ENGG 319 – Probability and Statistics for engineers
Probability exercise problems #1 (Chapter 2)

1- A fair coin is tossed until a tail appears. What is the sample space? What is the size of sample space?

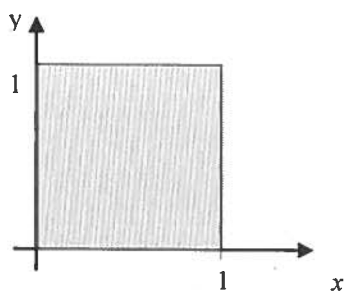
2- You are given a biased coin such that the probability of observing a head is twice as the probability of observing a tail. You toss the coin until a tail or three heads appear. Draw a tree diagram for this example. What is the sample space and its size? What is the probability of observing at least two heads in this experiment?

3- Let event A be the set of hearts in a deck of 52 cards and event B be the set of face cards (kings, queens, and jacks), and event C be the rest (No heart or face cards).

a. What is the size of event A, B, and C?
b. Are events A and B mutually exclusive?
c. What is the probability of $A \cup B$, $A \cap B \cap C$, and $A \cup B \cup C$?
d. Draw a Venn diagram to show the relationships between A, B, and C.

4- John is about to graduate by the end of 2016. However, there is still one course required for him to pass in order to meet the June convocation deadline. He can choose from Math, Physics, and Statistics as his last course to take. He takes statistics and Math with 60% and 25% probability, respectively. Depending on the difficulty and his interest in these courses, he passes Statistics, Math, and Physics with 85%, 70%, and 75% probability. What is his chance to attend June convocation?

5- You and your friend are playing dart, while the target is shown as the shaded area and it is guaranteed that any darts you are throwing falls within the target:



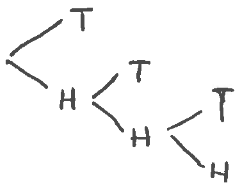
a. What is the sample space?
b. What is the probability of throwing a dart in the north-west quarter of the target?
c. What is the probability of throwing a dart at the point of $(\frac{1}{3}, \frac{1}{2})$?
d. What is the probability of event A described by the following rule:

$$A = \{(x, y) |\, x + y \leq \frac{1}{2}\}$$

6- In a classroom full of students, there are 30 students registered in the History class, 25 students registered in the Art class, and 20 students registered in the Math class. Among those, 4 students registered in both History and Art, 6 students registered in both History and Math, 10 students registered in both Math and Art, and 5 students registered in all three classes. How many students are in the classroom?

1. $S = \{T, HT, HHT, \ldots\}$ Sample Size $= |S| = \infty$

2. $\left.\begin{array}{l} P(H) = 2P(T) \\ P(H) + P(T) = 1 \end{array}\right\} \longrightarrow \begin{array}{l} P(H) = 2/3 \\ P(T) = 1/3 \end{array}$



$S = \{T, HT, HHT, HHH\}$

$|S| = 4$

$P(\text{at least } 2\ H) = P(HHT \cup HHH) = 2/4$

3. $A = \{\text{hearts}\} \to |A| = 13$

$B = \{\text{face cards}\} \to |B| = 12$

$C = \{10, 10, 10\} \to |C| = 30$

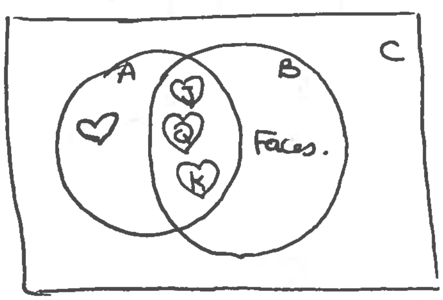
10 → diamonds, no faces; 10 → clubs, no faces; 10 → Spades, No faces

$A \cap B = \{K\heartsuit, Q\heartsuit, J\heartsuit\} \Longrightarrow$ A & B are not ME or disjoint.

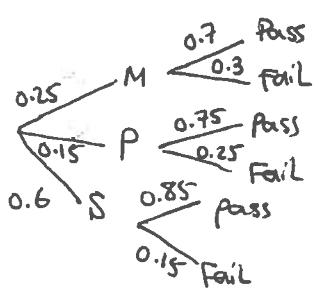
$P(A \cup B) = \frac{22}{52}$

$P(A \cap B \cap C) = 0$

$P(A \cup B \cup C) = 1$



4.



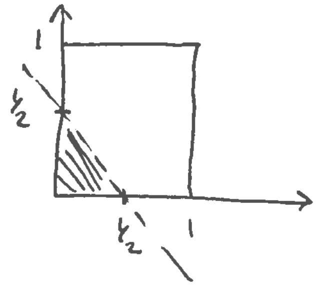
$P(\text{pass}) = 0.25 \times 0.7 + 0.15 \times 0.75 + 0.6 \times 0.85$

$= 0.7975$

5. a) $S = \{(x,y) \mid 0 \le x \le 1,\ 0 \le y \le 1\}$

b) $\frac{P(NW)}{P(S)} = \frac{\text{Area }(NW)}{P(S)} = \frac{1/4}{1} = 1/4$

c) $P(1/3, 1/2) = 0$

d. 

$$P(A) = \frac{\text{Area}(A)}{\text{Area}(S)} = \frac{\frac{1}{2}\left(\frac{1}{2} \times \frac{1}{2}\right)}{1} = \frac{1}{8}$$

6. Corrections: 5 students in History & Art

4 " " all three

N: number of students

$P(H) = \frac{30}{N}$, $P(A) = \frac{25}{N}$, $P(M) = \frac{20}{N}$, $P(H \cap A) = \frac{5}{N}$

$P(H \cap M) = \frac{6}{N}$, $P(M \cap A) = \frac{10}{N}$, $P(A \cap H \cap M) = \frac{4}{N}$

$$1 = P(H \cup M \cup A) = P(H) + P(M) + P(A) - P(A \cap H) - P(M \cap A) - P(M \cap H) + P(A \cap H \cap M)$$

$$\Rightarrow 1 = \frac{30}{N} + \frac{20}{N} + \frac{25}{N} - \frac{5}{N} - \frac{10}{N} - \frac{6}{N} + \frac{4}{N}$$

$$= \frac{58}{N} \Rightarrow \boxed{N = 58}$$

ENGG 319 (Fall 2016)

Lecture/ Tutorial # 2 (Videos 1-8, Chapter 2, Sections 2.1, 2.2, 2.4, 2.5, 2.6)

Sample space and event

Question # 1. A coin is tossed until a tail or three heads appear. List the elements of the sample space.

Question # 2. An engineering firm is hired to determine if certain waterways in Virginia are safe for fishing. Samples are taken from three rivers.

(a) List the elements of the sample space S, using the letters F for safe to fish and N for not safe to fish

(b) List the element of S corresponding to the event E that the second river was safe for fishing

Question # 3. Consider the sample space S = {Cu, Na, N, K, U, O, Zn} and the following events
A = {Cu, Na, Zn}
B = {Na, N, K}
C = {O}
List the elements of the sets corresponding to the event $(A' \cup B') \cap (A' \cap C)$

Question # 4. (a) Draw the sample space of the experiment of tossing two coins in a Venn diagram with the probabilities of each sample point. (b) Let A, B, C be events relative to the sample space S. Using a Venn diagram, shade the areas reprinting the event $(A \cap C) \cup B$. Assume that A, B, C share few common elements.

Probability of an event: application of additive rules

Question # 5. The probability that a Canadian industry will locate in Shanghai, China, is 0.7, the probability that it will locate in Beijing, China is 0.4, and the probability that it will locate in either Shanghai or Beijing or both is 0.8. What is the probability that the industry will locate
(a) in both cities
(b) in neither city (**Ans:** 0.2)

Question # 6. A pair of fair dice is tossed. Find the probability that
(a) the sum of two scores is 6;
(b) Niether die records a 6.

Question # 7. It is common in many industrial areas to use a filling machine to fill boxes full of product. These machines are not perfect, and indeed they may A, fill to specification, B, underfill, and C, overfill. Let P(B) =0.001 while P(A) =0.99
(a) Give P(C)
(b) What is the probability that the machine does not underfill ?
(c) What is the probability that the machine either overfills or underfills ? (**Ans:** 0.01)

Now suppose 50,000 boxes of detergent are produced per week and suppose also that those underfilled are "sent back," with customers requesting reimbursement of the purchase price. Suppose also that the cost of production is known to be $ 4 per box while the purchase price is $ 4.50 per box.
(d) What is the weekly profit under the condition of no defective boxes?
(e) What is the loss in profit expected due to underfilling ? (**Ans:** $ 225)

Probability of an event: Application of conditional probability, independence and product rule

Question # 8. In an experiment to study the relationship of hypertension and smoking habits, the following data are collected for 180 individuals (H = Hypertesnion, NH =Nonhypertension):

	Nonsmokers	Moderate smokers	Heavy smokers
H	21	36	30
NH	48	26	19

If one of these individuals is selected at random,
(a) find the probability that the person is experiencing hypertension, given that the person is a heavy smoker. (**Ans:** 30/49)
(b) a nonsmoker, given that the person is experiencing no hypertension. (**Ans:** 16/31)

Question # 9. An extrusion die is used to produce aluminium rods. Specifications are given for the length and the diameter of the rods. For each rod, the length is classified as too short, too long, or OK, and the diameter is classified as too thin, too thick, or OK. In a population of 1000 rods, the number of rods in each class is as follows:

	Diameter		
Length	**Too thin**	**OK**	**Too thick**
Too short	10	3	5
OK	38	900	4
Too long	2	25	13

A rod is sampled at random from this population.

(a)What is the probability that it is too short? (**Ans:** 0.018)
(b) What is the probability that it is either too short or too thick? (**Ans:** 0.035)
(c) Compute the conditional probability P (diameter OK | length too long). Is this the same as the unconditional probability P(diameter OK) ? (**Ans:** 0.625, 0.928)
(d) Find P(too long) and P (too long | too thin). Are the two events 'too long' and 'too thin' independent?

Question # 10. An electrical system consists of two components as illustrated in the figure below. The system works if either of the components B or C works. The reliability (probability of working of each component is also shown in the figure. What is the probability that the system works? Assume that the component operates (or fails) independently. (**Ans:** 0.94)

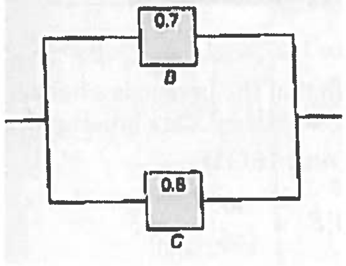


Figure: Circuit for problem 10

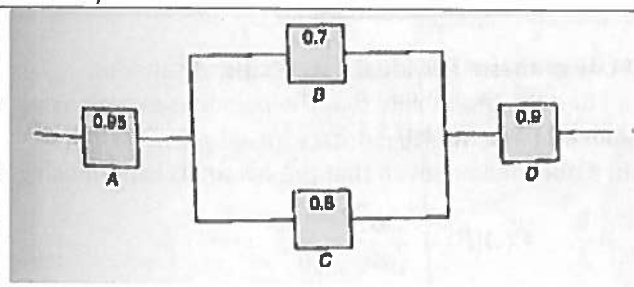


Figure: Circuit for problem 11

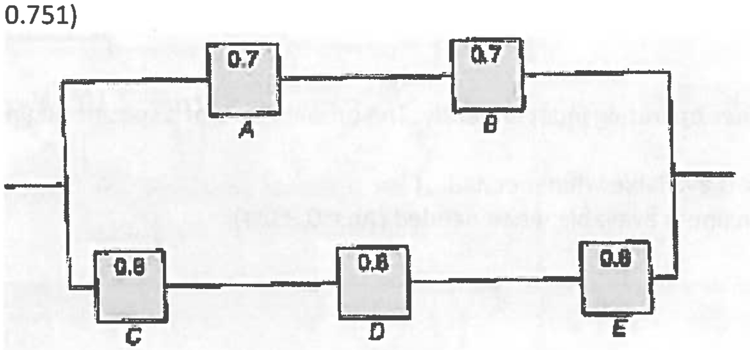
Question # 11. Consider an extension of the circuit in problem 10 that consists of four components as illustrated in the figure above. The system works if component A and D work and either of the components B or C works. The reliability (probability of working of each component is also shown in the figure. What is the probability that the system works? Assume that the component operates (or fails) independently.(**Ans:** 0.8037)

Question # 12. Consider a river, and the following three events: A: the river is polluted, B: a sample of water tested detects pollution, and C: fishing is permitted. Given, P(A) =0.3, P(B|A)=0.75, P(C|A∩B')=0.8, P(B'∩C)=0.564. Find the probability that the river is polluted, given that fishing is permitted and the sample of water tested did not detect pollution. (**Ans:** 0.1064)

Question # 17 Assume A is the event that a person is smoker and B is the event that a person develops cancer. Given $P(A \cap B) = 0.05$, $P(A \cap B') = 0.2$, $P(A' \cap B) = 0.03$, $P(A' \cap B') = 0.72$, Calculate the probability that (a) a smoker develops cancer, and (b) a nonsmoker develops cancer, and (c) Does the conditional probabilities obtained in parts (a) and (b), suggest that there is a link between smoking and cancer? (d) What is the probability that a nonsmoker does not develop cancer? (**Ans**: part (a) =0.2, part (d) =0.96))

Question # 18. Consider a river, and the following three events: A: the river is polluted, B: a sample of water tested detects pollution, and C: fishing is permitted. Given, $P(A) = 0.3$, $P(B|A) = 0.75$, $P(B|A') = 0.2$, $P(C|A \cap B) = 0.2$, $P(C|A' \cap B) = 0.15$, $P(C|A \cap B') = 0.8$, and $P(C|A' \cap B') = 0.9$. Find
(a) $P(A \cap B \cap C)$, and (b)The probability that the sample of water did not detect the pollution and fishing is permitted. (**Ans (b):** 0.564)

Question # 19 The following circuit works if at least one of the following two conditions are met (1) top route works, i.e., both A and B works (2) bottom route works, i.e., all of C, D and E works. The components A, B, C, D, E can function or fail independently. What is the probability that the entire system works? (**Ans:** 0.751)



Additional problems (miscellaneous)

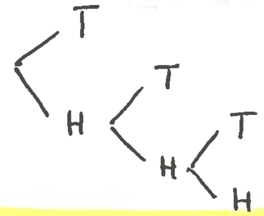
Question # 13 Consider the experiment of tossing two balanced coins. Find the probability of observing
(a) exactly one head, and
(b) at least one head (**Ans**: ¾)

Question # 14 Consider the experiment of tossing a fair die, and let A is the event that an even number is observed, and B is the event that a number less than or equal to 4 is observed. Is A and B independent event? [Hint: Calculate and compare P(A) and P(A|B)] (**Ans**: Independent)

Question # 15. The probability that a doctor correctly diagnoses a particular illness is 0.7. Given that the doctor makes an incorrect diagnosis, the probability that the patient files a lawsuit is 0.9. What is the probability that the doctor makes an incorrect diagnosis and the patient sues? (**Ans**: 0.27)

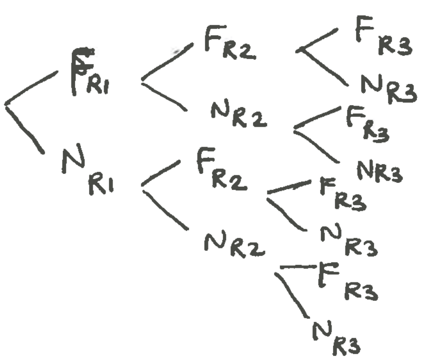
Question # 16 A town has two fire engines operating independently. The probability that a specific engine is available when needed is 0.96.
(a) What is the probability that neither is available when needed
(b) What is the probability that a fire engine is available when needed (**Ans**: 0.9984)

1.



$S = \{T, HT, HHT, HHH\}$

2. R_i : River i^{th} $i = 1, 2, 3$



$S = \{F_1F_2F_3, F_1F_2N_3, \ldots, N_1N_2N_3\}$
(8 elements)

Event A : second river is safe.

$A = \{F_1F_2F_3, F_1F_2N_3, N_1F_2F_3, N_1F_2N_3\}$

3. $A' = S - A = \{N, K, U, O\}$

$B' = \{Cu, U, O, Zn\}$ $\rightarrow$ $A' \cup B' = \{Cu, N, K, U, O, Zn\}$

$A' \cap C = \{O\}$

$(A' \cup B') \cap (A' \cap C) = \{O\}$

5. $P(\text{Shanghai}) = 0.7 = P(S)$

$P(\text{Beijing}) = P(B) = 0.4$

$P(B \cup S) = 0.8$

a). $P(B \cap S) = P(S) + P(B) - P(S \cup B)$
$= 1.1 - 0.8 = 0.3$

b) $P(S' \cap B') = 1 - P(S \cup B) = 1 - 0.8 = 0.2$

6. a). $A = \{(1,5), (2,4), (4,2), (3,3), (5,1)\}$ $P(A) = \frac{5}{36}$

Sum of two is 6.

$B' = \{(1,6), (2,6), (3,6), (4,6), (5,6), (6,6), (6,1), (6,2), (6,3), (6,4), (6,5)\}$

b) $P(B) = 1 - P(B') = 1 - 11/36 = 25/36$

Neither is a 6

either one or both are 6

7. a) $P(C) = 1 - P(A) - P(B) = 0.009$ A, B, and C are disjoint

b) $P(B') = 1 - P(B) = 0.999$

c) $P(C \cup B) = P(C) + P(B) - \underbrace{P(B \cap C)}_{0} = 0.01$

since they're disjoint

d) $\text{Profit} = 50{,}000 \times (4.5 - 4) = \$25000$

e)

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8. a) $P(H \mid \underbrace{\text{Heavy smoker}}_{HS}) = \dfrac{P(H \cap HS)}{P(HS)} = \dfrac{30/180}{49/180} = 30/49$

b) $P(\underbrace{\text{nonsmoker}}_{NS} \mid NH) = \dfrac{P(NS \cap NH)}{P(NH)} = \dfrac{48/180}{93/180} = 48/93$

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9. Too short = A

OK = B

Too long = C

Too thin = D

OK = E

Too thick = F

a) $P(A) = 18/1000 = 0.018$

b) $P(A \cup F) = P(A) + P(F) - P(A \cap F) = 18/1000 + 22/1000 - 5/1000 = 35/1000$

c) $P(E \mid C) = \dfrac{P(E \cap C)}{P(C)} = \dfrac{25/1000}{40/1000} = 25/40$

$P(E) = 928/1000$

d) $P(C) = 40/1000 = 4/100 = 2/50$

$P(C \mid D) = \dfrac{P(C \cap D)}{P(D)} = \dfrac{2/1000}{50/1000} = 2/50$

$\rightarrow$ Since $P(C \mid D) = P(C)$, C & D are independent

ALTERNATIVE APPROACH:

$P(C) = 2/50$

$P(D) = 50/1000 = 1/20$

$P(D \cap C) = 2/1000$

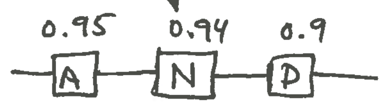
$\rightarrow$ Since $P(D \cap C) = P(C) \cdot P(D)$, C & D are independent.

10. $P(\text{system works}) = P(B \cup C) = P(B) + P(C) - P(B \cap C) = P(B) + P(C) - P(B) \cdot P(C)$

$= 0.7 + 0.8 - 0.56 = 0.94$

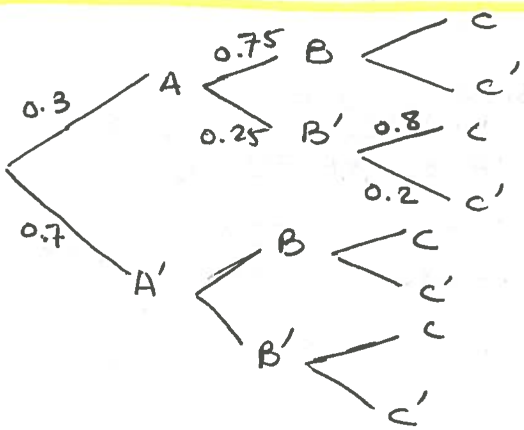
$P(\text{system fails}) = 1 - 0.94 = 0.06$

11. $P(\text{system works}) = P(A \cap N \cap D)$



$= P(A) \cdot P(N) \cdot P(D) = 0.95 \times 0.94 \times 0.9 = 0.8037$

12.



$P(A \mid C \cap B') = \dfrac{P(A \cap C \cap B')}{P(C \cap B')}$

$= \dfrac{P(A) \cdot P(B' \mid A) \cdot P(C \mid A \cap B')}{P(C \cap B')}$

$= \dfrac{0.3 \times 0.25 \times 0.8}{0.564} = 0.1064$

13. $S = \{HH, HT, TH, TT\}$ $P(HT \cup TH) = 1/4 + 1/4 = 1/2$

$P(HT) + P(TH) + P(HH) = 3/4$

$P(A) = 1 - P(B) = 1 - P(TT) = 1 - 1/4 = 3/4$

at least one head

No Head

14. $A = \{2, 4, 6\}$

$B = \{1, 2, 3, 4\}$

$S = \{1, 2, 3, 4, 5, 6\}$

$A \cap B = \{2, 4\}$

$P(A) = 3/6 = 1/2$

$P(B) = 4/6 = 2/3$

$P(A \cap B) = 2/6 = 1/3$

$\rightarrow$ Since $P(A \cap B) = P(A) \cdot P(B)$

A & B are independent

15. A: doctor correctly diagnose

B: patient files a lawsuit

$P(A' \cap B) = 0.9 \times 0.3 = 0.27$

input: $P(A) = 0.7 \rightarrow P(A') = 0.3$

$P(B \mid A') = 0.9$

$\rightarrow P(B \mid A') = \dfrac{P(A' \cap B)}{P(A')}$

$0.9 = \dfrac{P(A' \cap B)}{0.3}$

16. A : engine 1 works
B : engine 2 works

a) $P(A' \cap B') = \underbrace{P(A') \cdot P(B')}_{\text{Since } A \,\&\, B \text{ are independent}} = (1-0.96) \times (1-0.96) = 0.04 \times 0.04 = 0.0016$

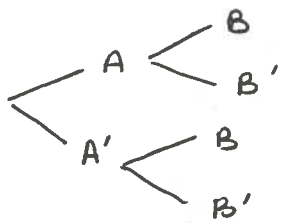
OR

$1 - P(A \cup B) = 1 - P(A) - P(B) + P(A \cap B)$

$= 1 - 0.96 - 0.96 + 0.96 \times 0.96 = 0.0016$

b) $P(A \cup B) = P(A) + P(B) - P(A) \cdot P(B) = 0.96 + 0.96 - 0.96 \times 0.96 = 0.9984$

17.

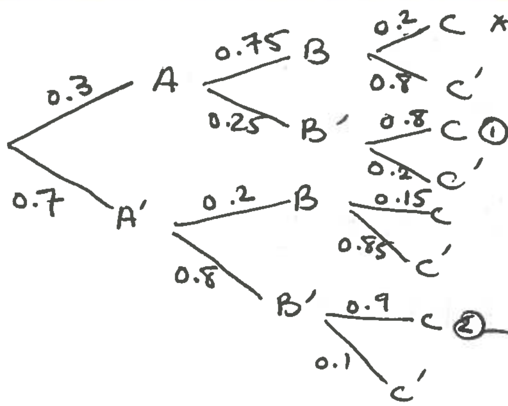


a) $P(B|A) = \dfrac{P(A \cap B)}{P(A)} = \dfrac{P(A \cap B)}{P(A \cap B) + P(A \cap B')} = \dfrac{0.05}{0.05 + 0.2} = 0.2$

b) $P(B|A') = \dfrac{P(A' \cap B)}{P(A')} = \dfrac{0.03}{1 - 0.25} = \dfrac{0.03}{0.75} = 0.04$

d) $P(B'|A') = 1 - P(B|A') = 1 - 0.04 = 0.96$

18.

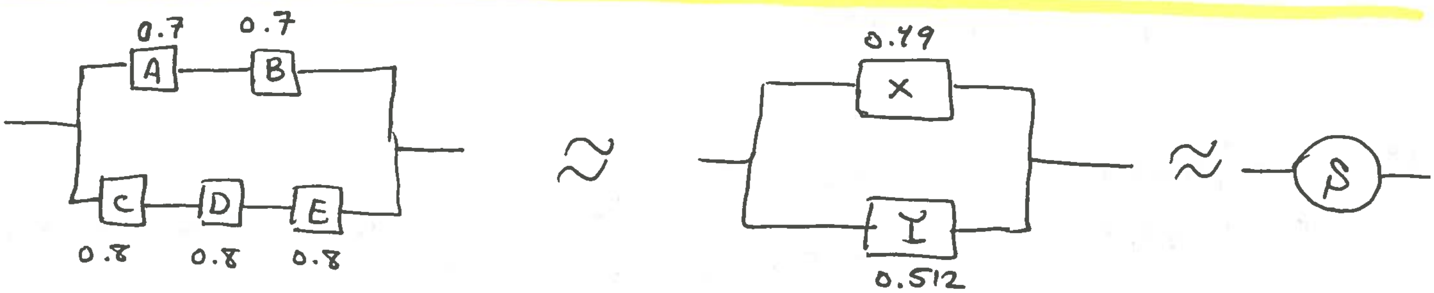


a) $P(A \cap B \cap C) = 0.3 \times 0.75 \times 0.2 = 0.045$

b) $P(B' \cap C) = P(B' \cap C \cap A) + P(B' \cap C \cap A')$

$= 0.3 \times 0.25 \times 0.8 + 0.7 \times 0.8 \times 0.9 = 0.564$

19.



$P(X) = P(A \cap B) = P(A) \cdot P(B) = 0.49$

$P(Y) = P(C \cap D \cap E) = P(C) \cdot P(D) \cdot P(E) = 0.512$

$P(S) = P(X \cup Y) = P(X) + P(Y) - P(X \cap Y) = 0.49 + 0.512 - 0.49 \times 0.512$

$= 0.751$