Amat 307 Final Test Fall 2014

1. The solution to the initial value problem $y' + y = e^{-t} \cos t, \quad y(0) = 3$, is

A) $3e^{-t} + e^{-t} \sin t$ *

B) $3e^{-t} - e^{-t} \sin t$

C) $e^{-t} + 2e^{-t} \cos t$

D) $2e^{t} + e^{t} \cos t$

E) $3e^{t} + e^{t} \sin t$

2. A tank contains 100 litres of water. A solution with salt concentration 0.004 kg per litre starts flowing into the tank at the rate 6 litres per minute, and well stirred mixture flows out of tank at the same rate 6 litres per minute. The amount of salt in the tank in the limit (as time grows and tends to infinity)

A) equals 0.1 kg

B) equals 0.2 kg

C) equals 0.3 kg

D) equals 0.4 kg *

E) depends on the initial amount of salt in the tank

3. Consider the exact equation $(2x - y)dx + (2y - x)dy = 0, \quad y(1) = 3$. The solution is

A) $2x^2 - xy + y^2 = 8$

B) $x^2 - 2xy + y^2 = 4$

C) $x^2 + xy + y^2 = 13$

D) $x^2 - xy + y^2 = 7$ *

E) $x^2 - xy + 2y^2 = 16$

4. Apply Euler's approximation method once to the initial value problem $y' = 2\cos(ty)$, $y(0) = 1$, to find the approximate value $y(t_1) = y(h) \approx 1.1$. The step size h is

A) $h = 0.05$ *

B) $h = 0.1$

C) $h = 0.5$

D) $h = 0.55$

E) $h = 1$

5. The solution to the initial value problem: $y^{(4)} - 5y'' + 4y = 0$; $y(0) = 1$, $y'(0) = 0$, $y''(0) = 0$, $y^{(3)}(0) = 0$ is given by:

A) $e^{-x} - e^{x} + \frac{1}{2}e^{-3x} + \frac{1}{2}e^{-3x}$

B) $e^{-x} - e^{x} + \frac{1}{2}e^{2x} + \frac{1}{2}e^{-2x}$

C) $\frac{2}{3}e^{-x} + \frac{2}{3}e^{x} - \frac{1}{6}e^{2x} - \frac{1}{6}e^{-2x}$ (*)

D) $\frac{1}{3}e^{-x} + \frac{1}{3}e^{x} + \frac{1}{6}e^{3x} + \frac{1}{6}e^{-3x}$

E) None of the above

6. The solution to the initial value problem: $x^{(3)} - x' = t$, $x(0) = 1$, $x'(0) = 0$, $x''(0) = 0$ is given by:

A) $\cos(t) - \sin(t) + t^2e^t$

B) $\frac{1}{2}e^{-t} + \frac{1}{2}e^{t} - \frac{1}{2}t^2$ (*)

C) $2e^{-t} - e^{t} + t^2$

D) $e^{-t} - e^{t} + 1 - \frac{1}{2}t^2$

E) None of the above

7. The smallest order m of a homogeneous real constant coefficient linear ordinary differential equation with a solution $x(t) = 5t^2 - \ln(10)e^2$ is

A) $m = 2$

B) $m = 3$ *

C) $m = 4$

D) $m = 5$

E) $m = 6$

8. Let y_1, y_2 and y_3 be three solutions of the differential equation $ty^{(3)} + t\sin(t)y'' + y' + \ln(t)y = 0$, $t > 0$. Assume that the Wronskian of y_1, y_2 and y_3 at $t = \dfrac{\pi}{2}$ is equal to 5; $W(y_1, y_2, y_3)(\frac{\pi}{2}) = 5$. Then $W(y_1, y_2, y_3)(\pi)$ is equal to:

A) $5e^{1-\pi}$

B) $-5e^{-1}$

C) $5e$

D) $-e^{-1}$

E) $5e^{-1}$ (*)

9. A particular solution of the differential equation: $y'' + y = \sec(t)$, $0 < t < \frac{\pi}{2}$, is given by:

A) $\cos(t)\ln(\cos(t)) + t\sin(t)$ (*)

B) $\cos(t)\ln(\cos(t)) + \sin(t)$

C) $-\cos(t)\ln(\cos(t)) - t\sin(t)$

D) $-\cos(t)\ln(\cos(t)) + t\sin(t)$

E) None of the above

10. According to the method of undetermined coefficients, a particular solution to the equation $y'' - 4y' + 5y = te^{2t}\sin(t) + e^{2t}$ is of the following form, where $A_0, A_1, B_0, B_1, C_0, C_1$ are real constants:

A) $(A_0t + A_1)e^{2t}\cos(t) + (B_0t + B_1)e^{2t}\sin(t) + C_0e^{2t}$

B) $(A_0t + A_1)e^{2t}\cos(t) + (B_0t + B_1)e^{2t}\sin(t) + (C_0t + C_1)e^{2t}$

C) $A_0te^{2t}\cos(t) + B_0te^{2t}\sin(t) + C_0e^{2t}$

D) $A_0t^2e^{2t}\cos(t) + B_0t^2e^{2t}\sin(t) + C_0te^{2t}$

E) $(A_0t^2 + A_1t)e^{2t}\cos(t) + (B_0t^2 + B_1t)e^{2t}\sin(t) + C_0e^{2t}$ (*)

11. The equation $y^{(4)} - 2y^{(2)} + y = te^{-t} + e^t\cos(t)$ has a particular solution to the equation is of the following form, where A_0, A_1, B_0, B_1, C_0 are real constants:

A) $(A_0t^2 + A_1t)e^{-t} + B_0t^2e^t\cos(t) + C_0t^2e^t\sin(t)$

B) $(A_0t^2 + A_1t)e^{-t} + (B_0t + B_1)e^t\cos(t) + (C_0t + C_1)e^t\sin(t)$

C) $(A_0t + A_1)e^{-t} + B_0e^t\cos(t) + C_0e^t\sin(t)$

D) $(A_0t^3 + A_1t^2)e^{-t} + B_0e^t\cos(t) + C_0e^t\sin(t)$ (*)

E) $(A_0t^3 + A_1t^2)e^{-t} + B_0t^2e^t\cos(t) + C_0t^2e^t\sin(t)$

12. Suppose $e^t\cos(2t) - 19e^{-t}$ is a solution of $y''' + ay'' + by' + cy = 0$, where a, b and c are real constants, then

A) $a = -3,\ b = 7,\ c = -5$

B) $a = -1,\ b = 3,\ c = 5$ *

C) $a = 1,\ b = 3,\ c = -5$

D) $a = 3,\ b = 7,\ c = 5$

E) None of the above

13. The **largest** interval on which a unique solution to $\overrightarrow{\mathbf{y}}' = \begin{bmatrix} t-1 & (t+2) \\ (t^2+9)^{-1} & (t-2)^{-1} \end{bmatrix} \overrightarrow{\mathbf{y}} + \begin{bmatrix} (t+1)^{-1}\ln|t| \\ 1 \end{bmatrix}$,
$\overrightarrow{\mathbf{y}}(1) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ is **guaranteed** to exist is

A) $(-1,\, 2)$

B) $(-3,\, 3)$

C) $(0,\, 2\,)$ *

D) $(-2,\, 2)$

E) The empty set

14. A 2×2 matrix A has an eigenvalue $2i$ with eigenvector $\begin{bmatrix} i \\ -1 \end{bmatrix}$. A particular solution $\overrightarrow{\mathbf{y}}_p$ to the system $\overrightarrow{\mathbf{y}}' = A\overrightarrow{\mathbf{y}} + \begin{bmatrix} 0 \\ 2 \end{bmatrix}$ is

A) $\mathbf{y}_p(t) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ *

B) $\mathbf{y}_p(t) = \begin{bmatrix} \cos^2(2t) - \sin^2(2t) \\ 0 \end{bmatrix}$

C) $\mathbf{y}_p(t) = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$

D) $\mathbf{y}_p(t) = \begin{bmatrix} -2\sin(2t)\cos(2t) \\ 1 \end{bmatrix}$

E) None of the above

15. Which of the following linear differential equations can be written as the linear system $\overrightarrow{x}' = \mathbf{A}\overrightarrow{x}$ where $\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \cos(t) & -t & \sin(t) \end{bmatrix}$ and $\overrightarrow{x} = \begin{bmatrix} y \\ y' \\ y'' \end{bmatrix}$ for some function $y = y(t)$?

A) $y''' - \cos(t)y'' + ty' - \sin(t)y = 0$

B) $y''' + \cos(t)y'' - ty' + \sin(t)y = 0$

C) $y''' - \sin(t)y'' + ty' - \cos(t)y = 0$ *

D) $y''' - \sin(t)y'' - ty' + \cos(t)y = 0$

E) $y''' + \sin(t)y'' - ty' + \cos(t)y = 0$

16. Solve $\overrightarrow{x}' = \mathbf{A}\overrightarrow{x}$ where $\mathbf{A} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & -1 & 3 \end{bmatrix}$, $\overrightarrow{x}(0) = \begin{bmatrix} 0 \\ 5 \\ 2 \end{bmatrix}$ where you already know A has eigenvalues $\{1, 1, 2\}$, and $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ are eigenvectors for 1 and 2 respectively. The unique solution $\overrightarrow{x}(t)$ is

A) $\begin{bmatrix} 2te^t + 3e^{2t} \\ 2e^t - 4te^t + 3e^{2t} \\ -e^t - 4te^t + 3e^{2t} \end{bmatrix}$

B) $\begin{bmatrix} e^t + 6te^t - e^{2t} \\ 6e^t - e^{2t} \\ 3e^t - e^{2t} \end{bmatrix}$ *

C) $\begin{bmatrix} -2e^t + 5te^t + e^{2t} \\ 5e^{2t} \\ 2e^{2t} \end{bmatrix}$

D) $\begin{bmatrix} e^t - e^{2t} \\ 6e^t - e^{2t} \\ 3e^t - e^{2t} \end{bmatrix}$

E) none of the above

17. The Laplace transform of $u_2(t)(t^2 - 4)$ is

A) $e^{-2s}(\frac{2}{s^3} - \frac{4}{s})$

B) $e^{-2s}(\frac{2}{s^3} - \frac{4}{s^2})$

C) $e^{-2s}(\frac{2}{s^3} + \frac{4}{s})$

D) $e^{-2s}(\frac{2}{s^3} + \frac{4}{s^2})$ *

E) None of the above

18. The inverse Laplace transform of $\dfrac{1}{s(s^2+1)}$ is

A) $1 + \sin(t)$

B) $1 - \sin(t)$

C) $1 + \cos(t)$

D) $1 - \cos(t)$ *

E) $\cos(t) - \sin(t)$

19. The inverse Laplace transform of $e^{-2s}\left(\dfrac{s+6}{s^3 - 3s^2}\right)$ is

A) $u_2(t)[-5 - 2t + e^{3(t+2)}]$

B) $u_2(t)[-1 - 2t + e^{3t}]$

C) $u_2(t)[3 - 2t + e^{3(t-2)}]$ *

D) $e^{-2t}[3 - 2t + e^{3(t-2)}]$

E) $e^{-2t}[-1 - 2t + e^{3(t-2)}]$

20. Using the functions $u_c(t)$ the following piece-wise continuous function

$$f(t) = \begin{cases} -1, & 0 \le t < 2 \\ t + \sin t - 1, & 2 \le t < 5 \\ t^3 + t + \sin t, & 5 \le t < 7 \\ e^t + t^3, & t \ge 7 \end{cases}$$

can be re-written as

A) $f(t) = -1 + u_2(t)[t + \sin t] + u_5(t)[t^3 + 1] + u_7(t)[e^t - t - \sin t]$ *

B) $f(t) = -1 + u_2(t)[t + \sin t - 1] + u_5(t)[t^3 + 1] + u_7(t)[e^t - t - \sin t]$

C) $f(t) = u_2(t)[t + \sin t - 1] + u_5(t)[t^3 + 1] + u_7(t)[e^t - t - \sin t]$

D) $f(t) = -1 + u_2(t)[t + \sin t - 1] + u_5(t)[t^3 + t + \sin t] + u_7(t)[e^t + t^3]$

E) $f(t) = u_2(t)[t + \sin t - 1] + u_5(t)[t^3 + t + \sin t] + u_7(t)[e^t + t^3]$

21. If $F(s)$ is the Laplace transform of the function

$$f(t) = \begin{cases} -1, & 0 \le t < 1 \\ 1, & 1 \le t < 2 \\ 3, & t \ge 2 \end{cases}$$

then

A) $F(s) = \dfrac{-1 + 2e^{-s} + 2e^{-2s}}{s - 1}$

B) $F(s) = \dfrac{-1 + e^{-s} + 3e^{-2s}}{s}$

C) $F(s) = \dfrac{-1 + 2e^{-s} + 2e^{-2s}}{s}$ *

D) $F(s) = \dfrac{-1 + e^{-s} + 2e^{-2s}}{s + 1}$

E) $F(s) = \dfrac{1 - 2e^{-s} - 2e^{-2s}}{s}$

22. The largest open interval on which the power series $\sum_{n=1}^{\infty} \frac{2^n x^n}{(n+2)!}$ converges is

A) $(-\frac{1}{2}, \frac{1}{2})$
B) $(-1, 1)$
C) $(-2, 2)$
D) $(-\infty, \infty)$ *
E) There is no interval, the series converges for $x = 0$ only

23. The largest open interval on which the power series $\sum_{n=0}^{\infty} \frac{(x-3)^{2n}}{4^n}$ converges is

A) $(-\infty, \infty)$
B) $(-4, 4)$
C) $(-2, 2)$
D) $(-1, 7)$
E) $(1, 5)$ *

24. For the series solution $\sum_{n=0}^{\infty} a_n t^n$ of the equation $y'' + (t-1)y' + 3y = 0$ the recurrence relation satisfied by the coefficients $a_0, a_1, a_2, a_3, \ldots$ is

A) $n(n-1)a_n + (t-1)a_n + 3a_n = 0$

B) $(n+2)(n+1)a_{n+2} - (n+1)a_{n+1} + (n+3)a_n = 0$ *

C) $(n+2)(n+1)a_n - a_{n+1} + (n+3)a_n = 0$

D) $(n+2)(n+1)a_{n+2} - na_{n+1} + 3a_n = 0$

E) $(n+2)(n+1)a_n - (n+1)a_n + (n+3)a_n = 0$

25. If the series $y = \sum_{k=0}^{\infty} a_k t^k$ then $y'' + ty' + 3y$ is the series $\sum_{k=0}^{\infty}(a_{k+2}(k+2)(k+1) + ka_k + 3a_k)t^k$. The first four terms $y = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + \ldots$ for the series solution of the initial value problem $y'' + ty' + 3y = 1 + 5t$, $y(0) = 1$, $y'(0) = 2$ are given by

A) $y(t) = 1 + 2t + 0t^2 + 3t^3 + \ldots$

B) $y(t) = 1 + 2t - \frac{3}{2}t^2 - \frac{4}{3}t^3 + \ldots$

C) $y(t) = 1 + 2t + \frac{3}{2}t^2 + \frac{4}{3}t^3 + \ldots$

D) $y(t) = 1 + 2t + t^2 + \frac{1}{2}t^3 + \ldots$

E) $y(t) = 1 + 2t - t^2 - \frac{1}{2}t^3 + \ldots$ *

Extra workspace

1.