Ch 10 Soln: - Part 1

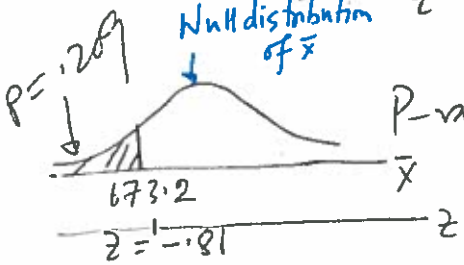
Q1. $H_0: \mu \geq 675$ $(\mu = 675)$

$H_1: \mu < 675$

$\bar{X} = 673.2,\ s = 14.9,\ n = 45$

$n \geq 30$, use z-test, $\sigma \simeq s = 14.9$

$$z = \frac{\bar{X} - \mu}{s/\sqrt{n}} = \frac{673.2 - 675}{14.9/\sqrt{45}} = -0.81$$



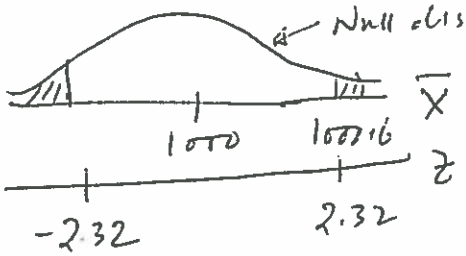
P-value $= P(z^* < -0.81) = .209 > .05$

H_0 is ~~not~~ credible. ~~Reject H_0.~~

Do not reject H_0

Q2. $H_0: \mu = 1000$, $H_1: \mu \neq 1000$

$\bar{X} = 1000.6$, $\sigma \simeq s = 2$, $z = \frac{1000.6 - 1000}{2/\sqrt{60}} = 2.32$ ($2/\sqrt{60} \approx .258$)



$P = 2 * P(z > 2.32)$

$= 2 * .0102$

$= .0204 < .05$

P is small, H_0 is less credible.

Reject H_0.

Q3-5: Please see hints provided in problem sheet.

Q6. P-val $= .209 < 0.25$, So, reject H_0 at 25% level

P-val $= .209 > .05$, So, we do not reject at 5% level.

Q7. t-test

Rejection reg: $|t| > t_{\alpha/2, n-1}$

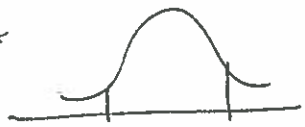
$t_{\alpha/2, n-1} = t_{.025, 59} = 2$

$\Rightarrow \frac{\bar{X} - 1000}{.258} = \pm 2$

$\Rightarrow \bar{X} = 1000 \pm 2(.258)$

$= 1000 \pm .516 = 1000.516$ or 999.484

z-test



$z_{\alpha/2} = 1.96$

$\frac{\bar{X} - 1000}{.258} = \pm 1.96$

$\Rightarrow \bar{X} = 1000 \pm 1.96 \times .258$

$= 1000.51, 999.49$

Q8
10.15

(a) $\mu = 200$, $n = 9$, $\sigma = 15$,

Acceptance region: $191 < \bar{x} < 209$ (satisfactory)

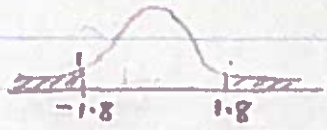
$H_0: \mu_0 = 200$, $H_a: \mu_0 \neq 200$

α = (Reject H_0 when $\mu_0 = 200$)

$= 1 - P(191 < \bar{x} < 209 \text{ when } \mu_0 = 200)$

$= 1 - P\left(\frac{191-200}{\frac{15}{3}} < \bar{z} < \frac{209-200}{15/3}\right)$

$= 1 - P(-1.8 < z < 1.8)$



$= 2P(z < -1.8) = 2 \times 0.359 = 0.0718$

(b) β = P('Do not reject H_0' when H_0 is false)

$= P(191 < \bar{x} < 209 \text{ when } \mu_{true} = 215)$

$= P\left(\frac{191-215}{5} < z < \frac{209-215}{5}\right)$

$= P(-4.8 < z < -1.2)$

$= 0.1151 - 0 = 0.1151$

Q9
10.17

$H_0: \mu = 5000$

$H_a: \mu < 5000$, Critical region: $\bar{x} < 4970$

(a) $\alpha = P(\bar{x} < 4970 \text{ when } \mu_0 = 5000)$

$= P\left(z < \frac{4970-5000}{120/\sqrt{50}}\right) = P(z < -1.7)$

$= 0.0384$

(b) If $\mu_{true} = 4970$

$\beta = P(\bar{x} \geq 4970 \text{ when } \mu_t = 4970)$

$= P(z \geq 0) = 0.5$

If $\mu_{true} = 4960$, $\beta = P(z > 0.59) = 0.2776$

0 → 0.5

7

Q10
10.19 $H_0: \mu = 40$ months

$H_a: \mu < 40$ months, $n = 64$, $\bar{x} = 38$, $s = 5.8$,

$$z = \frac{38 - 40}{\frac{5.8}{\sqrt{64}}} = -2.76$$

P-value $= P(z \leq -2.76) = 0.0029 < 0.05$

Decision: Reject H_0.

10.21 Q11 $H_0: \mu = 800$

$H_a: \mu \neq 800$

$$z = \frac{788 - 800}{44/\sqrt{30}} = -1.64,$$

P-value $= P(z \geq 1.64) + P(z \leq -1.64)$

$= 2P(z \leq -1.64) = 2 * 0.0505 = 0.101 > 0.05$

Decision: Do not reject H_0.

10.23 Q12 $H_0: \mu = 10$; $H_a: \mu \neq 10$

$\alpha = 0.01$, $n = 10$, $\bar{x} = 10.06$, $t_{.005,9} = 3.25$, $s = 0.246$

Critical region: $t < -3.25$ or $t > 3.25$

Computation: Observed $t = (10.06 - 10)/(.246/\sqrt{10})$

$= 0.77$, inside acceptance region.

Decision: Fail to reject H_0

10.48 Q13 From Exercise 10.19, $H_0: \mu = 40$, $H_a: \mu < 40$

Given, $\sigma = 5.8$, $\mu_{true} = 35.9$, $\beta = 0.1$,

For one-sided test, $n \simeq (z_\alpha + z_\beta)^2 \sigma^2 / \delta^2$

Assume $\alpha = 0.05$, $z_\alpha = z_{.05} = 1.645$, $\delta = 35.9 - 40 = -4.1$,

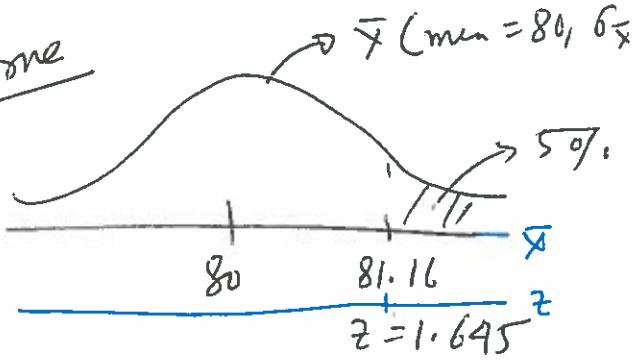
$z_\beta = z_{.01} = 1.28$

$$n = \frac{(1.645 + 1.28)^2}{(-4.1)^2} = 17.12$$

Hilbray

Q14

Ho true

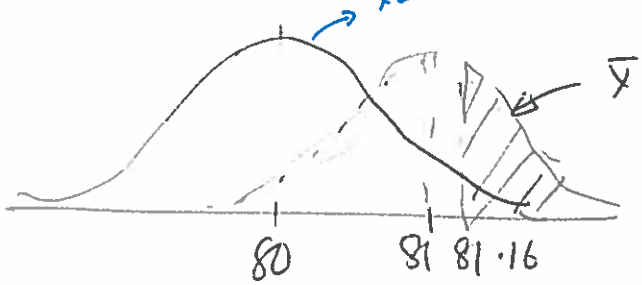


$$\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \geq z_{.05} = 1.645$$

$$\Rightarrow \bar{X} \geq 80 + \frac{5}{\sqrt{50}} \times 1.645$$

$$81.16$$

Ha true



$z_0 = 1.645$

$z_1 = 0.23$

$$z_1 = \frac{81.16 - 81}{.707} = .23$$

Area to the right of .23, $P(z > .23)$

$= .409 = \beta$

~~2.40%~~

too low!