Electricity and Magnetism

- Physics 259 – L02
- Lecture 22

UNIVERSITY OF CALGARY

Chapter 24.1: Electric Potential

Last time

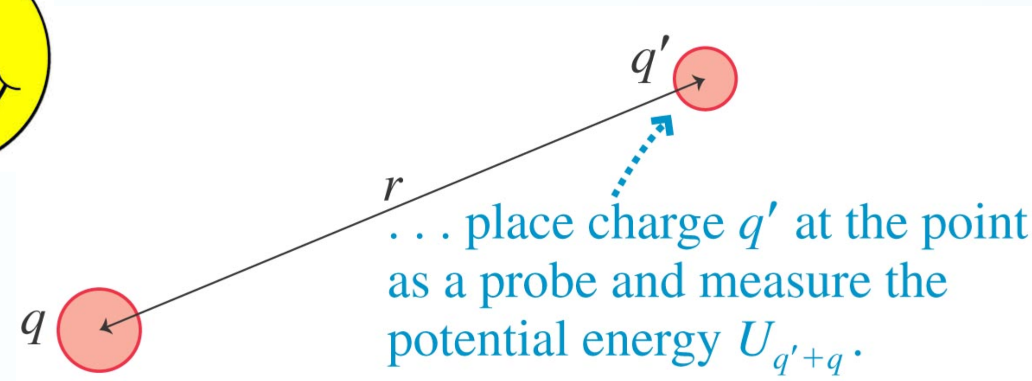
- Electric potential energy of a collection of charges
- Electric potential (very important concept)

This time

- Equipotential surfaces: visualizing electric potential
- Conductors and electric potential
- Interpreting equipotential surfaces

Starting from the end

The whole story is:



Electric force on q' from q

$$\vec{F}_{qq'} = \frac{1}{4\pi\varepsilon_0}\frac{qq'}{r^2}\hat{r}$$

Then the electric field of q is

$$\vec{E} = \frac{\vec{F}_{qq'}}{q'} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$$

Potential energy of q and q'

$$U_{q'+q} = \frac{1}{4\pi\varepsilon_0}\frac{qq'}{r}$$

Then the potential of q is

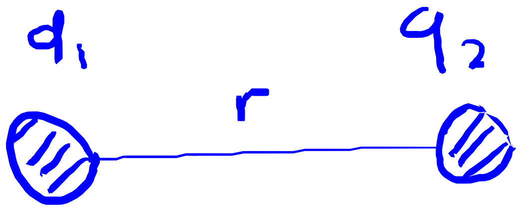
$$V = \frac{U_{q'+q}}{q'} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

Electric Potential

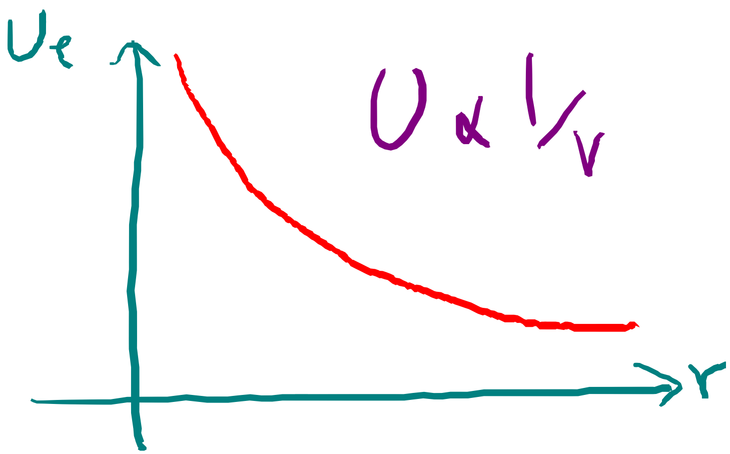
$$\boxed{V \equiv \frac{U_{q+sources}}{q}}$$

$$\boxed{U_{q+sources} = qV}$$

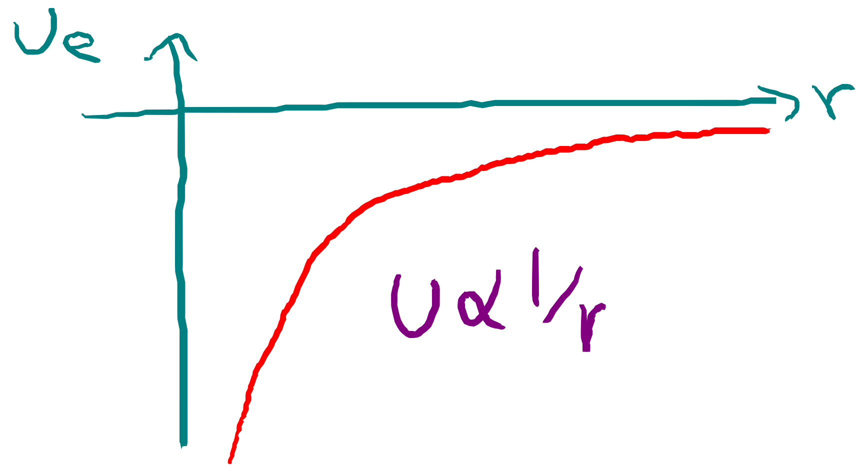
Electric potential energy ⇒



$$U_e = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}$$



if q_1 & q_2 have the same charge



if q_1 & q_2 have opposite charge

For conservative force $\Rightarrow$ there is a potential energy associated with

$$\vec{F} = -\vec{\nabla} U_e \stackrel{\downarrow}{=} -\frac{\partial U_e}{\partial x}\hat{i} - \frac{\partial U_e}{\partial y}\hat{j} - \frac{\partial U_e}{\partial z}\hat{k}$$

& $\underline{\vec{F} = q\vec{E}}$ & $\underline{U_e = qV}$

$$\cancel{q}\vec{E} = -\frac{\partial}{\partial x}\cancel{q}V\hat{i} - \frac{\partial}{\partial y}\cancel{q}V\hat{j} - \frac{\partial}{\partial z}\cancel{q}V\hat{k}$$

$$\vec{E} = -\frac{\partial}{\partial x}V\hat{i} - \frac{\partial}{\partial y}V\hat{j} - \frac{\partial}{\partial z}V\hat{k} \rightarrow$$

$$\rightarrow \vec{E} = -\nabla V = -\frac{\partial V}{\partial x}\hat{i} - \frac{\partial V}{\partial y}\hat{j} - \frac{\partial V}{\partial z}\hat{k}$$

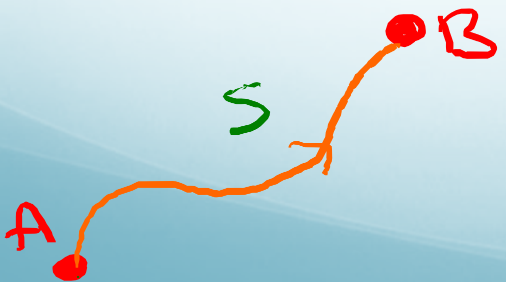
Potential Gradient -- E and V

Note: E is always ⊥ equipotential lines

$$\vec{E} = -\vec{\nabla}V = -\frac{\partial V}{\partial x}\hat{i} - \frac{\partial V}{\partial y}\hat{j} - \frac{\partial V}{\partial z}\hat{k}$$

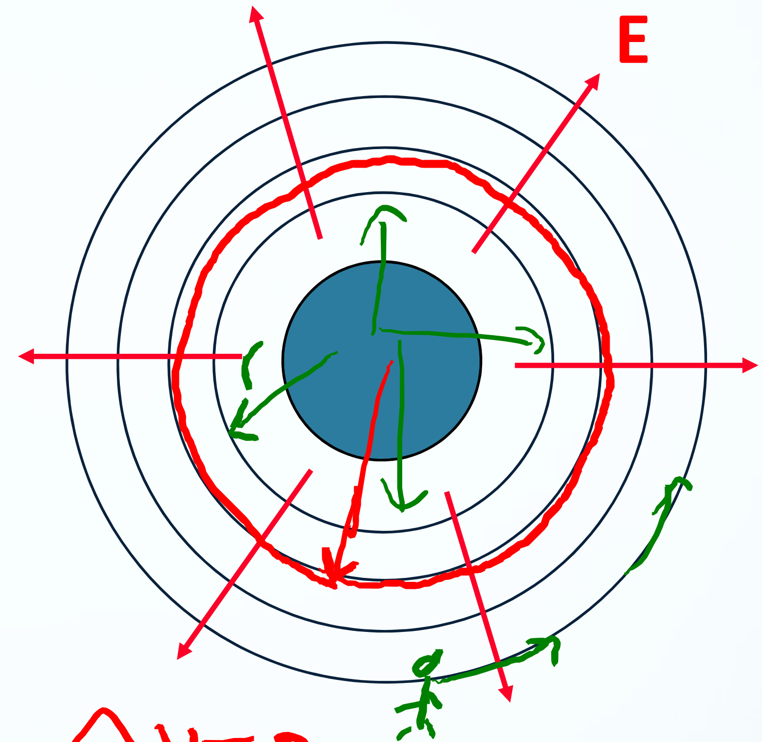
In 3 dimensions we must take 3 derivatives, then add them **VECTORIALLY**

Alternatively, the potential is found from the electric field integrated along any path connecting points A and B



$$V_{AB} = \int_A^B \vec{E} \cdot \vec{ds}$$

Equipotentials



Ex: For uniform spherical charge

$\longrightarrow$ $V(r) = k\,Q/r$

For each r, V(r) is constant→
V(r) is constant over any sphere concentric with the charged sphere

$\Delta V = 0$

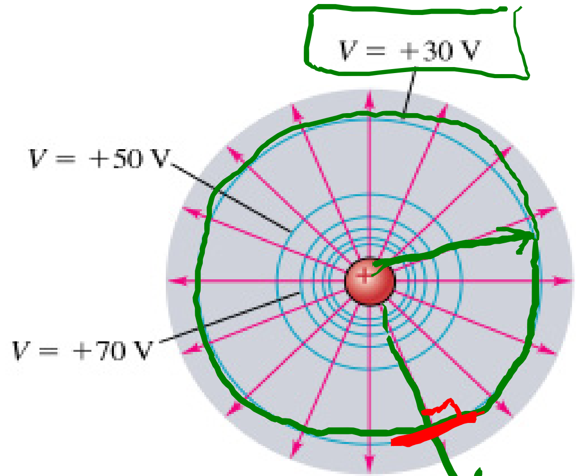
→ We have equipotential lines (or surfaces, actually, in 3-D)

Note that if move along equipotential surface→

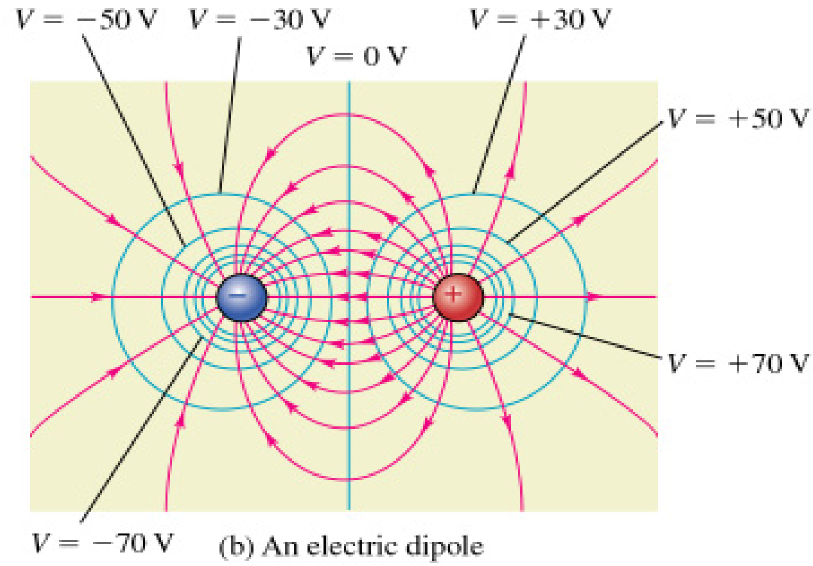
→ by definition $\Delta V = -\,\mathbf{E} \cdot \Delta \mathbf{r} = 0$ --> **E** is $\perp$ equipotential surface

$\Delta V = 0 \rightarrow \Delta V = -E \cdot \Delta r \rightarrow E \perp \Delta r$

Equipotential Surfaces



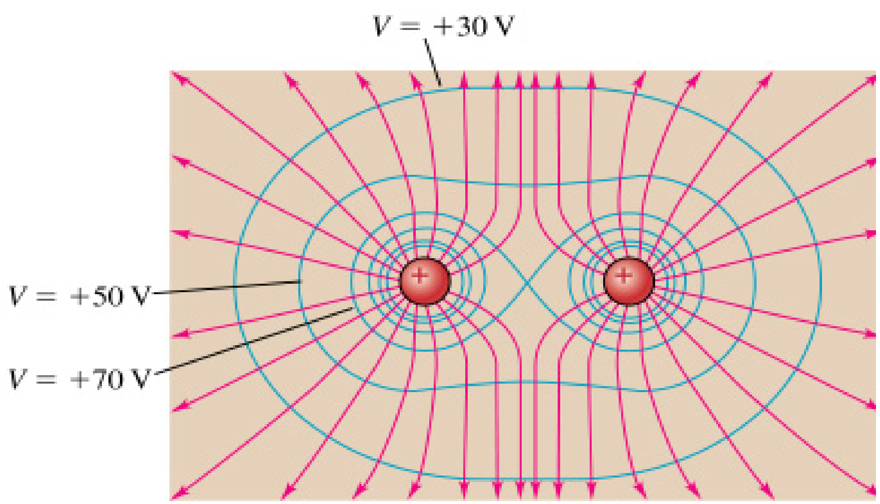
(a) A single positive charge



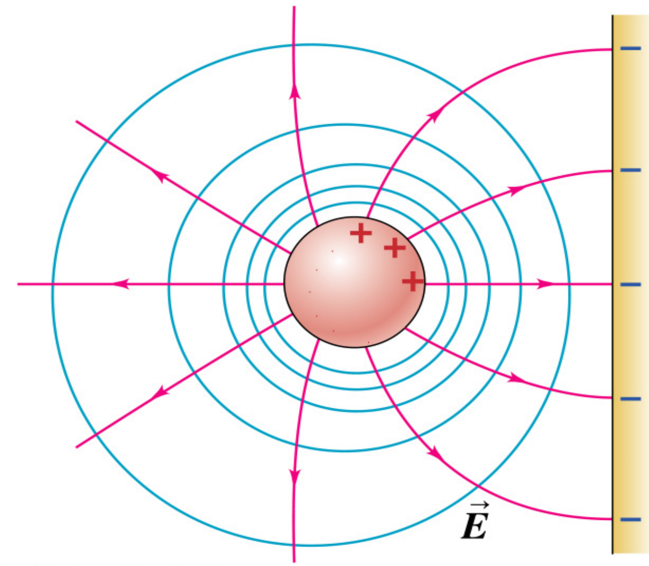
(b) An electric dipole

Note – E is always ⊥ V !!

Conducting sphere
+ sheet



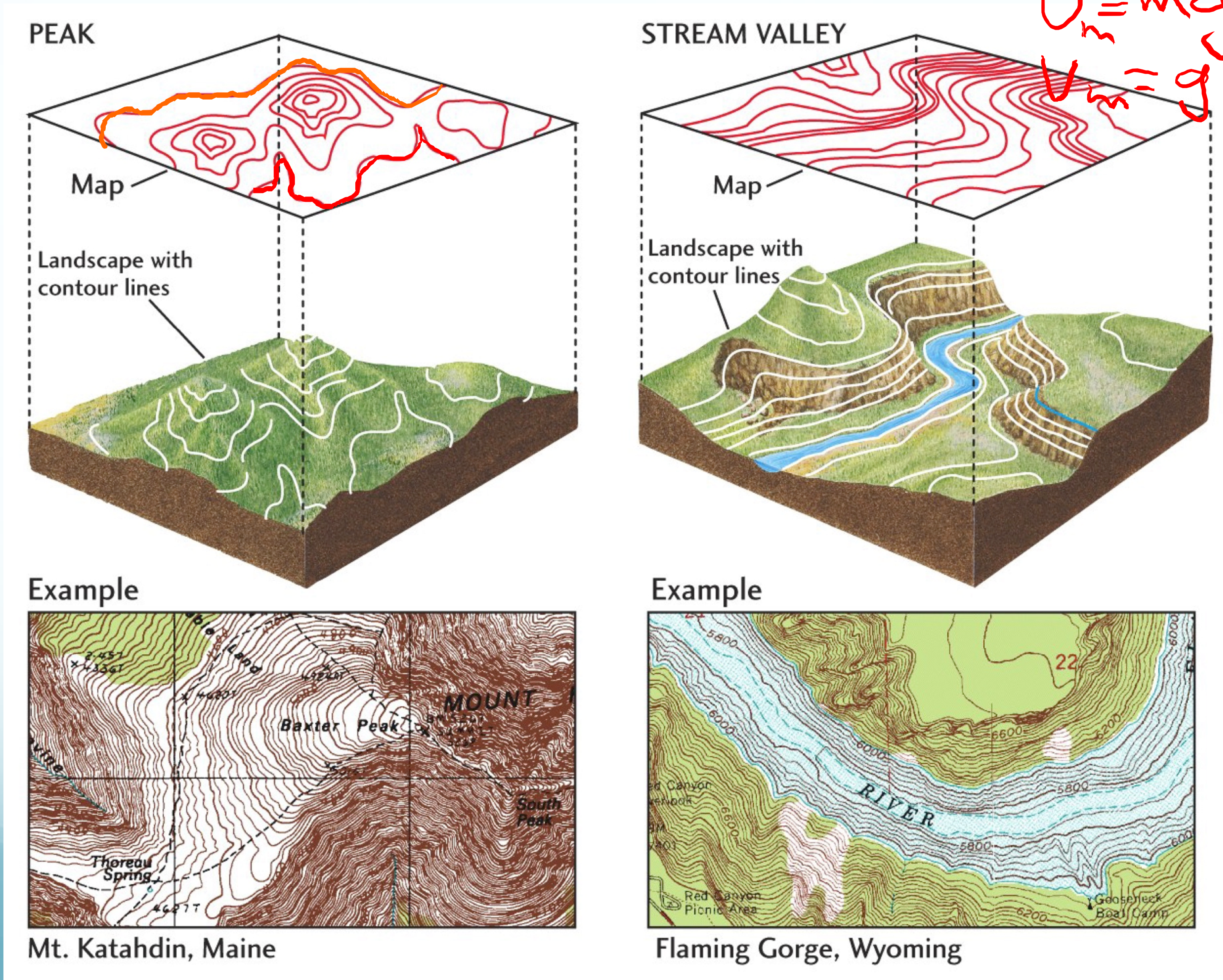
(c) Two equal positive charges



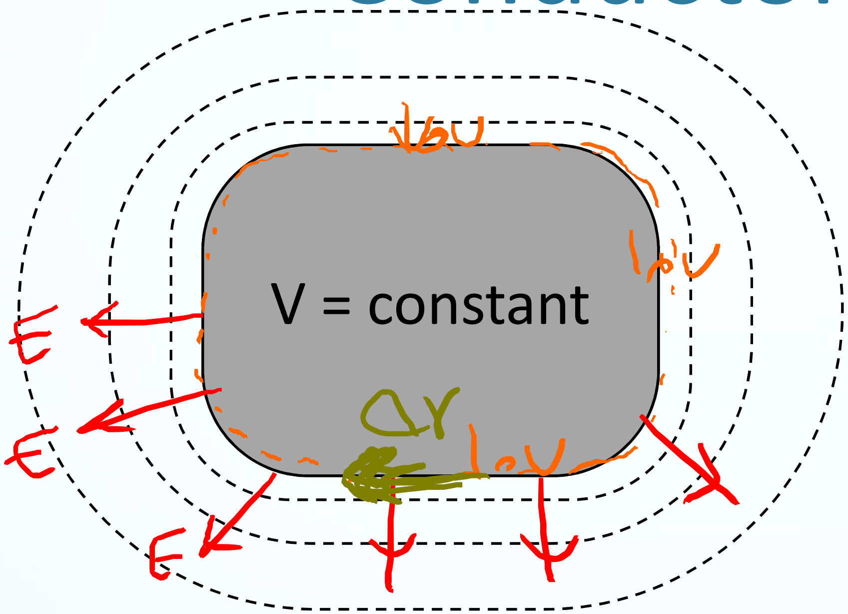
Where have you seen equipotentials before?

$U_m = mgh$

$V_m = gh$

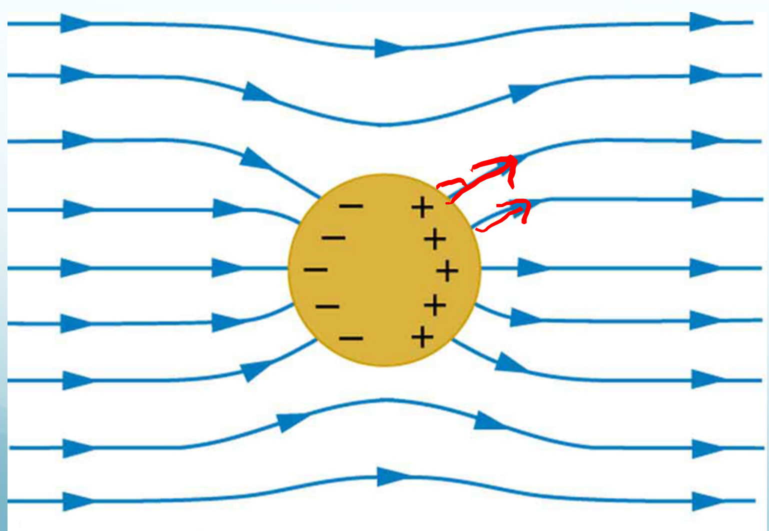


Conductors and E-fields



The surface of a conductor is an equipotential. If there was a potential difference across the surface of a conductor, the freely moving charges would move around until the potential is constant.

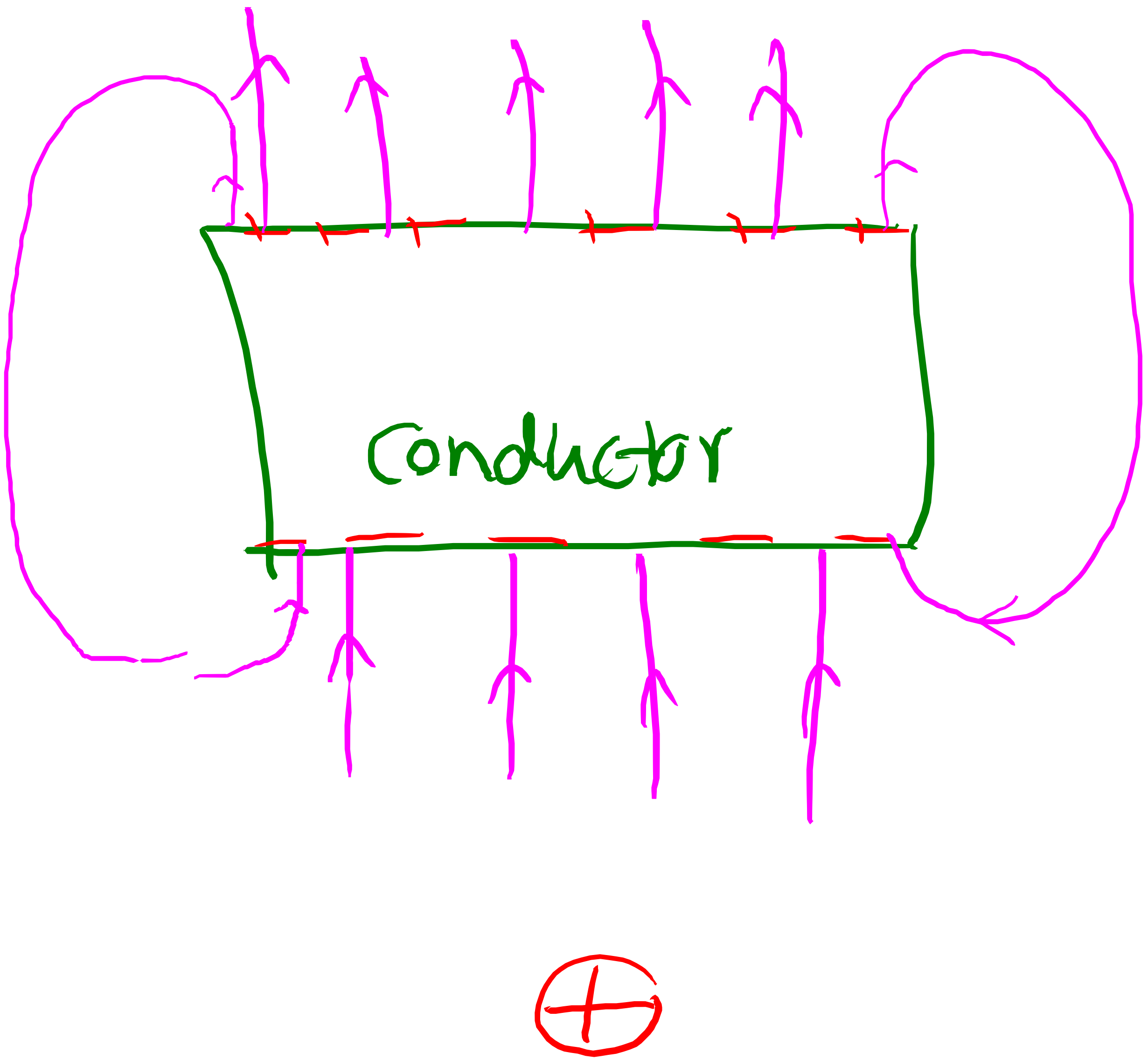
This means that electric field lines ALWAYS must meet a conducting surface at right angles (any tangential component would imply a tangential force on the free charges).



https://cnx.org/contents/Ax2o07Ul@8.8:a8tWDdDr@3/Conductors-and-Electric-Fields

$$\Delta V = -E \cdot \Delta r = 0$$

$$\cos\theta = 0 \rightarrow E \perp \Delta r$$

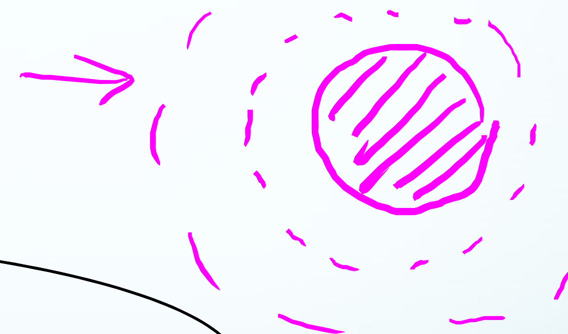

Conductor

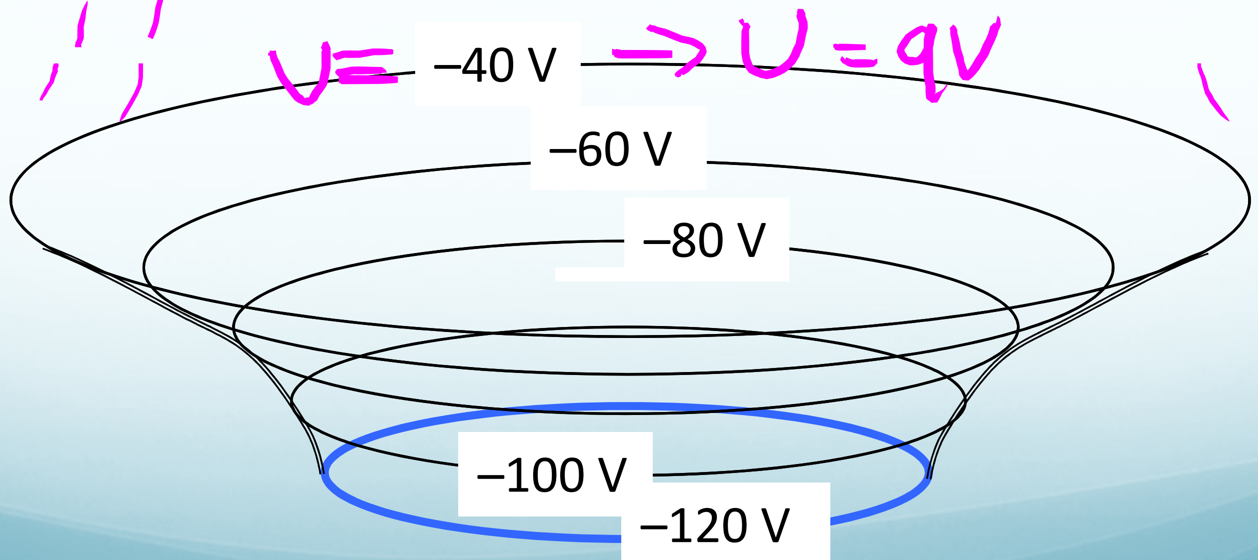
Equipotential surfaces for charged shell

Equipotential surfaces give you information about:

1. the potential energy that charged particles would have:

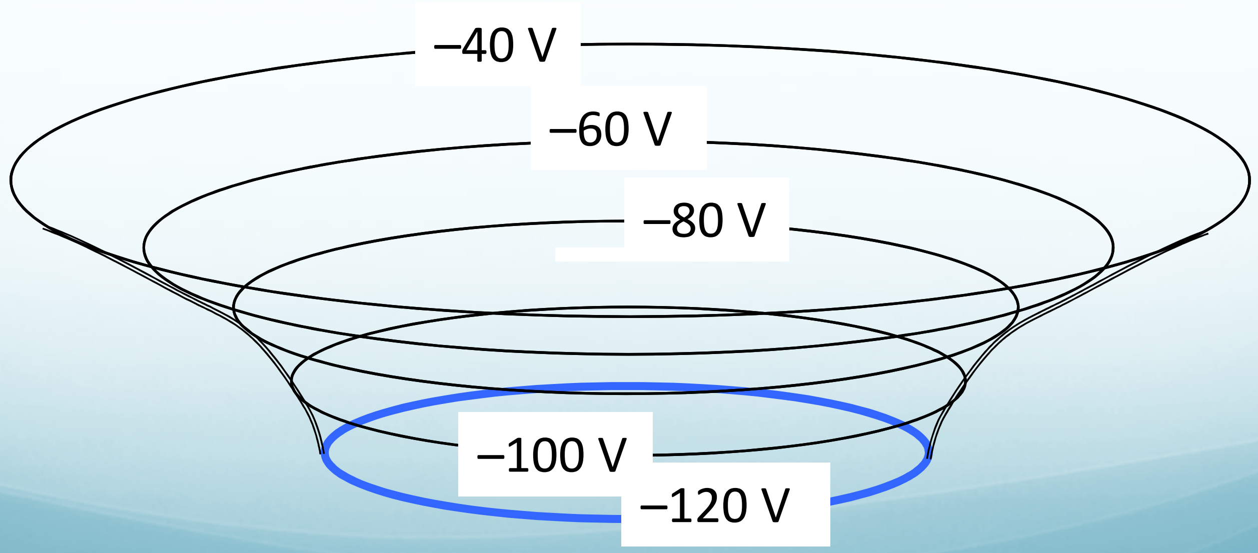
Think of the electric potential (V) the same way that gravitational potential (gh) is an altitude above sea level. The potential energy of a charge q is then just U = qV, while the potential energy of a mass is U = mgh.





Equipotential surfaces for charged shell

Equipotential surfaces give you information about
2. **the direction of the electric field:**
Just like in the gravitational analogy, objects roll downhill (to lower gravitational potential), positive charges move "downhill" to lower electric potential; the electric field always points "downhill".



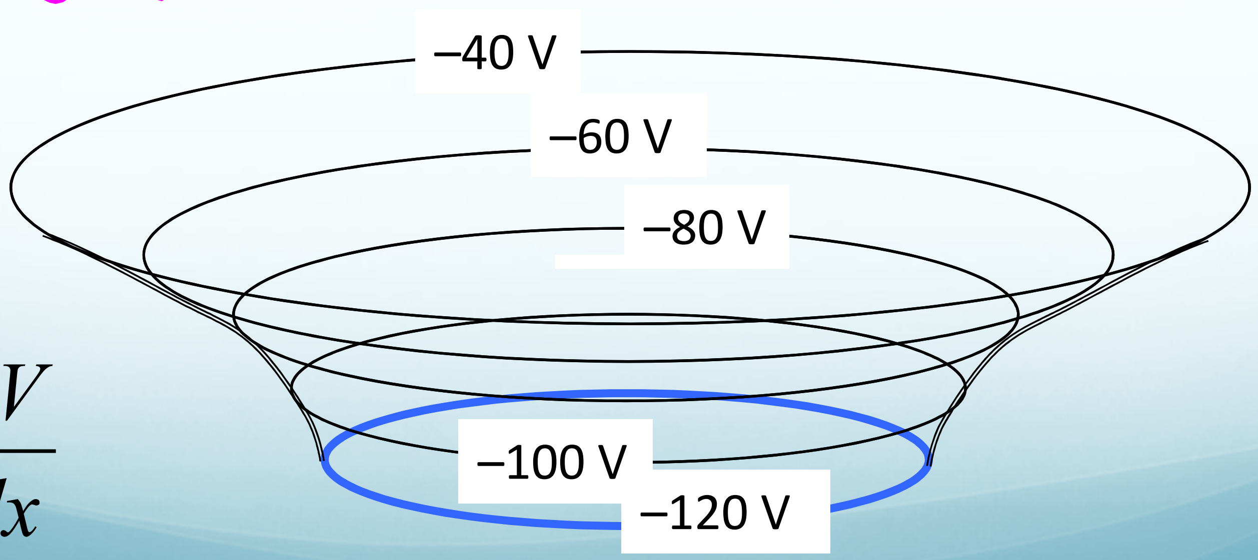
Equipotential surfaces for charged shell

Equipotential surfaces give you information about

3. **the strength of the electric field:**

We know that the in the gravitational case, objects on steeper slopes will accelerate faster. Similarly here, the strength of the electric field is related to the slope of V(x). The more bunched together the equipotential lines, the steeper the slope, the stronger the field.

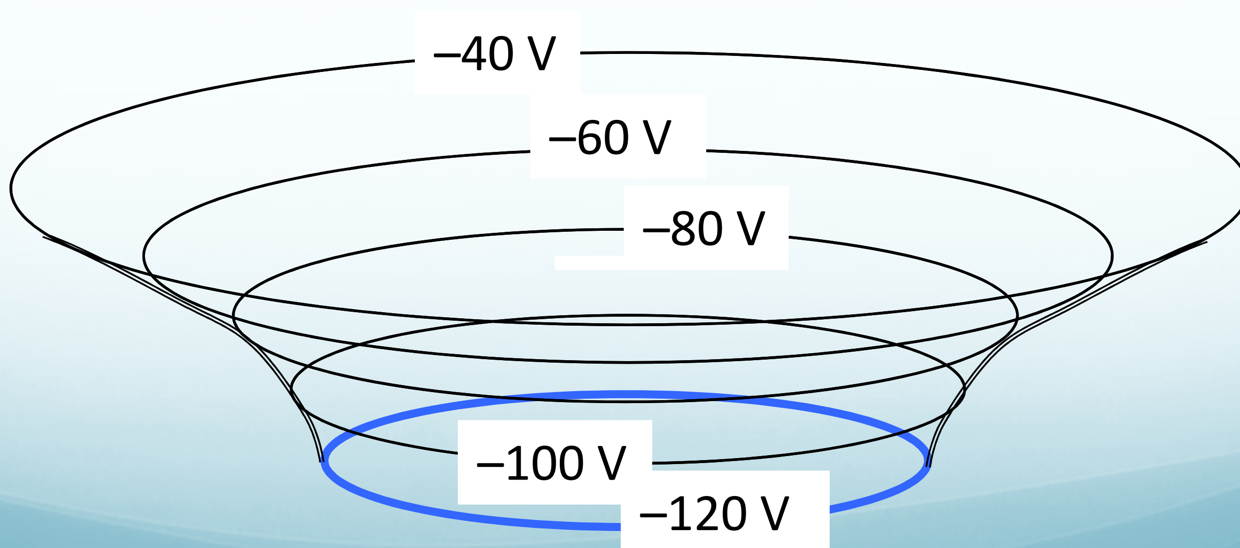
$\vec{E} = -\vec{\nabla} V$



$$E = -\frac{dV}{dx}$$

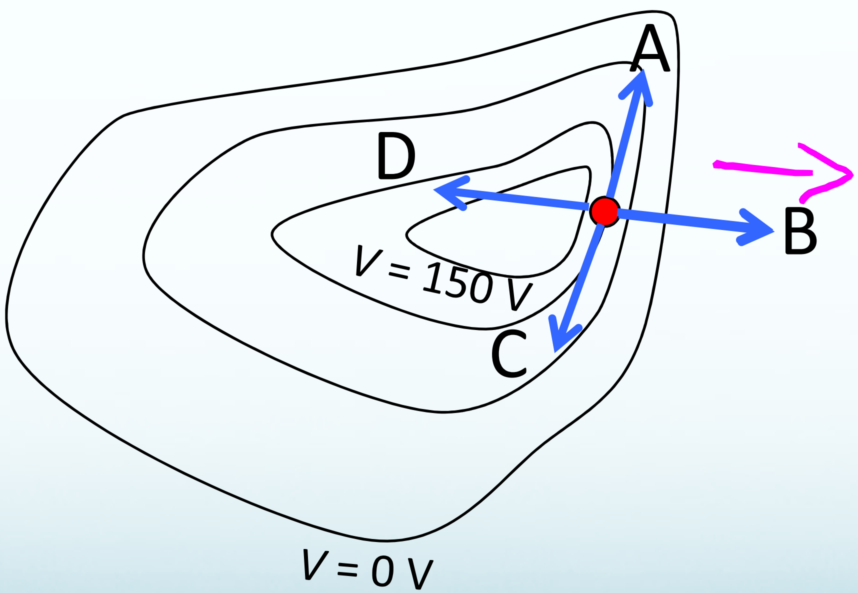
Equipotential surfaces for charged shell

Equipotential surfaces give you information about
4. **where a charged particle is allowed to go, based on its energy:**
If you release a marble in a bowl at some height *h*, it will never be able to reach a higher height. Similarly, if you release a positive charge from some potential, it can never reach a higher potential unless supplied with extra energy.



TopHat Question

Equipotential surfaces are shown below. If a positively charged particle were released from rest at the point indicated, in which direction would the particle begin to move?



A.
B.
C.
D.

TopHat Question

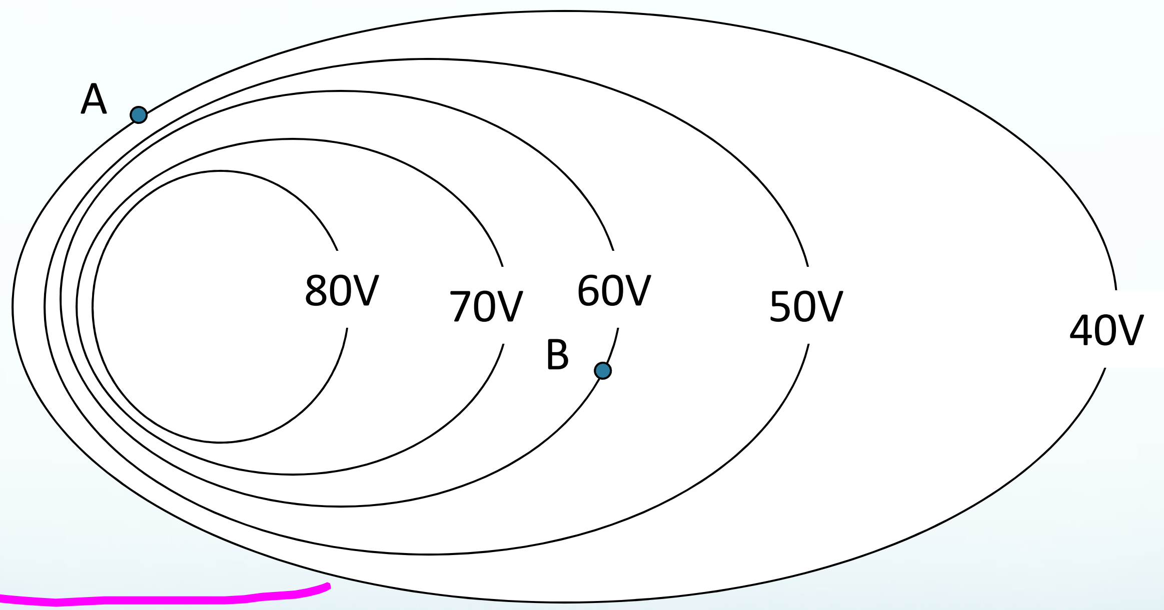
How much energy (in Joules) would 2C of charge gain if it was pushed from point A to point B?

$U_e = qV_e$

$2c \times 20v$

$= 40\ cv$

$= 40J$



$\boxed{\text{A. } 40 \text{ J}}$

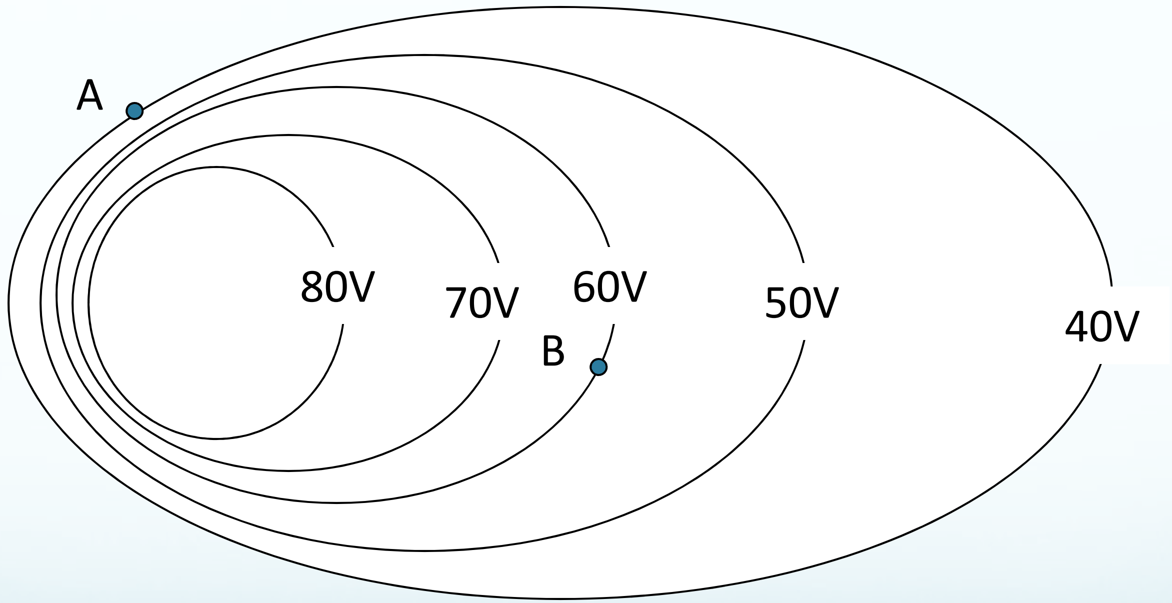
B. 60 J

C. 80 J

D. 120 J

TopHat Question

How much energy (in Joules) would 2C of charge gain if it was pushed from point A to point B, then to point A?



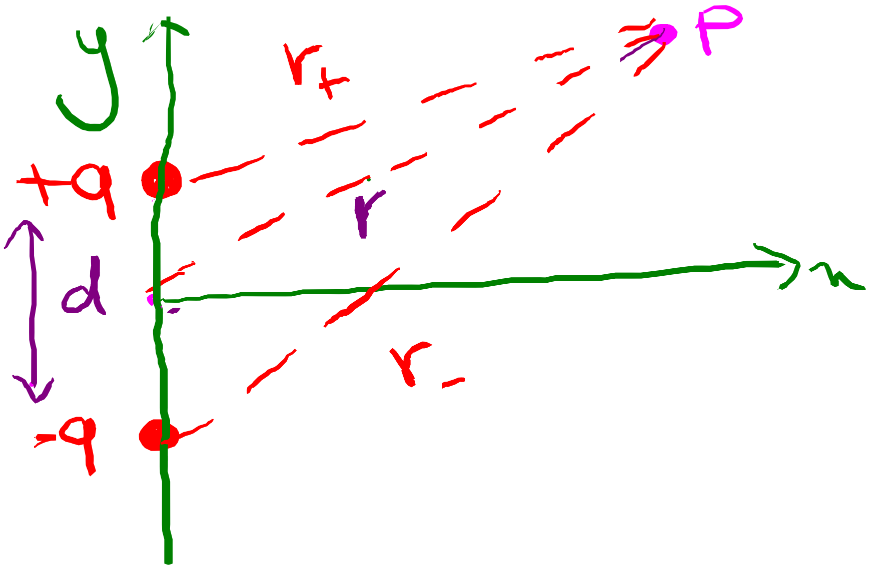
A. 80 J

B. 60 J

C. 40 J

D. 0 J

Electric potential of a dipole $\Rightarrow$ arbitrary P



$$V = V_+ + V_-$$

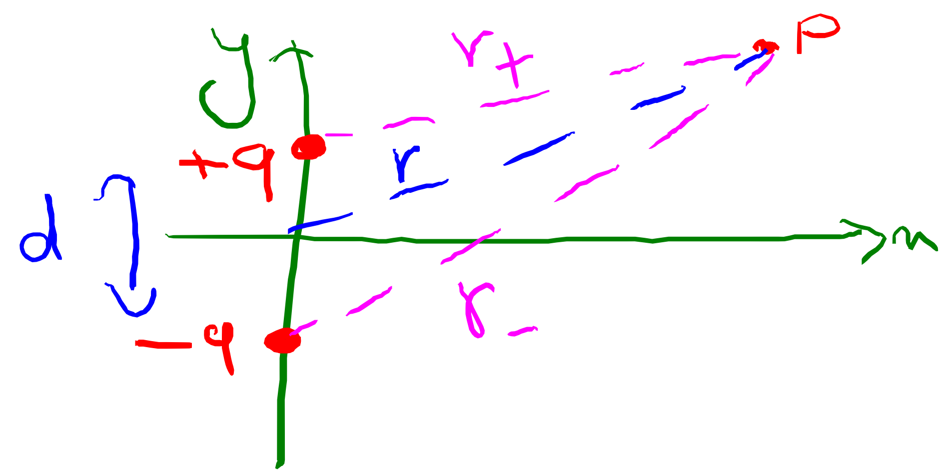
$$\begin{cases} V_+ = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r_+} \\[2ex] V_- = \dfrac{-1}{4\pi\varepsilon_0}\dfrac{q}{r_-} \end{cases}$$

$$\rightarrow V = V_+ + V_- = \frac{1}{4\pi\varepsilon_0}\frac{q}{r_+} + \frac{-1}{4\pi\varepsilon_0}\frac{q}{r_-}$$

$$V = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_+} - \frac{1}{r_-}\right)$$

$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{r_- - r_+}{r_- r_+}\right) \checkmark$$

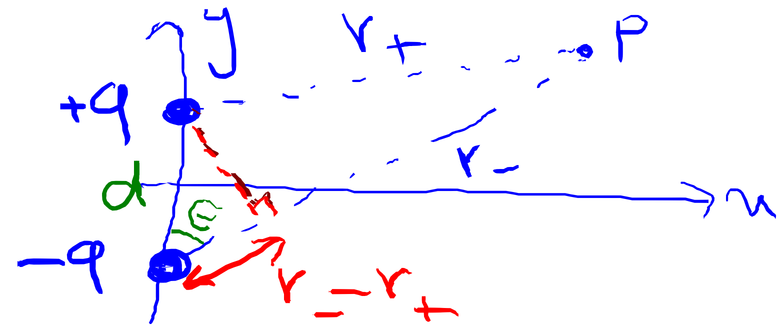
$$\Rightarrow V = \frac{q}{4\pi\varepsilon_0} \frac{r_- - r_+}{r_- r_+}$$



natural dipoles $\Rightarrow$

$r \gg d \Rightarrow \quad r_- \simeq r_+ \simeq r$

$\Rightarrow r_- r_+ \simeq r^2 \quad \& \quad r_- - r_+ \rightarrow$



$\rightarrow r_- - r_+ = d\cos\theta$

$\Rightarrow V =$

$$\rightarrow V = \frac{q}{4\pi\varepsilon_0} \frac{d\cos\theta}{r^2} = \frac{1}{4\pi\varepsilon_0} \frac{p\cos\theta}{r^2}$$

This section we talked about:

Chapter 24.1 and 24.2

See you on next Monday

Happy reading week