Review of MATH 275

(I) Find the derivative of each of the following functions :

$(i)\ y = \tan^{-1}(\sqrt{x})$ $(ii)\ f(x) = \sec(\sqrt{2x+35})$

$(iii)\ g(x) = \sqrt[3]{e^{2x}}$ $(iv)\ y = \cos(2\ln(x))$

$(v)\ f(x) = x^{\ln(x)}$ $(vi)\ h(x) = 3^{\cosh^{-1}(x)}$

$(vii)\ y = \ln(x^2+1)^x$ $(viii)\ y = \sec^{-1}(\frac{1}{x})\ ,\ 0 < x < 1$

(II) Find the domain of :

$(i)\ g(x) = \sqrt{\ln(x-2)}$ $(ii)\ f(x) = \sqrt[3]{\ln(x-2)}$

(III) Determine each of the following limits:

$(i)\ \lim_{x \to 0^+} \frac{\sin(2x)}{e^x - 1}$ $(ii)\ \lim_{x \to \infty} x\tan\left(\frac{2}{x}\right)$

$(iii)\ \lim_{x \to 1^-} \frac{\ln(2x-1)}{1-x}$ $(iv)\ \lim_{x \to 0^-} \tan^{-1}\left(\frac{3}{x}\right)$

$(v)\ \lim_{x \to 0^+} x\ln(x)$ $(vi)\ \lim_{x \to -\infty} x^2 e^{5x}$

(IV) Evaluate each of the following integrals using a suitable Trigonometric Identity or table :

$(1)\ \int \sin^2(\theta)\, d\theta$ $(2)\ \int \cot^2(3x)\, dx$

$(3)\ \int \sec^2(u)\, du$ $(4)\ \int [\cos^2(\theta) - \sin^2(\theta)]\, d\theta$

$(5) \int \frac{\sin(\theta)}{\cos^2(\theta)}\, d\theta$ $(6)\ \int \frac{\cos^2(t)}{\sin(t)}\, dt$

(V) Evaluate (Using Table !) :

$(1) \int \frac{2}{5x-3}\, dx$ $(2)\ \int \frac{1}{(7x-2)^2}\, dx$

$(3)\ \int \frac{1}{\sqrt{2-9x}}\, dx$ $(4) \int e^{3x-4}\, dx$

(5) $\int (2z - y + 3x)^{101} dy$, where x , z are constant real numbers.

(6) $\int x\cos(xy)\, dy$, where $x \neq 0$ is a constant real number.

(7) $\int_0^3 \frac{dx}{9 + x^2}$

(8) $\int_0^2 \frac{1}{\sqrt{16 - x^2}}\, dx$

(9) $\int \frac{1}{\sqrt{4 + t^2}} dt$

(10) $\int \frac{1}{\sqrt{t^2 - 4}}\, dt$, $t > 2$.

(VI) Evaluate using a suitable substitution :

(1) $\int \frac{\ln(x^2)}{x}\, dx$, $x > 0$.

(2) $\int \frac{1}{e^{2x} + 1}\, dx$. Hint : Multiply Numerator and Denominator by e^{-2x}.

(3) $\int x^3 e^{x^4}\, dx$

(4) $\int \frac{\sin(\sqrt{x})}{2\sqrt{x}}\, dx$

(VII) Use a suitable integration Technique to evaluate each of the following integrals :

(*a*) $\int x\cos(4x)\, dx$

(*b*) $\int 27x^2 e^{3x}\, dx$

(*c*) $\int \sqrt{x}\, \ln(\sqrt{x})\, dx$

(*d*) $\int 2e^x \sinh(x)\, dx$

(*e*) $\int \tan^{-1}(x)\, dx$

(*f*) $\int \frac{10}{x^2 + 4x - 21}\, dx$

(*g*) $\int \frac{x + 2}{x^2 + 4x + 20}\, dx$

(*h*) $\int \frac{2x + 5}{x^2 + 4x + 20}\, dx$

(*i*) $\int \sqrt{\cosh(2x) - \sinh(2x)}\, dx$

(*j*) $\int_{-2}^{-1} \sqrt{4t^2 - 4t + 1}\, dt$

(*k*) $\int_1^2 \sqrt{4t^2 + 12 + \frac{9}{t^2}}\, dx$

(*l*) $\int \frac{1}{x^3 + x}\, dx$