Electricity and Magnetism

- Physics 259 – L02
- Lecture 19

Chapter 23.3-4

Last time

- Chapter 23.2 

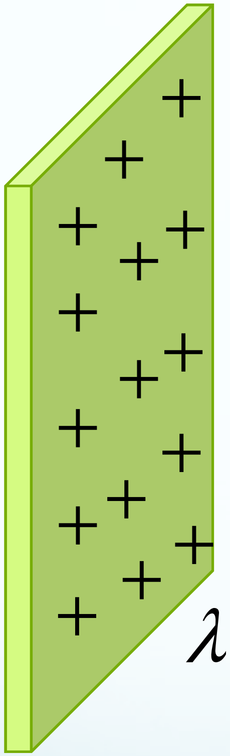
This time

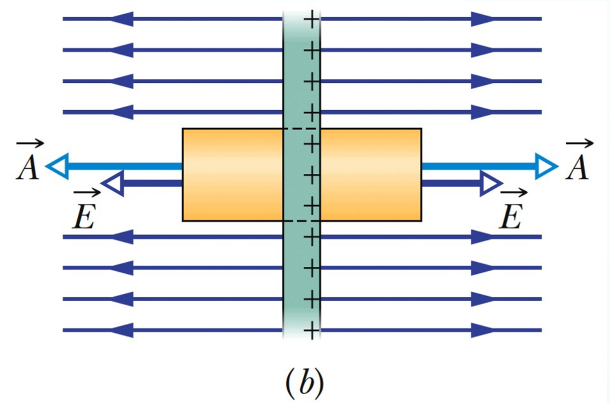
- Chapter 23. and 23.4 

23-5: Electric field of a plane of charge

Nonconduction infinite sheet

$$\Phi_e = \oint \vec{E}.d\vec{A} = \frac{Q_{in}}{\varepsilon_0}$$





$$E_{plane} = \frac{\sigma}{2\varepsilon_0}$$

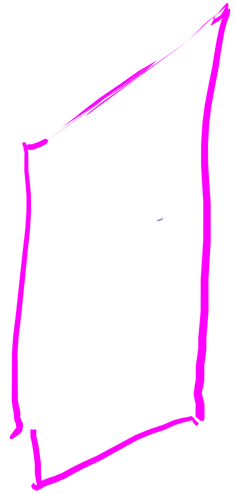
Q2) Two very thin infinite sheets are uniformly charged with surface charge densities -2η and $+5\ \eta$ as indicated in the figure. What is the magnitude and direction of the electric field at point P located between the sheets? (note the direction of +x in the figure)

a) $-3\eta/2\varepsilon_o$

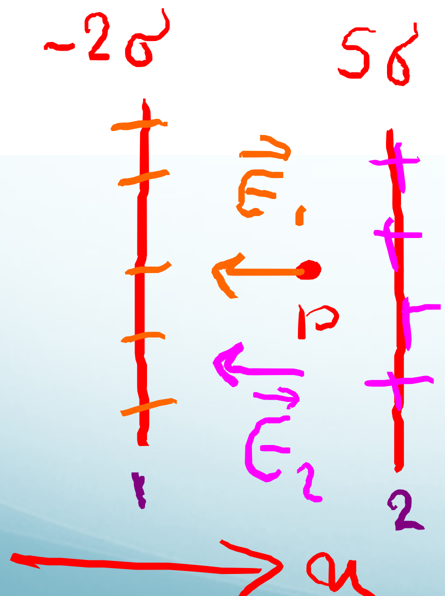
b) $+3\eta/2\varepsilon_o$

c) $-7\eta/2\varepsilon_o$

d) $+7\eta/2\varepsilon_o$



$$E = \frac{\sigma}{2\varepsilon_0}$$

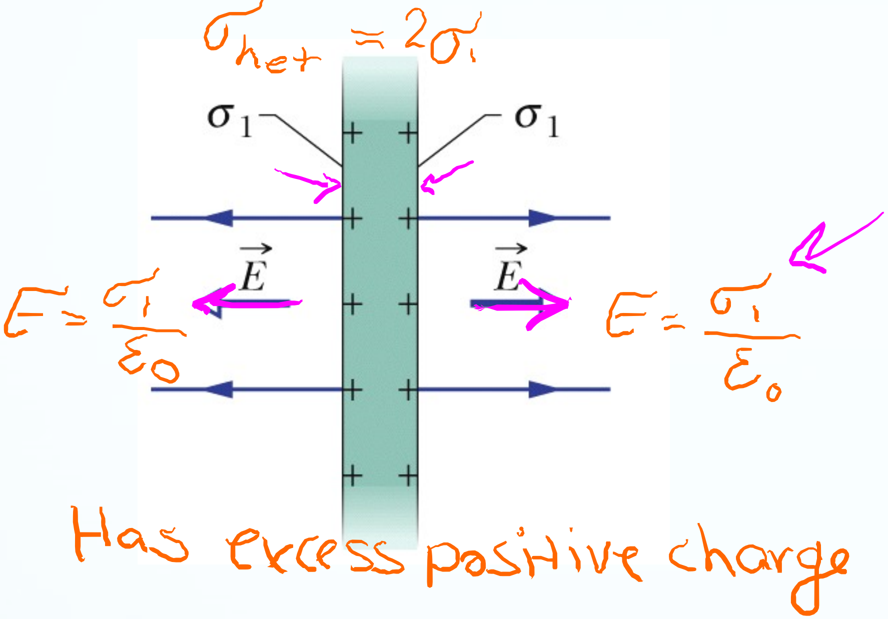


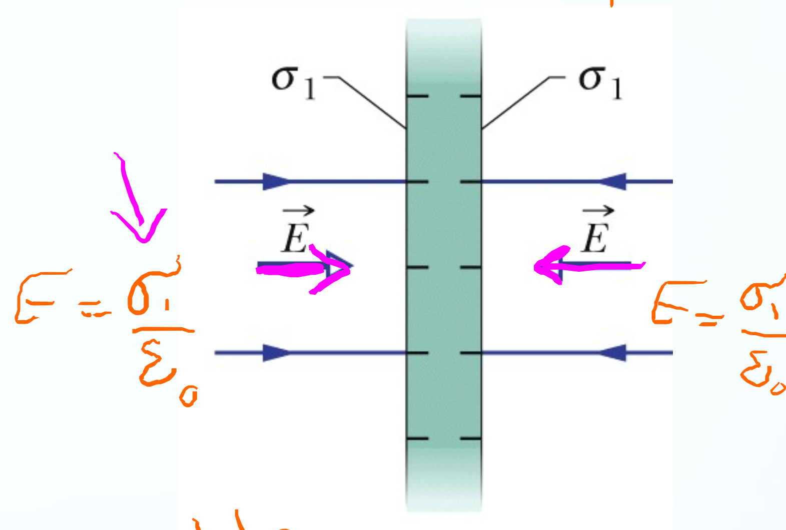
$$\vec{E}_1 = -\frac{2\sigma}{2\varepsilon_0}\hat{i}$$

$$\vec{E}_2 = -\frac{5\sigma}{2\varepsilon_0}\hat{i}$$

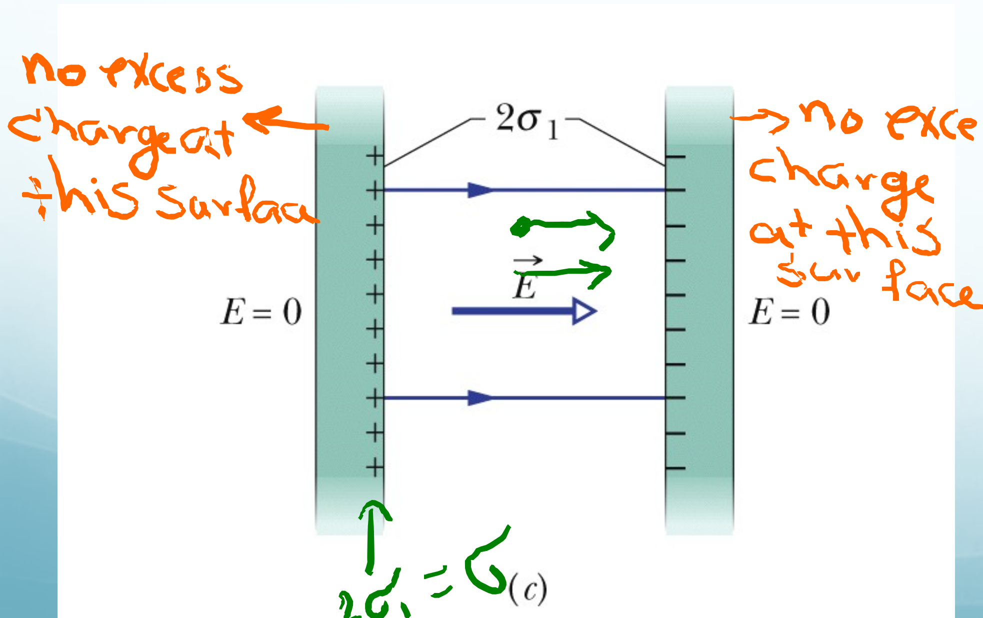
$$\rightarrow \vec{E}_{net} = -\frac{7\sigma}{2\varepsilon_0}\hat{i}$$

Two conducting Plates





$2\sigma_1 = \sigma$



$$E = \frac{2\sigma_1}{\varepsilon_0} = \frac{\sigma}{\varepsilon_0}.$$

TopHat Question 1

$Q = \lambda L$

4 surfaces are coaxial with an infinitely long line of charge with a uniform linear charge density = λ. Choose all the surfaces through which $\Phi_E = \lambda L / \varepsilon_0$

$\Phi_e = \frac{q_{enc}}{\varepsilon_0} = \frac{\lambda L}{\varepsilon_0}$

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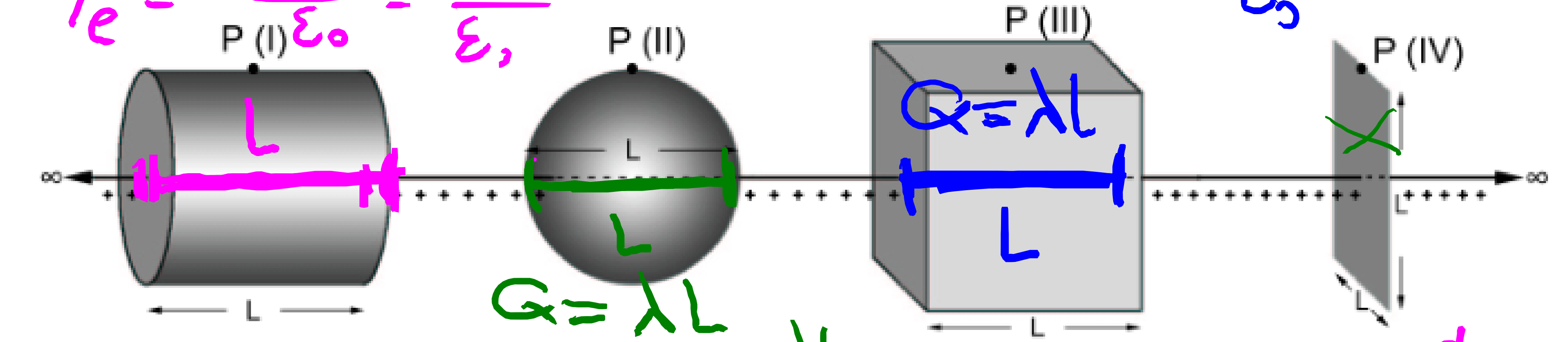


$Q = \lambda L$

$\Phi_e = \frac{q}{\varepsilon_0} = \frac{\lambda L}{\varepsilon_0}$

A) I only

B) I and II only

C) I and III only

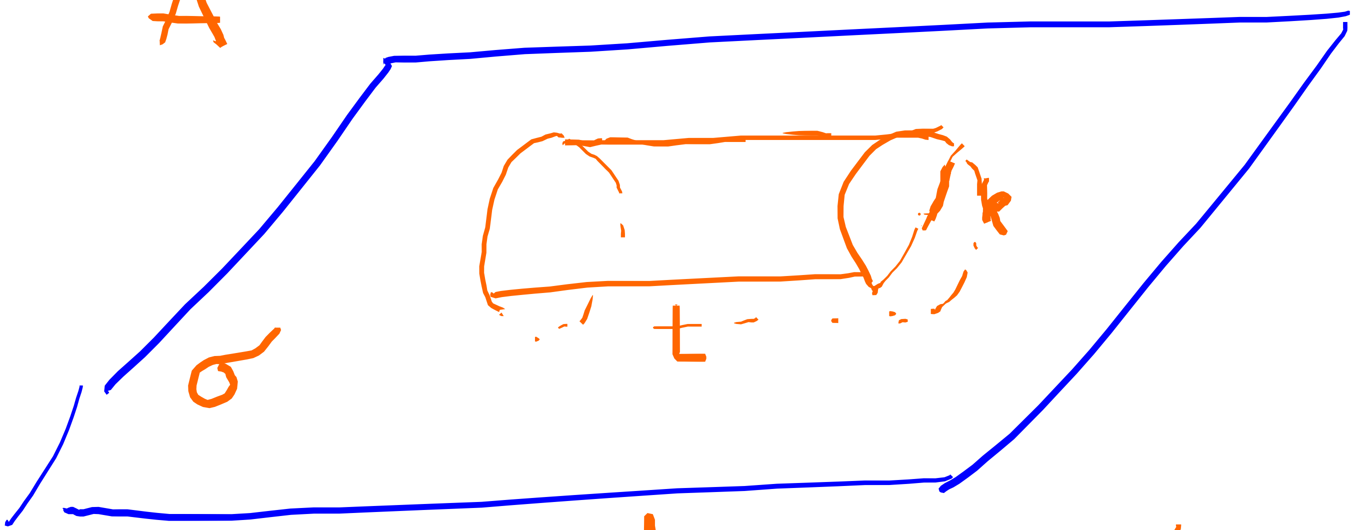
D) I, II, and III only

E) All four.

$\Phi_e = \oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$

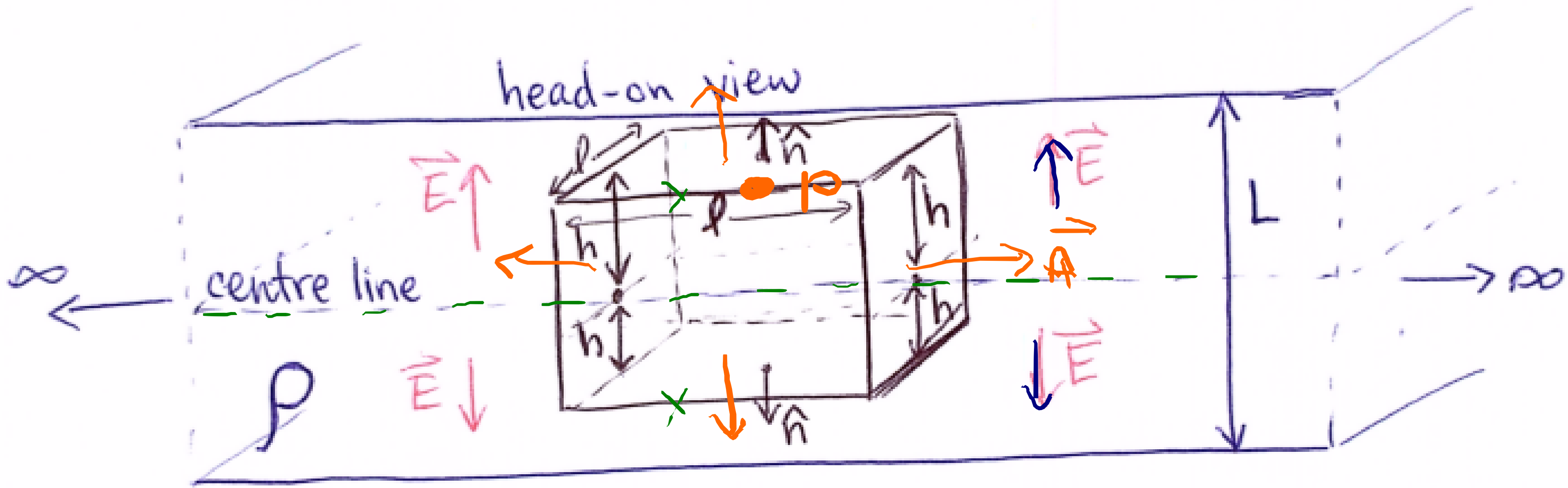
$\Phi_e = \frac{q_{enc}}{\varepsilon_0}$

$$\sigma = \frac{Q}{A} \rightarrow Q = \sigma A$$

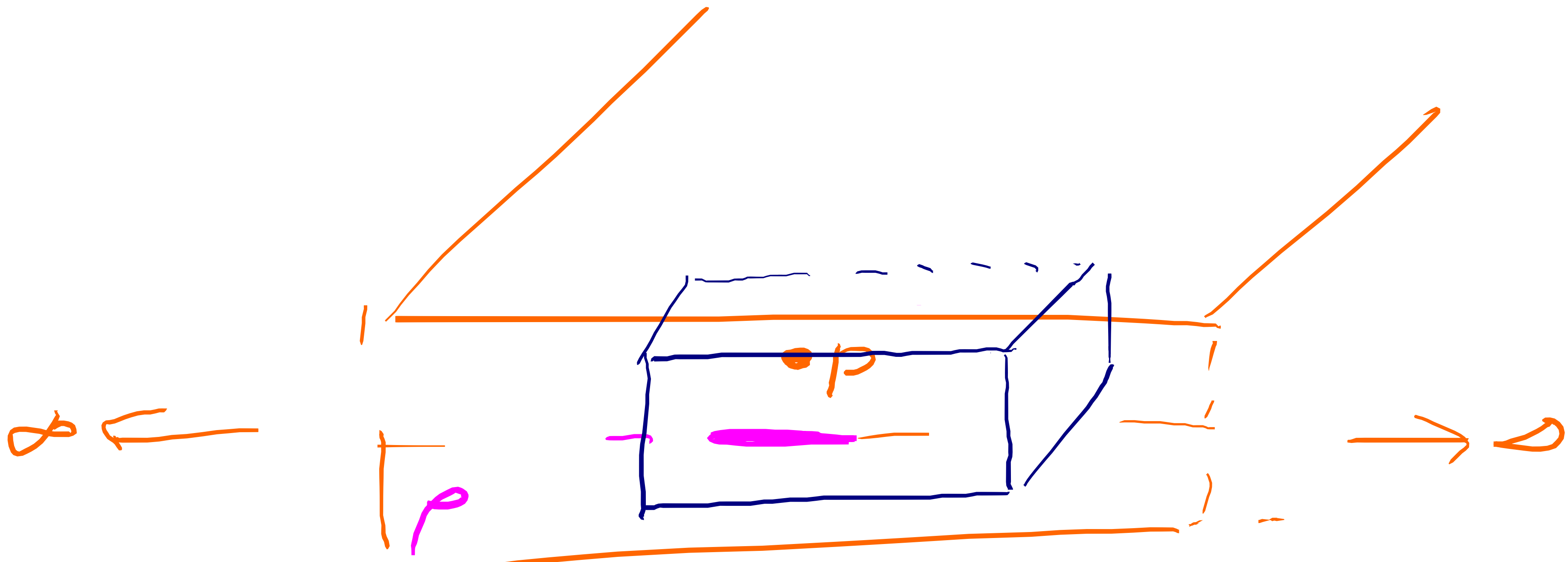


$$\Phi_e = \frac{q_{enc}}{\varepsilon_0} = \frac{\sigma A}{\varepsilon_0}$$

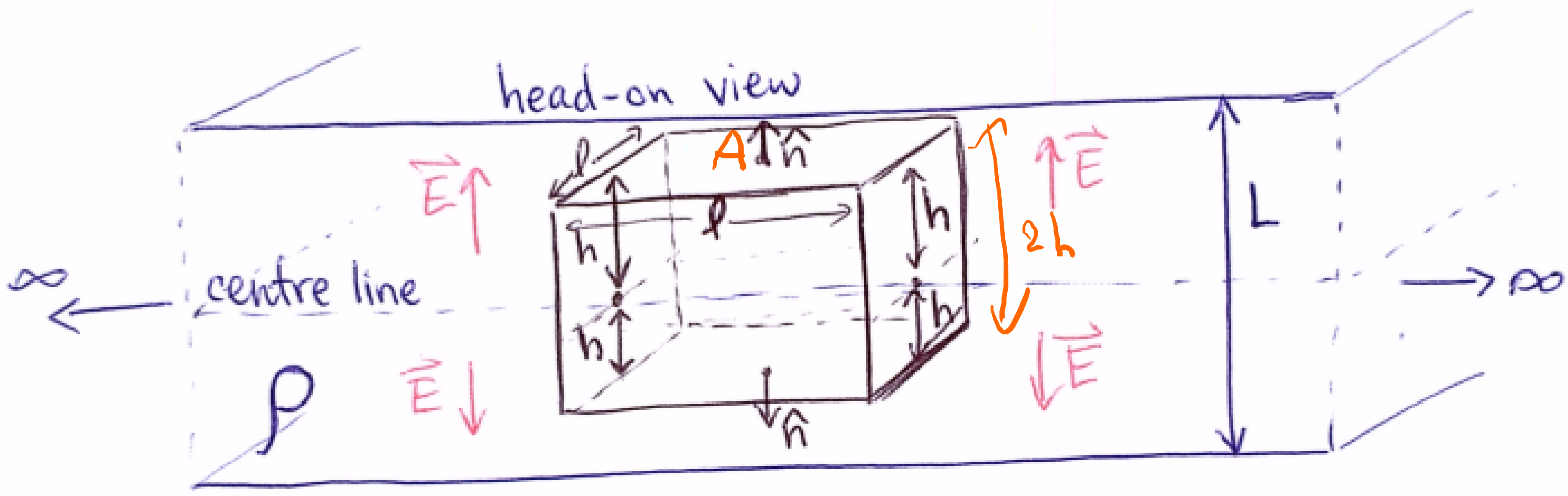
What is the field <u>inside</u> the slab?



The slab has thickness L, we have to choose a Gaussian surface with the same symmetries as the slab: choose a box whose centre coincides with the centre of the slab.



$$\rho = \frac{Q}{V}$$



$$\oint \vec{E}\cdot d\vec{A} = \int_{top} \vec{E}\cdot dA + \int_{bottom} \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0}$$

$$E\int_{top} dA + E\int_{bottom} dA = \frac{q_{enc}}{\epsilon_0}$$

$$\rho = \frac{q}{V}$$

$$q_{enc} = \rho V$$

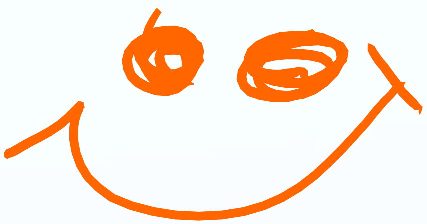
$$\underline{2EA = \frac{q_{enc}}{\epsilon_0}} \qquad q_{enc} = \underline{\rho}V = \underline{\rho A(2h)}$$

$$V = A \times 2h$$

$$\cancel{2}E\cancel{A} = \frac{\rho \cancel{A}(\cancel{2}h)}{\epsilon_0}$$

$$\boxed{E = \frac{\rho h}{\epsilon_0}}$$

What about cylinder?

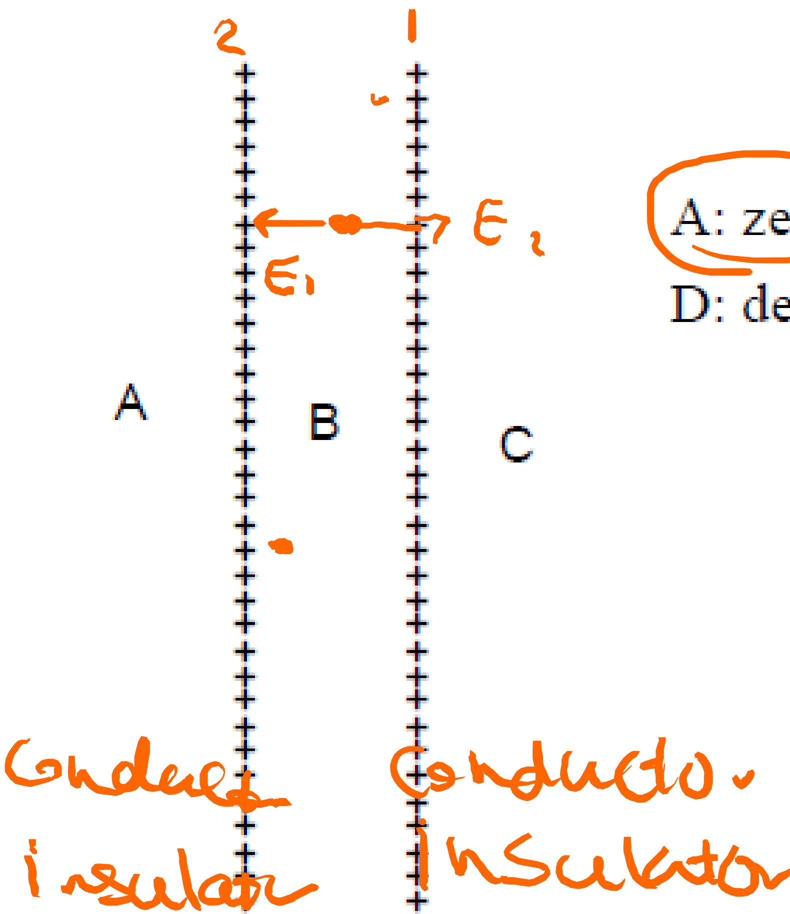
Homework 

Read appendix 1-chapter 23 posted on D2l.

TopHat Question

2

Two infinite planes are uniformly charged with the same charge per unit area σ (~~or η in your textbook~~). If <u>**one plane only**</u> were present, the E-field magnitude due to the **one** plane would be E. With both planes in place, the E-field magnitude in region B has magnitude:

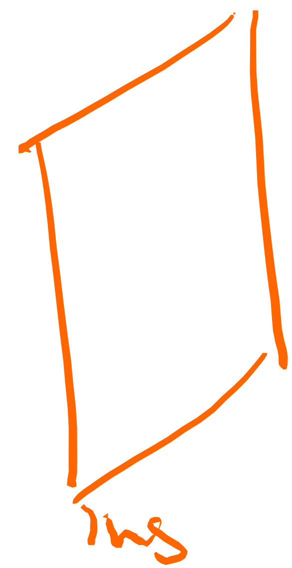


A: zero

B: E

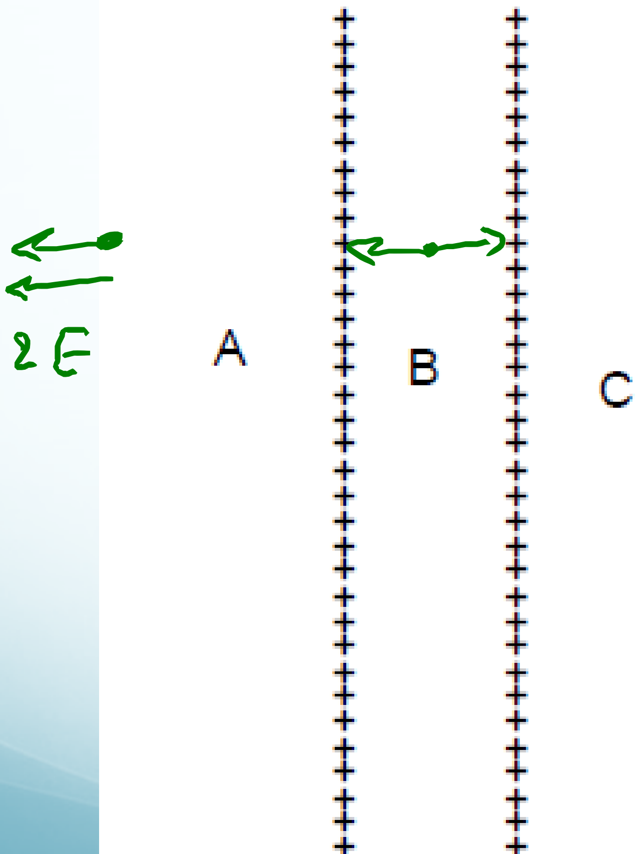
C: 2E

D: depends on exact position.



$$E = \frac{\sigma}{2\varepsilon_0}$$

Two infinite planes are uniformly charged with the same charge per unit area **σ** (~~or η in your textbook~~). If **one plane only** were present, the E-field magnitude due to the **one** plane would be E. With both planes in place, the E-field magnitude in region B has magnitude:



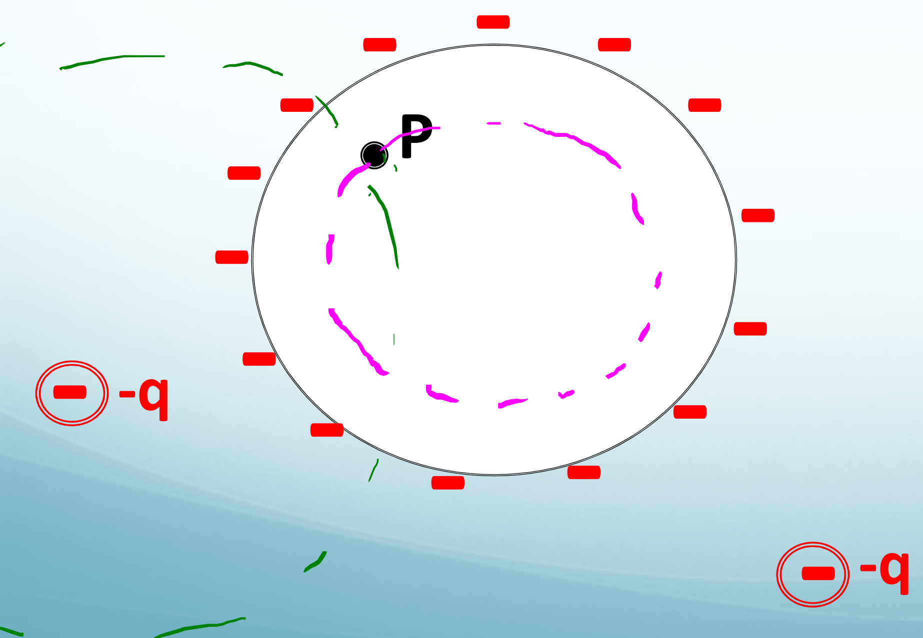
A: zero B: E C: 2E

D: depends on exact position.

TopHat Question 3

Negative charges are uniformly distributed on the surface of an insulating sphere with charge density σ. Two additional point charges -q are placed outside of the spherical charge distribution.

What is the magnitude of the E field at point P?



A: 0

B: non-zero

C: Not enough info given

Negative charges are uniformly distributed on the surface of an insulating sphere with charge density σ. Two additional point charges -q are placed outside of the spherical charge distribution.

What is the magnitude of the E field at point P?

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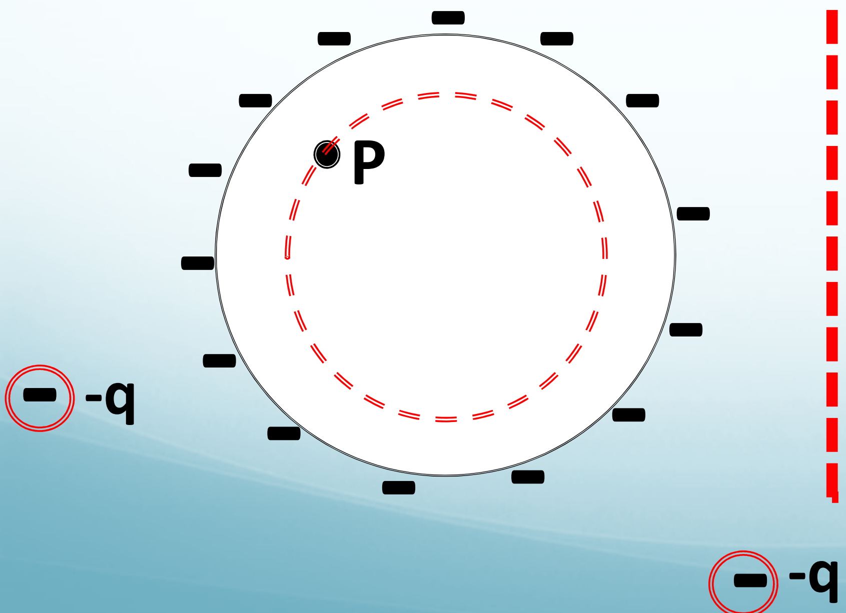
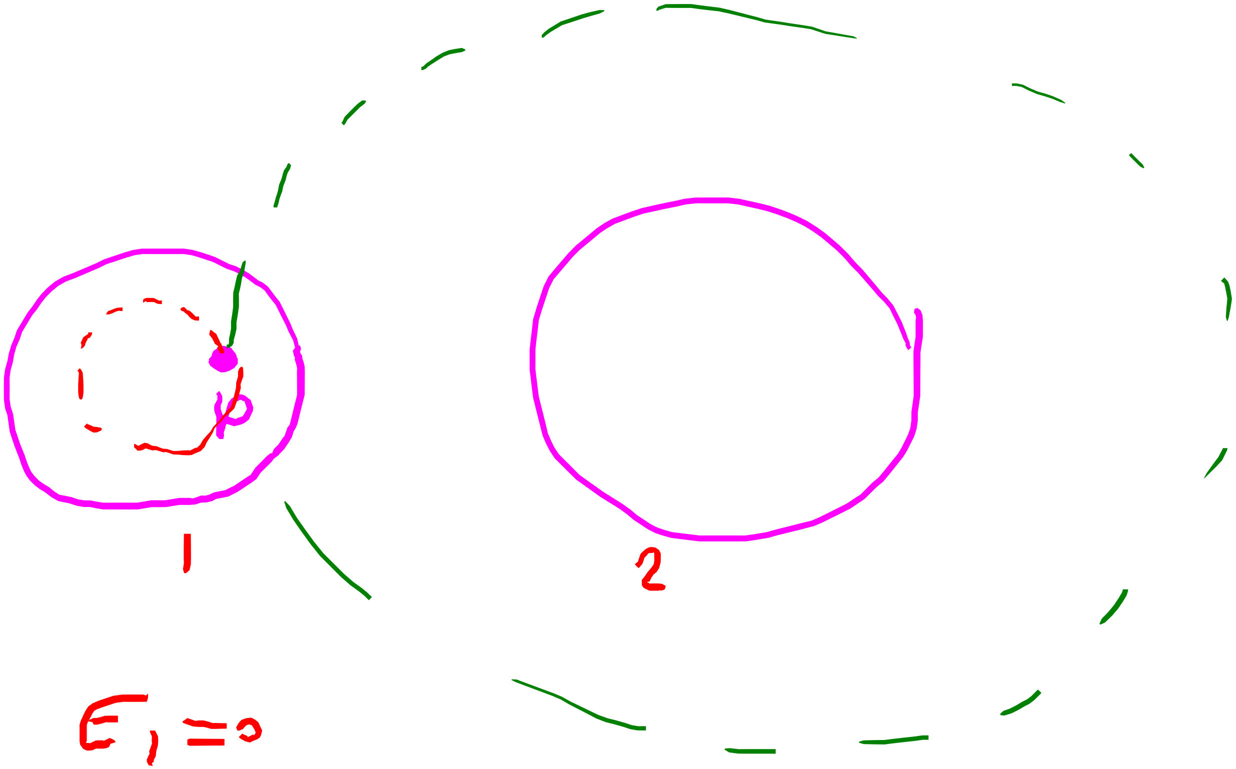


The two outside charges –q break the spherical symmetry.
If the 2 outside charges were **removed**,
the E field at point P would indeed be ZERO.

In order to find the E field at point P, we draw a Gaussian surface that goes through point P. If the 2 outside charges were removed (*just for a moment*), the spherical charge distribution would insure that the E field = 0 at point P (also anywhere inside the spherical charge distribution). THEN, once we **re-insert** the 2 outside charges –q, they would produce their own E field at point P...therefore resulting in a net E field at point P.

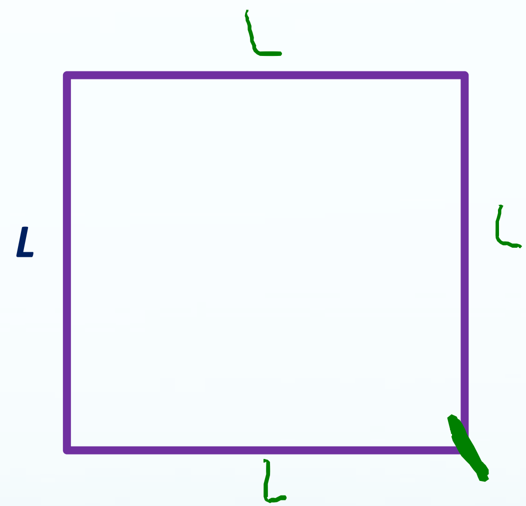


$E_1 = 0$

$E_2 = \cdot \; \text{—} \; \cdot$

TopHat Question 4

What is the charge of the insulating wire (the wire is reshaped to form a rectangle) with charge density $-\lambda$?



$$\lambda = \frac{q}{L}$$

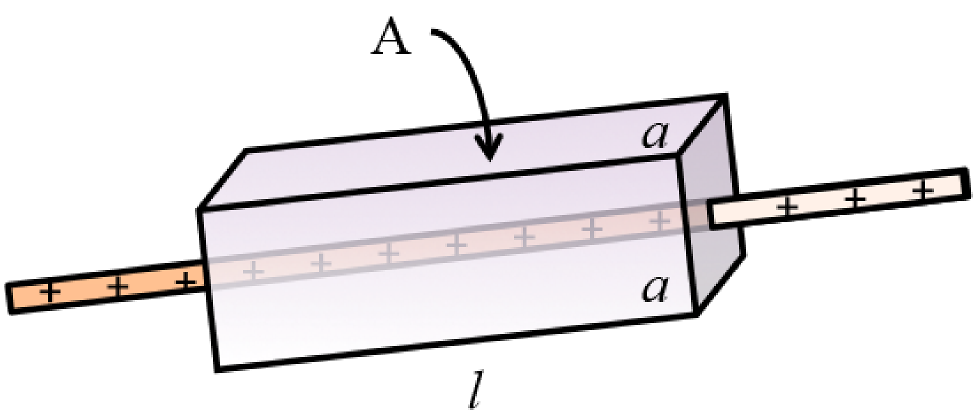
$$q = \lambda L$$

$$4 \lambda L$$

A) $L^2\lambda$
B) $-L^2\lambda$
C) $4L\lambda$
D) $-4L\lambda$

Field of a line charge

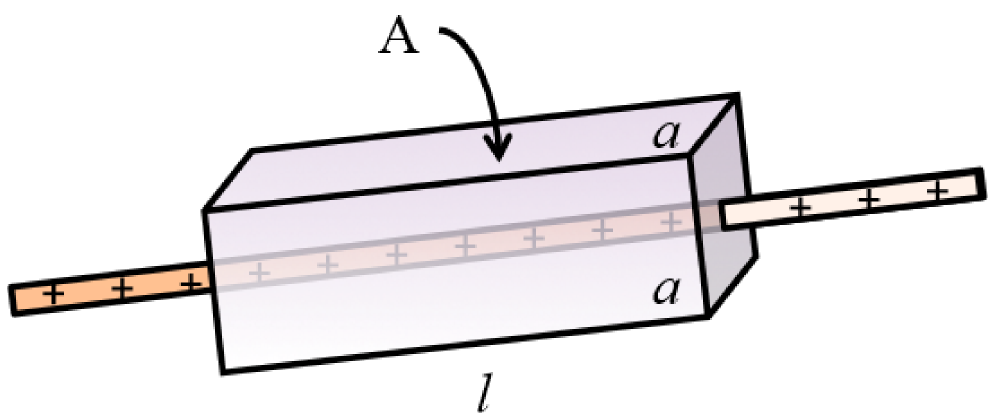
Consider an infinitely long, positively charged rod of linear charge density λ. How large is the flux through side A of the box? Suppose the values for l, a and λ are given.



Field of a line charge

- Consider an infinitely long, positively charged rod of linear charge density λ. How large is the flux through side A of the box? Suppose the values for l, a and λ are given.
- Gauss' law tells us that the total electric flux only depends on the enclosed charge – not the shape of the (closed) Gaussian surface:

  $\Phi_{tot} = Q_{encl}/\varepsilon_0 = \lambda l/\varepsilon_0$

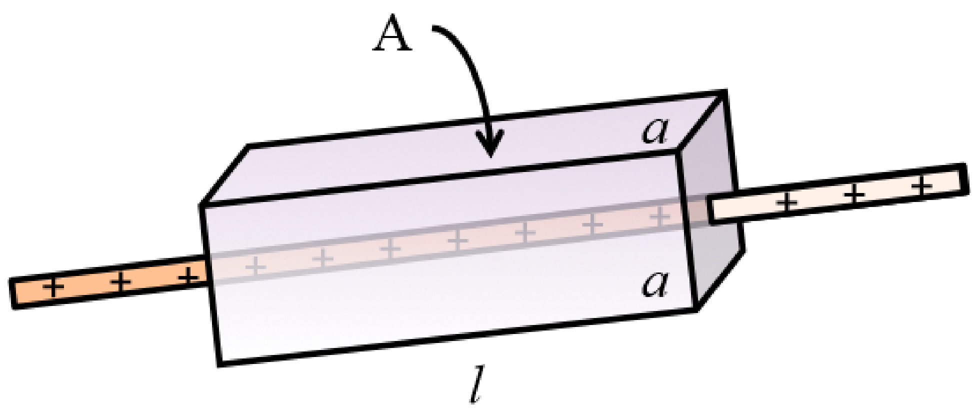


Field of a line charge

- Consider an infinitely long, positively charged rod of linear charge density λ. How large is the flux through side A of the box? Suppose the values for l, a and λ are given.
- Gauss' law tells us that the total electric flux only depends on the enclosed charge – not the shape of the (closed) Gaussian surface:

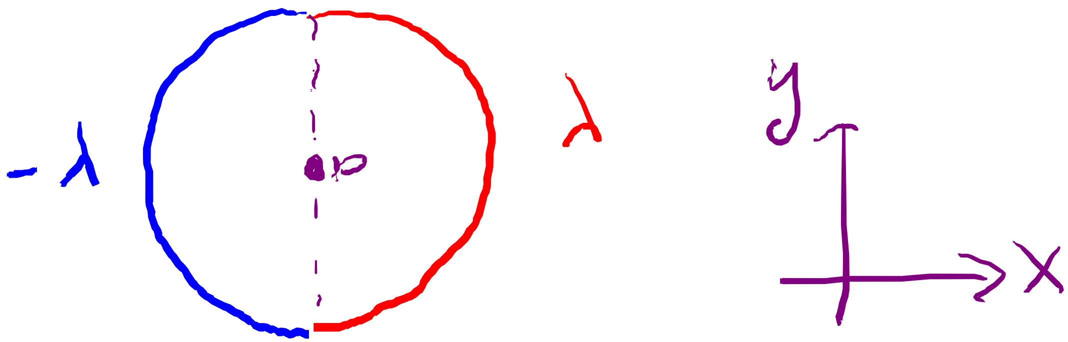
  $\Phi_{tot} = Q_{encl}/\varepsilon_0 = \lambda l/\varepsilon_0$
- The total flux must be equally partitioned into flux through the four surfaces whose area vectors are parallel to the electric field.

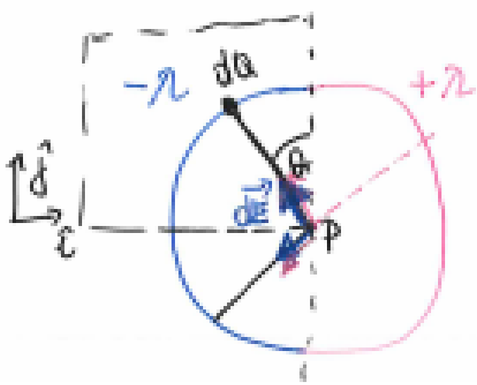
  Hence, $\Phi_A = \lambda l/4\varepsilon_0$



Practice question:

Find the field at point p at the centre of a ring of charge composed of two oppositely charged half rings.

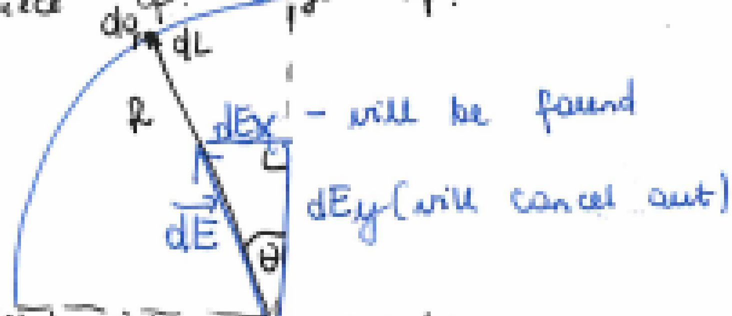




Find $\vec{E}$ at P.

Radius, R

1. Cut the distribution into a bunch of tiny pieces each with charge dQ
2. Look for a symetry → can use 1/4 circle (arc)
   Find $\vec{E}$ and multiply it by 4.
3. Calculate the magnitude of E-field due to ARBITRARY piece of charge dq.



4. Decompose field into components

$$\frac{dE_x}{dE} = \sin\theta \qquad \frac{dE_y}{dE} = \cos\theta$$

$$dE_x = dE\sin\theta$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{dq}{R^2}\cdot\sin\theta$$

$$dE_y = \frac{1}{4\pi\varepsilon_0}\frac{dq}{R^2}\cos\theta$$

⟹ not needed due to symetry

5. For each non-zero component, sum up all pieces dQ by integrating over the whole charge distribution

$$\int dE_x = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{R^2}\sin\theta$$

6. Express dQ in terms of a variable to be integrated over using linear / surface / volume density

$$\int dE_x = \frac{1}{4\pi\varepsilon_0}\int \frac{\lambda\, dL}{R^2}\cdot \sin\theta = \frac{1}{4\pi\varepsilon_0}\int \frac{\lambda \cdot \cancel{R} \sin\theta\, d\theta}{R^{\cancel{2}}}$$

$$dL = R\,d\theta$$

$\pi/2$ - quarter of the circle

$$E_x = \frac{\lambda}{4\pi\varepsilon_0 R}\int_0^{\pi/2} \sin\theta\, d\theta \qquad \int \sin x\, dx = -\cos x$$

$$E_x = \frac{\lambda}{4\pi\varepsilon_0 R}\left[-\cos\theta\right]_0^{\pi/2} = \frac{\lambda}{4\pi\varepsilon_0 R}\left[-\underbrace{\cos\left(\tfrac{\pi}{2}\right)}_{=0} - \left(-\underbrace{\cos 0^\circ}_{=1}\right)\right]$$

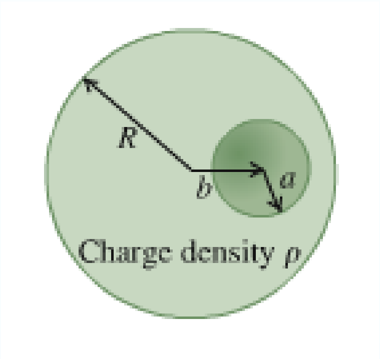
$$= \frac{\lambda}{4\pi\varepsilon_0 R}$$

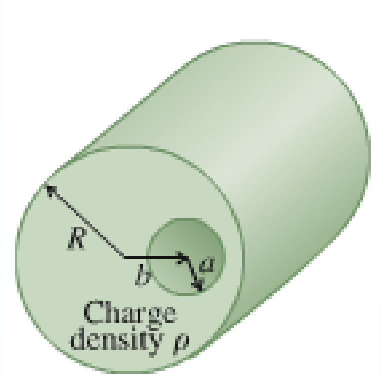
Total electric field

$$E_{net} = 4E_x = \frac{\cancel{4}\lambda}{\cancel{4}\pi\varepsilon_0 R} = \frac{\lambda}{4\pi\varepsilon_0 R^2}$$

$$\vec{E}_{net} = \frac{\lambda}{4\pi\varepsilon_0 R}\,(-\hat{i})$$

Superposition



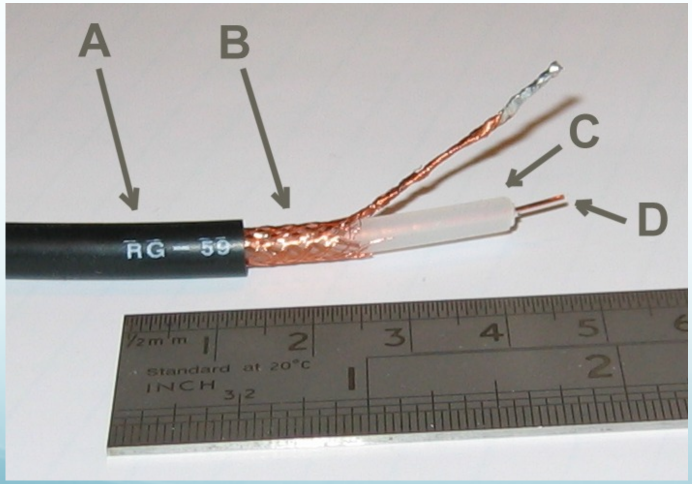


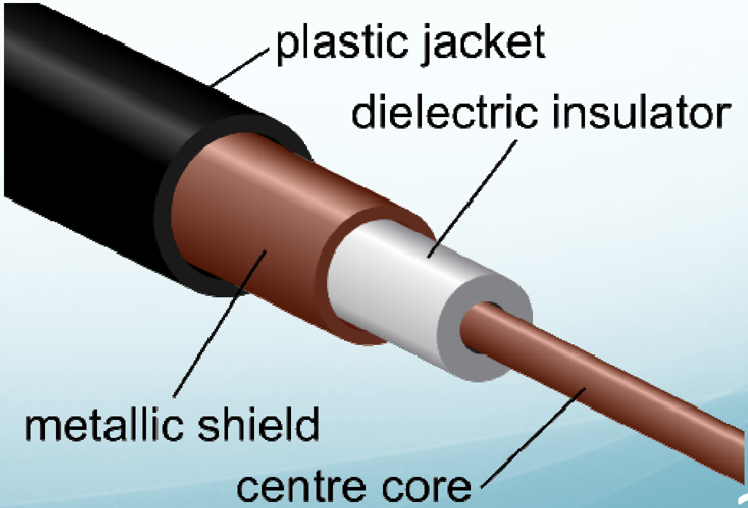
$$E_{total} = E_1 + E_2$$

Exercise: Coaxial Cable
Study appendix 1-Chapter 23

Assume there is a charge **+Q** on the centre core and **-Q** on the metallic shield. (Ignore the dielectric insulator and plastic jacket.)

Find the electric field outside the metallic shield (**E_2**) and just outside the central core (**E_1**).





This section we talked about:

Chapter 23 & Midterm Review

See you on Friday