Electricity and Magnetism

- Physics 259 – L02
- Lecture 43

Chapter 29: Magnetic field due to current

Last time:

- Biot-Savart Law (like Coulomb's Law for magnetism)
- B-field of a line of current
- Magnetic force between parallel current-carrying wires

Today:

- Ampere's law
- Applications of Ampere's law

For a single charge →

$$\boxed{\vec{F}_B = q\,\vec{v}_d \times \vec{B}}$$

For N charges moving through the wire (current carrying wire) →

$$\boxed{\vec{F}_B = i\,\vec{\ell} \times \vec{B}}$$

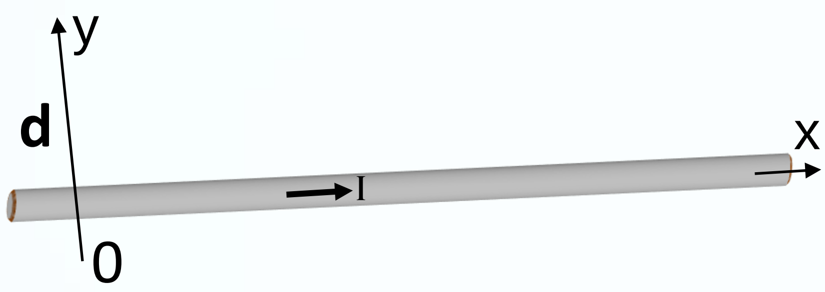
The Biot-Savart Law →

$$\boxed{\vec{B}_{\text{point charge}} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2}}$$

For an electric current →

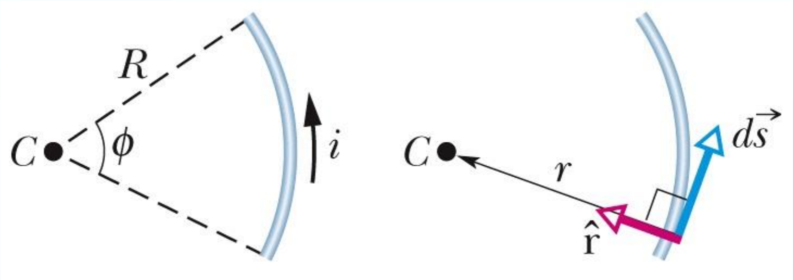
$$\boxed{\vec{B}_{\text{current segment}} = \frac{\mu_0}{4\pi}\frac{I\,\Delta\vec{s}\times\hat{r}}{r^2}}$$

Magnetic field due to current in long straight wire

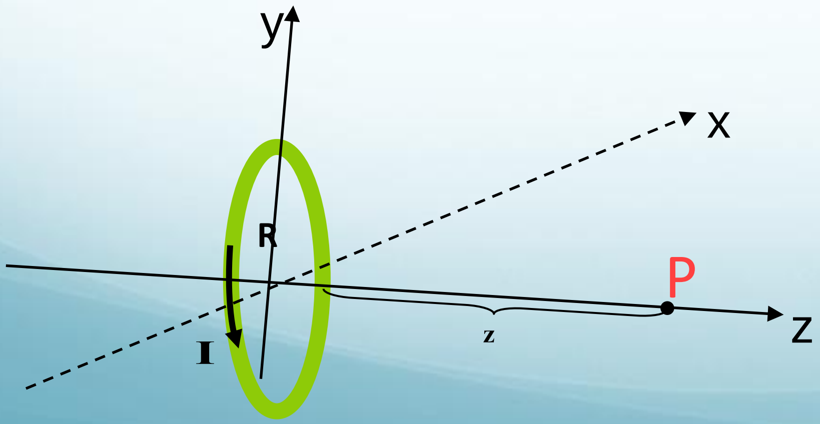


$$B_z = \frac{\mu_0}{2\pi}\frac{I}{d}$$

Non-infinite straight wire→ Appendix 1-chapter 22



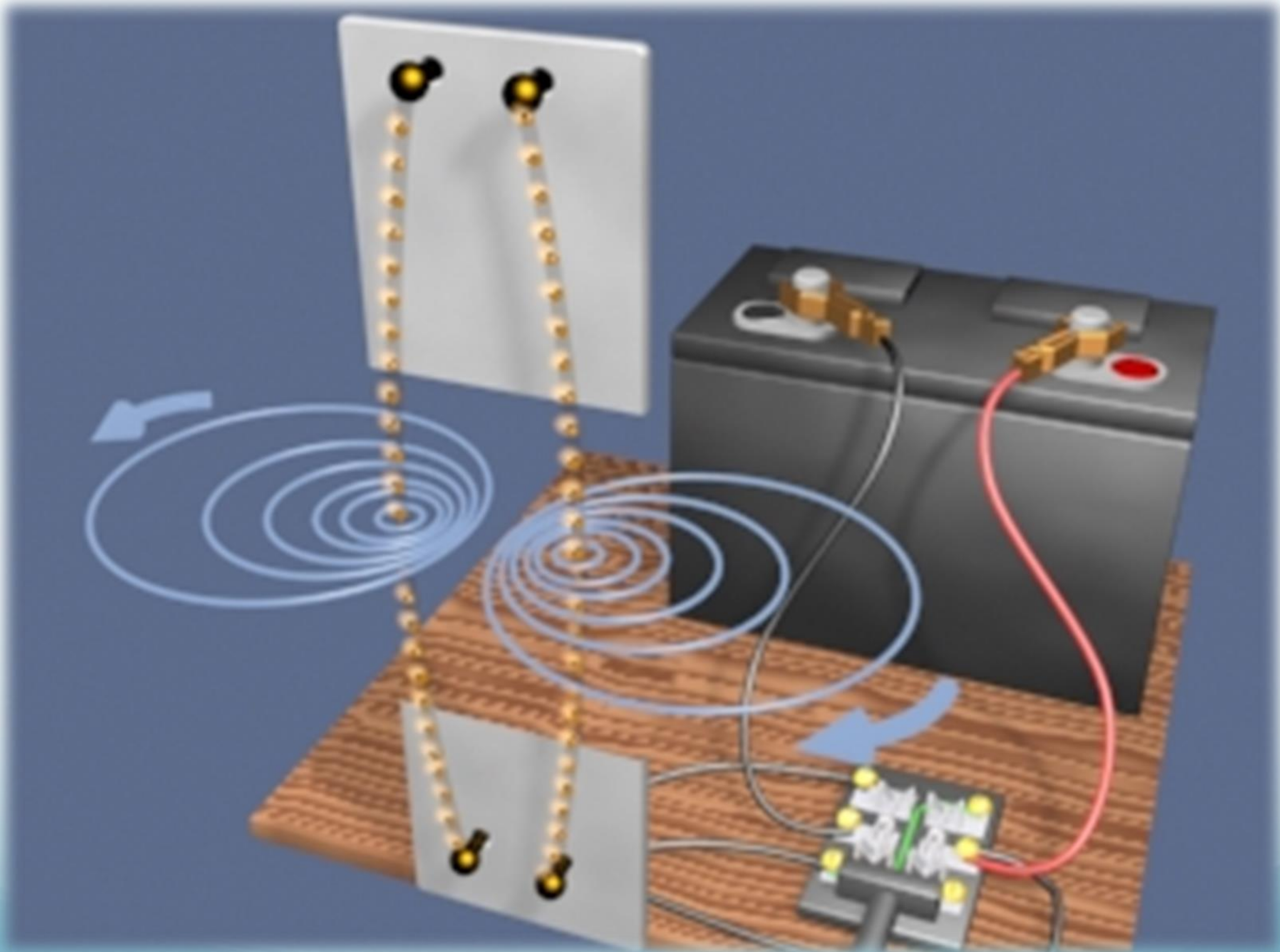
$$B = \frac{\mu_0 i \phi}{4\pi R}$$



$$\vec{B} = \frac{\mu_0}{2}\frac{IR^2}{\left(z^2 + R^2\right)^{3/2}}\hat{k}$$

if $z = 0$ $\quad \vec{B}_{center} = \frac{\mu_0}{2}\frac{I}{R}\hat{k}$

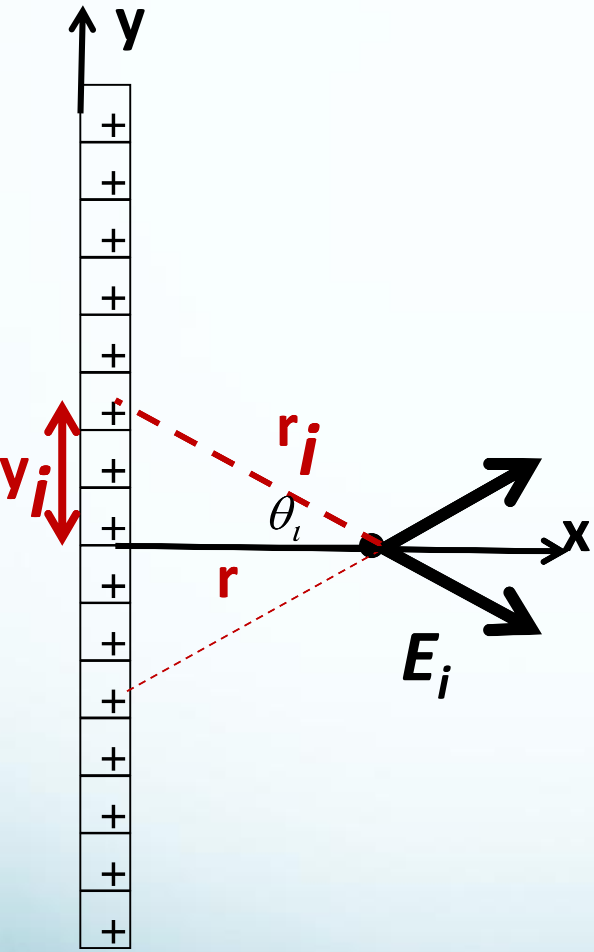
29.2: Force between two antiparallel currents

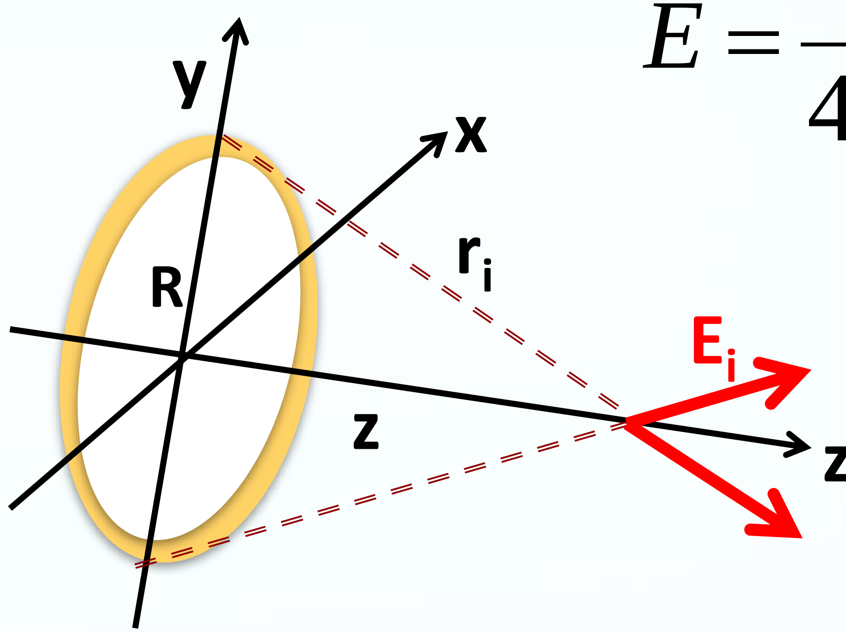


https://tinyurl.com/lghkxt8

Electrostatics

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$$





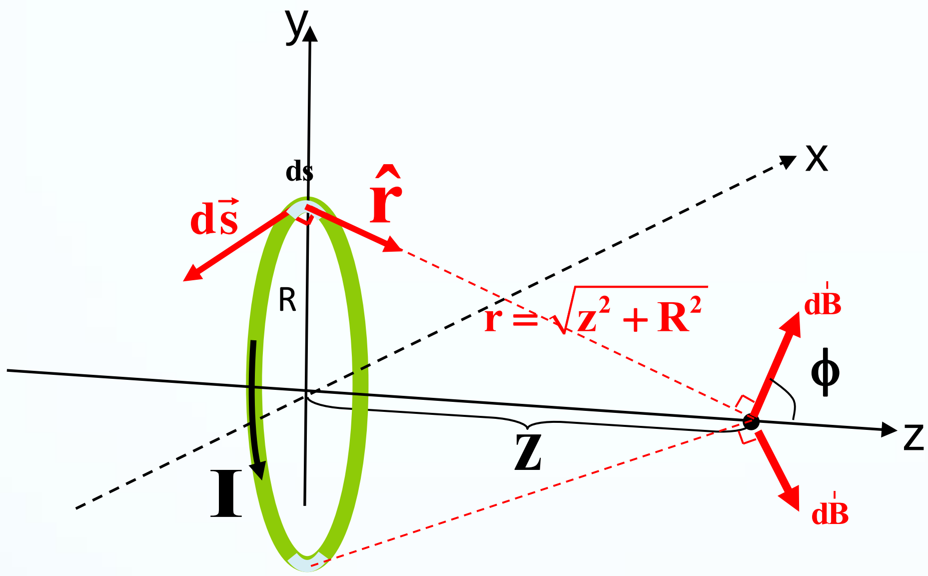
Savior:

Gausses' law

$$\oint \vec{E}.d\vec{A} = \frac{Q_{in}}{\varepsilon_0}$$

Magnetostatics

$$\vec{B}_{\text{current segment}} = \frac{\mu_o}{4\pi}\frac{I\Delta\vec{s}\times\hat{r}}{r^2}$$



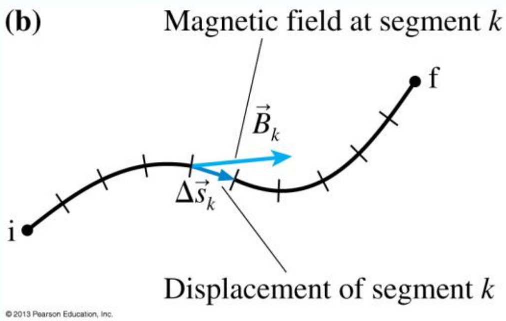
Savior:

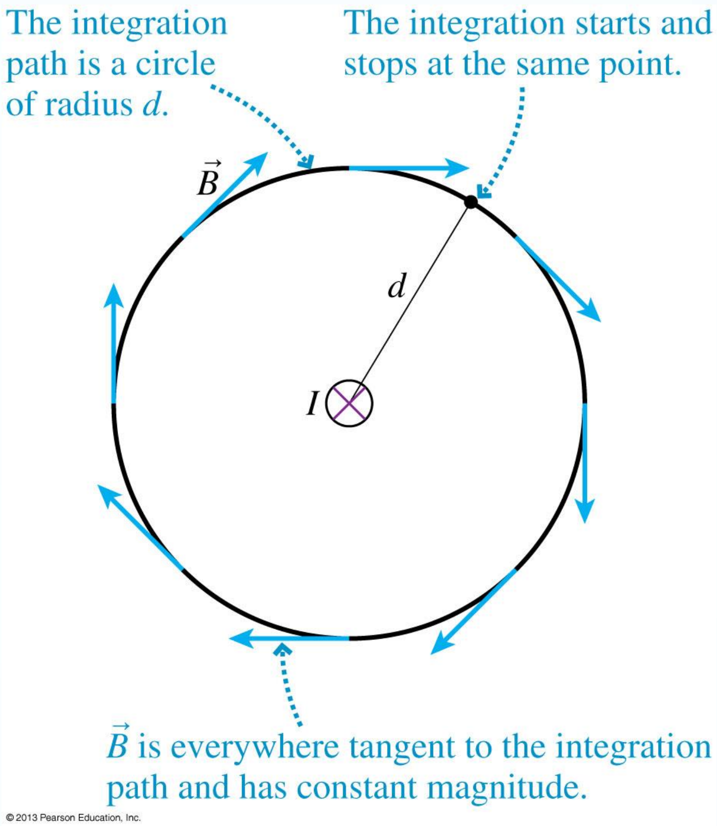
Ampere's law

Expression?

Ampère’s law

The line integral of **B** along the path:





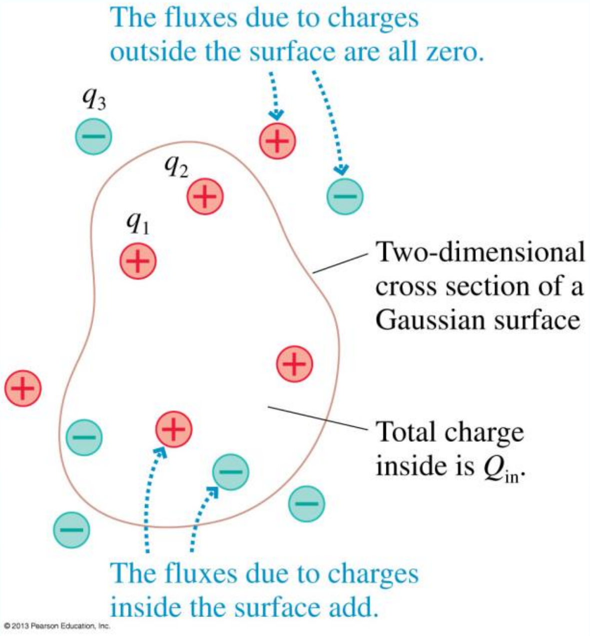
$$\int_i^f \vec{B}.d\vec{l}$$

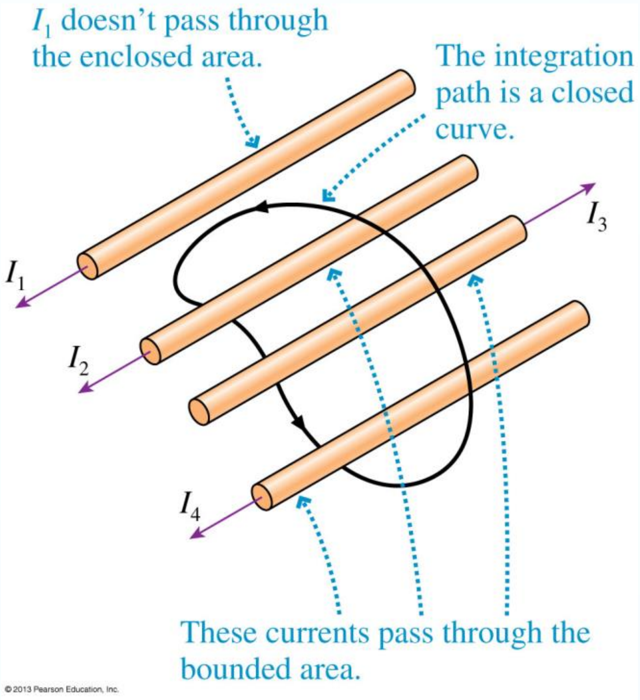
$$\oint \vec{B}\cdot d\vec{l} = (2\pi r)\left(\frac{\mu_0 I}{2\pi r}\right)$$

Infinite wire→ $B = \dfrac{\mu_0 I}{2\pi r}$

$$\boxed{\text{i.e.} \oint \vec{B}\cdot d\vec{l} = \mu_0 I_{encl}}$$

Ampère’s *Law is true for any* ***shape of path*** *and any current distribution*





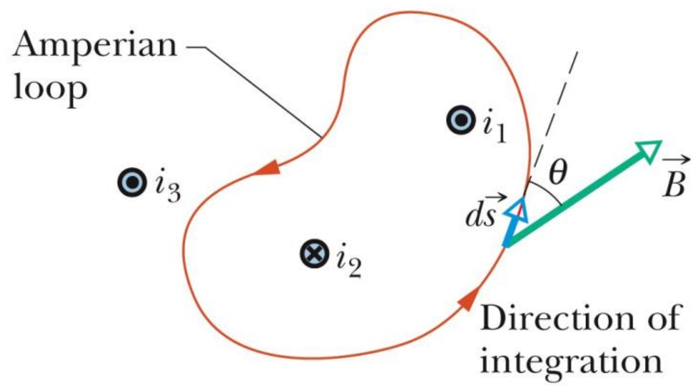
$$\oint \vec{E}.d\vec{A} = \frac{Q_{in}}{\varepsilon_0}$$

For a closed surface enclosing total Charge Q

$$\oint \vec{B}.d\vec{l} = \mu_0 I_{enclosing}$$

Current I passes through an area bounded by a closed curve

Only the currents encircled by the loop are used in Ampere's law.



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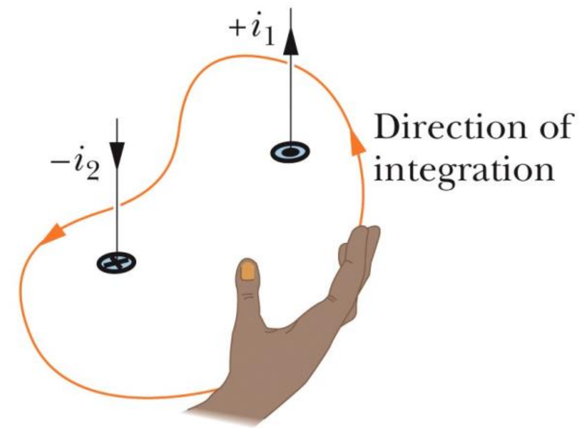
This is how to assign a sign to a current used in Ampere's law.



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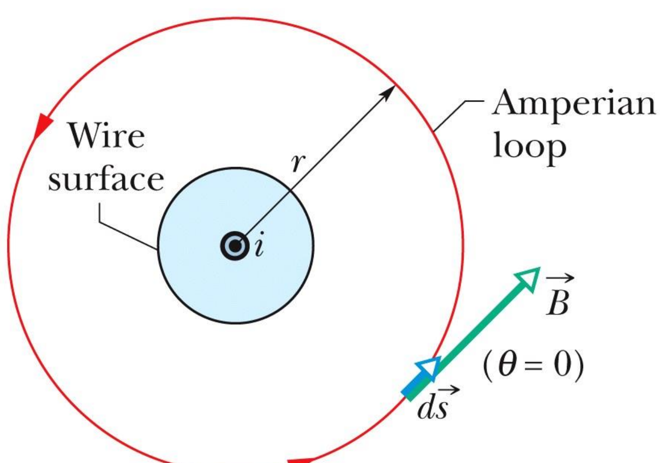
All of the current is encircled and thus all is used in Ampere's law.



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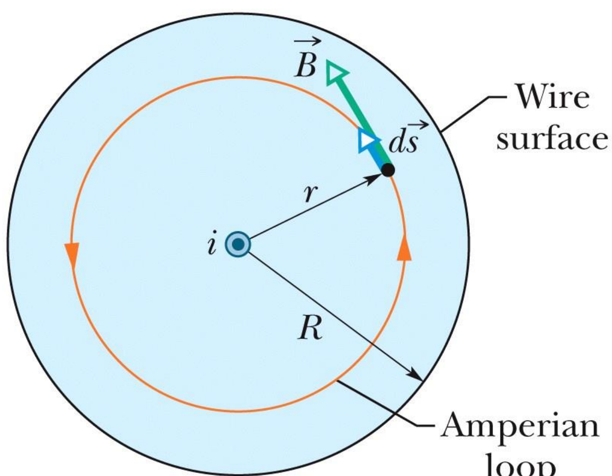
Only the current encircled by the loop is used in Ampere's law.



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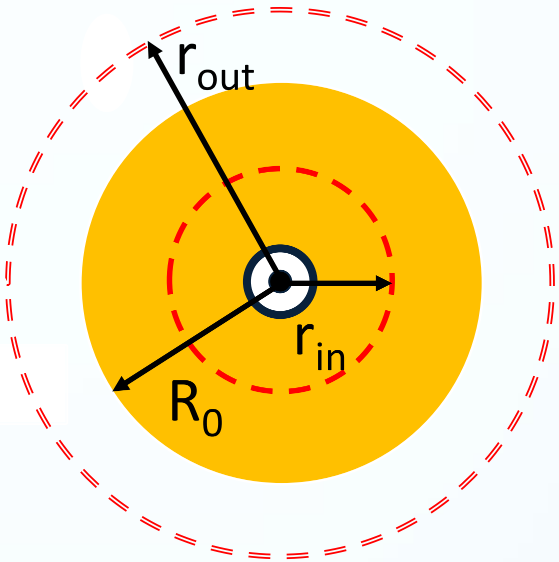
Ampère's law: application

(a) Using Ampère's law, calculate the magnetic field outside a solid current carrying wire a distance r_{out} from its axis

(The length of the solid wire is infinite and the current I is uniformly distributed throughout the solid wire)

b) Calculate the magnetic field inside a solid current carrying wire a distance r_{in} from its axis.

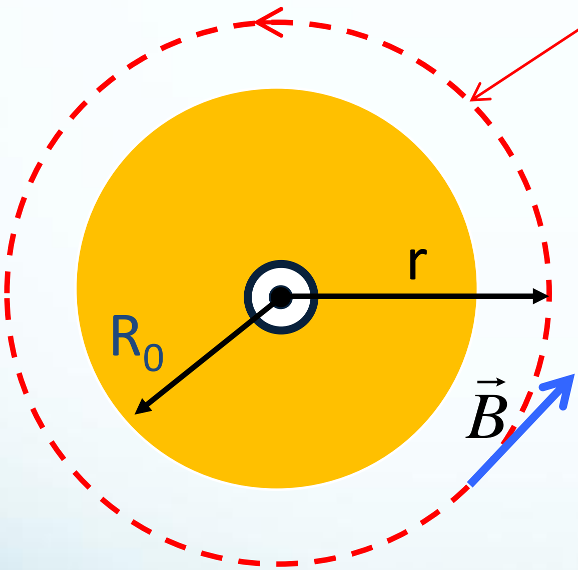


End view:
Wire with radius R
and current I

Ampère’s law: application(1)

(a) B-field outside

We want to know the B-field a distance r, so we choose an Ampèrian loop with radius r > R_0.



Ampère’s Law: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$

Left hand side: $\oint \vec{B} \cdot d\vec{l} = BL = B2\pi r$

Right hand side: $\mu_0 I_{enc} = \mu_0 I$

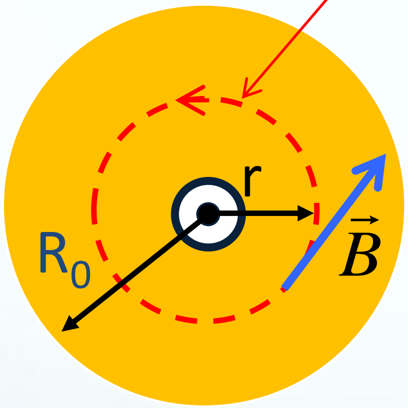
Combine together: $B2pr = \mu_0 I$

$$\boxed{B = \frac{\mu_0 I}{2pr}}$$

Ampère's law: application(1)

(b) B-field inside

We want to know the B-field a distance r, so we choose an Amperian circular loop with radius $r < R_0$.



Ampère's Law: $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc}$

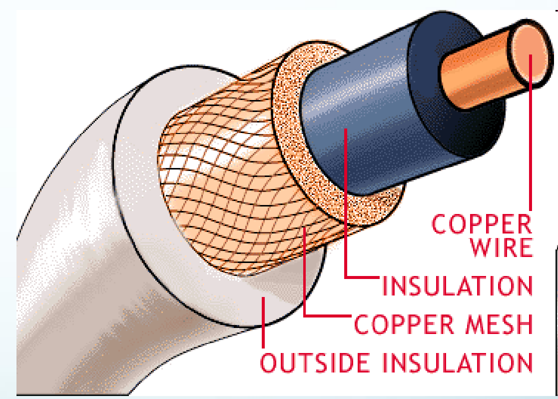
Left hand side: $\oint \vec{B}\cdot d\vec{l} = BL = B2\pi r$

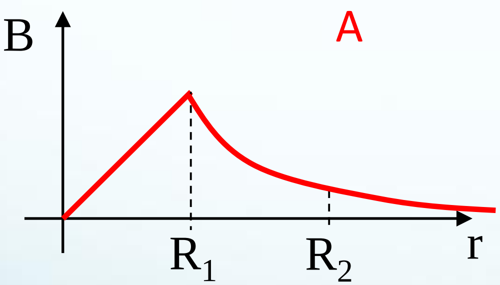
Right hand side: $\mu_0 I_{enc} = \mu_0 JA = \mu_0 \dfrac{I}{\pi R_0^2}\pi r^2$

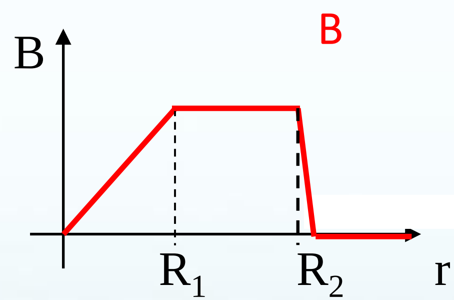
Combine together: $B2pr = \mu_0 \dfrac{I}{R_0^2} r^2$ $\boxed{B = \dfrac{\mu_0 I\, r}{2pR_0^2}}$

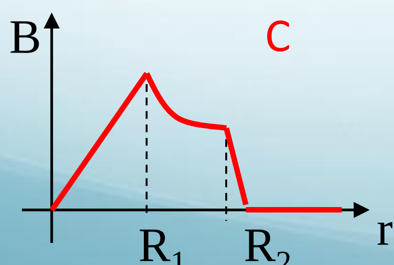
A coaxial cable consists of a wire (radius R_1) surrounded by an insulating sleeve and another cylindrical conducting shell (inner radius R_2) and finally another insulating sleeve. **The wire and the shell carry the same current I but in opposite directions**.

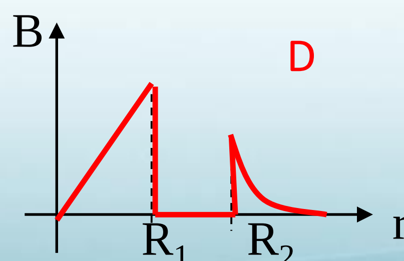
Which diagram best represents the **magnetic field** as a function of radial distance from the cable's axis?

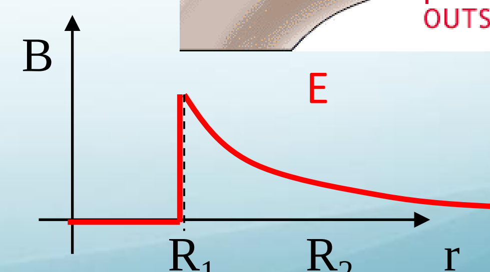












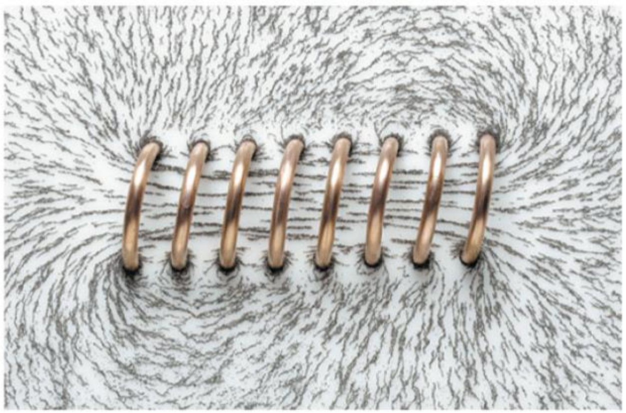
Ampère’s law: application (3)

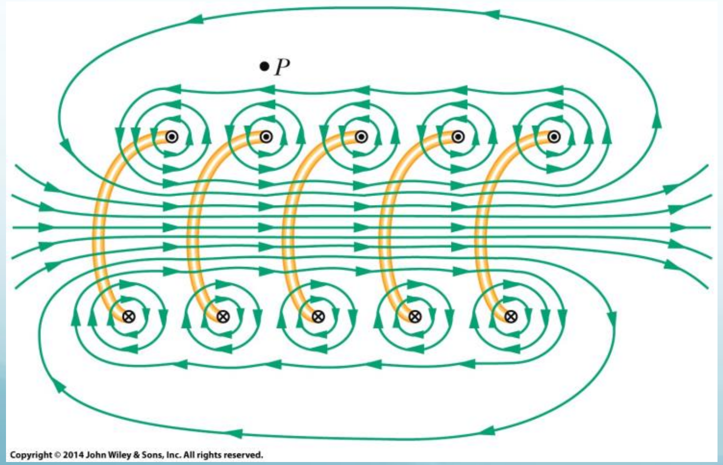
(a) Using Ampère’s law, calculate the magnetic field above the current carrying slab

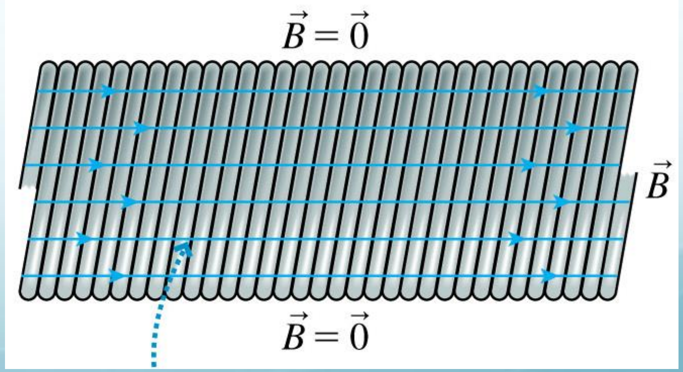
b) Calculate the magnetic field inside the current carrying slab

Ampère's law: application(2)

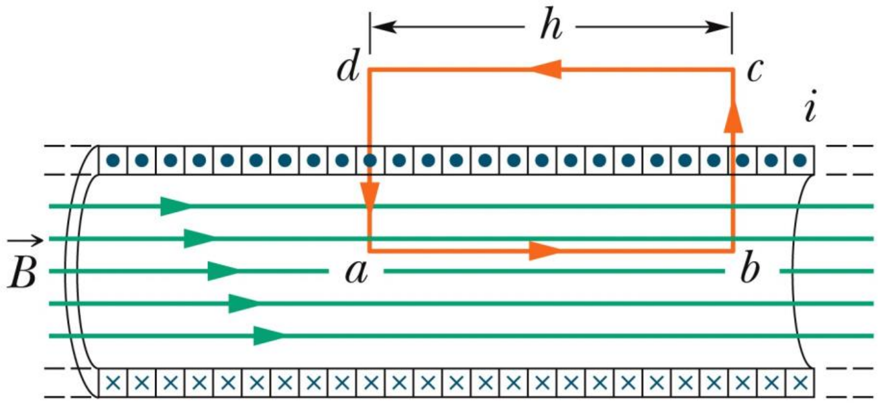
29.3: Solenoids and Toroids







$$\oint \vec{B}.d\vec{s} = \mu_0 i_{enc} = \mu_0 ni$$



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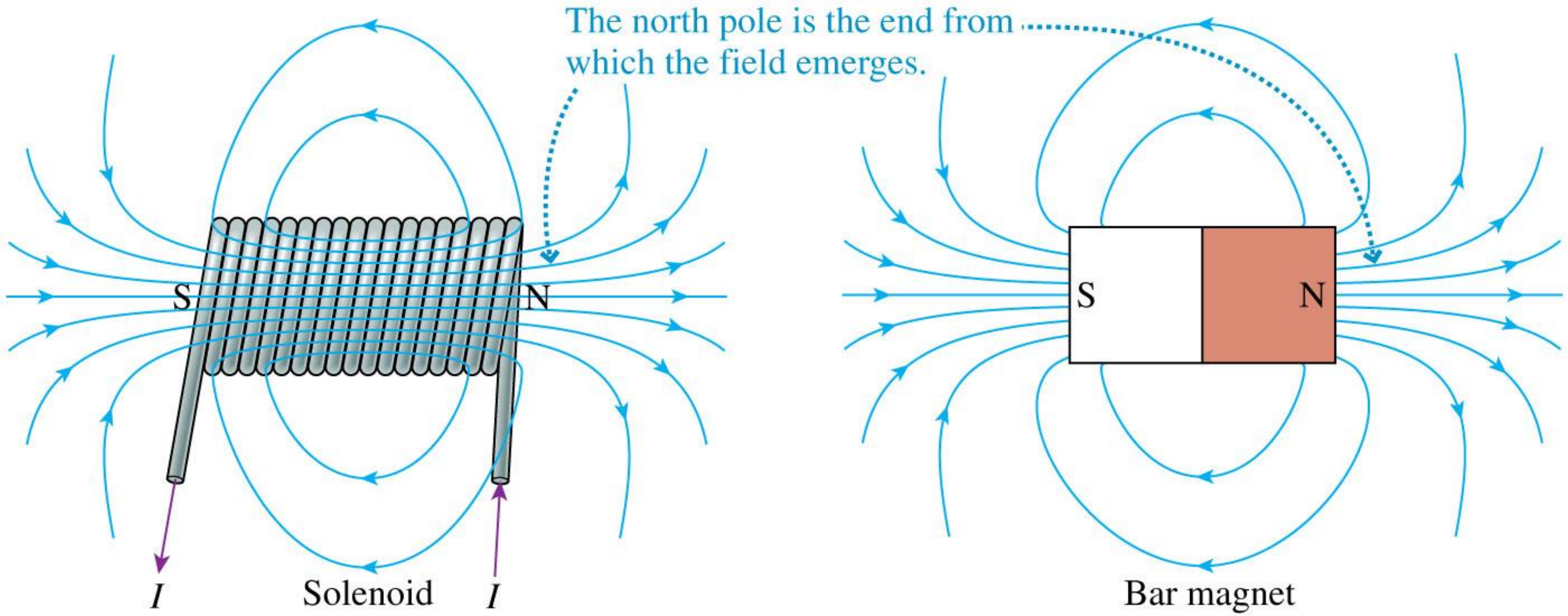
$\mathbf{n = \frac{N}{L}}$ **number of turns per unit length**

$$B_{Solenoid} = \mu_0 ni$$



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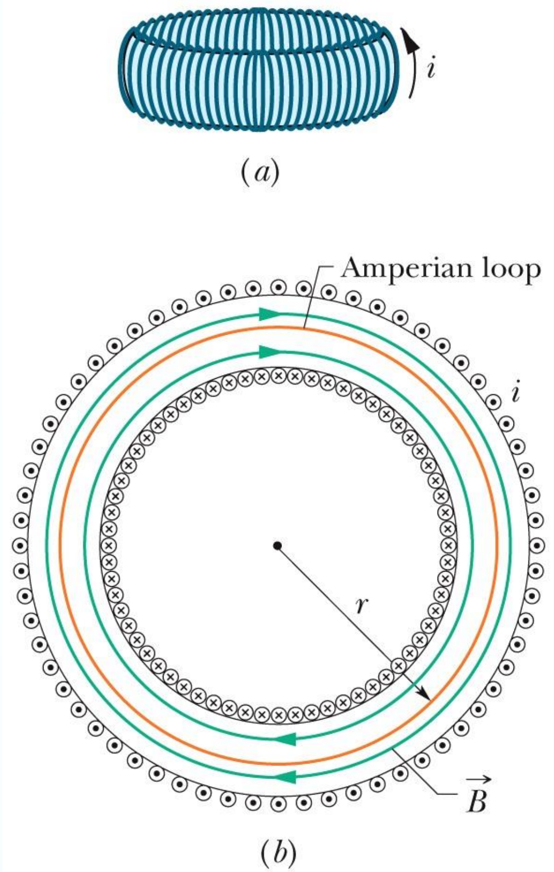
$$B_{Solenoid} = \mu_0 n i$$



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This section we talked about:

Chapter 29

See you on Friday