Electricity and Magnetism

- Physics 259 – L02
- Lecture 32

Chapter 25: Capacitance

Last time

- Energy in Capacitors
- Capacitors with a dielectric

This time

- Capacitors with a dielectric
- Class activity

Review: Calculating electric field and potential difference

1. To relate the electric field ***E*** between the plates of a capacitor to the charge *q* on either plate→ use Gauss' law:

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q.$$

2. the potential difference between the plates of a capacitor is related to the field ***E*** by

$$V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s},$$

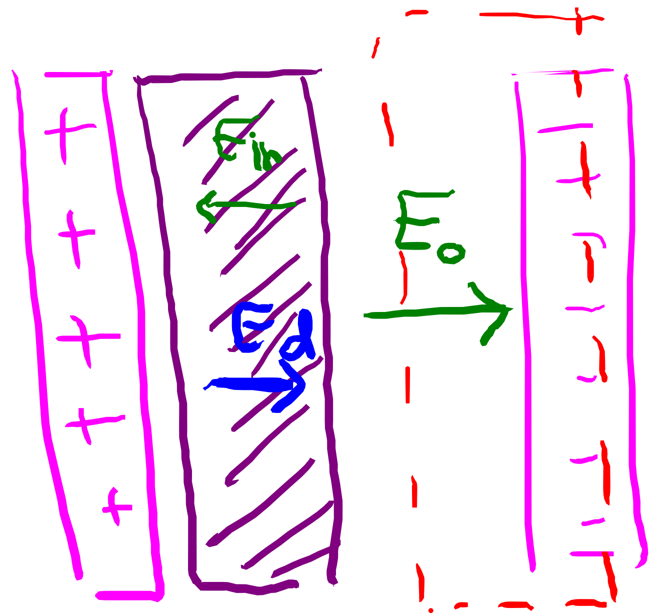
Letting V represent the difference $V_f = V_i$, we can then recast the above equation as:

$$V = -\int_{-}^{+} \vec{E} \cdot d\vec{s} = \underset{+}{\cancel{-}} \int_{-}^{+} E\, ds$$

$$V = \int_{-}^{+} E\, ds$$

3. Find Capacitance

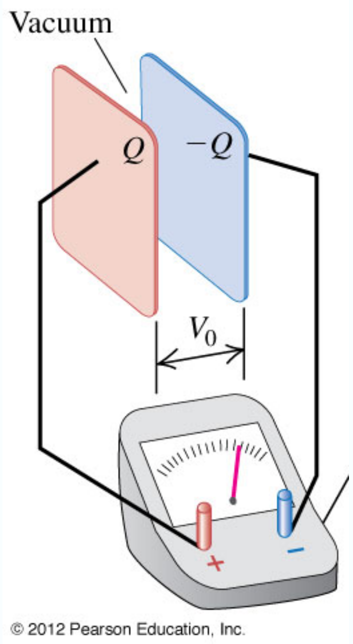
$$q = CV.$$

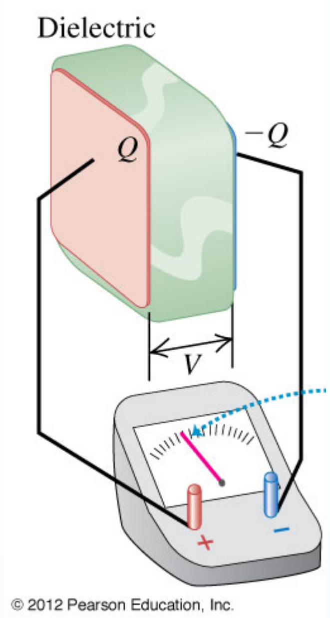


$$\oint \vec{E}\cdot d\vec{A} = \frac{q}{\varepsilon_0}$$

$E_d < E_0$ $\quad$ $\varepsilon_0 \to \kappa\,\varepsilon_0$ $\quad$ $E_0 = \dfrac{q}{A\varepsilon_0}$

25-5 Capacitor with a Dielectric





If the space between the plates of a capacitor is completely filled with a **dielectric material**, the capacitance C in vacuum (or, effectively, in air) is multiplied by the material's **dielectric constant** κ, which is a number greater than 1.

$$\boxed{C = \kappa C_{air}}$$

If the potential difference between the plates of a capacitor is maintained, as by the presence of battery B→

The effect of a dielectric is to increase the charge on the plates.

$$\boxed{q = \kappa q_{air}}$$

If the charge on the capacitor plates is maintained, as in this case by isolating the capacitor→

The effect of a dielectric is to reduce the potential difference between the plates.

$q = \frac{C}{V} \rightarrow V_0 = \frac{q_0}{C_0}$

$V = \frac{q}{\kappa C_0}$

$$\boxed{V = \frac{V_{air}}{\kappa}}$$

In a region completely filled by a dielectric material of dielectric constant κ, all electrostatic equations containing the permittivity constant ε_0 are to be modified by replacing ε_0 with $\kappa\varepsilon_0$.

Examples:

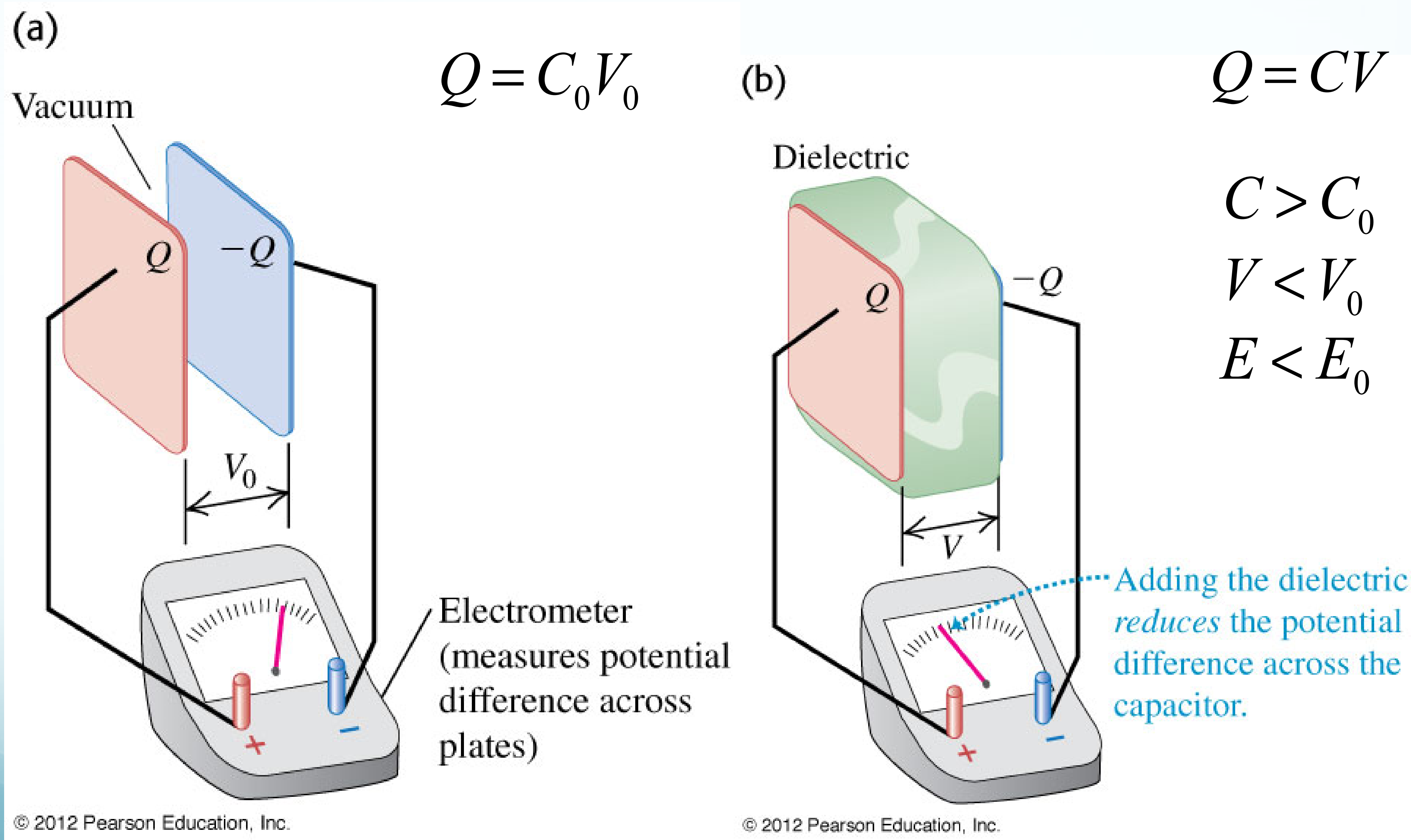
The magnitude of electric field produced by a point charge inside a dielectric→

$$\boxed{E = \frac{1}{4\pi\kappa\varepsilon_0}\frac{q}{r^2}}$$

The magnitude of electric field outside an isolated conductor immersed inside a dielectric→

$$\boxed{E = \frac{\sigma}{\kappa\varepsilon_0}}$$

To sum up →



$$Q = C_0 V_0$$

$$Q = CV$$

$$C > C_0$$

$$V < V_0$$

$$E < E_0$$

TopHat Question

A capacitor without a dielectric is charged up so that it stores potential energy U_0, and it is then disconnected so that **its charge remains the same**. A dielectric with constant $\kappa = 2$ is then inserted between the plates. What is the new potential energy stored in the capacitor **with the dielectric**?

$$U_C = \frac{Q^2}{2C} = \frac{V_C^2 C}{2}$$

A. $4U_0$

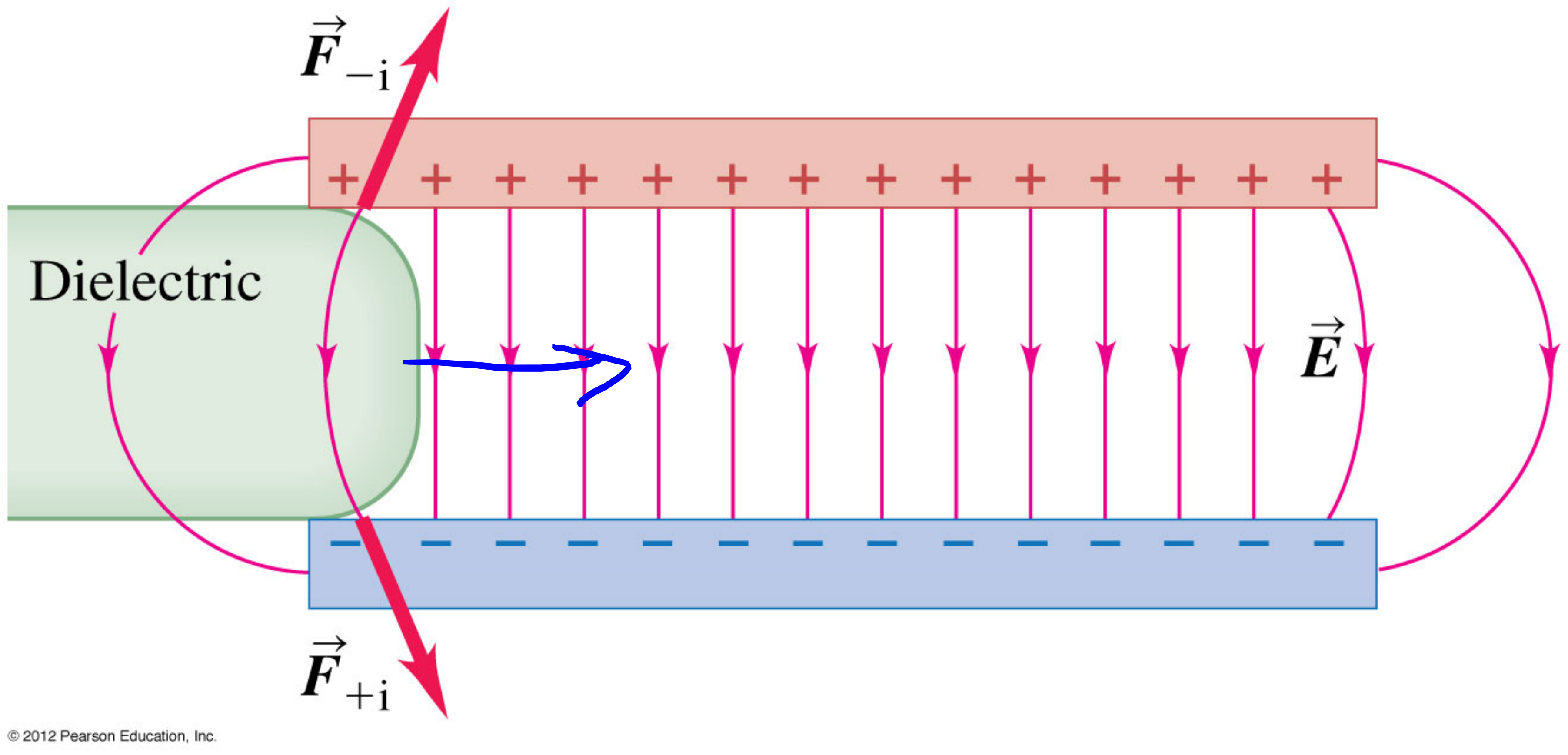
C. $\frac{1}{2}U_0$

A. $2U_0$

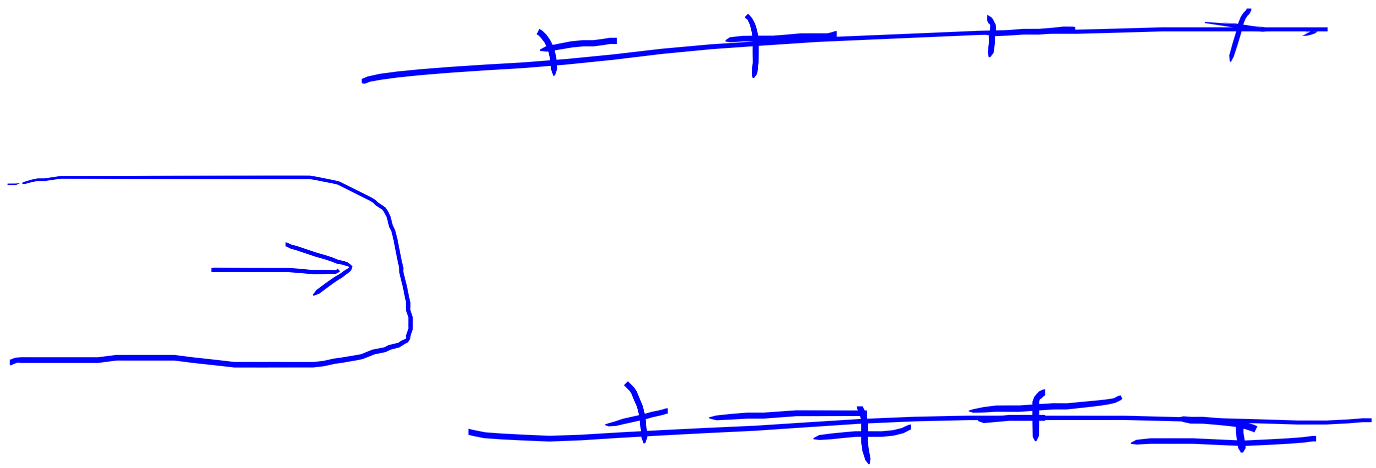
D. $\frac{1}{4}U_0$

The **potential energy lowers** when the dielectric is added, so it will feel a **force sucking it into the gap** between the plates.

Can find the force using $F_x = -dU/dx$ and



Fringe Electric Field pulls the dielectric into the gap



TopHat Question

A capacitor without a dielectric is charged up so that it stores potential energy U_0, and is kept connected **at constant voltage**. A dielectric with constant $\kappa = 2$ is then inserted between the plates. What is the new potential energy stored in the capacitor **with the dielectric**?

$$U_C = \frac{Q^2}{2C} = \frac{V_C^2 C}{2}$$

A. $4U_0$

C. $\frac{1}{2}U_0$

B. $2U_0$

D. $\frac{1}{4}U_0$

$C = \kappa C_0$

$U \uparrow \quad 2$

TopHat Question

A capacitor without a dielectric is charged up so that it stores potential energy U_0, and is kept connected **at constant voltage**. A dielectric with constant $\kappa = 2$ is then inserted between the plates. What is the new potential energy stored in the capacitor **with the dielectric**?

$$U_C = \frac{Q^2}{2C} = \frac{V_C^2 C}{2}$$

A. $4U_0$

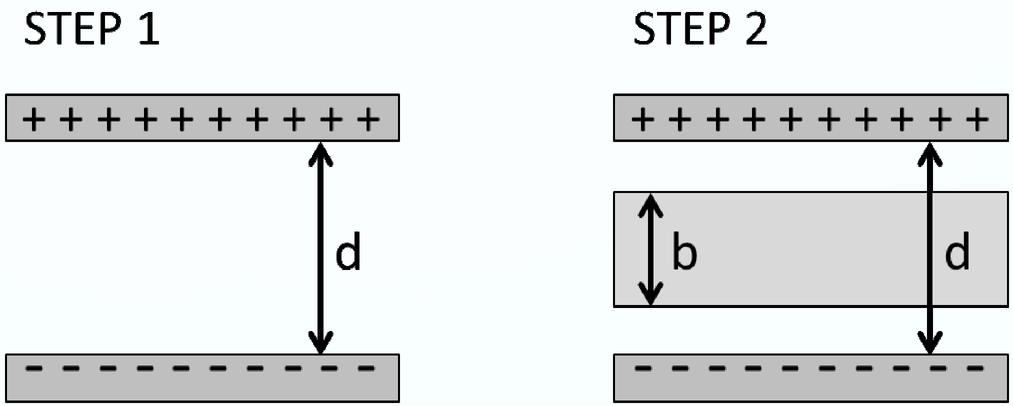
C. $\frac{1}{2}U_0$

A. $2U_0$

D. $\frac{1}{4}U_0$

The **potential energy raises** when the dielectric is added, so it will feel a **force pushing it out of the gap** between the plates.

(10 marks) Consider the following scenario depicted in the diagrams below. Step 1: A parallel-plate capacitor with surface area A and plate separation d is charged to an electric charge of q. Step 2: A dielectric slab of thickness b and dielectric constant κ is inserted between the capacitor plates. What is the ratio between the final and initial voltages measured across the capacitor?



1. **(1 mark)** What is the capacitance of the parallel-plate capacitor in Step 1 in terms of its dimensions and $\epsilon_\circ$?

2. **(1 mark)** What is the voltage, $V_\circ$, across the capacitor in Step 1 in terms of q, $\epsilon_\circ$ and the dimensions of the capacitor?

3. **(1 mark)** Is the charge on the capacitor in Step 2 the same as it was in Step 1? Explain.

4. **(2 marks)** If the electric field strength between the plates in Step 2 is $E_\circ$ in the region *outside* of the dielectric, and E_d *inside* the dielectric, what is the **voltage** across the capacitor in Step 2 in terms of $E_\circ$, E_d and the dimensions of the capacitor and the dielectric slab?

5. **(2 marks)** Use Gauss's law to find $E_\circ$ and E_d in Step 2, in terms of q, $\epsilon_\circ$, κ and the dimensions of the capacitor and the dielectric slab.

6. **(2 marks)** Using your result from Question 5, write the voltage, V, across the capacitor in Step 2 in terms of q, $\epsilon_\circ$, κ and the dimensions of the capacitor and the dielectric slab.

This section we talked about:

Chapter 25

See you on Monday