Phys 259, Group Activity 5, Winter 2017

Group #	Student	Last Name	First Name
	1		
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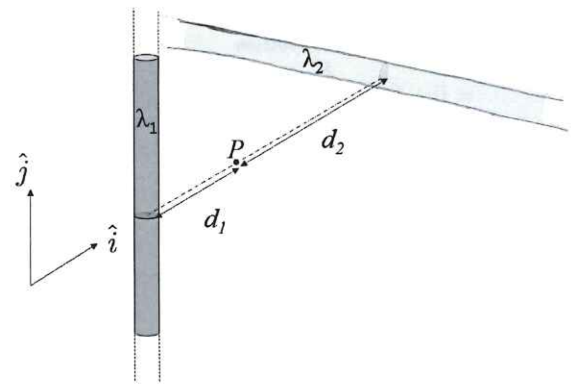
(10 marks) What is the electric field, $\vec{E}$, at point P due to two charged rods of infinite length, as presented in the figure below? Rod 1 has a positive, linear charge density λ_1 and is oriented vertically, a distance d_1 from P. Rod 2 has a positive, linear charge density λ_2 and is oriented horizontally, a distance d_2 from P.

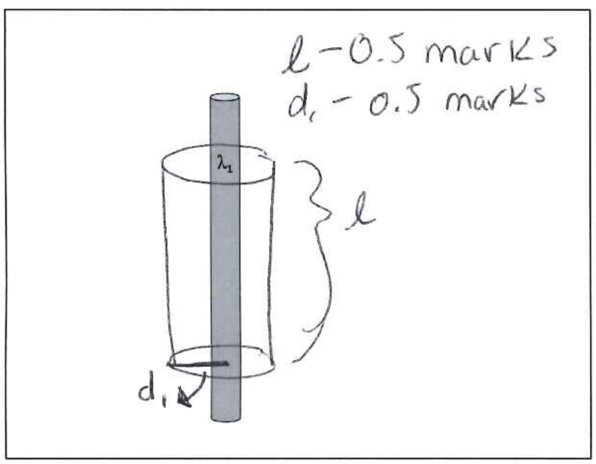


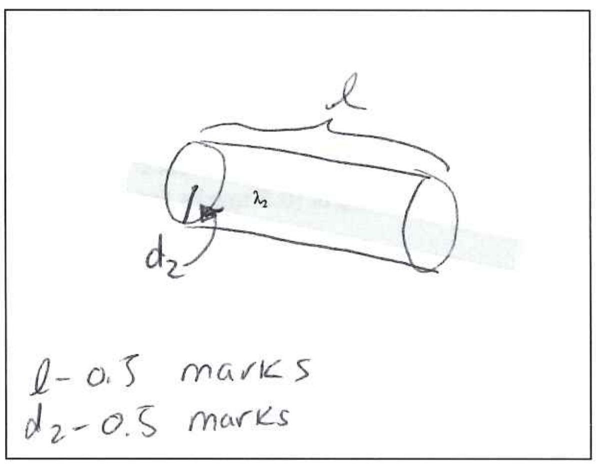
The parts below walk you through related questions, and the steps with which to solve this problem. Please ***show all work*** *in the boxes provided and then choose the correct answer at the bottom*

1. **(1 mark)** What Gaussian surface (i.e., what 3D shape) makes it **easiest** to apply Gauss' law for a rod? Explain why.

A cylinder (0.5 mark).
Both $\vec{E}$ of a rod and the surface area elements, dA, of a cylinder body point radially outward from the cylinder axis. This makes it easy to apply Gauss' law because $\vec{E} \cdot d\vec{A}$ becomes either 0 or EdA for the different faces of the Gaussian surface. (0.5 mark)

2. **(2 marks)** For each rod, **draw** the cylindrical Gaussian surface needed to find its electric field contribution at point P. **Label** each Gaussian surface with length l and the appropriate radius. → (i.e. d_1 & d_2)





3. **(1 mark)** Calculate the charge enclosed in **each** of your Gaussian surfaces, in terms of l, λ_1 and λ_2.
$\lambda = Q/l$
Rod 1 : $Q_1 = \lambda_1 l$ (0.5 marks)
Rod 2: $Q_2 = \lambda_2 l$ (0.5 marks)

4. **(2.5 marks)** Use Gauss' law ($\oint \overrightarrow{E} \cdot \overrightarrow{dA} = \frac{Q}{\epsilon_0}$) to calculate $\overrightarrow{E_1}$, the electric field at point P due to Rod 1, in terms of λ_1 and d_1. LHS of Gauss' law (0.5 marks to show work, 0.5 marks answer):

$$\oint \overrightarrow{E_1} \cdot \overrightarrow{dA} = \int\limits_{cylinder1} \overrightarrow{E_1} \cdot d\overrightarrow{A} \tag{1}$$

$$= \int\limits_{top} \overrightarrow{E_1} \cdot d\overrightarrow{A} + \int\limits_{bottom} \overrightarrow{E_1} \cdot d\overrightarrow{A} + \int\limits_{body} \overrightarrow{E_1} \cdot d\overrightarrow{A} \tag{2}$$

$$= 0 + 0 + \int\limits_{body} E_1 dA \tag{3}$$

$$= E_1 \int\limits_{body} dA \tag{4}$$

$$= E_1 2\pi d_1 l \tag{5}$$

Where $2\pi d_1 l$ is the surface area of the cylinder body.
RHS of Gauss' law (0.5marks):

$$E_1 2\pi d_1 l = \frac{Q}{\epsilon_0} \tag{6}$$

$$= \frac{\lambda_1 l}{\epsilon_0} \tag{7}$$

$$\tag{8}$$

With algera (0.5 marks):

$$E_1 = \frac{\lambda_1}{2\pi d_1 \epsilon_0} \tag{9}$$

In particular, E_1 is in the direction of $\hat{i}$ so (0.5 marks):

$$\overrightarrow{E_1} = \frac{\lambda_1}{2\pi d_1 \epsilon_0}\hat{i} \tag{10}$$

5. **(1.5 marks)** Use Gauss' law ($\oint \overrightarrow{E} \cdot \overrightarrow{dA} = \frac{Q}{\epsilon_0}$) to calculate $\overrightarrow{E_2}$, the electric field at point P due to Rod 2, in terms of λ_2 and d_2.

The math behind the electric field is the same for Rod 2, we just need to sub in from our answer in (4):

$\lambda_1 \rightarrow \lambda_2$; $d_1 \rightarrow d_2$ (0.5 marks - "work" is writing/explaining the substitution)
Thus, the magnitude E_2 is (0.5 marks for answer):

$$E_2 = \frac{\lambda_2}{2\pi d_2 \epsilon_0} \tag{11}$$

Looking again at the diagram, we see that E_2 will have to be in the **direction** of $-\hat{i}$ (0.5 marks).

$$\overrightarrow{E_2} = -\frac{\lambda_2}{2\pi d_2 \epsilon_0}\hat{i} \tag{12}$$

6. **(1 mark)** Write the total electric field ($\overrightarrow{E}$) at point P **in terms of** $\overrightarrow{E_1}$ **and** $\overrightarrow{E_2}$.
Using superposition: $\overrightarrow{E} = \overrightarrow{E_1} + \overrightarrow{E_2}$

(1 mark for the correct answer) What is the total electric field in terms of $\lambda_1, \lambda_2, d_1$ and d_2?

A. $\boxed{\overrightarrow{E} = \frac{1}{2\pi\epsilon_0}(\frac{\lambda_1}{d_1} - \frac{\lambda_2}{d_2})\hat{\imath}}$ B. $\overrightarrow{E} = \frac{1}{2\pi\epsilon_0}(\frac{\lambda_1}{d_1} + \frac{\lambda_2}{d_2})\hat{\imath}$

C. $\overrightarrow{E} = \frac{1}{2\pi\epsilon_0}(\frac{\lambda_1}{d_1}\hat{\jmath} - \frac{\lambda_2}{d_2}\hat{\imath})$ D. $\overrightarrow{E} = \frac{1}{2\pi\epsilon_0}(\frac{\lambda_1}{d_1}\hat{\jmath} + \frac{\lambda_2}{d_2}\hat{\imath})$