$$\lambda = \frac{q}{L}$$

$$q_{tot} = \lambda \cdot L$$

$$dq = \lambda \cdot dL$$

$$\rho = \frac{q}{V}$$

$$q_{tot} = \rho \cdot V$$

$$dq = \rho \cdot dV$$

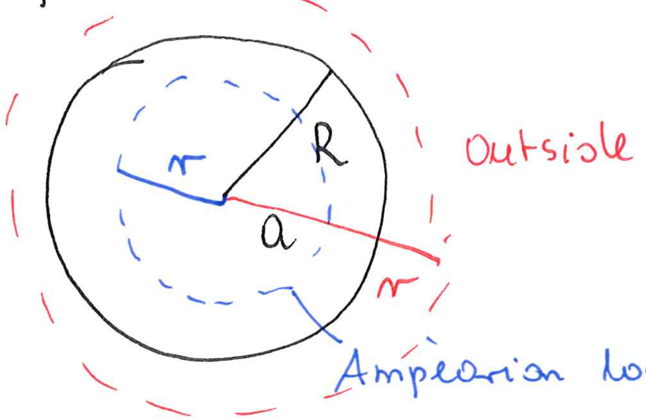
$$q_{enc} = \rho \cdot \frac{4}{3}\pi r^3$$

$$\rho = \frac{q}{V} = \frac{q}{\frac{4}{3}\pi R^3}$$

$$q_{enc} = \frac{q}{\cancel{\frac{4}{3}}\pi R^3} \cdot \cancel{\frac{4}{3}}\pi r^3$$

$$q_{enc} = \frac{q \cdot r^3}{R^3}$$

B- field inside solid wire



$$\oint \vec{B}\cdot d\vec{l} = \mu_0 i_{ncl}$$

At given r - B is constant (magnitude)

$$B\oint dl = B\cdot(2\pi r)$$

$\underline{\underline{i_{inc}}}$

$$J = \frac{I}{A}$$

cross sectional area

$a = \pi r^2$

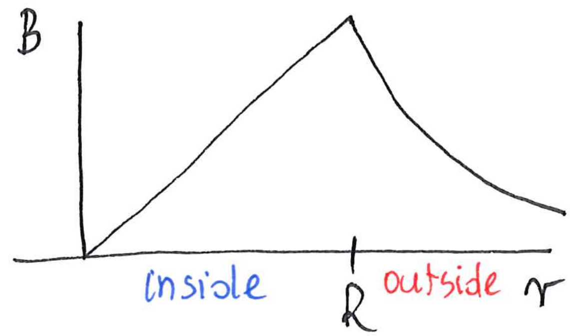
$$J = \frac{I}{\pi R^2}$$

$$i_{inc} = J\cdot a = J\cdot \pi r^2$$

$$i_{inc} = \frac{I}{\cancel{\pi} R^2}\cdot \cancel{\pi} r^2 = \frac{I\cdot r^2}{R^2}$$

$$B\cdot(2\pi \cancel{r}) = \mu_0\frac{I\cdot r^{\cancel{2}}}{R^2}$$

$$B = \frac{\mu_0 I}{2\pi R^2}\cdot r$$



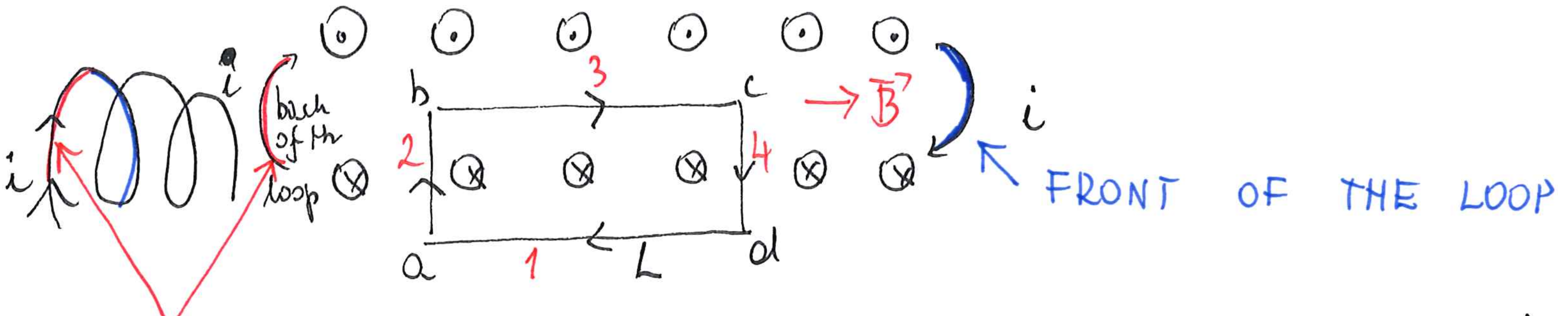
Outside

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 i_{inc}$$

$$B\cdot(2\pi r) = \mu_0 i$$

$$B = \frac{\mu_0 i}{2\pi r}$$

B- field of a solenoid (ideal) $L \gg d$



n = number of turns per length unit
e.g. 5 turns / cm

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \, i_{inc}$$

$$\oint \vec{B} \cdot d\vec{l} = \underbrace{\int_{ab} \vec{B} \cdot d\vec{l}}_{0} + \int_{bc} \vec{B} \cdot d\vec{l} + \underbrace{\int_{cd} \vec{B} \cdot d\vec{l}}_{0} + \underbrace{\int_{da} \vec{B} \cdot d\vec{l}}_{0}$$

$$\oint \vec{B} \cdot d\vec{l} = B \int_{bc} dl = B \cdot L$$

$$i_{inc} = N \cdot i = (nL) \cdot i$$

$$B \cdot \cancel{L} = \mu_0 n \cdot \cancel{L} \cdot i$$

$$\boxed{B = \mu_0 \cdot n \cdot i}$$ inside the solenoid

unifor inside

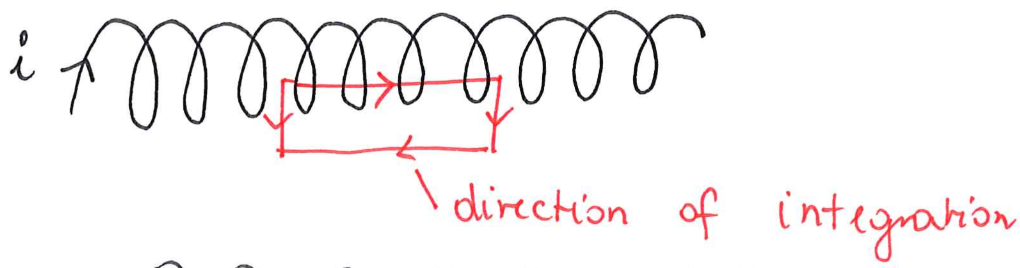
N - number of wires (loops) inclosed by Ampèorian loop

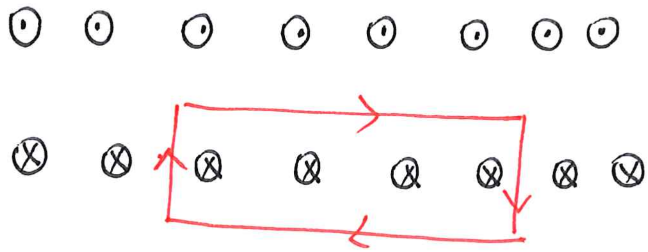
If the lenght of the loop is L

$$N = n \cdot L$$

↑ # turns per lenght unit

Orientation of Ampèarian loop in solenoid



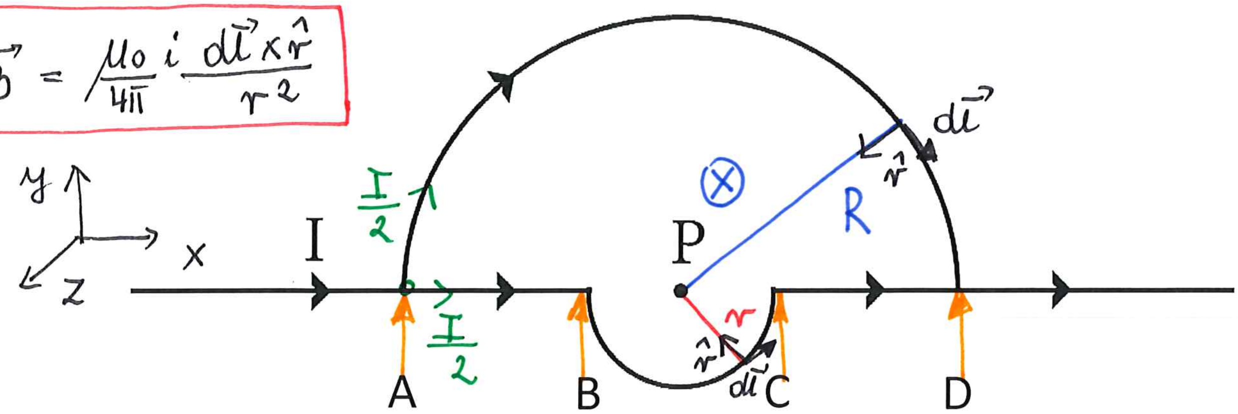


Same loop, but wires are shown as circles with direction of current going through it.

Activity # 9

The wire in the diagram below carries a current, I, as shown. The current splits equally at the junction. The top curved section of wire is a semi-circle of radius R, the bottom curved section of wire is a semi-circle of radius r, and point P is at the center of curvature of both section.

$$\boxed{d\vec{B} = \frac{\mu_0}{4\pi} i \frac{d\vec{l} \times \hat{r}}{r^2}}$$



Use the law of Biot and Savart to set up the equations which can be integrated to obtain the magnetic field strength, B, at point P due to the four sections of wire (AB, BC, CD & AD). For each section indicate the direction of $\vec{B}$. Start with the law of Biot and Savart, and show **all** variables on the figure provided (there is no need to re-draw it). [8 mark]

For straigth sections of the wire $d\vec{l} \parallel \hat{r}$ – they do NOT contribute to magnetic field at P, so $dB_{AB} = 0$ 2 $dB_{CD} = 0$.

Top section (AD).

$d\vec{l} \times \hat{r} = |dl||1| \cdot \sin 90^\circ = dl$

Using RHR (right hand rule) $d\vec{l} \times \hat{r}$ – into the page $(-\hat{k})$

All the segments have the same direction → calculate magnitude

$$dB = \frac{\mu_0 \frac{I}{2} dl}{4\pi R^2} \qquad B = \int_0^{\pi R} \frac{\mu_0 \frac{I}{2}}{4\pi R^2} dl = \frac{\mu_0 \frac{I}{2}}{4\pi R^2} \int_0^{\pi R} dl$$

$$B = \frac{\mu_0 \frac{I}{2}}{4\cancel{\pi} R^{\cancel{2}}} \cdot \cancel{\pi} \cancel{R} \qquad B = \frac{\mu_0 I}{8R}$$

(BC) Bottom section: $d\vec{l} \times \hat{r}$ – out of the page $(+\hat{k})$

$$dB = \frac{\mu_0 \frac{I}{2} dl}{4\pi r^2} \qquad B = \frac{\mu_0 \frac{I}{2}}{4\pi r^2} \int_0^{\pi r} dl \qquad B = \frac{\mu_0 I}{8\cancel{\pi} r^{\cancel{2}}} \cdot \cancel{\pi} \cancel{r} = \frac{\mu_0 I}{8r}$$

What is the magnitude and direction of $\vec{B}$ at point P due to the four sections of wire? [2 marks]

$$\vec{B}_P = \vec{B}_{AD} + \vec{B}_{BC} = \left(-\frac{\mu_0 I}{8R} + \frac{\mu_0 I}{8r}\right)\hat{k}$$

$$\vec{B} = \frac{\mu_0 I}{8}\left(\frac{1}{r} - \frac{1}{R}\right)\hat{k}$$

*