Wednesday, February 3, 2016

Multiply by 20: $2V_2 - 2V_1 + 10V_2 + V_3 - V_1 + 4V_3 = 0$

$$12V_2 - 3V_1 + 5V_3 = 0$$

and $V_1 = 10$, so $12V_2 - 30 + 5V_3 = 0$

$$12V_2 + 5V_3 = 30 \qquad (1)$$

The dependence: $V_3 - V_2 = 5v$

so $V_3 = V_2 + 5 \qquad (2)$

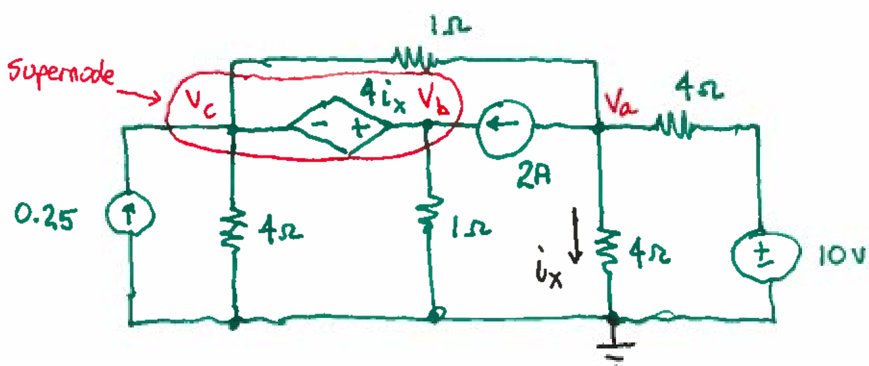
Substitute into (1) $12V_2 + 5(V_2 + 5) = 30$

$$17V_2 = 5$$

$$V_2 = 0.294\ v$$

and $V_3 = 5.294\ v.$

<u>Example 2</u>: Determine the node voltages (from 2014 midterm exam)



Node a: $\dfrac{V_a - 10}{4} + \dfrac{V_a}{4} + \dfrac{V_a - V_c}{1} + 2 = 0$

$$V_a - 10 + V_a + 4V_a - 4V_c + 8 = 0$$

$$6V_a - 4V_c = 2 \qquad (1)$$

Supernode: $\underbrace{\dfrac{V_c}{4} - 0.25 + \dfrac{V_c - V_a}{1}}_{V_c \text{ side}} + \underbrace{\dfrac{V_b}{1} - 2}_{V_b \text{ side}} = 0$

$$V_c - 1 + 4V_c - 4V_a + 4V_b - 8 = 0$$

$$-4V_a + 4V_b + 5V_c = 9 \qquad (2)$$

Supernode dependence: $V_b - V_c = 4i_x$ and $i_x = \frac{V_a}{4}$

So $V_b - V_c = \frac{4V_a}{4} = V_a$

$$-V_a + V_b - V_c = 0 \qquad (3)$$

Solve for 3 unknowns, 3 equations

Subtract 4 times equation (3) from (2)

$$\begin{array}{lr} (2) & -4V_a + 4V_b + 5V_c = 9 \\ -4\times \; (3) & -4V_a + 4V_b - 4V_c = 0 \\ \hline & 9V_c = 9, \end{array}$$

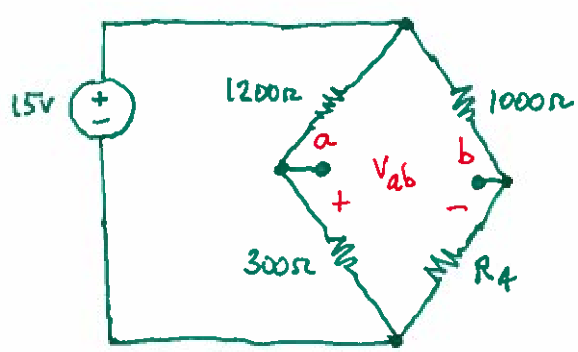
so $\boxed{V_c = 1}$

From (1) $6V_a - 4(1) = 2$

$6V_a = 6$ so $\boxed{V_a = 1}$

and from (3), $-1 + V_b - 1 = 0$, so $\boxed{V_b = 2}$

Example 3 — A Wheatstone bridge.

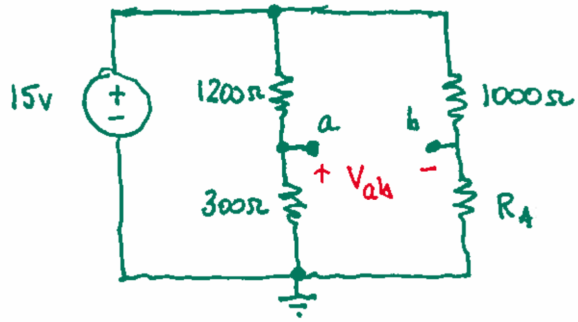


(a) Assume this bridge is balanced; that is, $V_{ab} = 0$.
Determine R_4.

(b) Now set $R_4 = 200\Omega$, and connect a and b with a 250Ω resistor. Find the power in the 250Ω resistor.

Solution

(a) Really just a pair of voltage dividers



When "balanced", a and b have equal voltages.

$$V_a = \frac{300}{300+1200} \times 15v = 3v.$$

→ Check by node-voltage method:

$$\frac{V_a - 15}{1200} + \frac{V_a}{300} = 0$$

$$V_a\left(\frac{1}{1200} + \frac{1}{300}\right) = \frac{15}{1200}$$

$$V_a\left(\frac{1}{1200} + \frac{4}{1200}\right) = \frac{15}{1200}$$

$$5V_a = 15$$

$$V_a = 3$$

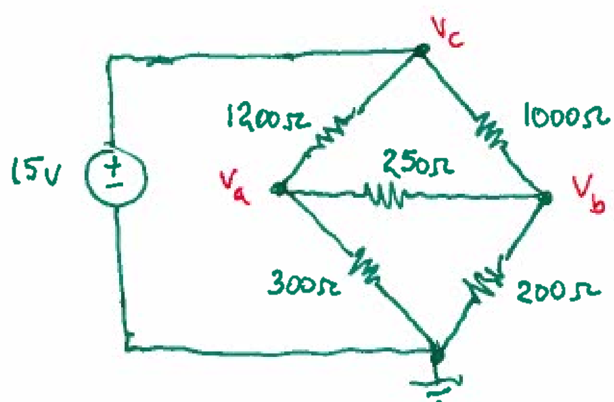
$$V_b = \frac{R_4}{R_4 + 1000} \times 15v = 3v, \quad \text{so} \quad V_{ab} = 0$$

so $15R_4 = 3(R_4 + 1000)$

$$12R_4 = 3000$$

$$R_4 = 250\Omega$$

(b)



$V_c = 15v$

Node a: $$\frac{V_a - 15}{1200} + \frac{V_a}{300} + \frac{V_a - V_b}{250} = 0.$$