MATHEMATICS 271 Fall 2015
Practice Problems 2

For each of the following statements, prove or disprove the statement. Note that you can use the fact that $\sqrt{2}$ is irrational. For all other irrational numbers, you must prove that they are in fact irrational.

1. $\forall x,\ y \in \mathbb{R}$, if x and y are irrational then $x + y$ is irrational.
2. $\forall x,\ y \in \mathbb{R}$, if x and y are irrational then xy is irrational.
3. $2 - \sqrt{2}$ is irrational.
4. $3\sqrt{2}$ is irrational.
5. $\forall x, y \in \mathbb{R},\ \lfloor x + y \rfloor = \lfloor x \rfloor + \lfloor y \rfloor$.
6. $\exists a \in \mathbb{R}$ so that $a \notin \mathbb{Z}$, $a > 2015$, and $\lfloor a^2 \rfloor = \lfloor a \rfloor^2$.
7. $\forall n \in \mathbb{Z}^+,\ \exists a \in \mathbb{R}$ so that $a \notin \mathbb{Z}$, $a > n$ and $\lfloor a^2 \rfloor = \lfloor a \rfloor^2$.
8. For all real numbers x, there exists a real number y so that $x + y$ is rational.
9. For all real numbers x, there exists a real number y so that $x + y$ is irrational.
10. For all real numbers x, there exists a real number y so that xy is irrational.
11. For all real numbers x, if x is irrational then $\sqrt{x}$ is irrational.