THE UNIVERSITY OF CALGARY
DEPARTMENT OF MATHEMATICS AND STATISTICS
MIDTERM EXAMINATION

MATH 307 Lec 01-04 FALL 2013

Friday, November 08, 2013 90 minutes

Last Name	First Name

Student I.D. Number	Tutorial Number

This examination consists of 22 equally weighted questions. Please attempt all problems and record your answer by circling your choice in the exam booklet, and filling in the appropriate circle in the scantron sheet.

This is a closed book examination and calculators are not permitted.

Please record your answers in the table below.

01	02	03	04	05	06	07	08	09	10	11
D	**C**	**D**	**T**	**C**	**B**	**D**	**C**	**C**	**B**	**A**
12	13	14	15	16	17	18	19	20	21	22
D	**C**	**A**	**C**	**B**	**B**	**D**	**D**	**B**	**C**	**D**

01 The differential equation $y'\,y'' + y'\,\mathrm{e}^y + 5\,y = t$ is

A. Linear second order

B. Nonlinear, third order

C. Linear, third order

D. Nonlinear second order

E. None of the above

02. The value of the real constant a for which the differential equation

$$\Big(\sin(2\,y) + a\,y\,\sin(3\,x) + x\Big)\,\mathrm{d}x + \Big(2\,x\,\cos(2\,y) - 2\,\cos(3\,x) + y^2\Big)\,\mathrm{d}y = 0$$

is exact in the xy-plane is

A. 3 **B.** -3 **C.** 6 **D.** -6 **E.** 2

03. If $y = y(t)$ is the solution of the initial value problem

$$\begin{cases} y' + (2\,t+1)\,y = 2\,\cos(t) \\ y(0) = 2 \end{cases} \quad \text{then}$$

A. $y''(0) = 2$ **B.** $y''(0) = -2$ **C.** $y''(0) = 4$

D. $y''(0) = -4$ **E.** $y''(0) = 0$

04. **True/False**
If $\cos(2\,x)$ is a solution of $a\,y'' + b\,y' + c\,y = 0$, where $a \neq 0, b, c$ are real constants, then $100\,\sin(2\,x)$ is also a solution of the same differential equation.

05. The unique solution of the initial value problem $\begin{cases} t\,y' + 2\,y = 4\,t^2 \\ y(1) = 2 \end{cases}$ is

A. $t^4 + t^{-4}$ **B.** $t^3 + t^{-3}$ **C.** $t^2 + t^{-2}$

D. $t + t^{-1}$ **E.** $\sqrt{t} + \dfrac{1}{\sqrt{t}}$

06. The unique solution of the initial value problem $\begin{cases} y' = \dfrac{2\,x}{5+2\,y} \\ y(2) = -1 \end{cases}$ is given by

A. $y = \dfrac{5}{2} - \dfrac{1}{2}\sqrt{4\,x^2 - 7}$

B. $y = -\dfrac{5}{2} + \dfrac{1}{2}\sqrt{4\,x^2 - 7}$

C. $y = \sqrt{6\,x - 3} - 4$

D. $y = \dfrac{1}{2} - \dfrac{1}{2}\sqrt{4\,x^2 - 7}$

E. None of the above

07. Let $y_1(t)$ and $y_2(t)$ be two solutions of the differential equation $y'' + \mathrm{e}^t\,y' + y = t^3$. Which of the following is a solution of the differential equation?

A. $y_1(t) - y_2(t)$

B. $y_1(t) + y_2(t)$

C. $y_1(t) - 5\,y_2(t)$

D. $2\,y_1(t) - y_2(t)$

E. $2\,y_1(t) + 3\,y_2(t)$

08. Consider the initial value problem $\begin{cases} y' + \dfrac{\ln(x)}{x^2 - 1}\,y = \dfrac{1}{x-3} \\ y(2) = 1 \end{cases}$ The largest open interval on which a unique soution is guaranteed to exist is

A. $(0\,,\,+\infty)$

B. $(-1\,,\,1)$

C. $(1\,,\,3)$

D. $(1\,,\,+\infty)$

E. $(-1\,,\,3)$

09. The solution of the exact differential equation

$$\Big(y\,\mathrm{e}^{x\,y}\,\cos(2\,x) - 2\,\mathrm{e}^{x\,y}\,\sin(2\,x) + 2\,x\Big) + \Big(x\,\mathrm{e}^{x\,y}\,\cos(2\,x) - 3\Big)\,y' = 0$$

is given by

A. $\mathrm{e}^{x\,y}\,\sin(2\,x) + x^2 - 3\,y = C$

B. $\dfrac{1}{2}\,\mathrm{e}^{x\,y}\,\sin(2\,x) + x^2 - 3\,x\,y = C$

C. $\mathrm{e}^{x\,y}\,\cos(2\,x) + x^2 - 3\,y = C$

D. $\dfrac{1}{2}\,\mathrm{e}^{x\,y}\,\cos(2\,x) + x^2 - 3\,x\,y = C$

E. None of the above

10. Let $y_p(x)$ be a particular solution of the differential equation

$$y'' + y' - 2\,y = x\,\mathrm{e}^{x} + \mathrm{e}^{-3\,x}\,\cos(x)$$

Using the method of undetermined coefficients, y_p will have the form

A. $\mathrm{e}^{x}\left(A_0 + A_1\,x\right) + \mathrm{e}^{-3\,x}\Big[\left(B_0\,x + B_1\,x^2\right)\,\cos(x) + \left(C_0\,x + C_1\,x^2\right)\,\sin(x)\Big]$

B. $\mathrm{e}^{x}\left(A_0\,x + A_1\,x^2\right) + \mathrm{e}^{-3\,x}\Big[B_0\,\cos(x) + C_0\,\sin(x)\Big]$

C. $\mathrm{e}^{x}\left(A_0\,x + A_1\,x^2\right) + \mathrm{e}^{-3\,x}\Big[B_0\,\cos(x)\Big]$

D. $\mathrm{e}^{x}\left(A_0 + A_1\,x\right) + \mathrm{e}^{-3\,x}\Big[\left(B_0 + B_1\,x\right)\,\cos(x) + \left(C_0 + C_1\,x\right)\,\sin(x)\Big]$

E. $\mathrm{e}^{x}\left(A_0 + A_1\,x\right) + \mathrm{e}^{-3\,x}\Big[B_0\,\cos(x) + C_0\,\sin(x)\Big]$

$A_0, A_1, B_0, B_1, C_0, C_1$ are real constants.

11. If $\mu(x)$ is an integrating factor of the differential equation

$$\left(2\,x\,y^3 - 2\,x^3\,y^3 - 4\,x\,y^2 + 2\,x\right)\,\mathrm{d}x + \left(3\,x^2\,y^2 + 4\,y\right)\,\mathrm{d}y = 0,$$

then

A. e^{-x^2} **B.** $\mathrm{e}^{-2\,x}$ **C.** $\mathrm{e}^{2\,x}$ **D.** e^{x} **E.** e^{-x}

12. The initial value problem $\begin{cases} y' - \mathrm{e}^{2\,t}\,\mathrm{e}^{-y} = 0 \\ y(0) = 0 \end{cases}$ has solution

A. $y = \mathrm{e}^{2\,t} - 1$ **B.** $y = \ln\left(2\,\mathrm{e}^{2\,t} - 1\right)$

C. $y = \ln\left(\mathrm{e}^{2\,t} + 2\right) - \ln(3)$ **D.** $y = \ln\left(\mathrm{e}^{2\,t} + 1\right) - \ln(2)$

E. $y = \ln\left(\mathrm{e}^{2\,t} + 3\right) - \ln(4)$

13. A radioactive element decays into non-radioactive substances. If in 12 days the radioactive sample of 80 grams, decreases to 10 grams, then the half-life of the element is

A. 2 days **B.** 3 days **C.** 4 days **D.** 5 days **E.** 6 days

14. A tank with infinite capacity, initially contains 100 litres of a salt solution with concentration of 0.5 kg per litre. A solution with a salt concentration of 1.5 kg/litre is added to the tank at 4 litres per minute, and the well-stired mixture is drained out at 4 litres per minute. The differential equation for the quantity $Q(t)$ of salt in the tank at time t, is given by

A. $\begin{cases} Q' + \frac{1}{25}\,Q = 6 \\ Q(0) = 50 \end{cases}$ **B.** $\begin{cases} Q' + \frac{1}{25}\,Q = \frac{3}{2} \\ Q(0) = 50 \end{cases}$

C. $\begin{cases} Q' + \frac{1}{25}\,Q = 6 \\ Q(0) = 100 \end{cases}$ **D.** $\begin{cases} Q' + \frac{1}{100}\,Q = \frac{3}{2} \\ Q(0) = 50 \end{cases}$

E. $\begin{cases} Q' + \frac{1}{25}\,Q = \frac{1}{2} \\ Q(0) = 100 \end{cases}$

15. The unique solution of the initial value problem

$$\begin{cases} y'' + \dfrac{t}{(t+2)\,(t+4)}\,y' + \dfrac{t-3}{(t+2)\,(2\,t+5)}\,y = \dfrac{\sqrt{3-t}}{t+2} \\ \\ y(1) = 0 \quad \text{and} \quad y'(1) = 9 \end{cases}$$

is guaranteed to exist in the open interval

A. $\big(-4\,,\,3\big)$ **B.** $\big(-5/2\,,\,3\big)$ **C.** $\big(-2\,,\,3\big)$

D. $\big(-\infty\,,\,3\big)$ **E.** $\big(-4\,,\,+\infty\big)$

16. The wronskian of the pair of functions $\Big\{\, y_1 = \mathrm{e}^x\,,\ y_2 = \mathrm{e}^x\,\cos(x)\,\Big\}$, is given by

A. $\mathrm{e}^{2\,x}\,\big(\cos(x) + \sin(x)\big)$ **B.** $-\mathrm{e}^{2\,x}\,\sin(x)$ **C.** $-\mathrm{e}^{2\,x}\,\cos(x)$

D. $\mathrm{e}^{x}\,\big(\cos(x) + \sin(x)\big)$ **E.** $\mathrm{e}^{x}\,\big(\cos(x) - \sin(x)\big)$

17. The solution of the initial value problem $\begin{cases} y'' + 4\,y' + 3\,y = 0 \\ \\ y(0) = 2 \ \text{ and } \ y'(0) = -1 \end{cases}$ is

A. $y = \frac{5}{2}\,\mathrm{e}^{t} - \frac{1}{2}\,\mathrm{e}^{-3\,t}$ **B.** $y = \frac{5}{2}\,\mathrm{e}^{-t} - \frac{1}{2}\,\mathrm{e}^{-3\,t}$

C. $y = \frac{5}{2}\,\mathrm{e}^{t} - \frac{1}{2}\,\mathrm{e}^{3\,t}$ **D.** $y = 3\,\mathrm{e}^{-t} - \mathrm{e}^{-3\,t}$

E. $y = 3\,\mathrm{e}^{t} - \mathrm{e}^{3\,t}$

18. The solution of the initial value problem $\begin{cases} y'' + 4\,y = 0 \\ y(0) = 0 \ \text{ and } \ y'(0) = 1 \end{cases}$ is

A. $y = \dfrac{1}{2}\,\mathrm{e}^t\,\sin(2\,t)$

B. $y = \mathrm{e}^t\,\big(\sin(2\,t) + \cos(2\,t)\big)$

C. $y = \dfrac{1}{2}\,\mathrm{e}^t\,\sin(t)$

D. $y = \dfrac{1}{2}\,\sin(2\,t)$

E. $y = \mathrm{e}^t\,\sin(t)$

19. Suppose that $y_1 = t$ is a solution of $t^2\,y'' - t\,(t+2)\,y' + (t+2)\,y = 0, \quad t > 0.$ Use the method of reduction of order to find a second solution y_2 of the differential equation.

A. $y_2 = \mathrm{e}^t$ **B.** $y_2 = \mathrm{e}^{2\,t}$ **C.** $y_2 = t^2\,\mathrm{e}^t$ **D.** $y_2 = t\,\mathrm{e}^t$

E. None of the above

20. If a linear homogeneous differential equation with constant coefficients has characteristic equation $(\lambda - 5)\,\big(\lambda^2 + 4\,\lambda + 4\big)\,\big(\lambda^2 - 2\,\lambda + 10\big) = 0$, then its general solution is given by

A. $y = C_1\,\mathrm{e}^{5\,t} + C_2\,\mathrm{e}^{-2\,t} + C_3\,\mathrm{e}^t\,\cos(3\,t)$

B. $y = C_1\,\mathrm{e}^{5\,t} + C_2\,\mathrm{e}^{-2\,t} + C_3\,t\,\mathrm{e}^{-2\,t} + C_4\,\mathrm{e}^t\,\cos(3\,t) + C_5\,\mathrm{e}^t\,\sin(3\,t)$

C. $y = C_1\,\mathrm{e}^{5\,t} + C_2\,\mathrm{e}^{2\,t} + C_3\,t\,\mathrm{e}^{2\,t} + C_4\,\mathrm{e}^t\,\cos(3\,t) + C_5\,\mathrm{e}^t\,\sin(3\,t)$

D. $y = C_1\,\mathrm{e}^{5\,t} + C_2\,\mathrm{e}^{-2\,t} + C_3\,t\,\mathrm{e}^{-2\,t} + C_4\,\mathrm{e}^{-t}\,\cos(3\,t) + C_5\,\mathrm{e}^{-t}\,\sin(3\,t)$

E. $y = C_1\,\mathrm{e}^{5\,t} + C_2\,\mathrm{e}^{2\,t} + C_3\,t\,\mathrm{e}^{2\,t} + C_4\,\mathrm{e}^{-t}\,\cos(3\,t) + C_5\,\mathrm{e}^{-t}\,\sin(3\,t)$

C_1, C_2, C_3, C_4, and C_5 are arbitrary real constants.

21. If the equation $y''' + a\,y'' + b\,y' + c\,y = 0$, where a, b, and c are real constants, has $6\,t^2\,\mathrm{e}^{-t}$ as one of its solutions, then

A. $a = 3, \ \ b = 2, \ \ c = 1$

B. $a = 2, \ \ b = 3, \ \ c = 1$

C. $a = 3, \ \ b = 3, \ \ c = 1$

D. $a = 2, \ \ b = 1, \ \ c = 1$

E. $a = 3, \ \ b = 1, \ \ c = 3$

22. If $t^2\,\mathrm{e}^{2t}\,\cos(t) + \mathrm{e}^{2t}$ is a solution of an n^{th} order, homogeneous, linear differential equation with constant coefficients, then the minimal value of n is

A. 4 **B.** 5 **C.** 6 **D.** 7 **E.** 8