Chapters 8 – 10 (Solutions)

1. $(1-\alpha)\%$ CI for σ^2 : $\left[\dfrac{(n-1)S^2}{\chi^2_{\alpha/2, n-1}} , \dfrac{(n-1)S^2}{\chi^2_{1-\alpha/2, n-1}} \right]$

$$S^2 = \frac{\sum (x_i - \bar{x})^2}{n-1} = 0.0525$$

$\bar{x} = 15.925$

$\Rightarrow 90\%$ for σ^2 : $\left[\dfrac{19 \times 0.0525}{\chi^2_{0.05, 19}} , \dfrac{19 \times 0.0525}{\chi^2_{0.95, 19}} \right] = [0.033, 0.098]$

$\chi^2_{0.05,19} = 30.144$, $\chi^2_{0.95,19} = 10.117$

$\Rightarrow 90\%$ CI for σ = $[0.182, 0.314]$

2. $\bar{x} = 683.3$

$S^2 = 4506.7$

(Assume the normality)

99% upper CI : $\left(-\infty, \bar{x} + t_{\alpha, n-1} \dfrac{S}{\sqrt{n}} \right] = \left(-\infty, 683.3 + 3.365 \times \dfrac{67.13}{\sqrt{6}} \right]$

σ^2 unknown $\rightarrow$ t distribution. (α: 1%, $n-1$: 5)

$= (-\infty, 775.56]$

3. a. Type I $= \alpha = P(\underbrace{\text{Reject } H_0}_{\text{Critical region}} \mid \mu \text{ is } \mu_0 \text{ in the } H_0) = P(\bar{x} < 11.5 \mid \mu = 12)$

$\bar{x} \sim N(12, \frac{0.25}{16})$ $\leftarrow \sigma^2/n$

$$= P\left(\frac{\bar{x} - \mu}{\sigma/\sqrt{n}} < \frac{11.5 - 12}{0.5/4} \right) = P(Z < -4) \approx 0$$

b. Type II $= \beta = P(\overbrace{\text{Accept } H_0}^{\text{Acceptance Region}} \mid \mu \text{ is } 11.25) = P(\bar{x} > 11.5 \mid \mu = 11.25)$

$\hookrightarrow \bar{x} \sim N(11.25, \frac{0.25}{16})$

$$= P\left(\frac{\bar{x} - \mu}{\sigma/\sqrt{n}} > \frac{11.5 - 11.25}{0.5/4} \right) = P(Z > 2) = 0.0228$$

4. $\mu = 300$, $n = 15$, $\bar{x} = 290$, $S = 50$ $\Rightarrow \dfrac{\bar{x} - \mu}{S/\sqrt{n}} \sim t_{n-1}$

$$P(\bar{x} < 290) = P\left(\frac{\bar{x} - \mu}{S/\sqrt{n}} < \frac{290 - 300}{50/\sqrt{15}} \right)$$

$$= P(t_{14} < -0.77) \cong 0.25$$

5. $\sigma = 0.12$

$\bar{x} \sim N(9, \frac{0.12}{6})$

a. Type I $= \alpha = P(\text{Reject } H_0 \mid \mu = 9)$

Acceptance Region $= 8.7 \leq \bar{x} \leq 9.1$

$\rightarrow$ Rejection or Critical Region $= \bar{x} \geq 9.1$ OR $\bar{x} \leq 8.7$

$\Rightarrow 1 - \alpha = P(\text{Accept } H_0 \mid \mu = 9)$

$= P(8.7 \leq \bar{x} \leq 9.1 \mid \mu = 9) = P\left(\frac{8.7 - 9}{\sqrt{\frac{0.12}{6}}} \leq \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} \leq \frac{9.1 - 9}{\sqrt{\frac{0.12}{6}}} \right)$

$= P(-2.12 < Z < 0.71) = 0.7611 - 0.0170 = 0.744$

$\bar{x} \sim N(9.2, \frac{0.12}{6})$

b. Type II $= \beta = P(\text{Accept } H_0 \mid \mu = 9.2)$

$= P(8.7 \leq \bar{x} < 9.1 \mid \mu = 9.2) = P\left(\frac{8.7 - 9.2}{\sqrt{\frac{0.12}{6}}} < \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} \leq \frac{9.1 - 9.2}{\sqrt{\frac{0.12}{6}}} \right) = P(-3.53 < Z < -0.71)$

$= 0.2389 - 0 \Longrightarrow$ power of test $= 1 - \beta = 0.7611$

6. $\bar{x} = 22.25$, $s^2 = 16.5$

Test/examining $\mu \longrightarrow$ either t or z test $\xrightarrow[\text{unknown}]{\sigma^2}$ t-test.

$$T_0 = \frac{\bar{x} - \mu}{S / \sqrt{n}} = \frac{22.25 - 20}{\sqrt{16.5} / \sqrt{8}} = 1.57$$

Acceptance region $= [-t_{\alpha/2, n-1}, t_{\alpha/2, n-1}] = [-t_{0.1, 7}, t_{0.1, 7}] = [-1.415, 1.415]$

Since $T_0 \notin AR \longrightarrow$ Reject H_0