
Fill in these boxes with your SURNAME and INITIAL.

University of Calgary

Faculty of Science

Midterm Test

PHYSICS 259 ALL LECTURE SECTIONS

February 15, 2013, 6:30–8:00 p.m. **Time: 90 minutes.**

DO NOT TEAR OFF THIS PAGE! You may, however, tear off the last page, which has an equation sheet and table of integrals.

This is a closed-book exam worth a total of 32 points. Please answer all questions. Use of the Schulich calculator or equivalent is allowed.

Write your Last Name and Initial on this top sheet in the grid above. (Do not write your ID number on this page.)

Make sure this question paper booklet contains 15 pages. If you are missing any pages, get a new booklet from the exam supervisor.

You should also have a **separate set of Answer Sheets**. This is where you enter Multiple Choice answers of Part I and also detailed solutions to the problems of Part II. Only work entered in the indicated spaces on the Answer Sheets will be marked.

IMPORTANT: YOUR ID NUMBER IS TO BE ENTERED AT THE TOP OF EACH AND EVERY ONE OF THE ANSWER SHEETS. DO THIS NOW.

Begin working on the examination when instructed to do so by the supervisor.

Part I: Multiple-Choice Questions (Total: 12 marks)

Enter answers to multiple choice questions on the first Answer Sheet using space provided in the upper right of the page. Each question in Part I is worth one point. You should complete Part I in about 30 minutes or less.

1) Three point charges are equally spaced along a line as shown in Figure 1. For this question assume that $q_1 = q_3 = -3q_2$. In which regions on the line will the potential be zero?

a) B and C
b) A and D
c) only B
d) only C
e) none of the above

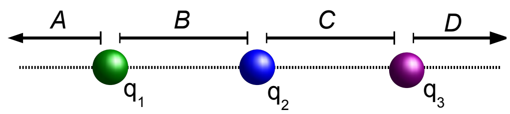

Figure 1: Three charges equally spaced along a line.

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2) In Figure 2, $q_1 = -5$ Coulombs and $q_4 = +5$ Coulombs. The electric flux through surface B is 10 Coulombs/ϵ_0 and that through surface A is zero. Which of the following statements is true?

a) $q_3 = +5$ Coulombs and $q_5 = -5$ Coulombs
b) $q_3 = +5$ Coulombs and $q_5 = +5$ Coulombs
c) $q_3 = -5$ Coulombs and $q_5 = -5$ Coulombs
d) $q_3 = -5$ Coulombs and $q_5 = +5$ Coulombs
e) none of the above

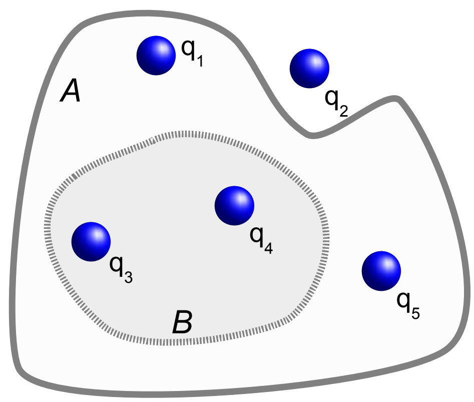

Figure 2: Two Gaussian surfaces (A & B) with five point charges (1-5).

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3) In Figure 3 the central point charge is $q_1 = +2Q$ and there is a net negative charge $-Q$ on the spherical conducting shell. Which of the following statements about the electric field magnitude E is true?

a) E is $\dfrac{Q}{4\pi\epsilon_0 r^2}$ for all $r > b$.
b) E is zero for all $r < b$.
c) E is $\dfrac{Q}{4\pi\epsilon_0 r^2}$ for all $r < a$.
d) E is $\dfrac{2Q}{4\pi\epsilon_0 r^2}$ for all $r > b$.
e) none of the above

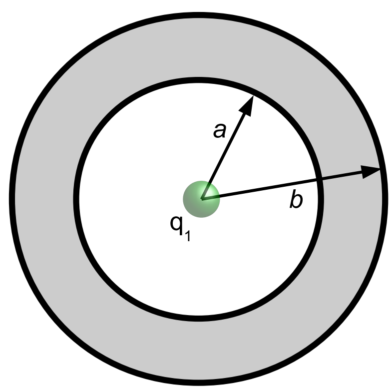

Figure 3: Spherical conducting shell with inner radius a and outer radius b. Point charge q_1 is located at the centre of the hollow shell.

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4) For the charged rod shown in Figure 4, assume that the total charge in the rod is positive ($Q > 0$) and uniformly distributed along the rod. Which of the following statements is true for the electric field on the y-axis?

a) for $y \gg L \quad E_y \approx \dfrac{Q}{4\pi\epsilon_0 y^2}$

b) for $y \gg L \quad E_y \approx \dfrac{Q}{2\pi\epsilon_0 y}$

c) for $y \gg L \quad E_y \approx \dfrac{Q}{2\pi\epsilon_0 y^3}$

d) for all $y > 0 \quad E_y = \dfrac{Q}{4\pi\epsilon_0}\dfrac{L}{y^3}$

e) none of the above

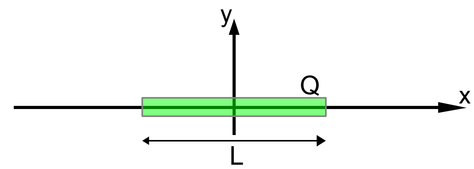

Figure 4: Uniformly charged rod.

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5) Four different sets of positive and negative point charges are placed on the x- and y-axes at equal distances from the origin as shown in Figure 5. Assume that all charges have the same magnitude. Which configuration produces $\vec{E} = 0$ <u>and</u> $V = 0$ at the origin?

a) configuration #2
b) configuration #3
c) configuration #1 and #4
d) configuration #1
e) none of the above

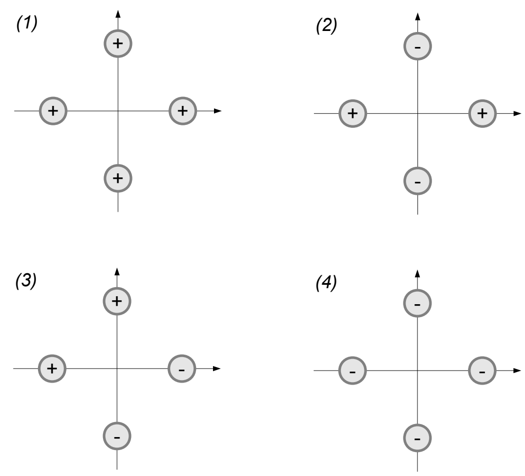

Figure 5: Four sets of four charges.

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6) Two flat metal plates are parallel to each other and separated by a gap of 2.00 mm. There is vacuum between the plates. An electron released at rest at one plate accelerates across the gap and reaches the other plate with a speed of 4.00×10^6 m/s. The electric field strength between the plates is closest to

a) 2.27×10^4 V/m

b) 4.40×10^7 V/m

c) 2.27×10^{-8} V/m

d) 4.54×10^4 V/m

e) 4.40×10^{-5} V/m

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7) Figure 6 shows a cube with edges of length L=2.00 m, and one corner at the origin. If there is a uniform electric field given by $\vec{E} = 4.25\hat{i} - 2.75\hat{j}$ in this region of space, what is the electric flux through the right-hand face (x=L) shown in the figure?

a) $+17\ Nm^2/C$
b) $-17\ Nm^2/C$
c) $-11\ Nm^2/C$
d) $+6\ Nm^2/C$
e) $+11\ Nm^2/C$

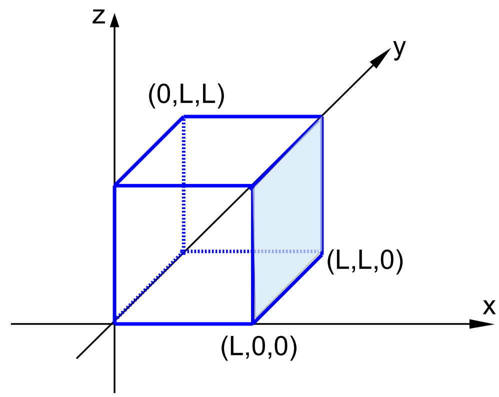

Figure 6: Cube with sides of length L.

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8) A solid insulating sphere of radius R carries a positive charge density ρ (C/m^3) distributed uniformly throughout its volume. What is the electric field strength E as a function of radius inside the sphere?

a) $E = \dfrac{\rho r}{3\epsilon_0}$
b) $E = \dfrac{\rho R}{3\epsilon_0 r}$
c) $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{\rho}{r^2}$
d) $E = \dfrac{4\pi\rho r^3}{3\epsilon_0 R^2}$
e) $E = \dfrac{\rho r^2}{3\epsilon_0}$

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9) The electric potential in a certain region of space varies only with distance from the origin r (spherically symmetric). If $V(r) = \dfrac{2Q_0 r^2}{3\epsilon_0 R_0^3}$, then what is the electric field in this region?

a) $\vec{E} = \dfrac{-4Q_0 r}{3\epsilon_0 R_0^3}\hat{r}$
b) $\vec{E} = \dfrac{+2Q_0 r^3}{9\epsilon_0 R_0^3}\hat{r}$
c) $\vec{E} = \dfrac{-2Q_0 r^3}{9\epsilon_0 R_0^3}\hat{r}$
d) $\vec{E} = \dfrac{+4Q_0 r}{3\epsilon_0 R_0^3}\hat{r}$
e) $\vec{E} = \dfrac{-2Q_0 r^2}{3\epsilon_0 R_0^3}\hat{r}$

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10) The electric potential at some point A is 10 volts. The electric potential at another point B is 5 volts. A proton moves from point A to point B, being acted on by only the electric force. The proton's kinetic energy

a) Increases by roughly 8×10^{-19} joules.
b) Increases by roughly 8×10^{-19} watts.
c) Increases by roughly 1.6×10^{-19} joules.
d) Decreases by roughly 8×10^{-19} joules.
e) cannot answer because we do not know the path taken by the proton.

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11) A point charge is placed at the centre of a spherical Gaussian surface. The net electric flux Φ_E through the surface is different if

a) the sphere is replaced by a cube of the same volume.
b) the sphere is replaced by a cube of one-tenth the volume.
c) a second charge is placed just outside the sphere.
d) the point charge is moved off-centre (but still inside the sphere).
e) none of the above

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12) In Figure 7, if $q_1 = 4Q$ and $q_2 = -Q$, and a=1 cm, then the electric field on the x-axis is zero at
a) x=3 cm
b) x=−3 cm
c) x=1/3 cm
d) x=0 cm
e) x=−1/3 cm

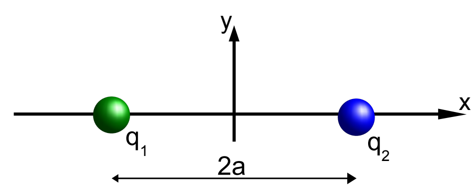

Figure 7: Two charges on a line.

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13) This is **version @V@** of the exam. Please select @V@ in the row marked "Version" in the Multiple Choice Answers area on the first Answer Sheet. **Be sure to do this now, before moving on to other questions on the paper**.

This is the end of the Multiple Choice part of the exam. Make sure that you have entered all your answers from this section on the Answer Sheet "bubble" page.

You may now proceed to Part II. All your answers for Part II must be written in the appropriate box on the Answer Sheet pages.

Part II: Written Answer Questions (Total: 20 marks)

IMPORTANT: Write your answers to the problems in Part II in the corresponding boxes on the Answer Sheets. Work must be shown for full marks. Rough work can be done on the back of this question paper, but only the work appearing on the Answer Sheets will be marked.

14) *[7 marks]* A circle of radius $R = 20.0$ metres is centered on the origin (x=0,y=0). Two charges $q_1 = +3.00\ \mu C$ and $q_2 = -7.00\ \mu C$ are located on the circle as shown in Figure 8.

a) Calculate the electric field $\vec{E}$ at the origin. *[2 marks]*

b) Calculate the electric potential V at the origin. *[2 marks]*

c) A third charge $q_3 = +4.00\ \mu C$ is brought in from very far away and placed at the origin (the other two charges do not move). How much work is required to make this happen? What is the total potential energy stored in the final configuration of three charges? *[3 marks]*

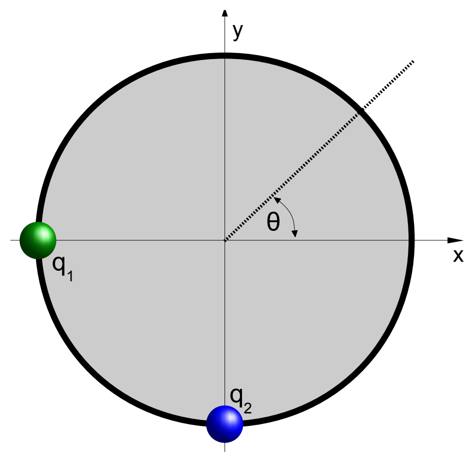

Figure 8: Two charges on a circle.

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15) *[8 marks]* A total charge Q is uniformly distributed along a rod of length L. The rod is located on the x-axis with the midpoint at $x = 0$.

a) Draw a sketch of the problem including axes and labels. *[1 mark]*

b) Consider a small segment of the rod dx. Write down an expression for the potential dV at an arbitrary location some distance r from the small segment. *[1 mark]*

c) Derive an integral expression for the electric field $\vec{E}(x = L, y = 0)$ produced by the entire rod at a point on the x-axis at $x = L$. Show your work, but you do not need to evaluate the final integral. *[2 marks]*

d) Use Gauss's law to derive a general expression for the electric field, assuming that the rod is very long ($L \to \infty$). Show all steps and clearly state any assumptions. *[3 marks]*

e) What is the linear charge density λ if $Q = -3.00$ Coulombs and $L = 2.00$ metres? *[1 mark]*

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16) *[5 marks]* The electric field in the $x-y$ plane ($z = 0$) is given by the formula $\vec{E} = 3.00y^2\,\hat{\imath} + 6.00xy\,\hat{\jmath}$. We are interested in finding the voltage differences between three locations: A ($x = 1, y = 1$), B ($x = 5, y = 1$), and C ($x = 5, y = 4$).

a) Calculate the potential difference between point A and point B. *[2 marks]*

b) Calculate the potential difference between point B and point C. *[2 marks]*

c) If the potential at point A is +62 volts, what is the potential at point C? *[1 mark]*

CONSTANTS AND USEFUL EQUATIONS

k = Coulomb constant = 8.99×10^{9} N m^2 C^{-2}
ϵ_0 = permittivity of free space = 8.85×10^{-12} C^2 N^{-1} m^{-2}
e = fundamental charge = 1.602×10^{-19} C
m_p = mass of proton = 1.67×10^{-27} kg
m_e = mass of electron = 9.11×10^{-31} kg

$\mathrm{m} = 10^{-3}$ $\mu = 10^{-6}$ $\mathrm{n} = 10^{-9}$ $\mathrm{p} = 10^{-12}$

Surface area of a sphere: $A = 4\pi r^2$ Volume of a sphere: $A = \frac{4}{3}\pi r^3$
Area of a circle: $A = \pi r^2$ Circumference of a circle: $C = 2\pi r$

$$x = x_0 + v_{x0}t + \frac{1}{2}a_x t^2 \qquad v_x = v_{x0} + a_x t \qquad v_x^2 = v_{x0}^2 + 2a_x x \qquad \vec{F} = m\vec{a}$$

$$\vec{F} = k\frac{q_1 q_2}{r^2}\,\hat{r} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}\,\hat{r} \qquad \vec{E} = \frac{\vec{F}}{q} \qquad \vec{E} = k\frac{q}{r^2}\,\hat{r} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\,\hat{r}$$

$$\Phi_E = \oint_A \vec{E}\cdot d\vec{A} = \oint_A E\,dA\,cos\theta = \frac{Q_{encl}}{\epsilon_0} \qquad V = \frac{U}{q} \qquad U = k\frac{q_1 q_2}{r} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r}$$

$$V = k\frac{q}{r} = \frac{1}{4\pi\epsilon_0}\frac{q}{r} \qquad W = qV_{ab} \qquad V_{ab} = V_a - V_b = \int_a^b \vec{E}\cdot d\vec{l}$$

$$\vec{E} = -\vec{\nabla}V = -\left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)V \qquad C = \frac{Q}{V_{ab}} \qquad C = \frac{\epsilon_0 A}{d}$$

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}QV = \frac{1}{2}CV^2 \qquad \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \qquad C = C_1 + C_2 + C_3$$

$$u = \frac{1}{2}\epsilon_0 E^2$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin\frac{x}{a} \qquad \int \frac{dx}{\sqrt{x^2 + a^2}} = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

$$\int \frac{dx}{x^2 + a^2} = \frac{1}{a}\arctan\frac{x}{a} \qquad \int \frac{dx}{(x^2 + a^2)^{3/2}} = \frac{1}{a^2}\frac{x}{\sqrt{x^2 + a^2}}$$

$$\int \frac{x\,dx}{(x^2 + a^2)^{3/2}} = -\frac{1}{\sqrt{x^2 + a^2}}$$

Multiple Choice Answer Sheet

#1 a
#2 a
#3 a
#4 a
#5 a
#6 a

#7 a
#8 a
#9 a
#10 a
#11 e
#12 a

Paul's bubble sheet goes here

Written Answer Sheets

1. You are responsible for marking the Problems, Part II. The multiple choice answers are scored automatically. The "Version" number applies only to the multiple-choice answers; there is only one version of Part II. You should also have these general instructions, plus a set of solutions with detailed instructions, and a copy of an exam question paper. The solutions are presented in “Answer Page format” for ease of reference.

2. You can pick up your papers from the drop boxes in Science Theatres basement. I believe Paul has written the number of papers on the envelope. Count your papers and make sure you turn in the same number to Paul when you are finished marking.

3. Markers must have all exam papers marked before **4:30 pm Wednesday afternoon, March 7.** Please return them either directly to Paul, or to the drop box where you picked up your envelope, and send Paul an e-mail letting him know the marked papers are there.

4. Print out these **Instructions**, and read them before starting to mark. You might be tempted to bypass these, but you will save us much time and trouble if you follow instructions.

5. **Marking**. Please make your comments on student work in RED INK and initial them. Bear in mind that the students will see a scanned black-and-white version, so your comment should stand out from their work…your initials on each comment will help to do this. Also, sometimes we are tempted to write in a correct answer or missing units in red pen. Please do not do that, because students may claim it as their own work. It is helpful, however, to mark wrong answers with a medium-large “X”.

6. **How to Enter the Mark on the first page of the Answer Sheets**. Enter the total points awarded for each part of a problem by **FILLING IN** the “bubble” (small circle) corresponding to that number of points. The Answer Sheets will be computer-read, so **FILL IN** the bubble; do not put a check mark or "x" there:

   Correct way ● **Wrong** way **Wrong** way ⊗

   USE PENCIL so that you can erase if you change your mind (this will happen frequently.) If you make an error entering the mark and can't erase it, cross it out: ●

8. On the following pages, I've given a suggested detailed marking scheme for each question. However, not all solutions will follow the one I've given, especially for the 3-mark questions, so use your judgment. The following broad guidelines may help:

   In 3-mark questions, the first half mark should be easy to get: if a student shows any reasonable insight at all, give a half mark. In an integration problem, If the student simply makes a valiant attempt at integrating, this might be enough.

   Give half of the allotted marks (e.g., 1.5 marks out of 3) if you feel that the student has got half way to the right answer. For example, if a student makes five mistakes but still has the solution half right, give 1.5; don't deduct five half-marks.

   In numerical questions, the **final** answer must have correct units. Deduct 0.5 mark if the units on the final answer are not shown or are incorrect.

   Be lenient on significant figures.

   If a lot of students are making the same "mistake", check my answer - I could be the one who is wrong! If you discover any errors in my solutions, please let me know as soon as possible so I can alert all other markers.

Bill Wilson
Phys 259 Coordinator
February 29, 2012

UCID: | | | | | | | | |

Q14a *[2 marks]*

-1/2 for inconsistent use of vectors ie. vector=scalar, -1 for adding $\vec{E}$ as scalars.

The electric field due to a point charge q is

$$\vec{E}(x,y) = \frac{1}{4\pi\epsilon_0}\frac{q}{R^2}\hat{i}$$

The vector pointing from charge #1 to the origin is $\vec{r} = (0\hat{i} + 0\hat{j}) - (-R\hat{i} + 0\hat{j}) = R\hat{i}$ so $|\vec{r}| = r = R$ and $\hat{r} = \hat{i}$. The electric field at the origin due to charge #1 is

$$\vec{E}_1(x{=}0, y{=}0) = \frac{1}{4\pi\epsilon_0}\frac{q_1}{R^2}\hat{i} = 8.99 \times 10^9\, m^2N/C^2\, \frac{+3.00 \times 10^{-6}\ C\hat{i}}{(20.0\ m)^2} = \boxed{67.425\hat{i}\ N/C}$$

For charge #2 the vector pointing from the source to the origin is $\vec{r} = (0\hat{i}+0\hat{j})-(0\hat{i}-R\hat{j}) = R\hat{j}$ so $r = R$ and $\hat{r} = \hat{j}$. The electric field at the origin is

$$\vec{E}_2(x{=}0, y{=}0) = \frac{1}{4\pi\epsilon_0}\frac{q_2}{R^2}\hat{j} = \boxed{-157.325\hat{j}\ N/C}$$

Total electric field using superposition (ie. add vectors)

$$\vec{E}(x{=}0, y{=}0) = \vec{E}_1 + \vec{E}_2 = \frac{1}{4\pi\epsilon_0}\frac{q_1\hat{i} + q_2\hat{j}}{R^2}$$

recalculating terms not required

$$= 8.99 \times 10^9\, m^2N/C^2\, \frac{(+3.00\hat{i} - 7.00\hat{j}) \times 10^{-6}\ C}{(20.0\ m)^2}$$

$$= \boxed{(67.4\hat{i} - 157.3\hat{j})\ V/m}$$

-1/2 for wrong or no units

Be flexible with significant figures | Conversion to magnitude and angle not required

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Q14b *[2 marks]*

-1 for treating V as vectors

Potential at the origin due to both point charges

$$V(x=0, y=0) = V_1 + V_2 = \frac{1}{4\pi\epsilon_0}\frac{q_1}{R} + \frac{1}{4\pi\epsilon_0}\frac{q_2}{R} = \frac{1}{4\pi\epsilon_0}\frac{q_1 + q_2}{R}$$
$$= 8.99 \times 10^9\, m^2N/C^2\ \frac{(+3.00 - 7.00) \times 10^{-6}\ C}{20.0\ m}$$
$$= (+1348.5 - 3146.5)\ Nm/C = \boxed{-1798\ Nm/C} = \boxed{-1.80 \times 10^3\ V}$$

Q14c *[3 marks]*

The energy is required to add charge q_3 is $\Delta U = q_3 \Delta V$ where the potential difference is between zero at infinity and V at the origin as found in part (b).

$$U_f - U_i = q_3(V_f - V_i) = (4.00 \times 10^{-6}\ C)(-1798\ V) = -7.19 \times 10^{-3}\ Joules$$

Question does not specify which "work done" (ie. by us or by the field), so either sign ($\pm$) is okay

Alternative is to add potential energy of two pairs (slightly more calculations)

For total configuration energy we need to add the energy due to the first two charges separated by distance $r = \sqrt{R^2 + R^2} = \sqrt{2}R$

$$U = \frac{q_1 q_2}{4\pi\epsilon_0 r} = \frac{q_1 q_2}{4\pi\epsilon_0 \sqrt{2}R}$$
$$= 8.99 \times 10^9\, m^2N/C^2\ \frac{(+3.00 \times 10^{-6}\ C)(-7.00 \times 10^{-6}\ C)}{\sqrt{2}(20.0\ m)}$$
$$= -6.67 \times 10^{-3}\ Nm$$

for a total configuration energy of

$$U_{total} = (-6.67\ mJ) + (-7.19\ mJ) = \boxed{-1.39 \times 10^{-2}\ J}$$ -1/2 for wrong sign

Alternative is to add potential energy of all three pairs (slightly more calculations)

$$U = U_{12} + U_{13} + U_{23} = \boxed{-1.39 \times 10^{-2}\ J}$$

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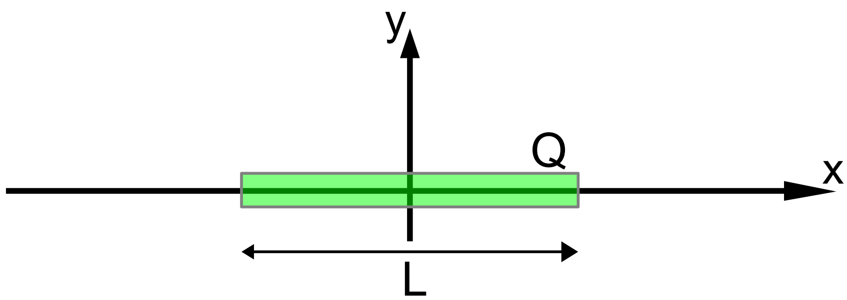
Q15a *[1 mark]*



Most students will get 1/1 for this.

Q15b *[1 mark]*

The electric potential produced by an infinitesimal segment of a charged rod is

$$dV = \frac{dq}{4\pi\epsilon_0 r} = \frac{\lambda dx}{4\pi\epsilon_0 r} \qquad \text{scalar } \underline{\text{not}} \text{ vector}$$

-1/2 mark for not saying $dq = \lambda dx$

Q15c *[2 marks]*

The electric field due to a small element of charge is

$$d\vec{E} = \frac{dq}{4\pi\epsilon_0 r^2}\hat{r} = \frac{\lambda dx}{4\pi\epsilon_0 (L-x)^2}\hat{i} \qquad \textbf{must indicate vectors}$$

where for a field point at $x = L$ the vector from the source point at x is

$$\vec{r} = (L\hat{i}) - (x\hat{i}) = (L-x)\hat{i}$$

The total electric field due to the entire rod

$$\vec{E} = \int d\vec{E} = \int_{-L/2}^{+L/2} \frac{\lambda dx}{4\pi\epsilon_0 (L-x)^2}\hat{i} = \boxed{\frac{\lambda}{4\pi\epsilon_0}\int_{-L/2}^{+L/2} \frac{dx}{(L-x)^2}\hat{i}}$$

Alternative ($u = x - L$)

$$\vec{E} = \int d\vec{E} = \boxed{\frac{\lambda}{4\pi\epsilon_0}\int_{L/2}^{3L/2} \frac{dx}{x^2}\hat{i}}$$

Be relaxed about direction of integral (left to right or right to left) and corresponding $\pm$ sign

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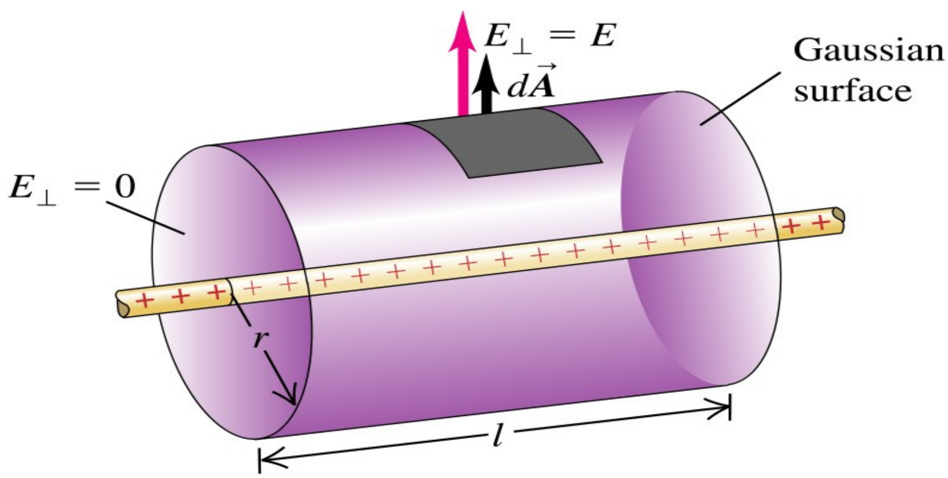
Q15d *[3 marks]*

Must draw or describe Gaussian cylinder



Must give Gauss's law $\int \vec{E} \cdot d\vec{a} = \frac{Q_{enc}}{\epsilon_0}$ where the charge enclosed is just $Q = \lambda L$.
The Gaussian surface has three parts. By symmetry we know that the electric field of a very long line of charge will be purely radial so the end-caps have no flux

$$\int \vec{E} \cdot d\vec{a} = \Phi_1 + \Phi_2 + \Phi_3 = \int (E\hat{r}) \cdot (+\hat{z}da) + \int (E\hat{r}) \cdot (-\hat{z}da) + \int (E\hat{r}) \cdot (\hat{r}da) = 0 + E \int da + 0$$

and the area of the "tube" is $2\pi rL$. As expected, the length cancels to give the radial field

$$E_r = \frac{\lambda}{2\pi\epsilon_0 r}$$

Q15e *[1 mark]*

For a uniformly distributed linear charge

$$\lambda = \frac{Q}{L} = \frac{-3.00\ C}{2.00\ m} = -1.50\ C/m$$

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UCID:

Q16a *[2 marks]*

Question asks for "potential difference" so either V_{AB} or V_{BA} is okay if it has the correct $\pm$ sign

Start with the definition $V_{AB} = \int_A^B \vec{E} \cdot d\vec{l}$. Then note that the path from A to B is just in the x-direction $d\vec{l} = dx\,\hat{i}$ and at constant $y = 1$ so $\vec{E}(x, y = 1) = 3.0\hat{i} + 6.0x\hat{j}$

$$V_A - V_B = V_{AB} = \int_A^B (3.0\hat{i} + 6.0x\hat{j}) \cdot (dx\,\hat{i})$$

$$= \int_{x=1}^{x=5} 3.0(\hat{i} \cdot \hat{i})dx = 3.0 \int_{x=1}^{x=5} dx$$

$$= 3.0[x|_1^5 = 3.0[5 - 1] = \boxed{12.0\ Volts}$$

Q16b *[2 marks]*

The path from B to C is just in the y-direction $d\vec{l} = dy\,\hat{j}$ and at constant $x = 5$ so $\vec{E}(x = 5, y) = 3.0y^2\hat{i} + 30.0y\hat{j}$.

$$V_B - V_C = V_{BC} = \int_B^C (3.0y^2\hat{i} + 30.0y\hat{j}) \cdot (dy\,\hat{j}) = \int_{y=1}^{y=4} 30.0(\hat{i} \cdot \hat{i})ydy$$

$$= 30.0 \int_{y=1}^{y=4} ydy = 30.0 \left[\frac{y^2}{2}\right|_1^4$$

$$= 15.0[16 - 1] = \boxed{225.0\ Volts}$$

Q16c *[1 mark]*

Potential differences add

$$V_{AB} + V_{BC} = V_{AC}$$

so the total potential difference is

$$V_A - V_C = V_{AC} = (+12\ V) + (+225\ V) = +237\ V$$

which means that point A is 237 Volts above point C. If $V_A = +62\ V$ then

$$V_C = V_A - V_{AC} = (+62\ V) - (+237\ V) = \boxed{-175\ V}$$

Q16a-c *[3 marks]*

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An alternative approach is to calculate or "guess" (by inspection) the potential function

$$V(x, y) = (-3xy^2 + 65)\ V$$

check that it gives the right electric field ($\vec{E} = -\nabla V$) and calculate the potential at each point

$$V(1,1) = 62\ V \qquad V(5,1) = 50\ V \qquad V(5,4) = -175\ V$$