Electricity and Magnetism

- Physics 259 – L02
- Lecture 22

Chapter 24.1: Electric Potential

Last time

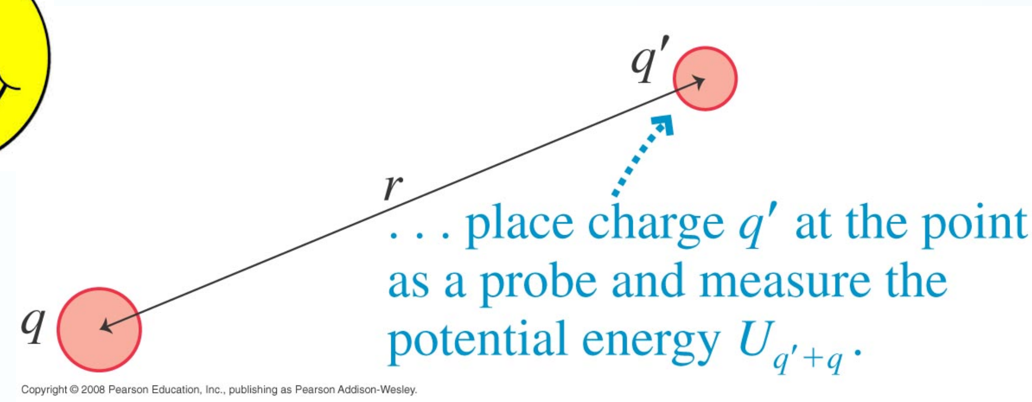
- Midterm review
- Honestly I'm just happy you showed up the day after your midterm

This time

- Electric potential energy: uniform E-field
- Electric potential energy: 2 point charges
- Electric potential energy of a collection of charges
- Electric potential (very important concept)

Starting from the end

The whole story is:



Electric force on q’ from q

$$\vec{F}_{qq'} = \frac{1}{4\pi\varepsilon_0}\frac{qq'}{r^2}\hat{r}$$

Then the electric field of q is

$$\vec{E} = \frac{\vec{F}_{qq'}}{q'} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$$

Potential energy of q and q’

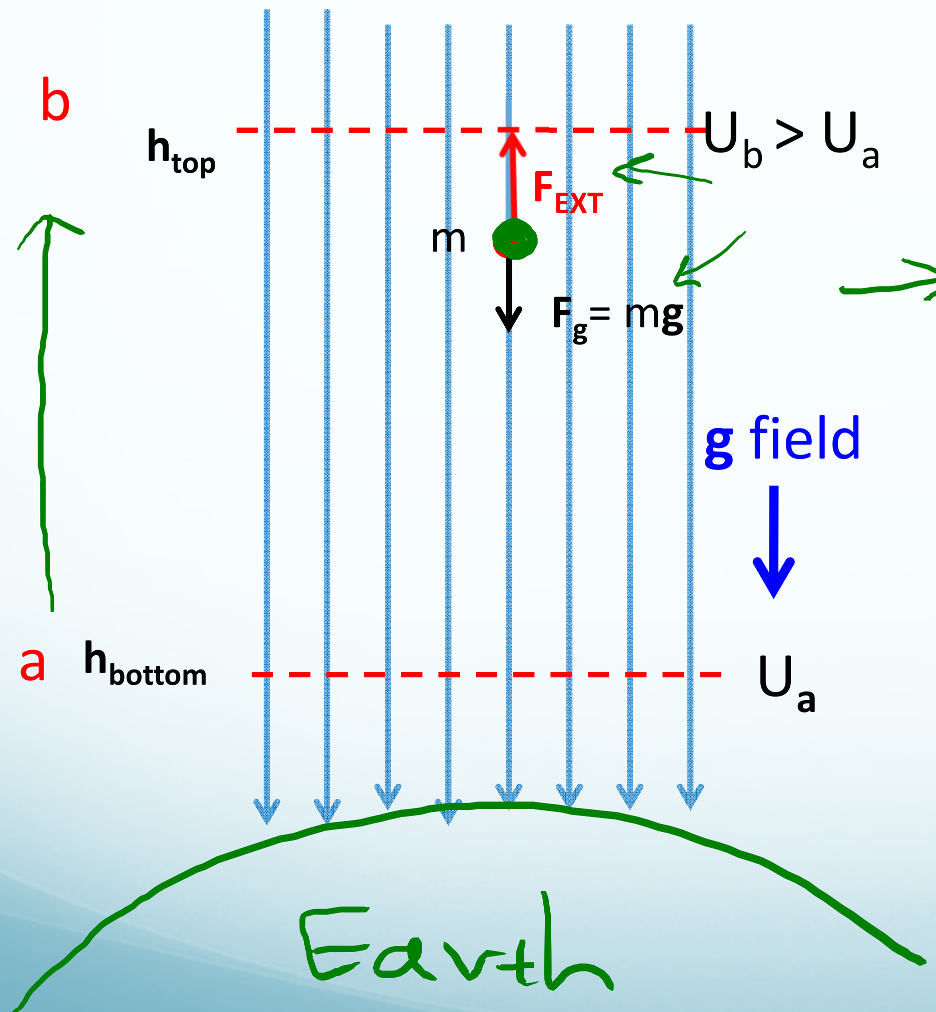
$$U_{q'+q} = \frac{1}{4\pi\varepsilon_0}\frac{qq'}{r}$$

Then the potential of q is

$$V = \frac{U_{q'+q}}{q'} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

Gravitational

(Simple case: uniform fields)



$$WORK_{EXT} = F_{EXT} \times \Delta h = mg\Delta h$$

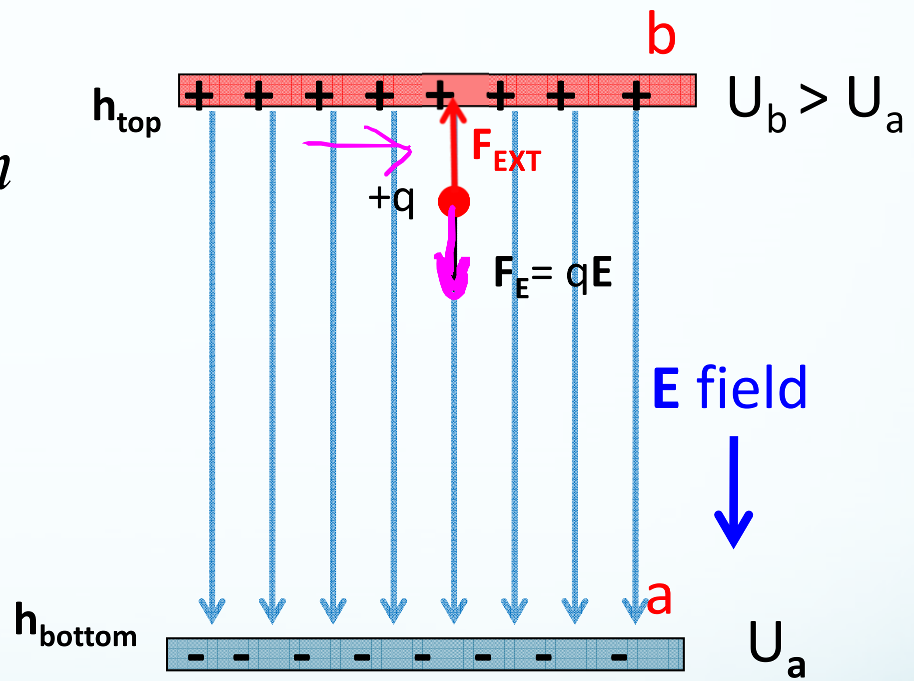
$$\boxed{WORK_{EXT}^{a->b} = U_b^g - U_a^g > 0}$$

$$\boxed{WORK_g^{a->b} = -(U_b^g - U_a^g)}$$

Electric Fields

(Simple case: uniform fields)

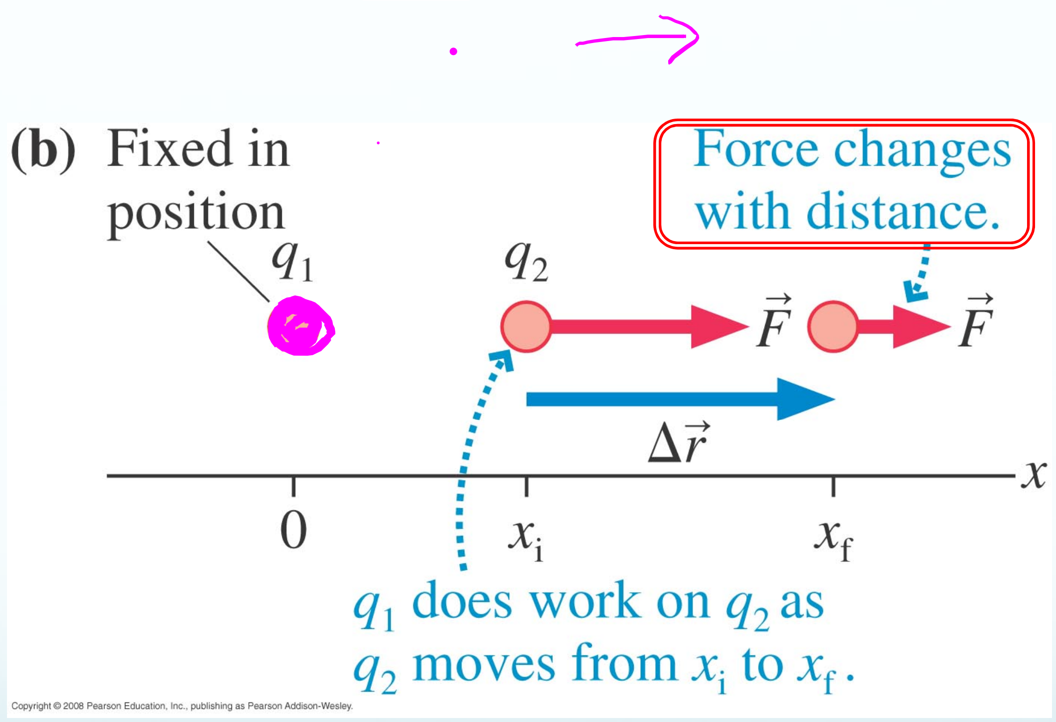
$$WORK_{EXT} = F_{EXT} \times \Delta h = qE\Delta h$$



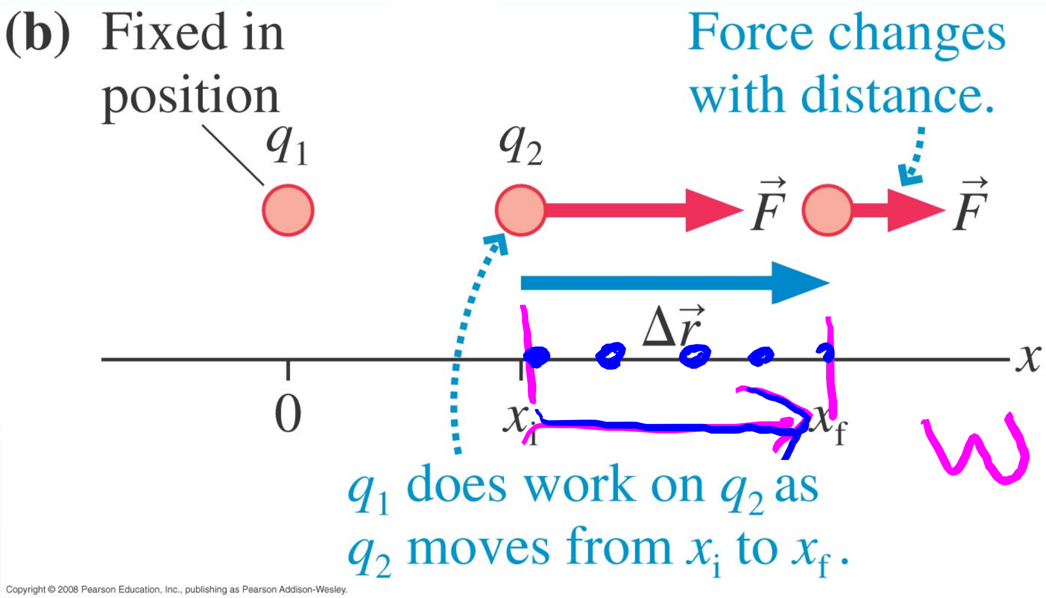
$$\boxed{WORK_{EXT}^{a->b} = U_b^E - U_a^E > 0}$$

$$\boxed{WORK_E^{a->b} = -(U_b^E - U_a^E)}$$

Finding Potential Energy of two point charges (more building blocks)



Finding Potential Energy of two point charges (more building blocks)



W = F x Δr

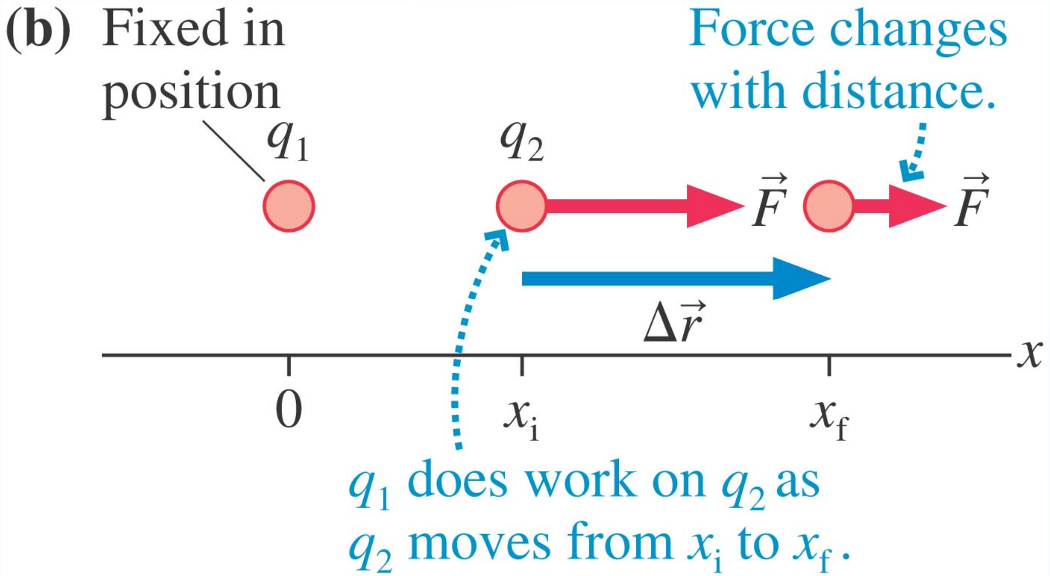
F(r) x Δr

$$W_{i\to f}^{ELEC} = -\Delta U$$

$$\cancel{W_{i\to f}^{ELEC} = F\Delta r}$$

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$$

Finding Potential Energy of two point charges (more building blocks)

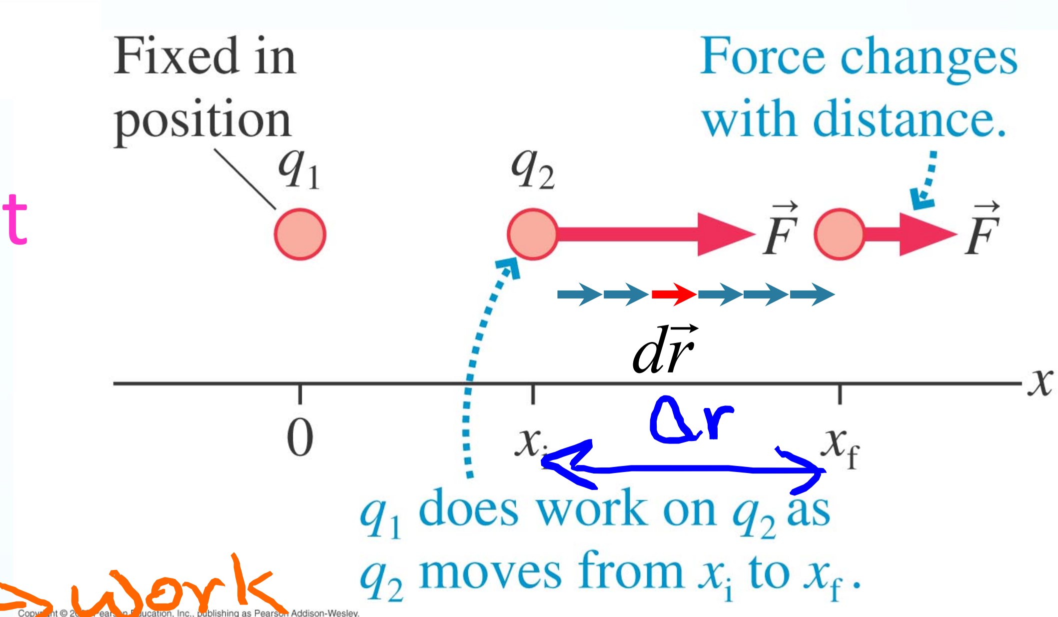


The field is **not** uniform so $\vec{F}$ is **not** constant over the displacement Δr and we **cannot** use

$$W_{i\rightarrow f}^{ELEC} = F\Delta r$$

Finding Potential Energy of two point charges (more building blocks)

Break the displacement $\Delta\vec{r}$ into many tiny displacements $d\vec{r}$.

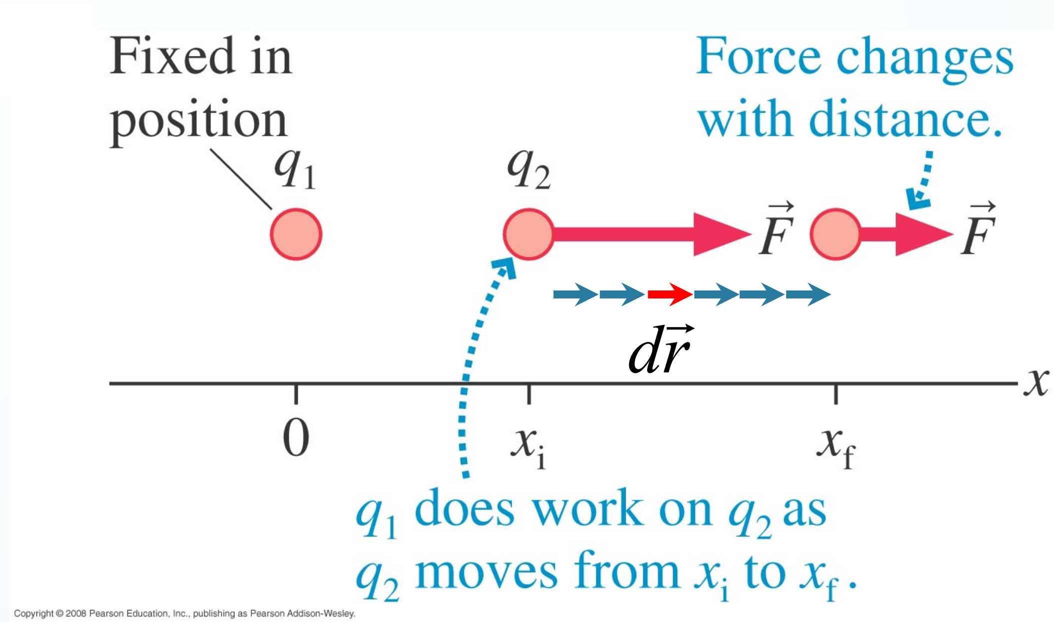


$dW = F\,dr$, w → work

work = force · displacement

$\vec{F}$ is essentially constant over such a small displacement, so the work done on q_2 in **each** displacement is $F\,dr$.

Finding Potential Energy of two point charges (more building blocks)



The total work is the sum of all the little bits of work:

$dW = Fdr$

$$W_{i\to f}^{ELEC} = \int_{r_i}^{r_f} F dr$$

$$W_{i\to f}^{ELEC} = \int_{r_i}^{r_f} \overbrace{\frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}}^{F} dr$$

Finding Potential Energy of two point charges (more building blocks)

Work done **by electric force**:

$$W_{i\to f}^{ELEC} = \int_{r_i}^{r_f} \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\,dr = \frac{1}{4\pi\varepsilon_0} q_1 q_2 \int_{r_i}^{r_f} \frac{dr}{r^2} = \frac{q_1 q_2}{4\pi\varepsilon_0}\int r^{-2}\,dr$$

Recall from integral calculus

$$\int_{x_i}^{x_f} x^n\,dx = \underline{\frac{1}{n+1}}\, x^{n+1}\Big|_{x_i}^{x_f} = \frac{1}{n+1}\left(x_f^{n+1} - x_i^{n+1}\right)$$

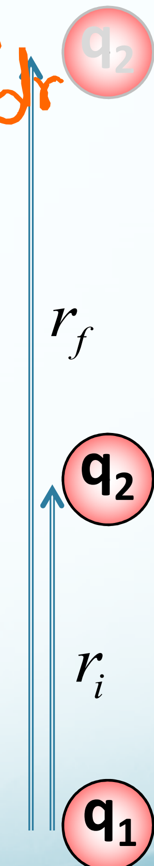
$n \to -2$

$x \to r$

In our case, let x → r, then we have

r^{n+1}

$$W_{i\to f}^{ELEC} = \underline{\frac{1}{4\pi\varepsilon_0} q_1 q_2 \int_{r_i}^{r_f} r^{-2}\,dr} = \underline{\frac{1}{4\pi\varepsilon_0} q_1 q_2}\left(\underline{\frac{1}{-2+1}}\, r^{-2+1}\right)\Bigg|_{r_i}^{r_f}$$



q_1 held fixed

Finding Potential Energy of two point charges (more building blocks)

$$W_{i\to f}^{ELEC} = -\frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}\Bigg|_{r_i}^{r_f}$$

final

initial

$$\rightarrow \quad W_{i\to f}^{ELEC} = -\left(\frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{\underline{r_f}} - \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{\underline{r_i}}\right)$$

$$W_{i\to f}^{ELEC} = -\Delta U = -(U_f - U_i) = U_i - U_f$$

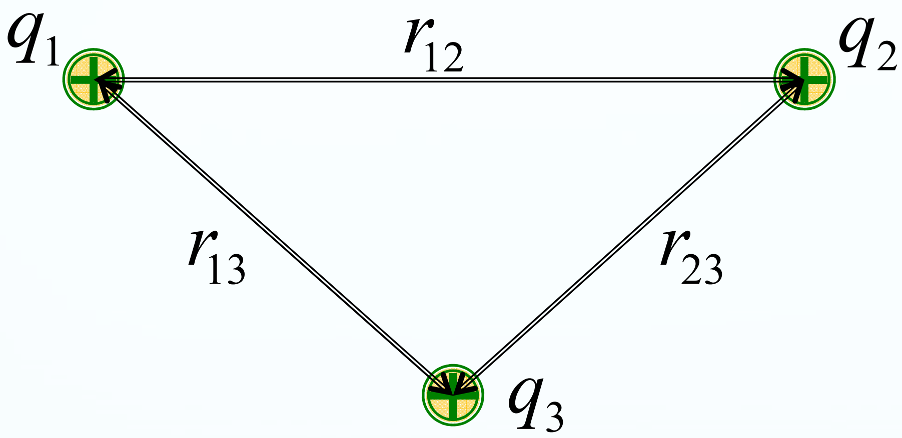
Then the potential energy of two point charges a distance r apart is

$$\boxed{\boxed{U_e = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}} + \cancel{U_0}}$$

(1) There is a U_0, but we normally set it to zero.

(2) The potential energy of two charges an infinite distance apart ($r = \infty$) is zero.

Superposition: Potential Energy due to Multiple Charges



$$U_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}}$$

$$U_{23} = \frac{1}{4\pi\varepsilon_0}\frac{q_2 q_3}{r_{23}}$$

$$U_{13} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_3}{r_{13}}$$

$$\boxed{U_{total} = U_{12} + U_{23} + U_{13}}$$

In general, the total potential energy is just the sum of the pairwise potential energies of all the charges present.

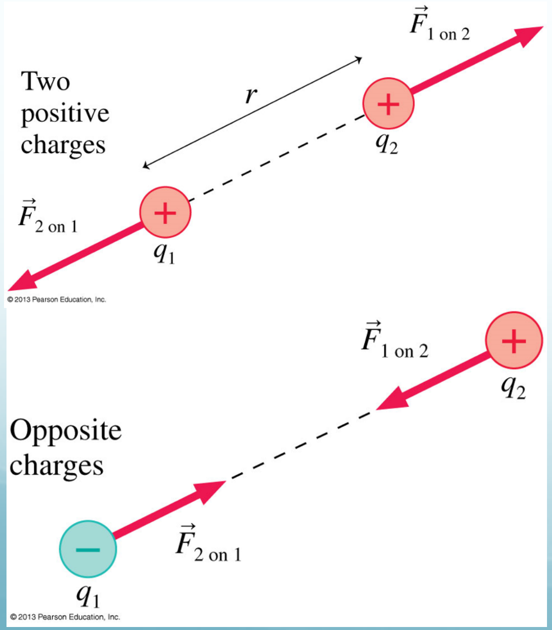
Calculate U between each pair, then sum all of them up.

Electric Force vs Electric Field

Electric Force $\vec{F}$

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2} = q\vec{E}$$

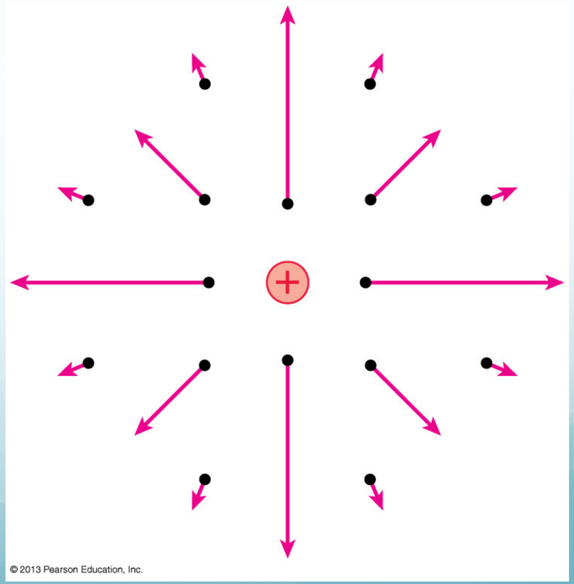
A physical property between two point charges



Electric Field $\vec{E}$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

A physical property around a single point charge



Electric Force vs Electric Field

Electric Force $\vec{F}$

Electri

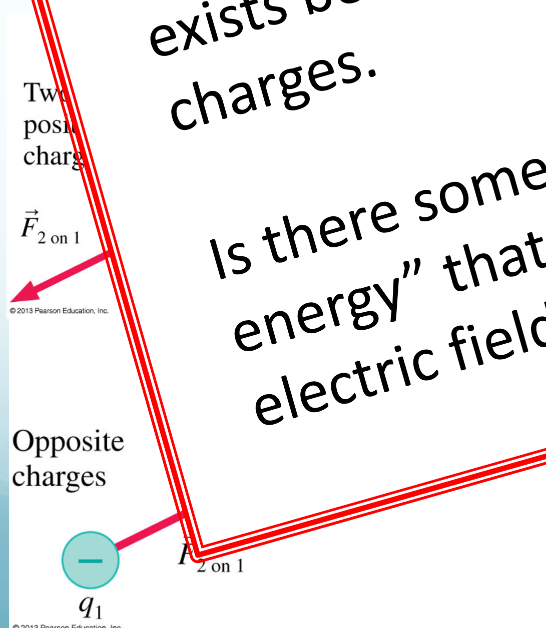
$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2} = q\vec{E}$$

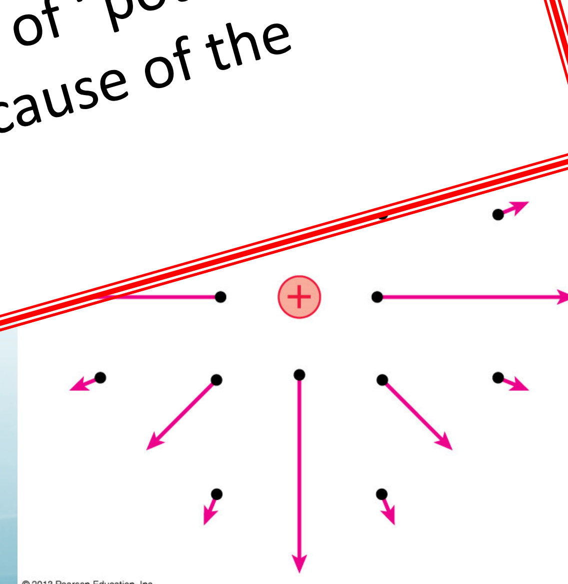
A p ... round

two

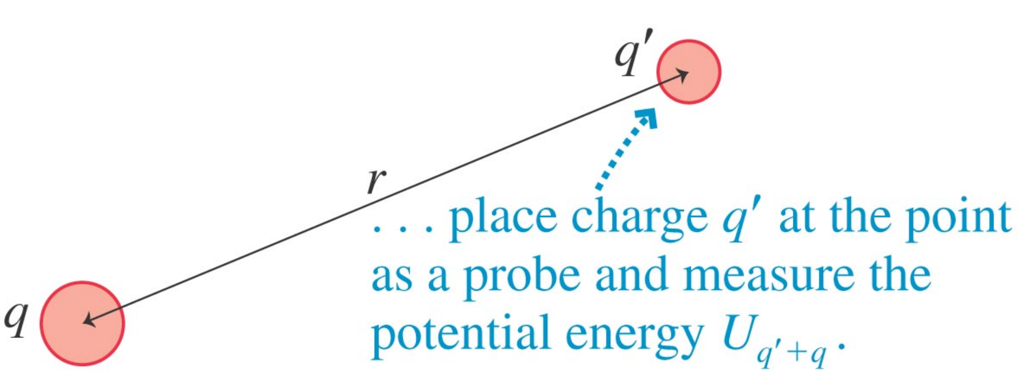
Potential energy is a physical property that exists because of the force between two charges.

Is there some similar notion of “potential energy” that exists only because of the electric field?





Yes, there is:



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Potential energy of q and q'

$$U_{q'+q} = \frac{1}{4\pi\varepsilon_0}\frac{qq'}{r}$$

Then the potential of q is

$$V = \frac{U_{q'+q}}{q'} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

Electric Potential

Here are some source charges and a point P.

If we place a charge q at point P, then q and the source charges interact with each other.

The interaction energy is the potential energy of q and the source charges,

$$U_{q+sources}$$

How does this interaction happen?

Electric Potential

Model:

The source charges create a **potential for interaction** everywhere, including at point P.

This potential for interaction is a **property of space**. Charge q does not need to be there.

We call this potential for interaction the **electric potential, V.** (Often just called “the potential”)

Electric Field of a point charge

Electric force on q' from q

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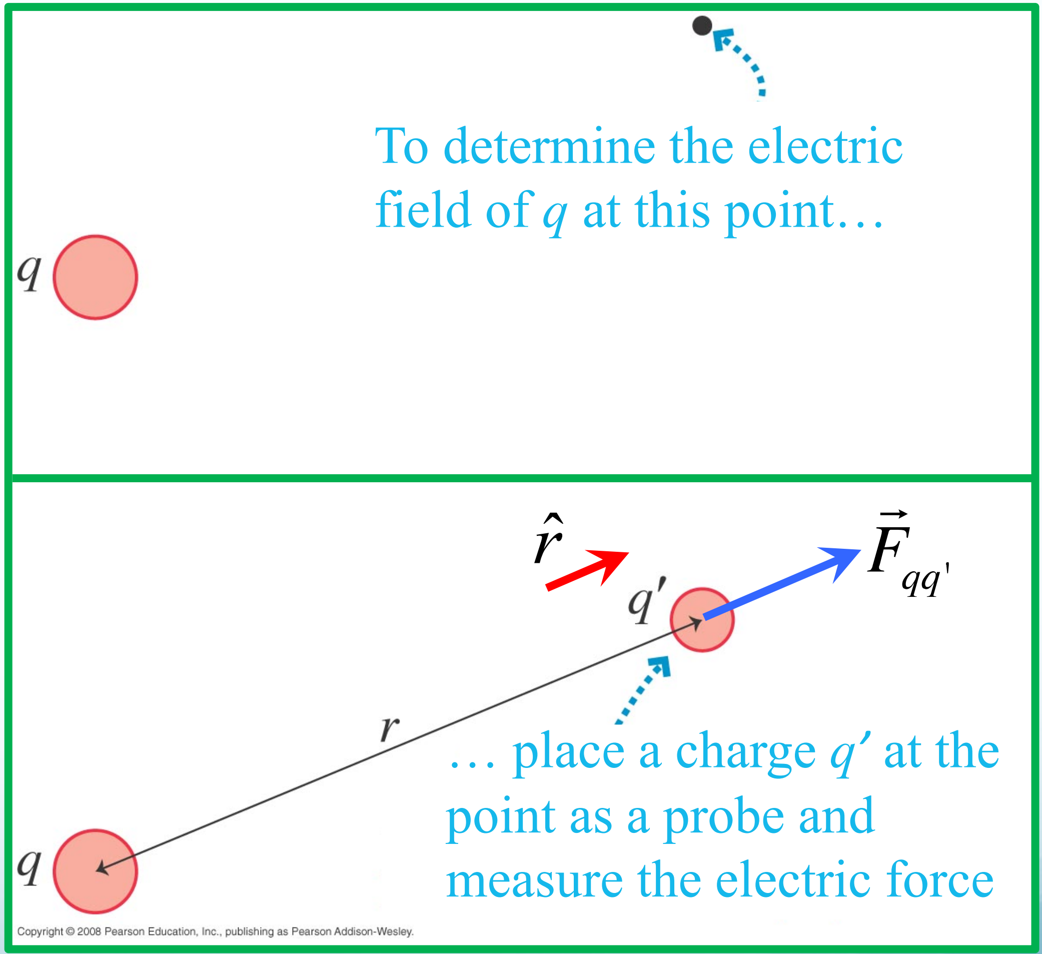

Electric Potential of a point charge

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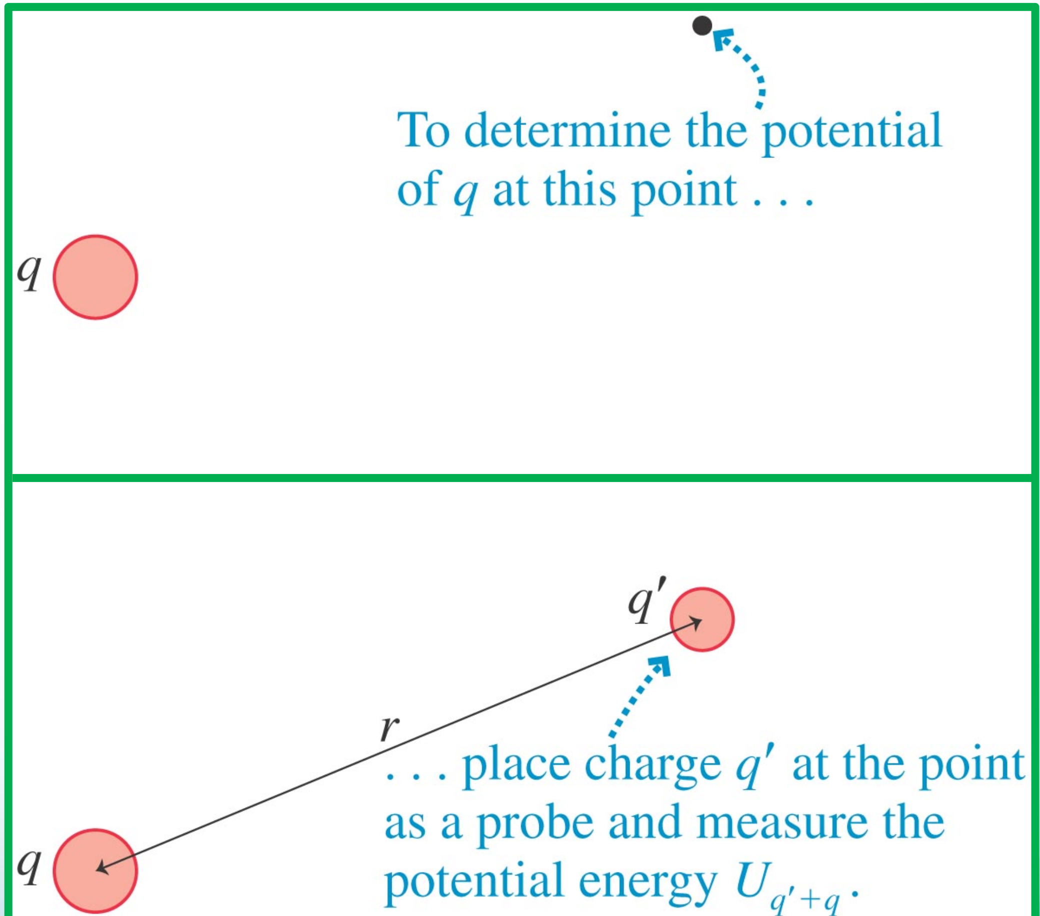

This section we talked about:

Chapter 24.1

See you on Thursday