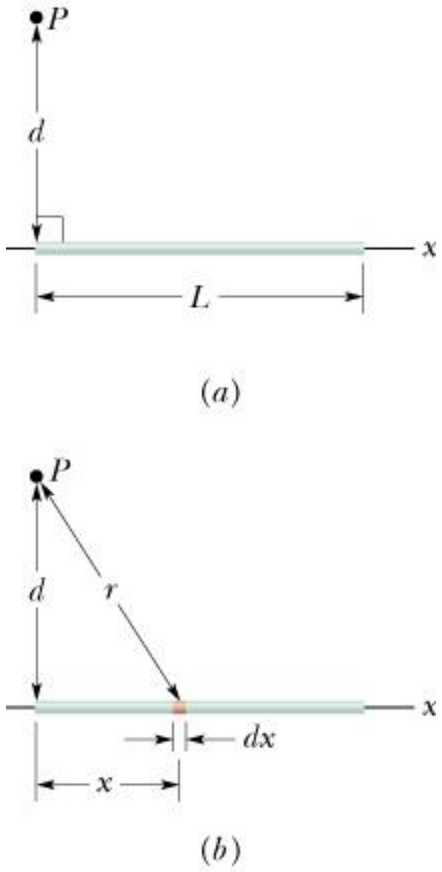
Last time

- More on equipotential surface

This time

- Electric potential for a line charge
- Electric field from electric potential for a line charge
- Class activity #6

Potential due to a line of charge



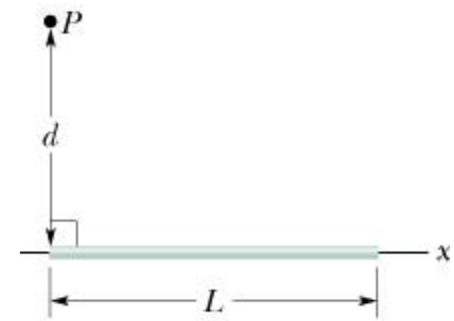
Potential due to a line of charge

We will calculate the electric potential for a point P as shown in the figure. Assuming a uniform charge density, we have

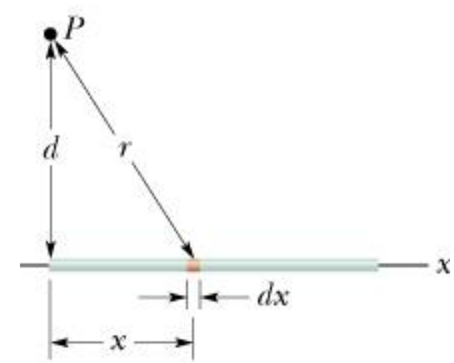
$$dV = \frac{1}{4\pi\epsilon_0}\frac{dq}{r}$$

with $dq = \lambda dx$ and $r = \sqrt{x^2 + d^2}$. The electric potential due to all the charge from $x = 0$ to $x = L$ is given by

$$\begin{aligned} V &= \int dV = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{r} \\ &= \frac{1}{4\pi\epsilon_0}\int_0^L \frac{\lambda dx}{\sqrt{x^2+d^2}} \\ &= \frac{\lambda}{4\pi\epsilon_0}\int_0^L \frac{dx}{\sqrt{x^2+d^2}} \\ &= \frac{\lambda}{4\pi\epsilon_0}\left[\ln\left\{x + \left(x^2+d^2\right)^{1/2}\right\}\right]_0^L \\ &= \frac{\lambda}{4\pi\epsilon_0}\left[\ln\left\{L + \left(L^2+d^2\right)^{1/2}\right\} - \ln d\right] \end{aligned}$$



(a)

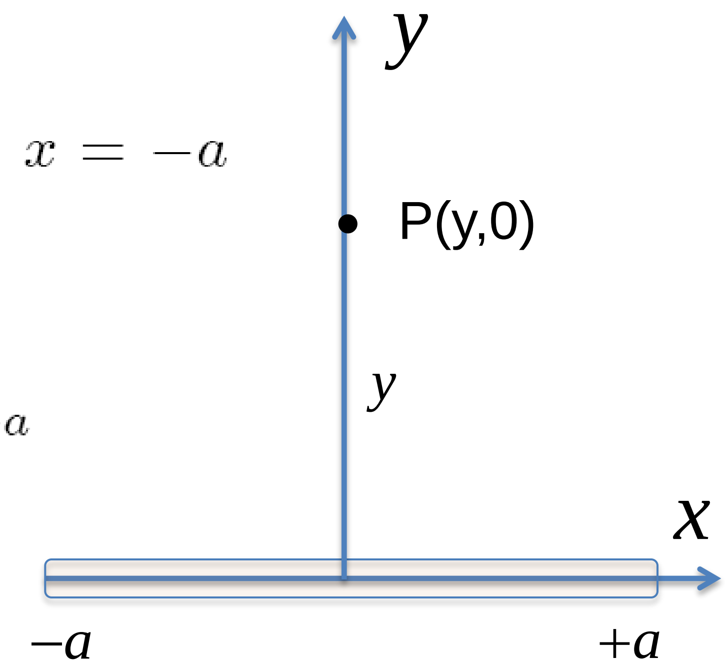


(b)

or

$$V = \frac{\lambda}{4\pi\epsilon_0} \ln \frac{\left[L + \left(L^2 + d^2\right)^{1/2}\right]}{d}$$

If however, we have a rod which extends from $x = -a$ to $x = +a$, we would get



$$V = \frac{\lambda}{4\pi\epsilon_0} \left[\ln\left\{x + \left(x^2 + y^2\right)^{1/2}\right\}\right]_{-a}^{a}$$

and

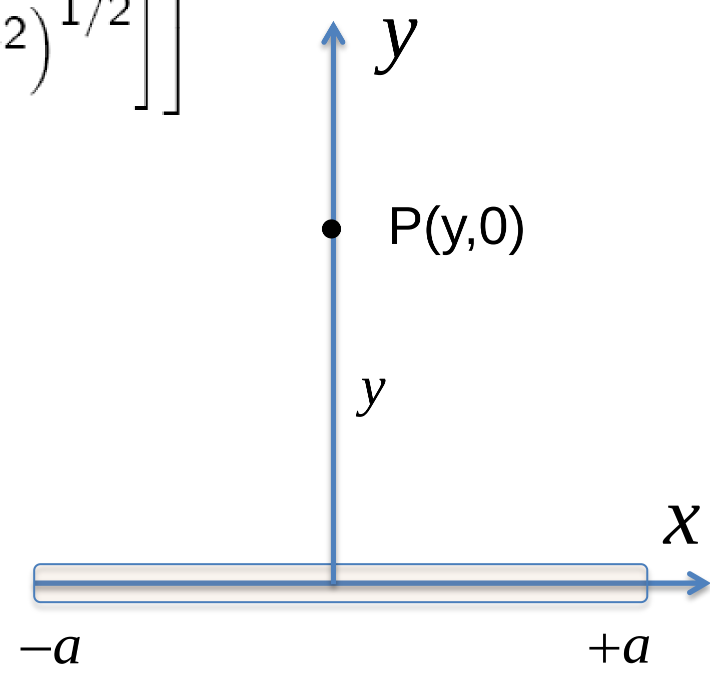
$$V = \frac{\lambda}{4\pi\epsilon_0} \ln \frac{\left[a + \left(a^2 + y^2\right)^{1/2}\right]}{\left[-a + \left(a^2 + y^2\right)^{1/2}\right]}$$

and

$$E_x = -\frac{\partial V}{\partial x} = 0$$

$$E_z = -\frac{\partial V}{\partial z} = 0$$

$$
\begin{aligned}
E_y &= -\frac{\partial V}{\partial y} = -\frac{\partial}{\partial y}\left[\frac{\lambda}{4\pi\epsilon_0}\ln\frac{\left[a+\left(a^2+y^2\right)^{1/2}\right]}{\left[-a+\left(a^2+y^2\right)^{1/2}\right]}\right] \\
&= \frac{2a\lambda}{4\pi\epsilon_0}\,\frac{1}{y\left(a^2+y^2\right)^{1/2}} \\
&= \frac{1}{4\pi\epsilon_0}\,\frac{Q}{y\left(a^2+y^2\right)^{1/2}}
\end{aligned}
$$



A result which we obtained before by direct integration using

$$
\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int_{-\infty}^{+\infty}\frac{dq}{r^2}\hat{r}
$$

(10 marks) The figure below shows a ring of charge with total charge dQ (Figure.1) and a solid disk of constant charge density σ (Figure.2). The points P are located a distance z above the center of both the ring and disk. Find the electrical potential at a point P above the center of the disk.

Useful formulas: $\int \frac{x\,dx}{\sqrt{x^2+a^2}} = \sqrt{x^2+a^2}$ $\qquad E_z^{disk} = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{z}{\sqrt{z^2+R^2}}\right]$

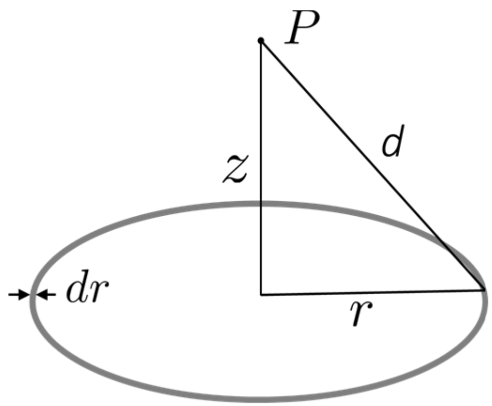


Figure 1. Ring

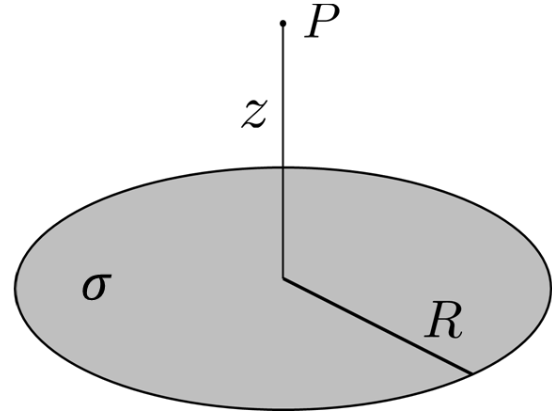


Figure 2. Disk

1. **(1 mark)** What is the distance d from some point on the ring of radius r to point P a distance z above the ring?
2. **(2 marks)** If you knew the potential at point P for a ring of thickness dr and charge dQ, how would go about calculating the potential at point P for a disk?
3. **(1 mark)** Considering the fact that all points on the ring are at the same distance from point P, write the expression for the small contribution to the potential at point P due to the ring of radius r and thickness dr shown in Figure 1?
4. **(2 marks)** What is the total potential at point P due to the disk (Figure.2). State explicitly what the limits of integration are and evaluate the integral.
5. **(1 mark)** Is there a direction associated with the electric potential in question 4? Why or why not?
6. **(2 marks)** Verify your expression for the potential of the disk (question 4) by calculating $E_z = -\frac{\partial V}{\partial z}$. Does this correspond with the electric field produced by a disk that you would expect?