Logic II — Richard Zach

Phil 379 L01 — Winter 2016

Problem Set #5

This assignment is due on **Tuesday, March 29, at 12:30 pm**. You can turn it in in class or in the dropbox labelled "Logic II (379 L01)—Richard Zach" in the Philosophy Department. The dropbox is cleared at 4 pm daily.

1. Problem 8.1: Complete the proof of Proposition 8.2.
2. Complete cases (3) and (6) of Lemma 8.9. Note that $\boldsymbol{M}(\Gamma^*)$ in the book is what I've called $\boldsymbol{M}^*$ in lecture. Case (6) is the hard part, especially the direction from $\boldsymbol{M}(\Gamma^*) \models \forall x\, B(x)$ to $\forall x\, B(x) \in \Gamma^*$.
3. Problem 8.4: Use Corollary 8.17 to prove Theorem 8.16.
4. Use the compactness theorem to show that any set of sentences in the language of arithmetic which are true in the standard model of arithmetic $\boldsymbol{N}$ are also true in a structure $\boldsymbol{N}'$ that contains an element greater than all natural numbers $\overline{n}^{N'}$ ($\overline{n}$ is $0^{\prime\cdots\prime}$ with n $\prime$'s). (Hint: add a new constant c to the language and consider the sentences $\overline{n} < c$ for all n.)
5. Problem 10.4: Design a Turing-machine with alphabet $\{\sqcup, A, B\}$ that takes as input any string α of As and Bs and duplicates them to produce an output of the form $\alpha\alpha$. (E.g. input $ABBA$ should result in output $ABBAABBA$).

Remember: this is not a test. You are allowed—indeed, encouraged—to work together, and to ask questions on the website and in office hours.