David Ng **F2016MATH375**

Assignment04 is due on Tuesday, December 06, 2016 at 11:59pm.

The number of attempts available for each question is noted beside the question. If you are having trouble figuring out your error, you should consult the textbook, or ask a fellow student, one of the TA's or your professor for help.
There are also other resources at your disposal, such as the Mathematics Continuous Tutorials. Don't spend a lot of time guessing – it's not very efficient or effective.

Make sure to give lots of significant digits for (floating point) numerical answers. For most problems when entering numerical answers, you can if you wish enter elementary expressions such as $2 \wedge 3$ instead of 8, $\sin(3*\text{pi}/2)$instead of -1, $e \wedge (\ln(2))$ instead of 2, $(2+\tan(3))*(4-\sin(5))\wedge 6-7/8$ instead of 27620.3413, etc.

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1. (1 point) Suppose

$$\begin{aligned}(t+2)y_1' &= 4ty_1+5y_2, & y_1(1)=0,\\ (t-3)y_2' &= 4y_1+8ty_2, & y_2(1)=2.\end{aligned}$$

(1) This system of linear differential equations can be put in the form $\vec{y}\,' = P(t)\vec{y}+\vec{g}(t)$. Determine $P(t)$ and $\vec{g}(t)$.

$$P(t) = \begin{bmatrix} \_\_\_\_ & \_\_\_\_ \\ \_\_\_\_ & \_\_\_\_ \end{bmatrix}$$

$$\vec{g}(t) = \begin{bmatrix} \_\_\_\_ \\ \_\_\_\_ \end{bmatrix}$$

(2) Is the system homogeneous or nonhomogeneous?
- Choose
- homogeneous
- nonhomogeneous

(3) Find the largest interval $a < t < b$ such that a unique solution of the initial value problem is guaranteed to exist.

Interval: ________ help (inequalities)

Answer(s) submitted:
- 4t/(t+2)
- 0
- homogeneous
- (-2, 3)

(correct)
Correct Answers:
- <table border='0' cellspacing='10'><tr><td> 4*t/(t+2) </td><td> 5/(t+2) </td></tr> <tr><td> 4/(t-3) </td><td> 8*t/(t-
- <table border='0' cellspacing='5'><tr><td> 0 </td></tr><tr><td> 0 </td></tr></table>
- homogeneous
- -2 < t < 3

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2. (1 point) The Differential Equation $2x''' - 2x'' - 18x' + 18x = 0$ can be expressed as a system of first order equations by letting $x_1 = x,\ x_2 = x',\ x_3 = x''$. The system has the form $\mathbf{X}' = A\mathbf{X}$ where A has entries:

$A_{11} =$ ____, $A_{12} =$ ____, $A_{13} =$ ____
$A_{21} =$ ____, $A_{22} =$ ____, $A_{23} =$ ____
$A_{31} =$ ____, $A_{32} =$ ____, $A_{33} =$ ____

The eigenvalues of A are (in increasing order) :

$\lambda_1 =$ ____
$\lambda_2 =$ ____
$\lambda_3 =$ ____

Answer(s) submitted:
- 0
- 1
- 0
- 0
- 0
- 1
- -9
- 9
- 1
- -3
- 1
- 3

(correct)
Correct Answers:
- 0
- 1
- 0
- 0
- 0
- 1
- -9
- 9
- 1
- -3
- 1
- 3

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3. (1 point) Consider the system of higher order differential equations

$$\begin{aligned} y'' &= t^{-1}y' + 5y - tz + (\sin t)z' + e^{4t},\\ z'' &= y - 6z'. \end{aligned}$$

Rewrite the given system of two second order differential equations as a system of four first order linear differential equations of the form $\vec{y}\,' = P(t)\vec{y} + \vec{g}(t)$. Use the following change of variables

$$\vec{y}(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \\ y_3(t) \\ y_4(t) \end{bmatrix} = \begin{bmatrix} y(t) \\ y'(t) \\ z(t) \\ z'(t) \end{bmatrix}.$$

$$\begin{bmatrix} y_1' \\ y_2' \\ y_3' \\ y_4' \end{bmatrix} = \begin{bmatrix} \_\_ & \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ & \_\_ \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} + \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}$$

Answer(s) submitted:

- 0
- 0

(correct)

Correct Answers:

- <table border='0' cellspacing='10'>
  <tr><td> 0 </td><td> 1 </td><td> 0 </td><td> 0 </td></tr>
  <tr><td> 5 </td><td> 1/t </td><td> -t </td><td> sin(t) </td></tr>
  <tr><td> 0 </td><td> 0 </td><td> 0 </td><td> 1 </td></tr>
  <tr><td> 1 </td><td> 0 </td><td> 0 </td><td> -6 </td></tr>
  </table>
- <table border='0' cellspacing='10'>
  <tr><td> 0 </td></tr>
  <tr><td> e^(4*t) </td></tr>
  <tr><td> 0 </td></tr>
  <tr><td> 0 </td></tr>
  </table>

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4. (1 point)

(1) Find the eigenvalues and eigenvectors of the matrix $\begin{bmatrix} 26 & -9 \\ 54 & -19 \end{bmatrix}$.

$\lambda_1 =$ ___, $\vec{v}_1 = \begin{bmatrix} \_\_ \\ \_\_ \end{bmatrix}$, and $\lambda_2 =$ ___, $\vec{v}_2 = \begin{bmatrix} \_\_ \\ \_\_ \end{bmatrix}$

(2) Solve the system of differential equations $\vec{x}' = \begin{bmatrix} 26 & -9 \\ 54 & -19 \end{bmatrix} \vec{x}$ satisfying the initial conditions $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$.

$x_1(t) =$ ________________
$x_2(t) =$ ________________

Answer(s) submitted:

- -1
- 2e^(-t)-e^(8t)
- 6e^(-t) -2e^(8t)

(correct)

Correct Answers:

- <table border='0' cellspacing='0'>
  <tr>
  <td> -1, </td>
  <td><table border='0' cellspacing='5'><tr><td> -1 </td></tr><tr><td> -3 </td></tr></table></td>
  <td>   8,</td>
  <td><table border='0' cellspacing='5'><tr><td> 1 </td></tr><tr><td> 2 </td></tr></table></td>
  </tr>
  </table>
- -2*-1*e^(-1*t)+-1*1*e^(8*t)
- -2*-3*e^(-1*t)+-1*2*e^(8*t)

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5. (1 point) The system of first order differential equations

$y_1' = -26y_1 - 15y_2$
$y_2' = 50y_1 + 29y_2$
where $y_1(0) = -9, y_2(0) = -4$ has solution
$y_1(t) =$ ____________
$y_2(t) =$ ____________

Answer(s) submitted:

- -66e^(-t)+57e^(4t)
- -5/3*(-66)e^(-t) + 57*(-2)e^(4t)

(correct)

Correct Answers:

- 57*1*e**(4*t) + -22*3*e**(-1*t)
- 57*-2*e**(4*t) + -22*-5*e**(-1*t)

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6. (1 point) The system of first order differential equations :

$y_1' = -1y_1 - 1y_2 + 1y_3$
$y_2' = 4y_1 - 1y_2 + 4y_3$
$y_3' = 4y_1 + 1y_2 + 2y_3$
where $y_1(0) = 1, y_2(0) = -1, y_3(0) = 2$

has the solution
$y_1(t) =$ ____________
$y_2(t) =$ ____________
$y_3(t) =$ ____________

Answer(s) submitted:

- -3e^(-2t)+4e^(-t)
- 3e^(3t) -4e^(-t)
- 3e^(3t) +3e^(-2t)-4e^(-t)

(correct)

Correct Answers:

- -3*1*e**(-2*t) + 4*1*e**(-1*t) + -3*0*e**(3*t)
- -3*0*e**(-2*t) + 4*-1*e**(-1*t) + -3*-1*e**(3*t)
- -3*-1*e**(-2*t) + 4*-1*e**(-1*t) + -3*-1*e**(3*t)

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7. (1 point) The system of first order differential equations:

$y_1' = 14y_1 + 3y_2$
$y_2' = -30y_1 - 4y_2$
where $y_1(0) = 3, y_2(0) = -7$

has solution:
$y_1(t) =$ ____________
$y_2(t) =$ ____________

Note You must express the answer in terms of real numbers only.

Answer(s) submitted:

- 3cos(3t)e^(5t) + 2e^(5t)sin(3t)
- 3[illegible]sin(3t)+cos(3t))

(correct)

Correct Answers:

- `e^(5*t)*[3*cos(3*t)+2*sin(3*t)]`
- `e^(5*t)*[-7*cos(3*t)+-9*sin(3*t)]`

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8. (1 point) Find the nth Fourier polynomial for

$$f(x) = \begin{cases} x, & -\pi < x \le 0 \\ -x, & 0 < x \le \pi \end{cases},$$

assuming that $f(x)$ is periodic with period 2π.

$F_n(x) = a_0 + \sum_{k=1}^{n} a_k \cos(kx) + b_k \sin(kx) =$ ______ $+ \sum_{k=1}^{n}$ ______ $\cos(kx) +$ ______ $\sin(kx)$.

Solution:

SOLUTION

To find the nth Fourier polynomial, we need to find a_0 and the different a_k and b_k (for $1 \le k \le n$). These are:

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{0} x\,dx + \frac{1}{2\pi}\int_{0}^{\pi} -x\,dx = -1.5708,$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{0} x\cos(kx)\,dx + \frac{1}{\pi}\int_{0}^{\pi} -x\cos(kx)\,dx = \frac{2(-1+\cos(k\pi))}{k^2\pi}(-1)$$

and

$$b_n = \frac{1}{\pi}\int_{-\pi}^{0} x\sin(kx)\,dx + \frac{1}{\pi}\int_{0}^{\pi} -x\sin(kx)\,dx = 0.$$

Thus,

$$F_n = -1.5708 + \sum_{k=1}^{n} \frac{2(-1+\cos(k\pi))}{k^2\pi}(-1)\cos(kx) + 0\sin(kx).$$

Answer(s) submitted:

- `-pi/2`
- `(2/pi)*((1-cos(k*pi))/k^2)`
- `0`

(correct)

Correct Answers:

- `-1.5708`
- `2*[-1+cos(k*pi)]/(k^2*pi)*(-1)`
- `0`

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9. (1 point) If f is the Fourier series of $g(x) = \begin{cases} 3, & -4 < x < 0 \\ 16 - x^2, & 0 \le x < 4 \end{cases}$, then

$$f(x) = \_\_\_ + \sum_{n=1}^{\infty}\left[\left(\_\_\_\_\_\_\right)\cos\left(\frac{n\pi}{4}x\right) + \left(\_\_\_\_\_\_\right)\sin\left(\frac{n\pi}{4}x\right)\right]$$

What does $f(-4)$ equal? $f(-4) =$___
What does $f(-2)$ equal? $f(-2) =$___
What does $f(0)$ equal? $f(0) =$___
What does $f(1)$ equal? $f(1) =$___
What does $f(4)$ equal? $f(4) =$___

Answer(s) submitted:

- `41/6`
- `-2*cos(n*pi)/(n^2(pi/4)^2)`
- `1/4*(4/(n*pi)(-3+3cos(n*pi)+16)+(4/(n*pi))^3(2-2cos(n*pi))`
- `1.5`
- `3`
- `19/2`
- `15`
- `1.5`

(correct)

Correct Answers:

- `6.83333`
- `-2*4^2*cos(n*3.14159)/(n^2*3.14159^2)`
- `(3*n^2*[cos(n*3.14159)-1]*3.14159^2-4^2*[2*cos(n*3.14159)-`
- `1.5`
- `3`
- `9.5`
- `15`
- `1.5`

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10. (1 point)

If f is the Fourier series of $g(x) = \begin{cases} 0, & -5 < x < 0 \\ 9x, & 0 \le x < 1 \\ 11, & 1 \le x < 5 \end{cases}$, then

$$f(x) = \_\_\_ + \sum_{n=1}^{\infty}\left[\left(\_\_\_\_\_\_\right)\cos\left(\frac{n\pi}{5}x\right) + \left(\_\_\_\_\_\_\right.\right.$$

What does $f(-5)$ equal? $f(-5) =$___
What does $f(-2.5)$ equal? $f(-2.5) =$___
What does $f(0)$ equal? $f(0) =$___
What does $f(1)$ equal? $f(1) =$___
What does $f(4)$ equal? $f(4) =$___
What does $f(5)$ equal? $f(5) =$___

Answer(s) submitted:

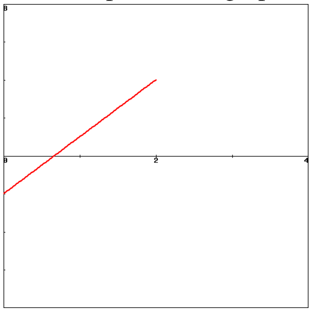
- `97/20`
- `(45cos(n*pi/5)-45-2*n*pi*sin(n*pi/5))/(n^2*pi^2)`
- `(45sin(n*pi/5)+2*n*pi*cos(n*pi/5)-11n*pi*cos(n*pi))/(n^2*p`
- `5.5`
- `0`
- `0`
- `10`
- `11`
- `11/2`

(correct)

Correct Answers:

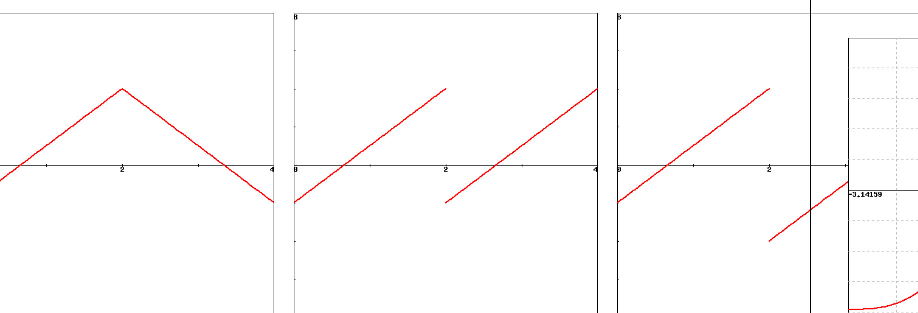
- `4.85`
- `[45*cos(3.14159*n/5)-3.14159*2*n*sin(3.14159*n/5)-45]/(9.8`
- `9*[sin(n*3.14159/5)*5-cos(n*3.14159/5)*n*3.14159]/(n^2*3.1`
- `5.5`
- `0`
- `0`
- `10`
- `11`
- `5.5`

11. (1 point) The graph of the function f is shown on $[0,2]$



Match the graphs with the corresponding extension of f to $[0,4]$

[?] 1. Fourier sine extension
[?] 2. Fourier cosine extension
[?] 3. Fourier expansion



A **B** **C**

Answer(s) submitted:

- C
- A
- B

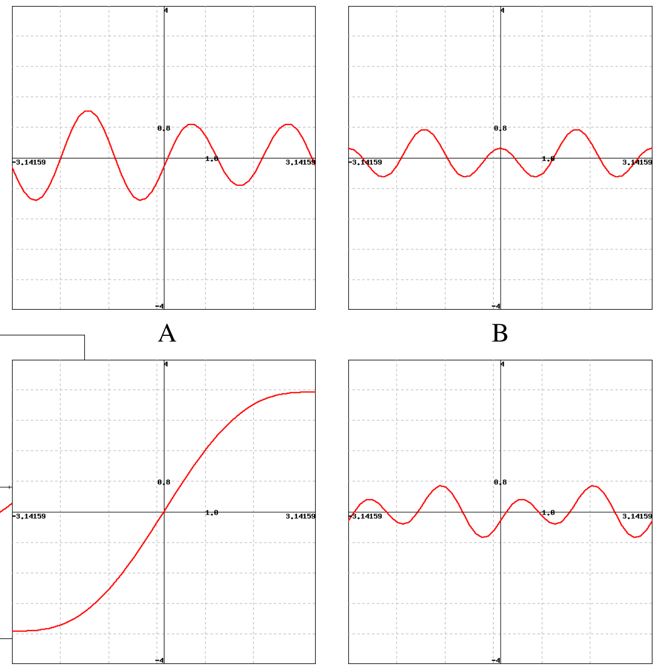
(correct)
Correct Answers:

- C
- A
- B

12. (1 point) f is a function with the following Fourier coefficients on the interval $[-\pi,\pi]$

$a_0 = 0$	
$a_1 = 0$	$b_1 = 0$
$a_2 = -\frac{1}{4}$	$b_2 = 0$
$a_3 = 0$	$b_3 = 1$

Which of the following is the graph of f? [?/A/B/C/D]



A B C D

(Click on a graph to enlarge it.)

Answer(s) submitted:

- A

(correct)
Correct Answers:

- A