Chapter 9 Questions:

1) $n = 100$ Boxes
$\bar{x} = 12.05$
$S = 0.1$

85 % Confidence Interval
$1 - \alpha = 0.85$
$\alpha = 0.15 \Rightarrow \alpha/2 = 0.075$

Z-Table:
$z_{0.075} = 1.44$

We approximate:
$\sigma_{\bar{x}} \approx s/\sqrt{n} = 0.01$

so, the 85 % confidence Interval is

Range: $\bar{x} \pm z_{\alpha/2} \cdot \frac{s}{\sqrt{n}} = 12.05 \pm (1.44)(0.01)$
$\Rightarrow [12.0356, 12.0644]$

2) $n = 50$
$\bar{x} = 12.68$
$S = 6.83$

(i) 95% CI
$1 - \alpha = 0.95 \Rightarrow \alpha = 0.05$
$\alpha/2 = 0.025$

z-table:
$z_{0.025} = 1.96$

we approximate:
$\sigma_{\bar{x}} \approx s/\sqrt{n}$
$= 0.9659$

Range: $12.68 \pm (1.96)(0.9659)$
$\Rightarrow [10.79, 14.57]$

(ii) 80 % CI
$1 - \alpha = 0.80 \Rightarrow \alpha = 0.20$
$\alpha/2 = 0.10$

$z_{0.10} = 1.28$

Range: $12.68 \pm (1.28)(0.9659)$
$\Rightarrow [11.44, 13.92]$

(iii) Range: $[11.09, 14.27]$
CI = ?

upper limit = 14.27; so $14.27 = 12.68 + z_{\alpha/2} \cdot \left(\frac{6.83}{\sqrt{50}}\right)$
$z_{\frac{\alpha}{2}} = 1.646$
$\Rightarrow$ from z-table $\frac{\alpha}{2} = 0.05$
$\alpha = 10\%$

so, $CI = 1 - \alpha = 90\%$

3) 90% CI

Range: [14.73, 14.91]

The probability that the mean diameter of rods manufactured by this process is between the range is 90%?

↳ False. (The mean is either in the interval or it isn't. The term probability is inappropriate).

4) 90% CI

$\bar{X} \pm 1.645 s/\sqrt{n}$

$1-\alpha = 0.90 \Rightarrow \alpha = 0.10 \Rightarrow \alpha/2 = 0.05$

z table: $z_{0.05} = 1.645$ so, True

5) $n = 250$ 95% CI

- Since the weighting procedure is unbiased, the true weight of a rock is equal to the population mean of its measurements.

- We may think of each 250 confidence intervals as a Bernoulli trial.

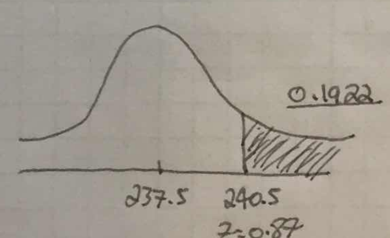
↳ the success probability for each trial is 0.95.

let, Y represent the number of CI's that cover the true weight.

so, $Y \sim Bin(250, 0.95) \approx N(237.5, 11.875)$

$\sigma = \sqrt{11.875} = 3.45$

- Using normal curve, the probability that $Y > 240$ is 0.1922.



6) CI width: ± 0.012

$\bar{x} = 12.05$
$s = 0.1$

CI = 90%
$\alpha = 10\%$
$\alpha/2 = 5\%$ $\Rightarrow$ $Z_{0.05} = 1.645$

$$1.645 \cdot \frac{(0.1)}{\sqrt{n}} = 0.012$$

$\Rightarrow$ $n = 188$ for CI 90%

$n = 463$ for CI 99%

7) 95% CI, CI width: ± 0.5
$s = 6.83$
$1-\alpha = 0.95$ $\Rightarrow$ $\alpha = 0.05$
$\alpha/2 = 0.025$ $\Rightarrow$ $Z_{0.025} = 1.96$

so,
$$1.96 \cdot \frac{(6.83)}{\sqrt{n}} = 0.5$$

$\Rightarrow$ $n = 717$

8) $n = 50$
$\bar{x} = 174.5$ $\quad$ <u>σ unknown</u>
$s = 6.9$
98% CI $\Rightarrow$ $\alpha = 2\%$
$\alpha/2 = 0.01$ $\Rightarrow$ t-table: $t_{0.01, 49} = 2.4$

CI: $\left[\bar{x} - t_{\frac{\alpha}{2}, n-1} \cdot \frac{s}{\sqrt{n}}, \bar{x} + t_{\frac{\alpha}{2}, n-1} \cdot \frac{s}{\sqrt{n}} \right]$

$\left[174.5 - (2.4)\frac{(6.9)}{\sqrt{50}}, 174.5 + (2.4)\frac{(6.9)}{\sqrt{50}} \right] = [172.16, 176.84]$

9) 95% CI $\quad$ width: $+15$
$\sigma = 40$
$\alpha = 5\%$ $\Rightarrow$ $\alpha/2 = 0.025$ $\Rightarrow$ $Z_{0.025} = 1.96$

CI: $\left[\bar{x} - Z_{\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}}, \bar{x} + Z_{\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}} \right]$ $\Rightarrow$ $Z_{\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}} = 15$

$$(1.96)\frac{(40)}{\sqrt{n}} = 15$$

$n \approx 28$

10) 95% CI $\quad$ $\underline{\sigma \text{ unknown}}$ $\quad$ $t_{0.025,14} = 2.145$

$\alpha = 5\% \Rightarrow \alpha/2 = 0.025$

$\bar{X} = 3.787$

$s^2 = 0.94$

$n = 15$

$$CI: \left[3.787 - t_{0.025,14} \cdot \sqrt{\frac{0.94}{15}},\ 3.787 + t_{0.025,14} \cdot \sqrt{\frac{0.94}{15}} \right]$$

$$CI: [3.25, 4.32]$$

11) $\bar{X}_A = 78.3$, $\underline{\sigma_A = 5.6}$, $n_A = 50$ $\quad$ $\underline{\sigma_A, \sigma_B \text{ known}}$

$\bar{X}_B = 87.2$, $\underline{\sigma_B = 6.3}$, $n_B = 50$ $\quad$ 95% CI

$z_{0.025} = 1.96$

$$(\bar{X}_1 - \bar{X}_2) \pm z_{\frac{\alpha}{2}} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$(87.2 - 78.3) \pm (1.96) \cdot \sqrt{\frac{(5.6)^2}{50} + \frac{(6.3)^2}{50}}$$

$$CI: [6.56, 11.24]$$

12) $n = 20$ $\quad$ 98% CI for σ^2

$\bar{x} = 72$

$s^2 = 16$

$$CI \text{ for } \sigma^2: \left[\frac{(n-1)s^2}{\chi^2_{0.99,19}},\ \frac{(n-1)s^2}{\chi^2_{0.01,19}} \right]$$

$\chi^2_{0.99,19} = 36.19$

$\chi^2_{0.01,19} = 7.633$

$$\left[\frac{(19)(16)}{36.19},\ \frac{(19)(16)}{7.633} \right] = [8.4, 39.83]$$

13) 90% CI for $\frac{\sigma_1^2}{\sigma_2^2}$

A: $\bar{X}_1 = 36{,}300$
$S_1 = 5000$

$F_{0.05,11,11} = 2.8$ (F-table)

B: $\bar{X}_2 = 38{,}100$
$S_2 = 6100$

$$CI \text{ for } \sigma_1^2/\sigma_2^2: \left[\frac{S_1^2}{S_2^2} \cdot \frac{1}{F_{\frac{\alpha}{2}, n_1-1, n_2-1}},\ \frac{S_1^2}{S_2^2} \cdot F_{\frac{\alpha}{2}, n_2-1, n_1-1} \right]$$

$$\left[\frac{(5000)^2}{(6000)^2} \cdot \frac{1}{2.8},\ \left(\frac{5000}{6000}\right)^2 \cdot 2.8 \right] = [0.24, 1.88]$$

We can justify the assumption $\sigma_1^2 = \sigma_2^2$.

14) $n = 27$ $\alpha = 5\%$, $\alpha/2 = 2.5\%$
$s = 5.86$

95% CI for σ^2: $\left[\frac{(n-1)s^2}{\chi^2_{1-\frac{\alpha}{2}, n-1}} , \frac{(n-1)s^2}{\chi_{\frac{\alpha}{2}, n-1}} \right]$

$\chi^2_{0.975, 26} = 41.92$ $\quad \left[\frac{26(5.86)^2}{41.92} , \frac{26(5.86)^2}{13.89} \right]$

$\chi^2_{0.025, 26} : 13.89$

$[20.86, 64.51]$

CI for σ : $[4.57, 8.03]$

15) group 1: $n_1 = 11$, $s_1 = 5.8$ $\alpha = 5\%$, $\alpha/2 = 2.5\%$

group 2: $n_2 = 4$, $s_2 = 3.4$

95% CI for $\frac{\sigma_1^2}{\sigma_2^2}$: $\left[\frac{(5.8)^2}{(3.4)^2} \cdot \frac{1}{F_{\frac{\alpha}{2}, 10, 3}} , \frac{(5.8)^2}{(3.4)^2} \cdot \frac{1}{F_{1-\frac{\alpha}{2}, 10, 3}} \right]$

$F_{0.025, 10, 3} = 14.42$ $\quad \left[\left(\frac{5.8}{3.4}\right)^2 \cdot \frac{1}{14.42} , \left(\frac{5.8}{3.4}\right)^2 \cdot \frac{1}{0.21} \right]$

$F_{0.975, 10, 3} = 0.21$

$[0.20, 14.06]$

16) $\bar{x}_1 = 4.5$, $s_1^2 = 4.9$, $n_1 = 12$ $\quad$ σ_1 & σ_2 unknown

$\bar{x}_2 = 3.4$, $s_2^2 = 5.6$, $n_2 = 15$ $\quad$ Assume $\sigma_1 = \sigma_2$

$\alpha = 5\%$, $\frac{\alpha}{2} = 2.5\%$, $t_{0.025, 25} = 2.056$

95% CI for $\mu_1 - \mu_2$:

$$\left[\bar{x}_1 - \bar{x}_2 - t_{\frac{\alpha}{2}, n_1+n_2-2} \cdot S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} , \bar{x}_1 - \bar{x}_2 + t_{\frac{\alpha}{2}, n_1+n_2-2} \cdot S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \right]$$

$$S_p^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2} = \frac{(11)(4.9) + (14)(5.6)}{12+15-2} = 5.29$$

$$\left[1.1 - (2.056)(\sqrt{5.29})\left(\sqrt{\frac{1}{12} + \frac{1}{15}}\right), 1.1 + (2.056)(\sqrt{5.29})\left(\sqrt{\frac{1}{12} + \frac{1}{15}}\right) \right.$$

$[-0.73, 2.93]$

17) $n_1 = 15$ $\quad$ $n_2 : 12$ $\quad$ 95% CI

$\bar{x}_1 = 3.84$ $\quad$ $\bar{x}_2 = 1.49$ $\quad$ σ_1 & σ_2 unknown

$s_1 = 3.07$ $\quad$ $s_2 = 0.8$ $\quad$ $\sigma_1 \neq \sigma_2$

$df = 16.3$

$t_{0.025, 16.3} = 2.12$

$$(\bar{x}_1 - \bar{x}_2) \pm t_{\frac{\alpha}{2}} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$$(3.84 - 1.49) \pm (2.12) \sqrt{\frac{(3.07)^2}{15} + \frac{(0.8)^2}{12}}$$

$$[0.6, 4.1]$$

18) Existing : $75/1500 = 0.05$ $\quad$ $p_1 = 0.05$ $\quad$ $n_1 = 1500$

$p_2 = 0.04$ $\quad$ $n_2 = 2000$

New : $80/2000 = 0.04$

$p_1 - p_2 = 0.01$

90% CI , $z_{0.05} = 1.645$

$\alpha = 10\%$

$\alpha/2 = 5\%$

$$(p_1 - p_2) - z_{\frac{\alpha}{2}} \sqrt{\frac{p_1 q_1}{n_1} + \frac{p_2 q_2}{n_2}} < p_1 - p_2 < (p_1 - p_2) + z_{\alpha/2} \sqrt{\frac{p_1 q_1}{n_1} + \frac{p_2 q_2}{n_2}}$$

$$0.01 \pm 1.645 \sqrt{\frac{(0.05)(0.95)}{1500} + \frac{(0.04)(0.96)}{2000}}$$

$$[-0.0017, 0.0217]$$