

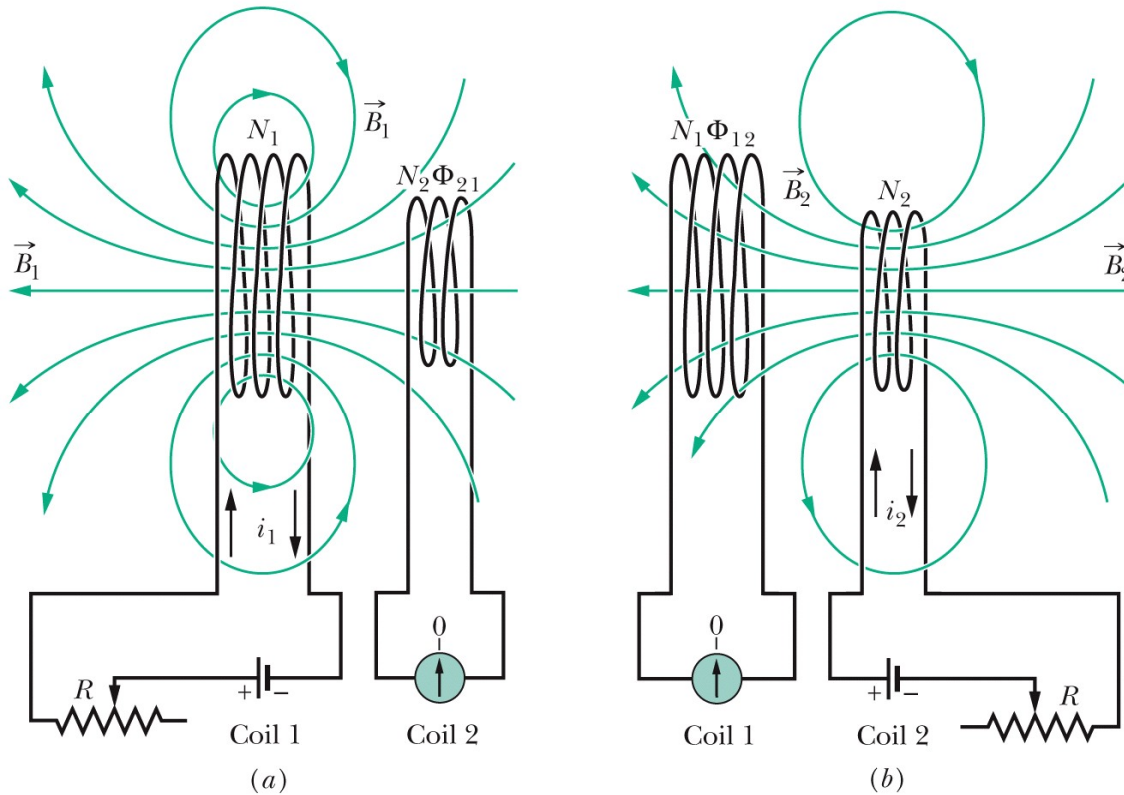
Electricity and Magnetism

- Physics 259 – L02
 - Lecture 48

Chapter 29: Magnetic field due to current



30-8 Mutual Induction



Mutual induction. (a) The magnetic field B_1 produced by current i_1 in coil 1 extends through coil 2. If i_1 is varied (by varying resistance R), an *emf* is induced in coil 2 and current registers on the meter connected to coil 2. (b) The roles of the coils interchanged.

If coils 1 and 2 are near each other, a changing current in either coil can induce an emf in the other. This mutual induction is described by

$$\mathcal{E}_2 = -M \frac{di_1}{dt}$$

$$\mathcal{E}_1 = -M \frac{di_2}{dt}$$

Mutual inductance M_{21} of coil 2 with respect to coil 1 \Rightarrow

$$M_{21} = \frac{N_2 \Phi_{21}}{i_1} \rightarrow M_{21} i_1 = N_2 \Phi_{21} \rightarrow M_{21} \frac{di_1}{dt} = N_2 \frac{d\Phi_{21}}{dt}$$

$$\rightarrow M_{21} \frac{di_1}{dt} = -\varepsilon_2 \rightarrow \varepsilon_2 = -M_{21} \frac{di_1}{dt}$$

we had: $\varepsilon_2 = -N_2 \frac{d\Phi}{dt}$

and the same for $\varepsilon_1 \rightarrow \varepsilon_1 = -M_{12} \frac{di_2}{dt}$

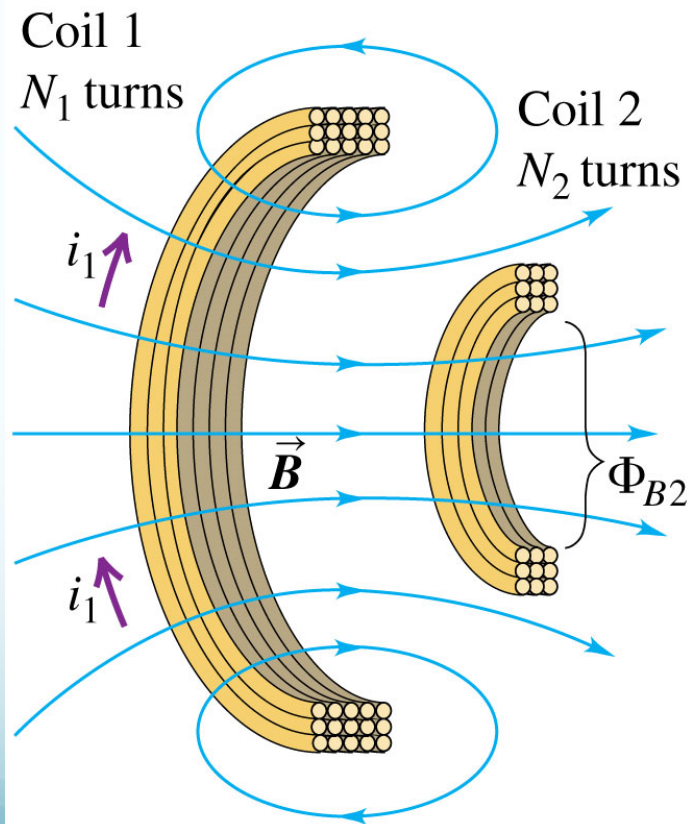
& $M_{12} = M_{21} = M \rightarrow$ we can not prove here 😊

$$\Rightarrow \begin{aligned} \varepsilon_2 &= -M \frac{di_1}{dt} \\ \varepsilon_1 &= -M \frac{di_2}{dt} \end{aligned}$$

M : mutual inductance, depends on the geometry of the two coils

$[M] = 1H = 1Wb/A = 1Vs/A = 1\Omega s = 1J/A^2$. Typical values: $M = \mu H - mH$

Mutual inductance: If the current in coil 1 is changing, the changing flux through coil 2 induces an emf in coil 2.



Induced EMF in coil 2:

$$\mathcal{E}_2 = -N_2 \frac{d\phi_{B2}}{dt} = -\frac{d(N_2\phi_{B2})}{dt}$$

Note: ϕ_{B2} is the magn. flux through a single loop of coil 2. N_2 is the number of loops.

The magnetic field in coil 2 is prop. to the current through coil 1:

$$d\mathbf{B} = \frac{\mu_0}{4\pi} \frac{i_1 d\mathbf{l} \times \mathbf{r}}{r^2} \quad \text{Biot-Savart}$$

Hence, the magnetic flux through coil 2 is proportional to i_1 :

$$N_2\phi_{B2} = M_{21}i_1$$

and

$$\mathcal{E}_2 = -\frac{d(N_2\phi_{B2})}{dt} = -M_{21} \frac{di_1}{dt}$$

M_{21} : *mutual inductance*, depends on the geometry of the two coils

Question 1 – Mutual inductance

The long solenoid will produce a magnetic field that is proportional to the current I_1 and the number of turns per unit length n_1

$$B_1 = \frac{\mu_0 N_1 I_1}{L} = \mu_0 n_1 I_1$$

and the total flux through each loop of the outer coil is

$$\Phi_{B2} = B_1 A_1$$

Why not A_2 ?

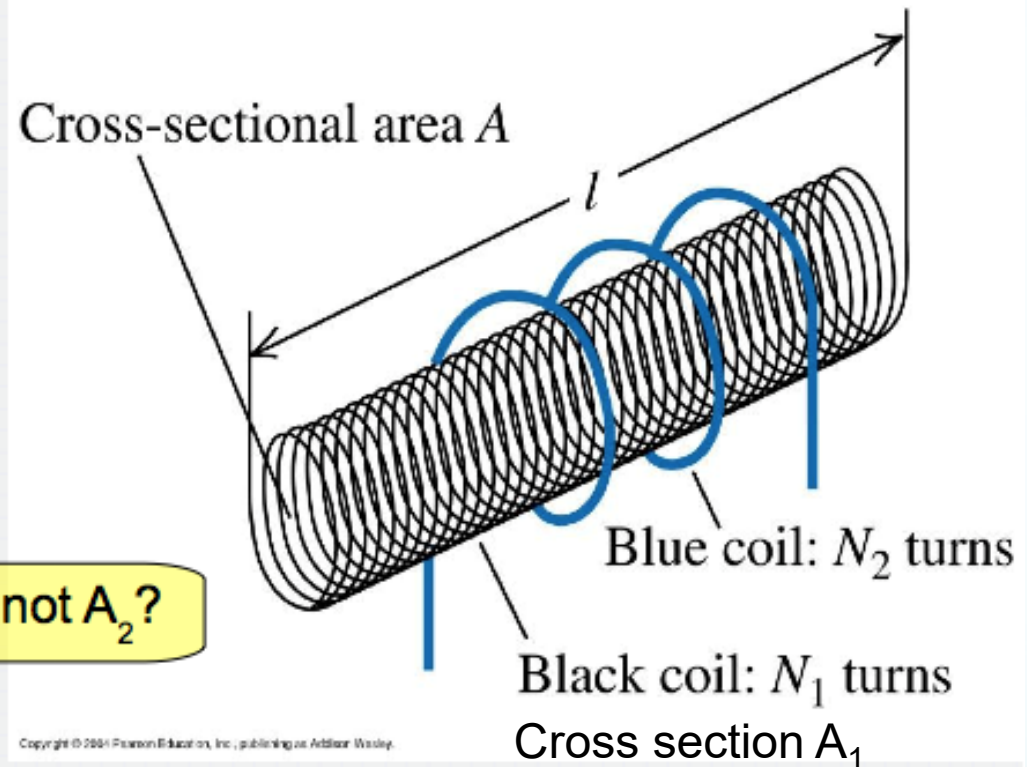
so the mutual inductance is

$$M = \frac{N_2 \Phi_{B2}}{I_1} = \frac{N_2 (B_1 A_1)}{I_1} = \frac{\mu_0 A N_1 N_2}{L}$$

does not depend on I !

For a 0.5m long coil with 10cm² area and $N_1=1000$, $N_2=10$ turns

$$M = \frac{(4\pi \times 10^{-7} \text{ T m/A})(1.0 \times 10^{-3} \text{ m}^2)(1000)(10)}{0.5\text{m}} = 2.5 \times 10^{-6} \text{ H} = 25 \mu\text{H}$$



Question 1 (b)

If a rapidly increasing current is driven through the outer coil

$$i_2(t) = (2.0 \times 10^6 \text{ A/s}) t$$

what EMF will be induced in the inner coil?

$$\mathcal{E}_1 = -M \frac{di_2}{dt}$$

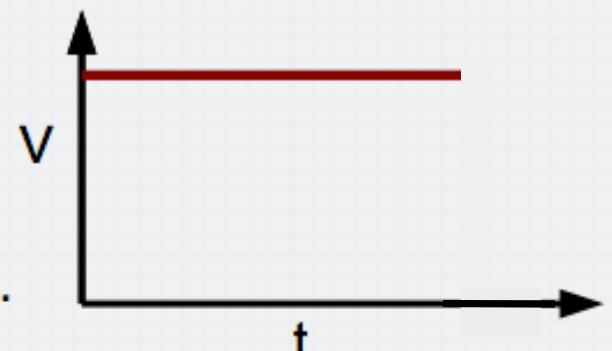
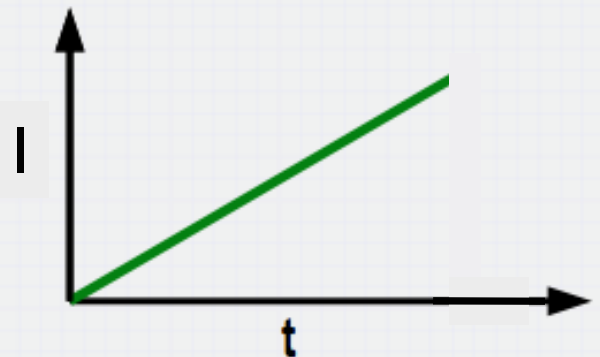
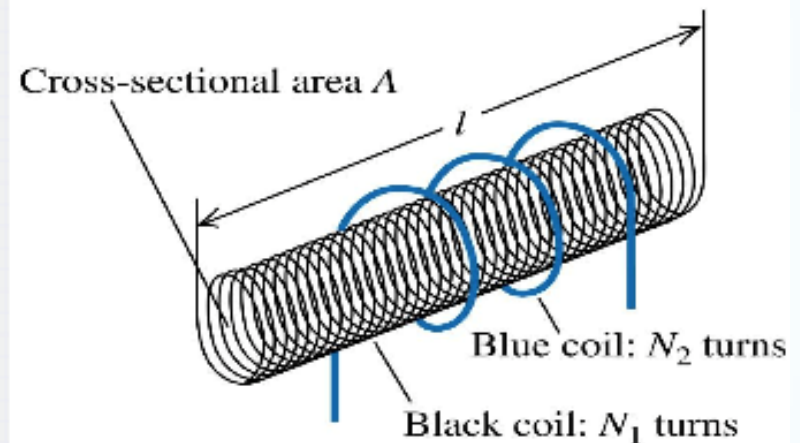
Note: M also allows calculating \mathcal{E}_2 if i_1 changes

$$= -(25 \times 10^{-6} \text{ H}) \frac{d}{dt} [(2.0 \times 10^6 \text{ A/s}) t]$$

$$= -(25 \times 10^{-6} \text{ H})(2.0 \times 10^6 \text{ A/s})$$

$$= -50 \text{ V}$$

This allows electrical energy in one circuit to be converted to electric energy in a separate device.

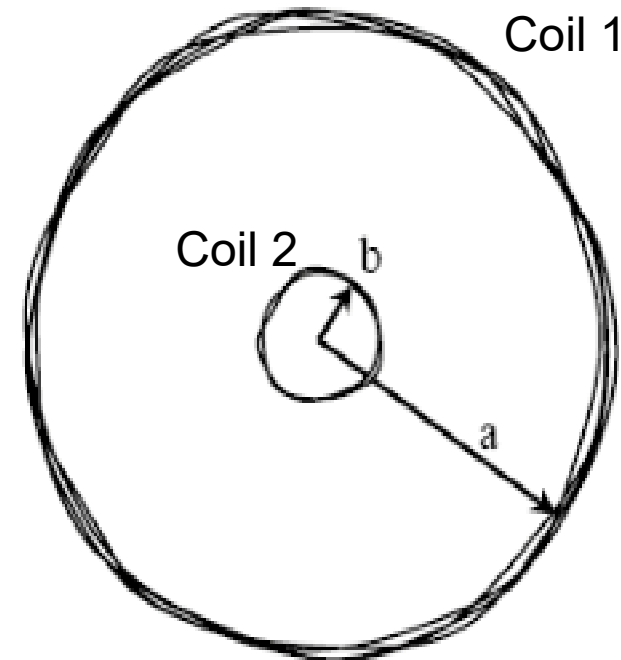


Question 2

16. The diagram below shows two nested, circular coils of wire. The larger coil has radius a and consists of N_1 turns. The smaller coil (radius b) consists of N_2 turns, and is both coplanar and coaxial with the larger coil. Assume $b \ll a$, so that the magnetic field of the larger coil is approximately uniform over the area of the smaller coil. The **mutual inductance** of this combination is given by the expression

$$M = \frac{N_2 \phi_{B2}}{i_1} = \frac{N_1 \phi_{B1}}{i_2}$$

Flux through one loop of coil 2 (area A_2) due to magnetic field generated by current in coil 1



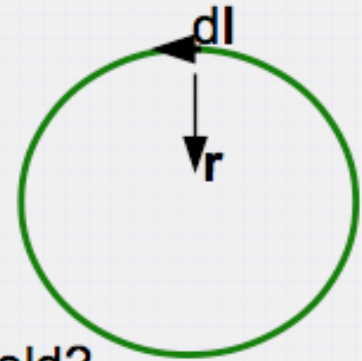
- a) $\frac{\mu_0 N_1 N_2}{2a}$.
- b) $\frac{\pi \mu_0 N_1 N_2 b}{a}$.
- c) $\frac{\pi \mu_0 N_1 N_2 b^2}{2a}$.
- d) $\frac{\mu_0 N_1 N_2 b^2}{2a}$.
- e) $\frac{\pi \mu_0 N_2 b^2}{2a}$.

We assume current in the larger coil (coil 1), which generates a roughly uniform field in the area covered by the much smaller coil.

How large is B?

How large is B ?

Calculate for one loop!



A circular loop of radius a carries a constant current I .
What is the magnetic field at the center of the loop?

What are the two methods we know for calculating magnetic field?
Biot-Savard law & Ampere's law.

Ampere's law isn't useful for a loop, so use the Biot-Savard law:

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{l} \times \hat{r}}{r^2} = \frac{\mu_0}{4\pi} \frac{I}{a^2} dl \hat{z}$$

$$\vec{B} = \int d\vec{B} = \frac{\mu_0}{4\pi} \frac{I}{a^2} \hat{z} \int dl = \frac{\mu_0}{4\pi} \frac{I}{a^2} (2\pi a) \hat{z} = \frac{\mu_0}{2} \frac{I}{a} \hat{z}$$

With the B-field direction directed out of the page, either from $d\mathbf{l} \times \mathbf{r}$, or right thumb in direction of current and fingers curl in direction of \mathbf{B} .

$$M = \frac{N_2 \phi_{B2}}{i_1} = \frac{N_2 BA}{i_1}$$

$$B = N_1 \frac{\mu_0 i_1}{2a}$$

$$A = \pi b^2$$

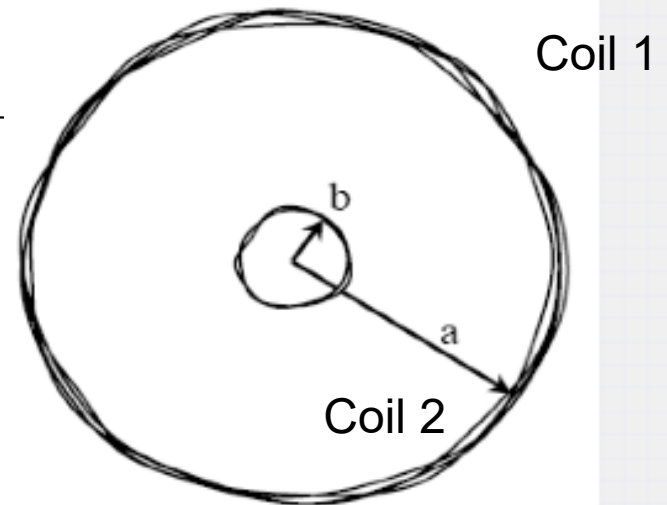
$$M = \mu_0 N_1 N_2 \frac{\pi b^2}{2a}$$

Question

16. The diagram below shows two nested, circular coils of wire. The larger coil has radius a and consists of N_1 turns. The smaller coil (radius b) consists of N_2 turns, and is both coplanar and coaxial with the larger coil. Assume $b \ll a$, so that the magnetic field of the larger coil is approximately uniform over the area of the smaller coil. The **mutual inductance** of this combination is given by the expression

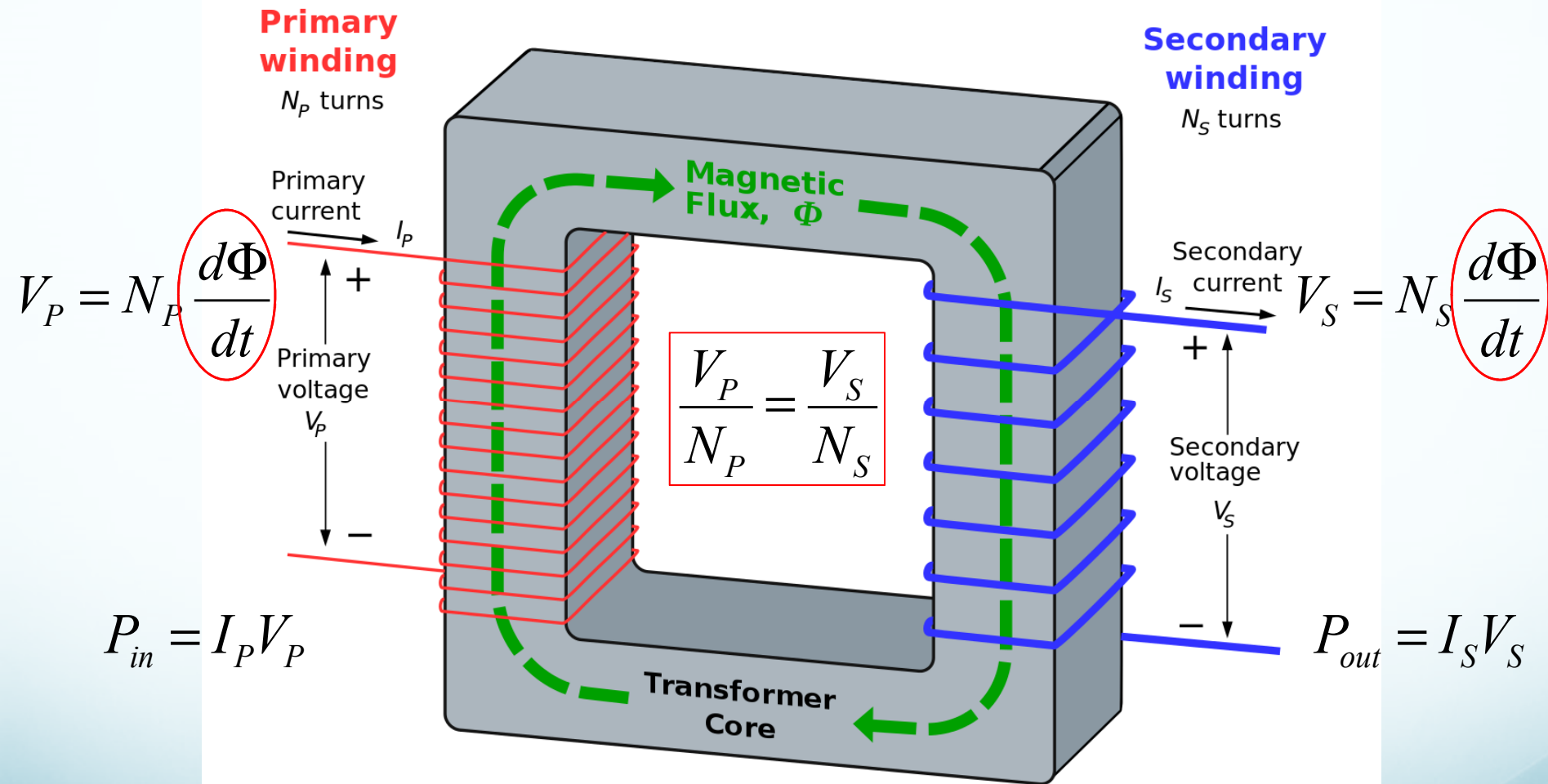
- a) $\frac{\mu_0 N_1 N_2}{2a}$.
- b) $\frac{\pi \mu_0 N_1 N_2 b}{a}$.
- c) $\frac{\pi \mu_0 N_1 N_2 b^2}{2a}$. ✓
- d) $\frac{\mu_0 N_1 N_2 b^2}{2a}$.
- e) $\frac{\pi \mu_0 N_2 b^2}{2a}$.

$$M = \frac{N_2 \phi_{B2}}{i_1} = \frac{N_1 \phi_{B1}}{i_2}$$



- ✓ We expect the result to be proportional to the area of the coil that sees the field of the other coil, i.e. πb^2 .
- ✓ Furthermore, we expect a dependence on N_1 and N_2 : the field depends on N_1 , and the flux on N_2 .
- ✓ This leaves only answer c).

Transformers

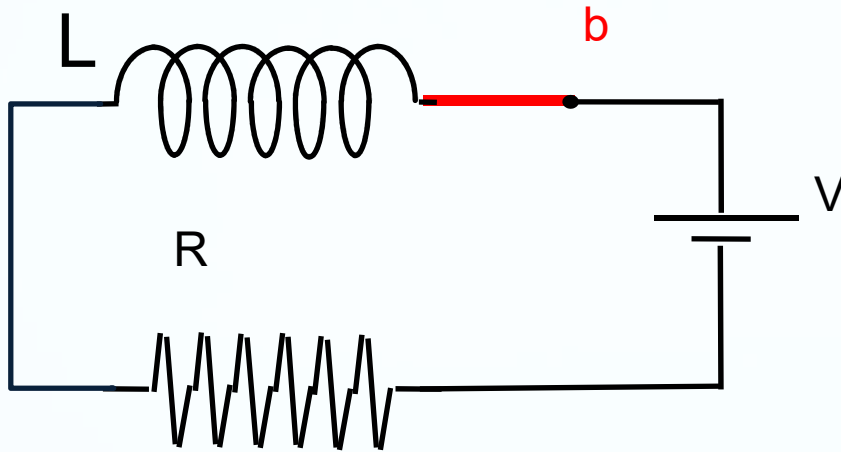


$$P_{in} = P_{out}$$

$$I_P V_P = I_P \frac{N_P}{N_S} V_S = I_S V_S$$

$$I_P N_P = I_S N_S$$

30.6: R-L Circuit



$$V - L \frac{di}{dt} - iR = 0$$

If the switch is moved to position **b**, to initiate the current flow, what happens?

Faraday's law applies and so the change in the Magnetic Field in the inductor L means there is a back EMF induced in L .

So in this case at $t = 0$, $i(0) = 0$.

Inductor acts like a BATTERY

After a long time, $i = V/R$

Inductor acts like a WIRE

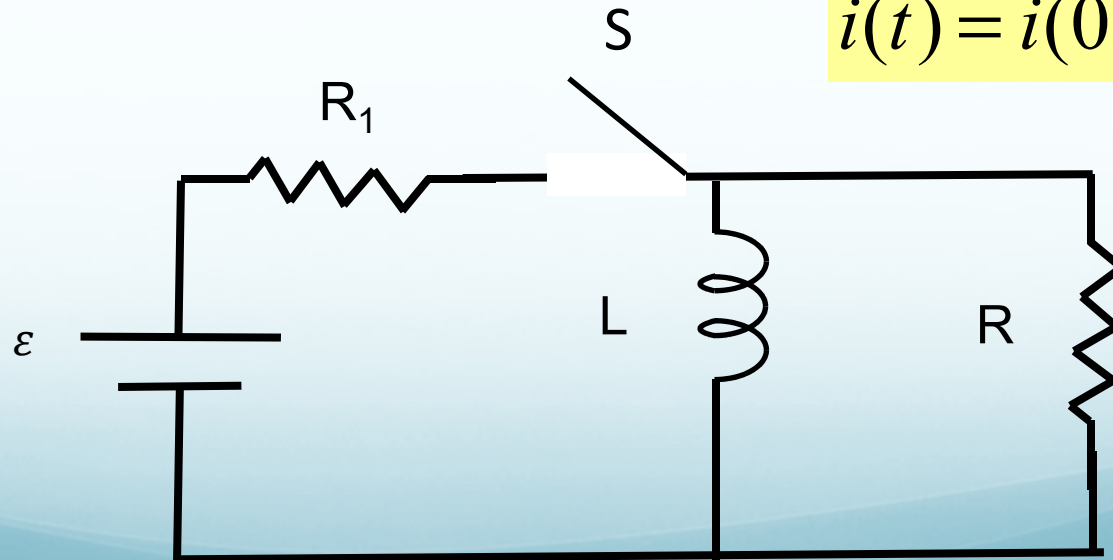
The components have all been connected for a very long time.
At $t=0$ the switch S is **opened**. The current through R_1 and R are 0 and ε/R

Using the loop rules

$$-L \frac{di}{dt} - iR = 0$$

Solving with the method we used for a **discharging capacitor**

$$i(t) = i(0) e^{-\left(\frac{Rt}{L}\right)}$$

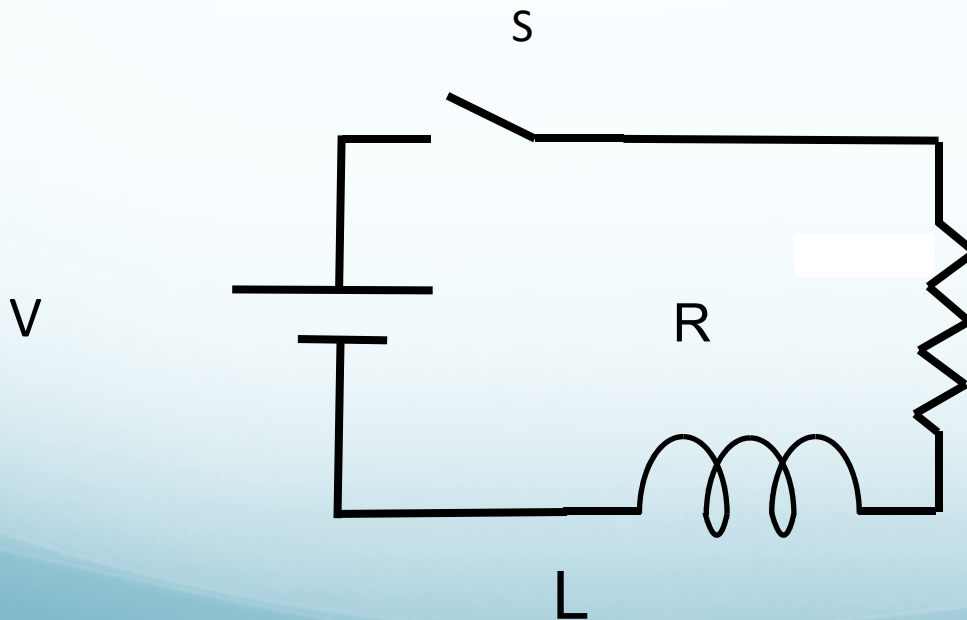


At $t=0$ the switch S is **closed**.

Using the loop rules

$$V - iR - L \frac{di}{dt} = 0$$

Solving using the method we used for the **charging capacitor**

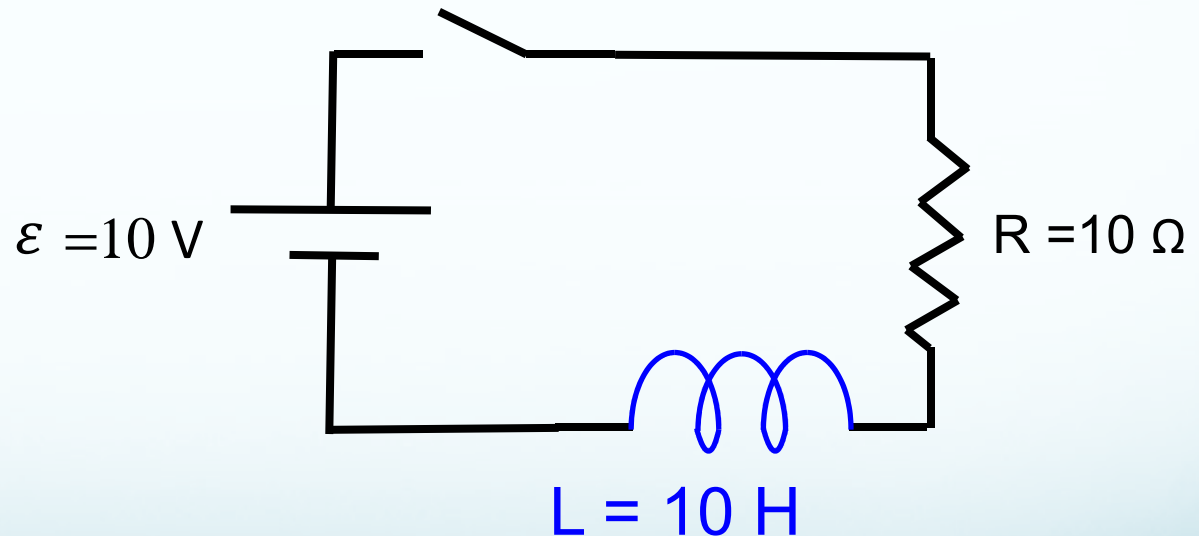


$$i(t) = i_{\max} \left(1 - e^{-\left(\frac{Rt}{L}\right)} \right)$$

Top Hat Question

The switch in the series circuit below is closed at $t=0$.

What is the **initial rate of change of current di/dt** in the **inductor**, immediately after the switch is closed (time = $0+$) ?



A. 0 A/s

B. 0.5 A/s

C. 1 A/s

D. 10 A/s

V_R

$V_L = L di/dt$

Top Hat Question

The switch in the series circuit below is closed at $t=0$.

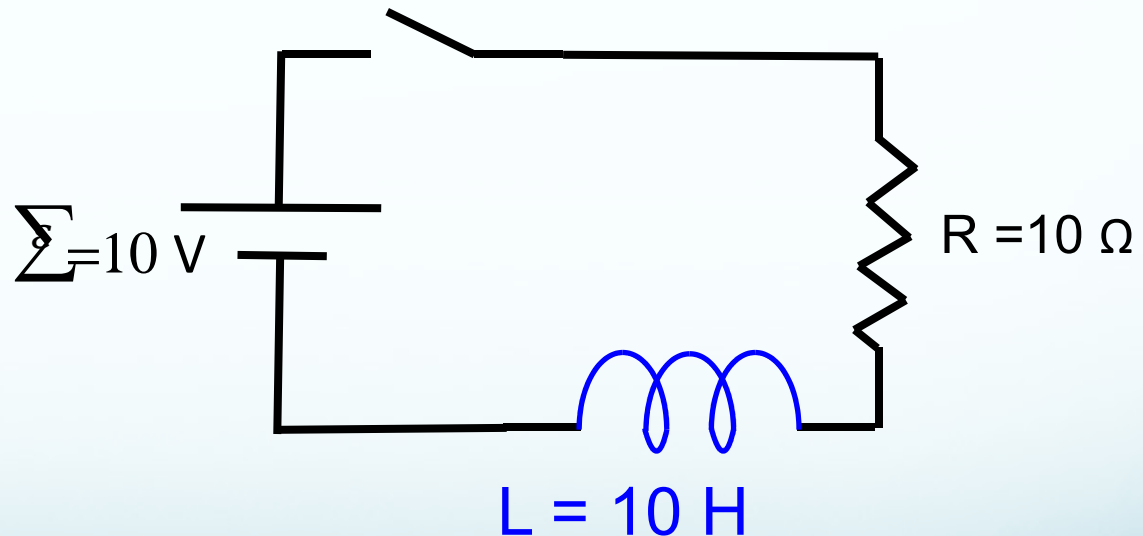
What is the **initial rate of change of current di/dt** in the **inductor**, immediately after the switch is closed (time = $0+$) ?

A. 0 A/s

B. 0.5 A/s

C. 1 A/s

D. 10 A/s



$i = 0$ at $t = 0$ so $V_R(0) = 0$ which means

$$10 \text{ V} = V_L = L di/dt \text{ so } di/dt = 10 \text{ V} / 10 \text{ H} = 1 \text{ A/s}$$

Top Hat Question

The switch in the series circuit below is closed at $t=0$.

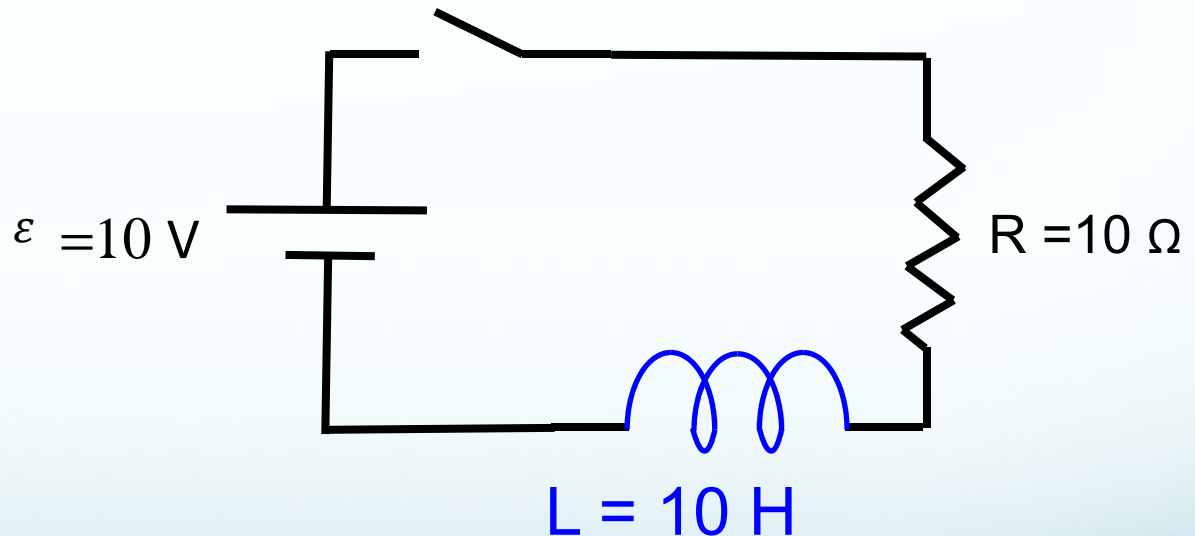
What is the current in the circuit after a time $t = 3.0$ s?

A. 0 A

B. 0.63 A

C. 0.86 A

D. 0.95 A



Top Hat Question

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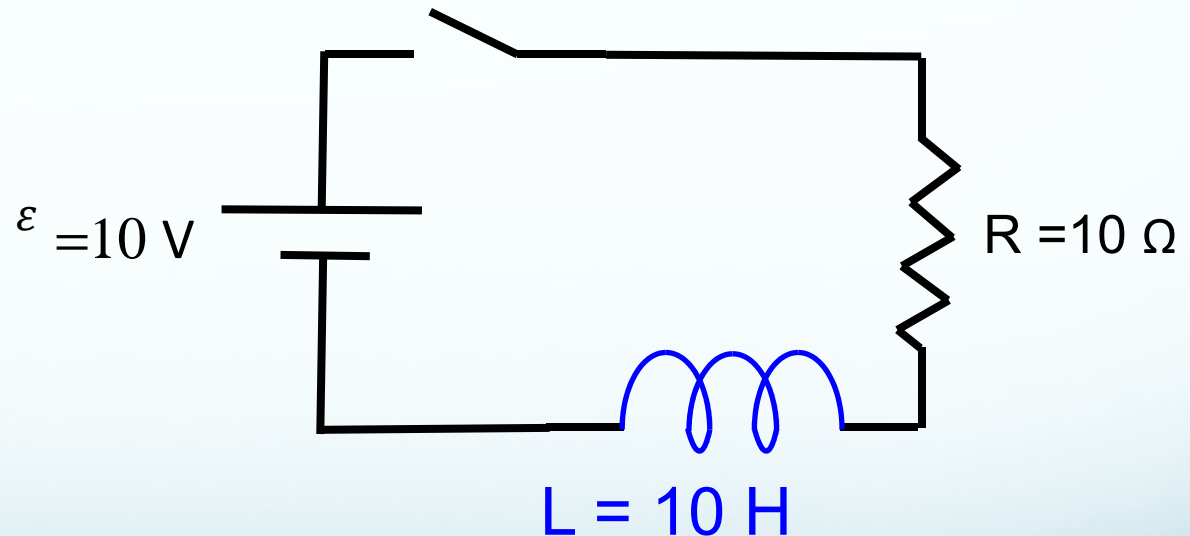
What is the current in the circuit after a time $t = 3.0$ s?

A. 0 A

B. 0.63 A

C. 0.86 A

D. 0.95 A



$$i(3\text{s}) = \frac{10\text{V}}{10\Omega} (1 - e^{-3})$$

This section we talked about:

Chapter 30

See you on Monday

