Wednesday April 5, 2017

Last time:

- Applying Ampère's Law:
  - Magnetic field of solenoid and toroid
- Faraday's Law of Induction

Today:

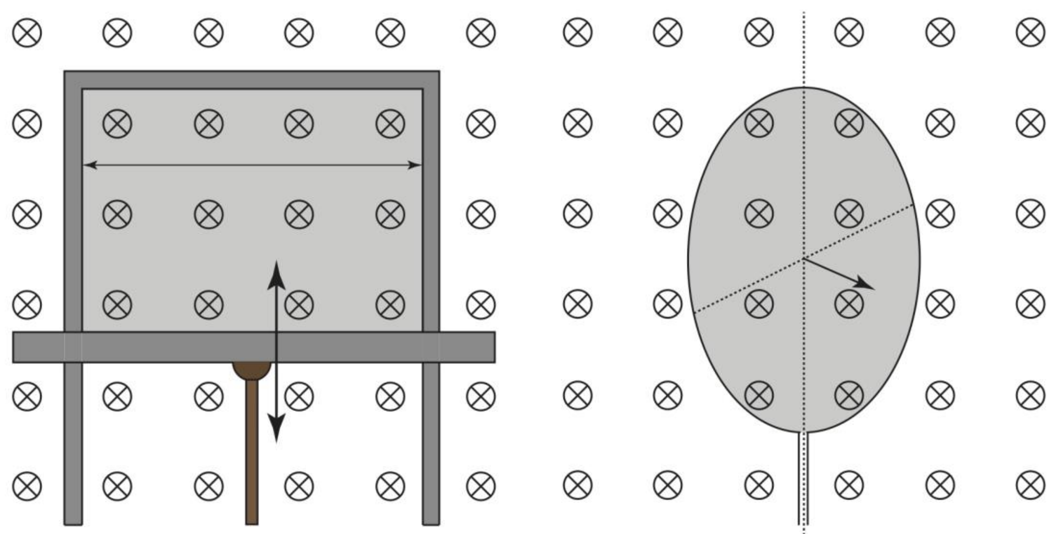
- Lenz's Law
- Non-conservative electric fields
- Motional emf

This is valid even if Φ_B changes because of a time dependent A or angle ϕ (without changing the magnetic field)!

$$\boxed{e = -\frac{d}{dt}\left(BA\cos f\right)}$$

→ **3 possible terms**

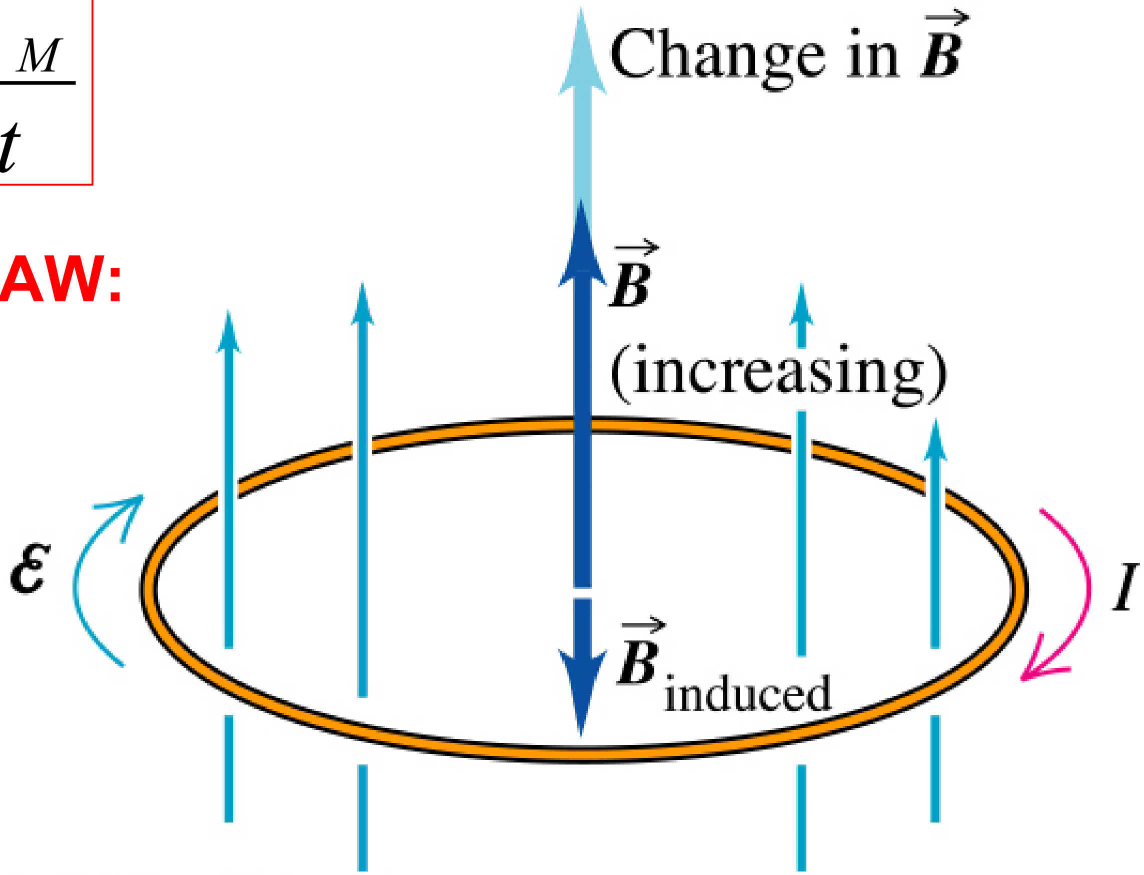
$$e = -\frac{dB}{dt}A\cos f - \frac{dA}{dt}B\cos f + \frac{df}{dt}BA\sin f$$



What about the **minus sign** in Faraday's law?

$$e = -\frac{dF_M}{dt}$$

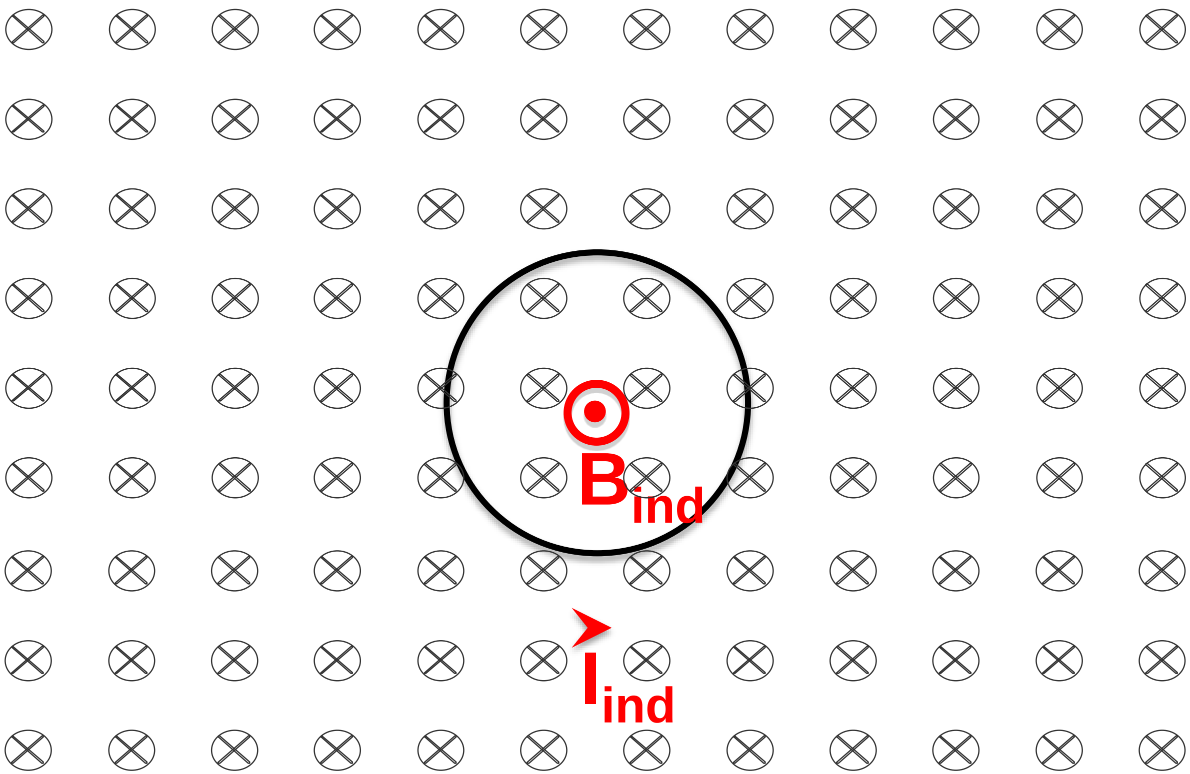
LENZ'S LAW:



The changing magnetic flux generates an induced current which creates an induced magnetic field which, in turn, resists the change in magnetic flux.

Lenz's Law

The induced current from Faraday's Law is always in a direction such that the induced magnetic field from the induced current opposes the change in the magnetic flux through the loop.



More B-field lines inside the loop: induced B-field from induced current must be out of the page to compensate.
Induced current is CCW

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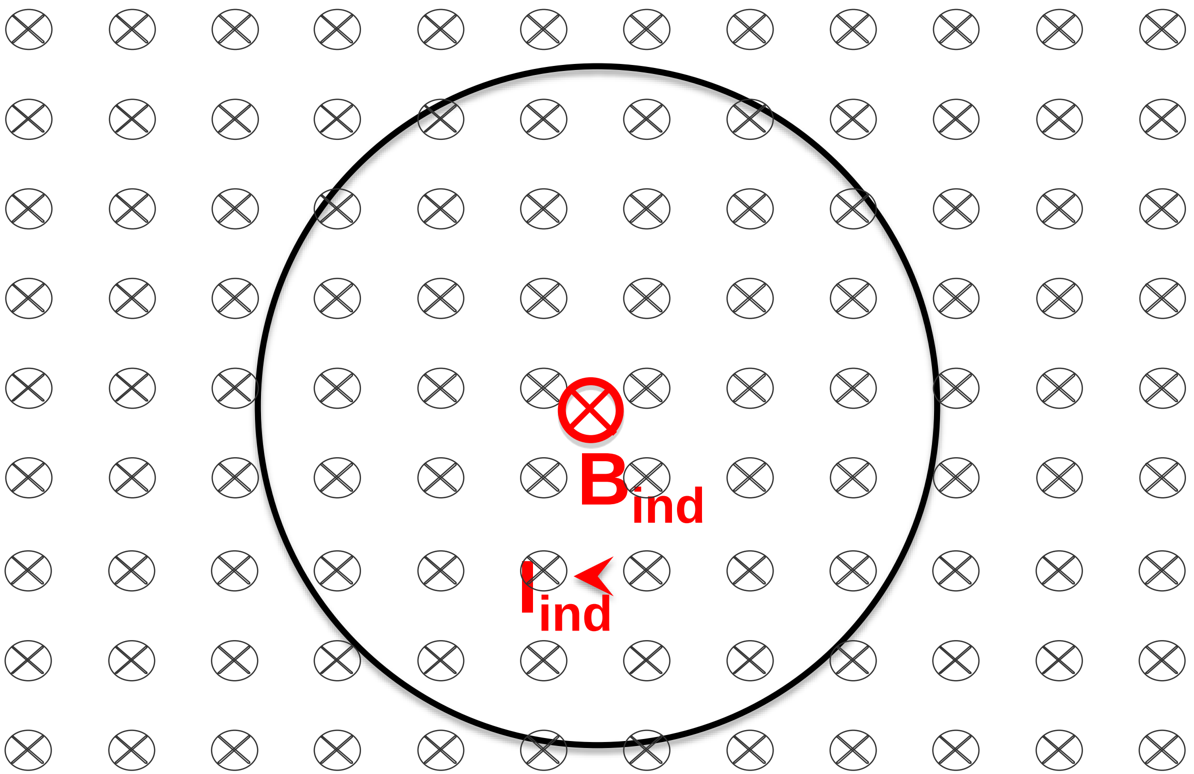
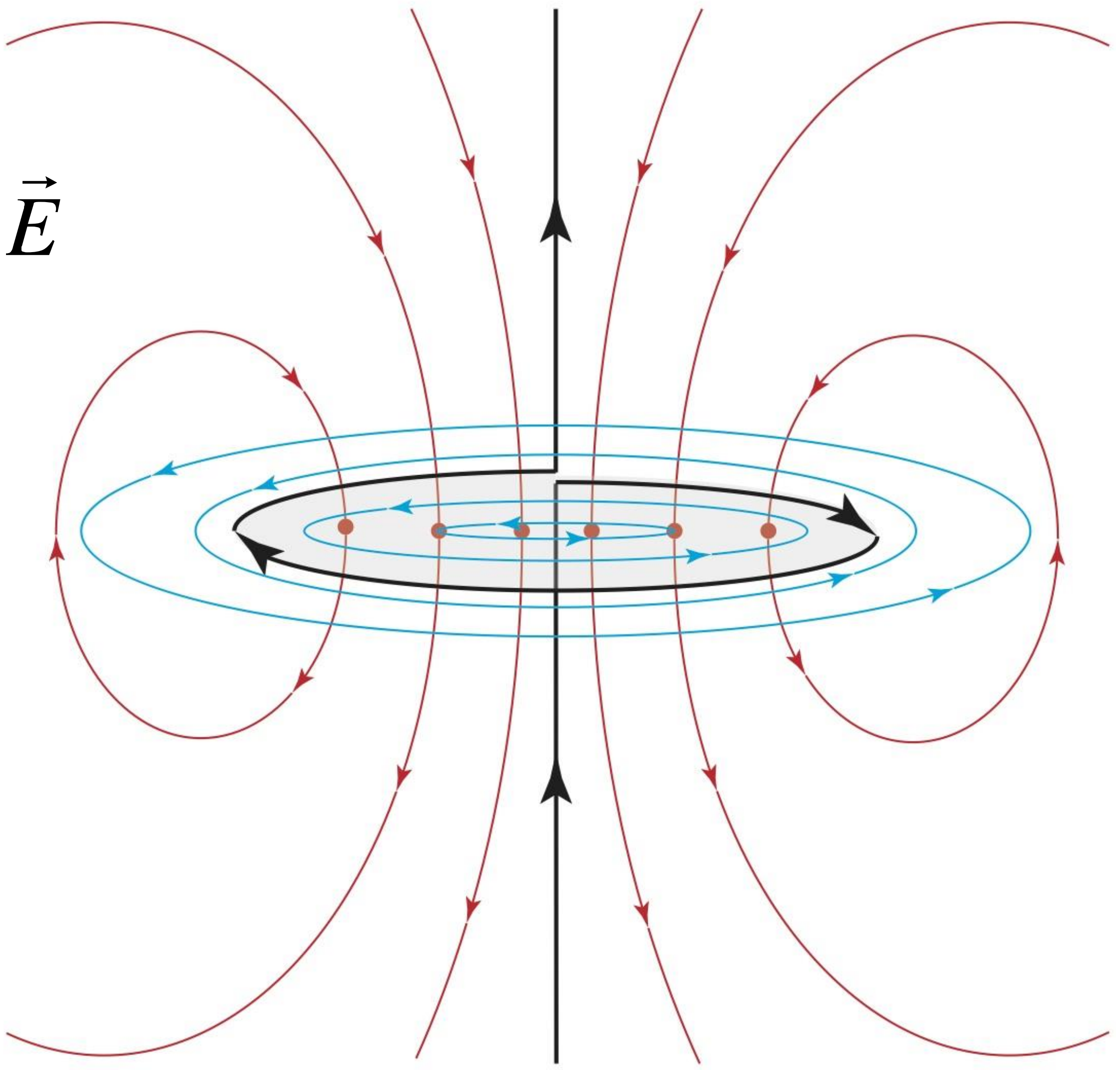


Fewer B-field lines inside the loop: induced B-field from induced current must be into the page to compensate. Induced current is CW

Imagine a loop in a wire carrying a current I_1. The current is then increased to $I_2 > I_1$, increasing the magnetic flux.
Changing B-fields induce non-conservative E-fields.

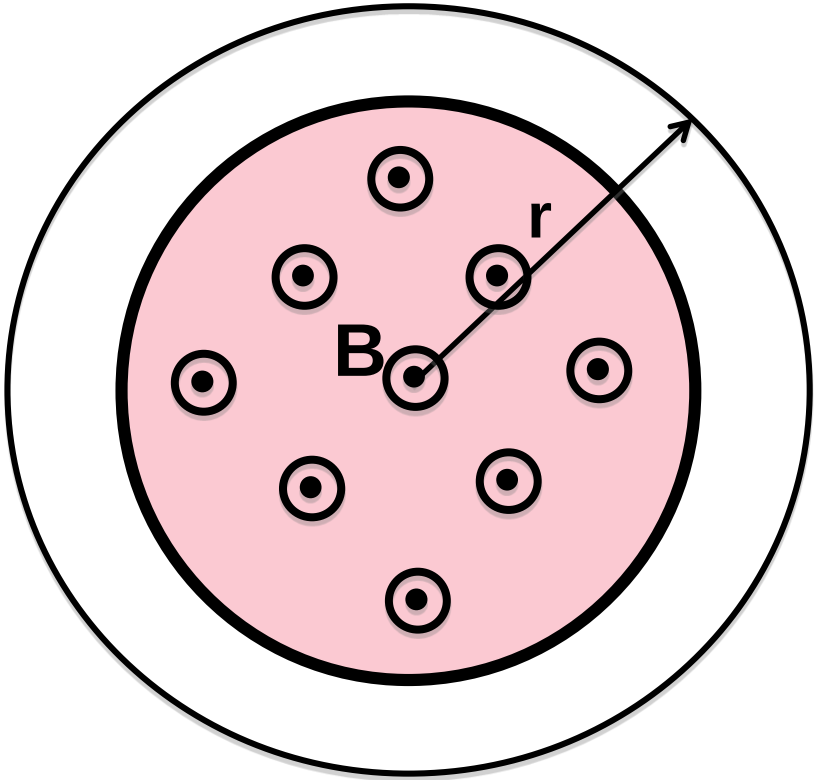
$$-\frac{d\vec{B}}{dt} = \nabla \times \vec{E}$$



Top Hat Question

The current in an infinitely long solenoid with uniform magnetic field B inside is increasing so that the magnitude B increases in time as $B=B_0+kt$. A circular loop of radius r is placed coaxially outside the solenoid as shown. In what direction is the induced **E**-field around the loop ?

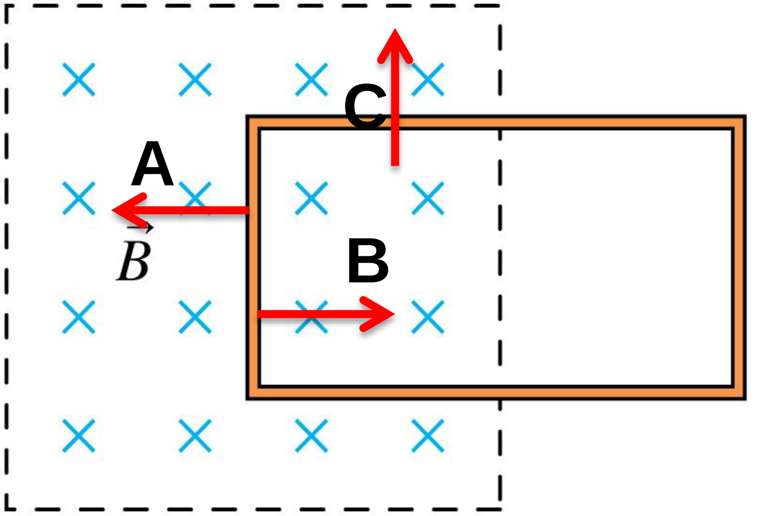
A. CW
B. CCW
C. The induced E is zero
D. Not enough information



Lenz' law: induced EMF around the loop is in the CW direction. The induced E-field must therefore be in the CW direction

Top Hat Question

A conducting loop is halfway inside a magnetic field. Suppose the magnetic field begins to increase rapidly in strength. What happens to the loop?



A. The loop is pulled to the left, into the magnetic field.

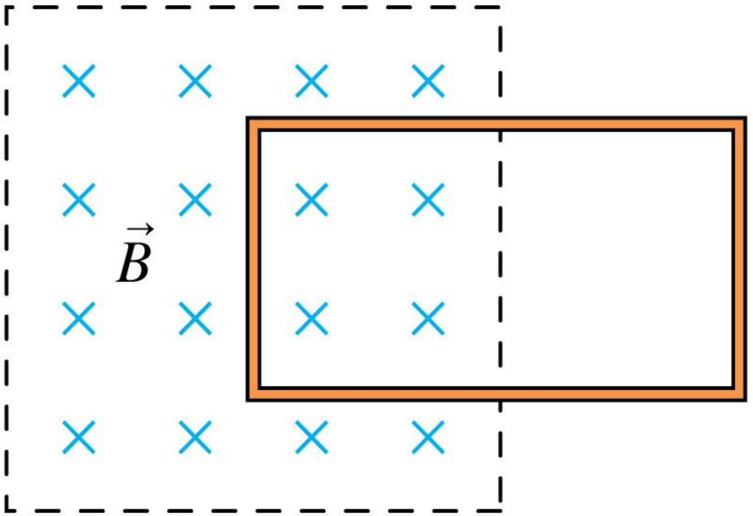
B. The loop is pushed to the right, out of the magnetic field.

C. The loop is pushed upward, out of the magnetic field

D. The tension in the wire increases but the loop does not move.

Top Hat Question Feedback

A conducting loop is halfway inside a magnetic field. Suppose the magnetic field begins to increase rapidly in strength. What happens to the loop?

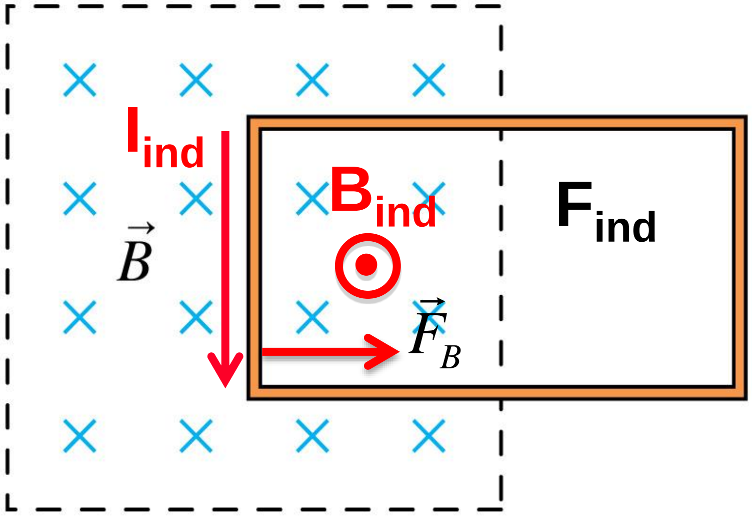


Qualitative argument

Lenz's Law: whatever happens must be such that it maintains the "amount of B-field" inside the loop. Since the strength of B is increasing, the loop must be pushed outside so that there are fewer B-field lines inside the loop.

Top Hat Question Feedback

A conducting loop is halfway inside a magnetic field. Suppose the magnetic field begins to increase rapidly in strength. What happens to the loop?



More rigorous argument

Lenz's Law: B_{ind} must point out, so I_{ind} is CCW

Recall: the Lorentz force on a current carrying wire

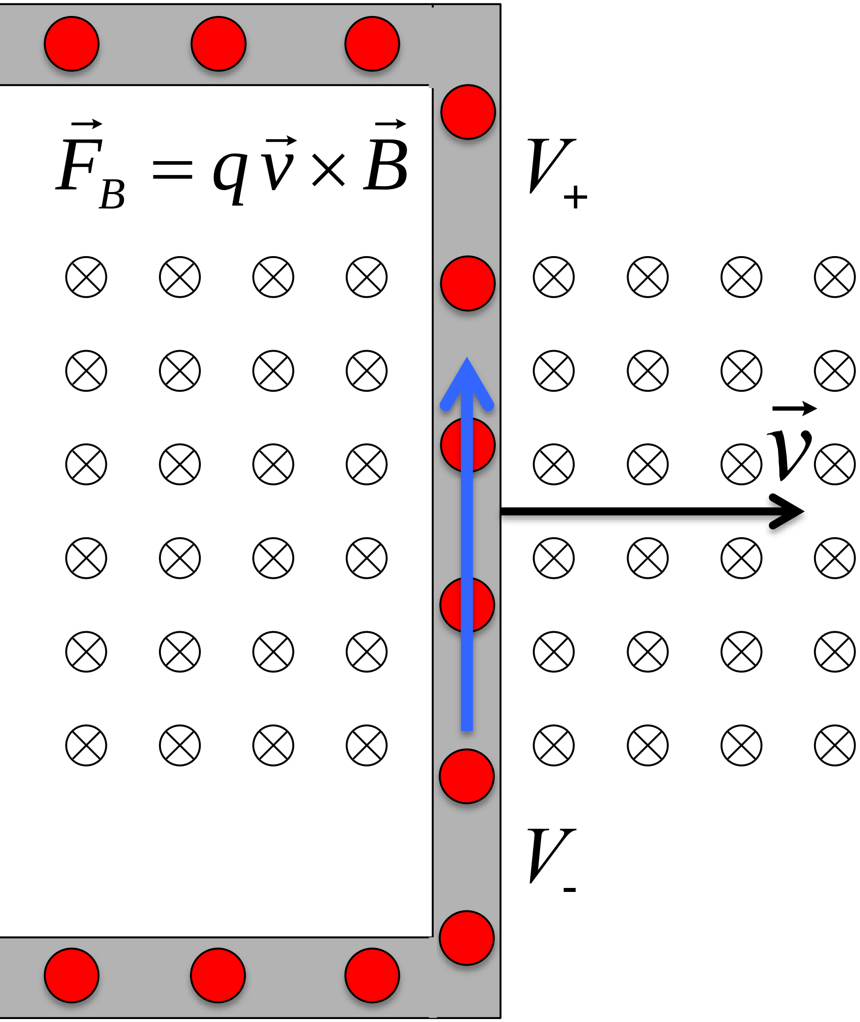
$$\vec{F}_B = I\vec{\ell} \times \vec{B}$$ ➔ points to the RIGHT

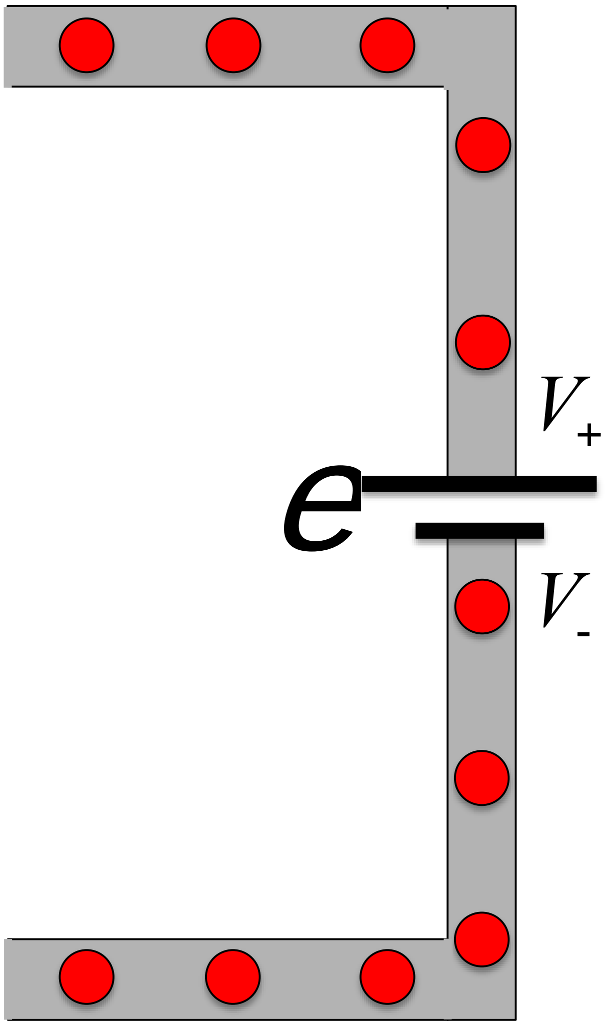
Recall there are 3 possible terms:

$$e = \underbrace{-\frac{dB}{dt}A\cos f}_{\text{Maxwell Equation}} \underbrace{- \frac{dA}{dt}B\cos f + \frac{df}{dt}BA\sin f}_{\text{What about these two terms?}}$$

$$-\frac{d\vec{B}}{dt} = \nabla \times \vec{E}$$

Motional EMF





There is an induced ΔV across the length of the conductor

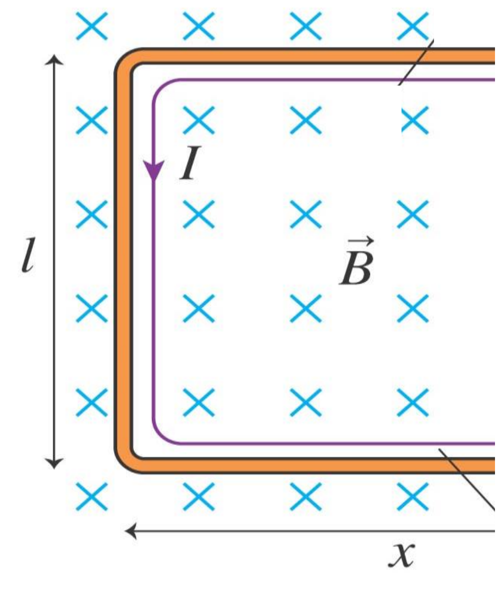
This is equivalent to having an EMF source: “motional EMF”

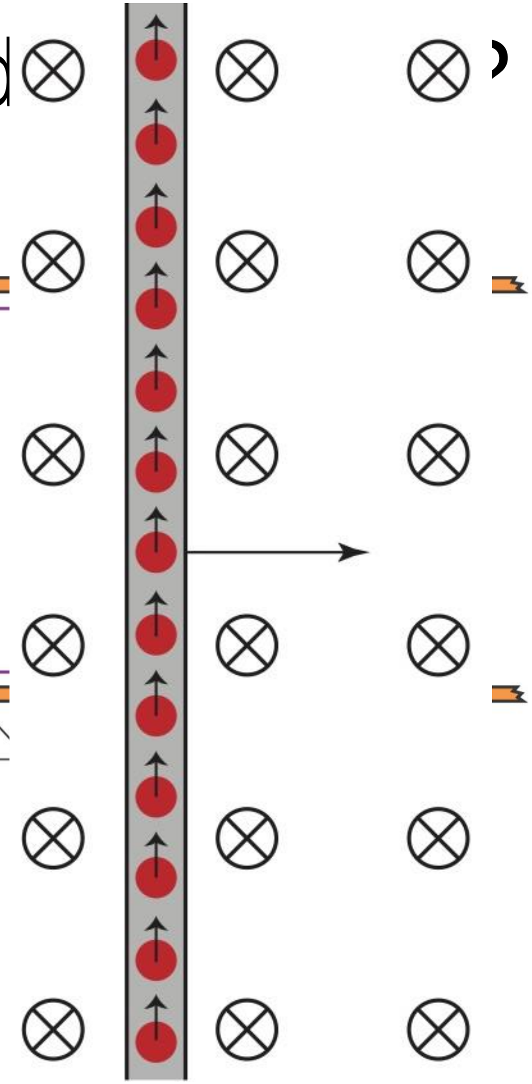
How can we quantify the ind

The free charges feel a magnetic force:

$$F = qvB$$

This induces a voltage difference (E-field), and therefore an electric force on the charges





$$F = qE \qquad E = \frac{\Delta V}{l}$$

$$\cancel{q}vB = \cancel{q}\frac{\Delta V}{l}$$

MOTIONAL EMF: $\Delta V \equiv \boxed{\varepsilon = vlB}$

Top Hat Question

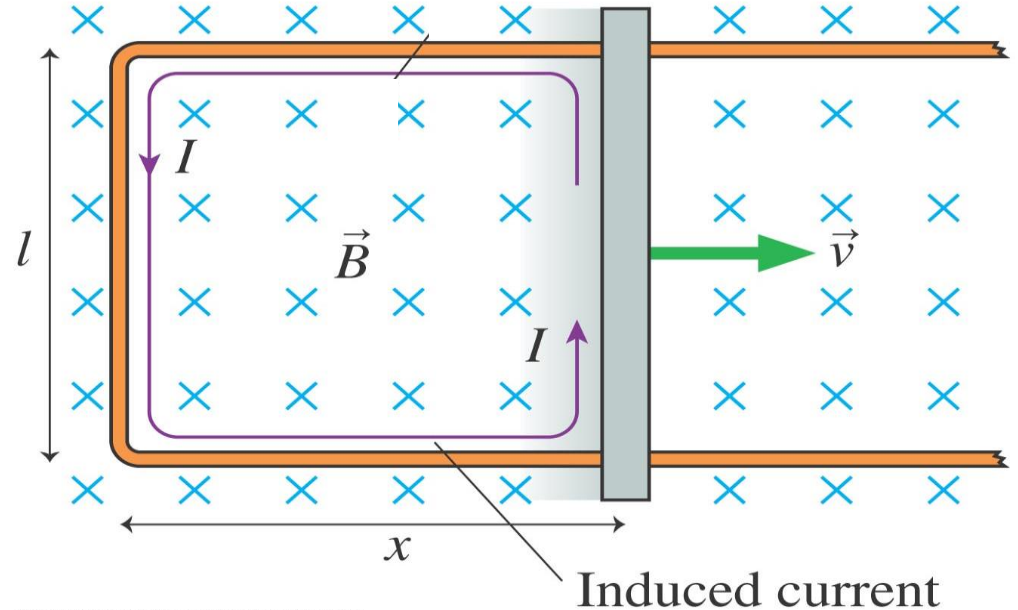
A U-shaped conductor with side length *l* = 1.0 m is sitting in a uniform magnetic field of field strength 1.0x10^{-2} T. A conducting cross bar is **moving with a constant velocity** of 1.0 m/s and has a resistance of R = 0.10 ohms. What is the **induced current** in the loop?

A. 0.0 A

B. 0.010 A

C. 0.10 A

D. 1.0 A



Top Hat Question

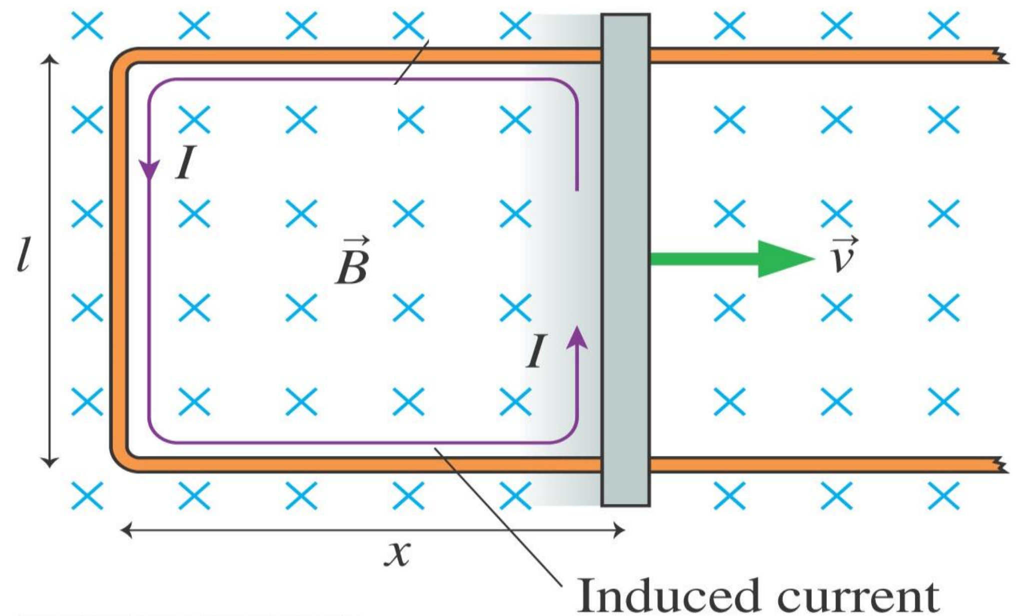
A U-shaped conductor with side length *l* = 1.0 m is sitting in a uniform magnetic field of field strength 1.0×10^{-2} T. A conducting cross bar is **moving with a constant velocity** of 1.0 m/s and has a resistance of R = 0.10 ohms. What is the **power dissipated by the bar's resistance?**

A. 0.0010 W

B. 0.010 W

C. 0.10 W

D. 1.0 W



Recall there are 3 possible terms:

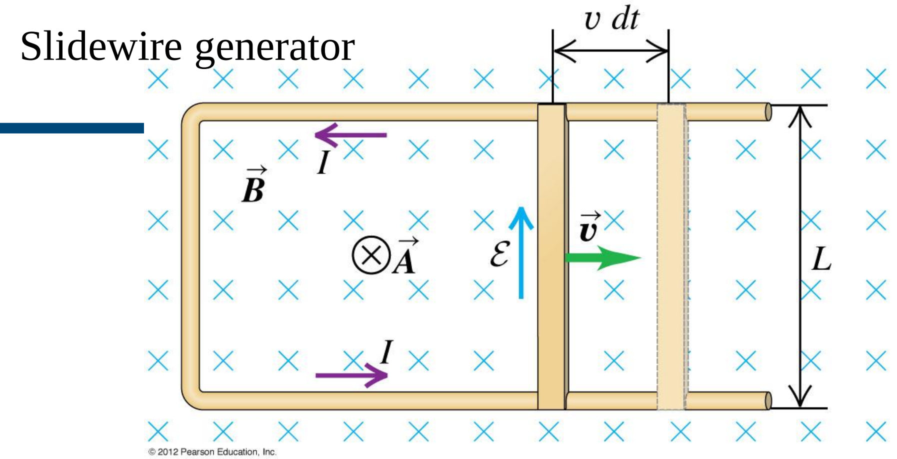
$$e = -\underbrace{\frac{dB}{dt}A\cos f}_{\text{Maxwell Equation}} - \underbrace{\frac{dA}{dt}B\cos f + \frac{df}{dt}BA\sin f}_{\text{Magnetic Force on free charges}}$$

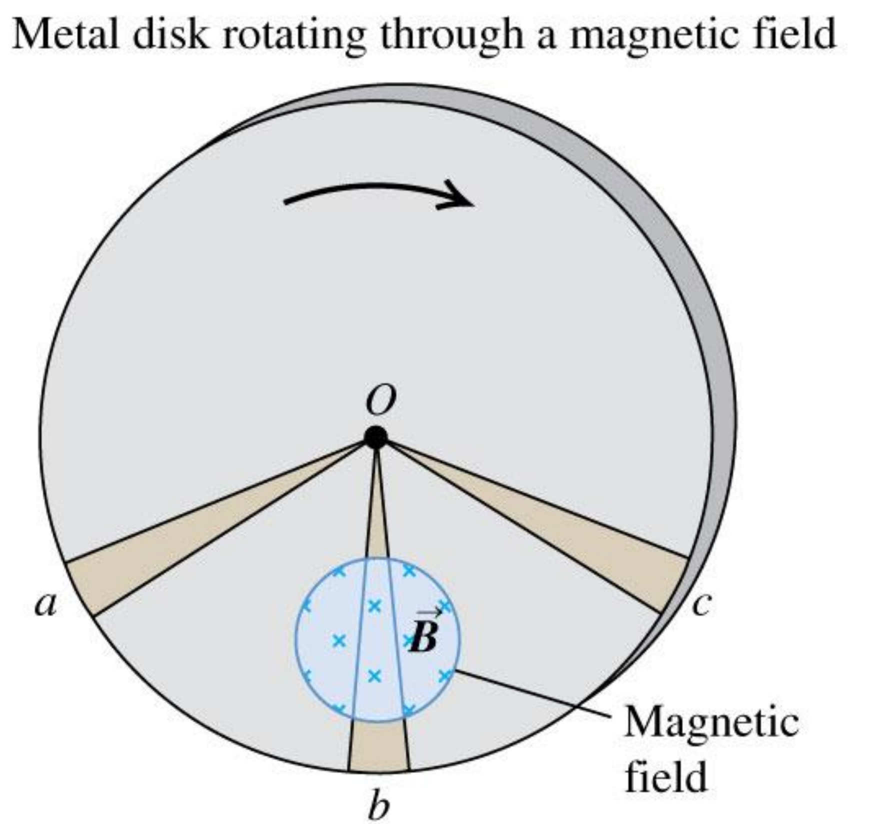
$$-\frac{d\vec{B}}{dt} = \nabla \times \vec{E} \qquad\qquad F = q\vec{v} \times \vec{B}$$

It is quite striking that drastically different sources for the induced EMF give an identical law. This makes Faraday's Law a particularly powerful tool from a practical engineering standpoint!

Eddy currents

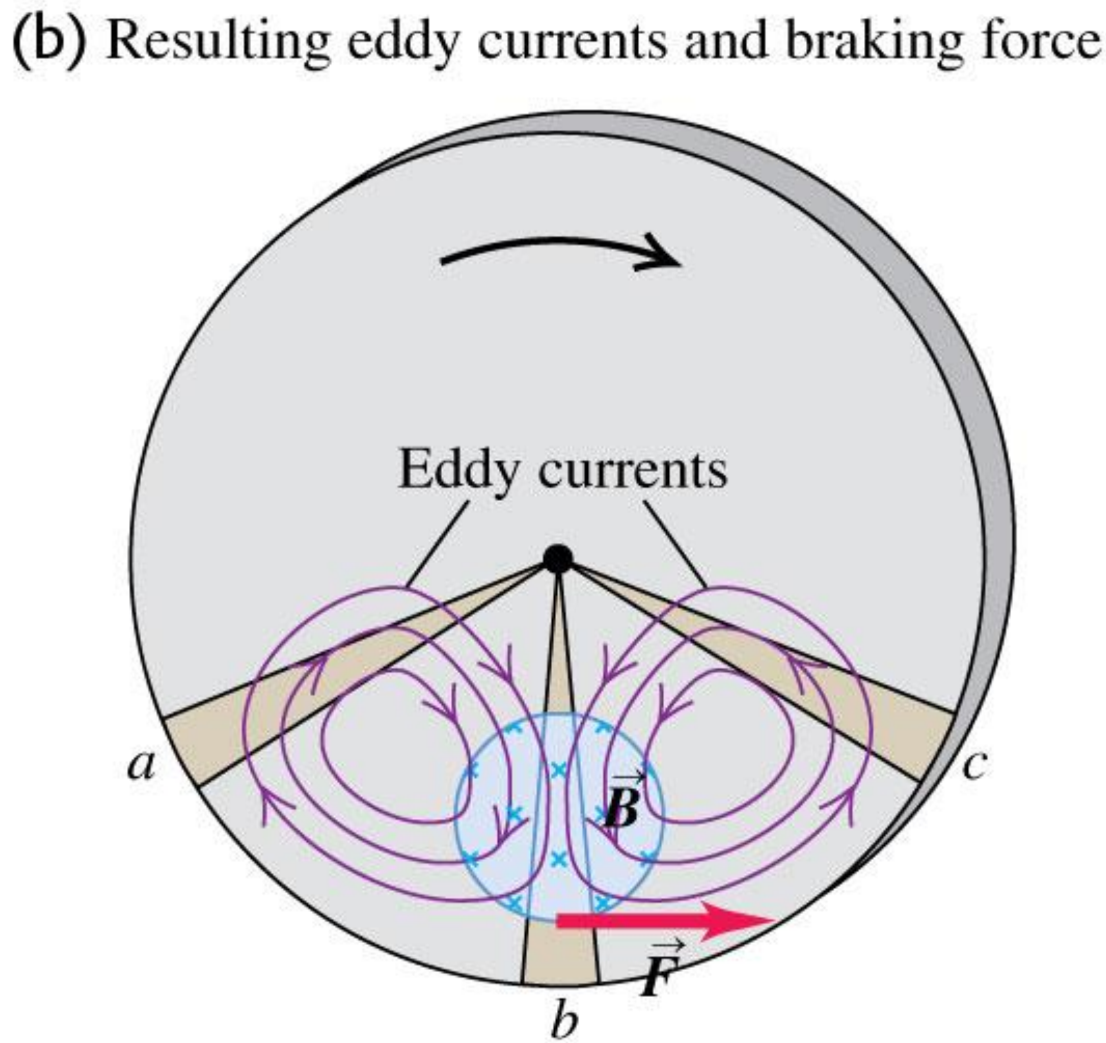
- So far we have considered induction in circuits, where the induced current is confined to wires
- Induction also happens if the magnetic flux through extended metallic objects changes
- As with wires, the induced currents attempt to keep the flux stable: *eddy* currents





Eddy currents

- The direction of the currents can be found using Lenz's law:
  - Without eddy currents, the magnetic flux at the leading (trailing) edge decreases (increases)
  - The induced Eddy currents circulate in a sense that prevents this from happening
  - Result: transformation of mechanical energy into heat!

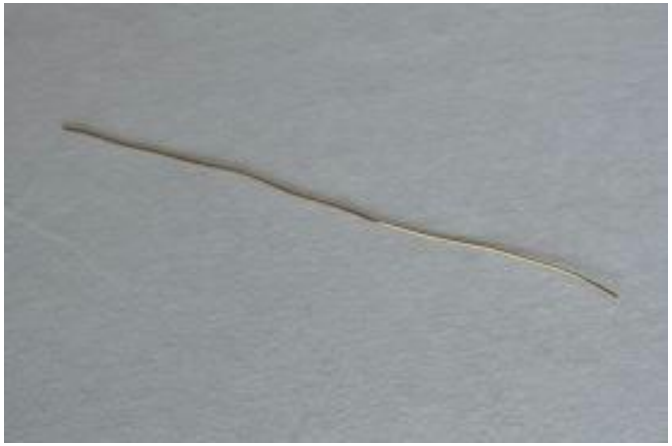


Question

Let’s consider a piece of wire in an electric circuit.

Does it matter if it is straight, or coiled?

a) It does not matter: both have the same resistance (if they have the same length)
b) It matters: the reaction to changing current is very different



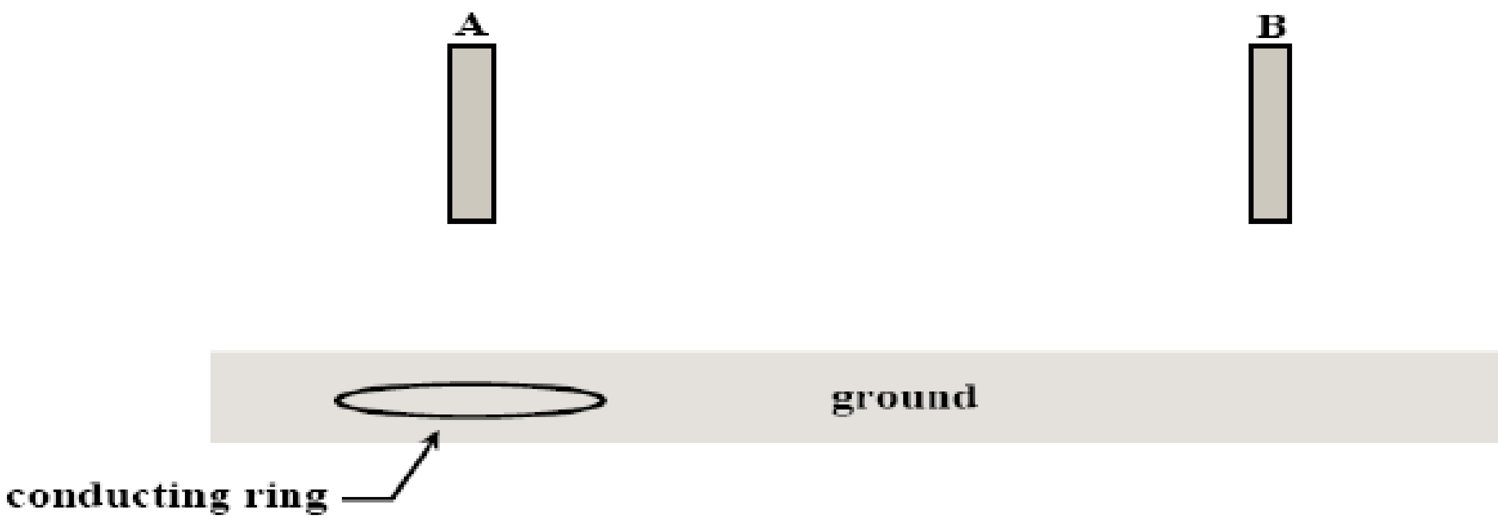
Wednesday April 5, 2017 – class 2

Examples

Examples

16. In the diagram below, two identical permanent magnets are dropped from equal heights above the ground at the same instant of time. There is a continuous ring of conducting material lying on the ground below magnet A. What happens? (Assume the two magnets are far enough apart that they do not influence each other, and the ground is nonconducting.)

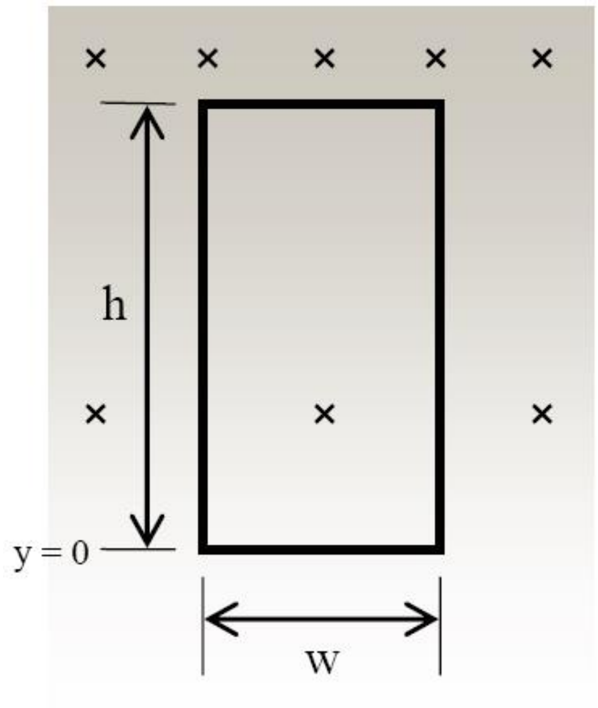
a. Magnet A hits the ground before magnet B.
b. Magnet B hits the ground before magnet A.
c. Both magnets hit the ground at the same time.
d. The answer depends on which pole of magnet A faces downwards.
e. None of the above.



Examples

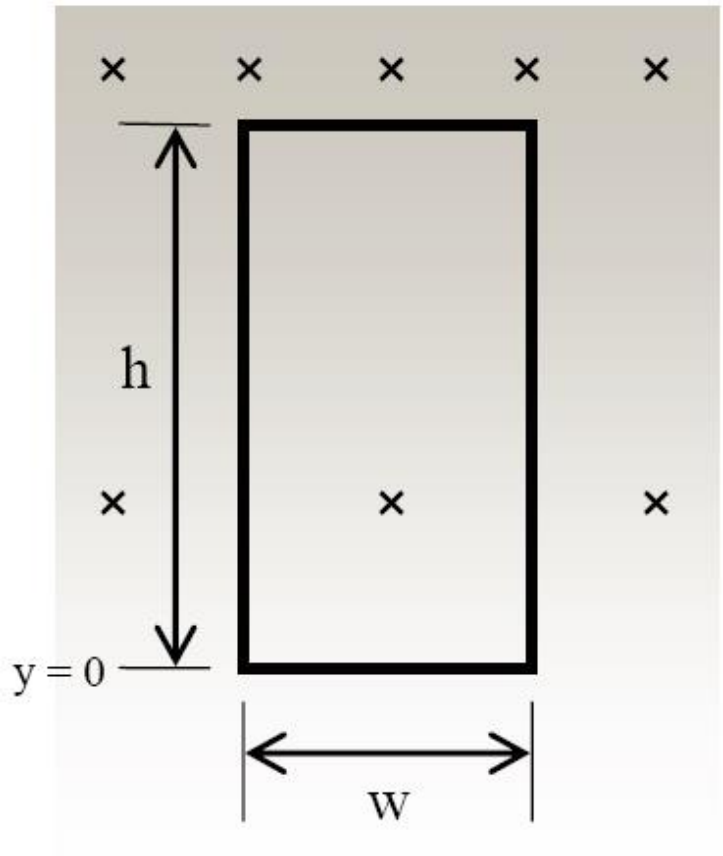
c. *(Parts c and d are not related to parts a or b, above.)* In the figure below, a rectangular loop of conductor (e.g., a metal wire) of width w and height h is immersed in a non-uniform magnetic field into the page. The strength of the magnetic field increases upwards linearly as $B = B_0 + Cy$, where B_0 is the magnetic field strength at the base of the loop (i.e., at $y = 0$), and C is a constant. In the diagram, the strength of the magnetic field is indicated by the amount of shading (darker = stronger field).

Derive an expression for the magnetic flux enclosed by the loop, in terms of B_0, C, and quantities given in the diagram.



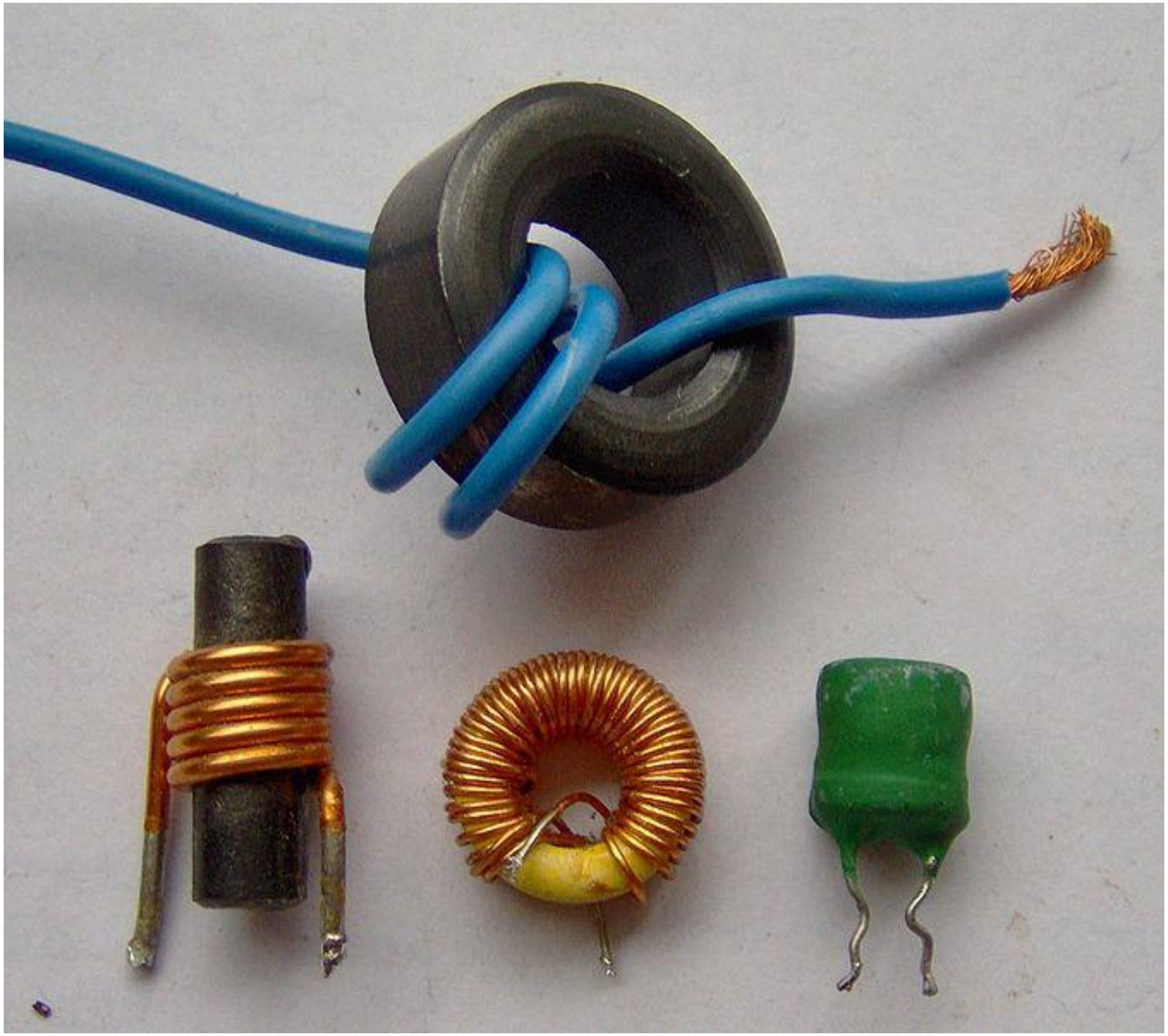
Examples

d. In which direction should you move the loop in the figure above in order to induce a current in the loop in the ***clockwise*** direction? Justify your answer clearly.



Inductors

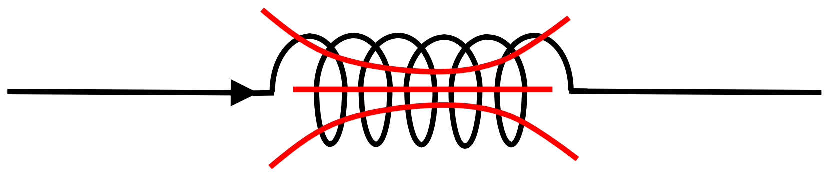
An inductor is a passive electrical component that can store energy in a magnetic field.



Inductance

Note that a changing Magnetic flux produces an induced EMF in a direction which "tries to oppose the change"

$$\frac{di}{dt}$$



$$\varepsilon = -\frac{d}{dt}\left(N\mu_0 n i A\right)$$

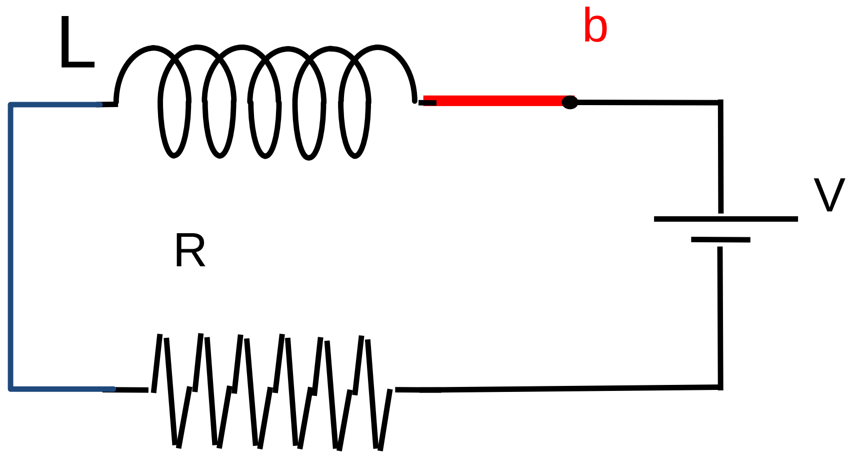
Changing the current changes the flux through the inductor, which creates a back-emf. Model inductor as perfect solenoid

$$\Delta V = -\left(\frac{N^2}{\ell}\mu_0 A\right)\frac{di}{dt} = -L\frac{di}{dt}$$

$$\boxed{L = \mu_0 \frac{N^2}{\ell} A}$$

L is a geometric quantity

R-L Circuit



$$V - L\frac{di}{dt} - iR = 0$$

If the switch is moved to position b, to initiate the current flow, what happens?

Faraday's law applies and so the change in the Magnetic Field in the inductor L means there is a back EMF induced in L.

So in this case at t = 0, $i(0) = 0$. Inductor acts like a BATTERY

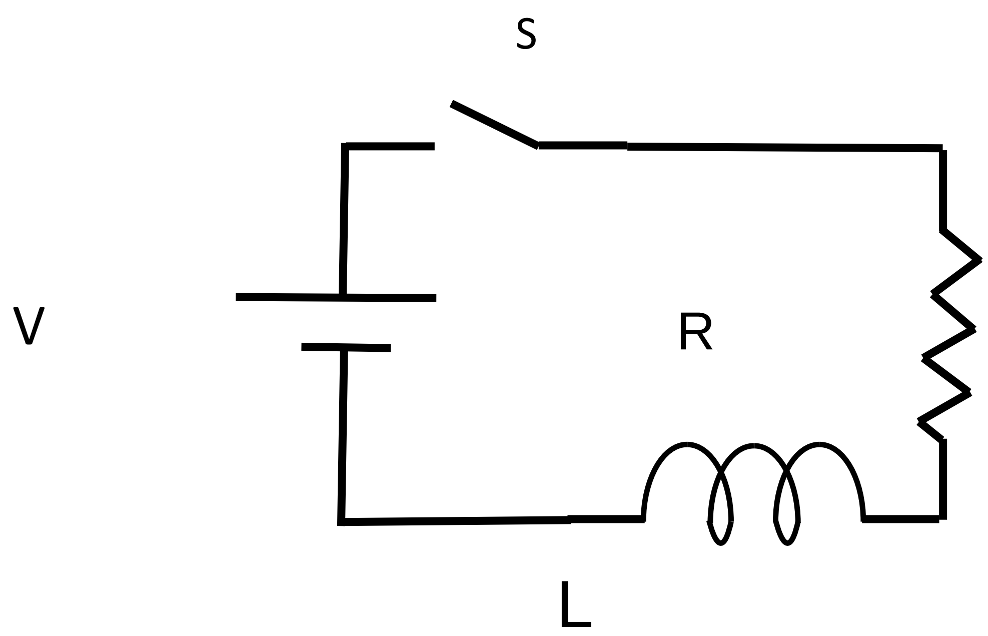
After a long time, $i = V/R$ Inductor acts like a WIRE

At t=0 the switch S is closed.

Using the loop rules

$$V - iR - L\frac{di}{dt} = 0$$

Solving using the method we used for the charging capacitor



$$i(t) = i_{\max}\left(1 - e^{-\left(\frac{Rt}{L}\right)}\right)$$

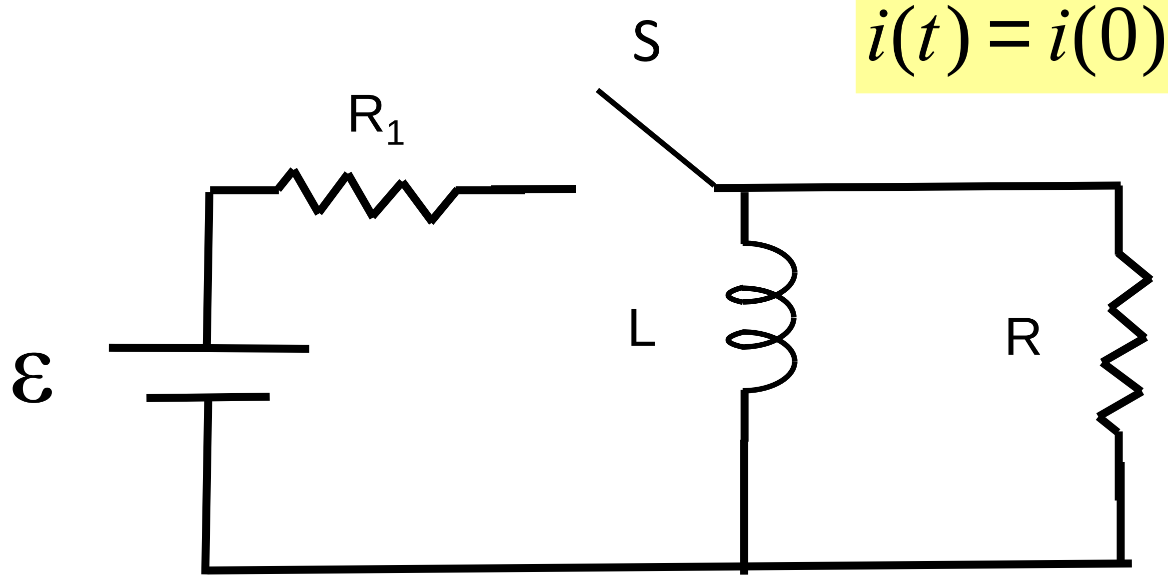
The components have all been connected for a very long time. At t=0 the switch S is opened. The current through R_1 and R are 0 and ε/R.

Using the loop rules

$$-L\frac{di}{dt} - iR = 0$$

Solving with the method we used for a discharging capacitor

$$i(t) = i(0)e^{-\left(\frac{Rt}{L}\right)}$$



Top Hat Question

The switch in the series circuit below is closed at t=0.

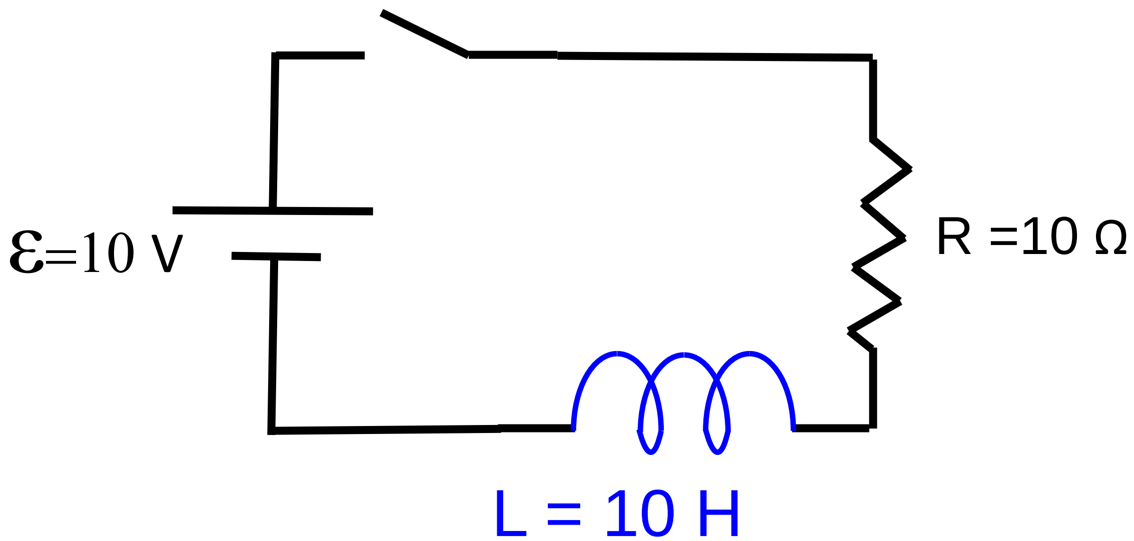
What is the initial rate of change of current di/dt in the inductor, immediately after the switch is closed (time = 0+) ?

A. 0 A/s

A. 0.5 A/s

B. 1 A/s

C. 10 A/s



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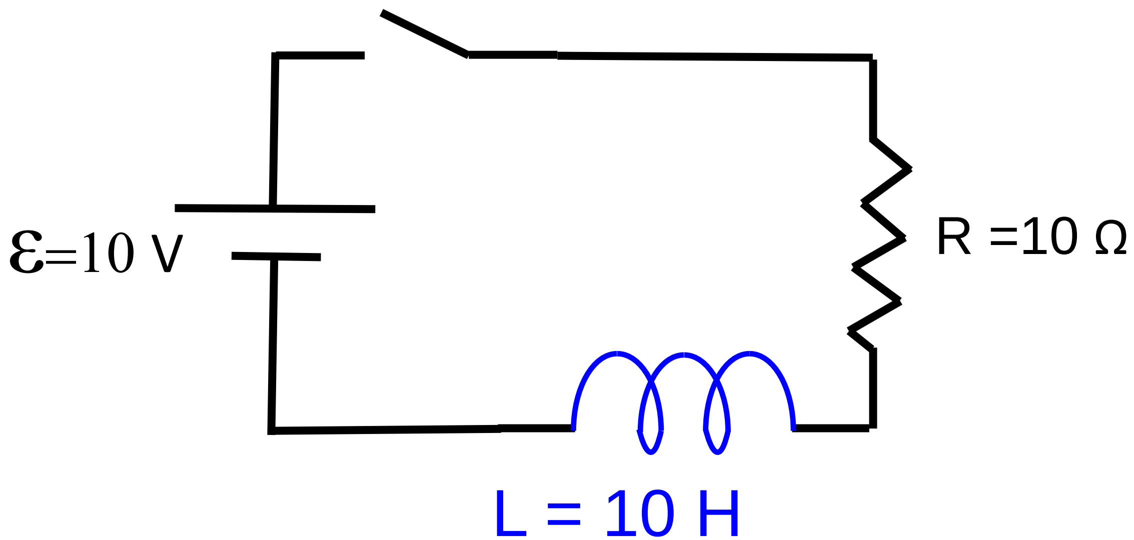


i = 0 at t = 0 so V_R (0)= 0 which means

10 V = V_L= Ldi/dt so di/dt = 10V/10H = 1 A/s

Top Hat Question

The switch in the series circuit below is closed at t=0.

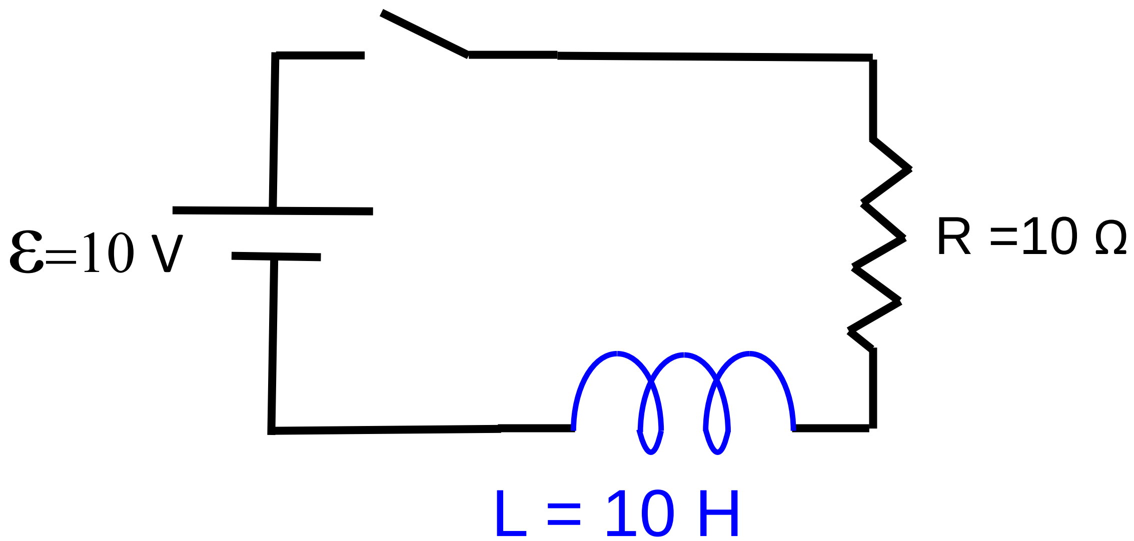
What is the current in the circuit after a time t = 3.0 s?

A. 0 A

A. 0.63 A

B. 0.86 A

C. 0.95 A



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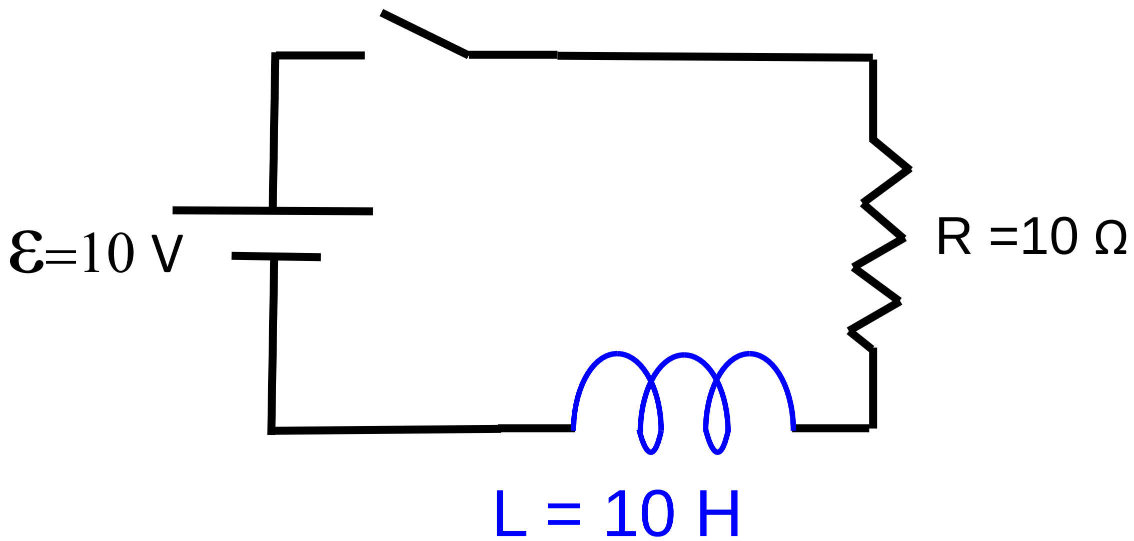
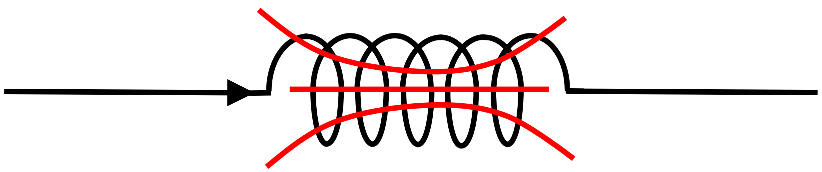


$$i(3\mathrm{s}) = \frac{10V}{10\Omega}\left(1 - e^{-3}\right)$$

Inductance

Note that a changing Magnetic flux produces an induced EMF in a direction which "tries to oppose the change"

$$\frac{di}{dt}$$



$$e = -\frac{d}{dt}\left(N \mu_0 n i A\right)$$

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$$\boxed{L = \mu_0 \frac{N^2}{\ell} A}$$

Energy in a Capacitor is stored in the Electric Field

Energy in an Inductor is stored in the Magnetic Field.

Energy storage in Inductors

If we build up the current, starting from $\mathbf{I_0}$ = 0 (initial) → $\mathbf{I_f}$,
at the time t when we have achieved a current $\mathbf{I}$, we have to work against an opposing EMF = Ld$\mathbf{I}$/dt in order to achieve a further increase in current, so our energy source is doing work per unit time

$$dP = IV = IL\frac{dI}{dt}$$

total work done: $$W = \int P\,dt = \int IL\frac{dI}{dt}dt$$

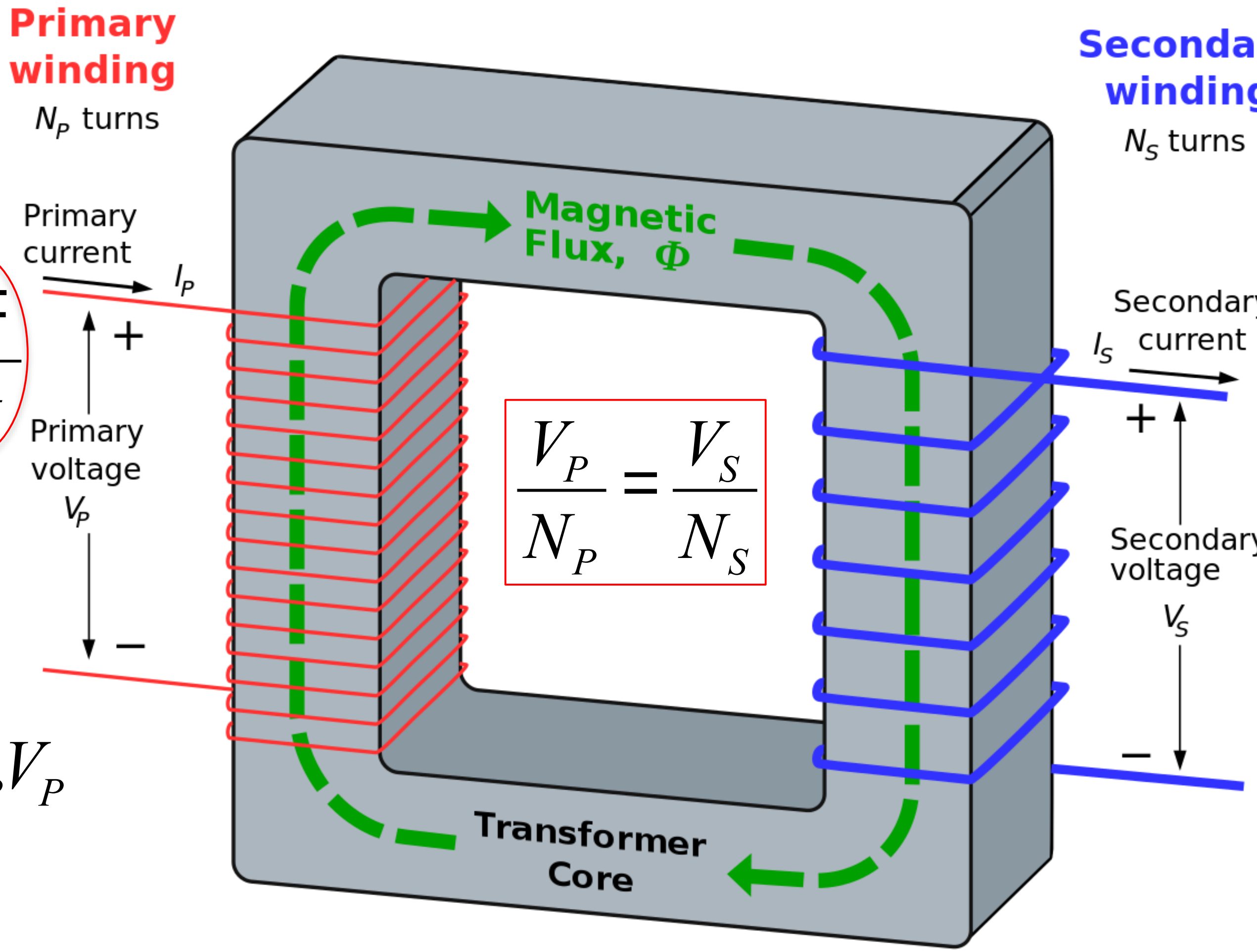
ie energy stored in system: $$U = \int_0^{I_f} LI\,dI$$

$$\boxed{U = \frac{1}{2}LI^2}$$

$$u = \frac{U}{V} = \frac{1}{2V}(\mu_0 nNA)I^2 = \frac{1}{2\mu_0}(\mu_0^2 n^2 I^2)\frac{A\ell}{V} = \boxed{\frac{1}{2\mu_0}B^2}$$

Transformers

$$V_P = N_P \frac{dF}{dt}$$



$$\boxed{\frac{V_P}{N_P} = \frac{V_S}{N_S}}$$

$$V_S = N_S \frac{dF}{dt}$$

$$P_{in} = I_P V_P$$

$$P_{out} = I_S V_S$$

$$P_{in} = P_{out}$$

$$I_P V_P = I_P \frac{N_P}{N_S} V_S = I_S V_S$$

$$\boxed{I_P N_P = I_S N_S}$$

Top Hat Question

The transformer for your laptop (the adaptor) has an output voltage of 18.5V. Your laptop uses about 85W of energy. The adaptor uses a step down transformer – what is the ratio of turns, primary to seconday, N_P/N_S ?

a) 0.065

b) 0.65

c) 6.5

d) 65

Top Hat Question

The transformer for your laptop (the adaptor) has an output voltage of 18.5V. Your laptop uses about 85W of energy. The adaptor uses a step down transformer– what is the resistive load of the laptop *R*?

a) 0.4Ω

b) 4Ω

c) 40Ω

d) 400Ω

That’s all for content!

Monday’s class: Review