MATH 277

Problem Set # 6 for Labs

Note : Problems marked with $(*)$ are left for students to do at home.

1. Find an equation of the plane tangent to the surface $z = \sqrt[5]{x^3 + y^2}$ at the point on surface where $x = -2$, and $y = 3$.

2. Find the coordinates of all points on the hyperboloid $2x^2 - y^2 + 5z^2 - 13 = 0$ where the tangent plane is parallel to the plane $2x - 3y - 10z + 17 = 0$.

3. Find an equation of the plane tangent to the paraboloid $2x + 3y^2 + 2z^2 = 31$ and is parallel to the plane $2x + 6y + 16z - 107 = 0$.

4* Find an equation of the plane tangent to the surface $4x^2 - y^2 + 3z^2 - 43 = 0$ at the point $(1,-3,4)$.

5. Find a unit vector orthogonal (normal) to the surface $xyz = -2$ at the point $(1,-2,1)$.

6. Find the parametric equation of the line normal to the surface $y = \ln\left(\dfrac{x+2y}{y+2z}\right) - 1$ at the point $P(3,-1,1)$ on the surface.

7. Let $P(x_0, y_0, 7)$ be a point on the surface $3x^2 + y^2 - 4z = 0$. Find the coordinates (x_0, y_0) so that the normal line to surface at P passes through the point $Q(1,2,8)$.

8. In Each case find the differential dF of the given function f at the specified point (a,b) :

   $(a)\ f(x,y) = e^{xy}$ $\qquad$ $(b)\ f(x,y,z) = xyz$

   $(c)\ f(x,y) = \sin^{-1}\left(\dfrac{y}{\sqrt{x^2+y^2}}\right)$ $\qquad$ $(d)^*\ f(x,y,z) = \dfrac{1}{x^2+y^2+z^2}$

9. In Each case find the linearization $L(x,y)$ of the given function f at the specified point (a,b) :

$(a)\ f(x,y) = \tan^{-1}\left(\frac{y}{x}\right)$, $(a,b) = (3,3)$ $\qquad$ $(b)\ f(x,y) = \sqrt{x^2+y^2+5}$, $(a,b) = (4,2)$

$(c)^*\ f(x,y) = ye^{x+y^2}$, $(a,b) = (-4,2)$

10. In Each case use a suitable linearization $L(x,y)$ to approximate the value of the given function f at the specified point (x,y) :

$(a)\ f(x,y) = \tan^{-1}(x+2y)$, $(x,y) = (-3.1, 1.09)$ $\quad$ $(b)\ f(x,y) = \ln(x^2+y^2-12)$, $(x,y) = (2.03, 2.99)$

$(c)^*\ f(x,y) = \dfrac{1}{\sqrt{2x+3y-4}}$, $(x,y) = (1.08, 2.16)$

11. Use differentials to determine by approximately what percentage the volume of a rectangular box with a square base change if its base length x, is increased by 1.5% and its and its height y is decreased by 1%.

12*. Use differentials to determine by approximately what percentage the total surface area of an open rectangular box with a square base change if its base length x, is increased from $10\ cm$ to $10.2\ cm$ and its and its height y is decreased from $5\ cm$ to $4.75\ cm$.

13. Use differentials to determine by approximately how many cubic centimeters should the volume of a right circular cone change if its radius is increased from $10\ cm$ to $10.2\ cm$ and its height is decreased from $8\ cm$ to $7.63\ cm$

14. The equivalent resistance R of Resistors R_1 and R_2 connected in parallel in an electrical circuit is given by $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$. Use differentials to determine by approximately what percentage the resistance R change if R_1 is increased from 100 ohms to 105 ohms and $R_2 = 25$ ohms is decreased by 1.5 ohms.

MATH 277

Solutions to Problem Set # 6

1. For an equation of a plane, we need:

(i) A point : At $x = -2$, $y = 3$,

$$z = \sqrt[5]{(-2)^3 + (3)^2} = \sqrt[5]{-8+9} = \sqrt[5]{1} = 1$$

$\therefore$ point is $P(-2, 3, 1) \Rightarrow \vec{r}_0 = (-2, 3, 1)$

(ii) A normal vector $\vec{N}$:

First, let us rewrite equation of surface

$$z = \sqrt[5]{x^3 + y^2}$$

in the simple form: $z^5 = x^3 + y^2$ or

$$F(x, y, z) = x^3 + y^2 - z^5 = 0$$

$$\therefore \vec{N} = \text{grad}\, F(P) = \left(\frac{\partial F}{\partial x}, \frac{\partial F}{\partial y}, \frac{\partial F}{\partial z}\right)\Big|_P$$

$$= (3x^2, 2y, -5z^4)\Big|_{(x,y,z)=(-2,3,1)}$$

$$= (12, 6, -5)$$

Equation of tangent plane is thus given by

$$\vec{r} \cdot \vec{N} = \vec{r}_0 \cdot \vec{N}, \quad \vec{r} = (x, y, z)$$

$$\therefore (x, y, z) \cdot (12, 6, -5) = (-2, 3, 1) \cdot (12, 6, -5)$$

$$\therefore 12x + 6y - 5z = -11$$

2. Let $P(x, y, z)$ be a point on the surface, and let

$$F(x, y, z) = 2x^2 - y^2 + 5z^2 - 13$$

A vector normal to tangent plane at P is thus given by

$$\vec{N} = \nabla F(P) = \left(\frac{\partial F}{\partial x}, \frac{\partial F}{\partial y}, \frac{\partial F}{\partial z} \right)\Big|_{P(x,y,z)}$$

$$= (4x, -2y, 10z)$$

But tangent plane is parallel to the plane

$$2x - 3y - 10z + 17 = 0$$

hence

$$(4x, -2y, 10z) = K(2, -3, -10)$$

for some scalar K.

Equating corresponding components, we get:

$$4x = 2K \Rightarrow x = \frac{K}{2},$$

$$-2y = -3K \Rightarrow y = \frac{3}{2}K, \text{ and}$$

$$10z = -10K \Rightarrow z = -K$$

Substituting $x = \frac{K}{2}$, $y = \frac{3}{2}K$, and $z = -K$ into equation of surface: $2x^2 - y^2 + 5z^2 - 13 = 0$,

one obtains:

$$2\left(\frac{K}{2}\right)^2 - \left(\frac{3}{2}K\right)^2 + 5(-K)^2 - 13 = 0$$

or $$2\frac{K^2}{4} - \frac{9K^2}{4} + 5K^2 - 13 = 0 \qquad (*4)$$

$$2K^2 - 9K^2 + 20K^2 - 52 = 0$$

$$\Rightarrow 13K^2 = 52 \Rightarrow K^2 = \frac{52}{13} = 4$$

$$\therefore \quad K = \pm 2$$

There are two points:

If $K = -2$, we have

$$x = \frac{-2}{2} = -1,$$

$$y = \frac{3}{2}(-2) = -3,$$

$$z = -(-2) = 2$$

$\therefore$ First point is $P_1(-1, -3, 2)$

If $K = +2$, we have

$$x = \frac{2}{2} = 1,$$

$$y = \frac{3}{2}(2) = 3,$$

$$z = -2$$

The second point is $P_2(1, 3, -2)$

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3. Find an equation of the plane tangent to the paraboloid $2x + 3y^2 + 2z^2 = 31$ and is parallel to the plane $2x + 6y + 16z - 107 = 0$.

Solution:

let $P(x, y, z)$ be a point on the paraboloid, and let $F(x, y, z) = 2x + 3y^2 + 2z^2 - 31$

A vector $\vec{N}$ normal to tangent plane at P is thus given by

$$\vec{N} = \nabla F \Big|_P = \left(\frac{\partial F}{\partial x}, \frac{\partial F}{\partial y}, \frac{\partial F}{\partial z} \right) \Big|_{P(x,y,z)}$$

$$= (2, 6y, 4z)$$

But the tangent plane is parallel to the plane $2x + 6y + 16z - 107 = 0$,

hence, we must have

$$(2, 6y, 4z) = K(2, 6, 16)$$

Equating Corresponding Components, we get,

$$2 = 2K \quad \text{--- (1)}$$

$$6y = 6K \quad \text{--- (2)}, \text{ and}$$

$$4z = 16K \quad \text{--- (3)}.$$

From (1), $K = 1$. Substituting into (2), (3), we respectively have

$$y = 1, \text{ and } z = 4$$

Substituting $y = 1$, $z = 4$ into Eq. of paraboloid:

$$2x + 3y^2 + 2z^2 = 31$$

we get:

$$2x + 3(1)^2 + 2(4)^2 = 31$$

$$2x + 3 + 32 = 31$$

$$2x = 31 - 3 - 32 = -4$$

$$\therefore \quad x = -2$$

$\therefore$ point $P(x, y, z) = (-2, 1, 4)$

Eq. of tangent plane is thus given by

$$\bar{r} \cdot \bar{N} = \vec{r}_0 \cdot \bar{N}$$

Here $\vec{r}_0 = (-2, 1, 4)$, $\vec{N} = (2, 6, 16)$ (or $(1, 3, 8)$).

$$\therefore \quad (x, y, z) \cdot (1, 3, 8) = (-2, 1, 4) \cdot (1, 3, 8)$$

$$\Rightarrow x + 3y + 8z = -2 + 3 + 32$$

$$\Rightarrow x + 3y + 8z = 33$$

4. For students to do at Home.

Answer: $4x + 3y + 12z - 43 = 0$

5. A vector $\vec{N}$ normal to surface at the point P is given by

$$\vec{N} = \vec{\nabla} F(P) \overset{\text{or}}{=} \text{grad } F(P)$$

where $F(x, y, z) = xyz + 2$

$$\therefore \vec{N} = \left(\frac{\partial F}{\partial x}, \frac{\partial F}{\partial y}, \frac{\partial F}{\partial z} \right) \Big|_P$$

$$= (yz, xz, xy) \Big|_{(x,y,z)=(1,-2,1)}$$

$$= (-2, 1, -2)$$

Required unit vector is thus given by

$$\vec{n} = \pm \frac{\vec{N}}{\|\vec{N}\|}$$

But $\|\vec{N}\| = \sqrt{(-2)^2 + (1)^2 + (-2)^2} = \sqrt{4+1+4} = 3$

$$\therefore \vec{n} = \pm \frac{(-2, 1, -2)}{3} \overset{\text{or}}{=} \pm \frac{1}{3} (-2, 1, -2)$$

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6. $y = \ln\left(\dfrac{x+2y}{y+2z}\right) - 1$; $P(3,-1,1)$

Rewrite equation of surface in the form

$$\ln\left(\frac{x+2y}{y+2z}\right) - 1 - y = 0$$

Take $F(x,y,z) = \ln\left(\dfrac{x+2y}{y+2z}\right) - 1 - y$

or easier:

$$F(x,y,z) = \ln(x+2y) - \ln(y+2z) - 1 - y$$

(Assuming $x+2y > 0$, $y+2z > 0$ which is the case at $P(3,-1,1)$!!)

$\therefore F_x = \dfrac{1}{x+2y} - 0 - 0 - 0 \Rightarrow F_x\big|_P = 1$

$F_y = \dfrac{2}{x+2y} - \dfrac{1}{y+2z} - 0 - 1 \Rightarrow F_y\big|_P = 2-1-1 = 0$

$F_z = 0 - \dfrac{2}{y+2z} - 0 - 0 \Rightarrow F_z\big|_P = -2$

$\therefore \vec{\nabla} F(P) = (F_x, F_y, F_z)\big|_P = (1, 0, -2)$

Equation of normal line at $P(3,-1,1)$ is given by

$$(x,y,z) = (3,-1,1) + t(1,0,-2), \quad t \in \mathbb{R}$$

or

$$\begin{cases} x = 3 + t \\ y = -1 \\ z = 1 - 2t \end{cases}, \quad t \in \mathbb{R}$$

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$$K(1-x_0) = 6x_0 \quad \text{-----} \quad (1)$$

$$K(2-y_0) = 2y_0 \quad \text{-----} \quad (2)$$

$$K = -4$$

substituting $K=-4$ into (1), (2) we obtain:

$$-4(1-x_0) = 6x_0$$

$$\Rightarrow -4+4x_0 = 6x_0 \Rightarrow \boxed{x_0 = -2}$$

and $$-4(2-y_0) = 2y_0$$

$$\Rightarrow -8+4y_0 = 2y_0 \Rightarrow \boxed{y_0 = 4}$$

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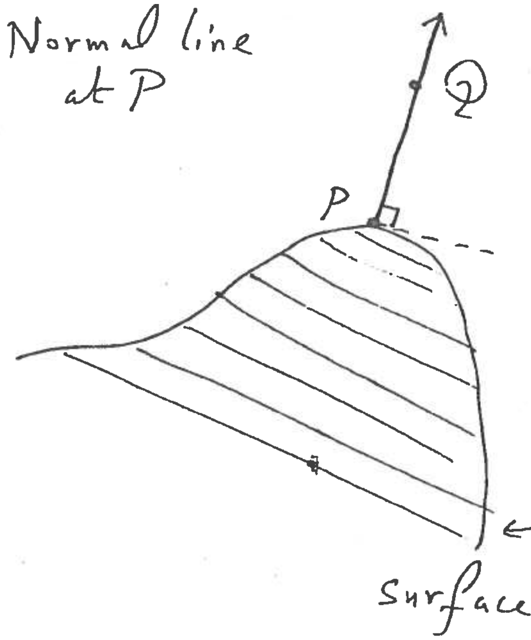
7. A vector Normal to surface at P is given by



$$\vec{N} = \vec{\nabla} F(P)$$
$$= \left(\frac{\partial F}{\partial x}, \frac{\partial F}{\partial y}, \frac{\partial F}{\partial z} \right) \Big|_P$$

Here $3x^2 + y^2 - 4z = 0$

$\therefore$ Take $F(x, y, z) = 3x^2 + y^2 - 4z$

$\therefore \vec{N} = (6x, 2y, -4) \Big|_{P(x_0, y_0, 7)} = (6x_0, 2y_0, -4)$

On the other hand, the line through the points $P(x_0, y_0, 7)$, and $Q(1, 2, 8)$ has a direction vector $\vec{v} = \vec{PQ} = (1,2,8) - (x_0, y_0, 7)$
$$= (1 - x_0, 2 - y_0, 1)$$

But $\vec{v} \parallel \vec{N}$, therefore

$\vec{v}$ = a scalar Multiple of $\vec{N}$, say $k\vec{N}$

$\therefore \vec{v} = K\vec{N}$ or $\vec{N} = k\vec{v}$ (easier)

$$K(1 - x_0, 2 - y_0, 1) = (6x_0, 2y_0, -4)$$

Equating Corresponding Components:

8. (a) $f(x, y) = e^{xy}$

By definition:

$$df = \frac{\partial f}{\partial x}\, dx + \frac{\partial f}{\partial y}\, dy$$

Now, $\frac{\partial f}{\partial x} = e^{xy} \cdot y$, and $\frac{\partial f}{\partial y} = e^{xy} \cdot x$

$$\therefore \; df = y e^{xy}\, dx + x e^{xy}\, dy$$

$$= e^{xy} \left[y\, dx + x\, dy \right]$$

— — — — —

(b) $f(x, y, z) = xyz$

$\frac{\partial f}{\partial x} = yz$, $\frac{\partial f}{\partial y} = xz$, and $\frac{\partial f}{\partial z} = xy$

$$\therefore \; df = \frac{\partial f}{\partial x}\, dx + \frac{\partial f}{\partial y}\, dy + \frac{\partial f}{\partial z}\, dx$$

$$= yz\, dx + xz\, dy + xy\, dz$$

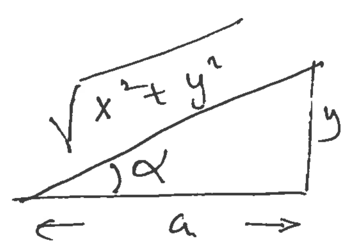
— — — — —

(c) $f(x, y) = \sin^{-1}\left(\frac{y}{\sqrt{x^2 + y^2}} \right)$

let us first simplify!

let $\alpha = \sin^{-1}\left(\frac{y}{\sqrt{x^2 + y^2}} \right)$

$$\therefore \; \sin(\alpha) = \frac{y}{\sqrt{x^2 + y^2}}$$

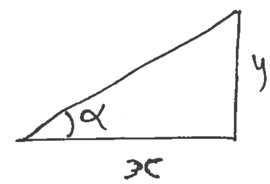


$(a > 0)$

Now, $a^2 + y^2 = \left(\sqrt{x^2+y^2}\right)^2$
$= x^2 + y^2$

$\Rightarrow \quad a^2 = x^2$, hence $a = x$ (Since $a>0$, $x>0$)

$\therefore \quad \tan(\alpha) = \dfrac{y}{x}$



It follows that

$$f(x,y) = \alpha = \tan^{-1}\left(\frac{y}{x}\right)$$

$$\therefore \quad \frac{\partial f}{\partial x} = \frac{1}{1+\left(\frac{y}{x}\right)^2}\,\frac{\partial}{\partial x}\left(\frac{y}{x}\right) = \frac{1}{1+\frac{y^2}{x^2}}\left(-\frac{y}{x^2}\right)$$

$$= \frac{-y}{x^2+y^2}$$

$$\frac{\partial f}{\partial y} = \frac{1}{1+\left(\frac{y}{x}\right)^2}\cdot\frac{\partial}{\partial y}\left(\frac{y}{x}\right) = \frac{1}{1+\frac{y^2}{x^2}}\cdot\frac{1}{x}$$

$$= \frac{1}{\left(1+\frac{y^2}{x^2}\right)}\cdot\frac{x}{x^2} = \frac{x}{x^2+y^2}$$

$$\therefore \quad df = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy = -\frac{y}{x^2+y^2}dx + \frac{x}{x^2+y^2}dy$$

$$\overset{\text{or}}{=} \frac{1}{x^2+y^2}\left[-y\,dx + x\,dy\right]$$

(d) For students to do at home. Answer:

$$df = \frac{-2}{(x^2+y^2+z^2)}\left[x\,dx + y\,dy + z\,dz\right]$$

9. Recall: The linearization of $f(x,y)$ at the point (a,b) is given by

$$L(x,y) = f(a,b) + \frac{\partial f}{\partial x}(a,b)(x-a) + \frac{\partial f}{\partial y}(a,b)(y-b)$$

(a) Here $f(x,y) = \tan^{-1}\left(\frac{y}{x}\right)$, $(a,b) = (3,3)$

$$\therefore \quad f_x(x,y) = \frac{1}{1+\left(\frac{y}{x}\right)^2}\left(-\frac{y}{x^2}\right)$$

$$f_y(x,y) = \frac{1}{1+\left(\frac{y}{x}\right)^2}\left(\frac{1}{x}\right)$$

No need to Simplify!

At $(x,y) = (3,3)$:

$$f(3,3) = \tan^{-1}\left(\frac{3}{3}\right) = \tan^{-1}(1) = \frac{\pi}{4},$$

$$f_x(3,3) = \frac{1}{1+\left(\frac{3}{3}\right)^2}\left(-\frac{3}{3^2}\right) = \frac{1}{2}\cdot\left(-\frac{1}{3}\right) = -\frac{1}{6}$$

$$f_y(3,3) = \frac{1}{1+\left(\frac{3}{3}\right)^2}\cdot\left(\frac{1}{3}\right) = \frac{1}{2}\cdot\frac{1}{3} = \frac{1}{6}$$

$$\therefore \quad L(x,y) = \frac{\pi}{4} - \frac{1}{6}(x-3) + \frac{1}{6}(y-3)$$

(b) $f(x,y) = \sqrt{x^2+y^2+5}$, $(a,b) = (4,2)$

$$f_x(x,y) = \frac{1}{2}\left(x^2+y^2+5\right)^{-\frac{1}{2}} \cdot 2x = \frac{x}{\sqrt{x^2+y^2+5}}$$

$$f_y(x,y) = \frac{1}{2}\left(x^2+y^2+5\right)^{-\frac{1}{2}} \cdot 2y = \frac{y}{\sqrt{x^2+y^2+5}}$$

$$\therefore f(4,2) = \sqrt{4^2+2^2+5} = \sqrt{25} = 5,$$

$$f_x(4,2) = \frac{4}{\sqrt{4^2+2^2+5}} = \frac{4}{5}, \text{ and } f_y(4,2) = \frac{2}{5}$$

$$\therefore L(x,y) = f(4,2) + f_x(4,2)(x-4) + f_y(4,2)(y-2)$$

$$= 5 + \frac{4}{5}(x-4) + \frac{2}{5}(y-2)$$

(c) For students to do at Home.

Answer:

$$L(x,y) = 2x + 9y - 8$$

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10. (a) To estimate $f(x,y) = \tan^{-1}(x+2y)$ at $(x,y) = (-3.1, 1.09)$ we use the linearization $L(x,y)$ of $\tan^{-1}(x+2y)$ at (a,b) where (a,b) is "close" to $(-3.1, 1.09)$.

Let us choose $(a,b) = (-3, 1)$

Now, $$L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$$
$$= f(-3,1) + f_x(-3,1)(x+3) + f_y(-3,1)(y-1)$$

Here $$f(x,y) = \tan^{-1}(x+2y)$$

$$\therefore f_x(x,y) = \frac{1}{1+(x+2y)^2}\cdot 1, \quad f_y(x,y) = \frac{1}{1+(x+2y)^2}\cdot 2$$

$$\therefore f(-3,1) = \tan^{-1}(-3+2) = \tan^{-1}(-1) = -\tan^{-1}(1) = -\frac{\pi}{4}$$
$$f_x(-3,1) = \frac{1}{1+(-3+2)^2} = \frac{1}{2}, \quad f_y(-3,1) = \frac{2}{1+(-3+2)^2} = 1$$

$$\therefore L(x,y) = -\frac{\pi}{4} + \frac{1}{2}(x+3) + 1(y-1) \leftarrow \text{Do \underline{not} expand!}$$

Recall $f(x,y) \approx L(x,y)$

$$\therefore f(-3.1, 1.09) \approx L(-3.1, 1.09)$$

$$\Rightarrow f(-3.1, 1.09) \approx -\frac{\pi}{4} + \frac{1}{2}(-3.1+3) + (1.09-1)$$
$$\tan^{-1}(-0.92) \approx -\frac{\pi}{4} - 0.05 + 0.09$$
$$\approx -\frac{\pi}{4} + 0.04 \approx 0.745$$

(b) To estimate $f(x,y) = \ln(x^2+y^2-12)$ at $(x,y) = (2.03, 2.99)$ we use the linearization $L(x,y)$ at $(a,b) = (2,3)$

Now, $f(x,y) = \ln(x^2+y^2-12) \Rightarrow f(2,3) = \ln(2^2+3^2-12) = \ln 1 = 0$,

$$f_x(x,y) = \frac{2x}{x^2+y^2-12} \Rightarrow f_x(2,3) = \frac{2(2)}{2^2+3^2-12} = \frac{4}{1} = 4,$$

and $$f_y(x,y) = \frac{2y}{x^2+y^2-12} \Rightarrow f_y(2,3) = \frac{2(3)}{2^2+3^2-12} = \frac{6}{1} = 6$$

$$\therefore L(x,y) = f(2,3) + f_x(2,3)(x-2) + f_y(2,3)(y-3)$$

$$= 0 + 4(x-2) + 6(y-3)$$

$$\overset{\text{or}}{=} 4(x-2) + 6(y-3) \leftarrow \text{Do } \underline{\text{not}} \text{ Expand!}$$

$$f(x,y) \approx L(x,y)$$

$$\therefore f(2.03, 2.99) \approx L(2.03, 2.99)$$

$$\Rightarrow \ln(1.061) \approx 4(2.03-2) + 6(2.99-3)$$

$$\approx 4(0.03) + 6(-0.01)$$

$$\approx 0.06$$

(c) For students to do at Home

<u>Answer</u>:

$$f(1.08, 2.16) = \frac{1}{\sqrt{4.64}} \approx 0.46$$

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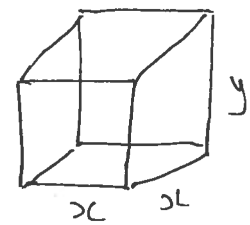
11. Volume of box:

$$V = x^2 y$$



Recall $\Delta V \approx dV$

where $dV = \frac{\partial V}{\partial x} dx + \frac{\partial V}{\partial y} dy \overset{\text{or}}{=} \frac{\partial V}{\partial x} \Delta x + \frac{\partial V}{\partial y} \Delta y$

$$\therefore \quad \Delta V \approx \frac{\partial}{\partial x}(x^2 y) \cdot \Delta x + \frac{\partial}{\partial y}(x^2 y) \cdot \Delta y$$

$$\approx (2xy)\,\Delta x + (x^2)\,\Delta y$$

Know:

$\frac{\Delta x}{x} = 1.5\%$

$\frac{\Delta y}{y} = -1\%$

want:

$\frac{\Delta V}{V}$?

Dividing both sides by $V = x^2 y$, we obtain

$$\frac{\Delta V}{V} \approx \frac{2xy}{x^2 y} \Delta x + \frac{x^2}{x^2 y} \Delta y$$

$$\approx 2\left(\frac{\Delta x}{x}\right) + \left(\frac{\Delta y}{y}\right)$$

$$\approx 2(1.5\%) + (-1\%)$$

$$\therefore \quad \frac{\Delta V}{V} \approx 2\%$$

The volume of box increases by approximately 2%

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12. For students to do at home.

<u>Answer</u>: Surface area decrease by 2 square centimetres

That is $\Delta S \approx -2 \text{ cm}^2$

<u>Hint</u>: Total surface area $= x^2 + 4xy$

$\Delta x = 10.2 - 10 = 0.2$ cm,

$\Delta y = 4.75 - 5 = -0.25$ cm.

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13. let r, h, and V be respectively radius, height and volume of cone.

$\therefore \quad V = \frac{1}{3}\pi r^2 h,$

$$\frac{\partial V}{\partial r} = \frac{2}{3}\pi r h, \quad \frac{\partial V}{\partial h} = \frac{1}{3}\pi r^2$$

Recall $\quad \Delta V \approx dV = \frac{\partial V}{\partial r}\Delta r + \frac{\partial V}{\partial h}\Delta h$

$$\therefore \quad \Delta V \approx \frac{2}{3}\pi r h \,\Delta r + \frac{1}{3}\pi r^2 \Delta h$$

$$\approx \frac{1}{3}\pi\left[2rh\,\Delta r + r^2 \Delta h\right]$$

Here $r = 10$, $\Delta r = 10.2 - 10 = 0.2$ cm,

$h = 8$, $\Delta h = 7.63 - 8 = -0.37$ cm

$$\therefore \Delta V \approx \frac{1}{3}\pi\left[2(10)(8)(0.2) + (10)^2(-0.37)\right]$$

$$\approx \frac{1}{3}\pi[-5] \approx -5.24 \text{ cm}^3$$

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14. $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$

$$d\left(\frac{1}{R}\right) = d\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$-\frac{1}{R^2}\,dR = \frac{\partial}{\partial R_1}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)\Delta R_1 + \frac{\partial}{\partial R_2}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)\Delta R_2$$

$$-\frac{1}{R^2}\,dR = -\frac{1}{R_1^2}\,\Delta R_1 - \frac{1}{R_2^2}\,\Delta R_2$$

Multiplying both sides by $-R$:

$$\frac{dR}{R} = \frac{R}{R_1^2}\,\Delta R_1 + \frac{R}{R_2^2}\,\Delta R_2$$

But $\Delta R \approx dR$

$$\therefore \quad \frac{\Delta R}{R} \approx \frac{R}{R_1^2}\,\Delta R_1 + \frac{R}{R_2^2}\,\Delta R_2$$

Here $\Delta R_1 = 105 - 100 = 5$ ohms,
$\Delta R_2 = -1.5$ ohms

and $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{100} + \frac{1}{25} = \frac{1}{100} + \frac{4}{100} = \frac{5}{100} = \frac{1}{20}$

$$\Rightarrow \boxed{R = 20}$$

$$\therefore \quad \frac{\Delta R}{R} \approx \frac{20}{(100)^2}(5) + \frac{20}{(25)^2}(-1.5)$$

$$\approx 0.01 - 0.048 = -0.038$$

$$\Rightarrow \quad \frac{\Delta R}{R} \approx -(0.038)(100)\ \%$$

$$\approx -3.8\ \%$$

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