

# Electricity and Magnetism

- Physics 259 – L02
  - Lecture 14



UNIVERSITY OF  
CALGARY

# Chapter 23

(please read chapter 22 of the textbook)



# Last time

- Chapter 23.1



# This time

- Chapter 23.2

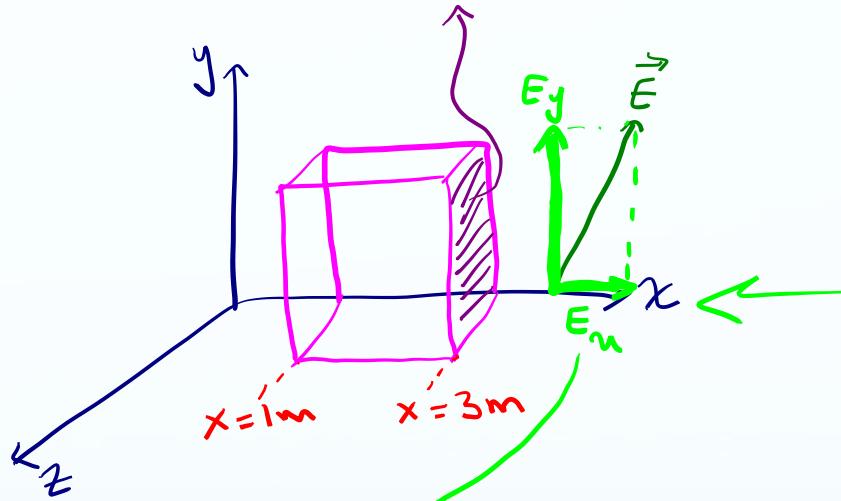


## Sample problem 23.02

$$\vec{E} = 2x \hat{i} + 3 \hat{j}$$

nonuniform  $\vec{E}$  field

find  $\phi_E$  through this face



$E_x$  depends  
on position  $x$

problem 23.02 of your textbook

$$\phi = \oint \vec{E} \cdot d\vec{A}$$

$$\vec{A} = A \hat{i} = 4 \hat{i}$$

$$\begin{aligned}
 \Phi_e &= \int (2x\hat{i} + 3\hat{j}) \cdot dA \hat{i} = \\
 &= \int (2x\hat{i}) \cdot dA \hat{i} + 3\hat{j} \cdot dA \hat{i} = \\
 &= \int \underbrace{2x dA}_{\text{O } 1511 \text{ i } | \text{CS } 20} \hat{i} \cdot \hat{i} + \underbrace{3 dA \hat{j}}_{\text{O } 1511 \text{ i } | \text{CS } 20} \cdot \hat{i} = 0 \\
 &= \int 2x dA = 2x \int dA = 2x A = 2 \times 3 \times 4 \\
 \Rightarrow \Phi_e &= \boxed{24 \frac{Nm^2}{C}}
 \end{aligned}$$

## 23-2: Gauss's Law



Gauss' law relates the net flux  $\phi$  of an electric field through a closed surface (a Gaussian surface) to the *net* charge  $q_{enc}$  that is enclosed by that surface.



$$\epsilon_0 \Phi = q_{enc} \quad (\text{Gauss' law}).$$

$$\Phi_e = \frac{q_{\text{inside}}}{\epsilon_0}$$

we can also write Gauss' law as



$$\epsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{enc} \quad (\text{Gauss' law}).$$

$$\Phi_e = \oint \vec{E} \cdot d\vec{A}$$

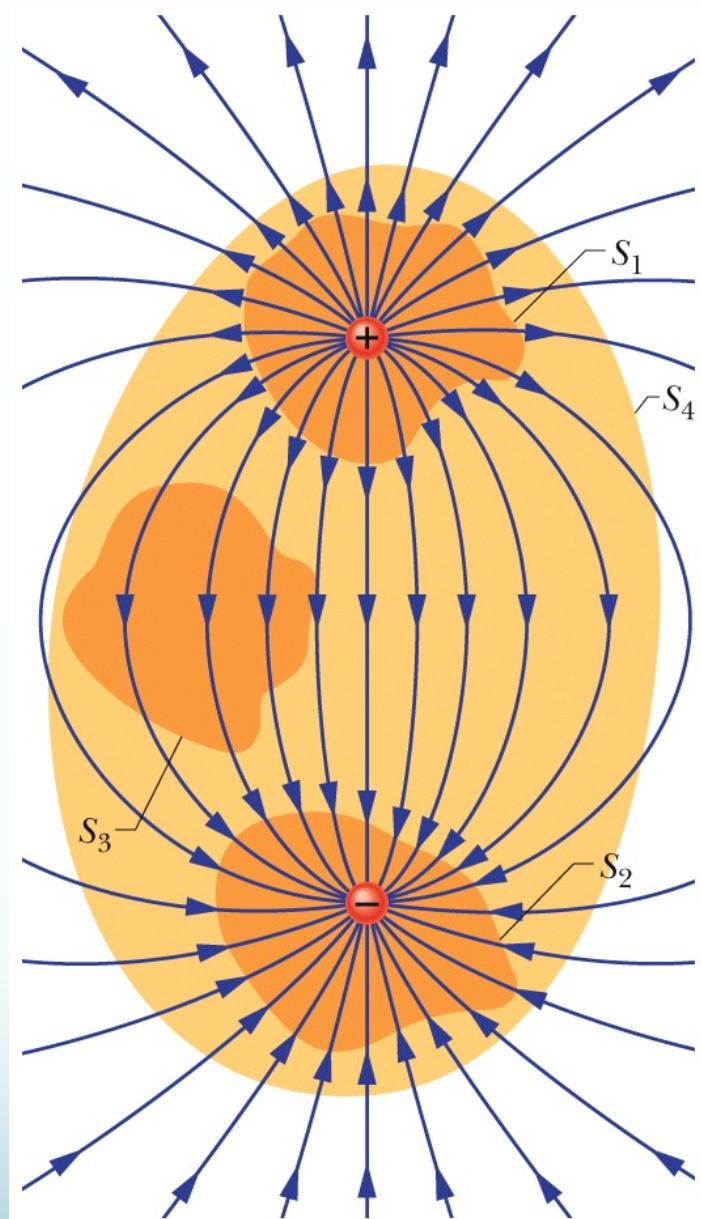
$$\Phi_e = \oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0}$$

**Surface S1.** The electric field is outward for all points on this surface. Thus, the flux of the electric field through this surface is positive, and so is the net charge within the surface, as Gauss' law requires

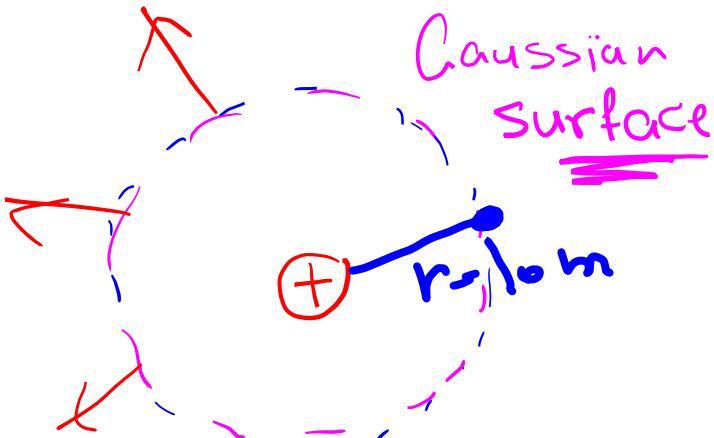
**Surface S2.** The electric field is inward for all points on this surface. Thus, the flux of the electric field through this surface is negative and so is the enclosed charge, as Gauss' law requires.

**Surface S3.** This surface encloses no charge, and thus  $q_{enc} = 0$ . Gauss' law requires that the net flux of the electric field through this surface be zero. That is reasonable because all the field lines pass entirely through the surface, entering it at the top and leaving at the bottom.

**Surface S4.** This surface encloses no net charge, because the enclosed positive and negative charges have equal magnitudes. Gauss' law requires that the net flux of the electric field through this surface be zero. That is reasonable because there are as many field lines leaving surface S4 as entering it.



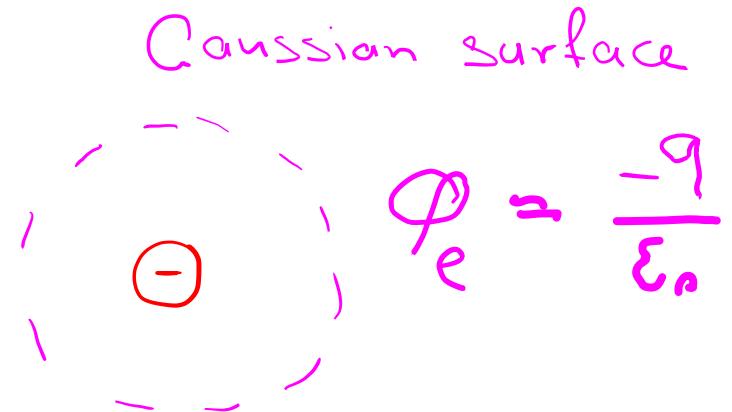
Copyright © 2014 John Wiley & Sons, Inc. All rights reserved.



$$\Phi_e = \oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0} = \frac{+q}{\epsilon_0}$$

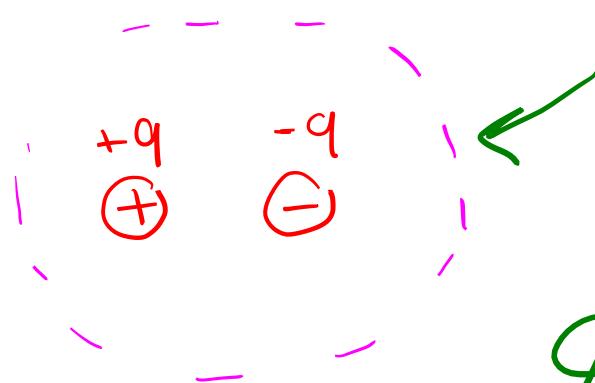
$$\Phi_e = \frac{+q}{\epsilon_0} > 0$$

$$\oint \vec{E} \cdot d\vec{A} = \frac{+q}{\epsilon_0}$$



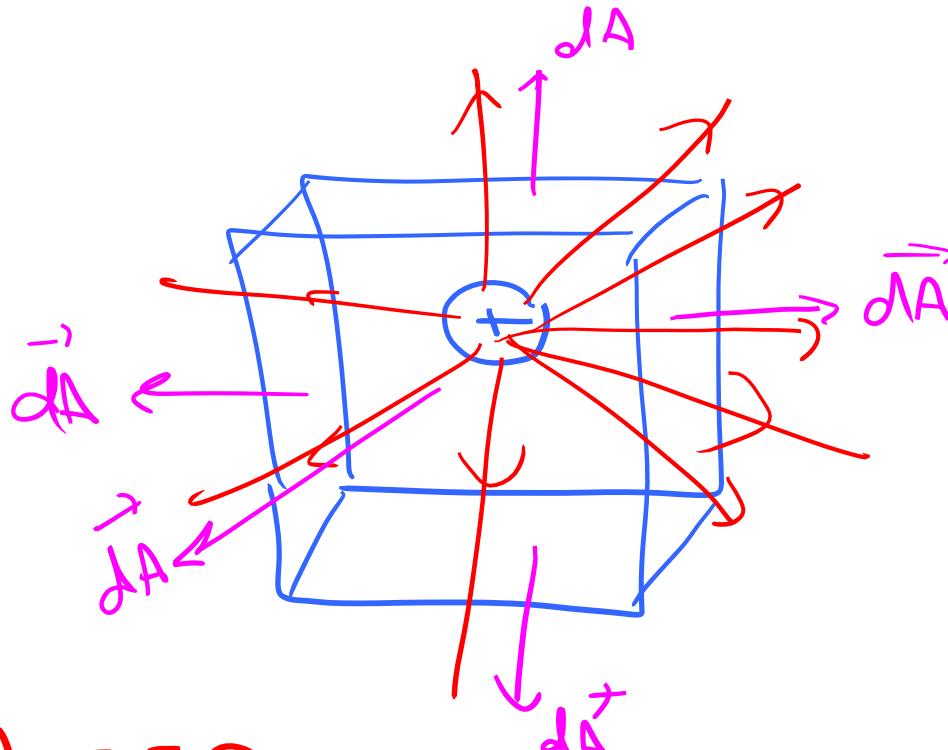
$$\Phi_e = \frac{-q}{\epsilon_0}$$

$$\epsilon_0 = 8.85 \times 10^{-12}$$

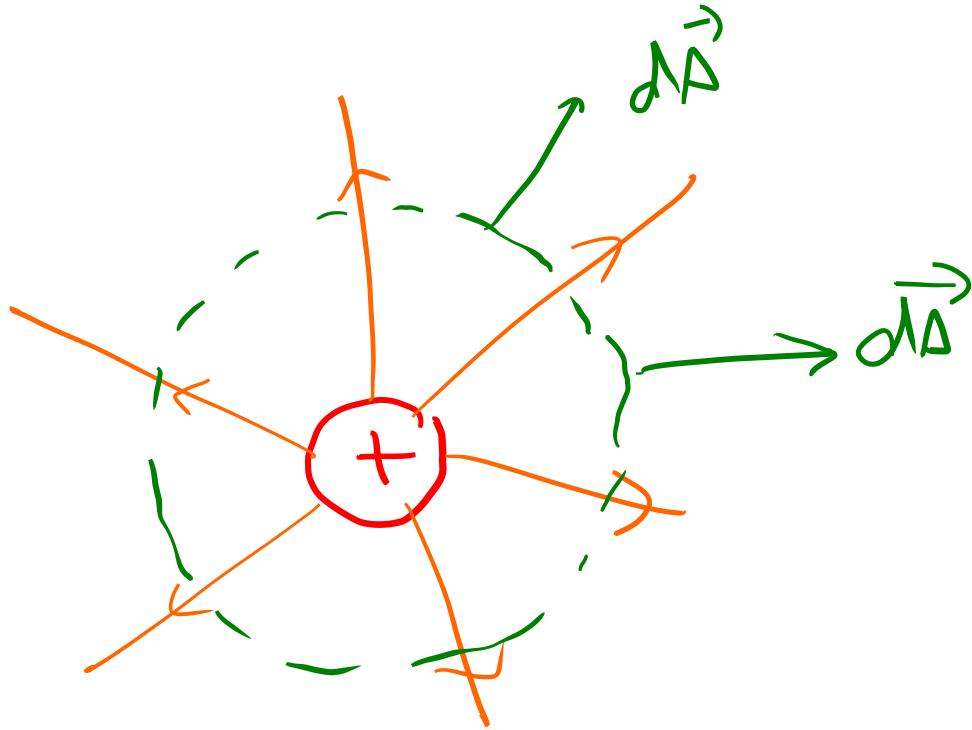


$$\Phi_e = \oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0}$$

$$\Phi_e = \oint \vec{E} \cdot d\vec{A} = 0$$



$\oint E \cdot dA$  cosaQ  $\rightarrow$  changing

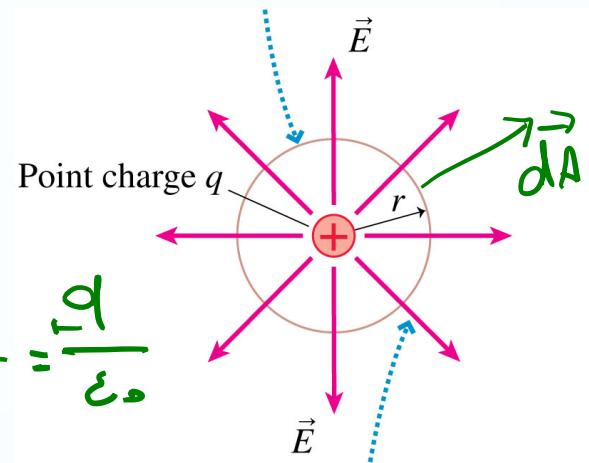


$$\oint \vec{E} \cdot d\vec{A} = \underbrace{\oint E dA \cos 0}_{1}$$

# Gauss's Law: Point Charge

Electric field of a point charge

$$\Phi_e = \oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\epsilon_0} \rightarrow \oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\epsilon_0} = \frac{q}{\epsilon_0}$$



$$\rightarrow E \oint dA \cos \theta = \frac{q}{\epsilon_0} \rightarrow E \oint dA = \frac{q_{\text{enc}}}{\epsilon_0} \rightarrow EA = \frac{q}{\epsilon_0}$$

$$\rightarrow E 4\pi r^2 = \frac{q}{\epsilon_0} \rightarrow E = \frac{q}{4\pi r^2 \epsilon_0}$$

$$\Phi_e = \oint \vec{E} \cdot d\vec{A} = \frac{q}{\epsilon_0}$$

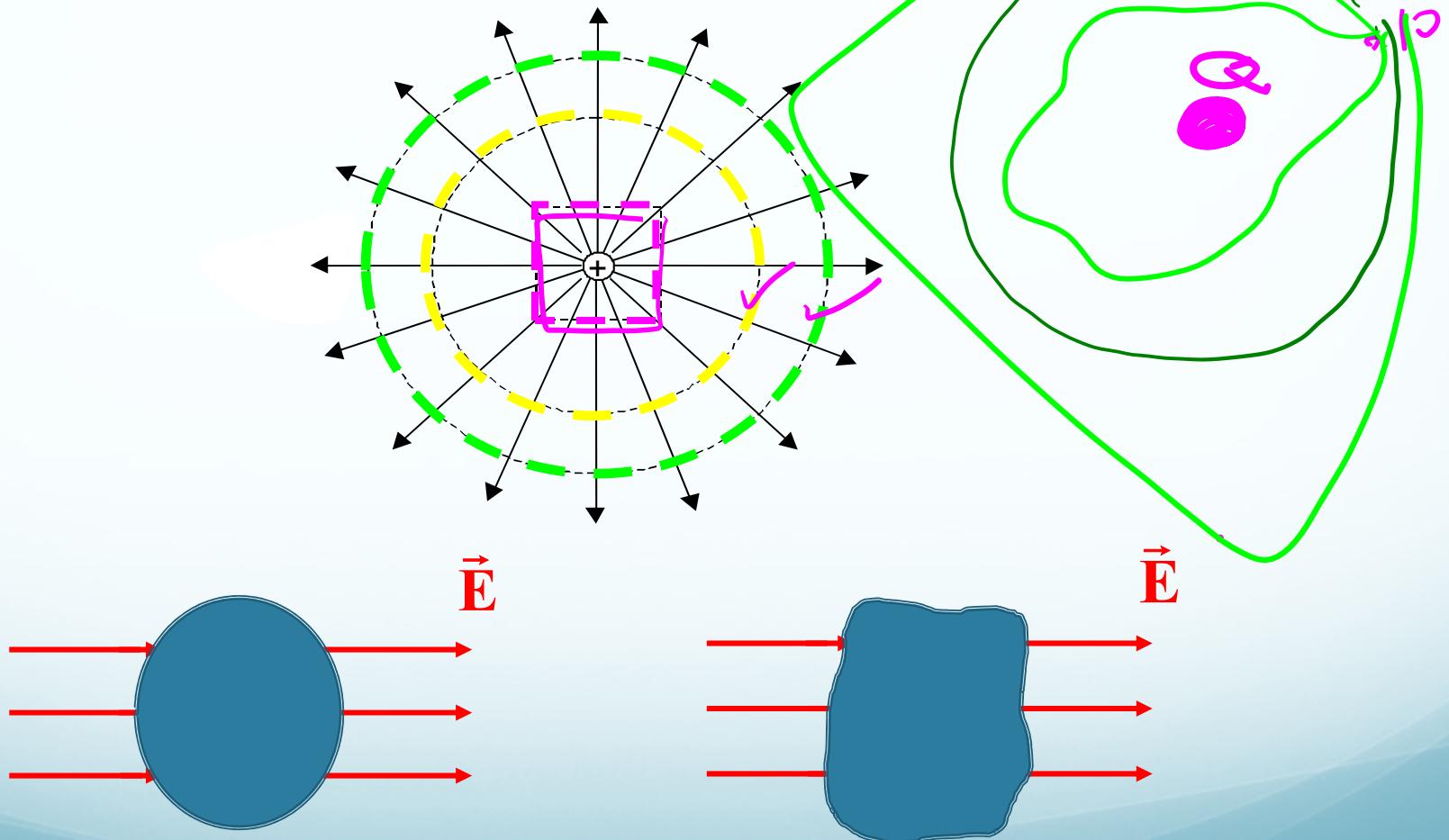
$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$

$$A_{\text{sph}} = 4\pi r^2$$

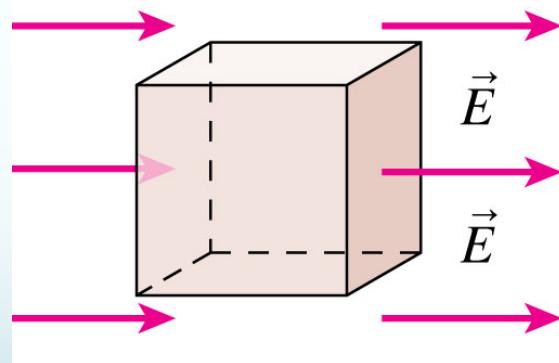
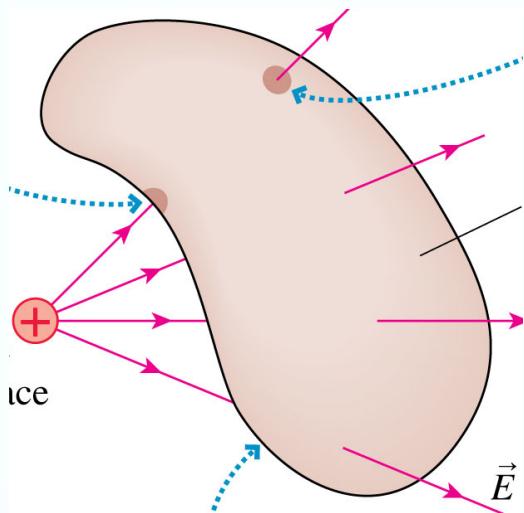
Gauss's law applies to closed surfaces

Electric flux is independent of surface shape and radius

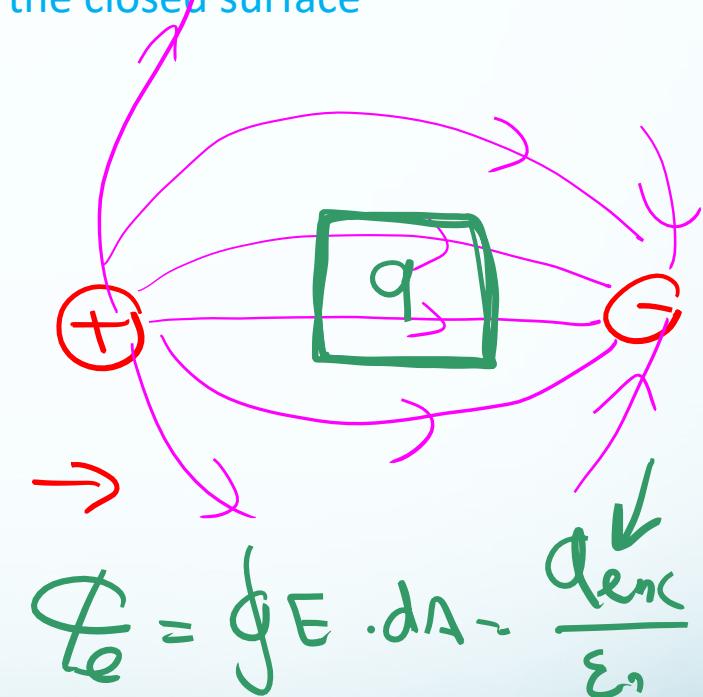
$$\Phi_e = \oint \vec{E} \cdot d\vec{A} = \frac{q}{\epsilon_0}$$



# Charge outside the surface



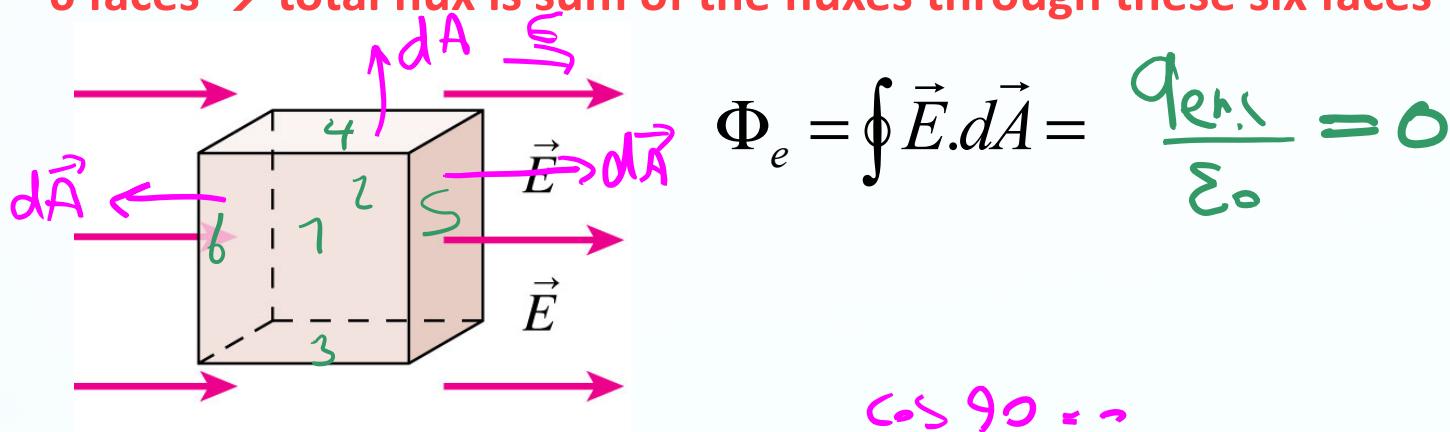
There is no net flow into or out of the closed surface



The net Flux through a Gaussian surface that does not contain any net charge is zero

# What is the total flux through the cube?

6 faces → total flux is sum of the fluxes through these six faces



1,2: the top and bottom faces:  $\theta = 90^\circ$

$$\theta = 90^\circ \Rightarrow \Phi_1 = \Phi_2 = 0$$

3,4: the front and back faces:  $\theta = 90^\circ$

$$\theta = 90^\circ \Rightarrow \Phi_3 = \Phi_4 = 0$$

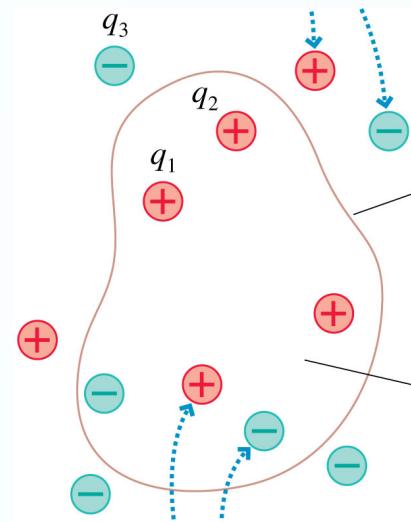
5: the left side face:  $\Phi_5 = \oint \vec{E} \cdot d\vec{A} \cos 90^\circ$   $\theta = 180^\circ \Rightarrow \Phi_5 = -EA$

6: the right side face:  $\Phi_6 = \oint \vec{E} \cdot d\vec{A} \cos 180^\circ$   $\theta = 0 \Rightarrow \Phi_6 = EA$

**total flux:**  $\Phi = \Phi_1 + \Phi_2 + \Phi_3 + \Phi_4 + \Phi_5 + \Phi_6 = 0 + 0 + 0 + 0 - EA + EA = 0$

## Multiple charges

$$\Phi_e = \oint \vec{E} \cdot d\vec{A} =$$



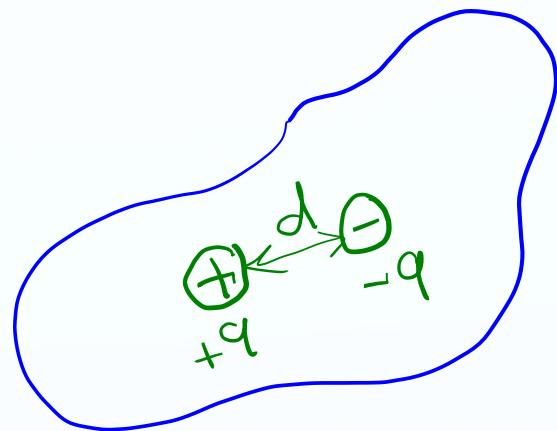
$$\Phi_e = \left( \frac{q_1}{\epsilon_0} + \frac{q_2}{\epsilon_0} + \dots \text{for all charges inside the surface} \right)$$

$$+ (0 + 0 + \dots \text{for all charges outside the surface})$$

$$\Phi_e = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\epsilon_0}$$

## Top Hat question 8

What is the electric flux of the surface?



A)  $\Phi_e = \frac{2q}{\epsilon_0}$

B)  $\Phi_e = \frac{q}{\epsilon_0}$

C)  $\Phi_e = 0$

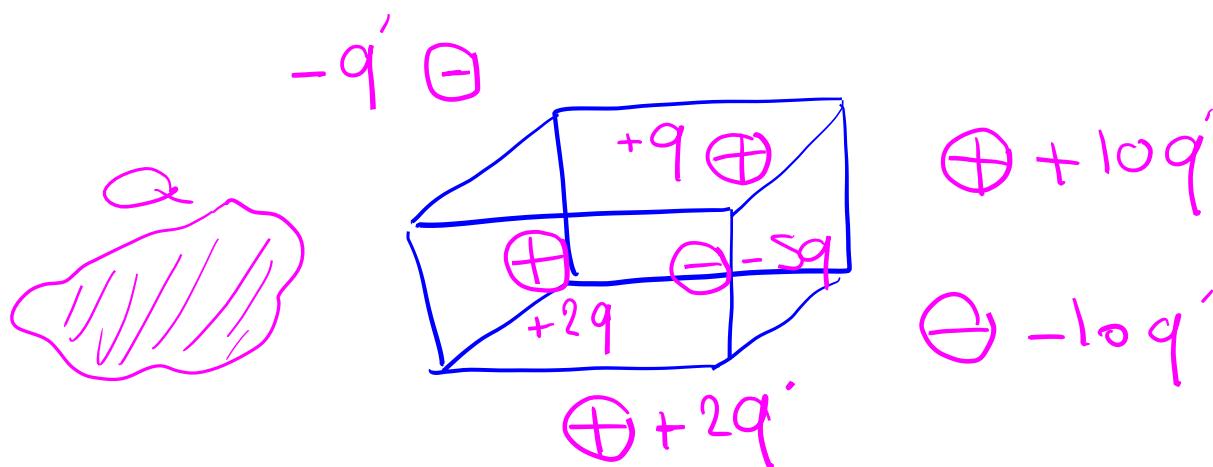
D)  $\Phi_e = -\frac{q}{\epsilon_0}$

$$\Phi = \frac{q_{\text{enc}}}{\epsilon_0}$$

# Top Hat Question



What is the electric flux at the surface?



A)  $\Phi_e = 0$

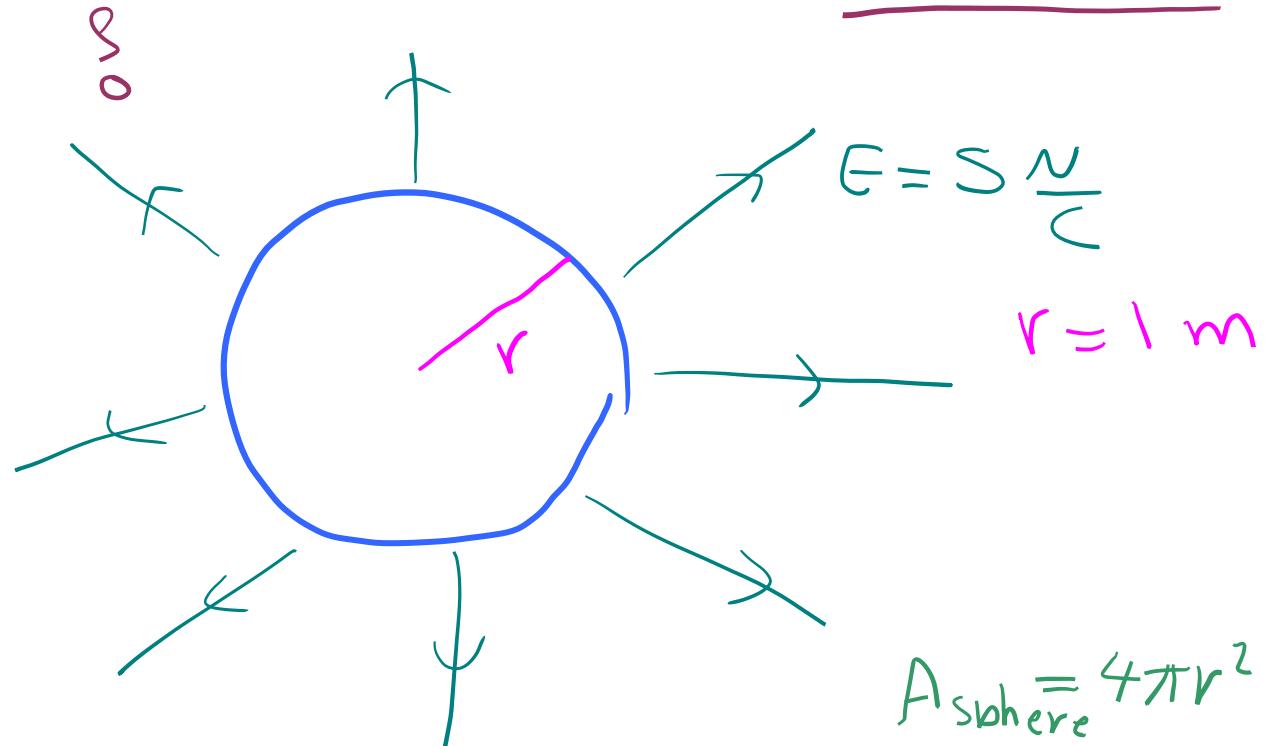
B)  $\Phi_e = \frac{q}{\epsilon_0}$

C)  $\Phi_e = \frac{-q}{\epsilon_0} + \frac{Q}{\epsilon_0} + \frac{q'}{\epsilon_0}$

D)  $\Phi_e = \frac{-2q}{\epsilon_0}$

# Top Hat Question

How much is the net charge INSIDE the sphere



- A) -0.25 nC
- B) -0.5 nC
- C) 0.5 nC
- D) 0.25 nC

$$A_{\text{sphere}} = 4\pi r^2$$

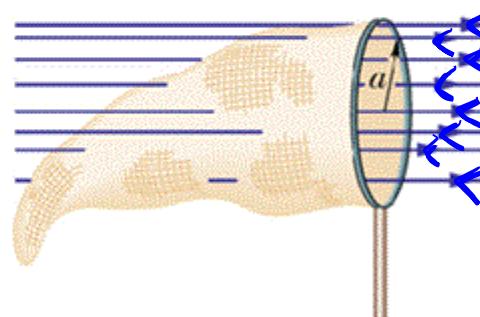
$$V_{\text{sphere}} = \frac{4}{3} \pi r^3$$

$$\epsilon_0 = 8.85 \times 10^{-12} \frac{\text{C}^2}{\text{Nm}^2}$$

Example: A butterfly net is in a uniform electric field of magnitude  $E = 3.0 \text{ mN/C}$ . The rim, a circle of radius  $a = 11 \text{ cm}$ , is aligned perpendicular to the field. The net contains no net charge. Find the electric flux through the net.

$$\Phi = \int \mathbf{E} \cdot d\mathbf{A} = E \cdot A$$

$$\begin{aligned}\Phi &= E \left( \overbrace{\pi a^2}^{\text{effective surface area}} \right) \\ &= (3.0 \times 10^{-3} \text{ N/C}) \pi (.11 \text{ m})^2 \\ &= 1.1404 \times 10^{-4} \text{ N/Cm}^2\end{aligned}$$





This section we talked about:

Chapter 23.2

*See you on Thursday*

