Last time

- Review of the last lecture
- Resistivity and temperature
- Semiconductors
- Superconductors
- Measuring temperature
- Introduction of magnetism
- Magnetic force

This time

- More on magnetic force
- Differences between electric and magnetic force
- Magnetic field lines
- Gauss's law for magnetism
- Cyclotron motion
- Hall effect

Differences between electric force and magnetic force

1. Electric force on a charged particle is parallel to the electric field.

$$\vec{F}_e = q\vec{E}$$

   Magnetic force is perpendicular to the magnetic field and velocity.

$$\vec{F}_B = q\vec{v} \times \vec{B}$$

2. Electric force applies to both stationary and moving charges. Magnetic force is proportional to velocity. Therefore, stationary charged particles don't experience magnetic force.

Differences between electric force and magnetic force

3. Electric force does work in displacing the particle. It can deliver/extract energy into/from the particle thereby increasing/decreasing the particle's velocity.

$$\vec{F}_e = q\vec{E} = m\vec{a}$$

$$\vec{a} = \frac{q\vec{E}}{m}$$

For constant acceleration

$$\vec{v} = \vec{v}_0 + \vec{a}t$$

$$W_e = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$$

Differences between electric force and magnetic force

Magnetic force does not deliver/extract energy into/from a charged particle. It can only change the direction of the velocity of the particle and not its magnitude.

Note that to change the energy of a particle we must change its kinetic energy which is proportional to $v^2 ; \frac{1}{2}mv^2$

$$dW_B = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 = \vec{F}_B \cdot d\vec{r} = \vec{F}_B \cdot \frac{d\vec{r}}{dt}dt$$

$$dW_B = \underbrace{\overbrace{\left(\vec{F}_B \cdot \vec{v}\right)}^{\vec{F}_B \perp \vec{v}}}_{0} dt = 0$$

$$v = v_0$$

Cyclotron Motion

$$\vec{F}_B = q\vec{v} \times \vec{B} \qquad\qquad \vec{F}_B \perp \vec{v} \;\&\; \vec{B}$$

The magnetic force on the particle is perpendicular to velocity. A force which is always perpendicular to velocity changes the direction of motion, by deflecting the particle sideways, but it cannot change the particle’s speed.

A particle moving perpendicular to a uniform magnetic field undergoes uniform circular motion at a constant speed. This is called Cyclotron motion.

A mathematical proof is given in the last four slides of this lecture.

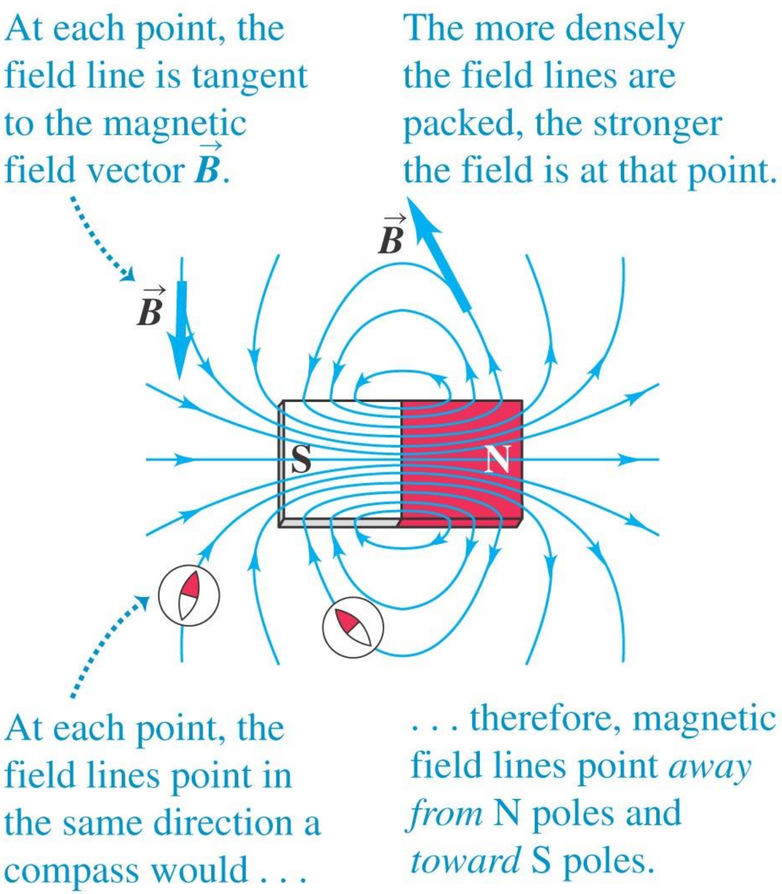
Differences between electric field and magnetic field lines

Magnetic field lines form closed loops. They have no beginning or ends.

$$\oint \vec{B} \bullet d\vec{A} = 0$$

Electric field lines originate from positive charge and terminate at negative charge.

$$\oint \vec{E} \bullet d\vec{A} = \frac{Q_{inside}}{\varepsilon_0}$$



Cyclotron Motion

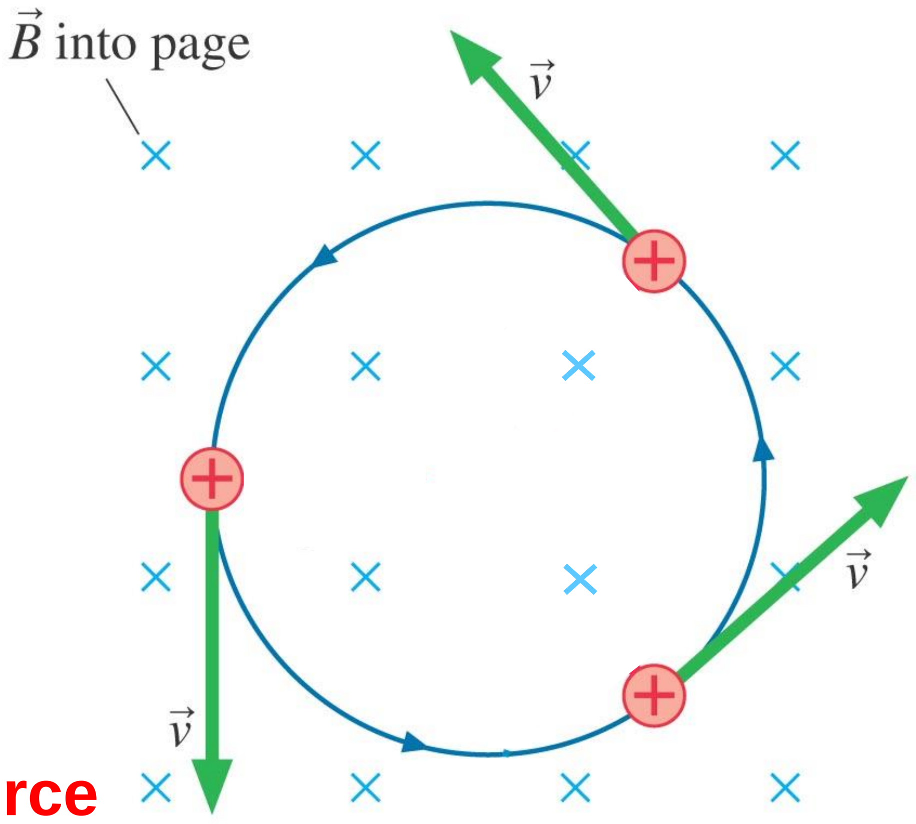
Charged particles in uniform magnetic fields undergo **uniform circular motion**. The radius of the circle depends on how fast the particle is moving:

$$\vec{F}_B = q\vec{v} \times \vec{B}$$

$$\left|\vec{F}_B\right| = \left|q\right| vB \sin\alpha = \left|q\right| vB$$

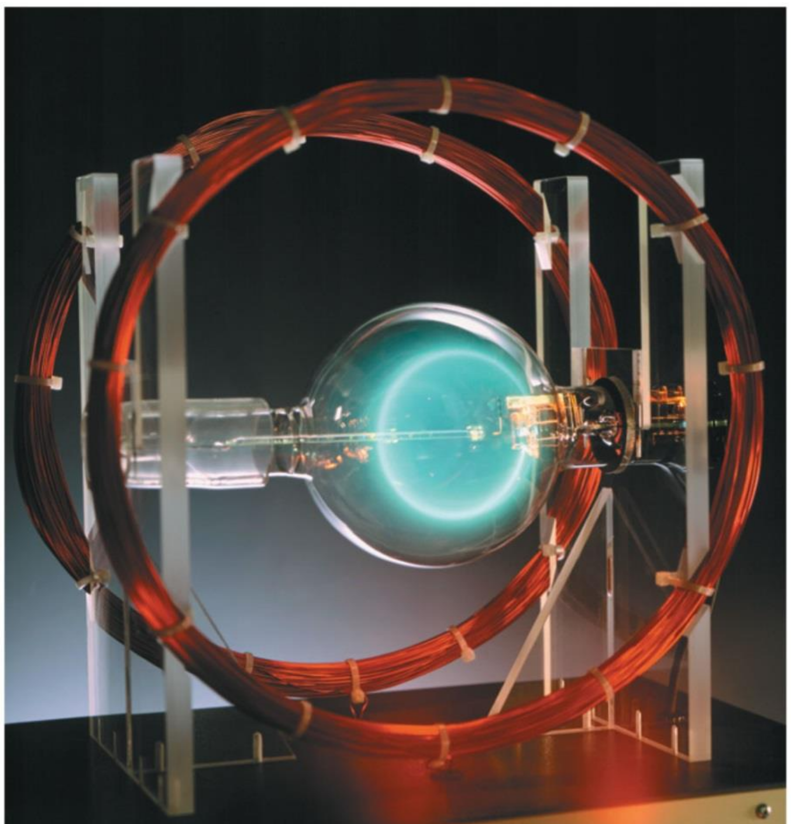
The magnetic force is the **net force**

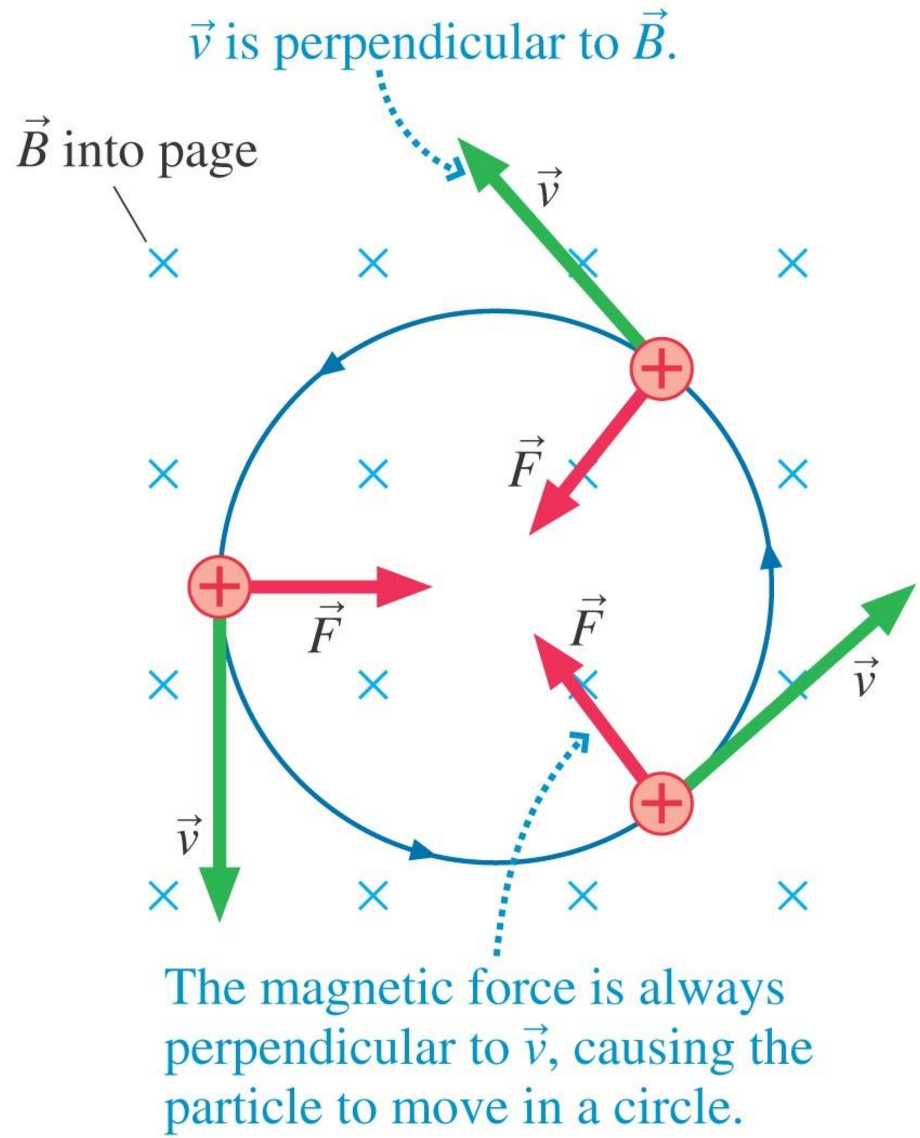
$$\left|\vec{F}_B\right| = m\frac{v^2}{R}$$



Cyclotron Motion

$$\left|\vec{F}_B\right| = |q|\cancel{v}B = m\frac{v^{\cancel{2}}}{R} \qquad \boxed{R = \frac{mv}{|q|B}}$$





Cyclotron Motion

$$v = \frac{2\pi R}{T_{cyc}}$$

T_{cyc} is the cyclotron period (time it takes to make one cycle)

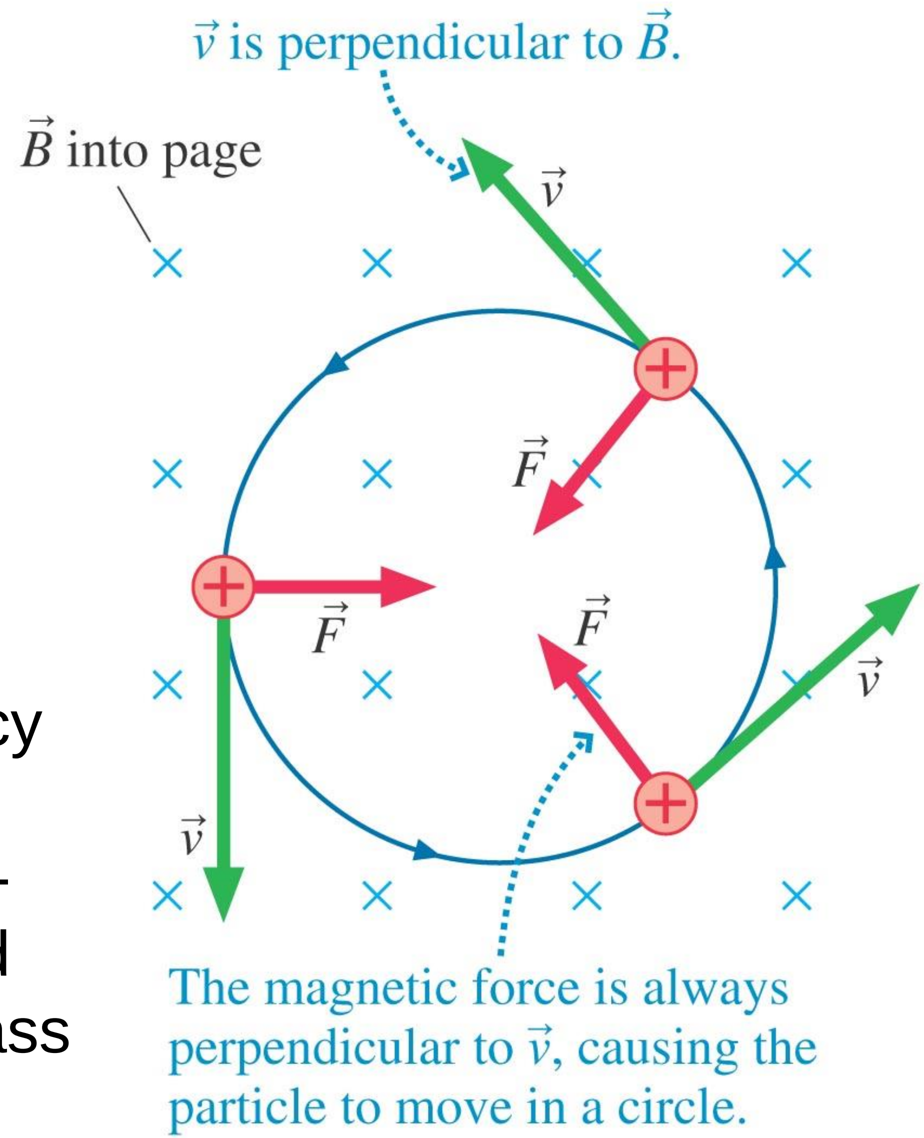
$$R = \frac{mv}{|q|B}$$

$$\cancel{R} = \frac{m}{|q|B}\frac{2\pi \cancel{R}}{T}$$

$$\boxed{T_{cyc} = \frac{2\pi m}{|q|B}}$$

$$\boxed{f_{cyc} = \frac{|q|B}{2\pi m}}$$

The period (and also the frequency of the cyclotron) depend on the B-field strength and the charge-to-mass ratio q/m

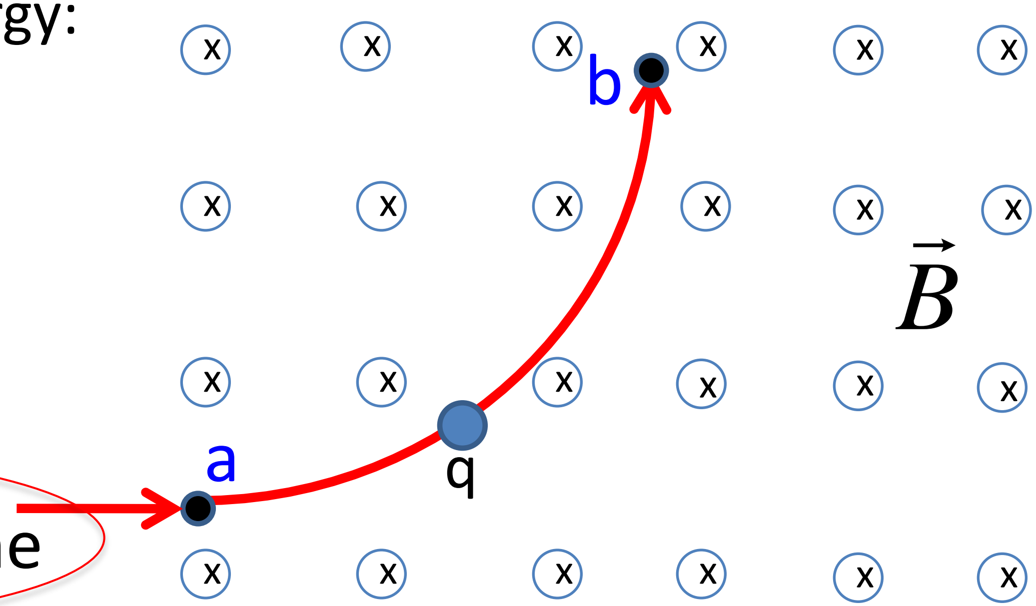


Top Hat Question

A charged particle q enters a region with a constant B-field pointing into the page. The force on the charged particle is

$$\boxed{\vec{F} = q\vec{v} \times \vec{B}}$$

As the particle travels from point **a** to point **b**, its kinetic energy:



A. Should increase

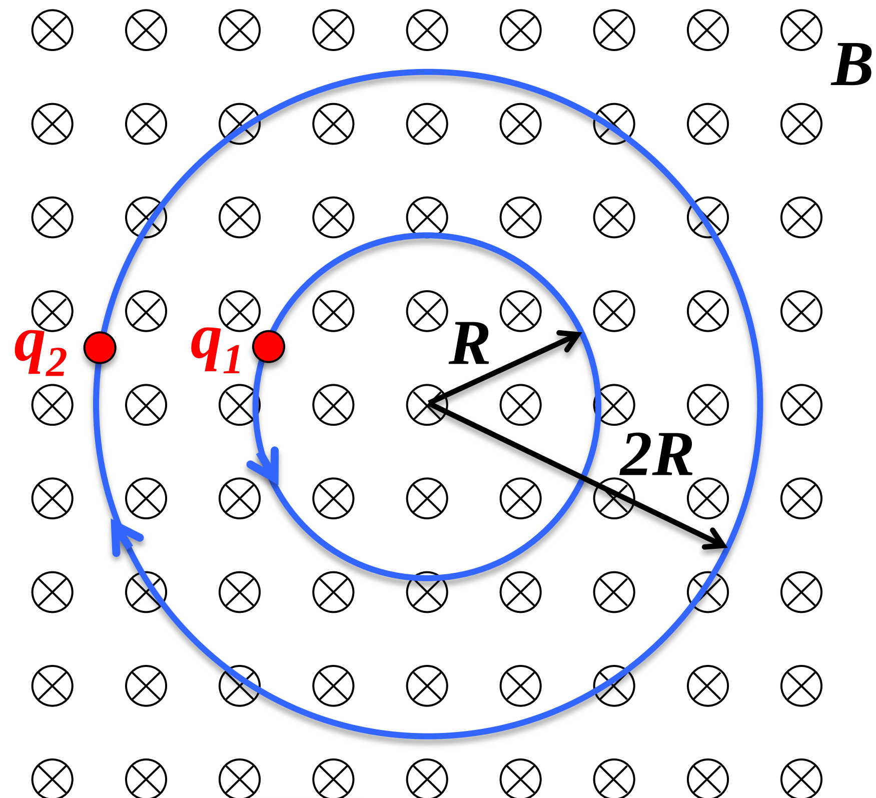
B. Should decrease

C. Should stay the same

D. Not enough info

Top Hat Question

Two charges q_1 and q_2 with the same mass m and the same magnitude of charge $|q|$ are undergoing cyclotron motion in a uniform B-field (into the page).



What are the **signs of the charges**?

A. Both positive

B. Both negative

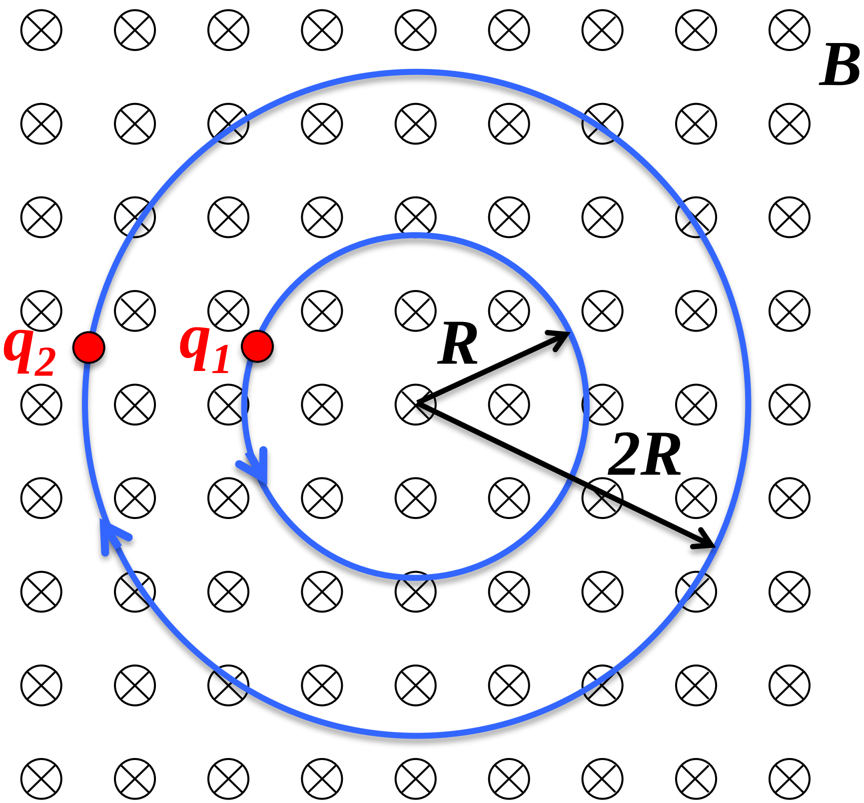
C. q_1 positive, q_2 negative

D. q_1 negative, q_2 positive

Top Hat Question

Two charges q_1 and q_2 with the same mass m and the same magnitude of charge $|q|$ are undergoing cyclotron motion in a uniform B-field (into the page).

If the speed of q_1 is v, what is the speed of q_2?



A. v

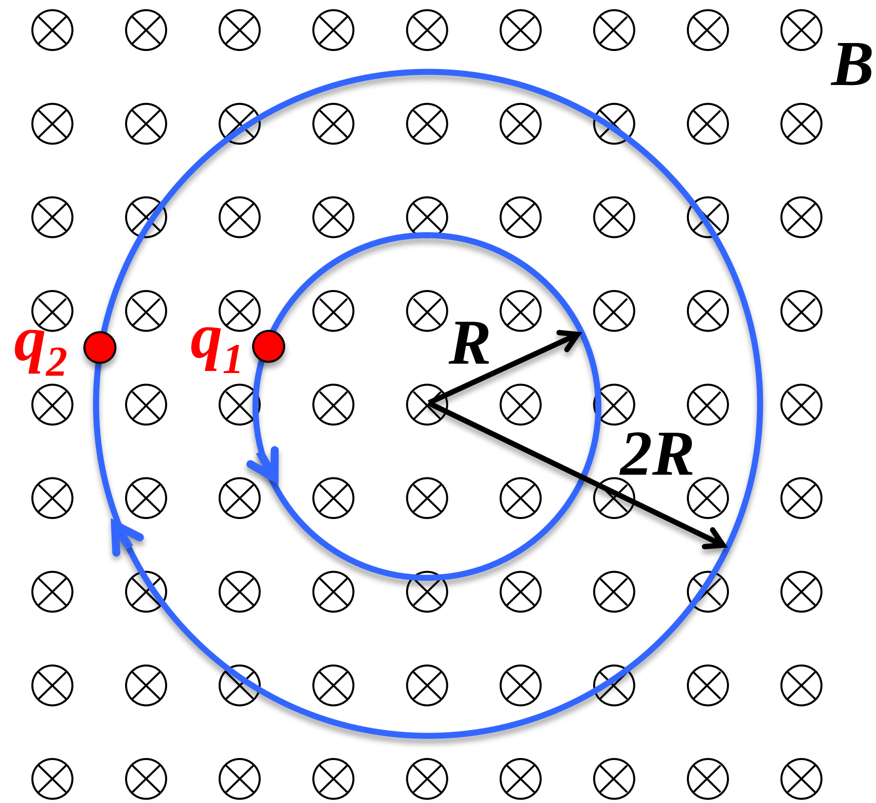
B. $2v$

C. $\frac{1}{2} v$

D. $4v$

Top Hat Question

Two charges q_1 and q_2 with the same mass m and the same magnitude of charge $|q|$ are undergoing cyclotron motion in a uniform B-field.



If the period of rotation of q_1 is T, what is the period of rotation of q_2?

A. T

B. $2T$

C. $\frac{1}{2}T$

D. $4T$

Lorentz force

Force on a charged particle moving through a region of space where both electric and magnetic fields are present.

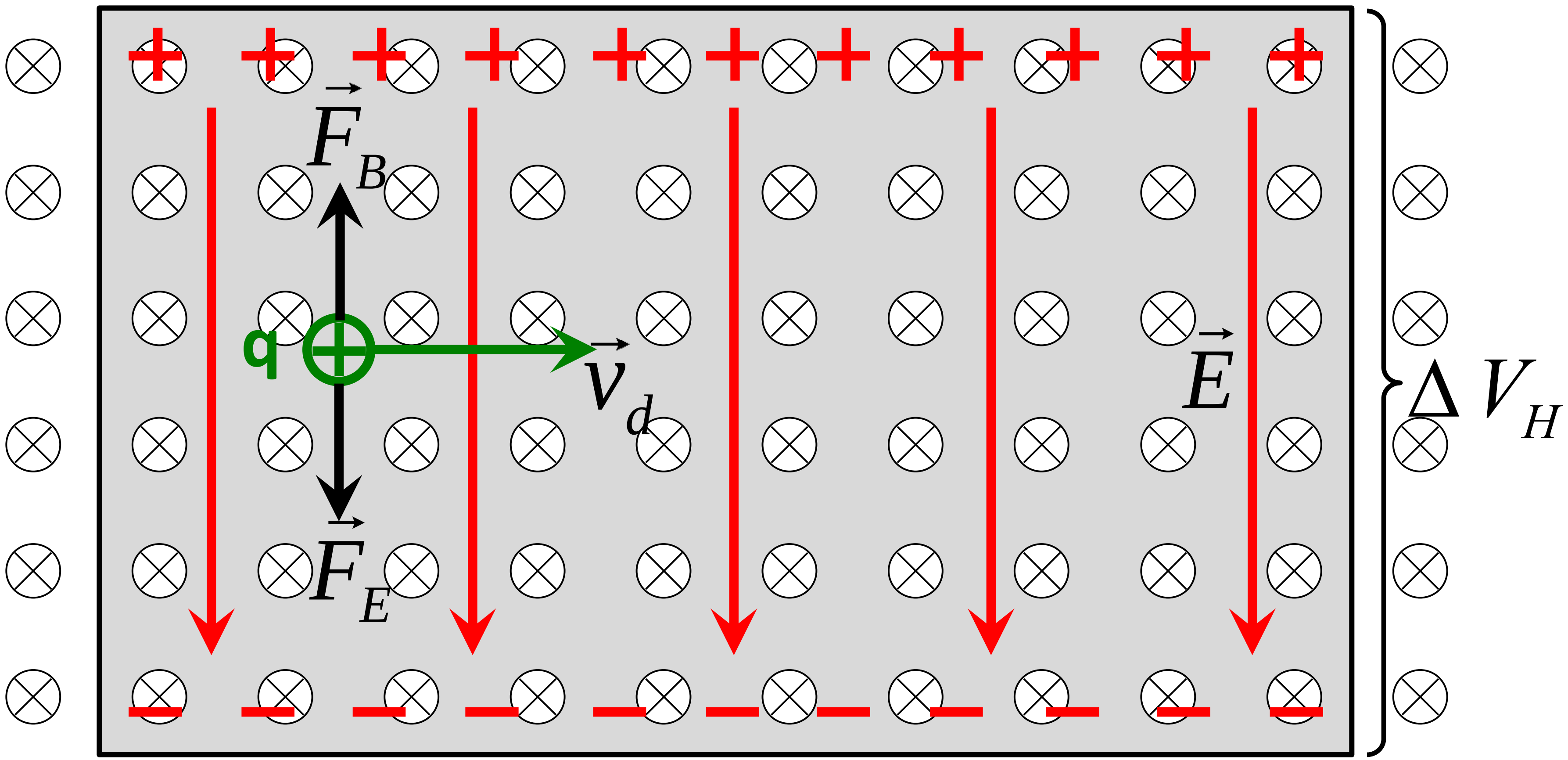
$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

Unit of magnetic field strength

1 tesla = 1 T =1 N/A m
1 Gauss = 10^{-4} T

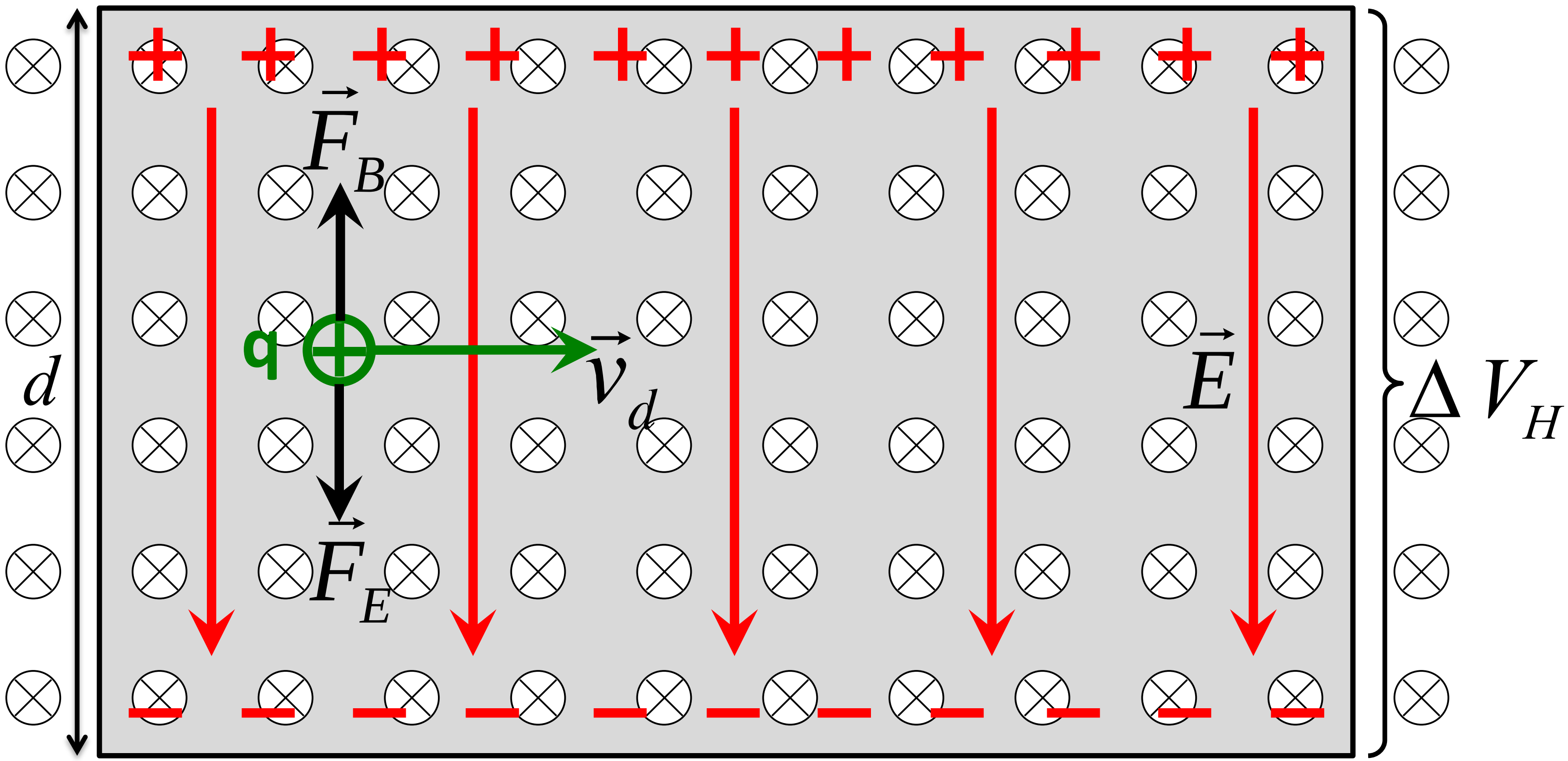
The Hall Effect

Due to the B-field, net charge build up on the edges.



In equilibrium, current still flows. Need to balance the magnetic and electric forces on the charge carriers.

The Hall Effect

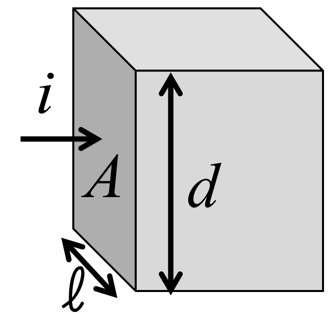


$$F_B = q v_d B \qquad F_E = q\frac{\Delta V_H}{d} \qquad q\frac{\Delta V_H}{d} = q v_d B \qquad \boxed{\Delta V_H = v_d B d}$$

The Hall Effect

We have just found that the voltage established across a conductor carrying a current in a magnetic field is

$$\Delta V_H = v_d B d$$



We previously related the drift speed to the current via

$$v_d = \frac{i}{neA}$$ where $A = \ell d$ and n is a material property

We can then relate the Hall voltage to known quantities:

$$\Delta V_H = \frac{i}{ne\ell \cancel{d}} B\cancel{d} = \frac{iB}{ne\ell}$$

In practical applications, you measure ΔV_H to find B:

$$\boxed{B = \frac{ne\ell}{i} \Delta V_H}$$

How the B-field probe used in the next lab works

Mathematical proof of cyclotron motion

Here we assume that velocity vector is in the xy-plane and magnetic field vector is along the positive z-axis.

$$\vec{F} = q\,\vec{v} \times \vec{B}$$

$$\vec{B} = B_z \hat{k}$$

$$\vec{v} = v_x \hat{i} + v_y \hat{j}$$

$$\vec{F} = q\left(v_x \hat{i} + v_y \hat{j}\right) \times B_z \hat{k}$$

$$\vec{F} = -q\,v_x B \hat{j} + q\,v_y B_z \hat{i}$$

$$F_x = q\,v_y B_z \Rightarrow m\frac{dv_x}{dt} = q\,v_y B_z \Rightarrow \frac{dv_x}{dt} = \frac{q}{m} v_y B_z$$

$$F_y = -q\,v_x B_z \Rightarrow m\frac{dv_y}{dt} = -q\,v_x B_z \Rightarrow \frac{dv_y}{dt} = -\frac{q}{m} v_x B_z$$

OR

$$\frac{d^2x}{dt^2} = \frac{q}{m}\frac{dy}{dt}B_z$$

$$\frac{d^2y}{dt^2} = -\frac{q}{m}\frac{dx}{dt}B_z$$

TRY

$$x = x_0 \cos(\omega t + \phi)$$
$$y = y_0 \sin(\omega t + \phi)$$

$$-x_0\omega^2\cos(\omega t+\phi) = q/m\, y_0 B_z \omega \cos(\omega t+\phi)$$

$$\omega = -\frac{q}{m}\frac{y_0}{x_0}B_z$$

SIMILARLY

$$-y_0\omega^2\sin(\omega t+\phi) = +\frac{q}{m}x_0\omega B_z \sin(\omega t+\phi)$$

$$\omega = -\frac{q}{m}\frac{x_0}{y_0}B_z$$

IF ① + ② ARE TO BE COMPATIBLE, THEN

$$\frac{y_o}{x_o} = 1 \Rightarrow x_o = y_o = a$$

$$x = a\cos(\omega t + \phi)$$

$$y = a\sin(\omega t + \phi)$$

$$x^2 = a^2\cos^2(\omega t + \phi)$$

$$y^2 = a^2\sin^2(\omega t + \phi)$$

$$\boxed{x^2 + y^2 = a^2}$$ EQUATION OF A CIRCLE.

SINCE

$$\omega = -\frac{q}{m}\frac{x_o}{y_o}B_z \quad + \quad x_o = y_o$$

$$\boxed{\omega = \frac{q}{m}B_z}$$