Wed Feb 1, 2017

Last time

- Electric field visualization applet - reminder
- Introduction to Gauss' Law: the first of the four Maxwell equations of electromagnetism
- Electric Flux, calculating flux

This time

- Applying Gauss' Law: start simple, verify what we expect
  - Electric field of a point charge
  - Electric field of a spherical shell of charge
  - Electric field of a uniformly charged ball
- More examples (afternoon lecture)

Electric Flux; Gauss' Law

Gauss' Law is equivalent to Coulomb's law. It will provide us:

(i) an **easier way to calculate the electric field** in specific circumstances (especially situations with a high degree of symmetry)

(ii) a better understanding of the properties of conductors in electrostatic equilibrium (more on this as we go)

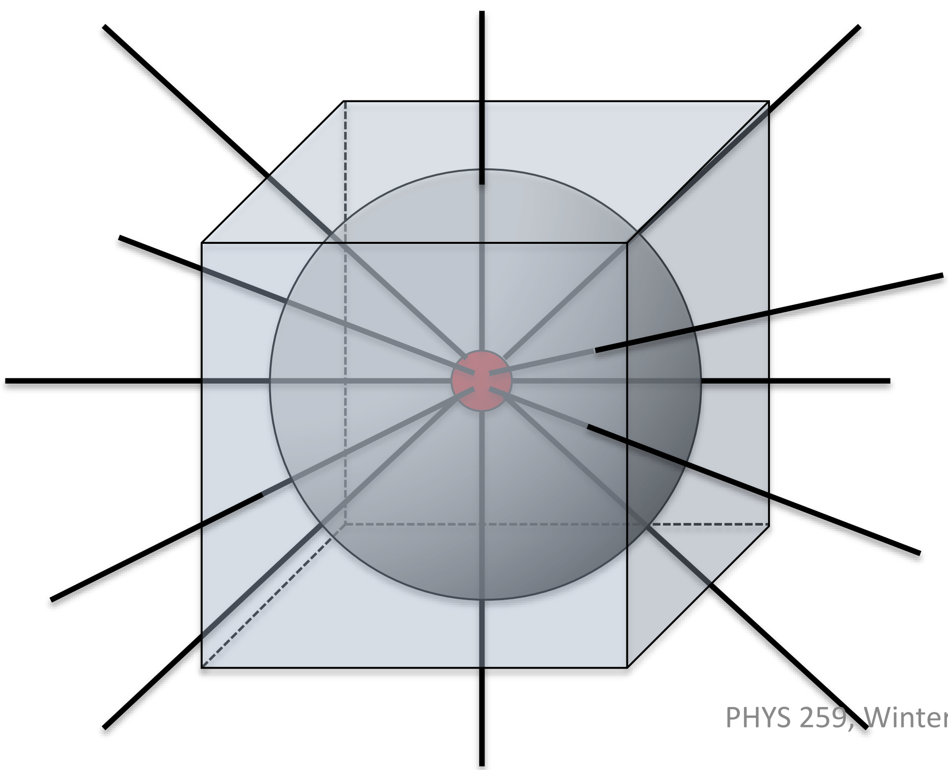
(iii) It is valid for moving charges – not limited to electrostatics.

Electric flux, passing through a **closed** Gaussian surface

$$\Phi_E = \oint\!\!\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$$

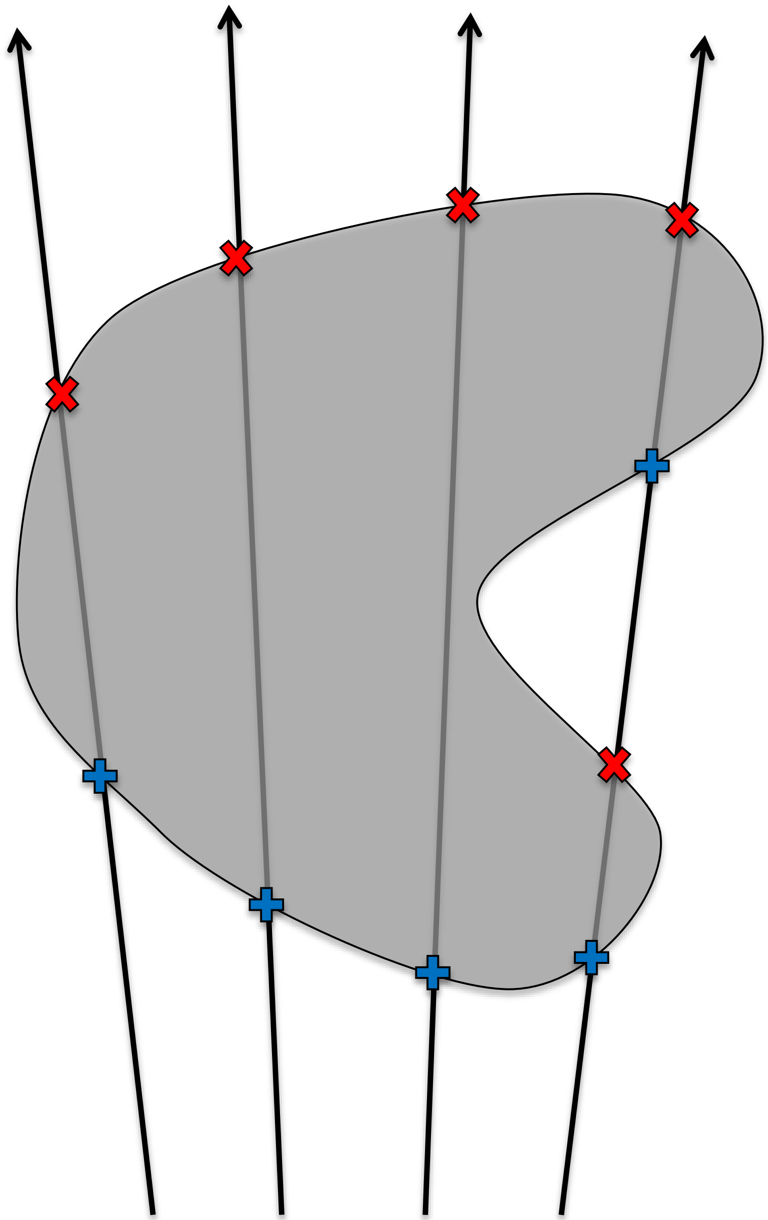
The **electric flux** through a closed surface does not depend on the **SHAPE** of the surface, it only depends on the **charges enclosed** by the surface.

This gives a nice interpretation of the flux as the **number of electric field lines passing through the surface**. Take the example of a point charge surrounded by a sphere, surrounded by a cube



The number of field lines passing through the sphere is the same as the number of field lines passing through the cube.

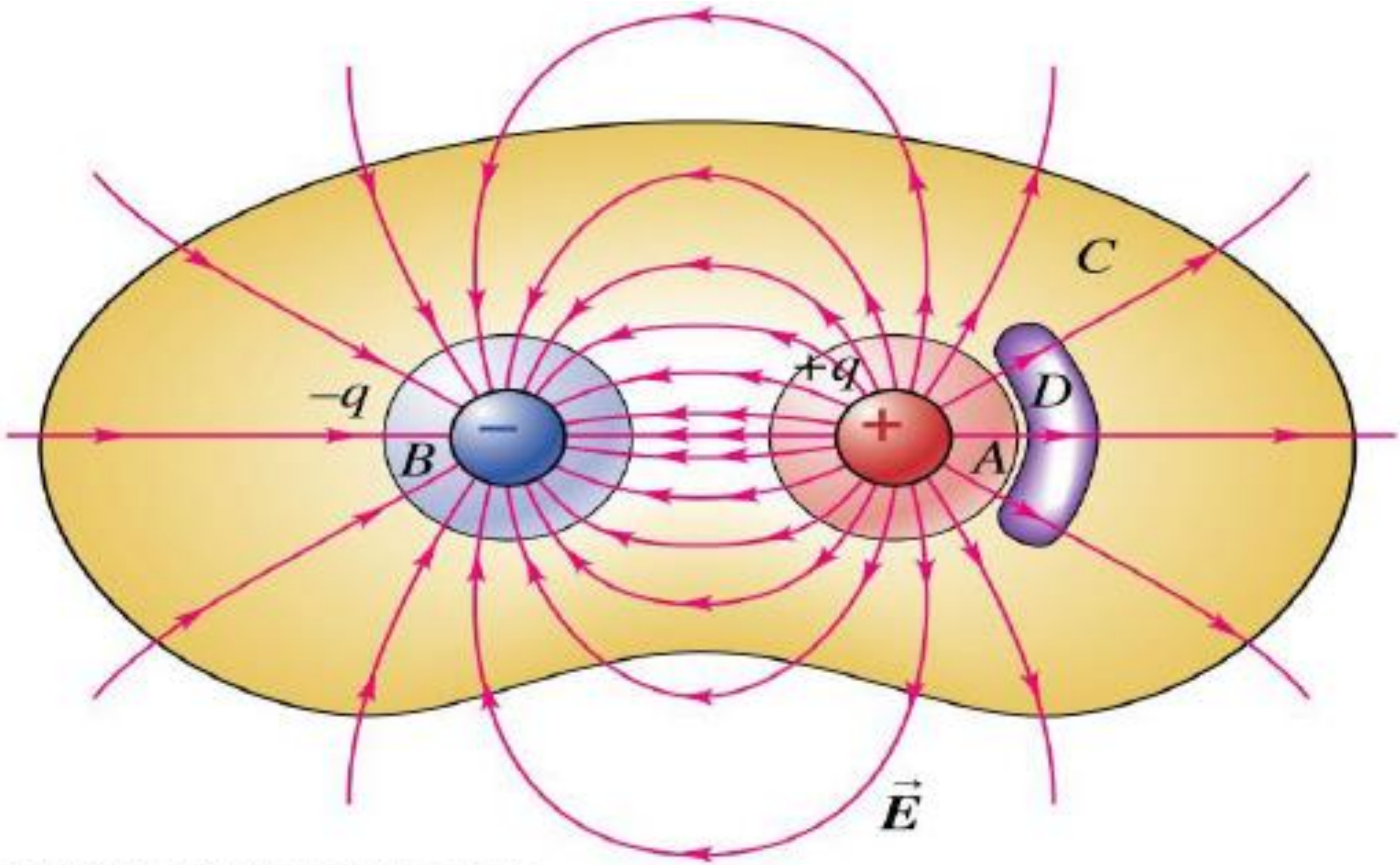
The electric flux through each surface is the same.



Using the idea that electric flux "counts the number of field lines" passing through a surface, we immediately see why external electric field lines contribute nothing to the net flux.

Any external field line that enters later has to leave.

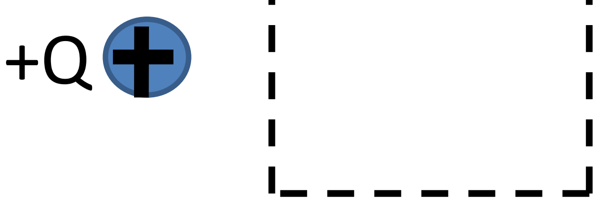
Something to think about: what is the flux through a surface surrounding an electric dipole? How can you think about it in terms of this idea?



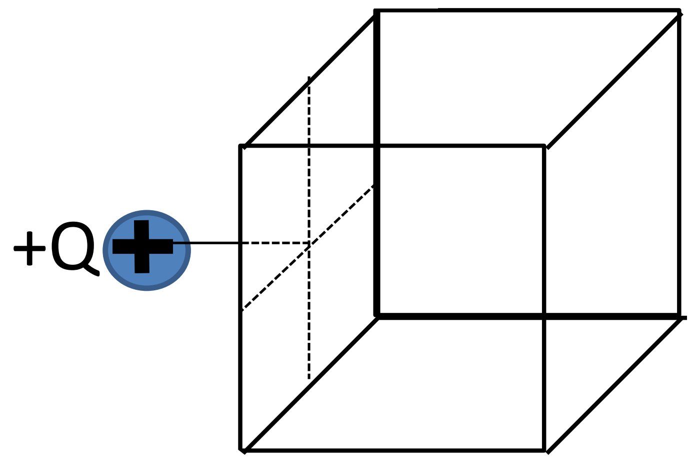
C
+q
−q
D
B
A
$\vec{E}$

TopHat Question

What is the NET electric flux passing through the cube?



Side view

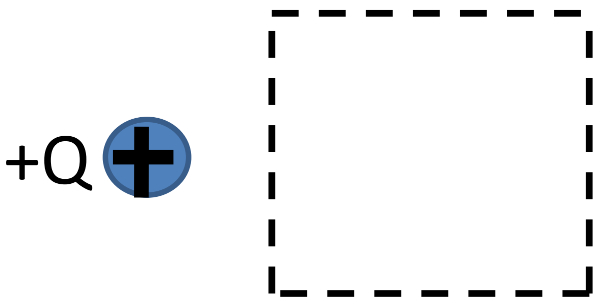


3-D view

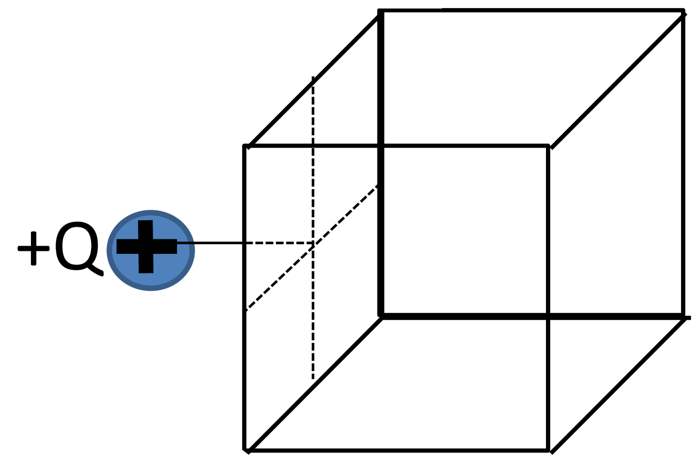
(A) Positive
(B) Negative
(C) Zero
(D) Not enough information

TopHat Question

What is the **sign** (positive or negative) of the NET electric flux passing through the **4 side surfaces**? (To be clear, the cube has 6 sides. It has a TOP, BOTTOM and 4 SIDE surfaces)



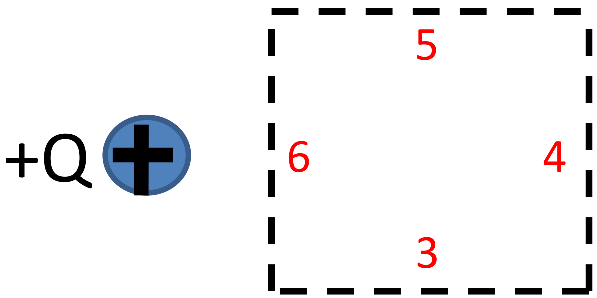
Side view



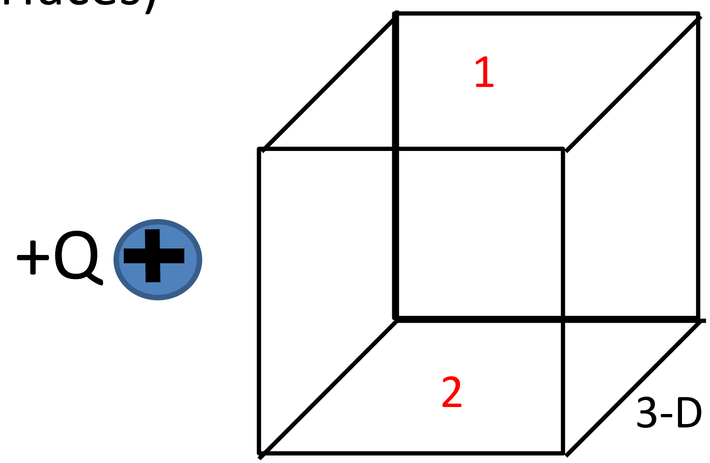
3-D view

(A) Positive
(B) Negative
(C) Zero
(D) Not enough information

What is the **sign** (positive or negative) of the NET electric flux passing through the **4 side surfaces**?
(To be clear, the cube has 6 sides. It has a TOP, BOTTOM and 4 SIDE surfaces)



Top view



3-D view

ANSWER: The **net** flux going thru the 4 sides **must be negative**

Reasoning: (1) Since no charge is enclosed in the Gaussian cube, **E Flux = 0 through the whole closed surface.**
(2) The flux going through the TOP(1) and BOTTOM(2) is positive. (the E field at the top and bottom surfaces is flowing **outward**, so the flux is positive).
(3) Therefore, the flux through the 4 sides MUST be a net negative
Basically, the flux going through the nearest side surface to the charge +Q is very large & negative because the E field at the nearest surface is strongest at that surface.

Electric Fields

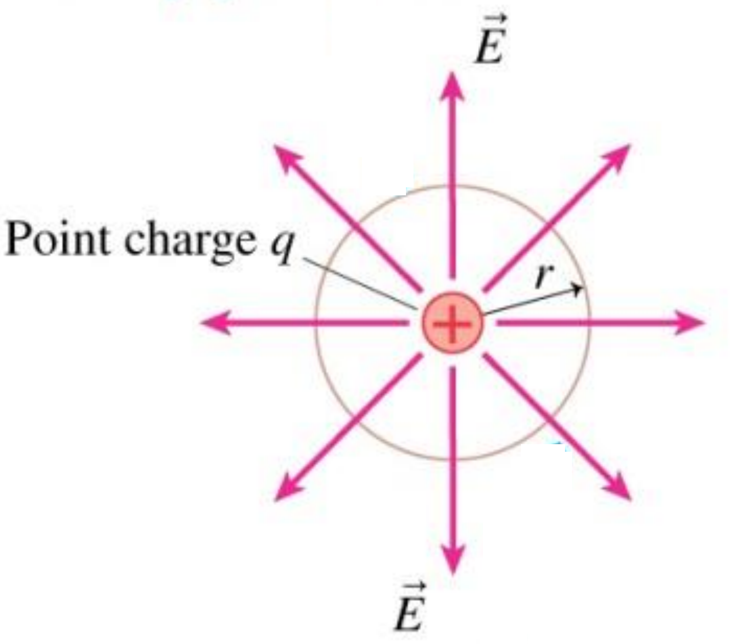
www.falstad.com/vector3de

shows electric field lines

1. Point source

Task

Use Gauss' law to compute the E field at a distance r from the positive charge



We will not assume anything about the E-field of a point charge but we can be guided by symmetry.

Symmetry argument:

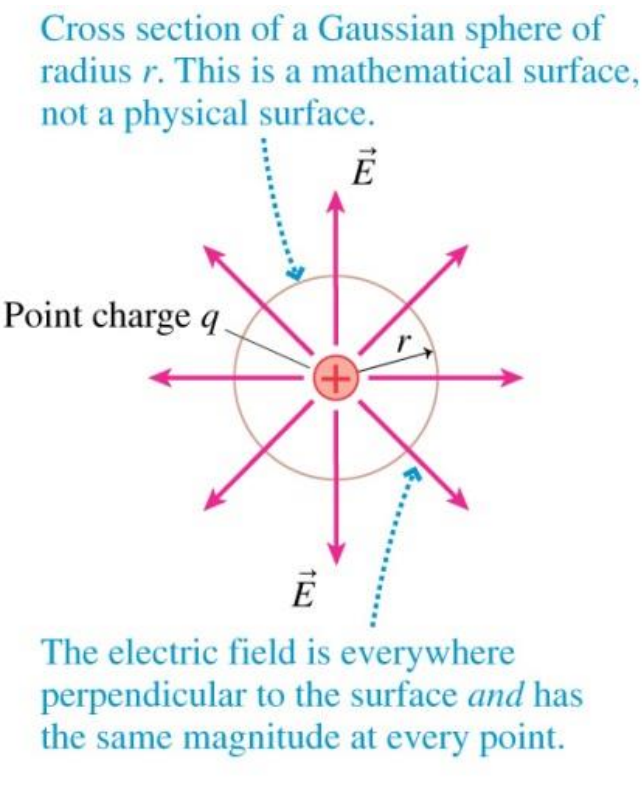
1. Electric field must point in the **radial direction** only.
2. The electric field must be the **same magnitude** at constant radius.

Using GAUSS's Law

1. Point source

(spherical symmetry → <u>choose</u> a sphere for Gaussian surface)

Use Gauss's law to compute the E field at a distance r from the positive charge



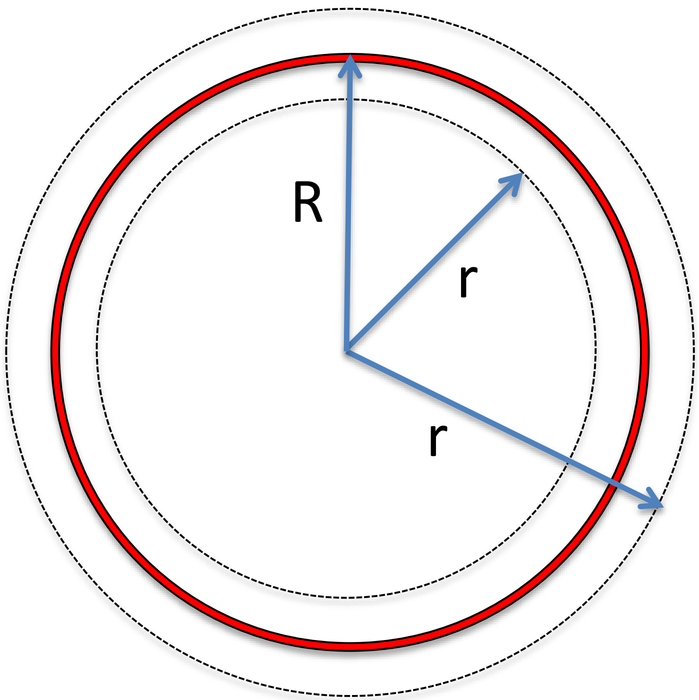
$$\Phi_E = \oiint \vec{E} \cdot d\vec{A} = \mathrm{E}A_{sphere} = \frac{q}{\varepsilon_0}$$

$$\boxed{E = \frac{q}{\varepsilon_0 \mathrm{A}_{\mathrm{sphere}}} = \frac{q}{\varepsilon_0 4\pi r^2}}$$

This is Coulomb Law!

The integral is very simple because the E field has constant magnitude everywhere on the surface and can therefore be factored outside of the integral. The area of a sphere is $4\pi r^2$, so the flux integral is equal to $E*4\pi r^2 = Q/\varepsilon_0$.

1. Shell of charge



Task

Use Gauss' law to compute the E field inside and outside a spherical shell of charge

Symmetry argument:

1. Electric field must point in the **radial direction** only.
2. The electric field must be the **same magnitude** at constant radius.

$$\Phi_E = \oiint \vec{E} \cdot d\vec{A} = \mathrm{E}A_{sphere} = \frac{q_{enc}}{\varepsilon_0}$$

Inside the shell: $q_{enc} = 0$

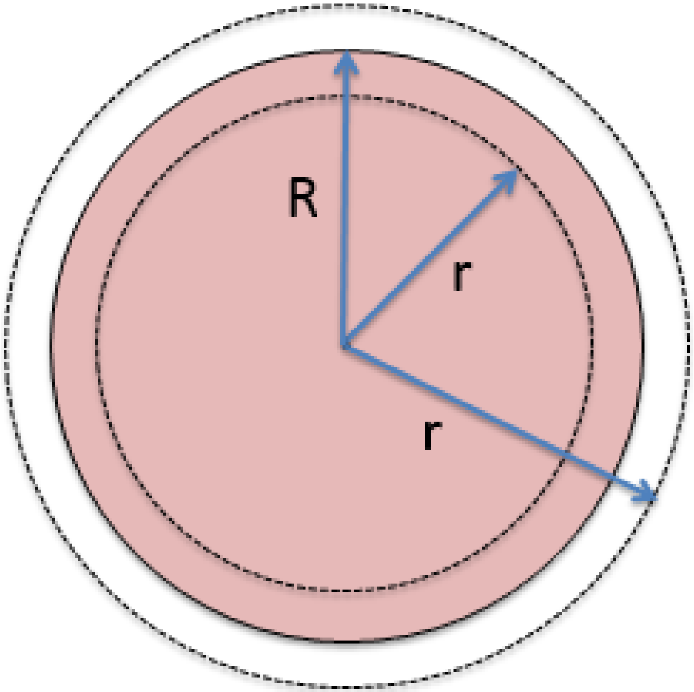
$$\boxed{E = 0 \quad \text{for} \quad r < R}$$

Outside the shell: $q_{enc} = Q$

$$\boxed{E = \frac{Q}{4pe_0 r^2} \quad \text{for} \quad r > R}$$

Using Gauss' Law

1. Solid ball of uniform charge density (charge Q)



Outside the ball: $q_{enc} = Q$

$$\boxed{E = \frac{Q}{4\pi\varepsilon_0 r^2} \quad \text{for} \quad r > R}$$

Task

Use Gauss' law to compute the E field inside and outside a uniformly charged ball

Symmetry argument:

1. Electric field must point in the **radial direction** only.

2. The electric field must be the **same magnitude** at constant radius.

$$\Phi_E = \oiint \vec{E} \cdot d\vec{A} = \mathrm{E}A_{sphere} = \frac{q_{enc}}{\varepsilon_0}$$

Inside the ball:

$$q_{enc} = \rho V_{ball} = \left(\frac{Q}{\cancel{\tfrac{4}{3}\pi} R^3}\right)\cancel{\tfrac{4}{3}\pi} r^3 = Q\frac{r^3}{R^3}$$

$$E = \frac{Qr^3}{\varepsilon_0 (4\pi r^2) R^3} = \boxed{\frac{Qr}{4\pi\varepsilon_0 R^3}} \quad \text{for} \quad r < R$$