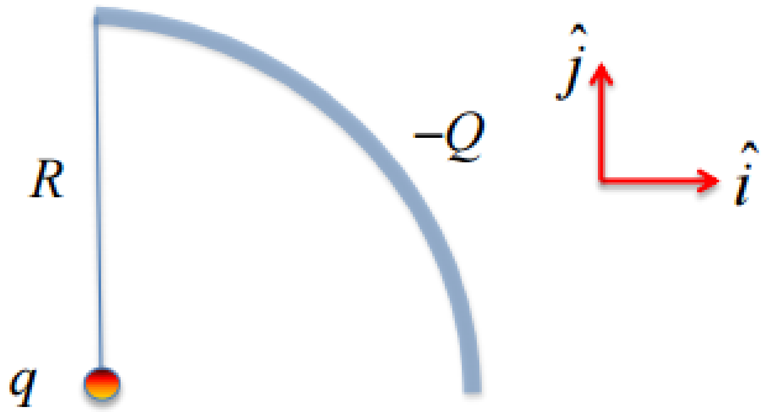
Chapter 21 Appendix

1. Electric force due to an arc at its centre

Electric force due to an arc at its center

Consider a arc of radius *R* with the total charge of -*Q* and a uniform charge distribution.

Compute the force due to the arc on a point charge *q* located at the center of the arc.



solution

Divide the arc into an infinite number of pieces each with a small amount of charge, then calculate the force for each and add up all the forces.

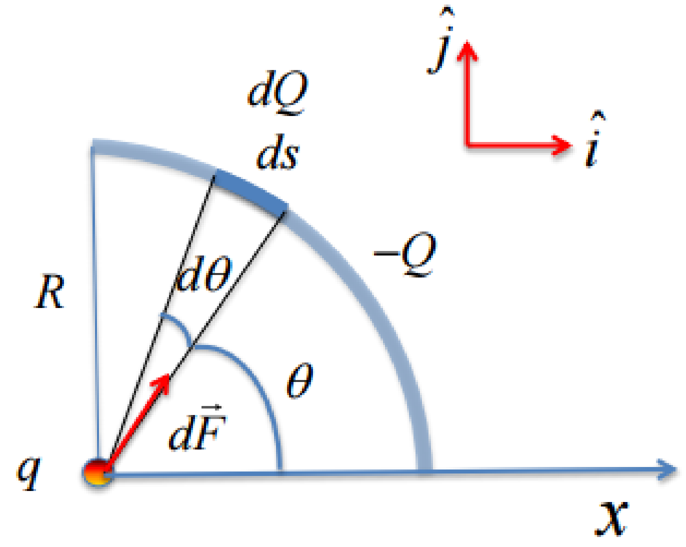
Electric force due to an arc at its center

Charge per unit length: $\lambda = \dfrac{|-Q|}{\pi R/2}$

ds Infinitesimal arc length

$d\theta$ Infinitesimal angle subtended by ds

dQ Infinitesimal charge on ds

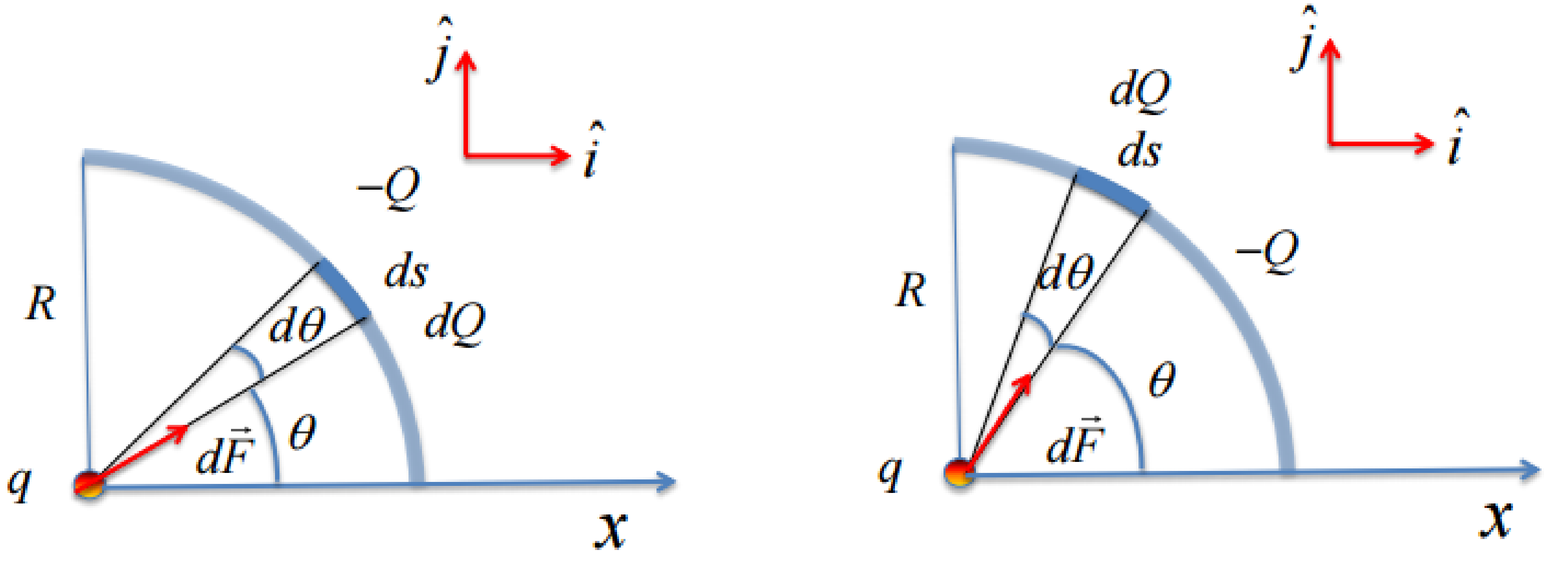


If ds is infinitely small then dQ is infinitely small and can be considered a point charge.

$$dQ = \lambda ds$$

$$d\vec{F} = \frac{k_e q dQ}{R^2}\hat{r} = \frac{k_e q \lambda R d\theta}{R^2}\hat{r}$$

Electric force due to an arc at its center



$$dQ = \lambda ds$$

$$d\vec{F} = \frac{k_e q dQ}{R^2}\hat{r} = \frac{k_e q \lambda R d\theta}{R^2}\hat{r}$$

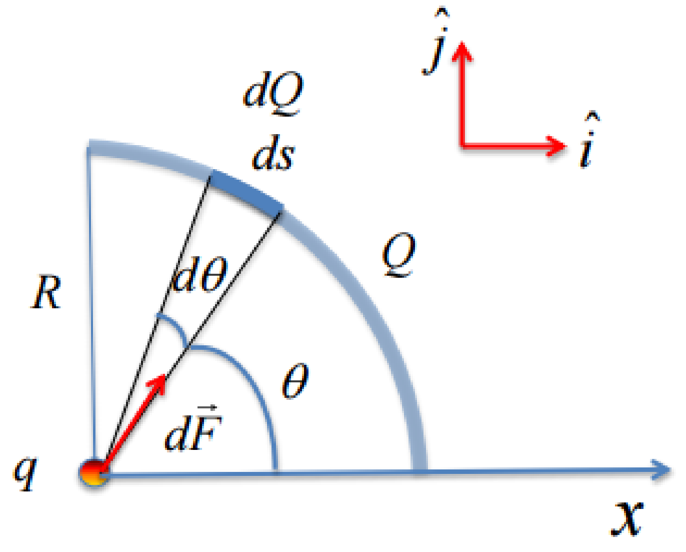
Electric force due to 45° arc at its center

Charge per unit length: $\lambda = \dfrac{|-Q|}{\pi R/2}$

$$d\vec{F} = \frac{k_e q dQ}{R^2}\hat{r} = \frac{k_e q \lambda R d\theta}{R^2}\hat{r}$$

$$dF_x = \frac{k_e q \lambda R d\theta}{R^2}\cos\theta$$

$$dF_y = \frac{k_e q \lambda R d\theta}{R^2}\sin\theta$$



$$F_x = \int dF_x = \int_0^{\pi/2} \frac{k_e q \lambda R d\theta}{R^2}\cos\theta = \frac{k_e q \lambda}{R}\int_0^{\pi/2}\cos\theta d\theta = \frac{k_e q \lambda}{R}\sin\theta\Big|_0^{\pi/2} = \frac{k_e q \lambda}{R} = \left(\frac{2}{\pi}\right)\frac{k_e q Q}{R^2}$$

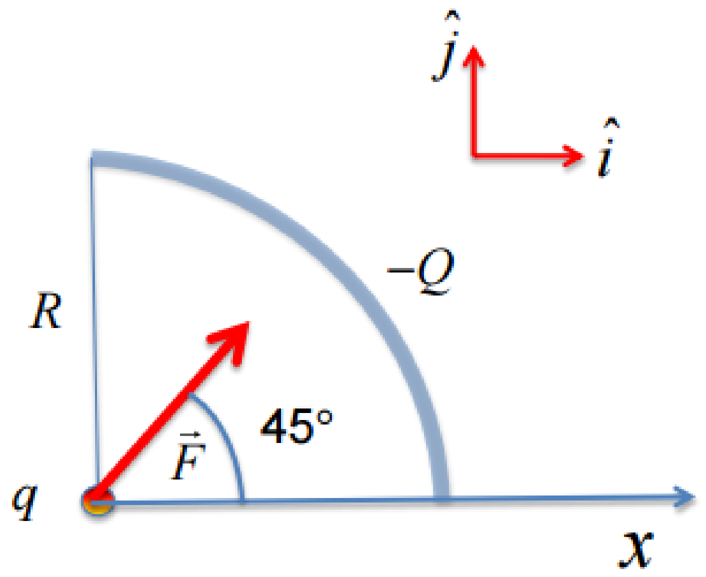
$$F_y = \int dF_y = \int_0^{\pi/2} \frac{k_e q \lambda R d\theta}{R^2}\sin\theta = \frac{k_e q \lambda}{R}\int_0^{\pi/2}\sin\theta d\theta = -\frac{k_e q \lambda}{R}\cos\theta\Big|_0^{\pi/2} = \frac{k_e q \lambda}{R} = \left(\frac{2}{\pi}\right)\frac{k_e q Q}{R^2}$$

Electric force due to 45° arc at its center

$$F_x = \left(\frac{2}{\pi}\right)\frac{k_e qQ}{R^2}$$

$$F_y = \left(\frac{2}{\pi}\right)\frac{k_e qQ}{R^2}$$

$$\vec{F} = \frac{2}{\pi}\frac{k_e qQ}{R^2}\left(\hat{i}+\hat{j}\right)$$



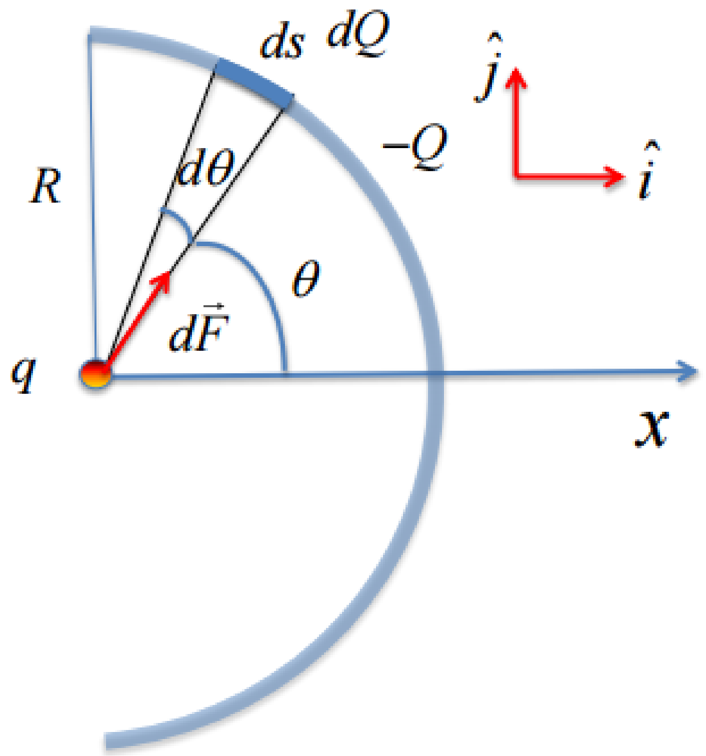
Electric force due to 90 arc at its centre

$$\lambda = \frac{|-Q|}{\pi R}$$

$$d\vec{F} = \frac{k_e q dQ}{R^2}\hat{r} = \frac{k_e q \lambda R d\theta}{R^2}\hat{r}$$

$$dF_x = \frac{k_e q \lambda R d\theta}{R^2}\cos\theta$$

$$dF_y = \frac{k_e q \lambda R d\theta}{R^2}\sin\theta$$



$$F_x = \int dF_x = \int_{-\pi/2}^{\pi/2} \frac{k_e q \lambda R d\theta}{R^2}\cos\theta = \frac{k_e q \lambda}{R}\int_{-\pi/2}^{\pi/2}\cos\theta d\theta = \frac{k_e q \lambda}{R}\sin\theta \Big|_{-\pi/2}^{\pi/2} = \frac{2k_e q \lambda}{R} = \left(\frac{2}{\pi}\right)\frac{k_e q Q}{R^2}$$

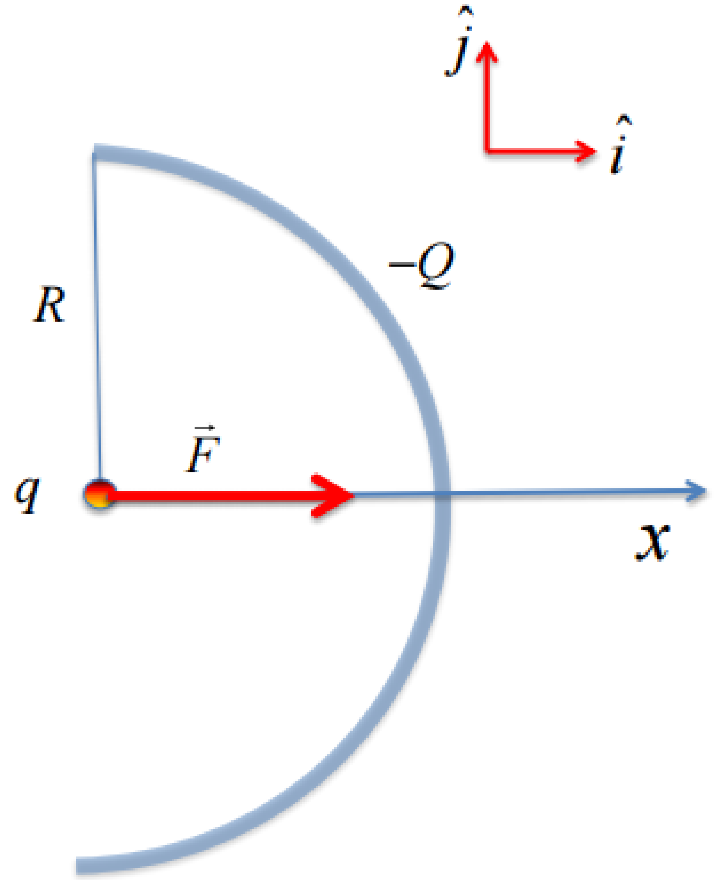
$$F_y = \int dF_y = \int_{-\pi/2}^{\pi/2} \frac{k_e q \lambda R d\theta}{R^2}\sin\theta = \frac{k_e q \lambda}{R}\int_{-\pi/2}^{\pi/2}\sin\theta d\theta = -\frac{k_e q \lambda}{R}\cos\theta \Big|_{-\pi/2}^{\pi/2} = 0$$

Electric force due to 90° arc at its center

$$F_x = \left(\frac{2}{\pi}\right)\frac{k_e qQ}{R^2}$$

$$F_y = 0$$

$$\vec{F} = \frac{2}{\pi}\frac{k_e qQ}{R^2}\hat{i}$$



Electric force due to 90° arc at its center

Or divide the arc into two 45° arcs
and use the superposition principle.

$$\vec{F}_{upper} = \left(\frac{2}{\pi}\right)\frac{k_e qQ/2}{R^2}(\hat{i}+\hat{j})$$

$$\vec{F}_{lower} = \left(\frac{2}{\pi}\right)\frac{k_e qQ/2}{R^2}(\hat{i}-\hat{j})$$

SYMMERTY ABOUT X-AXIS!

$$F_x = \frac{2}{\pi}\frac{k_e qQ}{R^2} \qquad F_y = 0$$

$$\vec{F} = \frac{2}{\pi}\frac{k_e qQ}{R^2}\hat{i}$$