

Concept of Random Variables

Question 1. Classify the following random variables as discrete or continuous

X: the number of automobile accidents per year in Virginia

Y: the length of time to play 18 holes of golf

M: the amount of milk produced yearly by a particular cow

N: Number of bacteria in per cubic centimeter in drinking water

T: Time spent by a school bus to reach school from a certain location

Developing Probability Distribution Function

Question 3. Suppose that a day's production of 850 manufactured parts contains 50 parts that do not conform to customer requirements. Two parts are randomly selected in succession, without replacement, from the batch. Let the random variable X equal the number of nonconforming parts in the sample. (a) Find the distribution function of X in a tabular form and show it graphically. (b) Find the probability distribution function and plot it. What is the cumulative distribution function of X? Graph the cumulative distribution function of X.

Question 4. From a box containing 4 black balls and 2 green balls, 3 balls are drawn in succession, each ball being replaced in the box before the next draw is made. Find the probability distribution for the number of green balls.

Question 5. (a) Suppose the measurement error X of a certain physical quantity is decided by the density function. Determine k that renders f(x) a valid probability density function.

$$f(x) = \begin{cases} k(3 - x^2), & -1 \leq x \leq 1 \\ 0, & \text{elsewhere} \end{cases}$$

(b) Determine the value of c so that the following function can serve as a probability distribution function of the discrete random variable X

$$f(x) = c \binom{2}{x} \binom{3}{3-x}, \quad \text{for } x = 0, 1, 2$$

Calculation of probability from distribution functions

Question 6. The total number of hours, measured in units of 100 hours, that a family runs a vacuum cleaner over a period of one year is a random variable X that has the density function

$$f(x) = \begin{cases} x, & 0 < x < 1 \\ 2 - x, & 1 \leq x < 2 \\ 0, & \text{elsewhere} \end{cases}$$

Find the probability that over a period of one year, a family runs their vacuum cleaner

- (a) Less than 120 hours
- (b) Between 50 and 100 hours
- (c) At least 200 hours

Question 7. The cumulative distribution function of a random variable T is given below

$$F(t) = \begin{cases} 0, & t < 1 \\ \frac{1}{4}, & 1 \leq t < 3 \\ \frac{1}{2}, & 3 \leq t < 5 \\ \frac{3}{4}, & 5 \leq t < 7 \\ 1, & t \geq 7 \end{cases}$$

Find

- (a) $P(T=5)$
- (b) $P(T>5)$
- (c) $P(1.4 < T < 6)$
- (d) $P(T \leq 5 | T \geq 2)$

Question 8. A shipment of 7 television contains 2 that are defective. A hotel makes a random purchase of 3 televisions. If x is the number of defective televisions purchased by the hotel, (a) find the probability distribution of X. (b) Express the results graphically as a histogram. (c) Find F(x) (d) Find P (X=1) (e) Find $P(0 < X \leq 2)$

Q1: X : Discrete
 Y : Continuous.

M : Continuous.
 N : Discrete

T : Continuous.

Q2: 800 C
50 N \leftarrow Conforming
 \leftarrow Nonconforming.

X : # of nonconforming parts in a sample of 2.

$$X = \{0, 1, 2\}$$

$$P(X=0) = \frac{\binom{800}{2}}{\binom{850}{2}} \leftarrow \begin{array}{l} \text{\# of ways choosing 2 conforming.} \\ \text{Total \# of way choosing any 2 parts.} \end{array}$$

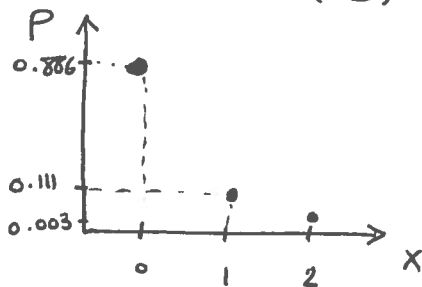
$$P(X=1) = \frac{\binom{800}{1}\binom{50}{1}}{\binom{850}{2}}$$

$$P(X=2) = \frac{\binom{50}{2}}{\binom{850}{2}}$$

$$P(X) = \frac{\binom{50}{x}\binom{800}{2-x}}{\binom{850}{2}} \quad x=0,1,2$$

$$P(X=0)=0.886, \quad P(X=1)=0.111, \quad P(X=2)=0.003$$

$$\sum P(x) = 1 \quad \checkmark$$



$$F_X(0) = P(X=0) = 0.886$$

Cumulative density function

$$F_X(1) = P(X \leq 1) = P(X=0) + P(X=1) = 0.997$$

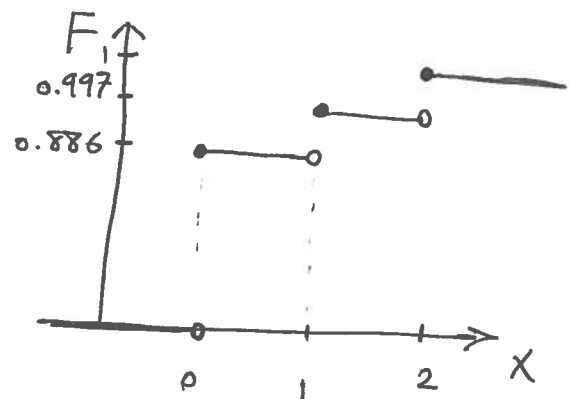
$$F_X(2) = P(X \leq 2) = P(X=0) + P(1) + P(2) = 1$$

ex.

$$F_X(5) = P(X \leq 5) = P(X=0) + P(1) + P(2) = 1$$

$$F_X(+\infty) = 1$$

\leftarrow Although 5 & $+\infty$ are not feasible values for X .



Always starts from 0
 goes to 1

Q4. $X = \#$ of green balls in 3 draws (with replacement)
 ↪ Not a combination.

$$X = \{0, 1, 2, 3\}$$

if $X = 0 \rightarrow$ all 3 are blacks

$$P(0) = \frac{4}{6} \times \frac{4}{6} \times \frac{4}{6} = \frac{64}{216} \quad \text{There are 3 ways to have}$$

$X = 1 \rightarrow$ 1 green & 2 black

$$P(1) = \frac{2}{6} \times \frac{4}{6} \times \frac{4}{6} \times 3 = \frac{96}{216} \quad \text{1 green & 2 black.}$$

$X = 2 \rightarrow$ 2 " & 1 "

$$P(2) = \frac{2}{6} \times \frac{2}{6} \times \frac{4}{6} \times 3 = \frac{48}{216}$$

$X = 3 \rightarrow$ 3 green

$$P(3) = \frac{2}{6} \times \frac{2}{6} \times \frac{2}{6} = \frac{8}{216}$$

$$\sum P(X) = 1 \quad \checkmark$$

~~P(X)~~

Q5. (a) $\int_{-\infty}^{+\infty} f_x dx = 1 \rightarrow \int_{-1}^1 k(3-x^2) dx = k(3x - \frac{x^3}{3}) \Big|_{-1}^1$

$$= 6k - \frac{2}{3}k = \frac{16k}{3} = 1 \rightarrow \boxed{k = \frac{3}{16}}$$

(b) - $f(0) = c \binom{2}{0} \binom{3}{3} = c \times \frac{2!}{0!(2-0)!} \times \frac{3!}{3!(3-3)!} = c$

$$f(1) = c \binom{2}{1} \binom{3}{2} = 6c$$

$$f(2) = c \binom{2}{2} \binom{3}{1} = 3c$$

$$\sum f(x) = 1 \rightarrow f(0) + f(1) + f(2) = 10c = 1 \rightarrow \boxed{c = \frac{1}{10}}$$

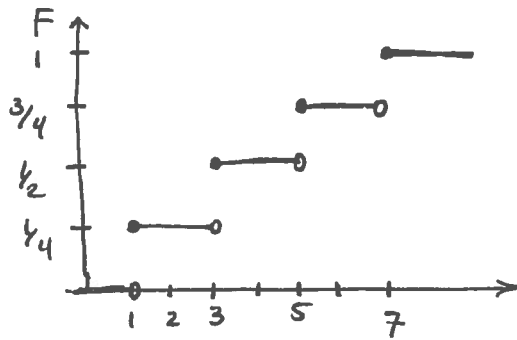
Q6. (a) $P(X \leq \frac{120}{100}) = P(X \leq 1.2) = \int_{-\infty}^{1.2} f_x dx = \int_0^1 x dx + \int_1^{1.2} (2-x) dx$
units of 100

$$= \frac{x^2}{2} \Big|_0^1 + 2x \Big|_1^{1.2} - \frac{x^2}{2} \Big|_1^{1.2} = \frac{1}{2} + 0.4 - 0.22 = \underline{0.68}$$

(b) $P(\frac{50}{100} \leq x \leq \frac{100}{100}) = P(0.5 \leq x \leq 1) = \int_{0.5}^1 x dx = \frac{x^2}{2} \Big|_{0.5}^1 = \underline{0.375}$

(c) $P(\underbrace{x \geq 200}_{\substack{\uparrow \\ \text{at least}}}) = P(x \geq 2) = \int_2^{+\infty} f_x dx = \int_2^{+\infty} 0 dx = \underline{0}$

Q7.



$$P(T=1) = 1/4, \quad P(T=2) = 0, \quad P(T=3) = 1/4, \quad P(T=4) = 0, \quad P(T=5) = 1/4$$

$$P(T=6) = 0, \quad P(T=7) = 1/4, \quad P(T \geq 8) = 0$$

$$(a) = P(T=5) = \boxed{1/4}, \quad (b) P(T > 5) = P(T=6) + P(T=7) + P(T \geq 8)$$

$$= 0 + 1/4 + 0 = \boxed{1/4}$$

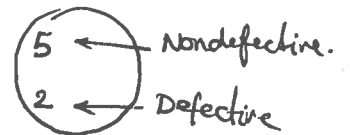
$$(c) P(1.4 < T < 6) = P(T=3) + P(T=4) + P(T=5) = \boxed{1/2}$$

$$(d) P(T \leq 5 | T \geq 2) = \frac{P(2 \leq T \leq 5)}{P(T \geq 2)} = \frac{1/4 + 1/4}{1/4 + 1/4 + 1/4} = \frac{2/4}{3/4} = \boxed{2/3}$$

Q8.

x : # of defective.

$$x = \{0, 1, 2\}$$



$$P(0) = \frac{\binom{5}{3}}{\binom{7}{3}} = \frac{10}{35}$$

$$P(1) = \frac{\binom{5}{2} \binom{2}{1}}{\binom{7}{3}} = \frac{20}{35}$$

$$P(2) = \frac{\binom{2}{2} \binom{5}{1}}{\binom{7}{3}} = \frac{5}{35}$$

$$P(x) = \frac{\binom{5}{3-x} \binom{2}{x}}{\binom{7}{3}} \quad x = 0, 1, 2$$

$$\sum P(x) = 1 \checkmark$$

$$P(0 < x \leq 2) = P(x=1) + P(x=2)$$

$$= \frac{25}{35}$$

$$F(0) = P(x \leq 0) = \frac{10}{35}$$

$$F(1) = P(x \leq 1) = \frac{30}{35}$$

$$F(2) = P(x \leq 2) = 1$$

