UNIVERSITY OF CALGARY

UNIVERSITY OF CALGARY
FACULTY OF SCIENCE
DEPARTMENT OF MATHEMATICS AND STATISTICS
MATH 277 MIDTERM TEST – ALL LECTURES
Winter 2015

DATE: 13/03 **Time: 90 Minutes**

Student ID Number:	Last Name:	Other Names:	Lecture Section

EXAMINATION RULES

1. This is a closed book examination.
2. No aids are allowed for this examination.
3. **RECORD your answers in the SCANTRON SHEET , using a Number 2 HB Pencil , and filling in the appropriate circles.**
   Make no stray marks on the scantron sheet; they may count against you!
4. Scantron sheets must be filled out during the exam time limit. No additional time will be granted to fill in scantron form.
5. The use of personal electronic or communication devices is prohibited.
6. Students late in arriving will not be permitted after one-half hour of the examination time has passed.
7. No student will be permitted to leave the examination room during the first 30 minutes, nor during the last 15 minutes of the examination. Students must stop writing and hand in their exam immediately when time expires.
8. All inquiries and requests must be addressed to the exam supervisor.
9. Students are strictly cautioned against:
   a. communicating to other students;
   b. leaving answer papers exposed to view;
   c. attempting to read other students' examination papers
10. During the examination, if a student becomes ill or receives word of domestic affliction, the student must report to the Invigilator, hand in the unfinished paper and request that it be cancelled. If ill, the student must report immediately to a physician/counselor for a medical note.
11. Once the examination has been handed in for marking, a student cannot request that the examination be cancelled. Retroactive withdrawals from the course will be denied.
12. Failure to comply with these regulations will result in rejection of the examination paper.

EXAM # 22

1. The arc length of the space curve given by $\vec{r}(t) = t^3\,\vec{i} + \sqrt{3}\,t^2\,\vec{j} + (2t+1)\vec{k}$, $0 \le t \le 2$ is equal to

(A) 12 (B) 14 (C) -12 (D) $14 + 4\sqrt{3}$ (E) $10 + \frac{8}{3}\sqrt{3}$

$\left(\frac{x-3}{4}\right)^2 + \left(\frac{y-5}{2}\right)^2 = 1$

2. Which of the following plane parametric curves is an equation of an ellipse centred at (4,−2) ?

(A) $\vec{r}(t) = (\underbrace{3-4\cos(t)}_{x})\,\vec{i} + (\underbrace{5+2\sin(t)}_{y})\,\vec{j}$, $0 \le t \le 2\pi$. $\left(\frac{x-4}{a}\right)^2 + \left(\frac{y+2}{b}\right)^2 = 1$

(B) $\vec{r}(t) = (3+4\cos(t))\,\vec{i} + (5-2\sin(t))\,\vec{j}$, $0 \le t \le 2\pi$. cos t sin t

(C) $\vec{r}(t) = (4+3\cos(t))\,\vec{i} + (-2+5\sin(t))\,\vec{j}$, $0 \le t \le 2\pi$. $x = a\cos t + 4$ $y = b\sin t - 2$

(D) $\vec{r}(t) = (-4+3\cos(t))\,\vec{i} + (2+5\sin(t))\,\vec{j}$, $0 \le t \le 2\pi$.

(E) $\vec{r}(t) = (4-2\cos(t))\,\vec{i} + (4-2\sin(t))\,\vec{j}$, $0 \le t \le 2\pi$.

Circle – centered at (4,4)
r = 2

2D

3. The Cartesian equation of the straight line tangent to the plane curve given parametrically by $x(t) = 2t^3 - 3t^2 + 1$, $y(t) = t^2 - 2t + 2$ at the point on the curve where $t = 1$ is given by

(A) $3x - y - 1 = 0$ (B) $x - 3y + 3 = 0$ (C) $y = 1$ (D) $y = x + 1$

$\vec{r}(t) = \langle 2t^3 - 3t^2 + 1,\ t^2 - 2t + 2 \rangle$ @ $t=1$ $(0,1)$

$r'(t) = \langle 6t^2 - 6t,\ 2t - 2 \rangle$ $\frac{2t-2}{6t^2-6t} = \text{slope} = \frac{2(t-1)}{6t(t-1)}$

$y - y_0 = m(x - x_0)$

$y = \frac{1}{3}(x+1)$ $3y = x + 3$ $x - 3y + 3 = 0$ $= \frac{2}{6t}$ @ $t=1$ $= \frac{1}{3} = m$

4. A parametric representation of the curve of intersection of the two surfaces $4x^2 + y^2 + z^2 = 8$ and $z = -\sqrt{4x^2 + y^2}$ is given by the vector equation :

(A) $\vec{r}(t) = 2\cos(t)\vec{i} + \sin(t)\vec{j} - 2\vec{k}, \quad 0 \le t \le 2\pi.$

(B) $\vec{r}(t) = 2\cos(t)\vec{i} + \sin(t)\vec{j} + 2\vec{k}, \quad 0 \le t \le 2\pi.$

(C) $\vec{r}(t) = \cos(t)\vec{i} + 2\sin(t)\vec{j} + 2\vec{k}, \quad 0 \le t \le 2\pi.$

(D) $\vec{r}(t) = \cos(t)\vec{i} + 2\sin(t)\vec{j} - 2\vec{k}, \quad 0 \le t \le 2\pi.$

5. Let $\vec{\mathbf{T}}, \vec{\mathbf{N}}$ and $\vec{\mathbf{B}}$ be the unit Tangent, the Principal unit Normal and the unit Binormal respectively.

Which of the following equations is correct?

(1) $\vec{\mathbf{T}} = \cancel{\vec{\mathbf{B}} \times \vec{\mathbf{N}}}$ (2) $\vec{\mathbf{N}} = \vec{\mathbf{B}} \times \vec{\mathbf{T}}$ (3) $\cancel{\vec{\mathbf{B}} = \vec{\mathbf{N}} \times \vec{\mathbf{T}}}$

(4) $\vec{\mathbf{B}} = \vec{\mathbf{T}} \times \vec{\mathbf{N}}$ (5) $\vec{\mathbf{T}} = \vec{\mathbf{N}} \times \vec{\mathbf{B}}$ (6) $\cancel{\vec{\mathbf{N}} = \vec{\mathbf{T}} \times \vec{\mathbf{B}}}$

(A) 4, 5, 6 (B) 1, 2, 4 (C) 1, 4, 6

(D) 1, 2, 3 (E) 2, 4, 5

6. The radius of curvature of the space curve $\vec{r}(t) = 2\ln(t)\,\vec{i} + t\,\vec{j} + \frac{2}{t}\vec{k}$ at $t = 1$ is equal to :

(A) $\frac{25}{2}$ (B) $\frac{9}{8}$ (C) $\frac{9}{2}$ (D) $\frac{25}{8}$ (E) $\frac{1}{2}$

7. The position of a moving object in space is given by $\vec{r}(t) = 2\cos(t)\vec{i} + 2\sin(t)\vec{j} + t\vec{k}$,

Tangential component of the acceleration at time t is equal to :

(A) $-\frac{2}{3}\sin(t)\vec{i} + \frac{2}{3}\cos(t)\vec{j} + \frac{1}{3}\vec{k}$ (B) 0 (C) 1

(D) $\frac{1}{\sqrt{2}}\sin(t)\vec{i} - \frac{1}{\sqrt{2}}\cos(t)\vec{j} + \frac{1}{\sqrt{2}}\vec{k}$ (E) 2

8. Let $W(x,y,z) = \sin(\sqrt{5}\,x + 2y)\cosh(kz)$, for some **positive** constant real number k.

If $W(x,y,z)$ is a **Harmonic function** in $\mathbf{R}^3$, then the value of k is

(A) $\sqrt{5}$ (B) 3 (C) 2 (D) $\sqrt{5}+2$ (E) 1

9. The domain of which of the following functions is the shaded region shown in figure below :

(A) $f(x,y) = \sqrt{\ln(x^2+4y^2-63)}$ >0 (B) $f(x,y) = \sqrt{\ln(x^2+4y^2-64)}$ $x^2+4y^2-64>0$

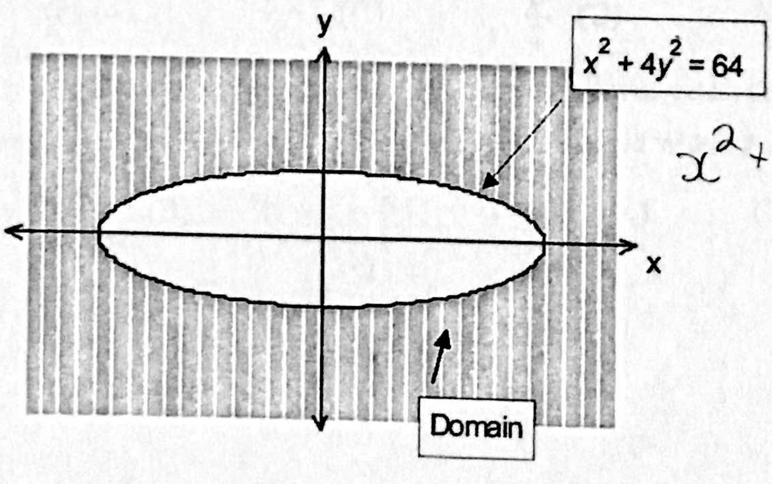
(C) $f(x,y) = \sqrt{\ln(64-x^2-4y^2)}$ (D) $f(x,y) = \sqrt{\ln(63-x^2-4y^2)}$

$\ln(x^2+4y^2-64) \geq 0$

$x^2+4y^2-63 \geq e^0$

$x^2+4y^2-65 \geq 0$

$x^2+4y^2-63 \geq 1$



$x^2+4y^2-64 \geq 0$

10. Which of the following is an equation of a **Hyperboloid of two Sheets?**

(A) $x^2 - y^2 - z^2 = 0$ (B) $x^2 - y^2 - z^2 = -1$ (C) $z^2 = 1 - x^2 - y^2$

(D) $x^2 + y^2 + 1 = z^2$ (E) $z = -x^2 - y^2$

11. If $z = y^x$, then $\frac{\partial^2 z}{\partial x \partial y}$ is equal to

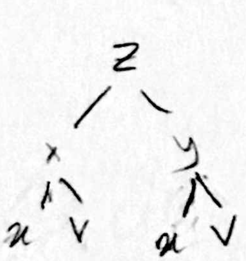
(A) $x(x-1)y^x$ (B) 0 (C) $y^{x-1}(1 + x\ln(y))$ (D) $xy^{2x-1}\ln(y)$ (E) $x(x-1)y^{x-2}$

$x = 2 \quad y = -2$

12. Let $z = f(x,y)$, where $x = uv^2$, and $y = \frac{u}{v}$. Use the chain rule to find $\frac{\partial z}{\partial u}$ at $(u,v) = (2,-1)$ given that $f_x(2,-1) = -7$, $f_y(2,-1) = 5$, $f_x(2,-2) = 3$, and $f_y(2,-2) = -2$. The value of $\frac{\partial z}{\partial u}$ is given by :

(x, y) (x, y) (x, y) (x, y)

(A) 5 (B) -12 (C) 4 (D) 2 (E) -5



$$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial u}$$

$$= \frac{\partial z}{\partial x}(v^2) + \frac{\partial z}{\partial y}\frac{1}{v} \quad (u, v) \ @ \ 2, -1$$

$$= 3(1) + -2(-1) = 5$$

13. If $W = e^{2x+y}$, where $x = t + \sin(t)$, and $y(t) = 2t - 1$, then the value of $\frac{dW}{dt}$ at $t = 0$ is equal to :

(A) $\frac{4}{e}$ (B) $\frac{3}{e}$ (C) $\frac{6}{e}$ (D) $\frac{2}{e}$ (E) 0

14. An equation of the plane tangent to the surface $5x^2 - 2y^2 + 2z = -9$ at the point $P(1,2,-3)$ on the surface is given by :

(A) $\vec{r}(t) = (1+5t)\,\vec{i} + (2-4t)\,\vec{j} + (-3+t)\vec{k}$, $t \in \mathbb{R}$ (B) $5x - 4y + z = -6$ (C) $10x - 4y - 2 = 0$

(D) $\vec{r}(t) = (5+t)\,\vec{i} + (-4+2t)\,\vec{j} + (1-3t)\vec{k}$, $t \in \mathbb{R}$ (E) $5x - 4y + z = -9$

15. Given that the relation $x^3y^2 + \sin(3xy - 2z) - \sqrt{10x^2 + 6} = 0$ implicitly defines y as a differentiable function of x and z. The value of $\frac{\partial y}{\partial z}$ at the point $P(1,2,3)$ is equal to :

(A) 0 (B) 5 (C) $\frac{7}{2}$ (D) $-\frac{7}{2}$ (E) $\frac{2}{7}$

16. Given $f(x,y,z) = xy + 3xz + 2yz$. The directional derivative of f at the point $P(1,1,-1)$ in the direction from $P(1,1,-1)$ toward the point $Q(-1,3,-2)$ is given by

then .

(A) 1 (B) -3 (C) $(4,-2,-5)$ (D) $(\frac{4}{3},-\frac{2}{3},-\frac{5}{3})$ (E) -1

$\vec{PQ} = \langle -2, 2, -1\rangle$ $\|\vec{PQ}\| = 3$

$\vec{u} = \langle \frac{-2}{3}, \frac{2}{3}, -\frac{1}{3}\rangle$ $\vec{\nabla} f = \langle y + 3z, x + 2z, 3x + 2y\rangle$ at $(1,1,-1)$

$\nabla f \cdot \vec{u} = \frac{4}{3} - \frac{2}{3} - \frac{5}{3} = -1$ $\vec{\nabla} f = \langle -1, -1, 5\rangle$

17. Let $f(x,y,z) = z^2 - 2x^3 - 3y + 4$, and let P be the point $(1,-1,3)$.

Which of the following statements is the only **False** statement?

$\vec{\nabla} f = \langle -6x^2, -3, 2z\rangle @ (1,-1,3)$

$\vec{\nabla} f = \langle -6, -3, 6\rangle$ $\|\nabla f\| = 9$

(A) The function f decreases most rapidly at P in the direction of the unit vector $(\frac{2}{3},\frac{1}{3},-\frac{2}{3})$.

(B) The maximum rate of change of f at the point P is 9.

(C) There is **no** direction $\vec{u}$ in which the rate of change of f at the point P is equal to -12.

(D) There is a direction $\vec{u}$ in which the rate of change of f at the point P is equal to -12.

18. The **Electric Resistance** R of a wire of **length** y and cross–sectional **radius** x is given by $\boxed{R = \frac{Ky}{x^2}}$, where K is a non – zero constant real number. By approximately what percentage the resistance R change if the length of the wire is increased by 1% and the **diameter** of the wire is **decreased** by 4 % ?

(A) 9 % (B) -7 % (C) 5 % (D) -2 % (E) 3 %

$\frac{dy}{y} \simeq 0.01$ $\frac{dx}{x} \simeq -0.04$

$= -2(-0.04) + (0.01)$

$= 0.09$

$dR = \frac{\partial R}{\partial x}dx + \frac{\partial R}{\partial y}dy$

$= -\frac{2Ky}{x^3}dx + \frac{K}{x^2}dy \Rightarrow \frac{dR}{R} = \frac{-2Ky/x^3}{Ky/x^2}dx + \frac{K/x^2}{Ky/x^2}dy$

$\frac{dR}{R} = -2\frac{dx}{x} + \frac{dy}{y}$

19. Which function $f(x,y)$ has the level curves corresponding to $c = -1$, 0 , and 1 shown in figure below ?

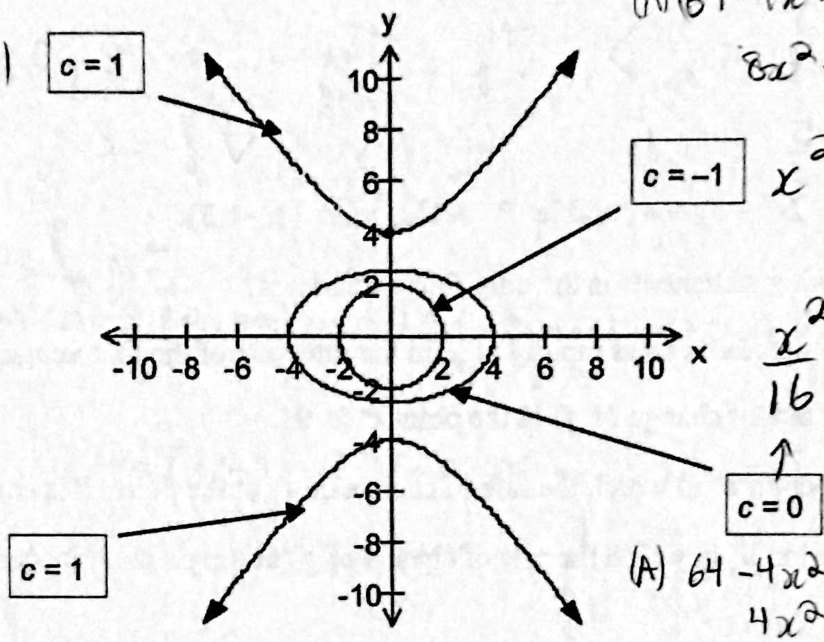
(A) $f(x,y) = \dfrac{64-4x^2-10y^2}{12x^2+6y^2}$ (B) $f(x,y) = \dfrac{64-4x^2+10y^2}{12x^2+6y^2}$ (C) $f(x,y) = \dfrac{4x^2+10y^2+64}{12x^2+6y^2}$

(D) $f(x,y) = \dfrac{4x^2+10y^2-64}{12x^2+6y^2}$ (E) $\cancel{f(x,y)} = \dfrac{4x^2-10y^2+64}{12x^2+6y^2}$

(A) $(64-4x^2-10y^2) = -12x^2-6y^2$

$8x^2-4y^2=-64$

$\frac{y^2}{16} - \frac{x^2}{m} = 1$



$x^2+y^2=4$

$\frac{x^2}{16} + \frac{y^2}{m} = 1$

(A) $64-4x^2-10y^2=0$

$4x^2+10y^2=64$

$\frac{x^2}{16} + \frac{y^2}{m} = 1$

20. A rocket has mass $40,000$ kilogram (kg), which includes $20,000$ kg of fuel mixture is fired vertically upward in a vacuum (that is Free Space where gravitational field is negligible) During the burning process the exhaust gases are ejected at a constant rate 1000 kg/s and at constant velocity with magnitude $\boxed{400 \textit{ metre/s}}$ relative to the rocket.

$m(t) = 20000 - 1000t$

If the rocket was initially at rest , then its speed after 30 seconds is equal to :

(A) $400\ln(2)$ *metre/s* (B) $400\ln(\frac{1}{2})$ *metre/s* (C) $400\ln(\frac{1}{4})$ *metre/s*

(D) $400\ln(4)$ *metre/s* (E) 400 *metre/s*

$V(20) = V(30)$

$= 400\ln\left(\frac{40000}{20000}\right) = 400\ln(2)$