Last time

- Sources of magnetic field (Biot-Savart Law)
- Magnetic field of a moving point charge
- Magnetic field of a current carrying conductor
- Magnetic field of an infinitely long straight current carrying conductor

This time

- Mutual force between two long straight current carrying conductors
- Magnetic field of a circular current loop on the axis of the loop
- Introduction of Maxwell's Equations

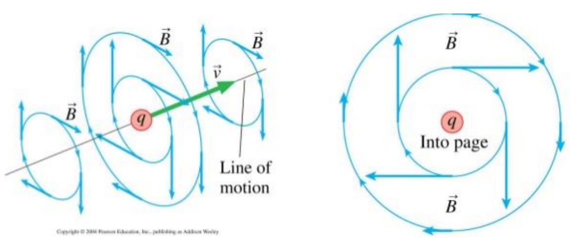
Sources of magnetic field

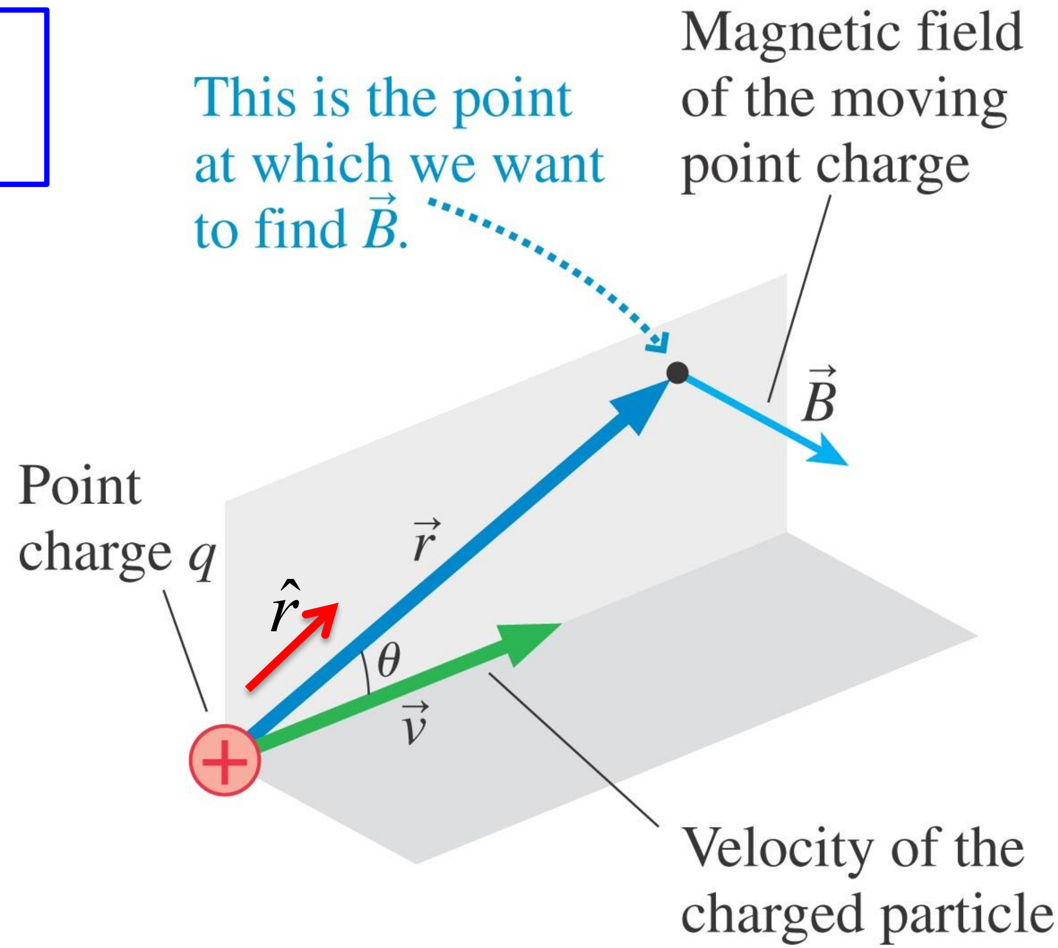
The Biot-Savart Law

(Bee-oh Sah-var)

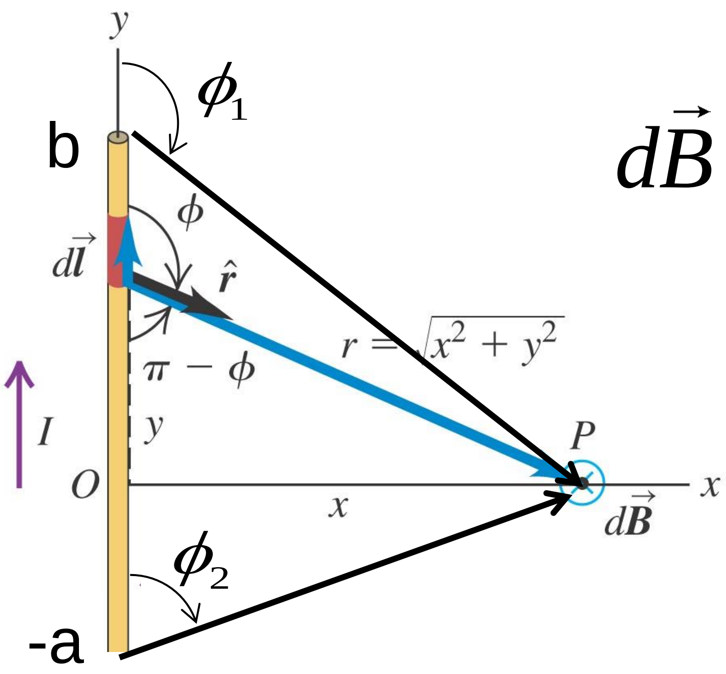
Magnetic fields are caused by **moving charges**.

$$\vec{B}_{\text{point charge}} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2}$$





Magnetic field of a straight current-carrying conductor



$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{l} \times \hat{r}}{r^2}$$

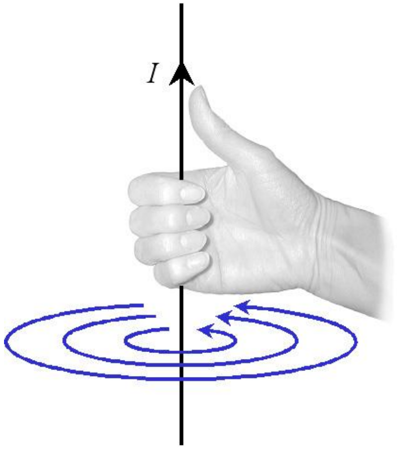
Out of the page to the left of the wire.

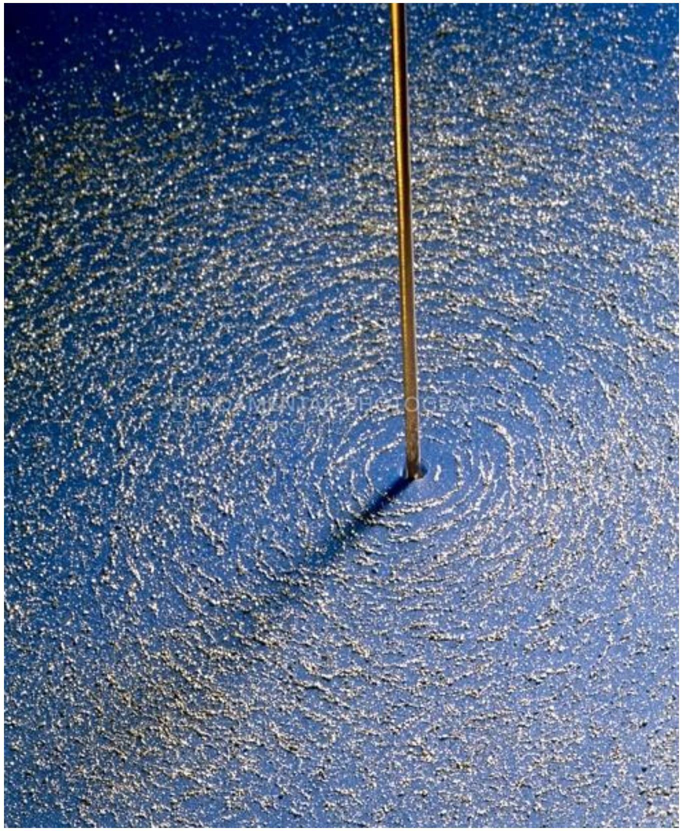
Into the page to the right of the wire.

$$B = \frac{\mu_0 I}{4\pi x}\left[\cos\phi_2 - \cos\phi_1\right]$$

Magnetic field of a infinitely long straight current carrying conductor

$$B = \frac{\mu_0 I}{2\pi x}$$





Magnetic field lines are circular loops.

TopHat Question

A long straight wire carries current I out of the page. An proton travels towards the wire from the right. What is the direction of the force on the electron?

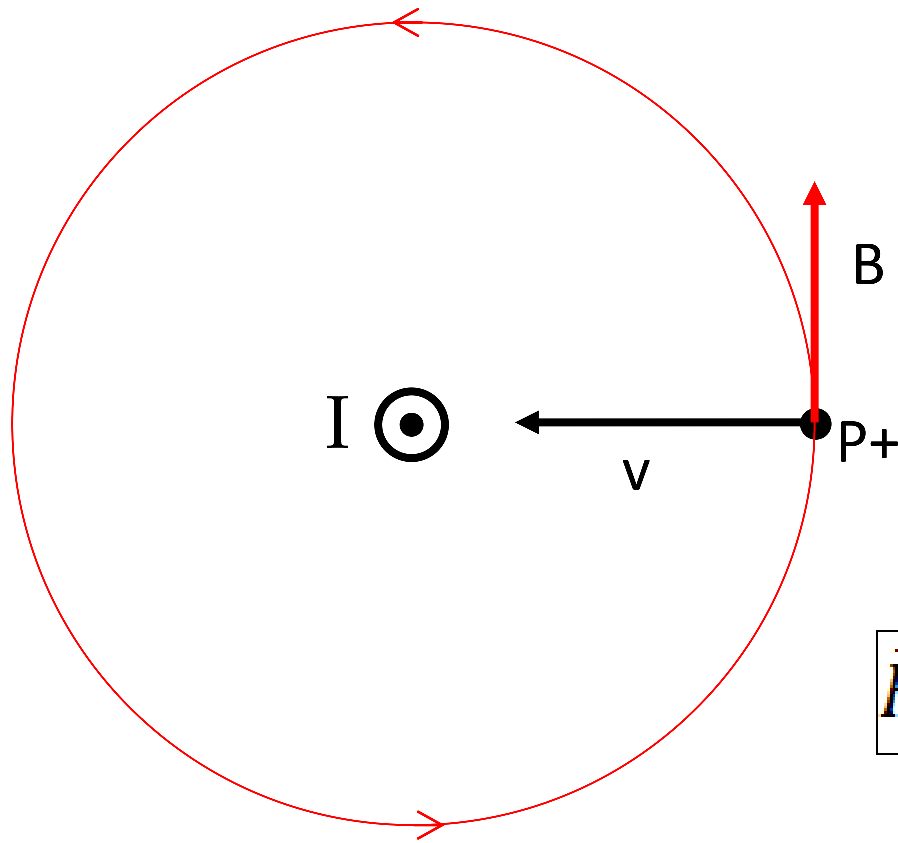
A. $\otimes$

B. $\odot$

C. $\leftarrow$

D. $\rightarrow$

E. $\downarrow$



$$\vec{F} = q\vec{v} \times \vec{B}$$

TopHat Question

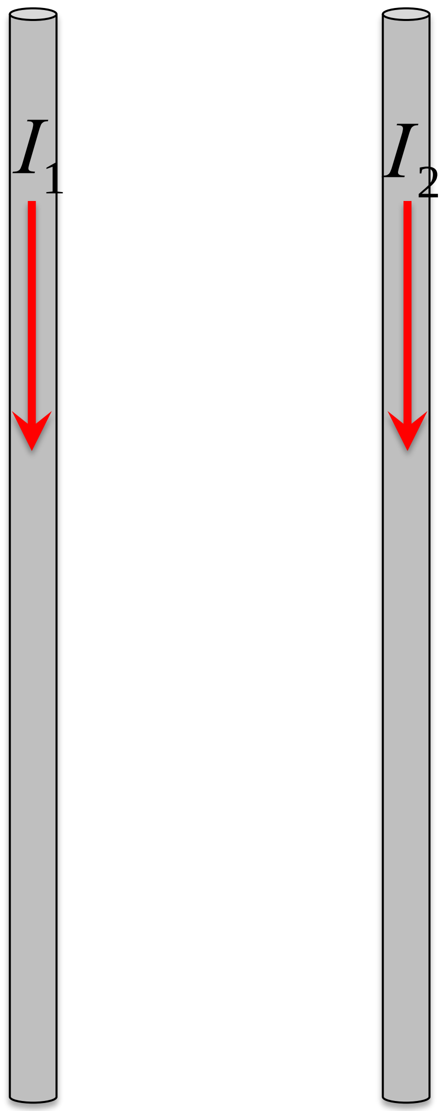
Two wires carry currents I_1 and I_2 as shown. What direction is the magnetic field produced by wire 2 at the location of wire 1?

A. Downward

B. Upward

C. Into the page

D. Out of the page



TopHat Question

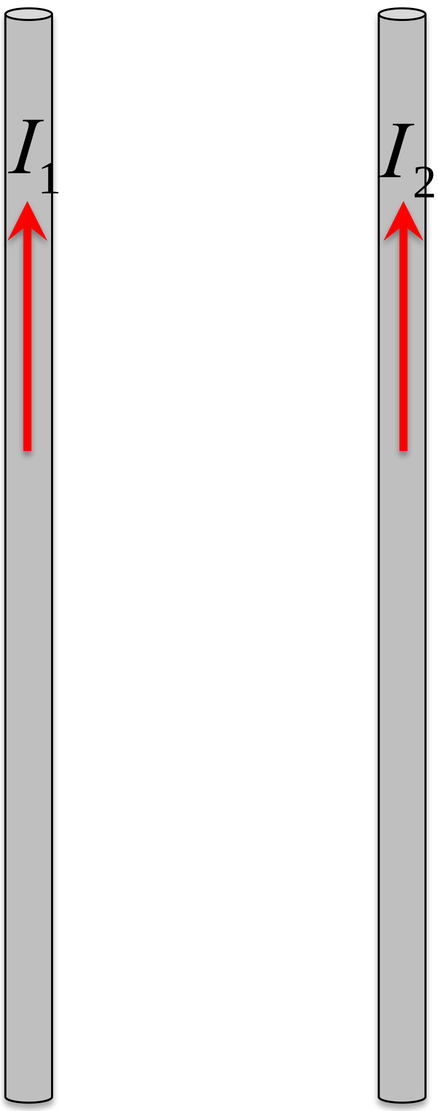
Two wires carry currents I_1 and I_2 as shown. What direction is the force of wire 1 on wire 2?

A. Left

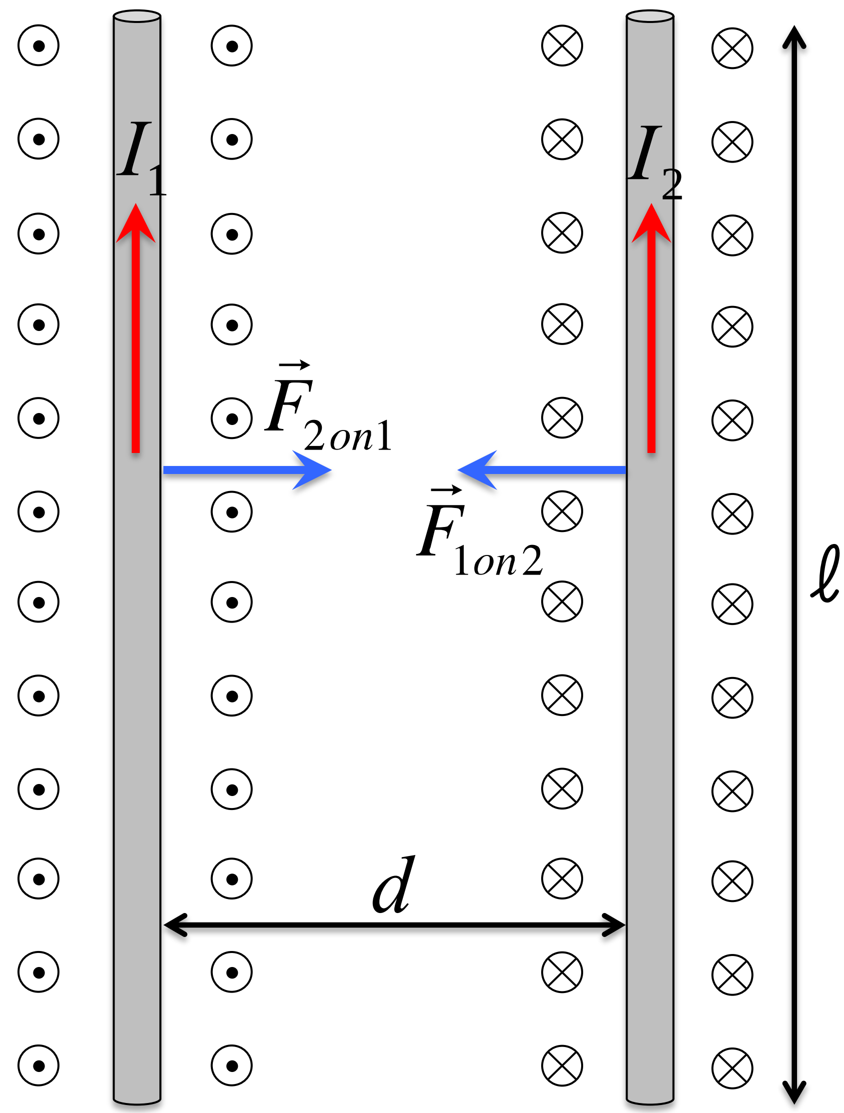
B. Right

C. Up

D. Down



$$\left|\vec{B}_2\right| = \frac{\mu_0 I_2}{2\pi d} \qquad \left|\vec{B}_1\right| = \frac{\mu_0 I_1}{2\pi d}$$



Wire 2 exerts a force on wire 1

$$\vec{F}_{2on1} = I_1 \vec{\ell} \times \vec{B}_2$$

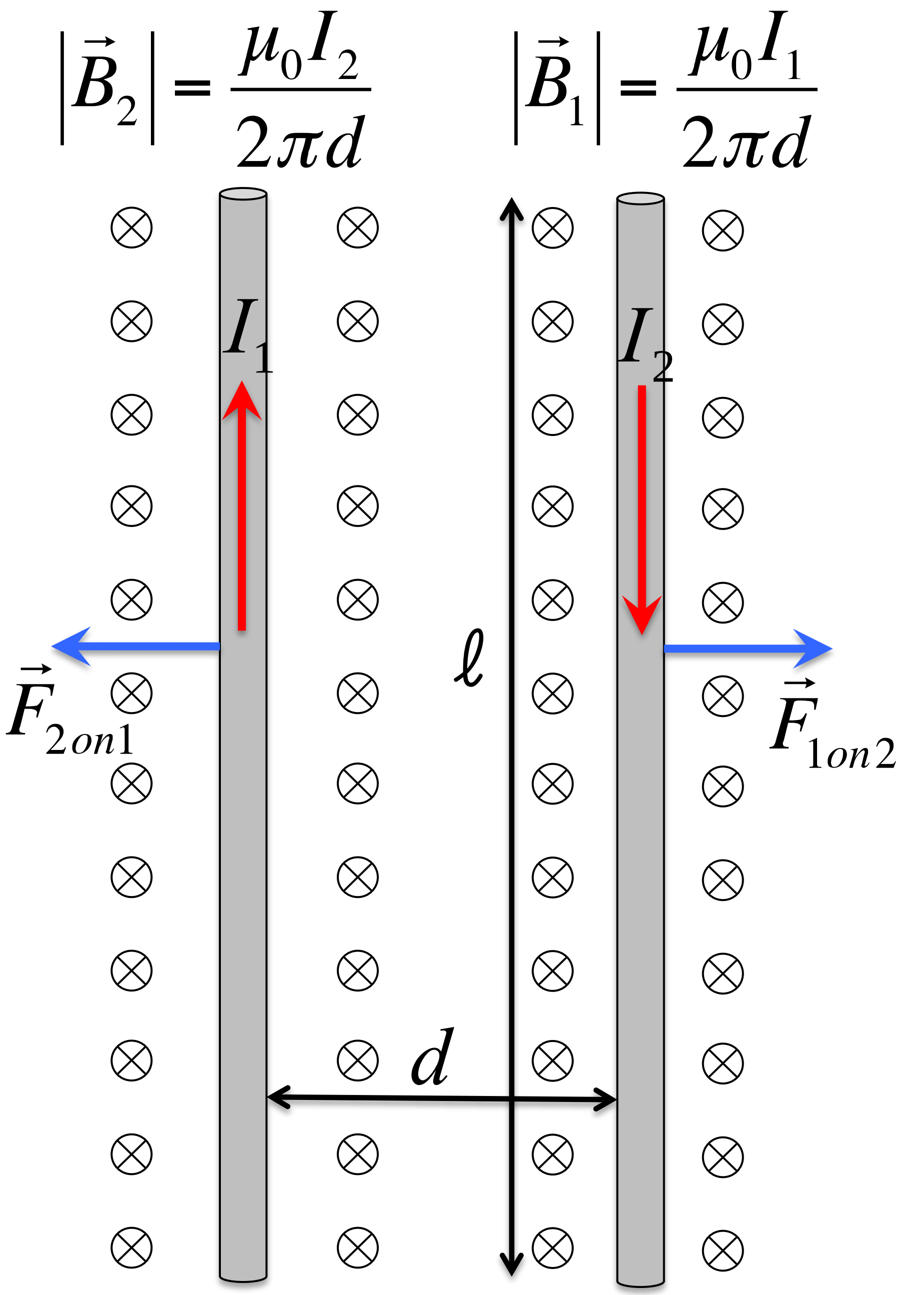
$$\left|\vec{F}_{2on1}\right| = I_1 \ell \frac{\mu_0 I_2}{2\pi d} = \boxed{\frac{\mu_0 \ell I_1 I_2}{2\pi d}}$$

Wire 1 exerts a force on wire 2

$$\vec{F}_{1on2} = I_2 \vec{\ell} \times \vec{B}_1$$

$$\left|\vec{F}_{1on2}\right| = I_2 \ell \frac{\mu_0 I_1}{2\pi d} = \boxed{\frac{\mu_0 \ell I_1 I_2}{2\pi d}}$$

Newton's third law!



Wire 2 exerts a force on wire 1

$$\vec{F}_{2on1} = I_1 \vec{\ell} \times \vec{B}_2$$

$$\left|\vec{F}_{2on1}\right| = I_1 \ell \frac{\mu_0 I_2}{2\pi d} = \boxed{\frac{\mu_0 \ell I_1 I_2}{2\pi d}}$$

Wire 1 exerts a force on wire 2

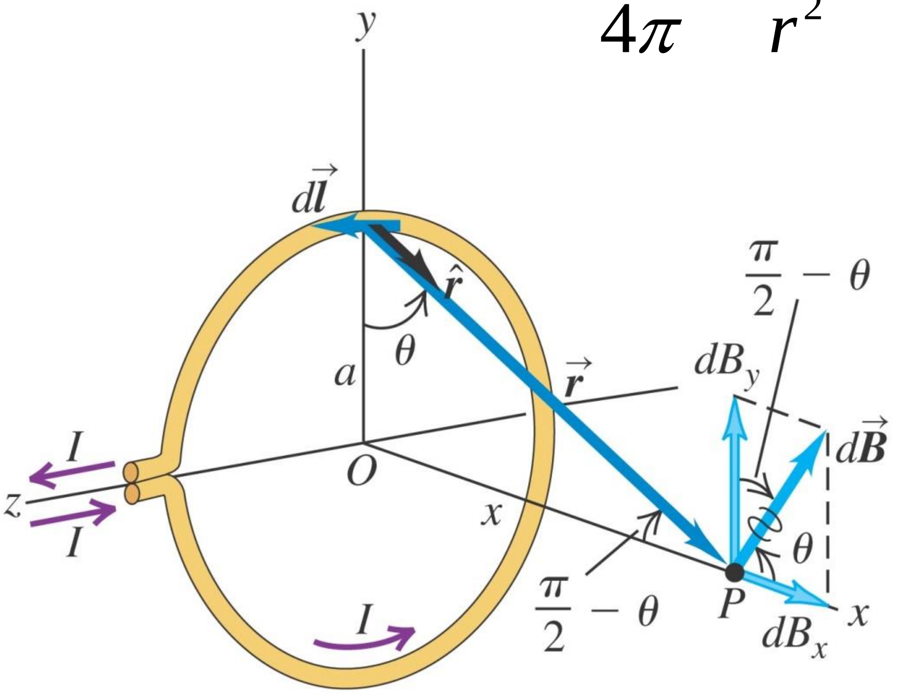
$$\vec{F}_{1on2} = I_2 \vec{\ell} \times \vec{B}_1$$

$$\left|\vec{F}_{1on2}\right| = I_2 \ell \frac{\mu_0 I_1}{2\pi d} = \boxed{\frac{\mu_0 \ell I_1 I_2}{2\pi d}}$$

Newton's third law!

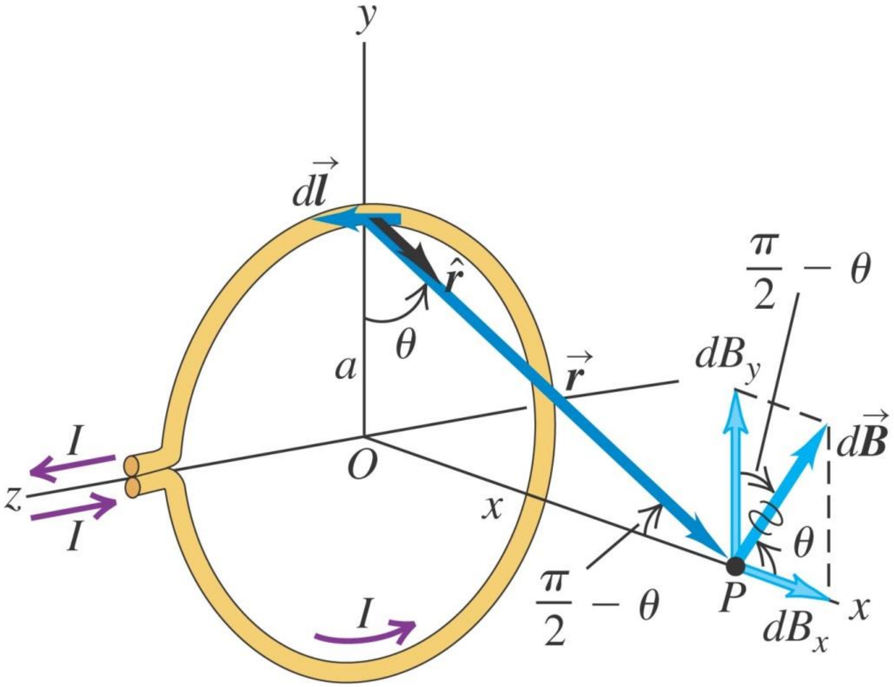
Magnetic field of a circular current loop

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{l} \times \hat{r}}{r^2}$$



$d\vec{l}$ and $\hat{r}$ are perpendicular to each other.

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{l} \times \hat{r}}{r^2}$$



$$dB = \frac{\mu_0}{4\pi}\frac{I\left|d\vec{l} \times \hat{r}\right|}{r^2}$$

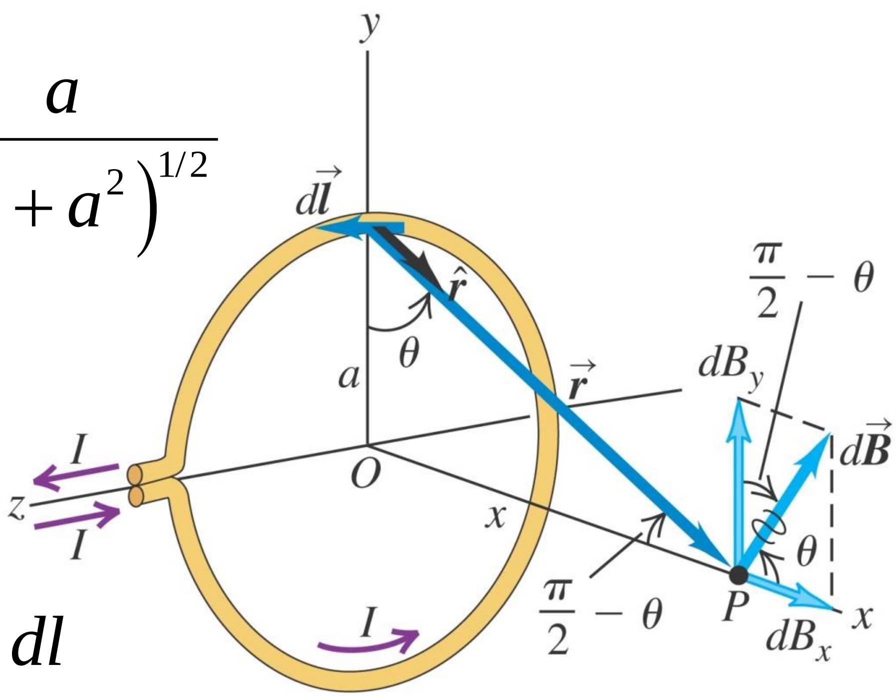
$$dB = \frac{\mu_0}{4\pi}\frac{Idl\sin(\pi/2)}{x^2+a^2} = \frac{\mu_0}{4\pi}\frac{Idl}{x^2+a^2}$$

$d\vec{l}$ and $\hat{r}$ are perpendicular to each other.

$$dB_x = dB\cos\theta = \frac{\mu_0}{4\pi}\frac{Idl}{x^2+a^2}\frac{a}{\left(x^2+a^2\right)^{1/2}}$$

$$dB_y = dB\sin\theta$$

$$dB_x = dB\cos\theta = \frac{\mu_0}{4\pi}\frac{Idl}{x^2+a^2}\frac{a}{\left(x^2+a^2\right)^{1/2}}$$



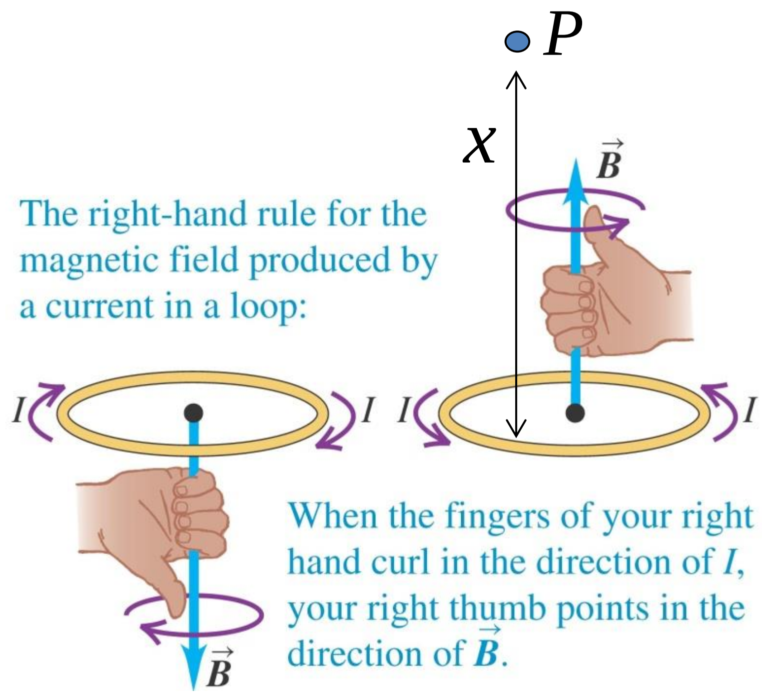
$$dB_y = dB\sin\theta$$

$$B_x = \frac{\mu_0 Ia}{4\pi\left(x^2+a^2\right)^{3/2}} \int_{\text{Around the loop}} dl$$

$$B_x = \frac{\mu_0 Ia\left(2\pi a\right)}{4\pi\left(x^2+a^2\right)^{3/2}} = \frac{\mu_0 Ia^2}{2\left(x^2+a^2\right)^{3/2}}$$

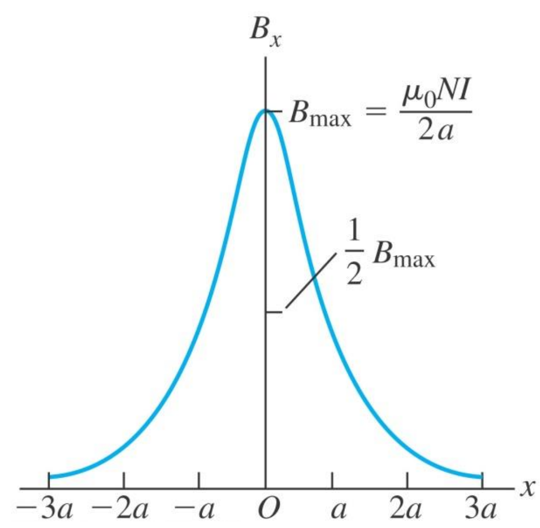
$B_y = 0$ by symmetry.

$$B_x = \frac{\mu_0 I a^2}{2\left(x^2 + a^2\right)^{3/2}}$$



$$B_x = \frac{\mu_0 I}{2a}$$

Magnetic field is strongest at the center of the loop.



$$B_x = \frac{\mu_0 NI}{2a}$$

Magnetic field at the center of N circular loops.

The wonderful Maxwell's Equations

$$\oint \vec{E} \bullet d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_0}$$

$$\oint \vec{B} \bullet d\vec{A} = 0$$

$$\oint \vec{B} \bullet d\vec{l} = \mu_0 \left(i_c + \varepsilon_0 \frac{d\Phi_E}{dt} \right)$$

$$\oint \vec{E} \bullet d\vec{l} = -\frac{d\Phi_B}{dt}$$

Among other things, they explain the behaviour of light.