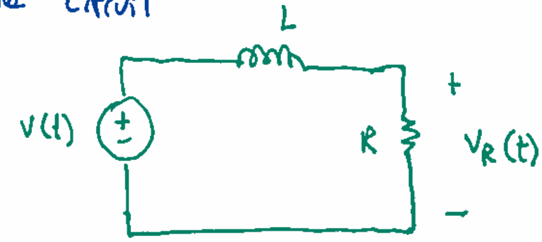
Frequency-dependent Circuits

Consider the circuit



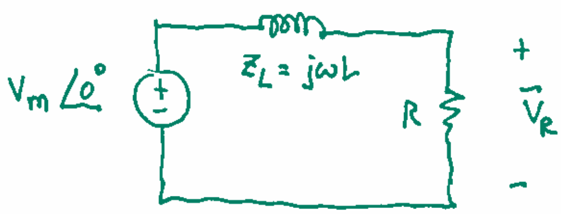
and let $v(t)$ be as follows

$$v(t) = V_m \cos(\omega t + 0^\circ)$$

V_m: constant value

ω: angular frequency, ω (keep as a variable)

In terms of phasors and complex impedances,



This is a simple voltage divider

$$\bar{V}_R = \frac{R}{R + j\omega L} \times V_m \angle 0^\circ$$

Z_L is frequency-dependent.

We may write $$\bar{V}_R = \frac{1}{1 + j\omega (L/R)} \times V_m$$

Here, the magnitude and phase of $\bar{V}_R$ are frequency-dependent. This dependence is called <u>frequency response</u>.

In the time domain,

$$v_R(t) = V_R \cos(\omega t + \theta_R)$$

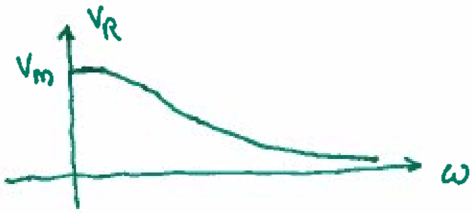
V_R: amplitude $|\bar{V}_R|$ at frequency ω

θ_R: phase of $v_R(t)$ at ω.

The amplitude $\quad |\vec{V}_R| = V_R = \left| \dfrac{1}{1 + j(\omega L/R)} \times V_m \right|$

$$V_R = \frac{V_m}{|1 + j\omega(L/R)|} = \frac{V_m}{\sqrt{(\omega L/R)^2 + 1^2}}$$

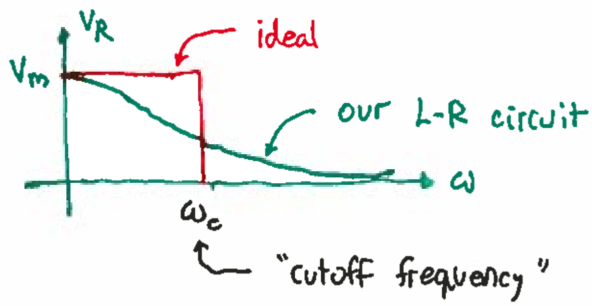
Graph this:



This is called a <u>lowpass filter</u>.

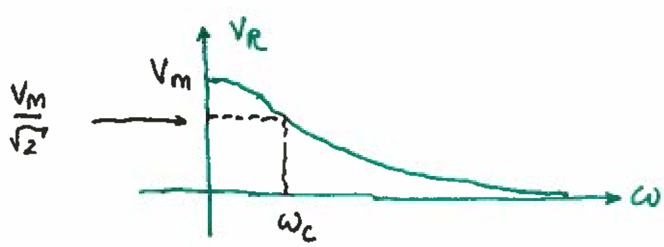
- Sinusoids with low frequencies come through strongly.
- higher frequencies come through at reduced amplitudes.

An ideal lowpass filter has the frequency response



For the ideal filter,
$$V_R = \begin{cases} V_m, & 0 \le \omega \le \omega_c \\ 0, & \omega > \omega_c \end{cases}$$

We usually define the cutoff frequency as the frequency at which $V_R = (1/\sqrt{2}) \times V_m$.



For our L-R circuit,

$$V_R = \frac{V_m}{\sqrt{(\omega_c L/R)^2 + 1}}$$

The cutoff frequency for this filter is when

$$V_R = \frac{V_m}{\sqrt{2}} = \frac{V_m}{\sqrt{(\omega_c L/R)^2 + 1}}$$

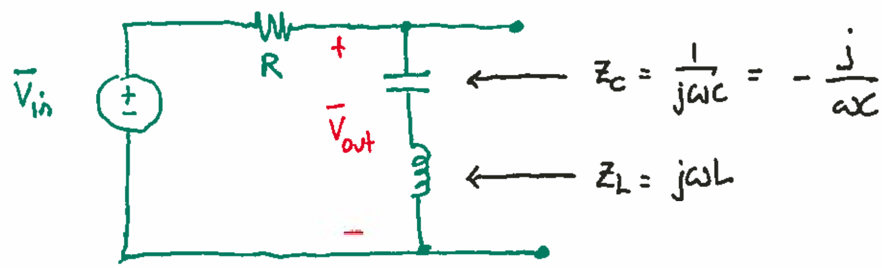
so $(\omega_c L/R)^2 + 1 = 2$

and $(\omega_c L/R) = 1$

Thus, $\omega_c = R/L$ rads/sec, and $f_c = \omega_c/2\pi$.

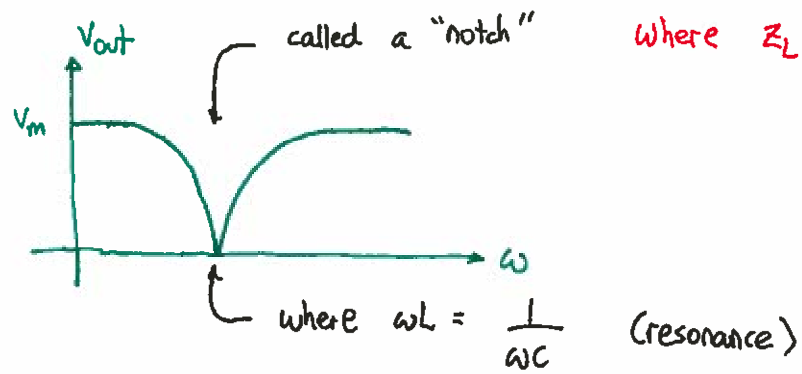
There are many other extremely useful filter circuits.

One such <u>bandstop</u> filter circuit that exploits the series cancellation of impedances is:



$$\bar{V}_{out} = \frac{Z_L + Z_C}{Z_L + Z_C + Z_R} \times \bar{V}_{in}$$

will cancel at the frequency where $Z_L = -Z_C$, so $\bar{V}_{out} = 0$



This is called "notch filter" and has lots of important applications.
E.g., "hum" in an audio system is due to 60 Hz power source.