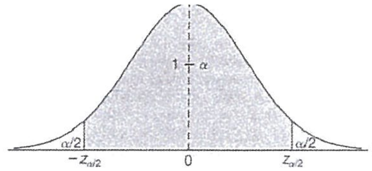
Section #9 (Chapter #9)

Question ~~8~~ #1



$$P(-z_{\alpha/2} < Z < z_{\alpha/2}) = 1 - \alpha$$

$$Z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

$$P\left(\bar{X} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}} < \mu < \bar{X} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right) = 1 - \alpha$$

$$\alpha = 1 - 0.9 = 0.1 \Rightarrow P(Z < -z_{\alpha/2}) = \alpha/2 = 0.05 \Rightarrow z_{\alpha/2} = 1.645$$

$$\sigma_{\bar{X}} = \frac{0.001}{\sqrt{9}}, \bar{X} = 8.05$$

$$8.05 - 1.645\frac{0.001}{\sqrt{9}} < \mu < 8.05 + 1.645\frac{0.001}{\sqrt{9}}$$

$$8.04945 < \mu < 8.05055$$

Question ~~9~~ 2

Similar to question 8, solve it yourself:

$$P\left(\bar{X} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}} < \mu < \bar{X} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right) = 1 - \alpha = 0.95 \Rightarrow \alpha = 0.05$$

$$P\left(\mu - z_{\alpha/2}\frac{\sigma}{\sqrt{n}} < \bar{X} < \mu + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right) = 0.95$$

$$P\left(20 - z_{\alpha/2}\frac{\sqrt{3}}{\sqrt{n}} < \bar{X} < 20 + z_{\alpha/2}\frac{\sqrt{3}}{\sqrt{n}}\right) = 0.95 = P(19.9 < \bar{X} < 20.1)$$

$$\Rightarrow z_{\alpha/2}\frac{\sqrt{3}}{\sqrt{n}} = 0.1, z_{\alpha/2} = z_{0.025} = 1.96 \Rightarrow 1.96\frac{\sqrt{3}}{\sqrt{n}} = 0.1$$

n=1152.48 => n=1153

Question ~~10~~ 3

- a) $\bar{X} = 715, S_X = 1.57$
- b) True standard deviation is unknown!

=> use t-distribution

- $T = \frac{\bar{X}-\mu}{S/\sqrt{n}}$

$$\bar{X} - t_{\alpha/2}\frac{S}{\sqrt{n}} < \mu < \bar{X} + t_{\alpha/2}\frac{S}{\sqrt{n}}$$

$$715 - 2.262 * \frac{1.57}{\sqrt{10}} < \mu < 715 + 2.262 * \frac{1.57}{\sqrt{10}}$$

$$715 - 1.123 < \mu < 715 + 1.123$$

$$713.877 < \mu < 716.123$$

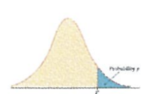


TABLE D

t distribution critical values

	Upper-tail probability p							
df	.25	.20	.15	.10	.05	.025	.02	.
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31
2	0.816	1.061	1.386	1.886	2.920	4.303	4.849	6.
3	0.765	0.978	1.250	1.638	2.353	3.182	3.482	4.
4	0.741	0.941	1.190	1.533	2.132	2.776	2.999	3.
5	0.727	0.920	1.156	1.476	2.015	2.571	2.757	3.
6	0.718	0.906	1.134	1.440	1.943	2.447	2.612	3.
7	0.711	0.896	1.119	1.415	1.895	2.365	2.517	2.
8	0.706	0.889	1.108	1.397	1.860	2.306	2.449	2.
9	0.703	0.883	1.100	1.383	1.833	2.262	2.398	2.
10	0.700	0.879	1.093	1.372	1.812	2.228	2.359	2.

Question ~~10~~ 3

c) According to chi-squared table with df=9

$\alpha = 1 - 0.95 = 0.05$

$$\chi^2_{1-\alpha/2} < \chi^2 = \frac{(n-1)S^2}{\sigma^2} < \chi^2_{\alpha/2}$$

$$\frac{(n-1)s^2}{\chi^2_{\alpha/2}} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_{1-\alpha/2}}$$

$$\chi^2_{1-\alpha/2=0.975} = 2.70, \quad \chi^2_{\alpha/2=0.025} = 19.023$$

$$\frac{9*(1.57)^2}{19.023} < \sigma^2 < \frac{9*(1.57)^2}{2.700} \Rightarrow 1.166 < \sigma^2 < 8.216 \quad 1.080 < \sigma < 2.866$$

Question ~~11~~ 4

- Two different populations $\sigma_1^2 = 1.5$, $\sigma_2^2 = 1.2$
- Two different samples:

$n_1 = 15, \bar{X}_1 = 89.6, \ S_1 = 1.8$

$n_2 = 20, \bar{X}_2 = 92.5, \ S_2 = 1.5$

- $\mu_{\bar{X}_1 - \bar{X}_2} = \mu_1 - \mu_2, \ \sigma^2_{\bar{X}_1 - \bar{X}_2} = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}$
- $Z = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{(92.5 - 89.6) - (\mu_1 - \mu_2)}{\sqrt{\frac{1.2}{20} + \frac{1.5}{15}}} = \frac{(2.90) - (\mu_1 - \mu_2)}{0.40}$
- $\alpha = 1 - 0.99 = 0.01 \Rightarrow P(Z < -z_{\alpha/2}) = 0.005, z_{\alpha/2} = 2.575$

$-2.575 < Z < +2.575 \Rightarrow -2.575 < \frac{(2.90) - (\mu_1 - \mu_2)}{0.40} < +2.575$

$\Rightarrow confidence\ interval \ 2.90 \pm 2.575 * 0.40$

Question ~~12~~ 5

- The true variance is unknown! => t-distribution
- $T = \frac{\bar{X} - \mu}{S/\sqrt{n}}$
- The mean of sample $\bar{X} = 1.35, S = 0.3385$
- According to t-distribution table:

df=16−1=15 → $t_{\alpha/2} = t_{0.025} = 2.131$

$$\bar{x} - t_{\alpha/2}\frac{s}{\sqrt{n}} < \mu < \bar{x} + t_{\alpha/2}\frac{s}{\sqrt{n}}$$

$$1.35 - 2.131 * \frac{0.3385}{\sqrt{16}} < \mu < 1.35 + 2.131 * \frac{0.3385}{\sqrt{16}}$$

$$1.35 - 0.1803 < \mu < 1.35 + 0.1803$$

$$1.170 < \mu < 1.530$$

Question ~~13~~ ~~7~~ 6

- Sample information: $\bar{X} = 17, S = 0.319$
- $T = \frac{\bar{X}-\mu}{S/\sqrt{n}}$ why ?
- $df = 6 - 1 = 5, \rightarrow t_{\alpha/2} = t_{0.01} = 3.365$

$$\bar{x} - t_{\alpha/2}\frac{s}{\sqrt{n}} < \mu < \bar{x} + t_{\alpha/2}\frac{s}{\sqrt{n}}$$

$$17 - 3.365 * \frac{0.319}{\sqrt{6}} < \mu < 17 + 3.365 * \frac{0.319}{\sqrt{6}}$$

$$173.365 * 0.130 < \mu < 17 + 3.365 * 0.130$$

$$16.562 < \mu < 17.438$$

Question ~~14~~ ~~8~~ 7

- Paired observations
- $d = \mu_1 - \mu_2$, $T = \frac{\bar{d}-\mu_D}{S_d/\sqrt{n}}$
- $\bar{d}$ = mean (commutator-Pinion) = -4.18
- S_d= STD (commutator-Pinion) = 35.85
- $df = 17 - 1 = 16, t_{\alpha/2} = t_{0.025} = 2.120$

$$\bar{d} - t_{\alpha/2}\frac{s_d}{\sqrt{n}} < \mu_D < \bar{d} + t_{\alpha/2}\frac{s_d}{\sqrt{n}}$$

$interval\ for\ \mu_D : -4.18 \pm 2.120 \times 35.85/\sqrt{17}$

Question ~~15~~ 8

- $\sigma_{standard} = 0.004,\ df_{standard} = 24$,
- $\sigma_{alloyed} = 0.005,\ df_{alloyed} = 15$
- $F_{\alpha/2=0.05}(24,15) = 2.29$
- $F_{\alpha/2=0.05}(15,24) = 2.11$

$$\frac{s_1^2}{s_2^2}\frac{1}{f_{\alpha/2}(df_1, df_2)} < \frac{\sigma_1^2}{\sigma_2^2} < \frac{s_1^2}{s_2^2} f_{\alpha/2}(df_2, df_1)$$

$$\Rightarrow \frac{(0.004)^2}{(0.005)^2}\frac{1}{2.29} < \frac{\sigma_{stand}^2}{\sigma_{alloyed}^2} < \frac{(0.004)^2}{(0.005)^2} \times 2.11$$

$$0.279 < \frac{\sigma_{stand}^2}{\sigma_{alloyed}^2} < 1.350$$

Question ~~16~~ ~~10~~ 9

See section 09 page 37:

$$\hat{p} - z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}} < p < \hat{p} + z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$0.01 \pm 1.96 * \sqrt{\frac{0.01*0.99}{500}} = 0.01 \pm 1.96*0.00445$$

$$0.0013 < p < 0.0187$$