Electricity and Magnetism

- Physics 259 – L02
- Lecture 45

Chapter 30: Induction and inductance

Last Time:

- Chapter 29

Today:

- Faraday’s Law of Induction
- Non-conservative electric fields
- Lenz’s Law

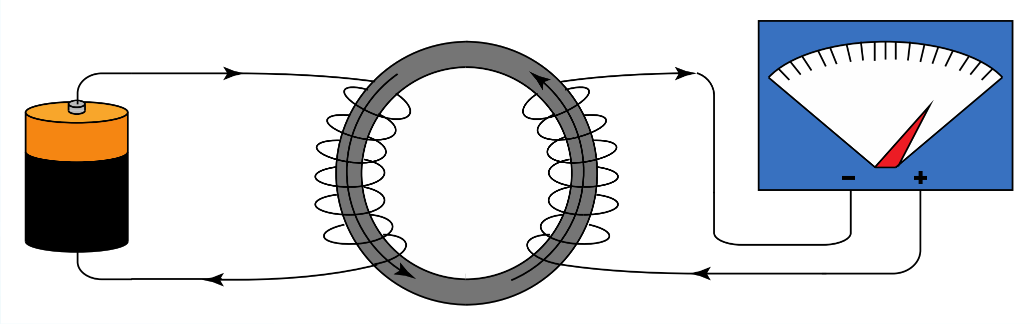
Section 3.1: Faraday’s law and Lens’s law

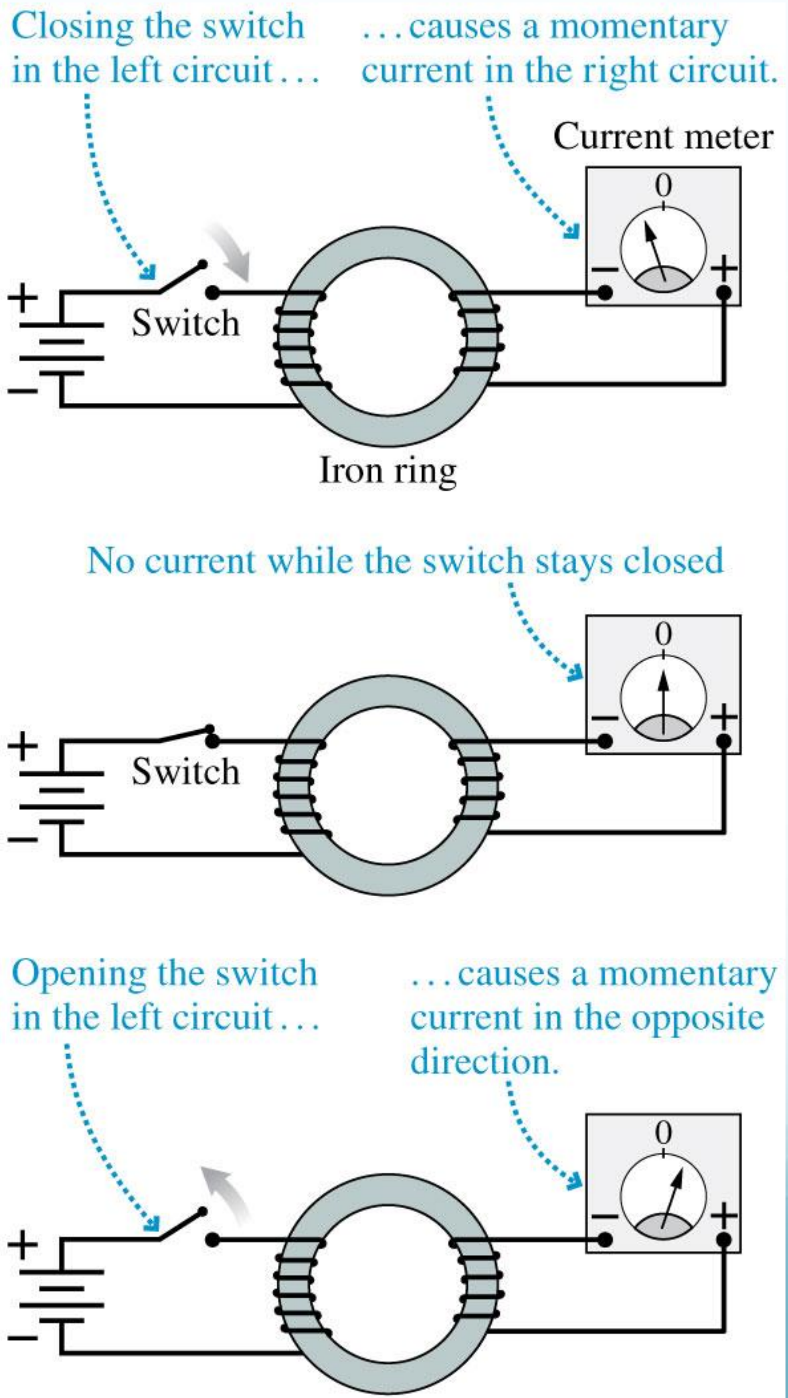
Electric currents cause magnetic fields

- Much of chapter 29 was about magnetic fields being caused by moving electrical charges

The reverse is true

- Magnetic fields can cause electrical currents
- Only changing magnetic fields can cause currents
- Which makes sense since only moving electrical charges can cause magnetic fields





Closing the switch in the left circuit . . .
. . . causes a momentary current in the right circuit.
Current meter
0
Switch
Iron ring
No current while the switch stays closed
0
Switch
Opening the switch in the left circuit . . .
. . . causes a momentary current in the opposite direction.
0

Changing magnetic field

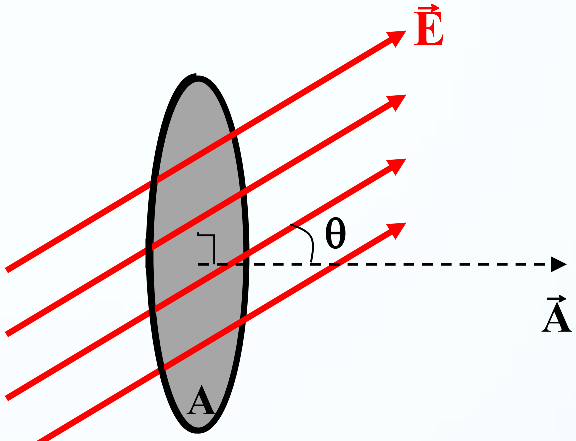
In order for the magnetic field to change

A) The strength of the field would change
B) The direction of the field would change
C) The size of the loop of wire would change
D) The direction of the loop of wire would change

Electric flux versus Magnetic flux

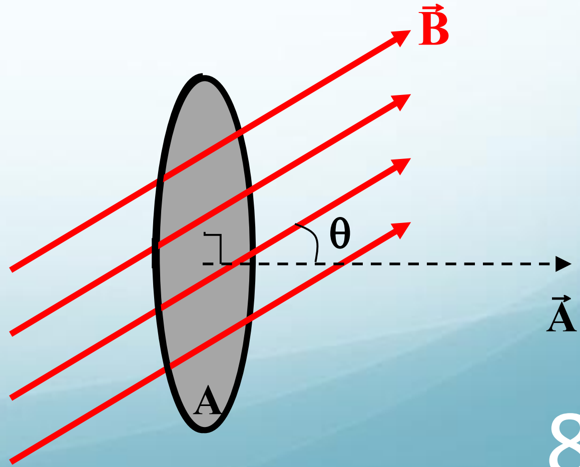
Electric flux of a constant electric field:

$$\Phi_e = \vec{E} \cdot \vec{A} = EA\cos\theta$$



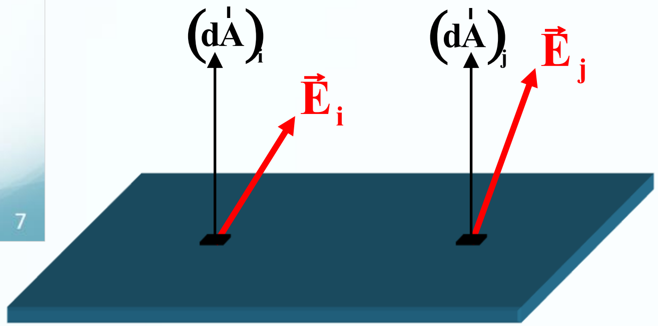
Magnetic flux of a uniform magnetic field:

$$\Phi_B = \vec{B} \cdot \vec{A} = BA\cos\theta$$

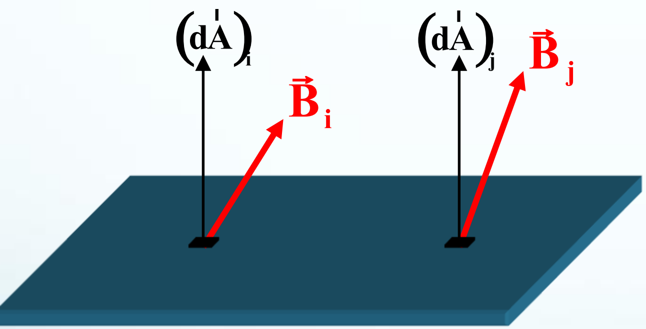


Unit: $\mathrm{T \cdot m^2 = Wb}$

More generally



$$\Phi_e = \oint \vec{E} \cdot d\vec{A}$$



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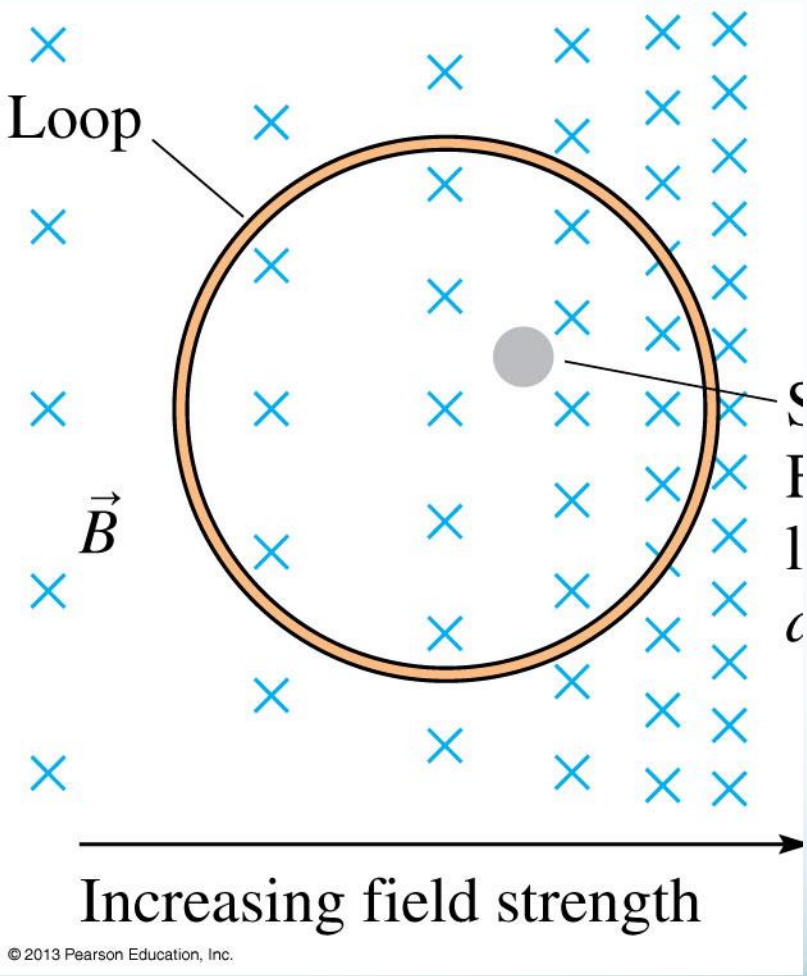


Magnetic flux can be thought of us counting the field lines

Charges don’t start moving spontaneously!

A current requires an emf to exist

$$I_{induced} = \frac{\varepsilon}{R}$$

emf associated with a changing magnetic flux→induced emf

The induced emf is the rate of change of magnetic flux through the loop.

Faraday discovered that there is an induced EMF in the secondary circuit given by

$$\varepsilon = -\frac{d\Phi_M}{dt}$$

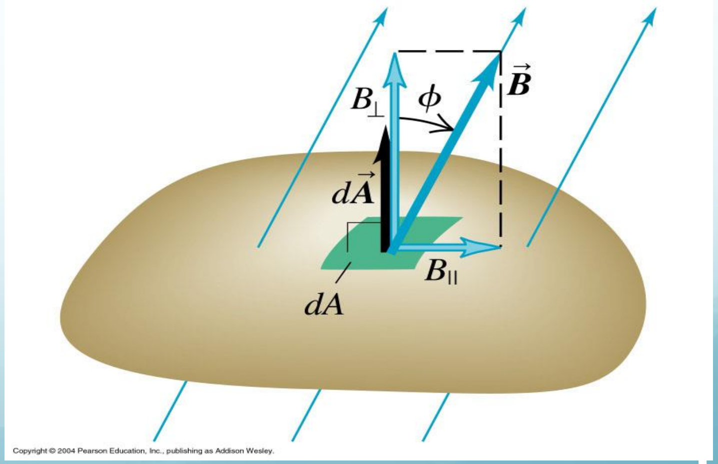
This is a new generalized law called **Faraday's Law**.

For a coil of N turns→ $$\varepsilon = -N\frac{d\Phi_M}{dt}$$

Recall the definition of magnetic flux:

$$\Phi_M = \int \vec{B} \cdot d\vec{A}$$

Not a closed surface!



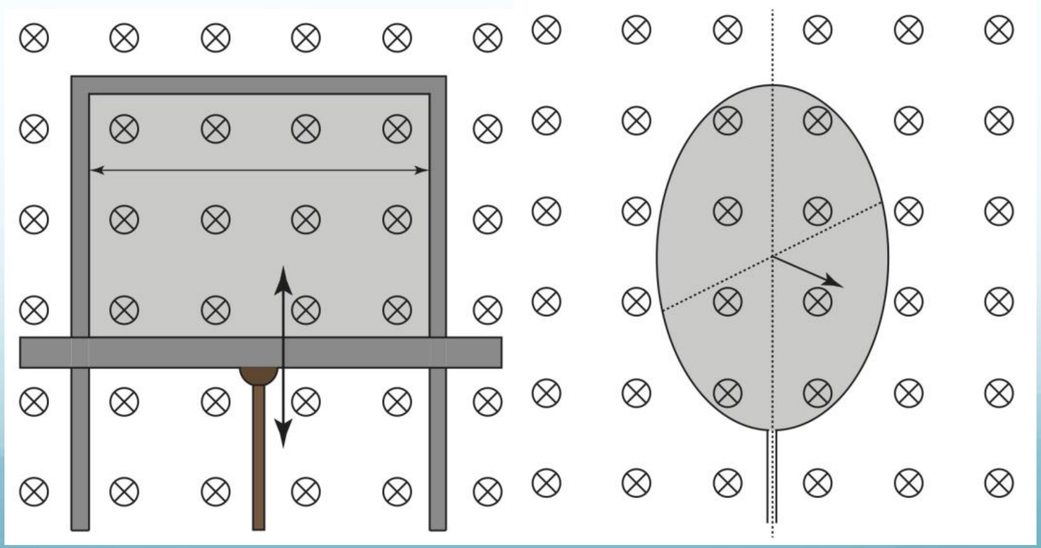
This is valid even if Φ_M changes because of a time dependent A or angle φ (without changing the magnetic field)!

$$\varepsilon = -\frac{d\Phi_M}{dt}$$

$$e = -\frac{d}{dt}\left(BA\cos f\right)$$

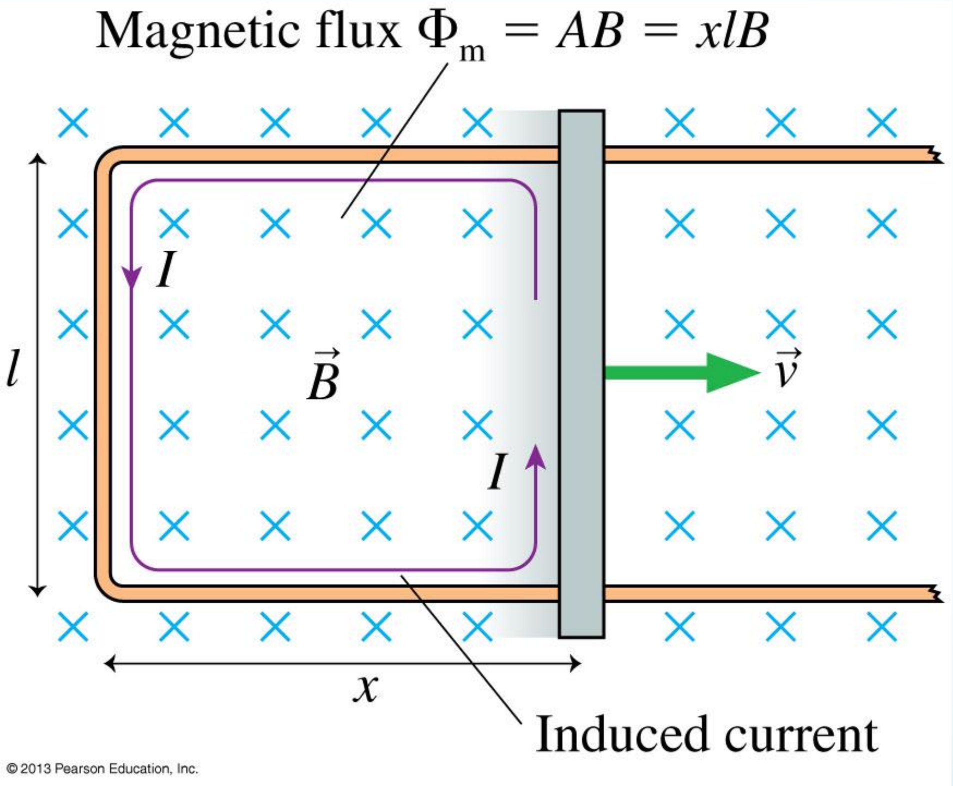
→ 3 possible terms

$$e = -\frac{dB}{dt}A\cos f - \frac{dA}{dt}B\cos f + \frac{df}{dt}BA\sin f$$



Example of using Faraday's law

$$\Phi_m = A.B$$



$$\varepsilon = \left|\frac{d\Phi_m}{dt}\right| = \frac{d}{dt}xlB = vlB$$

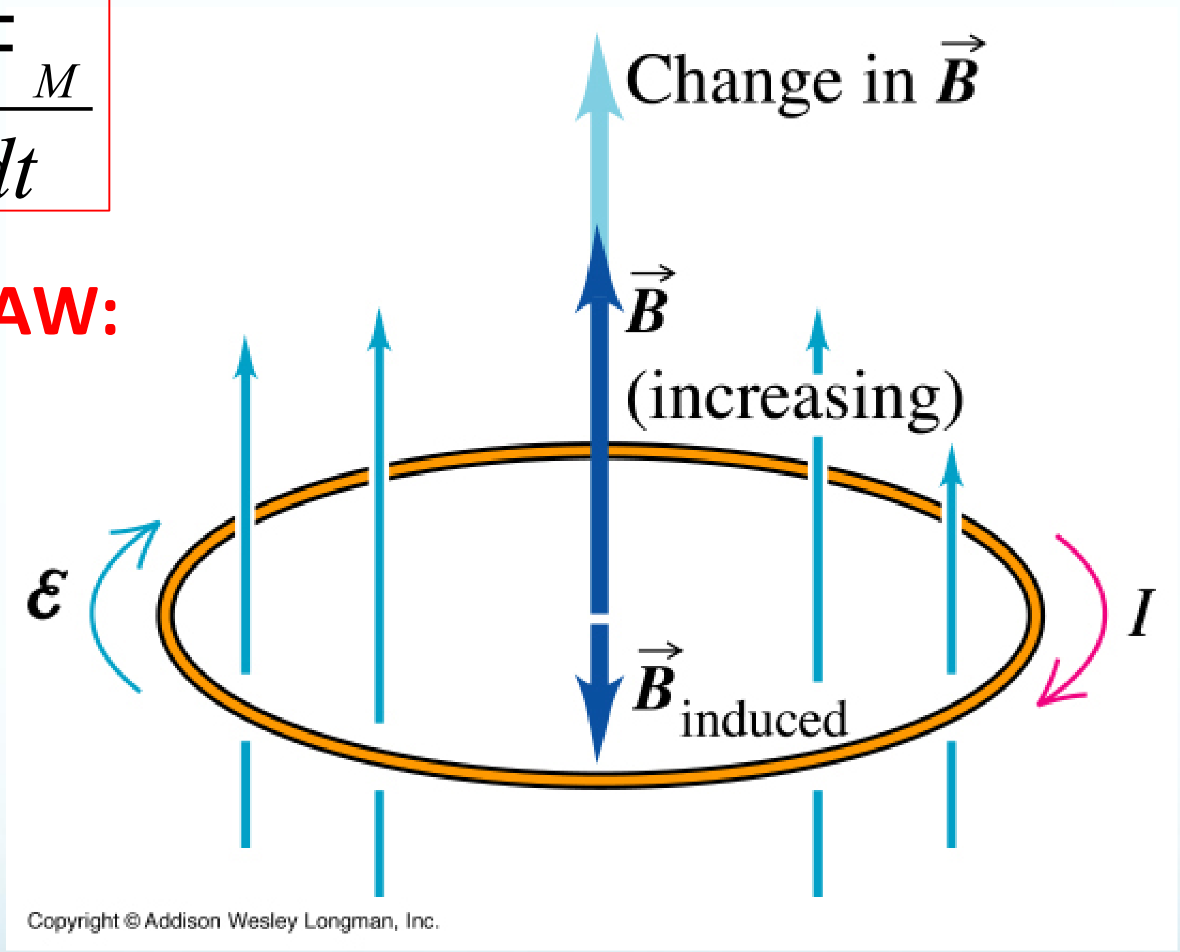
Use Ohm's law to find induced current

$$I = \frac{\mathcal{E}}{R} = \frac{vlB}{R}$$

What about the **minus sign** in Faraday's law?

$$\varepsilon = -\frac{d\Phi_M}{dt}$$

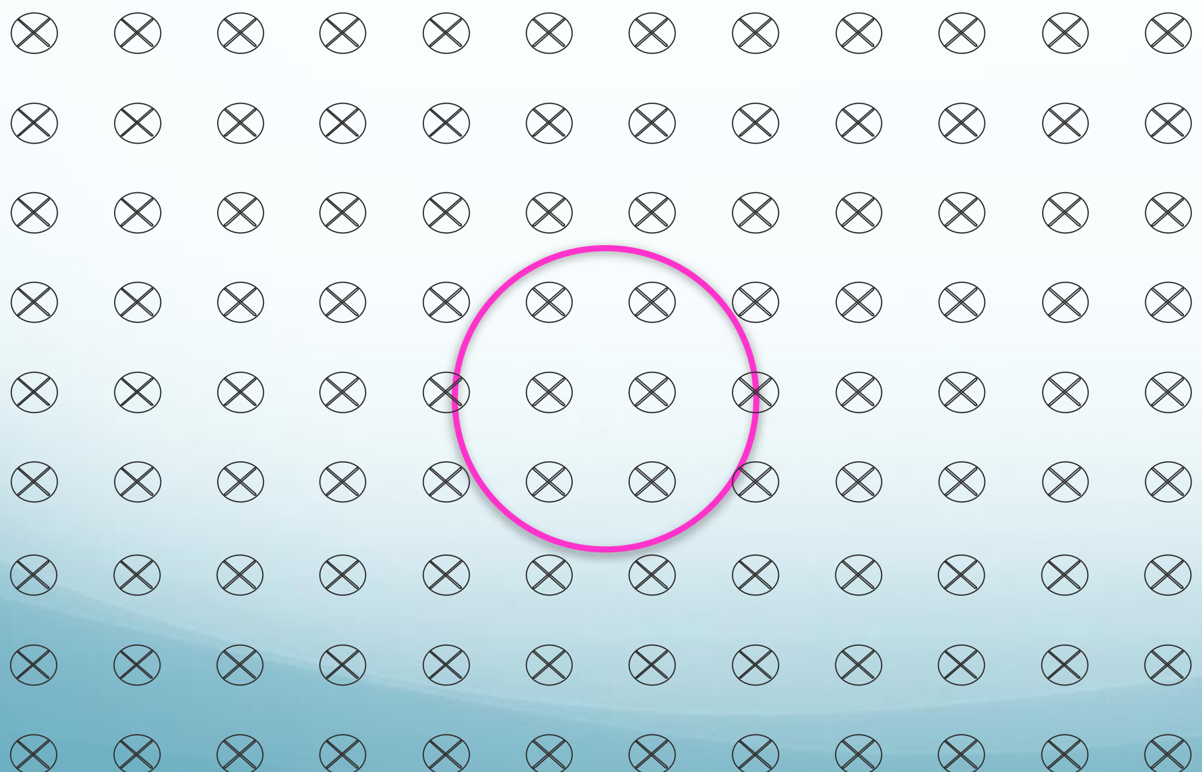
LENZ'S LAW:



The changing magnetic flux generates an induced current which creates an induced magnetic field which, in turn, resists the change in magnetic flux.

Lenz's Law

The induced current from Faraday's Law is always in a direction such that the induced magnetic field from the induced current opposes the change in the magnetic flux through the loop.



More B-field lines inside the loop: induced B-field from induced current must be out of the page to compensate. Induced current is CCW

I_{ind}

B_{ind}

Lenz's Law

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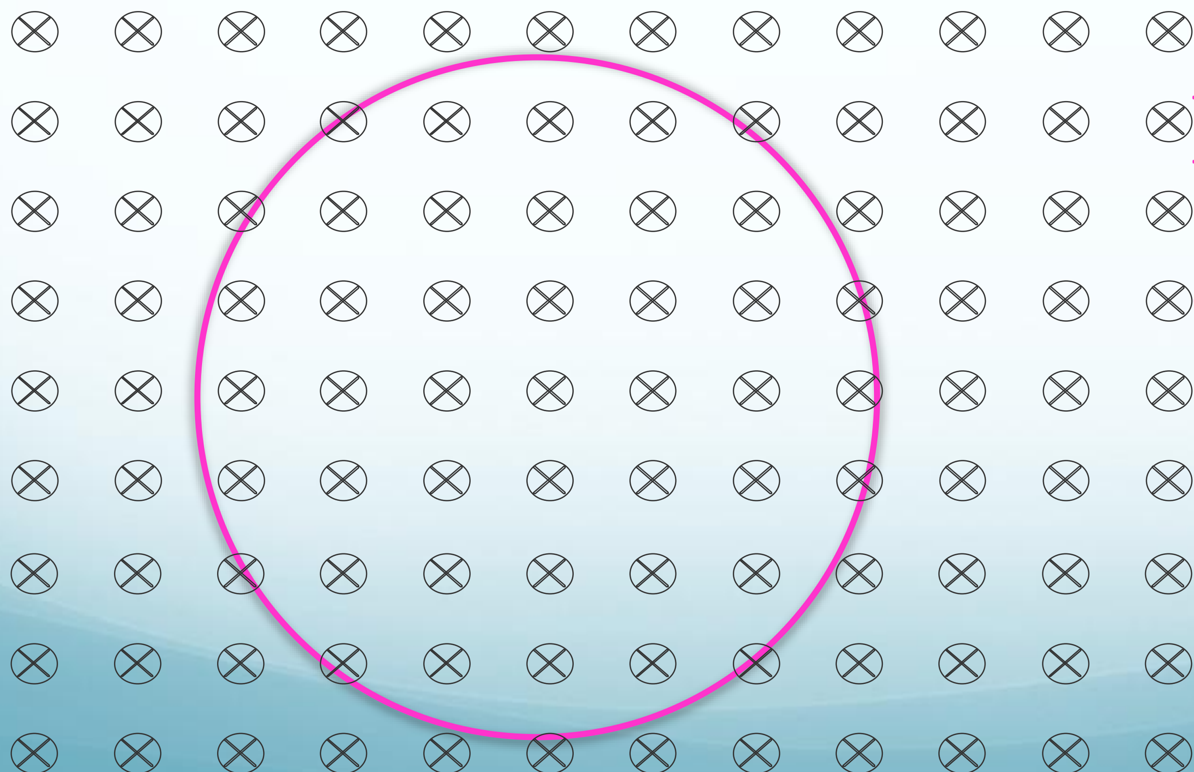
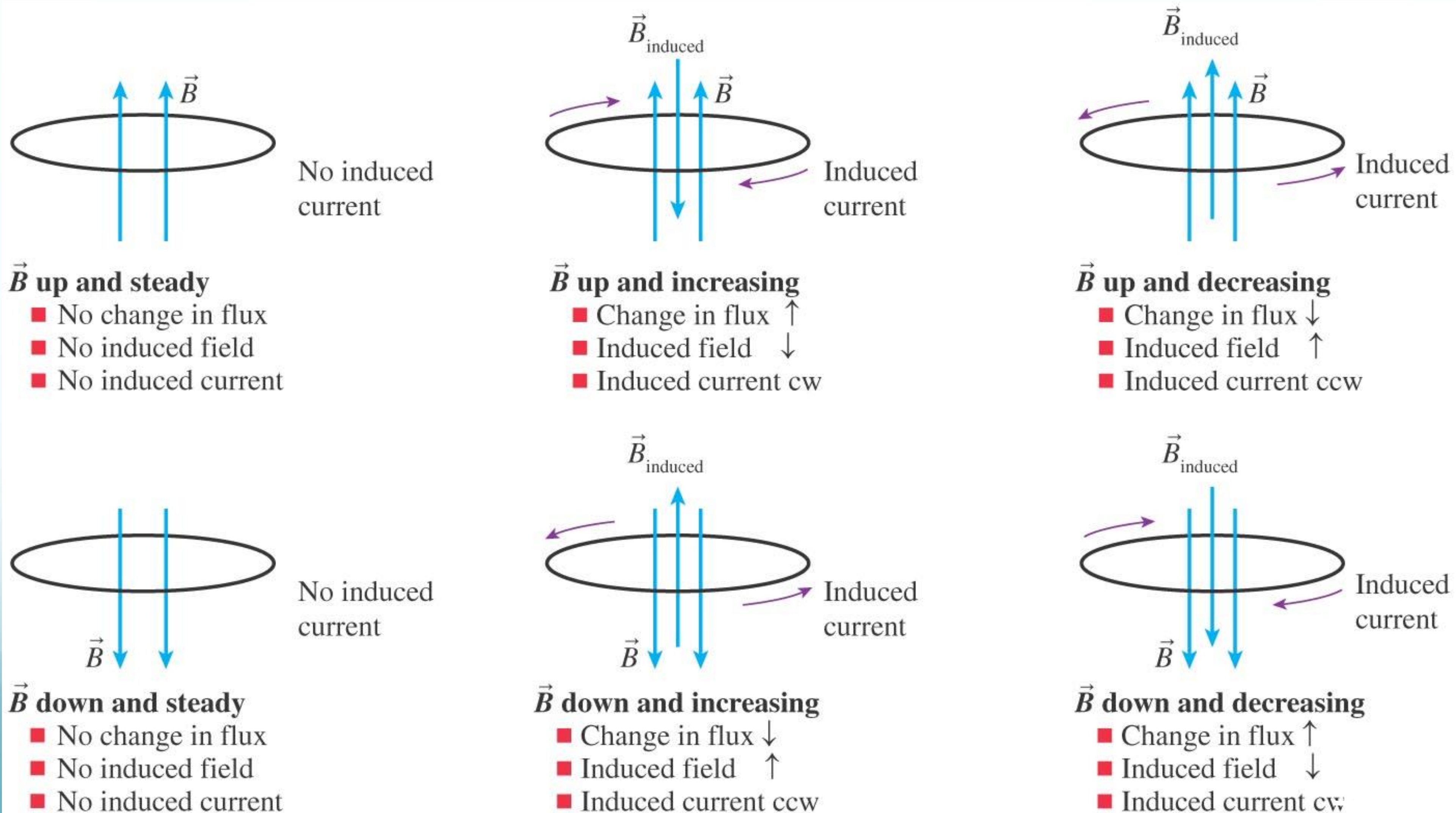


Fewer B-field lines inside the loop: induced B-field from induced current must be into the page to compensate.

Induced current is CW

Lenz's Law

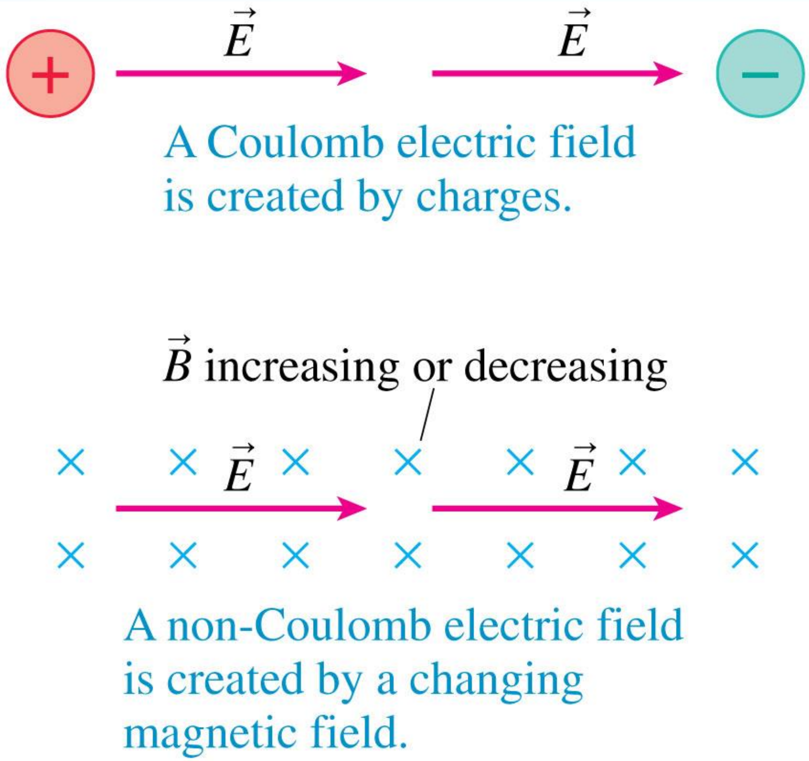
The currents always generate a field to oppose the change in flux.

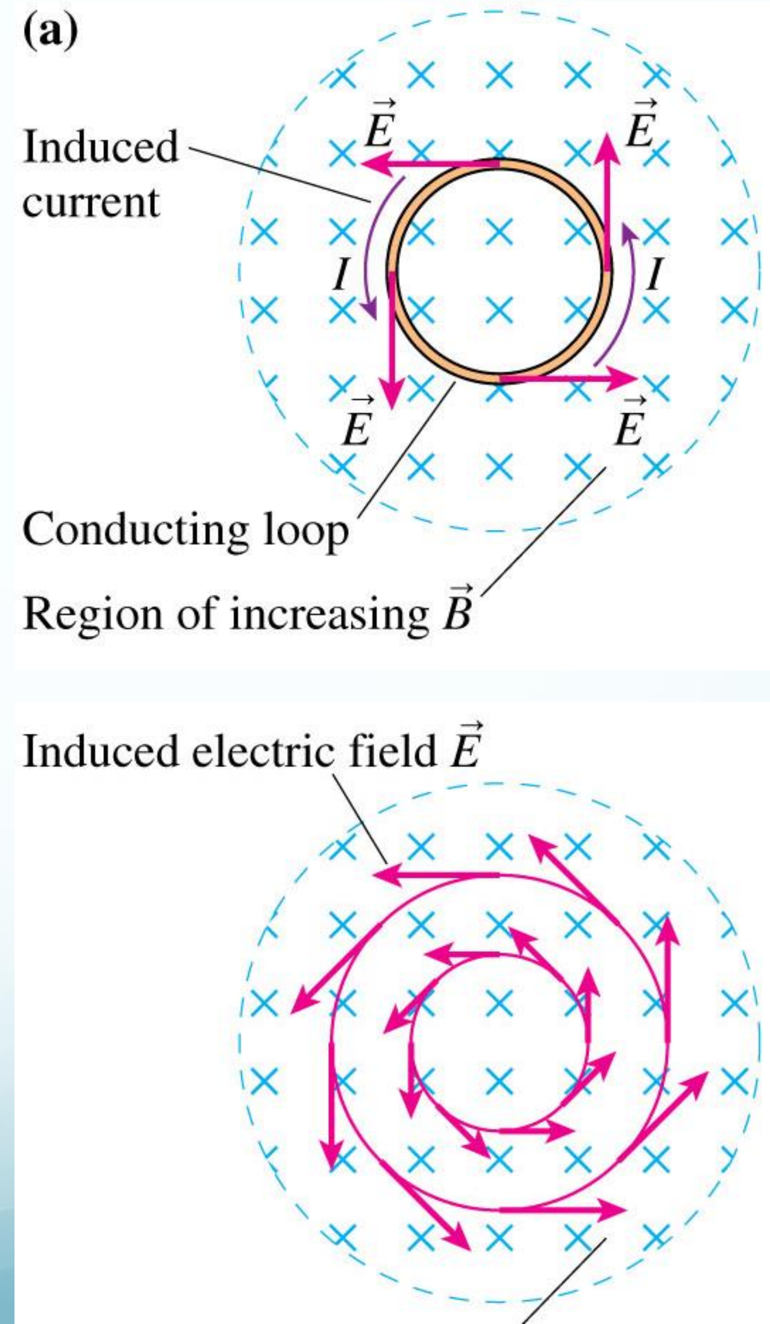


Faraday's law→ strength of induced current

What cause the current?

There is an electric field
caused by changing magnetic field
→ **Induced electric field**

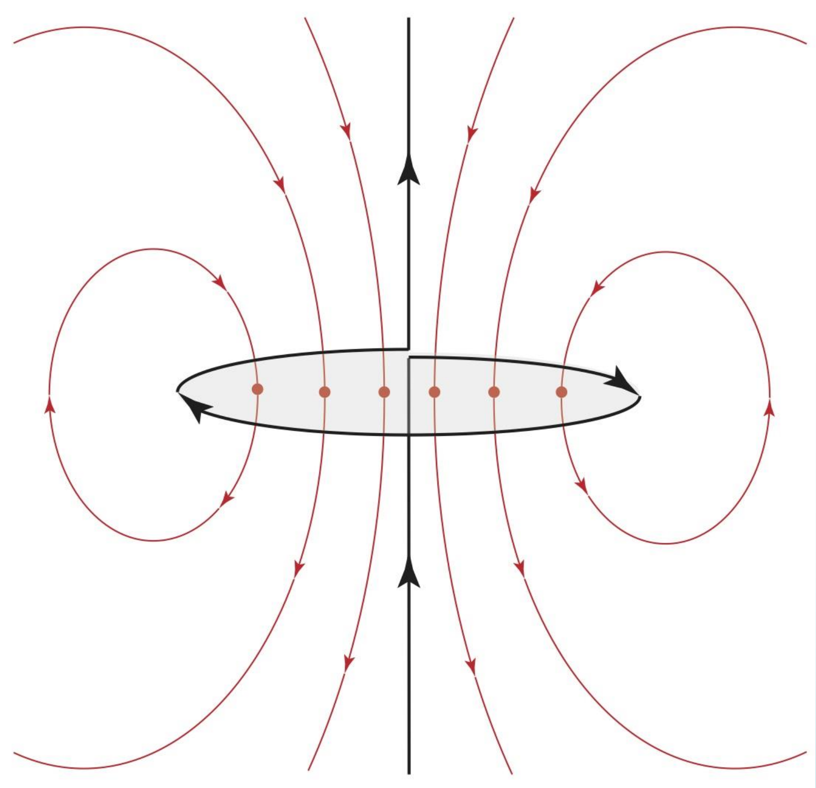


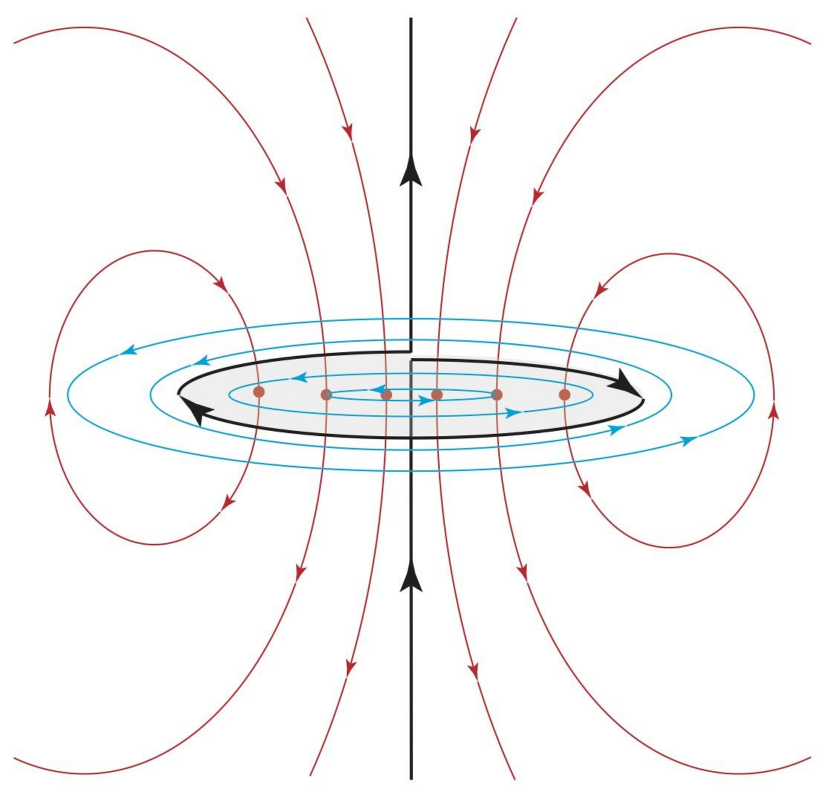


Imagine a loop in a wire carrying a current I_1. The current is then increased to $I_2 > I_1$, increasing the magnetic flux.

Changing B-fields induce non-conservative E-fields.

$$-\frac{d\vec{B}}{dt} = \nabla \times \vec{E}$$





This section we talked about:

Chapter 30

See you on Wednesday