So for the dependent source, $V_2 = 2V_x = 2(V_3 - V_1)$ (3)

Now solve for unknowns. Multiply (1) by 30

Monday, February 1, 2016.

$$V_1(6+2+3) - V_2(2) - V_3(3) = 30$$

$$11V_1 - 2V_2 - 3V_3 = 30$$

From (3), $$11V_1 - 2[2(V_3 - V_1)] - 3V_3 = 30$$

$$11V_1 - 4V_3 + 4V_1 - 3V_3 = 30$$

$$15V_1 - 7V_3 = 30 \qquad (4)$$

Multiply (2) by 10

$$-V_1 - V_2 + V_3(1+1+2) = 20$$

$$-V_1 - V_2 + 4V_3 = 20$$

From (3), $$-V_1 - 2(V_3 - V_1) + 4V_3 = 20$$

$$V_1 + 2V_3 = 20 \qquad (5)$$

From equations (3), (4), (5),

$$V_1 = 5.405\text{ v}$$

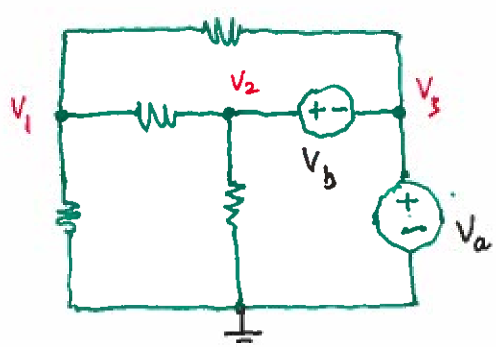
$$V_3 = 7.297\text{ v}$$

$$V_2 = 2(V_3 - V_1) = 3.784\text{ v}$$

Node-voltage method ~ special case

There is only one special case we need to handle, where we have voltage sources between nodes where neither node is a reference node

First, the easy case when voltage sources are connected directly to other voltage sources.



Notice that $V_3 = V_a$

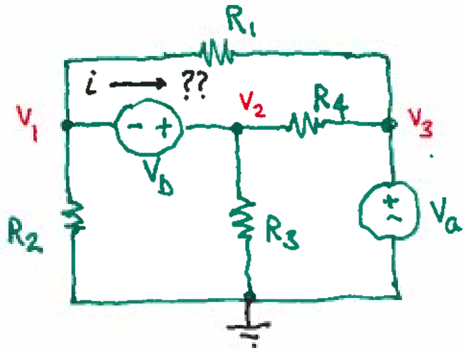
and also that $V_2 - V_3 = V_b$

giving $V_2 = V_3 + V_b$

$$V_2 = V_a + V_b$$

- V_2 and V_3 are known! Don't need to write equations there.
- leaves one unknown (and one equation) at node 1.

Now the trickier case when sources not directly connected to each other.



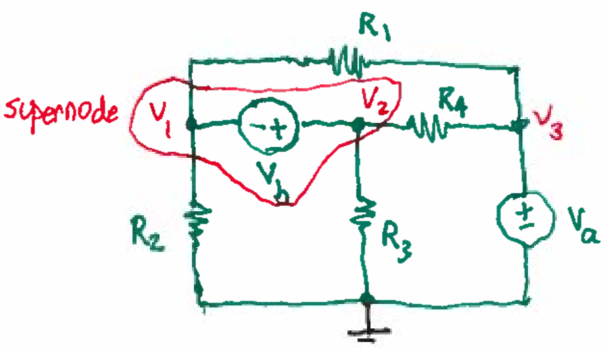
Note $V_3 = V_a$

Recall in writing node equations, we sum currents leaving nodes.
But what's i?

node 1: $\dfrac{V_1}{R_2} + \dfrac{V_1 - V_3}{R_1} + i = 0$ (1)

another unknown, along with V_1, V_2 (and we can't ignore it!)

The problem is handled by the concept of a supernode



Consider node 2 of the original circuit

$$\frac{V_2}{R_3} + \frac{V_2 - V_3}{R_4} - i = 0 \qquad (2)$$

Now, let's eliminate i by adding equations (1) and (2)

$$\frac{V_1}{R_2} + \frac{V_1 - V_3}{R_1} + i + \frac{V_2}{R_3} + \frac{V_2 - V_3}{R} - i = 0$$

The result is a supernode equation, which we can write in one step.

$$\boxed{\underbrace{\frac{V_1}{R_2} + \frac{V_1 - V_3}{R_1}}_{\text{left side of supernode}} + \underbrace{\frac{V_2}{R_3} + \frac{V_2 - V_3}{R_4}}_{\text{right side of supernode}} = 0}$$

SUPERNODE EQUATION

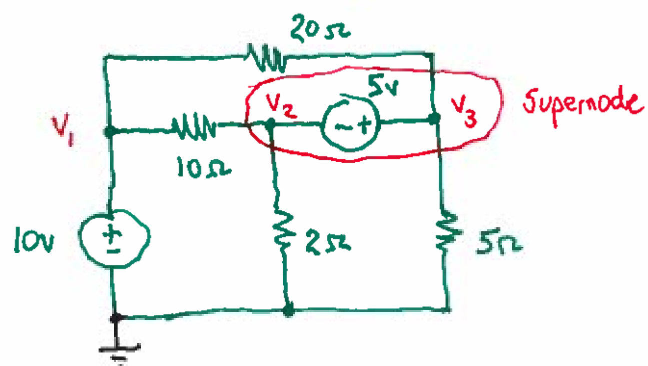
We also have a dependence equation for the two nodes within the supernode.

$$\boxed{V_2 - V_1 = V_b}$$

DEPENDENCE EQUATION

The result is that we still have two equations to describe the nodes V_1 and V_2: the supernode and dependence equations.

Example 1: Solve for node voltages



We know that $V_1 = 10v$.

Now write the supernode and dependence equation for V_2, V_3.

$$\underbrace{\frac{V_2 - V_1}{10} + \frac{V_2}{2}}_{V_2 \text{ side of supernode}} + \underbrace{\frac{V_3 - V_1}{20} + \frac{V_3}{5}}_{V_3 \text{ side of supernode}} = 0$$