MATH 375
Handout # 8
Partial Differential Equations

1. Find the eigenvalues and eigenfunctions of the Sturm-Liouville problem

$$X''(x) + \lambda X(x) = 0, \quad X'(0) = 0,\ X'(\pi/2) = 0.$$

2. a) Formulate the boundary value problem for heat transfer (conduction) in a slab of length π with $k = 4$ and the initial temperature distribution $u(x, 0) = x^2 - \pi x$ and the temperature at the ends kept at zero.
   b) Use the method of separation of variables to determine the temperature $u(t, x)$.

3. Use the method of separation of variables to find the solution to the heat conduction problem

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}, \quad 0 < x < \pi,\ t > 0,$$
$$u(0, t) = u(\pi, t) = 0,\ t > 0, \quad u(x, 0) = \sin(2x),\ 0 < x < \pi.$$

4. Use the method of separation of variables to find the solution to the Laplace equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \quad 0 < x < \pi, \quad 0 < y < 2$$

$$u(0, y) = 0,\ u(\pi, y) = 0, \quad 0 < y < 2, \quad \frac{\partial u}{\partial y}(x, 0) = 0, \quad \frac{\partial u}{\partial y}(x, 2) = 6\sin(3x),\ 0 < x < \pi$$

5. Use the method of separation of variables to find the solution to the Laplace equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \quad 0 < x < \pi, \quad 0 < y < 1$$

$$u(0, y) = 0,\ u(\pi, y) = 0, \quad 0 < y < 1, \quad \frac{\partial u}{\partial y}(x, 0) = 0, \quad \frac{\partial u}{\partial y}(x, 1) = x,\ 0 < x < \pi$$

6. Use the method of separation of variables to find the solution $u(t, x)$ of a vibrating string problem:

$$4\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}, \quad 0 < x < \pi,\ t > 0,$$
$$u(0, t) = 0,\ u(\pi, t) = 0,\ t > 0, \quad u(x, 0) = 0, \quad \frac{\partial u}{\partial t}(x, 0) = 8\sin(3x),\ 0 < x < \pi.$$

7. Use the method of separation of variables to find the displacement $u(t, x)$ of a vibrating elastic string which satisfies the conditions:

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}, \quad 0 < x < 2,\ t > 0,$$

$$u(t, 0) = 0,\ u(t, 2) = 0,\ t > 0, \quad u(0, x) = f(x) = \begin{cases} x, & 0 \le x \le 1, \\ 2 - x, & 1 < x \le 2, \end{cases}$$

$\frac{\partial u}{\partial t}(0, x) = 0$, $0 < x < 2$.