AMAT 375
Handout # 6
Systems of linear differential equations

1. Find the general solution of the following systems of differential equations

$$a)\ X'(t) = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} X(t) \qquad b)\ X'(t) = \begin{bmatrix} 1 & -3 \\ 3 & 1 \end{bmatrix} X(t)$$

$$c)\ X'(t) = \begin{bmatrix} 3 & 1 \\ -5 & -3 \end{bmatrix} X \qquad d)\ X' = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & -1 \\ 2 & -1 & 0 \end{bmatrix} X(t)$$

$$e)\ X' = \begin{bmatrix} 1 & -2 & -1 \\ -1 & 1 & 1 \\ 1 & 0 & -1 \end{bmatrix} X \qquad f)\ X' = \begin{bmatrix} 3 & -1 & 1 \\ 1 & 1 & 1 \\ 4 & -1 & 4 \end{bmatrix} X$$

$$g)\ X' = \begin{bmatrix} -1 & -5 \\ 1 & 1 \end{bmatrix} X(t)$$

2. Find the solution of the initial value problem

$$X' = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} X, X(0) = \begin{bmatrix} 1 \\ 3 \end{bmatrix}.$$

3. Find the general solution of the system $X'(t) = AX(t)$, where for the matrix $A = 2\begin{bmatrix} 2 & 1 & 0 \\ -1 & 2 & 0 \\ 0 & 1 & 7 \end{bmatrix}$ we have that the determinant of $A - \lambda I$ is $(7-\lambda)(\lambda^2 - 4\lambda + 5$.

4. For a system of two equations $X'(t) = AX(t)$ with a real constant matrix A which has an eigenvalue $\lambda = 6 + i$ and a corresponding eigenvector $X = \begin{bmatrix} 1-i \\ 1 \end{bmatrix}$, find the general solution.

5. For a homogeneous linear system of order three $X'(t) = AX(t)$, where a real constant matrix A has eigenvalues $3, -5, 0$ and corresponding eigenvectors $\begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 6 \end{bmatrix}, \begin{bmatrix} 4 \\ -1 \\ 6 \end{bmatrix}$, find the general solution.

6. Find the general solution of the system $x_1' = 2x_1 + 3x_3$, $x_2' = 2x_2$, $x_1' = 3x_1 + 2x_3$.