Math 375		Fall 2016
First Order Differential Equations		
Worksheet # 1	**Part 3**	**October 03 - 07**

The problems marked with $(*)$ are to be attempted during the tutorial time. Students are strongly encouraged to attempt the remaining problems on their own. Solutions to all the problems will be available on the course's D2L website Friday, October 07. Please report any typos, omissions and errors to `aiffam@ucalgary.ca`

Mixing Problems

01. A tank initially contains 40 gallons of pure water. A solution with 1 gram of salt per gallon of water is added to the tank at 3 gallons per minute, and the resulting well mixed solution leaves the tank at the same rate. Find the quantity $Q(t)$ of salt in the tank at time t, and $\lim_{t\to+\infty} Q(t)$

02*. A 500-liter tank initially contains 100 liters of a salt solution with concentration of 0.1 gram per liter. A salt solution with concentration of 0.3 gram per liter, is added to the tank at a rate of 5 liters per second. The resulting mixture is drained out at a rate of 3 liters per second. Find the concentration $c(t)$ of salt in the tank at any given time t. How much salt will the tank contain when it is on the point of overflowing.

Radioactive Decay

03. The half-life of a radioactive substance is 2 days. Find the time required for a given amount of that substance to decay to $\frac{1}{10}$ of its original mass.

04*. Initially, 100 grams of a radioactive material is present. After 3 days, only 75 grams remains. How much additional time will it take for radioactive decay to reduce the amount present to 30 grams?

Heating and Cooling

05*. A body of temperature $80^\circ F$ is placed at time $t = 0$ in a medium the temperature of which is maintained at $50^\circ F$. At the end of 5 minutes, the body has cooled to a temperature of $70^\circ F$.

- **a.** What is the temperature of the body at the end of 10 minutes?
- **b.** When will the temperature of the body be $60^\circ F$?

06. A student performs the following experiment using two identical cups of water. One cup is removed from a refrigerator at $34^\circ F$ and allowed to warm in its surroundings to room temperature $\left(72^\circ F\right)$. A second cup is simultaneously taken from room temperature surroundings and placed in the refregirator to cool. The time at which each cup of water reached a temperature of $53^\circ F$ is recorded. Are the two recorded times the same?

07. A generator having an emf of $100\,V$ is connected in series with a $10\,\Omega$ resistor and a $2\,H$ inductor. Find the current $I(t)$, if $I(0) = 0$.

08. Let $p \neq 0$, q, ω, y_0 be real constants. Show that the solution of the initial value problem $\begin{cases} y' + p\,y = q\,\cos(\omega\,t) \\ y(0) = y_0 \end{cases}$ is

$$y(t) = \frac{q}{p^2 + \omega^2}\Big(p\,\cos(\omega\,t) + \omega\,\sin(\omega\,t)\Big) + \left(y_0 - \frac{p\,q}{p^2 + \omega^2}\right)\,\mathrm{e}^{-p\,t}$$

Note: the solution above remains valid when $q = 0$ or $\omega = 0$.

09*. A generator having an emf $E(t) = 20\,\cos(5\,t)$, is connected in series with a $10\,\Omega$ resistor and a $2\,H$ inductor. Find the current $I(t)$, if $I(0) = 0$.

10. A decaying emf $E(t) = 200\,\mathrm{e}^{-5\,t}$ is connected in series with a $20\,\Omega$ resistor and a $\dfrac{1}{100}\,F$ capacitor.

a. Find the charge $Q(t)$ and the current $I(t)$ at any time, if $Q(0) = 0$

b. Find the maximum charge of the capacitor and the time it takes to reach that maximum.