1 January 11, 2017

1.1 Fundamental Particles

All fundamental particles can be classified according to two observable parameters, $\mathbf{mass}\ m$ and $\mathbf{charge}\ q$. We note that this simplified model of he universe is incredibly effective at explaining a wide range of physical phenomena. There are four fundamental forces in nature:

- 1. Strong Nuclear Force is responsible for holding together protons and neutrons, as well as holding atomic nuclei together. It is over a very short range, around 10^{-15} m.
- 2. Weak Nuclear Force is responsible for radioactive decay and fusion reactions in the sum. It has a very short range, around 10^{-17} m.
- 3. **Electromagnetic Force** is responsible for nearly everything we observe. It is an extremely important force to understand that ranges over a long range.
- 4. **Gravitational Force** is responsible for planetary orbits, holding together galaxies and maintaining an atmosphere. It has a long range.

An electric charge is an intrinsic property of particles. It is a quantity that determines the strength of the electric force between two objects. It cannot be created or destroyed, but can transfer from one object to another. Like charges repel, while opposite charges attract. Electric charge is always quantized. Charge always comes in some integer multiple of some fundamental charge e, which is the charge of the electron.

Almost all of the mass of an atom is contained in the nucleus, while almost all of the space is occupied by the electron cloud. Therefore, the diameter of the nucleus is much smaller than the diameter of its corresponding atom. Thus, electric charge comes in discrete packages, as does photons

Insulators do not conduct electricity, since the electrons are not free to move. The valence electrons are tightly bounded. **Conductors** on the other hand, do conduct, since the electrons are free to move. In conductors, valence electrons form a "sea of electrons". We have two kinds of charges, positive and negative. We always draw the force vector with the tail on the particle. We recall that same charges repel, while opposite charges attract.

1.2 Charge Experiment

When we rub a balloon on our hair, we cannot create charged particles. However, when rubbing, electrons may be transferred from one to the other. This results in a net transfer of charge. In an isolated atom, the electron cloud is centered on the nucleus. When an external charge polarizes the atom, the polarized atom becomes

an electric dipole. This results in polarized atoms. In the case of the balloon experiment, the local external charge on the side of the balloon that was rubbed agains the hair causes a net force on the wall, which is an insulator.

Negatively charged valence electrons inside a conductor are able to freely move around. As a result, while the positively charged atomic cores are fixed in place. When we bring a positive rod near a conductor, the metal's net charge is still zero, but it has been polarized by the charged rod. Free electrons are attracted to the positively charged rod, inducing a polarization. A deficit of electrons on the far side of the conductor results.

2 January 12, 2017

2.1 Coulomb's Law

We know that the force between charged objects varies with distance, and this force also depends on the amount of charge. **Coulomb's law** states that the electric field decreases with distances. The force that describes this is Coulomb's force. Coulomb's law describes the force that charged particles exert on each other. For point charges, the force always acts along the line joining the charges. We can formulate Coulomb's law in two different ways:

$$F = K \frac{q_1 q_2}{r^2},$$

where K, the electrostatic constant, is equal to $8.99 \times 10^9 \frac{N \cdot m^2}{C^2}$, and

$$F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2},$$

where ϵ_0 , the permittivity of free space, is $\frac{1}{4\pi K} = 8.85 \times 10^{-12} \frac{C^2}{N \cdot m^2}$.

To compute the magnitude and direction, we first follow the following steps:

- 1. Find the distance between the charges.
- 2. Draw a line passing through the two charges.
- 3. The force on q_1 due to q_2 has its tail at location 1, and points either towards q_2 or away from q_2 .
- 4. Pick the direction according to the basic rule of charges, where like charges repel and opposite charges attract.

The SI unit for charge is the **coulomb** (C). Fundamental charge is the smallest possible amount of free charge, which is equal to the charge of a proton,

$$e = 1.60 \times 10^{-19} C.$$

Therefore, we note that $1 C = 6.25 \times 10^{18}$ protons. We sometimes use microcoulombs $(10^{-6}C)$ and nanocoulombs $(10^{-9}C)$.

2.2 Superposition Principle

The total force on a point charge q_3 is the vector sum of the individual forces acting on the charge by both q_1 and q_2 . We can consider the building blocks of electric charge by considering the effect of a point charge on positive charges q located at any location relative to the point charge. If the point charge is also positive, the electric force on the charges q would be directed outwards, with a decreasing magnitude as q increases in distance from the point charge. In the case that the point charge is negative, the point charges would be attracted instead.

Remark. \vec{i} corresponds with the positive x axis, \vec{j} corresponds to the positive y axis, and \vec{k} corresponds to the positive z axis.

3 January 13, 2017

3.1 Van de Graff Generator

A Van de Graaf generator uses a moving belt to accumulate electric charge on a hollow metal globe on the top of an insulated column, creating high electric potentials. Without a potential difference, there is no discharge through the air to the ground. By slowing the discharge with a lightning rod, the discharge to ground slows, and no visible instant jump discharge occurs.

4 January 16, 2017

4.1 Coulomb's Law Examples

Example. Suppose that the force of q_1 on q_3 is $\vec{F}_{13} = +3N$, and the force of q_3 on q_1 is $\vec{F}_{31} = F_{31}\hat{i}$. Determine the component F_{31} .

According to Newton's Third Law, we recall that the force is equal in magnitude and opposite in direction. Therefore, $\vec{F}_{13} = \vec{F}_{31}$. Thus, the scalar factor by which we multiply the x component is -3N.

Example. Determine the total electric force on q_1 given that q_3 has the same charge and is d to the right of q_1 , and q_2 has the opposite charge and is 2d from q_1 .

We apply Coulomb's Law to determine the force of \vec{F}_{21} and \vec{F}_{31} . Since the charges are $\pm 1C$, and the distance between charges is 1m and 2m respectively, we can use $K = 8.99 \times 10^{-9} \frac{N \cdot m^2}{C^2}$ to find that the forces become

$$\|\vec{F}_{31}\| = 8.99 \times 10^{-9} N,$$

 $\|\vec{F}_{21}\| = 2.25 \times 10^{-9} N.$

Taking the direction into consideration, we note that the force towards q_1 is greater than that away from q_1 . Therefore, the net force becomes

$$\vec{F}_{net} = (-6.74 \times 10^{-9} N) \,\hat{i}.$$

5 January 28, 2017

5.1 Coulomb's Law Examples Cont'd

Example. Let $q_1 = +2C$, $q_2 = +5C$, and r = 10m. Determine the electrostatic force on the charges, given that they are positioned at 30° from each other.

We first determine the unit vector in this direction. Wr note that $\hat{r}_{12} = \cos(30^{\circ}) \hat{i} + \sin(30^{\circ}) \hat{j}$. We apply Coulomb's law to determine that

$$\vec{F}_{12} = K \frac{q_1 q_2}{r^2} \hat{r}_{12}$$

$$= (8.99 \times 10^9 N m^2 / C^2) \frac{(2C)(5C)}{(10m)^2} \hat{r}_{12}$$

$$= (8.99 \times 10^8 N) \hat{r}_{12}$$

$$= 8.99 \times 10^8 (\cos (30^\circ) + \sin (30^\circ)) N$$

Similarly, we note that $\vec{F}_{21} = 8.99 \times 10^8 (-\cos(30^\circ) - \sin(30^\circ)) N$.

Remark. We can use the Pythagorean Theorem and the definition of trigonometric functions to rewrite our expressions for electrostatic force.

5.2 Electric Force of Dipole

Consider a dipole consisting of a positive and negative charge, with charges q and -q located d apart. Another charge Q is located x away on the axis perpendicular to the center of the dipole. Since the horizontal components cancel due to symmetry, we can simply add the **vertical components** to determine an expression for the electric force,

$$F = 2\left(\frac{KqQ}{x^2 + \left(\frac{d}{2}\right)^2}\right)\sin\theta.$$

However, we can express

$$\sin \theta = \frac{\left(\frac{d}{2}\right)}{\sqrt{x^2 + \left(\frac{d}{2}\right)^2}},$$

so the final expression becomes

$$F = \frac{KqQd}{\left(x^2 + \left(\frac{d}{2}\right)^2\right)^{\frac{3}{2}}}.$$

6 January 19, 2017

6.1 Electric Force of Line of Charge

Consider a line of charge L with a total charge Q. That is, instead of a discrete number of charges, we have a continuous set of charges in a line. We note that the linear charge density is given as

 $\lambda = \frac{Q}{L}.$

In the case that the line of charge is positioned vertically, we note that the y component of the charge cancels out. We are therefore concerned only with determining the **horizontal component**. The electrostatic force due to a line of charge on a point charge is given by

 $F = \frac{KqQ}{d\sqrt{\left(\frac{L}{2}\right)^2 + d^2}},$

where K is the electrostatic constant, d is the distance from the point with charge q to the centre of the line of charge, Q is the total charge of the rod of length L.

We can also consider the limit as d >> L or when d << L. In the case that d >> L, the wire line of charge appears to be a point charge since $\left(\frac{L}{2}\right)^2 + d^2 \approx d^2$. Therefore, we can use the simplified expression

$$F = \frac{kqQ}{d^2}.$$

In the case that $d \ll L$, we note that since we are near the line of charge, the length appears to be infinite since $\left(\frac{L}{2}\right)^2 + d^2 \approx \left(\frac{L}{2}\right)^2$. Therefore, we can use the simplified expression

$$F = \frac{2kq\lambda}{d}.$$

7 January 20, 2017

7.1 Electric Force Examples

Example. Calculate the net force on particle 1 with a charge of +2q located at the bottom left corner, from particle 2 with a charge of -2q located at the bottom right corner, particle 3 with a charge of -q located at the top right corner, and particle 4 with a charge of +q located at the top left corner. They form a square with side lengths a.

We apply the superposition principle with Coulomb's law to determine that

$$\vec{F}_{21} = 4K \frac{q^2}{a^2} \hat{i},$$

$$\vec{F}_{41} = -2K \frac{q^2}{a^2} \hat{j},$$

$$\vec{F}_{31} = K \frac{q^2}{a^2} \left(\cos(45^\circ) \, \hat{i} + \sin(45^\circ) \, \hat{j} \right).$$

The overall net force is therefore

$$\vec{F}_{21} = K \frac{q^2}{a^2} \left((4 + \cos(45^\circ)) \,\hat{i} + (-2 + \sin(45^\circ)) \,\hat{j} \right).$$

7.2 Charge Geometries

We can consider the four main basic charge geometries. These geometries are basic because they are very symmetrical. We note that these charge geometries can be either positive or negative in charge. We have already considered the point charge, so we shall now consider the three others:

1. An infinitely long charged wire with length L and charge Q has a linear charge density of

 $\lambda = \frac{Q}{L}.$

2. An infinitely wide charged plane with area A and charge Q has a surface charge density of

$$\sigma = \frac{Q}{A}.$$

3. A charged sphere with volume V and charge Q has a volume charge density of

$$\rho = \frac{Q}{V}.$$

Electric dipole moment for charges +q and -q separated by a distance of s is given by

$$\vec{p} = qs$$
,

in the direction from -q to +q. Regarding charges, we note the principles of **charge quantization**, which states that

$$q = ne$$
,

where $n = \pm 1, \pm 2, \pm 3, ...$ and $e = 1.60 \times 10^{-19} C$, and **charge conservation**, as in the case of annihilation and pair production given respectively as

$$e^- + e^+ \rightarrow \gamma + \gamma,$$

$$\gamma \rightarrow e^- + e^+$$

where γ is the gamma ray, e^- is the electron, and e^+ is the positron.

8 January 23, 2017

8.1 Electric Fields

When a charge A exerts a force on charge B through empty space, where there is no contact and no apparent mechanism, we refer to this as an action-at-a-distance force. Gravity is an example of such a force, is the the electrostatic force. In our example, if A suddenly moved to a new position, the force on B varies to match this change. We consider the case where we only have one charge. This charge still affects the surrounding space. We quantify this by making use of the concept of an electric field. That is, charges create fields and the fields push the charges. A field is the ability to exert an electric force if a charge were present. The equation for an electric field is given as

$$\vec{E}(x,y,z) = \frac{\vec{F}(x,y,z)}{q}.$$

For a point charge for instance, the force on q' at a point in the field can be measured from the charge q. We note then that

$$\vec{F}(x,y,z) = \frac{1}{4\pi\epsilon_0} \frac{qq'}{r^2}$$

in the direction away from q'. The electric field is therefore given as

$$\vec{E}(x,y,z) = \frac{\vec{F}}{q'} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$$

in the direction away from q'. We can also add up the field like vectors by using the superposition principle. We can therefore add up all the forces on a charge and divide out the charge. That is,

$$\vec{E}(x, y, z) = \frac{\vec{F}}{q}$$

$$= \frac{\vec{F}_{1q}}{q} + \frac{\vec{F}_{2q}}{q} + \frac{\vec{F}_{3q}}{q} + \dots$$

$$= \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \dots$$

$$= \sum_{i} \vec{E}_i$$

Field lines show the density of a field. They are lines with arrows showing the direction of the field. The density of the field lines gives an idea of how strong the field is. Field lines also never cross each other. Field lines are drawn outwards from positive charges, and inwards towards negative charges.

8.2 Electric Field of a Dipole

9 January 25, 2017

9.1 Electric Field of Line of Charge

9.2 Electric Field of Ring of Charge

The electric field due to a ring of charge with radius R on a point P located directly z from the center of the ring so as to maintain symmetry with the ring is given as

$$E_{ring} = \frac{1}{4\pi\epsilon_0} \frac{zQ}{(z^2 + R^2)^{\frac{3}{2}}}.$$

We note the limiting cases where z approaches zero, and when z >> R. First we consider when z = 0, the numerator becomes 0, so

$$E_{ring} \approx \frac{1}{4\pi\epsilon_0} \frac{zQ}{(z^2 + R^2)^{\frac{3}{2}}} = 0.$$

In the case when z >> R, R becomes essentially 0 compared to z, so we obtain

$$E_{ring} \approx \frac{1}{4\pi\epsilon_0} \frac{zQ}{(z^2 + 0^2)^{\frac{3}{2}}} = \frac{1}{4\pi\epsilon_0} \frac{Q}{z^2}.$$

10 January 26, 2017

10.1 Electric Field of Charged Disk

Let the surface charge density of the disk of radius R to be $\sigma = \frac{Q}{A}$, and let point P be located directly z from the center of the disk so as to maintain symmetry with the disk. The electric field is therefore given as

$$E_{disk} = \frac{\sigma}{2\epsilon_0} \left(1 - \frac{z}{\sqrt{z^2 + R^2}} \right).$$

We note the limiting cases where z approaches zero, and when z >> R. First we consider when z = 0. Since we are positioned close to the disk, the area of the plane appears to extend indefinitely. Thus, the following is also the expression for the electric field of the plane,

$$E_{disk} \approx E_{plane} = \frac{\sigma}{2\epsilon_0}.$$

Note that when z > 0, the electric field for the plane is positive, whereas the field is negative when z < 0. In the case when z >> R, R becomes essentially 0 compared to z, so we obtain

$$E_{disk} \approx \frac{Q}{4\pi\epsilon_0 z^2}.$$

For disks with arbitrary widths composed of different densities, we evaluate

$$-\frac{\sigma z}{2\epsilon_0} \left(z^2 + r^2\right)^{-1/2} \Big|_{R_1}^{R_2},$$

where R_1 is inner radius and R_2 is outer radius. It follows that the electric field of a disk comes from this when $R_1 = 0$ and $R_2 = R$.

10.2 Electric Field Lines

We recall that field lines are less dense when the field is weaker, and field lines are more dense when the field is stronger. Electric field lines are continuous curves, while electric field vectors are tangent to the field lines. We also note that two electric field vectors cannot intersect, since the electric field at that point would be undefined. **Sources** of field lines are positive charges where field lines start, and **sinks** are negative charges where field lines end. We can consider this in the case of electric dipoles with two charges of equal magnitude and opposite sign.

When electric force is the only force acting on a particle, we can relate $\vec{F} = m\vec{a}$ with $\vec{F} = q\vec{E}$. Thus,

$$\vec{a} = \frac{q\vec{E}}{m}.$$

In a uniform field, \vec{E} is the same everywhere.

11 January 27, 2017

11.1 Electric Flux

Field strengths are measured in

$$\vec{E} = \frac{\vec{F}}{q},$$

where \vec{F} is measured in Newtons N and q is measured in Coulombs C. If the field is coming out of each face of the box, then there must be a positive charge in the box. If the field is going into each face of the box, then there must be a negative charge in the box. A field that passes through the box implies that there is no et charge in the box. Gauss' Law is equivalent to Coulomb's Law. It presents an easier way to calculate electric fields in specific circumstances (especially situations with a high degree of symmetry). It also provides a better understanding of the properties of conductors in electrostatic equilibrium and is valid for moving charges since it is not limited to electrostatics.

12 January 30, 2017

12.1 Electric Flux Cont'd

A closed surface through which an electric field passes is called a **Gaussian surface**. This is an imaginary mathematical surface that is closed around a charge. The Gaussian surface is most useful when it matches the shape of the field. Gauss' law relates the electric field at points on a closed Gaussian surface to the net charge enclosed by that surface. The **area vector** of a surface indicates the vector perpendicular to the surface, and always points outside. **Electric flux** Φ_e is defined by the amount of electric field going through a surface and the number of field lines coming through a surface. It is given by

$$\Phi_e = E \cdot A = EA\cos(\theta).$$

where E specifies the electric field, A is the area of the surface, and θ is the angle between E and A. This is the expression for a flat surface and a uniform field. If the field lines are going towards the surface, it is negative. If the field lines are extending from the surface, it is positive. We note that the electric flux through a parallel surface is 0, whereas the electric flux through a perpendicular surface is the entire magnitude of EA.

The **total flux** through a closed surface under a uniform field can be obtained by integrating the dot product over the full surface

$$\Phi = \int \vec{E} \cdot d\vec{A}.$$

The **net flux** through a closed surface (which is used in Gauss' law) is given as

$$\Phi = \oint \vec{E} \cdot d\vec{A}.$$

To determine the flux through a closed surface, we first divide the closed surface into pieces that are tangent to the electric field, perpendicular to the electric field, or with a certain specific angle to the field. We then evaluate the surface integral.

For instance, suppose we have a cylinder surface. We can divide it into sections a, b, and c, where a is the area of the top circle, b is the area of the side of the cylinder, and c is the area of the bottom circle. Thus, we obtain

$$\Phi_e = \oint_a \vec{E} \cdot d\vec{A} + \oint_b \vec{E} \cdot d\vec{A} + \oint_c \vec{E} \cdot d\vec{A}$$

13 February 1, 2017

13.1 Gauss' Law

Gauss' law relates the net flux Φ of an electric field through a closed surface (a Gaussian surface) to the net charge q that is enclosed by that surface. That is,

$$\epsilon_0 \Phi = q,$$

$$\epsilon_0 \oint \vec{E} \cdot d\vec{A} = q.$$

Therefore, charge outside of the surface is not considered. We note that this expression can be rearranged to find that

$$\oint \vec{E} \cdot d\vec{A} = \frac{q}{\epsilon_0}.$$

We consider the electric flux over a proton, electron, and then a proton along with an electron. We apply the equation directly above to find that $\frac{+q}{\epsilon_0}$, $\frac{-q}{\epsilon_0}$, and $\frac{0}{\epsilon_0}$. The last surface encloses no net charge, since the enclosed positive and negative charges have equal magnitudes. Gauss' lLw requires that the net flux of the electric field through this surface be zero. Gauss' Law us applied to closed surfaces, and the electric flux is independent of surface shape and radius.

Applying Gauss' Law to a point charge, we obtain the expression of an electric field for a point charge. Thus, we note that Gauss' Law provides equivalent solutions to Coulomb's Law. Furthermore, charges outside the surface do not affect the electric flux, since there is no net flow into our out of the surface due to the external charge. Therefore, the net flux through a Gaussian surface that does not contain any charge is zero. For multiple charges, the electric flux is given as

$$\Phi = \left(\frac{q_1}{\epsilon_0} + \frac{q_2}{\epsilon_0} + \dots + \frac{q_n}{\epsilon_0}\right),\,$$

for charges $q_1, q_2, ..., q_n$ enclosed by the surface.

14 February 2, 2017

14.1 Electric Field in Shell of Charge

We shall use Gauss' law to compute the electric field inside and outside a spherical shell of charge. By using a symmetrical argument, we note that the electric field must point in the radial direction only. Furthermore, the electric field must be the same magnitude at a constant radius. We note that inside the sphere, there is no enclosed charge, so

$$E_{shell} = 0$$

for r < R, where r is the imaginary radius of the sphere at point P and R is the radius of the shell of charge. For any radius outside the shell of charge, we have an enclosed charge of Q. Since the surface area of a sphere is $4\pi r^2$, we recall that $\oint \vec{E} \cdot d\vec{A} = EA = \frac{q}{\epsilon_0}$, so our expression is

$$E_{shell} = \frac{Q}{4\pi\epsilon_0 r^2}$$

for r > R.

Remark. When we want to determine the electric field at a certain point, we can consider the sphere with that point lying on the surface of the sphere. We then consider the charges enclosed by that imaginary sphere.

14.2 Electric Field in Shell of Charge

We note that the electric field outside a sphere of charge is the same as the electric field outside of a shell of charge Q. That is,

$$E_{sphere} = \frac{Q}{4\pi\epsilon_0 r^2}$$

for r > R. The field inside the sphere of charge can be determined by considering the volume charge density

$$\rho = \frac{Q}{\frac{4}{3}\pi R^3},$$

where Q is the total charge of the sphere and R is the radius of the sphere of charge. Thus, since the charge enclosed is $\rho * V$, we note that $V = \frac{4}{3}\pi r^3$, where r is the radius of the imaginary sphere are point P inside of the sphere. Therefore, the enclosed charge is

$$q = \frac{Qr^3}{R^3}.$$

Once again, $E = \frac{q}{A\epsilon_0}$ where $A = 4\pi r^2$. Thus, the electric field inside a sphere of charge is

$$E_{sphere} = Q \frac{r}{4\pi\epsilon_0 R^3}$$

for r < R.

15 February 3, 2017

15.1 Gauss' Law Cont'd

Remark. Different surfaces may be more suitable when the direction of the enclosed charge is not directed radially outwards. For instance, the flux could be more easily calculated using a cubic gaussian surface when presented a plane of charge.

16 February 6, 2016

16.1 Charged Isolated Conductor

If an excess charge is placed on an isolated conductor, that amount of charge will move entirely to the surface of the conductor. None of the excess charge will be found within the body of the conductor. That is, if we charge a conductor, the excess charges spread out on the exterior surface of the conductor, with the electric field at the surface perpendicular to the surface. A **conductor** is a material in which the charges are free to move. This means that

- 1. There is zero net charge inside a conductor ($Q_{net} = 0$). If there are two or more like charges inside a conductor, they will repel and push each other far away (to the surface).
- 2. There is zero electric field inside a conductor $(E_{in} = 0)$. If there is a non-zero field, the F = Eq implies that there is a net force which means charges would move until the force on them is zero, so we have static equilibrium.

Since the electric field inside a conductor is zero, this immediately implies that conductors are electrically neutral in their interiors. This also means that the surface of a hollow cavity inside a conductor cannot carry any excess charge. All excess charge must reside on the outside surface only. The electric field over the conducting surface is given as

$$E = \frac{\sigma}{\epsilon_0}$$
.

To summarize, we note that the electric field inside the conductor is zero. All excess charge is distributed to the surface, so that the conductor is neutral on the inside. In the event that there is a void completely enclosed by the conductor, the electric field inside the enclosed void is zero. Furthermore, the electric field that is distributed to the surface is perpendicular to the surface (and parallel to the area vector) with a magnitude of $E = \sigma/\epsilon_0$. This means that charges are closer together and the electric field is strongest at a pointed end of a surface. That is, the charge density is greatest where the radius of curvature is smallest.

Remark. We note however, that the electric field is not dependent on the distance.

17 February 8, 2017

17.1 Electric Field of Charged Wire and Plane

We note that for a long charged wire with a charge density of λ of length L and radius r, the electric field of the wire is given by

$$E_{wire} = \frac{\lambda}{2\pi\epsilon_0 r}.$$

18 February 9, 2017

18.1 Examples

Example. Two very thin infinite sheets are uniformly charged with surface charge densities of -2q and +5q. What is the magnitude and direction of the electric field between the two sheets?

We note that the electric field would travel from right to left, from the positive charge to the negative charge. We use the superposition principle to obtain a magnitude of +7q.