
Write your UCID in the space above

University of Calgary

Faculty of Science

Midterm Test

PHYSICS 259 ALL LECTURE SECTIONS

Part II: Written Answer Questions (Total: 16 marks)

IMPORTANT: Write your answers to the problems in Part II in boxes. Work must be shown for full marks. Rough work can be done on the back of this question paper, but only the work in the boxes be marked.

Question 22 (5 marks)

Six point charges are located in the x-y plane in a hexagon centered on the origin. The distance between adjacent charges is d. The charges all have the same magnitude q, but with signs as shown in Figure 22.

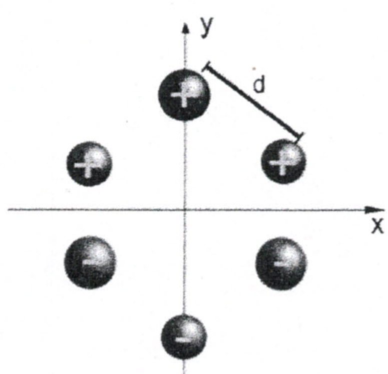
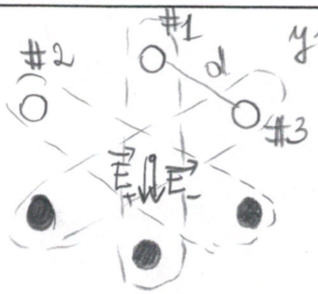


Figure 22: Charge hexagon

Write your last name and initials in the space above

22.1. What is the electric field $\vec{E}$ at the origin? Include diagram. (2 marks)

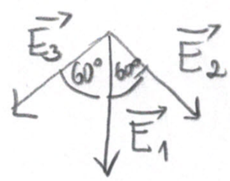


○ positive charge ● negative charge

Pick diametrically oposite pairs (1-3)

For each pair: $|\vec{E}_-| = |\vec{E}_+| = \frac{q}{4\pi\varepsilon_0 d^2}$

$|\vec{E}_1| = |\vec{E}_+ + \vec{E}_-| = \frac{2q}{4\pi\varepsilon_0 d^2}$ → the same for other two pairs.



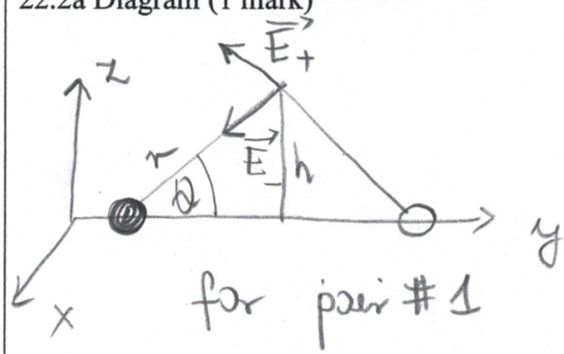
Then we consider three sets of charges, Horizontal (x) components cancel

$|\vec{E}_{net}| = |\vec{E}_{+-}| + 2|\vec{E}_{+-}|\cos 60°$

$|\vec{E}_{net}| = \frac{q}{\pi\varepsilon_0 d^2}$ $\quad \vec{E}_{net} = \frac{q}{\pi\varepsilon_0 d^2}(-\hat{j})$ negative y-direction

22.2. Calculate the electric field $\vec{E}$ on the z-axis at (x=0, y=0, z=h). (4 marks)

22.2a Diagram (1 mark)



for pair #1

Again, consider diametrically opposite pairs.

$\vec{E}_1$ will be in the negative y-direction $(-\hat{j})$

22.2b Calculation of x component (1 mark)

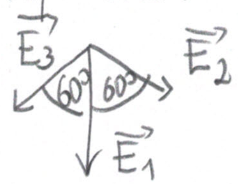
$E_x = 0$ due to symetry
(components cancel)

22.2c Calculation of y component (1 mark)

For each pair: $|\vec{E}_+ + \vec{E}_-| = \frac{2}{4\pi\varepsilon_0}\frac{q}{(h^2+d^2)}\cos\theta$

$\cos\theta = \frac{d}{\sqrt{h^2+d^2}} \Rightarrow |\vec{E}_+ + \vec{E}_-| = \frac{q\,d}{2\pi\varepsilon_0 (h^2+d^2)^{3/2}}$

Top view:



$\vec{E}_{net} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$

$|\vec{E}_{net}| = |\vec{E}_1| + 2|\vec{E}_1| \cdot \cos 60° = \frac{q\,d}{\pi\varepsilon_0 (h^2+d^2)^{3/2}}$

$\vec{E}_{net} = \frac{q\,d}{\pi\varepsilon_0 (h^2+d^2)^{3/2}}\,(-\hat{j})$

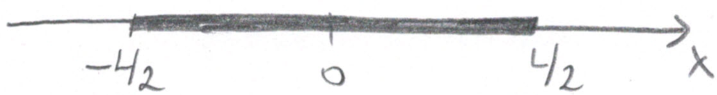
22.2d Calculation of z component (1 mark)

$E_z = 0$ due to symetry
(components cancel)

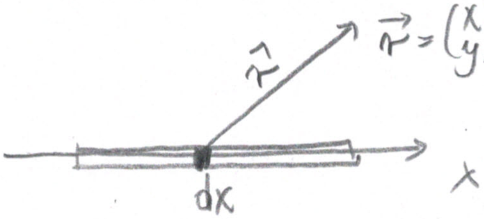
Question 23 (6 marks)

A total charge Q is uniformly distributed along a rod of length L. The rod is located on the x-axis with the midpoint at x = 0.

23.1 Draw a sketch of the problem including axes and labels. (1 mark)



23.2. Consider a small segment of the rod dx at the origin. Write down an expression for the electric field $d\vec{E}$ due to the small segment at an arbitrary location outside the rod. $\vec{r} = x\hat{\imath} + y\hat{\jmath}$.
(1 mark)



$\vec{r} = \begin{pmatrix} x \\ y \end{pmatrix}$ or $\vec{r} = x\hat{\imath} + y\hat{\jmath}$ $r = \sqrt{x^2+y^2}$

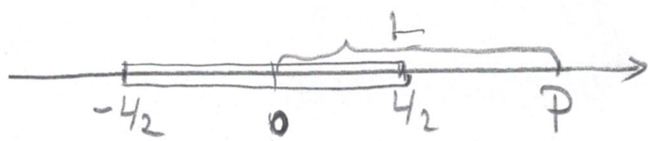
$$d\vec{E}(x,y) = \frac{1}{4\pi\epsilon_0}\frac{dq}{r^2}\hat{r} \quad \lambda = \frac{Q}{L}$$

$$d\vec{E}(x,y) = \frac{1}{4\pi\epsilon_0}\frac{\lambda dx}{(x^2+y^2)}\hat{r}$$

$$d\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{\lambda\, dx}{(x^2+y^2)^{3/2}}\begin{pmatrix} x \\ y \end{pmatrix}$$

23.3. Derive an integral expression for the electric field $\vec{E}$ (x = L, y = 0) produced by the entire rod the location on the x-axis where x = L. Show your work, but you do not need to evaluate the final integral. (3 marks)

Diagram showing the set up for integration (1 mark):



Calculations (2 marks):

Using a similar approach as in 23.2, but with $\hat{r} = \begin{pmatrix}1\\0\end{pmatrix}$ (along x-axis)

Using $r = L - x$

$$d\vec{E}(L,0) = \frac{\lambda\, dx}{4\pi\varepsilon_0 (L-x)^2}\,\hat{\imath}$$

$$\vec{E}(L,0) = \frac{\lambda}{4\pi\varepsilon_0}\int_{-L/2}^{L/2}\frac{dx}{(L-x)^2}\,\hat{\imath}$$

23.4. What is the linear charge density λ if Q = -3.00 Coulombs and L = 2.00 metres? (1 mark)

$$\lambda = \frac{Q}{\lambda} = \frac{-3.00\,C}{2.00\,m} = -1.50\,\frac{C}{m}$$

Question 24 (4 marks)

A very long line of charge density λ is located on the x-axis.

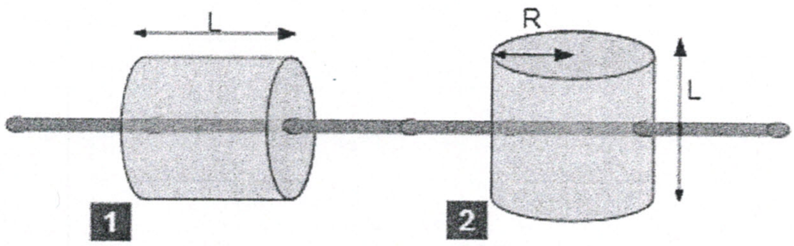
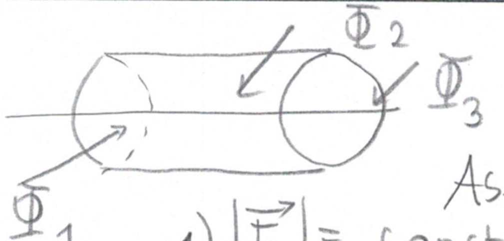


Figure. 24. Two Gaussian cylinders

24.1. Use Gauss's law to derive a general expression for the electric field (use cylinder 1), assuming that the rod is very long (L→ ∞). Show all steps and clearly (2 marks) state any assumptions. (1 mark)



The Gaussian surface has three parts (1-3).

Assume:

1) $|\vec{E}|$ = constant & distributed radially

2) End caps (1&3) have no flux

Gauss's law: $\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_0}$

$$\int \vec{E}\, dA = \underbrace{\Phi_1}_{=0} + \Phi_2 + \underbrace{\Phi_3}_{=0}$$

The area of the "tube" is $2\pi r \cdot L$

$$\int E dA = E \int dA = E \cdot 2\pi r \cdot L \qquad \lambda = \frac{Q}{L}$$

$$E\, 2\pi r \cancel{L} = \frac{Q_{encl}}{\varepsilon_0} = \frac{\lambda \cdot \cancel{L}}{\varepsilon_0} \qquad Q = \lambda \cdot L$$

$$E(r) = \frac{\lambda}{2\pi \varepsilon_0 \cdot r}$$

24.2. Two identical cylinders are placed as shown in Figure 24. If the total flux through both cylinders is identical, then what can we say about the cylinder dimensions? Explain. (1 mark)

Equal flux = equal charge (Q_{encl}) $\frac{Q_1}{\varepsilon_0} = \Phi$, $\frac{Q_2}{\varepsilon_0} = \Phi$

$Q_1 = \lambda \cdot L$ $Q_2 = \lambda 2R$ $Q_1 = Q_2$

$\cancel{\lambda} \cdot L = \cancel{\lambda} \cdot 2R$ $L = 2R$