Wednesday Mar 8, 2017

Last time:

- Capacitors
- Capacitance as a geometric quantity
- General Capacitors, relating Q to ΔV
- Setting up a process to find Capacitance

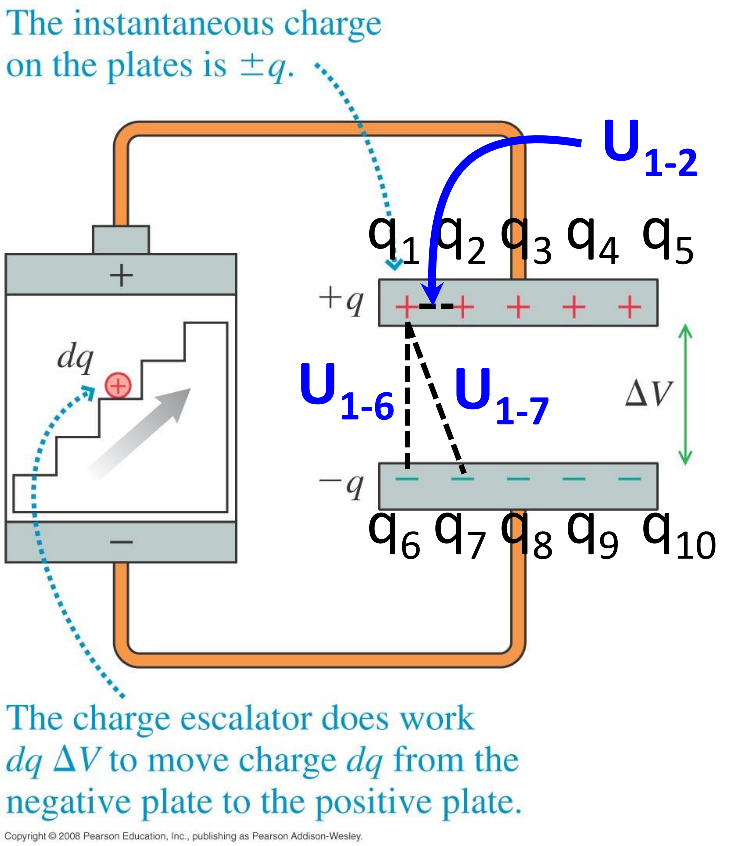
Today:

- Energy stored in parallel plate, spherical, and cylindrical capacitors
- Potential energy stored in the electric field itself.
- Capacitors in electric circuits: how charges move
- Kirchhoff's loop rule with capacitors
- Capacitors in series and parallel
- More complicated capacitor circuit

Energy Storage in Capacitors

We want to calculate this potential energy stored in the capacitor.

It is **way** too hard to add up all the potential energies of every pair of charges in the capacitor:



$$U = U_{1-2} + U_{1-3} + \ldots + U_{1-10} + U_{2-1}$$ **+ $U_{i\text{-}j}$ of every other pair**

Easier way!

Move a tiny charge, dq, from the negative plate to the positive plate.

It moves through a potential difference ΔV.

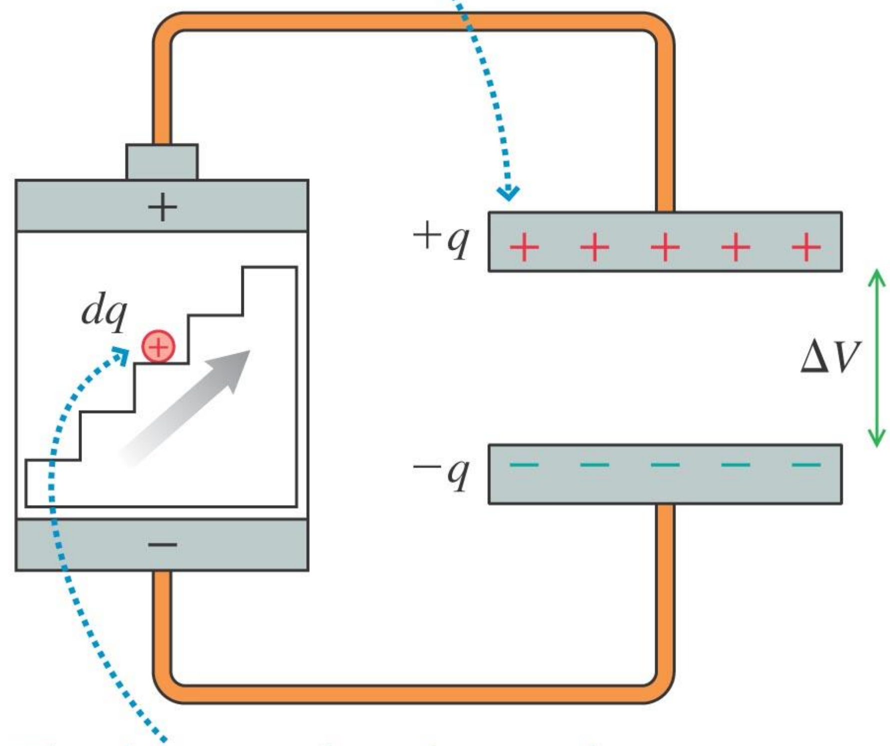
So its potential energy increases by an amount

$$dU = dq\,\Delta V_C$$

But we also know $\Delta V_C = \dfrac{q}{C}$

$$dU = dq\frac{q}{C} = \frac{q\,dq}{C}$$

The instantaneous charge on the plates is $\pm q$.



The charge escalator does work $dq\,\Delta V$ to move charge dq from the negative plate to the positive plate.

$$\boxed{U = \frac{1}{C}\int_0^Q q\,dq = \frac{1}{2}\frac{Q^2}{C}}$$

Potential Energy in a Capacitor

Energy storage in terms of the charge on the plates:

$$\boxed{U = \frac{1}{2}\frac{Q^2}{C}}$$

Use the general relation for a capacitor to swap charge for voltage

$$Q = C\Delta V_C$$

Energy storage in terms of the voltage across the plates:

$$U = \frac{1}{2}\frac{\left(C\Delta V_C\right)^2}{C}$$

$$\boxed{= \frac{1}{2}C\left(\Delta V_C\right)^2}$$

Where is the Energy Stored?

$$U = \frac{1}{2}C(\Delta V_C)^2$$

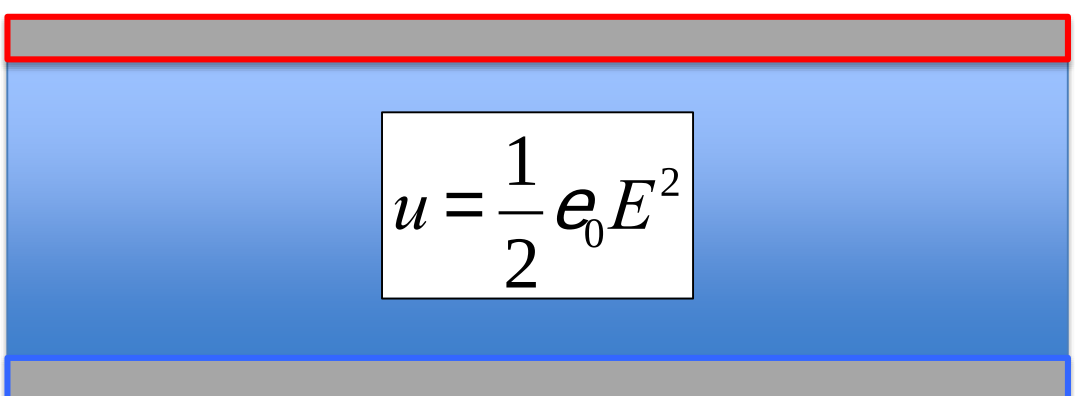
$$= \frac{1}{2}CE^2d^2$$

$$= \frac{1}{2}\frac{\epsilon_0 A}{d}E^2d^2 = \frac{1}{2}\epsilon_0 E^2(Ad)$$

$$\Delta V = Ed$$

$$C = \frac{\epsilon_0 A}{d}$$

$$u = \frac{U}{Ad}$$

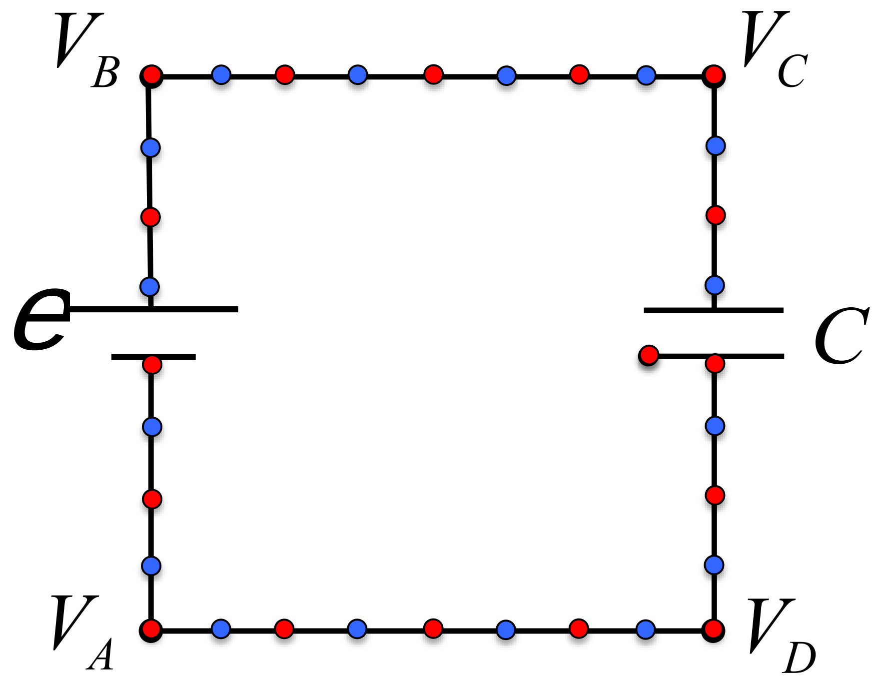


The capacitor's energy is stored in the electric field between the plates!

A Basic Circuit with Capacitor

The simplest capacitor circuit has an ideal battery, ideal wires, and a single capacitor.

The battery causes charges to flow from the bottom plate to the top plate. This creates a potential ΔV_C between the two plates. Remember charges never "jump the gap" between the two plates of a capacitor.



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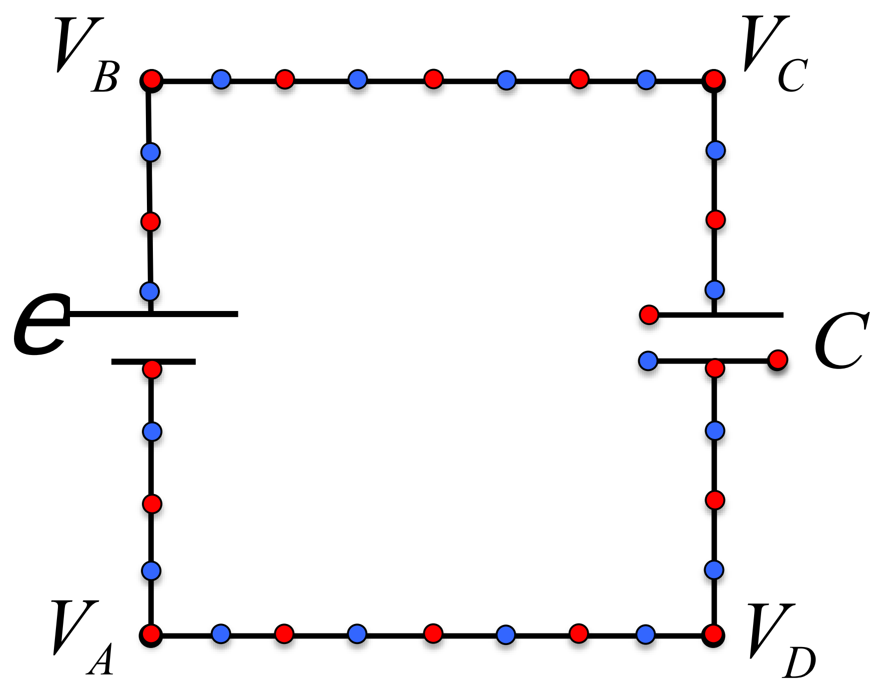

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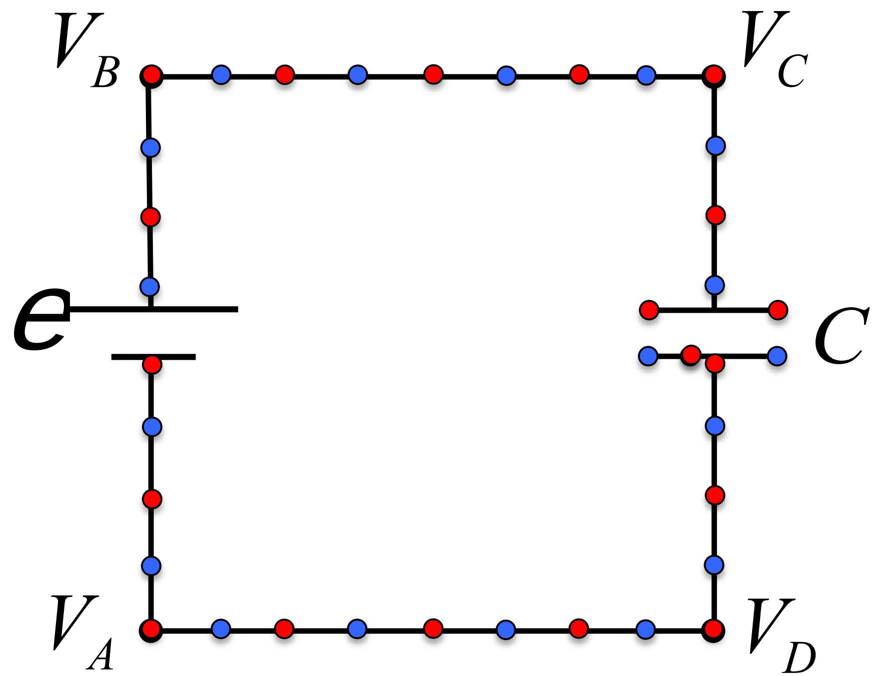

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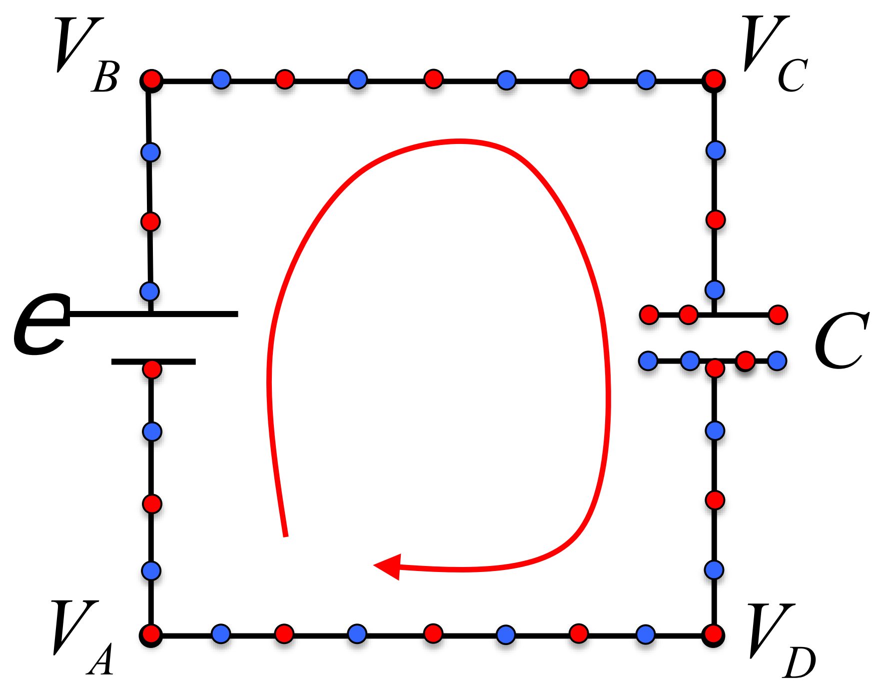
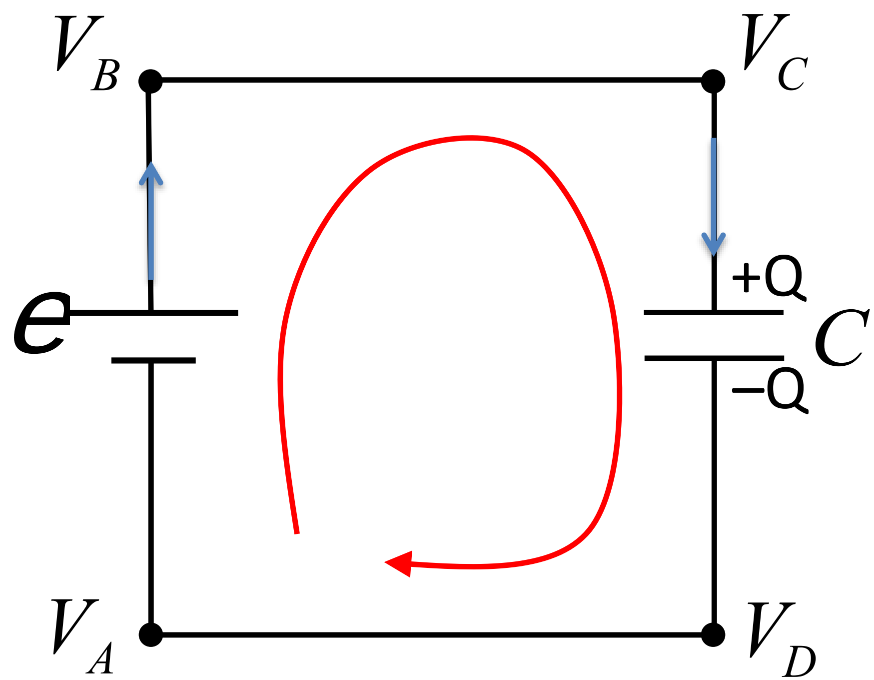


$$\boxed{\Delta V_{AB} + \Delta V_{BC} + \Delta V_{CD} + \Delta V_{DA} = 0}$$

A Basic Circuit

The voltage across a capacitor is **negative** if you are going around the loop in the direction **from the + plate to the – plate**. Current flows **from the negative** terminal **to the positive** terminal



ideal wires

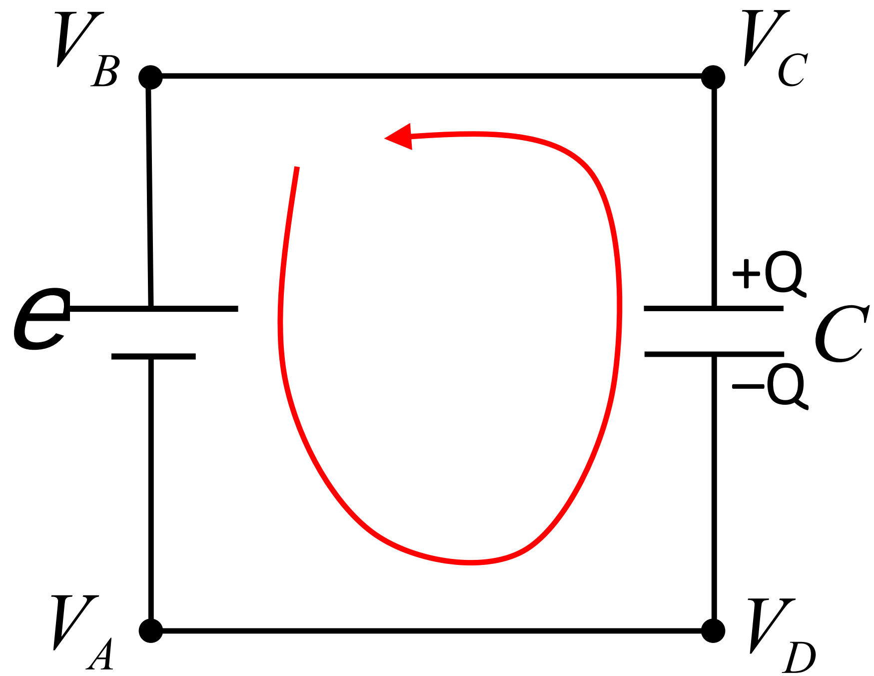
$$\Delta V_{AB} + \cancel{\Delta V_{BC}} + \Delta V_{CD} + \cancel{\Delta V_{DA}} = 0$$

$$\varepsilon - \frac{Q}{C} = 0$$

A Basic Circuit

The voltage across a capacitor is **positive** if you are going around the loop in the direction **from – plate to + plate**.

Voltage across a battery is **negative** going **from positive to negative**



ideal wires

$$\Delta V_{BA} + \cancel{\Delta V_{AD}} + \Delta V_{DC} + \cancel{\Delta V_{CB}} = 0$$

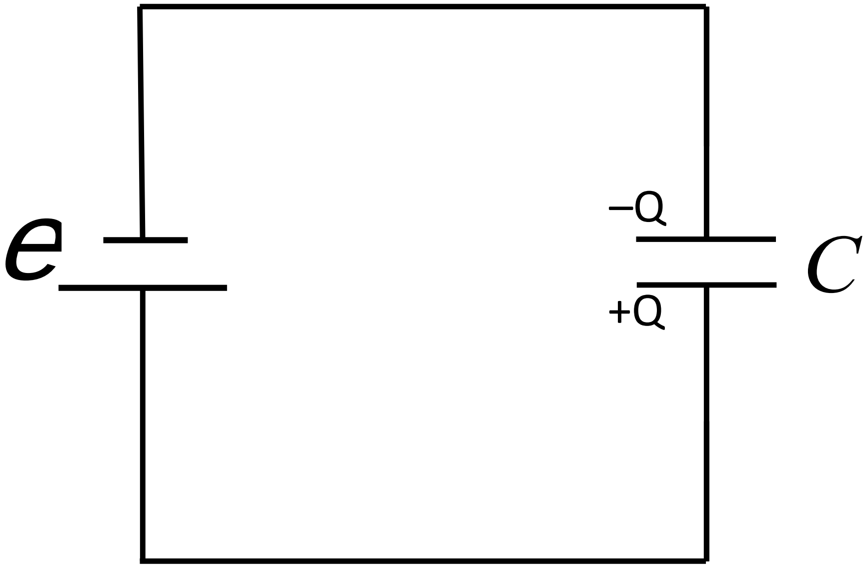
$$-e + \frac{Q}{C} = 0$$

Same as before

TopHat Question

TopHat Question

What is the charge on the top plate of the capacitor in the circuit shown?
$\mathcal{E}$ = 12 V and C = 0.25 µF.



A. Q = 3.0 µC

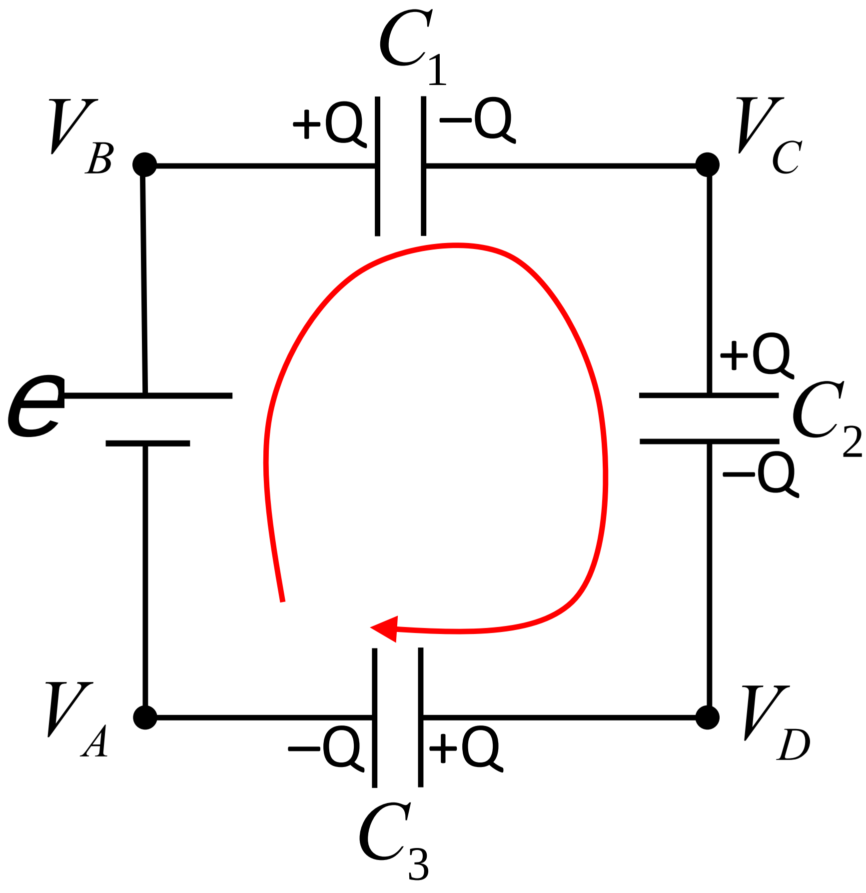
B. Q = 48 µC

C. Q = 21 nC

D. Q = –3.0 µC

Capacitors in Series

A slightly more complicated circuit has multiple capacitors in series



Kirchhoff's Loop Rule:

$$\Delta V_{AB} + \Delta V_{BC} + \Delta V_{CD} + \Delta V_{DA} = 0$$

Charge on each plate is the same

$$\varepsilon - \frac{Q}{C_1} - \frac{Q}{C_2} - \frac{Q}{C_3} = 0$$

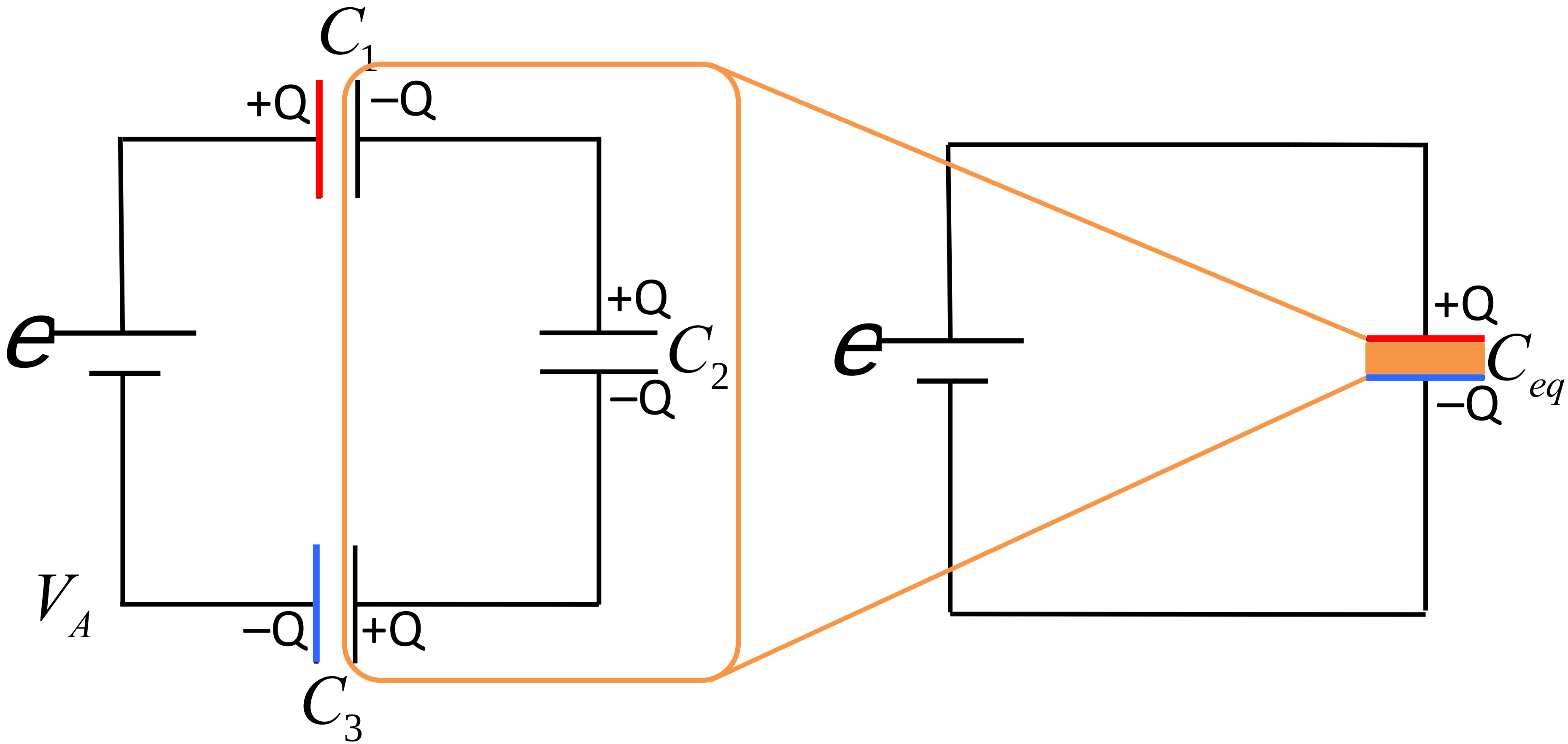
Rewrite this as

$$\varepsilon - Q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right) = 0$$

Define an equivalent capacitance

$$\varepsilon - \frac{Q}{C_{eq}} = 0$$

Capacitors in Series

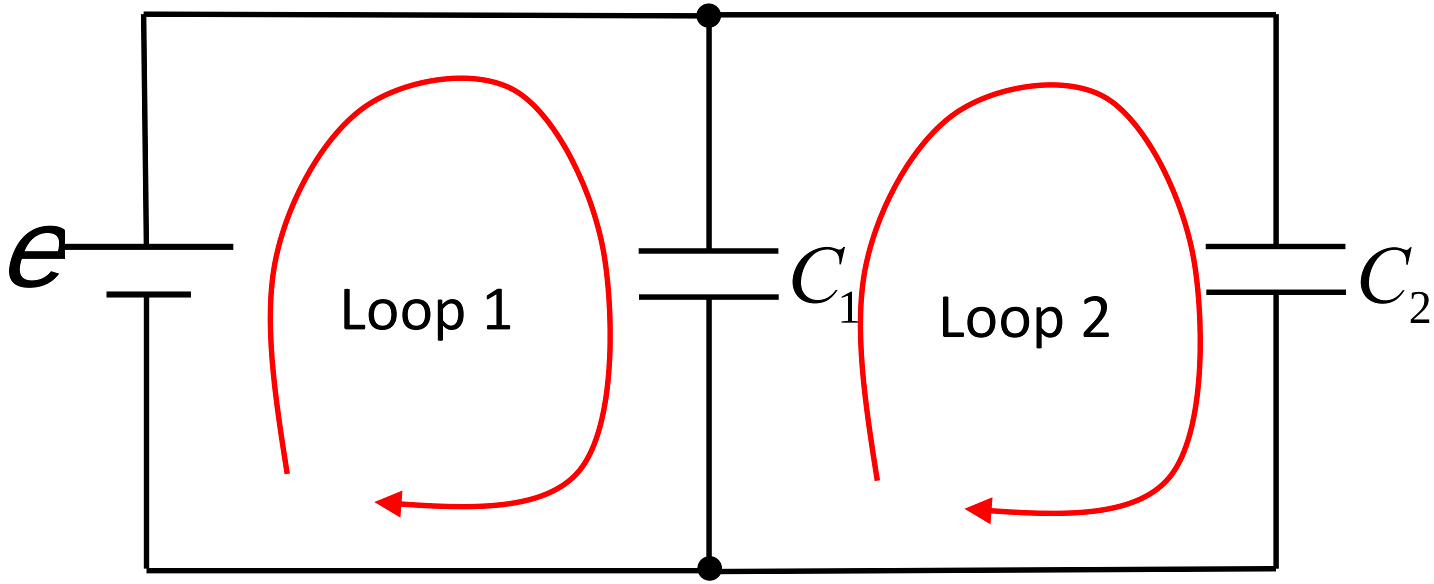


Capacitors in series act like a single equivalent capacitor:

$$C_{eq} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \right)^{-1}$$

Capacitors in Parallel

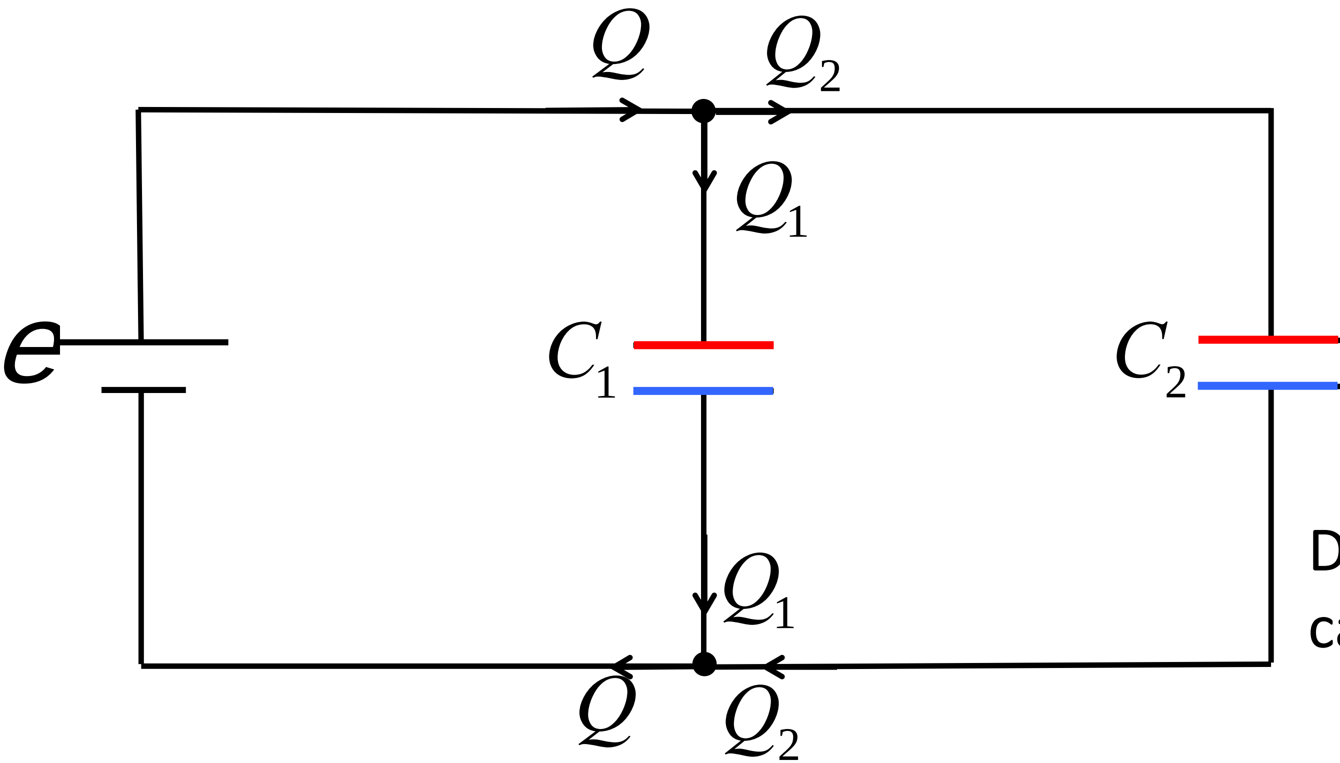
A slightly more complicated circuit has multiple branches with capacitors in parallel



Capacitors in parallel have the same voltage across their plates

Loop 1: $\mathcal{E} - \Delta V_{C_1} = 0$ Loop 2: $\Delta V_{C_1} - \Delta V_{C_2} = 0$

Capacitors in Parallel



$$Q = Q_1 + Q_2$$

$$Q_1 = \Delta V_{C_1} C_1$$

$$Q_2 = \Delta V_{C_2} C_2$$

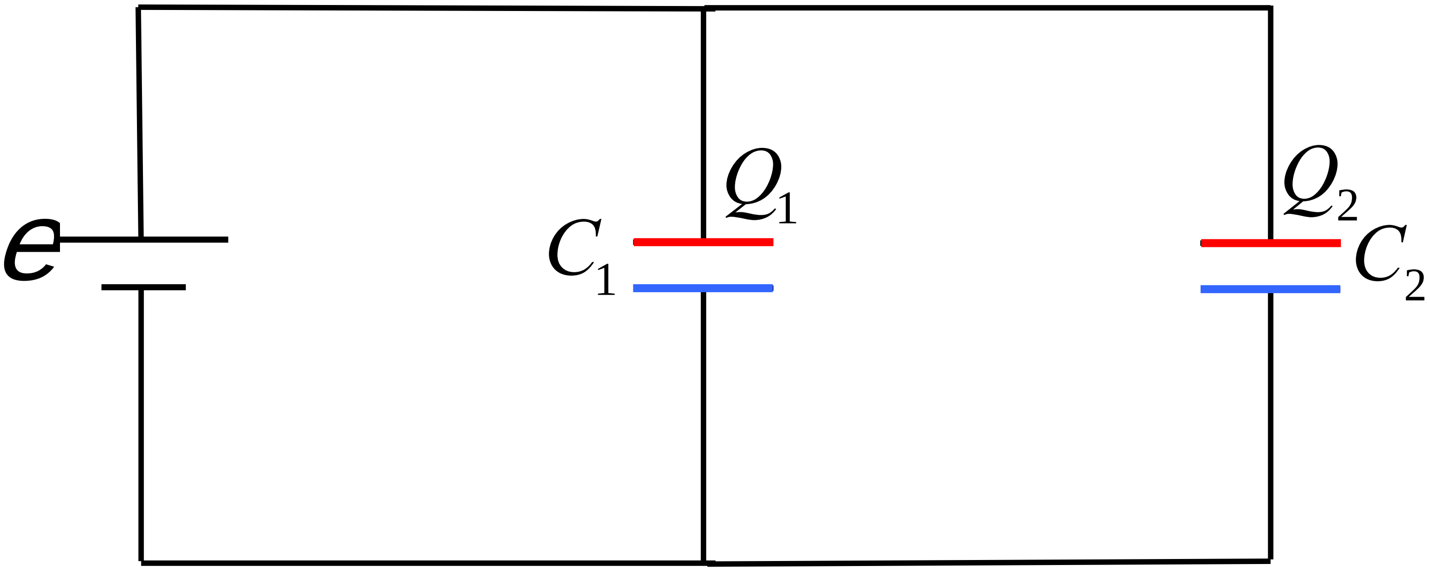
Define an equivalent capacitance:

$$Q = \varepsilon C_{eq}$$

From conservation of charge: $\varepsilon C_{eq} = \varepsilon C_1 + \varepsilon C_2$

For capacitors in parallel: $C_{eq} = C_1 + C_2$

Capacitors in Parallel



$$Q = Q_1 + Q_2 \qquad C_{eq} = C_1 + C_2$$

Summary of Capacitors

Relation between charge and voltage across plates

$$\Delta V_C = \frac{Q}{C}$$

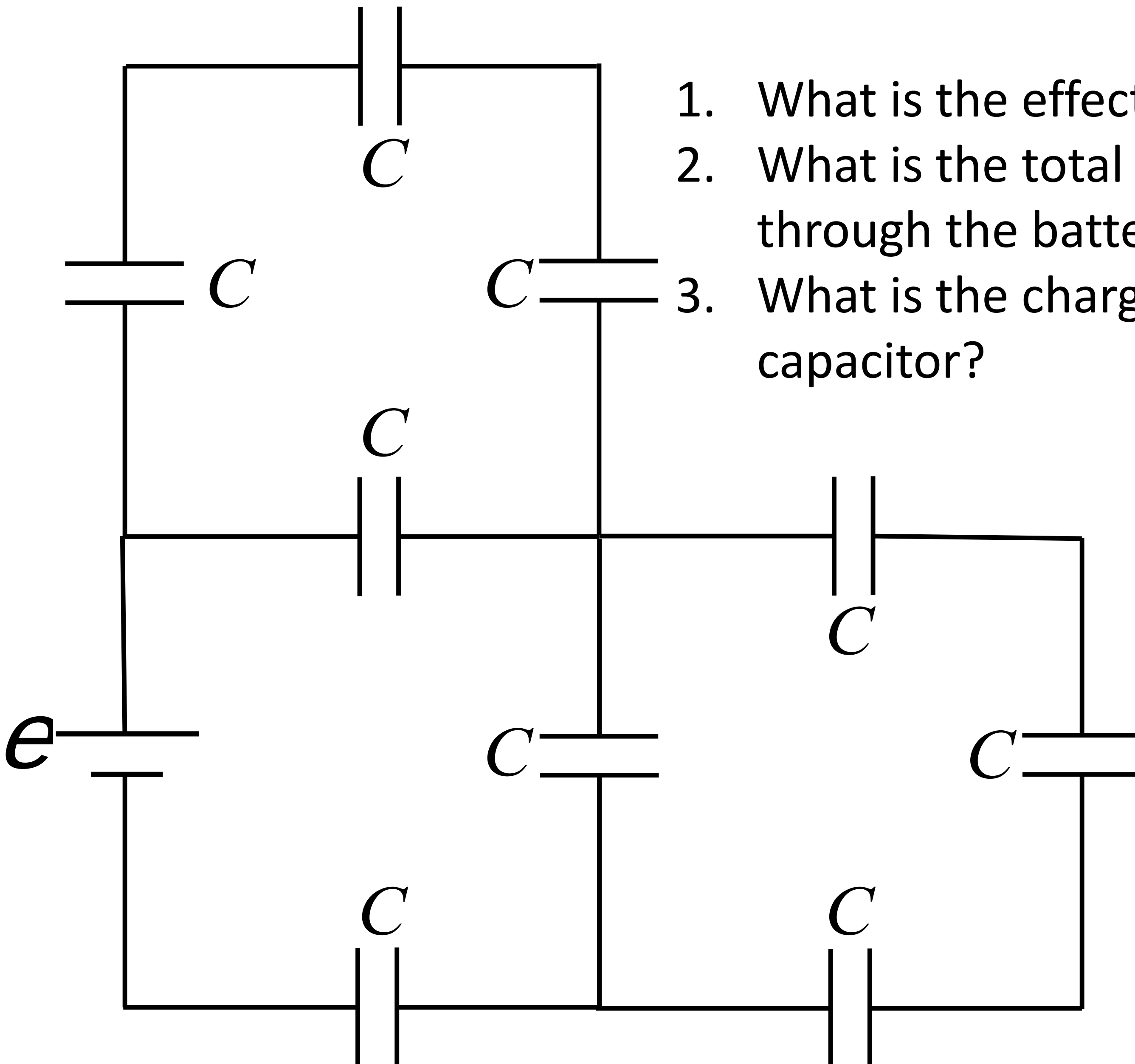
Capacitors in Series: store the same amount of charge

$$C_{eq} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \ldots + \frac{1}{C_N} \right)^{-1}$$

Capacitors in Parallel: have the same voltage across them

$$C_{eq} = C_1 + C_2 + \ldots + C_N$$

On document camera



1. What is the effective capacitance?
2. What is the total charge moved through the battery?
3. What is the charge on each capacitor?

Wednesday March 8, 2017 class 2

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Today:

- Subtlety with capacitors: series or parallel?
- Linear dielectric materials: an atomic perspective
- Effect of dielectrics on capacitance
- Applications of dielectrics and capacitors

Summary of Capacitors

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Capacitors in Parallel: have the same voltage across them

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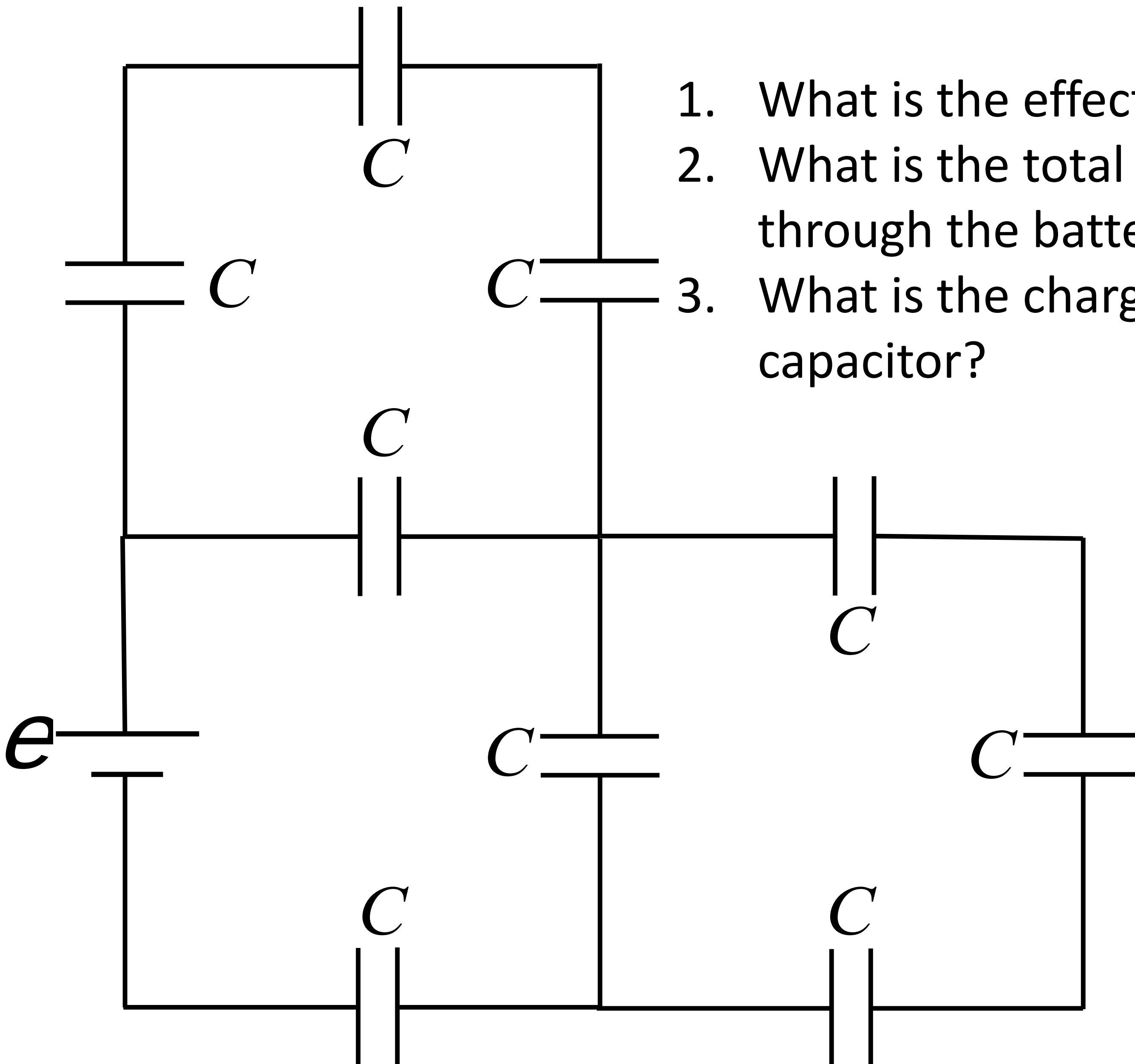
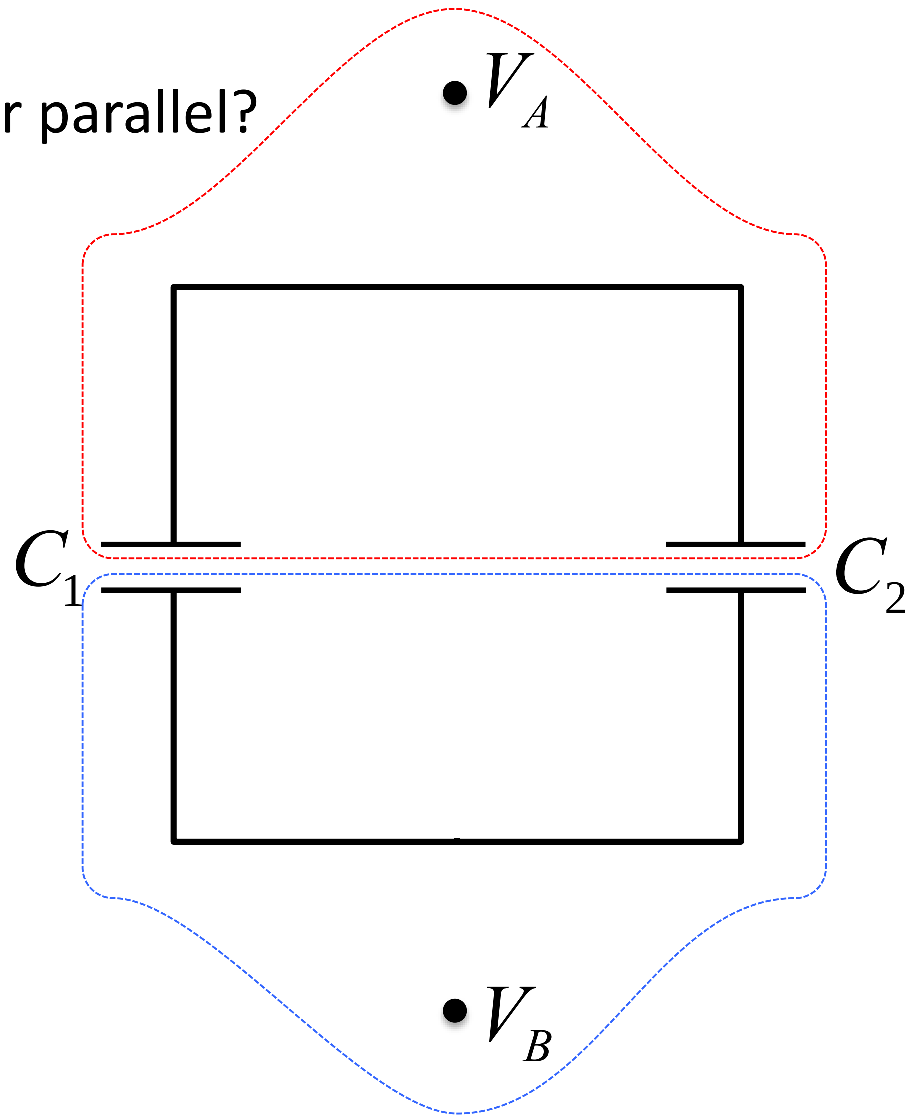


1. What is the effective capacitance?
2. What is the total charge moved through the battery?
3. What is the charge on each capacitor?

Capacitor Subtlety

Are these capacitors in series or parallel?

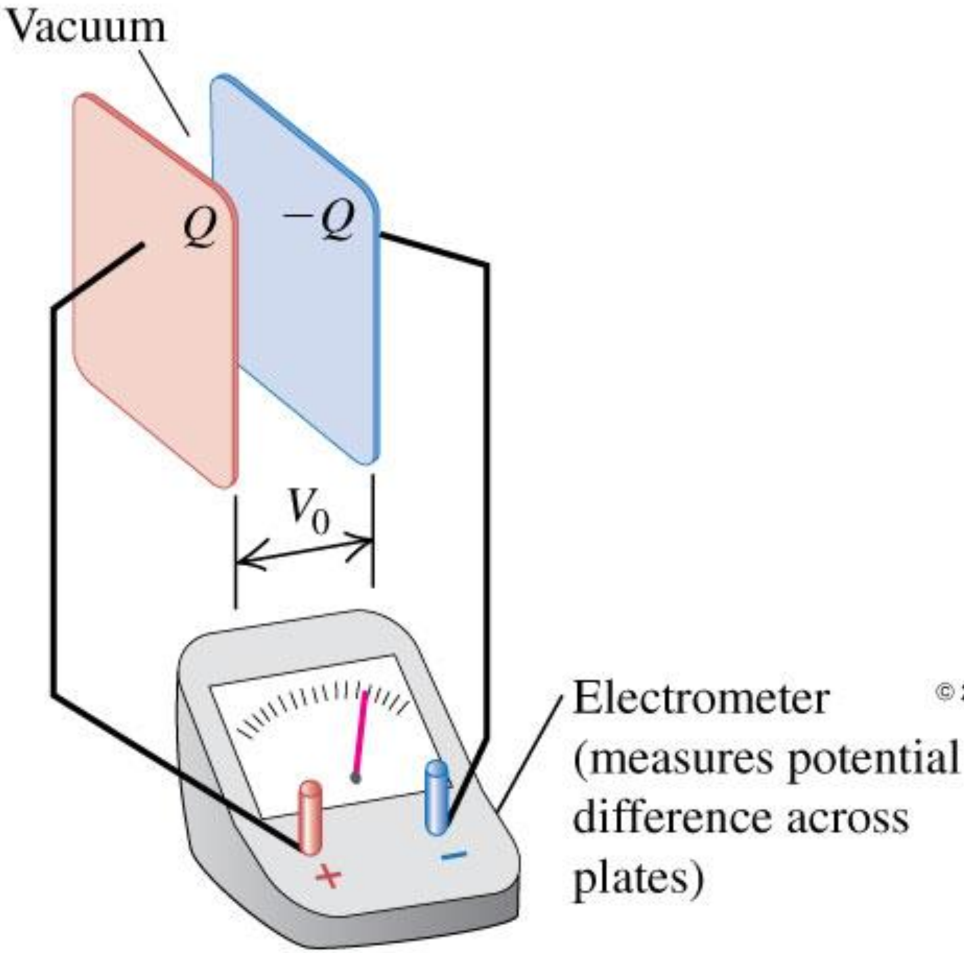


The plates are conductors, and so are the wires.
The top half of the circuit is electrically disconnected from the bottom half.
Top half is at potential V_A and the bottom half is at potential V_B.
These capacitors are in parallel! They have the same voltage across them

$$Q = C_0 V_0$$

$$V_0 = E_0 d$$

(a)

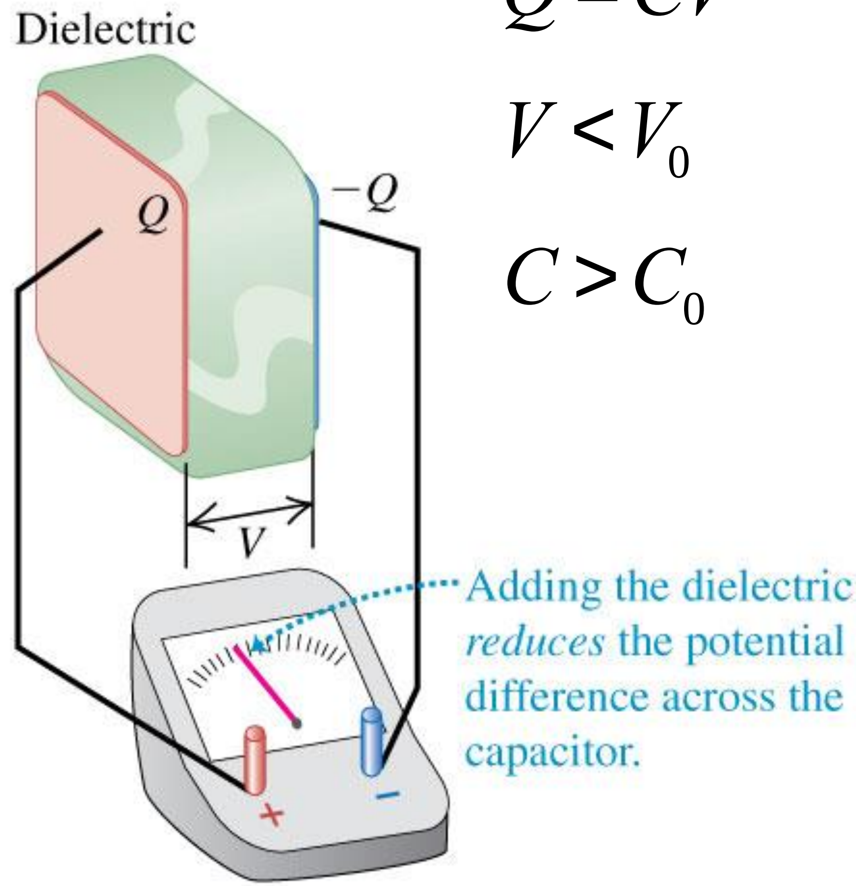


(b)

$$Q = CV$$

$$V < V_0$$

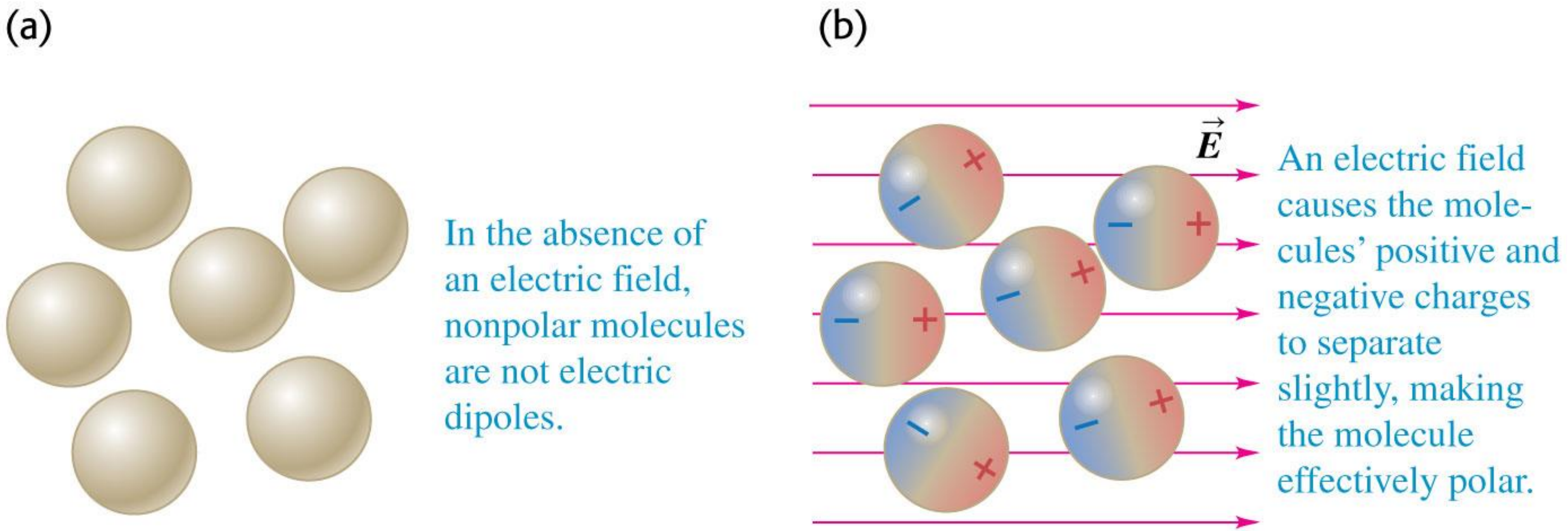
$$C > C_0$$



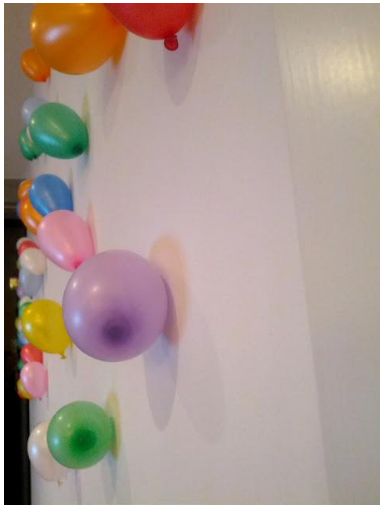
$$V = Ed$$

$$E < E_0$$

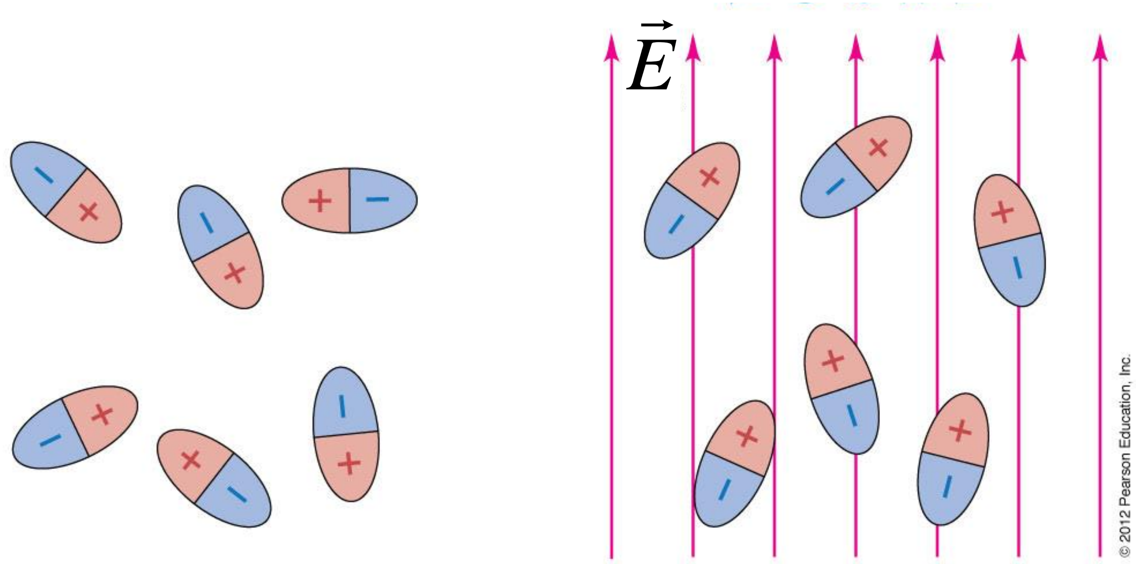
non-polar molecules

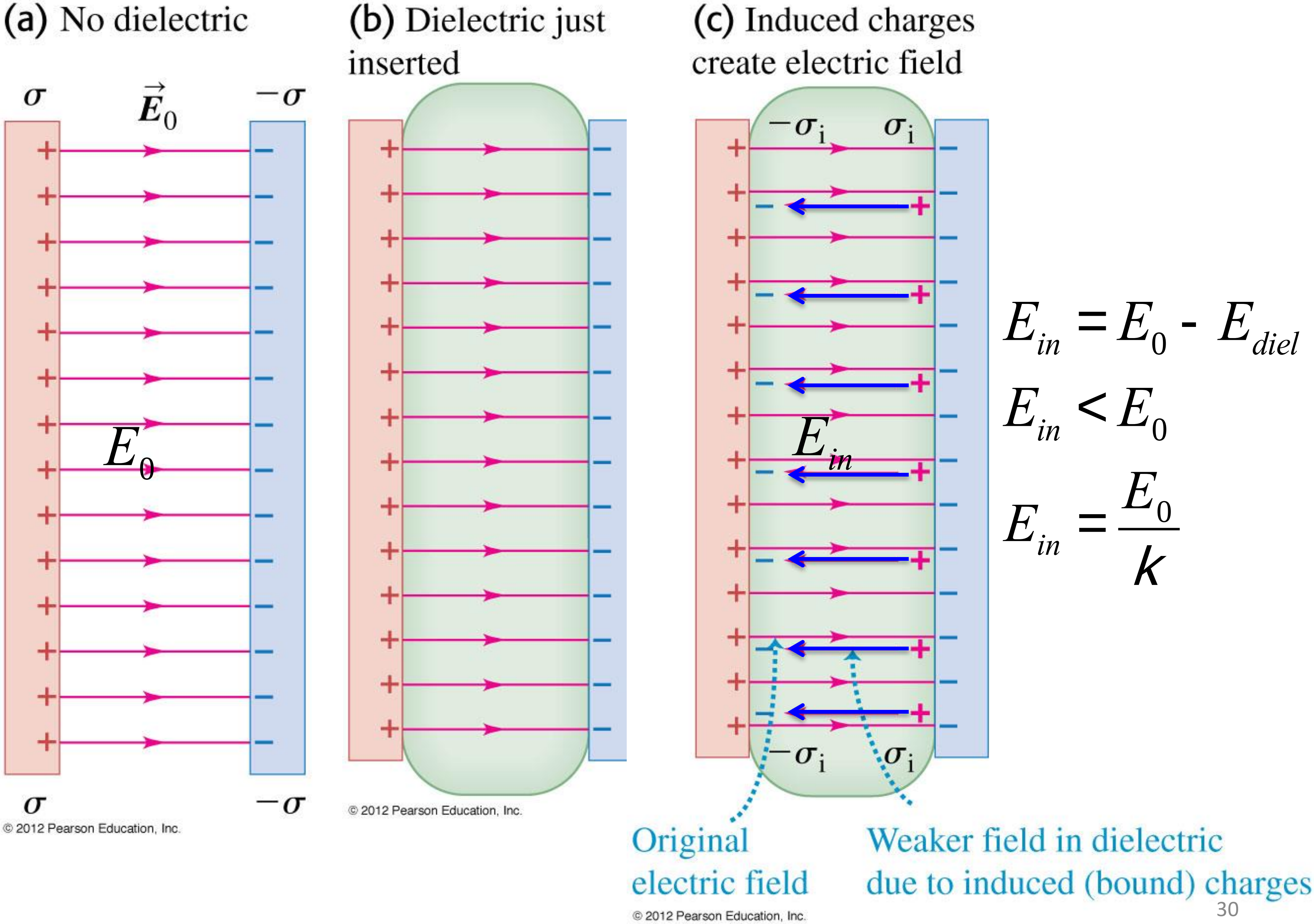


Recall the balloon on the wall example from week 1



polar molecules





$$E_{in} = E_0 - E_{diel}$$

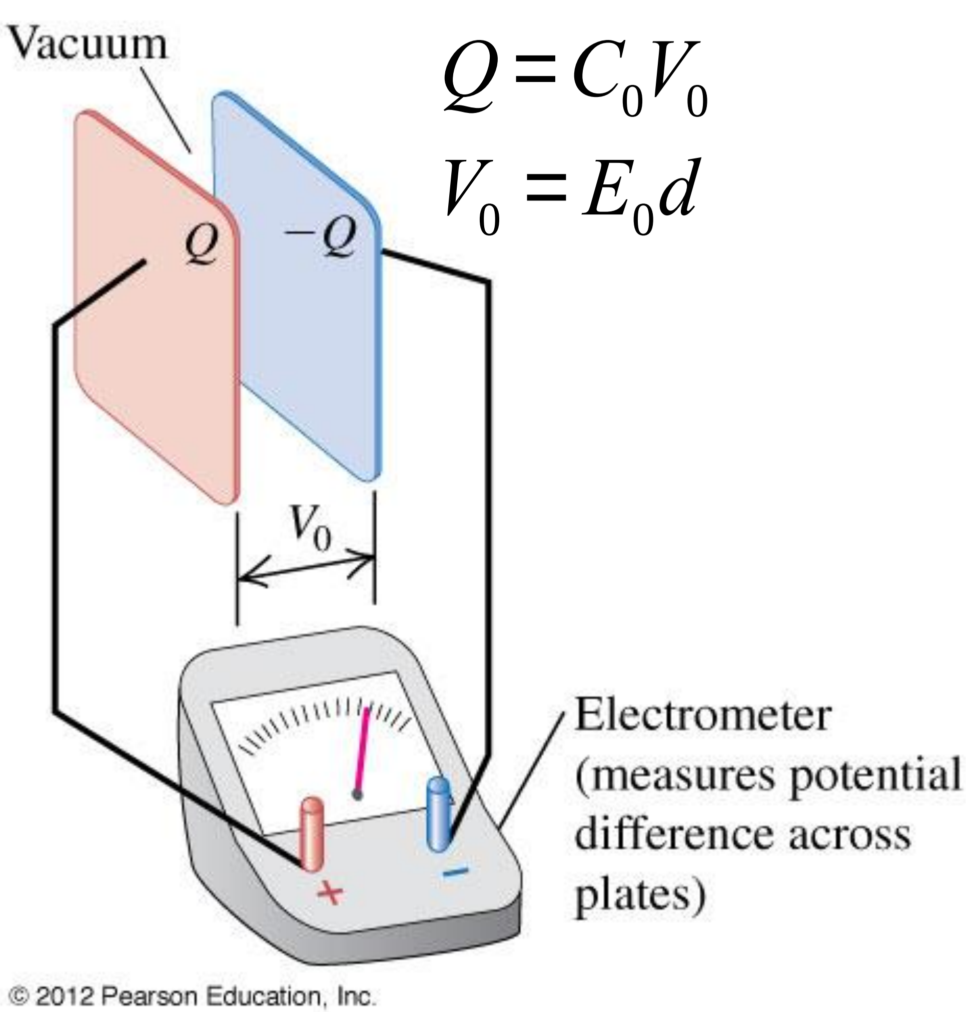
$$E_{in} < E_0$$

$$E_{in} = \frac{E_0}{k}$$

Linear Dielectrics

Materials in which the induced polarization is proportional to the electric field are called **linear dielectrics**.

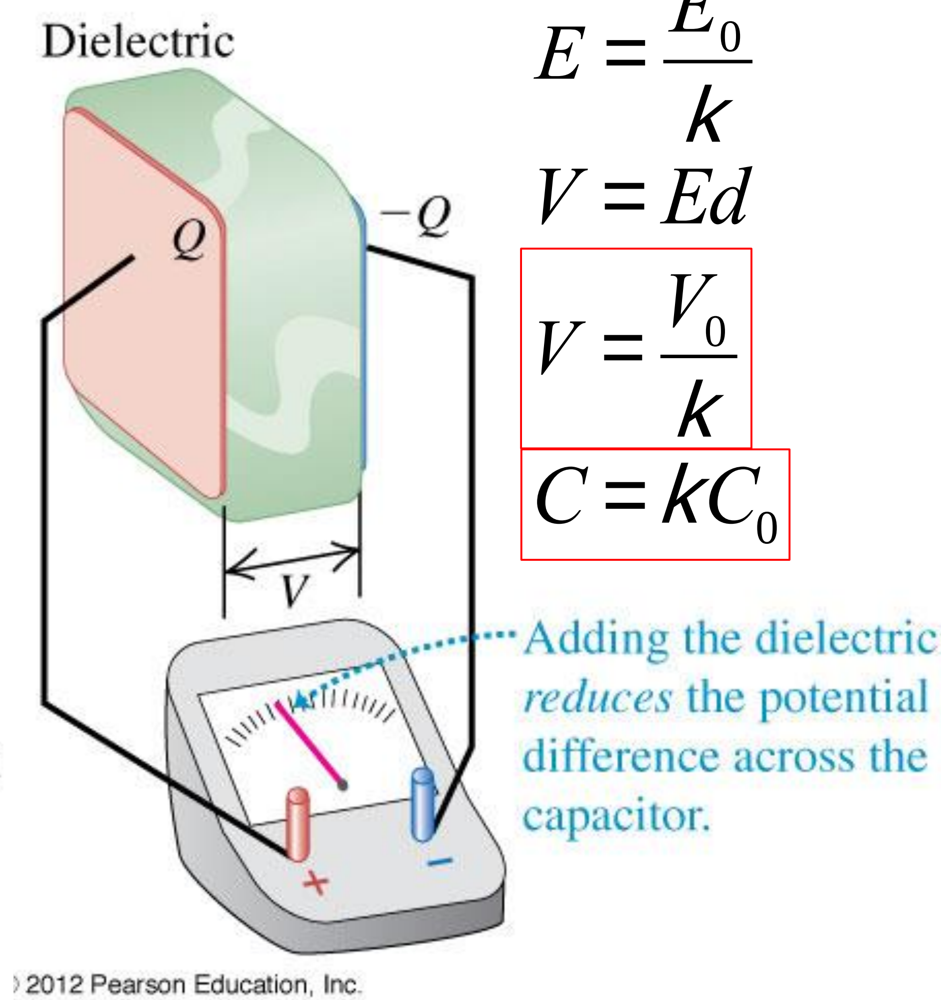
(a)



$$Q = C_0 V_0$$

$$V_0 = E_0 d$$

b)



$$E = \frac{E_0}{k}$$

$$V = Ed$$

$$\boxed{V = \frac{V_0}{k}}$$

$$\boxed{C = kC_0}$$

Parallel Plate Capacitors With and Without Dielectrics

$$E_0 = \frac{s}{e_0} \qquad\qquad E = \frac{E_0}{k} = \frac{s}{ke_0}$$

$$V_0 = E_0 d = \frac{s\,d}{e_0} \qquad\qquad V = \frac{V_0}{k} = \frac{s\,d}{ke_0}$$

$$C_0 = \frac{Q}{V_0} = \frac{s\,A}{s\,d}\,e_0 = \frac{Ae_0}{d} \qquad\qquad C = kC_0 = \frac{Ake_0}{d}$$

Conclusion: for a linear dielectric, all the regular electrostatic equations hold if $e_0 \rightarrow e \equiv ke_0$

Now you see why e_0 is called “permittivity of free space” (i.e. in vacuum)

Gauss' law for dielectrics

- $\oiint \vec{E} \cdot d\vec{A} = \frac{q_{f,\,enc}}{\kappa\varepsilon_0}$

TopHat Question

A capacitor without a dielectric is charged up so that it stores potential energy U_0, and it is then disconnected so that **its charge remains the same**. A dielectric with constant $\kappa = 2$ is then inserted between the plates. What is the new potential energy stored in the capacitor **with the dielectric**?

$$U_C = \frac{Q^2}{2C} = \frac{V_C^2 C}{2}$$

A. $4U_0$

C. $\frac{1}{2}U_0$

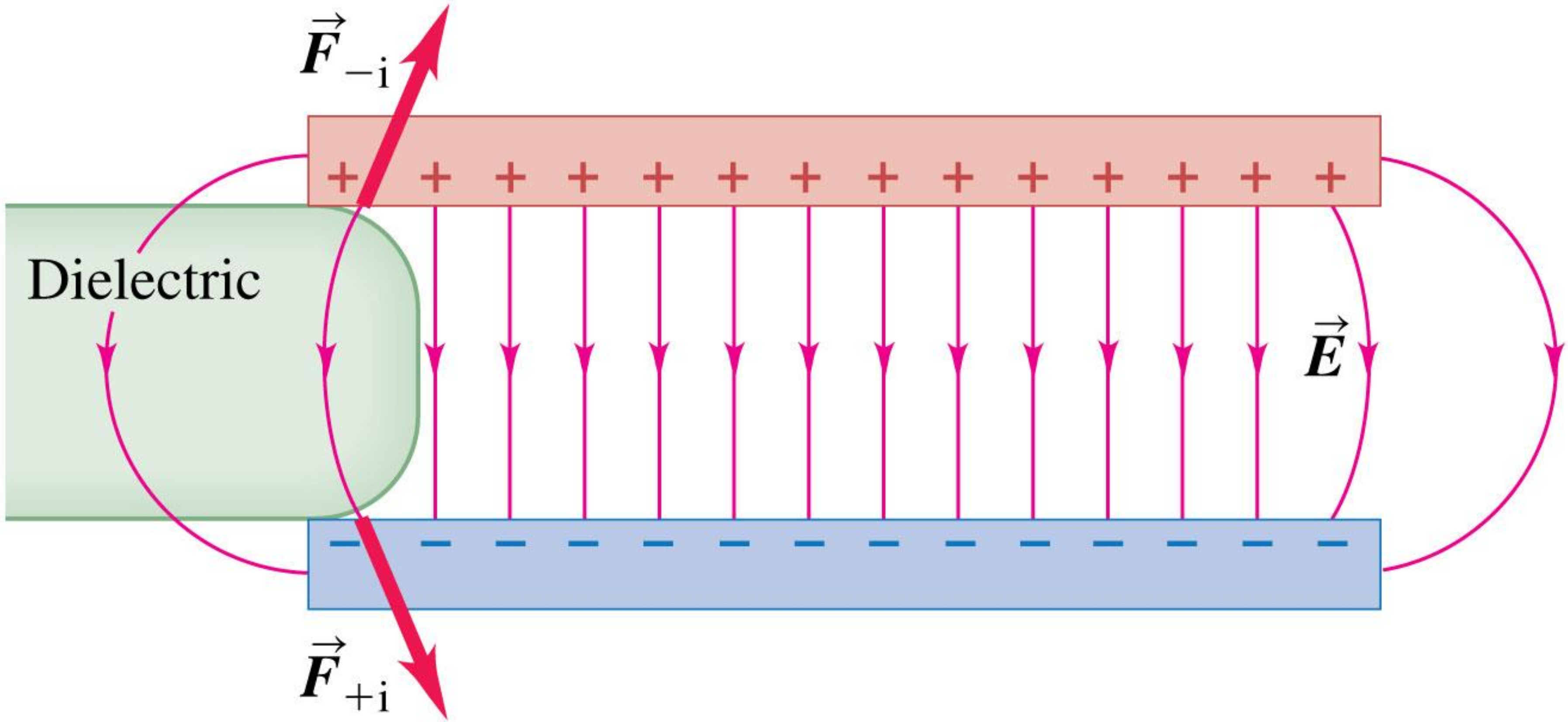
A. $2U_0$

D. $\frac{1}{4}U_0$

The **potential energy lowers** when the dielectric is added, so it will feel a **force sucking it into the gap** between the plates.

Can find the force using $F_x = -dU/dx$ and

$$W = \int dq \left[\frac{q}{C}\right] = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2$$



Fringe Electric Field pulls the dielectric into the gap

TopHat Question

A capacitor without a dielectric is charged up so that it stores potential energy U_0, and is kept connected **at constant voltage**. A dielectric with constant $\kappa = 2$ is then inserted between the plates. What is the new potential energy stored in the capacitor **with the dielectric**?

$$U_C = \frac{Q^2}{2C} = \frac{V_C^2 C}{2}$$

A. $4U_0$

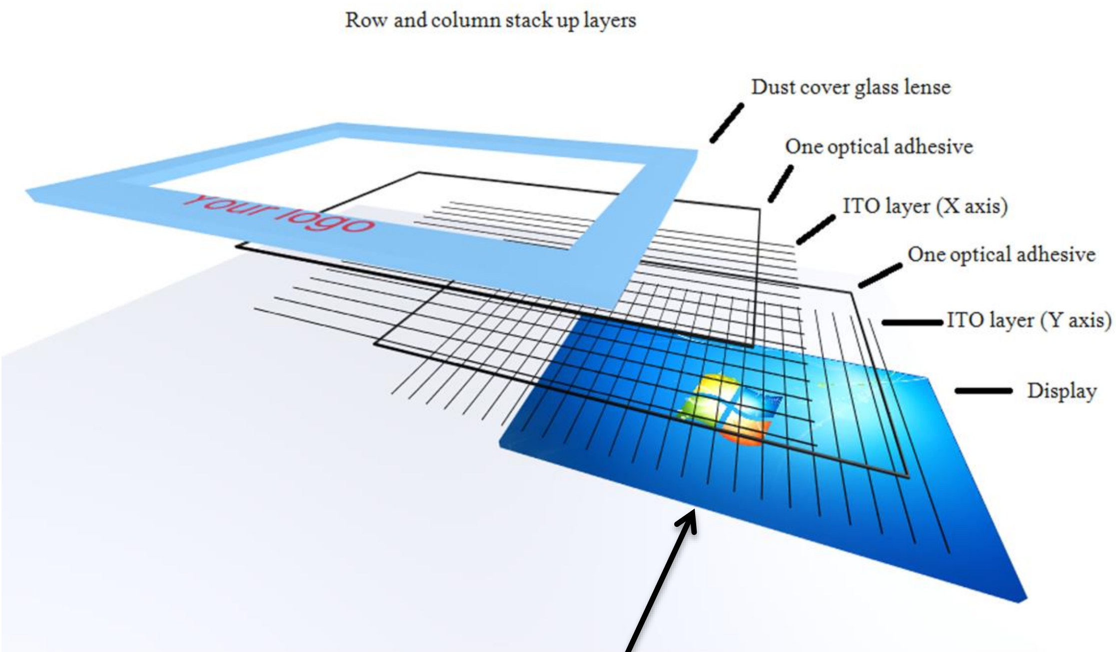
C. $\frac{1}{2}U_0$

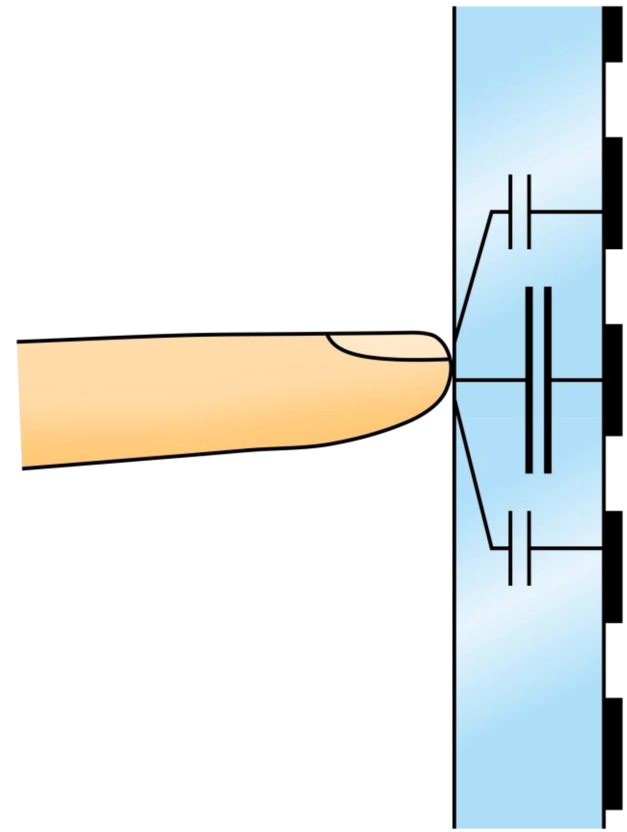
A. $2U_0$

D. $\frac{1}{4}U_0$

The **potential energy raises** when the dielectric is added, so it will feel a **force pushing it out of the gap** between the plates.

Application: Capacitive Touch Screen

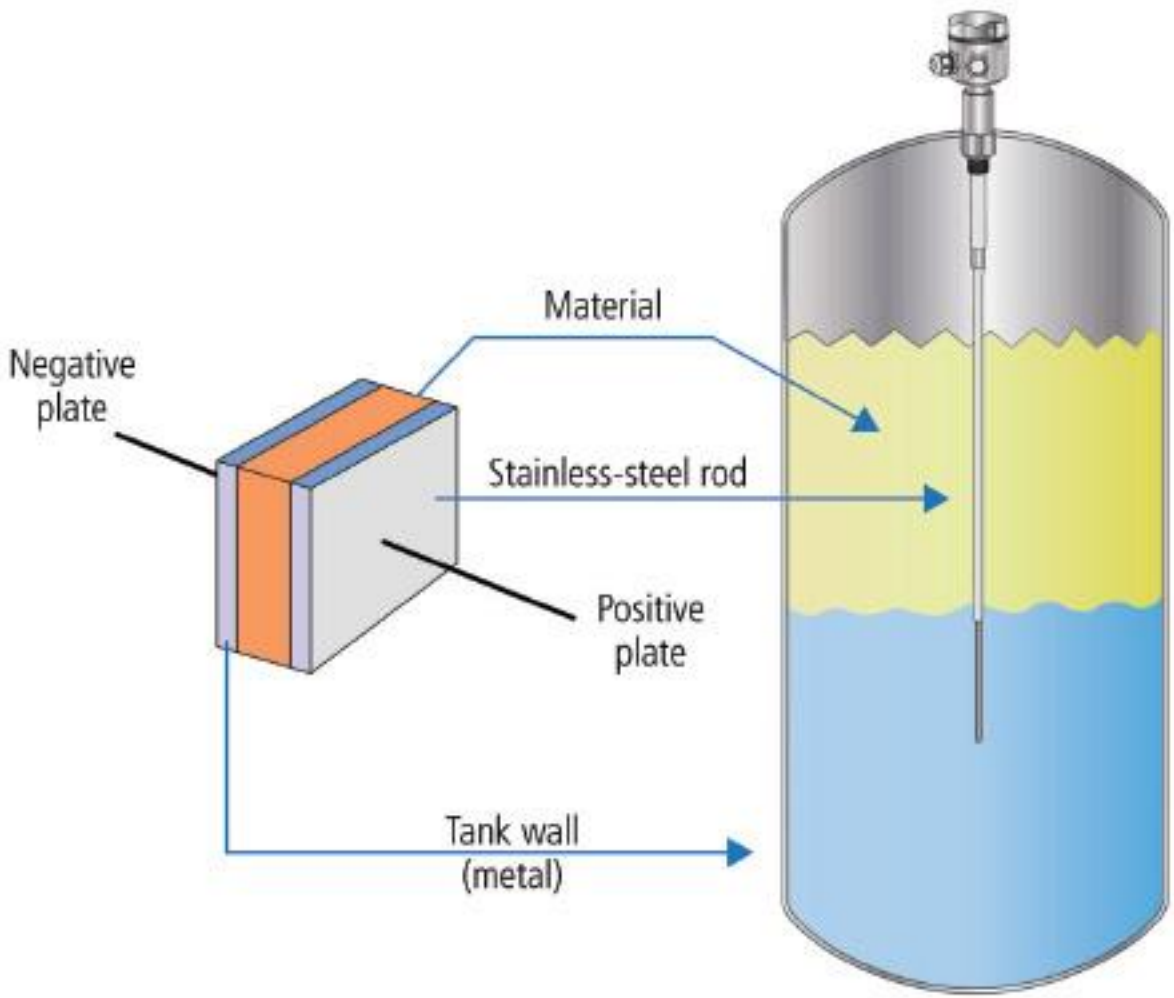




Should be

https://www.youtube.com/watch?v=qBbxSEp3-6o

Application: Capacitive Fuel Gauge



https://www.youtube.com/watch?v=0du-QU1Q0T4