Wednesday, March 30, 2016

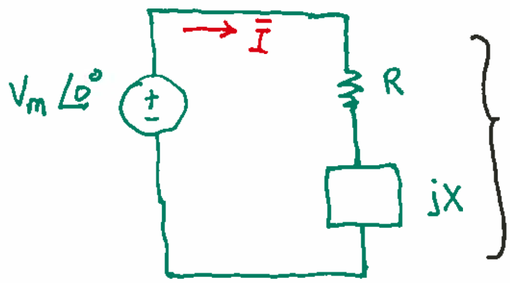
Superposition in AC circuits

As with other methods of AC circuit analysis, the procedure is identical, except using complex algebra!

- Important: this method is the only way to analyze circuits with sources with different frequencies.

Power in AC circuits

Consider an arbitrary complex impedance



load impedance

$$Z = R + jX$$

$$= |Z| \angle \theta$$

$$|Z| = \sqrt{R^2 + X^2}$$

$$\theta = \tan^{-1}\left(\frac{X}{R}\right)$$

R - resistive part

X - "reactive" part

We have phasor current $\bar{I}$

$$\bar{I} = \frac{V_m \angle 0°}{|Z| \angle \theta} = \frac{V_m}{|Z|} \angle -\theta$$

and let $I_m = \dfrac{V_m}{|Z|}$, so $\bar{I} = I_m \angle -\theta$

We will investigate four cases:

1. A resistor
2. Inductor
3. Capacitor
4. General load.

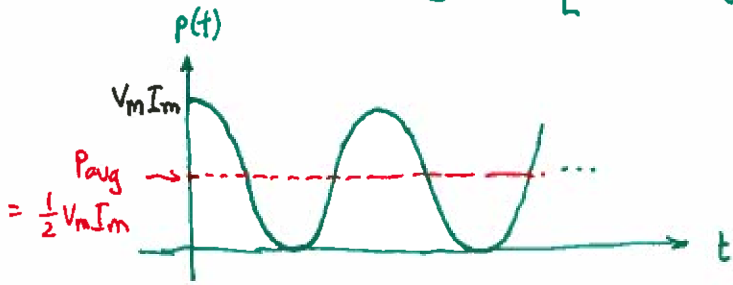
1. Purely resistive load ($X=0$)

We have $v(t) = V_m \cos(\omega t)$, $i(t) = I_m \cos(\omega t)$ } in phase $v(t) = R\, i(t)$

And power is $p(t) = v(t)\, i(t) = V_m I_m \cos^2(\omega t)$

Using the identity $\cos^2(x) = \frac{1}{2}\left[1 + \cos(2x)\right]$

$$p(t) = \frac{1}{2} V_m I_m \left[1 + \cos(2\omega t)\right]$$



← always positive – resistor only absorbs power.

2. Purely inductive $(R = 0,\ X > 0)$

For the inductor, $Z = j\omega L = \omega L \angle 90^\circ$, so $\theta = 90^\circ$

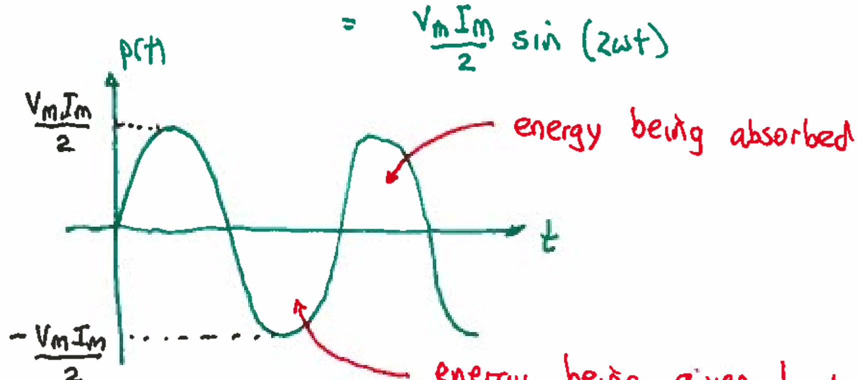
$$v(t) = V_m \cos(\omega t)$$
$$i(t) = I_m \cos(\omega t - 90^\circ) = I_m \sin(\omega t)$$

Power: $p(t) = v(t)\, i(t) = V_m I_m \cos(\omega t) \sin(\omega t)$

Using the identity $\cos(x)\sin(x) = \frac{1}{2}\sin(2x)$

so $p(t) = V_m I_m \cos(\omega t) \sin(\omega t)$

$$= \frac{V_m I_m}{2} \sin(2\omega t)$$



This is called reactive power – average is zero.