Laplace Transforms

01. $\mathcal{L}\{K_1\ f_1(t) + K_2\ f_2(t)\}(s) = K_1\ \mathcal{L}\{f_1(t)\}(s) + K_2\ \mathcal{L}\{f_2(t)\}(s)$

02. $\mathcal{L}\left\{y^{(n)}(t)\right\}(s) = s^n\,\mathcal{L}\{y(t)\}(s) - s^{n-1}\,y(0) - s^{n-2}\,y'(0) - \cdots - y^{(n-1)}(0) \quad n = 1, 2, 3, \cdots$

03. $\mathcal{L}\left\{\mathrm{e}^{a\,t}\ f(t)\right\}(s) = \mathcal{L}\{f(t)\}\big(s-a\big)$

04. $\mathcal{L}\{u_a(t)\ f(t)\}(s) = \mathcal{L}\left\{f\big(t+a\big)\right\}(s)\ \mathrm{e}^{-a\,s}$

05. $\mathcal{L}\{f(t)\}(s) = \dfrac{1}{1-\mathrm{e}^{-T\,s}}\displaystyle\int_0^T f(t)\,\mathrm{e}^{-s\,t}\,\mathrm{d}t, \quad f(t+T) = f(t)$

06. $\mathcal{L}\{t\ f(t)\}(s) = -\dfrac{\mathrm{d}}{\mathrm{d}s}\Big(\mathcal{L}\{f(t)\}(s)\Big)$

07. $\mathcal{L}\left\{\dfrac{f(t)}{t}\right\}(s) = \displaystyle\int_s^{+\infty}\mathcal{L}\{f(t)\}(r)\,\mathrm{d}r$

08. $\mathcal{L}\{t^n\}(s) = \dfrac{n!}{s^{n+1}}, \quad n = 0, 1, 2, \cdots$

09. $\mathcal{L}\left\{\mathrm{e}^{a\,t}\,t^n\right\}(s) = \dfrac{n!}{\big(s-a\big)^{n+1}}, \quad n = 0, 1, 2$

10. $\mathcal{L}\left\{\cos\big(b\,t\big)\right\}(s) = \dfrac{s}{s^2+b^2}$

11. $\mathcal{L}\left\{\mathrm{e}^{a\,t}\ \cos\big(b\,t\big)\right\}(s) = \dfrac{s-a}{(s-a)^2+b^2}$

12. $\mathcal{L}\left\{\sin\big(b\,t\big)\right\}(s) = \dfrac{b}{s^2+b^2}$

13. $\mathcal{L}\left\{\mathrm{e}^{a\,t}\ \sin\big(b\,t\big)\right\}(s) = \dfrac{b}{(s-a)^2+b^2}$

14. $\mathcal{L}\{u_a(t)\}(s) = \dfrac{\mathrm{e}^{-a\,s}}{s}$

15. $\mathcal{L}\left\{\mathrm{e}^{a\,t}\right\}(s) = \dfrac{1}{s-a}$

16. $\mathcal{L}\left\{\dfrac{1}{\sqrt{t}}\right\}(s) = \sqrt{\dfrac{\pi}{s}}$

Inverse Laplace Transforms

01. $\mathcal{L}^{-1}\{K_1\ F_1(s) + K_2\ F_2(s)\}(t) = K_1\ \mathcal{L}^{-1}\{F_1(s)\}(t) + K_2\ \mathcal{L}^{-1}\{F_2(s)\}(t)$

02. $\mathcal{L}^{-1}\left\{F\big(s-a\big)\right\}(t) = \mathrm{e}^{a\,t}\ \mathcal{L}^{-1}\{F(s)\}(t)$ or $\mathcal{L}^{-1}\left\{F\big(s+a\big)\right\}(t) = \mathrm{e}^{-a\,t}\ \mathcal{L}^{-1}\{F(s)\}(t)$

03. $\mathcal{L}^{-1}\left\{F(s)\ \mathrm{e}^{-a\,s}\right\}(t) = u_a(t)\ \mathcal{L}^{-1}\{F(s)\}\big(t-a\big)$ $\qquad \mathcal{L}^{-1}\left\{\dfrac{\mathrm{e}^{-a\,s}}{s}\right\}(t) = u_a(t)$

04. $\mathcal{L}^{-1}\{F'(s)\}(t) = -t\ \mathcal{L}^{-1}\{F(s)\}(t)$

05. $\mathcal{L}^{-1}\left\{\dfrac{1}{s^{n+1}}\right\}(t) = \dfrac{t^n}{n!}\ \ n = 0, 1, 2, \cdots$

06. $\mathcal{L}^{-1}\left\{\dfrac{1}{(s-a)^{n+1}}\right\}(t) = \mathrm{e}^{a\,t}\ \dfrac{t^n}{n!}\ \ n = 0, 1, 2$

07. $\mathcal{L}^{-1}\left\{\dfrac{s}{s^2+b^2}\right\}(t) = \cos\big(b\,t\big)$

08. $\mathcal{L}^{-1}\left\{\dfrac{s-a}{(s-a)^2+b^2}\right\}(t) = \mathrm{e}^{a\,t}\ \cos\big(b\,t\big)$

09. $\mathcal{L}^{-1}\left\{\dfrac{1}{s^2+b^2}\right\}(t) = \dfrac{1}{b}\ \sin\big(b\,t\big)$

10. $\mathcal{L}^{-1}\left\{\dfrac{1}{(s-a)^2+b^2}\right\}(t) = \dfrac{1}{b}\ \mathrm{e}^{a\,t}\ \sin\big(b\,t\big)$

Trigonometric Identities

1. $\cos^2(\theta) + \sin^2(\theta) = 1$

2. $\cos(2\,\theta) = \cos^2(\theta) - \sin^2(\theta)$

3. $\sin(2\,\theta) = 2\ \cos(\theta)\ \sin(\theta)$

4. $2\ \cos^2(\theta) = 1 + \cos(2\,\theta)$

5. $2\ \sin^2(\theta) = 1 - \cos(2\,\theta)$

6. $\cos(\theta \pm \pi) = -\cos(\theta)$

7. $\sin(\theta \pm \pi) = -\sin(\theta)$

8. $\cos(\theta \pm 2\,\pi) = \cos(\theta)$

9. $\sin(\theta \pm 2\,\pi) = \sin(\theta)$

1. If $f(x)$ is $2\,L$-periodic and piecewise continuous then its fourier series is

$$f(x) \sim \frac{a_0}{2} + \sum_{n=1}^{+\infty} a_n \cos\left(n\frac{\pi}{L}\,x\right) + b_n \sin\left(n\frac{\pi}{L}\,x\right)$$

where

$$a_n = \frac{1}{L}\int_I f(x)\cos\left(n\frac{\pi}{L}\,x\right)\,\mathrm{d}x \qquad b_n = \frac{1}{L}\int_I f(x)\sin\left(n\frac{\pi}{L}\,x\right)\,\mathrm{d}x$$

and I is an interval of length $2\,L$

2. The eigenvalues and corresponding eigenfunctions of the BVP

$$\Big\| \; U'' + \lambda\,U = 0, \quad U(0) = 0, \;\; U(L) = 0$$

are given by: $\lambda_n = \dfrac{n^2\,\pi^2}{L^2}$ and $U_n(x) = \sin\left(\dfrac{n\,\pi}{L}\,x\right)$, respectively.