Wednesday, March 9, 2016

INDUCTORS AND CAPACITORS

So far, we have considered the basic circuit elements

$v = iR$

Sources
(dependent and independent)

resistor

Inductors and capacitors are dependent on electromagnetic fields

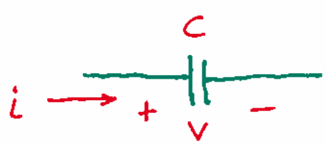
- capacitors: separation of charge produces an electric field
- inductors: motion of charge produces a magnetic field.

Unlike resistors, these devices can store energy and return stored energy (but are not producers of energy).

The capacitor

The circuit symbol is

Like all circuit elements, the capacitor has its own important voltage-current relationship



where

$$\boxed{i = C\frac{dv}{dt}}$$

v = voltage in Volts (V)
i = current in Amps (A)
t = time in secs (s)
C = capacitance in Farads (F)

Capacitor properties:

- Constant voltage across terminals results in zero current flow — capacitor looks like an open circuit

$$i = C\frac{dv}{dt} \longrightarrow = 0, \text{ when } v \text{ is constant.}$$

- voltage cannot change instantaneously; current would be infinite.

Capacitor voltage in terms of current

$$i = C\frac{dv}{dt}$$

$$i\,dt = C\,dv \quad \text{and} \quad \int_{t_0}^{t} i\,d\tau = C\int_{v(t_0)}^{v(t)} dx$$

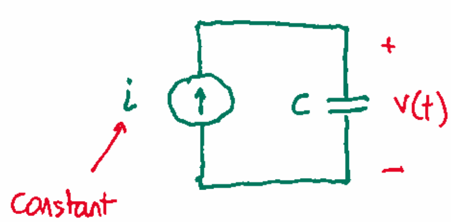
$$= C\left[v(t) - v(t_0)\right]$$

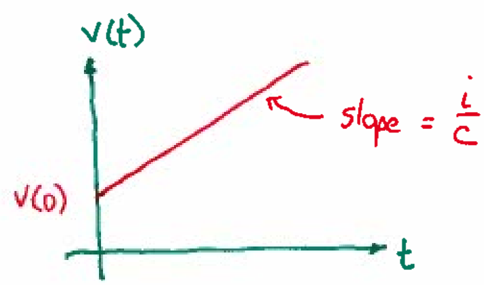
$$\text{so} \quad v(t) = \frac{1}{C}\int_{t_0}^{t} i\,d\tau + v(t_0)$$

We usually assume that $t_0 = 0$

$$v(t) = \frac{1}{C}\int_{0}^{t} i\,d\tau + v(0)$$

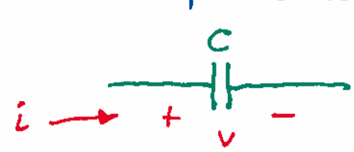
E.g.,





Power and energy in the capacitor

We again use the passive reference convention



$$p = vi$$

(Current in the direction of a voltage drop)

$$p = vi = v\left(C\frac{dv}{dt}\right)$$

$$\text{or} = i\left[\frac{1}{C}\int_0^t i\,d\tau + v(0)\right]$$

For energy, recall $p = \frac{dw}{dt}$

$$p = \frac{dw}{dt} = v\left(C\frac{dv}{dt}\right)$$

$$\text{so} \quad dw = C\,v\,dv$$

Integrate both sides

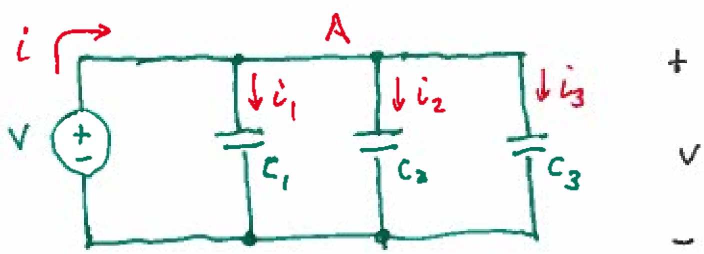
$$\int_0^w dx = C\int_0^v y\,dy$$

$$w = \left[\tfrac{1}{2}Cy^2\right]_0^v = \boxed{\tfrac{1}{2}Cv^2}$$ ENERGY IN A CAPACITOR

where w is energy in Joules.

<u>Capacitances and series and parallel</u>

In parallel:



KCL at node A gives: $i = i_1 + i_2 + i_3$

$$\text{or} \quad i = C_1\frac{dv}{dt} + C_2\frac{dv}{dt} + C_3\frac{dv}{dt}$$

$$\text{so} \quad i = (C_1 + C_2 + C_3)\frac{dv}{dt}$$

which we can write as $\quad i = C_{eq}\frac{dv}{dt}$

where $\boxed{C_{eq} = C_1 + C_2 + C_3}$ CAPACITANCES IN PARALLEL <u>ADD</u>