Q1: (a) binomial. X : # of wins. $P(x) = \binom{n}{x} p^x (1-p)^{n-x}$

$n_1 = 4 \quad P(X=3) = \binom{4}{3}\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^1 = 0.25$

$n_2 = 8 \quad P(X=5) = \binom{8}{5}\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^3 = 0.219$

(b)

$n_1 = 4 \quad P(x \geq 3) = \sum_{x=3}^{4} \binom{4}{x}\left(\frac{1}{2}\right)^x\left(\frac{1}{2}\right)^{4-x} = 0.312$

$n_2 = 8 \quad P(x \geq 5) = \sum_{x=5}^{8} \binom{8}{x}\left(\frac{1}{2}\right)^x\left(\frac{1}{2}\right)^{8-x} = 0.362$

Q2: Hypergemetric.

$n = 50$

$n_1 = 2$ defective

$n_2 = 48$ nondefective

Sample of size $K = 10$

$P(x) = \dfrac{\binom{2}{x}\binom{48}{10-x}}{\binom{50}{10}}$

$E(x) = \dfrac{K n_1}{n}$

(a) X = # of defective in the sample.

$P(X \geq 1) = 1 - P(X=0) = 1 - \dfrac{\binom{2}{0}\binom{48}{10}}{\binom{50}{10}} = 0.363$

(b) $\mu = E(X) = \dfrac{10 \times 2}{50} = 0.4$

(c) Binomial approximation.

$P(\text{defective}) = \dfrac{n_1}{n} = \dfrac{2}{50}$

$P(X \geq 1) = 1 - P(X=0) = 1 - \binom{10}{x}\left(\frac{2}{50}\right)^x\left(1 - \frac{2}{50}\right)^{10-x}$

$= 1 - \binom{10}{0}\left(\frac{2}{50}\right)^0\left(\frac{48}{50}\right)^{10} = 0.335$

Q3: $P(\text{success}) = P = 0.25$

(a) Binomial $P(x=1) = \binom{10}{1}(0.25)^1(0.75)^{10-1} = 0.1877$

(b) (i) Binomial $P(x=0) = \binom{10}{0}(0.25)^0(0.75)^{10-0} = 0.056$

(ii) Geometric $P(x=11) = P(1-P)^{11-1} = 0.25\,(0.75)^{10} = 0.014$

(c) Negative Binomial $P(x=6) = \binom{6-1}{2-1}(0.25)^2(0.75)^4 = 0.098$

second success

$K = 2$

Q4: Poisson $\lambda = 100/\text{hour} \longrightarrow \lambda_{adj} = 5/3\text{min}$ $\quad P(x) = \dfrac{e^{-\lambda_{adj}} (\lambda_{adj})^x}{x!}$

(a) $P(x=0) = \dfrac{e^{-5} (5)^0}{0!} = 0.0067$

(b) $P(x>5) = 1 - P(x=0) - P(x=1) - P(x=2) - P(x=3) - P(x=4) - P(x=5)$

$$= 1 - \frac{e^{-5} 5^0}{0!} - \frac{e^{-5} 5}{1!} - \frac{e^{-5} (5)^2}{2!} - \frac{e^{-5} (5)^3}{3!} - \frac{e^{-5} (5)^4}{4!} - \frac{e^{-5} (5)^5}{5!} = 0.384$$

Q5. Poisson. $\lambda = 5.4/2\text{min} \longrightarrow \lambda_{adj} = 2.7/\text{min}$

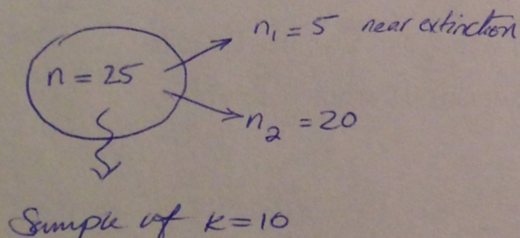
(a) $P(x \leq 4) = \displaystyle\sum_{x=0}^{4} \frac{e^{-2.7} (2.7)^x}{x!} = 0.8629$

(b) $P(x<2) = P(x=0) + P(x=1) = \dfrac{e^{-2.7} (2.7)^0}{0!} - \dfrac{e^{-2.7} (2.7)^1}{1!} = 0.2487$

(c) $\lambda_{adj} = 13.5/5\text{min}$

$$P(x<2) = P(x=0) + P(x=1) = \frac{e^{-13.5} (13.5)^0}{0!} + \frac{e^{-13.5} (13.5)^1}{1!} = 1.987 \times 10^{-6}$$

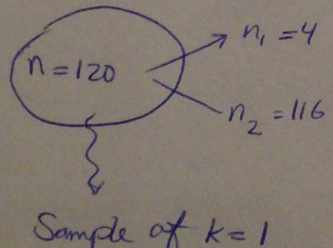
Q6: Hypergeometric



(a) $P(x=2) = \dfrac{\binom{5}{2}\binom{20}{10-2}}{\binom{25}{10}} = 0.385$

(b) $P(x \leq 2) = \displaystyle\sum_{x=0}^{2} \frac{\binom{5}{x}\binom{20}{10-x}}{\binom{25}{10}} = 0.699$

Q7: Hypergemetric.



$P(x=1) = \dfrac{\binom{4}{1}\binom{116}{1-0}}{\binom{120}{1}} = 0.033$

Q9: Geometric.

1 min 1 min 1 min

fail fail Success

$P(x=1) = (0.3)^2\, 0.7 = 0.063$

Q10: $\lambda = 0.5 / 1\,\text{sheet}$

(a) $P(X=0) = \dfrac{e^{-0.5}(0.5)^0}{0!} = 0.61$

(b) $P(X \geq 2) = 1 - P(X=0) - P(X=1) = 1 - \dfrac{e^{-0.5}(0.5)^0}{0!} - \dfrac{e^{-0.5}(0.5)^1}{1!}$

$= 0.09$

Q11:

Q12: Hypergeometric

$n = 5$

$n_1 = 3$ Sound

$n_2 = 2$ faulty

a random sample of $k = 3$

$E(x) = \dfrac{n_1 k}{n} = \dfrac{3 \times 3}{5} = 1.8$

$var(x) = k \dfrac{n_1}{n} \dfrac{(n_2)}{n} \times \dfrac{n-k}{n-1}$

$= 3 \times \dfrac{3}{5} \times \dfrac{2}{5} \times \dfrac{2}{4} = 0.36$