Chapter 9

1. The sample mean and standard deviation for the fill weights of 100 boxes are mean = 12.05, standard deviation = 0.1. Find an 85% confidence interval (C.I.) for the mean fill weight of the boxes. [Ans: (12.0356, 12.0644)]

2. In a sample of 50 microdrills, the average lifetime (expressed in number of holes drilled before failure) was 12.68 with a standard deviation of 6.83.
   (i) Find a 95% C.I. for the mean lifetime of microdrills. [Ans: (10.79, 14.57)]
   (ii) Find a 80% C.I. for the mean lifetime of microdrills. [Ans: (11.44, 13.92)]
   (iii) Based on the data given, an engineer reported a C.I. of (11.09,14.27), but neglected to specify the level. What is the level of this C.I.? [Ans: 90%]

3. A 90% C.I. for the mean diameter (in cm) of steel rods manufactured on a certain extrusion machine is computed to be (14.73, 14.91). True or false: The probability that the mean diameter of rods manufactured by this process is between 14.73 and 14.91 is 90%. [Ans: False]

4. An engineer plans to compute a 90% C.I. for the mean diameter (in cm) of steel rods. She will measure the diameters of a large sample of rods, compute sample mean $\bar{X}$ and standard deviation s, and then compute the interval

   $$\bar{X} \pm 1.645s/\sqrt{n}$$

   True or false: The probability that the population mean diameter will be in this interval is 90%. [Ans: True]

5. A team of geologists plans to measures the weights of 250 rocks. After weighing each rock a large number of times, they will compute a 95% C.I. for its weight. Assume there is no bias in the weighing procedure. What is the probability that more than 240 of the C.I. will cover the true weight of the rocks? [Ans: 0.1922]
6. The sample mean and standard deviation for the fill weights of 100 boxes are mean = 12.05, standard deviation = 0.1. How many boxes must be sampled to obtain a 90% C.I. of width $\pm$ 0.012[Ans: 463]
7. Determine the sample size necessary in question # 2 to produce a 95% C.I. with width $\pm$ 0.5 [Ans: 717]

8. The heights of a random sample of 50 college students showed a mean of 174.5 centimeters and a standard deviation of 6.9 centimeters. Construct a 98% confidence interval for the mean height of all college students. .[Ans: 172.16,176.84]

9. An efficiency expert wishes to determine the average time that it takes to drill three holes in a certain metal clamp. How large a sample will she need to be 95% confident that her sample mean will be within 15 seconds of the true mean? Assume that it is known from previous studies that σ = 40 seconds.

10. The following measurements were recorded for the drying time, in hours, of a certain brand of latex paint: 3.4, 2.5, 4.8, 2.9, 3.6, 2.8, 3.3, 5.6, 3.7, 2.8, 4.4, 4.0, 5.2, 3.0, 4.8. Assuming that the measurements represent a random sample from a normal population, find a 95% prediction interval for the drying time for the next trial of the paint. [Ans: 3.25,4.32]

11. Two kinds of thread are being compared for strength. Fifty pieces of each type of thread are tested under similar conditions. Brand A has an average tensile strength of 78.3 kilograms with a standard deviation of 5.6 kilograms, while brand B has an average tensile strength of 87.2 kilograms with a standard deviation of 6.3 kilograms. Construct a 95% confidence interval for the difference of the population means. [Ans: -11.27, 6.53]

12. A random sample of 20 students yielded a mean of $\bar{X}$ = 72 and a variance of s^2 = 16 for scores on a college placement test in mathematics. Assuming the scores to be normally distributed, construct a 98% confidence interval for σ^2.[Ans: 8.4, 39.83]

13. A taxi company is trying to decide whether to purchase brand 1 or brand 2 tires for its fleet of taxis. 12 tires from two brand 1 and brand 2 are collected and used until they wear out. The sample mean and variance for brand 1 are 36,300 km and 5000 km, respectively. The sample mean and variance for brand 2 are 38,100 km and 6100 km, respectively. Construct a 90% confidence interval for σ_1^2/σ_2^2.

14. A statistician chooses 27 randomly selected dates, and when examining the occupancy records of a particular motel for those dates, finds a standard deviation of 5.86 rooms rented. If the number of rooms rented is normally distributed, find the 95% confidence interval for the population standard deviation of the number of rooms rented. [Ans: 4.57, 8.03]

15. Among 11 patients in a certain study, the standard deviation of the property of interest was 5.8. In another group of 4 patients, the standard deviation was 3.4. We wish to construct a 95 percent confidence interval for the ratio of the variances of these two populations. [Ans: 0.2, 14.06]

16. A research team is interested in the difference between serum uric acid levels in patients with and without Down's syndrome. In a large hospital for the treatment of the mentally retarded, a sample of 12 individuals with Down's syndrome yielded a mean of $\bar{x}_1$ = 4.5 mg/100 ml and s_1^2 = 4.9. In a general hospital a sample of 15 normal individuals of the same age and sex were found to have a mean value of $\bar{x}_2$= 3.4 mg/100 ml and s_2^2 = 5.6. If it is reasonable to assume that the two populations of values are normally distributed, find the 95 percent confidence interval for $\mu_1 - \mu_2$. [Ans: -0.73, 2.93]

17. A study was conducted by the Department of Biological Sciences at the Virginia Tech to estimate the difference in the amounts of the chemical orthophosphorous measured (in mg/L) at two different stations on the James river.
Station 1: n_1=15, sample mean =3.84, s_1=3.07
Station 2: n_2=12, sample mean =1.49, s_2=0.8
Find a 95% C.I. for the difference in the true average orthophosphorous contents at these two stations, assuming that the average came from normal population with different variances.
[Solution Hint: df =16.3

Using alpha = 0.05, $t_{0.025,16} = 2.12$

Then, 95% C.I. = (0.6, 4.1)]

18. A certain change in a process for manufacturing component parts is being considered. Samples are taken under both the existing and the new process so as to determine if the new process results in an improvement. If 75 of 1500 items from the existing process are found to be defective and 80 of 2000 items from the new process are found to be defective, find a 90% confidence interval for the true difference in the proportion of defective components between the existing and the new process.

[Hint: Let p_1 and p_2 be the true proportion of defective components for the existing and new processes. Hence sample proportions for existing process= 75/1500 =0.05, and for new process =80/2000=0.04. Difference of sample proportions=0.01. For 90% C.I, $z_{0.05}$= 1.645, and the C.I. = (-0.0017, 0.0217).]

Reference:

1.Statistics for Engineers and Scientists, William Navidi, Third edition, McGraw Hill

2.Probability and Statistics for Engineers and Scienctists , R.E. Walpole, R.H. Myers, S. L. Myers, and K. Ye, 9e, Prentice Hall