Announcements

- Complete Assignment #2 before **11:59 pm, Wednesday, January 25**.

- No laboratorial this week.

Last time

- More on superposition principle
- An example involving four point charges
- Define electric dipole and force due to a dipole
- Line, surface and volume charge density

This time

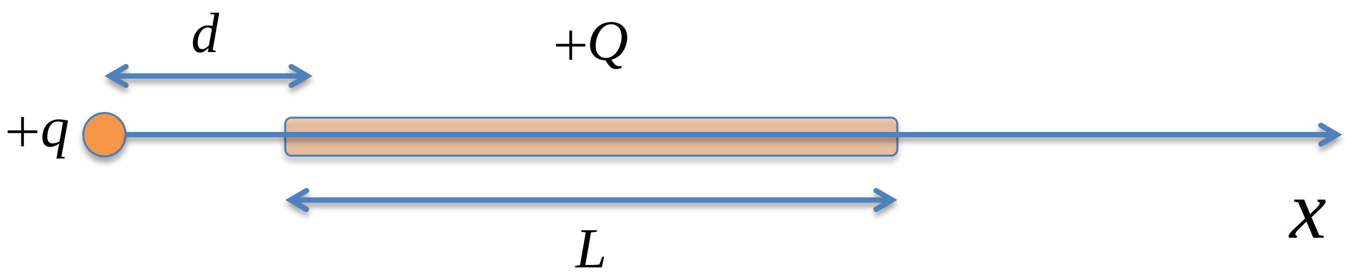
- Coulomb's force due to a line charge, making approximations
- Coulomb's force due to a line charge, exact solution

How to compute Coulomb’s force for a charge distribution?

How to compute Coulomb's force for a charge distribution?

Consider a rod of length *L* with the total charge of +*Q* and a uniform charge distribution.

Compute the force due to the entire rod on the point charge *q* located at the origin.



$$d = 10\ \text{cm},\ L = 40\ \text{cm},\ Q = 10\ \mu\text{C},\ q = 20\ \text{nC}$$

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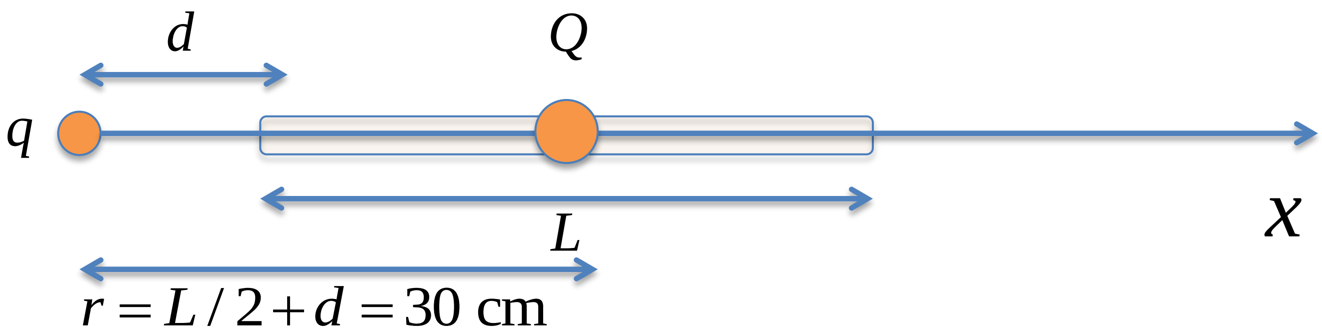


Terrible approximation:

Replace the entire rod by a point charge Q located at the center of the rod and calculate the force in the usual way between two point charges.

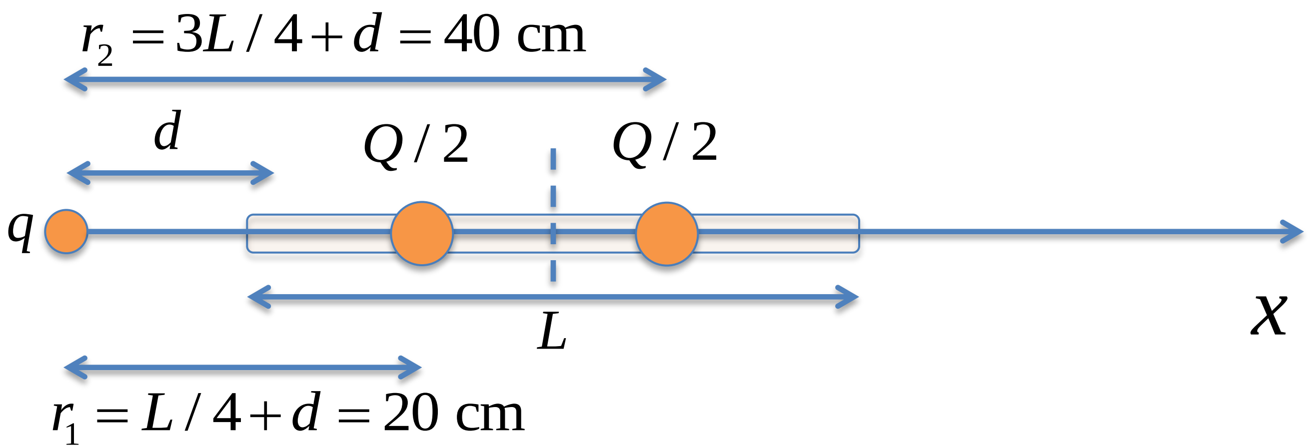
$$\vec{F} = -\frac{k_e qQ}{\left(d + L/2\right)^2}\hat{i}$$

$$F = \left(8.99\times10^9\ \text{Nm}^2/\text{C}^2\right)\frac{\left(10\mu\text{C}\right)\left(20\text{nC}\right)}{\left(0.30\text{ m}\right)^2}$$

$= 2.0\times10^{-2}\ \text{N}$ Off by 80%

Exact answer: $F = 3.6\times10^{-2}\ \text{N}$

$$d = 10 \text{ cm},\ L = 40 \text{ cm},\ Q = 10\ \mu\text{C},\ q = 20 \text{ nC}$$



Awful approximation:

Break the rod in half, then replace each half by a point charge of magnitude *Q*/2.

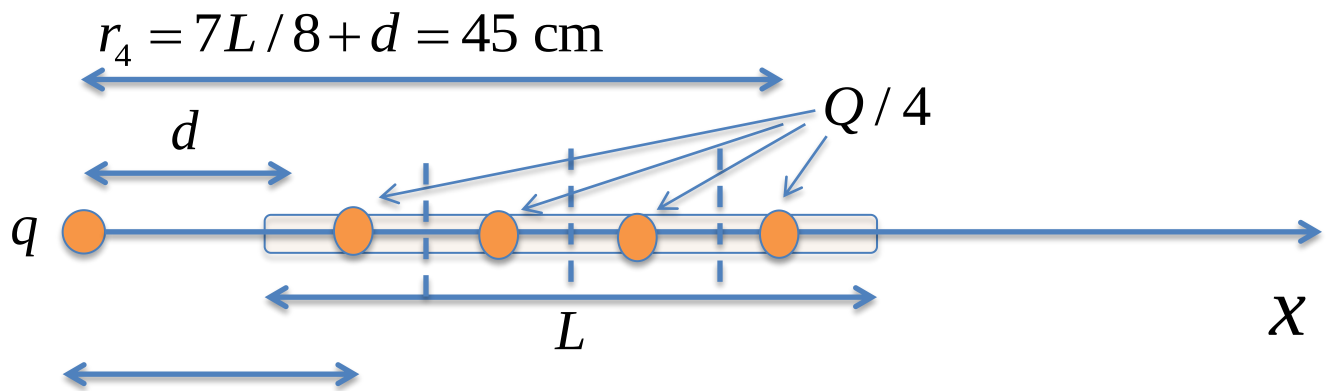
$$F = \frac{k_e q(Q/2)}{r_1^2} + \frac{k_e q(Q/2)}{r_2^2}$$

$$F = \left(8.99\times10^9\ \text{Nm}^2/\text{C}^2\right)(20\text{nC})(5\mu\text{C})\left[\frac{1}{(0.20\text{ m})^2} + \frac{1}{(0.40\text{ m})^2}\right]$$

$= 2.8\times10^{-2}$ N   Off by 28%

Exact answer: $F = 3.6\times10^{-2}$ N

$$d = 10 \text{ cm},\ L = 40 \text{ cm},\ Q = 10\ \mu\text{C},\ q = 20 \text{ nC}$$



$$r_1 = L/8 + d = 15 \text{ cm}$$

Bad approximation:

Break the rod in four equal pieces, then replace each piece by a point charge of magnitude *Q*/4.

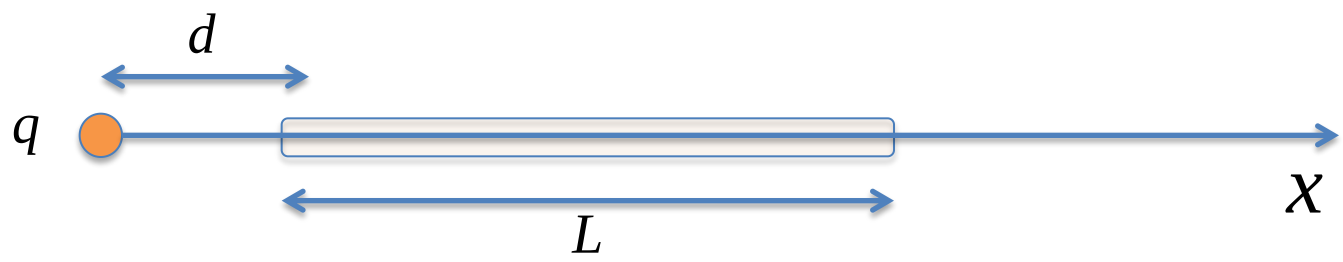
$$F = \frac{k_e q(Q/4)}{r_1^2} + \frac{k_e q(Q/4)}{r_2^2} + \frac{k_e q(Q/4)}{r_3^2} + \frac{k_e q(Q/4)}{r_4^2}$$

$$F = \left(8.99\times10^9 \text{ Nm}^2/\text{C}^2\right)(20\text{nC})(2.5\mu\text{C})\left[\frac{1}{(0.15 \text{ m})^2} + \frac{1}{(0.25 \text{ m})^2} + \frac{1}{(0.35 \text{ m})^2} + \frac{1}{(0.45 \text{ m})^2}\right]$$

$= 3.3\times10^{-2}$ N Off by 9%

Exact answer: $F = 3.6\times10^{-2}$ N

$$d = 10\ \text{cm},\ L = 40\ \text{cm},\ Q = 10\ \mu\text{C},\ q = 20\ \text{nC}$$



Good approximation:

Break the rod in 10 equal pieces, then replace each piece by a point charge of magnitude $Q/10$.

$$F = \frac{k_e q\left(Q/10\right)}{r_1^2} + \frac{k_e q\left(Q/10\right)}{r_2^2} + \frac{k_e q\left(Q/10\right)}{r_3^2} + \ldots + \frac{k_e q\left(Q/10\right)}{r_{10}^2}$$

$$F = \left(8.99\times10^9\ \text{Nm}^2/\text{C}^2\right)\left(20\text{nC}\right)\left(1.0\mu\text{C}\right)\left[\frac{1}{\left(0.12\ \text{m}\right)^2} + \frac{1}{\left(0.16\ \text{m}\right)^2} + \frac{1}{\left(0.20\ \text{m}\right)^2} + \frac{1}{\left(0.24\ \text{m}\right)^2}\right.$$

$$\left. + \frac{1}{\left(0.28\ \text{m}\right)^2} + \frac{1}{\left(0.32\ \text{m}\right)^2} + \frac{1}{\left(0.36\ \text{m}\right)^2} + \frac{1}{\left(0.40\ \text{m}\right)^2} + \frac{1}{\left(0.44\ \text{m}\right)^2} + \frac{1}{\left(0.48\ \text{m}\right)^2}\right]$$

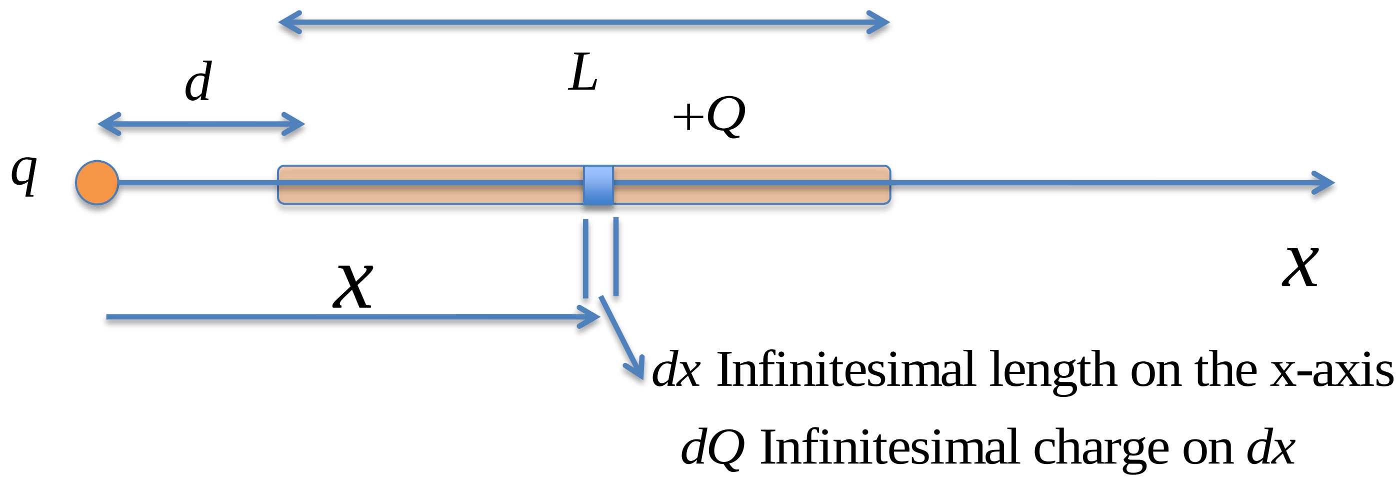
$= 3.54\times10^{-2}\ \text{N}$ Off by 1%

Exact answer: $F = 3.6\times10^{-2}\ \text{N}$

Exact solution

Divide the rod into an infinite number of pieces each with a small amount of charge, then calculate the force for each and add up all the forces.

Exact solution



$\lambda = \dfrac{Q}{L}$ Charge per unit length. $dQ = \lambda dx$

If *dx* is infinitely small, then we can treat it as a point charge and calculate the force on q in the usual way.

$$dF = \frac{k_e q dQ}{x^2} = \frac{k_e q \lambda dx}{x^2}$$

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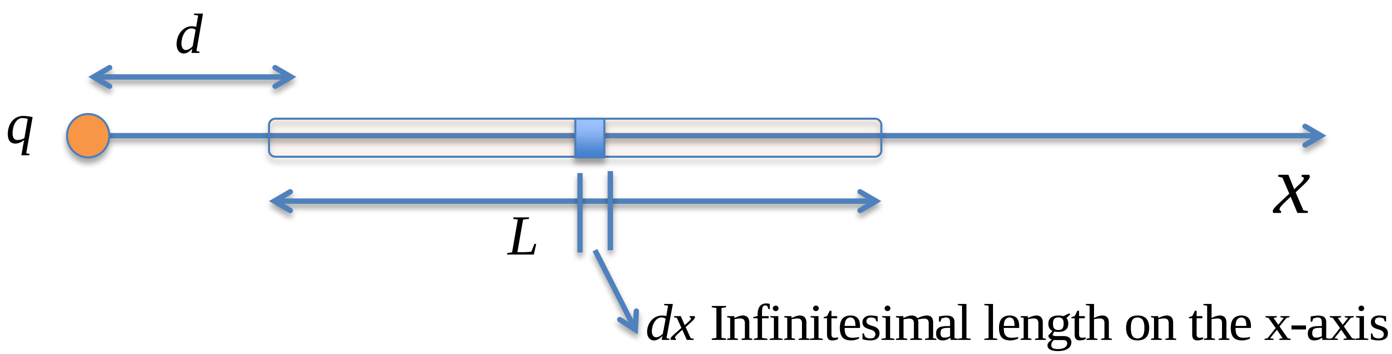
We now sum these infinitesimal forces for the charge distribution from one end of the rod to the other end. This is accomplished by integrating over the charge distribution.



$$\int dF = \int_d^{d+L} \frac{k_e q \lambda dx}{x^2} = k_e q \lambda \int_d^{d+L} \frac{dx}{x^2} = -\left.\frac{k_e q \lambda}{x}\right|_d^{d+L}$$

$$F = -\left.\frac{k_e q \lambda}{x}\right|_d^{d+L} = -\frac{k_e q \lambda}{d+L} + \frac{k_e q \lambda}{d} = k_e q \lambda \left(\frac{1}{d} - \frac{1}{d+L} \right)$$

$$
\begin{aligned}
F &= k_e q \frac{Q}{L}\left(\frac{1}{d} - \frac{1}{d+L}\right) \\
&= k_e q \frac{Q}{L}\left(\frac{d+L-d}{d\left(d+L\right)}\right) \\
&= k_e q \frac{Q}{L}\left(\frac{L}{d\left(d+L\right)}\right) = k_e \frac{qQ}{d\left(d+L\right)} \\
&= \left(8.99\times10^{9}\ \mathrm{Nm}^2/\mathrm{C}^2\right)\frac{\left(10\mu\mathrm{C}\right)\left(20\mathrm{nC}\right)}{\left(0.10\ \mathrm{m}\right)\left(0.50\ \mathrm{m}\right)} \\
&= 3.6\times10^{-2}\,\mathrm{N}
\end{aligned}
$$

$$
\vec{F} = -\left(3.6\times10^{-2}\,\mathrm{N}\right)\hat{i}
$$

When is the terrible approximation a good approximation?

When q is very far from the rod, that is

$$d \gg L$$

$$F_{exact} = k_e \frac{qQ}{d(d+L)} \simeq k_e \frac{qQ}{d(d)} = k_e \frac{qQ}{d^2} = F_{approx}$$

Say $d = 100L$

$$F_{exact} = k_e \frac{qQ}{d(d+L)} = k_e \frac{qQ}{100L(101L)} = k_e \frac{qQ}{10100L^2}$$

$$F_{aprrox} = k_e \frac{qQ}{d^2} = k_e \frac{qQ}{100L(100L)} = k_e \frac{qQ}{10000L^2}$$

$$\frac{F_{exact}}{F_{approx}} == \frac{10000}{10100} = .99$$ Off by 1%