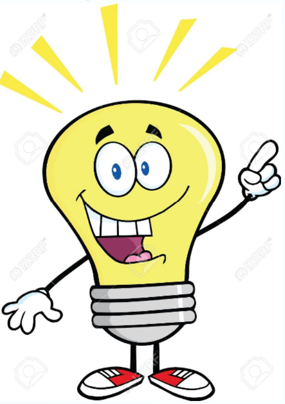
Electricity and Magnetism

- Physics 259 – L02
- Lecture 36

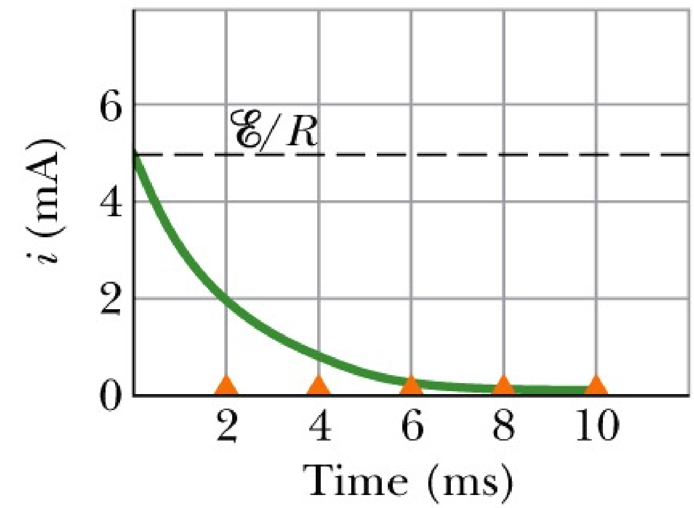
Chapter 27

27 Circuits

Case 1: Charging a capacitor



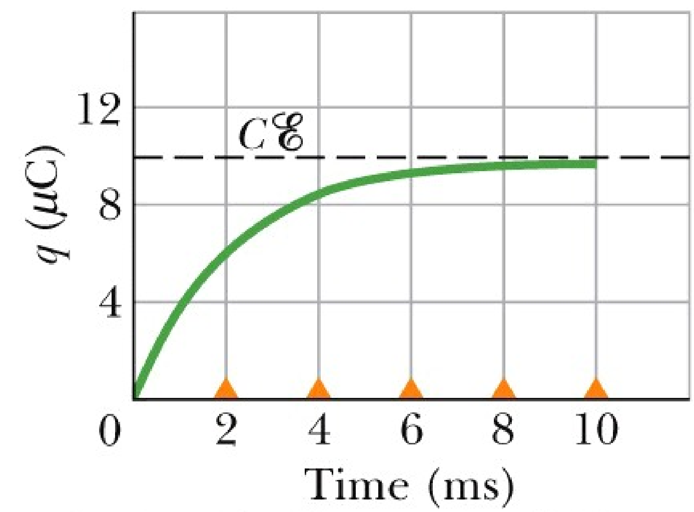
$$i = i_0 e^{-t/RC}$$



$= CV$

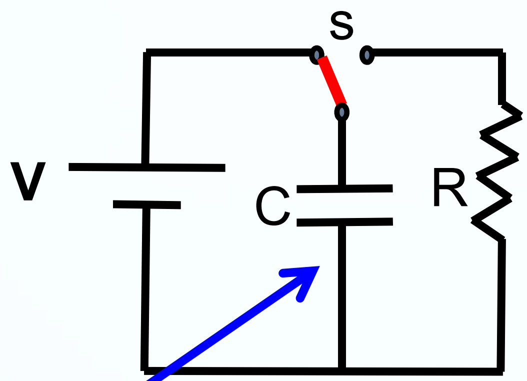
$$q = \varepsilon C\left(1 - e^{-t/RC}\right) = Q_f\left(1 - e^{-t/RC}\right)$$

charge of fully
charged capacitor

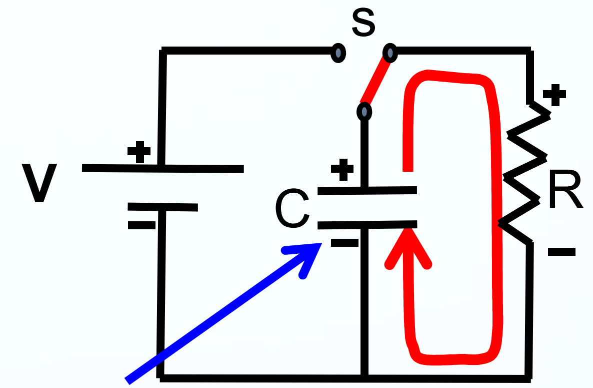


Case 2: Discharging a capacitor

Switch is connected to the left for a long time until t=0-



Switch is suddenly flipped to the right at t=0+

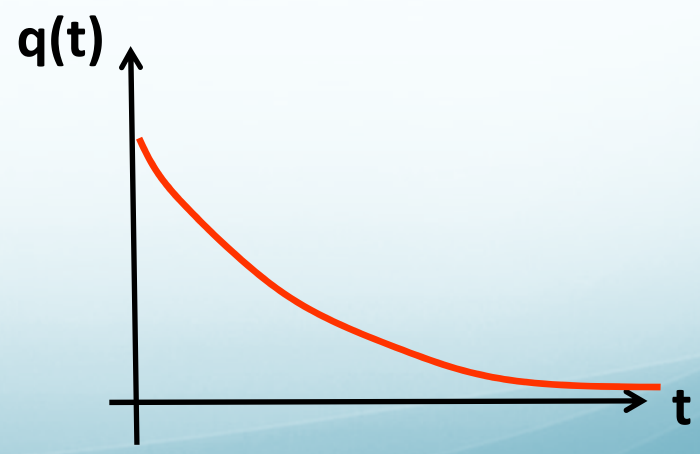


Capacitor charges up to voltage V

Capacitor discharges

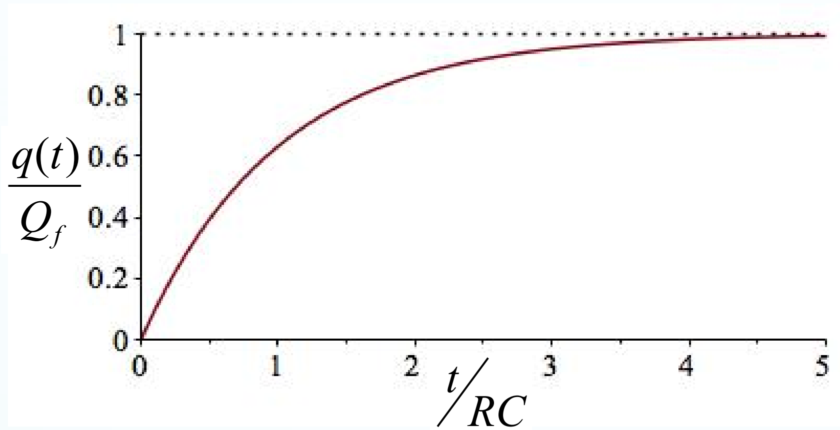
$$\boxed{q(t) = q_0 e^{-t/RC}}$$

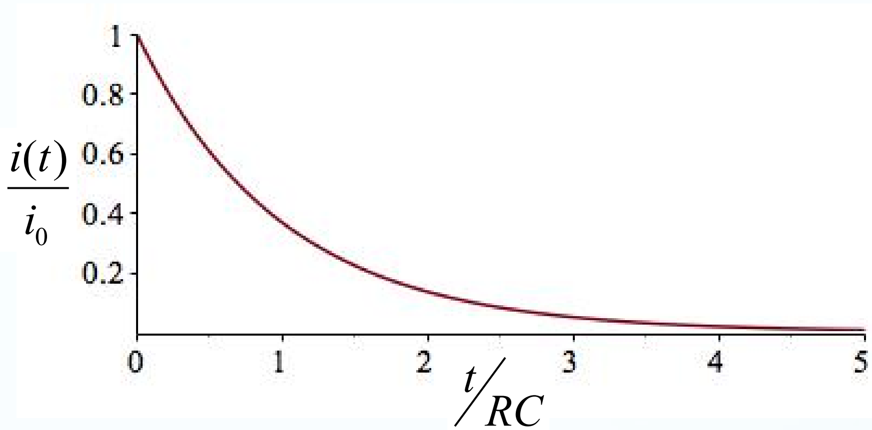
$$\boxed{i(t) = i_0 e^{-t/RC}}$$



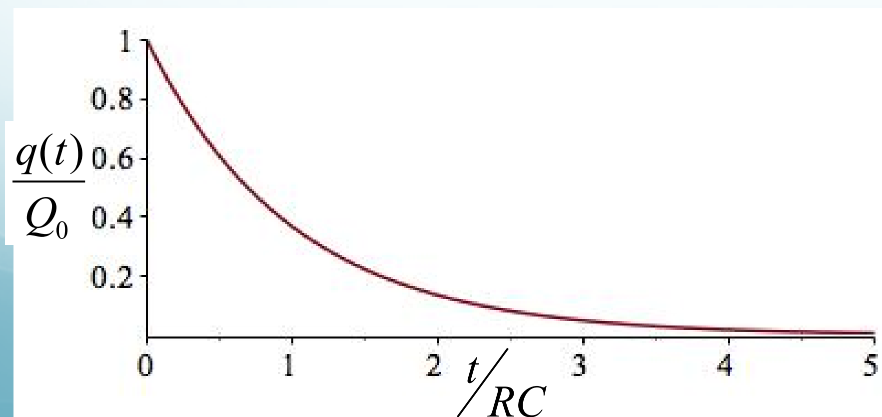
Charging/Discharging Capacitors

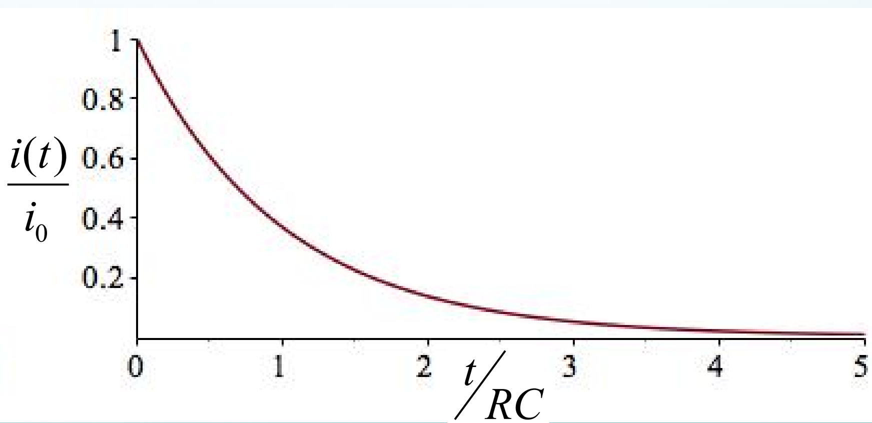
Charging: $q(t) = Q_f\left(1 - e^{-\frac{t}{RC}}\right)$ $\qquad$ $i(t) = i_0 e^{-\frac{t}{RC}}$

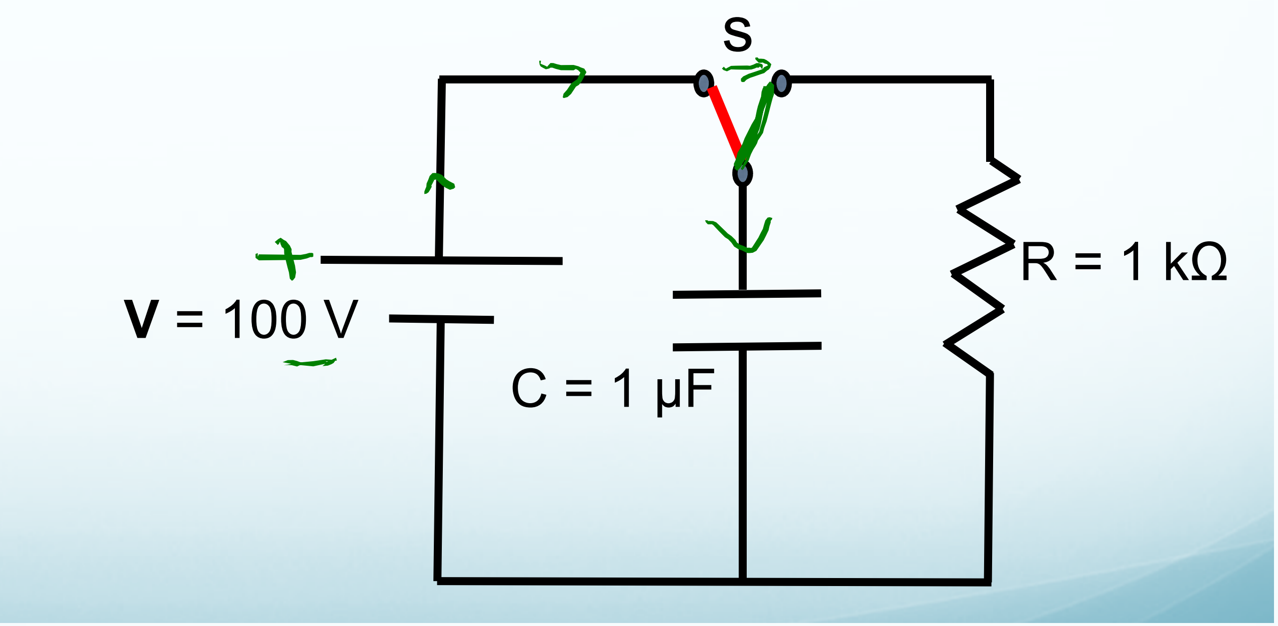




Discharging: $q(t) = Q_0 e^{-\frac{t}{RC}}$ $\qquad$ $i(t) = i_0 e^{-\frac{t}{RC}}$







I in this circuit

The RC time constant

The constant RC pops up in the exponential factor for both charging and discharging capacitors. What does it represent?

The units of RC is seconds: $$[RC] = \frac{V}{A}\frac{C}{V} = \frac{C}{C/s} = s$$

We call RC the "**RC time constant**" and it tells us how quickly a capacitor can charge or discharge.

$$\boxed{RC \equiv \tau}$$

After a time τ, the charge on a discharging capacitor is reduced by a factor of 1/e. After a time $N\tau$, it is reduced by a factor of $1/e^N$

$$\boxed{q(t) = Q_0 e^{-\frac{t}{\tau}}}$$

Top Hat Question

An RC circuit is shown below. Initially the switch is open and the capacitor is fully charged. At time t = 0, the switch is closed.

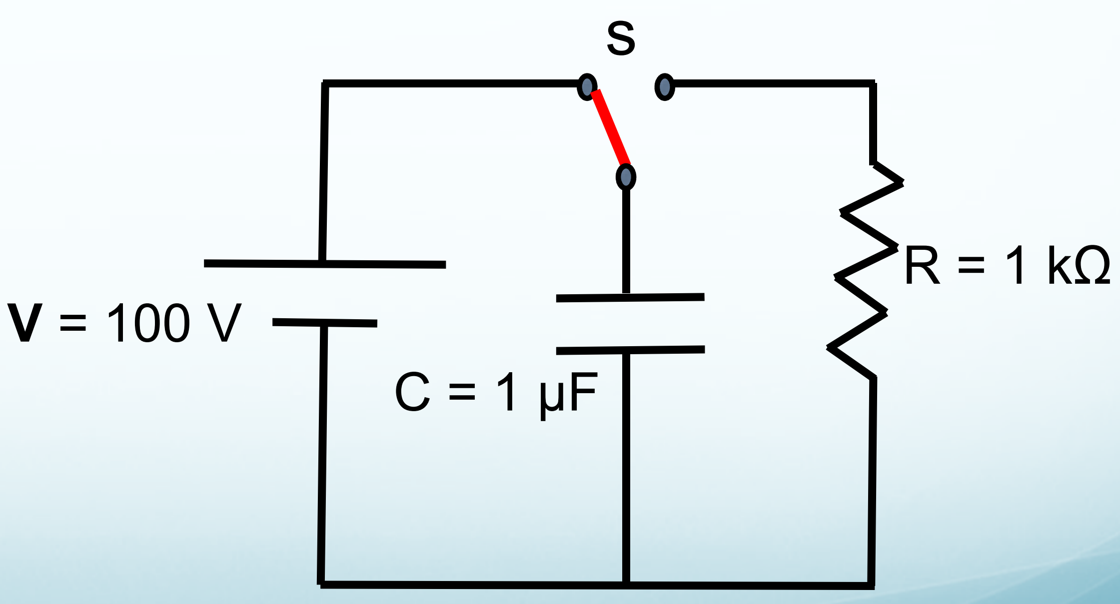
How much charge is left on the capacitor plates after t = 10 ms?

A. 0.67 nC

B. 14 μC

C. 37 μC

D. 4.5 nC



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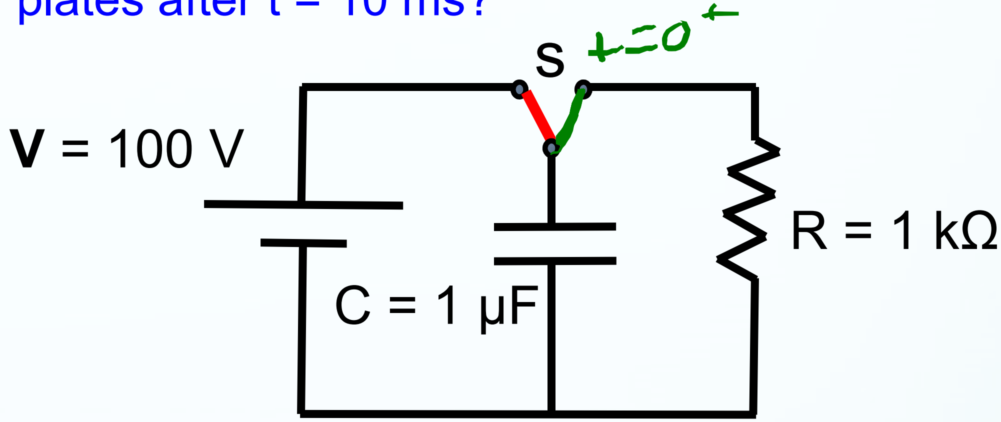


discharging $\Rightarrow q(t) = q_0 e^{-t/RC}$

$V = 100\,V$

$R = 1\,k\Omega$

$C = 1\,\mu F$

$q_0 \to q_f$ of charging $\Rightarrow q_0 = CV$!

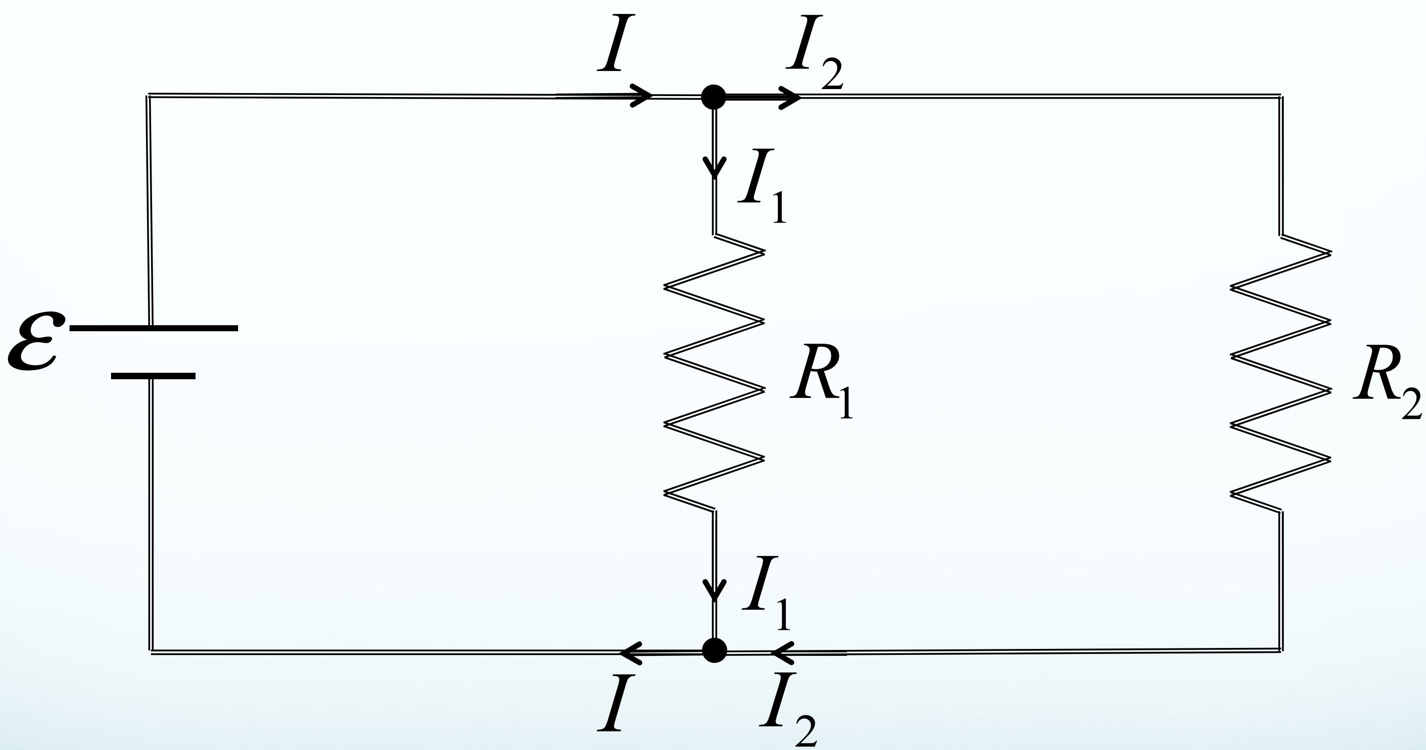
$= 1\mu F \times 100\,V = 100\,\mu C$

$\to q(t) = 100\,\mu C \times e^{-t/RC} = (100\,\mu C)\, e^{-\frac{10}{1ms}}$

$q(t) = (100\,\mu C)\, e^{-\frac{10}{1ms}}$ 4.54 nC

Kirchhoff's junction rule

A slightly more complicated circuit has multiple branches with resistors in parallel



Current is the flow of charges. Charge has to be conserved.

Current into junction = current out of junction

$$\boxed{I = I_1 + I_2}$$

This section we talked about:

Chapter 27

See you on Monday