



solid insulating ball

$$\rho = \frac{Q}{\frac{4}{3}\pi R^3}$$

$$E_r = \frac{\rho r}{3\epsilon_0}$$

$$V_{in}(r) = -\int_0^r \frac{\rho r}{3\epsilon_0} dr \quad (\vec{E} \cdot d\vec{r} = E_r dr = \frac{\rho r}{3\epsilon_0} dr)$$

$$= -\frac{\rho}{3\epsilon_0} \frac{1}{2} r^2 \Big|_0^r + V_0$$

$$V_{in}(r) = -\frac{\rho}{3\epsilon_0} \frac{r^2}{2} + V_0$$

potential has to be continuous.

V_0 is because we set $V=0$ at $r=\infty$

$$\text{At } r=R: \quad V_s(R) = \frac{1}{4\pi\epsilon_0} \frac{Q}{R^2} = \frac{1}{4\pi\epsilon_0} \frac{\frac{4}{3}\pi R^3 \rho}{R^2}$$

$$V_{in}(R) = V_s(R)$$

$$-\frac{\rho}{3\epsilon_0} \frac{R^2}{2} + V_0 = \frac{\rho}{3\epsilon_0} R^2 \Rightarrow V_0 = \frac{\rho}{2\epsilon_0} R^2$$

$$\boxed{V_{in}(r) = \frac{\rho}{2\epsilon_0} R^2 - \frac{\rho}{6\epsilon_0} r^2}$$

