David Ng **F2016MATH375 03**

Review is due : 09/25/2016 at 11:59pm MDT.

This assignment is for marks and is intended to review some topics from Math 211 and Amat 219 that will be used in this course (complex numbers, techniques of integration, eigenvalues/eigenvectors) If you are having trouble figuring out your errors, you should consult the textbook, or ask a fellow student, one of the TA's or your professor for help.
There are also other resources at your disposal, such as the Engineering Drop in Centre and the Mathematics Continuous Tutorials. Don't spend a lot of time guessing – it's not very efficient or effective.

Make sure to give lots of significant digits for (floating point) numerical answers. For most problems when entering numerical answers, you can if you wish enter elementary expressions such as $2 \wedge 3$ instead of 8, $sin(3*pi/2)$instead of -1, $e \wedge (ln(2))$ instead of 2, $(2+tan(3))*(4-sin(5)) \wedge 6-7/8$ instead of 27620.3413, etc.

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1. (1 point) For some practice working with complex numbers:

Calculate
$(2-2i)+(2+3i)=$ ________,
$(2-2i)-(2+3i)=$ ________,
$(2-2i)(2+3i)=$ ________.
The complex conjugate of $(1+i)$ is $(1-i)$. In general to obtain the complex conjugate reverse the sign of the imaginary part. (Geometrically this corresponds to finding the "mirror image" point in the complex plane by reflecting through the x-axis. The complex conjugate of a complex number z is written with a bar over it: $\bar{z}$ and read as "z bar".

Notice that if $z=a+ib$, then
$(z)(\bar{z})=|z|^2=a^2+b^2$
which is also the square of the distance of the point z from the origin. (Plot z as a point in the "complex" plane in order to see this.)

If $z=2-2i$ then $\bar{z}=$ ________ and $|z|=$ ________.

You can use this to simplify complex fractions. Multiply the numerator and denominator by the complex conjugate of the denominator to make the denominator real.
$\dfrac{2-2i}{2+3i}=$ _____ $+i$ _____.

Two convenient functions to know about pick out the real and imaginary parts of a complex number.

$Re(a+ib)=a$ (the real part (coordinate) of the complex number), and
$Im(a+ib)=b$ (the imaginary part (coordinate) of the complex number. Re and Im are linear functions – now that you know about linear behavior you may start noticing it often.

Answer(s) submitted:

- `4+i`
- `-5i`
- `10+2i`
- `2+2i`
- `sqrt(8)`
- `-2/13`
- `-10/13`

(correct)
Correct Answers:

- `4+i`
- `-5i`
- `10+2i`
- `2+2i`
- `2.82842712474619`
- `-0.153846153846154`
- `-0.769230769230769`

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2. (1 point) Solve the following equations for z, find all solutions :
(1) $2z^2 + z + 4 = 0$
Place all answers in the following blank, separated by commas:
____

(2) $z^2 - (3 - 2i)z + 1 - 3i = 0$
Place all answers in the following blank,separated by commas:
____

(3) $z^2 - 2z + i = 0$
Place all answers in the following blank, separated by commas:
____

Answer(s) submitted:

- `-1/4+sqrt(31)i/4, -1/4-sqrt(31)i/4`
- `1-i, 2-i`
- `1+sqrt(1-i), 1-sqrt(1-i)`

(correct)
Correct Answers:

- `-0.25+1.39194109070751i AND -0.25-1.39194109070751i`
- `1-i AND 2-i`
- `[2.14745938013009,-0.213539293196239] AND [0.4656665496226`

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3. (1 point) Let $z=4+9i$. Write the following numbers in $a+bi$ form:
(a) $7z=$ _____ $+$ _____ i,
(b) $\bar{z}=$ _____ $+$ _____ i,
(c) $\dfrac{1}{z}=$ _____ $+$ _____ i.
Answer(s) submitted:

- `28`
- `63`
- `4`
- `-9`
- `4/97`
- `-9/97`

(correct)
Correct Answers:

- `28`
- `63`
- `4`
- `-9`

- 0.0412371134020619
- -0.0927835051546392

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4. (1 point) Given

$$P = 3b^3 + 4b - 3,$$

$$Q = b^2 - 8b + 8,$$

$$R = b^3 - 7$$

Then $P+Q =$ ___ b^3+ ___ b^2+ ___ $b+$ ___

and $R(P+Q) =$ ___ b^6+ ___ b^5+ ___ b^4+ ___ b^3+ ___ b^2+ ___ $b+$ ___

Answer(s) submitted:

- 3
- 1
- -4
- 5
- 3
- 1
- -4
- -16
- -7
- 28
- -35

(correct)
Correct Answers:

- 3
- 1
- -4
- 5
- 3
- 1
- -4
- -16
- -7
- 28
- -35

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5. (1 point) The expression $2(3x^2 - 2x + 6) - (9x^2 + 8x - 7)$ equals
___x^2+___ $x+$___

Answer(s) submitted:

- -3
- -12
- 19

(correct)
Correct Answers:

- -3
- -12
- 19

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6. (1 point) The expression $4(6x^3 + 5x^2 - 6x + 3) - (4x^2 + 3x - 3)$ equals
___x^3+___x^2+___ $x+$___

Answer(s) submitted:

- 24
- 16
- -27
- 15

(correct)
Correct Answers:

- 24
- 16
- -27
- 15

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7. (1 point) The partial fraction decomposition of $\frac{66x}{8x^2 - 10x + 3}$ can be written in the form of $\frac{f(x)}{2x-1} + \frac{g(x)}{4x-3}$, where
$f(x) =$ _____,
$g(x) =$ _____.

Answer(s) submitted:

- -33
- 99

(correct)
Correct Answers:

- -33
- 99

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8. (1 point) Suppose

$$f(x) = \frac{3x^2 - 3x + 8}{x(x^2+4)}$$

Then

$$f(x) = \frac{A}{x} + \frac{Bx + C}{x^2+4}$$

where
A = _____ and B = _____, and C = _____.
Solution:
Solution: Since

$$\frac{A}{x} + \frac{Bx+C}{x^2+4} = \frac{A(x^2+4) + x(Bx+C)}{x(x^2+4)}$$

The numerator can be converted to the standard form of a polynomial and we must have

$$(A+B)x^2 + Cx + 4A = 3x^2 - 3x + 8.$$

Matching coefficients we see that

$$C = -3 \quad \text{and} \quad A = 2.$$

Moreover, considering the leading coefficients gives $A + B = 2 + B = 3$, i.e.,

$$B = 1.$$

Hence

$$f(x) = \frac{3x^2 - 3x + 8}{x(x^2+4)} = \frac{2}{x} + \frac{x-3}{x^2+4}.$$

It's amazing what Mathematics will do for you!

Answer(s) submitted:

- `2`
- `1`
- `-3`

(correct)

Correct Answers:

- `2`
- `1`
- `-3`

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9. (1 point) Find the quotient and remainder using synthetic division for

$$\frac{x^3 + 4x^2 + 5x + 5}{x+2}.$$

The quotient is ________
The remainder is ________

Answer(s) submitted:

- `x^2+2x+1`
- `3`

(correct)

Correct Answers:

- `x**2+2*x+1`
- `3`

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10. (1 point)

Let

$$p(x) = 6x^3 - 5x^2 - 2x + 1.$$

It is easy to check that $p(1) = 0$, i.e., 1 is a zero of p. p has two more real zeros. The smaller is $x =$ ___, and the larger is $x =$ ___.

Hint: Use synthetic division to divide p by $x-1$, and find the zeros of the quadratic quotient.

Solution:

Solution: Using synthetic division we obtain the table

$$\begin{array}{cccc} 6 & -5 & -2 & 1 \\ & 6 & 1 & -1 \\ 6 & 1 & -1 & 0 \end{array}$$

Thus

$$p(x) = (x-1)(6x^2 + x - 1).$$

The roots of the quadratic factor $(6x^2 + x - 1)$ can be found by completing the square or by applying the quadratic formula. They are $-\frac{1}{2}$ and $\frac{1}{3}$.

Answer(s) submitted:

- `-1/2`
- `1/3`

(correct)

Correct Answers:

- `-0.5`
- `0.333333333333333`

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11. (1 point) Factor $P(x) = x^3 + 5x^2 + 9x + 9$ into linear and irreducible quadratic factors with real coefficients.

Let $P(x) = (x+a)(x^2 + bx + c)$. Then

$a =$_____
$b =$_____
$c =$_____

Answer(s) submitted:

- `3`
- `2`
- `3`

(correct)

Correct Answers:

- `3`
- `2`
- `3`

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12. (1 point) Give **all** zeros of

$$P(x) = x^3 + 25x$$

as a comma separated list.

____

Answer(s) submitted:

- `0, 5i, -5i`

(correct)

Correct Answers:

- `0, -5i, 5i`

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13. (1 point)

Match the polynomial function to its correct roots

Place the letter of the list of correct roots next to each function listed below:

___1. $f(x) = x^4 + 6x^3 + 18x^2 + 54x + 81$
___2. $f(x) = x^4 - 6x^3 + 18x^2 - 54x + 81$
___3. $f(x) = x^4 - 18x^2 + 81$
___4. $f(x) = x^4 - 6x^3 + 54x - 81$

A. $x = -3$, with multiplicity 2, $x = 3i, x = -3i$
B. $x = 3$, with multiplicity 2, $x = 3i, x = -3i$
C. $x = 3$, with multiplicity 2, $x = -3$, with multiplicity 2
D. $x = -3$, $x = 3$, with multiplicity 3

Answer(s) submitted:

- `A`
- `B`
- `C`
- `D`

(correct)

Correct Answers:

- `A`
- `B`
- `C`
- `D`