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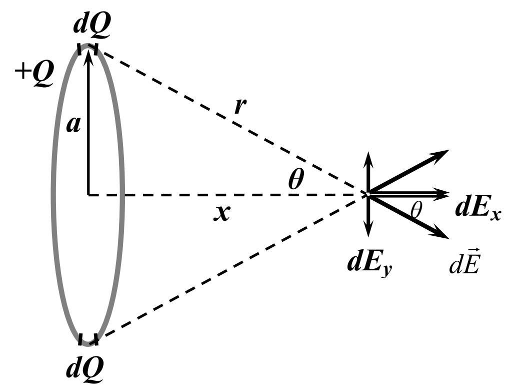
Q 12 a (1 mark)

An amount of charge Q is distributed evenly around a ring of radius a, so

$$\lambda = \frac{Q}{circumference} = \frac{Q}{2\,\pi\,a}$$

Q 12 b (3 marks)



$$\vec{E} = \int_{Ring} d\vec{E} \quad Taking\ components,$$

$$E_y = \int_{Ring} dE_y = 0 \quad by\ symmetry$$

$$E_z = \int_{Ring} dE_z = 0 \quad by\ symmetry$$

$$E_x = \int_{Ring} dE_x \quad where\ dE_x = dE\cos\theta, \quad dE = \frac{1}{4\pi\epsilon_0}\frac{dQ}{r^2}, \quad and \quad \cos\theta = \frac{x}{r}.$$

$$Then \qquad E_x = \int_{Ring} dE\cos\theta = \int_{Ring}\left(\frac{1}{4\pi\epsilon_0}\frac{dQ}{r^2}\right)\left(\frac{x}{r}\right) = \int_{Ring}\frac{1}{4\pi\epsilon_0}\frac{x\,dQ}{r^3}$$

$$But \quad r = \left(x^2+a^2\right)^{1/2} \quad so \quad E_x = \int_{Ring}\frac{1}{4\pi\epsilon_0}\frac{x\,dQ}{\left(x^2+a^2\right)^{3/2}} = \frac{1}{4\pi\epsilon_0}\frac{x}{\left(x^2+a^2\right)^{3/2}}\int_{Ring} dQ$$

$$and \quad \int_{Ring} dQ = Q$$

$$Then \qquad E_x = \frac{1}{4\pi\epsilon_0}\frac{x\,Q}{\left(x^2+a^2\right)^{3/2}}$$

Q12 c (1 mark)

$$At\ the\ centre,\ x=0,\ so \quad E_x = 0\times\frac{1}{4\pi\epsilon_0}\frac{Q}{\left(a^2\right)^{3/2}} = 0$$

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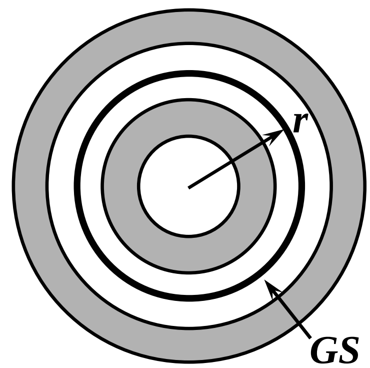
Q 12 d (0.5 mark)

$$E_x = \frac{1}{4\pi\epsilon_0}\frac{Q}{x^2} \quad or \quad E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$$

(When viewed from a long distance away, the ring should look like a point charge, so the electric field should be the same as for a point charge.)

Q13 a (3 marks)

Gauss's law: $\oint \vec{E}\cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$



Define a Gaussian surface (GS) of radius $r_b < r < r_c$*, as shown at the right. By symmetry,* $\vec{E}$ *is radial, and* $d\vec{A}$ *is also radial, so* $\vec{E}||d\vec{A}$*. Then* $\vec{E}\cdot d\vec{A} = E\,dA$ *and*

$$\oint EdA = \frac{Q_{encl}}{\epsilon_0}$$

Also, by symmetry, the magnitude, E, is uniform over the Gaussian surface. Then

$$\oint \vec{E}\cdot d\vec{A} = \oint E\,dA = E\oint dA = E\,A = E\,4\pi r^2 = \frac{Q_{encl}}{\epsilon_0} = \frac{2q}{\epsilon_0}$$

Dividing through by $4\pi r^2$ *gives* $E = \frac{1}{4\pi\epsilon_0}\frac{2q}{r^2}$

Q13 b (0.5 mark)

The enclosed charge is positive, so $\vec{E}$ *is* <u>*outward*</u>

Q13c(i) (1 mark)

$\vec{E} = 0$ *because no charge is enclosed within the central cavity.*

Q13c(ii) (1 mark)

$\vec{E} = 0$ *inside a conductor.*

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Q13c(iii) (1 mark)

$\vec{E} = 0$ *inside a conductor.*

Q13c(iv) (1 mark)

The enclosed charge is now $2q - 4q = -2q$*, so* $E = \frac{1}{4\pi\epsilon_0}\frac{2q}{r^2}$ *directed inward.*

Q14 (1 mark)

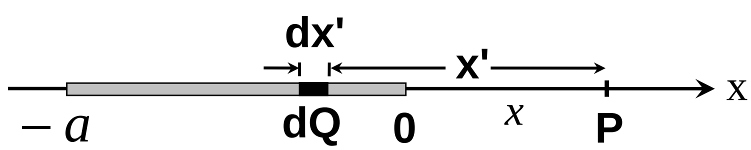
When the fifth charge is at infinity, its potential energy is 0.

When it is at the centre of the square, its potential energy is

$$U = \frac{1}{4\pi\epsilon_0}\frac{(+q)(+q)}{r} + \frac{1}{4\pi\epsilon_0}\frac{(+q)(-q)}{r} + \frac{1}{4\pi\epsilon_0}\frac{(+q)(+q)}{r} + \frac{1}{4\pi\epsilon_0}\frac{(+q)(-q)}{r} = 0$$

Therefore, the change in its potential energy is $\underline{\underline{\text{zero}}}$*.*

Q15 (3 mark)



x is a constant distance, and we need a variable for integrating, so maybe call it x'*, as in the figure above.*

Select an infinitesimal elelment of length dx' *and charge* $dQ = \lambda\, dx'$ *on the rod, at a distance* x' *from point P.*

Then dQ creates a potential at point P of $dV = \frac{1}{4\pi\epsilon_0}\frac{dQ}{x'} = \frac{1}{4\pi\epsilon_0}\frac{\lambda\, dx'}{x'}$

Then the potential at point P due to the entire rod is

$$V = \int_{x'=x}^{x+a} dV = \int_{x'=x}^{x+a} \frac{1}{4\pi\epsilon_0}\frac{\lambda\, dx'}{x'} = \frac{\lambda}{4\pi\epsilon_0}\int_{x'=x}^{x+a}\frac{dx'}{x'} = \frac{\lambda}{4\pi\epsilon_0}\, ln\, \frac{x+a}{x}$$

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Q 16 (3 marks)

The electric force is the only force acting, so mechanical energy is conserved:

$$K_f + U_f = K_i + U_i \qquad \mathbf{(1)}$$

$K_i = 0$ *because nothing is moving, and* $U_f \to 0$ *as the separations become very large.*

The initial potential energy is

$$U_i = U_{12} + U_{23} + U_{13} = \frac{1}{4\pi\epsilon_0}\frac{qq}{r_{12}} + \frac{1}{4\pi\epsilon_0}\frac{qq}{r_{23}} + \frac{1}{4\pi\epsilon_0}\frac{qq}{r_{13}}$$

$$= \frac{1}{4\pi\epsilon_0}\frac{(5\times10^{-6}\,C)^2}{0.01\,m} + \frac{1}{4\pi\epsilon_0}\frac{(5\times10^{-6}\,C)^2}{0.01\,m} + \frac{1}{4\pi\epsilon_0}\frac{(5\times10^{-6}\,C)^2}{0.02\,m}$$

where $\epsilon_0 = 8.85\times10^{-12}\frac{C^2}{N\,m^2}$. *Then* $U_i = 22.48\,J + 22.48\,J + 11.24\,J = 56.2\,J.$

After the two outer charges are released, they move in opposite directions away from the central charge. The initial configuration was symmetric about the central charge and they were released at the same instant, so it follows from symmetry that they have equal speeds at any subsequent time. Then as $t \to \infty$, *the final kinetic energy is*

$$K_f = 2\left(\frac{1}{2}mv_f^2\right) = mv_f^2$$

Then from **(1)**, $mv_f^2 + 0 = 0 + U_i.$

$$v_f = \sqrt{\frac{U_i}{m}} = \sqrt{\frac{56.2\,J}{0.01\,kg}} = \underline{\underline{75.0\,m/s}}$$

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