
UNDER CONSTRUCTION


UNDER
CONST-
RUCTION

Announcements

Last time

- Potential energy of a point charge in a constant electric field.
- Potential energy between two point charges, one moving along the electric field lines of the other
- Potential energy between two point charges, one moving along an arbitrary path in electric field of the other
- General results for potential energy of a static electric field.

This time

- Potential energy of a collection of point charges
- Electric potential due to a point charge
- Electric potential due to a collection of point charges
- Electric potential due to an arbitrary charge distribution

The potential energy of two point charges a distance r apart is

$$\boxed{U_e = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r} \cancel{+U_0}}$$

(1) There is a U_0, but we normally set it to zero.
(2) The potential energy of two charges an infinite distance apart ($r = \infty$) is zero.

The Coulomb force due to a point is a **conservative force** (independent of path taken).

Therefore the **electric field due to a point charge is a conservative field** like the gravitational field.

By superposition principle the electric field due to a collection of point charges is also a **conservative field**.

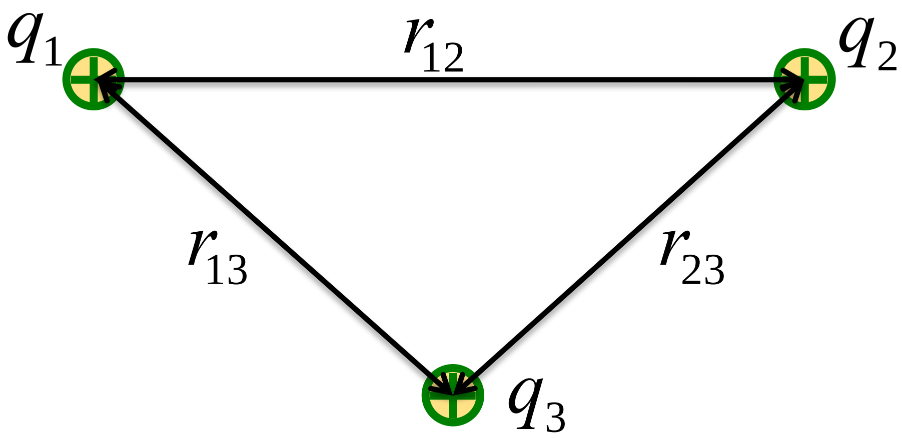
Because an arbitrary charges distribution is considered to be a collection of a large number of infinitesimal point charges, by superposition principle the electric **field for an arbitrary charge distribution is also a conservative field.**

The potential energy of two point charges a distance r_{12} apart is

$$\boxed{U_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}}}$$

Note that potential energy is a scalar quantity and easier to deal with unlike Coulomb's force and electric field which are vector quantities.

Superposition: Potential Energy due to Multiple Charges



$$\boxed{U_{total} = U_{12} + U_{23} + U_{13}}$$

$$U_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}}$$

$$U_{23} = \frac{1}{4\pi\varepsilon_0}\frac{q_2 q_3}{r_{23}}$$

$$U_{13} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_3}{r_{13}}$$

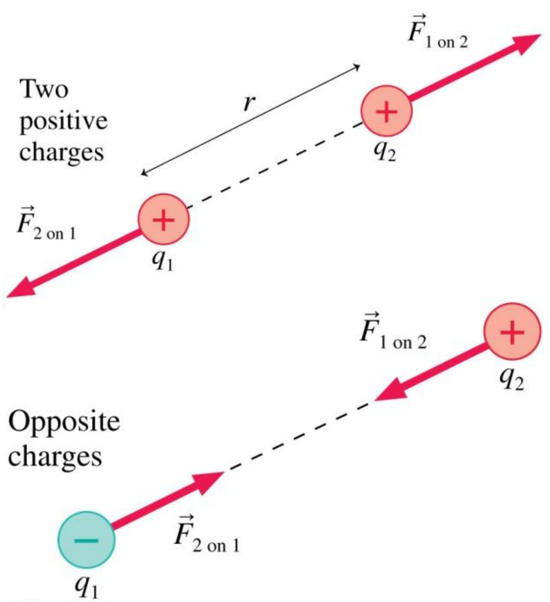
In general, the total potential energy is just the sum of the pairwise potential energies of all the charges present. Calculate U between each pair, then sum all of them up.

Electric Force vs Electric Field

Electric Force $\vec{F}$

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2} = q\vec{E}$$

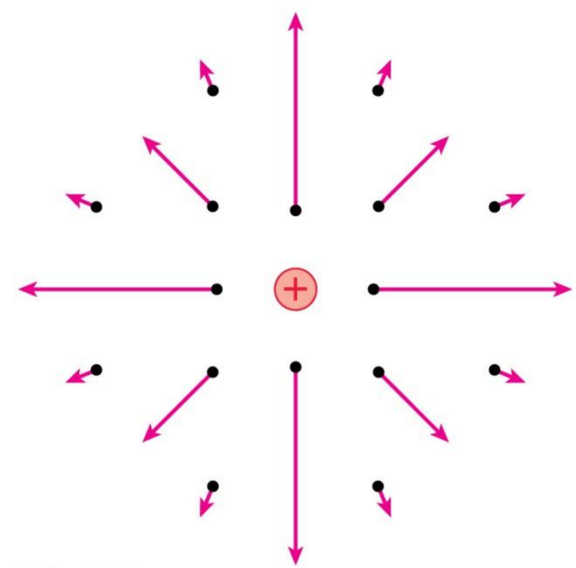
A physical property between two point charges



Electric Field $\vec{E}$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

A physical property around a single point charge



Electric Force vs Electric Field

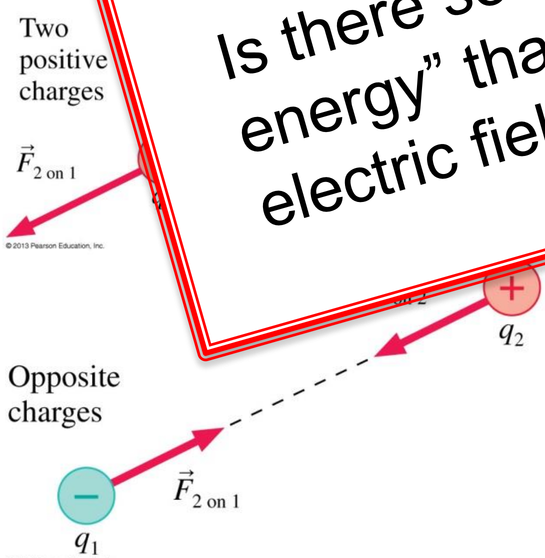
Electric Force $\vec{F}$

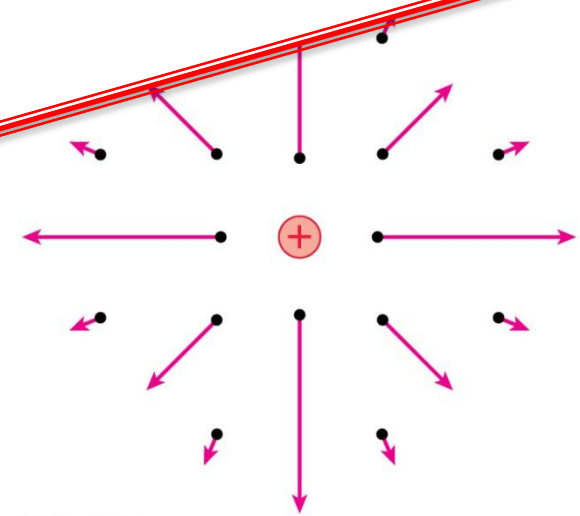
$$\vec{F} = \frac{1}{4\pi\ldots}\frac{Qq}{\ldots}$$

A p…
betw…
charg…

Potential energy is a physical property that exists because of the force between two charges.

Is there some similar notion of “potential energy” that exists only because of the electric field?





Electric Potential

Here are some source charges and a point P.

If we place a charge q at point P, then q and the source charges interact with each other.

The interaction energy is the potential energy of q and the source charges,

$$U_{q+sources}$$

How does this interaction happen?

Electric Potential

Model:

The source charges create a **potential for interaction** everywhere, including at point P.

This potential for interaction is a **property of space**. Charge q does not need to be there.

We call this potential for interaction the **electric potential, V.** (Often just called “the potential”)

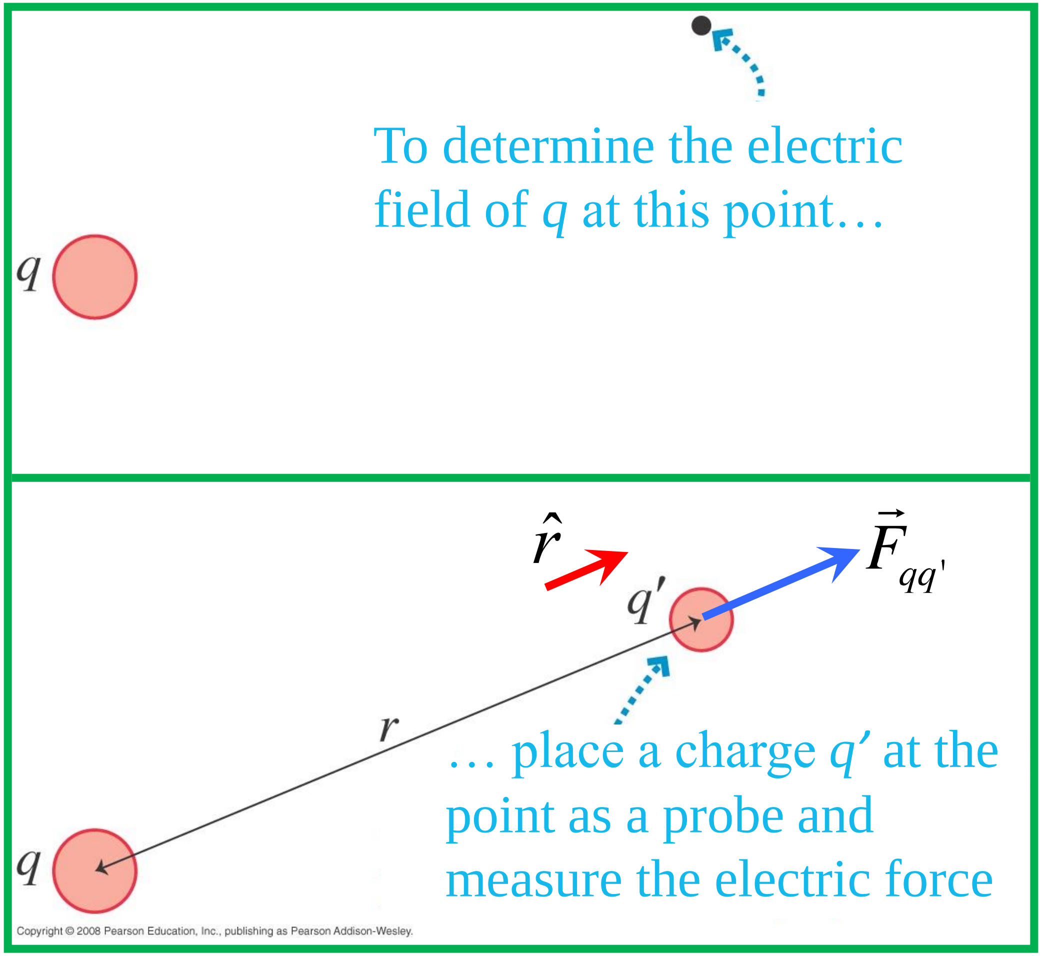
Electric Field of a point charge

Electric force on q' from q

$$\vec{F}_{qq'} = \frac{1}{4\pi\varepsilon_0}\frac{qq'}{r^2}\hat{r}$$

Then the electric field of q is

$$\vec{E} = \frac{\vec{F}_{qq'}}{q'} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$$



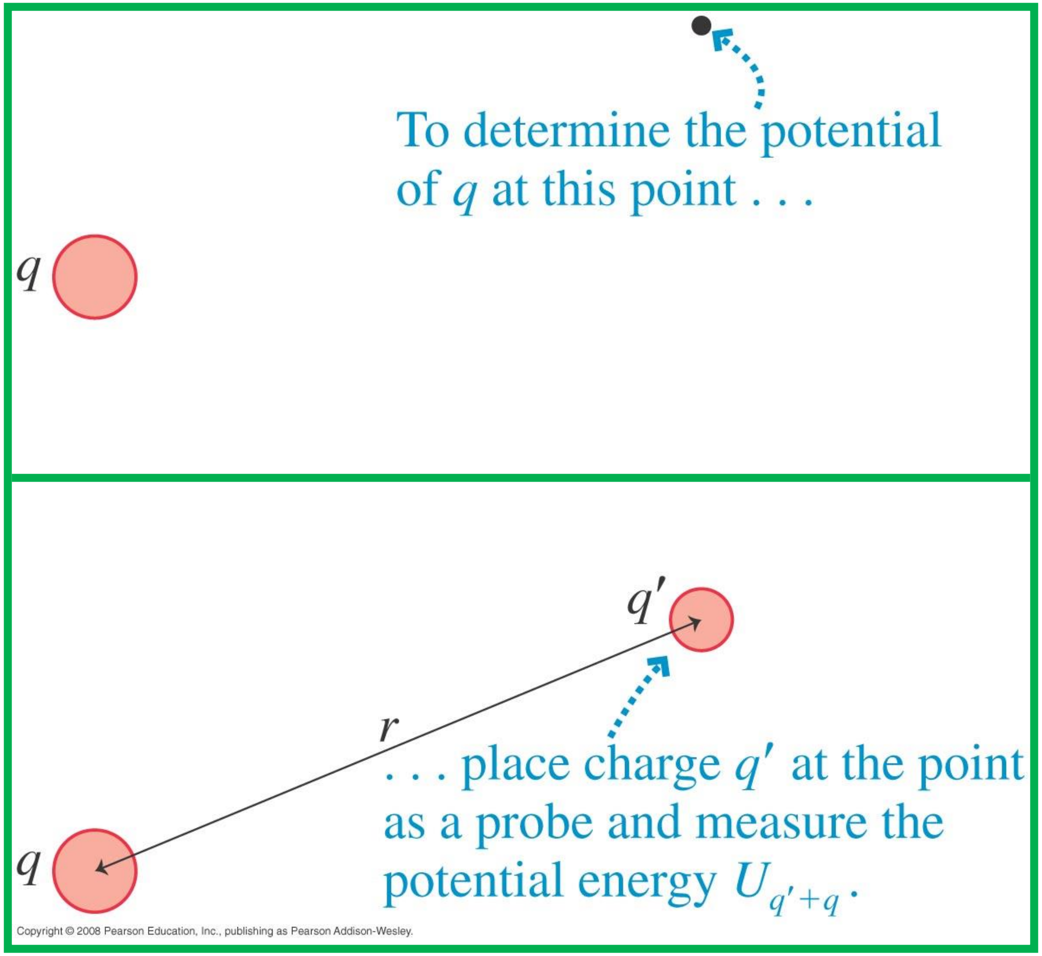
Electric Potential of a point charge

Potential energy of q and q'

$$U_{q'+q} = \frac{1}{4\pi\varepsilon_0}\frac{qq'}{r}$$

Then the potential of q is

$$V = \frac{U_{q'+q}}{q'} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$



Electric Potential

Definition of V: Place charge q at point P and measure its potential energy. Then

$$\boxed{V \equiv \frac{U_{q+sources}}{q}}$$

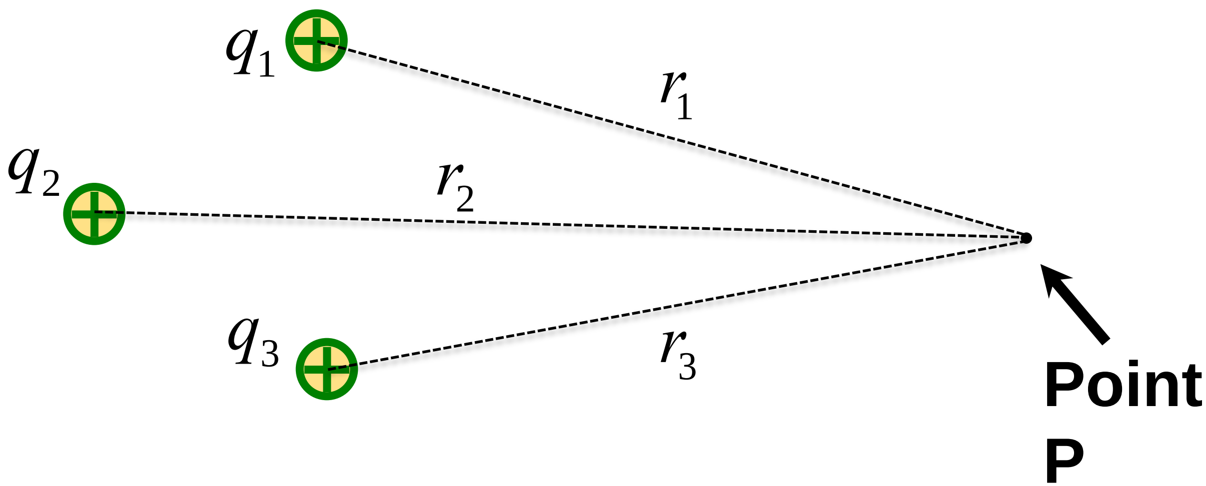
Unit: $1\,\mathrm{volt} = 1\,\mathrm{V} = 1\,\frac{\mathrm{J}}{\mathrm{C}}$

Electric Potential

Or, if we know the potential, V, at point P, then if we place a charge, q, at point P, the potential energy of q and the source charges is

$$\boxed{U_{q+sources} = qV}$$

Advantage of Electric Potential



V is a SCALAR! There is no direction associated with it. This makes it much easier to calculate!

$$V_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1} \qquad V_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2} \qquad V_3 = \frac{1}{4\pi\varepsilon_0}\frac{q_3}{r_3}$$

$$\boxed{V = V_1 + V_2 + V_3}$$

Vector quantities

$$\vec{F}_{qq'} = \frac{1}{4\pi\varepsilon_0}\frac{qq'}{r^2}\hat{r}$$

$$\vec{E} = \frac{\vec{F}_{qq'}}{q'} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$$

$$\vec{F} = q\vec{E}$$

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r^2}\hat{r}$$

Scalar quantities

$$U_{q'+q} = \frac{1}{4\pi\varepsilon_0}\frac{qq'}{r}$$

$$V = \frac{U_{q'+q}}{q'} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

$$U = qV$$

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r}$$

$$U_b - U_a = -q_0\int_a^b \vec{E}\bullet d\vec{l}$$

$$V_b - V_a = -\int_a^b \vec{E}\bullet d\vec{l} = \int_b^a \vec{E}\bullet d\vec{l}$$