MATHEMATICS 271 FALL 2015
ASSIGNMENT 1

Due at 12:00 noon on Friday, October 2, 2015. Please hand in your assignment to Mark Girard at the beginning of the lab on October 2. Assignments must be understandable to the marker (i.e., logically correct as well as legible), and must be done by the student in his / her own words. Answer all questions, but only one question per assignment will be marked for credit. Please make sure that: (i) the cover page has **only** your student ID number and your instructor's name, (ii) your name and ID number are on the top right corners of **all** the remaining pages, and (iii) your assignment is **STAPLED**.

Marked assignments will be handed back during your scheduled lab.

1. For each of the following statements, determine whether the statement is true or false. Prove the true statements, and for the false statements, write down their negations and prove them. Note that you are allowed to use $\sqrt{2}$ as an irrational number without clarification. However, for any irrational numbers other than $\sqrt{2}$, you must prove that they are irrational.

(a) For all real numbers x and y, if x and y are irrational then $x + y$ is irrational.

(b) For all real numbers x, there exists a real number y so that $x + y$ is rational.

(c) For all real numbers x, there exists a real number y so that $x - y$ is irrational.

(d) For all real numbers x, there exists a real number y so that $x + y$ is rational and $x - y$ is irrational.

2. For each of the following statements, determine whether the statement is true or false. Prove the true statements, and for the false statements, write down their negations and prove them.

(a) For all real numbers x and y, if x and y are not integers then $\lfloor xy \rfloor = \lfloor x \rfloor \lceil y \rceil$.

(b) For all real numbers x and y, if x and y are not integers then $\lfloor xy \rfloor \neq \lfloor x \rfloor \lceil y \rceil$.

(c) There exists a real number x so that x is not an integer, $x > 271$ and $\lfloor x^2 \rfloor = \lfloor x \rfloor^2$.

(d) For all positive integers N, there exists a real number x so that x is not an integer, $x > N$ and $\lfloor x^2 \rfloor = \lfloor x \rfloor^2$.

3. Let N be your University of Calgary ID number.

(a) Use the Euclidean Algorithm to find $\gcd(N, 271)$.

(b) Use the result in part (a) to find integers x and y so that $\gcd(N, 271) = Nx + 271y$.

(c) Suppose that M is an integer so that $\gcd(M, 271) = \gcd(M, 2015)$. Find $\gcd(M, 271)$. Explain how you get the answer.

(d) Suppose that K is an integer between 800,000 and 900,000 so that $\gcd(K, 271) > \gcd(K, 2015) > 100$. Find all possible values of K. Explain how you get the answers.