Logic II — Richard Zach

Phil 379 L01 — Winter 2016

Problem Set #2

This assignment is due on **Thursday, February 4, at 12:30 pm**. You can turn it in in class or in the dropbox labelled "Logic II (379 L01)—Richard Zach" in the Philosophy Department. The dropbox is cleared at 4 pm.

1. Show that if X is equinumerous with U and Y is equinumerous with V, and $X \cap Y = U \cap V = \emptyset$, then $X \cup Y$ is equinumerous with $U \cup V$.
2. Show that if X and Y are countable, then $X \cup Y$ is countable. Then use this to show (by induction on n) that if $X_1, \ldots, X_n$ are all countable, so is $\bigcup_{i=1}^{n} X_n$.
3. Show that if $X_1, X_2, \ldots$ are all countable, so is $\bigcup_{i=1}^{\infty} X_i$. (Note: induction does *not* help.)
4. Show that the set of all functions $f : \mathbb{Z}^+ \to \mathbb{Z}^+$ is not countable. Use a direct diagonal argument.
5. Show that $\mathbb{N}^\omega$ is not countable. Use a reduction argument (you may use anything proved in Chapter 4, or (4) above.)

Remember: this is not a test. You are allowed—indeed, encouraged—to work together, and to ask questions on the website and in office hours.