Electricity and Magnetism

- Physics 259 – L02
- Lecture 26

Chapter 24.4 and 24.5:

Potential due to an electric dipole
Potential due to a continuous charge distribution

Last time

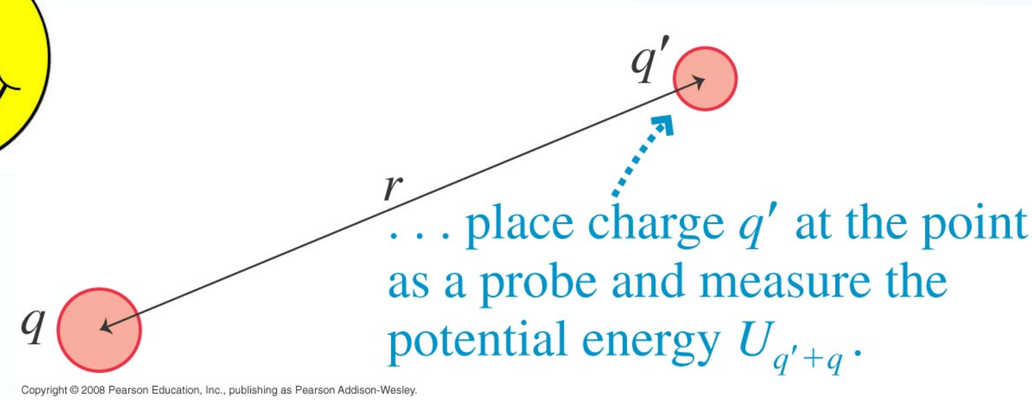
- Electric potential energy of a collection of charges
- Interpreting equipotential surfaces
- Equipotential surfaces: visualizing electric potential

This time

- Potential due to an electric dipole
- Potential due to a continuous charge distribution

Starting from the end

The whole story is:



Electric force on q’ from q

$$\vec{F}_{qq'} = \frac{1}{4\pi\varepsilon_0}\frac{qq'}{r^2}\hat{r}$$

Then the electric field of q is

$$\vec{E} = \frac{\vec{F}_{qq'}}{q'} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$$

Potential energy of q and q’

$$U_{q'+q} = \frac{1}{4\pi\varepsilon_0}\frac{qq'}{r}$$

Then the potential of q is

$$V = \frac{U_{q'+q}}{q'} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

Electric Potential

$$\boxed{V \equiv \frac{U_{q+sources}}{q}}$$

$$\boxed{U_{q+sources} = qV}$$

$$\vec{E} = \frac{\vec{F}}{q}$$

$$\vec{F} = q\vec{E}$$

Potential Gradient -- E and V

Note: E is always $\perp$ equipotential lines

$$\vec{E} = -\vec{\nabla} V = -\frac{\partial V}{\partial x}\hat{i} - \frac{\partial V}{\partial y}\hat{j} - \frac{\partial V}{\partial z}\hat{k}$$

In 3 dimensions we must take 3 derivatives, then add them **VECTORIALLY**

Alternatively, the potential is found from the electric field integrated along any path connecting points A and B

$$V_{AB} = \int_A^B \vec{E} \cdot ds$$

TopHat Question

A positive charge is located at origin. What is the direction of electric potential of the positive charge?

a) radially outward from the origin

b) radially inward from the origin

c) toward the positive x, y, and z directions

d) toward the negative x, y, and z directions

e) There is no direction since the electric potential is a scalar quantity.

24.3.2. A positive charge is located at the origin. What is the direction of the electric potential of the positive charge?

a) radially outward from the origin

b) radially inward from the origin

c) toward the positive x, y, and z directions

d) toward the negative x, y, and z directions

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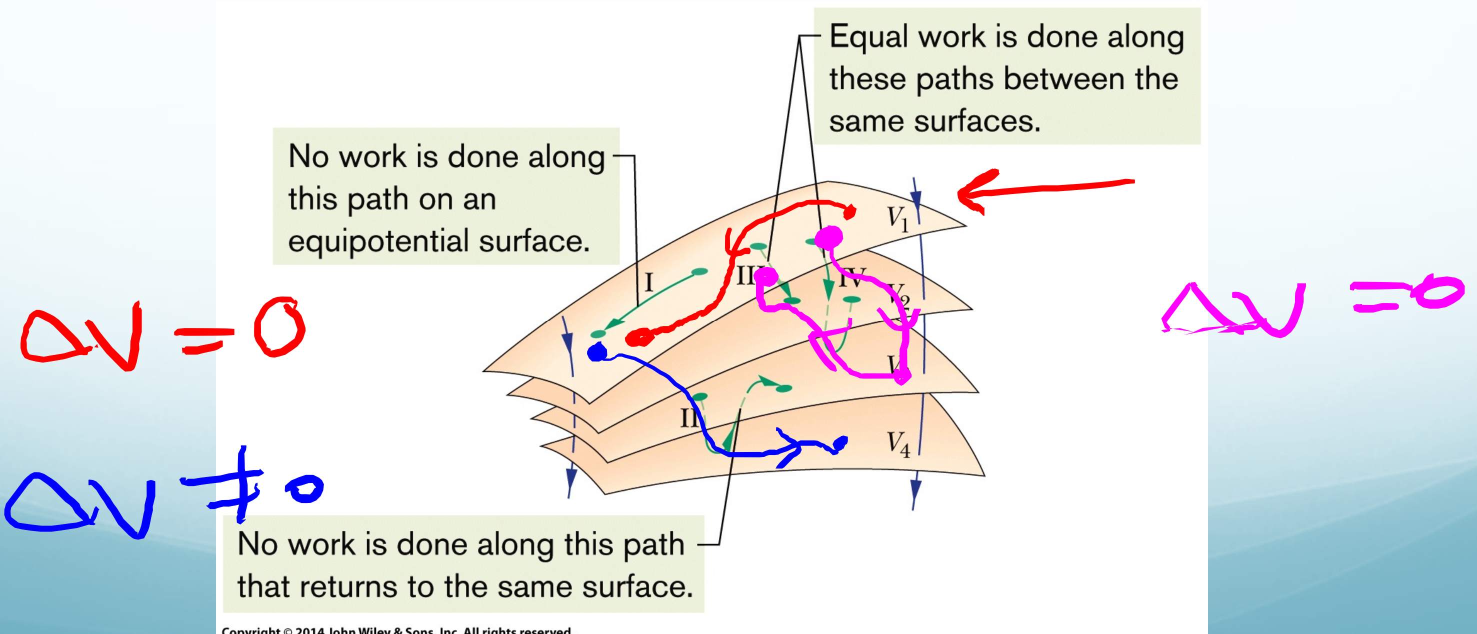
Equipotential Surfaces

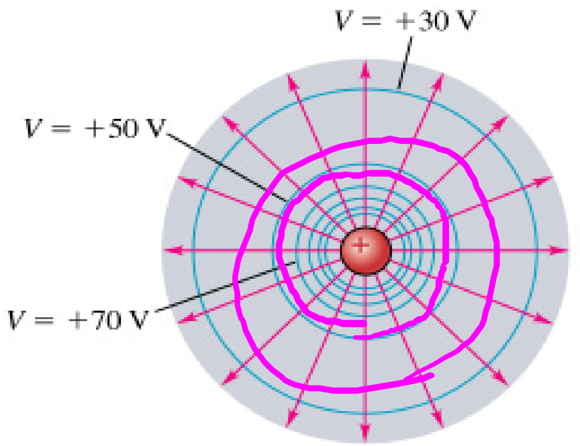
- **Adjacent points with the same electric potential from an equipotential surface**

- No net work W is done on a charged particle by an electric field when particle moves on the same equipotential surface

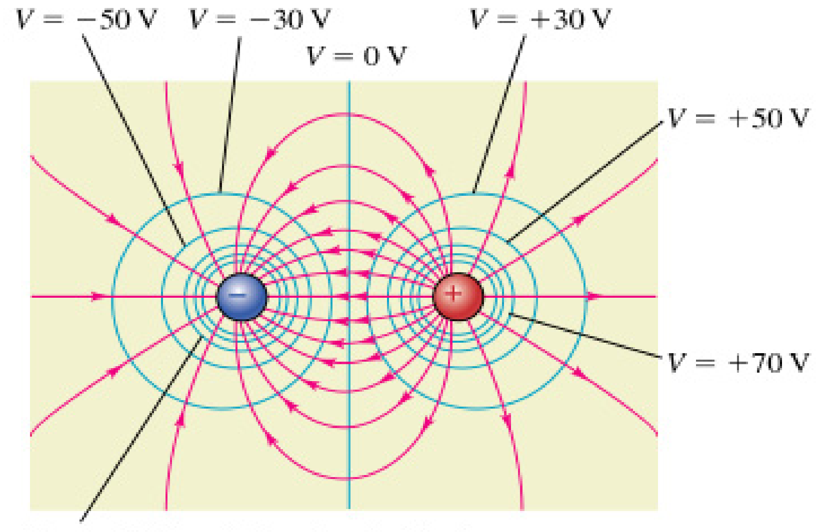
$$W = -\Delta U = -q\,\Delta V = -q(V_f - V_i).$$



Equipotential Surfaces



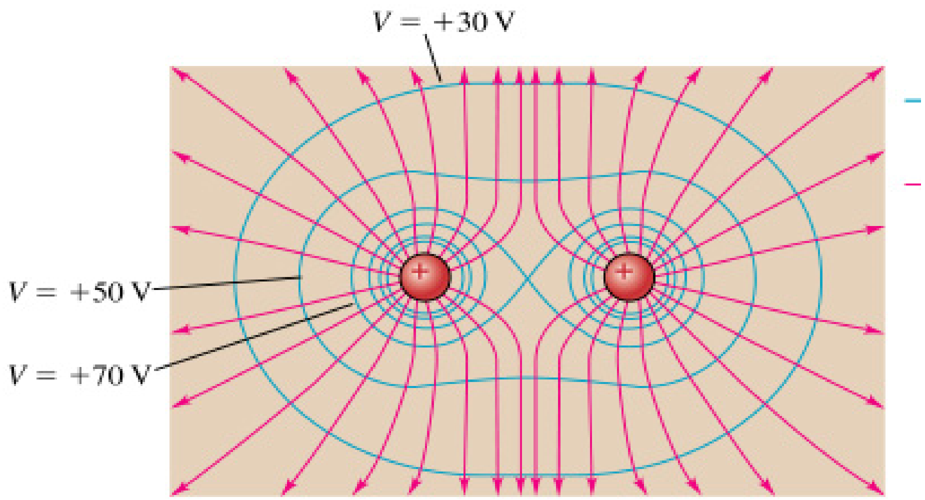
(a) A single positive charge



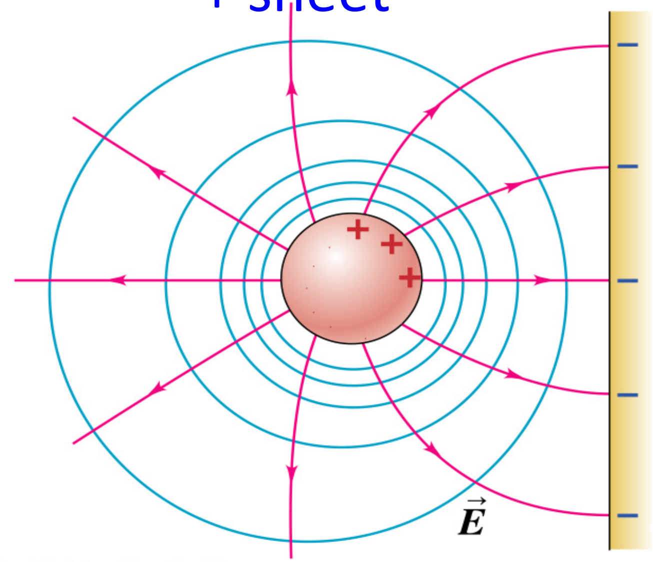
(b) An electric dipole

Note – E is always ⊥ V !!

Conducting sphere
+ sheet



(c) Two equal positive charges



Calculating Potential from the Field

Differential work dW done on a particle by force F→

$$dW = \vec{F}.d\vec{s} = q\vec{E}.d\vec{s} \rightarrow W = q\int_i^f \vec{E}.d\vec{s} \quad (1)$$

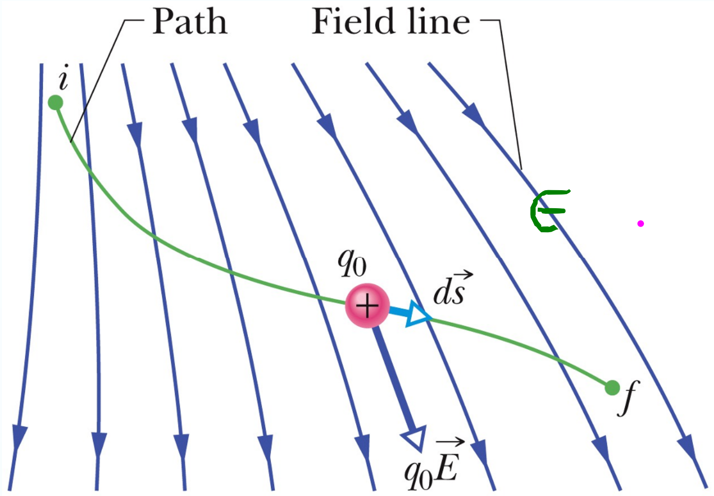
& $$W = -\Delta U = -q\,\Delta V = -q(V_f - V_i). \quad (2)$$

$(1) = (2) \Rightarrow q\int \vec{E}.d\vec{s} = -q(V_f - V_i)$

→ $$\boxed{V_f - V_i = -\int_i^f \vec{E}\cdot d\vec{s},}$$

$$\boxed{V = -\int_i^f \vec{E}\cdot d\vec{s}.}$$

← if $V_i = 0$

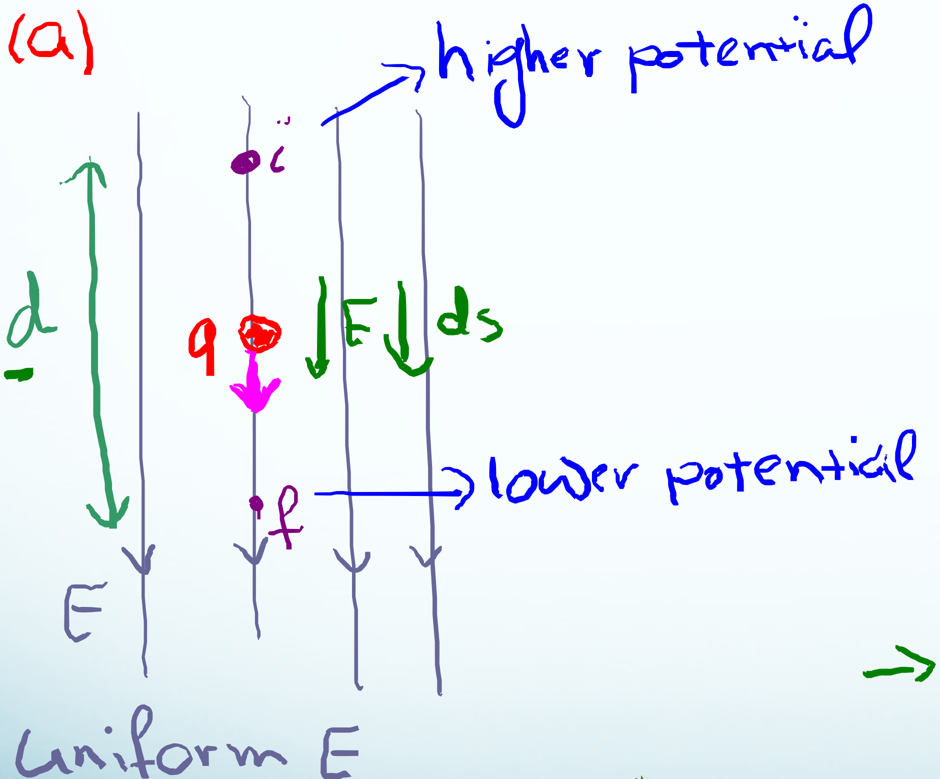


For uniform electric field→ $$\boxed{\Delta V = -E\,\Delta x.}$$

Problem 24.02 of textbook:
Finding potential change from the electric field

Find the potential difference $v_f - v_i$ by moving a positive test charge q from I to f along the path shown.

(a)



$$\Delta V = -\int_i^f \vec{E} \cdot d\vec{s}$$

$$\Delta V = -\int_i^f E\, ds \cos\theta$$

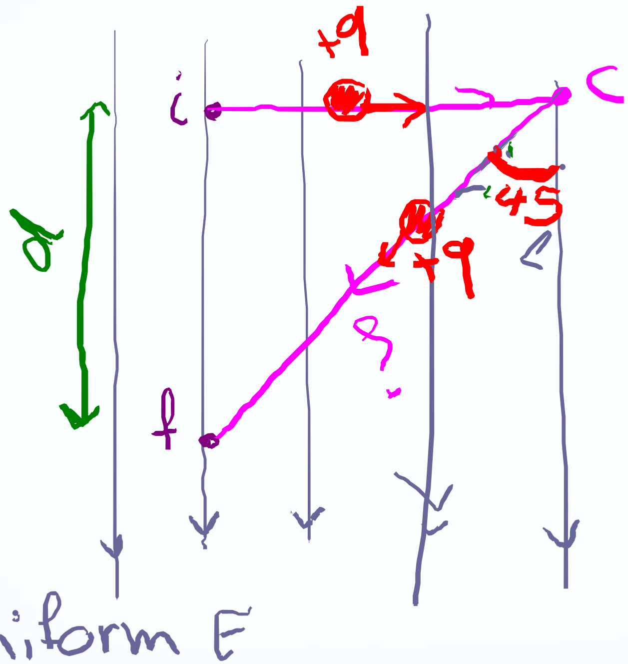
$$\theta = 0 \rightarrow \cos\theta = 1$$

$$\rightarrow \Delta V = -\int E\, ds \rightarrow$$

$$\rightarrow \Delta V = -E \int_i^f ds \Rightarrow \boxed{\Delta V = -Ed}$$

(b)

same d as part (a)



$$\Delta V = -\int_i^f \vec{E}\cdot d\vec{s}$$

$$V_f - V_i = -\int_i^f E\,ds\cos\theta$$

$\theta \rightarrow$ angle between ds & E

$i \rightarrow c$ $\quad\theta = 90$

$c \rightarrow f$ $\quad\theta = 45$

$$\rightarrow V_f - V_i = -\int_i^c E\,ds\,\cancel{\cos\theta} - \int_c^f E\,ds\cos\theta$$

$d = s\cos 45$

$$V_f - V_i = -\int_c^f E\,ds\cos 45 = -E\cos 45 \underbrace{\int_i^f ds}_{s}$$

$$S = \frac{d}{\cos 45}$$

$$\rightarrow V_f - V_i = -E\cos(45^\circ)\int_c^f ds = -E\cancel{\cos 45}\,\frac{d}{\cancel{\cos 45}} = -Ed$$

TopHat Question

24.2.3. Why is an electrostatic force considered a conservative force?

a) Charged particles do not experience friction, which is a non-conservative force.

b) The energy required to move a charged particle around a closed path is equal to zero joules.

c) The work required to move a charged particle from one point to another does not depend upon the path taken.

d) Answers (a) and (b) are both correct.

e) Answers (b) and (c) are both correct.

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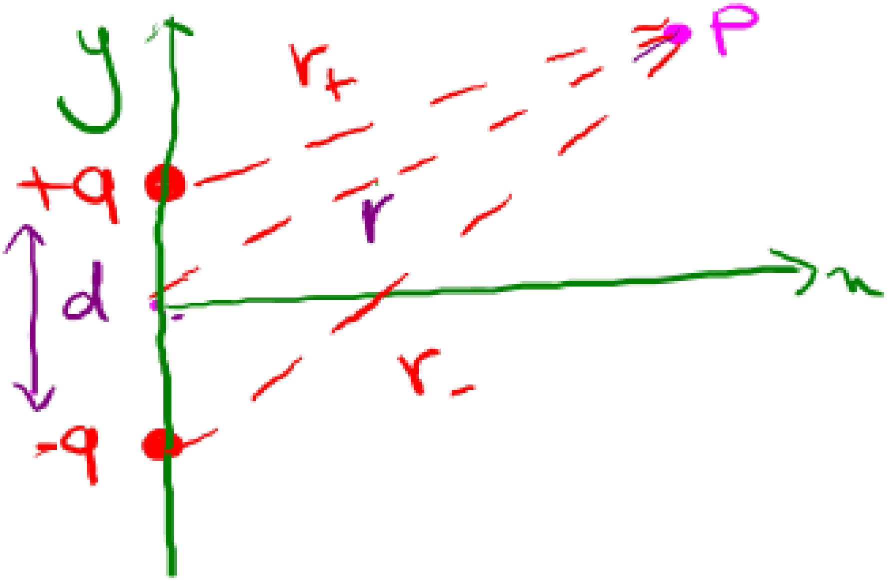
Electric potential of a dipole at arbitrary point p



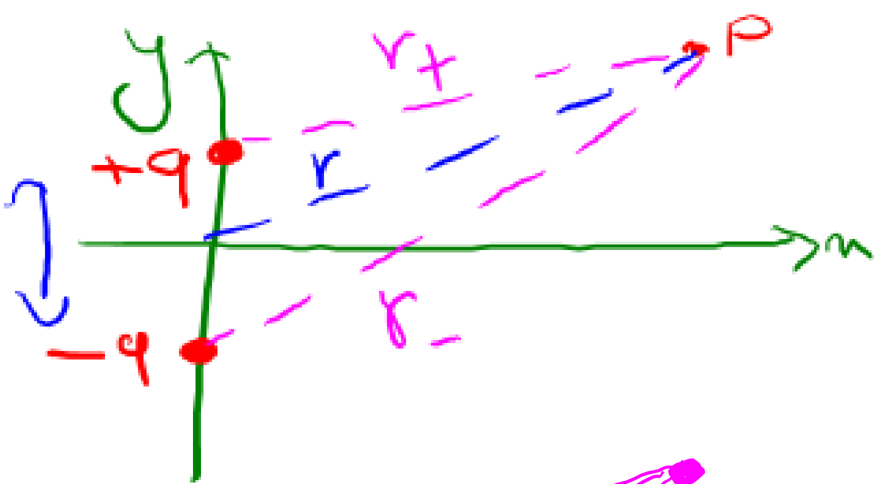
$$V = V_+ + V_-$$

$$\begin{cases} V_+ = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r_+} \\ V_- = \dfrac{-1}{4\pi\varepsilon_0}\dfrac{q}{r_-} \end{cases}$$

$$\rightarrow V = V_+ + V_- = \frac{1}{4\pi\varepsilon_0}\frac{q}{r_+} + \frac{-1}{4\pi\varepsilon_0}\frac{q}{r_-}$$

$$V = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_+} - \frac{1}{r_-}\right) = \frac{q}{4\pi\varepsilon_0}\left(\frac{r_- - r_+}{r_- r_+}\right)$$

$$\Rightarrow V = \frac{q}{4\pi\varepsilon_0}\,\frac{r_- - r_+}{r_- r_+}$$

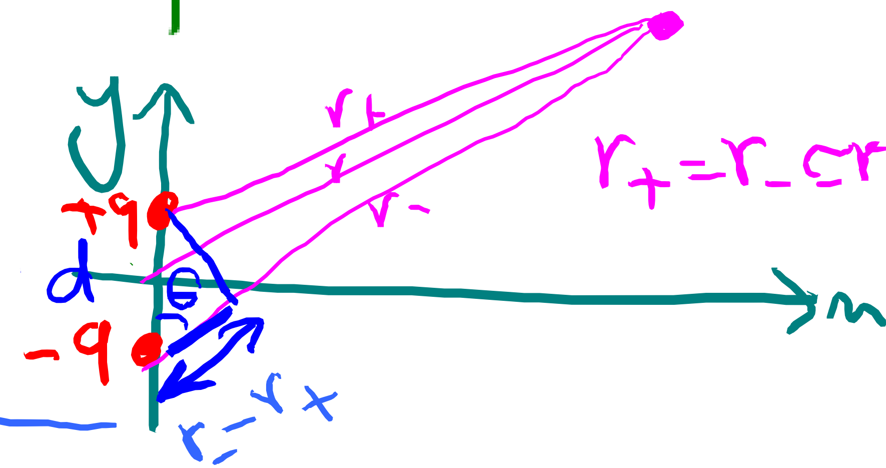


natural dipoles $\Rightarrow$ $r \gg d$

$r \gg d \Rightarrow r_- \simeq r_+ \simeq r$

$\Rightarrow r_- r_+ \simeq r^2$ & $r_- - r_+$?

$\rightarrow r_- - r_+ = d\cos\theta$



$$\Rightarrow V = \frac{d}{4\pi\varepsilon_0}\,\frac{d\cos\theta}{r^2}$$

& $p = qd$

$$\rightarrow V = \frac{q}{4\pi\varepsilon_0}\,\frac{d\cos\theta}{r^2} = \frac{1}{4\pi\varepsilon_0}\,\frac{p\cos\theta}{r^2}$$ Scalar

- **Go through "Appendix 1-chapter 24" in D2L (different approach)**

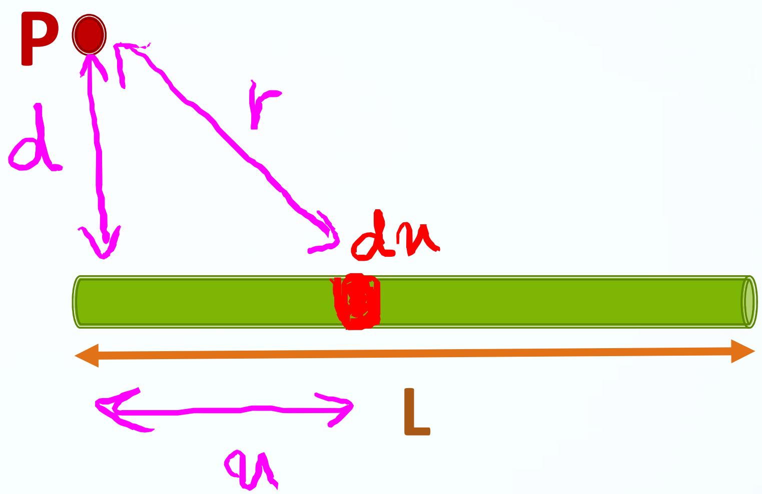
Electric potential of a line of charge at point p

Next Section

Thin nonconducting rod of length L with uniform positive charge with charge density λ.

Find electric potential V due to the rod at p, a perpendicular distance d from the left end of the rod.

$$V=\frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$



$$dV=\frac{1}{4\pi\varepsilon_0}\frac{dq}{r} \rightarrow V=\int dV$$

$$dq=\lambda\, dx \quad , r=\sqrt{x^2+d^2}$$

$$\rightarrow \quad V=$$

$$V = \int dV = \int_0^L \frac{1}{4\pi\varepsilon_0} \frac{\lambda}{(x^2+d^2)^{1/2}} \, dx$$

$$\int_0^L \frac{dx}{(x^2+d^2)^{1/2}} =$$

$$\ln\left(x + (x^2+d^2)^{1/2}\right)\Big|_0^L$$

$$\rightarrow V = \frac{\lambda}{4\pi\varepsilon_0} \ln\left(\frac{L + (L^2+d^2)^{1/2}}{d}\right)$$

This section we talked about:

Chapter 24.4 and 24.5

See you on Wednesday