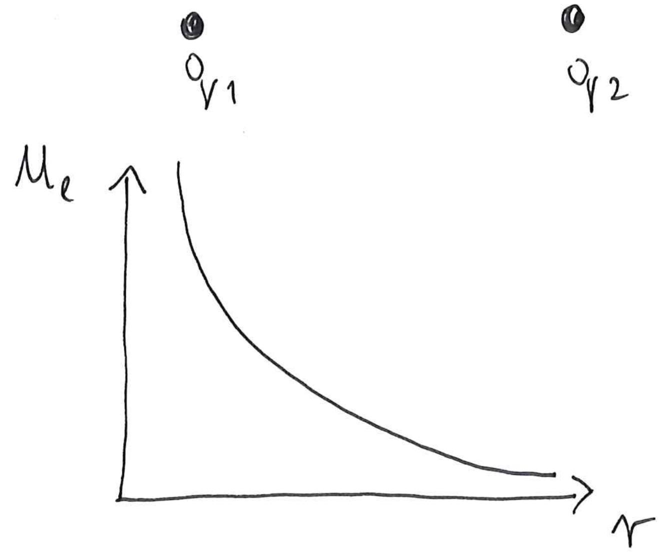
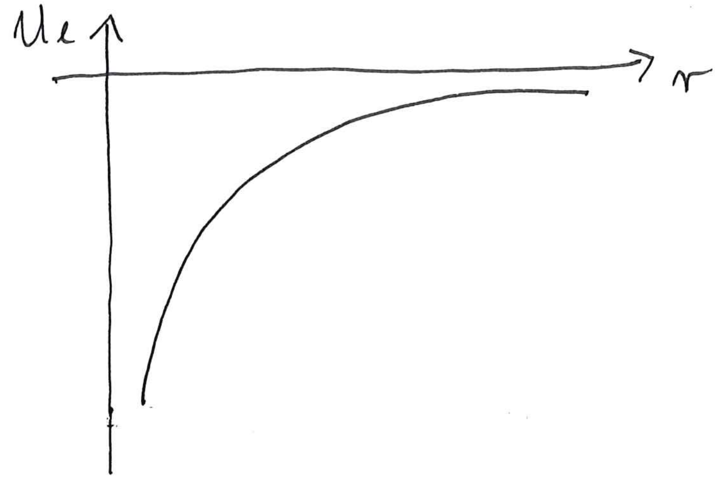
ELECTRIC POTENTIAL ENERGY

$$U = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r}$$





q_1 & q_2 have the same sign

"harder" to bring them together

q_1 & q_2 have opposite signs

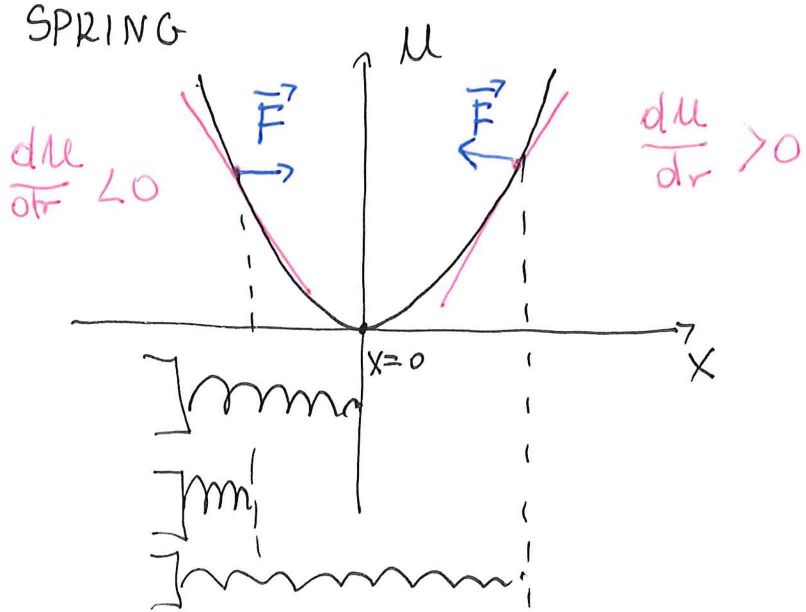
$$\boxed{\vec{F}_e = - \frac{dU_e}{dr}\hat{r}}$$

$$\vec{F_e} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2}\hat{r} \qquad U_e = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r}$$

$$\vec{F_e} = -\frac{d}{dr}\left(\frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r}\right)\hat{r}$$

$$\vec{F_e} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2}\hat{r}$$

SPRING

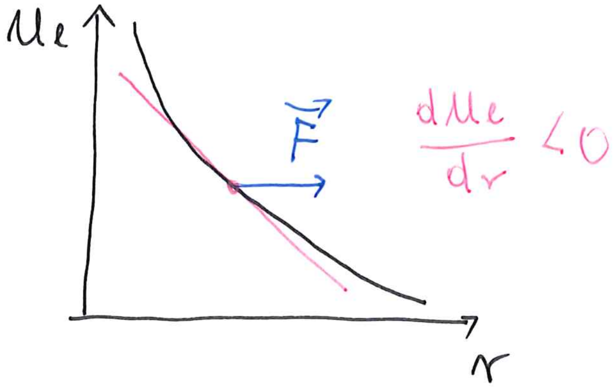


$$F = -\frac{dU}{dx}\hat{i}$$

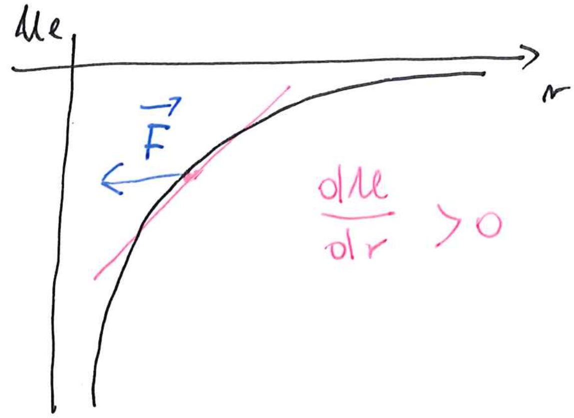
relaxed spring

compressed spring

stretched spring



q_1, q_2 same sign



q_1, q_2 opposite sign

For a conservative force $\vec{F}$, there is a potential energy associated with it, such that:

$$\vec{F} = -\underbrace{\vec{\nabla} U}_{\text{gradient}} \equiv -\frac{\partial U}{\partial x}\hat{i} - \frac{\partial U}{\partial y}\hat{j} - \frac{\partial U}{\partial z}\hat{k}$$

„defined as" or „equivalent to"

The force on a charge q sitting in an electric field is: $\vec{F}_e = q\vec{E}$ & $U_e = q V_{\text{potential}}$

$$\cancel{q}\vec{E} = -\frac{\partial}{\partial x}(\cancel{q}V)\hat{i} - \frac{\partial}{\partial y}(\cancel{q}V)\hat{j} - \frac{\partial}{\partial z}(\cancel{q}V)\hat{k}$$

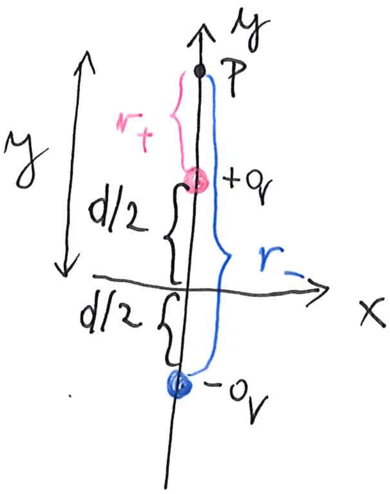
q is constant

$$\boxed{\vec{E} = -\frac{\partial V}{\partial x}\hat{i} - \frac{\partial V}{\partial y}\hat{j} - \frac{\partial V}{\partial z}\hat{k}}$$

relates electric field & electric potential

$$\vec{E} = -\vec{\nabla} V$$

Example: Electric potential of a dipole



Find V at P

$$V = V_{\oplus} + V_{\ominus}$$

$$V = \frac{1}{4\pi\varepsilon_0} \sum \frac{q_n}{r_n}$$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r_+} + \frac{1}{4\pi\varepsilon_0}\frac{-q}{r_-}$$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{(y-\frac{1}{2}d)} - \frac{1}{4\pi\varepsilon_0}\frac{q}{(y+\frac{1}{2}d)}$$

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{(y-\frac{1}{2}d)} - \frac{q}{(y+\frac{1}{2}d)}\right]$$

Find electric field!

$$E_y = -\frac{\partial V}{\partial y}\hat{j} \qquad E_x = 0, \quad E_z = 0$$

$$E_y = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{(y-\frac{1}{2}d)^2} - \frac{q}{(y+\frac{1}{2}d)^2}\right]$$

$y >> d$

binomial expenssion

$(1+x)^n \approx 1 + nx$ if x is smalle

$$E_y = \frac{q}{4\pi\varepsilon_0 y^2}\left[\left(1-\frac{d}{2y}\right)^{-2} - \left(1+\frac{d}{2y}\right)^{-2}\right]$$

$$E_y \approx \frac{q}{4\pi\varepsilon_0 y^2}\left[\left(\cancel{1} - (-2)\frac{d}{2y}\right) - \left(\cancel{1} + (-2)\frac{d}{2y}\right)\right]$$

$$E_y \approx \frac{q}{4\pi\varepsilon_0 y^2}\left(\frac{2d}{y}\right) = \frac{q \cdot d}{2\pi\varepsilon_0 y^3}$$

$q \cdot d$ — dipol moment

$$E_y \approx \frac{p}{2\pi\varepsilon_0 y^3}$$ it is NOT zero...