Electricity and Magnetism

- Physics 259 – L02
- Lecture 11

Chapter 23: Gauss's Law

Last time

- Chapters 21 and 22

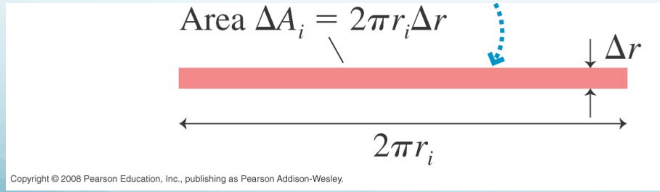
This time

Continue Disk of Charge
Chapter 23.1: Electric Flux

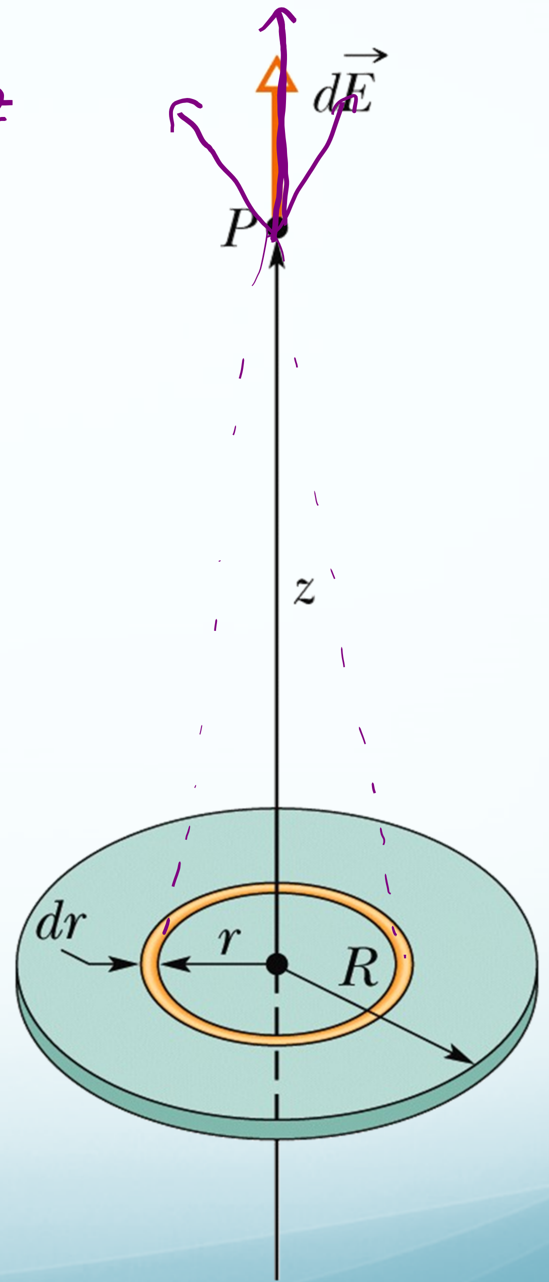
A disk of charge

$$\sigma = \frac{Q}{A} = \frac{Q}{\pi R^2} = \frac{\Delta Q}{\Delta A_i} = \frac{dQ}{dA_i}$$

$$dQ = \sigma dA_i = \sigma 2\pi r_i dr$$

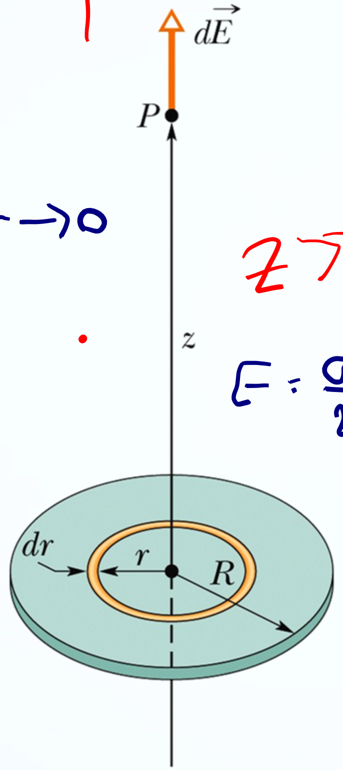


E_z



Limiting cases?

Z>>R



$\frac{R}{z} \to 0$

$z \gg R$

$E = \frac{\sigma}{2\varepsilon_o}\left(1 - \frac{1}{\sqrt{1+\frac{R^2}{z^2}}}\right)$

$$E_{disk,z} = \frac{\sigma}{2\varepsilon_o}\left[1 - \frac{z}{\sqrt{z^2+R^2}}\right]$$

$$E_{disk,z} = \frac{\sigma}{2\varepsilon_o}\left[1 - \frac{z}{\sqrt{z^2}}\right] = 0????$$

$\frac{R}{z} \to$ very small

$$E_{disk,z} = \frac{\sigma}{2\varepsilon_o}\left[1 - \frac{1}{\sqrt{1+R^2/z^2}}\right] \Rightarrow \frac{\sigma}{2\varepsilon_o}\left[1 - \left(1 + R^2/z^2\right)^{-\frac{1}{2}}\right] = \frac{\sigma}{2\varepsilon_o}\left[\cancel{1} - \left(\cancel{1} - \tfrac{1}{2}R^2/z^2\right)\right]$$

$$\approx \frac{\sigma}{2\varepsilon_o}\frac{R^2}{2z^2} = \frac{Q/A}{2\varepsilon_o}\frac{\pi R^2}{2\pi z^2} = \frac{Q}{4\pi\varepsilon_o z^2}$$

$(1+a)^{-1/2} = 1 - \frac{1}{2}a$

$a \ll$

$$E = \frac{\sigma}{2\varepsilon_0}\frac{R^2}{2z^2} = \frac{Q/A}{2\varepsilon_0}\frac{R^2}{2z^2} = \frac{Q/A}{4\varepsilon_0}\frac{R^2}{z^2}$$

$$= \frac{Q/A}{4\pi\varepsilon_0}\frac{\pi R^2}{z^2} = \frac{Q/\cancel{A}}{4\pi\varepsilon_0}\frac{\cancel{A}}{z^2} = \frac{Q}{4\pi\varepsilon_0}\frac{1}{z^2}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{Q}{z^2}$$

$$Z >> R$$

$$\frac{R}{Z} \to 0$$

$$E = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{Z}{\sqrt{Z^2 + R^2}}\right)$$

$$\to E = \frac{\sigma}{2\varepsilon_0}\left(1 - Z \frac{1}{\sqrt{Z^2\left(1 + \frac{R^2}{Z^2}\right)}}\right)$$

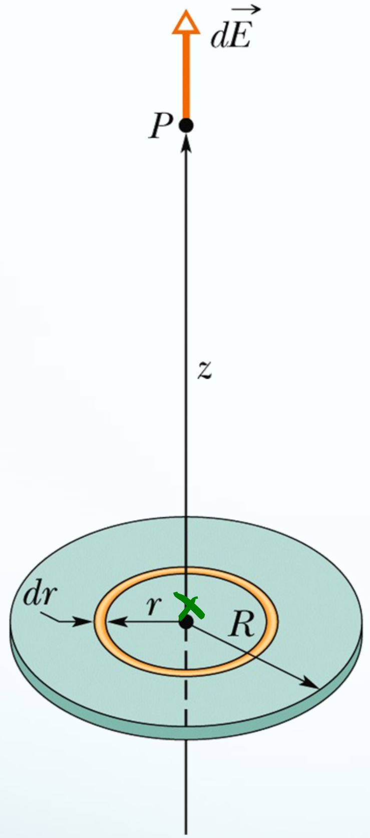
$$= \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{\cancel{Z}}{\cancel{Z}\sqrt{1 + \cancel{\frac{R^2}{Z^2}}}}\right) = \frac{\sigma}{2\varepsilon_0}(1 - 1) = 0$$

0

not very far

$\frac{R}{Z} \to$ verysmall $\to$ we reach to the field of the point charge

Limiting cases? $z \rightarrow 0$



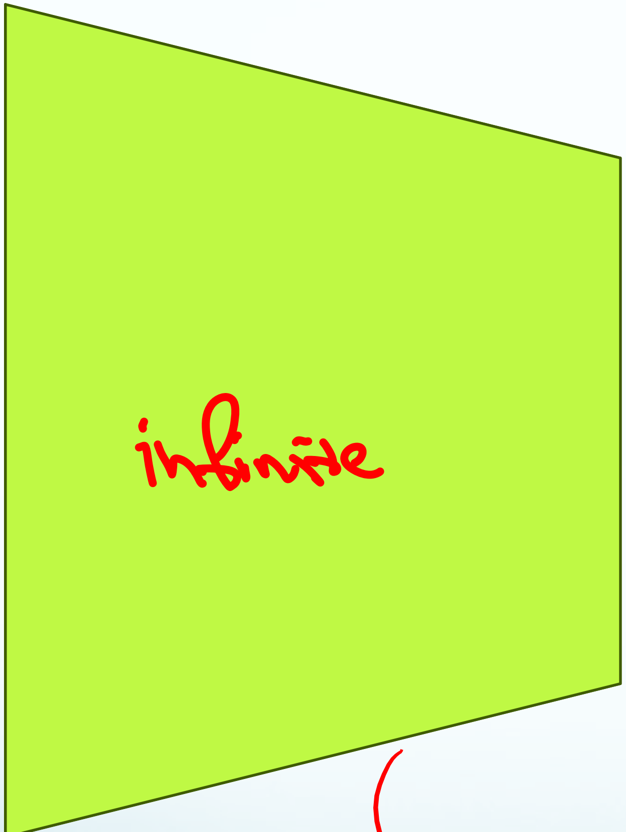
$$E_{disk,z} = \frac{\sigma}{2\varepsilon_o}\left[1 - \frac{z}{\sqrt{z^2 + R^2}}\right]$$

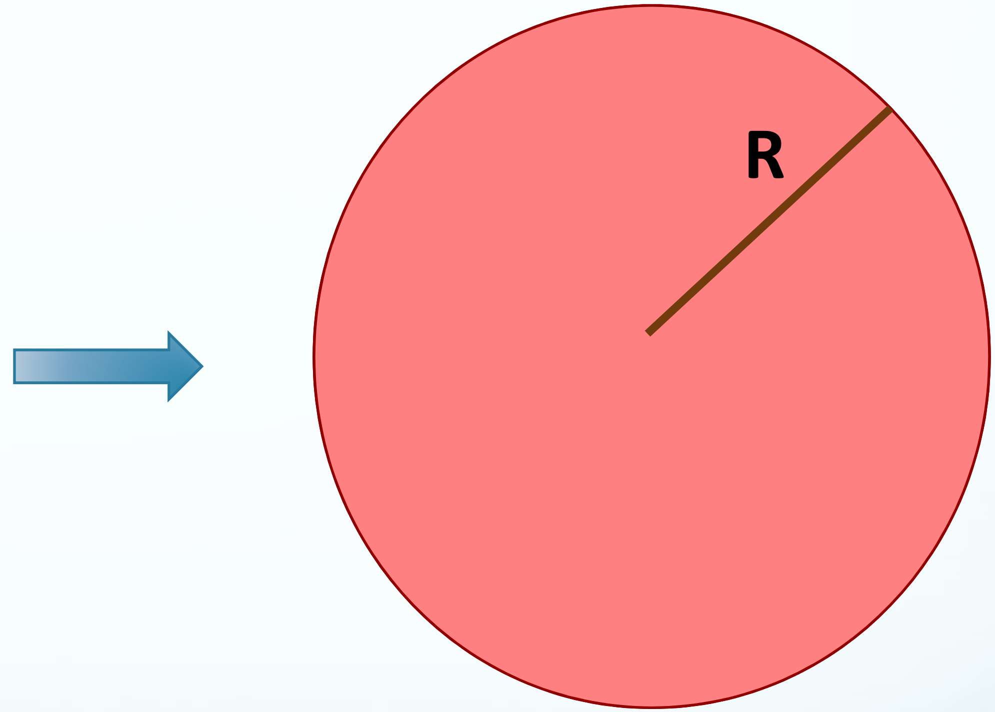
$z \rightarrow 0$

$E_{disk,z} = \frac{\sigma}{2\varepsilon_0}$

$$\boxed{E_{disk,z} = \frac{\sigma}{2\varepsilon_o}}$$

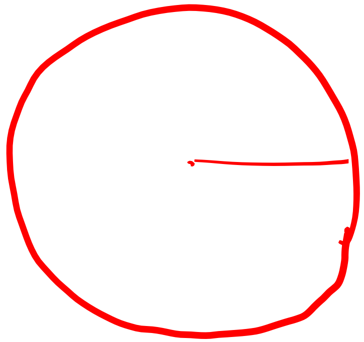
Plane of charge

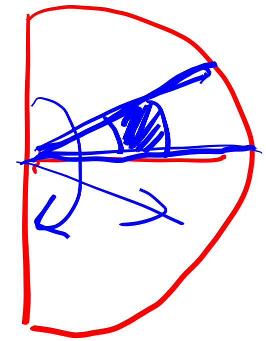




$$(E_{disk})_z = \frac{\sigma}{2\varepsilon_o}[1 - \frac{z}{\sqrt{z^2 + R^2}}]$$

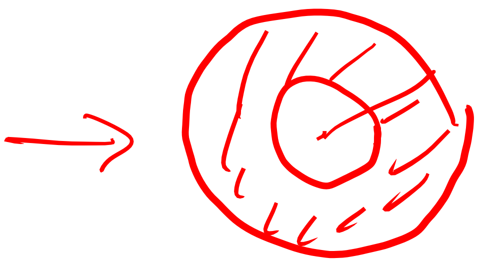
$$\boxed{E_{plane} = \frac{\sigma}{2\varepsilon_o}}$$

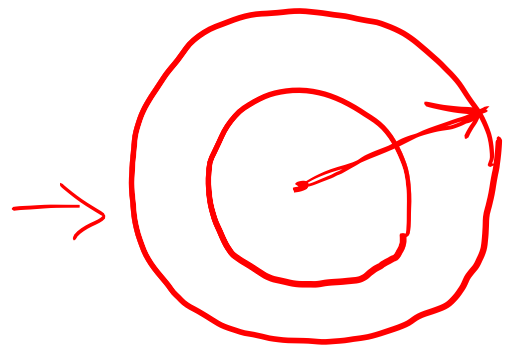


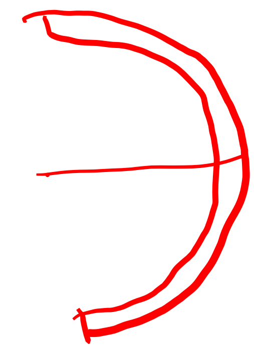


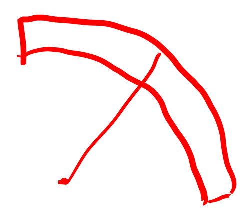
$\int_R$

$\int_\theta$

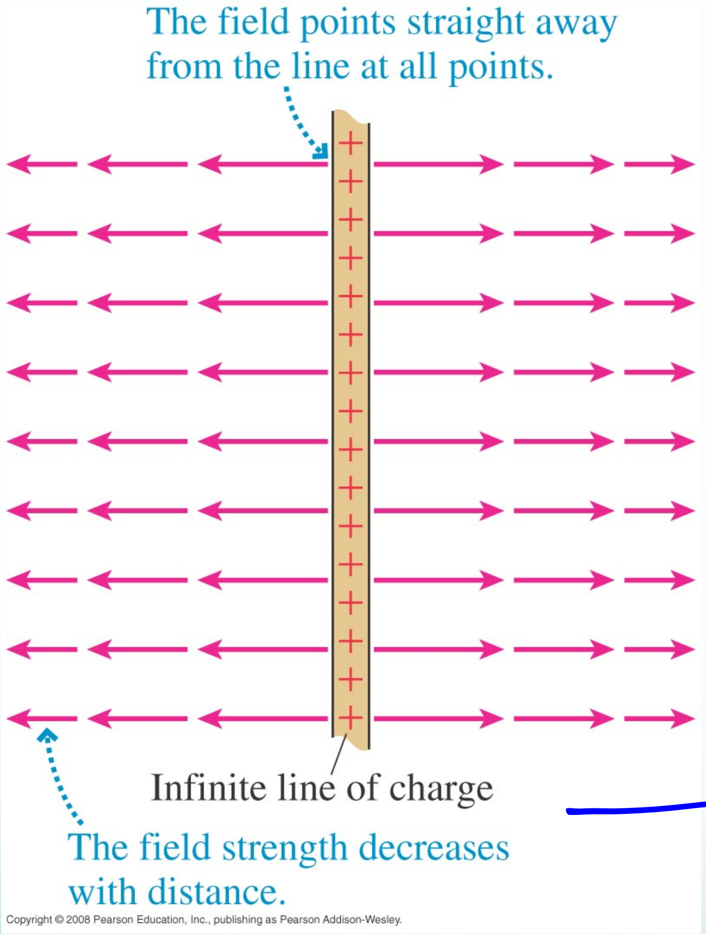








This is the result for a plane of charge

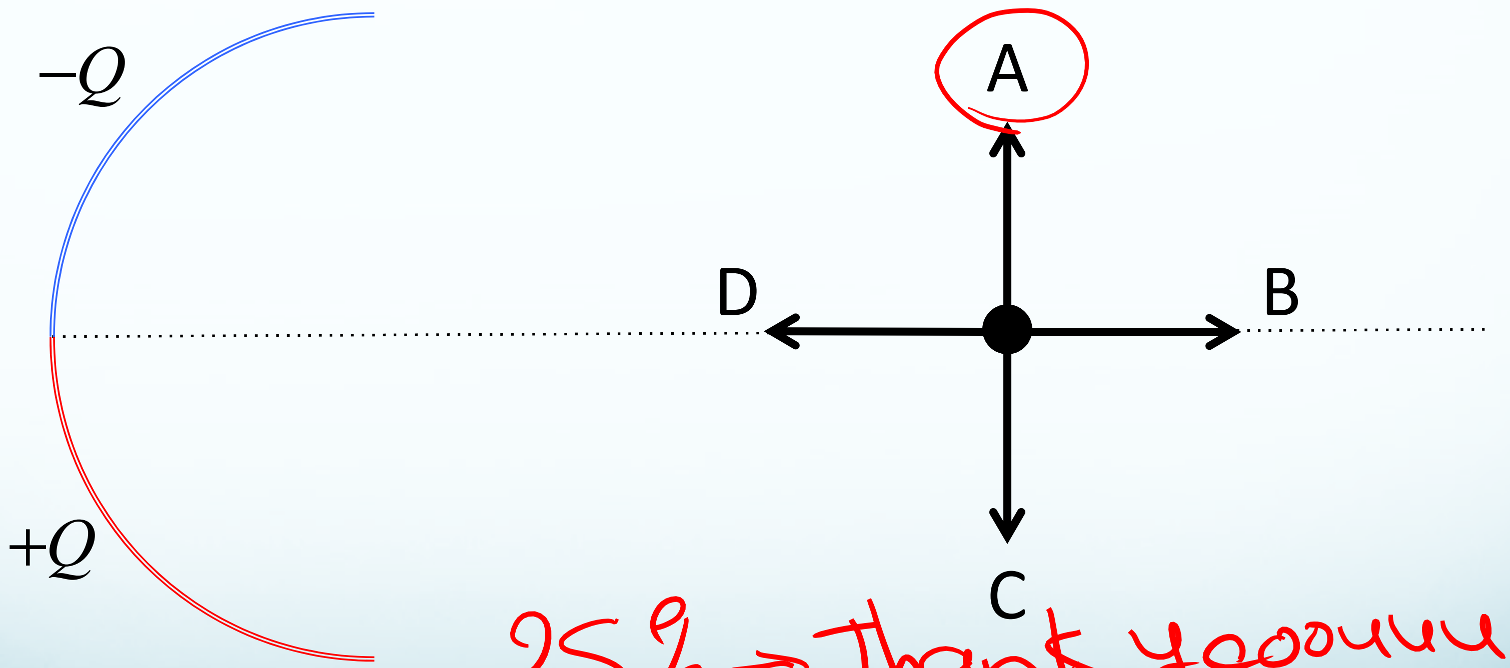


$$E_{plane,z} = \begin{cases} \dfrac{\sigma}{2\varepsilon_o}, z > 0 \\ -\dfrac{\sigma}{2\varepsilon_o}, z < 0 \end{cases}$$

z

TopHat Question

What is the direction of the electric field at the point indicated?

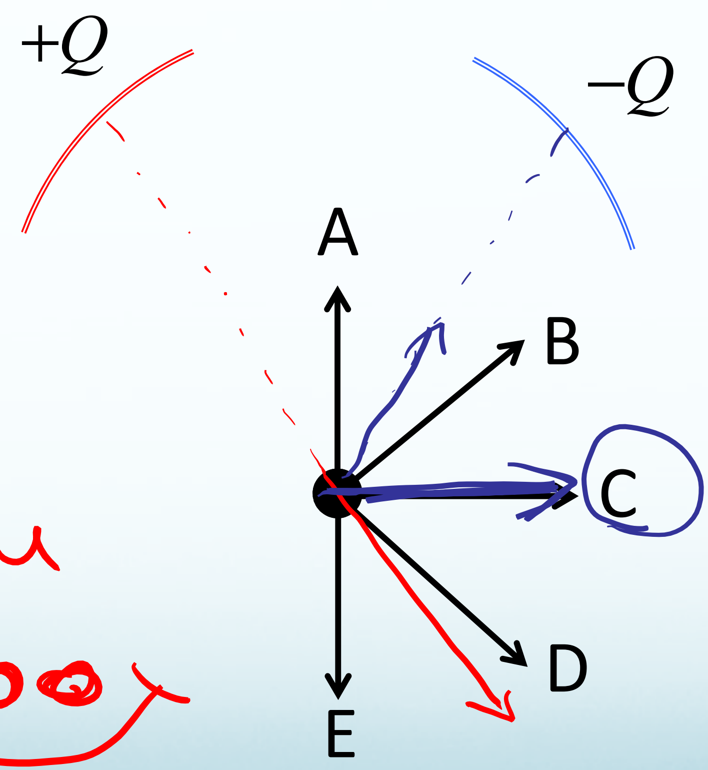


95% → thank yooouuu

correct

TopHat Question

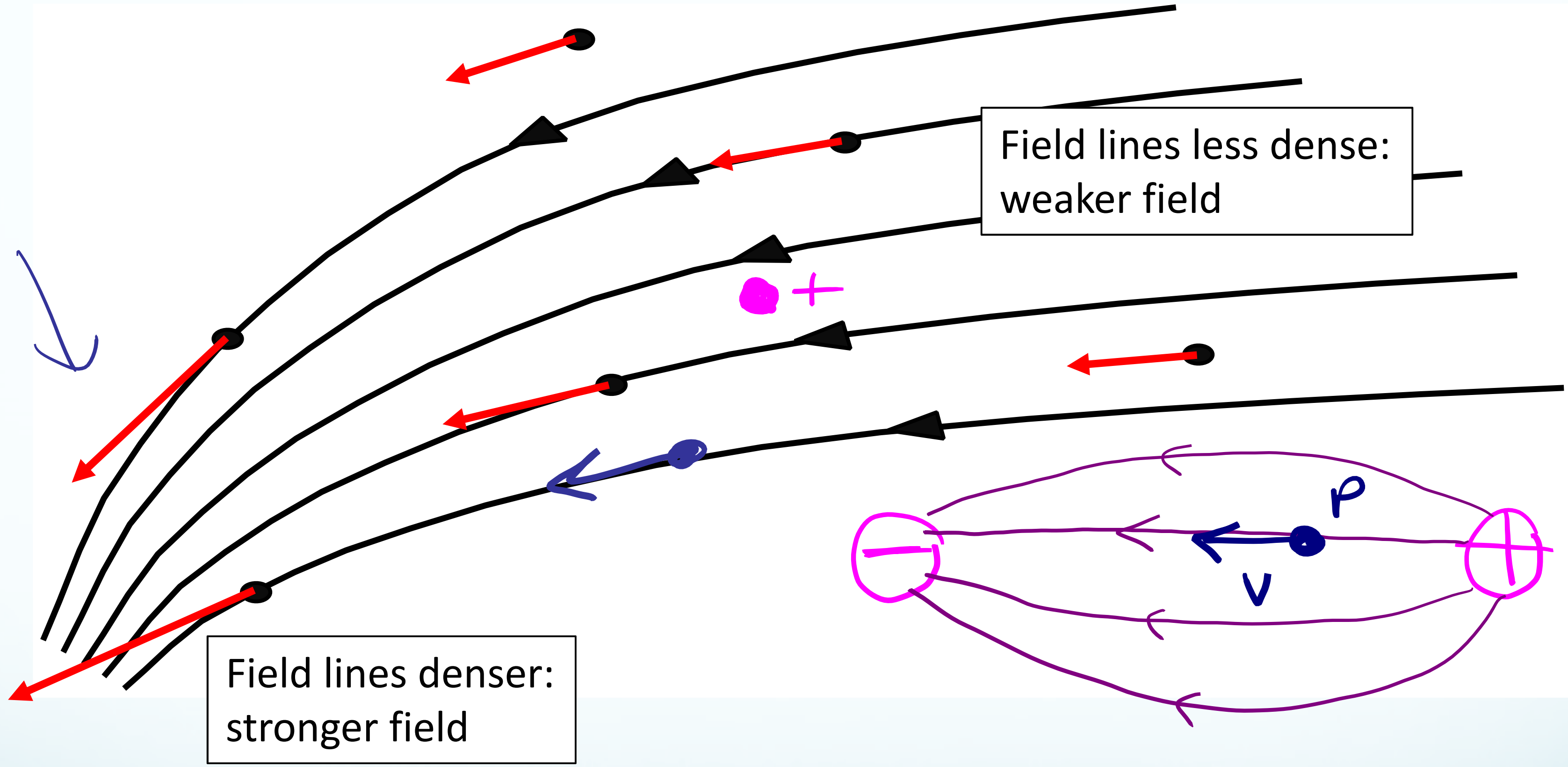
What is the direction of the electric field at the point indicated?



97% Correct

Thank youuuu

Electric Field Lines

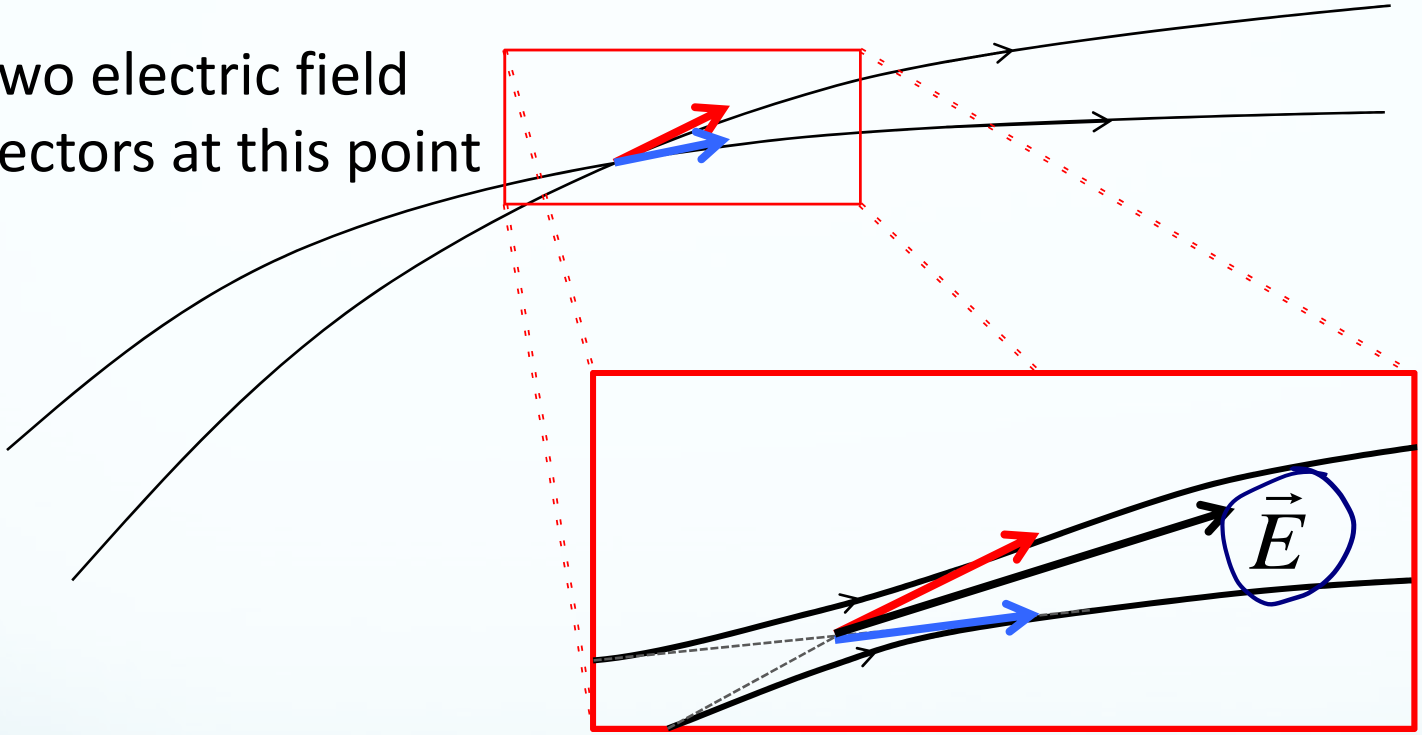


Electric field lines are continuous curves. The electric field vectors are tangent to the field lines

The denser the field lines, the stronger the field (magnitude of E)

Electric Field Lines Can't Cross

Two electric field vectors at this point

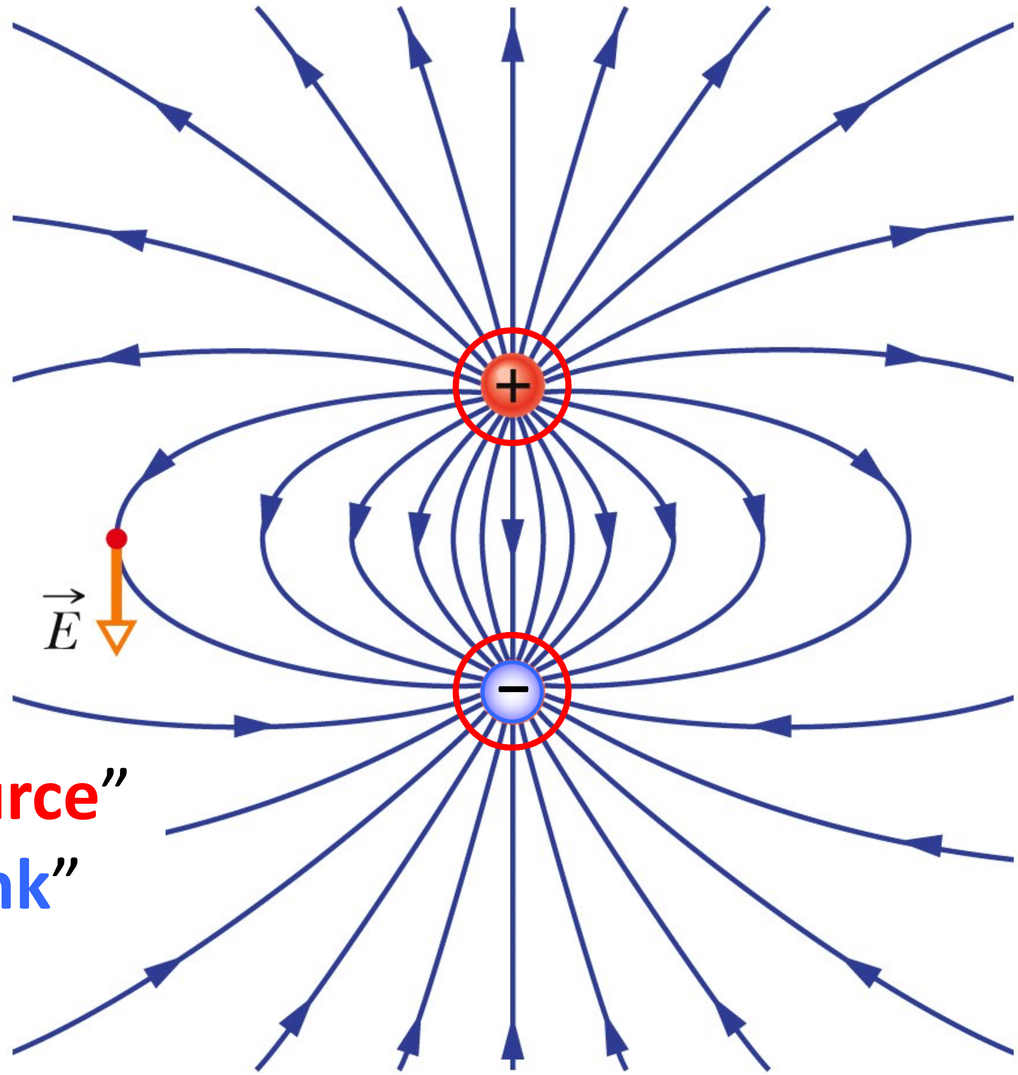


If field lines crossed, the electric field at that point would not be defined: superposition saves the day.

Sources and Sinks of Field Lines

Two charges of **equal magnitude** and **opposite sign**.

Field lines **start on +**
Field lines **end on –**



halliday_10e_fig_22_08

Positive charge called "**source**"
Negative charge called "**sink**"

Electric force on q: $\vec{F}_{on\ q} = q\vec{E}$ $\leftarrow$ $\vec{E} = \frac{\vec{F}}{q}$

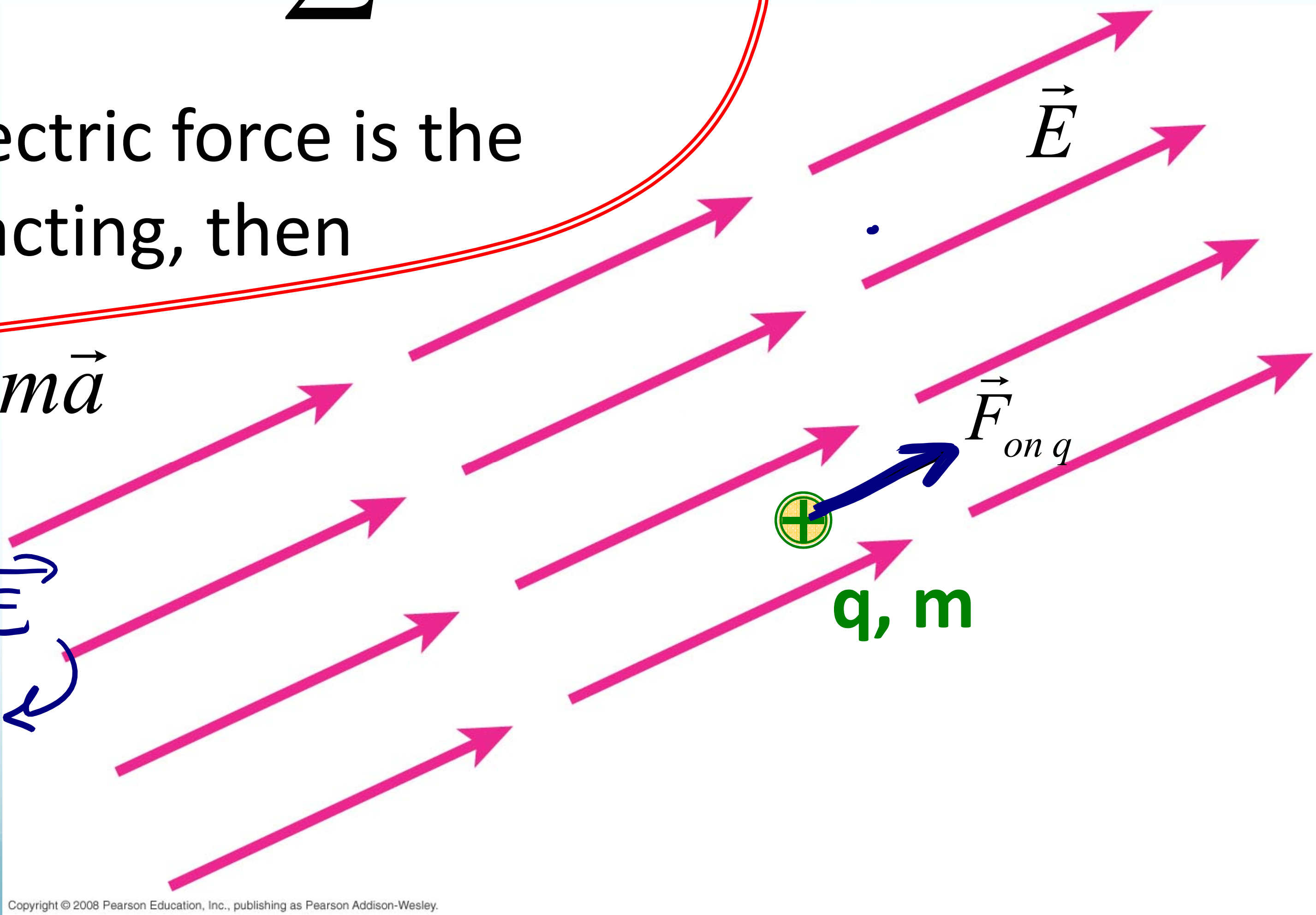
Newton's 2nd Law: $\sum \vec{F} = m\vec{a}$

So if the electric force is the only force acting, then

$$\vec{F}_{on\ q} = m\vec{a}$$

$\vec{F} = m\vec{a} = q\vec{E}$

$$q\vec{E} = m\vec{a}$$



$$q\vec{E} = m\vec{a} \quad \Rightarrow \quad \boxed{\vec{a} = \frac{q\vec{E}}{m}}$$

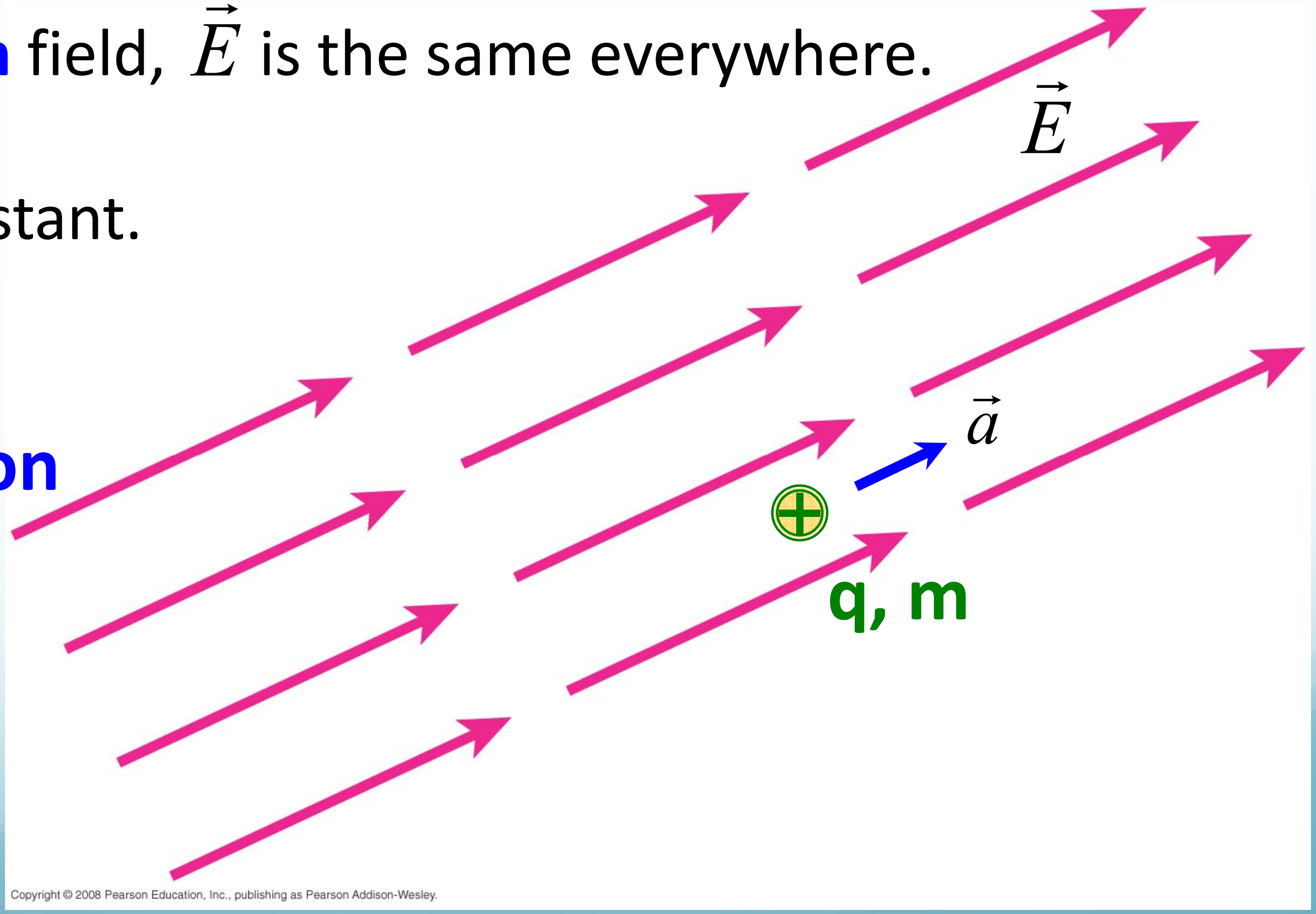
q = constant

m = constant

In a **uniform** field, $\vec{E}$ is the same everywhere.

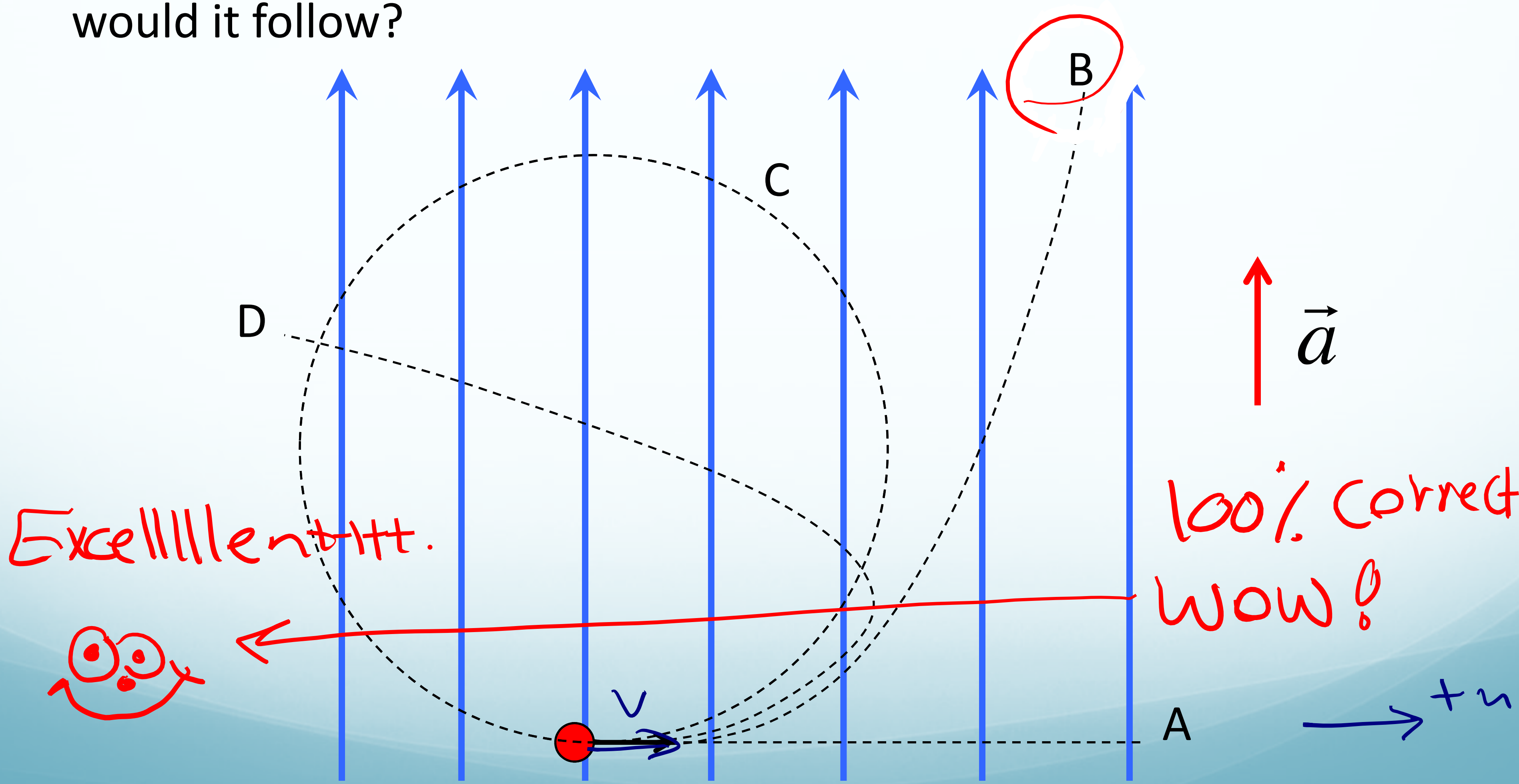
So $\vec{a}$ is constant.

Constant acceleration motion!



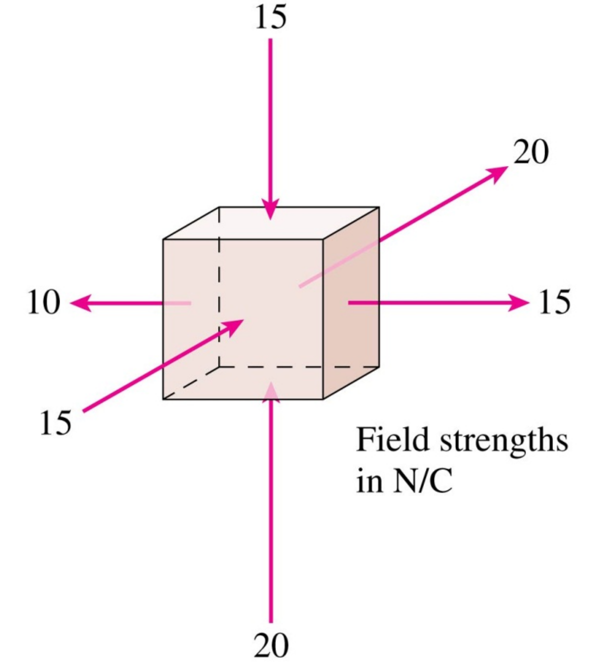
TopHat Question

A proton is moving through a uniform electric field. If its initial velocity is in the +x direction, which of the following trajectories would it follow?



next section

23-1: The Electric Flux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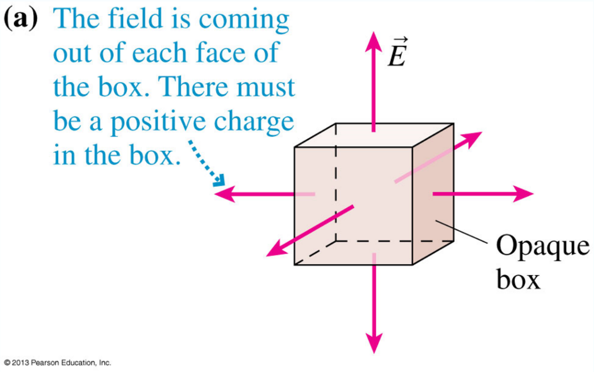
Field strengths in N/C


(a) The field is coming out of each face of the box. There must be a positive charge in the box.
$\vec{E}$
Opaque box
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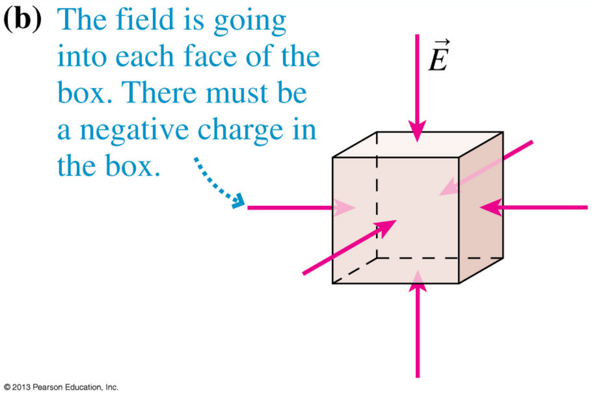
(b) The field is going into each face of the box. There must be a negative charge in the box.
$\vec{E}$
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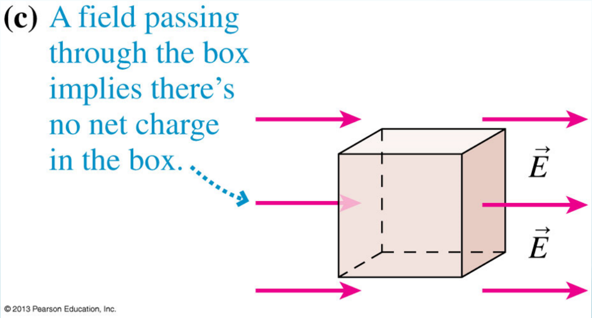
(c) A field passing through the box implies there's no net charge in the box.
$\vec{E}$
$\vec{E}$
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A closed surface through which an electric field passes is called **Gaussian surface**

An imaginary mathematical surface

Electric Flux; Gauss' Law

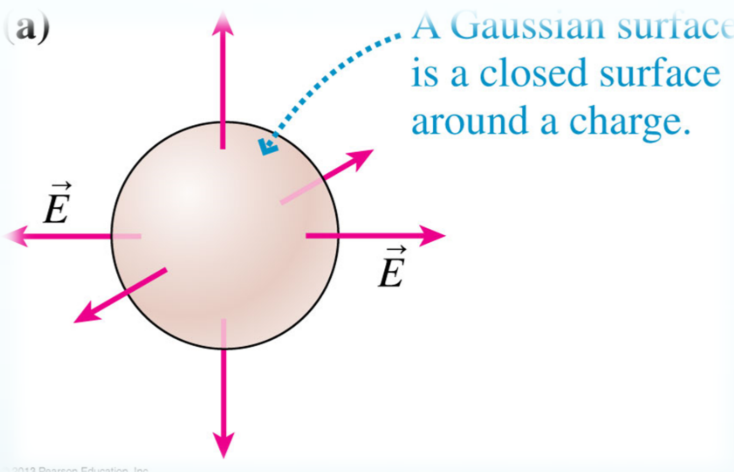
Gauss' Law is equivalent to Coulomb's law. It will provide us:

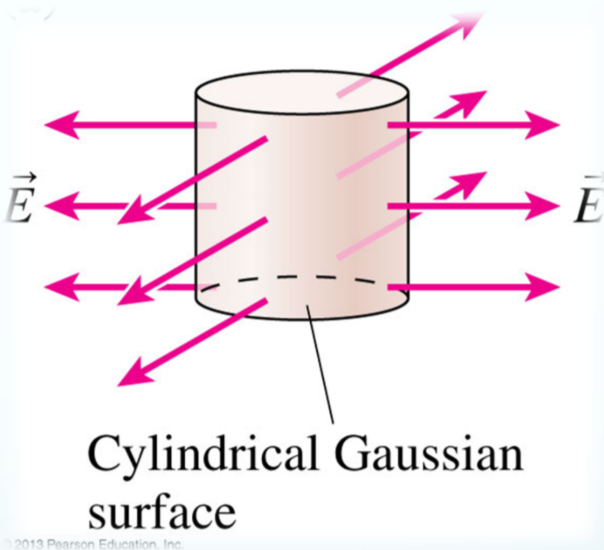
(i) an **easier way to calculate the electric field** in specific circumstances (especially situations with a high degree of symmetry)

(ii) a better understanding of the properties of conductors in electrostatic equilibrium (more on this as we go)

(iii) It is valid for moving charges – not limited to electrostatics.

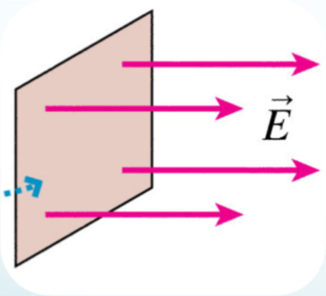
The Gaussian surface is most useful when it matches the shape of the field

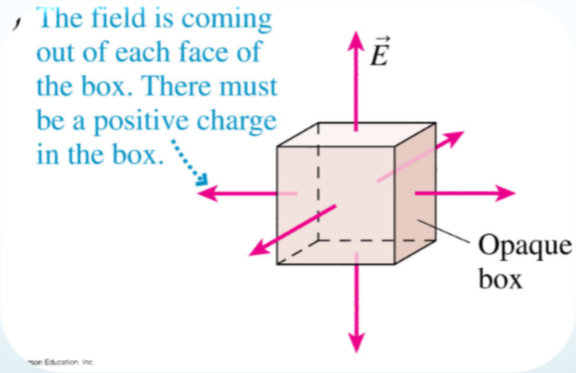


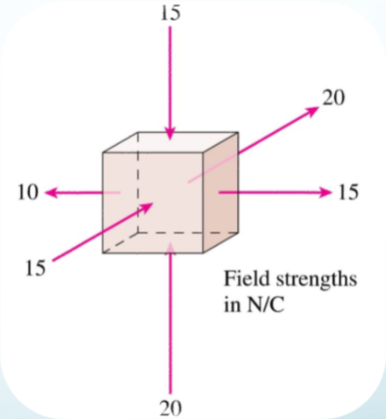


Electric Flux (Φ_e)

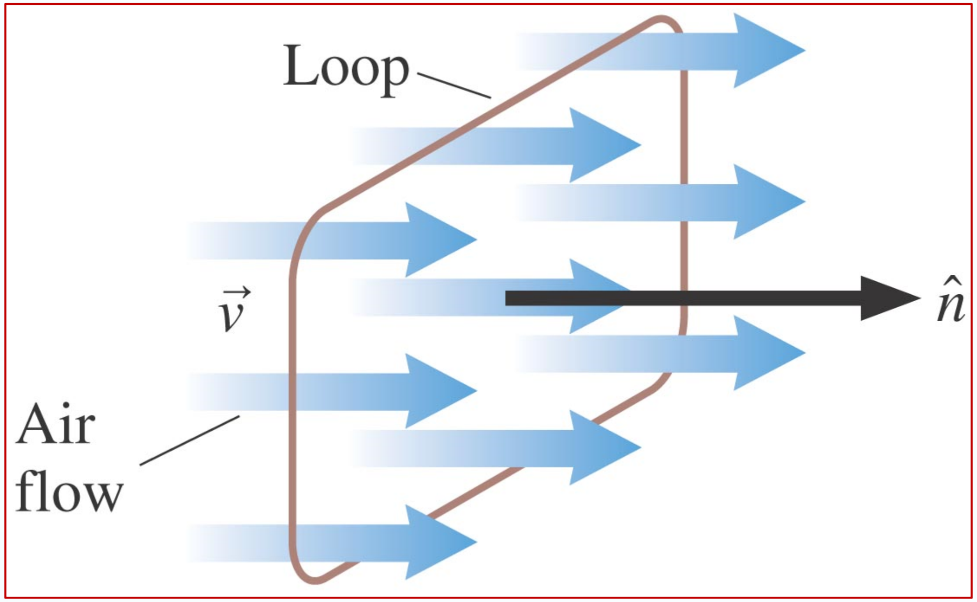
- **Amount of electric field going through a surface**
- **The number of field lines coming through a surface**

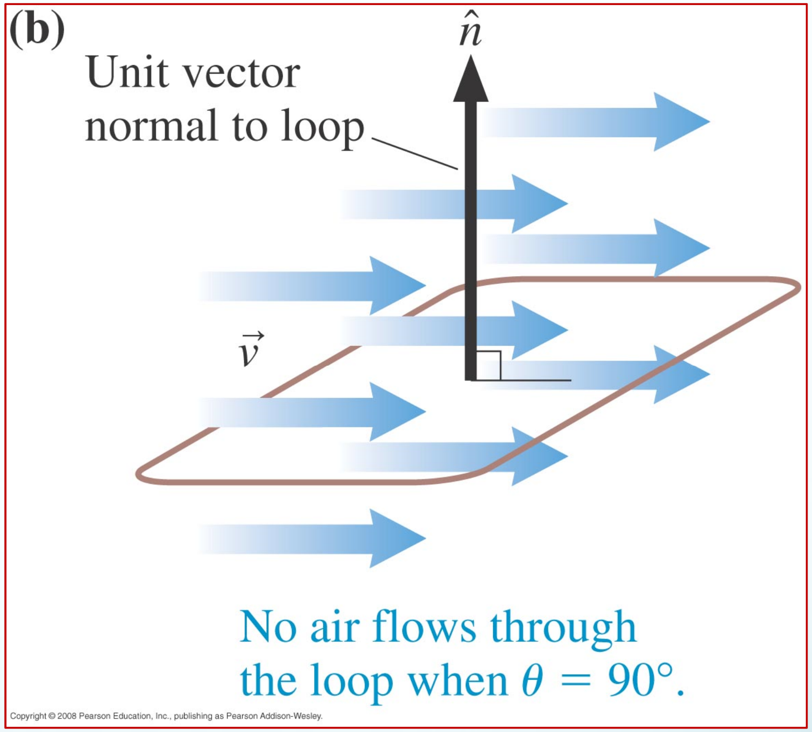


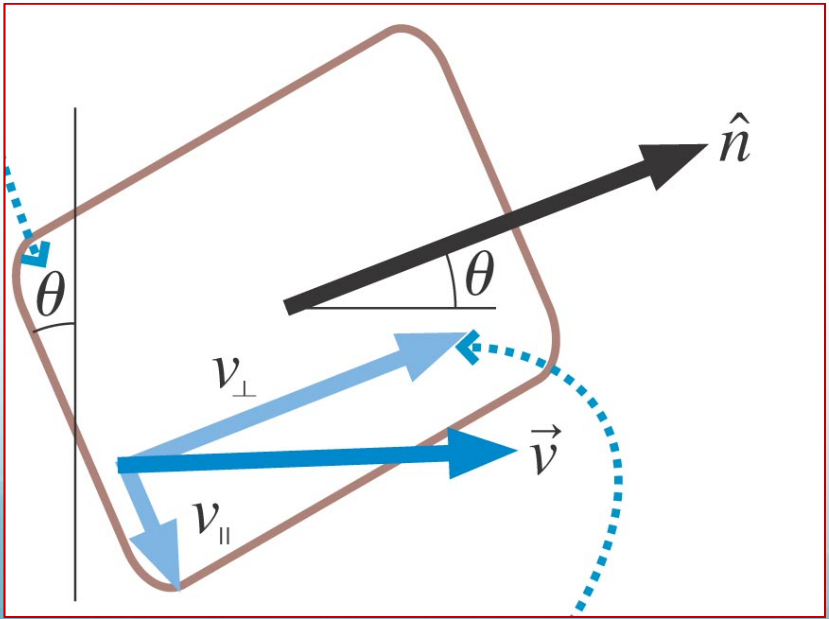




Wind going through a loop







The Electric Flux

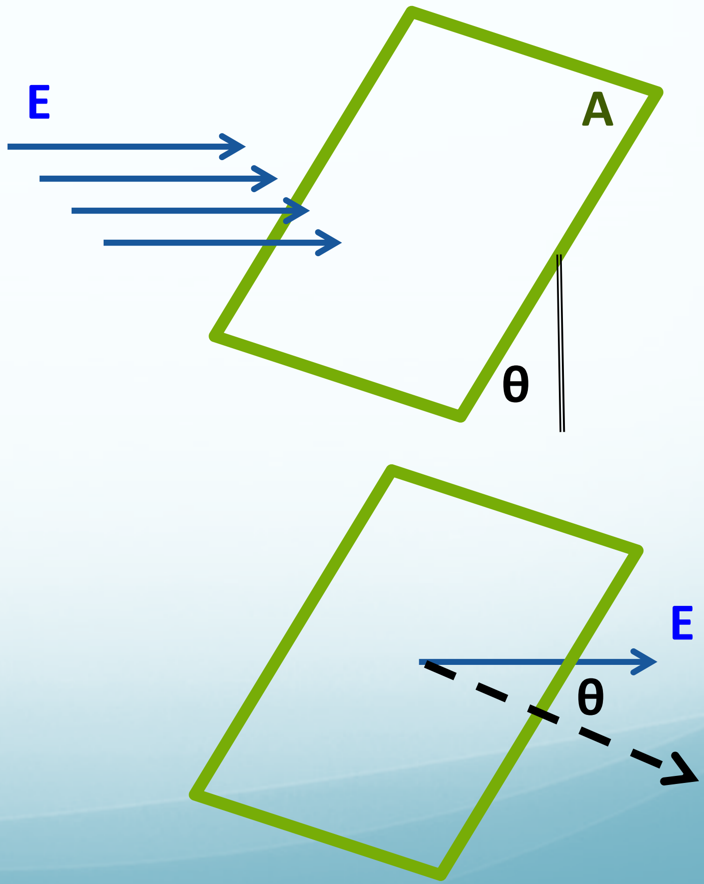
Amount of electric field going through a surface

$\Phi_e \propto E$

$\Phi_e \propto A$

$\Phi_e \propto \theta$

$$\boxed{\Phi_e = E_{\perp}A = EA\cos\theta}$$



QuickCheck 27.2

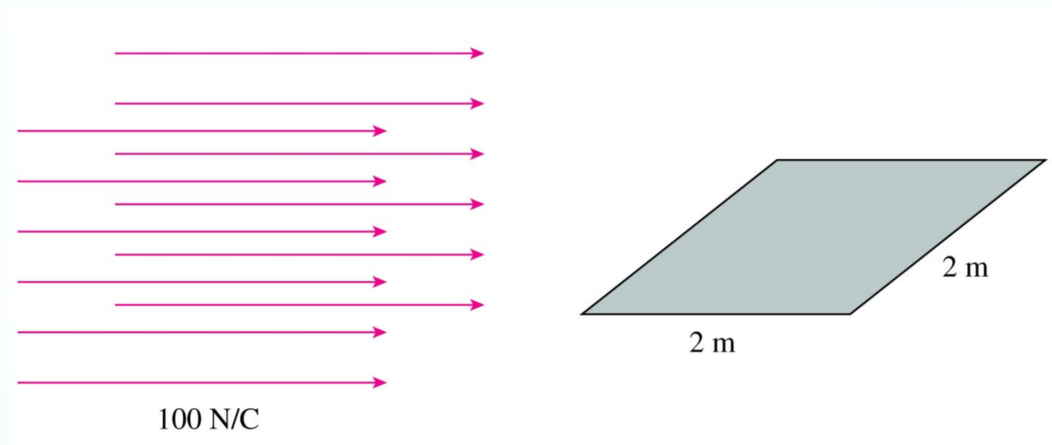
The electric flux through the shaded surface is

A. 0.

B. 200 N m/C.

C. 400 N m^2/C.

D. Some other value.



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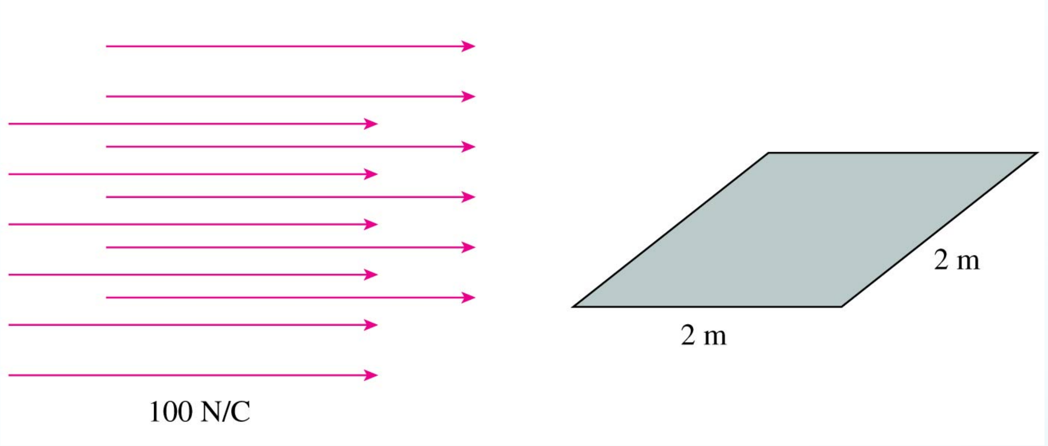

This section we talked about:

Chapter 23.1

See you on Friday