University of Calgary
Department of Physics and Astronomy
PHYS 259, Winter 2017

Labatorial 7: Electrical Resistivity of Play-Doh™

One of the most basic circuit elements is a resistor, which is a device that dissipates energy. Resistive devices come in all shapes and sizes as can be seen in the photo on the right, which is not surprising because there is a connection between a device's resistance and its geometry. The two are related by an intrinsic property of the material itself called the *resistivity*. For instance, any two pieces of copper will have the same resistivity but their individual resistances will depend on their shapes. In this labatorial, you will explore this connection between geometry and resistance by using objects made of Play-Doh™as resistors in a circuit.

Learning Goals:

To understand how the resistance of an object depends on its geometric properties.

Preparation:

Halliday, Resnick, and Walker, "Fundamentals of Physics" 10th edition, Wiley: 26.1–26.5

Equipment:

Anatek power supply, two digital multimeters, 3 banana plug connecting cables, 2 alligator clips, 2 aluminum plates, 2 voltage probes, 3 Play-Doh™containers, ruler, computer with LoggerPro software.

Note that there is an equation sheet at the end of this worksheet.

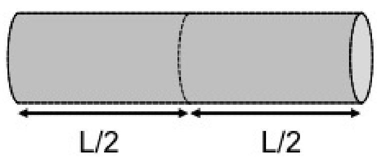
1 Resistance is a Geometrical Property

Every material has an intrinsic property known as the resistivity ρ. Objects made out of a material have a property called the resistance R that depends on the object's geometry. The relationship between resistance and resistivity is

$$R = \frac{\rho L}{A} \tag{1}$$

where L is the length of the object and A is its cross-sectional area perpendicular to its length. Consider a solid metal cylinder of length L and radius r. You measure the resistance of this cylinder and find it to have a value R.

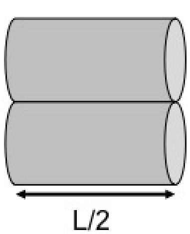
Question 1: If you cut the cylinder in half so that you had two pieces each of length $L/2$, as shown in the diagram, what would be the resistance of each piece according to Eq. (1)?



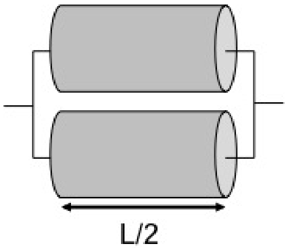
Question 2: If you took a wire and connected the two pieces end-to-end, what is the total resistance according to the addition of resistors in series?

Question 3: Based on your observations, is Eq. (1) consistent with what you know about how resistors add in series? Explain.

Question 4: If you took the two pieces arranged side-by-side as shown and considered them a single object of length $L/2$, what is the total resistance according to Eq. (1)?



Question 5: If you took the two pieces and connected them with wires as shown, what is the total resistance according to the addition of resistors in parallel?

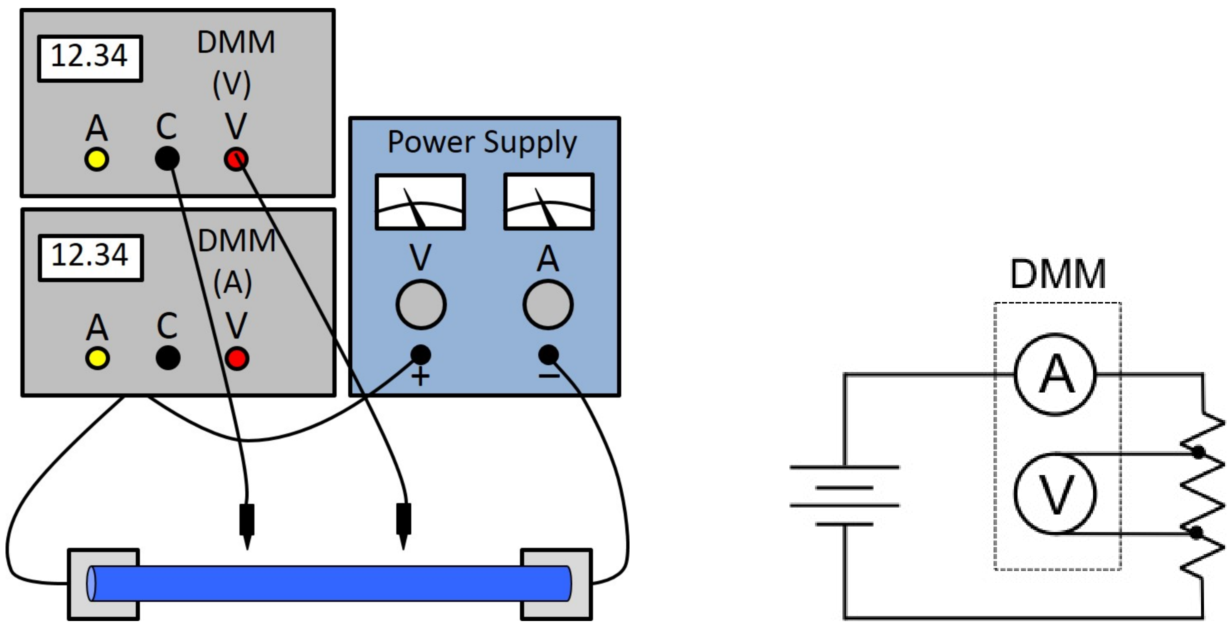


Question 6: Based on your observations, is Eq. (1) consistent with what you know about how resistors add in parallel? Explain.

CHECKPOINT 1: Before moving on to the next part, have your TA check the results you obtained so far.

2 Resistivity of Play-Doh™

You will now investigate the relationship between resistance and resistivity by using Play-Doh™, which can be moulded into numerous geometrical shapes. In particular, you will investigate how the resistance of a cylindrical object depends on its length and its cross-sectional area. To do this, you will use the experimental setup shown in the diagram below.



The equivalent circuit is shown as well. Notice that the digital multimeter (DMM) can act either as an ammeter (A) or a voltmeter (V) but not both at the same time; these separate functions are achieved by selecting the appropriate settings on the DMM. In this experiment you will use two digital multimeters.

Before turning on the power supply during any of the measurements that follow in this labatorial, ensure that your equipment is set up as shown:

1. the positive terminal of the power supply (+) is connected to the positive terminal of the ammeter (A) on the digital multimeter (DMM)
2. the negative terminal of the power supply (−) and the "common" (ground) terminal of the DMM (C) are connected to the two small square aluminum plates with alligator clips
3. the red voltage probe is connected to the positive terminal of the voltmeter on the DMM (V), and the black voltage probe is connected to the "common" (ground) terminal of the DMM (C)
4. the two ends of the Play-Doh™object are squished between the aluminum plates and the table

2.1 Constant Area, Variable Length

First we are going to examine how the resistance of a cylinder depends on length while keeping the cross-sectional area constant. Take the Play-Doh™ from two containers and roll it out into a cylinder of approximately constant diameter that is at least 12 cm long.

Question 7: Use the ruler provided to measure the diameter of your cylinder and record the value here, along with the cross-sectional area:

$D =$

$A =$

Question 8: Squish the ends of the cylinder down. Place the cylinder between the aluminum plates and the table. Now turn the power supply on and use the DMM to measure the current in your circuit (use the dials on the power supply to make sure the current is no greater than 85 mA) and record the value here:

$I =$

Turn the power supply back off. Take the ruler and lay it along the top of your cylinder so that you can mark off regularly spaced length intervals; instead of making equal area cylinders of different lengths, it is easier to use the voltage probes to measure voltage differences for different lengths of a single cylinder.

Please do not leave either probe in the Play-Doh™ for extended periods of time.

Question 9: Where should the zero mark be placed to ensure reliable results? Explain.

Question 10: Now lightly mark off with a pointed object (such as a pen or pencil) the zero mark, and regular intervals 15 mm apart up to a total length of 90 mm; these marks are where the voltage probes will be inserted. Record these length values in the first column of the table below.

Question 11: Turn the power supply back on and use the ammeter to make sure the current hasn't changed from its previous value. Switch to the voltmeter function on the DMM, and place the red probe into the zero mark and the black probe into the 15 mm mark. Measure the voltage difference and record this value in the appropriate place in the table below. Is the reading on the DMM positive or negative? Explain why. What would change if you switched the order of the red and black probes?

Question 12: What we need is the resistance of this 15 mm piece of cylinder. How can you find R from the ΔV that you just measured? Explain why you can do this and record the value in the appropriate place in the table below.

Question 13: Repeat this process for each of the lengths and record the corresponding ΔV and R values in the table below.

ΔL (mm)	ΔV (V)	R (Ω)

Question 14: Open LoggerPro and create a plot of your data in the form of R vs ΔL by putting your ΔL values in the x-column and your R values in the y-column. Does your data form a straight line? Explain why it does or does not based on what you know about the relationship between resistance and resistivity.

Question 15: Fit your data to a straight line and record the value for the slope below. Be careful to include appropriate units

$m_1 =$

2.2 Calculating Resistivty

You will now calculate the resistivity of Play-Doh using the measurements you have taken. Recall that the formula for resistance R as a function of resistivity ρ is given by

$$R = \rho \frac{L}{A}.$$

Question 16: Should the value of ρ change depending on whether you are keeping L or A fixed? Explain why or why not.

Question 17: How is the slope from your data set, m_1, related to the resistivity ρ? If there are geometric quantities involved, identify what they are.

Question 18: Calculate ρ using m_1 and the appropriate value of the relevant geometric quantity. Hint: what was kept constant in the data set?

CHECKPOINT 2: Before moving on to the next part, have your TA check the results you obtained so far.

2.3 Constant Length, Variable Area

Next we are going to examine how the resistance of a cylinder depends on cross-sectional area while keeping the length constant. This requires a bit more work because changing the area means we need cylinders of different diameters. To do this, follow these steps:

1. Take the entire ball of Play-Doh$^{\text{TM}}$(3 containers) and form it into a uniform object that is 15 cm long (this can be a constant diameter cylinder with flattened ends, a brick with constant dimensions and flat sides, etc.)
2. Take the ruler and place marks at 10 mm, 30 mm, 60 mm, and 100 mm
3. Take the edge of the ruler and cut the Play-Doh$^{\text{TM}}$along the marks so that you get five chunks of Play-Doh$^{\text{TM}}$of different sizes
4. Roll each of these chunks out into a uniform diameter cylinder that is approximately 6 cm long

Question 19: Measure the diameter of each of the 5 cylinders using the ruler, and record the values below:

$D_1 =$

$D_2 =$

$D_3 =$

$D_4 =$

$D_5 =$

Question 20: From these diameters, calculate the corresponding cross-sectional areas of each cylinder and record these values in the table below.

Question 21: Place the smallest cylinder onto the aluminum plates and squish the ends down. Now turn the power supply on and use the ammeter and the dials on the power supply to tune the current to 35.0 mA.

Question 22: Using the ruler, lightly mark off an interval of 30 mm on the cylinder. Switch the DMM to the voltmeter and insert the voltage probes into the two marks. Record the value of ΔV in the appropriate place in the table below.

Question 23: Calculate R from the ΔV that you just measured and record the value in the appropriate place in the table below.

Question 24: Repeat this process for each of the cylinders and record the corresponding ΔV and R values in the table below.

A (mm^2)	ΔV (V)	R (Ω)

Question 25: Open LoggerPro and create a plot of your data in the form of R vs A by putting your A values in the x-column and your R values in the y-column. Does your data form a straight line? Explain why it does or does not based on what you know about the relationship between resistance and resistivity.

Question 26: How does the resistance change with increasing diameter?

Question 27: Discuss which factors affected the measurements in the second part of the experiment.

Last Checkpoint! Clean up your area, and put the equipment back the way you found it. Call your TA over to check your work and your area before you can get credit for the labatorial.

Equations and constants

$$
\begin{aligned}
F_C(r) &= \frac{1}{4\pi\epsilon_0}\frac{|q_1 q_2|}{r^2} \\
\vec{E} &= \frac{\vec{F}}{q} \\
\vec{E} &= -\vec{\nabla} V \\
\Delta V &= -\int \vec{E}\cdot d\vec{l} \\
\Delta U &= q\Delta V \\
Q(t) &= Q_{max} e^{-t/RC} \\
Q(t) &= Q_{max}(1 - e^{-t/RC}) \\
C &= \frac{Q}{\Delta V} \\
C &= \epsilon_0 \frac{A}{d} \\
C &= KC_0 \\
U &= \frac{Q^2}{2C} \\
\Delta V &= RI \\
R &= \frac{\rho L}{A} \\
\rho &= \frac{E}{J} \\
\rho_{Cu} &= 1.72 \times 10^{-8}\,\Omega m \\
\rho_{Ag} &= 1.47 \times 10^{-8}\,\Omega m
\end{aligned}
$$

$$
\begin{aligned}
F_g(r) &= G\frac{m_1 m_2}{r^2} \\
U_{grav}(y) &= mgy \\
K &= \frac{1}{2}mv^2 \\
v_x(t) &= v_{0x} + a_x t \\
x(t) &= x_0 + v_{0x}t + \frac{1}{2}a_x t^2 \\
v_x^2(t) &= v_{0x}^2 + 2a_x(x(t) - x_0) \\
\omega &= \frac{d\theta}{dt} \\
v &= \frac{2\pi r}{T} = \omega r \\
a_{rad} &= \frac{v^2}{r} = \omega^2 r \\
g &= 9.81\frac{m}{s^2} \\
G &= 6.67 \times 10^{-11}\frac{Nm^2}{kg^2} \\
\frac{1}{4\pi\epsilon_0} &= 8.99 \times 10^{9}\, Nm^2C^{-2} \\
\epsilon_0 &= 8.85 \times 10^{-12}\, C^2N^{-1}m^{-2} \\
e &= 1.60 \times 10^{-19}\, C \\
m_e &= 9.11 \times 10^{-31}\, kg \\
m_p &= 1.67 \times 10^{-27}\, kg \\
m_n &= 1.67 \times 10^{-27}\, kg
\end{aligned}
$$