MATH 375
Handout # 9
The Laplace Transform

1. Find the Laplace Transform of the following functions

   a) $f(t) = 2\sin t - \cos t$ b) $f(t) = t^3 e^{-t}$ c) $f(t) = \sin(mt)\cos(nt)$

   d) $f(t) = (t+1)\sin 2t$ e) $f(t) = \int_0^t \cos^2(\omega u)\, du$ f) $f(t) = e^{3t}\sin^2 t$

   g) $f(t) = \sin(t-b)u_b(t)$

2. Find the Laplace Transform of the following functions:

   a) $f(t) = \begin{cases} e^{5t}, & 0 \le t < 3, \\ e^{5t} + 5t^2 - 2t + 7, & t \ge 3. \end{cases}$ b) $f(t) = \begin{cases} t-3, & 0 \le t < 1, \\ 2t^2 + t - 4, & 1 \le t < 2, \\ t^2 + 2t - 4, & t \ge 2. \end{cases}$

3. Find the inverse Laplace Transform of the following functions

   a) $F(s) = \dfrac{1}{s^2 + 4s + 5}$ b) $F(s) = \dfrac{s+2}{(s+1)(s-2)(s^2+4)}$ c) $F(s) = \dfrac{s}{(s^2+1)^2}$

   d) $F(s) = \dfrac{e^{-s}}{s(s-1)}$ e) $F(s) = \dfrac{1}{s^2+1}\left(e^{-2s} + 2e^{-3s} + 3e^{-4s}\right)$ f) $F(s) = \dfrac{e^{-s/3}}{s(s^2+1)}$

4. Using the Laplace Transform solve the initial value problems

   a) $x'' + 3x' = e^t,\ x(0) = 0,\ x'(0) = -1$

   b) $x'' + 2x' - 3x = e^{-t},\ x(0) = 0,\ x'(0) = 1$

   c) $x'' + 2x' + x = \sin t,\ x(0) = 0,\ x'(0) = -1$

   d) $x''' + 2x'' + 5x' = 0,\ x(0) = -1,\ x'(0) = 2,\ x''(0) = 0$

   e) $x'' - 2x' + x = t - \sin t,\ x(0) = 0,\ x'(0) = 0$

   f) $x''' + x'' = \cos t,\ x(0) = -2,\ x'(0) = 0,\ x''(0) = 0$

   g) $x''' + x' = e^{2t},\ x(0) = 0,\ x'(0) = 0,\ x''(0) = 0$

   h) $x'' + 4x = f(t)$, $x(0) = 0$, $x'(0) = -1$, $f(t) = \begin{cases} 4t, & 0 \le t < 2 \\ 5t - 2, & t \ge 2 \end{cases}$