Last time:

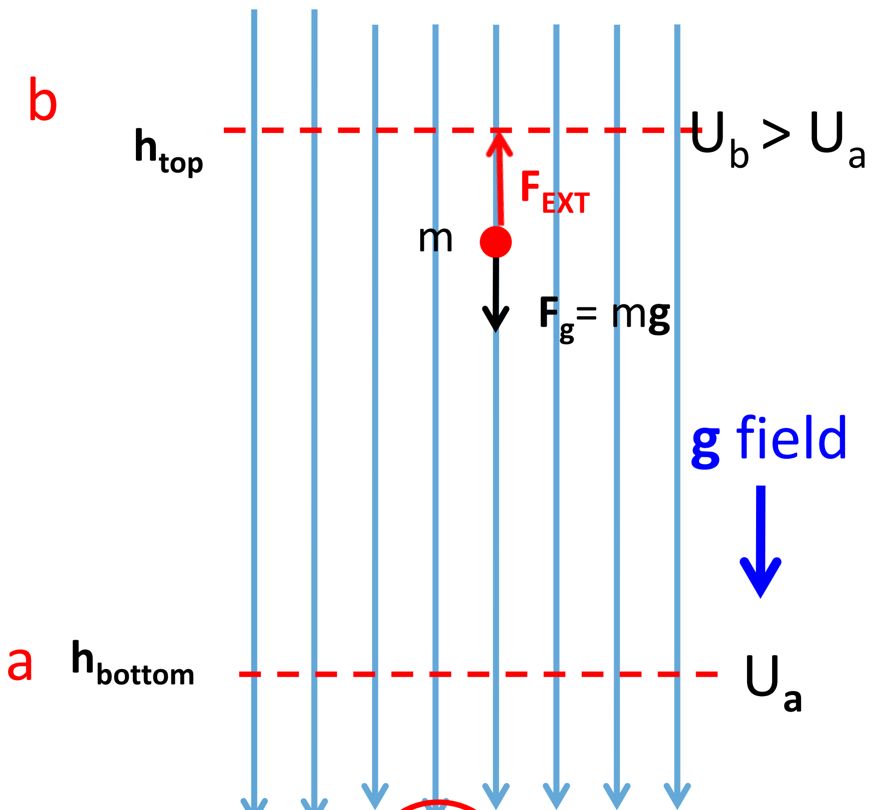
- Midterm review
- Honestly I’m just happy you showed up the day after your midterm

Today:

- Electric potential energy: uniform E-field
- Electric potential energy: 2 point charges
- Electric potential energy of a collection of charges
- Electric potential (very important concept)

Gravitational & Electric Fields

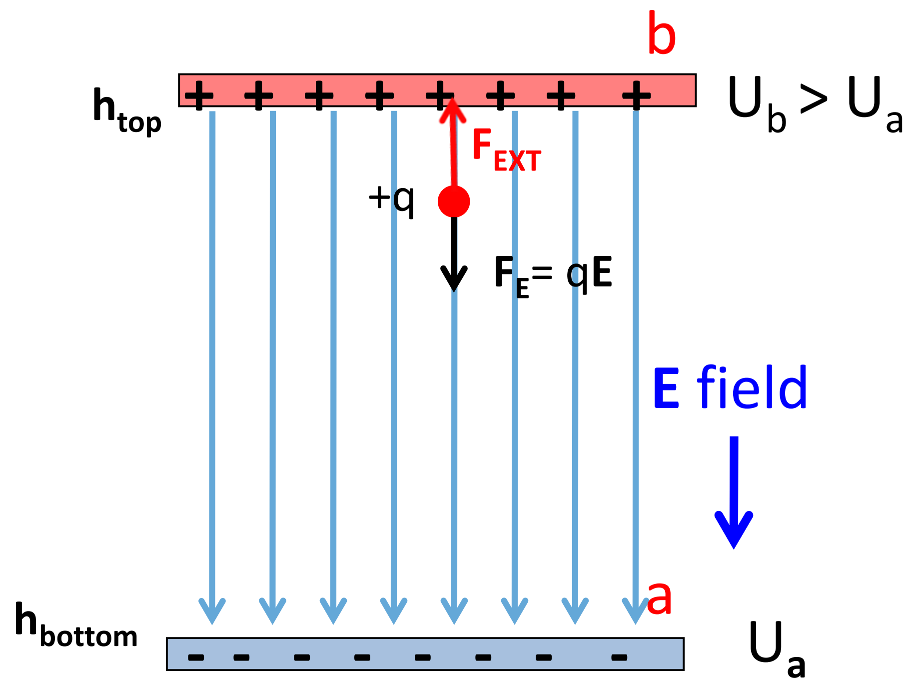
(Simple case: uniform fields)



$$WORK_{EXT} = F_{EXT} \times Dh = mgDh$$

$$WORK_{EXT}^{a->b} = U_b^g - U_a^g > 0$$

$$WORK_g^{a->b} = -(U_b^g - U_a^g)$$



$$WORK_{EXT} = F_{EXT} \times Dh = qEDh$$

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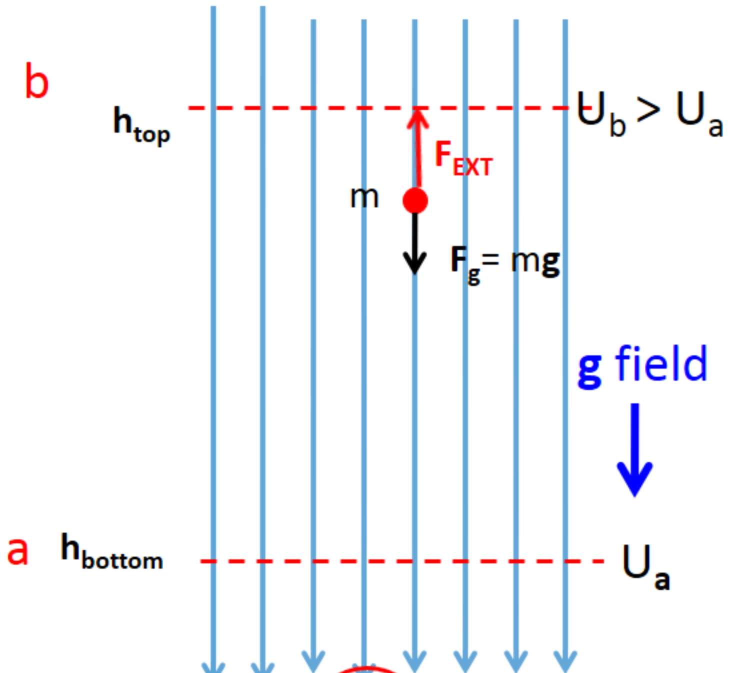
Gravitational & Electric Fields

(Simple case: uniform fields)



$$WORK_{EXT} = F_{EXT} \times \Delta h = mg\Delta h$$

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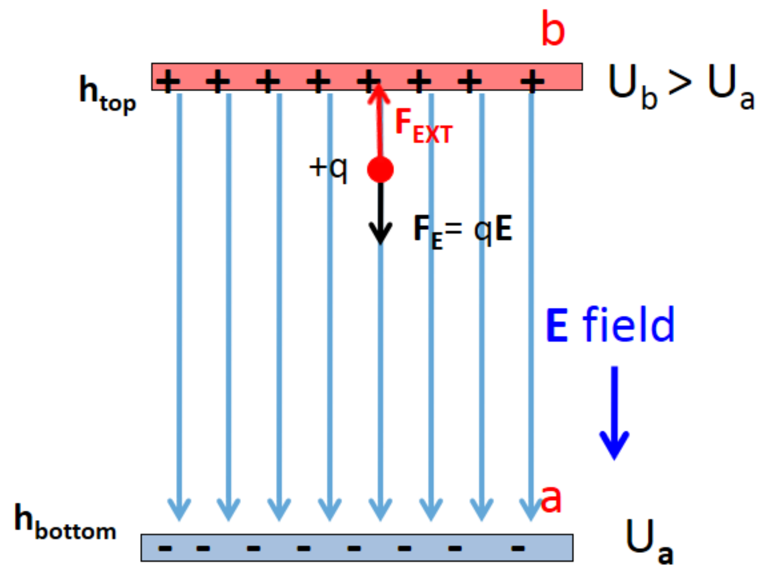

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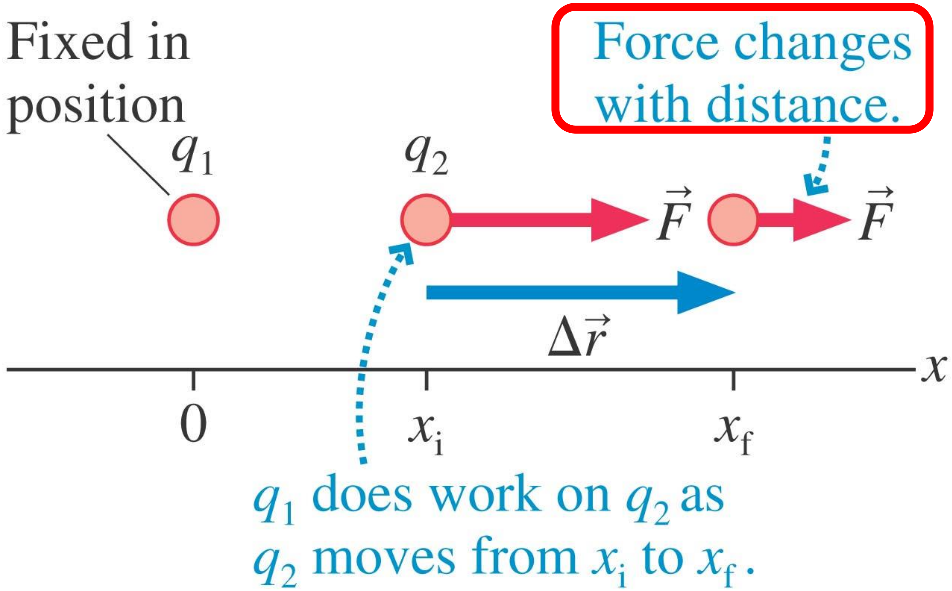
Finding Potential Energy of two point charges (more building blocks)

$$W_{i\to f}^{ELEC} = -\Delta U$$

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$$



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The field is **not** uniform so $\vec{F}$ is **not** constant over the displacement Δr and we **cannot** use

$$W_{i\to f}^{ELEC} = F\Delta r$$

Finding Potential Energy of two point charges (more building blocks)

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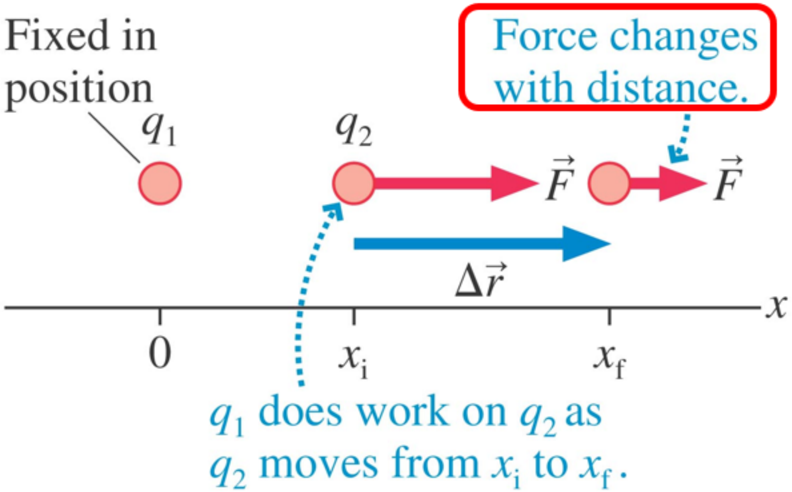

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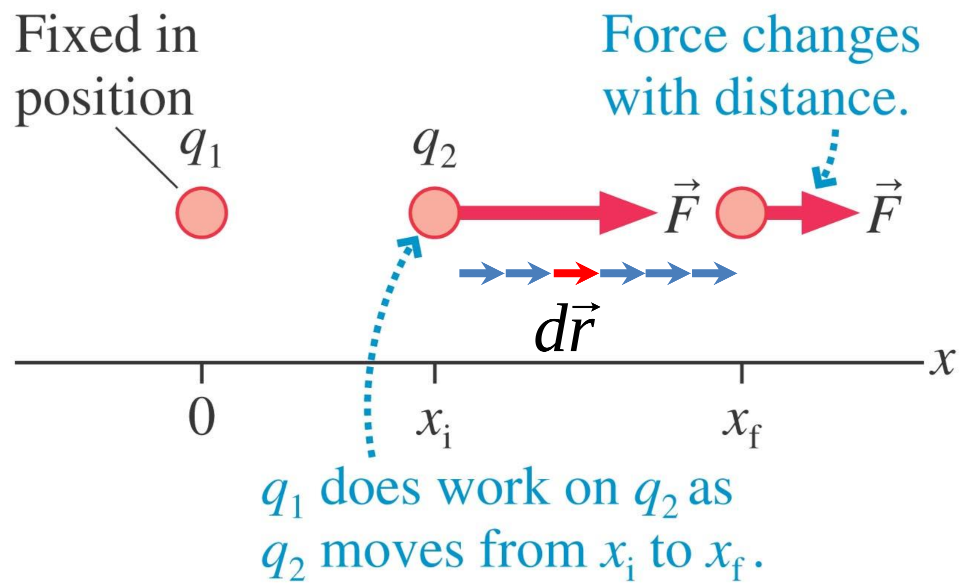
The field is **not** uniform so $\vec{F}$ is **not** constant over the displacement Δr and we **cannot** use

$$W_{i \to f}^{ELEC} = F\Delta r$$

Finding Potential Energy of two point charges (more building blocks)

Break the displacement $\Delta\vec{r}$ into many tiny displacements $d\vec{r}$.



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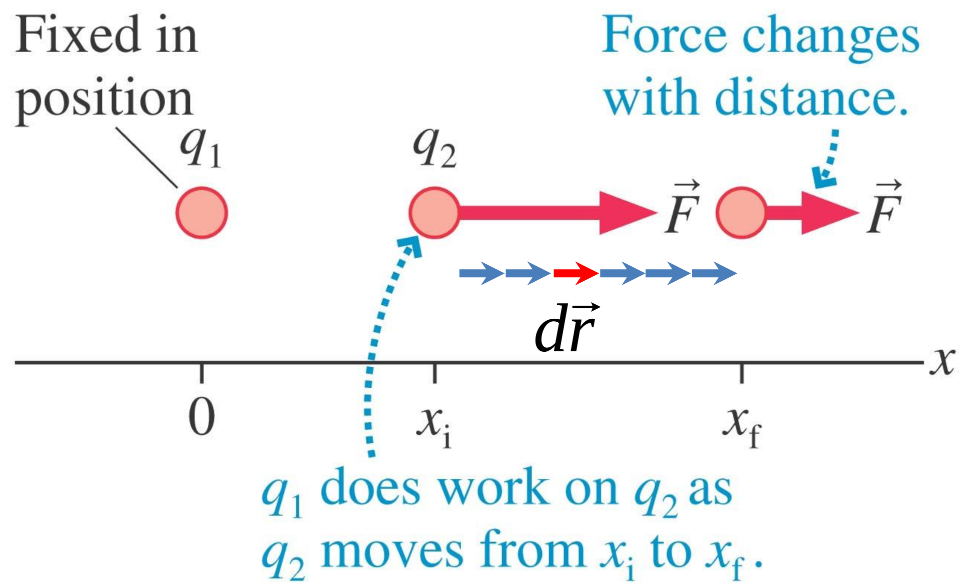
$\vec{F}$ is essentially constant over such a small displacement, so the work done on q_2 in **each** displacement is Fdr.

Finding Potential Energy of two point charges (more building blocks)

The total work is the sum of all the little bits of work:

$$W_{i\to f}^{ELEC} = \int_{r_i}^{r_f} F dr$$



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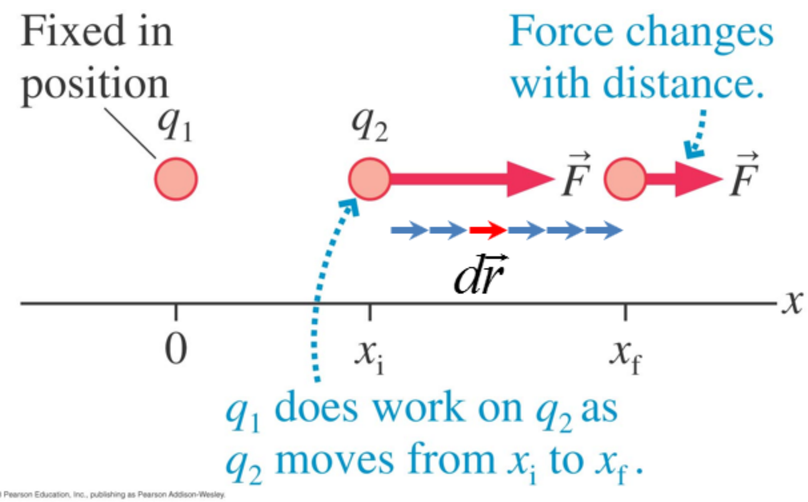
$$W_{i\to f}^{ELEC} = \int_{r_i}^{r_f} \overbrace{\frac{1}{4pe_0}\frac{q_1 q_2}{r^2}} dr$$

Finding Potential Energy of two point charges (more building blocks)

The total work is the sum of all the little bits of work:



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$$W_{i\to f}^{ELEC} = \int_{r_i}^{r_f} \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2} dr$$

Finding Potential Energy of two point charges (more building blocks)

Work done **by electric force**:

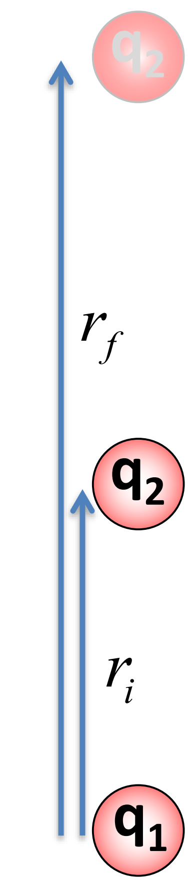
$$W_{i\to f}^{ELEC} = \int_{r_i}^{r_f} \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\,dr = \frac{1}{4\pi\varepsilon_0} q_1 q_2 \int_{r_i}^{r_f} r^{-2}\,dr$$

Recall from integral calculus

$$\int_{x_i}^{x_f} x^n\,dx = \frac{1}{n+1} x^{n+1}\Big|_{x_i}^{x_f} = \frac{1}{n+1}\left(x_f^{n+1} - x_i^{n+1}\right)$$

In our case, let x → r, then we have

$$W_{i\to f}^{ELEC} = \frac{1}{4\pi\varepsilon_0} q_1 q_2 \int_{r_i}^{r_f} r^{-2}\,dr = \frac{1}{4\pi\varepsilon_0} q_1 q_2 \left(\frac{1}{-2+1} r^{-2+1}\right)\Big|_{r_i}^{r_f}$$



q_1 held fixed

Finding Potential Energy of two point charges (more building blocks)

Work done **by electric force**:

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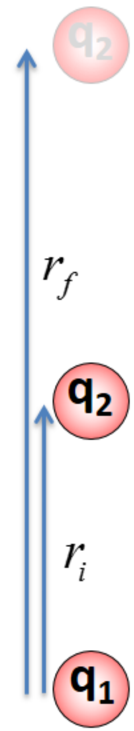


q_1 held fixed

Finding Potential Energy of two point charges (more building blocks)

$$W_{i\to f}^{ELEC} = -\frac{1}{4pe_0}\frac{q_1q_2}{r}\Bigg|_{r_i}^{r_f}$$

$$W_{i\to f}^{ELEC} = -\left(\frac{1}{4pe_0}\frac{q_1q_2}{r_f} - \frac{1}{4pe_0}\frac{q_1q_2}{r_i}\right)$$

$$W_{i\to f}^{ELEC} = -\Delta U = -(U_f - U_i) = U_i - U_f$$

Finding Potential Energy of two point charges (more building blocks)

$$W_{i \to f}^{ELEC} = -\frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}\bigg|_{r_i}^{r_f}$$

$$W_{i \to f}^{ELEC} = -\left(\frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_f} - \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_i}\right)$$

$$W_{i \to f}^{ELEC} = -\Delta U = -(U_f - U_i) = U_i - U_f$$

Then the potential energy of two point charges a distance r apart is

$$\boxed{U_e = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r} \cancel{+U_0}}$$

(1) There is a U_0, but we normally set it to zero.

(2) The potential energy of two charges an infinite distance apart ($r = \infty$) is zero.

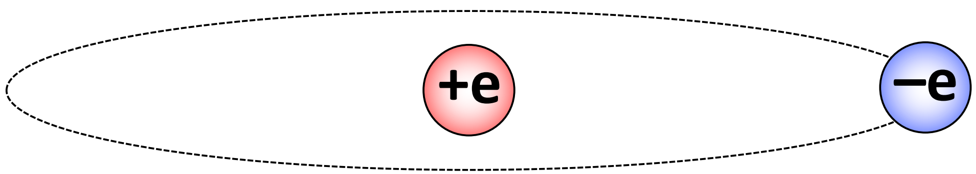
TopHat Question

The Bohr model of the hydrogen atom consists of an electron orbiting a proton with a radius of $r_B = 0.529 \times 10^{-10}$ m. What is the electric potential energy of a hydrogen atom in this model **in units of eV**?

$\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{C^2/N\,m^2}$

$e = 1.60 \times 10^{-19}\ \mathrm{C}$

$1\ \mathrm{eV} = 1.60 \times 10^{-19}\ \mathrm{J}$



A. –13.6 eV

B. –27.2 eV

C. –5.75 x 10^{11} eV

D. –9.21 x 10^{-8} eV

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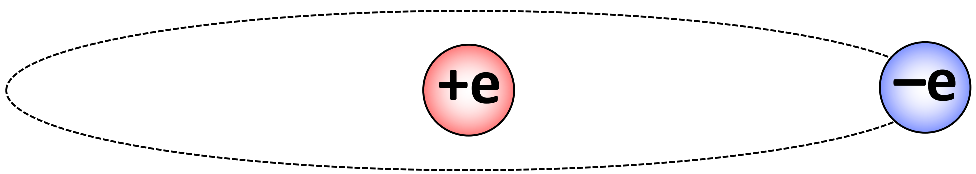
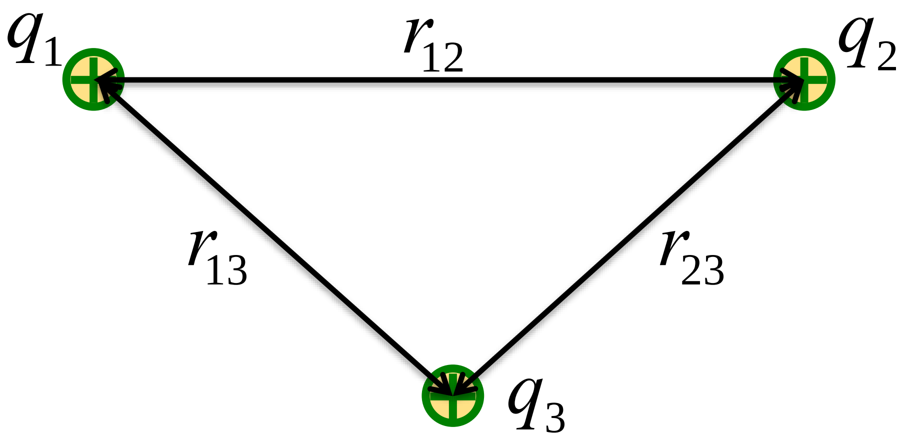


$$U_e = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r} = -\frac{1}{4\pi\epsilon_0}\frac{e^2}{r_B} \Longrightarrow U_e = -4.60 \times 10^{-18}\ \mathrm{J}$$

$$U_e = \left(-4.60 \times 10^{-18}\ \cancel{\mathrm{J}}\right)\left(\frac{1\ \mathrm{eV}}{1.60 \times 10^{-19}\ \cancel{\mathrm{J}}}\right) = -27.2\ \mathrm{eV}$$

The kinetic energy of the electron is +13.6 eV, so the binding energy of H is –13.6 eV

Superposition: Potential Energy due to Multiple Charges



$$U_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}}$$

$$U_{23} = \frac{1}{4\pi\varepsilon_0}\frac{q_2 q_3}{r_{23}}$$

$$U_{13} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_3}{r_{13}}$$

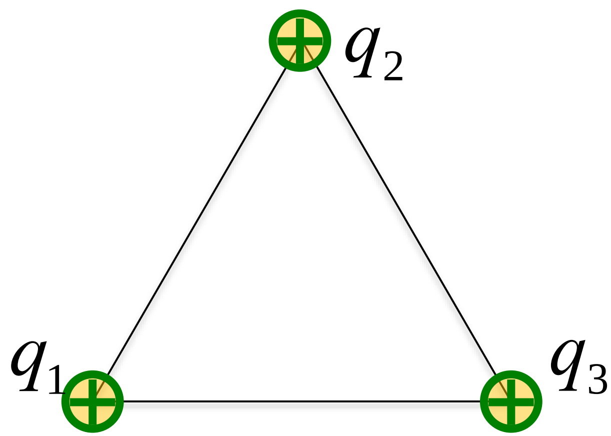
$$\boxed{U_{total} = U_{12} + U_{23} + U_{13}}$$

In general, the total potential energy is just the sum of the pairwise potential energies of all the charges present. Calculate U between each pair, then sum all of them up.

TopHat Question

Three charges q_1 = 1.0 nC, q_2 = –2.0 nC, and q_3 = 3.0 nC are fixed in an equilateral triangle of side length d = 5.0 cm. What is the electric potential energy of this configuration?

$$U_{ij} = \frac{1}{4\pi\varepsilon_0}\frac{q_i q_j}{r_{ij}}$$



$$\boxed{\varepsilon_0 = 8.85 \times 10^{-12}\ \mathrm{C^2/N\,m^2}}$$

$$U_{12} = -3.596 \times 10^{-7}\ \mathrm{J}$$
$$U_{23} = -1.079 \times 10^{-6}\ \mathrm{J}$$
$$U_{13} = +5.394 \times 10^{-7}\ \mathrm{J}$$

A. 2.0 x 10^{-6} J

B. 1.3 x 10^{-6} J

C. –9.0 x 10^{-7} J

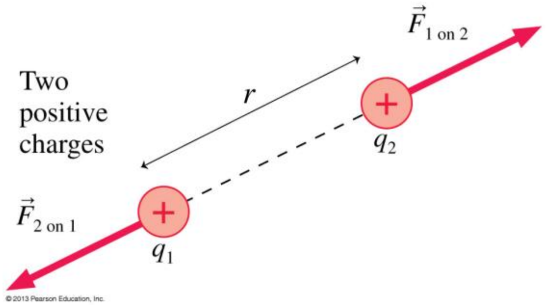
D. –1.8 x 10^{-7} J

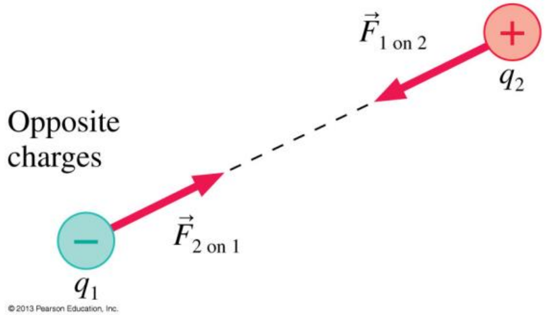
Electric Force vs Electric Field

Electric Force $\vec{F}$

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2} = q\vec{E}$$

A physical property between two point charges

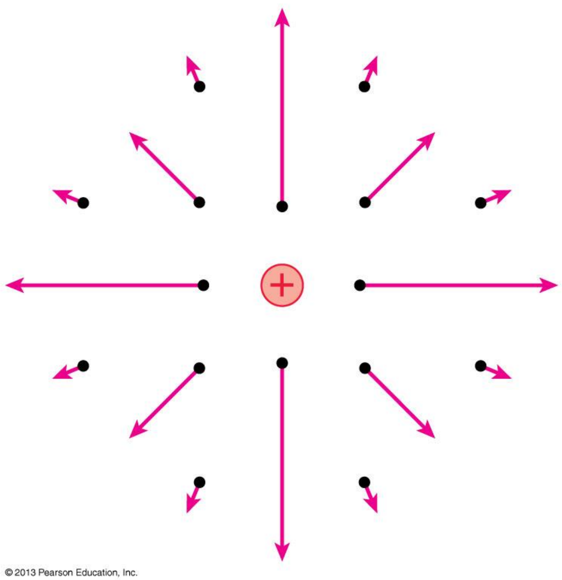




Electric Field $\vec{E}$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

A physical property around a single point charge



Electric Force vs Electric Field

Electric Force $\vec{F}$

Electri

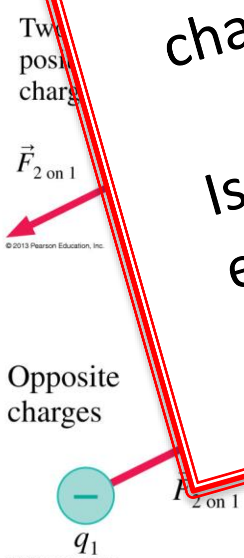
$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2} = q\vec{E}$$

A p ... round

two

Potential energy is a physical property that exists because of the force between two charges.

Is there some similar notion of "potential energy" that exists only because of the electric field?



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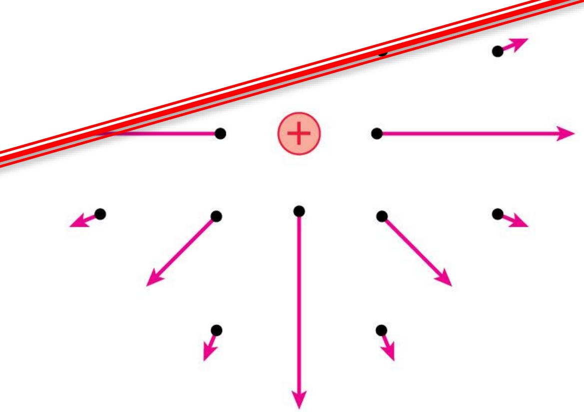

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Electric Potential

Here are some source charges and a point P.

If we place a charge q at point P, then q and the source charges interact with each other.

The interaction energy is the potential energy of q and the source charges,

$$U_{q+sources}$$

How does this interaction happen?

Electric Potential

Model:

The source charges create a **potential for interaction** everywhere, including at point P.

This potential for interaction is a **property of space**. Charge q does not need to be there.

We call this potential for interaction the **electric potential, V.** (Often just called "the potential")

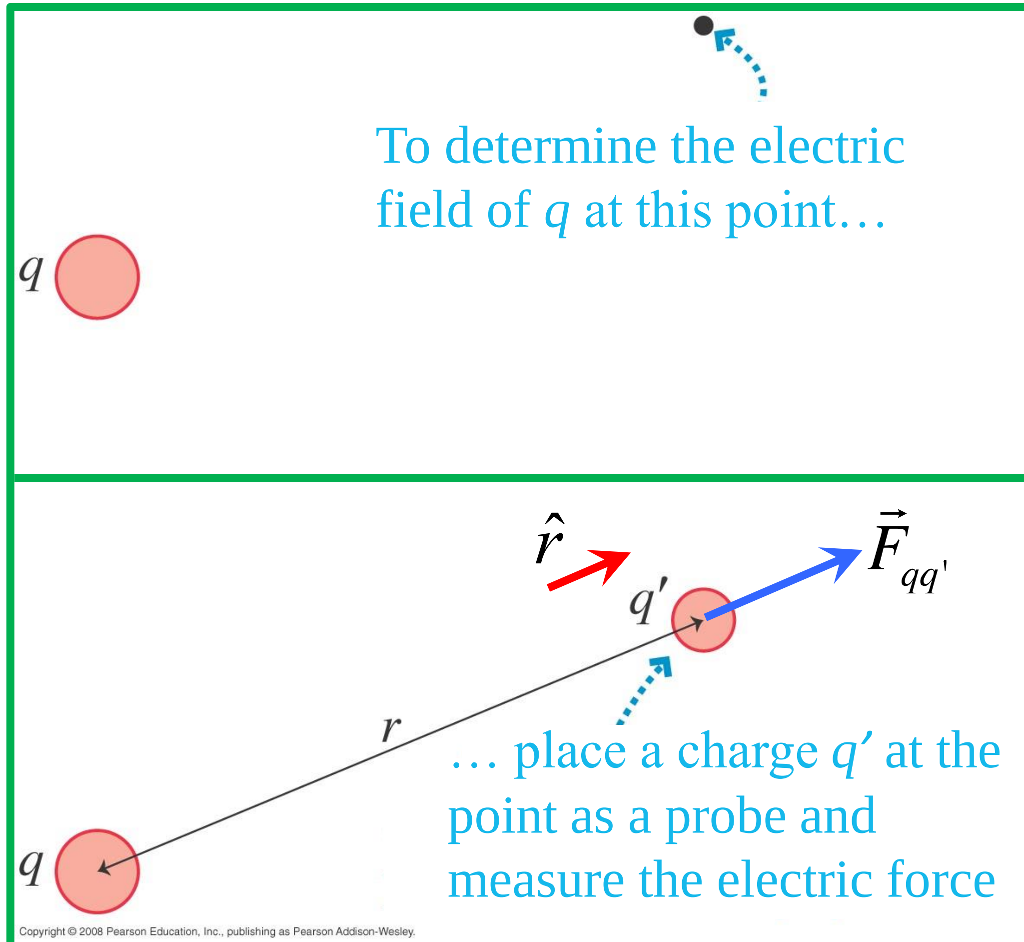
Electric Field of a point charge

Electric force on q' from q

$$\vec{F}_{qq'} = \frac{1}{4\pi\varepsilon_0}\frac{qq'}{r^2}\hat{r}$$

Then the electric field of q is

$$\vec{E} = \frac{\vec{F}_{qq'}}{q'} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$$



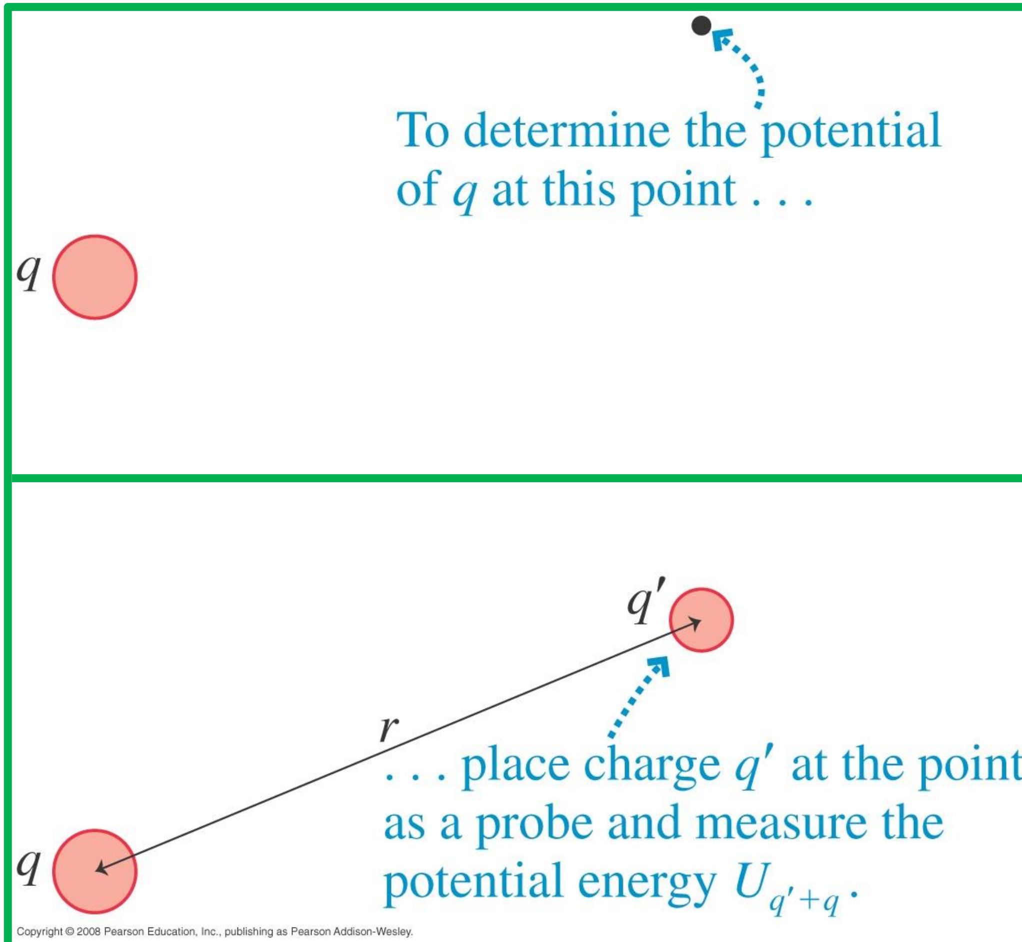
Electric Potential of a point charge

Potential energy of q and q’

$$U_{q'+q} = \frac{1}{4\pi\varepsilon_0}\frac{qq'}{r}$$

Then the potential of q is

$$V = \frac{U_{q'+q}}{q'} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$



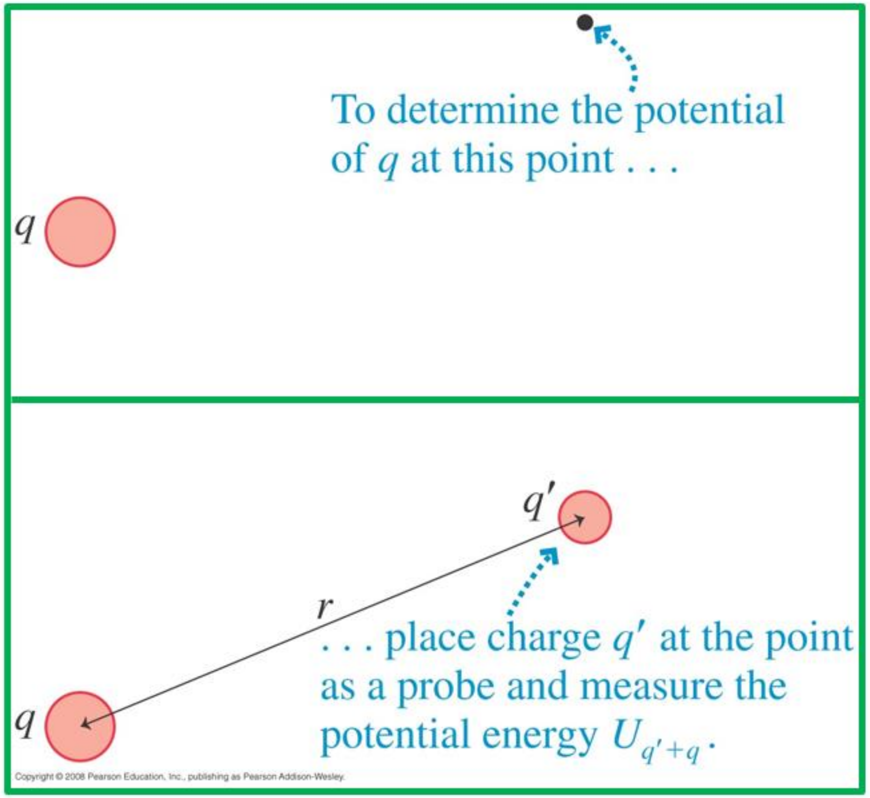
Electric Potential of a point charge

Potential energy of q and q’

$$U_{q'+q} = \frac{1}{4\pi\varepsilon_0}\frac{qq'}{r}$$

Then the potential of q is

$$V = \frac{U_{q'+q}}{q'} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$



Electric Potential

Definition of V: Place charge q at point P and measure its potential energy. Then

$$\boxed{V \equiv \frac{U_{q+sources}}{q}}$$

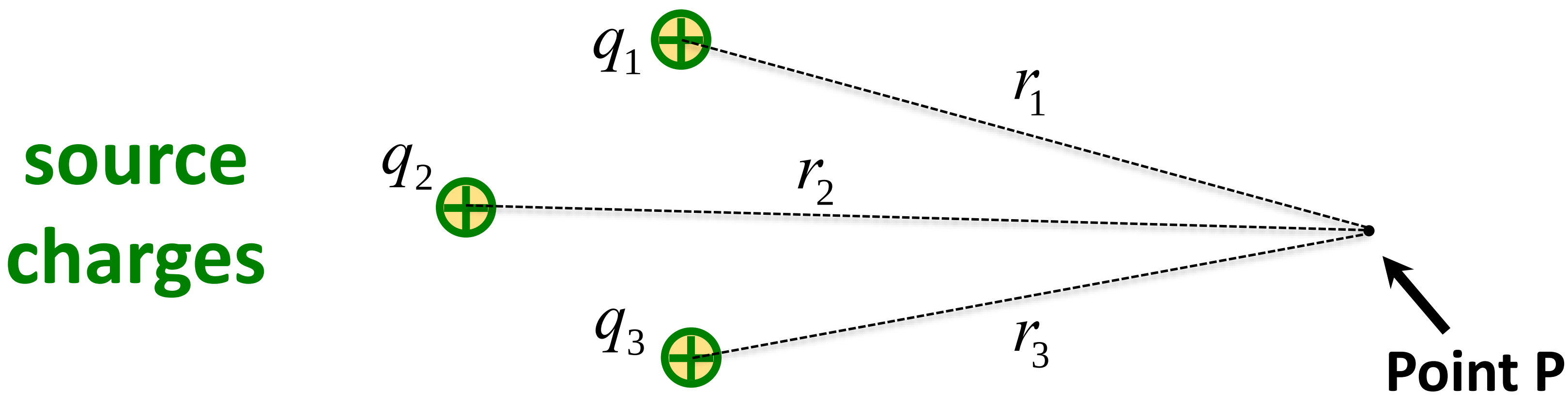
Unit: $1\,\mathrm{volt} = 1\,\mathrm{V} = 1\,\frac{\mathrm{J}}{\mathrm{C}}$

Electric Potential

Or, if we know the potential, V, at point P, then if we place a charge, q, at point P, the potential energy of q and the source charges is

$$\boxed{U_{q+sources} = qV}$$

Advantage of Electric Potential



V is a SCALAR! There is no direction associated with it. This makes it much easier to calculate!

$$V_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1} \qquad V_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2} \qquad V_3 = \frac{1}{4\pi\varepsilon_0}\frac{q_3}{r_3}$$

$$\boxed{V = V_1 + V_2 + V_3}$$

Look at my lecture notes called:
Feb_Appendix4_Potential of a Dipole

So far:

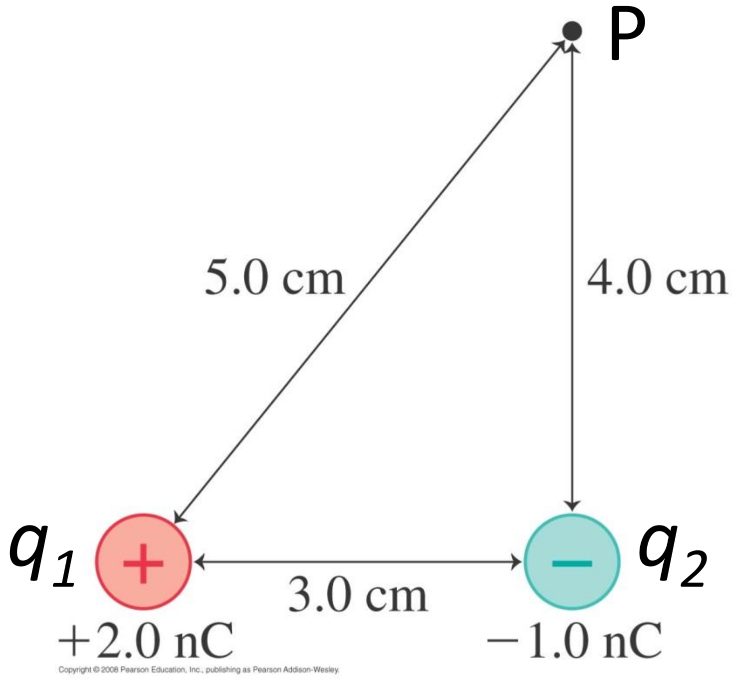
- Electric potential energy and electric force
- Electric potential and electric field
- Electric potential of a dipole

To be continued:

- Equipotential surfaces: visualizing electric potential
- Conductors and electric potential
- Interpreting equipotential surfaces

TopHat Question

What is the electric potential at point P for the arrangement of two charges shown to the right?



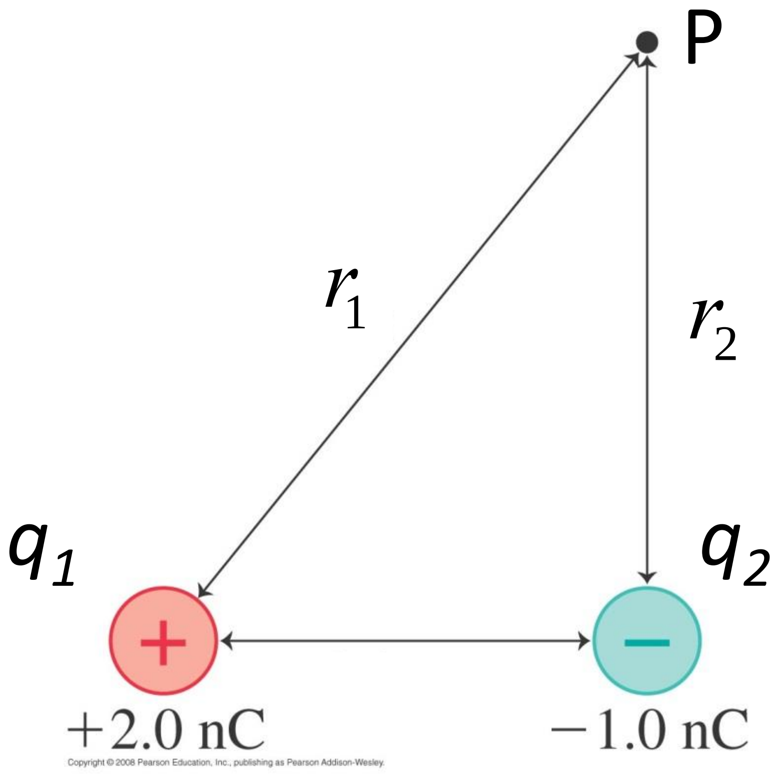
A. $585\,V$

B. $135\,V$

C. $1600\,V$

D. $-140\,V$

Finding V at point P.



Potential is a scalar

There are no components

Just add the potentials

V at P = (V_1 at P due to q_1) + (V_2 at P due to q_2).

Last time:

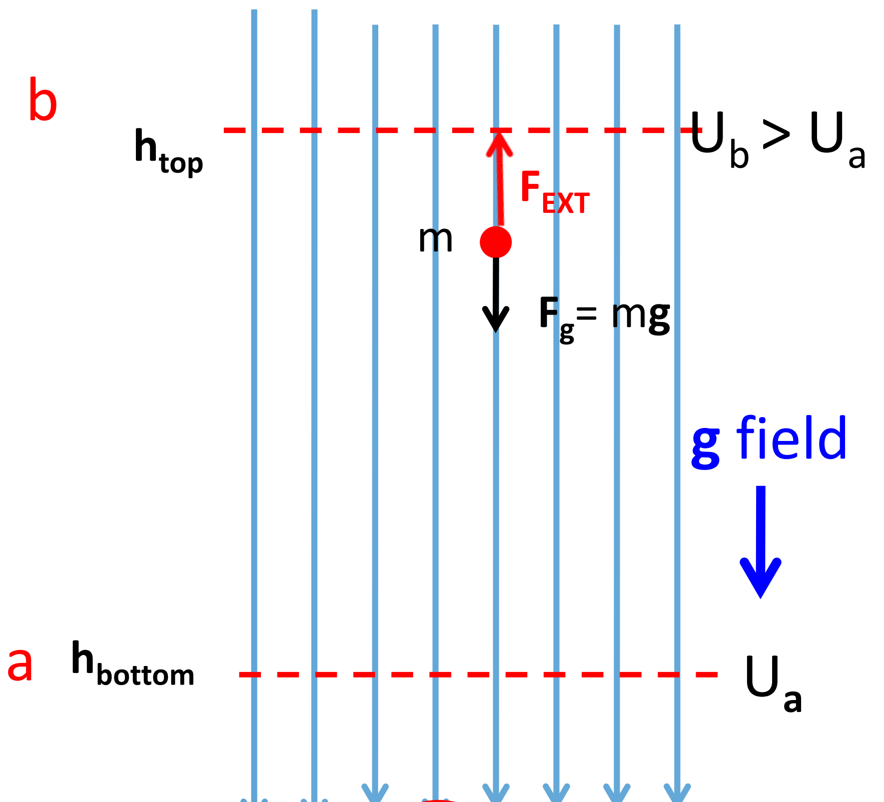
- Equipotential surfaces

This time:

- Equipotential surfaces: visualizing electric potential
- Conductors and electric potential
- Interpreting equipotential surfaces

Gravitational & Electric Fields

(Simple case: uniform fields)



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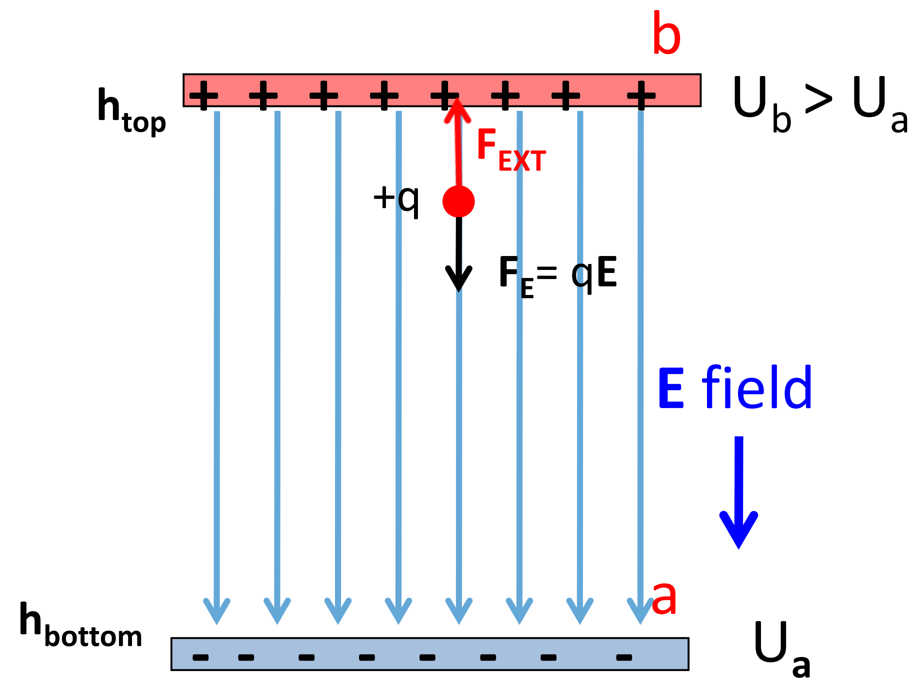

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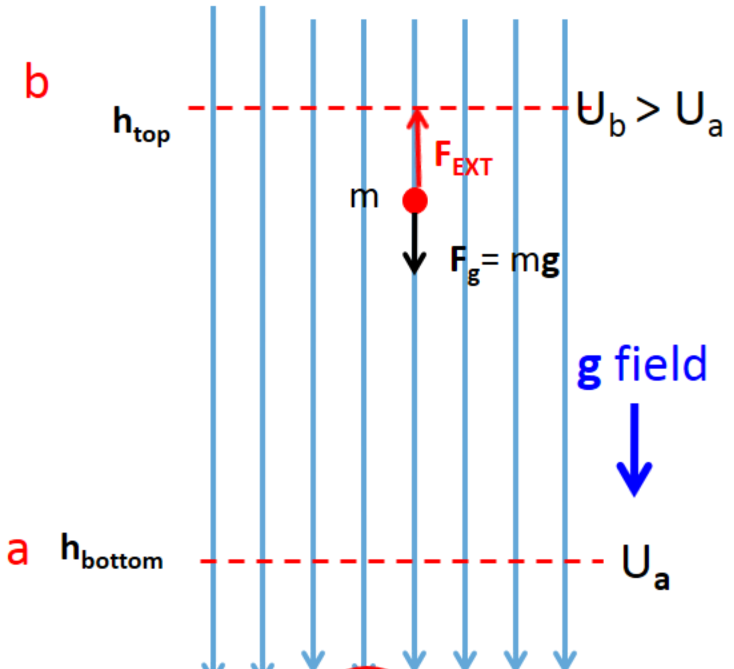

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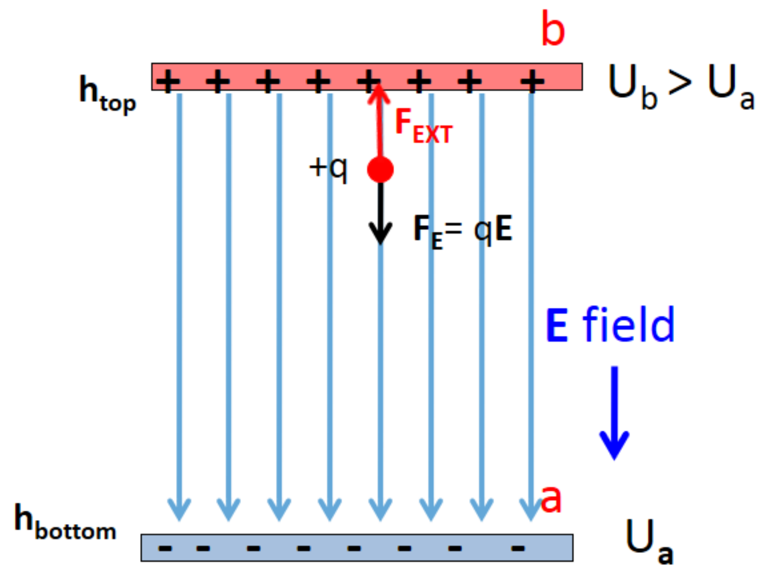

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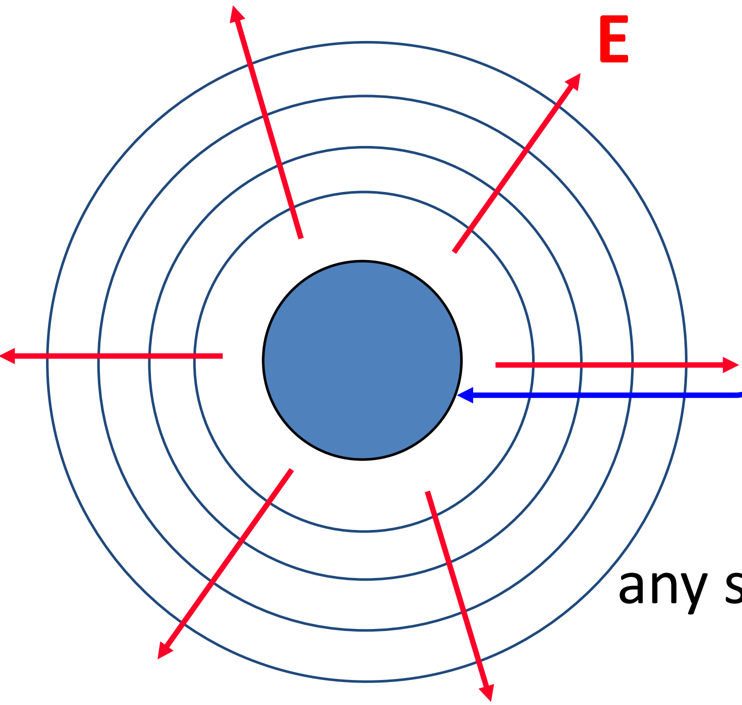
Equipotentials

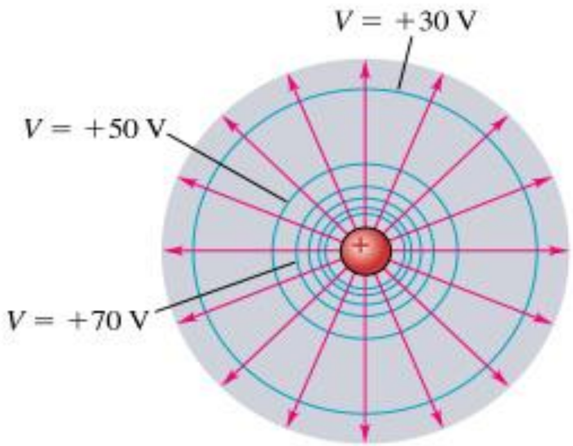


Consider equipotential lines
(or surfaces, actually, in 3-D):

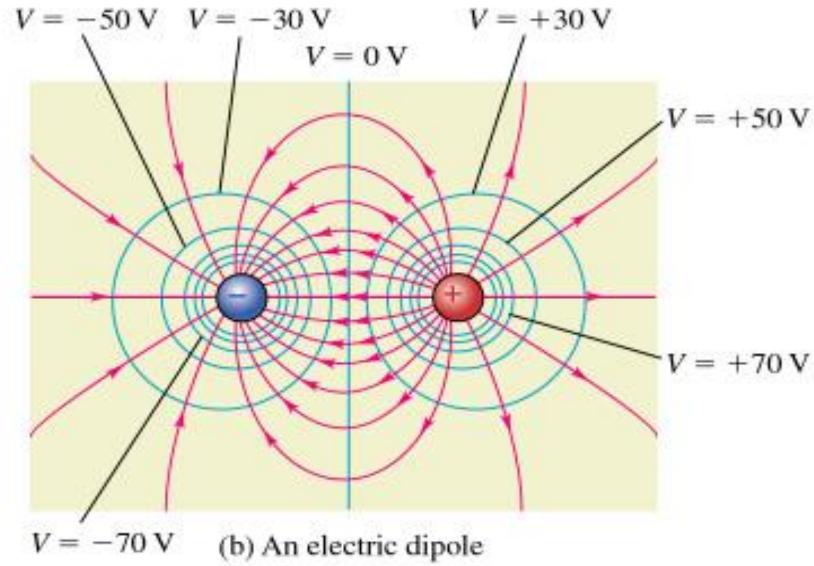
eg --for uniform spherical charge,
$V(r) = k\,Q/r$ is constant over
any sphere concentric with the charged sphere.

Note that if move along equipotential surface, by definition $\Delta V = 0$
but this $= -\vec{\mathbf{E}} \cdot \Delta\vec{\mathbf{r}} \Rightarrow \vec{\mathbf{E}}$ is $\perp$ equipotential surface

Equipotential Surfaces

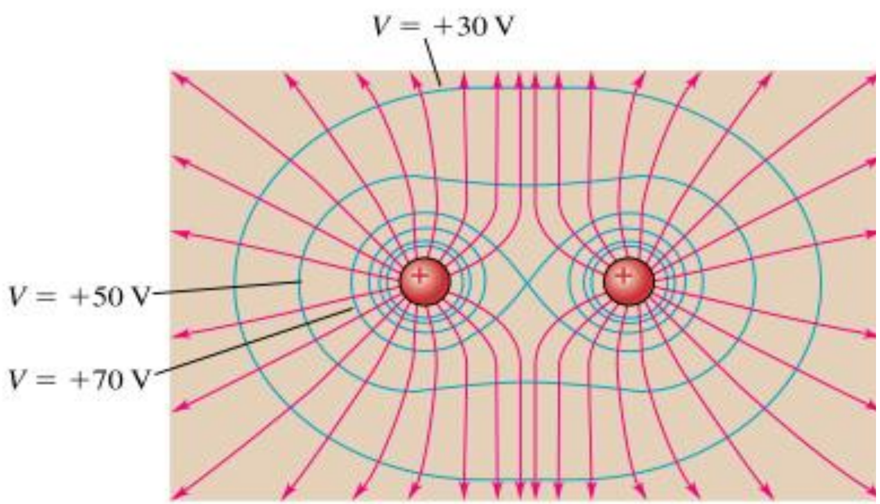


(a) A single positive charge



(b) An electric dipole

Note – E is always ⊥ V !!



(c) Two equal positive charges

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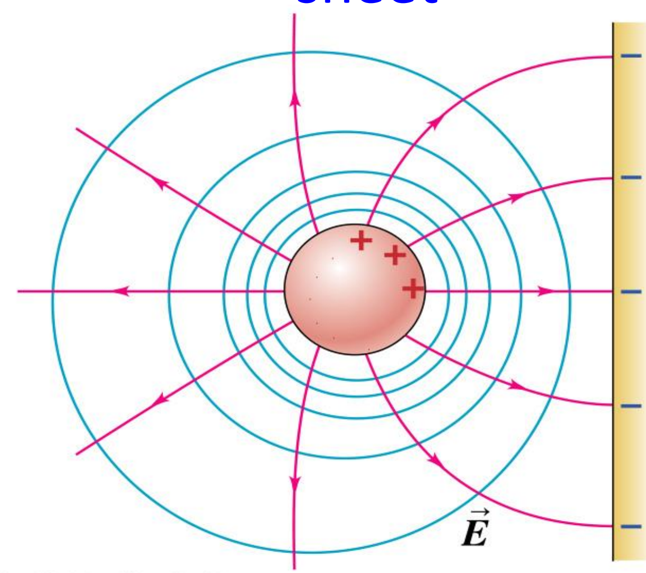
Conducting sphere
+ sheet



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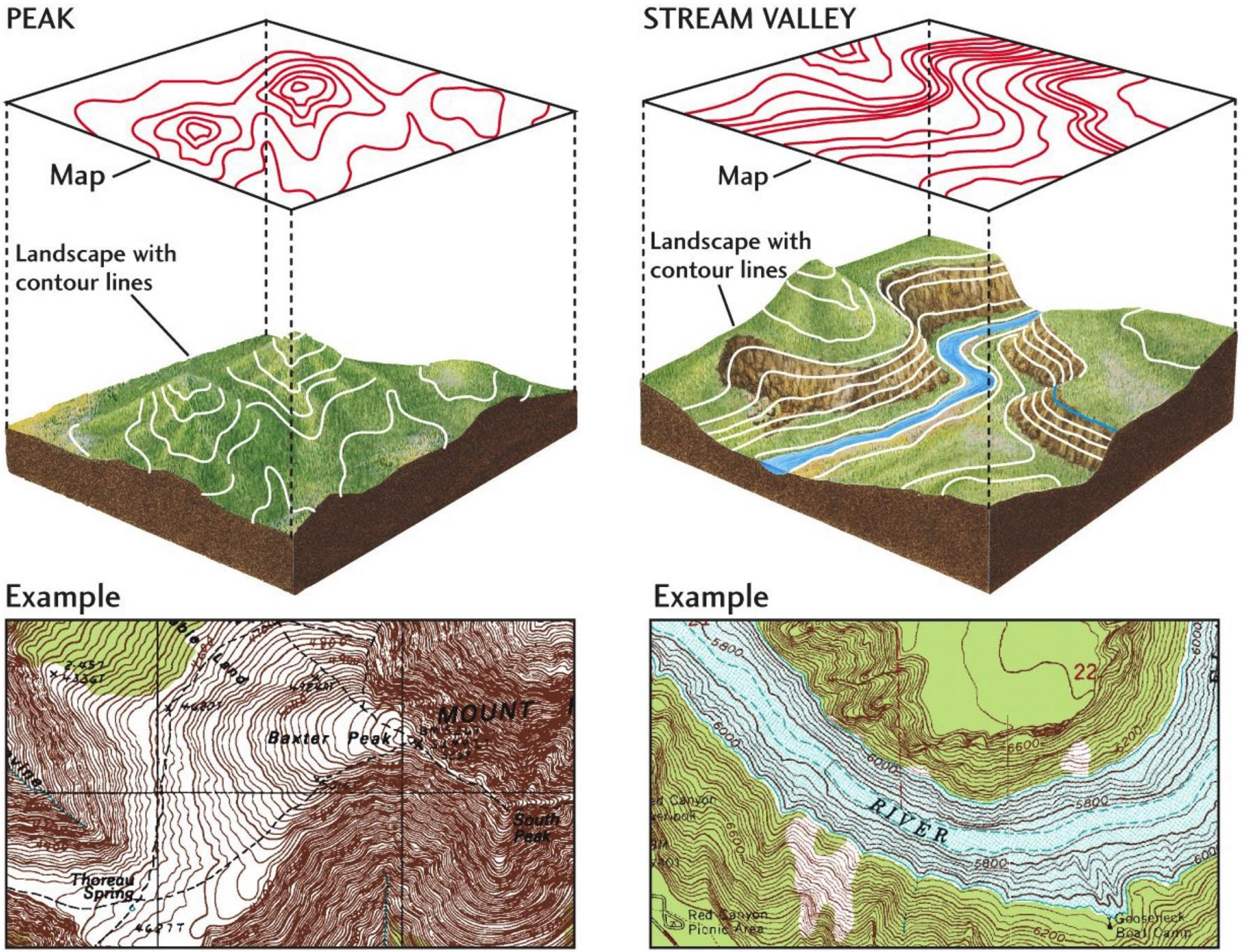
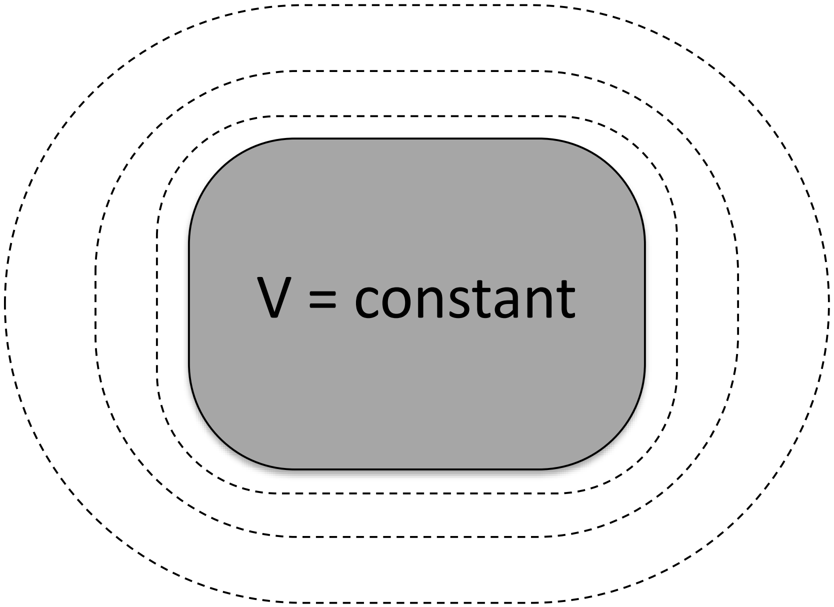
Where have you seen equipotentials before?

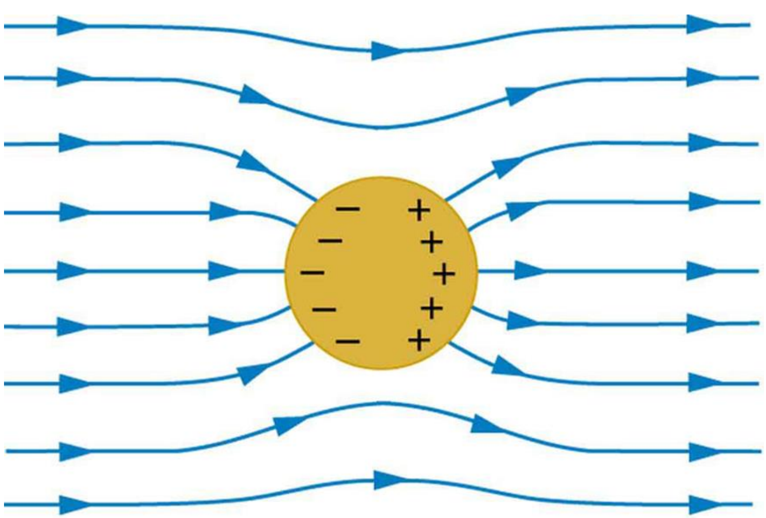


Mt. Katahdin, Maine

Flaming Gorge, Wyoming

Conductors and E-fields





https://cnx.org/contents/Ax2o07Ul@8.8:a8tWDdDr@3/Conductors-and-Electric-Fields

The surface of a conductor is an equipotential. If there was a potential difference across the surface of a conductor, the freely moving charges would move around until the potential is constant.

This means that electric field lines ALWAYS must meet a conducting surface at right angles (any tangential component would imply a tangential force on the free charges).

Potential Gradient -- E and V

Note: E is always $\perp$ equipotential lines

$$\vec{E} = -\vec{\nabla} V = -\frac{\partial V}{\partial x}\hat{i} - \frac{\partial V}{\partial y}\hat{j} - \frac{\partial V}{\partial z}\hat{k}$$

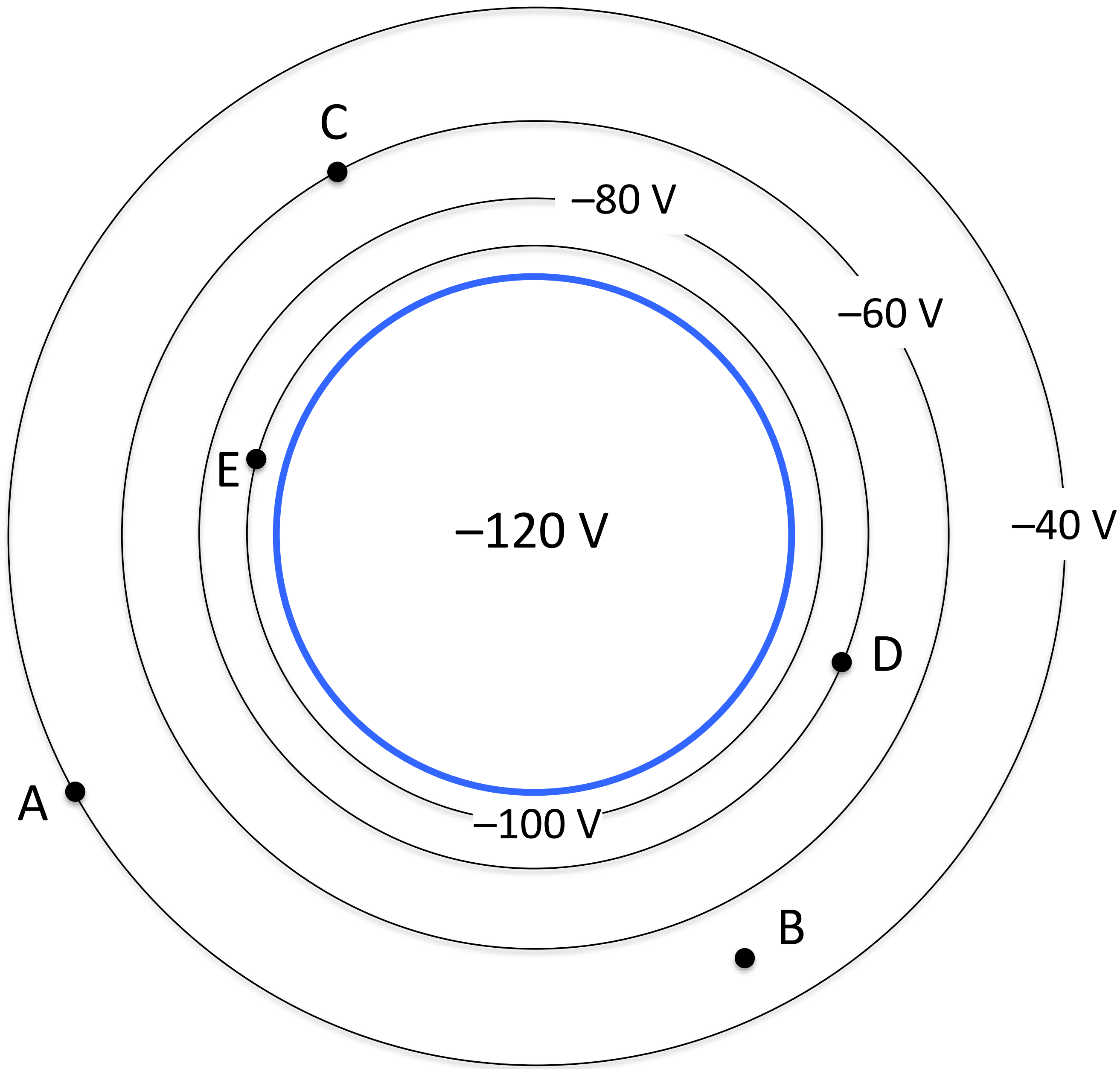
In 3 dimensions we must take 3 derivatives, then add them **VECTORIALLY**

Alternatively, the potential is found from the electric field integrated along any path connecting points A and B

$$\Delta V_{AB} = \int_A^B \vec{E} \cdot d\vec{\ell}$$

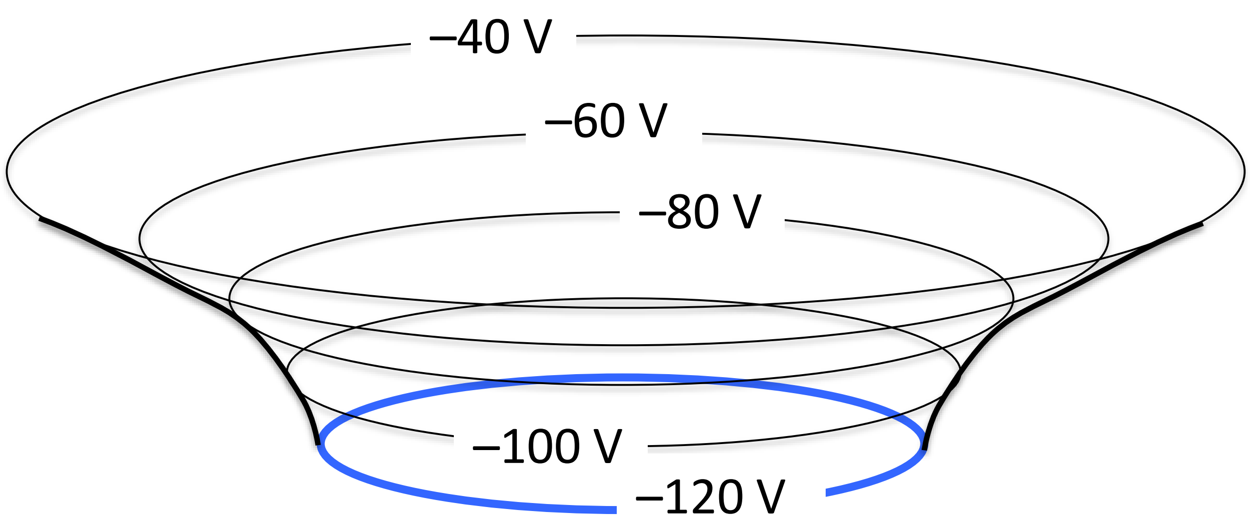
Equipotential surfaces for charged shell

Equipotential surfaces give you information about the potential energy that charged particles would have, the direction of the electric field, the strength of the electric field, and where a charged particle is allowed to go, based on its energy.



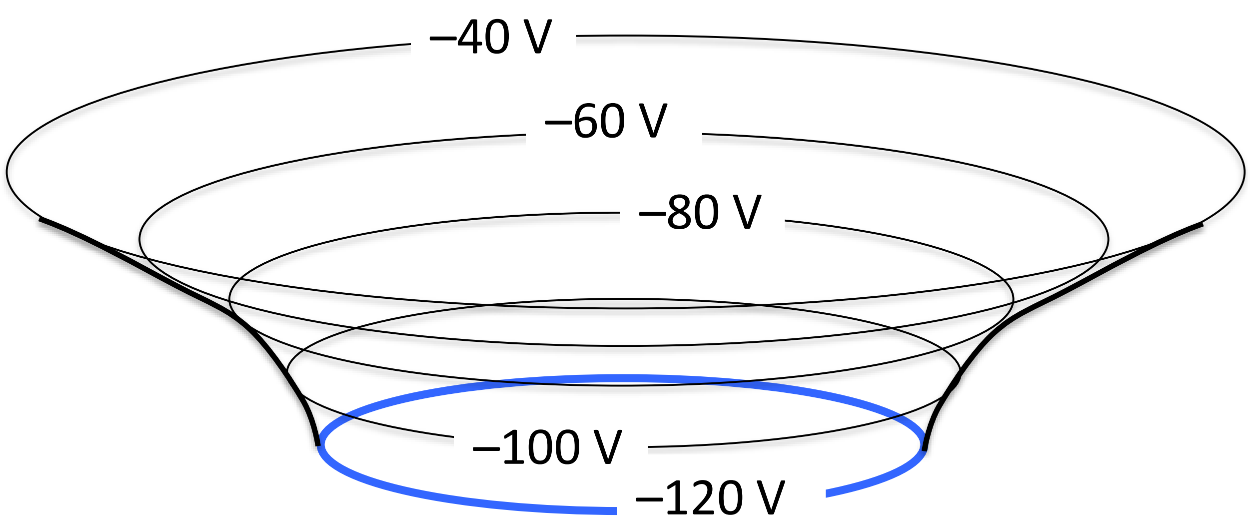
Equipotential surfaces for charged shell

Equipotential surfaces give you information about **the potential energy that charged particles would have**: Think of the electric potential (V) the same way that gravitational potential (gh) is an altitude above sea level. The potential energy of a charge q is then just U = qV, while the potential energy of a mass is U = mgh.



Equipotential surfaces for charged shell

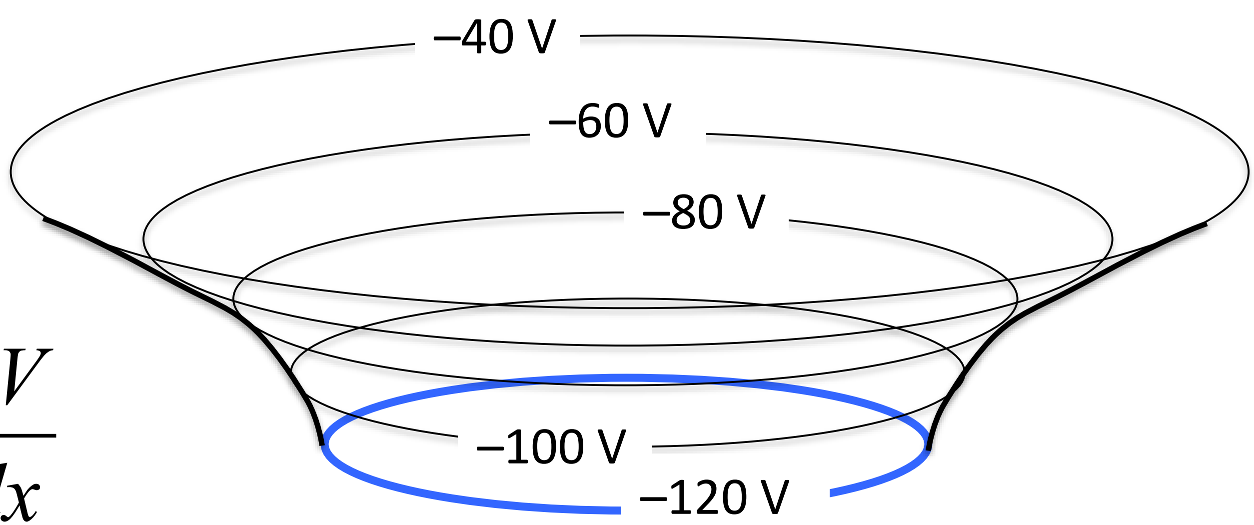
Equipotential surfaces give you information about **the direction of the electric field**. Just like in the gravitational analogy, objects roll downhill (to lower gravitational potential), positive charges move “downhill” to lower electric potential; the electric field always points “downhill”.



Equipotential surfaces for charged shell

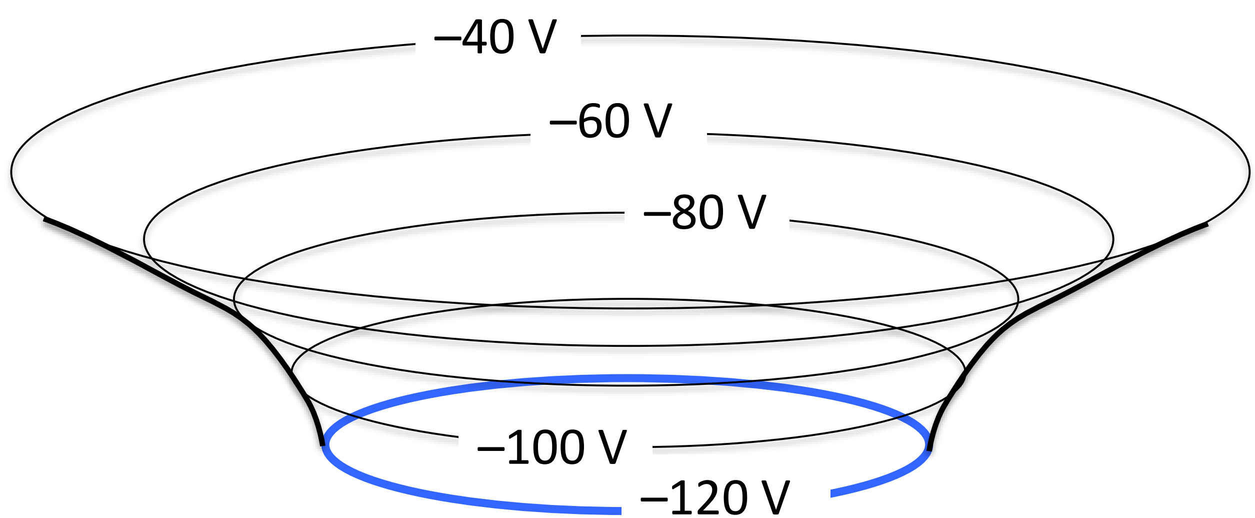
Equipotential surfaces give you information about **the strength of the electric field**. We know that the in the gravitational case, objects on steeper slopes will accelerate faster. Similarly here, the strength of the electric field is related to the slope of V(x). The more bunched together the equipotential lines, the steeper the slope, the stronger the field.

$$E = -\frac{dV}{dx}$$



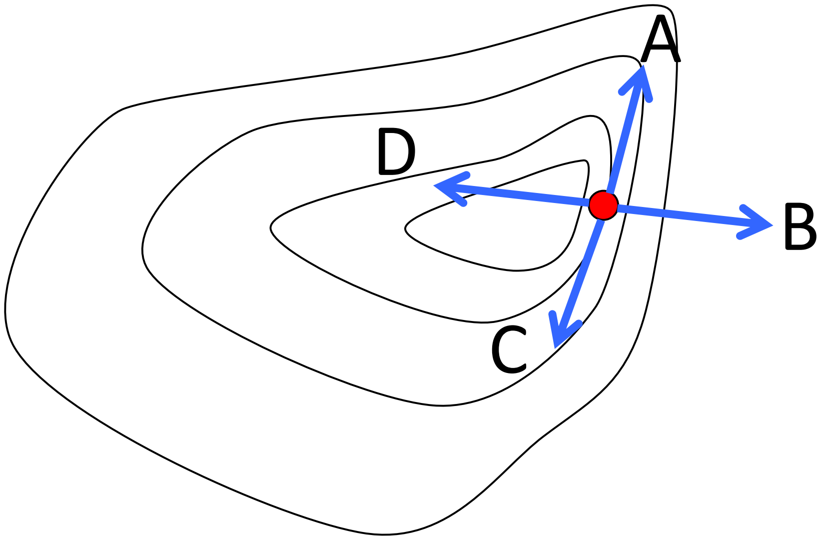
Equipotential surfaces for charged shell

Equipotential surfaces give you information about **where a charged particle is allowed to go, based on its energy**. If you release a marble in a bowl at some height *h*, it will never be able to reach a higher height. Similarly, if you release a positive charge from some potential, it can never reach a higher potential unless supplied with extra energy.



TopHat Question

Equipotential surfaces are shown below. If a positively charged particle were released from rest at the point indicated, in which direction would the particle begin to move?



A.

B.

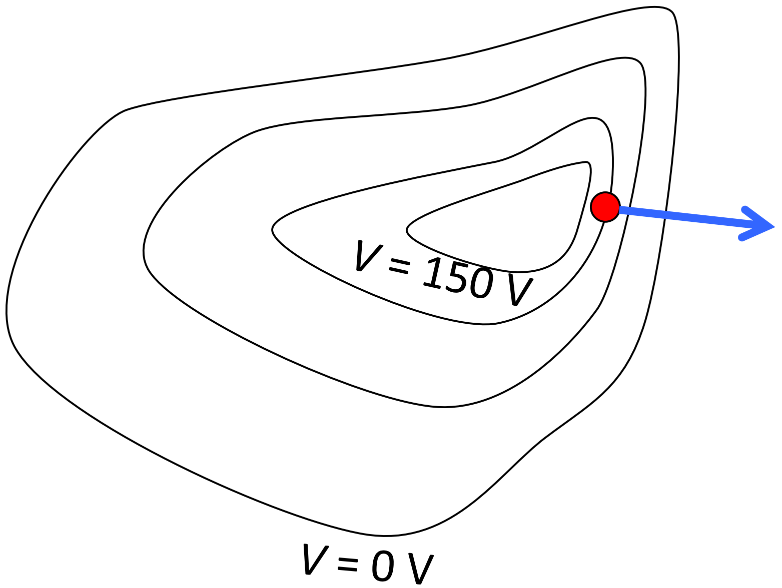
C.

D.

E. Not enough info

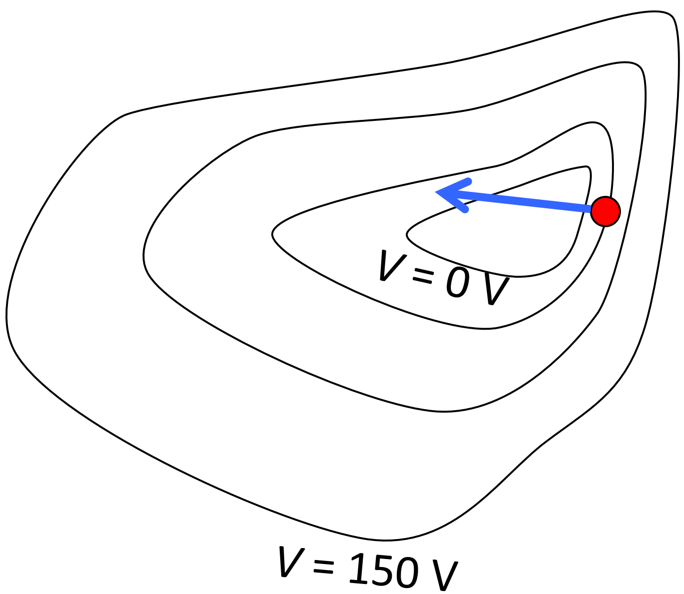
TopHat Question

Equipotential surfaces are shown below. If a positively charged particle were released from rest at the point indicated, in which direction would the particle begin to move?



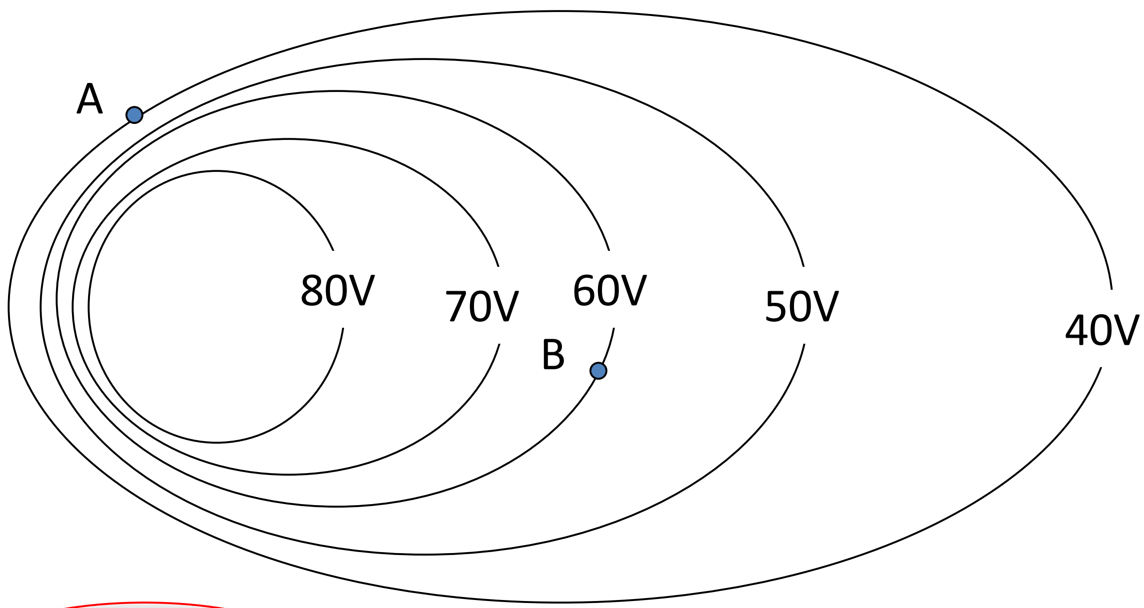
TopHat Question

Equipotential surfaces are shown below. If a positively charged particle were released from rest at the point indicated, in which direction would the particle begin to move?



TopHat Question

How much energy (in Joules) would 2C of charge gain if it was pushed from point A to point B?



A. 40 J

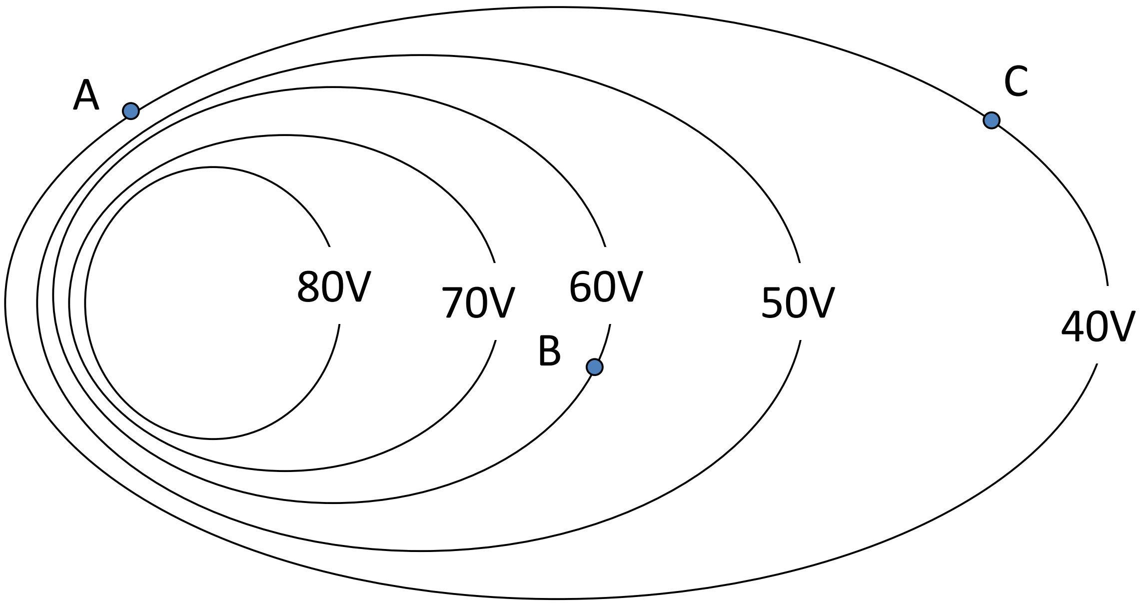
B. 60 J

C. 80 J

D. 120 J

TopHat Question

How much energy (in Joules) would 2C of charge gain if it was pushed from point A to point B, then to point C?



A. 80 J

B. 60 J

C. 40 J

D. 0 J

- Look at my lecture notes called: Feb_Appendix5_Potential of a line of charge