

MATHEMATICS 271 L02 FALL 2015
ASSIGNMENT 3

Due at 12:00 noon on Friday, November 20, 2015. Please hand in your assignment to Mark Girard at the beginning of the lab on November 20. Assignments must be understandable to the marker (i.e., logically correct as well as legible), and must be done by the student in his / her own words. Answer all questions, but only one question per assignment will be marked for credit. Please make sure that: (i) the cover page has **only** your student ID number and your instructor's name, (ii) your name and ID number are on the top right corners of **all** the remaining pages, and (iii) your assignment is **STAPLED**.

Marked assignments will be handed back during your scheduled lab.

1. Let $S = \{1000, 1001, 1002, \dots, 9999\}$. For each of the following questions, you must simplify your answers and explain how you got the answers.

- (a) How many numbers in S have at least one digit that is a 2 or a 5?
- (b) How many numbers in S have at least one digit that is a 2 and at least one digit that is a 5?
- (c) How many numbers in S have the property that the sum of its digits is even?
- (d) How many numbers in S have the property that the digits appear in increasing order (that is, the first digit is smaller than the second digit, the second digit is smaller than the third digit, and the third digit is smaller than the fourth digit)?

2. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be the function defined by $f(x) = 2 \lfloor x \rfloor - x$ for each $x \in \mathbb{R}$.

- (a) Prove that f is one-to-one.
- (b) Prove that f is onto.
- (c) From (a) and (b) we see that f is invertible. Find a formula for $f^{-1}(x)$ for $x \in \mathbb{R}$. Verify that $f \circ f^{-1} = f^{-1} \circ f = I_{\mathbb{R}}$.

3. Let $f : A \rightarrow B$ and $g : B \rightarrow C$ be functions. Prove or disprove the following statements:

- (a) If both f and g are onto then $g \circ f$ is onto.
- (b) If $g \circ f$ is onto then f is onto.
- (c) If $g \circ f$ is onto then g is onto.
- (d) If $g \circ f$ is onto and g is one-to-one then f is onto.