Problem 1

For the circuit shown,

$$\bar{I} = \frac{260\sqrt{2}\angle 50^\circ - 220\sqrt{2}\angle 30^\circ}{5 + j12}$$

$$= \frac{236.35 + j281.67 - 269.44 - j155.56}{5 + j12}$$

$$= \frac{-33.09 + j126.11}{5 + j12} = \frac{130.38\angle 104.70^\circ}{13\angle 67.38^\circ}$$

$$= 10.03\angle 37.32^\circ$$

The rms current is $I_{rms} = \frac{10.03}{\sqrt{2}} = 7.09\ A$

Average power absorbed by the resistor $P_R = I_{rms}^2 R = 251\ W$
Reactive power "absorbed" by the inductor $Q_L = I_{rms}^2 X = 603\ VAR$

Source $\bar{V}_1$:

$$\bar{S}_1 = -\tfrac{1}{2}\bar{V}_1 \bar{I}^* = -(\bar{V}_1)_{rms}(\bar{I}^*)_{rms}$$

thanks to passive reference convention

$$\bar{S}_1 = -260\angle 50^\circ \times 7.09\angle -37.32^\circ$$
$$= 260\angle 50^\circ - 180^\circ \times 7.09\angle -37.32^\circ$$

could add or subtract 180°!

$$\bar{S}_1 = 1843.4\angle -167.32^\circ = -1798 - j405\ VA$$

Average power delivered is 1798 W
Reactive power "delivered" is 405 VAR

Source $\bar{V}_2$:

$$\bar{S}_2 = (\bar{V}_2)_{rms}(\bar{I}^*)_{rms}$$

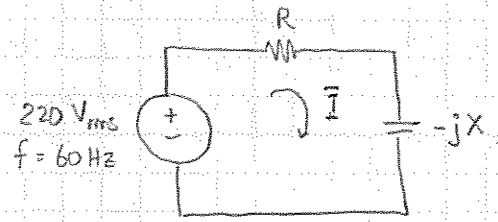
$$= 220\angle 30^\circ \times 7.09\angle -37.32^\circ$$

$$\bar{S}_2 = 1559.8 \angle -7.32^\circ = 1547 - j199 \text{ VA}$$

Average power absorbed is 1547 W
Reactive power "delivered" is −199 VAR

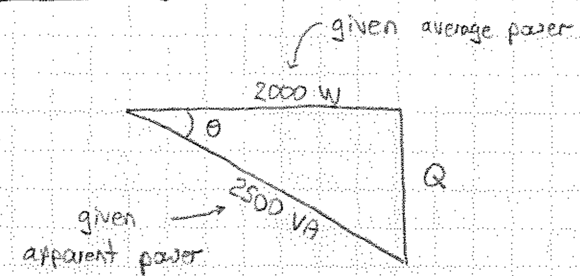
Energy balance is observed by adding all average power and all reactive power.

Problem 2



, where $X = \frac{1}{\omega C}$

The power triangle:



The power angle $\theta = \theta_V - \theta_I$ is the angle of the load's impedance, which is negative, so $Q < 0$.

$$Q = -\sqrt{2500^2 - 2000^2}$$
$$= -1500 \text{ VAR}$$

$$\bar{S} = (\bar{V}_{rms})(\bar{I}_{rms}^*) = 2000 - j1500 \text{ VA}$$

$$\bar{S} = (\bar{V}_{rms})\left(\frac{\bar{V}_{rms}}{Z}\right)^* = \frac{(\bar{V}_{rms})(\bar{V}_{rms})^*}{Z^*}$$

With $\bar{V}_{rms} = 220\angle 0^\circ$, then $\bar{V}_{rms}^* = 220\angle 0^\circ$

so $\bar{S} = \frac{(\bar{V}_{rms})^2}{Z^*} \implies$ so $Z^* = \frac{(\bar{V}_{rms})^2}{\bar{S}}$

$$Z^* = \frac{(220)^2}{2000 - 1500} = 15.49 + j\,11.62$$

giving $Z = 15.49 - j\,11.62\ \Omega$

From this value of Z, $R = 15.49\,\Omega$, and $X = 11.62\,\Omega$

$$X = \frac{1}{\omega C}, \quad \text{so} \quad C = \frac{1}{\omega X}$$

$$C = \frac{1}{2\pi \times 60 \times 11.62} = 228.3\ \mu F.$$

Problem 3

We are given information that will allow us to determine the machine constant

$$E_A = 200\ V \quad \text{at} \quad n_m = 1200\ \text{rpm}$$

We have $\omega_m = 1200 \times \frac{2\pi}{60} = 125.66$ rad/sec.

From one of the machine equations, $E_A = K\phi\,\omega_m$, so $K\phi = \frac{E_A}{\omega_m}$.

$$K\phi = \frac{200}{125.66} = 1.591$$

With the motor running at $n_{m2} = 1500$ rpm, $P_{dev} = 5$ HP.

$$\omega_m = 1500 \times \frac{2\pi}{60} = 157.08\ \text{rad/sec}$$

$$P_{dev} = 5\ \text{HP} \times 746\ \frac{W}{HP} = 3730\ W$$

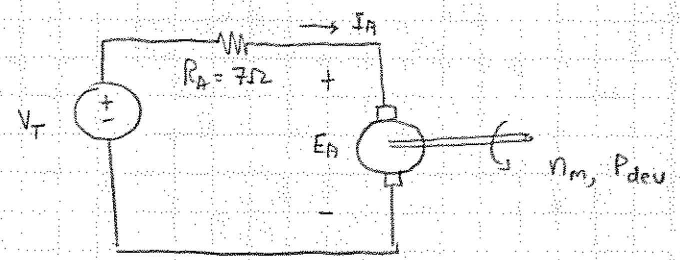
$$\text{So} \quad T_{dev} = \frac{P_{dev}}{\omega_m} = \frac{3730}{157.08} = \boxed{23.746\ \text{Nm}}$$

The other machine equation allows us to find I_A

$$T_{dev} = K\phi\, I_A, \quad \text{so} \quad I_A = \frac{T_{dev}}{K\phi} = \frac{23.746}{1.591} = \boxed{14.92\ A}$$

Problem 4

The simple circuit model for the motor is



From the information given, we may determine the machine constant and rotational losses

At no-load, $I_A = 1\,A$, $n_m = 1500$ rpm

We have $\omega_m = 1500 \times \frac{2\pi}{60} = 157.08$ rad/sec

and $E_A = V_T - I_A R_A = 240 - 1 \times 7 = 233$ V

From one of the machine equations, $K\phi = \frac{E_A}{\omega_m}$

$$K\phi = \frac{E_A}{\omega_m} = \frac{233}{157.08} = \boxed{1.483}$$

We also know that $P_{dev} = E_A I_A = 233 \times 1 = 233$ W. Since there is no mechanical load attached, P_{dev} is used entirely to overcome rotational losses at 1500 rpm.

Thus, $P_{rot} = P_{dev} = 233$ W.

Frictional torque loss $= T_{rot} = \frac{P_{rot}}{\omega_m} = \frac{233}{157.08}$

$$\boxed{T_{rot} = 1.483 \text{ Nm}}$$

With the load now attached, the speed drops to $n_m = 1300$ rpm. At the lower speed, P_{rot} reduces proportionally

$P_{rot} = T_{rot}\,\omega_m$, $\omega_m = 1300 \times \frac{2\pi}{60} = 136.14$ rad/sec so $P_{rot} = 1.483 \times 136.14 \text{ rad/sec} = 201.93\,W$

The induced voltage E_A will also decrease proportionally

$$E_A = \frac{1300}{1500} \times 233 = 201.93\,V$$

so the new $I_A = \frac{V_T - E_A}{R_A} = \frac{240 - 201.93}{7} = 5.439\,A$

For the efficiency calculation, $P_{in} = V_T \times I_A = 240 \times 5.439$
$= 1305.26\,W$

and $P_{out} = P_{dev} - P_{rot} = E_A I_A - 201.93$
$= 201.93 \times 5.439 - 201.93$
$= 896.4\,W$

Efficiency $\eta = \frac{P_{out}}{P_{in}} \times 100\% = \frac{896.4}{1305.26} \times 100\% = \boxed{68.7\%}$

Problem 5

From the information given, we may determine the machine constant

At 1200 rpm, $\omega_m = 1200 \times \frac{2\pi}{60} = 125.66 \text{ rad/sec}$

$$E_A = K\phi\,\omega_m, \quad \text{so } K\phi = \frac{E_A}{\omega_m} = \frac{175}{125.66} = 1.393$$

Rotational losses at 1200 rpm: $P_{rot} = 50\,W$.

$$\text{so } T_{rot} = \frac{P_{rot}}{\omega_m} = \frac{50}{125.66} = 0.398\,Nm$$

When the machine is run with no mechanical load, $T_{dev} = T_{rot}$.
From this, we can find I_A, E_A, then ω_m

$$I_A = \frac{T_{dev}}{K\phi} = \frac{0.398}{1.393} = 0.2857\,A$$

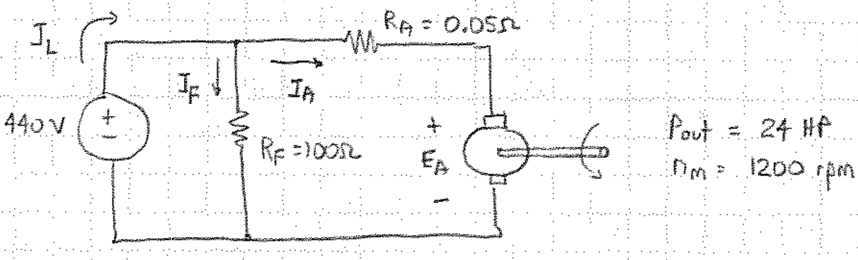
and $E_A = V_T - I_A R_A$
$= 200 - 0.2857 \times 1 = 199.714\,V$

so $\omega_m = \dfrac{E_A}{K\phi} = \dfrac{199.714}{1.393} = 143.41$ rad/sec

Finally, $n_m = 143.41 \times \dfrac{60}{2\pi} = \boxed{1369.5 \text{ rpm}}$

Problem 6

The electric circuit model for the motor:



I_L is given as 50 A.

From the information given, we may determine I_A, E_A, then the machine constant $K\phi$.

We have, by KCL, $I_L = I_F + I_A$. I_F will be $I_F = \dfrac{440}{100} = 4.4$ A

so $I_A = 50 - 4.4 = 45.6$ A

From KVL, $E_A = V_T - I_A R_A = 440 - 45.6 \times 0.05$

$E_A = 437.72$ V

At 1200 rpm, $\omega_m = 1200 \times \dfrac{2\pi}{60} = 125.66$ rad/sec

so $E_A = K\phi\,\omega_m$, and $K\phi = \dfrac{E_A}{\omega_m} = \dfrac{437.72}{125.66}$

$K\phi = 3.483$.

We can now find $T_{dev} = K\phi\, I_A = 3.483 \times 45.6 = \boxed{158.84 \text{ Nm}}$

For efficiency, we already know P_{out} = 24 HP

$$P_{out} = 24 \times 746 \frac{W}{HP} = 17904 \text{ W}$$

and $P_{in} = V_T I_L = 440 \times 50 = 22000 \text{ W}$

$$\eta = \frac{P_{out}}{P_{in}} \times 100\% = \frac{17904}{22000} \times 100\% = \boxed{81.4\%}$$