SCHULICH
School of Engineering
UNIVERSITY OF CALGARY

ENGG 225

Fundamentals of Electrical Circuits and Machines

Laboratory #2
DC Resistive Circuits

1 Introduction

In this laboratory, you will get some hands-on laboratory experience in the design and testing of some simple and important circuit configurations. The circuits contain DC sources and resistors. The design exercises involve direct application of Kirchhoff's voltage and current laws, Ohm's law, and more high-level circuit analysis methods such as the node-voltage and mesh-current methods.

1.1 Learning Outcomes

At the end of this laboratory session, you will be able to design and analyze the following resistive circuits:

- a voltage divider circuit that provides three outputs from a series connection of resistors, where each output has a prescribed voltage level;
- a current divider circuit, with prescribed current levels in each of three parallel resistor branches;
- a Wheatstone bridge circuit, for which you will explore a practical application in estimating temperatures from bridge voltage measurements.

1.2 Acknoweldgments

The course instructors for ENGG 225 greatly acknowledge the University of Calgary Engineering Endowment Fund for substantial funding toward the purchase of important high-quality laboratory test equipment and supplies needed to enhance the learning value of the laboratories in the course.

2 Background Review

2.1 The Voltage Divider

Consider the simple circuit shown in Fig. 1, which contains a series connection of two resistors and a voltage source.

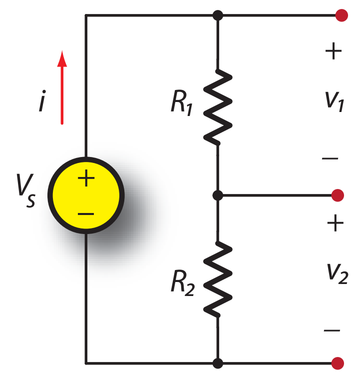

Fig. 1. A simple series resistor circuit

The total current i flowing in the circuit is given by:

$$i = \frac{V_s}{R_1 + R_2} \tag{1}$$

The voltages v_1 and v_2 across the resistors are therefore

$$v_1 \;=\; iR_1 \;=\; \left(\frac{V_s}{R_1 + R_2}\right) R_2 \;=\; \left(\frac{R_1}{R_1 + R_2}\right) V_s \tag{2}$$

and

$$v_2 = \left(\frac{R_2}{R_1 + R_2}\right) V_s. \tag{3}$$

Notice that the source voltage V_s is divided among the series resistors in direct proportion to their resistances. This is the *principle of voltage division*, and the circuit of Fig. 1 is called a *voltage divider*. This is a very common and useful circuit configuration.

In general, if a voltage divider has N series-connected resistors (R_1, R_2, …, R_N) in series with the voltage source V_s, the nth resistor R_n will have a voltage drop of

$$v_n = \left(\frac{R_n}{R_1 + R_2 + \ldots + R_N}\right) V_s. \tag{4}$$

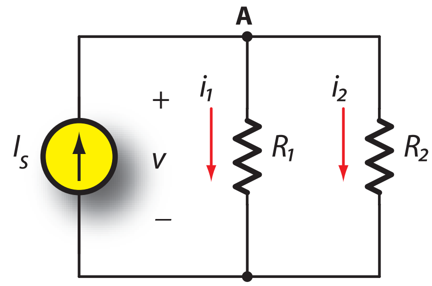

Fig. 2. A simple parallel resistor circuit

2.2 The Current Divider

Consider the simple circuit shown in Fig. 2, which contains a parallel connection of two resistors and a current source. By Ohm's Law for this circuit,

$$v = i_1 R_1 = i_2 R_2 \tag{5}$$

or

$$i_1 = \frac{v}{R_1}, \quad i_2 = \frac{v}{R_2} \tag{6}$$

Applying KCL at node **A**, we get

$$I_s = \frac{v}{R_1} + \frac{v}{R_2} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right) v = \frac{v}{R_{eq}} \tag{7}$$

where R_{eq} is the resistance of the two resistors in parallel and is given by

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}. \tag{8}$$

For a circuit such as this with two parallel resistors, equation (8) can be written as

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}. \tag{9}$$

It is often convenient to use *conductances* G_i rather than resistances R_i, where $G_i = R_i^{-1}$, when dealing with resistors in parallel. Equation (8) simplifies to

$$G_{eq} = G_1 + G_2. \tag{10}$$

From equation (7),

$$I_s = vG_{eq}, \quad \text{so} \quad v = \frac{I_s}{G_{eq}} = \frac{I_s}{G_1 + G_2}. \tag{11}$$

Combining equations (11) and (6), we obtain

$$i_1 = vG_1 = \left(\frac{G_1}{G_1 + G_2}\right) I_s, \quad \text{and} \quad i_2 = \left(\frac{G_2}{G_1 + G_2}\right) I_s. \tag{12}$$

Notice that the source current I_s is divided among the parallel resistors in direct proportion to their conductances. This is the *principle of current division*, and the circuit of Fig. 2 is called a *current divider*. Like the voltage divider, this is also a very common and useful circuit configuration.

In general, if a current divider has N parallel-connected resistors (R_1, R_2, ..., R_N) in parallel with the current source I_s, the nth resistor R_n will have a current of

$$i_n = \left(\frac{G_n}{G_1 + G_2 + \ldots + G_N} \right) I_s. \tag{13}$$

2.3 The Wheatstone Bridge

A resistor circuit called a *Wheatstone bridge* is shown in Fig. 3. The voltage between points **a** and **b** of the bridge is sensitive to a change in resistance of any one of the resistors in the circuit. Therefore, the bridge converts a change of resistance into a measurable voltage. There are many important practical engineering applications for the Wheatstone bridge. One common application is in the measurement associated with a transducer that undergoes a resistance change due to an external physical change. Examples of these are strain gauges (measurement of mechanical strain), thermistors (measurement of temperature) and photo-resistors (measurement of illumination).

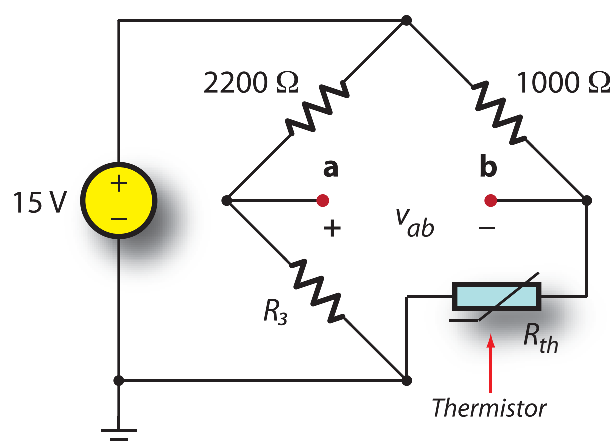

Fig. 3. Wheatstone bridge resistor circuit

Although many students find the appearance of this circuit somewhat intimidating, it can be analyzed in a straightforward manner using, for example, the node-voltage or mesh-current methods.

3 Pre-Lab Exercises

The pre-lab work must be completed prior to the laboratory session and will be checked prior to beginning the experimental exercises. *Without the required pre-lab work, there is not enough information to even attempt the experiment.*

3.1 Design the Voltage Divider

Design the voltage divider circuit shown in Fig. 4 such that it produces no-load voltages of +12V, +7V and +2V with respect to the reference node, as indicated. Choose a total series resistance in the circuit such that 500 mW of power is dissipated.

The output voltages should be accurate to within ±5% assuming that the resistors that you are provided are exact in value. *Use only resistor values that are listed in Table I.* For any values not in this table, determine an appropriate series-parallel combination of resistors from the table. (Note that there are many possible solutions for this problem.) For convenience, the resistor color-code chart is repeated in Appendix A.

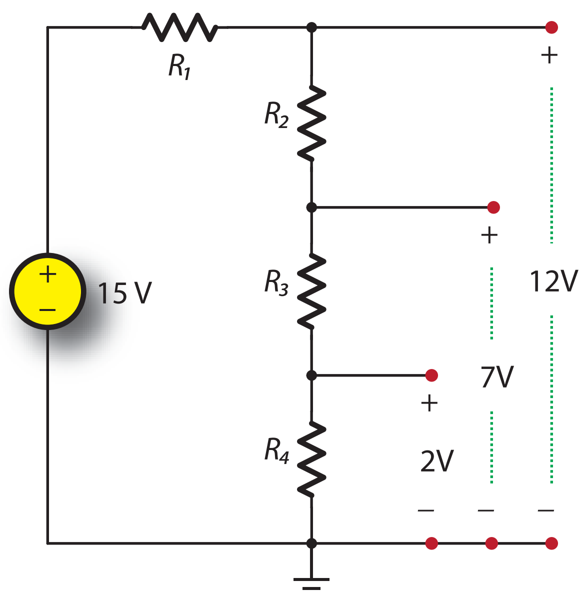

Fig. 4. Design this voltage divider circuit

3.2 Design the Current Divider

Design the current divider circuit shown in Fig. 5 such that it produces the branch currents of 16.67 mA, 10 mA, and 6.67 mA, as indicated. Since a true current source is not available, it is implemented here as a 15-volt source in series with a 100 Ω resistor. It is therefore necessary to choose R_1, R_2, and R_3 so that the total supplied current is $I_s = 33.33$ mA, and the three branch currents are as shown.

Table I - Available Resistors

100 Ω	390 Ω	1 $K\Omega$	8.2 $K\Omega$
120 Ω	470 Ω	1.2 $K\Omega$	10 $K\Omega$
220 Ω	680 Ω	2.2 $K\Omega$	51 $K\Omega$
330 Ω	820 Ω	4.7 $K\Omega$	100 $K\Omega$

The branch currents should be accurate to within ±5% assuming that the resistors that you are provided are exact in value. Use only resistor values that are listed in Table I and appropriate series-parallel combinations of resistors to approximate the values you need.

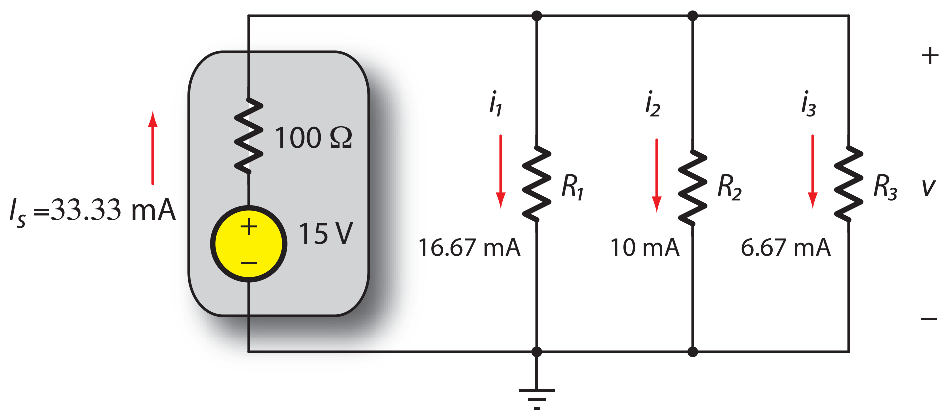

Fig. 5. Design this current divider circuit

3.3 Analyze the Wheatstone Bridge

Consider the Wheatstone bridge circuit in Fig. 6. Note the resistance in the lower-right branch of the circuit is a thermistor. The thermistor that you will use is designed to have a resistance of 75 Ω at 25°C (a Panasonic ERTD2FGL*750S device). This is the circuit that you will use to estimate a variety of temperatures.

1. Calculate the value of R_3 that would be required to balance or “null” the bridge (i.e., to cause $v_{ab} = v_a - v_b = 0$) in Fig. 6 at 25°C. With such a value of R_3, voltage readings of v_{ab} that are negative will indicate cooler temperatures, while positive readings will indicate warmer temperatures.

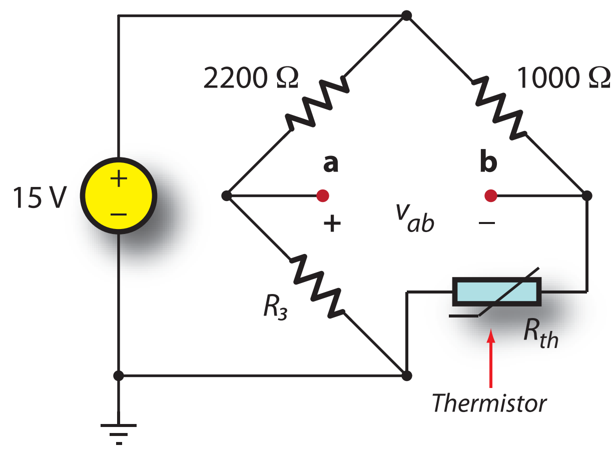

Fig. 6. Analyze this Wheatstone bridge circuit

2. Using your value for R_3 and the other component values given in Fig. 6, obtain an expression for v_{ab} in terms of R_{th}. Then, rearrange this equation to express R_{th} in terms of v_{ab}. You will use this to calculate R_{th} and and look up the corresponding temperature using the graph in Appendix B.

4 Procedure

One report per group is required, and must be completed during the lab session.

Breadboarding Circuits

The breadboard will be used to construct each circuit. For easy troubleshooting, we suggested that you lay out each circuit on the breadboard in the way it appears in the circuit diagram. For convenience, the diagram showing the internal electrical connection of the row-connected and column-connected sockets on the breadboard is reproduced from Lab #1 and given below in Fig. 7.

Recall that components may be connected together by inserting their leads into a common socket row or column. Wire jumpers may be used to connect socket rows or columns together.

4.1 Test the Voltage Divider

1. Construct the voltage divider circuit in Fig. 4 that you designed to give no-load voltages of +12V, +7V, and +2V. As in Lab #1, use the 15 V source on the breadboard. Using the DMM set to measure DC voltage, measure each of the output voltages.

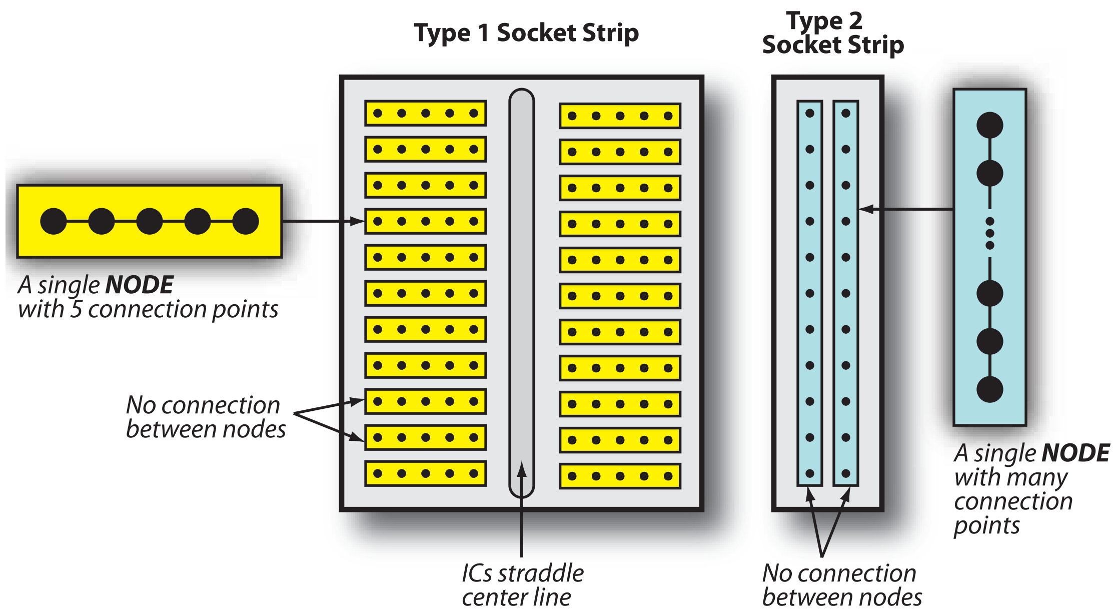

Fig. 7. Breadboard internal connections

> **IMPORTANT**: *Make sure the resistors that you obtain from the resistor drawers are correct in value. Check their color codes using the chart in the Appendix A, or use the DMM to measure their values. This can save you a lot of grief in debugging your circuit!*

2. Taking component tolerances and meter accuracy specifications into account, comment on the measured values compared with the design values.

4.2 Test the Current Divider

1. Construct the current divider circuit in Fig. 5 that you designed to give branch currents of 16.67 mA, 10 mA, and 6.67 mA.

2. Set up the DMM to measure current as you did in Lab #1. Measure each of i_1, i_2, and i_3 in your circuit by placing the multimeter *in series with* R_1, R_2, and R_3, respectively.

3. Taking component tolerances and meter accuracy specifications into account, comment on the measured values compared with the design values. Calculate and record the error in percent.

4.3 The Wheatstone Bridge

Connect the Wheatstone bridge circuit shown in Fig. 6 using a suitable series-parallel resistor combination for your pre-calculated value of R_3.

1. Measure and record v_{ab} and calculate R_{th}. Using R_{th}, estimate the ambient temperature in the lab by using the graph in Appendix B. *(You may notice that this graph is somewhat difficult to read, as both the resistance and temperature axes use a logarithmic scale. In this, and the measurements below, just give your best "eye-ball" estimates of temperature!)*

2. Firmly squeeze the thermistor between your thumb and index finger. Watch v_{ab} and record its value after it stabilizes (takes a few seconds). Estimate the temperature of your fingertips.

> **SAFETY WARNING**: *Be very careful with the cold spray in Step 3, below. If it comes in contact your skin, your skin can freeze instantly! The temperature of the spray can be as cold as -52^oC.*

3. Use the can of cold spray to *gently* chill the thermistor. The spray is extremely cold, so all you should need is a *really short burst* (a quarter-second at most). Avoid over-chilling the thermistor, because you will not be able to use the graph in Appendix B to estimate temperatures below -20^oC.

   - Hold the nozzle about 10 cm (4 in) away from the thermistor, and give a *very short burst* using the spray button. Make a measurement.
   - Repeat the measurement by changing the distance (either closer or further away), and give another *very short burst* of spray.

   Estimate the temperature on both measurements.

4. Use the electric hairdryer on the *low setting* to heat the thermistor. Try to direct the heat just at the thermistor without pointing the hairdryer downward to unnecessarily heat up the breadboard. Make a measurement, and estimate the temperature of the air that the hairdryer is blowing.

Appendix A - Resistor Color Codes

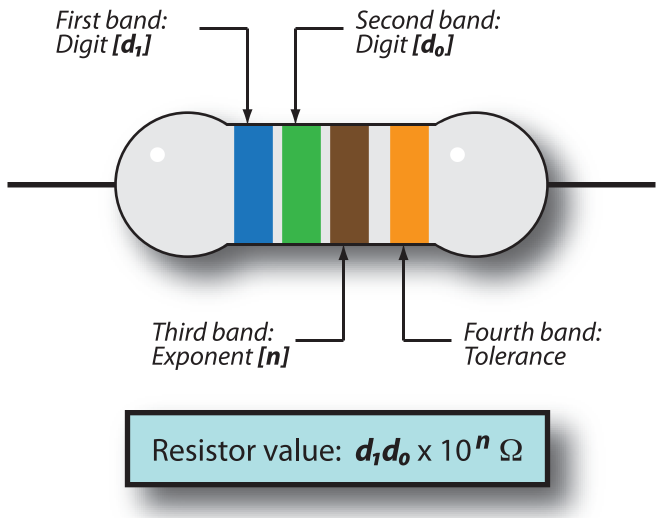

Fig. 8. Interpretation of resistor color bands

Table II - Resistor Color Codes

Color	1st Band - d_1	2nd Band - d_0	3rd Band - n $[10^n]$	4th Band - Tolerance
black	0	0	0 [1]	-
brown	1	1	1 [10]	±1%
red	2	2	2 [100]	±2%
orange	3	3	3 [1,000]	-
yellow	4	4	4 [10,000]	-
green	5	5	5 [100,000]	±0.5%
blue	6	6	6 [1,000,000]	±0.25%
violet	7	7	7 [10,000,000]	±0.1%
gray	8	8	8 [100,000,000]	-
white	9	9	9 [1,000,000,000]	-
gold	-	-	-1 [0.1]	±5%
silver	-	-	-2 [0.01]	±10%

Example:

Color bands are, from left-to-right, **Blue, Green, Brown, Gold**.

From the table above, $d_1 = 6$, $d_0 = 5$, $n = 1$, so that $R = 65 \times 10^1 = 650\Omega$, $\pm 5\%$.

Appendix B - Thermistor Operating Characteristic

The resistance of the thermistor is graphed in Fig. 9 as a function of temperature for a number of Panasonic's thermistor types. The particular thermistor that we are using has a resistance of 75 Ω at 25°C. Use the curve labeled "1".

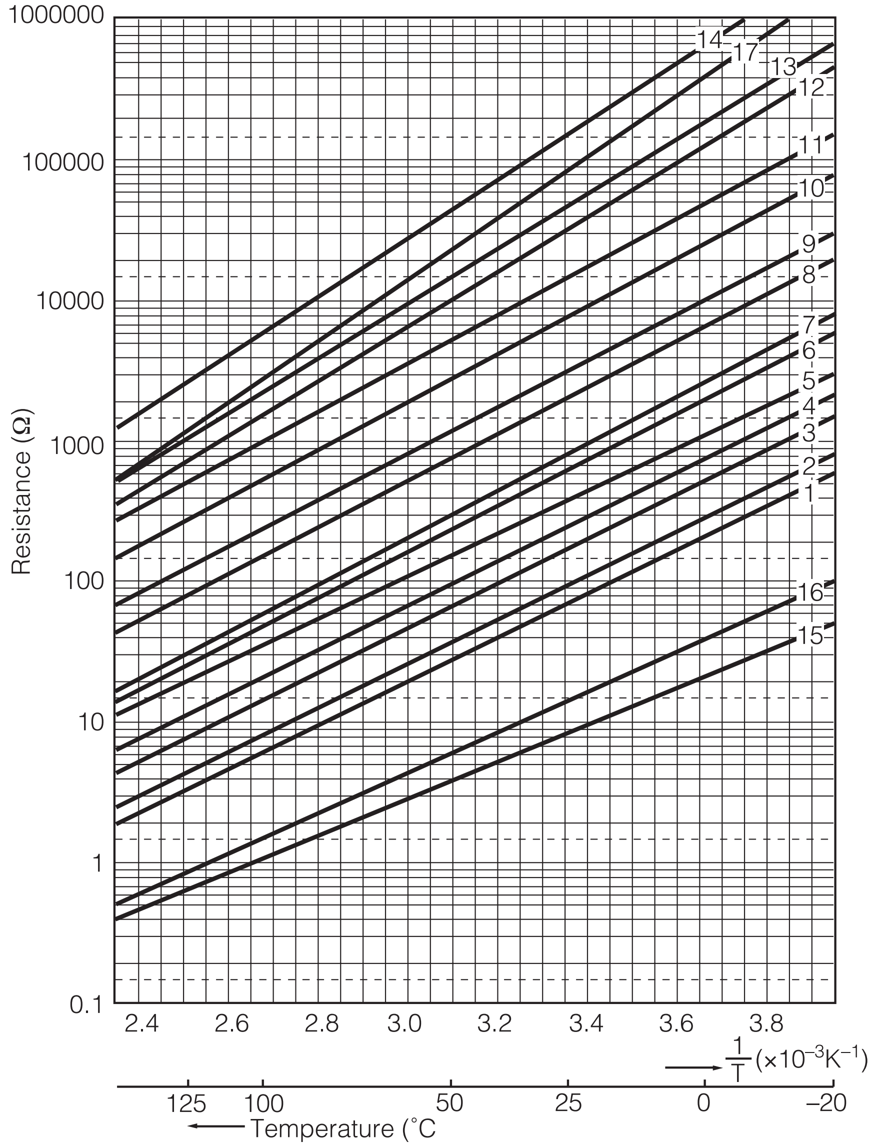

Fig. 9. Thermistor resistance vs. temperature; use curve "1"