# Monday Mar 6, 2017

# Last time:

- Capacitors – demonstrations
- Group activity- electric potential

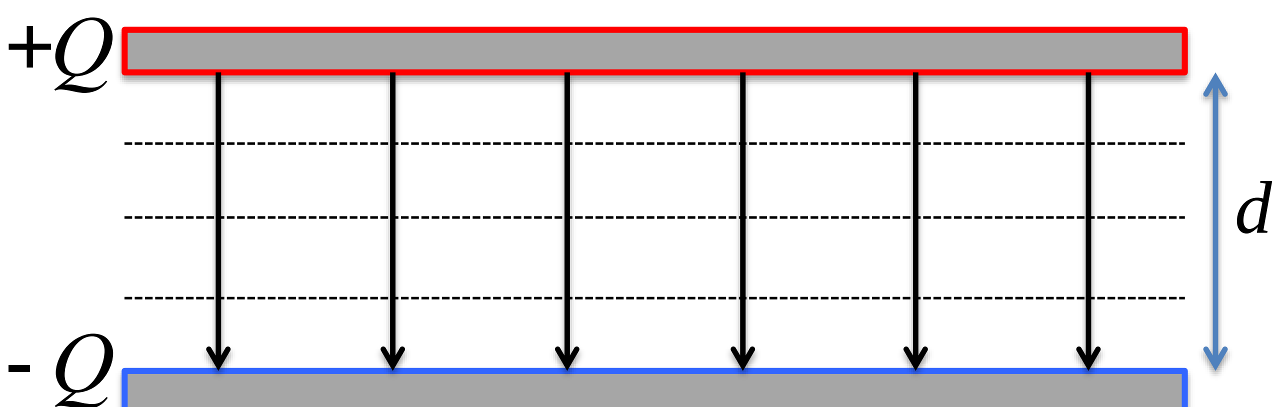
# Today:

- Capacitors
- Capacitance as a geometric quantity
- General Capacitors, relating Q to $\Delta V$
- Setting up a process to find Capacitance

# Parallel Plate Capacitors

- One plate carries a charge $+Q$, the other plate carries a charge $-Q$.
- This creates a uniform E-field between the plates.
- This E-field can be written as a potential difference.

$$E = \frac{\sigma}{\varepsilon_o} = \frac{\Delta V_C}{d} \qquad \sigma = \frac{Q}{A} \qquad Q = \left(\frac{\varepsilon_o A}{d}\right)\Delta V_C$$



# Capacitors and Capacitance

We find it useful to shorten that constant to just the letter *C*. This is a **geometric property** of the specific capacitor (not necessarily parallel plates)

$$Q = \left( \frac{e_o A}{d} \right) \mathrm{D} V = C \Delta V$$

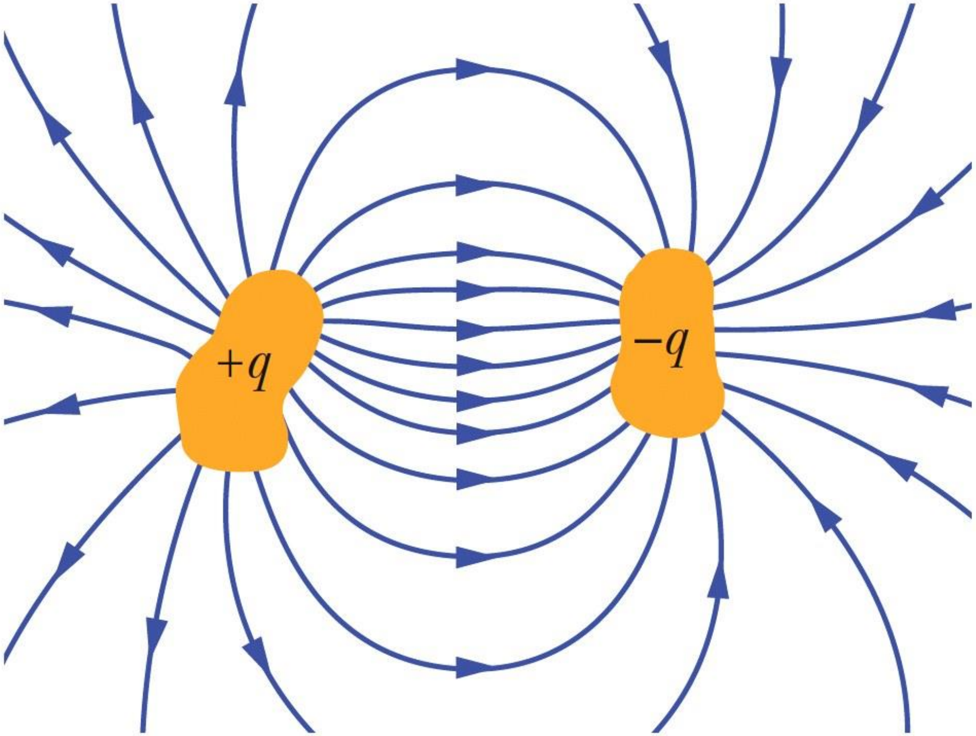
$$\boxed{C = \frac{e_o A}{d}}$$

*C* is called the **capacitance** and it represents the "**capacity to store charge**". For any capacitor, the relationship between its stored charge and the voltage across its electrodes is given by

$$Q = C \Delta V$$

# Capacitors in General

A capacitor is any two electrodes separated by some distance. Regardless of the geometry, we call the electrodes "plates".




Need to be electrodes (metal) in order to charge and discharge freely by the flow of charges.

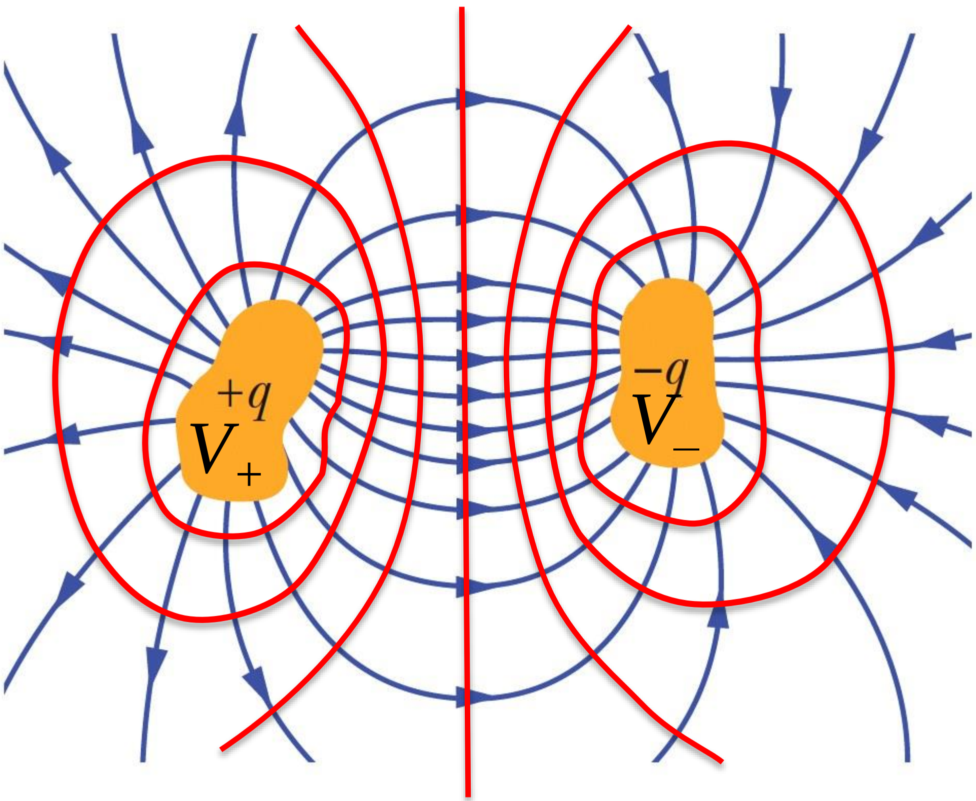
By convention, a capacitor has equal and opposite charges on its plates.



halliday_10e_fig_25_02

# Capacitors in General

For equal but opposite charges one the plates, this arbitrary set of electrodes creates an electric field. What are the equipotentials?




 halliday_10e_fig_25_02

The potential changes from $V_+$ on the positive plate to $V_-$ on the negative plate.

This is not as simple as for parallel plates: $\Delta V = Ed$, but the charge is still related to $\Delta V$

$$Q = C\Delta V$$

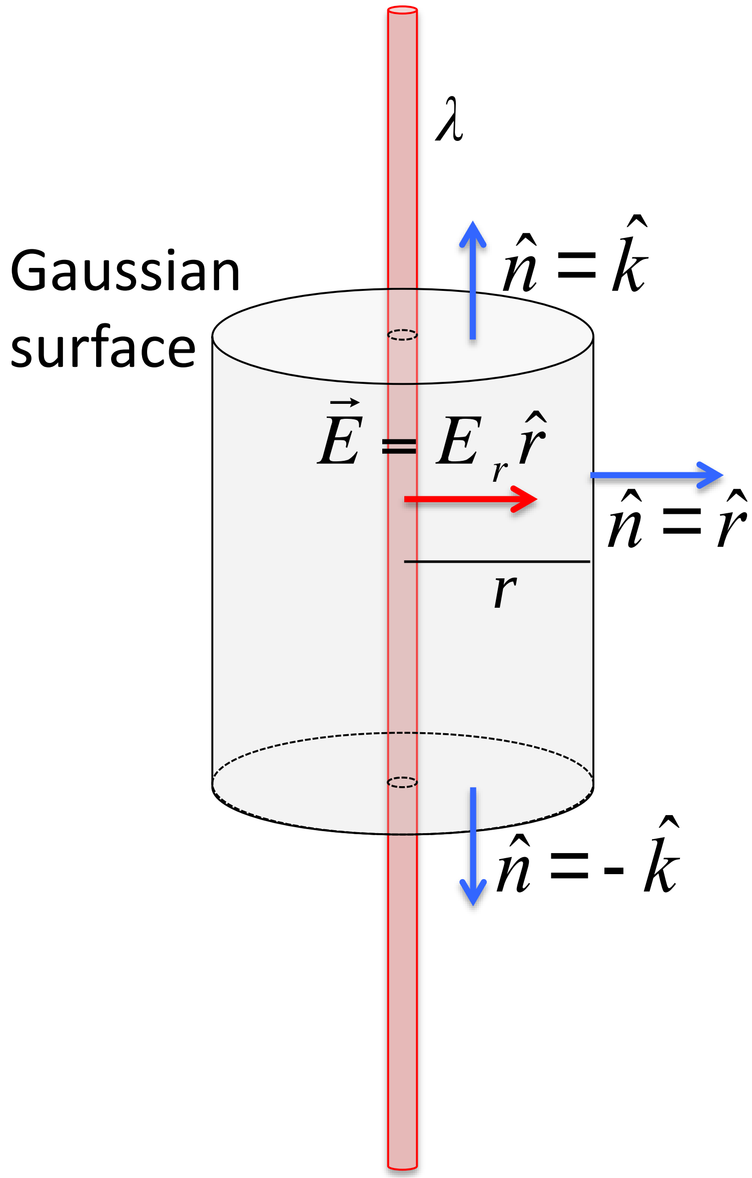
For some geometric quantity $C$

# TopHat question

# Finding capacitance – parallel plates

# Finding capacitance – Cylindrical Capacitor

# Calculate V from E




Gauss' Law: $\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$

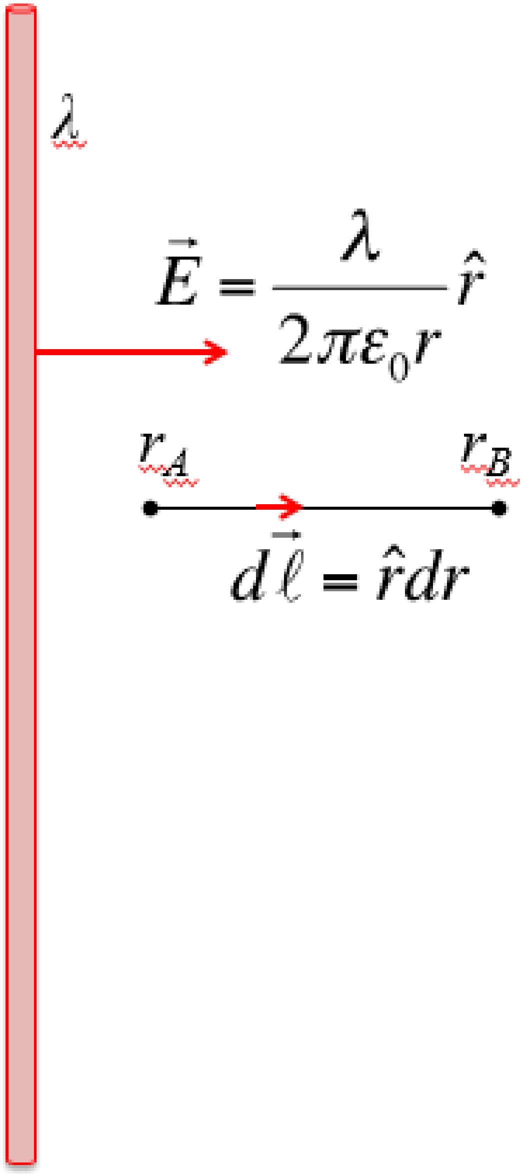
$$\oint \vec{E} \cdot d\vec{A} = \cancel{\int_{\text{top}} \vec{E} \cdot \hat{n}\, dA} + \cancel{\int_{\text{bottom}} \vec{E} \cdot \hat{n}\, dA}$$

$$+ \int_{\text{tube}} \vec{E} \cdot \hat{n}\, dA$$

$$E_r \oint dA = E_r 2\pi r L = \frac{\lambda L}{\varepsilon_0}$$

$$E_r = \frac{l}{2pe_0 r}$$

# Calculate V from E




$$\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 r}\hat{r}$$

$$d\vec{\ell} = \hat{r}dr$$

$$\Delta V_{AB} = -\int_A^B \vec{E}\cdot d\vec{\ell}$$

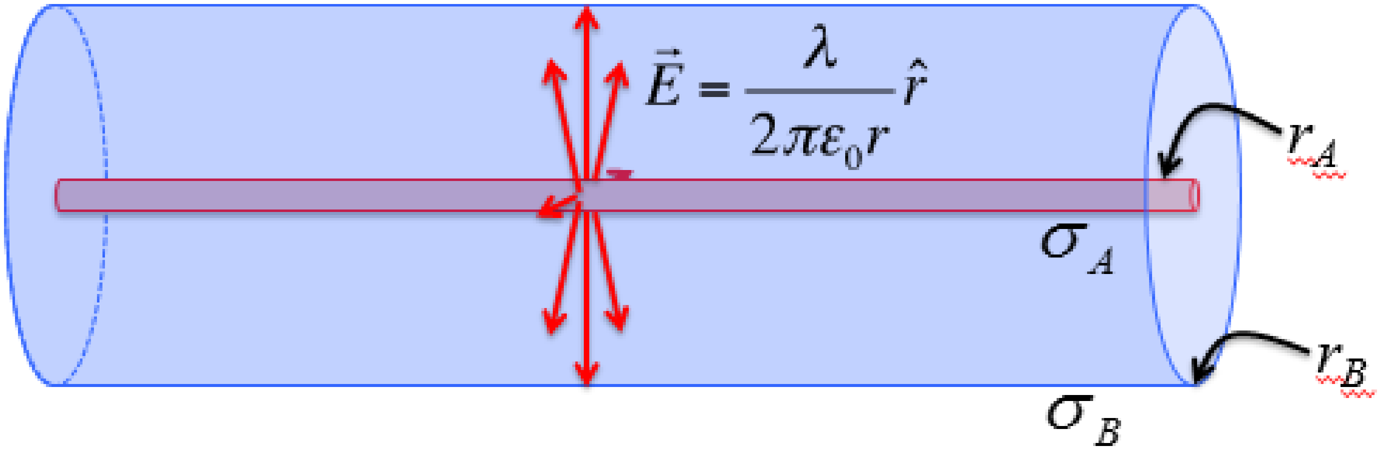
$$\Delta V_{AB} = -\int_A^B \frac{\lambda}{2\pi\varepsilon_0 r}\hat{r}\cdot\hat{r}dr$$

$$\Delta V_{AB} = -\frac{\lambda}{2\pi\varepsilon_0}\int_A^B \frac{dr}{r}$$

$$\Delta V_{AB} = -\frac{\lambda}{2\pi\varepsilon_0}\left(\ln(r_B) - \ln(r_A)\right)$$

$$\boxed{\Delta V_{AB} = -\frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{r_B}{r_A}\right)}$$

# Application: Cylindrical Capacitor



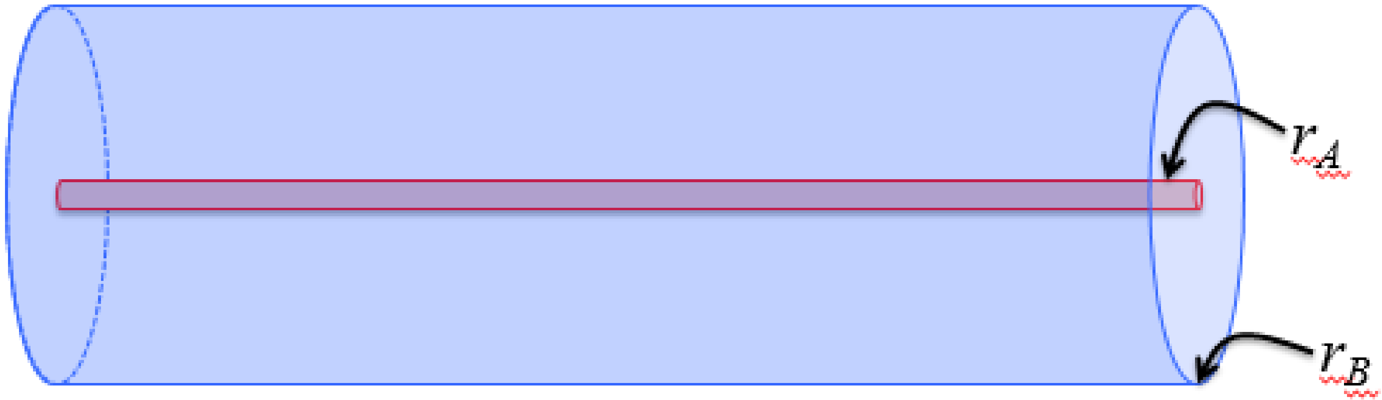

Inside, E is due to central cylinder

$$\Delta V_{12} = -\frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{r_2}{r_1}\right)$$

For some points $r_1$ and $r_2$ inside the bigger cylinder

Outside the cylinder, $E = 0$ because $q_{enc} = 0$

# Application: Cylindrical Capacitor




Voltage difference across the capacitor plates is obtained by taking $r_1 = r_A$ and $r_2 = r_B$:

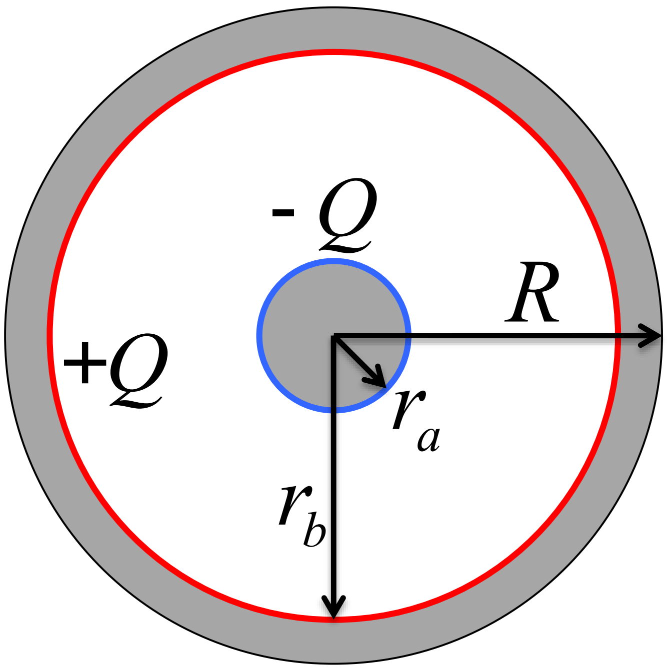
$$\Delta V_C = \frac{\lambda}{2\pi\varepsilon_0} \ln\left(\frac{r_B}{r_A}\right) = \frac{Q}{2\pi\varepsilon_0 L} \ln\left(\frac{r_B}{r_A}\right) \qquad Q = \boxed{\left(\frac{2\pi\varepsilon_0 L}{\ln\left(\frac{r_B}{r_A}\right)}\right)} \Delta V_C$$

Define capacitance per unit length:

$$C / L = \left(\frac{2\pi\varepsilon_0}{\ln\left(\frac{r_B}{r_A}\right)}\right)$$

# Finding capacitance – Spherical Capacitor

# Spherical Capacitor




1) What is the E-field everywhere?
2) What is V everywhere?
3) What is ΔV between the plates?
4) How can we relate ΔV to the charge on the plates?

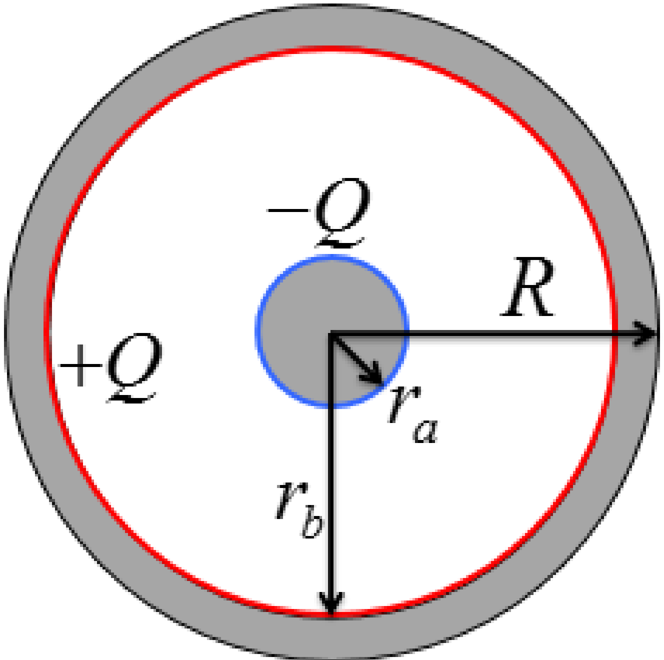
$$r > R, \quad \vec{E} = 0 \quad \text{(from Gauss' Law)}$$

$$R > r > r_b, \quad \vec{E} = 0 \quad \text{(inside a conductor)}$$

$$r < r_a, \quad \vec{E} = 0 \quad \text{(inside a conductor)}$$

$$\boxed{r_a > r > r_b, \quad \vec{E} = \frac{-Q}{4\pi\varepsilon_0 r^2}\hat{r}}$$

# Spherical Capacitor




1) What is the E-field everywhere?
2) What is V everywhere?
3) What is ΔV between the plates?

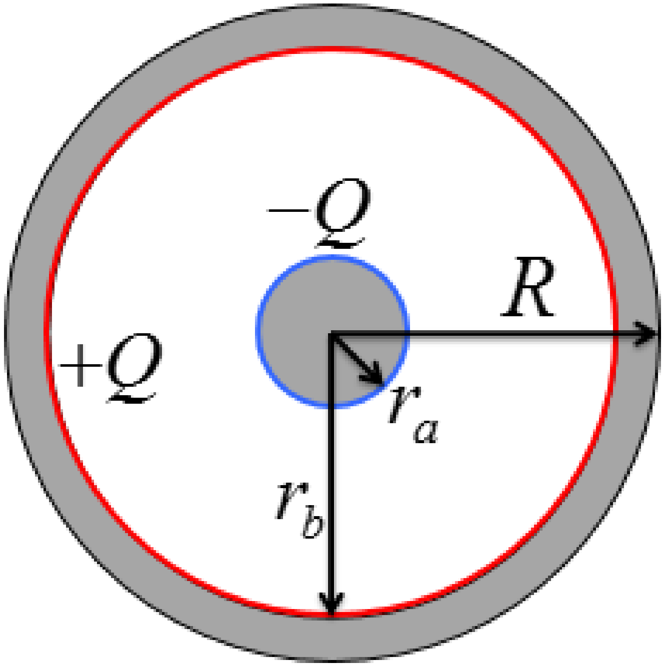
$$\boxed{V = 0 \quad \text{at infinity}}$$

$$r > r_b, \qquad \Delta V = -\int_{\infty}^{R} \vec{E} \cdot d\vec{r} = 0 \qquad \boxed{V_{r>r_b} = 0}$$

$$r_b > r > r_a, \qquad \Delta V = -\int_{r_b}^{r_a} \vec{E} \cdot d\vec{r} = -\int_{r_b}^{r_a} \frac{-Q}{4\pi\varepsilon_0 r^2} dr \boxed{= \frac{-Q}{4\pi\varepsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right)}$$

$$r < r_a, \qquad \Delta V = -\int_{\infty}^{R} \vec{E} \cdot d\vec{r} = 0 \qquad \boxed{V_{r<r_a} = \frac{-Q}{4\pi\varepsilon_0 r_a}}$$

# Spherical Capacitor




1) What is the E-field everywhere?
2) What is V everywhere?
3) What is ΔV between the plates?
4) How can we relate ΔV to the charge on the plates?

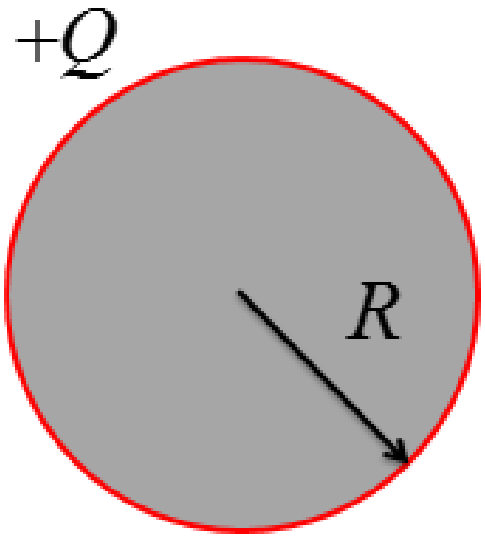
$$\Delta V_C = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right) = \frac{Q}{4\pi\varepsilon_0}\left(\frac{r_b - r_a}{r_b r_a}\right)$$

Rewrite this relation as

$$Q = \left(\frac{4\pi\varepsilon_0 r_b r_a}{r_b - r_a}\right)\Delta V_C$$

$$\boxed{Q = C\,\Delta V_C}$$

# Isolated Sphere as a Capacitor




Capacitors need two plates in general for the field lines to end. In the case of a sphere, we can consider the other plate to be at infinity and define the capacitance of an isolated sphere with charge Q. This will not work for an infinite cylinder as we will see later.

Start with expression for spherical capacitor with $r_a = R, r_b = \infty$:

$$Q = \left( \frac{4\pi\varepsilon_0 r_b r_a}{r_b - r_a} \right) \Delta V_C \rightarrow (4\pi\varepsilon_0 R)\Delta V_C \qquad \boxed{C = 4\pi\varepsilon_0 R}$$

# Capacitors

General relationship: $$Q = C\Delta V_C$$

Parallel plate capacitor: $$Q = \left(\frac{\varepsilon_o A}{d}\right)\Delta V_C$$

Cylindrical capacitor: $$Q = \left(\frac{2\pi\varepsilon_0 L}{\ln\left(\frac{r_B}{r_A}\right)}\right) V_C$$

Spherical capacitor: $$Q = \left(\frac{4\pi\varepsilon_0 r_b r_a}{r_b - r_a}\right)\Delta V_C$$

Isolated sphere: $$Q = (4\pi\varepsilon_0 R)\Delta V_C$$