Math 375		**Fall 2016**
	Higher Order Linear Differential Equations	
Worksheet # 2	**Part 1**	**October 10 - 14**

The problems on this worksheet refer to material from section §4.1 of your text. Solutions to all problems will be available on the course's D2L website Friday, October 14. Please report any typos, omissions and errors to `aiffam@ucalgary.ca`

Basics

01. Which of the following is a linear second order differential equation

a*. $y'' - 5\,t\,y = t\,y' - 25$ **b*.** $\big(\sin(t)\,y'\big)' + 2\,t\,y^2 = 0$

c*. $y'' + t\,y' - \dfrac{\sin(y)}{y} = \ln(t)$ **d*.** $\dfrac{y' + t\,y}{1 + y''} = \mathrm{e}^t$

02. Given that the general solution of $y'' + 4\,y = 0$ is $C_1\,\cos(2\,t) + C_2\,\sin(2\,t)$, solve the initial value problem $\begin{cases} y'' + 4\,y = 0 \\ y(\pi/2) = -1 \;\; \& \;\; y'(\pi/2) = 2\sqrt{3} \end{cases}$

Express your answer in the form $y = A\,\cos\big(2\,t + \phi\big)$

03*. Consider the differential equation $t^2\,y'' - 3\,t\,y' - 5\,y = 0$. Verify that $y_1 = t^5$ and $y_2 = \dfrac{1}{t}$ are solutions of the equation, then solve the I.V.P.

$$\begin{cases} t^2\,y'' - 3\,t\,y' - 5\,y = 0 \\ y(1) = 4 \;\; \& \;\; y'(1) = 2 \end{cases}$$

04. The 2^{nd} order linear differential equation $y'' + p(t)\,y' + q(t)\,y = 0$, has solutions $y_1(t) = \dfrac{1}{t}$ and $y_2(t) = \mathrm{e}^t$. Find $p(t)$ and $q(t)$.

Existence and Uniqueness

05. Determine the largest open interval over which the unique solution of the given initial value problem is guaranteed to exist.

a*. $\begin{cases} t\,y'' + \dfrac{1}{t^2 - 9}y' + y = t \\ y(1) = 0 \;\; \& \;\; y'(1) = 2 \end{cases}$ **b*.** $\begin{cases} t\,y'' + \dfrac{1}{t^2 - 9}y' + y = t \\ y(4) = 0 \;\; \& \;\; y'(4) = 2 \end{cases}$

c. $\begin{cases} y'' + y' + y = \sec(t) \\ y(\pi/4) = 1 \;\; \& \;\; y'(\pi/4) = -1 \end{cases}$ **d.** $\begin{cases} y'' + t\,y' + \ln(1 - t)\,y = \ln(2 + t) \\ y(0) = 1 \;\; \& \;\; y'(0) = 2 \end{cases}$

06*. Consider the 2^{nd} order linear differential equation $y'' - 4\,t\,y' + \left(4\,t^2 - 2\right) y = 0$

a. Verify that $y_1(t) = \mathrm{e}^{t^2}$ is a solution.

b. Use the reduction of order method to find the general solution.

07. Consider the 2^{nd} order linear differential equation $t^2\left(1 - \ln(t)\right) y'' + t\,y' - y = 0$

a. Verify that $y_1(t) = t$ is a solution

b. Use the reduction of order method to find the general solution.