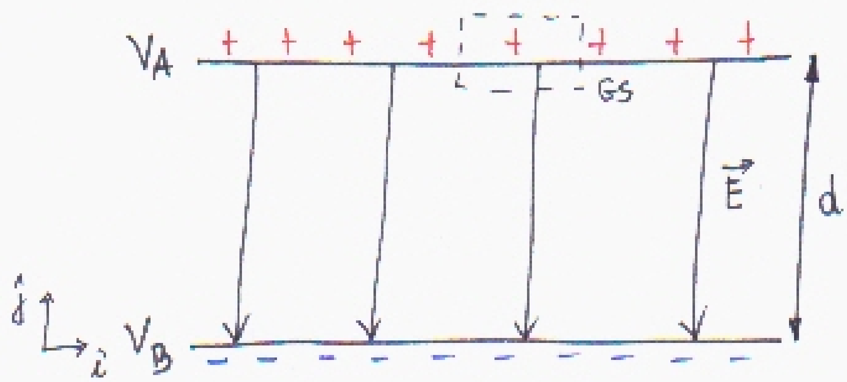
PARALLEL PLATES CAPACITOR



GS - Gaussian surface
cube, side area $= a^2$

d - separation of the plates

1. Find electric field (Gauss' Law)

$$\oint \vec{E} \circ d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$E \cdot \cancel{a^2} = \frac{\sigma \cancel{a^2}}{\varepsilon_0}$$

$$E = \frac{\sigma}{\varepsilon_0}$$

2. Find potential difference:

$$\Delta V_{AB} = -\int_A^B \vec{E} \circ d\vec{l}$$

$$\Delta V_{AB} = \int_A^B E\,dy = \int_A^B \frac{\sigma}{\varepsilon_0}\,dy = \frac{\sigma}{\varepsilon_0}\left[y\right]_A^B = \frac{\sigma}{\varepsilon_0}(d-0)$$

$$\Delta V_{AB} = \frac{\sigma}{\varepsilon_0} \cdot d = \frac{Q}{A}\frac{d}{\varepsilon_0} \qquad \left(\sigma = \frac{Q}{A}\right)$$

A - area of the plate

3. Calculate capacitance

$$\boxed{Q = C \cdot \Delta V}$$ true for all capacitors

$$C = \frac{Q}{\Delta V}$$

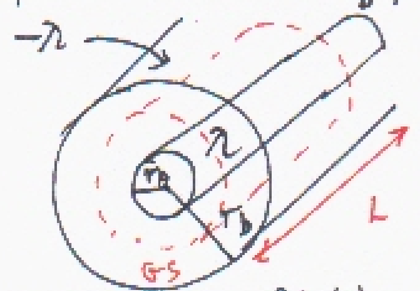
$$C = \frac{\cancel{Q}}{\frac{\cancel{Q} \cdot d}{A \cdot \varepsilon_0}} \qquad C = \frac{\varepsilon_0 A}{d}$$

Capacitance is a measure of how much charge is required to raise the electric potential by one volt.

$$C = \frac{Q}{\Delta V} \qquad \left[\frac{C}{V} = F\right]$$ F - farad

CYLINDRICAL CAPACITOR

(long cylindrical conductor of radius r_a surrounded by a larger cylindrical conducting shell with radius r_b)



1. Find electric field for $r_A < r < r_b$

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$E \cdot 2\pi r \cdot L = \frac{\lambda \cdot L}{\varepsilon_0}$$

$$E = \frac{\lambda}{2\pi \varepsilon_0 r}$$

2. Find ΔV_{AB}

$$\Delta V_{AB} = \int_A^B \vec{E} \cdot d\vec{l}$$

$$\Delta V_{AB} = \int_A^B E \cdot dr \qquad \vec{E} \parallel d\vec{r}$$

$$\Delta V_{AB} = \int_A^B \frac{\lambda}{2\pi\varepsilon_0 r} dr = \frac{\lambda}{2\pi\varepsilon_0} \int_A^B \frac{dr}{r} = \frac{\lambda}{2\pi\varepsilon_0} \Big[\ln r\Big]_A^B$$

$$\Delta V_{AB} = \frac{\lambda}{2\pi\varepsilon_0} \ln\left(\frac{r_B}{r_A}\right)$$

3. Find capacitance

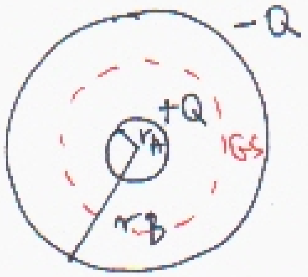
$$C = \frac{Q}{\Delta V}$$

$$C = \frac{\lambda \cdot L}{\frac{\lambda}{2\pi\varepsilon_0} \ln\left(\frac{r_B}{r_A}\right)}$$

$$C = \frac{2\pi \varepsilon_0 \cdot L}{\ln\left(\frac{r_B}{r_A}\right)}$$

Capacitance per length: $c = \frac{C}{L} = \frac{2\pi\varepsilon_0}{\ln\left(\frac{r_B}{r_A}\right)}$

SPHERICAL CAPACITOR



Inner conductor with radius r_A surrounded by a second concentric shell with radius r_B.

1. Find $\vec{E}$ (Gauss' Law)

$$E \cdot 4\pi r^2 = \frac{+Q}{\varepsilon_0}$$

$$E = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2}$$

2. Find ΔV_{AB}

$$\Delta V_{AB} = \int_A^B E \cdot dr = \frac{Q}{4\pi\varepsilon_0} \int_A^B \frac{dr}{r^2}$$

$$\Delta V_{AB} = \frac{Q}{4\pi\varepsilon_0} \left[-\frac{1}{r} \right]_A^B$$

$$\Delta V_{AB} = \frac{Q}{4\pi\varepsilon_0} \left[\frac{1}{r_A} - \frac{1}{r_B} \right]$$

Common denominator

$$\Delta V_{AB} = \frac{Q}{4\pi\varepsilon_0} \left[\frac{r_B - r_A}{r_A \cdot r_B} \right]$$

3. Find capacitance:

$$C = \frac{Q}{\Delta V}$$

$$C = \frac{Q}{\frac{Q}{4\pi\varepsilon_0} \left[\frac{r_B - r_A}{r_A \cdot r_B} \right]}$$

$$C = 4\pi\varepsilon_0 \frac{r_A \cdot r_B}{r_B - r_A}$$