Wednesday Feb 15, 2017

Last time:

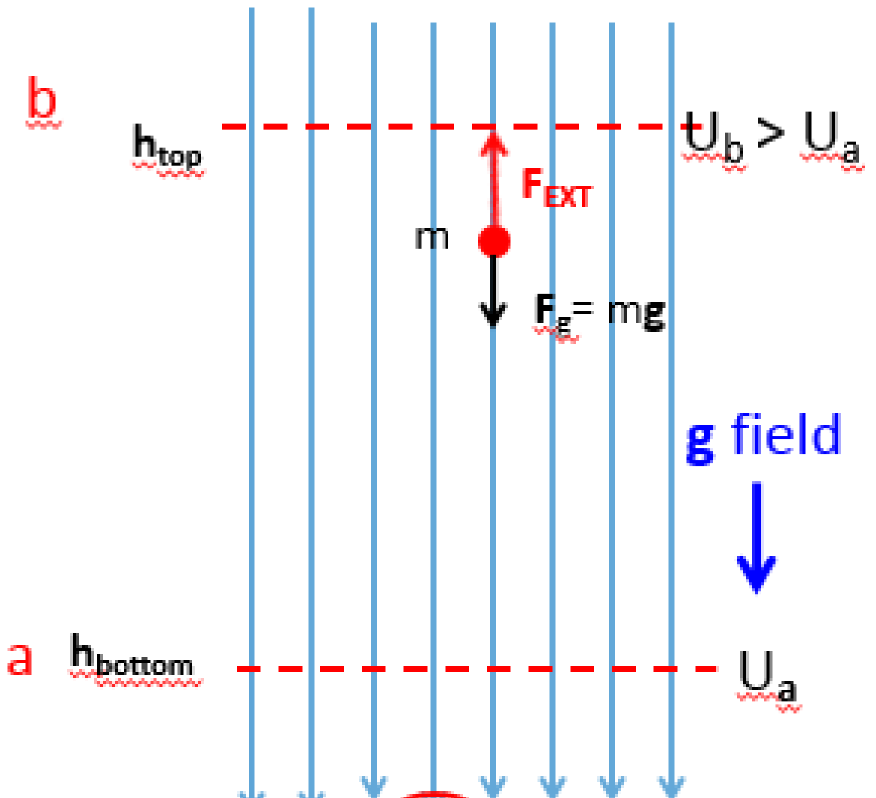
- Midterm review

Today:

- Electric potential energy: uniform E-field
- Electric potential energy: 2 point charges
- Electric potential energy of a collection of charges
- Electric potential (very important concept)

Gravitational & Electric Fields

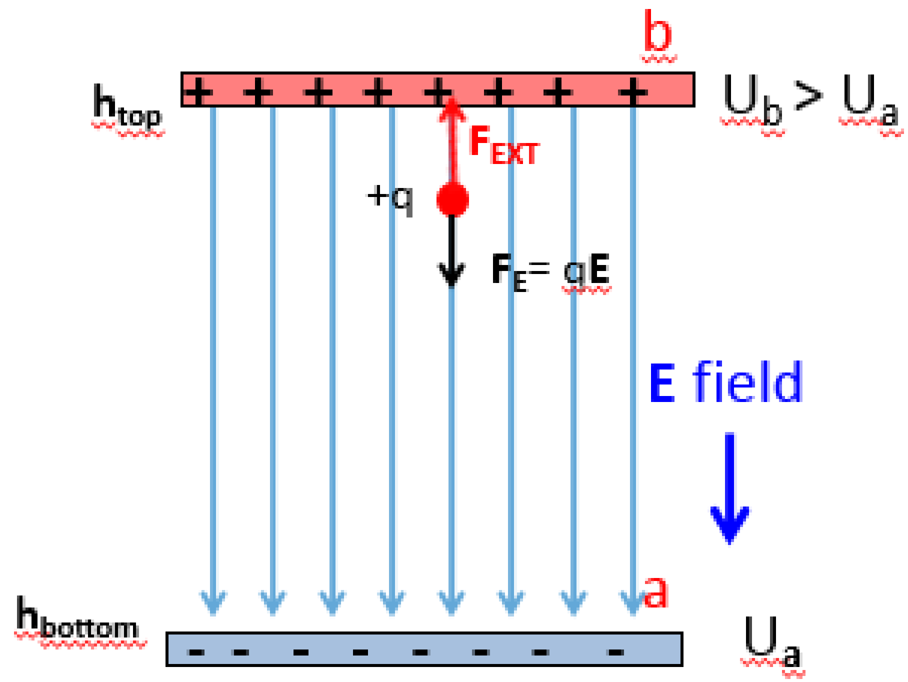
(Simple case: uniform fields)



$$WORK_{EXT} = F_{EXT} \times \Delta h = mg\Delta h$$

$$\boxed{WORK_{EXT}^{a->b} = U_b^g - U_a^g > 0}$$

$$\boxed{WORK_g^{a->b} = -(U_b^g - U_a^g)}$$



$$WORK_{EXT} = F_{EXT} \times \Delta h = qE\Delta h$$

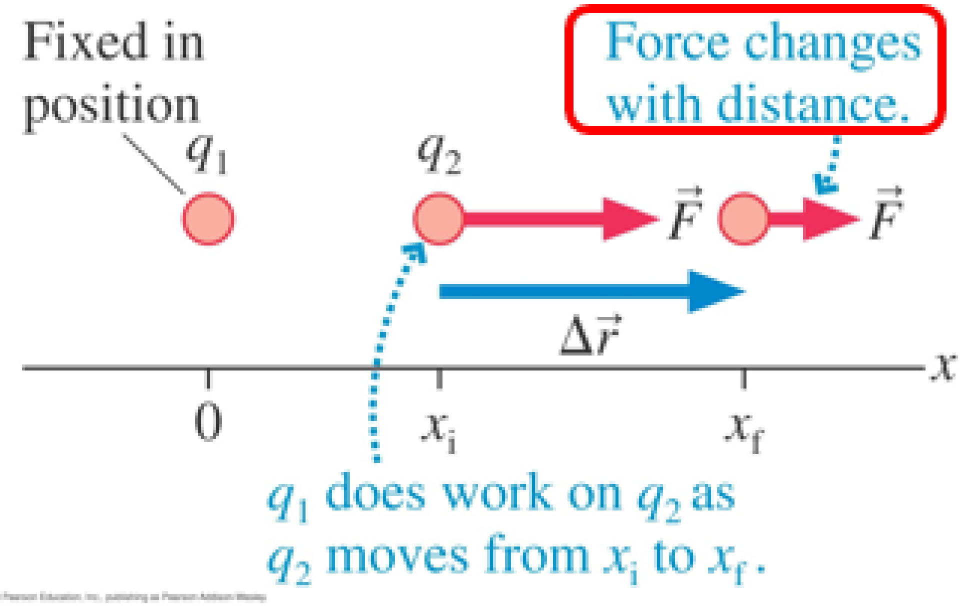
$$\boxed{WORK_{EXT}^{a->b} = U_b^E - U_a^E > 0}$$

$$\boxed{WORK_E^{a->b} = -(U_b^E - U_a^E)}$$

Finding Potential Energy of two point charges (more building blocks)

$$W_{i \to f}^{ELEC} = -\Delta U$$

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$$

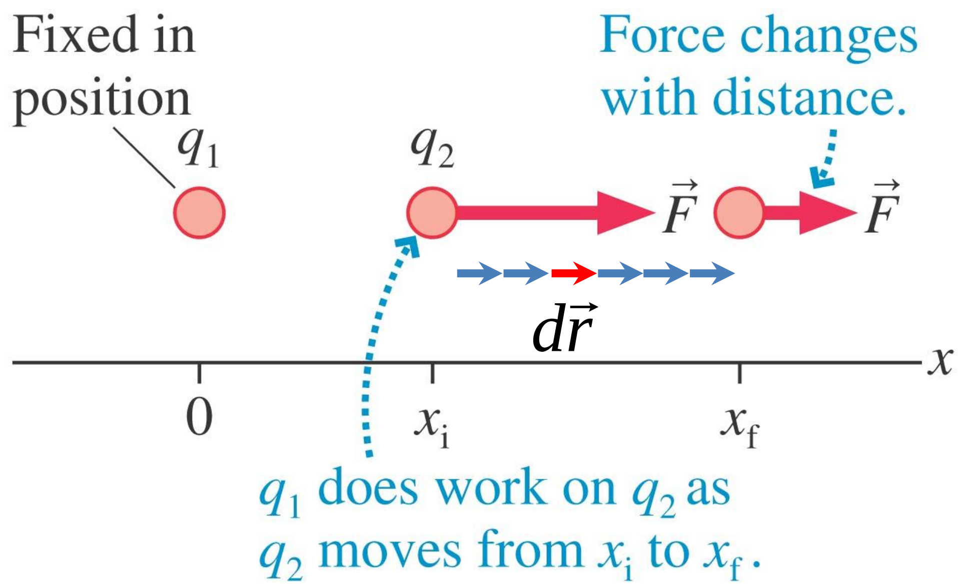


The field is **not** uniform so $\vec{F}$ is **not** constant over the displacement Δr and we **cannot** use

$$W_{i \to f}^{ELEC} = F\Delta r$$

Finding Potential Energy of two point charges (more building blocks)

Break the displacement $\Delta\vec{r}$ into many tiny displacements $d\vec{r}$.

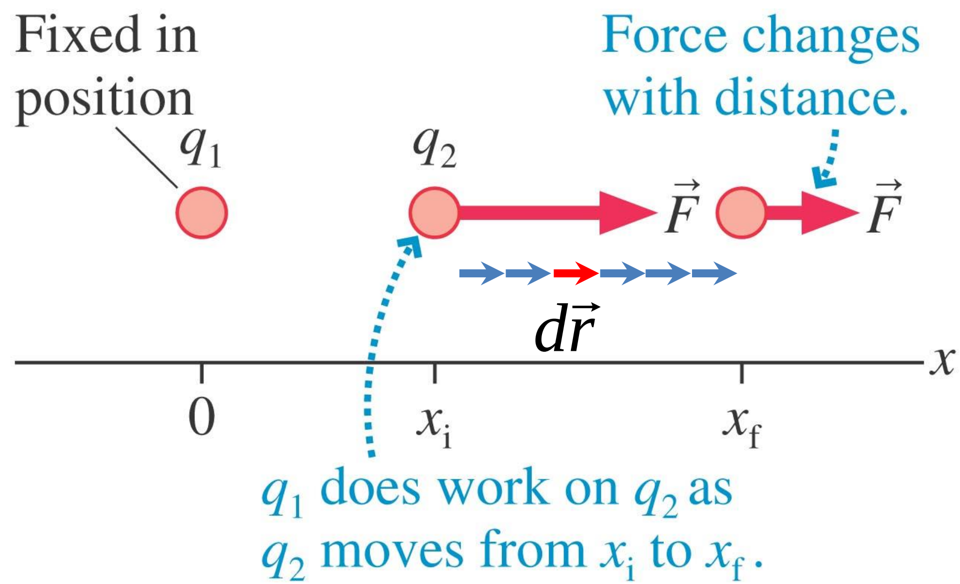


$\vec{F}$ is essentially constant over such a small displacement, so the work done on q_2 in **each** displacement is Fdr.

Finding Potential Energy of two point charges (more building blocks)

The total work is the sum of all the little bits of work:

$$W_{i \to f}^{ELEC} = \int_{r_i}^{r_f} F dr$$



$$W_{i \to f}^{ELEC} = \int_{r_i}^{r_f} \overbrace{\frac{1}{4pe_0} \frac{q_1 q_2}{r^2}} dr$$

Finding Potential Energy of two point charges (more building blocks)

Work done **by electric force**:

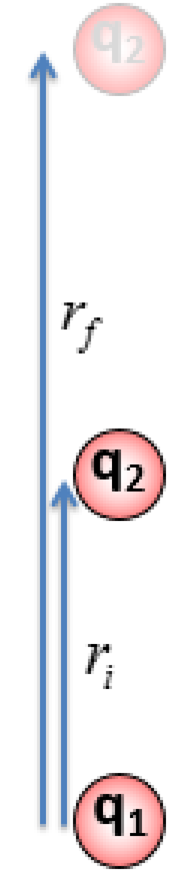
$$W_{i\to f}^{ELEC} = \int_{r_i}^{r_f} \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\,dr = \frac{1}{4\pi\varepsilon_0} q_1 q_2 \int_{r_i}^{r_f} r^{-2}\,dr$$

Recall from integral calculus

$$\int_{x_i}^{x_f} x^n\,dx = \frac{1}{n+1}x^{n+1}\Big|_{x_i}^{x_f} = \frac{1}{n+1}\left(x_f^{n+1} - x_i^{n+1}\right)$$

In our case, let x → r, then we have

$$W_{i\to f}^{ELEC} = \frac{1}{4\pi\varepsilon_0} q_1 q_2 \int_{r_i}^{r_f} r^{-2}\,dr = \frac{1}{4\pi\varepsilon_0} q_1 q_2 \left(\frac{1}{-2+1} r^{-2+1}\right)\Bigg|_{r_i}^{r_f}$$



q_1 held fixed

Finding Potential Energy of two point charges (more building blocks)

$$W_{i\to f}^{ELEC} = -\frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}\bigg|_{r_i}^{r_f}$$

$$W_{i\to f}^{ELEC} = -\left(\frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_f} - \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_i}\right)$$

$$W_{i\to f}^{ELEC} = -\Delta U = -(U_f - U_i) = U_i - U_f$$

Then the potential energy of two point charges a distance r apart is

$$\boxed{U_e = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r} \cancel{+U_0}}$$

(1) There is a U_0, but we normally set it to zero.

(2) The potential energy of two charges an infinite distance apart ($r = \infty$) is zero.

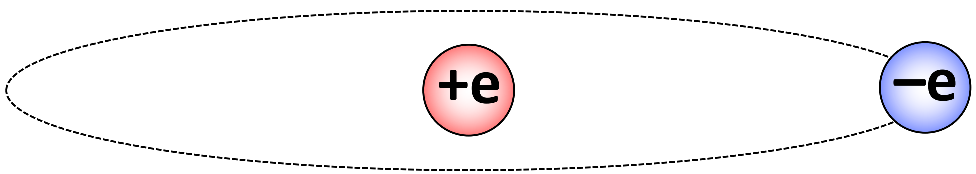
TopHat Question

The Bohr model of the hydrogen atom consists of an electron orbiting a proton with a radius of $r_B = 0.529 \times 10^{-10}$ m. What is the electric potential energy of a hydrogen atom in this model **in units of eV**?

$\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{C^2/N\,m^2}$

$e = 1.60 \times 10^{-19}\ \mathrm{C}$

$1\ \mathrm{eV} = 1.60 \times 10^{-19}\ \mathrm{J}$



A. –13.6 eV

B. –27.2 eV

C. –5.75 x 10^{11} eV

D. –9.21 x 10^{-8} eV

TopHat Question

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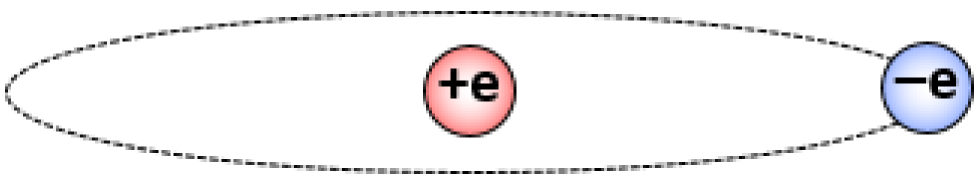
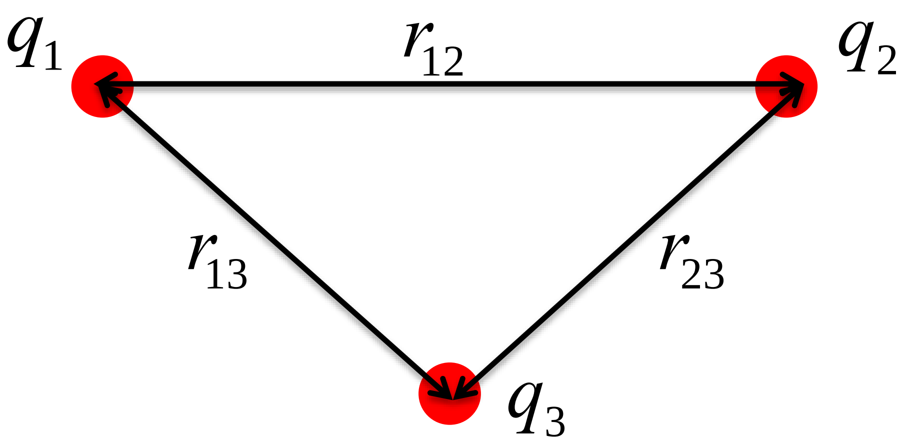


$$U_e = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r} = -\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_B} \Rightarrow U_e = -4.60 \times 10^{-18}\ \mathrm{J}$$

$$U_e = \left(-4.60 \times 10^{-18}\ \cancel{\mathrm{J}}\right)\left(\frac{1\ \mathrm{eV}}{1.60 \times 10^{-19}\ \cancel{\mathrm{J}}}\right) = -27.2\ \mathrm{eV}$$

The kinetic energy of the electron is +13.6 eV, so the binding energy of H is −13.6 eV

Superposition: Potential Energy due to Multiple Charges



$$U_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}}$$

$$U_{23} = \frac{1}{4\pi\varepsilon_0}\frac{q_2 q_3}{r_{23}}$$

$$U_{13} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_3}{r_{13}}$$

$$\boxed{U_{total} = U_{12} + U_{23} + U_{13}}$$

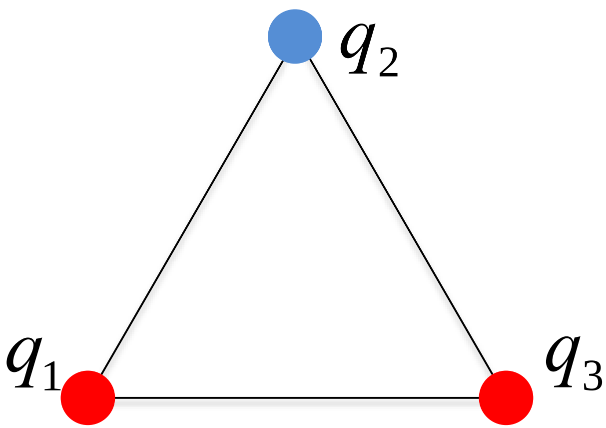
In general, the total potential energy is just the sum of the pairwise potential energies of all the charges present. Calculate U between each pair, then sum all of them up.

TopHat Question

Three charges q_1 = 1.0 nC, q_2 = –2.0 nC, and q_3 = 3.0 nC are fixed in an equilateral triangle of side length d = 5.0 cm. What is the electric potential energy of this configuration?

$$\epsilon_0 = 8.85\times 10^{-12}\ \mathrm{C^2/N\,m^2}$$

$$U_{ij} = \frac{1}{4\pi\epsilon_0}\frac{q_i q_j}{r_{ij}}$$



$$U_{12} = -3.596\times 10^{-7}\ \mathrm{J}$$
$$U_{23} = -1.079\times 10^{-6}\ \mathrm{J}$$
$$U_{13} = +5.394\times 10^{-7}\ \mathrm{J}$$

A. 2.0 x 10^{-6} J

B. 1.3 x 10^{-6} J

C. –9.0 x 10^{-7} J

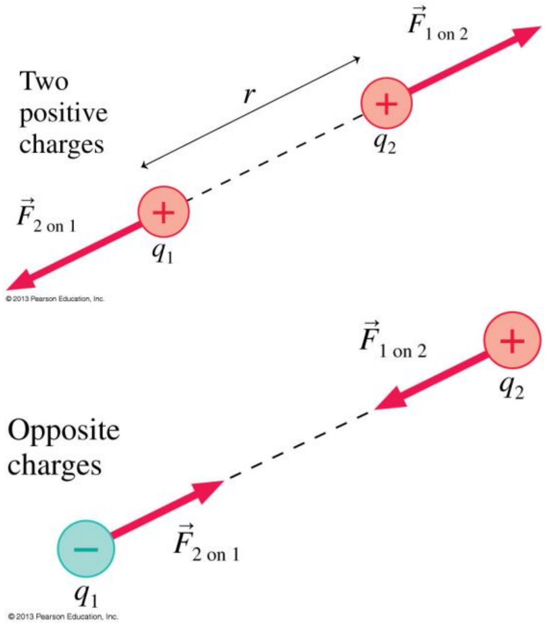
D. –1.8 x 10^{-7} J

Electric Force vs Electric Field

Electric Force $\vec{F}$

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2} = q\vec{E}$$

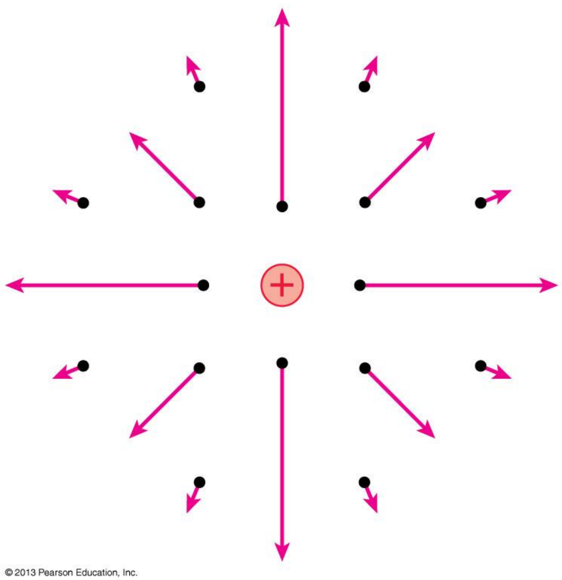
A physical property between two point charges



Electric Field $\vec{E}$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

A physical property around a single point charge



Electric Force vs Electric Field

Electric Force $\vec{F}$

Electric Fi

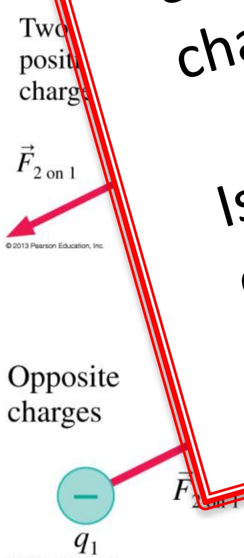
$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2} = q\vec{E}$$

A p ... round

two

Potential energy is a physical property that exists because of the force between two charges.

Is there some similar notion of "potential energy" that exists only because of the electric field?



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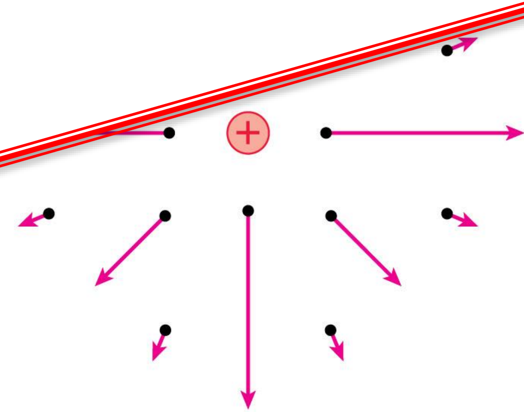

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Electric Potential

Here are some source charges and a point P.

If we place a charge q at point P, then q and the source charges interact with each other.

The interaction energy is the potential energy of q and the source charges,

$$U_{q+sources}$$

How does this interaction happen?

Electric Potential

Model:

The source charges create a **potential for interaction** everywhere, including at point P.

This potential for interaction is a **property of space**. Charge q does not need to be there.

We call this potential for interaction the **electric potential, V.** (Often just called “the potential”)

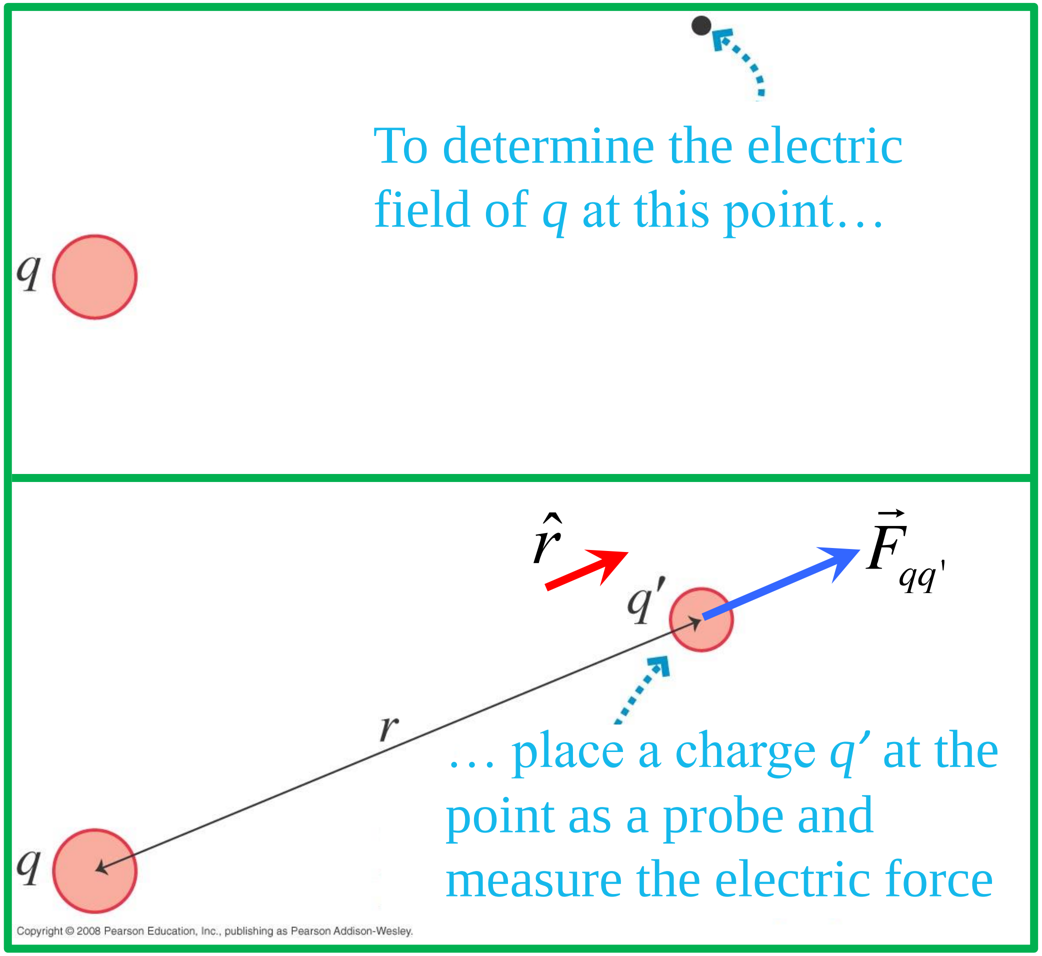
Electric Field of a point charge

Electric force on q’ from q

$$\vec{F}_{qq'} = \frac{1}{4\pi\varepsilon_0}\frac{qq'}{r^2}\hat{r}$$

Then the electric field of q is

$$\vec{E} = \frac{\vec{F}_{qq'}}{q'} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$$



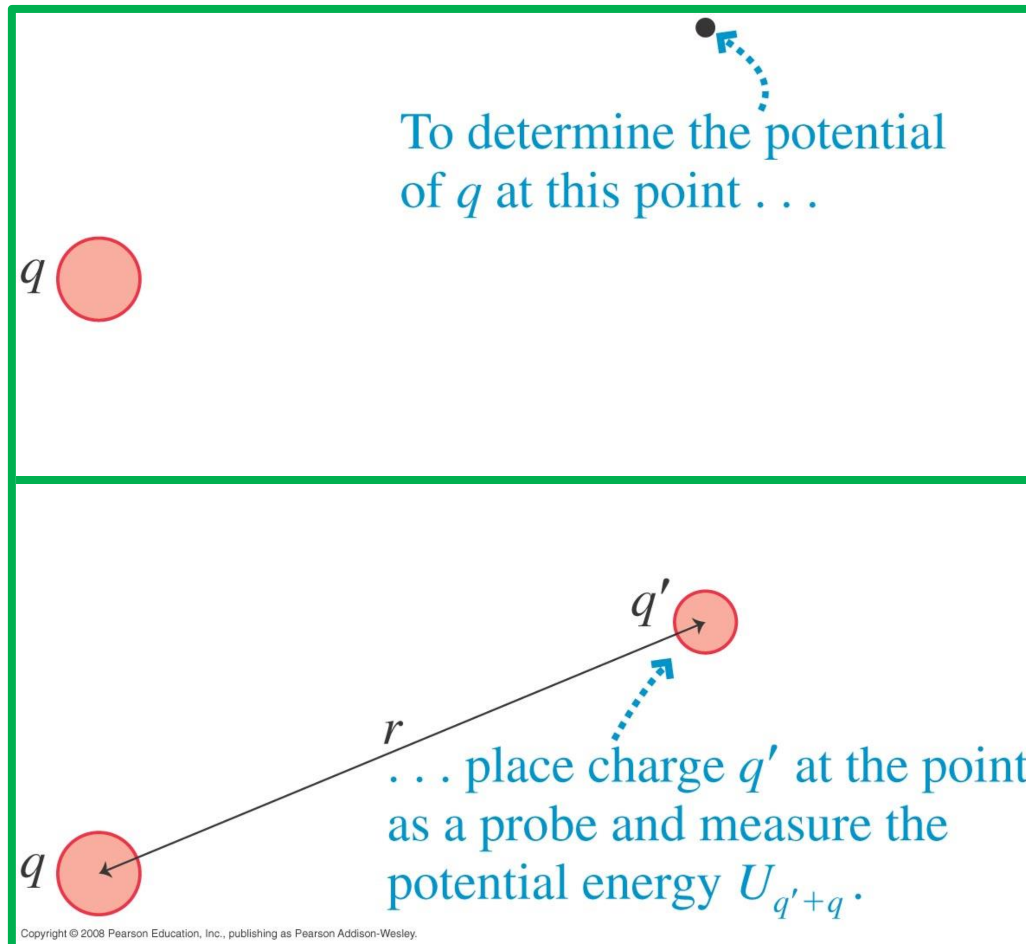
Electric Potential of a point charge

Potential energy of q and q’

$$U_{q'+q} = \frac{1}{4\pi\varepsilon_0}\frac{qq'}{r}$$

Then the potential of q is

$$V = \frac{U_{q'+q}}{q'} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$



Electric Potential

Definition of V: Place charge q at point P and measure its potential energy. Then

$$\boxed{V \equiv \frac{U_{q+sources}}{q}}$$

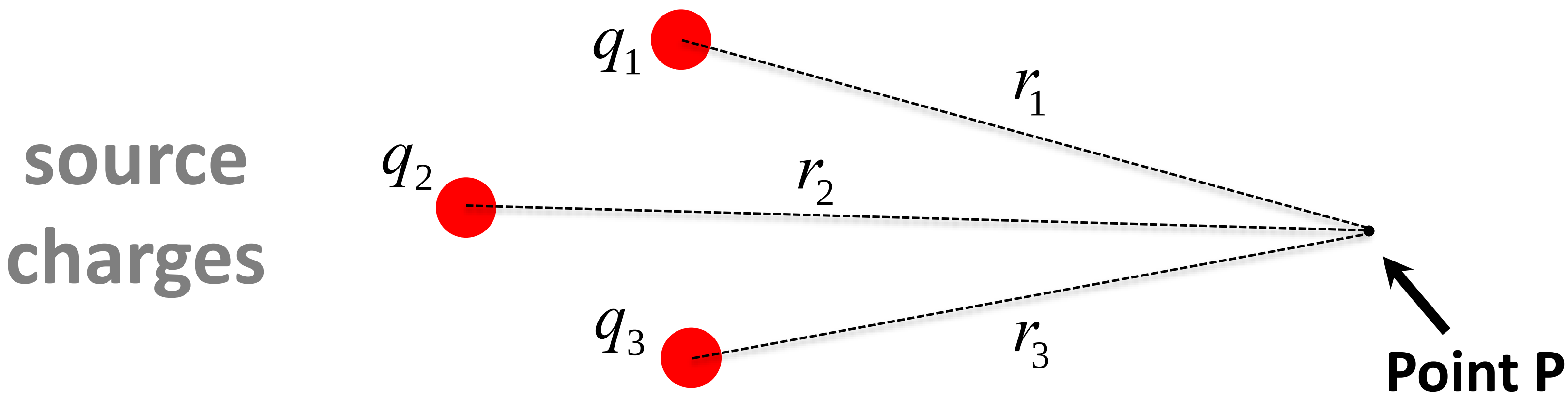
Unit: $1\,\text{volt} = 1\,\text{V} = 1\,\frac{\text{J}}{\text{C}}$

Electric Potential

Or, if we know the potential, V, at point P, then if we place a charge, q, at point P, the potential energy of q and the source charges is

$$\boxed{U_{q+sources} = qV}$$

Advantage of Electric Potential



V is a SCALAR! There is no direction associated with it.
This makes it much easier to calculate!

$$V_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1} \qquad V_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2} \qquad V_3 = \frac{1}{4\pi\varepsilon_0}\frac{q_3}{r_3}$$

$$\boxed{V = V_1 + V_2 + V_3}$$