MATHEMATICS 271 L01 FALL 2015

QUIZ 4 Friday, November 27, 2015 **Duration: 45 minutes.**

ID#_____ SOLUTION

[8] 1. Let $f : \mathbb{Z} \to \mathbb{Z}$ and $g : \mathbb{Z} \to \mathbb{Z}$ be functions defined by $f(x) = 2x + 1$ and $g(x) = \left\lfloor \frac{x}{3} \right\rfloor$.

(a) Is $g \circ f$ one-to-one? Prove your answer.

Solution: $g \circ f$ is not one-to-one.

This is because $g \circ f(1) = g \circ f(2)$ but $1 \neq 2$. Note that

$g \circ f(1) = g(f(1)) = g(3) = \left\lfloor \frac{3}{3} \right\rfloor = 1$

and $g \circ f(2) = g(f(2)) = g(5) = \left\lfloor \frac{5}{3} \right\rfloor = 1$.

(b) Is $f \circ g$ onto $\mathbb{Z}$? Prove your answer.

Solution: $f \circ g$ is not onto $\mathbb{Z}$.

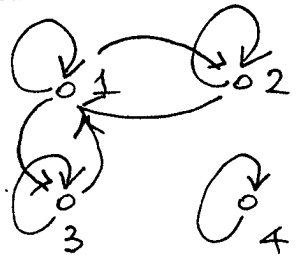
This is because for any $x \in \mathbb{Z}$, $f \circ g(x) = 2\left\lfloor \frac{x}{3} \right\rfloor + 1$ where $\left\lfloor \frac{x}{3} \right\rfloor$ is an integer, so $f \circ g(x)$ is always odd, and therefore, $f \circ g(x) \neq 0$ for all $x \in \mathbb{Z}$. Thus, $f \circ g$ is not onto.

LAST NAME_____ FIRST NAME_____

[7] **2.** Let $A = \{1, 2, 3, 4\}$. For each of the following, describe your relations as a set of ordered pairs and also draw the directed graph of your relation. No explanation is needed

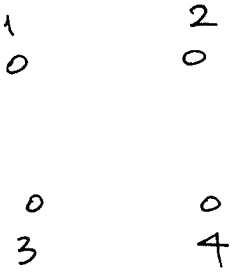
(a) Find a relation $\mathcal{R}$ on A so that $\mathcal{R}$ is reflexive and symmetric but not transitive.

$\mathcal{R} = \{(1,1), (2,2), (3,3), (4,4), (1,2), (2,1), (1,3), (3,1)\}$



(b) Find a relation $\mathcal{S}$ on A so that $\mathcal{S}$ is symmetric and transitive but not reflexive.

$\mathcal{S} = \phi$



(c) Find a relation $\mathcal{S}$ on A so that $\mathcal{S}$ is transitive and reflexive, but not symmetric.

$\mathcal{S} = \{(1,1), (2,2), (3,3), (4,4), (1,2)\}$