Group #	**Student**	**Last Name**	**First Name**
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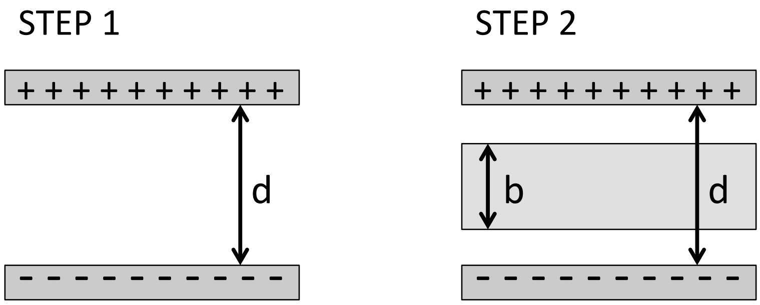
(10 marks) Consider the following scenario depicted in the diagrams below. Step 1: A parallel-plate capacitor with surface area A and plate separation d is charged to an electric charge of q. Step 2: A dielectric slab of thickness b and dielectric constant κ is inserted between the capacitor plates. What is the ratio between the final and initial voltages measured across the capacitor?



The parts below walk you through related questions, and the steps with which to solve this problem. Please show all work in the boxes provided and then choose the correct answer at the bottom

1. **(1 mark)** What is the capacitance of the parallel-plate capacitor in Step 1 in terms of its dimensions and $\epsilon_\circ$?

Answer:

$$C = \frac{\epsilon_\circ A}{d}$$

Note: derivation not necessary.

2. **(1 mark)** What is the voltage, $V_\circ$, across the capacitor in Step 1 in terms of q, $\epsilon_\circ$ and the dimensions of the capacitor?

Answer:

$$\begin{aligned} \text{formula:} \quad C &= \frac{q}{V_\circ} \\ \text{rearrange:} \quad V_\circ &= \frac{q}{C} \\ \text{plug in C from Q1:} \quad V_\circ &= \frac{qd}{\epsilon_\circ A} \end{aligned}$$

3. **(1 mark)** Is the charge on the capacitor in Step 2 the same as it was in Step 1? Explain.

Answer:

Yes, the charge remains the same. Inserting the dielectric slab does not change the charge on the plates, since the capacitor is not in contact with anything, so charges can't flow into or out of it.

4. **(2 marks)** If the electric field strength between the plates in Step 2 is $E_\circ$ in the region *outside* of the dielectric, and E_d *inside* the dielectric, what is the **voltage** across the capacitor in Step 2 in terms of $E_\circ$, E_d and the dimensions of the capacitor and the dielectric slab?

Answer:

$$\begin{aligned}
\text{equation relating field and voltage:} \quad V &= -\int_i^f \vec{E}\cdot d\vec{l} \\
\text{simplify:} \quad V &= \int_-^+ E(l)dl \\
\text{3 different regions:} \quad V &= \int_{below} E_\circ dl + \int_{above} E_\circ dl + \int_{inside} E_d dl \\
\text{plug in distances and fields:} \quad V &= E_\circ(d-b) + E_d b
\end{aligned}$$

5. **(2 marks)** Use Gauss's law to find $E_\circ$ and E_d in Step 2, in terms of q, $\epsilon_\circ$, κ and the dimensions of the capacitor and the dielectric slab.

Answer:

$$\begin{aligned}
\text{Gauss's law for outside of dielectric:} \quad E_\circ A &= \frac{q}{\epsilon_\circ} \\
\text{rearrange:} \quad E_\circ &= \frac{q}{\epsilon_\circ A} \\
\text{Gauss's law for inside dielectric:} \quad \kappa E_d A &= \frac{q}{\epsilon_\circ} \\
\text{rearrange:} \quad E_d &= \frac{q}{\kappa\epsilon_\circ A}
\end{aligned}$$

6. **(2 marks)** Using your result from Question 5, write the voltage, V, across the capacitor in Step 2 in terms of q, $\epsilon_\circ$, κ and the dimensions of the capacitor and the dielectric slab.

Answer:

$$\begin{aligned}
\text{plug into result from Q4:} \quad V &= \frac{q}{\epsilon_\circ A}(d-b) + \frac{q}{\kappa\epsilon_\circ A}b \\
\text{simplify:} \quad V &= \frac{qd}{\epsilon_\circ A}\left(1 - \frac{b}{d} + \frac{b}{\kappa d}\right)
\end{aligned}$$

(1 mark for the correct answer) Choose the correct expression for the ratio between the voltage across the capacitor in Step 2 and the voltage across the capacitor in Step 1.

A. $\frac{V}{V_\circ} = 1 - \frac{b}{d} + \frac{b}{\kappa d}$ B. $\frac{V}{V_\circ} = 1 - \frac{b}{d} + \frac{\kappa b}{d}$ C. $\frac{V}{V_\circ} = 1 + \frac{b}{\kappa d}$ D. $\frac{V}{V_\circ} = \frac{\kappa d - \kappa b + b}{\kappa d}$

Note: Options A and D are equivalent, and both were accepted as answers.