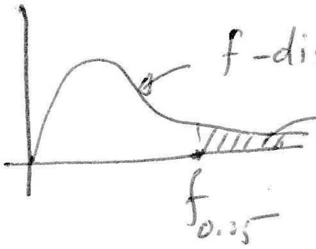
Ch 8 - Tutorial (Part 2)

Solution:

1. (a) $f_{0.05}$ with $\nu_1 = 7$ and $\nu_2 = 15$



f-distribution with $\nu_1 = 7$, $\nu_2 = 15$

tail area $= f_{0.05}(7, 15) = 2.71$ (from Table A.6)

Table A.6 (P741)

	$f_{0.05}(\nu_1, \nu_2)$					
	ν_1					
ν_2	1	2	...	7	8	9
1						
2						
⋮						
15	...	...	...	2.71		
⋮						

$2.71 \leftarrow f_{0.05}(7,15)$

b) $f_{0.95}(\nu_1 = 15, \nu_2 = 7)$

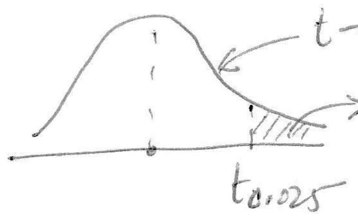
Using theorem 8.7 (P252): $\quad f_{1-\alpha}(\nu_1, \nu_2) = \dfrac{1}{f_\alpha(\nu_2, \nu_1)}$

$$f_{0.95}(\nu_1 = 15, \nu_2 = 7) = \frac{1}{f_{0.05}(\nu_1 = 7, \nu_2 = 15)}$$

$$= \frac{1}{2.71} = 0.369$$

c) $f_{0.01}(\nu_1 = 24, \nu_2 = 19) = 2.92$ (Table A.6, P744)

2. a) $t_{0.025}$ when $\nu = 14$ = ?



t-distribution for $\nu = 14$

tail area = $t_{0.025} = 2.145$ (Table A.4, P 737)

ν \ α	0.4	0.3	...	0.025
1				
2				
⋮				
14				2.145
⋮				

b) $-t_{0.1}$ when $\nu = 10$

$t_{0.1}$ when $(\nu = 10) = 1.372$ (Table A.4, P 737)

Therefore, $-t_{0.1}$ when $\nu = 10 = -1.372$

c) $P(-t_{0.005} < T < t_{0.01})$ for $\nu = 20$

$= P(T < t_{0.01}) - P(T < -t_{0.005})$

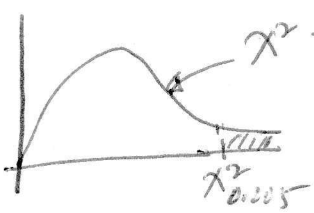
$= (1 - 0.01) - P(T < t_{.995})$

$= (1 - 0.01) - (1 - 0.995) = 0.99 - 0.005 = 0.985$

~~= 0.999 − .005 =~~

[Notes: By the definition of critical value, t_α = value of t containing an area α to the right. So, $P(T > t_\alpha) = \alpha$]
Also, by ~~sym~~ symmetry $-t_\alpha = t_{1-\alpha}$, see page 278.

3. (a) $\chi^2_{0.005}$ for $\nu = 5$ = 16.75 (Table A.5, P740)



χ^2 for $\nu = 5$

(b) $\chi^2_{0.05}$ for $\nu = 19$ = 30.144.

4. Let X = tare content

Given $x_1 = 7.3,\ x_2 = 8.6, \cdots, x_8 = 9.3$

(a) $\bar{x} = \frac{1}{8}(x_1 + x_2 + \cdots + x_8) = 11.69$ (P 228)

sample mean

(b) $s^2 = \frac{1}{n-1} \sum (x_i - \bar{x})^2$

$s^2 = \frac{1}{n-1} \sum (x_i - \bar{x})^2 = 10.77$ (sample variance)

5. $P(\mu_{\bar{x}} - 1.9\sigma_{\bar{x}} < \bar{x} < \mu_{\bar{x}} - 0.4\sigma_{\bar{x}}) = ?$

$= P(-1.9\sigma_{\bar{x}} < \bar{x} - \mu_{\bar{x}} < -0.4\sigma_{\bar{x}})$ Subtracting $\mu_{\bar{x}}$

$= P\left(-1.9 < \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} < -0.4\right)$ Dividing by $\sigma_{\bar{x}}$

$= P(-1.9 < Z < -0.4)$ Since $\bar{x}$ follows normal distribution, the quantity $\frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}}$, by definition is a standard normal variable, Z.

$= 0.3446 - 0.0287$ (Table A-3, P 735)

$= 0.316$

6. Population distribution is unknown.

However sample size $n = 36 \geq 30$, so we invoke CLT (central limit theorem) & say that ~~the standard deviation~~ $\bar{x}$ approximately follow normal distribution with mean $\bar{x} = \mu$ (population mean) & ~~standard~~ variance ~~deviation~~.

$$\sigma_{\bar{x}}^2 = \frac{\sigma^2}{n}$$

Given, $\sigma_{\bar{x}} = 2$ when $n = 36 \Rightarrow 2^2 = \frac{\sigma^2}{36}$

$\Rightarrow \sigma^2 = 36 \times 4$

Then, let, $1.2^2 = \frac{\sigma^2}{n} = \frac{36 \times 4}{n} \Rightarrow n = 100$

So, $\boxed{n \geq 100}$. to ~~b~~ reduce sample standard deviation to ~~1.2 or smaller~~ at least 1.2.

7. Both samples come from normal population. So the sample means exactly follow normal distribution. ~~Also~~, Moreover, their difference $(\bar{X}_1 - \bar{X}_2)$ also follow normal distribution. (P238)

$\mu_1 = 80$, $\sigma_1 = 5$ $\longrightarrow$ $n_1 = 25$, $\bar{x}_1 =$

Population ~~A~~ 1

$\mu_2 = 75$, $\sigma_2 = 3$ $\longrightarrow$ $n_2 = 36$, $\bar{x}_2 =$

Population ~~B~~ 2

$P(3.4 < \bar{X}_1 - \bar{X}_2 < 5.9) = ?$

Subtract $(\mu_1 - \mu_2)$ and then divide by $\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$

$$P\left(\frac{3.4 - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} < \underbrace{\frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}} < \frac{5.9 - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \right)$$

this statistic follows standard normal distribution, because $\mu_{\bar{X}_1 - \bar{X}_2} = \mu_1 - \mu_2$ and $\sigma^2_{\bar{X}_1 - \bar{X}_2} = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}$

$$= P\left(\frac{3.4 - 5}{\sqrt{25/25 + 9/36}} < Z < \frac{5.9 - 5}{\sqrt{25/25 + 9/36}} \right)$$ See Theorem 8.3, P238.

$= P(-1.43 < Z < 0.8)$

$= 0.7881 - 0.0764 = 0.7117$

8. The population is normal. So, $\bar{X}$ follows normal distribution. However, we cannot construct Z-statistic as σ = population standard deviation is unknown. In such situation, we can construct t-statistic that follows T-distribution.

$t = \frac{\bar{X} - \mu}{s/\sqrt{n}}$ $\quad \bar{X} = \frac{508.6}{16} = 31.78$, $s^2 = \frac{1}{n-1} \Sigma (x_i - \bar{x})^2 = 0.68$

$P(\bar{X} > 31) = P\left(\frac{\bar{X} - \mu}{s/\sqrt{n}} > \frac{31 - \mu}{s/\sqrt{n}} \right) = P(t > 2.428)$ for $\nu = 16 - 1 = 15$

$\simeq 0.014$

For $\alpha = 0.01$, $t_\alpha = t_{0.01} = 2.602$

For $\alpha = 0.015$, $t_\alpha = t_{0.015} = 2.397$

For $t = 2.428$, $\alpha = ?$

Interpolation:

$\frac{.015 - \alpha}{2.397 - t} = \frac{.015 - .01}{\cancel{.015} - .205}$

$= -0.0244$

$\Rightarrow .015 - \alpha = \cancel{.01295}$ $.000756$

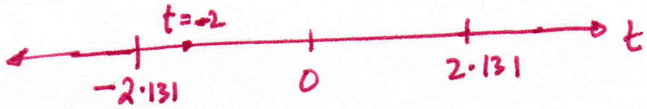
$\Rightarrow \alpha = 0.014$

9. Population approximately normal, but σ is unknown. We use t-statistic

$$t = \frac{\bar{X} - \mu}{s/\sqrt{n}} = \frac{27.5 - 30}{5/\sqrt{16}} = -2$$

$t_{0.025}$ for $\nu = 15$ $= 2.131$

$-t_{0.025}$ for $\nu = 15$ $= -2.131$, so, $-t_{0.025} < t < t_{0.025}$, satisfied.



10. $s_1^2 = 10.441$, $s_2^2 = 1.846$, $f = \dfrac{s_1^2/\sigma_1^2}{s_2^2/\sigma_2^2} = \dfrac{s_1^2}{s_2^2} \cdot \left(\dfrac{\sigma_2^2}{\sigma_1^2}\right)$

If we assume that $\sigma_1^2 = \sigma_2^2$, f becomes $f = \dfrac{s_1^2}{s_2^2} = 5.658$. Now we calculate the probability $P(f > 5.658)$ for $\nu_1 = 9$ and $\nu_2 = 7$.
From table A.6 for $\nu_1 = 9$, $\nu_2 = 7$, we obtain

$f_{0.01}(9,7) = 6.72$ i.e., $P(F > 6.72) = 0.01 = 1\%$

$f_{0.05}(9,7) = 3.68$ ie., $P(F > 3.68) = 0.05 = 5\%$

Therefore, the probability $P(F > 5.658)$ is greater than 1% but Less than 5%, which is a small. Therefore the assumption of equality of population variance lead to rare-event. So, it is likely that population variances are not equal.

Note: If $\sigma_2^2 > \sigma_1^2$, then, $f > 5.656$, which is also not likely.

11. $P(s_1^2/s_2^2 > 1.26) \Rightarrow P\left(\dfrac{s_1^2/\sigma_1^2}{s_2^2/\sigma_2^2} > 1.26 \dfrac{\sigma_2^2}{\sigma_1^2}\right) \Rightarrow P(F > 1.26 \times 1.5)$

$\Rightarrow = P(F > 1.89)$ for $\nu_1 = 24$ & $\nu_2 = 30$

From table A.6, $f_{0.05}(24, 30) = 1.89$ (P742)

Therefore, $P(F > 1.89) = 0.05$