Department of Mathematics and Statistics
MATH 375
Handout # 1
Differential equations in general; linear equations

1. For each of the following ordinary differential equations determine the order, the dependent variable (or the unknown function), and the independent variable. Determine whether the equation is linear or not.

   a) $\dfrac{dx}{dt} + 8x = e^{t+x}$ b) $\dfrac{d^2t}{dx^2} + xt = \sin(x^2)$

   c) $\left(\dfrac{dr}{ds}\right)^3 + \dfrac{d^2r}{ds^2} = 7$ d) $y^{(4)} - 2y''' + 5y' - 9y = \cos^2(t)$

   e) $\left(\dfrac{dp}{dq}\right)^6 = 2p$ f) $\dfrac{d^6p}{dq^6} = 2p$

   g) $t^2\ddot{s} - t\dot{s} + 4s = 1 - \sin(t)$

2. For each of the following partial differential equations determine the order, the dependent variable (or the unknown function), and the independent variables. Determine whether the equation is linear or not.

   a) $\dfrac{\partial x}{\partial s} + 4\dfrac{\partial x}{\partial t} + x^2 = 5e^{s+t}$ b) $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = \sin(x+y)$

3. Which of the following are solutions to differential equations $\dfrac{dx}{dt} = \dfrac{x}{t}$, $t > 0$?

   a) $x = 0$ b) $x = 2$ c) $x = 2t$ d) $x = -3t$ e) $x = t^2$

4. Given the differential equation $t^2\dfrac{d^2x}{dt^2} - t\dfrac{dx}{dt} + x = 0, \ \ t > 0$

   a) verify that $x(t) = t(C_1 + C_2\ln(t))$ for any $C_1, C_2 \in \mathbb{R}$ is a solution of the given differential equation; b) find the solution satisfying the initial conditions $x(1) = -3$, $x'(1) = 4$.

5. Find the general solution of the differential equation $y' = -2x + 2$. Sketch few members of the one-parameter family of solutions.

6. For $t > 0$, find the equivalent forms of the differential equation $e^{x'+x} = t$ among the following:

   a) $x' + x = \ln(t)$ b) $x' + x = e^{-t}$ c) $\dfrac{dx}{dt} = -x + \ln(t)$

   d) $dx + (x - e^{-t})dt = 0$ e) $dx + (x - \ln(t))dt = 0$

7. Find the general solution of the following differential equations

   a) $y' - \dfrac{y}{x} = x$, $x > 0$

   b) $(2x+1)y' = 4x + 2y$

   c) $xy' + (x+1)y = 3x^2e^{-x}$

8. Find the general solution of the equation

   $$xy' - 2y = x^3 + x;$$

   find the solution satisfying $y(1) = 3$.