Department of Mathematics and Statistics
MATH 375
Handout # 3
Applications of First Order Equations

1. Radium decomposes at a rate proportional to the amount present. If the half-life of Radium is 1600 years , find percentage lost in 4800 years.

2. Iodine-131 decomposes at a rate proportional to the amount present. Find the half-life of Iodine-131 if after 40 days , 96. 875% of the original amount has disintegrated.

3. A glass of boiling water (100°C) placed in a large room with constant temperature of 20°C cooled down to 60°C after 10 minutes. How much time will it take to cool water to 25°C?

4. If when the temperature of the air is 290 K , a certain substance cools from 370 K to 330 K in 10 minutes , find the temperature after 40 minutes.

5. A body cools from 60° C to 50° C in 15 minutes in air which is maintained at 30° C. How long will it take this body to cool from 100° C to 75° C in air that is maintained at 55° C? Assume Newton's Law of Cooling.

6. Assuming Newton's Law of Cooling. If the temperature of the air is 300 K , and a substance cools from 380 K to 340 K in 15 minutes , find the temperature of the substance after an additional 45 minutes.

7. A cup of boiling water (100°C) is placed outside. One minute later the temperature of the water is 70°C. and after another minute its temperature is 49°C. What is the outside temperature (the medium) ?

8. A tank contains 100 litres of water with 10 kg of salt. Pure tap water flows into the tank at a rate of 5 litres per minute, the mixture is well sturred and drains from the tank at the same rate. How much salt will be contained in the tank after an hour?

9. A tank contains 450 litres of brine made by dissolving 30 kg of salt in water. Saltwater containing $\frac{1}{9}$ kg of salt per litre runs in at the rate of 9 litre/min and the mixture kept uniform by stirring, runs out at the rate of 13.5 litre/min.
   a) Find the amount of salt in tank at time t.
   b) how much salt is in tank at the end of one hour?

10. A tank initially contains 40 litres of fluid in which there is dissolved 10 gram of salt. Starting at t 0, a brine containing 2 grams per litre of the dissolved salt flow into he tank at the rate of 2 litres/min. The mixture is kept uniform by continuous stirring nd the well-stirred mixture simultaneously flow out of the tank at the slower rate of 1 litre/min.

Let $x = x(t)$ be the amount of salt in the tank at time $t \geq 0$.
a) Set up the initial value problem satisfied by x. b) Find an expression for $x(t)$. c) How many grams of salt is in the tank at the end of 30 minutes ? d) what is the concentration of the brine after half an hour ? e) when is there 44 grams of salt in tank ?

11. A tank containing 0.5 m^3 of brine made by dissolving 40 kg of salt in water. Pure water runs into the tank at the rate of 3×10^{-4} m^3/s and the mixture , kept uniform by stirring , runs out at the same rate.
a) How much salt in tank at time $t > 0$? After 1 hour? b) How much salt in tank after a long time; that is as $t \to \infty$?

12. A tank of volume $0.5m^3$ is filled with brine containing 30 kg of dissolved salt. Water runs into the tank at the rate of 15×10^{-5} m^3/s and the mixture , kept uniform by stirring , runs out at the same rate.
a) How much salt is in tank after one hour? b) What is the concentration of the brine after one hour?

13. A large tank initially contains 100 litres of brine made by dissolving 10 kg of salt. Pure water runs into the tank at the rate of 5 litres / min and the mixture , kept uniform by stirring , runs out at the rate of 2 litres/min.
a) How much salt is in tank at the end of 15 min and what is the concentration at that time?
b) If the capacity of the tank is 250 litres, what is the concentration at the instant the tank overflows?

14. A 20 ohm resistor and a 5 henry inductor are connected in a series in an electric circuit in which there is initially a current flow of 20 amperes. Find expression for the current $I(t)$ at any time $t > 0$ if the emf is zero for $t > 0$.

15. A capacitor of 0.005 farad is connected in a series with a 25 ohm resistor and a generator of an emf of 50 volts. If the switch is closed at $t = 0$ and the initial charge on the capacitor is zero, find an expression for the charge and the current at any time $t > 0$.

16. An inductor of L henries, where $L = 0.05 + 0.001t$, $t \in [0, 100]$, is connected in a series with a 40 emf volts and a 10 ohm resistor. If the initial current in the circuit is zero, find an expression for the current $I(t)$, $t > 0$. What is the maximum current passing through the circuit?

17. A resistor of R ohms, where $R = 3 + 0.5t$, $t \in [0, 100]$ is connected in a series with a 0.5 farad capacitor and a generator of an emf of $(3 + 0.5t)^{-3}$ volts. If the initial charge on the capacitor is zero, find the charge and current as a function of time. What is the maximum charge?

18. An inductor of 0.1 henry, a resistor of 10 ohms and an emf $E(t)$ volts are connected in a series, where

$$E(t) = \begin{cases} 10, & 0 \leq t \leq 5, \\ 0, & t > 5 \end{cases}$$

Find the current $I(t)$, assuming $I(0) = 0$.