Solution to Quiz #4

1. $P(441 < \bar{x} < 446)$

$$= P\left(\frac{441-448}{\frac{21}{\sqrt{49}}} < z < \frac{446-448}{21/\sqrt{49}}\right) \qquad \left[z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}}\right]$$

$$\mu_{\bar{x}} = \mu$$
$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

$$= P(-2.33 < z < -0.667)$$
$$= \phi(-0.67) - \phi(-2.33)$$
$$= 0.2514 - 0.0099 = 0.2415$$

2.

A	B
$n_1 = 16$	$n_2 = 9$
$\sigma_1 = 8$	$\sigma_2 = 12$
$\mu_1 = 75$	$\mu_2 = 70$

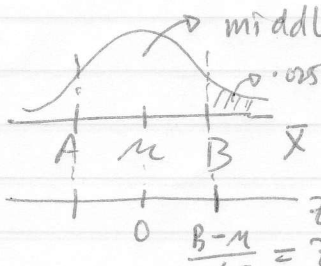
$$\mu_{\bar{x}_1-\bar{x}_2} = \mu_1 - \mu_2 = 5$$
$$\sigma_{\bar{x}_1-\bar{x}_2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} = \sqrt{4+16} = \sqrt{20}$$

$P(\bar{x}_1 - \bar{x}_2 \geq 4)$

$$= P\left[\frac{(\bar{x}_1-\bar{x}_2) - \mu_{\bar{x}_1-\bar{x}_2}}{\sigma_{\bar{x}_1-\bar{x}_2}} \geq \frac{4-5}{\sqrt{20}}\right] = P(z \geq -0.224)$$
$$= 1 - \phi(-0.224)$$
$$= 1 - 0.4129 = 0.5871$$

3.

middle 95% on line segment AB



Area above AB = 0.95

$$\frac{B-\mu}{\sigma/\sqrt{n}} = z_{.025} = 1.96 \Rightarrow B - \mu = 1.96\frac{\sigma}{\sqrt{n}}$$
$$\Rightarrow B = \mu + 1.96\frac{\sigma}{\sqrt{n}}$$
$$= 0.503 + 1.96 \times \frac{.004}{5}$$
$$= 0.5046 \simeq 0.505$$

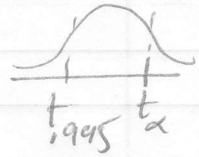
Solution to quiz 4

4. Given $P(-2.977 < T < K) = 0.045$ for $df = \nu = n-1 = 14$

Let $K = t_\alpha$

Also, $-2.977 = -t_{0.05} = t_{0.995}$

Then, $P(t_{0.995} < T < t_\alpha) = 0.045$



$\Rightarrow (1-\alpha) - (1-0.995) = 0.045$

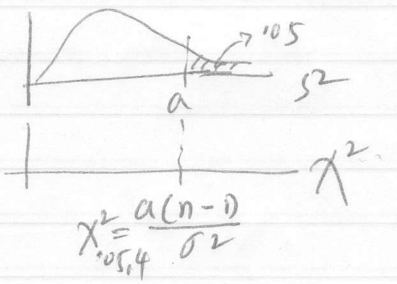
$\Rightarrow 0.995 - 0.045 = \alpha$

$\Rightarrow \alpha = 0.95$

$K = t_\alpha = t_{0.95} = -t_{0.05} = -1.761$

5. $\mu = 3$ yrs, $\sigma = 2.646$ months, $\sigma^2 = \cancel{6.0713}\ 7$ month^2, $n=5$

Let, s^2 = A random variable representing sample variance



$P(s^2 > a) = 0.05$

$\Rightarrow P\left[\frac{s^2(n-1)}{\sigma^2} > \frac{a(n-1)}{\sigma^2}\right] = 0.05$

$\Rightarrow P\left[\chi^2 > \frac{a(n-1)}{\sigma^2}\right] = .05$

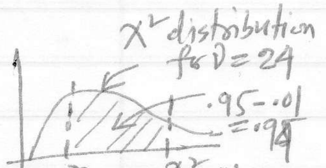
Therefore, $\frac{a(n-1)}{\sigma^2} = \chi^2_{0.05,4} = 9.488$

$\Rightarrow a = \frac{9.488 \times \sigma^2}{4} = \frac{9.488 \times 7}{4} = 16.6$

6. $P(3.463 < s^2 < 10.745)$

$= P\left[\frac{3.463 \times (n-1)}{\sigma^2} < \chi^2 < \frac{10.745 \times (n-1)}{\sigma^2}\right]$

$= P\left[\frac{3.463 \times 24}{6.0025} < \chi^2 < \frac{10.745 \times 24}{6.0025}\right]$



$= P[13.85 < \chi^2 < 42.98] = P[\chi^2_{0.95} < \chi^2 < \chi^2_{0.01}]$ for $\nu = 24$

$= 0.95 - 0.01$

$= \boxed{0.94}$