Q1.
$\Sigma x_i = 311.6 \quad \Sigma x_i^2 = 8134.26$
$\Sigma y_i = 297.2 \quad \Sigma x_i y_i = 7687.6$
$n = 12$

P396

$$b_1 = \frac{n \Sigma x_i y_i - \Sigma x_i \Sigma y_i}{n \Sigma x_i^2 - (\Sigma x_i)^2} = \frac{12 \times 7687.6 - 297.2 \times 311.6}{12 \times 8134.26 - 311.6^2}$$

$$= -0.6861$$

$$b_0 = \frac{\Sigma y_i - b_1 \Sigma x_i}{n} = 42.582$$

Hence, $\hat{y} = 42.582 - 0.6861x$

$$x = \frac{\hat{y} - 42.582}{-0.6861} = \frac{25.77 - 42.582}{-0.6861} = \boxed{+24.507}$$

~~Best Answer (a)~~

Q2. 99% confidence interval for the intercept of true line (β_0)

$$S_{xx} = \Sigma(x_i - \bar{x})^2 = \Sigma x_i^2 - 2 \Sigma x_i \bar{x} + \Sigma(\bar{x})^2$$

(P402)

$$= \Sigma x_i^2 - 2\frac{(\Sigma x_i)^2}{n} + \left(\frac{\Sigma x_i}{n}\right)^2$$

$$= \Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n}$$

Then, $S_{xx} = 8134.26 - 311.6^2/12$ (using Q1 data & results)
$= 43.0467$

$$S_{yy} = 7407.80 - 297.2^2/12 = 47.1467$$

$$S_{xy} = \Sigma(x_i - \bar{x})(y_i - \bar{y}) = \Sigma x_i y_i - \bar{y}\Sigma x_i - \bar{x}\Sigma y_i + \Sigma \bar{x}\bar{y}$$

$$= \Sigma x_i y_i - \frac{2\Sigma x_i \Sigma y_i}{n} + \frac{n \Sigma x_i y_i}{n^2}$$

$$= \Sigma x_i y_i - \frac{\Sigma x_i \Sigma y_i}{n}$$

So, $S_{xy} = 7687.76 - \frac{311.6 \times 297.2}{12} = -29.5333$

$$s^2 = \frac{SSE}{n-2} = \frac{S_{yy} - b_1 S_{xy}}{n-2} = \frac{47.1467 - (-.6861)(-29.5333)}{10} = 2.688$$

(P402)

$\Rightarrow s = 1.640, \quad t_{0.05} = 3.169$ for $\nu = n-1 = 10$

Then, 99% C.I.

$$b_0 \pm t_{\alpha/2} \frac{s}{\sqrt{n S_{xx}}} \sqrt{\sum x_i^2} \qquad (P406)$$

$$= b_0 \pm t_{\alpha/2}\, s \sqrt{\frac{\sum x_i^2}{n S_{xx}}}$$

$$= 42.5818 \pm 3.169 \times 1.64 \sqrt{\frac{8134.26}{12 \times 43.0467}}$$

$$= 42.5818 \pm 20.6236$$

C.I.: $\boxed{21.958 < \beta_0 < 63.205}$ ~~a [illegible]~~

Q3. $b_1 \pm t_{\alpha/2} \frac{s}{\sqrt{S_{xx}}} \qquad (P\ 403)$

$= -0.6861 \pm 3.169 \times 1.640 / \sqrt{43.0467}$ using previous calculations (Q1 & Q2)

$\Rightarrow \boxed{-1.478 < \beta_1 < 0.106}$ ~~a [illegible] answer~~

Q4. population variance of the error (ε) term $= \sigma^2$

Point estimate $= s^2$

from previous calculation Q2, $s = 1.64$

$\boxed{s^2 = 2.68}$ ~~[illegible]~~

Q5

$\Sigma x_i = 930$, $n=12$, $\bar{x} = 77.5$

$\Sigma y_i = 56.69$, $n=12$, $\bar{y} = \cancel{77.5}\ 4.724167$

$\Sigma x_i^2 = 73764$, $\Sigma y_i^2 = 270.9251$, $\Sigma x_i y_i = 72075$

$\Sigma x_i y_i = 4462.22$

$$S_{xx} = \Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n} = 73764 - \frac{930^2}{12} = 1689,$$

$$S_{yy} = 270.925 - \frac{56.69^2}{12} = 3.112092$$

$$b_1 = \frac{n\,\Sigma x_i y_i - \Sigma x_i \Sigma y_i}{n\,\Sigma x_i^2 - (\Sigma x_i)^2} = 0.0407016$$

$$b_0 = \frac{\Sigma y_i - b_1 \Sigma x_i}{n} = \bar{y} - b_1\bar{x} = 4.724167 - .040716 * 77.5$$
$$= 1.5686767$$

$$\hat{y} = .0407016x + 1.568677$$

$$SST = \Sigma(y_i - \bar{y})^2 = \cancel{3.112092}\ S_{yy} = 3.112092$$

$$SSE = \Sigma(y_i - \hat{y})^2 = \cancel{S_{yy} = 3.112092}$$
$$= S_{yy} - b_1 S_{xy} \cancel{= 3.112092 - .0407016 \times 72075}$$
$$= S_{yy} - b_1^2 S_{xx} = 3.112092 - (.040702)^2 \times 1689$$
$$= 0.314005$$

$$R^2 = 1 - \frac{SSE}{SST} = 1 - \frac{.31405}{3.11209} = \boxed{0.899084}$$

Q6.

$\hat{y} \approx 1.5687 + 0.0407x$

Test: $H_0: \beta_1 = 0$

$H_1: \beta_1 \neq 0$

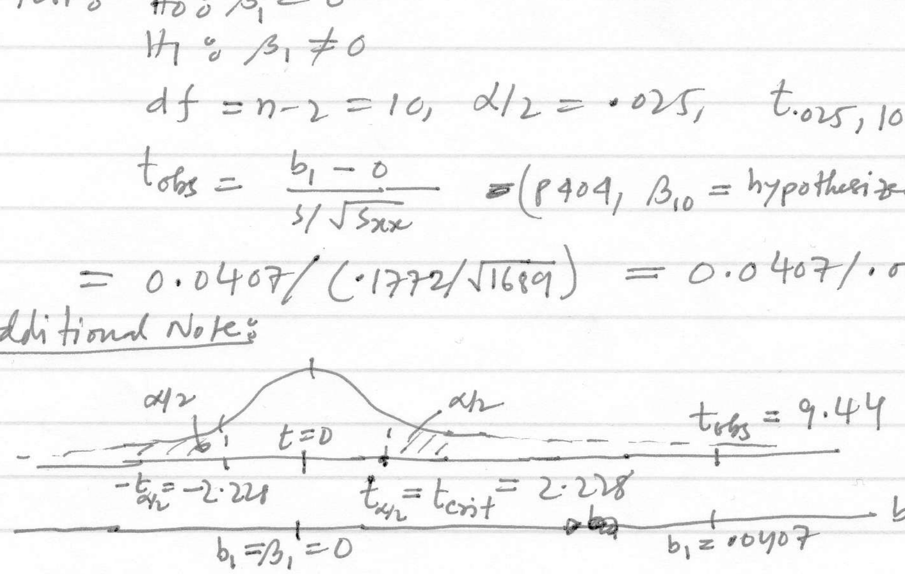
data from Q5

$df = n-2 = 10$, $\alpha/2 = .025$, $t_{.025,10} = 2.228$, $s = \sqrt{\frac{SSE}{10}} = .1772$

$$t_{obs} = \frac{b_1 - 0}{s/\sqrt{S_{xx}}} \quad (p404,\ \beta_{10} = \text{hypothesized value of } \beta_1 = 0)$$

$$= 0.0407/(.1772/\sqrt{1689}) = 0.0407/.004312 = \boxed{9.44}\ \text{Ans.}$$

Additional Note:



~~t_{obs} is in crit~~ Rejection region. H_0 is rejected.