MATHEMATICS 271 L02 FALL 2015
ASSIGNMENT 2

Due at 12:00 noon on Friday, October 16, 2015. Please hand in your assignment to Mark Girard at the beginning of the lab on October 16. Assignments must be understandable to the marker (i.e., logically correct as well as legible), and must be done by the student in his / her own words. Answer all questions, but only one question per assignment will be marked for credit. Please make sure that: (i) the cover page has **only** your student ID number and your instructor's name, (ii) your name and ID number are on the top right corners of **all** the remaining pages, and (iii) your assignment is **STAPLED**.

Marked assignments will be handed back during your scheduled lab.

1. Define the sequences $a_0, a_1, a_2, \ldots$ and $b_0, b_1, b_2, \ldots$ recursively as follows:
$a_0 = 0$, and for $n > 0$, $a_n = a_{\lfloor \frac{n}{5} \rfloor} + a_{\lfloor \frac{3n}{5} \rfloor} + n$, and
$b_0 = 2$, $b_1 = 3$ and for $n > 1$, $b_n = 3b_{n-1} - 2b_{n-2}$.
(a) Find $a_1,\ a_2,\ a_3,\ a_4$ and a_5.
(b) Prove that $a_n \leq 20n$ for all integers $n \geq 0$.
(c) Find $b_2,\ b_3,\ b_4$ and b_5.
(d) Guess a formula for b_n using part (c).
(e) Prove by induction on n that your guess in part (d) is correct for all integers $n \geq 0$.

2. Define the Fibonacci sequence $f_1, f_2, f_3, \ldots$ recursively as follows:
$f_1 = f_2 = 1$, and for $n \geq 3$, $f_n = f_{n-1} + f_{n-2}$.
(a) Prove that for all integers $n \geq 3$, $\gcd(f_n, f_{n-1}) = \gcd(f_{n-1}, f_{n-2})$. (You may want to use Lemma 4.8.2).
(b) Prove that $\gcd(f_n, f_{n-1}) = 1$ for all integers $n \geq 2$.
(c) Prove that $\sum_{i=1}^{n} f_i^2 = f_{n+1} f_n$ for all integers $n \geq 1$.
(d) Prove that $f_n < \left(\frac{7}{4}\right)^{n-1}$ for all integers $n \geq 2$.

3. For any sets A and B, we define the *symmetric difference* $A \triangle B$ by $A \triangle B = (A \cup B) - (A \cap B)$. Note that it is also true that $A \triangle B = (A - B) \cup (B - A)$. Let $\mathcal{S}$ be the statement: "*For all sets* A, B *and* C, *if* $A \subseteq B \cup C$ *and* $B \subseteq C \cup A$ *then* $A \triangle B = C$." and let $\mathcal{T}$ be the statement: "*For all sets* A, B *and* C, *if* $A \triangle B = A \triangle C$ *then* $B \subseteq C$."
(a) Is $\mathcal{S}$ true for all sets A, B and C? Prove your answer.
(b) Is $\mathcal{T}$ true? Prove your answer.
(c) Write the converse of $\mathcal{S}$. Is the converse of $\mathcal{S}$ true? Prove your answer.