Last Time:

- Applications of Ampère’s Law
  - Magnetic field of an infinite slab of current
  - Magnetic field of a solenoid and a toroid
  - Displacement current (parallel plate capacitors)

Today:

- Faraday’s Law of Induction
- Non-conservative electric fields
- Motional emf
- Applications to useful technologies

Faraday's Law of Induction

Electrostatics: E-field from motionless charges
Magnetostatics: B-field from charges in motion

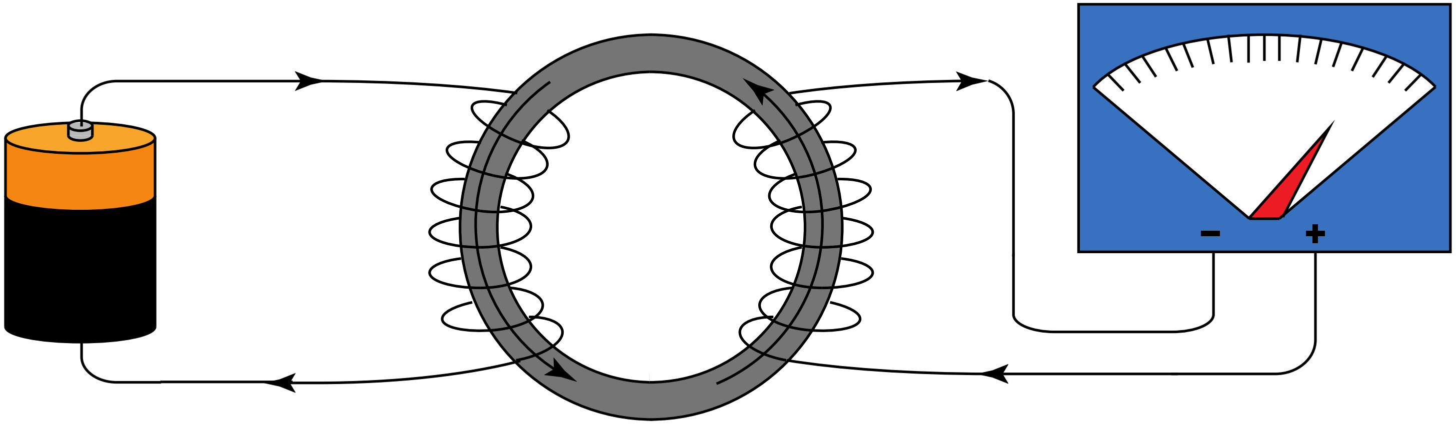
Changing electric fields (moving charges) create magnetic fields. Is the opposite true?

YES!

$$\varepsilon = -\frac{d\Phi_M}{dt}$$

i.e., A changing magnetic flux creates an induced EMF.

Faraday’s Initial Experiment (+ demo)



Faraday discovered that there is an **induced EMF** in the secondary circuit given by

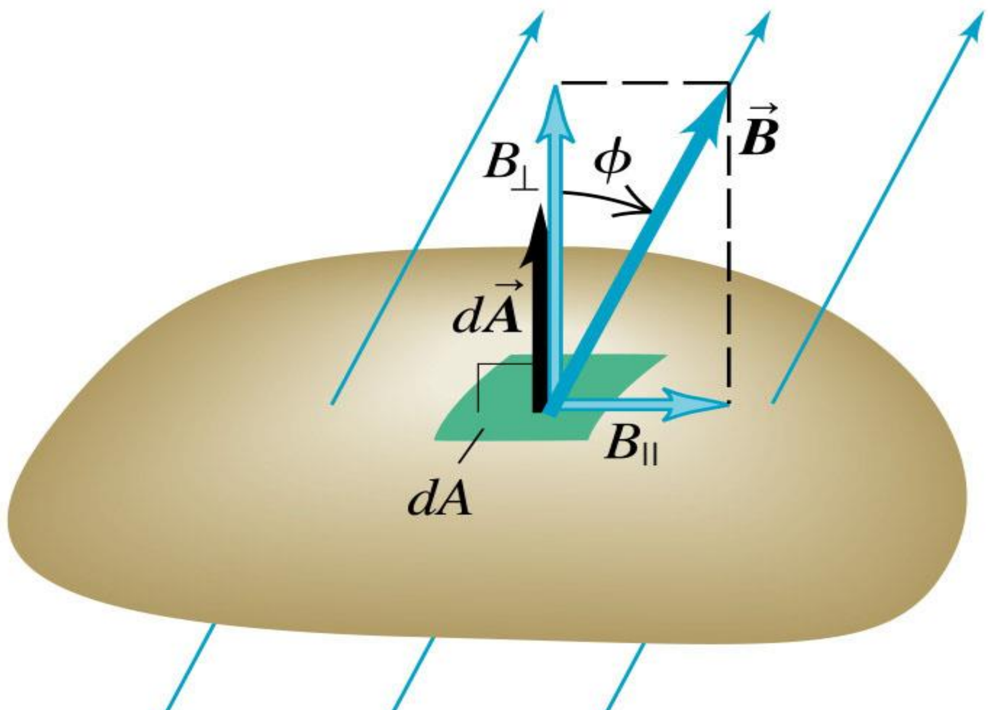
$$e = -\frac{dF_M}{dt}$$

This is a new generalized law called **Faraday's Law**.

Recall the definition of magnetic flux:

$$\Phi_M = \int \vec{B} \cdot d\vec{A}$$

Not a closed surface!



This is valid even if Φ_M changes because of a time dependent A or angle ϕ (without changing the magnetic field)!

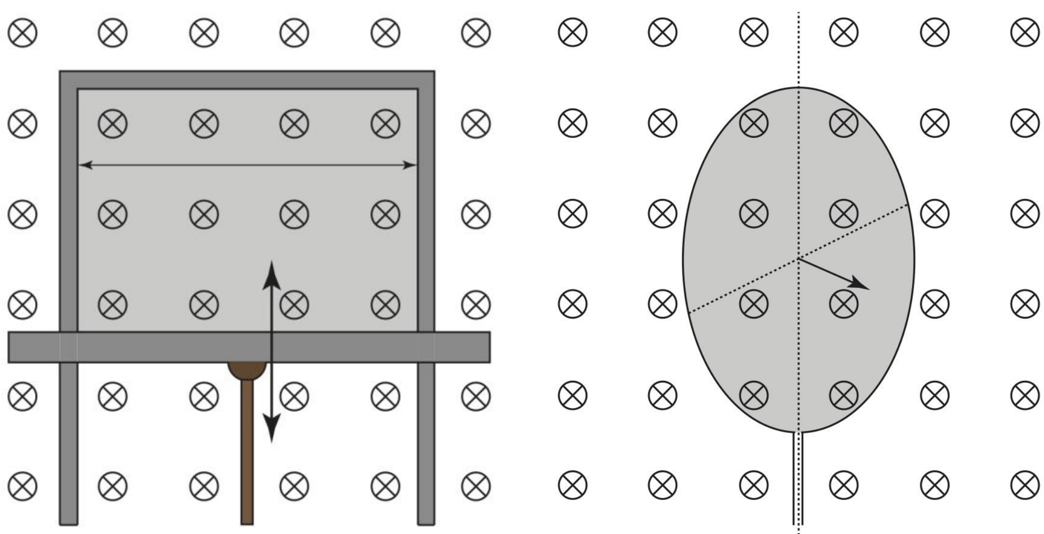
$$\boxed{\varepsilon = -\frac{d}{dt}\left(BA\cos\phi\right)}$$

→ 3 possible terms

$$\varepsilon = -\frac{dB}{dt}A\cos\phi - \frac{dA}{dt}B\cos\phi + \frac{d\phi}{dt}BA\sin\phi$$

From Maxwell Eq.

$$-\frac{d\vec{B}}{dt} = \nabla \times \vec{E}$$



Top Hat Question

A square loop of wire with a resistance of 1 Ω is moving with a constant velocity of 1 m/s through a uniform magnetic field as shown. What is the current induced in the loop? Pick the closest answer
(Note: 1 Ampere = 1 Coulomb/sec)

The magnetic flux through the loop is not changing, so there is no induced emf and hence no induced current

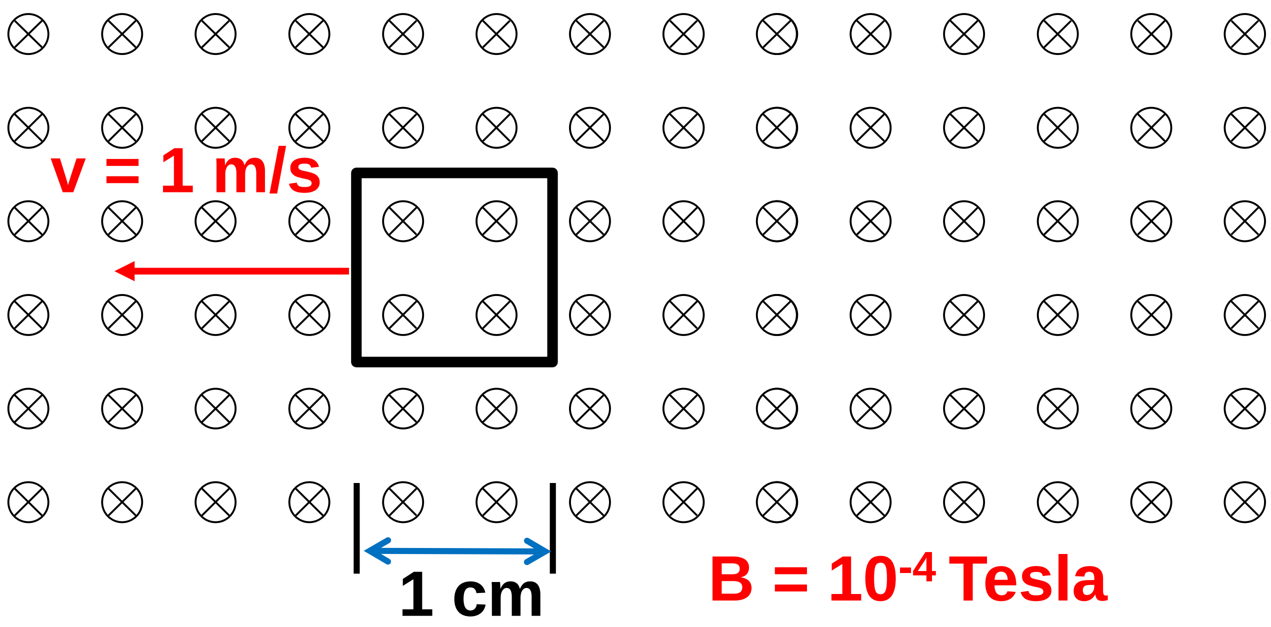
A. 0 A

B. 0.001 A

C. 0.01 A

D. 0.1 A

E. 1 A

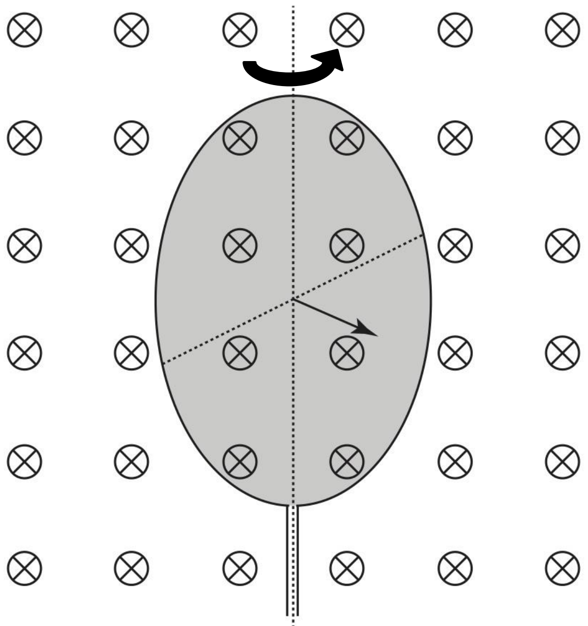


Top Hat Question

A loop of wire is spinning rapidly about a stationary vertical axis in a uniform B-field. Is there a current (or EMF) induced in the loop?

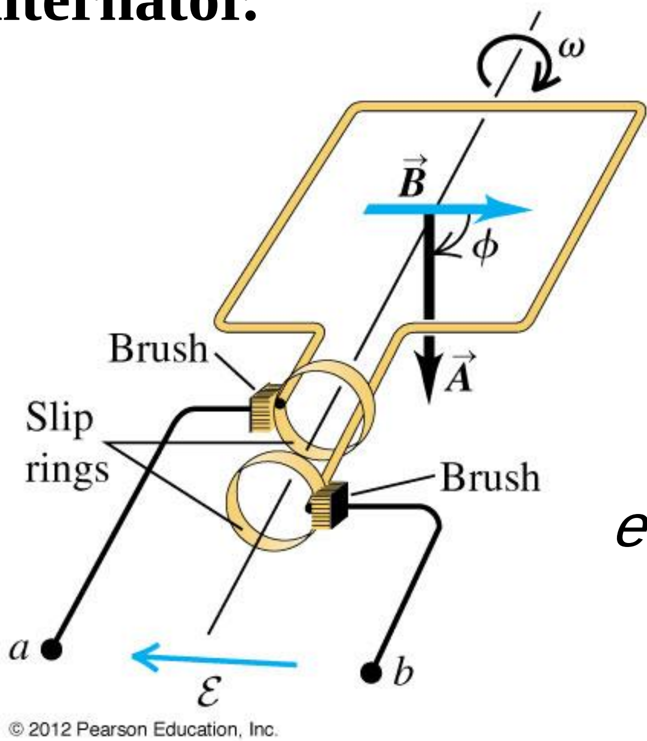
A. Yes, a DC current is induced
B. Yes, an AC (time varying) current is induced
C. The B-field is not changing, so no currents are induced

In this case, the flux through the loop is changing with time because of the **B•A** term, so there will be an induced current (or emf) in the loop. The normal vector is changing direction so half the time the flux is positive and half the time it is negative: i.e. an AC current is induced.



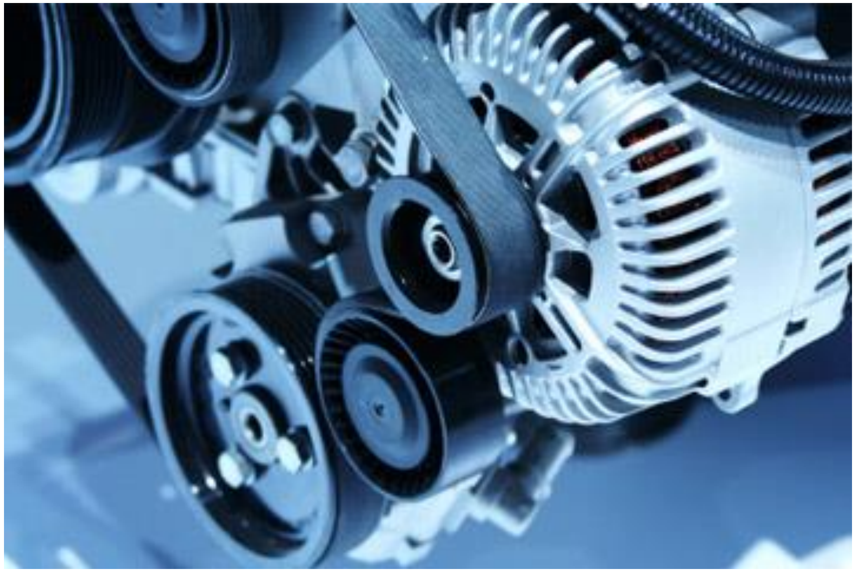
Application: a simple alternator

An *alternator* is an electromechanical device that *converts mechanical energy to electrical energy* in the form of alternating current. In principle, any *AC electrical generator* can be called an alternator.



$$\varepsilon = -\frac{d\phi_B}{dt}$$

$$\phi_B = \oint B\,dA$$

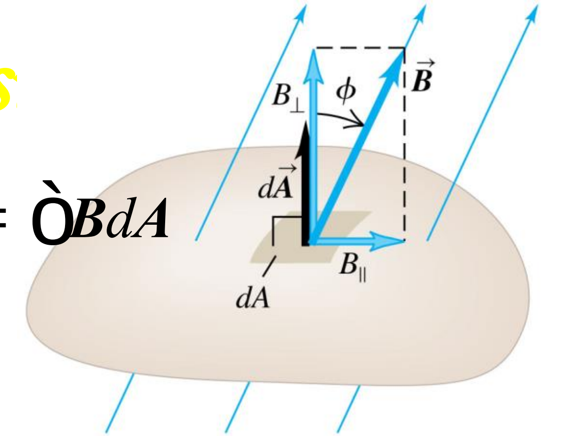


Alternators are used in cars to charge the battery and to power the electrical system when its engine is running. In practice, the loop is stationary and a magnet rotates.

Application: a s

$$\varepsilon = -\frac{d\phi_B}{dt}$$

$$\phi_B = \oint B\,dA$$



Magnetic flux through element of area $d\vec{A}$:
$d\Phi_B = \vec{B} \cdot d\vec{A} = B_\perp dA = B\,dA\cos\phi$

- **The magnetic field and the area are constant, but the angle between the two changes constantly**

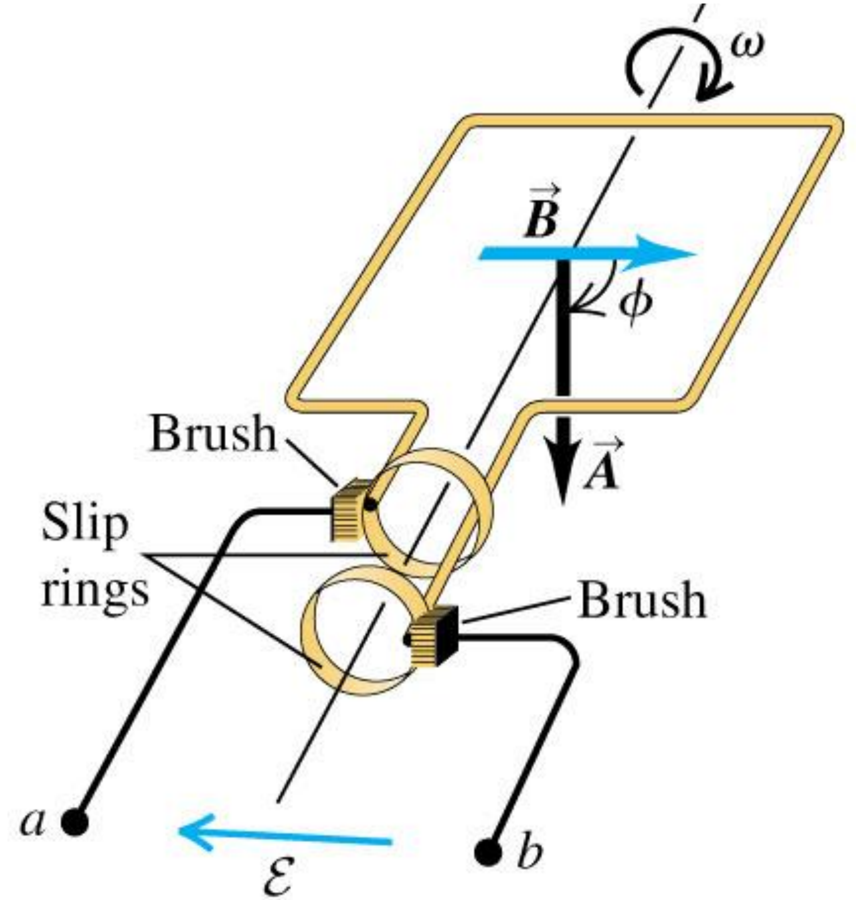
$$\phi = \omega t$$

Hence, the time-dependent magnetic flux is

$$\phi_B = BA\cos\phi = BA\cos\omega t$$

- **The alternator thus generates a sinusoidally varying EMF**

$$\varepsilon = -d\phi_B/dt$$
$$= -(-\omega BA\sin\omega t)$$
$$= \omega BA\sin\omega t$$



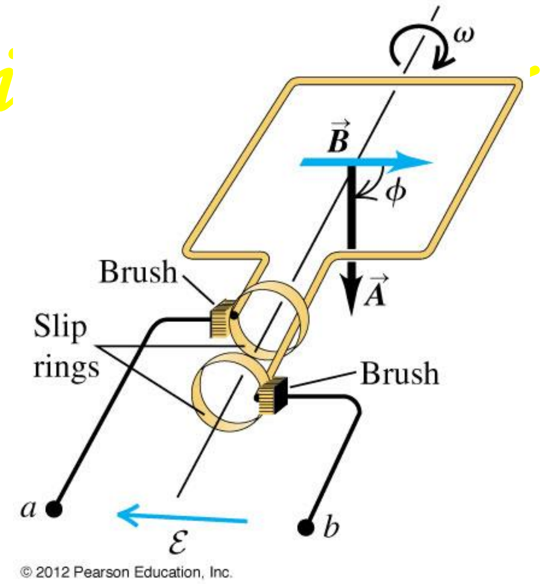
Application: a si

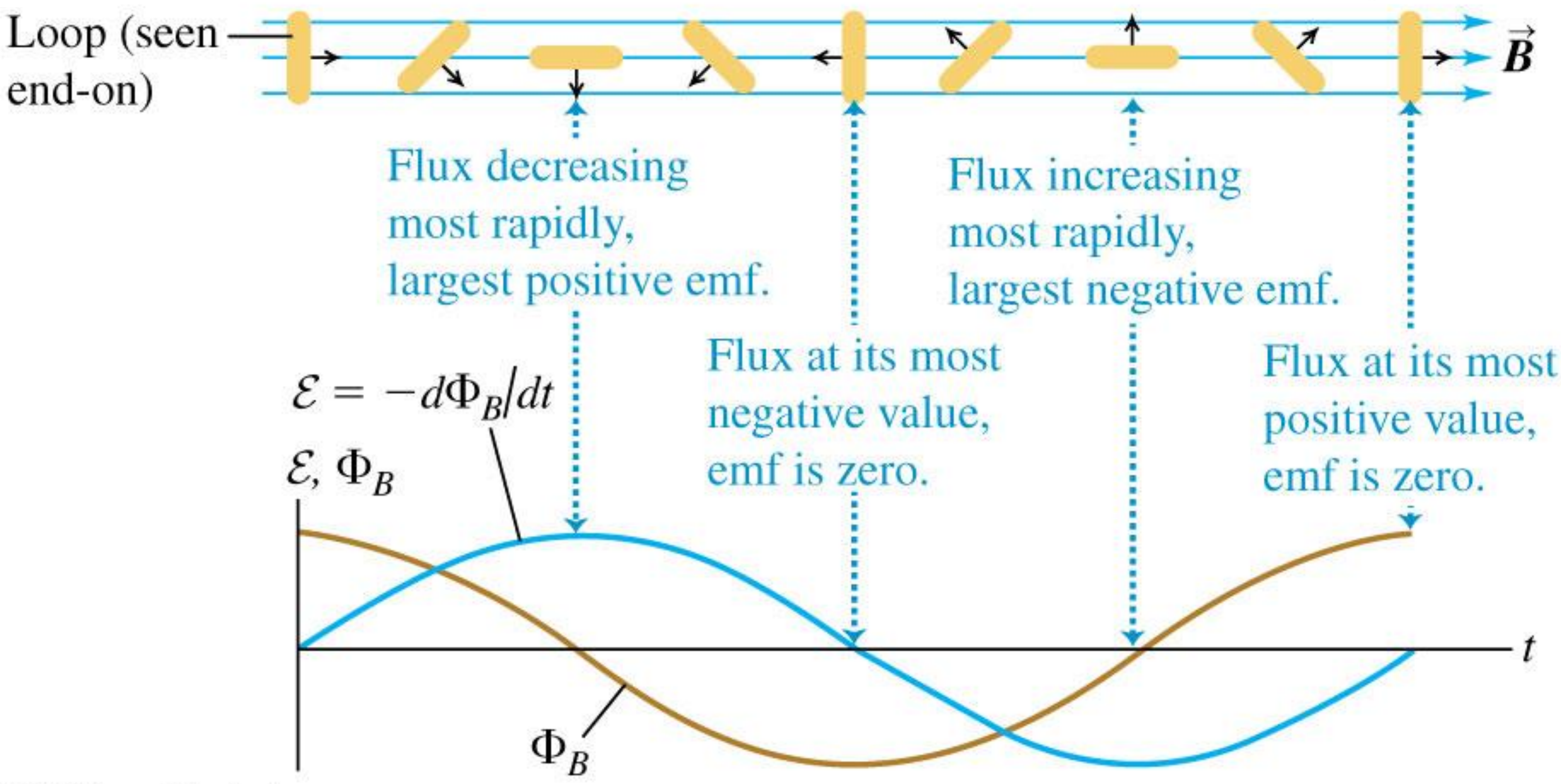
$\Phi = \omega t$

$\phi_B = BA \cos\phi = BA \cos\omega t$

$\varepsilon = - d\phi_B/dt$

$= \omega BA \sin\omega t$



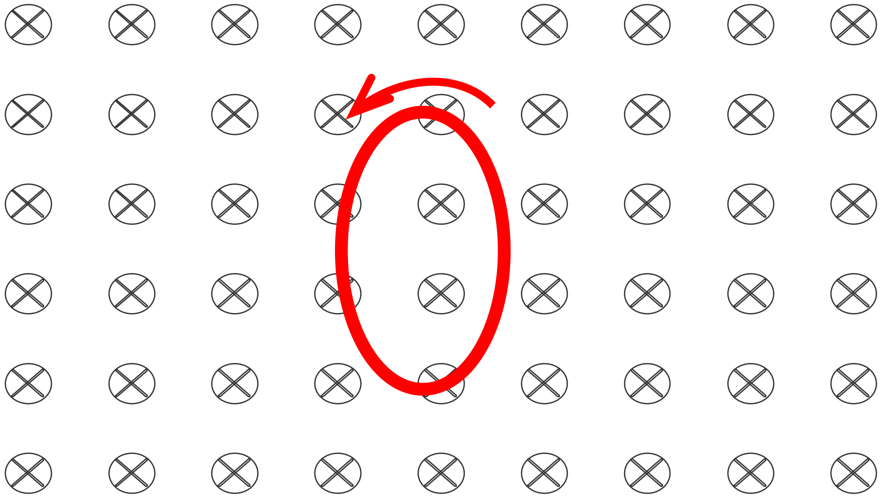


Top Hat Question

An oval shaped loop is spun around an axis pointing out of the page passing through the center of the loop. Is there a current (or EMF) induced in the loop?

A: Yes, there is

B: No, there is not



Last Time:

- Faraday's Law of Induction
- Non-conservative electric fields
- Lenz's Law

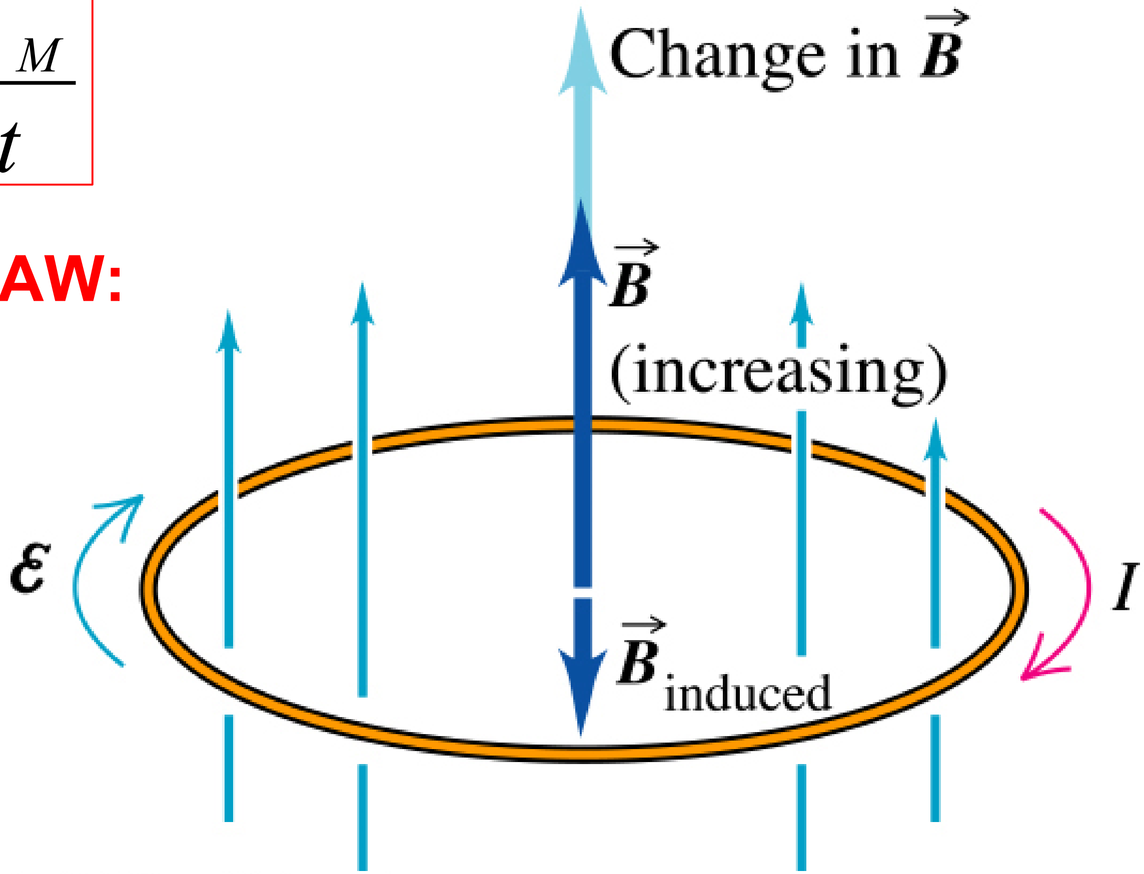
Today:

- Motional emf
- Applications to useful technologies
- Current loops as magnetic dipoles

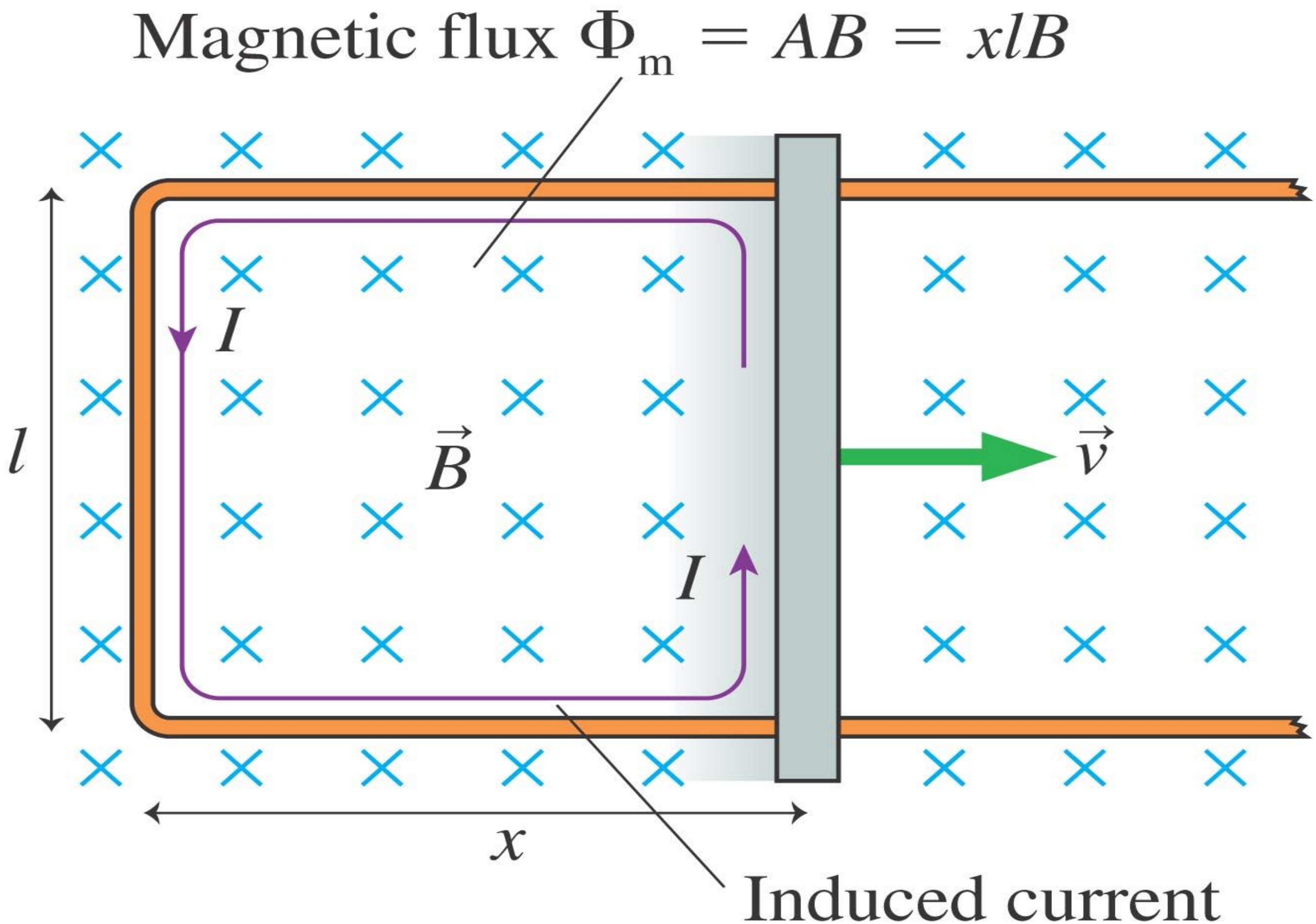
What about the minus sign in Faraday's law?

$$e = -\frac{dF_M}{dt}$$

LENZ'S LAW:

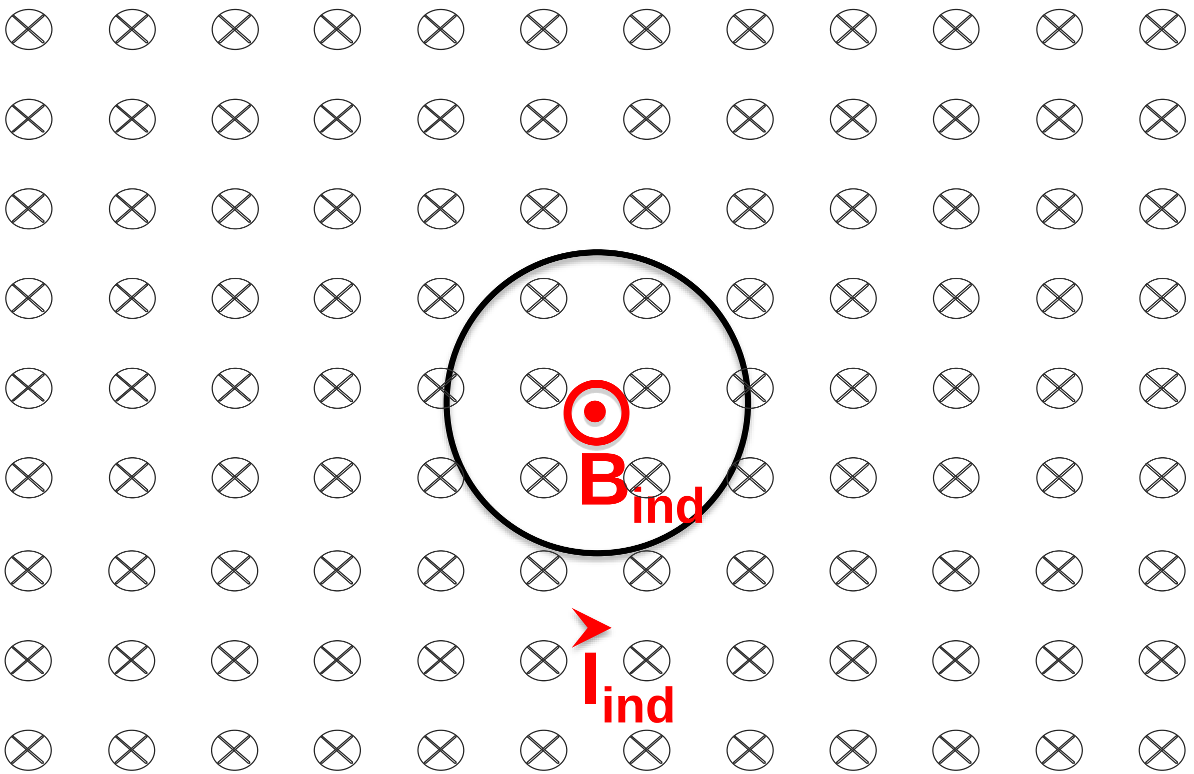


The changing magnetic flux generates an induced current which creates an induced magnetic field which, in turn, resists the change in magnetic flux.


Magnetic flux $\Phi_m = AB = xlB$
I
$\vec{B}$
I
l
$\vec{v}$
x
Induced current

Lenz's Law

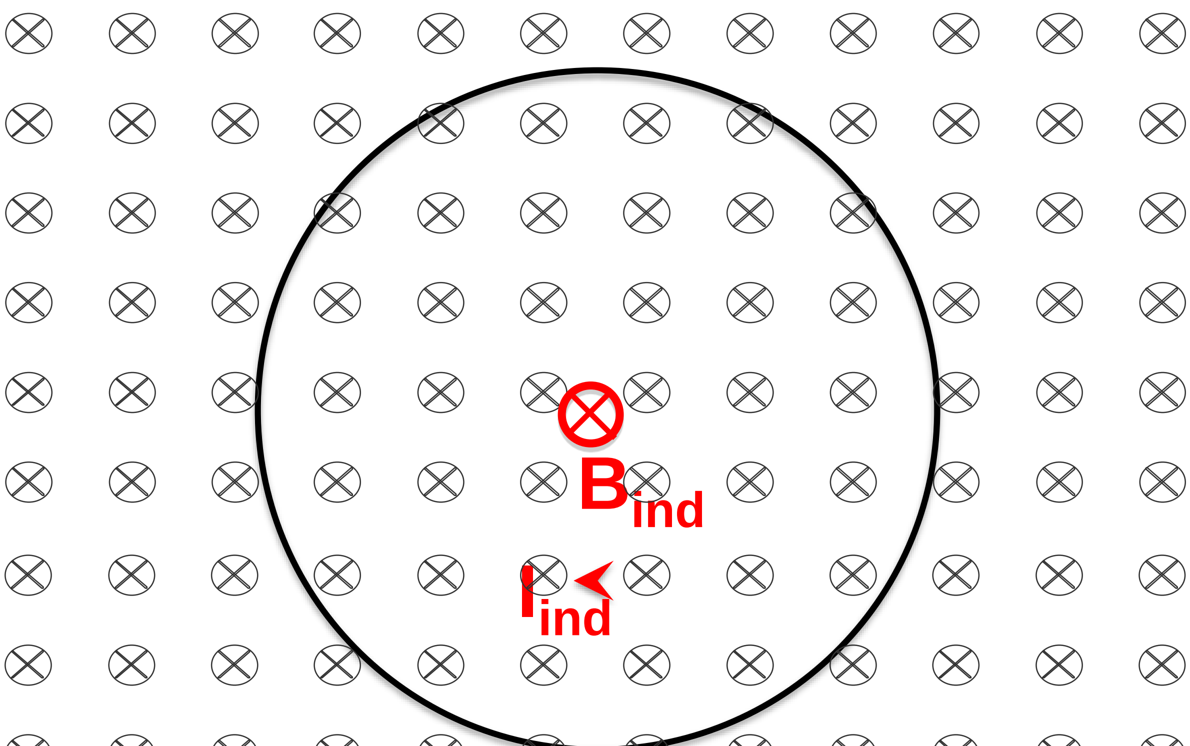
The induced current from Faraday's Law is always in a direction such that the induced magnetic field from the induced current opposes the change in the magnetic flux through the loop.



More B-field lines inside the loop: induced B-field from induced current must be out of the page to compensate.
Induced current is CCW

Lenz's Law

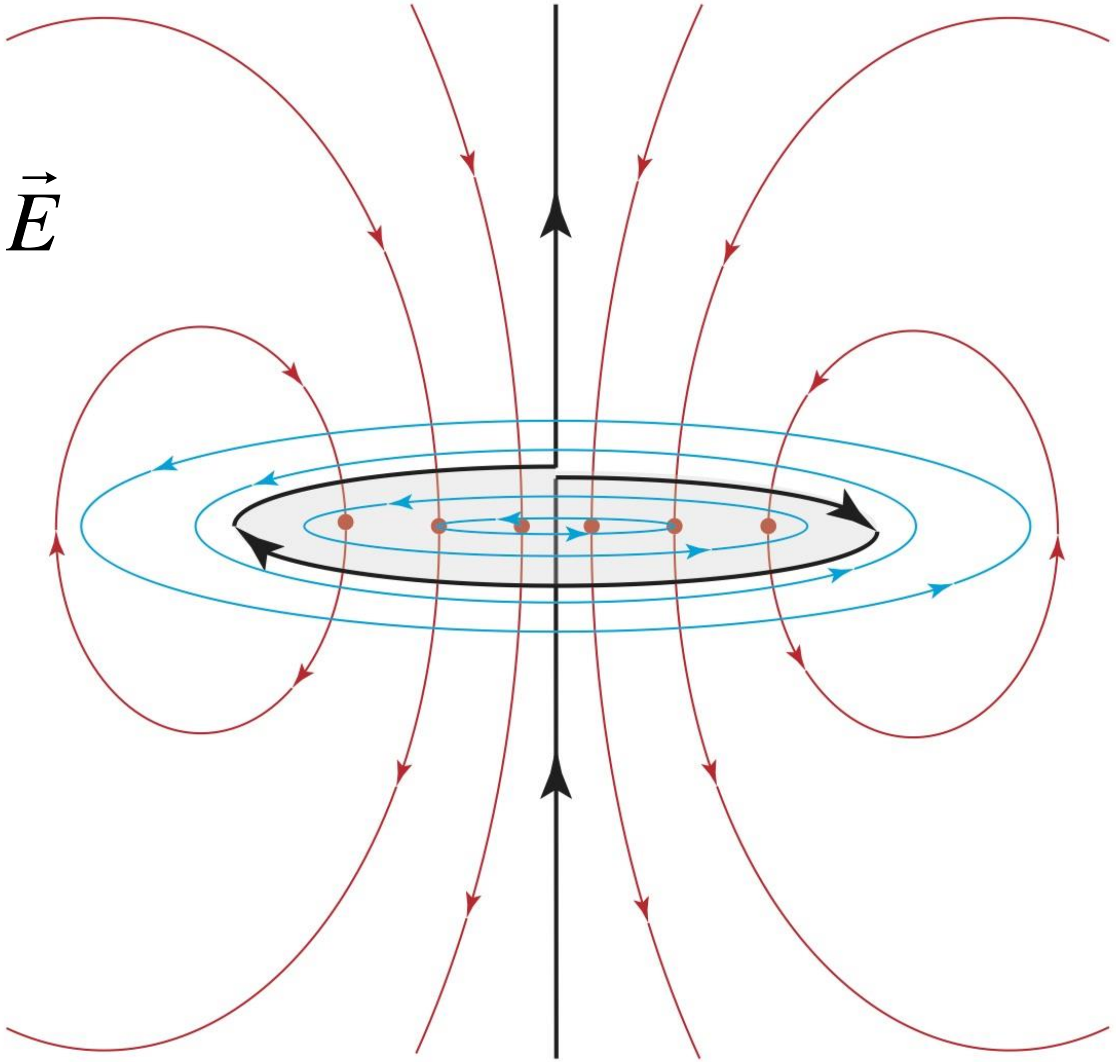
The induced current from Faraday's Law is always in a direction such that the induced magnetic field from the induced current opposes the change in the magnetic flux through the loop.



Fewer B-field lines inside the loop: induced B-field from induced current must be into the page to compensate.
Induced current is CW

Imagine a loop in a wire carrying a current I_1. The current is then increased to $I_2 > I_1$, increasing the magnetic flux.
Changing B-fields induce non-conservative E-fields.

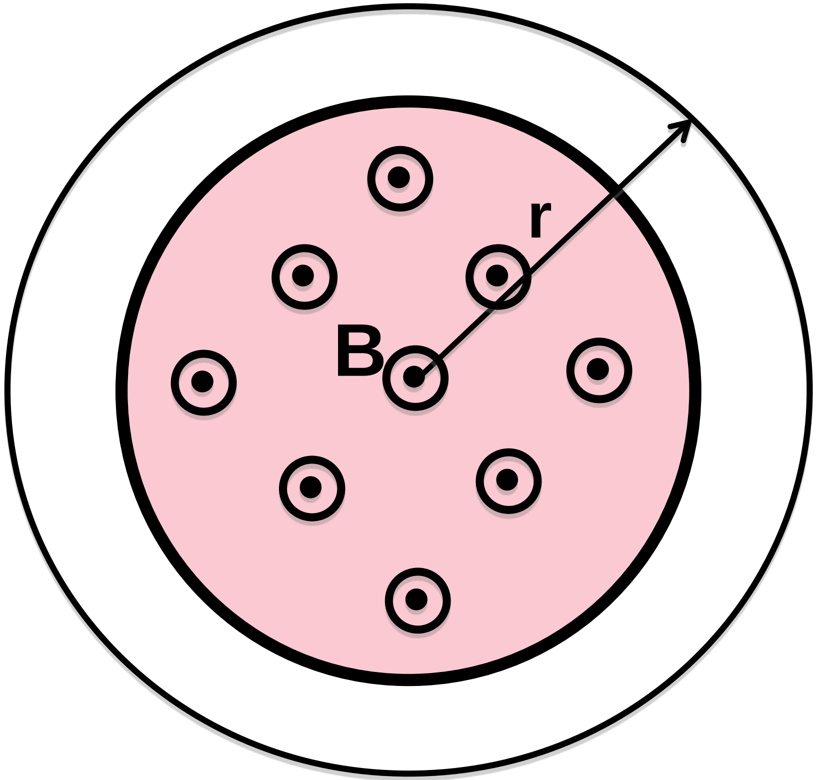
$$-\frac{d\vec{B}}{dt} = \nabla \times \vec{E}$$



Top Hat Question

The current in an infinitely long solenoid with uniform magnetic field B inside is increasing so that the magnitude B increases in time as $B=B_0+kt$. A circular loop of radius r is placed coaxially outside the solenoid as shown. In what direction is the induced **E**-field around the loop ?

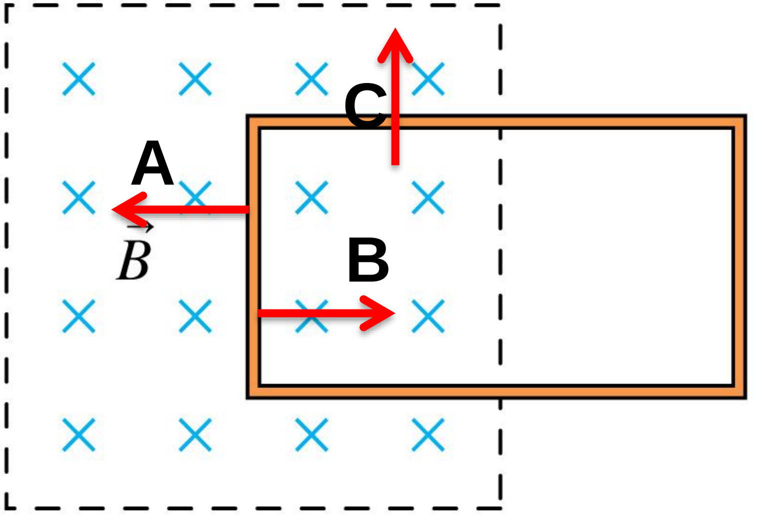
A. CW
B. CCW
C. The induced E is zero
D. Not enough information



Lenz' law: induced EMF around the loop is in the CW direction. The induced E-field must therefore be in the CW direction

Top Hat Question

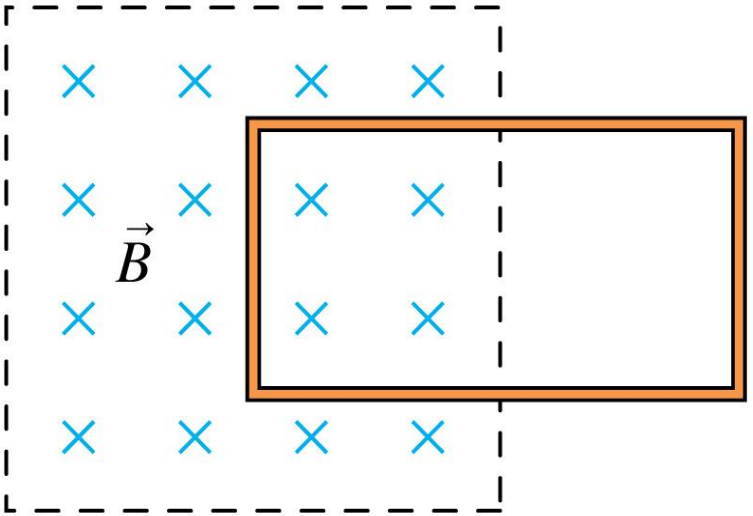
A conducting loop is halfway inside a magnetic field. Suppose the magnetic field begins to increase rapidly in strength. What happens to the loop?



A. The loop is pulled to the left, into the magnetic field.
B. The loop is pushed to the right, out of the magnetic field.
C. The loop is pushed upward, out of the magnetic field
D. The tension in the wire increases but the loop does not move.

Top Hat Question Feedback

A conducting loop is halfway inside a magnetic field. Suppose the magnetic field begins to increase rapidly in strength. What happens to the loop?

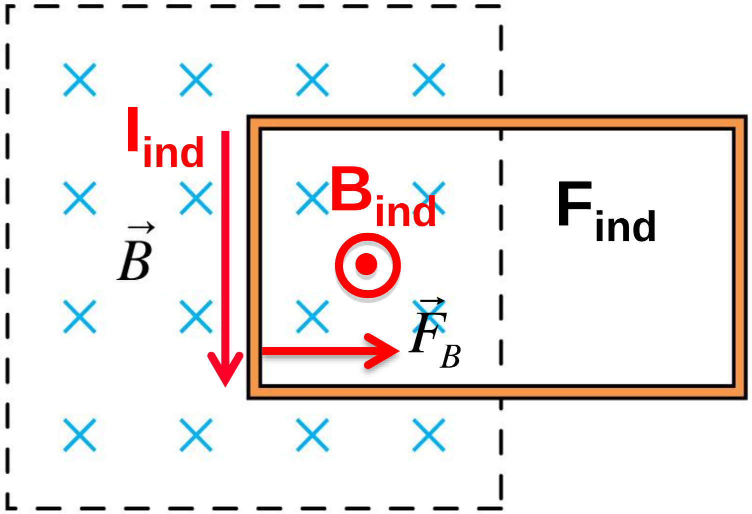


Qualitative argument

Lenz’s Law: whatever happens must be such that it maintains the “amount of B-field” inside the loop. Since the strength of B is increasing, the loop must be pushed outside so that there are fewer B-field lines inside the loop.

Top Hat Question Feedback

A conducting loop is halfway inside a magnetic field. Suppose the magnetic field begins to increase rapidly in strength. What happens to the loop?



More rigorous argument

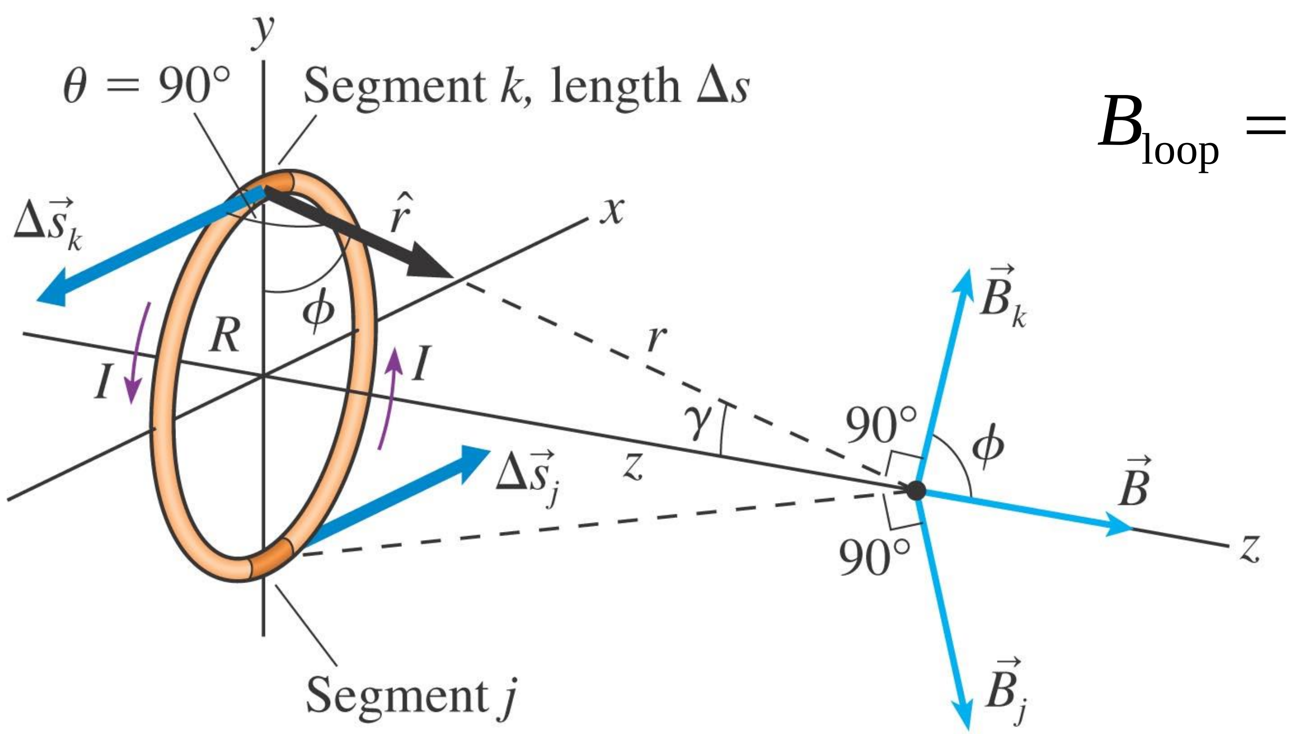
Lenz's Law: B_{ind} must point out, so I_{ind} is CCW

Recall: the Lorentz force on a current carrying wire

$$\vec{F}_B = I\vec{\ell} \times \vec{B} \;\; \Rightarrow$$ points to the RIGHT

Magnetic Field of Current Loop

What is the magnetic field strength **on the axis** of this loop, at **a distance z** from the centre of the loop?

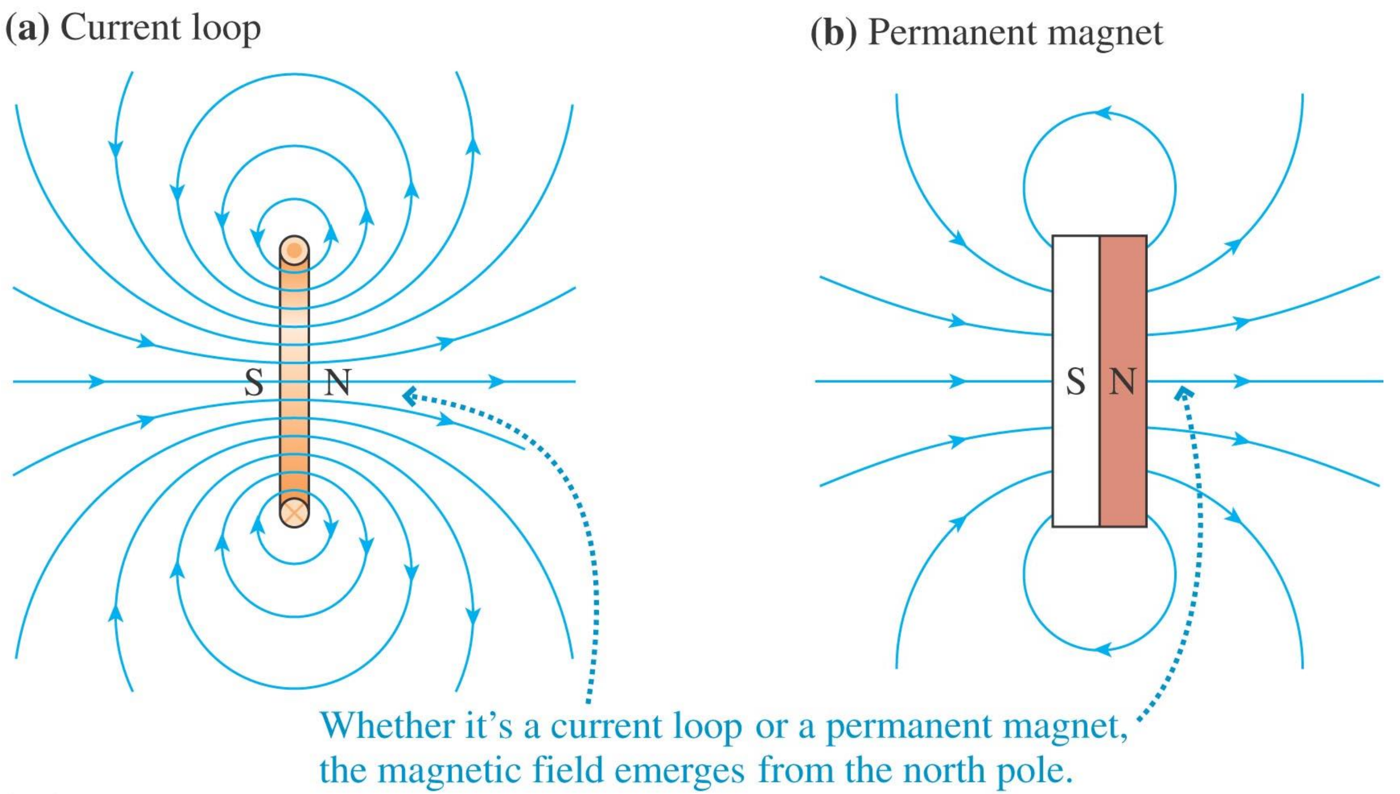


$$B_{loop} = \frac{\mu_0}{2} \frac{IR^2}{\left(z^2 + R^2\right)^{3/2}}$$

Right-hand rule:

Curl your fingers along I and your thumb gives $\vec{B}$ inside the loop.

Magnetic Dipoles

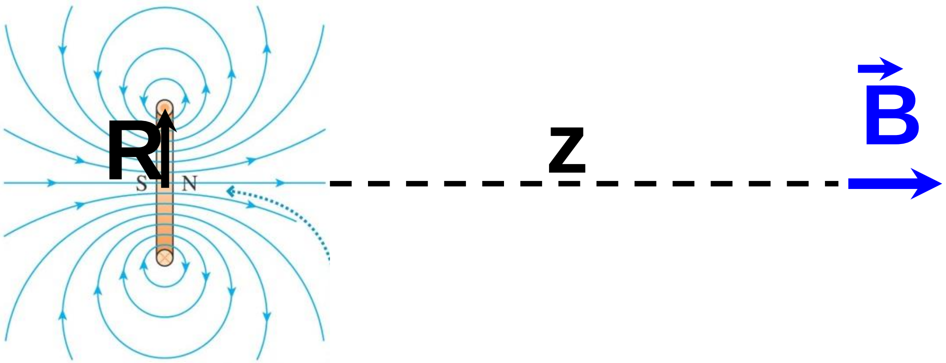


So the field outside a current loop looks like the field outside a permanent magnetic of similar shape.

Two poles (N & S): **magnetic dipole.**

Magnetic Dipoles

What is B on the axis of the loop, a long way away from the loop (z >> R)?



$$B_{\text{loop}} = \frac{m_0}{2}\frac{IR^2}{\left(z^2+R^2\right)^{3/2}}$$

$$= \frac{m_0}{2}\frac{IR^2}{\left(z^2+0\right)^{3/2}}$$

$$= \frac{m_0}{2}\frac{IR^2}{z^3}$$

$$B_{\text{loop}} = \frac{\mu_0}{2}\frac{IR^2}{z^3}\times\frac{2\pi}{2\pi} = \frac{\mu_0}{4\pi}\frac{2(\pi R^2)I}{z^3}$$

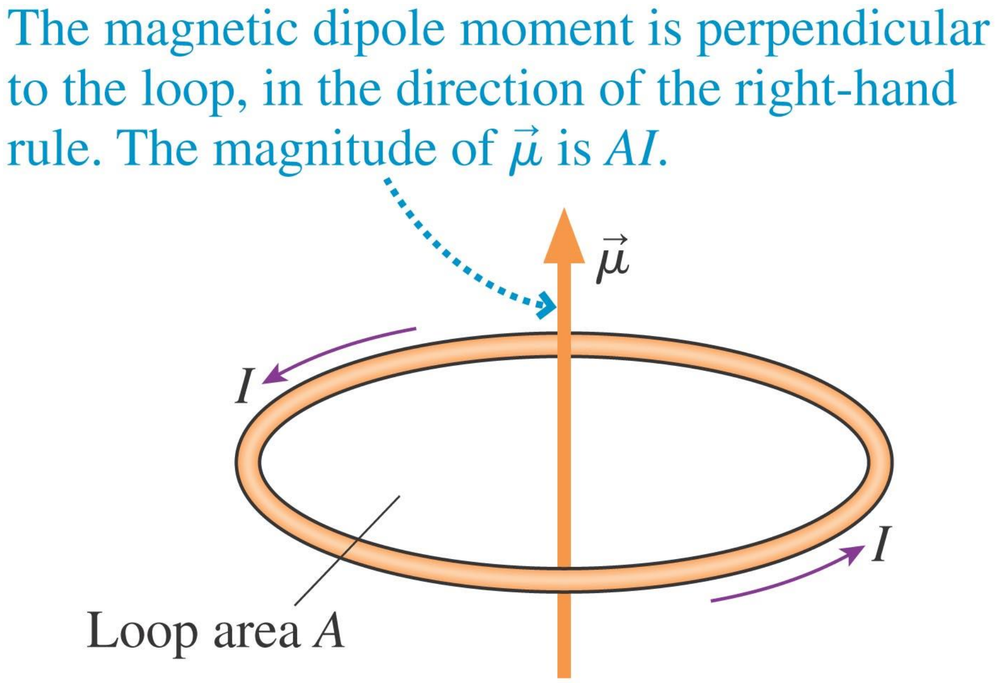
$$\boxed{B_{\text{loop}} = \frac{\mu_0}{4\pi}\frac{2AI}{z^3}}$$ **on the axis of a magnetic dipole.**

Magnetic Dipole Moment

$\vec{\mu} = IA$ **in the direction of the right-hand rule.**

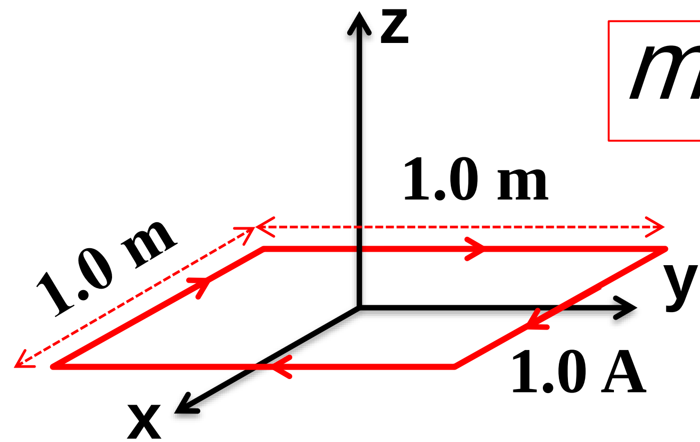
$$\vec{B}_{\text{loop}} = \frac{\mu_0}{4\pi}\frac{2\vec{\mu}}{z^3}$$

Don't confuse $\vec{\mu}$ with μ_0!!



Top Hat Question

A current of 1.0 A is going through a square loop of side length 1.0 m. What is the magnetic dipole moment of this current loop?



$\boxed{m = IA}$

A. $\vec{\mu} = \left(\pi \text{ A} \cdot \text{m}^2\right)\hat{k}$

C. $\vec{\mu} = \left(1.0 \text{ A} \cdot \text{m}^2\right)\hat{k}$

A. $\vec{\mu} = -\left(\pi \text{ A} \cdot \text{m}^2\right)\hat{k}$

D. $\vec{\mu} = -\left(1.0 \text{ A} \cdot \text{m}^2\right)\hat{k}$

Recall there are 3 possible terms:

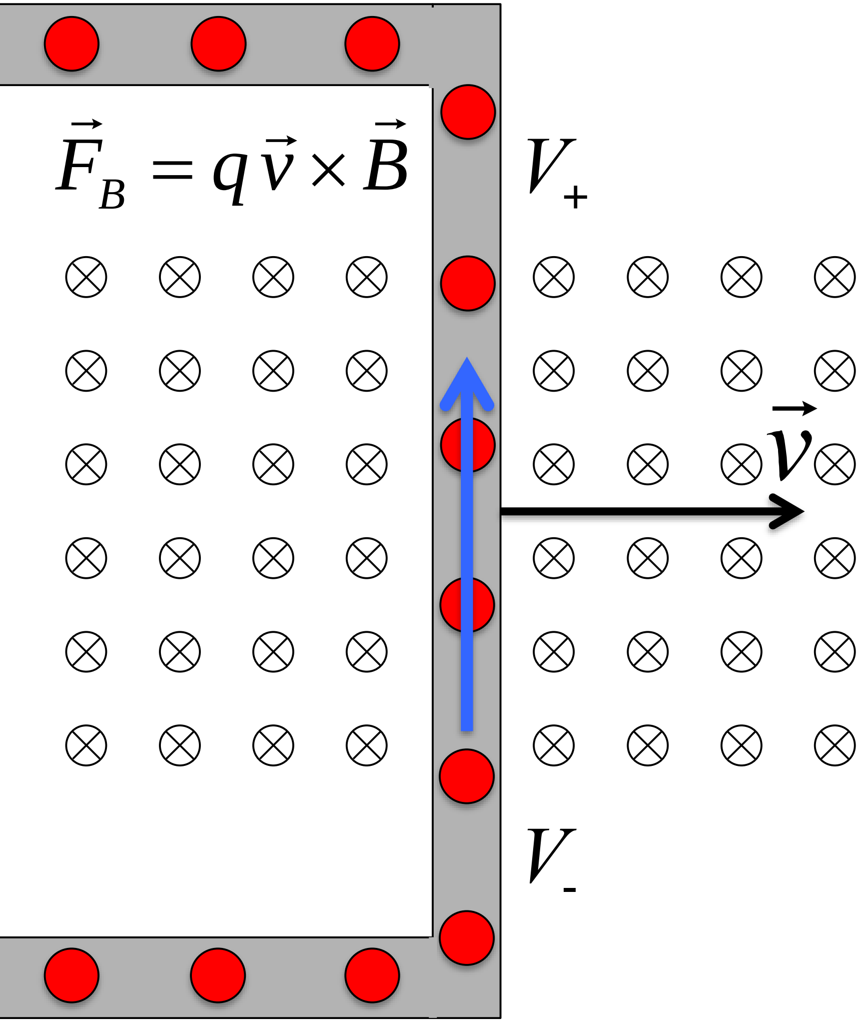
$$e = \underbrace{-\frac{dB}{dt}A\cos f}\;\underbrace{-\frac{dA}{dt}B\cos f + \frac{df}{dt}BA\sin f}$$

Maxwell Equation

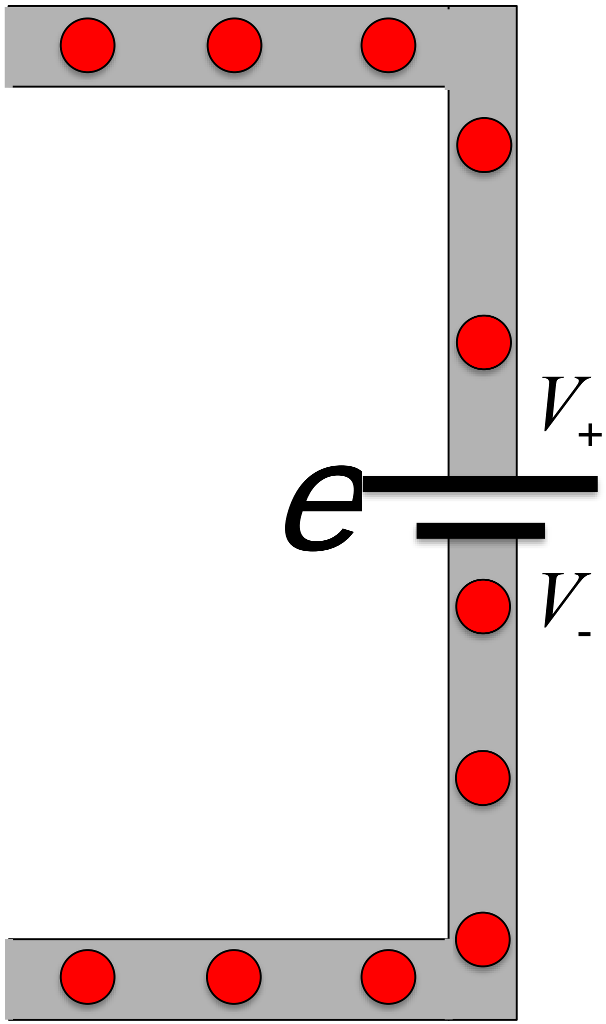
What about these two terms?

$$-\frac{d\vec{B}}{dt} = \nabla \times \vec{E}$$

Motional EMF



There is an induced ΔV across the length of the conductor

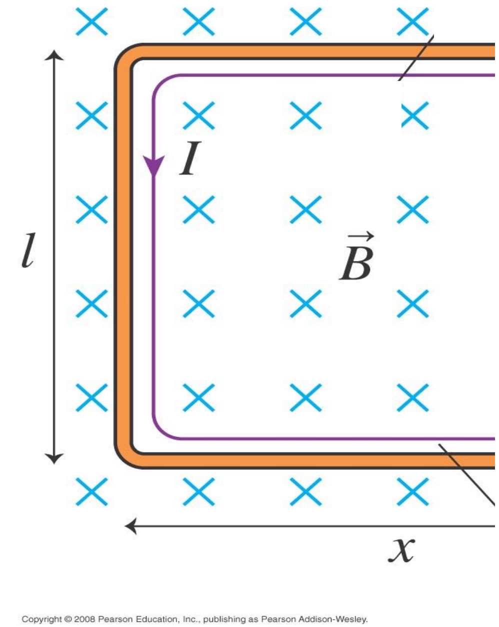


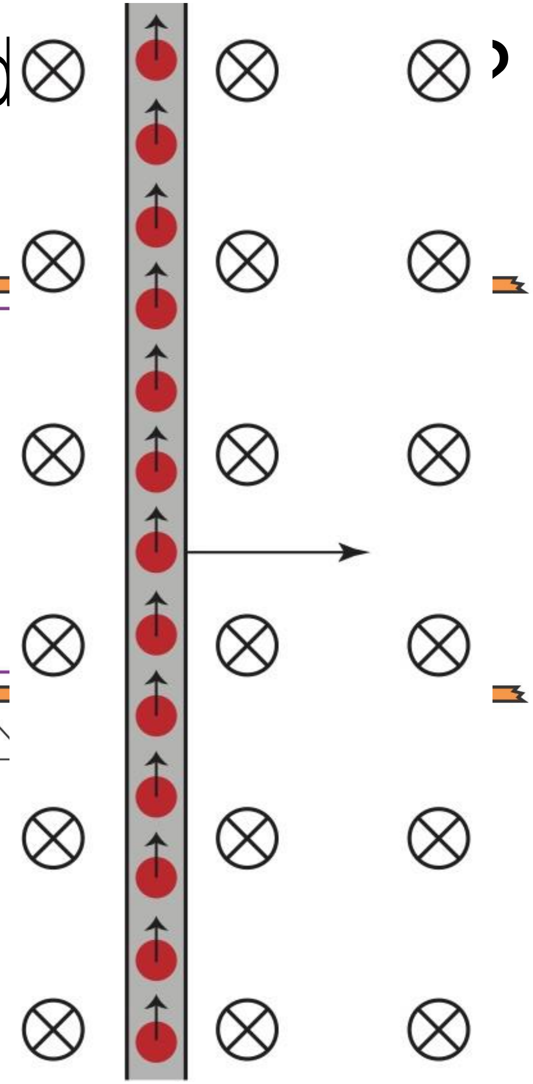
This is equivalent to having an EMF source: “motional EMF”

How can we quantify the ind

The free charges feel a magnetic force:

$$F = qvB$$





This induces a voltage difference (E-field), and therefore an electric force on the charges

$$F = qE \qquad E = \frac{\Delta V}{l}$$

$$\cancel{q}vB = \cancel{q}\frac{\Delta V}{l}$$

MOTIONAL EMF: $\Delta V \equiv \boxed{\varepsilon = vlB}$

Top Hat Question

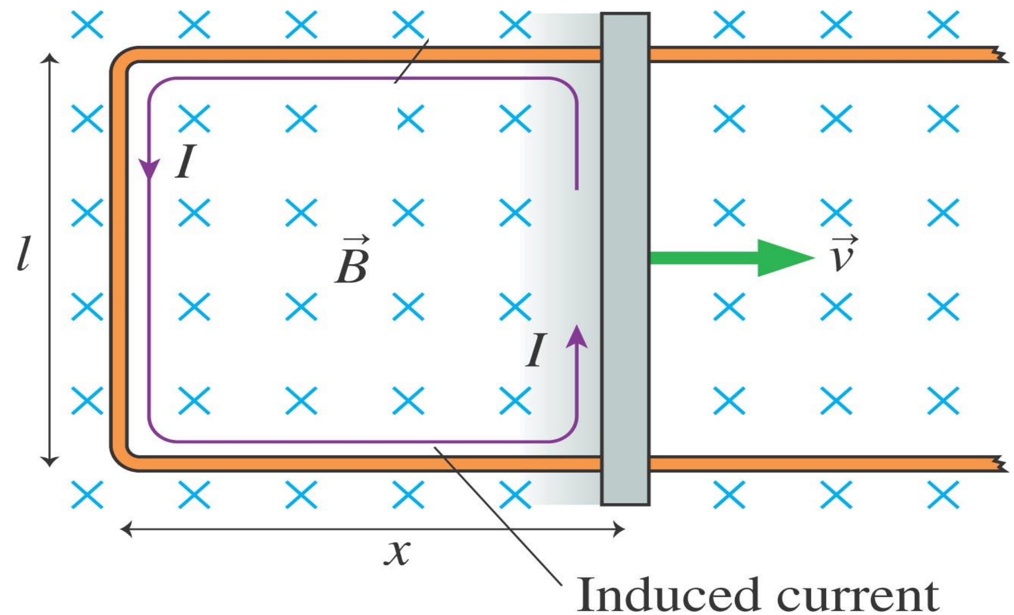
A U-shaped conductor with side length l = 1.0 m is sitting in a uniform magnetic field of field strength $1.0\text{x}10^{-2}$ T. A conducting cross bar is **moving with a constant velocity** of 1.0 m/s and has a resistance of R = 0.10 ohms. What is the **induced current** in the loop?

A. 0.0 A

B. 0.010 A

C. 0.10 A

D. 1.0 A



Top Hat Question

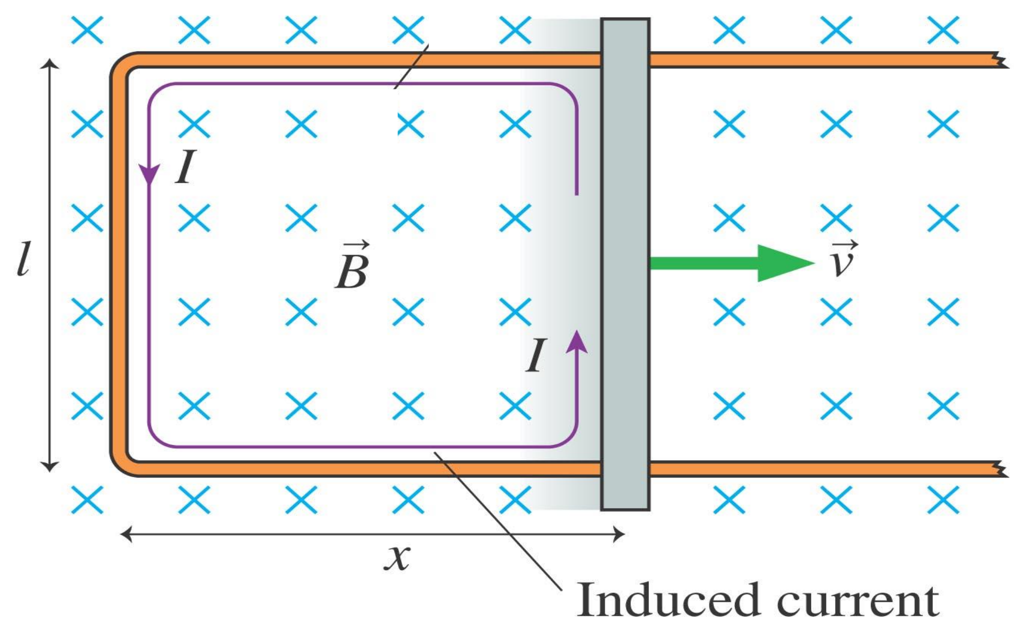
A U-shaped conductor with side length l = 1.0 m is sitting in a uniform magnetic field of field strength 1.0×10^{-2} T. A conducting cross bar is **moving with a constant velocity** of 1.0 m/s and has a resistance of R = 0.10 ohms. What is the **power dissipated by the bar's resistance?**

A. 0.0010 W

B. 0.010 W

C. 0.10 W

D. 1.0 W



Recall there are 3 possible terms:

$$e = \underbrace{-\frac{dB}{dt}A\cos f}_{\textbf{Maxwell Equation}} \underbrace{-\frac{dA}{dt}B\cos f + \frac{df}{dt}BA\sin f}_{\textbf{Magnetic Force on free charges}}$$

$$-\frac{d\vec{B}}{dt} = \nabla \times \vec{E} \qquad\qquad F = q\vec{v} \times \vec{B}$$

It is quite striking that drastically different sources for the induced EMF give an identical law. This makes Faraday's Law a particularly powerful tool from a practical engineering standpoint!

Applications of Faraday's Law:

Last Time:

- Lez`s Law
- Motional emf
- Applications to useful technologies
- Current loops as magnetic dipoles

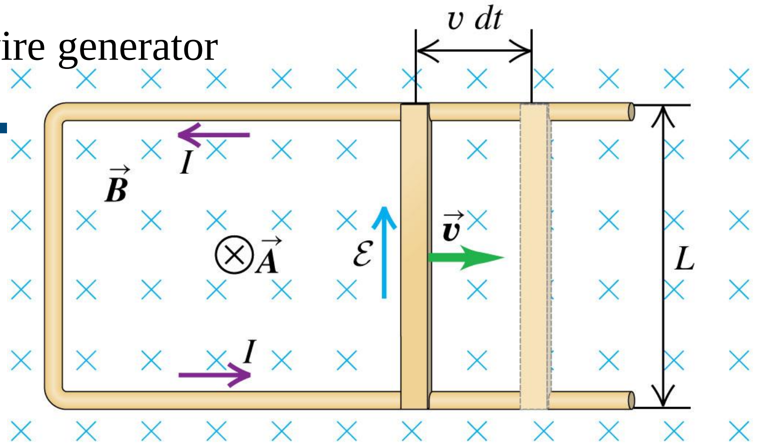
Today:

- Eddy Current
- Mutual Inductance
- Transformer

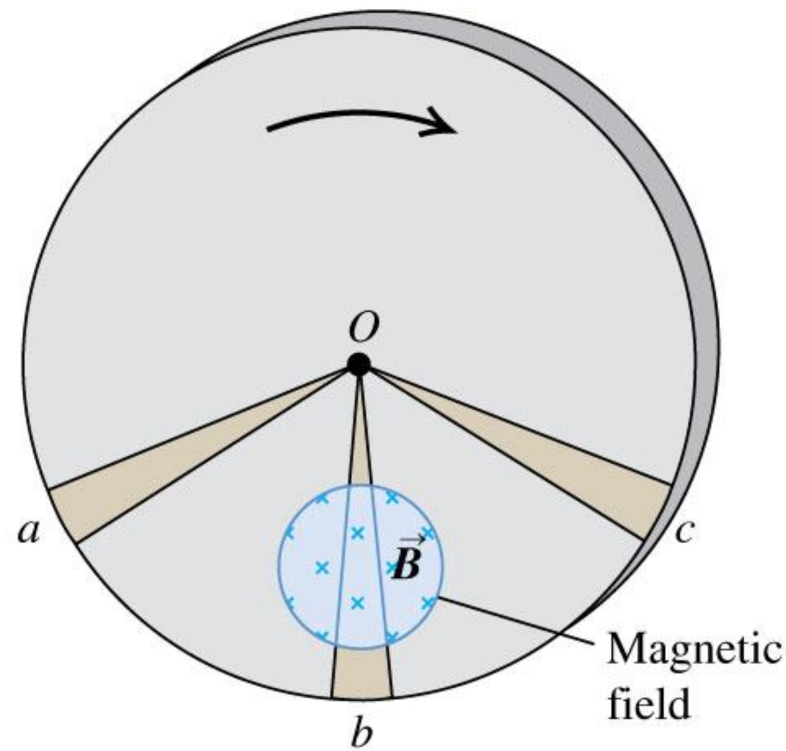
Eddy currents

- So far we have considered induction in circuits, where the induced current is confined to wires
- Induction also happens if the magnetic flux through extended metallic objects changes
- As with wires, the induced currents attempt to keep the flux stable: *eddy* currents

Slidewire generator



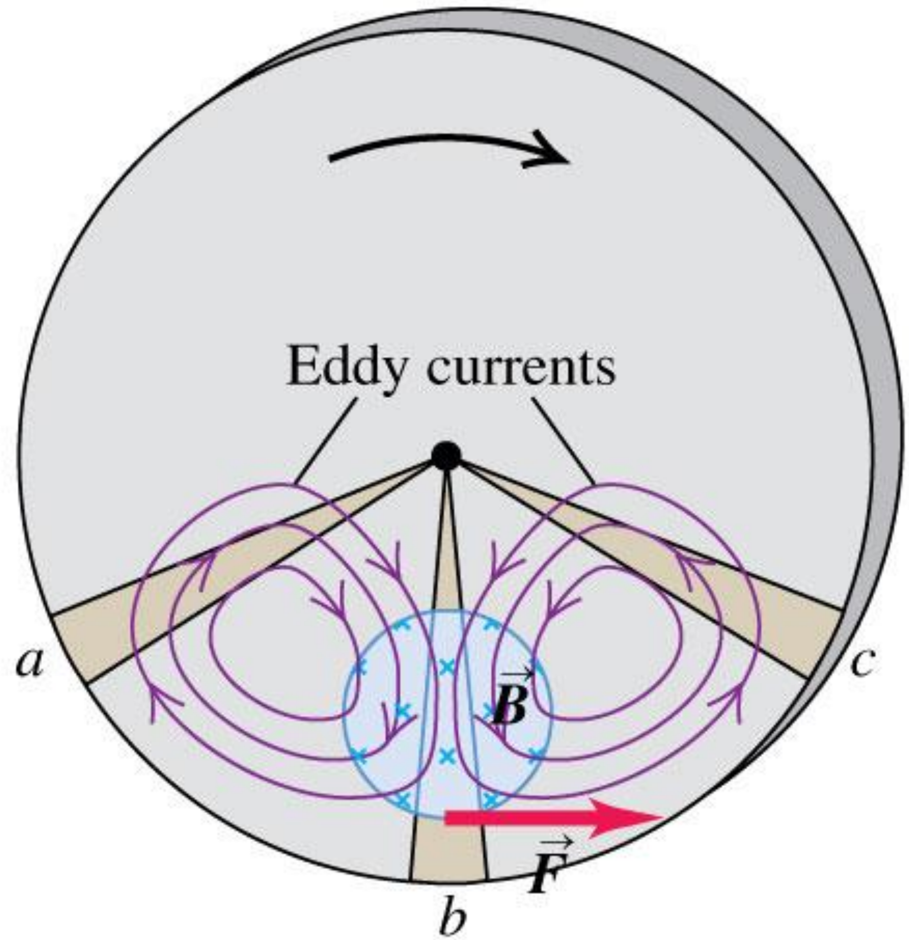
Metal disk rotating through a magnetic field



Eddy currents

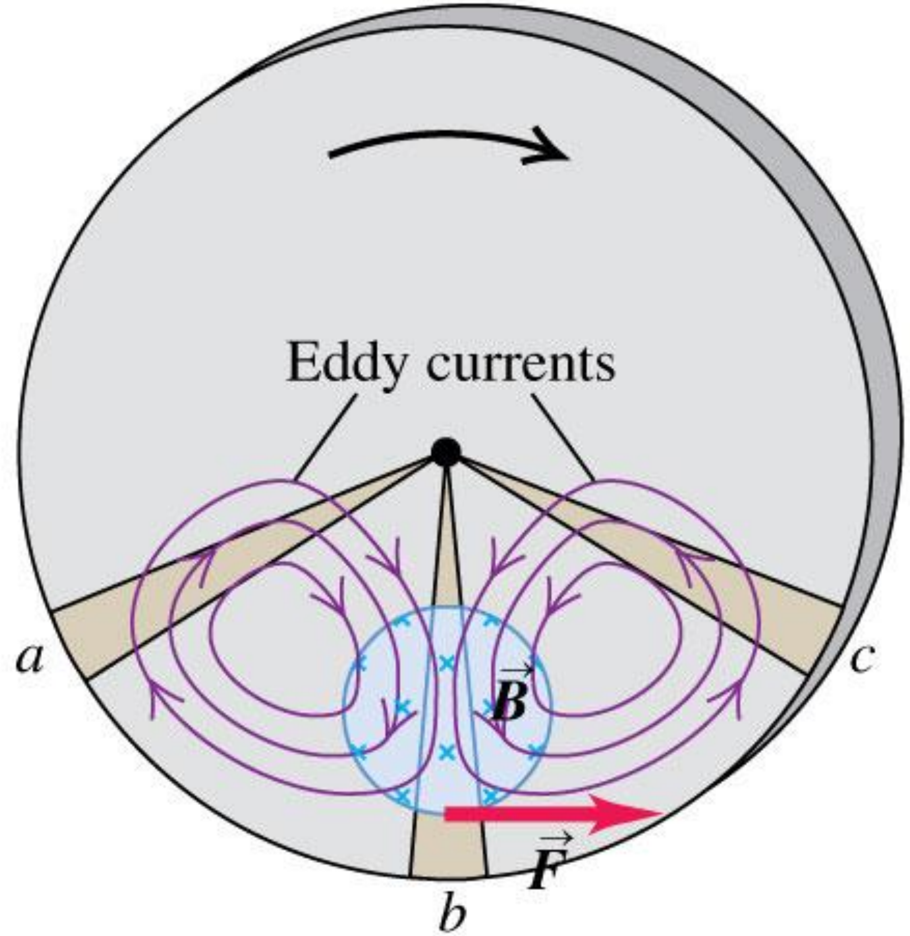
- The direction of the currents can be found using Lenz's law:
  - Without eddy currents, the magnetic flux at the leading (trailing) edge decreases (increases)
  - The induced Eddy currents circulate in a sense that prevents this from happening
  - Result: transformation of mechanical energy into heat!

(b) Resulting eddy currents and braking force



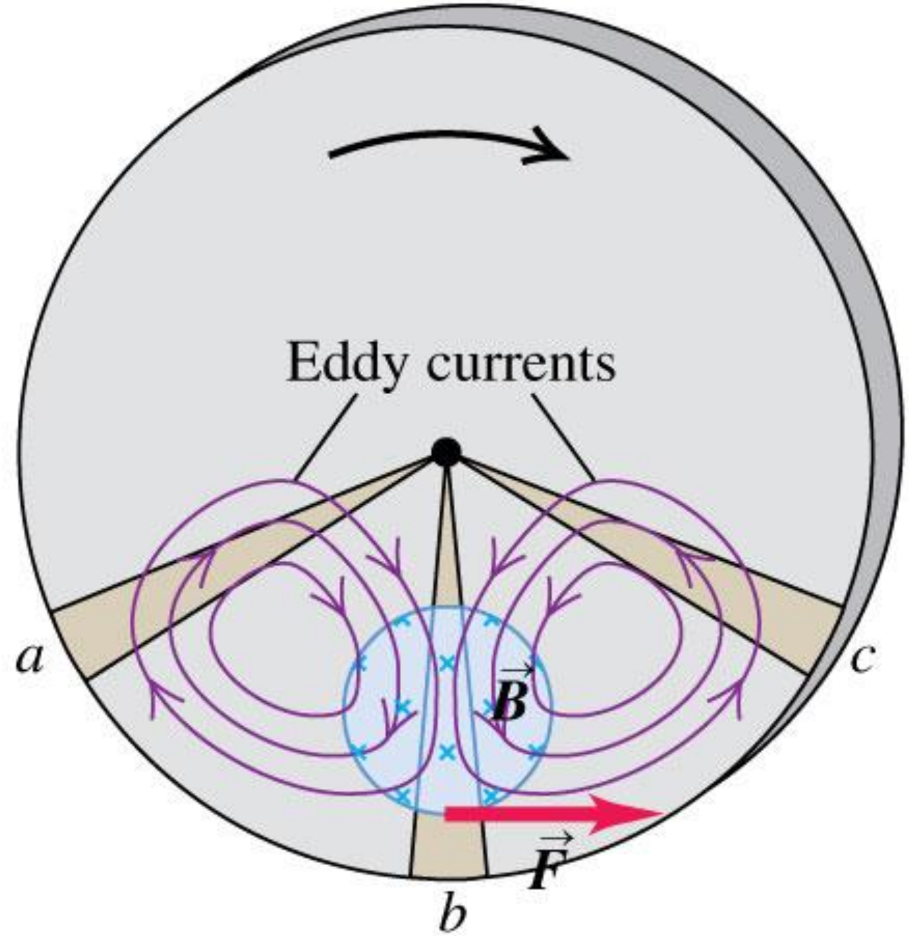
Eddy currents

- What changes if the wheel is slotted?



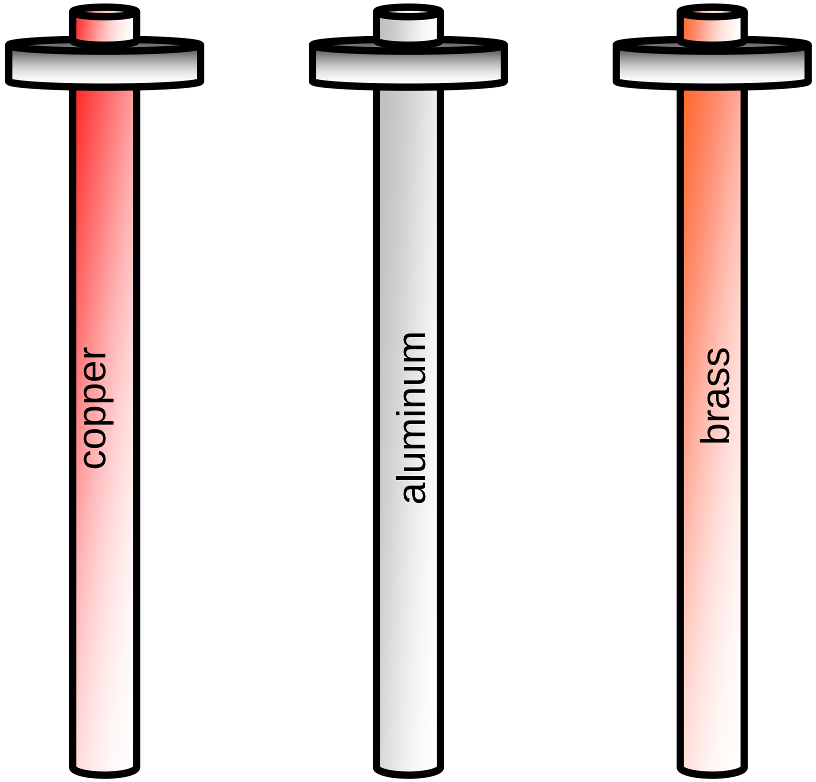
Eddy currents

- What changes if the wheel is slotted?
- Slots inhibit the generation of eddy currents, and the braking force is reduced



Question

- Three metal rods (brass, aluminum, copper) hold three ring magnets. The three magnets are dropped at the same time, and then slide (fall) down, guided by the rods
- Which magnet (if any) will reach the bottom first? *Note*: copper has the least resistivity, followed by aluminum and brass



Question

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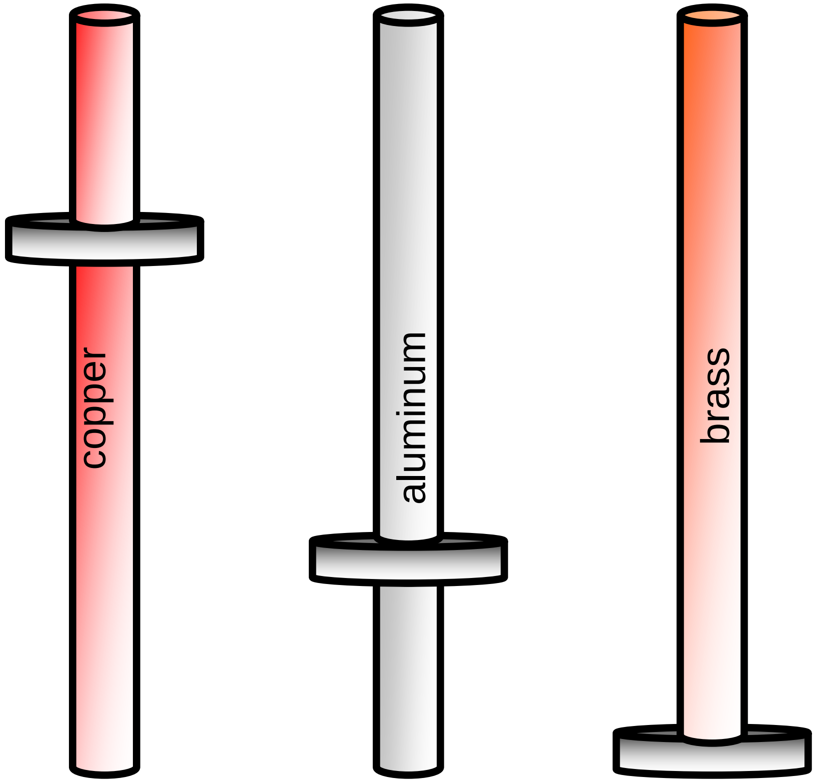
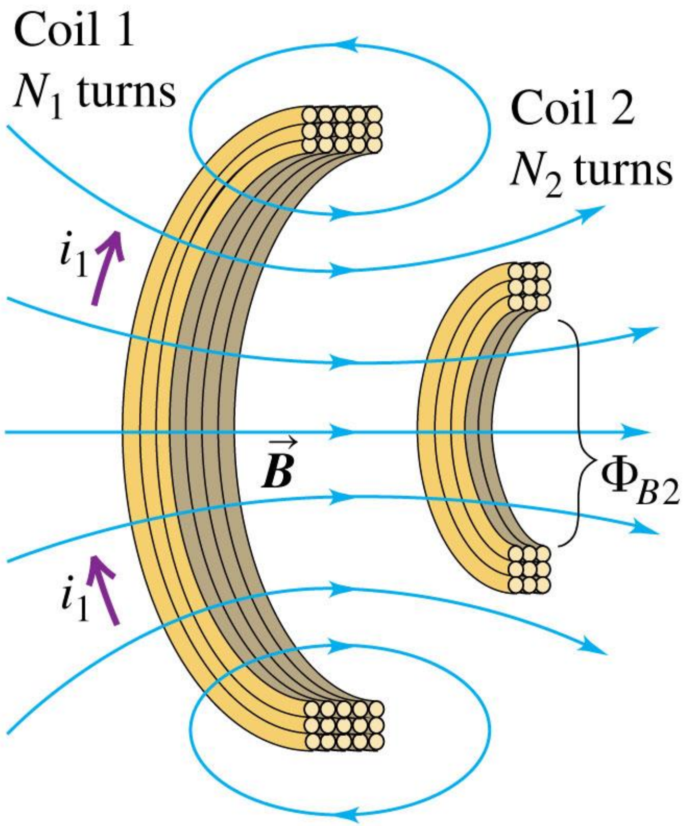


- The *magnet on the brass rod will fall fastest*, as the magnitude of the eddy currents, and hence their capability to slow the magnets down, depends on the material's resistivity.

Mutual Inductance

Mutual inductance: If the current in coil 1 is changing, the changing flux through coil 2 induces an emf in coil 2.



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Induced EMF in coil 2:

$$\varepsilon_2 = -N_2 \frac{d\phi_{B2}}{dt} = -\frac{d\left(N_2 \phi_{B2}\right)}{dt}$$

Note: ϕ_{B2} is the magn. flux through a single loop of coil 2. N_2 is the number of loops.

The magnetic field in coil 2 is prop. to the current through coil 1:

$$d\boldsymbol{B} = \frac{\mu_0}{4\pi} \frac{i_1 d\boldsymbol{l} \times \boldsymbol{r}}{r^2}$$

Biot-Savart

Hence, the magnetic flux through coil 2 is proportional to i_1:

$$N_2 \phi_{B2} = M_{21} i_1$$

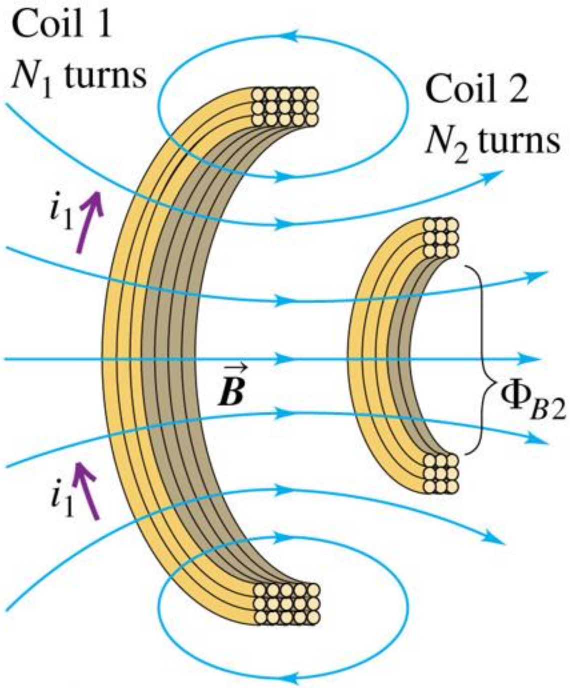
and

$$\varepsilon_2 = -\frac{d\left(N_2 \phi_{B2}\right)}{dt} = -M_{21} \frac{di_1}{dt}$$

M_{21}: *mutual inductance*, depends on the geometry of the two coils

Mutual Inductance

Mutual inductance: If the current in coil 1 is changing, the changing flux through coil 2 induces an emf in coil 2.



Induced EMF in coil 2:

$$\varepsilon_2 = -N_2 \frac{d\phi_{B2}}{dt} = -\frac{d(N_2\phi_{B2})}{dt}$$

Note: ϕ_{B2} is the magn. flux through a single loop of coil 2. N_2 is the number of loops.

The magnetic field in coil 2 is prop. to the current through coil 1:

$$d\boldsymbol{B} = \frac{\mu_0}{4\pi}\frac{i_1 d\boldsymbol{l} \times \boldsymbol{r}}{r^2}$$

Biot-Savart

Hence, the magnetic flux through coil 2 is proportional to i_1:

$$N_2\phi_{B2} = M_{21} i_1$$

and

$$\varepsilon_2 = -\frac{d(N_2\phi_{B2})}{dt} = -M_{21}\frac{di_1}{dt}$$

M_{21}: *mutual inductance*, depends on the geometry of the two coils

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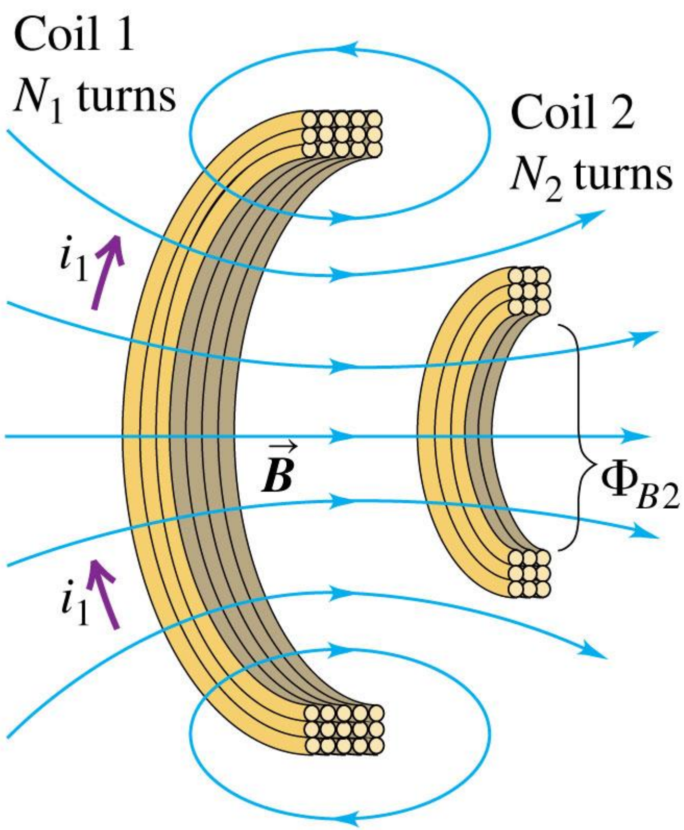


$$N_2 \Phi_{B2} = M_{21} i_1$$

$$\mathcal{E}_2 = -\frac{d(N_2 \Phi_{B2})}{dt} = -M_{21}\frac{di_1}{dt}$$

Similarly, if current flows through coil 2:

$$\mathcal{E}_1 = -\frac{d(N_1 \Phi_{B1})}{dt} = -M_{12}\frac{di_2}{dt}$$

One can show that $M_{21}=M_{12}$, hence

$$\mathcal{E}_2 = -M\frac{di_1}{dt} \qquad \mathcal{E}_1 = -M\frac{di_2}{dt}$$

(mutually induced EMF)

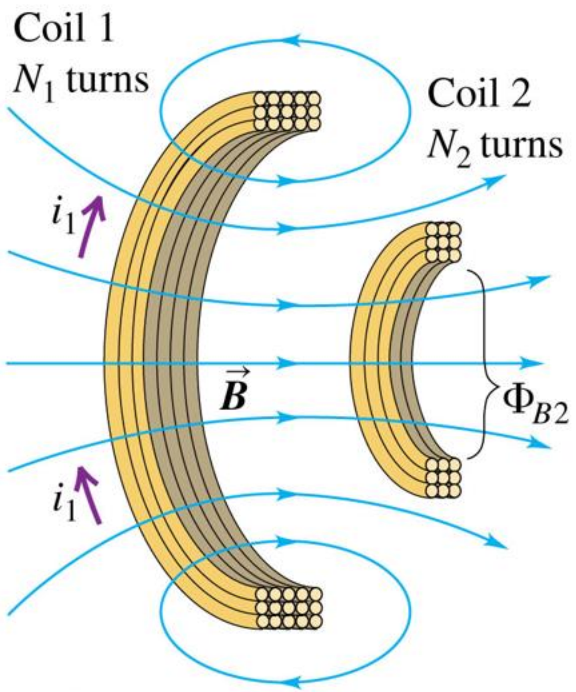
$$M = \frac{N_2 \Phi_{B2}}{i_1} = \frac{N_1 \Phi_{B1}}{i_2}$$

(mutual inductance, <u>can be calculated either way</u>)

$[M]=1H=1Wb/A=1Vs/A=1\Omega s=1J/A^2$. Typical values: $M=\mu H$-mH

Mutual Inductance

Mutual inductance: If the current in coil 1 is changing, the changing flux through coil 2 induces an emf in coil 2.



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$$\boxed{M = \frac{N_2\phi_{B2}}{i_1} = \frac{N_1\phi_{B1}}{i_2}}$$

(mutual inductance, <u>can be calculated either way</u>)

$[M]=1H=1Wb/A=1Vs/A=1\Omega s=1J/A^2$. Typical values: $M=\mu H$-mH

Example – Mutual inductance

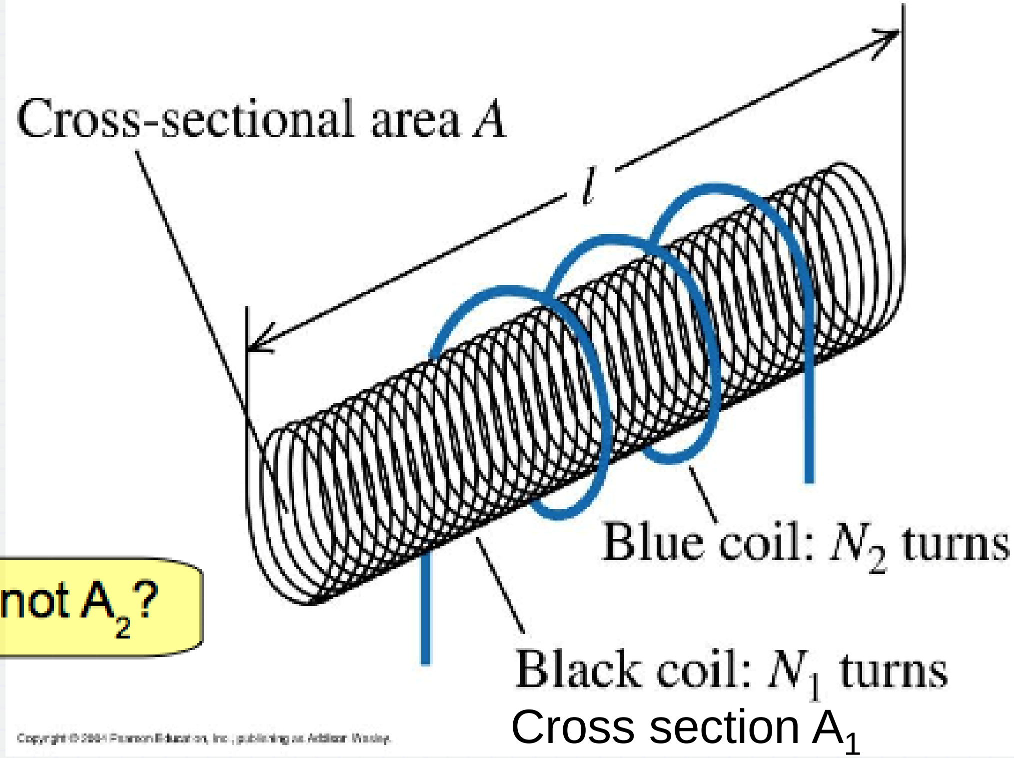
The long solenoid will produce a magnetic field that is proportional to the current I_1 and the number of turns per unit length n_1

$$B_1 = \frac{\mu_0 N_1 I_1}{L} = \mu_0 n_1 I_1$$

and the total flux through each loop of the outer coil is

$$\Phi_{B2} = B_1 A_1$$

Why not A_2?



so the mutual inductance is

$$M = \frac{N_2 \Phi_{B2}}{I_1} = \frac{N_2 (B_1 A_1)}{I_1} = \frac{\mu_0 A_1 N_1 N_2}{L}$$

does not depend on I!

For a 0.5m long coil with 10cm^2 area and N_1=1000, N_2=10 turns

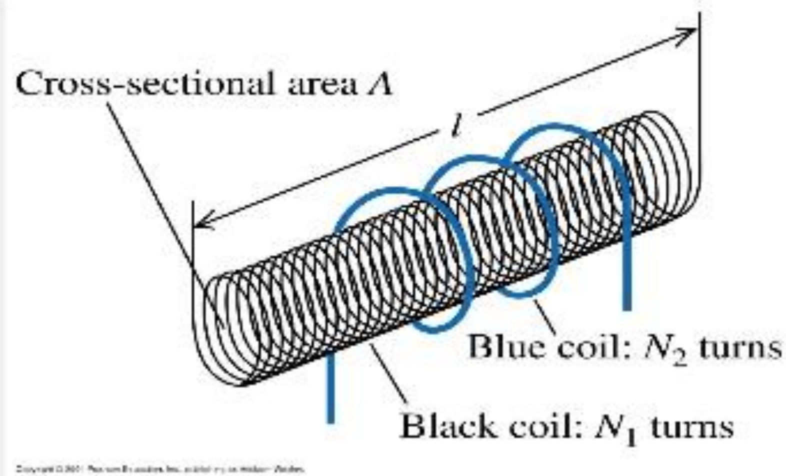
$$M = \frac{(4\pi \times 10^{-7} T\, m / A)(1.0 \times 10^{-3} m^2)(1000)(10)}{0.5\text{m}} = 2.5 \times 10^{-6} H = 25 \mu H$$

Example

If a rapidly increasing current is driven through the outer coil

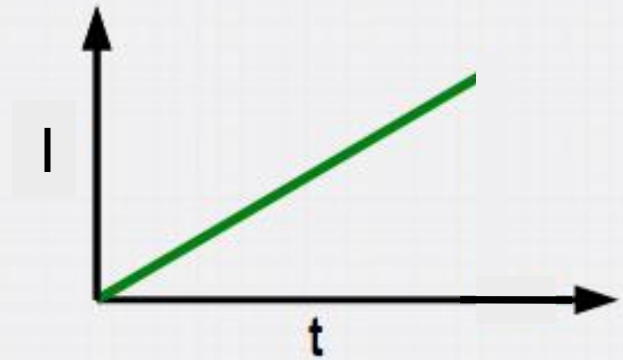
$$i_2(t) \;=\; (2.0\times 10^{6}\,A/s)\;t$$

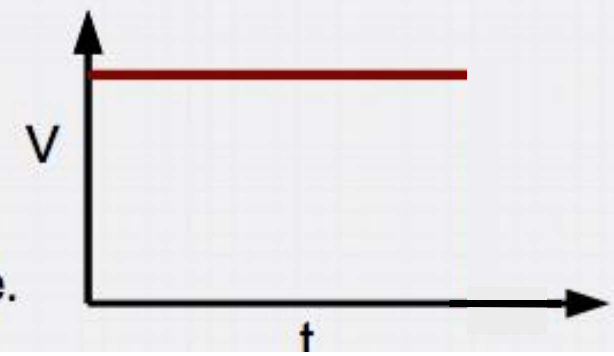
what EMF will be induced in the inner coil?



$$\begin{aligned}
\mathcal{E}_1 &= -M\,\frac{di_2}{dt} \\
&= -(25\times 10^{-6}\,H)\,\frac{d}{dt}\big[(2.0\times 10^{-6}\,A/s)\,t\big] \\
&= -(25\times 10^{-6}\,H)(2.0\times 10^{-6}\,A/s) \\
&= -50\,V
\end{aligned}$$

Note: M also allows calculating ε_2 if I_1 changes





This allows electrical energy in one circuit to be converted to electric energy in a separate device.

Question

16. The diagram below shows two nested, circular coils of wire. The larger coil has radius a and consists of N_1 turns. The smaller coil (radius b) consists of N_2 turns, and is both coplanar and coaxial with the larger coil. Assume b << a, so that the magnetic field of the larger coil is approximately uniform over the area of the smaller coil. The **mutual inductance** of this combination is given by the expression

a) $\dfrac{\mu_0 N_1 N_2}{2a}$.

b) $\dfrac{\pi\mu_0 N_1 N_2 b}{a}$.

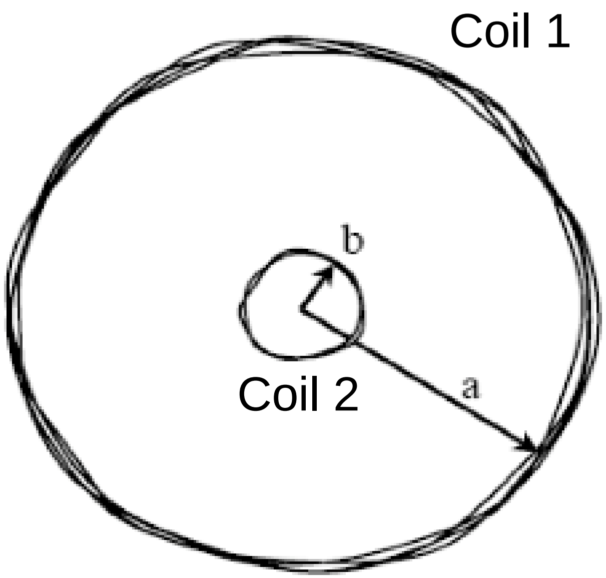
c) $\dfrac{\pi\mu_0 N_1 N_2 b^2}{2a}$.

d) $\dfrac{\mu_0 N_1 N_2 b^2}{2a}$.

e) $\dfrac{\pi\mu_0 N_2 b^2}{2a}$.

$$M = \frac{N_2 \phi_{B2}}{i_1} = \frac{N_1 \phi_{B1}}{i_2}$$

Flux through one loop of coil 2 (area A_2) due to magnetic field generated by current in coil 1



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e) $\frac{\pi\mu_0 N_2 b^2}{2a}$.

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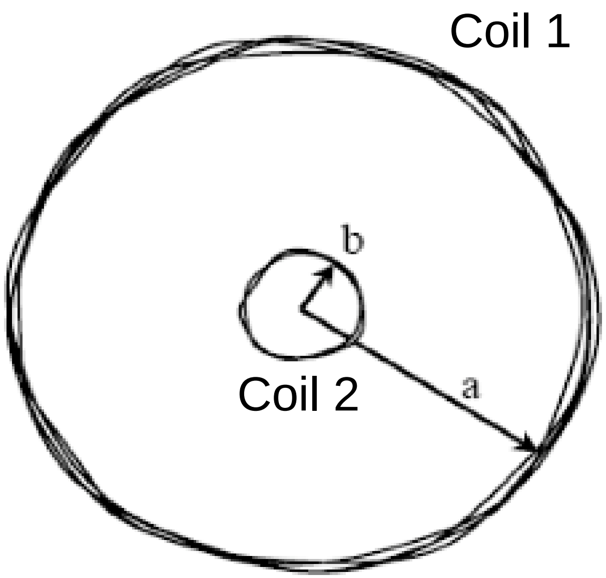
Flux through one loop of coil 2 (area A_2) due to Magnetic field generated by current in coil 1

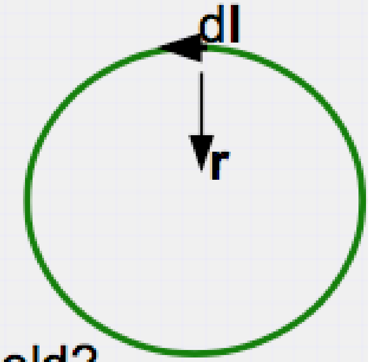


We assume current in the larger coil (coil 1), which generates a roughly uniform field in the area covered by the much smaller coil.

But how large is B?

Question

Calculate for one loop!



A circular loop of radius a carries a constant current I.
What is the magnetic field at the center of the loop?

What are the two methods we know for calculating magnetic field?
Biot-Savard law & Ampere's law.

Ampere's law isn't useful for a loop, so use the Biot-Savard law:

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{l}\times\hat{r}}{r^2} = \frac{\mu_0}{4\pi}\frac{I}{a^2}\,dl\,\hat{z}$$

$$\vec{B} = \int d\vec{B} = \frac{\mu_0}{4\pi}\frac{I}{a^2}\hat{z}\int dl = \frac{\mu_0}{4\pi}\frac{I}{a^2}(2\pi a)\hat{z} = \frac{\mu_0}{2}\frac{I}{a}\hat{z}$$

With the B-field direction directed out of the page, either from **dl x r**, or right thumb in direction of current and fingers curl in direction of **B**.

Question

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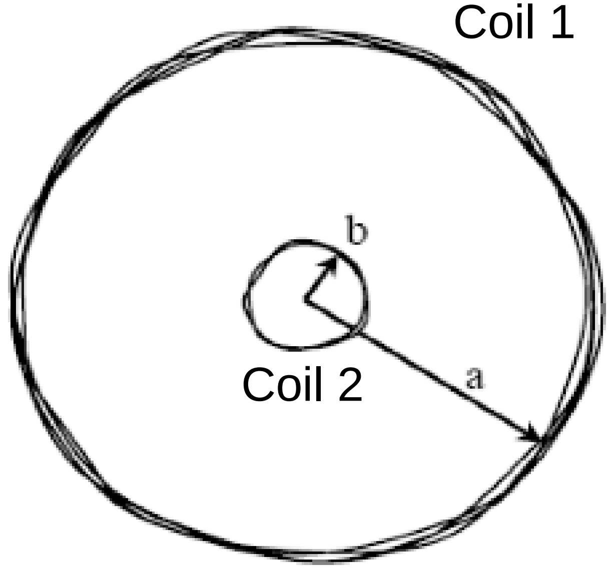
$$M = \frac{N_2 \Phi_{B2}}{i_1} = \frac{N_1 \Phi_{B1}}{i_2}$$



We assume current in the larger coil (coil 1), which generates a roughly uniform field in the area covered by the much smaller coil. Hence,

$$M = \frac{N_2 \Phi_{B2}}{i_1} = \frac{N_2}{i_1} \underbrace{N_1 \frac{\mu_0 i_1}{2a}}_{B_1} \underbrace{\pi b^2}_{A_2} = \mu_0 N_1 N_2 \frac{\pi b^2}{2a}$$

Question

16. The diagram below shows two nested, circular coils of wire. The larger coil has radius a and consists of N_1 turns. The smaller coil (radius b) consists of N_2 turns, and is both coplanar and coaxial with the larger coil. Assume b << a, so that the magnetic field of the larger coil is approximately uniform over the area of the smaller coil. The **mutual inductance** of this combination is given by the expression

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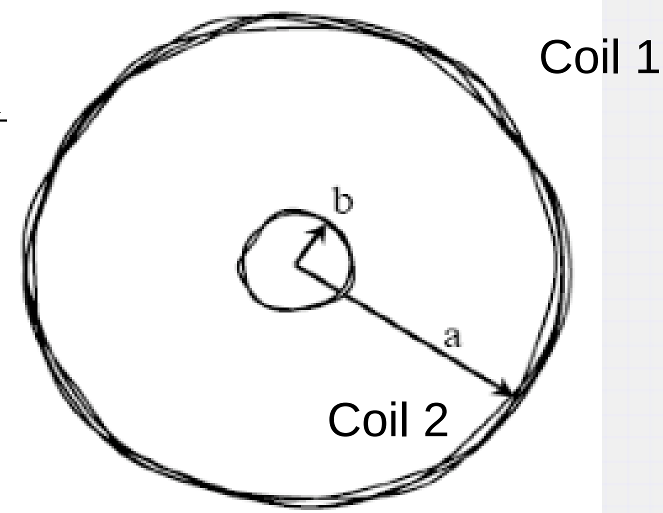
c) $\dfrac{\pi \mu_0 N_1 N_2 b^2}{2a}$.

d) $\dfrac{\mu_0 N_1 N_2 b^2}{2a}$.

e) $\dfrac{\pi \mu_0 N_2 b^2}{2a}$.

$$M = \frac{N_2 \Phi_{B2}}{i_1} = \frac{N_1 \Phi_{B1}}{i_2}$$

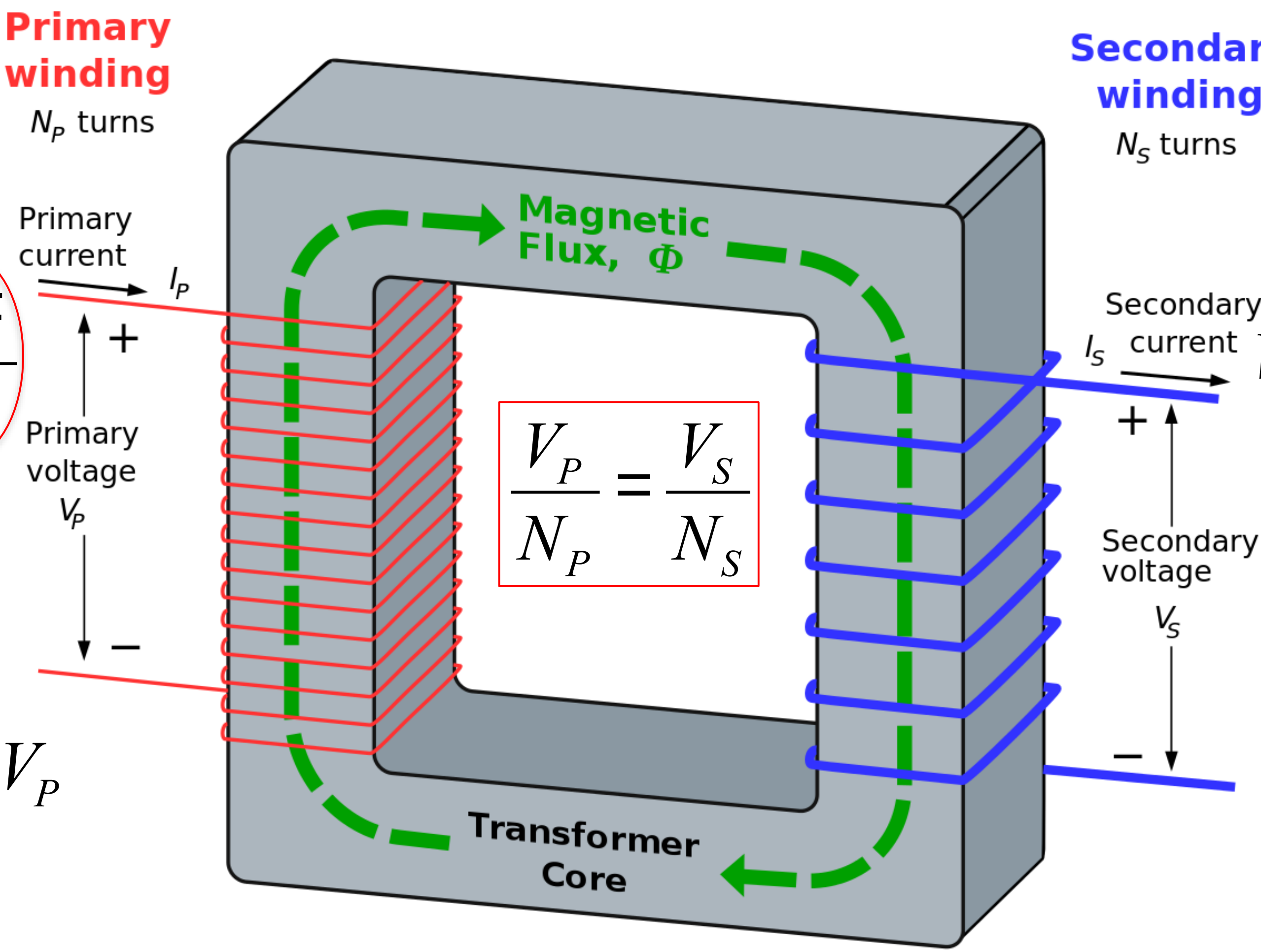


Makes sense?

We expect the result to be proportional to the area of the coil that sees the field of the other coil, i.e. πb^2. Furthermore, we expect a dependence on N_1 and N_2: the field depends on N_1, and the flux on N_2. This leaves only answer c).

Transformers

$$V_P = N_P \frac{dF}{dt}$$



$$\boxed{\frac{V_P}{N_P} = \frac{V_S}{N_S}}$$

$$V_S = N_S \frac{dF}{dt}$$

$$P_{in} = I_P V_P$$

$$P_{out} = I_S V_S$$

$$P_{in} = P_{out}$$

$$I_P V_P = I_P \frac{N_P}{N_S} V_S = I_S V_S$$

$$\boxed{I_P N_P = I_S N_S}$$

Top Hat Question

The transformer for your laptop (the adaptor) has an output voltage of 18.5V. Your laptop uses about 85W of energy. The adaptor uses a step down transformer – what is the ratio of turns, primary to seconday, N_P/N_S ?

a) 0.065

b) 0.65

c) 6.5

d) 65

Top Hat Question

The transformer for your laptop (the adaptor) has an output voltage of 18.5V. Your laptop uses about 85W of energy. The adaptor uses a step down transformer– what is the resistive load of the laptop *R*?

a) 0.4Ω

b) 4Ω

c) 40Ω

d) 400Ω

Last Time:

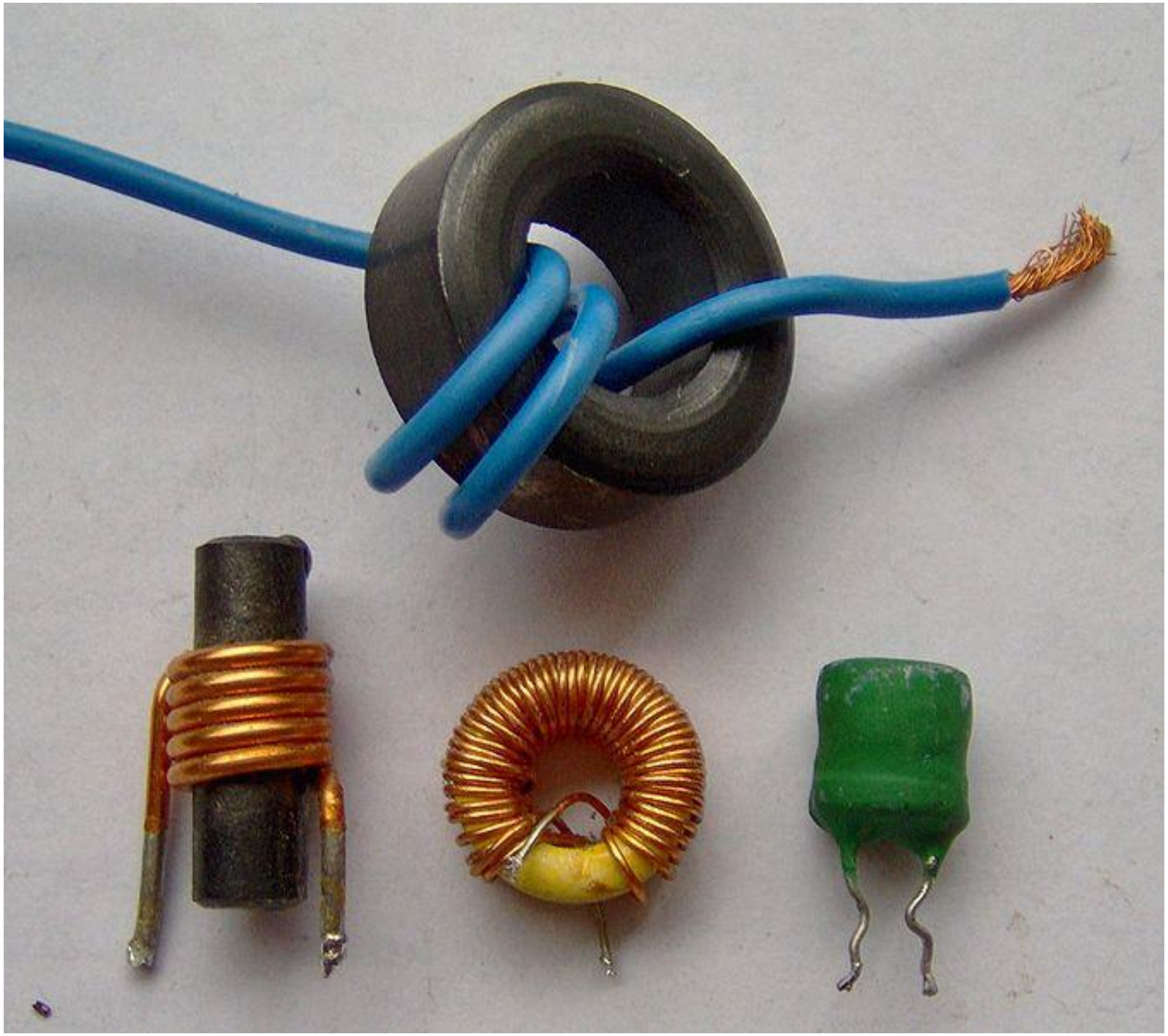
- Eddy Current
- Mutual Inductance
- Transformer

Today:

- Inductors
- R-L circuits

Inductors

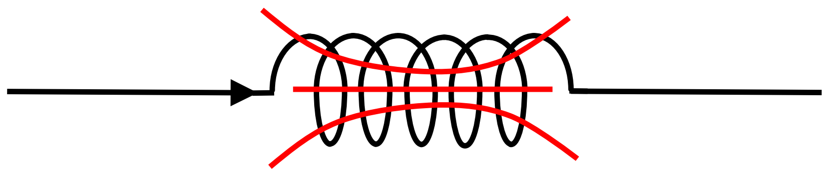
An inductor is a passive electrical component that can store energy in a magnetic field.



Inductance

Note that a changing Magnetic flux produces an induced EMF in a direction which “tries to oppose the change”

$$\frac{di}{dt}$$



$$\varepsilon = -\frac{d}{dt}\left(N\mu_0 niA\right)$$

Changing the current changes the flux through the inductor, which creates a back-emf. Model inductor as perfect solenoid

$$\Delta V = -\left(\frac{N^2}{\ell}\mu_0 A\right)\frac{di}{dt} = -L\frac{di}{dt}$$

$$\boxed{L = \mu_0 \frac{N^2}{\ell} A}$$

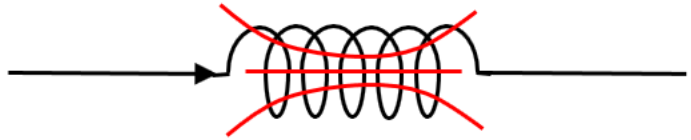
Energy in a Capacitor is stored in the Electric Field

Energy in an Inductor is stored in the Magnetic Field.

Inductance

Note that a changing Magnetic flux produces an induced EMF in a direction which "tries to oppose the change"

$$\frac{di}{dt}$$



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Energy in a Capacitor is stored in the Electric Field

Energy in an Inductor is stored in the Magnetic Field.

Energy storage in Inductors

If we build up the current, starting from $\mathbf{I_0}$ = 0 (initial) → $\mathbf{I_f}$, at the time t when we have achieved a current $\mathbf{I}$, we have to work against an opposing EMF = Ld$\mathbf{I}$/dt in order to achieve a further increase in current, so our energy source is doing work per unit time

$$P = IV = IL\frac{dI}{dt}$$

total work done: $$W = \int P\,dt = \int IL\frac{dI}{dt}dt$$

ie energy stored in system: $$U = \int_0^{I_f} LI\,dI$$

$$\boxed{U = \frac{1}{2}LI^2} \qquad u = \frac{U}{V} = \frac{1}{2V}\left(\mu_0 nNA\right)I^2 = \frac{1}{2\mu_0}\left(\mu_0^2 n^2 I^2\right)\frac{A\ell}{V} = \boxed{\frac{1}{2\mu_0}B^2}$$

Energy storage in Inductors

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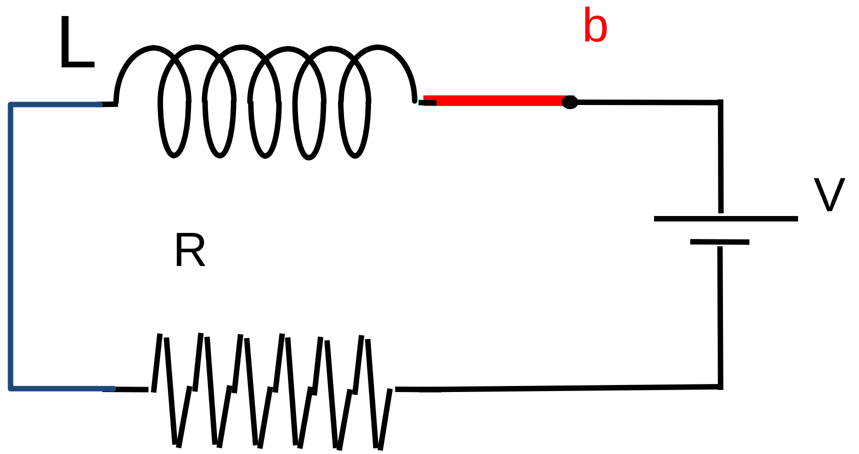
R-L Circuit



$$\boxed{V - L\frac{di}{dt} - iR = 0}$$

If the switch is moved to position b, to initiate the current flow, what happens?

Faraday's law applies and so the change in the Magnetic Field in the inductor L means there is a back EMF induced in L.

So in this case at t = 0, $i(0) = 0$.

Inductor acts like a BATTERY

After a long time, i= V/R

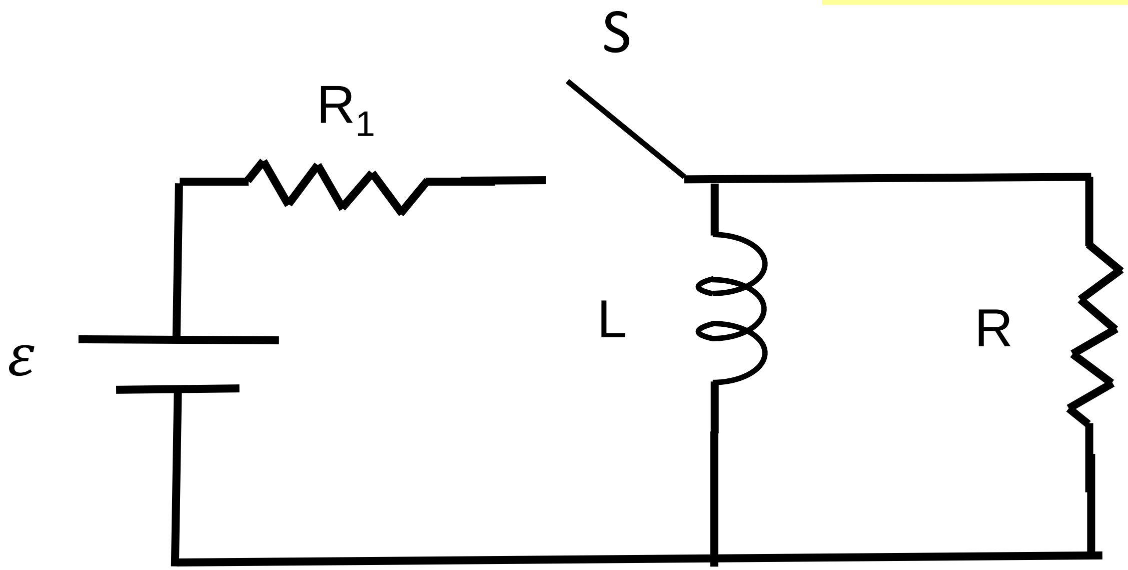
Inductor acts like a WIRE

The components have all been connected for a very long time. At t=0 the switch S is opened. The current through R_1 and R are 0 and $\varepsilon/_{R_1}$

Using the loop rules $$-L\frac{di}{dt}-iR=0$$

Solving with the method we used for a discharging capacitor

$$i(t)=i(0)e^{-\left(\frac{Rt}{L}\right)}$$



At t=0 the switch S is closed on a long enough for the equilibrium current

$\mathcal{E}$/R to be established, and then thrown to b. The current through R are at t=0 is $\varepsilon/_R$

$$- L\frac{di}{dt} - iR = 0$$

$$i(t) = i(0)\mathrm{e}^{-\left(\frac{Rt}{L}\right)}$$

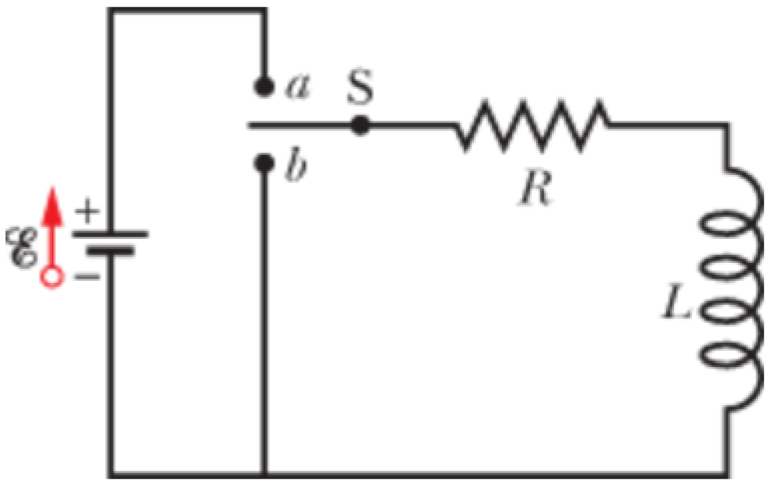


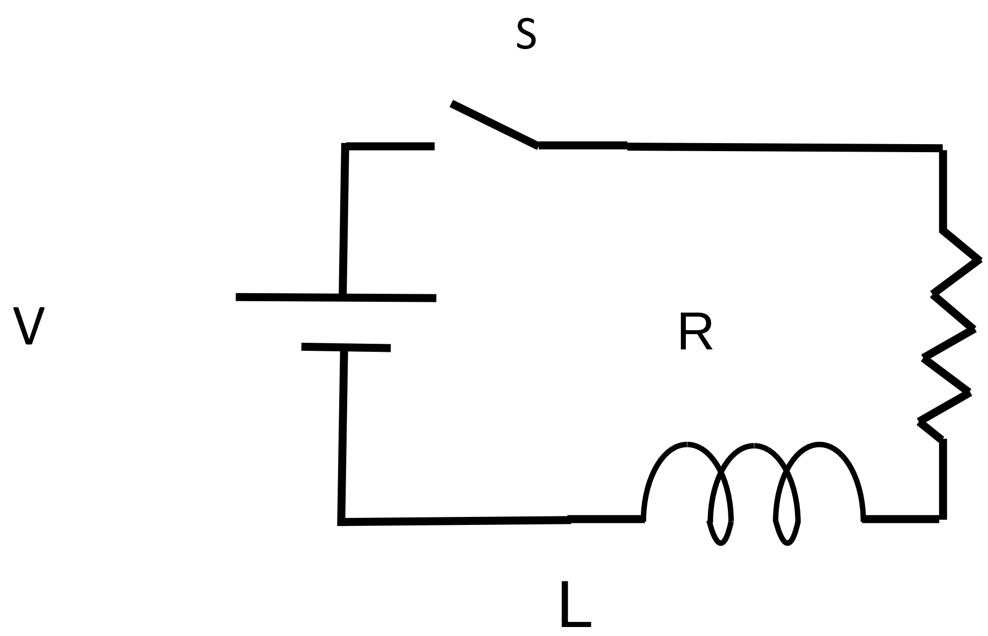
Figure 30-15
An RL circuit. When switch S is closed on *a*, the current rises and approaches a limiting value $\mathcal{E}/R$.

At t=0 the switch S is closed.

Using the loop rules

$$V - iR - L\frac{di}{dt} = 0$$

Solving using the method we used for the charging capacitor

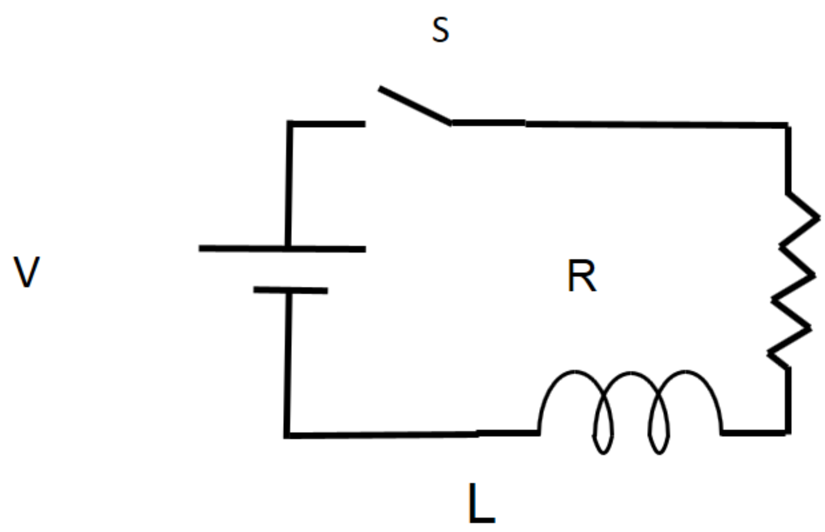


$$i(t) = i_{\max}\left(1 - e^{-\left(\frac{Rt}{L}\right)}\right)$$

At t=0 the switch S is closed.

Using the loop rules $$V - iR - L\frac{di}{dt} = 0$$

Solving using the method we used for the charging capacitor



$$i(t) = i_{\max}\left(1 - e^{-\left(\frac{Rt}{L}\right)}\right)$$

Top Hat Question

The switch in the series circuit below is closed at t=0.

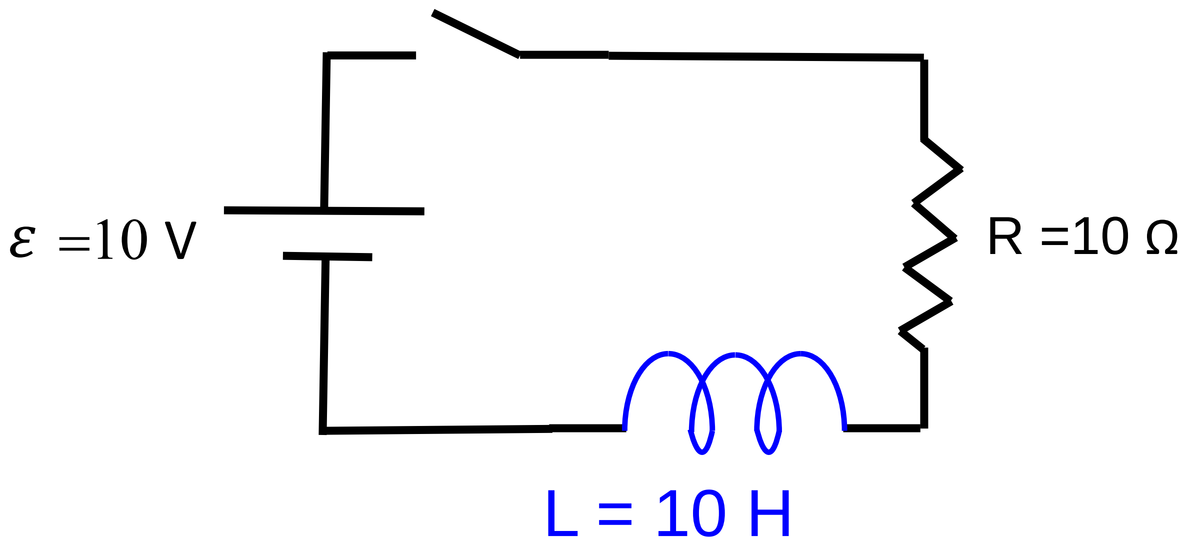
What is the initial rate of change of current di/dt in the inductor, immediately after the switch is closed (time = 0+) ?

A. 0 A/s

B. 0.5 A/s

C. 1 A/s

D. 10 A/s



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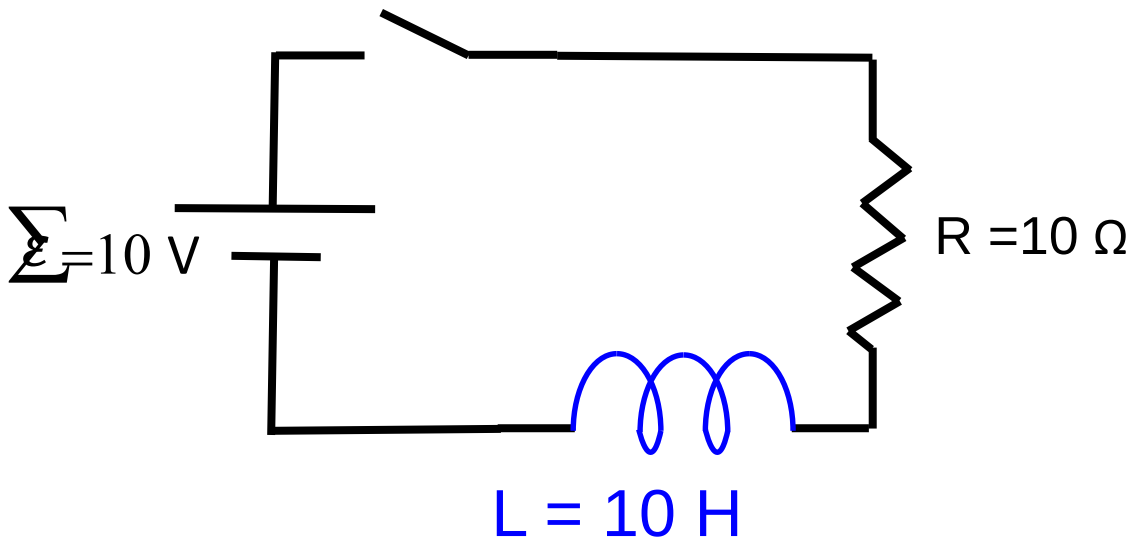


A. 0 A/s

B. 0.5 A/s

C. 1 A/s

D. 10 A/s

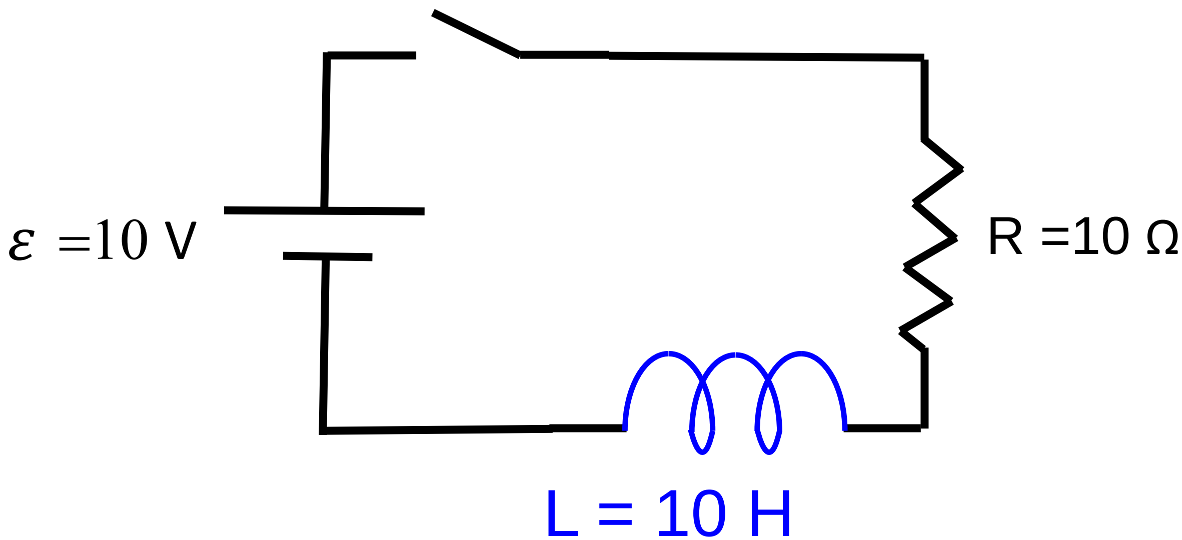
i = 0 at t = 0 so V_R (0)= 0 which means

10 V = V_L= Ldi/dt so di/dt = 10V/10H = 1 A/s

Top Hat Question

The switch in the series circuit below is closed at t=0.

What is the current in the circuit after a time t = 3.0 s?



A. 0 A

B. 0.63 A

C. 0.86 A

D. 0.95 A

Top Hat Question

The switch in the series circuit below is closed at t=0.

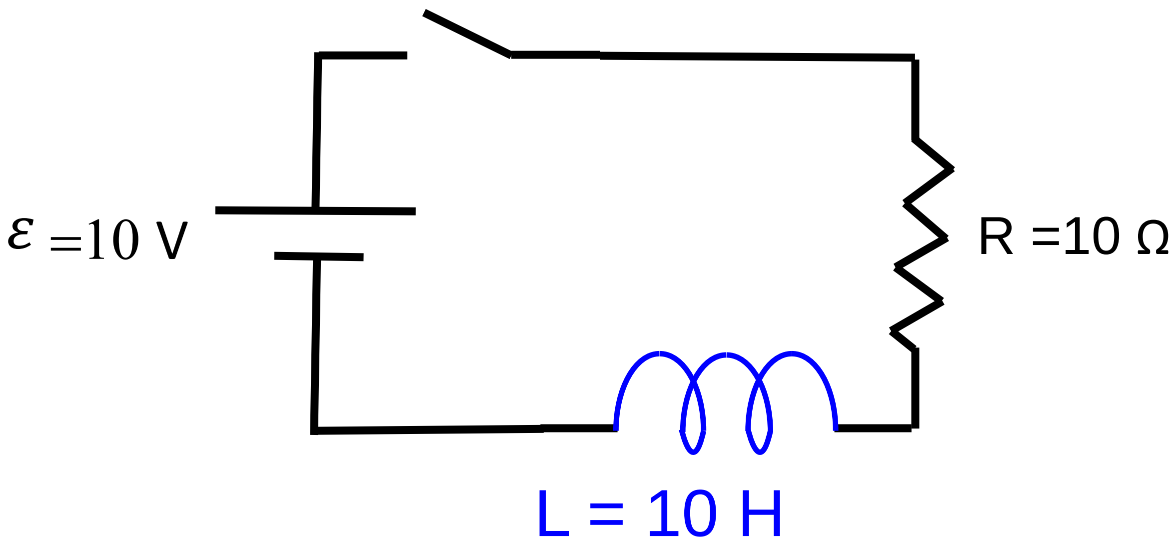
What is the current in the circuit after a time t = 3.0 s?

A. 0 A

B. 0.63 A

C. 0.86 A

D. 0.95 A



$$i(3s) = \frac{10V}{10\Omega}\left(1 - e^{-3}\right)$$