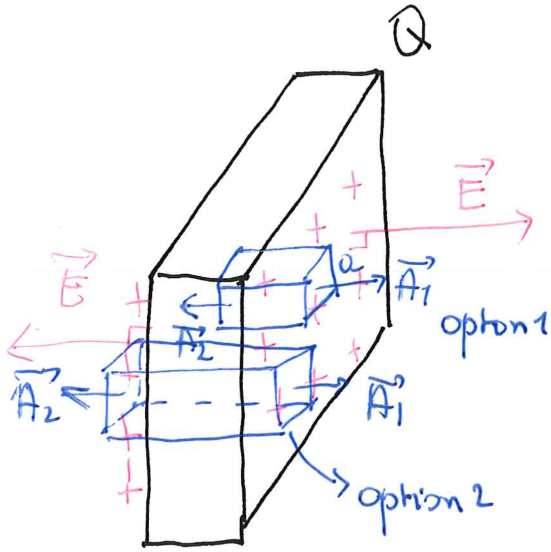
Find E-field due to the conducting plate



$$\sigma = \frac{Q}{2L^2} \qquad \sigma = \frac{Q}{Area}$$

Option 1

$$\oiint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$\Phi_E = \underbrace{\int \vec{E} \cdot \vec{A_1}}_{\text{front face}} + \underbrace{\int \vec{E} \cdot \vec{A_2}}_{\text{back face} \,=\, 0} + \underbrace{\int \vec{E} \cdot d\vec{A}}_{\text{Sides} \,=\, 0}$$

inside the conductor

$$\Phi_E = E \cdot A_1 = E \cdot a^2$$

$$q_{enc} = \sigma \cdot a^2$$

$$E \cdot \cancel{a^2} = \frac{\sigma \cdot \cancel{a^2}}{\varepsilon_0} \qquad E = \frac{\sigma}{\varepsilon_0}$$

Option 2

$$\Phi_E = \underbrace{\int \vec{E} \cdot \vec{A_1}}_{\text{front}} + \underbrace{\int \vec{E} \cdot \vec{A_2}}_{\text{back}} = E \cdot a^2 + E \cdot a^2$$

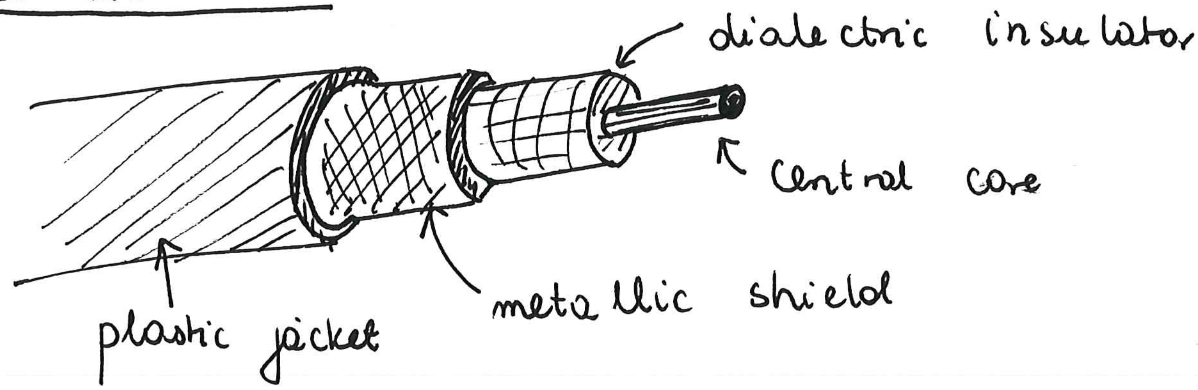
$$= 2E \cdot a^2$$

$$q_{enc} = 2 \cdot \sigma \cdot a^2$$

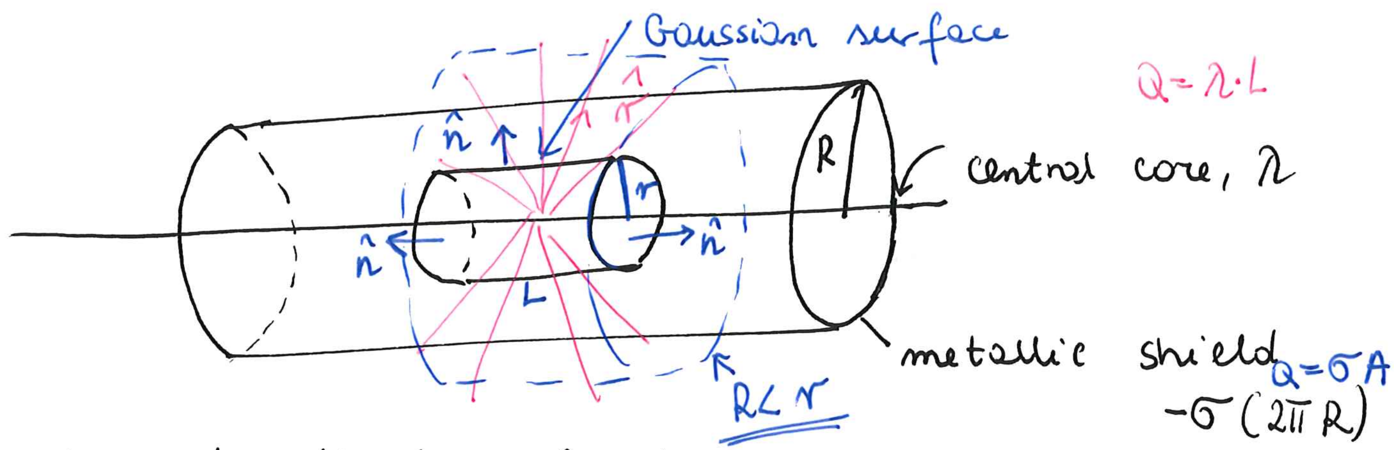
$$\cancel{2} E \cancel{a^2} = \cancel{2} \sigma \cancel{a^2} / \varepsilon_0$$

$$E = \frac{\sigma}{\varepsilon_0}$$

Co-axial cable



Ignore the dielectric insulator and plastic jacket.



In a given length L of the coaxial cable, the central core carries $+Q = \lambda L$ and the metallic shield carries $-Q = -\sigma(2\pi R L)$

INSIDE METALLIC SHIELD $r < R$

Symetry: the E-field can only point radially (assuming cable is infinite) and must be constant magnitude at constant radius $\Rightarrow$ cylindrical Gaussian surface.

$\boxed{\hat{n} \perp \hat{r}}$

$$\oint\!\!\oint \vec{E} \cdot d\vec{A} = \underbrace{\int_{\text{cup 1}} \vec{E} \cdot d\vec{A}}_{=0} + \underbrace{\int_{\text{cup 2}} \vec{E} \cdot d\vec{A}}_{=0} + \int_{\text{tube}} \vec{E} \cdot d\vec{A} = E \cdot 2\pi r \cdot L$$

$q_{enc} = \lambda \cdot L$

$$\Rightarrow \quad E \cdot 2\pi r \cdot \cancel{L} = \lambda \cdot \cancel{L} \qquad E = \frac{\lambda}{2\pi \varepsilon_0 r} \quad \text{for } r < R$$

Inside the cable, the field is that of a line of charge λ from the central core.

OUTSIDE THE CABLE:

$\vec{E} = 0$

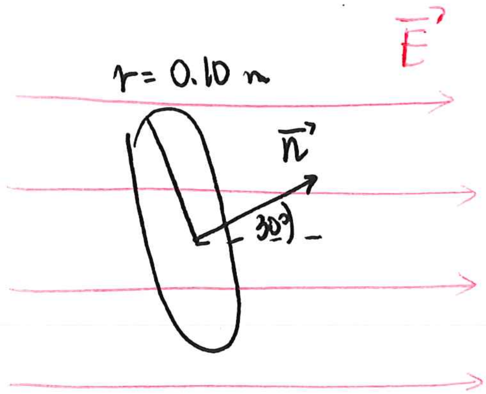
- by symetry E must still be radial and constant magnitude at constant radius

$$\oint \vec{E} \cdot d\vec{A} = \int_{tube} E\,dA = E\,A_{tube} = \frac{q_{enc}}{\varepsilon_0}$$

but $q_{enc} = \underset{\text{shield}}{-Q} + \underset{\text{core}}{Q} = 0$

This is why coaxial cables are used! They shield the outside from the fields inside. We will see these again when we talk about magnetic fields.

Q1 Circular disk



$E = 2.30 \times 10^5$ N/C

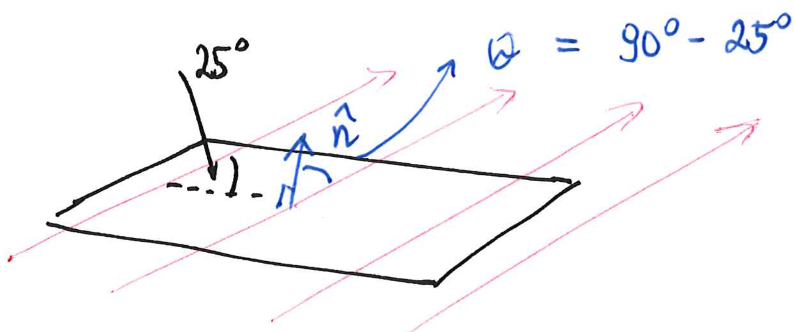
$\Phi_E = \vec{E} \cdot \vec{A}$

$= E \cdot A \cdot \cos \measuredangle(E, A)$

$\Phi_E = 2.30 \times 10^5 \, N/C \cdot \pi (0.10\,m)^2 \cdot \cos 30°$

$\Phi_E = 6257.6078 \, \frac{N \cdot m^2}{C} \approx 6.26 \times 10^3 \, \frac{N \cdot m^2}{C}$

Q2 Electric flux (square surface)



$E = 8.50 \times 10^4$ V/m

$A = 17.0 \, m^2$

$\alpha = 25°$
(angle b/w the surface & E-field vector)

θ (angle b/w normal to the surface & $\vec{E}$)

$\theta = 90° - 25° = 65°$

$\Phi_E = \vec{E} \cdot \vec{A} = E \cdot A \cos \theta$

$= 8.50 \times 10^4 \, N/C \cdot 17.0\,m^2 \cdot \cos 65°$

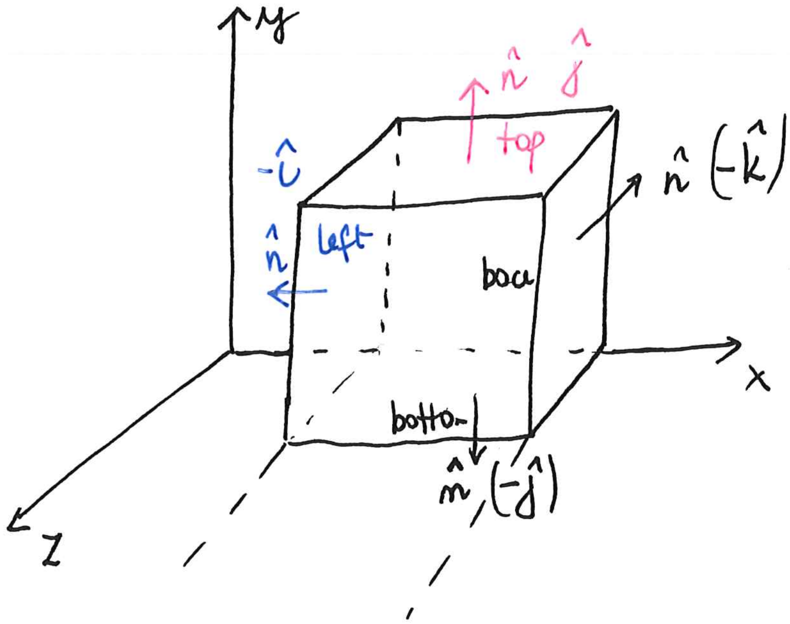
$= 8.50 \times 10^4 \, N/C \times 17.0\,m^2 \cdot (0.4226)$

$= 6.11 \times 10^5 \, \frac{N \cdot m^2}{C}$

Q3 Non-uniform electric field

$$\vec{E} = 6.1\,\hat{i} - 9.1(y^2 + 8.5)\,\hat{j} \quad N/C$$

$$l = 0.260\ m$$



top

$$\Phi_{E\,top} = \int \vec{E} \circ d\vec{A}$$

$$y = 0.260\ m$$

$$\vec{E} = 6.1\,\hat{i} - 9.1\left((0.260)^2 + 8.5\right)\hat{j} \qquad E_z = 0$$

$$\vec{E} = \underbrace{6.1}_{Ex}\,\hat{i} - 77.96\,\hat{j} = \underbrace{6.1}_{Ex}\hat{i} - \underbrace{78}_{Ey}\hat{j}$$

$$\vec{A}_{top} = (0.260\,m)^2\,\hat{j} = \underbrace{0.0676}_{Ay}\,\hat{j}\ m^2 \qquad A_x = A_z = 0$$

$$\int \vec{E} \circ \vec{A} = E_x A_x + E_y A_y + E_z A_z$$

$$\int \vec{E} \circ \vec{A} = 6.1 \times 0 + -78 \times 0.0676 + 0 \times 0 = -5.27\ N \cdot m^2/C$$

botton

$$y = 0$$

$$\vec{E} = 6.1\,\hat{i} - 9.1 \times 8.5\,\hat{j} = 6.1\,\hat{i} - 77.4\,\hat{j}$$

$$\vec{A} = 0.0676\ -\hat{j}$$

$$\int \vec{E} \circ \vec{A} = -77.4 \cdot 0.0676\,\hat{j} = +5.23\ N \cdot m^2/C$$

Left face $\vec{A} = 0.0676\ -\hat{i}$

$$\int \vec{E} \cdot \vec{A} = 6.1 \times (-0.0676)\hat{i} + E_y \times 0 + 0 \times 0 = -0.412\ N \cdot m^2/C$$

$$\vec{E} = 6.1\,\hat{i} - (9.1y^2 + 8.5)\hat{j}$$

Right face $\Phi_E = +\ 0.412\ N \cdot m^2/C$

Back face

$$\vec{A} = 0.0676\ -\hat{k}$$

$\vec{E}$ has no z component $\Phi_E = 0$

The same is true for the front face. $\Phi_E = 0$

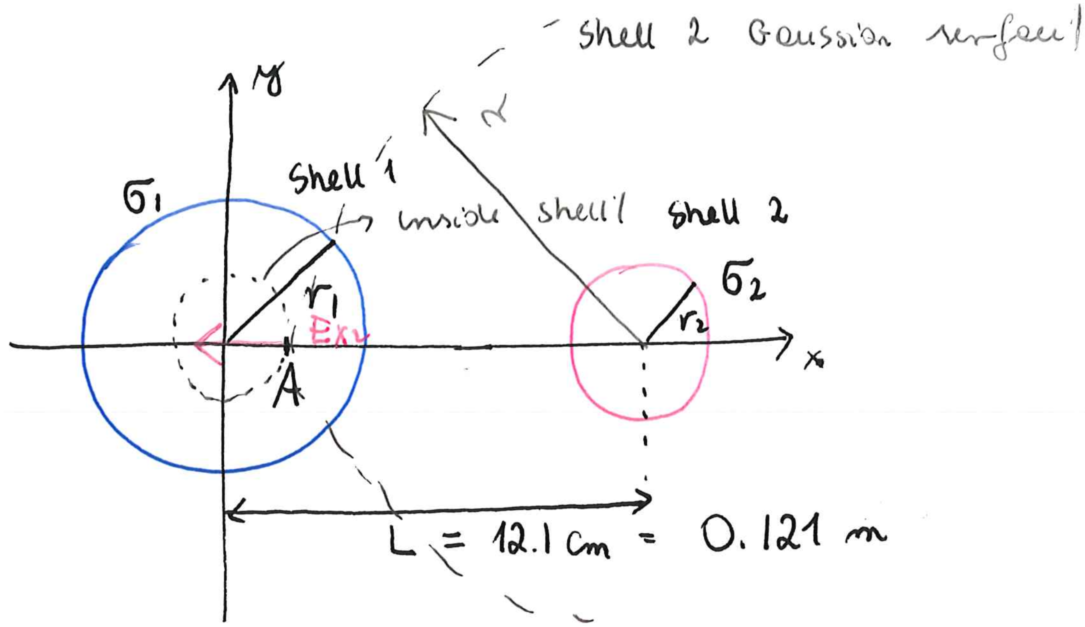
Total (net) flux

$$\Phi_{E,net} = \Phi_{top} + \Phi_{bottom} + \Phi_{left} + \underbrace{\Phi_{right}}_{= \Phi_{left}} + \underbrace{\Phi_{back}}_{=0} + \underbrace{\Phi_{front}}_{=0}$$

$$\Phi_{E,net} = \Phi_{top} + \Phi_{bottom} + \underbrace{\Phi_{left} - \Phi_{left}}_{=0}$$

$$\Phi_{E,net} = -5.27\ N \cdot m^2/C + 5.23\ N \cdot m^2/C$$

$$= -0.04\ N/m^2/C$$



$\sigma_1 = +5.2\ \mu C/m^2 = 5.2 \times 10^{-6} C/m$ | $\sigma_2 = +3.3\ \mu C/m^2 = 3.3 \times 10^{-6}\ C/m^2$

$r_1 = 4.2\ cm = 0.042\ m$ | $r_2 = 2.2\ cm = 0.022\ m$

$E_x = ?$ $\underline{A}$: $x = 2.0\ cm = 0.2\ m$

Use superposition:

$$E_x = \underset{\substack{\parallel \\ 0}}{E_{x1}} + E_{x2}$$

inside shell 1

$E_{x2} \Rightarrow$ $\oiint \vec{E} \cdot d\vec{A} = E \cdot 4\pi r^2$ $\qquad q_{enc} = \sigma_2 \cdot 4\pi r_2^2$

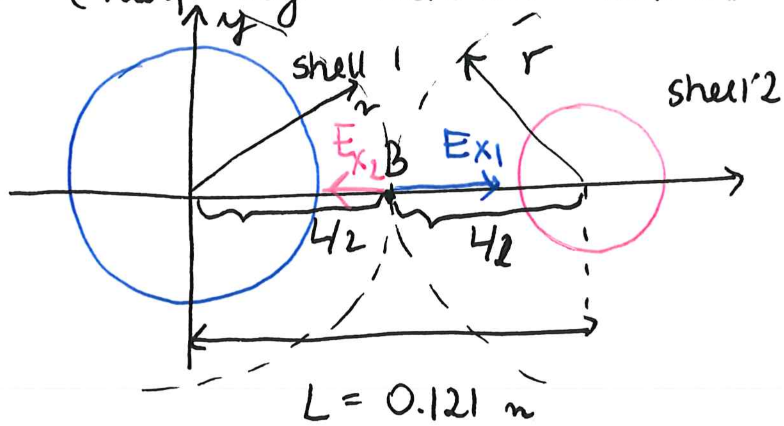
E - constant, spherical symety

$$E \cdot \cancel{4\pi} r^2 = \frac{\sigma_2 \cancel{4\pi} r_2^2}{\varepsilon_0} \Rightarrow$$

$$E = \frac{\sigma_2 r_2^2}{\varepsilon_0 r^2} = \frac{\sigma_2 r_2^2}{\varepsilon_0 (L-x)^2} = \frac{3.3 \times 10^{-6} C/m^2 \cdot (0.022\ m)^2}{(8.85 \times 10^{-12} C^2/N \cdot m^2)(0.121 - 0.02)^2}$$

$$= 17700\ N/C\ (\hat{i})$$

At point B (half way between centers of shell 1 and 2



$$E_x = E_{x1} + E_{x2}$$

Spherical symetry, E constant at given r.

$$\oiint \vec{E} \circ d\vec{A} = E \cdot 4\pi r^2 \qquad r = \frac{L}{2}$$

shell 1

$$q_{enc} = \sigma_1 \cdot 4\pi r_1^2$$

$$E_{x_1} \cdot \cancel{4\pi} \left(\frac{L}{2}\right)^2 = \frac{\sigma_1 \cdot \cancel{4\pi}\, r_1^2}{\varepsilon_0}$$

$$E_{x_1} = \frac{\sigma_1 r_1^2}{\varepsilon_0 \left(\frac{L}{2}\right)^2} \;(\hat{\imath})$$

shell 2

$$q_{enc} = \sigma_2 \cdot 4\pi r_2^2$$

$$E_{x2} \cdot \cancel{4\pi} \left(\frac{L}{2}\right)^2 = \frac{\sigma_2 \cdot \cancel{4\pi}\, r_2^2}{\varepsilon_0}$$

$$E_{x2} = \frac{\sigma_2 r_2^2}{\varepsilon_0 \left(\frac{L}{2}\right)^2} \;(-\hat{\imath})$$

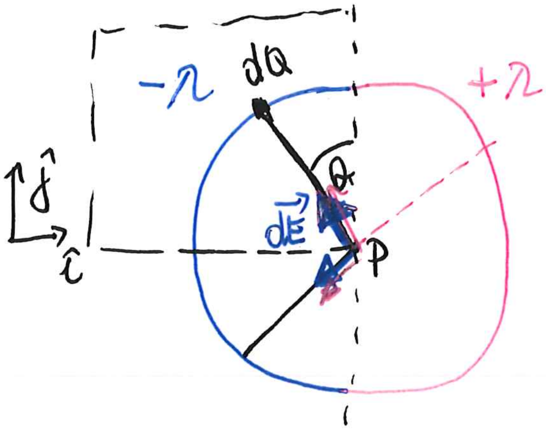
at B: $\Rightarrow$ $$E_x = \frac{1}{\varepsilon_0 \left(\frac{L}{2}\right)^2} \left(\sigma_1 r_1^2 - \sigma_2 r_2^2\right)$$

$$E_x = \frac{1}{8.85 \times 10^{-12}\, C^2/N \cdot m^2 \left(\frac{L}{2}\right)^2} \left(5.2 \times 10^{-6}\, C/m \cdot (0.042\, m)^2 - 3.3 \times 10^{-6}\, C/m \times 0.022\, m\right)$$

$$E_x = \frac{9.17 \times 10^{-9} - 1.59 \times 10^{-9}}{8.85 \times 10^{-12} \cdot \left(\frac{0.121}{2}\, m\right)^2} \; N/C$$

$$E_x = \frac{7.58 \times 10^{-9}}{8.85 \times 10^{-12} \cdot 3.6 \times 10^{-3}} = 0.238 \times 10^{6}\, N/C = 238\, kN/C \;(\hat{\imath})$$

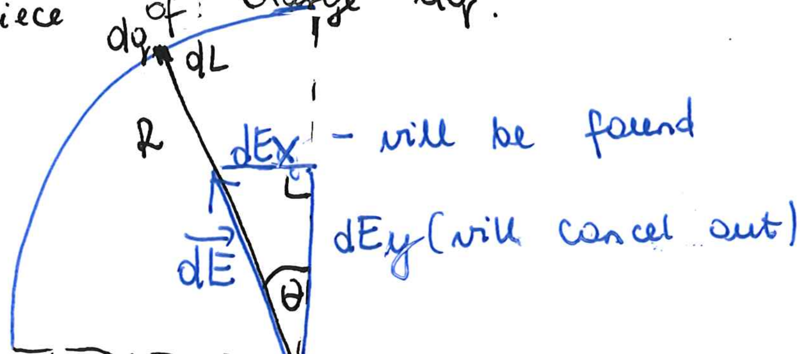
2



Find $\vec{E}$ at P.

Radius, R

1. Cut the distribution into a bunch of tiny pieces each with charge dQ
2. Look for a symetry → can use 1/4 circle (arc) Find $\vec{E}$ and multiply it by 4.
3. Calculate the magnitude of E-field due to ARBITRARY piece of charge dq.



4. Decompose field into components

$$\frac{dE_x}{dE} = \sin\theta \qquad \frac{dE_y}{dE} = \cos\theta$$

$$dE_x = dE\sin\theta$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{dq}{R^2}\cdot\sin\theta$$

$$dE_y = \frac{1}{4\pi\varepsilon_0}\frac{dq}{R^2}\cos\theta$$

⟹ not needed due to symetry

5. For each non-zero component, sum up all pieces dQ by integrating over the whole charge distribution

$$\int dE_x = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{R^2}\sin\theta$$

6. Express dQ in terms of a variable to be integrated over using linear / surface / volume density

$$\int dE_x = \frac{1}{4\pi\varepsilon_0} \int \frac{\lambda\, dL}{R^2} \cdot \sin\theta = \frac{1}{4\pi\varepsilon_0} \int \frac{\lambda \cdot \cancel{R} \sin\theta\, d\theta}{R^{\cancel{2}}}$$

$$dL = R\,d\theta$$

$\pi/2$ - quarter of the circle

$$E_x = \frac{\lambda}{4\pi\varepsilon_0 R} \int_0^{\pi/2} \sin\theta\, d\theta \qquad \int \sin x\, dx = -\cos x$$

$$E_x = \frac{\lambda}{4\pi\varepsilon_0 R} \left[-\cos\theta\right]_0^{\pi/2} = \frac{\lambda}{4\pi\varepsilon_0 R} \left[\underbrace{-\cos\left(\tfrac{\pi}{2}\right)}_{0} - \underbrace{\left(-\cos 0^\circ\right)}_{1}\right]$$

$$= \frac{\lambda}{4\pi\varepsilon_0 R}$$

Total electric field

$$E_{net} = 4E_x = \frac{\cancel{4}\lambda}{\cancel{4}\pi\varepsilon_0 R} = \frac{\lambda}{4\pi\varepsilon_0 R^2}$$

$$\vec{E}_{net} = \frac{\lambda}{4\pi\varepsilon_0 R} (-\hat{\imath})$$