Sections #5 and #6

(1) An electronics firm manufactures a certain type of computer microchip, which according to specifications has an operational capacity with a mean of 100 MHz and a standard deviation of 15 MHz. Indicate which of the following is the percentage of the microchips that have an operational capacity greater than 79 MHz (to 2 significant digits).

a. 92%
b. 8.0%
c. 84%
d. 74%
e. None of the above

Solution: Normal Distribution:
$Z = (79-100)/15 = -1.4$
$P(X>79)=1-P(Z<-1.4)=1-0.0808$ so the right answer is (a)

(2) A large shovel at an open pit mine picks up an average of 9.4 m^3 per scoop. Assume the volume of the scoop is normally distributed with a standard deviation of 2.0 m^3. What is the probability of a scoop exceeding 12 m^3? (three significant digits)

a. 0.995
b. 0.00471
c. 0.903
d. 0.0968
e. None of the above

Solution: $Z = (12-9.4)/2.0 = 1.3$
$P(x>12) = P(Z>1.3) = 1-P(Z<1.3) = 1-0.9032 = 0.0968$

(3) Suppose that the lifetime of television tubes is normally distributed. A study of the output of one manufacturer shows that 15% of tubes fail before 2 years, while 5% last longer than 6 years. Find the variance of the lifetime distribution to two digits after the decimal.

a) 0.67
b) 0.45
c) 2.22
d) 1.49
e) None of the above

Solution: $P(X > 6) = 0.95$ so, $P(X < 6) = 0.05$ => $Z = 1.64$
$P(X < 2) = 0.15$ => $Z = -1.04$
$X = Z\ \sigma_x + \mu_x$

We have two points in this line with slope $\sigma_x = \dfrac{6-2}{1.64-(-1.04)} = 1.49$

So, $\sigma_x{}^2 = 2.22$
Answer is c.

(4) A switchboard at a consultant's office receives, on average, 0.9 calls per minute. What is the probability that the time between two successive calls will exceed 3 minutes, to three digits after the decimal?

a) 0.933
b) 0.964
c) 0.067
d) 0.036
e) None of the above

Solution: $\lambda = 0.9$ calls/ minute

$$\beta = \frac{1}{\lambda} = 1.111$$

$$P(X > 3) = 1 - P(X < 3)$$

$$= 1\text{-}\left(1\text{-}\ e^{\frac{-x}{\beta}}\right) = e^{-3*0.9} = 0.067$$

Answer is c.

(5) A survey of construction workers indicates that 35% wear their helmets during lunch at work. Assuming this is true, what is the probability that 3, 4 or 5 workers in a group of 6 are wearing helmets?

a) 0.256
b) 0.351
c) 0.452
d) 0.146
e) None of the above

Solution: Binomial. Let X be the number of workers in a group of 6 that are wearing their helmets.

$$P(3 \le X \le 5) = \binom{6}{3}(0.35^3)(0.65^3) + \binom{6}{4}(0.35^4)(0.65^2) + \binom{6}{5}(0.35^5)(0.65^1)$$

$$= 0.351$$

Answer is b.

(6) A retail store has ten computers of a particular brand, out of which four are defective. If a person makes a random purchase of two of the ten computers, find the probability that exactly one of the purchased computers is defective.

a) 8/9
b) 2/15
c) 8/15
d) 5/10
e) None of the above

Solution: $f(1) = P(X = 1) = \dfrac{\binom{4}{1}\binom{6}{1}}{\binom{10}{2}} = \dfrac{4*6}{\frac{10*9}{2*1}} = \dfrac{24}{45} = \dfrac{8}{15}$

Answer is c.

(7) The lifespan of a electrical bulb is a random variable with cumulative distribution function

$$F(x)=\begin{cases}1-e^{\frac{-x}{40}} & x>0\\ 0 & elsewhere\end{cases}$$

What is the probability that the lifespan of the bulb will exceed 75 hours?

a) 0.15
b) 0.30
c) 0.45
d) 0.55
e) None of the above

Solution: Exponential. P (X > 75) = 1 – P (X ≤ 75) = 1 – F (75) = 1 – (1 - $e^{\frac{-75}{40}}$) = 0.15
Answer is a.

(8) There are 13 hearts, 13 diamonds, 13 spades and 13 clubs in a deck of playing cards. What is probability of being dealt a bridge hand of 13 cards containing 5 spades, 2 hearts, 3 diamonds and 3 clubs?

a. 0.013
b. 0.014
c. 0.025
d. 0.026
e. None of the above

Using the extension of the hypergeometric distribution, the probability is

$$\frac{\binom{13}{5}\binom{13}{2}\binom{13}{3}\binom{13}{3}}{\binom{52}{13}}=0.0129$$ So, answer is a.

(9) For the LotoLoto Lottery, five extra prizes were added. The five prizes were won by five different people. Each winner can select an all-expense paid trip to one of three destinations. Independently of each other, winners select destinations 1, 2 or 3 with probabilities 0.5, 0.3 and 0.2, respectively. What is the probability that exactly 1 person selects destination 2 and exactly one person selects destination 3, or that exactly two people select destinations 2 and exactly two people select destination 3.

a. 0.204
b. 0.261
c. 0.291
d. 0.321
e. None of the above

Multinomial

$$P(X_1=3,X_2=1,X_3=1)+P(X_1=1,X_2=2,X_3=2)$$

$$=\binom{5}{3,1,1}p_1^3p_2^1p_3^1+\binom{5}{1,2,2}p_1^1p_2^2p_3^2=20(0.5)^3(0.3)^1(0.2)^1+30(0.5)^1(0.3)^2(0.2)^2=0.204$$

So a is correct.

(10) An archer hits a bull's-eye with a probability of 0.09, and misses the target completely with a probability of 0.12. If the archer shoots eight arrows whose performance are independent of each other, calculate the probability of scoring at least two bull's-eyes.

(a) 0.1111
(b) 0.1254
(c) 0.1577
(d) 0.2781
(e) None of the above

MULTINOMIAL or BINOMIAL. Binomial solution:

P(at least 2 bull's-eyes) = 1- P(X=0) – P(X=1) = 1 – b(0;8,0.09) – b(1;8,0.09)

(c) 0.1577