Last time

- Force on a point charge in an external electric field
- Calculation of electric flux for constant electric fields and flat surfaces.
- Example

This time

- Calculation of electric flux for non-uniform electric fields and arbitrary shape surfaces
- Properties of electric flux for a closed surface
- Introducing Gauss's law
- Activity #4

Flux for a flat surface and uniform electric field

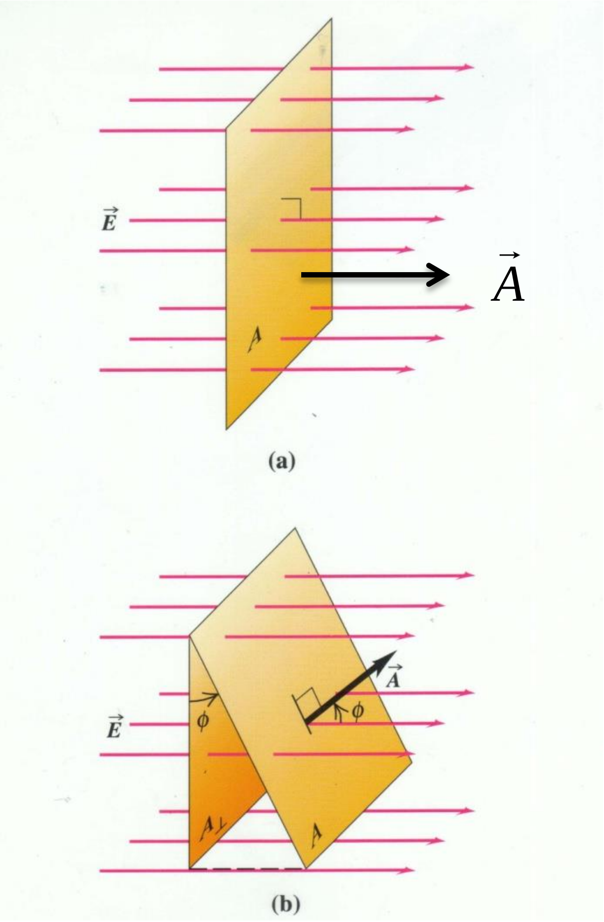
$$\Phi_E = \vec{E} \bullet \vec{A}$$

(a)

$$\Phi_E = EA\cos 0 = EA$$

(b)

$$\Phi_E = \vec{E} \bullet \vec{A} = EA\cos\phi$$



General case

1. Non-uniform electric field

2. Arbitrary shape surface

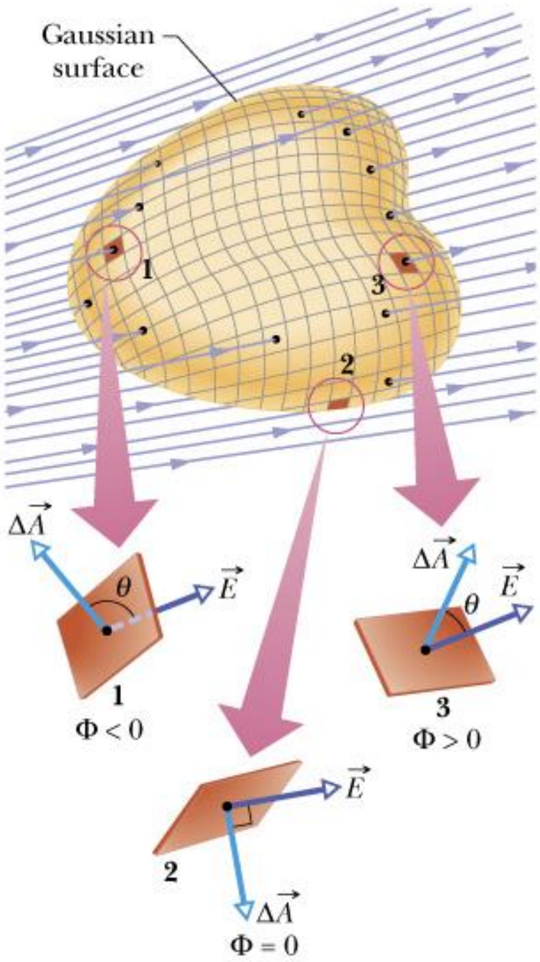
Step 1: Divide the surface into small pieces.
How small?

Small enough that each piece is flat.

Small enough that the electric field across the small piece is a constant.

Step 2: Calculate the flux for each piece.

Step 3: Obtain the total flux by summing over the flux for all the small pieces.



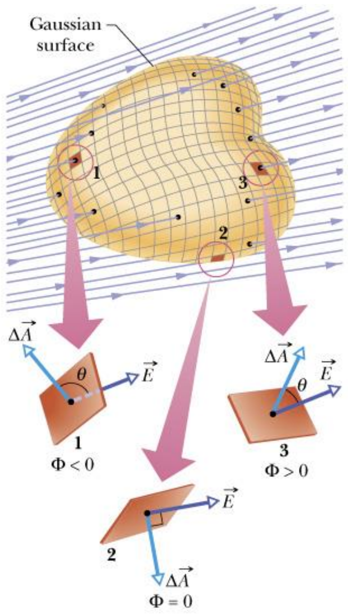
Flux for a given piece

$$\Delta\Phi_i = \vec{E}_i \bullet \Delta\vec{A}_i$$

Add the contribution to flux for all pieces

$$\sum_i \Delta\Phi_i = \sum_i \vec{E}_i \bullet \Delta\vec{A}_i$$

To obtain the exact flux we must divide the surface into a large number of pieces.



$$\Phi_E = \lim_{\Delta A_i \to 0} \lim_{N \to \infty} \sum_{i=1}^{N} \vec{E}_i \bullet \Delta\vec{A}_i$$

$$\Phi_E = \int_{\text{Over the entire surface}} \vec{E} \bullet d\vec{A}$$

This applies to open and closed surfaces.

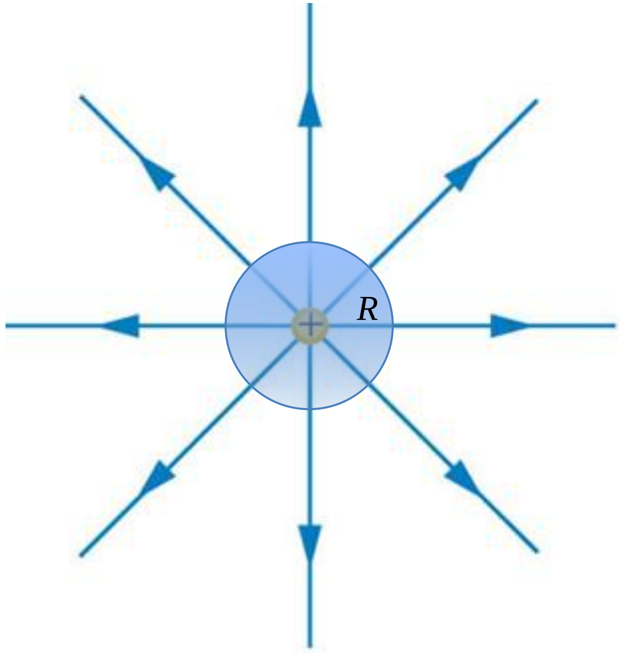
Some properties of electric flux for a closed surface

1. The number of electric field lines (flux) crossing a closed surface is independent of the shape and size of the surface and where the charges are located inside the closed surface.

2. Charge located outside a closed surface does not contribute to net flux.

Flux though a closed surface

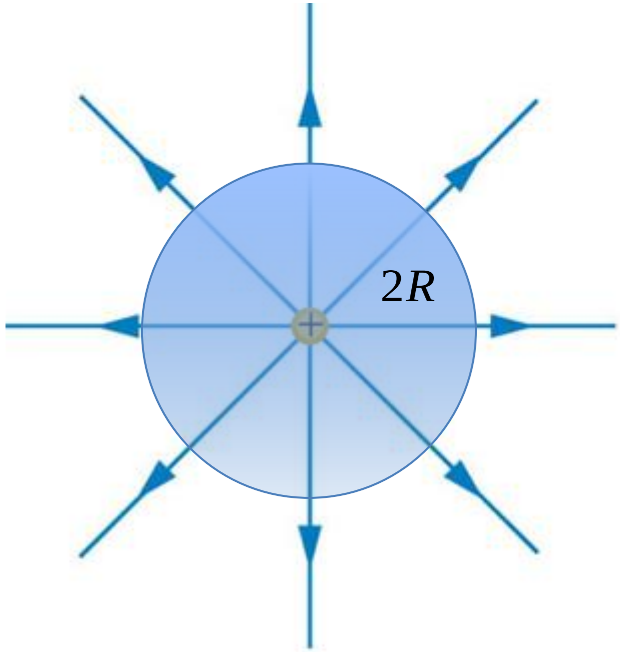
In this 3D problem only 8 electric field lines are drawn for a positive charge +q



How many field lines will cross the closed spherical surface of radius R?

Answer: all 8 field lines

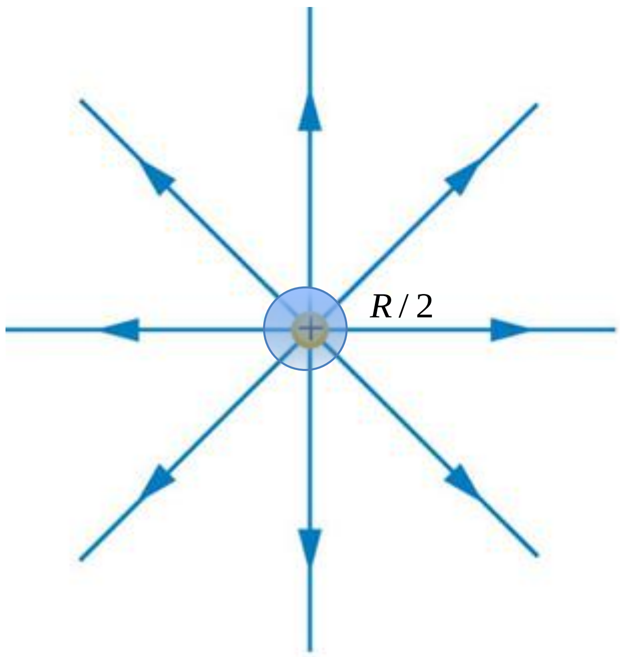
Considering the same charge +q as before



How many field lines will cross the closed spherical surface 2R?

Answer: all 8 field lines

Considering the same charge +q as before



How many field lines will cross the closed spherical surface of radius R/2?

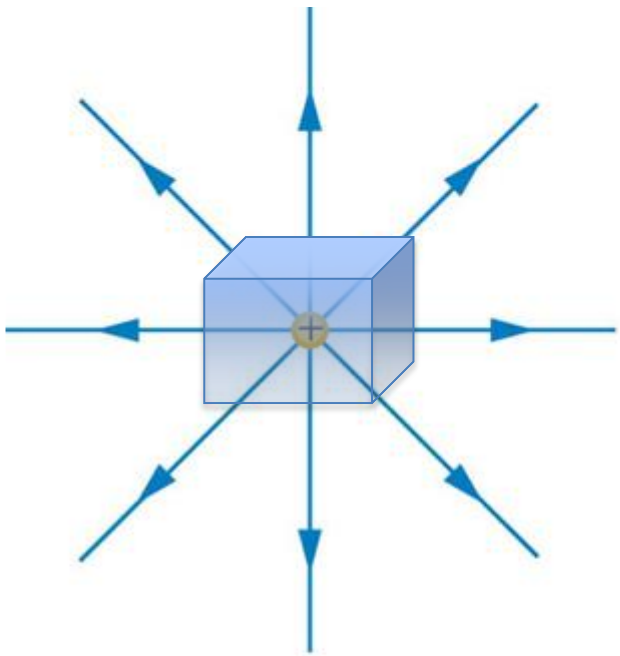
Answer: all 8 field lines

Conclusion:

The number of electric field lines crossing the spherical surface is independent of radius of the sphere.

Does it matter if the closed surface has another shape?

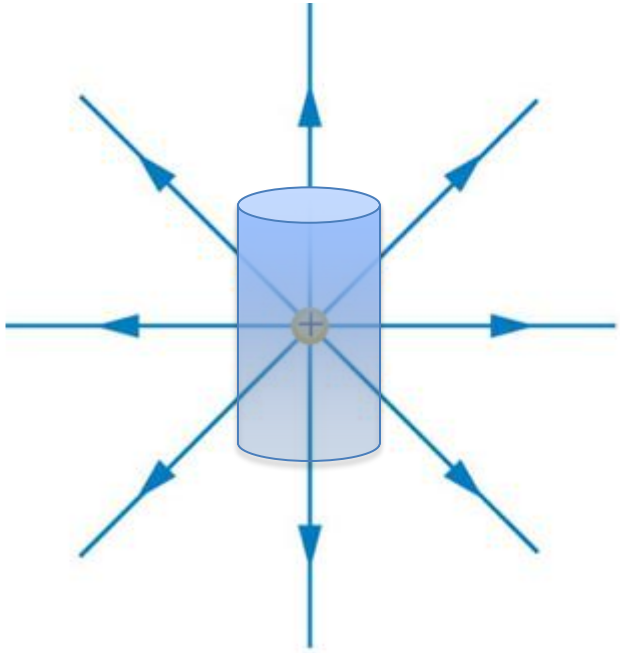
Considering the same charge +q as before



How many field lines will cross the closed rectangular surface?

Answer: all 8 field lines

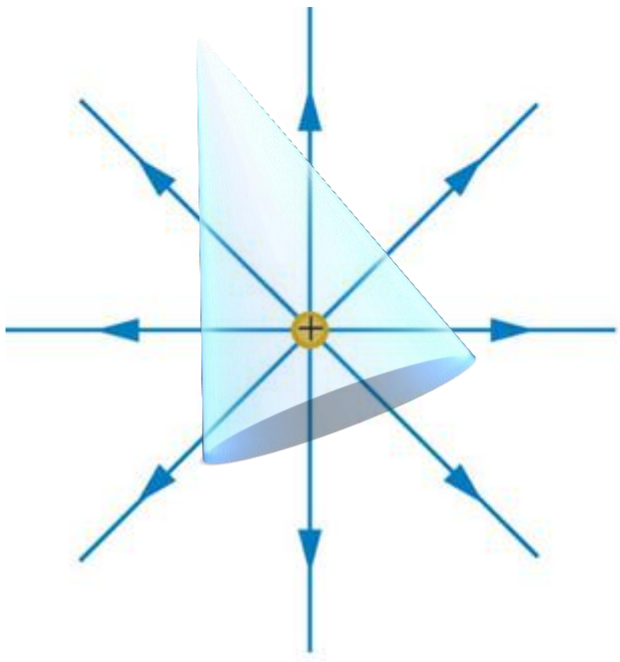
Considering the same charge +q as before



How many field lines will cross the closed cylindrical surface?

Answer: all 8 field lines

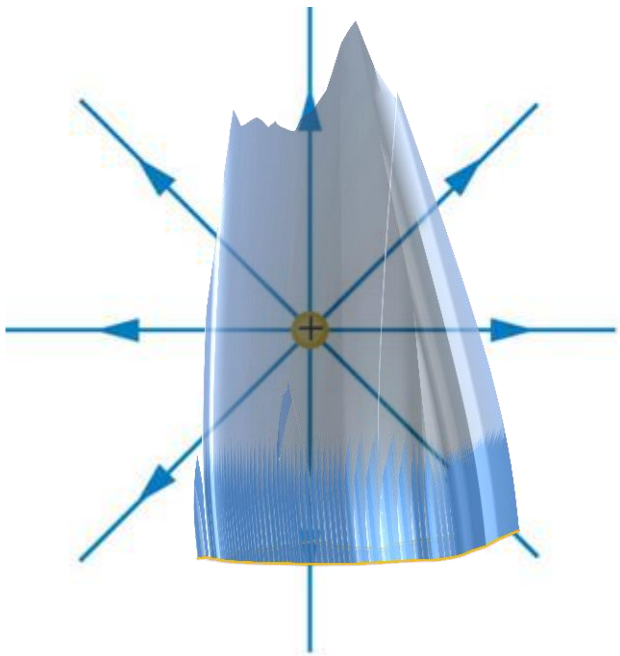
Considering the same charge +q as before



How many field lines will cross the closed conical surface?

Answer: all 8 field lines

Considering the same charge +q as before



How many field lines will cross the closed irregular shaped surface?

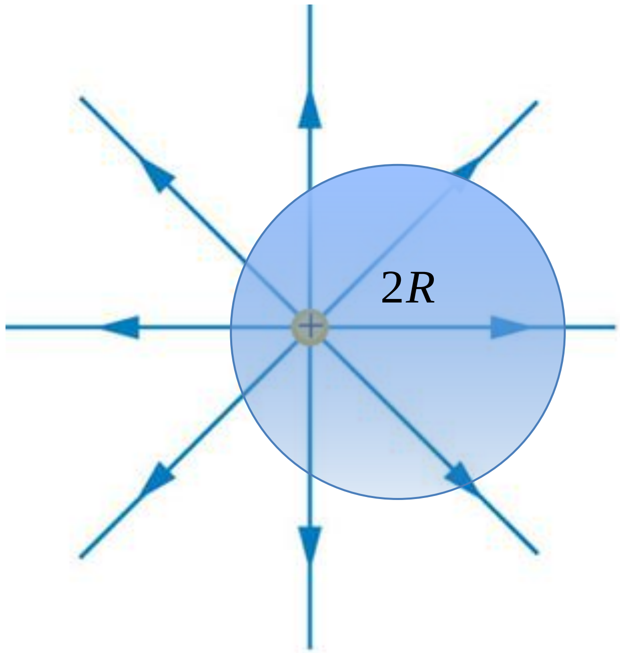
Answer: all 8 field lines

Conclusion:

The number of electric field lines crossing the closed surface is independent of shape of the closed surface.

Does it matter where the charge reside inside the closed surface?

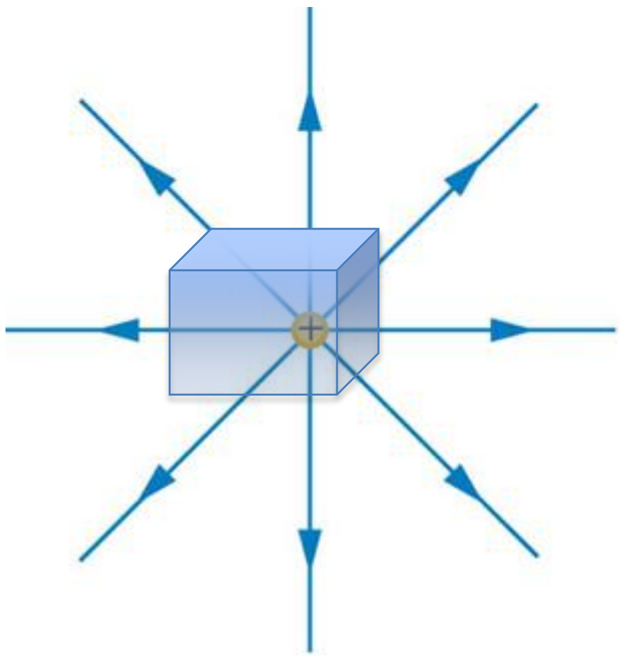
Considering the same charge +q as before



How many field lines will cross the closed spherical surface 2R?

Answer: all 8 field lines

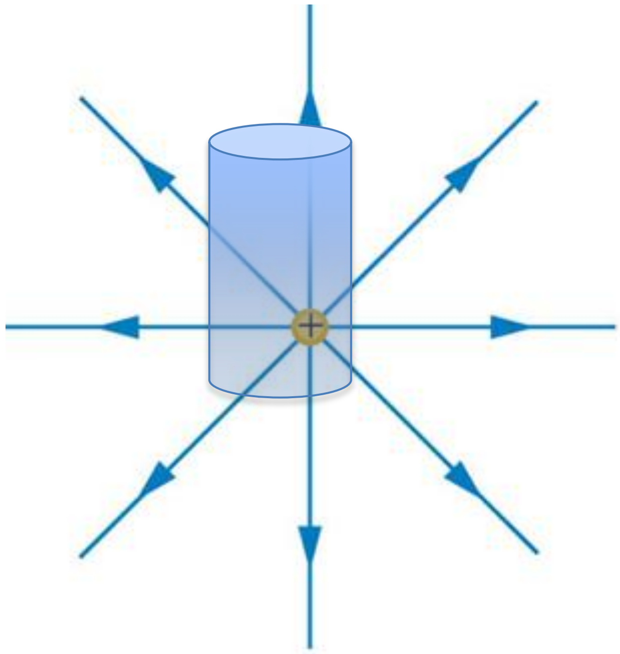
Considering the same charge +q as before



How many field lines will cross the closed rectangular surface?

Answer: all 8 field lines

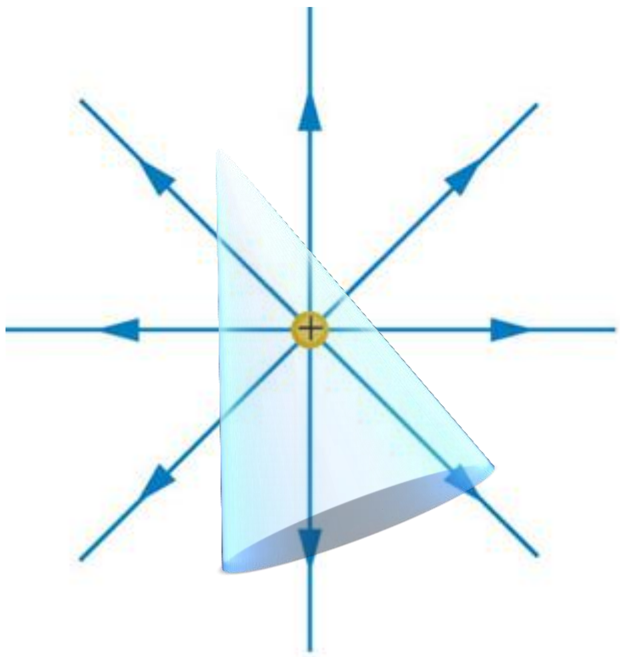
Considering the same charge +q as before



How many field lines will cross the closed cylindrical surface?

Answer: all 8 field lines

Considering the same charge +q as before



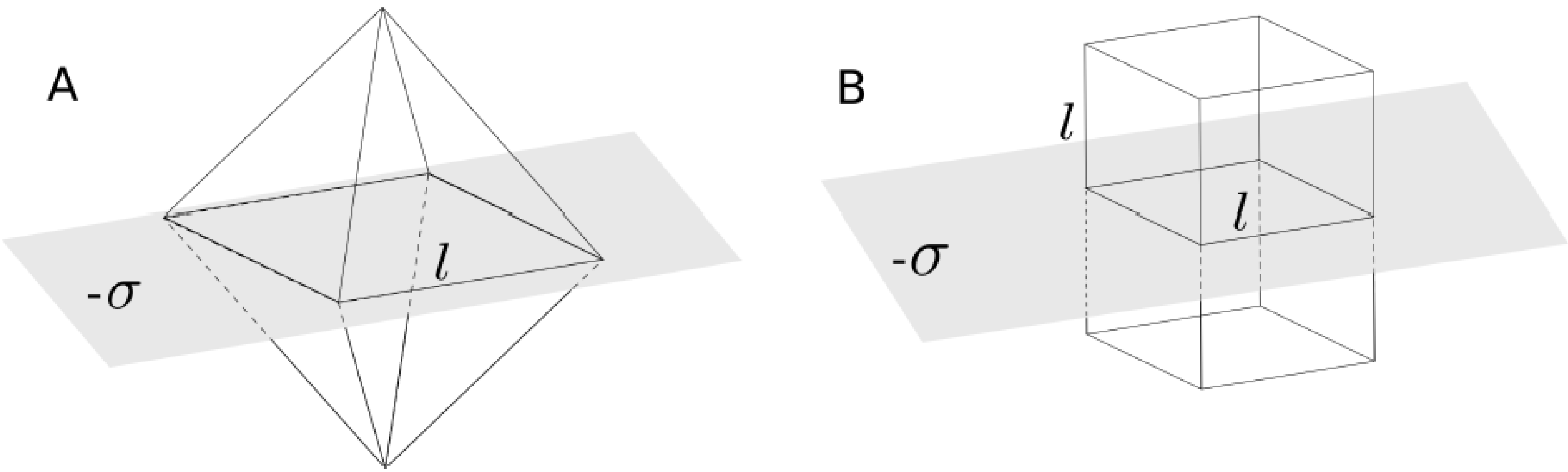
How many field lines will cross the closed conical surface?

Answer: all 8 field lines

Conclusion:

The number of electric field lines crossing the closed surface is independent of the shape and size of the surface and where the positive charge is located inside the closed surface.

(10 marks) The figure below shows an octahedral Gaussian surface made up of equilateral triangles with a square base of length l and a pill-box with a square base of length l. The horizontal plane represents a large thin sheet with uniform surface charge density $-\sigma$. Find the electric flux through the octahedral Gaussian surface.



1. **(1 mark)** In one sentence, state the meaning of Gauss' relation $\oiint \vec{E} \cdot d\vec{A} = \frac{Q}{\epsilon_0}$.

2. **(3 marks)** Find the total electric flux through the Gaussian surface in figure B and draw the surface area vectors for each face.

3. **(2 marks)** For the pill-box in figure B, using Gauss's relation; $\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\epsilon_0}$, find an expression for the electric field in terms of the charge density σ.

4. **(1 mark)** What is the difference between the electric field flux through the pill-box and the octahedron? Explain.

5. **(2 marks)** Find the total electric flux through the octahedral Gaussian surface?

(1 mark for the correct answer) What is the electric flux through the octahedron? Express your answer in terms of σ using the relation $E = \sigma/2\epsilon_0$.

A. $-\frac{\sigma}{2\epsilon_0}l^2$ B. $-\frac{\sigma}{\epsilon_0}l^2$ C. $\frac{\sigma}{2\epsilon_0}l^2$ D. $\frac{\sigma}{\epsilon_0}l^2$