THE UNIVERSITY OF CALGARY
DEPARTMENT OF MATHEMATICS AND STATISTICS

MATH 375 L01-04 PRACTICE FINAL EXAM FALL 2012

The exam has **25** equally weighted questions. Please attempt all questions and record your answer by circling your choice in the exam booklet, and filling in the appropriate circle in the scantron sheet.
This is a closed book examination and calculators are not permitted. A formula sheet is attached to this booklet.

Key

01	02	03	04	05	06	07	08	09	10	11	12	13
B	**A**	**B**	**C**	**A**	**E**	**C**	**A**	**B**	**B**	**D**	**E**	**B**

14	15	16	17	18	19	20	21	22	23	24	25	
D	**E**	**E**	**A**	**C**	**D**	**C**	**C**	**B**	**A**	**E**	**A**	

01. Consider a nonexact differential equation

$$M(x,y)\ \mathrm{d}x + N(x,y)\ \mathrm{d}y = 0$$

where $M(x,y)$ and $N(x,y)$ have continuous first partial derivatives in the xy-plane. Then the equation must have at least one integrating factor μ that either depends on x only or on y only.

Circle your choice in the box below

T	**F**

02. If $y_1(t)$ and $y_2(t)$ are solutions of the nonhomogeneous differential equation

$$y'' + p(t)\,y' + q(t)\,y = g(t), \qquad a < t < b$$

then so is $\dfrac{7}{5}\,y_1(t) - \dfrac{2}{5}\,y_2(t)$

Circle your choice in the box below

T	**F**

03. There is no second order linear homogeneous differential equation with constant coefficients that has $y(t) = \pi^t$ as a solution.

Circle your choice in the box below

T	**F**

04. The initial value problem $\begin{cases} \left(2\,x\,\ln(y) - 3\,x^2\right)\,\mathrm{d}x + \left(\dfrac{x^2}{y} + 2\,y\right)\,\mathrm{d}y = 0 \\ y(-1) = 1 \end{cases}$

has solution

A. $x^2\,\ln(y) + y^2 - x^3 = 0$
B. $x^2\,\ln(y) + 2\,x\,y - x^3 = -1$
C. $x^2\,\ln(y) + y^2 - x^3 = 2$
D. $\dfrac{x^2}{y} + 2\,y + 2\,x\,\ln(y) - 3\,x^2 = 0$
E. none of the above

05. The solution of the ivp $\begin{cases} y''' - 4\,y' = 4\,t \\ y(0) = 0, \quad y'(0) = 2 \quad y''(0) = -5 \end{cases}$ is

A. $y(t) = 1 - \dfrac{t^2}{2} - \mathrm{e}^{-2\,t}$
B. $y(t) = \dfrac{5}{4} - \dfrac{t}{2} - \dfrac{5}{4}\,\mathrm{e}^{-2\,t}$
C. $y(t) = -3 - \dfrac{t^2}{2} + \left(3 - 4\,t\right)\mathrm{e}^{2\,t}$
D. $y(t) = \dfrac{5}{4} + \dfrac{9\,t}{2} - \dfrac{5}{4}\,\mathrm{e}^{2\,t}$
E. none of the above

06. If an n^{th} order linear homogeneous differential equation with constant coefficients has $t^2\,\mathrm{e}^t\left(3 - 2\,\cos(t) + \sin(2\,t)\right)$, as a solution, then the smallest value of n is

A. 7 **B.** 9 **C.** 10 **D.** 11 **E.** 15

07. The differential equation for which the method of undetermined coefficients can not be used to find a particular solution $y_p(t)$ is

A. $y'' + 2\,y' - 3\,y = 2^t + \dfrac{1}{2^t}$
B. $y'' + 2\,y' - 3\,y = \dfrac{\sin(t)}{\mathrm{e}^{3\,t}}$
C. $y'' + 2\,y' - 3\,y = t^3 + 3\,t^2 - 2\,t^{-1} + 5$
D. $y'' + 2\,y' - 3\,y = \dfrac{t}{\sec(t)}$
E. $y'' + 2\,y' - 3\,y = \mathrm{e}^{t+2\,\ln(t)}$

08. If $f(t) = \begin{cases} 1 & \text{if} \quad 0 \le t < 2 \\ 2\,t - 3 & \text{if} \quad 2 \le t < 4 \\ 5 & \text{if} \quad 4 \le t \end{cases}$ then

A. $f(t) = 1 + (2\,t - 4)\,u_2(t) - (2\,t - 8)\,u_4(t)$
B. $f(t) = 1 + (2\,t - 3)\,u_2(t) + 5\,u_4(t)$
C. $f(t) = 1 + (2\,t - 2)\,u_2(t) + (2\,t + 2)\,u_4(t)$
D. $f(t) = 1 - (2\,t - 4)\,u_2(t) + (2\,t - 8)\,u_4(t)$
E. none of the above

09. $\mathcal{L}\Big\{(\pi - t)\,\sin(t)\,u_\pi(t)\Big\}(s)$ is equal to

A. $\left(\dfrac{\pi}{s} - \dfrac{1}{s^2}\right)\dfrac{1}{s^2+1}\,\dfrac{e^{-\pi s}}{s}$
B. $\dfrac{2\,s}{\left(s^2+1\right)^2}\,e^{-\pi s}$
C. $\left(\dfrac{\pi}{s^2+1} - \dfrac{2\,s}{\left(s^2+1\right)^2}\right)e^{-2\,s}$
D. $\left(\dfrac{2\,s}{\left(s^2+1\right)^2} - \dfrac{2\,\pi}{s^2+1}\right)e^{-\pi s}$
E. none of the above

10. $\mathcal{L}\Big\{\,|t - 5|\,\Big\}(s)$ is equal to

A. $\dfrac{5}{s} - \dfrac{1}{s^2} + \left(-\dfrac{5}{s} + \dfrac{2}{s^2}\right)e^{-5\,s}$
B. $\dfrac{5}{s} - \dfrac{1}{s^2} + \dfrac{2}{s^2}\,e^{-5\,s}$
C. $\dfrac{5}{s} - \dfrac{1}{s^2} + \left(-\dfrac{5}{s} + \dfrac{1}{s^2}\right)e^{-5\,s}$
D. $\left|\dfrac{1}{s^2} - \dfrac{5}{s}\right|$
E. none of the above

11. $\mathcal{L}^{-1}\left\{\dfrac{12\,s}{s^2-2\,s-3}\right\}(t)$ is equal to

A. $9\,\mathrm{e}^{-3\,t}+3\,\mathrm{e}^{t}$
B. $-9\,\mathrm{e}^{3\,t}+3\,\mathrm{e}^{-t}$
C. $3\,\mathrm{e}^{3\,t}+9\,\mathrm{e}^{-t}$
D. $9\,\mathrm{e}^{3\,t}+3\,\mathrm{e}^{-t}$
E. $3\,\mathrm{e}^{-3\,t}+9\,\mathrm{e}^{t}$

12. $\mathcal{L}^{-1}\left\{\dfrac{s-1}{s^2+2\,s+5}\right\}(t)$ is equal to

A. $\mathrm{e}^{t}\left(\cos(4\,t)-\dfrac{1}{2}\sin(4\,t)\right)$
B. $\mathrm{e}^{-t}\left(\cos(4\,t)-\dfrac{1}{2}\sin(4\,t)\right)$
C. $\mathrm{e}^{-t}\left(\cos(2\,t)-2\sin(2\,t)\right)$
D. $\mathrm{e}^{t}\left(\cos(2\,t)-\sin(2\,t)\right)$
E. $\mathrm{e}^{-t}\left(\cos(2\,t)-\sin(2\,t)\right)$

13. $\mathcal{L}^{-1}\left\{\dfrac{s\,\mathrm{e}^{-2\,s}}{s^2+\pi^2}\right\}(t)$ is equal to

A. $\dfrac{1}{\pi}\,u_2(t)\,\sin\left(\pi\,t\right)$
B. $u_2(t)\,\cos\left(\pi\,t\right)$
C. $u_\pi(t)\,\cos\left(2\,t\right)$
D. $\dfrac{1}{\pi}\,u_2(t)\,\cos\left(\pi\,t\right)$
E. none of the above

14. A particular solution of the differential equation $y''+2\,y'+y=\dfrac{\mathrm{e}^{-t}}{t+1}$, is

A. $y_p(t)=\dfrac{\mathrm{e}^{-t}}{t+1}$
B. $y_p(t)=(t+1)\,\ln(t+1)$
C. $y_p(t)=t\,\ln(t+1)\,\mathrm{e}^{-t}$
D. $y_p(t)=(t+1)\,\ln(t+1)\,\mathrm{e}^{-t}$
E. none of the above

15. Consider the differential equation

$$y'' + 4\,y = \begin{cases} 1 & \text{if} \quad 0 \le t < 2 \\ \mathrm{e}^{2-t} & \text{if} \quad 2 \le t \end{cases} \qquad y(0) = 0, \quad y'(0) = 0$$

Then $\mathcal{L}\left\{y(t)\right\}(s)$, is equal to

A. $\dfrac{1-\mathrm{e}^{-s}}{s^2\left(s^2+4\right)}$ **B.** $\dfrac{1-\mathrm{e}^{-s}}{s^2\left(s^2-4\right)}$ **C.** $\dfrac{1-\mathrm{e}^{-s}}{s\left(s^2+4\right)}$

D. $\dfrac{1+\mathrm{e}^{-s}}{s\left(s^2+4\right)}$ **E.** $\dfrac{s+1-\mathrm{e}^{-2\,s}}{s\,(s+1)\left(s^2+4\right)}$

16. The Laplace transform of $\mathrm{e}^{3\,t}\left(t^2+\sin(t)\right)$, is

A. $\dfrac{2}{(s-3)^3} + \dfrac{s-3}{(s-3)^2+1}$ **B.** $\dfrac{1}{(s-3)^2} + \dfrac{s}{(s-3)^2+1}$

C. $\dfrac{2}{(s-3)^2} + \dfrac{s-1}{(s-3)^2+1}$ **D.** $\dfrac{2}{(s-3)^2} + \dfrac{3}{(s-3)^2+1}$

E. $\dfrac{2}{(s-3)^3} + \dfrac{1}{(s-3)^2+1}$

17. The Laplace transform of $2\,\mathrm{e}^t\,u_2(t)\,\cos^2(t-2)$, is

A. $\mathrm{e}^{-2\,(s-1)}\left[\dfrac{1}{s-1} + \dfrac{s-1}{(s-1)^2+4}\right]$ **B.** $\mathrm{e}^{-(s-1)}\left[\dfrac{1}{s-1} + \dfrac{s-1}{(s-1)^2+1}\right]$

C. $\mathrm{e}^{-2\,s}\left[\dfrac{1}{s-1} + \dfrac{s}{(s-1)^2+4}\right]$ **D.** $\mathrm{e}^{s-1}\left[\dfrac{1}{s-1} + \dfrac{s-1}{(s-1)^2+4}\right]$

E. $\mathrm{e}^{-2\,(s-1)}\left[\dfrac{1}{s} + \dfrac{1}{s^2+4}\right]$

18. The solution of the initial value problem

$$y^{(4)} - y = 0, \quad y(0) = 1,\ y'(0) = 0,\ y''(0) = 1,\ y'''(0) = 0$$

is given by

A. $\big(2\,\mathrm{e}^t + \mathrm{e}^{-t}\big)/3$ **B.** $\big(-\,\mathrm{e}^t + 3\,\mathrm{e}^{-t}\big)/2$ **C.** $\big(\mathrm{e}^t + \mathrm{e}^{-t}\big)/2$

D. $\big(\mathrm{e}^t + 2\,\mathrm{e}^{-t}\big)/3$ **E.** $\big(4\,\mathrm{e}^t - \mathrm{e}^{-t}\big)/3$

19. Given that the matrix $\begin{bmatrix} 2 & 3 \\ -3 & 2 \end{bmatrix}$ has a complex eigenvalue $2 + 3\,i$, and a corresponding eigenvector $\begin{bmatrix} 1 \\ i \end{bmatrix}$, the initial value problem

$$\begin{cases} y_1' = 2\,y_1 + 3\,y_2 \\ y_2' = -3\,y_1 + 2\,y_2 \\ y_1(0) = 1 \ \ y_2(0) = 2 \end{cases}$$

has solution

A. $y_1(t) = \mathrm{e}^{2\,t}\,\big(\cos(3\,t) - 2\,\sin(3\,t)\big), \qquad y_2(t) = \mathrm{e}^{2\,t}\,\big(2\,\cos(3\,t) - \sin(3\,t)\big)$

B. $y_1(t) = \mathrm{e}^{t}\,\big(\cos(3\,t) - 2\,\sin(3\,t)\big), \qquad y_2(t) = \mathrm{e}^{t}\,\big(2\,\cos(3\,t) + \sin(3\,t)\big)$

C. $y_1(t) = \mathrm{e}^{t}\,\big(\cos(3\,t) + \sin(3\,t)\big), \qquad y_2(t) = \mathrm{e}^{t}\,\big(2\,\cos(3\,t) - 2\,\sin(3\,t)\big)$

D. $y_1(t) = \mathrm{e}^{2\,t}\,\big(\cos(3\,t) + 2\,\sin(3\,t)\big), \qquad y_2(t) = \mathrm{e}^{2\,t}\,\big(2\,\cos(3\,t) - \sin(3\,t)\big)$

E. $y_1(t) = \mathrm{e}^{-t}\,\big(\cos(3\,t) - \sin(3\,t)\big), \qquad y_2(t) = \mathrm{e}^{-t}\,\big(2\,\cos(3\,t) - 2\,\sin(3\,t)\big)$

20. Suppose the matrix $\begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 1 \end{bmatrix}$ has a eigenvalues $\lambda_1 = 4,\ \lambda_2 = 1,\ \lambda_3 = -1,$

with corresponding eigenvectors $\vec{V}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix},\ \vec{V}_2 = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix},\ \vec{V}_3 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix},$

then the unique solution of $\vec{Y}' = \boldsymbol{A}\,\vec{Y}$, subject to $\vec{Y}(0) = \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}$, is given by

A. $\vec{Y}(t) = \begin{bmatrix} e^{4t} + e^{t} + e^{-t} \\ e^{4t} - 2\,e^{t} \\ e^{4t} - e^{t} + e^{-t} \end{bmatrix}$

B. $\vec{Y}(t) = \begin{bmatrix} 2\,e^{4t} + 2\,e^{t} - e^{-t} \\ e^{4t} - 3\,e^{t} + e^{-t} \\ e^{4t} + e^{t} - e^{-t} \end{bmatrix}$

C. $\vec{Y}(t) = \begin{bmatrix} e^{4t} + e^{t} + e^{-t} \\ e^{4t} - 2\,e^{t} \\ e^{4t} + e^{t} - e^{-t} \end{bmatrix}$

D. $\vec{Y}(t) = \begin{bmatrix} e^{4t} + e^{t} + e^{-t} \\ e^{4t} - 2\,e^{t} \\ -e^{4t} + e^{t} + e^{-t} \end{bmatrix}$

E. $\vec{Y}(t) = \begin{bmatrix} e^{4t} + e^{t} + e^{-t} \\ e^{4t} - 2\,e^{-t} \\ -e^{4t} + e^{t} + e^{-t} \end{bmatrix}$

21. The Fourier cosine series of $f(x) = e^{x},\ 0 < x < 2\pi$, is given by

A. $\dfrac{a_0}{2} + \sum_{i=1}^{+\infty} a_n \cos(n\,x)$, with $a_n = \dfrac{1}{2\pi}\displaystyle\int_0^{2\pi} e^{x} \cos(n\,x)\,dx$

B. $\dfrac{a_0}{2} + \sum_{i=1}^{+\infty} a_n \cos(2\,n\,x)$, with $a_n = \dfrac{1}{2\pi}\displaystyle\int_0^{2\pi} e^{x} \cos(2\,n\,x)\,dx$

C. $\dfrac{a_0}{2} + \sum_{i=1}^{+\infty} a_n \cos(n\,x/2)$, with $a_n = \dfrac{1}{\pi}\displaystyle\int_0^{2\pi} e^{x} \cos(n\,x/2)\,dx$

The upper limit of the integral above, has been changed from π to 2π

D. $\dfrac{a_0}{2} + \sum_{i=1}^{+\infty} a_n \cos(n\,x)$, with $a_n = \dfrac{2}{\pi}\displaystyle\int_0^{\pi} e^{x} \cos(n\,x)\,dx$

E. $\dfrac{a_0}{2} + \sum_{i=1}^{+\infty} a_n \cos(n\,x)$, with $a_n = \dfrac{1}{\pi}\displaystyle\int_{-\pi}^{\pi} e^{x} \cos(n\,x)\,dx$

22. The Fourier sine series of $f(x) = x,\ 0 < x < 2\pi,$ is given by

A. $\displaystyle\sum_{i=1}^{+\infty} -\frac{2}{n}\ \sin(n\,x)$

B. $\displaystyle 4\sum_{i=1}^{+\infty} \frac{(-1)^{n-1}}{n}\ \sin(n\,x/2)$

C. $\displaystyle\sum_{i=1}^{+\infty} -\frac{1}{n}\ \sin(2\,n\,x)$

D. $\displaystyle \pi - 2\sum_{i=1}^{+\infty} \frac{1}{n}\ \sin(n\,x)$

E. $\displaystyle\sum_{i=1}^{+\infty} \frac{2\,(1-(-1)^n)}{n}\ \sin(n\,x/2)$

23. Let $f(x) = \begin{cases} x^2+4 & \text{if} \quad 0 < x < 1 \\ -7 & \text{if} \quad x = 1 \\ 2\,x+9 & \text{if} \quad 1 < x < 3 \end{cases}$ with $f(x+3) = f(x)$

The value of the Fourier series of $f(x)$, at $x = 1$ is

A. 8 **B.** 3 **C.** -7 **D.** 16 **E.** -3

24. The solution of the Initial Boundary Value Problem

$$\begin{cases} 2\,u_{xx} = u_t, \quad 0 < x < 4, \quad t > 0 \\ u(0,t) = 0, \quad u(4,t) = 0, \quad t > 0 \\ u(x,0) = 5\,\sin(\pi\,x/4) - 2\,\sin(\pi\,x), \quad 0 < x < 4 \end{cases}$$

is given by

A. $u(x,t) = \Big(5\,\sin\big((\pi/4)\ x\big) - 2\,\sin(\pi\,x)\Big)\ \mathrm{e}^{-(\pi^2/8)\ t}$

B. $u(x,t) = 5\,\sin\big((\pi/4)\ x\big)\,\mathrm{e}^{-(\pi^2/8)\ t} - 2\,\sin(\pi\,x)\ \mathrm{e}^{-(\pi^2/2)\ t}$

C. $u(x,t) = 5\,\sin\big((\pi/2)\,x\big)\,\mathrm{e}^{-(\pi^2/2)\ t} - 2\,\sin(\pi\,x)\ \mathrm{e}^{-2\,\pi^2\ t}$

D. $\displaystyle u(x,t) = 5\sum_{n=1}^{+\infty} \sin\left(n\,\frac{\pi}{4}\,x\right)\ \mathrm{e}^{-\left(n^2\,\pi^2/8\right)\ t}$

E. $u(x,t) = 5\,\sin\big((\pi/4)\ x\big)\ \mathrm{e}^{-(\pi^2/8)\ t} - 2\,\sin(\pi\,x)\ \mathrm{e}^{-2\,\pi^2\ t}$

25. The solution of the Initial Boundary Value Problem

$$\begin{cases} u_{xx} = u_{tt}, \quad 0 < x < 3\,\pi, \quad t > 0 \\ u(0,t) = 0, \quad u(3\,\pi, t) = 0, \quad t > 0 \\ u(x,0) = 3\,\sin(2\,x/3), \quad u_t(x,0) = 0 \quad 0 < x < 3\,\pi \end{cases}$$

is given by

A. $u(x,t) = 3\,\sin(2\,x/3)\ \cos(2\,t/3)$

B. $u(x,t) = 3\,\sin(2\,x/3)\ \sin(2\,t/3)$

C. $u(x,t) = 3\,\cos(2\,x/3)\ \sin(2\,t/3)$

D. $u(x,t) = 3\displaystyle\sum_{n=1}^{+\infty} \sin(2\,n\,x/3)\ \cos(2\,n\,t/3)$

E. $u(x,t) = 3\displaystyle\sum_{n=1}^{+\infty} \sin(2\,x/3)\ \cos(2\,t/3)$

Laplace Transforms

01. $\mathcal{L}\{K_1\ f_1(t) + K_2\ f_2(t)\}(s) = K_1\ \mathcal{L}\{f_1(t)\}(s) + K_2\ \mathcal{L}\{f_2(t)\}(s)$

02. $\mathcal{L}\left\{y^{(n)}(t)\right\}(s) = s^n\,\mathcal{L}\{y(t)\}(s) - s^{n-1}\,y(0) - s^{n-2}\,y'(0) - \cdots - y^{(n-1)}(0) \quad n = 1, 2, 3, \cdots$

03. $\mathcal{L}\left\{e^{a\,t}\ f(t)\right\}(s) = \mathcal{L}\{f(t)\}\left(s-a\right)$

04. $\mathcal{L}\{u_a(t)\ f(t)\}(s) = \mathcal{L}\left\{f\left(t+a\right)\right\}(s)\ e^{-a\,s}$

05. $\mathcal{L}\{f(t)\}(s) = \dfrac{1}{1-e^{-T\,s}}\displaystyle\int_0^T f(t)\,e^{-s\,t}\,dt, \quad f(t+T) = f(t)$

06. $\mathcal{L}\{t\ f(t)\}(s) = -\dfrac{d}{ds}\Big(\mathcal{L}\{f(t)\}(s)\Big)$

07. $\mathcal{L}\left\{\dfrac{f(t)}{t}\right\}(s) = \displaystyle\int_s^{+\infty}\mathcal{L}\{f(t)\}(r)\,dr$

08. $\mathcal{L}\{t^n\}(s) = \dfrac{n!}{s^{n+1}}, \quad n = 0, 1, 2, \cdots$

09. $\mathcal{L}\left\{e^{a\,t}\,t^n\right\}(s) = \dfrac{n!}{\left(s-a\right)^{n+1}}, \quad n = 0, 1, 2$

10. $\mathcal{L}\left\{\cos\left(b\,t\right)\right\}(s) = \dfrac{s}{s^2+b^2}$

11. $\mathcal{L}\left\{e^{a\,t}\ \cos\left(b\,t\right)\right\}(s) = \dfrac{s-a}{(s-a)^2+b^2}$

12. $\mathcal{L}\left\{\sin\left(b\,t\right)\right\}(s) = \dfrac{b}{s^2+b^2}$

13. $\mathcal{L}\left\{e^{a\,t}\ \sin\left(b\,t\right)\right\}(s) = \dfrac{b}{(s-a)^2+b^2}$

14. $\mathcal{L}\{u_a(t)\}(s) = \dfrac{e^{-a\,s}}{s}$

15. $\mathcal{L}\left\{e^{a\,t}\right\}(s) = \dfrac{1}{s-a}$

16. $\mathcal{L}\left\{\dfrac{1}{\sqrt{t}}\right\}(s) = \sqrt{\dfrac{\pi}{s}}$

Inverse Laplace Transforms

01. $\mathcal{L}^{-1}\{K_1\ F_1(s) + K_2\ F_2(s)\}(t) = K_1\ \mathcal{L}^{-1}\{F_1(s)\}(t) + K_2\ \mathcal{L}^{-1}\{F_2(s)\}(t)$

02. $\mathcal{L}^{-1}\left\{F\left(s-a\right)\right\}(t) = e^{a\,t}\ \mathcal{L}^{-1}\{F(s)\}(t)$ or $\mathcal{L}^{-1}\left\{F\left(s+a\right)\right\}(t) = e^{-a\,t}\ \mathcal{L}^{-1}\{F(s)\}(t)$

03. $\mathcal{L}^{-1}\left\{F(s)\ e^{-a\,s}\right\}(t) = u_a(t)\ \mathcal{L}^{-1}\{F(s)\}\left(t-a\right)$ $\qquad \mathcal{L}^{-1}\left\{\dfrac{e^{-a\,s}}{s}\right\}(t) = u_a(t)$

04. $\mathcal{L}^{-1}\{F'(s)\}(t) = -t\ \mathcal{L}^{-1}\{F(s)\}(t)$

05. $\mathcal{L}^{-1}\left\{\dfrac{1}{s^{n+1}}\right\}(t) = \dfrac{t^n}{n!}\ \ n = 0, 1, 2, \cdots$

06. $\mathcal{L}^{-1}\left\{\dfrac{1}{(s-a)^{n+1}}\right\}(t) = e^{a\,t}\ \dfrac{t^n}{n!}\ \ n = 0, 1, 2$

07. $\mathcal{L}^{-1}\left\{\dfrac{s}{s^2+b^2}\right\}(t) = \cos\left(b\,t\right)$

08. $\mathcal{L}^{-1}\left\{\dfrac{s-a}{(s-a)^2+b^2}\right\}(t) = e^{a\,t}\ \cos\left(b\,t\right)$

09. $\mathcal{L}^{-1}\left\{\dfrac{1}{s^2+b^2}\right\}(t) = \dfrac{1}{b}\ \sin\left(b\,t\right)$

10. $\mathcal{L}^{-1}\left\{\dfrac{1}{(s-a)^2+b^2}\right\}(t) = \dfrac{1}{b}\ e^{a\,t}\ \sin\left(b\,t\right)$

Trigonometric Identities

1. $\cos^2(\theta) + \sin^2(\theta) = 1$

2. $\cos(2\,\theta) = \cos^2(\theta) - \sin^2(\theta)$

3. $\sin(2\,\theta) = 2\,\cos(\theta)\,\sin(\theta)$

4. $2\,\cos^2(\theta) = 1 + \cos(2\,\theta)$

5. $2\,\sin^2(\theta) = 1 - \cos(2\,\theta)$

6. $\cos(\theta \pm \pi) = -\cos(\theta)$

7. $\sin(\theta \pm \pi) = -\sin(\theta)$

8. $\cos(\theta \pm 2\,\pi) = \cos(\theta)$

9. $\sin(\theta \pm 2\,\pi) = \sin(\theta)$

Boundary Value Problems

1. If $f(x)$ is $2\,L$-periodic and piecewise continuous then its fourier series is

$$f(x) \sim \frac{a_0}{2} + \sum_{n=1}^{+\infty} a_n \cos\left(n\frac{\pi}{L}\,x\right) + b_n \sin\left(n\frac{\pi}{L}\,x\right)$$

where

$$a_n = \frac{1}{L}\int_I f(x)\cos\left(n\frac{\pi}{L}\,x\right)\,\mathrm{d}x \qquad b_n = \frac{1}{L}\int_I f(x)\sin\left(n\frac{\pi}{L}\,x\right)\,\mathrm{d}x$$

and I is an interval of length $2\,L$

2. The eigenvalues and corresponding eigenfunctions of the BVP

$$\Big\| \; U'' + \lambda\,U = 0, \quad U(0) = 0, \;\; U(L) = 0$$

are given by: $\lambda_n = \dfrac{n^2\,\pi^2}{L^2}$ and $U_n(x) = \sin\left(\dfrac{n\,\pi}{L}\,x\right)$, respectively.