Last time

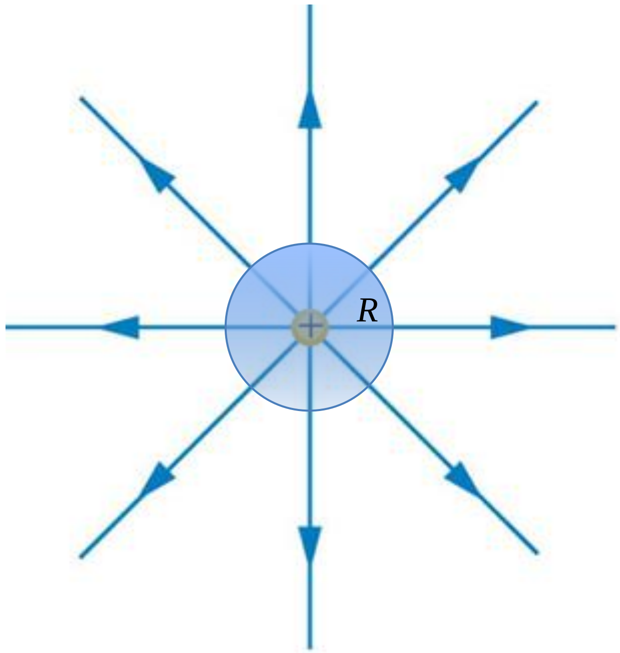
- Calculation of electric flux for non-uniform electric fields and arbitrary shape surfaces
- Properties of electric flux for a closed surface
- Introducing Gauss's law
- Activity #4

This time

- More on properties of electric flux for a closed surface.
- More on Gauss's law
- Examples of calculation of flux for open surfaces
- Application of Gauss's law for a point charges
- Application of Gauss's law for a spherical charge distribution

Flux though a closed surface

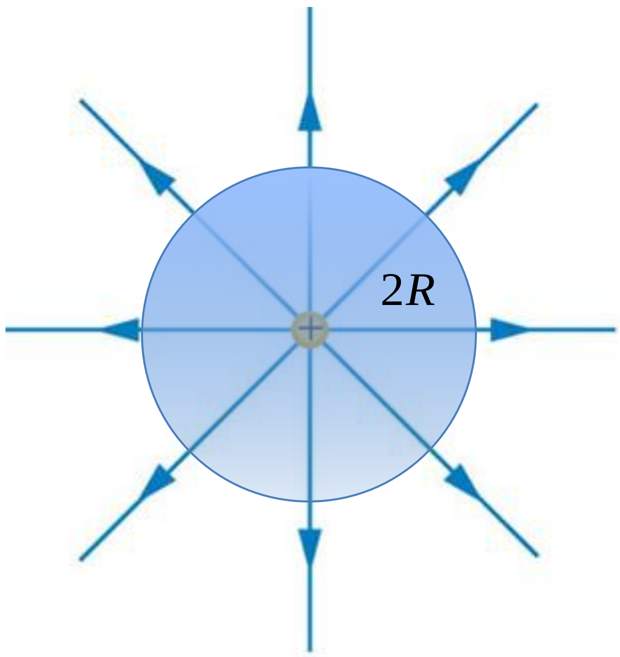
In this 3D problem only 8 electric field lines are drawn for a positive charge +q



How many field lines will cross the closed spherical surface of radius R?

Answer: all 8 field lines

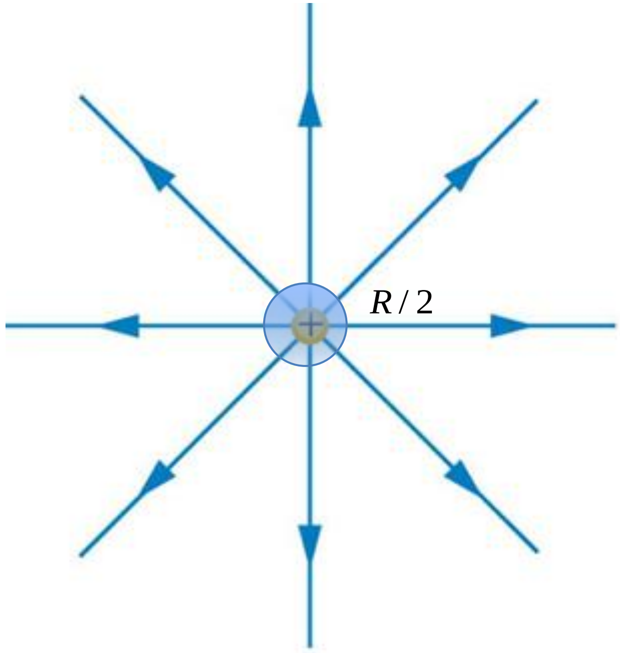
Considering the same charge +q as before



How many field lines will cross the closed spherical surface 2R?

Answer: all 8 field lines

Considering the same charge +q as before



How many field lines will cross the closed spherical surface of radius R/2?

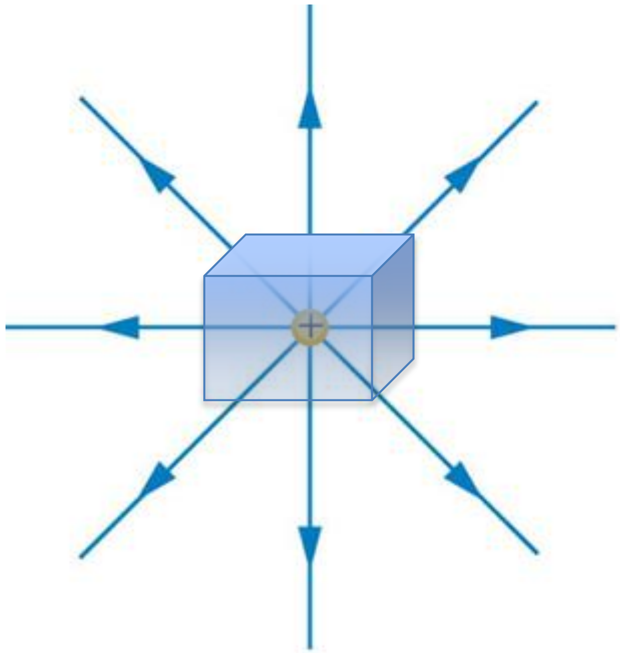
Answer: all 8 field lines

Conclusion:

The number of electric field lines crossing the spherical surface is independent of radius of the sphere as long as the charge resides inside the spherical surface.

Does it matter if the closed surface has another shape?

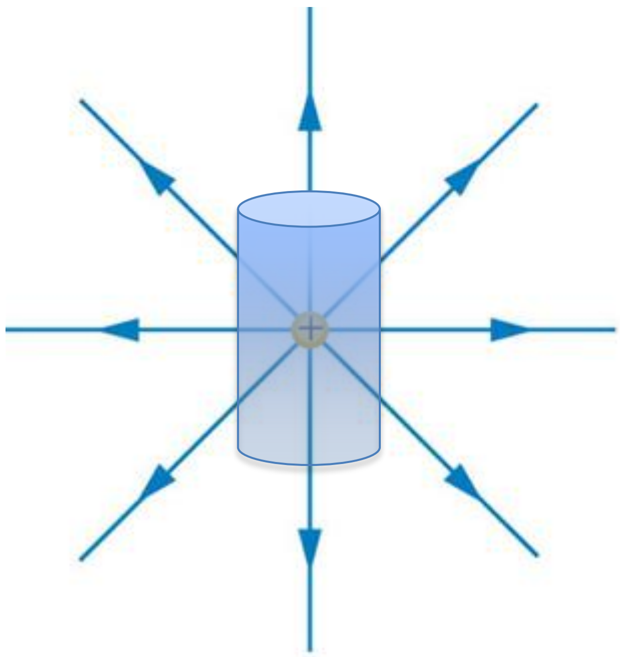
Considering the same charge +q as before



How many field lines will cross the closed rectangular surface?

Answer: all 8 field lines

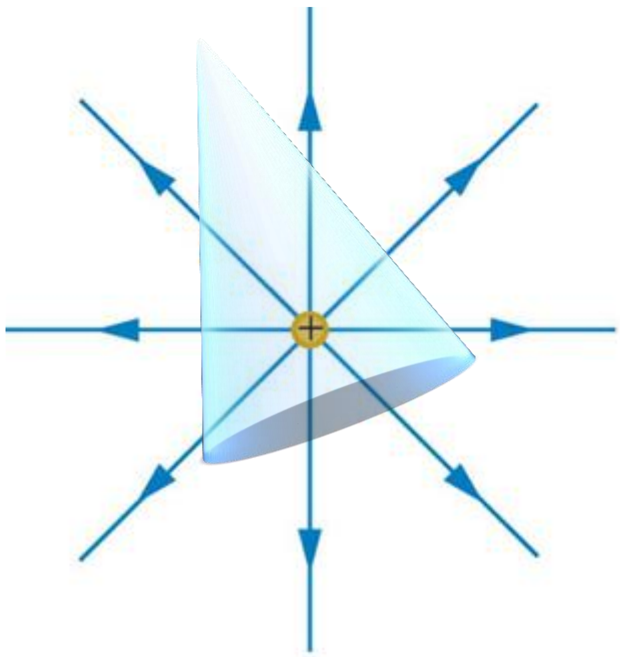
Considering the same charge +q as before



How many field lines will cross the closed cylindrical surface?

Answer: all 8 field lines

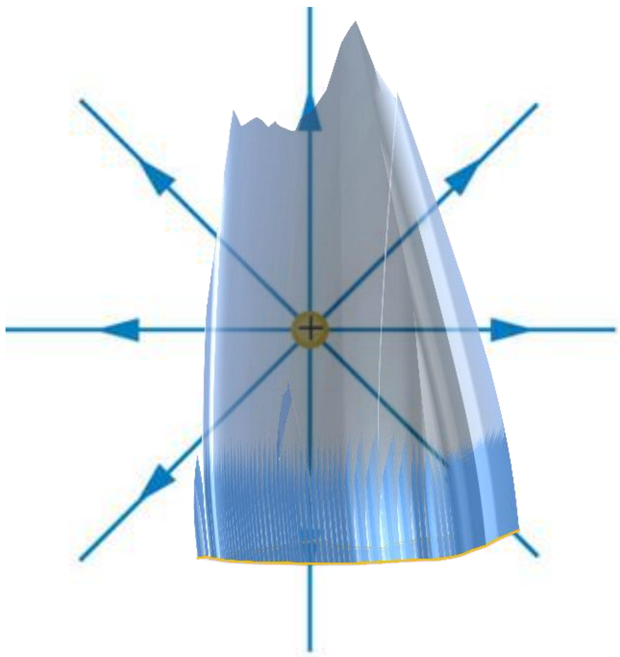
Considering the same charge +q as before



How many field lines will cross the closed conical surface?

Answer: all 8 field lines

Considering the same charge +q as before



How many field lines will cross the closed irregular shaped surface?

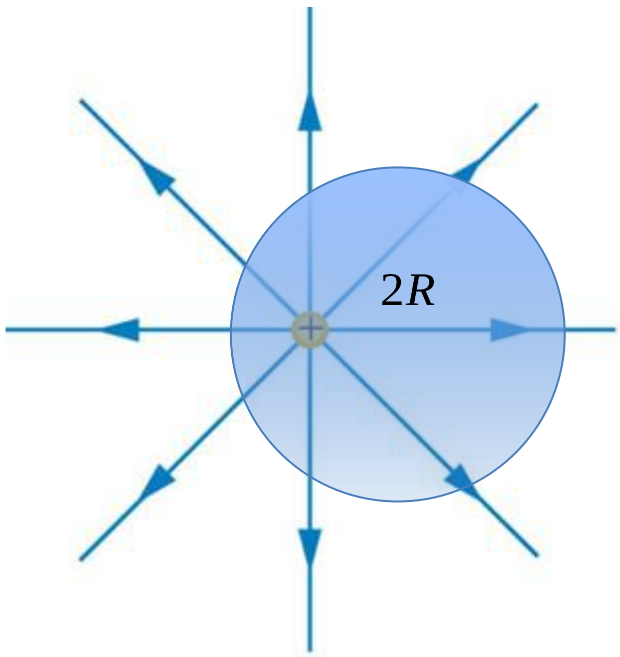
Answer: all 8 field lines

Conclusion:

The number of electric field lines crossing the closed surface is independent of shape of the closed surface as long as the charge resides inside the closed surface.

Does it matter where the charge reside inside the closed surface?

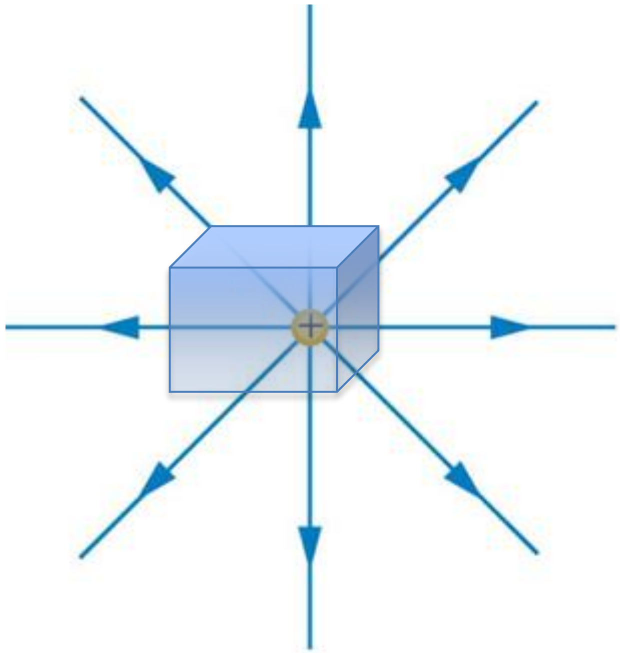
Considering the same charge +q as before



How many field lines will cross the closed spherical surface 2R?

Answer: all 8 field lines

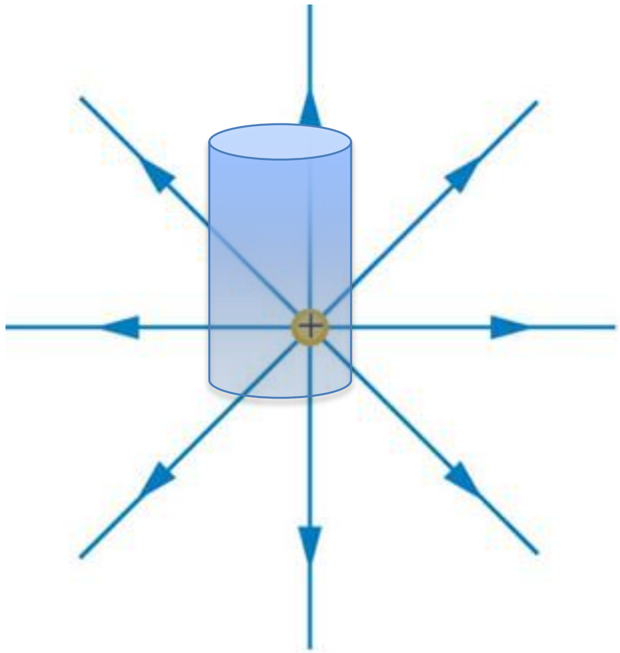
Considering the same charge +q as before



How many field lines will cross the closed rectangular surface?

Answer: all 8 field lines

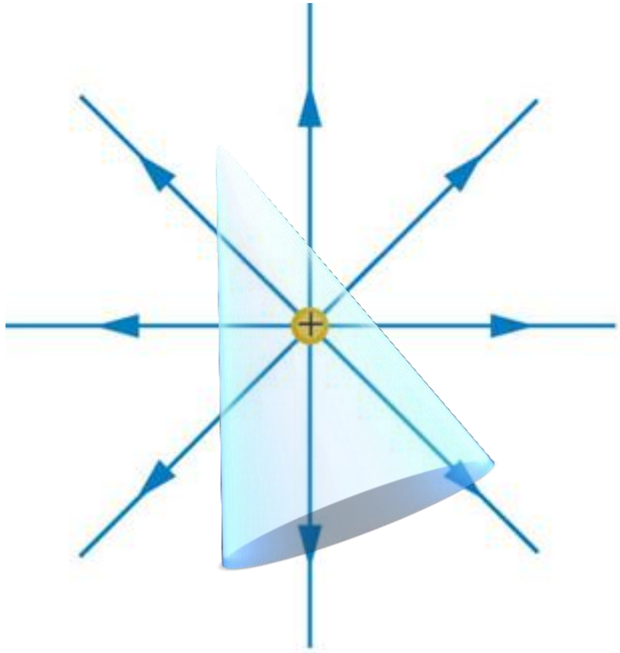
Considering the same charge +q as before



How many field lines will cross the closed cylindrical surface?

Answer: all 8 field lines

Considering the same charge +q as before



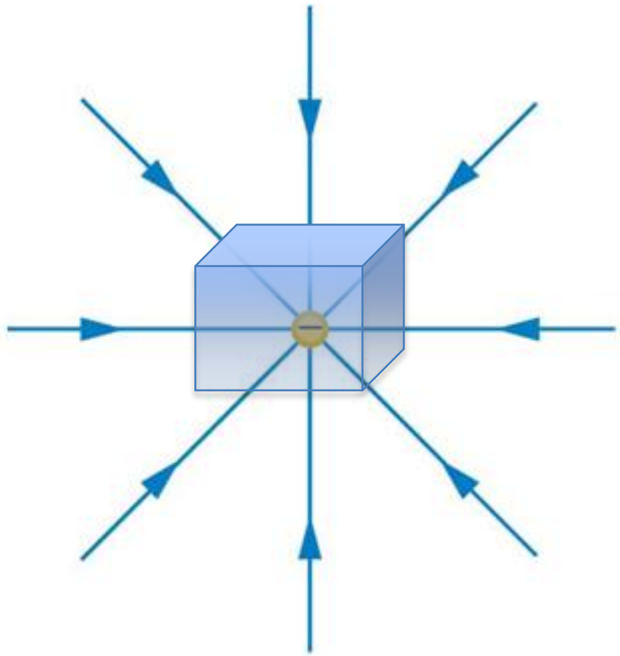
How many field lines will cross the closed conical surface?

Answer: all 8 field lines

Conclusion:

The number of electric field lines crossing the closed surface is independent of the shape, size of the closed surface, and where the positive charge is located inside the surface.

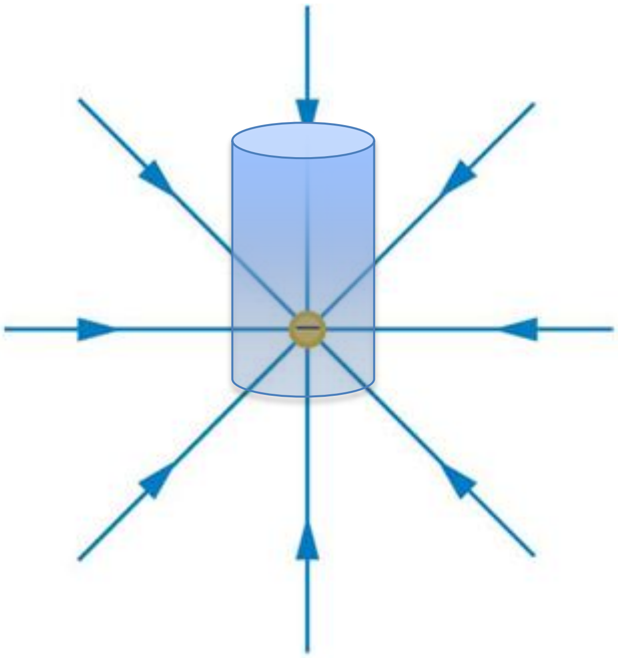
Consider a negative charge of the same magnitude



How many field lines will cross the closed rectangular surface?

Answer: all 8 field lines are going inside

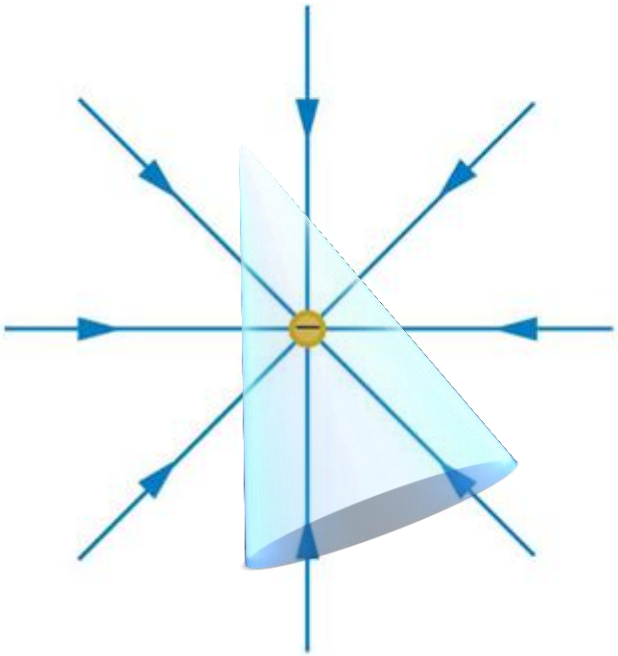
Consider a negative charge of the same magnitude



How many field lines will cross the closed cylindrical surface?

Answer: all 8 field lines are going inside

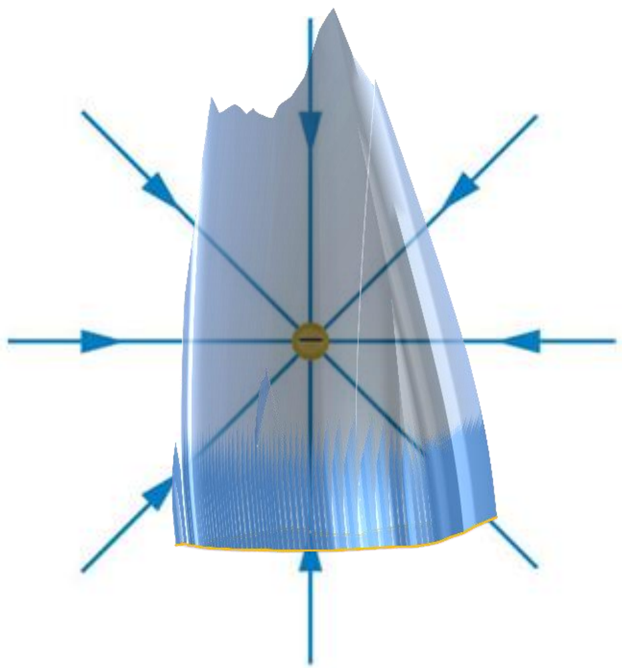
Consider a negative charge of the same magnitude



How many field lines will cross the closed conical surface?

Answer: all 8 field lines going inside

Consider a negative charge of the same magnitude



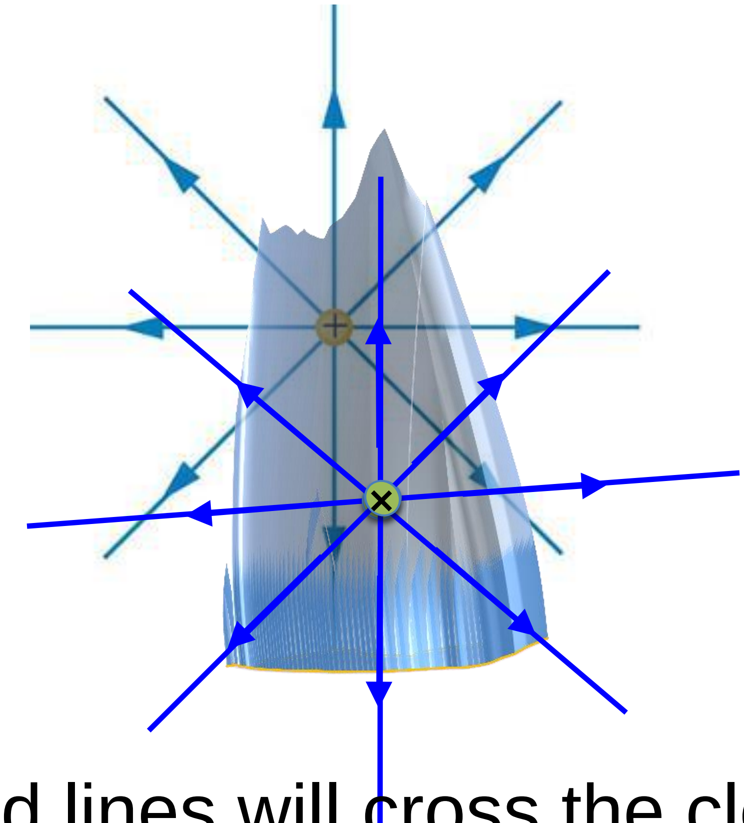
How many field lines will cross the closed irregular shaped surface?

Answer: all 8 field lines going in

Conclusion:

The number of electric field lines crossing the closed surface is independent of the shape, size of the closed surface, and where the negative charge is located inside the surface.

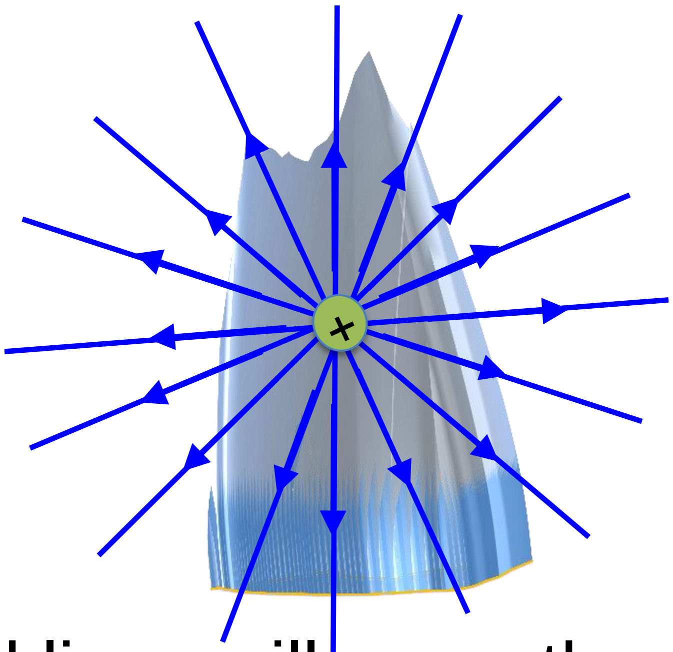
Considering two point charges with +q each



How many field lines will cross the closed irregular shaped surface?

Answer: 16 field lines going out, because net charge is doubled.

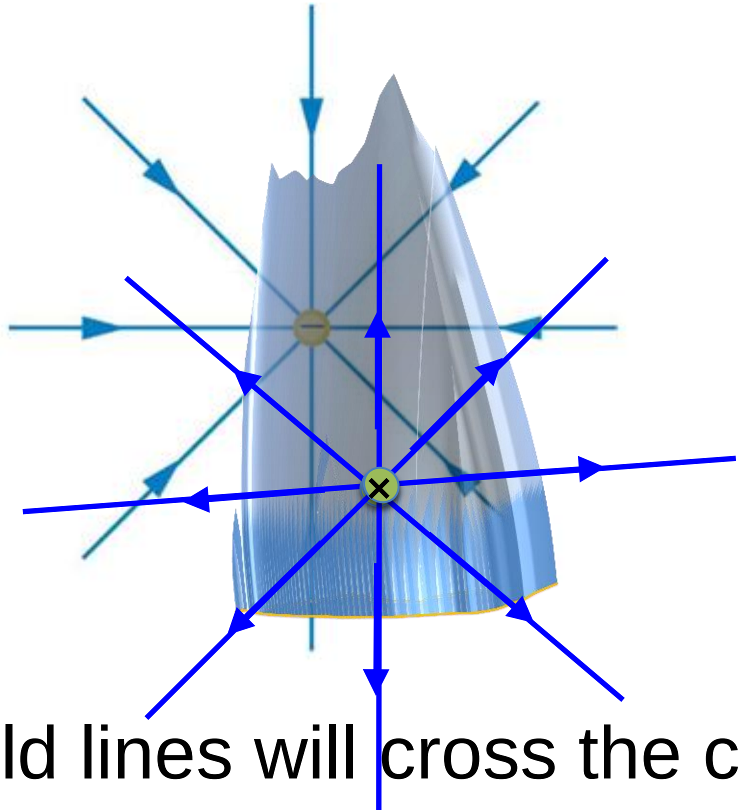
Considering a point charge with +2q



How many field lines will cross the closed irregular shaped surface?

Answer: 16 field lines going out, because net charge is doubled.

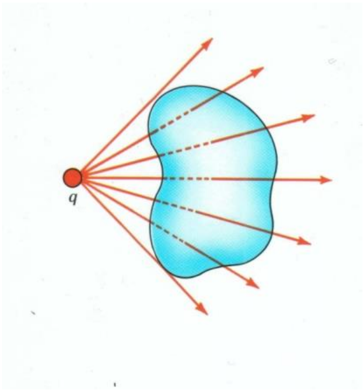
Considering two point charges one with +q and the other with -q



How many field lines will cross the closed irregular shaped surface?

Answer: Zero. 8 lines going in, 8 lines coming out. Because net charge is zero.

What about a charge which resides outside the closed surface of interest?



What is the flux through the surface due to a charge outside?

Charge located outside a closed surface does not contribute to net flux. Electric field lines that cross the surface cross it twice, once on their way in and another time on their way out.

Conclusion:

1. The number of electric field lines (flux) crossing a closed surface is independent of the shape and size of the surface, and where the charges are located inside the surface.

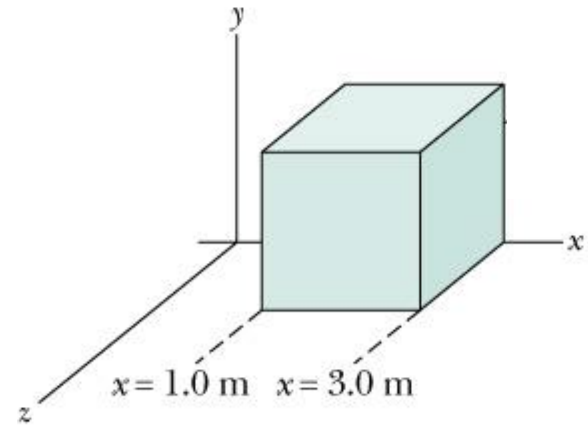
2. Flux depends only on the amount of net charge enclosed by the surface.

3. Larger net charge enclosed leads to larger flux.

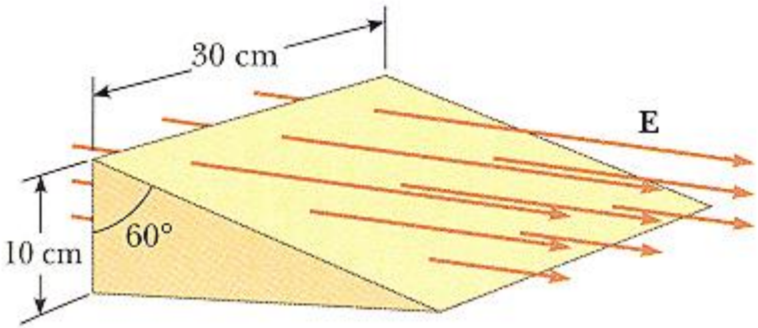
4. Charge located outside a closed surface does not contribute to net flux.

Example: Calculate the flux for the electric field expression and the closed surface given bellow.

$$\vec{E} = (3\ \text{N/Cm})\,x\hat{i} + (4\ \text{N/C})\,\hat{j}$$



Calculate the flux through the incline plane, assuming a constant electric field 3 mN/C.



Note that the flux through the back surface is the same of the flux through the incline plane.

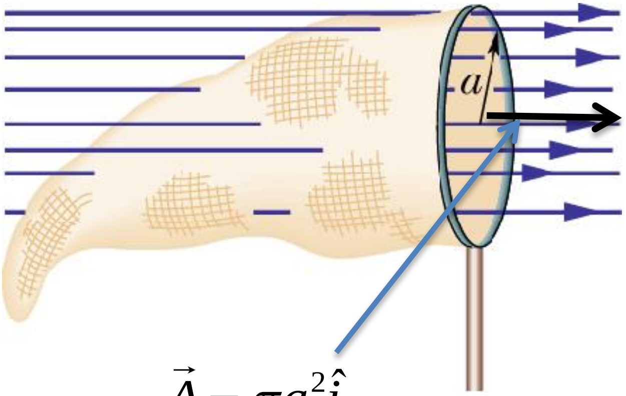
$$\Phi_E = \left(3\times10^{-3}\,N/C\right)\left(0.1m\right)\left(0.3m\right) = 9\times10^{-5}\,Nm^2/C$$

Example: A butterfly net is in a uniform electric field of magnitude $E = 3.0$ mN/C. The rim, a circle of radius $a = 11$ cm, is aligned perpendicular to the field. The net contains no net charge. Find the electric flux through the net.

$$\Phi = \vec{E} \bullet \vec{A} = EA\cos 0 = EA$$

$$\begin{aligned}
\Phi &= E\left(\overbrace{\pi a^2}^{\text{effective surface area}}\right) \\
&= \left(3.0 \times 10^{-3}\ \text{N/C}\right)\pi\,(.11\ \text{m})^2 \\
&= 1.1404 \times 10^{-4}\ \text{N/Cm}^2
\end{aligned}$$

$$\vec{E} = \left(3.0\ \text{mN/C}\right)\hat{i}$$



$$\vec{A} = \pi a^2 \hat{i}$$

Electric Flux; Gauss' Law

Gauss' Law is equivalent to Coulomb's law. It will provide us:

(i) an **easier way to calculate the electric field** in specific circumstances (especially situations with a high degree of symmetry)

(ii) a better understanding of the properties of conductors in electrostatic equilibrium (more on this as we go)

(iii) It is valid for moving charges – not limited to electrostatics.

Electric flux passing through a **closed** surface (**Gaussian**)

$$\Phi_E = \oint \vec{E} \bullet d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_0}$$

Gauss' Law

Electric flux passing through a **closed** surface (**Gaussian**)

$$\Phi_E = \oint \vec{E} \bullet d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_0}$$

$Q_{enclosed}$ is the net charge inside the closed surface (Gaussian)

Φ_E is the net electric flux crossing the closed surface (Gaussian)

$\vec{E}$ is the net electric field at the surface (Gaussian)

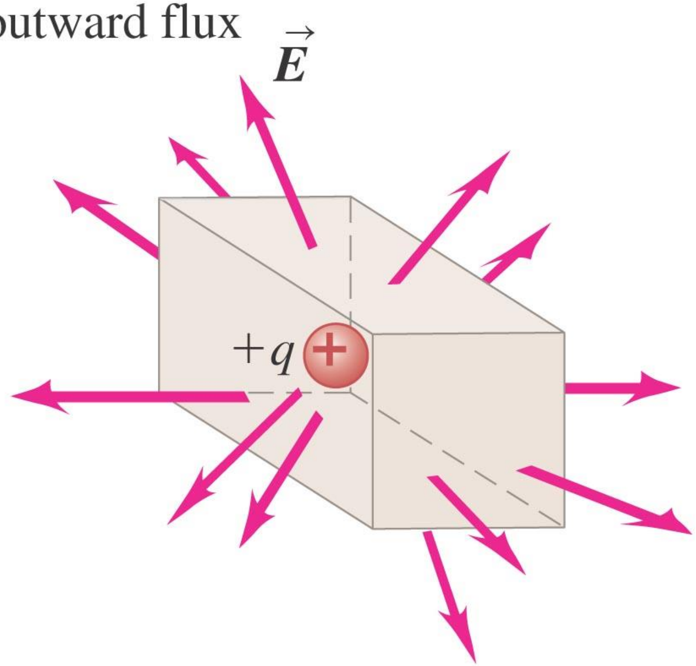
$d\vec{A}$ is the infinitesimal area vector at the surface (Gaussian) where $\vec{E}$ is measured.

Note that any charge outside the Gaussian does not contribute to flux.

Just interested in calculating flux from information on the charge enclosed

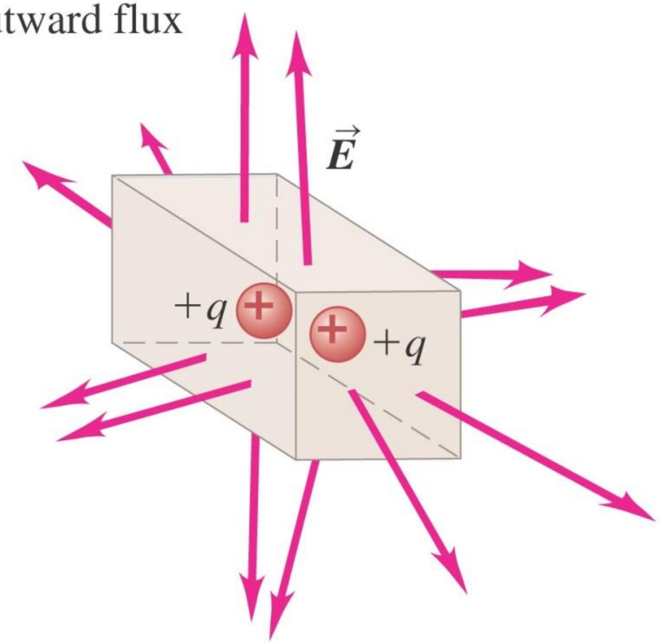
$$\Phi_E = \frac{Q_{enclosed}}{\varepsilon_0}$$

(a) Positive charge inside box, outward flux



$$\Phi_E = \frac{+q}{\varepsilon_0}$$

(b) Positive charges inside box, outward flux

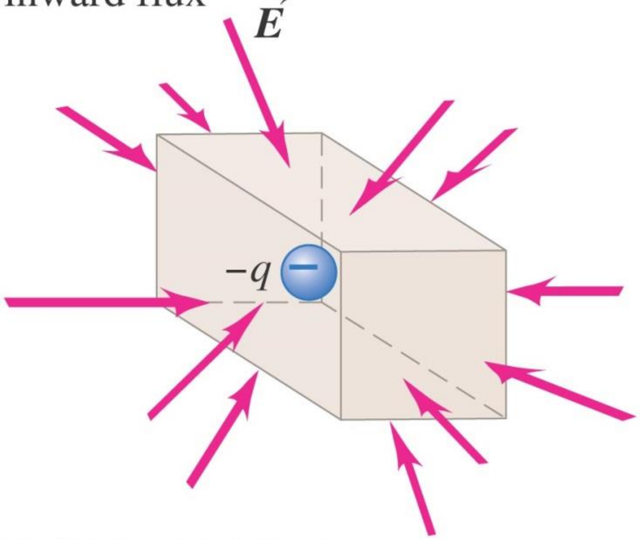


$$\Phi_E = \frac{+q+q}{\varepsilon_0} = \frac{2q}{\varepsilon_0}$$

Just interested in calculating flux from information on the charge enclosed

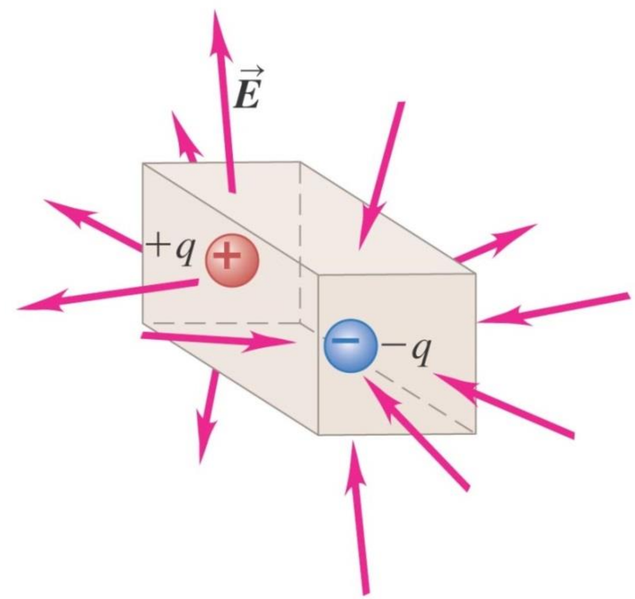
$$\Phi_E = \frac{Q_{enclosed}}{\varepsilon_0}$$

(c) Negative charge inside box, inward flux



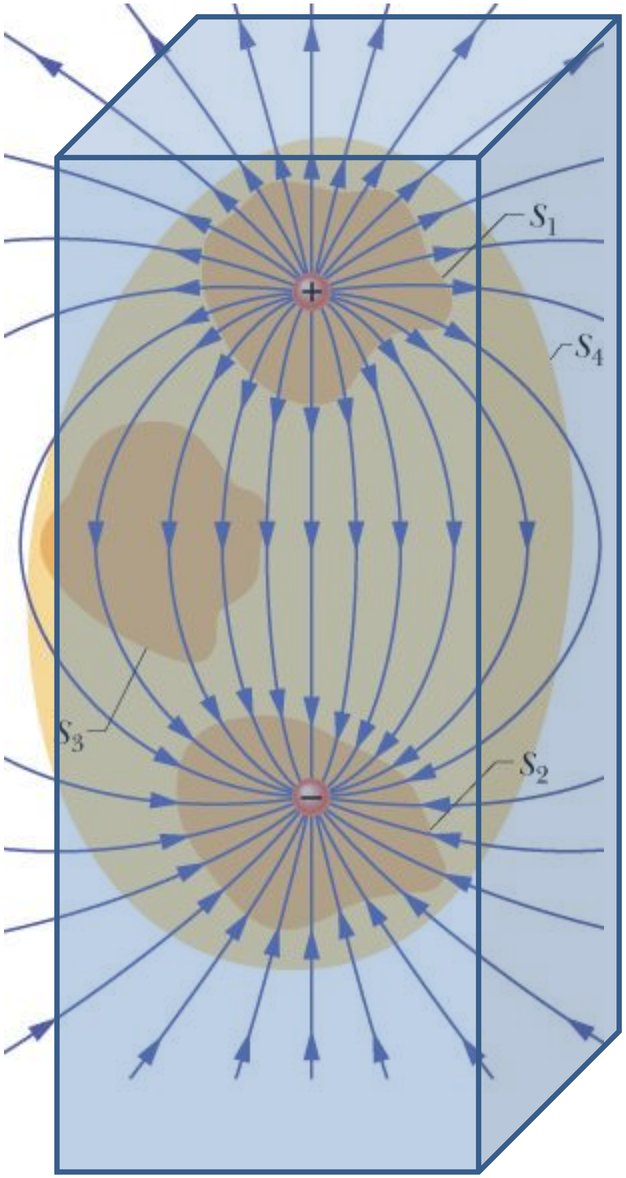
$$\Phi_E = \frac{-q}{\varepsilon_0}$$

(b) Zero *net* charge inside box, inward flux cancels outward flux.



$$\Phi_E = \frac{+q-q}{\varepsilon_0} = 0$$

The surfaces S_1, S_2, S_3, S_4 and the box are closed surfaces, they each define a volume.



$$\Phi_{S_1} = \frac{+q}{\varepsilon_0}$$

$$\Phi_{S_2} = \frac{-q}{\varepsilon_0}$$

$$\Phi_{S_3} = \frac{0}{\varepsilon_0} = 0$$

$$\Phi_{S_4} = \frac{+q-q}{\varepsilon_0} = 0$$

$$\Phi_{\text{box}} = \frac{+q-q}{\varepsilon_0} = 0$$

Solid spherical volume of constant charge density

Calculate the electric field for points inside (r<R) and out side (r>R) of the solid sphere of radius R.

Insulating solid sphere with uniform charge density and total charge Q

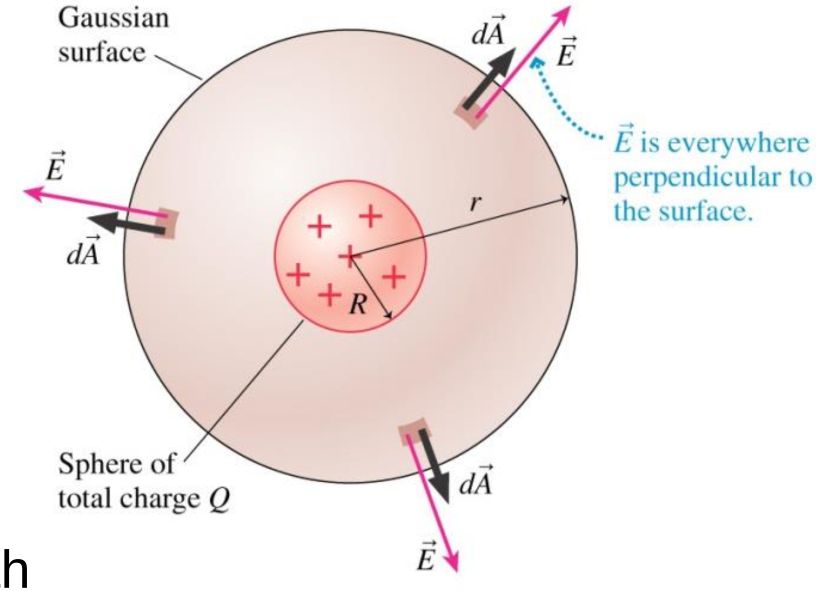
$$\rho = \frac{Q}{\frac{4}{3}\pi R^3}$$

Solid spherical volume of constant charge density

For points outside the sphere

$r > R$

$$\oint \vec{E} \bullet d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_0}$$



Gaussian is drawn as a spherical surface with r>R and concentric with the solid sphere. It encloses all of the charge.

$$Q_{enclosed} = Q$$

Electric field on the surface of the Gaussian is radial and has a constant magnitude.

$$\vec{E} = E\hat{r}$$

Infinitesimal area of the Gaussian is also radial.

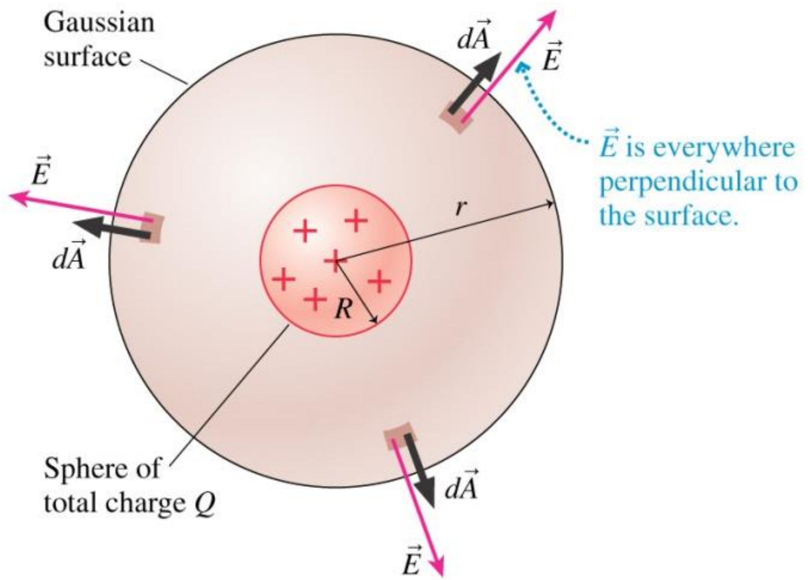
$$d\vec{A} = dA\hat{r}$$

$$\vec{E} \bullet d\vec{A} = (E\hat{r}) \bullet (dA\hat{r}) = EdA$$

$$\oint \vec{E} \bullet d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_0}$$

$$\oint \underbrace{E}_{\text{constant}} dA = \frac{Q}{\varepsilon_0}$$

$$\underbrace{E}_{\text{constant}} \underbrace{\oint dA}_{\text{surface area of the Gaussian}} = \frac{Q}{\varepsilon_0}$$



$$E4\pi r^2 = \frac{Q}{\varepsilon_0} \Rightarrow E = \frac{Q}{4\pi r^2 \varepsilon_0} \Rightarrow \vec{E} = \frac{Q}{4\pi r^2 \varepsilon_0}\hat{r}$$

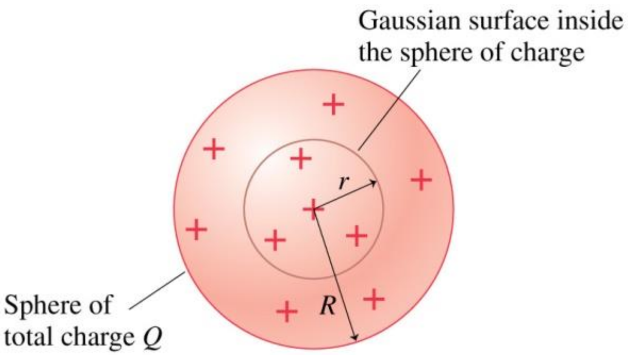
$$\vec{E} = \frac{Q}{4\pi r^2 \varepsilon_0}\hat{r} \qquad \text{For } r > R$$

Solid spherical volume of constant charge density

For points inside the sphere

$r < R$

$$\oint \vec{E} \bullet d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_0}$$



Gaussian is drawn as a spherical surface with r<R. It encloses the charge

$$Q_{enclosed} = \rho \frac{4}{3} \pi r^3$$

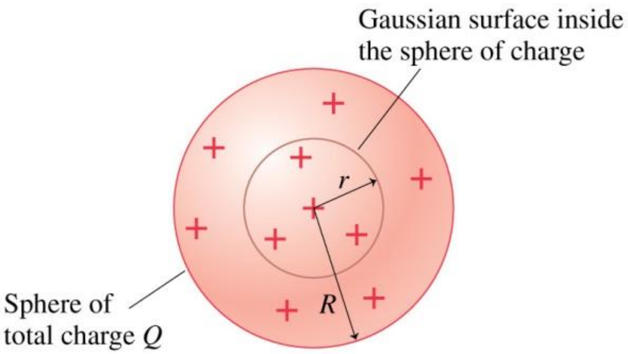
Electric field on the surface of the Gaussian is radial and has a constant magnitude.

$$\vec{E} = E\hat{r}$$

Infinitesimal area of the Gaussian is also radial.

$$d\vec{A} = dA\hat{r}$$

$$\vec{E} \bullet d\vec{A} = \left(E\hat{r}\right) \bullet \left(dA\hat{r}\right) = EdA$$



$$\oint \vec{E} \bullet d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_0}$$

$$\oint \underbrace{E}_{\text{constant}} \, dA = \frac{Q}{\varepsilon_0}$$

$$\underbrace{E}_{\text{constant}} \underbrace{\oint dA}_{\text{surface area of the Gaussian}} = \frac{Q}{\varepsilon_0}$$

$$E 4\pi r^2 = \frac{\rho \frac{4}{3}\pi r^3}{\varepsilon_0} \Rightarrow E = \frac{\rho r}{3\varepsilon_0} \Rightarrow \vec{E} = \frac{\rho r}{3\varepsilon_0}\hat{r}$$

$$\vec{E} = \frac{\rho r}{3\varepsilon_0}\hat{r} \qquad \text{For } r < R$$