Last time

- The electric field: conceptually difficult but much more useful
- TopHat questions related to direction of net E-field
- Finishing up Coloumb’s Law: Group Activity

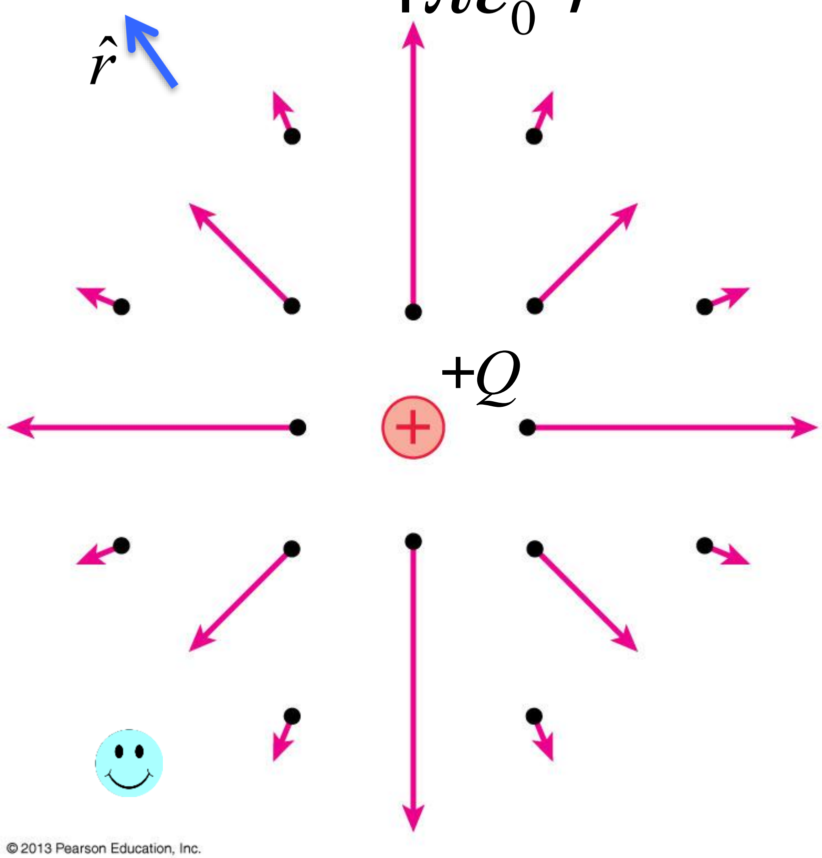
This time

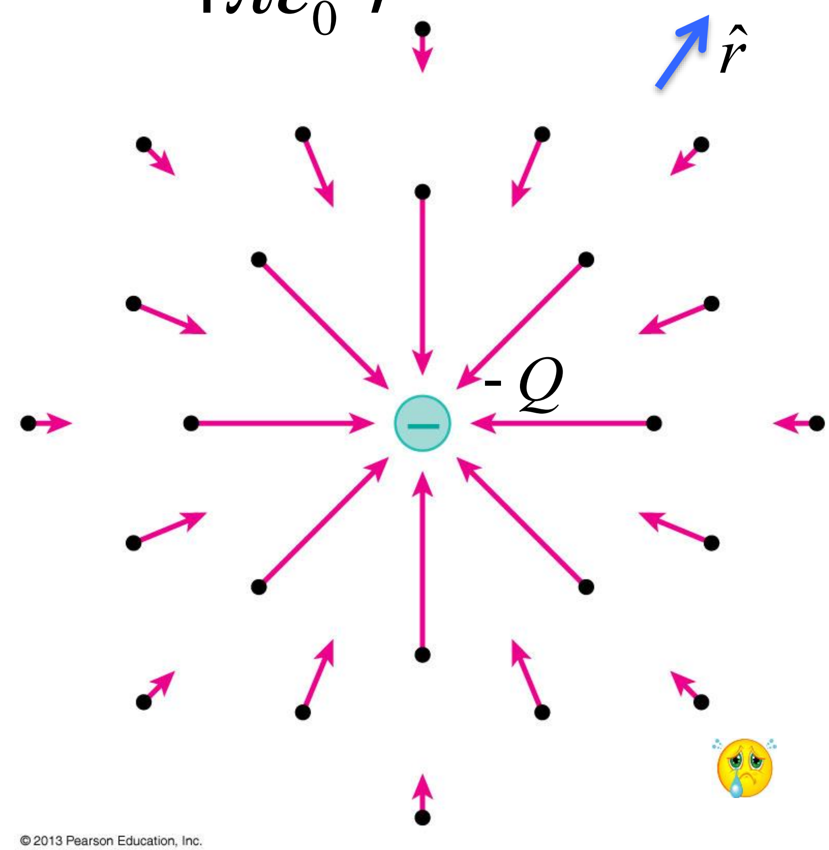
- More on electric fields: how to calculate them
- Visualizing electric fields: electric field lines
- Example: electric field of a dipole.

Electric field of point charges

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}$$

$$\vec{E} = \frac{-1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}$$





+ charge: $\vec{E}$ points away from Q

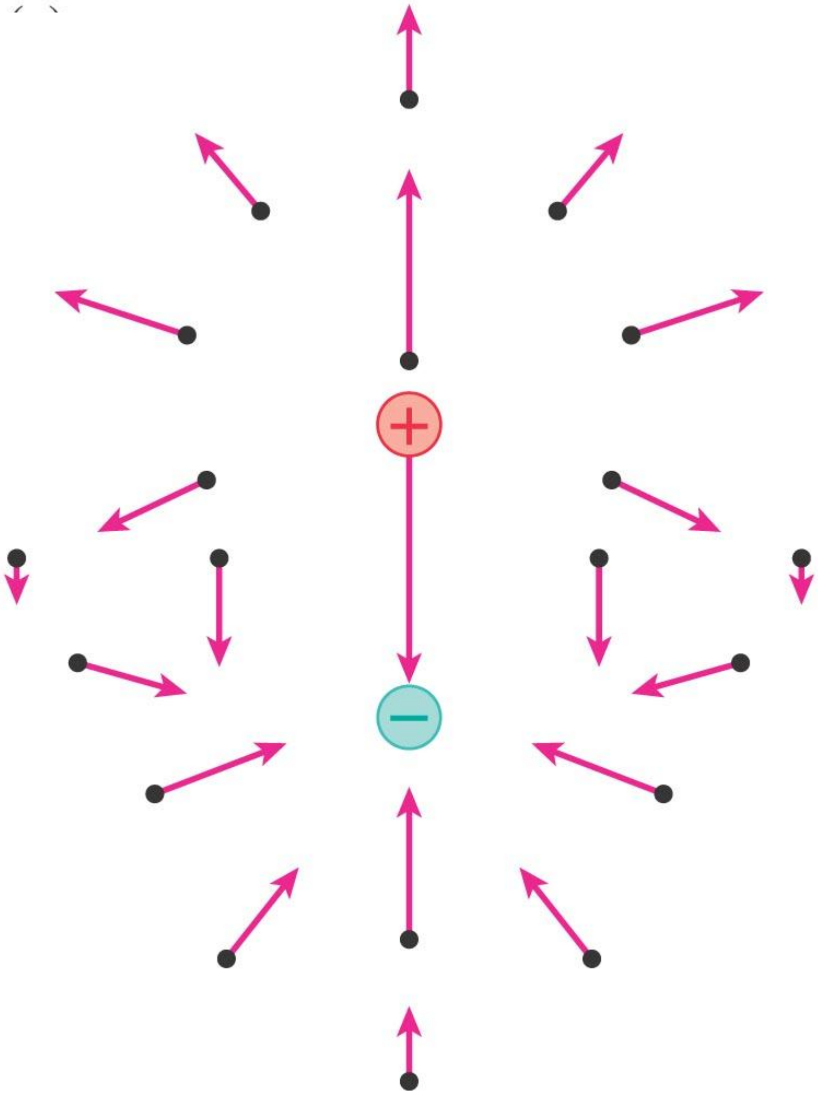
– charge: $\vec{E}$ points toward –Q

Electric Field Vectors

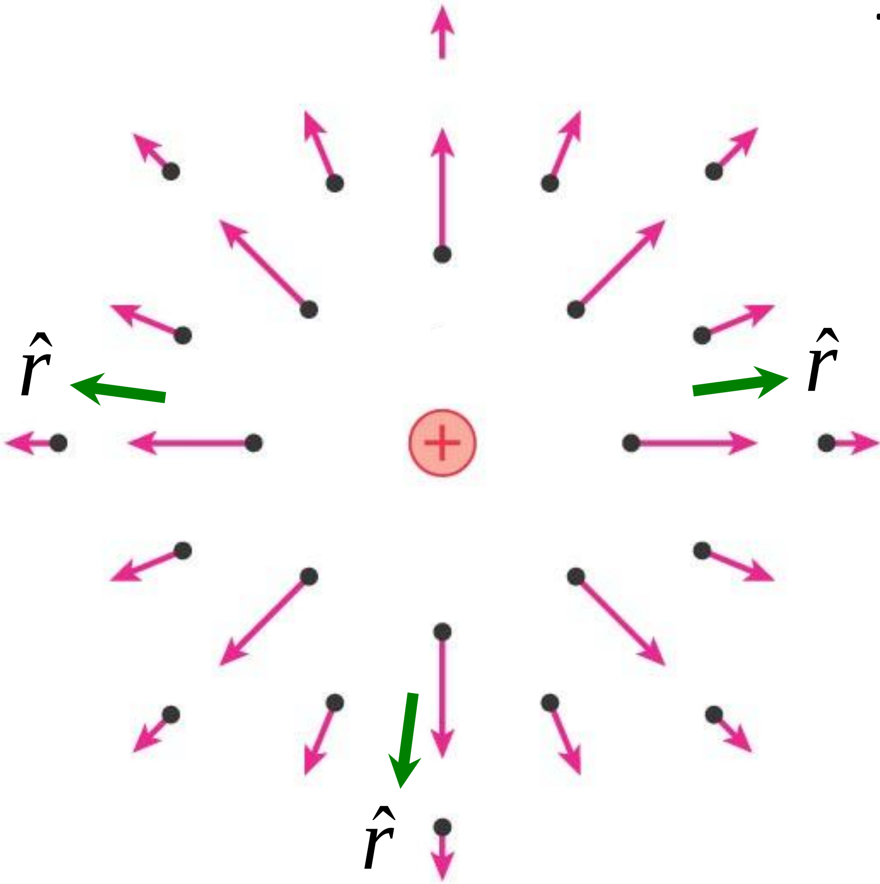
The vector represents the magnitude and direction of the electric field **at that point**.

But $\vec{E}$ is not a spatial quantity that stretches from one end of the arrow to the other.

Instead, think of $\vec{E}$ as a spatial quantity at every point with a direction at that point given by the arrow.



Electric Field building blocks



The electric field around a **point charge, q,** is given by

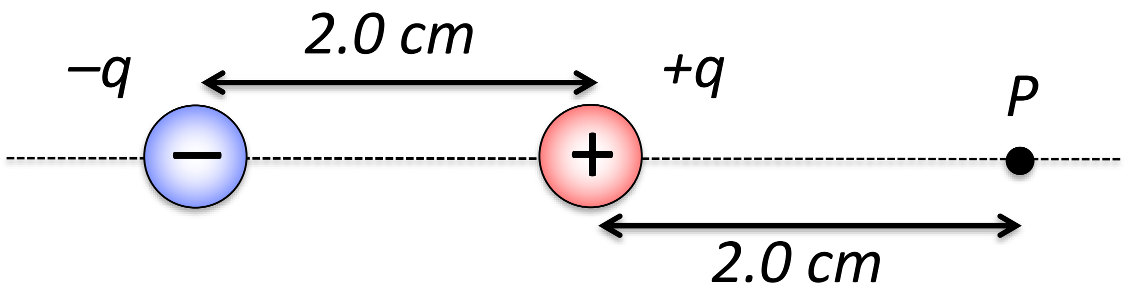
$$\boxed{\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}}$$

$\hat{r}$ is a **unit vector** that always points **away from** q.

We can use this with superposition to find the electric field of more complicated objects.

TopHat Question

An electric dipole: if the electric field strength at point P is E = 6068 N/C, what is the charge q?



For single point charge:

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

$$\varepsilon_0 = 8.85 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2}$$

A. 0.36 nC

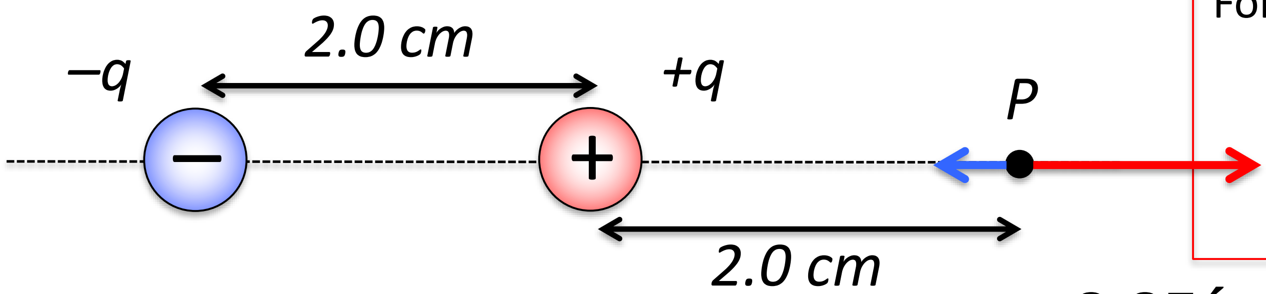
B. 0.27 nC

C. 0.22 nC

D. 0.13 nC

TopHat Question Feedback

An electric dipole: if the electric field strength at point P is E = 6068 N/C, what is the charge q?



For single point charge:

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

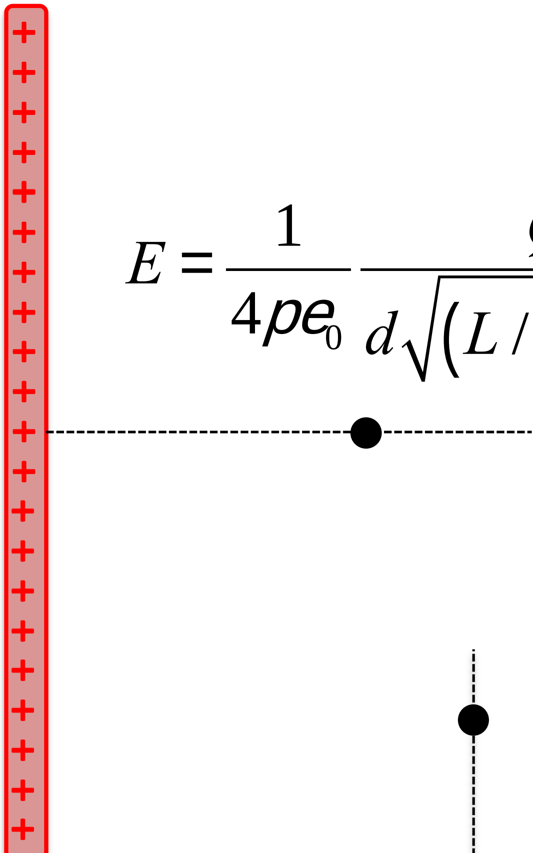
$$\varepsilon_0 = 8.85\times 10^{-12}\ \mathrm{C^2/N\cdot m^2}$$

$$E_+ = \frac{1}{4\pi\varepsilon_0}\frac{q}{r_+^2} \qquad E_- = -\frac{1}{4\pi\varepsilon_0}\frac{q}{(2r_+)^2}$$

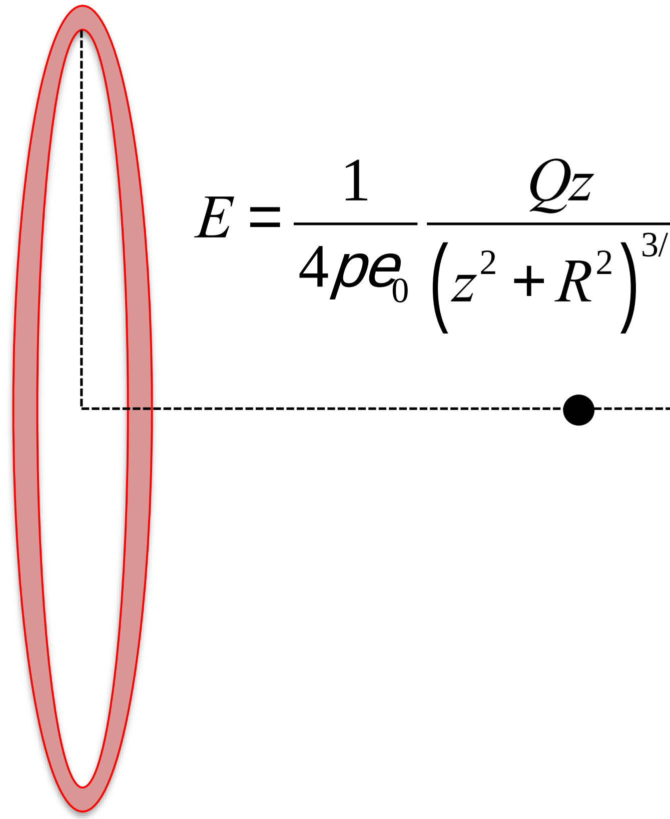
$$E_{net} = \frac{1}{4\pi\varepsilon_0} q \left(\frac{1}{r_+^2} - \frac{1}{4r_+^2}\right) = \frac{1}{4\pi\varepsilon_0}\frac{3q}{4r_+^2}$$

$$\boxed{q = 4\pi\varepsilon_0 \frac{4r_+^2 E_{net}}{3}}$$

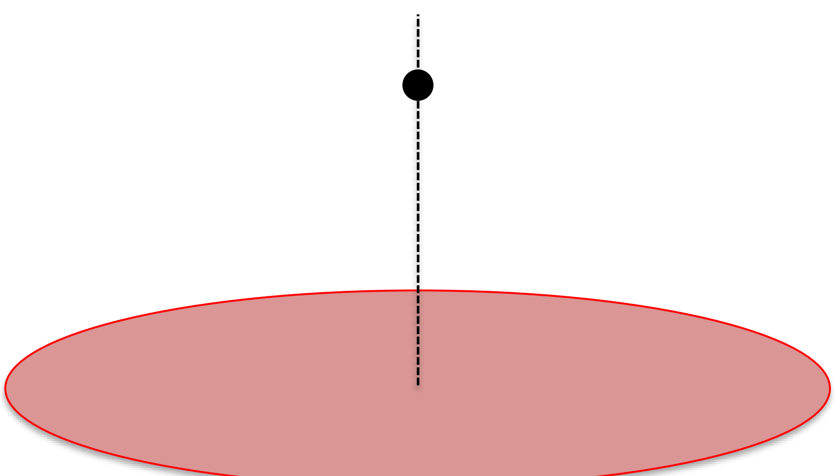
Cases we’ve already seen



$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{d\sqrt{(L/2)^2 + d^2}}$$



$$E = \frac{1}{4\pi\varepsilon_0}\frac{Qz}{\left(z^2 + R^2\right)^{3/2}}$$



$$E = \frac{\eta}{2\varepsilon_0}\left[1 - \frac{z}{\sqrt{z^2 + R^2}}\right]$$

TopHat Question

An infinitely long wire: if the electric field strength at point P is E = 3670 N/C, how much charge is contained in a 0.500 m length of the wire?

$$\boxed{E_{wire} = \frac{1}{4\pi\epsilon_0}\frac{2\lambda}{r}}$$

$$\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2}$$

A. 18.37 nC

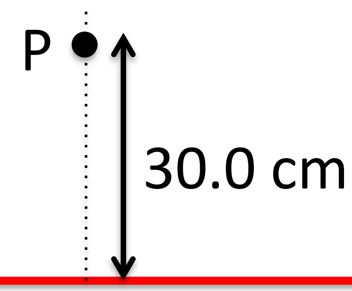
B. 30.62 nC

C. 61.23 nC

D. 9.19 nC

TopHat Question Feedback

An infinitely long wire: if the electric field strength at point P is E = 3670 N/C, how much charge is contained in a 0.500 m length of the wire?



$$\boxed{E_{wire} = \frac{1}{4\pi\varepsilon_0}\frac{2\lambda}{r}}$$

$$\varepsilon_0 = 8.85\times 10^{-12}\ \mathrm{C^2/N\cdot m^2}$$

$$\lambda = \frac{E_{wire}r}{2K} \qquad \lambda = \frac{\Delta Q}{\Delta L} \qquad \boxed{\Delta Q = 4\pi\varepsilon_0\frac{E_{wire}r\Delta L}{2}}$$

Summary so far

If we think there is an electric field somewhere in space, then we can measure it by placing a charge q in the field. If q feels an electric force, then

$$\vec{E} = \frac{\vec{F}_{on\,q}}{q}$$

(How we have proceeded so far)

Or, if we know the electric field, then the electric force on any charge q placed in this field is

$$\vec{F}_{on\,q} = q\vec{E}$$

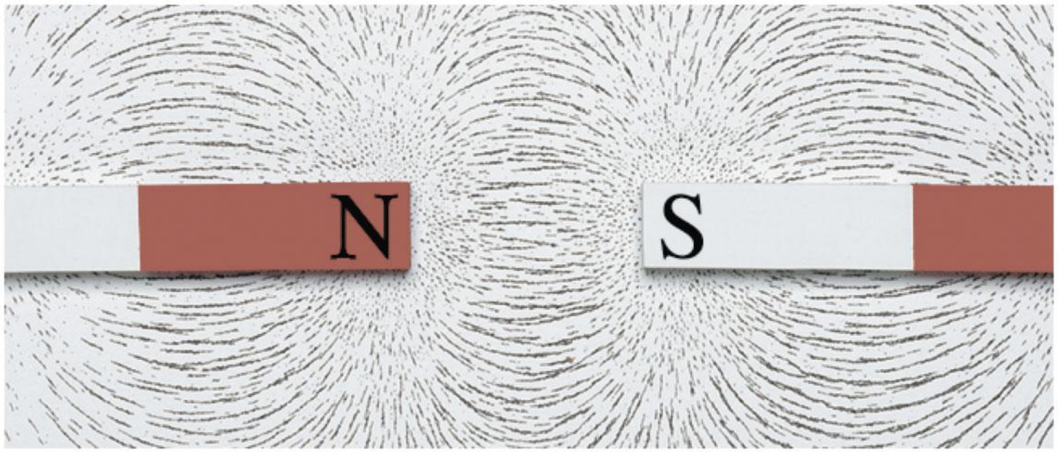
(How nature really works)

We'll come back to this

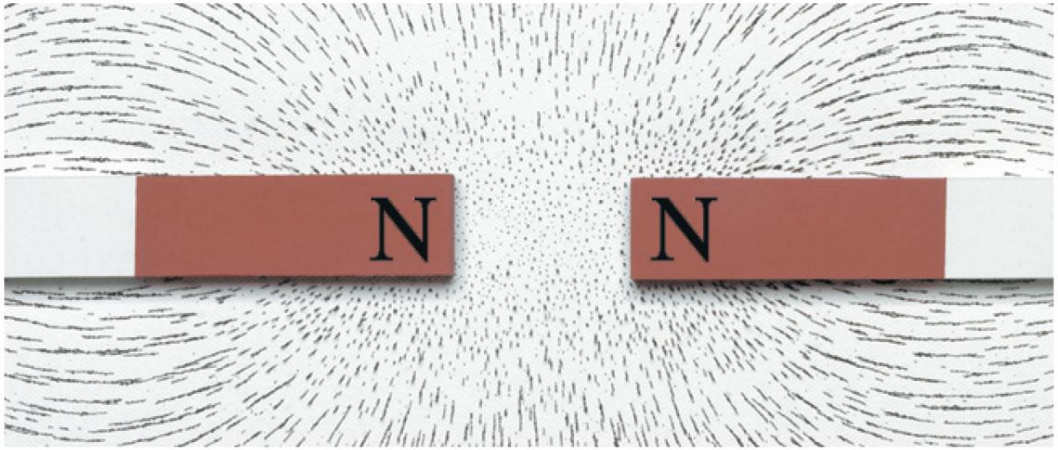
Full force on charged particle due to electromagnetic field:

$$\boxed{\vec{F}_{on\,q} = q\vec{E} + q\vec{v}\times\vec{B}}$$

Visualizing E-field: field lines



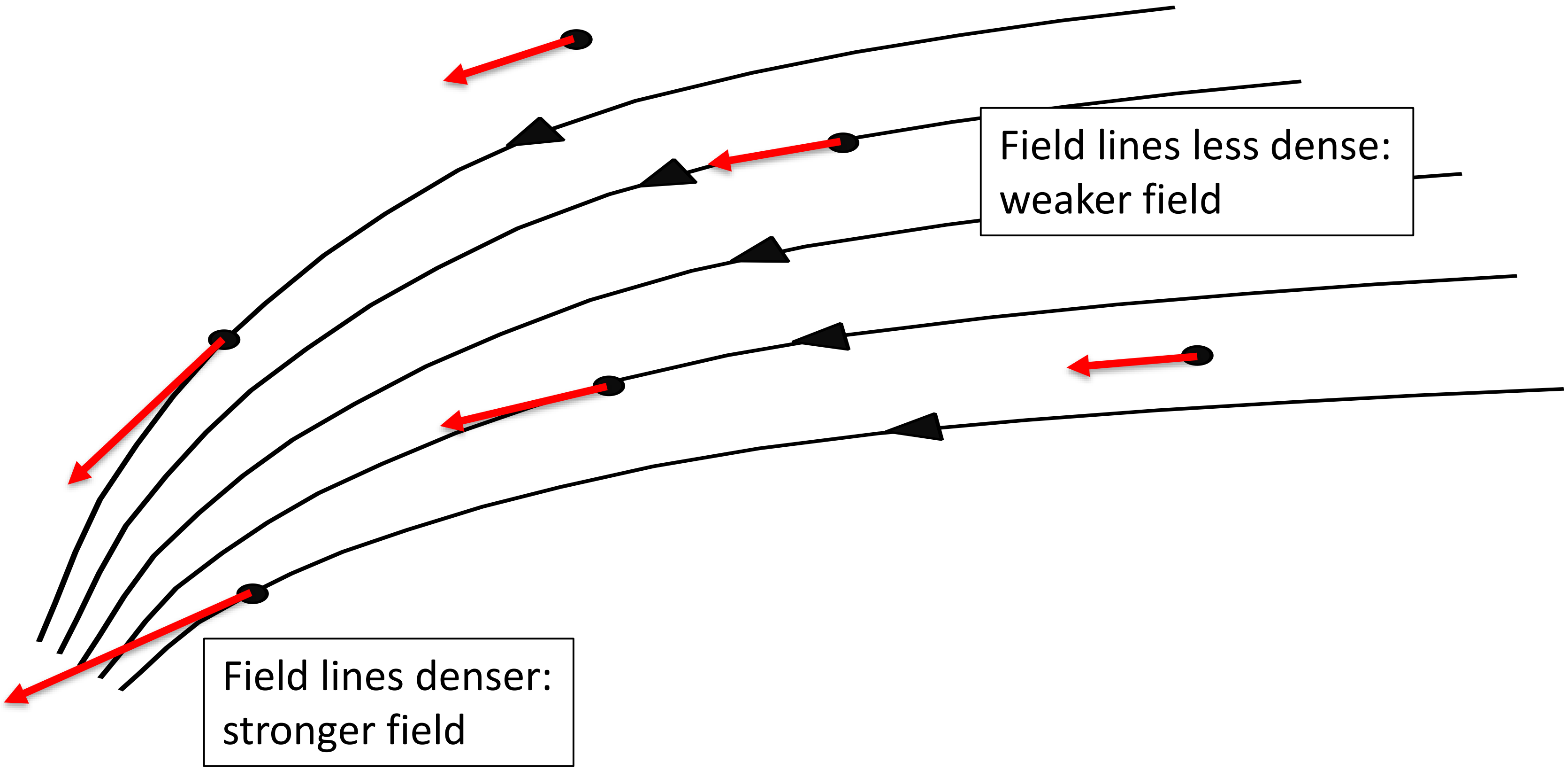
Magnetic field lines



You are already familiar with the idea that magnets set up a magnetic field. This can be demonstrated with iron filings on paper over top of a magnet.

Electric fields also have “field lines” but we have less intuition about them from everyday experience

Electric Field Lines

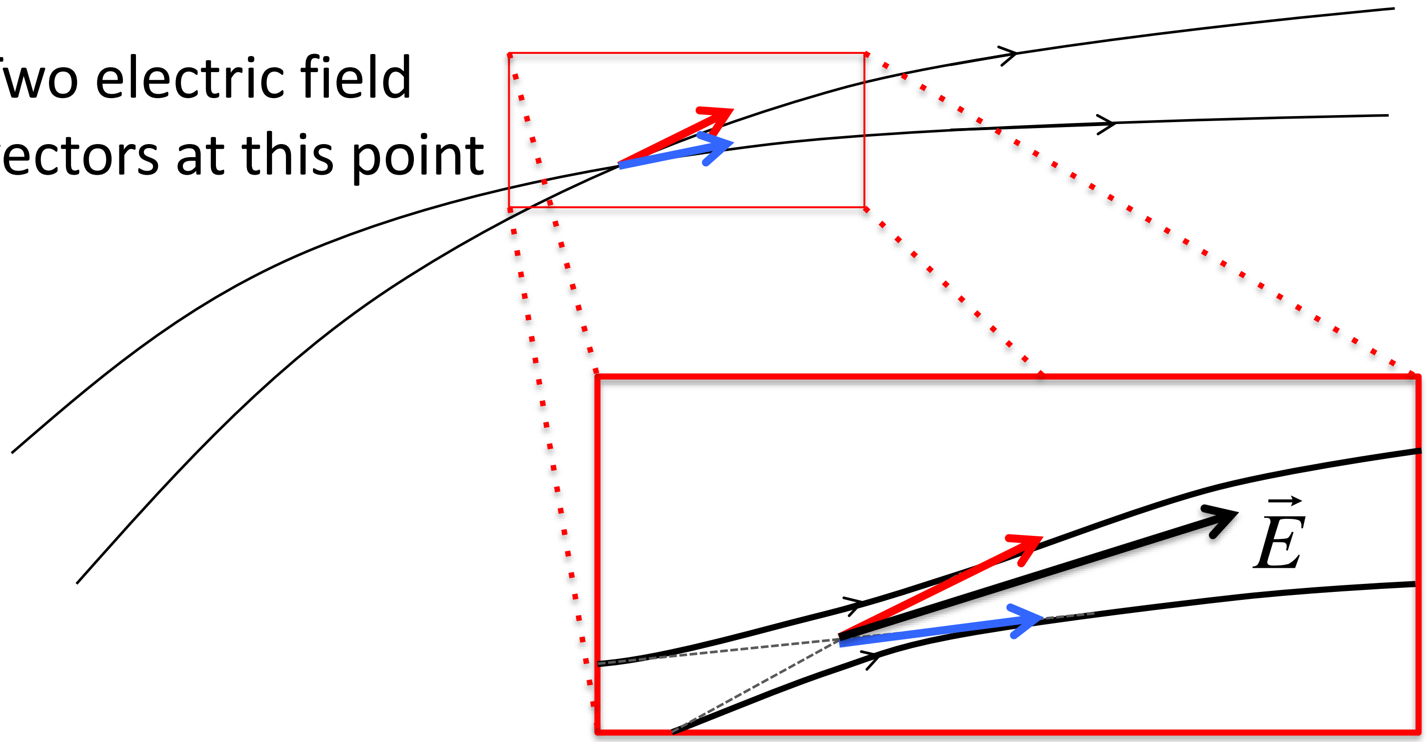


Electric field lines are continuous curves. The electric field vectors are tangent to the field lines

The denser the field lines, the stronger the field (magnitude of E)

Electric Field Lines Can't Cross

Two electric field vectors at this point



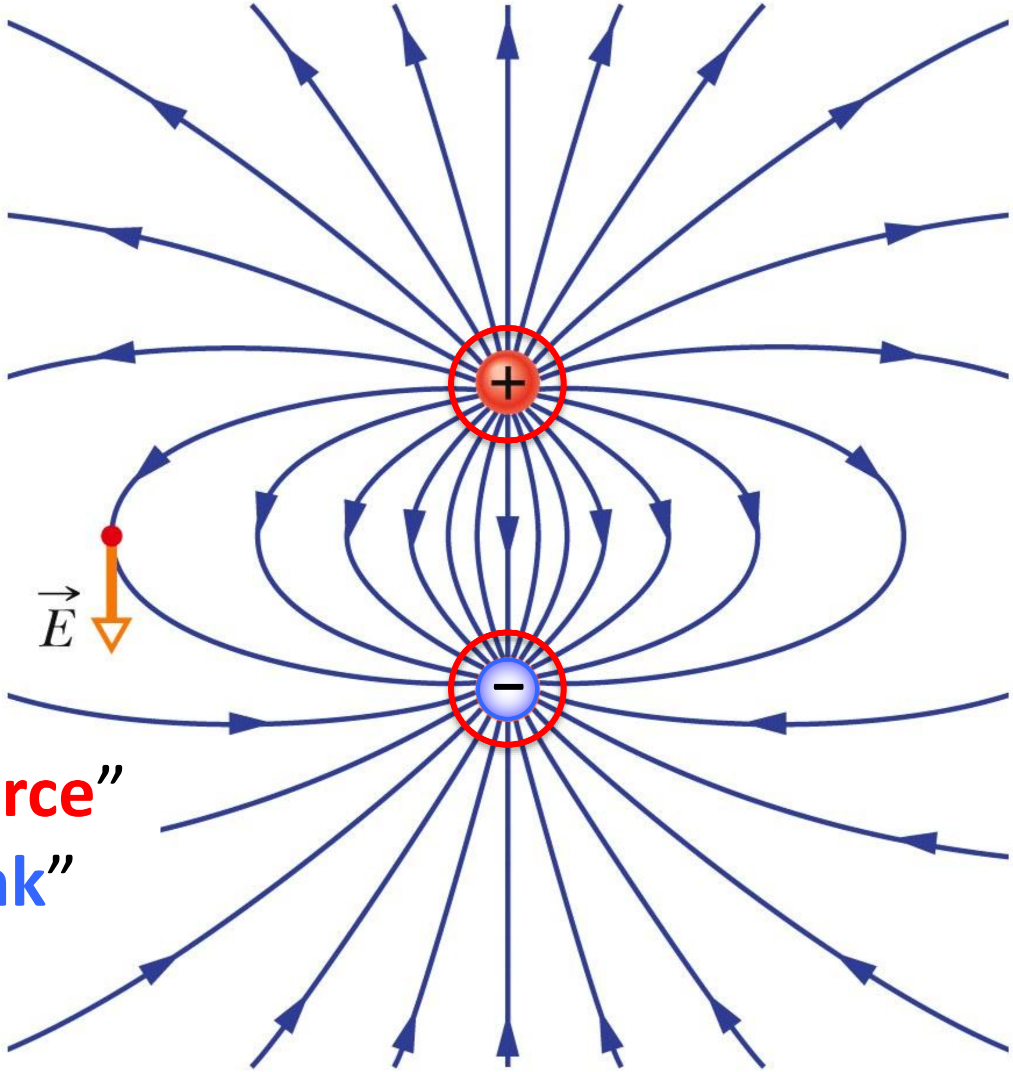
If field lines crossed, the electric field at that point would not be defined: superposition saves the day.

Sources and Sinks of Field Lines

Two charges of **equal magnitude** and **opposite sign**.

Field lines **start on +**
Field lines **end on –**

Positive charge called "**source**"
Negative charge called "**sink**"



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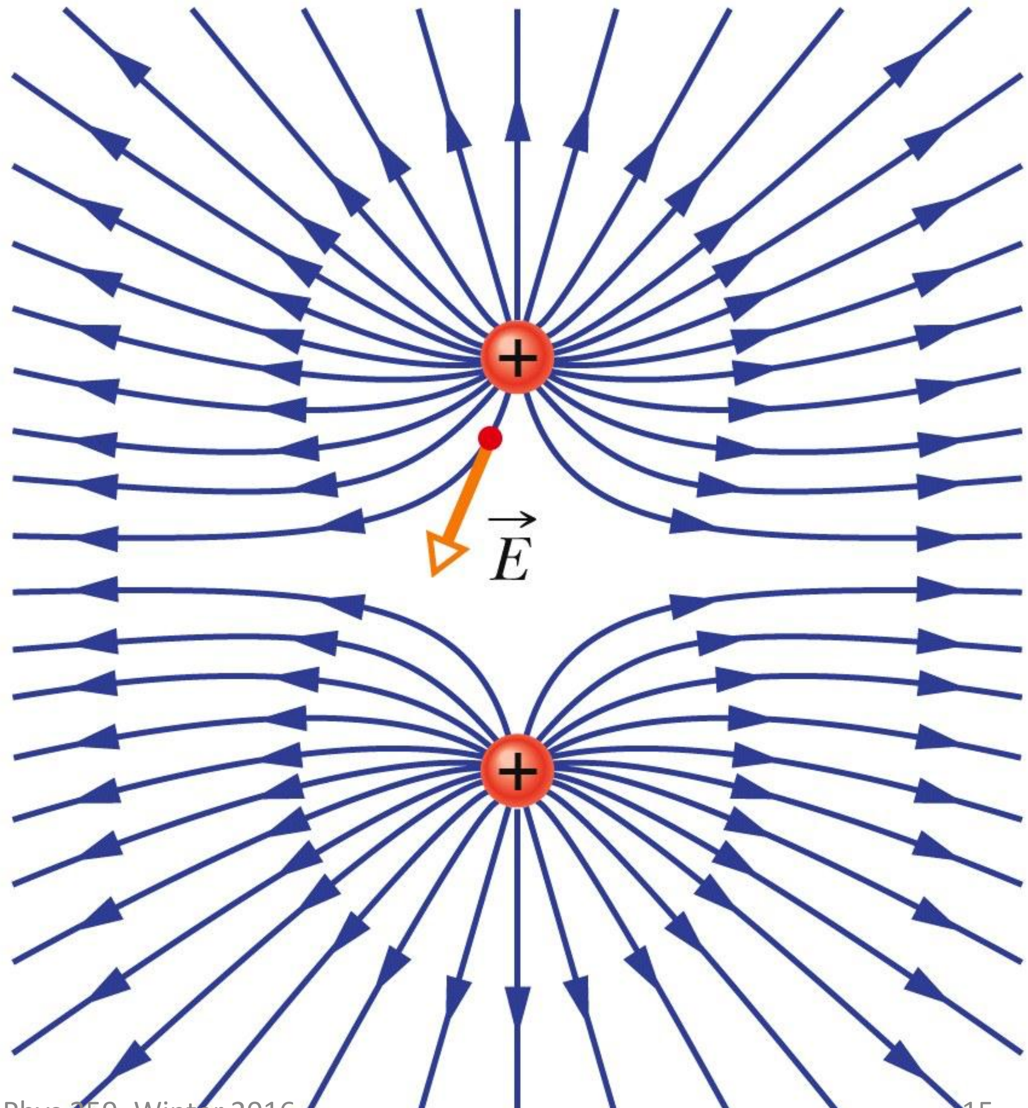
Electric Field Lines

The electric field lines around a pair of equal positive charges.

No negative charges for the field lines to end on. The field lines "repel" each other and all point outward.

Direction comes from superposition!



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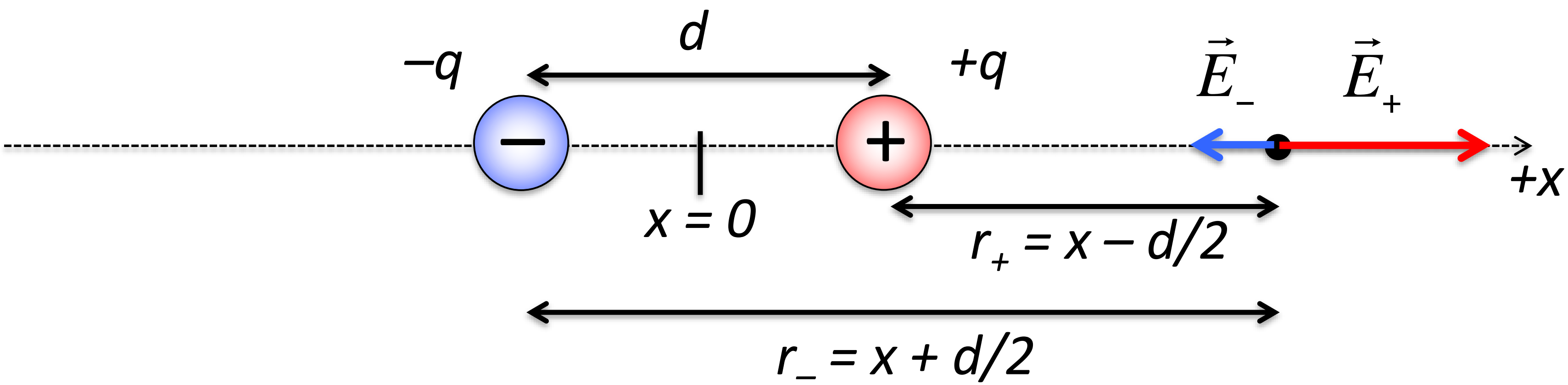
Electric Field of a Dipole Along Axis

What direction is the electric field at a point along the axis of an electric dipole?



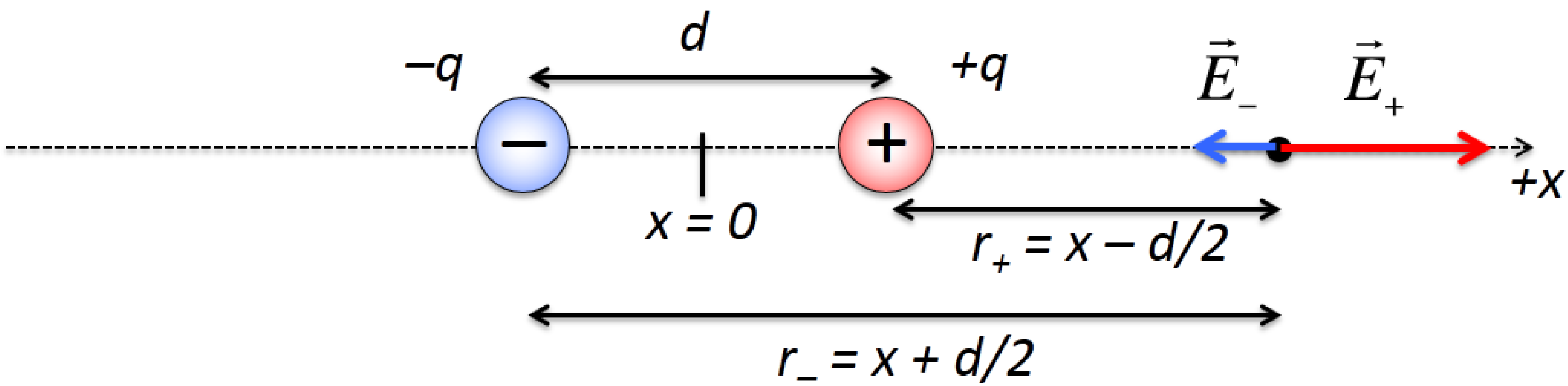
Step 1: What are the distances r_+ and r_-?

Step 2: What are the individual fields E_+ and E_-?

Step 3: Use superposition to find the net field E_x.

$$\vec{E}_+ = \frac{1}{4\pi\varepsilon_0}\frac{q}{r_+^2}\hat{i} \qquad \vec{E}_- = -\frac{1}{4\pi\varepsilon_0}\frac{q}{r_-^2}\hat{i}$$

Electric Field of a Dipole: Along Axis



$$E_- = -\frac{1}{4\pi\varepsilon_0}\frac{q}{(x+d/2)^2} \qquad E_+ = \frac{1}{4\pi\varepsilon_0}\frac{q}{(x-d/2)^2}$$

$$E_x = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{(x-d/2)^2} - \frac{1}{(x+d/2)^2}\right)$$

Can simplify this further

Electric Field of a Dipole: Along Axis

$$E_x = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{(x-d/2)^2} - \frac{1}{(x+d/2)^2}\right)$$

$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{(x+d/2)^2-(x-d/2)^2}{(x-d/2)^2(x+d/2)^2}\right)$$

(Get a common denominator)

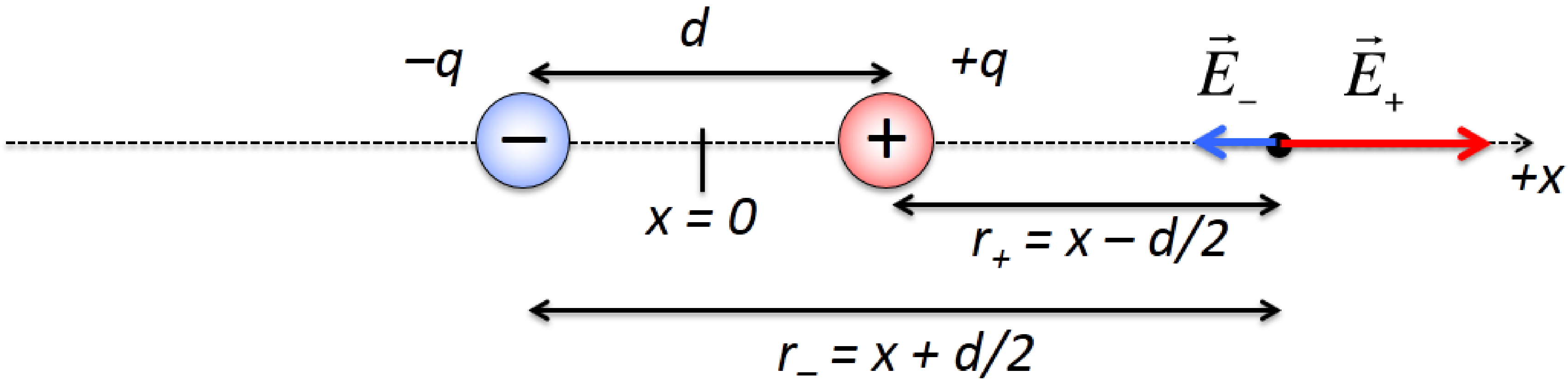
$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{(\cancel{x^2}+xd+\cancel{d^2/4})-(\cancel{x^2}-xd+\cancel{d^2/4})}{(x^2-d^2/4)^2}\right)^2$$

(expand and cancel)
Use (a+b)(a-b)=(a^2–b^2)
in denominator

$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{2xd}{(x^2-d^2/4)^2}\right)$$

$$\boxed{E_x = \frac{1}{4\pi\varepsilon_0}\frac{2qdx}{(x^2-d^2/4)^2}}$$

Electric Field of a Dipole: Along Axis



$$\boxed{E_x = \frac{1}{4\pi\varepsilon_0}\frac{2px}{\left(x^2 - d^2/4\right)^2}}$$

Dipole moment: $p \equiv qd$

"perfect dipole": keep *p* fixed but let *d* → 0 (or equivalently x>>d)

$$E_x = \frac{1}{4\pi\varepsilon_0}\frac{2p}{x^3}$$

A monopole has a $1/r^2$ falloff
A dipole has a $1/r^3$ falloff

Last time

- Calculating E-fields (same calculations as electrostatic force)
- Visualizing electric fields: electric field lines
- Electric field of a dipole on axis: $1/r^3$ falloff of field

This time

- Motion of charged particles in electric fields
- E-fields of other objects: using superposition to avoid doing more work.
- One more E-field calculation: charged arc of a circle

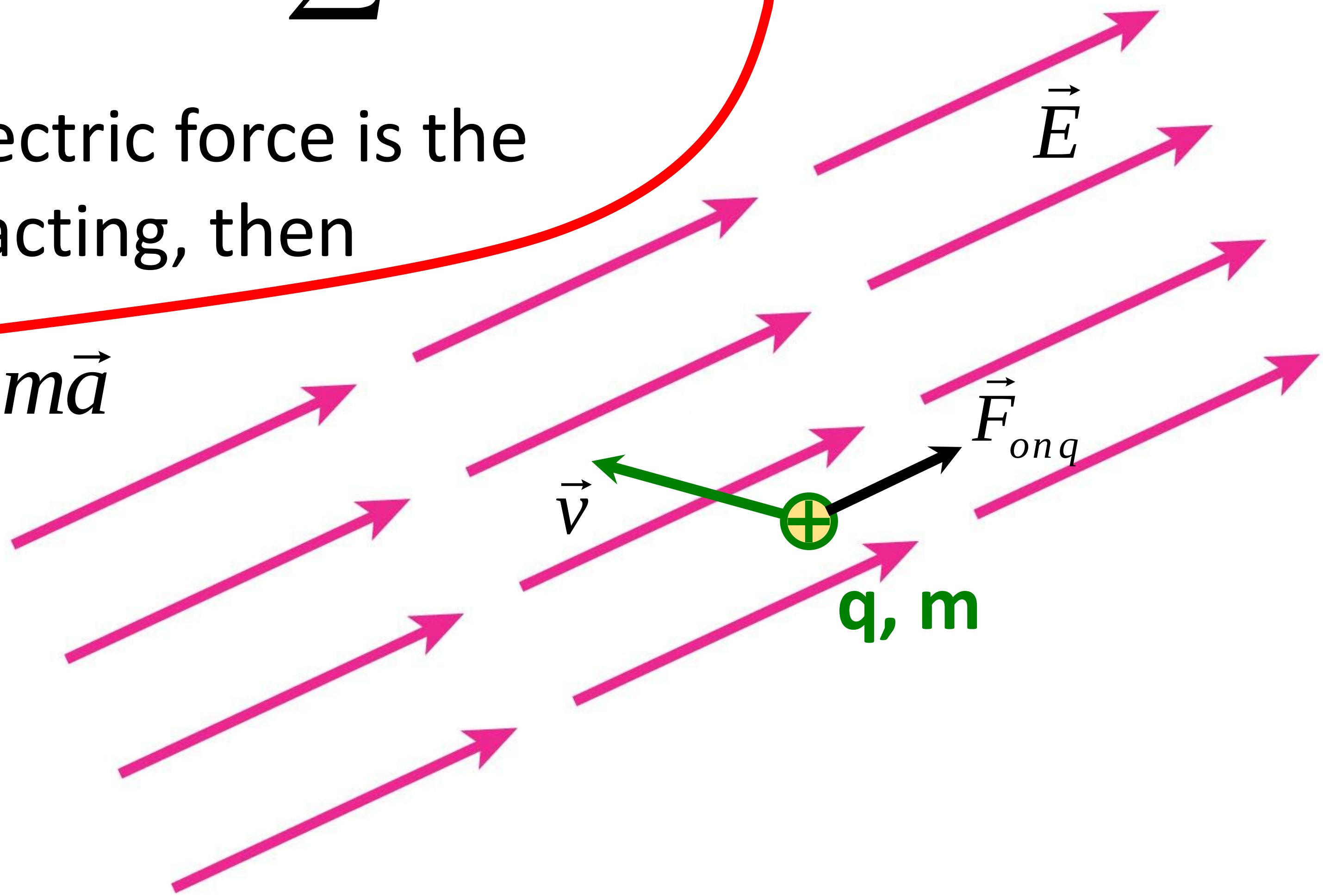
Electric force on q: $\vec{F}_{on\,q} = q\vec{E}$

Newton's 2nd Law: $\sum \vec{F} = m\vec{a}$

So if the electric force is the only force acting, then

$$\vec{F}_{on\,q} = m\vec{a}$$

$$q\vec{E} = m\vec{a}$$



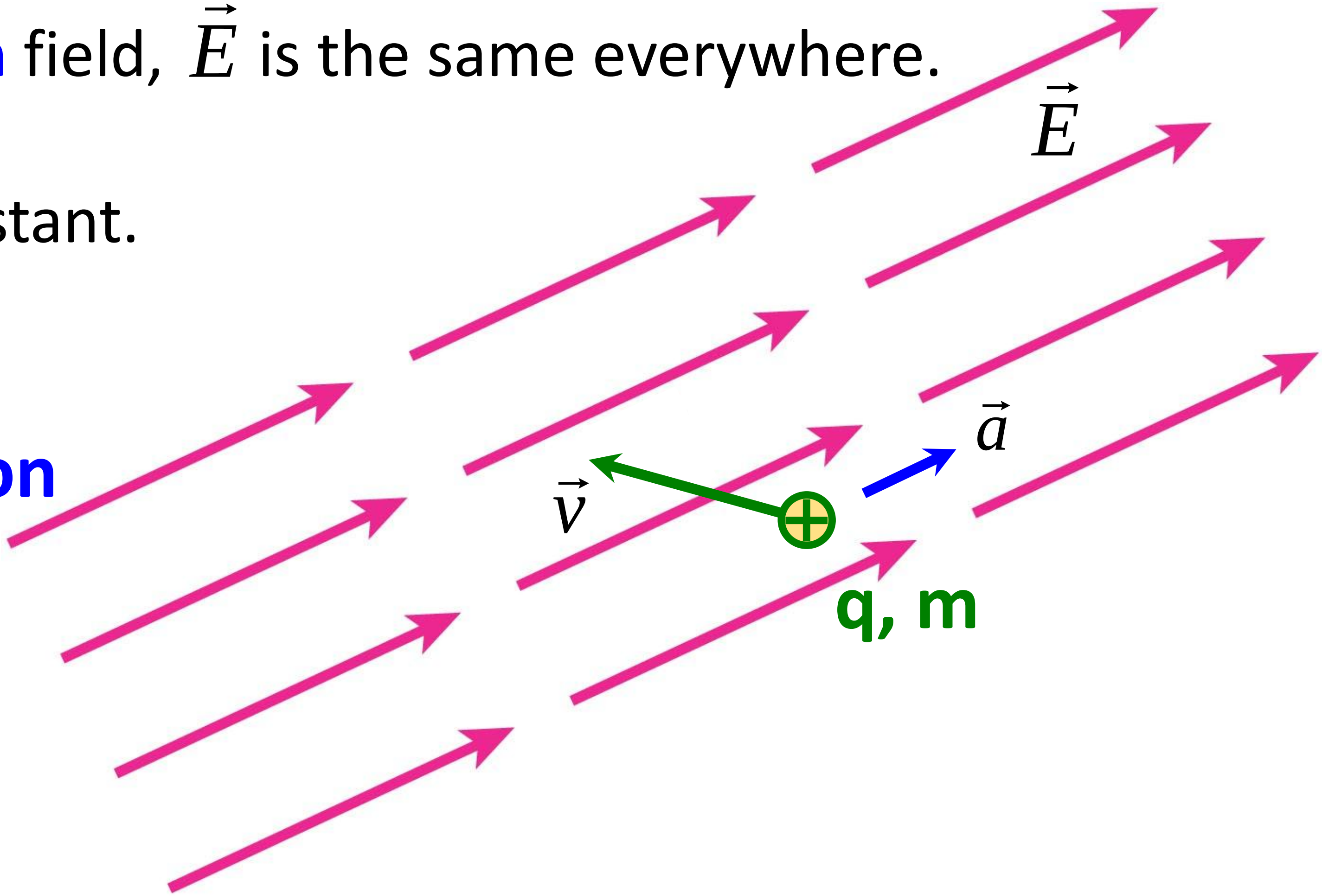
$$q\vec{E} = m\vec{a} \quad \Rightarrow \quad \boxed{\vec{a} = \frac{q\vec{E}}{m}}$$

q = constant
m = constant

In a **uniform** field, $\vec{E}$ is the same everywhere.

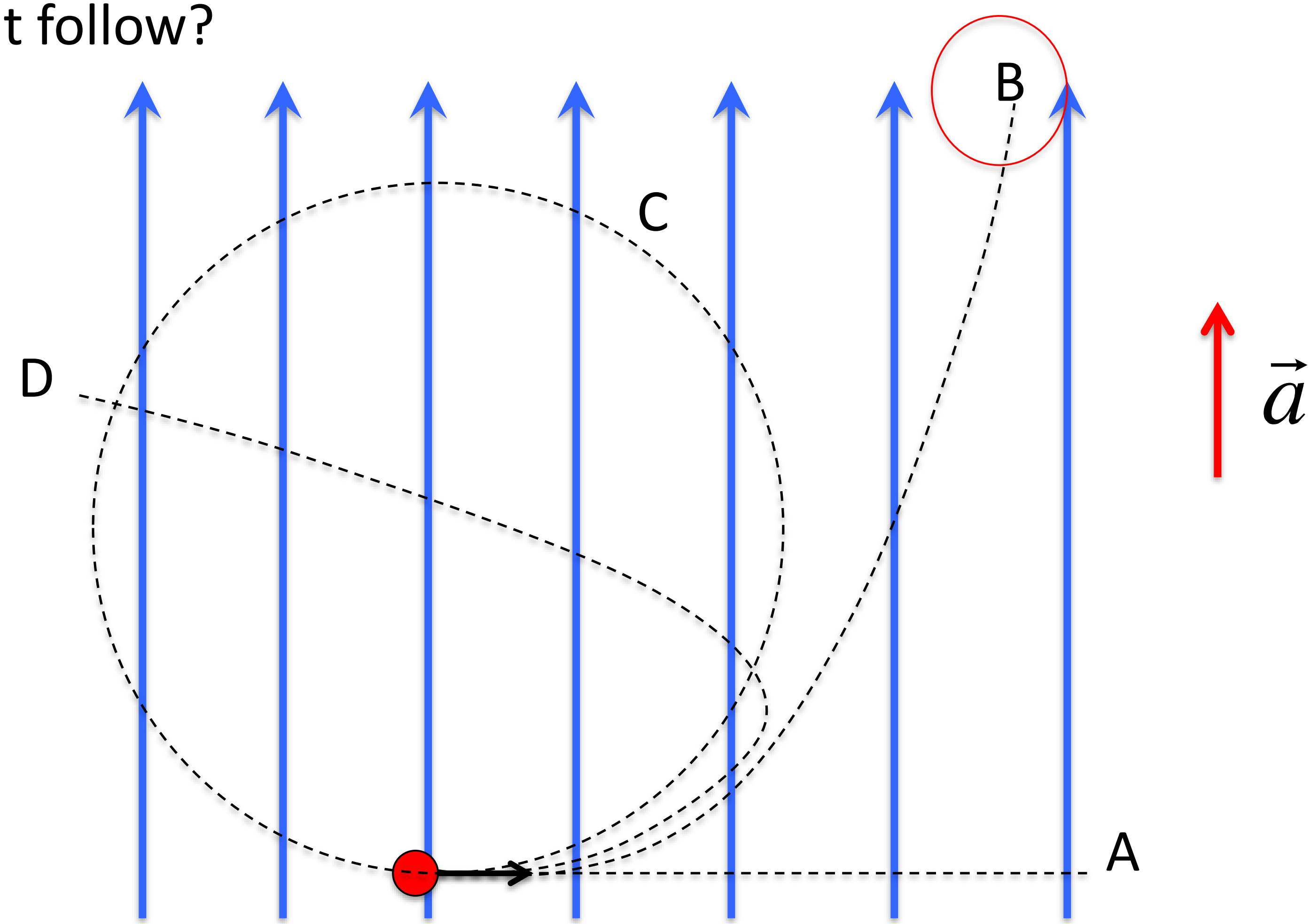
So $\vec{a}$ is constant.

Constant acceleration motion!



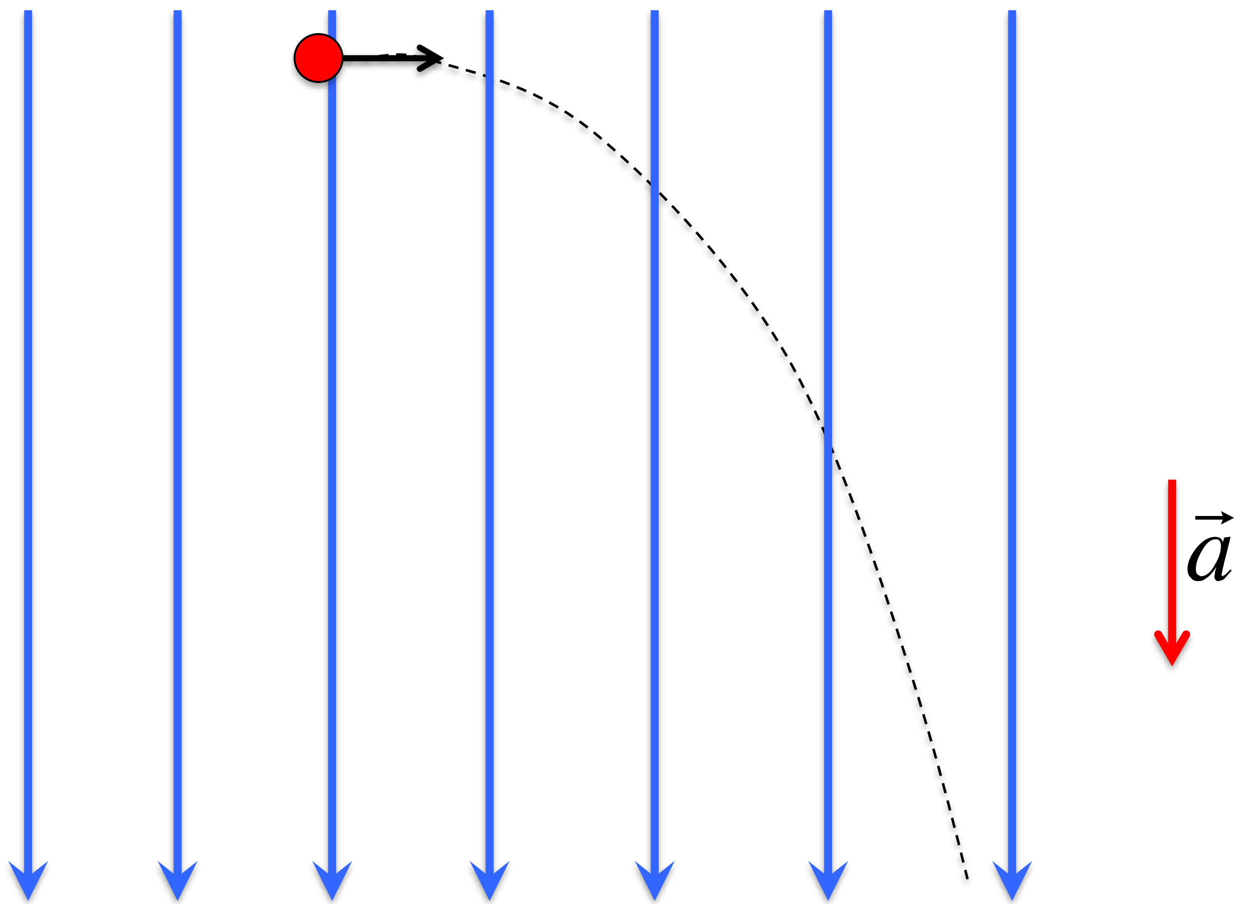
TopHat Question

A proton is moving through a uniform electric field. If its initial velocity is in the +x direction, which of the following trajectories would it follow?



TopHat Question Feedback

When a projectile is shot horizontally in a uniform gravitational field, it follows a parabolic trajectory because of its constant acceleration.



Uniform E-field: projectile motion

$$\vec{F}_{net} = q\vec{E}$$

Take E to point along +x-direction

$$a_x = \frac{qE}{m}$$

If q is +, a_x is +
If q is −, a_x is −

$$a_y = 0$$

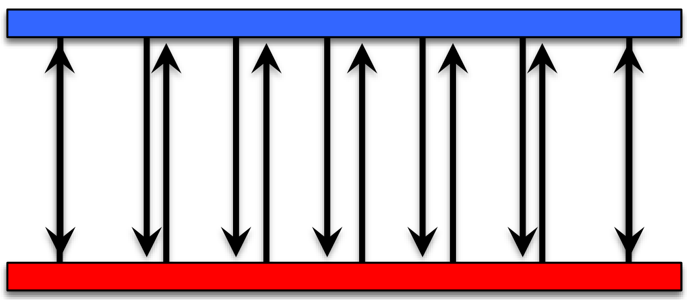
$$x_f = x_i + v_{ix}\Delta t + \frac{1}{2}\frac{qE}{m}\Delta t^2$$

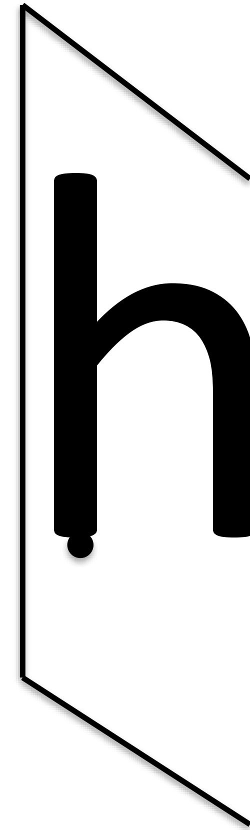
$$y_f = y_i + v_{iy}\Delta t$$

$$v_{fx} = v_{ix} + \frac{qE}{m}\Delta t$$

$$v_{fy} = v_{iy}$$

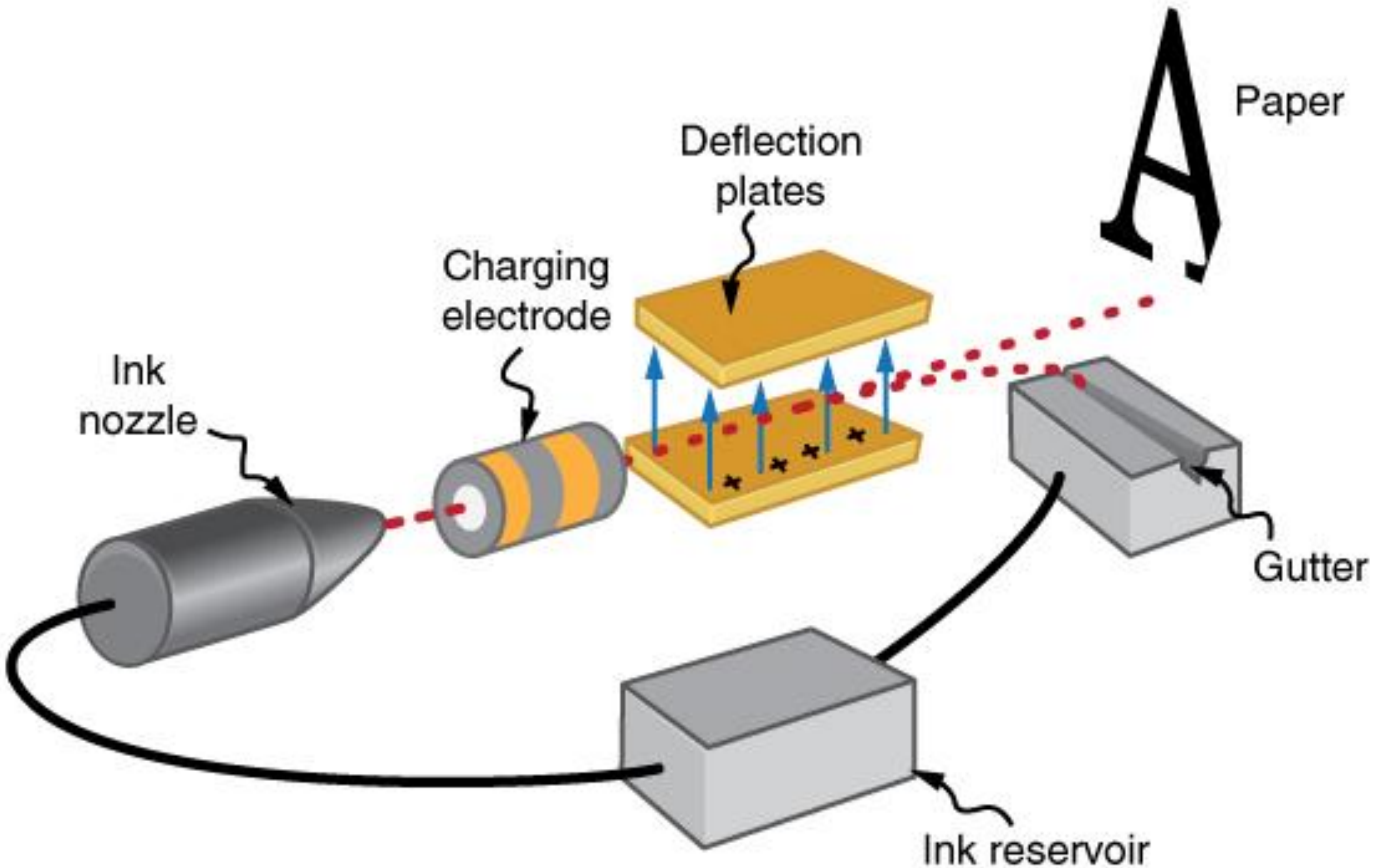
Application: Inkjet Printers





By controlling the strength of the electric field between the plates, you control the deflection of the charged ink droplets. This allows you to feed an electronic signal to the plates, thereby allowing you to create images out of ink droplets (in this case, letters)

Application: Inkjet Printers



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