Wednesday April 5, 2017

Last time:

- Applying Ampère’s Law:
  - Magnetic field of solenoid and toroid
- Faraday’s Law of Induction

Today:

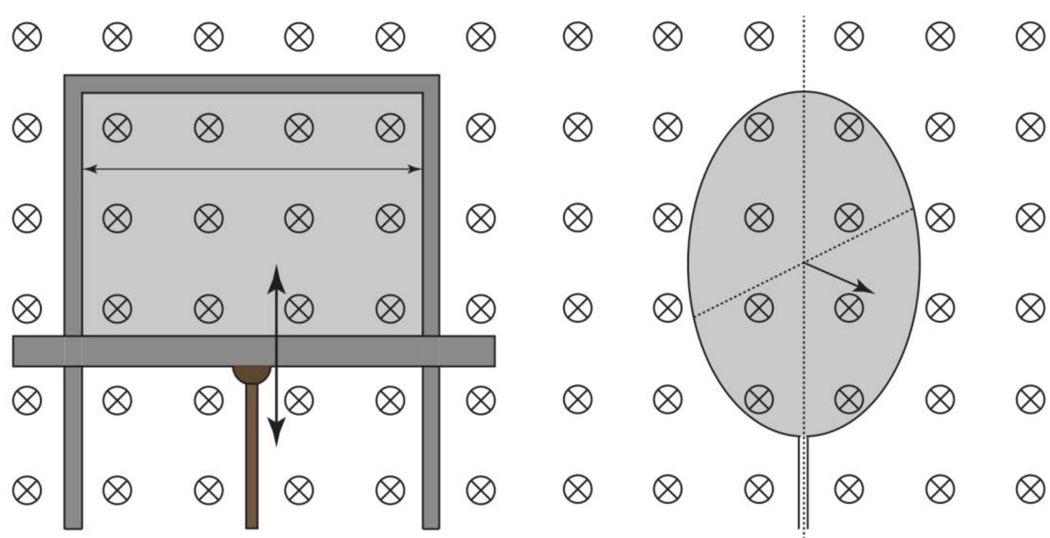
- Lenz’s Law
- Non-conservative electric fields
- Motional emf

This is valid even if Φ_B changes because of a time dependent A or angle ϕ (without changing the magnetic field)!

$$e = -\frac{d}{dt}\left(BA\cos f\right)$$

→ **3 possible terms**

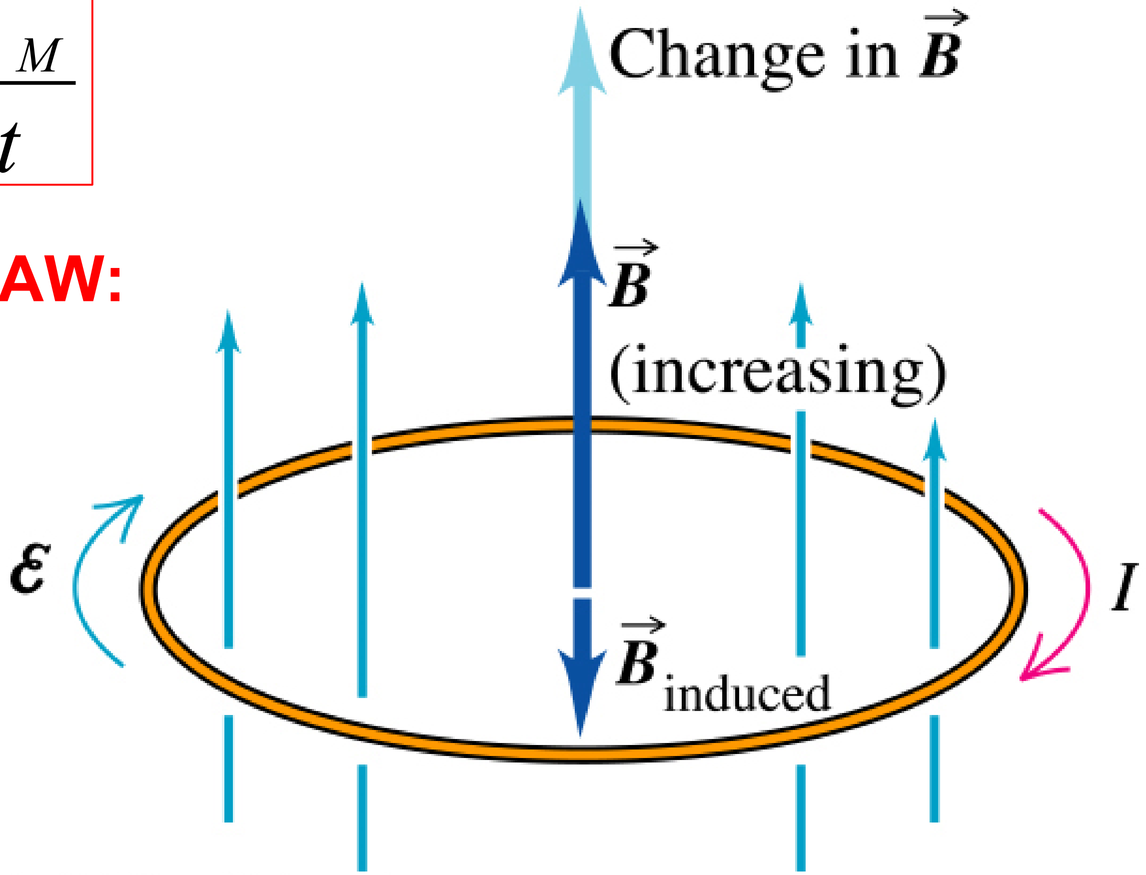
$$e = -\frac{dB}{dt}A\cos f - \frac{dA}{dt}B\cos f + \frac{df}{dt}BA\sin f$$



What about the minus sign in Faraday's law?

$$\boxed{e = -\frac{d\Phi_M}{dt}}$$

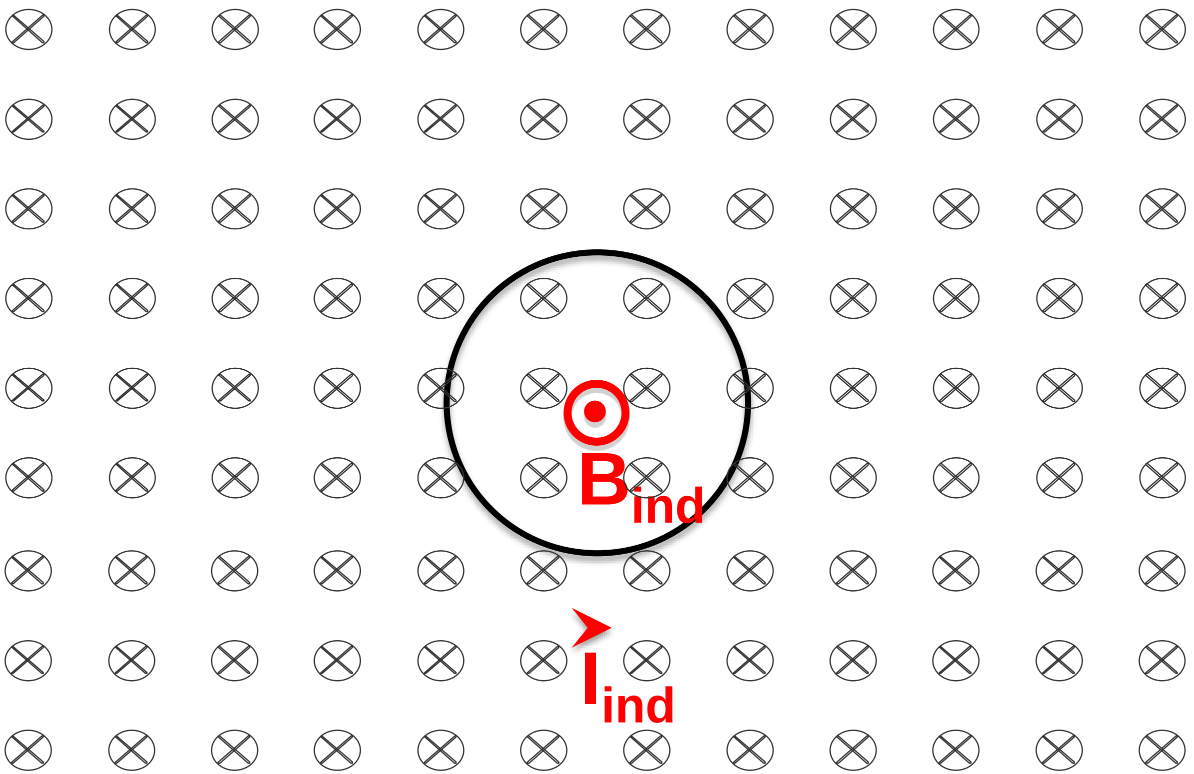
LENZ'S LAW:



The changing magnetic flux generates an induced current which creates an induced magnetic field which, in turn, resists the change in magnetic flux.

Lenz's Law

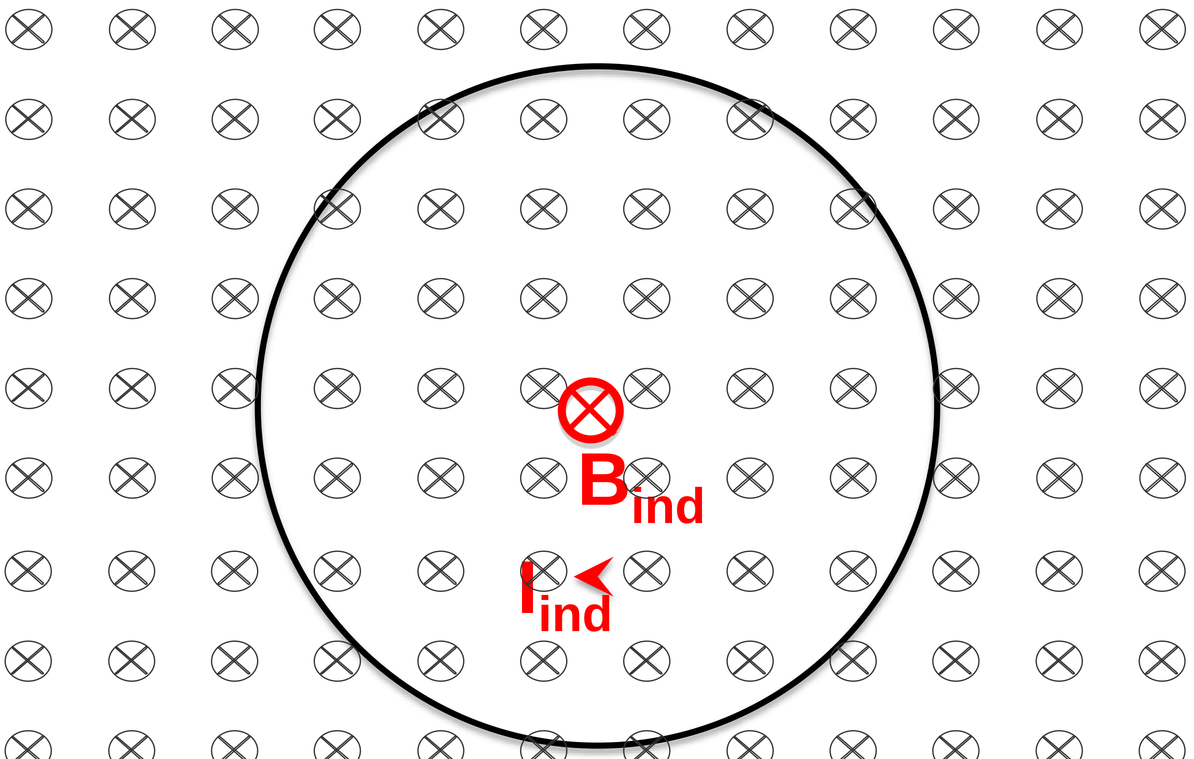
The induced current from Faraday's Law is always in a direction such that the induced magnetic field from the induced current opposes the change in the magnetic flux through the loop.



More B-field lines inside the loop: induced B-field from induced current must be out of the page to compensate.
Induced current is CCW

Lenz's Law

The induced current from Faraday's Law is always in a direction such that the induced magnetic field from the induced current opposes the change in the magnetic flux through the loop.

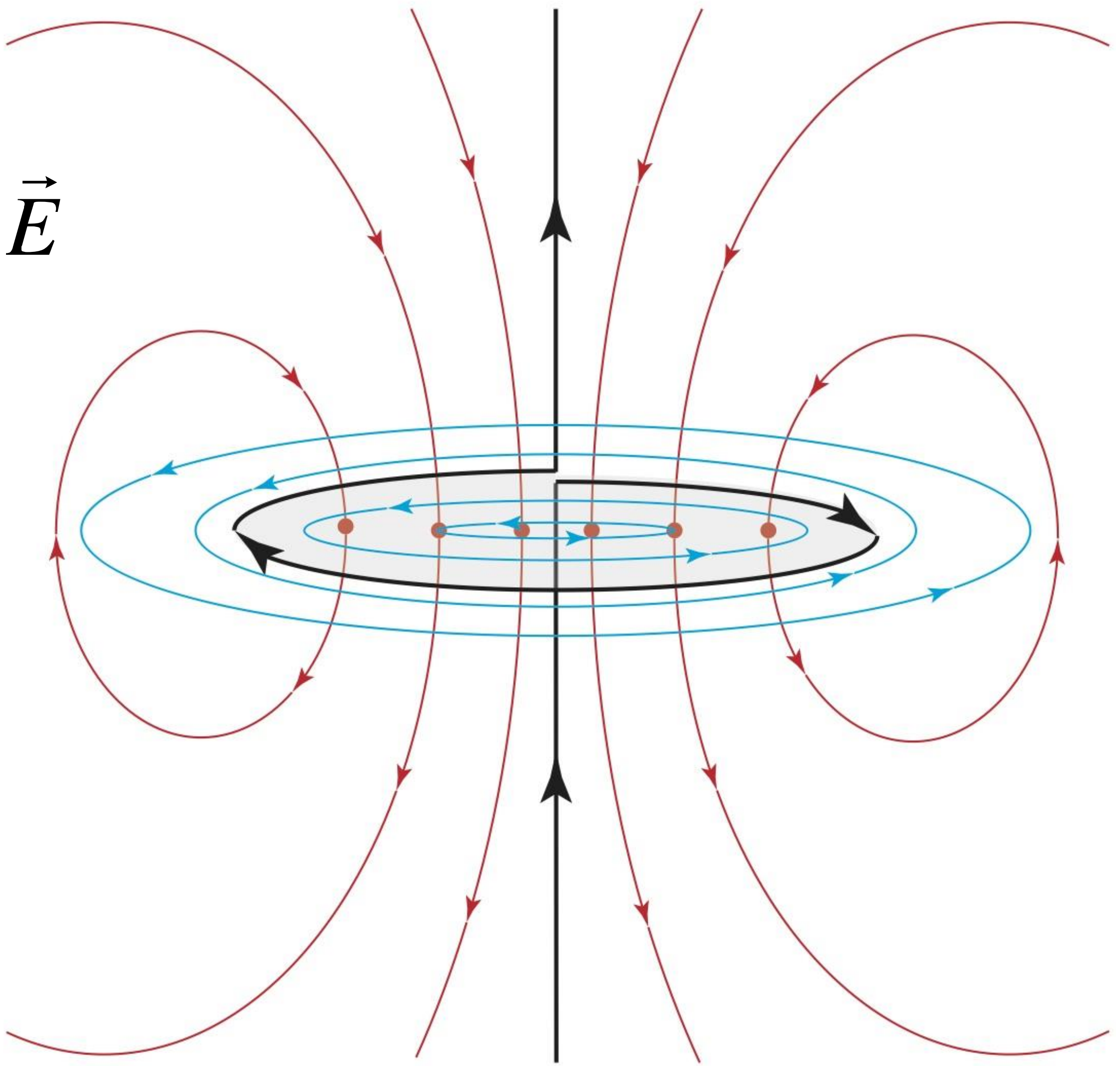


Fewer B-field lines inside the loop: induced B-field from induced current must be into the page to compensate. Induced current is CW

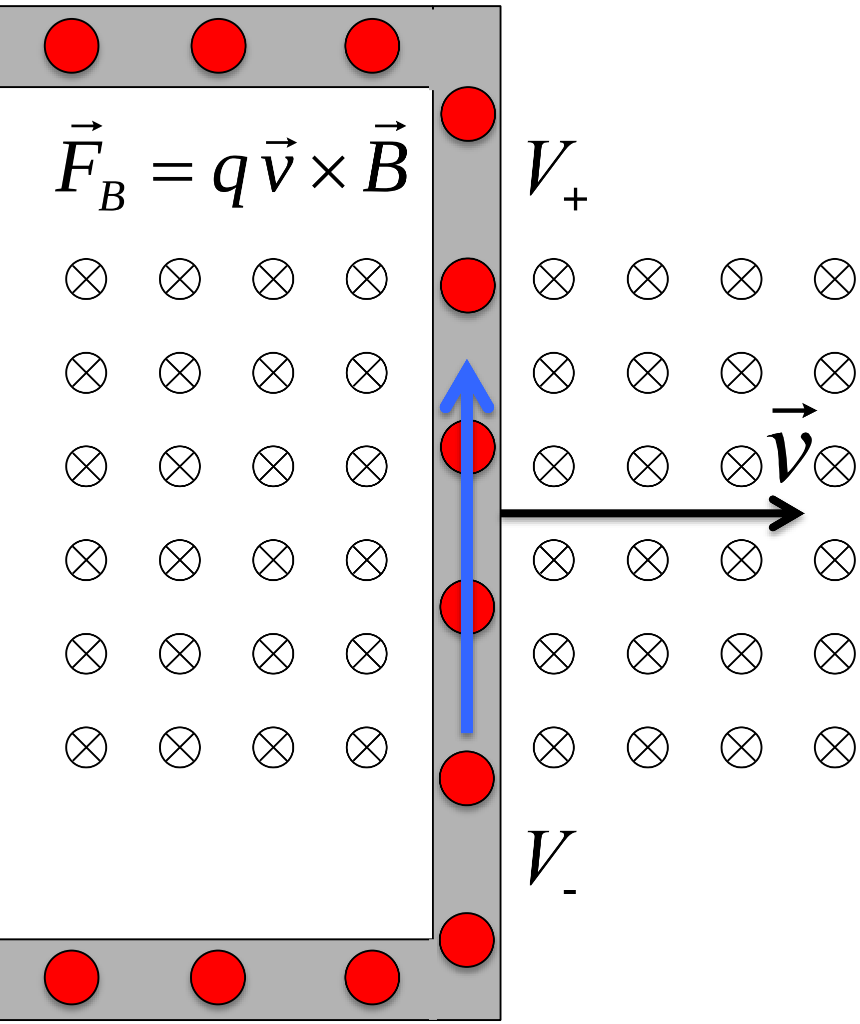
Imagine a loop in a wire carrying a current I_1. The current is then increased to $I_2 > I_1$, increasing the magnetic flux.

Changing B-fields induce non-conservative E-fields.

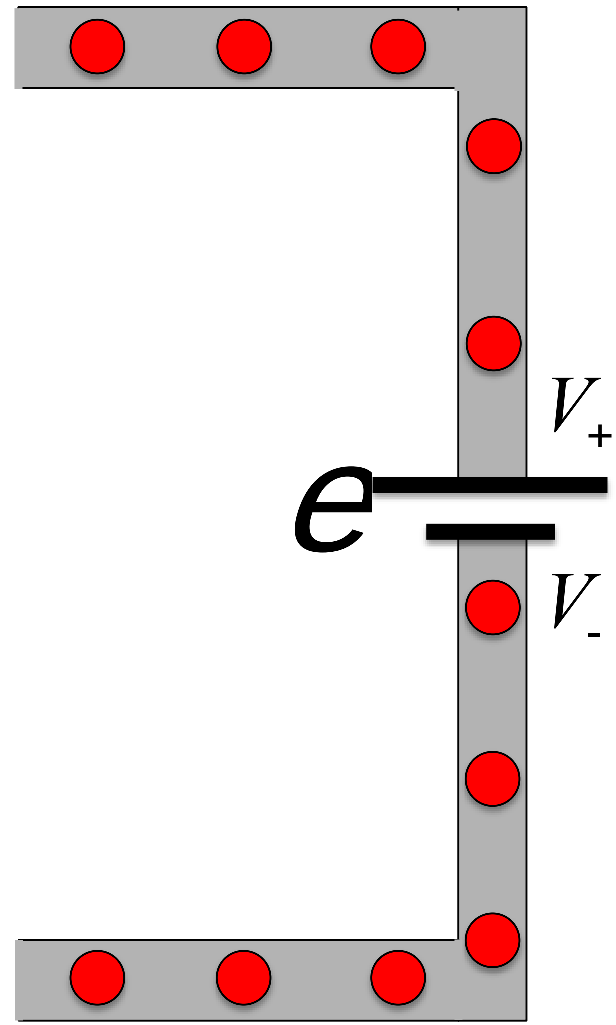
$$-\frac{d\vec{B}}{dt} = \nabla \times \vec{E}$$



Motional EMF

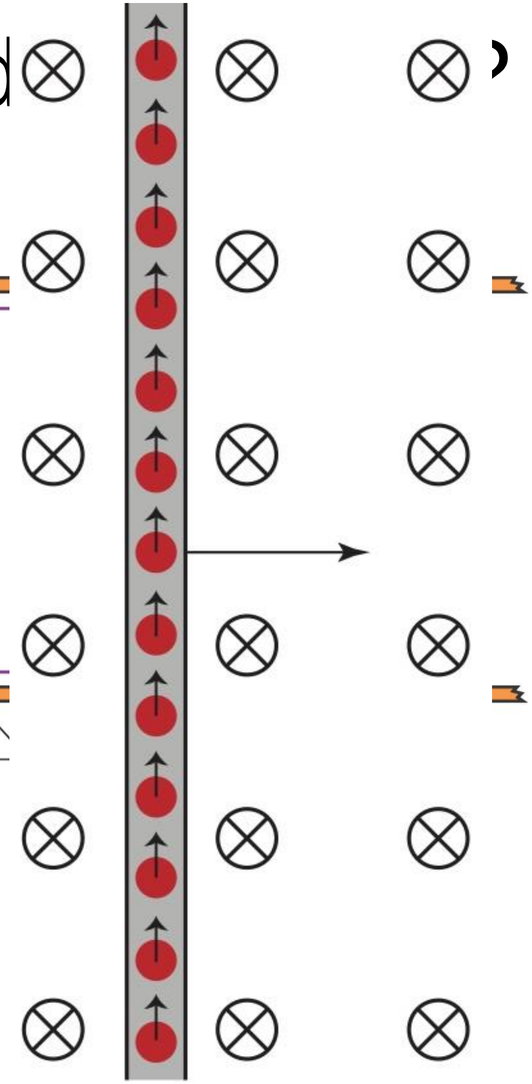


There is an induced ΔV across the length of the conductor



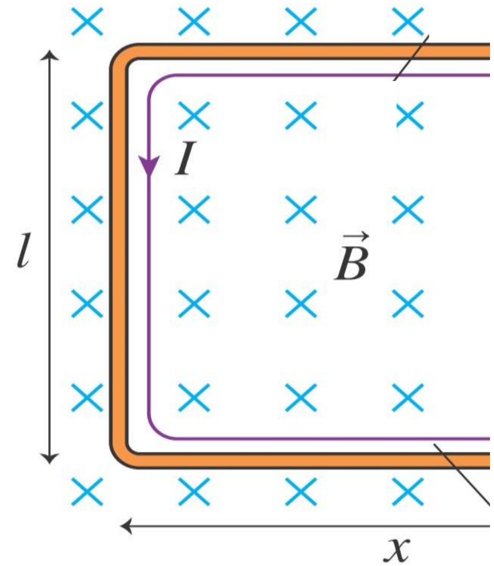
This is equivalent to having an EMF source: “motional EMF”

How can we quantify the ind



The free charges feel a magnetic force:

$$F = qvB$$



This induces a voltage difference (E-field), and therefore an electric force on the charges

$$F = qE \qquad E = \frac{\Delta V}{l}$$

$$\cancel{q}vB = \cancel{q}\frac{\Delta V}{l}$$

MOTIONAL EMF: $\Delta V \equiv \boxed{\varepsilon = vlB}$

Recall there are 3 possible terms:

$$e = \underbrace{-\frac{dB}{dt}A\cos f}_{} \underbrace{-\frac{dA}{dt}B\cos f + \frac{df}{dt}BA\sin f}_{}$$

Maxwell Equation

$$-\frac{d\vec{B}}{dt} = \nabla \times \vec{E}$$

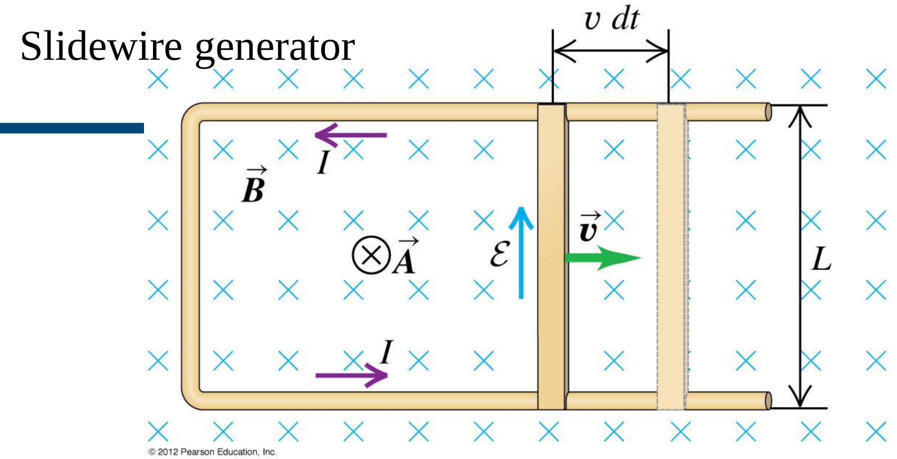
Magnetic Force on free charges

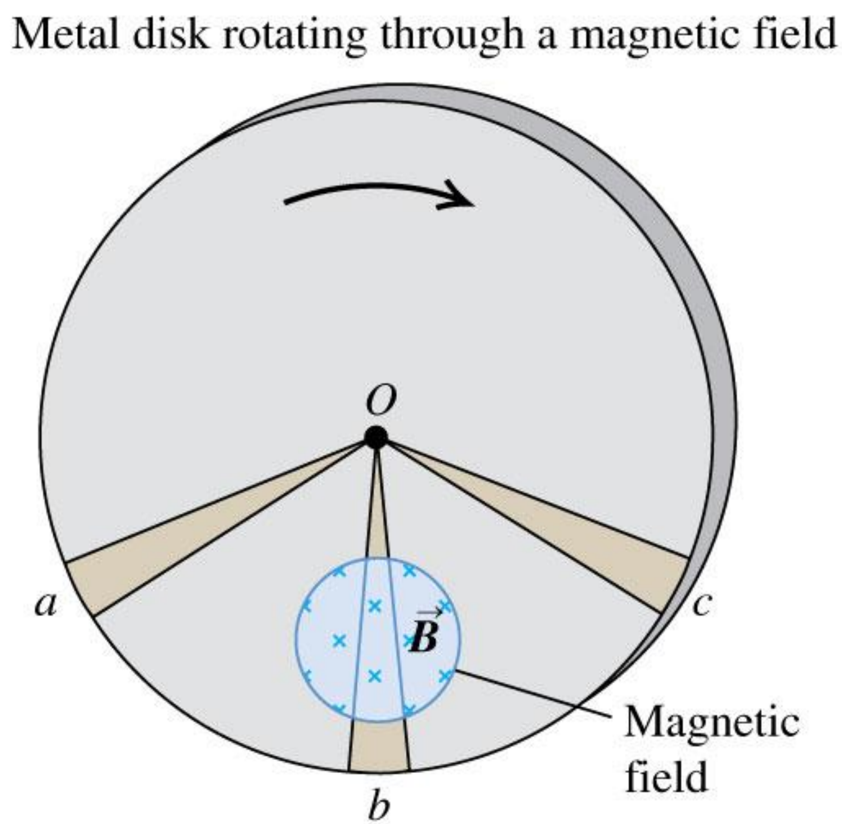
$$F = q\vec{v} \times \vec{B}$$

It is quite striking that drastically different sources for the induced EMF give an identical law. This makes Faraday's Law a particularly powerful tool from a practical engineering standpoint!

Eddy currents

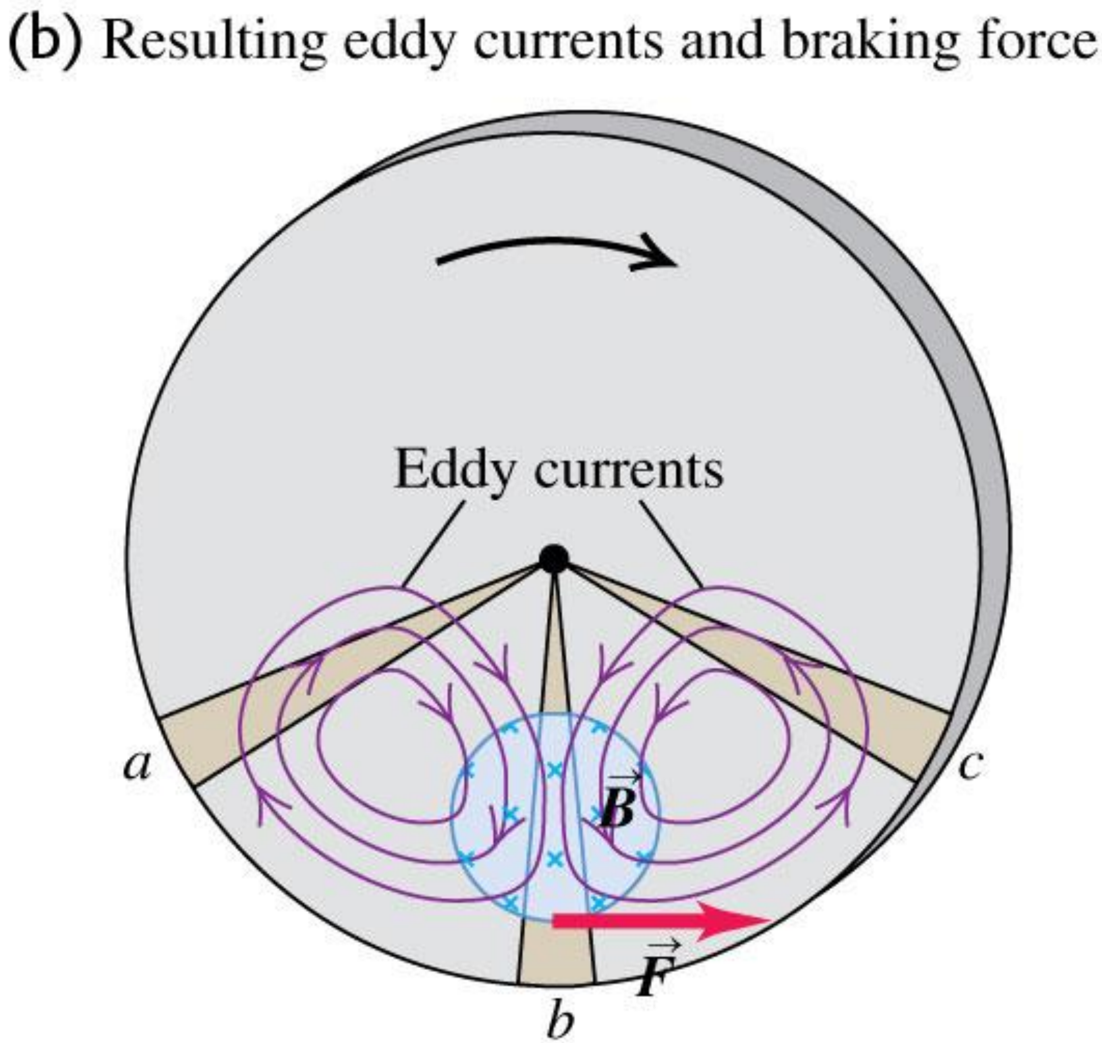
- So far we have considered induction in circuits, where the induced current is confined to wires
- Induction also happens if the magnetic flux through extended metallic objects changes
- As with wires, the induced currents attempt to keep the flux stable: *eddy* currents





Eddy currents

- The direction of the currents can be found using Lenz's law:
  - Without eddy currents, the magnetic flux at the leading (trailing) edge decreases (increases)
  - The induced Eddy currents circulate in a sense that prevents this from happening
  - Result: transformation of mechanical energy into heat!



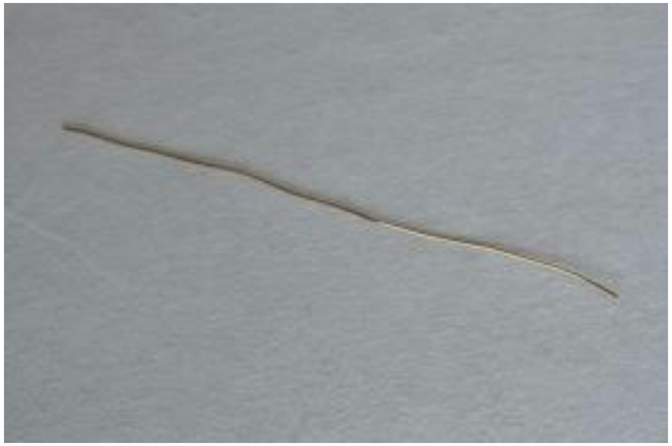
Question

Let's consider a piece of wire in an electric circuit.

Does it matter if it is straight, or coiled?

a) It does not matter: both have the same resistance (if they have the same length)

b) It matters: the reaction to changing current is very different

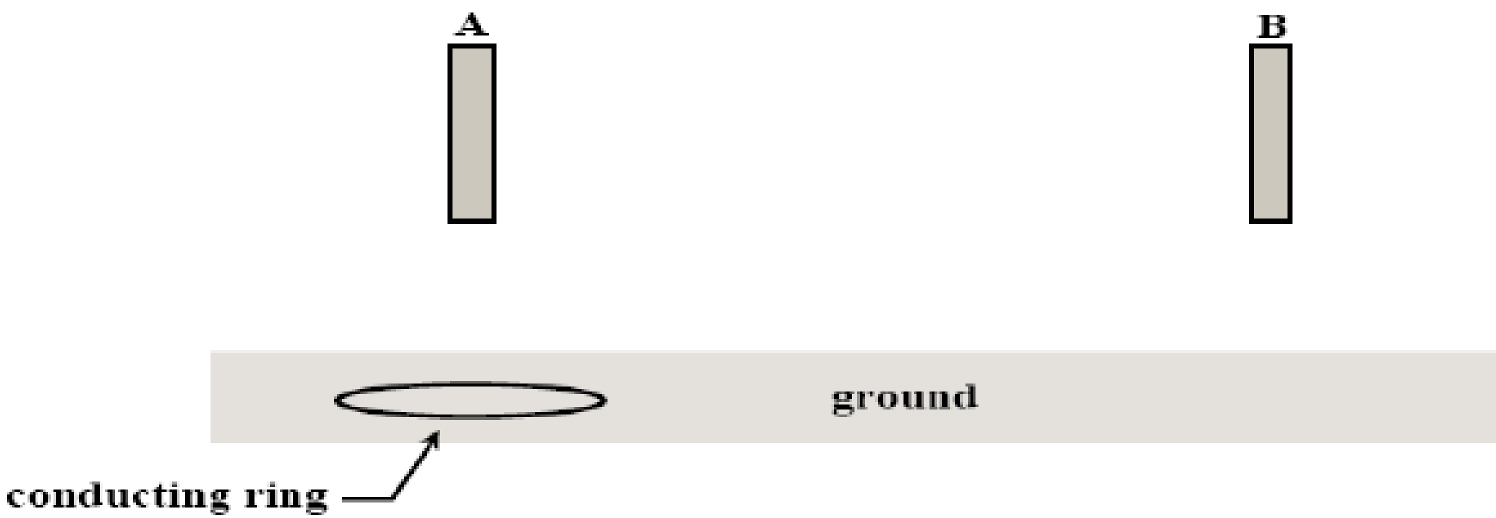


Examples

Examples

16. In the diagram below, two identical permanent magnets are dropped from equal heights above the ground at the same instant of time. There is a continuous ring of conducting material lying on the ground below magnet A. What happens? (Assume the two magnets are far enough apart that they do not influence each other, and the ground is nonconducting.)

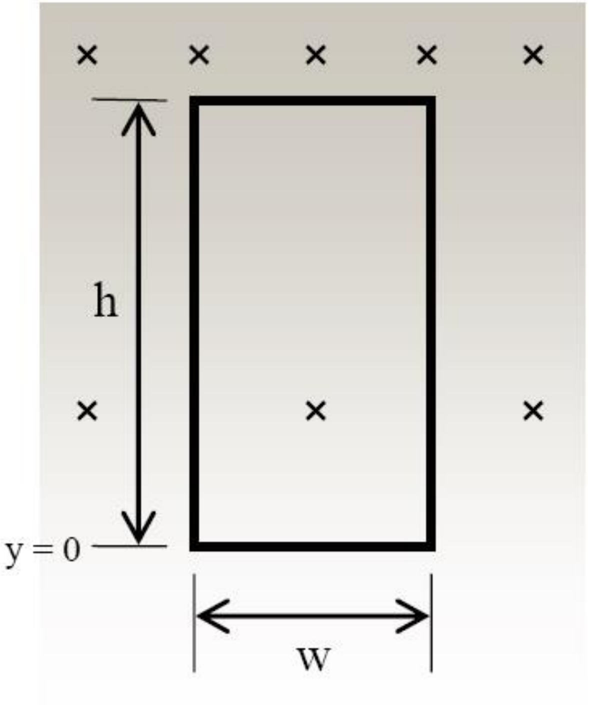
a. Magnet A hits the ground before magnet B.
b. Magnet B hits the ground before magnet A.
c. Both magnets hit the ground at the same time.
d. The answer depends on which pole of magnet A faces downwards.
e. None of the above.



Examples

c. *(Parts c and d are not related to parts a or b, above.)* In the figure below, a rectangular loop of conductor (e.g., a metal wire) of width w and height h is immersed in a non-uniform magnetic field into the page. The strength of the magnetic field increases upwards linearly as $B = B_0 + Cy$, where B_0 is the magnetic field strength at the base of the loop (i.e., at $y = 0$), and C is a constant. In the diagram, the strength of the magnetic field is indicated by the amount of shading (darker = stronger field).

Derive an expression for the magnetic flux enclosed by the loop, in terms of B_0, C, and quantities given in the diagram.



Examples

d. In which direction should you move the loop in the figure above in order to induce a current in the loop in the ***clockwise*** direction? Justify your answer clearly.