

Last time:

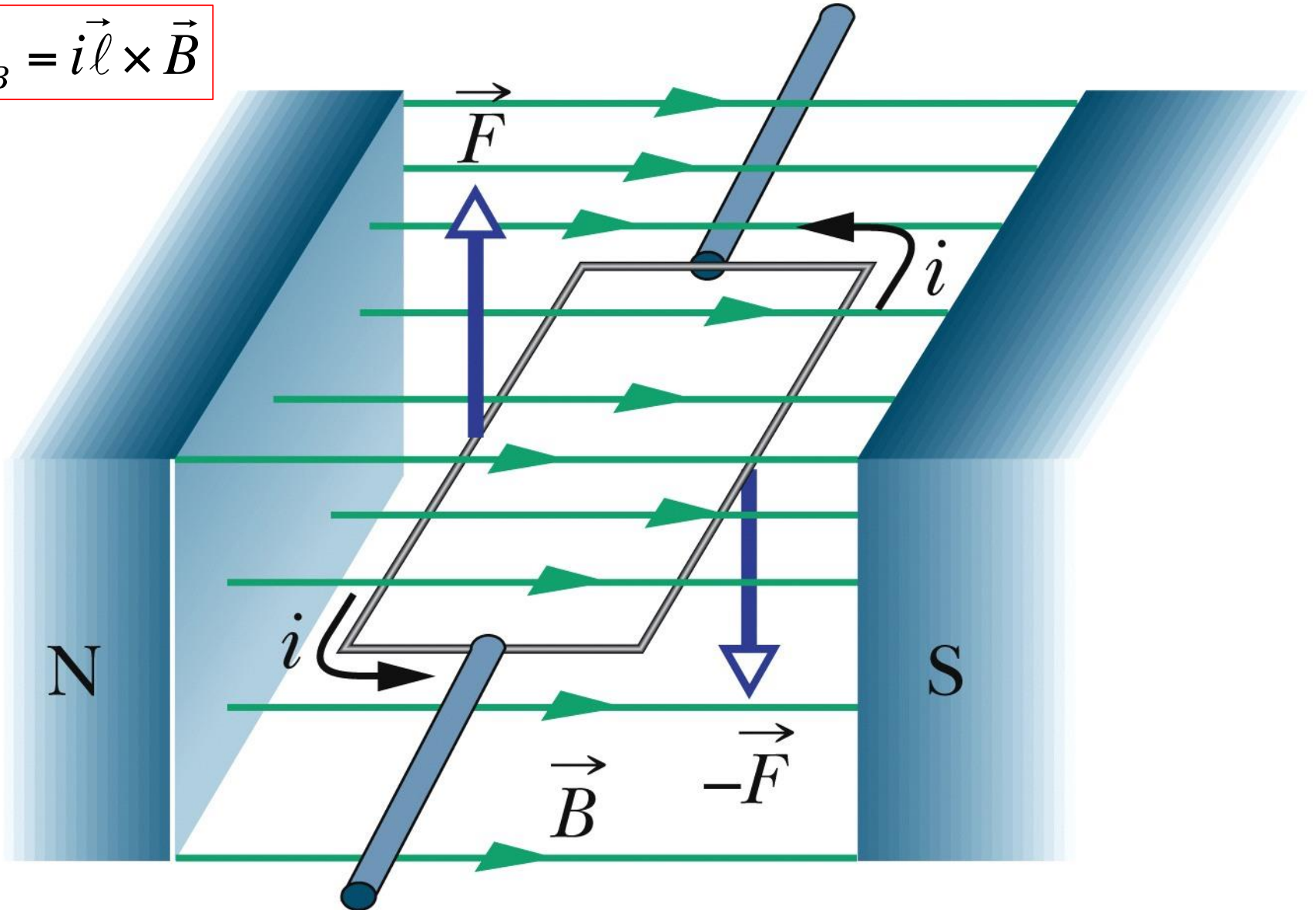
- Conductors moving through B-fields: Hall(ish) Effect
- Magnetic force on current carrying wires

Today:

- Torque on a current loop
- Biot-Savart Law (like Coulomb's Law for magnetism)
- B-field of a line of current
- Magnetic force between parallel current-carrying wires

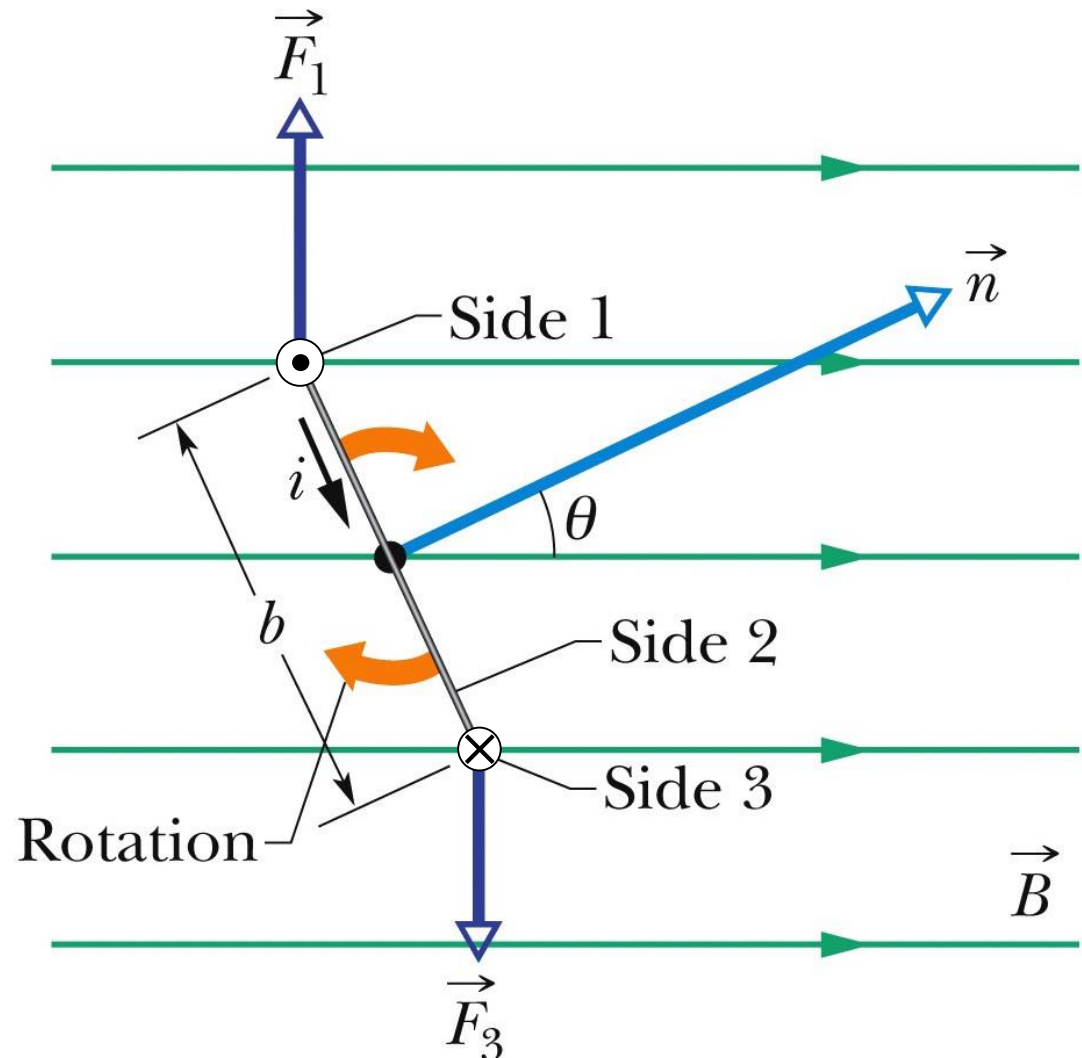
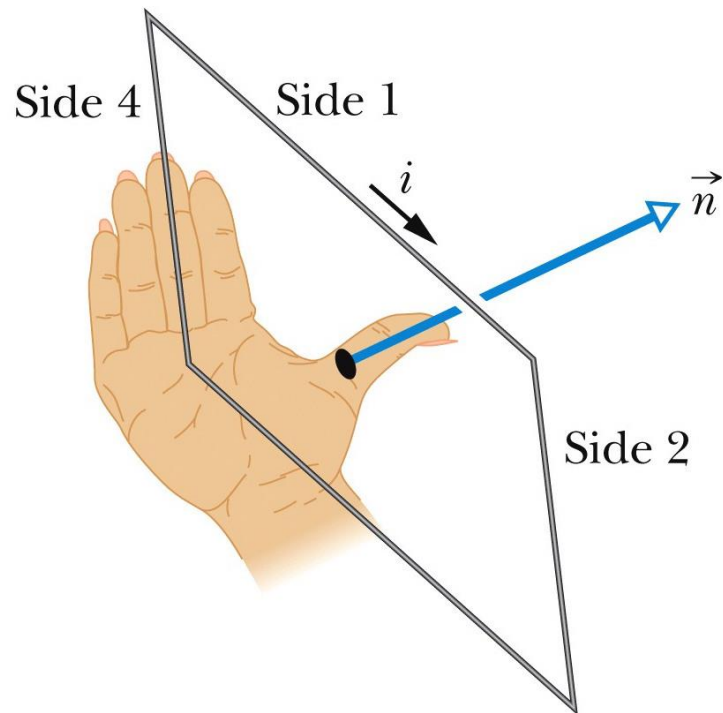
Torque on a current loop

$$\vec{F}_B = i\vec{\ell} \times \vec{B}$$

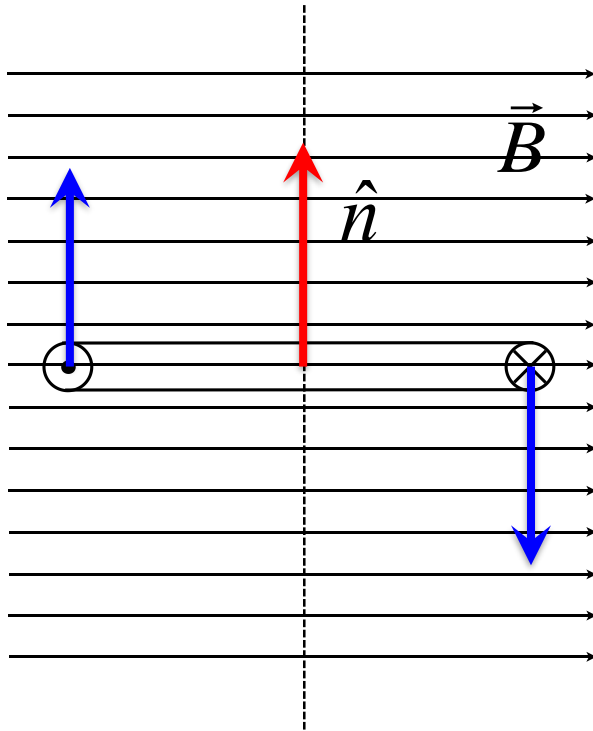


Torque on a current loop

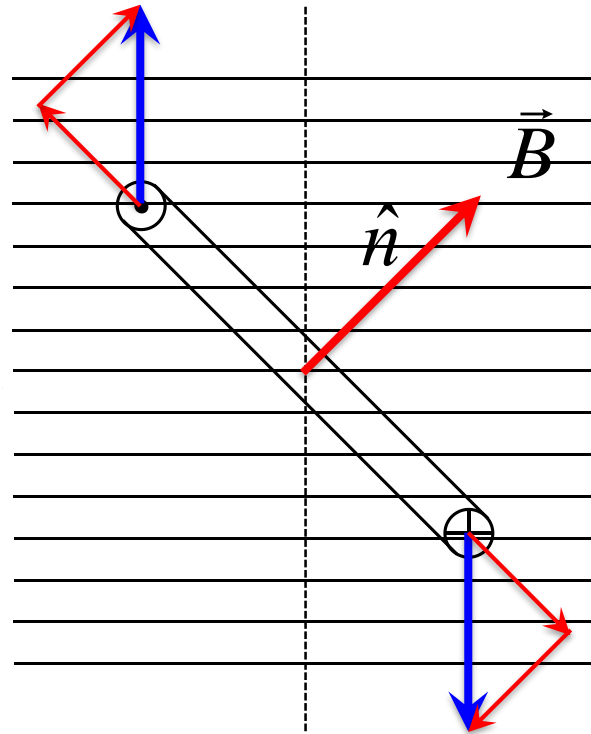
Pick the normal vector to the loop area by RHR: curl your fingers in the direction of i , thumb points in direction of \vec{n}



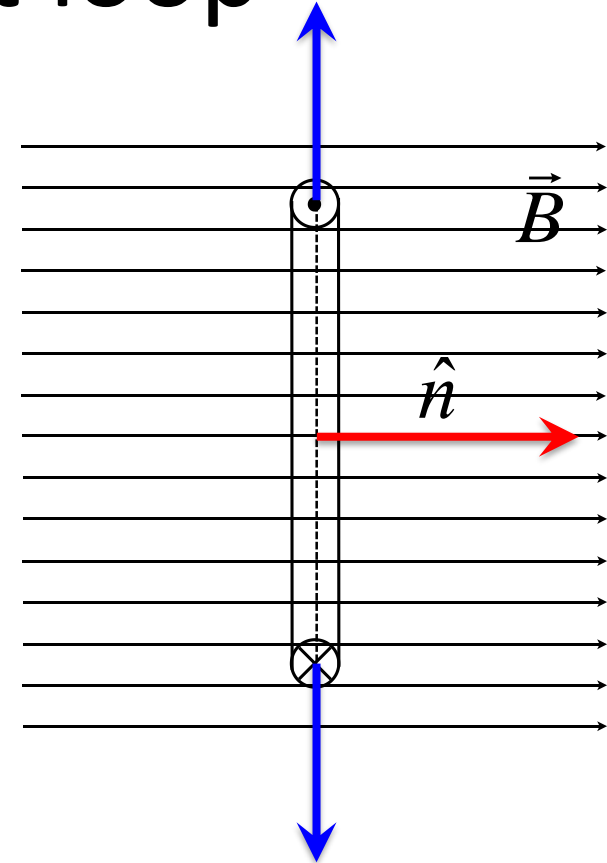
Torque on a current loop



The normal vector is at right angles to the B-field: all magnetic force causes rotation of the loop



The normal vector is at some angle to the B-field: some of the magnetic force causes rotation of the loop



The normal vector is parallel to the B-field: none of the magnetic force causes rotation of the loop

Conclusion: components of magnetic force (anti)parallel to normal vector that cause torque

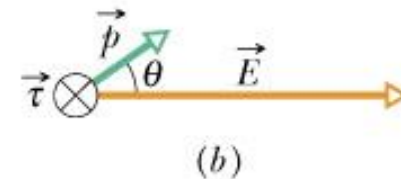
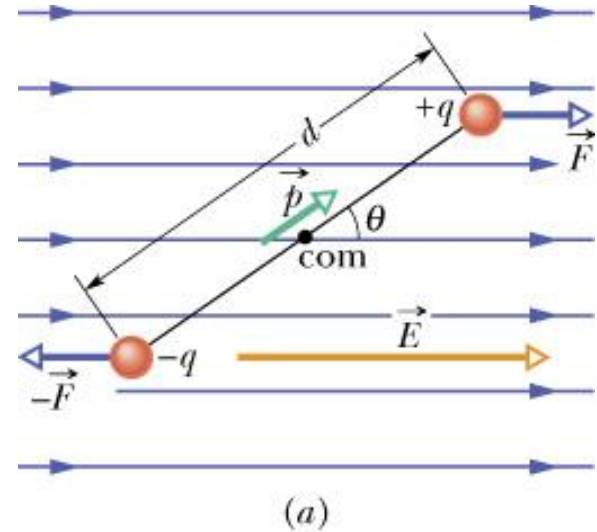
Electric dipole in a uniform electric field

$$\vec{F}_+ + \vec{F}_- = +q\vec{E} - q\vec{E} = 0$$

$$\begin{aligned}\tau &= qE \frac{d}{2} \sin \theta + (-qE) \left(-\frac{d}{2} \right) \sin \theta \\ &= qdE \sin \theta = pE \sin \theta\end{aligned}$$

$$\vec{\tau}_E = \vec{p} \times \vec{E}$$

$$U = -\vec{p} \cdot \vec{E} = -pE \cos \theta$$



Force and torque on a current loop

- This basis of electric motors

Maximum torque:

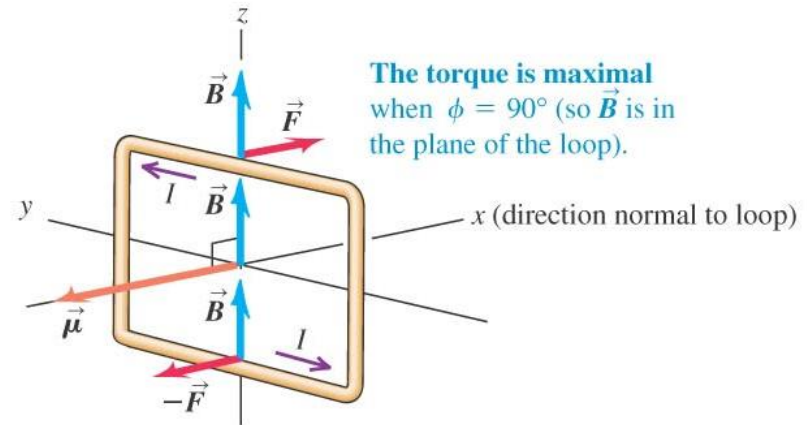
$$\tau = 2F(b/2) = IBab = (Iab)B = \mu B$$

Magnetic Dipole Moment

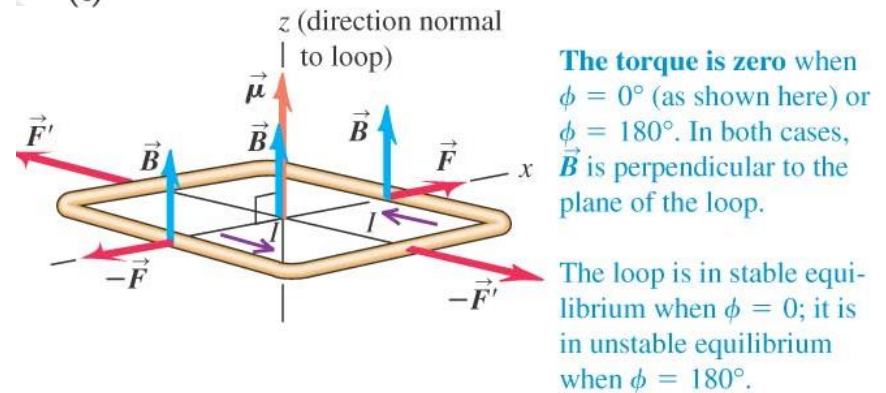
Minimum torque:

$$\tau_{\min} = 0$$

(b)



(c)



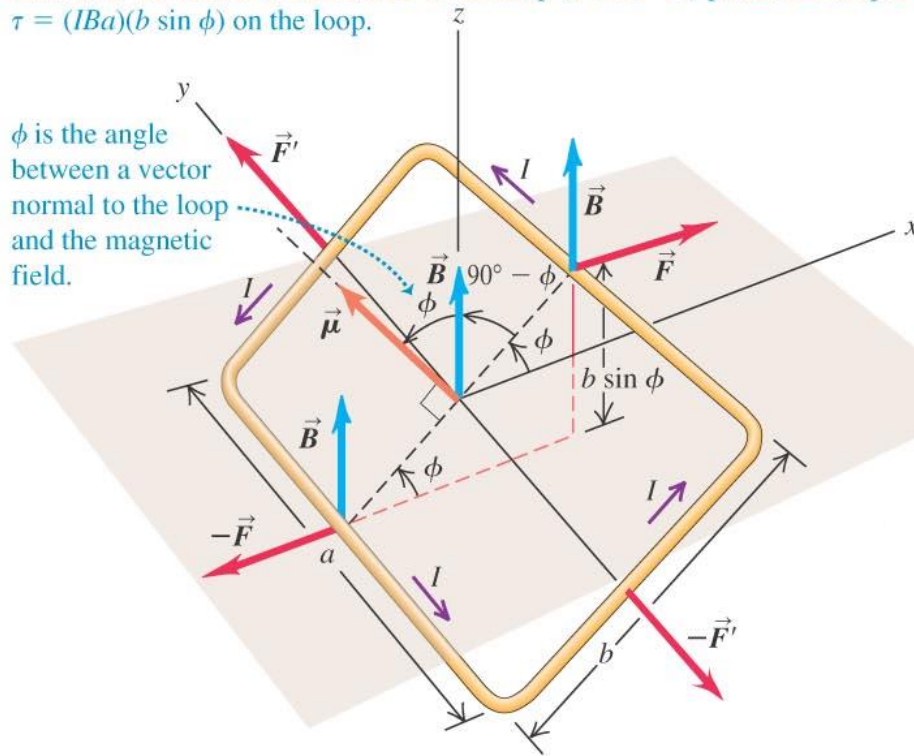
Force and torque on a current loop

- This basis of electric motors

(a)

The two pairs of forces acting on the loop cancel, so no net force acts on the loop.

However, the forces on the a sides of the loop (\vec{F} and $-\vec{F}$) produce a torque $\tau = (IBa)(b \sin \phi)$ on the loop.

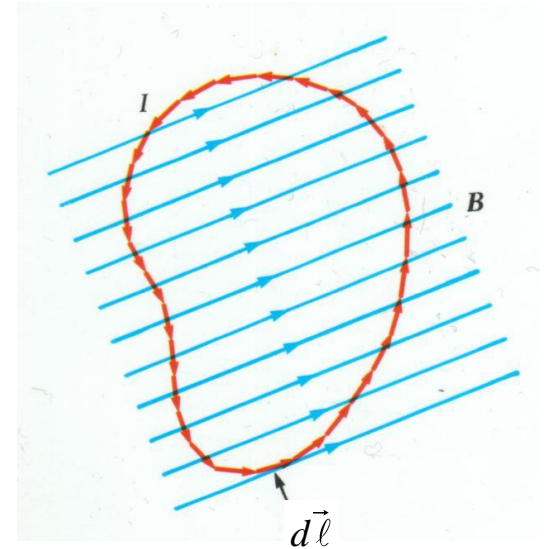


$$\tau = 2F(b/2)\sin\phi$$

$$= IBab \sin \phi = (Iab) B \sin \phi = \mu B \sin \phi$$

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

A current loop in magnetism is equivalent to an electric dipole. There is therefore a **magnetic moment** associated with a current loop as there is an electric moment associated with an electric dipole.



Magnetic

$$\vec{F}_B = 0$$

$$\vec{\tau}_B = \vec{\mu} \times \vec{B}$$

$$\vec{\mu} = I\vec{A}$$

$$U_B = -\vec{\mu} \cdot \vec{B} = -\mu B \cos \theta$$

Electric

$$\vec{F}_E = 0$$

$$\vec{\tau}_E = \vec{p} \times \vec{E}$$

$$\vec{p} = q\vec{d}$$

$$U = -\vec{p} \cdot \vec{E} = -pE \cos \theta$$

The Biot-Savart Law

(Bee-oh Sah-var)

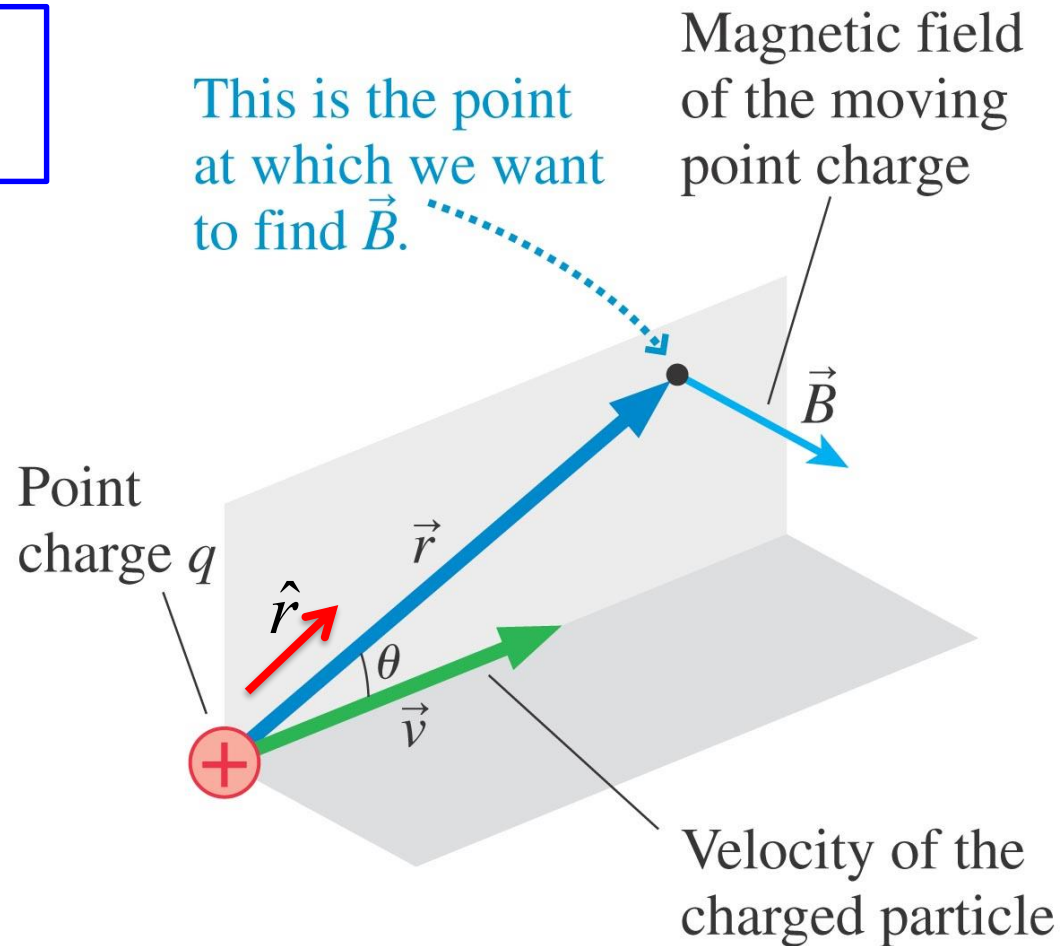
Magnetic fields are caused by **moving charges**.*

$$\vec{B}_{\text{point charge}} = \frac{\mu_0}{4\pi} \frac{q\vec{v} \times \hat{r}}{r^2}$$

Or, using the definition

$$\hat{r} = \frac{\vec{r}}{|\vec{r}|}$$

$$\vec{B}_{\text{point charge}} = \frac{\mu_0}{4\pi} \frac{q\vec{v} \times \vec{r}}{r^3}$$



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*One exception is due to quantum mechanics: charged particles with “spin” produce B fields

Constants of nature

“Permittivity of free space”

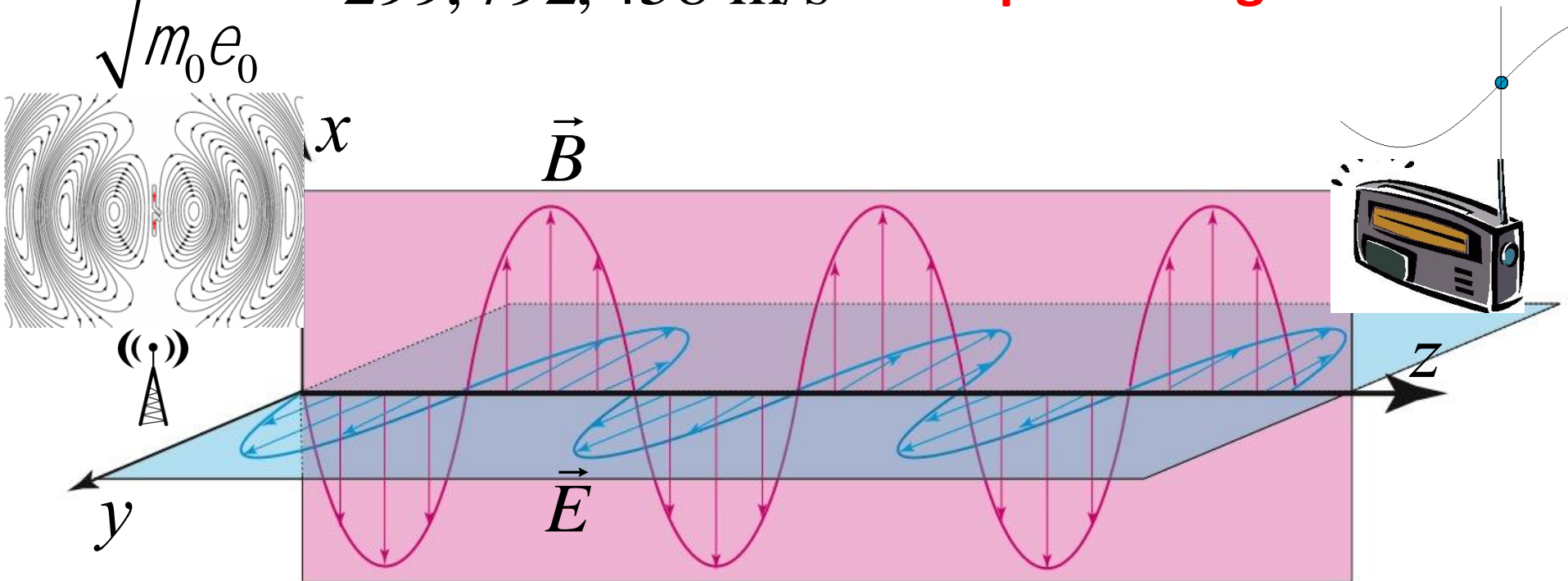
$$\epsilon_0 = 8.85418781719 \times 10^{-12} \frac{C^2}{N \times m^2}$$

“Permeability of free space”

$$\mu_0 = 4\pi \times 10^{-7} \frac{N \times s^2}{C^2}$$

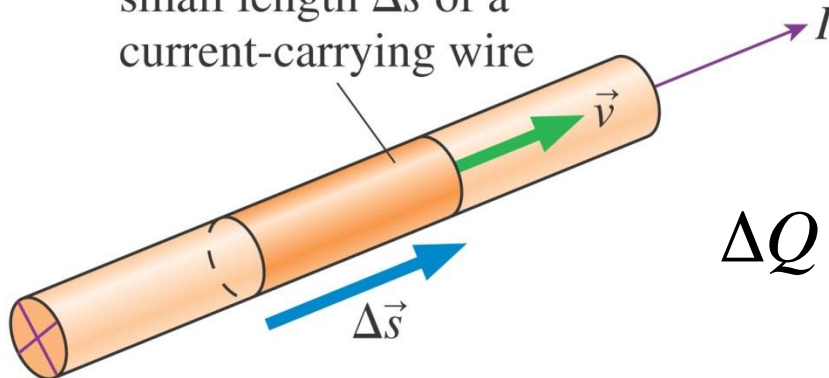
$$\frac{1}{\sqrt{\mu_0 \epsilon_0}} = 299,792,458 \text{ m/s}$$

Speed of light!



What if we have a whole bunch of moving charges (an electric current)?

(a) Charge ΔQ in a small length Δs of a current-carrying wire



$$\Delta Q \vec{v} = \Delta Q \frac{\Delta \vec{s}}{\Delta t} = \frac{\Delta Q}{\Delta t} \Delta \vec{s} = I \Delta \vec{s}$$

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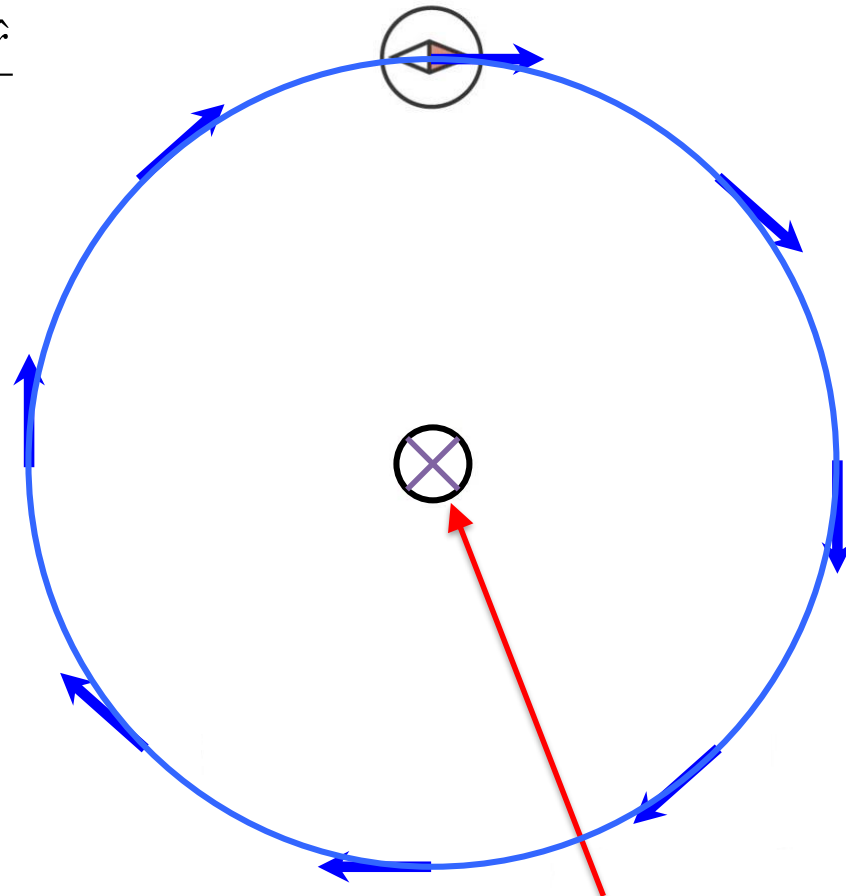
Then

$$\vec{B}_{\text{point charge}} = \frac{\mu_0}{4\pi} \frac{q \vec{v} \times \hat{r}}{r^2}$$

becomes

$$\vec{B}_{\text{current segment}} = \frac{\mu_0}{4\pi} \frac{I \Delta \vec{s} \times \hat{r}}{r^2}$$

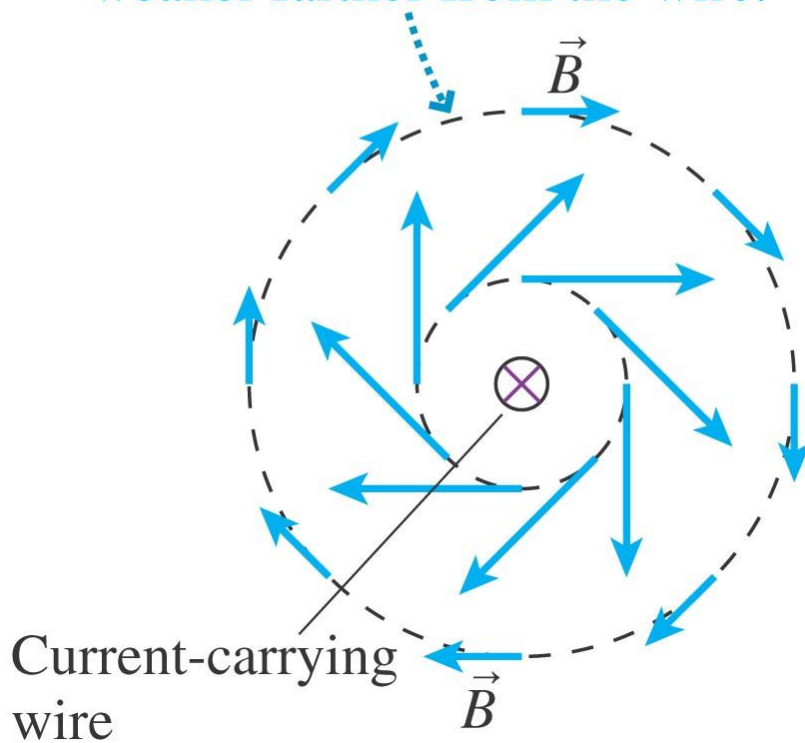
$$\vec{B}_{\text{current segment}} = \frac{\mu_0}{4\pi} \frac{I \Delta \vec{s} \times \hat{r}}{r^2}$$



Current going into page

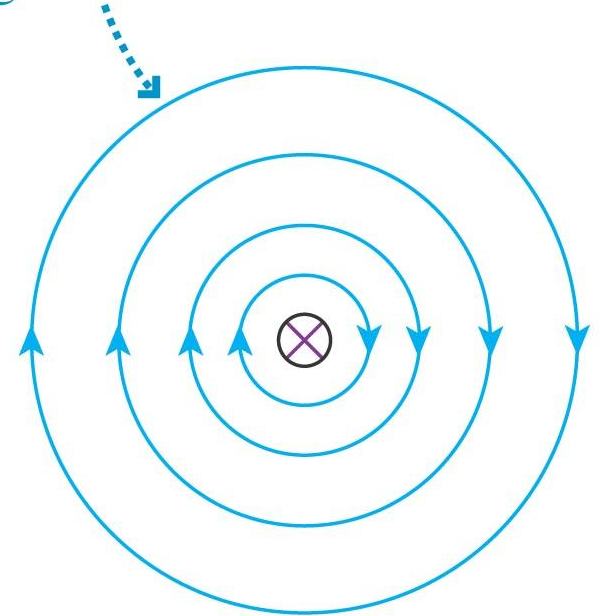
The magnetic field vector points in the direction of the north pole of the compass magnet.

- (a) The magnetic field vectors are tangent to circles around the wire, pointing in the direction given by the right-hand rule. The field is weaker farther from the wire.



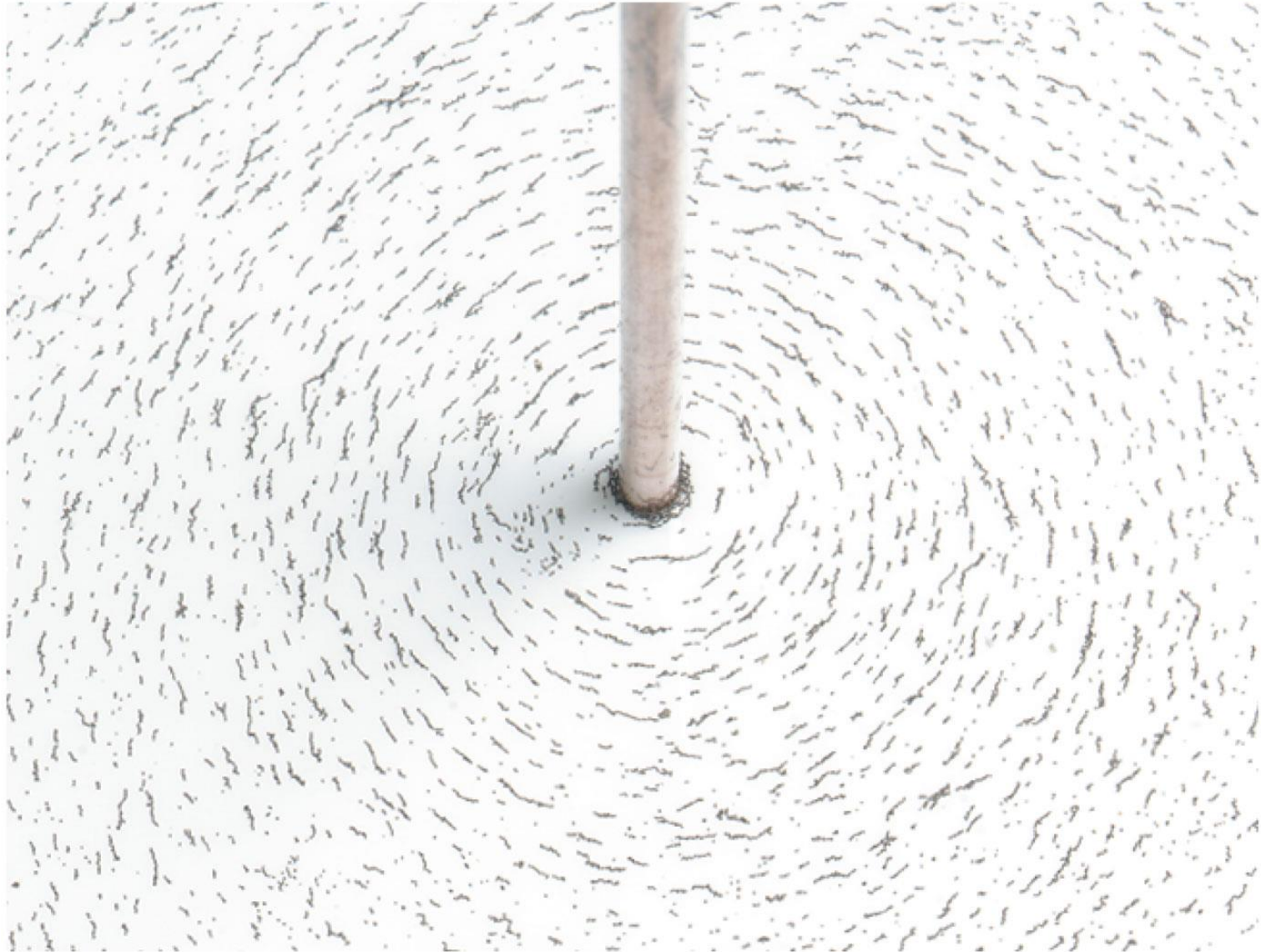
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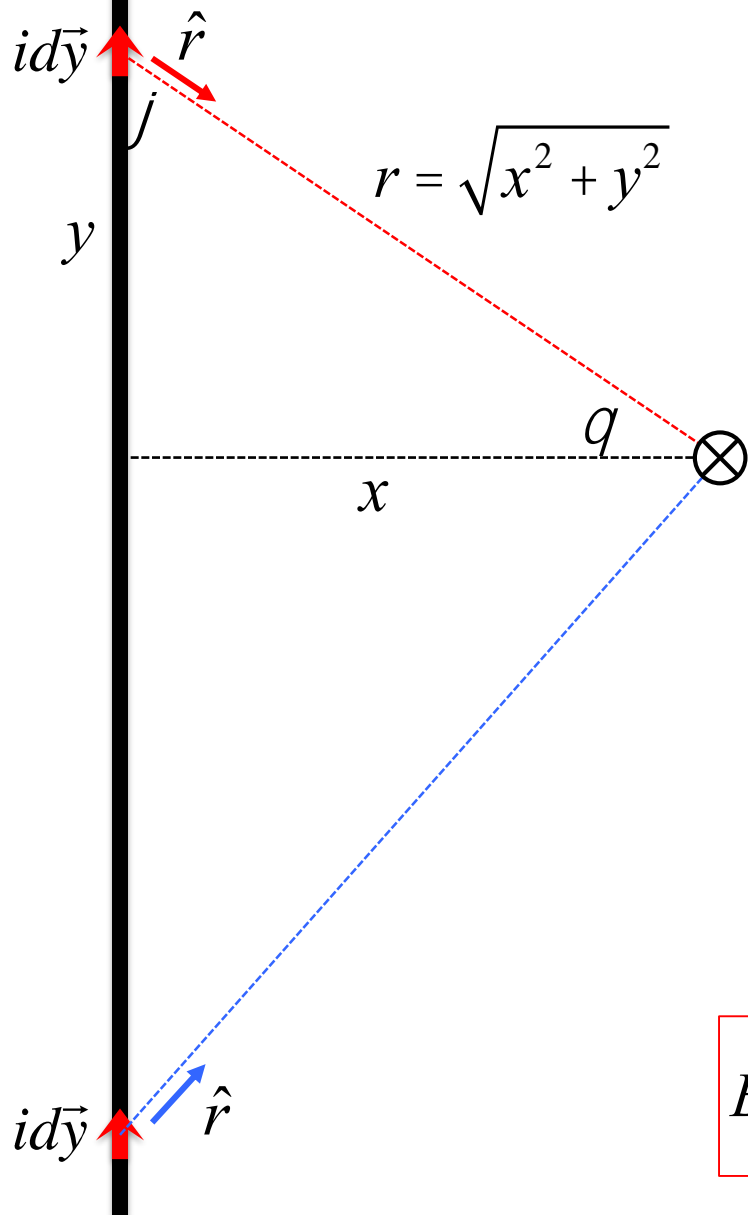
- (b) Magnetic field lines are circles.



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Magnetic Field of a Long, Straight Wire





$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{i d\vec{y} \times \hat{r}}{r^2}$$

$$i d\vec{y} \times \hat{r} = i dy \sin \varphi (-\hat{k}) = -i dy \frac{x}{\sqrt{x^2 + y^2}} \hat{k}$$

$$d\vec{B} = -\frac{\mu_0}{4\pi} \frac{i x dy}{(x^2 + y^2)^{3/2}} \hat{k}$$

All contributions are in the same direction

$$B = \int_{-\infty}^{\infty} \frac{\mu_0}{4\pi} \frac{i x dy}{(x^2 + y^2)^{3/2}}$$

Can just worry about the magnitude

$$= \frac{\mu_0 i x}{4\pi} \int_{-\infty}^{\infty} \frac{dy}{x^3 \left(1 + (y/x)^2\right)^{3/2}}$$

Pull out factors of x to make sub of y/x easier

$$= \frac{\mu_0 i}{4\pi x} \int_{-\infty}^{\infty} \frac{\sec^2 q dq}{(1 + \tan^2 q)^{3/2}}$$

Let $y/x = \tan \theta$, so that $dy/x = \sec^2 \theta d\theta$

$$= \frac{\mu_0 i}{4\pi x} \int_{-\infty}^{\infty} \cos q dq$$

Integrand reduces to $\sec^2 \theta / \sec^3 \theta = \cos \theta$

$$B_{\text{wire}} = \frac{\mu_0 i}{2\pi x}$$

Magnetic field strength of a long straight wire. Direction from RHR

Last time:

- Torque on a current loop
- Biot-Savart Law (like Coulomb's Law for magnetism)
- B-field of a line of current

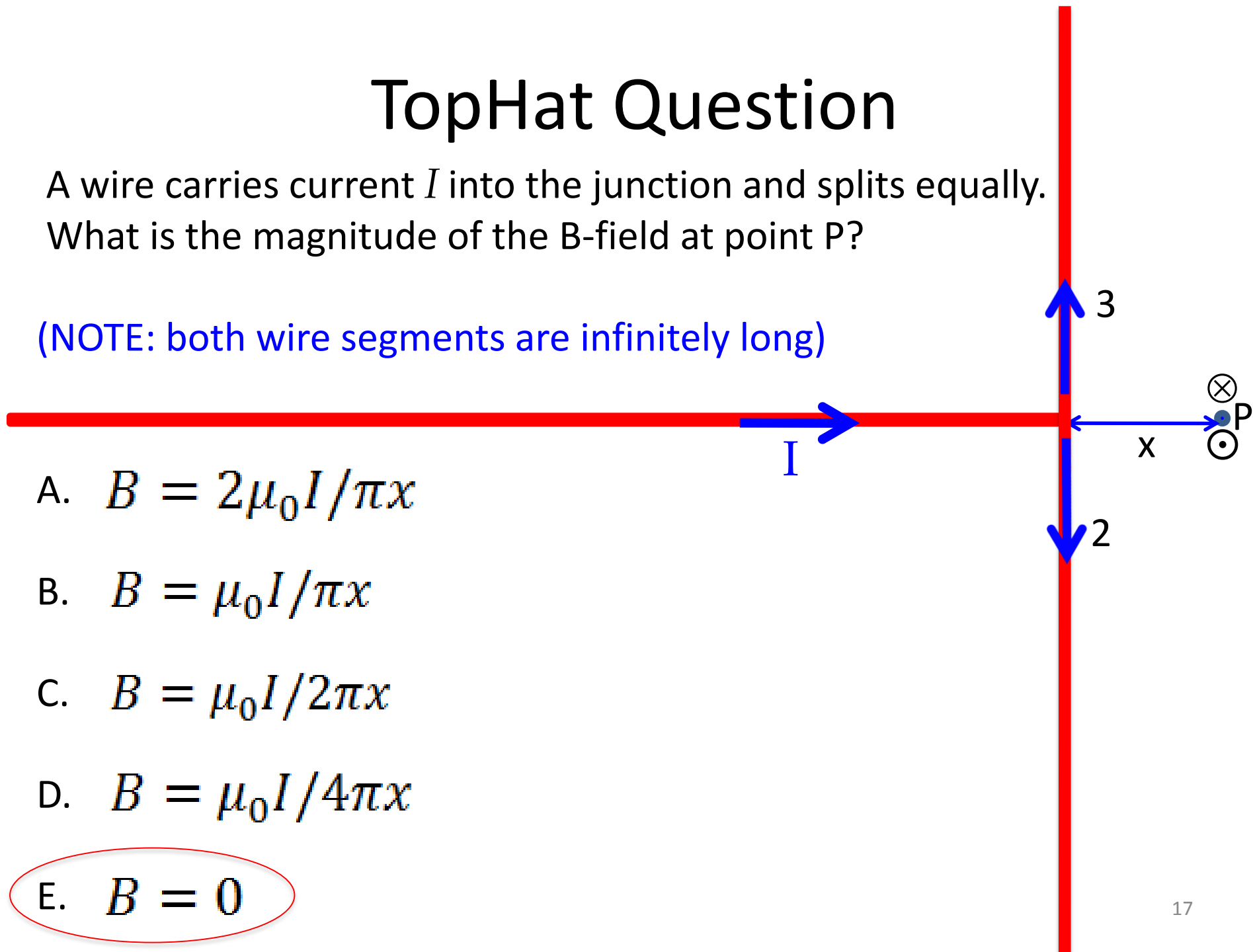
Today:

- Magnetic force between parallel current-carrying wires
- Applying the Biot-Savart Law:
 - Circular arc of current
 - Finite line of current

TopHat Question

A wire carries current I into the junction and splits equally. What is the magnitude of the B-field at point P?

(NOTE: both wire segments are infinitely long)



A. $B = 2\mu_0 I / \pi x$

B. $B = \mu_0 I / \pi x$

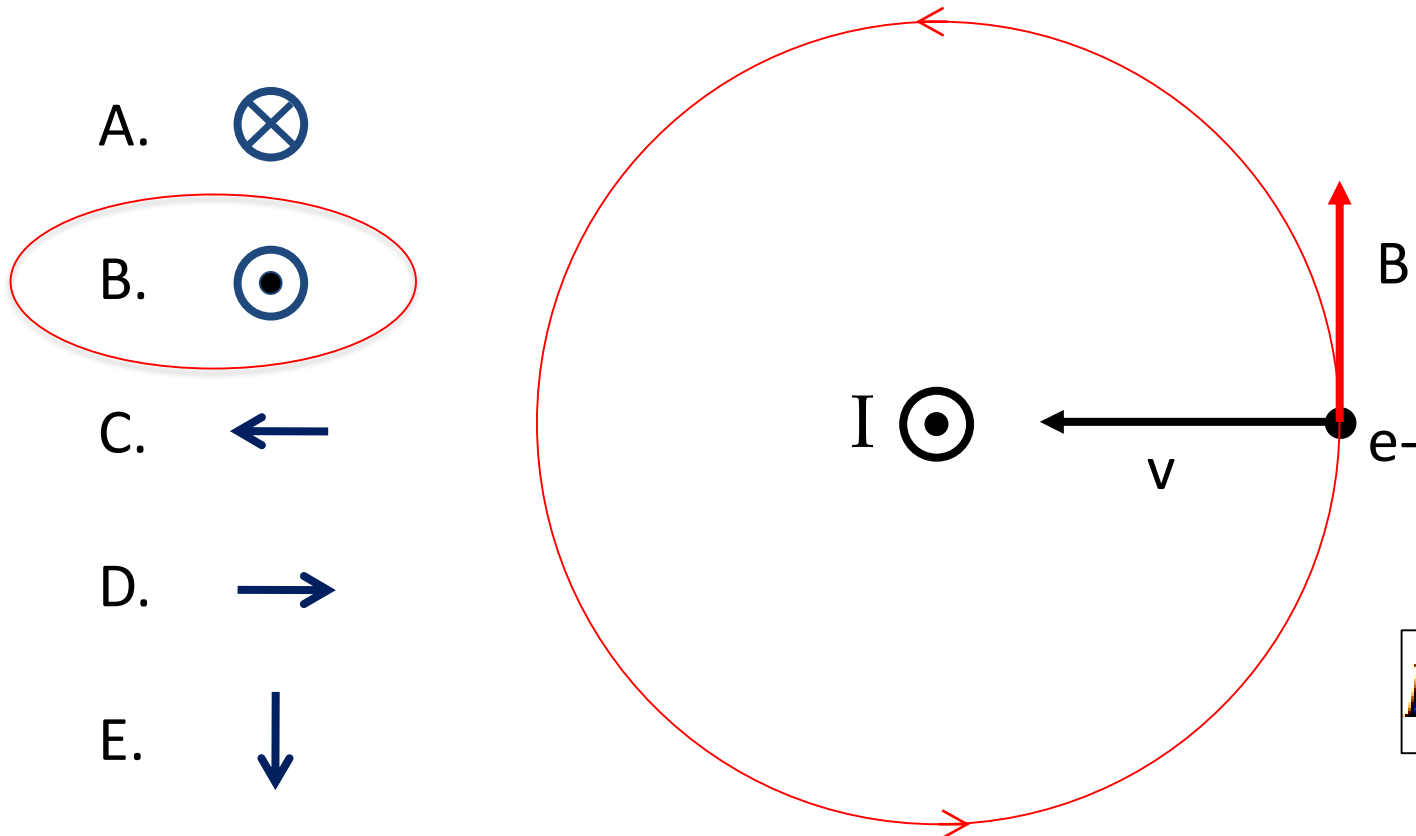
C. $B = \mu_0 I / 2\pi x$

D. $B = \mu_0 I / 4\pi x$

E. $B = 0$

TopHat Question

A long straight wire carries current I out of the page. An electron travels towards the wire from the right. What is the direction of the force on the electron?



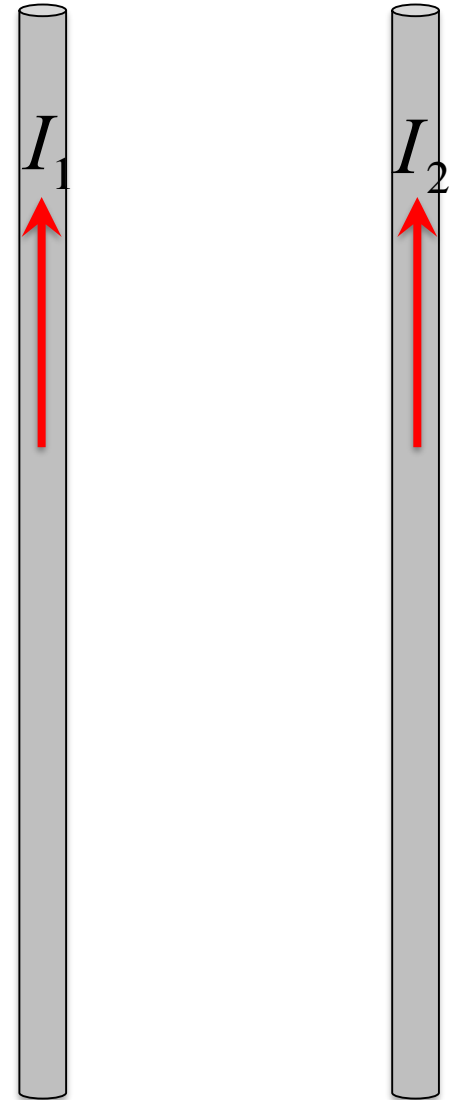
$$\vec{F} = q\vec{v} \times \vec{B}$$

TopHat Question

Two wires carry currents I_1 and I_2 as shown. What direction is the magnetic field produced by wire 2 at the location of wire 1?

- A. Downward
- B. Upward
- C. Into the page

D. Out of the page



TopHat Question

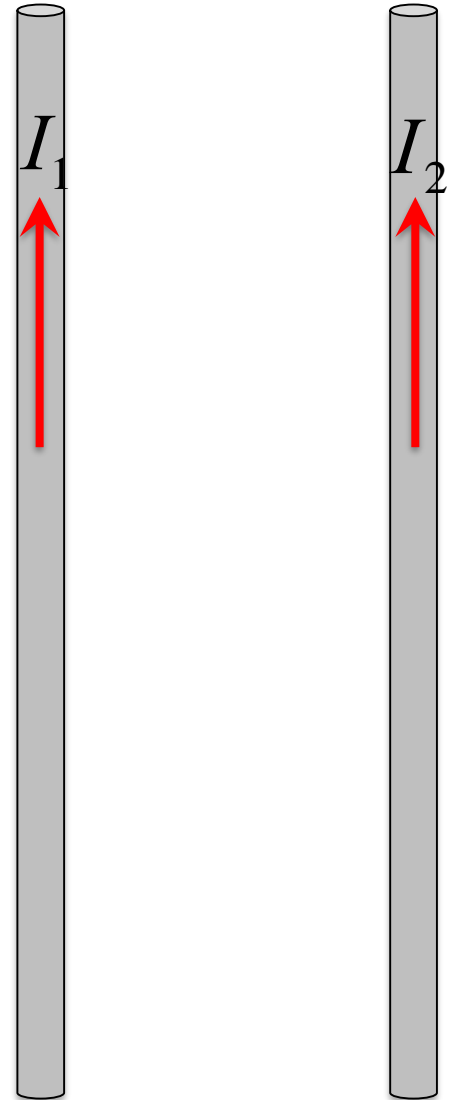
Two wires carry currents I_1 and I_2 as shown. What direction is the force of wire 2 on wire 1?

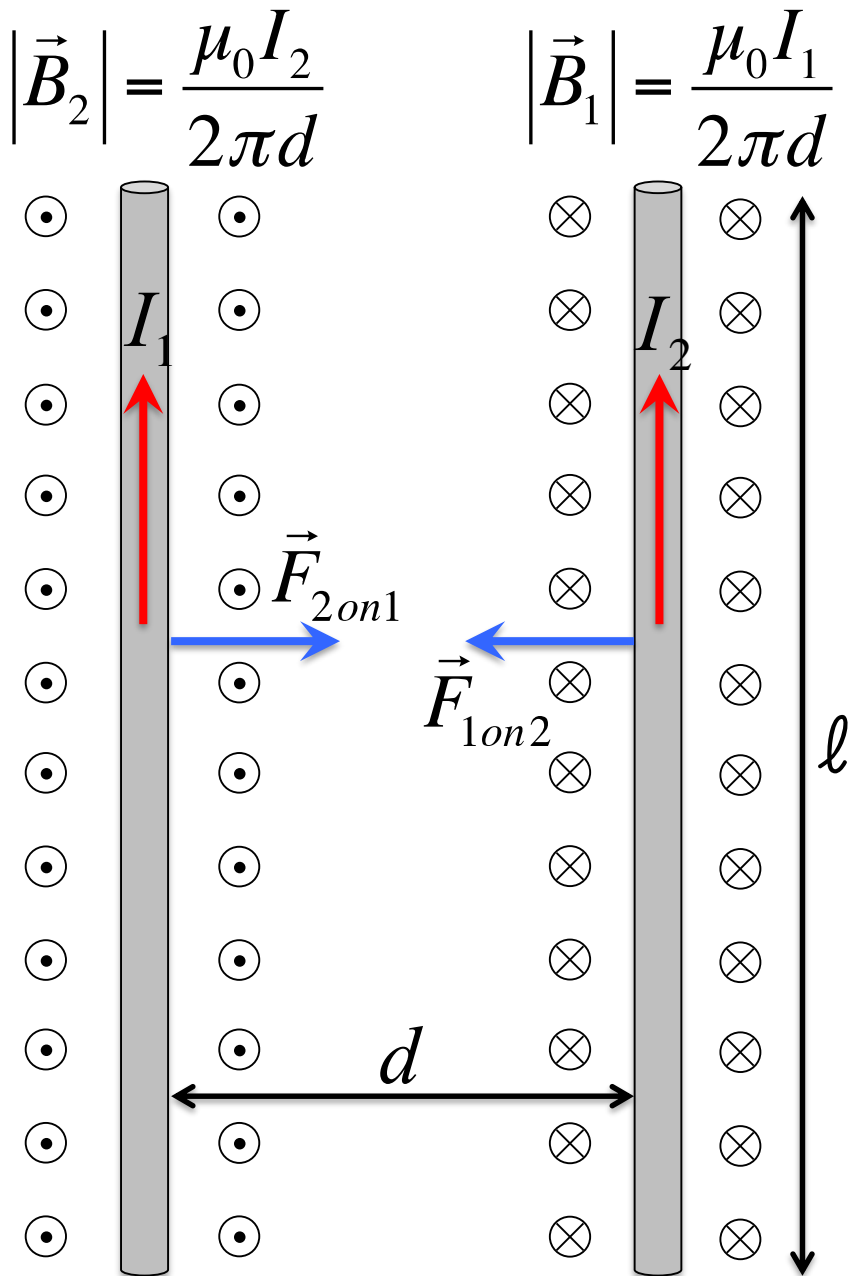
A. Left

B. Right

C. Up

D. Down





Wire 2 exerts a force on wire 1

$$\vec{F}_{2on1} = I_1 \vec{\ell} \times \vec{B}_2$$

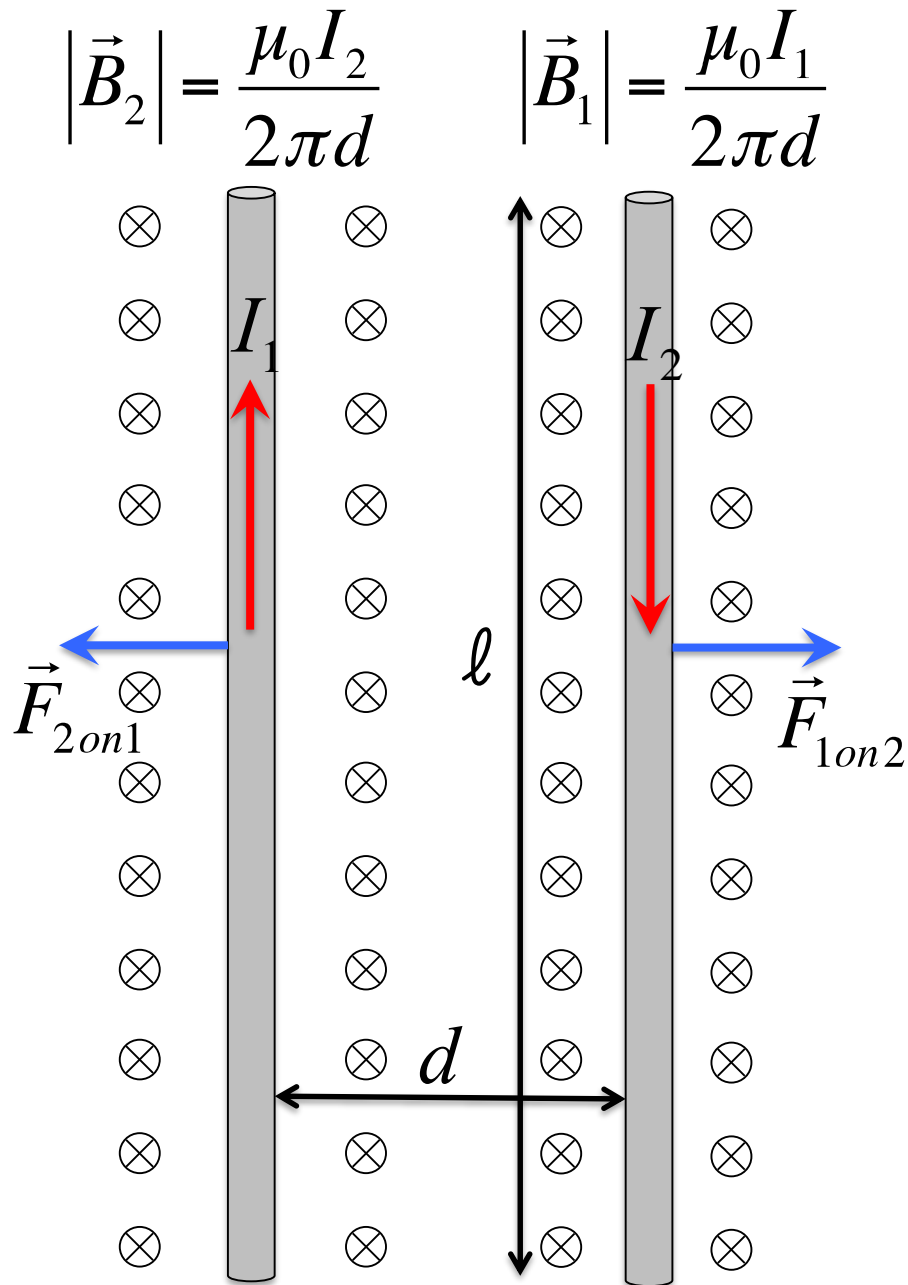
$$|\vec{F}_{2on1}| = I_1 \ell \frac{\mu_0 I_2}{2\pi d} = \boxed{\frac{\mu_0 \ell I_1 I_2}{2\pi d}}$$

Wire 1 exerts a force on wire 2

$$\vec{F}_{1on2} = I_2 \vec{\ell} \times \vec{B}_1$$

$$|\vec{F}_{1on2}| = I_2 \ell \frac{\mu_0 I_1}{2\pi d} = \boxed{\frac{\mu_0 \ell I_1 I_2}{2\pi d}}$$

Newton's third law!



Wire 2 exerts a force on wire 1

$$\vec{F}_{2on1} = I_1 \vec{\ell} \times \vec{B}_2$$

$$|\vec{F}_{2on1}| = I_1 \ell \frac{\mu_0 I_2}{2\pi d} = \boxed{\frac{\mu_0 \ell I_1 I_2}{2\pi d}}$$

Wire 1 exerts a force on wire 2

$$\vec{F}_{1on2} = I_2 \vec{\ell} \times \vec{B}_1$$

$$|\vec{F}_{1on2}| = I_2 \ell \frac{\mu_0 I_1}{2\pi d} = \boxed{\frac{\mu_0 \ell I_1 I_2}{2\pi d}}$$

Newton's third law!

Document Camera Calculations

My notes called:

“[Mar_Chapter29_Appendix3_FiniteLineMagneticField.PDF](#)”

&

“[Mar_Chapter29_Appendix4_ArcMagneticField.PDF](#)”

On D2L

Last time:

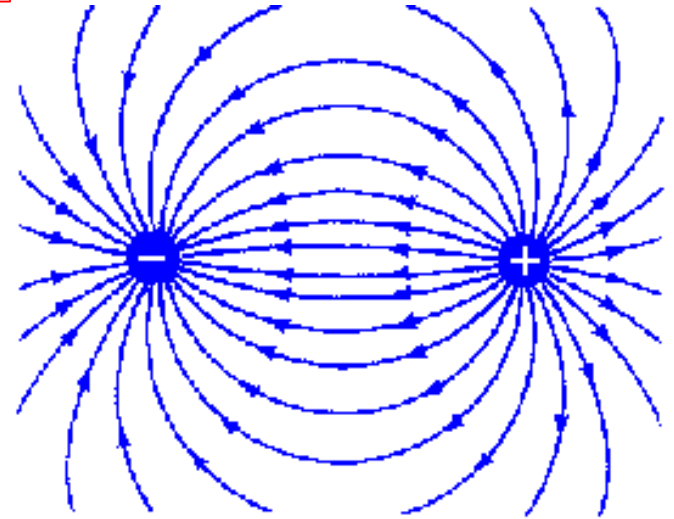
- Magnetic force between parallel current-carrying wires
- Applying the Biot-Savart Law:
 - Circular arc of current
 - Finite line of current (I totally botched this...)

Today:

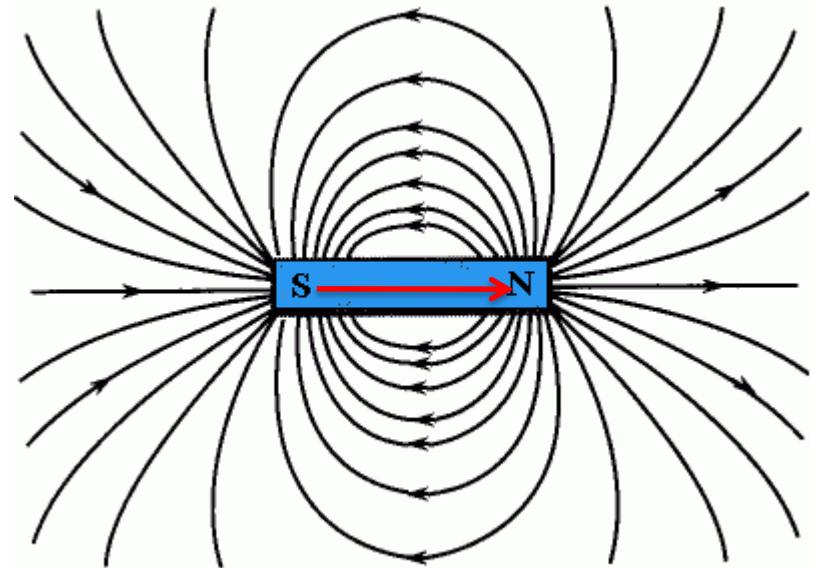
- Finite line of current (I've totally got this now...)
- Ampère's Law: Like Gauss' Law, but named after Ampère
- Magnetic field of a long wire (inside and outside)
- Magnetic field of a coaxial cable

Dipole Fields

Electric field from an electric dipole



Magnetic field from a magnetic dipole. **Note** that the magnetic field lines are **continuous** – they do **NOT** stop at the poles!



Both fields have the same shape!

Not a Top Hat Question

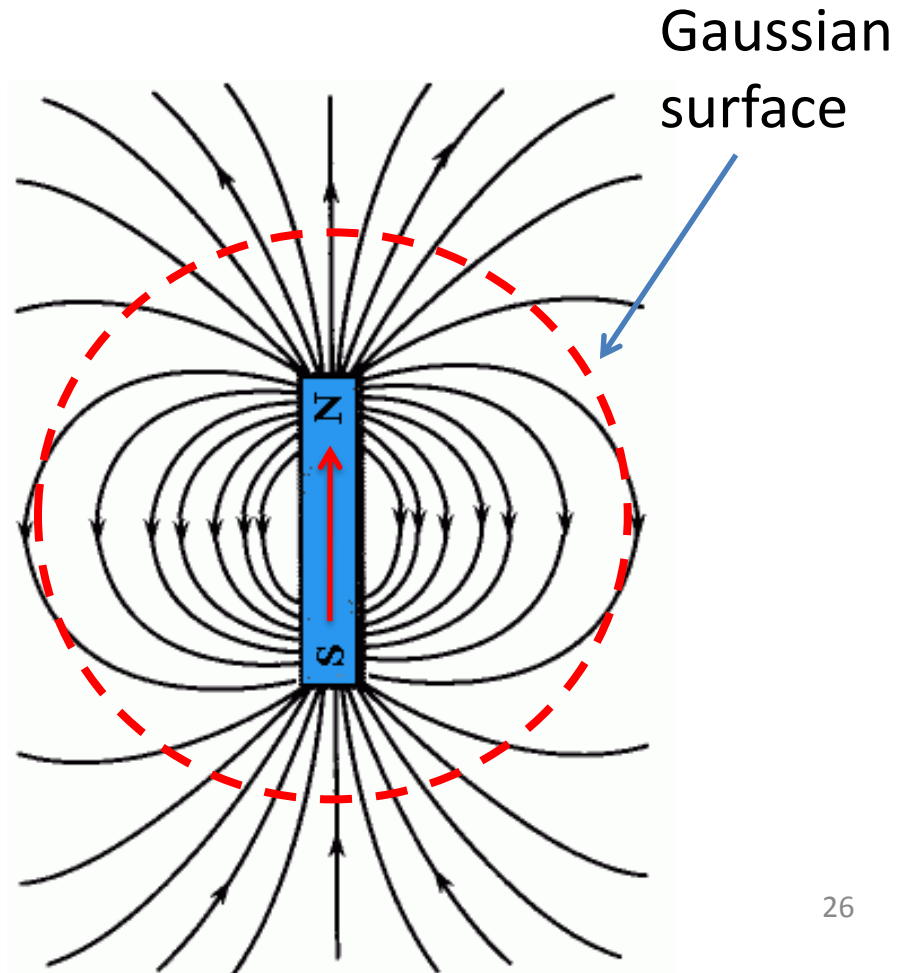
The magnetic field lines from a magnet point out of the North pole and point into the South pole.

What can you say about the magnetic flux passing through this Gaussian surface?

$$\Phi_B = \oint \vec{B} \cdot d\vec{a}$$

- A. Magnetic flux is zero
- B. Magnetic flux is greater than zero
- C. Magnetic flux is smaller than zero
- D. Can't tell without computing the integral

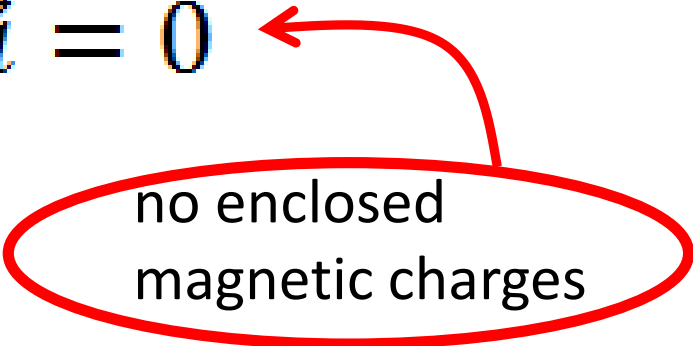
By symmetry the same number of flux lines enter and leave the spherical Gaussian surface



Gauss' Law for Magnetism

The magnetic flux through a closed surface is ALWAYS zero.

$$\Phi_B = \oint \vec{B} \cdot d\vec{a} = 0$$



no enclosed
magnetic charges

There is no way to isolate a North or South magnetic pole

The simplest **E-field** is from a **point charge**, while the simplest **B-field** is from a **magnetic dipole** (e.g. Bar Magnet)

Maxwell's equations

Essentially all of Electricity & Magnetism can be described by a set of 4 equations, referred to as Maxwell's equations.

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

We now have **two** of them!

$$\Phi_E = \oint \vec{E} \cdot d\vec{a} = \frac{Q_{encl}}{\epsilon_0}$$

$$\Phi_B = \oint \vec{B} \cdot d\vec{a} = 0$$

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 i_{encl} + \frac{1}{c^2} \frac{d\Phi_E}{dt}$$

We will learn about these other two Maxwell equations today and next week

Maxwell's equations

And God Said

$$\nabla \cdot \vec{D} = \rho_{\text{free}}$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

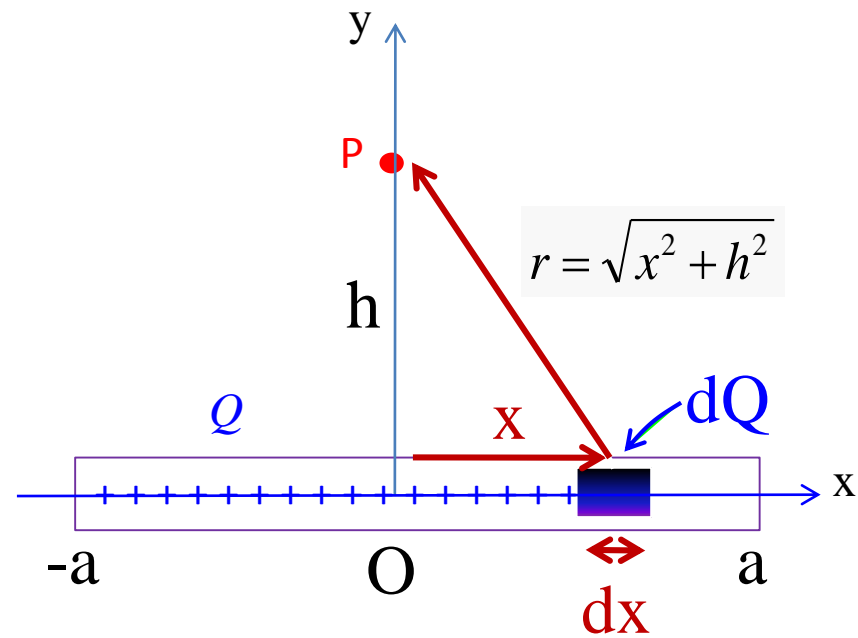
$$\nabla \times \vec{H} = \vec{J}_{\text{free}} + \frac{\partial \vec{D}}{\partial t}$$

and *then* there was
light.

Remember this activity?
 Solving for E_p for an infinitely
 long line of charge (i.e. $a \gg h$)
 using Coulomb's Law was harder
 than using

GAUSS'S LAW

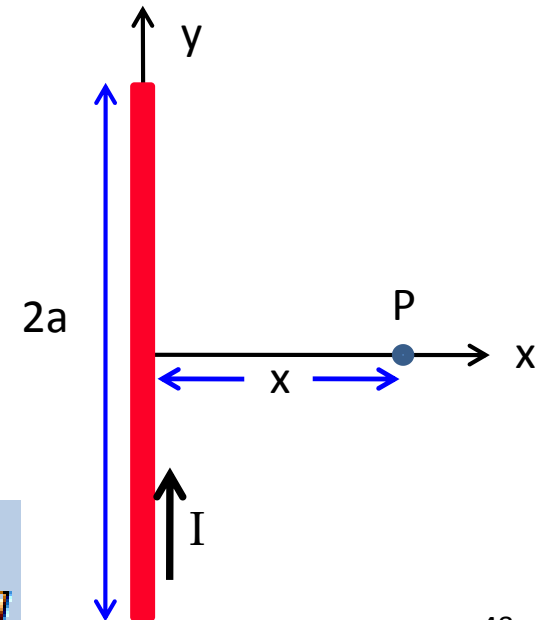
$$\oint \vec{E} \cdot d\vec{a} = \frac{Q_{encl}}{\epsilon_0}$$



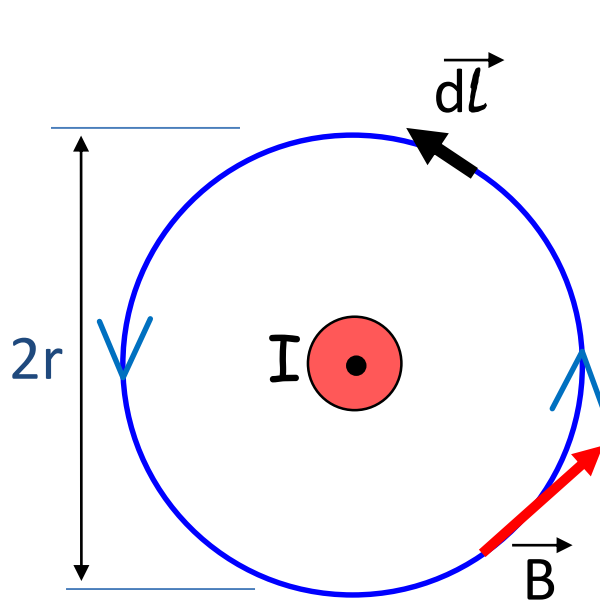
Solving for B_p for an infinitely
 long current carrying wire (i.e. $a \gg x$)
 using Biot-Savart's Law was also hard,
 but there is a MUCH easier alternative!

AMPERE'S LAW

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 i_{encl}$$



Ampère's Law



Suppose we calculate $\oint \vec{B} \cdot d\vec{l}$ around path shown for the simple case of an infinitely long straight line of current

Using our previous result we obtain:

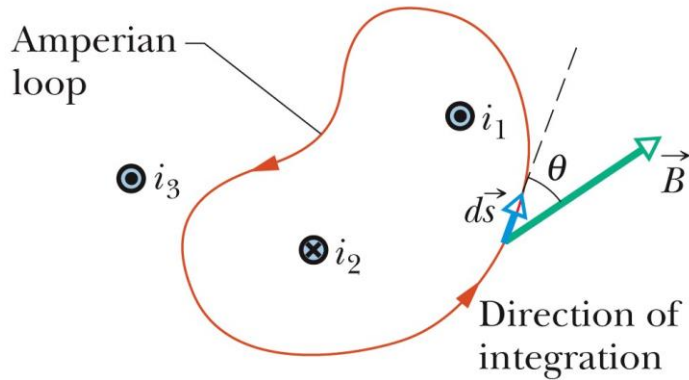
$$B = \frac{\mu_0 I}{2\pi r}$$

$$\oint \vec{B} \cdot d\vec{l} = (2\pi r) \left(\frac{\mu_0 I}{2\pi r} \right)$$

$$\text{i.e. } \oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$$

Ampère's Law is true for any shape of path and any current distribution

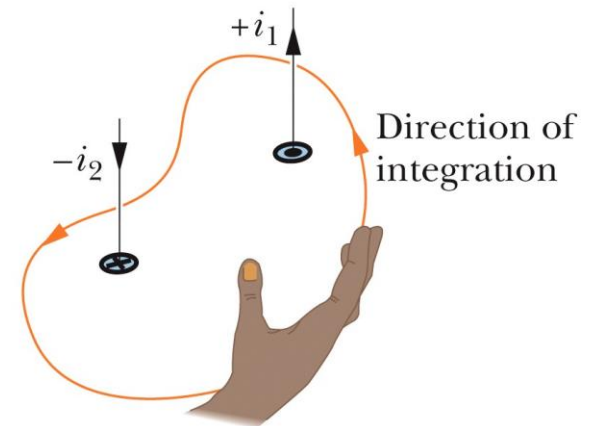
Only the currents encircled by the loop are used in Ampere's law.



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halliday_10e_fig_29_12

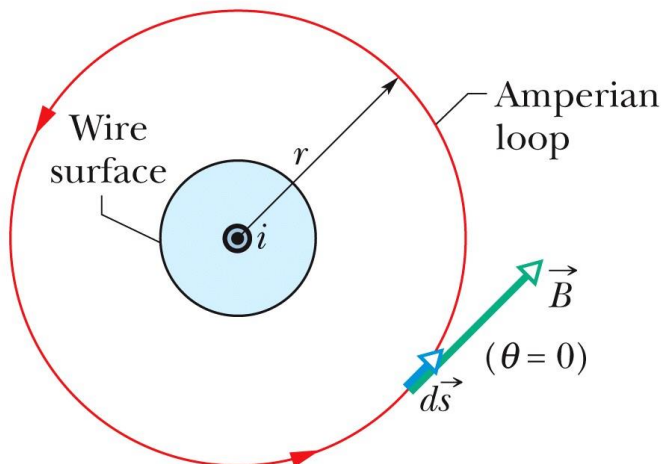
This is how to assign a sign to a current used in Ampere's law.



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halliday_10e_fig_29_13

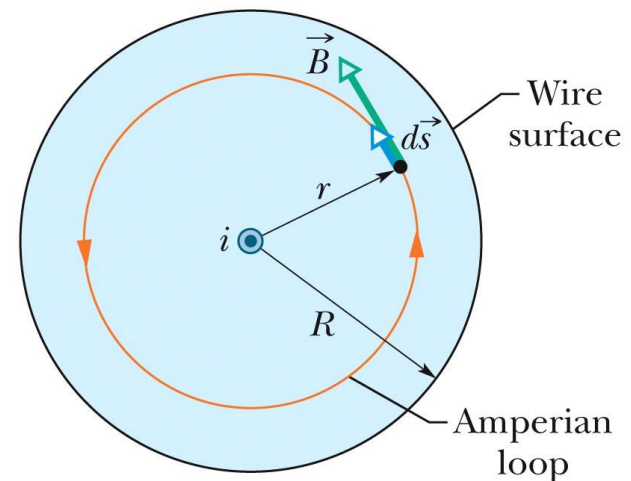
All of the current is encircled and thus all is used in Ampere's law.



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halliday_10e_fig_29_14

Only the current encircled by the loop is used in Ampere's law.



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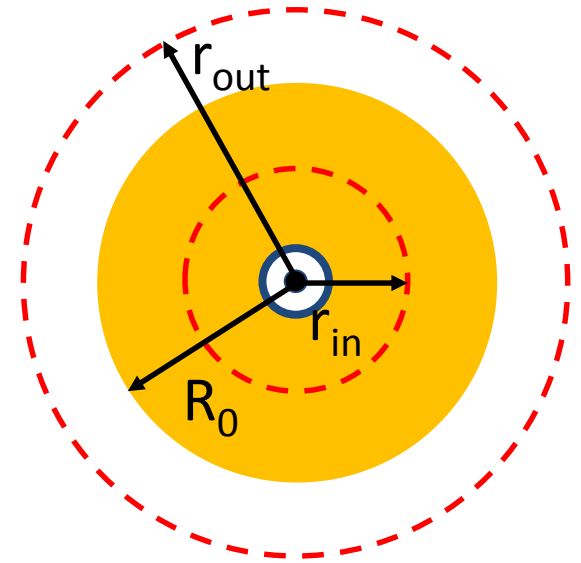
halliday_10e_fig_29_15

Ampère's law: application

- (a) Using Ampère's law, calculate the magnetic field **inside** a solid current carrying wire a distance r_{in} from its axis.

(The length of the solid wire is infinite and the current I is uniformly distributed throughout the solid wire)

- b) Calculate the magnetic field **outside** a solid current carrying wire a distance r_{out} from its axis.

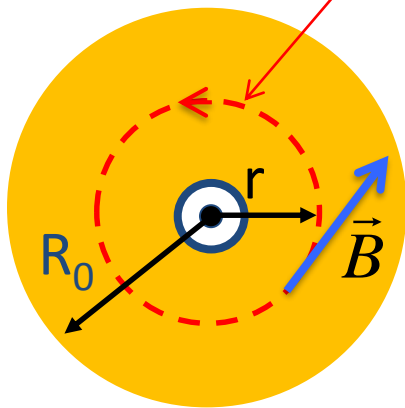


End view:
Wire with radius R
and current I

Ampère's law: application(1)

(a) B-field **inside**

We want to know the B-field a distance r , so we choose an Amperian circular loop with radius $r < R_0$.



Ampère's Law: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$

Left hand side: $\oint \vec{B} \cdot d\vec{l} = BL = B2\pi r$

Right hand side: $\mu_0 I_{enc} = \mu_0 JA = \mu_0 \frac{I}{\rho R_0^2} \rho r^2$

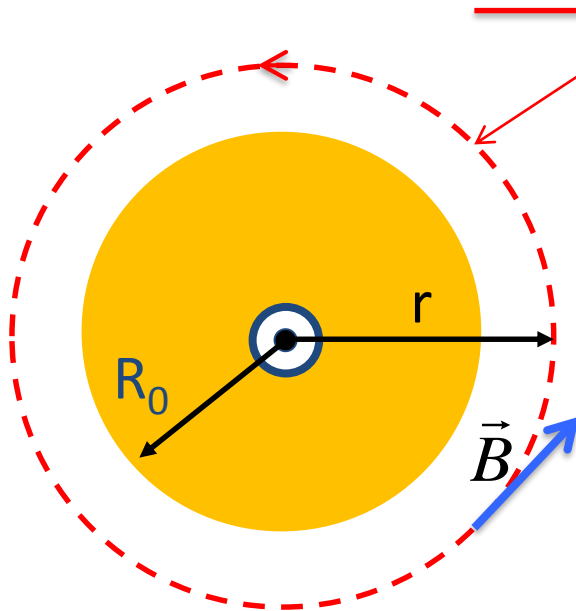
Combine together: $B2\pi r = \mu_0 \frac{I}{R_0^2} r^2$

$$B = \frac{\mu_0 I r}{2\rho R_0^2}$$

Ampère's law: application(1)

(a) B-field **outside**

We want to know the B-field a distance r , so we choose an Ampèrian loop with radius $r > R_0$.



Ampère's Law:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$$

Left hand side:

$$\oint \vec{B} \cdot d\vec{l} = BL = B2\pi r$$

Right hand side:

$$\mu_0 I_{enc} = \mu_0 I$$

Combine together:

$$B2\pi r = \mu_0 I$$

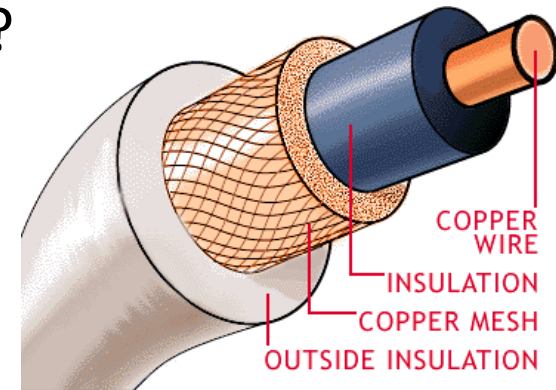
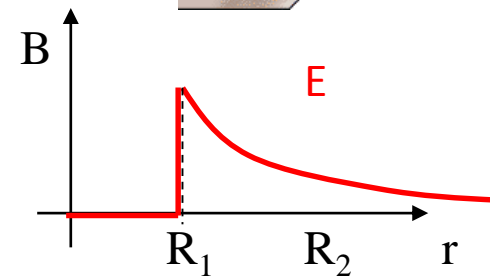
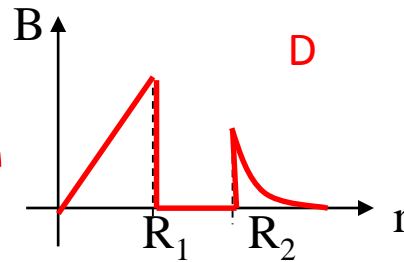
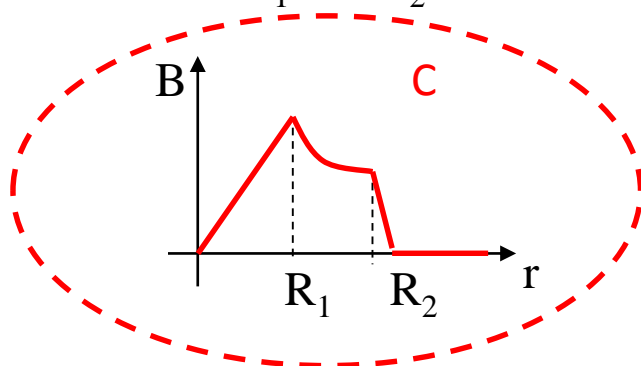
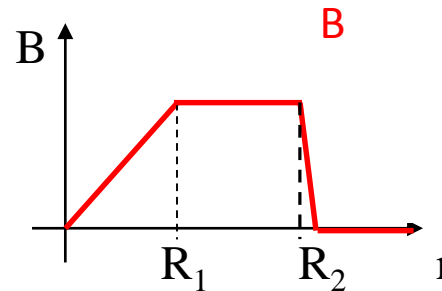
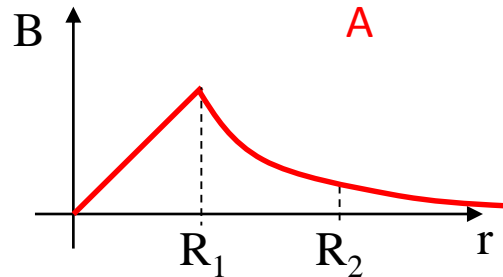
$$B = \frac{\mu_0 I}{2\pi r}$$

Top Hat (not really)



A coaxial cable consists of a wire (radius R_1) surrounded by an insulating sleeve and another cylindrical conducting shell (inner radius R_2) and finally another insulating sleeve. **The wire and the shell carry the same current I but in opposite directions.**

Which diagram best represents the **magnetic field** as a function of radial distance from the cable's axis?



Last time:

- Finite line of current
- Ampère's Law: Like Gauss' Law, but for magnetism
- Magnetic field of a long wire (inside and outside)
- Magnetic field of a coaxial cable

Today:

- More on Ampère's Law
 - Magnetic field of a sheet of current
 - Magnetic field of a solenoid
 - Magnetic field of a toroid
 - Superposition and Ampère's Law

- Look at my notes named:
“Mar_Chapter29_Appendix5_AmperLawApplica
tion.PDF”