MATH 213 – ASSIGNMENT 4

DUE MONDAY, DECEMBER 7 IN CLASS.

(1) Let A be a 3×4 matrix and suppose that

$$N(A) = \operatorname{span}\left\{ \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \right\}.$$

Let

$$B = \begin{pmatrix} A & A \end{pmatrix} \quad \text{and let} \quad C = \begin{pmatrix} A \\ A \end{pmatrix}.$$

Compute the dimensions of $C(X)$, $R(X)$, $N(X)$ and $N(X^T)$ for $X = A$, B, or C.

(2) (a) Let $\mathbf{u}$ and $\mathbf{v}$ be nonzero vectors in $\mathbf{R}^{n\times 1}$. What is the size of the matrix $\mathbf{u}\mathbf{v}^T$? What is the rank of the matrix $\mathbf{u}\mathbf{v}^T$?

(b) Find vectors $\mathbf{u}$ and $\mathbf{v}$ in $\mathbf{R}^{3\times 1}$ such that

$$\mathbf{u}\mathbf{v}^T = \begin{pmatrix} 1 & 3 & 7 \\ -2 & -6 & -14 \\ 0 & 0 & 0 \end{pmatrix}.$$

(c) Show that if A is an $n \times n$ matrix of rank 1 then there is are vectors $\mathbf{u}$ and $\mathbf{v}$ in $\mathbf{R}^{n\times 1}$ such that $A = \mathbf{u}\mathbf{v}^T$.

(3) Let P_2 be the space of polynomials of degree ≤ 2 and consider the transformation

$$T : P_2 \longrightarrow \mathbf{R}^3 \quad \text{given by} \quad T(f) = \begin{pmatrix} f(2) \\ f'(2) \\ f''(2) \end{pmatrix}$$

(The $'$ means derivative.)

(a) Show that T is linear. Feel free to invoke properties of the derivative that you know from calculus.

(b) Let $B = (1, x, x^2)$ be the standard basis of P_2. Find a 3×3 matrix A such that

$$T(f) = A[f]_B \quad \text{for all } f \in P_2.$$