Last time:

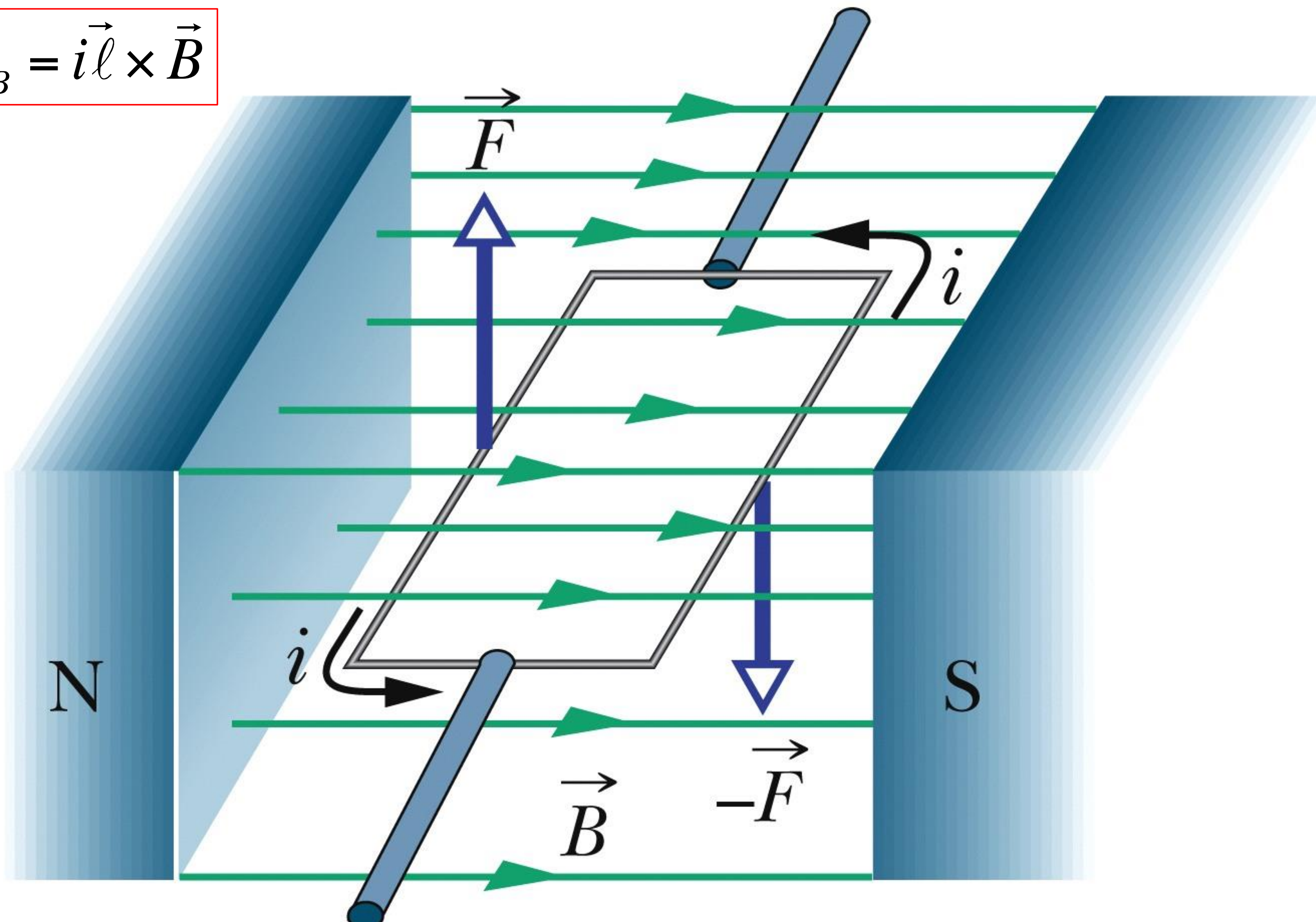
- Conductors moving through B-fields: Hall(ish) Effect
- Magnetic force on current carrying wires

Today:

- Torque on a current loop
- Biot-Savart Law (like Coulomb’s Law for magnetism)
- B-field of a line of current
- Magnetic force between parallel current-carrying wires

Torque on a current loop

$$\vec{F}_B = i\vec{\ell} \times \vec{B}$$



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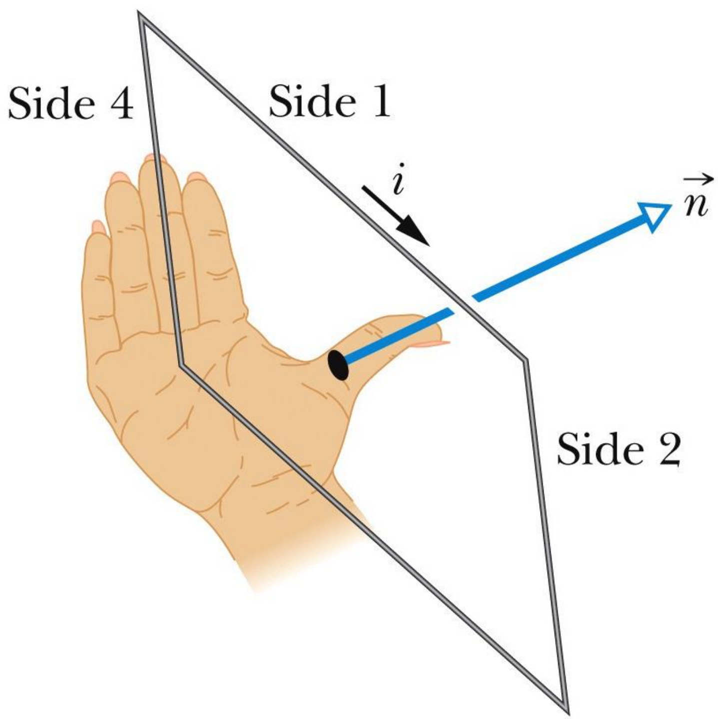
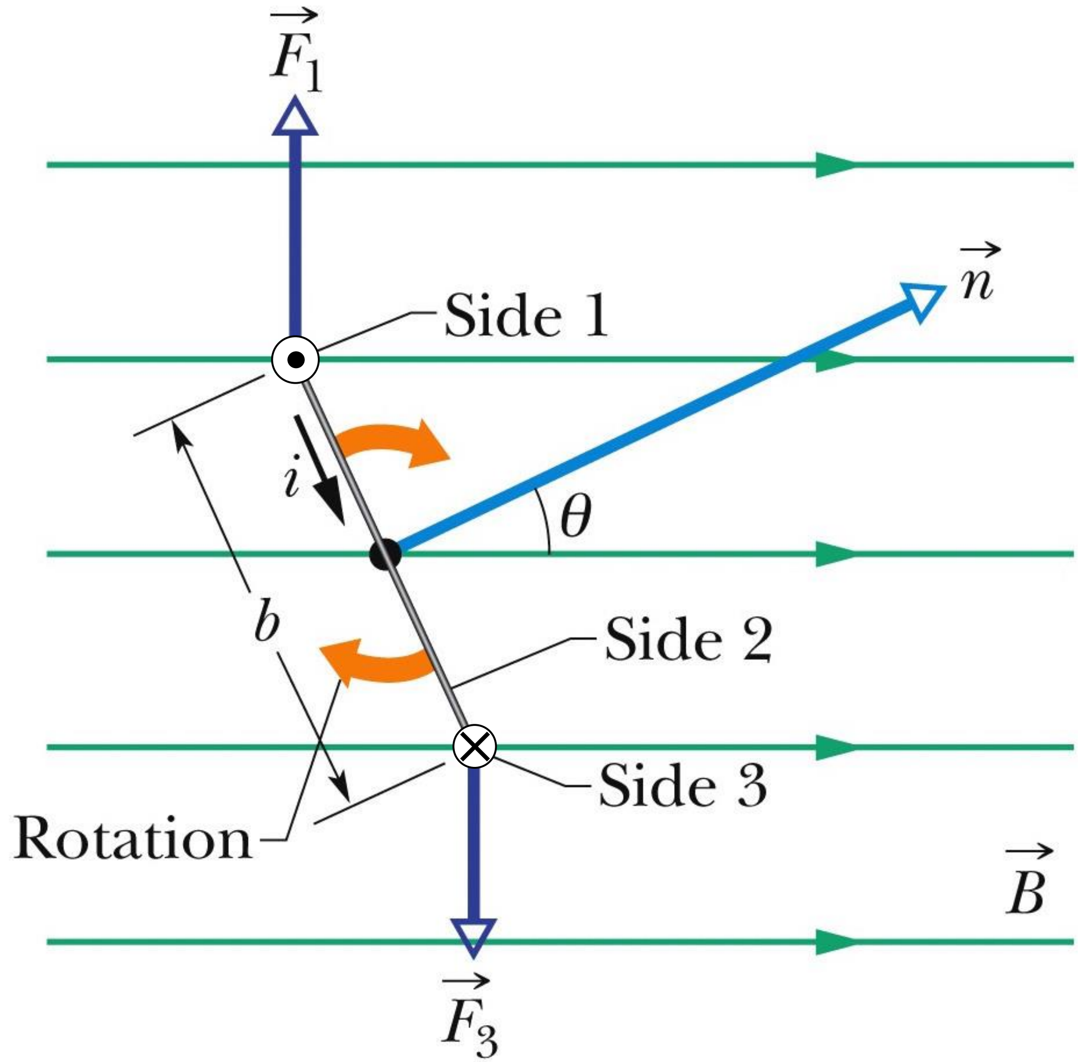
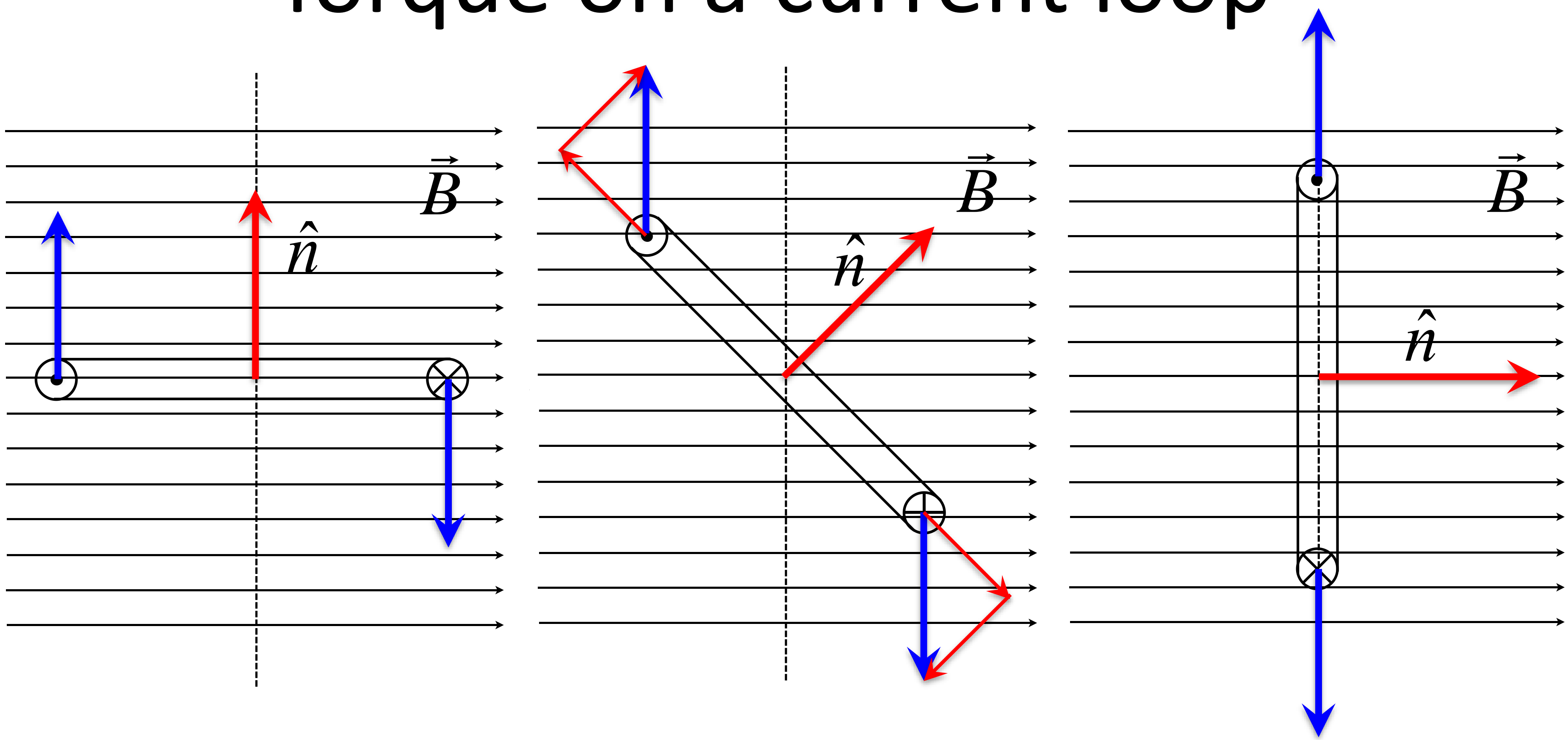
Torque on a current loop

Pick the normal vector to the loop area by RHR: curl your fingers in the direction of i, thumb points in direction of n





Torque on a current loop



The normal vector is at right angles to the B-field: all magnetic force causes rotation of the loop

The normal vector is at some angle to the B-field: some of the magnetic force causes rotation of the loop

The normal vector is parallel to the B-field: none of the magnetic force causes rotation of the loop

Conclusion: components of magnetic force (anti)parallel to normal vector that cause torque

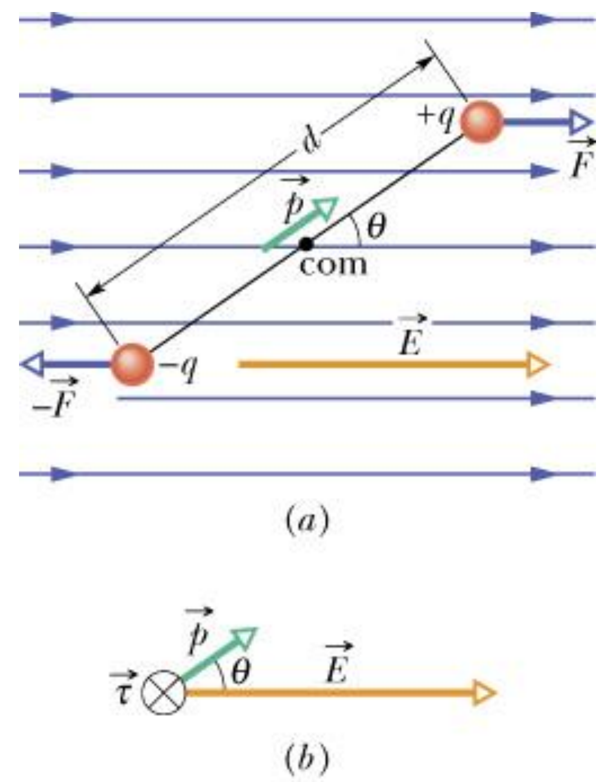
Electric dipole in a uniform electric field

$$\vec{F}_+ + \vec{F}_- = +q\vec{E} - q\vec{E} = 0$$

$$\tau = qE\frac{d}{2}\sin\theta + (-qE)\left(-\frac{d}{2}\right)\sin\theta$$

$$= qdE\sin\theta = pE\sin\theta$$

$$\vec{\tau}_E = \vec{p}\times\vec{E}$$



$$U = -\vec{p}\cdot\vec{E} = -pE\cos\theta$$

Force and torque on a current loop

- This basis of electric motors

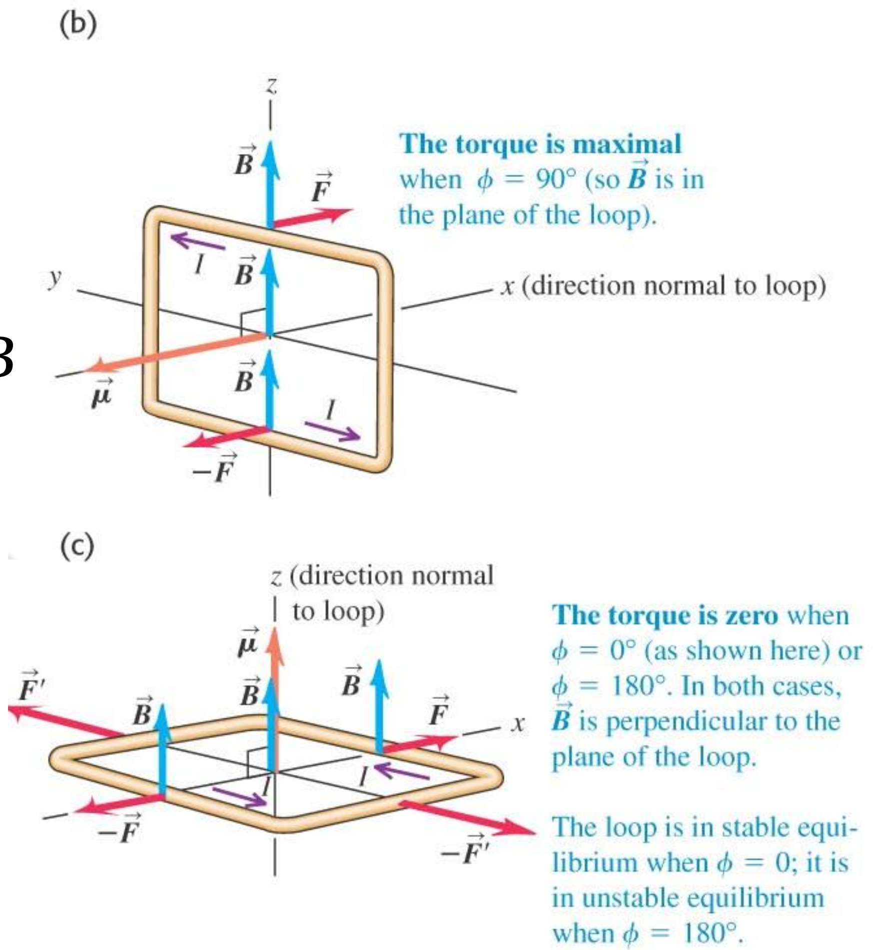
Maximum torque:

$$\tau = 2F(b/2) = IBab = \left(Iab\right)B = \mu B$$

Magnetic Dipole Moment

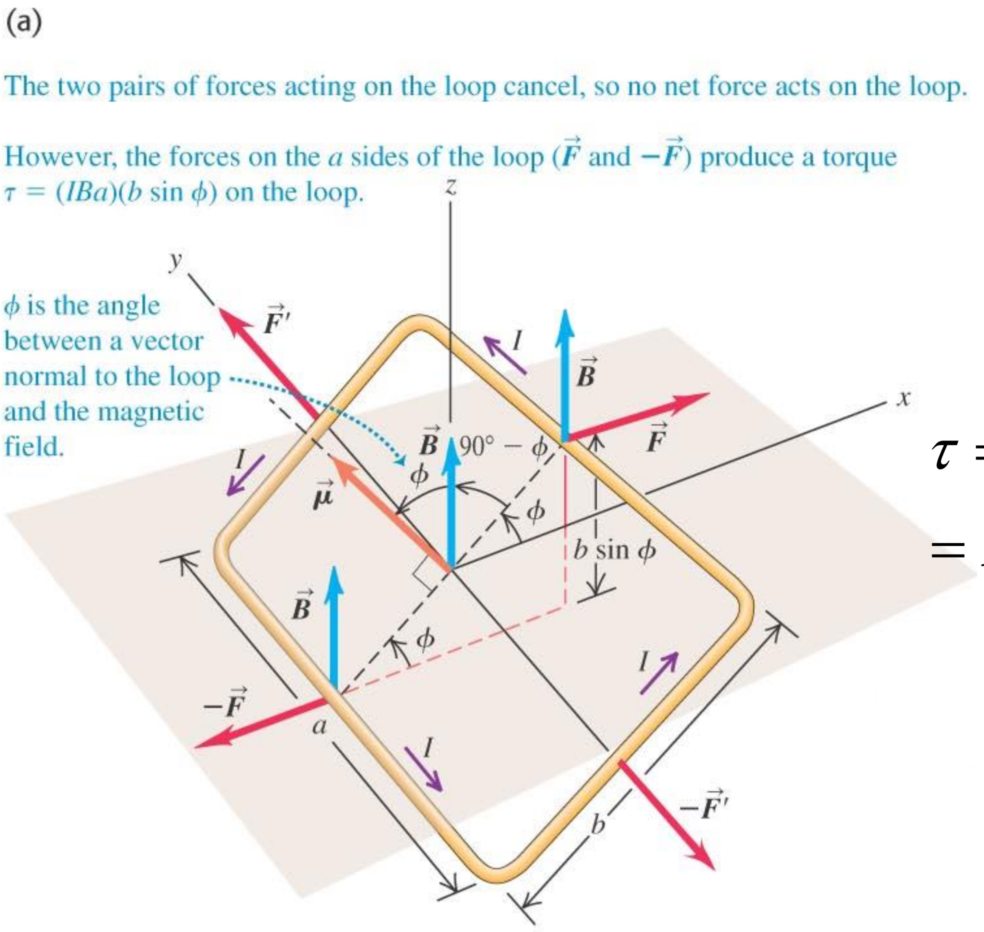
Minimum torque:

$$\tau_{\min} = 0$$



Force and torque on a current loop

- This basis of electric motors

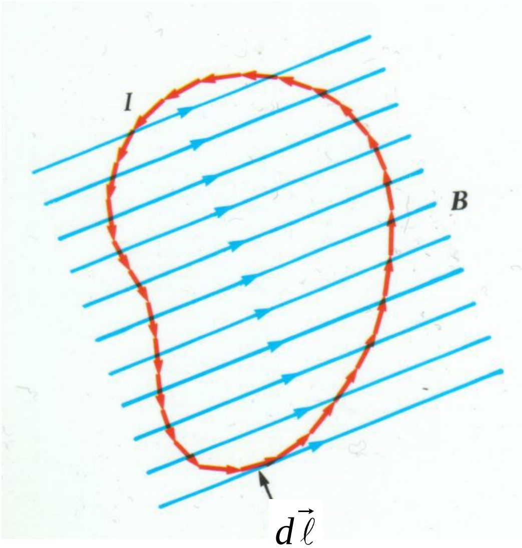


$$\tau = 2F(b/2)\sin\phi$$

$$= IBab\sin\phi = \left(Iab\right)B\sin\phi = \mu B\sin\phi$$

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

A current loop in magnetism is equivalent to an electric dipole. There is therefore a magnetic moment associated with a current loop as there is an electric moment associated with an electric dipole.



Magnetic	Electric
$\vec{F}_B = 0$	$\vec{F}_E = 0$
$\vec{\tau}_B = \vec{\mu} \times \vec{B}$	$\vec{\tau}_E = \vec{p} \times \vec{E}$
$\vec{\mu} = I\vec{A}$	$\vec{p} = q\vec{d}$
$U_B = -\vec{\mu} \cdot \vec{B} = -\mu B \cos\theta$	$U = -\vec{p} \cdot \vec{E} = -pE\cos\theta$

The Biot-Savart Law

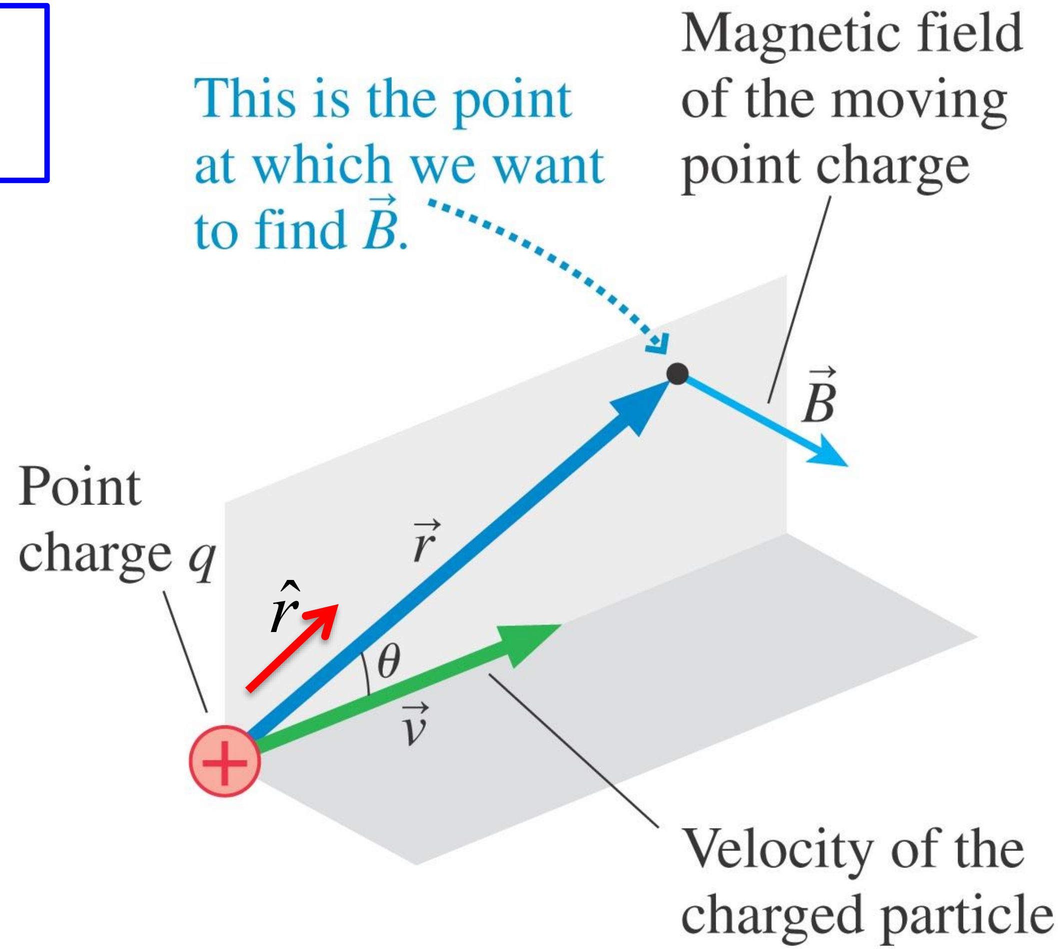
(Bee-oh Sah-var)

Magnetic fields are caused by **moving charges**.*

$$\vec{B}_{\text{point charge}} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2}$$

Or, using the definition $\hat{r} = \frac{\vec{r}}{|\vec{r}|}$

$$\vec{B}_{\text{point charge}} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\vec{r}}{r^3}$$



*One exception is due to quantum mechanics: charged particles with "spin" produce B fields

Constants of nature

"Permittivity of free space"

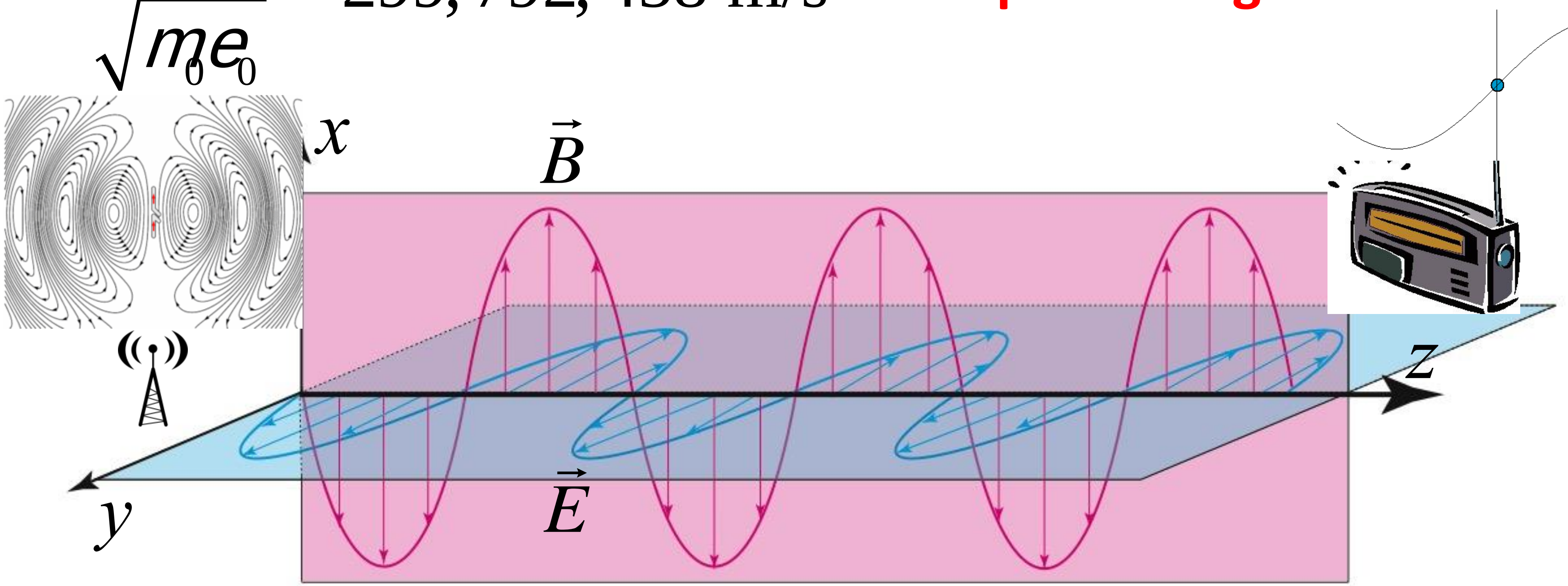
$$\varepsilon_0 = 8.85418781719 \times 10^{-12} \frac{C^2}{N \cdot m^2}$$

"Permeability of free space"

$$\mu_0 = 4\pi \times 10^{-7} \frac{N \cdot s^2}{C^2}$$

$$\frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 299,792,458 \text{ m/s}$$

Speed of light!



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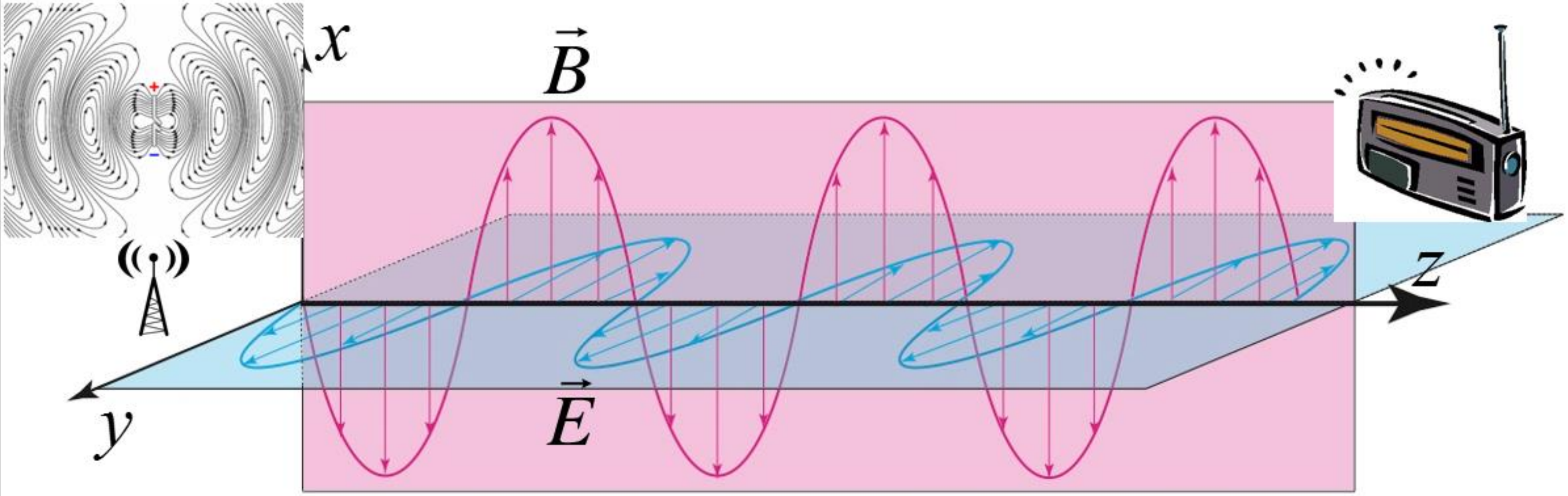
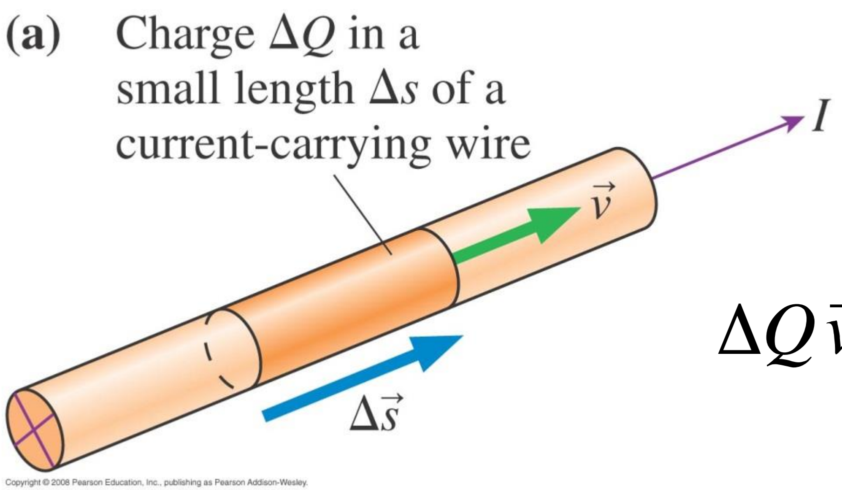


What if we have a whole bunch of moving charges (an electric current)?

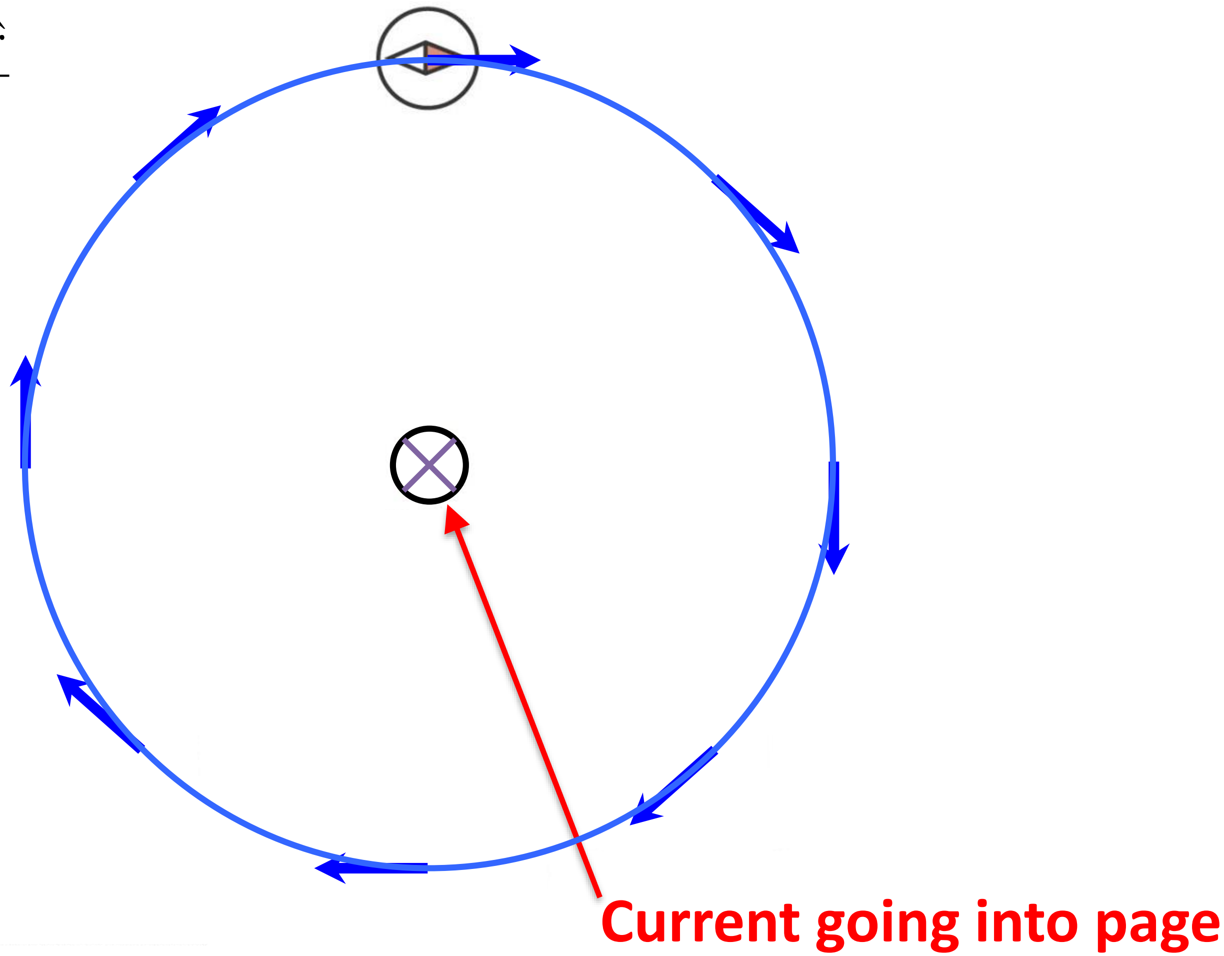


$$\Delta Q\,\vec{v} = \Delta Q\,\frac{\Delta\vec{s}}{\Delta t} = \frac{\Delta Q}{\Delta t}\Delta\vec{s} = I\Delta\vec{s}$$

Then becomes

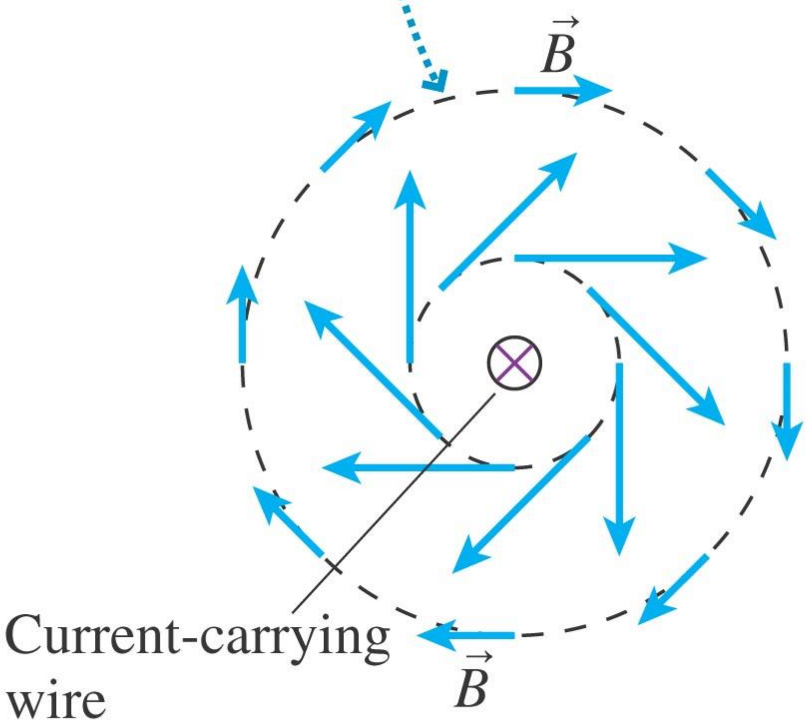
$$\vec{B}_{\text{point charge}} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2} \qquad \vec{B}_{\text{current segment}} = \frac{\mu_0}{4\pi}\frac{I\,\Delta\vec{s}\times\hat{r}}{r^2}$$

$$\vec{B}_{\text{current segment}} = \frac{\mu_0}{4\pi}\frac{I\,\Delta\vec{s}\times\hat{r}}{r^2}$$

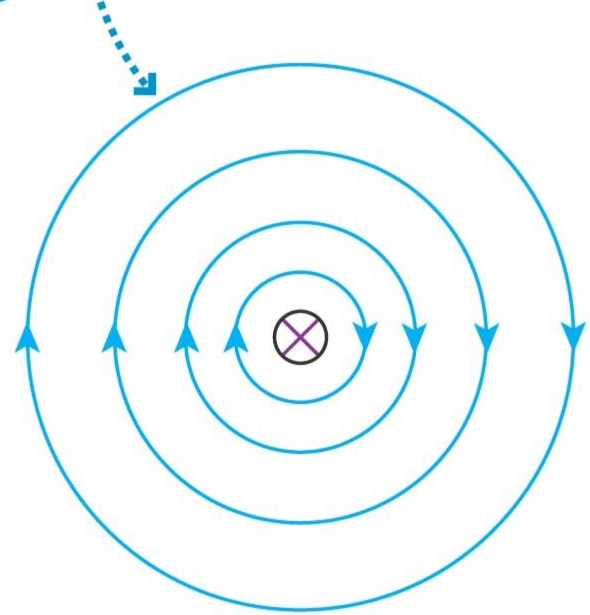


The magnetic field vector points in the direction of the north pole of the compass magnet.

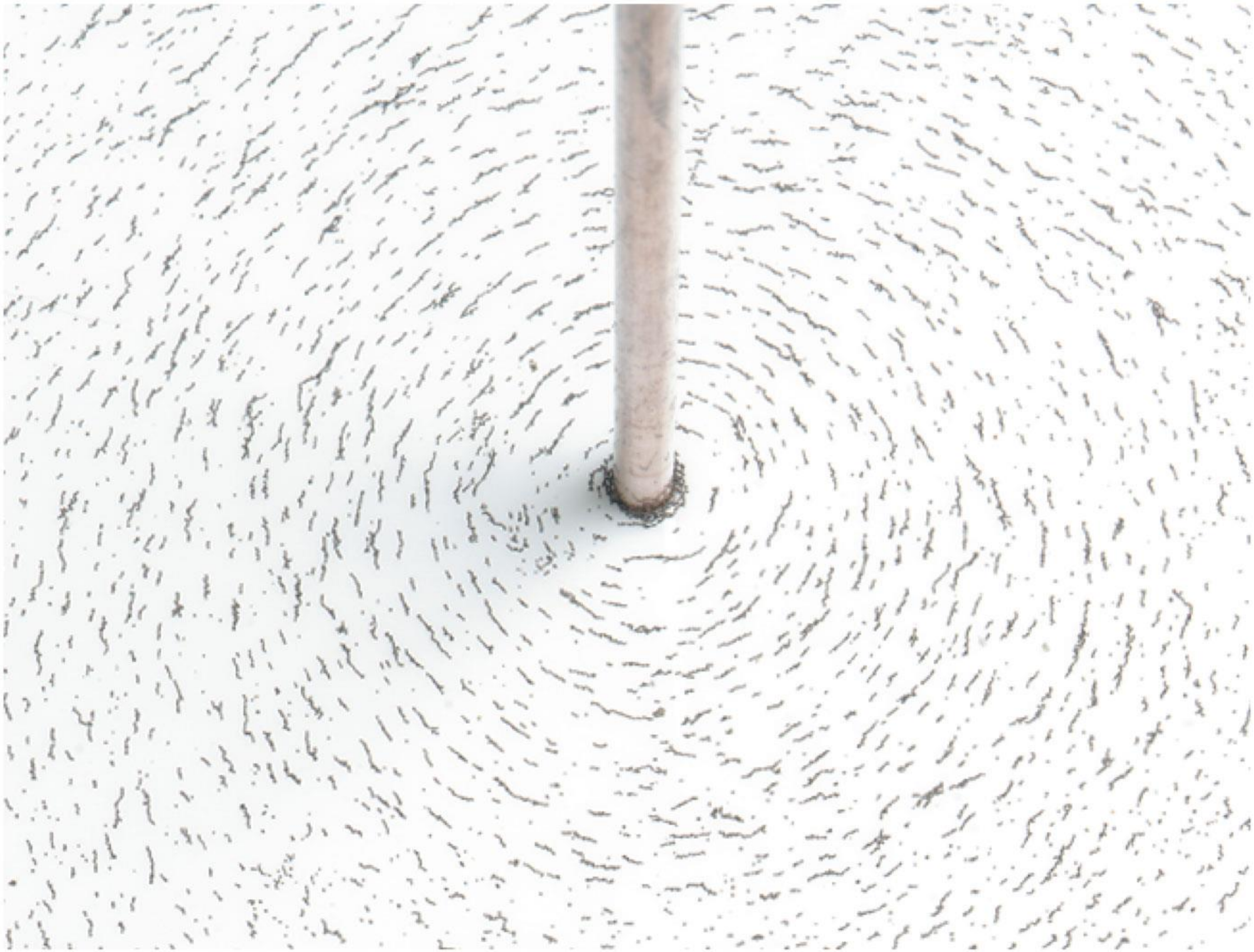
(a) The magnetic field vectors are tangent to circles around the wire, pointing in the direction given by the right-hand rule. The field is weaker farther from the wire.

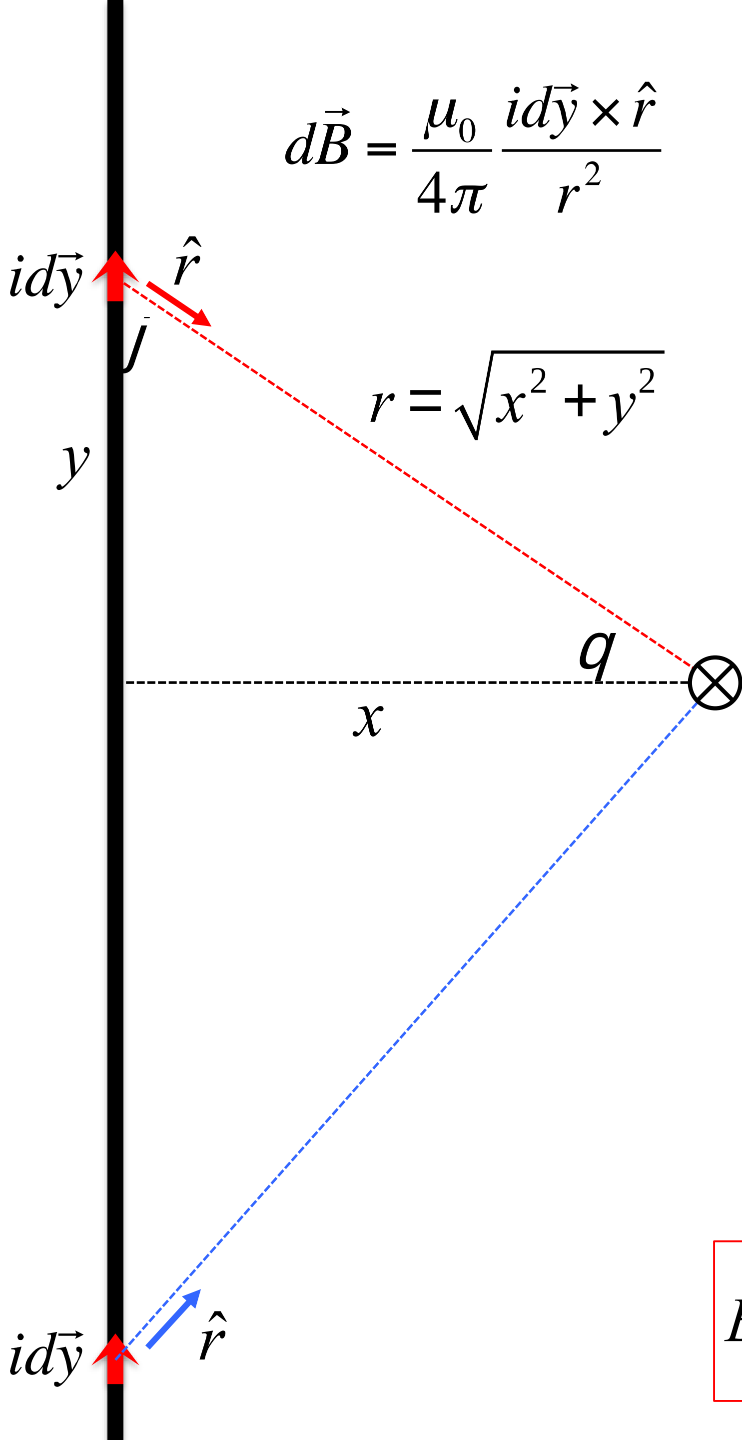


(b) Magnetic field lines are circles.



Magnetic Field of a Long, Straight Wire





$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{id\vec{y}\times\hat{r}}{r^2}$$

$$r = \sqrt{x^2+y^2}$$

$$id\vec{y}\times\hat{r} = idy\sin\varphi\left(-\hat{k}\right) = -\,idy\frac{x}{\sqrt{x^2+y^2}}\hat{k}$$

$$d\vec{B} = -\frac{\mu_0}{4\pi}\frac{ixdy}{\left(x^2+y^2\right)^{3/2}}\hat{k}$$

All contributions are in the same direction

$$B = \int_{-\infty}^{\infty}\frac{\mu_0}{4\pi}\frac{ixdy}{\left(x^2+y^2\right)^{3/2}}$$

Can just worry about the magnitude

$$= \frac{\mu_0 ix}{4\pi}\int_{-\infty}^{\infty}\frac{dy}{x^3\left(1+\left(y/x\right)^2\right)^{3/2}}$$

Pull out factors of x to make sub of y/x easier

$$= \frac{\mu_0 i}{4\pi x}\int_{-\pi/2}^{\pi/2}\frac{\sec^2\theta d\theta}{\left(1+\tan^2\theta\right)^{3/2}}$$

Let y/x = tanθ, so that dy/x = $\sec^2\theta$ dθ

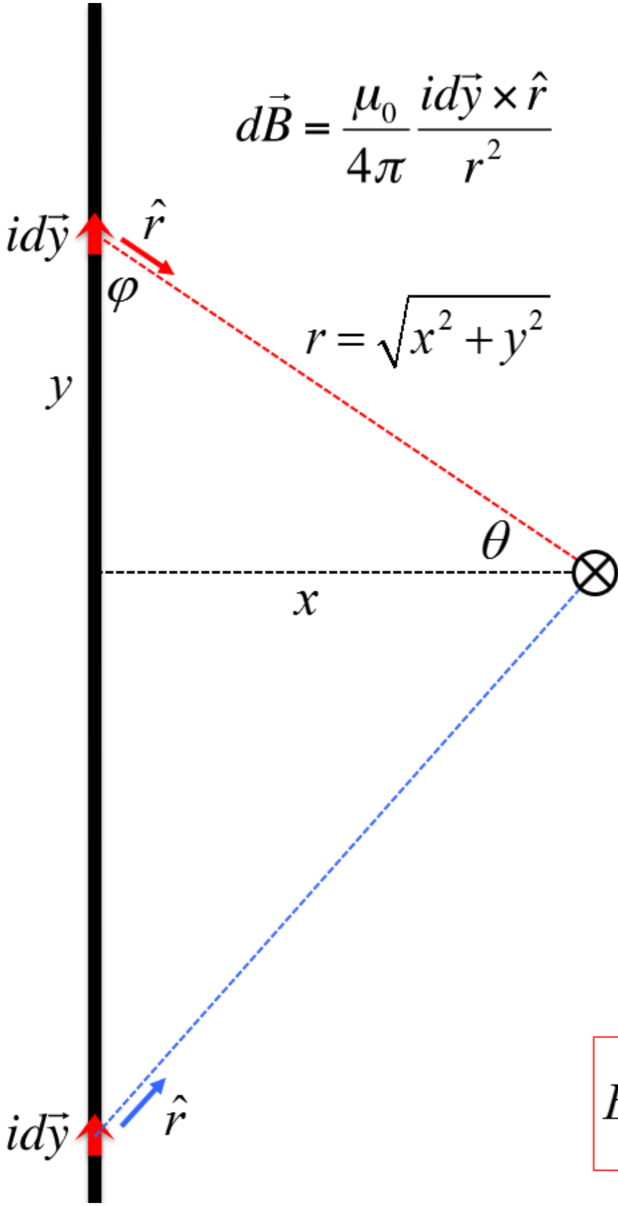
$$= \frac{\mu_0 i}{4\pi x}\int_{-\pi/2}^{\pi/2}\cos\theta\, d\theta$$

Integrand reduces to $\sec^2\theta/\sec^3\theta = \cos\theta$

$$\boxed{B_{wire} = \frac{\mu_0 i}{2\pi x}}$$

Magnetic field strength of a long straight wire. Direction from RHR

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{id\vec{y}\times\hat{r}}{r^2}$$



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Pull out factors of x to make sub of y/x easier

$$= \frac{\mu_0 i}{4\pi x}\int_{-\pi/2}^{\pi/2}\frac{\sec^2\theta d\theta}{\left(1+\tan^2\theta\right)^{3/2}}$$

Let $y/x = \tan\theta$, so that $dy/x = \sec^2\theta\, d\theta$

$$= \frac{\mu_0 i}{4\pi x}\int_{-\pi/2}^{\pi/2}\cos\theta\, d\theta$$

Integrand reduces to $\sec^2\theta/\sec^3\theta = \cos\theta$

$$\boxed{B_{wire} = \frac{\mu_0 i}{2\pi x}}$$

Magnetic field strength of a long straight wire. Direction from RHR

Last time:

- Torque on a current loop
- Biot-Savart Law (like Coulomb's Law for magnetism)
- B-field of a line of current

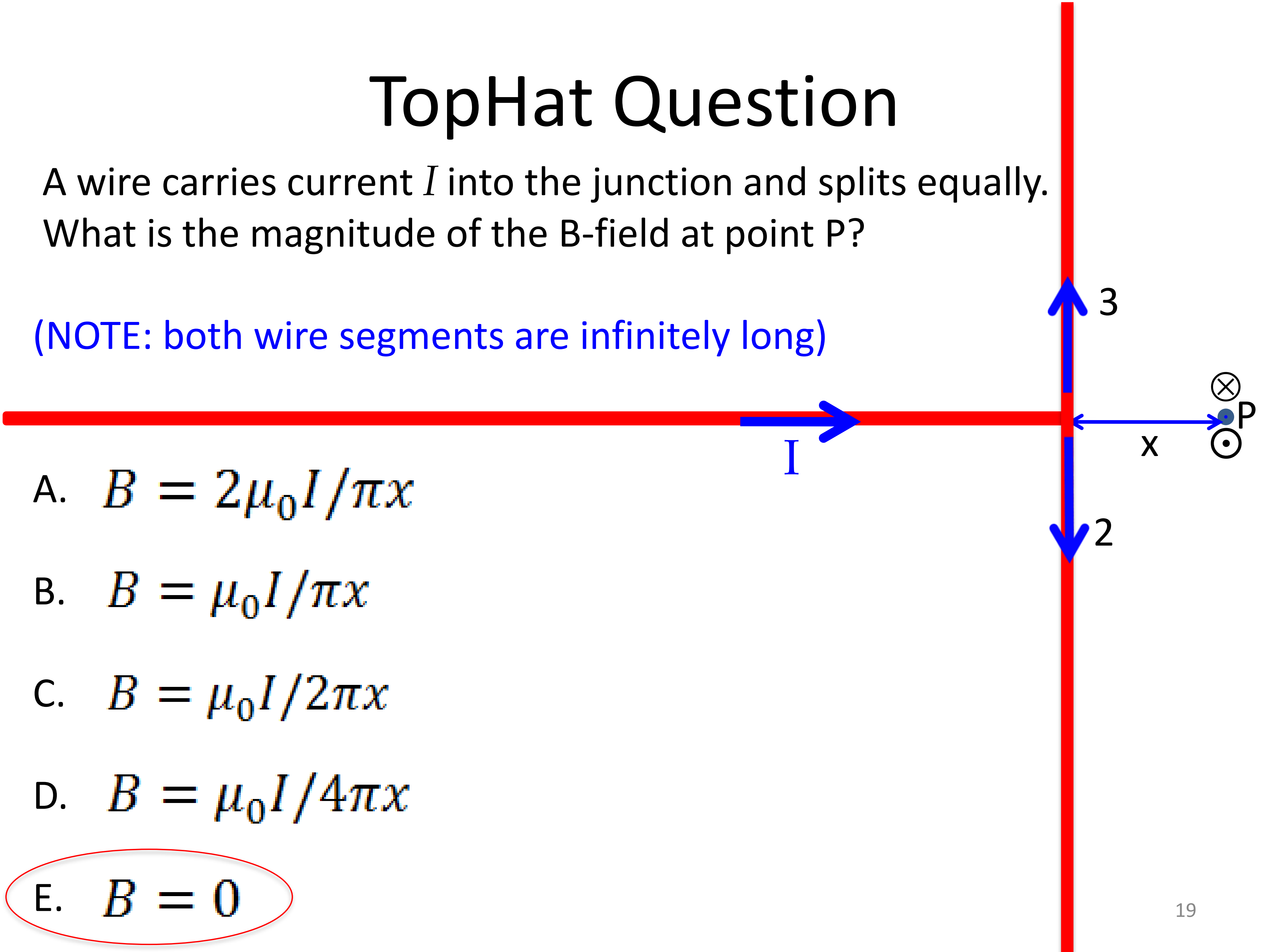
Today:

- Magnetic force between parallel current-carrying wires
- Applying the Biot-Savart Law:
  - Circular arc of current
  - Finite line of current

TopHat Question

A wire carries current I into the junction and splits equally. What is the magnitude of the B-field at point P?

(NOTE: both wire segments are infinitely long)



A. $B = 2\mu_0 I/\pi x$

B. $B = \mu_0 I/\pi x$

C. $B = \mu_0 I/2\pi x$

D. $B = \mu_0 I/4\pi x$

E. $B = 0$

TopHat Question

A long straight wire carries current I out of the page. An electron travels towards the wire from the right. What is the direction of the force on the electron?

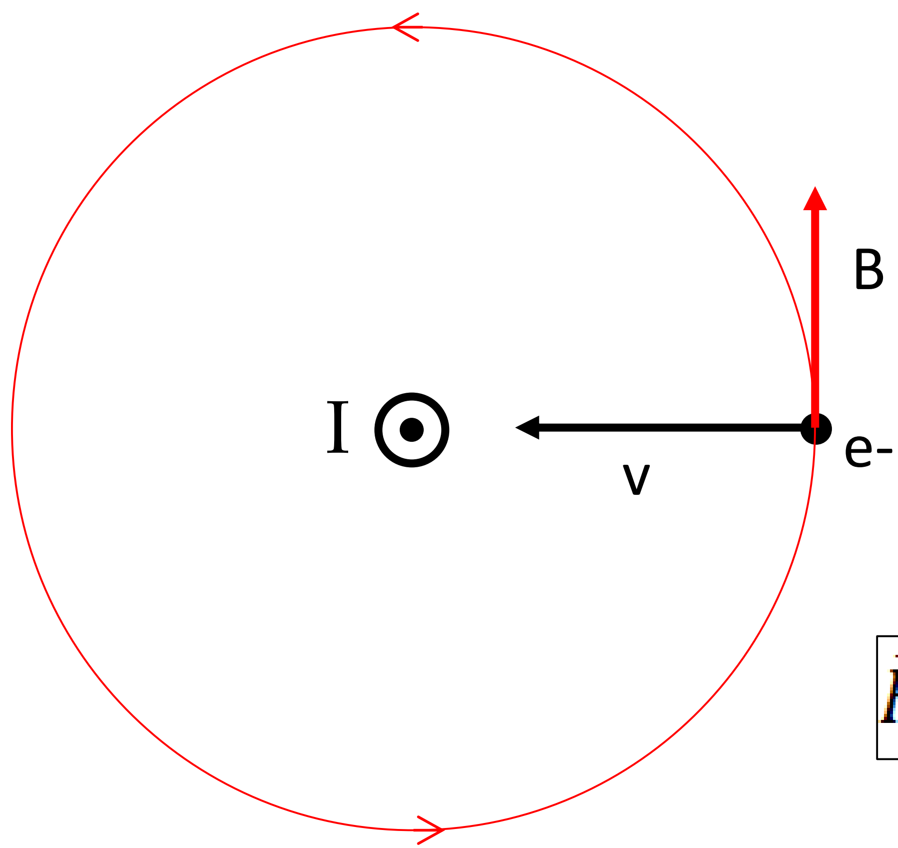
A. ⊗

B. ⊙

C. ←

D. →

E. ↓



$$\vec{F} = q\vec{v} \times \vec{B}$$

TopHat Question

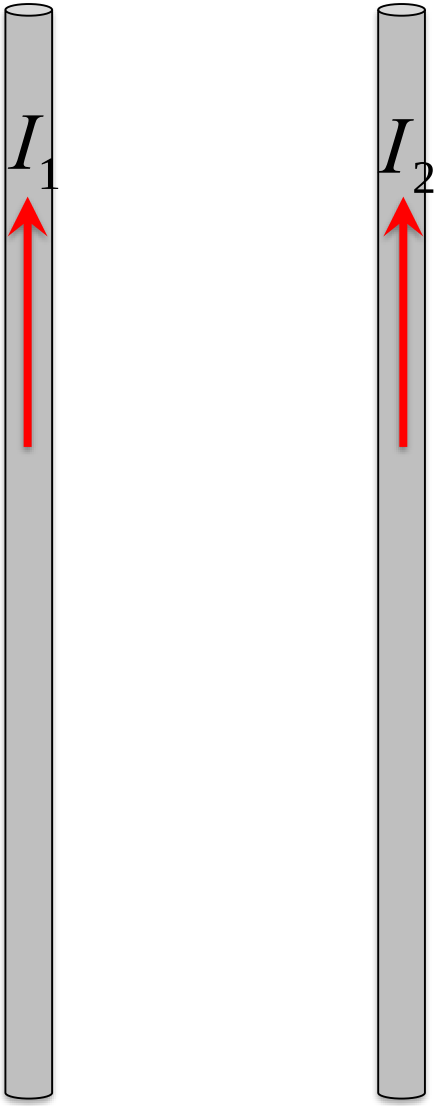
Two wires carry currents I_1 and I_2 as shown. What direction is the magnetic field produced by wire 2 at the location of wire 1?

A. Downward

B. Upward

C. Into the page

D. Out of the page



TopHat Question

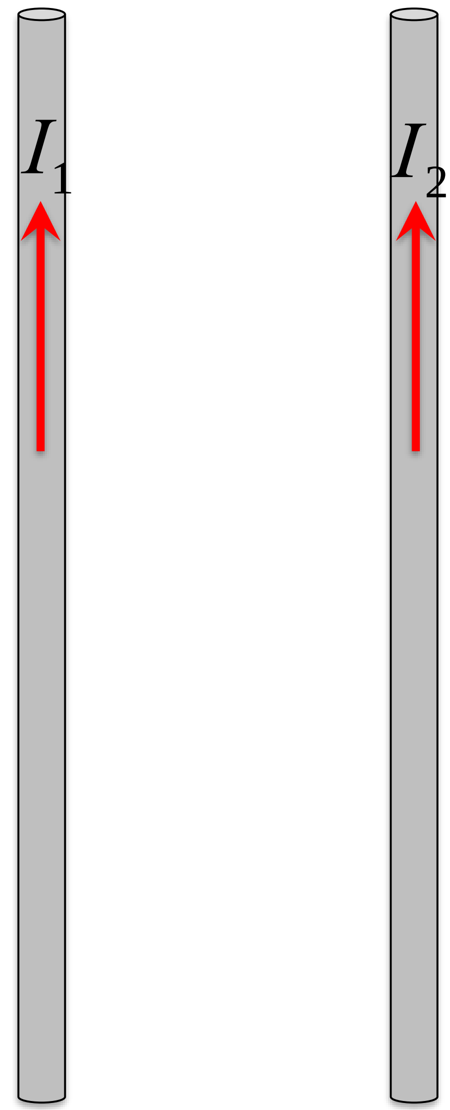
Two wires carry currents I_1 and I_2 as shown. What direction is the force of wire 2 on wire 1?

A. Left

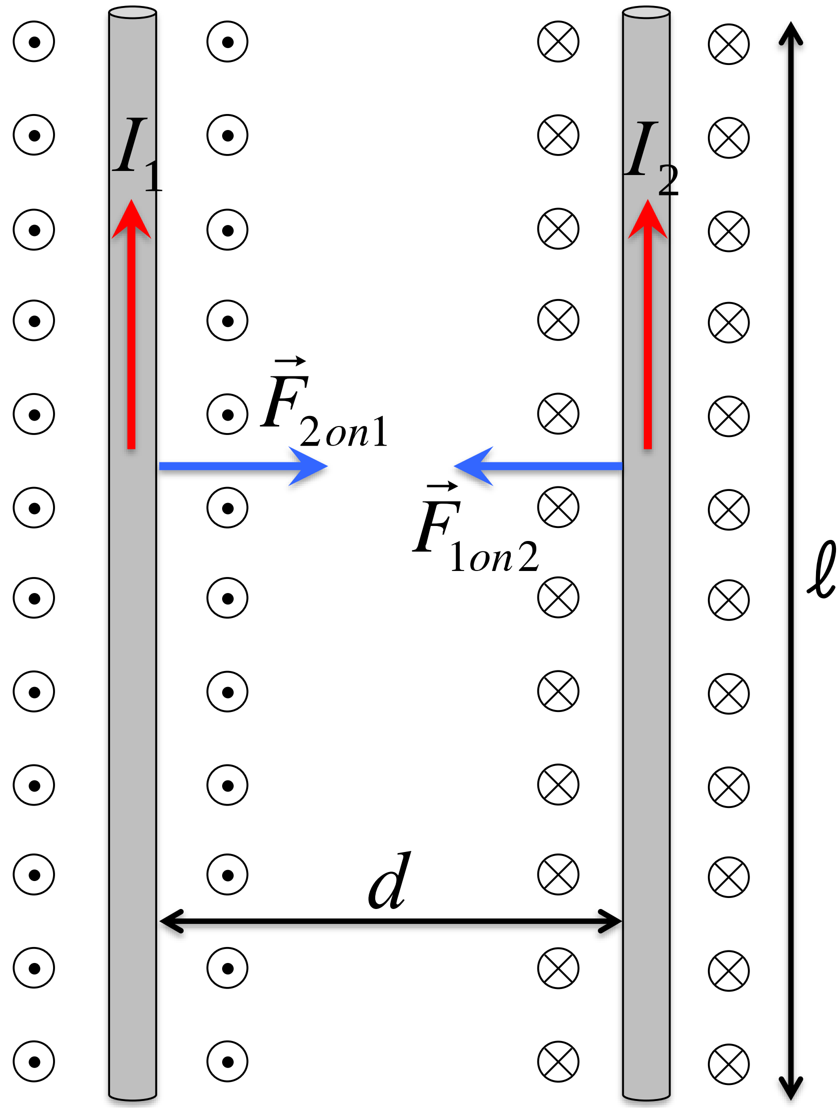
B. Right

C. Up

D. Down



$$\left|\vec{B}_2\right| = \frac{\mu_0 I_2}{2\pi d} \qquad \left|\vec{B}_1\right| = \frac{\mu_0 I_1}{2\pi d}$$



Wire 2 exerts a force on wire 1

$$\vec{F}_{2on1} = I_1 \vec{\ell} \times \vec{B}_2$$

$$\left|\vec{F}_{2on1}\right| = I_1 \ell \frac{\mu_0 I_2}{2\pi d} = \boxed{\frac{\mu_0 \ell I_1 I_2}{2\pi d}}$$

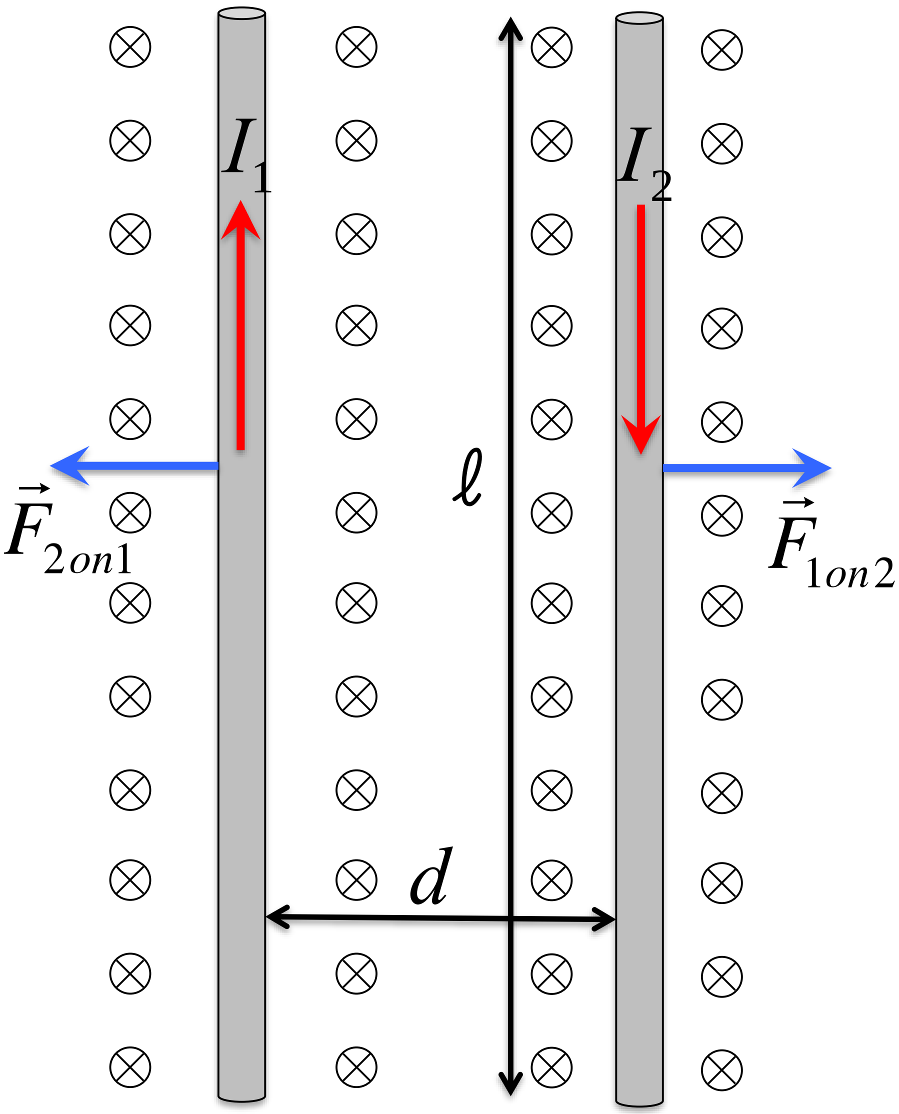
Wire 1 exerts a force on wire 2

$$\vec{F}_{1on2} = I_2 \vec{\ell} \times \vec{B}_1$$

$$\left|\vec{F}_{1on2}\right| = I_2 \ell \frac{\mu_0 I_1}{2\pi d} = \boxed{\frac{\mu_0 \ell I_1 I_2}{2\pi d}}$$

Newton's third law!

$$\left|\vec{B}_2\right| = \frac{\mu_0 I_2}{2\pi d} \qquad \left|\vec{B}_1\right| = \frac{\mu_0 I_1}{2\pi d}$$



Wire 2 exerts a force on wire 1

$$\vec{F}_{2on1} = I_1 \vec{\ell} \times \vec{B}_2$$

$$\left|\vec{F}_{2on1}\right| = I_1 \ell \frac{\mu_0 I_2}{2\pi d} = \boxed{\frac{\mu_0 \ell I_1 I_2}{2\pi d}}$$

Wire 1 exerts a force on wire 2

$$\vec{F}_{1on2} = I_2 \vec{\ell} \times \vec{B}_1$$

$$\left|\vec{F}_{1on2}\right| = I_2 \ell \frac{\mu_0 I_1}{2\pi d} = \boxed{\frac{\mu_0 \ell I_1 I_2}{2\pi d}}$$

Newton's third law!

Document Camera Calculations

My notes called:
"Mar_Chapter29_Appendix3_FiniteLineMagneticField.PDF"
&
"Mar_Chapter29_Appendix4_ArcMagneticField.PDF"
On D2L

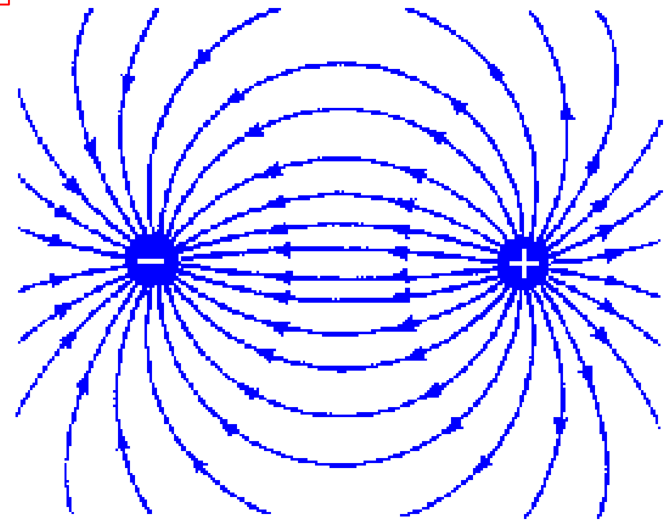
Last time:

- Magnetic force between parallel current-carrying wires
- Applying the Biot-Savart Law:
  - Circular arc of current
  - Finite line of current (I totally botched this…)

Today:

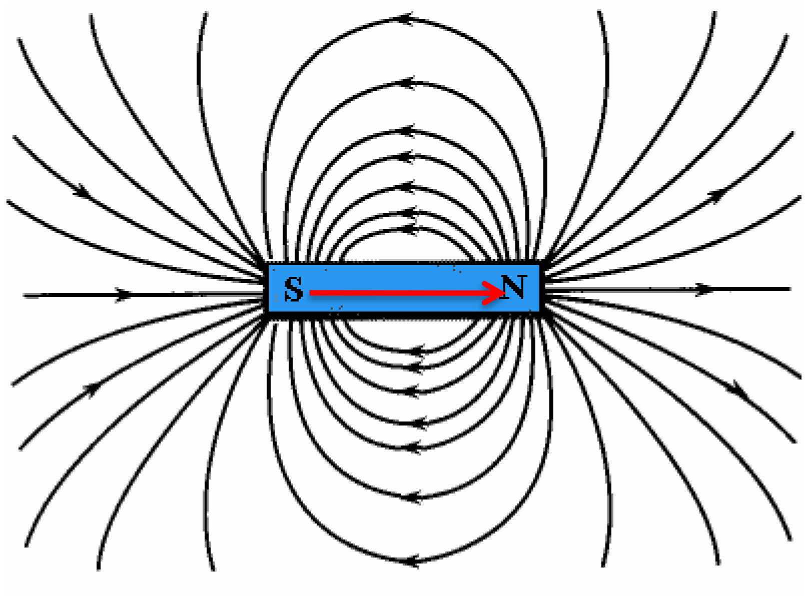
- Finite line of current (I've totally got this now…)
- Ampère's Law: Like Gauss' Law, but named after Ampère
- Magnetic field of a long wire (inside and outside)
- Magnetic field of a coaxial cable

Dipole Fields



Electric field from an electric dipole

Magnetic field from a magnetic dipole. Note that the magnetic field lines are continuous – they do NOT stop at the poles!



Both fields have the same shape!

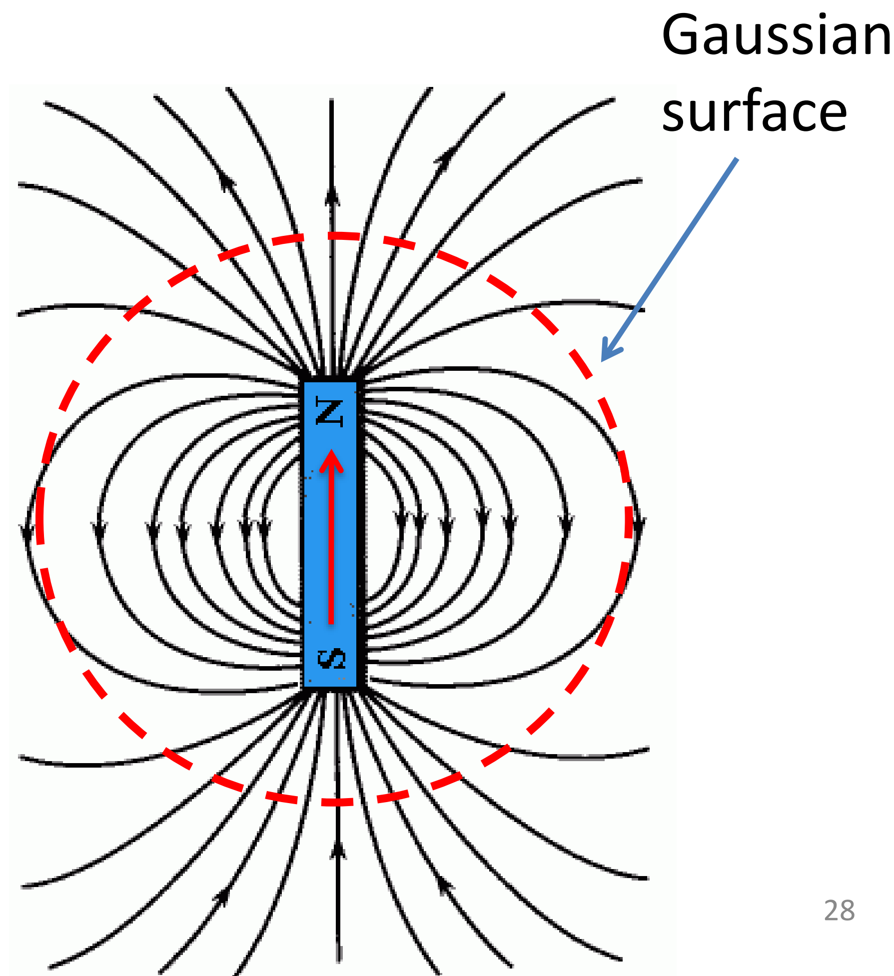
Not a Top Hat Question

The magnetic field lines from a magnet point out of the North pole and point into the South pole.

What can you say about the magnetic flux passing through this Gaussian surface?

$$\Phi_B = \oint \vec{B} \cdot d\vec{a}$$

A. Magnetic flux is zero
B. Magnetic flux is greater than zero
C. Magnetic flux is smaller than zero
D. Can’t tell without computing the integral



By symmetry the same number of flux lines enter and leave the spherical Gaussian surface

Gauss' Law for Magnetism

The magnetic flux through a closed surface is ALWAYS zero.

$$\Phi_B = \oint \vec{B} \cdot d\vec{a} = 0$$

no enclosed
magnetic charges

There is no way to isolate a North or South magnetic pole

The simplest E-field is from a point charge, while the simplest B-field is from a magnetic dipole (e.g. Bar Magnet)

Maxwell's equations

Essentially all of Electricity & Magnetism can be described by a set of 4 equations, referred to as Maxwell's equations.

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\boxed{c = \frac{1}{\sqrt{m_0 e_0}}}$$

We now have two of them!

$$\Phi_E = \oint \vec{E} \cdot d\vec{a} = \frac{Q_{encl}}{\epsilon_0} \qquad \Phi_B = \oint \vec{B} \cdot d\vec{a} = 0$$

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt} \qquad \oint \vec{B} \cdot d\vec{l} = \mu_0 i_{encl} + \frac{1}{c^2}\frac{d\Phi_E}{dt}$$

We will learn about these other two Maxwell equations today and next week

Maxwell's equations

And God Said

$$\nabla \cdot \vec{D} = \rho_{\text{free}}$$

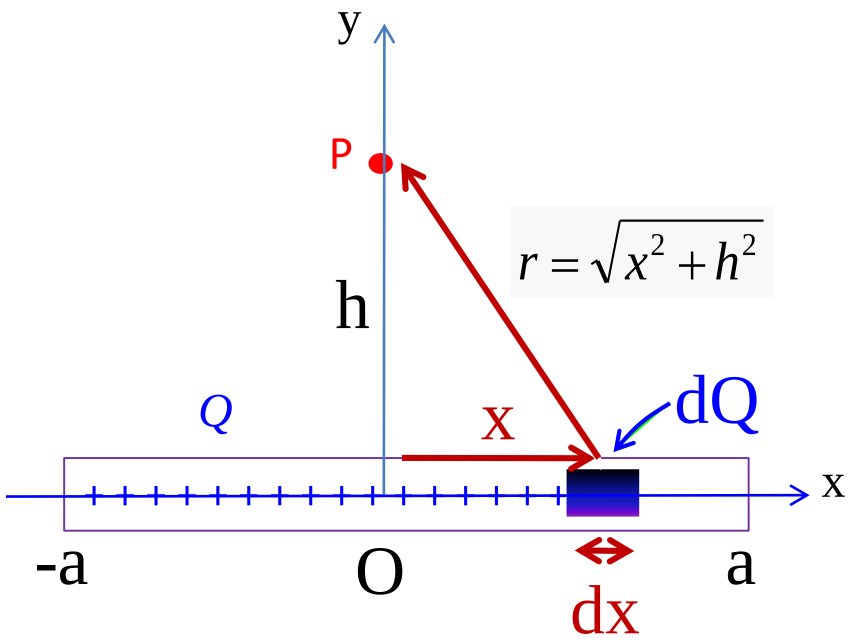
$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{H} = \vec{J}_{\text{free}} + \frac{\partial \vec{D}}{\partial t}$$

and *then* there was light.

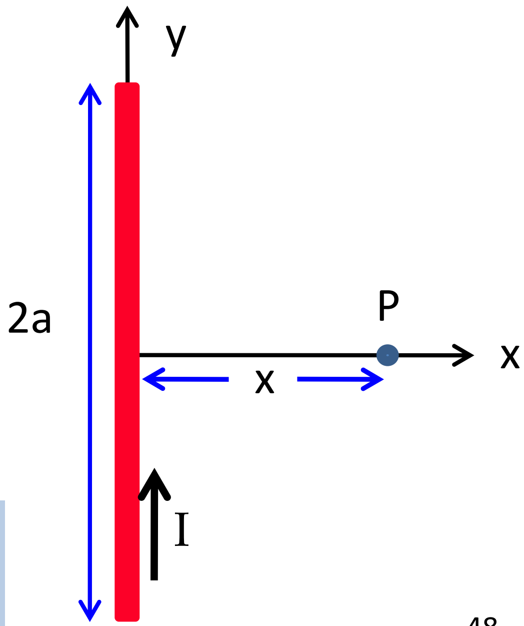
Remember this activity?
Solving for E_P for an infinitely long line of charge (i.e. a >> h) using Coulomb's Law was harder than using



GAUSS'S LAW

$$\oint \vec{E} \cdot d\vec{a} = \frac{Q_{encl}}{\epsilon_0}$$

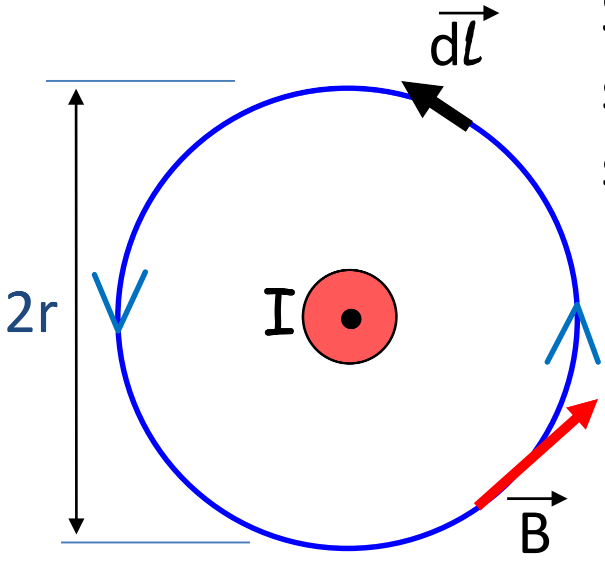
Solving for B_P for an infinitely long current carrying wire (i.e. a >> x) using Biot-Savart's Law was also hard, but there is a MUCH easier alternative!



AMPERE'S LAW

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 i_{encl}$$

Ampère’s Law



Suppose we calculate å $\vec{B} \cdot d\vec{l}$ around path shown for the simple case of an infinitely long straight line of current

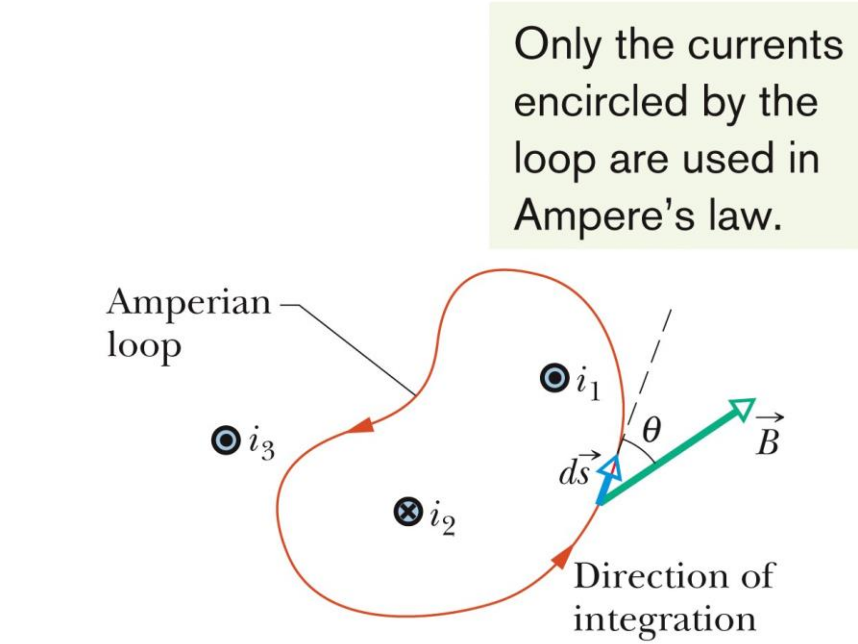
Using our previous result we obtain:

$$B = \frac{\mu_0 I}{2\pi r}$$

$$\oint \vec{B} \cdot d\vec{l} = (2\pi r)\left(\frac{\mu_0 I}{2\pi r}\right)$$

i.e. $\boxed{\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}}$

Ampère’s Law is true for any <u>shape of path</u> and any current distribution



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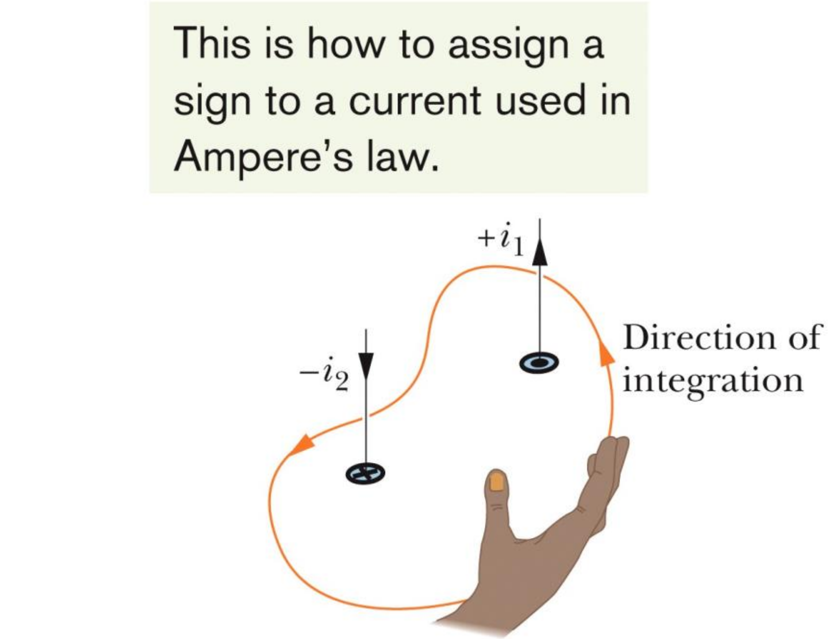

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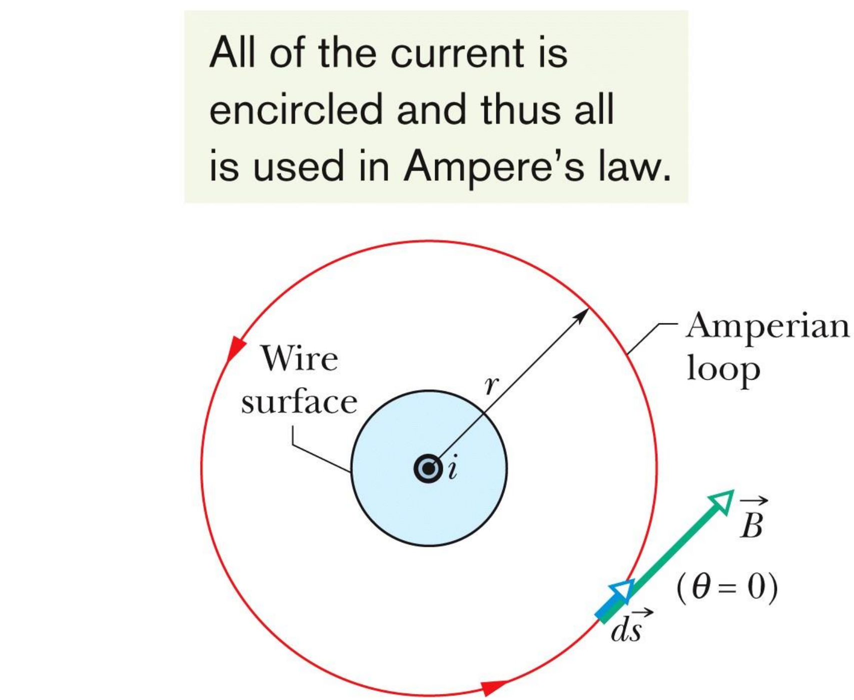

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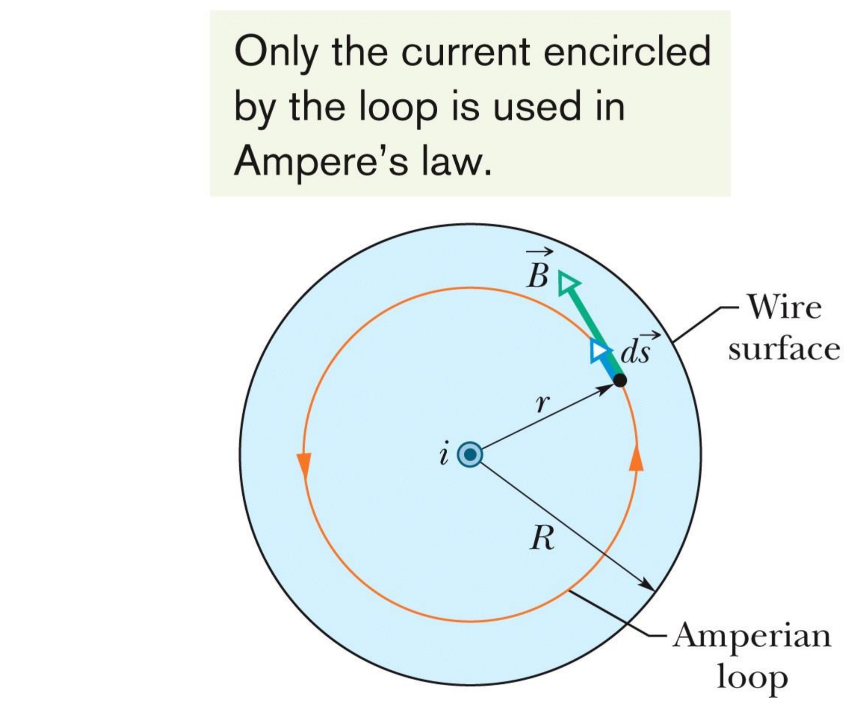

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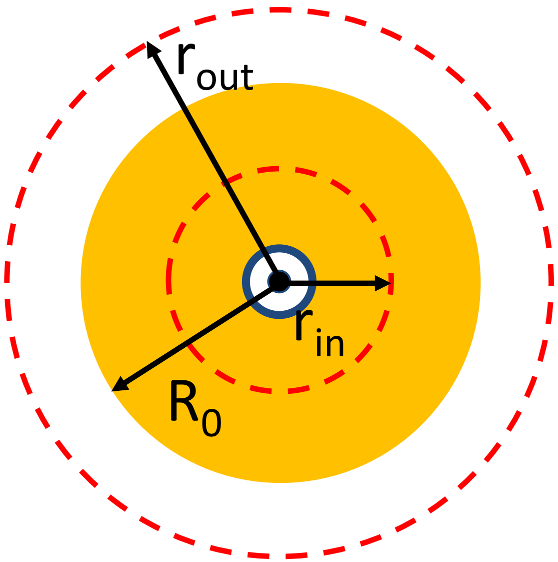
Ampère's law: application

(a) Using Ampère's law, calculate the magnetic field inside a solid current carrying wire a distance r_{in} from its axis.

(The length of the solid wire is infinite and the current I is uniformly distributed throughout the solid wire)



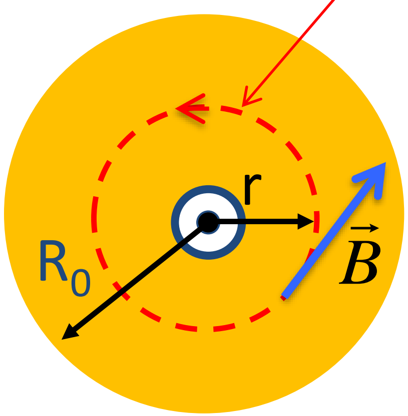
End view:
Wire with radius R
and current I

b) Calculate the magnetic field outside a solid current carrying wire a distance r_{out} from its axis.

Ampère's law: application(1)

(a) B-field inside

We want to know the B-field a distance r, so we choose an Amperian circular loop with radius r < R_0.



Ampère's Law: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$

Left hand side: $\oint \vec{B} \cdot d\vec{l} = BL = B2\pi r$

Right hand side: $\mu_0 I_{enc} = \mu_0 JA = \mu_0 \dfrac{I}{pR_0^2} pr^2$

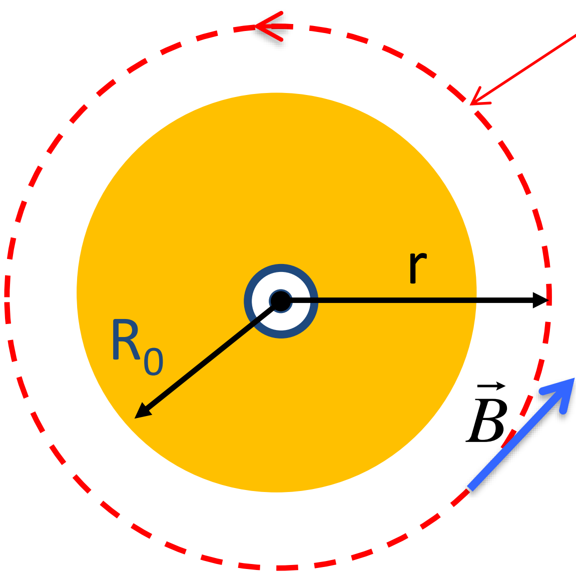
Combine together: $B2pr = \mu_0 \dfrac{I}{R_0^2} r^2$

$$\boxed{B = \frac{\mu_0 I\, r}{2pR_0^2}}$$

Ampère's law: application(1)

(a) B-field outside

We want to know the B-field a distance r, so we choose an Ampèrian loop with radius $r > R_0$.



Ampère's Law: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$

Left hand side: $\oint \vec{B} \cdot d\vec{l} = BL = B2\pi r$

Right hand side: $\mu_0 I_{enc} = \mu_0 I$

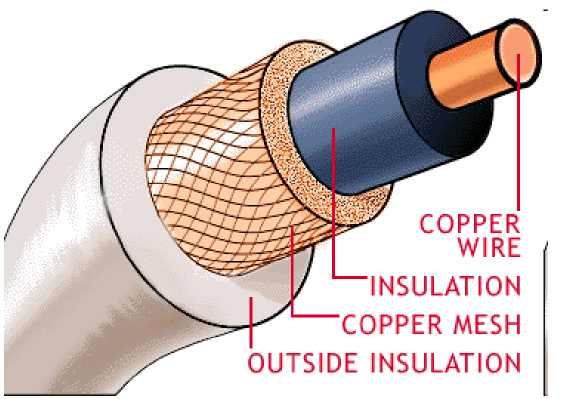
Combine together: $B2pr = \mu_0 I$

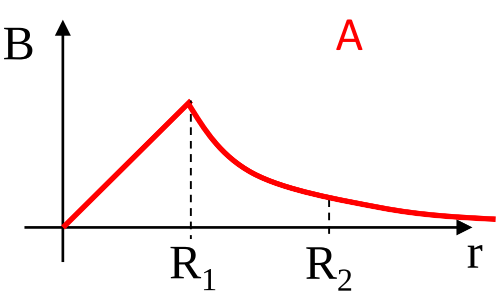
$$\boxed{B = \frac{\mu_0 I}{2pr}}$$

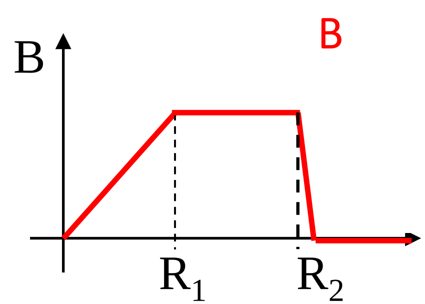
Top Hat (not really)

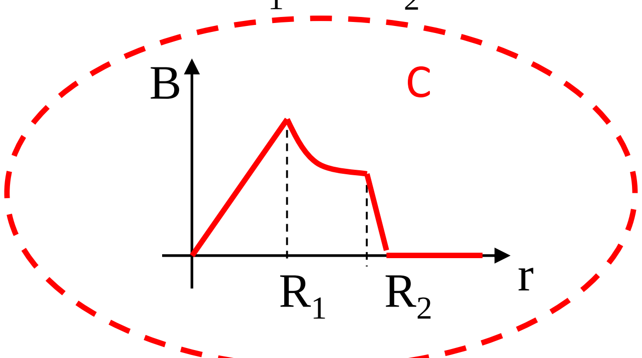
A coaxial cable consists of a wire (radius R_1) surrounded by an insulating sleeve and another cylindrical conducting shell (inner radius R_2) and finally another insulating sleeve. **The wire and the shell carry the same current I but in opposite directions.**

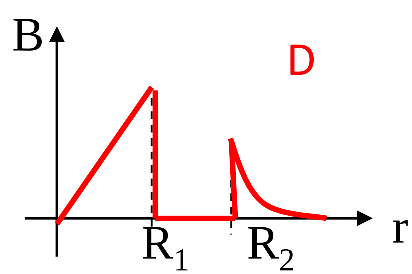
Which diagram best represents the **magnetic field** as a function of radial distance from the cable's axis?

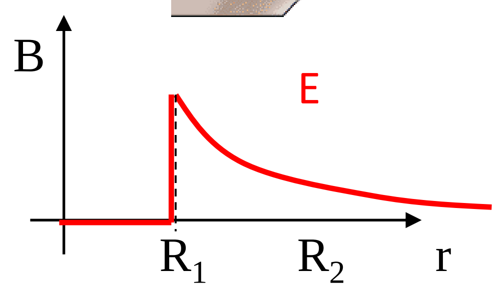












Last time:

- Finite line of current
- Ampère’s Law: Like Gauss’ Law, but for magnetism
- Magnetic field of a long wire (inside and outside)
- Magnetic field of a coaxial cable

Today:

- More on Ampère’s Law
  - Magnetic field of a sheet of current
  - Magnetic field of a solenoid
  - Magnetic field of a toroid
  - Superposition and Ampère’s Law

- Look at my notes named:

“Mar_Chapter29_Appendix5_AmperLawApplication.PDF”