**THE UNIVERSITY OF CALGARY**
**DEPARTMENT OF MATHEMATICS AND STATISTICS**
**MIDTERM EXAMINATION**

**AMAT 307 Lec 01 - 04 FALL 2012**

**Friday, November 09, 2012 90 minutes**

| **Last Name** | **First Name** |
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| **Student I.D. Number** | **Section Number** |
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*This examination consists of 20 equally weighted questions: 3 True/False, and 17 multiple choice. Please attempt all problems and record your answer by circling your choice in the exam booklet and filling in the appropriate circle in the scantron sheet.*

*This is a closed book examination, but the use of a Schulich calculator is permitted.*

*Fill in your name and lecture number on the scantron sheet and the test booklet.*

*When finished, turn in both the answer sheet and this exam booklet.*

**01.** It is possible for a first order differential equation to be linear, separable, and exact, all at the same time.

**T** **F**

**02.** It is possible for a nonexact differential equation

$$M(x,y)\ \mathrm{d}x + N(x,y)\ \mathrm{d}y = 0$$

to have two integrating factors: one that depends on $x$ only, and another one that depends on $y$ only.

**T** **F**

**03.** If $y_1(t) = 3\,t\,\mathrm{e}^{2\,t} - 5\,\cos(t)$ is a solution of a linear homogeneous constant coefficient differential equation, then $y_2(t) = 7\,\mathrm{e}^{2\,t} + 3\,\sin(t)$ is a solution of the differential equation as well.

**T** **F**

**04.** Let $y_1(t)$ and $y_2(t)$ be two solutions of the differential equation $y'' + t\,y' + y = t$. Only one of the following statements is correct. Which one is it?

**A.** $y_1(t) - y_2(t)$ is a solution of the differential equation.

**B.** $3\,y_1(t) + 2\,y_2(t)$ is a solution of the differential equation.

**C.** $2\,y_1(t) - 3\,y_2(t)$ is a solution of the differential equation.

**D.** $2\,y_1(t) + 3\,y_2(t)$ is a solution of the differential equation.

**E.** $3\,y_1(t) - 2\,y_2(t)$ is a solution of the differential equation.

**05** Only one of the following statements is incorrect. Which one is it?

**A.** $y' + t\,y = 3\,t\,\mathrm{e}^y + t$ is a separable equation.

**B.** $2\,y' + t = \dfrac{t}{y} - \dfrac{y}{t}$ is a Bernoulli equation.

**C.** $y' + y = \dfrac{y^3 + 3\,y}{3\,t + t\,y^2}$ is a linear equation.

**D.** $\dfrac{y^2 + 1}{2\,x}\,\mathrm{d}x + y\,\ln(2\,x)\,\mathrm{d}y = 0$ is an exact equation.

**06.** The solution of the initial value problem $\begin{cases} y' = 2\,t\,y + 2\,t\,\mathrm{e}^{t^2} \\ y(3) = 0 \end{cases}$ is

**A.** $y = t^2 - 9$

**B.** $y = \left(t^2 - 9\right)\mathrm{e}^{-t^2}$

**C.** $y = \left(t^2 - 9\right)\mathrm{e}^{t^2}$

**D.** $\ln\left(y + \mathrm{e}^{t^2}\right) = t^2$

**E.** none of the above

**07.** The unique solution of the initial value problem

$$\begin{cases} y' + \dfrac{1}{t^2 - 9}\,y \;=\; \dfrac{\ln(2-t)}{t+2} \\ y(1) = 3 \end{cases}$$

is guaranteed to exist in the open interval

**A.** $\left(-2\,,\,3\right)$

**B.** $\left(-3\,,\,3\right)$

**C.** $\left(-3\,,\,2\right)$

**D.** $\left(-2\,,\,2\right)$

**E.** $\left(-\infty\,,\,+\infty\right)$

**08.** The solution of the initial value problem $\begin{cases} 3\,t\,y' - 2\,y = 3\,t^{-1}\,y^{-2} \\ y(1) = 1 \end{cases}$ is given by

**A.** $y = \left(\dfrac{2\,t^3 - 1}{t}\right)^{1/3}$

**B.** $y = \dfrac{\left(2\,t^3 - 1\right)^{1/3}}{t}$

**C.** $y = \dfrac{2\,t^3 - 1}{t}$

**D.** $y = \dfrac{t}{3 - 2\,t^{1/3}}$

**E.** $y = \dfrac{3 - 2\,t^{1/3}}{t}$

**09.** The initial value problem $\begin{cases} y' - 3\,t^2\,\mathrm{e}^{-y} = 0 \\ y(1) = \ln(3) \end{cases}$ has solution

**A.** $y = \ln\left(3\,t\right)$

**B.** $y = \ln\left(t^3 + 2\right)$

**C.** $y + \ln\left(\dfrac{4}{3} - t^3\right) = 0$

**D.** $3\,y + 3\,t^3\,\mathrm{e}^{-y} = 1 + 3\,\ln(3)$

**E.** None of the above

**10.** The general solution of $\left(3\,x^2 + y\,\mathrm{e}^{x\,y}\right)\,\mathrm{d}x + \left(2\,y^3 + x\,\mathrm{e}^{x\,y}\right)\,\mathrm{d}y = 0$ is

**A.** $x^3 + 2\,\mathrm{e}^{x\,y} + \dfrac{y^4}{2} = C$

**B.** $x^3 + \mathrm{e}^{x\,y} + \dfrac{y^4}{2} = C$

**C.** $x^3 + \mathrm{e}^{x\,y} = C$

**D.** $\mathrm{e}^{x\,y} + \dfrac{y^4}{2} = C$

**E.** None of the above

**11.** An integrating factor for the nonexact differential equation

$$\left(x^4\,\ln(x) - 2\,x\,y^3\right)\,\mathrm{d}x + 3\,x^2\,y^2\,\mathrm{d}y = 0$$

is given by

**A.** $\mu = y^{-4}$

**B.** $\mu = x^4$

**C.** $\mu = -4\,x$

**D.** $\mu = 4\,y$

**E.** $\mu = x^{-4}$

**12.** A large tank initially contains 10 kg of salt dissolved in 500 litres of water. Starting at $t = 0$, a brine solution with 0.05 kg of salt per litre, starts flowing into the tank at a rate of 3 litres per minute. The brine solution in the tank is kept well stirred and flows out of the tank at the rate of 5 litres per minute. If $Q(t)$ is the total mass of salt (in kg) in the tank at time $t$ (in minutes), then $Q(t)$ is solution of the initial value problem

**A.** $\begin{cases} Q' + \dfrac{5}{500 - 2\,t}\,Q = 0.15 \\ Q(0) = 10 \end{cases}$

**B.** $\begin{cases} Q' + \dfrac{3}{500 + 2\,t}\,Q = 0.25 \\ Q(0) = 10 \end{cases}$

**C.** $\begin{cases} Q' + \dfrac{5}{500 - 2\,t}\,Q = 0.15 \\ Q(0) = 0 \end{cases}$

**D.** $\begin{cases} Q' + \dfrac{3}{500 + 2\,t}\,Q = 0.25 \\ Q(0) = 0 \end{cases}$

**E.** $\begin{cases} Q' + \dfrac{5}{500 - 2\,t}\,Q = 0.15 \\ Q(0) = 25 \end{cases}$

**13.** Consider the initial value problem $\begin{cases} y' - y^2 = t \\ y(0) = 2 \end{cases}$

Using Euler's method with step size $h = 0.1$, an approximate value $y_2$ of $y(0.2)$, is

**A.** 42.1

**B.** 1.354

**C.** 2.986

**D.** $-5.9$

**E.** 2.876

**14.** The unique solution of the initial value problem

$$\begin{cases} (t-3)\,y'' + \dfrac{t}{t+4}\,y' + \dfrac{t-3}{2\,t+5}\,y = \sqrt{3-2\,t} \\ y(0) = 0 \quad \text{and} \quad y'(0) = 1 \end{cases}$$

is guaranteed to exist in the open interval

**A.** $\big(-4\,,\,3\big)$

**B.** $\big(-5/2\,,\,3/2\big)$

**C.** $\big(-5/2\,,\,3\big)$

**D.** $\big(-\infty\,,\,3\big)$

**E.** $\big(-4\,,\,+\infty\big)$

**15.** The solution of the initial value problem $\begin{cases} 3\,y'' + 2\,y' - 5\,y = 0 \\ y(0) = 9 \quad \text{and} \quad y'(0) = 1 \end{cases}$ is

**A.** $y = -15\,\mathrm{e}^{-5\,t/3} + 24\,\mathrm{e}^{-t}$

**B.** $y = \dfrac{21}{4}\,\mathrm{e}^{-t} - \dfrac{17}{4}\,\mathrm{e}^{5\,t/3}$

**C.** $y = 3\,\mathrm{e}^{-5\,t/3} + 6\,\mathrm{e}^{t}$

**D.** $y = -12\,\mathrm{e}^{5\,t/3} + 21\,\mathrm{e}^{t}$

**E.** none of the above

**16.** If a linear homogeneous differential equation with constant coefficients has characteristic equation $\left(\lambda^2 - 2\,\lambda + 1\right)\left(\lambda^2 - 2\,\lambda + 2\right) = 0$, then its general solution is

**A.** $C_1\,e^t + C_2\,t\,e^t + C_3\,e^t\,\cos(t) + C_4\,e^t\,\sin(t)$

**B.** $C_1\,e^t + C_2\,e^t + C_3\,e^t\,\cos(t) + C_4\,e^t\,\sin(t)$

**C.** $C_1\,e^{-t} + C_2\,t\,e^{-t} + C_3\,e^{-t}\,\cos(t) + C_4\,e^{-t}\,\sin(t)$

**D.** $C_1\,e^t + C_2\,t\,e^t + C_3\,t^2\,e^t\,\cos(t) + C_4\,t^3\,e^t\,\sin(t)$

**E.** $C_1\,e^{-t} + C_2\,t\,e^{-t} + C_3\,t^2\,e^{-t}\,\cos(t) + C_4\,t^3\,e^{-t}\,\sin(t)$

where $C_1$, $C_2$, $C_3$, and $C_4$ are arbitrary real constants.

**17.** The equation $2\,y'' + b\,y' + c\,y = 0$, where $b$ and $c$ are real constants, has $2\,t\,e^{-5\,t}$ as one of its solutions. Then

**A.** $b = -10$ and $c = 25$

**B.** $b = 10$ and $c = 25$

**C.** $b = -20$ and $c = 50$

**D.** $b = 20$ and $c = 50$

**E.** $b = -10$ and $c = -25$

**18** The general solution of $y'' + y' - 6\,y = 2\,\mathrm{e}^{-2\,t} - 12\,t$ is

**A.** $C_1\,\mathrm{e}^{-2\,t} + C_2\,\mathrm{e}^{3\,t} - \dfrac{1}{2}\,\mathrm{e}^{-2\,t} + 2\,t + \dfrac{1}{3}$

**B.** $C_1\,\mathrm{e}^{-3\,t} + C_2\,\mathrm{e}^{2\,t} + \dfrac{2}{5}\,t\,\mathrm{e}^{2\,t} + 2\,t + \dfrac{1}{3}$

**C.** $C_1\,\mathrm{e}^{-3\,t} + C_2\,\mathrm{e}^{2\,t} + 2\,\mathrm{e}^{-2\,t} - 12\,t$

**D.** $C_1\,\mathrm{e}^{-3\,t} + C_2\,\mathrm{e}^{2\,t} + \dfrac{2}{5}\,\mathrm{e}^{2\,t} + 2\,t + \dfrac{1}{3}$

**E.** $C_1\,\mathrm{e}^{-3\,t} + C_2\,\mathrm{e}^{2\,t} - \dfrac{1}{2}\,\mathrm{e}^{-2\,t} + 2\,t + \dfrac{1}{3}$

**19.** If the method of undetermined coefficients were to be used to find a particular solution $y_p(t)$ to the differential equation $y''' - 2\,y'' = 3\,t^2 - 5\,\mathrm{e}^{2\,t} + \mathrm{e}^{2\,t}\,\sin(t)$, then $y_p(t)$ should have the form

**A.** $A\,t^2 + B\,t + C + D\,\mathrm{e}^{2\,t} + E\,\mathrm{e}^{2\,t}\,\cos(t) + F\,\mathrm{e}^{2\,t}\,\sin(t)$

**B.** $A\,t^3 + B\,t^2 + C\,t + D\,t\,\mathrm{e}^{2\,t} + E\,\mathrm{e}^{2\,t}\,\cos(t) + F\,\mathrm{e}^{2\,t}\,\sin(t)$

**C.** $A\,t^4 + B\,t^3 + C\,t^2 + D\,t\,\mathrm{e}^{2\,t} + E\,t\,\mathrm{e}^{2\,t}\,\cos(t) + F\,t\,\mathrm{e}^{2\,t}\,\sin(t)$

**D.** $A\,t^4 + B\,t^3 + C\,t^2 + D\,t\,\mathrm{e}^{2\,t} + E\,\mathrm{e}^{2\,t}\,\cos(t) + F\,\mathrm{e}^{2\,t}\,\sin(t)$

**E.** none of the above

where $A$, $B$, $C$, $D$, $E$, and $F$ are real constants to be determined.

**20.** The general solution of the equation $y'' + 2\,y' + y = \dfrac{\mathrm{e}^{-t}}{t}$, is

**A.** $y = C_1\,\mathrm{e}^{-t} + C_2\,t\,\mathrm{e}^{-t} + t\,\ln|t|$

**B.** $y = C_1\,\mathrm{e}^{-t} + C_2\,t\,\mathrm{e}^{-t} + \dfrac{\mathrm{e}^{-t}}{t}$

**C.** $y = C_1\,\mathrm{e}^{-t} + C_2\,t\,\mathrm{e}^{-t} + t\,\mathrm{e}^{-t}\,\ln|t|$

**D.** $y = C_1\,\mathrm{e}^{-t} + C_2\,t\,\mathrm{e}^{-t}$

**E.** $y = C_1\,\mathrm{e}^{-t} + C_2\,t\,\mathrm{e}^{-t} + \ln|t|$