Quiz # 3

Question # 1. A committee has 7 members, ~~find~~ find the probability of having at least four female members. Assume that the probability of having a male or a female member is equal.

Solution:

The probability of having a female member = 0.5
The probability of having a male member = 0.5
X = number of female members
X follows binomial distributions.

$$P(X \geq 4) = P(X = 4) + P(X = 5) + P(X = 6) + P(X = 7)$$

$$P(X = x) = b(x; n, p) = {}^{n}C_{x} p^{x} q^{n-z} \qquad \boxed{P145}$$

$$P(X \geq 4) = {}^{7}C_{4}(0.5)^{4}(0.5)^{3} + {}^{7}C_{5}(0.5)^{5}(0.5)^{2} + {}^{7}C_{6}(0.5)^{6}(0.5)^{1} + {}^{7}C_{7}(0.5)^{7}(0.5)^{0}$$

$$= \boxed{0.5} \qquad = \cancel{0.\ldots}$$

Question # 2 The average number of calls coming per minute into a hotel reservation center is 3. Find the probability that at least two calls will arrive in a given two-minute period.

Solution:

Let *X* be a Poisson random variable defining the number of calls coming in that given two-minute period. The average of X is 3*2=6

$$\boxed{P162} \quad P(X = x) = \frac{e^{-\lambda t}(\lambda t)^{x}}{x!}$$

$$P(X = x) = \frac{e^{-6}(6)^{x}}{x!}$$

$$P(x \geq 2) = 1 - P(x=0) - P(x=1)$$
$$= 1 - e^{-6}\left[\frac{6^0}{0!} + \frac{6^1}{1}\right] = 1 - 7e^{-6} = \boxed{0.983}$$
$$= 1 - 0.01735$$

Question # 3: Four different prizes are randomly put into boxes of cereal. One of the prizes is a free ticket to the local zoo. Suppose that a family of four ~~(called Family A)~~ decides to buy this cereal until obtaining four free tickets to the zoo. What is the probability that the family will have to buy 10 boxes of cereal to obtain the four free tickets to the zoo?

Solution:

The success is a box of cereal with a free ticket to the zoo. So getting a ticket to the zoo is considered a success. Any one of the other three prizes is considered undesirable or a failure. Any one of the four prizes is equally likely.

Let X is the number of trial on which the k-th success occur. If the family has to buy 10 boxes to get 4 ticket, then they will have the 4th success on the 10th trial. The negative binomial distribution in this example has $k = 4$ and $p = 0.25$.

$$b^*(x; k, p) = \binom{x-1}{k-1} p^k q^{x-k} \quad \boxed{P159}$$

$$P(X = 10) = \binom{10-1}{3}(0.25)^4(0.75)^{10-4}$$

$$= \binom{9}{3}(0.25)^4(0.75)^6 = 84(0.25)^4(0.75)^6 = \boxed{0.0583992}$$

Question # 4 The amount of mustard dispensed from a machine ~~at~~ is normally distributed with a mean of 0.9 ounce and a standard deviation of 0.1 ounce. If the machine is used 500 times, how many times (approximately) will it be expected to dispense 1 or more ounces of mustard?

Solution:

X = amount of mustard dispensed

$$P(X > 1 \text{ ounce}) = P\left(Z > \frac{1-0.9}{0.1}\right) = P(Z > 1)$$

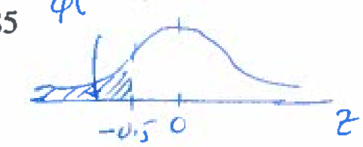
$$= 1 - P(Z < 1) = 1 - \phi(1) = 1 - .8413$$
$$= 0.1587$$

$$500 \times .1587 = 79.35$$

Question # 5 The chickens of the Ortnithes farm are processed when they are 20 weeks old. The distribution of their weights is normal with mean 3.8 lb, and standard deviation 0.6 lb. The farm has created three categories for these chickens according to their weight: petite (weight less than 3.5 lb), standard (weight between 3.5 lb and 4.9 lb), and big (weight above 4.9 lb). Suppose that 5 chickens are selected at random. What is the probability that 3 of them will be petite?

Solution:

$\phi(-0.5) = 0.3085$



Petite: $P(X < 3.5) = P(Z < \frac{3.5-3.8}{0.6}) = P(Z < -0.50) = 0.3085$

This is binomial with $n = 5$, $p = 0.3085$ Thus:

$$P(Y = 3) = \binom{5}{3}(0.3085)^3(1-0.3085)^2 = \boxed{0.1404}$$

Question # 6 Suppose that the lifetime of a component is modeled with a Weibull distribution with beta = 0.5 and the mean life of 400 hours. What is the probability that the system is still working (functioning) after 6000 hours if there is no failure in the first 3000 hours?

$$P(X > 6000 \mid X > 3000 \text{ hr}) = \frac{P(X > 6000 \text{ hrs})}{P(X > 3000 \text{ hrs})} \qquad X = \text{lifetime in hrs.}$$

$$= \frac{1 - F(6000)}{1 - F(3000)} = \frac{e^{-\alpha 6000^\beta}}{e^{-\alpha \cdot 3000^\beta}} = e^{\alpha[3000^\beta - 6000^\beta]}$$

$$\boxed{\beta = 0.5}, \quad \mu = \alpha^{-1/\beta}\,\Gamma\left(1 + \frac{1}{\beta}\right) = \alpha^{-2}\,\Gamma(3) = \frac{2}{\alpha^2} \qquad \Gamma(3) = (3-1)! = 2! = 2.$$

$$\Rightarrow 400 \text{ hrs} = \frac{2}{\alpha^2} \Rightarrow \alpha^2 = \frac{1}{200} \quad \boxed{\alpha = 0.07071}$$

Substituting α & β, required prob $= e^{0.07071(\sqrt{3000} - \sqrt{6000})}$

$$= e^{-1.6072} = \boxed{0.201}$$