Write your UCID in the space above

University of Calgary

Faculty of Science

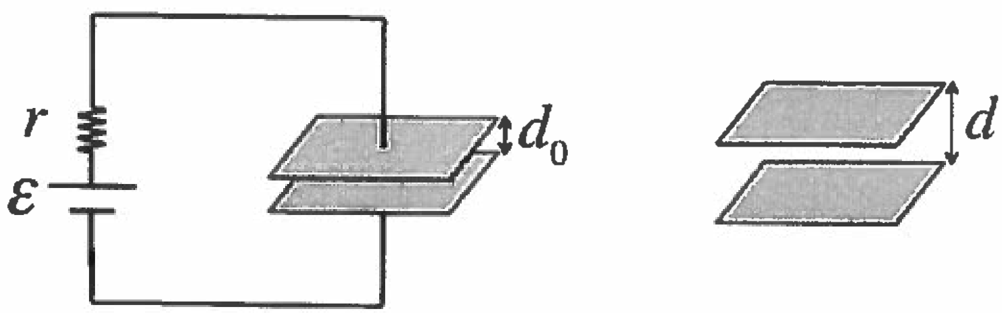
Final Test

PHYSICS 259 ALL LECTURE SECTIONS

Part II: Written Answer Questions (Total: 28 marks)

IMPORTANT: Write your answers to the problems in Part II in boxes. Work must be shown for full marks. Rough work can be done on the back of this question paper, but only the work in the boxes be marked.

[5 marks] A parallel plate capacitor with capacitance C_0 and a plate separation of $d_0 = 0.50$ mm is fully charged by a 12 V battery with an internal resistance of $r = 1.0\ \Omega$. The battery is then disconnected and the plates of the capacitor are pulled apart to a new separation of $d = 3.0$ mm so that the capacitance is now C. The amount of work required to pull the plates apart is 3.6 μJ.



26. Which quantities remain constant when the plates are pulled apart? Explain. [1 mark]

The charge on the plates remains constant (the battery is disconnected).
The area of the plates remains constant.

Write your last name and initials in the space above

27. What is the final capacitance C in terms of the initial capacitance C_0? [1 mark]

$d \rightarrow 6d_0$ so $C = \frac{\epsilon_0 A}{d} = \frac{\epsilon_0 A}{6d_0} = \frac{1}{6} C_0$

$\boxed{C = \frac{1}{6} C_0}$

28. What is the initial capacitance C_0? [~~1~~ 2 mark]

$W = + \Delta U = U_f - U_i \Rightarrow W = \left(3 - \frac{1}{2}\right) \frac{Q^2}{C_0} = \frac{5}{2} \frac{Q^2}{C_0}$

$U_f = \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} \frac{Q^2}{\frac{1}{6} C_0} = 3 Q^2/C_0$ $\quad U_i = \frac{1}{2} \frac{Q^2}{C_0}$

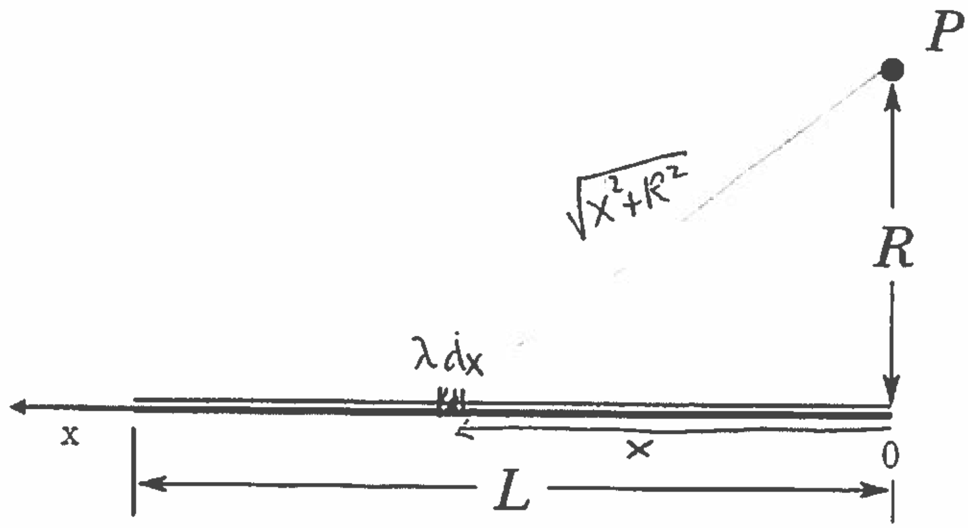
but $Q = (12V) C_0 \Rightarrow W = \frac{5}{2} (12V)^2 C_0$

$C_0 = \frac{2W}{5(12V)^2} = \boxed{10 nF}$

29. What is the final charge on the capacitor plates? [1 mark]

$Q = (12V)(10nF) = \boxed{120 nC}$

[4 marks] In the figure below, point P is at perpendicular distance R from the end of a finite line of charge with a constant charge distribution, λ.



30. Write down an expression for the contribution to the electric potential at point P from a small fragment of the line of charge. Label all relevant variables in the diagram provided. [2 marks]

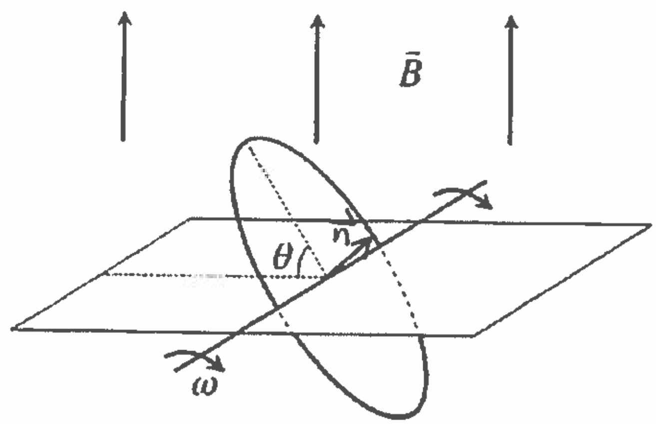
$$dV = \frac{1}{4\pi\epsilon_0} \frac{\lambda dx}{\sqrt{x^2+R^2}}$$

31. Calculate the electric potential at point P from the entire line of charge (evaluate the integral) [2 marks]

$$V_p = \frac{\lambda}{4\pi\epsilon_0} \int_0^L \frac{dx}{\sqrt{x^2+R^2}} = \frac{\lambda}{4\pi\epsilon_0} \ln\left(x + \sqrt{x^2+R^2}\right)\Big|_0^L$$

$$V_p = \frac{\lambda}{4\pi\epsilon_0} \ln\left(\frac{L+\sqrt{L^2+R^2}}{R}\right)$$

[5 marks] The figure below shows a circular loop rotating on an axis that is attached to a horizontal wire frame. The loop has radius r = 24 cm and is rotating with an angular speed (or angular frequency) ω = 250 rad/s in the clockwise direction as indicated by the curved arrows. A uniform magnetic field of 0.60 T is directed upward, as indicated. At the instant shown, the loop makes an angle θ with the horizontal. Time t = 0 occurs when θ = 0.



32. Calculate the flux for θ = 0° and θ = 90°. [1 mark]

$\theta = 0^\circ$ $\Phi_B = \int \vec{B} \cdot d\vec{A}$ $\vec{B}$ and $\vec{A}$ are $\parallel$ so

$\Phi_B = BA = (0.60T)(\pi(0.24m)^2) = 0.109 \, Tm^2$

$\theta = 90^\circ$: $\vec{B}$ and $\vec{A}$ are $\perp$ so $\Phi_B = 0$

33. Write down the flux as a function of time. [1 mark]

$\Phi_B = \Phi_{max} \cos(\omega t + \phi_0)$

where $\Phi_{max} = 0.109 \, Tm^2$

$t=0$ when $\theta = 0$ so $\phi_0 = 0$

$\Rightarrow \Phi_B = (0.109 \, Tm^2)\cos(\omega t) = B\pi r^2 \cos\omega t$

34. Write down Faraday's law, and use it to find an equation for the emf, ε, induced in the loop as a function of time, t. Express your answer in terms of r, ω, B, t and constants. [2 marks]

1

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -B\pi r^2(-\omega \sin\omega t)$$

$$\boxed{\mathcal{E} = +B\pi r^2 \omega \sin\omega t}$$

+ sign means emf is in ccw direction.

35. At the instant shown in the figure above, what is the direction of the induced current on the near (left) side of the loop (upward to the left or downward to the right)? [1 mark]

according to previous Q, induced current is downward and to the right

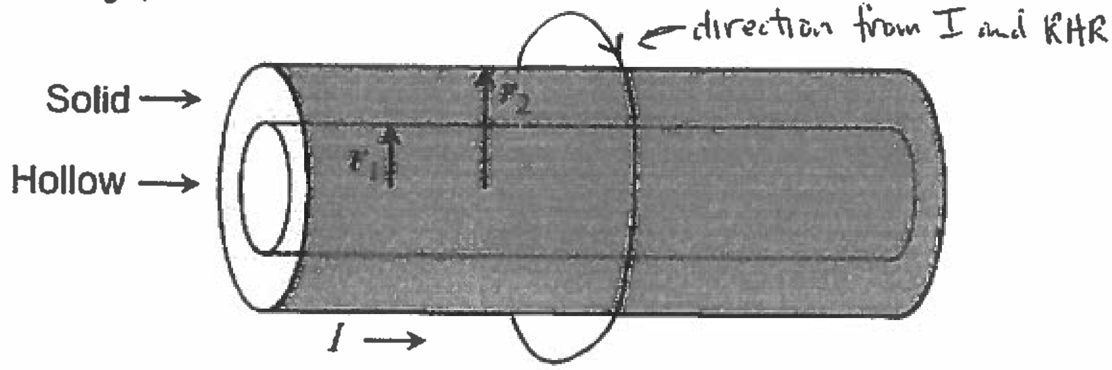
36. For what angle θ does the induced emf have maximum magnitude? Two angles are possible; either one will be accepted as correct. [1 mark]

max $\mathcal{E}$ when $\frac{d\mathcal{E}}{dt} = 0$: $\frac{d\mathcal{E}}{dt} = B\pi r^2\omega^2 \cos\omega t$

$\frac{d\mathcal{E}}{dt} = 0$ when $\omega t = \frac{\pi}{2}, \frac{3}{2}\pi$ $\Rightarrow$ $\boxed{\theta = \frac{\pi}{2} \text{ or } \frac{3\pi}{2}}$

Source 2012 Q 22

[5 marks] The figure below shows a portion of a very long, hollow, cylindrical conductor of inner radius $r_1 = 1.00$ cm and outer radius $r_2 = 2.00$ cm. The solid portion of the cylinder carries a current $I = 15.0$ A uniformly distributed over the area between r_1 and r_2, with positive charge flow toward the right, as indicated.



37. Write down Ampere's Law, and use it to find the magnetic field strength as a function of r outside the cylinder ($r > r_2$). Draw ~~a diagram to illustrate~~ your Amperean loop in the diagram above, and show all steps and reasoning. [2 marks]

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 i_{enc} \quad \text{(Amperean loop shown above)}$$

B is constant along loop and $\parallel$ to $d\vec{l}$ everywhere on loop

$$\Rightarrow B(2\pi r) = \mu_0 I$$

$$\therefore \boxed{B = \frac{\mu_0 I}{2\pi r}}$$

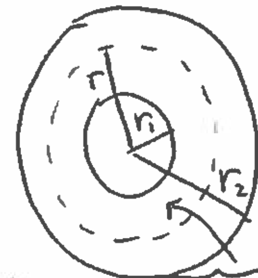
38. Calculate the current density in the solid part of the cylinder ($r_1 < r < r_2$). [1 mark]

$$J = \frac{I}{A} \quad \text{where } A = \pi r_2^2 - \pi r_1^2$$

$$J = \frac{I}{\pi(r_2^2 - r_1^2)}$$

39. Find the magnetic field strength as a function of r in the solid part of the cylinder ($r_1 < r < r_2$). You may use relevant results from above without re-deriving them, provided you state clearly what you are doing. [2 mark]

$$\underbrace{B(2\pi r)}_{\text{same reasoning as before}} = \mu_0 i_{enc}$$

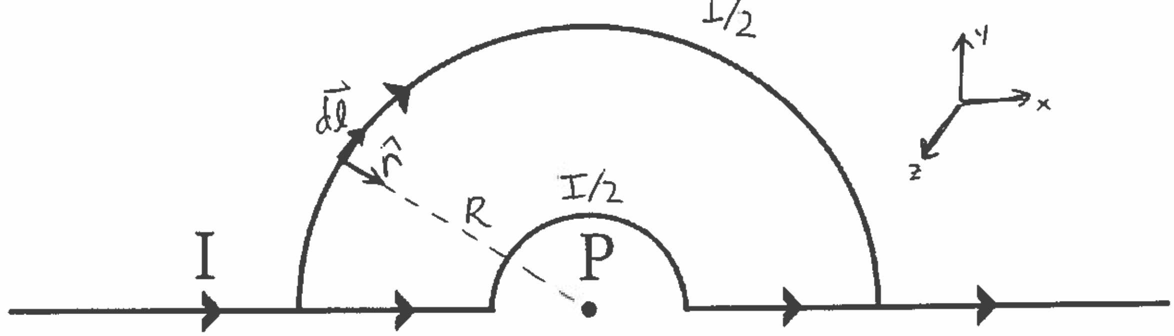


$$B = \frac{\mu_0 J(\pi r^2 - \pi r_1^2)}{2\pi r} = \boxed{\frac{\mu_0 I (r^2 - r_1^2)}{2\pi r (r_2^2 - r_1^2)}}$$

$$i_{enc} = J(\pi r^2 - \pi r_1^2)$$

Source 2012 Q 23

[5 marks] The wire in the diagram below carries a current, I, as shown. The current splits equally at the junction. The top curved section of wire is a semi-circle of radius R, the bottom curved section of wire is a semi-circle of radius r, and point P is at the center of curvature of both sections.



40. What is the contribution to the magnetic field from the straight sections of the wire? Explain.[1 mark]

From Biot-Savart law, on straight wire segments $d\vec{B} = \frac{\mu_0}{4\pi} \frac{i\, d\vec{l} \times \hat{r}}{r^2}$ but $d\vec{l}$ and $\hat{r}$ are (anti)parallel so $d\vec{l} \times \hat{r} = \vec{0}$, hence the straight segments do not contribute.

41. For a small segment of one of the two curved sections of the wire, label on the diagram provided all the relevant variables for **use in the Biot-Savart Law**. Derive the equation for the contribution to the magnetic field strength, B, at point P due to this curved segment. [2 marks]

Top segment $d\vec{B} = \frac{\mu_0}{4\pi} \frac{I}{2} \frac{d\vec{l} \times \hat{r}}{R^2}$

$d\vec{l} \times \hat{r} = dl\,(-\hat{k})$ ← can also write as ⊗

all segments produce field into page.

$$\vec{B} = \int_0^{\pi R} \frac{\mu_0}{4\pi} \frac{I}{2} \frac{dl}{R^2} (-\hat{k})$$

$$\vec{B} = \frac{\mu_0 I}{8\pi R^2} \int_0^{\pi R} dl\,(-\hat{k}) \Rightarrow \boxed{\vec{B} = -\frac{\mu_0 I}{8R} \hat{k}}$$

42. What is the magnitude and direction of $\vec{B}$ at point P due to all the sections of the wire? [2 marks]

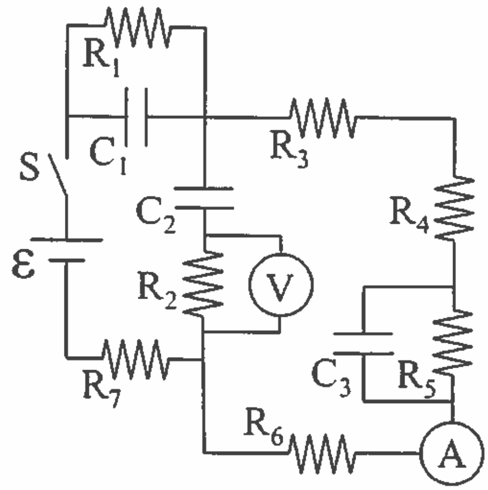
The calculation for the bottom segment is identical but $R \rightarrow r$

$$\vec{B}_{top} = -\frac{\mu_0 I}{8R}\hat{k} \qquad \vec{B}_{bottom} = -\frac{\mu_0 I}{8r}\hat{k}$$

$$\vec{B}_{net} = \vec{B}_{top} + \vec{B}_{bottom} = -\frac{\mu_0 I}{8R}\hat{k} - \frac{\mu_0 I}{8r}\hat{k}$$

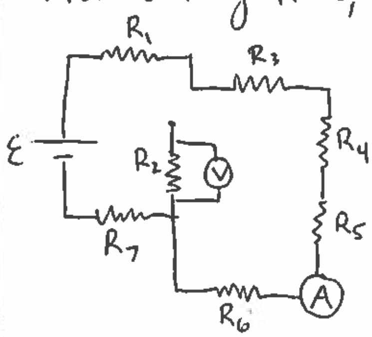
$$\boxed{\vec{B}_{net} = -\frac{\mu_0 I}{8}\left(\frac{1}{R} + \frac{1}{r}\right)\hat{k}}$$

[4 marks] In the following circuit, all capacitors are initially uncharged, $C_1 = 10\ \mu F$, $C_2 = 15\ \mu F$, $C_3 = 20\ \mu F$, $R_1 = 16\ \Omega$, $R_2 = 22\ \Omega$, $R_3 = 7.0\ \Omega$, $R_4 = 25\ \Omega$, $R_5 = 12\ \Omega$, $R_6 = 9.0\ \Omega$, and $R_7 = 30\ \Omega$. At time $t = 0$ the switch S is closed, connecting the $\varepsilon = 10.0$ V battery to the circuit. Hint: How do capacitors act when fully charged and uncharged?



43. After the switch has been closed for a long time, what is the reading of the ammeter? [1 mark]

After a long time, capacitors act like breaks in the circuit:



Eq. resistance (all Rs in series) $R_{eq} = R_1 + R_3 + R_4 + R_5 + R_6 + R_7$

$R_{eq} = 99\Omega$

$\varepsilon = I R_{eq}$

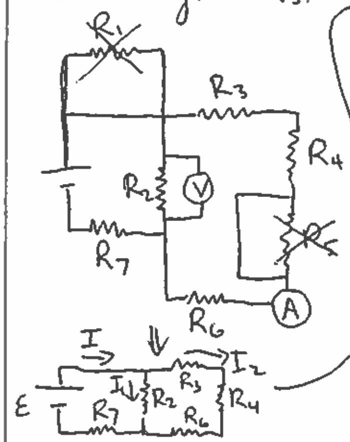
$I = \dfrac{10\text{ V}}{99\Omega} = \boxed{0.10\text{A}}$

44. After the switch has been closed for a long time, what is the reading of the voltmeter? [1 mark]

After a long time no current flows through R_2, so $\Delta V_{R2} = 0$. The voltmeter is in parallel with R_2, so its reading will also be zero

45. **Immediately after the switch is closed**, what is the reading of the ammeter? [1 mark]

Immediately after the switch is closed, capacitors act like wire segments.



$I = I_1 + I_2$

$R_{eq} = R_7 + \left(\frac{1}{R_2} + \frac{1}{R_3+R_4+R_6}\right)^{-1}$

$R_{eq} = 44.317\Omega$

$I = \frac{10V}{44.317\Omega} = 0.22565A$

loop rule: $\varepsilon - I_2(R_3+R_4+R_6) - IR_7 = 0$

$I_2 = \frac{10V - (0.22565A)(30\Omega)}{7\Omega + 25\Omega + 9\Omega}$

$\boxed{I_2 = 0.079A}$

46. **Immediately after the switch is closed**, what is the reading of the voltmeter? [1 mark]

Voltmeter reading $= \Delta V_{R_2} = I_1 R_2$

$= (0.22565A - 0.079A)\, 22\Omega$

$\boxed{= 3.23V}$