

MATHEMATICS 271 FALL 2015
Practice Problems 1

For each of the following statements, prove or disprove the statement. For the false statement, write out its negation and prove that. Also, for the conditional statements, write out the converse and the contrapositive, and determine whether they are true or false.

1. $\forall n \in \mathbb{Z}$, $n^2 + 2n$ is even.
2. $\exists n \in \mathbb{Z}$ such that $n^3 + n$ is odd.
3. $\forall x \in \mathbb{R}$, $x^2 - x \geq 0$.
4. $\forall x \in \mathbb{Z}$, $x^2 - x \geq 0$.
5. $\forall x, y \in \mathbb{Z}$, if $x^2 + 2x = y^2 + 2y$ then $x = y$.
6. $\forall x, y \in \mathbb{Z}$, if $2x^2 + x = 2y^2 + y$ then $x = y$.
7. $\forall a, b, c \in \mathbb{Z}$, if $a \mid b + c$ and $a \mid b - c$ then $a \mid b$ and $a \mid c$.
8. $\forall a, b, c \in \mathbb{Z}$, if $a \mid b + c$ and $a \mid 2b + c$ then $a \mid b$ and $a \mid c$.
9. $\forall n \in \mathbb{Z}, \exists m \in \mathbb{Z}$ such that $n + m$ is even.
10. $\exists m \in \mathbb{Z}$ such that $\forall n \in \mathbb{Z}$, $n + m$ is even.
11. $\forall r \in \mathbb{Q}, \exists m \in \mathbb{Z}$ such that $rm \in \mathbb{Z}$.
12. $\exists m \in \mathbb{Z}$ such that $\forall r \in \mathbb{Q}, rm \in \mathbb{Z}$.
13. For all positive integers n , there exists a positive integer m so that $3 \mid n + m$.
14. There exists a positive integer m so that for all positive integers n , $3 \mid n + m$.