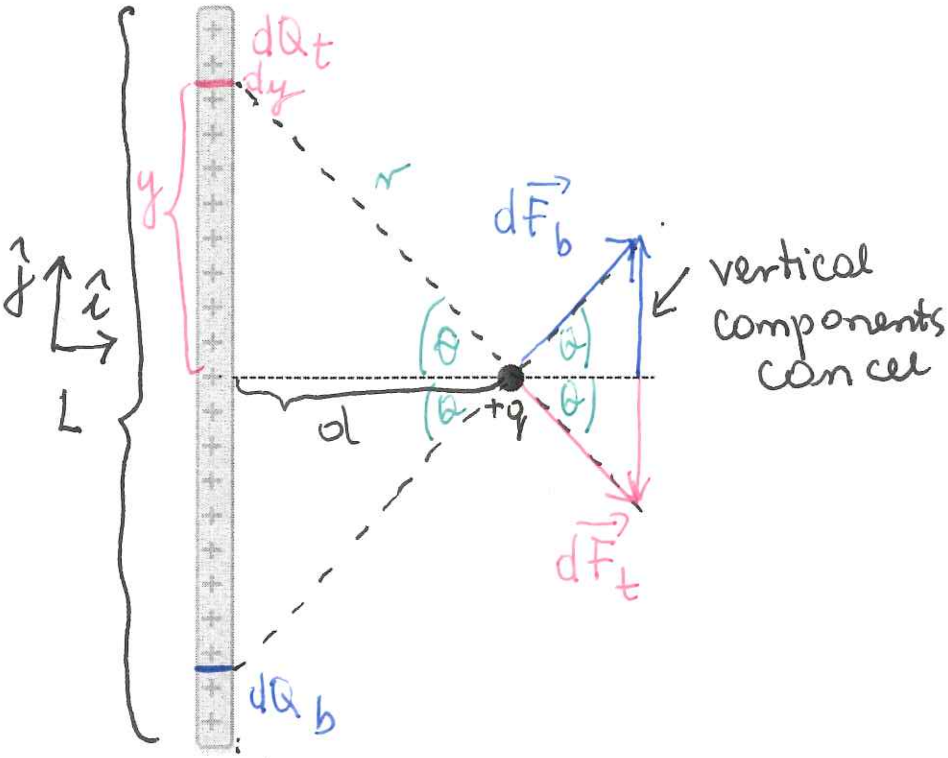
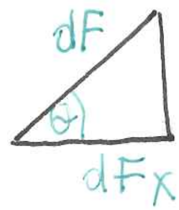
Force from a line of charge





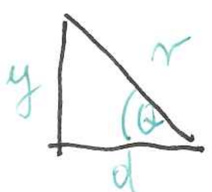
$$\frac{dF_x}{dF} = \cos\theta$$

$$dF_x = \underbrace{dF}_{\text{Coulomb's law}} \cdot \underbrace{\cos\theta}_{\text{geometry}}$$

Step 1: Choose coordinate system

Step 2: Consider a small charge dQ

Note: For each dQ in the top half, there is a dQ in the bottom half that will cancel the vertical component of the force.



$$\cos\theta = \frac{d}{r} = \boxed{\frac{d}{\sqrt{d^2+y^2}}}$$

$$dF = \frac{kq\,dQ}{r^2} = \boxed{\frac{kq\,dQ}{(d^2+y^2)}}$$

$$dF_x = \frac{kq\,dQ \cdot d}{(d^2+y^2)^{3/2}}$$

To find $F_{net,x}$ we need to integrate

Solution: use LINEAR DENSITY, λ

$$\boxed{\lambda = \frac{Q}{L}}$$ total charge (uniform)
length

How much charge is located at given length of the wire?

e.g. For $\Delta L = \frac{1}{2}L$ $\quad \Delta Q = \lambda \cdot \Delta L$

$\Delta Q = ?$ $\quad \Delta Q = \frac{Q}{\cancel{L}} \cdot \frac{1}{2} \cdot \cancel{L} = \frac{1}{2}Q$

$$dQ = \lambda \cdot dy$$

$$\int dF_x = \int \frac{kq \cdot d\,\lambda\, dy}{(d^2+y^2)^{3/2}}$$ integration variable

$$F_{net\,x} = kq\,d\lambda \int_{-L/2}^{L/2} \frac{dy}{(d^2+y^2)^{3/2}}$$

Integration limits b/c coordinate origin at the line center

$$F_{net,x} = kq\cancel{d}\lambda \left[\frac{y}{d^{\cancel{2}}(d^2+y^2)^{1/2}}\right]_{-L/2}^{L/2}$$

$$F_{net,x} = \frac{kq\lambda}{d}\left[\left(\frac{\frac{L}{2}}{(d^2+(\frac{L}{2})^2)^{1/2}}\right) - \left(\frac{(-\frac{L}{2})}{(d^2+(-\frac{L}{2})^2)^{1/2}}\right)\right] = \frac{kq\lambda \cdot L}{d\sqrt{(\frac{L}{2})^2+d^2}} = \frac{kqQ}{d\sqrt{(\frac{L}{2})^2+d^2}}$$

$\lambda \cdot L = Q$

$$\vec{F}_{net,x} = \frac{kqQ}{d\sqrt{(\frac{L}{2})^2+d^2}}\,\hat{i}$$

$$\vec{F}_{net,x} = \frac{kQ\cdot q}{d\sqrt{\left(\frac{L}{2}\right)^2 + d^2}}\ \hat{i}$$

Far limit $\quad d \gg L$

what do you expect?

For $d \gg L \qquad \left(\frac{L}{2}\right)^2 + d^2 \approx d^2$

$$F_{net,x} = \frac{kQq}{d\sqrt{d^2}} = \frac{kQq}{d^2}$$

WIRE LOOKS LIKE A POIN CHARGE Q!

Near limit : what is the force close to the wire?

for $d \ll L \qquad \left(\frac{L}{2}\right)^2 + d^2 \approx \left(\frac{L}{2}\right)^2$

$$F_{net,x} = \frac{kQq}{d\sqrt{\left(L/2\right)^2}} = \frac{2k\overset{\lambda}{Q}q}{d\cdot L} = \frac{2kq\lambda}{d}$$

wire looks like INFINIETELY LONG wire.

$$F_{net,x} = \frac{2k\lambda q}{d}$$