Wednesday,
March 16, 2016

rms voltage : $V_{rms} = \dfrac{V_m}{\sqrt{2}} = 7.071 \text{ V}$

and average power

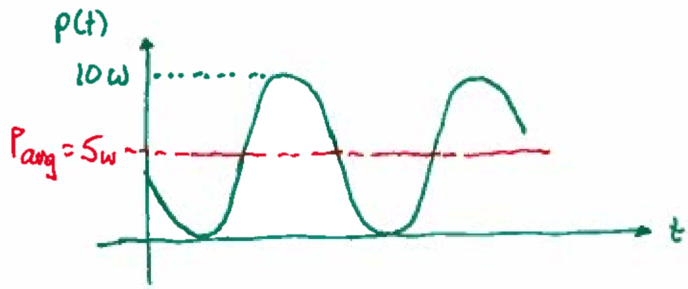
$$P_{avg} = \frac{V_{rms}^2}{R} = \frac{(7.071)^2}{10} = 5 \text{ W}$$

Let's also sketch the instantaneous power $p(t)$

$$p(t) = \frac{v^2(t)}{R} = \frac{100}{10} \cos^2(1000\pi t - 60^\circ)$$

Use the trig identity $\cos^2(x) = \frac{1}{2}\left[1 + \cos(2x)\right]$

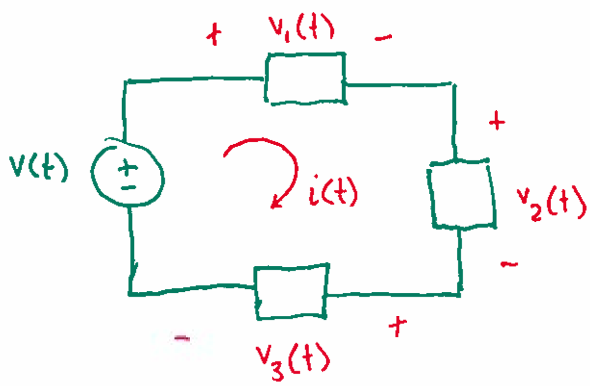
$$p(t) = 5 + 5\cos(2000\pi t - 120^\circ)$$



Phasors

When dealing with sinusoidal voltages and currents, we need a convenient way to add them to satisfy KVL, KCL.

Consider :



Let $v_1(t) = 10\cos(\omega t)$

$v_2(t) = 5\cos(\omega t - 30^\circ)$

$v_3(t) = 5\cos(\omega t + 90^\circ)$

Find $v(t) = V_m \cos(\omega t + \theta)$

KVL must be satisfied by this circuit over all time

$$-v(t) + v_1(t) + v_2(t) + v_3(t) = 0$$

so $v(t) = 10\cos(\omega t) + 5\cos(\omega t - 30^\circ) + 5\cos(\omega t + 90^\circ)$

How do we manipulate this to get the desired form $v(t) = V_m \cos(\omega t + \theta)$? What are V_m and θ? Help!

For this, we instead express voltages and currents in terms of phasors.

Let $v_1(t) = V_1 \cos(\omega t + \theta_1)$

usually fixed in value throughout the circuit for an analysis problem.

We have a pair of independent parameters describing the voltage:

V_1 – magnitude (amplitude)
θ_1 – phase angle

BASIC IDEA: Represent as a vector on a plane: then add the vector lengths.

Examples of phasor representation:

Voltage	Phasor representation
$v_a(t) = V_a \cos(\omega t + \theta_a)$	$\bar{V}_a = V_a \angle \theta_a$
$v_b(t) = V_b \sin(\omega t + \theta_b)$ $= V_b \cos(\omega t + \theta_b - 90^\circ)$	$\bar{V}_b = V_b \angle \theta_b - 90^\circ$

magnitude
phase angle.

Similarly, for currents

$$i_c(t) = I_c \cos(\omega t + \theta_c) \longleftrightarrow \bar{I}_c = I_c \angle \theta_c$$

To manipulate phasors, we need to use complex numbers.

Complex numbers – review

We express and manipulate phasors as <u>complex numbers</u>.

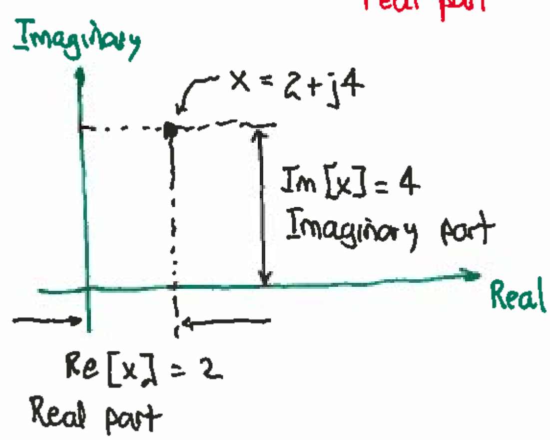
Complex numbers involve "imaginary" numbers.

Mathematicians: $i = \sqrt{-1}$

Engineers: i is current, so $\boxed{j = \sqrt{-1}}$

or $j^2 = -1$

E.g., complex number $x = \underbrace{2}_{\text{real part}} + \underbrace{j4}_{\text{imaginary part}}$



x is a point on the <u>complex plane</u>.

The complex conjugate of x: $x^{*} = 2 - j4$

$*$ denotes complex conjugate

$-$ flip sign of Im[x]

Rectangular and polar forms of complex numbers

$x = a + jb$ (rectangular)

$x = M\angle\theta$ (polar)