Inductor properties (continued)

Friday, March 11, 2016

$$v = L\frac{di}{dt} \longrightarrow = 0, \text{ when } i \text{ is constant}$$

- current cannot change instantaneously ; this would produce an infinite voltage.

Inductor current in terms of voltage

$$v = L\frac{di}{dt}$$

$$v\,dt = L\,di$$

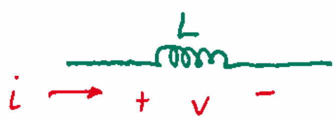
Integrate both sides

$$\int_{t_0}^{t} v\,d\tau = L\int_{i(t_0)}^{i(t)} d\tau = L\left[i(t) - i(t_0)\right]$$

$$\text{so} \quad i(t) = \frac{1}{L}\int_{t_0}^{t_0} v\,d\tau + i(t_0)$$

As before, usually $t_0 = 0$.

Power and energy in the inductor



$$p = vi$$

$$p = \frac{dw}{dt} = Li\frac{di}{dt}$$

$$dw = Li\,di$$

Integrate both sides

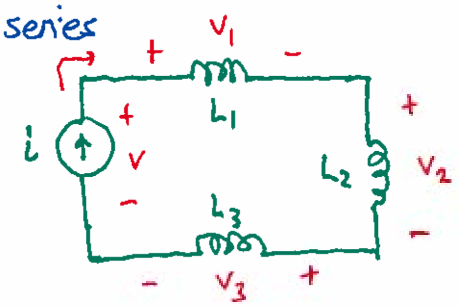
$$\int_0^w dx = L\int_0^i y\,dy$$

$$\boxed{w = \frac{1}{2}Li^2}$$

ENERGY IN INDUCTOR

Inductors in series and parallel

In series



By KVL,

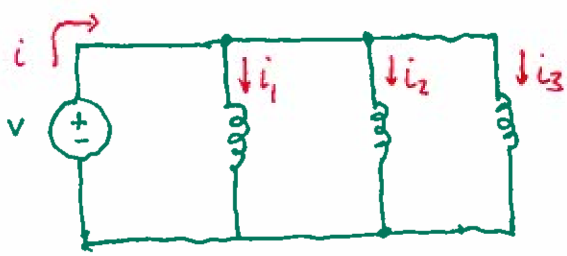
$$v = v_1 + v_2 + v_3$$

so $$v = L_1\frac{di}{dt} + L_2\frac{di}{dt} + L_3\frac{di}{dt}$$

$$= (L_1 + L_2 + L_3)\frac{di}{dt}$$

so $$\boxed{L_{eq} = L_1 + L_2 + L_3}$$ INDUCTANCES IN SERIES ADD

In parallel



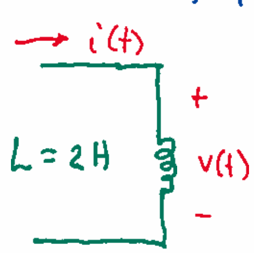
By KCL,

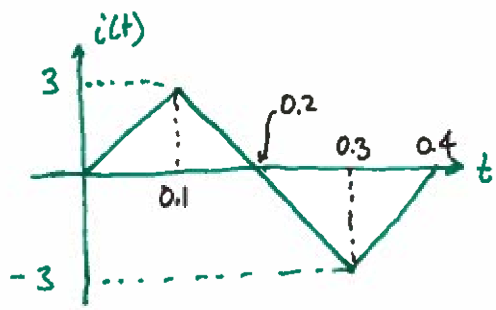
$$i = i_1 + i_2 + i_3$$

so $$i = \frac{1}{L_1}\int_0^t v\,dx + \frac{1}{L_2}\int_0^t v\,dx + \frac{1}{L_3}\int_0^t v\,dx$$

so $$\boxed{\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}}$$ INDUCTANCES IN PARALLEL ARE LIKE PARALLEL RESISTANCES.

Example: Given an inductor with $i(t)$ sketched below, find $v(t)$, $p(t)$, $w(t)$.





For the inductor $$v(t) = L\frac{di(t)}{dt}$$

Interval: $0 < t \leq 0.1$ $$\frac{di(t)}{dt} = \text{slope} = \frac{3}{0.1} = 30$$

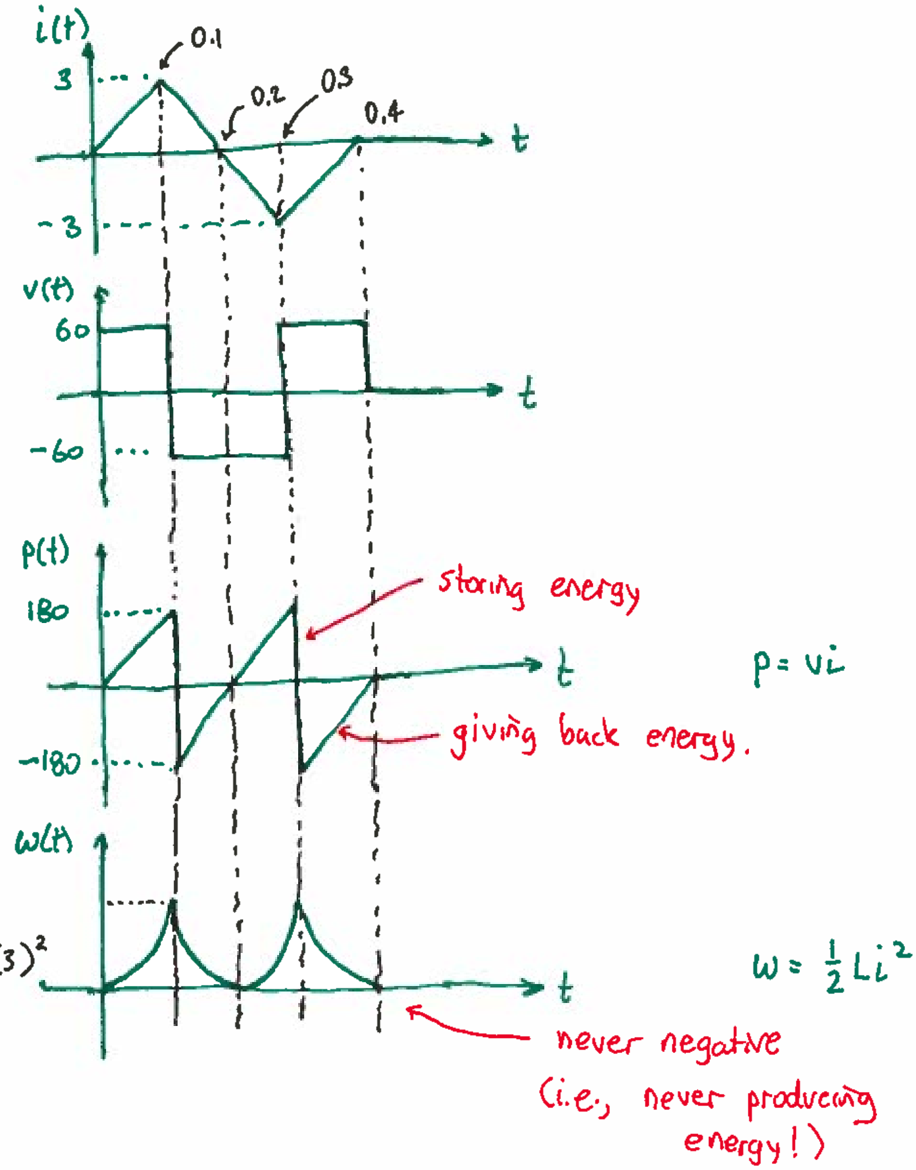
so $$v(t) = L \times 30 = 60\,v$$

Interval $0.1 < t \leq 0.3$ $\quad \dfrac{di(t)}{dt} = \dfrac{-6}{0.2} = -30$

so $\quad v(t) = L \times -30 = -60\,v$

Interval $0.3 < t \leq 0.4$ $\quad \dfrac{di(t)}{dt} = 30$, so $V = 60\,v$.

Sketch:



$w = \frac{1}{2} L i^2$

$= \frac{1}{2}(2)(3)^2$

$= 9\,J$