Electricity and Magnetism

- Physics 259 – L02
- Lecture 22

Chapter 24.1: Electric Potential

Last time

- Electric potential energy and electric force
- Electric potential and electric field
- Electric potential of a dipole

This time

- Electric potential energy of a collection of charges
- Electric potential (very important concept)
- Equipotential surfaces: visualizing electric potential
- Conductors and electric potential
- Interpreting equipotential surfaces

Last section we talked about:

If we release particle 1 at p, it begins to move→ has kinetic energy

ENERGY CAN NOT APPEAR BY MAGIC

It comes from electric potential energy U associated with the force between two particles

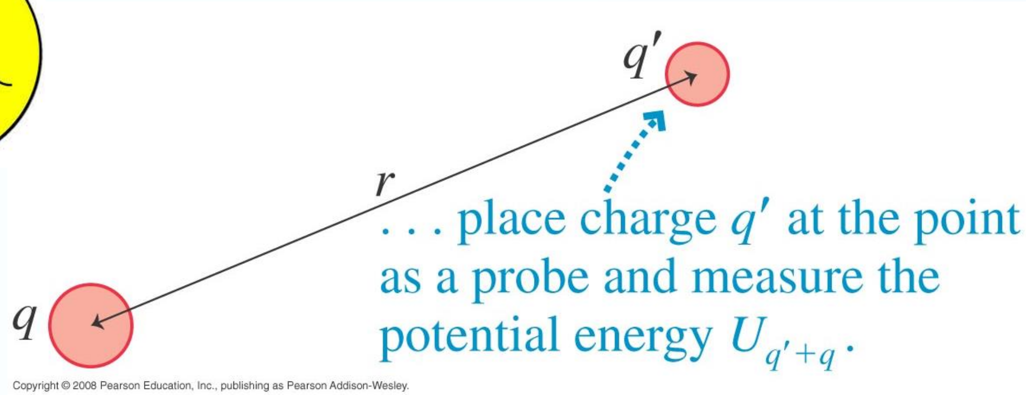
We also defined→

Electric potential V that is set up at point p by particle 2.

The electric potential exists regardless of whether particle 1 is at p.

Starting from the end

The whole story is:



Electric force on q' from q

$$\vec{F}_{qq'} = \frac{1}{4\pi\varepsilon_0}\frac{qq'}{r^2}\hat{r}$$

Then the electric field of q is

$$\vec{E} = \frac{\vec{F}_{qq'}}{q'} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$$

Potential energy of q and q'

$$U_{q'+q} = \frac{1}{4pe_0}\frac{qq'}{r}$$

Then the potential of q is

$$V = \frac{U_{q'+q}}{q'} = \frac{1}{4pe_0}\frac{q}{r}$$

Electric Potential

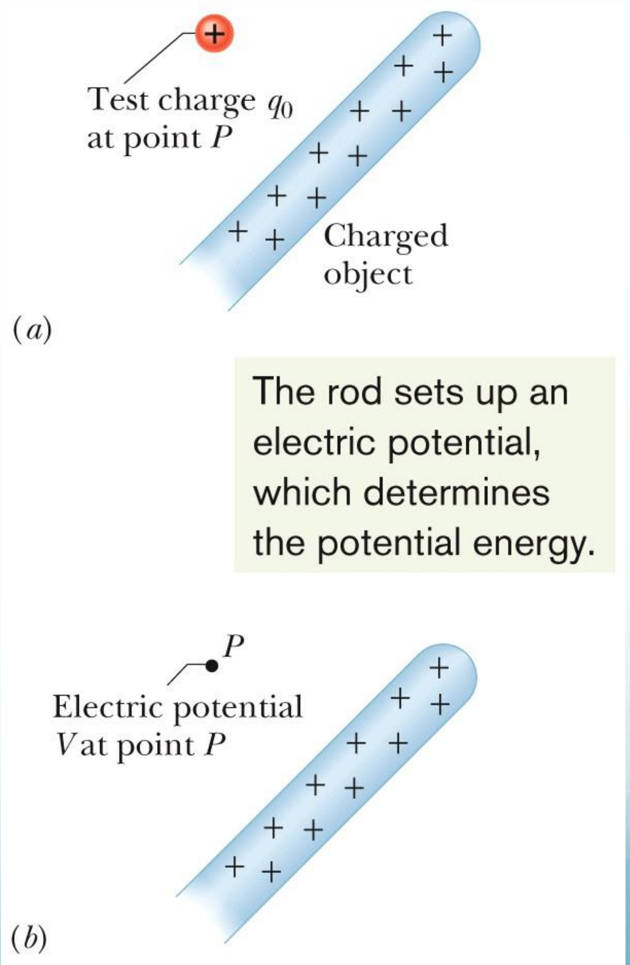
Electric potential V at a point P in the electric field of a charged object →

$$V = \frac{-W_\infty}{q_0} = \frac{U}{q_0}$$

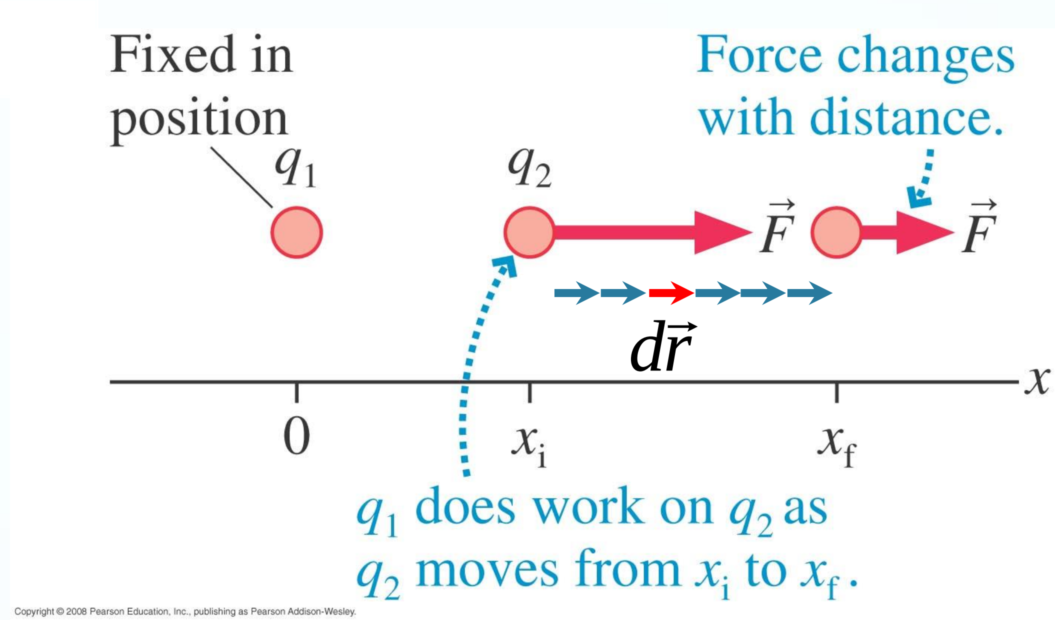
W_∞ → work that would be done by the electric force on a positive test charge q_0 were it brought from an infinite distance to *P*, and *U* is the electric potential energy that would then be stored in the test charge–object system.

The electric potential energy *U* of the particle–object system is →

$$U = qV.$$



Review: Finding Potential Energy of two point charges



The total work is the sum of all the little bits of work:

$$W_{i\to f}^{ELEC} = \int_{r_i}^{r_f} F dr$$

$$W_{i\to f}^{ELEC} = \int_{r_i}^{r_f} \frac{1}{4pe_0} \frac{q_1 q_2}{r^2} dr$$

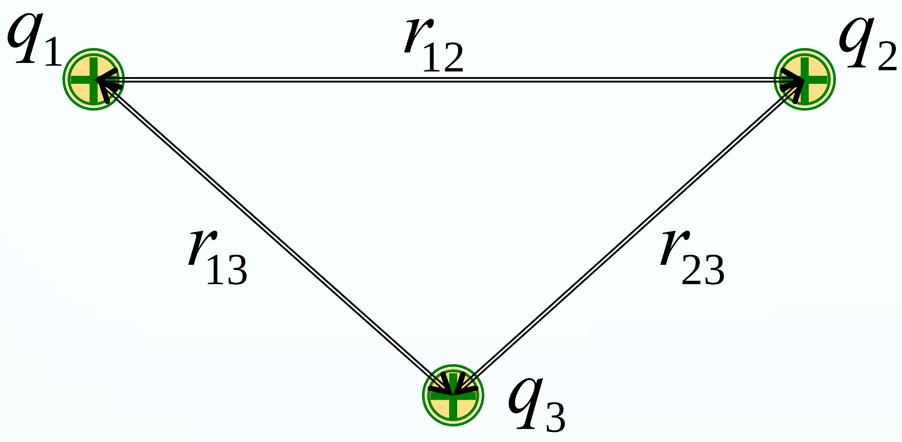
Review: Finding Potential Energy of two point charges

$$W_{i\to f}^{ELEC} = -\frac{1}{4pe_0}\frac{q_1 q_2}{r}\Bigg|_{r_i}^{r_f}$$

$$W_{i\to f}^{ELEC} = -\Delta U = -(U_f - U_i) = U_i - U_f$$

$$\boxed{U_e = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}}$$

Superposition: Potential Energy due to Multiple Charges



$$\boxed{U_{total} = U_{12} + U_{23} + U_{13}}$$

$$U_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}}$$

$$U_{23} = \frac{1}{4\pi\varepsilon_0}\frac{q_2 q_3}{r_{23}}$$

$$U_{13} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_3}{r_{13}}$$

In general, the total potential energy is just the sum of the pairwise potential energies of all the charges present.

Calculate U between each pair, then sum all of them up.

Electric Potential

Here are some source charges and a point P.

If we place a charge q at point P, then q and the source charges interact with each other.

The interaction energy is the potential energy of q and the source charges,

$$U_{q+sources}$$

How does this interaction happen?

Electric Potential

Model:

The source charges create a **potential for interaction** everywhere, including at point P.

This potential for interaction is a **property of space**. Charge q does not need to be there.

We call this potential for interaction the **electric potential, V.** (Often just called “the potential”)

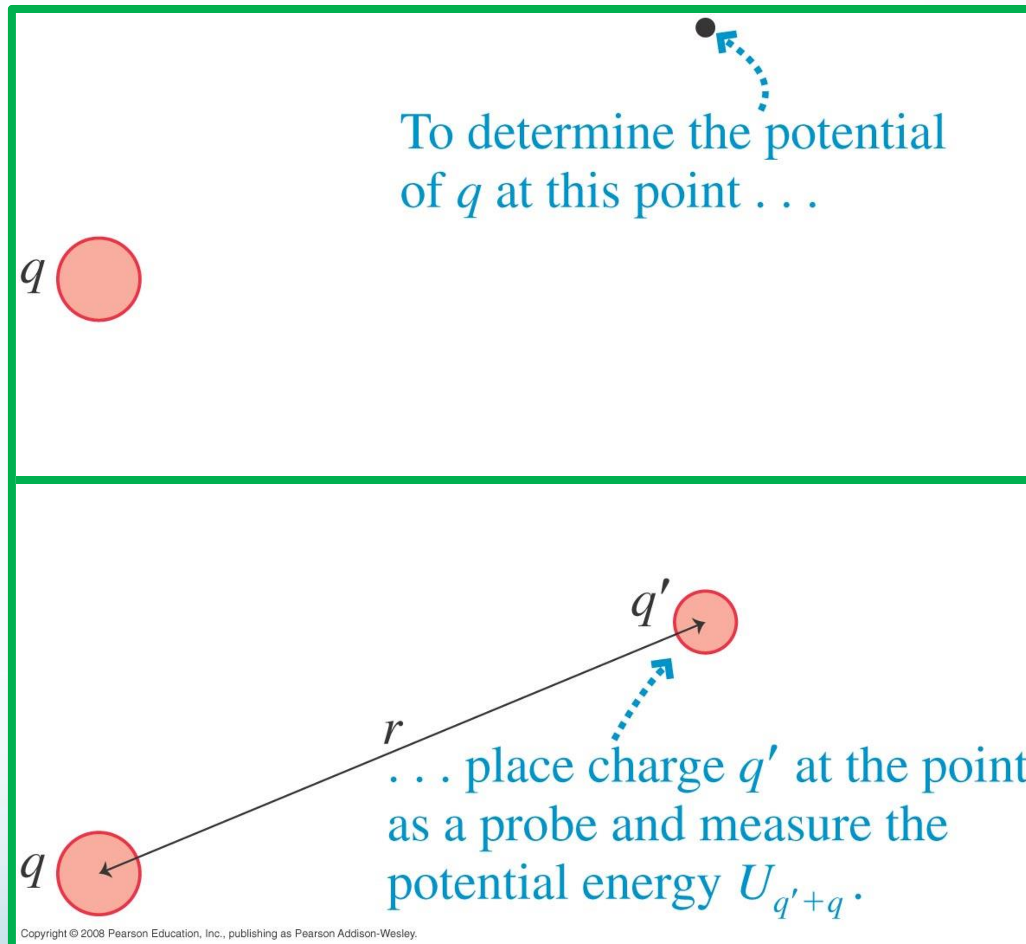
Electric Potential of a point charge

Potential energy of q and q’

$$U_{q'+q} = \frac{1}{4\pi\varepsilon_0}\frac{qq'}{r}$$

Then the potential of q is

$$V = \frac{U_{q'+q}}{q'} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$



Electric Potential

Definition of V: Place charge q at point P and measure its potential energy. Then

$$\boxed{V \equiv \frac{U_{q+sources}}{q}}$$

Unit: $1\,\text{volt} = 1\,\text{V} = 1\,\dfrac{\text{J}}{\text{C}}$

Electric Potential

Or, if we know the potential, V, at point P, then if we place a charge, q, at point P, the potential energy of q and the source charges is

$$\boxed{U_{q+sources} = qV}$$

Change in Electric Potential.

If the particle moves through a potential difference *ΔV*, the change in the electric potential energy is

$$\Delta U = q\,\Delta V = q(V_f - V_i).$$

Work by the Field.

The work W done by the electric force as the particle moves from *i* to *f:*

$$W = -\Delta U = -q\,\Delta V = -q(V_f - V_i).$$

Conservation of Energy.

If a particle moves through a change *ΔV* in electric potential without an applied force acting on it, applying the conservation of mechanical energy gives the change in kinetic energy as

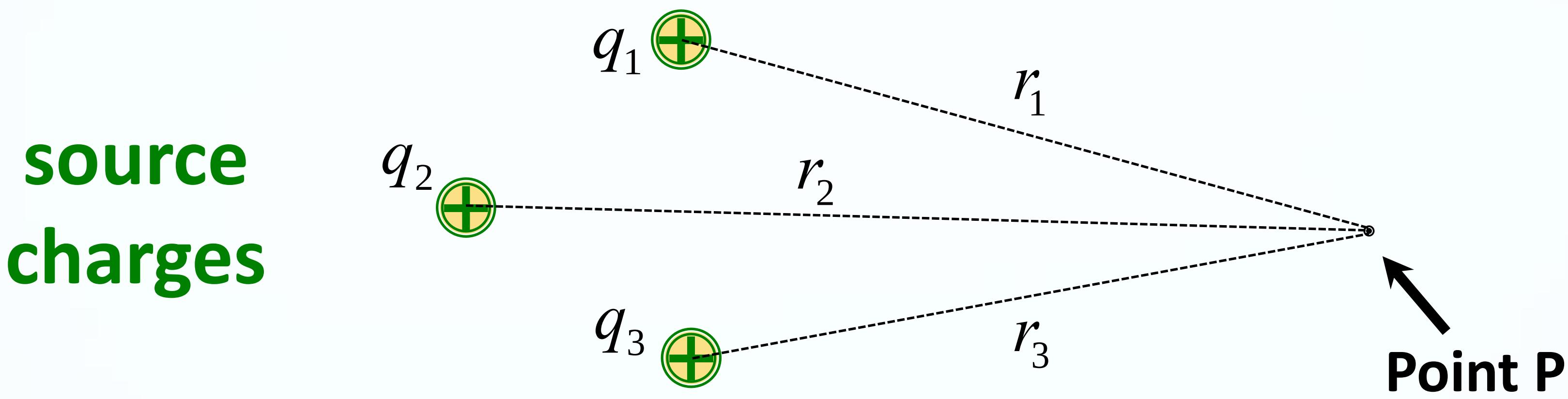
$$\Delta K = -q\,\Delta V = -q(V_f - V_i).$$

Work by an Applied Force.

If some force in addition to the electric force acts on the particle, we account for that work

$$\Delta K = -\Delta U + W_{\text{app}} = -q\,\Delta V + W_{\text{app}}.$$

Advantage of Electric Potential

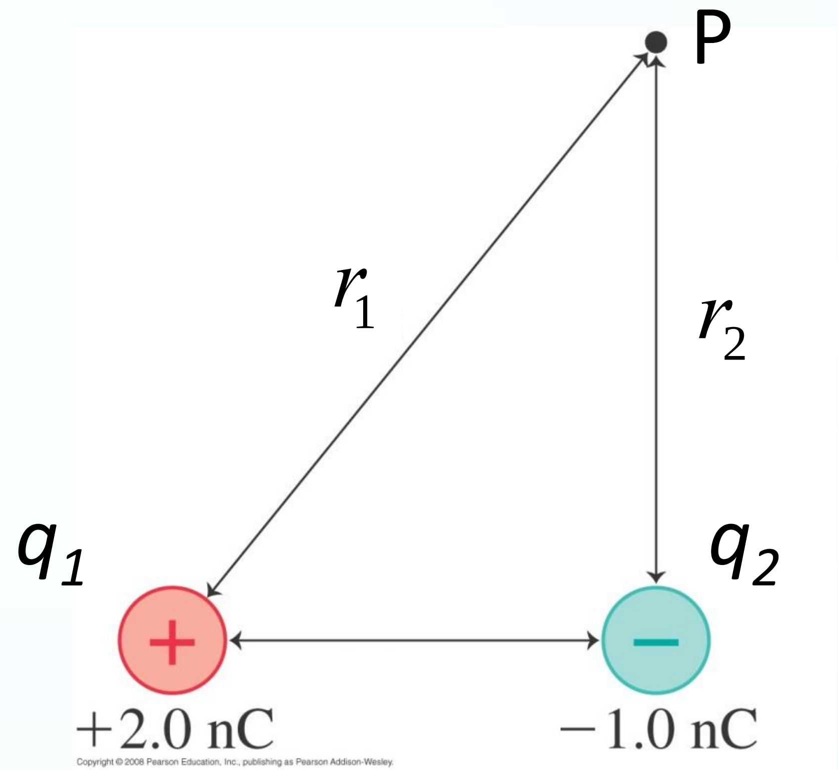


V is a SCALAR! There is no direction associated with it. This makes it much easier to calculate!

$$V_1 = \frac{1}{4\pi\epsilon_0}\frac{q_1}{r_1} \qquad V_2 = \frac{1}{4\pi\epsilon_0}\frac{q_2}{r_2} \qquad V_3 = \frac{1}{4\pi\epsilon_0}\frac{q_3}{r_3}$$

$$\boxed{V = V_1 + V_2 + V_3}$$

Finding V at point P.



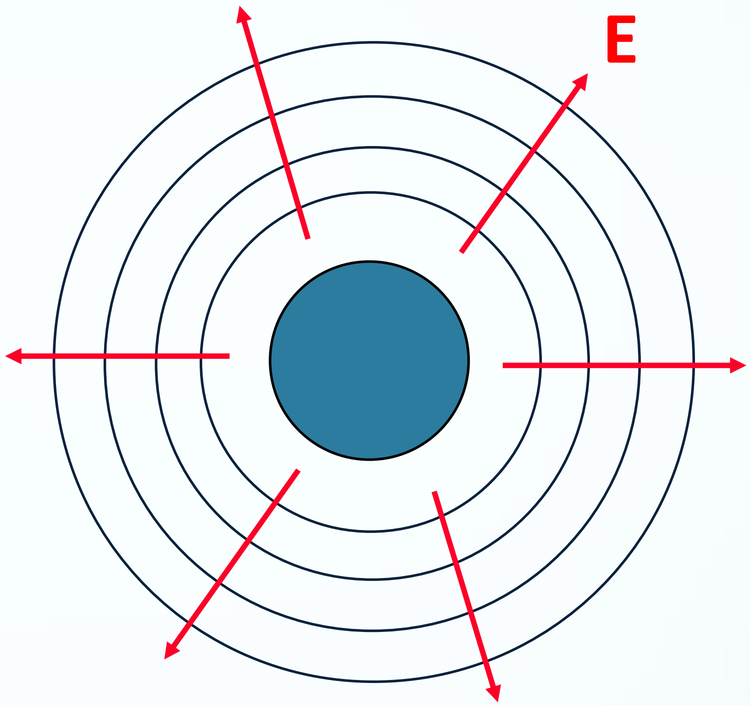
Potential is a scalar

There are no components

Just add the potentials

V at P = (V_1 at P due to q_1) + (V_2 at P due to q_2).

Equipotentials



Ex: For uniform spherical charge

$$V(r) = k\,Q/r$$

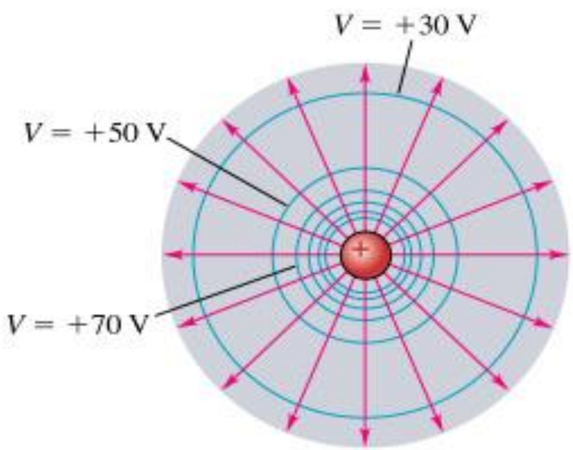
For each r, V(r) is constant→
V(r) is constant over any sphere concentric with the charged sphere

→ We have equipotential lines (or surfaces, actually, in 3-D)

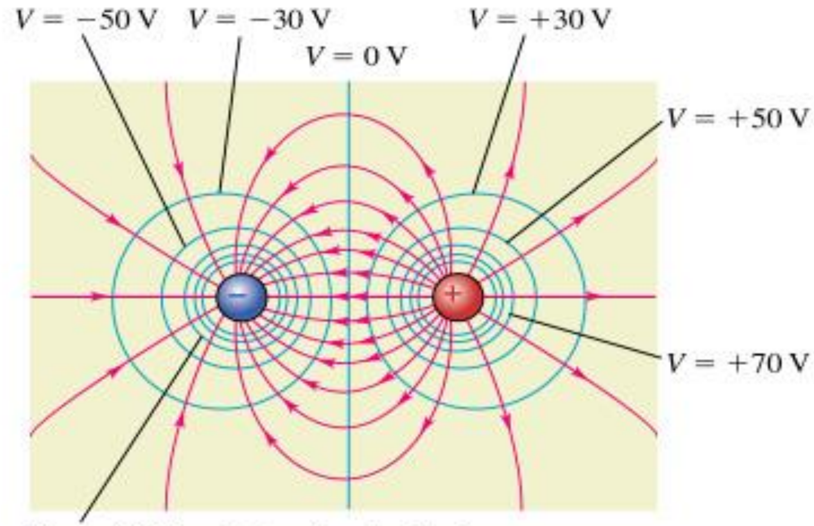
Note that if move along equipotential surface→

→ by definition $\Delta V = -\mathbf{E} \cdot \Delta \mathbf{r} = 0$ --> **E** is ⊥ equipotential surface

Equipotential Surfaces



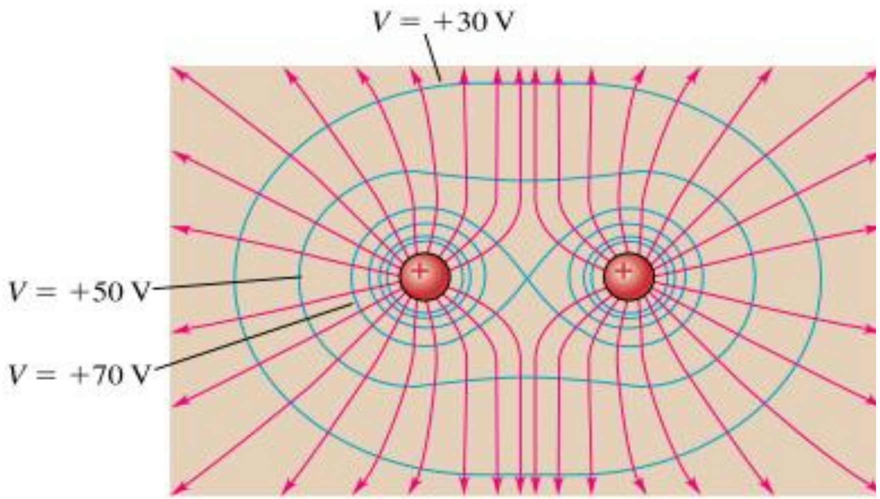
(a) A single positive charge



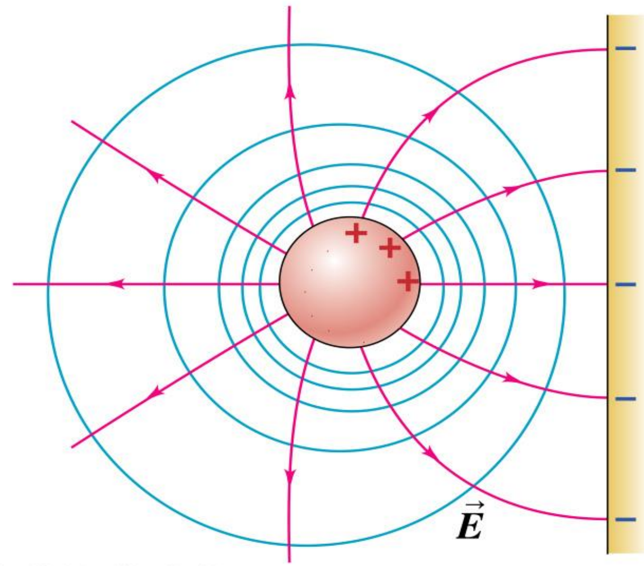
(b) An electric dipole

Note – E is always ⊥ V !!

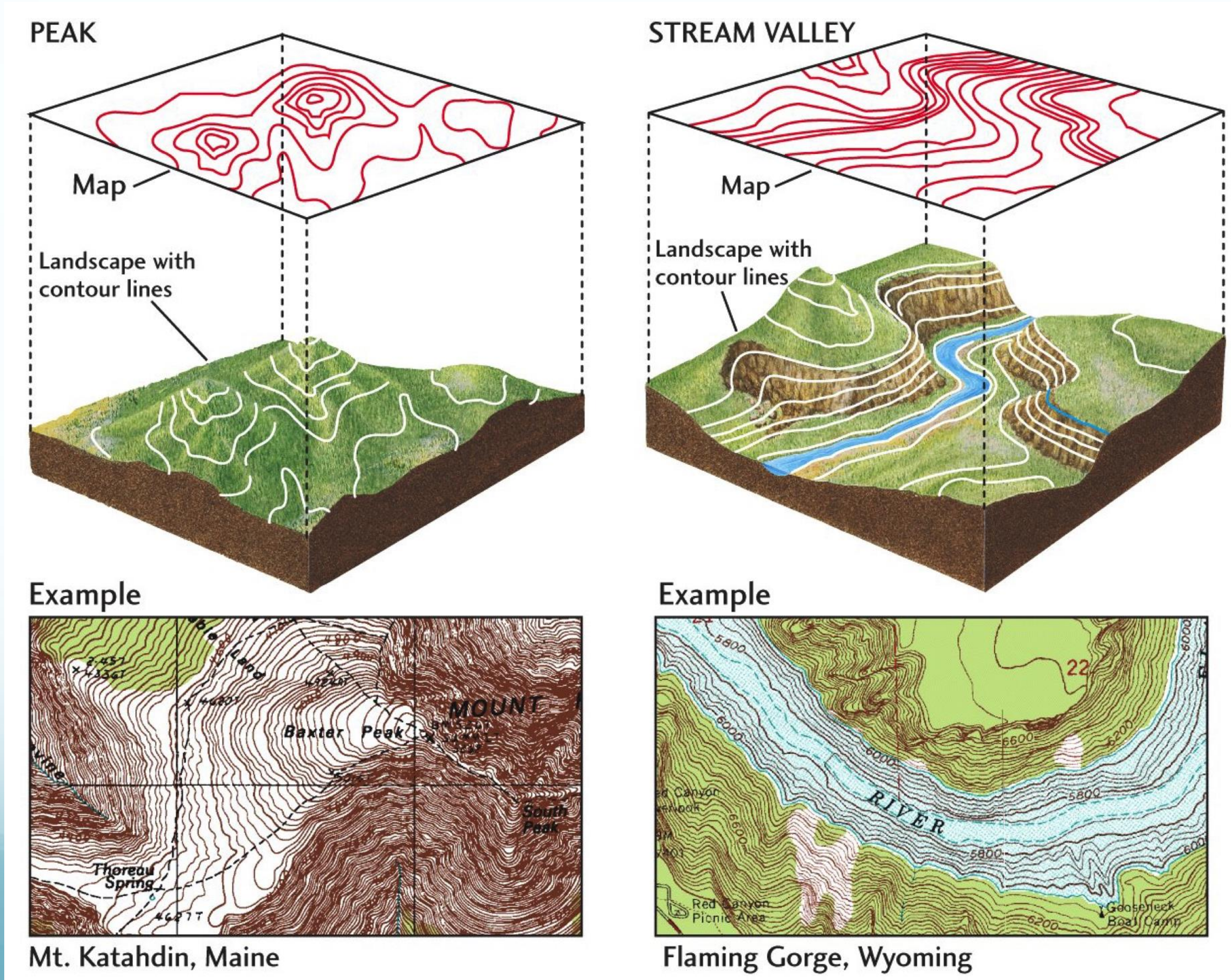
Conducting sphere
+ sheet



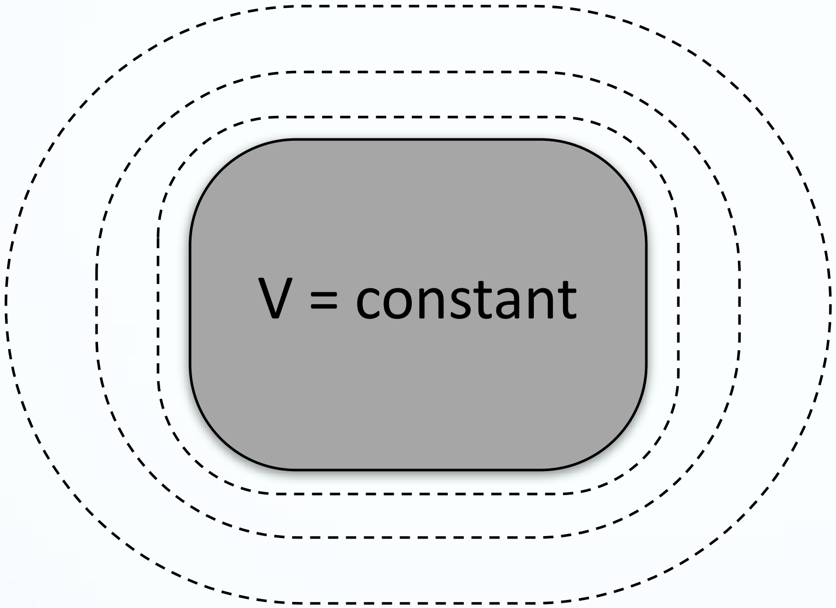
(c) Two equal positive charges



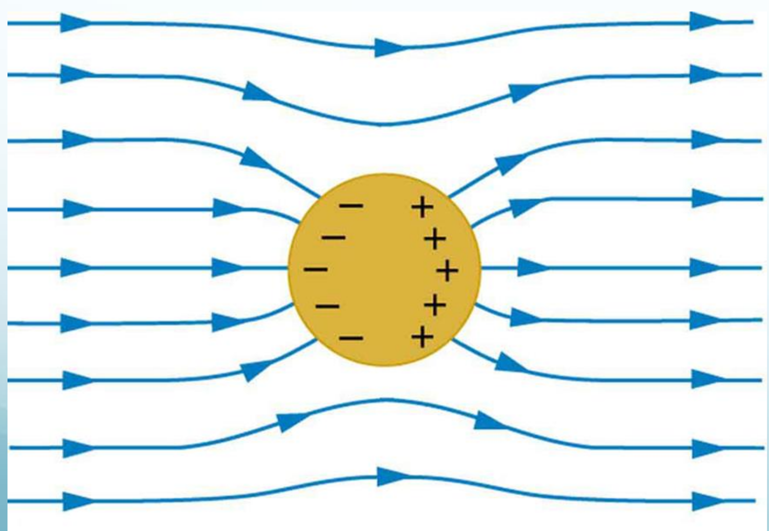
Where have you seen equipotentials before?



Conductors and E-fields



The surface of a conductor is an equipotential. If there was a potential difference across the surface of a conductor, the freely moving charges would move around until the potential is constant.



https://cnx.org/contents/Ax2o07Ul@8.8:a8tWDdDr@3/Conductors-and-Electric-Fields

This means that electric field lines ALWAYS must meet a conducting surface at right angles (any tangential component would imply a tangential force on the free charges).

This section we talked about:

Chapter 24.1

See you on Friday