David Ng **F2016MATH375**

Assignment01 is due on Sunday, October 09, 2016 at 11:59pm.

The number of attempts available for each question is noted beside the question. If you are having trouble figuring out your error, you should consult the textbook, or ask a fellow student, one of the TA's or your professor for help.
There are also other resources at your disposal, such as the Mathematics Continuous Tutorials. Don't spend a lot of time guessing – it's not very efficient or effective.

Make sure to give lots of significant digits for (floating point) numerical answers. For most problems when entering numerical answers, you can if you wish enter elementary expressions such as $2\wedge 3$ instead of 8, $\sin(3*\mathrm{pi}/2)$instead of -1, $e\wedge(\ln(2))$ instead of 2, $(2+\tan(3))*(4-\sin(5))\wedge 6-7/8$ instead of 27620.3413, etc.

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1. (1 point) Find the solution of the differential equation

$$xy' + 6y = 245x^1 \ln x \quad (x > 0)$$

that satisfies the initial condition $y(1) = -6$.
Answer: $y(x) =$ ____________________

Answer(s) submitted:

- `5x(7ln(x)-1)-1/x^6`

(correct)
Correct Answers:

- `35 * x^1*ln(x) - 5 * x^1 + -1/(x^6)`

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2. (1 point) Find the general solution solution of the differential equation

$$\begin{cases} y' + \dfrac{1}{6}\sec\left(\dfrac{t}{6}\right) y = 7\cos\left(\dfrac{t}{6}\right) \\ y(0) = 4 \end{cases} \qquad 0 < \frac{t}{6} < \frac{\pi}{2}$$

$y(t) =$ ________________________

Answer(s) submitted:

- `((46+7 t-42 cos(t/6)) (cos(t/12)-sin(t/12)))/(cos(t/12)+sin(t/12))`

(correct)
Correct Answers:

- `((7)*t - (42)*cos(t/6)+46)/(sec(t/6)+tan(t/6))`

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3. (1 point)

Solve the initial value problem $\begin{cases} y' = \dfrac{1+x}{xy^4} \\ y(1) = 6 \end{cases} \qquad x > 0$

$y^5 =$ ____________________.
Answer(s) submitted:

- `5ln(x) +5x+7771`

(correct)
Correct Answers:

- `5*ln(x) + 5*x + 7771`

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4. (1 point)
Find the solution of the initial value problem

$$\begin{cases} \dfrac{x^2}{y^2-5}\, y' = \dfrac{1}{2y} \\ y(1) = \sqrt{6} \end{cases}$$

$y =$ ____________________

Your answer should be a function of x.
Answer(s) submitted:

- `sqrt(e^(-1/x+1)+5)`

(correct)
Correct Answers:

- `sqrt(e^(1-1/x)+5)`

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5. (1 point) Solve the initial value problem:

$$ty' + y = (32t^2 + 28t + 6)y^{-5}, \quad y(1) = 2 \quad (t > 0)$$

$y(t) =$ ____________________
Your answer should be a function of t.
Answer(s) submitted:

- `(24t^2+24t+6+10/t^6)^(1/6)`

(correct)
Correct Answers:

- `(24 * t^2 + 24 * t + 6 + 10/(t^6))^(1/6)`

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6. (1 point) Solve the following exact differential equation

$$(ye^{xy} + 5x^4)dx + (xe^{xy} - 7)dy = 0$$

Express your answer in the form $F(x,y) = C$, where $F(x,y)$ has no constant term.
$F(x,y) =$ ______________________ $= C$
Answer(s) submitted:

- `e^(xy)+x^5-7y`

(correct)
Correct Answers:

- `k( e**(x*y) + x**5 - 7*y )`

7. (1 point) The values of A and B which make the differential equation

$$\left(A\frac{e^{3y}}{\sqrt{x}} - 8\ln(4y)\right)dx + \left(8\sqrt{x}e^{3y} + B\frac{x}{y} - 2\right)dy = 0$$

exact are : $A =$ _____, $B =$ _____

The exact differential equation which results from that choice of A and B has solution $F(x,y) = C$ where
$F(x,y) =$ ______________________________
Please write your answer with no constant term.

Answer(s) submitted:

- `4/3`
- `-8`
- `8/3e^(3y)sqrt(x)-8ln(4y)x-2y`

(correct)

Correct Answers:

- `8/6`
- `-8`
- `8/3*sqrt(x)*e^(3*y)+-8*x*ln(4*y)+-2*y`

8. (1 point) Consider the nonexact differential equation

$$\left(5x^6 - y^3\right)dx + \left(3xy^2\right)dy = 0.$$

(a) Find an integrating $\mu = \mu(x)$ for the differential equation.
$\mu(x) =$ __________.

(b) Solve the differential equation and express your answer as $F(x,y) = C$, where $F(x,y)$ has no constant term.
$F(x,y) =$ ______________________________.

Answer(s) submitted:

- `1/x^2`
- `x^5+y^3/x`

(correct)

Correct Answers:

- `k(1/(x**2))`
- `k(x**5 + y**3/x)`

9. (1 point) Find the general solution of the differential equation

$$y - 1y^2 = \left(y^7 + 1x\right)y'.$$

Write your solution in the form $F(x,y) = C$, where C is an arbitrary constant.

__________________________ $= C$.

Answer(s) submitted:

- `(y-y^2)x/y^2 -y^6/6`

(correct)

Correct Answers:

- `-y^6/6+x/(y^1)+-1*x`

10. (1 point)

For what values of m and n will $u = x^n y^m$ be an integrating factor for the differential equation

$$(-4y - 1x)dx + (8x + 3x^2y^{-1})dy = 0$$

Answer: $n =$ _____, $m =$ _____

The exact differential equation which results from multiplying by this integrating factor has solution $F(x,y) = C$ where
$F(x,y) =$ ___________________________________

Answer(s) submitted:

- `-3`
- `3`
- `2x^(-2)y^4+x^(-1)y^3`

(correct)

Correct Answers:

- `-3`
- `3`
- `(-4)*(x**(-3+1)*y**(3+1))/(-3+1) + (-1)*(x**(-3+2)*y**(3))`

11. (1 point)

Solve the Initial Value Problem $\begin{cases} y' = \dfrac{3xy}{3x^2+4y^2} \\ y(1) = 1 \end{cases}$

Express your answer in the form $F(x,y) = \dfrac{3}{8}$, where
$F(x,y) =$ __________________________________

Hint: Use the substitution $y = xu$

Answer(s) submitted:

- `3/8(y/x)^(-2) -ln(y/x)-ln(x)`

(correct)

Correct Answers:

- `(3*x^2/(8*y^2)-ln(abs(y/x))-ln(abs(x)))`

12. (1 point) A bacteria culture starts with 640 bacteria and grows at a rate proportional to its size. After 5 hours there will be 3200 bacteria.

(a) Express the population after t hours as a function of t.
population: ______________________ (function of t)

(b) What will be the population after 4 hours?
____

(c) How long will it take for the population to reach 2790 ?
____

Answer(s) submitted:

- `640*5^(t/5)`
- `640e^(4* ln(5)/5)`
- `5ln(2790/640)/ln(5)`

(correct)

Correct Answers:

- 640*(2.71828182845905^(0.32188758248682*t))
- 2319.29492376863
- 4.57404627754467

13. (1 point) An unknown radioactive element decays into non-radioactive substances. In 680 days the radioactivity of a sample decreases by 29 percent.

(a) What is the half-life of the element?
half-life: ______________ (days)

(b) How long will it take for a sample of 100 mg to decay to 61 mg?
time needed: ___________________ (days)

Answer(s) submitted:

- 680ln(.5)/ln(.71)
- 680ln(.61)/ln(.71)

(correct)
Correct Answers:

- 1376.21436422603
- 981.404407814309

14. (1 point) A tank contains 1440 L of pure water. A solution that contains 0.01 kg of sugar per liter enters a tank at the rate 4 L/min The solution is mixed and drains from the tank at the same rate.

(a) How much sugar is in the tank initially?
_________ (kg)

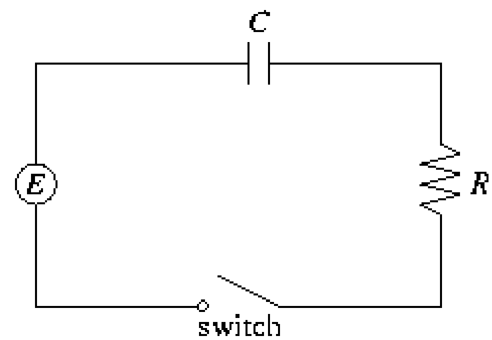
(b) Find the amount of sugar in the tank after t minutes.
amount = ___________________ (kg)
(your answer should be a function of t)

(c) Find the concentration of sugar in the solution in the tank after 30 minutes.
concentration = ___________________ (kg/L)
Answer(s) submitted:

- 0
- 14.4 -14.4e^(-t/360)
- (14.4 -14.4e^(-30/360))/1440

(correct)
Correct Answers:

- 0
- 14.4 *(1-e^(-0.00277777777777778 * t))
- (14.4 *(1- e^(-0.00277777777777778 * 30)))/1440



15. (1 point)

The figure above shows a circuit containing an electromotive force (a battery), a capacitor with a capacitance of C farads (F), and a resistor with a resistance of R ohms (Ω). The voltage drop across the capacitor is Q/C, where Q is the charge (in coulombs), so in this case Kirchhoff's Law gives

$$RI + \frac{Q}{C} = E(t).$$

Since the current is $I = \frac{dQ}{dt}$, we have

$$R\frac{dQ}{dt} + \frac{1}{C}Q = E(t).$$

Suppose the resistance is $20\,\Omega$, the capacitance is 0.2F, a battery gives a constant voltage of $E(t) = 30$V, and the initial charge is $Q(0) = 0$C.
Find the charge and the current at time t.
$Q(t) =$ ___________________,
$I(t) =$ ___________________.
Answer(s) submitted:

- 6-6e^(-t/4)
- 3/2e^(-t/4)

(correct)
Correct Answers:

- 30*0.2*(1-2.71828182845905**(-t/20/0.2))
- 30/20*2.71828182845905**(-t/20/0.2)