David Ng **F2016MATH375**

Assignment03 is due on Sunday, November 20, 2016 at 11:59pm.

The number of attempts available for each question is noted beside the question. If you are having trouble figuring out your error, you should consult the textbook, or ask a fellow student, one of the TA's or your professor for help.
There are also other resources at your disposal, such as the Mathematics Continuous Tutorials. Don't spend a lot of time guessing – it's not very efficient or effective.

Make sure to give lots of significant digits for (floating point) numerical answers. For most problems when entering numerical answers, you can if you wish enter elementary expressions such as $2 \wedge 3$ instead of 8, $\sin(3*\text{pi}/2)$instead of -1, $e \wedge (\ln(2))$ instead of 2, $(2+\tan(3))*(4-\sin(5)) \wedge 6 - 7/8$ instead of 27620.3413, etc.

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1. (1 point) Consider the piecewise function

$$f(t) = \begin{cases} -5 & t < -10 \\ -6\sin(5t) & -10 \le t < 5 \\ -5t^2 - 6t + 5 & 5 \le t < 9 \\ -6 - 5e^{5t} & t \ge 9 \end{cases}.$$

This can be written using step functions as :
$f(t) =$ __________ $+ u_{-10}(t)\cdot$ __________ $+$ $u_5(t)\cdot$ __________ $+u_9(t)\cdot$ __________.

Answer(s) submitted:

- `-5`
- `-6sin(5t)+5`
- `-5t^2-6t+5+6sin(5t)`
- `-6-5e^(5t)-(-5t^2-6t+5)`

(correct)

Correct Answers:

- `-5`
- `-6 sin(5 t) - (-5)`
- `-5 t^{2} + -6 t + 5 - (-6 sin(5 t))`
- `-6-5 e^{5 t} - (-5 t^{2} + -6 t + 5)`

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2. (1 point) Use an appropriate shift or differentiation property to calculate the Laplace Transform of $f(t) = t^2\sin(-2t)$.
$L(f(t)) = F(s)$ where $F(s) =$ __________

Answer(s) submitted:

- `(4(s^2+4)-16s^2)/(s^2+4)^3`

(correct)

Correct Answers:

- `(6*(-2)*s^2 - 2*(-2)^3)/(s^2 + (-2)^2)^3`

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3. (1 point) Find the Laplace transform $F(s) = \mathcal{L}\{f(t)\}$ of the function $f(t) = (5-t)\,(u_3(t) - u_6(t))$

$F(s) = \mathcal{L}\{f(t)\} =$ ____________________ help (formulas)

Answer(s) submitted:

- `e^(-3s)(-1/s^2+2/s)-e^(-6s)(-1/s^2-1/s)`

(correct)

Correct Answers:

- `2*e^(-3*s)/s+e^(-6*s)/s-e^(-3*s)/(s^2)+e^(-6*s)/(s^2)`

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4. (1 point)
Consider the periodic function $f(t)$ defined as follows:

$$f(t) = 5 - 4e^{-3t} \text{ for } 0 \le t < 2, \quad \text{and} \quad f(t+2) = f(t).$$

Sketch a graph of $f(t)$ over several periods and compute its Laplace transform.

$$F(s) = \mathcal{L}\{f(t)\} = \_\_\_\_ \cdot \int_A^B \_\_\_\_$$

where $A =$ ___ and $B =$ ___

$F(s) = \mathcal{L}\{f(t)\} =$ ____________________
help (formulas)

Answer(s) submitted:

- `1/(1-e^(-2s))`
- `(5-4e^(-3t))e^(-st)dt`
- `0`
- `2`
- `((5-5 e^(-2 s))/s+(4 (-1+e^(-2 (3+s))))/(3+s))/(1-e^(-2 s)`

(correct)

Correct Answers:

- `1/[1-e^(-2*s)]`
- `[5-4*e^(-3*t)]*e^(-s*t)*dt`
- `0`
- `2`
- `1/[1-e^(-2*s)]*([-e^(-2*s)+1]/s*5+(e^[-(s+3)*2]-1)/(s+3)*4`

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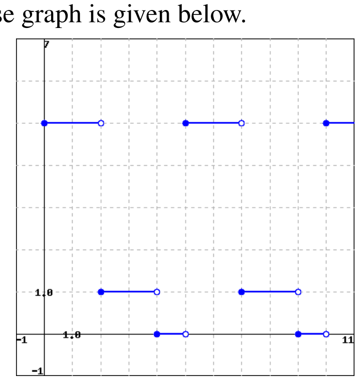
5. (1 point) Find the Laplace transform of the periodic function $f(t)$ whose graph is given below.



(Click on graph to enlarge)

$$F(s) = \mathcal{L}\{f(t)\} = \underline{\qquad\qquad} \cdot \left(\int_0^2 \underline{\qquad\qquad} + \int_2^4 \underline{\qquad\qquad} + \int_4^5 \underline{\qquad\qquad} \right)$$

$$= \underline{\qquad\qquad\qquad\qquad\qquad\qquad}$$

help (formulas)

Answer(s) submitted:

- `1/(1-e^(-5s))`
- `5e^(-st)dt`
- `e^(-st)dt`
- `0e^(-st)dt`
- `((5-5 e^(-2 s))/s+(e^(-4 s) (-1+e^(2 s)))/s)/(1-e^(-5 s))`

(correct)

Correct Answers:

- `1/[1-e^(-5*s)]`
- `5*e^(-s*t)*dt`
- `e^(-s*t)*dt`
- `0`
- `[5-5*e^(-2*s)-e^(-4*s)+e^(-2*s)]/(s*[1-e^(-5*s)])`

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6. (1 point) Given that

$$\mathcal{L}\left\{\frac{\cos(8t)}{\sqrt{\pi t}}\right\} = \frac{e^{-8/s}}{\sqrt{s}}$$

find the Laplace transform of $\sqrt{\frac{t}{\pi}}\cos(8t)$.

$\mathcal{L}\left\{\sqrt{\frac{t}{\pi}}\cos(8t)\right\} = \underline{\qquad\qquad\qquad\qquad}$.

Answer(s) submitted:

- `(e^(-8/s) (-16+s))/(2 s^(5/2))`

(correct)

Correct Answers:

- `exp(-8/s)*(s-2*8)/(2*s^(5/2))`

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7. (1 point) Consider the differential equation $1y''' - 1y'' - 5y' - 2y = 0$ with $y(0) = 3$, $y'(0) = -1$, $y''(0) = 1$. Taking the Laplace transform and solving for $L(y)$ yields :

$L(y) = F(s)$ where $F(s) = \underline{\qquad\qquad\qquad\qquad\qquad}$

Note: don't forget to substitute your initial conditions.

Answer(s) submitted:

- `(3s^2-s+1-3s+1-15)/(s^3-s^2-5s-2)`

(correct)

Correct Answers:

- `(1*((s**2)*3 + s*-1 + 1) + -1*(s*3 + -1) + -5*3) /(1*(s**3) + -1*(s**2) + -5*s + -2)`

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8. (1 point) Find a solution to the initial value problem

$$y' + \sin(t)y = g(t), \quad y(0) = 9,$$

that is continuous on the interval $[0, 2\pi]$ where

$$g(t) = \begin{cases} \sin(t) & \text{if } 0 \le t \le \pi, \\ -\sin(t) & \text{if } \pi < t \le 2\pi. \end{cases}$$

$$y(t) = \begin{cases} \underline{\qquad\qquad\qquad\qquad} & \text{if } 0 \le t \le \pi, \\ \underline{\qquad\qquad\qquad\qquad} & \text{if } \pi < t \le 2\pi. \end{cases}$$

Answer(s) submitted:

- `1+8e^(cos(t))/e`
- `8e^(cos(t)-1)+2e^(cos(t)+1)-1`

(correct)

Correct Answers:

- `1+8*e^[cos(t)-1]`
- `-1+8*e^[cos(t)-1]+2*e^[cos(t)+1]`

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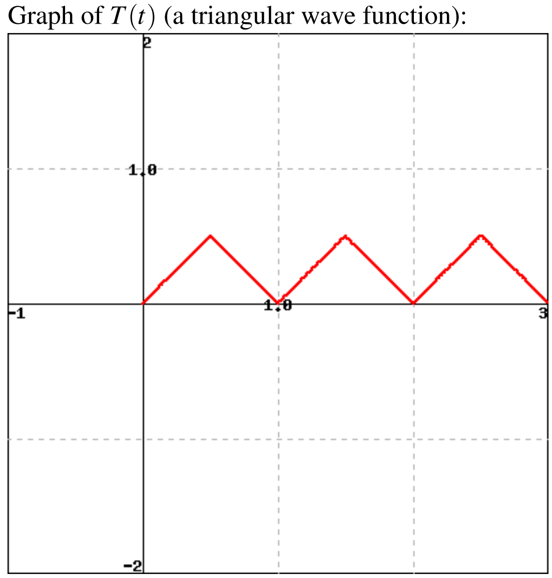
9. (1 point) Take the Laplace transform of the following initial value problem and solve for $Y(s) = \mathcal{L}\{y(t)\}$:

$$y'' + 6y' + 18y = T(t) \qquad y(0) = 0,\ y'(0) = 0$$

Where $T(t) = \begin{cases} t, & 0 \le t < 1/2 \\ 1-t, & 1/2 \le t < 1 \end{cases}$, $\quad T(t+1) = T(t)$.

$Y(s) = \underline{\qquad\qquad\qquad\qquad}$.

Graph of $T(t)$ (a triangular wave function):



Answer(s) submitted:

- `(((e^(-s) (2+e^(s/2) (-2+s)))/(2 s^2)+(1-1/2 e^(-s/2) (2+s`

(correct)

Correct Answers:

- `[tanh(s/4)]/[s^2*(s^2+6*s+18)]`

10. (1 point) Find the inverse Laplace transform $f(t) = \mathcal{L}^{-1}\{F(s)\}$ of the function $F(s) = \dfrac{e^{-2s}(s-9)}{s^2+25}$.
You may use $h(t)$ for the Heaviside step function.

$f(t) = \mathcal{L}^{-1}\left\{\dfrac{e^{-2s}(s-9)}{s^2+25}\right\} =$ ______________________ help (formulas)

Answer(s) submitted:

- `(cos(5(t-2))-9/5sin(5(t-2)))h(t-2)`

(correct)

Correct Answers:

- `h(t-2)*cos(5*(t-2))-1.8*sin(5*(t-2))*h(t-2)`

11. (1 point) Find the inverse Laplace transform $f(t) = \mathcal{L}^{-1}\{F(s)\}$ of the function $F(s) = \dfrac{8s-34}{s^2-8s+17}$.

$f(t) = \mathcal{L}^{-1}\left\{\dfrac{8s-34}{s^2-8s+17}\right\} =$ ______________________ help (formulas)

Answer(s) submitted:

- `e^(4t)(8cos(t)-2sin(t))`

(correct)

Correct Answers:

- `8*e^(4*t)*cos(t)-2*e^(4*t)*sin(t)`

12. (1 point) Consider the function $F(s) = \dfrac{s+8}{s^3-3s^2+3s-1}$.

(1) Find the partial fraction decomposition of $F(s)$:

$$\frac{s+8}{s^3-3s^2+3s-1} = \underline{\qquad\qquad} + \underline{\qquad\qquad}$$

(2) Find the inverse Laplace transform of $F(s)$.

$f(t) = \mathcal{L}^{-1}\{F(s)\} =$ ______________________ help (formulas)

Answer(s) submitted:

- `9/(-1+s)^3`
- `1/(-1+s)^2`
- `te^t+9e^tt^2/2`

(correct)

Correct Answers:

- `9/[(s-1)^3]`
- `1/[(s-1)^2]`
- `9/2*e^t*t^2+1*e^t*t`

13. (1 point) The inverse Laplace transform of $\dfrac{-64-4s^2+20s^3}{16s^3+s^5}$ is ____________

Answer(s) submitted:

- `5sin(4t) -2t^2`

(correct)

Correct Answers:

- `-2*t^2+5*sin(4*t)`

14. (1 point) The inverse Laplace transform of $F(s) = \dfrac{e^{-8s}}{s^2+0s-25}$ is

$f(t) =$ ______________________

Use *step(t-c)* **for** $u_c(t)$

Answer(s) submitted:

- `step(t-8)(e^(5(t-8))/10-e^(-5(t-8))/10)`

(correct)

Correct Answers:

- `step(t-8)*(-0.1*exp(-5*(t-8))+0.1*exp( + 5*(t-8)))`

15. (1 point)
Take the Laplace transform of the following initial value and solve for $Y(s) = \mathcal{L}\{y(t)\}$:

$$y'' + 1y = \begin{cases} \sin(\pi t), & 0 \le t < 1 \\ 0, & 1 \le t \end{cases} \qquad y(0) = 0,\ y'(0) = 0$$

$Y(s) =$ ________________

Next take the inverse transform of $Y(s)$ to get

$y(t) =$ ________________

Use *step(t-c)* **for** $u_c(t)$

Note: $\dfrac{\pi}{(s^2+\pi^2)(s^2+1)} = \dfrac{\pi}{\pi^2-1}\left(\dfrac{1}{s^2+1} - \dfrac{1}{s^2+\pi^2}\right)$

Answer(s) submitted:

- `(pi+pi*e^(-s))/((s^2+pi^2)*(s^2+1))`
- `pi/(1-pi^2)(-sin(t)+sin(pi*t)/pi-step(t-1)sin(pi(t-1))/pi`

(correct)

Correct Answers:

- `pi*(1+exp(-s))/((s^2+pi^2)*(s^2+1))`
- `0.354197606964742*(sin(1*t)/1-sin(pi*t)/pi + step(t-1)*(si`

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