01. About the differential equation

$$y''y' + y'e^y + 5y = t,$$

which of the following statement is correct?

(A) Linear, second order.

(B) Nonlinear, third order.

(C) Linear, third order.

(D) Nonlinear, second order.

(E) None of the above.

02. If $y = y(t)$ is the solution of the initial value problem

$$\begin{cases} y' + (2\,t + 1)\,y = 2\,\cos(t) \\ y(0) = 2 \end{cases}$$

then

(A) $y''(0) = 2$ (B) $y''(0) = -2$

(C) $y''(0) = 4$ (D) $y''(0) = 0$

(E) $y''(0) = -4$

03. (**True** or **False**) If $\cos(2x)$ is a solution of $ay'' + by' + cy = 0$, where $a \neq 0$, b and c are real constants, then $100\sin(2x)$ is also a solution of this differential equation.

(A) True

(B) False

04. The unique solution of the initial value problem

$$ty' + 2y = 4t^2, \quad y(1) = 2$$

is

(A) $t^4 + t^{-4}$

(B) $t^3 + t^{-3}$

(C) $t^2 + t^{-2}$

(D) $t + t^{-1}$

(E) $\sqrt{t} + 1/\sqrt{t}$

05. Let $y_1(t)$ and $y_2(t)$ be two solutions of the differential equation

$$y'' + e^t y' + y = t^3.$$

Which of the following is a solution of the differential equation?

(A) $y_1(t) - y_2(t)$

(B) $y_1(t) + y_2(t)$

(C) $y_1(t) - 5\,y_2(t)$

(D) $2y_1(t) - y_2(t)$

(E) $2y_1(t) + 3y_2(t)$

06. Consider the initial-value problem:

$$y' + \frac{\ln(x)}{x^2 - 1} y = \frac{1}{x - 3}, \quad y(2) = 1,$$

The largest interval on which a unique solution exists is

(A) $(0, \quad +\infty)$

(B) $(-1, \quad 1)$

(C) $(-1, \quad 3)$

(D) $(1, \quad +\infty)$

(E) $(1, \quad 3)$

07. The solution of the exact differential equation:

$$(ye^{xy}\cos(2x) - 2e^{xy}\sin(2x) + 2x) + (xe^{xy}\cos(2x) - 3)y' = 0$$

is given by

(A) $e^{xy}\sin(2x) + x^2 - 3y = C$

(B) $\frac{1}{2}e^{xy}\sin(2x) + x^2 - 3xy = C$

(C) $e^{xy}\cos(2x) + x^2 - 3y = C$

(D) $\frac{1}{2}e^{xy}\cos(2x) + x^2 - 3xy = C$

(E) None of the above

08. Let y_p be a particular solution of the differential equation:

$$y'' + y' - 2y = xe^x + e^{-3x}\cos(x),$$

then y_p is in the following form, where $A_0, A_1, B_0, B_1, C_0, C_1$ are real constants.

(A) $e^x(A_0 + A_1x) + e^{-3x}[(B_0x + B_1x^2)\cos(x) + (C_0x + C_1x^2)\sin(x)]$

(B) $e^x(A_0x + A_1x^2) + e^{-3x}[B_0\cos(x) + C_0\sin(x)]$

(C) $e^x(A_0x + A_1x^2) + e^{-3x}[B_0\cos(x)]$

(D) $e^x(A_0 + A_1x) + e^{-3x}[(B_0 + B_1x)\cos(x) + (C_0 + C_1x)\sin(x)]$

(E) $e^x(A_0 + A_1x) + e^{-3x}[B_0\cos(x) + C_0\sin(x)]$

09. If $\mu(x)$ is an integrating factor of the differential equation

$$(2xy^3 - 2x^3y^3 - 4xy^2 + 2x)dx + (3x^2y^2 + 4y)dy = 0,$$

then $\mu(x)$ is

(A) e^{-x^2}

(B) e^{-2x}

(C) e^{2x}

(D) e^{x}

(E) None of the above

10. The initial value problem $\begin{cases} y' - e^{2t}e^{-y} = 0 \\ y(0) = 0 \end{cases}$ has solution

(A) $y = e^{2t} - 1$

(B) $y = \ln(2e^{2t} - 1)$

(C) $y = \ln(e^{2t} + 2) - \ln(3)$

(D) $y = \ln(e^{2t} + 1) - \ln(2)$

(E) None of the above

11. A tank with infinite capacity initially contains 100 liters of a salt solution with a concentration of 0.5 kg/liter. A solution with a salt concentration of 1.5 kg/liter is added to the tank at 4 liters/minute, and the well-stirred mixture is drained out at 4 liters/minute. The differential equation for the quantity $Q(t)$ of salt in the tank at time t is given by

(A) $\begin{cases} Q' + \dfrac{1}{25}Q = 6, \\ Q(0) = 50. \end{cases}$

(B) $\begin{cases} Q' + \dfrac{1}{25}Q = 1.5, \\ Q(0) = 50. \end{cases}$

(C) $\begin{cases} Q' + \dfrac{1}{25}Q = 6, \\ Q(0) = 100. \end{cases}$

(D) $\begin{cases} Q' + \dfrac{1}{100}Q = 1.5, \\ Q(0) = 50. \end{cases}$

(E) $\begin{cases} Q' + \dfrac{1}{25}Q = 0.5, \\ Q(0) = 100. \end{cases}$

12. A radioactive element decays into non-radioactive substances. In 12 days the radioactive sample of 80g decreases to 10g, the half-life of the element is

(A) 2 days (B) 3 days (C) 4 days

(D) 5 days (E) 6 days

13. Consider the initial value problem

$$y' = y + t, \quad y(0) = 1.$$

Using Euler's method with step size $h = 0.1$, an approximate value y_2 of $y(0.2)$ is

(A) 1.11

(B) 1.22

(C) 1.32

(D) 1.44

(E) 1.68

14. The unique solution of the initial value problem

$$\begin{cases} y'' + \dfrac{t}{(t+2)(t+4)}\, y' + \dfrac{t-3}{(t+2)(2\,t+5)}\, y = \dfrac{\sqrt{3-t}}{t+2}, \\ y(1) = 0 \quad \text{and} \quad y'(1) = 9. \end{cases}$$

is guaranteed to exist in the open interval

(A) $\big(-4\,,\,3\big)$

(B) $\big(-5/2\,,\,3\big)$

(C) $\big(-2\,,\,3\big)$

(D) $\big(-\infty\,,\,3\big)$

(E) $\big(-4\,,\,+\infty\big)$

15. The Wronskian of the pair of functions

$$y_1(x) = e^x, \quad y_2(x) = e^x \cos(x)$$

is given by

(A) $e^{2x}(\cos(x) + \sin(x))$

(B) $-e^{2x}\sin(x)$

(C) $-e^{2x}\cos(x)$

(D) $e^x(\cos(x) + \sin(x))$

(E) $e^x(\cos(x) - \sin(x))$

16. The solution of the initial value problem $\begin{cases} y'' + 4\,y' + 3\,y = 0 \\ y(0) = 2 \quad \text{and} \quad y'(0) = -1 \end{cases}$ is

(A) $\frac{5}{2}e^t - \frac{1}{2}e^{-3t}$

(B) $3e^t - e^{3t}$

(C) $\frac{5}{2}e^t - \frac{1}{2}e^{3t}$

(D) $3e^{-t} - e^{-3t}$

(E) $\frac{5}{2}e^{-t} - \frac{1}{2}e^{-3t}$

17. The solution of the initial value problem $\begin{cases} y'' + 4\,y = 0 \\ y(0) = 0 \quad \text{and} \quad y'(0) = 1 \end{cases}$ is

(A) $\frac{1}{2}e^t \sin(2t)$

(B) $e^t(\sin(2t) + \cos(2t))$

(C) $\frac{1}{2}e^t \sin(t)$

(D) $\frac{1}{2}\sin(2t)$

(E) $e^t \sin(t)$

18. If a linear homogeneous differential equation with constant coefficients has characteristic equation

$$(r-5)(r^2+4r+4)(r^2-2r+10) = 0,$$

then its general solution is

(A) $C_1\, \mathrm{e}^{5t} + C_2 e^{-2t} + C_3 e^t \cos(3t)$

(B) $C_1\, \mathrm{e}^{5t} + C_2 e^{-2t} + C_3 te^{-2t} + C_4 e^t \cos(3t) + C_5 e^t \sin(3t)$

(C) $C_1\, \mathrm{e}^{5t} + C_2 e^{2t} + C_3 te^{2t} + C_4 e^t \cos(3t) + C_5 e^t \sin(3t)$

(D) $C_1\, \mathrm{e}^{5t} + C_2 e^{-2t} + C_3 te^{-2t} + C_4 e^{-t} \cos(3t) + C_5 e^{-t} \sin(3t)$

(E) $C_1\, \mathrm{e}^{5t} + C_2 e^{2t} + C_3 te^{2t} + C_4 e^{-t} \cos(3t) + C_5 e^{-t} \sin(3t)$

$C_1,\ C_2,\ C_3,\ C_4,$ and C_5 are arbitrary real constants.

19. The equation $y''' + ay'' + by' + cy = 0$, where a, b and c are real constants, has $6t^2e^{-t}$ as one of its solutions. Then

(A) $a = 3, \quad b = 3, \quad c = 1.$

(B) $a = 2, \quad b = 3, \quad c = 1.$

(C) $a = 3, \quad b = 2, \quad c = 1.$

(D) $a = 2, \quad b = 1, \quad c = 1.$

(E) $a = 3, \quad b = 1, \quad c = 3.$

20. The general solution of the equation $y'' + 4\,y' + 4y = t^{-2}e^{-2t}, \quad t > 0$, is

(A) $y = C_1\,\mathrm{e}^{-2t} + C_2 e^{2t} - e^{-2t}\ln(t)$

(B) $y = C_1\,\mathrm{e}^{-2t} + C_2 te^{-2t} - e^{-2t}\ln(t)$

(C) $y = C_1\,\mathrm{e}^{-2t} + C_2 e^{2t} - \ln(t)$

(D) $y = C_1\,\mathrm{e}^{-2t} + C_2 te^{-2t} - t^2e^{-2t}$

(E) $y = C_1\,\mathrm{e}^{-2t} + C_2 te^{-2t} - \ln(t)$