Electricity and Magnetism

- Physics 259 – L02
- Lecture 10

Sections 22.4-5

Last time

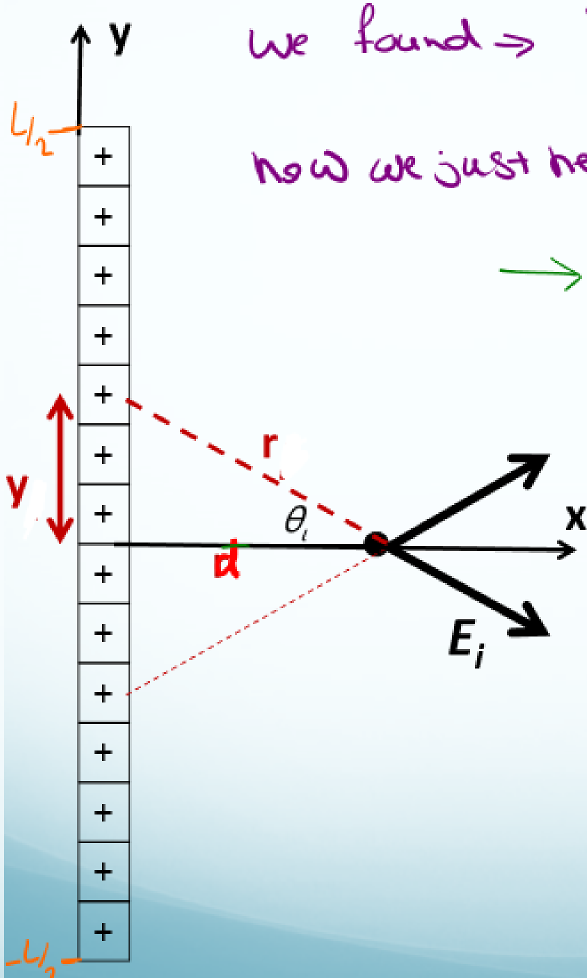
- Chapter 21
- Van De Graaff Generator Experiment 
- Electric Ping Pong Experiment 

This time

- Chapter 22
- Field of a Ring of Charge and a Disk

22-4: The Electric Field Due To a Line of Charge

For a line of charge we found the force as:



we found ⇒ $F_{net,x} = \int_{-L/2}^{L/2} \frac{d\,k\,q\,\lambda\,dy}{(d^2+y^2)^{3/2}}$

now we just need to solve the integral ⇒

$\rightarrow \vec{F}_{net} = \frac{kQq}{d\sqrt{(L/2)^2+d^2}}$ ¿

limiting cases ⇒

① $d \gg L \Rightarrow d^2 + \underbrace{(L/2)^2}_{0} \simeq d^2$

$\vec{F}_{net} = \vec{F}_{net} = k\frac{Qq}{d^2}$

② $d \ll L \Rightarrow \overbrace{\cancel{d^2}}^{0} + (\frac{L}{2})^2 \simeq (\frac{L}{2})^2$

$\vec{F}_{net} = k\frac{qQ}{L/2} = k\frac{2qQ}{L}$

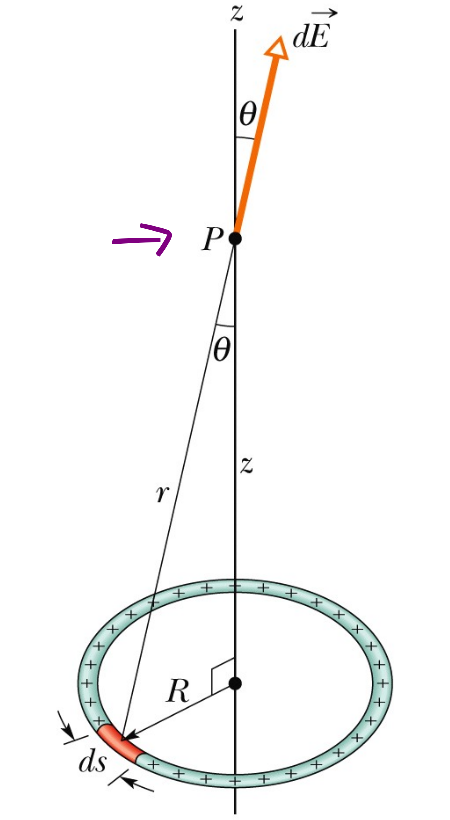
infinite long wire

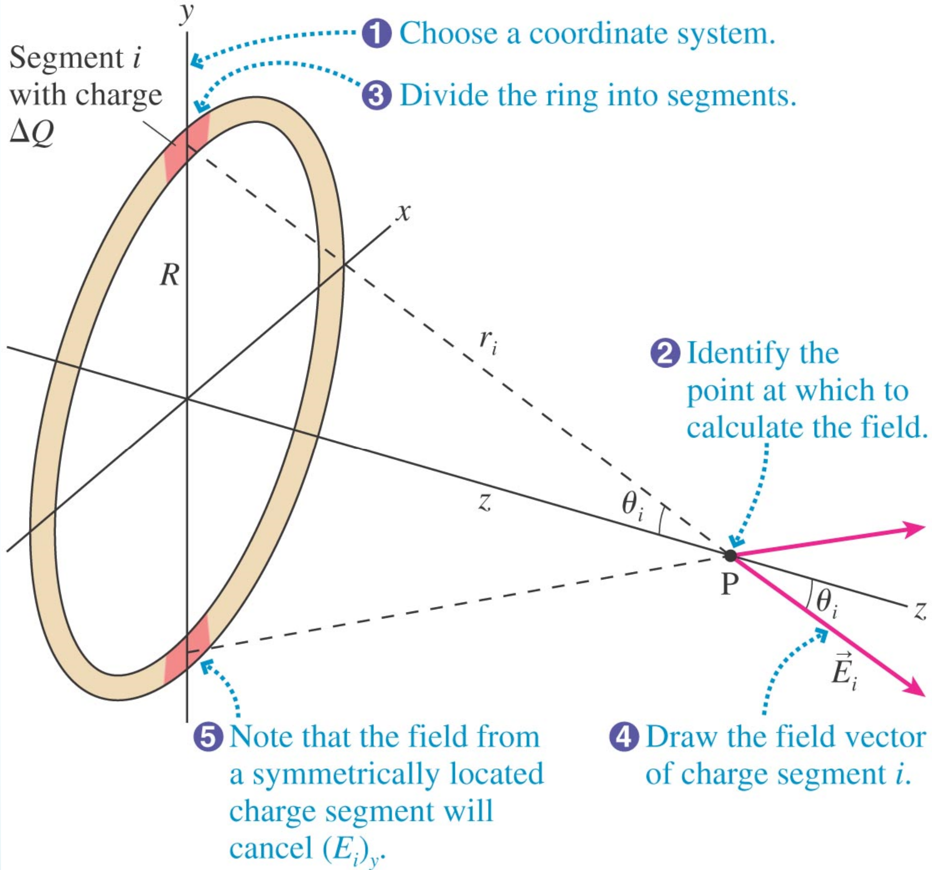
$\vec{E} = \frac{\vec{F}}{q}$

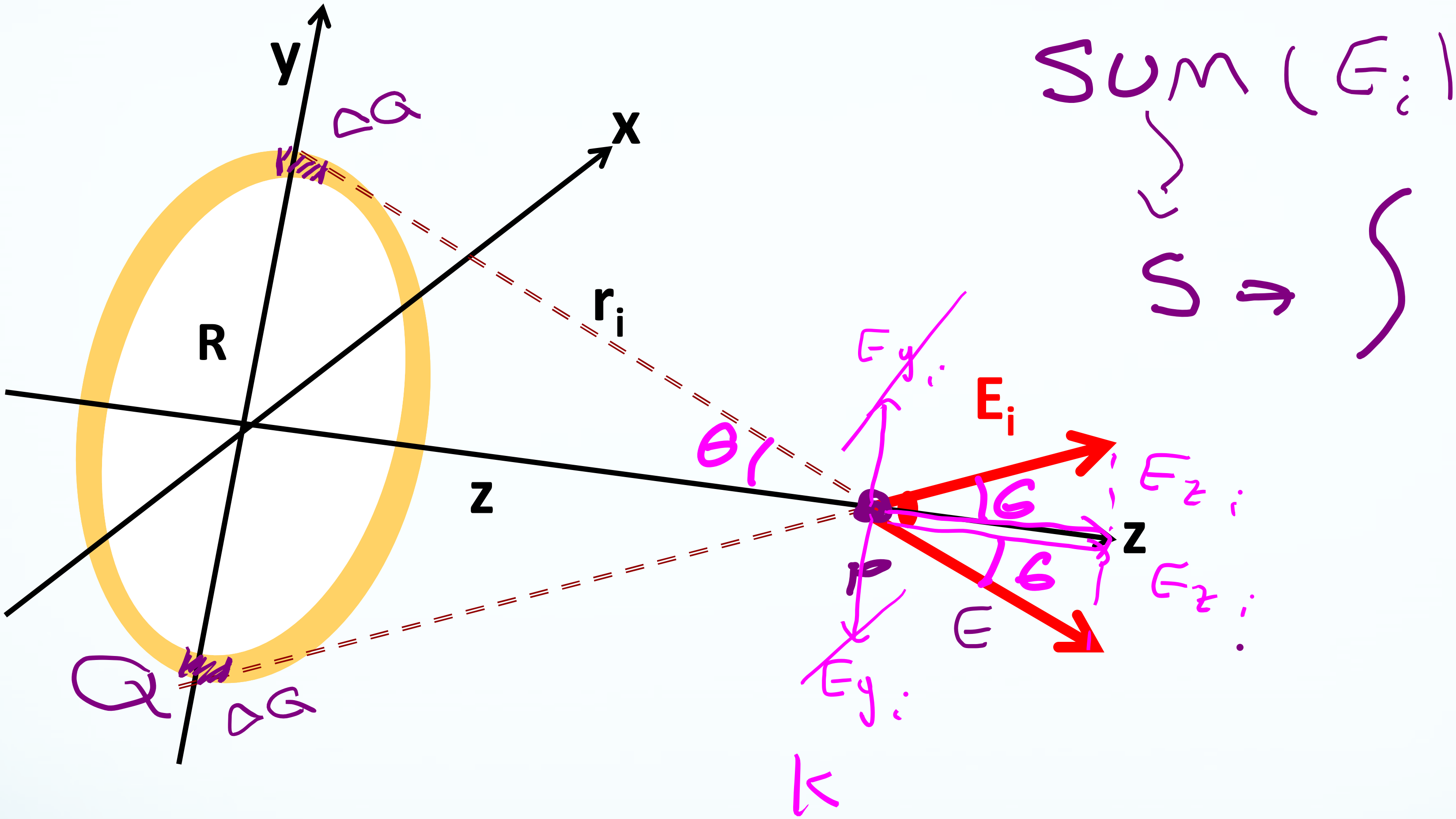
$\vec{E} = \frac{kQ}{d\sqrt{(\frac{L}{2})^2+d^2}}$

Therefore, the field →

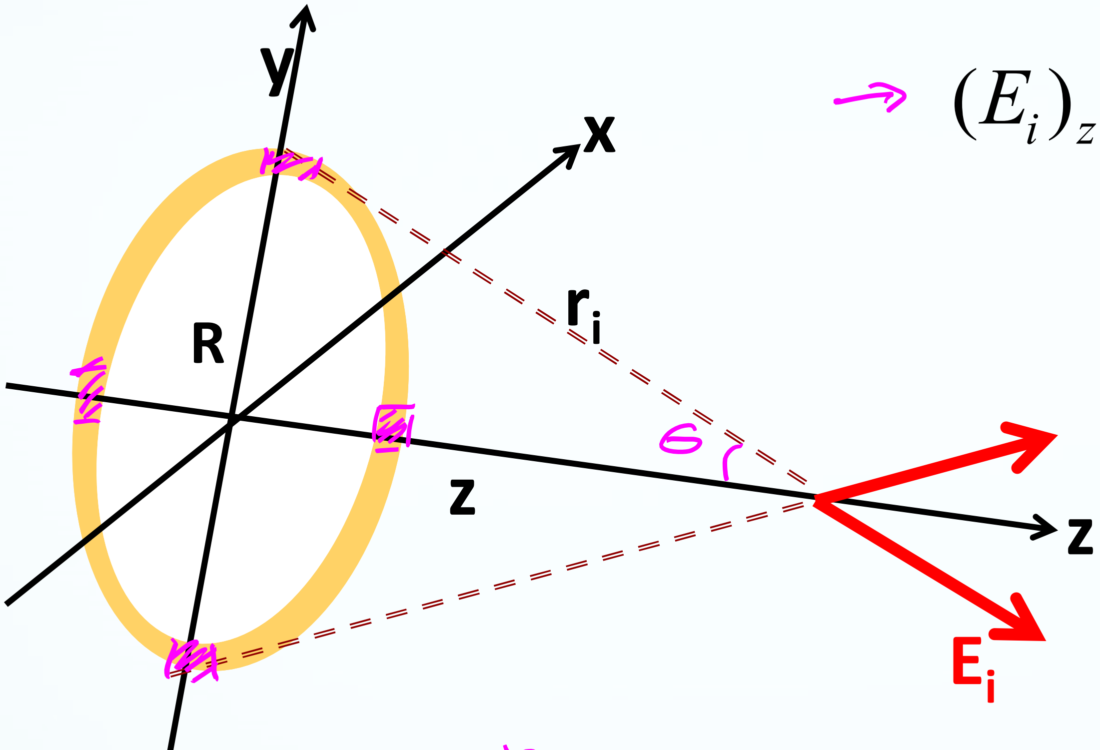
A ring of charge, similar derivation







$$(E_i)_z = \frac{1}{4\pi\varepsilon_o}\frac{\Delta Q}{r_i^2}\cos\theta_i$$

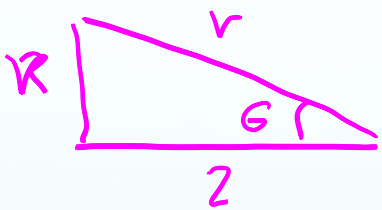


$$\rightarrow (E_i)_z = \frac{1}{4\pi\varepsilon_o}\frac{\Delta Q}{r_i^2}\cos\theta_i \quad (7)$$

$\cos\theta = \frac{z}{r}$

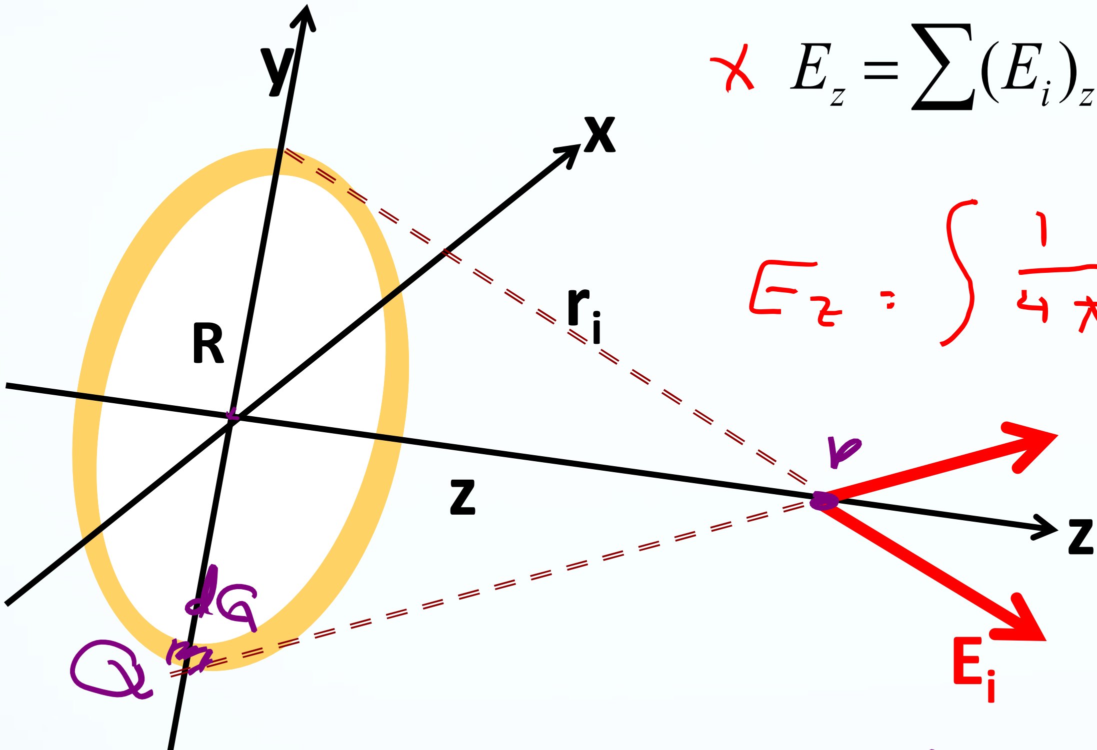
$r = \sqrt{R^2 + z^2}$

$\cos\theta = \frac{z}{\sqrt{R^2+z^2}}$



$$(7) \rightarrow (E_i)_z = \frac{1}{4\pi\varepsilon_o}\frac{\Delta Q}{R^2+z^2}\cdot\frac{z}{\sqrt{R^2+z^2}}$$

$$\rightarrow (E_i)_z = \frac{1}{4\pi\varepsilon_o}\frac{z\Delta Q}{\left(z^2+R^2\right)^{\frac{3}{2}}}$$

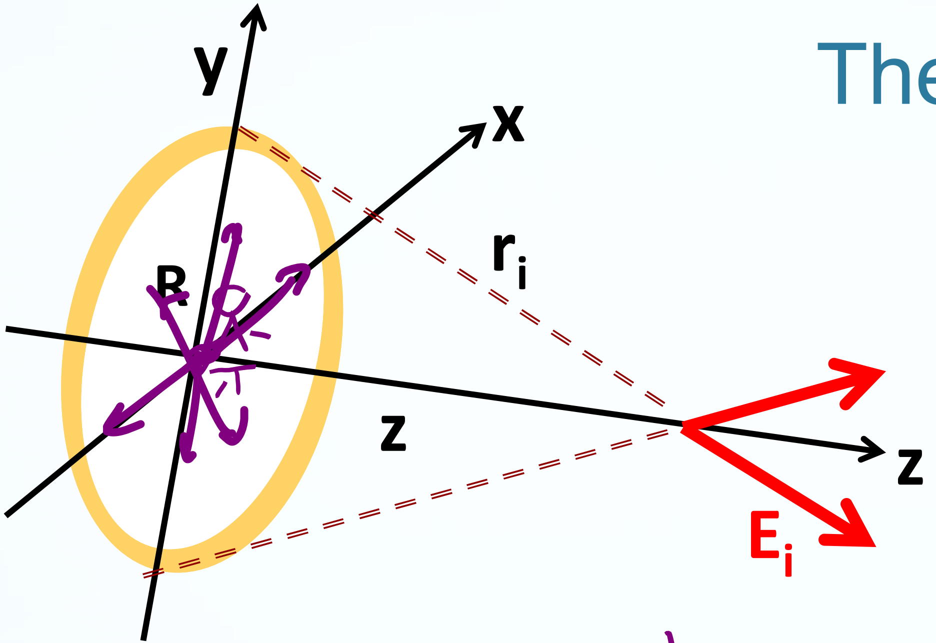


$$E_z = \sum (E_i)_z = \sum \frac{1}{4\pi\varepsilon_o}\frac{z\Delta Q}{\left(z^2+R^2\right)^{\frac{3}{2}}} \to dQ$$

$$E_z = \int \frac{1}{4\pi\varepsilon_o}\frac{z\,dQ}{(z^2+R^2)^{3/2}}$$

$$E_z = \frac{1}{4\pi\varepsilon_o}\frac{z}{(z^2+R^2)^{3/2}} \underbrace{\int dQ}_{Q}$$

$$(E_{ring})_z = \frac{1}{4\pi\varepsilon_o}\frac{zQ}{\left(z^2+R^2\right)^{\frac{3}{2}}}$$



The limiting cases:

Z=0 →

$$E_{ring} = \overbrace{\frac{1}{4\pi\varepsilon_o}}^{k}\frac{\cancelto{0}{z}Q}{\left(z^2+R^2\right)^{3/2}}$$

→ E=0 ✓

Z>>R →

$\frac{R}{z} \to 0$

$$E_{ring} = \overbrace{\frac{1}{4\pi\varepsilon_o}}^{k}\frac{\cancel{z}Q}{\left(z^2+0^2\right)^{3/2}} = \frac{1}{4\pi\varepsilon_o}\frac{Q}{z^2}$$ ✓

R

z^3

22-5: The Electric Field Due To a Charged Disk

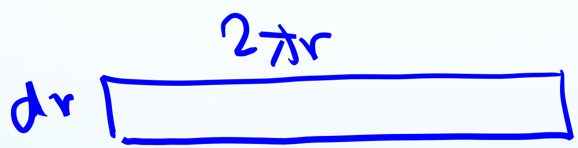
A disk of charge

$$\sigma = \frac{Q}{A} = \frac{Q}{\pi R^2} = \frac{\Delta Q}{\Delta A_i} = \frac{dQ}{dA_i}$$

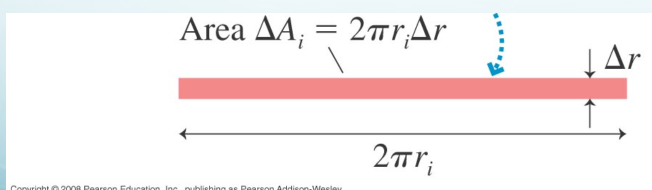
A

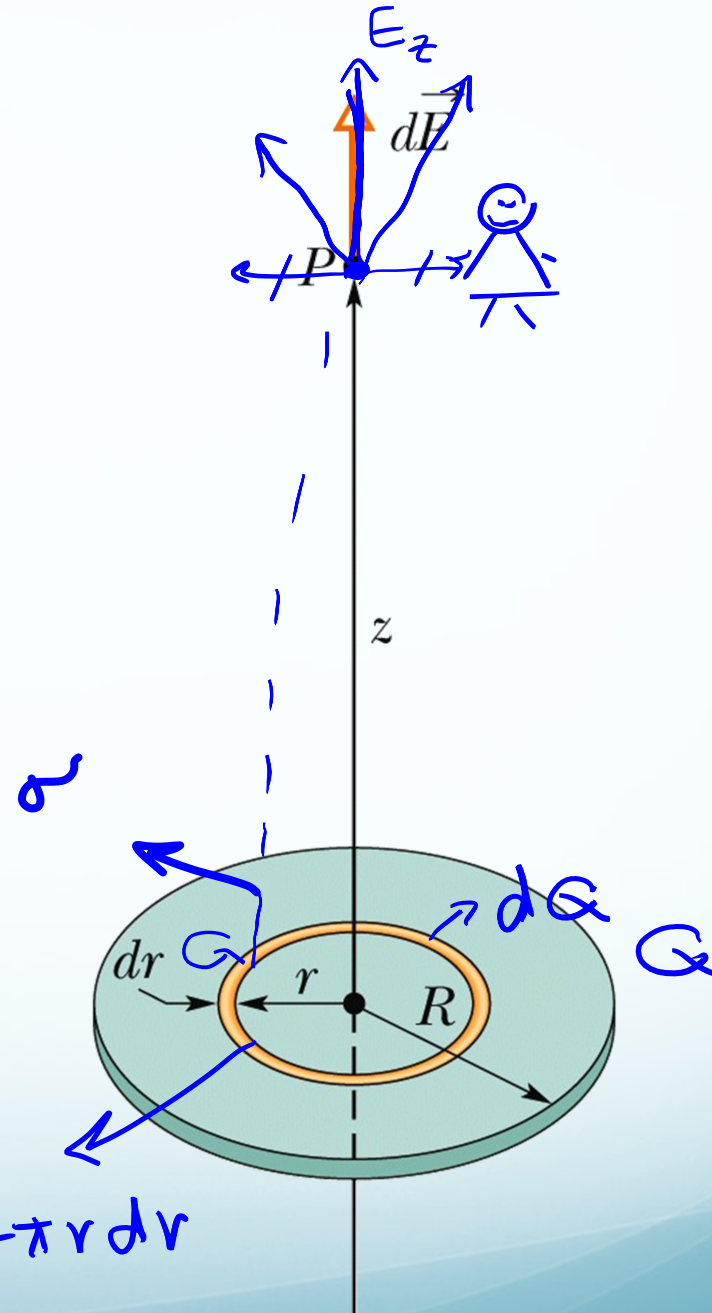
$$\sigma = \frac{dQ}{dA}$$

$$dQ = \sigma dA_i = \sigma 2\pi r_i dr$$



$A = 2\pi r dr$



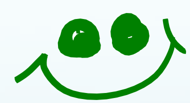


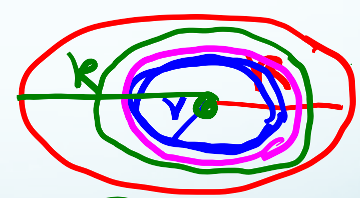
$A = 2\pi r dr$

$$\underline{dE} = dE_z = (E_i)_z = \frac{1}{4\pi\varepsilon_o}\frac{z\,dQ}{\left(z^2 + r_i^2\right)^{\frac{3}{2}}} \qquad dQ = \sigma 2\pi r_i\,dr \checkmark$$

$$(E_{\text{ring}})_z = dE_z = \frac{1}{\cancel{4}_{2}\cancel{\pi}\varepsilon_o}\frac{z\sigma\cancel{2}\cancel{\pi} r\,dr}{(z^2+r^2)^{3/2}} = \frac{\sigma z}{2\varepsilon_o}\frac{r\,dr}{(z^2+r^2)^{3/2}}$$

$$E_{disk} = \int E_{ring} = \int_{r=0}^{R}\frac{\sigma z}{2\varepsilon_o}\frac{r\,dr}{(z^2+r^2)^{3/2}} = \frac{\sigma z}{2\varepsilon_o}\int\frac{r\,dr}{(z^2+r^2)^{3/2}}$$



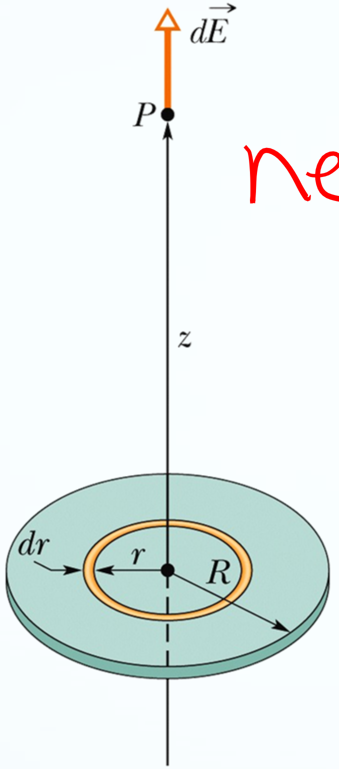


$$(E_{disk})_z = \frac{\sigma z}{2\varepsilon_o}\int_0^R \frac{r\,dr}{\left(z^2 + r_i^2\right)^{\frac{3}{2}}}$$

$$(E_{disk})_z = \frac{\sigma}{2\varepsilon_o}\left[1 - \frac{z}{\sqrt{z^2 + R^2}}\right]$$

Limiting cases?

Z>>R



next section

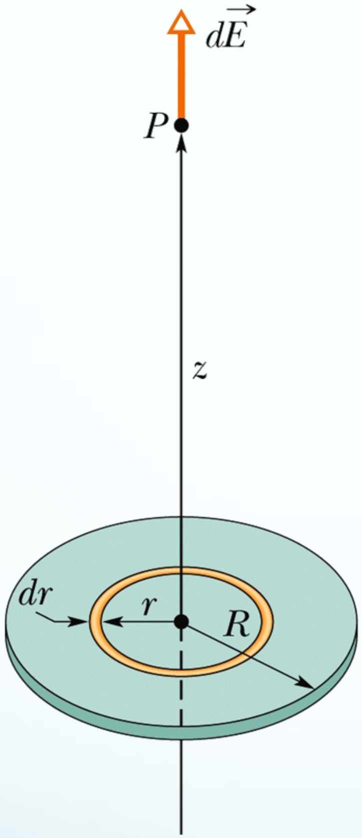
$$E_{disk,z} = \frac{\sigma}{2\varepsilon_o}\left[1 - \frac{z}{\sqrt{z^2 + R^2}}\right]$$

$$E_{disk,z} = \frac{\sigma}{2\varepsilon_o}\left[1 - \frac{z}{\sqrt{z^2}}\right] = 0\,????$$

$$E_{disk,z} = \frac{\sigma}{2\varepsilon_o}\left[1 - \frac{1}{\sqrt{1 + R^2/z^2}}\right] = \frac{\sigma}{2\varepsilon_o}\left[1 - \left(1 + R^2/z^2\right)^{-\frac{1}{2}}\right] = \frac{\sigma}{2\varepsilon_o}\left[1 - \left(1 - \tfrac{1}{2}R^2/z^2\right)\right]$$

$$\approx \frac{\sigma}{2\varepsilon_o}\frac{R^2}{2z^2} = \frac{Q/A}{2\varepsilon_o}\frac{\pi R^2}{2\pi z^2} = \frac{Q}{4\pi\varepsilon_o z^2}$$

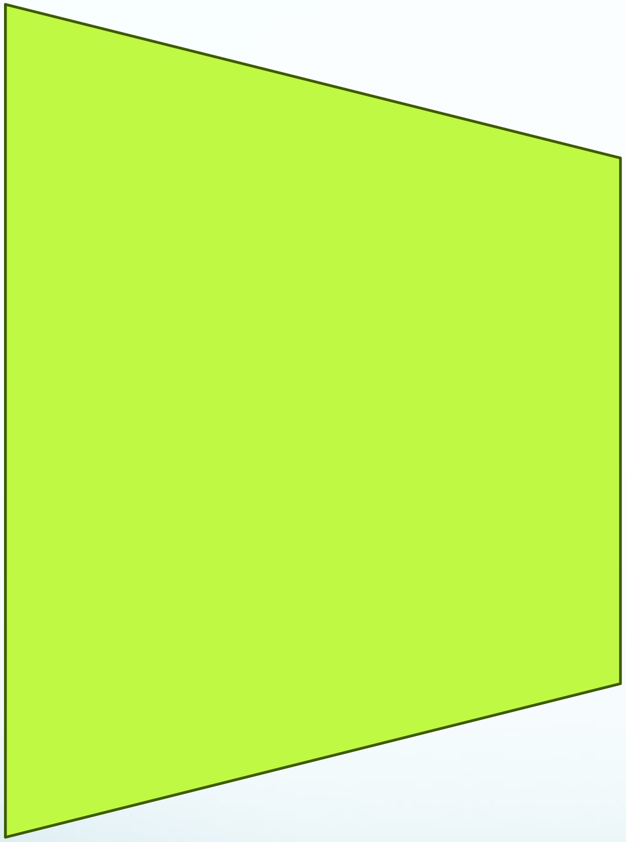
Limiting cases? $z \rightarrow 0$

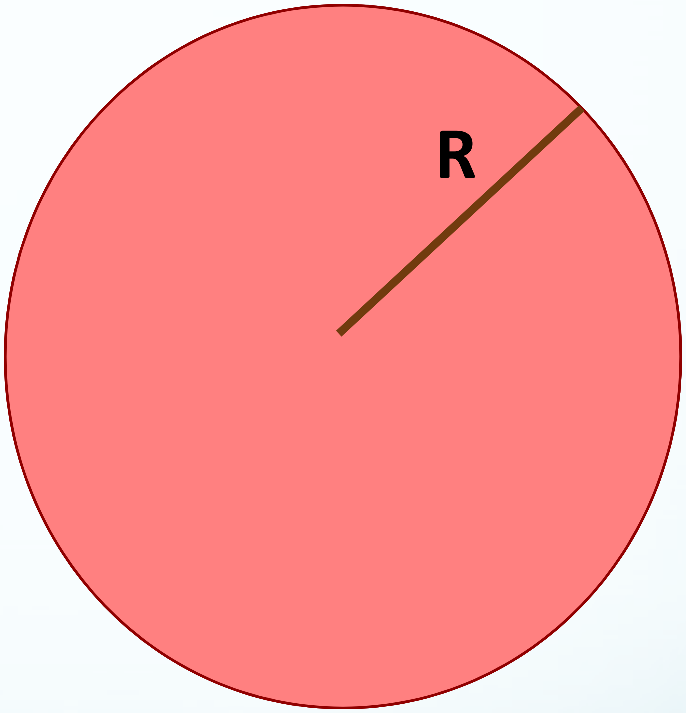


$$E_{disk,z} = \frac{\sigma}{2\varepsilon_o}\left[1 - \frac{z}{\sqrt{z^2 + R^2}}\right]$$

$$E_{disk,z} = \frac{\sigma}{2\varepsilon_o}$$

Plane of charge

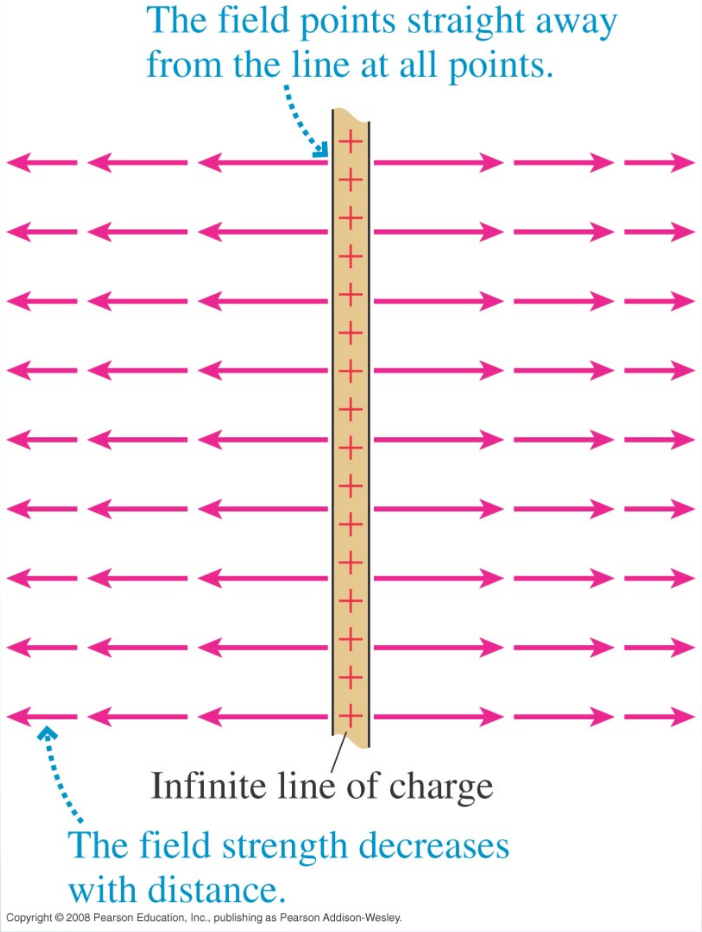




$$(E_{disk})_z = \frac{\sigma}{2\varepsilon_o}[1 - \frac{z}{\sqrt{z^2 + R^2}}]$$

$$E_{plane} = \frac{\sigma}{2\varepsilon_o}$$

This is the result for a plane of charge



$$E_{plane,z} = \begin{cases} \dfrac{\sigma}{2\varepsilon_o}, z > 0 \\ -\dfrac{\sigma}{2\varepsilon_o}, z < 0 \end{cases}$$

This section we talked about:

Chapter 22.4-5

See you on Thursday