TABLES OF DERIVATIVES INTEGRALS IDENTITIES

A: TABLE OF BASIC DERIVATIVES

Let $u = u(x)$ be a differentiable function of the independent variable x, that is $u'(x)$ exists.

(A) The Power Rule :	Examples :
$\frac{d}{dx}\{u^n\} = nu^{n-1}.\ u'$	$\frac{d}{dx}\{(x^3+4x+1)^{3/4}\} = \frac{3}{4}(x^3+4x+1)^{-1/4}.(3x^2+4)$
$\frac{d}{dx}\{\sqrt{u}\} = \frac{1}{2\sqrt{u}}.u'$	$\frac{d}{dx}\{\sqrt{2-4x^2+7x^5}\} = \frac{1}{2\sqrt{2-4x^2+7x^5}}(-8x+35x^4)$
$\frac{d}{dx}\{c\} = 0$, c is a constant	$\frac{d}{dx}\{\pi^6\} = 0$, since $\pi \cong 3.14$ is a constant.

(B) The Six Trigonometric Rules :	Examples :
$\frac{d}{dx}\{\sin(u)\} = \cos(u).\ u'$	$\frac{d}{dx}\{\sin(x^3)\} = \cos(x^3).\ 3x^2$
$\frac{d}{dx}\{\cos(u)\} = -\sin(u).\ u'$	$\frac{d}{dx}\{\cos\sqrt{x})\} = -\sin(\sqrt{x}).\ \frac{1}{2\sqrt{x}}$
$\frac{d}{dx}\{\tan(u)\} = \sec^2(u).\ u'$	$\frac{d}{dx}\{\tan\{\frac{5}{x^2})\} = \sec^2(5x^{-2}).\ (-10x^{-3})$
$\frac{d}{dx}\{\cot(u)\} = -\csc^2(u).\ u'$	$\frac{d}{dx}[\cot\{\sin(2x)\}] = -\csc^2\{\sin(2x)\}.\ 2\cos(2x).$
$\frac{d}{dx}\{\sec(u)\} = \sec(u)\tan(u).\ u'$	$\frac{d}{dx}\{\sec(\sqrt[4]{x})\} = \sec(\sqrt[4]{x})\tan(\sqrt[4]{x}).\ \frac{1}{4}x^{-3/4}$
$\frac{d}{dx}\{\csc(u)\} = -\csc(u)\cot(u).\ u'$	$\frac{d}{dx}\{\csc(8x-7)\} = -\csc(8x-7)\cot(8x-7).\ 8$

(C) The Six Hyperbolic Rules :	Examples :
$\frac{d}{dx}\{\sinh(u)\} = \cosh(u).\ u'$	$\frac{d}{dx}\{\sinh(\sqrt[3]{x})\} = \cosh(\sqrt[3]{x}).\ \frac{1}{3}x^{-2/3}$
$\frac{d}{dx}\{\cosh(u)\} = \sinh(u).\ u'$	$\frac{d}{dx}\{\cosh(\sec(x)\} = \sinh\{\sec(x)\}.\ \sec(x)\tan(x)$
$\frac{d}{dx}\{\tanh(u)\} = \mathrm{sech}^2(u).\ u'$	$\frac{d}{dx}[\tanh\{x^3+\sin(x^2)\}] = \mathrm{sech}^2\{x^3+\sin(x^2)\}.\ (3x^2+2x\cos(x^2))$
$\frac{d}{dx}\{\coth(u)\} = -\mathrm{csch}^2(u).\ u'$	$\frac{d}{dx}\{\coth(\frac{1}{x}+2x)\} = -\mathrm{csch}^2(\frac{1}{x}+2x).\ (-\frac{1}{x^2}+2)$
$\frac{d}{dx}\{\mathrm{sech}(u)\} = -\mathrm{sech}(u)\tanh(u).\ u'$	$\frac{d}{dx}\{\mathrm{sech}\{9x)\} = -\mathrm{sech}(9x)\tanh(9x).\ 9$
$\frac{d}{dx}\{\mathrm{csch}(u)\} = -\mathrm{csch}(u)\coth(u).\ u'$	$\frac{d}{dx}[\mathrm{csch}\{\sinh(3x)\}] = -\mathrm{csch}\{\sinh(3x)\}\coth\{\sinh(3x)\}.\ 3\cosh(3x)$

(D) The Exponential & Logarithmic Rule :	Examples :
$\frac{d}{dx}\{e^u\} = e^u.\ u'$	$\frac{d}{dx}\{e^{-x^3}\} = e^{-x^3}.\ (-3x^2)$
$\frac{d}{dx}\{\ln\|u\|\} = \frac{u'}{u}$	$\frac{d}{dx}\{\ln\|x^3+5x+6\|\} = \frac{3x^2+5}{x^3+5x+6}$
$\frac{d}{dx}\{a^u\} = a^u.\ln(a).\ u'$, $a \in \mathbb{R}$, $a > 0$, $a \neq 1$	$\frac{d}{dx}\{2^{\sec(x)}\} = 2^{\sec(x)}.\ln(2).\ \sec(x)\tan(x)$
$\frac{d}{dx}\{\log_a\|u\|\} = \frac{1}{\ln(a)}\frac{u'}{u}$, $a \in \mathbb{R}$, $a > 0$, $a \neq 1$	$\frac{d}{dx}\{\log_4\|\tan(x)\|\} = \frac{1}{\ln(4)}\frac{\sec^2(x)}{\tan(x)}$

(E) The Six Inverse Trigonometric Functions :	Examples :
$\frac{d}{dx}\{\sin^{-1}(u)\} = \frac{u'}{\sqrt{1-u^2}}$	$\frac{d}{dx}\{\sin^{-1}(4x^2)\} = \frac{8x}{\sqrt{1-16x^4}}$
$\frac{d}{dx}\{\cos^{-1}(u)\} = -\frac{u'}{\sqrt{1-u^2}}$	$\frac{d}{dx}\{\cos^{-1}(3x)\} = -\frac{3}{\sqrt{1-9x^2}}$
$\frac{d}{dx}\{\tan^{-1}(u)\} = \frac{u'}{1+u^2}$	$\frac{d}{dx}\{\tan^{-1}(\sqrt{x})\} = \frac{\frac{1}{2\sqrt{x}}}{1+x} = \frac{1}{2\sqrt{x}\,(1+x)}$
$\frac{d}{dx}\{\cot^{-1}(u)\} = -\frac{u'}{1+u^2}$	$\frac{d}{dx}\{\cot^{-1}(e^x)\} = -\frac{e^x}{1+e^{2x}}$
$\frac{d}{dx}\{\sec^{-1}(u)\} = \frac{u'}{\lvert u\rvert\sqrt{u^2-1}}$	$\frac{d}{dx}[\sec^{-1}(x^4)\} = \frac{4x^3}{\lvert x^4\rvert\sqrt{x^8-1}} = \frac{4x^3}{x^4\sqrt{x^8-1}}$
$\frac{d}{dx}\{\csc^{-1}(u)\} = -\frac{u'}{\lvert u\rvert\sqrt{u^2-1}}$	$\frac{d}{dx}\{\csc^{-1}(2x)\} = -\frac{2}{\lvert 2x\rvert\sqrt{4x^2-1}} = -\frac{1}{\lvert x\rvert\sqrt{4x^2-1}}$

(F) The Inverse Hyperbolic Functions :	Examples :
$\frac{d}{dx}\{\sinh^{-1}(u)\} = \frac{u'}{\sqrt{1+u^2}}$	$\frac{d}{dx}\{\sinh^{-1}(\ln(x)\} = \frac{1/x}{\sqrt{1+\ln^2(x)}}$
$\frac{d}{dx}\{\cosh^{-1}(u)\} = \frac{u'}{\sqrt{u^2-1}}$	$\frac{d}{dx}\{\cosh^{-1}(5x)\} = \frac{5}{\sqrt{25x^2-1}}$
$\frac{d}{dx}\{\tanh^{-1}(u)\} = \frac{u'}{1-u^2}$	$\frac{d}{dx}\{\tanh^{-1}(\frac{2}{x})\} = \frac{-\frac{2}{x^2}}{1-\frac{4}{x^2}} = \frac{-2}{x^2-4}$

(G) The Product and Quotient Rules :	Examples :
$\frac{d}{dx}\{uv\} = u'v + uv'$	$\frac{d}{dx}\{x^3\ln(5x+1)\} = 3x^2\ln(5x+1) + x^3\frac{5}{5x+1}$
$\frac{d}{dx}\{ku\} = ku'$, k is a constant	$\frac{d}{dx}\{\frac{x^3}{4}\} = \frac{1}{4}\frac{d}{dx}\{x^3\} = \frac{1}{4}.\ 3x^2 = \frac{3x^2}{4}$
$\frac{d}{dx}\{\frac{u}{v}\} = \frac{u'v - uv'}{v^2}$	$\frac{d}{dx}\{\frac{\tan(2x)}{x^3}\} = \frac{2\sec^2(2x)\ .x^3 - \tan(2x)\ .3x^2}{x^6}$

In table above it is assumed that $u = u(x)$ and $v = v(x)$ are differentiable functions

B: TABLE OF BASIC INTEGRALS

Let r, a, b, and $\beta \in \mathbb{R}$, $r \neq -1$, $a \neq 0$, and $\beta > 0$.

(A) The Power Rule :	Examples :
$\int (ax+b)^r dx = \frac{(ax+b)^{r+1}}{a(r+1)} + C$	$\int x^{-5} dx = -\frac{1}{4}x^{-4} + C$, $\int (3x-1)^{-2} dx = \frac{(3x-1)^{-1}}{-3} + C$
$\int dx = \int 1\, dx = x + C$	$\int 7dx = 7\int dx = 7x + C$
$\int \frac{1}{\sqrt{ax+b}}\, dx = \frac{2}{a}\sqrt{ax+b} + C$	$\int \frac{1}{\sqrt{x+4}} dx = 2\sqrt{x+4} + C$.

(B) The Six Trigonometric Rules :	Examples :
$\int \sin(ax+b))dx = -\frac{1}{a}\cos(ax+b) + C$	$\int \sin(9x-2)dx = -\frac{1}{9}\cos(9x-2) + C$
$\int \cos(ax+b)dx = \frac{1}{a}\sin(ax+b) + C$	$\int \cos(3x)dx = \frac{1}{3}\sin(3x) + C$
$\int \tan(ax+b)dx = \frac{1}{a}\ln\lvert\sec(ax+b)\rvert + C$	$\int \tan(5w-1)dw = \frac{1}{5}\ln\lvert\sec(5w-1)\rvert + C$
$\int \cot(ax+b)dx = \frac{1}{a}\ln\lvert\sin(ax+b)\rvert + C$	$\int \cot(1-7u)du = -\frac{1}{7}\ln\lvert\sin(1-7u)\rvert + C$
$\int \sec(ax+b)dx = \frac{1}{a}\ln\lvert\sec(ax+b) + \tan(ax+b)\rvert + C$	$\int \sec(3x)dx = \frac{1}{3}\ln\lvert\sec(3x) + \tan(3x)\rvert + C$
$\int \csc(ax+b)dx = \frac{1}{a}\ln\lvert\csc(ax+b) - \cot(ax+b)\rvert + C$	$\int \csc(2t)dt = \frac{1}{2}\ln\lvert\csc(2t) - \cot(2t)\rvert + C$

(C) Additional Trigonometric Rules :	Examples
$\int \sec^2(ax+b)dx = \frac{1}{a}\tan(ax+b) + C$	$\int \sec^2(2u/3)du = \frac{3}{2}\tan(2u/3) + C$
$\int \csc^2(ax+b)dx = -\frac{1}{a}\cot(ax+b) + C$	$\int \csc^2(\frac{w}{2})dw = -\frac{1}{1/2}\cot(\frac{w}{2}) + C = -2\cot(\frac{w}{2}) + C$
$\int \sec(ax+b)\tan(ax+b)dx = \frac{1}{a}\sec(ax+b) + C$	$\int \sec(3u)\tan(3u)du = \frac{1}{3}\sec(3u) + C$
$\int \csc(ax+b)\cot(ax+b)dx = -\frac{1}{a}\csc(ax+b) + C$	$\int \csc(5x)\cot(5x)dx = -\frac{1}{5}\csc(5x) + C$

(D) The Six Hyperbolic Rules :	Examples
$\int \sinh(ax+b)dx = \frac{1}{a}\cosh(ax+b) + C$	$\int \sinh(2x-7)dx = \frac{1}{2}\cosh(2x-7) + C$
$\int \cosh(ax+b)dx = \frac{1}{a}\sinh(ax+b) + C$	$\int \cosh(\frac{2x}{5})dx = \frac{5}{2}\sinh(\frac{2x}{5}) + C$
$\int \tanh(ax+b)dx = \frac{1}{a}\ln[\cosh(ax+b)] + C$	$\int \tanh(2u)du = \frac{1}{2}\ln[\cosh(2u)] + C$
$\int \coth(ax+b)dx = \frac{1}{a}\ln\lvert\sinh(ax+b)\rvert + C$	$\int \coth(x+3)dx = \ln\lvert\sinh(x+3)\rvert + C$
$\int \operatorname{sech}(ax+b)dx = \frac{2}{a}\tan^{-1}(e^{ax+b}) + C$	$\int \operatorname{sech}(3x-6)dx = \frac{2}{3}\tan^{-1}(e^{3x-6}) ++C$
$\int \operatorname{csch}(ax+b)dx = \frac{1}{a}\ln\lvert\tanh(ax+b)/2\rvert + C$	$\int \operatorname{csch}(10t)dt = \frac{1}{10}\ln\lvert\tanh(5t)\rvert + C$

(E) Additional Hyperbolic Rules :	Examples
$\int \operatorname{sech}^2(ax+b)dx = \frac{1}{a}\tanh(ax+b)+C$	$\int \operatorname{sech}^2(4w)dw = \frac{1}{4}\tanh(4w)+C$
$\int \operatorname{csch}^2(ax+b)dx = -\frac{1}{a}\coth(ax+b)+C$	$\int \operatorname{csch}^2(2u)du = -\frac{1}{2}\coth(2u)+C$
$\int \operatorname{sech}(ax+b)\tanh(ax+b)dx = -\frac{1}{a}\operatorname{sech}(ax+b)+C$	$\int \operatorname{sech}(3x)\tanh(3x)dx = -\frac{\operatorname{sech}(3x)}{3}+C$
$\int \operatorname{csch}(ax+b)\coth(ax+b)dx = -\frac{1}{a}\operatorname{csch}(ax+b)+C$	$\int \operatorname{csch}(\frac{x}{3})\coth(\frac{x}{3})dx = -3\operatorname{csch}(x/3)+C$

(F) Exponential /Logarithmic Rules :	Examples :
$\int e^{ax+b}dx = \frac{1}{a}e^{ax+b}+C$	$\int e^{7x}dx = \frac{1}{7}e^{7x}+C$
$\int k^{\alpha x+\beta}dx = \frac{1}{a\ln(k)}.k^{ax+b}+C$, $0<k\in\mathbb{R}$, $k\neq 1.$	$\int 2^{10x-17}dx = \frac{1}{10\ln 2}\, 2^{10x-17}+C$
$\int \frac{1}{ax+b}dx = \frac{1}{a}\ln\lvert ax+b\rvert+C$	$\int \frac{1}{2x-3}dx = \frac{1}{2}\ln\lvert 2x-3\rvert+C$

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(G) The Three Inverse Trigonometric Functions :	Examples :
$\int \frac{1}{\sqrt{\beta^2-x^2}}dx = \sin^{-1}(\frac{x}{\beta})+C$	$\int \frac{1}{\sqrt{16-x^2}}dx = \sin^{-1}(x/4)+C$
$\int \frac{1}{\beta^2+x^2}dx = \frac{1}{\beta}\tan^{-1}(\frac{x}{\beta})+C$	$\int \frac{1}{3+x^2}dx = \frac{1}{\sqrt{3}}\tan^{-1}(\frac{x}{\sqrt{3}})+C$
$\int \frac{1}{x\sqrt{x^2-\beta^2}}dx = \frac{1}{\beta}\sec^{-1}(\frac{x}{\beta})+C$, $x>\beta$	$\int \frac{1}{x\sqrt{x^2-4}}dx = \frac{1}{2}\sec^{-1}(\frac{x}{2})+C$, $x>2.$

(H) The Three Inverse Hyperbolic Functions :	Examples :
$\int \frac{1}{\sqrt{\beta^2+x^2}}dx = \sinh^{-1}(\frac{x}{\beta})+C$	$\int \frac{1}{\sqrt{1+x^2}}dx = \sinh^{-1}(x)+C$
$\int \frac{1}{\sqrt{x^2-\beta^2}}dx = \cosh^{-1}(\frac{x}{\beta})+C$, $x>\beta$	$\int \frac{1}{\sqrt{x^2-5}}dx = \cosh^{-1}(x/\sqrt{5})+C$
$\int \frac{1}{\beta^2-x^2}dx = \frac{1}{\beta}\tanh^{-1}(\frac{x}{\beta})+C$, $\lvert x\rvert<\beta$	$\int \frac{1}{36-x^2}dx = \frac{1}{6}\tanh^{-1}(\frac{x}{6})+C$, $\lvert x\rvert<6$

(I) The Fundamental Theorems	Examples :
$\int_a^b f(x)dx = g(x)\rvert_{x=a}^{x=b} = g(b)-g(a)$	$\int_e^{e^3}\frac{1}{x}dx = \ln\lvert x\rvert\,\rvert_{x=e}^{x=e^3} = \ln(e^3)-\ln(e) = 3-1 = 2$
$\frac{d}{dx}\{\int_{u(x)}^{v(x)} F(t)\,dt = F(v(x)).v'(x)-F(u(x)).u'(x)$	$\frac{d}{dx}\{\int_x^{x^2}\cos(t^2)dt = \cos(x^4).2x-\cos(x^2).1$

In table above it is assumed that :

(1) The function $f(x)$ is continuous on $[a,b]$ and $\int f(x)\,dx = g(x)+C.$

(2) The functions $u(x)$ and $v(x)$ are differentiable and $\int_{u(x)}^{v(x)} F(t)\,dt$ exists.

C: BASIC TRIGONOMETRIC IDENTITIES

GROUP (A) :

$(i)\ \tan(\theta) = \dfrac{\sin(\theta)}{\cos(\theta)}$ $\qquad$ $(ii)\ \cot(\theta) = \dfrac{\cos(\theta)}{\sin(\theta)}$

$(iii)\ \sec(\theta) = \dfrac{1}{\cos(\theta)}$ $\qquad$ $(iv)\ \csc(\theta) = \dfrac{1}{\sin(\theta)}$

GROUP (B) :

$(i)\ \cos^2(\theta) + \sin^2(\theta) = 1$ $\qquad$ $(ii)\ 1 + \tan^2(\theta) = \sec^2(\theta)$ $\qquad$ $(iii)\ \cot^2(\theta) + 1 = \csc^2(\theta)$

GROUP (C) :

$(i)\ \sin(2\theta) = 2\sin(\theta)\cos(\theta)$ $\qquad$ $(ii)\ \cos(2\theta) = 2\cos^2(\theta) - 1$ $\qquad$ $(iii)\ \cos(2\theta) = 1 - 2\sin^2(\theta)$

$(iv)\ \cos^2(\theta) = \dfrac{1}{2}[1 + \cos(2\theta)]$ $\qquad$ $(v)\ \sin^2(\theta) = \dfrac{1}{2}[1 - \cos(2\theta)]$

GROUP (D)

$(i)\ \sin(-\theta) = -\sin(\theta)$ $\qquad$ $(ii)\ \cos(-\theta) = \cos(\theta)$ $\qquad$ $(iii)\ \tan(-\theta) = -\tan(\theta).$

GROUP(E)

$(i)\ \cos(\theta \pm \phi) = \cos(\theta)\cos(\phi) \mp \sin(\theta)\sin(\phi)$

$(ii)\ \sin(\theta \pm \phi) = \sin(\theta)\cos(\phi) \pm \cos(\theta)\sin(\phi)$

GROUP (F)

$(i)\ \cos(\theta)\cos(\phi) = \dfrac{1}{2}[\cos(\theta - \phi) + \cos(\theta + \phi)]$

$(ii)\ \sin(\theta)\sin(\phi) = \dfrac{1}{2}[\cos(\theta - \phi) - \cos(\theta + \phi)]$

$(iii)\ \sin(\theta)\cos(\phi) = \dfrac{1}{2}[\sin(\theta - \phi) + \sin(\theta + \phi)]$

D: SPECIAL TRIGONOMETRIC EQUATIONS

(i) $\sin(x) = 0 \quad \Rightarrow x = n\pi$ $\qquad$ (ii) $\cos(x) = 0 \quad \Rightarrow x = \frac{(2n-1)}{2}\pi$

where n is an integer : $n = 0, \pm 1, \pm 2, \pm 3, \ldots$

E: HYPERBOLIC FUNCTIONS

(i) $\sinh(x) = \frac{1}{2}[e^x - e^{-x}]$ $\qquad$ (ii) $\cosh(x) = \frac{1}{2}[e^x + e^{-x}]$ $\qquad$ (iii) $\tanh(x) = \frac{\sinh(x)}{\cosh(x)}$

(iv) $\coth(x) = \frac{\cosh(x)}{\sinh(x)}$ $\qquad$ (v) $\mathrm{sech}(x) = \frac{1}{\cosh(x)}$ $\qquad$ (vi) $\mathrm{csch}(x) = \frac{1}{\sinh(x)}$

(vii) $\cosh^2(x) - \sinh^2(x) = 1$ $\qquad$ $(viii)$ $1 - \tanh^2(x) = \mathrm{sech}^2(x)$ $\qquad$ (ix) $\coth^2(x) - 1 = \mathrm{csch}^2(x)$

F:PROPERTIES OF LOGARITHMS

Let x and y be positive real numbers.

(i) $\ln(x) + \ln(y) = \ln(xy)$ $\qquad$ (ii) $\ln(x) - \ln(y) = \ln(\frac{x}{y})$ $\qquad$ (iii) $\ln(x^m) = m\ln(x)$.

(iv) $\ln(e^k) = k$ $\qquad$ (v) $e^{\ln(x)} = x$ $\qquad$ (vi) $\ln(1) = 0$, $\ln(e) = 1$.

G:SPECIAL VALUES

(i) $\sin(0) = 0$ $\qquad$ (ii) $\cos(0) = 1$ $\qquad$ (iii) $\tan(0) = 0$

(iv) $\sinh(0) = 0$ $\qquad$ (v) $\cosh(0) = 1$ $\qquad$ (vi) $\tanh(0) = 0$

(vii) $\sin(n\pi) = 0$ and $\cos(n\pi) = (-1)^n$, provided that " n " is an integer.

END