MATH 277 Midterm (Fall 2015)

Thank you for attending my **PREP101** session! Best of luck!

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1. $\vec{r}(t) = \langle t^3, \sqrt{3}t^2, (2t+1) \rangle \qquad 0 \le t \le 2$

$\vec{r}'(t) = \langle 3t^2, 2\sqrt{3}t, 2 \rangle$

$\|\vec{r}'(t)\| = \sqrt{6t^4 + 12t^2 + 4} = (3t^2+2)$

$L = \int_0^2 (3t^2+2)\,dt = (t^3+2t)\Big|_0^2 = 8+4 = 12 \Rightarrow (A)$

2. Ellipse centered at $(4,-2)$ $\qquad \dfrac{(x-4)^2}{a^2} + \dfrac{(y+2)^2}{b^2} = 1$

$x = a\cos t + 4 \;;\; y = b\sin t - 2 \Rightarrow (C)$

3. $\vec{r}'(t) = \langle x'(t), y'(t) \rangle = \langle 6t^2-6t, 2t-2 \rangle$

Slope: $\dfrac{2(t-1)}{6(t^2-t)} = \dfrac{2(t-1)}{6t(t-1)} = \dfrac{1}{3t}$ when $t=1$ $m = 1/3$

when $t=1$ $(x,y) = (0,1)$ $\Rightarrow y = \frac{1}{3}x + 1$

or $x - 3y + 3 = 0 \Rightarrow (B)$

4. $4x^2 + y^2 + z^2 = 8$

$z = -\sqrt{4x^2+y^2}$

$4x^2 + y^2 + 4x^2 + y^2 = 8$

$8x^2 + 2y^2 = 8$

$x^2 + \dfrac{y^2}{4} = 1$

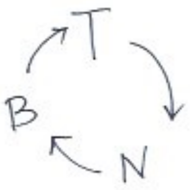
$x = \cos t$

$y = 2\sin t$

$z = -\sqrt{4\cos^2 t + 4\sin^2 t}$

$= -2$

5.



$$\hat{T} \times \hat{N} = \hat{B}$$
$$\hat{N} \times \hat{B} = \hat{T} \Rightarrow (E)$$
$$\hat{B} \times \hat{T} = \hat{N}$$

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6. $\vec{r}'(t) = \left\langle \frac{2}{t}, 1, -\frac{2}{t^2} \right\rangle \quad @ \ t=1 \quad \vec{r}'(1) = \langle 2, 1, -2 \rangle$

$$\|r'(1)\| = \sqrt{9} = 3$$

$$\vec{r}''(t) = \left\langle -\frac{2}{t^2}, 0, \frac{4}{t^3} \right\rangle$$

$$\vec{r}' \times \vec{r}'' = \left\langle \frac{4}{t^3}, -\frac{4}{t^4}, \frac{2}{t^2} \right\rangle \quad @ \ t=1 \quad \langle 4, -4, 2 \rangle$$

$$\| r' \times r'' \| = 2\sqrt{4+4+1} = 2\sqrt{9} = 6$$

$$K = \frac{6}{27} = \frac{2}{9} \Rightarrow \rho = 9/2. \rightarrow (C)$$

7. $\vec{r}'(t) = \langle -2\sin t, 2\cos t, 1 \rangle \quad \|\vec{r}'(t)\| = \sqrt{5} \quad a_{\hat{T}} = 0 \Rightarrow (C)$

8. $W_x = \sqrt{5}\cos(\sqrt{5}x + 2y)\cosh z \qquad W_{xx} = -5\sin(\sqrt{5}x + 2y)\cosh z$

$W_y = 2\cos(\sqrt{5}x + 2y)\cosh z \qquad W_{yy} = -4\sin(\sqrt{5}x + 2y)\cosh z$

$W_z = K\sin(\sqrt{5}x + 2y)\sinh z \qquad W_{zz} = K^2\sin(\sqrt{5}x + 2y)\cosh z$

$$W_{xx} + W_{yy} + W_{zz} = 0$$
$$\Rightarrow K^2 - 9 = 0 \Rightarrow K = 3 \text{ or } -3$$

9. (A) $\left.\begin{array}{l} x^2 + 4y^2 - 63 > 0 \\ \& \ x^2 + 4y^2 - 63 \geq 1 \end{array}\right\} \Rightarrow x^2 + 4y^2 \geq 64$

10. (D)

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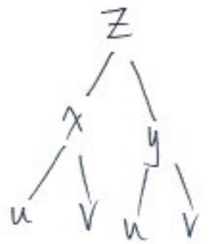
11. $\dfrac{\partial z}{\partial y} = x\, y^{x-1}$

$$\frac{\partial^2 z}{\partial x \partial y} = x\, y^{x-1} \cdot \ln y + y^{x-1} = y^{x-1}(x \ln y + 1)$$

12.



$$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial u}$$

$$= (z_x)\, v^2 + (z_y)\left(\frac{1}{v}\right)$$

$$= (3)(-1)^2 + (-2)(-1) = 5$$

when $(u,v) = (2,-1)$
$(x,y) = (2,-2)$

13. $\dfrac{dW}{dt} = \dfrac{\partial W}{\partial x}\dfrac{dx}{dt} + \dfrac{\partial W}{\partial y}\dfrac{dy}{dt} = 2e^{2x+y} \cdot (1+\cos t) + e^{2x+y}(2)$

when $t=0$; $x=0, y=-1$ $\Rightarrow$ $\dfrac{dW}{dt} = 2e^{0-1}(1+1) + e^{2(0)-1}(2)$

$$= \frac{4}{e} + \frac{2}{e} = \frac{6}{e}$$

14. $\vec{\nabla F} = \langle 10x - 4y, 2 \rangle$ @ $(1,2,-3)$ $\quad \langle 10, -8, 2 \rangle$ choose $\vec{n} = \langle 5, -4, 1 \rangle$

Pt $(1,2,-3)$

$5(x-1) - 4(y-2) + 1(z+3) = 0$

$5x - 4y + z + 6 = 0$

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15. $$\frac{\partial y}{\partial z} = -\frac{F_z}{F_y} = \frac{+2\cos(3xy-2z)}{2x^3y + 3x\cos(3xy-2z)}$$

@ $(1,2,3)$

$$\frac{\partial y}{\partial z} = \frac{2\cos(0)}{4+3\cos(0)} = \frac{2}{7}.$$

16. $\vec{\nabla} f = \langle y+3z,\ x+2z,\ 3x+2y\rangle$ @ $(1,1,-1)$ $\quad \vec{\nabla} f = \langle -2,-1,5\rangle$

$\vec{PQ} = \langle -2,2,-1\rangle \qquad \|\vec{PQ}\| = 3 \quad \Rightarrow \vec{u} = \langle -\frac{2}{3}, \frac{2}{3}, -\frac{1}{3}\rangle$

$$D_{\hat{u}} f = -2\left(-\tfrac{2}{3}\right) - 1\left(\tfrac{2}{3}\right) + 5\left(-\tfrac{1}{3}\right) = \frac{4-2-5}{3} = -1.$$

17. $\vec{\nabla} f = \langle -6x^2, -3, 2z\rangle$ @ $(1,-1,3)$ $\quad \vec{\nabla} f = \langle -6,-3,6\rangle = 3\langle -2,1,2\rangle$

$\|\vec{\nabla} f\| = 3\sqrt{9} = 9.$

(D) is the FALSE Statement.

18. $R = \frac{Ky}{x^2} \qquad \frac{\Delta y}{y} = 0.01 \qquad \frac{\Delta x}{x} = -0.04$

$$dR = \frac{\partial R}{\partial y}dy + \frac{\partial R}{\partial x}dx = \frac{K}{x^2}\cdot dy + \frac{-2Ky}{x^3}dx$$

$$\frac{dR}{R} = \frac{K/x^2}{Ky/x^2}dy + \frac{-2Ky/x^3}{Ky/x^2}dx = \frac{dy}{y} - \frac{2dx}{x} \approx 0.01 + 0.08$$

$$= 0.09 \quad (9\%)$$

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19. for $c=1$ $\qquad \frac{y^2}{16} - \frac{x^2}{K^2} = 1$

for $c=-1$ $\qquad x^2+y^2=4$

for $c=0$ $\qquad \frac{x^2}{16} + \frac{y^2}{C^2} = 1.$

Check (D) $c=1$ $\qquad 4x^2+10y^2-64 = 12x^2+6y^2$

$$4y^2-8x^2 = 64$$

$$\frac{y^2}{16} - \frac{x^2}{8} = 1 \checkmark$$

$c=0$ $\qquad 4x^2+10y^2=64$

$$\frac{x^2}{16} + \frac{y^2}{64/10} = 1$$

$c=-1$ $\qquad 4x^2+10y^2-64 = -12x^2-6y^2$

$$16x^2+16y^2=64$$

$$x^2+y^2=4$$

20. $m(0) = 40{,}000$ $\qquad 1000$ Kg/s , $v_e = 400$ m/s $\qquad v(0)=0$ $\qquad t=30$ secs.

$m(t) = 40{,}000 - 1000t$ $\qquad 0 \le t \le 20 \leftarrow$ (all fuel is used)

$$v(20) = v(30) = 400 \ln\left(\frac{40{,}000}{m(20)}\right) = 400 \ln 2. \Rightarrow (A)$$