choosing one side of a voltage source.

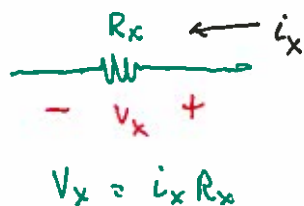
Note that reference node as above, node voltage V_1 automatically becomes

$$V_1 = V_s$$

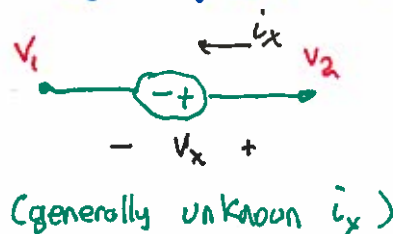
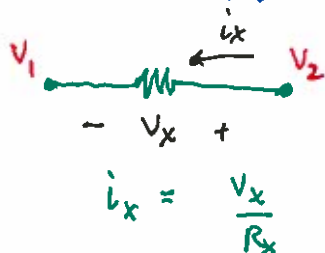
Step 2 Apply KCL at nodes

Node voltages vs. branch voltages

So far, we have always labeled branch voltages for circuit elements



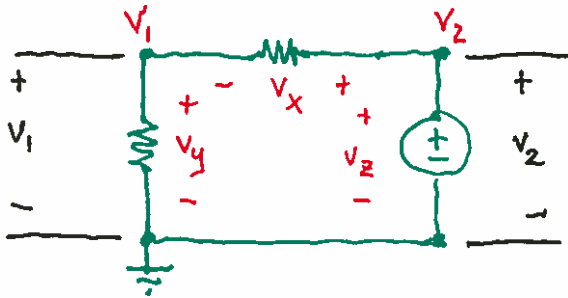
Here, node voltages are our only variables, so we must express V_x and i_x in terms of node voltages, V_1 and V_2 .



Common-sense interpretation: Voltage V_2 appears to be at a higher potential than V_1 . Therefore, the branch voltage V_x is the difference between the higher voltage (V_2) and lower voltage (V_1).

$$V_x = V_2 - V_1$$

KVL interpretation: Consider a loop



KVL in terms of branch voltages

$$-V_y - V_x + V_z = 0$$

But: $V_1 = V_y$, and $V_2 = V_z$

$$\text{so } -V_1 - V_x + V_2 = 0$$

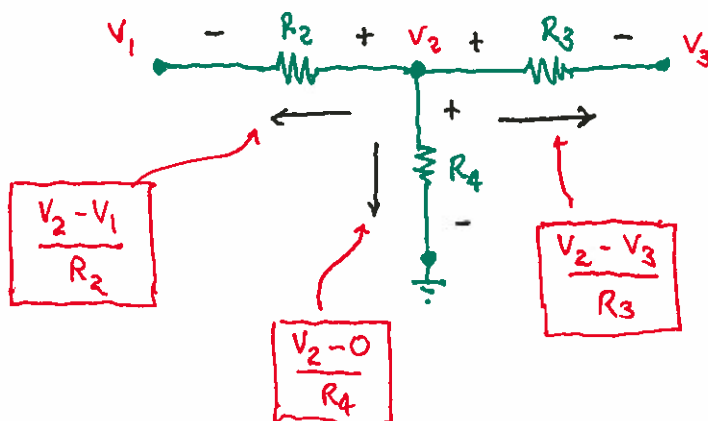
$$V_x = V_2 - V_1$$

In a resistor:

$$i_x = \frac{V_2 - V_1}{R_x}$$

current in terms of node voltages

Now consider node V_2 in our original circuit



By convention, always point the arrows away from node of interest

Note: each boxed current term is found using KVL and Ohm's Law.

Now sum currents at node 2 by KCL

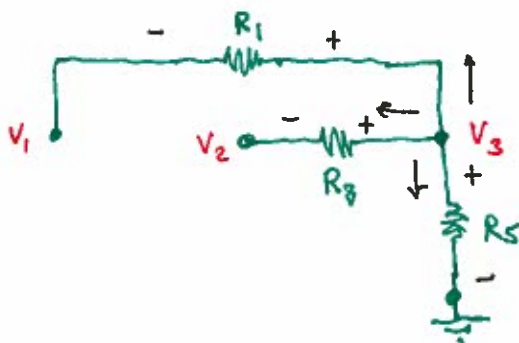
$$-\left(\frac{V_2 - V_1}{R_2}\right) - \left(\frac{V_2 - 0}{R_4}\right) - \left(\frac{V_2 - V_3}{R_3}\right) = 0$$

or, more conveniently, multiply by -1 to get

$$\boxed{\frac{V_2 - V_1}{R_2} + \frac{V_2 - 0}{R_4} + \frac{V_2 - V_3}{R_3} = 0}$$

NODE EQUATION
FOR NODE 2.

Now, forget about node 2, and repeat analysis at node 3.



← currents pointing away
from node 3.

$$\boxed{\frac{V_3 - V_1}{R_1} + \frac{V_3 - V_2}{R_3} + \frac{V_3 - 0}{R_5} = 0}$$

NODE EQUATION
FOR NODE 3

Notice the pattern when writing a node equation with resistor branches

$$\frac{V_3 - V_1}{R_1} + \frac{V_3 - V_2}{R_3} + \frac{V_3 - 0}{R_5} = 0$$

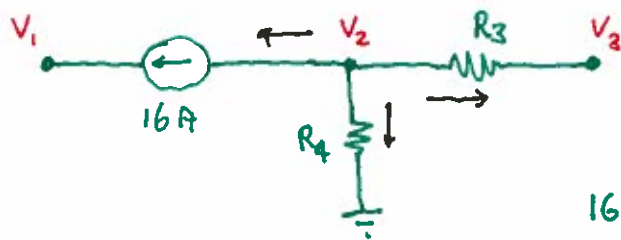
node of interest comes first

subtract connecting node

connecting resistance

- All terms in this equation represent current leaving a node, summed to zero by KCL (and multiplied by -1 for convenience!)

Things other than resistors connected to nodes:



$$16 + \frac{V_2 - 0}{R_4} + \frac{V_2 - V_3}{R_3} = 0$$

a constant
(positive if leaving a node)