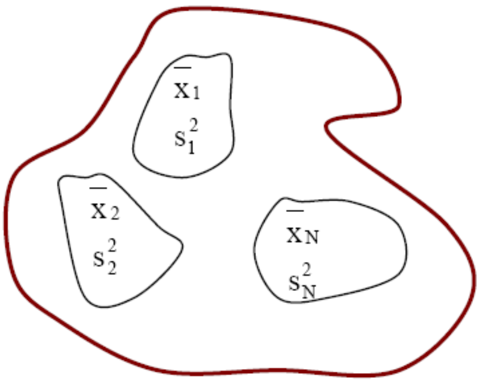
Chapter 8 (Fundamental Sampling Distributions and Data Descriptions)

Population with mean μ and variance σ^2

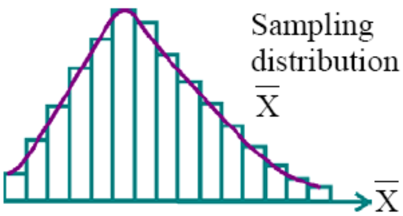


$\bar{x}_1, \bar{x}_2, \ldots \bar{x}_N$ are observed means of samples of size n, or alternatively, are observations of the random variable $\bar{X}$

$$\boxed{\bar{X} = \frac{\sum_{i=1}^{n} X_i}{n}}$$

$s_1^2, s_2^2, \ldots s_N^2$ are the variances of samples of size n, or alternatively, are observations of the random variable S^2

$$\boxed{S^2 = \frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n-1} = \frac{n\sum_{i=1}^{n} X_i^2 - \left(\sum_{i=1}^{n} X_i\right)^2}{n(n-1)}}$$



- A sample is a subset of a population.
- A population consists of the totality of the observations with which are concerned.
- Statistics are measures of a random sample.

Draw Conclusions About μ

If population of X is normal, then the sampling distribution of the mean $\bar{X}$ will be normal and $\mu = \mu_{\bar{X}}$ and $\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$

Central Limit Theorem → Regardless of original population distribution, if n ≥ 30, and σ is known, then $Z = \frac{\bar{X} - \mu_{\bar{X}}}{\sigma_{\bar{X}}}$ follows a standard normal distribution. Also applies to, $Z = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\left(\frac{\sigma_1^2}{n_1}\right) + \left(\frac{\sigma_2^2}{n_2}\right)}}$ and

$\mu_{\bar{X}_1 - \bar{X}_2} = \mu_1 - \mu_2$ and $\sigma^2_{\bar{X}_1 - \bar{X}_2} = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}$

If σ is unknown, then use the **t-distribution** with ν = n-1 degrees of freedom and $T = \frac{\bar{X} - \mu}{S/\sqrt{n}}$ and the original population must be normal.

Draw Conclusions About σ

Need to describe the sampling distribution of the variance S^2. Note that if original population of X is normal, then $\chi^2 = \frac{(n-1)s^2}{\sigma^2}$ follows a **chi-squared distribution** with $\upsilon = n - 1$ degrees of freedom.

If you want to compare two sample variances, then use the **f-distribution** where $F = \frac{(S_1^2/\sigma_1^2)}{(S_2^2/\sigma_2^2)}$ with $\nu_1 = n_1 - 1$, $\nu_2 = n_2 - 1$ and original population must be normal. Note that $f_{1-\alpha}(\nu_1, \nu_2) = 1/[f_\alpha(\nu_2, \nu_1)]$