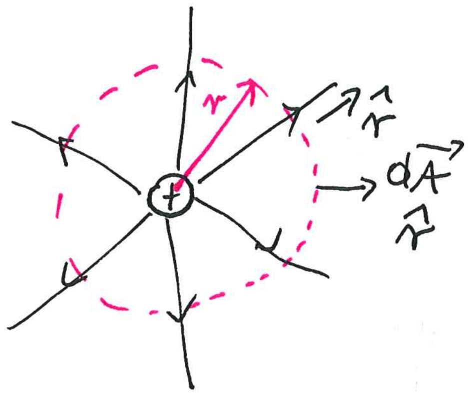
POINT CHARGE



$$\Phi_E = \oiint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$\vec{E}$ points in radian direction only

$|\vec{E}|$ constant for a given radius

$$\vec{E} \parallel d\vec{A} \Rightarrow \oiint \vec{E} \cdot d\vec{A} = \oiint E \cdot dA$$

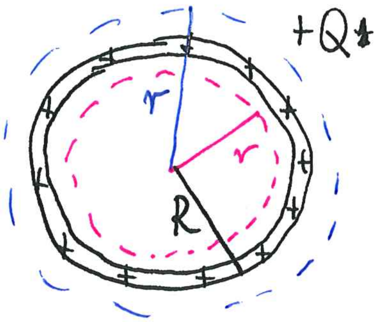
$$\Phi_E = E \oiint_{\text{sphere}} dA = E \cdot 4\pi r^2$$

$$q_{enc} = q$$

$$E\, 4\pi r^2 = \frac{q}{\varepsilon_0} \Longrightarrow E = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2}$$

$$\boxed{\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \hat{r}}$$

SHELL OF CHARGE



Compute E field
inside & outside
a spherical shell.

Charge is uniformly distributed.

INSIDE $\Phi_E = \oiint \vec{E} \circ d\vec{A} = 0 \Rightarrow E = 0$

OUTSIDE

$\vec{E}$ direction $\hat{r}$

$d\vec{A}$ direction $\hat{r}$

$\hat{r} \circ \hat{r} = 1$

$$\oiint \vec{E} \circ d\vec{A} = \oiint E \cdot dA = E \oiint dA = E \cdot 4\pi r^2$$

$$q_{enc} = +Q$$

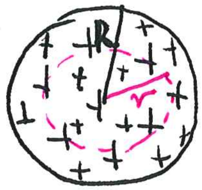
$$E \cdot 4\pi r^2 = \frac{+Q}{\varepsilon_0}$$

$$\boxed{E = \frac{1}{4\pi\varepsilon_0} \frac{+Q}{r^2}}$$

SAME AS FOR THE POINT CHARGE

SOLID BALL OF UNIFORM CHARGE

Compute E field inside & outside of the ball.



INSIDE $r < R$

$$\Phi_E = \oint\!\!\oint \vec{E} \cdot d\vec{A} = E \cdot 4\pi r^2$$

still spherical symetry

HOW TO FIND CHARGE ENCLOSED ?

$\lambda = \frac{Q}{L}$, $\sigma = \frac{Q}{A}$, $\underline{\rho} = \frac{Q}{V}$

$$\rho = \frac{m}{V}$$

Object $\rho = 1.2\ kg/m^3$, $m = ?$

$V = 0.5\ m^3$

$$m = \rho \cdot V = 1.2\ kg \cdot \cancel{m^3} \cdot 0.5\ \cancel{m^3} = 0.6\ kg$$

$$dQ = \rho\, dV$$

Volume of the Gaussian surface

$$\frac{4}{3}\pi r^3$$

Charge enclosed $q_{enc} = \underline{\rho} \cdot \frac{4}{3}\pi r^3$

$$\underline{\rho} = \frac{Q}{\frac{4}{3}\pi R^3}$$

$$q_{enc} = \frac{Q}{\cancel{\frac{4}{3}\pi} R^3} \cdot \cancel{\frac{4}{3}\pi} r^3 = \frac{Q \cdot r^3}{R^3}$$

to be continued...

SOLID BALL OF UNIFORM CHARGE – continued.

$$\rho = \frac{Q}{V} \quad \begin{matrix}\text{total charge} \\ \text{total volume}\end{matrix}$$

For the solid ball of uniform charge:

$$\rho = \frac{Q}{\frac{4}{3}\pi R^3}$$

Volume of small Gaussian sphere:

$$V_G = \frac{4}{3}\pi r^3$$

$$dQ = \rho \, dV$$

$$\underset{\substack{\text{inside} \\ \text{the} \\ \text{G-sphere}}}{Q} = \rho \cdot V_G = \frac{Q}{\frac{4}{3}\pi R^3} \cdot \frac{4}{3}\pi r^3 = Q\frac{r^3}{R^3}$$

Electric flux $= q_{enc}/\varepsilon_0$

$$E \cdot 4\pi \cancel{r^2} = \frac{Q r^{\cancel{3}}}{\varepsilon_0 R^3}$$

$$\Rightarrow E = \underbrace{\frac{1}{4\pi\varepsilon_0}\frac{Q}{R^3}}_{C} r$$

constant

$$E(r)_{\text{inside}} = C r$$

→ linearly increases with r

$$E(r)_{\text{inside}} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

→ decreases $\sim \frac{1}{r^2}$