Math 375		**Fall 2016**
	Variation of Parameters and Laplace Transform	
Worksheet # 2	**Part 4**	**October 31-November 04**

The problems on this worksheet refer to material from sections §3.6, §4.4, and §6.1, of your text. Solutions to all problems will be available on the course's D2L website Friday, November 4. Please report any typos, omissions and errors to aiffam@ucalgary.ca

The Variation of Parameters Method

01. Find the general solution of the following equations.

a. $y'' + y = \sec^3(t)$ **b.** $y'' - 2\,y' + y = \dfrac{\mathrm{e}^t}{t^2+1}$

Laplace Transform of Basic Functions

02. Compute $\mathcal{L}\{f(t)\}(s)$, if

a. $f(t) = (t^2+1)^2$ **b.** $f(t) = 3\,\cos(5\,t) - 2\,\sin(2\,t)$

c. $f(t) = \cos^4(t) - \sin^4(t)$ **d*.** $f(t) = \cosh(2\,t)\,\sinh(3\,t)$

e*. $f(t) = \cos^3(t)$ **f.** $f(t) = \sin^3(t)$

03. Use the substitution $x = \sqrt{s\,t}$ to show

$$\mathcal{L}\left\{\frac{1}{\sqrt{t}}\right\}(s) = \frac{2}{\sqrt{s}}\int_0^{+\infty} \mathrm{e}^{-x^2}\,\mathrm{d}x, \quad \text{for any} \quad s > 0$$

Given that $\displaystyle\int_0^{+\infty} \mathrm{e}^{-x^2}\,\mathrm{d}x = \frac{\sqrt{\pi}}{2}$, compute $\mathcal{L}\left\{\dfrac{1}{\sqrt{t}}\right\}(s)$

04. Use integration by parts to show that $\mathcal{L}\left\{\sqrt{t}\right\}(s) = \dfrac{1}{2\,s}\,\mathcal{L}\left\{\dfrac{1}{\sqrt{t}}\right\}(s)$, $s > 0$, then compute $\mathcal{L}\left\{\sqrt{t}\right\}(s)$

First Shift Formula

05. Find $\mathcal{L}\{f(t)\}(s)$, if

a. $f(t) = t^2\,\mathrm{e}^{-3\,t}$ **b*.** $f(t) = (t+1)^2\,\mathrm{e}^t$ **c*.** $f(t) = \big(\sin(2\,t) + \cos(3\,t)\big)\,\mathrm{e}^{-t}$

d. $f(t) = \dfrac{\mathrm{e}^{-2\,t}}{\sqrt{t}}$ **e.** $f(t) = \sqrt{t}\,\mathrm{e}^{2\,t}$