Electricity and Magnetism

- Physics 259 – L02
- Lecture 26

Chapter 24.5-24.8:

Potential due to a continuous charge distribution

Potential of isolated conductors

Last time

- Electric potential energy of a collection of charges
- Interpreting equipotential surfaces
- Equipotential surfaces: visualizing electric potential
- Potential due to an electric dipole

This time

- Potential due to an electric dipole
- Potential due to a continuous charge distribution

Vector quantities

$$\vec{F}_{qq'} = \frac{1}{4\pi\varepsilon_0}\frac{qq'}{r^2}\hat{r}$$

$$\vec{E} = \frac{\vec{F}_{qq'}}{q'} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$$

$$\vec{F} = q\vec{E}$$

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r^2}\hat{r}$$

Scalar quantities

$$U_{q'+q} = \frac{1}{4\pi\varepsilon_0}\frac{qq'}{r}$$

$$V = \frac{U_{q'+q}}{q'} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

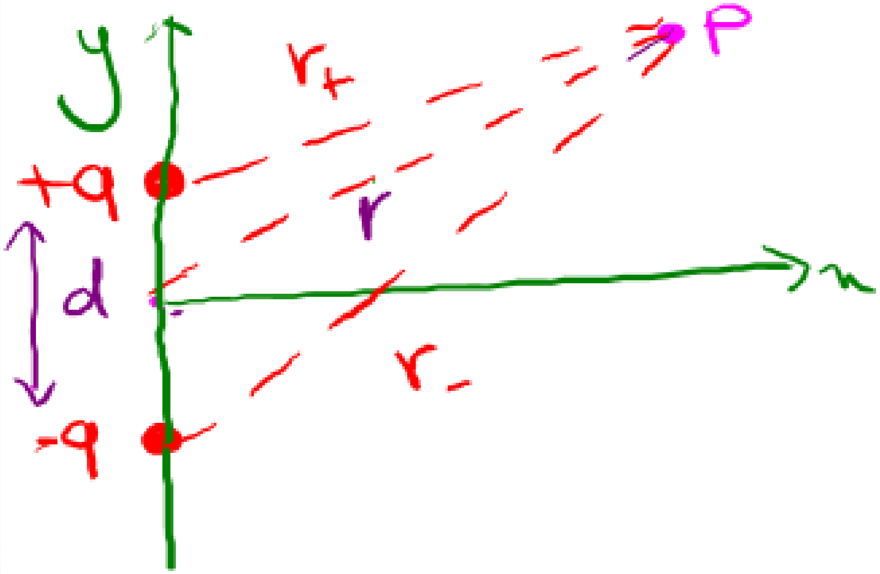
$$U = qV$$

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r}$$

$$\left\{\begin{aligned} U_b - U_a &= -q_0\int_a^b \vec{E}\bullet d\vec{l} \\ V_b - V_a &= -\int_a^b \vec{E}\bullet d\vec{l} = \int_b^a \vec{E}\bullet d\vec{l} \end{aligned}\right.$$

Electric potential of a dipole at arbitrary point p

Electric potential of a dipole $\Rightarrow$ arbitrary P



$$V = V_+ + V_-$$

$$\begin{cases} V_+ = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r_+} \\ V_- = \dfrac{-1}{4\pi\varepsilon_0}\dfrac{q}{r_-} \end{cases}$$

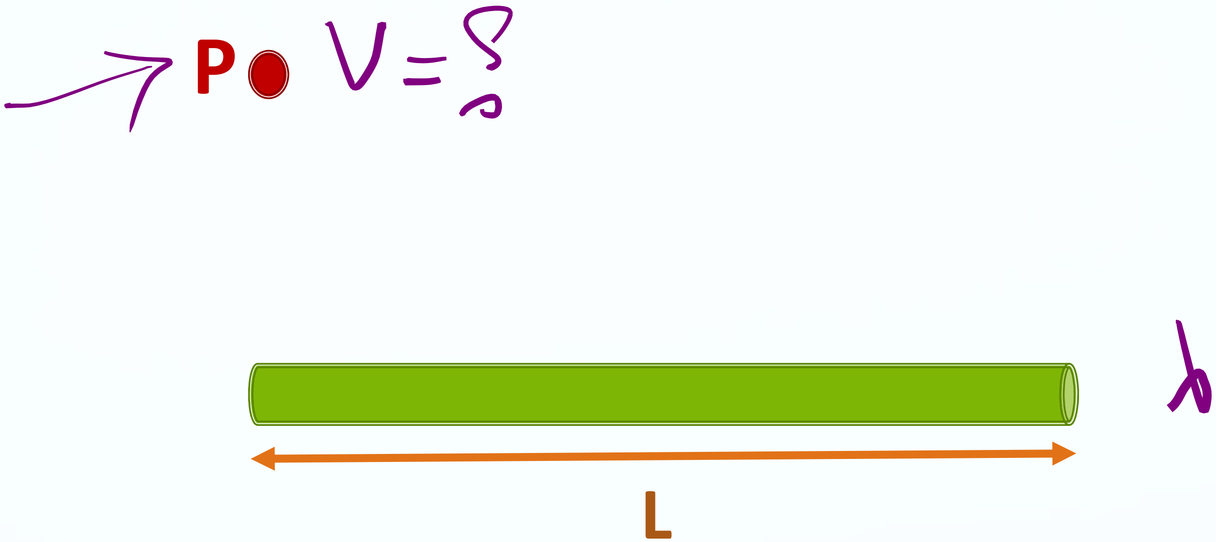
$$\rightarrow V = V_+ + V_- = \frac{1}{4\pi\varepsilon_0}\frac{q}{r_+} + \frac{-1}{4\pi\varepsilon_0}\frac{q}{r_-}$$

$$\rightarrow V = \frac{q}{4\pi\varepsilon_0}\frac{d\cos\theta}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}$$

$$E = -\nabla V$$

- **Go through "Appendix 1-chapter 24" in D2L (different approach)**

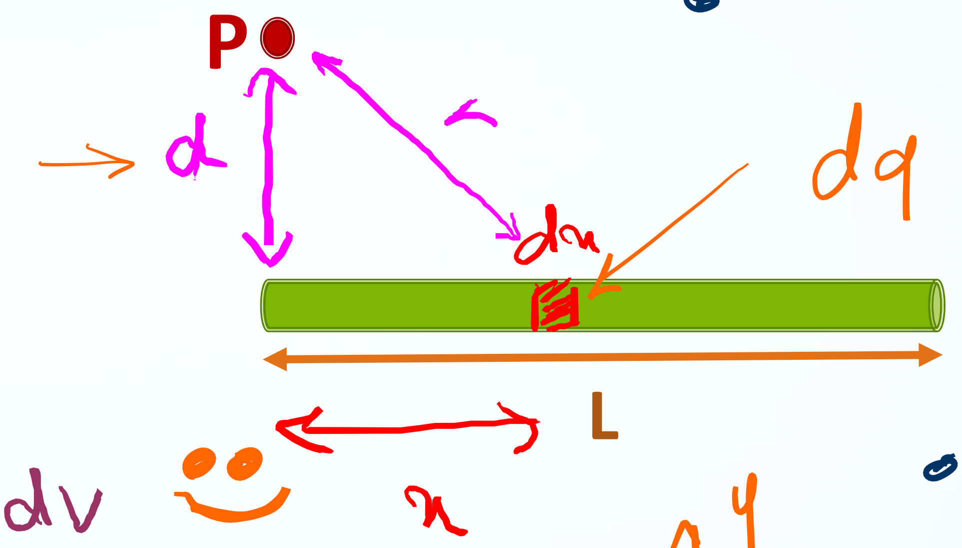
Electric potential of a line of charge at point p



Thin nonconducting rod of length L with uniform positive charge with charge density λ.

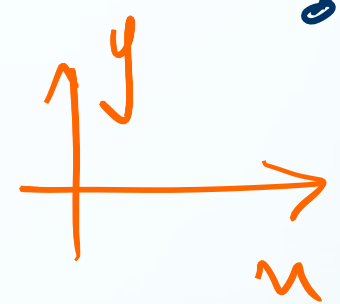
Find electric potential V due to the rod at p, a perpendicular distance d from the left end of the rod.

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} \leftarrow$$



potential of dq

$$\underline{dV} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r} \rightarrow V = \int dV$$



$$dq = \lambda\, dx \quad \& \quad r = \sqrt{d^2 + x^2}$$

$$\rightarrow V = \int dV = \int \frac{1}{4\pi\varepsilon_0}\frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}\int \frac{\lambda\, dx}{\sqrt{d^2+x^2}}$$

$$\rightarrow \quad V = \frac{\lambda}{4\pi\varepsilon_0}\int \frac{dx}{\sqrt{d^2+x^2}}$$

$$V = \int dV = \int_0^L \frac{1}{4\pi\varepsilon_0} \frac{\lambda}{(x^2+d^2)^{1/2}} dx \longleftarrow$$

$$V = \frac{\lambda}{4\pi\varepsilon_0} \int_0^L \frac{dx}{(x^2+d^2)^{1/2}}$$

$$\int_0^L \frac{dx}{(x^2+d^2)^{1/2}} =$$

$$\ln\left(x+(x^2+d^2)^{1/2}\right)\Big|_0^L$$

$$\Rightarrow V = \frac{\lambda}{4\pi\varepsilon_0} \ln\left(x+(x^2+d^2)^{1/2}\right)\Big|_0^L$$

$$\rightarrow V = \frac{\lambda}{4\pi\varepsilon_0}\left(\ln\left(L+(L^2+d^2)^{1/2}\right) - \ln(d^2)^{1/2}\right)$$

$$\rightarrow V = \frac{\lambda}{4\pi\varepsilon_0}\left[\ln\left(L+(L^2+d^2)^{1/2}\right) - \ln d\right]$$

$$\Longrightarrow \rightarrow V = \frac{\lambda}{4\pi\varepsilon_0} \ln\left(\frac{L+(L^2+d^2)^{1/2}}{d}\right)$$

$$\ln(a) - \ln(b) = \ln(a/b)$$

Electric potential of a line of charge at arbitrary point p

Appendix 2 _ chapter 24

TopHat question →

Electric potential at any point on the central axis of a uniformly charged disk is

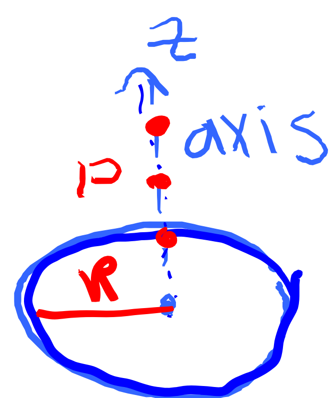
$$V = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{z^2+R^2} - z\right)$$

derive an expression for electric field.

A) $E_z = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{z^2+R^2} - z\right)$

B) $E_z = \frac{\sigma}{2\varepsilon_0}\left(1 + \frac{z}{\sqrt{z^2+R^2}}\right)$

C) $E_z = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2+R^2}}\right)$



$\vec{E} = -\nabla V$

$$\vec{E} = -\nabla V = -\frac{\partial V}{\partial x}\hat{i} - \frac{\partial V}{\partial y}\hat{j} - \frac{\partial V}{\partial z}\hat{k}$$

$$E_x = -\frac{\partial V}{\partial x}$$

$$E_y = -\frac{\partial V}{\partial y}$$

$$E_z = -\frac{\partial V}{\partial z}$$

Electric potential of a ring along its axes

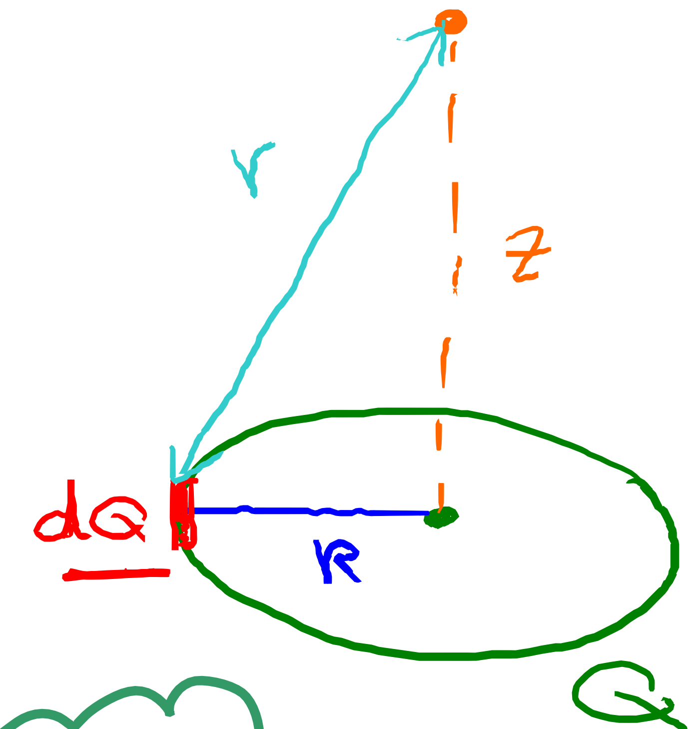
$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dQ}{r}$$

$$V = \int dV = \int \frac{1}{4\pi\varepsilon_0}\frac{dQ}{r}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{1}{r}\int dQ$$

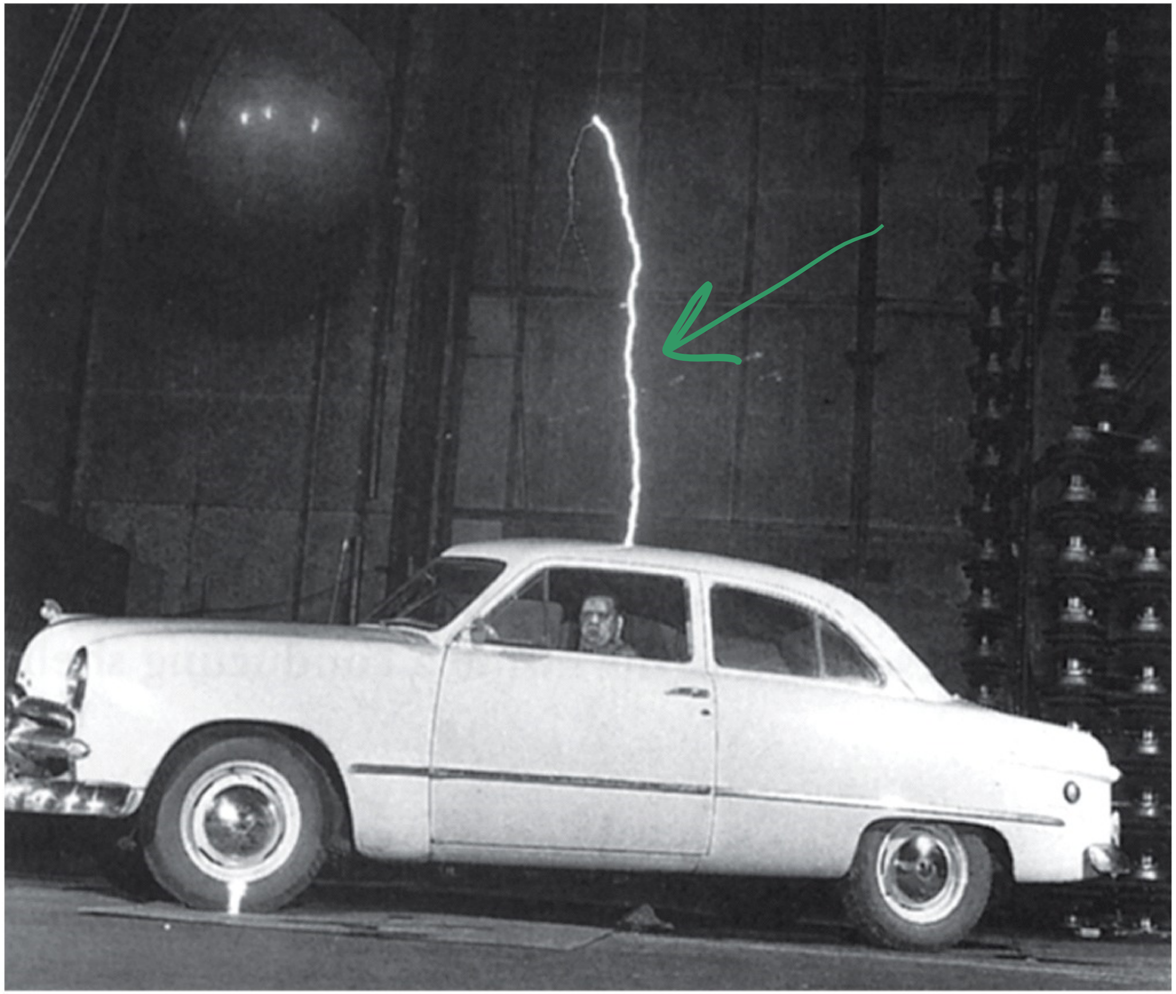
$$\Rightarrow V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{\sqrt{R^2+z^2}}$$



Potential of a charged isolated conductor

Conductor

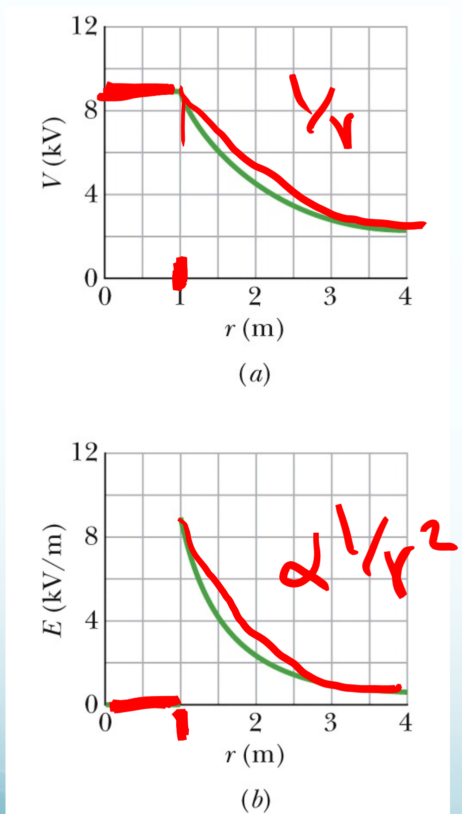
$E = 0$



Courtesy Westinghouse Electric Corporation

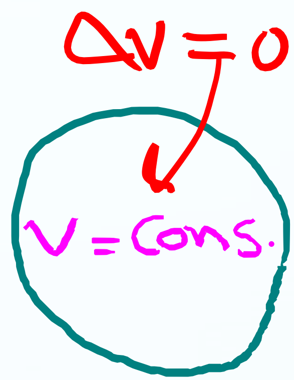
It is wise to enclose yourself in a cavity inside a conducting shell, where the electric field is guaranteed to be zero. A car (unless it is a convertible or made with a plastic body) is almost ideal.

For a spherical conducting shell $\Rightarrow$

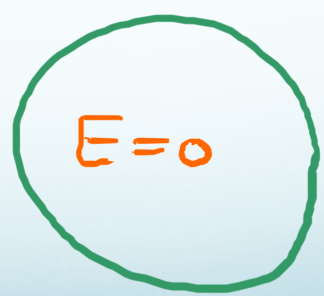


(a) A plot of *V(r)* both inside and outside a charged spherical shell of radius *1.0 m*.

(b) A plot of *E(r)* for the same shell.



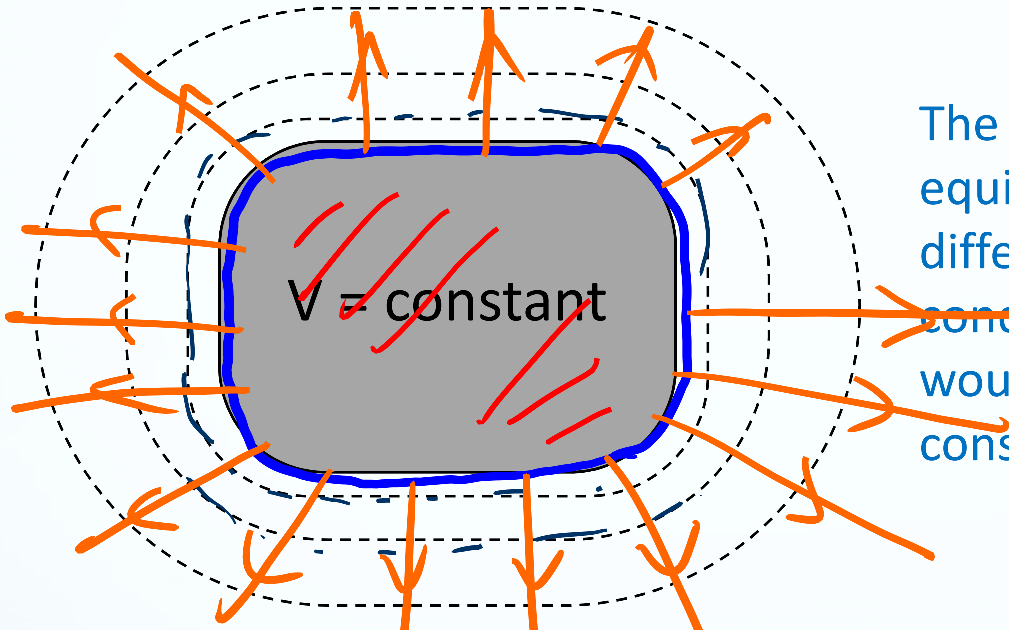
$V \propto \frac{1}{r}$



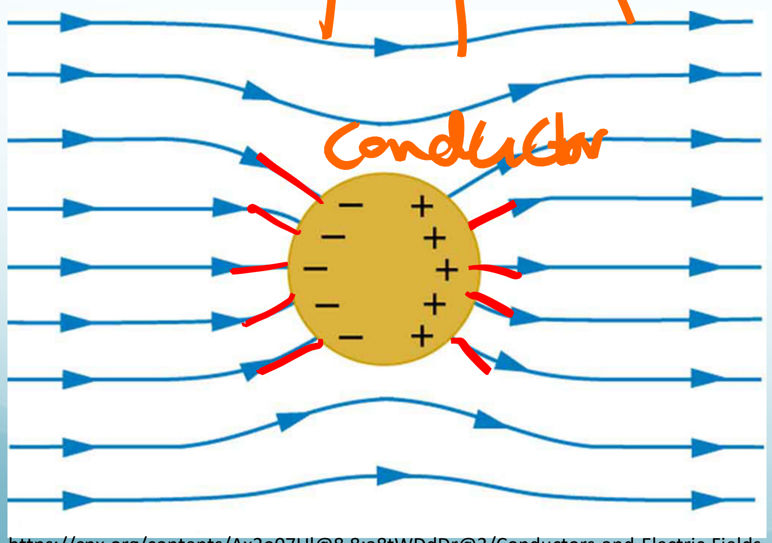
$E \propto \frac{1}{r^2}$

$E = -\nabla V$

Potential of a charged isolated conductor



The surface of a conductor is an equipotential. If there was a potential difference across the surface of a conductor, the freely moving charges would move around until the potential is constant.



https://cnx.org/contents/Ax2o07Ul@8.8:a8tWDdDr@3/Conductors-and-Electric-Fields

This means that electric field lines ALWAYS must meet a conducting surface at right angles (any tangential component would imply a tangential force on the free charges).

This section we talked about:

Chapter 24.5 and 24.8

See you on next Thursday