University of Calgary

Department of Physics and Astronomy

PHYS 259, Winter 2017

Labatorial 3: Gauss' Law

When you calculate the electric field of a charge distribution by using superposition (with a summation or integration), it takes quite a bit of work. Fortunately, there exists a powerful tool that allows you to calculate electric fields much faster, if they are sufficiently symmetric: Gauss' Law. It relates the electric flux through a closed (imaginary) surface to the charge enclosed by the surface. The math is already "built in", and you just have to apply it properly to the given physical situation, by making use of the symmetry.
From a more fundamental point of view, Gauss' law states that when there is an electric field in space, there must be a source for it – and when there is a charge, there will be an electric field in space, generated by that charge.

Carl Friedrich Gauss (1777-1855)
Before the introduction of the Euro as currency, Gauss' image – and even some of his work – was shown on the 10 DM (Deutsche Mark) bill.

Goals:

To study various symmetries of charge configurations and fields. To understand Gauss' law, and the conditions under which it is useful for applications.

Preparation:

Halliday, Resnick, and Walker, "Fundamentals of Physics" 10th edition, Wiley: 23.1 – 23.6.

Equipment:

None.

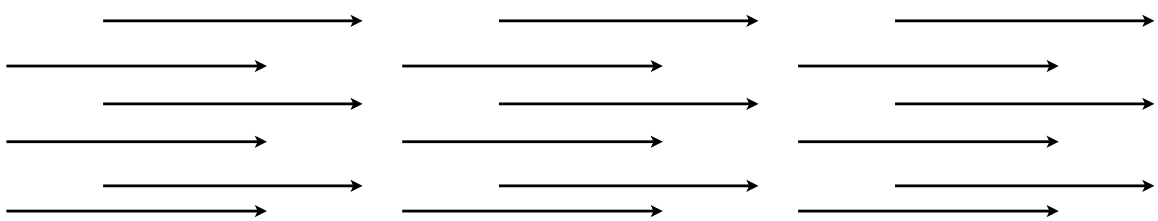
1 Electric Flux and Gauss' Law

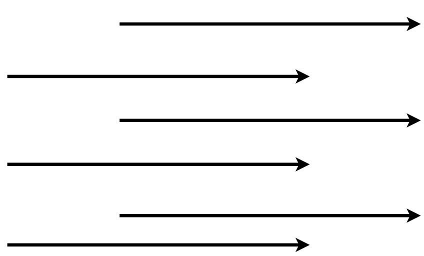
At its core, Gauss' law is a relationship between the electric field lines passing through a closed surface and the electric charges that are contained within that surface. To quantify what we mean by the electric field lines passing through a surface, we need to introduce the idea of the electric flux, which is given by the integral

$$\Phi_e = \int \vec{E} \cdot d\vec{A} \tag{1}$$

This integral is taken over the entire area of the surface; the dot product means that the directions of $\vec{E}$ and of the infinitesimal area element $d\vec{A}$ at every place along the surface both matter. The direction associated with $d\vec{A}$ is always perpendicular to the surface: for a closed surface (like a sphere) the direction is always chosen to point outward, while for an open surface (like a disk) there is no notion of "inside" and "outside" so the direction can point either way.

Question 1: The figures below show a constant electric field. For the figure on the left, draw an open surface that maximizes the flux; for the middle figure, draw an open surface that results in zero flux; and for the figure on the right, draw an open surface that yields a flux between these two extremes. For each, indicate the direction of the area vector $\vec{A} = A\hat{n}$ where $\hat{n}$ is the unit vector normal to the surface.

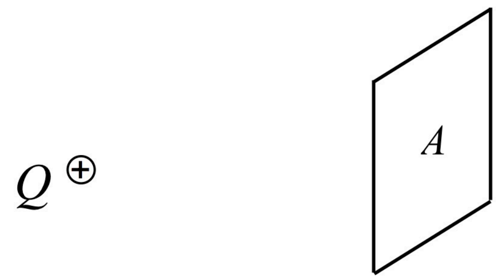




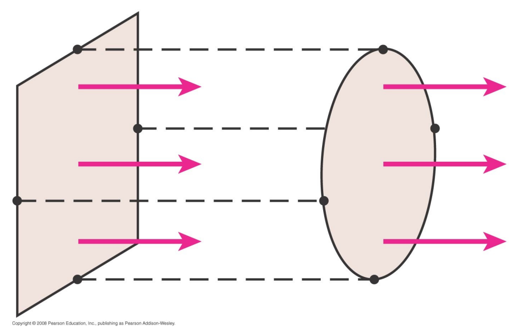
Question 2: Now draw a closed surface submersed in the field. Is the flux through the surface positive, negative, or zero? Justify your answer.

Question 3: Is the flux through a closed surface always zero? If yes, explain why. If no, sketch a counterexample.

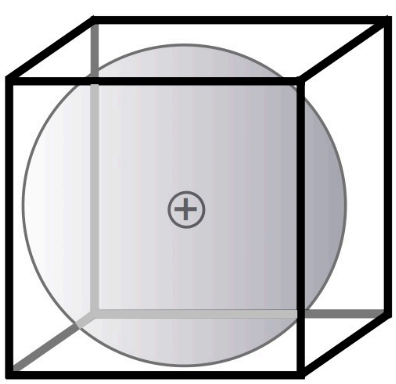
Question 4: The figure below shows a point charge Q and a surface of size A. Is the electric flux through this surface given by $\Phi_e = \int \vec{E} \cdot d\vec{A} = \frac{Q}{\epsilon_0}$? Why, or why not?



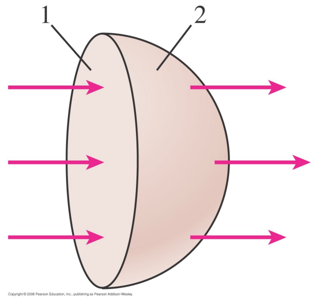
Question 5: A square and a circle are in the same uniform field. The diameter of the circle is equal to the length of the edges of the square. Is the flux through the circle larger than, smaller than, or the same as that through the square? Explain your answer.



Question 6: This figure shows a positive point charge Q surrounded by a spherical surface which is surrounded by a cube. Is the flux through the surface of the cube smaller, larger, or the same as the flux through the surface of the sphere? Explain.



Question 7: Two surfaces, a disk (1) and a hemisphere (2), are located in a uniform electric field. Is the magnitude of the flux through the hemisphere larger than, smaller than, or the same as the flux through the disk? Explain your answer.



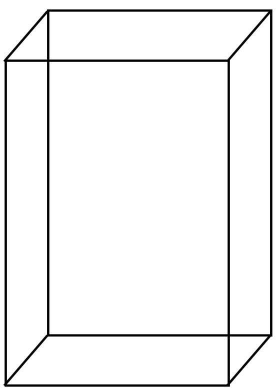
CHECKPOINT 1: Before moving on to the next part, have your TA check the results you obtained so far.

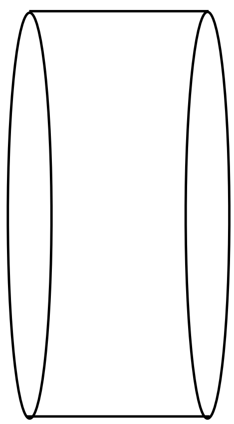
2 Using Gauss' Law to calculate the electric field

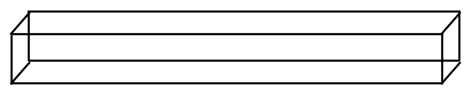
Question 8: Use Gauss' Law to calculate the electric field $E(r)$ of a positive point charge Q, using a sphere as the Gaussian surface. Explain every step in the calculation as if you were teaching this to a fellow student who missed class.

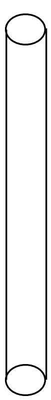
Question 9: Instead of using a sphere as the Gaussian surface, your friend suggests using a cube to calculate the electric field of a point charge. Will this be a suitable Gaussian surface to use? Explain why or why not, including a sketch.

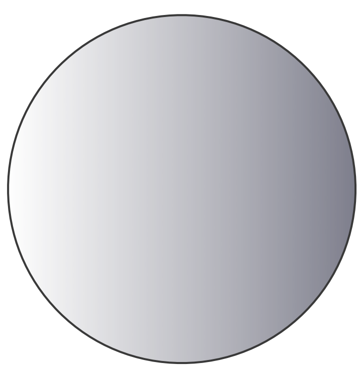
Question 10: For each of the surfaces below, state whether you could use it as a Gaussian surface to calculate the electric field close to a very large plane. If you think you can use a surface, draw the plane into the figure. If you think you cannot, explain why not.











Question 11: Can you calculate the electric field of a dipole using Gauss' Law? If yes, sketch the Gaussian surface you would use. If no, explain why not, including a sketch.

CHECKPOINT 2: Before moving on to the next part, have your TA check the results you obtained so far.

3 Using Gauss' Law to calculate the electric field of a spherical object

Question 12: a) What is the volume charge density $\rho = \frac{Q}{V}$ for a uniformly charged solid sphere of radius R and with total charge Q? Sketch a graph of ρ as a function of radius; mark the radius R in your graph.

b) For some radius $r < R$ inside the uniformly charged solid sphere, how much charge is contained within the sphere of radius r?

c) For some radius $r > R$ outside the uniformly charged solid sphere, how much charge is contained within the sphere of radius r?

Gauss' Law states $\oiint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0}$, where Q_{enc} is the charge contained within the Gaussian surface.

Question 13: Use your results from parts (b) and (c) to calculate the electric field $E(r)$ inside and outside the uniformly charged sphere.

Question 14: Sketch your result for the electric field inside and outside the solid sphere below.

STOP **Last Checkpoint! Clean up your area, and put the equipment back the way you found it. Call your TA over to check your work and your area before you can get credit for the labatorial.**

Equations and Constants

$$
\begin{aligned}
F_g(r) &= G\frac{m_1 m_2}{r^2} \\
U_{grav}(y) &= mgy \\
K &= \frac{1}{2}mv^2 \\
v_x(t) &= v_{0x} + a_x t \\
x(t) &= x_0 + v_{0x}t + \frac{1}{2}a_x t^2 \\
v_x^2(t) &= v_{0x}^2 + 2a_x(x(t) - x_0) \\
\omega &= \frac{d\theta}{dt} \\
v &= \frac{2\pi r}{T} = \omega r \\
a_{rad} &= \frac{v^2}{r} = \omega^2 r \\
F_C(r) &= \frac{1}{4\pi\epsilon_0}\frac{|q_1 q_2|}{r^2} \\
\vec{E} &= \frac{\vec{F}}{q}
\end{aligned}
$$

$$
\begin{aligned}
g &= 9.81\frac{m}{s^2} \\
G &= 6.67 \times 10^{-11}\frac{Nm^2}{kg^2} \\
V(r) &= \frac{4}{3}\pi r^3 \\
\frac{1}{4\pi\epsilon_0} &= 8.99 \times 10^9\, Nm^2C^{-2} \\
\epsilon_0 &= 8.85 \times 10^{-12}\, C^2N^{-1}m^{-2} \\
e &= 1.60 \times 10^{-19}\, C \\
m_e &= 9.11 \times 10^{-31}\, kg \\
m_p &= 1.67 \times 10^{-27}\, kg \\
m_n &= 1.67 \times 10^{-27}\, kg
\end{aligned}
$$

$$
\begin{aligned}
\int \frac{dx}{(x^2 \pm a^2)^{3/2}} &= \frac{\pm x}{a^2\sqrt{x^2 \pm a^2}} \\
\int \frac{xdx}{(x^2 \pm a^2)^{3/2}} &= -\frac{1}{\sqrt{x^2 \pm a^2}} \\
\int \frac{dx}{x^2 + z^2} &= \frac{1}{z}\arctan(\frac{x}{z})
\end{aligned}
$$