Last time

- Helical paths and Aurora
- Spiral paths and particle physics
- Magnetic bottle
- Velocity selector
- Mass spectrometers
- High vacuum leak detector
- Force on a current carrying conductor

This time

- Force on a current carrying conductor
- Force on a current loop
- Torque on a current loop
- Magnetic moment
- Symmetry between electric and magnetic field
- DC motor

Forces on Current-Carrying Wires

Current in wires is nothing ordered flow of charges. It doesn't matter if we consider $-q$ moving opposite i or $+q$ moving in the same direction as i.

In a magnetic field, these charges feel a force and get deflected from their normal straight path. For a single charge:

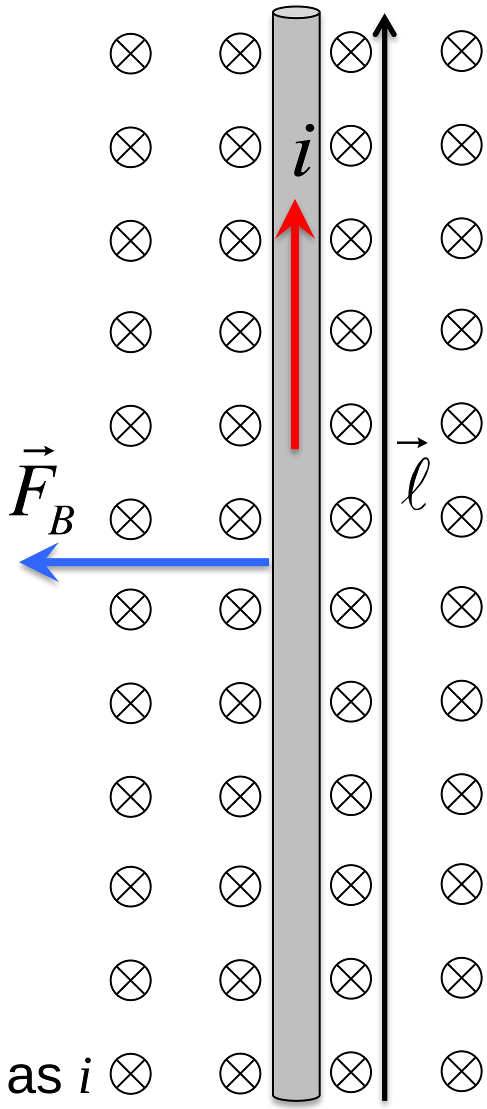
$$\vec{F}_B = q\vec{v}_d \times \vec{B}$$

For N charges moving through the wire:

$$Nq\vec{v}_d = (nAq\vec{v}_d)\ell = i\vec{\ell}$$

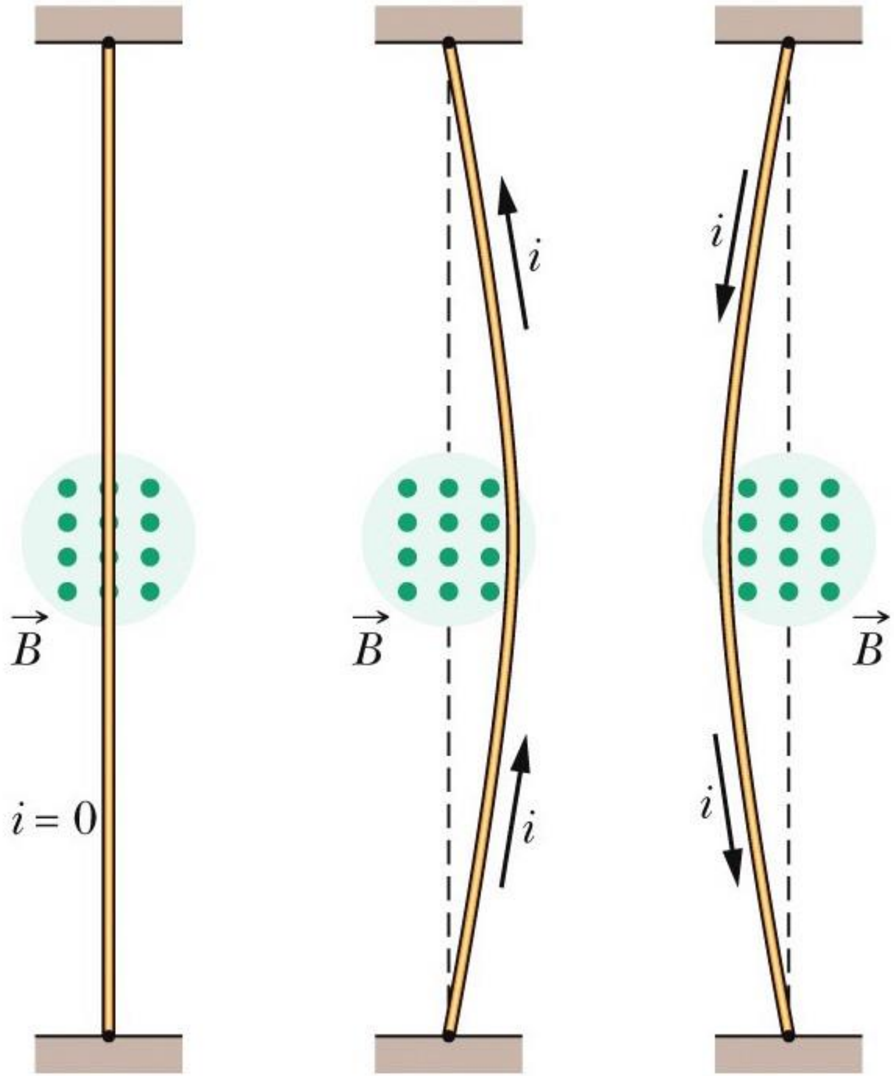
$$\boxed{\vec{F}_B = i\vec{\ell} \times \vec{B}}$$

Length of wire, the same direction as i



A force acts on a current through a B field.

$$\vec{F}_B = i\vec{\ell} \times \vec{B}$$



TopHat Question

A wire of length l cm is carrying a current i and is sitting in a uniform magnetic field B as shown. What is the magnitude and direction of the magnetic force on the wire?

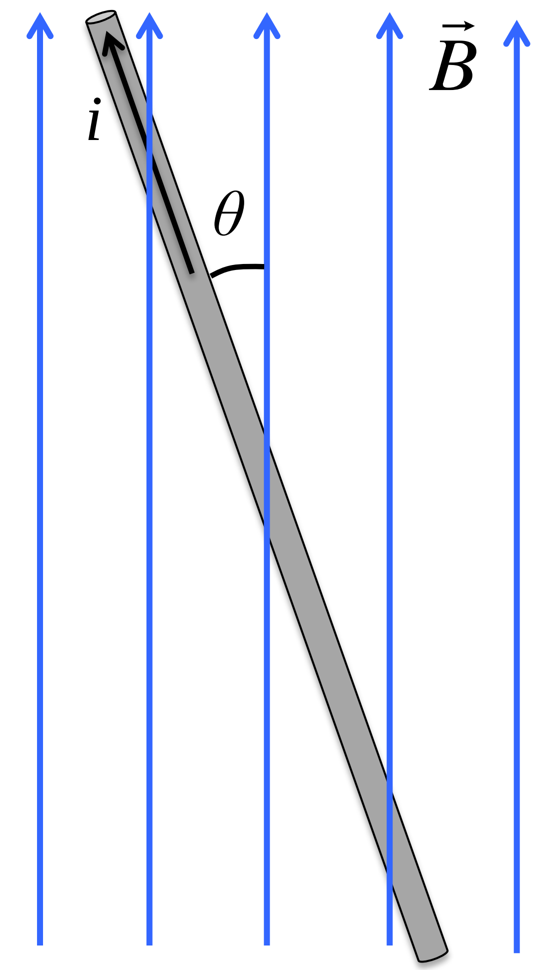
$$\vec{F}_B = i\vec{\ell} \times \vec{B}$$

A. ilB $\odot$

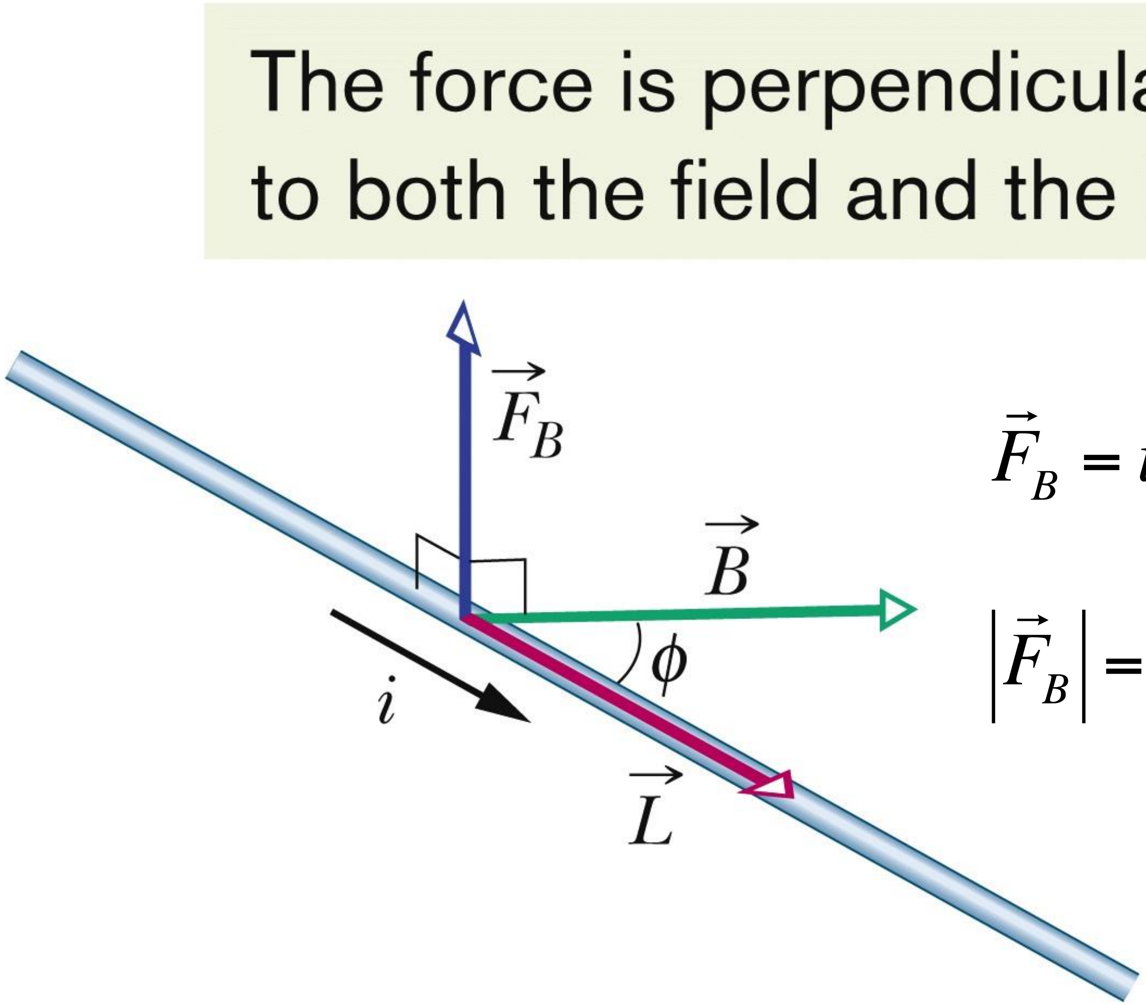
B. $ilB\sin\theta$ $\otimes$

C. $ilB\sin\theta$ $\swarrow$

D. ilB $\nearrow$



Forces on Current-Carrying Wires: B and L not perpendicular



$$\vec{F}_B = i\vec{L} \times \vec{B}$$

$$\left|\vec{F}_B\right| = \left|i\vec{L} \times \vec{B}\right| = iLB\sin\phi$$

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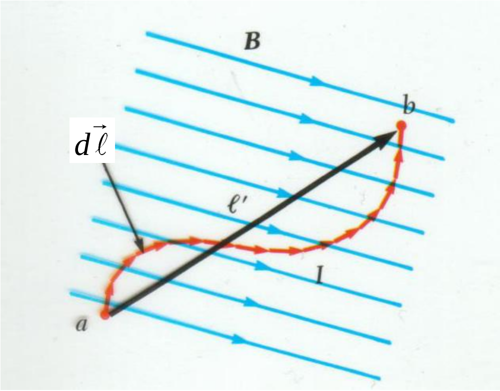
Arbitrary shaped wire in an arbitrary magnetic field

Divide the wire into small segments. If small enough, each segment can be considered straight, the magnetic field and the cross sectional area for the small segments will also be uniform.

$$\boxed{d\vec{F}_B = I d\vec{\ell} \times \vec{B}}$$

$$\boxed{\vec{F}_B = \int_a^b I d\vec{\ell} \times \vec{B}}$$

Applies to all cases.



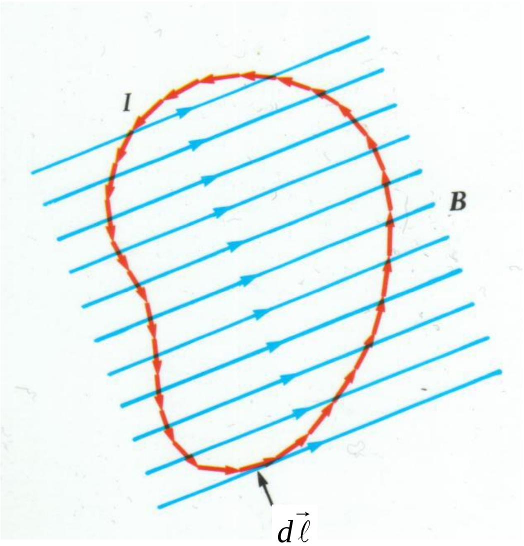
Assume the wire has a uniform cross sectional area and it is placed in a uniform magnetic field.

$$\boxed{\vec{F}_B = I\left[\int_a^b d\vec{\ell}\right] \times \vec{B}}$$

For a current loop of arbitrary shape in a uniform magnetic field

$$\oint d\vec{\ell} = 0$$

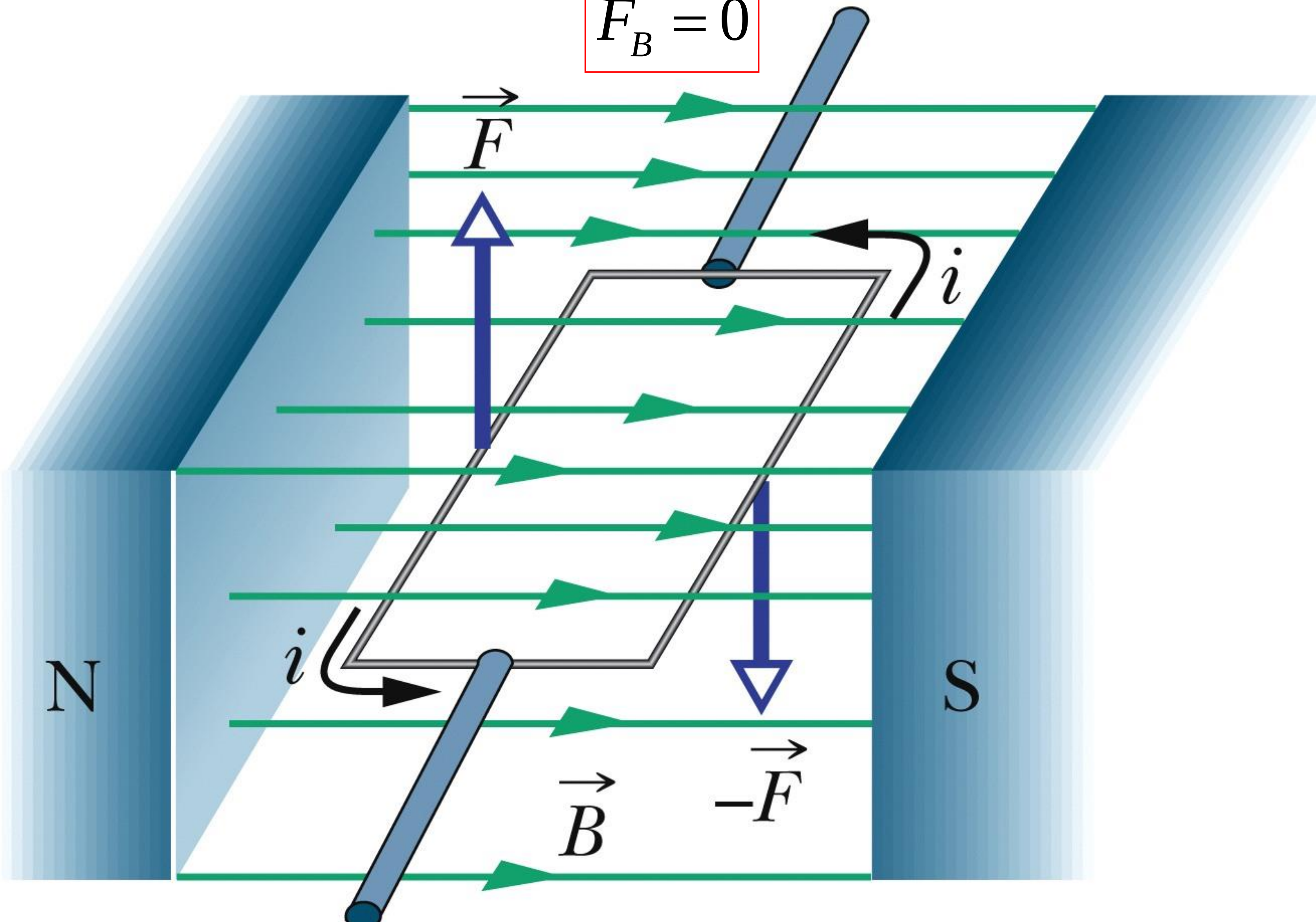
$$\boxed{\vec{F}_B = I\left[\oint d\vec{\ell}\right]\times\vec{B} = 0}$$



Total magnetic force on a closed current loop of uniform cross sectional area in a uniform magnetic field is zero.

Force on a current loop

$$\boxed{\vec{F}_B = 0}$$



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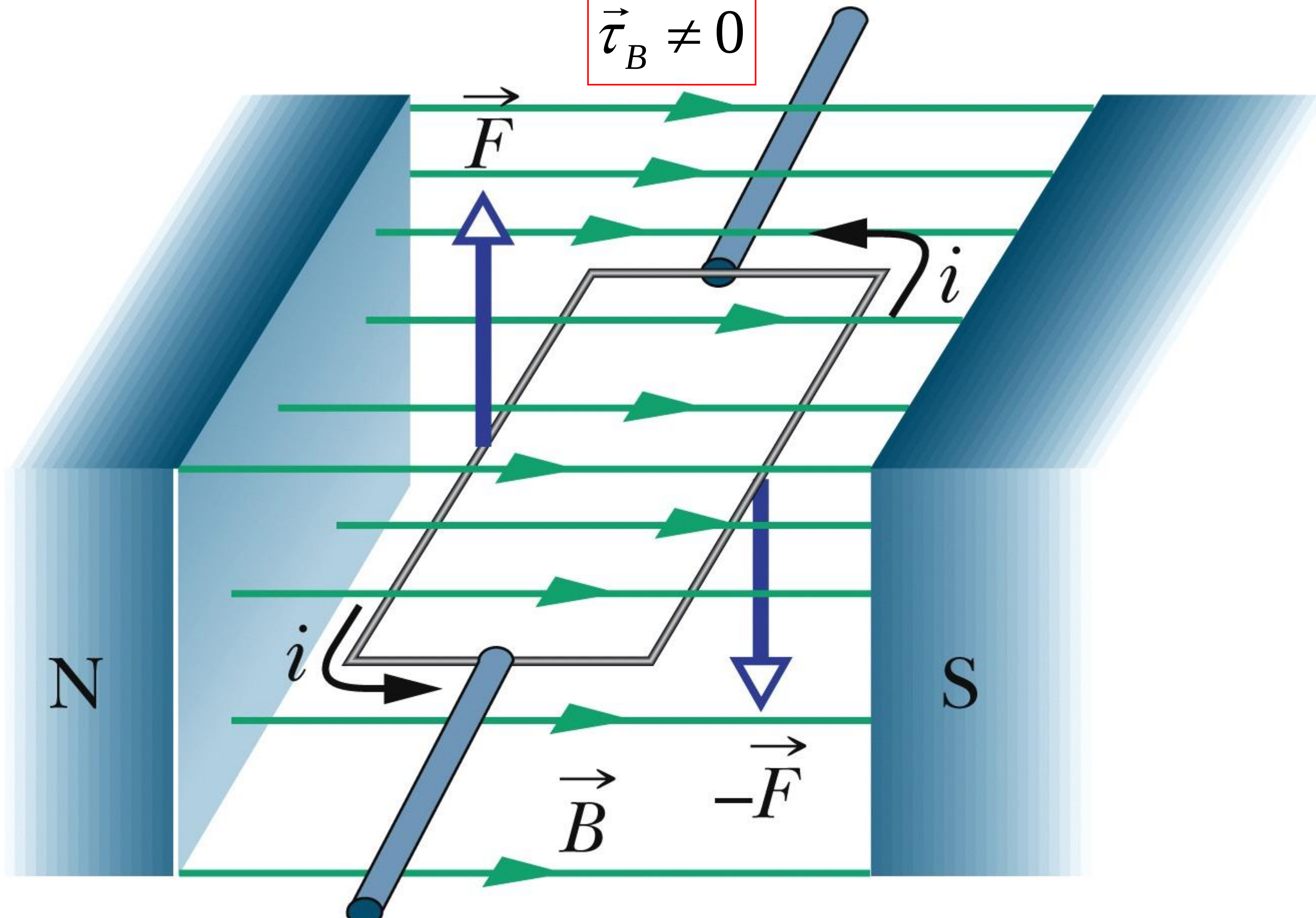
Torque on a current loop

$$\boxed{\vec{\tau}_B \neq 0}$$



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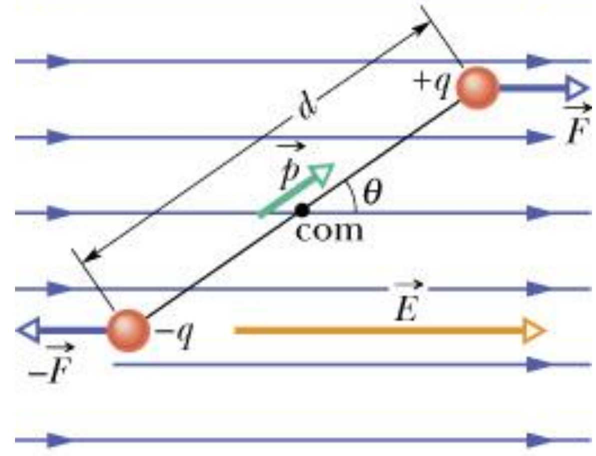
Electric dipole in a uniform electric field

$$\vec{F}_+ + \vec{F}_- = +q\vec{E} - q\vec{E} = 0$$

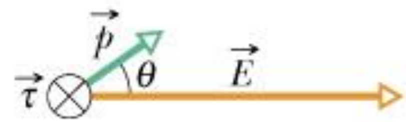
$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\tau = \frac{d}{2} qE \sin\theta + \left(-\frac{d}{2}\right)(-qE)\sin\theta$$
$$= qdE\sin\theta = pE\sin\theta$$

$$\vec{\tau}_E = \vec{p} \times \vec{E}$$



(a)



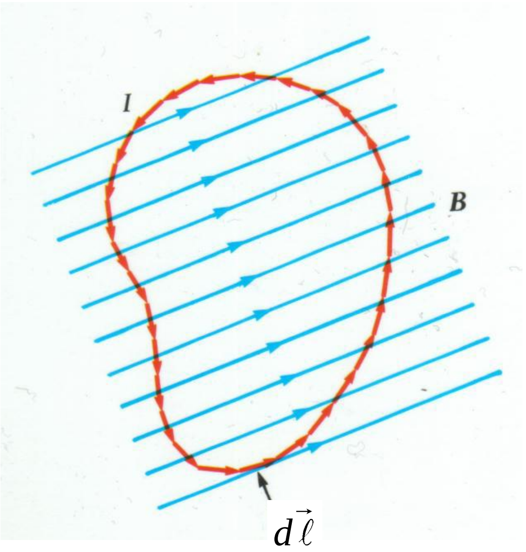
(b)

$$U_E = -\vec{p} \bullet \vec{E} = -pE\cos\theta$$

The minimum energy state of dipole moment is when it lines up with the electric field. The maximum energy state is when its direction is anti-parallel with the E field. Given the chance a dipole moment would line up with the E field.

Magnetic moment

A current loop in magnetism is equivalent to an electric dipole. There is therefore a magnetic moment associated with a current loop as there is an electric moment associated with an electric dipole.



$$\vec{\mu} = I\vec{A}$$

For a planar loop, the magnitude of the area vector is the area of the loop with the direction defined by right hand rule and perpendicular to the plane of the loop.

Symmetry between E and B

Electrostatic and electric dipole moment

$$\vec{p} = q\vec{d}$$

$$\vec{F}_E = 0$$

$$\vec{\tau}_E = \vec{p} \times \vec{E}$$

$$U_E = -\vec{p} \bullet \vec{E} = -pE\cos\theta$$

Magneto-static and magnetic moment

$$\vec{\mu} = I\vec{A}$$

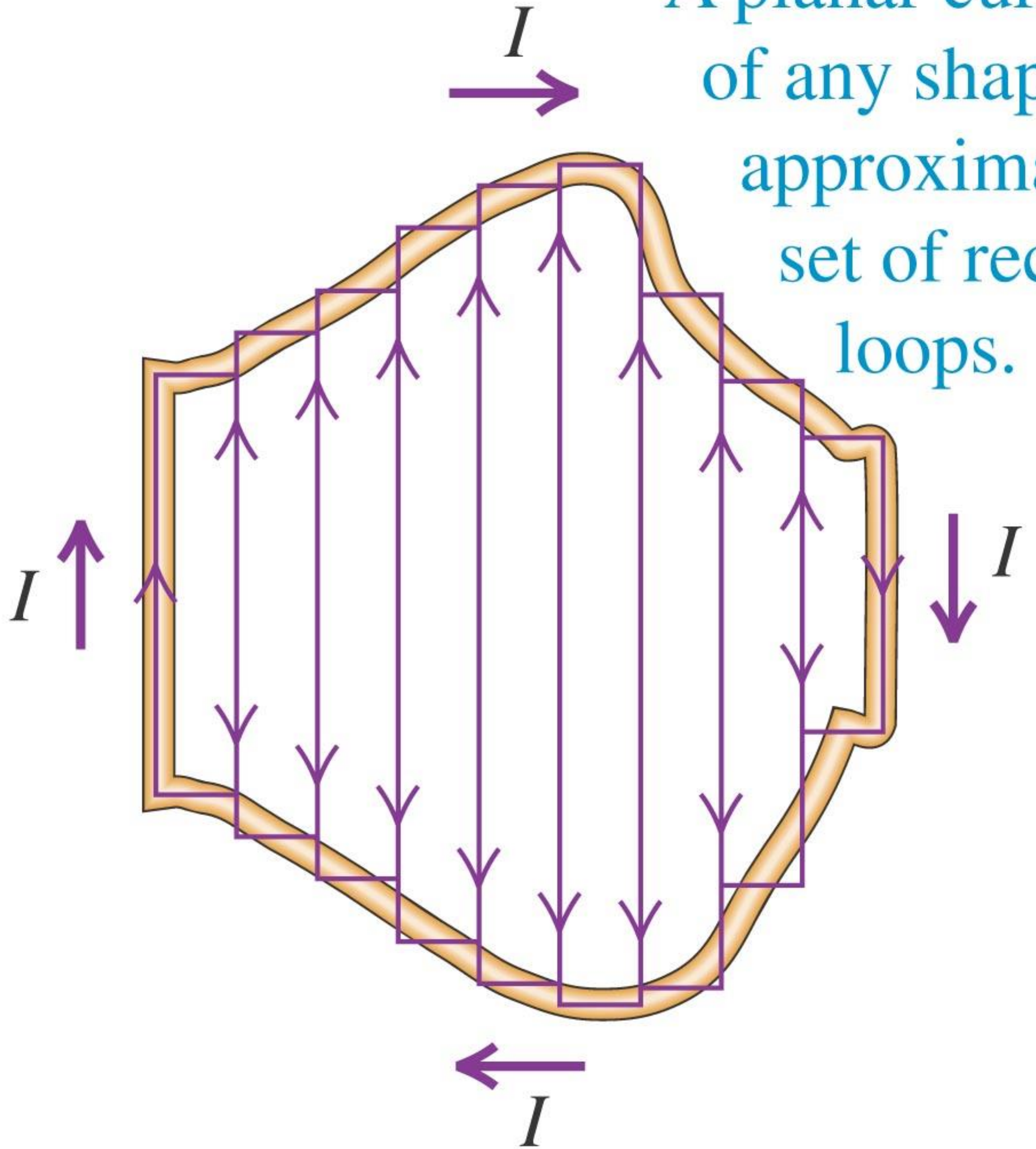
$$\vec{F}_B = 0$$

$$\vec{\tau}_B = \vec{\mu} \times \vec{B}$$

$$U_B = -\vec{\mu} \bullet \vec{B} = -\mu B\cos\theta$$

The minimum energy state of current loop is when its magnetic moment lines up with the magnetic field. The maximum energy state is when its magnetic moment is anti-parallel with the B field. Given the chance a magnetic moment would line up with the B field.

A planar current loop of any shape can be approximated by a set of rectangular loops.



$$\vec{\mu} = I\vec{A}$$

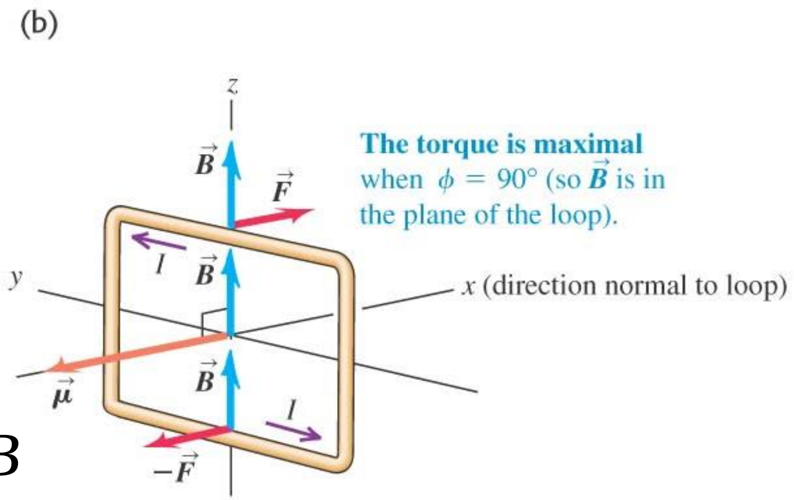
Force and torque on a current loop

This forms the basis of electric motors

Maximum torque:

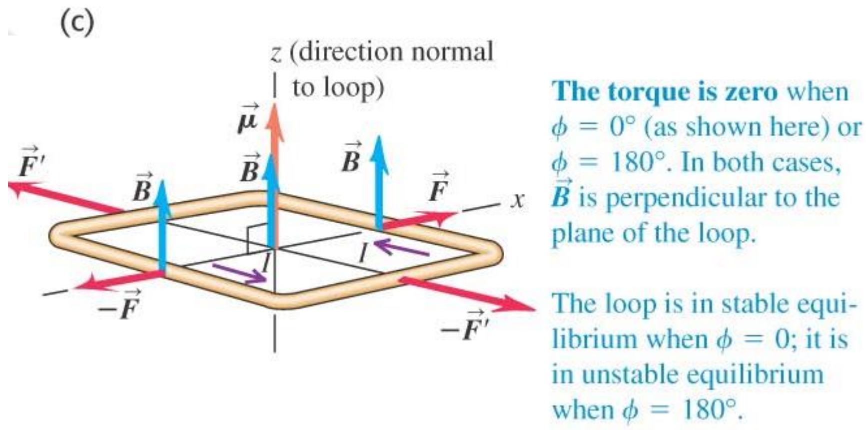
$$\left|\vec{\tau}_B\right| = \left|\vec{\mu}\times\vec{B}\right| = \mu B \sin\phi$$

$$\tau_{\max} = 2F(b/2) = IBab = (Iab)B = \mu B$$



Minimum torque:

$$\tau_{\min} = 0$$



$$\vec{\tau}_B = \vec{\mu} \times \vec{B}$$

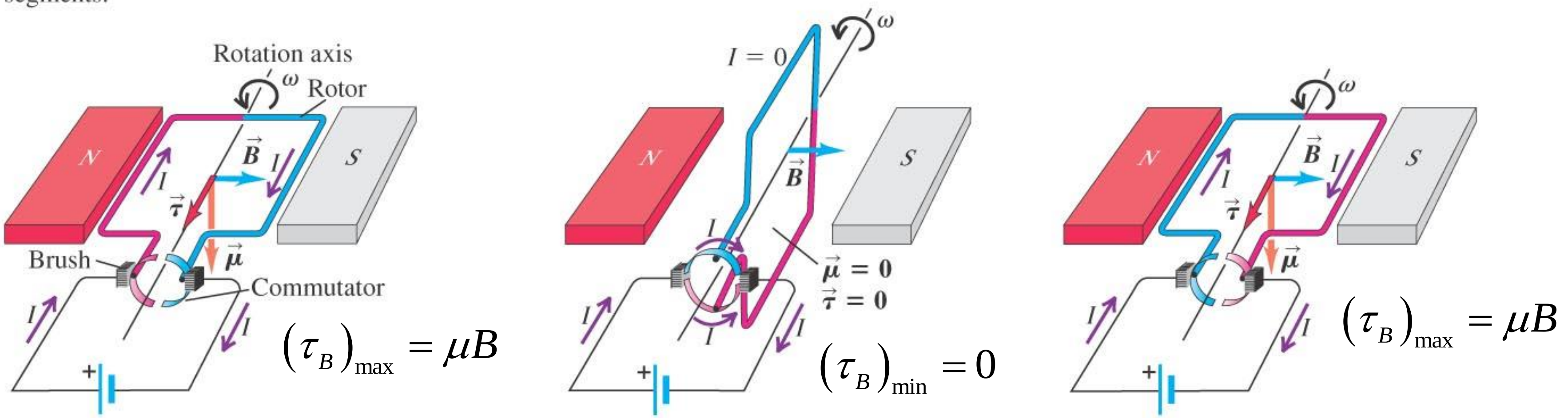
Direct current motor

$$\tau_B = \mu B \sin\phi$$

(a) Brushes are aligned with commutator segments.

(b) Rotor has turned 90°.

(c) Rotor has turned 180°.



$$(\tau_B)_{\max} = \mu B$$

$$(\tau_B)_{\min} = 0$$

$$(\tau_B)_{\max} = \mu B$$

- Current flows into the red side of the rotor and out of the blue side.
- Therefore the magnetic torque causes the rotor to spin counterclockwise.

- Each brush is in contact with both commutator segments, so the current bypasses the rotor altogether.
- No magnetic torque acts on the rotor.

- The brushes are again aligned with commutator segments. This time the current flows into the blue side of the rotor and out of the red side.
- Therefore the magnetic torque again causes the rotor to spin counterclockwise.

Pendulum

$$\vec{\tau}_g = m\vec{l} \times \vec{g}$$