Electricity and Magnetism

- Physics 259 – L02
- Lecture 29

Chapter 25: Capacitance

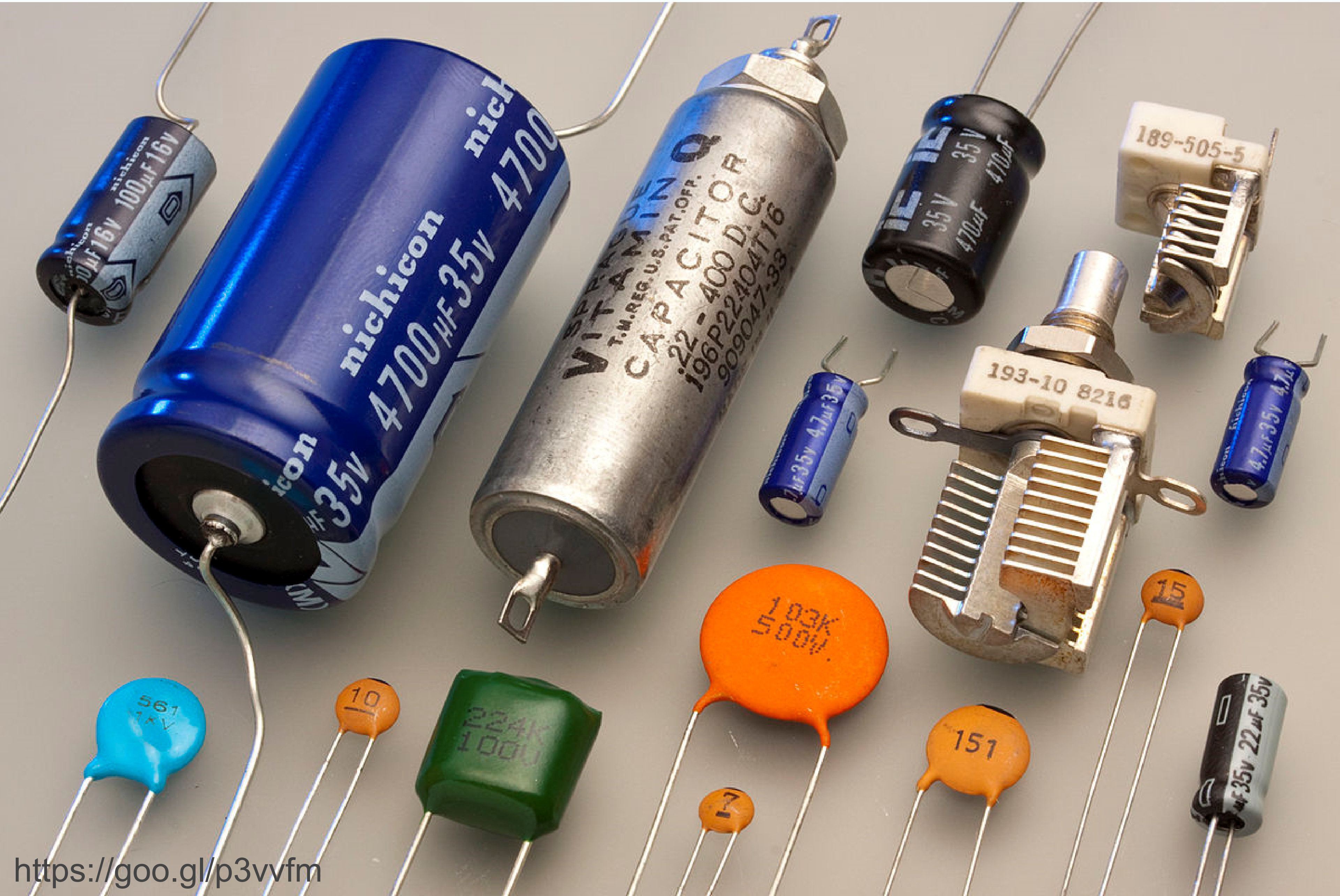
Last time

- Chapter 24: Electric potential

This time

- Capacitance

Capacitance



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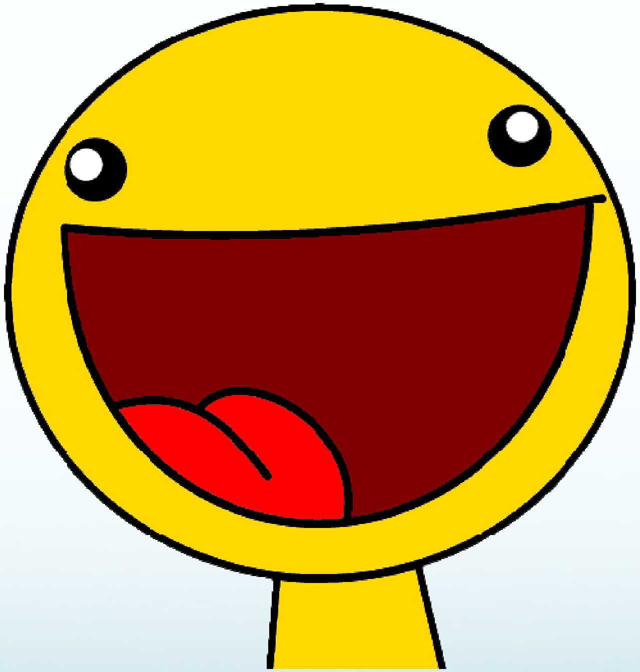
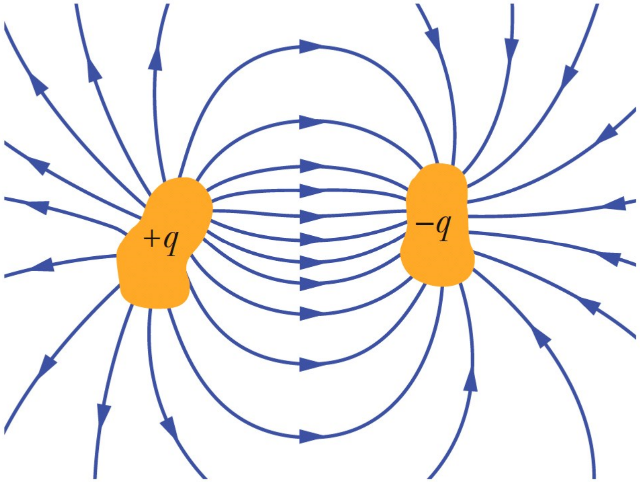
25-1 Capacitance



One goal of physics →
Provide the basic science for practical devices designed by engineers

25-1 Capacitance



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A capacitor is any two electrodes separated by some distance. Regardless of the geometry, we call the electrodes “plates”.

Capacitors in General

A capacitor consists of two isolated conductors (the plates) with charges *+q* and *-q*. Its **capacitance** *C* is defined from

$$\boxed{q = CV.} \rightarrow C = \frac{q}{V} \rightarrow \frac{C}{V}$$

V → potential difference between the plates.

$$1\,\frac{C}{V} = 1\,F$$

↓

Farad

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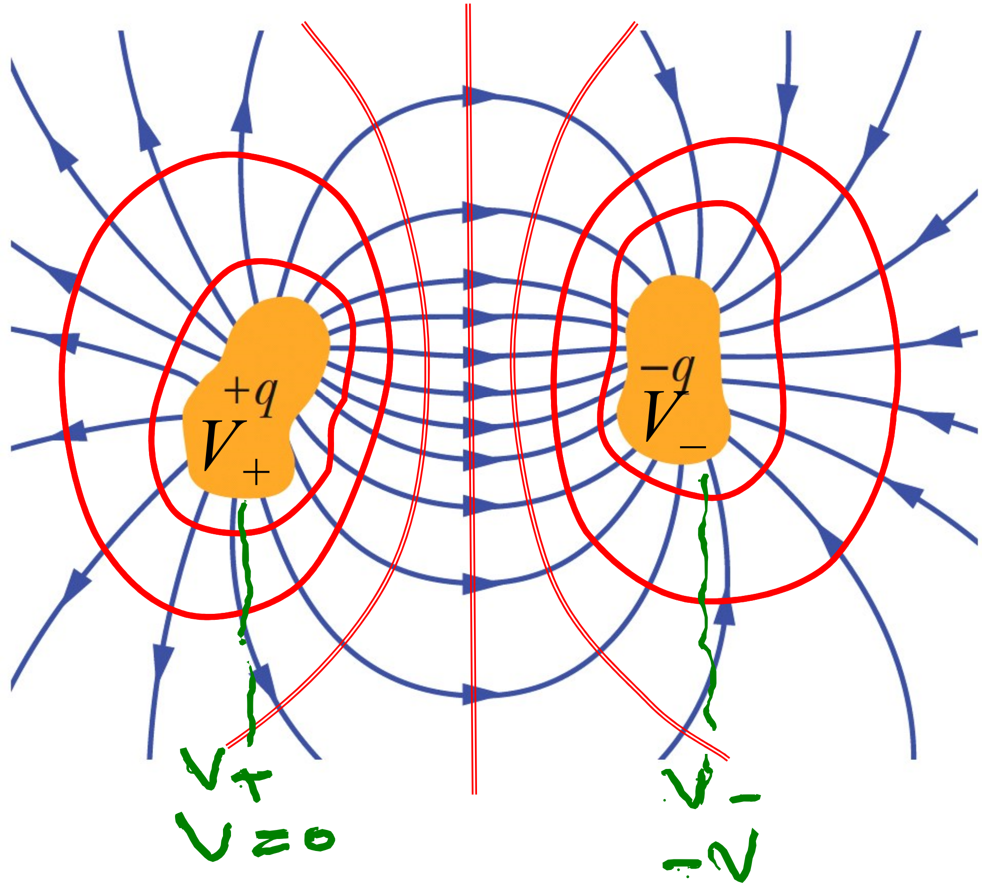
Capacitors in General

For equal but opposite charges one the plates, this arbitrary set of electrodes creates an electric field. What are the equipotential lines?



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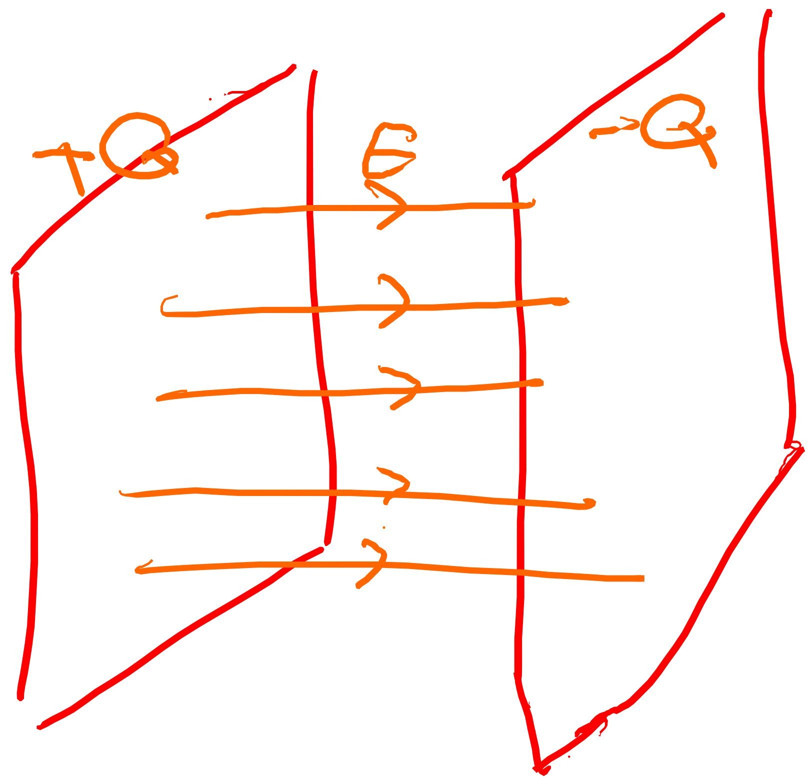
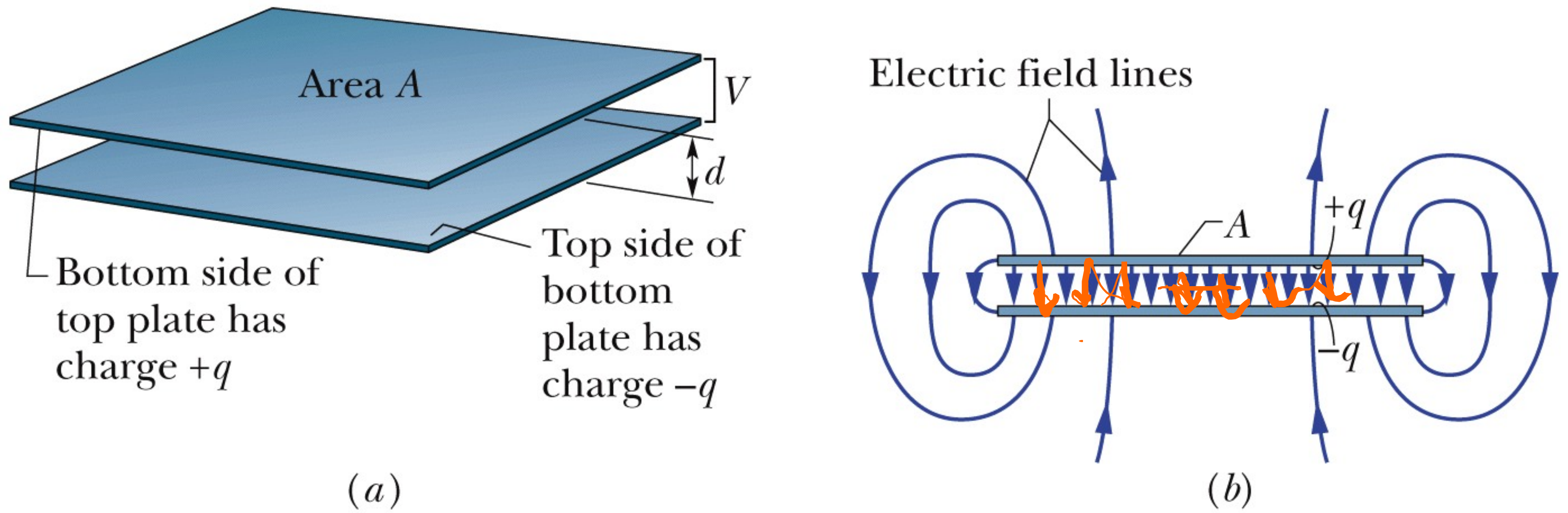
The potential changes from V_+ on the positive plate to V_- on the negative plate.



+Q
E
-Q

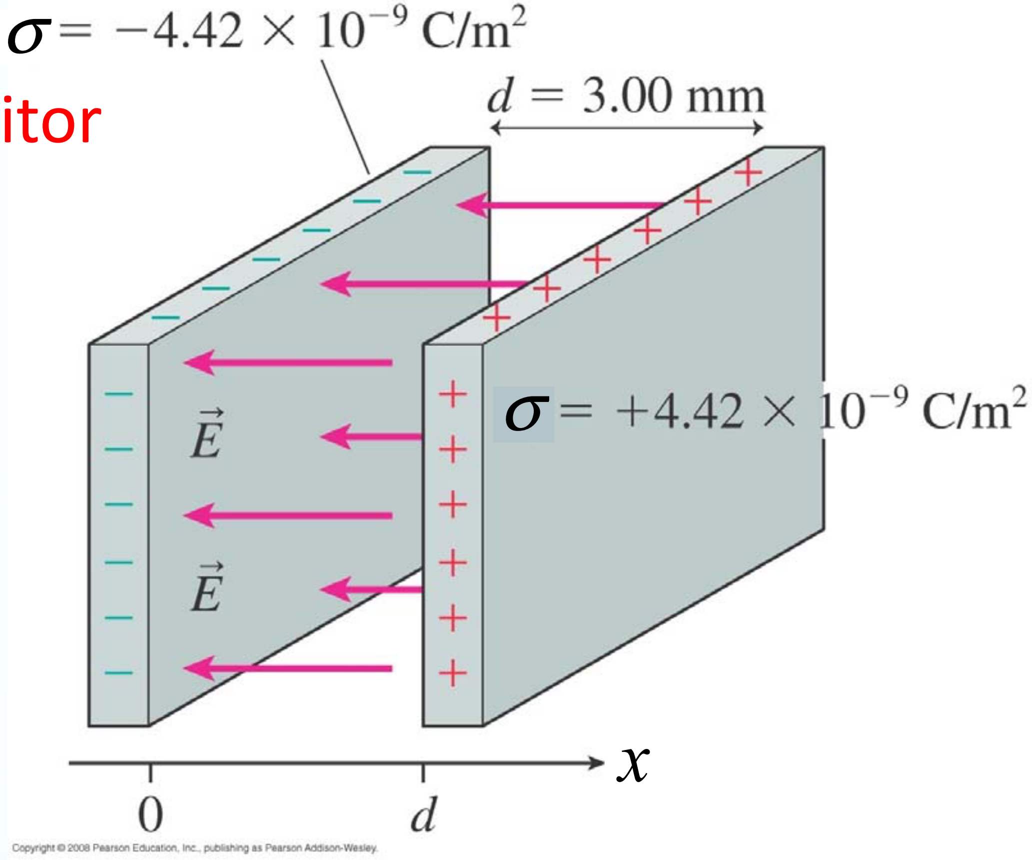
Example: Parallel plate capacitor



Made up of two plates of:
Area A separated by a distance *d*.
The charges on the facing plate surfaces have the same magnitude *q* but opposite signs.

Electric field due to the charged plates is uniform in the central region between the plates.
The field is not uniform at the edges of the plates, as indicated by the "fringing" of the field lines there.

Example: Parallel plate capacitor

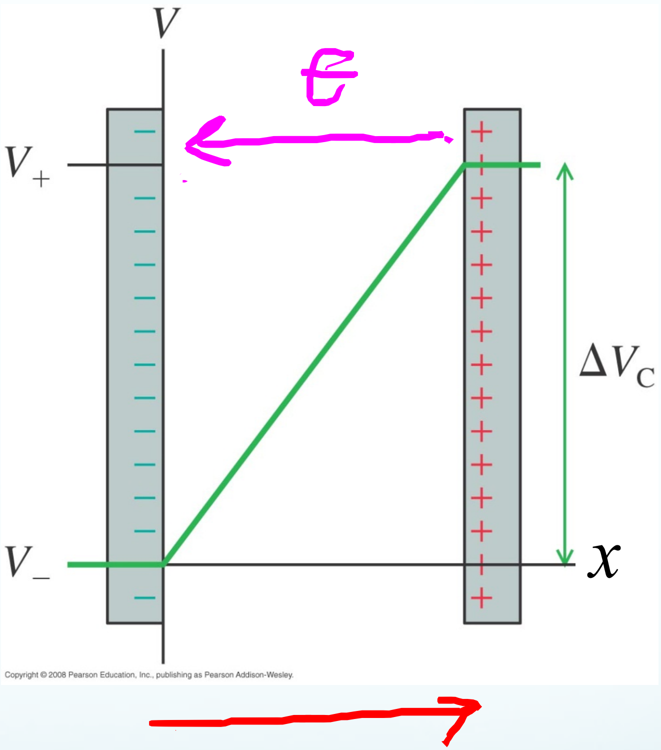


The source charges on the capacitor plates create a uniform electric field between the plates of

$$\vec{E} = \frac{\sigma}{\varepsilon_0} \text{ from positive to negative}$$

$$V = -\int \vec{E} \cdot \vec{ds}$$

The electric potential inside a charged capacitor increases linearly from the negative to the positive plate.

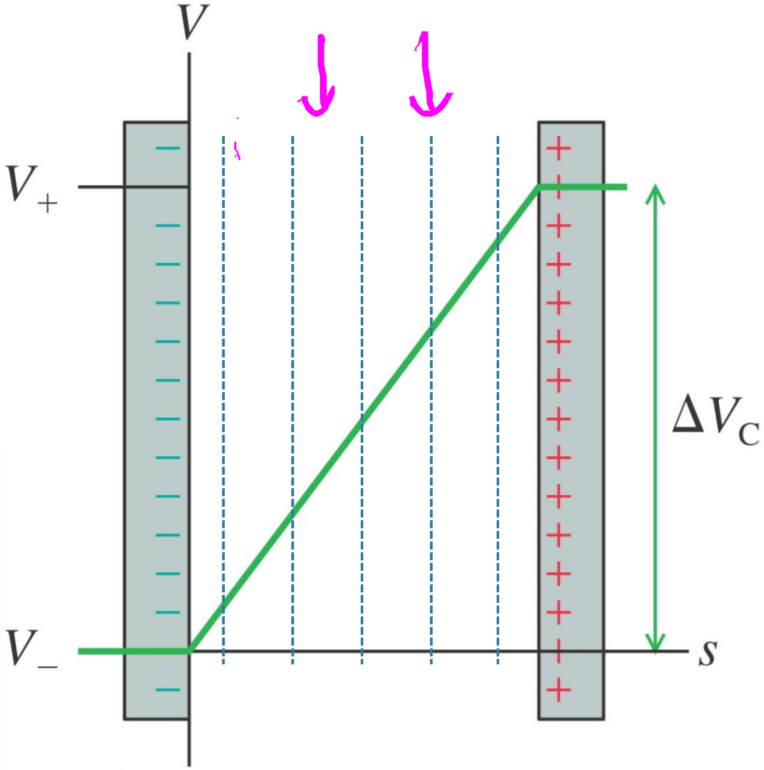


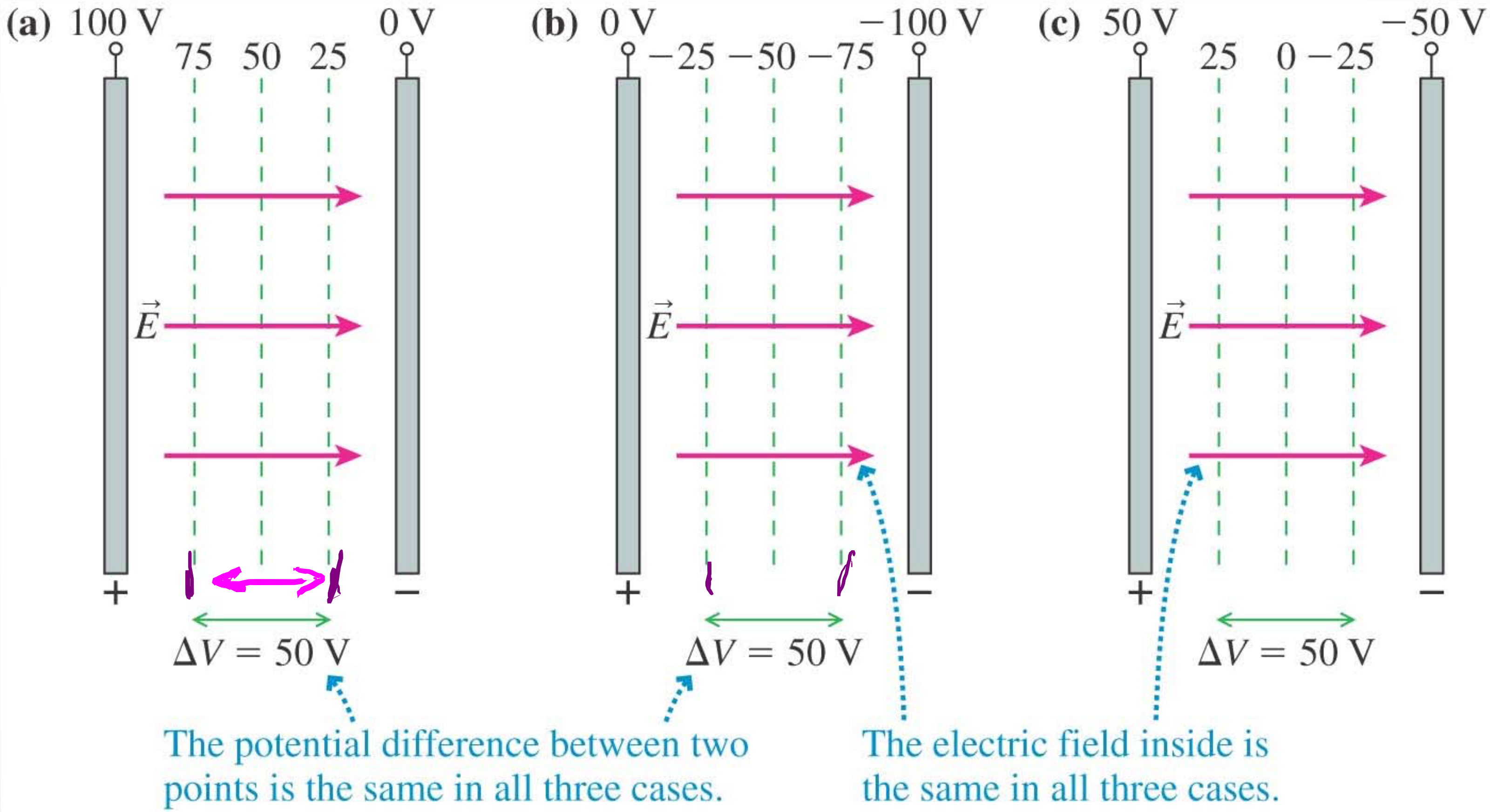
Top Hat Question

What are the lines of equipotential inside the parallel plate capacitor?

A. .Vertical lines
B. Horizontal lines
C. Diagonal lines slanting to the right
D. Not enough info

$c = \frac{q}{V}$



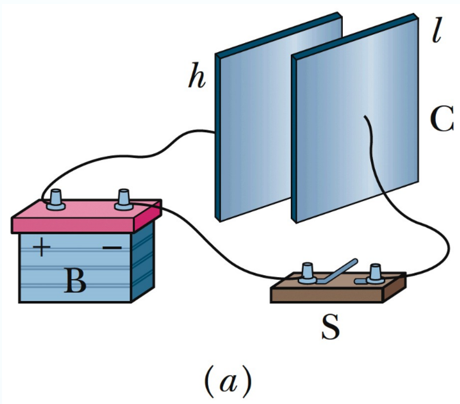


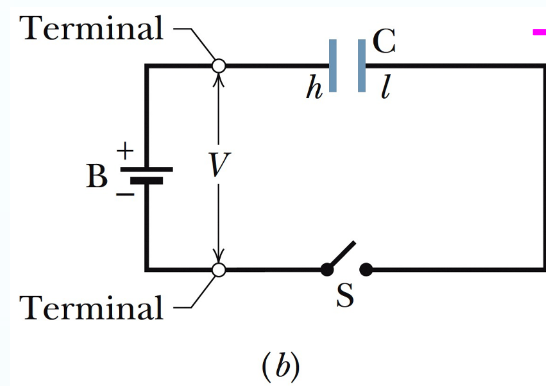
We can define V = 0 anywhere we want. Our choice of V = 0 does not affect any potential differences or the electric field.

Charging Capacitor

When a circuit with a battery, an open switch, and an uncharged capacitor is completed by closing the switch, conduction electrons shift, leaving the capacitor plates with opposite charges.

$-q \quad +q$





The battery maintains potential difference V between its terminals.

When the plates are uncharged, the potential difference between them is zero→ As the plates become oppositely charged, that potential difference increases until it equals the potential difference V between terminals of the battery.

25-2 Calculating the Capacitance

Calculating electric field and potential difference

1. To relate the electric field ***E*** between the plates of a capacitor to the charge *q* on either plate→ use Gauss' law:

1 $$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q.$$

2. the potential difference between the plates of a capacitor is related to the field ***E*** by

2 $$V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s},$$

Letting V represent the difference $V_f = V_i$, we can then recast the above equation as:

$$V = \int_-^+ E\, ds$$

3. Find Capacitance

3 $$q = CV.$$

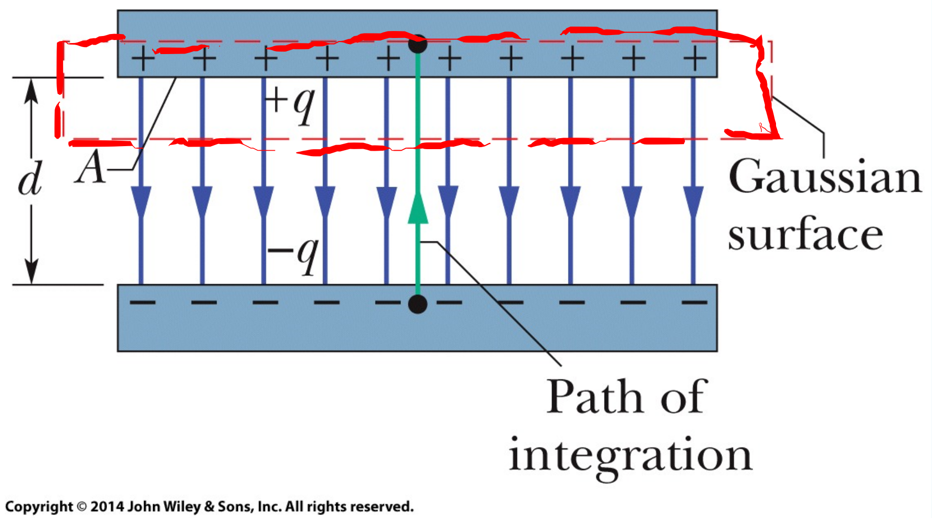
25-2 Calculating the Capacitance: Parallel-Plate Capacitor

Very large plates and very close together→ ***E*** constant throughout the region between the plates.

1. Use Gauss's law q

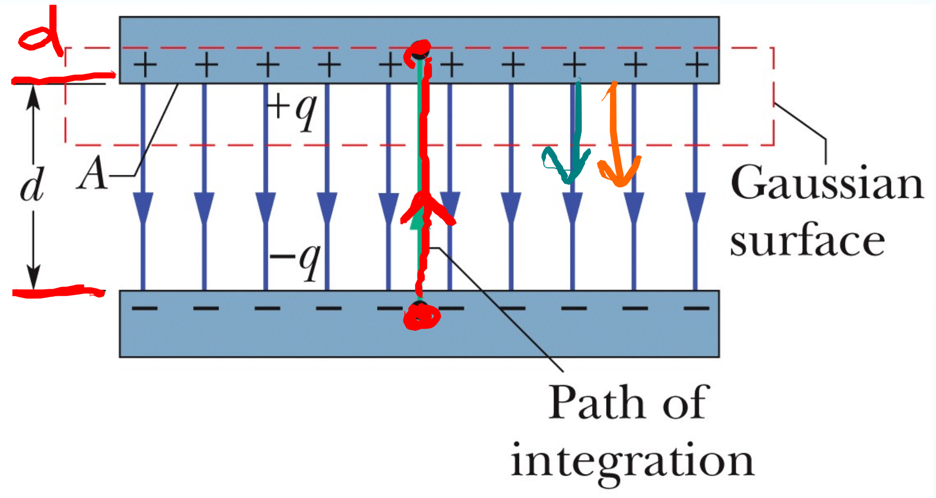
2. Find potential V

3. Find Capacitance $C = \frac{q}{V}$



Gaussian surface that encloses just charge *q* on positive plate

25-2 Calculating the Capacitance: Parallel-Plate Capacitor



1. Use Gauss's law

$\oint \vec{E}\cdot dA = \frac{q}{\varepsilon_0} \rightarrow E\int dA = \frac{q}{\varepsilon_0}$

$\rightarrow EA = q/\varepsilon_0 \Rightarrow$

$$\boxed{q = \varepsilon_0 EA}$$

2. Find potential

$$\boxed{V = \int_-^+ E\,ds = E\int_0^d ds = Ed.}$$

3. Find Capacitance

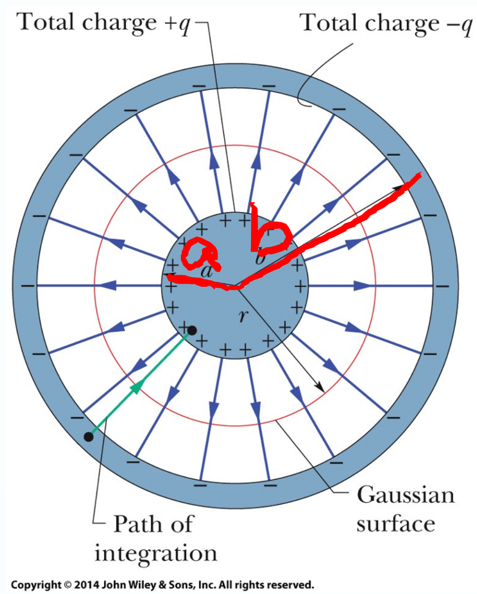
$$\boxed{q = CV.}$$

$\rightarrow C = \frac{q}{V} = \frac{\varepsilon_0 \cancel{E} A}{\cancel{E} d} = \varepsilon_0 \frac{A}{d}$

$\Longrightarrow$

$$\boxed{C = \frac{\varepsilon_0 A}{d} \quad \text{(parallel-plate capacitor).}}$$

25-2 Calculating the Capacitance: Cylindrical Capacitor



- ✓ cylindrical capacitor of length L formed by two coaxial cylinders of radii a and b.
- ✓ $L \gg b$ → neglect fringing of electric field that occurs at ends of the cylinders.
- ✓ Each plate contains a charge of magnitude q.

1. Use Gauss's law q

2. Find potential V

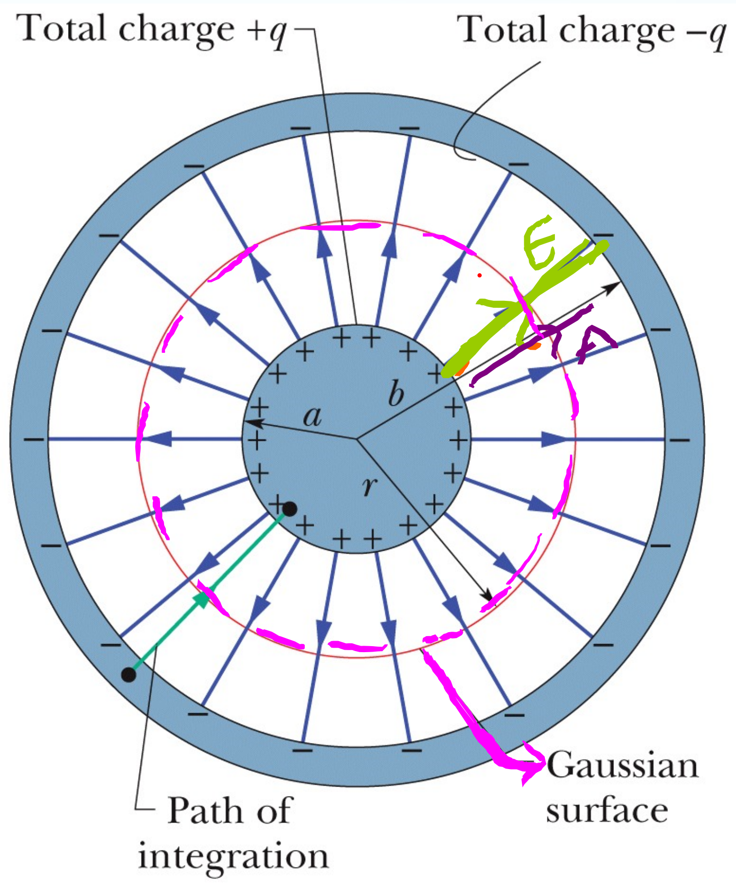
3. Find Capacitance $C = \frac{q}{V}$

25-2 Calculating the Capacitance: Cylindrical Capacitor

1. Use Gauss's law

$\oint \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0} \rightarrow EA = q/\varepsilon_0$

$\rightarrow q = \varepsilon_0 E A \rightarrow$ $\boxed{q = \varepsilon_0 EA = \varepsilon_0 E(2\pi r L)}$



2. Find potential

$$\boxed{V = \int_{-}^{+} E\,ds = -\frac{q}{2\pi\varepsilon_0 L}\int_b^a \frac{dr}{r} = \frac{q}{2\pi\varepsilon_0 L}\ln\left(\frac{b}{a}\right)}$$

3. Find Capacitance

$$C = 2\pi\varepsilon_0 \frac{L}{\ln(b/a)} \quad \text{(cylindrical capacitor).}$$

25-2 Calculating the Capacitance

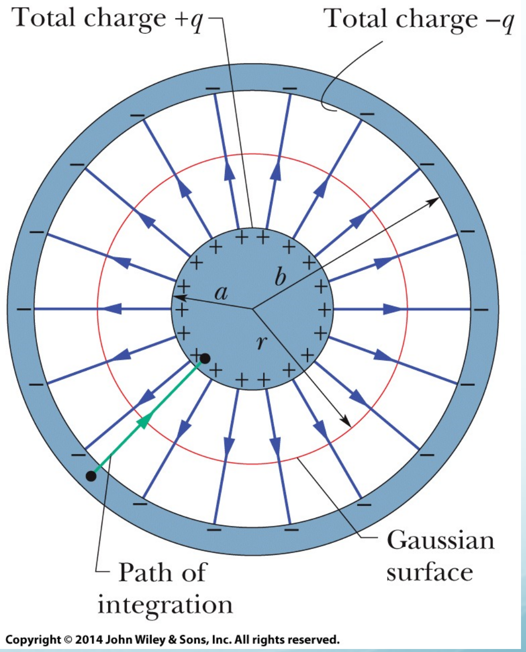
Others…

For **spherical capacitor** the capacitance is:

$$C = 4\pi\varepsilon_0 \frac{ab}{b-a} \quad \text{(spherical capacitor).}$$

Capacitance of an **isolated sphere:**

$$C = 4\pi\varepsilon_0 R \quad \text{(isolated sphere).}$$



Capacitors

General relationship: $$Q = C\Delta V_C$$

Parallel plate capacitor: $$Q = \left(\frac{\varepsilon_o A}{d}\right)\Delta V_C$$

Spherical capacitor: $$Q = \left(\frac{4\pi\varepsilon_0 r_b r_a}{r_b - r_a}\right)\Delta V_C$$

Isolated sphere: $$Q = \left(4\pi\varepsilon_0 R\right)\Delta V_C$$

Cylindrical capacitor: $$Q = \left(\frac{2\pi\varepsilon_0 L}{\ln\left(\frac{r_B}{r_A}\right)}\right)\Delta V_C$$

This section we started:

Chapter 24

See you on next Wednesday