THE UNIVERSITY OF CALGARY
DEPARTMENT OF MATHEMATICS AND STATISTICS
MIDTERM EXAMINATION

MATH 375 Lec 01-04 FALL 2015

Monday, November 02, 2015 90 minutes

Last Name	First Name

Student I.D. Number	Tutorial Number

This examination consists of 20 equally weighted questions. Please attempt all problems and record your answer by circling your choice in the exam booklet, and filling in the appropriate circle in the scantron sheet.

This is a closed book examination and calculators are not permitted.

Please record your answers in the table below.

01	02	03	04	05	06	07	08	09	10
D	**B**	**C**	**E**	**C**	**B**	**A**	**D**	**B**	**A**
11	12	13	14	15	16	17	18	19	20
C	**E**	**A**	**E**	**C**	**D**	**E**	**B**	**A**	**D**

01 The equation $y'' + k\,y'\left(y^2 - 1\right) + 3\,y''' = -2\,\cos(t)\,y^2$ is

A. second order, linear

B. second order, nonlinear

C. third order, linear

D. third order, nonlinear

E. nonlinear, and the order depends on k

02. The general solution of the equation $t\,y\,y' = 1 - t^2$ for $t > 0$, can be written as

A. $y = 2\,\ln(t) - t^2 + C$

B. $y = \pm\sqrt{C + 2\,\ln(t) - t^2}$

C. $y = \pm\sqrt{2\,\ln(t) - t^2} + C$

D. $y = \sqrt{\ln(t^2)} + t + C$

E. $y = \ln(t) - \dfrac{t^2}{2} + C$

03. The solution of the initial value problem $x' + \dfrac{5}{t}\,x = \dfrac{3}{t^3}$, $x(1) = 4$ is

A. $x = 3\,\mathrm{e}^{-5\,t} + 4 - 3\,\mathrm{e}^{-5}$

B. $x = 3\,(t-1)\mathrm{e}^{-5\,t} + 1$

C. $x = \dfrac{1}{t^2} + \dfrac{3}{t^5}$

D. $x = \dfrac{2}{t} + \dfrac{2}{t^2}$

E. $x = t^3 + 3$

04. The general solution of the equation $\dfrac{dy}{dt} + y = t\,y^2$ is

A. $y = C\,\mathrm{e}^{t} + t + 1$

B. $y = C\,\mathrm{e}^{-t} + \dfrac{t^2}{2} + \mathrm{e}^{t}$

C. $y = \left(C\,\mathrm{e}^{-2\,t} + \dfrac{t^2}{2}\right)^{-1}$

D. $y = C\,\mathrm{e}^{-t} + \dfrac{1}{t} + 1$

E. $y = \left(C\,\mathrm{e}^{t} + t + 1\right)^{-1}$

05. The equation $\mathrm{e}^{\alpha x+\beta y}\left(3+\dfrac{dy}{dx}\right)=0,$ is exact

A. for any $\alpha, \beta \in \mathbb{R}$

B. for $\alpha = 6$, $\beta = 2$ only

C. for any $\alpha = 3\beta \in \mathbb{R}$

D. for $\alpha = 3$, $\beta = 1$ only

E. never: there are no $\alpha \in \mathbb{R}$, and $\beta \in \mathbb{R}$, such that the equation is exact

06. A tank, containing 1 litre of liquid, has a brine solution entering at a constant rate of 1 litres per minute. The well-stirred solution leaves the tank at the same rate. The concentration within the tank is monitored and found to be $c(t) = \mathrm{e}^{-t/10}$ kg/L (kilograms per litre). Then the inflow concentration is

A. $c_{in}(t) = \mathrm{e}^{-t/10}$ kg/L

B. $c_{in}(t) = \dfrac{9}{10}\mathrm{e}^{-t/10}$ kg/L

C. $c_{in}(t) = \dfrac{1}{10}$ kg/L

D. $c_{in}(t) = 0$ kg/L

E. $c_{in}(t)$ cannot be found: not enough data

07. The solution of the initial value problem $x\,y' + 6\,y = -7\,x\,\cos(x^7)$, $y\left(\sqrt[7]{\dfrac{\pi}{2}}\right) = 0$ is

A. $y = \dfrac{1-\sin(x^7)}{x^6}$

B. $y = \dfrac{\sin(x^7)}{x^6} - 1$

C. $y = \cos(x^7)$

D. $y = \dfrac{\cos(x^7)}{x^6}$

E. $y = \dfrac{(x-\pi/2)\sin(x^7)}{x^6}$

08. According to the existence and uniqueness theorem, the largest open interval (a,b) on which the unique solution of the initial value problem $(t-4)\,y' + t\,y = \ln\left(t - \frac{1}{t}\right)$, $y(2) = 5$, is guaranteed to exist is

A. $(-\infty,\infty)$ **B.** $(-1,\infty)$ **C.** $(1,\infty)$ **D.** $(1,4)$ **E.** $(1,3)$

09. **Please ignore: this topic is no longer part of MATH 375**
We solve the initial value problem $y' = x^3 + \ln(y+1) + x\,\sin(y)$, $y(1) = 0$ using Euler's method with step size $h = 0.1$. Then

A. $y(1.1) \approx y_1 = 1$ **B.** $y(1.1) \approx y_1 = 0.1$ **C.** $y(1.1) \approx y_1 = 0$

D. $y(1.1) \approx y_1 = 0.2$ **E.** $y(1.1) \approx y_1 = 2$

10. **Please ignore: this topic is not part of our midterm**
The general solution of the equation $x^2\,y'' + 11\,x\,y' + 25\,y = 0$, $x > 0$, is

A. $y = x^{-5}\,(C_1 + C_2\,\ln(x))$

B. $y = C_1\,e^{(-5.5+\sqrt{21}/2)\,x} + C_2\,e^{(-5.5+\sqrt{21}/2)\,x}$

C. $y = C_1\,x^{-5} + C_2\,x^{-5}$

D. $y = C_1\,e^{-5\,x} + C_2\,x\,e^{-5\,x}$

E. $y = C_1\,x^{-5} + C_2\,x^{-6}$

11. A particular solution of the equation $y'' + y = 4\sin(t)$ is

A. $y = 4\,\sin(t)$

B. $y = -t\,\cos(t) + t\,\sin(t)$

C. $y = -2\,t\,\cos(t)$

D. $y = 2\,t\,\sin(t)$

E. $y = 4\,\cos(t) + 4\,\sin(t)$

12. A freshly poured coffee has a temperature of 92 degrees Celsius, and it is brought to the room where the temperature is kept at 22 degrees. Then, for a constant k, the coffee temperature is described by the initial value problem

A. $T'(t) = k\,(92 - T(t)), \qquad T(0) = 22$

B. $T'(t) = -k\,T(t), \qquad T(0) = 92$

C. $T'(t) = k\,(22 + T(t)), \qquad T(0) = 92$

D. $T'(t) = k\,(T(t) - 22), \qquad T(0) = 22$

E. $T'(t) = k\,(22 - T(t)), \qquad T(0) = 92$

13. The general solution of the equation $\cos(x+y) + 3\,x^2 + \cos(x+y)\,\dfrac{dy}{dx} = 0$ is

A. $\sin(x+y) + x^3 = C$

B. $\cos(x+y) = C$

C. $\sin(x+y) = C$

D. $y = \sin(x+y) + x^3 + C$

E. $\cos(x+y) + x^3 = C$

14. A fundamental solution set of the equation $y''' - 3\,y'' + 3\,y' - y = 0$, is

A. $\{e^t, e^{-t}, e^{3t}, e^{-3t}\}$

B. $\{e^t, e^{-t}, e^{3t}\}$

C. $\{e^t, t\,e^t, e^{-t}\}$

D. $\{e^t, e^{-t}, t\}$

E. $\{e^t, t\,e^t, t^2\,e^t\}$

15. The Wronskian for any two solutions y_1 and y_2 of the equation

$$y'' - \frac{1}{1+t}\, y'(t) + q(t)\, y(t) = 0$$

can be equal to

A. $\dfrac{1}{1+t}$ **B.** e^{1+t} **C.** $(1+t)$ **D.** $\mathrm{e}^{t+t^2/2}$

E. none of the above

16. The **minimum order** m of the linear homogeneous differential equation with constant coefficients which can have a function $y(t) = t^2\, \mathrm{e}^{5\,t} \cos(2\,t)$ as a solution is

A. $m = 3$ **B.** $m = 4$ **C.** $m = 5$ **D.** $m = 6$ **E.** $m = 7$

17. According to **the method of undetermined coefficients**, a particular solution of the equation $y'' + 9\,y = 4\,\cos(3\,x) + \mathrm{e}^{3\,x}$ has the form:

A. $C_1\, x\, \mathrm{e}^{3\,x} + C_2\, \cos(3\,x)$

B. $C_1\, x\, \mathrm{e}^{3\,x} + C_2\, \cos(3\,x) + C_3\, \sin(3\,x)$

C. $C_1\, \mathrm{e}^{3\,x} + C_2\, \sin(3\,x)$

D. $C_1\, x\, \mathrm{e}^{3\,x} + C_2\, x\, \cos(3\,x) + C_3\, x\, \sin(3\,x)$

E. $C_1\, \mathrm{e}^{3\,x} + C_2\, x\, \cos(3\,x) + C_3\, x\, \sin(3\,x)$

18. If y_1, and y_2 are solutions of the second order linear equation $y'' + p(t)\,y' + q(t)\,y = t\,\ln(t)$, $t > 0$, then the function $y(t) = a\,y_1(t) + b\,y_2(t)$ also satisfies this equation

A. for any $a, b \in \mathbb{R}$

B. for $a, b \in \mathbb{R}$ satisfying $a + b = 1$ only

C. for $a, b \in \mathbb{R}$ satisfying $a - b = 1$ only

D. for any $a = b \in \mathbb{R}$ only

E. never

19. If $x\,e^{2x} + 3\,\cos(x)$ is a solution of the fourth-order differential equation

$$y^{(4)} + a\,y''' + b\,y'' + c\,y' + d\,y = 0$$

then the equation is

A. $y^{(4)} - 4\,y''' + 5\,y'' - 4\,y' + 4\,y = 0$

B. $y^{(4)} - 2\,y''' + y'' - 2\,y' = 0$

C. $y^{(4)} + 5\,y'' + 4\,y = 0$

D. $y^{(4)} - 5\,y''' + 7\,y'' - 5\,y' + 6\,y = 0$

E. cannot be found: not enough data

20. **Please ignore: the topice of this problem is not part of our midterm**

If the linear equation $y'' + p(t)\,y' + q(t)\,y = 0$, $t > 0$, has solutions $y_1(t) = \dfrac{1}{t}$ and $y_2 = \dfrac{1}{t^2}$, then a particular solution of the non-homogeneous equation $y'' + p(t)\,y' + q(t)\,y = \dfrac{1}{t}$, is given by

A. $\dfrac{1}{4\,t^3}$ **B.** $\dfrac{t^2}{2} - \dfrac{t^3}{3}$ **C.** $\dfrac{1}{6\,t^2}$ **D.** $\dfrac{t}{6}$

E. cannot be found without knowing $p(t)$ and $q(t)$

End of examination