

Department of Mathematics and Statistics
MATH 375
Handout # 2
First Order Ordinary Differential Equations

1. Find the general solution of the following differential equations
 - a) $xy' - y = y^3$
 - b) $xyy' = 1 - x^2$
 - c) $y' \tan x = y$
 - d) $y' = 10^{x+y}$
2. Find the solution of the following initial value problems
 - a) $(x^2 - 1)y' + 2xy^2 = 0, \quad y(0) = 1$
 - b) $y' \sin x = y \ln y, \quad y(\frac{\pi}{2}) = 1$
3. Find the general solution of the following differential equations
 - a) $y' + y = xy^3$
 - b) $x^2y' = y(x + y)$
 - c) $y' + x\sqrt[3]{y} = 3y$
4. Find the general solution of the following differential equations
 - a) $y' = \frac{2xy}{x^2 + y^2}$
 - b) $x - y \cos\left(\frac{y}{x}\right) + x \cos\left(\frac{y}{x}\right) y' = 0$
5. Find the solution of the initial value problem $2xyy' + x^2 - y^2 = 0, \quad y(1) = 0$.
6. If possible, find the values of α and β such that the equation $xy^\beta y' = 3x^\alpha + x^3 y^3$ is
 - a) linear;
 - b) separable;
 - c) homogeneous;
 - d) Bernoulli?
7. Find α and β such that the following equations are exact:
 - a) $x^\alpha y^2 + x^3 y^\beta y' = 0$
 - b) $6x^\beta e^y dx + x^{\beta+\alpha} e^y dy = 0$
 - c) $e^{\alpha x + \beta y} (3 + y') = 0$
8. Find the general solution of the following differential equations
 - a) $(2 - 9xy^2)x dx + (4y^2 - 6x^3)y dy = 0$
 - b) $1 + y^2 \sin 2x - 2yy' \cos^2 x = 0$
 - c) $x dx + y dy = \frac{xdy - ydx}{x^2 + y^2}$
9. Find the general solution of the following differential equations.
 - a) $y' = 2xy + x$
 - b) $2xy' = y$
 - c) $x^2 + 2xy^3 + (y^2 + 3x^2y^2)y' = 0$
10. Use the appropriate existence and uniqueness theorem to find the largest interval (a, b) on which the solution to each the following equations is guaranteed to exist:

a) $y' + \frac{t}{t^2 - 1}y = \sqrt{5 - t}, \quad y(4) = -3$

b) $(t - 6)y' + ty = \ln\left(t - \frac{4}{t}\right), \quad y(3) = 7$

11. Find the general solution of the following differential equations

a) $(1 + 2y)y' + 2(y + y^2) = 0;$

b) $\left(x^2 + \frac{x}{\cos^2 y}\right)y' + 3xy + 2tany = 0.$