THE UNIVERSITY OF CALGARY, FACULTY OF SCIENCE
MATHEMATICS 271 L01, L02
FINAL EXAMINATION, WINTER 2014
WEDNESDAY, APRIL 23, 2014 FROM 3:30 TO 6:30

LAST NAME____________________ FIRST NAME________________ ID________________

LECTURE NUMBER____________________ INSTRUCTOR NAME________________

EXAMINATION RULES

1. This is a closed book examination.
2. No aids are allowed for this examination.
3. The use of personal electronic or communication devices is prohibited.
4. A University of Calgary Student ID card is required to write the Final Examination and could be requested for midterm examinations. If adequate ID isn't present the student must complete an Identification Form.

Question	Total Marks	Actual Marks
1	6	
2	9	
3	10	
4	10	
5	12	
6	10	
7	10	
8	6	
9	7	
Total	80	

[6] 1. (a) Use the Euclidean algorithm to find $\gcd(102, 47)$. Also use the algorithm to find integers u and v such that $\gcd(102, 47) = 47u + 102v$.

(b) Use part (a) to find an inverse a for 47 modulo 102 so that $0 \leq a \leq 101$; that is, find an integer $a \in \{0, 1, \ldots, 101\}$ so that $47a \equiv 1 (\bmod 102)$.

[10] 2. For this problem, use no facts about $|$ ("divides into") other than its definition. Recall that $\mathbb{Z}$ denotes the set of all integers. Let $\mathcal{P}$ be the statement:

"For all $a, b \in \mathbb{Z}$, if $a \mid b$ then $(10a) \mid (2b)$."

(a) Is $\mathcal{P}$ true? Prove your answer.

(b) State the *converse* of $\mathcal{P}$. Is the converse of $\mathcal{P}$ true? Explain.

(c) State the *contrapositive* of $\mathcal{P}$. Is the contrapositive of $\mathcal{P}$ true? Explain.

[9] 3. Of the following statements, one is true and one is false. Use the "element method" to prove the true statement. For the false statement, write out its negation and prove that.

(a) For all sets A, B and C, if $B \subseteq C$ then $A - C \subseteq A - B$.

(b) For all sets A, B and C, if $A - B = A - C$ then $B = C$.

[10] 4. Let f and g be functions from $\mathbb{Z}$ to $\mathbb{Z}$ defined by $f(x) = 2x$ and $g(x) = \left\lfloor \frac{x}{2} \right\rfloor$ for any $x \in \mathbb{Z}$.

(a) Find $f \circ g(1)$, $f \circ g(2)$ and $f \circ g(3)$.

(b) Is $f \circ g$ onto $\mathbb{Z}$? Explain.

(c) Find $g \circ f(1)$, $g \circ f(2)$ and $g \circ f(3)$.

(d) Is $g \circ f$ one-to-one? Explain.

[12] 5. Let $A = \{1, 2, 3, ..., 2014\} = \{x \mid 1 \leq x \leq 2014\}$. Let $\mathcal{P}$ be the set of all **non-empty** subsets of A. Define the relation R on $\mathcal{P}$ by:

for any $X, Y \in \mathcal{P}$, XRY if and only if the largest element of X equals the largest element of Y.

(a) Prove that R is an equivalence relation on $\mathcal{P}$.

(b) List all the elements of the equivalence class $[\{3\}]$ (the equivalence class of $\{3\}$).

(c) How many equivalence classes does R have? Explain.

(d) How many elements does the equivalence class $[\{271\}]$ (the equivalence class of $\{271\}$) have? Explain.

[10] 6. Let $A = \{1, 2, 3, ..., 2014\} = \{x \mid 1 \leq x \leq 2014\}$. Define a relation R on A by:

for any $x, y \in A$, xRy if and only if there exists a prime p so that $p \mid x$ and $p \mid y$.

(a) Is R reflexive? symmetric? transitive? Explain.

(b) Find three elements $a,\ b,\ c$ of A so that $271Ra$, $271Rb$ and $271Rc$.

(c) How many elements x of A are there so that $271Rx$? Explain.

[10] 7. Only one of the following statements is true. Prove the true statement. For the other two statements, write out their negations and prove them. You can use the fact that $\sqrt{2}$ is irrational. For irrational numbers other than $\sqrt{2}$, you must explain why they are irrational.

(a) For all non-zero real numbers a and b, if a is rational and b is irrational then ab is irrational.

(b) For all real numbers a and b, if both a and b are irrational then $a + b$ is irrational.

(c) For all real numbers a and b, if both a and b are irrational then ab is irrational.

[6] 8. (a) Draw a simple graph with exactly six vertices and exactly nine edges.

(b) Draw a simple graph with exactly six vertices and exactly nine edges that is not bipartite but has an Euler circuit.

(c) Draw a simple graph with exactly six vertices and exactly nine edges that is bipartite but does not have an Euler circuit.

[7] 9. Prove by induction on n that $\sum_{i=1}^{n} i = \frac{n\,(n+1)}{2}$ for all integers $n \geq 1$.