MATHEMATICS 271 L01 FALL 2015
QUIZ 1 SOLUTIONS

1. Write the *negation* (in good English) of each of the following statements. The answer " It is not the case that ..." is **not** acceptable.
(a) For all real numbers x and y, if x is rational and y is irrational then $x+y$ and xy are irrational.
Solution: There are real numbers x and y so that x is rational and y is irrational, but $x+y$ or xy are rational.

(b) For all integers x and y, if $x^2+3x=y^2+3y$ then $x=y$ or $x=-y$.
Solution: There are integers x and y so that $x^2+3x=y^2+3y$, but $x \neq y$ and $x \neq -y$.

(c) For all integers y, there exists an integer x so that $y=x^2-2x$.
Solution: There is an integer y so that for all integers x, $y \neq x^2-2x$.

(d) There exists an integer n so that for all integers m, $2 \nmid n-m$ or $3 \nmid n-m$.
Solution: For all integers n, there exists an integers m so that $2 \mid n-m$ and $3 \mid n-m$.

2. Prove that the statement: "There exists an integer n so that for all integers m, $n+m$ is odd and $n-m$ is even." is **false** by writing out its negation and prove that.
Solution: Its negation is: "For all integers n, there exists an integer m so that $n+m$ is even or $n-m$ is odd." and below is a proof of the negation.

Suppose that n is an integer. Let $m=-n$. Then m is an integer and $n+m=0$ which is even.

3. Let $\mathcal{P}$ be the statement: "For all integers a and b, if $a \mid bc$ then $a \mid b$ or $a \mid c$."
(a) Prove that $\mathcal{P}$ is false.
Solution: $\mathcal{P}$ is false because with the integers $a=4$ and $b=c=2$, we have $bc=6=a=a \times 1$ where $1 \in \mathbb{Z}$, but $a \nmid b$ and $a \nmid c$ since $4 \nmid 2$.

(b) Write out the converse of $\mathcal{P}$. Prove that the converse of $\mathcal{P}$ is true.
Solution: The converse of $\mathcal{P}$ is: "For all integers a and b, if $a \mid b$ or $a \mid c$ then $a \mid bc$." and below is a proof of the converse.

Let a and b be integers so that $a \mid b$ or $a \mid c$. Then we have two cases:

Case 1: $a \mid b$. Then $b=ak$ for some integer k, and therefore $bc=(ak)\,c=a\,(kc)$ where kc is an integer. This implies $a \mid bc$.

Case 2: $a \mid c$. Then $c=am$ for some integer m, and therefore $bc=b\,(am)=a\,(bm)$ where bm is an integer. This implies $a \mid bc$.

(c) Write out the contrapositive of $\mathcal{P}$. Is the contrapositive of $\mathcal{P}$ true? Explain.
Solution: The contrapositive of $\mathcal{P}$ is: "For all integers a and b, if $a \nmid b$ and $a \nmid c$ then $a \nmid bc$."

The contrapositive of $\mathcal{P}$ is false because it is logically equivalent to P which is proven to be false in part (a).