Chapter 9: Classical Estimation: Estimates of μ and σ in the Form of Confidence Intervals

Confidence interval for μ:

$\bar{x} - z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}$	· σ is known; if population is not normal, invoke CLT for large samples ($n \geq 30$)
$\bar{x} - t_{\frac{\alpha}{2}}\frac{s}{\sqrt{n}} < \mu < \bar{x} + t_{\frac{\alpha}{2}}\frac{s}{\sqrt{n}}$	· σ is unknown and population must be approximately normal, df = n-1
$\bar{x} - z_{\frac{\alpha}{2}}\frac{s}{\sqrt{n}} < \mu < \bar{x} + z_{\frac{\alpha}{2}}\frac{s}{\sqrt{n}}$	"Large-Sample Confidence Interval" · σ is unknown; normality cannot be assumed, but ($n \geq 30$)

Confidence interval for (μ_1 - μ_2):

$(\bar{x}_1 - \bar{x}_2) \pm z_{\frac{\alpha}{2}}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$	· σ_1, σ_2 known, if pop. is not normal, invoke CLT for $n \geq 30$
$(\bar{x}_1 - \bar{x}_2) \pm t_{\frac{\alpha}{2}} s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$	· $\sigma_1 = \sigma_2$ and unknown, populations are approx. normal, df = $n_1 + n_2 - 2$ and $s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$
$(\bar{x}_1 - \bar{x}_2) \pm t_{\frac{\alpha}{2}}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$	· $\sigma_1 \neq \sigma_2$ and unknown, populations are approx. normal and $df = \frac{\left(s_1^2/n_1 + s_2^2/n_2\right)^2}{\left(s_1^2/n_1\right)^2/(n_1 - 1) + \left(s_2^2/n_2\right)^2/(n_2 - 1)}$
$\bar{d} - t_{\frac{\alpha}{2}}\frac{s_d}{\sqrt{n}} < \mu_D < \bar{d} + t_{\frac{\alpha}{2}}\frac{s_d}{\sqrt{n}}$	· $\sigma_1 \neq \sigma_2$ and unknown, populations are normal, samples are not independent, $\mu_D = \mu_1 - \mu_2$, df = n-1

Confidence interval for σ:

$$\frac{(n-1)s^2}{\chi^2_{\frac{\alpha}{2}}} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_{1-\frac{\alpha}{2}}} \quad \text{df} = \text{n-1}$$

Confidence interval for σ_1^2/σ_2^2:

$$\frac{s_1^2}{s_2^2}\frac{1}{f_{\frac{\alpha}{2}}(df_1, df_2)} < \frac{\sigma_1^2}{\sigma_2^2} < \frac{s_1^2}{s_2^2} f_{\frac{\alpha}{2}}(df_2, df_1), \; df_1 = n_1\text{-1}, \; df_2 = n_2\text{-1}$$

Confidence interval for proportion p:

$$\hat{p} - z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}\hat{q}}{n}} < p < \hat{p} + z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}\hat{q}}{n}}, \text{ with } \hat{q} = 1 - \hat{p}$$

Errors/Sample Sizes

Thm 9.1: For $\bar{x}$ as an estimate of μ, upper bound on **error** ($|\bar{x} - \mu|$) is $e = z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}$ with $(1-\alpha)100\%$ confidence; and *Thm 9.2*: $n = \left(\frac{z_{\alpha/2}\sigma}{e}\right)^2$ will give $(1-\alpha)100\%$ confidence that error will not exceed e.

Thm 9.3: For $\hat{p}$ as an estimate of *p*, upper bound on error is $z_{\frac{\alpha}{2}}\sqrt{\hat{p}\hat{q}/n}$; and *Thm 9.4*: $n = \frac{z_{\alpha/2}^2\hat{p}\hat{q}}{e^2}$ at $(1-\alpha)100\%$ confidence, and **at least** $(1-\alpha)100\%$ confident when $n = \frac{z_{\alpha/2}^2}{4e^2}$.