Amat 307 Midterm Test Fall 2014

1. The following differential equation $y'' \cdot y' + t^2 y'' = y^3 \cos t$ is

   A) Third order, nonlinear

   B) Third order, linear

   C) Second order, nonlinear (*)

   D) Second order, linear

   E) Second order, Bernoulli

2. The solution of the differential equation $ty' + y = 0$ satisfying the initial condition $y(1) = 2$ is

   A) $y = 1 + t$

   B) $y = \dfrac{2}{t}$ (*)

   C) $y = 3 - t$

   D) $y = 2(\ln(t) + 1)$

   E) $y = \ln(e^{t-1} + 1)$

3. The solution of the differential equation $2ty' - y = 6t$, $t > 0$, satisfying the initial condition $y(1) = 4$ is

   A) $y = 3t^{1/2} + t^{-1/2}$

   B) $y = 3t^{3/2} + t^{1/2}$

   C) $y = 2(t + t^{-1/2})$

   D) $y = 6t - 2t^{1/2}$ (*)

   E) None of the above.

4. The equation $\left(y^2 - 3x^a y^b\right) \dfrac{dy}{dx} = 3x^2 y^3 - x$ is exact

   A) for any a, b

   B) never

   C) for $a = 2$, $b = 3$ only

   D) for $a = 3$, $b = 2$ only (*)

   E) for $a = b = 3$ only

5. A tank initially contains 1000 litres of pure water. A solution with a salt concentration of 2.5 g/litre is added to the tank at 4 litres/minute, and the resulting mixture is drained out at 4 litres/minute. Which of the following initial value problems describe the amount $Q(t)$ of salt in the tank at time t?

   A) $Q'(t) + \dfrac{1}{250} Q(t) = 10, \quad Q(0) = 1000$

   B) $Q'(t) + \dfrac{1}{250} Q(t) = 10, \quad Q(0) = 0$ (*)

   C) $Q'(t) + \dfrac{1}{1000 + 4t} Q(t) = 10, \quad Q(0) = 0$

   D) $Q'(t) + \dfrac{1}{25} Q(t) = 10, \quad Q(0) = 1000$

   E) None of the above.

6. The mass of a radioactive substance is 20 g at $t = 0$. After 100 hours, 10 g of the radioactive material remains. What is the mass in grams $m(t)$ after t hours?

   A) $Q(t) = 20 \cdot e^{-\frac{\ln 2}{100}t}$ (*)

   B) $Q(t) = 20 \cdot e^{\frac{\ln 2}{100}t}$

   C) $Q(t) = 10 \cdot e^{-\frac{\ln 2}{100}t}$

   D) $Q(t) = 10 \cdot e^{\frac{\ln 2}{100}t}$

   E) $Q(t) = 20 \cdot e^{-\frac{\ln 2}{50}t}$

7. A solution for the exact differential equation $(2x + y^3)dx + (3xy^2 + 1)dy = 0$, with initial condition $y(1) = -1$ is

   A) $3xy^2 + y + x^2 = 3$

   B) $x^2 + y^3 + x = 1$

   C) $x^2 + xy^3 + y = -1$ (*)

   D) $x^2 + xy^3 = 0$

   E) $x^2 + y^3 = 0$

8. According to the Euler numerical approximation method for $y'(t) = \cos(t)y + e^t$ with initial condition $y(0) = 1$ and step size $\Delta t = 0.5$, the first two approximate values $y(0.5)$ and $y(1)$ obtained are

   A) $y(0.5) \approx 3, \quad y(1) \approx 3 + 3\cos(0.5) + e^{0.5}$

   B) $y(0.5) \approx 2, \quad y(1) \approx 2 + 2\cos(0.5) + e^{0.5}$

   C) $y(0.5) \approx 3, \quad y(1) \approx 7$

   D) $y(0.5) \approx 3, \quad y(1) \approx 3 + \cos(0.5) + 1$

   E) $y(0.5) \approx 2, \quad y(1) \approx 2 + 0.5(2\cos(0.5) + e^{0.5})$ (*)

9. After the substitution $u(t) = y^a$ the differential equation $ty' + y = (ty)^{-3/2}$ becomes a linear differential equation in $u = u(t)$ when

   A) $a = 5/2$ (*)

   B) $a = 1/2$

   C) $a = -5/2$

   D) $a = -1/2$

   E) $a = 3/2$

10. The **largest** interval on which a unique solution to the initial value problem $(x+3)y'' + \frac{x}{x-3}y' + y = \frac{1}{x+2}, \quad y(1) = 2$ is **guaranteed** to exist is

    A) $(-3, 0)$

    B) $(-2, \infty)$

    C) $(-3, 3)$

    D) $(0, 3)$

    E) $(-2, 3)$ (*)

11. The solution to the initial value problem: $3y'' - 3y' - 6y = 0$, $y(0) = 1$, $y'(0) = 1$ is given by:

A) $\dfrac{1}{3}e^{-x} + \dfrac{2}{3}e^{2x}$ (*)
B) $\dfrac{1}{3}e^{x} + \dfrac{2}{3}e^{-2x}$
C) $3e^{-x} - 2e^{-2x}$
D) $\dfrac{4}{3}e^{-x} - \dfrac{1}{3}e^{2x}$
E) None of the above.

12. The solution to the initial value problem: $2y'' - 4y' + 10y = 0$, $y(\frac{\pi}{4}) = 1$, $y'(\frac{\pi}{4}) = 0$ is given by:
A) $-e^{-\frac{\pi}{2}}e^{2t}\cos(4t) + e^{-\frac{\pi}{2}}e^{2t}\sin(4t)$
B) $-e^{-\frac{\pi}{4}}e^{t}\cos(4t) + e^{-\frac{\pi}{4}}e^{t}\sin(4t)$
C) $\dfrac{1}{2}e^{-\frac{\pi}{4}}e^{t}\cos(2t) + e^{-\frac{\pi}{4}}e^{t}\sin(2t)$ (*)
D) $-\dfrac{1}{2}e^{-\frac{\pi}{4}}e^{t}\cos(2t) + \dfrac{1}{2}e^{-\frac{\pi}{4}}e^{t}\sin(2t)$
E) $\dfrac{1}{2}\cos(2t) + \sin(2t)$

13. If $2 + \ln(2)te^{3t}$ is a solution to a homogeneous constant coefficient linear differential equation, then so also must be
A) $\ln(2) - 2t^2e^{3t}$
B) $2(e^{3t} - \sin(2))$ (*)
C) $te^{3t}\cos(2) - t^2$
D) $2t(e^{3t} + 1)$
E) None of the above.

14. Let $y_1(t)$ and $y_2(t)$ be two solutions of the differential equation $ty'' + \cos(t)y' - 3y = t\ln t$, $t > 0$. Which of the following is also a solution of this equation?
A) $y_1(t) + y_2(t)$
B) $2y_1(t) - 3y_2(t)$
C) $y_1(t) - y_2(t)$
D) $3y_2(t) - 2y_1(t)$ (*)
E) $y_1(t) - 2y_2(t)$

15. According to the method of undetermined coefficients, a particular solution to the equation $y'' - 2y' + 2y = t^2e^t\cos(t) + te^{-3t}$ is of the form (where $A_0, A_1, A_2, B_0, B_1, B_2, C_0, C_1$ are real constants)
A) $(A_0t^2 + A_1t + A_2)e^t\cos(t) + C_0te^{-3t}$
B) $(A_0t^3 + A_1t^2 + A_2t)e^t\cos(t) + (B_0t^3 + B_1t^2 + B_2t)e^t\sin(t) + (C_0t + C_1)e^{-3t}$ (*)
C) $(A_0t^2 + A_1t + A_2)e^t\cos(t) + (B_0t^2 + B_1t + B_2)e^t\sin(t) + (C_0t + C_1)e^{-3t}$
D) $(A_0t^3 + A_1t^2 + A_2t)e^t\cos(t) + (C_0t + C_1)e^{-3t}$
E) $(A_0t^3 + A_1t^2 + A_2t)e^t\cos(t) + (B_0t^3 + B_1t^2 + B_2t)e^t\sin(t) + C_0te^{-3t}$

16. According to the method of undetermined coefficients, a particular solution to the equation $y^{(4)} + 2y'' + y = (x-2)e^{2x} + (x-1)\cos(x)$ is of the form (where $A_0, A_1, B_0, B_1, C_0, C_1$ are real constants)
A) $e^{2x}(A_0x + A_1) + [(B_0x + B_1)\cos(x) + (C_0x + C_1)\sin(x)]$
B) $e^{2x}(A_0x^2 + A_1x) + [B_0\cos(x) + C_0\sin(x)]$
C) $e^{2x}(A_0x^3 + A_1x^2) + (B_0x^3 + B_1x^2)\cos(x)$
D) $e^{2x}(A_0x + A_1) + [(B_0x^3 + B_1x^2)\cos(x) + (C_0x^3 + C_1x^2)\sin(x)]$ (*)
E) $e^{2x}(A_0x + A_1) + [B_0\cos(x) + C_0\sin(x)]$

17. The equation $y'' + by' + cy = 0$, where y is a function of t, has a solution $e^{2t}\cos(3t)$

A) for $b = 4$, $c = 5$ only

B) for $b = -4$, $c = 9$ only

C) for $b = -4$, $c = 13$ only (*)

D) for $b = 4$, $c = 13$ only

E) None of the above

18. Let y_1 and y_2 be solutions of $y'' - 3y + q(t)y = 0$ such that their Wronskian at $t = 0$ equals 1: $W(y_1, y_2)(0) = 1$. Then the Wronskian at $t = 2$

A) cannot be found: insufficient data

B) $W(y_1, y_2)(2) = 2$

C) $W(y_1, y_2)(2) = e^{-3}$

D) $W(y_1, y_2)(2) = 3$

E) $W(y_1, y_2)(2) = e^6$ (*)

19. There is a homogeneous constant coefficient linear ordinary differential equation of order m for which the function $y(t) = 5t\cos(t) + 10$ is a solution. This is true when

A) $m = 2$

B) $m = 3$

C) $m = 4$

D) $m = 5$ (*)

E) None of the above.

20. The unique solution to the differential equation $y'' - 2y' - 3y = 9t$ with initial conditions $y(0) = 2$, and $y'(0) = 1$ is:

A) $y = e^{3t} - e^{-t} - 3t + 2$ (*)

B) $y = \frac{3}{4}e^{3t} + \frac{5}{4}e^{-t} - 3t$

C) $y = -e^{-3t} + e^{t} - 3t + 2$

D) $y = \frac{3}{4}e^{-3t} + \frac{5}{4}e^{t} - 3t$

E) None of the above.