# MATH 375
# Handout # 5
## Higher Order Linear Differential Equations

1. In each case , determine whether the set **S** is linearly independent on the interval $(-\infty, \infty)$. Use the Wronskian test or the definition.
   a) $\mathbf{S} = \{x^2 + x, x^2 + 1, x^2 - 1\}$
   b) $\mathbf{S} = \{\cos(2x), -3, 2\sin^2 x\}$
   c) $\mathbf{S} = \{1, t, e^t\}$

2. Find the general solution of the following differential equations
   a) $y^{(4)} - 17y'' + 16y = 0$
   b) $y^{(3)} - 64y = 0$
   c) $y^{(4)} - 81y = 0$
   d) $y^{(4)} + 12y'' + 11y = 0$
   e) $y^{(5)} - 10y^{(3)} + 9y' = 0$
   f) $y^{(3)} + 6y'' + 12y' + 8y = 0$

3. Find the general solution to each of the following differential equations. The primes denote derivatives with respect to $t$, and $D \equiv \dfrac{d}{dt}$.

   *a)* $\dfrac{d^3x}{dt^3} - 6\dfrac{d^2x}{dt^2} + 11\dfrac{dx}{dt} - 6x = 0$ *b)* $x''' - 6x'' + 2x' + 36x = 0$

   *c)* $(D^4 + 8D^3 + 24D^2 + 32D + 16)x = 0$ *d)* $(D^5 - D^4 - 2D^3 + 2D^2 + D - 1)x = 0$

   *e)* $D^3(D^2 + 6D + 25)(D^4 - 16)x = 0$ *f)* $(D^4 - 16)^3 x = 0$

4. According to the method of undetermined coefficients, find the form of the particular solution for the following equations. Do not compute the coefficients.

   a) $y^{(4)} - 17y'' + 16y = x\cos x$
   b) $y^{(3)} - 64y = 7e^{4x}$
   c) $y^{(4)} - 81y = x^2e^{2x} + 6\cos(3x)$
   d) $y^{(4)} + 12y'' + 11y = 3x^2 + e^x$
   e) $y^{(5)} - 10y^{(3)} + 9y' = 3x^2 + e^x$
   f) $y^{(3)} + 6y'' + 12y' + 8y = xe^{-2x}$

5. Find a homogeneous linear differential equation with constant coefficients whose characteristic equation has the roots :
   a) $\{1, i, -i\}$ b) $\{7, 7, 0, 0, 0, 2, 5i, 2, 5i, 2, 5i, 2, 5i\}$

6. Find the homogeneous linear differential equation with constant coefficients of the minimal order which has the following solution:
   a) $x\cos x$ b) $x^2e^{2x}$ c) $x^5 + e^{5x}$