Electricity and Magnetism

- Physics 259 – L02
- Lecture 47

Chapter 30: Induction and inductance

Review

Faraday discovered that there is an induced EMF in the secondary circuit given by

$$\varepsilon = -\frac{d\Phi_M}{dt}$$

This is a new generalized law called Faraday's Law.

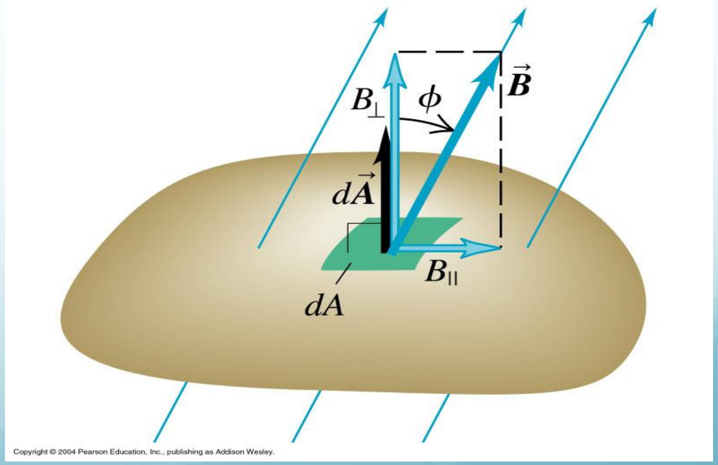
For a coil of N turns→

$$\varepsilon = -N\frac{d\Phi_M}{dt}$$

Recall the definition of magnetic flux:

$$\Phi_M = \int \vec{B} \cdot d\vec{A}$$

Not a closed surface!

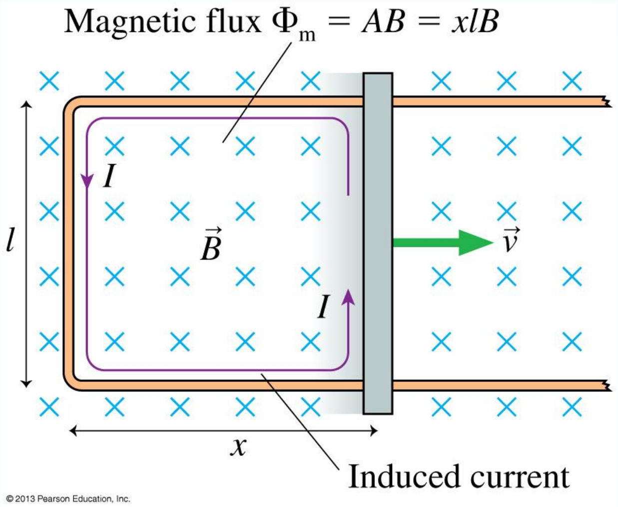


Review

Example of using Faraday's law→

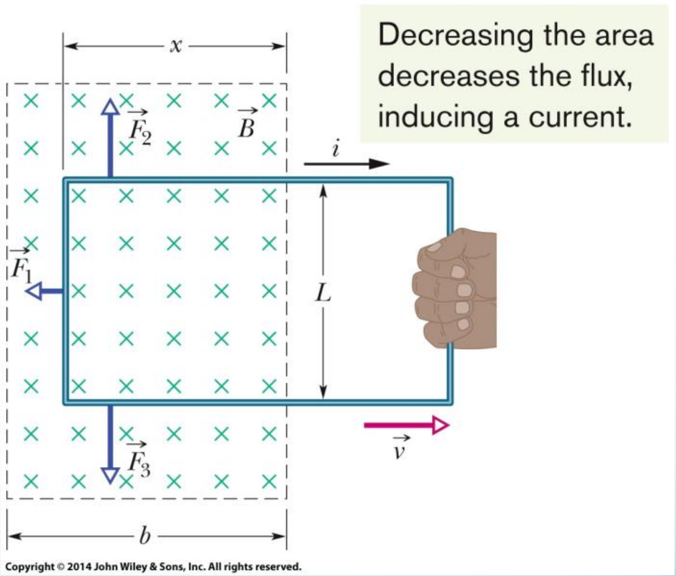
$$\Phi_m = A.B \qquad \varepsilon = \left|\frac{d\Phi_m}{dt}\right| = \frac{d}{dt}xlB = vlB$$

induced current→ $I = \dfrac{\mathcal{E}}{R} = \dfrac{vlB}{R}$



Now→

$$F = iL \times B = iLb\sin(90)$$



Rate of Work: rate at which you do work on the loop as you pull it from the magnetic field:

$$F = \frac{B^2L^2v}{R}.$$

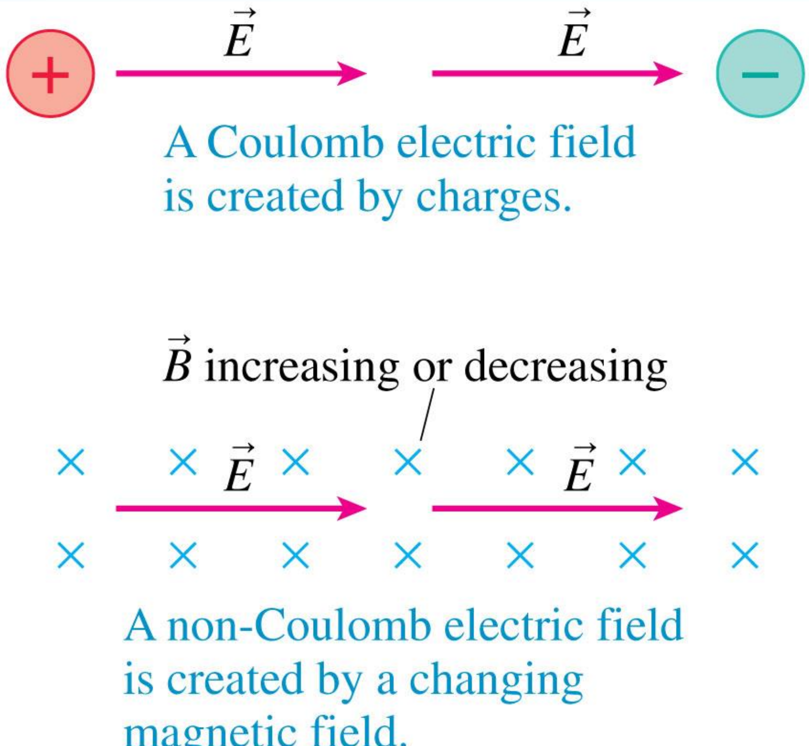
NOTE: The work that you do in pulling the loop through the magnetic field appears as thermal energy in the loop→ $P = Ri^2$

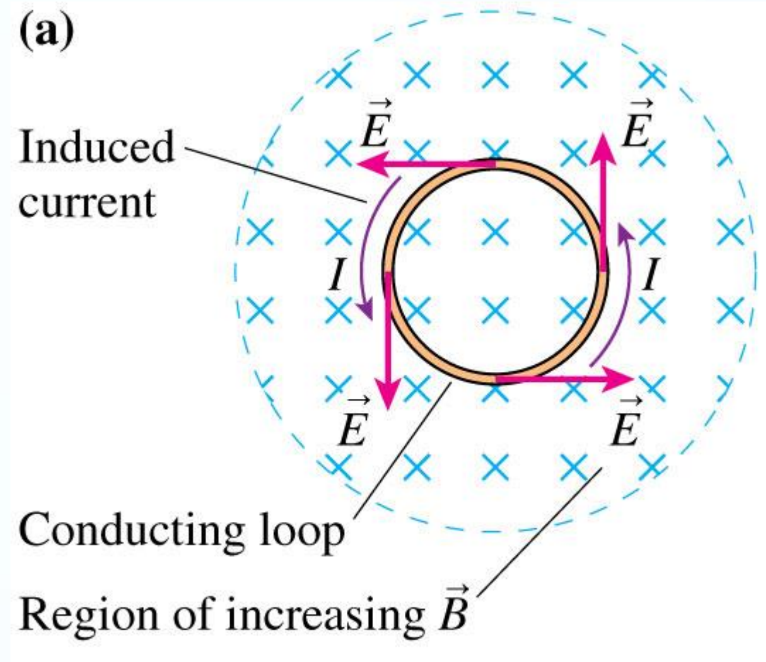
Review 30.3

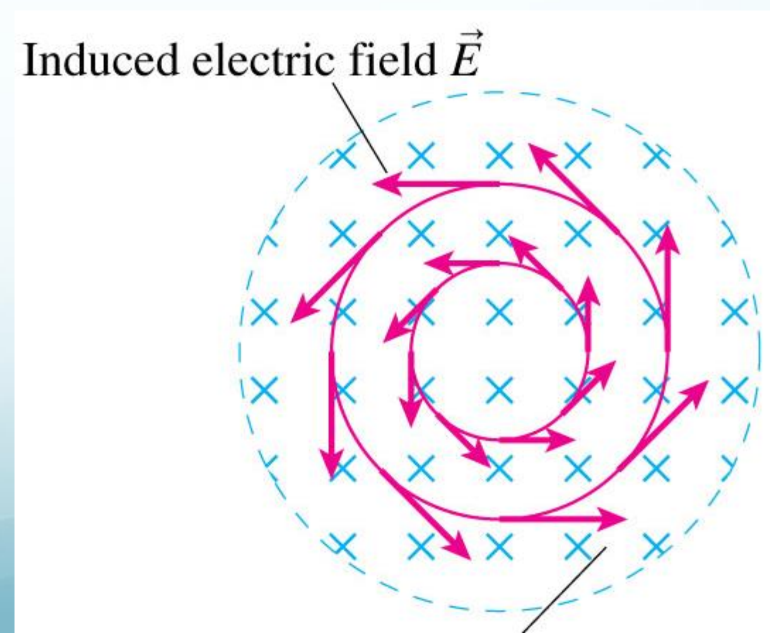
Faraday's law→ strength of induced current

What cause the current?

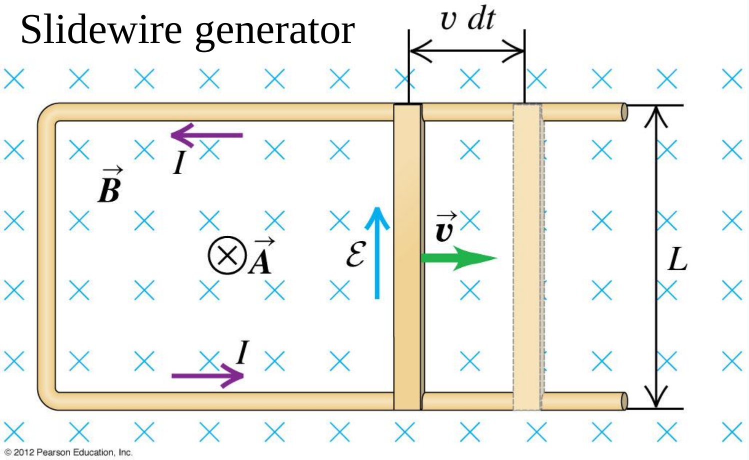
There is an electric field
caused by changing magnetic field
→ **Induced electric field**







30.2 (continue):Eddy currents



- So far we have considered induction in circuits, where the induced current is confined to wires
- Induction also happens if the magnetic flux through extended metallic objects changes
- As with wires, the induced currents attempt to keep the flux stable: *eddy* currents

$$I = -\frac{1}{R}\frac{d\phi_B}{dt}$$

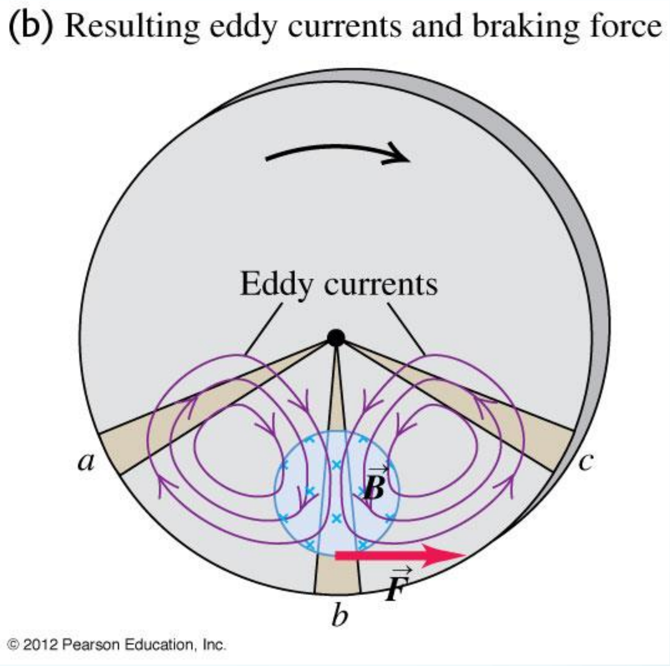
The direction of the currents can be found using Lenz's law

Suppose we replace conducting loop with a Solid conducting plate→

The relative motion of the field and the conductor again induces a current in the conductor→ again an opposing force

Example: Braking system

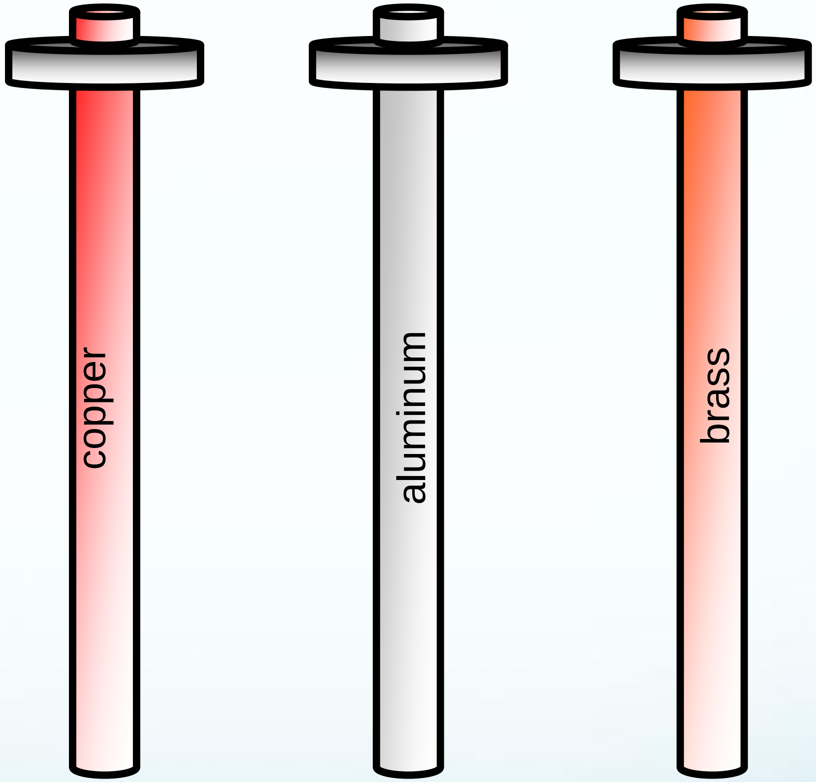
- Without eddy currents, the magnetic flux at the leading (trailing) edge decreases (increases)
- The induced Eddy currents circulate in a sense that prevents this from happening
- Result: transformation of mechanical energy into heat!

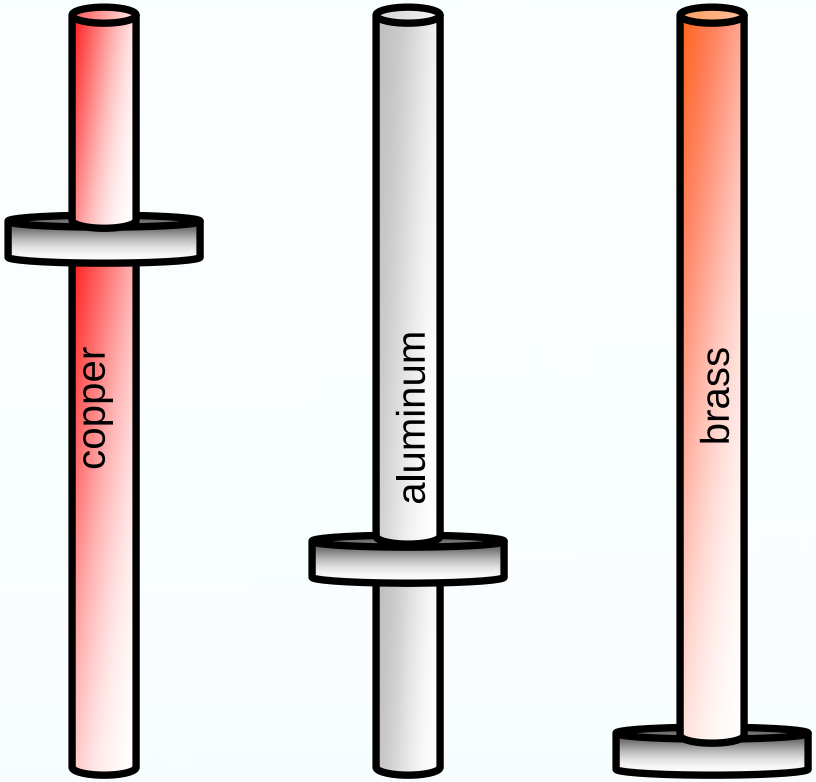


- What changes if the wheel is slotted?
- Slots inhibit the generation of eddy currents, and the braking force is reduced

Question

- Three metal rods (brass, aluminum, copper) hold three ring magnets. The three magnets are dropped at the same time, and then slide (fall) down, guided by the rods
- Which magnet (if any) will reach the bottom first?
- *Note*: copper has the least resistivity, followed by aluminum and brass





- The *magnet on the brass rod will fall fastest*, as the magnitude of the eddy currents, and hence their capability to slow the magnets down, depends on the material’s resistivity.

Recall there are 3 possible terms:

$$e = \underbrace{-\frac{dB}{dt} A \cos f}_{\text{Maxwell Equation}} \underbrace{- \frac{dA}{dt} B \cos f + \frac{df}{dt} BA \sin f}_{\text{Magnetic Force on free charges}}$$

Maxwell Equation

$$-\frac{d\vec{B}}{dt} = \nabla \times \vec{E}$$

Magnetic Force on free charges

$$F = q\vec{v} \times \vec{B}$$

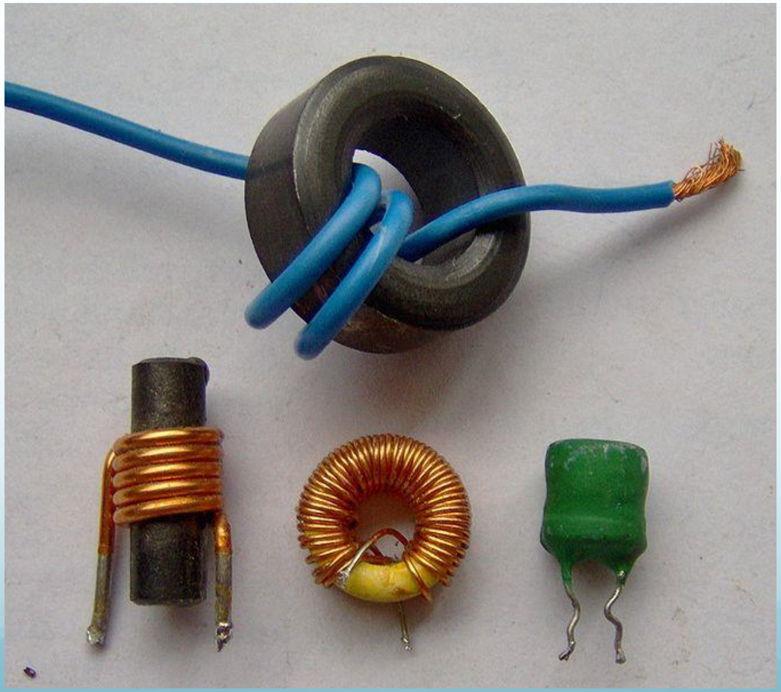
This makes Faraday's Law a particularly powerful tool from a practical engineering standpoint!

Applications of Faraday's Law:

30.4: Inductors and Inductance

An inductor is a device that can be used to produce a known magnetic field in a specified region.

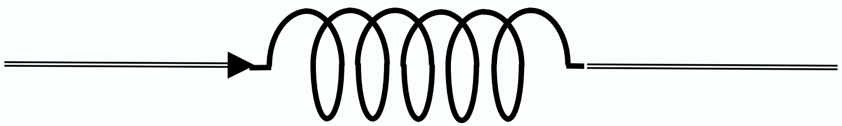
An inductor is a passive electrical component that can store energy in a magnetic field.



Inductance

Note that a changing Magnetic flux produces an induced EMF in a direction which “tries to oppose the change”

$$\frac{di}{dt}$$



$$\varepsilon = -\frac{d\phi}{dt}$$

Changing the current changes the flux through the inductor, which creates a back-emf.

If a current *i* is established through each of the *N* windings of an inductor, a magnetic flux Φ_B links those windings. The inductance *L* of the inductor is

$$\boxed{L = \frac{N\Phi_B}{i}}$$

The SI unit of inductance is the *henry (H)*, where *1 henry = 1H=1T•m²/A*

Energy in a Capacitor is stored in the Electric Field
Energy in an Inductor is stored in the Magnetic Field.

Inductance of a solenoid

The inductance per unit length near the middle of a long solenoid of cross-sectional area *A* and *n* turns per unit length is

$$L = \frac{N\Phi_B}{i}$$

$$\frac{L}{l} = \mu_0 n^2 A$$

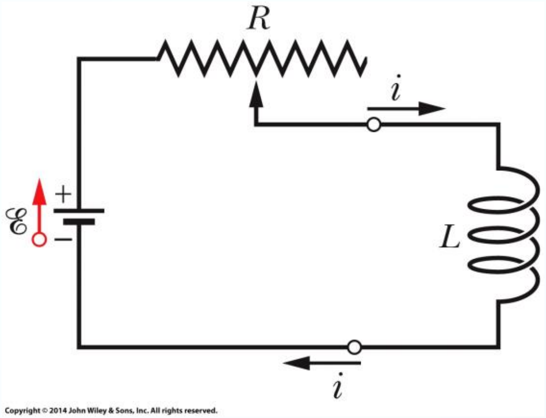
Potential difference across an inductor→ $\boxed{\Delta V = -L\frac{di}{dt}}$

If two coils — which we can now call inductors — are near each other, a current i in one coil produces a magnetic flux Φ_B through the second coil. We have seen that if we change this flux by changing the current, an induced *emf* appears in the second coil according to Faraday's law. An induced *emf* appears in the first coil as well.

30-5 Self-Induction

An induced emf $\mathscr{E}_L$ appears in any coil in which the current is changing.

This process is called self-induction, and the *emf* that appears is called a self-induced *emf*.



It obeys Faraday's law of induction just as other induced emfs do. For any inductor,

$$N\Phi_B = Li.$$

Faraday's law tells us that

$$\mathscr{E}_L = -\frac{d(N\Phi_B)}{dt}.$$

By combining these equations, we can write

$$\mathscr{E}_L = -L\frac{di}{dt} \quad \text{(self-induced emf).}$$

Note: a self-induced *emf* appears whenever the current changes with time. The magnitude of the current has no influence on the magnitude of the induced *emf*; only the rate of change of the current counts.

30.7: Energy storage in Inductors

If we build up the current, starting from $\mathbf{I}_0$ = 0 (initial) → $\mathbf{I}_f$,

at the time t when we have achieved a current $\mathbf{I}$, we have to work against an opposing EMF = Ld$\mathbf{I}$/dt in order to achieve a further increase in current, so our energy source is doing work per unit time

$$dP = IV = IL\frac{dI}{dt}$$

$$W = \int P\,dt = \int IL\frac{dI}{dt}dt$$

total work done:

ie energy stored in system: $U = \int_0^{I_f} LI\,dI$

$$\boxed{U = \frac{1}{2}LI^2}$$

& for solenoid→ $\frac{L}{l} = \mu_0 n^2 A$

$$\boxed{U = \frac{1}{2}Li^2}$$

$$\boxed{\frac{L}{l} = \mu_0 n^2 A}$$

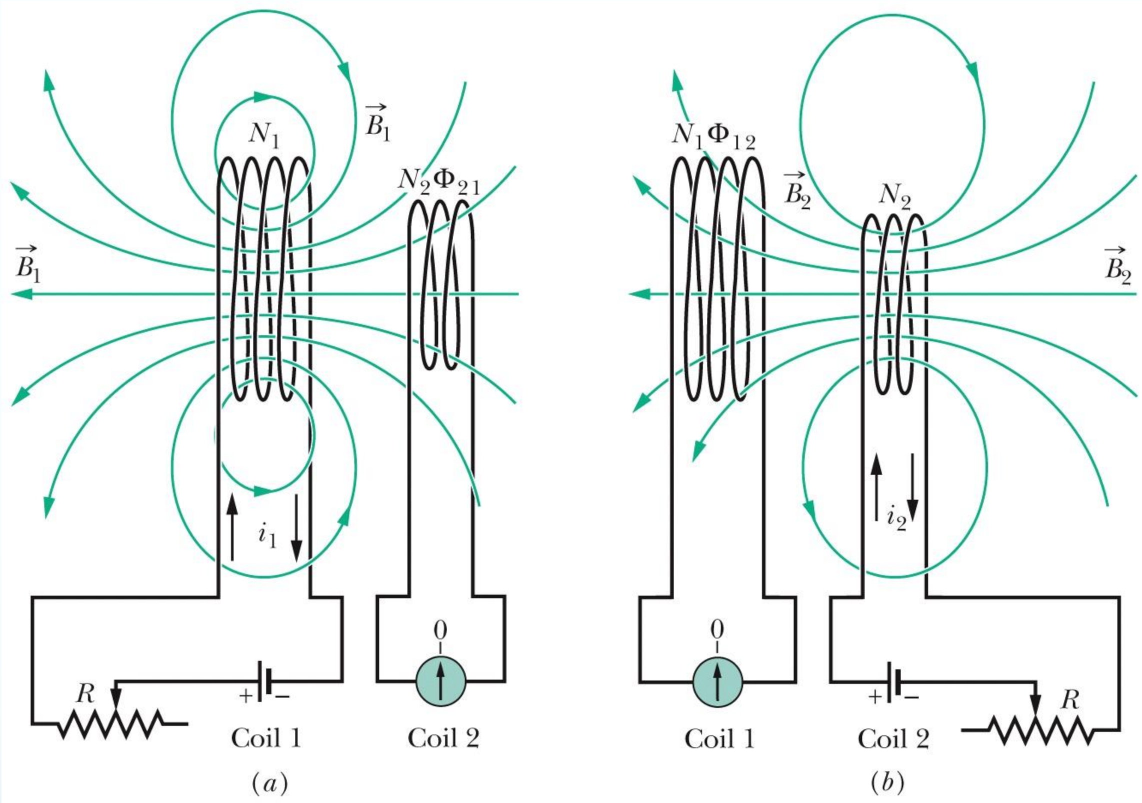
Energy density→

$$u = \frac{U}{V} = \frac{U}{Al} =$$

$$B = \mu_0 ni$$

$$\boxed{= \frac{1}{2\mu_0}B^2}$$

30-8 Mutual Induction



Mutual induction. (a) The magnetic field B_1 produced by current i_1 in coil 1 extends through coil 2. If i_1 is varied (by varying resistance R), an *emf* is induced in coil 2 and current registers on the meter connected to coil 2. (b) The roles of the coils interchanged.

If coils 1 and 2 are near each other, a changing current in either coil can induce an emf in the other. This mutual induction is described by

$$\mathscr{E}_2 = -M\frac{di_1}{dt}$$

and

$$\mathscr{E}_1 = -M\frac{di_2}{dt}.$$

This section we talked about:

Chapter 30

See you on Friday