Math 375 **Fall 2016**

First Order Differential Equations

Worksheet # 1 **Part 1** **September 19 - 23**

The problems marked with $(*)$ are to be attempted during the tutorial time. Students are strongly encouraged to attempt the remaining problems on their own. Solutions to all the problems will be available on the course's D2L website Friday, September 23. Please report any typos, omissions and errors to `aiffam@ucalgary.ca`

Basics

01*. For each of the differential equations, determine its order and whether it is linear or nonlinear.

a. $\big(t\,y''+y\big)\,(t^2+\mathrm{e}^t)=1$ **b.** $\big((t^2+1)\,y'\big)'+y^3=t$ **c.** $t\,y'+|y|=\tan(t)$

02. Verify that $u(t)=\dfrac{t^2}{3}+\dfrac{C}{t}$, is a solution of the differential equation $t\,y'+y=t^2$

03*. Verify that $y^2-2\,x\,y-x^2-8\,y+4\,x=C$, is an implicit solution of the differential equation

$$y'=\frac{y+x-2}{y-x-4}$$

04. Show that $u(t)=\begin{cases}1-\mathrm{e}^{-t} & \text{if} \quad t<0\\ \mathrm{e}^t-1 & \text{if} \quad t\geq 0\end{cases}$ is a solution of the differential equation

$$y'-|y|=1 \ \text{ in } \ (-\infty,+\infty)$$

Linear First Order

05. Find the general solution of the given differential equation.

a. $y'+\dfrac{1}{t}\,y=\dfrac{7}{t^2}+3$

b*. $t\,y'+\big(2\,t^2+1\big)\,y=t^3\,\mathrm{e}^{-t^2}$

c. $x^2y'+3\,x\,y=\mathrm{e}^x$

d*. $y'=y\,\cos(x)+\sin(2\,x)$

06. Solve the initial value problems.

a. $\begin{cases}y'+2\,x\,y=x\\ y(0)=3\end{cases}$

b*. $\begin{cases}x\,y'+3\,y=\dfrac{2}{x(x^2+1)}\\ y(-1)=0\end{cases}$

c. $\begin{cases}(t^2-5)\,y'-2\,t\,y=-2\,t\,(t^2-5)\\ y(2)=7\end{cases}$

07. Find the largest interval $\big(a\,,\,b\big)$ on which the solution of the given initial value problem is guaranteed to exist.

a*. $\begin{cases}(t+5)\,y'+t\,y=\ln\left(t-\dfrac{4}{t}\right)\\ y(-1)=1\end{cases}$

b*. $\begin{cases}(t+5)\,y'+t\,y=\ln\left(t-\dfrac{4}{t}\right)\\ y(3)=1\end{cases}$

c. $\begin{cases}y'+\dfrac{t}{t^2-4}\,y=\sqrt{5-t}\\ y(3)=0\end{cases}$

d. $\begin{cases}y'+\dfrac{t}{t^2-4}\,y=\sqrt{5-t}\\ y(1)=0\end{cases}$

Separable Equations

08. Solve the following differential equations.

a*. $t\,y' + y + y^2 = 0$ **b.** $(x-1)\,y\,\mathrm{d}y + x\,(y-3)\,\mathrm{d}x = 0$

c. $y' + 2\,x\big(y^2 - 3\,y + 2\big) = 0$ **d.** $x^2\,y\,y' = \big(y^2-1\big)^{3/2}$

09. Solve the initial value problem $\begin{cases} y' + 2\,t\,\big(y^2 - 3\,y + 2\big) = 0 \\ y(0) = 3 \end{cases}$ and find the largest interval of validity of the solution.

10. Solve the initial value problem, and find the largest interval of validity of the solution.

a. $\begin{cases} (1+2\,y)y' - 2\,x = 0 \\ y(2) = 3 \end{cases}$ **b*.** $\begin{cases} y' - t\,y^2 = 0 \\ y(0) = 1 \end{cases}$