



The University of Calgary
Schulich School of Engineering

ENGG 319: Probability and Statistics for Engineers (Fall 2015)

Midterm Examination

Tuesday, 20 October, 18:00 - 20:00

Instructions:

- Write your name, student ID, and lecture section on your exam booklet.
- This is an open textbook and open notes exam. Only the sanctioned Schulich School of Engineering calculators can be used.
- No students will be permitted to share calculators.
- The duration of examination is 120 minutes.
- Full marks are 35.

1. Four multiple choice questions below. Please write your answer for each question on the booklet, *not on this sheet*.

(i). (2 marks) There are 7 red balls and 6 blue balls in a bin. How many ways are there to arrange them so that the first, third, and fifth are of the same color?

- (a) 120. (b) 225. (c) 330. (d) 435. (e) 540.

(ii). (2 marks) You randomly select 12 balls without replacement from a bin containing 28 red and 33 blue balls. What is the probability of getting at most 2 red balls?

- (a) 0.02. (b) 0.04. (c) 0.06. (d) 0.08. (e) 0.10.

(iii). (2 marks) The probability of obtaining a high paying job without a university degree is 20%, while the probability of obtaining a high paying job with a university degree is 60%. If 30% of jobs in Canada are high paying, what is the percentage of workers that have a high paying job and a university degree?

- (a) 0.10. (b) 0.15. (c) 0.20. (d) 0.25. (e) 0.30.

(iv). (2 marks) What is the probability of answering at least 2 multiple choice questions correctly if you randomly select from 5 possible answers for 4 different multiple choice questions?

- (a) 0.06. (b) 0.11. (c) 0.16. (d) 0.21. (e) 0.26.

2. The reliability of light-sensitive polymer components degrades over time. Its reliability at time x is the probability that its lifetime will be greater than x . The probability of failing after a certain time is found to obey Weibull distribution with shape parameter $\beta = 1.8$ and scale parameter $\delta = 2.5$ years. Note: Given that the remaining survival probability is equal to reliability, the failure probability is equal to 100% minus the reliability.

(a) (3 marks) A mechanical device requires 4 of these components to be installed as sensors. At least 3 sensors have to work in order for the device to function properly. Determine the device reliability after 2 years.

(b) (3 marks) How many sensors should be installed in the device so that the reliability of the device after 2 years, i.e., the probability of failing after 2 years, is at least 75%? Remember that at least 3 sensors have to work in order for the device to function properly. Hint: You need to iteratively find the solution.

(c) (3 marks) The component manufacturer produces 125,000 components each year. The average lifetime of the polymer component is found to be 2.23 years. Using the reliability at the device's average lifetime as the survival probability for each of the 125,000 components, determine the probability that at most 35,000 of them are defective.

Hint: The continuous random variable X has a Weibull distribution, with parameters β and δ , with the probability density function $f(x) = (\beta/\delta)(x/\delta)^{\beta-1} \exp[-(x/\delta)^\beta]$ for $x \geq 0$. The cumulative distribution function is $1 - \exp[-(x/\delta)^\beta]$. The mean is $\delta\Gamma(1+\beta^{-1})$ and variance is $\delta^2\Gamma(1+2\beta^{-1}) - \delta^2[\Gamma(1+\beta^{-1})]^2$. *Using Notation of Walpole Textbook:*

$f(x; \alpha, \beta) = \begin{cases} \alpha \beta x^{\beta-1} e^{-\alpha x^\beta}, & x > 0 \\ 0, & \text{otherwise} \end{cases}$ See page 203-204

3. One factory produces two products ("widgets" and "gizmos"). Samples of products leaving this factory are randomly selected and tested non-destructively for quality assurance purposes. Sixty four percent of the products leaving this factory are widgets.

(a) (4 marks) Five products are sampled randomly. What is the probability that three or more products sampled is a gizmo?

(b) (5 marks) During monthly quality assurance testing, what is the probability that the 5th product selected is the second observed widget?

4. Hairline fractures in a long metal rod are distributed according to a Poisson process such that the distance between adjacent fractures in meters follows an exponential distribution with a 0.5 m average.

(a) (3 marks) Find the probability that the distance between two adjacent fractures is greater than its mean distance.

(b) (3 marks) Find the probability that the distance between the first and the fifth fracture is longer than 3 m.

(c) (3 marks) Find the probability that at least two fractures are located within a distance of 3 meters.

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