

22. An experiment was conducted to determine whether the firing temperature affects the density of the bricks produced. The following data relates to the density of bricks manufactured at Temperature 1 and Temperature 2, respectively.

$$\begin{aligned}\bar{x}_1 &= 21.6 & \bar{x}_2 &= 23.9 \\ s_1 &= 0.9 & s_2 &= 1.2 \\ n_1 &= 7 & n_2 &= 9\end{aligned}$$

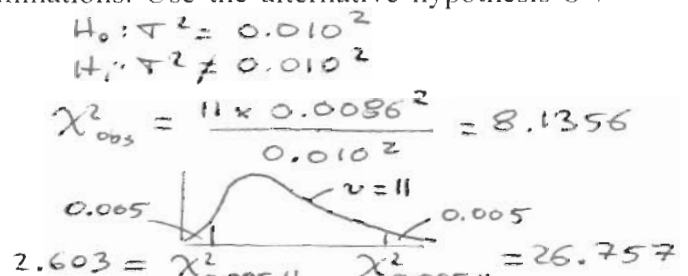
Using a significance level of 5% and assuming that the true standard deviation at both temperatures is 1.0, what is the conclusion to a two-tailed hypothesis test $H_0: \mu_1 - \mu_2 = -2$?

- a) $z_{\text{obs}} = -0.595 > z_{\text{cri}} = -1.960$; Accept H_0
b) $z_{\text{obs}} = -8.533 < z_{\text{cri}} = -1.645$; Reject H_0
c) $t_{\text{obs}} = -0.571 > t_{\text{cri}} = -1.645$; Accept H_0
d) $t_{\text{obs}} = -0.300 > t_{\text{cri}} = -1.960$; Accept H_0

$$\begin{aligned}\bar{x}_1 - \bar{x}_2 &\sim N(-2, \sqrt{\frac{1^2}{7} + \frac{1^2}{9}}) \\ \bar{x}_1 - \bar{x}_2 &\sim N(-2, 0.50) \\ z_{\text{obs}} &= \frac{(21.6 - 23.9) - (-2)}{0.50} \\ z_{\text{obs}} &= -0.595 \quad \pm z_{0.025} = \pm 1.960\end{aligned}$$

23. If 12 determinations of the specific heat of iron have a standard deviation of 0.0086, test the null hypothesis that $\sigma = 0.010$ for such determinations. Use the alternative hypothesis $\sigma \neq 0.010$ and the level of significance $\alpha = 0.01$.

- a) $\chi^2_{\text{obs}} = 8.1356 > \chi^2_{0.995} = 2.603$; Accept H_0
b) $\chi^2_{\text{obs}} = 8.1356 < \chi^2_{0.005} = 26.757$; Reject H_0
c) $\chi^2_{\text{obs}} = 8.1356 > \chi^2_{0.995} = 3.503$; Accept H_0
d) $\chi^2_{\text{obs}} = 8.1356 < \chi^2_{0.005} = 24.727$; Reject H_0



24. A soft drink bottler is studying the internal pressure strength of 1-litre glass non-returnable bottles. A random sample of 5 bottles obtained from Manufacturer 1 yielded a mean strength of 193 psi and a standard deviation of $s_1 = 34$ psi. A sample mean of 200 psi and a sample standard deviation of $s_2 = 23$ psi were computed from a sample of 8 bottles provided by Manufacturer 2. What is the test statistic for the hypothesis test: $H_0: \sigma_1 = \sigma_2$, $H_1: \sigma_1 > \sigma_2$ with $\alpha = 0.05$?

$$H_0: \sigma_1^2 / \sigma_2^2 = 1$$

$$H_1: \sigma_1^2 / \sigma_2^2 > 1$$

- a) -0.41
b) +1.25
c) +1.48
d) +2.19

$$f_{\text{obs}} = \frac{s_1^2}{s_2^2} \frac{\sigma_2^2}{\sigma_1^2} = \frac{34^2}{23^2} \times 1 = 2.19$$

25. A study of two types of materials used in electrical conduits, tubes used to house electrical wires, has been undertaken. The purpose of the study was to compare the strength of the two materials, defined as the load in kg required to crush a 25 cm diameter tube to 40% of its original diameter. A one-tailed hypothesis test is to be performed to determine if Material 1 is stronger than Material 2 (based on mean loads applied). The following data have been collected:

σ_1^2 and σ_2^2 unknown

$\sigma_1^2 \neq \sigma_2^2$

$x_1 = 380 \text{ kg}$ $x_2 = 370 \text{ kg}$

$s_1^2 = 100$ $s_2^2 = 400$

$n_1 = 12$ $n_2 = 7$

$\bar{x}_1 - \bar{x}_2 \sim t_v \left(0, \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \right)$

$\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = 8.09$

Using a significance level of 5% and assuming that the population variances are not equal, what is the conclusion of the hypothesis test $H_0: \mu_1 - \mu_2 = 0$?

$H_1: \mu_1 - \mu_2 > 0$

$t_{obs} = [(380 - 370) - 0] / 8.09 = 1.24$

$v = 7.79 \text{ say } 7$

$+ t_{0.05, 7} = + 1.895$

a) $t_{obs} = 1.24 < t_{crit} = 1.740$; Accept H_0

b) $t_{obs} = 1.24 < t_{crit} = 1.895$; Accept H_0

c) $t_{obs} = 1.47 < t_{crit} = 1.740$; Accept H_0

d) $t_{obs} = 1.47 < t_{crit} = 1.895$; Accept H_0

26. As part of an industrial training program, some trainees are instructed by Method A, which is straight teaching-machine instruction, and some are instructed by Method B, which also involves the personal attention of an instructor. If random samples of size 7 are taken from large groups of trainees instructed by the different methods, and the scores which they obtained in an appropriate achievement test are:

Method A	71	75	65	69	73	66	68
Method B	72	77	84	78	69	70	77

Use the 0.05 level of significance to test the claim that Method B is more effective. Assume that populations sampled can be approximated closely with normal distributions having the same variance. Note that the higher the score the more effective the method.

σ_1^2 and σ_2^2 unknown

$\sigma_1^2 = \sigma_2^2$

a) $t_{obs} = 2.356 < t_{crit} = 3.055$; Accept H_0

b) $t_{obs} = 2.356 > t_{crit} = 1.812$; Reject H_0

c) $t_{obs} = -2.356 < t_{crit} = 1.812$; Accept H_0

d) $t_{obs} = 2.356 > t_{crit} = 1.782$; Reject H_0

$\bar{x}_1 - \bar{x}_2 \sim t_p \left(0, s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \right)$

$\bar{x}_A = 69.6$ $\bar{x}_B = 75.3$

$s_A = 3.64$ $s_B = 5.28$

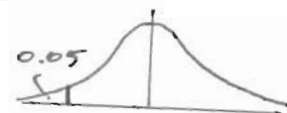
$H_0: \bar{x}_A - \bar{x}_B = 0$

$H_1: \bar{x}_A - \bar{x}_B < 0$

$s_p^2 = (6 \times 3.64^2 + 6 \times 5.28^2) / 12 = 20.56$

$s_p \sqrt{\frac{1}{7} + \frac{1}{7}} = 2.42$

$t_{obs} = \frac{(69.6 - 75.3) - 0}{2.42} = -2.352$



$- t_{0.05, 12} = - 1.782$

or $t_{obs} = 2.352 > t_{0.05, 12} = 1.782$
 $H_0: \bar{x} = \dots$

27. The following are the number of sales which a sample of 8 salespeople of industrial chemicals in California and a sample of 6 salespeople of industrial chemicals in Oregon, have made over a certain fixed period of time.

California (1)	59	68	44	71	63	46	69	54
Oregon (2)	50	36	62	52	70	41		

Assuming that the populations samples can be approximated closely with normal distributions with unequal variances, determine the appropriate critical value of the statistic for determining whether to reject the null hypothesis $H_0: \mu_1 - \mu_2 = 0$ against the alternative $H_1: \mu_1 - \mu_2 > 0$ at the 0.01 level of significance.

- a) 3.250
b) 2.681
c) 2.764
d) 2.821

$$\bar{x}_1 = 59.25 \quad \bar{x}_2 = 51.83$$

$$s_1 = 10.42 \quad s_2 = 12.69$$

$$\sigma_1^2 \neq \sigma_2^2$$

$$v = 9.6 \text{ say } 9$$

$$t_{0.01, 9} = 2.821$$

28. The following pH values were measured for various samples of deionized water:

4.39 5.18 5.19 5.29 5.33 5.34 5.34 5.42 5.45

Compute the sample mean.

- a) 5.87
b) 5.21
c) 4.69
d) 5

$$\bar{x} = 5.21$$

29. What is the variance for the following data:

0.78 0.97 0.65 0.71

- a) 0.014
b) 0.019
c) 0.120
d) 0.139

$$s = 0.1389$$

$$s^2 = 0.019$$

For Questions 30, 31, 32, and 33, use the information provided in the question below.

The cost of manufacturing a lot of a certain product depends on the lot size, as shown by the following sample data:

Cost in \$ (y)	50	100	200	500	750	1000	2000	4000
Lot Size in hectares (x)	1	5	10	25	50	100	250	500

From the above data, the following quantities are computed:

$$\sum x = 941$$

$$\sum y = 8600$$

$$\sum xy = 2,652,550$$

$$\sum x^2 = 325751$$

$$S_{xx} = 215,066$$

$$S_{yy} = 12,620,000$$

$$S_{xy} = 1,640,975$$

30. Compute the least squares estimates of the intercept a and slope b .

- a) $a = 7.63, b = 177.52$
- ☒ b) $a = 177.52, b = 7.63$
- c) $a = -22.157, b = 0.13$
- d) $a = 0.13, b = -22.157$

$$b = \frac{S_{xy}}{S_{xx}} = \frac{1,640,975}{215,066} = 7.63$$

$$a = (8600 - 7.63 \times 941) / 8 = 177.52$$

31. Using the information in Question 30, predict what cost would be observed when the lot size is 75 hectares.

- a) 5
- b) 747
- c) 550
- ☒ d) 750

$$\hat{y}_0 = 177.52 + 7.63 \times 75 = 749.77 \text{ say } 750$$

32. Using the information in Question 30, give a point estimate of the mean cost when the lot size is 200 hectares.

- a) 1,526
- b) 1,667
- ☒ c) 1,704
- d) 2,000

$$\hat{y}_0 = 177.52 + 7.63 \times 200 = 1704$$

33. What proportion of the cost is accounted for by the lot size?

- a) 99%
- b) 98%
- c) 96%
- d) 95%

$$r^2 = \frac{S_{xy}^2}{S_{xx} S_{yy}} = 0.99$$

34. Let x be the number of weeks that students have spent in a speed-reading program, and y is the speed gained (in words per minute) by the students enrolled in the program. For the linear regression model $y = bx + a$, the coefficient of determination for this model is found to be 0.73. Which of the following statements is true?

- a) 27% of the variability in the data is explained by the regression model
- b) 85% of the variability in the data is explained by the regression model
- c) 53% of the variability in the data is explained by the regression model
- d) 73% of the variability in the data is explained by the regression model

35. The strength of paper used in the manufacture of cardboard boxes (y) is related to the percentage of hardwood concentration in the original pulp (x). Under controlled conditions, a pilot plant manufactures 6 samples, each from a different batch of pulp, and measures the tensile strength. The data are provided below.

x	1.0	1.5	2.2	2.8	3.0	3.0
y	101.4	117.4	136.8	145.2	153.1	148.7

From the above data, the following quantities are computed:

$$\begin{aligned} \sum x &= 13.5 & \sum y &= 802.6 & \sum xy &= 1890.42 & \bar{x} &= 2.25 \\ \sum x^2 &= 33.93 & S_{xx} &= 3.56 & S_{yy} &= 2052.2 \\ S_{xy} &= 84.57 & s^2 &= 10.06 & s &= 3.17 \end{aligned}$$

The least squares regression model has been computed as $y = 80.24 + 23.79x$.

What is the 90% confidence interval for the mean tensile strength of the cardboard boxes when the hardwood concentration is 2.0?

$$\hat{y}_0 = 80.24 + 23.79 \times 2.0 = 127.8$$

- a) $127.8 \pm 2.015 \times 1.36$
- b) $127.8 \pm 2.132 \times 1.36$
- c) $127.8 \pm 2.015 \times 3.45$
- d) $127.8 \pm 2.132 \times 3.45$

$$\begin{aligned} \hat{y}_0 \pm t_{0.05, 4} s \sqrt{\frac{1}{n} + \frac{(x_0 - \bar{x})^2}{S_{xx}}} \\ 127.8 \pm 2.132 \times 3.17 \times \left(\frac{1}{6} + \frac{(2.0 - 2.25)^2}{3.56} \right)^{1/2} \\ 127.8 \pm 2.132 \times 1.36 \end{aligned}$$