Electricity and Magnetism

- Physics 259 – L02
- Lecture 33

Chapters 26 & 27

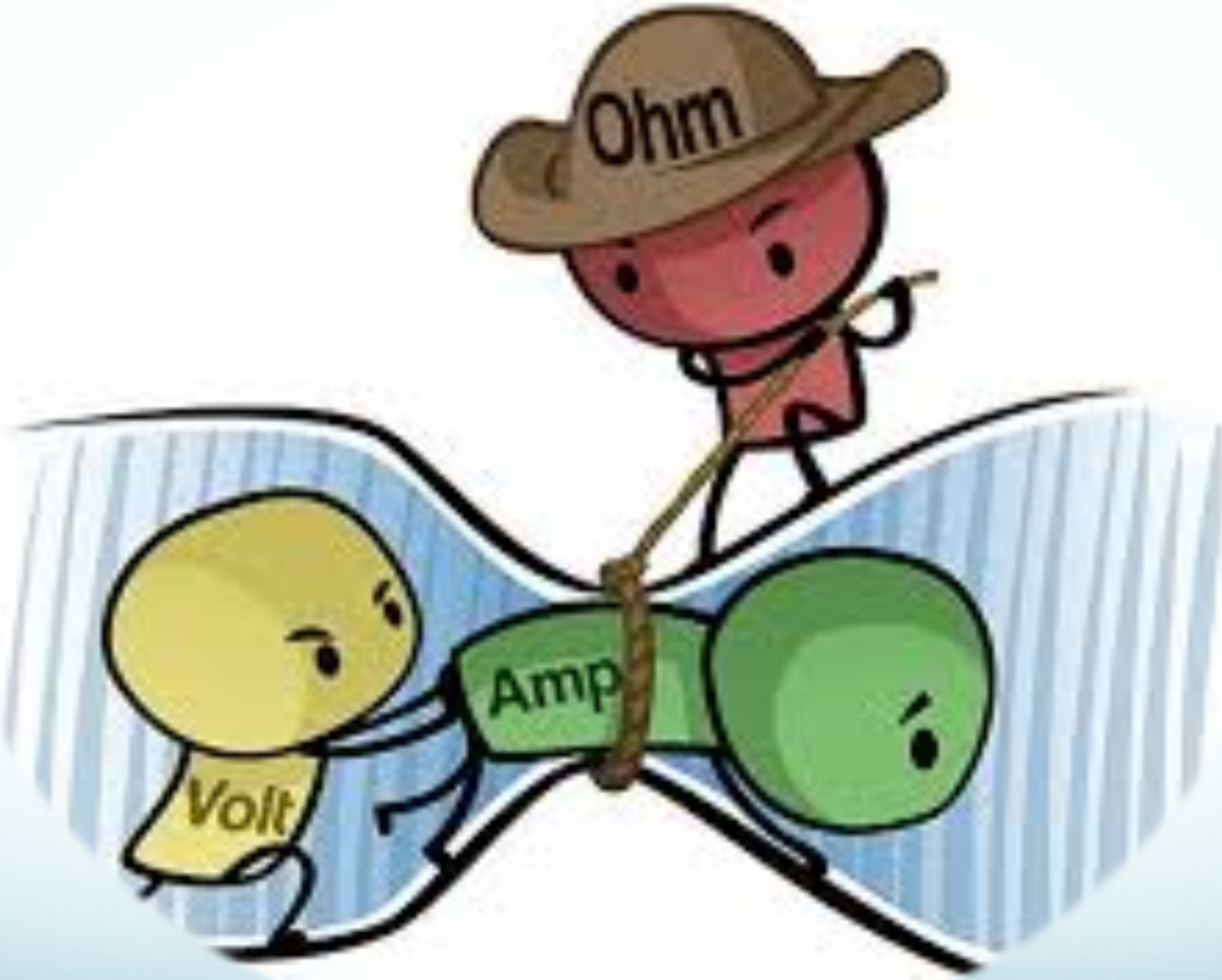
Last time

- Chapter 25- Capacitance

This time

- Chapters 26 and 27

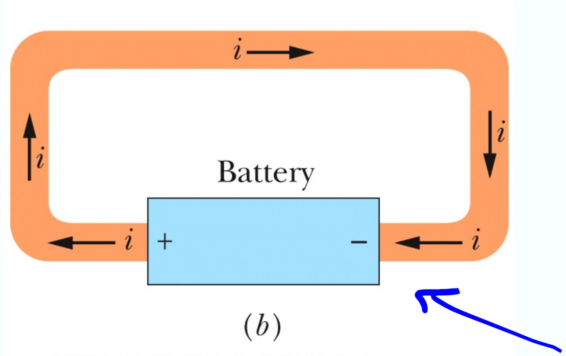
26-1 Current and Resistance



https://tinyurl.com/jo6fspq

26-1 Electric Current

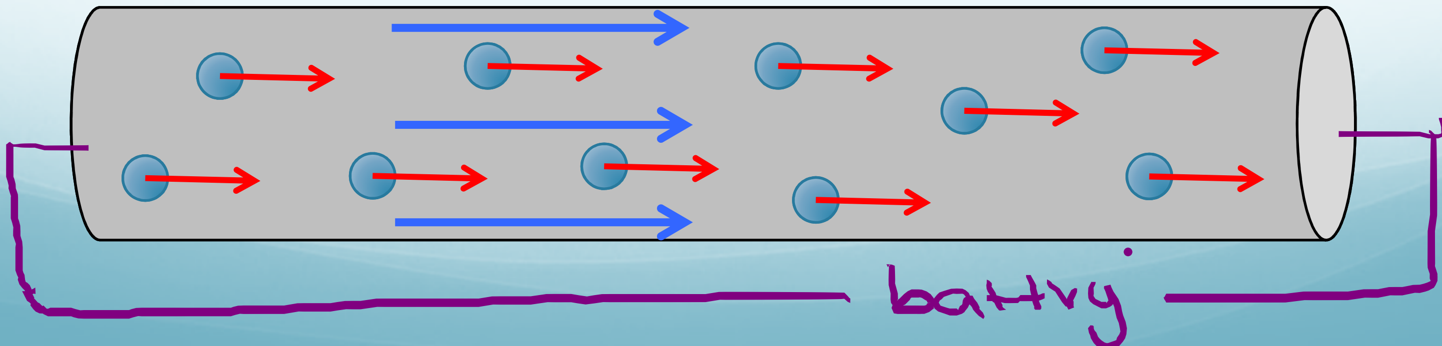
Where we're going?



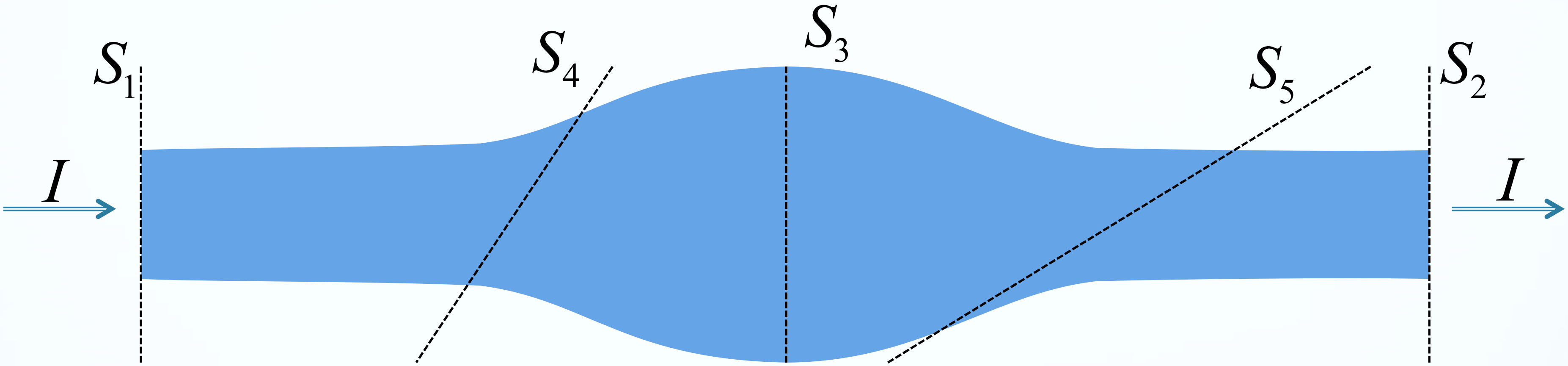
Moving charges in electric circuits → no electrostatic equilibrium→

conductors are allowed to have non-zero electric field inside

(this is what causes the charges to move).



Definition of current



Total amount of charge flowing past this surface in a time Δt

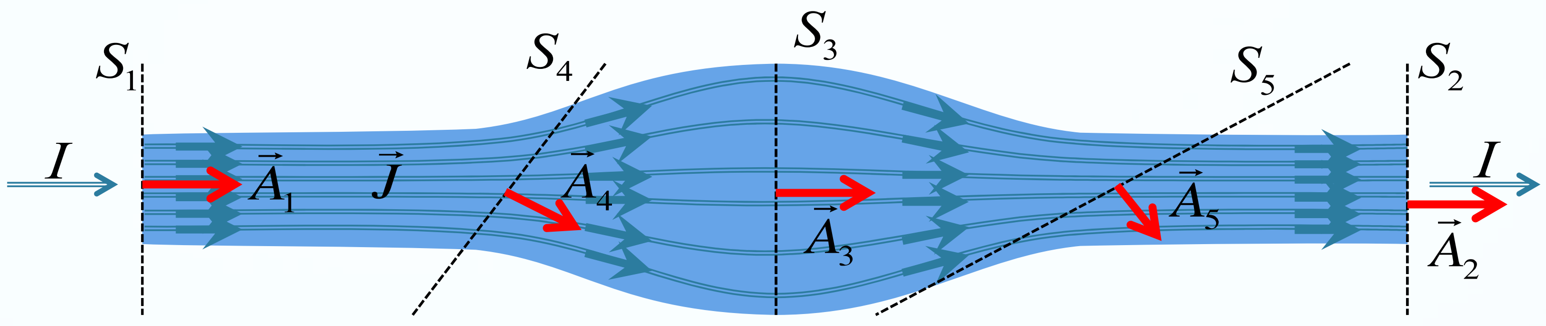
Total amount of charge flowing past this surface in the same time Δt

$$i = \boxed{I = \frac{dq}{dt}} = \frac{\Delta q}{\Delta t}$$

Total amount of charge flowing through **ANY** surface in a time Δt must be constant, otherwise charges would begin to accumulate.

Current in a wire is constant.

26-2 Current Density



$$\Phi = \oint_S \vec{E} \cdot d\vec{A}$$

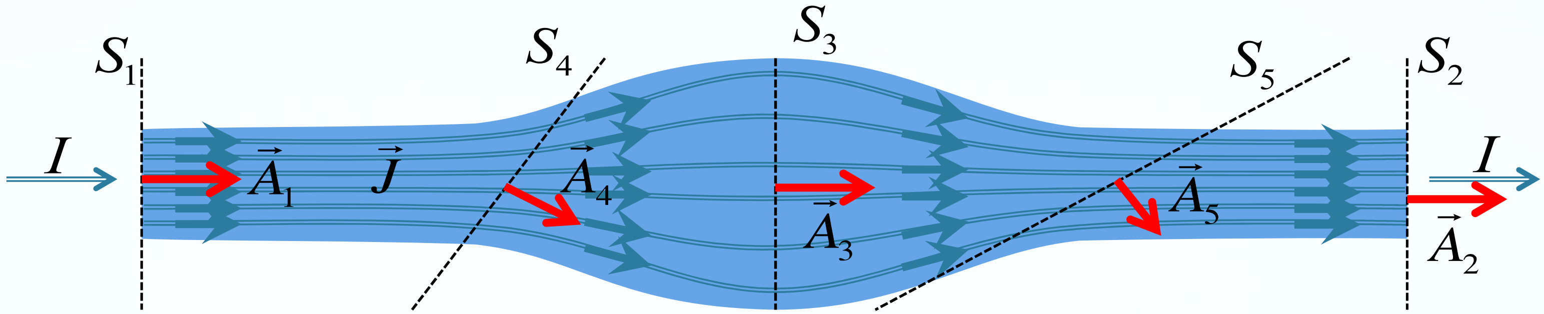
The current in a wire is the flux of charge carriers (i.e. electrons) through a surface.

$$\boxed{I = \oint_S \vec{J} \cdot d\vec{A}}$$

$\vec{J}$ → current density

$$J = \frac{I}{A}$$

The current I is then interpreted as the number of charges passing through a surface in a specified direction.



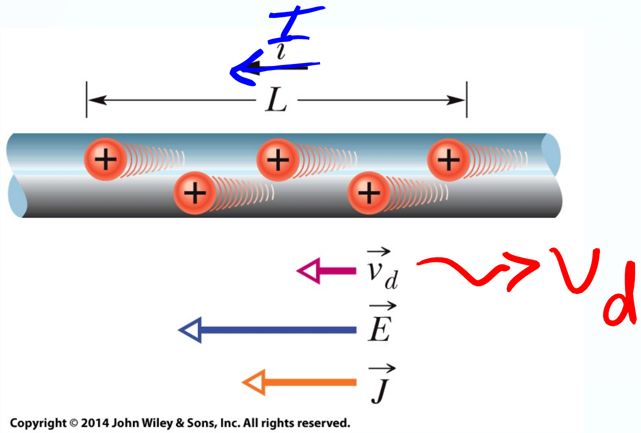
$J = \frac{I}{A}$

$\vec{J}$ encodes information about:

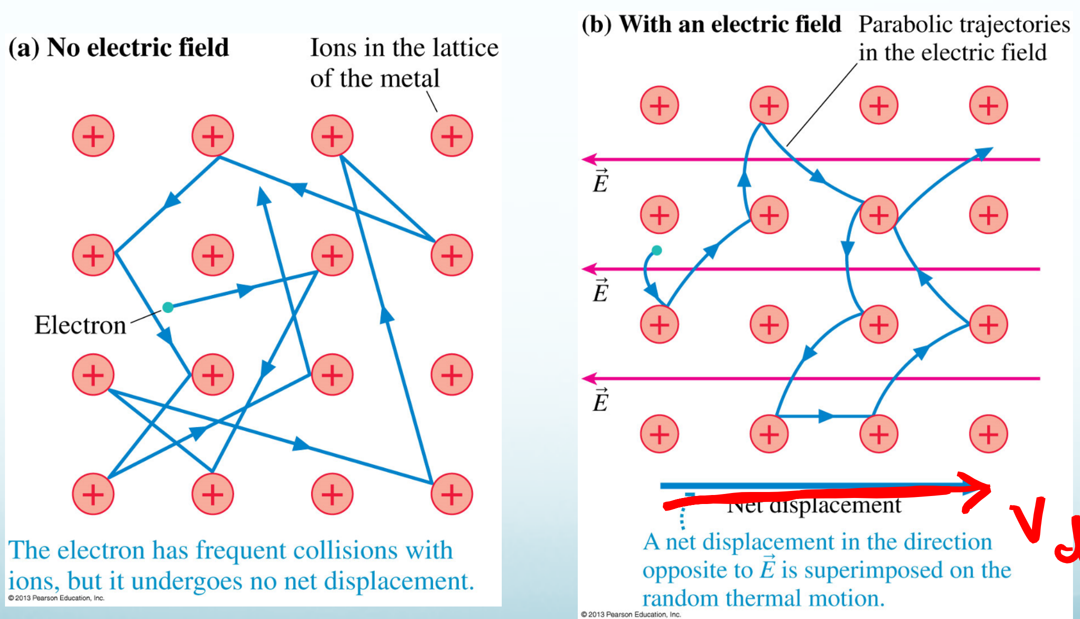
- The density of conduction electrons in the conductor
- The net velocity of these conduction electrons

Current density J → same direction as the velocity of the positive moving charges and opposite direction if the moving charges are negative.

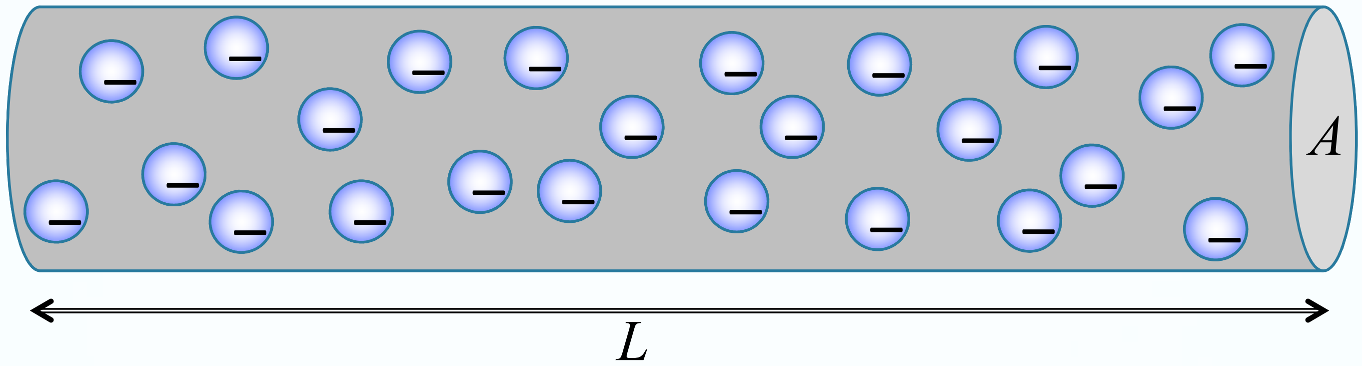
Inside a conductor



Conduction electrons are actually moving to the right but the conventional current *i* is said to move to the left.



Net result: electrons move at an average net "drift speed" v_d



Current → $i = \dfrac{q}{t} = JA$

I

$I = \dfrac{q}{t}$

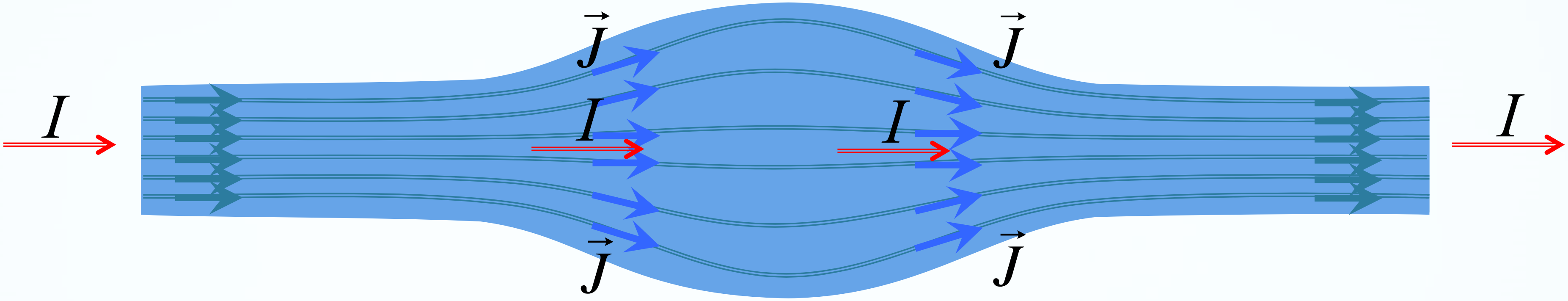
- ✓ volume density of conduction electrons → ~~ne~~ n_e

$q = n_e * V$

- ✓ Amount of charge contained in a length L of the wire → $q = n_e(AL)e$
- ✓ Time it takes each charge to travel a distance L → $t = L/v_d$

$L \leftarrow x = vt \rightarrow t = \dfrac{x}{v}$

Current → $\boxed{i = \dfrac{q}{t} = \dfrac{n_e(A\cancel{L})e}{\cancel{L}/v_d} = nAev_d}$ → $J = \dfrac{i}{A}$ → $\boxed{\vec{J} = n_e e\vec{v}_d}$



$J = \frac{I}{A}$ → const.

$\vec{J}$ **is a vector→** is always in the direction of the "streamlines" of the electrons at any given location in the wire. → direction ⇒ + charge

I **is a scalar→** just has a magnitude.

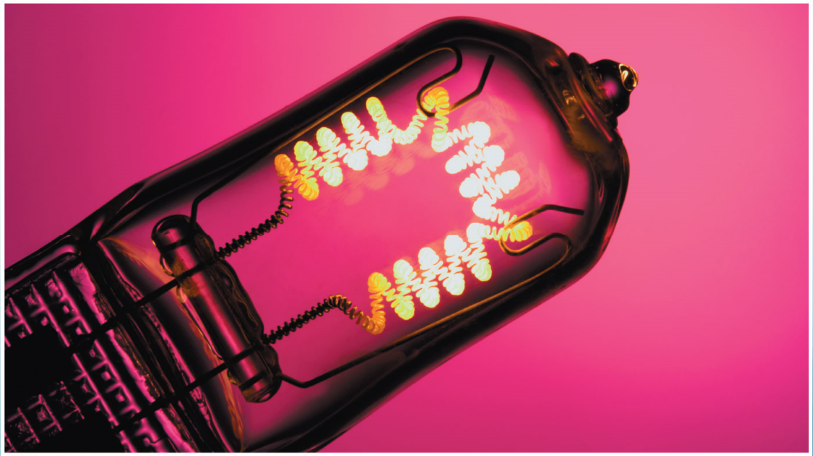
Direction→ is the average displacement of all the charges in the wire, and so always points along the general direction of the wire.

26-3 Resistance and Resistivity

Resistance is a property of conductors that are not ideal:

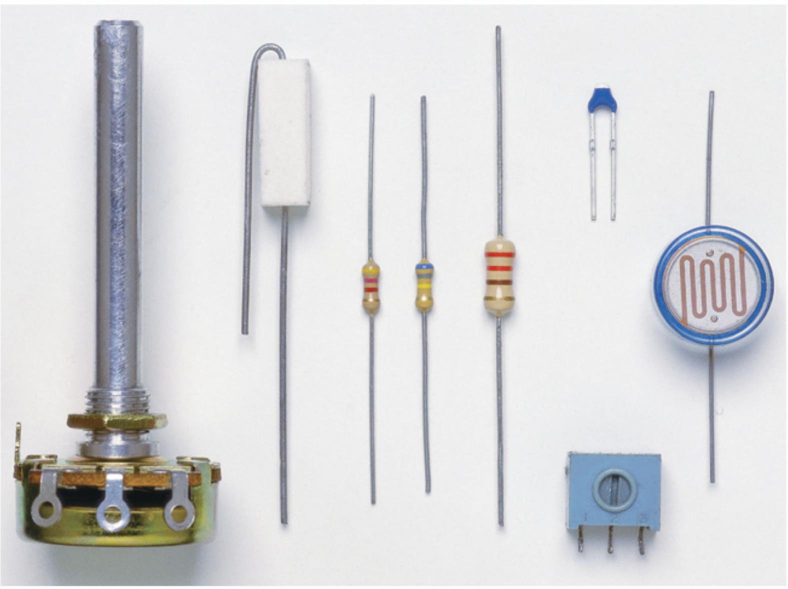
- Electrons have frequent collisions with atomic nuclei.
- When a voltage difference is created across the conductor, this accelerates the electrons, making their collisions more energetic.
- This gets dissipated as heat inside the metal

Tungsten filament:



Resistors

A resistor is any circuit element that dissipates energy. Light bulbs are the classic example, but there are others:

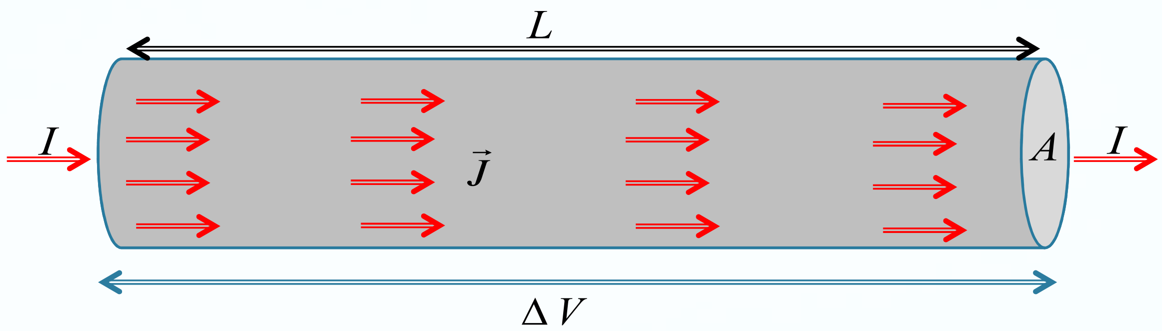


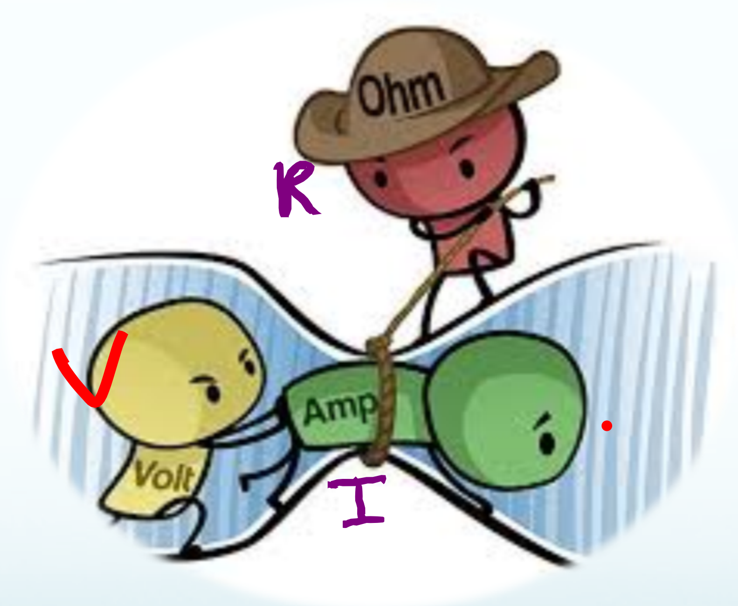
How much energy is dissipated by a given resistor is encoded in a property called its resistance *R*.

R

The resistance is dependent on the particular material used as well as the geometry.

How can we quantify resistance?





$$C = \frac{q}{V}$$

$$R = \frac{V}{I}$$

Ohm's law

$$\boxed{\Delta V = IR}$$

Proof→ Appendix 21-1

Resistance is a property of an object. Resistivity is a property of a material.

Instead of the resistance R of an **object**, we may deal with the **resistivity** ρ of the **material**:

$$\rho = \frac{E}{J} \quad \text{(definition of } \rho\text{).}$$

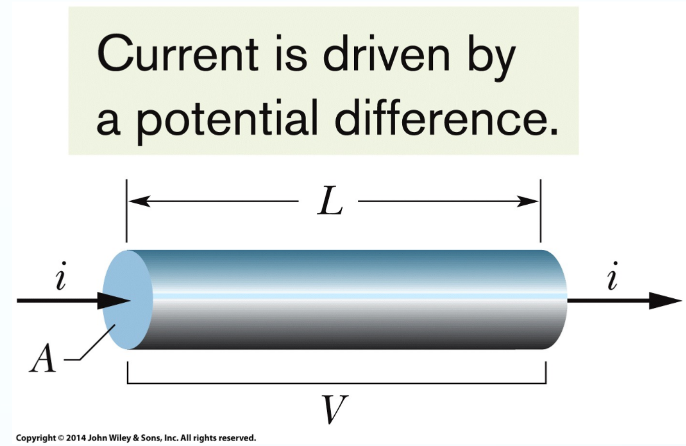
The reciprocal of resistivity is **conductivity** σ of the material:

$$\sigma = \frac{1}{\rho} \quad \text{(definition of } \sigma\text{).}$$

The resistance *R* of a conducting wire of length *L* and uniform cross section is

$R = \frac{V}{I}$ &

$$R = \rho \frac{L}{A}.$$



Here *A* is the cross-sectional area.

The resistivity ρ for most materials changes with temperature.

$$\rho - \rho_0 = \rho_0 \alpha (T - T_0).$$

→ reference temperature

→ temperature coefficient

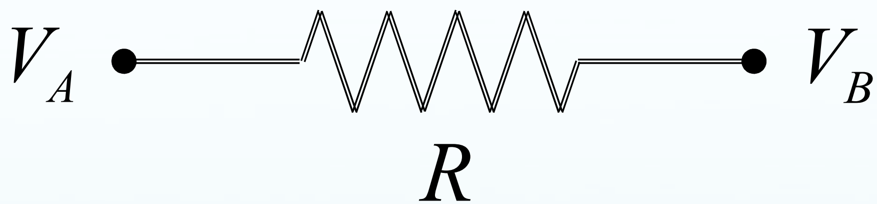
Here T_0 is a reference temperature, ρ_0 is the resistivity at T_0, and α is the temperature coefficient of resistivity for the material.

Proof→ Appendix 26-1

27 Circuits

Ohm's Law

When a voltage difference ΔV is applied across a resistor R, the voltage difference causes electrons to flow through the resistor



This flow of electrons is the electric current I. These quantities are related by Ohm's Law:

$$\Delta V = IR$$

Ideal wires & batteries

In this class we will usually treat wires as **ideal**, meaning $\Delta V = 0$ across any wire segment even if there is a current flowing.

$\Delta V = IR \rightarrow \Delta V = 0$

A battery is any **source** that supplies a **voltage difference** in an electric circuit. The voltage is either specified by *V* or by the symbol ε which stands for **electromotive force** (EMF) $\varepsilon = IR$ (V)

Real batteries also have a resistance to them and we will see later how to account for this.

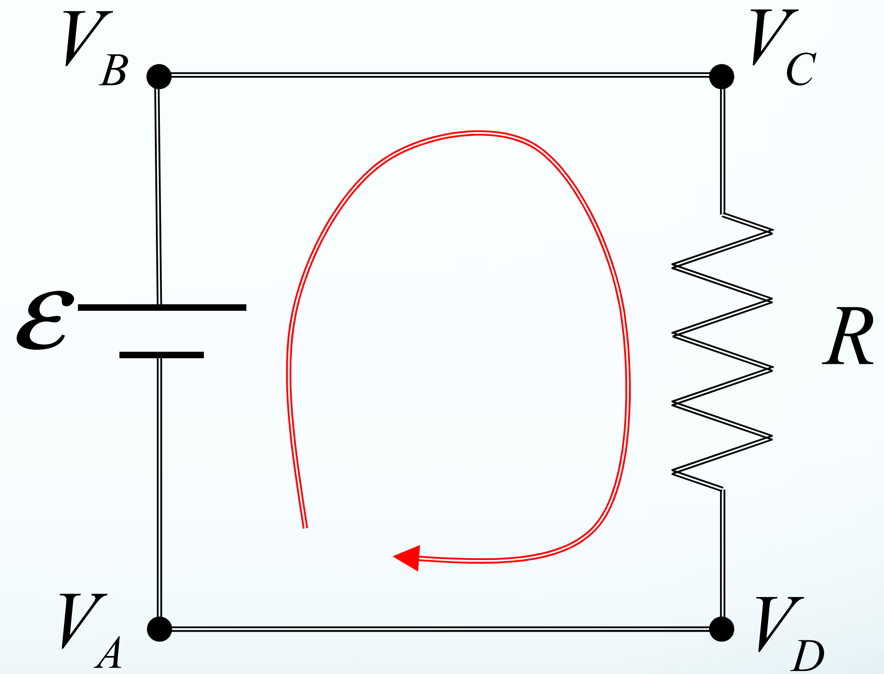
Current convention→ the flow of positive charge (opposite the flow of negative charge)

A Basic Circuit

The simplest circuit has an ideal battery, ideal wires, and a single resistor

Kirchhoff's Loop Rule:

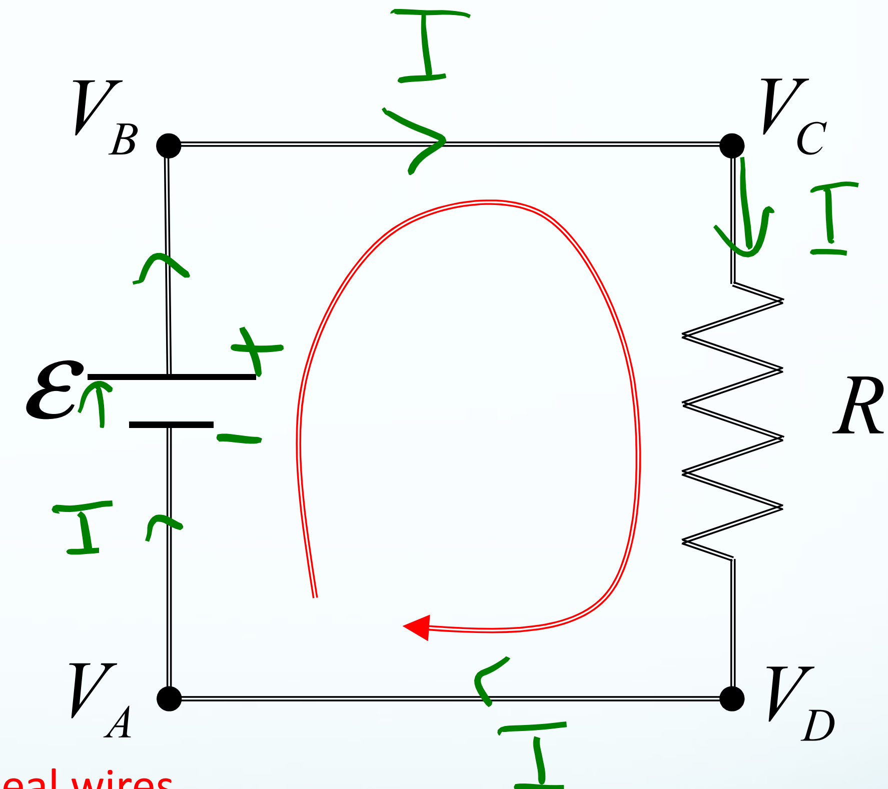
The sum of the voltage differences around a closed loop in a circuit must be zero. (conservation of energy)



$$\boxed{\Delta V_{AB} + \Delta V_{BC} + \Delta V_{CD} + \Delta V_{DA} = 0}$$

The voltage across a resistor is **negative** if you are going around the loop in the **direction of the flow of current**.

Current flows **from the negative** terminal **to the positive** terminal



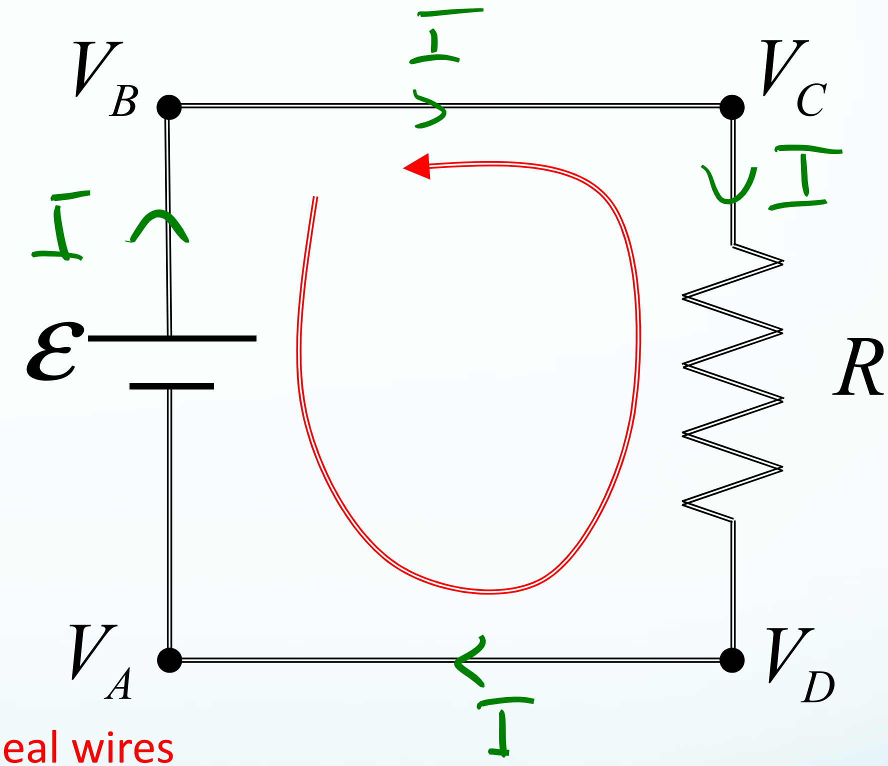
ideal wires

$$\Delta V_{AB} + \cancel{\Delta V_{BC}} + \Delta V_{CD} + \cancel{\Delta V_{DA}} = 0$$

$$\mathcal{E} - IR = 0$$

Ohm's Law

The voltage across a resistor is **positive** if you are going around the loop in the **opposite direction of the flow of current**. Voltage across a battery is **negative** going **from positive to negative**



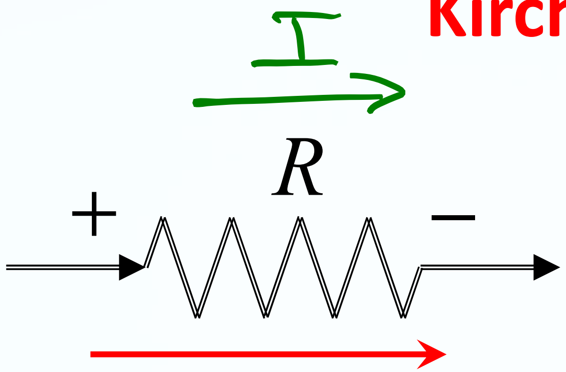
ideal wires

$$\Delta V_{BA} + \cancel{\Delta V_{AD}} + \Delta V_{DC} + \cancel{\Delta V_{CB}} = 0$$

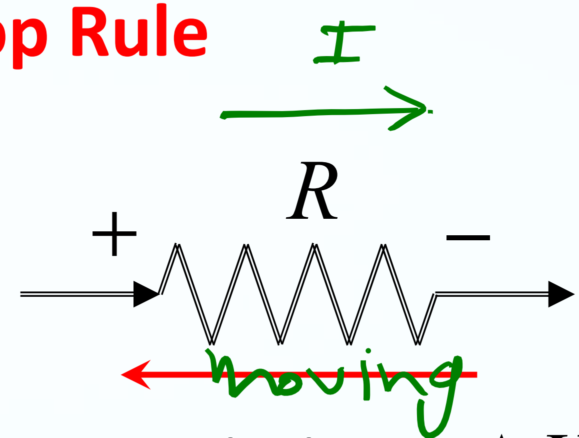
$$-\mathcal{E} + IR = 0$$

Same as before

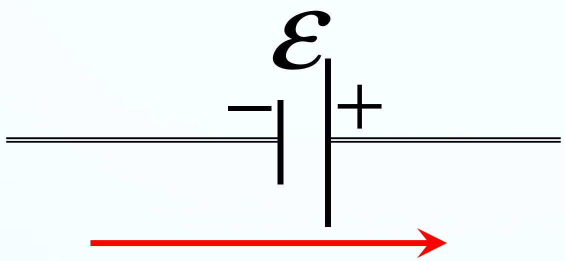
Kirchhoff's Loop Rule



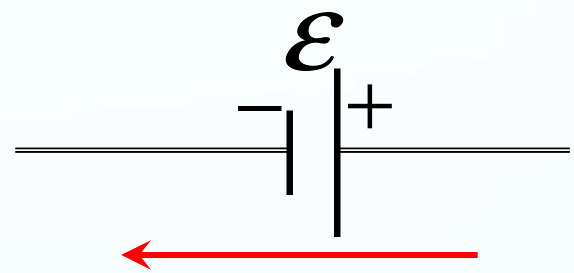
Higher to lower V: $\Delta V = -IR$



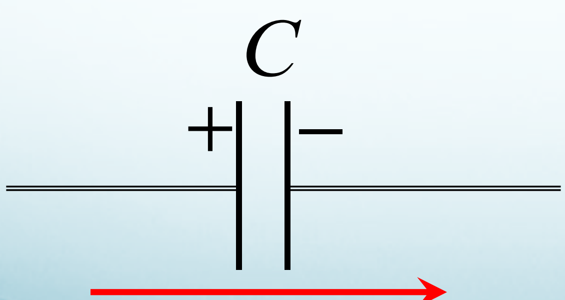
Lower to higher V: $\Delta V = +IR$



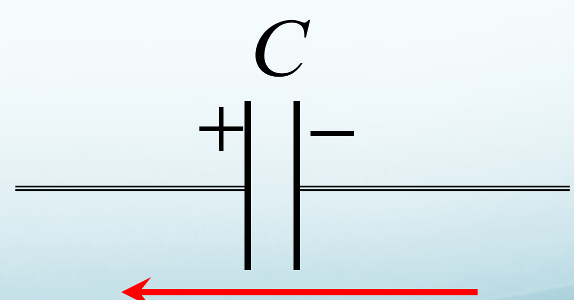
Lower to higher V: $\Delta V = +\mathcal{E}$



Higher to lower V: $\Delta V = -\mathcal{E}$



Higher to lower V: $\Delta V = -\frac{Q}{C}$

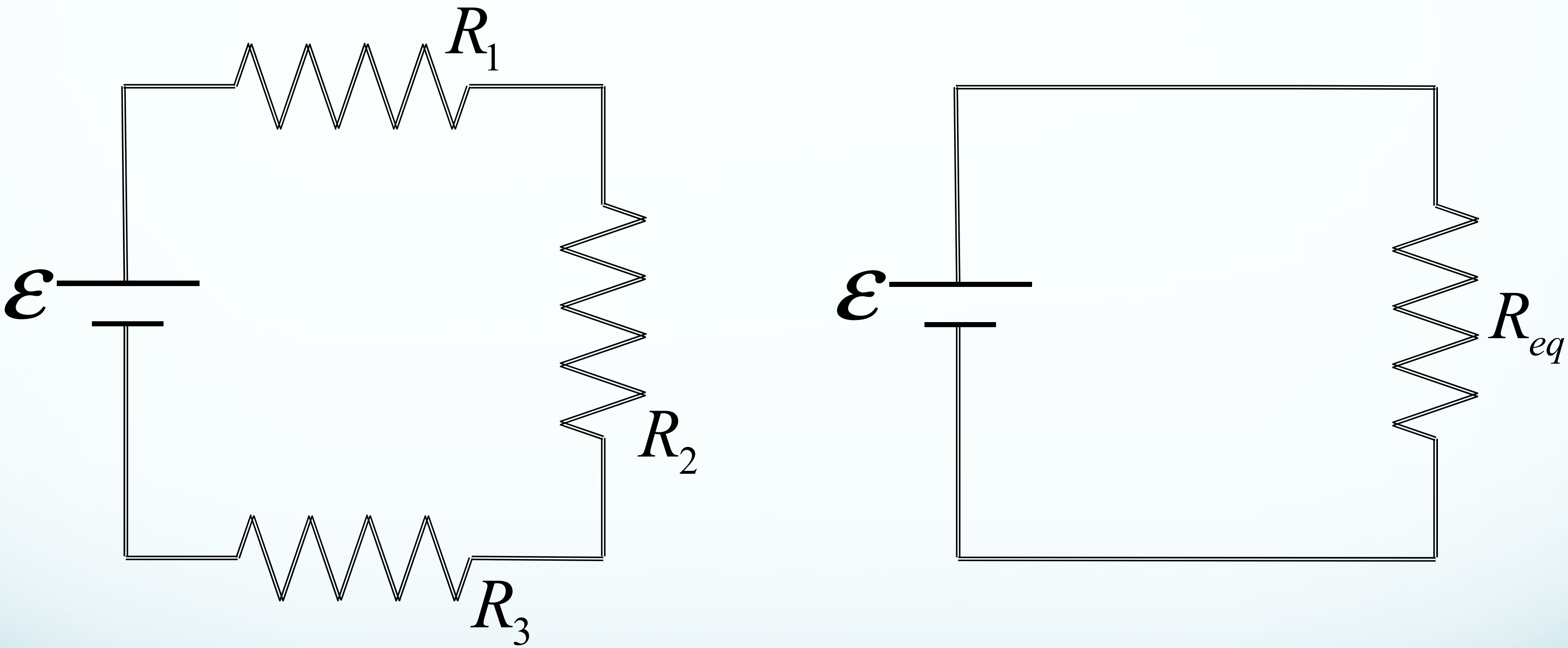


Lower to higher V: $\Delta V = +\frac{Q}{C}$

Resistors in Series

https://tinyurl.com/j6cb8sr

Resistors in Series

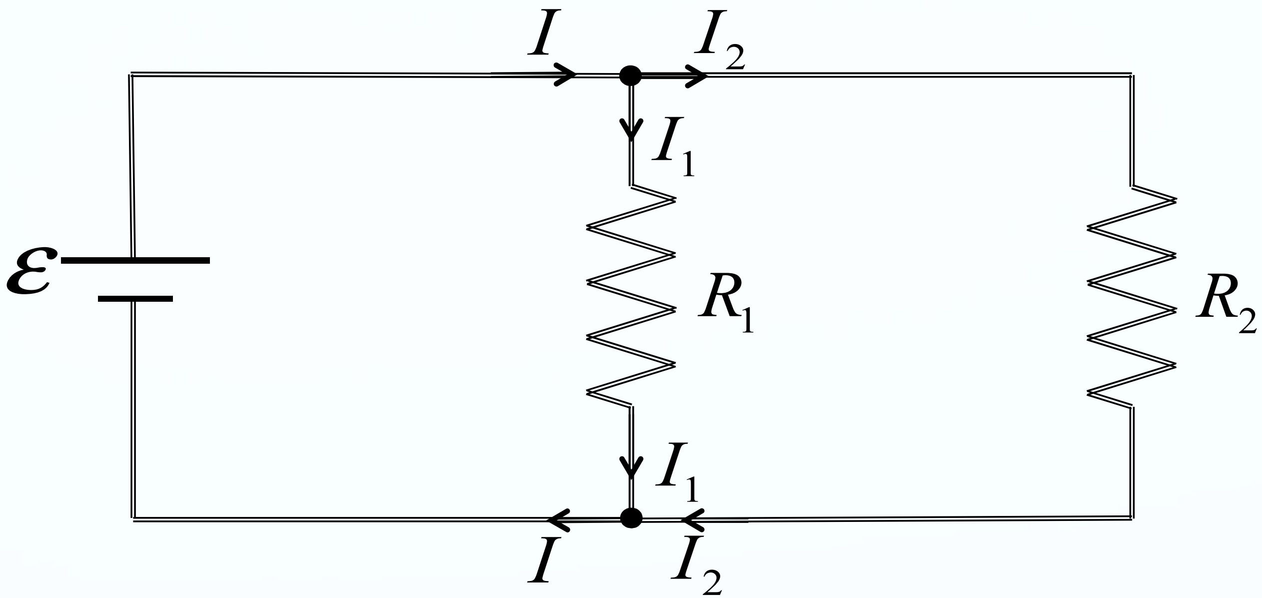


Resistors in series act like a single equivalent resistor:

$$\boxed{R_{eq} = R_1 + R_2 + R_3}$$

Resistors in Parallel

Resistors in Parallel and Kirchhoff's junction rule



$$\boxed{R_{eq} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1}}$$

Current is the flow of charges. Charge has to be conserved.
Current into junction = current out of junction

$$\boxed{I = I_1 + I_2}$$

Summary of Resistors

Ohm's Law

$$\Delta V_R = IR$$

Resistors in Series: have the same current running through them

$$R_{eq} = R_1 + R_2 + \ldots + R_N$$

Resistors in Parallel: have the same voltage across them

$$R_{eq} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_N} \right)^{-1}$$

This section we talked about:

Chapters 26 and 27

See you on Wednesday