

Wednesday March 22, 2017

# Last time:

- Introduction to magnetism
- Electric force vs magnetic force on charges
- Vector cross product
- Consequences of magnetic force
- Motion of charges in magnetic fields
- Cyclotron motion, cyclotron frequency,  $q/m$
- Mass spectrometers

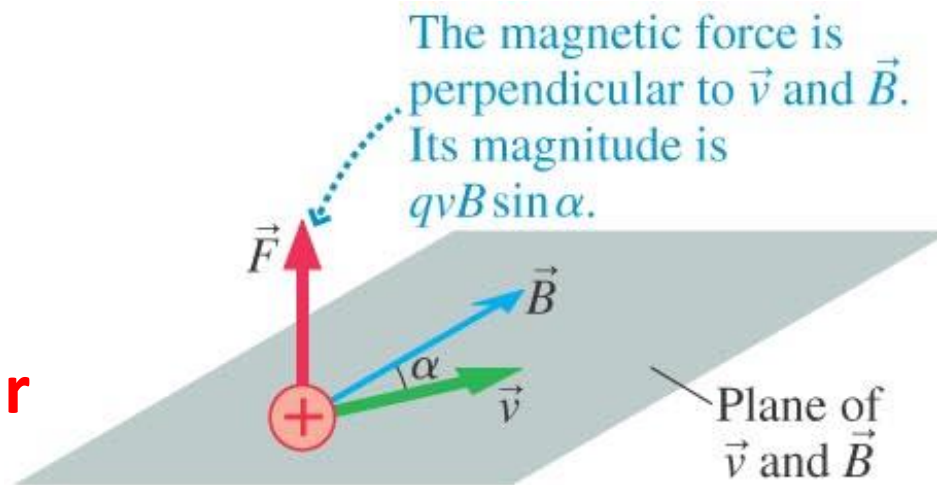
# Today:

- Charges on helical paths in B-field (aurora)
- The Hall Effect: underpinning of a B-field probe
- Velocity selector via crossed E- and B-fields
- Bainbridge Mass Spectrometer

# Magnetic Force on Charges

Magnetic force acts only on a moving charge.

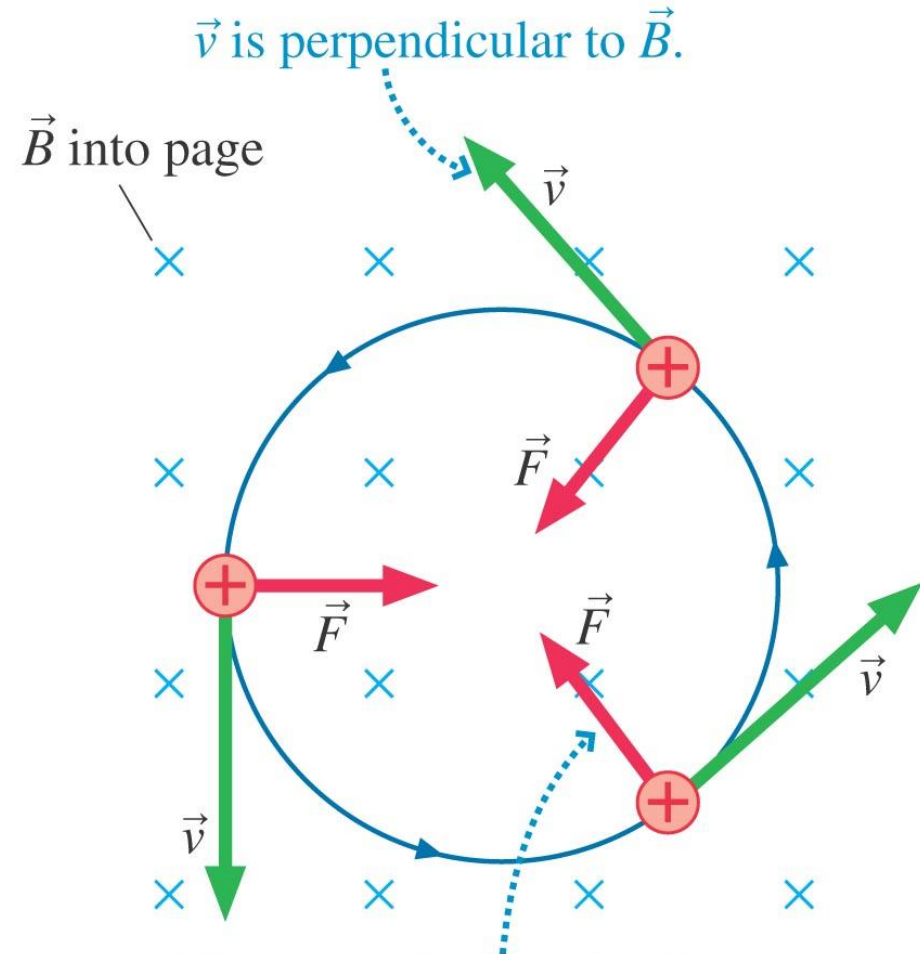
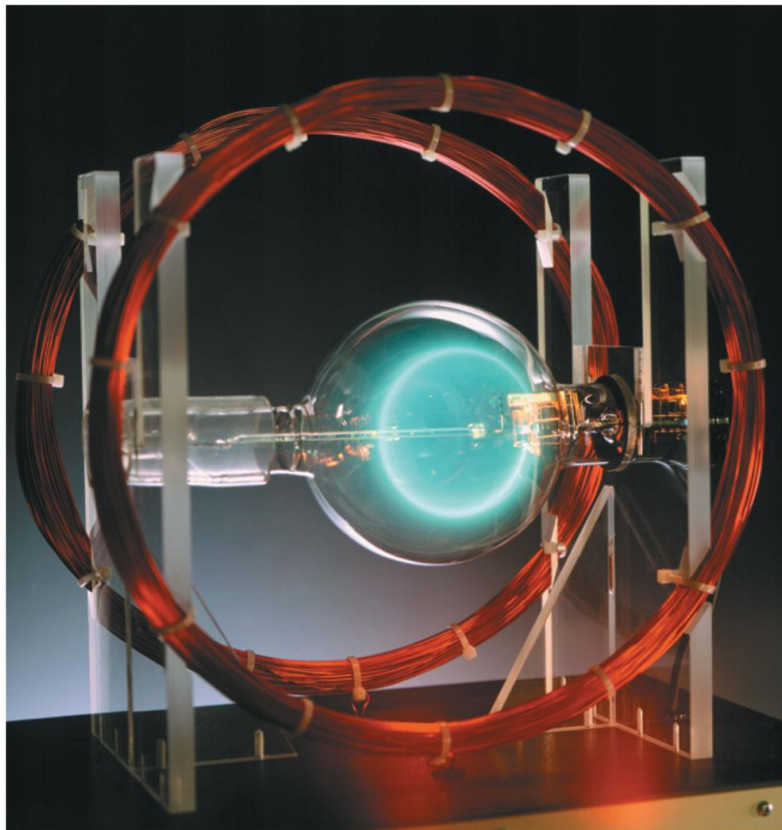
**It is perpendicular to both  $\vec{B}$  and  $\vec{v}$ .**



$$\vec{F}_B = q \vec{v} \times \vec{B} \quad \left\{ \begin{array}{l} \text{Magnitude: } F_B = qvB \sin \alpha \\ \text{Direction: RH rule} \end{array} \right.$$

# Cyclotron Motion

$$|\vec{F}_B| = |q| \cancel{v} B = m \frac{v^{\cancel{2}}}{R} \quad \boxed{R = \frac{mv}{|q|B}}$$



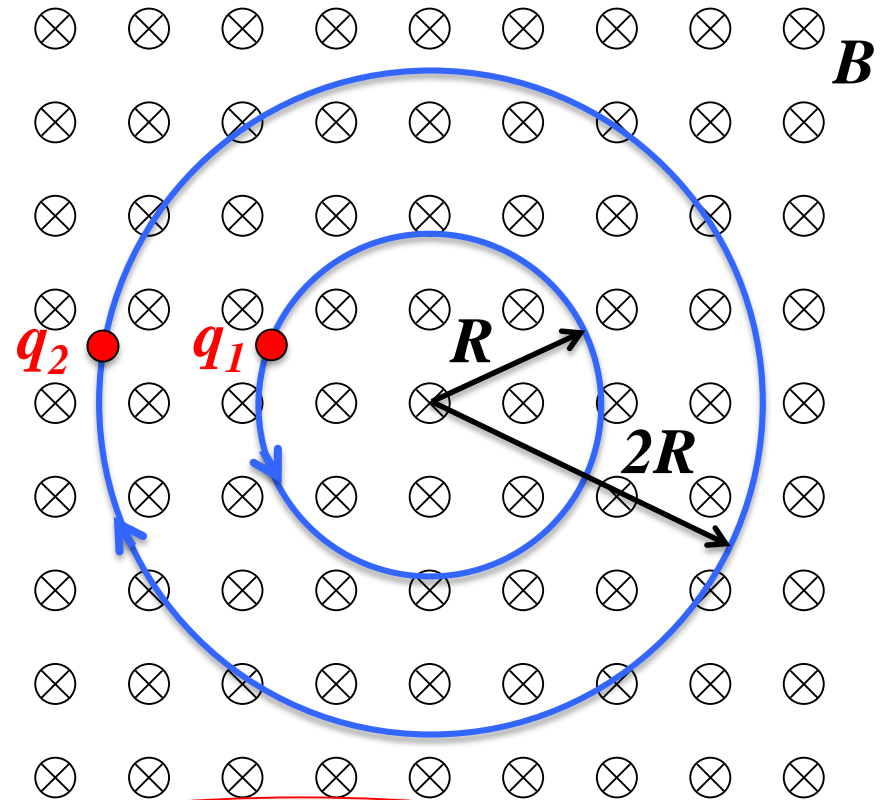
The magnetic force is always perpendicular to  $\vec{v}$ , causing the particle to move in a circle.

# Top Hat Question

# Top Hat Question

Two charges  $q_1$  and  $q_2$  with the same mass  $m$  and the same magnitude of charge  $|q|$  are undergoing cyclotron motion in a uniform B-field.

What are the **signs of the charges**?



A. Both positive

B. Both negative

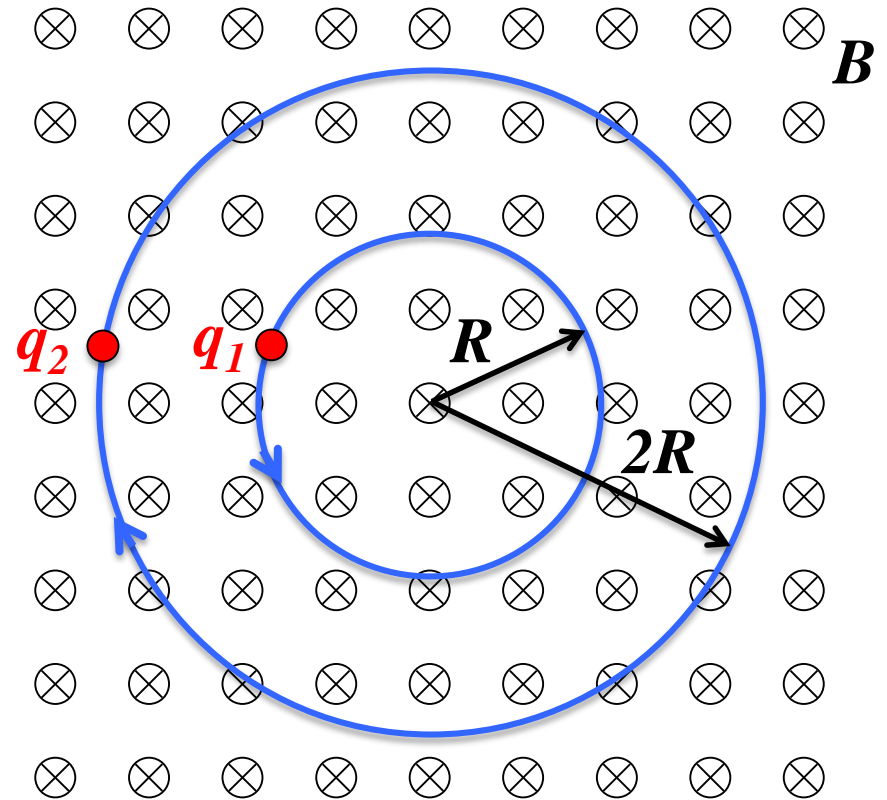
C.  $q_1$  positive,  $q_2$  negative

D.  $q_1$  negative,  $q_2$  positive

# Top Hat Question

Two charges  $q_1$  and  $q_2$  with the same mass  $m$  and the same magnitude of charge  $|q|$  are undergoing cyclotron motion in a uniform B-field.

If the speed of  $q_1$  is  $v$ , what is the speed of  $q_2$ ?



A.  $v$

C.  $\frac{1}{2} v$

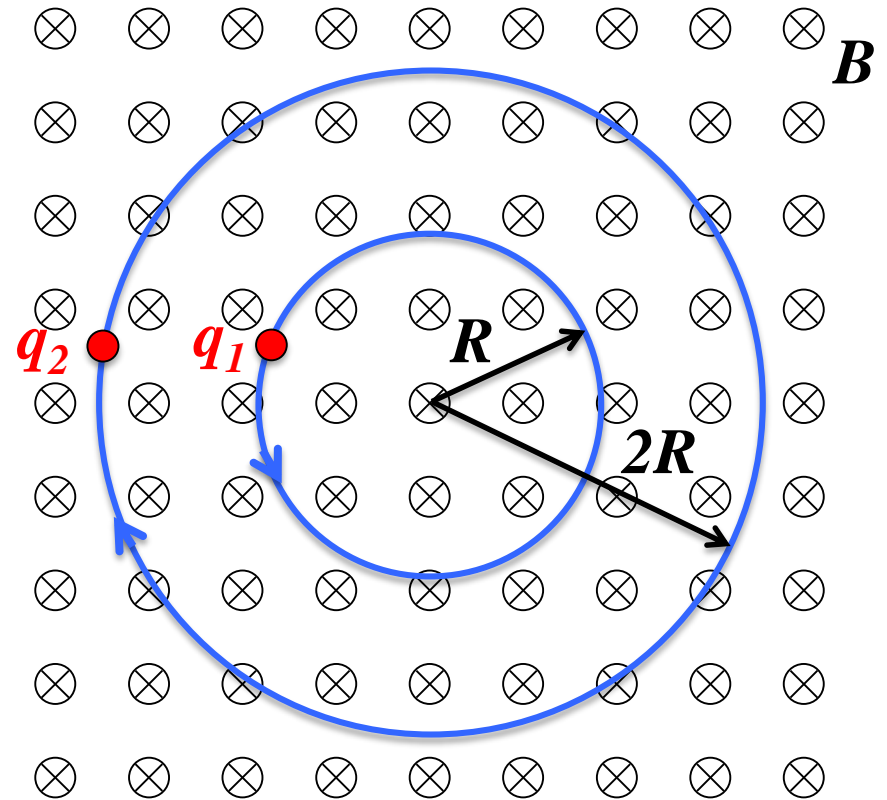
B.  $2v$

D.  $4v$

# Top Hat Question

Two charges  $q_1$  and  $q_2$  with the same mass  $m$  and the same magnitude of charge  $|q|$  are undergoing cyclotron motion in a uniform B-field.

If the period of rotation of  $q_1$  is  $T$ , what is the period of rotation of  $q_2$ ?



A.  $T$

B.  $2T$

C.  $\frac{1}{2}T$

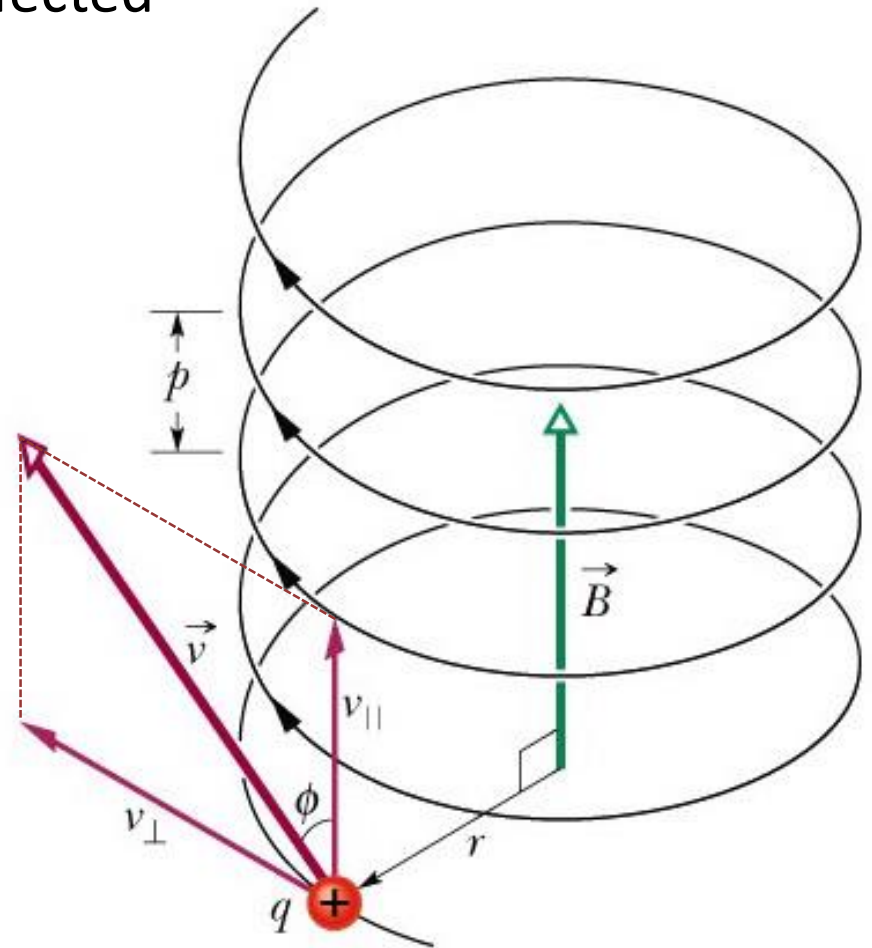
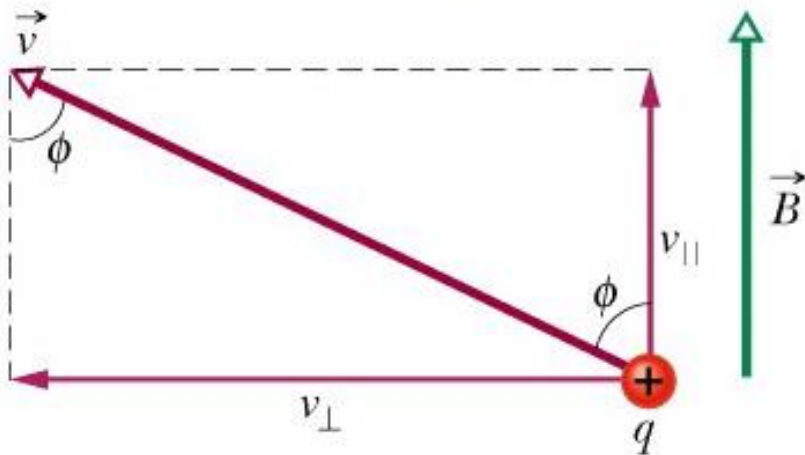
D.  $4T$



# Helical Paths Through a B-field

Splitting up the velocity into a component parallel to B-field and a component perpendicular to B-field immediately leads to helical motion: parallel component unaffected

The velocity component perpendicular to the field causes circling, which is stretched upward by the parallel component.

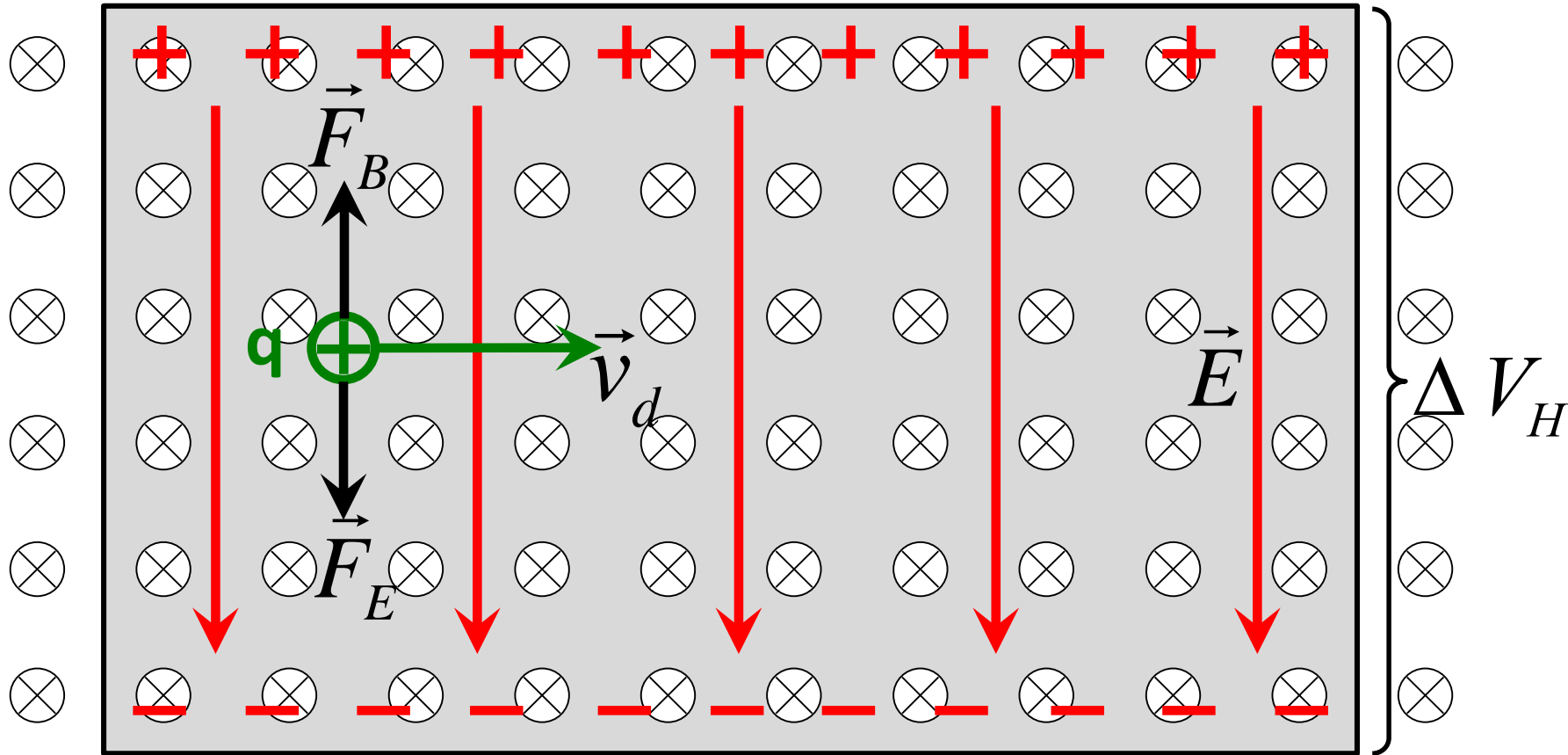


# Helical Paths: document camera

We can analyze and specify the motion exactly as the charge moves in a helix.

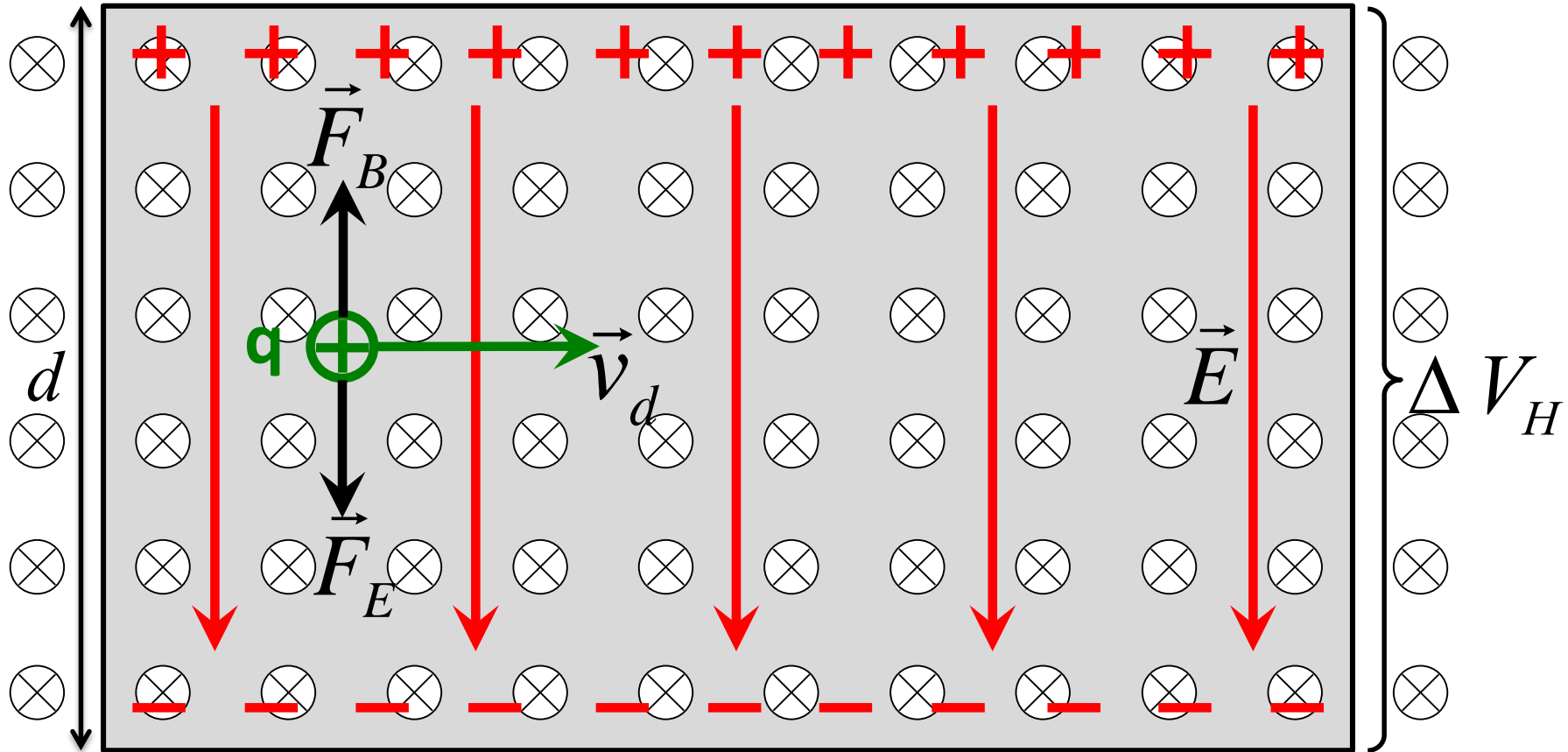
# The Hall Effect

Due to the B-field, net charge build up on the edges.



In equilibrium, current still flows. Need to balance the magnetic and electric forces on the charge carriers.

# The Hall Effect



$$F_B = q v_d B \quad F_E = q \frac{\Delta V_H}{d} \quad q \frac{\Delta V_H}{d} = q v_d B \quad \boxed{\Delta V_H = v_d B d}$$

# The Hall Effect

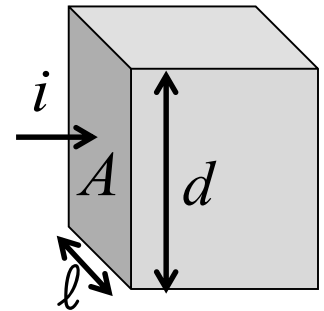
We have just found that the voltage established across a conductor carrying a current in a magnetic field is

$$\Delta V_H = v_d B d$$

We previously related the drift speed to the current via

$$v_d = \frac{i}{neA}$$

where  $A = \ell d$  and  $n$  is a material property



We can then relate the Hall voltage to known quantities:

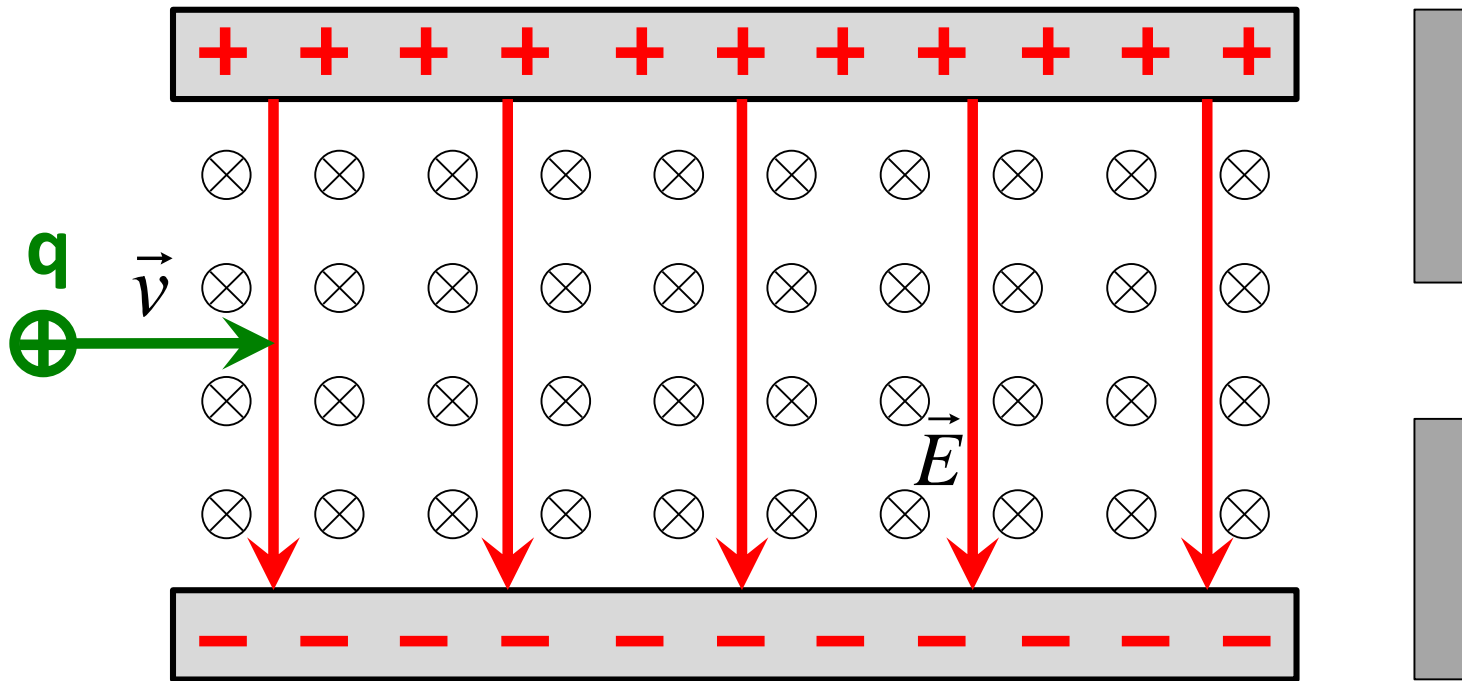
$$\Delta V_H = \frac{i}{ne\cancel{\ell d}} B \cancel{d} = \frac{iB}{ne\ell}$$

In practical applications, you measure  $\Delta V_H$  to find  $B$ :

$$B = \frac{ne\ell}{i} \Delta V_H$$

How the B-field probe used in the next lab works

# Similar concept: velocity selector

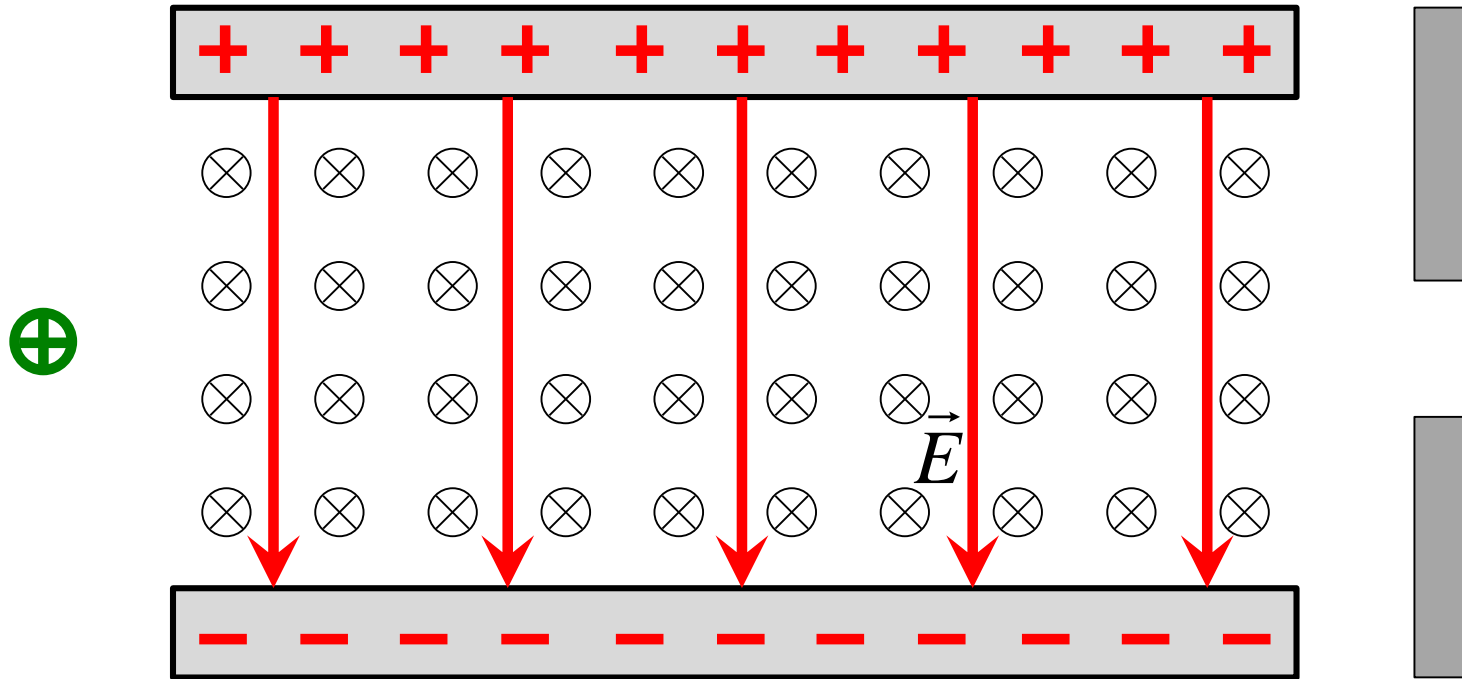


In a velocity selector, you send a charge through a region with crossed E and B fields, which leads to electric and magnetic forces:

$$\vec{F}_e = q\vec{E} \quad \vec{F}_B = q\vec{v} \times \vec{B} \quad qE = qvB \quad v = \frac{E}{B}$$

If the forces balance ( $F_{\text{net}} = 0$ ) the charge makes it through the slit

# Similar concept: velocity selector



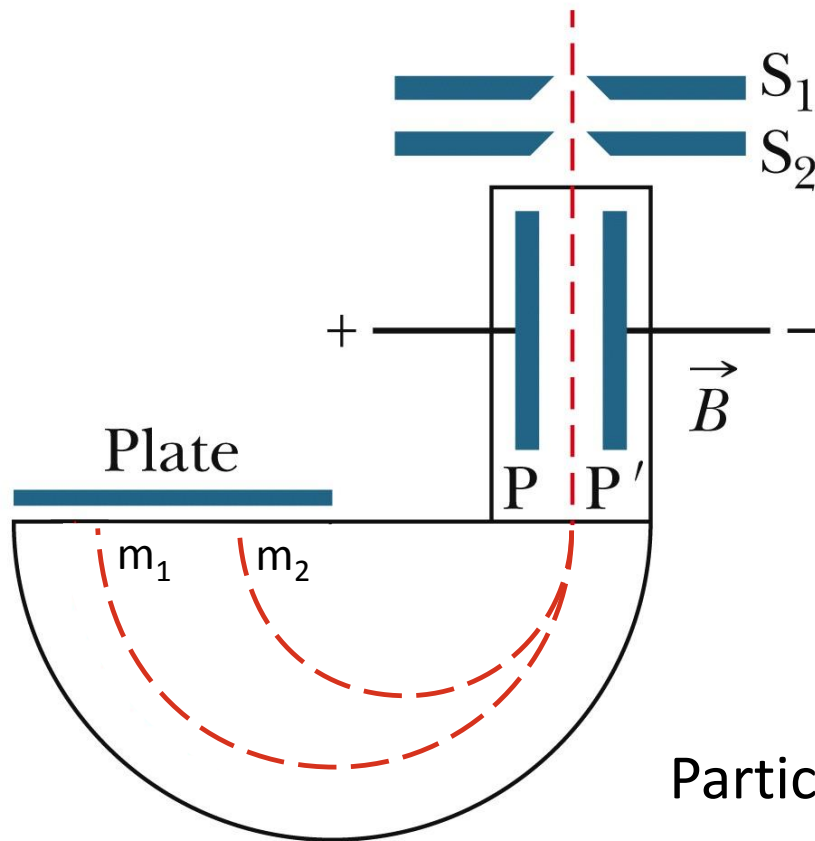
If the forces don't balance the charge hits the wall

$$qE - qvB = ma$$

We pick the E and B magnitudes to select the speeds we want

# Bainbridge Mass Spectrometer

Accelerate charges through  $\Delta V$  so they all have same Kinetic Energy



The slits  $S_1$  and  $S_2$  ensure the beam of particles is collimated.

The beam enters a region of crossed E and B-fields

A narrow slit ensures only particles with a specific speed enter

Particles with same KE but different masses and charges will have different radius in B field



Wednesday March 22, 2017 –  
lecture 2

# Last time:

- Charges on helical paths in B-field (aurora)
- The Hall Effect: underpinning of a B-field probe
- Velocity selector via crossed E- and B-fields
- Bainbridge Mass Spectrometer

# Today:

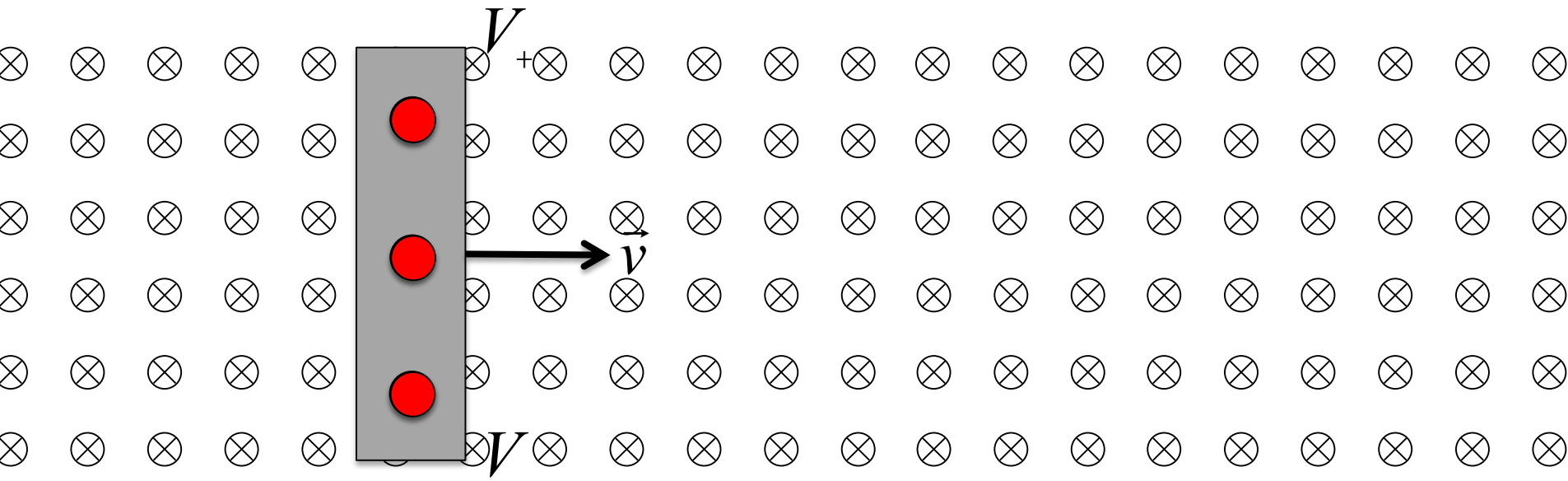
- Conductors moving through B-fields: Hall(ish) Effect
- Magnetic force on current carrying wires
- Torque on a current loop

# Conductors moving in B-fields

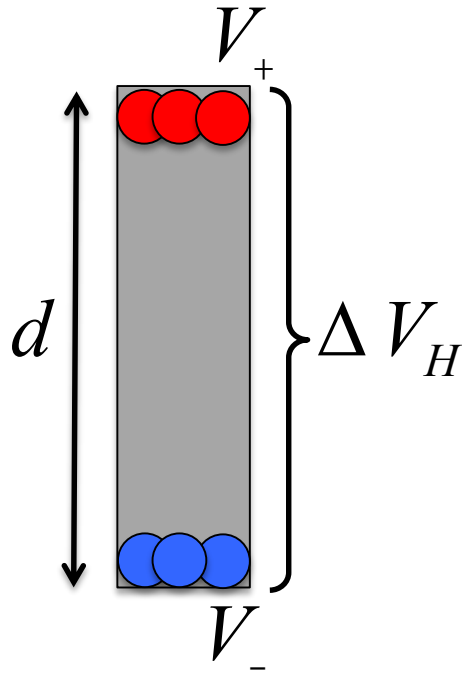
We've seen that free charges moving in a B-field feel a force perpendicular to the field and the charge's velocity:

$$\vec{F}_B = q \vec{v} \times \vec{B}$$

Conductors are full of charges that are free to move around (yet they have to stay confined to the conductor itself). If a conductor moves in a magnetic field, these charges also feel a magnetic force



# Conductors moving in B-fields



$$F_B = qvB$$

A red circle representing a positive charge is shown. An upward-pointing arrow is labeled  $F_B = qvB$  and a downward-pointing arrow is labeled  $F_E = q \frac{\Delta V_H}{d}$ .

$$F_E = q \frac{\Delta V_H}{d}$$

$$F_E = q \frac{\Delta V_H}{d}$$

A blue circle representing a negative charge is shown. An upward-pointing arrow is labeled  $F_E = q \frac{\Delta V_H}{d}$  and a downward-pointing arrow is labeled  $F_B = qvB$ .

$$F_B = qvB$$

In equilibrium, forces balance, leading to a constant voltage

$$q \frac{\Delta V_H}{d} = qvB$$

$$\Delta V_H = vBd$$

# Forces on Current-Carrying Wires

Current in wires is nothing more than charges in motion. It doesn't matter if we consider  $-q$  moving opposite  $i$  or  $+q$  moving in the same direction as  $i$

In a magnetic field, these charges feel a force and get deflected from their normal straight path. For a single charge:

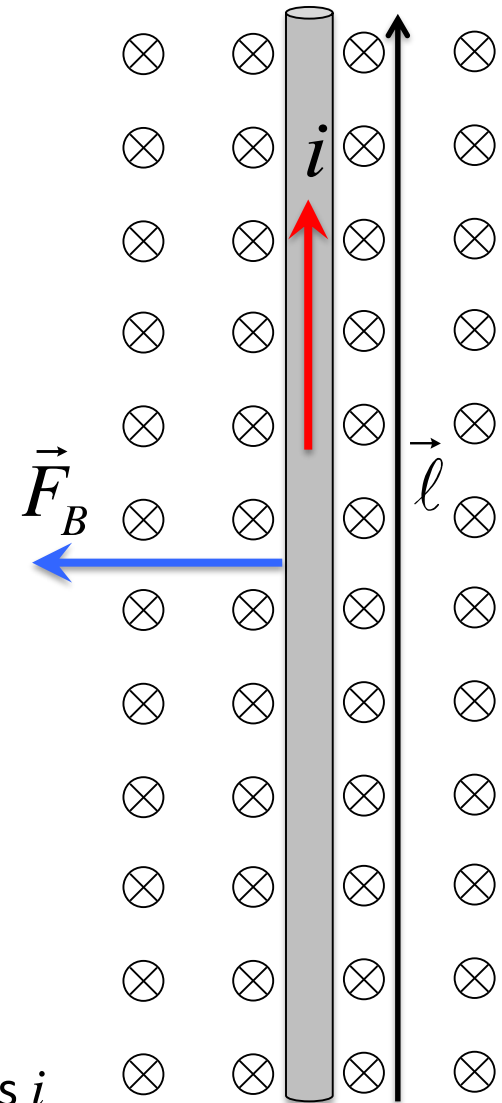
$$\vec{F}_B = q \vec{v}_d \times \vec{B}$$

For  $N$  charges moving through the wire:

$$Nq\vec{v}_d = (nAq\vec{v}_d)\ell = i\vec{\ell}$$

$$\vec{F}_B = i\vec{\ell} \times \vec{B}$$

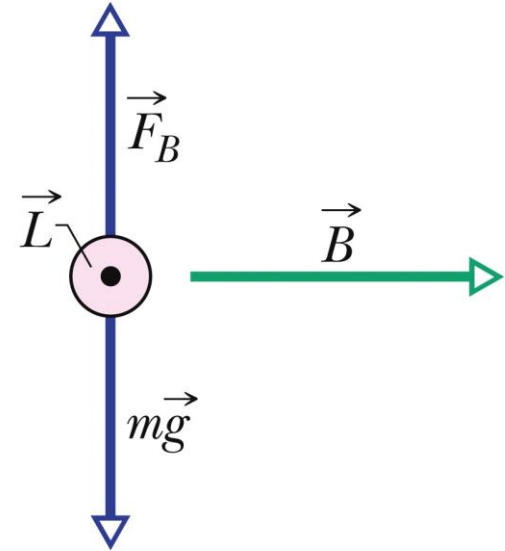
Length of wire, direction same as  $i$



# TopHat Question

A wire of length 50 cm is carrying a current  $i$  out of the page and is sitting in a uniform magnetic field of 500 mT pointing to the right. If the wire has a mass of 25 g, what current  $i$  is needed to support its weight?

$$\vec{F}_B = i\vec{\ell} \times \vec{B}$$



Copyright © 2014 John Wiley & Sons, Inc. All rights reserved.

halliday\_10e\_fig\_28\_17

A.  $9.81 \times 10^{-3} \text{ A}$

C.  $0.981 \text{ A}$

B.  $1.02 \text{ A}$

D.  $1.02 \times 10^{-3}$

# TopHat Question

A wire of length 50 cm is carrying a current  $i$  and is sitting in a uniform magnetic field  $B$  as shown. What is the magnitude and direction of the magnetic force on the wire?

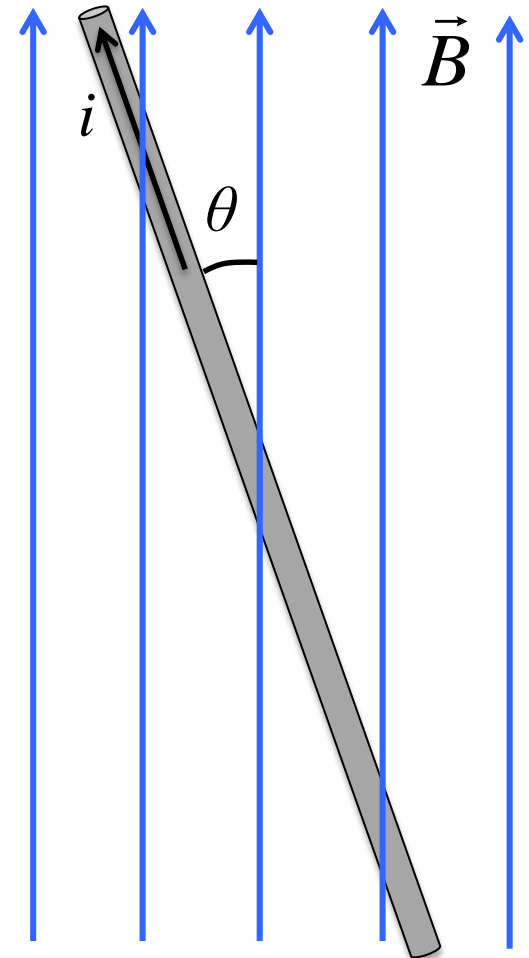
$$\vec{F}_B = i\vec{\ell} \times \vec{B}$$

A.  $ilB$   $\odot$

C.  $ilB\sin\theta$   $\nwarrow$

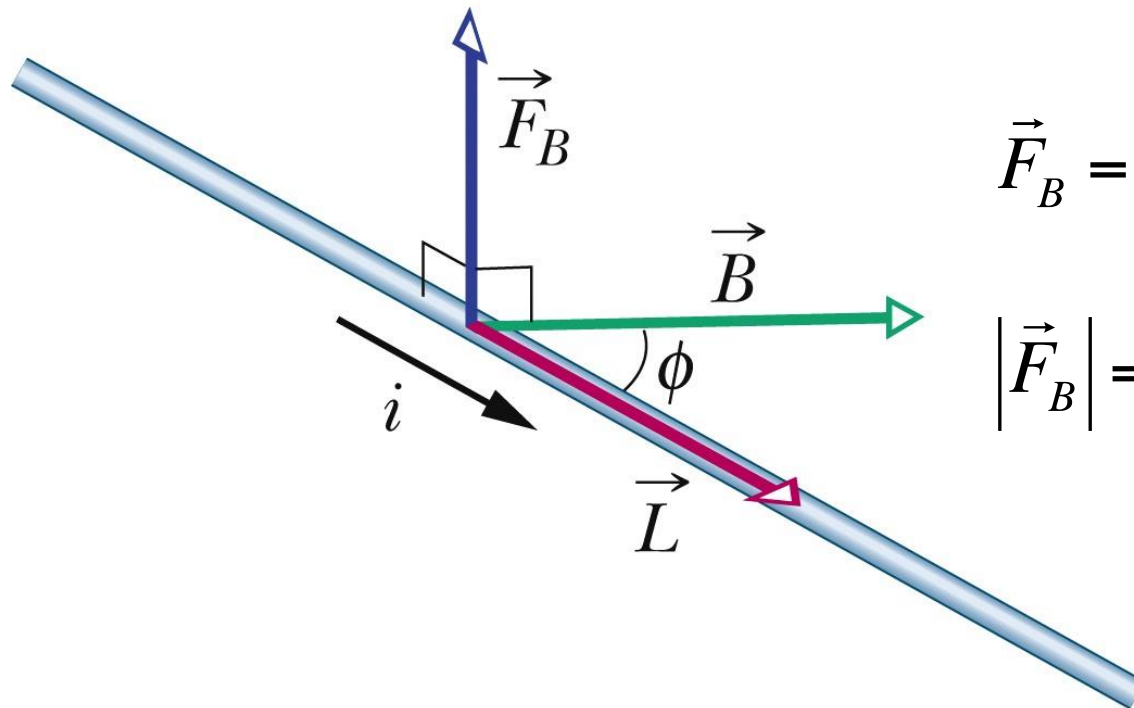
B.  $ilB\sin\theta$   $\otimes$

D.  $ilB$   $\nearrow$



# Forces on Current-Carrying Wires: B and L not perpendicular

The force is perpendicular to both the field and the length.



$$\vec{F}_B = i\vec{L} \times \vec{B}$$

$$|\vec{F}_B| = |i\vec{L} \times \vec{B}| = iLB \sin \phi$$



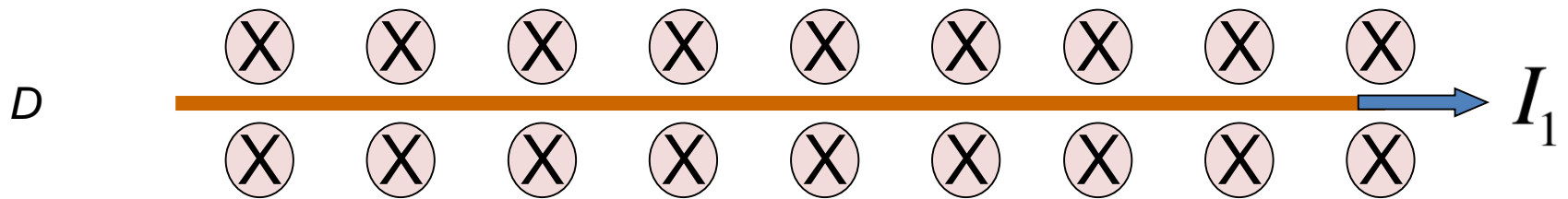
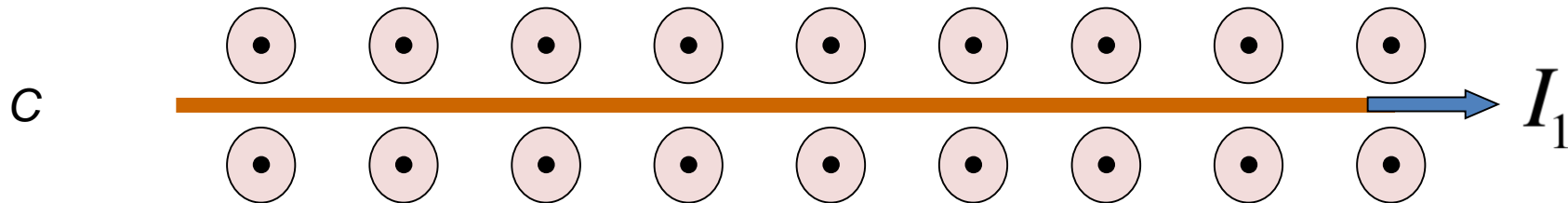
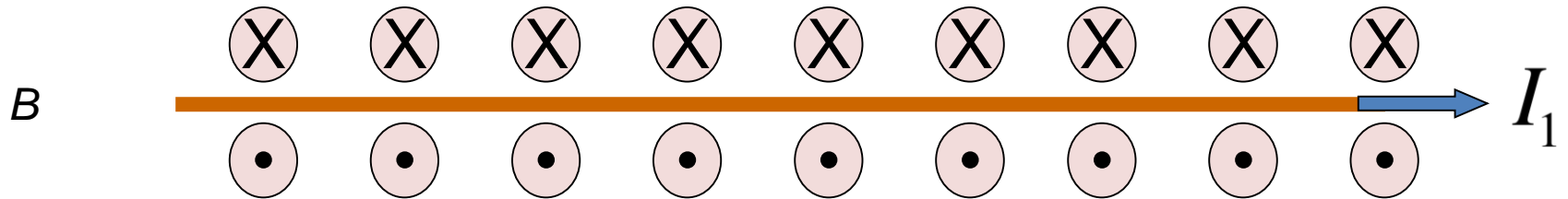
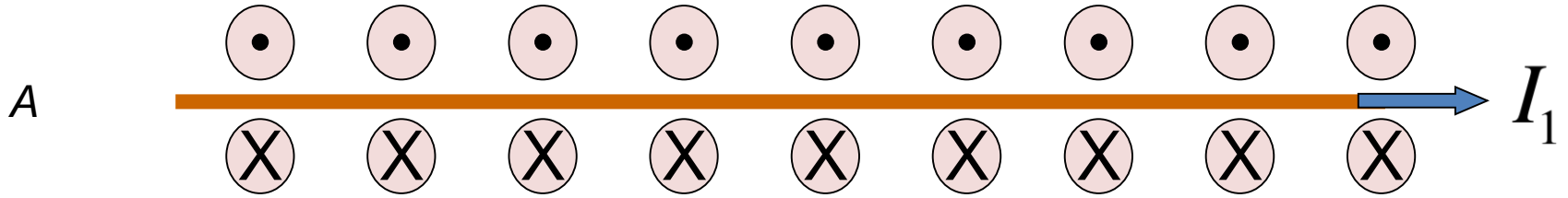
# Force Between Two Parallel Wires



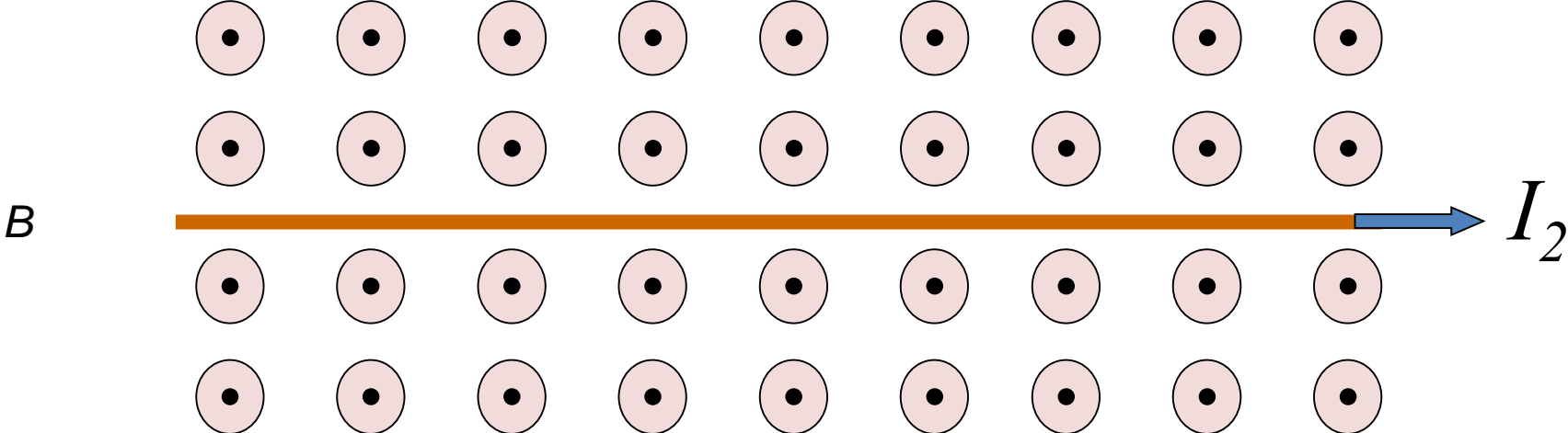
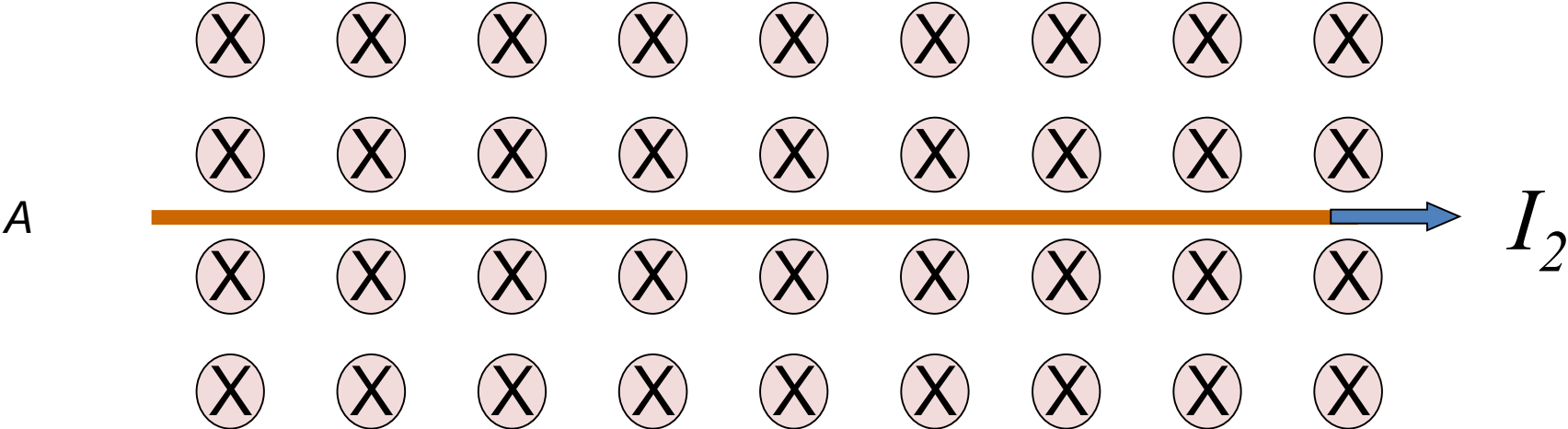
Derive an expression for the magnetic force on wire #2 due to the magnetic field created by the current flowing in wire #1.

# Top Hat Question

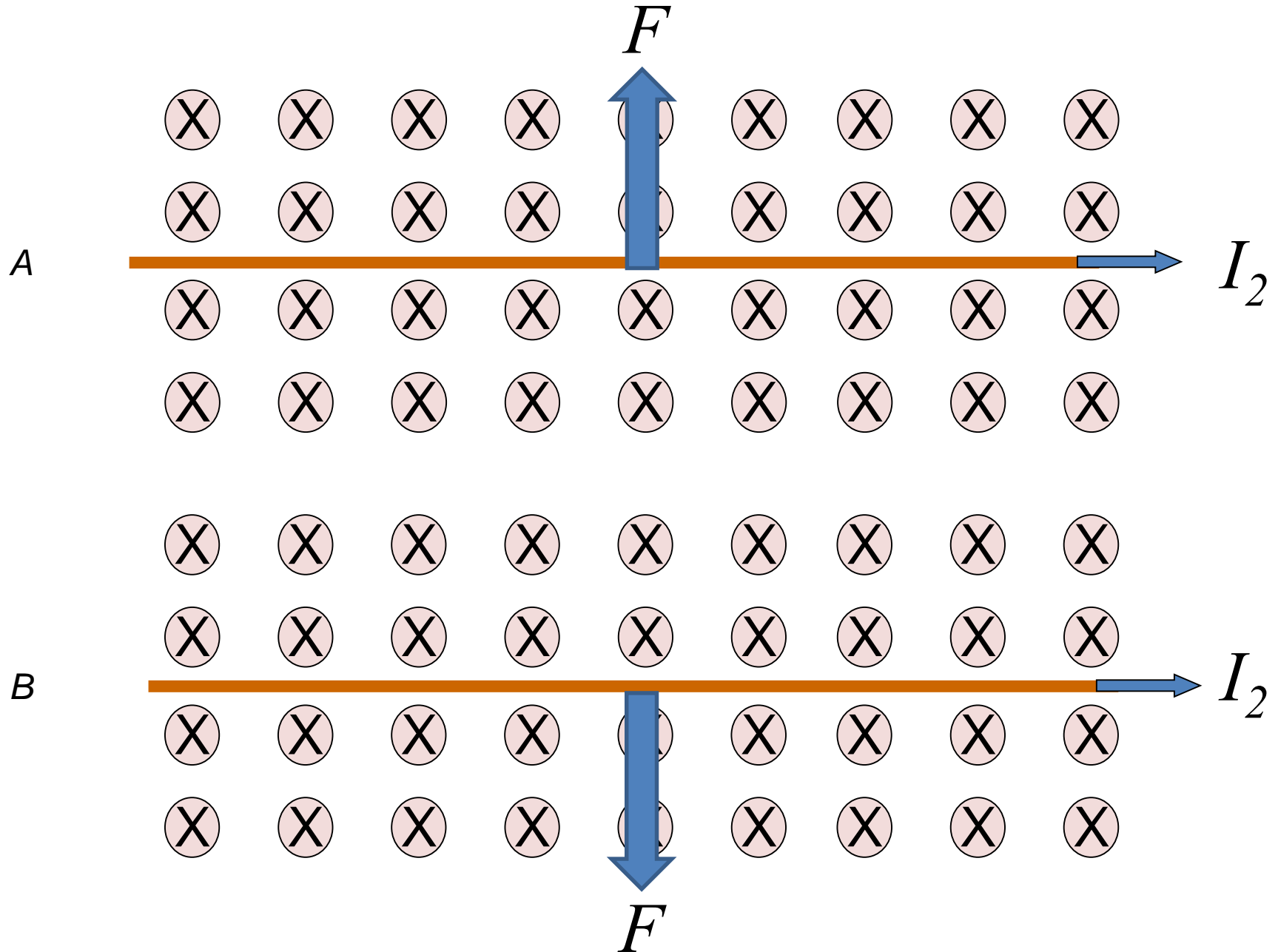
Which figure correctly represents magnetic field created by  $I_1$  flowing in upper wire?



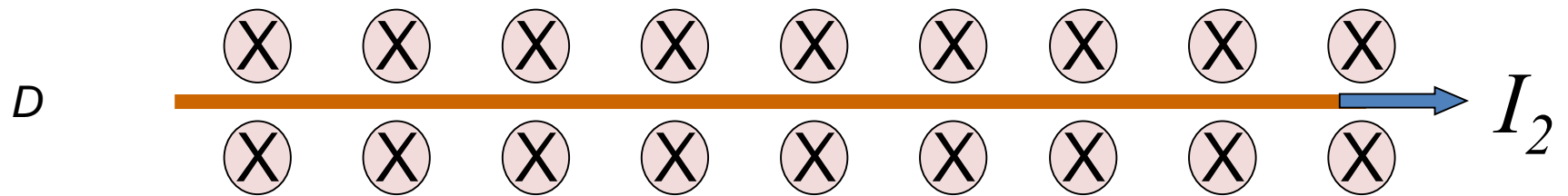
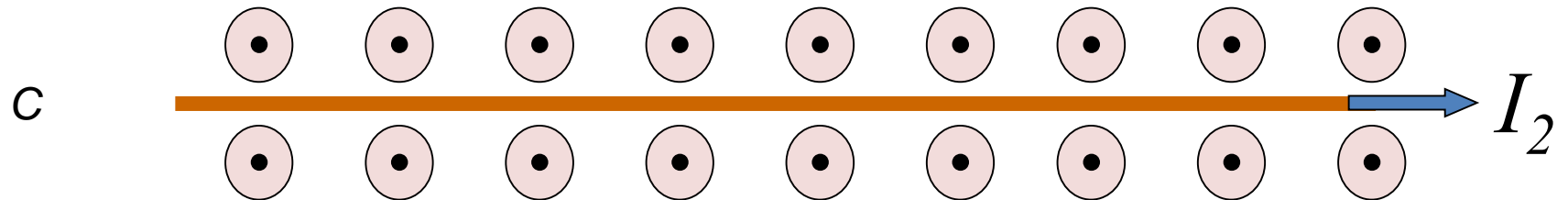
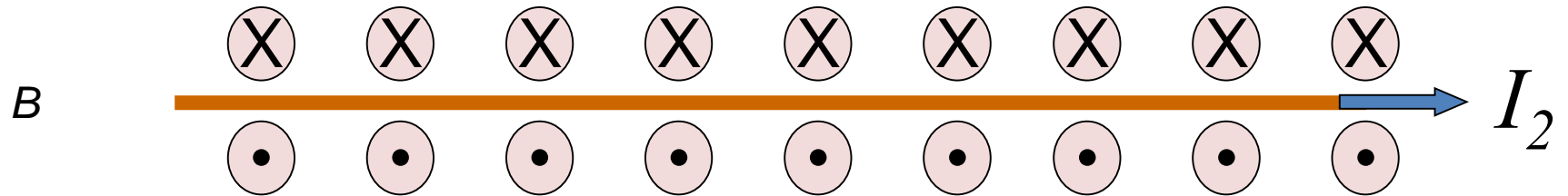
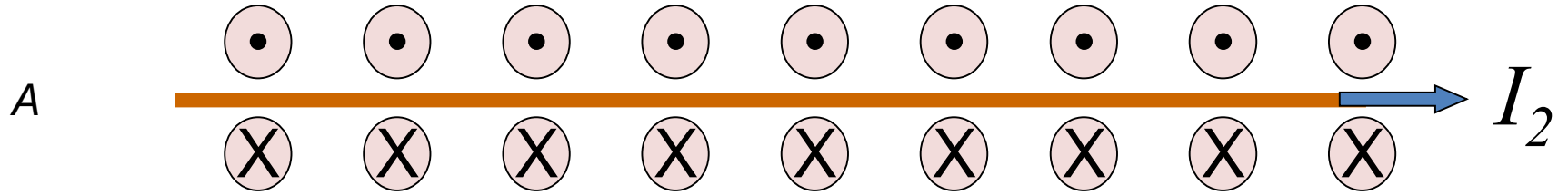
Which figure correctly represents magnetic field in which the lower wire is immersed?



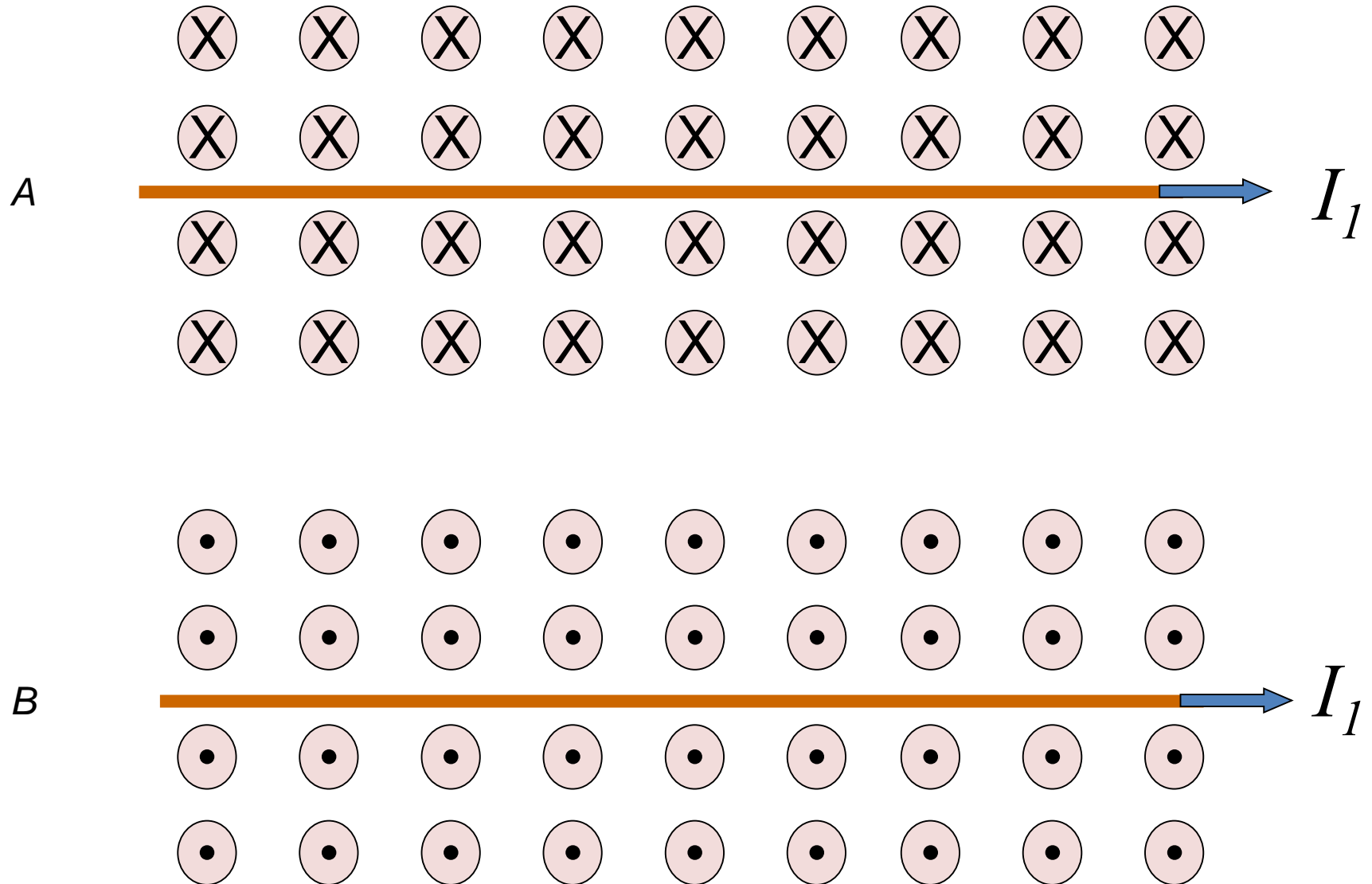
Which figure correctly represents **force** on the **lower wire**?



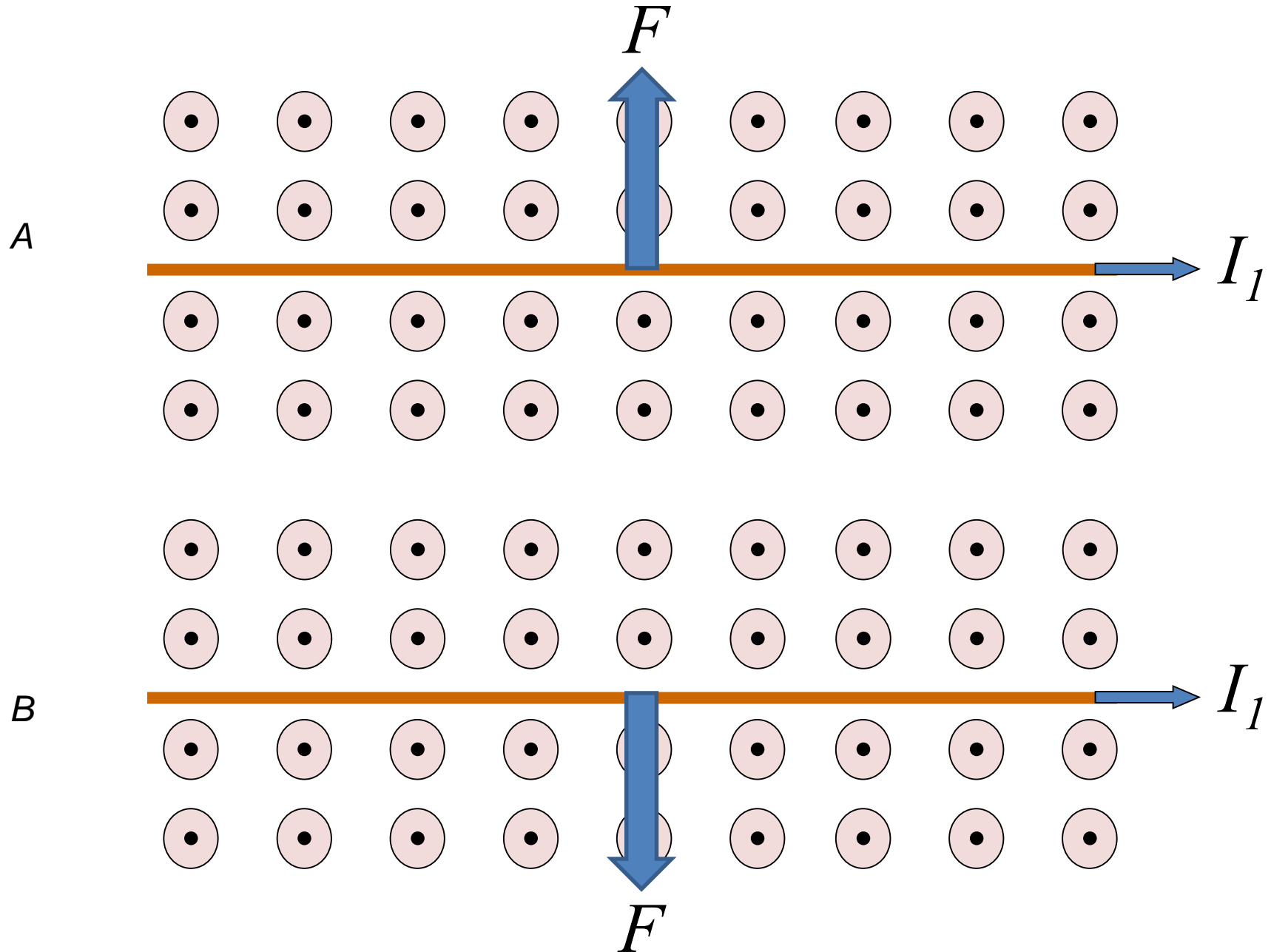
Which figure correctly represents magnetic field created by  $I_2$  flowing in lower wire?



Which figure correctly represents **magnetic field** in which **upper wire** is immersed?

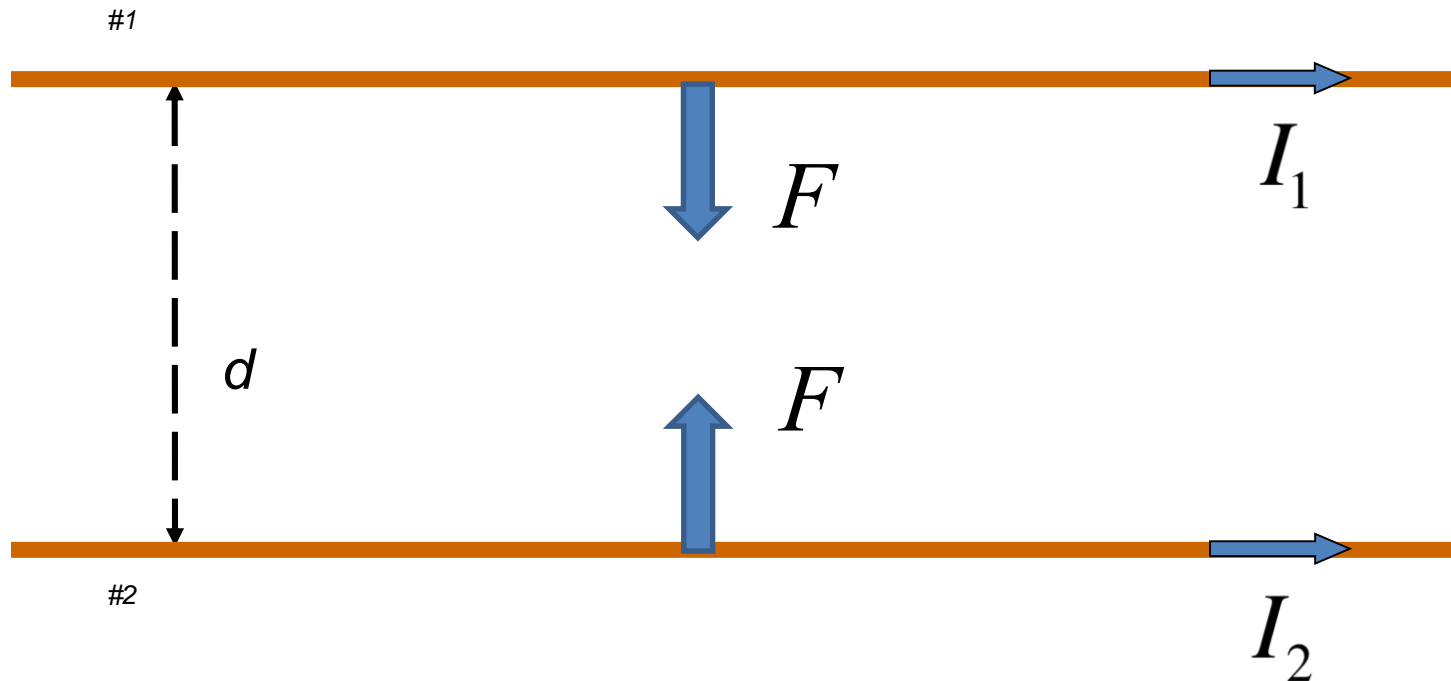


Which figure correctly represents **force** on the **upper** wire?





# Force Between Two Parallel Wires



Derive an expression for the magnetic force on wire #2 due to the magnetic field created by the current flowing in wire #1.