Math 375		Fall 2016
	Laplace Transform	
Worksheet # 3	**Part 2**	**November 14-18**

The problems on this worksheet refer to material from sections §6.1, §6.2, 6.3, and §6.4 of your text. Please report any typos, omissions and errors to aiffam@ucalgary.ca

Inverse Transform of Rational Functions

01. Compute the inverse Laplace transform of each of the following

a. $\dfrac{2}{s^2-6\,s}$ **b.** $\dfrac{3\,s-14}{s^2-4\,s+8}$ **c*.** $\dfrac{8\,s+20}{s^2-12\,s+32}$

d*. $\dfrac{3\,s+2}{(s+1)\,(s+2)\left(s^2+1\right)}$ **e.** $\dfrac{3\,s^2+2\,s+1}{\left(s^2+1\right)\left(s^2+2\,s+2\right)}$ **f.** $\dfrac{4}{s\left(s^4-1\right)}$

g. $\dfrac{s}{\left(s^2+1\right)\left(s^2+4\right)}$ **h.** $\dfrac{3\,s^2+24\,s+15}{s\left(s^2+8\,s+15\right)}$

The Second Shift Formula for the Inverse Transform

02. Determine the inverse Laplace transform of

a. $\dfrac{\mathrm{e}^{-5\,s}}{(s-2)^4}$ **b.** $\dfrac{8\,\mathrm{e}^{-3\,s}}{s^2+4}$ **c*.** $\dfrac{(s+2)\,\mathrm{e}^{-\pi\,s}}{s^2-4\,s+13}$

Inverse Transform of a Derivative

03. Compute the inverse Laplace transform of

a*. $\dfrac{s}{(s^2+1)^2}$ **b.** $\dfrac{s^2-1}{\left(s^2+1\right)^2}$ **c.** $\ln\left(\dfrac{s^2+1}{s^2+4}\right)$

d. $\tan^{-1}\left(\dfrac{3}{s+2}\right)$

Solving Initial Value Problems

04. Use Laplace transform to solve the initial value problems.

a. $\begin{cases} y''+5\,y'+6\,y=2\,\mathrm{e}^{-t} \\ y(0)=1,\ \ y'(0)=3 \end{cases}$ **b.** $\begin{cases} y''+4\,y=8\,\sin(2\,t)+9\,\cos(t) \\ y(0)=1,\ \ y'(0)=0 \end{cases}$

c*. $\begin{cases} y''-3\,y'+2\,y=g(t) \\ y(0)=-3,\ \ y'(0)=1 \end{cases}$ with $g(t)=\begin{cases} 0 & \text{if} \quad 0\le t<1 \\ 1 & \text{if} \quad 1\le t<2 \\ -1 & \text{if} \quad t\ge 2 \end{cases}$

05. Use Laplace transform method to solve the initial value problems.

a. $\begin{cases} y'' + 2\,y' + y = g(t) \\ y(0) = 3, \;\; y'(0) = -1 \end{cases}$ with $g(t) = \begin{cases} \mathrm{e}^t & \text{if} \quad 0 \le t < 1 \\ \mathrm{e}^t - 1 & \text{if} \quad t \ge 1 \end{cases}$

b. $\begin{cases} y'' + 9\,y = g(t) \\ y(0) = 0, \;\; y'(0) = 0 \end{cases}$ with $g(t) = \begin{cases} \cos(t) & \text{if} \quad 0 \le t < 3\pi/2 \\ \sin(t) & \text{if} \quad t \ge 3\pi/2 \end{cases}$