ANSWERS TO MT REVIEW SHEET

ANSWERS

1. (a) $\vec{V}(t) = (-a\sin(t),\ a\cos(t),\ 0)$

$\vec{a}(t) = (-a\cos(t),\ -a\sin(t),\ 0)$

speed $v = a$

(b) $\vec{V}(t) = (2t, -2t, 0)$, $\vec{a}(t) = (2, -2, 0)$, $v = 2\sqrt{2}\,t$

(c) $\vec{V}(t) = \left(\frac{1}{t},\ \sin(2t),\ \cos(2t)\right)$,

$\vec{a}(t) = \left(-\frac{1}{t^2},\ 2\cos(2t),\ -2\sin(2t)\right)$

$$v = \frac{\sqrt{1+t^2}}{t}$$

2(i) Equation of Tangent line

$$y+1 = \frac{1}{2}(x-4)$$

and Equation of Normal line

$$y+1 = -2(x-4)$$

(ii) Equation of Tangent line

$$y-3 = \frac{2}{3}(x-39)$$

Equation of Normal line

$$y-3 = -\frac{3}{2}(x-39)$$

3. parametric equations are

$$x = 2 + 2s, \quad y = 1 - s, \quad z = 4 + 8s, \quad s \in \mathbb{R}$$

or $\vec{r}(s) = (2+2s)\vec{i} + (1-s)\vec{j} + (4+8s)\vec{k}, \quad s \in \mathbb{R}$

4. (a) 52 (b) $\frac{5}{2}$

(c) $3e - \frac{3}{e}$ (d) 4

5. (a) $x = 1 + t, \quad y = -4 + t, \quad 0 \leq t \leq 1$

(b) $x = t, \quad y = 1, \quad z = 2 - 3t, \quad 0 \leq t \leq 1$

(c) $x = 1 + 4\cos(t), \quad y = 4\sin(t), \quad t \in [0, 2\pi]$

6. (i) $\vec{r}(t) = \left(-\frac{1}{3} + 10\cos(t)\right)\vec{i} + \left(\frac{2}{5} + 6\sin(t)\right)\vec{j}, \quad t \in [0, 2\pi]$

(ii) $\vec{r}(t) = (1 + 5\cos(t))\vec{i} + (-3 + 5\sin(t))\vec{j}, \quad t \in [0, 2\pi]$

7. (a) $x = 2\cos(t), \quad y = 4\sin(t), \quad z = \frac{1}{2} - 2\cos(t) - 6\sin(t),$
$t \in [0, 2\pi]$

(b) $x = t, \quad y = \dfrac{t^3 - 3t - 2}{1 - 2t}, \quad z = \dfrac{7 - t^2}{1 - 2t}, \quad t \in \mathbb{R}, \ t \neq \frac{1}{2}$

(c) $x = 1 + 3\cos(t), \quad y = -2 + 3\sin(t),$
$z = 14 + 6\cos(t) - 12\sin(t), \quad t \in [0, 2\pi]$

(d) $\vec{r}(t) = -3\vec{i} + (-2 - t)\vec{j} + t\vec{k}, \quad t \in \mathbb{R}$

(e) $\vec{r}(t) = \cos(t)\vec{i} + \sin(t)\vec{j} + (1 - 2\sin^2(t))\vec{k}, \quad t \in [0, 2\pi]$

8. $V(15) = 500 \ln(1.6) \cong 235 \text{ m/s}$
$V(20) = 500 \ln(2) \cong 347 \text{ m/s}$
$V(30) = 500 \ln(4) \cong 693 \text{ m/s}$
$V(35) = 500 \ln(4) \cong 693 \text{ m/s}$

9. (a) $\vec{T} = \frac{1}{\sqrt{2}}(1,-1,0)$ $\left(\text{or } \frac{1}{\sqrt{2}}(1,-1) \text{ in } \mathbb{R}^2\right)$

$\vec{N} = -\frac{1}{\sqrt{2}}(1,1,0)$ $\left(\text{or } -\frac{1}{\sqrt{2}}(1,1) \text{ in } \mathbb{R}^2\right)$

$\kappa = \frac{1}{\sqrt{2}}$

(b) $\vec{T} = \frac{1}{2}(1,-\sqrt{3},0)$ $\left(\text{or } \frac{1}{2}(1,-\sqrt{3}) \text{ in } \mathbb{R}^2\right)$

$\vec{N} = -\frac{1}{2}(\sqrt{3},1,0)$ $\left(\text{or } -\frac{1}{2}(\sqrt{3},1) \text{ in } \mathbb{R}^2\right)$

$\kappa = \frac{1}{16}$

10. (a) $\vec{T} = \frac{1}{5}(3,0,4)$, $\vec{N} = (0,-1,0)$, $\vec{B} = \frac{1}{5}(4,0,-3)$,

$\kappa = \frac{3}{25}$, $\rho = \frac{25}{3}$, $\tau = -\frac{4}{25}$

(b) $\vec{T} = \frac{1}{2}(1,-\sqrt{2},1)$, $\vec{N} = -\frac{1}{2}(1,\sqrt{2},1)$, $\vec{B} = \frac{1}{\sqrt{2}}(1,0,-1)$

$\kappa = \frac{1}{\sqrt{2}}$, $\rho = \sqrt{2}$, $\tau = 0$

(c) $\vec{T} = \frac{1}{\sqrt{2}}(0,-1,1)$, $\vec{N} = (1,0,0)$, $\vec{B} = \frac{1}{\sqrt{2}}(0,1,1)$

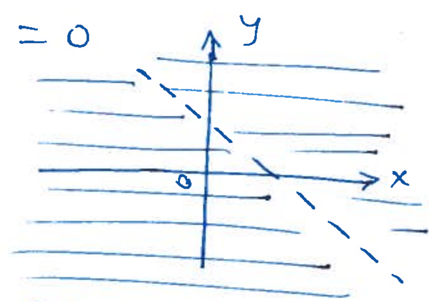
$\kappa = \frac{1}{2}$, $\rho = 2$, $\tau = -\frac{1}{2}$

11. (a) $a_T = \frac{20}{9}$, $a_N = \frac{\sqrt{5}}{9}$

(b) $a_T = 0$, $a_N = 2/5$

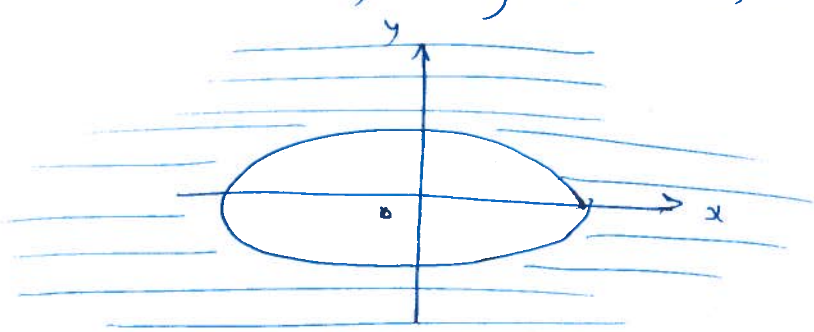
(c) $a_T = 0$, $a_N = 2\sqrt{2}$

12. (a) Domain D consists of all points (x,y) in $\mathbb{R}^2$ except points on the line $x+y-5=0$



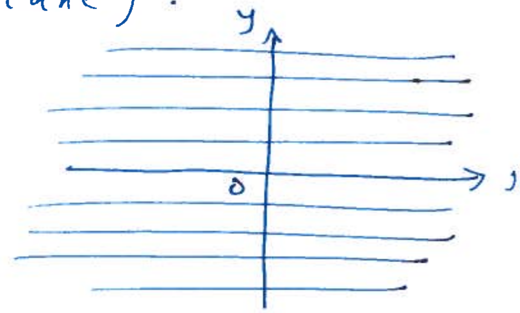
Domain f $\equiv$

(b) Domain f consists of all points (x,y) in $\mathbb{R}^2$ such that $4x^2+9y^2-36 \geq 0$. That is to say: All points outside and on the ellipse with centre $(0,0)$ and semi-axes of length $a=3$, $b=2$



Domain f $\equiv$

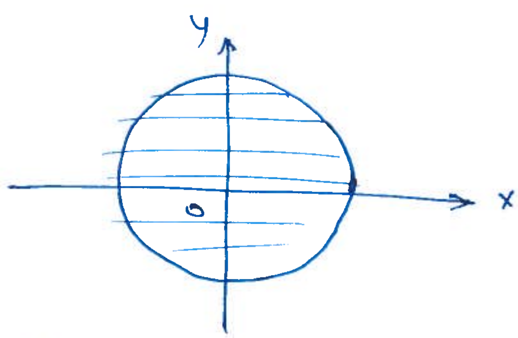
(c) The Domain D consists of all points (x, y) in $\mathbb{R}^2$ (The Entire xy-plane).



Domain f

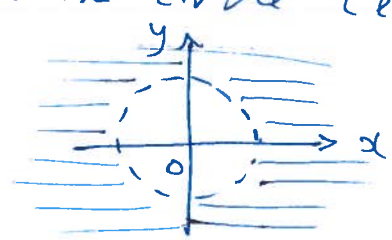
(d) The Domain D consists of all points (x, y) in $\mathbb{R}^2$ such that $x^2 + y^2 \leq 4$.

That is to say: D consists of all points inside and on the circle centred at $(0,0)$ and has radius 2 units



Domain f

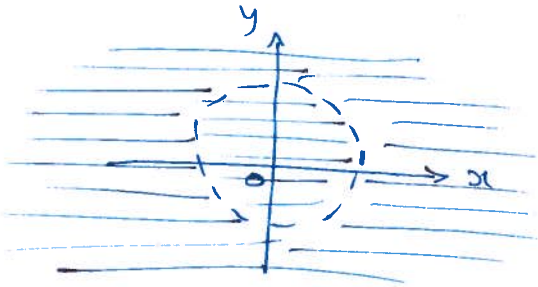
(e) The Domain D consists of all points (x, y) in $\mathbb{R}^2$ such that $x^2 + y^2 > 4$. That is to say: D consists of all points strictly outside the circle centred at $(0,0)$ and has radius 2 units



Domain f

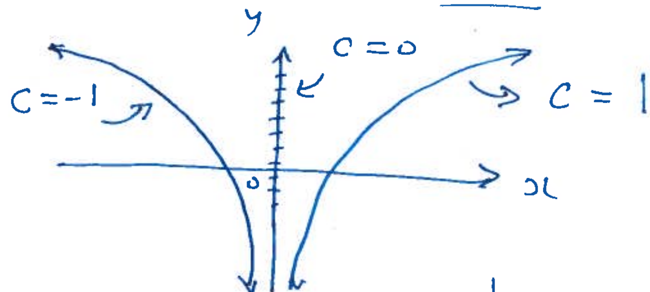
(f) The domain D consists of all points (x, y) in $\mathbb{R}^2$ except where $x^2 + y^2 = 4$.

That is to say: D consists of all points in $\mathbb{R}^2$ except those that lie on the circle $x^2 + y^2 = 4$ (with centre $(0,0)$, radius 2).

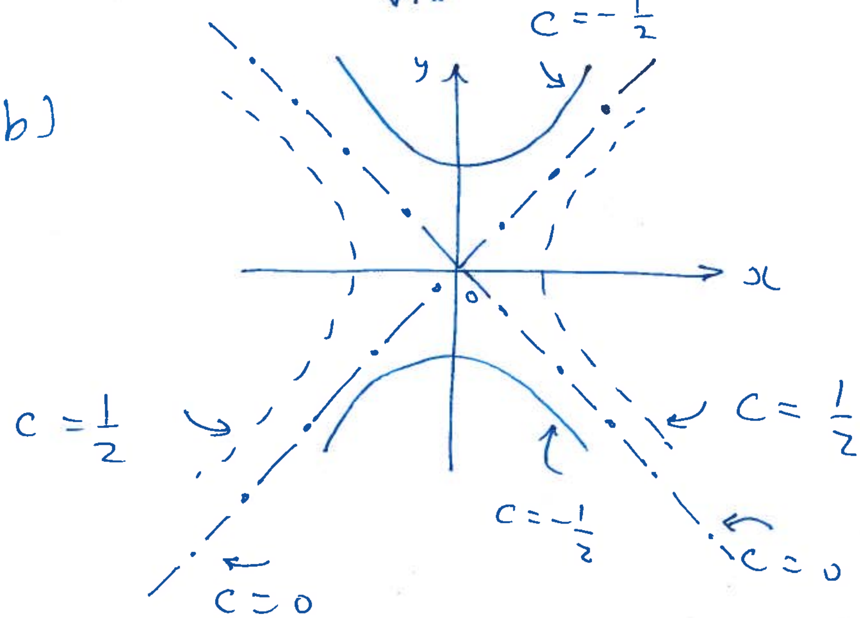


Domain f :

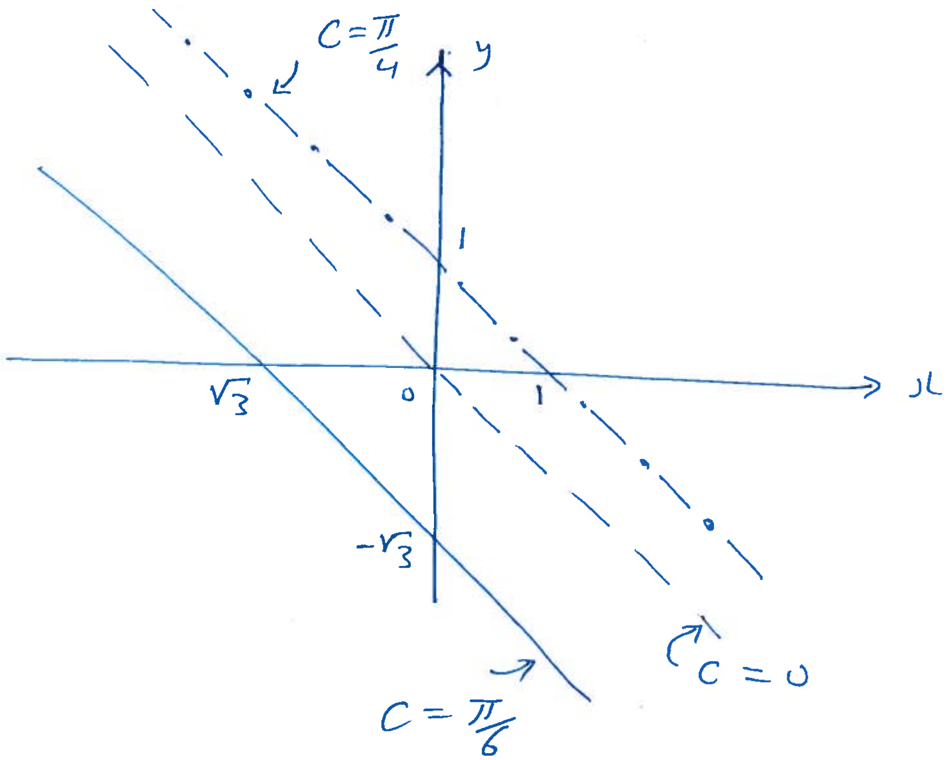
13. (a)



(b)



(c)



14. (i) The upper nappe of a Cone

(ii) Circular paraboloid

(iii) An Ellipsoid

(iv) A Hyperboloid of <u>one</u> sheet

(v) A cylindre

(vi) A plane (vii) A sphere

(viii) A Hyperboloid of <u>two</u> sheets

15. (a) $\frac{\partial z}{\partial y} = x\cos(xy)\ln(xy) + \frac{\sin(xy)}{y}$

(b) $f_{yx}(x,y) = \sec^2(x)\, y^{\tan(x)-1}\left[1+\tan(x)\cdot\ln(y)\right]$

16. $A = -3$

17. $m = \pm 2$

18. (a) Equation of tangent plane

$$3x - 4y - 5z = 0$$

and parametric Eq. of normal line

$$(x,y,z) = (3,-4,5) + t(6,-8,-10), \quad t \in \mathbb{R}$$

(b) Equation of tangent plane

$$3x + 4z = 7$$

and equation of normal line

$$(x,y,z) = (1,2,1) + t(3,0,4), \quad t \in \mathbb{R}$$

19. (a) $\frac{dz}{dt} = -2$

(b) $\frac{\partial z}{\partial v} = -12$

(c) $\frac{\partial w}{\partial s} = -3f_x(x,y,z) + 3t^{-1}s^2 f_y(x,y,z) + 3f_z(x,y,z)$

where $x = t^2 - 3s$, $y = t^{-1}s^3$, and $z = t + 3s$

(d) $\frac{\partial z}{\partial r} = \frac{1}{\sqrt{2}}, \quad \frac{\partial z}{\partial \theta} = -\frac{\sqrt{6}}{2}$

(e) $\frac{\partial z}{\partial y} = -4$

(f) $\frac{\partial w}{\partial u} = -1, \quad \frac{\partial w}{\partial v} = 2$

20. (a) Equation of Tangent plane

$$4x + 6y - 3z = 25$$

(b) Equation of Tangent plane

$$x + 3y + 8z = 33$$

(c) Unit vector normal to surface are

$$\vec{n} = \pm \frac{1}{7}(6, 3, 2)$$

21. (a) $df = [3e^{3x}\cos(2y) + 2]\,dx + [-2e^{3x}\sin(2y) - 1]\,dy$

(b) $dg = -\frac{y}{x\sqrt{x^2 - y^2}}\,dx + \frac{1}{\sqrt{x^2 - y^2}}\,dy$

(c) $dF = e^{x+2y+3z}[dx + 2dy + 3dz]$

(4) $dG = \frac{1}{x^2 + 2y - z}[2x\,dx + 2dy - dz]$

22. (a) $L(x,y) = 6 + \frac{1}{12}(x-4) - \frac{1}{6}(y+1)$

(b) $L(x,y) = x - y - 2$

(c) $L(x,y,z) = 2x + 2y + 2z - 3$

23. (a) $\sqrt{35.88} \approx 5.99$

(b) $\ln(1.0819) \approx 0.08$

24. $L(x,y) = 1 - 6(x-3) - 8(y+1)$

$f(2.9, -0.9) = \frac{1}{1.21} \approx 0.80$

25. $\frac{\Delta P}{P} \approx 8.75\%$

26. $\frac{\Delta V}{V} \approx -1.3\%$

27. $\frac{\Delta F}{F} \approx -5\%$

28. $K = 0.01\ m^{-1}$

29. $\theta \approx 13^{\circ}$

30. $\vec{r}(t) = \sqrt[3]{t}\ \vec{i} + \sqrt[3]{2-t}\ \vec{j} + 3t\vec{k},\quad t \in \mathbb{R}$

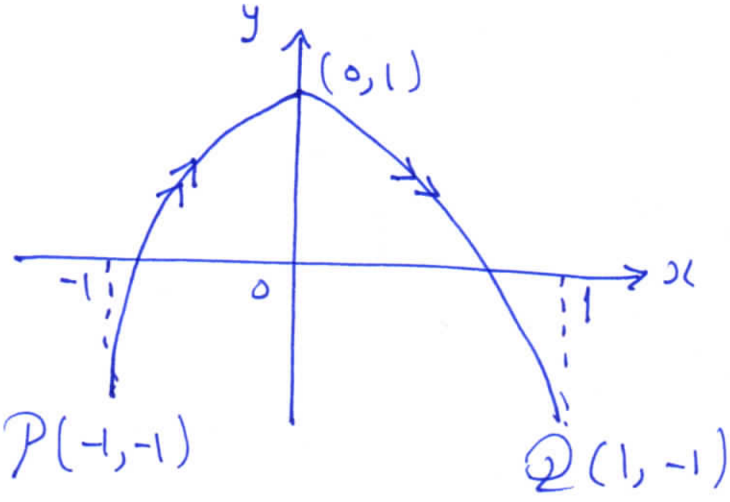
31. $\frac{(x+3)^2}{4} + \frac{(y-1)^2}{9} = 1$ .. An Ellipse with Centre at $(h,k) = (-3,1)$, semi-axes of length $a=2$, $b=3$

32. Cartesian Equation

$$y = 1 - 2x^2$$

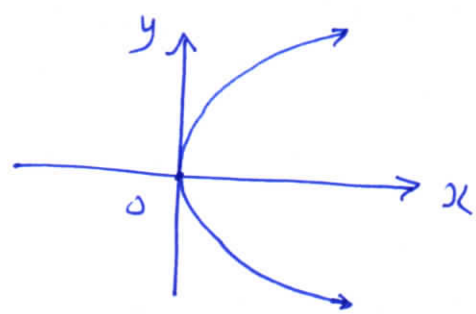
This is an equation of a parabola with vertex

at (0,1) and which opens downward.
Only the part from $P(-1,-1)$ to $Q(1,-1)$.



33. Cartesian Equation $y^2 = 8x$.
This is an equation of a parabola with vertex at (0,0), axis of symmetry is the x-axis and which opens to the right.

<u>Graph</u>: Entire Parabola



34. (a) $100\left(1-\frac{1}{e^2}\right) \approx 86.5\%$

(b) 366.5 m/s (or $400 \ln(2.5)$ m/s)

(c) $400 \ln\left(\frac{5}{3}\right)$ m/s (or 204.3 m/s)

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