Electricity and Magnetism

- Physics 259 – L02
- Lecture 41

UNIVERSITY OF CALGARY

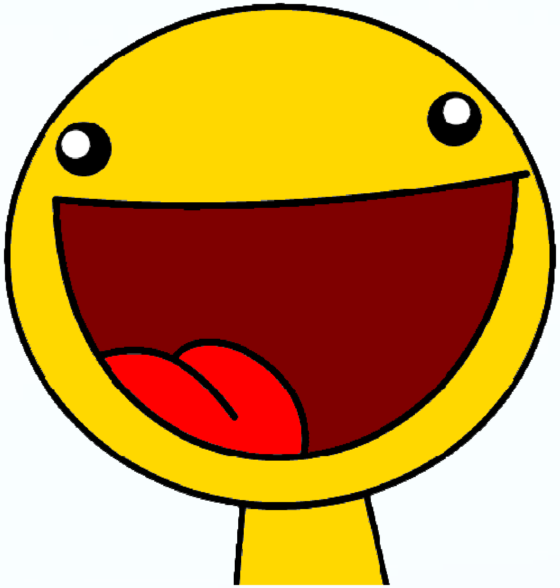
Chapter 29: magnetic fields due to currents

Last time:

- Chapter 28

Today:

- Biot-Savart Law (like Coulomb's Law for magnetism)
- B-field of a line of current
- Magnetic force between parallel current-carrying wires



For a single charge→ $\boxed{\vec{F}_B = q\vec{v}_d \times \vec{B}}$

For N charges moving through the wire (current carrying wire)→ $\boxed{\vec{F}_B = i\vec{\ell} \times \vec{B}}$

The Biot-Savart Law

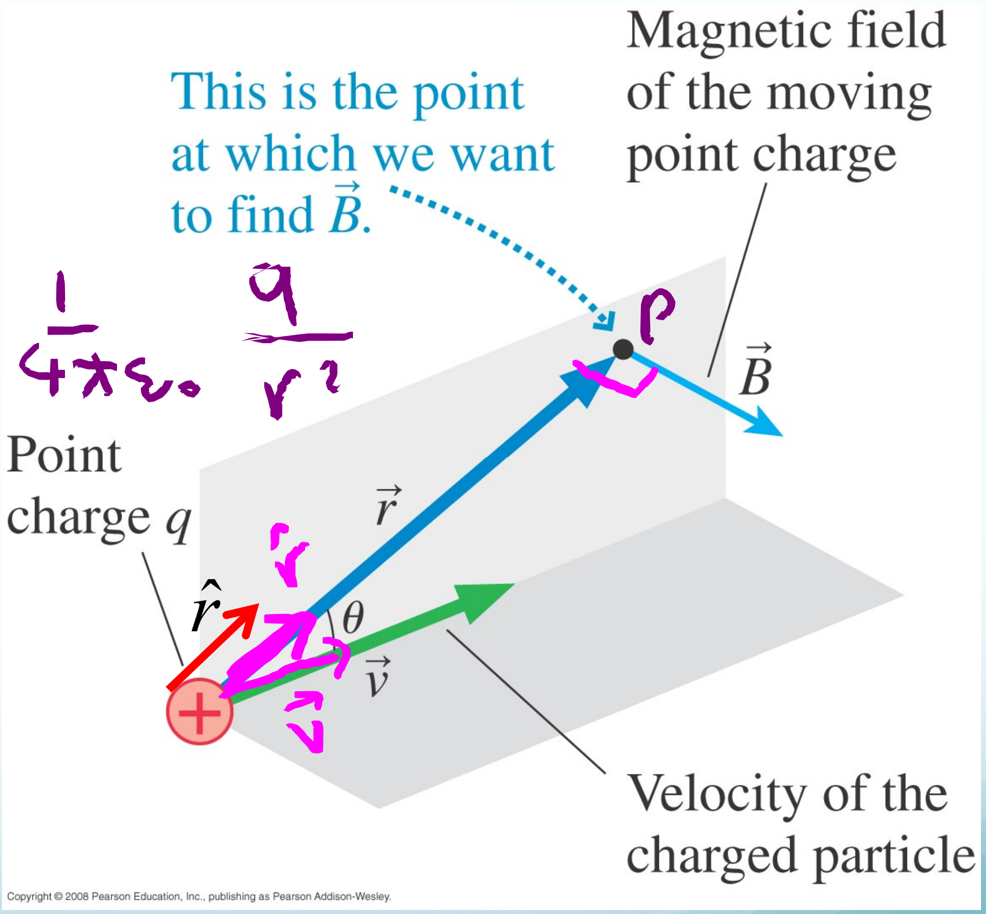
Magnetic fields are caused by **moving charges**.*

$$\boxed{\vec{B}_{\text{point charge}} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2}}$$

Or, using the definition $\hat{r} = \dfrac{\vec{r}}{|\vec{r}|}$

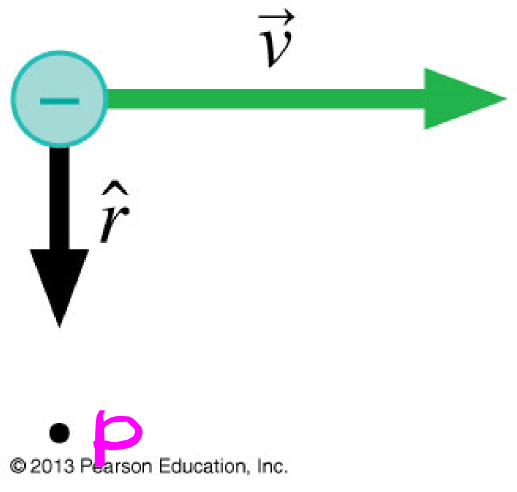
$$\boxed{\vec{B}_{\text{point charge}} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\vec{r}}{r^3}}$$

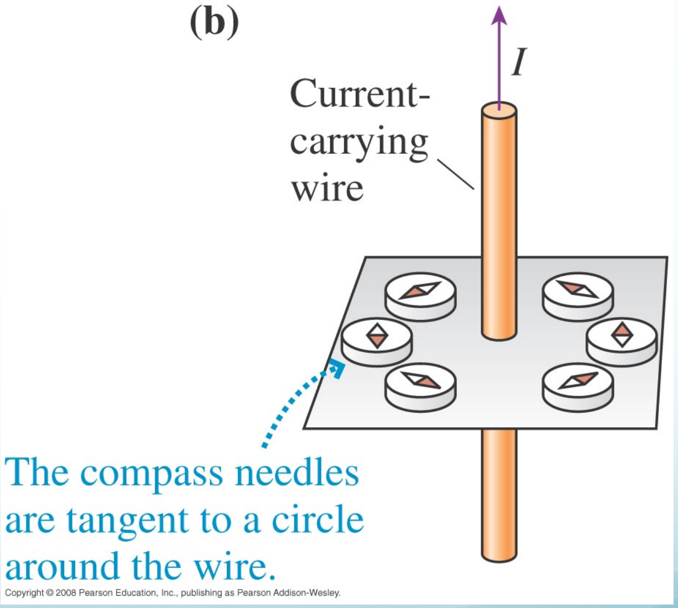
$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$



*One exception is due to quantum mechanics: charged particles with "spin" produce B fields

$$\vec{B}_{\text{point charge}} = \frac{\mu_o}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2}$$

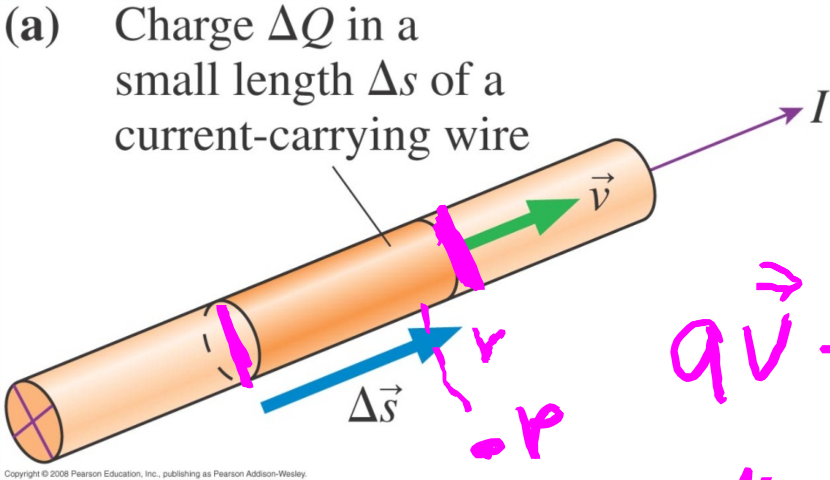




Current?

A bunch of moving charges

For a whole bunch of moving charges (an electric current)?



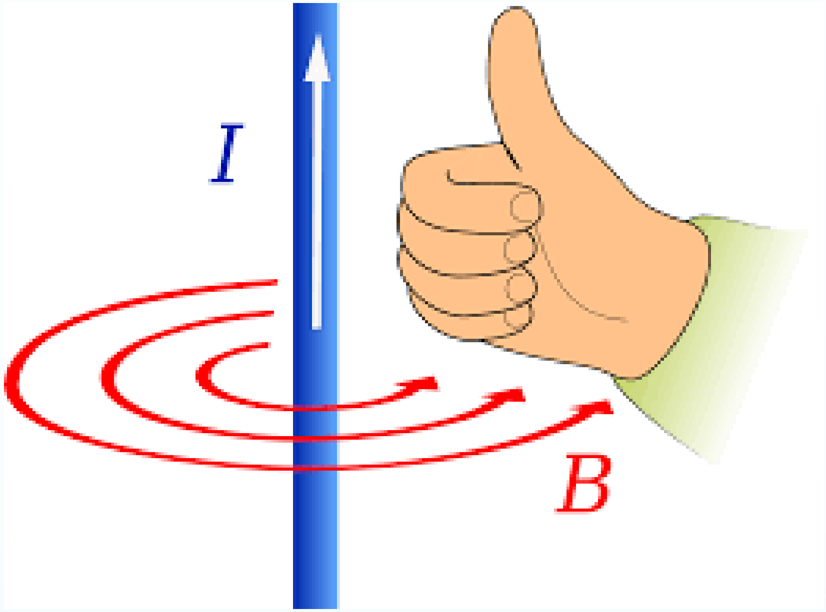
$$\rightarrow \vec{B}_{\text{point charge}} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\vec{r}}{r^3}$$

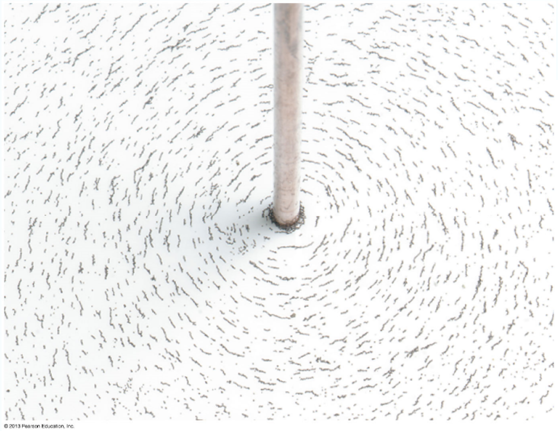
$$q\vec{v} \rightarrow \quad \Delta Q\,\vec{v} = \Delta Q\frac{\Delta\vec{s}}{\Delta t} = \frac{\Delta Q}{\Delta t}\Delta\vec{s} = I\Delta\vec{s}$$

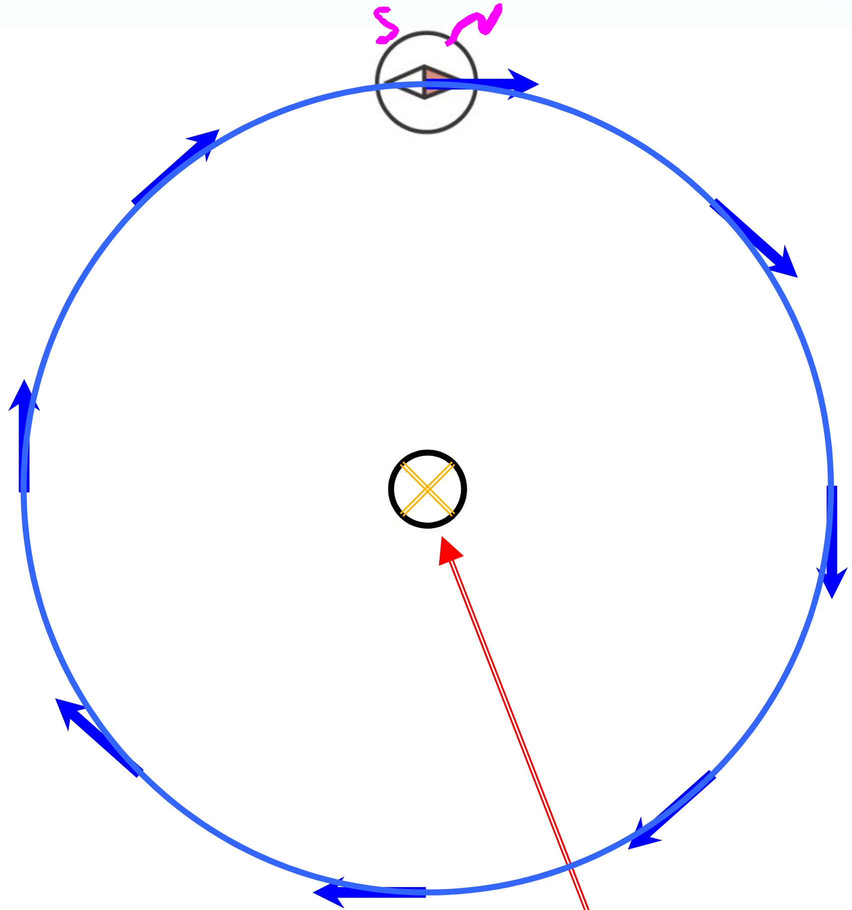
$$\rightarrow \vec{B}_{\text{current}} = \frac{\mu_0}{4\pi}\frac{I\Delta\vec{s}\times\vec{r}}{r^3} = \frac{\mu_0}{4\pi}\frac{I\Delta\vec{s}\times\hat{r}}{r^2}$$

$$\boxed{\vec{B}_{\text{current segment}} = \frac{\mu_0}{4\pi}\frac{I\,\Delta\vec{s}\times\hat{r}}{r^2}}$$

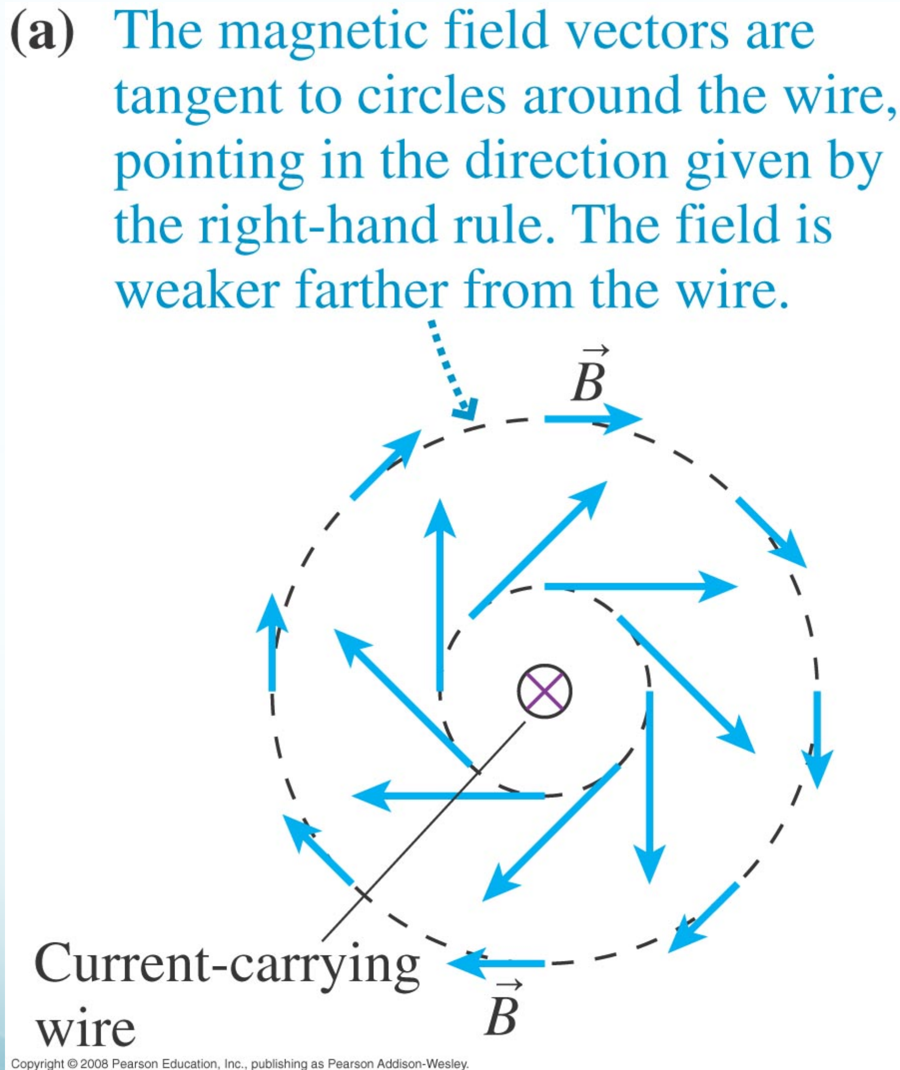
Right hand rule







The magnetic field vector points in the direction of the north pole of the compass magnet.


(a) The magnetic field vectors are tangent to circles around the wire, pointing in the direction given by the right-hand rule. The field is weaker farther from the wire.
$\vec{B}$
$\vec{B}$
Current-carrying wire
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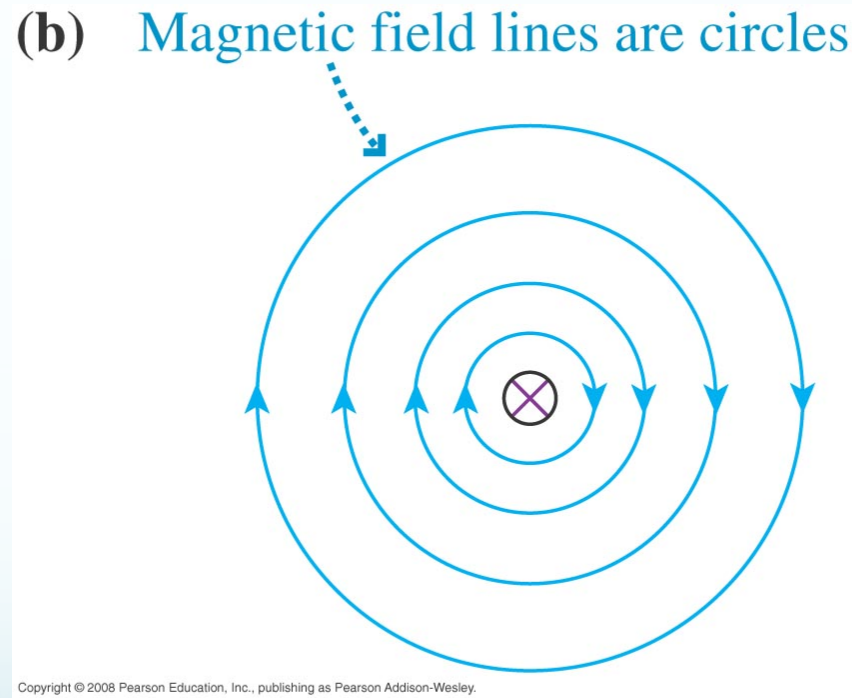
(b) Magnetic field lines are circles.
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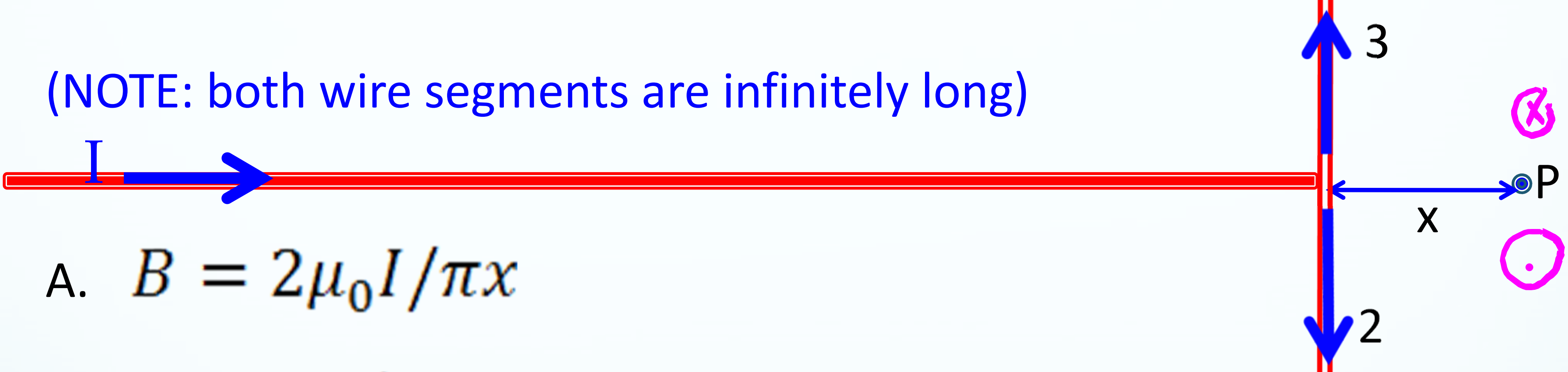
TopHat Question

A wire carries current I into the junction and splits equally.
What is the magnitude of the B-field at point P?

(NOTE: both wire segments are infinitely long)



A. $B = 2\mu_0 I/\pi x$

B. $B = \mu_0 I/\pi x$

C. $B = \mu_0 I/2\pi x$

D. $B = \mu_0 I/4\pi x$

E. $B = 0$

90%

TopHat Question

A long straight wire carries current I out of the page. An electron travels towards the wire from the right. What is the direction of the force on the electron?

$$\boxed{\vec{F} = q\vec{v} \times \vec{B}}$$

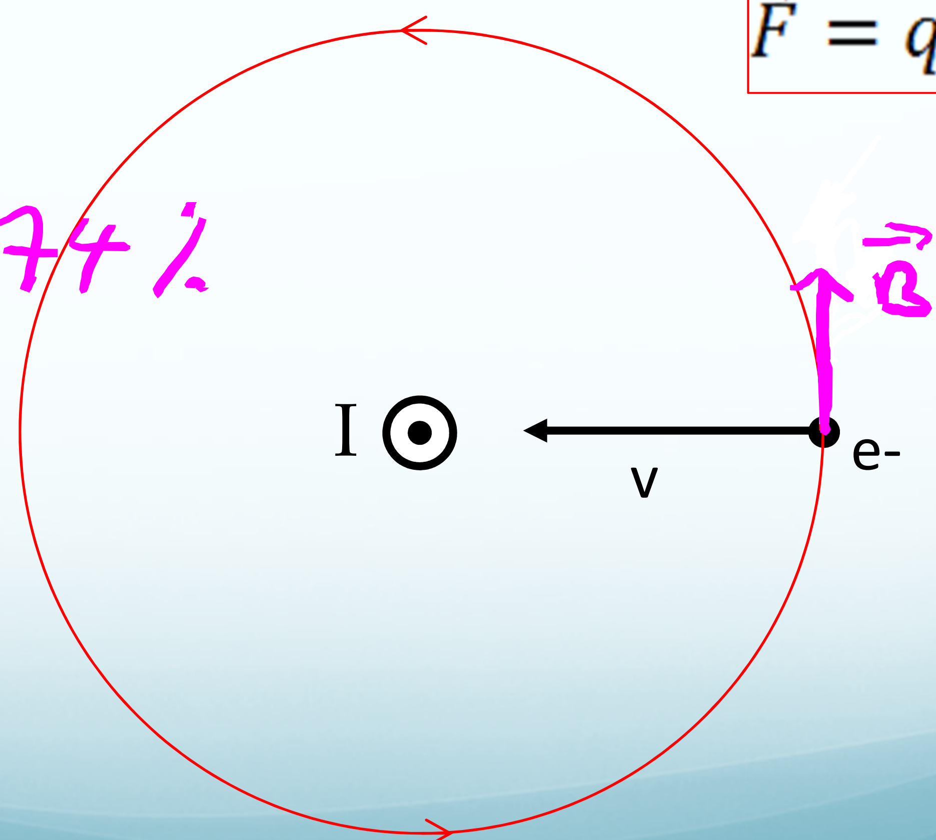
A. $\otimes$

B. $\odot$ 74%

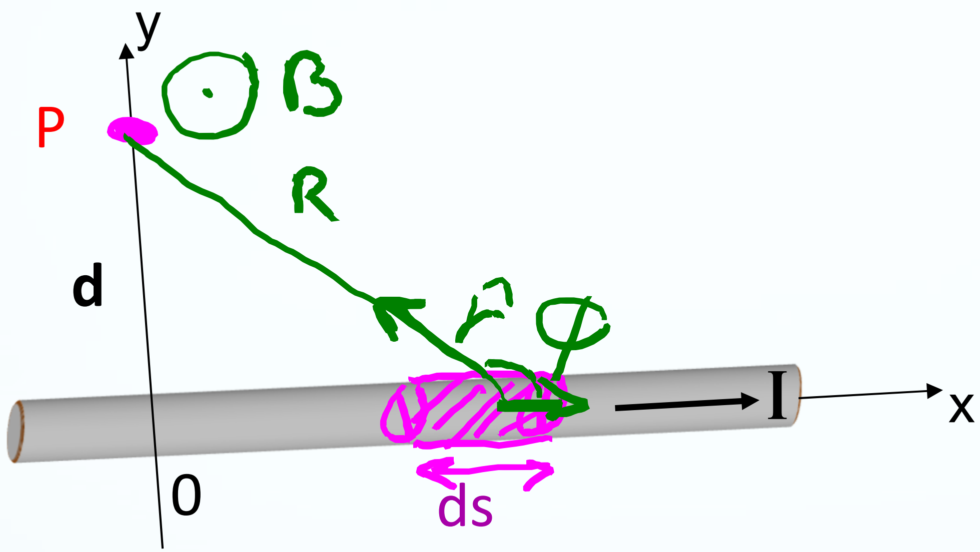
C. $\leftarrow$

D. $\rightarrow$

E. $\downarrow$



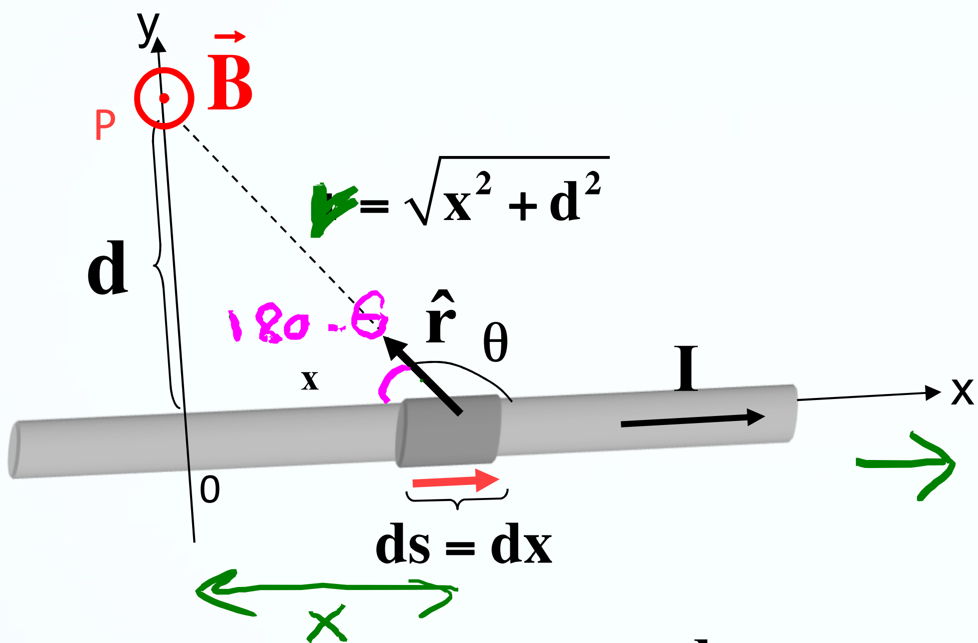
Magnetic field due to current in long straight wire



$$\vec{B} = \frac{\mu_0}{4\pi}\frac{I\Delta\vec{s}\times\hat{r}}{r^2}$$

$$\text{Segment} \rightarrow d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{s}\times\hat{r}}{R^2}$$

$$\rightarrow d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,ds\,\overbrace{|\hat{r}|}^{1}\sin\phi}{R^2}$$



$$\rightarrow dB = \frac{\mu_0}{4\pi}\frac{I dx \sin\theta}{r^2}$$

$$dB_x = dB_y = 0$$

$$dB_z = \checkmark$$

$$\sin\theta = \sin(180° - \theta) = \frac{d}{\sqrt{x^2 + d^2}}$$

$\&\ r^2 = x^2 + d^2$

$$\Rightarrow dB_z = \frac{\mu_0}{4\pi}\frac{I dx\, d}{\sqrt{x^2+d^2}\,(x^2+d^2)} = \frac{\mu_0}{4\pi}\frac{d\, I dx}{(x^2+d^2)^{3/2}}$$

$$\Rightarrow dB = \frac{\mu_0}{4\pi}\frac{I dx}{x^2 + d^2}\frac{d}{\sqrt{x^2 + d^2}} \quad \Rightarrow dB = \frac{\mu_0 I d}{4\pi}\frac{dx}{\left(x^2 + d^2\right)^{3/2}}$$

Integrate components of dB

$$dB = \frac{\mu_0 Id}{4\pi}\frac{dx}{\left(x^2+d^2\right)^{3/2}}$$

$$B_z = \int_{-\infty}^{\infty} dB_z = \int_{-\infty}^{\infty} \frac{\mu_0}{4\pi} Id \frac{dx}{(x^2+d^2)^{3/2}} = \frac{\mu_0}{4\pi} Id \int_{-\infty}^{\infty} \frac{dx}{(x^2+d^2)^{3/2}}$$

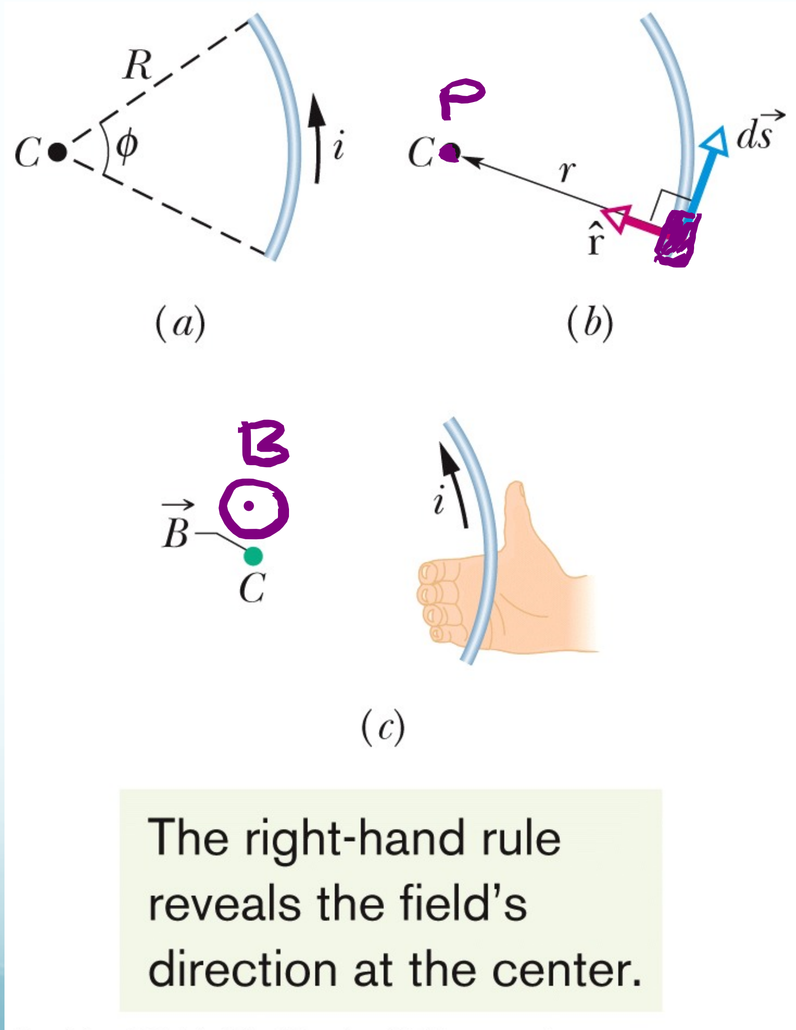
$$= \frac{\mu_0 Id}{4\pi}\frac{x}{d^2\left(x^2+d^2\right)^{1/2}}\Bigg|_{-\infty}^{\infty} = 2\times \frac{\mu_0 I \cancel{d}}{d^{\cancel{2}}\, 4\pi} \frac{x}{(x^2+d^2)^{1/2}}\Bigg|_0^\infty$$

$$\boxed{B_z = \frac{\mu_0}{2\pi}\frac{I}{d}}$$, tangent to a circle around the wire in the right-hand direction

Semi infinite wire → $B_z = \frac{1}{2}\frac{\mu_0}{2\pi}\frac{I}{d}$

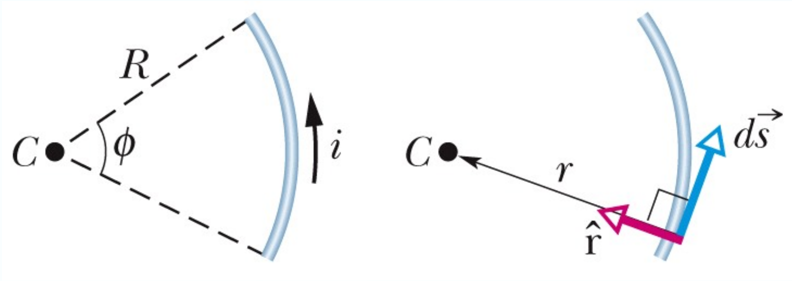
Magnetic field due to a current in a circular arc of wire

The magnitude of the **magnetic field at the center of a circular arc**, of radius *R* and central angle ϕ (in radians), carrying current *I* →



$$dB = \frac{\mu_0}{4\pi} \frac{I\,d\vec{s} \times \hat{r}}{r^2}$$

$$\rightarrow dB = \frac{\mu_0}{4\pi} \frac{I\,ds\,|\hat{r}|\sin\phi}{R^2}$$

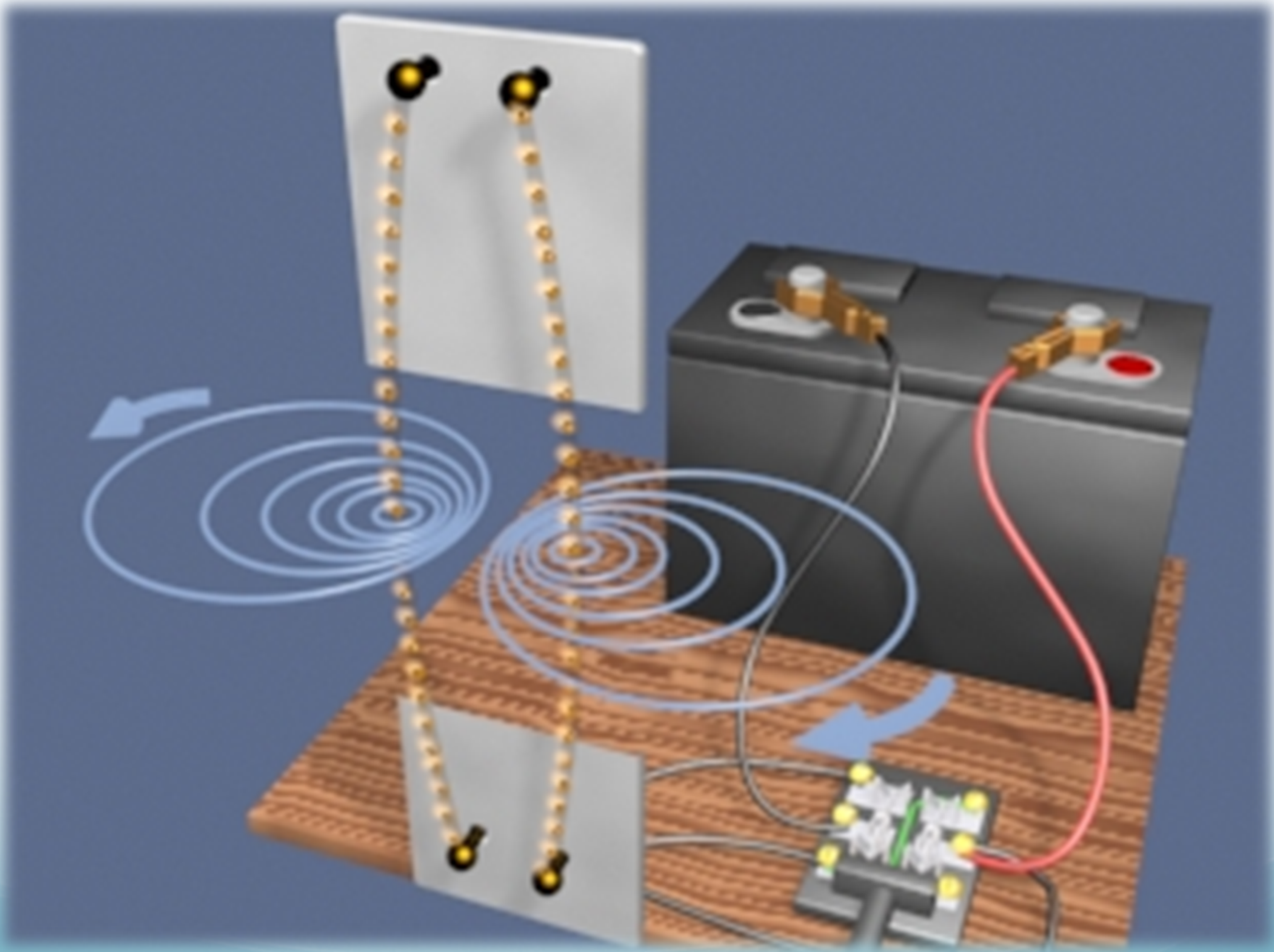


ϕ between $\hat{r}$ and ds

$$B = \int dB = \int_0^{\phi} \frac{\mu_0}{4\pi} \frac{iR\,d\phi}{R^2}$$

$$B = \frac{\mu_0 i \phi}{4\pi R}$$

29.2: Force between two antiparallel currents



https://tinyurl.com/lghkxt8

TopHat Question

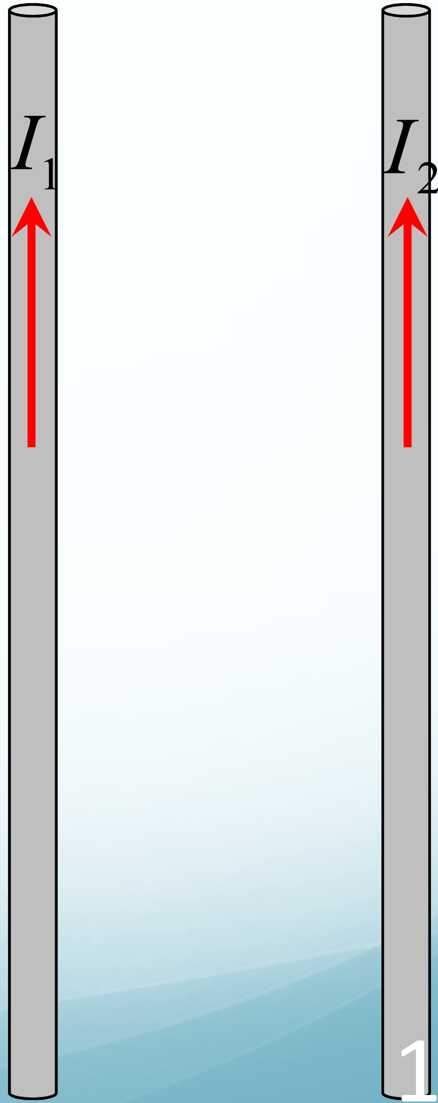
Two wires carry currents I_1 and I_2 as shown. What direction is the magnetic field produced by wire 2 at the location of wire 1?

A. Downward

B. Upward

C. Into the page

D. Out of the page



TopHat Question

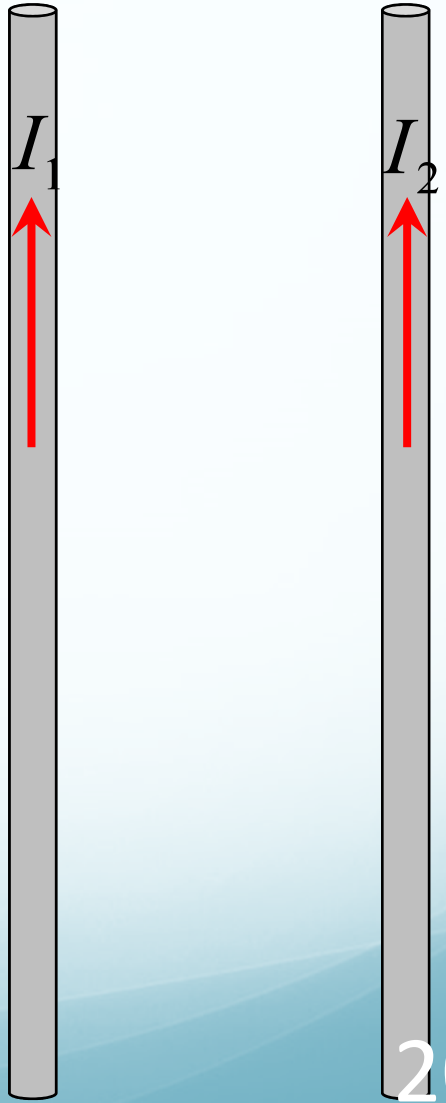
Two wires carry currents I_1 and I_2 as shown. What direction is the force of wire 2 on wire 1?

A. Left

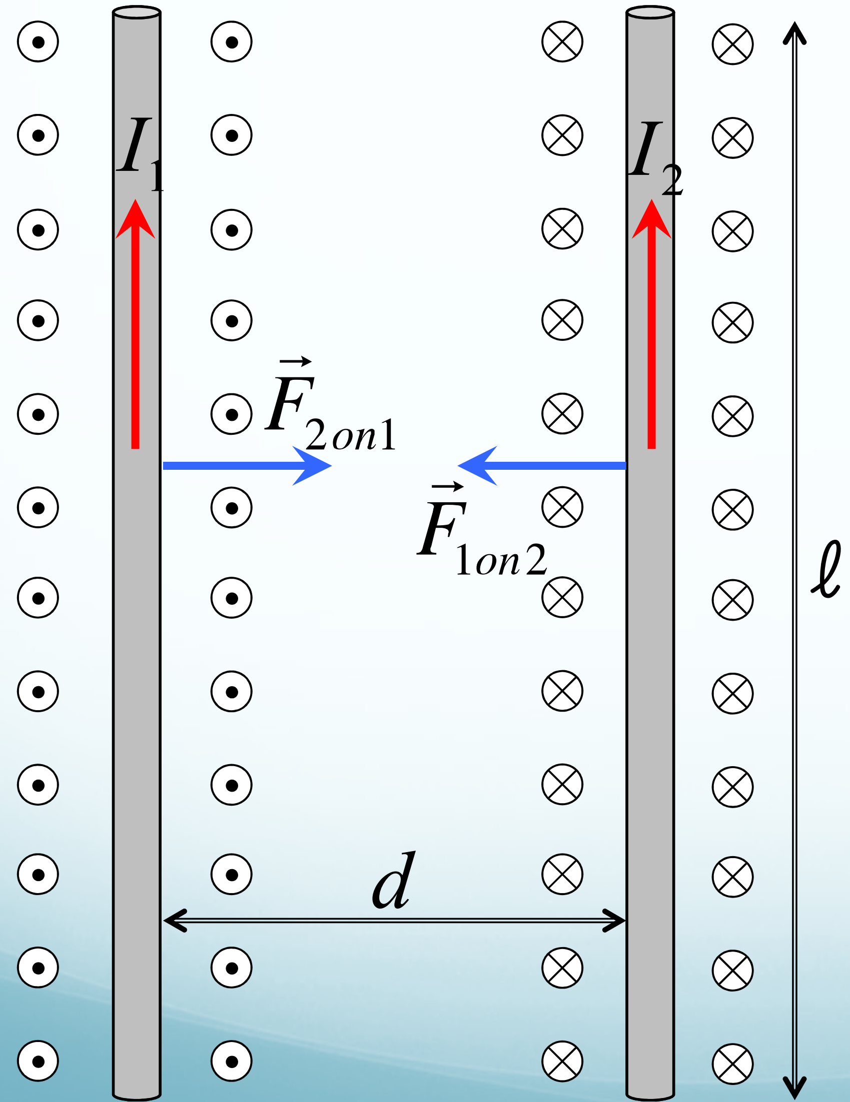
B. Right

C. Up

D. Down



$$\left|\vec{B}_2\right| = \frac{\mu_0 I_2}{2\pi d} \qquad \left|\vec{B}_1\right| = \frac{\mu_0 I_1}{2\pi d}$$



Wire 2 exerts a force on wire 1

$$\vec{F}_{2on1} = I_1 \vec{\ell} \times \vec{B}_2$$

$$\left|\vec{F}_{2on1}\right| = I_1 \ell \frac{\mu_0 I_2}{2\pi d} = \boxed{\frac{\mu_0 \ell I_1 I_2}{2\pi d}}$$

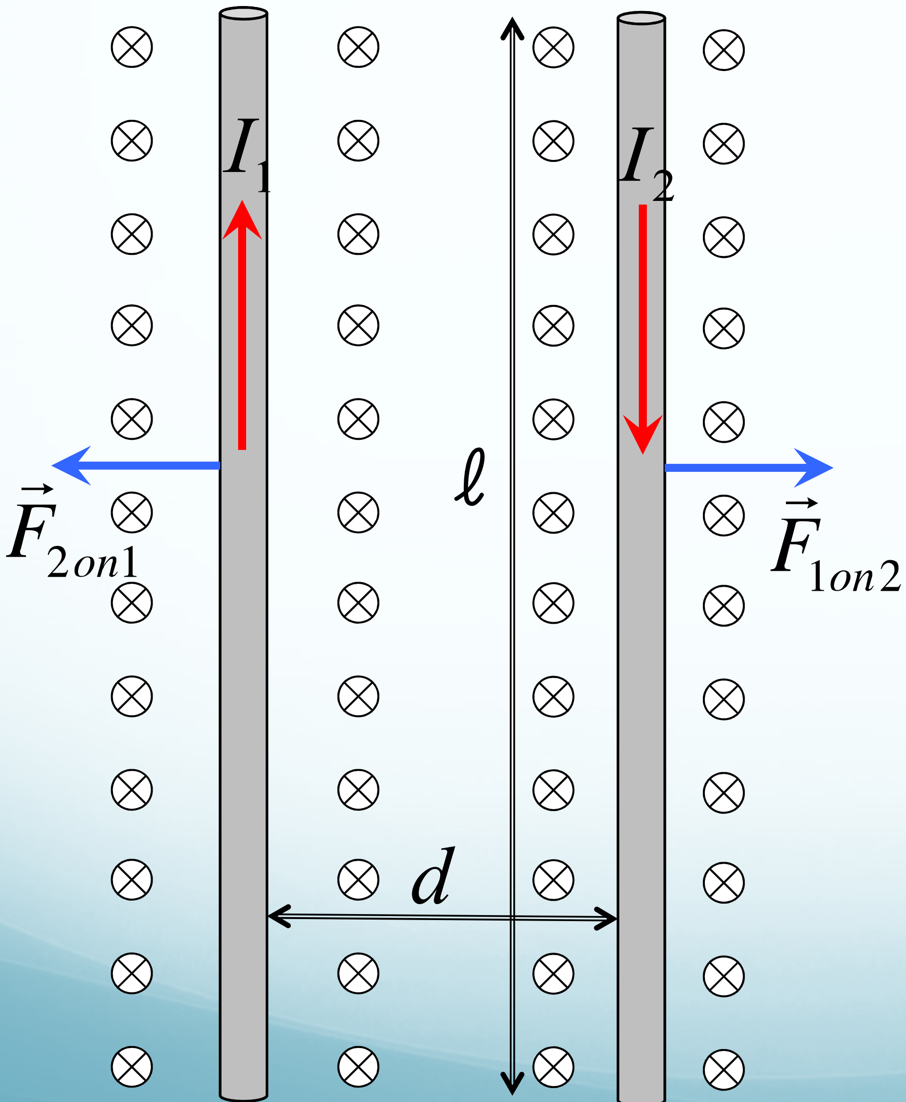
Wire 1 exerts a force on wire 2

$$\vec{F}_{1on2} = I_2 \vec{\ell} \times \vec{B}_1$$

$$\left|\vec{F}_{1on2}\right| = I_2 \ell \frac{\mu_0 I_1}{2\pi d} = \boxed{\frac{\mu_0 \ell I_1 I_2}{2\pi d}}$$

Newton's third law!

$$\left|\vec{B}_2\right| = \frac{\mu_0 I_2}{2\pi d} \qquad \left|\vec{B}_1\right| = \frac{\mu_0 I_1}{2\pi d}$$



Wire 2 exerts a force on wire 1

$$\vec{F}_{2on1} = I_1 \vec{\ell} \times \vec{B}_2$$

$$\left|\vec{F}_{2on1}\right| = I_1 \ell \frac{\mu_0 I_2}{2\pi d} = \boxed{\frac{\mu_0 \ell I_1 I_2}{2\pi d}}$$

Wire 1 exerts a force on wire 2

$$\vec{F}_{1on2} = I_2 \vec{\ell} \times \vec{B}_1$$

$$\left|\vec{F}_{1on2}\right| = I_2 \ell \frac{\mu_0 I_1}{2\pi d} = \boxed{\frac{\mu_0 \ell I_1 I_2}{2\pi d}}$$

Newton's third law!

This section we talked about:

Chapter 29

See you on Wednesday