Monday, March 21, 2016

$$v_L(t) = -\omega L I_m \sin(\omega t + \theta)$$
$$= -\omega L I_m \cos(\omega t + \theta - 90^\circ)$$

or $$v_L(t) = \omega L I_m \cos(\omega t + \theta - 90^\circ + 180^\circ)$$
$$= \omega L I_m \cos(\omega t + \theta + 90^\circ)$$

Now express in phasor form

$$i_L(t) = I_m \cos(\omega t + \theta) \longleftrightarrow \bar{I}_L = I_m \angle \theta$$

$$v_L(t) = \omega L I_m \cos(\omega t + \theta + 90^\circ) \longleftrightarrow \bar{V}_L = \omega L I_m \angle \theta + 90^\circ$$

We may write $\bar{V}_L$ as

$$\bar{V}_L = \omega L \angle 90^\circ \times I_m \angle \theta = \omega L I_m \angle \theta + 90^\circ$$
$$= \underbrace{\omega L \angle 90^\circ}_{\omega L e^{j90^\circ}} \times \bar{I}_L$$

$$\omega L e^{j90^\circ} = \omega L \cancelto{0}{\cos(90^\circ)} + j\omega L \cancelto{1}{\sin(90^\circ)}$$
$$= j\omega L$$

Finally,

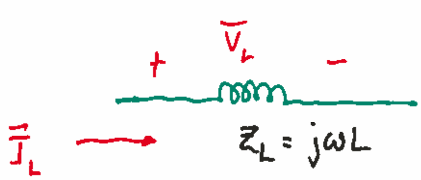
$$\boxed{\bar{V}_L = j\omega L \times \bar{I}_L}$$

The term $j\omega L$ is the inductor <u>impedance</u> Z_L

$$\boxed{Z_L = j\omega L = \omega L \angle 90^\circ}$$ IMPEDANCE OF AN INDUCTOR, IN OHMS

and we have an equivalent expression for Ohm's Law

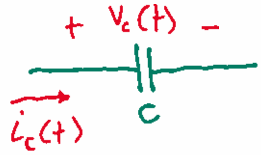
$$\boxed{\bar{V}_L = Z_L \bar{I}_L}$$ PHASOR EQUIVALENT OF OHM'S LAW.



Note that the voltage <u>leads</u> the current by $90°$ (i.e., $\bar{V}_L$ is $90°$ higher in phase than $\bar{I}_L$).

<u>Capacitance</u>

Recall for the capacitor,



$$i_c(t) = C\frac{dv_c(t)}{dt}$$

By the same analysis, we may write

$$\bar{V}_c = Z_c \bar{I}_c$$

where Z_c is the capacitor <u>impedance</u> given by

$$\boxed{Z_c = \frac{1}{j\omega C} = \frac{1}{\omega C}\angle -90°}$$

IMPEDANCE OF A CAPACITOR, IN OHMS

Note that voltage <u>lags</u> current by $90°$ (i.e., $\bar{V}_c$ is $90°$ <u>less</u> in phase than $\bar{I}_c$)

<u>Resistance</u>

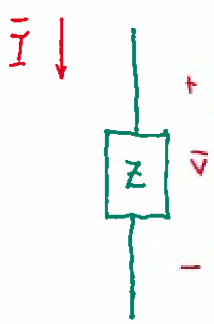
Nothing new about the resistor!

$$\bar{V}_R = R\bar{I}_R$$

real-valued constant (resistance)

$\bar{V}_R$ and $\bar{I}_R$ are exactly in phase.

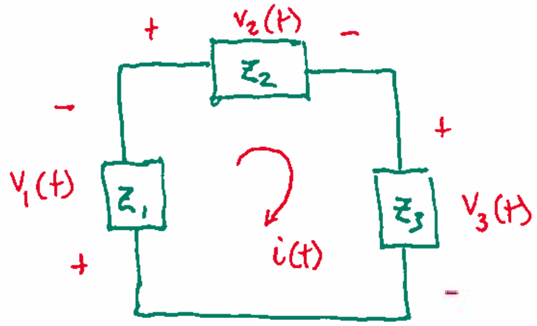
<u>Summary of impedances</u>



<u>Inductor</u>	<u>Capacitor</u>	<u>Resistor</u>
$\bar{V} = Z_L\bar{I}_L$	$\bar{V} = Z_c\bar{I}_c$	$\bar{V} = R\bar{I}$
$Z_L = j\omega L$	$Z_c = \frac{1}{j\omega C}$	$Z_R = R$.

Circuit analysis with phasors and complex impedances

KVL and KCL must always be satisfied, whether AC or DC



$$v_1(t) + v_2(t) + v_3(t) = 0$$

$$i_1(t) + i_2(t) + i_3(t) = 0$$

For sinusoidal AC circuits, we express KVL and KCL in terms of phasors.

$$\bar{V}_1 + \bar{V}_2 + \bar{V}_3 = 0$$
$$\bar{I}_1 + \bar{I}_2 + \bar{I}_3 = 0$$

and we use phasor representation of voltage-current relationships

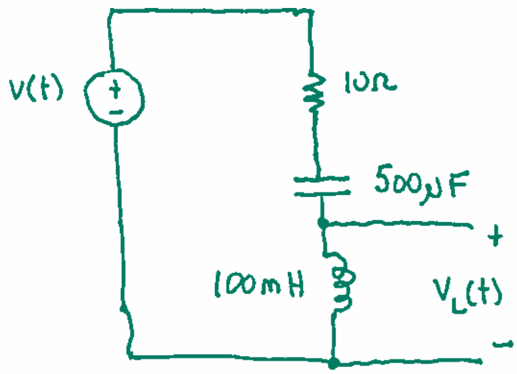
$$\bar{V} = Z\bar{I}$$

where Z is the complex impedance of an inductor, capacitor, or a resistor.

Analysis procedure:

- Use phasor for voltages and currents
- Use complex impedance Z
- then circuit analysis as usual!

<u>Example 1</u> : A complex voltage divider

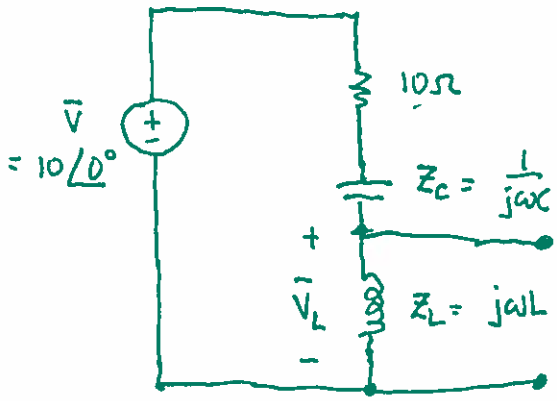


where

$v(t) = 10\cos(100t)$

Find $v_L(t)$

Like resistors in series, impedances <u>add</u>. First, we need the impedances of everything using $\omega = 100$ rads/sec.



$$Z_C = \frac{1}{j\omega C} = \frac{1}{j(100 \times 500 \times 10^{-6})} = -j20\Omega$$

$$Z_L = j\omega L = j(100 \times 0.1) = j10\Omega$$

Reminder: $\frac{1}{j} = -j$ $\rightarrow \frac{1}{j} \cdot \frac{j}{j} = \frac{j}{j^2} = \frac{j}{-1} = -j$