UNIVERSITY OF CALGARY

EXAMINATION VERSION

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UNIVERSITY OF CALGARY
FACULTY OF SCIENCE
DEPARTMENT OF MATHEMATICS AND STATISTICS
FINAL EXAMINATION
AMAT 307, LECTURE 01-04 – FALL, 2013

DATE: 11/DECEMBER **Time: 2 hours**

STUDENT ID NUMBER

EXAMINATION RULES

1. This is a closed book examination.
2. No aids are allowed for this examination except for a Schulich calculator
3. Students should put their answers on the scantron sheet and on the examination paper.
4. Scantron sheets must be filled out during the exam time limit. No additional time will be granted to fill in scantron form.
5. The use of personal electronic or communication devices is prohibited.
6. A University of Calgary Student ID card is required to write the Final Examination and could be requested for midterm examinations. If adequate ID isn't present the student must complete an Identification Form.
7. Students late in arriving will not be permitted after one-half hour of the examination time has passed.
8. No student will be permitted to leave the examination room during the first 30 minutes, nor during the last 15 minutes of the examination. Students must stop writing and hand in their exam immediately when time expires.
9. All inquiries and requests must be addressed to the exam supervisor.
10. Students are strictly cautioned against:
    a. communicating to other students;
    b. leaving answer papers exposed to view;
    c. attempting to read other students' examination papers
11. During the final examination, if a student becomes ill or receives word of domestic affliction, the student must report to the Invigilator, hand in the unfinished paper and request that it be cancelled. If ill, the student must report immediately to a physician/counselor for a medical note to support a deferred examination application.
12. Once the examination has been handed in for marking, a student cannot request that the examination be cancelled. Retroactive withdrawals from the course will be denied.
13. Failure to comply with these regulations will result in rejection of the examination paper.

01. The unique solution of the initial value problem

$$y' = \frac{2x}{5+2y}, \quad y(2) = -1$$

is given by

(A) $y = \frac{5}{2} - \frac{1}{2}\sqrt{4x^2 - 7}$

(B) $y = -\frac{5}{2} + \frac{1}{2}\sqrt{4x^2 - 7}$

(C) $y = \sqrt{6x - 3} - 4$

(D) $y = \frac{1}{2} - \frac{1}{2}\sqrt{4x^2 - 7}$

(E) $y = \frac{1}{2} - \frac{1}{2}\sqrt{3x^2 - 3}$

02. If y(t) is the solution of the initial value problem

$$ty' + 3y = 5t^2, \quad y(2) = 5$$

then

(A) $y(1) = -15/8$

(B) $y(1) = -25/2$

(C) $y(1) = 5/8 - 5\ln(2)$

(D) $y(1) = -7$

(E) $y(1) = 9$

03. Consider the initial-value problem:

$$y' + \frac{\ln(x)}{x^2 - 1} y = \frac{1}{x - 3}, \quad y(2) = 1,$$

The largest interval on which a unique solution exists is

(A) $(0, \quad +\infty)$

(B) $(-1, \quad 1)$

(C) $(1, \quad 3)$

(D) $(1, \quad +\infty)$

(E) $(-1, \quad 3)$

04. The solution of the initial value problem $\begin{cases} 3\,t\,y' - 2\,y = 3\,t^{-1}\,y^{-2} \\ y(1) = 1 \end{cases}$ is given by

(A) $y = \left(\dfrac{2\,t^3 - 1}{t}\right)^{1/3}$

(B) $y = \dfrac{\left(2\,t^3 - 1\right)^{1/3}}{t}$

(C) $y = \dfrac{2\,t^3 - 1}{t}$

(D) $y = \dfrac{t}{3 - 2\,t^{1/3}}$

(E) $y = \dfrac{3 - 2\,t^{1/3}}{t}$

05. A general solution of the exact differential equation

$$2\,x\,\ln(3\,y)\;\mathrm{d}x + \frac{x^2}{y}\;\mathrm{d}y = 0$$

is

(A) $x^2\,\ln(3\,y) + x^2\,\ln(y) = C$

(B) $x^2\,\ln(3\,y) = C$

(C) $2\,x\,y\,\ln(3\,y) + x^2 = C\,y$

(D) $x^3 - 6\,x\,y^2 + 6\,x\,y^2\,\ln(3\,y) = C\,y$

(E) $xy\ln(xy) + x^2 - y^2 = C$

06. A particular solution of the differential equation

$$y'' - y = t - 4e^{-t}$$

is

(A) $y_p(t) = t - 4e^{-t}$

(B) $y_p(t) = -t + 2e^{-t}$

(C) $y_p(t) = -t + 2te^{-t}$

(D) $y_p(t) = -t - 2te^{-t}$

(E) $y_p(t) = 2t - 4e^{t}$

07. Suppose $y_1(t) = t$ is a solution of

$$t^2 y'' - t(t+2)y' + (t+2)y = 0, \quad t > 0,$$

use the method of reduction of order to find a second solution of the differential equation

(A) e^t

(B) e^{2t}

(C) $t^2 e^t$

(D) te^t

(E) te^{3t}

08. Consider the equation

$$y'' + 9y = 7\cos(3x) + 1 + e^{3x},$$

according to the method of **Undetermined Coefficients**, the particular solution has the form

(A) $C_1 x \sin(3x) + C_2 x + C_3 x e^{3x} + C_4 x e^{-3x}$

(B) $C_1 x \cos(3x) + C_2 x \sin(3x) + C_3 + C_4 x e^{3x}$

(C) $C_1 \cos(3x) + C_2 \sin(3x) + C_3 + C_4 e^{3x}$

(D) $C_1 x \sin(3x) + C_2 x \cos(3x) + C_3 e^{3x}$

(E) $C_1 x \cos(3x) + C_2 x \sin(3x) + C_3 + C_4 e^{3x}$

09. If $y_1(t) = t^{-1}$ and $y_2(t) = t^{-5}$ are two solutions of the differential equation

$$t^2y'' + 7ty' + 5y = 0, \quad t > 0,$$

then a particular solution y_p of the non-homogeneous equation

$$t^2y'' + 7ty' + 5y = 12t, \quad t > 0,$$

is

(A) $y_p = t^3/3$

(B) $y_p = t^2/2$

(C) $y_p = t^2$

(D) $y_p = t$

(E) $y_p = t/2$

10. Let $y_1(t)$ and $y_2(t)$ be two solutions of $y'' - \dfrac{1}{t+1}y' + \dfrac{1}{t+2}\,y = 0$ such that $\begin{cases} y_1(0) = 1 \\ y_1'(0) = 0 \end{cases}$ and $\begin{cases} y_2(0) = 1 \\ y_2'(0) = 3 \end{cases}$, the wronskian $W(t)$ of $y_1(t)$ and $y_2(t)$ is

(A) $W(t) = 3t + 3$

(B) $W(t) = \dfrac{3}{2}t + 3$

(C) $W(t) = \dfrac{3}{t+1}$

(D) $W(t) = \dfrac{6}{t+2}$

(E) $W(t) = \dfrac{2}{t+2}$

11. If $2xe^{2x}\cos(3x) + 5x^2$ is a solution of a higher-order linear homogeneous equation with constant coefficients, then which of the following is guaranteed to be a solution of this differential equation?

(A) $e^{2x}\sin(3x) + x^3 + 2$

(B) $e^{2x}\sin(3x) + 3x + 2$

(C) $x^3e^{2x}\cos(3x) + 3x + 2$

(D) $2xe^{x}\cos(2x) + 3x + 2$

(E) $e^{2x}\cos(2x) + x^3 + 2x^2 - 6x + 1$

12. If the method of undetermined coefficients is used to find a particular solution $y_p(t)$ to the differential equation $y''' - y' = te^{-t} + 2\cos(t)$, then $y_p(t)$ should have the form

(A) $t(A_0t + A_1)e^{-t} + B_0\cos(t) + C_0\sin(t)$

(B) $t^2(A_0t + A_1)e^{-t} + B_0\cos(t) + C_0\sin(t)$

(C) $t(A_0t + A_1)e^{-t} + (B_0t + B_1)\cos(t) + (C_0t + C_1)\sin(t)$

(D) $(A_0t + A_1)e^{-t} + B_0\cos(t) + C_0\sin(t)$

(E) $t^2(A_0t + A_1)e^{-t} + (B_0t + B_1)\cos(t) + (C_0t + C_1)\sin(t)$

Where $A_0, A_1, B_0, B_1, C_0, C_1$ are real constants to be determined.

13. Consider the differential equation

$$y'' + 4y = \begin{cases} 1 & 0 \le t < 2, \\ e^{-(t-2)} & t \ge 2; \end{cases} \qquad y(0) = 0, \quad y'(0) = 0$$

Let $Y(s)$ be the Laplace transform of $y(t)$, then

(A) $Y(s) = \dfrac{1 - e^{-s}}{s^2(s^2+4)}$

(B) $Y(s) = \dfrac{1 - e^{-s}}{s^2(s^2-4)}$

(C) $Y(s) = \dfrac{1 - e^{-s}}{s(s^2+4)}$

(D) $Y(s) = \dfrac{1 + e^{-s}}{s(s^2+4)}$

(E) $Y(s) = \dfrac{s + 1 - e^{-2s}}{s(s+1)(s^2+4)}$

14. The inverse Laplace transform of the given function

$$F(s) = \frac{3e^{-2s}}{s^2 + s - 2}$$

is

(A) $f(t) = u_2(t)\left[e^{t-2} - e^{-2(t-2)}\right]$

(B) $f(t) = u_2(t)\left[e^{t-1} - e^{-2(t-1)}\right]$

(C) $f(t) = u_1(t)\left[e^{t-1} - e^{-(t-2)}\right]$

(D) $f(t) = u_1(t)\left[e^{t-2} - e^{-(t-1)}\right]$

(E) $f(t) = u_{-2}(t)\left[e^{t-2} - e^{-(t-1)}\right]$

15. The Laplace transform of $e^{3t}(t^2 + \sin(t))$ is

(A) $\dfrac{2}{(s-3)^3} + \dfrac{s-3}{(s-3)^2+1}$

(B) $\dfrac{1}{(s-3)^2} + \dfrac{s}{(s-3)^2+1}$

(C) $\dfrac{2}{(s-3)^2} + \dfrac{s-1}{(s-3)^2+1}$

(D) $\dfrac{2}{(s-3)^3} + \dfrac{3}{(s-3)^2+1}$

(E) $\dfrac{2}{(s-3)^3} + \dfrac{1}{(s-3)^2+1}$

16. The Laplace transform of $2e^t\, u_2(t)\, \cos^2(t-2)$ is

(A) $e^{-2(s-1)}\left[\dfrac{1}{s-1} + \dfrac{s-1}{(s-1)^2+4}\right]$

(B) $e^{-(s-1)}\left[\dfrac{1}{s-1} + \dfrac{s-1}{(s-1)^2+1}\right]$

(C) $e^{-2s}\left[\dfrac{1}{s-1} + \dfrac{s}{(s-1)^2+4}\right]$

(D) $e^{(s-1)}\left[\dfrac{1}{s-1} + \dfrac{s-1}{(s-1)^2+4}\right]$

(E) $e^{-2(s-1)}\left[\dfrac{1}{s} + \dfrac{1}{s^2+4}\right]$

17. The Laplace transform of

$$f(t) = \begin{cases} t & 0 \le t < 1, \\ t^2 + t - 1 & 1 \le t < 2, \\ t + 3 & t \ge 2, \end{cases}$$

is

(A) $\dfrac{1}{s^2} + e^{-s}\left(\dfrac{2}{s^3} + \dfrac{2}{s^2}\right) + e^{-2s}\left(\dfrac{2}{s^3} + \dfrac{4}{s^2}\right)$

(B) $\dfrac{1}{s^2} + e^{-s}\left(\dfrac{2}{s^3} - \dfrac{1}{s}\right) + e^{-2s}\left(\dfrac{2}{s^3} - \dfrac{4}{s}\right)$

(C) $\dfrac{1}{s^2} + e^{-s}\left(\dfrac{2}{s^3} - \dfrac{1}{s^2}\right) + e^{-2s}\left(\dfrac{2}{s^3} - \dfrac{4}{s^2}\right)$

(D) $\dfrac{1}{s^2} + e^{-s}\left(\dfrac{2}{s^3} + \dfrac{2}{s^2}\right) - e^{-2s}\left(\dfrac{2}{s^3} + \dfrac{4}{s^2}\right)$

(E) $\dfrac{1}{s^2} - e^{-s}\left(\dfrac{2}{s^3} + \dfrac{2}{s^2}\right) - e^{-2s}\left(\dfrac{2}{s^3} + \dfrac{4}{s^2}\right)$

18. The solution of the initial value problem

$$y^{(4)} - y = 0, \quad y(0) = 1, \quad y'(0) = 0, \quad y''(0) = 1, \quad y'''(0) = 0$$

is

(A) $(2e^t + e^{-t})/3$

(B) $(-e^t + 3e^{-t})/2$

(C) $(e^t + e^{-t})/2$

(D) $(e^t + 2e^{-t})/3$

(E) $(4e^t - e^{-t})/3$

19. The largest open interval on which the following power seires

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(n+1)^2(x+2)^n}{3^n}$$

converges is

(A) $(-4, \quad 2)$

(B) $(-3, \quad 3)$

(C) $(-2, \quad 2)$

(D) $(0, \quad 3)$

(E) $(-5, \quad 1)$

20. Consider the initial value problem $y'' - xy' - y = 0,\ y(0) = 2,\ y'(0) = 1$. The first **5** non-zero terms of the power series solution about x=0 are given by

(A) $y = 2 + x + x^2 + \frac{1}{3}x^3 + \frac{1}{4}x^4$

(B) $y = 2 + x + x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4$

(C) $y = 2 + x + x^2 - \frac{1}{3}x^3 + \frac{1}{4}x^4$

(D) $y = 2 + x - x^2 + \frac{1}{3}x^3 + \frac{1}{4}x^4$

(E) $y = 2 + x - x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4$

21. Consider the differential equation $3\,y'' + (t+1)\,y' + 4\,y = 0$. If we look for a power series solution $y(t) = \sum_{n=0}^{\infty} a_n\, t^n$, then the recurrence relation satisfied by the coefficients $a_0,\ a_1,\ a_2,\ a_3,\ \cdots$ is

(A) $3\,n\,(n-1)\,a_n + (t+1)\,n\,a_n + 4\,a_n = 0$

(B) $3\,(n+2)\,(n+1)\,a_{n+2} + 3\,n\,a_{n+1} + (n+4)\,a_n = 0$

(C) $3\,(n+2)\,(n+1)\,a_{n+2} + 3\,(n+1)\,a_{n+1} + (n+4)\,a_n = 0$

(D) $3\,(n+2)\,(n+1)\,a_{n+2} + (n+1)\,a_{n+1} + (n+4)\,a_n = 0$

(E) $3\,(n+2)\,(n+1)\,a_{n+2} + 2(n+1)\,a_{n+1} + (n+4)\,a_n = 0$

22. The unique solution of the initial value problem

$$x^2y'' - 4xy' + 4y = 0, \quad y(1) = 2, \quad y'(1) = -1$$

is given by

(A) $x + x^{-2}$

(B) $3x - x^4$

(C) $\frac{3}{4}x^2 + \frac{5}{4}x^{-2}$

(D) $\frac{7}{4}x^{-1} + \frac{1}{4}x^3$

(E) $\frac{5}{4}x^{-1} + \frac{3}{4}x$

23. Given that the marix $\begin{bmatrix} 2 & 3 \\ -3 & 2 \end{bmatrix}$ has a complex eigenvalue $\lambda = 2 + 3i$ and corresponding eigenvector V $= \begin{bmatrix} 1 \\ i \end{bmatrix}$, the initial value problem of the system of first-order differential equations

$$\begin{aligned} y_1' &= 2y_1 + 3y_2, \quad y_1(0) = 1 \\ y_2' &= -3y_1 + 2y_2, \quad y_2(0) = 2 \end{aligned}$$

has solution

(A) $y_1(t) = e^{2t}[\cos(3t) - 2\sin(3t)], \quad y_2(t) = e^{2t}[2\cos(3t) - \sin(3t)]$

(B) $y_1(t) = e^{t}[\cos(3t) - 2\sin(3t)], \quad y_2(t) = e^{t}[2\cos(3t) + \sin(3t)]$

(C) $y_1(t) = e^{t}[\cos(3t) + \sin(3t)], \quad y_2(t) = e^{t}[2\cos(3t) - 2\sin(3t)]$

(D) $y_1(t) = e^{2t}[\cos(3t) + 2\sin(3t)], \quad y_2(t) = e^{2t}[2\cos(3t) - \sin(3t)]$

(E) $y_1(t) = e^{-t}[\cos(3t) - \sin(3t)], \quad y_2(t) = e^{-t}[2\cos(3t) - 2\sin(3t)]$

24. Consider the initial value problem for the system of first-order differential equations

$$\begin{aligned} y_1' &= -2y_2 + 1, \quad y_1(0) = 2 \\ y_2' &= -8y_1 + 2, \quad y_2(0) = -1. \end{aligned}$$

If the matrix $\begin{bmatrix} 0 & -2 \\ -8 & 0 \end{bmatrix}$ has eigenvalues $\lambda_1 = -4$, $\lambda_2 = 4$ and corresponding eigenvectors $V_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $V_2 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$, then it has solution

(A) $y_1(t) = \frac{1}{4}e^{4t} + \frac{3}{2}e^{-4t} + \frac{1}{4}, \quad y_2(t) = -\frac{1}{2}e^{4t} - e^{-4t} + \frac{1}{2},$

(B) $y_1(t) = \frac{5}{4}e^{4t} + \frac{1}{2}e^{-4t} + \frac{1}{4}, \quad y_2(t) = -\frac{5}{2}e^{4t} + e^{-4t} + \frac{1}{2},$

(C) $y_1(t) = \frac{1}{4}e^{4t} + e^{-4t} + \frac{3}{4}, \quad y_2(t) = -\frac{5}{2}e^{4t} + 2e^{-4t} - \frac{1}{2},$

(D) $y_1(t) = \frac{5}{4}e^{4t} + \frac{1}{2}e^{-4t} + \frac{1}{4}, \quad y_2(t) = -\frac{5}{4}e^{4t} + e^{-4t} - \frac{3}{4},$

(E) $y_1(t) = \frac{1}{4}e^{4t} + \frac{1}{2}e^{-4t} + \frac{5}{4}, \quad y_2(t) = -\frac{5}{2}e^{4t} + \frac{1}{2}e^{-4t} + 1.$

25. Suppose the matrix $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 1 \end{bmatrix}$ has three distinct eigenvalues $\lambda_1 = 4$, $\lambda_2 = 1$ and $\lambda_3 = -1$ with the corresponding eigenvectors as $V_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $V_2 = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$, $V_3 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$, then the unique solution of $Y' = A\,Y$ subject to $Y(0) = \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}$ is given by

$(A)\quad Y(t) = \begin{pmatrix} e^{4t} + e^{t} + e^{-t} \\ e^{4t} - 2e^{t} \\ e^{4t} - e^{t} + e^{-t} \end{pmatrix}$

$(B)\quad Y(t) = \begin{pmatrix} 2e^{4t} + 2e^{t} - e^{-t} \\ e^{4t} - 3e^{t} + e^{-t} \\ e^{4t} + e^{t} - e^{-t} \end{pmatrix}$

$(C)\quad Y(t) = \begin{pmatrix} e^{4t} + e^{t} + e^{-t} \\ e^{4t} - 2e^{t} \\ e^{4t} + e^{t} - e^{-t} \end{pmatrix}$

$(D)\quad Y(t) = \begin{pmatrix} e^{4t} + e^{t} + e^{-t} \\ e^{4t} - 2e^{t} \\ -e^{4t} + e^{t} + e^{-t} \end{pmatrix}$

$(E)\quad Y(t) = \begin{pmatrix} e^{4t} + e^{t} + e^{-t} \\ e^{4t} - 2e^{-t} \\ -e^{4t} + e^{t} + e^{-t} \end{pmatrix}$

Laplace Transforms

01. $\mathcal{L}\Big\{K_1\ f_1(t)+K_2\ f_2(t)\Big\}(s)=K_1\ \mathcal{L}\Big\{f_1(t)\Big\}(s)+K_2\ \mathcal{L}\Big\{f_2(t)\Big\}(s)$

02. $\mathcal{L}\Big\{y^{(n)}(t)\Big\}=s^n\,\mathcal{L}\Big\{y(t)\Big\}(s)-s^{n-1}\,y(0)-s^{n-2}\,y'(0)-\cdots-y^{(n-1)}(0) \qquad n=1,\,2,\,3,\,\cdots$

03. $\mathcal{L}\Big\{e^{a\,t}\ f(t)\Big\}(s)=\mathcal{L}\Big\{f(t)\Big\}\big(s-a\big)$

04. $\mathcal{L}\Big\{u_a(t)\ f(t)\Big\}(s)=\mathcal{L}\Big\{f\big(t+a\big)\Big\}(s)\ e^{-a\,s}$

05. $\mathcal{L}\Big\{t\ f(t)\Big\}(s)=-\dfrac{d}{ds}\bigg(\mathcal{L}\Big\{f(t)\Big\}(s)\bigg)$

06. $\mathcal{L}\Big\{t^n\Big\}(s)=\dfrac{n!}{s^{n+1}},\quad n=0,\,1,\,2,\,\cdots$

07. $\mathcal{L}\Big\{e^{a\,t}\,t^n\Big\}(s)=\dfrac{n!}{\big(s-a\big)^{n+1}},\quad n=0,\,1,\,2$

08. $\mathcal{L}\Big\{\cos\big(b\,t\big)\Big\}(s)=\dfrac{s}{s^2+b^2}$

09. $\mathcal{L}\Big\{e^{a\,t}\ \cos\big(b\,t\big)\Big\}(s)=\dfrac{s-a}{(s-a)^2+b^2}$

10. $\mathcal{L}\Big\{\sin\big(b\,t\big)\Big\}(s)=\dfrac{b}{s^2+b^2}$

11. $\mathcal{L}\Big\{e^{a\,t}\ \sin\big(b\,t\big)\Big\}(s)=\dfrac{b}{(s-a)^2+b^2}$

12. $\mathcal{L}\Big\{u_a(t)\Big\}(s)=\dfrac{e^{-a\,s}}{s}$

13. $\mathcal{L}\Big\{e^{a\,t}\Big\}(s)=\dfrac{1}{s-a}$

Inverse Laplace Transforms

01. $\mathcal{L}^{-1}\Big\{K_1\ F_1(s)+K_2\ F_2(s)\Big\}(t)=K_1\ \mathcal{L}^{-1}\Big\{F_1(s)\Big\}(t)+K_2\ \mathcal{L}^{-1}\Big\{F_2(s)\Big\}(t)$

02. $\mathcal{L}^{-1}\Big\{F\big(s\big)\Big\}(t)=e^{a\,t}\ \mathcal{L}^{-1}\Big\{F(s+a)\Big\}(t)$ or $\mathcal{L}^{-1}\Big\{F\big(s-a\big)\Big\}(t)=e^{a\,t}\ \mathcal{L}^{-1}\Big\{F(s)\Big\}(t)$

03. $\mathcal{L}^{-1}\Big\{F(s)\ e^{-a\,s}\Big\}(t)=u_a(t)\ \mathcal{L}^{-1}\Big\{F(s)\Big\}\big(t-a\big)$ $\qquad \mathcal{L}^{-1}\Big\{\dfrac{e^{-a\,s}}{s}\Big\}(t)=u_a(t)$

04. $\mathcal{L}^{-1}\left\{F'(s)\right\}(t)=-t\,\mathcal{L}^{-1}\Big\{F(s)\Big\}(t)$

05. $\mathcal{L}^{-1}\Big\{\dfrac{1}{s^{n+1}}\Big\}(t)=\dfrac{t^n}{n!}\quad n=0,\,1,\,2,\,\cdots$

06. $\mathcal{L}^{-1}\Big\{\dfrac{1}{(s-a)^{n+1}}\Big\}(t)=e^{a\,t}\ \dfrac{t^n}{n!}\quad n=0,\,1,\,2$

07. $\mathcal{L}^{-1}\Big\{\dfrac{s}{s^2+b^2}\Big\}(t)=\cos\big(b\,t\big)$

08. $\mathcal{L}^{-1}\Big\{\dfrac{s-a}{(s-a)^2+b^2}\Big\}(t)=e^{a\,t}\ \cos\big(b\,t\big)$

09. $\mathcal{L}^{-1}\Big\{\dfrac{1}{s^2+b^2}\Big\}(t)=\dfrac{1}{b}\ \sin\big(b\,t\big)$

10. $\mathcal{L}^{-1}\Big\{\dfrac{1}{(s-a)^2+b^2}\Big\}(t)=\dfrac{1}{b}\ e^{a\,t}\ \sin\big(b\,t\big)$

Trigonometric Identities

1. $\cos^2(\theta)+\sin^2(\theta)=1$

2. $\cos(2\,\theta)=\cos^2(\theta)-\sin^2(\theta)$

3. $\sin(2\,\theta)=2\,\cos(\theta)\,\sin(\theta)$

4. $2\,\cos^2(\theta)=1+\cos(2\,\theta)$

5. $2\,\sin^2(\theta)=1-\cos(2\,\theta)$

6. $\cos(\theta\pm\pi)=-\cos(\theta)$

7. $\sin(\theta\pm\pi)=-\sin(\theta)$

8. $\cos(\theta\pm 2\,\pi)=\cos(\theta)$

9. $\sin(\theta\pm 2\,\pi)=\sin(\theta)$