MATH 375
Handout # 7
Fourier Series

1. Find the Fourier series of each of the following functions
   a) $f(x) = x$, $x \in [-\pi, \pi]$
   b) $f(x) = 3\pi^2 + 5x - 12x^2$, $-\pi < x < \pi$
   c) $f(x) = 3x^2 + 1$, $x \in [-\pi, \pi]$
   d) $f(x) = \begin{cases} 1, & -\frac{1}{2} < x \leq 0 \\ -1, & 0 < x < \frac{1}{2} \end{cases}$

2. Find the Fourier sine series of $f(x) = \begin{cases} 1, & 0 < x \leq \frac{\pi}{2} \\ 0, & \frac{\pi}{2} < x \leq \pi \end{cases}$

3. Find the Fourier cosine series of $f(x) = \begin{cases} 1, & 0 < x \leq 1 \\ 0, & 1 < x \leq \pi \end{cases}$

4. Find the Fourier sine series and the Fourier cosine series for each of the following functions
   a) $f(x) = x$, $0 < x < 1$
   b) $f(x) = 1$, $0 < x < \pi$.

5. Define and sketch the even and the odd extensions of f if
   a) $f(x) = x$, $0 < x < 1$
   b) $f(x) = \sin(x)$, $0 < x < \pi$
   c) $f(x) = 1 - x$, $0 < x < 1$
   d) $f(x) = x^2$, $0 < x < 1$

6. Consider the function
$$f(x) = \begin{cases} x^2 - 1 & 0 \leq x < 1 \\ x & 1 \leq x < 2 \\ -1 & 2 \leq x < 4 \end{cases}$$
   Determine the values to which the Fourier series of f converges at $x = \frac{1}{2}$, $x = 1$, $x = 2$, and $x = 4$.

7. For the function
$$f(x) = \begin{cases} 0 & -\pi < x < 0 \\ x + 1 & 0 \leq x < \frac{\pi}{2} \\ 2x - 1 & \frac{\pi}{2} < x < \pi \end{cases}$$
   determine the values to which the Fourier series of f converges at $x = 0$, $x = 1$, $x = \frac{\pi}{2}$, and $x = \pi$.

8. If $f(x) = \begin{cases} x^2 + c^2 & 0 < x < 2 \\ 3c + 2x & 2 < x < 3 \end{cases}$, determine all possible values of the constant real number c such that the Fourier series of $f(x)$ converges to 6 at $x = 2$.