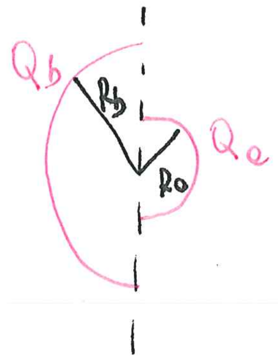
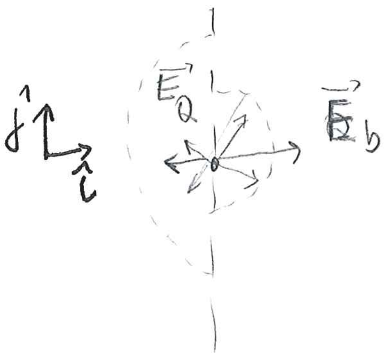
PHYS 253 Midterm Q from 2015



Two semicircular lines of charge, one with Q_a and radius R_a, the other with Q_b and radius R_b, are aligned as shown with a common centre.
What is the ratio Q_b / Q_a needed to have zero E-field at the centre?

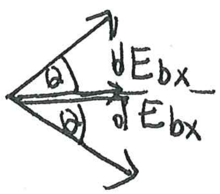


- line „b" creates an E-field to the right and „a" creates an E-field to the left.

$$dE_b = 2\,dE_x = 2\,dE\cos\theta$$

For a single half ring, each small charge element produces a field

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dQ}{R^2}$$



and we need the x-component which introduces $\cos\theta$.
R is the same for all elements dQ

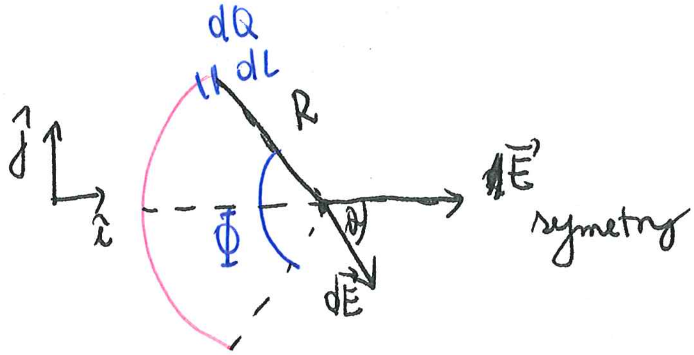
$$\int dE \approx \underset{\text{numerical factors}}{\cancel{\text{constant}}} \times \frac{Q}{R^2}$$

$$\vec{E}=0 \quad \text{if} \quad \vec{E}_a = \vec{E}_b$$

$$E_a = E_b \qquad \text{numerical factor} \times \frac{Q_a}{R_a^2} = \# \frac{Q_b}{R_b^2}$$

$$Q_a \cdot R_b^2 = Q_b \cdot R_a^2$$

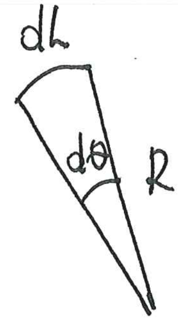
$$\frac{Q_b}{Q_a} = \frac{R_b^2}{R_a^2}$$



$$\vec{E} = \int d\vec{E}$$

$$dE_x = dE \cdot \cos\theta$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{dQ}{R^2}\cos\theta$$



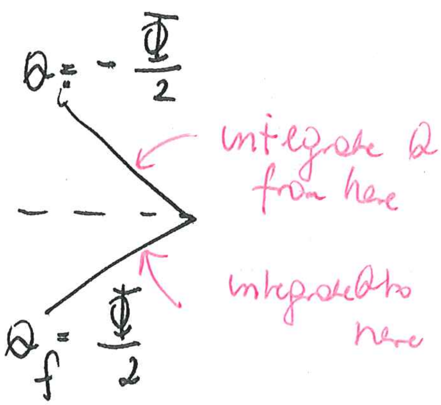
$$dL = R\,d\theta$$

$$\lambda = \frac{Q}{L} = \frac{Q}{R\Phi}$$

$$dQ = \lambda \cdot dL = \lambda \cdot R\,d\theta$$

$$dQ = \frac{Q}{\cancel{R}\,\Phi}\circ \cancel{R}\,d\theta = \frac{Q}{\Phi}\,d\theta$$

$$dE_x = \frac{1}{4\pi\varepsilon_0}\frac{Q}{\Phi R^2}\,d\theta\,\cos\theta$$



Sum up over every dQ

$$E_x = \int_{\theta_i}^{\theta_f} \underbrace{\frac{1}{4\pi\varepsilon_0}\frac{Q}{\Phi R^2}}_{\text{constant}} \cos\theta\,d\theta$$

$$E_x = \frac{1}{4\pi\varepsilon_0}\frac{Q}{\Phi R^2}\int_{-\frac{\Phi}{2}}^{\Phi/2}\cos\theta\,d\theta$$

$$E_x = \frac{1}{4\pi\varepsilon_0}\frac{Q}{\Phi R^2}\left(\sin\theta\Big|_{-\frac{\Phi}{2}}^{\frac{\Phi}{2}}\right) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{\Phi R^2}\left(\sin(\Phi/2) - \sin(-\Phi/2)\right)$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{Q}{\Phi R^2}\left(\sin(\Phi/2) + \sin(\Phi/2)\right)$$

$$E_x = \underbrace{\frac{1}{4\pi\varepsilon_0}\left(2\,\frac{\sin(\Phi/2)}{\Phi}\right)}_{\text{numerical factors}}\frac{Q}{R^2}$$

Special cases:

1) $\Phi = 0$

$$\lim_{\Phi \to 0} \frac{2\sin(\Phi/2)}{\Phi} = \frac{2(\Phi/2)}{\Phi} = 1$$

$$E_x = \frac{1}{4\pi\varepsilon_0} \frac{Q}{R^2} \quad (\text{point charge } Q)$$

2) $\Phi = 2\pi$ $\quad E_x = \frac{1}{4\pi\varepsilon_0}\left(\frac{2\sin(\pi)}{2\pi}\right)\frac{Q}{R^2} = 0$

(full ring)

$\sin \pi = 0$

3) Exam question $\quad \Phi = \pi$

$$E_x = \frac{1}{\cancel{4}\pi\varepsilon_0} \frac{\cancel{2}\overbrace{\sin(\pi/2)}^{=1}}{\pi} \frac{Q}{R^2}$$

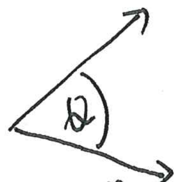
$$E_x = \frac{1}{2\pi^2\varepsilon_0}\frac{Q}{R^2} \Rightarrow \text{numerical factors} = \frac{1}{2\pi^2\varepsilon_0}$$

DOT PRODUCT

$$\vec{A} = A_x \hat{i} + A_y \hat{j}$$

$$\vec{B} = B_x \hat{i} + B_y \hat{j}$$

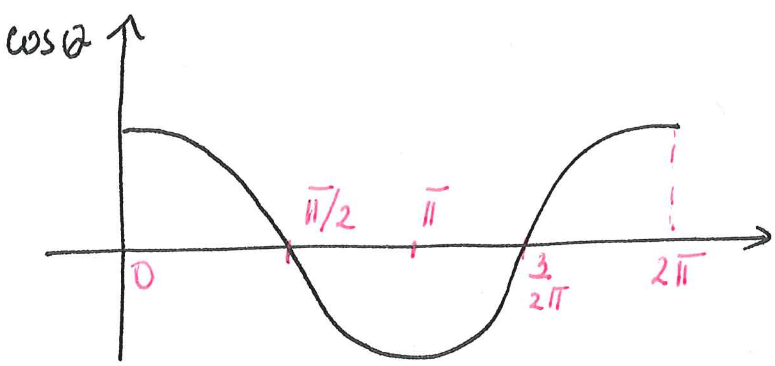
$$\vec{A} \circ \vec{B} = |\vec{A}|\,|\vec{B}| \cdot \cos\theta \rightarrow \text{symetric}$$

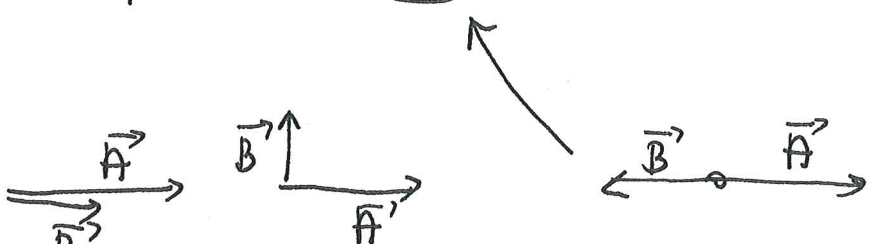


$$\vec{A} \circ \vec{B} = A_x B_x \overset{=1}{\hat{i} \circ \hat{i}} + A_x B_y \overset{=0}{\hat{i} \circ \hat{j}} + A_y B_x \overset{=0}{\hat{j} \circ \hat{i}} + A_y B_y \overset{=1}{\hat{j} \circ \hat{j}}$$

$$\vec{A} \circ \vec{B} = A_x B_x + A_y B_y$$

$$\overrightarrow{\text{vector}} \circ \overrightarrow{\text{vector}} = \text{SCALAR}$$





$\vec{A} \circ \vec{B}$
$= |\vec{A}|\,|\vec{B}|$

$\vec{A} \circ \vec{B}$
$= 0$

$\vec{A} \circ \vec{B} =$
$|\vec{A}|\,|\vec{B}| \cdot (-1)$

$$\vec{A} \circ \vec{B} = A_x B_x + A_y B_y$$