Math 375		**Fall 2016**
	First Order Differential Equations	
Worksheet # 1	**Part 2**	**September 26 - 30**

The problems marked with $(*)$ are to be attempted during the tutorial time. Students are strongly encouraged to attempt the remaining problems on their own. Solutions to all the problems will be available on the course's D2L website Friday, September 30. Please report any typos, omissions and errors to `aiffam@ucalgary.ca`

Bernoulli Equations

01. Solve the Bernoulli differential equations

a. $t\,y' + y = y^2\ \ln t$

b. $y' + \dfrac{1}{t}\,y + t\,y^3 = 0$

c*. $y' = \dfrac{t^3 + y^3}{t\,y^2}$

d. $y' - t\,y = t\,y^{3/2}$

02. Solve the initial value problems

a. $\begin{cases} t\,y' + y = t^4\,y^4 \\ y(1) = 1/2 \end{cases}$

b*. $\begin{cases} y' - 2y = 2\sqrt{y} \\ y(0) = 1 \end{cases}$

Exact Differential Equations

03. Determine whether the given differential equation is exact or not in its domain.

a. $2\,x\,\sin(y)\ \mathrm{d}x + x^2\,\cos(y)\ \mathrm{d}y = 0$

b. $\big(\,3 + \mathrm{e}^{x}\,\cos(y)\,\big)\ \mathrm{d}x + \mathrm{e}^{-x}\,\sin(y)\ \mathrm{d}y = 0$

c. $\big(\,\mathrm{e}^{-y} - y\,\sin(x\,y)\,\big)\ \mathrm{d}x = \big(\,x\mathrm{e}^{-y} - x\,\sin(x\,y)\,\big)\ \mathrm{d}y$

04. Find all functions $M(x,y)$ such that the differential equation

$$M(x,y)\ \mathrm{d}x + \Big(x\,\cos(y) - 2\,y\,\cos(x)\Big)\ \mathrm{d}y = 0$$

is exact in $\mathbb{R}^2$.

05. Solve the following exact differential equations

a. $\big(\,3\,x^2 + y^2 - 4\,x\,y - 3\,y\,\big)\ \mathrm{d}x + \big(\,-2\,x^2 + 6\,y^2 + 2\,x\,y - 3\,x\,\big)\ \mathrm{d}y = 0$

b. $\big(\,2\,x\,\sin(y) + \mathrm{e}^{x}\,\cos(y)\,\big)\ \mathrm{d}x + \big(\,x^2\,\cos(y) - \mathrm{e}^{x}\,\sin(y)\,\big)\ \mathrm{d}y = 0$

c*. $\big(\,x\,\ln(y) + y\,\ln(x)\,\big)\ \mathrm{d}x + \left(\,\dfrac{x^2}{2\,y} + x\,\ln(x) - x\,\right)\ \mathrm{d}y = 0$

06. Solve the following initial value problems.

a. $\Big\{\ \big(y^2 + 2\,x\big)\ \mathrm{d}x + \big(2\,x\,y + \dfrac{1}{y}\big)\ \mathrm{d}y = 0, \qquad y(1) = 1$

b*. $\Big\{\ \big(x\,y^2 + \cos(x)\big)\ \mathrm{d}x + \big(\mathrm{e}^{2\,y} + x^2\,y\big)\ \mathrm{d}y = 0, \qquad y(\pi/2) = 0$

Integrating Factors For Non Exact Equations

07. Each of the following differential equations has an integrating factor μ that depends on either x alone or on y alone. Find μ, then find the general solution of the differential equation.

a*. $\big(5\,x\,y + 4\,y^2 + 1\big)\ \mathrm{d}x + \big(x^2 + 2\,x\,y\big)\ \mathrm{d}y = 0$

b. $x\,y^3\ \mathrm{d}x + \big(x^2\,y^2 + 1\big)\ \mathrm{d}y = 0$

c*. $\big(2\,x + \tan(y)\big)\ \mathrm{d}x + \big(x - x^2\ \tan(y)\big)\ \mathrm{d}y = 0$

d. $\big(y^2 - x\big)\ \mathrm{d}x + 4\,x\,y\ \mathrm{d}y = 0,\ x > 0$

08. Verify that the differential equation is not exact in $\mathbb{R}^2$, then find an integrating factor in the form $\mu(x, y) = x^m\,y^n$

a*. $\big(\,4\,x\,y^2 + 6\,y\,\big)\ \mathrm{d}x + \big(\,5\,x^2\,y + 8\,x\,\big)\ \mathrm{d}y = 0$

b. $\big(\,3\,x\,y - 2\,y^2\,\big)\ \mathrm{d}x + \big(\,2\,x^2 - 3\,x\,y\,\big)\ \mathrm{d}y = 0$

Homogeneous Equations

09. Find the general solution of the differential equations

a. $x\,y' = y + x\cos^2\left(\dfrac{y}{x}\right)$

b*. $y' = \dfrac{x - y}{x + y}$

10. Solve the differential equations

a. $3x\,y\,y' = x^2 + 4y^2$

b. $xy' = \sqrt{x^2 - y^2} + y$

11. Find the general solution of $y' = -\dfrac{8\,x + 4\,y + 1}{4\,x + 2\,y + 1}$

Hint: Use the substitution $u = 4\,x + 2\,y$ to convert the equation into a separable equation.