Chapter 10 (Nov 17)

Problem # 1

In one experiment, 45 still balls lubricated with purified paraffin were subjected to a 40 kg load at 600 rpm for 60 minutes. The average wear, measured by the reduction in diameter, was 673.2 micrometer, and the standard deviation was 14.9 micrometer. Assume that the specification for a lubricant is that the mean wear be less than 675 micrometer. Find the P-value for testing: $H_0: \mu \geq 675$ vs $H_1: \mu < 1000$

Solution:

1. Define H_0 and H_1 (given in this case)

2. Assume

3. Compute a test statistic.

4. Compute the P-value test of the test statistic.

5. State a conclusion about the strength of the evidence against Ho

Problem # 2

A scale is to be calibrated by weighing a 1000 g test weight 60 times. The 60 scale readings have mean 1000.6 g and standard deviation 2 g. Find the P-value for testing:
$H_0: \mu = 1000$ vs $H_1: \mu \neq 1000$. [**Ans:** P-value = 0..0204, Reject H0 and recalibrate]

[**Statistical Significance:** Whenever, the P-value is less than a particular threshold, the result is said to be "statistically significant" at that level. So for example, if P is less than or equal to 0.05, the result is statistically significant at the 5% level, etc. If a result is statistically significant at the 100alpha% level, we can also say that the null hypothesis is "rejected at level 100alpha%".]

Problem # 3

A hypothesis test is performed of the null hypothesis $H_0: \mu = 0$. The P-value turns out to be 0.03. Is the result statistically significant at the 10% level? The 5% level? The 1% level? Is the null hypothesis rejected at the 10% level? The 5% level? The 1% level?

Problem # 4

Let alpha be any value between 0 and 1. Then if P-value is less than or equal to alpha, which of the following is/are true?

(a) The result of the test is said to be statistically significant at the 100alpha% level

(b) The null hypothesis is rejected at the 100alpha% level

[Ans: Both are true]

Problem # 5

Specifications for a water pipe call for a mean breaking strength μ of more than 2000 lb per linear foot. Engineers will perform a hypothesis test to decide whether or not to use a certain kind of pipe. They will select a random sample of 1 ft sections of pipe, measure their breaking strengths, and perform a hypothesis test. The pipe will not be used unless the engineers can conclude that $\mu > 2000$.

(i) Assume that they test

$H_0: \mu \leq 2000$ versus $H_1: \mu > 2000$. Will the engineers decide to use the pipe if H0 is rejected? What if H0 is not rejected?

(ii) Assume the engineers test $H_0: \mu \geq 2000$ versus $H_1: \mu < 2000$. Will the engineers decide to use the pipe if H0 is rejected? What if H0 is not rejected?

[Ans: (i) if Ho rejected, they will use the pipe; if not rejected, they will not use the pipe

(i) if Ho rejected, they will not use the pipe; if not rejected, they will not use the pipe]

Chapter 10 – Part 2

<u>Fixed-level testing</u>

[Steps: 1. Choose the fixed level alpha 2. Compute P-value 3. If $\leq \alpha$, reject H0, If $P > \alpha$ do not reject H0. Please see figure below]

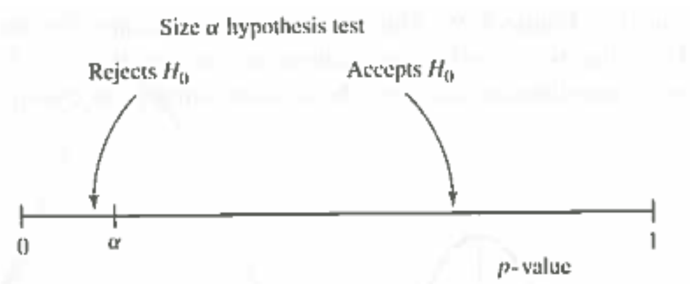


FIGURE 8.28

Decision rules for a size α hypothesis test

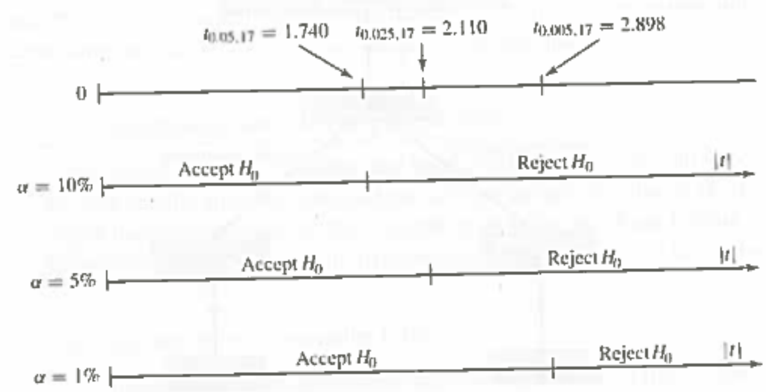


FIGURE 8.31

Hypothesis tests at fixed significance levels

Problem # 6

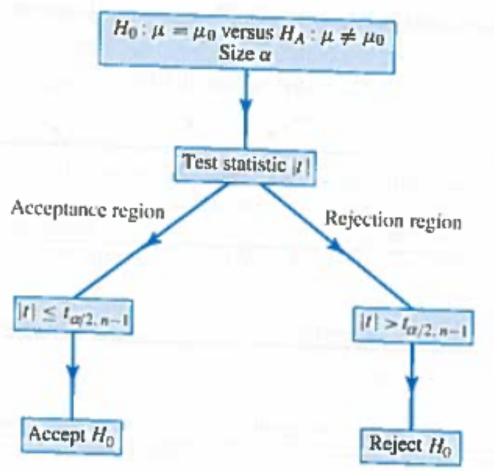
In one experiment, 45 still balls lubricated with purified paraffin were subjected to a 40 kg load at 600 rpm for 60 minutes. The average wear, measured by the reduction in diameter, was 673.2 micrometer, and the standard deviation was 14.9 micrometer. Let μ denote the population mean wear. A test of $H_0: \mu \geq 675$ versus $H_1: \mu < 675$ is performed. Can we reject H0 at the 25% significance level? Can we reject H0 at the 5% level? [Ans: Yes, No]

Problem # 7

A scale is to be calibrated by weighing a 1000 g test weight 60 times. The 60 scale readings have mean 1000.6 g and standard deviation 2 g. Find the rejection region if the test will be conducted at a significance level of 5%. [999.49, 1000.51] [Note: z-tesr or t-test? We should do a z-test as we do not know the population distribution. Since sample size is large, we can replace sigma with s. If we know/ or assume that the population of scale readings follow normal distribution, we can perform a t-test]

FIGURE 8.30

Size α two-sided t-test



<u>Type I and II Error</u>

	H_0 true	H_0 false
H_0 accepted	No error ☺ ✓	Type II error ☹ ✗
H_0 rejected	Type I error ☹ ✗	No error ☺ ✓

FIGURE 8.29

Error classification for hypothesis tests

Problem # 8

A soft drink machine at a steak house is regulated so that the amount of drink dispensed is approximately normally distributed with a mean of 200 millimeters and a standard deviation of 15 millimeters. The machine is checked periodically by taking a sample of 9 drinks and computing the average content. If the sample mean is greater than 191, but smaller than 209, the machine is thought to be operating satisfactorily; otherwise, we conclude that the population mean is not 200 milliliters.

a) Find the probability of committing a type I error when population mean is 200 milliliters

b) Find the probability of committing a type II error when population mean is 215 milliliters

Problem # 9

A new curing process developed for a certain type of cement results in a mean compressive strength of 5000 kilograms per square centimeter with a standard deviation of 120 kilograms. To test the hypothesis that $\mu = 5000$ versus $H_1: \mu < 5000$, a random sample of 50 pieces of cement is tested. The critical region is defined to be $\bar{x} < 4970$.

(a) Find the probability of committing a type I error when is true.

(b) Evaluate β for the alternative $\mu = 4970$

Problem # 10

In a research report, Tichard H. Weindruch of the UCLA Medical School claims that mice with an average life span of 32 months will live to be about 40 months old when 40% of the calories in their diet are replaced by vitamins and protein. Is there any reason to believe that $\mu < 40$ if 64 mice that are placed on this diet have an average life of 38 months with a standard deviation of 5.8 months. Use a P-value in your conclusion.

Problem # 11

An electrical firm manufactures light bulbs that have a lifetime that is approximately normally distributed with a mean of 800 hours and a standard deviation of 40 hours. Test the hypothesis that $\mu = 800\ hours$ versus $H_1: \mu \neq 800\ hours$, if a random sample of 30 bulbs has an average life of 788 hours. Use a P-value in your answer.

Problem # 12

Test the hypothesis that the average content of containers of a particular lubricant is 10 liters if the contents of a random sample of 10 containers are 10.2, 9.7, 10.1, 10.3, 10.1, 9.8, 9.9, 10.4, 10.3, and 9.8 liters. Use a 0.01 level of significance and assume that the distribution of contents is normal.

Problem # 13

If the distribution of life spans in Problem 11 is approximately normal, how large a sample is required in order that the probability of committing a type II error be 0.1 when the true mean is 35.9 months? Assume that population standard deviation is 5.8 months.

Problem # 14

A new chemical process has been developed that may increase the yield over that of the current process. The current process is known to have a mean yield of 80 and a standard deviation of 5. If the mean yield of the new process is shown to be greater than 80, the new process will be put into production. Let μ denote the mean yield of the new process. It is proposed to run the new process 50 times and then to test the hypothesis $H_0: \mu \leq 80$ versus $H_1: \mu > 80$ at a significance level of 5%. If Ho is rejected, it will be concluded that $\mu > 80$ and the new process will be put into production.

Let us assume that if the new process had a mean of 81, then it will be of substantial benefit to put this process into production. If it is in fact the case that μ =81, what is the power of the test, that is, the probability that Ho will be rejected? Assume that the population standard deviation for the new process is similar to that of current process. [Ans: 0.409]

[Power =1-P(Type II error). Power of the test is the probability of rejecting Ho when it is false. In the present case it is the probability that the sample mean will fall into the rejection region if the alternate hypothesis is true]

Reference: *Probability and Statistics for Engineers and Scienctists* , R.E. Walpole, R.H. Myers, S. L. Myers, and K. Ye, 9e, Prentice Hall