

ENGG 319 (Fall 2016) Lecture/ Tutorial # 3 (Videos 1-14, Chapter 2)

Counting techniques (Section 2.3)

Question # 1. How many ways are there that no two students will have the same birth date in a class of size 60? Consider for simplicity that no one's birthday is on Feb 29, and 1 year = 365 days.

Question # 2. In how many ways are there to select 3 candidates from 8 equally qualified recent graduates for openings in an accounting firm? [Ans: 56]

Question # 3. (a) In how many ways 6 teachers can be assigned to 4 sections of an introductory psychology course if no teacher is assigned to more than one section? [Ans: 360] **(b).** In an experiment a die is thrown and then a letter from the English alphabet is drawn at random. How many points are there in the sample space? [Ans: 156]

Question # 4. (a) How many three-digit numbers can be formed from the digits 0,1,2,3,4,5,6 if each digit can be used only once?

(b) How many of these are odd numbers? [Ans: 75]

(c) How many are greater than 330 ? [Ans: 105]

Question # 5. If a multiple-choice test consists of 5 questions, each with 4 possible answers of which only 1 is correct.

(a) in how many different ways can a student check off one answer to each question ? [Ans: 1024]

(b) in how many different ways can a student check off one answer to each question and get all the answers wrong ? [[Ans: 243]

Question # 6. (a) In how many ways can 6 people be lined up to get on a bus ? [Ans: 720]

(b) If 3 specific persons, among 6, insist on following each other, how many ways are possible? [A:144]

(c) If 2 specific persons, among 6, refuse to follow each other, how many ways are possible? [Ans: 480]

Question # 7. A bag contains 6 red and 8 green balls.

(a) If one ball is drawn at random, then what is the probability of the ball being green? [Ans: 4/7]

(b) If two balls are drawn at random, what is the probability that one is red and the other is green?

[Ans: 48/91]

Question # 8. A bag contains 7 red, 12 white and 4 green balls. What is the probability that

(a) 3 balls drawn are all white [Ans: 0.1242]

(b) 3 balls drawn are one of each color? [Ans: 0.1897]

Question # 9. There are 7 red balls and 6 black balls in a bin. In how many ways they can be arranged in a line so that the first, third, and fifth balls are of same color? [Ans: 330]

Bayes Rule and Theorem of Total Probability (Section 2.7)

Question # 10. In a bolt manufacturing factory, machines A, B, C manufacture 25%, 35%, and 40% of the total output respectively. Of the outputs from A, B, C, there are 5%, 4%, and 2% defective bolts. A bolt is drawn from all produced bolts and if found to be defective. What is the probability that it was manufactured by (a) machine A (b) machine B, and (c) machine C? [Ans: 0.362,0.406,0.232]

Question # 11. In a certain region of the country it is known from past experience that the probability of selecting an adult over 40 years of age with cancer is 0.05. If the probability of a doctor correctly diagnosing a person with cancer as having the disease is 0.78 and the probability of incorrectly diagnosing a person without cancer as having the disease is 0.06, (a) what is the probability that an adult over 40 years of age is diagnosed as having cancer? (b) What is the probability that a person diagnosed as having cancer actually has the disease? [Ans: (a) 0.096 (b) 0.406]

Additional problems (miscellaneous)

Question # 12 Two computers A and B are to be marketed. The probability that a salesperson will sell the computers are 0.6 for computer A and 0.4 for computer B. The two computers can be sold independently. Given that the salesperson was able to sell at least one of the computers, what is the probability that computer A was sold? [Ans: 0.7895]

Question # 13 Three research groups are working on a problem. The probability that group A, group B, and group C solves the problem are $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ respectively. What is the probability that the problem is solved by at least one group? [Ans: 0.75]

Question # 14. A box contains 8 red, 3 blue, and 9 green balls. If 3 balls are drawn at random. What is the probability that at least one is blue? (Ans: 0.4035)

Question # 15. The probability of obtaining a high-paying job without a university degree is 20%, while the probability of obtaining a high-paying job with a degree is 60%. If 30% of jobs in Canada are high-paying, Find the percentage of workers that have a high-paying job and a university degree.[Ans:15%]

Question # 16. An allergist claims that 50% of her patient she tests are allergic to some type of weed. What is the probability that exactly 3 of her next four patients are allergic to that weed? [Ans: 0.25]

Question # 17. A producer of a certain type of electronic component ships to suppliers in lots of twenty. Suppose that 60% of all such lots contain no defective components, 30% contain one defective component, and 10% contain two defective components. A lot is picked, two components from the lot are randomly selected and tested, and neither is defective. What is the probability that zero defective components exist in the lot? [Ans: 0.9505]

Question # 18. A shipment of 12 televisions contains 3 defective ones. In how many ways can a hotel purchase 5 of these ones and receive at least two defective ones? [Ans: 288]

Question # 19. The famous Monty Hall problem

Suppose a contestant is on a game show, and s/he is given the choice of three doors: Behind one door is a car; behind the others, goats. The contestant pick a door, say No. 1, and the host, who knows what's behind the doors, opens another door, say No. 2, which has a goat. He then says to you, "Do you want to pick door No. 3?" Is it advantageous to switch your choice?

[Hint: Define: A = the car is behind door 1, B = the car is behind door 2, and C = the car is behind door 3. Also define the following event, b = the host opens the door 2 after the contestant selects door A.

$$P(b) = P(b \cap A) + P(b \cap B) + P(b \cap C) = P(A)P(b|A) + P(B)P(b|B) + P(C)P(b|C) =$$

Since the host has opened door 2, the car is behind either door 1 or door 3. Based on the information that despite having two choices, the host chose to open door 2, you now revise the probability of events A and C. Find

$$P(A|b) = \frac{P(A)*P(b|A)}{P(b)} = \quad \text{and } P(C|b) =$$

Should the contestant change the door? The conclusion is counterintuitive. See: https://en.wikipedia.org/wiki/Monty_Hall_problem]