$-\exists x$ — x —

$\forall x -$ — x —

$-\forall x$ — x —

$\exists x -$ — x —

EI [Must check]

✓ $\exists x$ — x —

— n —

(n is a new name)

UI [Do not check]

$\forall x$ — x —

— n —

(n is an old name) unless no name occurred on the path

1. apply truth function
2. negative quantifiers
3. existential instantiation (EI)
4. Universal instantiation (UI)

ex) $\dfrac{\forall x(Fx \wedge Gx)}{(\forall x Fx \wedge \forall x Gx)}$ valid

$\forall x(Fx \wedge Gx)$

✓ $-(\forall x Fx \wedge \forall x Gx)$

Left branch: ✓ $-\forall x Fx$; $\exists x -Fx$; $-Fa$; $(Fa \wedge Ga)$; Fa; Ga; x

Right branch: ✓ $-\forall x Gx$; ✓ $\exists x -Gx$; $-Ga$; $(Fa \wedge Ga)$; Fa; Ga; x

ex) $\dfrac{\forall x(Fx \vee Gx)}{(\forall x Fx \vee \forall x Gx)}$

$\forall x(Fx \vee Gx)$

✓ $-(\forall x Fx \vee \forall x Gx)$

✓ $-\forall x Fx$

✓ $-\forall x Gx$

$\exists x -Fx$

$\exists x -Gx$

$-Ga$

$-Fb$

$(Fa \vee Ga)$

$(Fb \vee Gb)$

Branches: Fa / Ga x; under Fa: Fb x / Gb

satisfied

Domain {1, 2}

Ext of 'a' = 1

Ext of 'b' = 2

Ext of 'F' = {1}

Ext of 'G' = {2}

Ga, Gb

ex) $\dfrac{\exists x(Fx \rightarrow Gb)}{(\exists x Fx \rightarrow Gb)}$

$(\exists x Fx \rightarrow Gb)$: true → false ⟹ false

$(Fx \rightarrow Gb)$: false

there is 1 case where Fx is true with which turn is false

$\exists x(Fx \rightarrow Gb)$

✓ $-(\exists x Fx \rightarrow Gb)$

✓ $\exists x Fx$

$-Gb$

Fa

$(Fc \rightarrow Gb)$

Branches: $-Fc$ / Gb x

Domain {1, 2, 3}

ext of 'a' = 1

ext of 'b' = 2

ext of 'c' = 3

ext of 'F' = {1}

ext of 'G' = ϕ

Fa Fb Fc

Ga Gb Gc

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