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Solution to Quiz #5

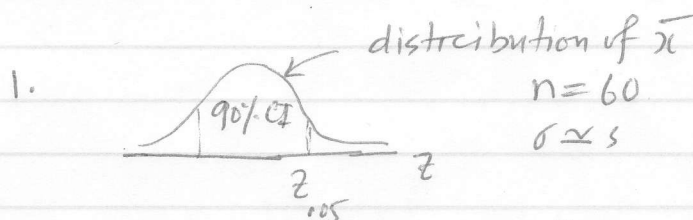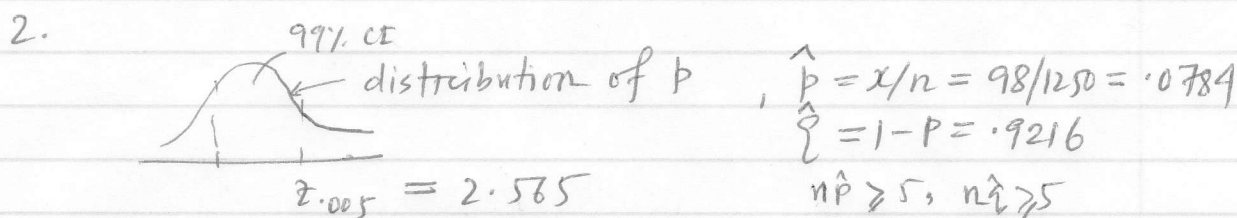1.

distribution of $\bar{x}$

$n = 60$

$\sigma \approx s$

90% CI

$z_{.05}$

$z_{.05} = 1.645$, $\quad z_{\alpha/2}\dfrac{\sigma}{\sqrt{n}} = (1.645) * \dfrac{0.134}{\sqrt{60}} = 0.0285$

$\bar{x} \pm z_{\alpha/2}\dfrac{\sigma}{\sqrt{n}} = 5 \pm 0.0285 = (4.9715, 5.0285)$

2.

99% CI

distribution of p, $\quad \hat{p} = x/n = 98/1250 = .0784$

$\hat{q} = 1 - \hat{p} = .9216$

$z_{.005} = 2.565$

$n\hat{p} \geqslant 5, \; n\hat{q} \geqslant 5$

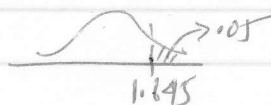$z_{.005} * \sqrt{\dfrac{\hat{p}\hat{q}}{n}} = 2.565 * \sqrt{\dfrac{.0784 \times .9216}{1250}}$

$= 2.565 \times .0076 = 0.0195$

$C.I. = .0784 \pm .0195 = (.0589, 0.0979)$

$C.I. \text{ of for Number of votes} = (5 \times 10^6 \times .0589, \; 5 \times 10^6 \times .0979)$
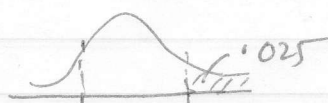
$= (294,500, \; 489,500)$.

3. $\alpha = P(\text{type I error})$

$= P(\text{Reject } H_0 \text{ when } H_0 \text{ is true})$

$= P(z > 1.645) \quad$ Acceptance region given: $z \leq 1.645$
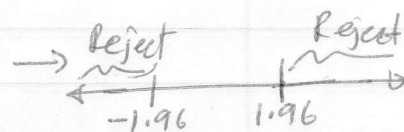
$= 0.05$

$> .05$

1.645

1/2

Solution to Quiz 5

Q4. $H_0: \mu = 3$

$H_1: \mu \neq 3$

Given: $\bar{x} = 2.15$, $n = 11,160$, $\sigma = 1.05$, $\alpha = 0.05$

$z_{\alpha/2} = z_{.05/2} = z_{.025} = 1.96$

$z_{obs} = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \dfrac{2.15 - 3}{\dfrac{1.05}{\sqrt{11,160}}} = \dfrac{-0.85}{.0099} = -85.52 < -1.96$

falls in Rejection region

Conclusion: Reject H_0

Answer is (c). Both statements (1) & (2) are true.

Q5. $H_0: \mu = 1.2$

$H_1: \mu \neq 1.2$ (Two-tailed test) | Given: $\sigma^2 = 25$

$\bar{x} = 1.02$, $n = 100$

$z_{obs} = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \dfrac{1.05 - 1.2}{5/\sqrt{100}}$

$= -0.15/.5 = -0.3$

P-value $= 2 * P(z < -0.3) = 2 \times 0.3821 = 0.7642$

Q6. $n \simeq \dfrac{(z_{\alpha/2} + z_\beta)^2 \sigma^2}{\delta^2}$

$z_{\alpha/2} = z_{.005} = 2.565$, $z_\beta = z_{.01} = 1.28$

$\delta = \mu_{true} - \mu_0 = 1.1 - 1.2 = -0.1$

$\sigma^2 = .025$

$n = \dfrac{(2.565 + 1.28)^2 \times .025}{(-0.1)^2} \simeq 37$