Electricity and Magnetism

- Physics 259 – L02
- Lecture 15

Chapter 23.2 and 23.3

$$\varphi_e = \oint \vec{E} \cdot d\vec{A} = \frac{q_{enc.}}{\varepsilon_0}$$

$$\vec{E} \cdot \vec{A}$$

Last time

- Chapter 23.1
- Chapter 23.2

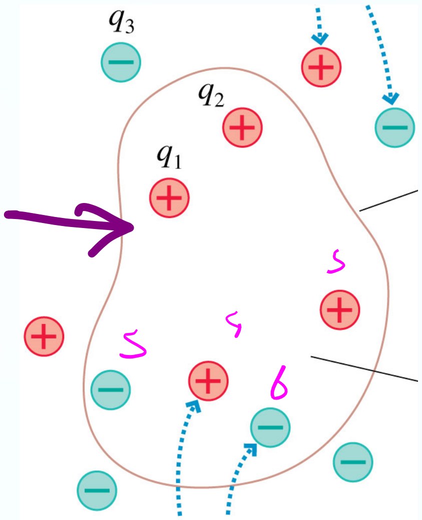
This time

- Chapter 23.2: Continue
- Chapter 23.3

Multiple charges

$$\Phi_e = \oint \vec{E}.d\vec{A} = \frac{q_1}{\varepsilon_0} + \frac{q_2}{\varepsilon_0} + \dots + \frac{q_6}{\varepsilon_0} = \frac{Q_{net}}{\varepsilon_0} \rightarrow q_{enc}$$

Gaussian s



$$\Phi_e = \left(\frac{q_1}{\varepsilon_0} + \frac{q_2}{\varepsilon_0} + \dots \text{for all charges inside the surface}\right)$$

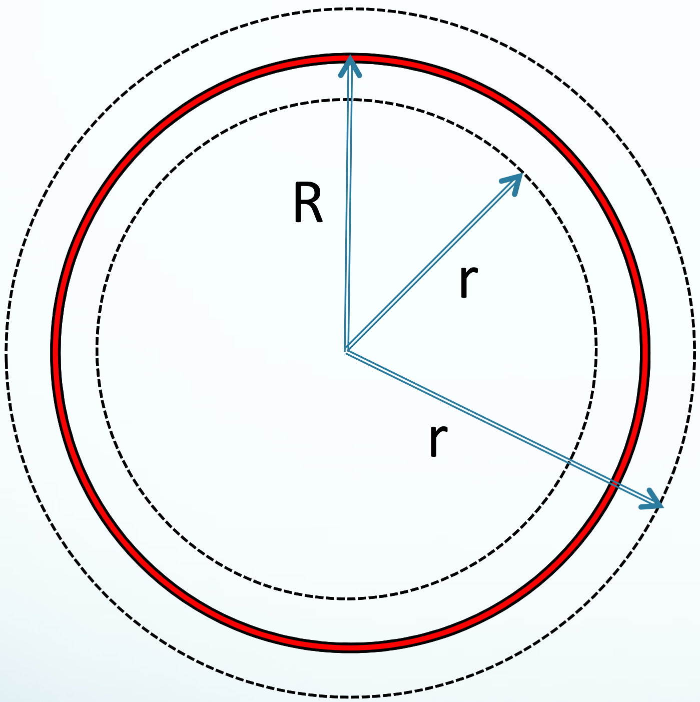
$$+ (0 + 0 + \dots \text{for all charges outside the surface})$$

$$\boxed{\Phi_e = \oint \vec{E}.d\vec{A} = \frac{Q_{in}}{\varepsilon_0}} \rightarrow q_{enc}$$

→ enclosing q of Gaussian surface

Using Gauss' Law

Shell of charge



Task

Use Gauss' law to compute the E field inside and outside a spherical shell of charge

Symmetry argument:

1. Electric field must point in the **radial direction** only.
2. The electric field must be the **same magnitude** at constant radius.

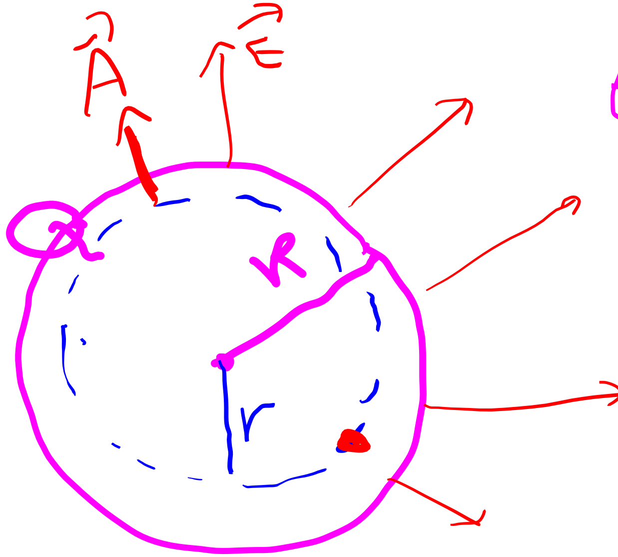
$$\Phi_E = \oint \vec{E} \cdot (d\vec{A}) = EA_{\text{sphere}} = \frac{q_{enc}}{\varepsilon_0}$$

Inside the shell: $q_{enc} = 0$

$$\boxed{E = 0 \qquad \text{for} \quad r < R}$$

Outside the shell: $q_{enc} = Q$

$$\boxed{E = \frac{Q}{4\pi\varepsilon_0 r^2} \qquad \text{for} \quad r > R}$$



$E = ?$

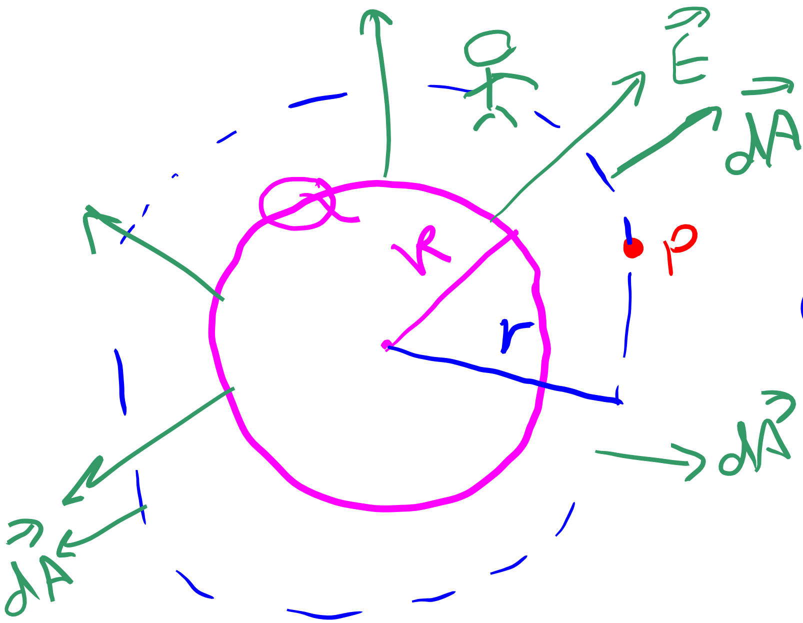
$r<R$

$r>R$

$q_{enc}=0$

$$r<R \rightarrow \Phi_e = \oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0} \rightarrow$$

$$\vec{E}\,\underbrace{A}_{\neq 0}\,\underbrace{\cos\theta}_{\neq 0} = 0 \rightarrow \vec{E} = 0 \quad r<R$$



$r > R$

$$\Phi_e = \oint \vec{E} \cdot \vec{dA} = \frac{q_{enc}}{\varepsilon_0}$$

$q_{enc} = Q$

$\theta = 0°$

$$\Rightarrow \oint E\, dA \overbrace{\cos 0}^{1} = \frac{Q}{\varepsilon_0}$$

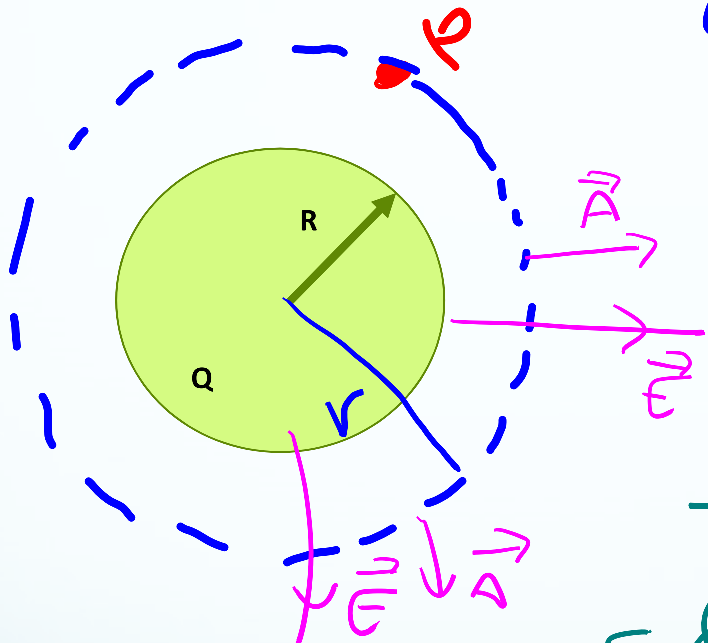
$$E \oint dA = EA = \frac{Q}{\varepsilon_0}$$

$$\rightarrow EA = \frac{Q}{\varepsilon_0}, \quad A = 4\pi r^2 \rightarrow E = \frac{Q}{\varepsilon_0} \frac{1}{4\pi r^2}$$

$$\rightarrow \boxed{E = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2}}$$

Field outside a sphere of charge

$$\Phi_e = \oint \vec{E}.d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$



$q_{enc} = Q.$

$\theta = 0$

$$\Phi_e = \oint \vec{E}.d\vec{A} = \frac{Q}{\varepsilon_0}$$

$$\rightarrow \oint E\,dA\,\underbrace{\cos 0}_{1} = \frac{Q}{\varepsilon_0}$$

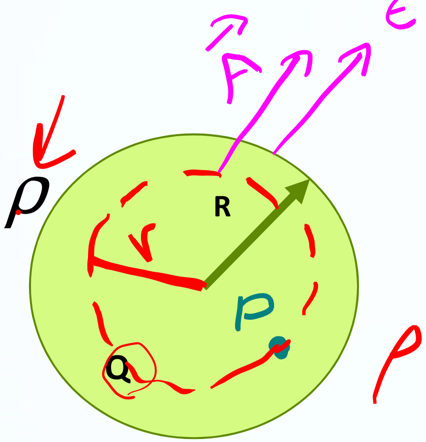
$$E \oint dA = \frac{Q}{\varepsilon_0} \rightarrow EA = \frac{Q}{\varepsilon_0}$$

$$A = 4\pi r^2 \longrightarrow E\,4\pi r^2 = \frac{Q}{\varepsilon_0} \rightarrow E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

$$\longrightarrow \vec{\mathbf{E}} = \frac{\mathbf{Q}}{4\pi\varepsilon_0 \mathbf{r}^2}\hat{\mathbf{r}} \quad , \text{for } r > R$$

Field inside a sphere of charge

$$\Phi_e = \oint \vec{E}.d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$



$$\rho = \frac{Q}{V} = \frac{q_{enc}}{V_g} \rightarrow q_{enc} = \rho V_g = \rho \frac{4}{3}\pi r^3$$

$$\rightarrow q_{enc} = \frac{Q}{V}\frac{4}{3}\pi r^3 = \frac{Q}{\cancel{\frac{4}{3}\pi} R^3}\cancel{\frac{4}{3}\pi} r^3$$

$$q_{enc} = \rho V_{\text{ball}}^{g} = \left(\frac{Q}{\frac{4}{3}\pi R^3}\right)\frac{4}{3}\pi r^3 = Q\frac{r^3}{R^3}$$

$$\boxed{q_{enc} = Q\frac{r^3}{R^3}} \longleftarrow$$

$$\theta = 0 \Rightarrow \Phi_e = \oint E\,dA\cos 0 = \frac{Q r^3}{R^3 \varepsilon_0} \rightarrow EA = \frac{Q r^3}{R^3 \varepsilon_0}$$

$$E = \frac{Q r^3}{R^3 \varepsilon_0}\frac{1}{4\pi r^2} \Rightarrow \vec{E} = \frac{Qr}{4\pi\varepsilon_0 R^3}\hat{r}, \text{for } r < R \qquad A = 4\pi r^2$$

Problem 23.03 of textbook:
Using Gauss' law to find the electric field

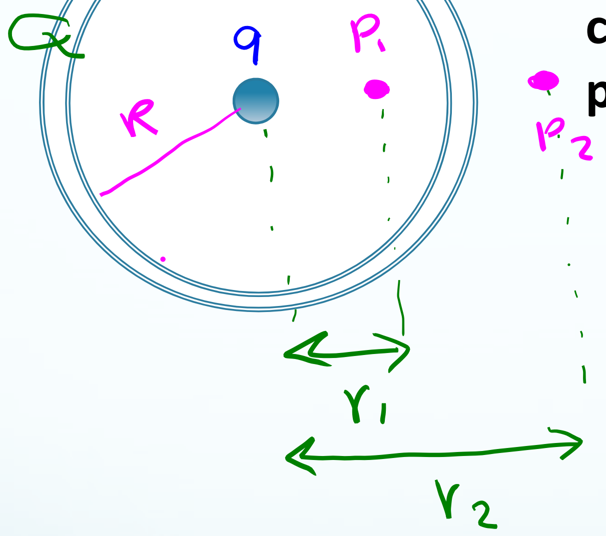
$R = 10cm$

$r_1 = 6\ cm$

$r_2 = 14\ cm$

$q = +5e$

$Q = -16e$



A Plastic Spherical shell with uniform charge Q and radius R, a particle with charge q is at the centre of the sphere. What is the electric field at points p1 at radial distance r1=6cm ?

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

inside →

Solve at home

~~TopHat Question: Homework~~

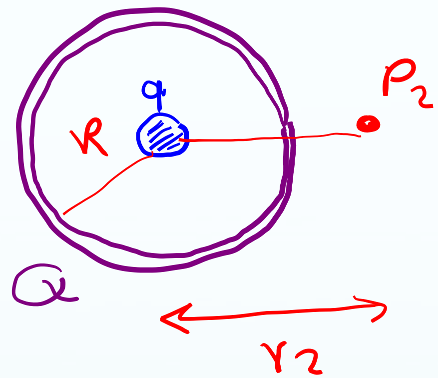
- What is the electric field at point p2 at radial distance r2=14cm?

$R = 10\,cm$

$r_2 = 14\,cm$

$q = +5e$

$Q = -16e$



Read the explanations from 23.3 of text book

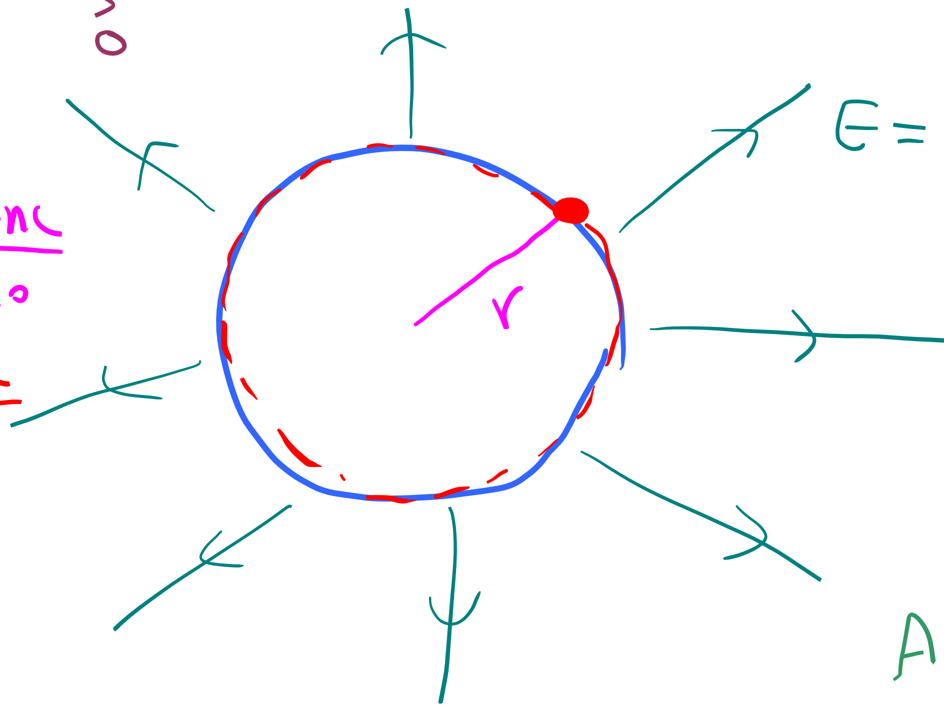
(numbers are changed in this example)

TopHat Question

How much is the net charge **INSIDE** the sphere ?

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$\oint E\,dA\cos\theta = \frac{q_{enc}}{\varepsilon_0}$$



$E = 5\ \frac{N}{C}$

$r = 1\ m$

A) $-0.25\ nC$

B) $-0.5\ \mu C$

C) $0.5\ nC$ ← 98% Correct → ☺

D) $0.25\ \mu C$

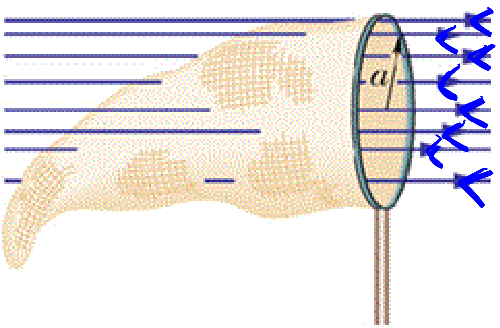
$A_{sphere} = 4\pi r^2$

$V_{sphere} = \frac{4}{3}\pi r^3$

$\varepsilon_0 = 8.85\times 10^{-12}\ \frac{C^2}{N m^2}$

Example: A butterfly net is in a uniform electric field of magnitude $E = 3.0$ mN/C. The rim, a circle of radius $a = 11$ cm, is aligned perpendicular to the field. The net contains no net charge. Find the electric flux through the net.

$$\Phi = \int E \cdot dA = E \cdot A$$

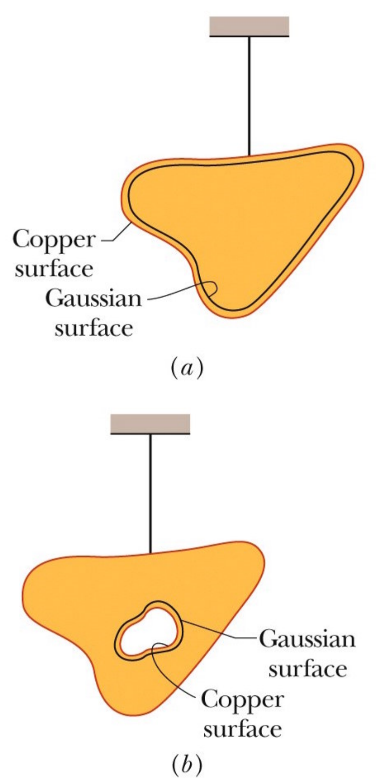


$$\begin{aligned}\Phi &= E(\overbrace{\pi a^2}^{\text{effective surface area}}) \\ &= \left(3.0 \times 10^{-3}\ \text{N/C}\right) \pi\,(.11\ \text{m})^2 \\ &= 1.1404 \times 10^{-4}\ \text{N/Cm}^2\end{aligned}$$

23-3: A Charges Isolated Conductor

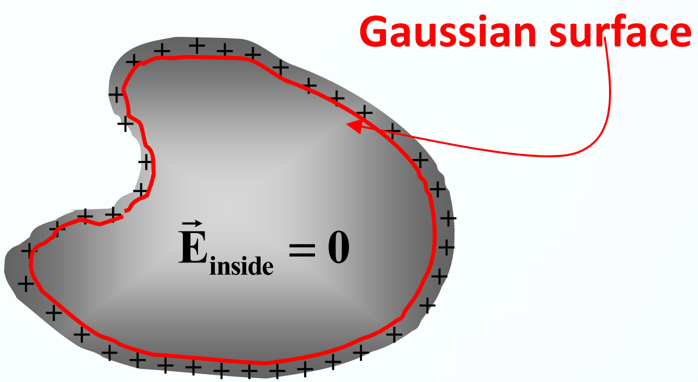
23-3 A Charged Isolated Conductor

If an excess charge is placed on an isolated conductor, that amount of charge will move entirely to the surface of the conductor. None of the excess charge will be found within the body of the conductor.



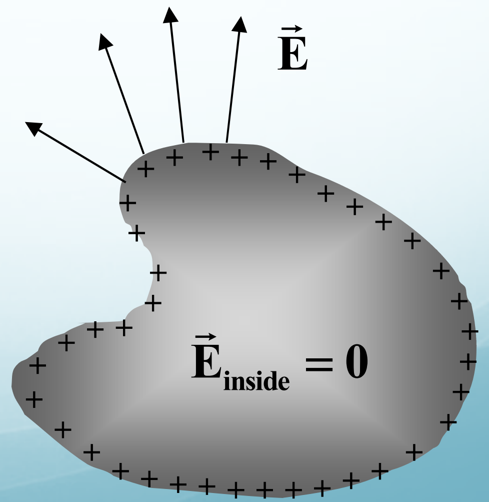
If we charge a conductor, where do the excess charges go?

A) The center of the conductor
B) The exterior surface of the conductor
C) Evenly inside the conductor
D) None of the above



The E-field right at the surface of the conductor is:

A) zero
B) parallel to the surface
C) perpendicular to the surface
D) in an arbitrary direction



This section we talked about:

Chapter 23.1 & 2

See you on Friday