MATHEMATICS 271 L02 FALL 2015
ASSIGNMENT 4

Due at 12:00 noon on Friday, December 4, 2015. Please hand in your assignment to Mark Girard at the beginning of the lab on November 20. Assignments must be understandable to the marker (i.e., logically correct as well as legible), and must be done by the student in his / her own words. Answer all questions, but only one question per assignment will be marked for credit. Please make sure that: (i) the cover page has **only** your student ID number and your instructor's name, (ii) your name and ID number are on the top right corners of **all** the remaining pages, and (iii) your assignment is **STAPLED**.

Marked assignments will be handed back during your scheduled lab.

1. Let $\mathcal{T}$ be the relation on $\mathbb{Z}$ defined by:

For all $x, y \in \mathbb{Z}$, $x\mathcal{T}y$ if and only if $3 \mid x + 2y$.

(a) Prove that $\mathcal{T}$ is an equivalence relation on $\mathbb{Z}$.
(b) List three negative elements and three positive elements of $[3]$.
(c) How many equivalence classes are there? Explain.

2. Let $\mathbb{Z}^+$ be the set of all positive integers. Let $\mathcal{S}$ be the relation on $\mathbb{Z}^+ \times \mathbb{Z}^+$ defined by:

For all $(a, b)\,,\ (c, d) \in \mathbb{Z}^+ \times \mathbb{Z}^+$, $(a, b)\,\mathcal{S}\,(c, d)$ if and only if $a + 2b = c + 2d$.

(a) Prove that $\mathcal{S}$ is an equivalence relation on $\mathbb{Z}^+ \times \mathbb{Z}^+$.
(b) List all elements of $[(3, 3)]$.
(c) List all elements of $[(4, 4)]$.
(d) Is there an equivalence class that has exactly 271 elements? Explain.

3. Let $S = \{1, 2, 3, ..., 2015\}$. Let $\mathcal{R}$ be the relation on $\mathcal{P}(S)$, the power set of S, defined by

For all $A, B \in \mathcal{P}(S)$, $A\mathcal{R}B$ if and only if $A \cup B = S$.

(a) Is $\mathcal{R}$ reflexive, symmetric, transitive? Prove your answers.
(b) Is it true that for all $X \in \mathcal{P}(S)$, there exists $Y \in \mathcal{P}(S)$ so that $(X, Y) \notin \mathcal{R}$? Prove your answers.
(c) Let $A = \{1, 2, 3, ..., 271\}$. How many elements X of $\mathcal{P}(S)$ are there so that $X\mathcal{R}A$? Explain.