Last time

- Applications of Ampere’s Law
- Solenoids
- Toroids

This time

- Faraday’s law of induction
- Lenz’s law

Faraday’s Law of Induction

Electrostatics: E-field from motionless charges
Magnetostatics: B-field from charges in motion

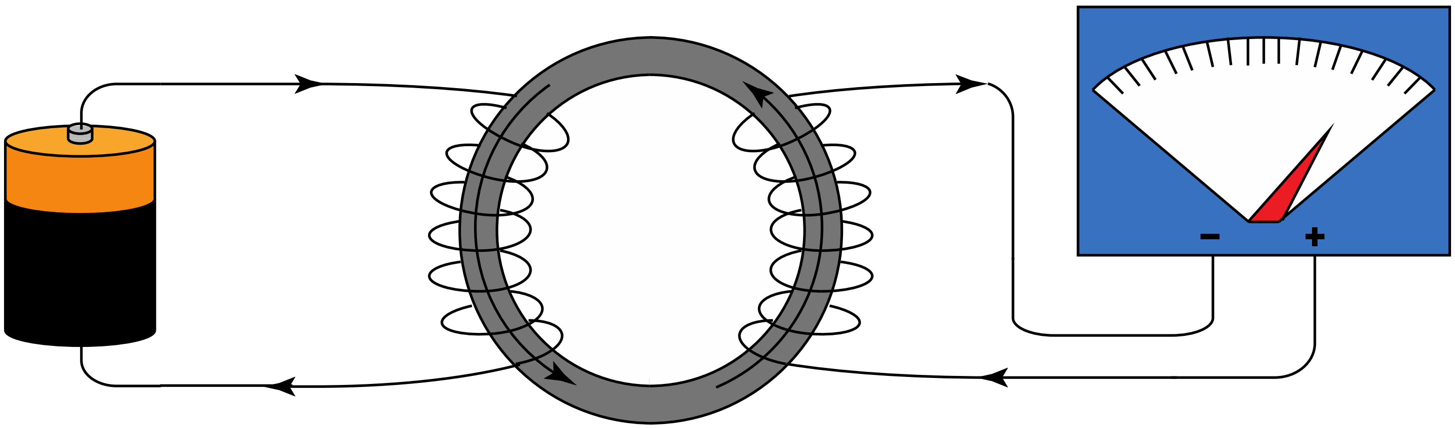
Changing electric fields (moving charges) create magnetic fields. Is the opposite true?

YES!

$$\boxed{\varepsilon = -\frac{d\Phi_M}{dt}}$$

i.e., A changing magnetic flux creates an induced EMF.

Faraday’s Initial Experiment (+ demo)



Faraday discovered that there is an **induced EMF** in the secondary circuit given by

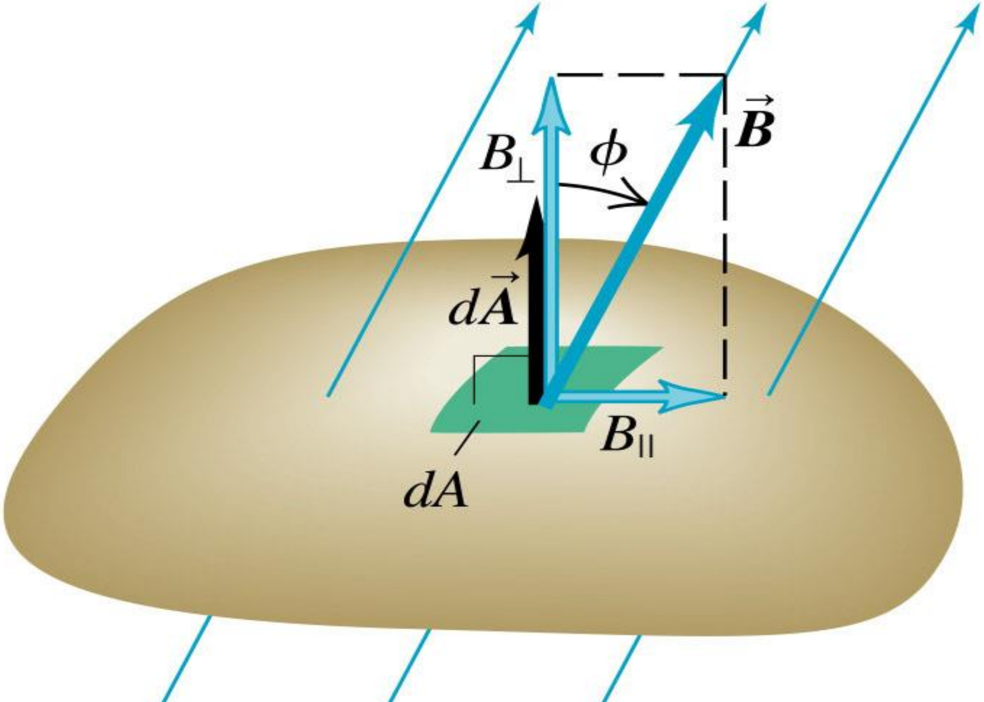
$$\varepsilon = -\frac{d\Phi_M}{dt}$$

This is a new generalized law called **Faraday's Law**.

Recall the definition of magnetic flux:

$$\Phi_M = \int \vec{B} \cdot d\vec{A}$$

Not a closed surface!



This is valid even if Φ_M changes because of a time dependent A or angle φ (without changing the magnetic field)!

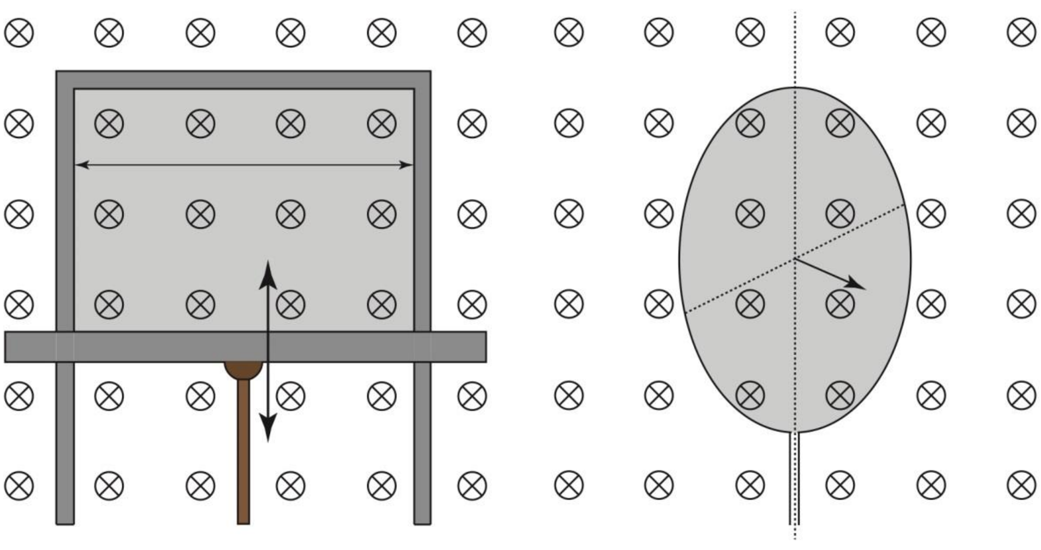
$$\boxed{\varepsilon = -\frac{d}{dt}\left(BA\cos\phi\right)}$$

→ **3 possible terms**

$$\varepsilon = -\frac{dB}{dt}A\cos\phi - \frac{dA}{dt}B\cos\phi + \frac{d\phi}{dt}BA\sin\phi$$

From Maxwell Eq.

$$-\frac{d\vec{B}}{dt} = \nabla \times \vec{E}$$

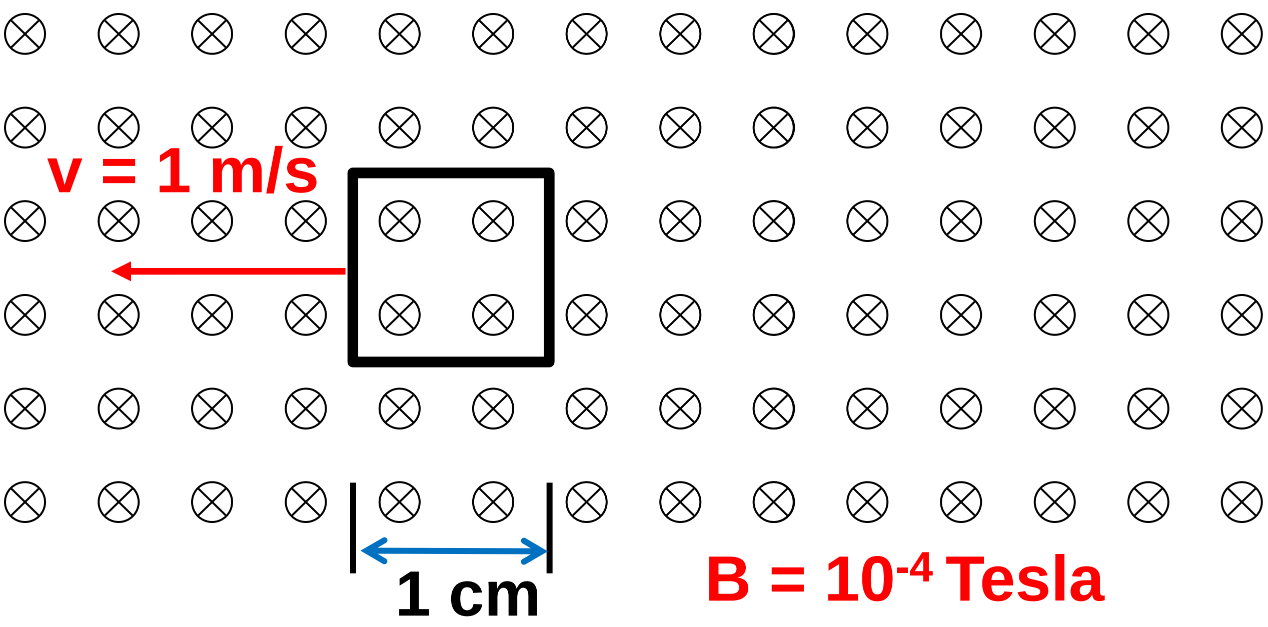


Top Hat Question

A square loop of wire with a resistance of 1 Ω is moving with a constant velocity of 1 m/s through a uniform magnetic field as shown. What is the current induced in the loop? Pick the closest answer
(Note: 1 Ampere = 1 Coulomb/sec)

The magnetic flux through the loop is not changing, so there is no induced emf and hence no induced current

A. 0 A
B. 0.001 A
C. 0.01 A
D. 0.1 A
E. 1 A

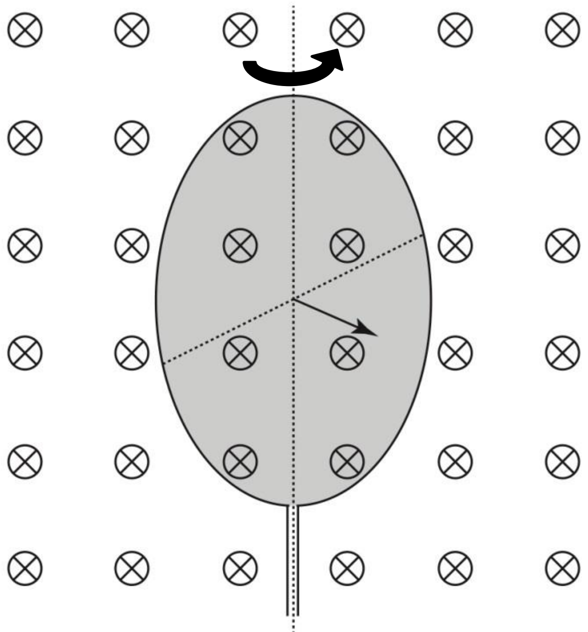


Top Hat Question

A loop of wire is spinning rapidly about a stationary vertical axis in a uniform B-field. Is there a current (or EMF) induced in the loop?

A. Yes, a DC current is induced
B. Yes, an AC (time varying) current is induced
C. The B-field is not changing, so no currents are induced

In this case, the flux through the loop is changing with time because of the **B•A** term, so there will be an induced current (or emf) in the loop. The normal vector is changing direction so half the time the flux is positive and half the time it is negative: i.e. an AC current is induced.

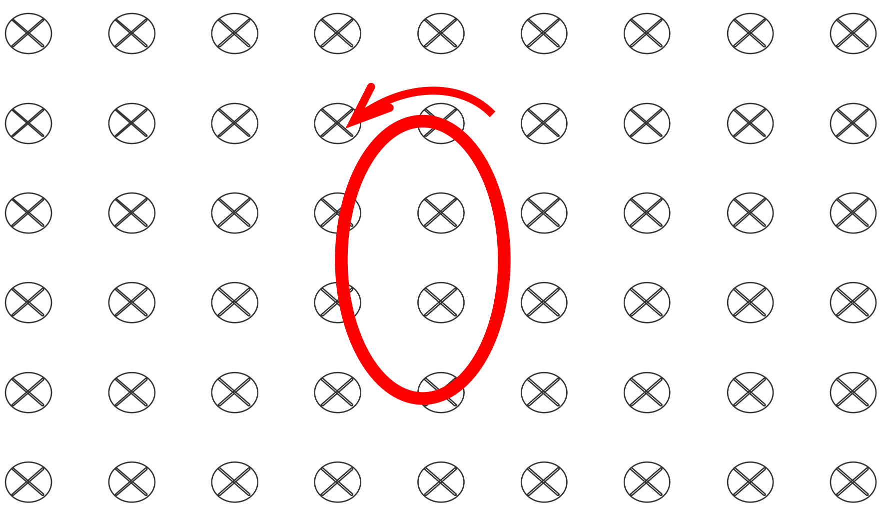


Top Hat Question

An oval shaped loop is spun around an axis pointing out of the page passing through the center of the loop. Is there a current (or EMF) induced in the loop?

A: Yes, there is

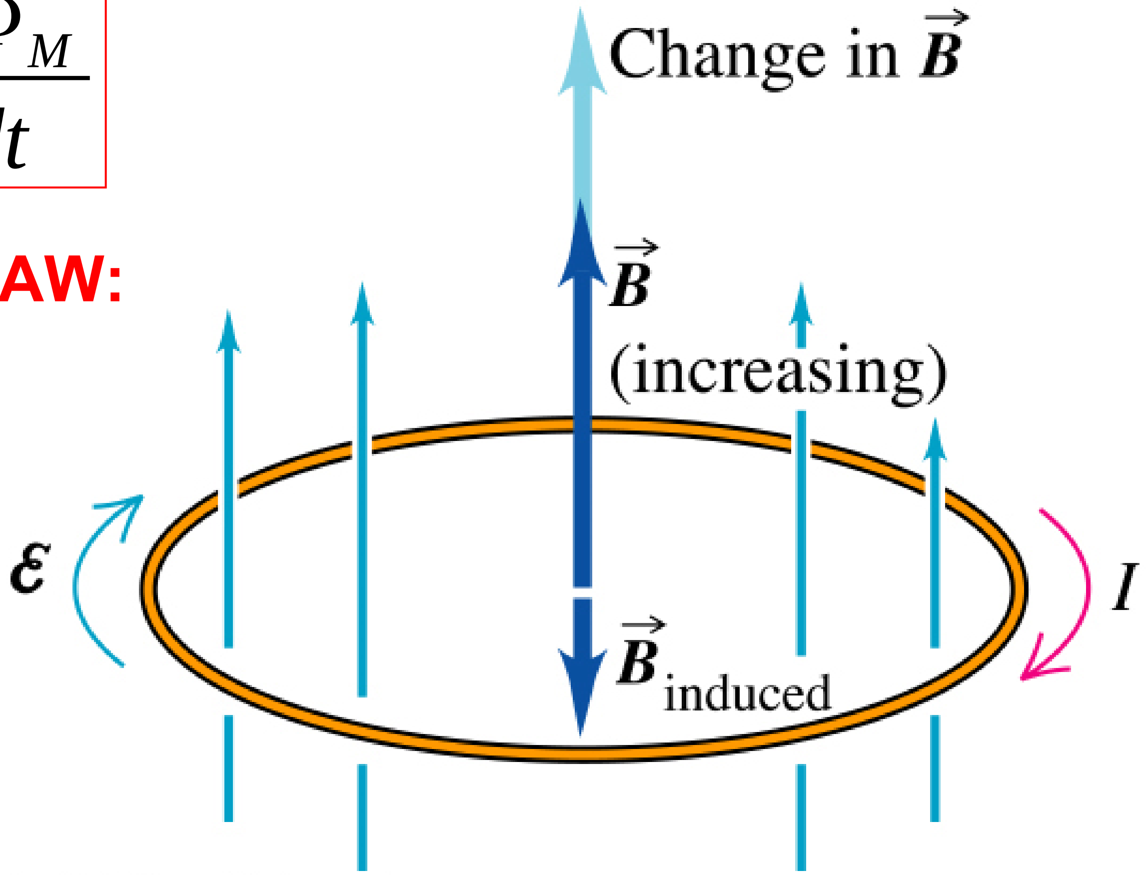
B: No, there is not



What about the minus sign in Faraday's law?

$$\boxed{\varepsilon = -\frac{d\Phi_M}{dt}}$$

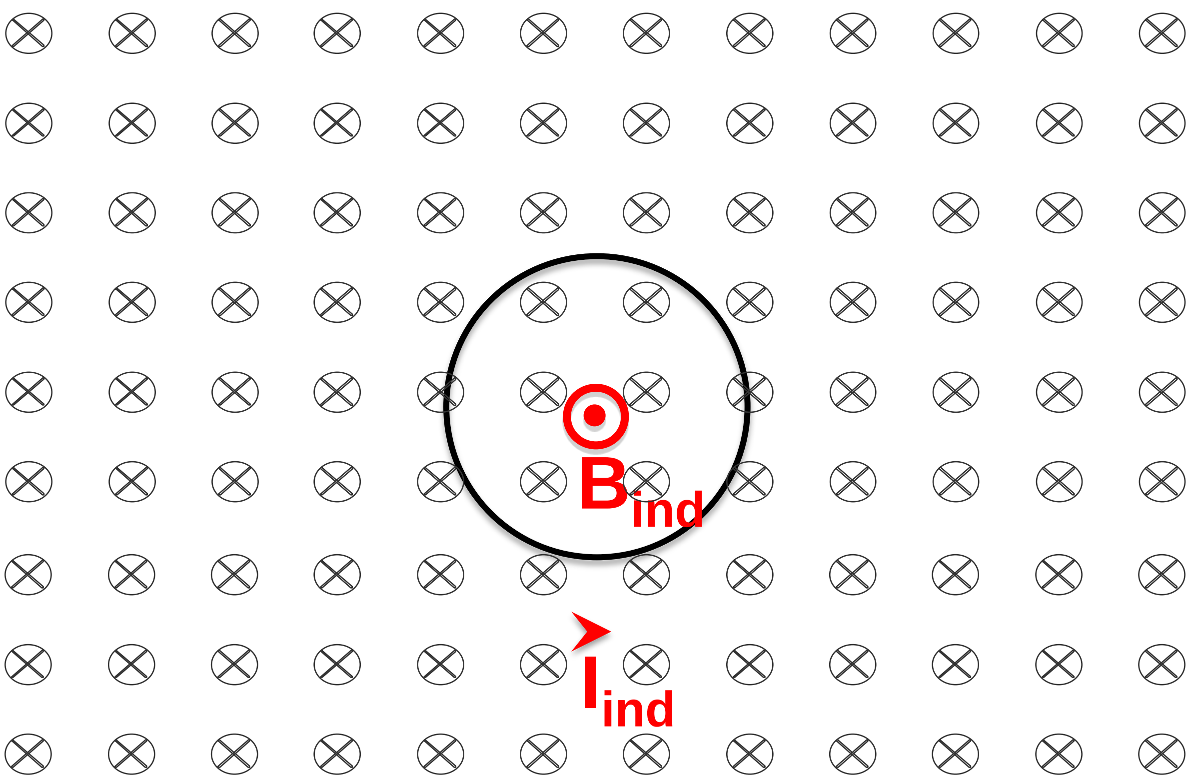
LENZ'S LAW:



The changing magnetic flux generates an induced current which creates an induced magnetic field which, in turn, resists the change in magnetic flux.

Lenz's Law

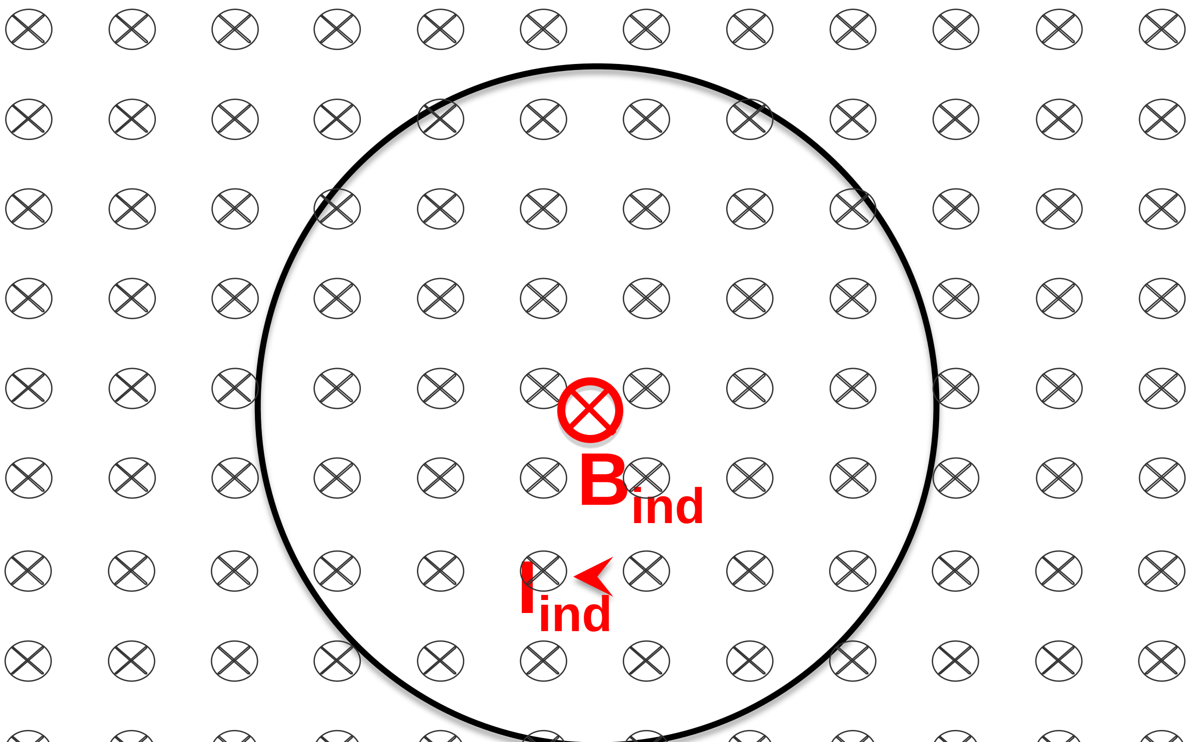
The induced current from Faraday's Law is always in a direction such that the induced magnetic field from the induced current opposes the change in the magnetic flux through the loop.



More B-field lines inside the loop: induced B-field from induced current must be out of the page to compensate.
Induced current is CCW

Lenz's Law

The induced current from Faraday's Law is always in a direction such that the induced magnetic field from the induced current opposes the change in the magnetic flux through the loop.



Fewer B-field lines inside the loop: induced B-field from induced current must be into the page to compensate.
Induced current is CW

How to find the direction of an induced emf?

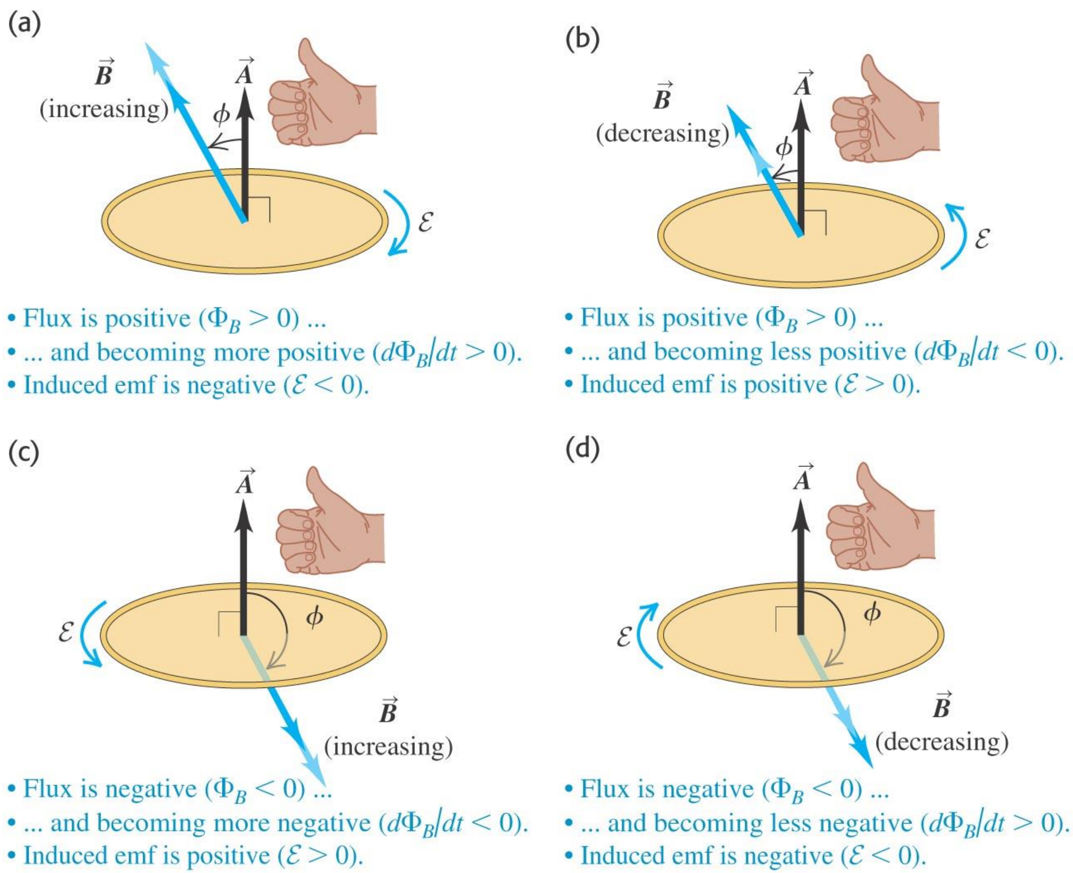
$$\varepsilon = -\frac{d\Phi_B}{dt}$$

Faraday's law of induction

Define a direction for $\vec{A}$.

Determine the sign of flux from the directions of area vector and magnetic field.

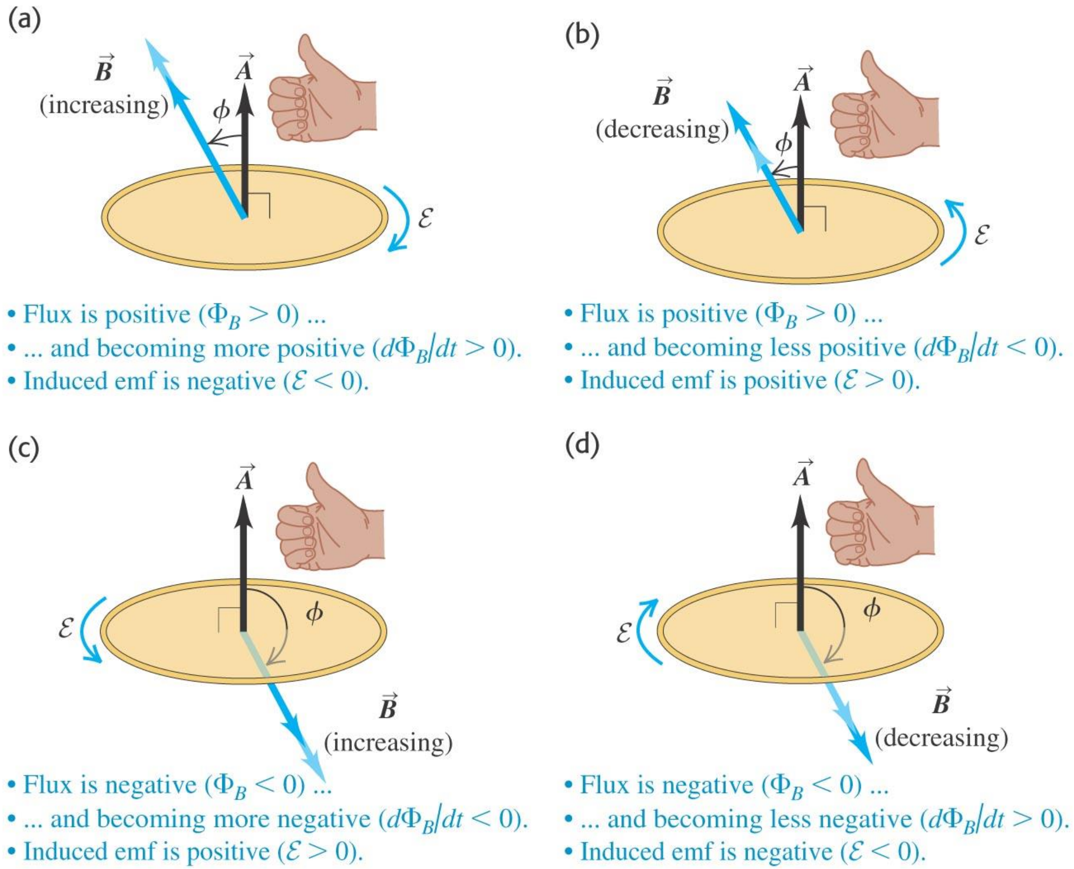
Determine the rate of change of flux. Is it increasing or decreasing? If flux is increasing the induced emf is negative. If flux is decreasing the induced emf is positive.



How to find the direction of an induced emf?

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

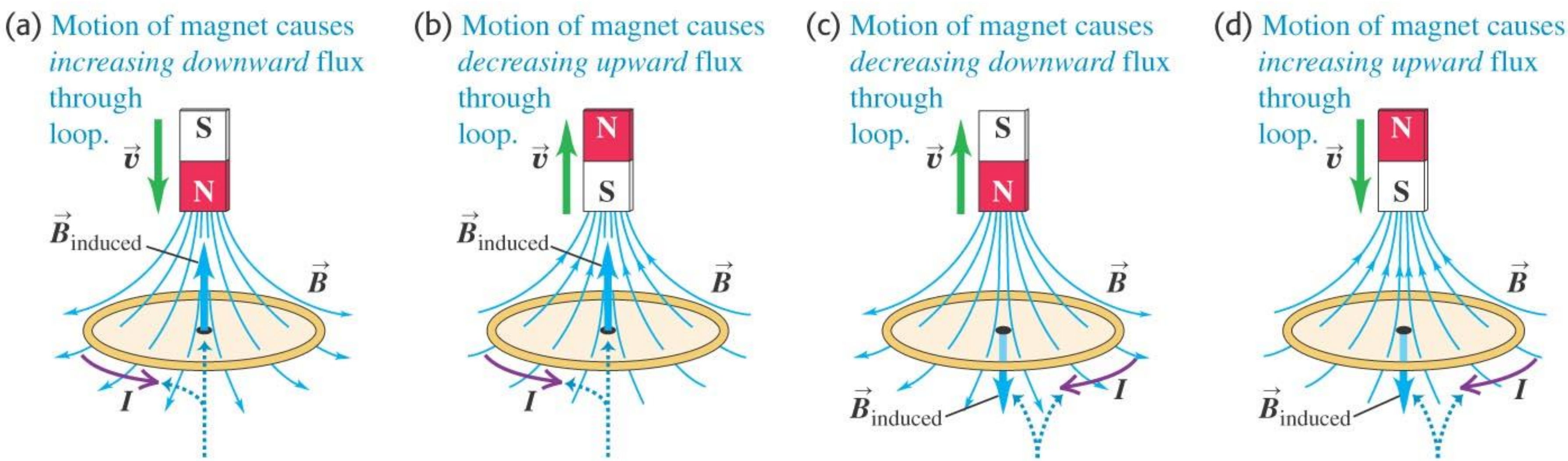
Faraday's law of induction



Finally determine the direction of current using the right hand rule by curling your fingers around the area vector with your thumb in the direction of the area vector. If the induced emf or current in the circuit is positive, it is in the same direction as your curled fingers; if the induce emf or current is negative, it is in the opposite direction.

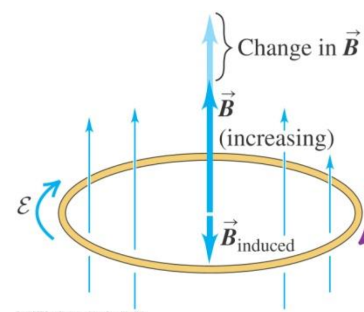
Lenz's Law

The direction of any magnetic induction is such as to oppose the cause of the effect.



The induced magnetic field is *upward* to oppose the flux change. To produce this induced field, the induced current must be *counterclockwise* as seen from above the loop.

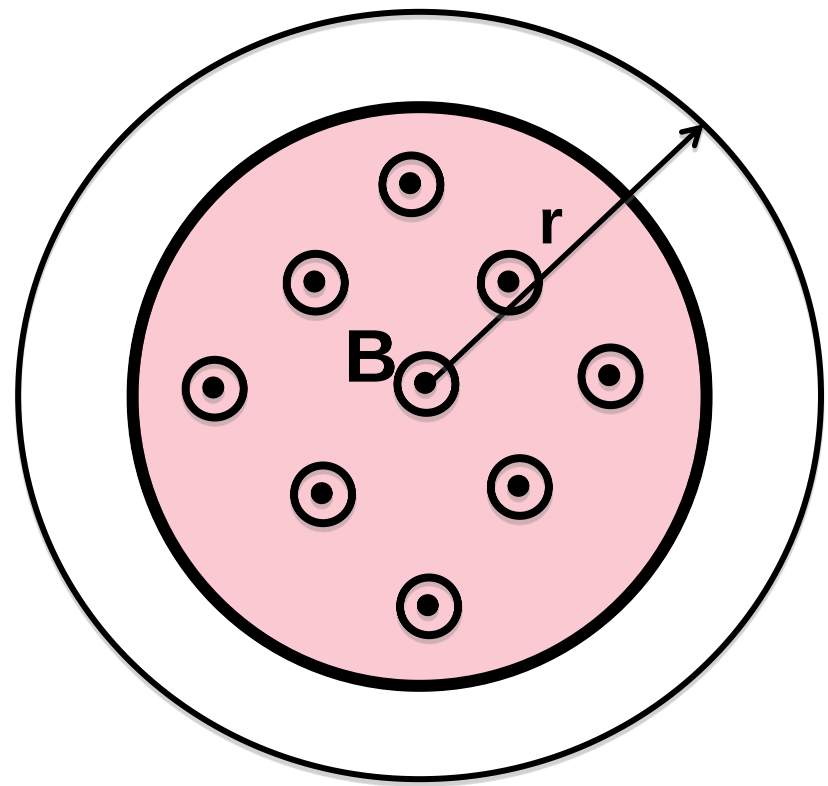
The induced magnetic field is *downward* to oppose the flux change. To produce this induced field, the induced current must be *clockwise* as seen from above the loop.



Top Hat Question

The current in an infinitely long solenoid with uniform magnetic field B inside is increasing so that the magnitude B increases in time as $B=B_0+kt$. A circular loop of radius r is placed coaxially outside the solenoid as shown. In what direction is the induced **E**-field around the loop ?

A. CW
B. CCW
C. The induced E is zero
D. Not enough information



Lenz' law: induced EMF around the loop is in the CW direction. The induced E-field must therefore be in the CW direction