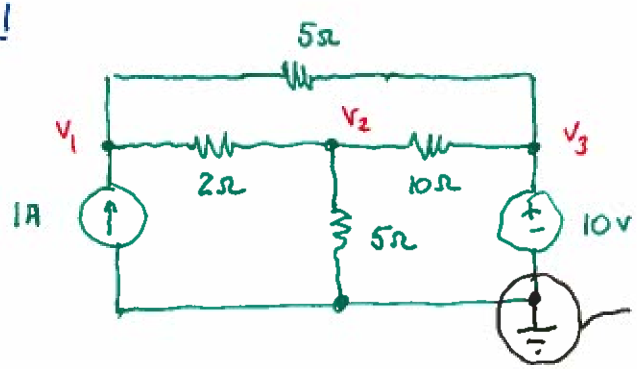
Example 1

Friday, January 29, 2016



reference node attached to a voltage source.

Looks like 3 nodes, with one (V_3) fixed at 10V.

We may write

1A forced into node 1.

Node V_1: $\dfrac{V_1 - V_2}{2} + \dfrac{V_1 - 10}{5} + (-1) = 0$

Node V_2: $\dfrac{V_2 - V_1}{2} + \dfrac{V_2 - 0}{5} + \dfrac{V_2 - 10}{10} = 0$

Can simplify to standard form:

node 1 ($\times 10$) $\quad 5V_1 - 5V_2 + 2V_1 - 20 + (-10) = 0$

$$7V_1 - 5V_2 = 30 \qquad (1)$$

node 2 ($\times 10$) $\quad 5V_2 - 5V_1 + 2V_2 + V_2 - 10 = 0$

$$-5V_1 + 8V_2 = 10 \qquad (2)$$

Solving ...

From (1), $\quad V_1 = \dfrac{30 + 5V_2}{7} \qquad (3)$

And from (2) $\quad -5\left(\dfrac{30 + 5V_2}{7}\right) + 8V_2 = 10$

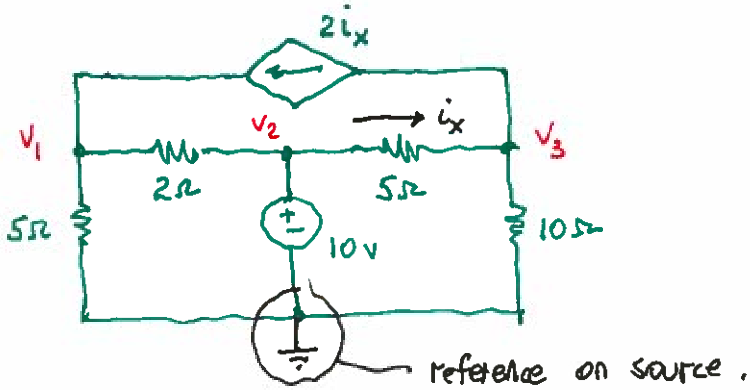
$$-\frac{150}{7} - \frac{25}{7}V_2 + 8V_2 = 10$$

$$4.429\ V_2 = 31.429$$

$$\text{so } V_2 = 7.097\ \text{V}$$

And from (3), $\quad V_1 = 9.35$

Example 2: Solve for node voltages



Three nodes again, one fixed by the voltage source $V_2 = 10v$.
At nodes 1, 3:

Node 1: $\dfrac{V_1 - 0}{5} + \dfrac{V_1 - V_2}{2} + (-2i_x) = 0$

Node 3: $\dfrac{V_3 - 0}{10} + \dfrac{V_3 - V_2}{5} + 2i_x = 0$

this will be $-i_x$

Simplifying with $V_2 = 10$

1 (x 10) $2V_1 + 5(V_1 - 10) - 20i_x = 0$

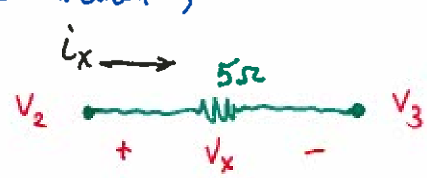
$7V_1 - 50 - 20i_x = 0$ (1)

2 (x 10) $V_3 + 2(V_3 - 10) + 20i_x = 0$

$3V_3 - 20 + 20i_x = 0$ (2)

We also have the dependent current source's controlling current i_x.
Need to express this in terms of node voltages.

For the direction indicated,



$$i_x = \frac{V_x}{5} = \frac{V_2 - V_3}{5} \quad (3)$$

Substitute (3) into (1) and (2)

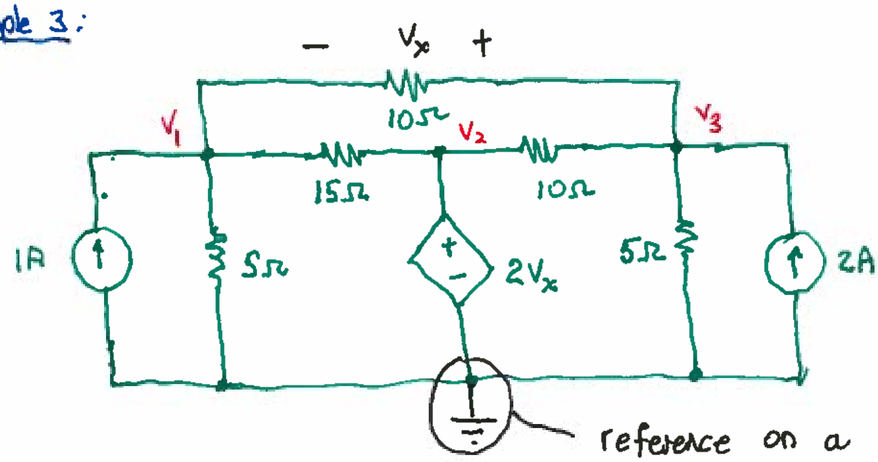
$$7V_1 - 50 \quad - 4(10 - V_3) = 0$$
$$7V_1 + 4V_3 = 90$$
$$3V_3 - 20 + 4(10 - V_3) = 0$$
$$-V_3 = -20$$

equations fully in terms of node voltages.

so $V_3 = 20$ V

and $V_1 = 10/7$ V.

Example 3:



reference on a dependent voltage source — same reason!

Three node voltages, where we know $V_2 = 2V_x$.

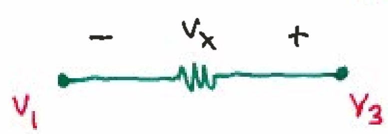
By rearranging, we see another important pattern in this equation:

Node 1: $$-1 + \frac{V_1}{5} + \frac{V_1 - V_2}{15} + \frac{V_1 - V_3}{10} = 0$$

$$\underbrace{V_1\left(\frac{1}{5} + \frac{1}{15} + \frac{1}{10}\right)}_{\text{sum of all conductances connected to node 1}} - V_2\underbrace{\left(\frac{1}{15}\right)}_{\text{conductance connecting nodes 1 and 2}} - V_3\underbrace{\left(\frac{1}{10}\right)}_{\text{conductance connecting nodes 1 and 3}} \underbrace{-1}_{\text{current sources connected to node 1.}} = 0$$

Node 3: $$\frac{V_3 - V_1}{10} + \frac{V_3 - V_2}{10} + \frac{V_3}{5} - 2 = 0$$

And we also have the dependence.



$$V_x = V_3 - V_1$$