Electricity and Magnetism

- Physics 259 – L02
- Lecture 35

Chapter 27

Last time

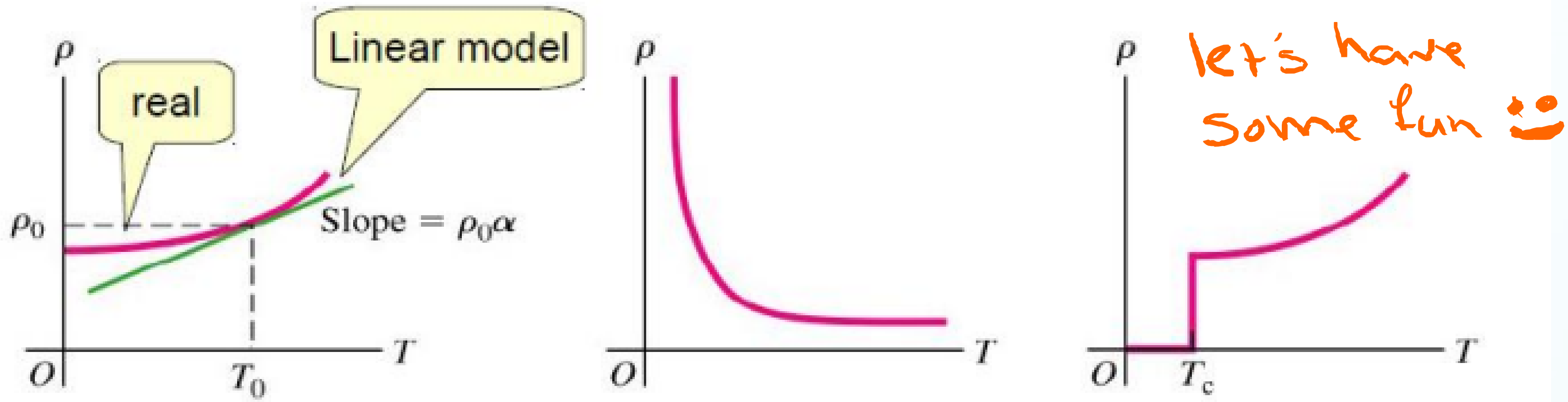
- Chapters 26 and 27

This time

- Solve some examples

27 Circuits:
continue of last section

Resistivity and temperature



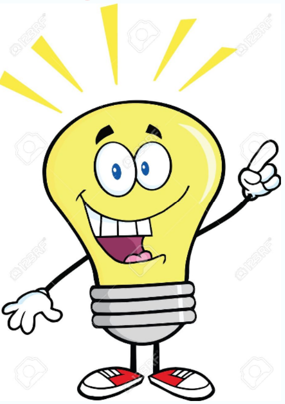
(a) Metal: ρ increases with increasing T

(b) Semiconductor: ρ decreases with increasing T

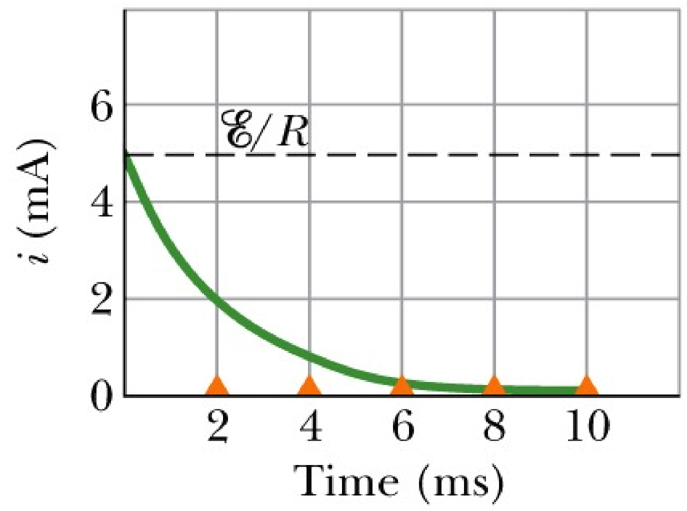
(c) Superconductor: $\rho = 0$ for $T < T_c$

https://www.youtube.com/watch?v=hTT6nktJQBE

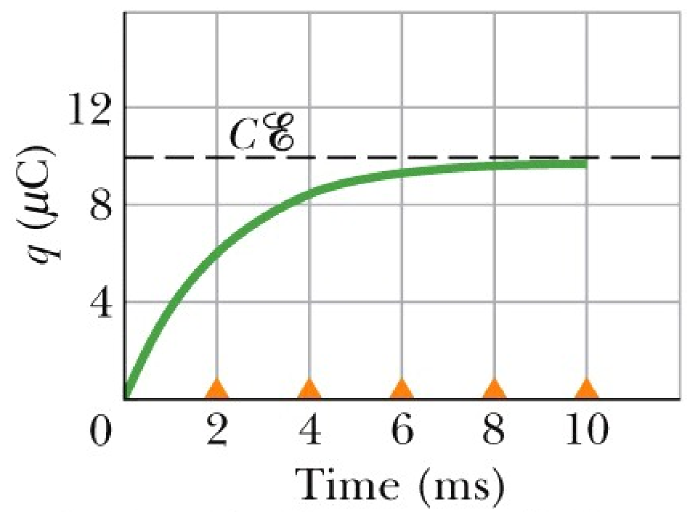
Case 1: Charging a capacitor



$$i = i_0 e^{-t/RC}$$

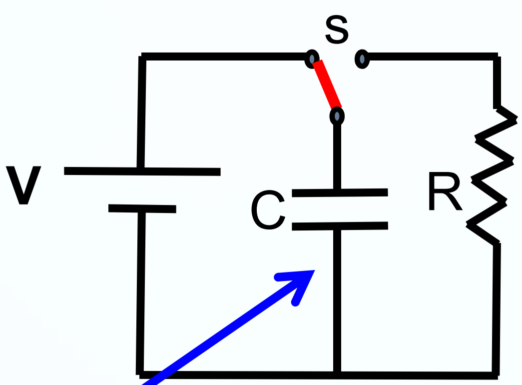


$$q = \varepsilon C\left(1 - e^{-t/RC}\right) = Q_f\left(1 - e^{-t/RC}\right)$$



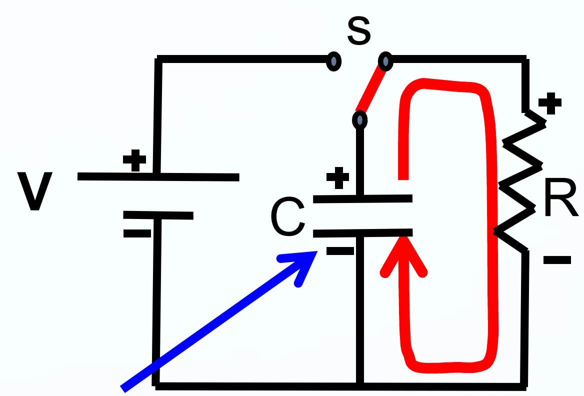
Case 2: Discharging a capacitor

Switch is connected to the left for a long time until t=0-



Capacitor charges up to voltage V

Switch is suddenly flipped to the right at t=0+

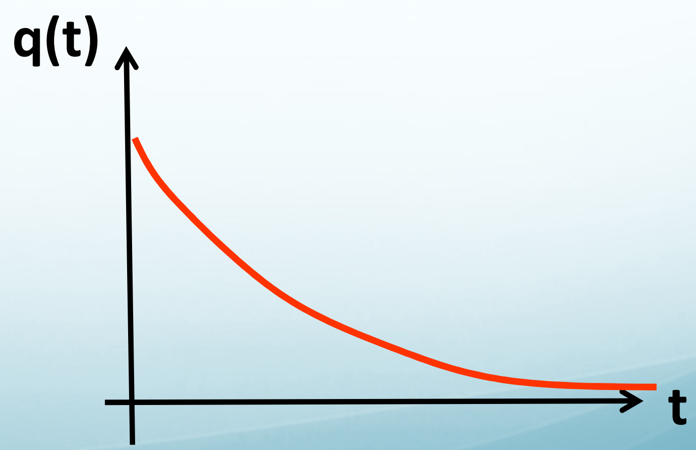


Capacitor discharges

$$\boxed{q(t) = q_0 e^{-t/RC}}$$

$$\boxed{i(t) = i_0 e^{-t/RC}}$$

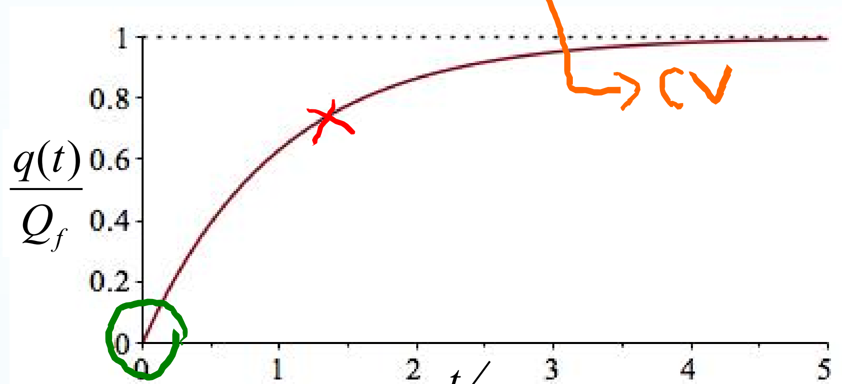
$$q_0 = CV$$



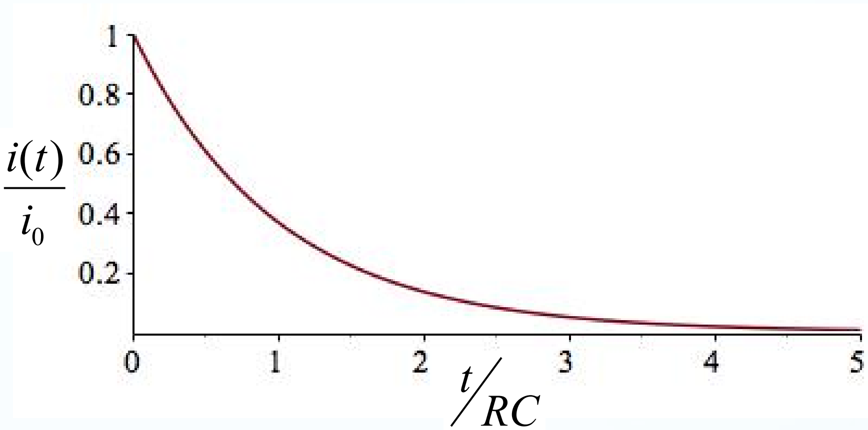
Charging/Discharging Capacitors

Charging: $q(t) = Q_f\left(1 - e^{-\frac{t}{RC}}\right)$ $\quad i(t) = i_0 e^{-\frac{t}{RC}}$

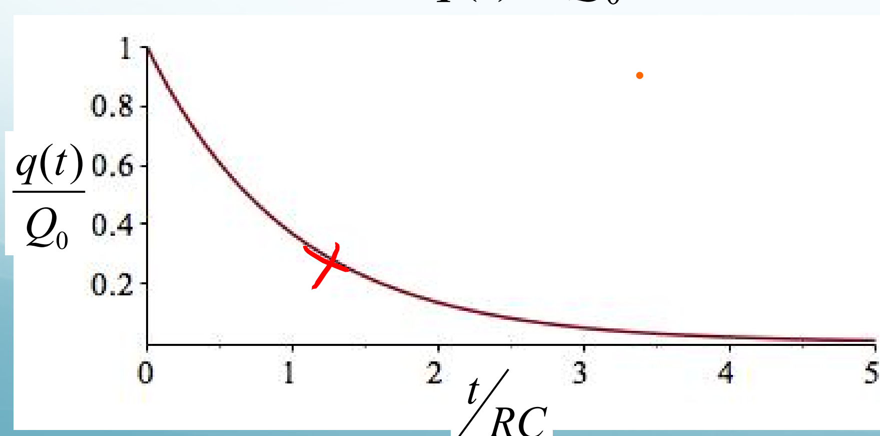
→ CV

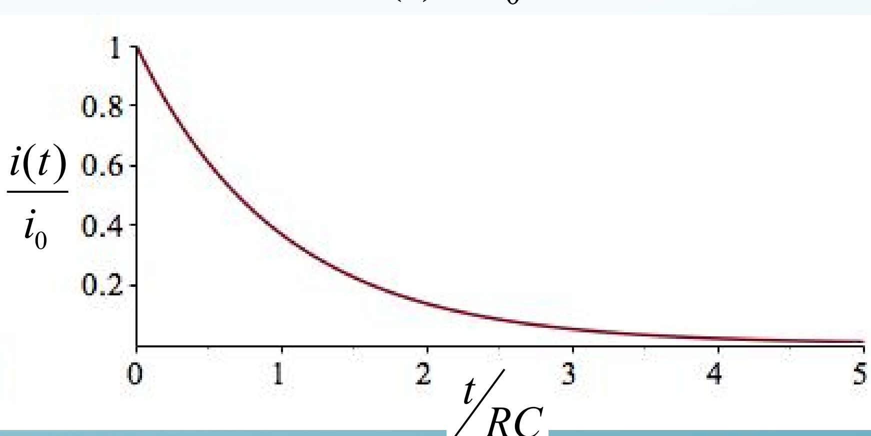


$t = 0^+ \to q = 0$



Discharging: $q(t) = Q_0 e^{-\frac{t}{RC}}$ $\quad i(t) = i_0 e^{-\frac{t}{RC}}$

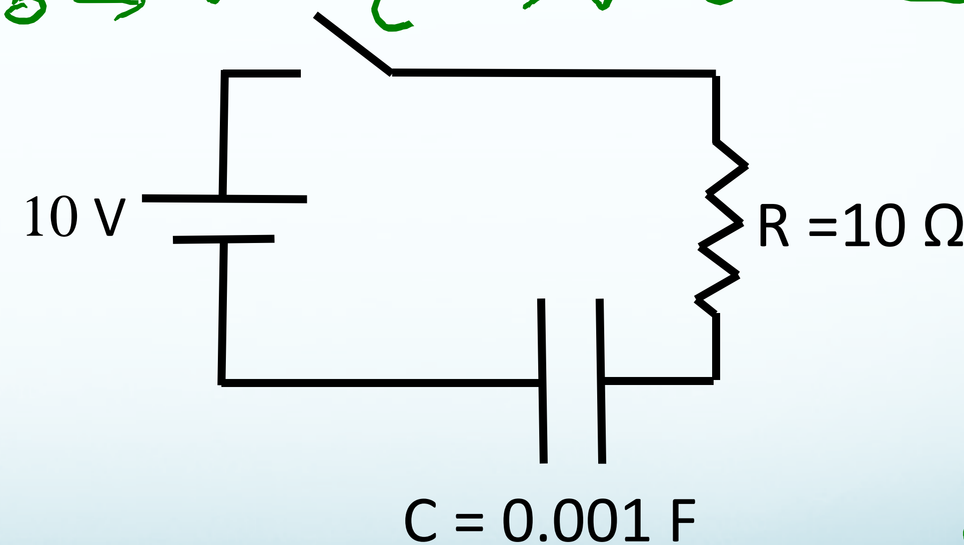




Top Hat Question

An RC circuit is shown below. Initially the switch is open and the capacitor is uncharged. At time t=0, the switch is closed. What is the voltage across the capacitor *immediately* after the switch is closed (time = 0+)?

at $t=0^+ \rightarrow q=0 \rightarrow V=\frac{q}{C} \rightarrow V=0$



A. 0.0 V
B. 10 V
C. 5.0 V
D. 1.0 V

Hint $\rightarrow C=\frac{q}{V} \rightarrow V=\frac{q}{C} \rightarrow$ what is q at $t=0+$?

The RC time constant

The constant RC pops up in the exponential factor for both charging and discharging capacitors. What does it represent?

The units of RC is seconds: $$[RC] = \frac{V}{A}\frac{C}{V} = \frac{C}{C/s} = s$$

We call RC the "**RC time constant**" and it tells us how quickly a capacitor can charge or discharge.

$$\boxed{RC \equiv \tau} \rightarrow \text{RC time constant}$$

After a time τ, the charge on a discharging capacitor is reduced by a factor of 1/e. After a time $N\tau$, it is reduced by a factor of $1/e^N$

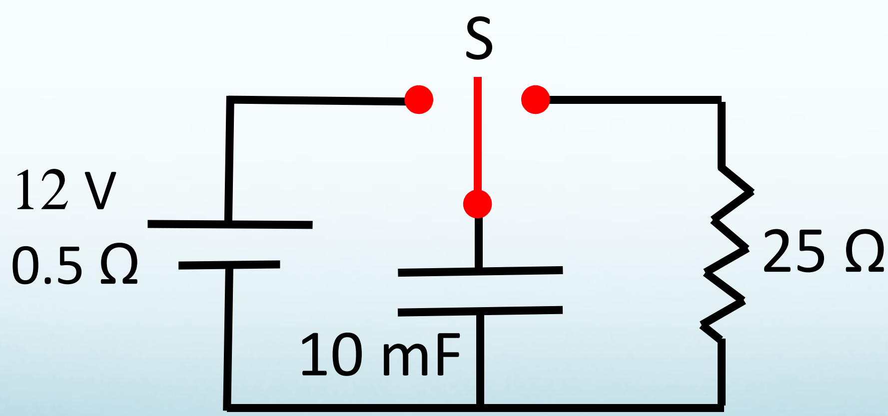
$$\boxed{q(t) = Q_0 e^{-\frac{t}{\tau}}}$$

$$q(t) = Q_0 e^{-t/\tau}$$

Sample question

An RC circuit is shown below. Initially the switch is open and the capacitor is uncharged. At time t = 0 s, the switch is thrown to the left, connecting the capacitor to the battery. At time t = 15 ms the switch is thrown to the right, connecting the capacitor to the resistor.

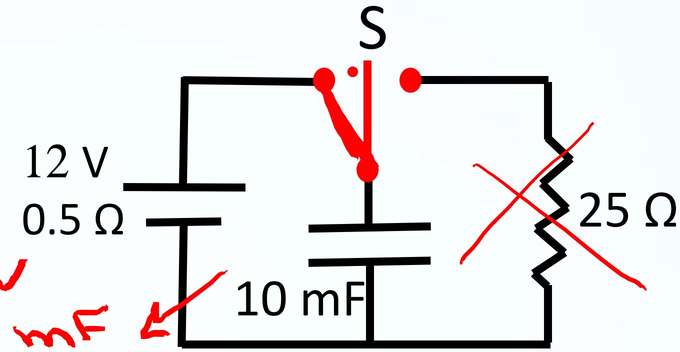
1) How much charge builds up on the capacitor while it is connected to the battery?
2) What is the voltage across the resistor as a function of time as the capacitor discharges?
3) What is the ratio of the charging time to discharging time?



$1 \to q(t) = Q_f(1 - \ldots)$

$Q_f = CV$

Initially the switch is open and the capacitor is uncharged. At time t = 0 s, the switch is thrown to the left, connecting the capacitor to the battery. At time t = 15 ms the switch is thrown to the right, connecting the capacitor to the resistor.



$V = 12\,V$

$C = 10\,mF$

$R = 0.5\,\Omega$

1) How much charge builds up on the capacitor while it is connected to the battery?

charging capacitor → $q(t) = Q_f\,(1 - e^{-t/RC})$

Q_f → fully charged capacitor → $Q_f = CV = 10\,mF \times 12 = \boxed{120\,mC}$

$\tau = RC = 0.5\,\Omega \times 10\,mF = 5\,ms$

→ at time $t = 15\,ms$ → $q(15\,ms) = Q_f\,(1 - e^{-t/RC})$

→ $q(t) = 120\,mC\,(1 - e^{-15/5}) = 114\,mC$

→ $q(15\,sec) = 114\,mC$

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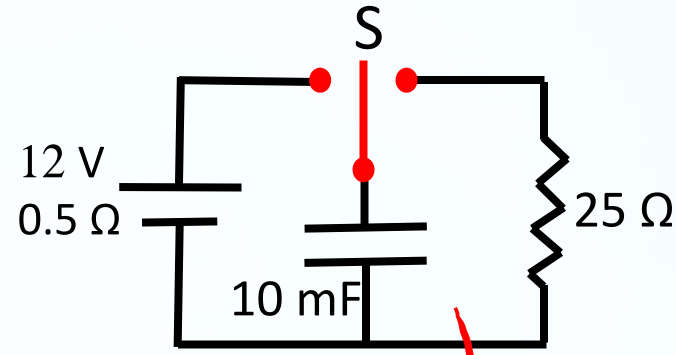

2) What is the voltage across the resistor as a function of time as the capacitor discharges?

$q_0 = 114\ mC$

$R = 25\ \Omega$

$C = 10\ mF$

discharging capacitor $\Rightarrow$

$$V = R\,I(t) = R\,I_0 e^{-t/RC} \qquad I_0 = ?$$

$$I(t) = -\frac{d}{dt} q(t) = -\frac{d}{dt} q_0 e^{-t/RC} \rightarrow I(t) = -q_0 \left(\frac{-1}{RC}\right) e^{-t/RC}$$

$$\rightarrow I(t) = \frac{q_0}{RC} \cancel{e^{-t/RC}} = I_0 \cancel{e^{-t/RC}} \Rightarrow I_0 = \frac{q_0}{RC}$$

$$\rightarrow V = R I_0 e^{-t/RC} = \cancel{R} \frac{q_0}{\cancel{R}C} e^{-t/RC} = \frac{q_0}{C} e^{-t/RC}$$

$$RC = 25 \times 10\,mF = 250\ ms$$

$$\rightarrow V = (11.4\ V)\, e^{-t/0.25}$$

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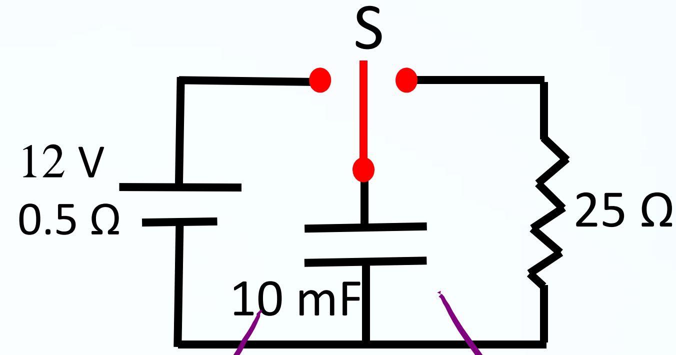


3) What is the ratio of the charging time to discharging time?

τ_{ch} τ_{dis}

$$\tau_{charging} = rC$$

$$\tau_{discharging} = RC$$

$$\Rightarrow \frac{\tau_{ch}}{\tau_{dis}} = \frac{r\cancel{C}}{R\cancel{C}} = \frac{r}{R} = \frac{0.5}{25}$$

$$\rightarrow \frac{\tau_{charging}}{\tau_{discharging}} = \frac{1}{50}$$

Top Hat Question

An RC circuit is shown below. Initially the switch is open and the capacitor is fully charged. At time t = 0, the switch is closed.

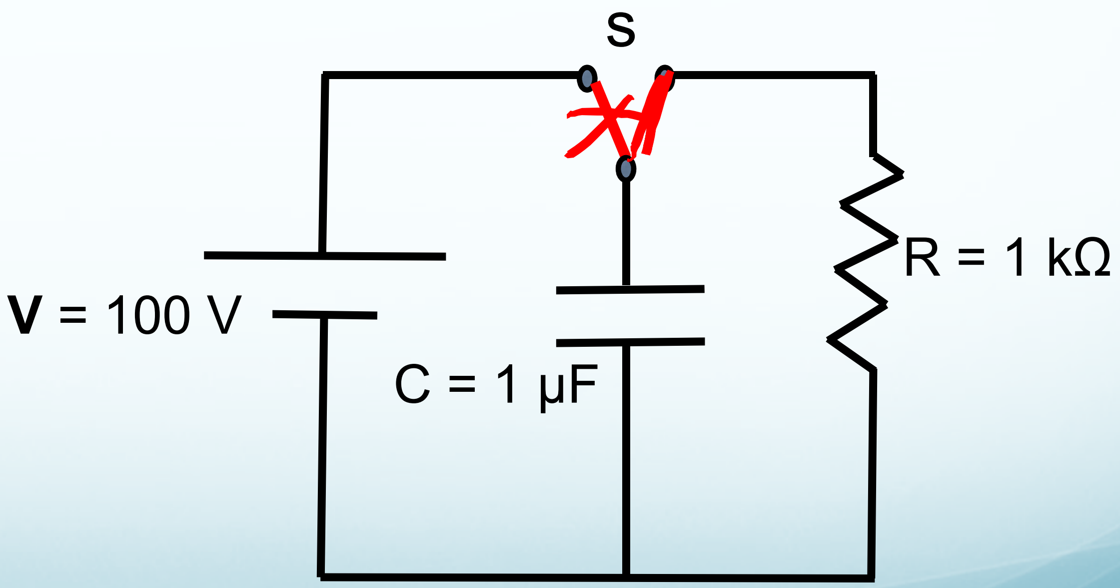
How much charge is left on the capacitor plates after t = 10 ms?

A. 0.67 nC

B. 14 μC

C. 37 μC

D. 4.5 nC



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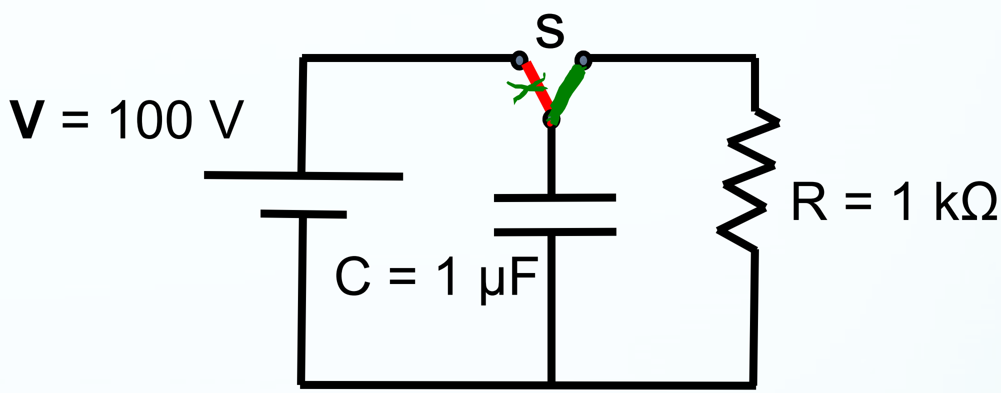

discharging $\rightarrow q(t) = q_0 e^{-t/RC}$

$q_0 \rightarrow q_f$ of charging $\Rightarrow q_0 = CV =$

$V = 100\,V$

$R = 1\,k\Omega$

$C = 1\,\mu F$

$\rightarrow q(t) = (100\,\mu C)\, e^{-\frac{10}{1\,ms}} = 4.54\,nC$

This section we talked about:

Chapter 27

See you on Friday