David Ng **F2016MATH375**

Assignment05 is due on Friday, December 16, 2016 at 11:59pm.

The number of attempts available for each question is noted beside the question. If you are having trouble figuring out your error, you should consult the textbook, or ask a fellow student, one of the TA's or your professor for help.
There are also other resources at your disposal, such as the Mathematics Continuous Tutorials. Don't spend a lot of time guessing – it's not very efficient or effective.

Make sure to give lots of significant digits for (floating point) numerical answers. For most problems when entering numerical answers, you can if you wish enter elementary expressions such as $2 \wedge 3$ instead of 8, $\sin(3*\text{pi}/2)$instead of -1, $e \wedge (\ln(2))$ instead of 2, $(2+\tan(3))*(4-\sin(5)) \wedge 6-7/8$ instead of 27620.3413, etc.

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1. (1 point) The left end of a rod of length L is held at temperature 160, and there is heat transfer from the right end into the surrounding medium at temperature zero. The initial temperature at any point x is given by $f(x)$.
Select the partial differential equation that can be used to model this scenario.

- A. $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial t^2} = 0, \quad 0 < x < L, t > 0$
- B. $k\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}, \quad 0 < x < L, t > 0$
- C. $\alpha^2 \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}, \quad 0 < x < L, t > 0$

Select ALL boundary/initial conditions that apply to this scenario

- A. $u(x,160) = f(x), \quad 0 < x < L$
- B. $u(0,t) = L, \quad t > 0$
- C. $u(x,L) = f(x), \quad 0 < x < L$
- D. $u(L,t) = 0, \quad t > 0$
- E. $u(x,0) = f(x), \quad 0 < x < L$
- F. $u(x,0) = 0, \quad 0 < x < L$
- G. $u(0,t) = 160, \quad t > 0$
- H. $u(0,t) = 0, \quad t > 0$
- I. $u(x,0) = 160, \quad 0 < x < L$

Answer(s) submitted:

- B
- (D, E, G)

(correct)

Correct Answers:

- B
- DEG

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2. (1 point) The left end of a rod of length L is held at temperature 0, and the right end is insulated. The initial temperature at any point x is given by $f(x)$.
Select the partial differential equation that can be used to model this scenario.

- A. $k\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}, \quad 0 < x < L, t > 0$
- B. $\alpha^2 \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}, \quad 0 < x < L, t > 0$
- C. $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial t^2} = 0, \quad 0 < x < L, t > 0$

Select ALL boundary/initial conditions that apply to this scenario

- A. $u(x,L) = f(x), \quad 0 < x < L$
- B. $u(x,0) = 0, \quad 0 < x < L$
- C. $u(0,t) = L, \quad t > 0$
- D. $u(0,t) = 0, \quad t > 0$
- E. $\left.\frac{\partial u}{\partial x}\right|_{x=L} = 0, \quad t > 0$
- F. $u(x,0) = f(x), \quad 0 < x < L$
- G. $u(L,t) = 0, \quad t > 0$
- H. $\left.\frac{\partial u}{\partial x}\right|_{t=L} = 0, \quad 0 < x < L$
- I. $\left.\frac{\partial u}{\partial t}\right|_{x=L} = 0, \quad t > 0$
- J. $\left.\frac{\partial u}{\partial t}\right|_{t=L} = 0, \quad 0 < x < L$

Answer(s) submitted:

- A
- (D, E, F)

(correct)

Correct Answers:

- A
- DEF

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3. (1 point) A string of length L is secured at both ends. The string is released from rest with initial displacement $f(x)$ at any point x.
Choose the PDE and boundary/initial conditions that model this scenario.
Select the partial differential equation that can be used to model this scenario.

- A. $k\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}, \quad 0 < x < L, t > 0$

- B. $\alpha^2 \dfrac{\partial^2 u}{\partial x^2} = \dfrac{\partial^2 u}{\partial t^2}, \quad 0 < x < L, t > 0$
- C. $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial t^2} = 0, \quad 0 < x < L, t > 0$

Select ALL boundary/initial conditions that apply to this scenario

- A. $\left.\dfrac{\partial u}{\partial x}\right|_{x=L} = 0, \quad t > 0$
- B. $u(0,t) = L, \quad t > 0$
- C. $u(x,0) = 0, \quad 0 < x < L$
- D. $u(x,0) = f(x), \quad 0 < x < L$
- E. $u(L,t) = 0, \quad t > 0$
- F. $u(x,L) = f(x), \quad 0 < x < L$
- G. $\left.\dfrac{\partial u}{\partial t}\right|_{x=L} = 0, \quad t > 0$
- H. $\left.\dfrac{\partial u}{\partial t}\right|_{x=0} = 0, \quad t > 0$
- I. $\left.\dfrac{\partial u}{\partial t}\right|_{t=0} = f(x), \quad 0 < x < L$
- J. $\left.\dfrac{\partial u}{\partial x}\right|_{t=0} = f(x), \quad 0 < x < L$
- K. $u(0,t) = 0, \quad t > 0$
- L. $\left.\dfrac{\partial u}{\partial x}\right|_{x=0} = 0, \quad t > 0$
- M. $u(x,0) = L, \quad 0 < x < L$

Answer(s) submitted:

- `B`
- `( D, E, K )`

(correct)
Correct Answers:

- `B`
- `DEK`

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4. (1 point) A string of length L is secured at both ends. The string has no initial displacement, but has initial velocity $f(x)$ at any point x.
Choose the PDE and boundary/initial conditions that model this scenario.
Select the partial differential equation that can be used to model this scenario.

- A. $k \dfrac{\partial^2 u}{\partial x^2} = \dfrac{\partial u}{\partial t}, \quad 0 < x < L, t > 0$
- B. $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial t^2} = 0, \quad 0 < x < L, t > 0$
- C. $\alpha^2 \dfrac{\partial^2 u}{\partial x^2} = \dfrac{\partial^2 u}{\partial t^2}, \quad 0 < x < L, t > 0$

Select ALL boundary/initial conditions that apply to this scenario

- A. $\left.\dfrac{\partial u}{\partial t}\right|_{x=0} = 0, \quad t > 0$
- B. $\left.\dfrac{\partial u}{\partial x}\right|_{x=L} = 0, \quad t > 0$
- C. $u(x,0) = L, \quad 0 < x < L$
- D. $\left.\dfrac{\partial u}{\partial x}\right|_{t=0} = f(x), \quad 0 < x < L$
- E. $u(x,L) = f(x), \quad 0 < x < L$
- F. $\left.\dfrac{\partial u}{\partial t}\right|_{x=L} = 0, \quad t > 0$
- G. $u(x,L) = L, \quad 0 < x < L$
- H. $u(0,t) = 0, \quad t > 0$
- I. $u(L,t) = 0, \quad t > 0$
- J. $u(0,t) = L, \quad t > 0$
- K. $u(x,0) = f(x), \quad 0 < x < L$
- L. $u(x,0) = 0, \quad 0 < x < L$
- M. $\left.\dfrac{\partial u}{\partial x}\right|_{x=0} = 0, \quad t > 0$
- N. $\left.\dfrac{\partial u}{\partial t}\right|_{t=0} = f(x), \quad 0 < x < L$

Answer(s) submitted:

- `C`
- `( H, I, L, N )`

(correct)
Correct Answers:

- `C`
- `HILN`

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5. (1 point) Solve the Boundary-Initial Value Problem

$$k \frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}, \quad 0 < x < L, \quad t > 0$$
$$\frac{\partial u}{\partial x}(0,t) = 0, \quad \frac{\partial u}{\partial x}(L,t) = 0, \quad t > 0$$
$$u(x,0) = x^2, \quad 0 < x < L$$

that models the temperature of a heated wire with insulated ends. The series solution of the boundary value problem is

$$u(x,t) = \frac{2}{L} \sum_{n=0}^{\infty} a_n e^{\left(-k \frac{n^2 \pi^2}{L^2} t\right)} \cos\left(\frac{n\pi}{L} x\right)$$

where $a_n =$ ________

Remember that $\frac{2}{L}$ has already been factored out!
Answer(s) submitted:

- `2Lcos(n*pi)/(n*pi/L)^2`

(correct)
Correct Answers:

- `2*L^3*(-1)^n/[(pi*n)^2]`

6. (1 point) Consider the Boundary-Initial Value problem

$$2\,\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}, \quad 0 < x < 5, \quad t > 0$$
$$u(0,t) = 0, \quad u(5,t) = 0, \quad t > 0$$
$$u(x,0) = x(5-x), \quad 0 < x < 5$$

This models a heated wire, with zero endpoints temperatures. The solution $u(x,t)$ of the initial-boundary value problem is given by the series

$$u(x,t) = \frac{8}{\pi^3}\sum_{n=1}^{\infty} b_n \,\sin\left((2n-1)\frac{\pi}{5}x\right)\, e^{-c_n t}$$

where
$b_n =$ ____________________
and
$c_n =$____________________

Answer(s) submitted:

- `5^2/(2n-1)^3`
- `2*(2n-1)^2*pi^2/5^2`

(correct)

Correct Answers:

- `25/[(2*n-1)^3]`
- `2*(2*n-1)^2*pi^2/25`

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7. (1 point) Consider the Boundary-Initial Value problem

$$16\,\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}, \quad 0 < x < 5, \quad t > 0$$
$$u(0,t) = 0, \quad u(5,t) = 0, \quad t > 0$$
$$u(x,0) = x(5-x), \quad \frac{\partial u}{\partial t}(x,0) = 0, \quad 0 < x < 5$$

This models the displacement $u(x,t)$ of a freely vibrating string, with fixed ends, initial profile $x\,(5-x)$, and zero initial velocity.
The solution $u(x,t)$, is given by the series

$$u(x,t) = \frac{4}{\pi^3}\sum_{n=1}^{\infty} b_n\,\sin\left(n\,\frac{\pi}{5}\,x\right)\,\cos(c_n\,t)$$

where
$b_n =$ ____________________
and
$c_n =$____________________

Answer(s) submitted:

- `25*(1-cos(n*pi))/n^3`
- `4*n*pi/5`

(correct)

Correct Answers:

- `25/(n^3)*[1-cos(n*pi)]`
- `n*pi/5*4`

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8. (1 point) Consider the Boundary Value problem

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \quad 0 < x, y < 7$$
$$u(x,0) = x(7-x), \quad u(x,7) = 0, \quad 0 < x < 7$$
$$u(0,y) = 0, \quad u(7,y) = 0, \quad 0 < y < 7$$

The solution $u(x,y)$, is given by the series

$$u(x,y) = \frac{4}{\pi^3}\sum_{n=1}^{\infty} b_n\,\sin\left(n\,\frac{\pi}{7}\,x\right)\,\frac{\sinh\left(n\,\frac{\pi}{7}(7-y)\right)}{\sinh(n\pi)}$$

where
$b_n =$ ____________________

Answer(s) submitted:

- `(7/n)^3(1-cos(n*pi))`

(correct)

Correct Answers:

- `49/(n^3)*[1-cos(n*pi)]`