Electricity and Magnetism

- Physics 259 – L02
- Lecture 10

Sections 22.4-5

Last time

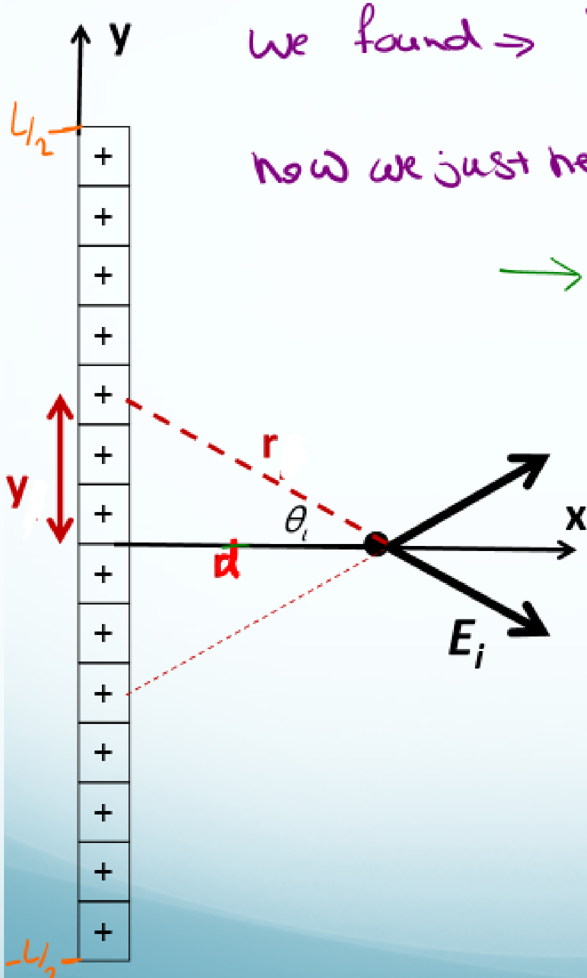
- Chapter 21
- Van De Graaff Generator Experiment 
- Electric Ping Pong Experiment 

This time

- Chapter 22
- Field of a Ring of Charge and a Disk

22-4: The Electric Field Due To a Line of Charge

For a line of charge we found the force as:



we found $\Rightarrow$ $F_{net,x} = \int_{-L/2}^{L/2} \frac{d\, k q \lambda\, dy}{(d^2+y^2)^{3/2}}$

now we just need to solve the integral =>

$$\rightarrow \vec{F}_{net} = \frac{kQq}{d\sqrt{(L/2)^2+d^2}} \quad ¿?$$

limiting cases =>

① $d \gg L \Rightarrow d^2 + \cancel{(L/2)^2} \approx d^2$, $\cancel{(L/2)^2} \to 0$

$$\vec{F}_{net} = \vec{F}_{net} = k\frac{Qq}{d^2}$$

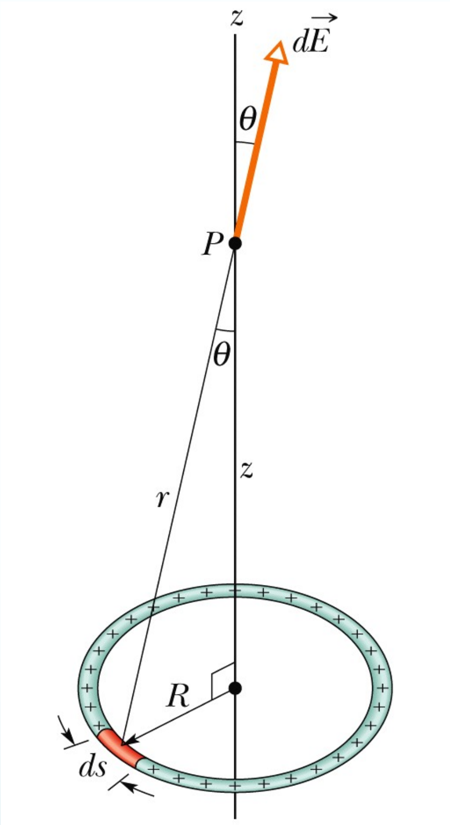
② $d \ll L \Rightarrow \cancel{d^2} + (L/2)^2 \approx (L/2)^2$, $\cancel{d^2} \to 0$

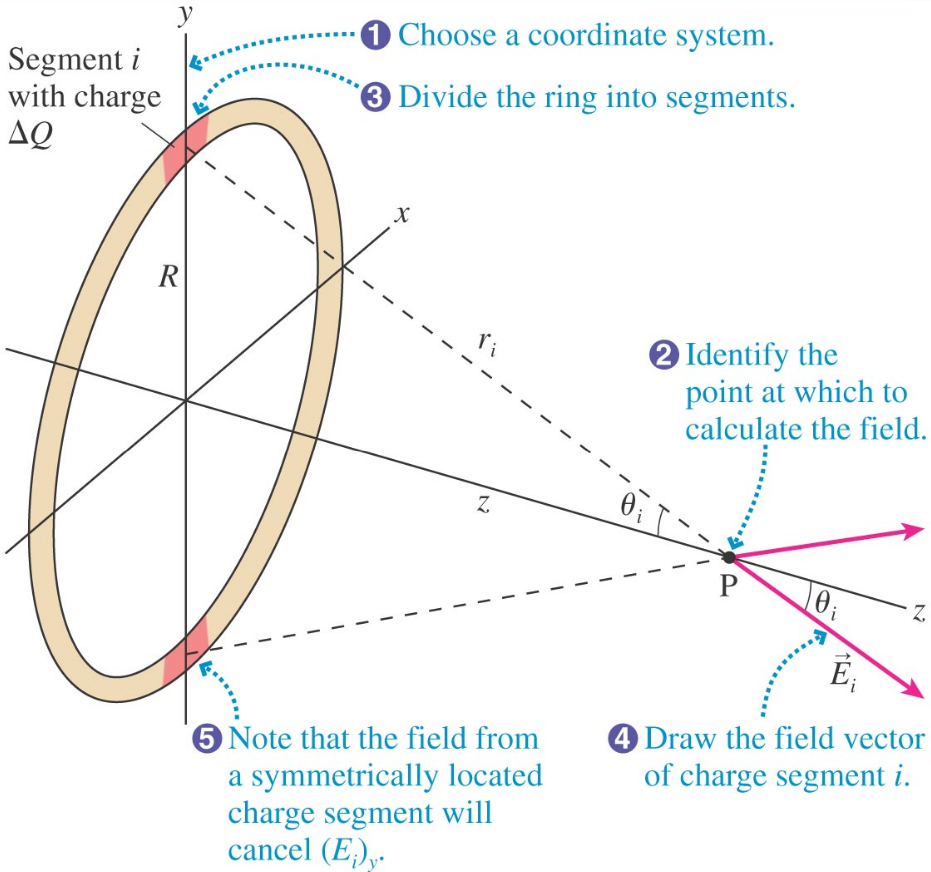
$$\vec{F}_{net} = k\frac{qQ}{L/2} = k\frac{2qQ}{L}$$

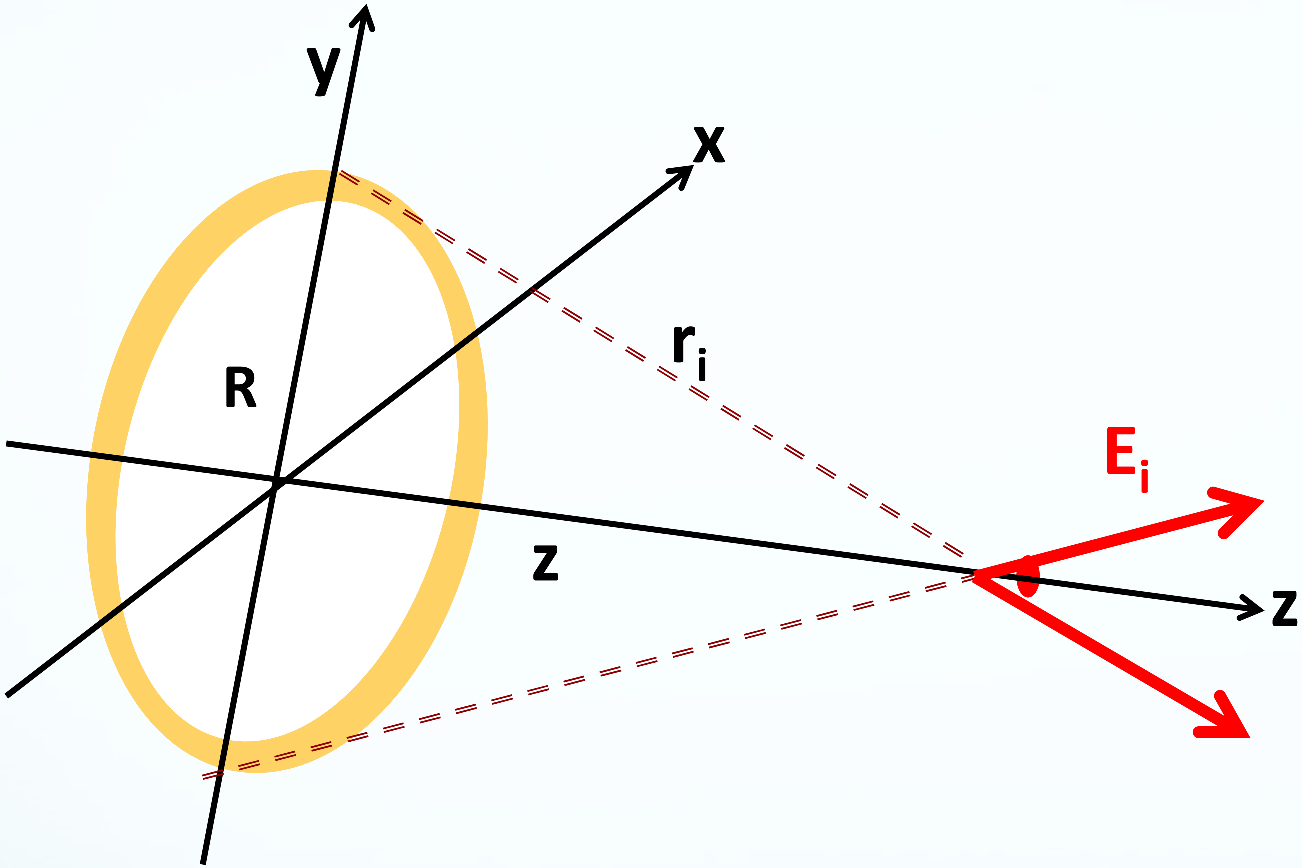
infinite long wire

Therefore, the field→

A ring of charge, similar derivation

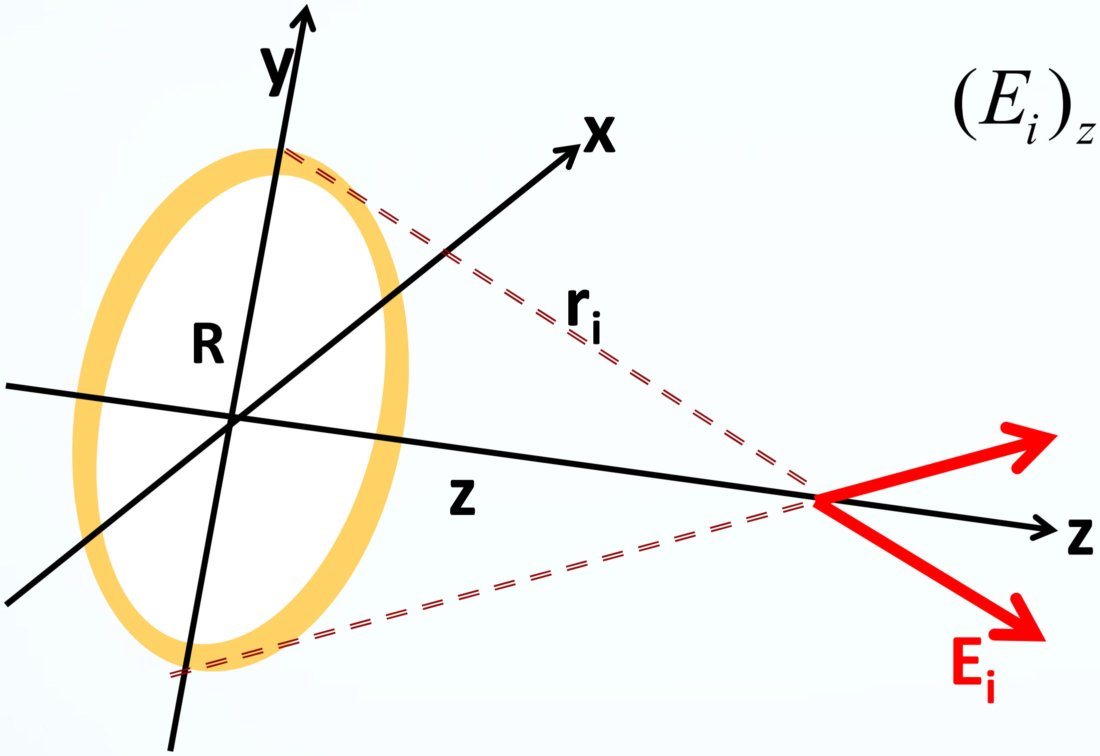






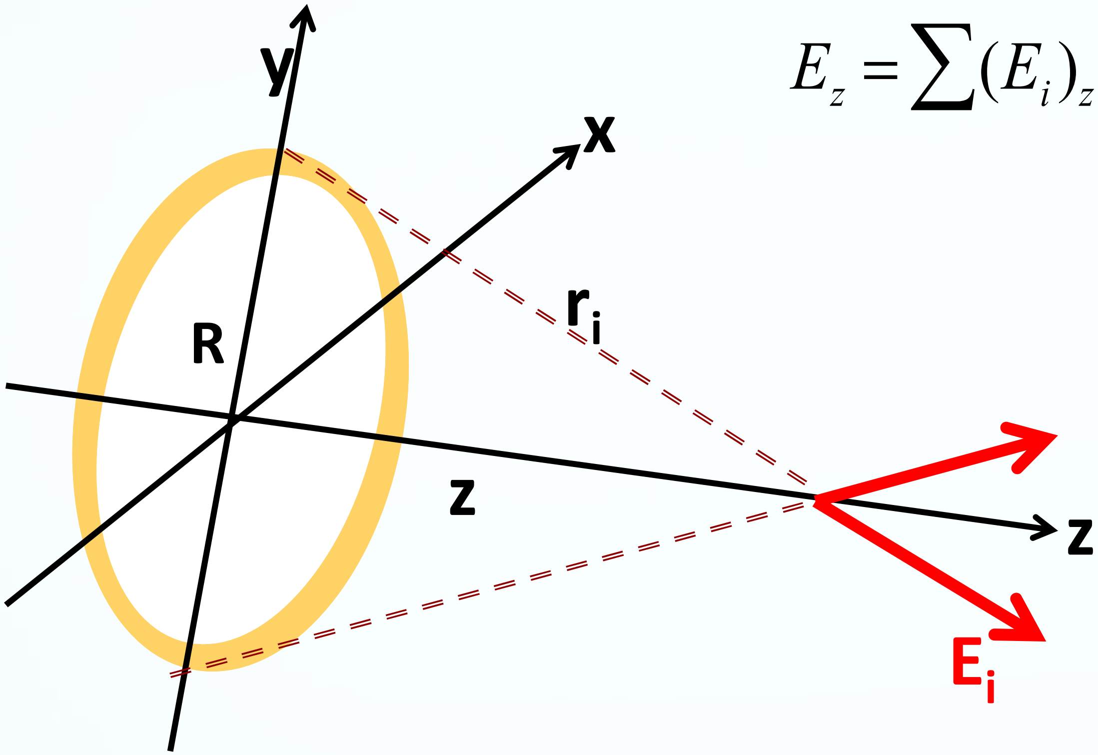
$$(E_i)_z = \frac{1}{4\pi\varepsilon_o}\frac{\Delta Q}{r_i^2}\cos\theta_i$$

$$(E_i)_z = \frac{1}{4\pi\varepsilon_o}\frac{\Delta Q}{r_i^2}\cos\theta_i$$



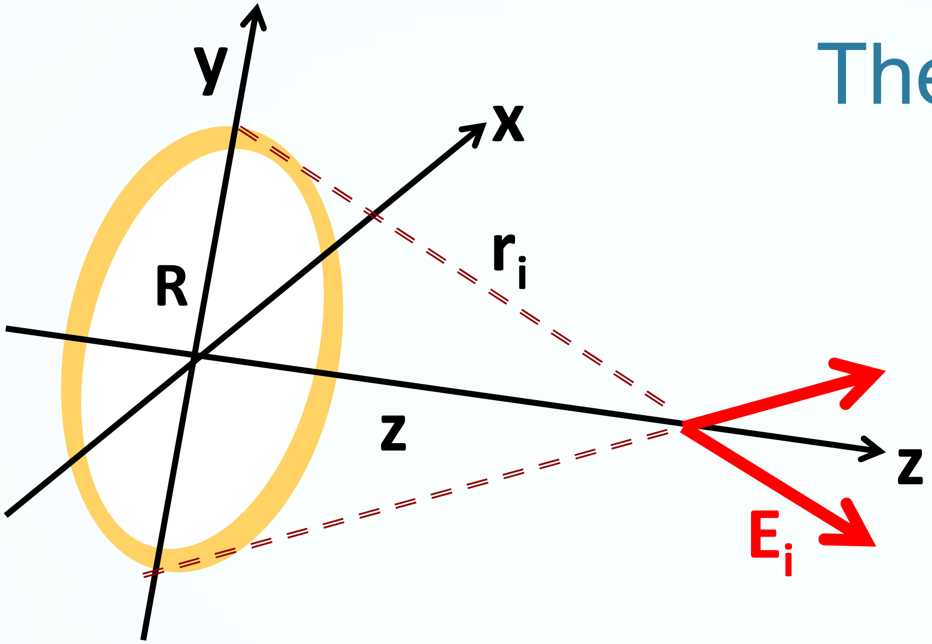
$$(E_i)_z = \frac{1}{4\pi\varepsilon_o}\frac{z\Delta Q}{\left(z^2+R^2\right)^{\frac{3}{2}}}$$

$$E_z = \sum (E_i)_z = \sum \frac{1}{4\pi\varepsilon_o} \frac{z\Delta Q}{\left(z^2 + R^2\right)^{\frac{3}{2}}}$$



$$(E_{ring})_z = \frac{1}{4\pi\varepsilon_o} \frac{zQ}{\left(z^2 + R^2\right)^{\frac{3}{2}}}$$

The limiting cases:



Z=0 → $$E_{ring} = \frac{1}{4\pi\varepsilon_o}\frac{zQ}{\left(z^2+R^2\right)^{3/2}}$$ → E=0

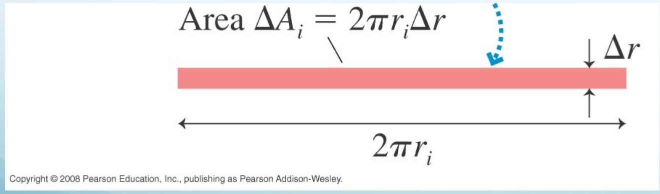
Z>>R → $$E_{ring} = \frac{1}{4\pi\varepsilon_o}\frac{zQ}{\left(z^2+0^2\right)^{3/2}} = \frac{1}{4\pi\varepsilon_o}\frac{Q}{z^2}$$

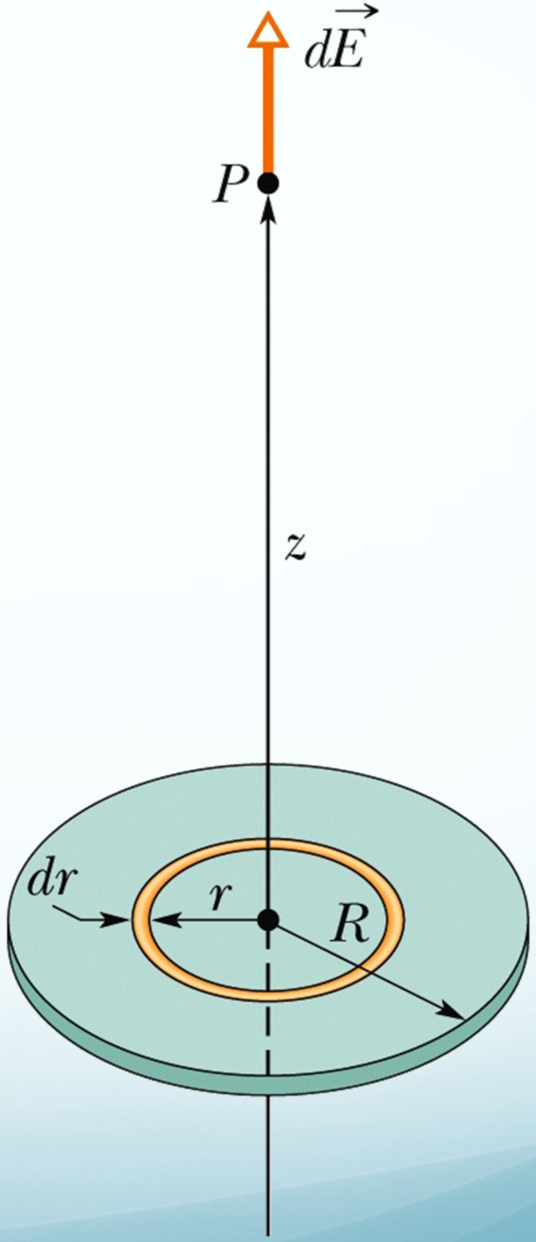
22-5: The Electric Field Due To a Charged Disk

A disk of charge

$$\sigma = \frac{Q}{A} = \frac{Q}{\pi R^2} = \frac{\Delta Q}{\Delta A_i} = \frac{dQ}{dA_i}$$

$$dQ = \sigma dA_i = \sigma 2\pi r_i dr$$





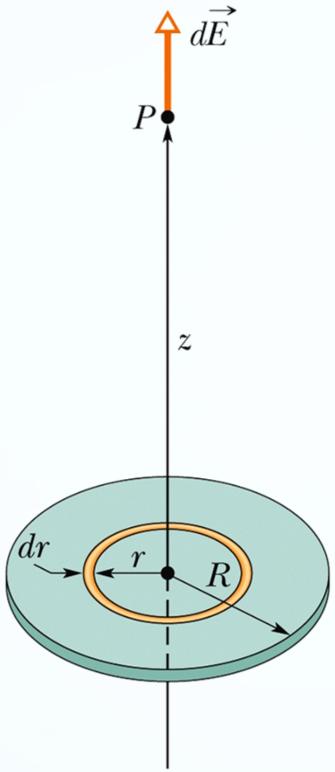
$$dE = dE_z = (E_i)_z = \frac{1}{4\pi\varepsilon_o} \frac{z\,dQ}{\left(z^2 + r_i^2\right)^{\frac{3}{2}}} \qquad dQ = \sigma 2\pi r_i dr$$

$$(E_{disk})_z = dE_z =$$

$$(E_{disk})_z = \frac{\sigma z}{2\varepsilon_o} \int_0^R \frac{r\,dr}{\left(z^2 + r_i^2\right)^{\frac{3}{2}}} \qquad (E_{disk})_z = \frac{\sigma}{2\varepsilon_o} [1 - \frac{z}{\sqrt{z^2 + R^2}}]$$

Limiting cases?

Z>>R



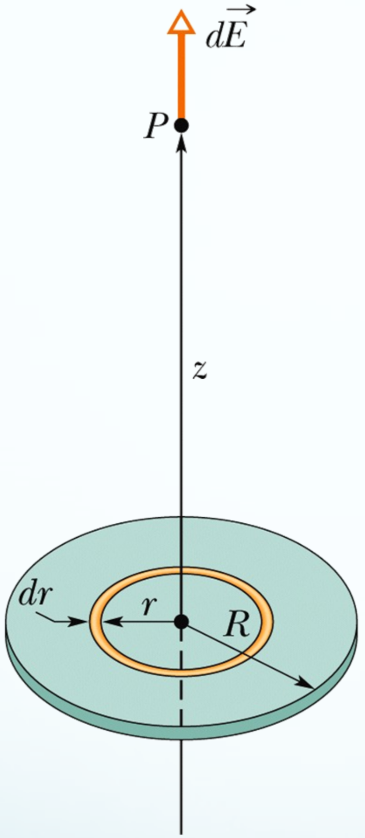
$$E_{disk,z} = \frac{\sigma}{2\varepsilon_o}\left[1 - \frac{z}{\sqrt{z^2 + R^2}}\right]$$

$$E_{disk,z} = \frac{\sigma}{2\varepsilon_o}\left[1 - \frac{z}{\sqrt{z^2}}\right] = 0????$$

$$E_{disk,z} = \frac{\sigma}{2\varepsilon_o}\left[1 - \frac{1}{\sqrt{1 + R^2/z^2}}\right] = \frac{\sigma}{2\varepsilon_o}\left[1 - \left(1 + R^2/z^2\right)^{-\frac{1}{2}}\right] = \frac{\sigma}{2\varepsilon_o}\left[1 - \left(1 - \tfrac{1}{2}R^2/z^2\right)\right]$$

$$\approx \frac{\sigma}{2\varepsilon_o}\frac{R^2}{2z^2} = \frac{Q/A}{2\varepsilon_o}\frac{\pi R^2}{2\pi z^2} = \frac{Q}{4\pi\varepsilon_o z^2}$$

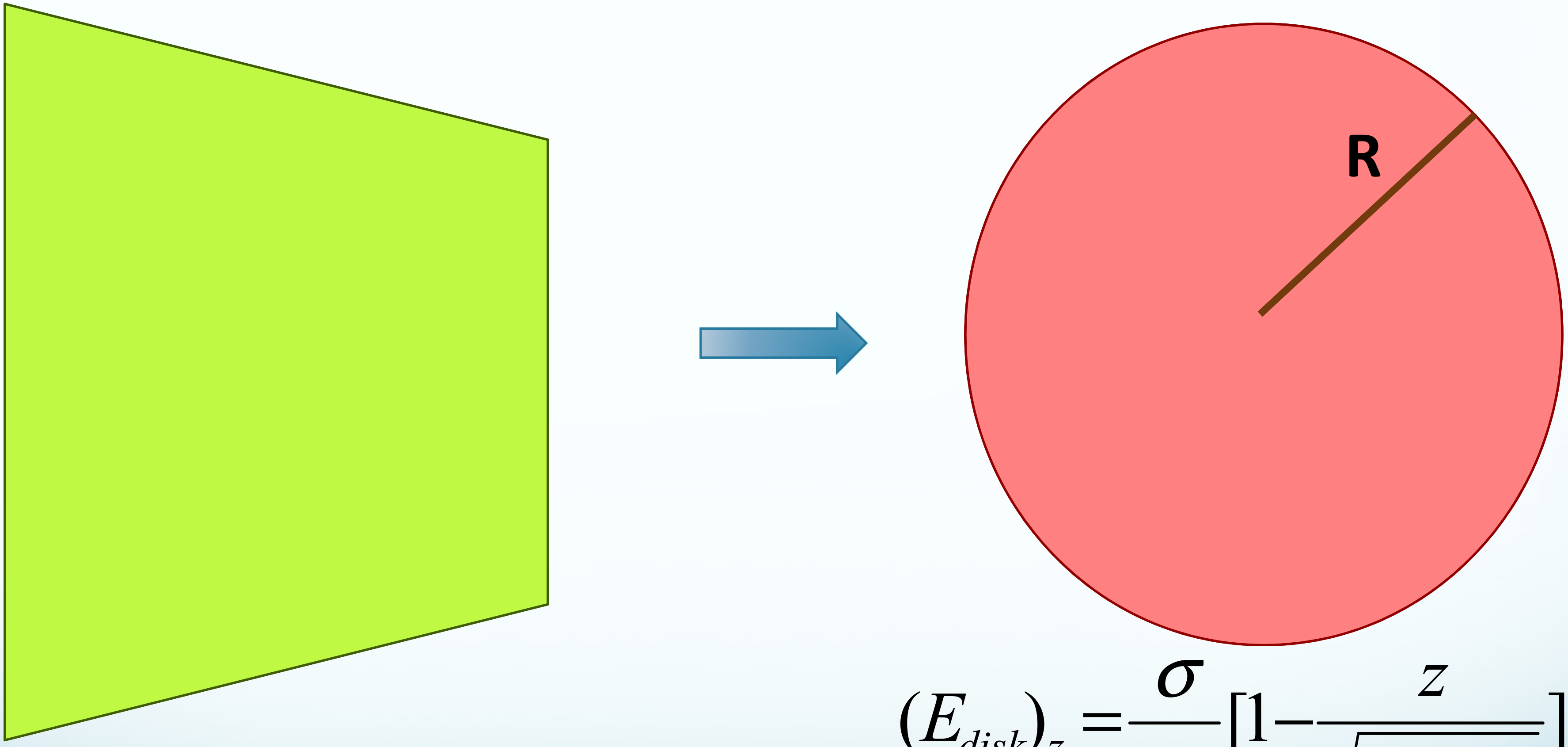
Limiting cases? z→0



$$E_{disk,z} = \frac{\sigma}{2\varepsilon_o}\left[1 - \frac{z}{\sqrt{z^2 + R^2}}\right]$$

$$E_{disk,z} = \frac{\sigma}{2\varepsilon_o}$$

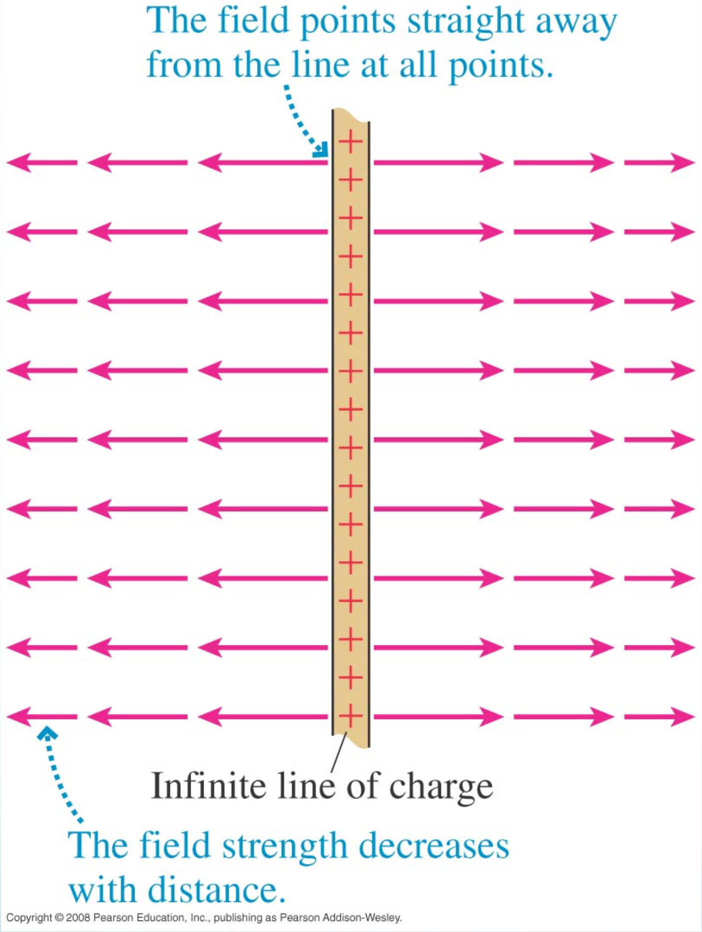
Plane of charge



$$(E_{disk})_z = \frac{\sigma}{2\varepsilon_o}\left[1 - \frac{z}{\sqrt{z^2 + R^2}}\right]$$

$$E_{plane} = \frac{\sigma}{2\varepsilon_o}$$

This is the result for a plane of charge



$$E_{plane,z} = \begin{cases} \dfrac{\sigma}{2\varepsilon_o}, z > 0 \\ -\dfrac{\sigma}{2\varepsilon_o}, z < 0 \end{cases}$$

This section we talked about:

Chapter 22.4-5

See you on Thursday