MATHEMATICS 271 WINTER 2015
Practice Problems 3

For each of the statements from1 to 10, prove or disprove the statement.

1. $\forall A \subseteq \mathbb{Z},\ \exists B \subseteq \mathbb{Z}$ so that $1 \in B - A$.
2. $\forall A \subseteq \mathbb{Z},\ \exists B \subseteq \mathbb{Z}$ so that $1 \notin B - A$.
3. For all sets $A,\ B$ and C, if $A \cup B = C$ then $C \in \mathcal{P}(A) \cup \mathcal{P}(B)$.
4. For all sets A, B and C, $(A \cup B) \cap C \subseteq A \cup (B \cap C)$.
5. For all sets A, B and C, $A \cup (B \cap C) \subseteq (A \cup B) \cap C$.
6. For all sets A, B and C, if $A \times B = A \times C$ then $B = C$.
7. For all sets A, B and C, if $A - B \subseteq C$ then $A - C \subseteq B$.
8. For all sets A, B and C, if $A \cap B \subseteq C$ and $B \cap C \subseteq A$ then $C \cap A \subseteq B$.
9. For all sets A, B and C, if $A - (B \cap C) = \emptyset$ then $A - C \subseteq B$.
10. For all sets A, B and C, if $A - C \subseteq B$ then $A - (B \cap C) = \emptyset$.

11. You are given that A and B are arbitrary subsets of the set $\mathbb{Z}$ of all integers such that $A \cap B = \{1\}$.
(a) Find an element of $A \times B$. Explain why it is an element of $A \times B$.
(b) Find an element of the complement $(A \times B)^c$. (Here, assume the universal set is $\mathbb{Z} \times \mathbb{Z}$.) Explain.