Last time

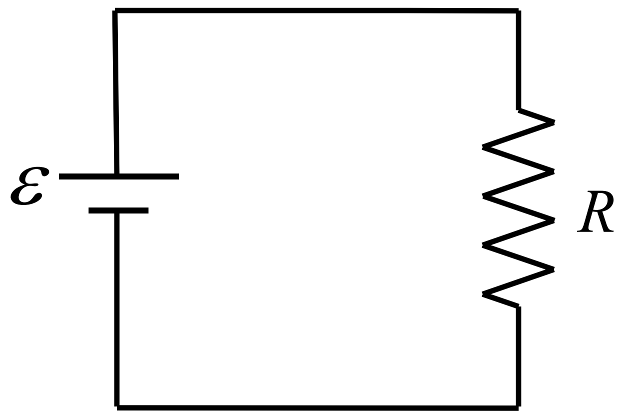
- Capacitors in electric circuits: how charges move
- Capacitors in series

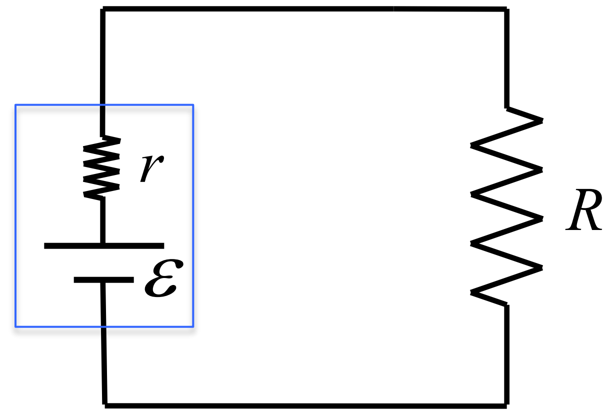
This time

- Non-ideal batteries and electromotive force
- Kirchhoff's loop rule with capacitors
- Capacitors in parallel
- More complicated capacitor circuit (tutorial)

Non-ideal Batteries: internal resistance

Every voltage source has **some** internal resistance to it. Usually this can be ignored but not always





The internal resistance simply acts as a resistor in series with the rest of the circuit.

$$\varepsilon - Ir - IR = 0$$

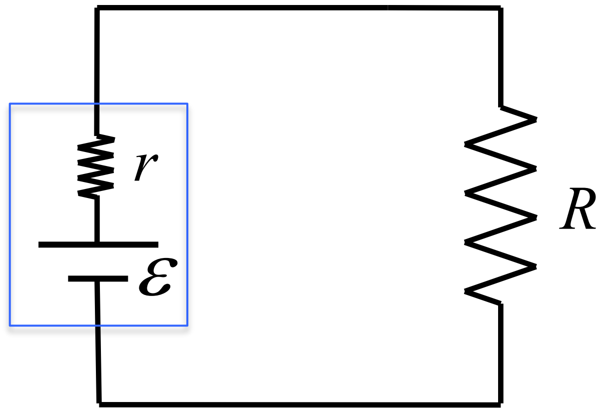
$$I = \frac{\varepsilon}{(r+R)}$$

$$P_\varepsilon = I\varepsilon = \frac{\varepsilon^2}{R+r}$$

Power output by the emf source

$$P_R = I^2 R = \frac{\varepsilon^2 R}{(R+r)^2}$$

Power dissipated by the resistive load



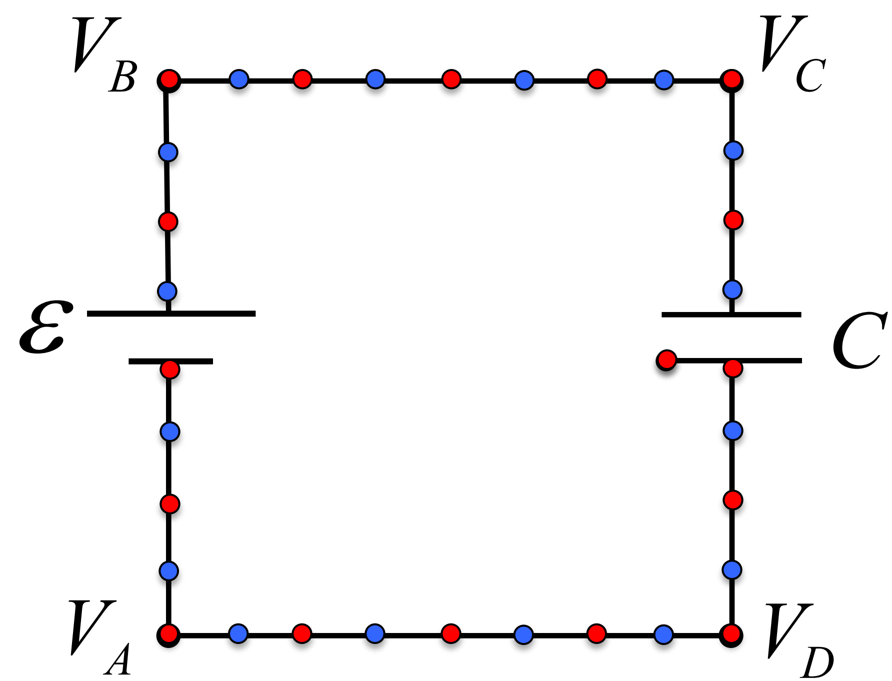
There is some energy loss due to r when the battery is charged and some energy loss when the battery is discharged.

This indicates that efficiency for charging and discharging a battery is less than 100% as required by the second law of thermodynamics.

A Basic Circuit with Capacitor

The simplest capacitor circuit has an ideal battery, ideal wires, and a single capacitor.

The battery causes charges to flow from the bottom plate to the top plate. This creates a potential ΔV_C between the two plates. Remember charges never "jump the gap" between the two plates of a capacitor.



$$\Delta V_C = QC$$

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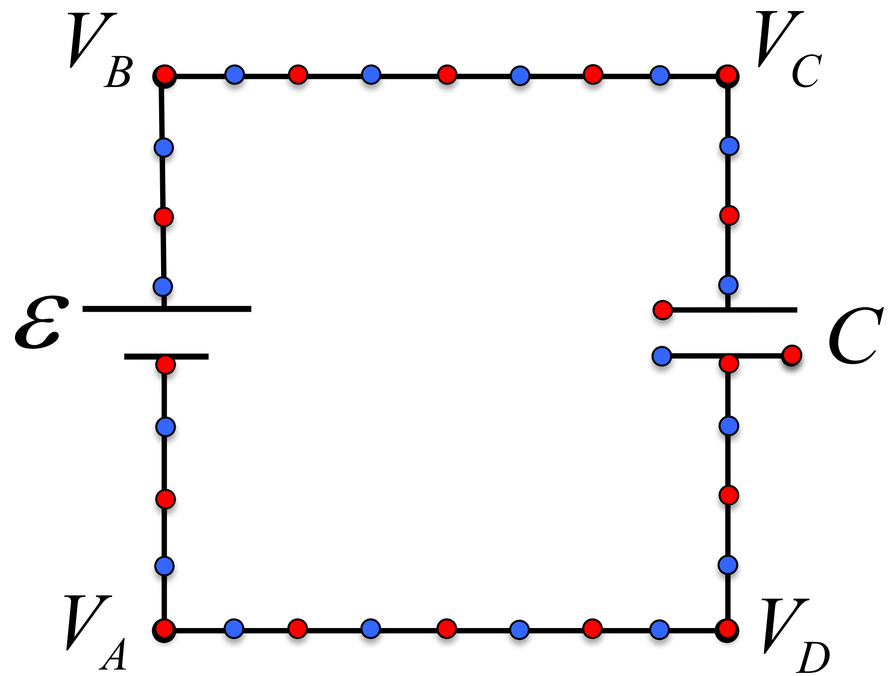

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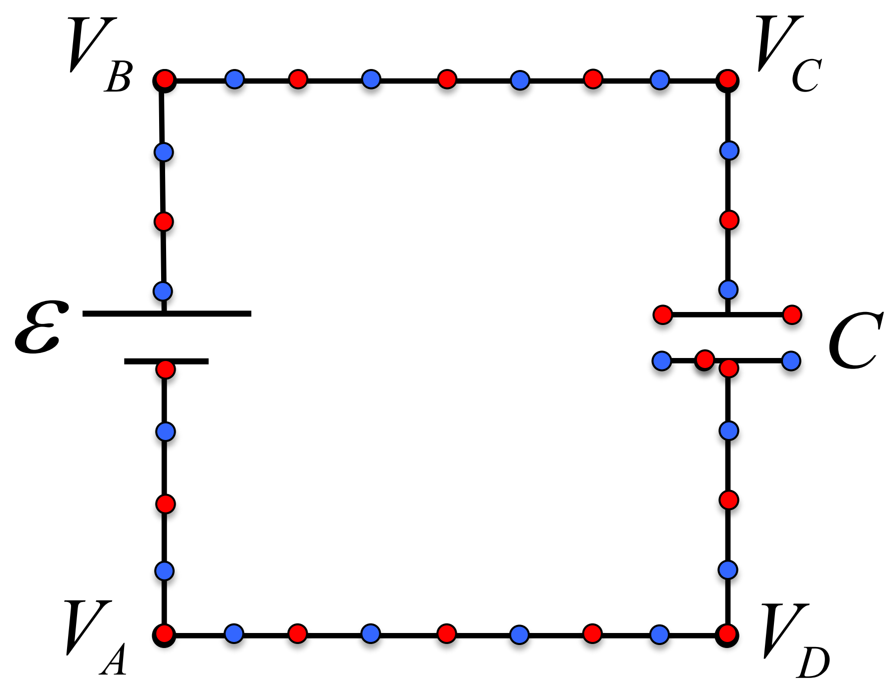

A Basic Circuit with Capacitor

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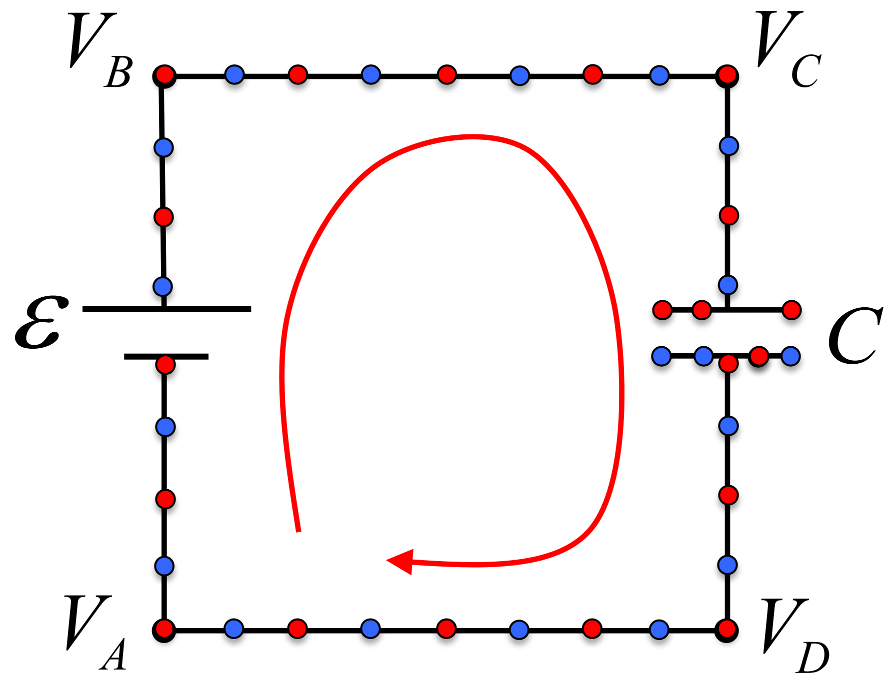
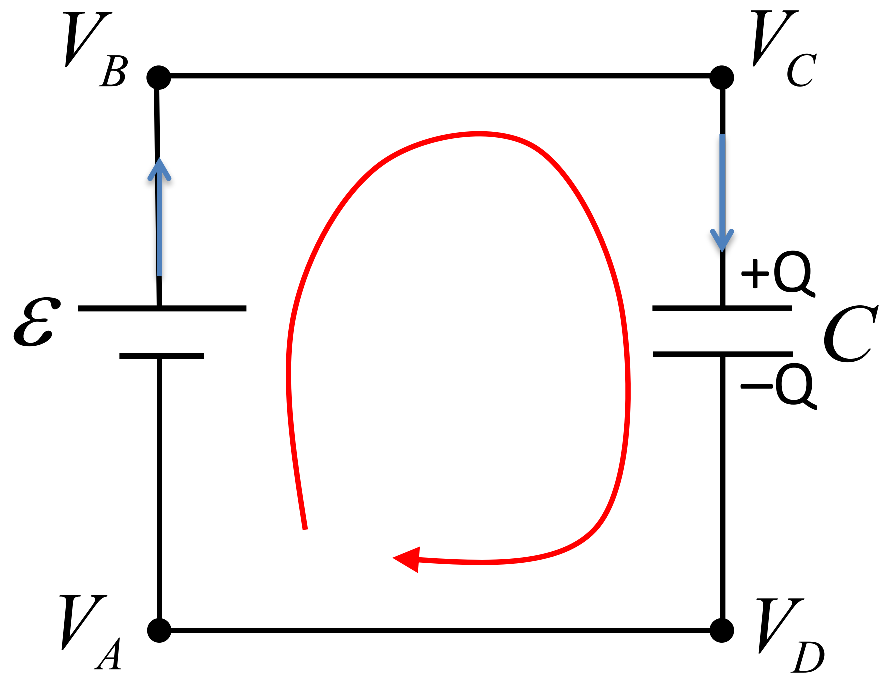


$$\boxed{\Delta V_{AB} + \Delta V_{BC} + \Delta V_{CD} + \Delta V_{DA} = 0}$$

A Basic Circuit

The voltage across a capacitor is **negative** if you are going around the loop in the direction **from the + plate to the – plate**. Current flows **from the negative** terminal **to the positive** terminal



ideal wires

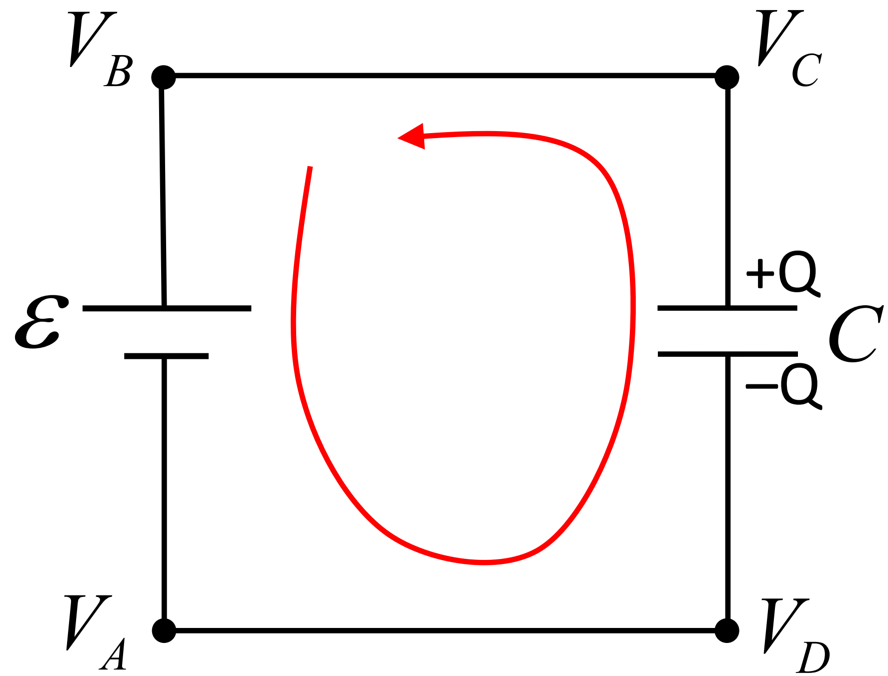
$$\Delta V_{AB} + \cancel{\Delta V_{BC}} + \Delta V_{CD} + \cancel{\Delta V_{DA}} = 0$$

$$\mathcal{E} - \frac{Q}{C} = 0$$

A Basic Circuit

The voltage across a capacitor is **positive** if you are going around the loop in the direction **from – plate to + plate**.

Voltage across a battery is **negative** going **from positive to negative**



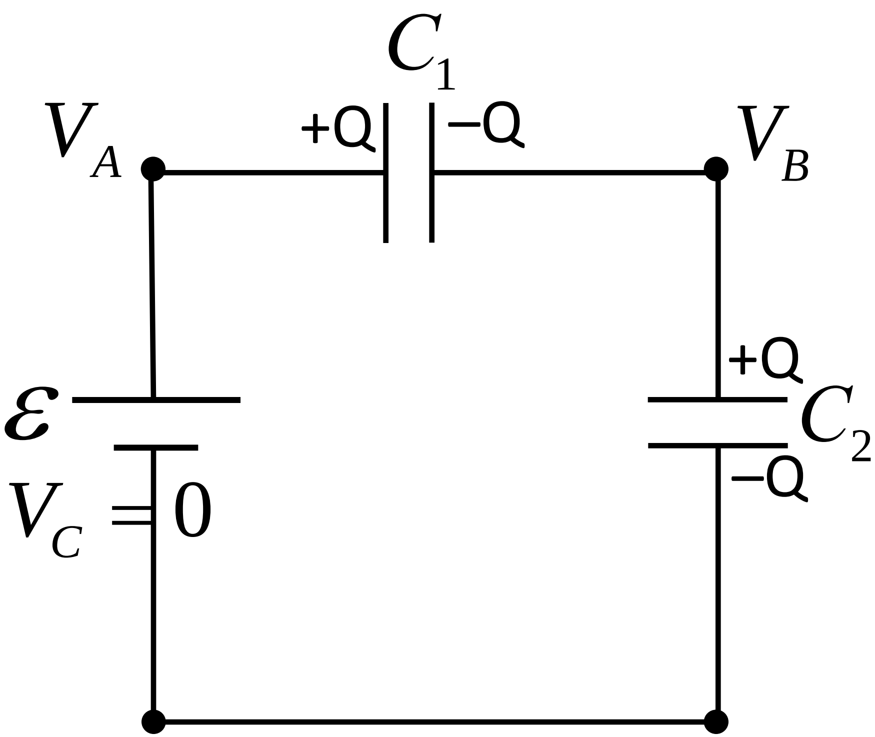
ideal wires

$$\Delta V_{BA} + \cancel{\Delta V_{AD}} + \Delta V_{DC} + \cancel{\Delta V_{CB}} = 0$$

$$-\mathcal{E} + \frac{Q}{C} = 0$$

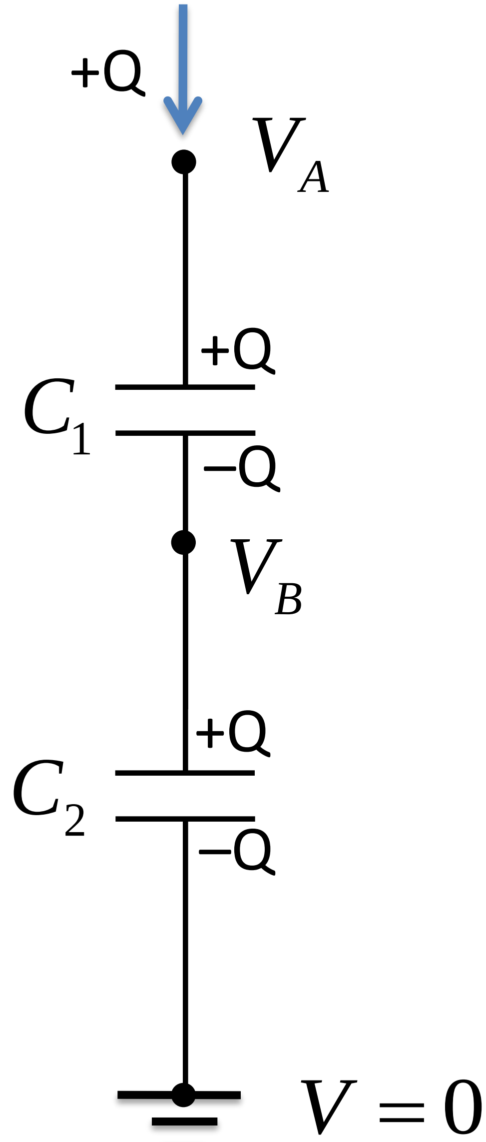
Same as before

(a) and (c) are drawn differently,
otherwise they are equivalent.

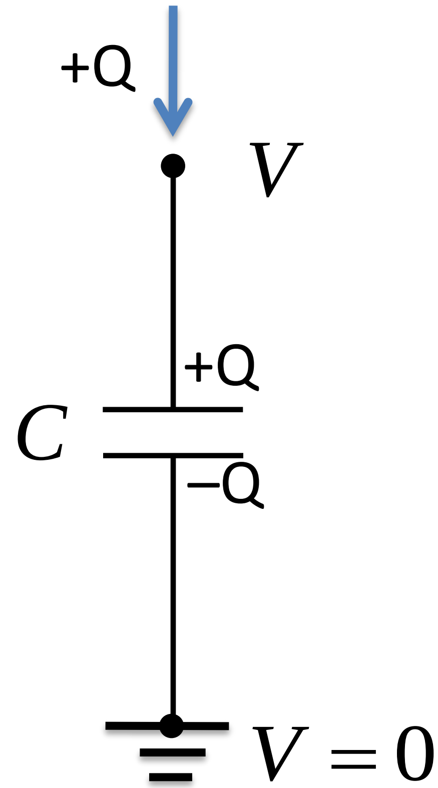


(c)

$$V_1 = V_A - V_B$$
$$V_2 = V_B - V_C$$



(a)



(b)

Capacitors in series

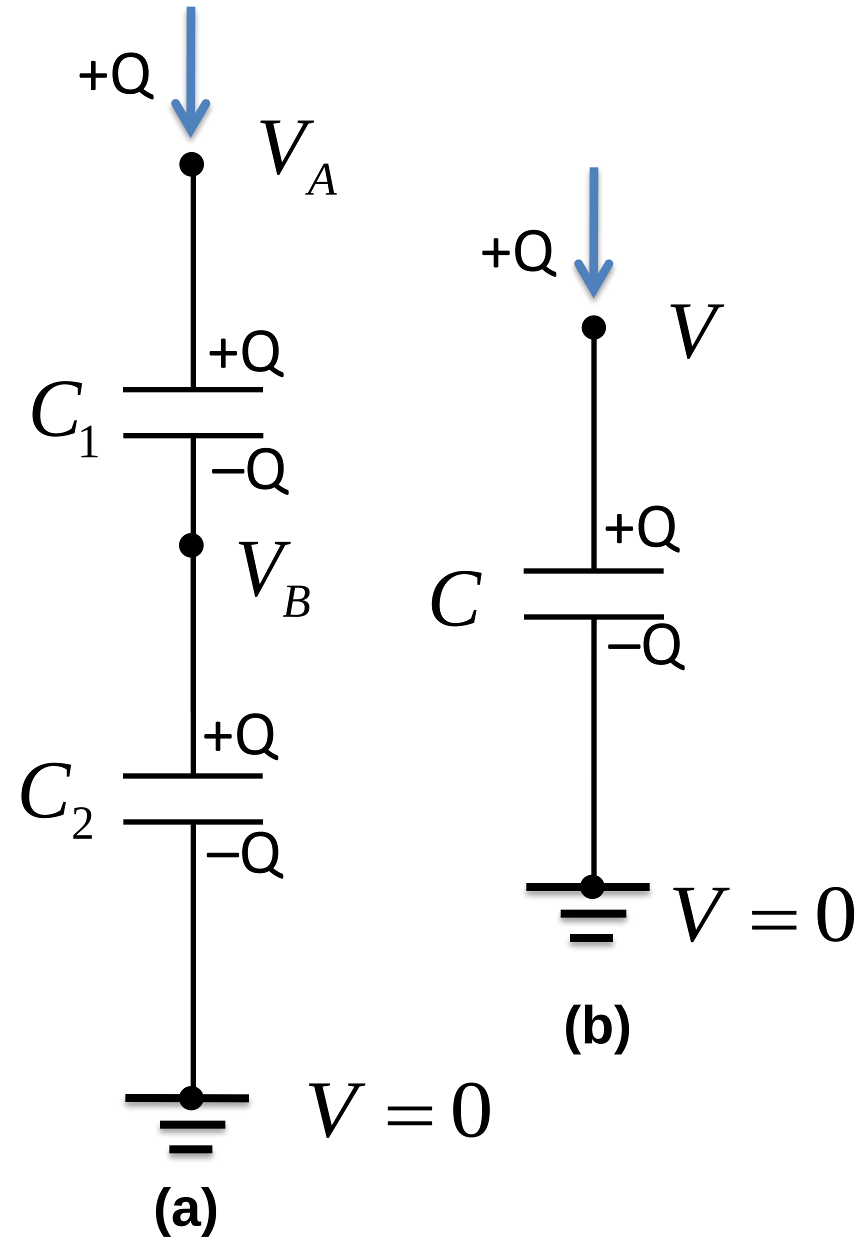
Inject charge +Q at point A.

1. +Q on the top plate of C_1 will induce a charge –Q on the lower plate.

2. –Q on the lower plate of C_1 will induce a charge +Q on the top plate of C_2.

3. +Q on the top plate of C_2 will induce a charge –Q on the lower plate.

4. This forces a charge of +Q to move to ground, leaving a charge of –Q on the lower plate of C_2.



For (a) and (b) to be equivalent

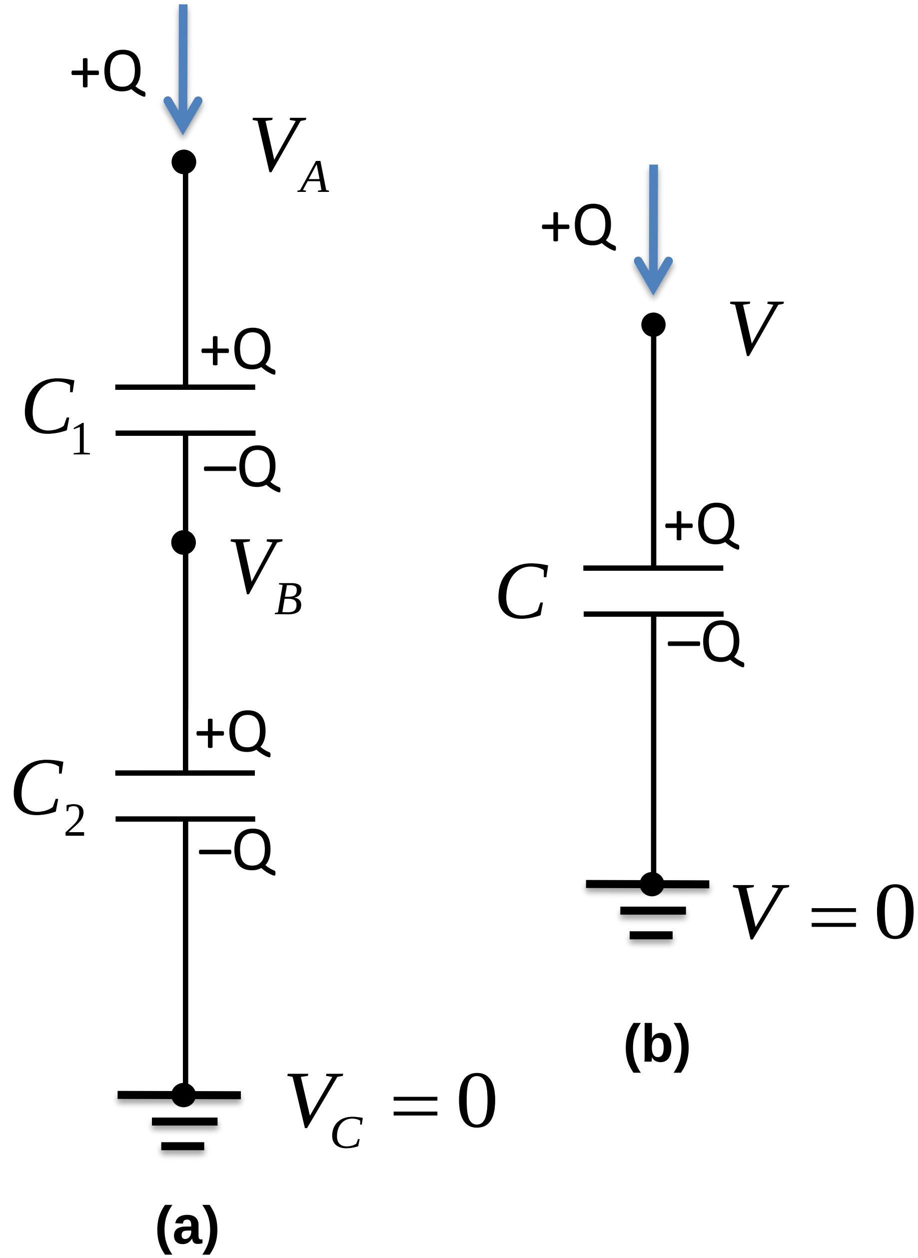
$$U_a = U_b$$

$$U_a = \frac{Q^2}{2C_1} + \frac{Q^2}{2C_2}$$

$$U_b = \frac{Q^2}{2C}$$

$$\frac{Q^2}{2C_1} + \frac{Q^2}{2C_2} = \frac{Q^2}{2C}$$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$



$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

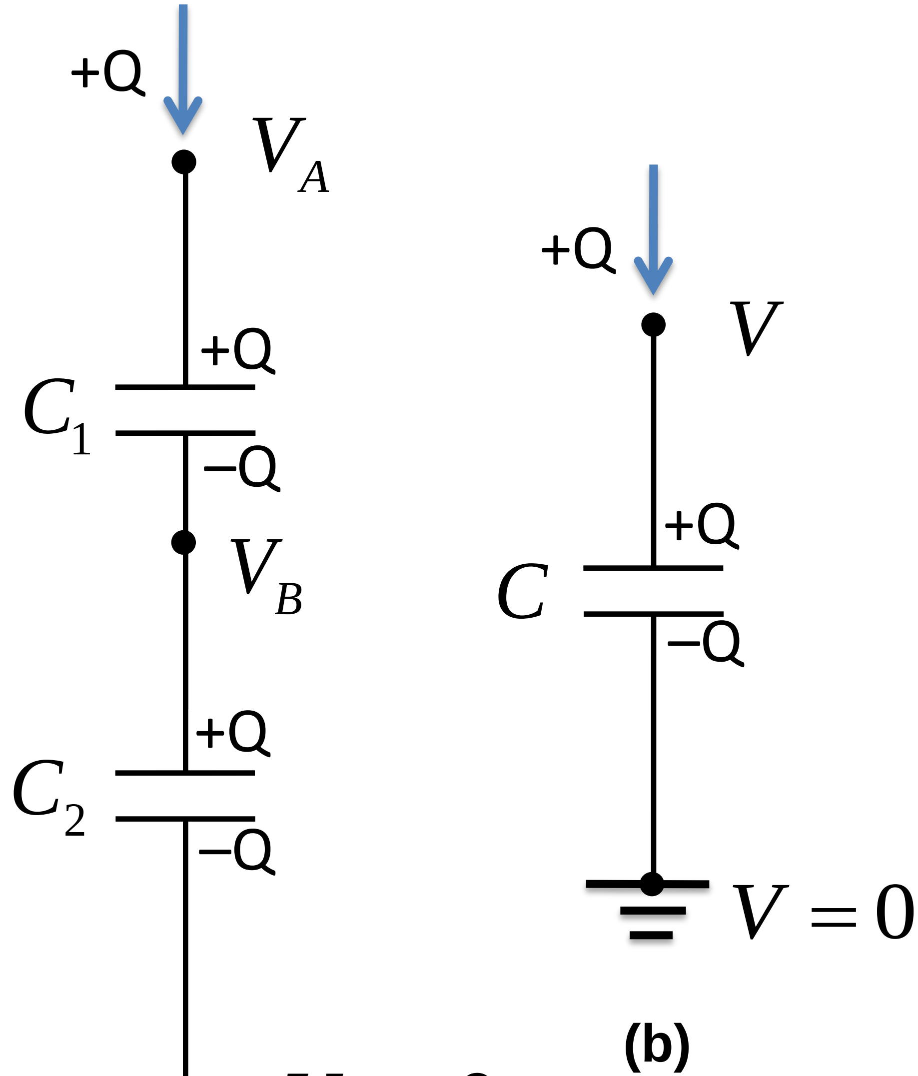
$$C_1 = \frac{Q}{V_1} \Rightarrow \frac{1}{C_1} = \frac{V_1}{Q}$$

$$C_2 = \frac{Q}{V_2} \Rightarrow \frac{1}{C_2} = \frac{V_2}{Q}$$

$$C = \frac{Q}{V} \Rightarrow \frac{1}{C} = \frac{V}{Q}$$

$$\frac{V}{Q} = \frac{V_1}{Q} + \frac{V_2}{Q}$$

$$V = V_1 + V_2$$



$$V_1 = V_A - V_B$$

$$V_2 = V_B - V_C$$

Capacitors in series

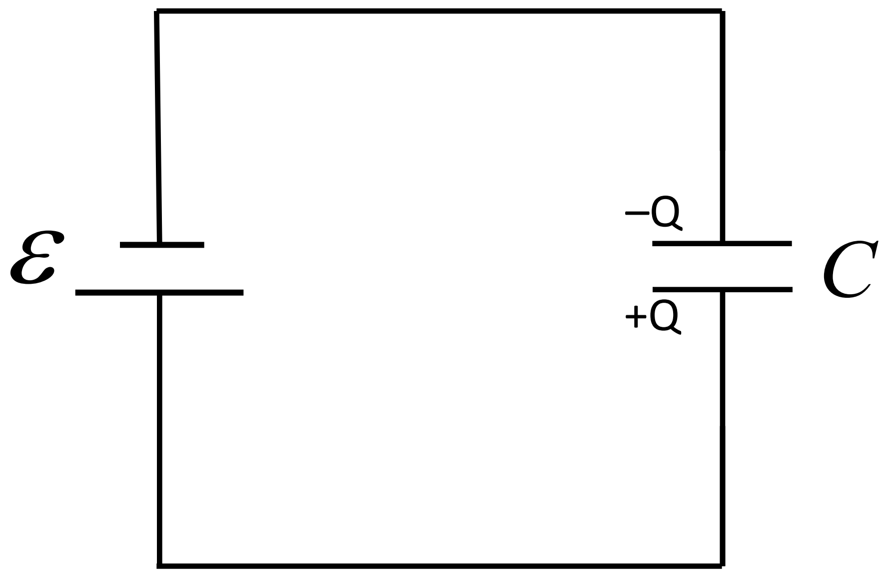
$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots$$

$$Q = Q_1 = Q_2 = \ldots$$

$$V = V_1 + V_2 + \ldots$$

TopHat Question

What is the charge on the top plate of the capacitor in the circuit shown?
$\mathcal{E}$ = 12 V and C = 0.25 μF.



A. Q = 3.0 μC

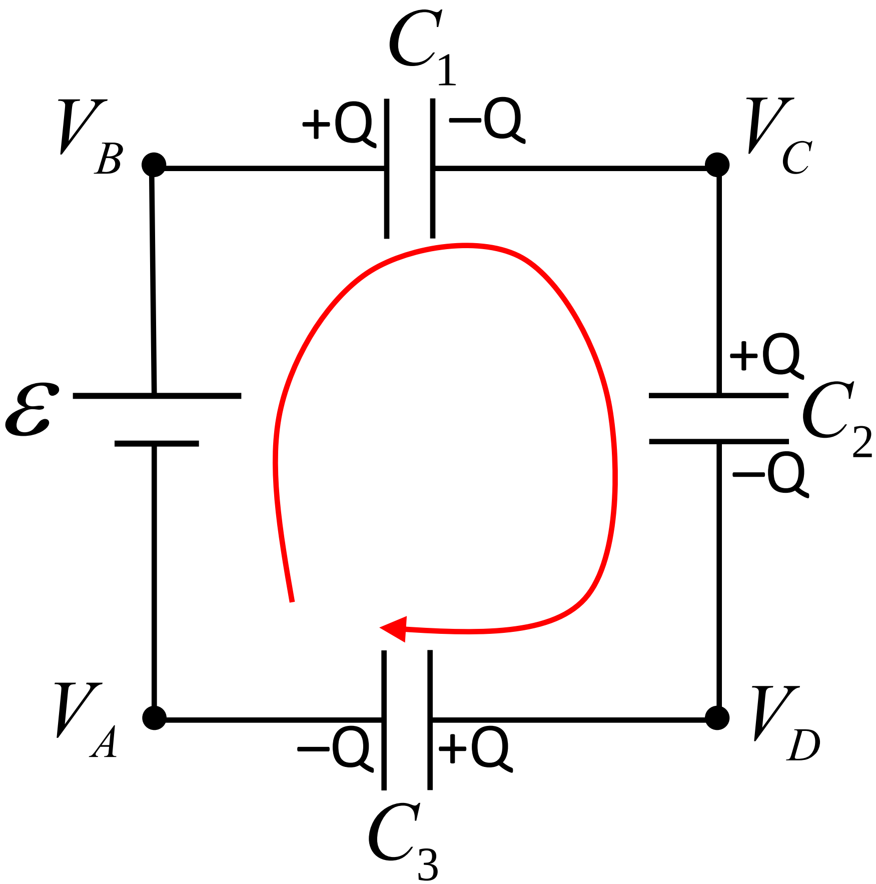
A. Q = 48 μC

C. Q = 21 nC

D. Q = –3.0 μC

Capacitors in Series

A slightly more complicated circuit has multiple capacitors in series



Kirchhoff's Loop Rule:

$$\Delta V_{AB} + \Delta V_{BC} + \Delta V_{CD} + \Delta V_{DA} = 0$$

Charge on each plate is the same

$$\mathcal{E} - \frac{Q}{C_1} - \frac{Q}{C_2} - \frac{Q}{C_3} = 0$$

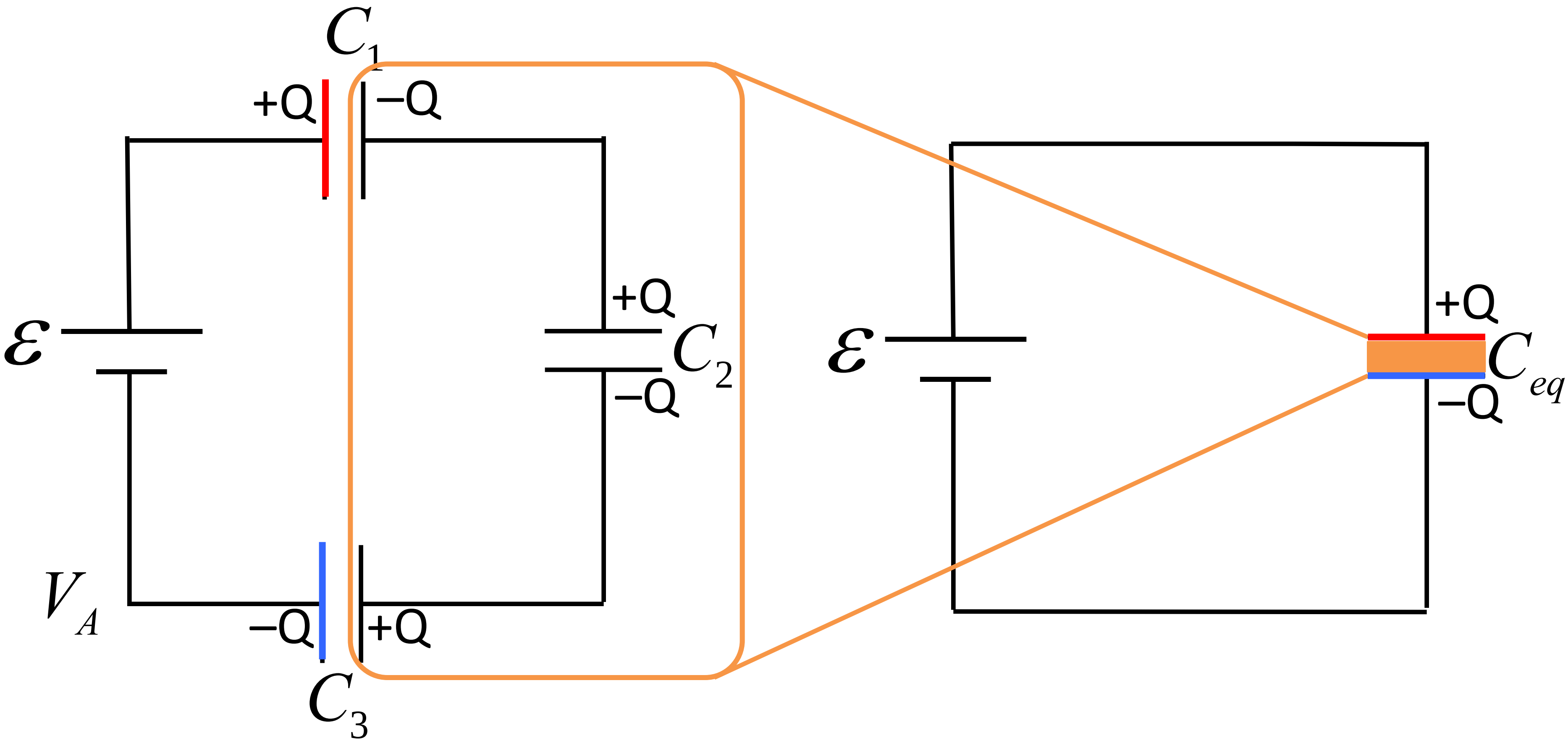
Rewrite this as

$$\mathcal{E} - Q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right) = 0$$

Define an equivalent capacitance

$$\mathcal{E} - \frac{Q}{C_{eq}} = 0$$

Capacitors in Series



Capacitors in series act like a single equivalent capacitor:

$$C_{eq} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \right)^{-1}$$

TopHat Question

In the circuit shown in Figure 1, points A, B, C, D, E, and F are connected by conducting wires with virtually no resistance. Which of the choices given below is correct?

a. $V_A = V_B < V_C$

b. $V_A = V_B = V_C$

c. $V_A = V_D$

d. $V_D = V_E = V_F$

e. (b) and (d)

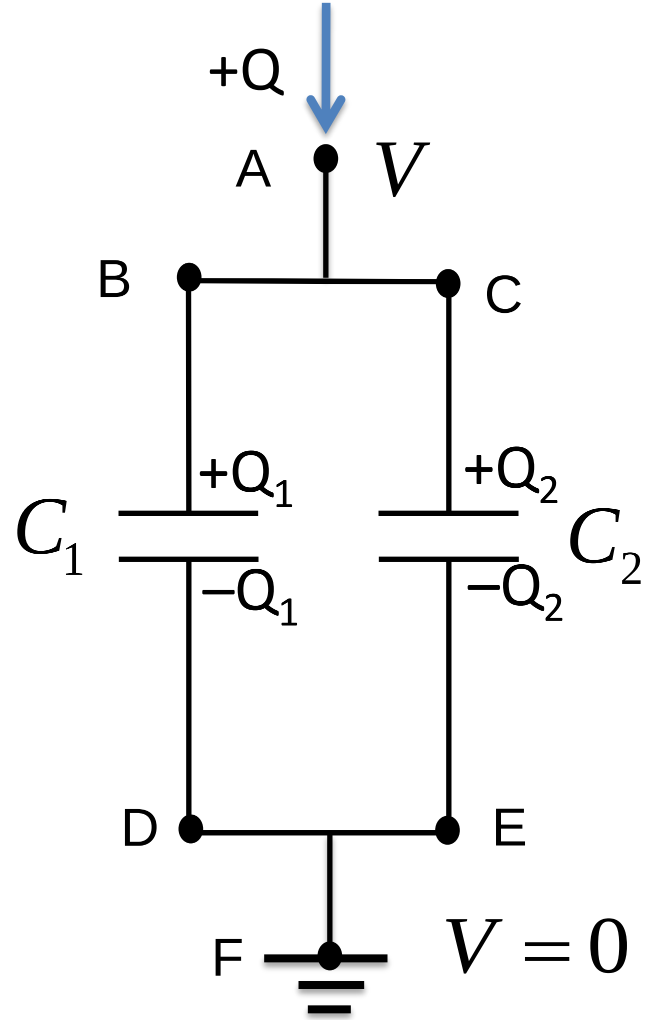
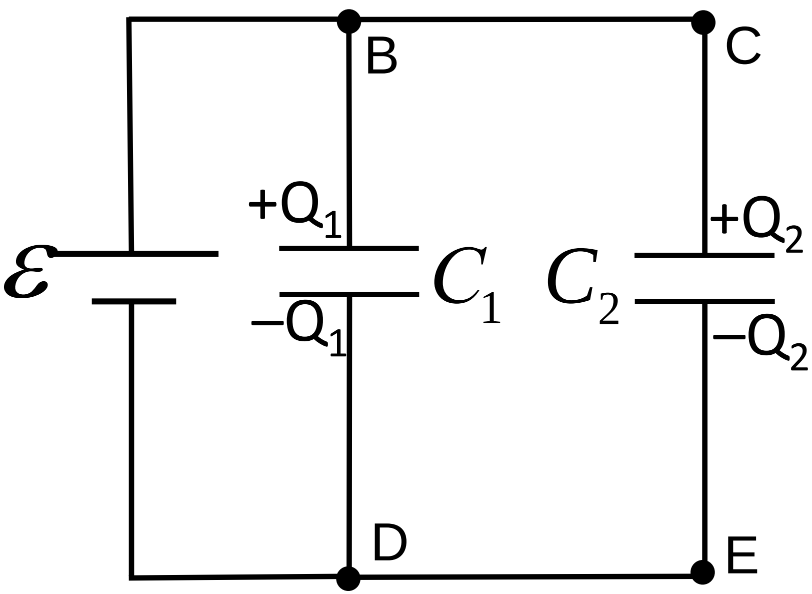


Figure 1

Capacitors in Parallel

(a) and (c) are drawn differently, otherwise they are equivalent.

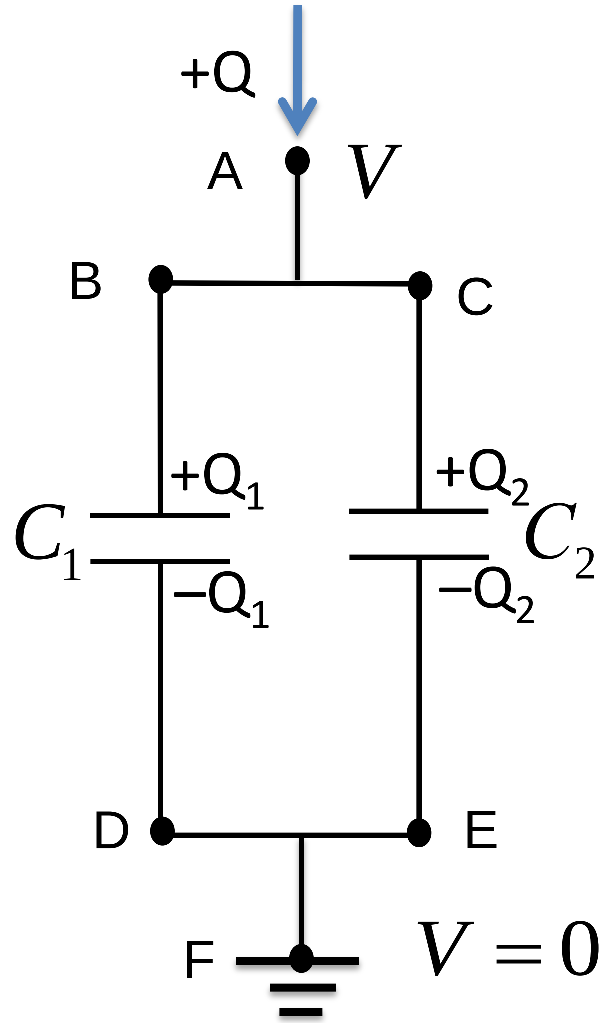


(c)

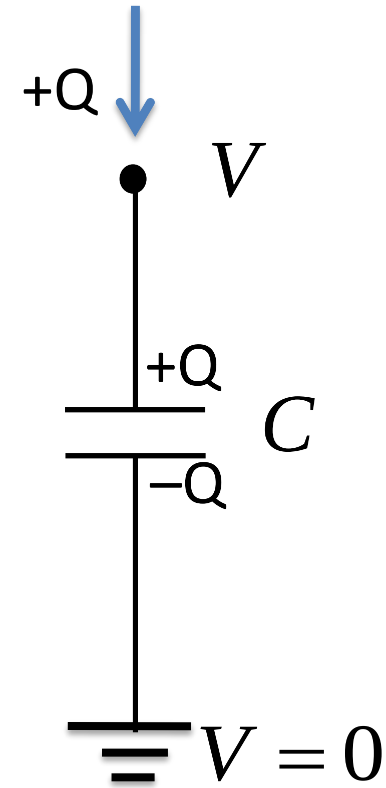
$$V_1 = V_B - V_D$$

$$V_2 = V_C - V_E$$

$$V_1 = V_2$$



(a)



(b)

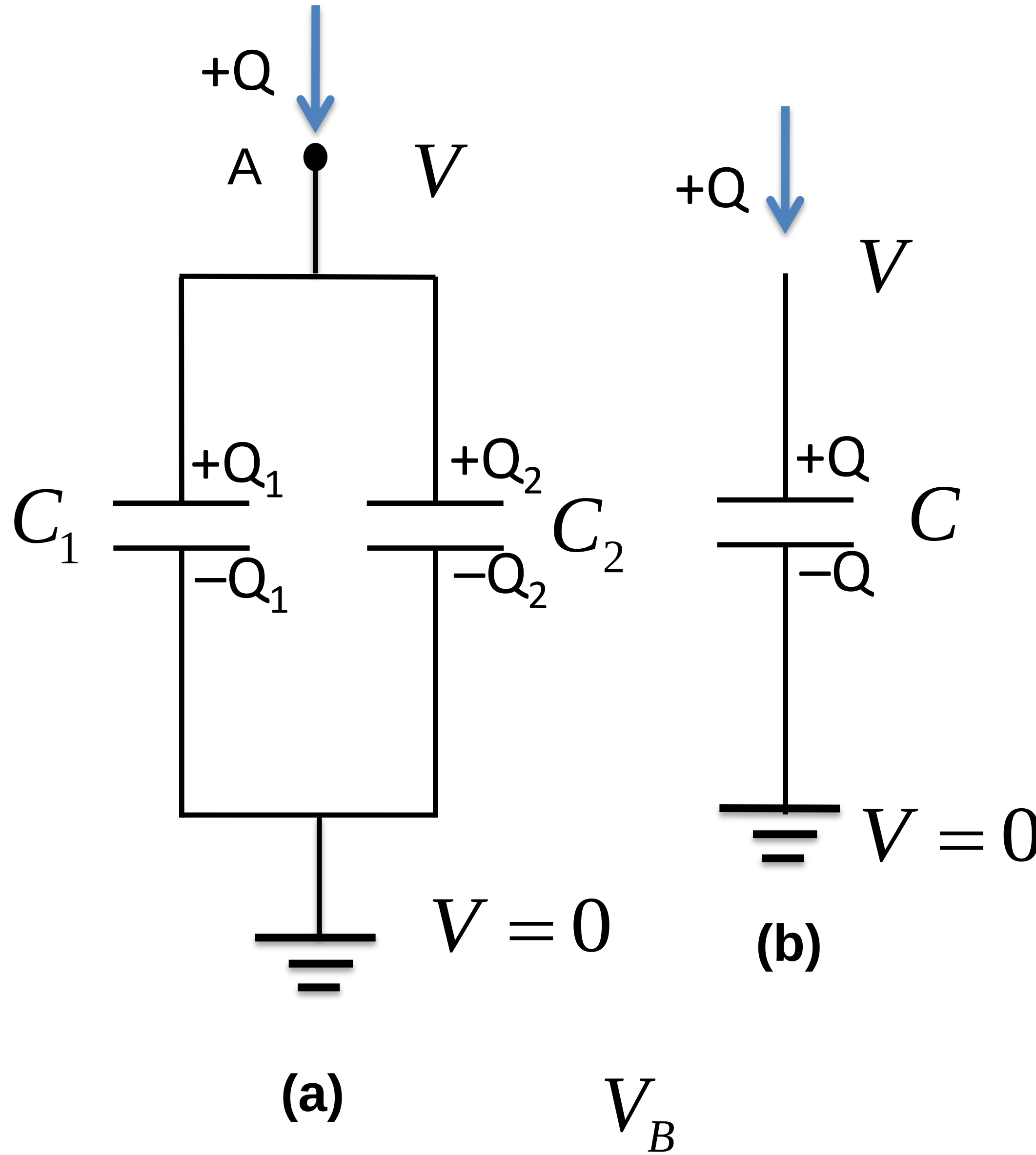
Capacitors in Parallel

Inject charge +Q at point A.

1. $+Q_1$ will reside on the top plate of C_1 and it will induce a charge $-Q_1$ on the lower plate.

2. $+Q_2$ will reside on the top plate of C_2 and it will induce a charge $-Q_2$ on the lower plate.

3. This forces a charge of +Q to move to ground, leaving a charge of $-Q_1$ on the lower plate of C_1 and $-Q_2$ on the lower plate of C_2.



Capacitors in Parallel

From conservation of charge we have

$$Q = Q_1 + Q_2$$

$$V = V_1 = V_2$$

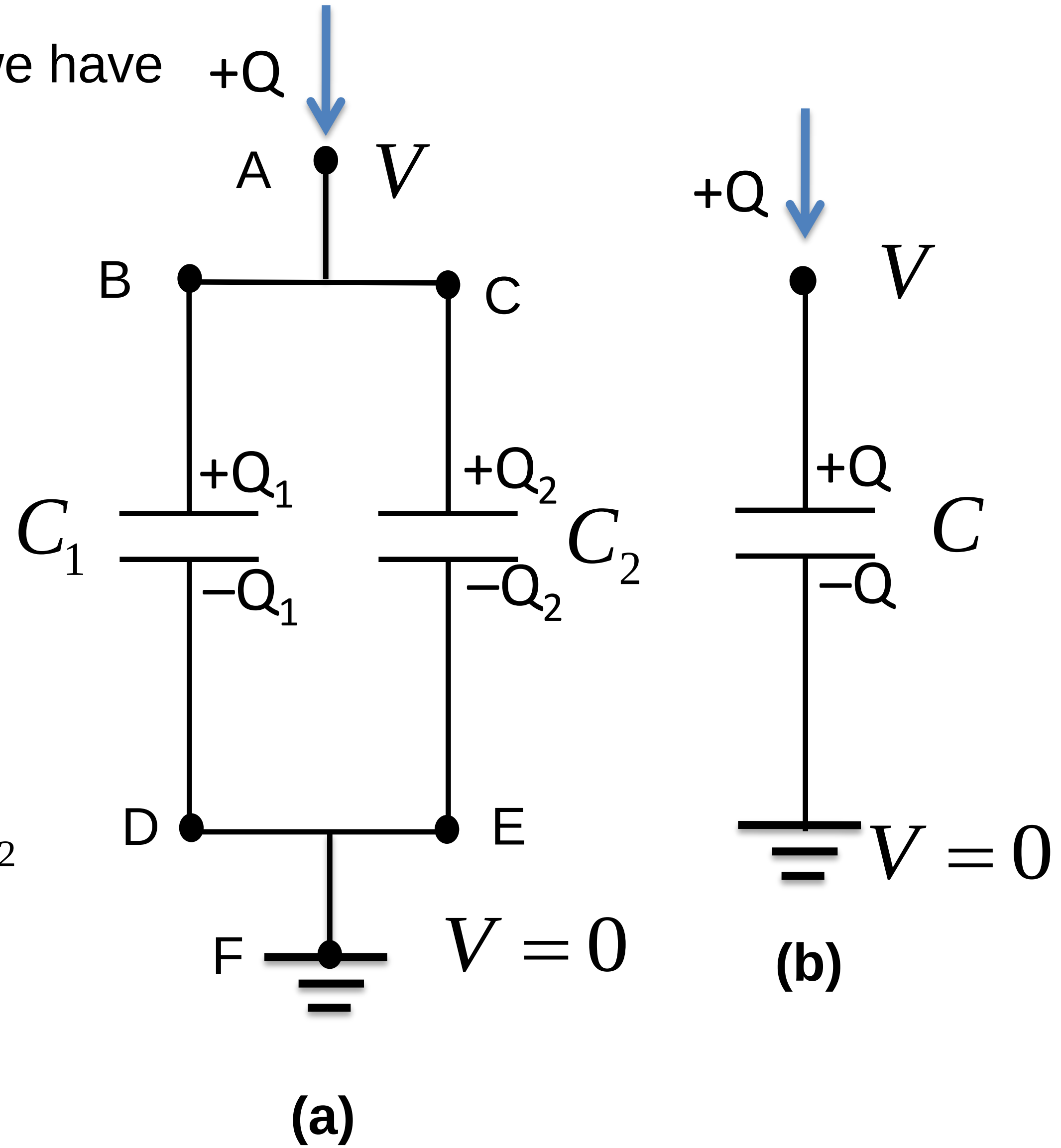
$$U_a = \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2$$

$$U_b = \frac{1}{2}CV^2$$

$$U_a = U_b$$

$$\frac{1}{2}CV^2 = \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2$$

$$\frac{1}{2}CV^2 = \frac{1}{2}(C_1 + C_2)V^2$$



$$C = C_1 + C_2$$

Capacitors in Parallel

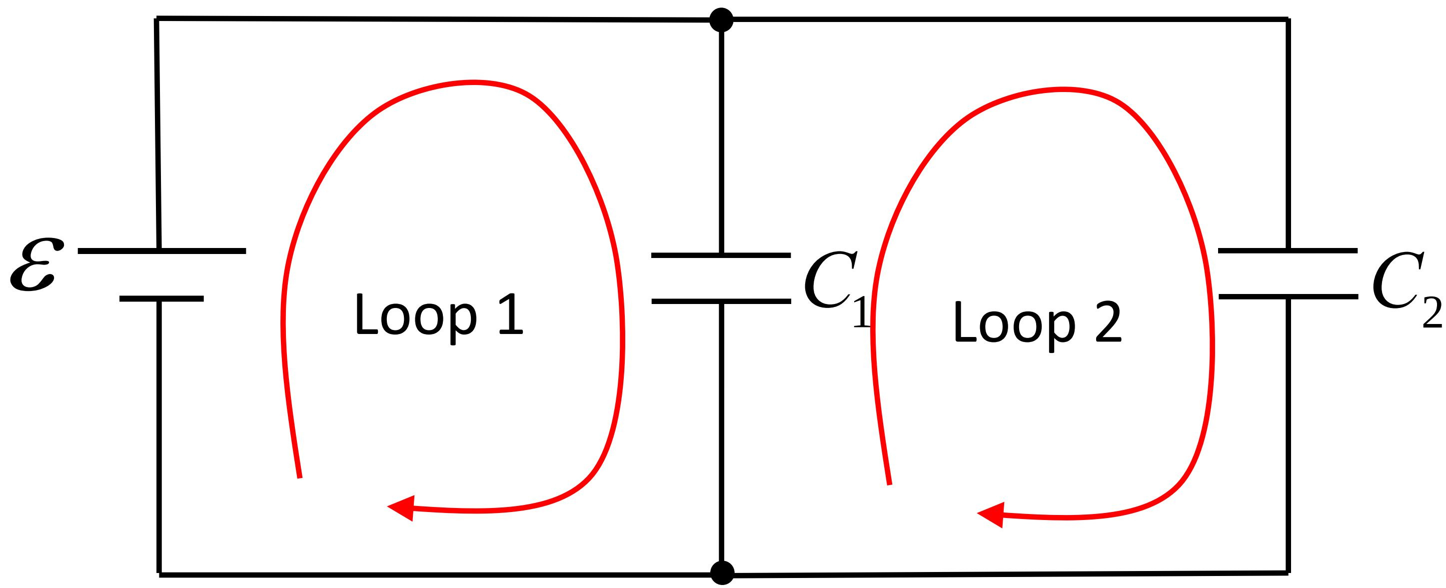
$$C = C_1 + C_2 + \ldots$$

$$Q = Q_1 + Q_2 + \ldots$$

$$V = V_1 = V_2 = \ldots$$

Capacitors in Parallel

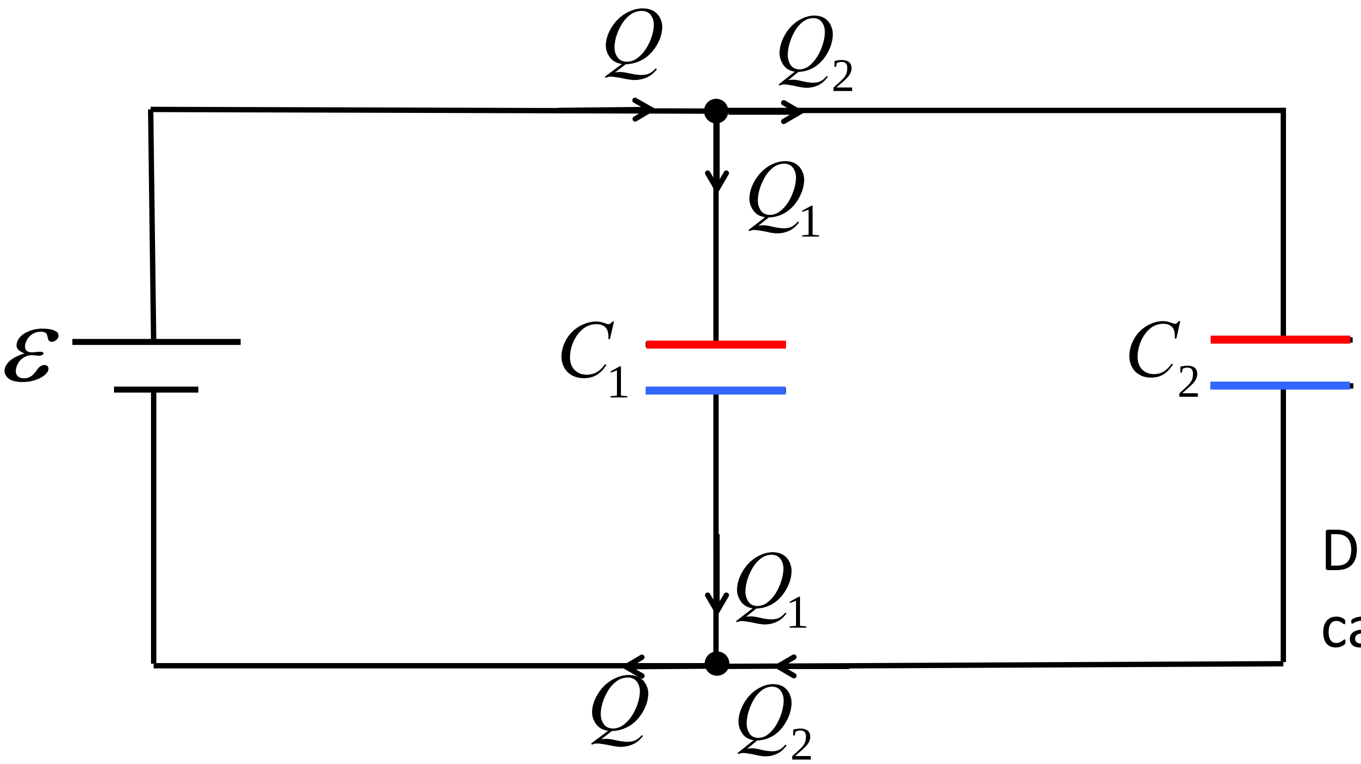
A slightly more complicated circuit has multiple branches with capacitors in parallel



Capacitors in parallel have the same voltage across their plates

Loop 1: $\mathcal{E} - \Delta V_{C_1} = 0$ Loop 2: $\Delta V_{C_1} - \Delta V_{C_2} = 0$

Capacitors in Parallel



$$Q = Q_1 + Q_2$$

$$Q_1 = \Delta V_{C_1} C_1$$

$$Q_2 = \Delta V_{C_2} C_2$$

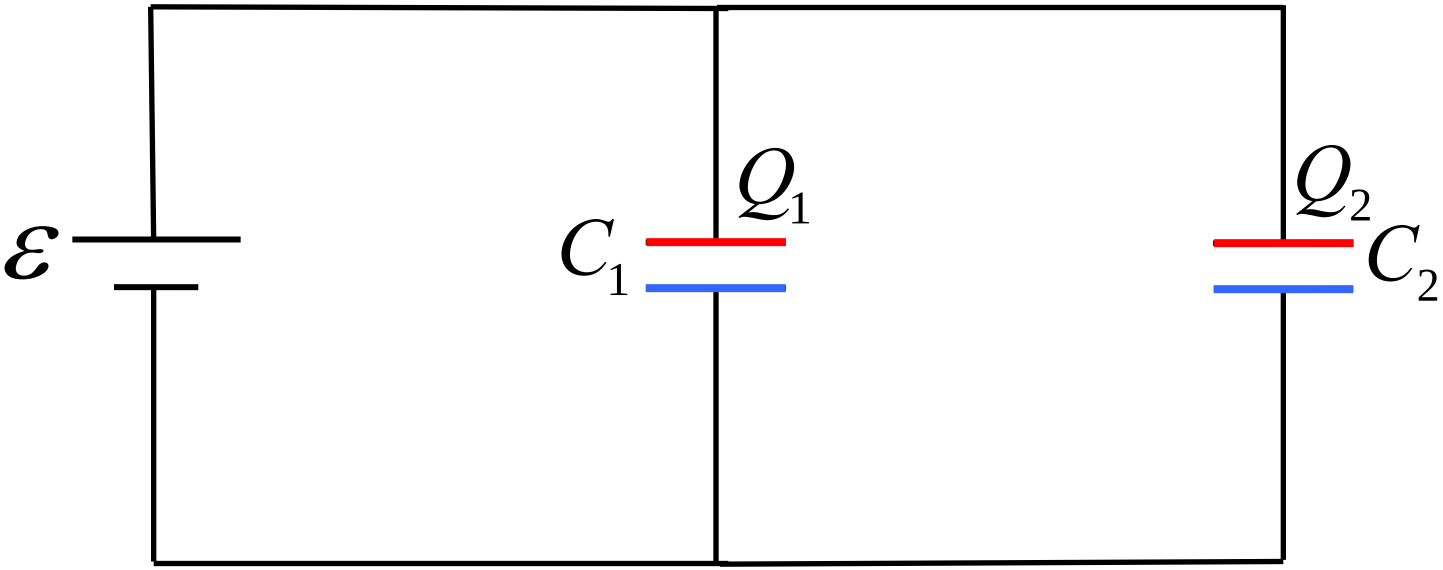
Define an equivalent capacitance:

$$Q = \mathcal{E}\, C_{eq}$$

From conservation of charge: $\mathcal{E} C_{eq} = \mathcal{E} C_1 + \mathcal{E}\, C_2$

For capacitors in parallel: $C_{eq} = C_1 + C_2$

Capacitors in Parallel



$$Q = Q_1 + Q_2$$

$$C_{eq} = C_1 + C_2$$

Summary of Capacitors

Relation between charge and voltage across plates

$$\Delta V_C = \frac{Q}{C}$$

Capacitors in Series: store the same amount of charge

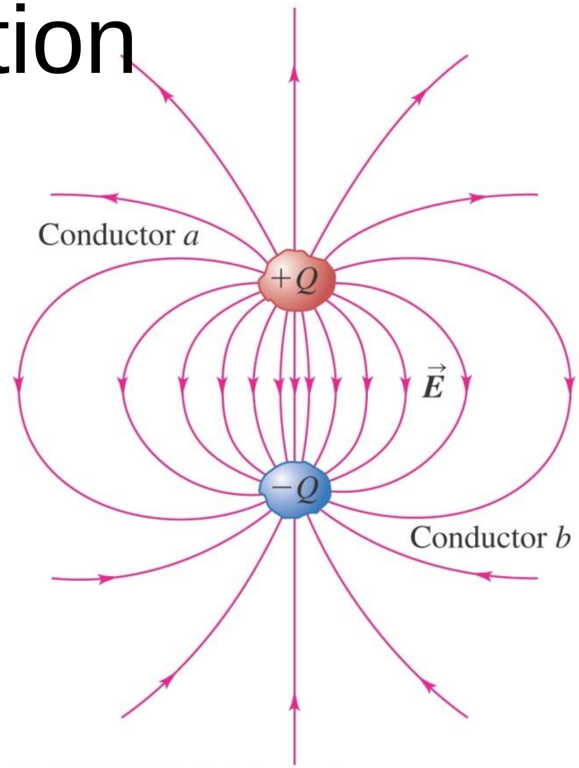
$$C_{eq} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \ldots + \frac{1}{C_N} \right)^{-1}$$

Capacitors in Parallel: have the same voltage across them

$$C_{eq} = C_1 + C_2 + \ldots + C_N$$

TopHat Question

The two conductors *a* and *b* are insulated from each other, forming a capacitor. You increase the charge on *a* to $+2Q$ and increase the charge on *b* to $-2Q$, while keeping the conductors in the same positions.



As a result of this change, the capacitance C of the two conductors

A. becomes 4 times great.

B. becomes twice as great.

C. remains the same.

D. becomes 1/2 as great.

E. becomes 1/4 as great.

TopHat Question

You reposition the two plates of a capacitor so that the capacitance doubles. There is vacuum between the plates.

If the charges $+Q$ and $-Q$ on the two plates are kept constant in this process, what happens to the potential difference V_{ab} between the two plates?

A. V_{ab} becomes 4 times as great

B. V_{ab} becomes twice as great

C. V_{ab} remains the same

D. V_{ab} becomes 1/2 as great

E. V_{ab} becomes 1/4 as great

TopHat Question

You reposition the two plates of a capacitor so that the capacitance doubles. There is vacuum between the plates.

If the charges $+Q$ and $-Q$ on the two plates are kept constant in this process, the energy stored in the capacitor

A. becomes 4 times greater.

B. becomes twice as great.

C. remains the same.

D. becomes 1/2 as great.

E. becomes 1/4 as great.

TopHat Question

You slide a slab of dielectric between the plates of a parallel-plate capacitor. As you do this, the *charges* on the plates remain constant.

What effect does adding the dielectric have on the *potential difference* between the capacitor plates?

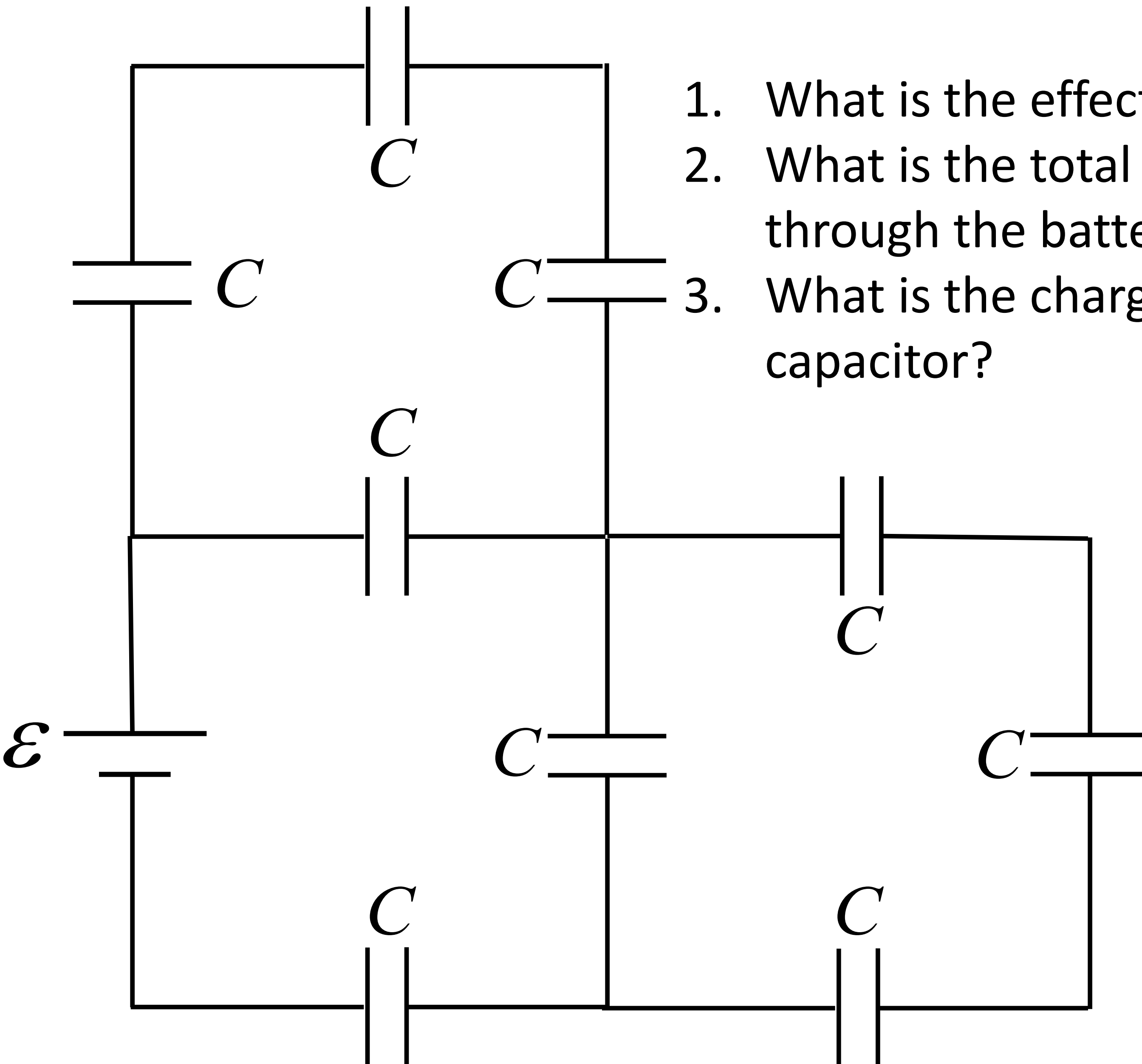
A. The potential difference increases.

B. The potential difference remains the same.

C. The potential difference decreases.

D. not enough information given to decide

Tutorial



1. What is the effective capacitance?
2. What is the total charge moved through the battery?
3. What is the charge on each capacitor?