MATH 213 – LAB 1

1. Terminology

- The leftmost nonzero entry in a row is called its *leading entry*. If this entry is 1 it is called a *leading 1*.

- A matrix is in *row echelon form* if
  (1) All zero rows are at the bottom of the matrix.
  (2) All nonzero rows have leading 1s.
  (3) A leading 1 lies to the right of the leading 1s in all the rows above it.

- An *elementary row operation* on a matrix is an operation of one of the following three types:
  (1) Swap two rows.
  (2) Multiply or divide a row by a nonzero number.
  (3) Add or subtract a multiple of one row to a different row.

- We say that a matrix A is *row equivalent* to a matrix B if A can be transformed into B by a sequence of elementary row operations.

Theorem: There is a sequence of elementary row operations that transforms a matrix into row echelon form.

2. Exercises

(1) Which of the following matrices are in row echelon form?

(a) $\begin{pmatrix} 1 & 3 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}$ (c) $\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \end{pmatrix}$ (e) $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

(b) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 4 & 0 \end{pmatrix}$ (d) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ (f) $\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$

(2) For each matrix in (1) that isn't in row echelon form, find a single elementary row operation transforming it into row echelon form.

(3) Let

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}.$$

Apply the following sequence of elementary row operations to A. (That is, apply (i) to A, apply (ii) to the resulting matrix, apply (iii) to the result of that...)

(i) Subtract $4 \times$ (row 1) from row 2.
(ii) Subtract $7 \times$ (row 1) from row 3.
(iii) Subtract $2 \times$ (row 2) from row 3.
(iv) Divide row 2 by -3.
(v) Subtract $2 \times$ (row 2) from row 1.

The matrix you get should be in row echelon form. Are any of the intermediate matrices in your calculation also in row echelon form? What does that say about the uniqueness of row echelon form?

(4) Consider the system of equations:

$$\begin{array}{rcrcl} x & + & 2y & = & 3 \\ 4x & + & 5y & = & 6 \\ 7x & + & 8y & = & 9 \end{array}$$

(a) Write down the augmented matrix of the system.
(b) Solve the system. (Hint: Use (3).)

(5) The elementary row operation "add $3 \times$ (row 1) to row 2" transforms a matrix A into the matrix

$$B = \begin{pmatrix} 1 & -2 & 5 & -7 \\ 0 & \frac{1}{2} & 1 & 1 \end{pmatrix}.$$

(a) Find A.
(b) Find an elementary row operation that transforms B back into A.

(6) Consider the following sequence of elementary row operations:

(i) Swap rows 1 and 3.
(ii) Divide row 1 by 5.
(iii) Add $2 \times$ (row 1) to row 3.

Suppose that applying the sequence of operations (i), (ii) and (iii) to a matrix A transforms it into

$$B = \begin{pmatrix} 1 & 0 & 3 \\ 0 & 5 & 0 \\ -3 & 1 & -2 \end{pmatrix}.$$

(a) Find A.
(b) Find a sequence of three elementary row operations that transforms B back into A.

(7) Explain why every elementary row operation can be undone by applying another elementary row operation of the same type. Conclude that if A is row equivalent to B then B is row equivalent to A.

(8) ($*$) Why do we include the word "different" in the description of the type (3) elementary row operation? (That is, why don't we allow adding a multiple of a row to itself?)