

**Assignment02 is due on Tuesday, November 01, 2016 at 11:59pm.**

The number of attempts available for each question is noted beside the question. If you are having trouble figuring out your error, you should consult the textbook, or ask a fellow student, one of the TA's or your professor for help.

There are also other resources at your disposal, such as the Mathematics Continuous Tutorials. Don't spend a lot of time guessing – it's not very efficient or effective.

Make sure to give lots of significant digits for (floating point) numerical answers. For most problems when entering numerical answers, you can if you wish enter elementary expressions such as  $2 \wedge 3$  instead of 8,  $\sin(3 * \pi / 2)$  instead of -1,  $e \wedge (\ln(2))$  instead of 2,  $(2 + \tan(3)) * (4 - \sin(5)) \wedge 6 - 7 / 8$  instead of 27620.3413, etc.

**1. (1 point)**

The initial value problem

$$(t^2 - 36)y'' - 5ty' + 4y = \ln(t^2)$$

$$y(-5) = 5$$

$$y'(-5) = 8$$

has a unique solution defined on the interval \_\_\_\_\_

Type **-inf** for  $-\infty$  and **inf** for  $+\infty$

Answer(s) submitted:

- (-6, 0)

(correct)

Correct Answers:

- (-6, 0)

**2. (1 point)**

You can verify that  $y_1 = x^{-1}$ ,  $y_2 = x^{-8}$  and  $y_3 = 3$  are all solutions of the differential equation

$$x^2 y''' + 12x y'' + 18 y' = 0$$

in  $(0, +\infty)$ . Compute the wronskian of the solutions  $y_1, y_2, y_3$

$$W(x) = \text{_____} \quad x \in (0, +\infty)$$

☐ 1. Is  $\{y_1, y_2, y_3\}$  a fundamental set for  $x^2 y''' + 12x y'' + 18 y' = 0$  in  $(0, +\infty)$ ?

Answer(s) submitted:

- -168/x^(12)
- Yes

(correct)

Correct Answers:

- $3 * [8 * x^{(-9)} * 2 * x^{(-3)} - x^{(-2)} * 72 * x^{(-10)}]$
- YES

**3. (1 point)**

Consider the initial value problem

$$(*) \begin{cases} t y'' + (2t + 5) y' + (t + 5) y = 0 \\ y(1) = \frac{6}{e} \quad \text{and} \quad y'(1) = \frac{7}{e} \end{cases}$$

Given that  $y_1(t) = e^{-t}$  is a solution of the differential equation, solve the initial value problem (\*).

$$y(t) = \text{_____}$$

Your answer should be a function of  $t$ .

Answer(s) submitted:

- $e^{(-t)} (37t^4 - 13) / (4t^4)$

(correct)

Correct Answers:

- $(-3.25 * t^{(-4)} + 9.25) * e^{(-t)}$

**4. (1 point) Find the solution of the differential equation**

$$y'' - 11y' + 30y = 0$$

satisfying the initial conditions  $y(0) = 3$ ,  $y'(0) = 20$ .

$$\text{Answer: } y(t) = \text{_____}$$

Your answer should be a function of  $t$ .

Answer(s) submitted:

- $e^{(5t)} (5e^{t-2})$

(correct)

Correct Answers:

- $-2 * e^{(5 * t)} + 5 * e^{(6 * t)}$

**5. (1 point) Find the solution of the differential equation**

$$y'' + 4y' + 4y = 0$$

satisfying the initial conditions  $y(0) = -3$ ,  $y'(0) = 13$ .

$$\text{Answer: } y(t) = \text{_____}$$

Your answer should be a function of  $t$

Answer(s) submitted:

- $e^{(-2t)} (7t - 3)$

(correct)

Correct Answers:

- $-3 * e^{(-2 * t)} + 7 * t * e^{(-2 * t)}$

6. (1 point) Find the solution of the constant coefficient differential equation

$$y'' + 6y' + 18y = 0$$

satisfying the initial conditions  $y(0) = -7$ ,  $y'(0) = 12$ .

Answer:  $y(t) =$  \_\_\_\_\_

Your answer should be a function of  $t$ .

Answer(s) submitted:

- $-e^{(-3t)}(3\sin(3t) + 7\cos(3t))$

(correct)

Correct Answers:

- $e^{(-3t)} * (-7\cos(3t) + -3\sin(3t))$

7. (1 point) Find  $y$  as a function of  $x$  if

$$y''' - 8y'' - y' + 8y = 0,$$

$y(0) = -1$ ,  $y'(0) = 3$ ,  $y''(0) = -1$ .

$y(x) =$  \_\_\_\_\_

Answer(s) submitted:

- $e^x - 2e^{-x}$

(correct)

Correct Answers:

- $1 * e^x + -2 * e^{-x} + 0 * e^{(8x)}$

8. (1 point) The differential equation

$$y'''' + ay''' + by'' + cy' + dy = 0$$

has a solution  $y = -2te^{-3t} + 6e^{-3t}\sin(2t)$  for real numbers  $a, b, c$  and  $d$ .

Then:

$a =$  \_\_\_\_\_,

$b =$  \_\_\_\_\_,

$c =$  \_\_\_\_\_,

$d =$  \_\_\_\_\_.

Note: it is possible that the correct answers may be 3- or 4-digit numbers

Answer(s) submitted:

- 12
- 58
- 132
- 117

(correct)

Correct Answers:

- 12
- 58
- 132
- 117

9. (1 point) Find a particular solution to the differential equation

$$y'' + 5y' + 4y = -250\sin(3t)$$

Answer:  $y(t) =$  \_\_\_\_\_

Your answer should be a function of  $t$ .

Answer(s) submitted:

- $5\sin(3t) + 15\cos(3t)$

(correct)

Correct Answers:

- $c * e^{(-1t)} + d * e^{(-4t)} + 15 * \cos(3t) + 5 * \sin(3t)$

10. (1 point)

Use the method of undetermined coefficients to find a particular solution  $y_p(t)$  of

$$y'' + 2y' + 2y = (10t + 7)e^{-t}\cos(t) + (11t + 25)e^{-t}\sin(t).$$

$y =$  \_\_\_\_\_

Answer(s) submitted:

- $5/2 e^{(-t)} t^2 \sin(t) - 11/4 e^{(-t)} t^2 \cos(t) + 25/4 e^{(-t)} t$

(correct)

Correct Answers:

- $(-11/4) * t * \exp(-t) * \cos(t) + (-10) * t * \exp(-t) * \cos(t)$

11. (1 point) Consider the initial value problem

$$y'' + 4y = e^{-t}, \quad y(0) = y_0, \quad y'(0) = y'_0.$$

Suppose we know that  $y(t) \rightarrow 0$  as  $t \rightarrow \infty$ . Determine the solution and the initial conditions.

$y(t) =$  \_\_\_\_\_ help (formulas)

$y(0) =$  \_\_\_\_\_ help (numbers)

$y'(0) =$  \_\_\_\_\_ help (numbers)

Answer(s) submitted:

- $e^{(-t)} / 5$
- $1/5$
- $-1/5$

(correct)

Correct Answers:

- $e^{(-t)} / 5$
- $1/5$
- $-1/5$

12. (1 point) Consider the equation  $y'' - 4y = 2\cos(2t) + 2 + e^{2t}$ . According to the method of **Undetermined Coefficients**, the particular solution has the form:

- A.  $C_1 \cos(2t) + C_2 2 + C_3 e^{2t}$
- B.  $C_1 t \cos(2t) + C_2 t \sin(2t) + C_3 t + C_4 t e^{2t} + C_5 t e^{-2t}$
- C.  $C_1 \cos(2t) + C_2 \sin(2t) + C_3 + C_4 e^{2t}$
- D.  $C_1 \cos(2t) + C_2 \sin(2t) + C_3 + C_4 t e^{2t}$
- E. none of the above

Answer(s) submitted:

- D

(correct)

Correct Answers:

- D

13. (1 point) Consider the equation  $y'' + 9y = 3\sin(3x) + 3 + e^{3x}$ . According to the method of **Undetermined Coefficients**, the particular solution has the form:

- A.  $C_1 \cos(3x) + C_2 \sin(3x) + C_3 + C_4 e^{3x}$
- B.  $C_1 x \cos(3x) + C_2 x \sin(3x) + C_3 + C_4 e^{3x}$
- C.  $C_1 x \sin(3x) + C_3 x + C_4 x e^{3x} + C_5 x e^{-3x}$
- D.  $C_1 \sin(3x) + C_2 3 + C_3 e^{3x}$
- E. none of the above

Answer(s) submitted:

- B

(correct)

Correct Answers:

- B

14. (1 point) Consider the equation  $y'' + 2y' - 3y = 9x^2 - 4e^{1x} + 2xe^{1x} + 9e^{-2x}$ . According to the method of **Undetermined Coefficients**, the particular solution has the form:

- A.  $C_1 + C_2 x + C_3 x^2 + C_4 x e^{1x} + C_5 x^2 e^{1x} + C_6 e^{-2x}$
- B.  $C_1 + C_2 x + C_3 x^2 + C_4 x e^{1x} + C_5 e^{-2x}$
- C.  $C_1 x^2 + C_2 x e^{1x} + C_3 e^{-2x}$
- D.  $C_1 x + C_2 x^2 + C_3 x^3 + C_4 x^2 e^{1x} + C_5 x e^{-2x}$
- E. none of the above

Answer(s) submitted:

- A

(correct)

Correct Answers:

- A

15. (1 point) Find a particular solution to the differential equation

$$y'' + 16y = \sec^2(4t)$$

y= \_\_\_\_\_

Your answer should be a function of  $t$ .

Answer(s) submitted:

- $-1/16 \sin(4t) \ln(\cos(2t) - \sin(2t)) + 1/16 \sin(4t) \ln(\sin(2t))$

(correct)

Correct Answers:

- $1/16 * (\ln(\sec(4 * t) + \tan(4 * t))) * \sin(4 * t) - 1 + C * \sin(4 * t)$