# Department of Mathematics and Statistics
# MATH 375
# Handout # 4
# Second Order Linear Equations

1. Solve the initial value problem

$$\frac{d^2y}{dx^2} = e^x - \frac{1}{(1+x)^2}, \quad y(0) = 1, \quad y'(0) = -2.$$

2. Prove that the following functions are linearly independent:

   a) $e^{2x}$ and $e^{-2x}$
   b) $e^{2x}$ and $e^{5x}$
   c) $\cos(3x)$ and $\sin(3x)$

3. Two functions $y_1 = x$ and $y_2 = x^3$ are solutions of the differential equation

$$y'' + p(x)y' + q(x) = 0.$$

   Can $p(x)$ and $q(x)$ be continuous on $(-\infty, \infty)$? If not, what is the discontinuity point?

4. Find the general solution of the following differential equations

   a) $y'' - 5y' + 6y = 0$
   b) $y'' - 25y = 0$
   c) $y'' + y' = 0$
   d) $y'' + 2y' + y = 0$
   e) $y'' - 6y' + 9y = 0$
   f) $y'' + 5y = 0$
   g) $y'' + 2y' + 5y = 0$
   h) $y'' - y' + y = 0$

5. Find the solution of $y'' + y = 0$ satisfying $y(\frac{\pi}{2}) = 1$, $y'(\frac{\pi}{2}) = 0$.

6. Find an equation $y'' + by' + cy = 0$, such that it has following solutions:
   a) $y_1 = e^x$, $y_2 = e^{-x}$
   b) $y_1 = e^{2x}$, $y_2 = xe^{2x}$

7. For which $c$ all solutions of the equation $y'' + cy = 0$ are periodic?

8. Find the general solution of the following differential equations
   a) $y'' - 2y' - 3y = e^{4x}$
   b) $y'' - y = -x^2$

c) $y'' + 3y' - 4y = xe^{-x}$
d) $y'' - 4y' + 3y = 20\cos 2x + 35\sin 2x$
e) $y'' - 4y' + 4y = 2\sin 2x + x$
f) $y'' - y = 2e^{x}$

9. Find the solution of $y'' + y = -\sin 2x$ satisfying $y(\pi) = 1$, $y'(\pi) = 1$.

10. Find the general solution of the following equations
a)$y'' + 2y' + y = \frac{1}{x}e^{-x}$
b)$y'' - 4y' + 5y = \frac{1}{\cos x}e^{2x}$

11. Find $b$ and $c$ such that, for some real $A$, the function $y = Ax^2e^{4x}$ is a solution of the differential equation
$$y'' + by' + cy = 5e^{4x}.$$

12. Find the Wronskian for the two solutions $y_1$ and $y_2$ of the equation
$$y'' + \frac{1}{1+t}y'(t) + q(t)y(t) = 0,$$
where $q(t)$ is continuous for $t \geq 0$, corresponding to the initial values
a) $y_1(0) = 2,\ y_1'(0) = -3, \quad y_2(0) = -1,\ y_2'(0) = 2$
b) $y_1(0) = 2,\ y_1'(0) = -3, \quad y_2(0) = -4,\ y_2'(0) = 6$

13. Find $p$ such that the Wronskian for the two solutions $y_1$ and $y_2$ of the equation
$$y'' + p(t)y'(t) + qy(t) = 0,$$
where $q$ is a constant, corresponding to the initial values $y_1(0) = 1,\ y_1'(0) = 0,\ \ y_2(0) = 0,\ y_2'(0) = 1$ is
$$W[y_1, y_2](t) = e^{7t^2}.$$

14. Find all solutions of the following boundary value problems for $y(t)$:
a) $y'' + 16y = 0,\ y(0) = 0,\ y\left(\frac{\pi}{8}\right) = 0$
b) $y'' + 16y = 0,\ y(0) = 0,\ y\left(\frac{\pi}{4}\right) = 0$
c) $y'' + y = 0,\ y(0) = 0,\ y(\pi) = 2$
d) $y'' + y = 0,\ y'(0) = 0,\ y'(\pi) = 0$

15. a) Check that $y_1 = \cosh(kt)$ and $\sinh(kt)$ are solutions of the equation $y'' - k^2y = 0$.
b) Solve the boundary value problem $y'' - 9y = 0,\ y(0) = 0,\ y\left(\frac{\pi}{3}\right) = 1$
c) Solve the boundary value problem $y'' - 25y = 0,\ y'(0) = 0,\ y'(3) = 10$