Monday March 13, 2017

Last time:

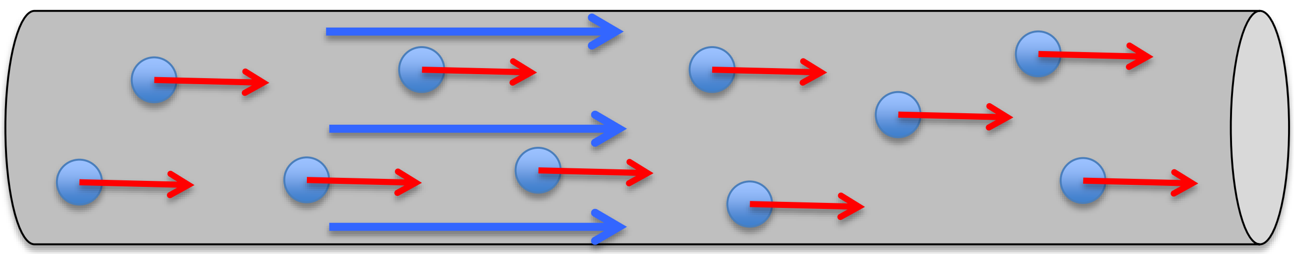
- Calculation of charge
- Applications of dielectrics and capacitors
- Group activity

Today:

- Electric current: a microscopic picture
- Current density (a vector) vs current (a scalar)
- Electric fields in conductors and electron drift speed
- Resistance as geometric quantity
- Resistors in series
- Resistors in parallel

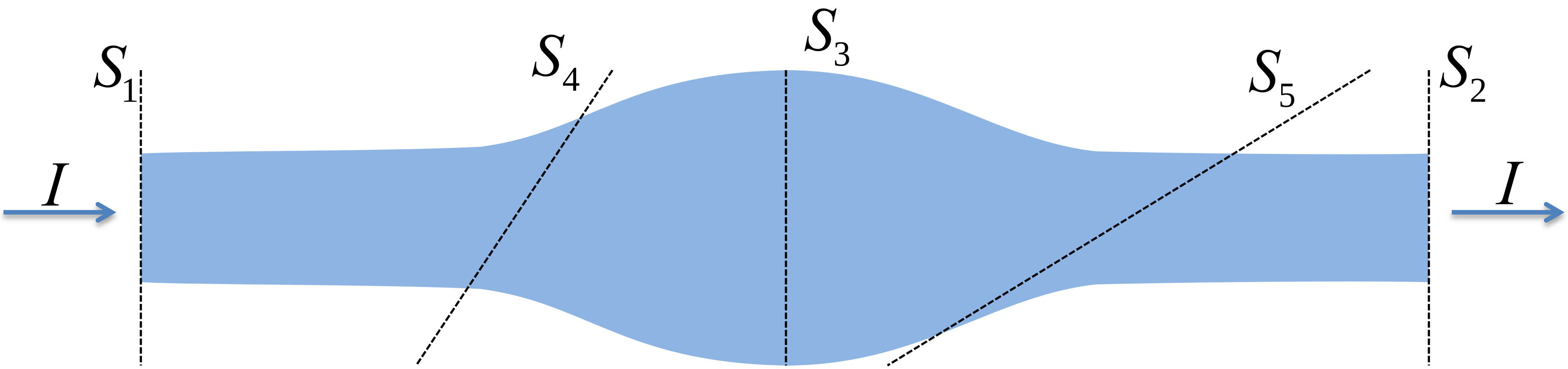
Where we're going

We will be using what we have been building up to talk about **moving charges** in electric circuits. This is no longer electrostatic equilibrium, so **conductors are allowed to have non-zero electric field inside** (this is what causes the charges to move).



First, we will take a closer look at what happens inside conductors and use this to define what an electric current is

Definition of current



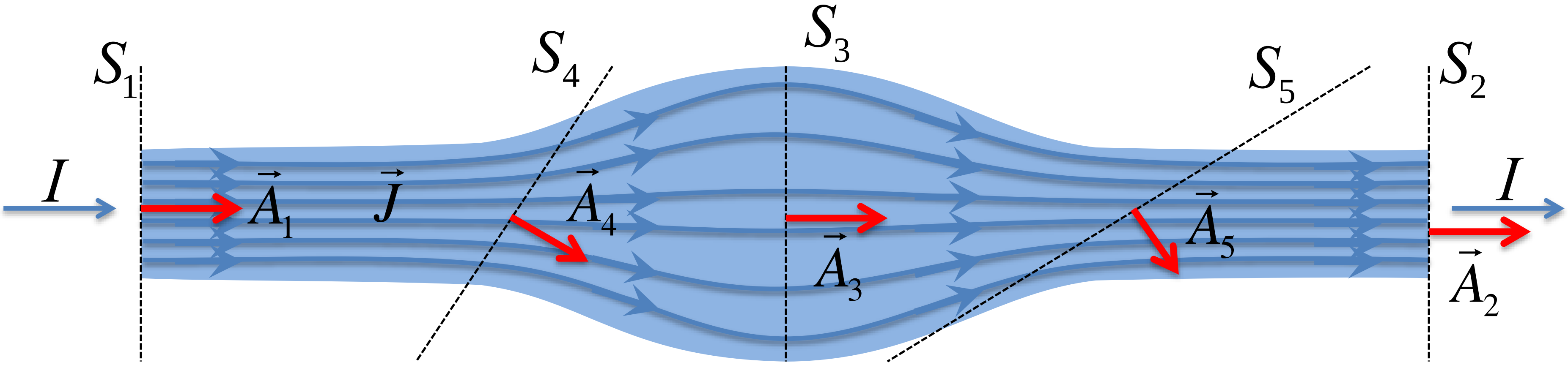
Total amount of charge flowing past this surface in a time Δt

$$\boxed{I = \frac{dq}{dt}}$$

Total amount of charge flowing past this surface in the same time Δt

Total amount of charge flowing through **ANY** surface in a time Δt must be constant, otherwise charges would begin to accumulate. **Current in a wire is constant**.

Current and Current Density



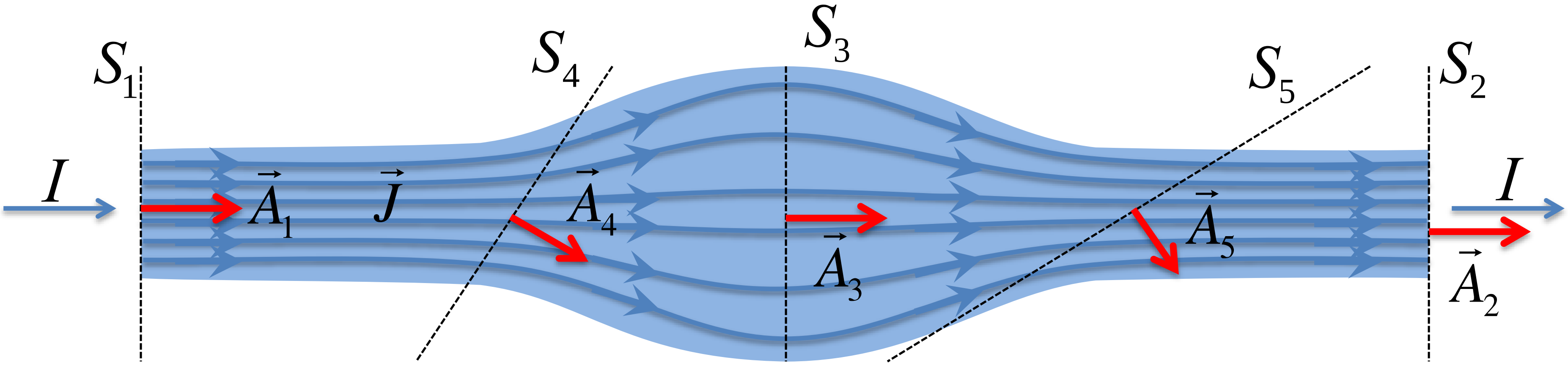
Total amount of charge flowing through **ANY** surface in a time Δt must be constant. This should be reminiscent of **FLUX**.

$$I = \oint_S \vec{J} \cdot d\vec{A}$$

The current in a wire is the flux of charge carriers (i.e. electrons) through a surface.

Since the current is constant, the flux through any cross-sectional surface must be the same.

Current and Current Density

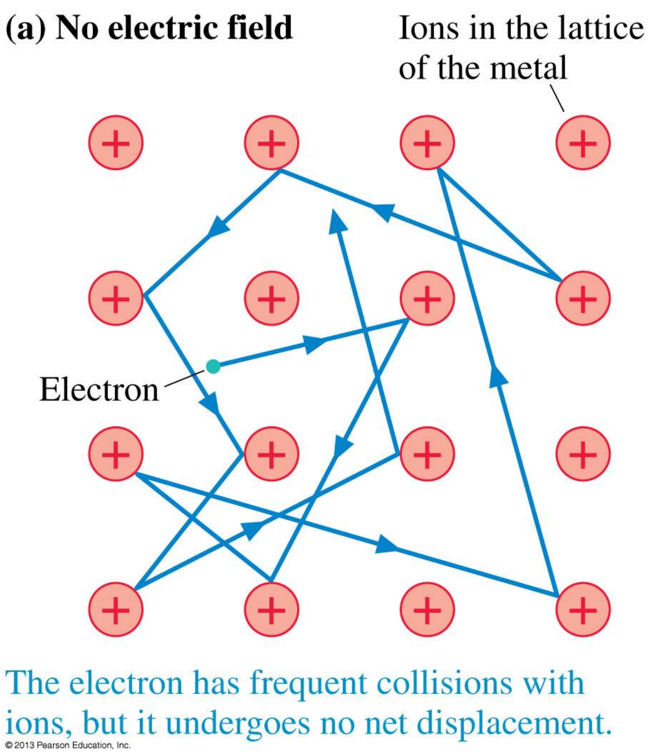


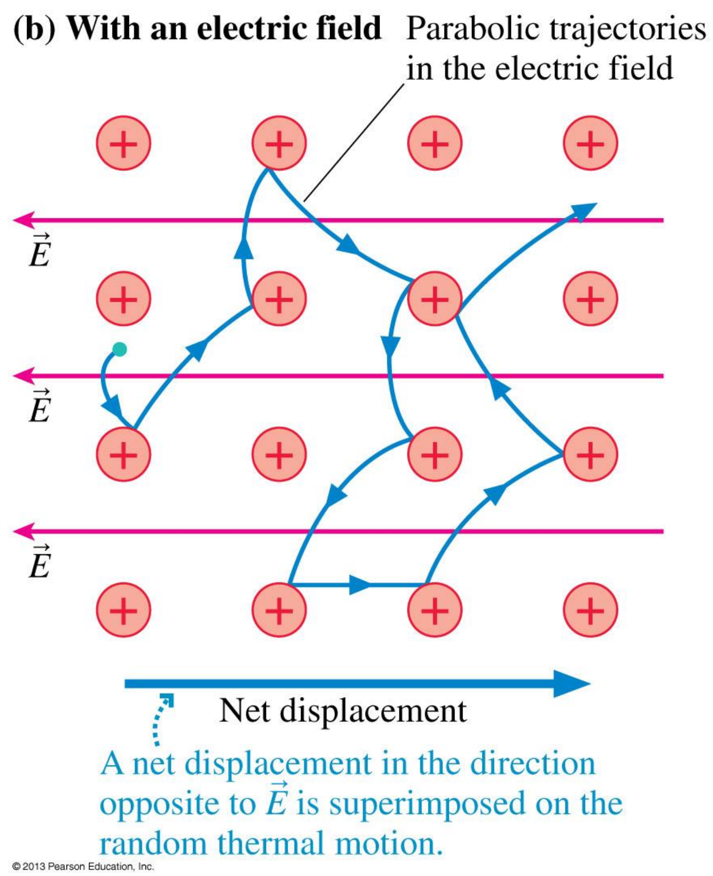
We call $\vec{J}$ the **current density**. It encodes information about:

- The density of conduction electrons in the conductor
- The net velocity of these conduction electrons

The current I is then interpreted as the number of charges passing through a surface in a specified direction. Note: current density is a vector, current is a scalar.

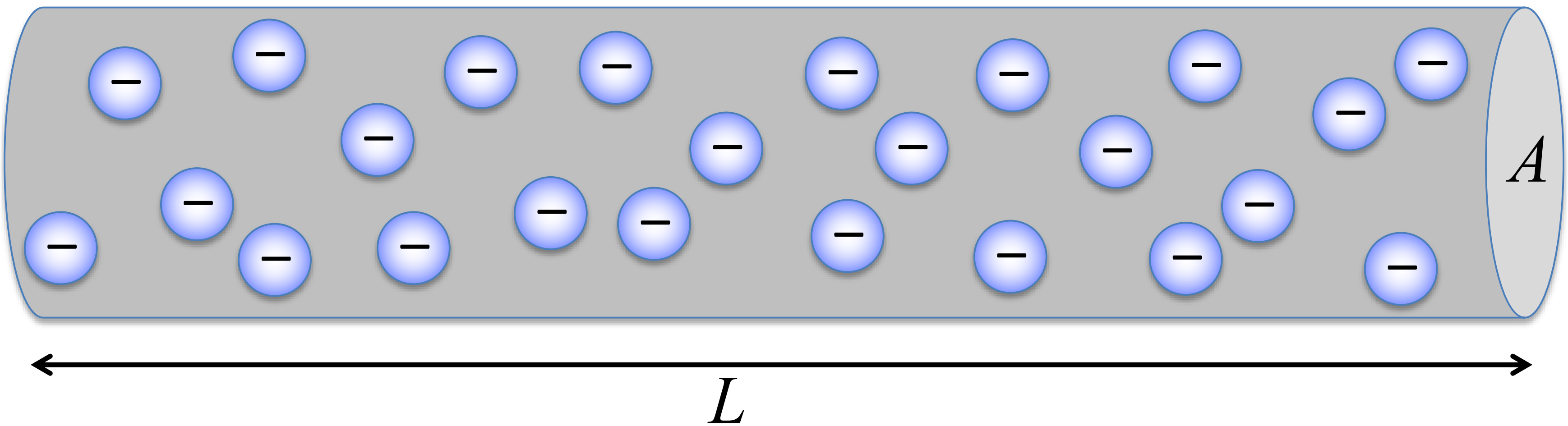
Inside a conductor





Net result: electrons move at an average net "drift speed" v_d

Current Density



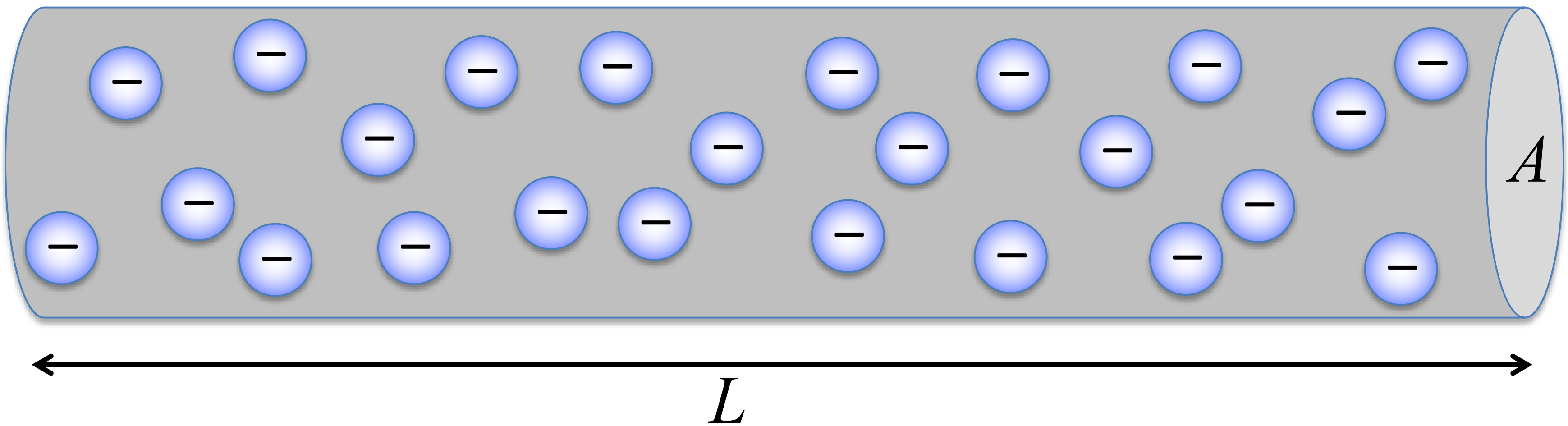
If the volume density of conduction electrons is n_e, then the amount of charge contained in a length L of the wire is

$$q = n(AL)e$$

The time it takes each charge to travel a distance L is $t = L/v_d$, so the current is

$$i = \frac{q}{t} = \frac{n(AL)e}{L/v_d} = nAev_d$$

Current Density

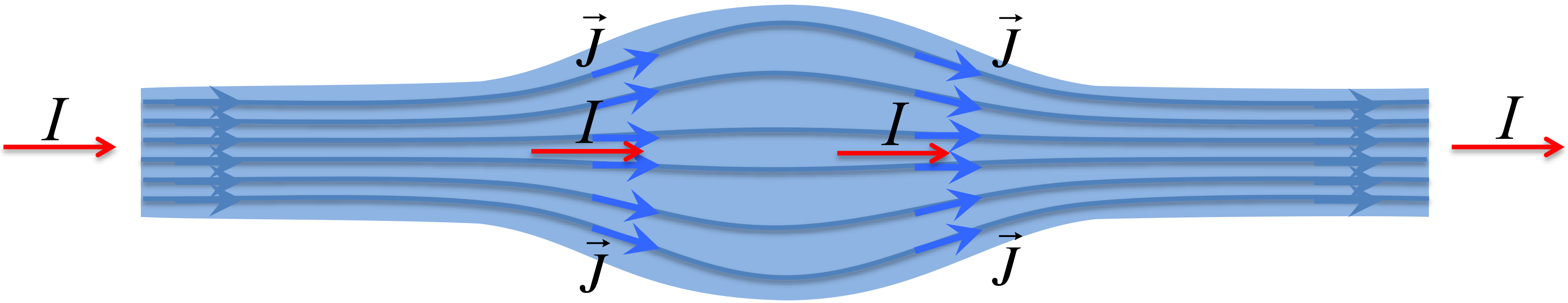


$$i = nAev_d = JA$$

The current density is then seen to be given by the **charge density** ne and the **drift velocity** (average velocity of the electrons)

$$\vec{J} = ne\vec{v}_d$$

Current and Current Density



Because it is a vector, $\vec{J}$ is always in the direction of the "streamlines" of the electrons at any given location in the wire.

The current I, on the other hand, is a scalar and so it just has a magnitude. The direction we typically associate with it is the average displacement of all the charges in the wire, and so always points along the general direction of the wire.

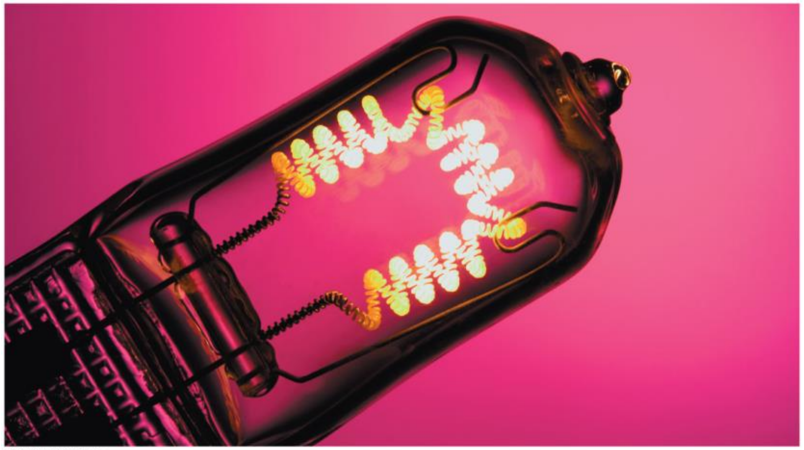
TopHat questions

Resistance

Resistance is a property of conductors that are not ideal:

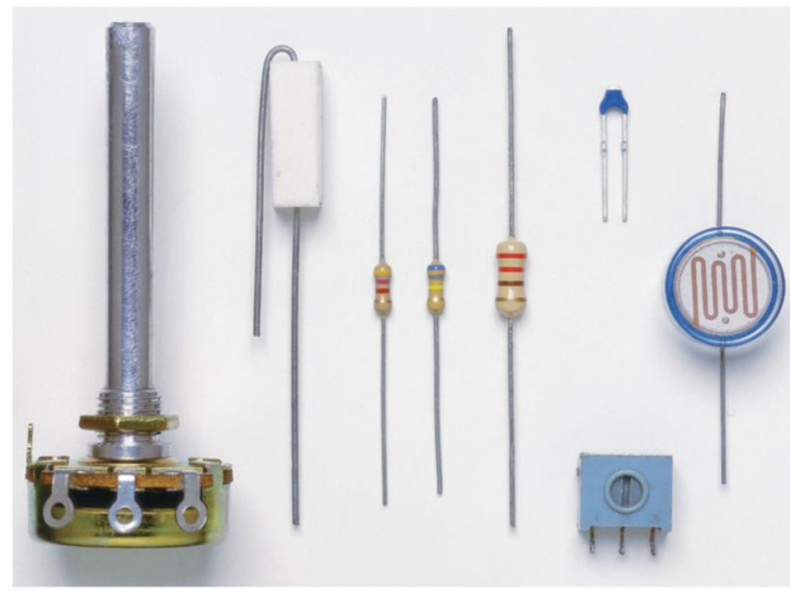
- Electrons have frequent collisions with atomic nuclei. At constant temperature, this process is in thermal equilibrium.
- When a voltage difference is created across the conductor, this accelerates the electrons, making their collisions more energetic.
- This gets dissipated as heat inside the metal

Tungsten filament:



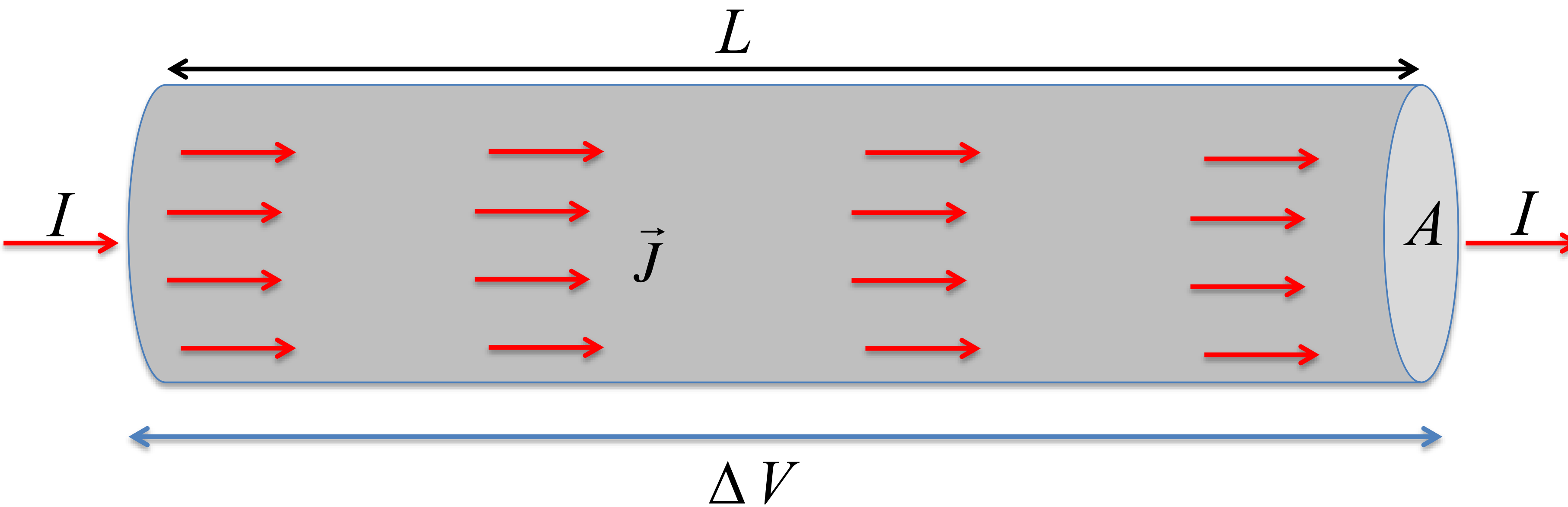
Resistors

A resistor is any circuit element that dissipates energy. Light bulbs are the classic example, but there are others:



How much energy is dissipated by a given resistor is encoded in a property called its resistance *R*. The resistance is dependent on the particular material used as well as the geometry.

How can we quantify this



Ohm’s Law states that the current density inside the conductor is related to the electric field causing the charges to move via

$$\vec{E} = \rho\vec{J}$$

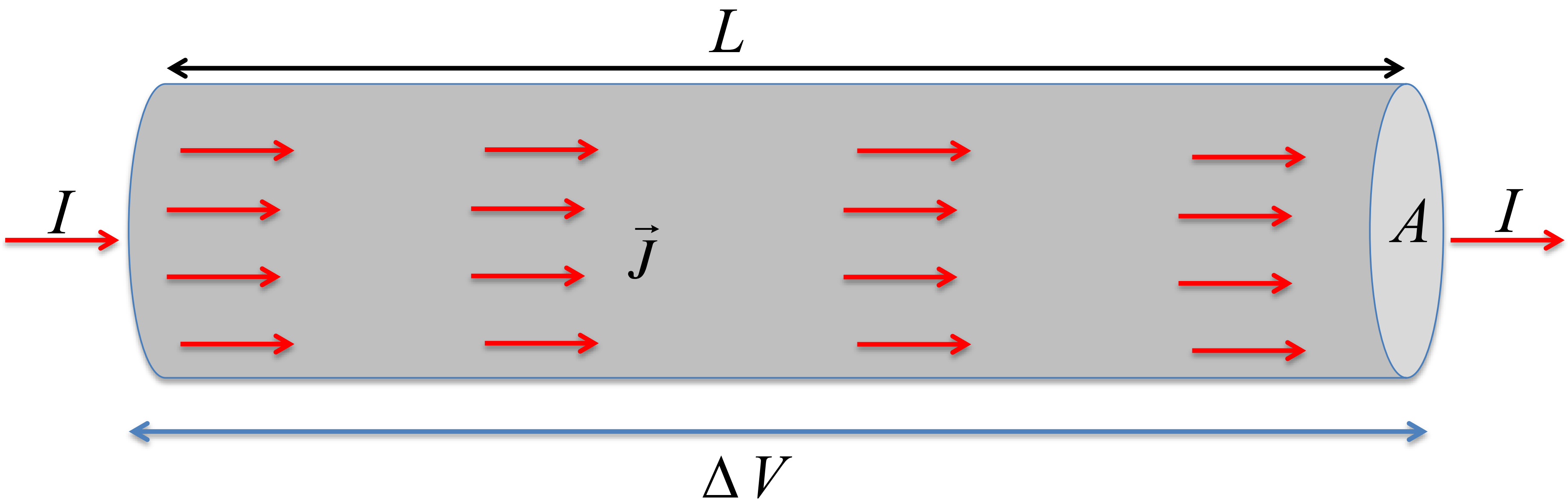
The resistivity is a physical property of the material that makes up the resistor

$$r = \frac{E}{J} = \frac{\Delta V/L}{I/A}$$

Using the resistivity, we can define a geometric quantity of the resistor:

$$r\frac{L}{A} = \frac{\Delta V}{I}$$

How can we quantify this



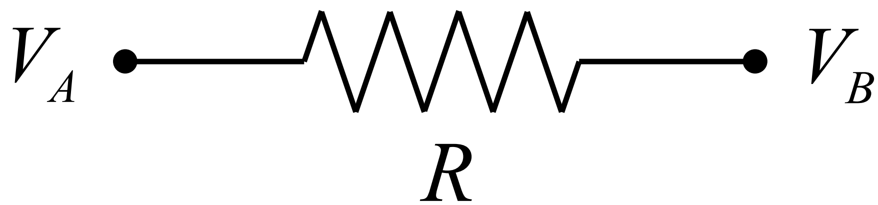
We introduce the resistance, which is dependent on ρ of the material and on the geometry of the resistor

$$\rho \frac{L}{A} \propto R \qquad \rho \frac{L}{A} = \frac{\Delta V}{I} \qquad R = \frac{\Delta V}{I}$$

This gives us the familiar form of Ohm's Law: $\boxed{\Delta V = IR}$

Ohm's Law

When a voltage difference Δ*V* is applied across a resistor *R*, the voltage difference causes electrons to flow through the resistor



This flow of electrons is the electric current *I*. These quantities are related by Ohm's Law:

$$\Delta V = IR$$

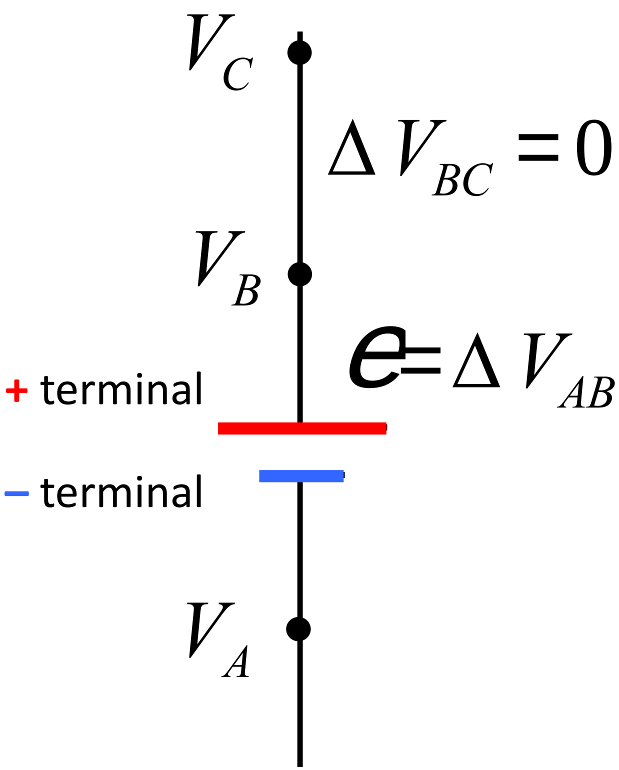
Current convention: the flow of positive charge (opposite the flow of negative charge)

$$(+e)(+\vec{v}) = (-e)(-\vec{v})$$

Both define a current I pointing to the right, so it is more convenient to think in terms of + charges moving

Ideal wires & batteries

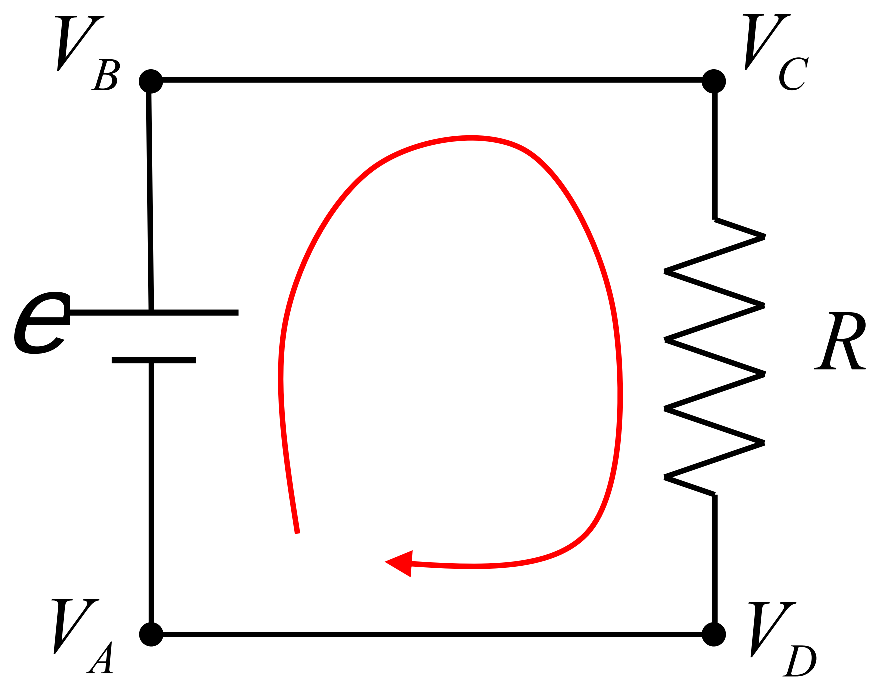
Real wires always have some **resistance** to them, but it is usually **small enough** that we can **ignore** it.
In this class we will <u>usually</u> treat wires as **ideal**, meaning $\Delta V = 0$ across any wire segment even if there is a current flowing.



A battery is any **source** that supplies a **voltage difference** in an electric circuit. The voltage is either specified by *V* or by the symbol e which stands for **electromotive force** (EMF)
Real batteries also have a resistance to them and we will see later how to account for this.

A Basic Circuit

The simplest circuit has an ideal battery, ideal wires, and a single resistor.



Kirchhoff's Loop Rule:
The sum of the voltage differences around a closed loop in a circuit must be zero.
(conservation of energy)

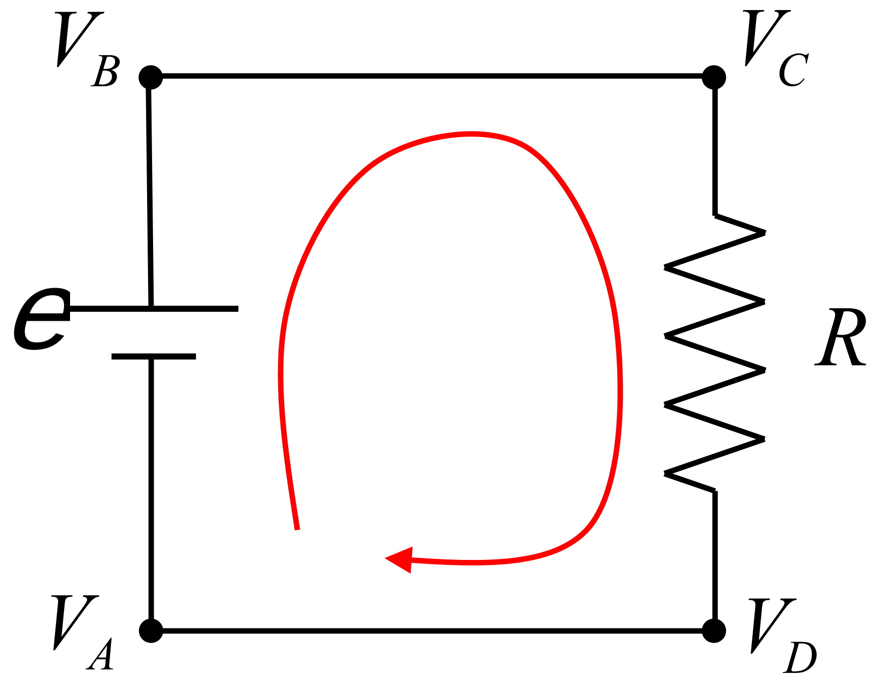
$$\boxed{\Delta V_{AB} + \Delta V_{BC} + \Delta V_{CD} + \Delta V_{DA} = 0}$$

$$(V_B - V_A) + (V_C - V_B) + (V_D - V_C) + (V_A - V_D) = 0$$

A Basic Circuit

The voltage across a resistor is **negative** if you are going around the loop in the **direction of the flow of current**.

Current flows **from the negative** terminal **to the positive** terminal



ideal wires

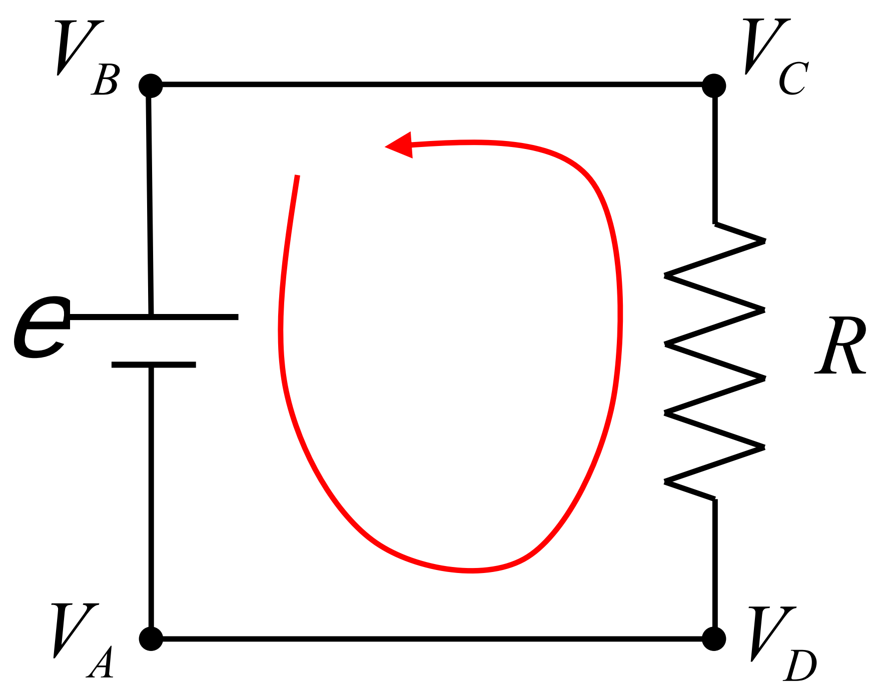
$$\Delta V_{AB} + \cancel{\Delta V_{BC}} + \Delta V_{CD} + \cancel{\Delta V_{DA}} = 0$$

$$e - IR = 0$$

Ohm's Law

A Basic Circuit

The voltage across a resistor is **positive** if you are going around the loop in the **opposite direction of the flow of current**. Voltage across a battery is **negative** going **from positive to negative**



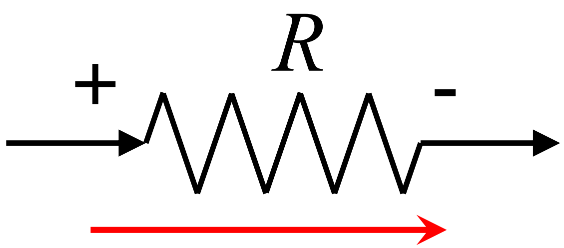
ideal wires

$$\Delta V_{BA} + \cancel{\Delta V_{AD}} + \Delta V_{DC} + \cancel{\Delta V_{CB}} = 0$$

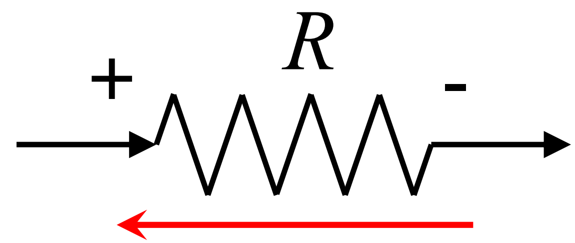
$$-e + IR = 0$$

Same as before

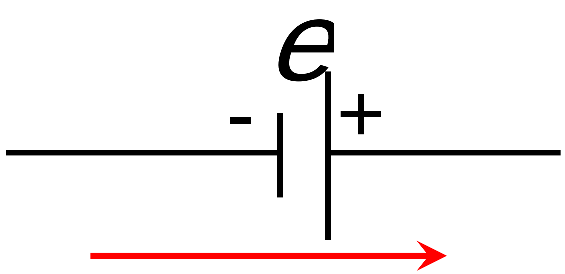
Kirchhoff’s Loop Rule



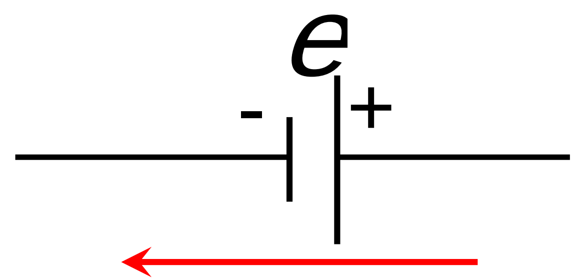
Higher to lower V: $\Delta V = -IR$



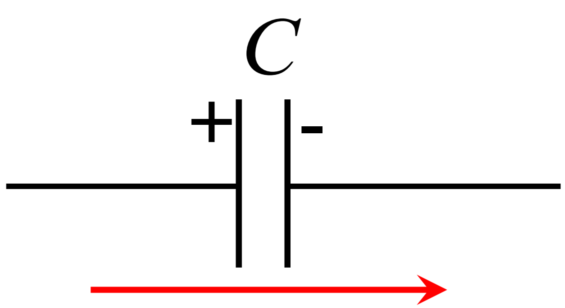
Lower to higher V: $\Delta V = +IR$



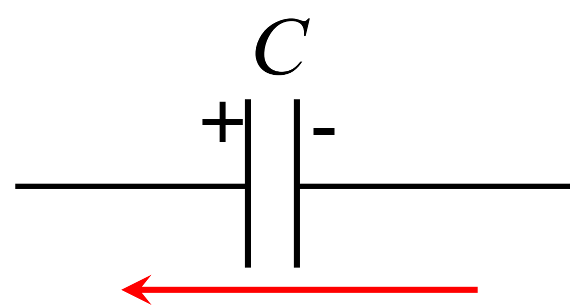
Lower to higher V: $\Delta V = +\varepsilon$



Higher to lower V: $\Delta V = -\varepsilon$



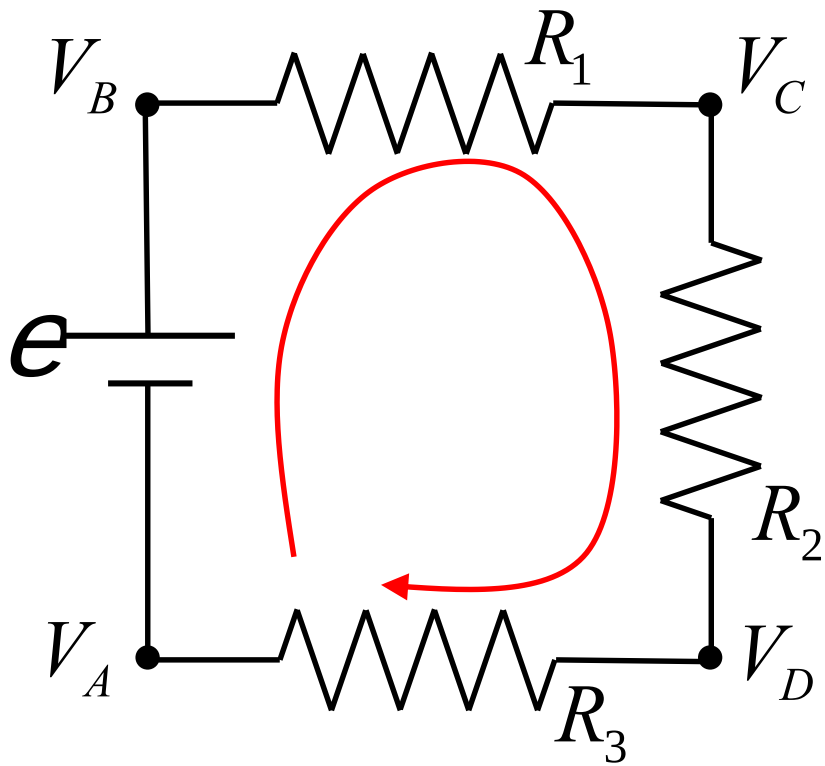
Higher to lower V: $\Delta V = -\dfrac{Q}{C}$



Lower to higher V: $\Delta V = +\dfrac{Q}{C}$

Resistors in Series

A slightly more complicated circuit has multiple resistors in series



Kirchhoff's Loop Rule:

$$\Delta V_{AB} + \Delta V_{BC} + \Delta V_{CD} + \Delta V_{DA} = 0$$

Current through each R is same

$$\mathcal{E} - IR_1 - IR_2 - IR_3 = 0$$

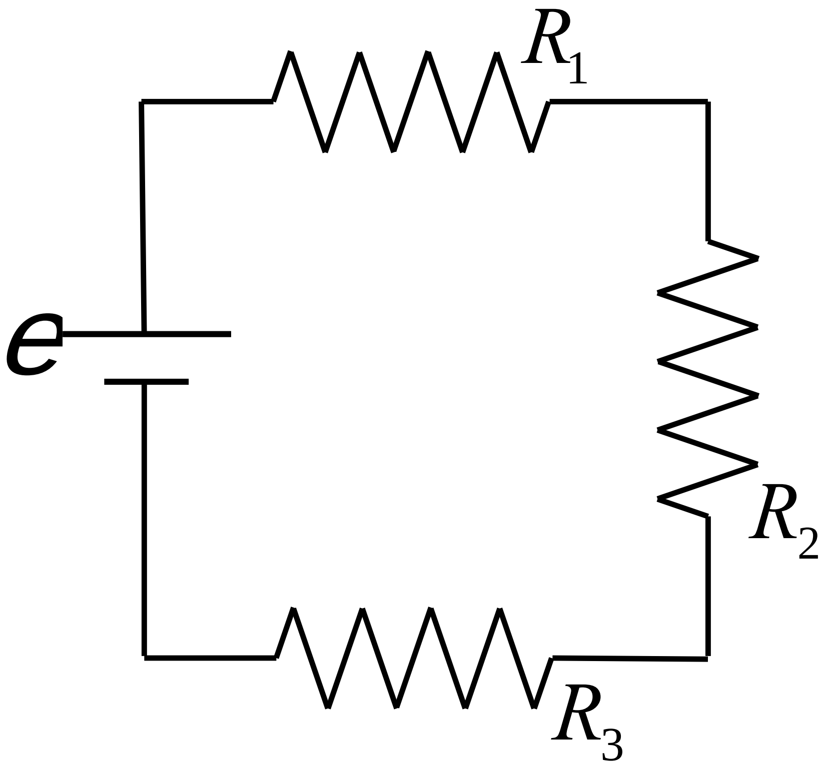
Rewrite this as

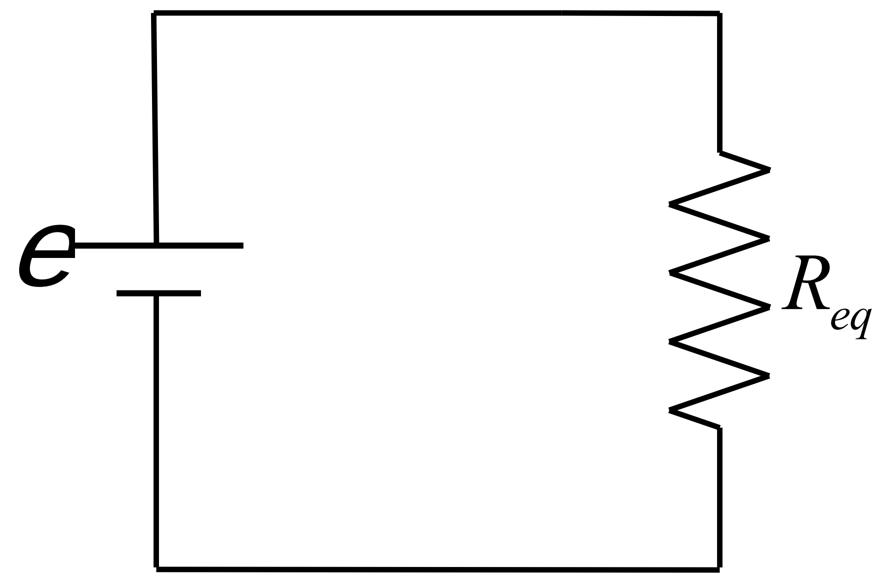
$$\mathcal{E} - I\left(R_1 + R_2 + R_3\right) = 0$$

Define an equivalent resistance

$$\mathcal{E} - IR_{eq} = 0$$

Resistors in Series



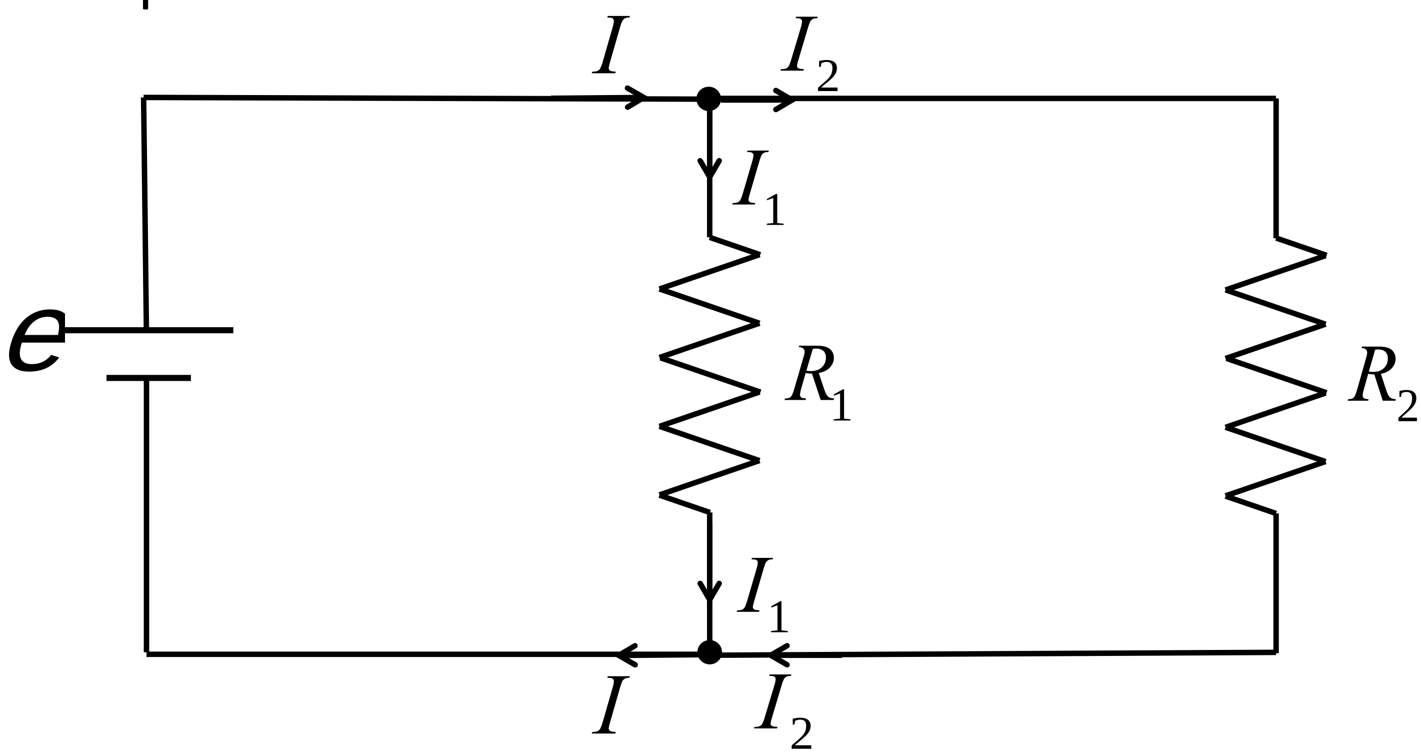


Resistors in series act like a single equivalent resistor:

$$R_{eq} = R_1 + R_2 + R_3$$

Resistors in Parallel and Kirchhoff's junction rule

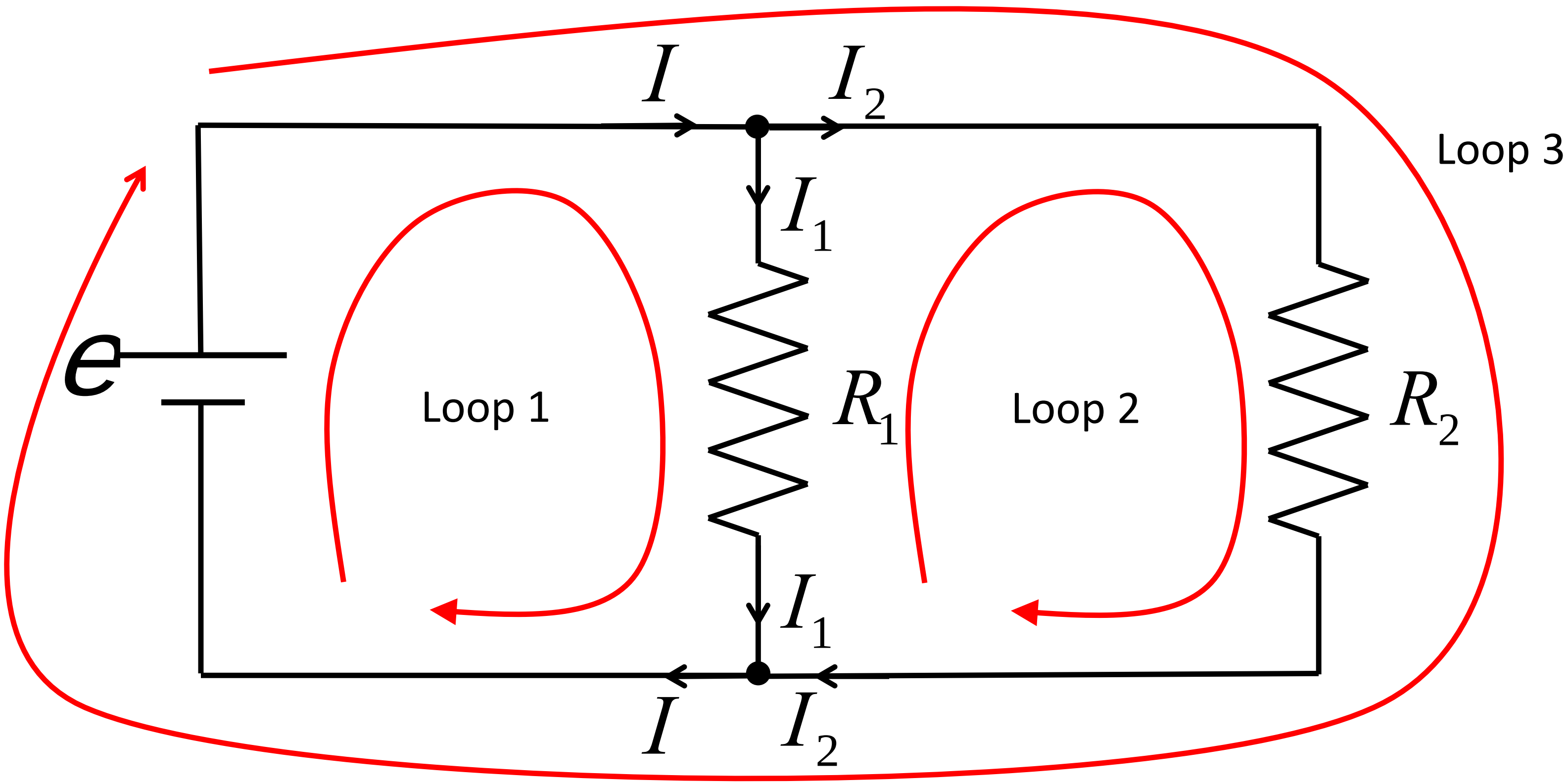
A slightly more complicated circuit has multiple branches with resistors in parallel



Current is the flow of charges. Charge has to be conserved.

Current into junction = current out of junction $\boxed{I = I_1 + I_2}$

Resistors in Parallel



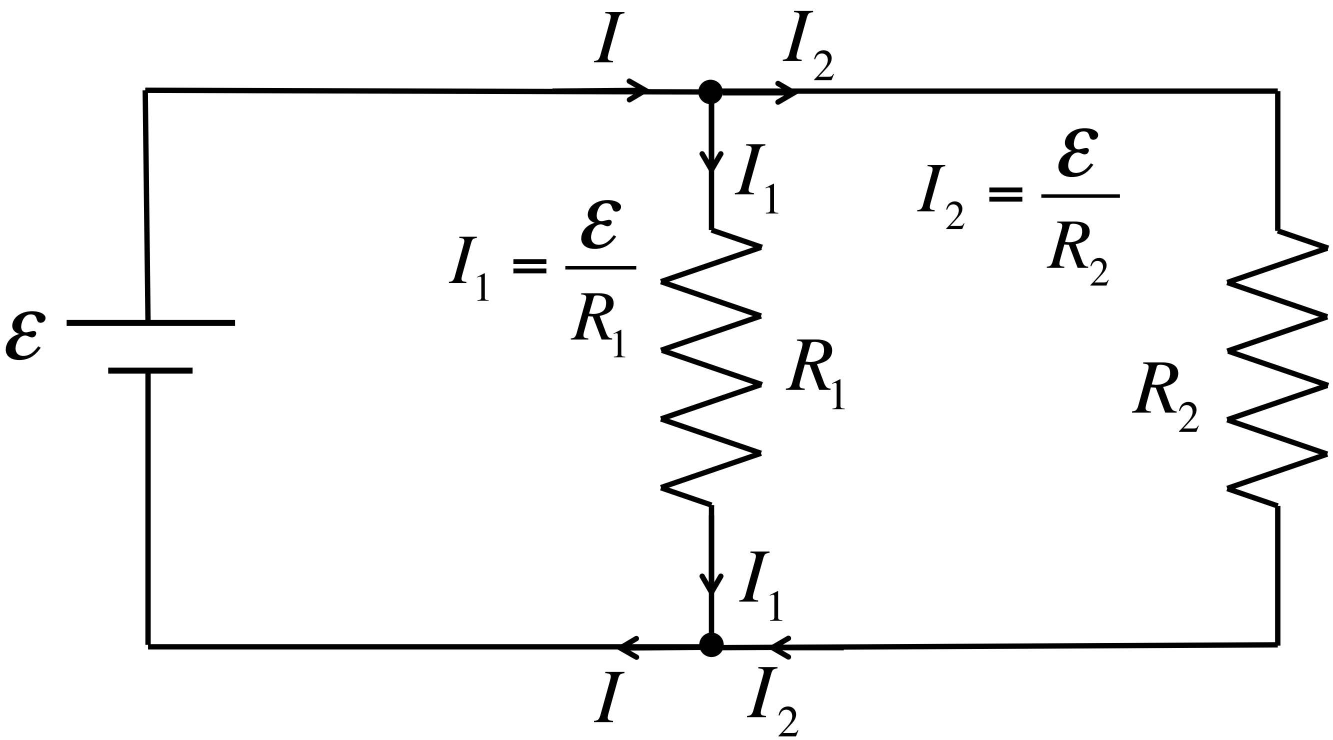
Loop 1: $e - I_1R_1 = 0$

Loop 2: $I_1R_1 - I_2R_2 = 0$

Loop 3: $e - I_2R_2 = 0$

$$I_1 = \frac{e}{R_1} \qquad I_2 = \frac{e}{R_2}$$

Resistors in Parallel



$$I = I_1 + I_2$$

$$I = \frac{\mathcal{E}}{R_1} + \frac{\mathcal{E}}{R_2}$$

$$= \mathcal{E}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$= \frac{\mathcal{E}}{R_{eq}}$$

Resistors in parallel: $R_{eq} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1}$

Summary of Resistors

Ohm's Law

$$\Delta V_R = IR$$

Resistors in Series: have the same current running through them

$$R_{eq} = R_1 + R_2 + \ldots + R_N$$

Resistors in Parallel: have the same voltage across them

$$R_{eq} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_N} \right)^{-1}$$