

Electricity and Magnetism

- Physics 259 – L02
 - Lecture 3

Section 21.1



What we know

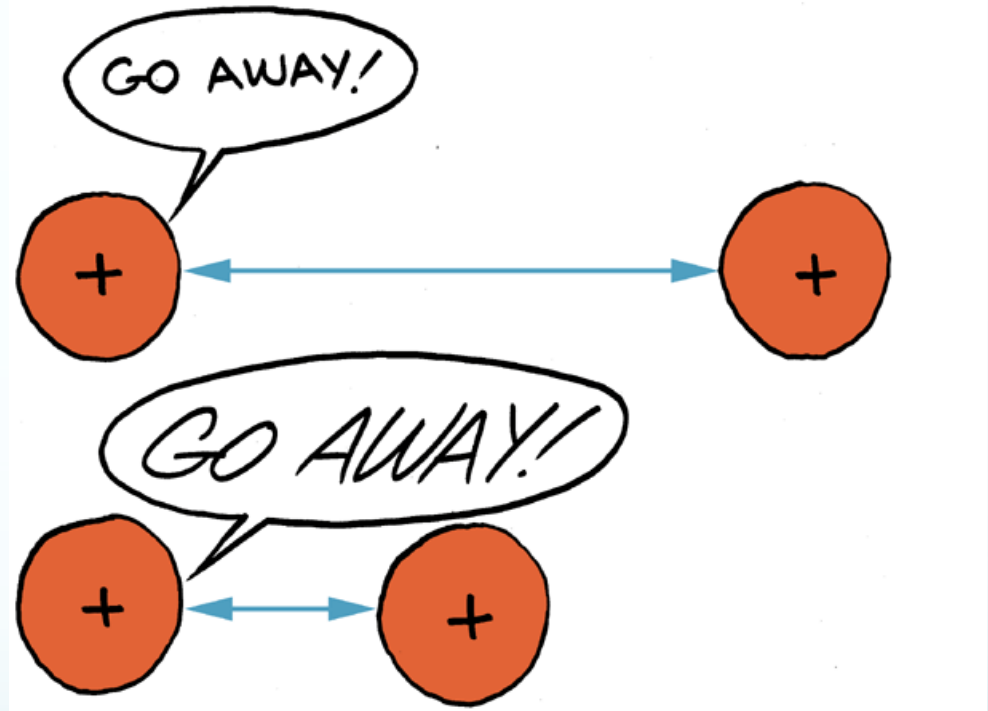
- There are **positive** and **negative** charges
- Like charges **repel** each other
- Opposite charges **attract** each other
- The force between charged objects **varies with distance**
- The force between charged objects depends on the **amount of charge**



HOW CAN WE QUANTIFY THIS?

Coulomb's Law

- Electric field decreases with distance



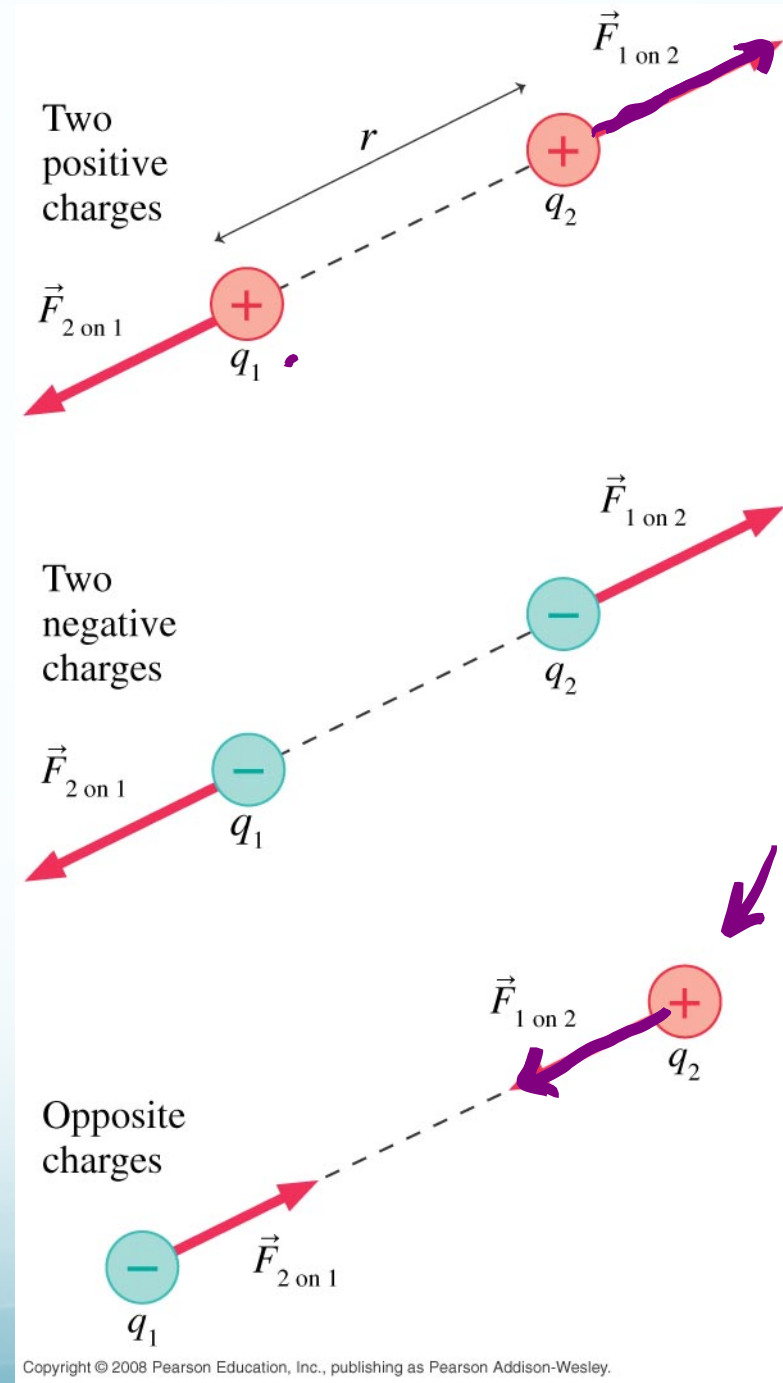
- The force that describes this behavior is known as **Coulomb's law**

Coulomb's Law

Describes the forces that charged **particles** exert on each other:

point charges

The forces always act along the line joining the charges.



Two Ways of Writing Coulomb's Law

$$F_{1 \text{ on } 2} = F_{2 \text{ on } 1} = K \frac{|q_1||q_2|}{r^2}$$

$$\underline{K} = 8.99 \times 10^9 \frac{N \cdot m^2}{C^2}$$

$$F_{1 \text{ on } 2} = F_{2 \text{ on } 1} = \frac{1}{\underline{4\pi\epsilon_0}} \frac{|q_1||q_2|}{r^2}$$

$$\epsilon_0 = \frac{1}{4\pi K} = 8.85 \times 10^{-12} \frac{C^2}{N \cdot m^2}$$

K = electrostatic constant

\vec{F} vector

$|\vec{F}|$ scalar

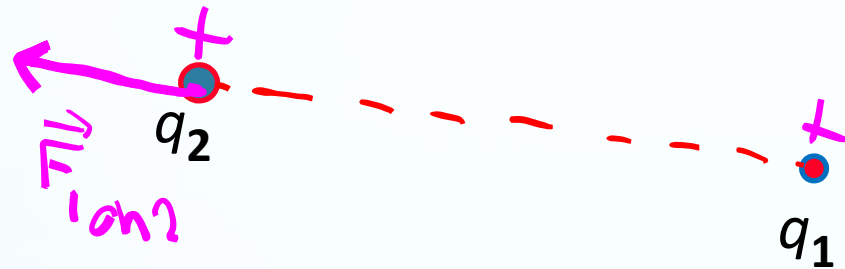
ϵ_0 = permittivity of free space

$$K = \frac{1}{4\pi\epsilon_0}$$

Coulomb's Law

How to compute the magnitude and direction properly?

\vec{F} has both magnitude and direction



$$|\vec{F}_{21}| = K \frac{|q_1||q_2|}{r_{21}^2}$$

↑
constant

- 1) Find the distance between the charges.
- 2) Draw a line passing through the two charges.
- 3) The force on q_1 due to q_2 has its tail at location 1 and points either towards q_2 or away from q_2 .
- 4) Pick the direction according to basic rule of charges:
Like charges repel, Opposite charges attract

SI unit of charge: the **coulomb** (C)

Fundamental charge:

the smallest possible amount of free charge

= charge of one proton: $e = 1.60 \times 10^{-19} \text{ C}$

Then 1 C is approximately 6.25×10^{18} protons.

1 C is **BIG!!**

$$\rightarrow 1 \mu\text{C} = 1 \text{ microcoulomb} = 10^{-6} \text{ C}$$

$$\rightarrow 1 \text{ nC} = 1 \text{ nanocoulomb} = 10^{-9} \text{ C}$$

1 Coulomb is a great deal of charge

An average bolt of lightning

charge = 5 Coulombs

current = 50,000 Amperes

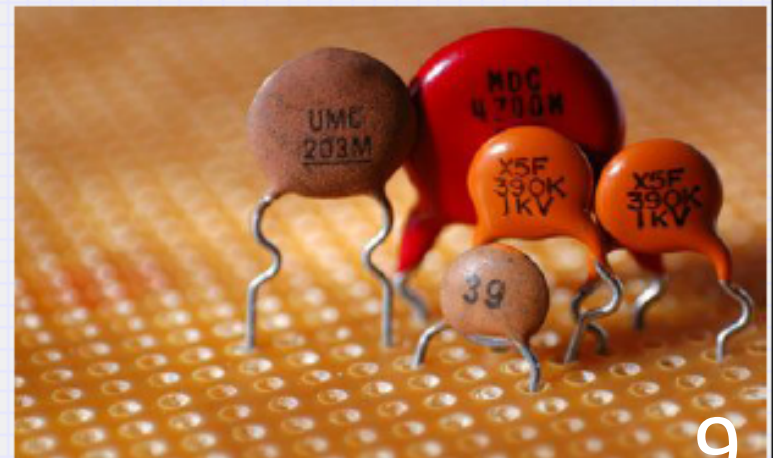
power = 500,000,000 Joules

so all the electrons in a copper penny have a total charge equivalent to 30,000 lightning bolts.

A single electron has $1.6\text{E-}19$ Coulombs of charge.

Capacitors in circuits typically hold charges on the order of $10\text{E-}9$ to $10\text{E-}3$ Coulombs.

All materials contain very large numbers of charges, but they are usually in nearly perfect balance ($N_+ = N_-$).



Scalars vs. Vectors

A **scalar** is any physical quantity that can be described by a **single number**.

- The temperature in the room is **20°C**.

$$\begin{array}{l} m_1 = 2 \text{ kg} \\ m_2 = 1 \text{ kg} \end{array} \rightarrow m_T = m_1 + m_2 = 3 \text{ kg}$$

A **vector** is a physical quantity that has both a **magnitude** and a **direction**.

- Edmonton is **300 km north** of Calgary.

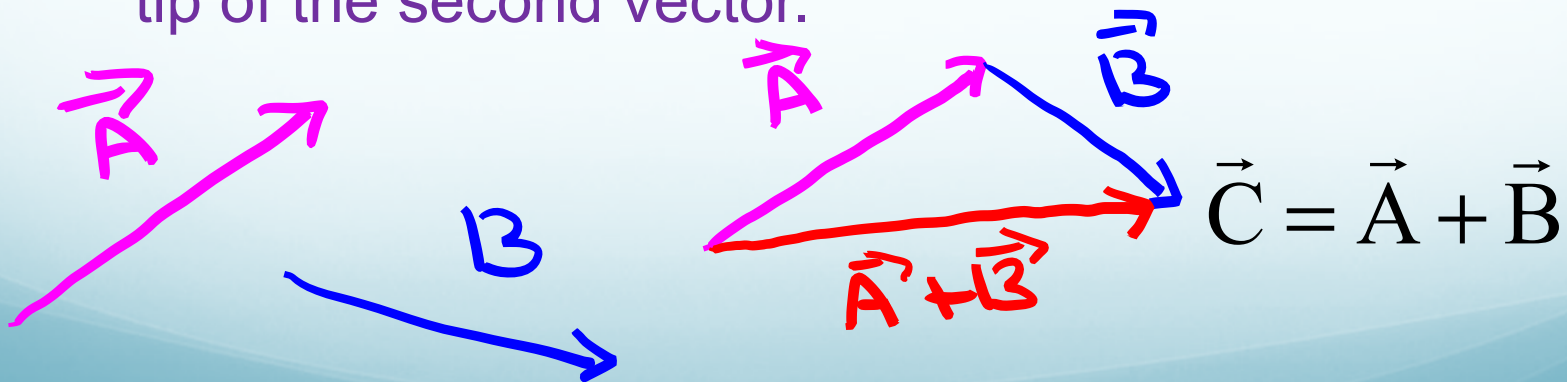


Vector Addition

Adding vectors requires taking not only their magnitudes into account, but also their directions.

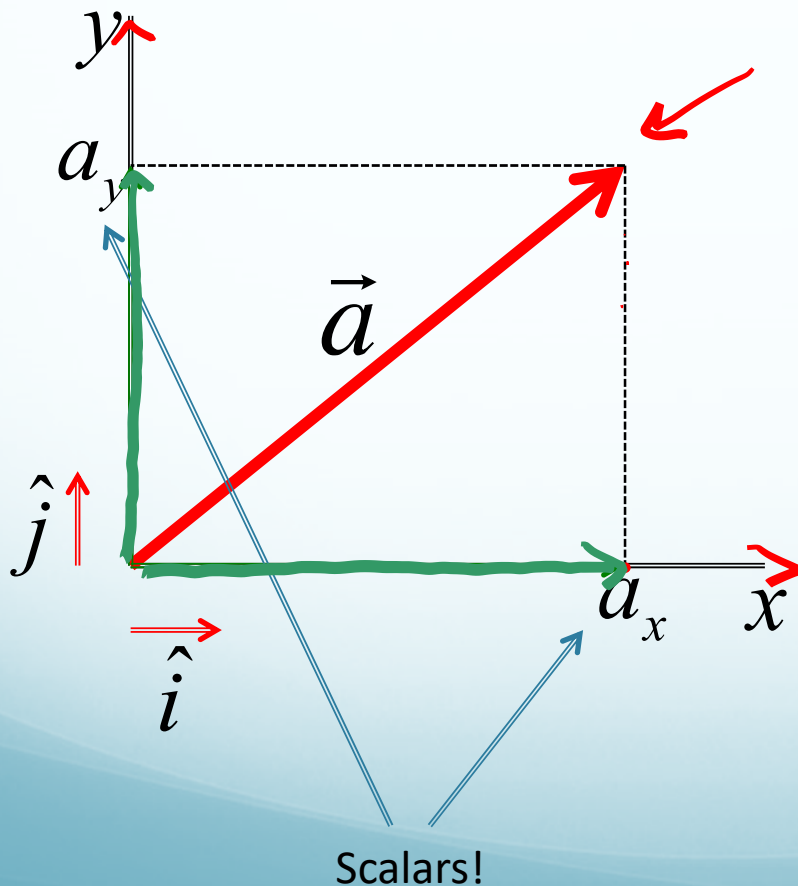
To find the sum of two vectors:

- Draw the first vector.
- Draw the second vector with the tail starting where the tip of the first vector ended.
- Draw a final vector from the tail of the first vector to the tip of the second vector.



Vector Components

Scalars are usually easier to use than **vectors**. So let's replace our vectors with scalar quantities called **vector components**.



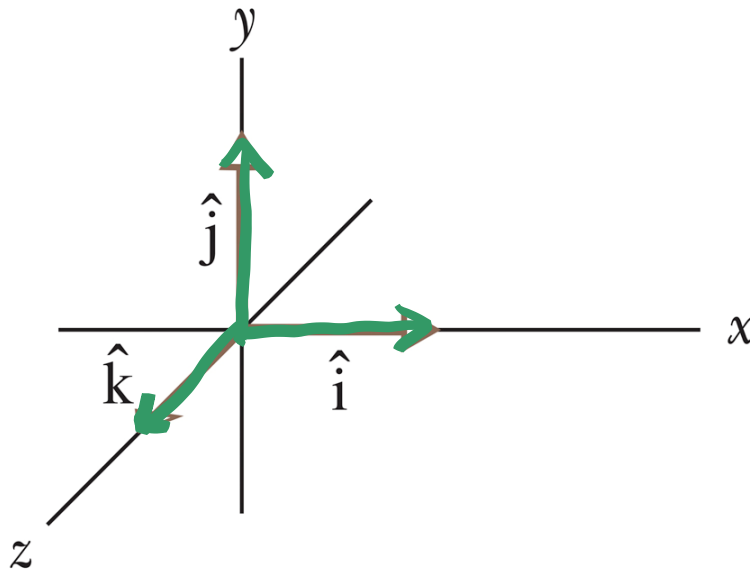
$$\vec{a} = a_x \hat{i} + a_y \hat{j}$$

$$|\vec{a}| = \sqrt{a_x^2 + a_y^2}$$

magnitude is always positive

Unit vectors

The unit vectors point along axes.



Unit \rightarrow *no unit*

Size \rightarrow *1*

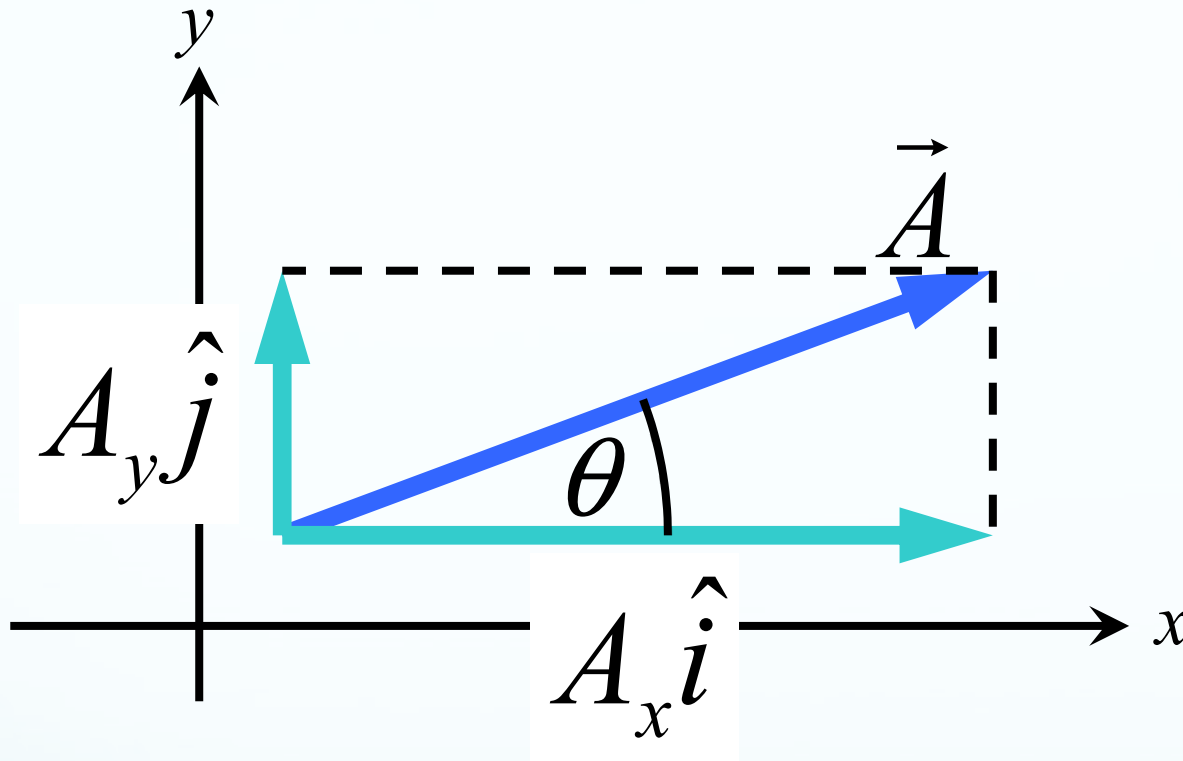
Direction \rightarrow *has direction*

$$|\hat{i}| = 1$$

$$|\hat{j}| = 1$$

$$|\hat{k}| = 1$$

Finding Components of Vectors

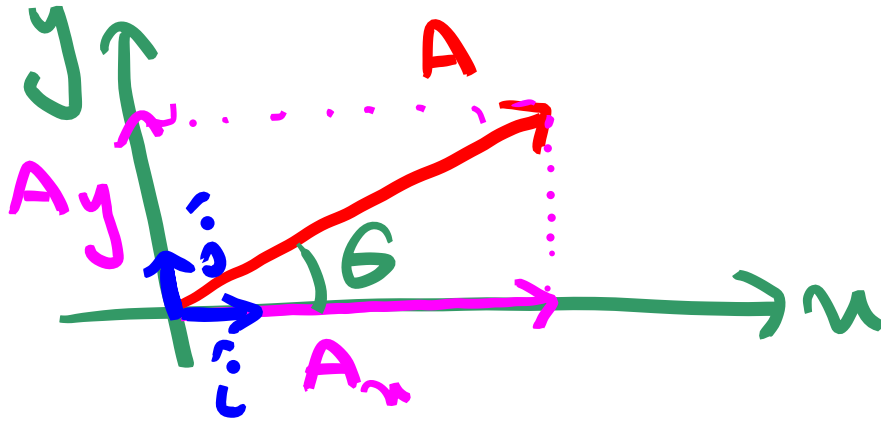


$$A_x = A \cos \theta$$

$$A_y = A \sin \theta$$

$$A^2 = A_x^2 + A_y^2$$

$$\theta = \tan^{-1} \left| \frac{A_y}{A_x} \right|$$



$$\vec{A} = A_x \hat{i} + A_y \hat{j}$$

$$\left. \begin{matrix} A_x \\ A_y \end{matrix} \right\} \Rightarrow \text{Scalar}$$

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2} = A$$

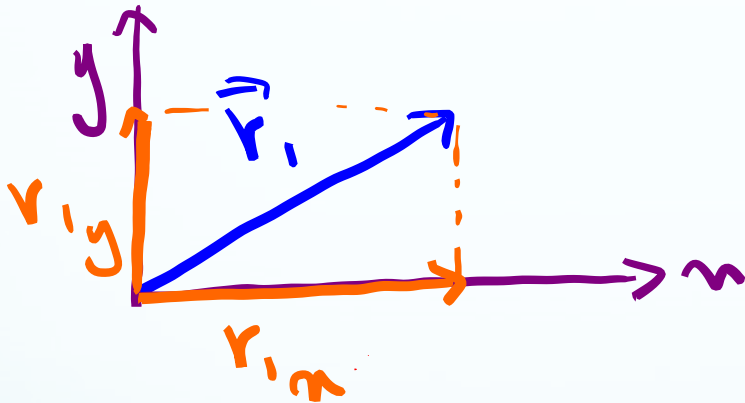
$$A_x = A \cos \theta$$

$$A_y = A \sin \theta$$

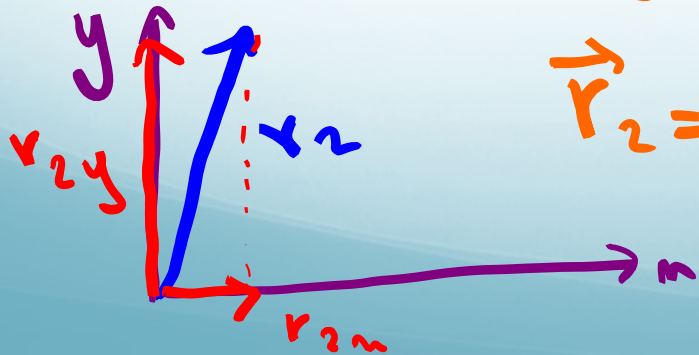
$$\vec{A} = A_x \hat{i} + A_y \hat{j} = \underbrace{A \cos \theta}_{\text{Scalar}} \hat{i} + \underbrace{A \sin \theta}_{\text{Scalar}} \hat{j}$$

Vector Addition using Components

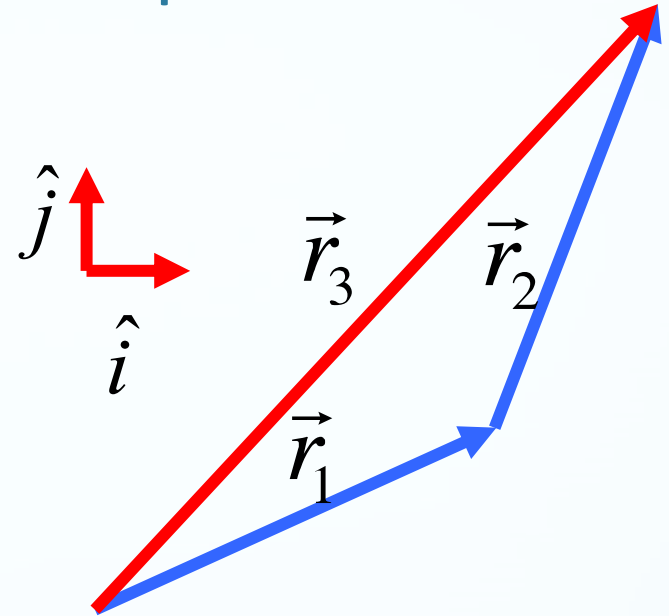
$$\vec{r}_3 = \vec{r}_1 + \vec{r}_2$$



$$\vec{r}_1 = r_{1x} \hat{i} + r_{1y} \hat{j}$$



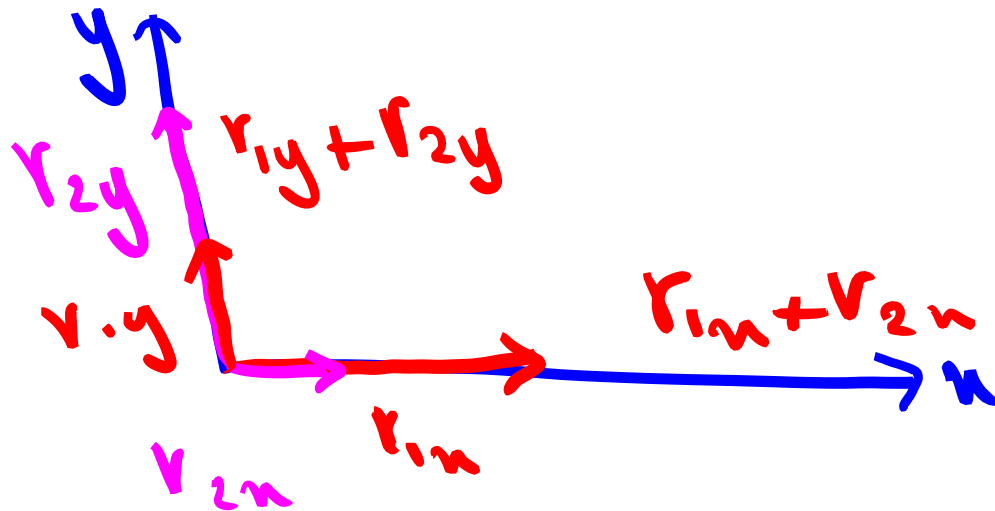
$$\vec{r}_2 = r_{2x} \hat{i} + r_{2y} \hat{j}$$

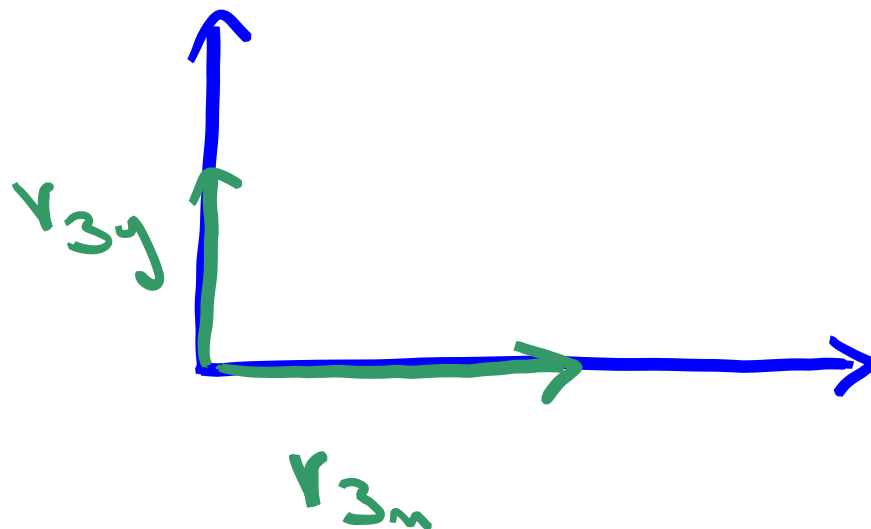


$$\vec{r}_3 = \vec{r}_1 + \vec{r}_2$$

$$= (r_{1x}\hat{i} + r_{1y}\hat{j}) + (r_{2x}\hat{i} + r_{2y}\hat{j})$$

$$= (r_{1x} + r_{2x})\hat{i} + (r_{1y} + r_{2y})\hat{j} \quad \leftarrow$$





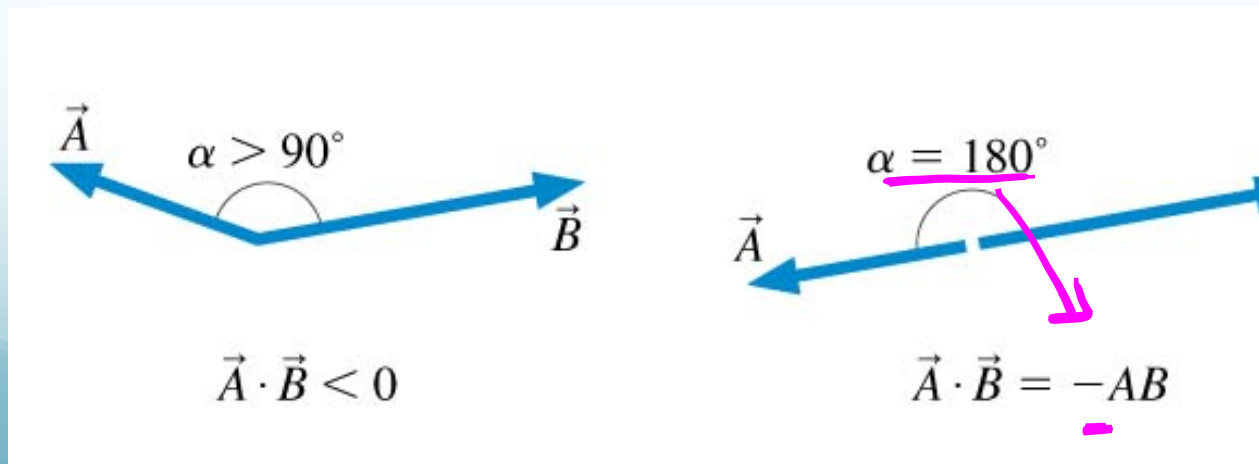
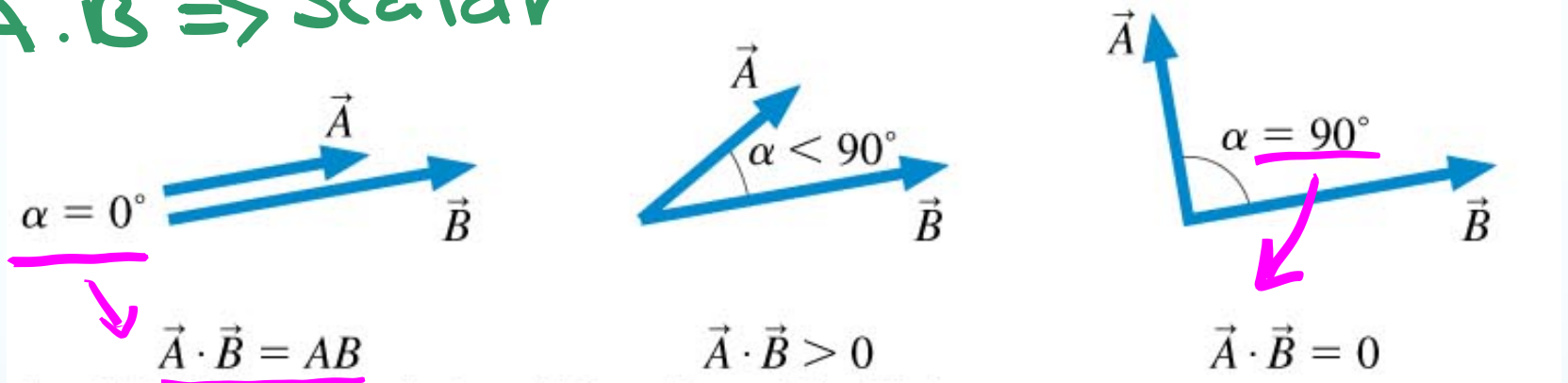
$$\vec{r}_3 = r_{3x} \hat{i} + r_{3y} \hat{j}$$

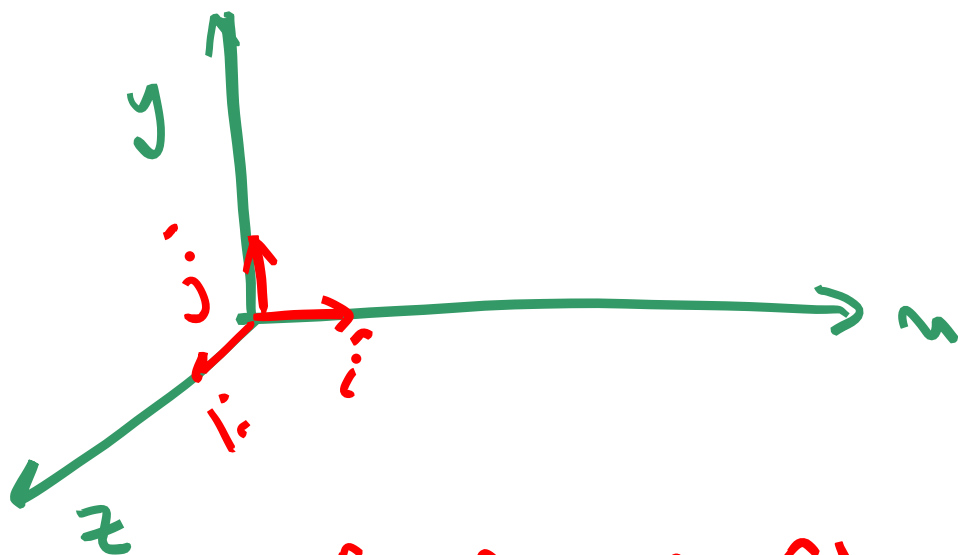
$$\rightarrow |\vec{r}| = \sqrt{r_{3x}^2 + r_{3y}^2}$$

Dot Product of Two Vectors

Scalar $\leftarrow \vec{A} \cdot \vec{B} = AB \cos \alpha$

$\vec{A} \cdot \vec{B} \Rightarrow \text{Scalar}$



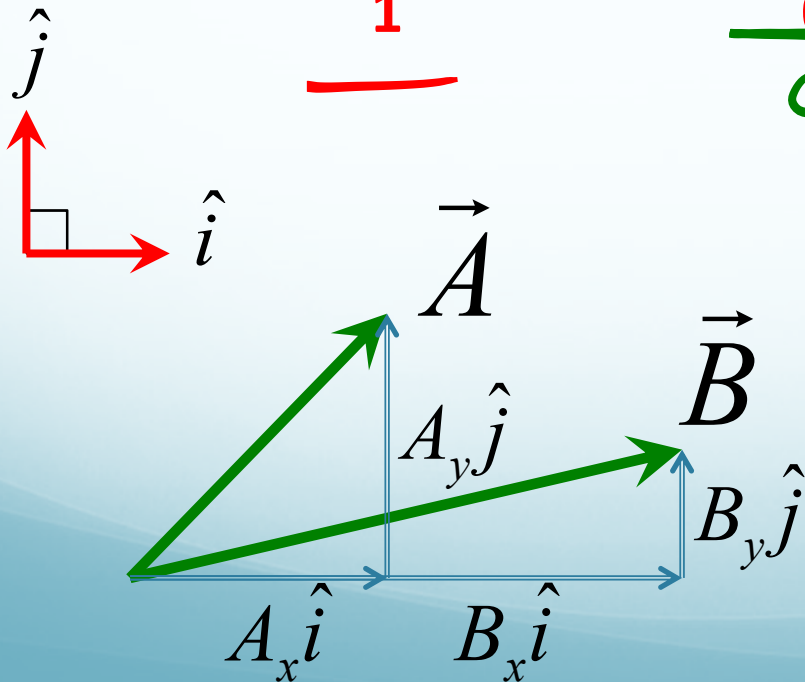


$$\hat{i} \cdot \hat{i} = |\hat{i}| |\hat{i}| \cos(0) = 1$$

$$\hat{i} \cdot \hat{j} = |\hat{i}| |\hat{j}| \cos(90) = 0$$

Dot Product of Two Vectors

$$\begin{aligned}\vec{A} \cdot \vec{B} &= (A_x \hat{i} + A_y \hat{j}) \cdot (B_x \hat{i} + B_y \hat{j}) \\ &= A_x B_x \underbrace{\hat{i} \cdot \hat{i}}_1 + A_x B_y \underbrace{\hat{i} \cdot \hat{j}}_0 + A_y B_x \underbrace{\hat{j} \cdot \hat{i}}_0 + A_y B_y \underbrace{\hat{j} \cdot \hat{j}}_1\end{aligned}$$

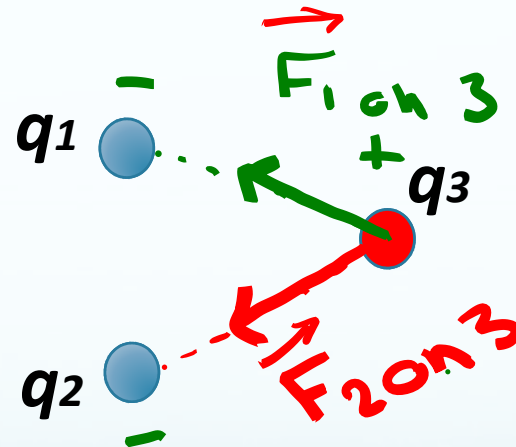


$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y$$

Superposition Principle

The total force on q_3 is the vector sum of the individual forces:

$$\vec{F}_{on\ 3} = \vec{F}_{1\ on\ 3} + \vec{F}_{2\ on\ 3}$$



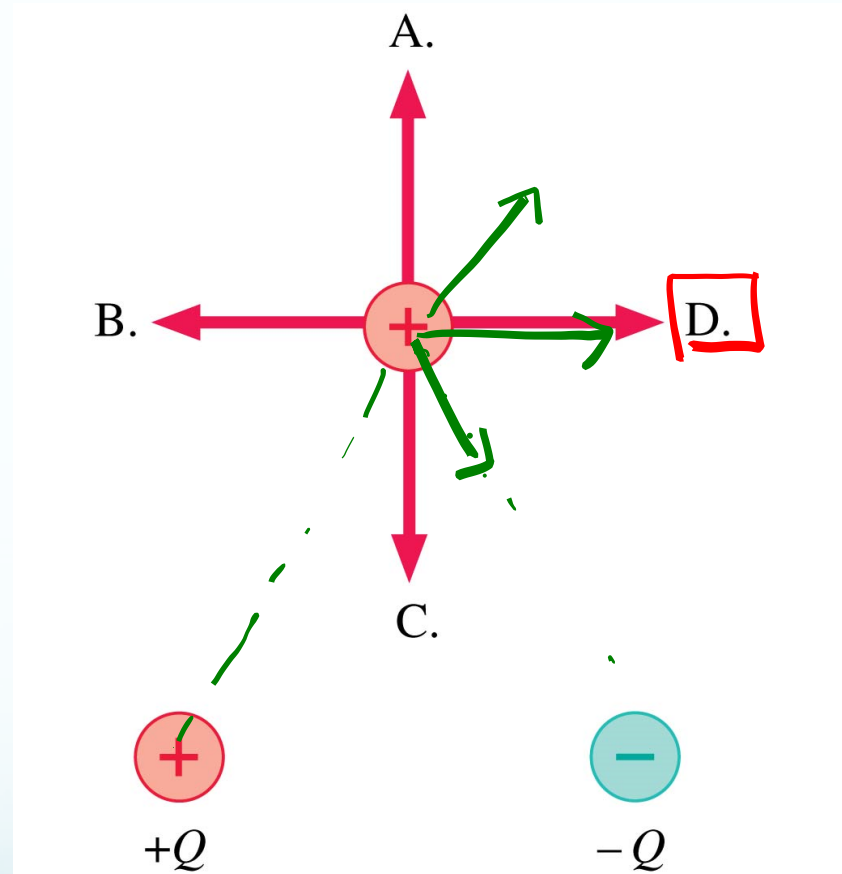
QuickCheck 25.10

TOPHAT QUESTION

Which is the direction of the net force on the charge at the top?

92% correct 😊

thank you



E. None of these.

This section we talked about:

Chapter 21.1

See you on Friday

