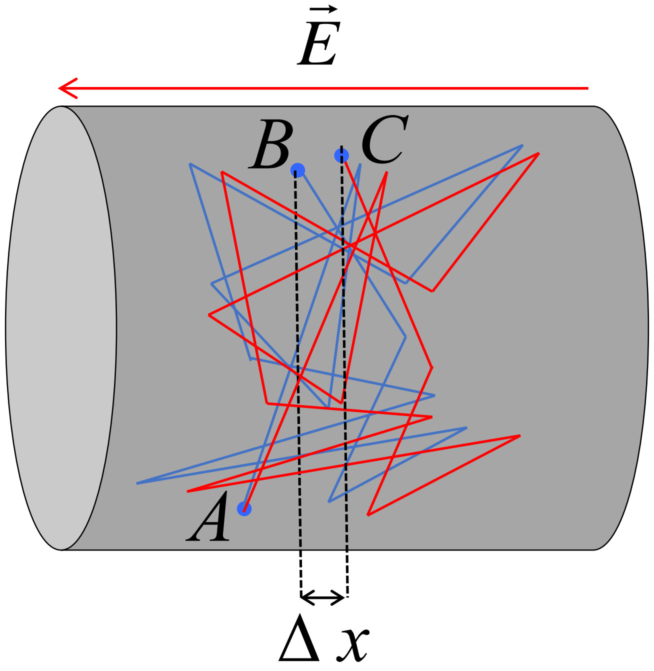
Appendix 2-Chapter 26

Microscopic view of Ohm's law (resistivity)



Electrons bounce around inside the metal at speeds very high speeds on the order of 0.5% light speed.

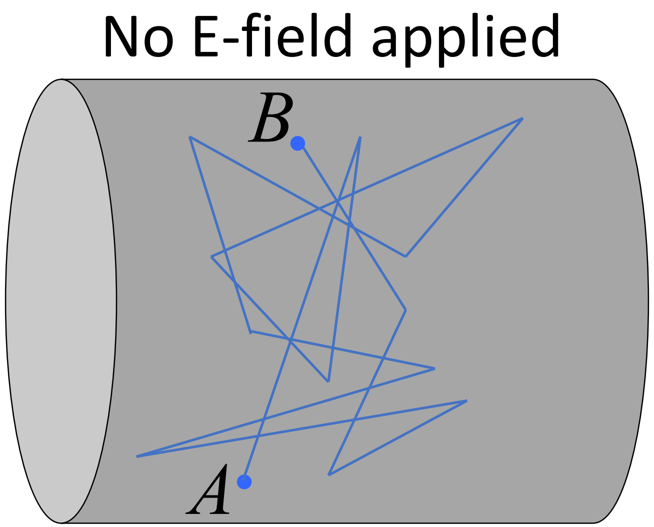
When an electric field is applied in the conductor, there is a net force on the electrons, leading to an average "drift speed" of $v_d = 0.5\ \mu\text{m/s}$

The acceleration felt by the electrons from the E-field is

$$a_x = \frac{eE}{m}$$

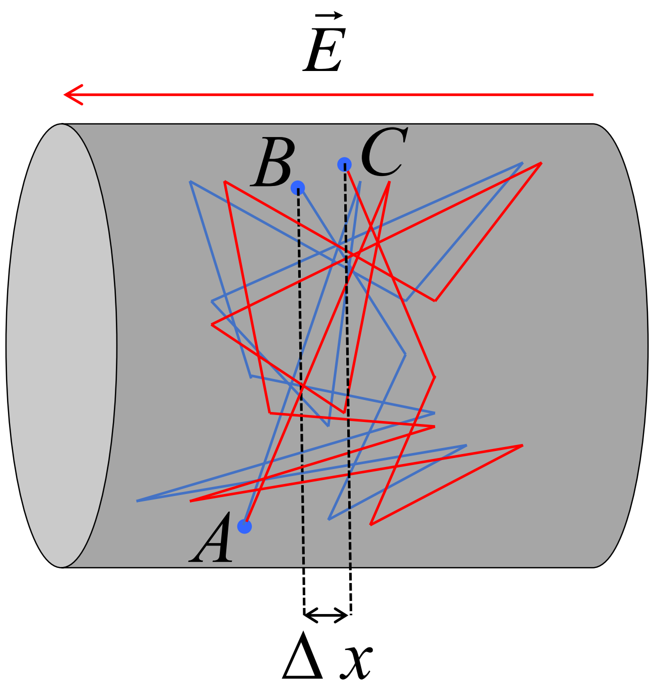
So the average drift speed of the electrons will be given by

$$v_d = at = \frac{eEt}{m}$$ but we found before: $$v_d = \frac{J}{ne}$$



$$v_d = \frac{eE\tau}{m}$$

$$v_d = \frac{J}{ne}$$



The average time between collisions is τ and is called the *mean free time*. Equating the two expressions for the drift speed, we get:

$$\frac{eE\tau}{m} = \frac{J}{ne}$$

Rearrange this to find $E = \left(\frac{m}{ne^2\tau}\right)J$

This gives a microscopic picture of resistivity: $\boxed{\rho = \frac{m}{ne^2\tau}}$

Consequence of this microscopic view

When the temperature of a metal increases, its volume increases (thermal expansion) according to

$$\frac{\Delta V}{V_0} = \alpha_V \frac{\Delta T}{T_0} \qquad \alpha_V = \text{vol. coefficient of thermal expansion}$$

The resistivity depends on the conduction electron number *density* and hence implicitly depends on the volume of the metal

$$\rho = \frac{m}{ne^2\tau} = \frac{mV}{Ne^2\tau}$$

m, N, e, and τ are unaffected by T

The resistivity is a temperature dependent property

$$\boxed{\alpha = \frac{\alpha_V}{T_0}}$$

$$\Delta\rho = \frac{m\Delta V}{Ne^2\tau} = \frac{mV_0}{Ne^2\tau}\left(\frac{\Delta V}{V_0}\right) \qquad \rho - \rho_0 = \rho_0\alpha(T - T_0)$$

This is why the resistance of a device depends on temperature

Temperature Dependent Resistance

$$\frac{\Delta A}{A_0} = a_A \frac{\Delta T}{T_0} = \tfrac{2}{3} a_V \frac{\Delta T}{T_0}$$

$$\frac{\Delta L}{L_0} = a_L \frac{\Delta T}{T_0} = \tfrac{1}{3} a_V \frac{\Delta T}{T_0}$$

$$\frac{\Delta R}{R_0} = \frac{\Delta r}{r_0} + \frac{\Delta L}{L_0} - \frac{\Delta A}{A_0}$$

$T_0 \quad L_0 \quad A_0$

$T \quad L \quad A$

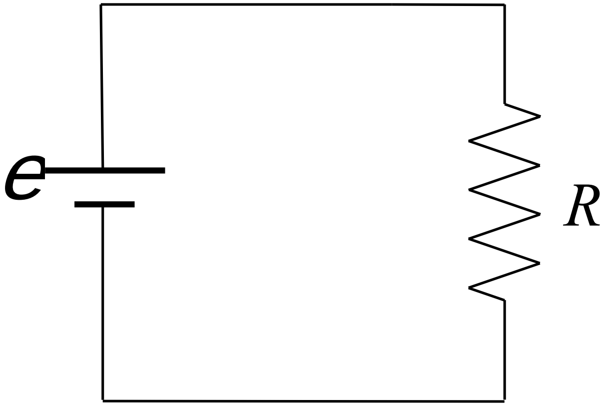
$$\frac{\Delta R}{R_0} = a_V \left(1 + \tfrac{1}{3} - \tfrac{2}{3}\right) \frac{\Delta T}{T_0}$$

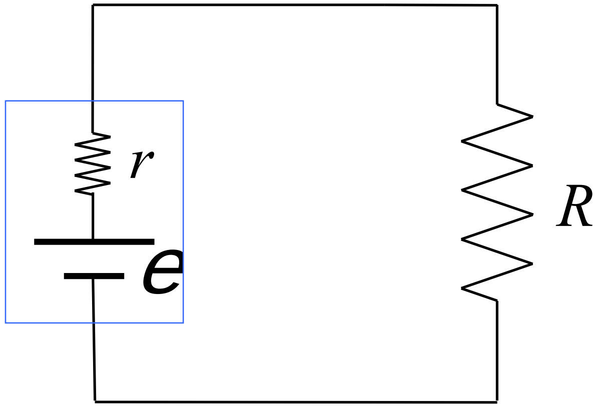
$$\frac{\Delta R}{R_0} = \tfrac{2}{3} a_V \frac{\Delta T}{T_0}$$

$$\boxed{R - R_0 = R_0 \left(\tfrac{2}{3} a\right)\left(T - T_0\right)}$$

Non-ideal Batteries: internal resistance

Every voltage source has **some** internal resistance to it. Usually this can be ignored but not always





The internal resistance simply acts as a resistor in series with the rest of the circuit.

$$\varepsilon - Ir - IR = 0$$

$$I = \frac{\varepsilon}{(r+R)}$$

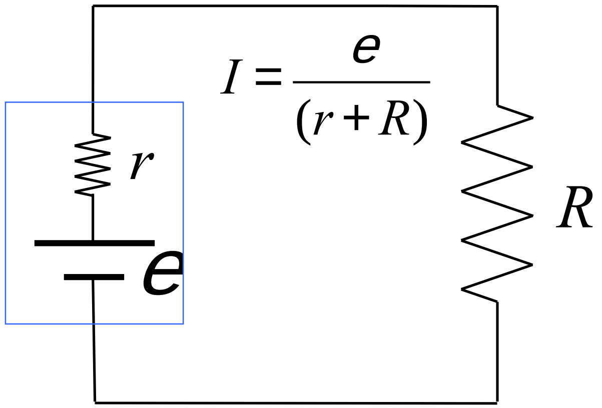
$$P_{\varepsilon} = I\varepsilon = \frac{\varepsilon^2}{R+r}$$

Power output required by the emf source

$$P_R = I^2 R = \frac{\varepsilon^2 R}{\left(R+r\right)^2}$$

Power dissipated by the resistive load

Non-ideal Batteries: internal resistance



Conservation of energy requires power in must be equal to power out, but power by emf is not power dissipated by R.

$$P_e = Ie = \frac{e^2}{R+r} \qquad P_r = I^2 R = \frac{e^2 R}{(R+r)^2}$$

Resolution: power dissipated by emf

$$P_r = I^2 r = \frac{e^2 r}{(R+r)^2}$$

The emf must do more work because it fights against its own internal resistance

Now we can verify that power in = power out

$$P_e = P_r + P_R = \frac{e^2 r}{(R+r)^2} + \frac{e^2 R}{(R+r)^2} = \frac{e^2 (R+r)}{(R+r)^2} = \frac{e^2}{R+r}$$

CASE 1: Charging the capacitor

$$\left(-R\frac{di}{dt}-\frac{1}{C}i=0\right)\times(-1)$$

$$R\frac{di}{dt}=\frac{-1}{C}i$$

$$\int\frac{di}{i}=\int-\frac{1}{RC}\,dt \qquad \text{use } \int\frac{dx}{x}=\ln x$$

$$\ln i=-\frac{1}{RC}t+C \qquad \text{use } e^{\ln x}=x$$

$$i(t)=e^{-t/RC}\underbrace{e^{C}}_{A}$$

$$i(t)=Ae^{-t/RC} \qquad \text{at } t=0 \;\; i(0)=V/R$$

$$i(0)=A\underbrace{e^{-0/RC}}_{1}=V/R$$

$$\boxed{i(t)=\frac{V}{R}e^{-t/RC}}$$

$$i=\frac{dq}{dt}\Rightarrow q(t)=\int i\,dt$$