Tutorial # 5 (Chapter 5): Discrete distributions

Question # 1. What is more likely to win when playing against an equally strong opponent: (a) three games out of four, or five games out of eight? (b) at least three games out of four, or at least five games out of eight? [Hint: (a) Find the probability that a player will win (i) 3 out of 4 games, (ii) 5 out of 8 games and compare the probabilities]

Question # 2. A production process outputs items in lots of 50. The current inspection plan is to periodically sample randomly 10 out of 50 items in a lot.

(a) Suppose in a lot randomly chosen, 2 out of 50 are defective. What is the probability that at least 1 in the sample of 10 from the lot is defective?

(b) What is the mean number of defectives found out of 10 items sampled?

(c) Recompute part (a) using binomial distribution. Comment.

Question # 3. An oil drilling company ventures into various locations, and its success or failure is independent form one location to another. Suppose the probability of a success at any specific location is 0.25.

(a) What is the probability that the driller drills at 10 locations and has 1 success?

(b) (i)The dealer will go bankrupt if it drills 15 times before the first success occurs. What is the probability that the driller has no success in the first 10 drills? (ii) Also find the chance that the driller will have the first success at the 11^{th} location if they drill 11 locations?

(c) The drilling company thinks that it will “hit it big” if the second success occurs on the sixth attempt. What is the probability that the driller will hit big?

Question # 4. A local drugstore owner knows that, on average, 100 people enter the store each hour.

(a) Find the probability that in a given 3-minute period nobody enters the store

(b) Find the probability that in a given 3-minute period more than 5 people enter the store.

Question # 5 On average, 5.4 calls are received in two minutes in a service center. Find the probability that (a) no more than 4 calls come in any minute

(b) fewer than 2 calls come in any minute

(c) fewer than 2 calls come in a 5-minute period

Question # 6. Five individuals from an animal population thought to be near extinction in a certain region have been caught, tagged, and released to mix into the population. After they had an opportunity to mix, a random sample of 10 of these animals is selected. Let X = number of tagged animal in the second sample. If there are actually 25 animals of this type in this region, what is the probability that (a) $X = 2$, (b) $X \leq 2$?

Question # 7. At a given segment of a ship’s route, the wave climatology is such that two out of sixty days of the season are characterized by wave conditions that might be dangerous to her safety. Based on the ship’s speed, it is envisioned that she will be in the dangerous region of the ocean for half a day. What is the probability that the ship will encounter dangerous wave conditions? [**Hint**: In total we have 120 ‘half-day’ in the season and 4 of those half-days are dangerous. Find the probability that the ‘half-day’ stay of ship in the dangerous segment of the path falls in those unsafe 4 ‘half-day’s of the season][Ans: 0.033]

Question # 8. The outer space particles hit the exposed surface area of satellites at an average rate of 2 particles per cm^2 of exposed surface. Consider a satellite which has 2 cm^2 exposed area where particles may hit. If no particles hit, the satellite will not fail. If one particles hit, the probability that the satellite fails is p =0.2 . The satellite fails if two or more of the particles hit the satellite. Find the probability that the satellite will fail. [**Hint**: Define: event H_0 = no hit, H_1= hit by one particle, H_2= hit by two or more particles. Find $P(H_0)$, $P(H_1)$, $P(H_2)$. Define A= satellite fails. Find P(A) using the law of total probability] [Ans: 0.922]

Solution:

Let X = number of hit on the surface area of 2 cm^2. X follows Poisson distribution with mean = 2*2 =4.

Then $P(H_0)$ = exp(-4)*4^0/0! = 0.0183

Then $P(H_1)$ = exp(-4)*4^1/1! = 0.0183*4=0.0733

Then $P(H_2) = 1- P(H_0) - P(H_1) = 0.908$

P (A) = P(A∩no hit) + P(A∩one hit) + P(A∩two or more hit) = $P(A\cap H_0) + P(A\cap H_1) + P(A\cap H_2)$

= $P(H_0) * P(A | H_0) + P(H_1) * P(A | H_1) + P(H_2) * P(A | H_2)$ = 0 + 0.0733*0.2 + 0.908*1 = **0.923**

Question # 9. A number of attempts were made to start an engine. Each attempt is independent of the other attempts. The probability that the attempt is successful is 0.7. The duration of each attempt is 1 minute. What is the probability that it will take 3 minutes to start the engine? [Ans: .063]

Question # 10. A quality inspector at a glass manufacturing company inspects sheets of glass to check for any slight imperfections. Suppose that the expected number of flaws per sheet is only 0.5. (a) What percentage of sheets are flawless? (b) Sheets with two or more flaws are scrapped by the company. What percentage of sheets are scrapped and recycled? (c) If X is the random variable that gives the number of flaws in a randomly chosen sheet, and it is given that P(X>t) =0.001, calculate the value of t by trial and error. [Ans: (a) 60.7% (b) 9% (c) 3]

Question # 11. A river dam has a projected life of 50 years. What is the probability that a 100-year flood will occur during its lifetime? [Hint: A 100-year flood is a flood that occurs once in 100 year.][Ans: 0.306]

Solution: The chance for the flood to occur in a random year = 0.01. Hence the probability that the flood will occur in any random year of the total fifty years = (50 choose 1)*(0.01)^1 * (0.99)^49= 0.31.

Question # 12. There are 5 devices in a box. Three of them are sound and 2 of them are faulty. One takes at random 3 devices. Find the mean and variance of the number X of the sound devices among those that were taken out? [Ans: mean = 1.8, variance = 0.36]

References:

(1) Probability and Statistics for Engineers and Scientists, 9 th edition, Walpole, Myers, Myers, Ye, Pearson
(2) Applied Probability for Engineers and Scientists, Ephraim Suhir, McGraw Hill