Electricity and Magnetism

- Physics 259 – L02
- Lecture 5

Section 21.1

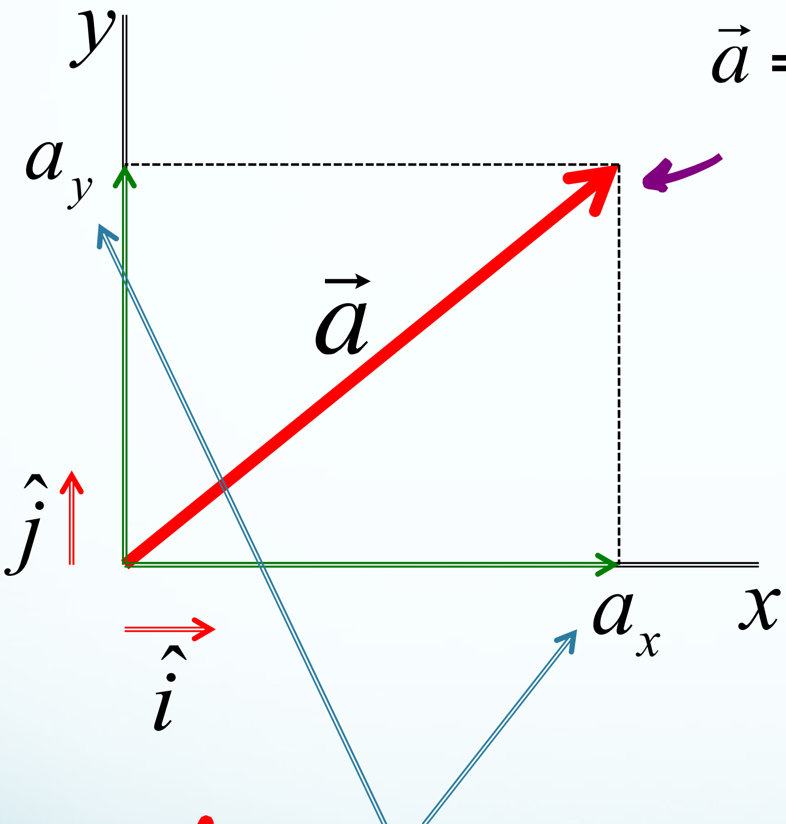
Last time

- Charges and Force Between Charges
- Conductors and Insulators
- Basics of Coulomb’s Law
- Van De Graaff Generator Experiment
- Class Activity

This time

- Solve Class Activity Question
- Coulomb’s Law
- Examples

Vector Components



$$\vec{a} = a_x\hat{i} + a_y\hat{j}$$

$$|\vec{a}| = \sqrt{a_x^2 + a_y^2}$$

$$a_y = a\sin\theta$$

$$a_x = a\cos\theta$$

$$\vec{a} = a_x\hat{i} + a_y\hat{j}$$

$$= \underbrace{a\cos\theta}_{\text{scalar}}\hat{i} + \underbrace{a\sin\theta}_{\text{scalar}}\hat{j}$$

Scalars!

$$|\vec{a}| = \sqrt{\underbrace{|a_x|^2}_{(a\cos\theta)^2} + \underbrace{|a_y|^2}_{(a\sin\theta)^2}}$$

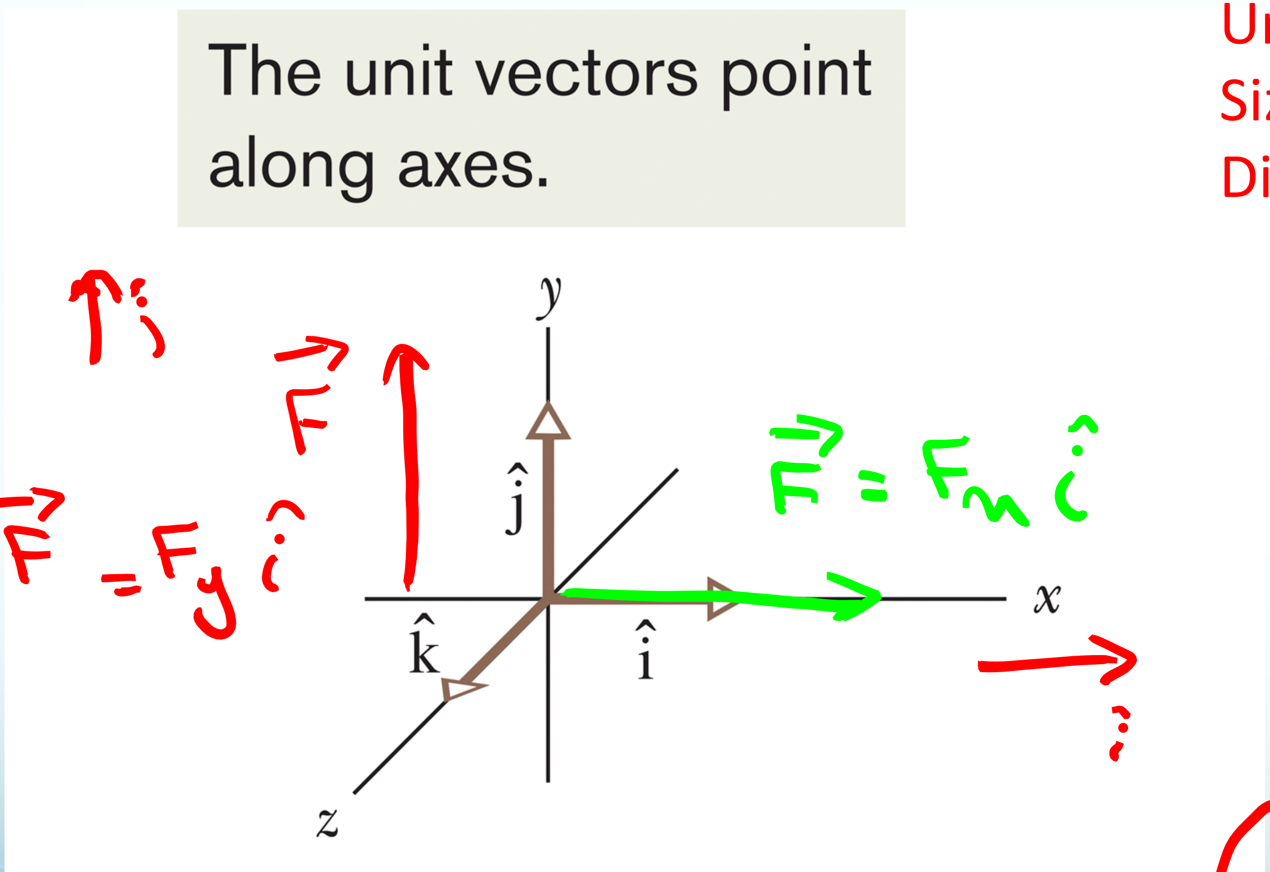
$$\cancel{\sqrt{a_x\hat{i}^2 + a_y\hat{j}^2}}$$

Unit vectors

Unit→

Size→

Direction→



 halliday_10e_fig_03_13

$\uparrow \hat{j}$

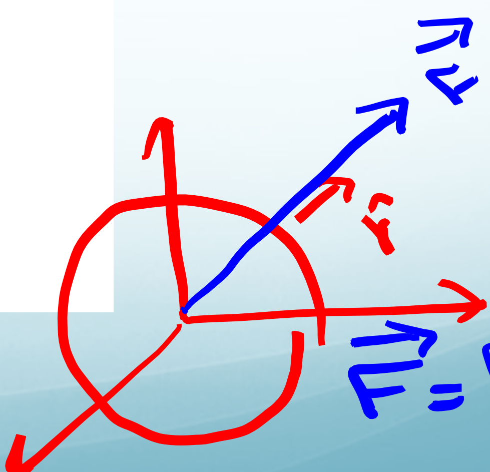
$\vec{F}$

$\vec{F} = F_y \hat{i}$

$\vec{F} = F_x \hat{i}$

$\rightarrow \hat{i}$

$\rightarrow \hat{i}$



$\vec{F} = F_r \hat{r}$

$\hat{r} \rightarrow$ unit vector

$\vec{r} \rightarrow$ vector

Class Activity

(10 marks) In a two dimensional Cartesian system $\hat{r}$ is located 30° north of east. What is the mathematical expression for $\hat{r}$ in terms of Cartesian unit vectors?

1. **(3 mark)** Draw a two dimensional Cartesian coordinate system (1 mark). Draw $\hat{r}$ and unit vectors which represent the direction in the positive x-axis and y-axis (1 mark each).

$$\hat{r} = r_x\hat{i} + r_y\hat{j} \rightarrow \hat{r} = \overbrace{|\hat{r}\cos\theta|}^{\text{scalar}}\hat{i} + \overbrace{|\hat{r}\sin\theta|}^{\text{scalar}}\hat{j}$$

↳ r_x is x comp. of $\hat{r}$ ($\hat{r}_x$)

$|r_x| = |\hat{r}_x| = 1$

2. **(1 mark)** Do unit vectors have physical units (such as metres)? Explain.

No ....

3. **(3 marks)** Write a mathematical expression for $\hat{r}$ in terms of Cartesian unit vectors.

$$\hat{r} = |\hat{r}_x|\,\hat{i} + |\hat{r}_y|\,\hat{j} = r_x\,\hat{i} + r_y\,\hat{j}$$

$$= |\hat{r}\cos\theta|\,\hat{i} + |\hat{r}\sin\theta|\,\hat{j}$$

$$\rightarrow \hat{r} = \cos\theta\,\hat{i} + \sin\theta\,\hat{j}$$

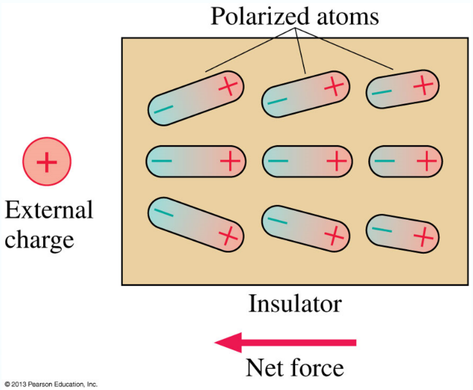
4. **(2 marks)** Show that $\hat{r}$ is indeed a unit vector.

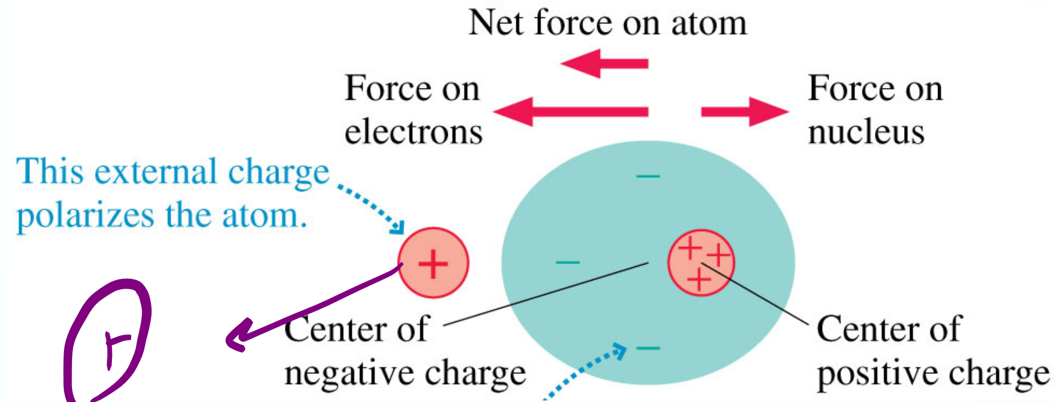
$$|\hat{r}| = \sqrt{|\hat{r}_x|^2 + |\hat{r}_y|^2} = \sqrt{\cos\theta^2 + \sin\theta^2}$$

$$\rightarrow |\hat{r}| = \boxed{1}$$

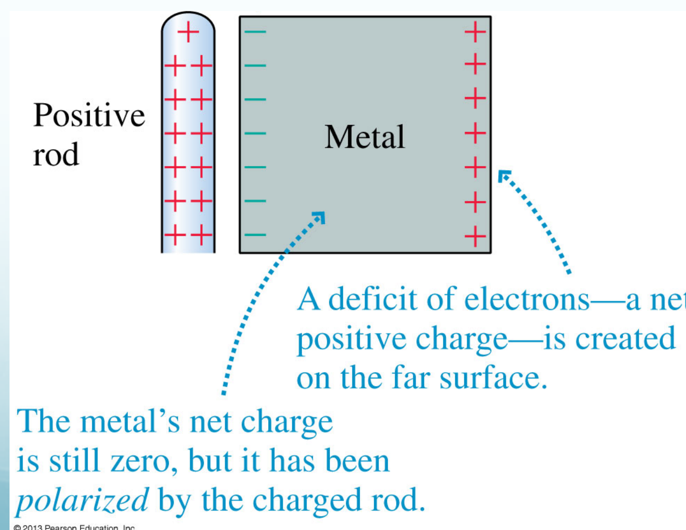
last section => we did the experiment of 8

Charge Polarization





Metal



Coulomb's Law

$$\boxed{F_{1\,on\,2} = F_{2\,on\,1} = K\frac{|q_1||q_2|}{r^2}}$$

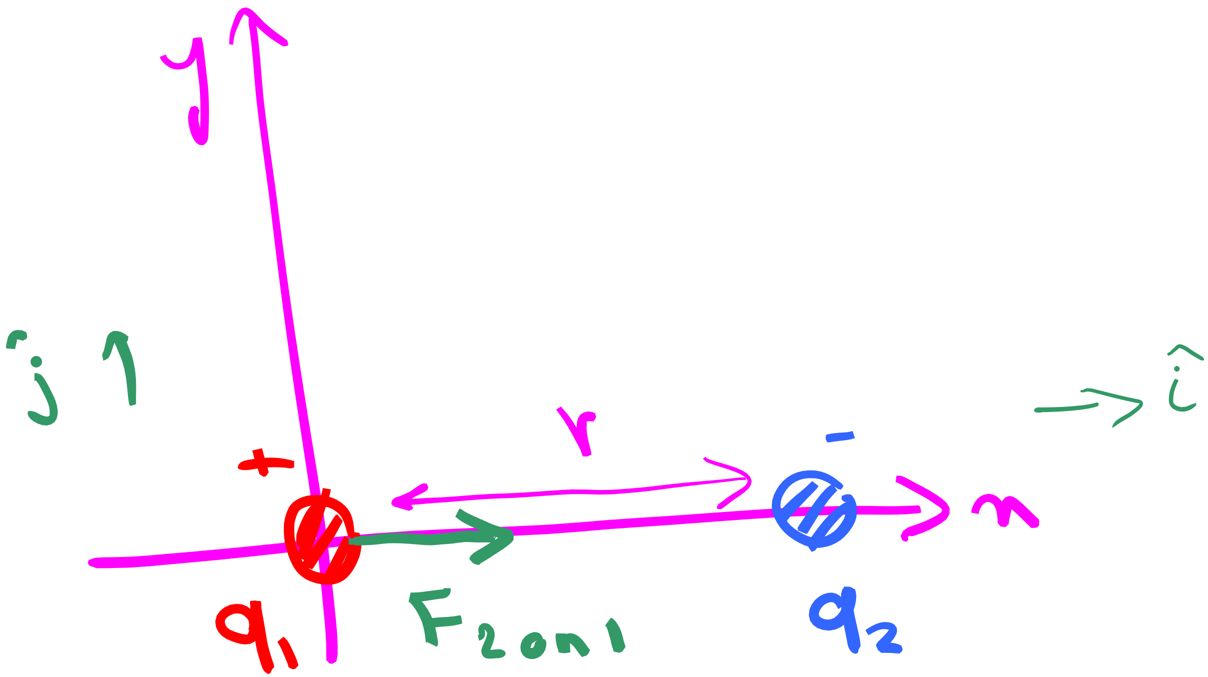
K = electrostatic constant

$$K = 8.99\times10^{9}\,\frac{N\cdot m^2}{C^2}$$

$$\boxed{F_{1\,on\,2} = F_{2\,on\,1} = \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_2|}{r^2}}$$

ε_0 = permittivity of free space

$$\varepsilon_0 = \frac{1}{4\pi K} = 8.85\times10^{12}\,\frac{C^2}{N\cdot m^2}$$



$$|F_{2on1}| = k\frac{|q_1||q_2|}{r^2}$$

$$\vec{F}_{2on1} = k\frac{|q_1||q_2|}{r^2}\,\hat{i}$$

First find the magnitude of force, then find the direction from diagram

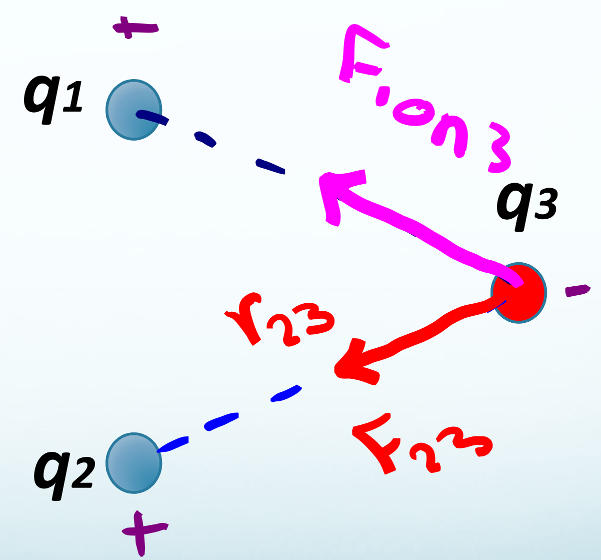
Superposition Principle

The total force on q_3 is the vector sum of the individual forces:

$$\vec{F}_{on\,3} = \vec{F}_{1\,on\,3} + \vec{F}_{2\,on\,3}$$

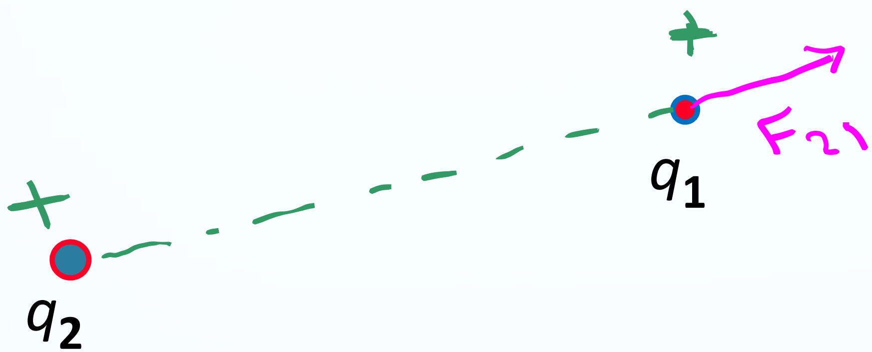
$$F_{1on3} = k\frac{|q_1||q_3|}{r_{13}^2}$$

$$F_{2on3} = k\frac{|q_2||q_3|}{r_{23}^2}$$

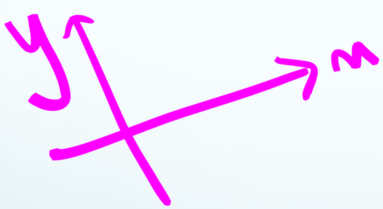


Coulomb's Law

How to compute the magnitude and direction properly.

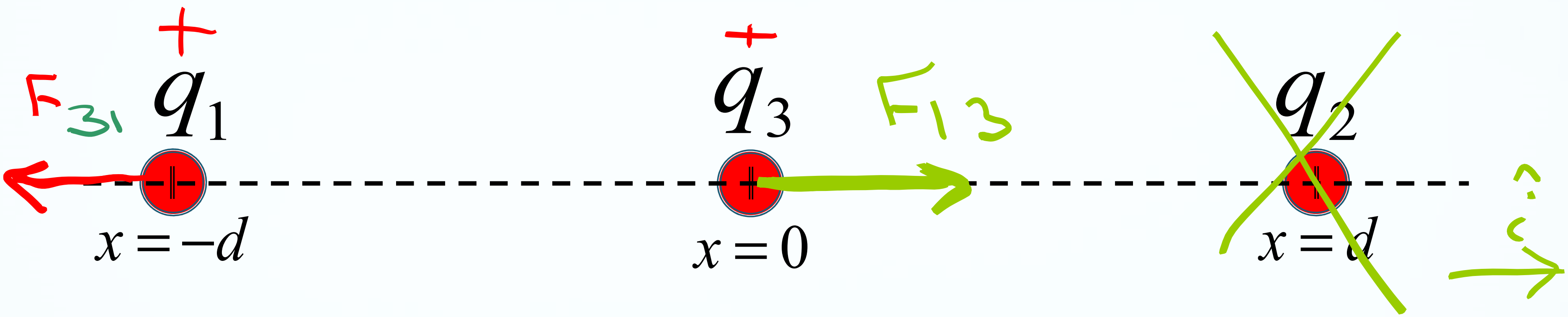


$$\left|\vec{F}_{21}\right| = K\frac{|q_1||q_2|}{r_{21}^2}$$



$$\vec{F}_{21} = K\frac{|q_1||q_2|}{r_{21}^2}\,\hat{i}$$

Example #1:



If the force of q_1 on q_3 is $\vec{F}_{13} = (+3\text{N})\hat{i}$ (i.e., in the +x direction), and the force of q_3 on q_1 is $\vec{F}_{31} = F_{31}\hat{i}$, what is the component F_{31}?

$F_{1\,on\,3} = +3N\,\hat{i} \rightarrow F_{3\,on\,1} = ?$

Newton's Third Law→ $\vec{F}_{13} = -\vec{F}_{31}$
Forces are equal and opposite

$$\left.\begin{array}{l} |F_{31}| = |F_{13}| \\ \vec{F}_{31} = -\vec{F}_{13} \end{array}\right\} \rightarrow$$

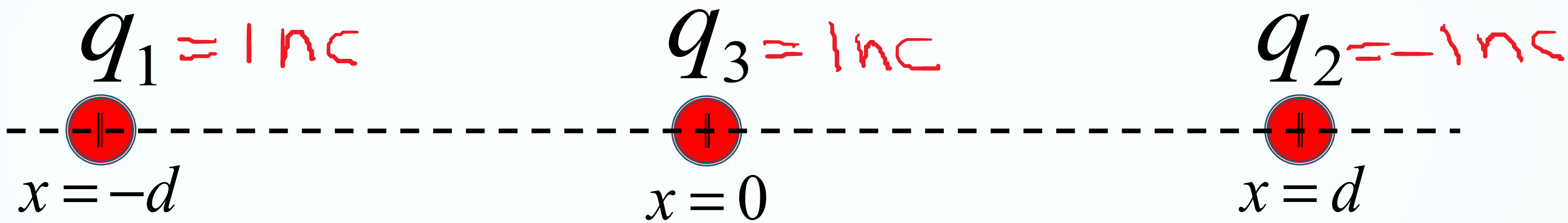
Coulomb's Law→ $\left|\vec{F}_{13}\right| = \left|\vec{F}_{31}\right|$
Pick direction using charge rules.

$\vec{F}_{31} = -3N\,\hat{i}$

$\boxed{F_{31} = -3\text{N}}$

Example #2:

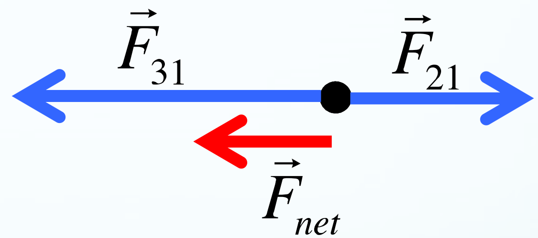
$$K = 8.99 \times 10^{9}\,\frac{N \cdot m^{2}}{C^{2}}$$



d = 1m

What is the total electric force on q_1? Note: nC = 10^{-9} C.

Free Body Diagram:



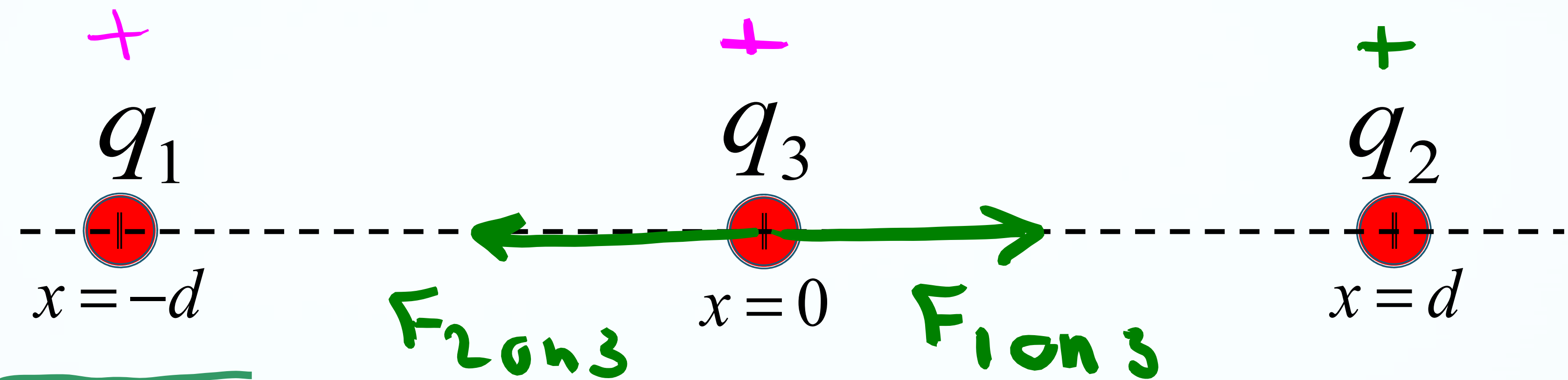
Do at Home

$$\left|\vec{F}_{31}\right| = 8.99 \times 10^{-9}\ \mathrm{N}$$

$$\left|\vec{F}_{21}\right| = 2.25 \times 10^{-9}\ \mathrm{N}$$

$$\boxed{\vec{F}_{net} = \left(-6.74 \times 10^{-9}\ \mathrm{N}\right)\hat{i}}$$

TOPHAT QUESTION



If $\boxed{q_1=+1.0\text{ nC}}$ and the total electric force on q_3 is exactly zero, what is the value of q_2 in nC?

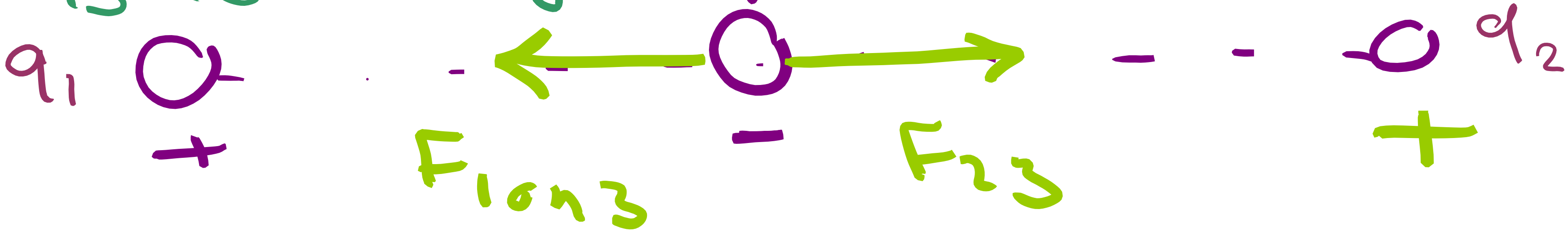
if q_3 is + charge $\Rightarrow$

$|F_{1 on 3}| = |F_{2 on 3}| \rightarrow$

$$\cancel{k}\frac{|q_1||\cancel{q_3}|}{\cancel{d^2}} = \cancel{k}\frac{|q_2||\cancel{q_3}|}{\cancel{d^2}} \Rightarrow |q_1| = |q_2|$$

$q_2 = +1$ nC

if q_3 is - charge → q_3



$$|F_{1on3}| = |F_{2on3}|$$

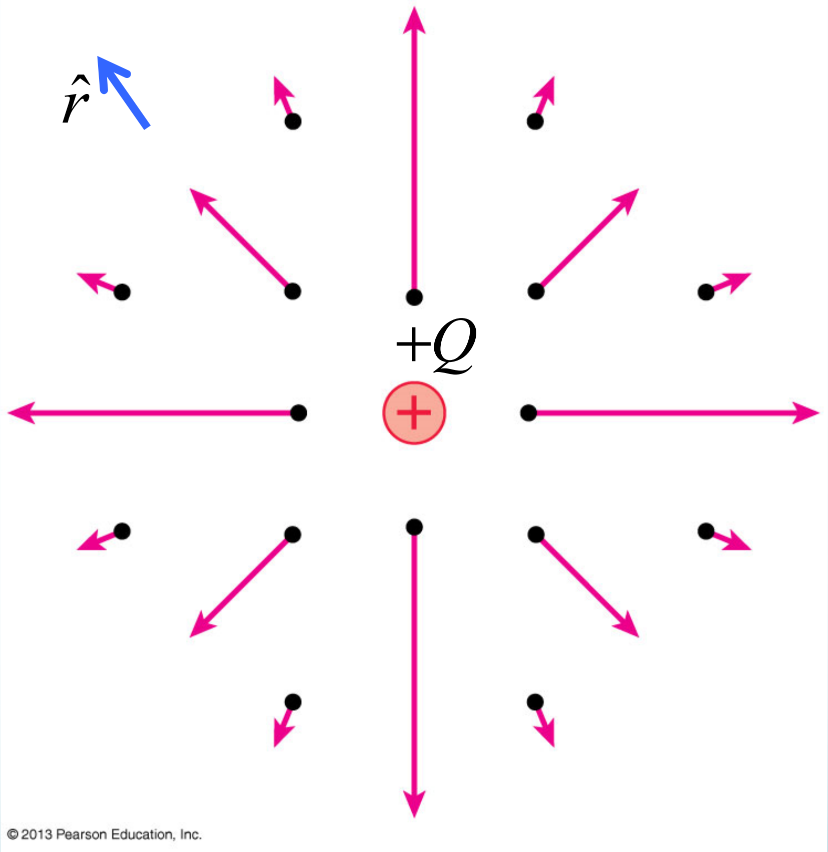
$$\cancel{k}\frac{|q_1||\cancel{q_3}|}{\cancel{d^2}} = \cancel{k}\frac{|q_2||\cancel{q_3}|}{\cancel{d^2}} \to |q_1| = |q_2|$$

$$\to q_2 = +1\ nC \to$$ same result

Next section

Building blocks of electric force

$$\vec{F} = \frac{KQq}{r^2}\hat{r}$$

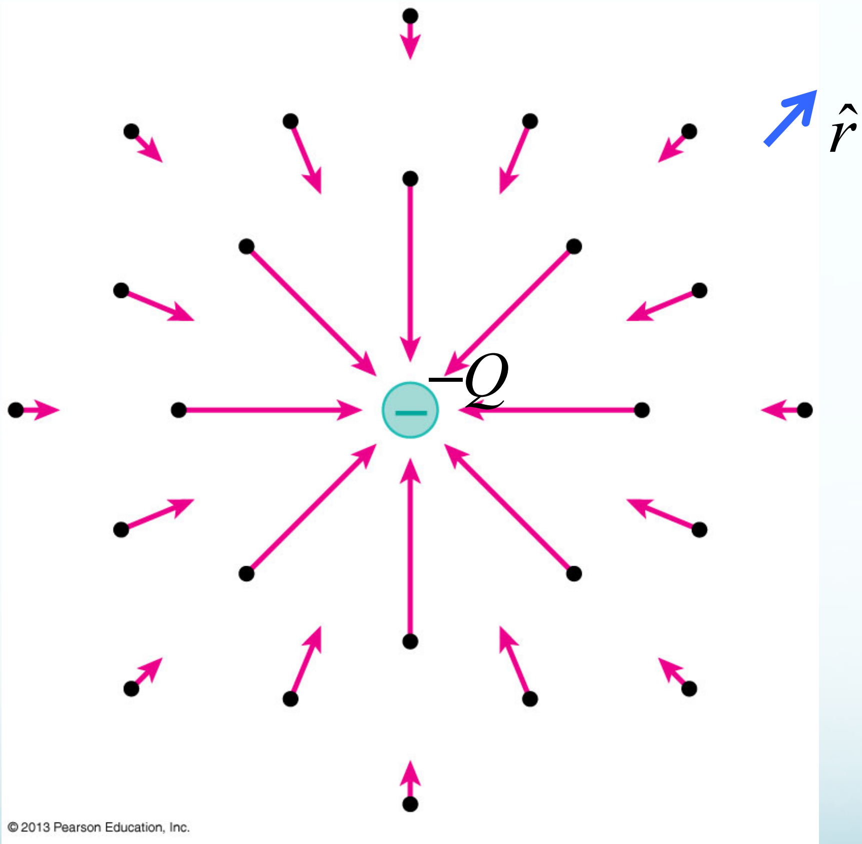


● = positive charge q at the position indicated

Next Section

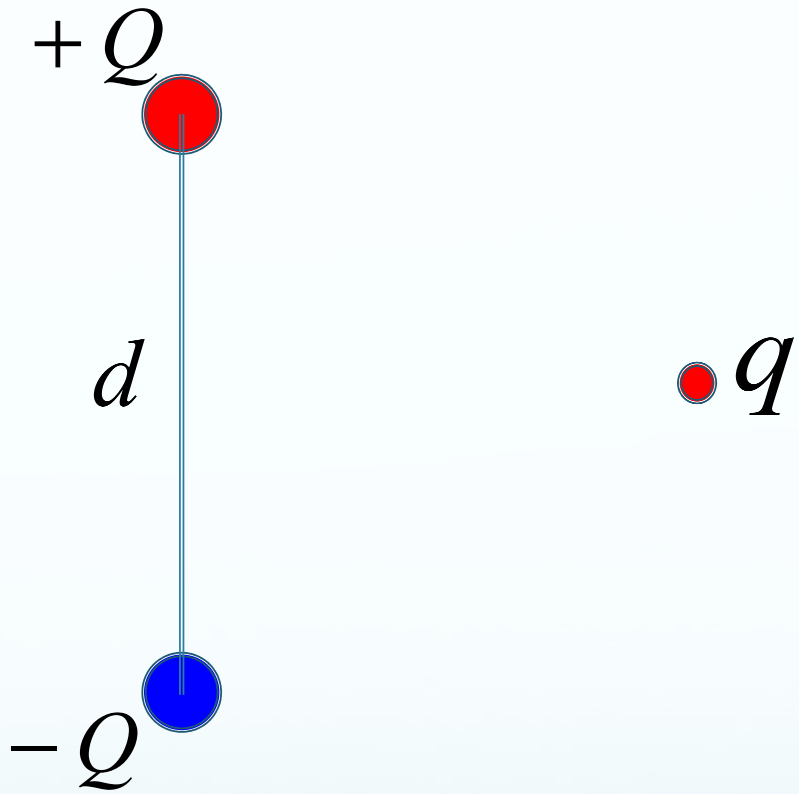
Building blocks of electric force

$$\vec{F} = \frac{-KQq}{r^2}\hat{r}$$



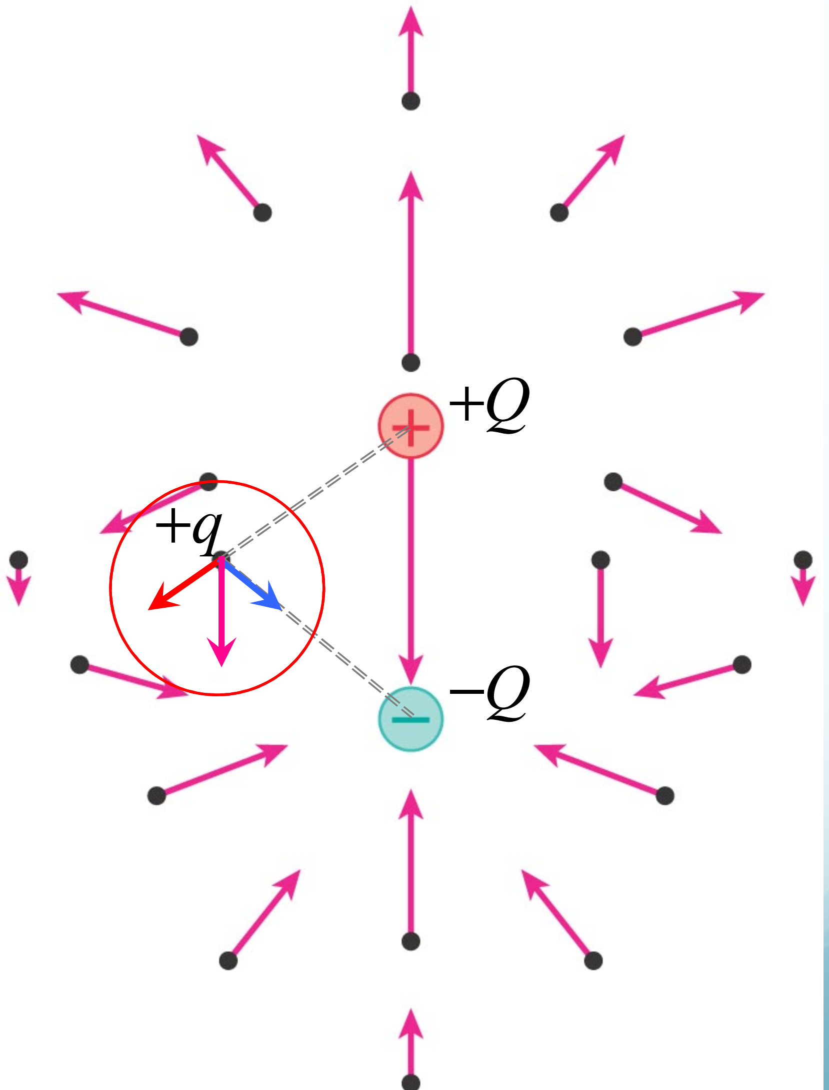
● = positive charge q at the position indicated

Next Section



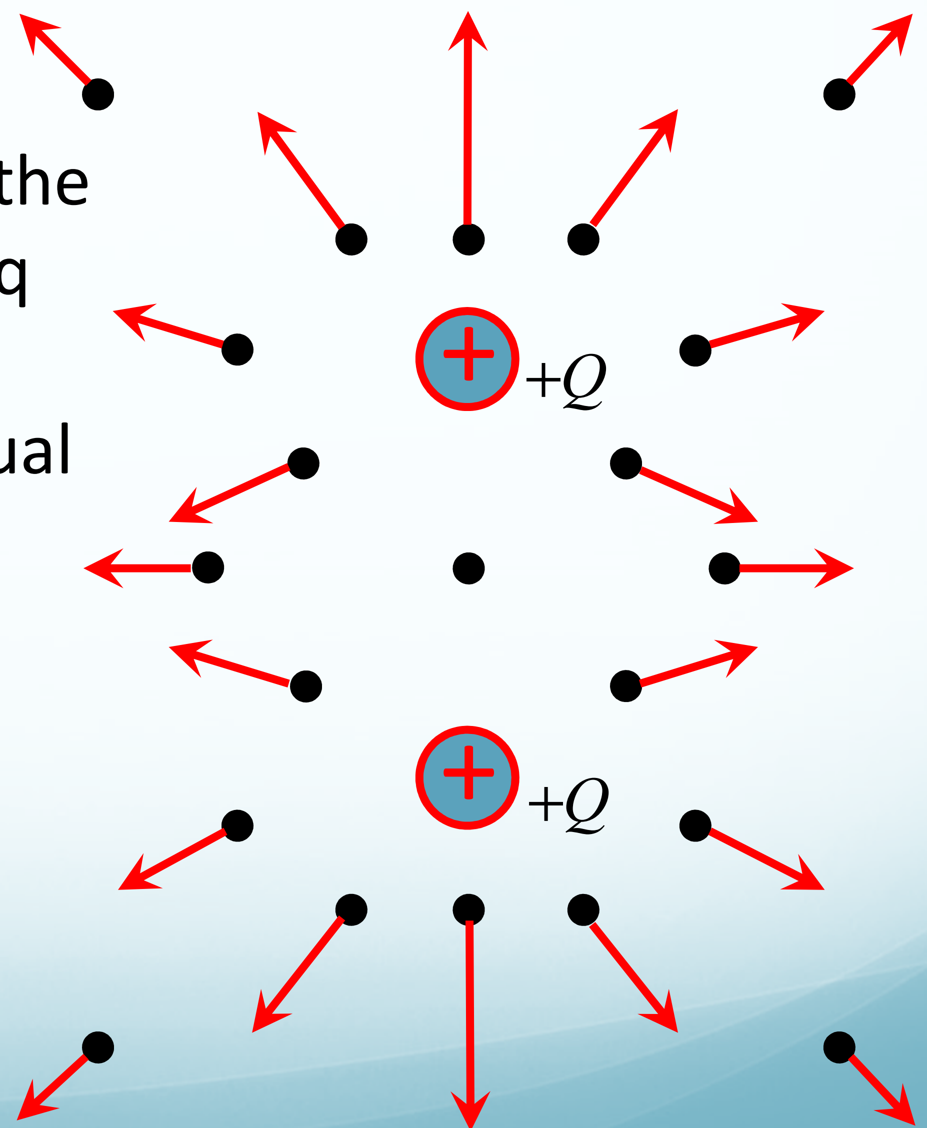
Superposition with Building Blocks

The vector represents the magnitude and direction of the electric force on the charge q **at that point**. It comes from superposition of the individual forces from +Q and –Q.



Superposition with Building Blocks

The vector represents the magnitude and direction of the electric force on the charge q **at that point**. It comes from superposition of the individual forces from +Q and +Q.



Direction again comes from superposition! Same steps as previous apply here too.

This section we talked about:

Chapter 21.1

See you on Wednesday

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