## **Laplace Transforms**

**01.** 
$$\mathcal{L}\left\{K_{1} f_{1}(t) + K_{2} f_{2}(t)\right\}(s) = K_{1} \mathcal{L}\left\{f_{1}(t)\right\}(s) + K_{2} \mathcal{L}\left\{f_{2}(t)\right\}(s)$$

**02.** 
$$\mathcal{L}\left\{y^{(n)}(t)\right\}(s) = s^n \mathcal{L}\left\{y(t)\right\}(s) - s^{n-1}y(0) - s^{n-2}y'(0) - \dots - y^{(n-1)}(0) \quad n = 1, 2, 3, \dots$$

**03.** 
$$\mathcal{L}\left\{e^{at} f(t)\right\}(s) = \mathcal{L}\left\{f(t)\right\}(s-a)$$

**04.** 
$$\mathcal{L}\{u_a(t) f(t)\}(s) = \mathcal{L}\{f(t+a)\}(s) e^{-as}$$

**05.** 
$$\mathcal{L}\left\{f(t)\right\}(s) = \frac{1}{1 - e^{-Ts}} \int_{0}^{T} f(t) e^{-st} dt, \quad f(t+T) = f(t)$$

**06.** 
$$\mathcal{L}\left\{t|f(t)\right\}(s) = -\frac{\mathrm{d}}{\mathrm{d}s}\left(\mathcal{L}\left\{f(t)\right\}(s)\right)$$
 **07.**  $\mathcal{L}\left\{\frac{f(t)}{t}\right\}(s) = \int_{s}^{+\infty} \mathcal{L}\left\{f(t)\right\}(r) \, \mathrm{d}r\right\}$ 

**08.** 
$$\mathcal{L}\left\{t^{n}\right\}(s) = \frac{n!}{s^{n+1}}, \quad n = 0, 1, 2, \cdots$$
 **09.**  $\mathcal{L}\left\{e^{at} t^{n}\right\}(s) = \frac{n!}{\left(s - a\right)^{n+1}}, \quad n = 0, 1, 2$ 

**10.** 
$$\mathcal{L}\left\{\cos\left(b\,t\right)\right\}(s) = \frac{s}{s^2 + b^2}$$
 **11.**  $\mathcal{L}\left\{e^{a\,t}\,\cos\left(b\,t\right)\right\}(s) = \frac{s - a}{(s - a)^2 + b^2}$  **12.**  $\mathcal{L}\left\{\sin\left(b\,t\right)\right\}(s) = \frac{b}{s^2 + b^2}$  **13.**  $\mathcal{L}\left\{e^{a\,t}\,\sin\left(b\,t\right)\right\}(s) = \frac{b}{(s - a)^2 + b^2}$ 

**12.** 
$$\mathcal{L}\{\sin(bt)\}(s) = \frac{b}{s^2 + b^2}$$
 **13.**  $\mathcal{L}\{e^{at} \sin(bt)\}(s) = \frac{b}{(s-a)^2 + b^2}$ 

**14.** 
$$\mathcal{L}\{u_a(t)\}(s) = \frac{e^{-as}}{\underline{s}}$$
 **15.**  $\mathcal{L}\{e^{at}\}(s) = \frac{1}{s-a}$ 

**16.** 
$$\mathcal{L}\left\{\frac{1}{\sqrt{t}}\right\}(s) = \sqrt{\frac{\pi}{s}}$$

## **Inverse Laplace Transforms**

**01.** 
$$\mathcal{L}^{-1}\left\{K_{1} F_{1}(s) + K_{2} F_{2}(s)\right\}(t) = K_{1} \mathcal{L}^{-1}\left\{F_{1}(s)\right\}(t) + K_{2} \mathcal{L}^{-1}\left\{F_{2}(s)\right\}(t)$$

**02.** 
$$\mathcal{L}^{-1}\left\{F(s-a)\right\}(t) = e^{at} \mathcal{L}^{-1}\left\{F(s)\right\}(t)$$
 or  $\mathcal{L}^{-1}\left\{F(s+a)\right\}(t) = e^{-at} \mathcal{L}^{-1}\left\{F(s)\right\}(t)$ 

**03.** 
$$\mathcal{L}^{-1}\left\{F(s) e^{-as}\right\}(t) = u_a(t) \mathcal{L}^{-1}\left\{F(s)\right\}(t-a)$$
  $\mathcal{L}^{-1}\left\{\frac{e^{-as}}{s}\right\}(t) = u_a(t)$ 

**04.** 
$$\mathcal{L}^{-1} \{ F'(s) \} (t) = -t \mathcal{L}^{-1} \{ F(s) \} (t)$$

**05.** 
$$\mathcal{L}^{-1}\left\{\frac{1}{s^{n+1}}\right\}(t) = \frac{t^n}{n!} \ n = 0, 1, 2, \cdots$$
 **06.**  $\mathcal{L}^{-1}\left\{\frac{1}{(s-a)^{n+1}}\right\}(t) = e^{at} \frac{t^n}{n!} \ n = 0, 1, 2$ 

**07.** 
$$\mathcal{L}^{-1}\left\{\frac{s}{s^2+b^2}\right\}(t) = \cos(bt)$$
 **08.**  $\mathcal{L}^{-1}\left\{\frac{s-a}{(s-a)^2+b^2}\right\}(t) = e^{at}\cos(bt)$ 

**07.** 
$$\mathcal{L}^{-1} \left\{ \frac{s}{s^2 + b^2} \right\} (t) = \cos (b t)$$
 **08.**  $\mathcal{L}^{-1} \left\{ \frac{s - a}{(s - a)^2 + b^2} \right\} (t) = e^{a t} \cos (b t)$  **09.**  $\mathcal{L}^{-1} \left\{ \frac{1}{s^2 + b^2} \right\} (t) = \frac{1}{b} \sin (b t)$  **10.**  $\mathcal{L}^{-1} \left\{ \frac{1}{(s - a)^2 + b^2} \right\} (t) = \frac{1}{b} e^{a t} \sin (b t)$ 

## Trigonometric Identities

**1.** 
$$\cos^2(\theta) + \sin^2(\theta) = 1$$
 **2.**  $\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$  **3.**  $\sin(2\theta) = 2\cos(\theta)\sin(\theta)$ 

**5.** 
$$2\sin^2(\theta) = 1 - \cos(2\theta)$$
 **6.**  $\cos(\theta \pm \pi) = -\cos(\theta)$ 

**4.** 
$$2\cos^2(\theta) = 1 + \cos(2\theta)$$
 **5.**  $2\sin^2(\theta) = 1 - \cos(2\theta)$  **6.**  $\cos(\theta \pm \pi) = -\cos(\theta)$  **7.**  $\sin(\theta \pm \pi) = -\sin(\theta)$  **8.**  $\cos(\theta \pm 2\pi) = \cos(\theta)$  **9.**  $\sin(\theta \pm 2\pi) = \sin(\theta)$