

1. $\forall x Fx$
 2. $\exists x Fx$
 3. Fa
- Suppose that I is an interpretation of 2. in which 2 is False
- Then there is no object in the domain of I of which Fx is true

s: The extension of F in I is \emptyset
 s: 1 is not true in I

so in any interpretation in which 1 is true, 2 is true

so, 1 implies 2

and the argument: $\frac{\forall x Fx}{\exists x Fx}$ is valid

$\frac{Fa}{\exists x Fx}$

Consider an interpretation I in which Fa is true
 this means the extension of ' F ' in I is not the extension of ' F ' in I
 so the ext. of ' F ' is not \emptyset
 $\exists x Fx$ is true in I

4 new rules for truth trees.

$\checkmark -\exists x \text{ --- } x \text{ ---}$
$\forall x \text{ --- } x \text{ ---}$
$\checkmark -\forall x \text{ --- } x \text{ ---}$
$\exists x \text{ --- } x \text{ ---}$

$-\exists x Fx$
 $\forall x Fx$
 $\forall x$
 $\exists x$

Existential Instantiation EI

if $\exists x \text{ --- } x \text{ ---}$ is on an open path, then
 check it, and write an instance of it with
 a name n that does not already occur
 anywhere on the path, ($-\text{---}$),
 on every path on which $\exists x \text{ --- } x \text{ ---}$ occurred
 $\checkmark \exists x \text{ --- } x \text{ ---}$

$-\text{---}$
 (n is a new name)

$\checkmark \exists x Fx$
 $\textcircled{a} Fa$

Universal Introduction UI

If $\forall x \text{---} x \text{---}$ is on an open path, then when I get path then Do NOT CHECK IT, but make/assume of it with \forall or all that already occur on that path, unless no names occurred on that path in which case use a new name.

$\forall x \text{---} x \text{---}$
 $\text{---} a \text{---}$
 (a is an old name)

$\frac{\forall x Fx}{\exists x Fx}$ is this valid?

make list
 $\{ \forall x Fx, \exists x Fx \}$

$\exists \forall x Fx$
 $\checkmark \exists x Fx$
 $\forall x Fx$

$\neg Fa$
 Fa
 \times

the initial list is unsatisfiable

\therefore valid

$\frac{\exists x Fx}{Fa}$ $\checkmark \exists x Fx$ $\neg Fa$] satisfiable
 Fb
 \hookrightarrow this is invalid

$\frac{Fa}{\exists x Fx}$ $\checkmark Fa$ $\neg \exists x Fx$] unsatisfiable
 $\checkmark \forall x Fx$
 $\neg Fa$
 \times
 \hookrightarrow valid

$\forall x (Fx \rightarrow Gx)$

$\frac{\forall x (Gx \rightarrow Hx)}{\forall x (Fx \rightarrow Hx)}$

\vdash \vdash

$\forall x (Fx \rightarrow Gx)$
 $\forall x (Gx \rightarrow Hx)$
 $\checkmark \forall x (Fx \rightarrow Hx)$
 $\checkmark \exists x \neg (Fx \rightarrow Hx)$] unsatisfiable

$\vee \neg (Fa \rightarrow Ha)$
 $\checkmark (Fa \rightarrow Ga)$
 $\vee (Ga \rightarrow Ha)$

Fa
 $\neg Ha$
 $\swarrow \searrow$
 $\neg Fa \quad Ga$
 $\times \quad \swarrow \searrow$
 $\neg Ga \quad Ha$
 $\times \quad \times$

1. TF
 2. $\neg Q$
 3. EI
 4. UI
- with \neg in $\neg Q$
 \neg in $\neg Q$