Last time

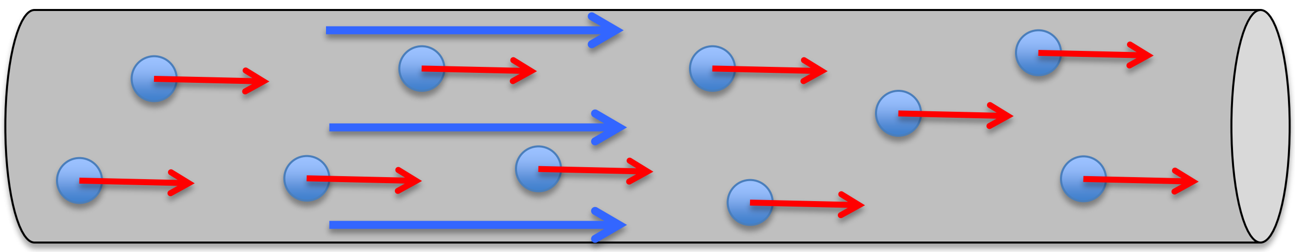
- Electric current (time dependent processes)
- Activity #8

This time

- Electric current: a microscopic picture
- Current density (a vector) vs current (a scalar)
- Electric fields in conductors and electron drift speed
- Resistance as a geometrical factor
- Resistivity: a microscopic picture

Current

Conductors are allowed to have non-zero electric field inside for non-steady state situations (this is what causes the charges to move).



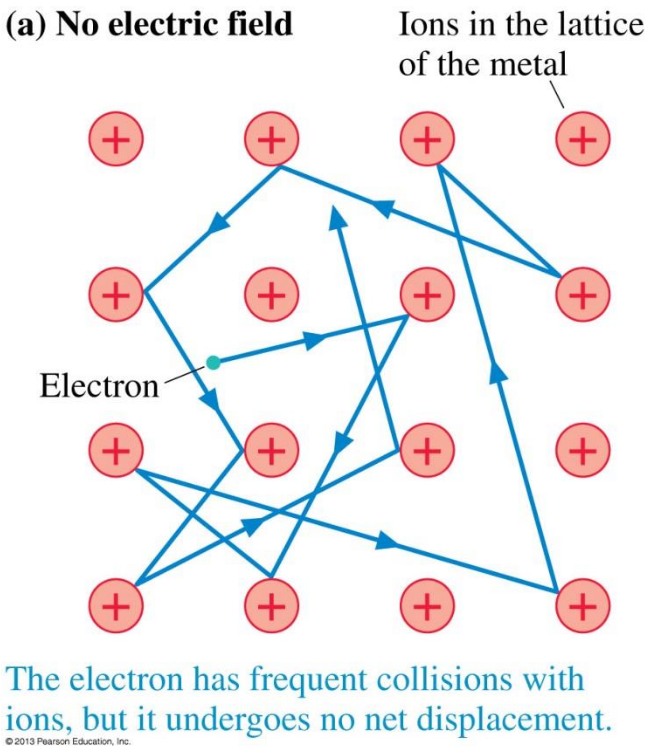
The ordered flow of charges is called the electric current. What is the ordered flow of charges?

Inside a conductor with no electric field

The zigzag motion (random motion) of a charge in a conductor

The change in the direction is due to collisions with atoms and other electrons in the conductor.

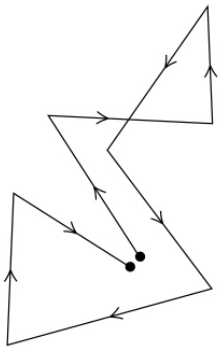
In the absence of an electric field the drift velocity due to random motion is zero.



Like molecules of air in our class room in the absence of any draft.

Conduction electrons are charge carriers. They collide with each other and with the atoms in the conductor. Atoms vibrate, because of thermal agitation and exchange energy with conduction electrons.

$$\frac{1}{2} m_e \mathrm{v}_{\mathrm{th}}^2 = \frac{3}{2} kT$$



$$T = 300\ K$$

$$\mathrm{v}_{\mathrm{th}} = \sqrt{\frac{3kT}{m_e}} \approx 10^5\ \mathrm{m/s}$$

But

$$\overline{\mathrm{v}}_{\mathrm{th}} = 0$$

Between collisions

Averaged over a long time

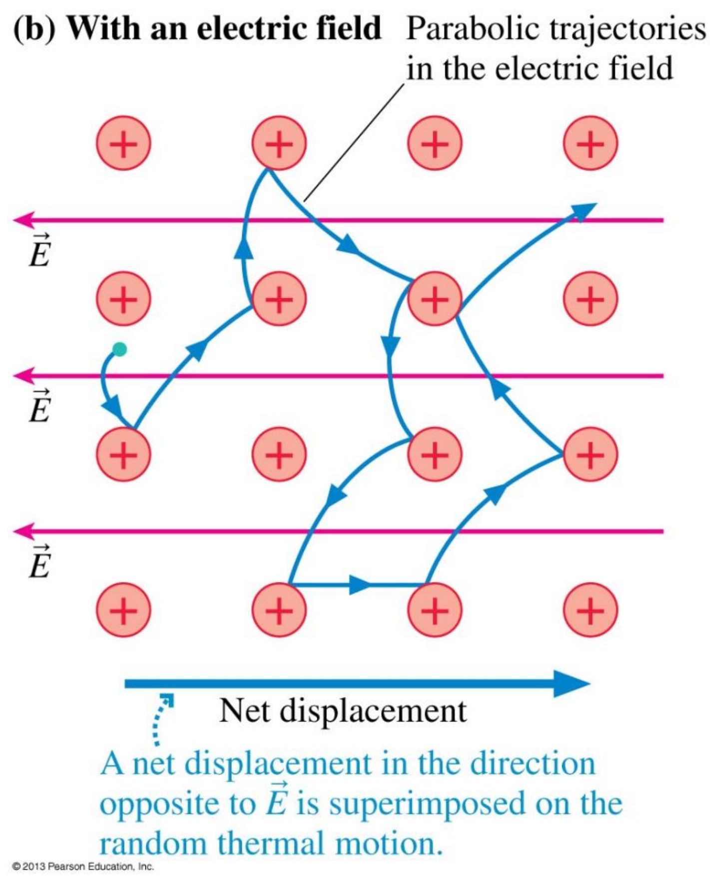
Inside a conductor with a constant electric field

Recall

$$\vec{F} = -e\vec{E} = m\vec{a}$$

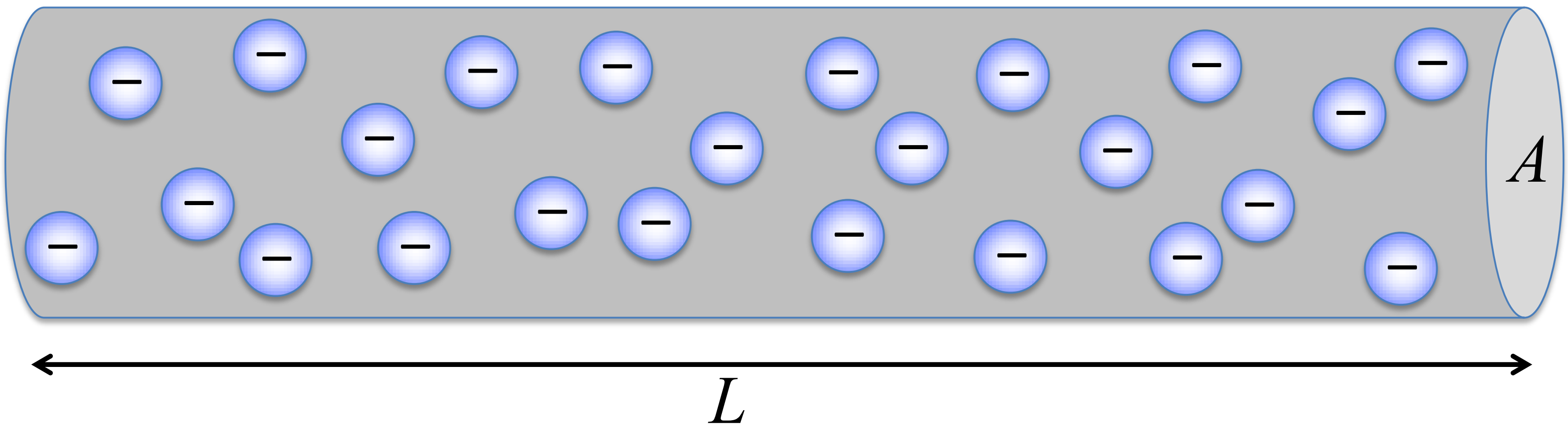
$$\vec{a} = -e\vec{E} / m$$

For a constant electric field, electrons experience parabolic trajectories.



Net result: electrons move at an average net "drift speed" v_d

Current Density and Drift Velocity



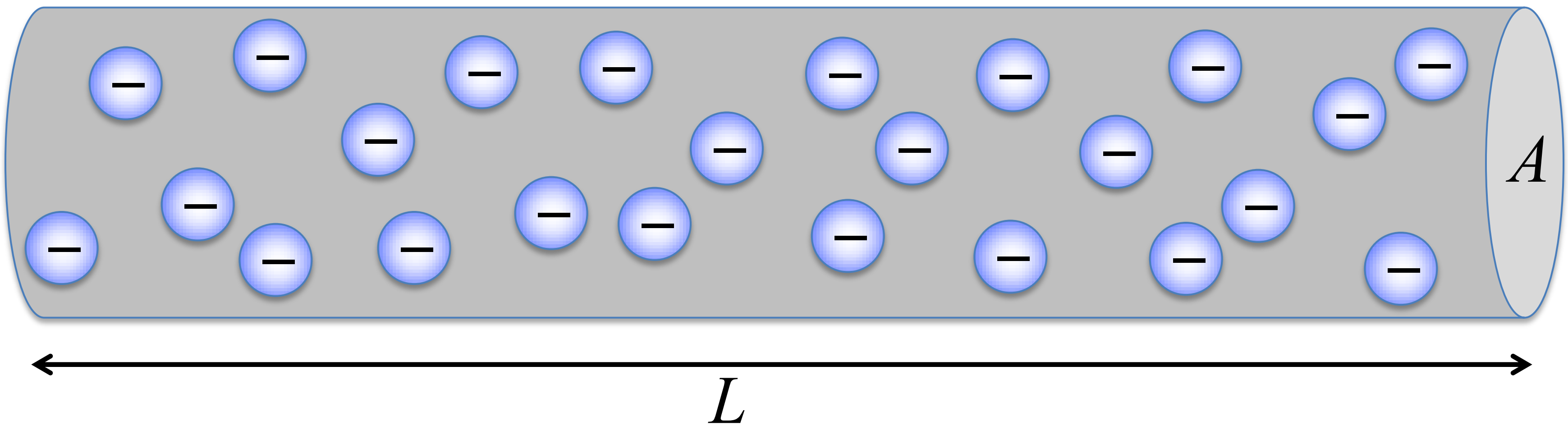
If the volume density of conduction electrons is n_e, then the amount of charge contained in a length L of the wire is

$$q = n_e(AL)e$$

The time it takes each charge to travel a distance L is $t = L/v_d$, so the current is

$$i = \frac{q}{t} = \frac{n_e(AL)e}{L/v_d} = n_e A e v_d$$

Current Density



$$i = n_e A e v_d = JA$$

The current density is then seen to be given by the **charge density** *ne* and the **drift velocity** (average velocity of the electrons)

$$\vec{J} = n_e e \vec{v}_d$$

Determine the order of magnitude of V_d in copper

Assume a current of 1 A, cross sectional area of 1.0 mm^2 and availability of one conduction electron per copper atom.

$$\rho = 8.9 \text{ g/cm}^3 \qquad \text{Atomic mass} = 63.5 \text{ g/mol}$$

$$n_e = \frac{\left(8.9 \text{ g/cm}^3\right)}{\left(63.5 \text{ g/mol}\right)}\left(6.0\times10^{23} \text{ atoms/mol}\right)\left(1 \text{ electron/atom}\right)$$

$$n_e \simeq 6.0 \times 10^{22} \text{ electrons/cm}^3$$

$$n_e \simeq 6.0 \times 10^{28} \text{ electrons/m}^3$$

$$\mathrm{v}_d = \frac{I}{n_e e A} \approx 0.1 \text{ mm/s}$$

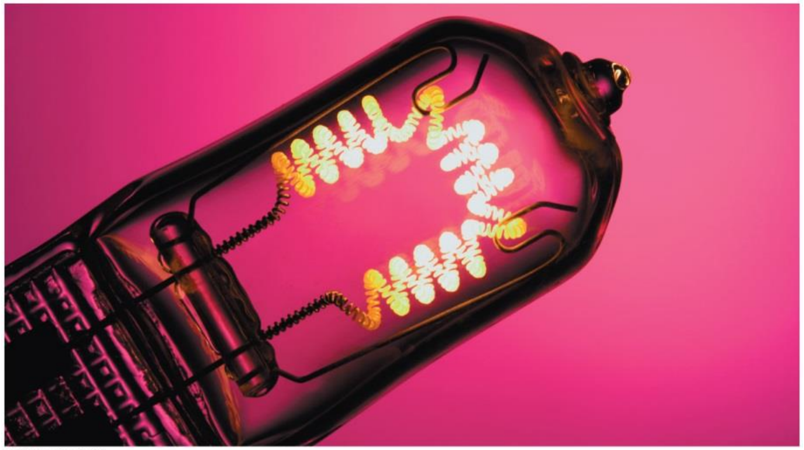
The relatively low drift velocity is much the same like the wind powered by pressure difference. The molecules are randomly zigzagging around at about 1 Km/s, while the organized breeze is much slower.

Resistance

Resistance is a property of conductors that are not ideal:

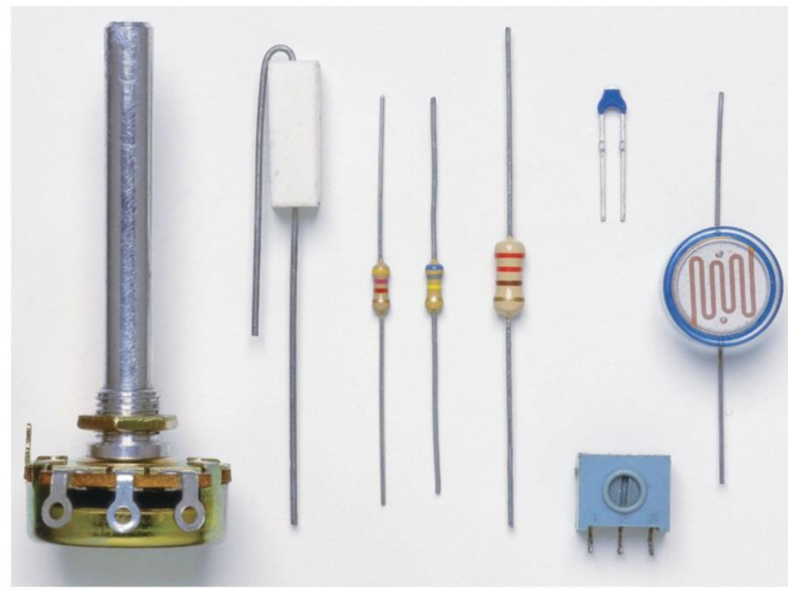
- Electrons have frequent collisions with atomic nuclei. At constant temperature, this process is in thermal equilibrium.
- When a voltage difference is created across the conductor, this accelerates the electrons, making their collisions more energetic.
- This gets dissipated as heat inside the metal

Tungsten filament:



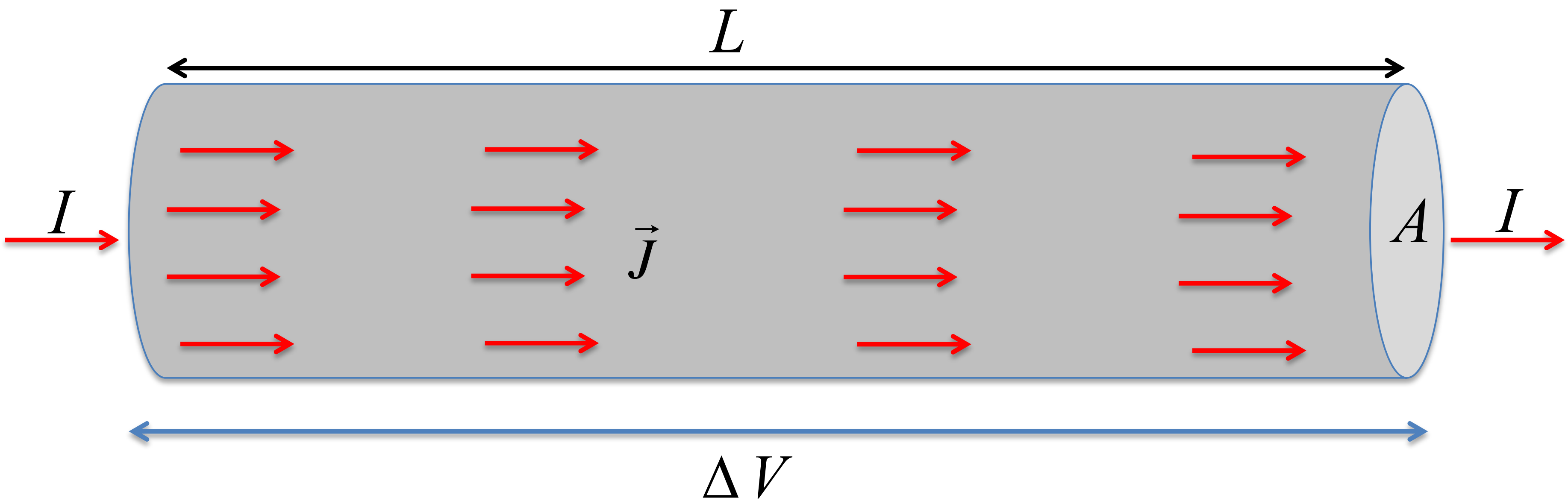
Resistors

A resistor is any circuit element that dissipates energy. Light bulbs are the classic example, but there are others:



How much energy is dissipated by a given resistor is encoded in a property called its resistance *R*. The resistance is dependent on the particular material used as well as the geometry.

How can we quantify this

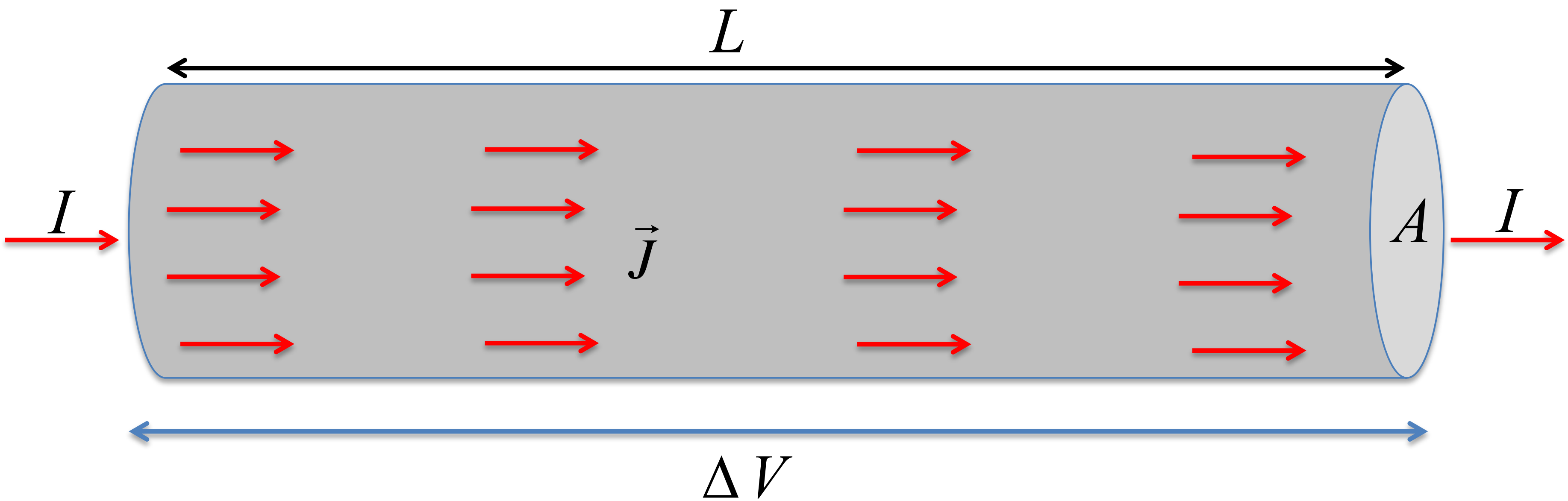


For Ohmic materials (linear)

$$\vec{J} = \sigma \vec{E} \qquad\qquad \vec{J} = \sigma \vec{E} = \frac{1}{\rho} \vec{E}$$

σ Is the conductivity (constant) and ρ is the resistivity (constant). This is a physical property of the material that makes up the resistor.

How can we quantify this

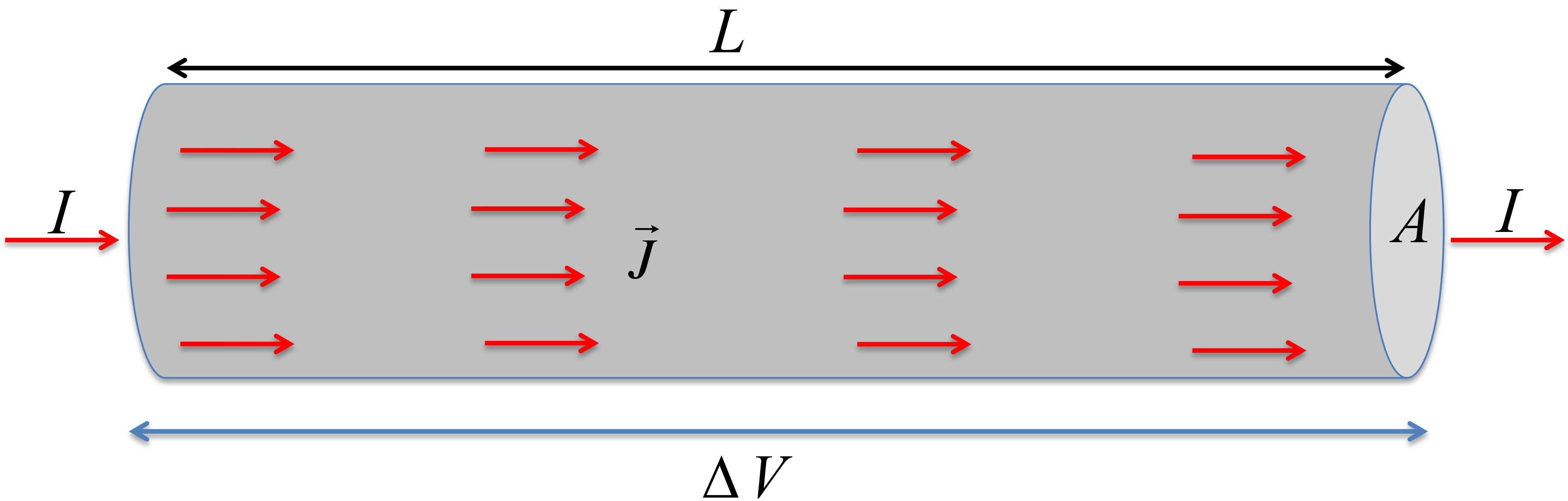


$$\rho = \frac{E}{J} = \frac{\Delta V / L}{I / A}$$

Using the resistivity, we can define a geometric quantity of the resistor:

$$\rho \frac{L}{A} = \frac{\Delta V}{I}$$

How can we quantify this

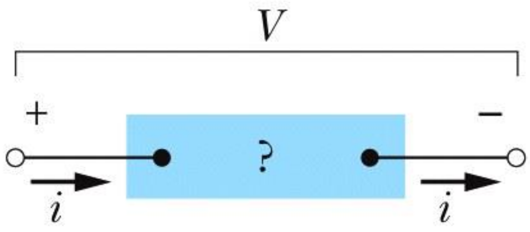


We introduce the resistance, which is dependent on ρ of the material and on the geometry of the resistor

$$\rho \frac{L}{A} \equiv R \qquad \rho \frac{L}{A} = \frac{\Delta V}{I} \qquad R = \frac{\Delta V}{I}$$

This gives us the familiar form of Ohm's Law: $\boxed{\Delta V = IR}$

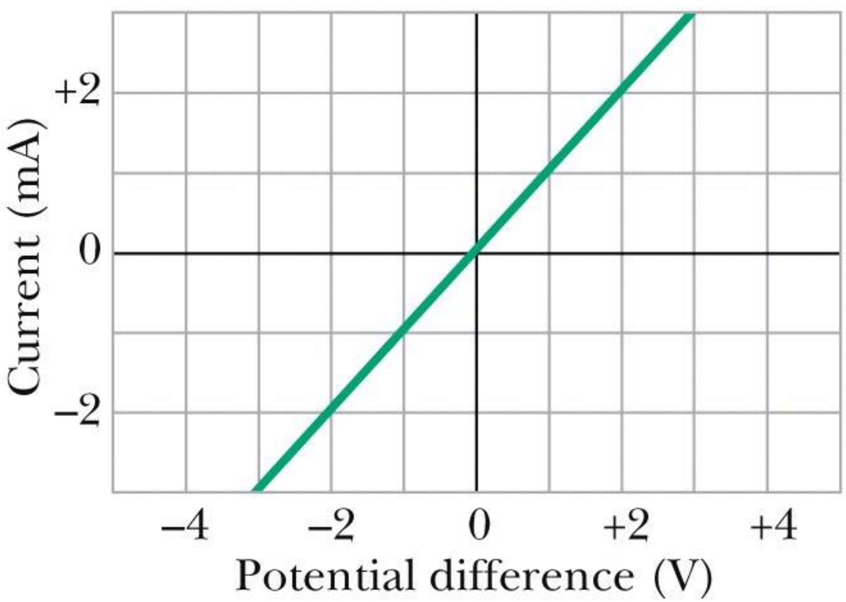
Ohmic vs non-Ohmic devices



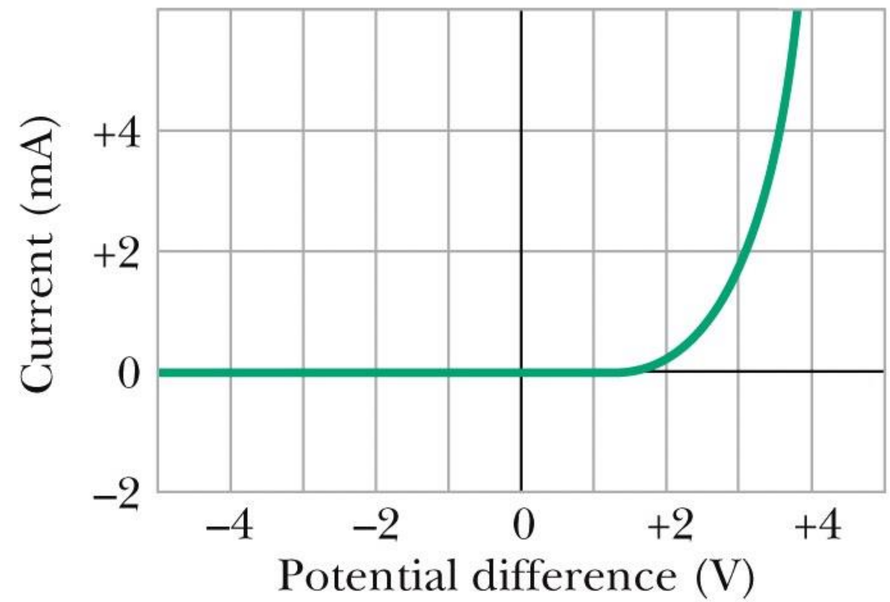
(a)

Materials with isotropic electrical properties

Materials with anisotropic electrical properties (pn junction diode)



(b)



(c)

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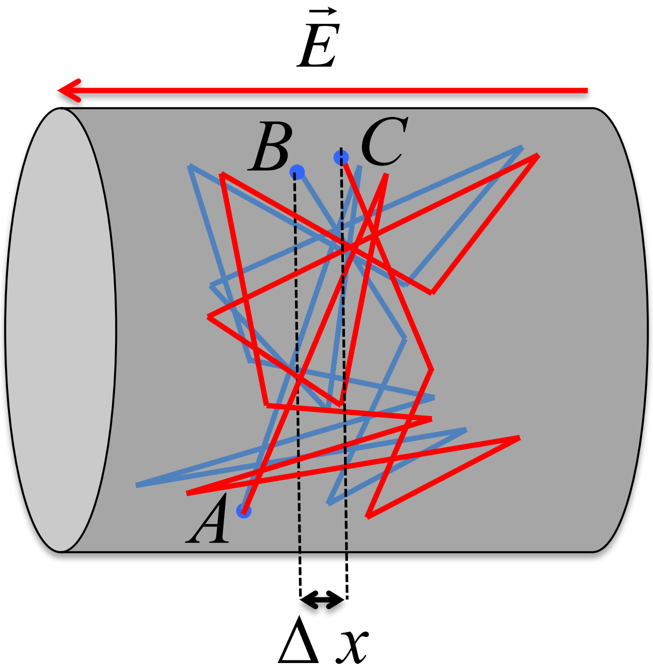
Microscopic view of resistivity



Electrons bounce around inside the metal at speeds very high speeds on the order of 0.5% light speed.

When an electric field is applied in the conductor, there is a net force on the electrons, leading to an average "drift speed" of $v_d = 0.5\ \mu\text{m/s}$

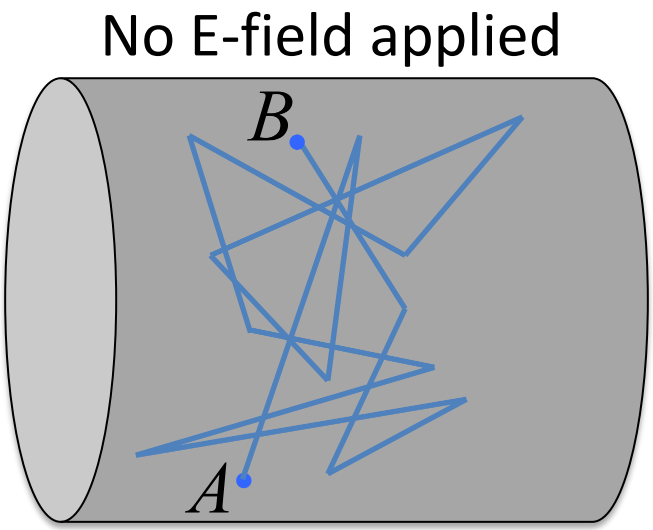
The acceleration felt by the electrons from the E-field is

$$a_x = \frac{eE}{m}$$

So the average drift speed of the electrons will be given by

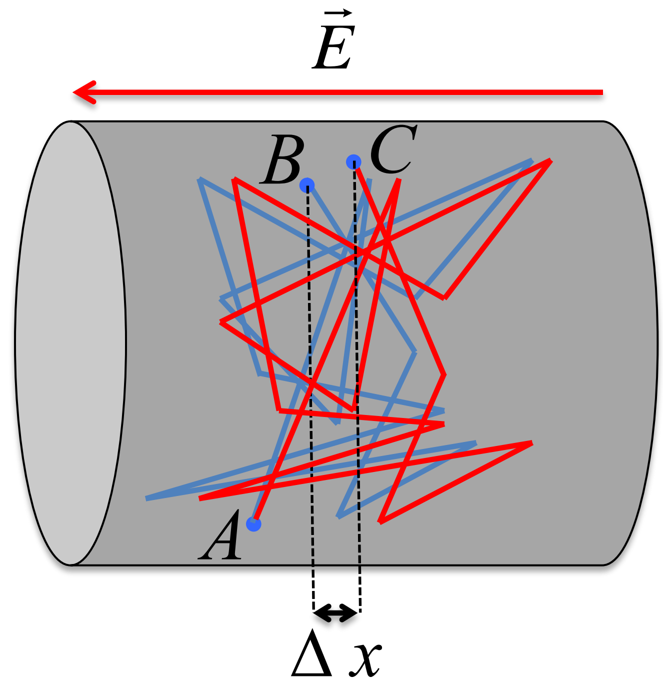
$$v_d = a\tau = \frac{eE\tau}{m}$$ but we found before: $$v_d = \frac{J}{n_e e}$$

Microscopic view of resistivity



$$v_d = \frac{eEt}{m}$$

$$v_d = \frac{J}{n_e e}$$



The average time between collisions is τ and is called the *mean free time*. Equating the two expressions for the drift speed, we get:

$$\frac{eE\tau}{m} = \frac{J}{n_e e}$$

Rearrange this to find $E = \left(\dfrac{m}{n_e e^2 \tau} \right) J$

This gives a microscopic picture of resistivity: $\boxed{\rho = \dfrac{m}{n_e e^2 \tau}}$

$$\tau = \frac{m}{n_e e^2 \rho}$$

For copper

$$n_e = 8.5 \times 10^{28} \ /\mathrm{m}^3 \qquad e = 1.6 \times 10^{-19} \ \mathrm{C}$$

$$\rho = 1.72 \times 10^{-8} \ \Omega.\mathrm{m} \qquad m_e = 9.11 \times 10^{-31} \ \mathrm{Kg}$$

$$\tau = 2.4 \times 10^{-14} \ \mathrm{s}$$ Averaged time between collisions

$$\frac{1}{\tau} = 4 \times 10^{13} \ 1/\mathrm{s}$$ Number of collisions per second (collision frequency)

Consequence of this microscopic view

When the temperature of a metal increases, its volume increases (thermal expansion) according to

$$\frac{\Delta V}{V_0} = \alpha_V \frac{\Delta T}{T_0} \qquad \alpha_V = \text{vol. coefficient of thermal expansion}$$

The resistivity depends on the conduction electron number *density* and hence implicitly depends on the volume of the metal

$$\rho = \frac{m}{n_e e^2 \tau} = \frac{mV}{Ne^2 \tau}$$

m, N, e, and τ are unaffected by T

The resistivity is a temperature dependent property

$$\boxed{\alpha = \frac{\alpha_V}{T_0}}$$

$$\Delta\rho = \frac{m\Delta V}{Ne^2\tau} = \frac{mV_0}{Ne^2\tau}\left(\frac{\Delta V}{V_0}\right) \qquad \rho - \rho_0 = \rho_0 \alpha \left(T - T_0\right)$$

This is why the resistance of a device depends on temperature