Wednesday, February 24, 2016

Node a: $\frac{V_a - 5i_x}{10} - 2 + \frac{V_a}{15} = 0$

(x30) $3V_a - 15i_x - 60 + 2V_a = 0$

$3V_a - 15i_x = 60$

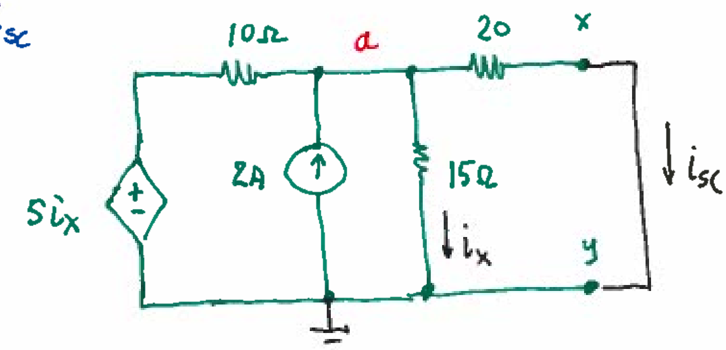
We know that $i_x = V_a/15$, so

$5V_a - 15\left(\frac{V_a}{15}\right) = 60$

$4V_a = 60$

so $V_a = V_t = V_{oc} = 15V$

Now, i_{sc}



Again solve for V_a, now with $i_{sc} = V_a/20$.

$$\frac{V_a - 5i_x}{10} - 2 + \frac{V_a}{15} + \underbrace{\frac{V_a}{20}}_{i_{sc}} = 0$$

(x60) $6V_a - 30i_x - 120 + 4V_a + 3V_a = 0$

$13V_a - 30i_x = 120$

Using $i_x = V_a/15$,

$13V_a - 30\left(\frac{V_a}{15}\right) = 120$

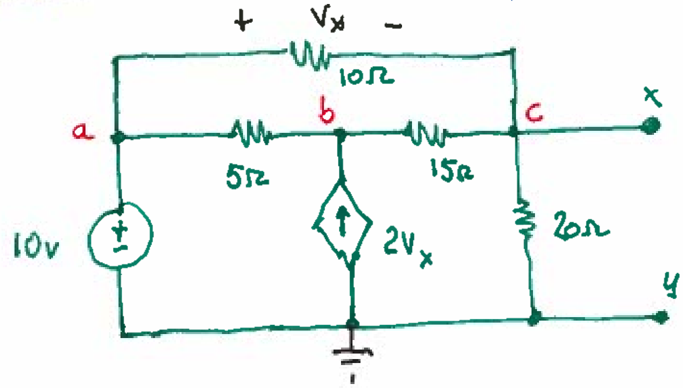
$11V_a = 120$

$\therefore V_a = 10.91$ v.

then $i_{sc} = \frac{V_a}{20} = 0.545$ A

And $R_t = \frac{V_t}{i_{sc}} = \frac{15}{0.545} = 27.5\Omega$

Example 2 : Find the Thevenin equivalent



$V_a = 10$

$V_t = V_{oc} = V_c$

Find $V_t = V_c$, so node-voltage method seems appropriate.

Node b: $\frac{V_b - 10}{5} - 2V_x + \frac{V_b - V_c}{15} = 0$

For the dependent current source, we have $V_x = V_a - V_c = 10 - V_c$, so

$$\frac{V_b - 10}{5} - 2(10 - V_c) + \frac{V_b - V_c}{15} = 0$$

$(\times 15)$ $3V_b - 30 - 30(10 - V_c) + V_b - V_c = 0$

$$4V_b + 29V_c = 330 \qquad (1)$$

Node c: $\frac{V_c - V_b}{15} + \frac{V_c}{20} + \frac{V_c - 10}{10} = 0$

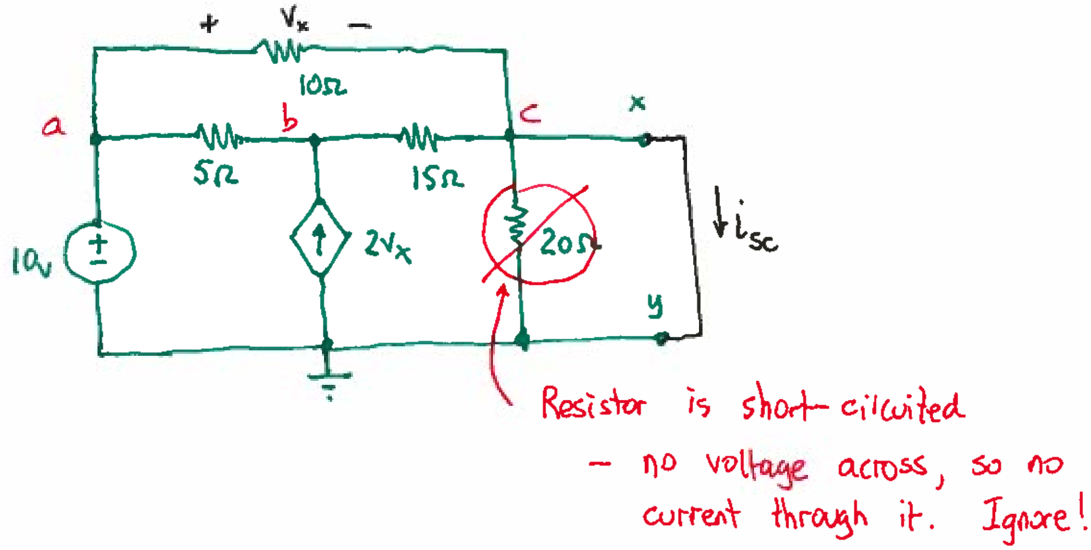
$(\times 60)$ $4V_c - 4V_b + 3V_c + 6V_c - 60 = 0$

$$13V_c - 4V_b = 60 \qquad (2)$$

Solve (1) and (2) for V_c.

$$V_c = V_{oc} = V_t = 9.29 \text{ v.}$$

Now find R_t. Has a dependent source, so no shortcuts.



Node b: $\dfrac{V_b - 10}{5} - 2V_x + \dfrac{V_b - V_c}{15} = 0$

Using $V_x = V_a - V_c$, equation (1) from previous analysis still applies

$$4V_b + 29V_c = 330, \quad \text{where } V_c = 0$$

$$V_b = 82.5\text{v}$$

Node c: $\dfrac{V_c - V_b}{15} + \cancel{\dfrac{V_c}{20}} + \dfrac{V_c - 10}{10} + i_{sc} = 0$

so $-\dfrac{V_b}{15} - 1 + i_{sc} = 0$

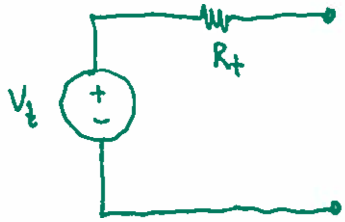
$$i_{sc} = 1 + \frac{82.5}{15} = 6.5\text{ A}$$

Therefore,

$$R_t = \frac{V_t}{i_{sc}} = \frac{9.29}{6.5} = 1.43\Omega$$

<u>Quick wrap-up to Thevenin equivalent circuits</u>

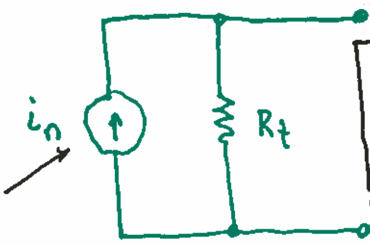
The <u>Norton equivalent</u> circuit provides an alternative form to the Thevenin equivalent



THEVENIN

$$\boxed{\begin{aligned} V_{oc} &= V_t \\ i_{sc} &= \frac{V_t}{R_t} \end{aligned}}$$

$$i_n = \frac{V_t}{R_t}$$



$$\boxed{\begin{aligned} V_{oc} &= i_n R_t = V_t \\ i_{sc} &= i_n = \frac{V_t}{R_t} \end{aligned}}$$