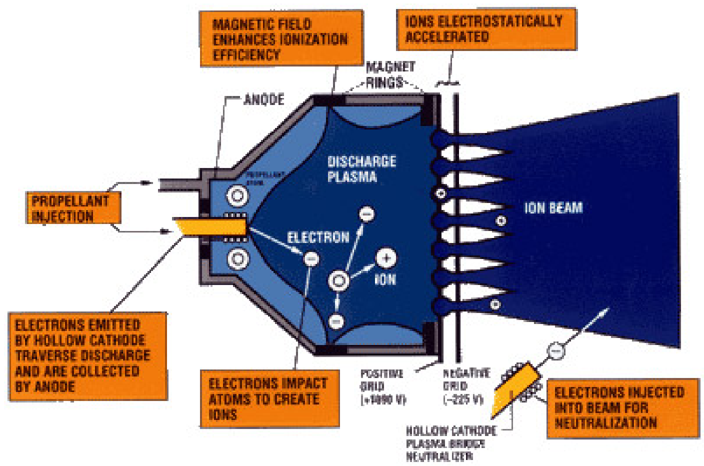
University of Calgary
Department of Physics and Astronomy
PHYS 259, Winter 2017

Labatorial 4: Motion of electric charges in electric fields

A charged particle in an electric field experiences a force, and therefore an acceleration. As a result, the particle gains kinetic energy and/or is deflected from its original direction of motion. These effects are used in many technical applications, from ink jet printers to particle accelerators.

The figure shows a technical application of electrostatic acceleration that you don't see every day: ion propulsion. Here, xenon gas is ionized and then electrically accelerated to a speed of about 25 miles per second (or 144810 km/h). When the xenon ions are emitted from the spacecraft, they push it in the opposite direction (more information at http://www.nasa.gov/mission_pages/dawn/news/dawn-20070913f2.html)



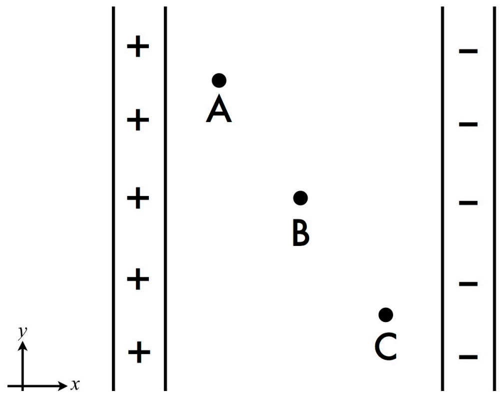
Goals:
To understand and be able to construct the trajectories of electric charges in various electric fields, both uniform and non-uniform.

Preparation:
Halliday, Resnick, and Walker, "Fundamentals of Physics" 10th edition, Wiley: 22.1–22.2; Labatorial 1.

Equipment:
Computer with browser to use the simulations at http://www.falstad.com/mathphysics.html; the "Electric Field Hockey" simulation (**electric-hockey_en.jar**), developed by the PhET Interactive Simulations Project at the University of Colorado, http://phet.colorado.edu.

Note that gravity can be ignored in the problems on this worksheet.

1 Motion of a charged particle in a uniform electric field I: Initial velocity parallel to the field



Question 1: The figure shows two infinitely large parallel charged plates.

a) How does the electric field $\vec{E}(x,y)$ in the space between the plates depend on position? What is the electric field to the left and to the right of both plates?

b) Draw the electric force that a positively charged particle would experience if it were placed at point A, B, or C.

c) Assume that the positively charged particle q with mass m starts at point A. If the electric field between the plates is E_x, what is the acceleration experienced by the charged particle?

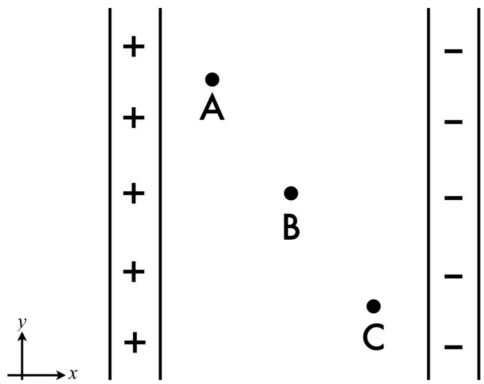
d) If the charged particle starts at point (x_0, y_0) with an initial velocity v_{0x} in the positive x-direction, what are the x- and y-components of the position of the particle as functions of time, $x(t)$ and $y(t)$? Ignore the force of gravity.

e) Using the third kinematic equation (see formula sheet), find an expression for the change in the kinetic energy of the charged particle after it has moved through a distance Δx.

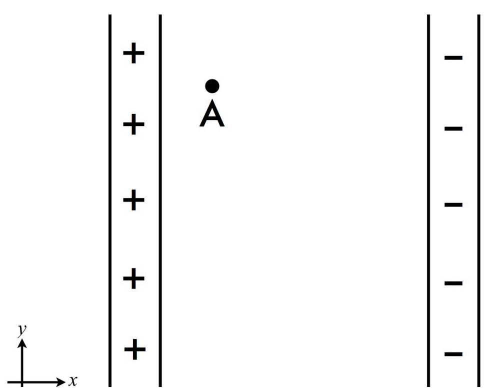
f) Use the relation $\Delta K = -q\Delta V$, where ΔV is the difference in potential between two points, to find the potential difference between the two plates if they are separated by a distance d.

2 Motion of a charged particle in a uniform electric field II: Initial velocity orthogonal to the field

Question 2: Draw the force that a positively charged particle with initial velocity in the negative y-direction would experience if it were placed at point A, B, or C. Explain how you determined this.

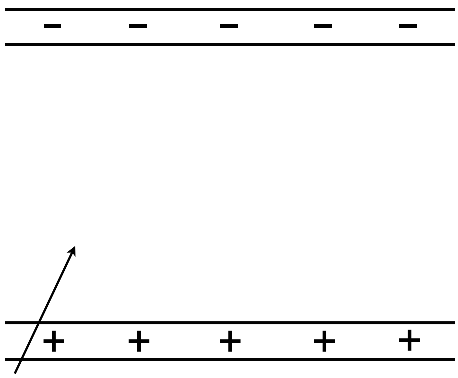


Question 3: Draw the trajectory that the positively charged particle would follow in this field if it started at point A with an initial velocity in the negative y-direction. Explain how you constructed the trajectory.



CHECKPOINT 1: Before moving on to the next part, have your TA check the results you obtained so far.

3 Motion of a charged particle in a uniform electric field III: Initial velocity at an arbitrary angle to the field



Question 4: Now consider the case that an electron is shot through a tiny hole in one of the plates so that its initial velocity is at some angle with respect to the electric field vector.

a) What direction is the net force on the electron? Sketch the trajectory that the electron will follow.

b) What type of motion is this analogous to? Assuming the electron travels slowly enough so that it does not reach the negative plate, which equation(s) would you use to find the closest point to the negative plate along the trajectory?

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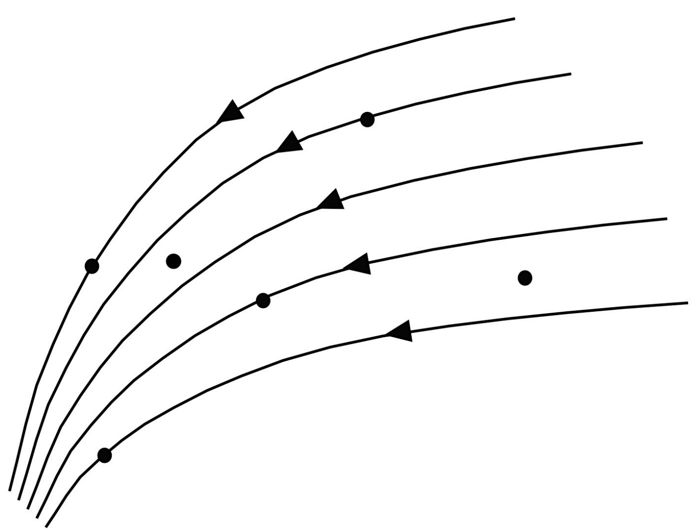
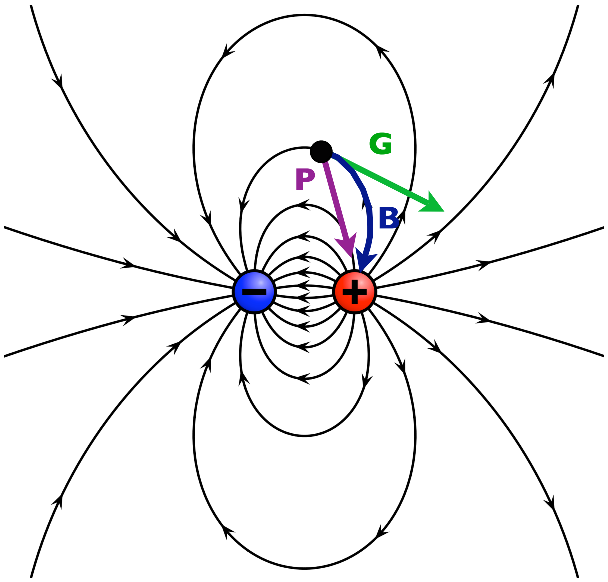
4 Motion of a charged particle in a non-uniform electric field

Question 5: For each point shown in the figure, draw the force that a positively charged particle would experience. Make sure to clearly show the directions and the relative magnitudes of these forces.



Question 6: A group of students was asked to draw the force on a negatively charged particle at the point indicated by the solid dot in the figure. They came up with three different answers G, B, and P shown. For each of the answers, state whether it is correct or not, and why.

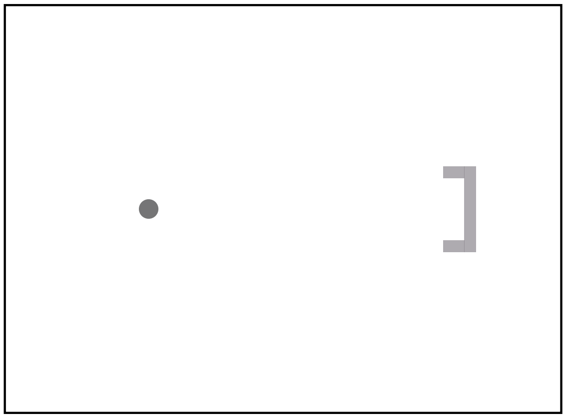


CHECKPOINT 2: Before moving on to the next part, have your TA check the results you obtained so far.

5 Electric Field Hockey

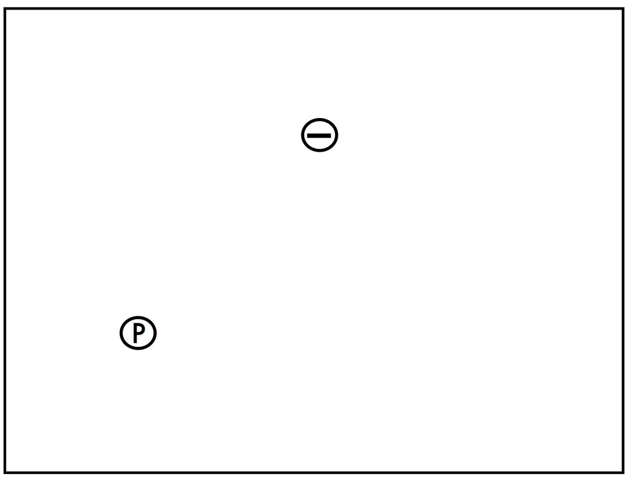
Open the simulation **electric-hockey_en.jar**. The object of the game is to place as few charges as possible at fixed positions on the screen in order to attract or repel the test charge (the "puck") so that it goes into the net on the right side of the screen. Make sure that the box "Puck is positive" is selected while you are playing.

Question 7: First select "Practice" at the bottom of the screen. Find a goal-scoring strategy involving a single negative fixed charge. Record your strategy in the diagram by indicating with a minus sign where you placed the fixed charge. Also draw the path the puck followed (you can trace the trajectory in the simulation)

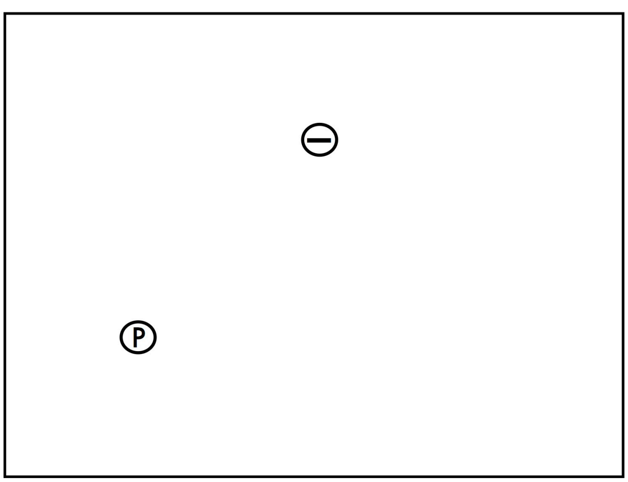


Question 8: Now find a goal-scoring strategy involving a single positive fixed charge. Record your strategy in the diagram by indicating with a plus sign where you placed the fixed charge. Also draw the path the puck followed.

Question 9: Before moving on to higher difficulty levels, we will practice some "shots" and to get a feel for how to control the puck:



a) Set up a negative charge half way between the puck and the hockey goal (above the puck), as shown in the figure. Draw the trajectory.



b) How can you use a positive charge to make the puck swing around the negative charge instead of colliding with it? Draw your solution in the figure, both the position of the positive charge and the trajectory of the test particle.

Question 10: a) In your solution to the previous question, what is the direction of the force acting on the puck when it is close to the negative charge?

b) Given that direction, why does the puck not hit the negative charge?

Question 11: As you move to higher difficulty levels in the game, you will have to make the puck detour around solid obstacles before it enters the net. You can select the "Trace" and "Field" settings to see what happens when you restart the simulation. This will help you in placing the fixed charges and recording the puck's path. Find a goal-scoring strategy at Difficulty 1 level. Record your strategy in the diagram by indicating where you placed the fixed charges. Also draw the path the puck followed. Feel free to try higher difficulty levels, if time permits.

Last Checkpoint! Clean up your area, and put the equipment back the way you found it. Call your TA over to check your work and your area before you can get credit for the labatorial.

Equations and Constants

$$F_g(r) = G\frac{m_1 m_2}{r^2}$$
$$U_{grav}(y) = mgy$$
$$K = \frac{1}{2}mv^2$$
$$v_x(t) = v_{0x} + a_x t$$
$$x(t) = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$$
$$v_x^2(t) = v_{0x}^2 + 2a_x(x(t) - x_0)$$
$$\omega = \frac{d\theta}{dt}$$
$$v = \frac{2\pi r}{T} = \omega r$$
$$a_{rad} = \frac{v^2}{r} = \omega^2 r$$
$$F_C(r) = \frac{1}{4\pi\epsilon_0}\frac{|q_1 q_2|}{r^2}$$
$$\vec{E} = \frac{\vec{F}}{q}$$

$$g = 9.81\frac{m}{s^2}$$
$$G = 6.67 \times 10^{-11}\frac{Nm^2}{kg^2}$$
$$\frac{1}{4\pi\epsilon_0} = 8.99 \times 10^9\, Nm^2C^{-2}$$
$$\epsilon_0 = 8.85 \times 10^{-12}\, C^2N^{-1}m^{-2}$$
$$e = 1.60 \times 10^{-19}\, C$$
$$m_e = 9.11 \times 10^{-31}\, kg$$
$$m_p = 1.67 \times 10^{-27}\, kg$$
$$m_n = 1.67 \times 10^{-27}\, kg$$

$$\int \frac{dx}{(x^2 \pm a^2)^{3/2}} = \frac{\pm x}{a^2\sqrt{x^2 \pm a^2}}$$
$$\int \frac{xdx}{(x^2 \pm a^2)^{3/2}} = -\frac{1}{\sqrt{x^2 \pm a^2}}$$
$$\int \frac{dx}{x^2 + z^2} = \frac{1}{z}\arctan(\frac{x}{z})$$
$$\arctan(x) = -\arctan(-x)$$