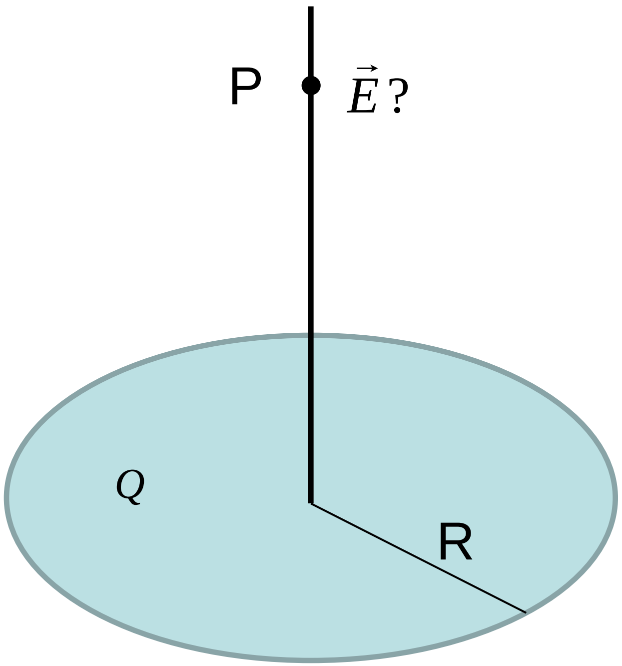
Last time

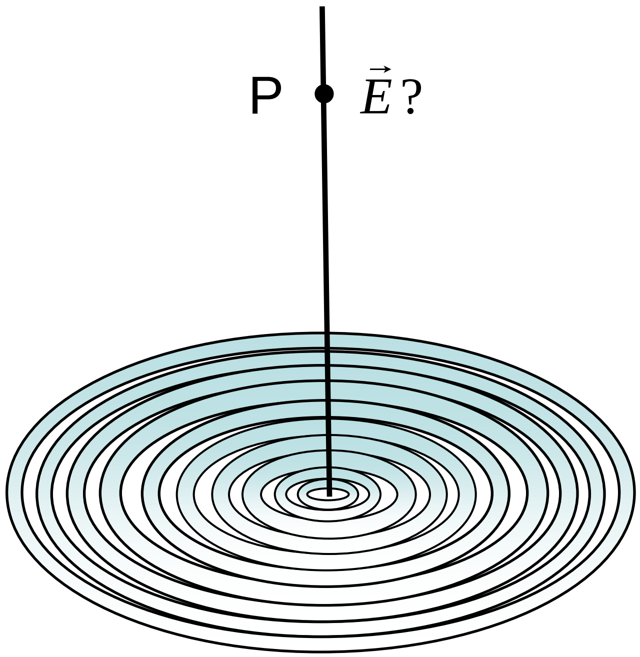
- Electric field of a thin charged ring at a point along the axis of the ring
- Activity #3

This time

- Electric field of a thin charged disk at a point along the axis of the disk
- Electric field of a dipole on its axis
- Electric field vectors and lines for a point charge

Infinitely thin charged disk



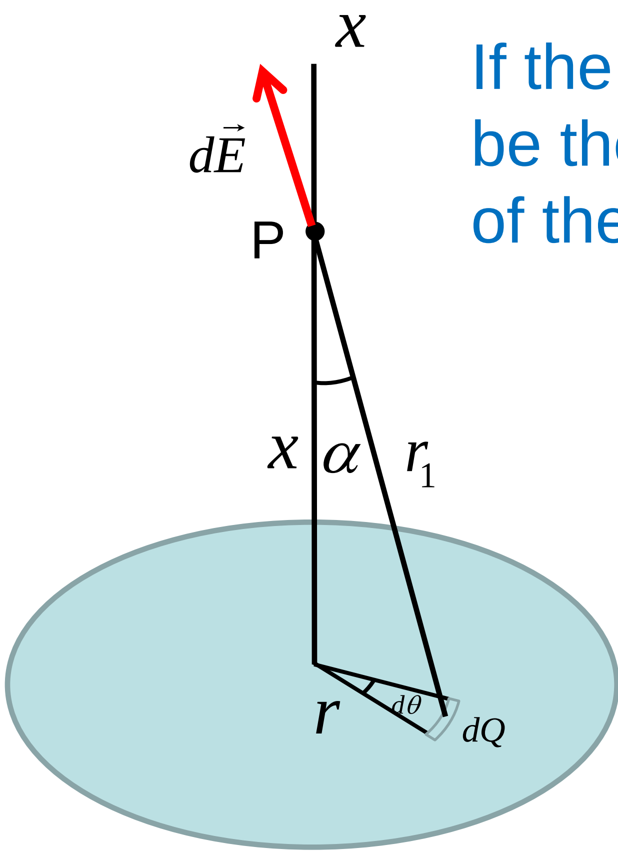


Step 1: A disk is essentially an infinite collection of infinitely thin rings.

Step 2: From previous lecture we learned that the electric field at a point on the axis of an infinitely thin ring is directed along its axis.

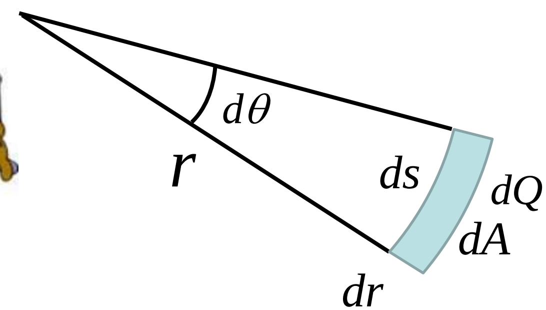
Step 3: By superposition principle the electric field for an infinitely thin disk is also along the axis of the disk.

Infinitely thin charged disk



If the axis of the disk is chosen to be the x-axis, only the x-component of the electric field is non-zero.

$$E_y = 0, E_z = 0$$



$$dA = dsdr = rd\theta dr$$

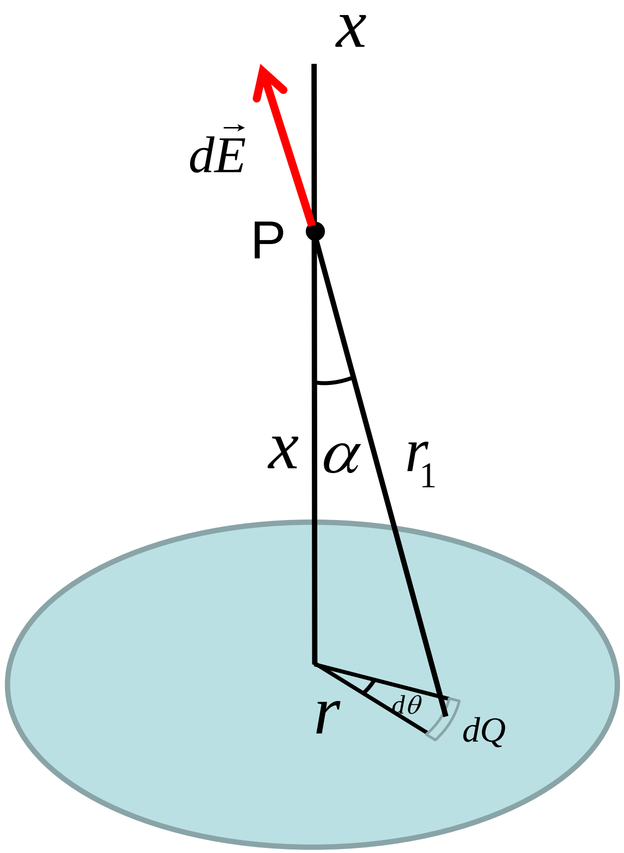
$$r_1^2 = x^2 + r^2$$

$$\sigma = \frac{Q}{\pi R^2}$$ Surface charge density

$$\cos\alpha = \frac{x}{r_1} = \frac{x}{\left(x^2 + r^2\right)^{1/2}}$$

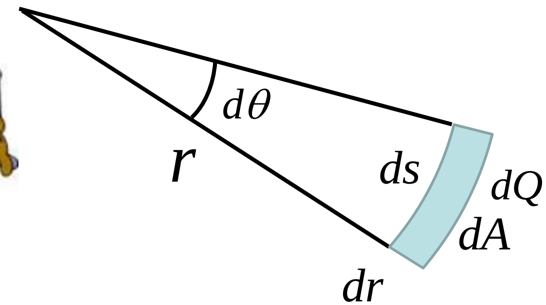
$$dQ = \sigma dA = \sigma rd\theta dr$$

Infinitely thin charged disk



$$dQ = \sigma dA = \sigma r d\theta dr$$

$$\cos\alpha = \frac{x}{\left(x^2 + r^2\right)^{1/2}} \qquad r_1^2 = x^2 + r^2$$

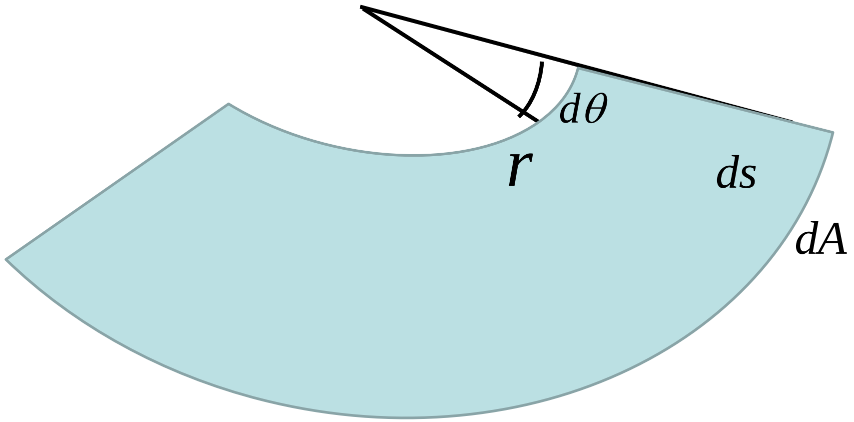


$$dE_x = dE\cos\alpha = k_e \frac{dQ}{r_1^2}\cos\alpha = k_e \frac{\sigma x r dr d\theta}{\left(x^2 + r^2\right)^{3/2}}$$

To calculate the net electric field for all of the charge on the disk we must integrate over θ from zero to 2π and over r from 0 to R.

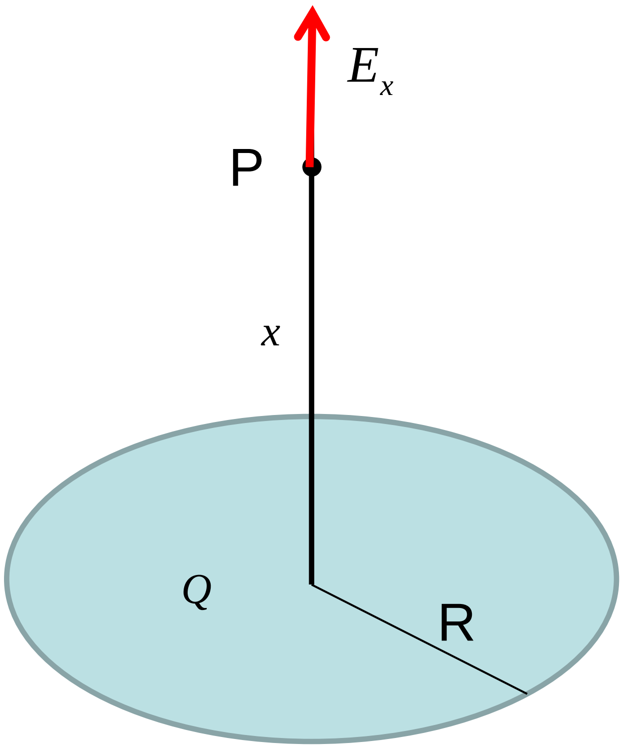
We say that θ and r are independent variables, changing one doesn't affect the value of the other.

A disk of radius R can be constructed from the strip shown below by independently increasing θ from zero to 2π and r from 0 to R.



We say that θ and r are independent variables, changing one doesn't affect the value of the other.

$$E_x = \int_{r=0}^{R} \int_{\theta=0}^{2\pi} dE_x = \int_{r=0}^{R} \int_{\theta=0}^{2\pi} k_e \frac{\sigma x r dr d\theta}{\left(x^2 + r^2\right)^{3/2}} = k_e \sigma x \int_{r=0}^{R} \frac{r dr}{\left(x^2 + r^2\right)^{3/2}} \int_{\theta=0}^{2\pi} d\theta$$



$$E_x = \frac{1}{4\pi\varepsilon_0} 2\pi\sigma x \int_{r=0}^{R} \frac{r dr}{\left(x^2 + r^2\right)^{3/2}}$$

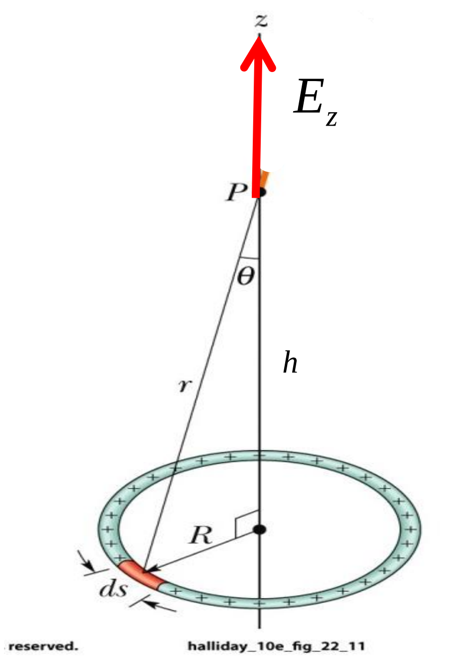
$$E_x = \frac{\sigma x}{2\varepsilon_0} \int_{r=0}^{R} r\left(x^2 + r^2\right)^{-3/2} dr = -\frac{\sigma x}{2\varepsilon_0} (x^2 + r^2)^{-1/2} \Big|_0^R$$

$$E_x = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{x}{\sqrt{x^2 + R^2}}\right) \qquad E_y = 0, E_z = 0$$

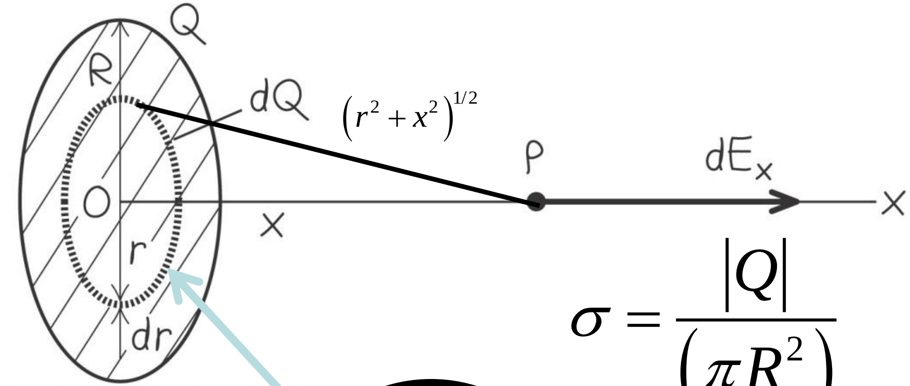
Alternate method

Infinitely thin charged disk

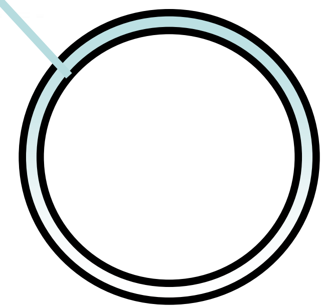
Adapt the electric field expression for the thin ring to the dashed strip on the disk.



$$E_z = k_e \frac{hQ}{\left(R^2 + h^2\right)^{3/2}}$$



$$\sigma = \frac{|Q|}{\left(\pi R^2\right)}$$



$$z-\text{axis} \Rightarrow x-\text{axis}$$

$$E_z \Rightarrow dE_x$$

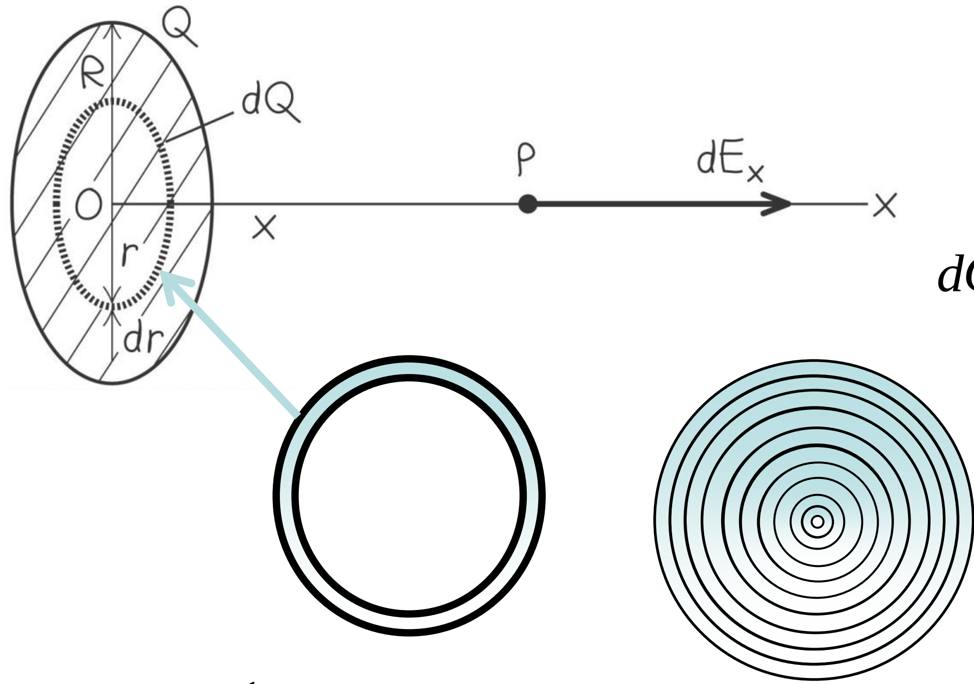
$$Q \Rightarrow dQ \text{ The charge on the strip}$$

$$h \Rightarrow x$$

$$R \Rightarrow r$$

$$dE_x = k_e \frac{x(dQ)}{\left(r^2 + x^2\right)^{3/2}}$$

Infinitely thin disk of charge



$$dE_x = k_e \frac{x\,dQ}{\left(r^2 + x^2\right)^{3/2}}$$

$$dQ = \sigma dA = \underset{\text{Chrage per unit area}}{\sigma} \quad \underset{\text{Inner circumference}}{(2\pi r)} \quad \underset{\text{Thickness}}{dr}$$

Add up the contributions for all the rings for r = 0 to r = R.

$$E_x = \int dE_x = k_e \sigma \pi x \int_0^R \frac{2r\,dr}{(x^2 + r^2)^{3/2}}$$

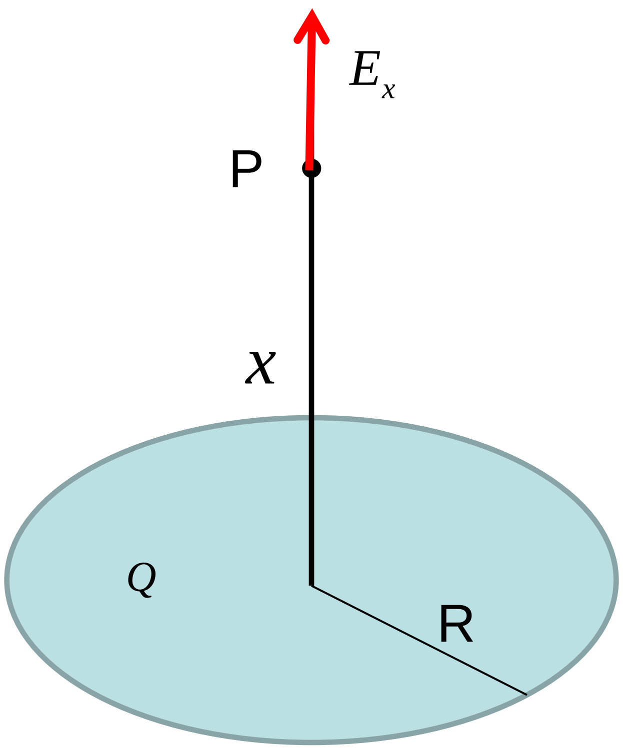
$$k_e = \frac{1}{4\pi\varepsilon_0}$$

$$E_x = \frac{2\sigma\pi x}{4\pi\varepsilon_0} \int_0^R r(x^2 + r^2)^{-3/2} dr = -\frac{\sigma x}{2\varepsilon_0}(x^2 + r^2)^{-1/2}\Big|_0^R$$

$$E_x = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{x}{\sqrt{x^2 + R^2}}\right) \qquad E_y = 0, E_z = 0$$

A large sheet of charge

How would I use the results from a charged disk?



$$E_x = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{x}{\sqrt{x^2 + R^2}}\right)$$

$R \rightarrow \infty$ and more importantly $R >> x$

Note that the question of whether the infinitely large sheet is circular, rectangular, a square or any other shape is irrelevant.

Physics is wonderful?

A large sheet of charge

$$E_x = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{x}{\sqrt{x^2 + R^2}}\right)$$ Result from the charged disk

$$R >> x$$

The second term in the bracket vanishes:

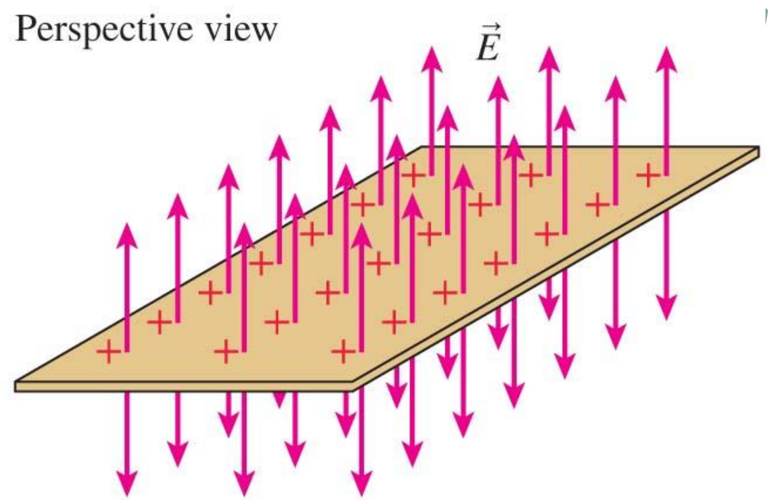
$$\lim_{R\to\infty} \frac{x}{\sqrt{x^2 + R^2}} \to 0$$

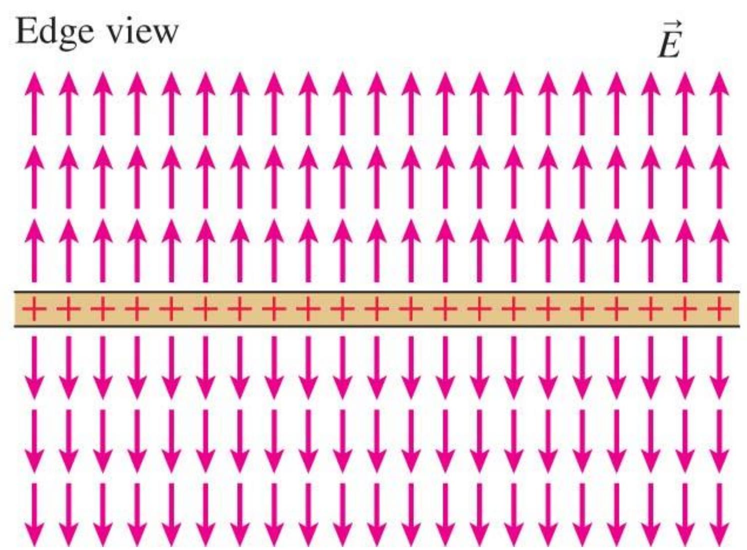
$$E_x = \frac{\sigma}{2\varepsilon_0}$$

Remember x is the axis perpendicular to the sheet.

The direction and magnitude of *E* are constant.

Two views of the electric field of an infinite plane of charge





Very far away from a thin disk of charge

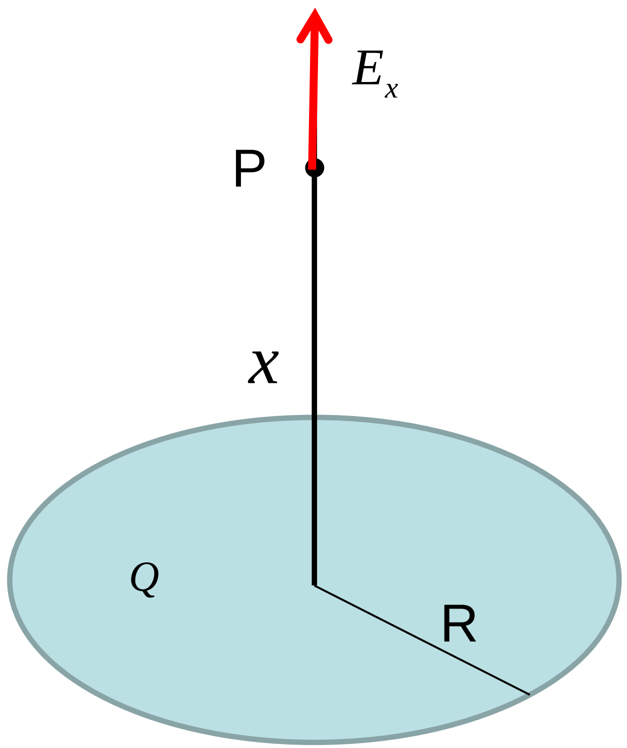
$$x \gg R \rightarrow \infty \text{ or } R/x \ll 1$$

$$E_x = \frac{\sigma}{2\varepsilon_0}\left[1 - x\left(x^2 + R^2\right)^{-1/2}\right]$$

$$E_x = \frac{\sigma}{2\varepsilon_0}\left[1 - \left(1 + \frac{R^2}{x^2}\right)^{-1/2}\right]$$

Use the binomial expansion

$$(1+a)^n \approx 1 + na \text{ For } a < 1.$$

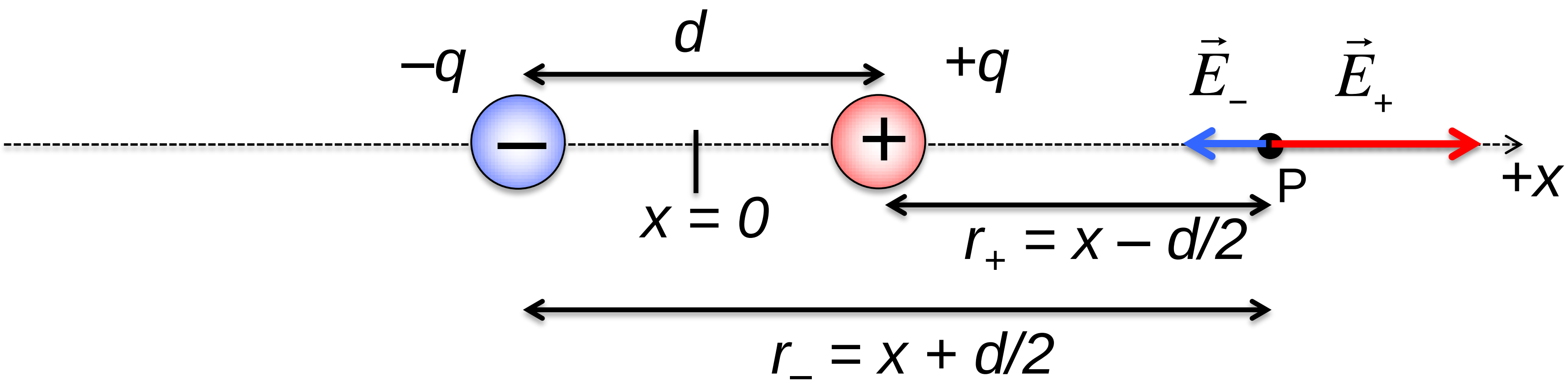


$$E_x \approx \frac{\sigma}{2\varepsilon_0}\left[1 - \left(1 - \frac{1}{2}\frac{R^2}{x^2}\right)\right] = \frac{\sigma}{4\varepsilon_0}\frac{R^2}{x^2} = \frac{\sigma}{4\pi\varepsilon_0}\frac{\pi R^2}{x^2} = \frac{Q}{4\pi\varepsilon_0 x^2}$$

Appears as a point charge Q located at the center of the disk.

Electric field of a dipole along its axis

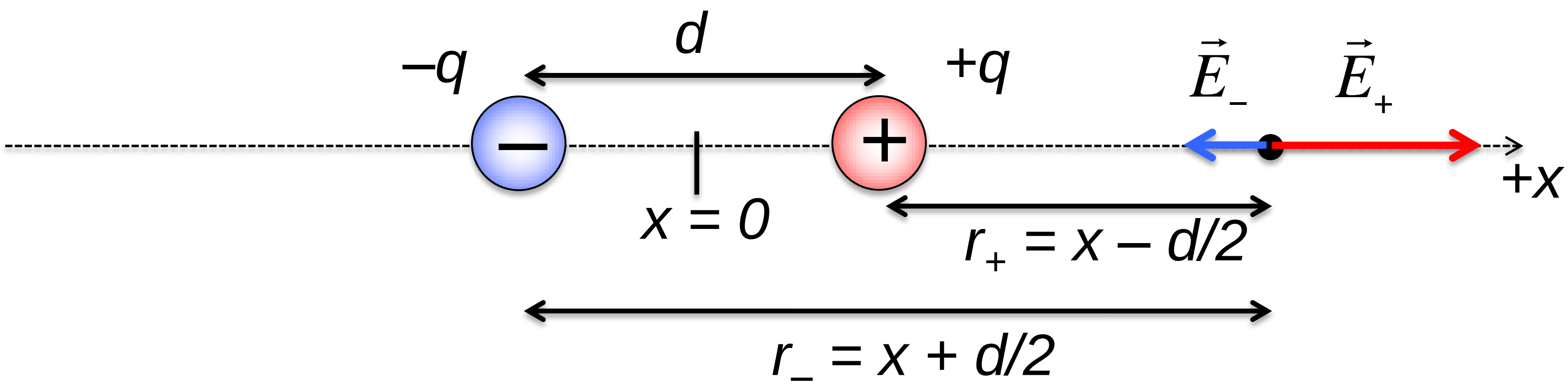
What direction is the electric field at a point P along the axis of an electric dipole?



Step 1: What are the distances r_+ and $r_–$?
Step 2: What are the individual fields E_+ and $E_–$?
Step 3: Use superposition to find the net field E_x.

$$\vec{E}_+ = k_e \frac{q}{r_+^{\,2}}\hat{i} \qquad \vec{E}_- = -k_e \frac{q}{r_-^{\,2}}\hat{i}$$

Electric field of a dipole along its axis



$$E_- = -k_e \frac{q}{(x+d/2)^2}$$

$$E_+ = k_e \frac{q}{(x-d/2)^2}$$

$$E_x = k_e q\left(\frac{1}{(x-d/2)^2} - \frac{1}{(x+d/2)^2}\right)$$

Can simplify this further

Electric field of a dipole along its axis

$$E_x = k_e q\left(\frac{1}{(x-d/2)^2} - \frac{1}{(x+d/2)^2}\right)$$

$$= k_e q\left(\frac{(x+d/2)^2 - (x-d/2)^2}{(x-d/2)^2(x+d/2)^2}\right)$$

(Get a common denominator)

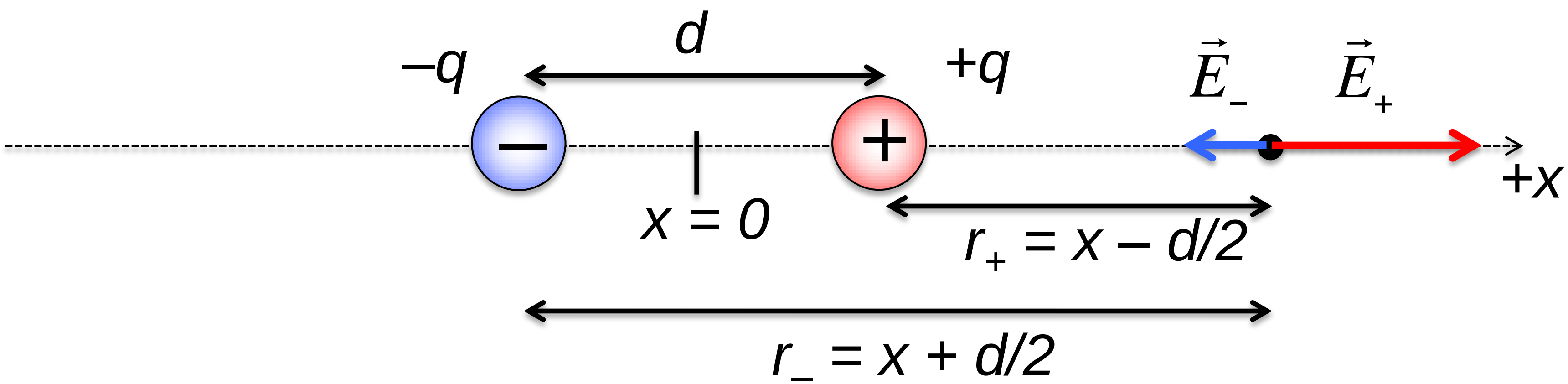
$$= k_e q\left(\frac{(x+\cancel{d/2}+x-\cancel{d/2})(\cancel{x}+d/2-\cancel{x}+d/2)}{(x^2-d^2/4)^2}\right)$$

(expand and cancel)
Use $a^2 - b^2 = (a+b)(a-b)$ in denominator

$$= k_e q\left(\frac{2xd}{(x^2-d^2/4)^2}\right)$$

$$\boxed{E_x = k_e \frac{2(qd)x}{(x^2-d^2/4)^2}}$$

Electric field of a dipole along its axis



$$\boxed{E_x = k_e \frac{2px}{\left(x^2 - d^2/4\right)^2}}$$

Dipole moment: $p \,^{\circ}\, qd$

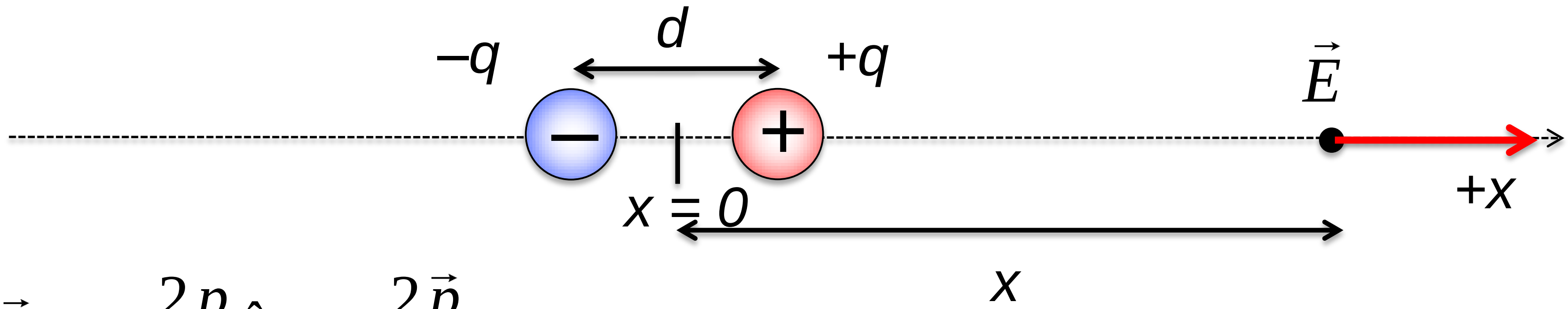
$$\vec{p} \equiv qd\hat{i} = p\hat{i}$$

"perfect dipole": keep *p* fixed but let *d* → 0 (or equivalently x>>d)

$$E_x = k_e \frac{2p}{x^3}$$

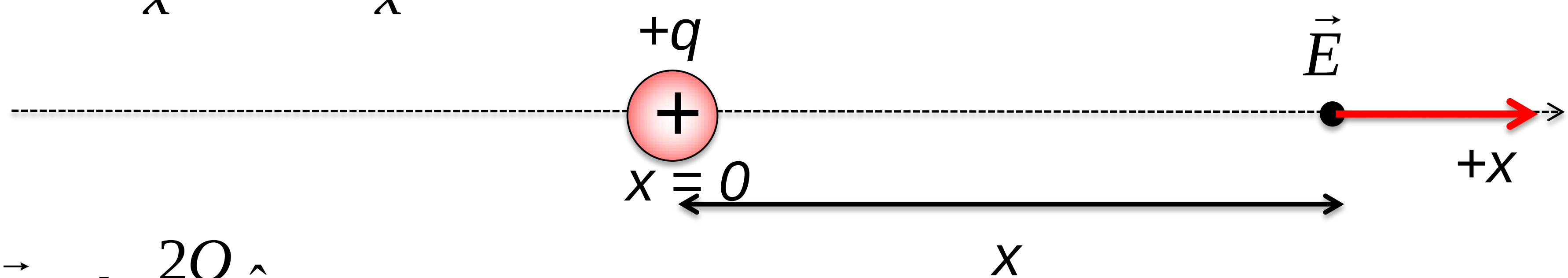
$$\vec{E} = k_e \frac{2p}{x^3}\hat{i} = k_e \frac{2\vec{p}}{x^3}$$

Electric field of a dipole along its axis



$$\vec{E} = k_e \frac{2p}{x^3}\hat{i} = k_e \frac{2\vec{p}}{x^3}$$

When $x >> d$



$$\vec{E} = k_e \frac{2Q}{x^2}\hat{i}$$

A dipole has a 1/*distance*^{3} falloff

A monopole has a 1/*distance*^{2} falloff

Why should I care?

Applications in molecular and material sciences where electric field of a neutral molecule with non-zero dipole moment (typically a size of 1-2 angstroms) is experienced by other molecules located tens of angstroms away. For example, helps in understanding condensation/solvation processes.