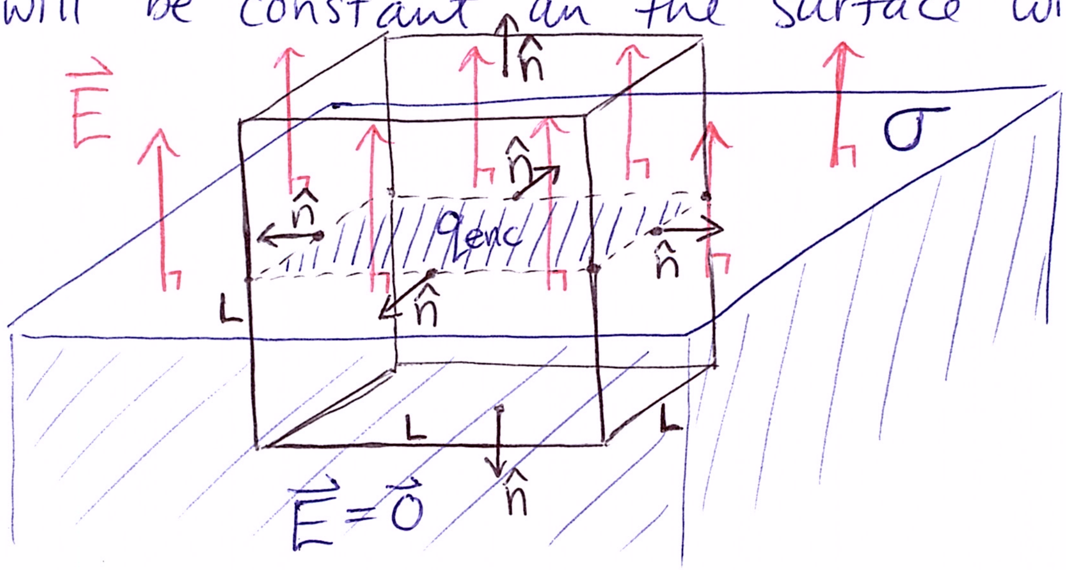
Electric field near a conductor

In general a conductor can have a highly non-uniform charge distribution across its surface, but for a small enough area, σ will be constant an the surface will be flat.



This represents a small chunk of the conductor. $\vec{E} = \vec{0}$ inside and $\vec{E}$ is perpendicular to the surface outside, and has constant magnitude at constant height.

Choose a Gaussian surface half inside and half outside the conductor.

$$\underbrace{\oint \vec{E} \cdot d\vec{A}} = \frac{q_{enc}}{\epsilon_0}$$

$$\underbrace{\int_{bottom} \vec{E} \cdot d\vec{A}}_{\substack{\text{zero because} \\ \vec{E} = \vec{0} \text{ inside}}} + \int_{top} \vec{E} \cdot d\vec{A} + \underbrace{4 \int_{side} \vec{E} \cdot d\vec{A}}_{\substack{\text{zero because} \\ \vec{E} \perp d\vec{A} \text{ on sides}}} = EA$$

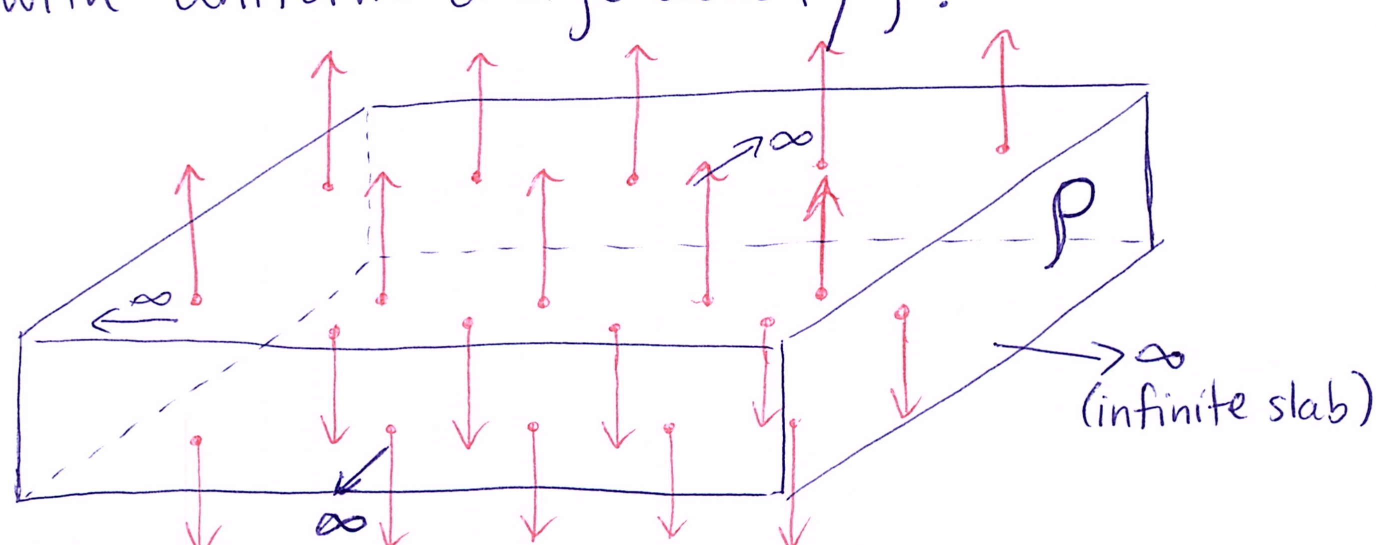
q_{enc} comes from the charge contained inside the Gaussian surface: $q_{enc} = \sigma A$

The top area is L^2 and the area for the enclosed charge is also L^2

$$\Rightarrow \quad E\cancel{A} = \frac{\sigma \cancel{A}}{\epsilon_0} \qquad \boxed{E = \frac{\sigma}{\epsilon_0}}$$

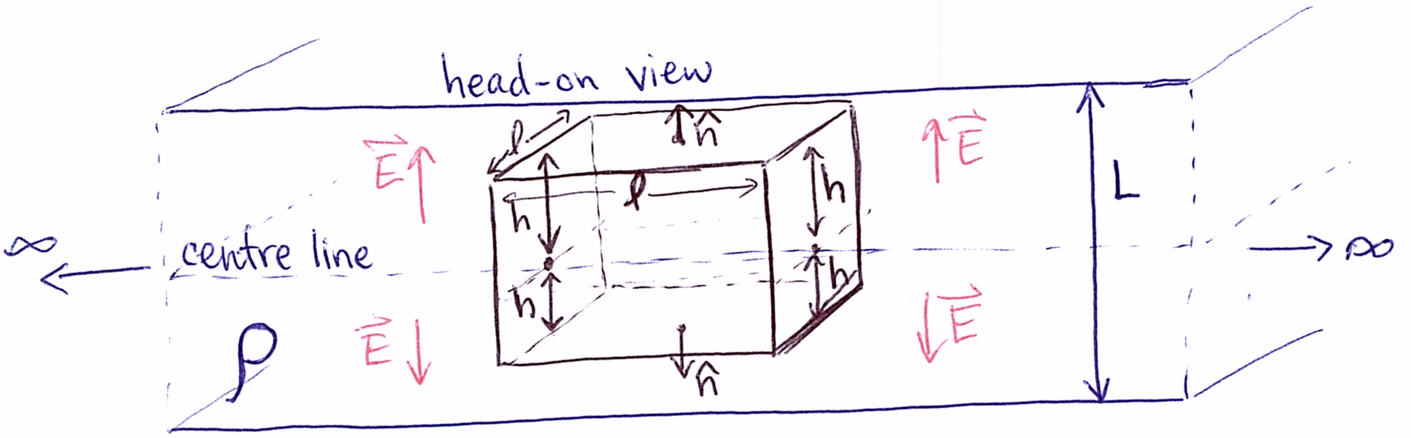
This gives the relationship between the strength of the electric field near a conductor and the local surface charge density. This relationship is what ensures that $\vec{E} = \vec{0}$ inside the conductor!

What about a slab of insulating material with uniform charge density ρ?



outside this just looks like an infinite sheet of charge

What is the field inside the slab?



The slab has thickness L, we have to choose a Gaussian surface with the same symmetries as the slab: choose a box whose centre coincides with the centre of the slab.

By symmetry, the E-field must be pointing upward above the centre line and downward below the centre line. The magnitude must also be constant at constant height h above and below the centre line. The normal vectors on all 4 sides are perpendicular to the E-field, so contribute nothing to the flux.

$$\oint \vec{E}\cdot d\vec{A} = \int_{top} \vec{E}\cdot dA + \int_{bottom} \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0}$$

$$E\int_{top} dA + E\int_{bottom} dA = \frac{q_{enc}}{\epsilon_0}$$

Since the top and bottom of the Gaussian box are the same distance away from the centre line, the strength of E is the same on both, so the two terms combine to give

$$2EA = \frac{q_{enc}}{\epsilon_0} \qquad q_{enc} = \rho V = \rho A(2h)$$

$$\cancel{2}E\cancel{A} = \frac{\rho \cancel{A}(\cancel{2}h)}{\epsilon_0}$$

$$\boxed{E = \frac{\rho h}{\epsilon_0}}$$

At the surface of the slab $E = \frac{\rho L}{2\epsilon_0}$

but $\rho L = \left(\frac{Q}{A\cancel{L}}\right)\cancel{L} = \frac{Q}{A} = \sigma$ (surface charge density of an equivalent sheet of charge.)

so $E = \frac{\sigma}{2\epsilon_0}$ and the slab looks just like an infinite sheet.