1. Suppose the cumulative distribution of the random variable X is

$$F(x) = \begin{cases} 0 & x < 0 \\ 0.2x & 0 \le x < 5 \\ 1 & x \ge 5 \end{cases}$$

   Determine the following:
   (a) P(X<2.8)
   (b) P(X>1.5)
   (c) P(X<-2)
   (d) P(X>6)

2. Suppose $f(x) = 1.5x^2$ for $-1 < x < 1$. Determine the mean and variance of X.
3. The thickness of a flange on an aircraft component is uniformly distributed between 0.95 and 1.05 millimeters.
   (a) Determine the cumulative distribution function of flange thickness.
   (b) Determine the proportion of flanges that exceeds 1.02 millimeters.
   (c) What thickness is exceeded by 90% of the flanges?
   (d) Determine the mean and variance of flange thickness

4. The bus you take randomly and uniformly arrive between 2 and 2:30pm. What is the probability that you take the bus between 2:15 and 2:20pm?
5. The line width of semiconductor manufacturing is assumed to be normally distributed with a mean of 0.5 micrometer and a standard deviation of 0.05 micrometer.
   (a) What is the probability that a line width is greater than 0.62 micrometer?
   (b) What is the probability that a line width is between 0.47 and 0.63 micrometer?
   (c) The line width of 90% of samples is below what value?

6. The length of an injection molded plastic case that holds magnetic tape is normally distributed with a length of 90.2 millimeters and a standard deviation of 0.1 millimeter. What is the probability that a part is longer than 90.3 millimeters or shorter than 89.7 millimeters?

7. A call center receives on average 5 calls per 10 minutes.
   (a) What is the probability that time until the first call is less than 5 minutes?
   (b) What is the probability that there are 2 calls in the first minute?

8. A supplier ships a lot of 1000 electrical connectors. A sample of 25 is selected at random, without replacement. Assume the lot contains 100 defective connectors.
   (a) What is the probability that there are no defective connectors in the sample?
   (b) Use binomial approximation to answer part (a)?
   (c) Use normal distribution to answer part (a)?

9. A fisher man expects to get a fish one every half an hour. Compute the probability that he will wait between 2 and 4 hours before he catches 4 fish.

10. In a company, user logons are modelled such that a mean of 25 logons per hour is received. What is the probability that there are no logons in an interval of 6 minutes?

11. Calls to the help line of a large computer distributor follow a Possion distribution with a mean of 20 calls per minutes.
    (a) What is the mean time until the 100^{th} call?
    (b) What is the mean time between call numbers 50 and 80?
    (c) What is the probability that three or more calls occur within 15 seconds?

12. Assume the life of a packaged magnetic disk exposed to corrosive gases has a Weibull distribution with $\beta = 0.5$ and the mean life is 600 hours.
    (a) Determine the probability that a packaged disk lasts at least 500 hours?
    (b) Determine the probability that a packaged disk fails before 400 hours?

1. (a) $P(x<2.8) = F(2.8) = 0.2 \times 2.8 = 0.56$

(b) $P(x>1.5) = 1 - P(x<1.5) = 1 - F(1.5) = 1 - 0.2 \times 1.5 = 0.7$

(c) $P(x<-2) = F(-2) = 0$

(d) $P(x>6) = 1 - P(x<6) = 1 - F(6) = 1 - 1 = 0$

2. $E(x) = \int x f(x)\,dx = \int_{-1}^{1} 1.5x^3\,dx = \frac{1.5}{4} x^4 \Big|_{-1}^{1} = 0$

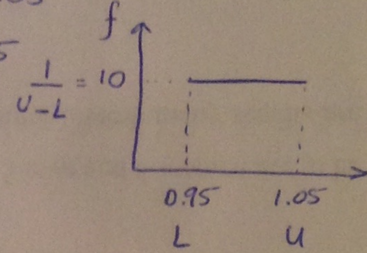
$var(x) = E(x^2) - E(x))^2 = \int_{-1}^{1} 1.5x^4\,dx - 0 = 0.3x^5 \Big|_{-1}^{1} = 0.3 - (-0.3) = 0.6$

3. $X \sim U[0.95, 1.05]$

$$f(x) = \begin{cases} 0 & x<0.95 \\ \frac{1}{1.05-0.95} & 0.95<x<1.05 \\ 0 & x>1.05 \end{cases} = \begin{cases} 0 & x<0.95 \\ 10 & 0.95<x<1.05 \\ 0 & x>1.05 \end{cases}$$

$U = 1.05$

$L = 0.95$



(a) $F(x) = \int f(x) = \begin{cases} 0 & x<0.95 \\ 10x & 0.95<x<1.05 \\ 0 & x>1.05 \end{cases}$

(b) $P(x>1.02) = \int_{1.02}^{1.05} 10\,dx = 10x \Big|_{1.02}^{1.05} = 0.3$

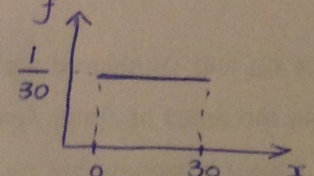
(c) $P(x>a) = \int_{a}^{1.05} 10\,dx = 10x \Big|_{a}^{1.05} = 10.5 - 10a = 0.90 \rightarrow 10a = 9.6 \rightarrow \boxed{a = 0.96}$

(d) $E(x) = \frac{U+L}{2} = \frac{2}{2} = 1$, $var(x) = \frac{(U-L)^2}{12} = 0.00083$

4. $X \sim U[2, 2{:}30]$ equivalent to $U[0, 30]$

$U = 30$

$L = 0$



$P(15<x<20) = \int_{15}^{20} \frac{1}{30}\,dx = \frac{20-15}{30} = \frac{1}{6}$

5. $X \sim N[0.5, (0.05)^2]$

(a) $P(X>0.62) = P(Z > \frac{0.62-0.5}{0.05}) = P(Z>2.4) = 0.0082$

(b) $P(0.47 < X < 0.63) = P(\frac{0.47-0.5}{0.05} < Z < \frac{0.63-0.5}{0.05}) = P(-0.6 < Z < 2.6)$

$= 0.9953 - 0.2743 = 0.721$

(c) $P(X<a) = 0.9 \longrightarrow P(Z < \frac{a-0.5}{0.05}) = 0.9 \Rightarrow \frac{a-0.5}{0.05} = 1.285$

$\rightarrow \boxed{a = 0.5642}$

6. $X \sim N[90.2, (0.1)^2]$

$P(X>90.3) + P(X<89.7) = 1 - P(89.7<X<90.3) = 1 - P(\frac{89.7-90.2}{0.1} < Z < \frac{90.3-90.2}{0.1})$

$= 1 - P(-5<Z<1) = 1 - (0.8413 - 0) = 0.1587$

7. $\lambda = 5/10\text{min} \rightarrow \lambda = 0.5/\text{min.}$

(a) Exponential $P(X<5) = \int_0^5 0.5e^{-0.5} dx = 0.918$

Poisson

(b) $P(X=2) = \frac{e^{-0.5}(0.5)^2}{2!} = 0.076$

8. (a) Hypergeometric

$P(X=0) = \frac{\binom{100}{0}\binom{900}{25}}{\binom{1000}{25}} = 0.069$

(b) Binomial $p(\text{defective}) = \frac{100}{1000} = \frac{1}{10}$

$P(X=0) = \binom{25}{0} (\frac{1}{10})^0 (\frac{9}{10})^{25} = 0.072$

(c) $E(X) = np = 25 \times \frac{1}{10} = 2.5$

$\text{var}(X) = npq = 25 \times \frac{1}{10} \times \frac{9}{10} = 22.5$

Attention:

The CORRECT variance is 2.25. Please correct the variance and standard deviation accordingly and solve the questions with the correct parameters.

$P(X=0) = P(-0.5 < X < 0.5) = P(\frac{-0.5-2.5}{\sqrt{22.5}} < Z < \frac{0.5-2.5}{\sqrt{22.5}}) =$ ~~P(...)~~

$P(-0.63 < Z < -0.42) =$

$0.3372 - 0.2643 = 0.0729$

9. $\lambda = 1/_{0.5} = 2/_{hour}$. $k = 4$ $\qquad \dfrac{\lambda^k x^{k-1} e^{-\lambda x}}{\Gamma(k)}$

$$P(2 \leq x \leq 4) = \int_2^4 \frac{e^{-2x} x^{4-1} 2^4 \, dx}{\Gamma(4)} = \int_2^4 e^{-2x} x^3 \frac{16}{3!} \, dx = 0.1238$$

To students: Don't worry about the integral

10. $\lambda = \dfrac{25}{hour} \rightarrow \lambda_{adj} = 2.5/_{6min}$

poisson.
$$P(X=0) = \frac{e^{-2.5}(2.5)^0}{0!} = 0.082$$

Using exponential.

$\lambda = 25/_{hour}$ $\qquad P(x > 6^{min}) = P(x > \frac{1}{10}^{hour}) = \int_{1/10}^{\infty} 25 e^{-25x} dx = 0.082$

11. $\lambda = 20/_{min}$ $\qquad$ (a) $E(X) = \dfrac{k}{\lambda} = \dfrac{100}{20} = 5$ min.

(b) $E(80) = \dfrac{80}{20} = 4$

$E(50) = \dfrac{50}{20} = 2.5$ $\rightarrow$ Time between $= 4 - 2.5 = 1.5$ min.

(c) Poisson: $\lambda_{adj} = 5/_{15sec}$ $\qquad P(x \geq 3) = 1 - \sum_{x=0}^{2} \dfrac{e^{-5}(5)^x}{x!}$

12. $X \sim$ Weibul $(0.5, 300)$

$E(X) = 600 = \delta \times 2 \rightarrow \delta = 30$

(a) $P(x > 500) = e^{-(5/3)^{0.5}} = 0.275$

(b) $P(x < 400) = 1 - e^{-(4/3)^{0.5}} = 0.68$