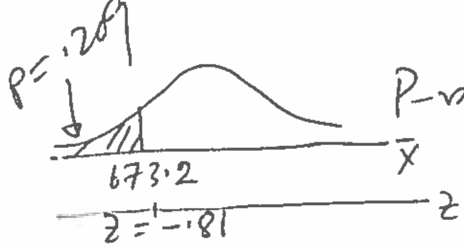
Q1. $H_0: \mu \geq 675$ $(\mu = 675)$

$H_1: \mu < 675$

$\bar{X} = 673.2$, $s = 14.9$, $n = 45$

$n \geq 30$, use z-test, $\sigma \simeq s = 14.9$

$$z = \frac{\bar{X} - \mu}{s/\sqrt{n}} = \frac{673.2 - 675}{14.9/\sqrt{45}} = -0.81$$



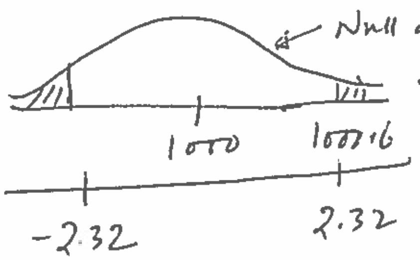
P-value $= P(z < -0.81) = .209 > .05$

H_0 is ~~not~~ credible. ~~Reject H_0.~~

Do not reject H_0

Q2. $H_0: \mu = 1000$, $H_1: \mu \neq 1000$

$\bar{X} = 1000.6$, $\sigma \simeq s = 2$, $z = \dfrac{1000.6 - 1000}{\underbrace{2/\sqrt{60}}_{.258}} = 2.32$



$P = 2 * P(z > 2.32)$

$= 2 * .0102$

$= .0204 < .05$

P is small, H_0 is less credible.

Reject H_0.

Q6. P-value $= .209 < 0.25$, so, reject H_0 at 25% level

P-val $= .209 > .05$, so, we do not reject at 5% level.

Q7. t-test

Rejection region: $|t| > t_{\alpha/2, n-1}$

$t_{\alpha/2, n-1} = t_{.025, 59}$

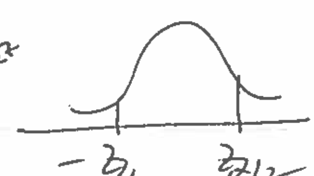
$= 2$

$\Rightarrow \dfrac{\bar{X} - 1000}{.258} = \pm 2$

$\Rightarrow \bar{X} = 1000 \pm 2(.258)$

$= 1000 \pm .516 = 1000.511$ & 999.48

z-test



$z_{\alpha/2} = 1.96$

$\dfrac{\bar{X} - 1000}{.258} = \pm 1.96$

$\Rightarrow \bar{X} = 1000 \pm 1.96 \times .258$

$= 1000.51, 999.49$

Q8

(a) $\mu = 200,\ n = 9,\ \sigma = 15,$

Acceptance region: $191 < \bar{x} < 209$ (satisfactory)

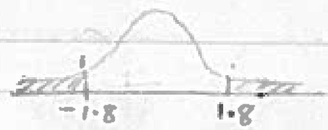
$H_0: \mu_0 = 200$, $H_a: \mu_0 \neq 200$

$\alpha = P(\text{Reject } H_0 \text{ when } \mu_0 = 200)$

$= 1 - P(191 < \bar{x} < 209 \text{ when } \mu_0 = 200)$

$= 1 - P\left(\dfrac{191-200}{\frac{15}{3}} < z < \dfrac{209-200}{15/3}\right)$

$= 1 - P(-1.8 < z < 1.8)$



$= 2P(z < -1.8) = 2 \times 0.359 = 0.0718$

(b) $\beta = P(\text{'Do not reject } H_0\text{' when } H_0 \text{ is false})$

$= P(191 < \bar{x} < 209 \text{ when } \mu_{true} = 215)$

$= P\left(\dfrac{191-215}{5} < z < \dfrac{209-215}{5}\right)$

$= P(-4.8 < z < -1.2)$

$= 0.1151 - 0 = 0.1151$

Q9

$H_0: \mu = 5000$

$H_a: \mu < 5000$, Critical region: $\bar{x} < 4970$

(a) $\alpha = P(\bar{x} < 4970 \text{ when } \mu_0 = 5000)$

$= P\left(z < \dfrac{4970-5000}{120/\sqrt{50}}\right) = P(z < -1.77)$

$= 0.0384$

(b) If $\mu_{true} = 4970$

$\beta = P(\bar{x} \geq 4970 \text{ when } \mu_t = 4970)$

$= P(z \geq 0) = 0.5$

If $\mu_{true} = 4960$, $\beta = P(z > 0.59) = 0.2776$

70

$P < .05$, Reject H_0

$P > .05$, Do not ~~rej~~ it

p-value $\propto$ credibility of H_0

p-value $\uparrow$ credi $\uparrow$

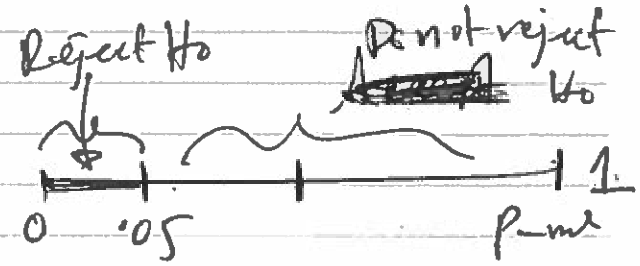
Q10.~~19~~ $H_0: \mu = 40$ months $\mu \geq 40$

$H_a: \mu < 40$ months , $n = 64$, $\bar{x} = 38$, $s = 5.8$,

$$z = \frac{38 - 40}{\frac{5.8}{\sqrt{64}}} = -2.76$$

$\underline{\text{P-value}} = P(Z \leq -2.76) = \underline{0.0029} < 0.05$

Decision: $\underline{\text{Reject } H_0}$.



~~10.21~~ Q 11 $H_0: \mu = 800$

$H_a: \mu \neq 800$

$$z = \frac{788 - 800}{40/\sqrt{30}} = -1.64.$$

$$\text{P-value} = P(Z \geq 1.64) + P(Z \leq -1.64)$$
$$= 2P(Z \leq -1.64) = 2 * 0.0505 = \underline{0.101} > \underline{0.05}$$

Decision: $\underline{\text{Do not reject}}$ H_0.

Define critical region

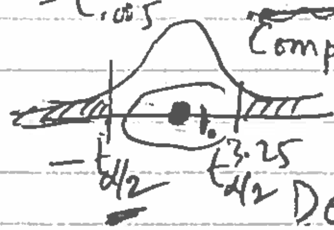
~~10.23~~ Q 12 $H_0: \mu = 10$; $H_a: \mu \neq 10$

$\alpha = \underline{0.01}$, $\underline{n = 10}$, $\bar{x} = 10.06$, $\underline{t_{.005,9}} = \underline{3.25}$, $s = 0.246$

$= t_{.005}$ $t_{\alpha/2}$ $\underline{\text{Critical region}}$: $\underline{t < -3.25 \text{ or } t > 3.25}$

Computation: $\underline{\text{Observed}}$ $t = (10.06 - 10)/(.246/\sqrt{10})$

$= 0.77$ inside acceptance region.



Decision: Fail to reject H_0

Q 13 ~~10.28~~ From Exercise 10.19, $H_0: \mu = 40$, $H_a: \mu < 40$

Given, $\sigma = 5.8$, $\mu_{true} = 35.9$, $\beta = 0.1$,

For one-sided test, $n \simeq (z_\alpha + z_\beta)^2 \sigma^2 / \delta^2$

Assume $\alpha = 0.05$, $z_\alpha = z_{.05} = 1.645$, $\delta = 35.9 - 40 = -4.1$,

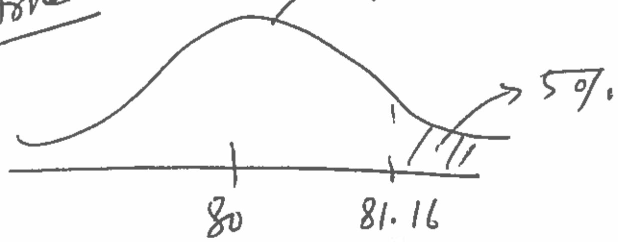
$z_\beta = z_{.01} = 1.28$

$$n = \frac{(z_\alpha + z_\beta)^2 \sigma^2}{\delta^2} = \frac{(1.645 + 1.28)^2}{(-4.1)^2} 5.8^2 = 17.12$$

Q14

H_0 true

$\bar{x}$ (mean = 80, $\sigma_{\bar{x}} = 5/\sqrt{50} = .707$)

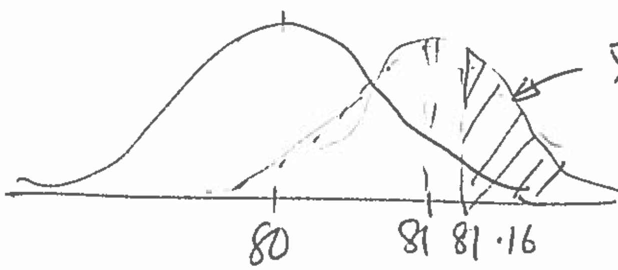


$z = 1.645$

$\frac{\bar{x}-\mu}{\sigma/\sqrt{n}} \geq z_{.05} = 1.645$

$\Rightarrow \bar{x} \geq \underbrace{80 + \frac{5}{\sqrt{50}} \times 1.645}_{81.16}$

H_1 true



$\bar{x}$ (mean = 81, $\sigma_{\bar{x}} = .707$) same

$z_0 = 1.645$

$z_1 = 0.23$

$z_1 = \frac{81.16 - 81}{.707} = .23$

Area to the right of .23, $P(z > .23)$

$= .409 = \beta$

≈ ~~40%~~