
Fill in these boxes with your LAST NAME and FIRST INITIAL.

University of Calgary

Faculty of Science

Midterm Test - grading guide

PHYSICS 259 ALL LECTURE SECTIONS

February 12, 2015, 7:00–8:30 p.m. **Time: 90 minutes.**

DO NOT TEAR OFF THIS PAGE! You may, however, tear off the last page, which has an equation sheet and table of integrals.

This is a closed-book exam worth a total of 35 points. Please answer all questions. Use of the Schulich calculator or equivalent is allowed.

Write your Last Name and First Initial on this top sheet in the grid above. (Do not write your ID number on this page.)

Make sure this question paper booklet contains 19 pages. If you are missing any pages, get a new booklet from the exam supervisor.

You should also have a **separate set of Answer Sheets**. This is where you enter Multiple Choice answers of Part I and also detailed solutions to the problems of Part II. Only work entered in the indicated spaces on the Answer Sheets will be marked.

IMPORTANT: YOUR ID NUMBER IS TO BE ENTERED AT THE TOP OF EACH AND EVERY ONE OF THE ANSWER SHEETS. DO THIS NOW.

Begin working on the examination when instructed to do so by the supervisor.

Part I: Multiple-Choice Questions (Total: 15 marks)

Enter answers to multiple choice questions on the first Answer Sheet using space provided in the upper right of the page. Each question in Part I is worth one point. You should complete Part I in about 40 minutes or less.

1) In all situations, the electric field lines in a given region of space

a) point in the direction that a negative charge will move.
b) point in the direction of the force on a positive charge. $\Leftarrow \checkmark$
c) point in the direction of the force on a negative charge.
d) point in the direction that a positive charge will move.
e) none of the above

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2) A conducting sphere of radius 0.01 m has a charge of 1.0×10^{-11} C deposited on it. The magnitude of the electric field in V/m just outside the surface of the sphere is closest to

a) 90,000
b) 900 $\Leftarrow \checkmark$
c) 4,500
d) zero
e) 450

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3) A solid insulating sphere of radius R carries a positive charge density ρ (C/m^3) distributed uniformly throughout its volume. What is the electric field strength E as a function of radius r inside the sphere?

a) $E = \dfrac{\rho R}{3\epsilon_0 r}$

b) $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{\rho}{r^2}$

c) $E = \dfrac{4\pi\rho r^3}{3\epsilon_0 R^2}$

d) $E = \dfrac{\rho r^2}{3\epsilon_0}$

e) $E = \dfrac{\rho r}{3\epsilon_0}$ $\Leftarrow \checkmark$

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4) Two very large thin parallel charge sheets of area A are separated by a small distance $d \ll \sqrt{A}$. A total charge $-3Q$ is evenly distributed over sheet #1, while $+Q$ is evenly distributed over sheet #2. What is the electric field magnitude E at a point halfway between the charge sheets?

a) $\dfrac{2Q}{\epsilon_0 A}$ $\Leftarrow \checkmark$

b) $\dfrac{4Q}{\epsilon_0 A}$

c) $\dfrac{3Q}{\epsilon_0 A}$

d) $\dfrac{5Q}{\epsilon_0 A}$

e) none of the above

---

5) A solid material carries a static excess charge distributed throughout its volume (i.e., the material has a volume charge density ρ). Which one of the following statements correctly describes this situation?

a) The material is a conductor.
b) The material is an insulator. $\Leftarrow \checkmark$
c) The material can be either a conductor or an insulator, but it is not possible to tell which from the information given.
d) The situation described is impossible.
e) none of the above

---

6) A total charge Q is uniformly distributed over a very thin disk of radius R. The electric field at a distance d along the disk axis is given by $\vec{E} = \dfrac{\sigma}{2\epsilon_0}\left[1 - \dfrac{1}{\sqrt{(R^2/d^2)+1}}\right]\hat{n}$ where $\hat{n}$ points away from the disk. At what distance along the axis will the electric field be closest to half of its peak value $E_{max} = \dfrac{\sigma}{2\epsilon_0}$ if the radius is $R = 40$ centimetres?

a) $d = 23.09$ cm $\Leftarrow \checkmark$
b) $d = 34.64$ cm
c) $d = 20.00$ cm
d) $d = 40.00$ cm
e) none of the above

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7) Which of the following statements about Gauss's law are correct?

- **a)** Only charge enclosed within a Gaussian surface can produce an electric field at points on that surface.
- **b)** Gauss's law is valid only for symmetric charge distributions, such as spheres and cylinders.
- **c)** The normal to a closed surface points inward for negatively charged objects.
- **d)** Gauss's law requires that the electric field be exactly perpendicular or parallel to the surface normal.
- **e)** none of the above $\Leftarrow \checkmark$

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8) Consider two lightweight metal spheres, each hanging from its own insulating nylon thread. One of the spheres has been negatively charged, while the other sphere is neutral. If the spheres are close together then which of the following statements will be true?

- **a)** they attract each other. $\Leftarrow \checkmark$
- **b)** they repel each other.
- **c)** the negatively charged sphere attracts the neutral one, but the neutral sphere does not attract the negatively charged one.
- **d)** they do not exert an electrostatic force on each other.
- **e)** the neutral sphere attracts the negatively charged one, but the negatively charged sphere does not attract the neutral one.

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9) A test charge q feels a force F when located a distance d away from a point charge Q. The amount of charge Q now changes and at the same time the test charge is moved 10 times further away, while keeping q constant. At this new location the test charge still feels the same force F. How much has the charge Q increased/decreased?

- **a)** It has increased by a factor of 10
- **b)** It has decreased by a factor of 10
- **c)** It has increased by a factor of 100 $\Leftarrow \checkmark$
- **d)** It has decreased by a factor of 100
- **e)** none of the above

---

10) Three point charges q_1, q_2, and q_3 are located on the x-axis at x_1 = -3.0 cm, $x_2 = 0$, and x_3 = +5.0 cm. The force on q_1 equals

- **a)** The charge of q_1 multiplied by the scalar sum of the electric fields of q_1, q_2, and q_3.
- **b)** The vector sum of the forces exerted by q_1 on q_2 and q_3.
- **c)** The charge of q_1 multiplied by the vector sum of the electric fields of q_1, q_2, and q_3.
- **d)** The vector sum of the forces exerted by q_2 and q_3. $\Leftarrow \checkmark$
- **e)** The sum of the charges q_1, q_2, and q_3 divided by ϵ_0.

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11) Figure 1 shows a thin insulating rod of length L that carries a uniform linear charge density λ. Which of the following integrals do we need to evaluate to find the x-component of the electric field at point P due to this rod?

a) $E_x = \int_{y=0}^{L} \frac{1}{4\pi\epsilon_0} \frac{\lambda}{x^2+y^2} dy$

b) $E_x = \int_{y=0}^{L} \frac{1}{4\pi\epsilon_0} \frac{\lambda x}{(x^2+y^2)^{3/2}} dy$ $\Leftarrow \checkmark$

c) $E_x = \int_{y=0}^{L} \frac{1}{4\pi\epsilon_0} \frac{\lambda y}{(x^2+y^2)^{3/2}} dy$

d) $E_x = \int_{y=0}^{L} \frac{1}{4\pi\epsilon_0} \frac{\lambda xy}{(x^2+y^2)^{3/2}} dy$

e) $E_x = \int_{y=0}^{L} \frac{1}{4\pi\epsilon_0} \frac{\lambda L}{x^2+y^2} dy$

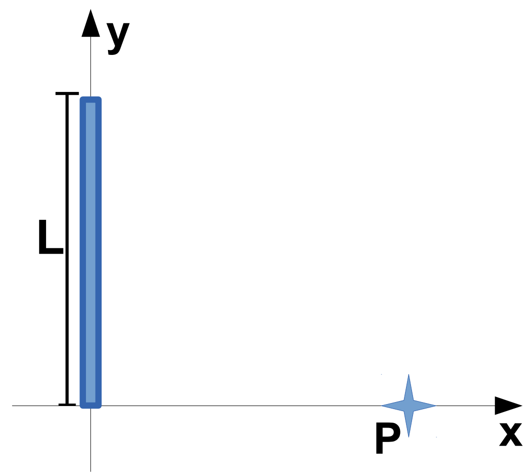

Figure 1: Line of charge along positive y-axis.

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12) In Figure 2 the electric potential is zero at infinity. Where else is the electric potential zero?

a) Somewhere in the region $0 < x < a$, only

b) Somewhere in each of the regions $x < 0$ and $0 < x < a$ $\Leftarrow \checkmark$

c) Somewhere in the region $x < 0$, only

d) Somewhere in each of the regions $0 < x < a$ and $x > a$

e) Somewhere in the region $x > a$, only

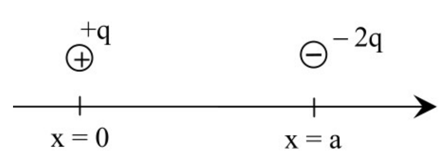

Figure 2: Two charges on a line.

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13) The SI units for electric flux Φ_E are

a) Nm/C

b) Nm^2/C $\Leftarrow \checkmark$

c) Nm^2/C^2

d) Nm/C^2

e) V/C

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14) A half-circular segment with radius R has total charge Q distributed uniformly along length L. The total electric field at the origin due to this charge segment is $E_0\hat{i}$. What is the electric field $\vec{E}$ at the origin produced by a quarter-circular segment with the same radius R and total charge Q?

a) $-E_0\hat{i} + E_0\hat{j}$ $\Leftarrow \checkmark$
b) $-\sqrt{2}E_0\hat{i} + \sqrt{2}E_0\hat{j}$
c) $-2E_0\hat{i} + 2E_0\hat{j}$
d) $+E_0\hat{i} - E_0\hat{j}$
e) none of the above

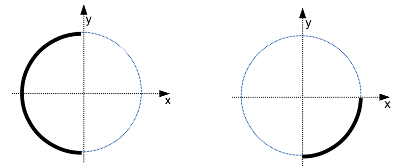

Figure 3: Half circle and quarter circle of charge.

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15) An electric dipole consists of two opposite charges $+q$ and $-q$ separated by a fixed distance d. An electric quadrupole is a combination of two dipoles locked at right angles to each other. An electric dipole and quadrupole are initially located as shown in Figure 4. The electric quadrupole is then released so that it can move freely in the plane of the page. What statement best describes what will happen?

a) The quadrupole will rotate by 90° and will move toward the dipole. $\Leftarrow \checkmark$
b) The quadrupole will rotate by 90° and will move away from the dipole.
c) The quadrupole will rotate by 45° and will move toward the dipole.
d) The quadrupole will rotate by 45° and will move away from the dipole.
e) none of the above

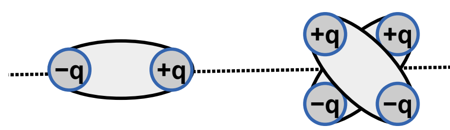

Figure 4: Dipole and quadrupole.

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16) This is **version @V@** of the exam. Please select @V@ in the row marked "Version" in the Multiple Choice Answers area on the first Answer Sheet. **Be sure to do this now, before moving on to other questions on the paper**.

This is the end of the Multiple Choice part of the exam. Make sure that you have entered all your answers from this section on the Answer Sheet "bubble" page.

You may now proceed to Part II. All your answers for Part II must be written in the appropriate box on the Answer Sheet pages.

Part II: Written Answer Questions (Total: 20 marks)

IMPORTANT: Write your answers to the problems in Part II in the corresponding boxes on the Answer Sheets. Work must be shown for full marks. Rough work can be done on the back of this question paper, but only the work appearing on the Answer Sheets will be marked.

17) *[6 marks]* Eight point charges are evenly spaced around a circle of radius $R = 1$ mm as shown in Figure 5. All of the even-numbered charges (2,4,6,8) are protons and all of the odd-numbered charges (1,3,5,7) are electrons.

- **a)** Calculate the force that charge #1 applies on charge #5.
- **b)** Calculate the force on charge #1 due to charges #3 and #7.
- **c)** What is the total electric field $\vec{E}$ at the center of the circle due to all eight charges?
- **d)** If charge #8 is removed, what is the total electric field $\vec{E}$ at the center of the circle?

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18) *[7 marks]* Total charge $+Q_a$ is uniformly distributed on a half-circle of radius R_a and total charge $+Q_b$ is uniformly distributed on a smaller half-circle of radius R_b as shown in Figure 10.

- **a)** What is the linear charge density λ_a on the larger segment?
- **b)** Determine the electric potential V at the center of the circle.
- **c)** Determine the ratio of charge Q_b/Q_a required to produce zero electric field $\vec{E}$ at the center of the circle.

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19) *[7 marks]* For an infinite line of charge density λ we normally use a "Gaussian cylinder" to find the electric field $\vec{E} = \dfrac{\lambda}{2\pi\epsilon_0 r}\hat{r}$. What happens if we use a cube of volume $L{\times}L{\times}L$? Assume that the line charge enters exactly through the center of one face, passes through the center of the cube, and exits through the center of the opposite side (see Figure 7). Note that the electric field magnitude E will not be constant everywhere on the surface of the cube. All of the following questions refer to the top face with normal $\hat{n}$ pointing in the $+\hat{k}$ direction.

- **a)** Consider just the top face: what is the maximum electric field magnitude E and where does it occur? Provide an equation for E with a brief explanation and a quick sketch including all the essential details.
- **b)** Consider just the top face: what is the maximum electric field component E_z parallel to the surface normal $\hat{n}$ and where does it occur? Provide an equation, a brief explanation, and a quick sketch.
- **c)** What is the minimum value of E_z on the top face? Where is it?
- **d)** Determine the total electric flux Φ_E through the top surface.
- **e)** Determine the average of E_z at the top surface.

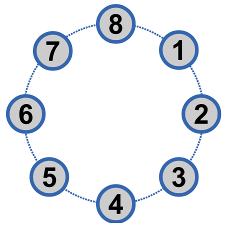


Figure 5: Eight charges on a circle.

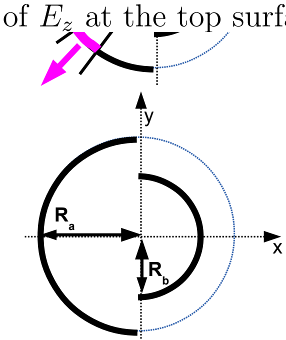


Figure 6: Two half-circles of charge.

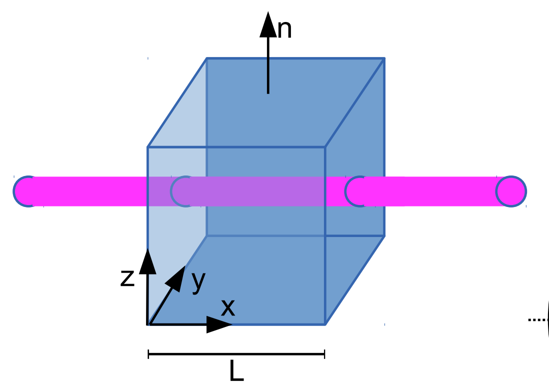


Figure 7: Line charge and Gaussian cube.

CONSTANTS AND USEFUL EQUATIONS

k = Coulomb constant = 8.99×10^{9} N m^2 C^{-2}
ϵ_0 = permittivity of free space = 8.85×10^{-12} C^2 N^{-1} m^{-2}
e = fundamental charge = 1.602×10^{-19} C
m_p = mass of proton = 1.67×10^{-27} kg
m_e = mass of electron = 9.11×10^{-31} kg

m = 10^{-3} $\mu = 10^{-6}$ n = 10^{-9} p = 10^{-12}

Surface area of a sphere: $A = 4\pi r^2$
Volume of a sphere: $A = \frac{4}{3}\pi r^3$
Area of a circle: $A = \pi r^2$
Circumference of a circle: $C = 2\pi r$

$$x = x_0 + v_{x0}t + \frac{1}{2}a_x t^2 \qquad v_x = v_{x0} + a_x t \qquad v_x^2 = v_{x0}^2 + 2a_x x \qquad \vec{F} = m\vec{a}$$

$$\vec{F} = k\frac{q_1 q_2}{r^2}\,\hat{r} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}\,\hat{r} \qquad \vec{E} = \frac{\vec{F}}{q} \qquad \vec{E} = k\frac{q}{r^2}\,\hat{r} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\,\hat{r}$$

$$\Phi_E = \oint_A \vec{E}\cdot d\vec{A} = \oint_A E\,dA\,cos\theta = \frac{Q_{encl}}{\epsilon_0} \qquad V = \frac{U}{q} \qquad U = k\frac{q_1 q_2}{r} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r}$$

$$V = k\frac{q}{r} = \frac{1}{4\pi\epsilon_0}\frac{q}{r} \qquad W = qV_{ab} \qquad V_{ab} = V_a - V_b = \int_a^b \vec{E}\cdot d\vec{l}$$

$$\vec{E} = -\vec{\nabla}V = -\left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)V \qquad C = \frac{Q}{V_{ab}} \qquad C = \frac{\epsilon_0 A}{d}$$

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}QV = \frac{1}{2}CV^2 \qquad \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \qquad C = C_1 + C_2 + C_3$$

$$u = \frac{1}{2}\epsilon_0 E^2$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin\frac{x}{a} \qquad \int \frac{dx}{\sqrt{x^2 + a^2}} = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

$$\int \frac{dx}{x^2 + a^2} = \frac{1}{a}\arctan\frac{x}{a} \qquad \int \frac{dx}{(x^2 + a^2)^{3/2}} = \frac{1}{a^2}\frac{x}{\sqrt{x^2 + a^2}}$$

$$\int \frac{x\,dx}{(x^2 + a^2)^{3/2}} = -\frac{1}{\sqrt{x^2 + a^2}}$$

Multiple Choice Answer Sheet - Version A

Multiple Choice Answer Sheet

Version A: d c b c c d e a c c b c e a a

Version B: b c e b b c e a a a b a c a b

Version C: b b e a b a e a c d b b b a a

Paul's bubble sheet goes here

Written Answer Sheets

Phys 259 Midterm Exam (Winter 2015): Instructions to Markers

You are responsible for marking the Problems, Part II. The multiple choice answers are scored automatically. The "Version" number applies only to the multiple-choice answers; there is only one version of Part II. You should also have these general instructions, plus a set of solutions with detailed instructions, and a copy of an exam question paper. The solutions are presented in "Answer Page format" for ease of reference.

Print out these Instructions, and read them before starting to mark. You might be tempted to bypass these, but you will save us much time and trouble if you follow instructions.

On the following pages, I've given a suggested detailed marking scheme for each question. However, not all solutions will follow the one I've given, especially for the 2-mark and 3-mark questions, so use your judgment. The following broad guidelines may help:

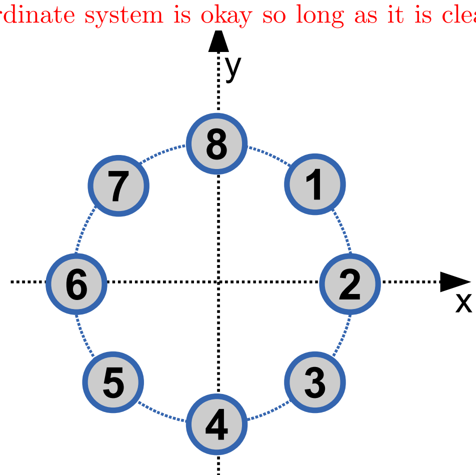
- In 3-mark questions, the first half mark should be easy to get: if a student shows any reasonable insight at all, give a half mark. In an integration problem, If the student simply makes a valiant attempt at integrating, this might be enough.
- Give half of the allotted marks (e.g., 1.5 marks out of 3) if you feel that the student has got half way to the right answer. For example, if a student makes five mistakes but still has the solution half right, give 1.5; don't deduct five half-marks.
- In numerical questions, the final answer must have correct units. Deduct 0.5 mark if the units on the final answer are not shown or are incorrect. Don't worry about units for intermediate calculations.
- Only subtract 1/2 mark once for each class of mistake. For example, missing vectors in one equation would be the same as missing them in two or three equations.
- Be lenient on significant figures. Final answer can have anything between 1 and 5 digits, take off 1/2 mark if 6 or more.
- If a lot of students are making the same "mistake", check my answer - I could be the one who is wrong! If you discover any errors in my solutions, please let me know as soon as possible (physics259@ucalgary.ca) so I can alert all other markers.

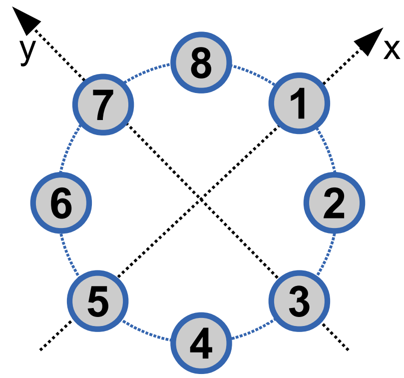
UCID:

Q17

All parts of this problem involve a vector $\vec{r}$ pointing from one charge to another, or the length $r = |\vec{r}|$, or the unit vector $\hat{r}$. This requires that we specify the location of each charge. There are 8 charges evenly spaced around the circle, so the angular separation is $360°/8 = 45°$ from one to the next. We will define the usual x,y coordinate system with $\hat{i}$ pointing to the right side of the page and $\hat{j}$ pointing toward the top of the page.

Any coordinate system is okay so long as it is clearly defined eg. $\hat{i}$ pointing towards #1, $\hat{j}$ pointing towards #7.





For simplicity, place the circle at the origin. We could define the usual angle θ going counter-clockwise from the $x-$axis or instead define a "clock" angle ϕ so that $\phi_8 = 0$ and $\phi_1 = 45°$. For a radius $R = 1$ mm the charge locations are given by

$$\vec{r} = \left[+\sin\phi\hat{i} + \cos\phi\hat{j}\right] mm = (\sin\phi, \cos\phi)\ mm$$

so using the standard convention for $\hat{i}$ and $\hat{j}$ the displacement vector

$$\vec{r}_1 = (\sin\phi_1 + \cos\phi_1)\ mm = \left[\sin 45°\hat{i} + \cos 45°\hat{j}\right]\ mm = \frac{1}{\sqrt{2}}\left[+\hat{i} + \hat{j}\right] mm$$

should have length 1 mm

$$|\vec{r}| = \sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2} = \sqrt{1/2 + 1/2} = 1\, mm$$

and unit vector

$$\hat{r} = \frac{\vec{r}}{r} = \frac{\frac{1}{\sqrt{2}}\left[+\hat{i} + \hat{j}\right] mm}{1mm} = \frac{1}{\sqrt{2}}\left[+\hat{i} + \hat{j}\right]$$

All the question parts refer to odd numbered charges which are all electrons with charge $q = 1.602 \times 10^{-19}\, C$. Electric fields should point towards the charges and pairs of charges should repel.

The largest distance between charges is $r = 2R = 2mm$.

There needs to be something that shows or explains what unit vectors were selected and what are the resulting displacement vectors but this level of detail is not required.

Vectors may be expressed in terms of magnitude (length) and angle.

Continued on next page

UCID:

Q17a *[2.0 marks]*

The force of charge #1 on charge #5 is given by Coulomb's law $\vec{F} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_5}{r^2}\hat{r}$.

+0.5 for using the correct vector equation

The displacement vector $\hat{r}$ points from #1 to #5 which will be down and to the left:

$$\vec{r} = \vec{r}_5 - \vec{r}_1 = \frac{\left[-\hat{i} - \hat{j}\right]}{\sqrt{2}} - \frac{\left[+\hat{i} + \hat{j}\right]}{\sqrt{2}} = -2\frac{\left[+\hat{i} + \hat{j}\right]}{\sqrt{2}} mm$$

so the distance r is the circle diameter $r = 2R$ (twice the radius).

+0.5 for getting the correct $\vec{r}$

$$\vec{F} = 8.99 \times 10^9\,\mathrm{Nm^2C^{-2}}\frac{(-1.6 \times 10^{-19}C)^2}{(2\ mm)^2}\frac{\left[-\hat{i} - \hat{j}\right]}{\sqrt{2}} \tag{1}$$

$$= \boxed{-4.068 \times 10^{-23}\left[+\hat{i} + \hat{j}\right]\ N} \tag{2}$$

$$= \boxed{5.768 \times 10^{-23}\,N\ \ 225^\circ \text{ counter-clockwise from the x-axis}} \tag{3}$$

+1.0 for correct result with magnitude, direction, and units. +0.5 if calculation error or no units.

Q17b *[2.0 marks]*

The force of charge #3 and #7 on charge #1 is given by a vector sum of forces from Coulomb's law $\vec{F} = \vec{F}_{3\to 1} + \vec{F}_{7\to 1}$ where both cases have the same distance $r_{31} = r_{71} = r$

$$\vec{F} = \frac{1}{4\pi\epsilon_0}\frac{q_3\, q_1}{r_{31}^2}\hat{r}_{31} + \frac{1}{4\pi\epsilon_0}\frac{q_7\, q_1}{r_{71}^2}\hat{r}_{71} \tag{4}$$

$$= \frac{1}{4\pi\epsilon_0}\left[\frac{(-q)^2}{r^2}\hat{j} + \frac{(-q)^2}{r^2}\hat{i}\right] = \frac{q^2}{4\pi\epsilon_0 r^2}\left[\hat{i} + \hat{j}\right] \tag{5}$$

+0.5 for words or equations indicating vector sum of two contributions.

In this case the distance between charges is $r = \sqrt{2}R = 1.414\ mm$.

+0.5 for getting the correct $\vec{r}$ magnitudes and directions

$$\vec{F} = 8.99 \times 10^9\,\mathrm{Nm^2C^{-2}}\frac{(1.6 \times 10^{-19}C)^2}{(1.41\ mm)^2}\left[+\hat{i} + \hat{j}\right] \tag{6}$$

$$= \boxed{1.15 \times 10^{-22}\left[+\hat{i} + \hat{j}\right]\ N} \tag{7}$$

$$= \boxed{1.63 \times 10^{-22}\,N\ \ 45^\circ \text{ counter-clockwise from the x-axis}} \tag{8}$$

+1.0 for correct result with magnitude, direction, and units. +0.5 if calculation error or no units.

Continued on next page

UCID:

Q17c *[1.0 marks]*

The electric field at the origin is a sum over 8 charges (superposition)

$$\vec{E} = k\frac{q}{r^2}\hat{r} \quad \rightarrow \quad \Sigma\vec{E} = \Sigma k\frac{q}{r^2}\hat{r} = k\frac{q}{r^2}\Sigma\hat{r}$$

but for each charge A there will be a corresponding charge B located exactly 180° away that has the same magnitude and sign, so the electric field will point in the opposite direction: $\hat{r}_A = -\hat{r}_B$ giving a net electric field of zero. With an even number of charges ($N = 8$) there will be perfect cancellation.

$$\vec{E}_1 = -\vec{E}_5\,,\ \vec{E}_2 = -\vec{E}_6\,,\ \vec{E}_3 = -\vec{E}_7\,,\ \vec{E}_4 = -\vec{E}_8\,,$$

Or note that the kq/r^2 term is the same for each case, so we just need to add up the unit vectors (or displacement vectors)

$$\begin{aligned}\Sigma\hat{r} = &\frac{1}{\sqrt{2}}(\hat{\imath}+\hat{\jmath}) + (1\hat{\imath}+0\hat{\jmath}) + \frac{1}{\sqrt{2}}(\hat{\imath}-\hat{\jmath}) + (0\hat{\imath}-1\hat{\jmath}) \\ &+\frac{1}{\sqrt{2}}(-\hat{\imath}-\hat{\jmath}) + (-1\hat{\imath}+0\hat{\jmath}) + \frac{1}{\sqrt{2}}(-\hat{\imath}+\hat{\jmath}) + (0\hat{\imath}+1\hat{\jmath}) = 0 \end{aligned} \tag{9}$$

to get

$$\boxed{\Sigma\vec{E} = 0}$$

"Zero due to symmetry" is correct, but needs a little more detail for full grade.

+0.5 for correct result, and +0.5 for reasonable argument or proof.

Q17d *[1.0 marks]*

The electric field due to charges #1 through #7

$$\Sigma\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \vec{E}_4 + \vec{E}_5 + \vec{E}_6 + \vec{E}_7$$

is the same as the field due to all the charges, minus charge #8.

$$\Sigma\vec{E} = \left[\vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \vec{E}_4 + \vec{E}_5 + \vec{E}_6 + \vec{E}_7 + \vec{E}_8\right] - \vec{E}_8$$

and since the field due to all eight charges is zero, the result is just minus the field of charge #8

$$\Sigma\vec{E} = 0 - \vec{E}_8 = -\vec{E}_8 = -k\frac{q}{r^2}(-\hat{\jmath}) = k\frac{q}{r^2}\hat{\jmath} \tag{10}$$

$$= 8.99\times10^9\,\mathrm{Nm^2C^{-2}}\frac{1.6\times10^{-19}C}{(1\ mm)^2}\hat{\jmath} = \boxed{1.438\times10^{-3}\ N/C\ \hat{\jmath}} \quad \text{V/m also valid} \tag{11}$$

+0.5 for correct result, and +0.5 for reasonable argument or proof. If they got the wrong answer from the previous question but then used it correctly, give full marks for this question.

Continued on next page

UCID:

Q18a *[1.0 marks]*

For a charge Q uniformly distributed along some length L the linear charge density is $\lambda = \frac{Q}{L}$. The length of a half circle is $L = \pi R$, so the linear charge density of the larger segment is

$$\lambda_a = \frac{Q_a}{L_a} = \boxed{\frac{Q_a}{\pi R_a}}$$

+1.0 for the correct result, or +0.5 for just the correct definition of linear charge density.

Q18b *[3.0 marks]*

A point charge produces a potential given by

$$V = k\frac{q}{r} \qquad \rightarrow \qquad dV = k\frac{dq}{r} = k\frac{\lambda\, R\, d\theta}{R} = \frac{1}{4\pi\epsilon_0}\lambda\, d\theta$$

1.0 for correct potential due to a point charge, either q or dq. Subtract 0.5 if use vectors, or $1/r^2$. Give 0.0 for $\int \vec{E}\cdot d\vec{l}$

All points on a (semi) circle are equally far from the center, so $r = R$ can be brought outside the integral. The total charge is $Q = \lambda L = \pi R \lambda$

$$V = \int dV = k/R \int dq = \frac{kQ}{R} = \frac{\lambda}{4\epsilon_0}$$

1.0 for correct result with details (eg. $r = R$), and just 0.5 for simply just writing the right result.

Combining both half-circles (superposition)

$$V = V_a + V_b = kQ_a/R_a + kQ_b/R_b \tag{12}$$

$$= \boxed{k\left[\frac{Q_a}{R_a} + \frac{Q_b}{R_b}\right]} = \boxed{\frac{1}{4\epsilon_0}\left[\lambda_a + \lambda_b\right]} \tag{13}$$

+1.0 for correct result, or just +0.5 for basic idea of adding two potentials.

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UCID:

Q18c *[3.0 marks]*

+1.0 for the general idea of balancing electric fields vectors $\vec{E}$

One (longer) way to solve this involves integrating each ring to find the fields at the origin eg.

$$E_x = \frac{1}{4\pi\epsilon_0}\frac{2\lambda}{R} = \frac{Q}{2\epsilon_0\pi^2 R^2}$$

then solve for Q_a and Q_b.

+1.0 for recognizing or computing that $E_x \neq 0$ and $E_y = 0$ by symmetry.

Another (easier) way is to recognize that each arc segment from one semi-circle needs to balance a corresponding arc segment from the other semicircle. If one segment is closer to the origin (smaller radius) then the other must have more charge.

The field magnitude E due to a small charge segment dq is

$$E = k\frac{dq}{r^2} = k\frac{\lambda R d\theta}{R^2} = k\lambda\frac{d\theta}{R}$$

with the linear charge densities given by

$$\lambda/R = \frac{Q}{\pi R}/R = Q/\pi R^2$$

In order for equal arc segments $d\theta$ to produce cancelling fields

$$\frac{\lambda_a}{R_a} = \frac{\lambda_b}{R_b}$$

but the question asks for a ratio of charges, not charge densities

$$Q_a/\pi R_a^2 = Q_b/\pi R_b^2$$

and the charges must be the same sign (both positive or both negative)

$$\boxed{\frac{Q_b}{Q_a} = \frac{R_b^2}{R_a^2}}$$

and the ratio is proportional to distance (radius) squared.

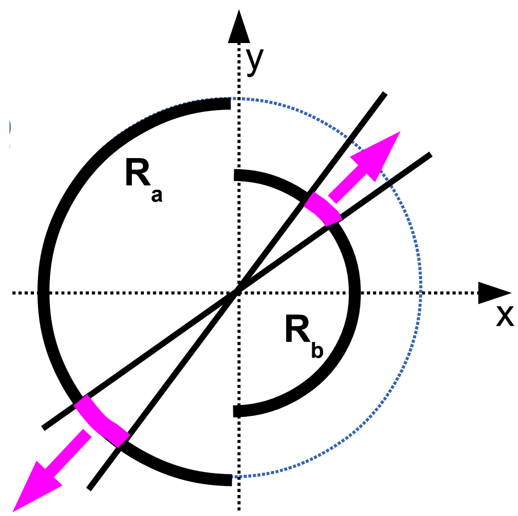

Figure 8: Balancing arc lengths.

+1.0 for correct result with details

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UCID:

Q19

This configuration is not perfectly symmetric, but 4 of the 6 cube faces are unchanged by a rotation of the charge line. We should also note that there is no physical variation in x, so none of the fields or potentials will depend on x. This means that we can draw everything as a single 2D slice in the y, z plane to get a square (cross-section of the cube) and the cross-section of the charge line at the origin. We are interested in the electric field just for the top part of the square (top face of the cube).

A circle (constant radius R1=L/2) just barely touches the closest part of the square. The electric field magnitude is constant everwhere on that circle and is directed radially outward. A larger circle of radius R2=L/sqrt(2) is just big enough to include the corners of the square. The field on the larger circle is constant, but with smaller magnitude than the inner circle.

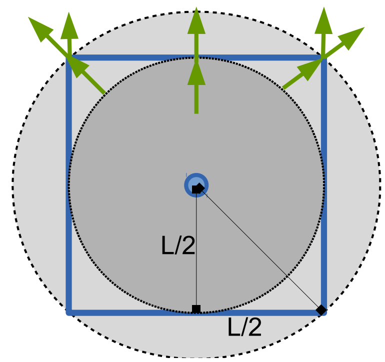

Figure 9: Side view of line charge passing through a cube.

All points along the top line fall between these two extremes

$$R_1 \leq r \leq R_2$$

and so do the electric field magnitudes

$$E_1 \geq E(r) \geq E_2$$

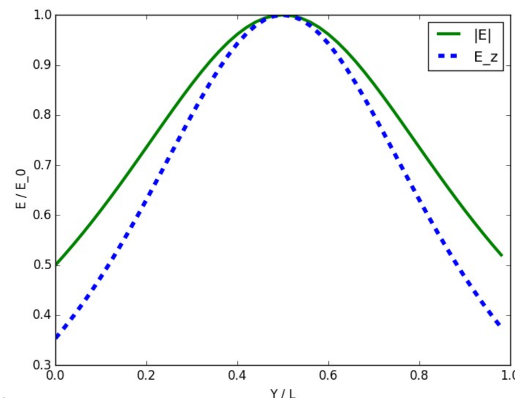

Figure 10: Profile of $|\vec{E}|$ and $E_z = E\cos\theta$ along the top surface.

The normal to the surface is parallel to the electric field vector at the point directly above the wire (and below, and left, and right). At the corners $\hat{n} = \hat{k}$ is 45° to $\vec{E}$, so even if E were constant the E_z component would be smaller.

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UCID: | | | | | | | | |

Q19a *[2.0 marks]*

+0.5 for rough sketch showing wire below top face with $\vec{E}$ arrows directed radially out from wire.

+0.5 for brief description The top face is an $L \times L$ square located $L/2$ above the line charge. If the field of a line charge falls off as $1/r$ then the largest electric field will be at the closest part of the cube. This will be at a distance $r = L/2$ corresponding to a line (x, y=L/2, z=L) along the top face running parallel to the line charge.

+0.5 for correct line, either equation or drawn in figure or described

The vector electric field everywhere along the line

$$\vec{E} = \frac{\lambda}{2\pi\epsilon_0 r}\hat{r} = \frac{\lambda}{\pi\epsilon_0 L}\hat{k}$$

has magnitude E

$$\boxed{E = \frac{\lambda}{\pi\epsilon_0 L}}$$

+0.5 for correct result

Q19b *[2.0 marks]*

+0.5 for rough sketch of $\vec{E}$ and $\hat{n}$. Can be included in same figure as Q19a.

+0.5 for brief description

The electric field magnitude is largest directly above the line of charge. It is also parallel to the surface normal, so $\vec{E} \cdot \hat{n}$ will be a maximum there. At all other points the field magnitude will be smaller and the field direction will rotate away from $\hat{n}$.

+0.5 for correct line, either equation or drawn in figure or described

$$E_z = \vec{E} \cdot \hat{k} = \frac{\lambda}{\pi\epsilon_0 L}\hat{k} \cdot \hat{k} = \boxed{\frac{\lambda}{\pi\epsilon_0 L}}$$

+0.5 for correct result

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UCID:

Q19c *[1.5 marks]*

No figure required

The electric field magnitude is smallest at the furthest distance from the line of charge ie. the corners. It is also rotated by 45°, so the E_z component is even smaller.

0.5 for identifying the location(s) of the E_z minima, words are fine.

The furthest points are along the edges (x, y=0 or L, z=L) so the distance is

$$\sqrt{(L/2)^2 + (L/2)^2} = \sqrt{2L^2/4} = \frac{\sqrt{2}L}{2} = \frac{L}{\sqrt{2}}$$

and the angle in the dot product is

$$\hat{z} \cdot \hat{r} = \hat{z} \cdot \left[\frac{\hat{k}}{\sqrt{2}} \pm \frac{\hat{j}}{\sqrt{2}}\right] = \frac{1}{\sqrt{2}} = \cos 45^\circ$$

so the smallest value for E_z is

$$E_z = \vec{E} \cdot \hat{n} = \frac{\lambda}{2\pi\epsilon_0 r} \cos\theta = \frac{\sqrt{2}\lambda}{2\pi\epsilon_0 L} \cos 45^\circ = \boxed{\frac{\lambda}{2\pi\epsilon_0 L}}$$

1.0 for correct result

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UCID:

Q19d *[1.0 marks]*

Total flux through the entire cube is equal to total charge enclosed, divided by ϵ_0

$$\Phi_E = Q_{\text{enc}}/\epsilon_0 = \lambda L/\epsilon_0$$

0.5 for Gauss's law in equation or words

The cube has 6 surfaces, all $L \times L$ squares. Two of the faces have surface normals pointing parallel to the line of charge, perpendicular to the electric field, so the electric flux through them will be zero. It might be difficult to calculate the flux through each of the other 4 faces, but since they are all identically located (orientation & distance) with respect to the wire, they will all have the same flux Φ_0.

$$\Phi_E = \Phi_{left} + \Phi_{right} + \Phi_{front} + \Phi_{back} + \Phi_{top} + \Phi_{bottom}$$

Flux is zero through two faces with normal vectors $\pm\hat{i}$, flux through other four faces is equal (symmetry)

$$\Phi_E = 0 + 0 + \Phi_0 + \Phi_0 + \Phi_0 + \Phi_0$$

$$\Phi_{\text{top}} = \Phi_0 = \frac{1}{4}\Phi_E = \boxed{\frac{\lambda L}{4\epsilon_0}}$$

0.5 marks for the right answer

Valid alternative: use $\vec{E}$ to calculate flux through a Gaussian cylinder, which must be the same as flux through the cube.

Q19e *[0.5 marks]*

Total flux through top face is an integral of E_z as it varies over the surface. It is also equal to the average value $\bar{E}_z$ times the surface area of the top face $A = L^2$

$$\Phi_E = \int \vec{E} \cdot d\vec{A} = \int E_z\,\hat{k} \cdot dA\,\hat{k} = \int E_z\,dA = \int \bar{E}_z\,dA = \bar{E}_z\,A$$

so the average $\bar{E}_z$ is total flux divided by total area

$$\bar{E}_z = \boxed{\frac{\Phi_0}{A}} = \frac{\lambda L}{4\epsilon_0}/L^2 = \boxed{\frac{\lambda}{4\epsilon_0 L}}$$

+0.5 for either answer, or if used wrong result from previous question but did this one correctly