Tutorial #6

Chapter 6 – Continuous Probability Distributions

Question 1. Suppose the cumulative distribution of the random variable X is

$$F(x) = \begin{cases} 0 & x < 0 \\ 0.2x & 0 \le x < 5 \\ 1 & x \ge 5 \end{cases}$$

Determine the following:

(a) P(X<2.8)
(b) P(X>1.5)
(c) P(X<-2)
(d) P(X>6)
(e) What is the probability that X is greater than 6 given that X is greater than 1.5

Question 2. Suppose $f(x) = 1.5x^2$ for $-1 < x < 1$. Determine the mean and variance of X.

Question 3. The thickness of a flange on an aircraft component is uniformly distributed between 0.95 and 1.05 millimeters.
(a) Determine the cumulative distribution function of flange thickness.
(b) Determine the proportion of flanges that exceeds 1.02 millimeters.
(c) What thickness is exceeded by 90% of the flanges?
(d) Determine the mean and variance of flange thickness

Question 4. The bus you take randomly and uniformly arrive between 2 and 2:30pm. What is the probability that you take the bus between 2:15 and 2:20pm?

Question 5. The line width of semiconductor manufacturing is assumed to be normally distributed with a mean of 0.5 micrometer and a standard deviation of 0.05 micrometer.
(a) What is the probability that a line width is greater than 0.62 micrometer?
(b) What is the probability that a line width is between 0.47 and 0.63 micrometer?
(c) The line width of 90% of samples is below what value?

Question 6. The length of an injection molded plastic case that holds magnetic tape is normally distributed with a length of 90.2 millimeters and a standard deviation of 0.1 millimeter. What is the probability that a part is longer than 90.3 millimeters or shorter than 89.7 millimeters?

Question 7. A call center receives on average 5 calls per 10 minutes.
(a) What is the probability that time until the first call is less than 5 minutes?
(b) What is the probability that there are 2 calls in the first minute?

Question 8. A supplier ships a lot of 1000 electrical connectors. A sample of 25 is selected at random, without replacement. Assume the lot contains 100 defective connectors.
(a) What is the probability that there are no defective connectors in the sample?

(b) Use binomial approximation to answer part (a)?
(c) Use normal distribution to answer part (a)?

Question 9. A fisher man expects to get a fish one every half an hour. Compute the probability that he will wait between 2 and 4 hours before he catches 4 fish.

Question 10. Assume the life of a packaged magnetic disk exposed to corrosive gases has a Weibull distribution with β=0.5 and the mean life is 600 hours.
(a) Determine the probability that a packaged disk lasts at least 500 hours? [Ans: 0 .275]
(b) Determine the probability that a packaged disk fails before 400 hours? [Ans: 0.685]

Question 11. In a company, user logons are modelled such that a mean of 25 logons per hour is received. What is the probability that there are no logons in an interval of 6 minutes?

Question 12. Assume that arrival times at a drive-through window follow a Poisson process with mean rate 0.2 arrivals per minute. Let T be the time until the third arrival. Find ($T \leq 20$). [Ans: 0.762]

Question 13.
(a) The lifetime in weeks of a certain type of transistor is known to follow gamma distribution with mean 10 weeks and standard deviation $\sqrt{50}$ weeks.
(i) What is the probability that a transistor of this type will last at most 50 weeks?
(ii) What is the probability that a transistor of this type will not survive the first 10 weeks?
(b) Now assume that the distribution in part (a) is normal instead of gamma with mean 10 weeks and standard deviation $\sqrt{50}$ weeks, and calculate
(i) What is the probability that a transistor of this type will last at most 50 weeks?
(ii) What is the probability that a transistor of this type will not survive the first 10 weeks?
(c) Comment on the results in part (a) and (b)

Question 14.
A certain type of device has an advertised failure rate of 0.01 device per hour. The failure rate is constant.
(a) What is the mean time to failure?
(b) What is the probability that 200 hours will pass before a failure is observed.
(c) Calculate reliability at 200 h and at 400 h.

Question 15. Suppose that a system contains a certain type of component whose time, in years, to failure is given by T. The random variable T is modeled by exponential distribution with mean time to failure = 5 years. If 5 of these components are installed in different systems, what is the probability that at least 2 are still functioning at the end of 8 years?