Last time

- Review of electrostatic equations
- Obtaining electric field from electric potential
- Equipotential surface

This time

- Electric potential due to a dipole
- Electric field from electric potential for a dipole
- Electric potential of a solid spherical conductor

Moving in the same direction of electrical field

$$dV = -\vec{E} \bullet d\vec{l} = -Edl\cos 0 = -Edl < 0$$

Moving in the opposite direction of electrical field

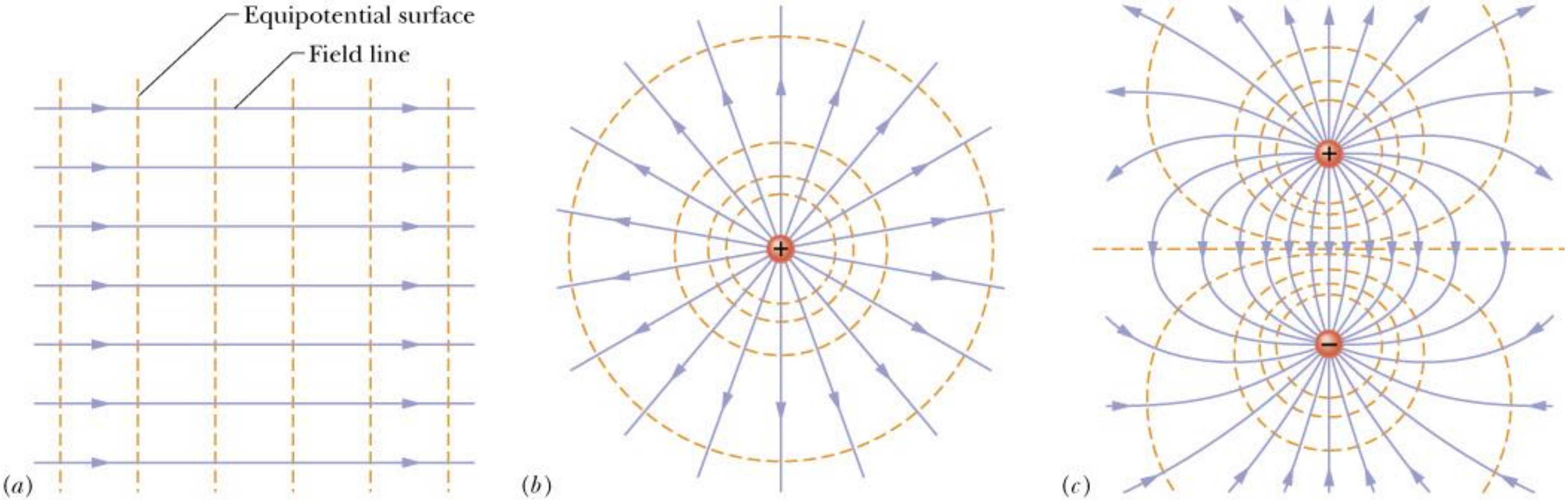
$$dV = -\vec{E} \bullet d\vec{l} = -Edl\cos 180 = Edl > 0$$

Moving perpendicular to the direction of electrical field

$$dV = -\vec{E} \bullet d\vec{l} = -Edl\cos 90 = 0$$

Moving with the electrical field decreases the electrical potential. Moving against the field increases it. Moving perpendicular to it does not change the electric potential.

$$V_f - V_i = -\int_i^f \vec{E} \bullet d\vec{l} = -\int_i^f E dl \cos 90 = 0$$



Electric potential doesn't change. A surface with this property is called an **equipotential surface**.

Electric field and electric potential

$$V_f - V_i = -\int_i^f \vec{E} \bullet d\vec{l}$$

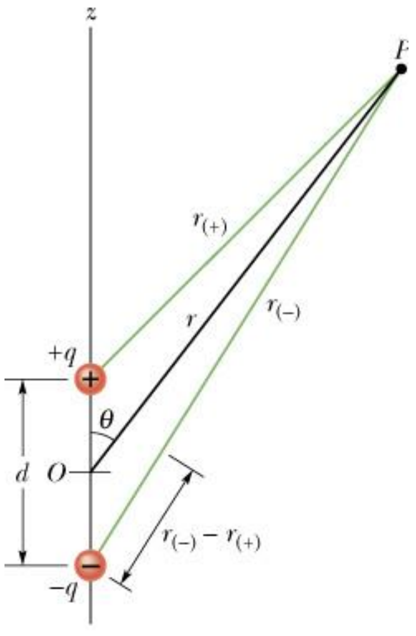
$$\frac{\partial V}{\partial x} = -E_x$$

$$\vec{E} = -\vec{\nabla} V = -\frac{\partial V}{\partial x}\hat{i} - \frac{\partial V}{\partial y}\hat{j} - \frac{\partial V}{\partial z}\hat{k}$$

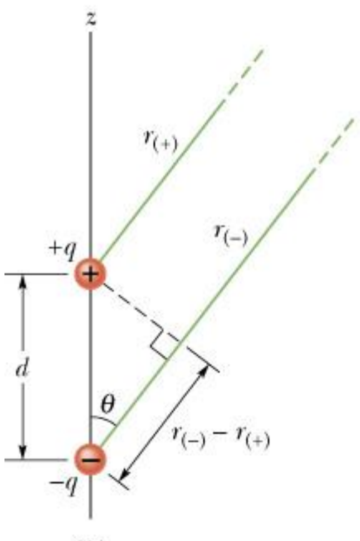
$$\frac{\partial V}{\partial y} = -E_y$$

$$\frac{\partial V}{\partial z} = -E_z$$

Potential due to an electric dipole



(a)



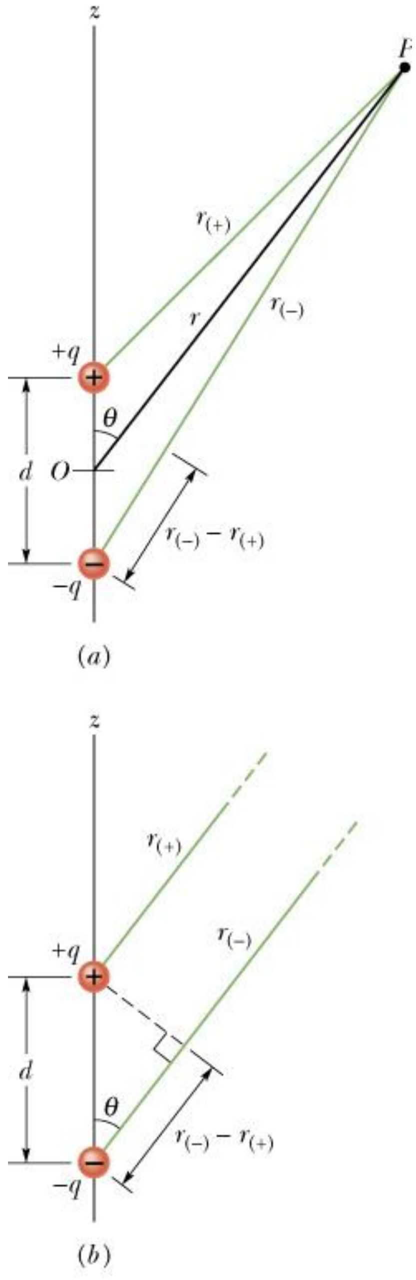
(b)

Remember V is a scalar

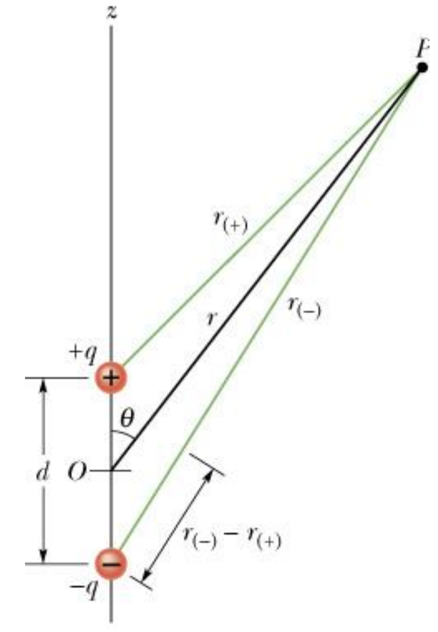
$$V(r) = V_+(r) + V_-(r)$$

$$V(r) = \frac{q}{4\pi\varepsilon_0 r_+} - \frac{q}{4\pi\varepsilon_0 r_-}$$

$$V(r) = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_+} - \frac{1}{r_-}\right) = \frac{q}{4\pi\varepsilon_0}\left(\frac{r_- - r_+}{r_+ r_-}\right)$$



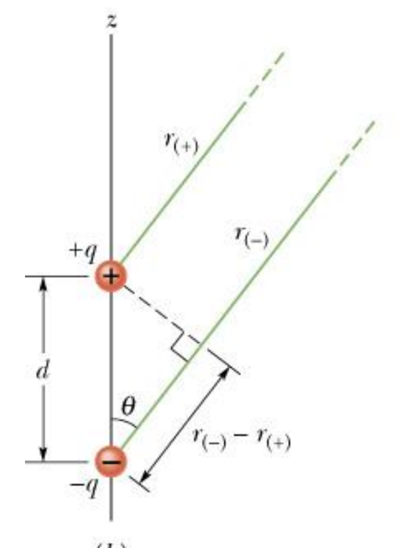
$$V(r)=\frac{q}{4\pi\varepsilon_0}\left(\frac{r_- - r_+}{r_+ r_-}\right)$$



(a)

$$\text{For } r \gg d \;\Rightarrow r_+ \approx r_- \approx r \Rightarrow r_+ r_- \approx r^2$$

$$r_- - r_+ \approx d\cos\theta$$



(b)

$$V(r)=\frac{qd\cos\theta}{4\pi\varepsilon_0 r^2} \qquad \cos\theta = \hat{k}\cdot\hat{r}$$

$$\vec{p}=qd\hat{k} \qquad V(r)=\frac{\vec{p}\bullet\hat{r}}{4\pi\varepsilon_0 r^2}$$

Now we can calculate the electric filed for an arbitrary point P at a distance r from the center of the dipole.

$$
\begin{aligned}
V(r) &= \frac{p \cos\theta}{4\pi\epsilon_0 r^2} \\
&= \frac{pz/r}{4\pi\epsilon_0 r^2} \\
&= \frac{pz}{4\pi\epsilon_0 r^3} \\
&= \frac{pz}{4\pi\epsilon_0 \left(x^2+y^2+z^2\right)^{3/2}} \\
&= \frac{pz}{4\pi\epsilon_0} \left(x^2+y^2+z^2\right)^{-3/2}
\end{aligned}
$$

Note that $z = r\cos\theta$.

$$
\begin{aligned}
E_x &= -\frac{\partial V}{\partial x} = -\frac{\partial}{\partial x}\left[\frac{pz}{4\pi\epsilon_0}\left(x^2+y^2+z^2\right)^{-3/2}\right] \\
&= -\frac{pz}{4\pi\epsilon_0}\left(-\frac{3}{2}2x\right)\left(x^2+y^2+z^2\right)^{-5/2} \\
&= \frac{3pzx}{4\pi\epsilon_0 r^5}
\end{aligned}
$$

Similarly

$$\begin{aligned}
E_y &= -\frac{\partial V}{\partial y} = -\frac{\partial}{\partial y}\left[\frac{pz}{4\pi\epsilon_0}\left(x^2+y^2+z^2\right)^{-3/2}\right] \\
&= -\frac{pz}{4\pi\epsilon_0}\left(-\frac{3}{2}2y\right)\left(x^2+y^2+z^2\right)^{-5/2} \\
&= \frac{3pzy}{4\pi\epsilon_0 r^5}
\end{aligned}$$

and

$$\begin{aligned}
E_z &= -\frac{\partial V}{\partial z} = -\frac{\partial}{\partial z}\left[\frac{pz}{4\pi\epsilon_0}\left(x^2+y^2+z^2\right)^{-3/2}\right] \\
&= -\frac{p}{4\pi\epsilon_0}\left[\frac{1}{r^3} + z\left(-\frac{3}{2}2z\right)\left(x^2+y^2+z^2\right)^{-5/2}\right] \\
&= -\frac{p}{4\pi\epsilon_0}\left[\frac{1}{r^3} - \frac{3z^2}{r^5}\right] \\
&= -\frac{p}{4\pi\epsilon_0 r^5}\left[r^2 - 3z^2\right] \\
&= -\frac{p}{4\pi\epsilon_0 r^5}\left[x^2+y^2-2z^2\right]
\end{aligned}$$

For a point $P(0,0,z)$, we have

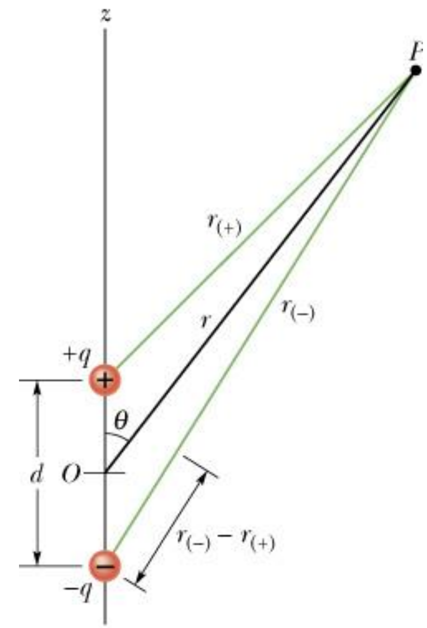
$$
\begin{aligned}
E_x &= \frac{3pzx}{4\pi\epsilon_0 r^5} \\
&= 0
\end{aligned}
$$

$$
\begin{aligned}
E_y &= \frac{3pzy}{4\pi\epsilon_0 r^5} \\
&= 0
\end{aligned}
$$

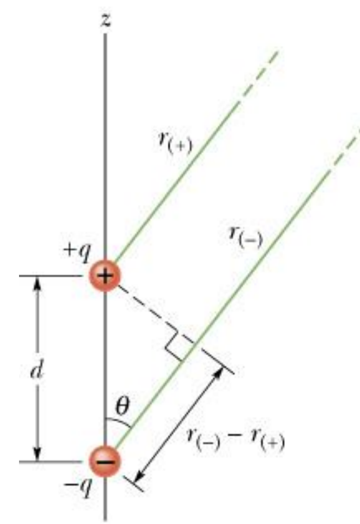
$$
\begin{aligned}
E_z &= -\frac{p}{4\pi\epsilon_0 r^5}\left[x^2 + y^2 - 2z^2\right] \\
&= \frac{p}{2\pi\epsilon_0 z^3}
\end{aligned}
$$

or

$$
\begin{aligned}
\overrightarrow{E} &= \frac{p\widehat{z}}{2\pi\epsilon_0 z^3} \\
&= \frac{\overrightarrow{p}}{2\pi\epsilon_0 z^3}
\end{aligned}
$$



(a)



(b)

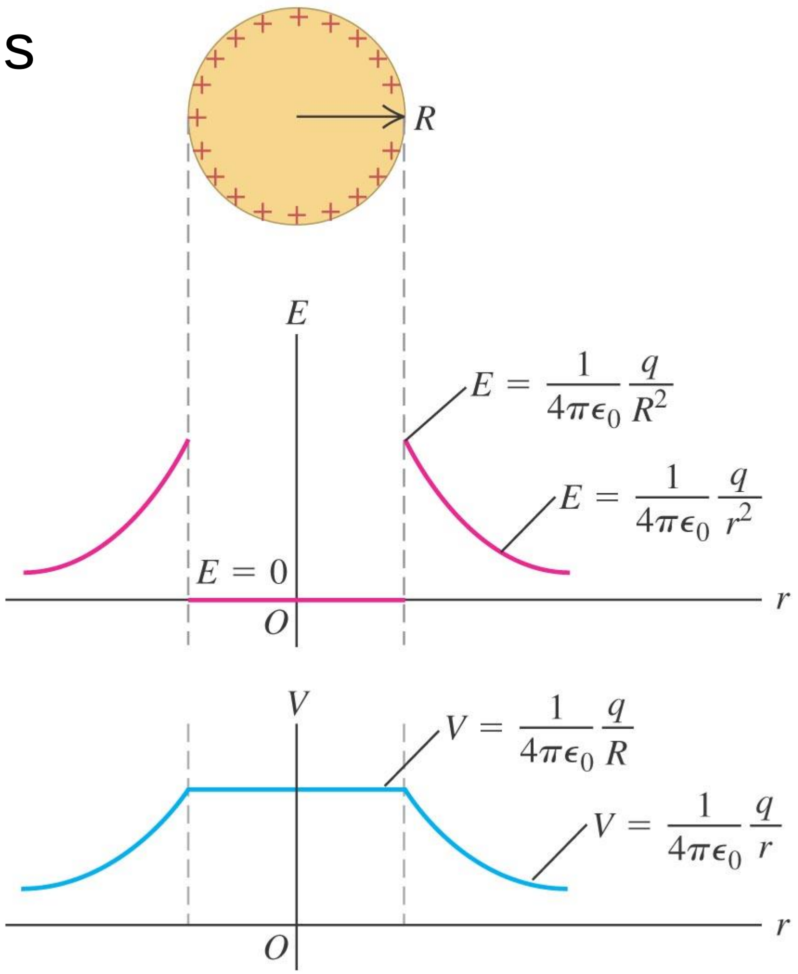
Calculation of electrical potential

The electric field everywhere inside the conductor and on the surface is zero.

$$\vec{E} = 0 \qquad \text{For } r \le R$$

$$V(r \le R) = -\int \vec{E} \bullet d\vec{l} = C_1$$

We therefore conclude the all points inside and on the surface of the solid conductor are at the same potential, the conductor is an equipotential object.



The electric field outside the conductor is given by

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r} \qquad \text{For } r > R$$

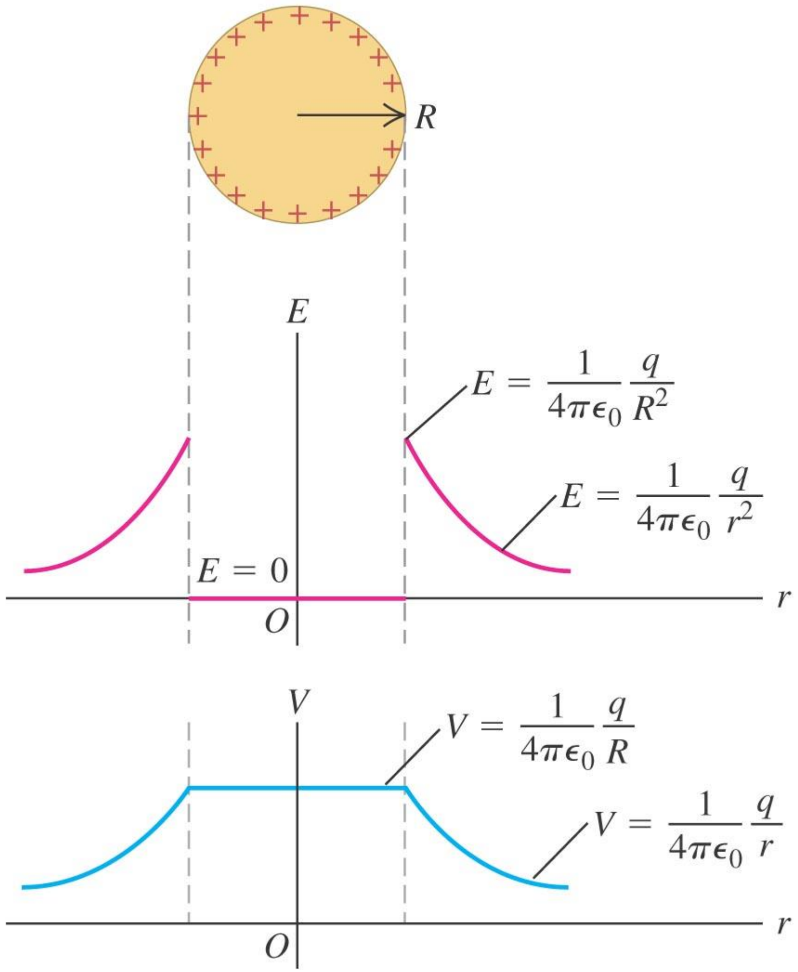
$$V(r > R) = -\frac{q}{4\pi\varepsilon_0}\int \frac{\hat{r}}{r^2}\bullet d\vec{l}$$

Because the electrostatic field is a conservative field, we can choose

$$d\vec{l} = dr\hat{r}$$

$$V(r > R) = -\frac{q}{4\pi\varepsilon_0}\int \frac{\hat{r}}{r^2}\bullet\left(dr\hat{r}\right)$$

$$V(r > R) = -\frac{q}{4\pi\varepsilon_0}\int \frac{dr}{r^2} = \frac{q}{4\pi\varepsilon_0}\frac{1}{r} + C_2$$

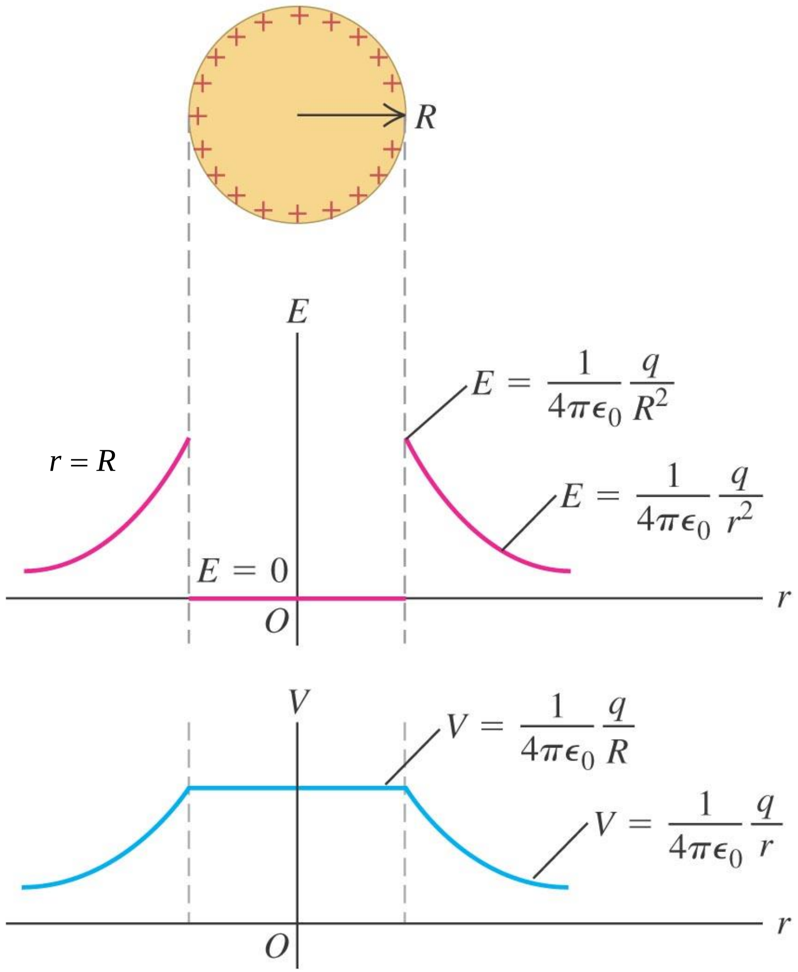


Boundary condition I:

$$V(r \to \infty) = 0$$

$$0 = \frac{q}{4\pi\varepsilon_0} \frac{1}{r}\Big|_{r=\infty} + C_2 \Rightarrow C_2 = 0$$

$$V(r > R) = \frac{q}{4\pi\varepsilon_0} \frac{1}{r}$$



Boundary condition II:
The electric potential must be continuous at r = R. Why?

$$V\left(r < R\right)\Big|_{r=R} = V(r > R)\Big|_{r=R}$$

$$C_1 = \frac{q}{4\pi\varepsilon_0} \frac{1}{R}$$

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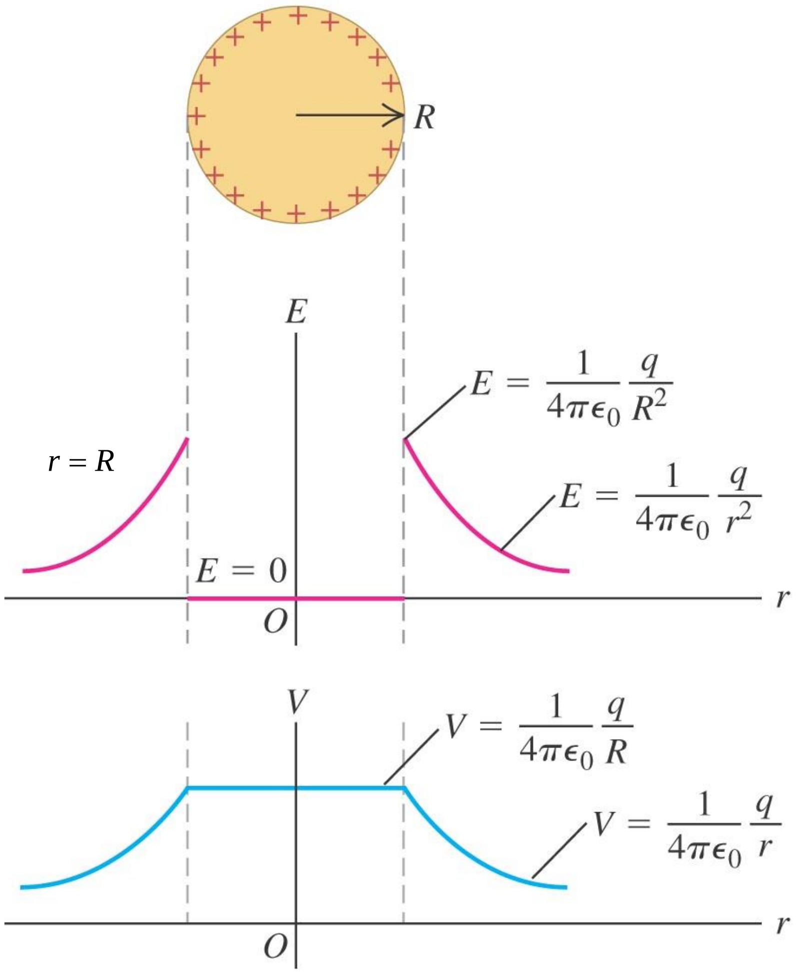

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Calculation of electrical potential

$$V(r) = \frac{q}{4\pi\varepsilon_0}\frac{1}{R} \quad \text{For } r \le R$$

$$V(r) = \frac{q}{4\pi\varepsilon_0}\frac{1}{r} \quad \text{For } r \ge R$$