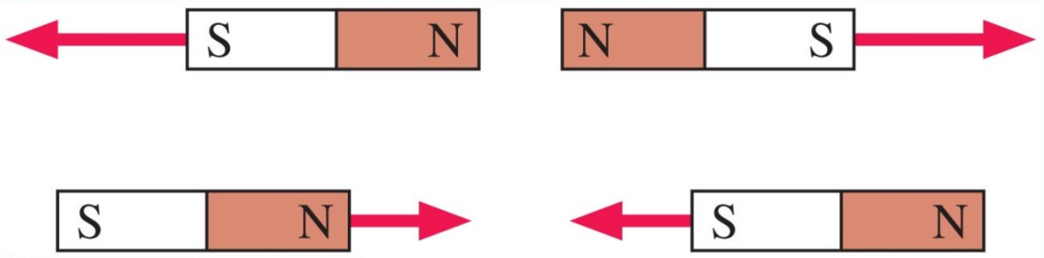
Electricity and Magnetism

- Physics 259 – L02
- Lecture 50

Final review

28.1: Magnetic fields

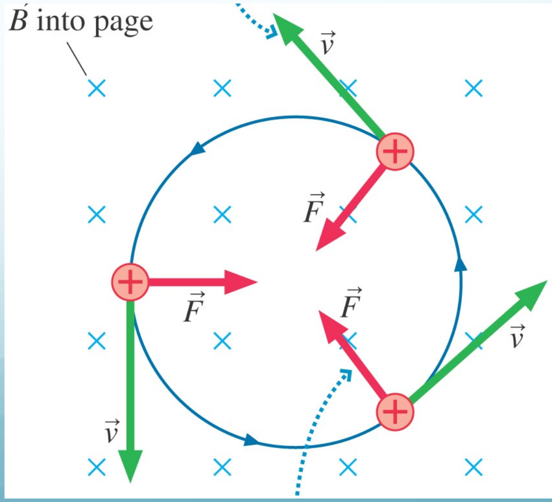


Magnetic force acts only on a moving charge.

$$\vec{F}_B = q\,\vec{v} \times \vec{B}$$

The Tesla
The Gauss

28.4: A circulating charged particle

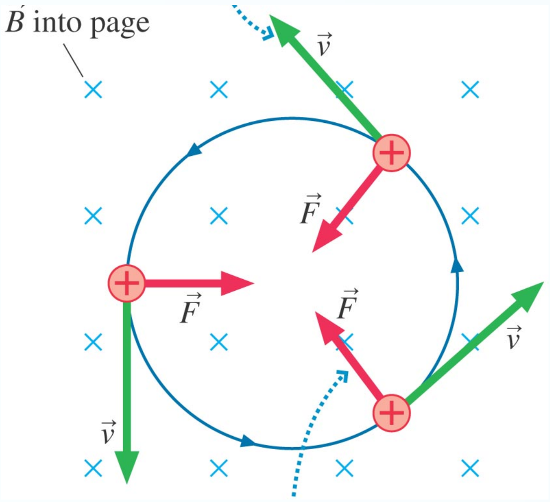


28.4: A circulating charged particle

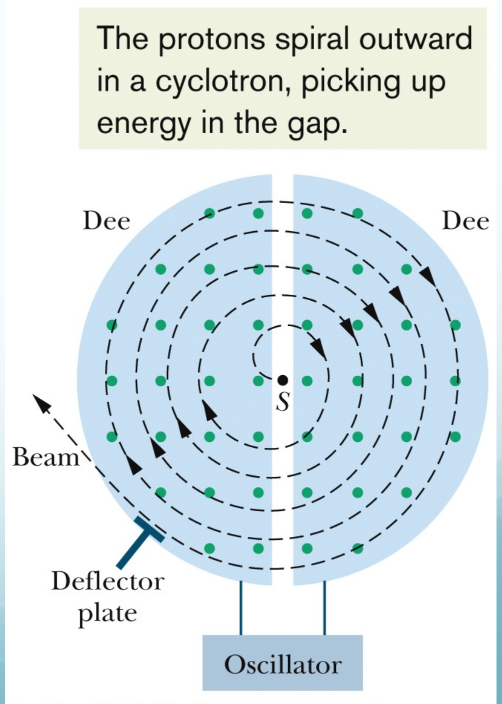
$$\boxed{R = \frac{mv}{|q|B}}$$

$$\boxed{T_{cyc} = \frac{2\pi m}{|q|B}}$$

$$\boxed{f_{cyc} = \frac{|q|B}{2\pi m}}$$



28.5: Cyclotrons and Synchrotrons

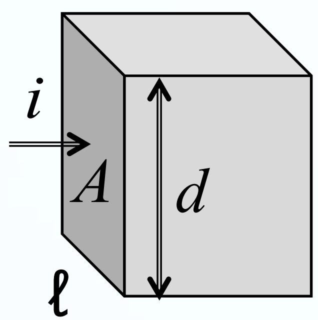


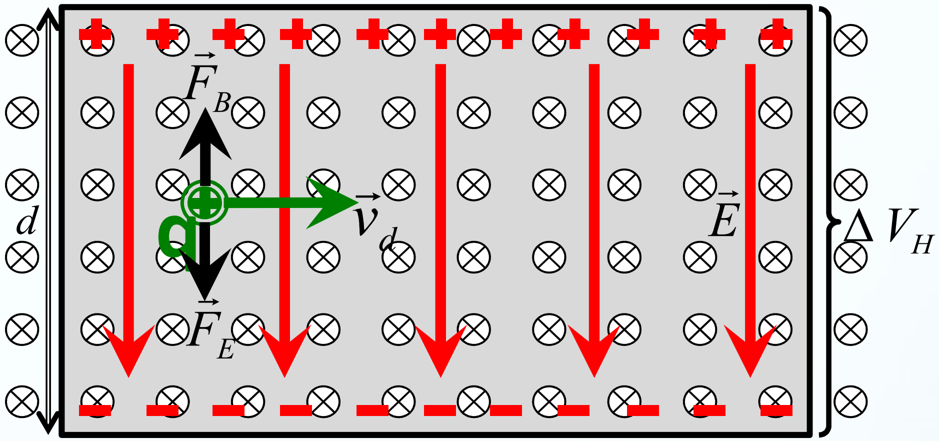
Application: Mass Spectrometer

$$\boxed{R = \frac{mv}{|q|B}}$$

28-2 Crossed Fields: Discovery of The Electron

28-3 Crossed Fields: The Hall Effect





$$F_B = q v_d B$$

$$v_d = \frac{i}{neA} \qquad A = \ell d$$

$$\boxed{\Delta V_H = v_d B d}$$

$$\boxed{B = \frac{ne\ell}{i} \Delta V_H}$$

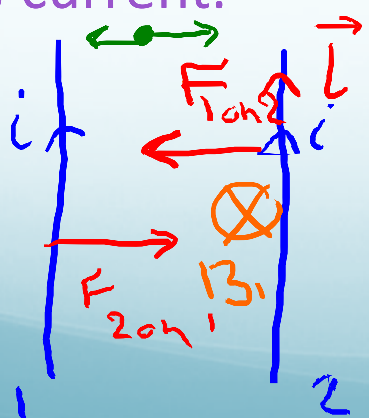
n is a material property

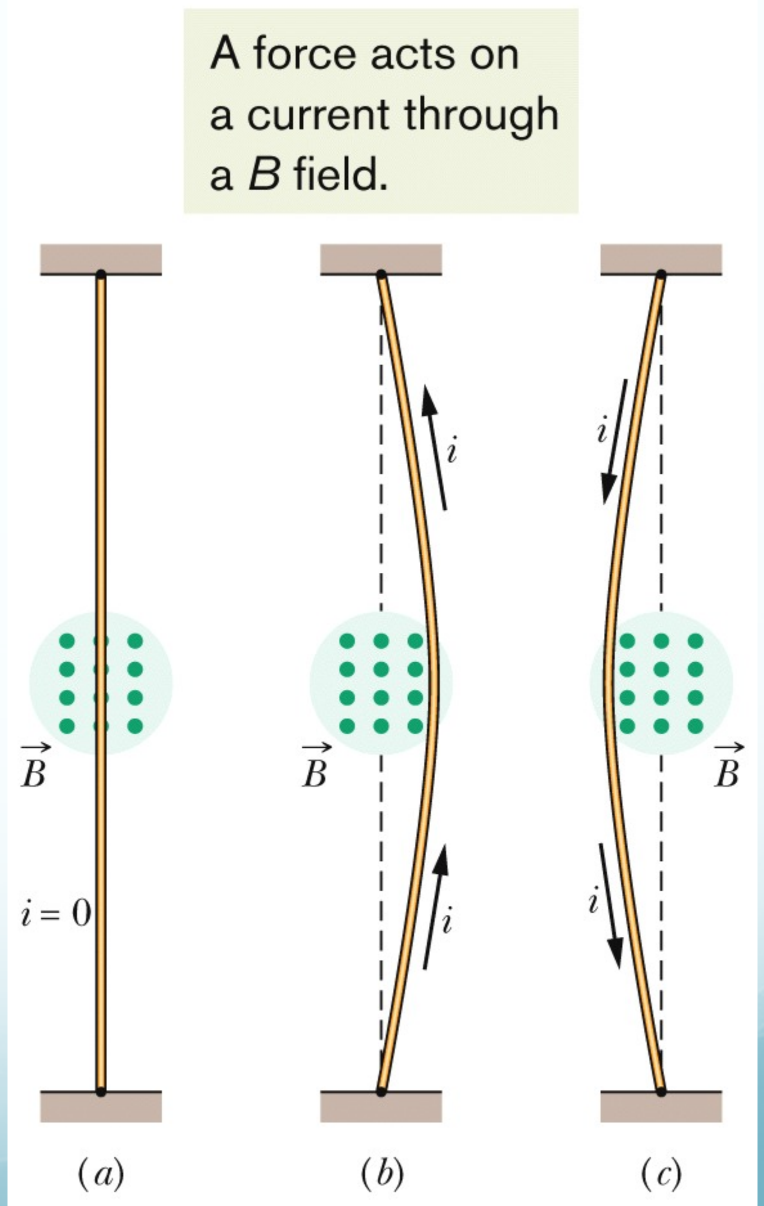
28-6 Magnetic Force on a Current-Carrying Wire

A straight wire carrying a current *i* in a uniform magnetic field experiences a sideways force

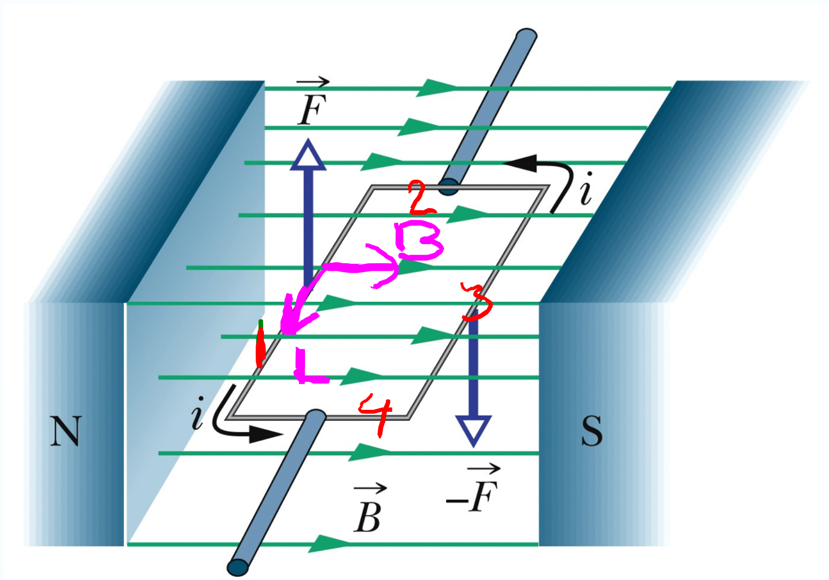
$$\vec{F}_B = i\vec{L} \times \vec{B} \quad \text{(force on a current).}$$

Here ***L*** is a length vector that has magnitude *L* and is directed along the wire segment in the direction of the (conventional) current.





28-7 Torque on a Current Loop



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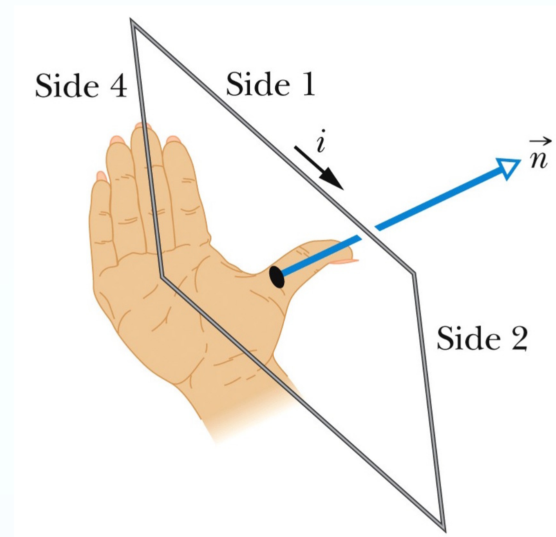
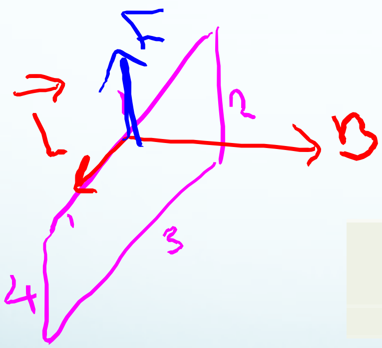
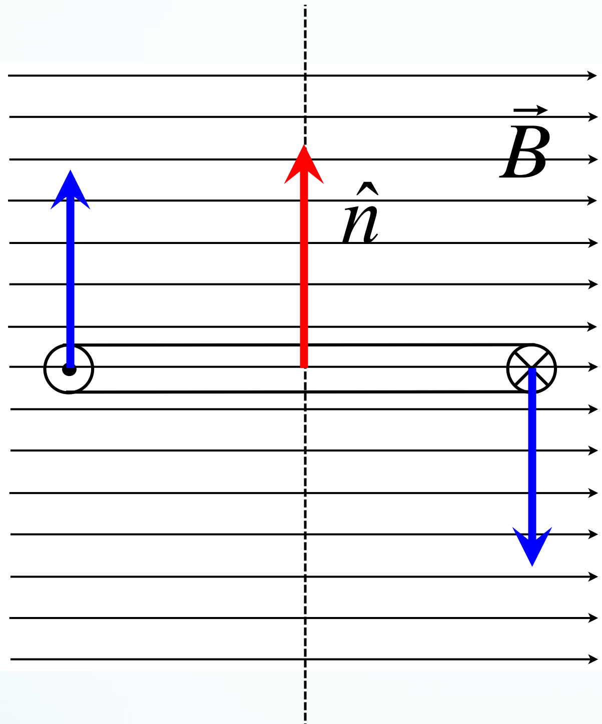


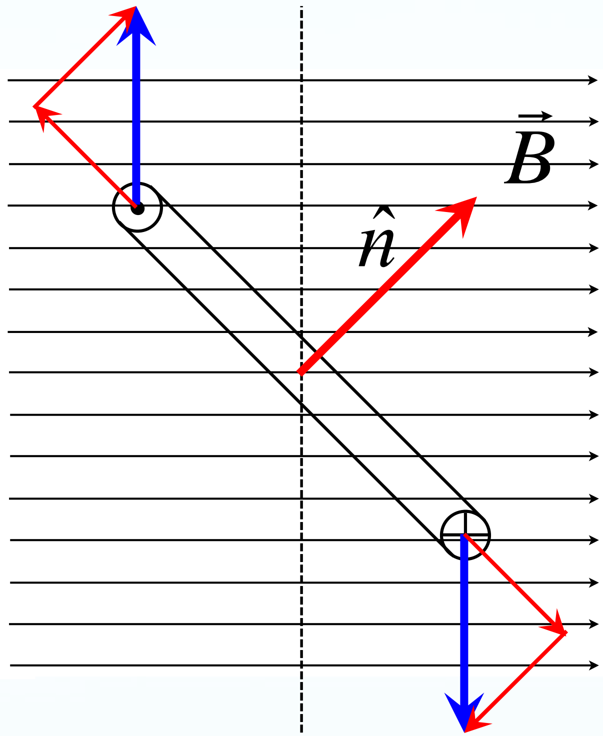


$$\vec{F}_B = i\vec{L} \times \vec{B} \quad \text{(force on a current).} = iLB\sin\theta$$

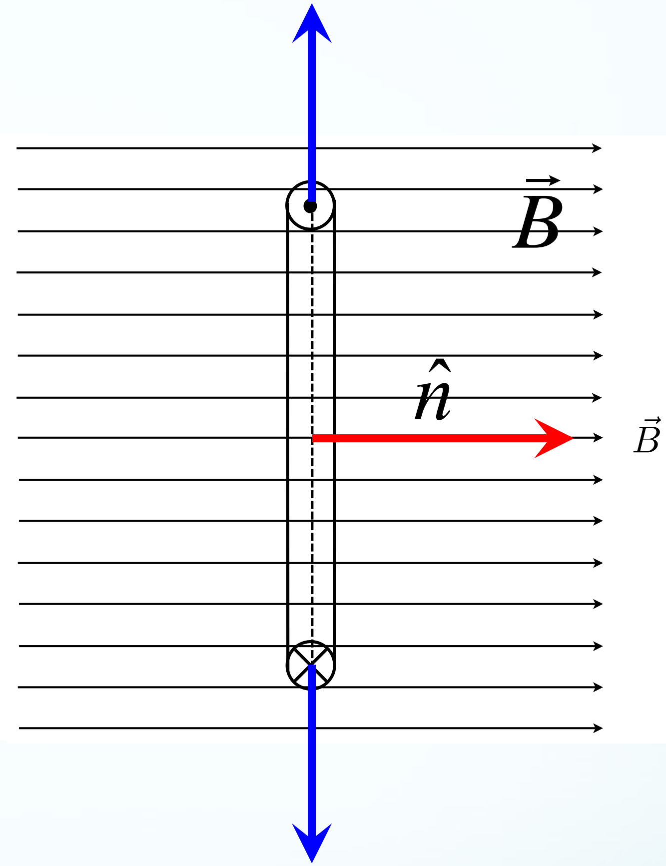
$$\tau = NiAB\sin\theta,$$



The normal vector is at right angles to the B-field: all magnetic force causes rotation of the loop

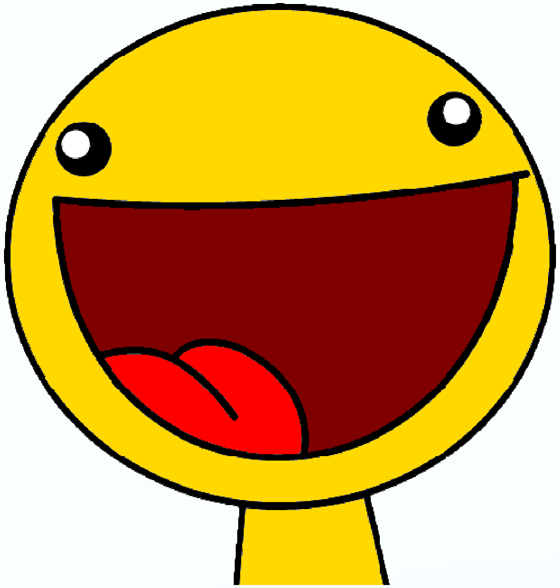


The normal vector is at some angle to the B-field: some of the magnetic force causes rotation of the loop



The normal vector is parallel to the B-field: none of the magnetic force causes rotation of the loop

Conclusion: components of magnetic force (anti)parallel to normal vector cause torque



For a single charge→ $\boxed{\vec{F}_B = q\vec{v}_d \times \vec{B}}$

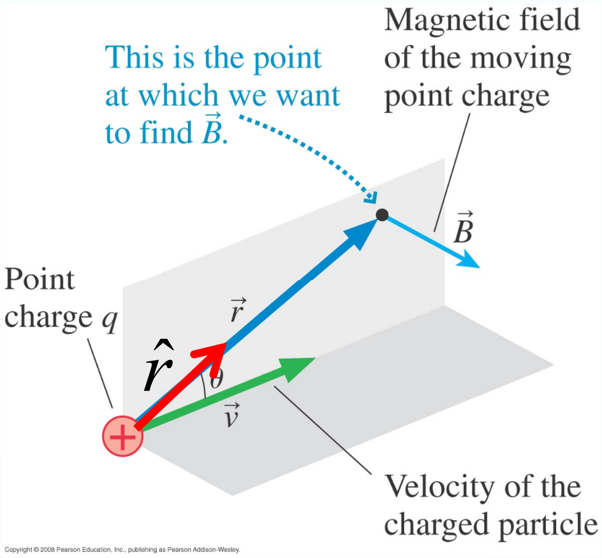
For N charges moving through the wire (current carrying wire)→ $\boxed{\vec{F}_B = i\vec{\ell} \times \vec{B}}$

The Biot-Savart Law

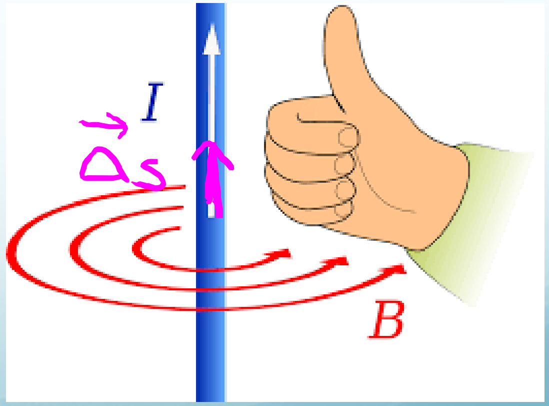
Magnetic fields are caused by **moving charges**.*

$$\vec{B}_{\text{point charge}} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2}$$

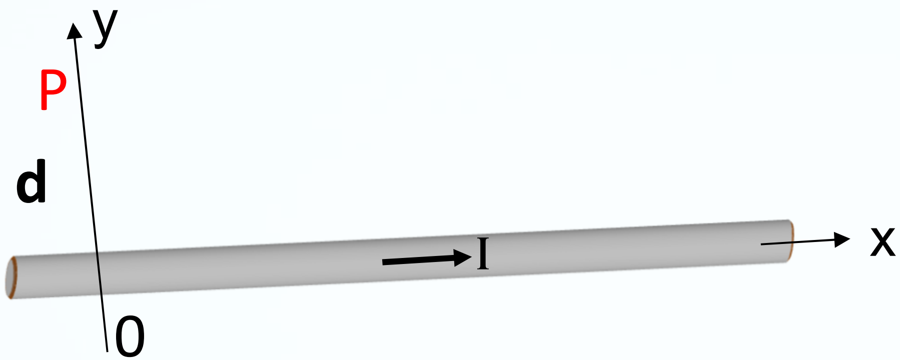
$$\vec{B}_{\text{point charge}} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\vec{r}}{r^3}$$



$$\vec{B}_{\text{current segment}} = \frac{\mu_0}{4\pi}\frac{I\Delta\vec{s}\times\hat{r}}{r^2}$$

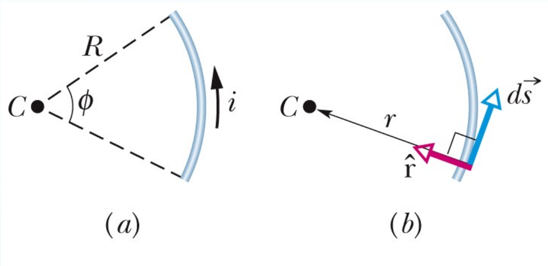


Magnetic field due to current in long straight wire



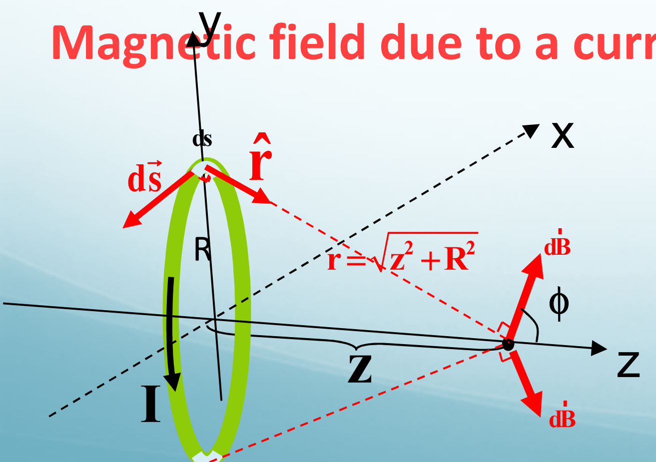
$$B_z = \frac{\mu_0}{2\pi}\frac{I}{d}$$

Magnetic field due to a current in a circular arc of wire



$$B = \frac{\mu_0 i \phi}{4\pi R}$$

Magnetic field due to a current in a circular loop (at distance z from the loop)



$$\vec{B} = \frac{\mu_0}{2}\frac{IR^2}{\left(z^2+R^2\right)^{3/2}}\hat{k}$$

Ampère's law

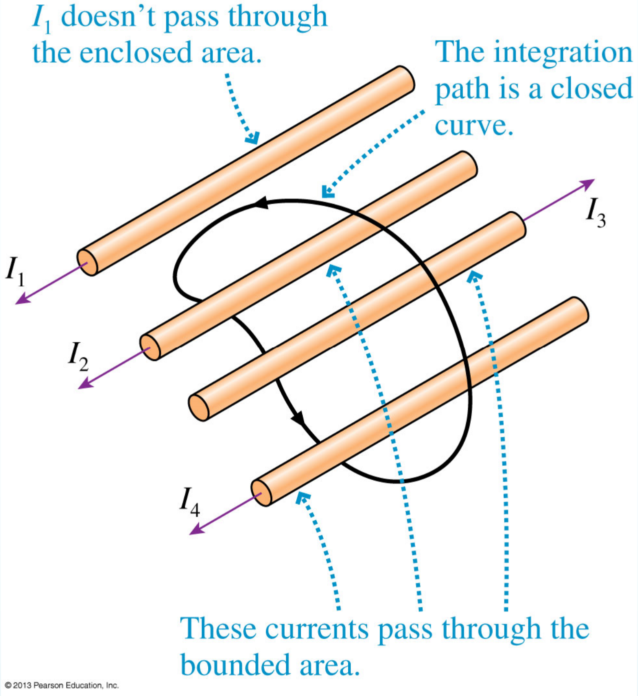
$$\text{i.e.} \boxed{\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}}$$

closed loop

Gauss's law

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

closed surface

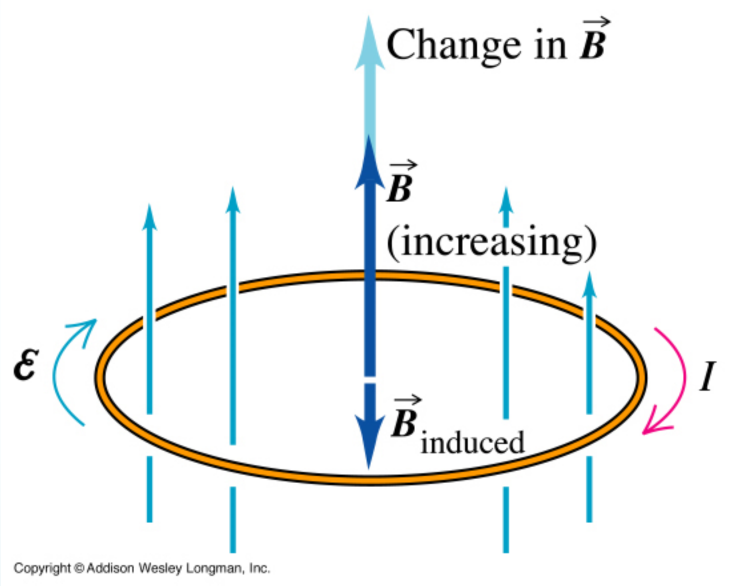


- **Sphere**
- **Solenoid**
- **…**

Induction and inductance

Faraday's law

Lenz's law



$$\mathcal{E} = -\frac{d\Phi_M}{dt}$$

$$\mathcal{E} = -N\frac{d\Phi_M}{dt}$$

$$\Phi_M = \int \vec{B} \cdot d\vec{A}$$

The changing magnetic flux generates an induced current which creates an induced magnetic field which, in turn, resists the change in magnetic flux.

Inductance

Changing the current changes the flux through the inductor, which creates a back-emf.

If a current *i* is established through each of the *N* windings of an inductor, a magnetic flux Φ_B links those windings. The inductance *L* of the inductor is

$$\boxed{L = \frac{N\Phi_B}{i}}$$

Self-Induction

$$\mathscr{E}_L = -L\frac{di}{dt} \quad \text{(self-induced emf).}$$

Mutual Induction

$$\mathscr{E}_2 = -M\frac{di_1}{dt}$$

$$\mathscr{E}_1 = -M\frac{di_2}{dt}.$$

Preparing for the exam

- Understand the concepts in Lecture notes/Text book
- Assignments (WileyPlus)
- Top Hat Questions
- Practice on Previous Years Finals
- & more
- Stay Positive, Calm & Confident

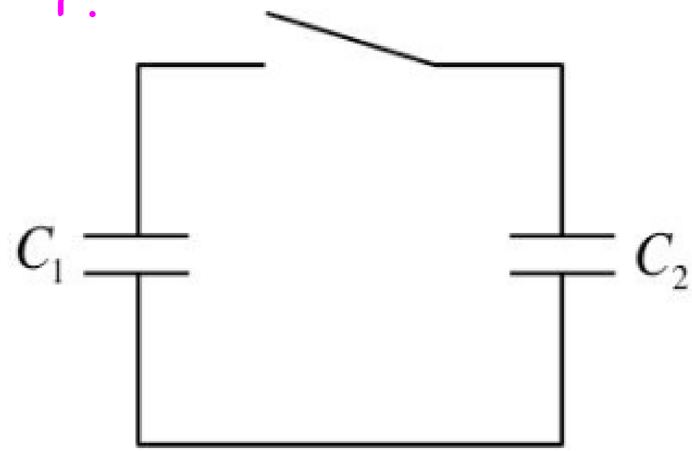
Some questions from 2016 final exam

Solutions of these questions =>
Appendix_Final review_Answeres
posted on D2L

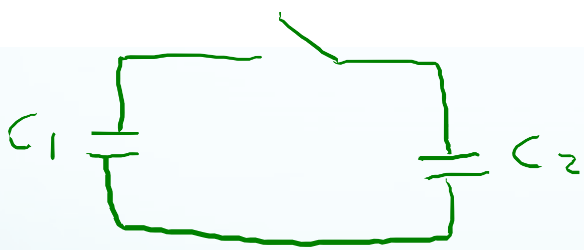
1. Capacitors C_1 and C_2 are identical. Initially, capacitor C_1 is charged and stores 4.0 J of potential energy and capacitor C_2 is uncharged. After the switch is closed, what will be the [redacted] energy stored in this [redacted]? capacitor 1.

$$U = \frac{1}{2}\frac{q^2}{C}$$

a) 16 J
b) 2.0 J
→ c) 1.0 J
d) 8.0 J
e) 4.0 J



$C_1 = C_2$



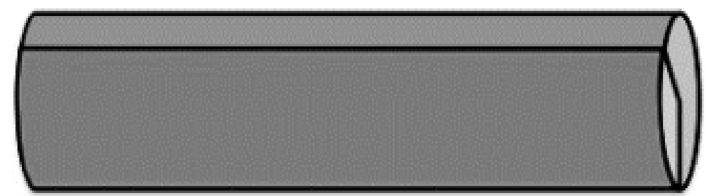
$$U_i = \frac{1}{2}\frac{q^2}{C_1} + 0 = \frac{1}{2}\frac{q^2}{C}$$

$$U_f = \frac{1}{2}\frac{\left(\frac{q}{2}\right)^2}{C_1} = \frac{1}{2}\left(\frac{1}{4}\frac{q^2}{C}\right) \rightarrow U_f = \frac{1}{4}U_i$$

the total energy stored in the system before & after closing switch is the same → energy close

2. A cylindrical resistor is composed of 1/3 gold and 2/3 iron as shown ($\rho_{Au} = 2.35\text{x}10^{-8}$ Ωm, and $\rho_{Fe} = 9.68\text{x}10^{-8}$ Ωm). The radius of the cylinder is $r = 55$ µm, and its total resistance is $R = 1.5\ \Omega$. What is its length L?

a) 6.6 cm
b) 12 cm
c) 23 cm
d) 30 cm
e) 75 cm



$\frac{1}{3}$ gold 1

$\frac{2}{3}$ iron 2

$R_1 = \rho \frac{L_1}{A_1} = \rho \frac{L}{\pi a^2/3}$ $= 7.42L$

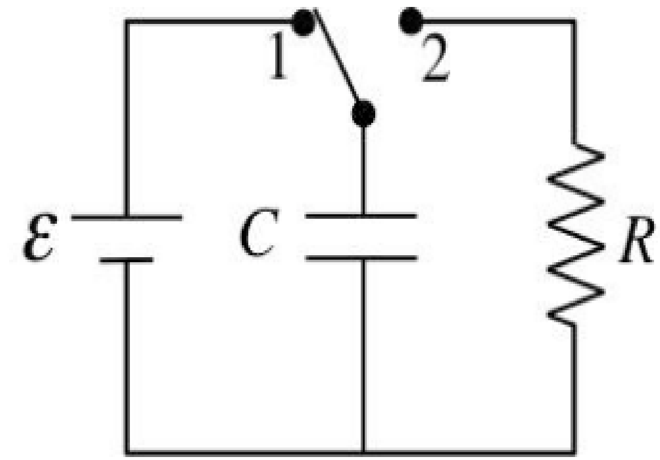
$R_2 = \rho \frac{L_2}{A_2} = \rho \frac{L}{2\pi a^2/3}$ $= 15.27L$

$R_T = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1} =$ $5L$

$5L = 1.5\Omega \rightarrow \boxed{L = 0.3m}$

3. In the RC circuit shown below, $\varepsilon = 100$ V, $C = 1.0\ \mu$F, and $R = 1.0$ kΩ. The switch has been in position 1 for a long time. At time $t = 0$, the switch is flipped to position 2. How much charge is left on the capacitor plates after $t = 10$ ms?

a) 0.67 nC
b) 45 nC
c) 14 nC
d) 37 nC
e) 4.5 nC



$$V_c - iR = 0 \rightarrow \frac{q}{c} - iR = 0 \rightarrow \frac{q}{c} = iR$$

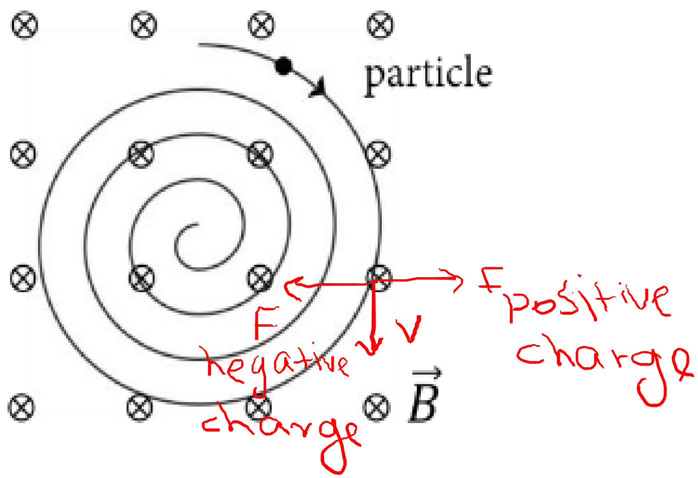
$$\rightarrow \frac{q}{c} = \frac{dq}{dt}R \rightarrow q = q_0 e^{-t/RC}$$

$$q_0 = CV = C\varepsilon$$

$$\rightarrow \text{at } t = 10 \text{ ms} \rightarrow q = (1\times10^{-6}\,\text{F})(100\,\text{V})\, e^{-10\times10^{-3}/(1\times10^{3}\,\Omega)(1\times10^{-6}\,\text{F})}$$

$$\rightarrow q = 4.5 \text{ nc}$$

7. A uniform magnetic field is directed into the page. A charged particle, moving in the plane of the page, follows a clockwise spiral of decreasing radius as shown. Which of the following is a reasonable explanation?

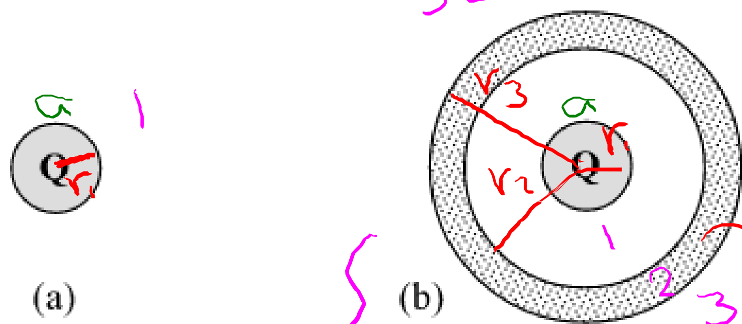


a) the charge is negative and slowing down
b) the charge is positive and slowing down
c) the charge is positive and speeding up
d) the charge is negative and speeding up
e) none of the above

R ↓ → loosing energy → loosing speed

10. In Figure (a) below, a small solid sphere has been given a uniform positive charge Q. The electric potential at the surface of the sphere is V_a, relative to $V = 0$ at infinity. In Figure (b), a thick, uncharged conducting shell (stippled in the figure) has been placed around the charged, solid sphere, without touching it. The electric potential at the surface of the solid sphere (relative to $V = 0$ at infinity) is now V_b. Which of the following is true?

$\Delta V = -\int \vec{E}\cdot d\vec{l}$



(a) (b)

$E = 0 \rightarrow \Delta V = \int E\,dl = 0$
$\rightarrow$ work $= 0$ inside the shell

$\rightarrow E = 0$ inside the conductor

a) $V_b = V_a$
b) $V_b < V_a$
c) $V_b > V_a$
d) $V_b = \infty$
e) $V_b = 0$

$\Delta V_a = -\int_1 = -\int_{\infty}^{a}$

$= -\int_{r_1}^{r_2} - \int_{r_2}^{r_3} - \int_{r_3}^{\infty}$

$\Delta V_b = -\int_3 - \int_2 - \int_1$

$= -\int_{r_1}^{r_2} - \cancel{\int_{r_2}^{r_3}}\,0 - \int_{r_3}^{\infty}$

$\Delta V = V_\infty - V_0 \rightarrow$ Work that we do to bring a unit charge from infinite to point a.

$\Rightarrow V_a > V_b$ since in b, $E = 0$ inside conducting shell

14. Two equal positive point charges, each of charge q, are separated by distance L. There are no other charges anywhere. The potential at the midpoint of the line joining the charges is defined to be zero. The electrostatic potential at an infinite distance from the two charges is:

a) $-\frac{1}{4\pi\varepsilon_0}\frac{2q}{L}$

b) $+\frac{1}{4\pi\varepsilon_0}\frac{2q}{L}$

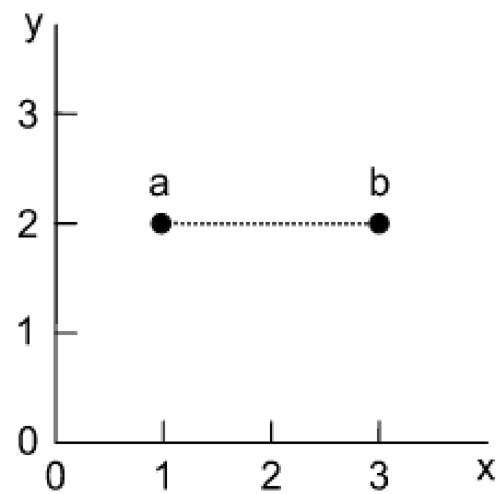
c) $-\frac{1}{4\pi\varepsilon_0}\frac{4q}{L}$

d) $+\frac{1}{4\pi\varepsilon_0}\frac{4q}{L}$

e) Zero

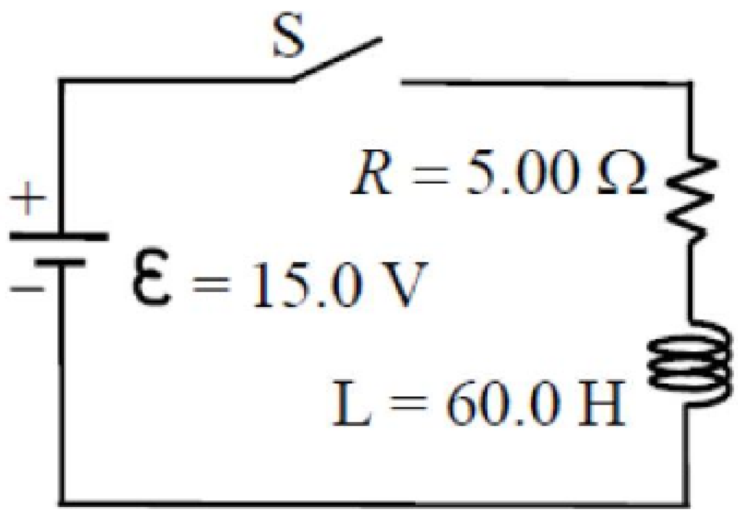
Solutions of these questions $\Rightarrow$
Appendix_Final review_Answeres
posted on D2L

16. The electric field in a particular region of space is given by $\vec{E} = y^2\hat{\imath} + 2xy\hat{\jmath}$. What is the electric potential difference, $V_{ab} = V_b - V_a$, between point a at $(x_a, y_a) = (1, 2)$ and point b at $(x_b, y_b) = (3, 2)$?



a) $V_{ab} = -8$ V
b) $V_{ab} = +24$ V
c) $V_{ab} = +8$ V
d) $V_{ab} = -24$ V
e) $V_{ab} = 0$ V

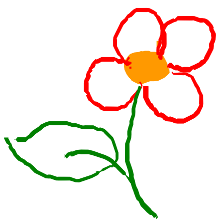
24. The switch, S, in the figure below is closed at time $t = 0$. At what time is the current in the circuit equal to 2.40 A? (Select the closest answer.)



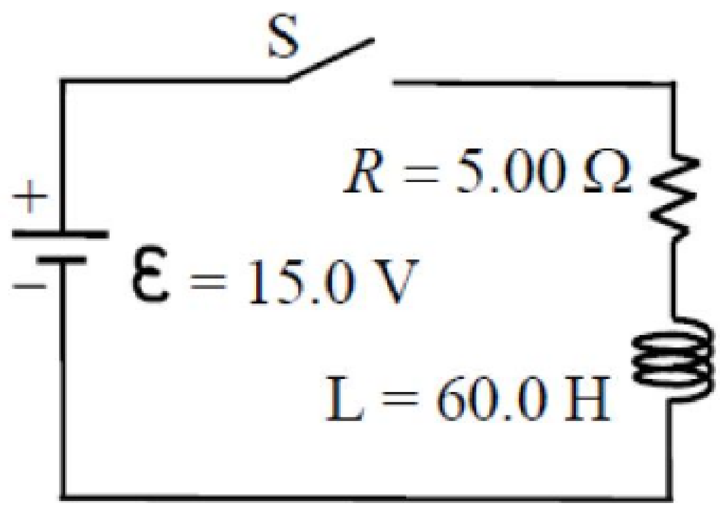
a) 0.134 s
b) 19.3 s
c) 12.0 s
d) 4.80 s
e) 1.61 s

Today is our last lecture

Hope you the Bests

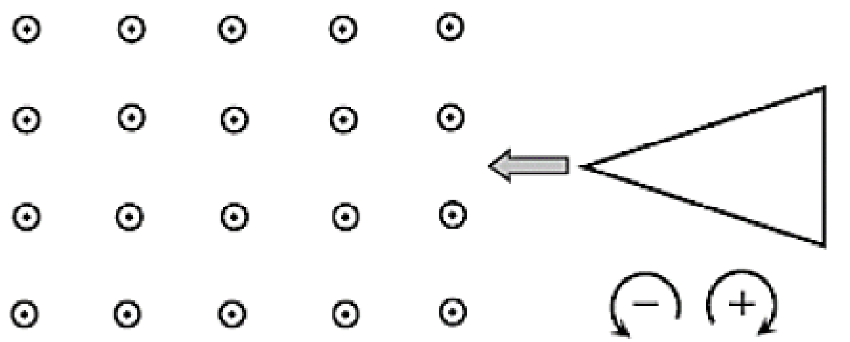


24. The switch, S, in the figure below is closed at time $t = 0$. At what time is the current in the circuit equal to 2.40 A? (Select the closest answer.)



a) 0.134 s
b) 19.3 s
c) 12.0 s
d) 4.80 s
e) 1.61 s

25. The diagram at the right shows a triangular loop of wire of resistance R moving at constant speed towards a region of magnetic field directed out of the page. (The magnetic field is zero everywhere outside the region shown.) Take current as positive when it is directed clockwise around the loop, as indicated by the arrows below the triangle.



The leading edge of the loop first encounters the magnetic field at time t_1, and the loop becomes fully immersed in the field at time t_2. Which one of the five graphs shown below correctly describes the current, i, induced in the loop as a function of time, t?