THE UNIVERSITY OF CALGARY
DEPARTMENT OF MATHEMATICS AND STATISTICS
FINAL EXAMINATION
VERSION # 1
AMAT 307 LEC 01-04 FALL 2012
TUESDAY, DECEMBER 11, 2012 3 HOURS

Last Name	First Name

Student I.D. Number	Section Number

This examination consists of 29 equally weighted questions. Please attempt all problems and record your answer by circling your choice in the exam booklet and filling in the appropriate circle in the scantron sheet.

This is a closed book examination, but the use of a Schulich calculator is permitted. A table of Laplace transforms is attached to this booklet.

Fill in your name and lecture number on the scantron sheet and the test booklet.

When finished, turn in both the answer sheet and this exam booklet.

01. Consider the differential equation

$$M(x, y)\ \mathrm{d}x + N(x, y)\ \mathrm{d}y = 0$$

where $M(x, y)$ and $N(x, y)$ have continuous first partial derivatives in the xy-plane.
If the equation is not exact, then it must have at least one integrating factor μ that either depends on x only or on y only.

Circle your choice in the box below

T	**F**

02. If $y_1(t)$ and $y_2(t)$ are solutions of $y'' + p(t)\, y' + q(t)\, y = g(t)$ in the interval $\big(a\,,\, b\big)$, then so is $\dfrac{3}{5}\, y_1(t) + \dfrac{2}{5}\, y_2(t)$

Circle your choice in the box below

T	**F**

03. There is a 15^{th} order linear homogeneous differential equation with constant coefficients that has $y(t) = t + t^{-1}$ as a solution.

Circle your choice in the box below

T	**F**

04. If $\overrightarrow{Y}\ (t) = \begin{bmatrix} 3\,\mathrm{e}^{t} \\ 2\,\mathrm{e}^{-t} - \mathrm{e}^{t} \end{bmatrix}$ is a solution to the system $\overrightarrow{Y}\ ' = \mathbf{A}\,\overrightarrow{Y}$, where $\mathbf{A}$ is a 2×2 constant matrix, then $\mathbf{A}$ has eigenvalues $\lambda_1 = -1$ and $\lambda_2 = 1$

Circle your choice in the box below

T	**F**

05. The initial value problem $\begin{cases} \left(\dfrac{y^2}{x} + 2\,x\right)\,\mathrm{d}x + \left(2\,y\,\ln(x) - 3\,y^2\right)\,\mathrm{d}y = 0 \\ y(1) = -1 \end{cases}$ has solution

A. $y^2\,\ln(x) + x^2 - y^3 = 0$

B. $y^2\,\ln(x) + 2\,x\,y - y^3 = -1$

C. $y^2\,\ln(x) + x^2 - y^3 = 2$

D. $\dfrac{y^2}{x} + 2\,x + 2\,y\,\ln(x) - 3\,y^2 = 0$

E. none of the above

06. An object is placed in a room where the temperature is $21°\mathrm{C}$. After 5 minutes the temperature of the object is $37°\mathrm{C}$, and after another 5 minutes its temperature drops to $29°\mathrm{C}$. Assuming that the temperature $y(t)$ of the object obeys Newton's Law of Cooling, i.e., it satisfies the first order differential equation

$$y' = k\,(21 - y)$$

then the value of the constant k is

A. $k = \dfrac{\ln(5)}{2}$

B. $k = -\dfrac{\ln(2)}{5}$

C. $k = 2\,\ln(5)$

D. $k = \dfrac{\ln(2)}{5}$

E. $k = 5\,\ln(2)$

07. The solution of the initial value problem $\begin{cases} y'' - 2\,y' = 4\,t\,\mathrm{e}^{2\,t} \\ y(0) = 5 \quad \text{and} \quad y'(0) = 3 \end{cases}$ is

A. $y(t) = \dfrac{7}{2} + \left(t^2 - t + \dfrac{3}{2}\right)\,\mathrm{e}^{2\,t}$

B. $y(t) = 3 + \left(t^2 - t + 2\right)\mathrm{e}^{2\,t}$

C. $y(t) = 4 + \left(t + 1\right)\mathrm{e}^{2\,t}$

D. $y(t) = \dfrac{7}{2} + \left(t + \dfrac{1}{2}\right)\,\mathrm{e}^{2\,t}$

E. none of the above

08. If an n^{th}-order linear homogeneous differential equation with constant coefficients has $t^2\left(3\,\mathrm{e}^t - 2\,\cos(t)\right)$ as a solution, then the smallest value of n is

A. 4 **B.** 6 **C.** 7 **D.** 9 **E.** 12

09. The differential equation for which the method of undetermined coefficients can not be used to find a particular solution $y_p(t)$ is

A. $y'' + 2\,y' - 3\,y = 3^t + 3^{-t}$

B. $y'' + 2\,y' - 3\,y = \dfrac{\sin(t)}{\mathrm{e}^{3\,t}}$

C. $y'' + 2\,y' - 3\,y = t^3 + 3\,t^2 - 2\,t^{1/2} + 5$

D. $y'' + 2\,y' - 3\,y = \dfrac{t}{\sec(t)}$

E. $y'' + 2\,y' - 3\,y = \left(t + \cos(2\,t - 1) + \mathrm{e}^t\right)^2$

10. The third order linear differential equation $y''' + t\,y'' - e^t\,y' + \left(1 - t^2\right) y = \ln(t)$ is equivalent to the first order linear system $\vec{Y}\,' = \mathbf{P}(t)\,\vec{Y} + \vec{G}\,(t)$, with $\vec{Y}\,(t) = \begin{bmatrix} y(t) \\ y'(t) \\ y''(t) \end{bmatrix}$,

A. $\mathbf{P}(t) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ t^2-1 & e^t & -t \end{bmatrix}$ and $\vec{G}\,(t) = \begin{bmatrix} 0 \\ 0 \\ \ln(t) \end{bmatrix}$

B. $\mathbf{P}(t) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1-t^2 & -e^t & t \end{bmatrix}$ and $\vec{G}\,(t) = \begin{bmatrix} 0 \\ 0 \\ \ln(t) \end{bmatrix}$

C. $\mathbf{P}(t) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ t^2-1 & e^t & -t \end{bmatrix}$ and $\vec{G}\,(t) = \begin{bmatrix} \ln(t) \\ \ln(t) \\ \ln(t) \end{bmatrix}$

D. $\mathbf{P}(t) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -t & e^t & t^2-1 \end{bmatrix}$ and $\vec{G}\,(t) = \begin{bmatrix} 0 \\ 0 \\ -\ln(t) \end{bmatrix}$

E. $\mathbf{P}(t) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ t^2-1 & e^t & -t \end{bmatrix}$ and $\vec{G}\,(t) = \begin{bmatrix} 0 \\ 0 \\ \ln(t) \end{bmatrix}$

11. The largest open interval where the solution of the initial value problem

$$\begin{cases} \vec{Y}\,' = \begin{bmatrix} \dfrac{1}{t-4} & \dfrac{1}{t+3} \\ \ln(3-t) & t+1 \end{bmatrix} \vec{Y}\,, \quad \vec{Y}\,(1) = \begin{bmatrix} 2 \\ -1 \end{bmatrix} \end{cases}$$

is guaranteed to be defined is

A. $(-3\,,\,4)$

B. $(-3\,,\,3)$

C. $(-1\,,\,3)$

D. $(-\infty\,,\,3)$

E. $\left(-3\,,\,+\infty\right)$

12. The general solution of the system $\vec{Y}\,' = \begin{bmatrix} 3 & 1 \\ 4 & 3 \end{bmatrix} \vec{Y}$ is given by

A. $C_1\, e^{t} \begin{bmatrix} 1 \\ 2 \end{bmatrix} + C_2\, e^{5t} \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

B. $C_1\, e^{t} \begin{bmatrix} 1 \\ -2 \end{bmatrix} + C_2\, e^{5t} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

C. $C_1\, e^{-t} \begin{bmatrix} 1 \\ -2 \end{bmatrix} + C_2\, e^{-5t} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

D. $C_1\, e^{t} \begin{bmatrix} -2 \\ 1 \end{bmatrix} + C_2\, e^{5t} \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

E. none of the above

13. The coefficient matrix of the system $\vec{Y}\,' = \begin{bmatrix} 1 & -1 & -2 \\ 1 & 3 & 2 \\ 1 & -1 & 2 \end{bmatrix} \vec{Y}$ has eigenvalues $\lambda_1 = 2$, $\lambda_2 = 2 + 2i$, and corresponding eigenvectors $\vec{V}_1 = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$, and $\vec{V}_2 = \begin{bmatrix} 1 \\ -1 \\ -i \end{bmatrix}$, respectively. A fundamental set of solutions is given by

A. $\left\{ \begin{bmatrix} e^{2t} \\ e^{2t} \\ -e^{2t} \end{bmatrix}, \begin{bmatrix} 2e^{2t}\cos(t) \\ -2e^{2t}\cos(t) \\ 2e^{2t}\sin(t) \end{bmatrix}, \begin{bmatrix} 2e^{2t}\sin(t) \\ -2e^{2t}\sin(t) \\ -2e^{2t}\cos(t) \end{bmatrix} \right\}$

B. $\left\{ \begin{bmatrix} e^{2t} \\ e^{2t} \\ -e^{2t} \end{bmatrix}, \begin{bmatrix} e^{2t}\cos(2t) \\ -e^{2t}\cos(2t) \\ -e^{2t}\sin(2t) \end{bmatrix}, \begin{bmatrix} e^{2t}\sin(2t) \\ -e^{2t}\sin(2t) \\ e^{2t}\cos(2t) \end{bmatrix} \right\}$

C. $\left\{ \begin{bmatrix} e^{2t} \\ e^{2t} \\ -e^{2t} \end{bmatrix}, \begin{bmatrix} e^{2t}\cos(2t) \\ e^{2t}\cos(2t) \\ e^{2t}\cos(2t) \end{bmatrix}, \begin{bmatrix} e^{2t}\sin(2t) \\ e^{2t}\sin(2t) \\ e^{2t}\cos(2t) \end{bmatrix} \right\}$

D. $\left\{ \begin{bmatrix} e^{2t} \\ e^{2t} \\ -e^{2t} \end{bmatrix}, \begin{bmatrix} e^{2t}\cos(2t) \\ -e^{2t}\cos(2t) \\ e^{2t}\sin(2t) \end{bmatrix}, \begin{bmatrix} e^{2t}\sin(2t) \\ -e^{2t}\sin(2t) \\ -e^{2t}\cos(2t) \end{bmatrix} \right\}$

E. none of the above

14. Given that $\left\{ \begin{bmatrix} e^t \\ e^t \\ 3\,e^t \end{bmatrix}, \begin{bmatrix} -e^{2\,t} \\ e^{2\,t} \\ 0 \end{bmatrix}, \begin{bmatrix} e^{2\,t} \\ 0 \\ e^{2\,t} \end{bmatrix} \right\}$ is a fundamental set of solutions for the system $\overrightarrow{Y}\,' = \mathbf{A}\,\overrightarrow{Y}$, the solution of the initial value problem

$$\overrightarrow{Y}\,' = \mathbf{A}\,\overrightarrow{Y}\,, \quad \overrightarrow{Y}\,(0) = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$$

is given by

A. $\overrightarrow{Y}\,(t) = \begin{bmatrix} 3\,e^t - e^{2\,t} \\ 3\,e^t - 3\,e^{2\,t} \\ 9\,e^t - 4\,e^{2\,t} \end{bmatrix}$

B. $\overrightarrow{Y}\,(t) = \begin{bmatrix} 3\,e^t - e^{2\,t} \\ e^t - e^{2\,t} \\ 4\,e^t - 3\,e^{2\,t} \end{bmatrix}$

C. $\overrightarrow{Y}\,(t) = \begin{bmatrix} 3\,e^{2\,t} - e^t \\ e^{2\,t} - e^t \\ 4\,e^{2\,t} - 3\,e^t \end{bmatrix}$

D. $\overrightarrow{Y}\,(t) = \begin{bmatrix} 2\,e^{2\,t} \\ 0 \\ e^{2\,t} \end{bmatrix}$

E. None of the above

15. Given that the matrix $\mathbf{A} = \begin{bmatrix} 1 & -1 \\ 1 & 3 \end{bmatrix}$ has eigenvalues $\lambda_1 = \lambda_2 = 2$, the solution of the initial value problem $\overrightarrow{Y}\,' = \mathbf{A}\,\overrightarrow{Y}\,, \ \overrightarrow{Y}\,(0) = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$ is given by

A. $\overrightarrow{Y}\,(t) = \begin{bmatrix} (3-t)\,e^{2\,t} \\ (t-2)\,e^{2\,t} \end{bmatrix}$

B. $\overrightarrow{Y}\,(t) = \begin{bmatrix} 3\,e^{2\,t} \\ -2\,e^{2\,t} \end{bmatrix}$

C. $\overrightarrow{Y}\,(t) = \begin{bmatrix} \left(3 - t^2/2\right)e^{2\,t} \\ \left(-2 + t^2/2\right)e^{2\,t} \end{bmatrix}$

D. $\overrightarrow{Y}\,(t) = \begin{bmatrix} \left(3 - t - t^2/2\right)e^{2\,t} \\ \left(-2 + t + t^2/2\right)e^{2\,t} \end{bmatrix}$

E. None of the above

16. Let $\mathbf{A} = \begin{bmatrix} 5 & 6 \\ -3 & -4 \end{bmatrix}$, $\vec{G}\,(t) = \begin{bmatrix} \mathrm{e}^{-t}/t \\ -\mathrm{e}^{-t}/t \end{bmatrix}$, and consider the system

$$\vec{Y}\,' = \mathbf{A}\,\vec{Y} + \vec{G}\,(t)$$

Given that $\mathbf{A}$ has eigenvalues $\lambda_1 = -1$ and $\lambda_2 = 2$, with corresponding eigenvectors $\vec{V}_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ and $\vec{V}_2 = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$, the general solution $\vec{Y}\,(t)$ of the nonhomogeneous system is

A. $C_1 \begin{bmatrix} \mathrm{e}^{2t} \\ -\mathrm{e}^{2t} \end{bmatrix} + C_2 \begin{bmatrix} 2\mathrm{e}^{-t} \\ -\mathrm{e}^{-t} \end{bmatrix} + \begin{bmatrix} 2\,\mathrm{e}^{2t}\,\ln|t| \\ -\mathrm{e}^{2t}\,\ln|t| \end{bmatrix}$

B. $C_1 \begin{bmatrix} \mathrm{e}^{-t} \\ -\mathrm{e}^{-t} \end{bmatrix} + C_2 \begin{bmatrix} 2\mathrm{e}^{2t} \\ -\mathrm{e}^{2t} \end{bmatrix} + \begin{bmatrix} \mathrm{e}^{-t}/t \\ -\mathrm{e}^{-t}/t \end{bmatrix}$

C. $C_1 \begin{bmatrix} \mathrm{e}^{-t} \\ -\mathrm{e}^{-t} \end{bmatrix} + C_2 \begin{bmatrix} 2\mathrm{e}^{2t} \\ -\mathrm{e}^{2t} \end{bmatrix}$

D. $C_1 \begin{bmatrix} \mathrm{e}^{-t} \\ -\mathrm{e}^{-t} \end{bmatrix} + C_2 \begin{bmatrix} 2\mathrm{e}^{2t} \\ -\mathrm{e}^{2t} \end{bmatrix} + \begin{bmatrix} 2\,\mathrm{e}^{2t}\,\ln|t| \\ -\mathrm{e}^{2t}\,\ln|t| \end{bmatrix}$

E. $C_1 \begin{bmatrix} \mathrm{e}^{-t} \\ -\mathrm{e}^{-t} \end{bmatrix} + C_2 \begin{bmatrix} 2\mathrm{e}^{2t} \\ -\mathrm{e}^{2t} \end{bmatrix} + \begin{bmatrix} \mathrm{e}^{-t}\,\ln|t| \\ -\mathrm{e}^{-t}\,\ln|t| \end{bmatrix}$

17. If $f(t) = \begin{cases} t & \text{if} \quad 0 \le t < 1 \\ 2\,t - 1 & \text{if} \quad 1 \le t < 2 \\ 3\,t & \text{if} \quad 2 \le t \end{cases}$ then

A. $f(t) = t + (t-1)\,u_1(t) + (t+1)\,u_2(t)$

B. $f(t) = t + (t+1)\,u_1(t) + (t-1)\,u_2(t)$

C. $f(t) = t - (t+1)\,u_1(t) - (t+1)\,u_2(t)$

D. $f(t) = t + (2\,t-1)\,u_1(t) + 3\,t\,u_2(t)$

E. none of the above

18. $\mathcal{L}\left\{\big(\cos(t)+\sin(t)\big)^2\right\}$ is equal to

A. $\dfrac{1}{s}+\dfrac{s}{s^2+4}$

B. $\dfrac{1}{s}+\dfrac{2}{s^2+4}$

C. $\left(\dfrac{s+1}{s^2+1}\right)^2$

D. $\dfrac{1}{s}+\dfrac{2\,s}{\big(s^2+1\big)^2}$

E. none of the above

19. $\mathcal{L}\left\{(t-2)\,\mathrm{e}^{2-t}\,u_2(t)\right\}$ is equal to

A. $\left(\dfrac{1}{(s-1)^2}-\dfrac{4}{s-1}\right)\mathrm{e}^{-2\,s}$

B. $\dfrac{1}{(s-1)^2}\,\mathrm{e}^{-2\,s}$

C. $\left(\dfrac{1}{(s+1)^2}-\dfrac{4}{s+1}\right)\mathrm{e}^{-2\,s}$

D. $\dfrac{1}{(s+1)^2}\,\mathrm{e}^{-2\,s}$

E. $\dfrac{1-2\,s}{s^3\,(s+1)}\,\mathrm{e}^{2-2\,s}$

20. $\mathcal{L}\Big\{\, |t-1| \,\Big\}$ is equal to

A. $\dfrac{1}{s^2} - \dfrac{1}{s}$

B. $\left|\dfrac{1}{s^2} - \dfrac{1}{s}\right|$

C. $\dfrac{1}{s} - \dfrac{1}{s^2} + \dfrac{2}{s^2}\,\mathrm{e}^{-s}$

D. $-\dfrac{1}{s} + \dfrac{1}{s^2} + \left(-\dfrac{1}{s} + \dfrac{2}{s^2}\right)\,\mathrm{e}^{-s}$

E. none of the above

21. $\mathcal{L}^{-1}\left\{\dfrac{12\,s}{s^2 - 2\,s - 3}\right\}$ is equal to

A. $9\,\mathrm{e}^{3\,t} + 3\,\mathrm{e}^{-t}$

B. $9\,\mathrm{e}^{-3\,t} + 3\,\mathrm{e}^{t}$

C. $-9\,\mathrm{e}^{3\,t} + 3\,\mathrm{e}^{-t}$

D. $3\,\mathrm{e}^{3\,t} + 9\,\mathrm{e}^{-t}$

E. $3\,\mathrm{e}^{-3\,t} + 9\,\mathrm{e}^{t}$

22. $\mathcal{L}^{-1}\left\{\dfrac{s-1}{s^2+2\,s+5}\right\}$ is equal to

A. $e^{-t}\Big(\cos(2\,t)-\sin(2\,t)\Big)$

B. $e^{t}\Big(\cos(2\,t)-\sin(2\,t)\Big)$

C. $e^{-t}\Big(\cos(2\,t)-2\,\sin(2\,t)\Big)$

D. $e^{t}\Big(\cos(4\,t)-\dfrac{1}{2}\,\sin(4\,t)\Big)$

E. $e^{-t}\Big(\cos(4\,t)-\dfrac{1}{2}\,\sin(4\,t)\Big)$

23. $\mathcal{L}^{-1}\left\{\dfrac{e^{-2\,s}}{s^2+\pi^2}\right\}$ is equal to

A. $u_2(t)\,\sin\big(\pi\,t\big)$

B. $\dfrac{1}{\pi}\,u_2(t)\,\sin\big(\pi\,t\big)$

C. $u_\pi(t)\,\sin\big(2\,t\big)$

D. $\dfrac{1}{\pi}\,u_2(t)\,\cos\big(\pi\,t\big)$

E. none of the above

24. If Laplace transform is used to solve the initial value problem

$$\begin{cases} y'' - 2\,y' + y = (t-1)^2\, u_1(t) \\ y(0) = 1 \quad \text{and} \quad y'(0) = 1 \end{cases}$$

then

A. $y = \mathcal{L}^{-1}\left\{ \dfrac{s+2}{(s-1)^2} + \dfrac{2\,e^{-s}}{s^3\,(s-1)^2} \right\}$

B. $y = \mathcal{L}^{-1}\left\{ \dfrac{2\,e^{-s}}{s^3\,(s-1)^2} \right\}$

C. $y = \mathcal{L}^{-1}\left\{ \dfrac{1}{s-1} + \dfrac{2\,e^{-s}}{s^3\,(s-1)^2} \right\}$

D. $y = \mathcal{L}^{-1}\left\{ \dfrac{1}{s-1} + \dfrac{2\,(s+1)\,e^{-s}}{s^3\,(s-1)^2} \right\}$

E. none of the above

25. The largest open interval where the power series $\displaystyle\sum_{n=0}^{\infty} \frac{n^4}{(n^2+1)\,4^n}\,\big(3\,t+1\big)^n$ converges is

A. $(-3\,,\,5)$

B. $\left(-\dfrac{13}{12}\,,\,\dfrac{5}{12}\right)$

C. $(-5\,,\,3)$

D. $\left(-\dfrac{1}{3}\,,\,\dfrac{7}{3}\right)$

E. $\left(-\dfrac{5}{3}\,,\,1\right)$

26. Consider the differential equation $y'' + \dfrac{3\,t}{t-5}\,y' + \dfrac{1}{t\,(t+2)}\,y = 0$

If $y(t) = \sum_{n=0}^{\infty} a_n\,(t-2)^n$ is a power series solution of the differential equation, then its radius of convergence is at least equal to

A. 3

B. 4

C. 5

D. ∞

E. none of the above

27. Consider the differential equation $(t+1)\,y'' + y' - 2\,y = 0$. If we look for a solution $y(t) = \sum_{n=0}^{\infty} a_n\,t^n$, then the recurrence relation satisfied by the coefficients $a_0,\,a_1,\,a_2,\,a_3,\,a_4,\,\cdots$ is

A. $(n+2)\,(n+1)\,a_{n+2} + (n+1)\,a_{n+1} - 2\,a_n = 0,$ $\quad n = 0,\,1,\,2,\,3,\,\cdots$

B. $(n+2)\,(n+1)\,a_{n+2} + (n+1)^2\,a_{n+1} - 2\,a_n = 0,$ $\quad n = 0,\,1,\,2,\,3,\,\cdots$

C. $(t+1)\,(n+2)\,(n+1)\,a_{n+2} + (n+1)\,a_{n+1} - 2\,a_n = 0,$ $\quad n = 0,\,1,\,2,\,3,,\,\cdots$

D. $n\,(n-1)\,a_{n+2} + n^2\,a_{n+1} - 2\,a_n = 0,$ $\quad n = 0,\,1,\,2,\,3,,\,\cdots$

E. none of the above

28. Consider the initial value problem $\begin{cases} y'' + t\,y' + 2\,t\,y = 0 \\ y(0) = 2 \quad \text{and} \quad y'(0) = 12 \end{cases}$ and suppose that

$$\begin{cases} a_2 = 0 \\ (n+2)\,(n+1)\,a_{n+2} + n\,a_n + 2\,a_{n-1} = 0, \quad n = 1,\ 2,\ 3,\ \cdots \end{cases}$$

is the recurrence relation satisfied by the coefficients of the solution $y(t) = \sum_{n=0}^{\infty} a_n\, t^n$ of the initial value problem. Then

A. $y(t) = 2 + 12\,t + \frac{8}{3}\,t^3 + 2\,t^4 - \frac{2}{5}\,t^5 + \frac{4}{9}\,t^6 + \frac{1}{21}\,t^7 + \cdots$

B. $y(t) = 2 + 12\,t - \frac{8}{3}\,t^3 - 2\,t^4 + \frac{2}{5}\,t^5 + \frac{4}{9}\,t^6 - \frac{1}{7}\,t^7 + \cdots$

C. $y(t) = 2 + 12\,t + \frac{8}{3}\,t^3 + 2\,t^4 + \frac{2}{5}\,t^5 + \frac{4}{9}\,t^6 - \frac{1}{21}\,t^7 + \cdots$

D. $y(t) = 2 + 12\,t - \frac{8}{3}\,t^3 - 2\,t^4 + \frac{2}{5}\,t^5 + \frac{4}{9}\,t^6 + \frac{1}{21}\,t^7 + \cdots$

E. none of the above

29. Consider Euler's equation

$$t^2\,y'' + 8\,t\,y' + 12\,y = 0, \quad t > 0$$

Given that the equation has solutions in the form t^r, where r is a real parameter, its general solution is given by

A. $C_1\,t^{-3} + C_2\,t^4$

B. $C_1\,t^3 + C_2\,t^4$

C. $C_1\,t^3 + C_2\,t^{-4}$

D. $C_1\,t^{-3} + C_2\,t^{-4}$

E. none of the above

Laplace Transforms

01. $\mathcal{L}\Big\{K_1\ f_1(t) + K_2\ f_2(t)\Big\}(s) = K_1\ \mathcal{L}\Big\{f_1(t)\Big\}(s) + K_2\ \mathcal{L}\Big\{f_2(t)\Big\}(s)$

02. $\mathcal{L}\Big\{y^{(n)}(t)\Big\} = s^n\,\mathcal{L}\Big\{y(t)\Big\}(s) - s^{n-1}\,y(0) - s^{n-2}\,y'(0) - \cdots - y^{(n-1)}(0)$ $\qquad n = 1,\ 2,\ 3,\ \cdots$

03. $\mathcal{L}\Big\{\mathrm{e}^{a\,t}\ f(t)\Big\}(s) = \mathcal{L}\Big\{f(t)\Big\}\big(s-a\big)$

04. $\mathcal{L}\Big\{u_a(t)\ f(t)\Big\}(s) = \mathcal{L}\Big\{f\big(t+a\big)\Big\}(s)\ \mathrm{e}^{-a\,s}$

05. $\mathcal{L}\Big\{t\ f(t)\Big\}(s) = -\dfrac{\mathrm{d}}{\mathrm{d}s}\Big(\mathcal{L}\Big\{f(t)\Big\}(s)\Big)$

06. $\mathcal{L}\Big\{t^n\Big\}(s) = \dfrac{n!}{s^{n+1}}$ $\qquad n = 0,\ 1,\ 2,\ \cdots$

07. $\mathcal{L}\Big\{\mathrm{e}^{a\,t}\ t^n\Big\}(s) = \dfrac{n!}{\big(s-a\big)^{n+1}}$ $\qquad n = 0,\ 1,\ 2,\ \cdots$

08. $\mathcal{L}\Big\{\cos\big(b\,t\big)\Big\}(s) = \dfrac{s}{s^2+b^2}$

09. $\mathcal{L}\Big\{\mathrm{e}^{a\,t}\ \cos\big(b\,t\big)\Big\}(s) = \dfrac{s-a}{(s-a)^2+b^2}$

10. $\mathcal{L}\Big\{\sin\big(b\,t\big)\Big\}(s) = \dfrac{b}{s^2+b^2}$

11. $\mathcal{L}\Big\{\mathrm{e}^{a\,t}\ \sin\big(b\,t\big)\Big\}(s) = \dfrac{b}{(s-a)^2+b^2}$

12. $\mathcal{L}\Big\{u_a(t)\Big\}(s) = \dfrac{\mathrm{e}^{-a\,s}}{s}$

13. $\mathcal{L}\Big\{\mathrm{e}^{a\,t}\Big\}(s) = \dfrac{1}{s-a}$

Inverse Laplace Transforms

01. $\mathcal{L}^{-1}\Big\{K_1\ F_1(s) + K_2\ F_2(s)\Big\}(t) = K_1\ \mathcal{L}^{-1}\Big\{F_1(s)\Big\}(t) + K_2\ \mathcal{L}^{-1}\Big\{F_2(s)\Big\}(t)$

02. $\mathcal{L}^{-1}\Big\{F\big(s-a\big)\Big\}(t) = \mathrm{e}^{a\,t}\ \mathcal{L}^{-1}\Big\{F(s)\Big\}(t)$ $\qquad$ or $\qquad$ $\mathcal{L}^{-1}\Big\{F\big(s+a\big)\Big\}(t) = \mathrm{e}^{-a\,t}\ \mathcal{L}^{-1}\Big\{F(s)\Big\}(t)$

03. $\mathcal{L}^{-1}\Big\{F(s)\ \mathrm{e}^{-a\,s}\Big\}(t) = u_a(t)\ \mathcal{L}^{-1}\Big\{F(s)\Big\}\big(t-a\big)$ $\qquad$ $\mathcal{L}^{-1}\Big\{\dfrac{\mathrm{e}^{-a\,s}}{s}\Big\}(t) = u_a(t)$

04. $\mathcal{L}^{-1}\left\{F'(s)\right\}(t) = -t\ \mathcal{L}^{-1}\Big\{F(s)\Big\}(t)$

05. $\mathcal{L}^{-1}\Big\{\dfrac{1}{s^{n+1}}\Big\}(t) = \dfrac{t^n}{n!}$ $\qquad n = 0,\ 1,\ 2,\ \cdots$

06. $\mathcal{L}^{-1}\Big\{\dfrac{1}{(s-a)^{n+1}}\Big\}(t) = \mathrm{e}^{a\,t}\ \dfrac{t^n}{n!}$ $\qquad n = 0,\ 1,\ 2,\ \cdots$

07. $\mathcal{L}^{-1}\Big\{\dfrac{s}{s^2+b^2}\Big\}(t) = \cos\big(b\,t\big)$

08. $\mathcal{L}^{-1}\Big\{\dfrac{s-a}{(s-a)^2+b^2}\Big\}(t) = \mathrm{e}^{a\,t}\ \cos\big(b\,t\big)$

09. $\mathcal{L}^{-1}\Big\{\dfrac{1}{s^2+b^2}\Big\}(t) = \dfrac{1}{b}\ \sin\big(b\,t\big)$

10. $\mathcal{L}^{-1}\Big\{\dfrac{1}{(s-a)^2+b^2}\Big\}(t) = \dfrac{1}{b}\ \mathrm{e}^{a\,t}\ \sin\big(b\,t\big)$

Trigonometric Identities

1. $\cos^2(\theta) + \sin^2(\theta) = 1$ $\qquad$ **2.** $\cos(2\,\theta) = \cos^2(\theta) - \sin^2(\theta)$ $\qquad$ **3.** $\sin(2\,\theta) = 2\ \cos(\theta)\ \sin(\theta)$

4. $\cos(\theta \pm \pi) = -\cos(\theta)$ $\qquad$ **5.** $\sin(\theta \pm \pi) = -\sin(\theta)$

6. $\cos(\theta \pm 2\,\pi) = \cos(\theta)$ $\qquad$ **7.** $\sin(\theta \pm 2\,\pi) = \sin(\theta)$