Friday, March 18, 2016

Summary of phasor summation method

1. Express cosine functions as phasors
2. Add the phasors
3. Convert the result, if necessary, back to a cosine function.

Example 1: Determine $v(t) = v_1(t) + v_2(t)$ and draw a phasor diagram.

where: $v_1(t) = 5\sin(\omega t + 45^\circ)$

$v_2(t) = 10\cos(\omega t + 90^\circ)$

"Time-domain" representation (cosine function)	Phasor notation
$v_1(t) = 5\sin(\omega t + 45^\circ)$ $= 5\cos(\omega t + 45^\circ - 90^\circ)$	$\bar{V}_1 = 5\angle{-45^\circ}$
$v_2(t) = 10\cos(\omega t + 90^\circ)$	$\bar{V}_2 = 10\angle{90^\circ}$

Thus, $\bar{V} = \bar{V}_1 + \bar{V}_2 = 5\angle{-45^\circ} + 10\angle{90^\circ}$

$$5\angle{-45^\circ} = 5\cos(-45^\circ) + j5\sin(-45^\circ)$$
$$= 3.54 - j3.54$$

$$10\angle{90^\circ} = 10\cos(90^\circ) + j10\sin(90^\circ)$$
$$= j10$$

so $\bar{V} = 3.54 - j3.54 + j10$

$= 3.54 + j6.46$

back to polar:

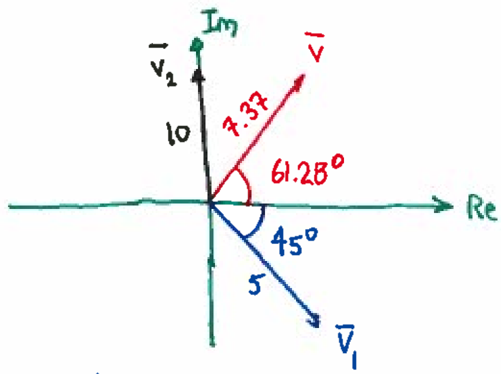
$$\sqrt{3.54^2 + 6.46^2}\ \angle \tan^{-1}\left(\frac{6.46}{3.54}\right)$$
$$= 7.37\angle{61.28^\circ}$$

Back to the time-domain expression

$v(t) = 7.37 \cos(\omega t + 61.28°)$

A common misconception: $\bar{V} = 3.54 + j6.46$. The voltage is not a complex number. The voltage is a real-valued cosine with a magnitude of 7.37 and a phase angle of 61.28°.

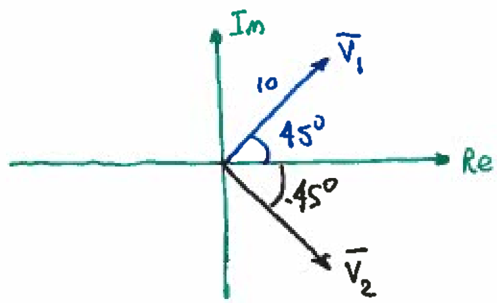
The phasor diagram:



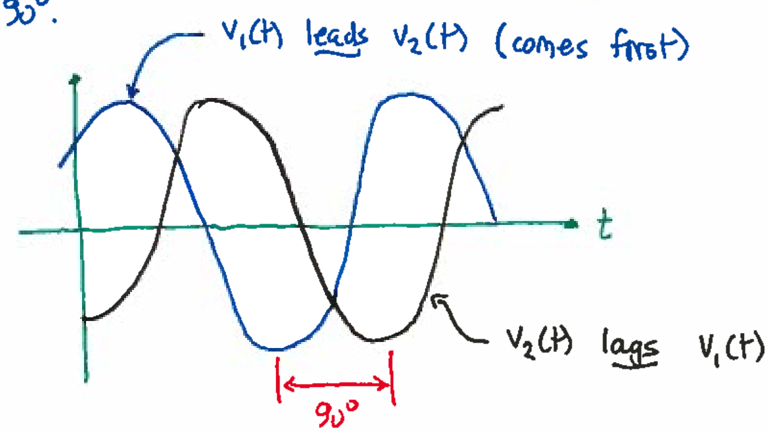
Phase relationship between sinusoids – important terminology

Let $v_1(t) = 10 \cos(\omega t + 45°) \longleftrightarrow \bar{V}_1 = 10 \angle 45°$

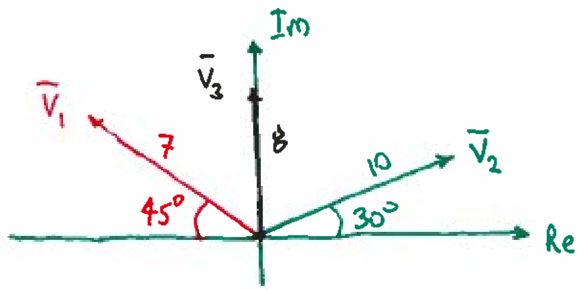
$v_2(t) = 8 \cos(\omega t - 45°) \longleftrightarrow \bar{V}_2 = 8 \angle -45°$



We say that $v_1(t)$ is 90° higher in phase than $v_2(t)$, and therefore $v_1(t)$ leads $v_2(t)$ by 90°; likewise, $v_2(t)$ lags $v_1(t)$ by 90°.



Example 2: Consider the phasor diagram below and let $f = 100$ Hz. Express each phasor voltage in the time domain as $V_m \cos(\omega t + \theta)$



We have $\bar{V}_1 = 7 \angle 180° - 45° = 7 \angle 135°$

always use angular distance from the positive real axis.

and $\bar{V}_2 = 10 \angle 30°$

$\bar{V}_3 = 8 \angle 90°$

We also have $f = 100$ Hz, so $\omega = 2\pi f = 200\pi$ rad/sec.

Therefore,
$$v_1(t) = 7 \cos(200\pi t + 135°)$$
$$v_2(t) = 10 \cos(200\pi t + 30°)$$
$$v_3(t) = 8 \cos(200\pi t + 90°)$$

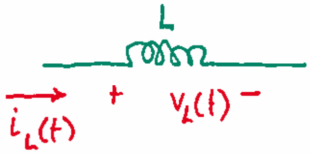
Note that:
- $v_1(t)$ leads $v_2(t)$ by 105° and leads $v_3(t)$ by 45°
- $v_3(t)$ leads $v_2(t)$ by 60° and lags $v_1(t)$ by 45°.

Complex impedances

Now we need to revisit the voltage-current relationship for resistors, capacitors, and inductors when voltages and currents are sinusoidal.

Inductance

Recall for the inductor



$$v_L(t) = L \frac{d i_L(t)}{dt}$$

Where we have $i_L(t) = I_m \cos(\omega t + \theta)$. The voltage will be

$$v_L(t) = L\, I_m \frac{d}{dt}\left[\cos(\omega t + \theta)\right]$$