Electricity and Magnetism

- Physics 259 – L02
- Lecture 6

Section 21.1-3

Last time

- Charges and Force Between Charges
- Conductors and Insulators
- Van De Graaff Generator Experiment 
- Solve Class Activity Question
- Coulomb’s Law

This time

- Examples for Coulomb’s law

Coulomb's Law

$$\boxed{F_{1\,on\,2} = F_{2\,on\,1} = K\frac{|q_1||q_2|}{r^2}}$$

K = electrostatic constant

$$K = 8.99\times10^{9}\,\frac{N\cdot m^2}{C^2}$$

$$\boxed{F_{1\,on\,2} = F_{2\,on\,1} = \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_2|}{r^2}}$$

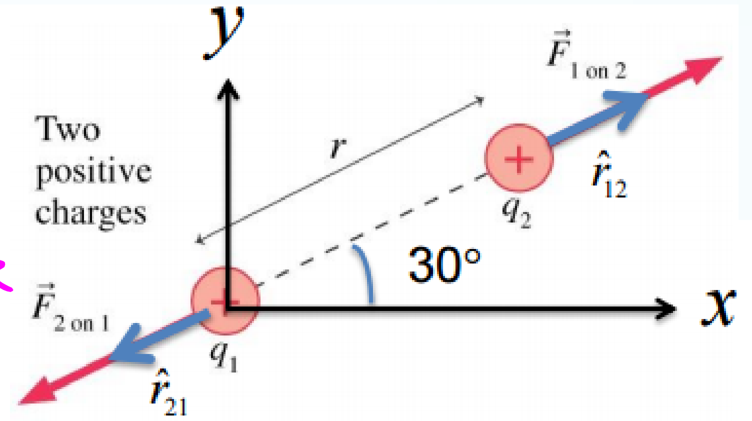
ε_0 = permittivity of free space

$$\varepsilon_0 = \frac{1}{4\pi K} = 8.85\times10^{12}\,\frac{C^2}{N\cdot m^2}$$

Example #1: Two point charges

Step 1 → draw the diagram and find direction of force

Step 2 → find the magnitude of force



$q_1 = +2\text{ C},\ q_2 = +5\text{ C},\ r = 10\text{ m}$

$$\vec{F}_{1\,on\,2} = k_e \frac{|q_1||q_2|}{r^2}\hat{r}_{12}$$

$$= \left(8.99\times10^9\ \text{Nm}^2/\text{C}^2\right)\frac{(\cancel{2}\text{C})(\cancel{5}\text{C})}{(\cancel{1}0\text{ m})^2}\hat{r}_{12}$$

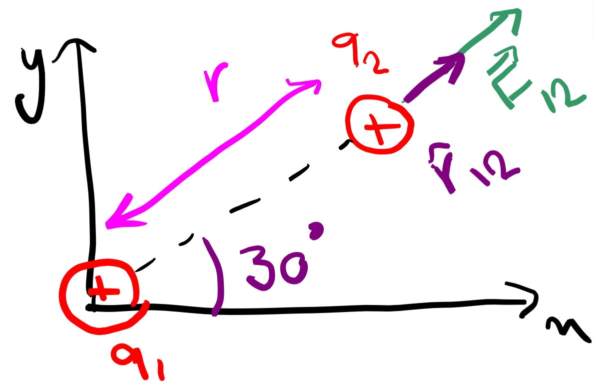
$$= \left(8.99\times10^8\,\text{N}\right)\hat{r}_{12}$$

direction

$$= 8.99\times10^8\left(\cos 30\ \hat{i} + \sin 30\ \hat{j}\right)\text{N}$$

magnitude of force

unit of force



$\hat{r}_{12} = |\hat{r}|\cos\theta\,\hat{i} + |\hat{r}|\sin\theta\,\hat{j}$

$\hat{r}_{12} = \cos 30\,\hat{i} + \sin 30\,\hat{j}$

$\vec{F}_{12} = F_{12}\,\hat{r}_{12}$

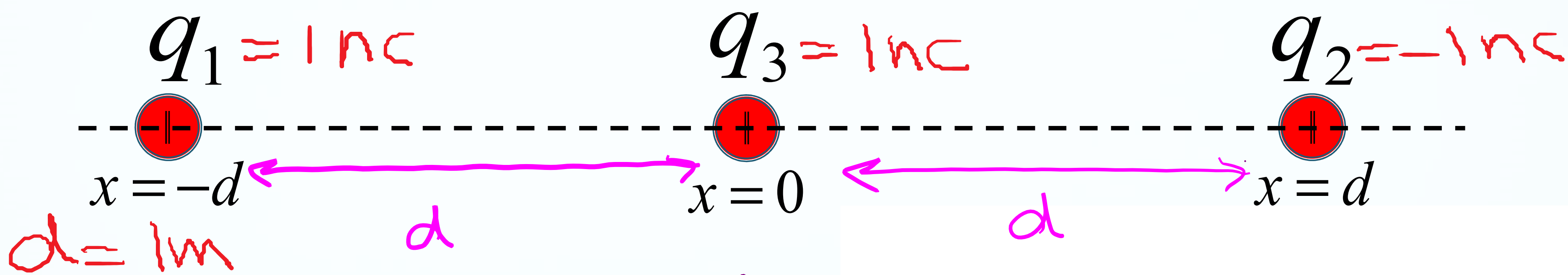
$\vec{F}_{12} = F_{12}(\cos 30\,\hat{i} + \sin 30\,\hat{j})$

$$\vec{F}_{2\,on\,1} = k_e \frac{|q_1||q_2|}{r^2}\hat{r}_{21}$$

$$= \left(8.99\times10^8\,\text{N}\right)\hat{r}_{21}$$

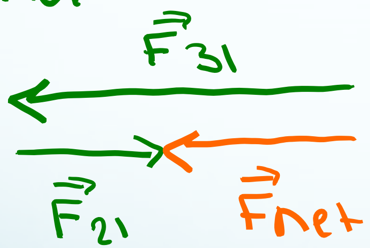
$$= 8.99\times10^8\left(-\cos 30\ \hat{i} - \sin 30\ \hat{j}\right)\text{N}$$

Example #2 (last section's homework): Three point Charges

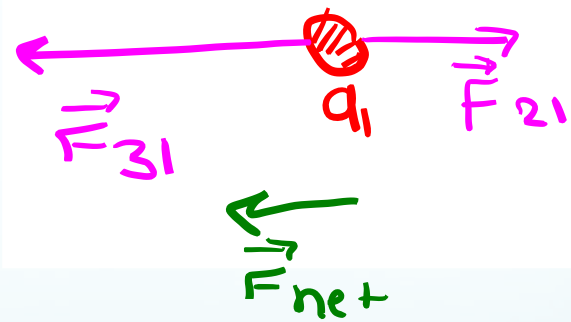


Total electric force on q_1?

$\vec{F}_{net}$ => from vector addition



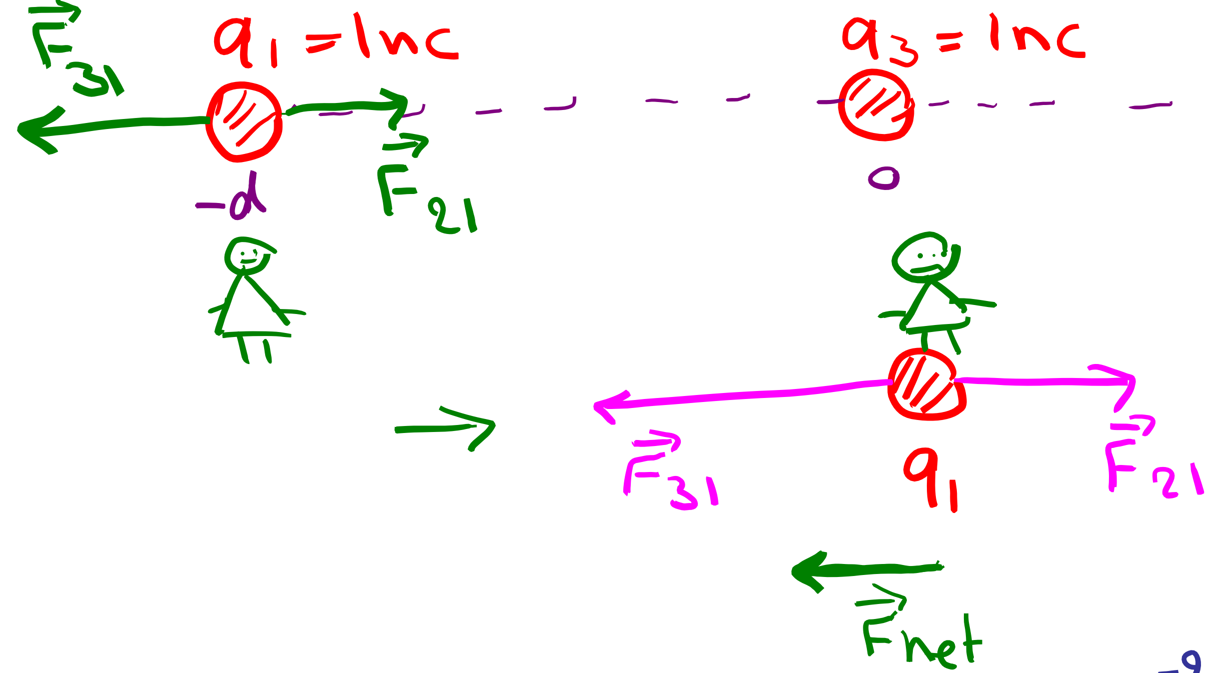
free body diagram =>

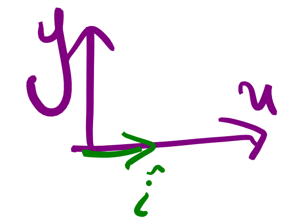


Final answers =>

$$\left|\vec{F}_{31}\right| = 8.99\times 10^{-9}\ \mathrm{N} \qquad \left|\vec{F}_{21}\right| = 2.25\times 10^{-9}\ \mathrm{N} \qquad \boxed{\vec{F}_{net} = \left(-6.74\times 10^{-9}\ \mathrm{N}\right)\hat{i}}$$

Calculations on next page =>





$$\vec{F}_{net} = F_{net}(-\hat{i})$$

$$\vec{F}_{21} = k\frac{|q_1||q_2|}{r_{21}^2}\hat{i} = 8.99\times10^9 \frac{1\times10^{-9}\times1\times10^{-9}}{(2d)^2} = (A)\,\hat{i}$$

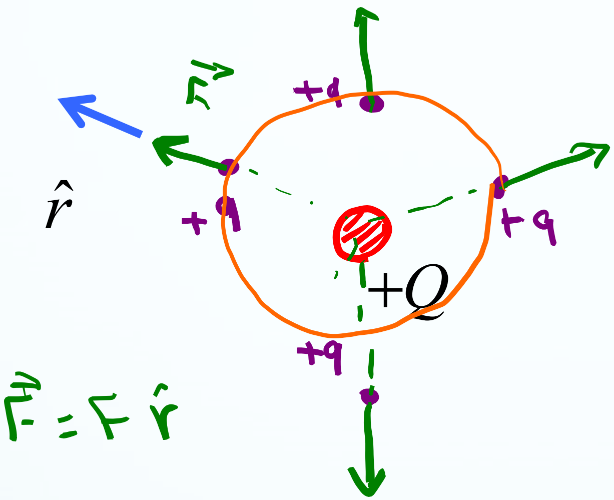
$$\vec{F}_{31} = k\frac{|q_3||q_1|}{r_{31}^2}(-\hat{i}) = 8.99\times10^9 \frac{1\times10^{-9}\times1\times10^{-9}}{d^2} = (B)(-\hat{i})$$
$$= -(B)\,\hat{i}$$

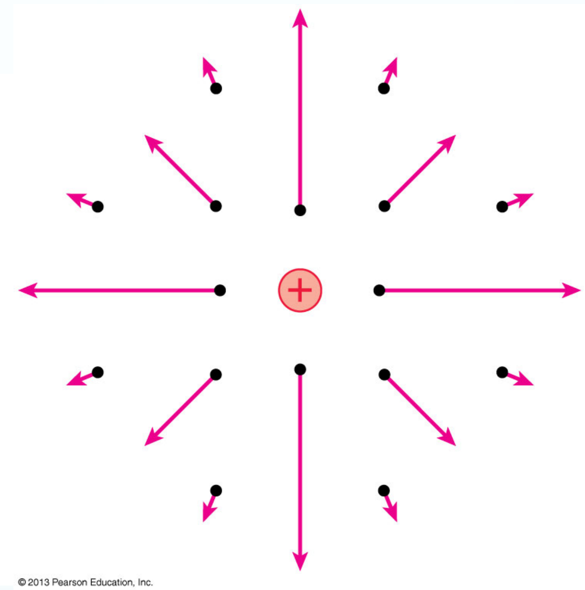
$$\vec{F}_{net} = \vec{F}_{21} + \vec{F}_{31} = A\hat{i} + (-B)\,\hat{i} = (A-B)\,\hat{i}$$

Example #3: Building blocks of electric force

Suppose we have charge $+Q$ and put a charge $+q$ beside it $\Rightarrow$

$$\vec{F} = \frac{KQq}{r^2}\hat{r}$$





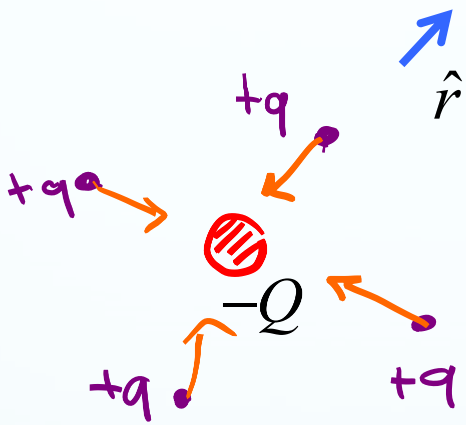
all force vectors are outward

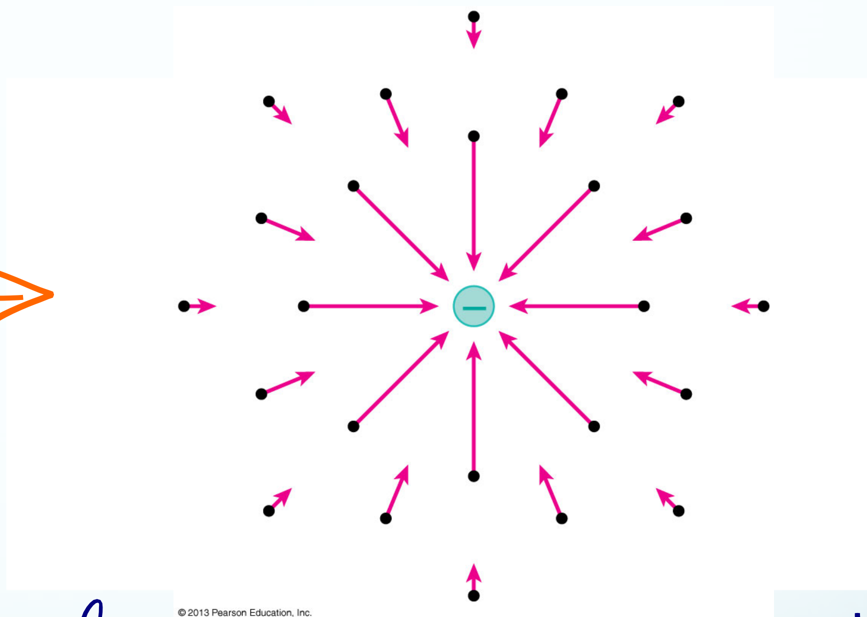
(you can consider sphere around the point charge)

● = positive charge q at the position indicated

Example #3: Building blocks of electric force

$$\vec{F} = \frac{-KQq}{r^2}\hat{r}$$





for negative charge (-Q) all force vectors are toward the -Q.

● = positive charge *q* at the position indicated

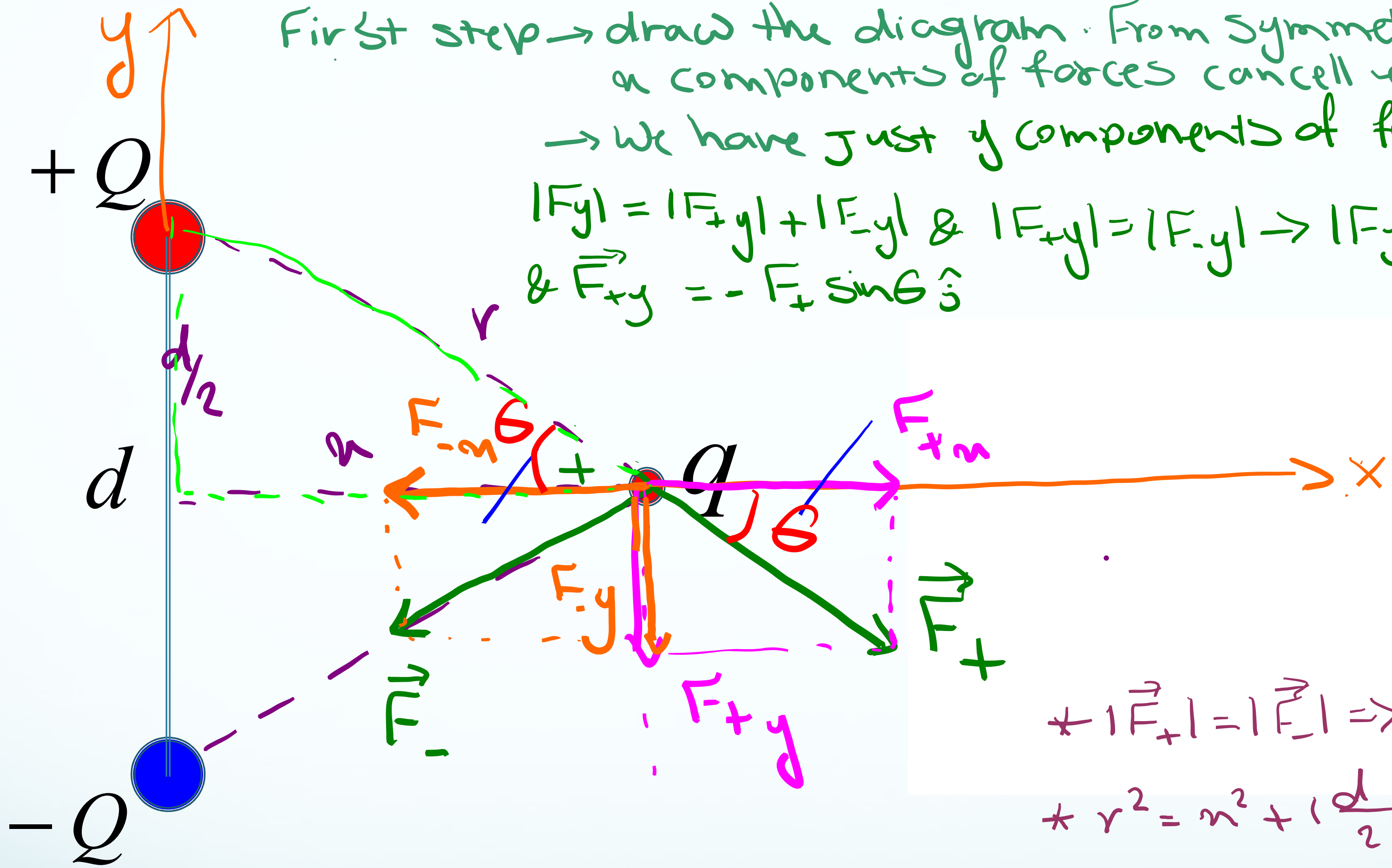
Example #4: Two point charges (Electric Dipole)

First step → draw the diagram. From symmetry we see that x components of forces cancell eachother.

→ we have just y components of forces ⇒

$|F_y| = |F_{+y}| + |F_{-y}|$ & $|F_{+y}| = |F_{-y}| \rightarrow |F_y| = 2|F_{+y}|$

& $\vec{F}_{+y} = -F_+ \sin\theta \,\hat{j}$



$\ast\ |\vec{F}_+| = |\vec{F}_-| \Rightarrow \begin{cases} |\vec{F}_{+x}| = |\vec{F}_{-x}| \\ |\vec{F}_{+y}| = |\vec{F}_{-y}| \end{cases}$

$\ast\ r^2 = x^2 + (\frac{d}{2})^2$

$\vec{F}_{net} = \underset{0}{\cancel{F_x}}\,\hat{i} + F_y\,\hat{j} = F_{-y}(-\hat{j}) + F_{+y}(-\hat{j}) \Rightarrow \vec{F}_{net} = -(F_{-y} + F_{+y})\,\hat{j}$

$\rightarrow \vec{F}_{net} = -2F_{+y}\,\hat{j}$

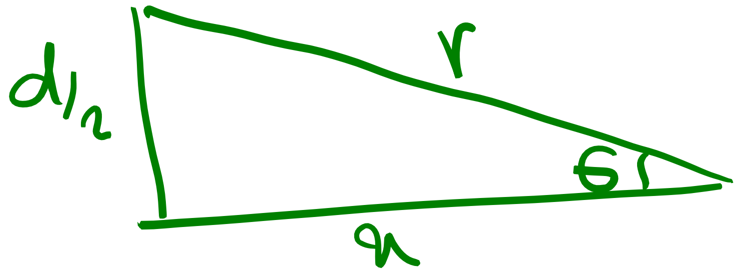
now problem converts to find the magnitude of F_{+y}

$$\vec{F}_{net} = -2(F_+ \sin\theta)\,\hat{j} \rightarrow |\vec{F}_{net}| = 2F_+\sin\theta$$

$$F_+ = k\frac{Q_+ q}{r^2} \rightarrow \vec{F}_{net} = -2k\frac{Q_+ q}{r^2}\sin\theta\,\hat{j}$$

$$r = \sqrt{x^2 + \left(\frac{d}{2}\right)^2}$$

$$\sin\theta = \frac{d/2}{\sqrt{x^2 + \left(\frac{d}{2}\right)^2}}$$



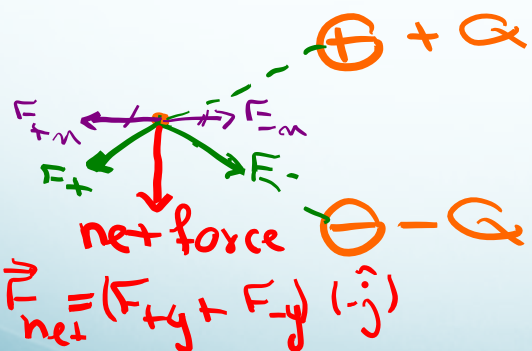
$$\sin\theta = \frac{d/2}{r}$$

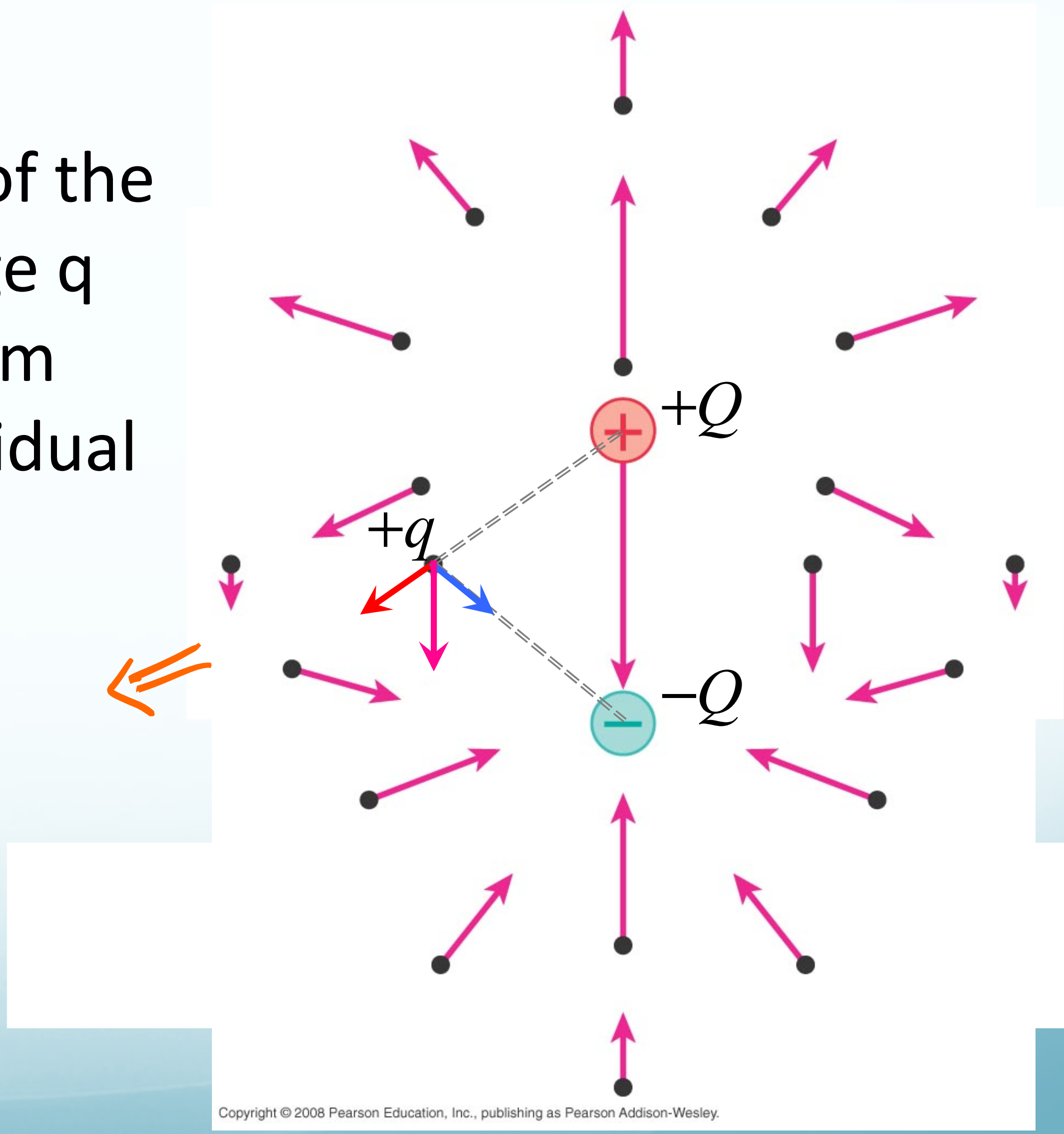
$$\Rightarrow \vec{F}_{net} = -2k\frac{Q_+ q}{x^2 + (d/2)^2}\cdot\frac{d/2}{\sqrt{x^2 + (d/2)^2}}\,\hat{j}$$

$$\rightarrow \left|\vec{F}_{net}\right| = 2\left(\frac{KqQ}{\left(x^2 + (d/2)^2\right)^{3/2}}\right)\sin\theta = \frac{KqQd}{\left(x^2 + (d/2)^2\right)^{3/2}}$$

Superposition with Building Blocks

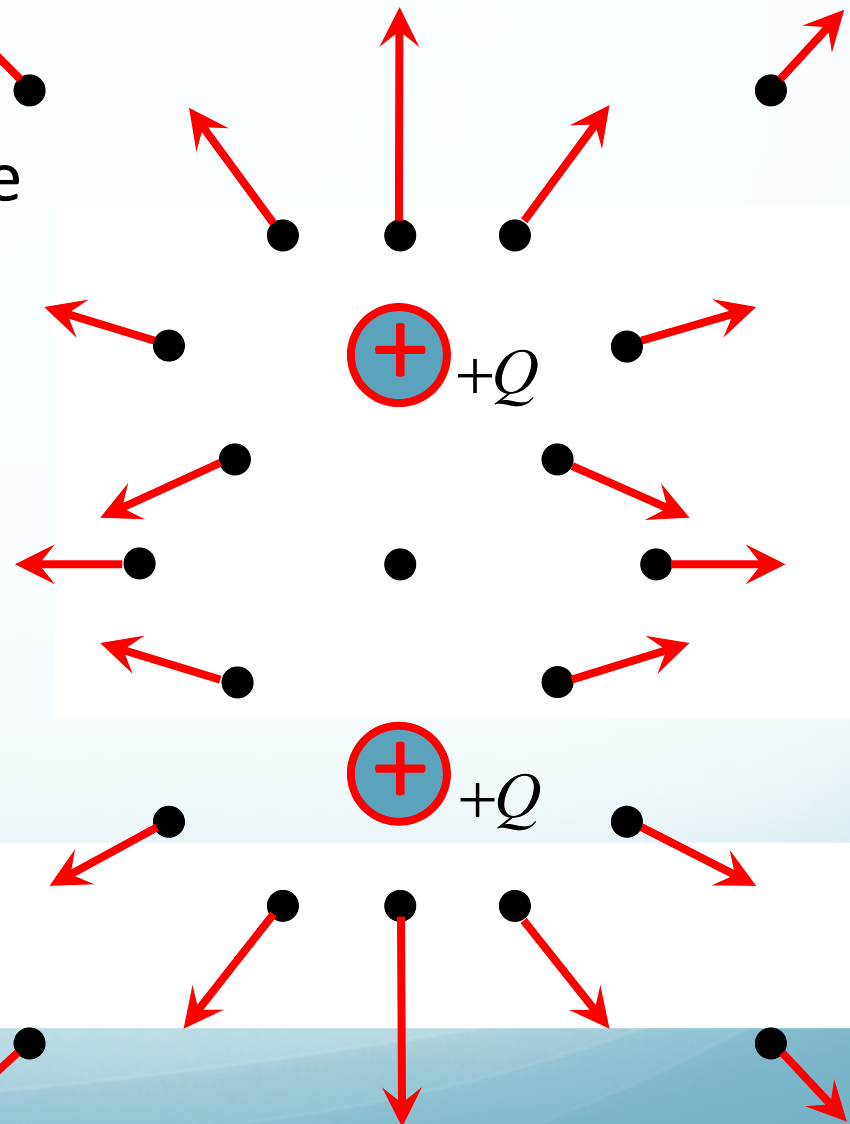
The vector represents the magnitude and direction of the electric force on the charge q **at that point**. It comes from superposition of the individual forces from +Q and –Q.





Superposition with Building Blocks

The vector represents the magnitude and direction of the electric force on the charge q **at that point**. It comes from superposition of the individual forces from +Q and +Q.



This section we talked about:

Chapter 21.1-3: Examples

See you on Wednesday