Sections #3 and #4

(1) Let X be a discrete random variable with probability mass function f(x) = k(x+2k); for x = 0, 1, 2, 3, 4, 5. Find P(1<X<4).

Discrete probabilities must sum to 1 so $\sum_{x=0}^{5} k(x+2k) = \sum_{x=0}^{5} kx + 2k^2 \sum_{x=0}^{5} 1 = 15k + 12k^2$ =1

The positive root of the quadratic equation is k =0.06345. The probability mass function can be tabulated:

x	0	1	2	3	4	5
f(x)	0.00805	0.07149	0.13494	0.19838	0.26184	0.32528

P(1<X<4) = 0.13494+0.19838 = 0.33332

(2) The length of time required by students to complete a 1-hour exam is a random variable with a probability density function given by

$$f(y) = \begin{cases} cy^2 + y, & \text{for } 0 \le y \le 1 \\ 0, & \text{elsewhere} \end{cases}$$

(a) Find c.
(b) Find F(y) and compute F(-1), F(0), and F(1).
(c) Find the probability that a student finishes in less than a half hour.
(d) Find the probability that a student finishes in more than 45 minutes.

a. $F(y) = \int_0^y (ct^2 + t)dt = \frac{c}{3}y^3 + \frac{1}{2}y^2$. Set F(1) =1 to find c = 3/2.
b. $F(y) = (y^3 + y^2)/2$. F(-1) = 0 since f(y)=0 for y < 0.
F(0) = 0. F(1) =1.
c. P(Y < ½) = F(1/2) =3/16
d. P(Y > 45 minutes) = 1 – P(Y < 45 min) =1-F(3/4) = $1-[(3/4)^3+(3/4)^2]/2 = 65/128$.

(3) The number of line painting errors per km of a new highway is given by

X	0	1	2	3	4	5
f(x)	0.30	0.38	0.16	0.11	0.03	0.02

What is the probability that X is at least 1 and less than 5?

$P(1 \leqslant X < 5) = f(1) + f(2) + f(3) + f(4) = 0.68$

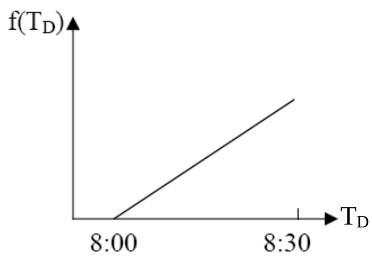
(4) The shelf life, in days, of bottles of a certain prescription medicine is a random variable having the density function $f(x) = 20{,}000/(x+100)^3$ for $x > 0$ and 0 elsewhere. Find the probability that a bottle of medicine will have a shelf life of
a. at least 200 days
b. anywhere from 80 to 120 days.

a. $F(x) = \int_0^x \frac{20{,}000}{(t+100)^3} dt = \left[\frac{-10{,}000}{(t+100)^2} \right]_0^x = 1 - \frac{10{,}000}{(x+100)^2}$

$P(X \geq 200) = 1 - F(200) = 1 - \left[1 - \frac{10{,}000}{(200+100)^2} \right] = 1/9.$

b. $P(80 \leq X \leq 120) = F(120) - F(80) = 10{,}000/180^2 - 10{,}000/220^2 = 0.102$

(5) A person leaves for work between 8:00 am and 8:30 am. The probability density function of his departure time T_D can be represented as shown in the figure below:



Regardless of the time the person leaves for work, it takes that person between 30 and 40 minutes to get to work (T_W), any length of time being equally likely. What is the expected time this person will be at work?

(a) 8:55 am
(b) 8:45 am
(c) 9:15 am
(d) 8:50 am
(e) none of the above

$f(x) = kx$ (straight line from 0) $\quad (X = T_D$ (in min) $- 8{:}00$

$\rightarrow \int_0^{30} F(x)\,dx = 1 \rightarrow \int_0^{30} kx\,dx = 1 \rightarrow k\frac{x^2}{2}\Big|_0^{30} = 1$

$\rightarrow \frac{k}{2}[900] = 1 \rightarrow \boxed{k = \frac{1}{450}} \rightarrow \boxed{f(x) = \frac{x}{450}}$

$\rightarrow E(X) = \int_0^{30} x\,f(x)\,dx \rightarrow E(x) = \int_0^{30} \frac{x^2}{450}\,dx$

$= \frac{1}{450} \times \frac{1}{3} x^3 \Big|_0^{30} = \frac{(30)^3}{450 \times 3} \rightarrow \boxed{E(X) = 20 \text{ min}}$

$\therefore \rightarrow T_D = 8.00 + 20$ min $= 8{:}20$ # (3)

$E(T_W) = T_W = \left(\frac{30+40}{2}\right) = 35$ min

$T_{arrival} = (T_D + T_W) \rightarrow E(T_{Arr}) = E(T_D) + E(T_W)$

$\rightarrow E(T_A) = 8{:}20 + 00{:}35 = 8{:}55$ min # $\rightarrow$ (a)

(6)

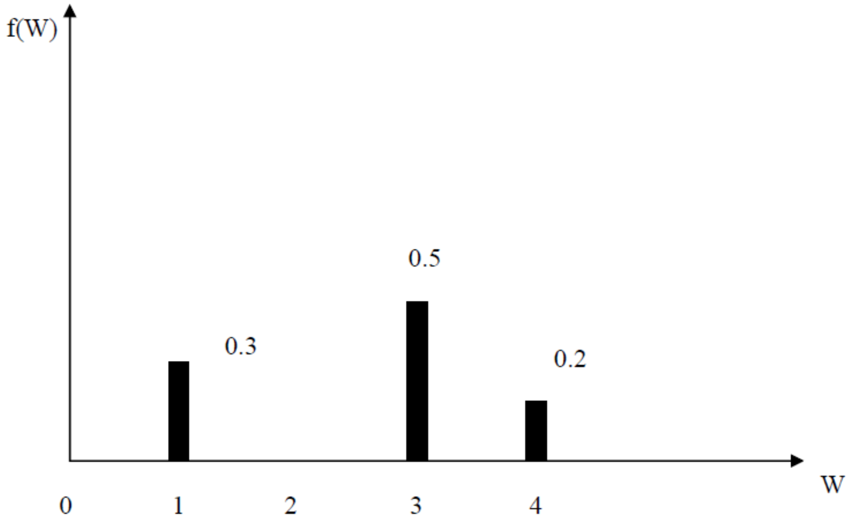


Figure 1

The discrete random variable W has the probability mass function shown in Figure 1. What is the variance of W?

a. 5.40
b. 1.24
c. 1.10
d. 8.00
e. None of the above

$E(W) = 0.3+1.5+0.8=2.6$
$E(W^2) = 0.3+4.5+3.2=8.0$
$\sigma^2_W = 8-(2.6)^2=1.24$

(7) Given a discrete random variable X that has the following probabilities associated with its outcomes:

k	$x=0$
$2k$	$x=1$
$3k$	$x=2$
0	Otherwise

find the variance of X

a. 7/9
b. 2/9
c. 7/3
d. 5/3
e. None of the above

Solution: None of the above, since k=1/6

$$\sigma_x^2 = E\left(x^2\right) - u^2$$

$$E\left(x^2\right) = 0^2\frac{1}{6} + 1^2\left(\frac{1}{3}\right) + 2^2\left(\frac{1}{2}\right) = \frac{7}{3}$$

$$E(x) = \frac{1}{3}(1) + \frac{1}{2}(2) = \frac{4}{3}$$

$$\sigma^2 = E\left(x^2\right) - \left[E(x)\right]^2 = \tfrac{7}{3} - \left(\tfrac{4}{3}\right)^2 = \tfrac{5}{9}$$