Electricity and Magnetism

- Physics 259 – L02
- Lecture 22

Chapter 24.1: Electric Potential

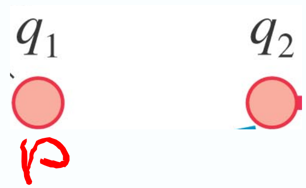
Last time

- Electric potential energy and electric force
- Electric potential and electric field
- Electric potential of a dipole

This time

- Electric potential energy of a collection of charges
- Electric potential (very important concept)
- Equipotential surfaces: visualizing electric potential
- Conductors and electric potential
- Interpreting equipotential surfaces

Last section we talked about:



If we release particle 1 at p, it begins to move→ has kinetic energy

ENERGY CAN NOT APPEAR BY MAGIC

It comes from electric potential energy U associated with the force between two particles

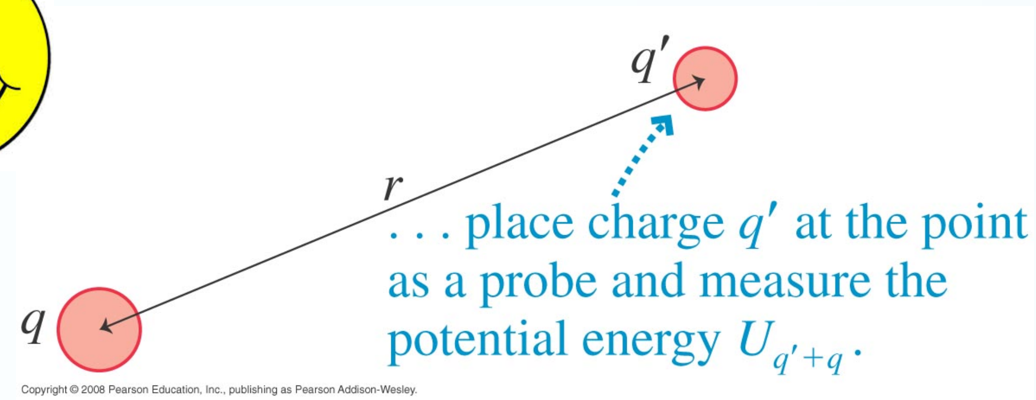
We also defined→

Electric potential V that is set up at point p by particle 2.

The electric potential exists regardless of whether particle 1 is at p.

Starting from the end

The whole story is:



Electric force on q' from q

$$\vec{F}_{qq'} = \frac{1}{4\pi\varepsilon_0}\frac{qq'}{r^2}\hat{r}$$

Then the electric field of q is

$$\vec{E} = \frac{\vec{F}_{qq'}}{q'} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$$

Potential energy of q and q'

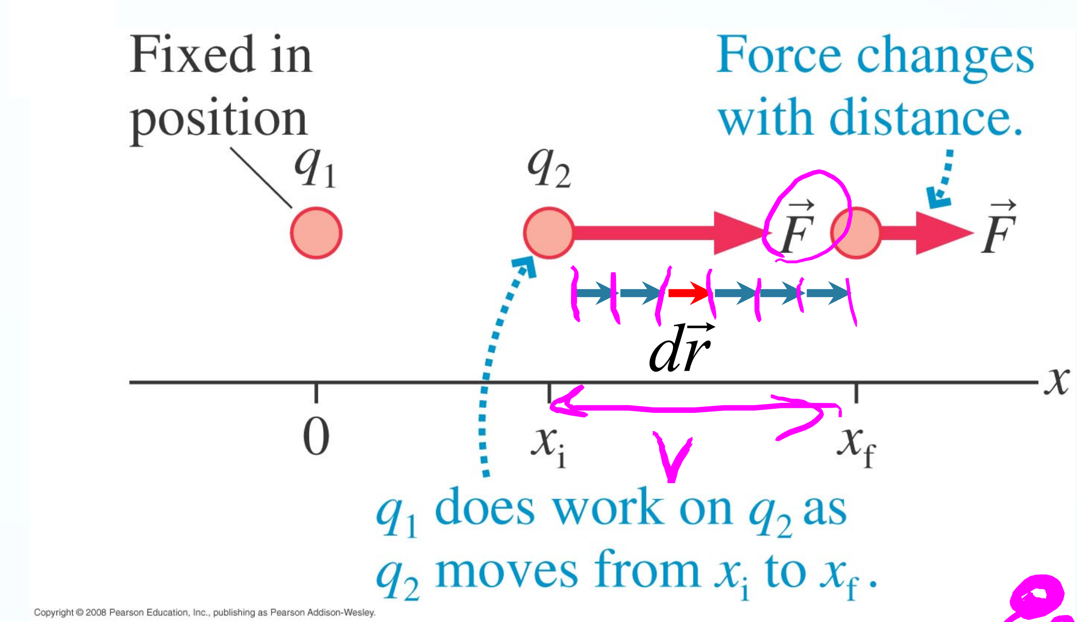
$$U_{q'+q} = \frac{1}{4\pi\varepsilon_0}\frac{qq'}{r}$$

Then the potential of q is

$$V = \frac{U_{q'+q}}{q'} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

Review: Finding Potential Energy of two point charges

W = F.R



The total work is the sum of all the little bits of work:

$$W_{i\to f}^{ELEC} = \int_{r_i}^{r_f} F dr$$

F dr

$$W_{i\to f}^{ELEC} = \int_{r_i}^{r_f} \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2} dr$$

Review: Finding Potential Energy of two point charges

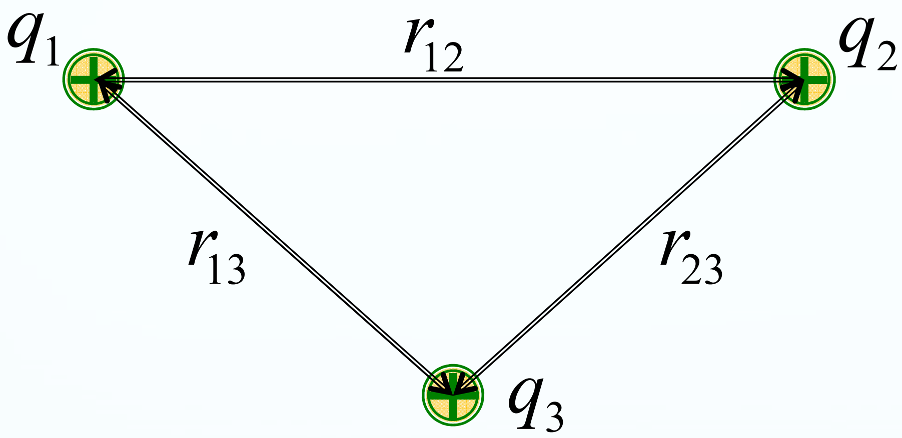
$$W_{i\to f}^{ELEC} = -\frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}\Bigg|_{r_i}^{r_f}$$

$$W_{i\to f}^{ELEC} = -\Delta U = -(U_f - U_i) = U_i - U_f$$

$W = -\nabla U$

$$\boxed{U_e = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}}$$

Superposition: Potential Energy due to Multiple Charges



$$U_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}}$$

$$U_{23} = \frac{1}{4\pi\varepsilon_0}\frac{q_2 q_3}{r_{23}}$$

$$U_{13} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_3}{r_{13}}$$

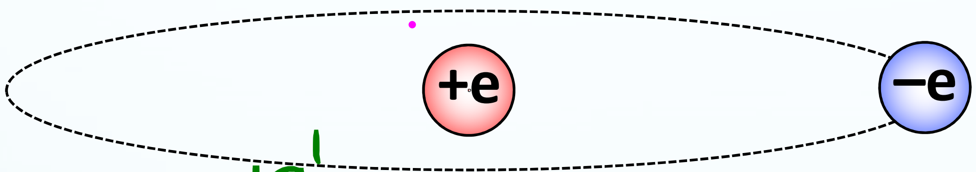
$$\boxed{U_{total} = U_{12} + U_{23} + U_{13}}$$

In general, the total potential energy is just the sum of the pairwise potential energies of all the charges present.

Calculate U between each pair, then sum all of them up.

TopHat Question

The Bohr model of the hydrogen atom consists of an electron orbiting a proton with a radius of $r_B = 0.529 \times 10^{-10}$ m. What is the electric potential energy of a hydrogen atom in this model **in units of eV**?



$$\varepsilon_0 = 8.85\times10^{-12}\ \mathrm{C^2/N\,m^2}$$
$$e = 1.60\times10^{-19}\ \mathrm{C}$$
$$1\ \mathrm{eV} = 1.60\times10^{-19}\ \mathrm{J}$$

$$U = \frac{1}{4\pi\varepsilon_0}\frac{qq'}{r} \quad J$$

A. –13.6 eV

B. –27.2 eV

C. –5.75 x 10^{11} eV

D. –9.21 x 10^{-8} eV

TopHat Question

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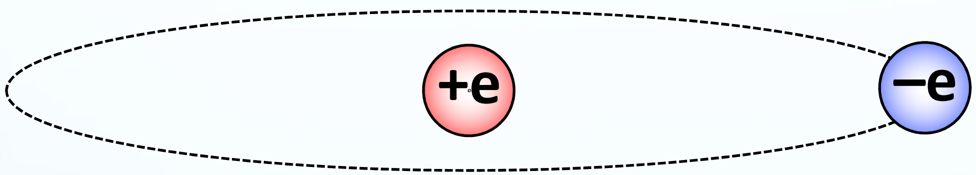


$$U_e = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r} = -\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_B} \Rightarrow U_e = -4.60\times10^{-18}\ \mathrm{J}$$

$$U_e = \left(-4.60\times10^{-18}\ \cancel{\mathrm{J}}\right)\left(\frac{1\ \mathrm{eV}}{1.60\times10^{-19}\ \cancel{\mathrm{J}}}\right) = -27.2\ \mathrm{eV}$$

Electric Potential

Here are some source charges and a point P.

If we place a charge q at point P, then q and the source charges interact with each other.

The interaction energy is the potential energy of q and the source charges,

$$U_{q+sources}$$

How does this interaction happen?

Electric Potential

Model: electric potential → $\underline{\underline{V}}$

The source charges create a **potential for interaction** everywhere, including at point P.

This potential for interaction is a **property of space**. Charge q does not need to be there.

We call this potential for interaction the **electric potential, V.** (Often just called "the potential")

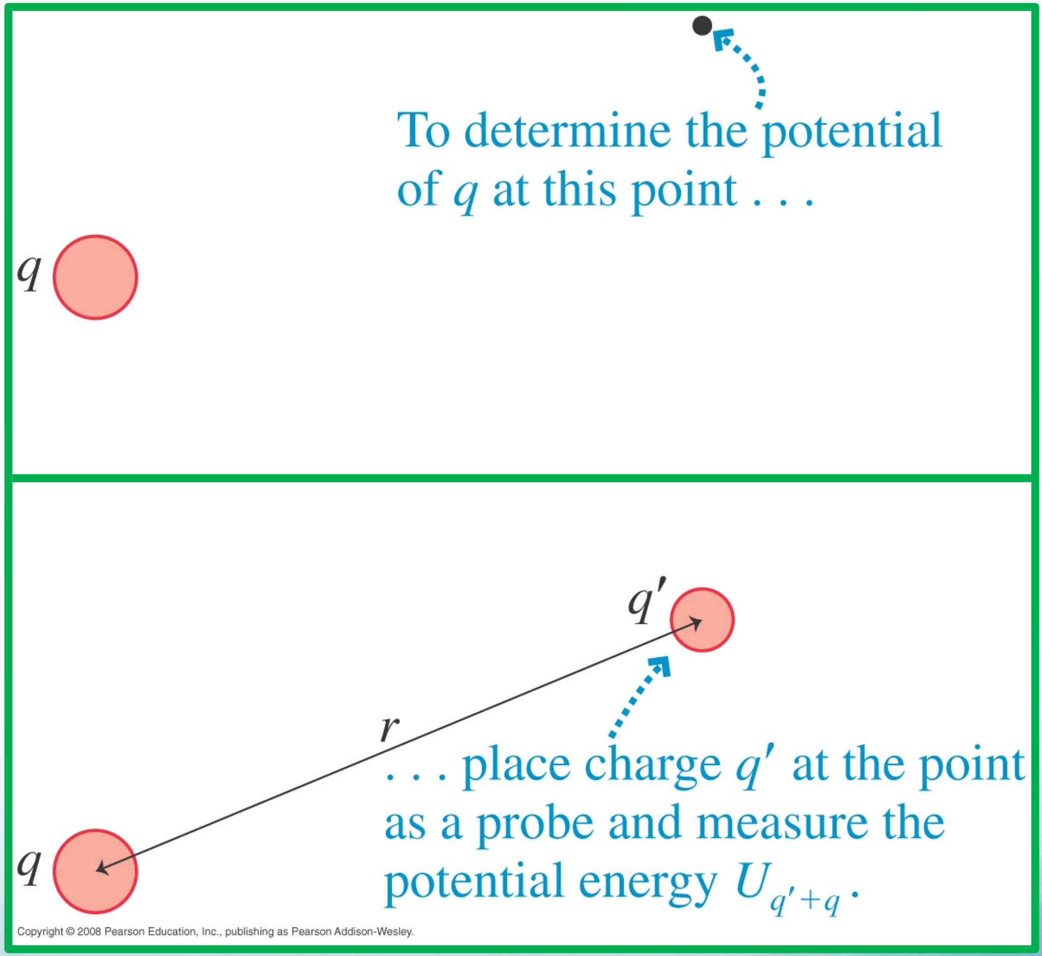
Electric Potential of a point charge

Potential energy of q and q'

$$U_{q'+q} = \frac{1}{4\pi\varepsilon_0}\frac{qq'}{r}$$

Then the potential of q is

$$V = \frac{U_{q'+q}}{q'} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$



Electric Potential

Electric potential V at a point P in the electric field of a charged object →

$V = \frac{U}{q_0}$ &

$$\boxed{V = \frac{-W_\infty}{q_0} = \frac{U}{q_0}}$$

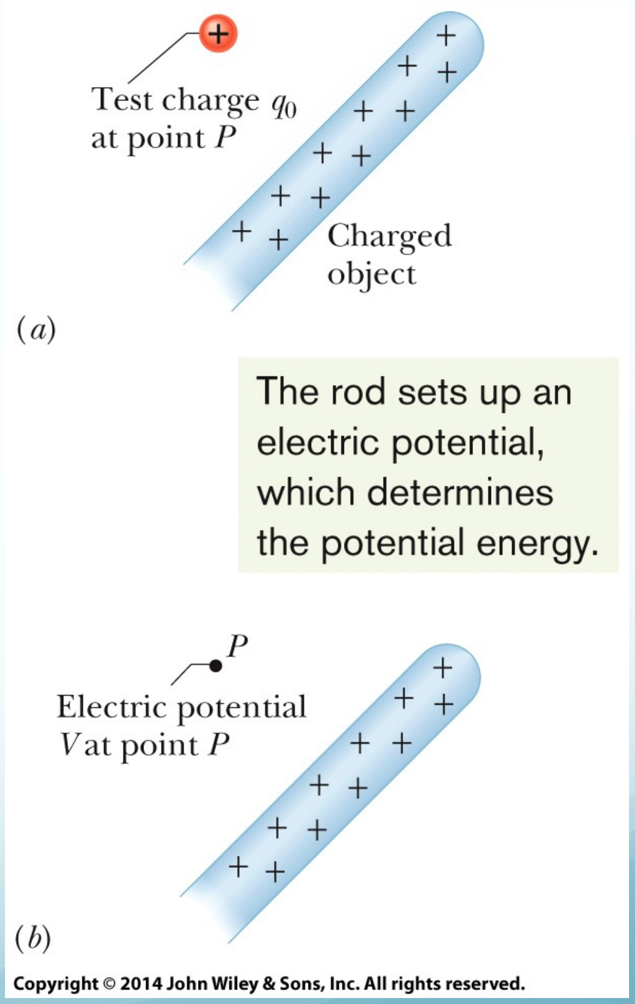
$W = -\Delta U$

W_∞ → work that would be done by the electric force on a positive test charge q_0 were it brought from an infinite distance to P, and U is the electric potential energy that would then be stored in the test charge–object system.

The electric potential energy U of the particle–object system is →

$$\boxed{U = qV.}$$

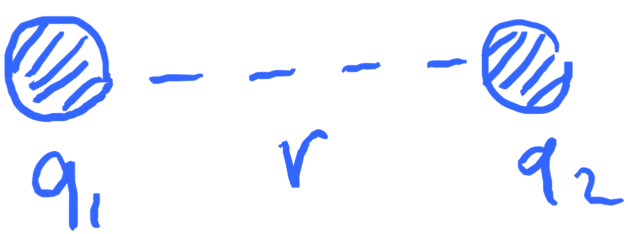
$F = qE$



$$F = k\frac{q_1 q_2}{r^2} \quad \text{(vector)}$$

$$E = k\frac{q_1}{r^2} = \frac{F}{q}$$

$$U = k\frac{q_1 q_2}{r}$$



$$V = k\frac{q_1}{r} = \frac{U}{q_2}$$

Electric Potential

Definition of V: Place charge q at point P and measure its potential energy. Then

$$V = \frac{U}{q} \rightarrow \frac{J}{C} \rightarrow V \rightarrow J/C$$

$$\boxed{V \equiv \frac{U_{q+sources}}{q}}$$

Unit: $1\ \text{volt} = 1\ \text{V} = 1\ \frac{\text{J}}{\text{C}}$

Electric Potential

Or, if we know the potential, V, at point P, then if we place a charge, q, at point P, the potential energy of q and the source charges is

$$\vec{F} = q\vec{E}$$

$$\boxed{U_{q+sources} = qV}$$

Change in Electric Potential.

If the particle moves through a potential difference *ΔV*, the change in the electric potential energy is

$U = qV \rightarrow$

$$\Delta U = q\,\Delta V = q(V_f - V_i). \leftarrow ①$$

Work by the Field.

The work W done by the electric force as the particle moves from *i* to *f:*

$$① \rightarrow W = -\Delta U = -q\,\Delta V = -q(V_f - V_i). \quad ②$$

Conservation of Energy.

If a particle moves through a change *ΔV* in electric potential without an applied force acting on it, applying the conservation of mechanical energy gives the change in kinetic energy as

change in kinetic energy

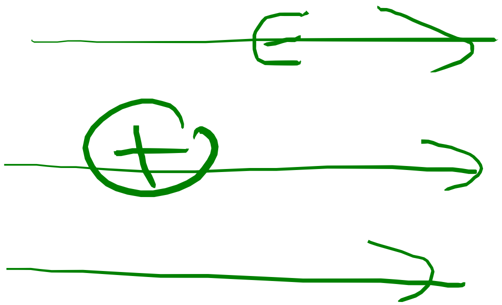
$$\Delta K = -q\,\Delta V = -q(V_f - V_i).$$

$$\Delta K = -\Delta U$$

Work by an Applied Force.

If some force in addition to the electric force acts on the particle, we account for that work

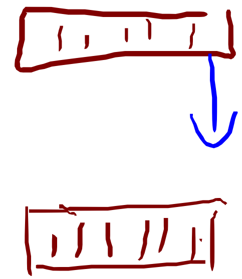
$$\Delta K = -\Delta U + W_{app} = -q\,\Delta V + W_{app}.$$



U_m

gravitational

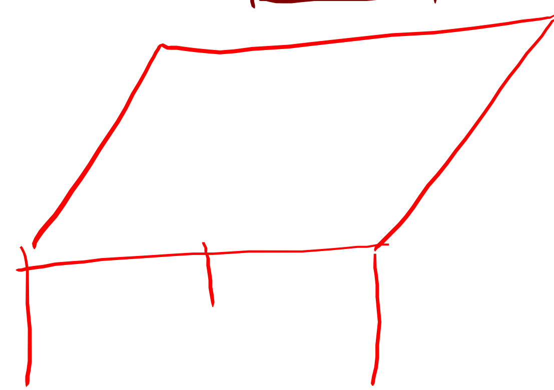
$\Rightarrow K_m$



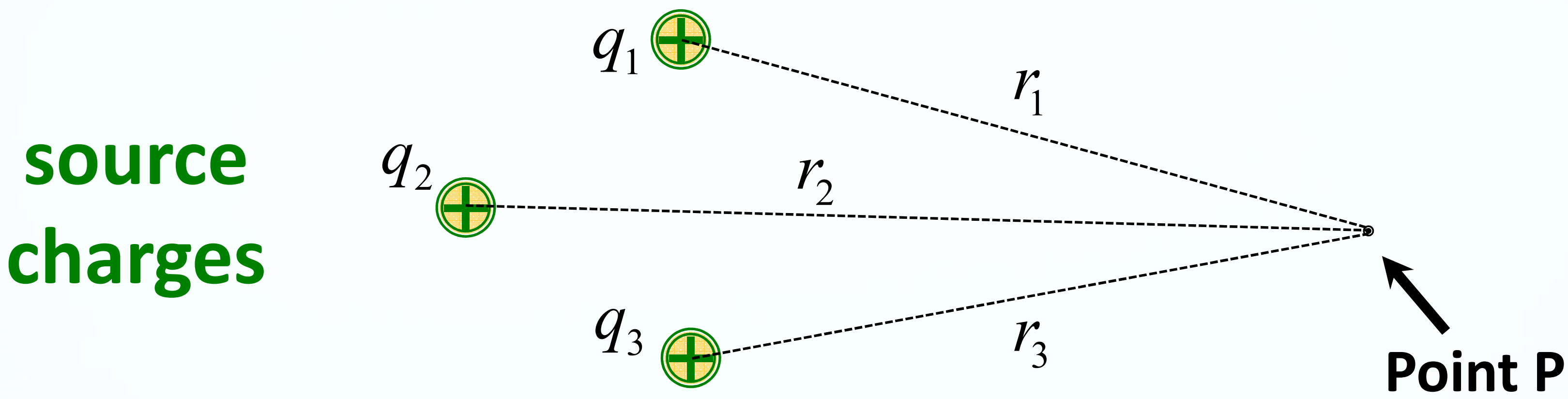
$U_e \rightarrow K_e$

potential energy

kinetic energy



Advantage of Electric Potential

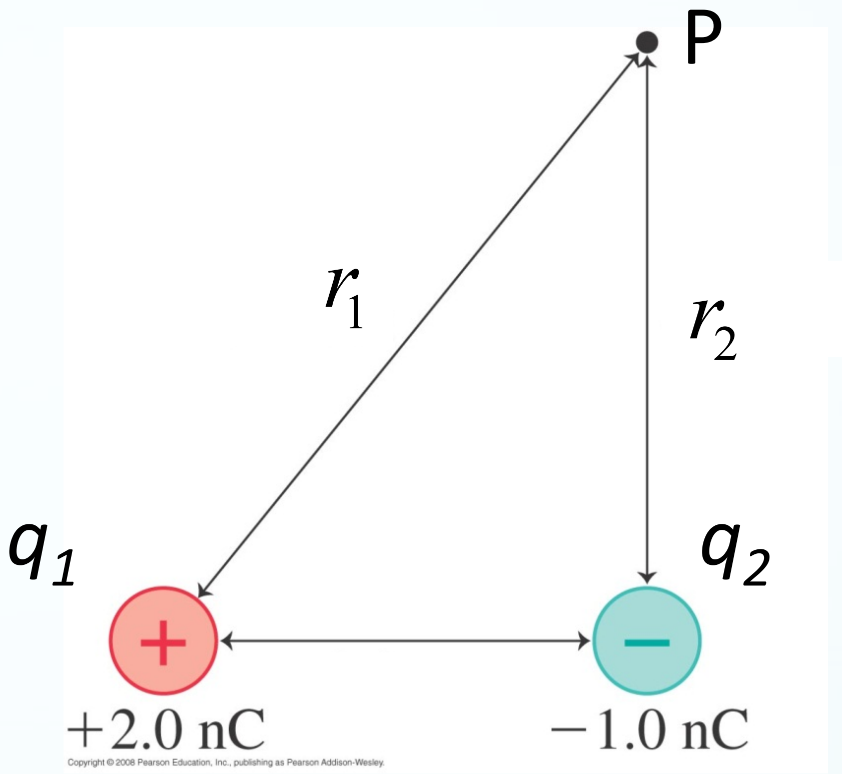


V is a SCALAR! There is no direction associated with it. This makes it much easier to calculate!

$$V_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1} \qquad V_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2} \qquad V_3 = \frac{1}{4\pi\varepsilon_0}\frac{q_3}{r_3}$$

$$\boxed{V = V_1 + V_2 + V_3}$$

Finding V at point P.



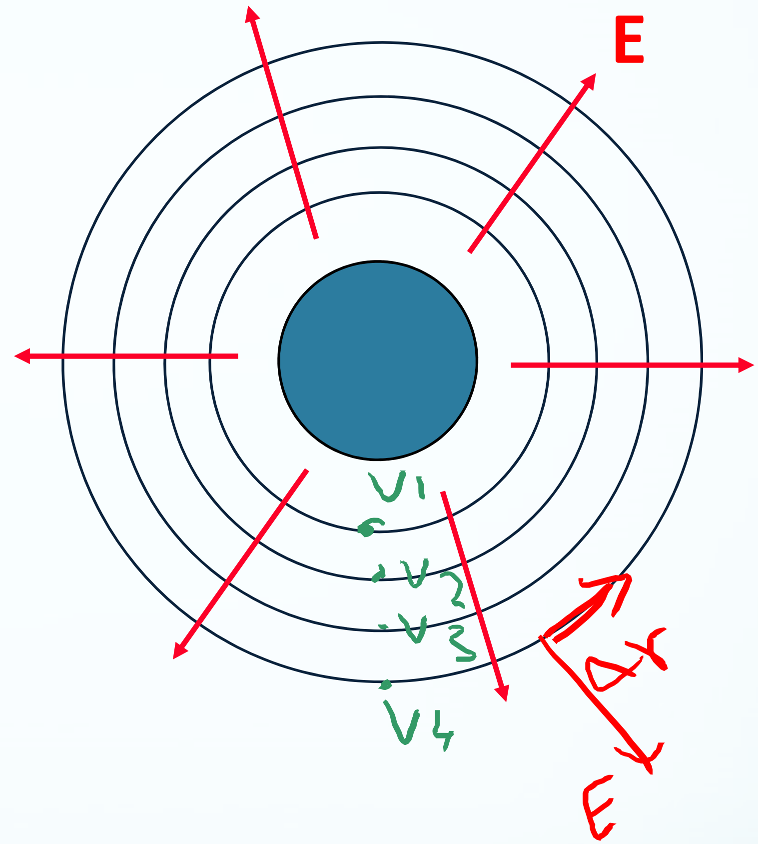
Potential is a scalar

There are no components

Just add the potentials

V at P = (V_1 at P due to q_1) + (V_2 at P due to q_2).

Equipotentials



Ex: For uniform spherical charge

$\longrightarrow$ $V(r) = k\,Q/r$

For each r, V(r) is constant→
V(r) is constant over any sphere concentric with the charged sphere

→ We have equipotential lines (or surfaces, actually, in 3-D)

Note that if move along equipotential surface→

→ by definition $\Delta V = -\mathbf{E}\cdot\Delta\mathbf{r} = 0$ --> **E** is ⊥ equipotential surface

$V =$ constan $\rightarrow \Delta V = 0 \rightarrow \Delta V = -E\cdot\Delta r$

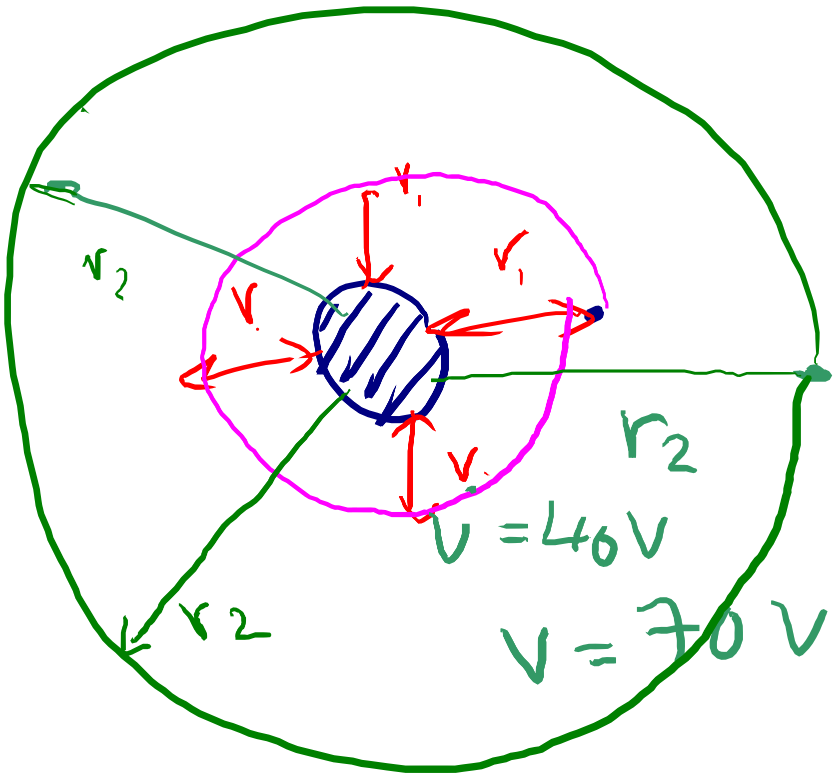
$U = -F \cdot \Delta r$

$V = -E \cdot \Delta r$

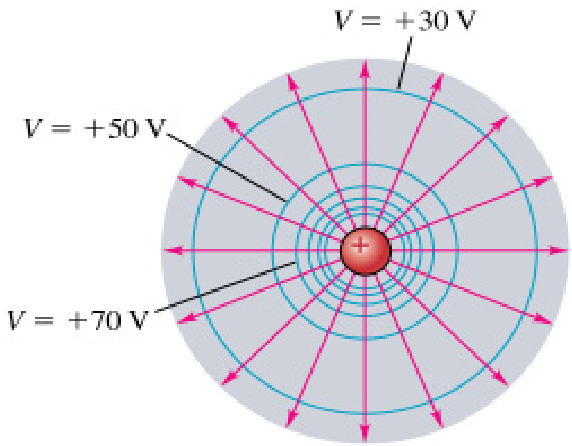
$U \longleftrightarrow F$

$V \longleftrightarrow E$

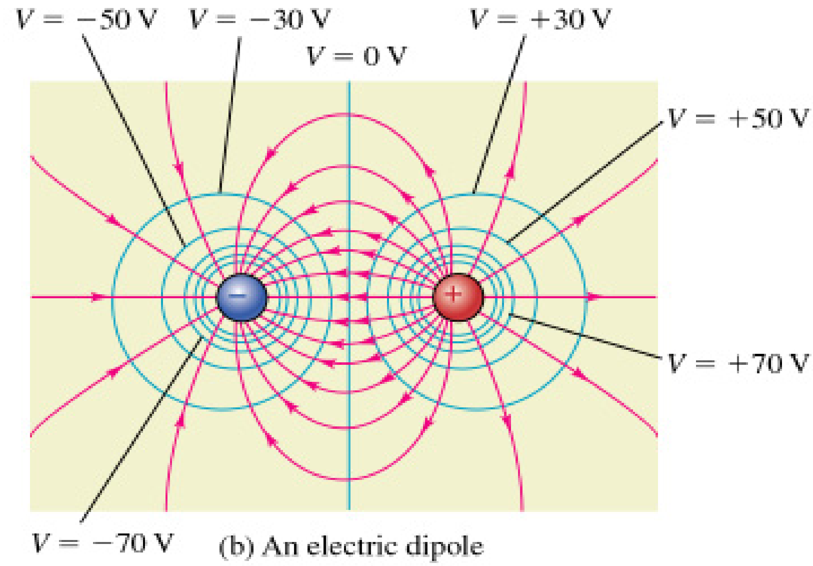
$$V = k\frac{q}{r}$$



Equipotential Surfaces



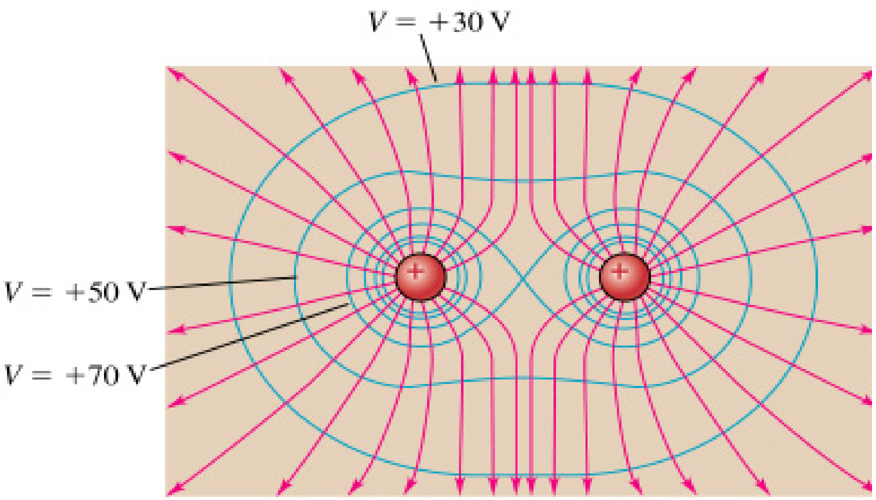
(a) A single positive charge



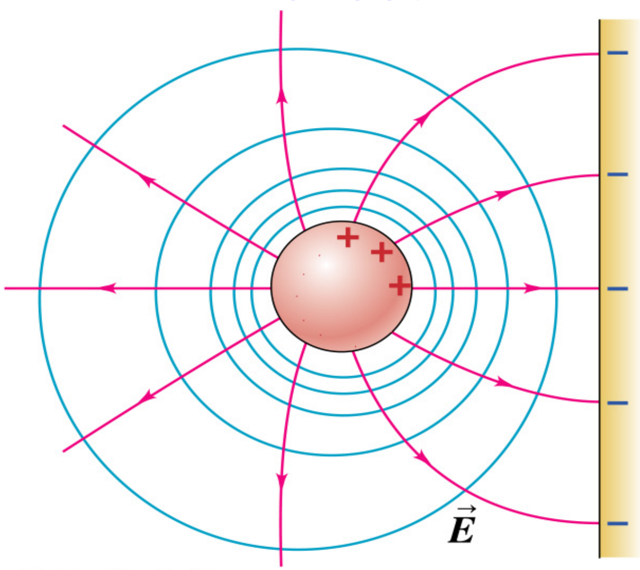
(b) An electric dipole

Note – E is always ⊥ V !!

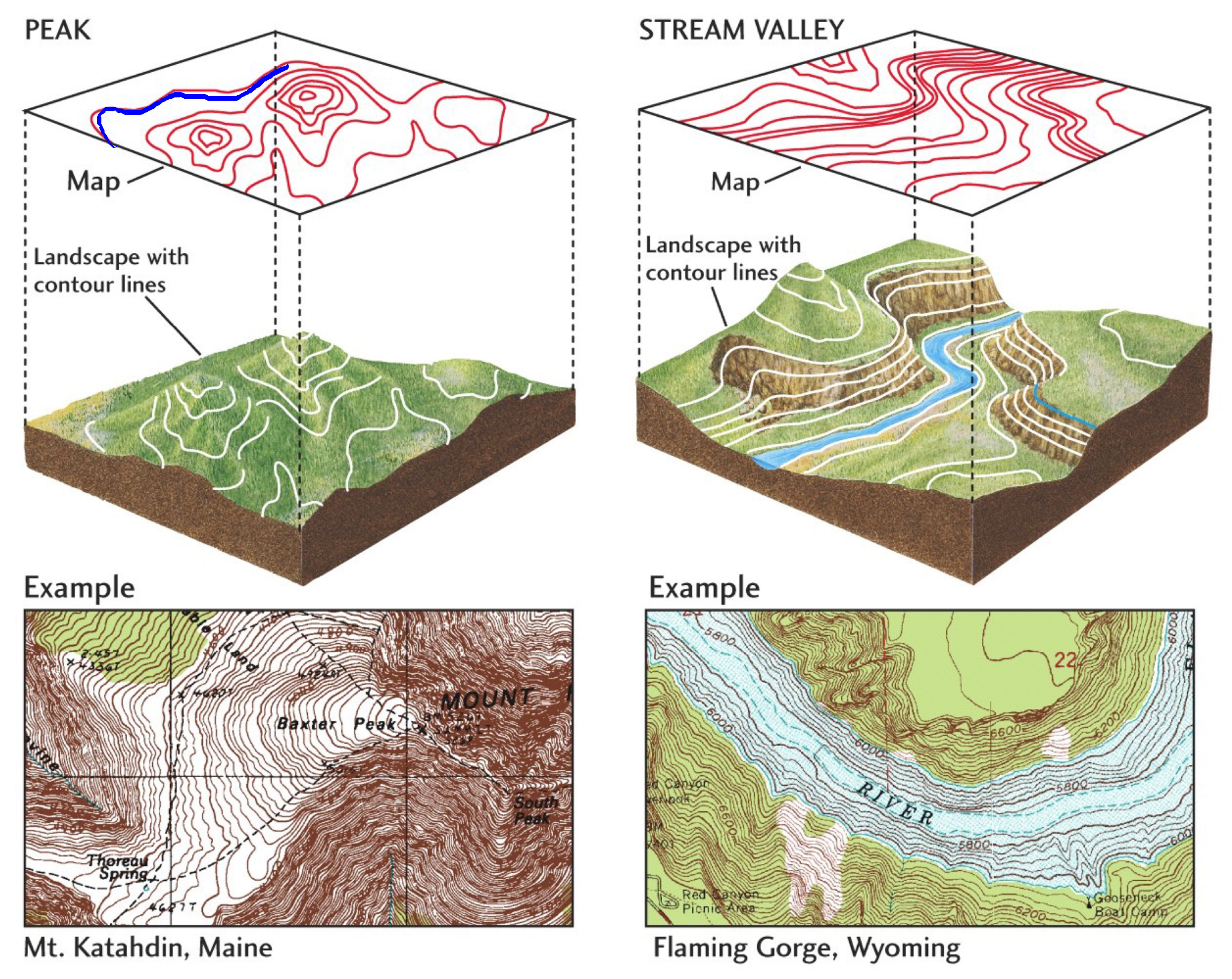
Conducting sphere
+ sheet



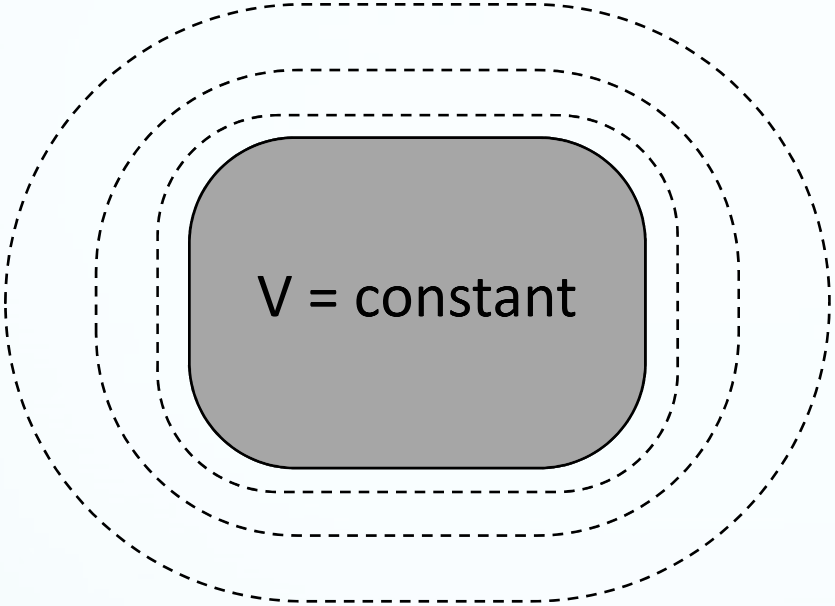
(c) Two equal positive charges



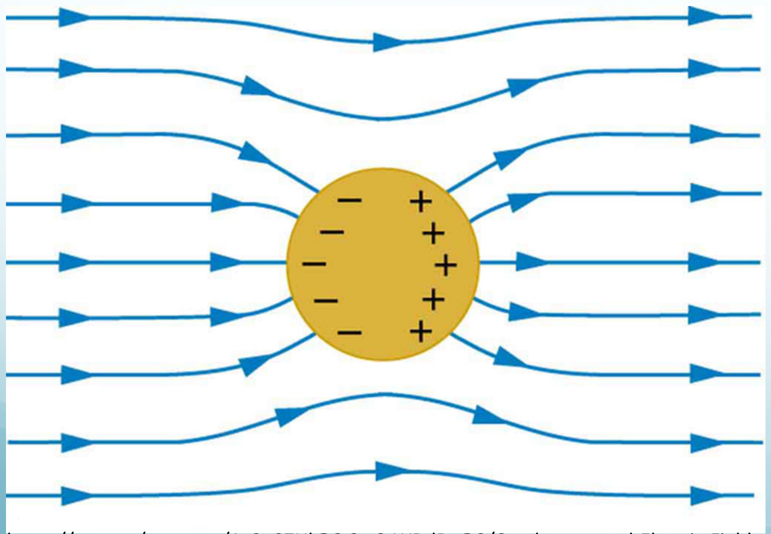
Where have you seen equipotentials before?



Conductors and E-fields



The surface of a conductor is an equipotential. If there was a potential difference across the surface of a conductor, the freely moving charges would move around until the potential is constant.



https://cnx.org/contents/Ax2o07Ul@8.8:a8tWDdDr@3/Conductors-and-Electric-Fields

This means that electric field lines ALWAYS must meet a conducting surface at right angles (any tangential component would imply a tangential force on the free charges).

This section we talked about:

Chapter 24.1

See you on Friday