We have $\bar{V} = 10\angle 0^\circ$. Phasor current is

Wednesday, March 23, 2016

$$\bar{I} = \frac{\bar{V}}{Z_R + Z_C + Z_L} = \frac{10\angle 0^\circ}{10 - j20 + j10}$$

$$= \frac{10}{10 - j10}$$

Expressing in standard complex form $(a + jb)$

$$\bar{I} = \frac{10}{10-j10} \times \frac{10+j10}{10+j10} = \frac{100 + j100}{100 - j100 + j100 - j^2 100}$$

$$= \frac{100 + j100}{200}$$

$$= 0.5 + j0.5$$

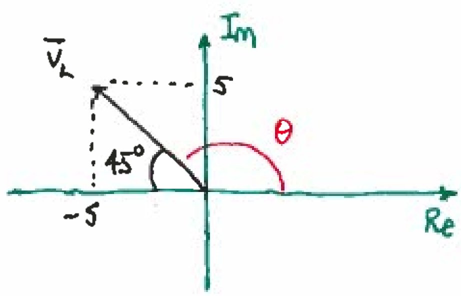
Now, we need $\bar{V}_L$. By the phasor equivalent of Ohm's law

$$\bar{V}_L = Z_L \bar{I} = j10 \times (0.5 + j0.5)$$

$$= j5 + j^2 5$$

$$= -5 + j5$$

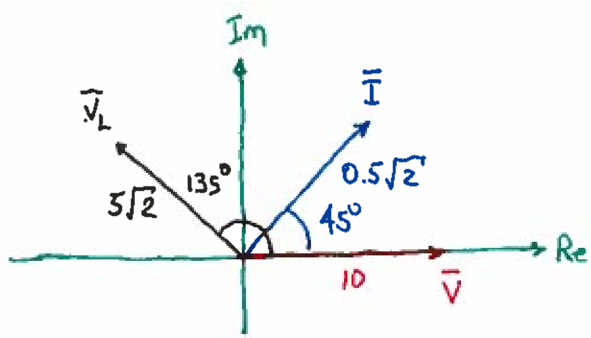
Expressed in polar form $V_L = \sqrt{5^2 + 5^2} = 5\sqrt{2}$



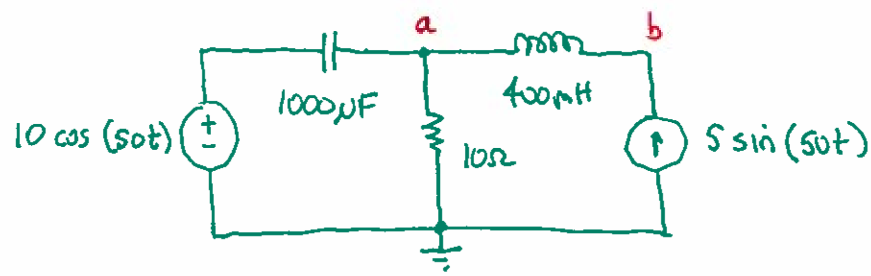
$$\theta = 180^\circ - 45^\circ = 135^\circ$$

so $V_L(t) = 5\sqrt{2} \cos(100t + 135^\circ)$

Phasor diagram:



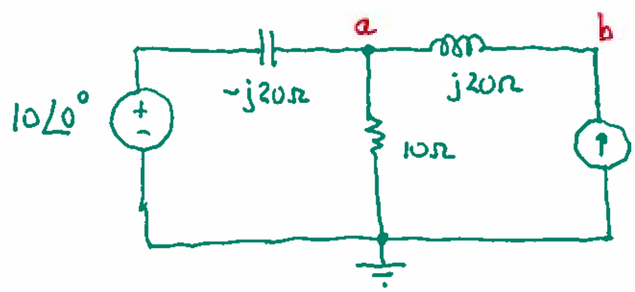
Example 2: Node-voltage method. Find $V_a(t)$ and $V_b(t)$



In terms of complex impedances,

$$Z_C = \frac{1}{j\omega C} = \frac{1}{j(50 \times 1000 \times 10^{-6})} = -j20\Omega$$

$$Z_L = j\omega L = j(50 \times 400 \times 10^{-3}) = j20\Omega$$



$$5\angle{-90^\circ} = 5\overset{0}{\cancel{\cos}}(-90^\circ) + j5\overset{-1}{\cancel{\sin}}(-90^\circ)$$
$$= -j5$$

Node a: $$\frac{\bar{V}_a - 10}{-j20} + \frac{\bar{V}_a}{10} + \frac{\bar{V}_a - \bar{V}_b}{j20} = 0$$

$(\times j20)$ $$-(\bar{V}_a - 10) + 2j\bar{V}_a + \bar{V}_a - \bar{V}_b = 0$$

$$2j\bar{V}_a - \bar{V}_b = -10 \qquad (1)$$

Node b: $$\frac{\bar{V}_b - \bar{V}_a}{j20} - 5\angle{-90^\circ} = 0$$

$$\frac{\bar{V}_b - \bar{V}_a}{j20} + j5 = 0$$

$(\times j20)$ $$\bar{V}_b - \bar{V}_a + j^2 100 = 0$$

$$\bar{V}_b - \bar{V}_a = 100 \qquad (2)$$

Add (1) and (2)

$$\begin{aligned} 2j\bar{V}_a - \bar{V}_b &= -10 \\ -V_a + \bar{V}_b &= 100 \\ \hline (-1 + j2)\bar{V}_a &= 90 \end{aligned}$$

Therefore, $\bar{V}_a = \dfrac{90}{-1+j2}$

Put in standard complex form,

$$\bar{V}_a = \frac{90}{-1+j2} \times \frac{-1-j2}{-1-j2} = \frac{-90-j180}{1+j2-j2-j^2 4}$$

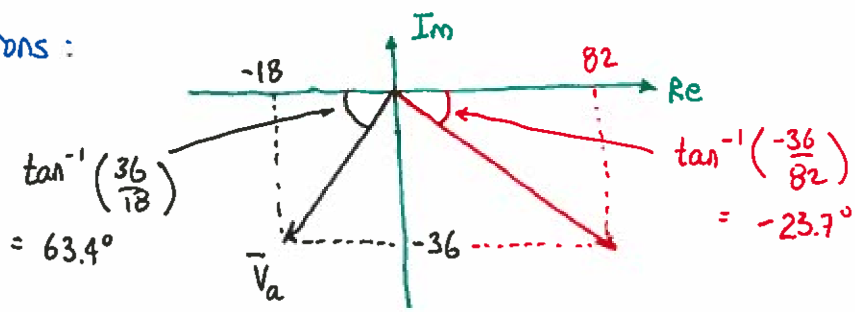
$$= \frac{-90-j180}{5} = -18-j36$$

And from (2) $\bar{V}_b - \bar{V}_a = 100$

$$\bar{V}_b + 18 + j36 = 100$$

so $\bar{V}_b = 82 - j36$

Phasor solutions :



$$\bar{V}_a = -18 - j36 = 40.25 \underline{/-(180-63.4°) = -116.6°}$$

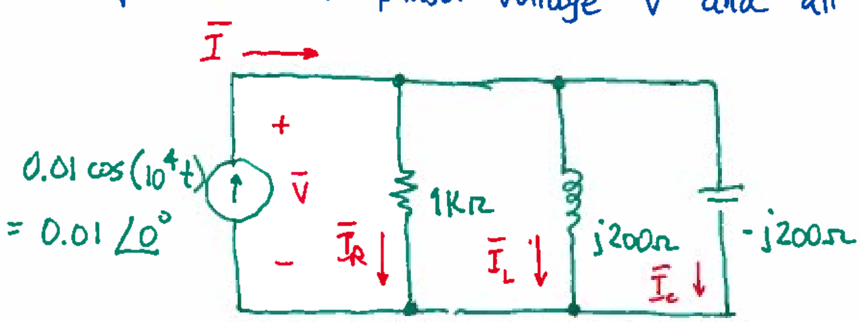
$$\bar{V}_b = 82 - j36 = 89.55 \underline{/-23.7°}$$

So back in the time domain

$$V_a(t) = 40.25 \cos(50t - 116.6°)$$

$$V_b(t) = 89.55 \cos(50t - 23.7°)$$

Example 3: Find phasor voltage $\bar{V}$ and all phasor currents.



We could find the total impedance Z_{eq}, starting with $Z_C // Z_L$

$$Z_C // Z_L = \frac{j200 \times -j200}{j200 - j200} = \infty$$

– Inductive impedance cancels capacitive impedance! (called resonance)