# ENGG319 L01 Fall 2016 – Section #10 (Hypothesis Testing) Summary Sheet

| | | | |
|---|---|---|---|
| **$H_0$: $\mu=\mu_0$; $\sigma$ known and $\alpha$ given. Data are either normally distributed or you are invoking CLT ($n \geq 30$)** | | | |
| If $H_1$: $\mu < \mu_0$ | Compute critical value $z_\alpha$ where $P(Z< -z_\alpha) = \alpha$ | Compute statistic $z = \dfrac{\bar{x}-\mu_0}{\sigma/\sqrt{n}}$ | Reject $H_0$ if $z < -z_\alpha$ |
| If $H_1$: $\mu > \mu_0$ | Compute critical value $z_\alpha$ where $P(Z> z_\alpha) = \alpha$ | | Reject $H_0$ if $z > z_\alpha$ |
| If $H_1$: $\mu \neq \mu_0$ | Compute critical value $z_{\alpha/2}$ where $P(Z< -z_{\alpha/2}) = \alpha/2$ and $P(Z> z_{\alpha/2}) = \alpha/2$ | | Reject $H_0$ if $z < -z_{\alpha/2}$ or $z > z_{\alpha/2}$ |
| **$H_0$: $\mu=\mu_0$; $\sigma$; unknown but $n \geq 30$ so you invoke CLT. This is a "large sample test for a single mean"; $\alpha$ given.** | | | |
| If $H_1$: $\mu < \mu_0$ | Compute critical value $z_\alpha$ where $P(Z< -z_\alpha) = \alpha$ | Compute statistic $z = \dfrac{\bar{x}-\mu_0}{s/\sqrt{n}}$ | Reject $H_0$ if $z < -z_\alpha$ |
| If $H_1$: $\mu > \mu_0$ | Compute critical value $z_\alpha$ where $P(Z> z_\alpha) = \alpha$ | | Reject $H_0$ if $z > z_\alpha$ |
| If $H_1$: $\mu \neq \mu_0$ | Compute critical value $z_{\alpha/2}$ where $P(Z< -z_{\alpha/2}) = \alpha/2$ and $P(Z> z_{\alpha/2}) = \alpha/2$ | | Reject $H_0$ if $z < -z_{\alpha/2}$ or $z > z_{\alpha/2}$ |
| **$H_0$: $\mu=\mu_0$; $\sigma$ unknown and data is normally distributed; cannot invoke CLT; $\alpha$ given.** | | | |
| If $H_1$: $\mu < \mu_0$ | Compute critical value $t_\alpha$ where $P(T< -t_\alpha) = \alpha$ with $df = n$-1 | Compute statistic $t = \dfrac{\bar{x}-\mu_0}{s/\sqrt{n}}$ | Reject $H_0$ if $t < -t_\alpha$ |
| If $H_1$: $\mu > \mu_0$ | Compute critical value $t_\alpha$ where $P(T> t_\alpha) = \alpha$ with $df = n$-1 | | Reject $H_0$ if $t > t_\alpha$ |
| If $H_1$: $\mu \neq \mu_0$ | Compute critical value $t_{\alpha/2}$ where $P(T< -t_{\alpha/2}) = \alpha/2$ and $P(T> t_{\alpha/2}) = \alpha/2$; $df = n$-1 | | Reject $H_0$ if $t < -t_{\alpha/2}$ or $t > t_{\alpha/2}$ |
| **$H_0$: $\mu_1-\mu_2=d_0$; $\sigma_1$, $\sigma_2$ known and $\alpha$ given. Data are either normally distributed or you invoke CLT ($n \geq 30$)** | | | |
| If $H_1$: $\mu_1-\mu_2 < d_0$ | Compute critical value $z_\alpha$ where $P(Z< -z_\alpha) = \alpha$ | Compute statistic $z = \dfrac{(\bar{x}_1-\bar{x}_2)-d_0}{\sqrt{(\sigma_1^2/n_1)+(\sigma_2^2/n_2)}}$ | Reject $H_0$ if $z < -z_\alpha$ |
| If $H_1$: $\mu_1-\mu_2 > d_0$ | Compute critical value $z_\alpha$ where $P(Z> z_\alpha) = \alpha$ | | Reject $H_0$ if $z > z_\alpha$ |
| If $H_1$: $\mu_1-\mu_2 \neq d_0$ | Compute critical value $z_{\alpha/2}$ where $P(Z< -z_{\alpha/2}) = \alpha/2$ and $P(Z> z_{\alpha/2}) = \alpha/2$ | | Reject $H_0$ if $z < -z_{\alpha/2}$ or $z > z_{\alpha/2}$ |
| **$H_0$: $\mu_1-\mu_2=d_0$; $\sigma_1$, $\sigma_2$ unknown, but $n \geq 30$: use "large sample test"; $\alpha$ given.** | | | |
| If $H_1$: $\mu_1-\mu_2 < d_0$ | Compute critical value $z_\alpha$ where $P(Z< -z_\alpha) = \alpha$ | Compute statistic $z = \dfrac{(\bar{x}_1-\bar{x}_2)-d_0}{\sqrt{(s_1^2/n_1)+(s_2^2/n_2)}}$ | Reject $H_0$ if $z < -z_\alpha$ |
| If $H_1$: $\mu_1-\mu_2 > d_0$ | Compute critical value $z_\alpha$ where $P(Z> z_\alpha) = \alpha$ | | Reject $H_0$ if $z > z_\alpha$ |
| If $H_1$: $\mu_1-\mu_2 \neq d_0$ | Compute critical value $z_{\alpha/2}$ where $P(Z< -z_{\alpha/2}) = \alpha/2$ and $P(Z> z_{\alpha/2}) = \alpha/2$ | | Reject $H_0$ if $z < -z_{\alpha/2}$ or $z > z_{\alpha/2}$ |
| **$H_0$: $\mu_1-\mu_2=d_0$; $\sigma_1=\sigma_2$ and both are unknown; data are normally distributed; $\alpha$ given.** | | | |
| If $H_1$: $\mu_1-\mu_2 < d_0$ | Compute critical value $t_\alpha$ where $P(T< -t_\alpha) = \alpha$ with $df=n_1+n_2$-2 | Compute statistic $t = \dfrac{(\bar{x}_1-\bar{x}_2)-d_0}{s_p\sqrt{(1/n_1)+(1/n_2)}}$ with $s_p^2 = \dfrac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2}$ | Reject $H_0$ if $t < -t_\alpha$ |
| If $H_1$: $\mu_1-\mu_2 > d_0$ | Compute critical value $t_\alpha$ where $P(T> t_\alpha) = \alpha$ with $df=n_1+n_2$-2 | | Reject $H_0$ if $t > t_\alpha$ |
| If $H_1$: $\mu_1-\mu_2 \neq d_0$ | Compute critical value $t_{\alpha/2}$ where $P(T< -t_{\alpha/2}) = \alpha/2$ and $P(T> t_{\alpha/2}) = \alpha/2$; $df=n_1+n_2$-2 | | Reject $H_0$ if $t < -t_{\alpha/2}$ or $t > t_{\alpha/2}$ |

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| **$H_0$: $\mu_1-\mu_2=d_0$; $\sigma_1 \neq \sigma_2$ and both are unknown; data are normally distributed; $\alpha$ given.** | | | |
|---|---|---|---|
| If $H_1$: $\mu_1-\mu_2<d_0$ | Compute critical value $t_\alpha$ where $P(T< -t_\alpha) = \alpha$ | $t=\dfrac{(\bar{x}_1-\bar{x}_2)-d_0}{\sqrt{(s_1^2/n_1)+(s_2^2/n_2)}}$; with $df=\dfrac{(s_1^2/n_1+s_2^2/n_2)^2}{\dfrac{(s_1^2/n_1)^2}{n_1-1}+\dfrac{(s_2^2/n_2)^2}{n_2-1}}$ | Reject $H_0$ if $t<-t_\alpha$ |
| If $H_1$: $\mu_1-\mu_2>d_0$ | Compute critical value $t_\alpha$ where $P(T> t_\alpha) = \alpha$ | | Reject $H_0$ if $t>t_\alpha$ |
| If $H_1$: $\mu_1-\mu_2\neq d_0$ | Compute critical value $t_{\alpha/2}$ where $P(T< -t_{\alpha/2}) = \alpha/2$ and $P(T> t_{\alpha/2}) = \alpha/2$ | | Reject $H_0$ if $t<-t_{\alpha/2}$ or $t>t_{\alpha/2}$ |
| **$H_0$: $\mu_D=d_0$; $\sigma_1 \neq \sigma_2$ and both are unknown; data are normally distributed AND PAIRED; $\alpha$ given.** | | | |
| If $H_1$: $\mu_D<d_0$ | Compute critical value $t_\alpha$ where $P(T< -t_\alpha) = \alpha$, $df = n-1$ | Compute statistic $t=\dfrac{\bar{d}-d_0}{s_d/\sqrt{n}}$ | Reject $H_0$ if $t<-t_\alpha$ |
| If $H_1$: $\mu_D>d_0$ | Compute critical value $t_\alpha$ where $P(T> t_\alpha) = \alpha$, $df = n-1$ | | Reject $H_0$ if $t>t_\alpha$ |
| If $H_1$: $\mu_D\neq d_0$ | Compute critical value $t_{\alpha/2}$ where $P(T< -t_{\alpha/2}) = \alpha/2$ and $P(T> t_{\alpha/2}) = \alpha/2$, $df = n-1$ | | Reject $H_0$ if $t<-t_{\alpha/2}$ or $t>t_{\alpha/2}$ |

## Formulating Hypothesis Testing Problems

Hypotheses about a *random* variable ***x*** are often formulated in terms of its distributional properties. Example, if property is a:

Null hypothesis **$H_0$: a = *$a_0$***

Alternative hypothesis **$H_1$: a < *$a_0$*** || **a > *$a_0$*** || **a ≠ *$a_0$***

Objective of hypothesis testing is to decide whether or not to reject this hypothesis.
Decision is based on estimator $\hat{a}$ of a:

***Reject*** **$H_0$**: If observed estimate $\hat{a}$ lies in *rejection region* $R_{a0}$ (*i.e.* $\hat{a} \in R_{a0}$)

***Do not reject*** **$H_0$**: Otherwise (*i.e.* $\hat{a} \notin R_{a0}$)

Select rejection region to obtain desired error properties:

| | | Test Result | |
|---|---|---|---|
| | | Do not reject H0 $\hat{a} \notin R_{a0}$ | Reject H0 $\hat{a} \in R_{a0}$ |
| True situation | H0 true | $P(H0\|H0) = 1 - \alpha$ | $P(\sim H0\|H0) = \alpha$ (**Type I Error**) |
| | H0 false | $P(H0\|\sim H0) = \beta$ (**Type II Error**) | $P(\sim H0\|\sim H0) = 1 - \beta$ |

Type I Error probability α is called the test **significance level**.
1-β is said to be the **power** of a test; and β is the Type II Error probability

## Computing β and choosing sample size

Suppose that we wish to test the hypothesis ($H_0$: $\mu=\mu0$; H1: $\mu>\mu0$) with a significance level $\alpha$ when $\sigma^2$ is known. For a specific alternative, say $\mu=\mu_0+\delta$, the power of our test is: $1-\beta = P[\,\bar{x}>a$ when $\mu=\mu_0+\delta]$, which is equivalent to:

$$\beta = P\left[Z < z_\alpha - \frac{\delta}{\sigma/\sqrt{n}}\right] = P[Z < -z_\beta]$$

$$\text{Hence}: n = \frac{(z_\alpha + z_\beta)^2\sigma^2}{\delta^2} \quad \text{or} \quad n \approx \frac{(z_{\alpha/2} + z_\beta)^2\sigma^2}{\delta^2} (\text{two}-\text{tailed})$$

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<table>
<tr><th colspan="4">$H_0$: $\sigma^2 = \sigma_0^2$</th></tr>
<tr><td>If $H_1$: $\sigma^2 < \sigma_0^2$</td><td>Compute critical value $\chi^2_{1-\alpha}$ where $P(\chi^2 < \chi^2_{1-\alpha}) = \alpha$</td><td rowspan="3">Compute statistic $$\chi^2 = \frac{(n-1)s^2}{\sigma_0^2}$$ with $df = n-1$</td><td>Reject $H_0$ if $\chi^2 < \chi^2_{1-\alpha}$</td></tr>
<tr><td>If $H_1$: $\sigma^2 > \sigma_0^2$</td><td>Compute critical value $\chi^2_{\alpha}$ where $P(\chi^2 > \chi^2_{\alpha}) = \alpha$</td><td>Reject $H_0$ if $\chi^2 > \chi^2_{\alpha}$</td></tr>
<tr><td>If $H_1$: $\sigma^2 \neq \sigma_0^2$</td><td>Compute critical values $\chi^2_{\alpha/2}$ and $\chi^2_{1-\alpha/2}$ where $P(\chi^2 < \chi^2_{1-\alpha/2}) = \alpha/2$ and $P(\chi^2 > \chi^2_{\alpha/2}) = \alpha/2$</td><td>Reject $H_0$ if $\chi^2 < \chi^2_{1-\alpha/2}$ or $\chi^2 > \chi^2_{\alpha/2}$</td></tr>
<tr><th colspan="4">$H_0$: $\sigma_1^2 = \sigma_2^2$</th></tr>
<tr><td>If $H_1$: $\sigma_1^2 \neq \sigma_2^2$</td><td>Compute critical values $F_{\alpha/2}(df_1, df_2)$ where $P(F > F_{\alpha/2}) = \alpha/2$</td><td rowspan="2">Compute statistic $$F = \frac{s_1^2}{s_2^2} \text{ if } s_1^2 > s_2^2$$ with $df_1 = n_1 - 1$ and $df_2 = n_2 - 1$<br>or $$F = \frac{s_2^2}{s_1^2} \text{ if } s_1^2 < s_2^2$$ with $df_1 = n_2 - 1$ and $df_2 = n_1 - 1$</td><td>Reject $H_0$ if $F > F_{\alpha/2}(df_1, df_2)$</td></tr>
<tr><td>If $H_1$: $\sigma_1^2 > \sigma_2^2$ (□or $\sigma_1^2 < \sigma_2^2$)</td><td>Compute critical values $F_{\alpha}(df_1, df_2)$ where $P(F > F_{\alpha}) = \alpha$</td><td>Reject $H_0$ if $F > F_{\alpha}(df_1, df_2)$</td></tr>
<tr><th colspan="4">(Goodness-for-Fit Test) $H_0$: observation x follows a specified distribution f(x)</th></tr>
<tr><td colspan="2">$H_1$: x does not follow the distribution f(x)</td><td>Compute statistic $$\chi^2 = \sum_{i=1}^{k} \frac{(o_i - e_i)^2}{e_i}$$ with $df = k-1$</td><td>Reject $H_0$ if $\chi^2 > \chi^2_{\alpha}$</td></tr>
<tr><td colspan="4">***Note:*** Expected frequencies $e_i$ must be $\geq 5$ => collapse classes where necessary</td></tr>
</table>