Thursday, March 31, 2016

3. Purely capacitive load ($R = 0,\ X < 0$)

We have $Z = 1/j\omega C = \frac{1}{\omega C}\angle -90^\circ$, so $\theta = -90^\circ$

$$v(t) = V_m \cos(\omega t)$$
$$i(t) = I_m \cos(\omega t + 90^\circ) = -I_m \sin(\omega t)$$

Therefore,
$$p(t) = -V_m I_m \cos(\omega t)\sin(\omega t)$$
$$= -\frac{V_m I_m}{2} \sin(2\omega t)$$

This is also reactive power. For the capacitor and inductor, no average power is consumed or generated.

4. For a general load ($R \neq 0,\ X \neq 0$)

Here, we allow for both resistance and capacitance or inductance. We allow θ in the range.

$$-90^\circ \leq \theta \leq 90^\circ$$

purely capacitive (-90°)   purely inductive (90°).

We have
$$v(t) = V_m \cos(\omega t)$$
$$i(t) = I_m \cos(\omega t - \theta)$$

and
$$p(t) = V_m \cos(\omega t)\, I_m \cos(\omega t - \theta)$$

This can be manipulated to get

$$p(t) = \frac{V_m I_m}{2} \cos(\theta)\left[1 + \underbrace{\cos(2\omega t)}_{\text{average} = 0}\right] + \underbrace{\frac{V_m I_m}{2} \sin(\theta)\sin(2\omega t)}_{\text{average} = 0}$$

The average power is

$$P = P_{avg} = \frac{V_m I_m}{2} \cos(\theta)$$

POWER, IN WATTS, ABSORBED BY THE RESISTIVE COMPONENT OF TOTAL IMPEDANCE

Recall that $V_{rms} = \frac{V_m}{\sqrt{2}}$, $I_{rms} = \frac{I_m}{\sqrt{2}}$

so $\boxed{P = V_{rms} I_{rms} \cos(\theta)}$ AVERAGE, OR REAL, POWER

For a resistor, $\theta = 0°$, so $P = V_{rms} I_{rms}$. The $\cos(\theta)$ term is very important, and is called

$\boxed{PF = \cos(\theta)}$ POWER FACTOR

where, in the more general case,

$\boxed{\theta = \theta_V - \theta_I}$ POWER ANGLE

We often state the PF and specify whether current leads or lags voltage.

Example: A load has a leading power factor of 0.707. Is this capacitive or inductive, and what's the power angle?

→ A leading PF means current is leading (has a higher phase than) the voltage.

Reviewing what we know:

We have $Z = R + jX = |Z|\angle\theta$

Let $\bar{V} = V_m \angle\theta_V$
and $\bar{I} = I_m \angle\theta_I$

where we are given that $\theta_I > \theta_V$ (current leads voltage)

We know $\bar{I} = \frac{\bar{V}}{Z}$, so $Z = \frac{\bar{V}}{\bar{I}} = \frac{V_m \angle\theta_V}{I_m \angle\theta_I}$

so $Z = |Z| \angle \theta_V - \theta_I$

power angle negative, because $\theta_I > \theta_V$

And what we have ...

We are given that $PF = \cos(\theta) = \cos(\theta_V - \theta_I) = 0.707$

and $\theta < 0$, so $\theta = -45°$

This suggest that $Z = R + jX$, where $X < 0$.

The load has a capacitance $(z_c = -j/\omega C)$

Reactive power

Average reactive power is always zero. However, its instantaneous value is sinusoidal with peak value Q.

$$\boxed{Q = V_{rms} I_{rms} \sin(\theta)}$$ REACTIVE POWER.

This is flowing back and forth between inductors/capacitors and the source.

- might be a problem in large-scale systems
- power companies might penalize you for reactive power.

Units of reactive power

VOLT - AMPERES - REACTIVE : VARs

Apparent power

This is a measure of the total power (average and reactive)

$$\boxed{\text{apparent power} = V_{rms} I_{rms}}$$