
Fill in these boxes with your SURNAME, a space, and your INITIALS.

University of Calgary

Faculty of Science

Final Exam Test - Solutions

PHYSICS 259 ALL LECTURE SECTIONS

April 21 2014, 12:00-15:00 **Time: 3 hours.**

DO NOT TEAR OFF THIS PAGE!

Answer all questions. There is an equation sheet and table of integrals on the last two pages. You may tear these two pages off if you wish.

This is a closed-book exam worth a total of 52 points. Use of the Schulich calculator or equivalent is allowed.

Write your Last Name and Initial on this top sheet in the grid above. (Do not write your ID number on this page.) **Also write your ID in the grid at top right of Page 2 of the Question paper. DO THIS NOW**.

Make sure this question paper booklet contains 27 pages. If you are missing any pages, get a new booklet from the exam supervisor.

You should also have a **separate set of Answer Sheets**. This is where you enter Multiple Choice answers of Part I and also detailed solutions to the problems of Part II. Only work entered in the indicated spaces on the Answer Sheets will be marked.

IMPORTANT: YOUR ID NUMBER IS TO BE ENTERED AT THE TOP OF EACH AND EVERY ONE OF THE ANSWER SHEETS. DO THIS NOW.

Begin working on the examination when instructed to do so by the supervisor.

UCID: | | | | | | | |

Part I: Multiple-Choice Questions (Total: 25 marks)

Enter answers to multiple choice questions on the first Answer Sheet using space provided in the upper right of the page. Each question in Part I is worth one point.

1) Two equal positive point charges, each of charge q, are separated by distance a. There are no other charges anywhere. The potential at the midpoint of the line joining the charges is defined to be zero. The electrostatic potential at an infinite distance from the two charges is

a) $-\frac{1}{4\pi\epsilon_0}\frac{2q}{a}$

b) $+\frac{1}{4\pi\epsilon_0}\frac{2q}{a}$

c) $-\frac{1}{4\pi\epsilon_0}\frac{4q}{a}$ $\Leftarrow \checkmark$

d) $+\frac{1}{4\pi\epsilon_0}\frac{4q}{a}$

e) Zero

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2) When the switch S in Figure 2 is closed, the current in the circuit reaches a value of 0.015 A (amperes) after a time of 0.040 s. What is the EMF of the battery? (Select the closest answer, to two significant figures.)

a) 8.9 V
b) 2.3 V $\Leftarrow \checkmark$
c) 1.8 V
d) 1.6 x 10 V
e) 0.040 V

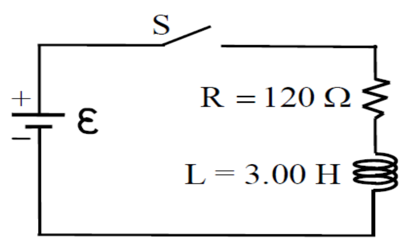

Figure 2: Battery, inductor, and resistor.

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3) The electric potential in a particular region of space is given by $V = 5x^2 - 3y^2$. What are the magnitude and direction of the electric field at the point (x,y) = (3 m, 3 m)?

a) 35 N/C at 31° below the $+x$ direction
b) 35 N/C at 31° above the $-x$ direction $\Leftarrow \checkmark$
c) 35 N/C at 59° below the $-x$ direction
d) 35 N/C at 59° above the $-x$ direction
e) 35 N/C at 59° below the $+x$ direction

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4) A jet airliner with a wingspan of 30 m is flying in a direction perpendicular to the magnetic field lines of the Earth. The jet's speed is 800 km/hour and the Earth's magnetic field strength is 5×10^{-4} T. What EMF is developed between the wingtips? Pick the closest answer.

a) 100 millivolts
b) 400 millivolts
c) 950 millivolts
d) 2.5 volts
e) 3.5 volts $\Leftarrow \checkmark$

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5) In Figure 5, two particles of the same charge q are executing cyclotron motion in a region of uniform magnetic field B directed into the page. The upper particle has mass m and travels at speed v. The lower particle has mass $2m$ and travels at speed $2v$. If the period of motion of the upper particle is T, then the period of motion of the lower particle is

a) T/2
b) T/4
c) T
d) 2T $\Leftarrow \checkmark$
e) 4T

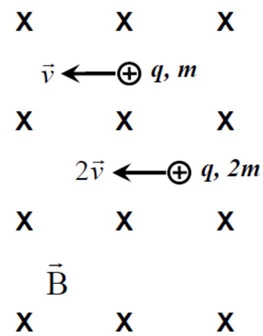

Figure 5: Two charges in a uniform magnetic field.

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6) You have a conducting sphere to which you are adding excess electrons. The air around the sphere will become conducting if the electric field strength exceeds 3.0×10^6 N/C. If you wish to attain an electric potential on the sphere of 900,000 volts, how large can the sphere be?

a) The radius of the sphere must be 0.30 m, no bigger and no smaller. $\Leftarrow \checkmark$
b) The radius of the sphere can be 0.30 m or bigger.
c) The radius of the sphere can be 0.60 m or smaller.
d) It depends on whether the sphere is solid or just a thin shell.
e) The radius of the sphere must be 0.60 m, no bigger and no smaller.

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7) Figure 7 shows a cube of side 4.00 m with its base at $z = 3.00$ m and its top at $z = 7.00$ m above the x,y plane. In the region of space shown, there is an electric field directed upwards with a magnitude given by $E_z = az^2$, where $a = 2.00 V \cdot m^{-3}$. How much charge is contained within the cube? Select the closest answer.

a) 7.08×10^{-10} C
b) 1.13×10^{-8} C $\Leftarrow \checkmark$
c) 1.74×10^{-7} C
d) 4.77×10^{-7} C
e) 5.66×10^{-9} C

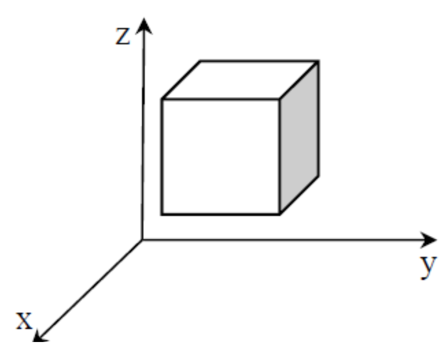

Figure 7: Cube in an electric field.

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8) A small object of mass 2.5×10^{-6} kg and charge -5×10^{-6} C is moving at a constant speed of 3.2×10^5 m/s in the -z direction. It enters a region where there is an electric field E and it experiences an acceleration of 4.0×10^5 m/s^2 in the -x direction. What is the electric field?

a) 6.4×10^{10} N/C in the +x direction.
b) 6.4×10^{10} N/C in the -z direction.
c) 2.0×10^5 N/C in the +x direction. $\Leftarrow \checkmark$
d) 2.0×10^5 N/C in the -z direction.
e) 2.0×10^{17} N/C in the -x direction.

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9) In Figure 9, a particle of negative charge q is fired into a velocity selector (a region of crossed electric and magnetic fields), with the electric field is directed downwards between two charged parallel metal plates, and the magnetic field (denoted by the symbol X) is directed into the page. The velocity selector allows particles of one specific speed, $v_0 = E/B$, to pass through undeflected (in a straight line), but this particle is traveling faster than the required speed (i.e., $v > v_0$). What happens to this particle?

a) It is deflected toward the bottom of the page. $\Leftarrow \checkmark$
b) It is deflected toward the top of the page
c) It is deflected out of the page
d) It is deflected into the page
e) It is slowed until its speed equals the required v_0

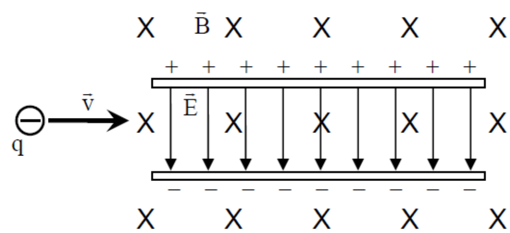

Figure 9: A velocity selector.

10) A small solid sphere (Figure 10a) has been given a uniform positive charge Q. The electric potential at the surface of this sphere is then V_1, relative to $V = 0$ at infinity. A thick uncharged hollow metal sphere (stippled in Figure 10b) is then placed around the small, charged, solid sphere, without touching it. In this new configuration shown in Figure 10b, the electric potential at the surface of the small, charged, solid sphere (relative to V=0 at infinity) is

a) zero.
b) less than before (but not zero). $\Leftarrow \checkmark$
c) the same as it was before.
d) greater than before (but not infinite).
e) infinite.

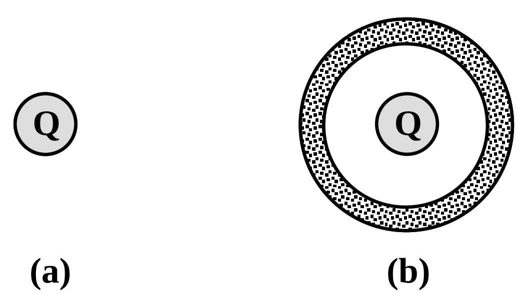

Figure 10 a) Charged solid sphere. b) thick uncharged hollow metal sphere.

11) One side of a thin, insulating sheet (vertical line in Figure 11) carries a uniform surface charge density $\sigma = +35.4\,\mathrm{nC/m^2}$. There are also charges outside the figure (not shown) that influence the electric field in the figure. The electric field, $\vec{E}_1$, to the left of the plate is 5,000 N/C, and is directed toward the right. The electric field, $\vec{E}_2$, to the right of the plate is closest to

a) 6,000 N/C toward the left.
b) 6,000 N/C toward the right.
c) 7,000 N/C toward the right.
d) 7,000 N/C toward the left.
e) 9,000 N/C toward the right. $\Leftarrow \checkmark$

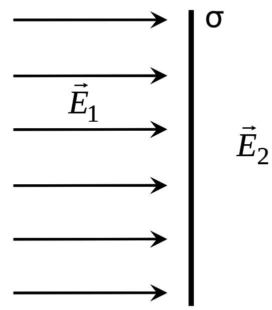

Figure 11 Insulating charge sheet with uniform surface charge density.

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12) The magnetic field strength in the analyzer portion of a particular Bainbridge mass spectrometer is 0.280 T. Singly-ionized selenium ions produced by the velocity selector have speeds of 1.55×10^4 m/s. These ions follow a half-circular path in the uniform magnetic field and strike the ion detector at a distance of 8.49 cm away where they entered the analyzer. The mass of each selenium ion, in unified atomic mass units ($1\ \mathrm{u} = 1.67 \times 10^{-27}$ kg), is closest to

a) 74.0 u. $\Leftarrow \checkmark$
b) 37.0 u.
c) 148 u.
d) 122 u.
e) 56.8 u.

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13) Figure 13 shows a Hall probe 5.0 cm in length, 1.8 cm in width and 0.12 cm in thickness in a 0.40 T magnetic field directed upward. The drift speed of the electrons in the Hall probe is 0.82 mm/s. The strength of the electrostatic field generated in the Hall probe is (select the closest answer)

a) $5.2 \times 10^{-23}\, V/m$
b) $6.7 \times 10^{-6}\, V/m$
c) $3.3 \times 10^{-4}\, V/m$ $\Leftarrow \checkmark$
d) $1.4 \times 10^{-2}\, V/m$
e) Zero

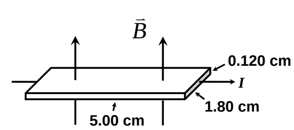

Figure 13 Hall probe in magnetic field.

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14) Figure 14 shows two capacitors that are initially not connected. The stored energy in capacitor $C_1 = 12mF$ is 96.0 mJ, and capacitor $C_2 = 3mF$ is uncharged. Now suppose the capacitors are connected together as indicated by the arrows. What will the final stored energy in the combination of the two capacitors be?

- **a)** 384 mJ
- **b)** 96.0 mJ
- **c)** 76.8 mJ ⇐ ✓
- **d)** 24.0 mJ
- **e)** 120 mJ

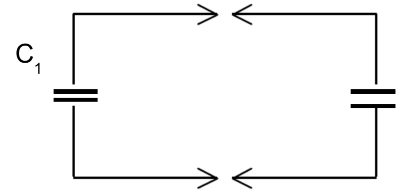

Figure 14 Two capacitors.

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15) Figure 15 shows an electron initially moving to the right when it enters a uniform electric field directed upwards. Which trajectory shown in the figure will the electron follow?

- **a)** Trajectory V
- **b)** Trajectory W
- **c)** Trajectory X
- **d)** Trajectory Y
- **e)** Trajectory Z ⇐ ✓

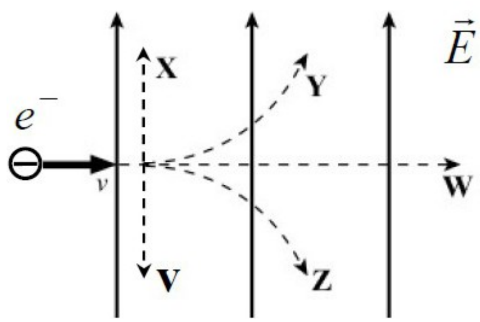

Figure 15 Electron moving in a uniform electric field.

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16) The circles in Figure 16 represent three identical light bulbs. When the switch, S, is closed, what happens to the two bulbs A and B?

- **a)** A remains at the same brightness while B becomes dimmer.
- **b)** Both A and B become brighter.
- **c)** Both A and B become dimmer.
- **d)** A becomes dimmer and B becomes brighter.
- **e)** A becomes brighter and B becomes dimmer. ⇐ ✓

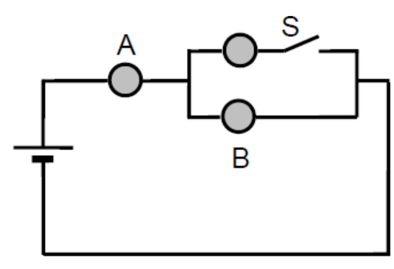

Figure 16 Battery, switch, and three lightbulbs.

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17) Four very long, current-carrying wires in the same plane intersect to form a square 20.0 cm on each side, as shown in Figure 17. The currents in three of the wires are specified in the diagram. What magnitude and direction of current, I, is required in the fourth wire so that the magnetic field at the centre of the square is zero?

a) I = 4.0 A, toward the top of the page
b) I = 2.0 A, toward the top of the page
c) I = 4.0 A, toward the bottom of the page
d) I = 2.0 A, toward the bottom of the page ⇐ ✓
e) I = 12.0 A, toward the top of the page

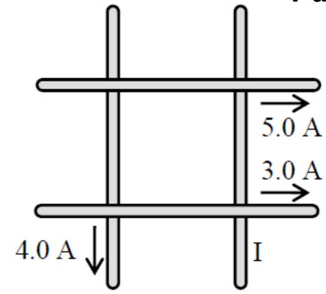

Figure 17 Four current-carrying wires.

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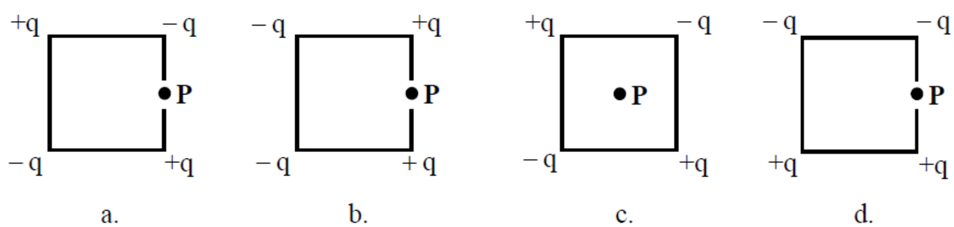
18) Four charges of equal magnitude but opposite sign are arranged at the corners of a square, as shown here. For which arrangement is the magnitude of the electric field at point P the largest? ⇐ ✓ D



e) The largest electric field occurs in more than one of these arrangements

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19) The marks in Figure 19 on both the x and y axes are 0.200 m apart. A charge $q_1 = +27.0$ nC is located at the origin, and a charge $q_2 =$ -27.0 nC is located at (x,y) = (0.800, 0) m. Points A and B are located at (x,y) = (0, 0.600) m and (0.800, 0.600) m, respectively. If a proton (charge = +e) is transported from point A to point B as indicated by the dashed arrow, the work done by the electric force is

a) +324 eV ⇐ ✓
b) 0
c) -864 eV
d) +864 eV
e) -134 eV

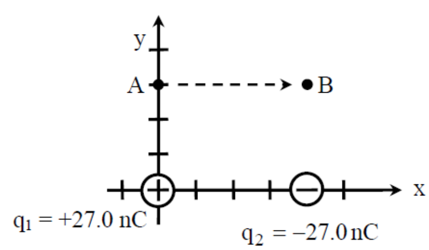

Figure 19 Two charges.

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20) Figure 20 shows a portion of two parallel, metal plates a distance of 1.50 cm apart. (Assume the plates are actually large enough to be considered infinite.) A proton (charge q = +e, mass m = 1.67×10^{-27} kg) leaves the left-hand plate with a speed of 1.25×10^5 m/s and arrives at the right-hand plate with a speed of 3.20×10^5 m/s. The electric field strength, E, between the plates is closest to

a) 450 V/m.
b) 0.07 V/m.
c) 3.0×10^4 V/m. $\Leftarrow\checkmark$
d) 5400 V/m.
e) 1.3×10^4 V/m.

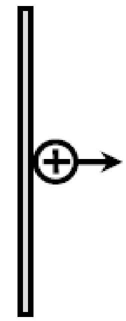

Figure 20 Proton moving between two charged plates.

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21) Three capacitors are connected to a battery of potential difference V, as shown in the Figure 21. The charge on capacitor C_1 is q_1 = 270 nC. The charge on capacitor C_2 is closest to

a) 27 nC.
b) 405 nC. $\Leftarrow\checkmark$
c) 270 nC.
d) 1160 nC.
e) 1 nC.

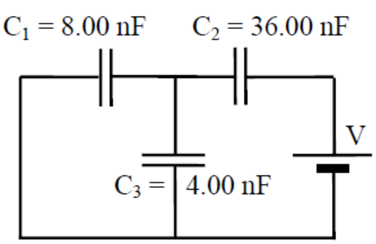

Figure 21 Three capacitors and an ideal battery.

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22) Figure 22 shows a capacitor C containing a dielectric of K = 4, with a capacitance of $25.0\mu F$ (with the dielectric present). V = 12 V, and the capacitor is fully charged. If the dielectric is pulled completely out of the capacitor while the battery remains connected, how much charge flows through the battery?

a) 300 μC
b) 1200 μC
c) 900 μC
d) 225 μC $\Leftarrow\checkmark$
e) 625 μC

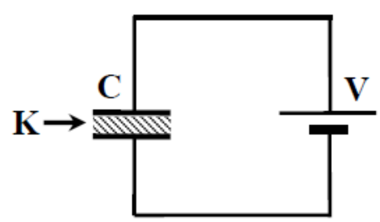

Figure 22 Capacitor and dielectric.

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23) The potential difference across the circuit in Figure 23 is 48.0 V. If we close the switch, S, and leave it closed for a long time, what will be the charge on capacitor C_2? Select the closest answer.

- **a)** 115 μC
- **b)** 144 μC $\Leftarrow \checkmark$
- **c)** 192 μC
- **d)** 480 μC
- **e)** 720 μC

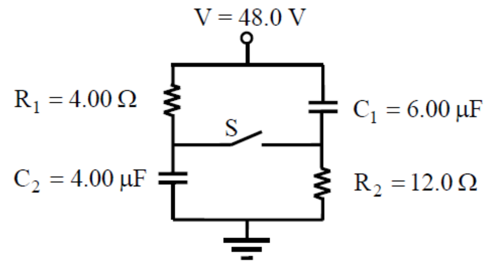

Figure 23 Capacitors and resistors.

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24) Figure 24 shows a wire carrying a strong current I through many parallel segments. The force between long segments of wire

- **a)** tends to pull the segments together.
- **b)** tends to push segments apart. $\Leftarrow \checkmark$
- **c)** is perpendicular to the page: out of the page for currents upwards and into the page for currents downwards.
- **d)** is zero, because a current segment feels no force from a $\vec{B}$ field produced by the same wire.
- **e)** is zero, because the force between wire segments is proportional to the sine of the angle between $\vec{\ell}$ vectors (where $\vec{\ell}$ is a vector parallel to the wire in the direction of the current).

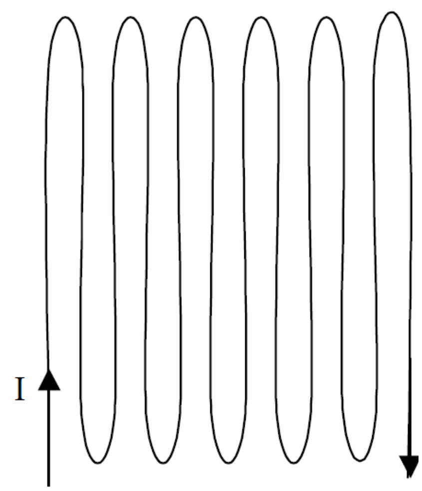

Figure 24 A long wire that is not in a straight line.

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25) Figure 25a shows a portion of a thin but very long and wide conducting sheet of thickness t. (For points close to the surface, this may be regarded as an infinite current sheet of thickness t.) The current density $\vec{J}$ vectors point as shown. Figure 25b is a view of the same sheet from directly in front; the dotted circles represent current directed out of the page toward the viewer. The magnetic field lines are parallel to the sheet, as indicated, with directions given by the right-hand rule. What is the magnetic field strength at point P, a distance z above the centre of the sheet?

a) $\dfrac{\mu_0 J}{4\pi}$

b) $\dfrac{\mu_0 J t}{2z}$

c) $\dfrac{\mu_0 J t}{2}$ $\Leftarrow \checkmark$

d) $\dfrac{\mu_0 J z}{4\pi t}$

e) $\dfrac{\mu_0 J t}{4\pi z}$

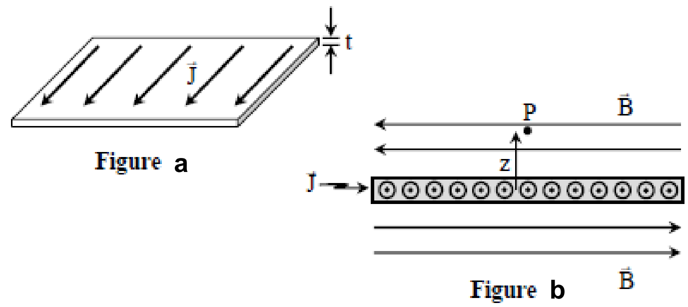

Figure 25 A current sheet.

This is the end of the Multiple Choice part of the exam. You may now proceed to Part II. Remember that your answers to Part II are to be entered on the Answer Sheet pages.

Part II: Written Answer Questions (Total: 25 marks)

IMPORTANT: Write your answers to the problems in Part II in the corresponding boxes on the Answer Sheets. Work must be shown for full marks. Rough work can be done on the back of this question paper, but only the work appearing on the Answer Sheets will be marked.

26) *[7 points]* Figure 26 (not to scale) shows the cross-section of a very long, solid, non-conducting circular cylinder of radius R_1 containing uniform positive volume charge density ρ_0. It is surrounded by a co-axial conducting cylindrical tube of inner radius $R_2 = 4R_1$ and outer radius $R_3 = 5R_1$. The net charge on the outer conductor is zero. A radial coordinate r is used to measure distance perpendicularly outward from the axis of symmetry of this arrangement.

a) Use Gauss's Law to derive an expression for the magnitude of the electric field for r in the range $0 < r < R_1$.

b) Derive an expression for the magnitude of the electric field for $R_1 < r < R_2$.

c) Determine the charge per unit area σ_2 on the inside surface of the conductor (your answer will depend on ρ_0 and other symbolic constants).

d) If the potential at $r = 0$ is 0 V, determine the potential at $r = R_1$ (again, your answer will depend on ρ_0 and other symbolic constants).

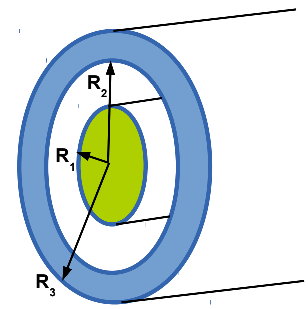

Figure 26 Insulating cylinder inside a conducting cylindrical shell.

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27) *[8 points]* A distant relative in Sri Lanka (near the equator) emails to ask your help with their design of a magnetic SkyHook™. Their goal is to lift objects (people, cars etc.) with the power of magnetism. They would like your help building a demonstration system from some copper wire and batteries. Assume that the magnetic field is horizontal and points directly North, but the strength decreases with height according to the formula $B(z) = B_0 - az$ where B_0 is the magnetic field near the ground ($z = 0$) and $a = 0.022$ nanoTesla per meter.

a) What is the first derivative of magnetic field with respect to height dB/dz?

b) Given the (slightly) non-uniform magnetic field, how could you arrange a $1m \times 1m$ square current loop to provide the largest vertical force? Draw axes, the loop, current, and the forces F on each side of the loop.

c) Calculate the maximum net force on a single loop for a 1 A current. What mass could this lift if $g = 9.81 m/s^2$?

d) How would the force change if the square loop is bent out into a circle (with the same perimeter)?

e) During testing one of the loops is dropped from an airplane and reaches a maximum speed of 90 m/s before crashing into the ground. Calculate the largest EMF that could have been induced.

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28) *[5 points]* An electron with mass m_e and charge $-q$ is moving parallel to the x-axis with speed $v = 500m/s$. A proton with mass m_p and charge $+q$ is also moving parallel to the x-axis with the same speed. The proton is located $d = 3$ metres away the electron in the $\hat{j}$ direction.

a) Draw and label a figure showing charge locations and velocities in a Cartesian coordinate system.

b) What is the electric field (magnitude and direction) due to the proton at the location of the electron?

c) What is the magnetic field (magntitude and direction) due to the electron at the location of the proton?

d) What is the electric force acting on the electron?

e) What is the magnetic force acting on the proton?

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29) *[5 points]* Figure 29 (not to scale) shows the cross-section of a very long, solid, cylindrical conductor of radius R_1. It is surrounded by a co-axial conducting cylindrical tube of inner radius $R_2 = 4R_1$ and outer radius $R_3 = 5R_1$. A current I flows along the central conductor (into the page) and back along the outer conductor (out of the page). The current density J is uniformly distributed across each conductor.

a) Derive expressions for the current density J for the inner conductor and the outer conductor.

b) Use Ampere's law to derive an expression for the magnetic field $\vec{B}$ for r in the range $r > R_3$.

c) Derive an expression for the magnetic field for $R_1 < r < R_2$.

d) Derive an expression for the magnetic field for $0 < r < R_1$.

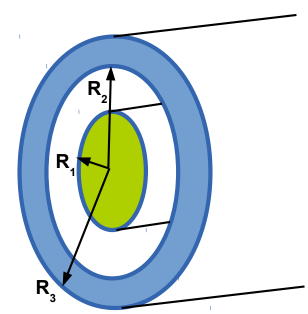

Figure 29 Conducting cylinder inside a conducting cylindrical shell.

CONSTANTS AND USEFUL EQUATIONS

k = Coulomb constant = 8.99×10^{9} N m^2 C^{-2}
ϵ_0 = permittivity of free space = 8.85×10^{-12} C^2 N^{-1} m^{-2}
μ_0 = permeability of free space = $4\pi \times 10^{-7}$ Wb A^{-1} m^{-1}
e = fundamental charge = 1.602×10^{-19} C
m_e = mass of electron = 9.11×10^{-31} kg
m_p = mass of proton = 1.67×10^{-27} kg
K_{air} = dielectric constant of air = 1.00059

m = 10^{-3} $\mu = 10^{-6}$ n = 10^{-9} p = 10^{-12}

Area of a circle: $A = \pi r^2$
Surface area of a sphere: $A = 4\pi r^2$
Surface area of a cylinder: $A = 2\pi r L$
Circumference of a circle: $C = 2\pi r$
Volume of a sphere: $V = \frac{4}{3}\pi r^3$
Volume of a cylinder: $V = \pi r^2 L$

$$x = x_0 + v_{x0}t + \frac{1}{2}a_x t^2 \qquad v_x = v_{x0} + a_x t \qquad v_x^2 = v_{x0}^2 + 2a_x x \qquad \vec{F} = m\vec{a}$$

$$\vec{F} = k\frac{q_1 q_2}{r^2}\,\hat{r} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}\,\hat{r} \qquad \vec{E} = \frac{\vec{F}}{q} \qquad \vec{E} = k\frac{q}{r^2}\,\hat{r} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\,\hat{r}$$

$$\Phi_E = \oint_A \vec{E}\cdot d\vec{A} = \oint_A E dA cos\theta = \frac{Q_{encl}}{\epsilon_0} \qquad V = \frac{U}{q} \qquad U = k\frac{q_1 q_2}{r} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r}$$

$$V = k\frac{q}{r} = \frac{1}{4\pi\epsilon_0}\frac{q}{r} \qquad W = qV_{ab} \qquad V_{ab} = V_a - V_b = \int_a^b \vec{E}\cdot d\vec{l}$$

$$\vec{E} = -\vec{\nabla}V = -\left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)V \qquad C = \frac{Q}{V_{ab}} \qquad C = \frac{\epsilon_0 A}{d}$$

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}QV = \frac{1}{2}CV^2 \qquad \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \qquad C = C_1 + C_2 + C_3$$

$$C = KC_0 = K\epsilon_0\frac{A}{d} = \epsilon\frac{A}{d} \qquad u = \frac{1}{2}\epsilon_0 E^2 \qquad P = I^2 R = \frac{V^2}{R}$$

More Equations on Next Page

CONSTANTS AND USEFUL EQUATIONS (Continued)

$$V = IR \qquad P = \mathcal{E}I \qquad \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \qquad R = R_1 + R_2 + R_3$$

$$R = \frac{\rho L}{A} \qquad \sum i = 0 \qquad \sum (\mathcal{E} + iR) = 0 \qquad nq = -\frac{J_x B_y}{E_z} \qquad \vec{J} = \sum_i n_i q_i \vec{v}_i$$

$$\vec{F} = q\vec{v} \times \vec{B} \qquad \Phi_B = \int \vec{B} \cdot d\vec{A} \qquad r = \frac{mv}{qB} \qquad \vec{F} = I\vec{l} \times \vec{B} \qquad \vec{\mu} = I\vec{A}$$

$$\vec{\tau} = \vec{\mu} \times \vec{B} \qquad \vec{B} = \frac{\mu_0}{4\pi} \frac{q\vec{v} \times \hat{r}}{r^2} \qquad d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{l} \times \hat{r}}{r^2} \qquad U = -\vec{\mu} \cdot \vec{B}$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl} \qquad \mathcal{E}_2 = -N_2 \frac{d\Phi_{B2}}{dt} \qquad \mathcal{E}_2 = -M \frac{di_1}{dt} \qquad \mathcal{E} = -L \frac{di}{dt}$$

$$\mathcal{E} = \int_a^b \left(\vec{v} \times \vec{B}\right) \cdot d\vec{l} \qquad M = \frac{N_2 \Phi_2}{i_1} = \frac{N_1 \Phi_1}{i_2} \qquad L = \frac{N\Phi}{i} \qquad u = \frac{B^2}{2\mu_0}$$

$$\tau = RC \qquad \tau = \frac{L}{R} \qquad x = x_0 e^{-\frac{t}{\tau}} \qquad x = x_0 \left(1 - e^{-\frac{t}{\tau}}\right) \qquad U = \frac{1}{2} L I^2$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} \qquad \int \frac{dx}{\sqrt{x^2 + a^2}} = ln\left(x + \sqrt{x^2 + a^2}\right)$$

$$\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \arctan \frac{x}{a} \qquad \int \frac{dx}{\left(x^2 + a^2\right)^{3/2}} = \frac{1}{a^2} \frac{x}{\sqrt{x^2 + a^2}}$$

$$\int \frac{x dx}{\left(x^2 + a^2\right)^{3/2}} = -\frac{1}{\sqrt{x^2 + a^2}}$$

Multiple Choice Answer Sheet - Version A

#1 c
#2 b
#3 b
#4 e
#5 d

#6 a
#7 b
#8 c
#9 a
#10 b

#11 e
#12 a
#13 c
#14 c
#15 e

Paul's bubble sheet goes here

#16 e
#17 d
#18 d
#19 a
#20 c

#21 b
#22 d
#23 b
#24 b
#25 c

Part II: Written Answer Questions (Total: 25 marks)

IMPORTANT: Write your answers to the problems in Part II in the corresponding boxes on the Answer Sheets. Work must be shown for full marks. Rough work can be done on the back of this question paper, but only the work appearing on the Answer Sheets will be marked.

30) *[7 points]* Figure 30 (not to scale) shows the cross-section of a very long, solid, non-conducting circular cylinder of radius R_1 containing uniform positive volume charge density ρ_0. It is surrounded by a co-axial conducting cylindrical tube of inner radius $R_2 = 4R_1$ and outer radius $R_3 = 5R_1$. The net charge on the outer conductor is zero. A radial coordinate r is used to measure distance perpendicularly outward from the axis of symmetry of this arrangement.

a) Use Gauss's Law to derive an expression for the magnitude of the electric field for r in the range $0 < r < R_1$.

b) Derive an expression for the magnitude of the electric field for $R_1 < r < R_2$.

c) Determine the charge per unit area σ_2 on the inside surface of the conductor (your answer will depend on ρ_0 and other symbolic constants).

d) If the potential at $r = 0$ is 0 V, determine the potential at $r = R_1$ (again, your answer will depend on ρ_0 and other symbolic constants).

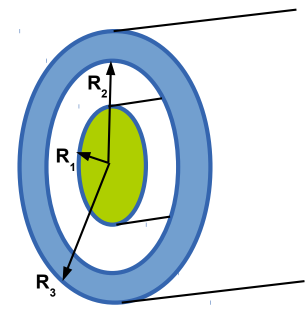

Figure 30 Insulating cylinder inside a conducting cylindrical shell.

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31) *[8 points]* A distant relative in Sri Lanka (near the equator) emails to ask your help with their design of a magnetic SkyHook^{TM}. Their goal is to lift objects (people, cars etc.) with the power of magnetism. They would like your help building a demonstration system from some copper wire and batteries. Assume that the magnetic field is horizontal and points directly North, but the strength decreases with height according to the formula $B(z) = B_0 - az$ where B_0 is the magnetic field near the ground ($z = 0$) and $a = 0.022$ nanoTesla per meter.

a) What is the first derivative of magnetic field with respect to height dB/dz?

b) Given the (slightly) non-uniform magnetic field, how could you arrange a $1m \times 1m$ square current loop to provide the largest vertical force? Draw axes, the loop, current, and the forces F on each side of the loop.

c) Calculate the maximum net force on a single loop for a 1 A current. What mass could this lift if $g = 9.81 m/s^2$?

d) How would the force change if the square loop is bent out into a circle (with the same perimeter)?

e) During testing one of the loops is dropped from an airplane and reaches a maximum speed of 90 m/s before crashing into the ground. Calculate the largest EMF that could have been induced.

---

32) *[5 points]* An electron with mass m_e and charge $-q$ is moving parallel to the x-axis with speed $v = 500m/s$. A proton with mass m_p and charge $+q$ is also moving parallel to the x-axis with the same speed. The proton is located $d = 3$ metres away the electron in the $\hat{j}$ direction.

a) Draw and label a figure showing charge locations and velocities in a Cartesian coordinate system.

b) What is the electric field (magnitude and direction) due to the proton at the location of the electron?

c) What is the magnetic field (magntitude and direction) due to the electron at the location of the proton?

d) What is the electric force acting on the electron?

e) What is the magnetic force acting on the proton?

---

33) *[5 points]* Figure 33 (not to scale) shows the cross-section of a very long, solid, cylindrical conductor of radius R_1. It is surrounded by a co-axial conducting cylindrical tube of inner radius $R_2 = 4R_1$ and outer radius $R_3 = 5R_1$. A current I flows along the central conductor (into the page) and back along the outer conductor (out of the page). The current density J is uniformly distributed across each conductor.

a) Derive expressions for the current density J for the inner conductor and the outer conductor.

b) Use Ampere's law to derive an expression for the magnetic field $\vec{B}$ for r in the range $r > R_3$.

c) Derive an expression for the magnetic field for $R_1 < r < R_2$.

d) Derive an expression for the magnetic field for $0 < r < R_1$.

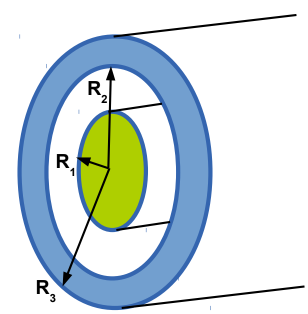


Figure 33 Conducting cylinder inside a conducting cylindrical shell.

Written Answer Sheets

Phys 259 Final Exam (Winter, 2015): Instructions to Markers

You are responsible for marking the Problems, Part II. The multiple choice answers are scored automatically. The "Version" number applies only to the multiple-choice answers; there is only one version of Part II. You should also have these general instructions, plus a set of solutions with detailed instructions, and a copy of an exam question paper. The solutions are presented in Answer Page format for ease of reference.

Print out these Instructions, and read them before starting to mark. You might be tempted to bypass these, but you will save us much time and trouble if you follow instructions.

On the following pages, I've given a suggested detailed marking scheme for each question. However, not all solutions will follow the one I've given, especially for the 2-mark and 3-mark questions, so use your judgment. The following broad guidelines may help:

- In 3-mark questions, the first half mark should be easy to get: if a student shows any reasonable insight at all, give a half mark. In an integration problem, if the student simply makes a valiant attempt at integrating, this might be enough.
- Give half of the allotted marks (e.g., 1.5 marks out of 3) if you feel that the student has got half way to the right answer. For example, if a student makes five mistakes but still has the solution half right, give 1.5; don't deduct five half-marks.
- In numerical questions, the final answer must have correct units. Deduct 0.5 mark if the units on the final answer are not shown or are incorrect.
- Be lenient on significant figures.
- vectors $\neq$ scalars, subtract 0.5 (once for any problem) eg. $\vec{E} = kqr$ or $E = kq\hat{r}$
- If a lot of students are making the same "mistake", check my answer - I could be the one who is wrong! If you discover any errors in my solutions, please let me know as soon as possible (physics259@ucalgary.ca) so I can alert all other markers.

UCID:

Q26a *[2.0 mark]*

Starting with Gauss's Law

$$\oint \vec{E} \cdot d\vec{a} = \frac{Q_{enc}}{\epsilon_0}$$

we note that cylindrical symmetry means that the electric field must be purely radial, and can depend only on radial distance r. Use a cylindrical Gaussian surface with $\vec{E}$ constant in magnitude and pointing parallel to $\hat{n}$ for the "tube", and perpendicular at the endcaps ($\vec{E} \cdot \hat{n} = 0$). The total flux is just the electric field times the area of the tube.

$$\Phi_E = \int_{tube} E da + 0 + 0 = EA = E 2\pi r L$$

A segment L of the inner insulator contains constant charge density times volume

$$Q = \rho_0 V = \rho_0 \pi R_1^2 L$$

For a Gaussian surface inside the inner insulator ($r < R_1$) the total charge enclosed is a fraction of the total

$$Q_{enc} = \rho \pi r^2 L$$

which allows us to solve for the electric field

$$E = \frac{\rho r}{2\epsilon_0}$$

which does not depend on the length L of the Gaussian cylinder. Reality check: $E = 0$ at $r = 0$ and increases linearly to a maximum at $r = R_1$ when we're outside all the charge.
0.5 marks for writing down Gauss's law, 0.5 for surface integral (must mention that endcaps are zero), 0.5 for correct total enclosed charge, 0.5 for final answer (direction not required).

Q26b *[2.0 mark]*

We still have cylindrical symmetry and should use a cylindrical Gaussian surface. For $R_1 < r < R_2$ the amount of charge enclosed is constant

$$Q_{enc} = \rho \pi R_1^2 L$$

while the surface area increases with r (flux through the endcaps is still zero)

$$\Phi_E = 0 + 0 + E 2\pi r L$$

Consequently, the electric field will fall off as $1/r$

$$E = \frac{\rho R_1^2}{2\epsilon_0 r}$$

just like a thin line of charge. 0.5 for Gauss's law, 0.5 for surface integral, 0.5 for enclosed charge, 0.5 for correct answer (direction not required).

Continued on next page

UCID: | | | | | | | | |

Q26c *[1.0 mark]*

The electrostatic field $\vec{E}$ inside the outer conductor must always be zero. This means that a Gaussian cylinder which is just barely larger that the inner surface at R_2 must have net enclosed charge of zero. *0.5 for stating zero enclosed charge*

$$\boxed{Q_1 + Q_2 = 0}$$

In other words, there will be a shielding charge on the inner surface that will exactly cancel the central charged cylinder

$$\rho_0 \pi R_1^2 L = \sigma_2 2\pi R_2 L$$

which lets us solve for the inner surface charge density

$$\sigma_2 = \frac{\rho_0 R_1^2}{2R_2} = \frac{\rho_0 R_1^2}{8R_1} = \boxed{\frac{\rho_0 R_1}{8}}$$

0.5 for correct answer in terms of R_1

Q26d *[2.0 mark]*

Start with the definition of potential difference in terms of the electric field integrated along a path between two points

$$\Delta V = V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{l}$$

Integrate from the reference point ($r = 0$) to the final point ($r = R_1$). (We can leave off the minus sign for now but must remember to put it back.)

$$\int_0^{R_1} dr E = \frac{\rho}{2\epsilon_0} \int_0^{R_1} dr\ r = \frac{\rho}{2\epsilon_0}\left[\frac{R_1^2}{2} - 0\right] = \frac{\rho R_1^2}{4\epsilon_0}$$

then calculate the potential difference relative to the axis

$$V_{R_1} = V_0 - \frac{\rho_0 R_1^2}{4\epsilon_0} = -\frac{\rho_0 R_1^2}{4\epsilon_0}$$

Does this make sense? A positively charged rod will have an electric field pointing radially away. A positive test charge will move away from the rod, which should also correspond to motion from a higher potential to a lower potential.

0.5 for the first equation with correct signs, 0.5 for setting up the integral properly (ie. from 0 to R_1), 1.0 for the correct result (0.5 for simply noting why the result should be negative)

Q27

Continued on next page

UCID:

The magnetic field is horizontal (pointing North) and decreases with height as $B(z) = B_0 - az$ where $a = 0.022\ nT/m$. The first derivative with respect to height is

$$\boxed{\frac{d}{dz}B(z) = -a = -0.022nT/m}$$

1.0 for either "–a" or the correct numerical value with units. Subtract 0.5 for one or more errors or omissions

Q27b *[2.0 mark]*

Current I flowing along a straight wire segment of length L in a magnetic field B will feel a force

$$\vec{F} = I\vec{L} \times \vec{B} = ILB \sin\phi$$

If the magnetic field is uniform then the net force on all 4 sides of a square loop will add up to zero. (There may be a torque). If the magnetic field is non-uniform then the net force can be non-zero. The maximum force will be produced when L and B are perpendicular and there is the largest possible difference in magnetic field between two sides of the loop.
We assume that North is in the x direction $\vec{B} = B\hat{i}$ and that up (+z) is in the $\hat{k}$ direction. In this case loop should be aligned in the y, z plane (x, z if $B\hat{j}$). The current should be flowing in the $+y$ direction in the upper part of the loop ($z = 1m$) to create a downward force

$$IL(+\hat{j}) \times B(+\hat{i}) = ILB\ (-\hat{k})$$

with the $-y$ current in the lower part giving an upward force

$$IL(-\hat{j}) \times B(+\hat{i}) = ILB\ (+\hat{k})$$

that will be slightly larger

$$\vec{F}_1 + \vec{F}_2 = IL(B_0 - aL)(-\hat{k}) + IL(B_0)(\hat{k}) = IL^2\, a\, \hat{k}$$

Forces on the vertical segments will point toward the center of the loop and exactly cancel.

Q27a *[1.0 mark]*

The magnetic field is horizontal (pointing North) and decreases with height as $B(z) = B_0 - az$ where $a = 0.022\ nT/m$. The first derivative with respect to height is

$$\boxed{\frac{d}{dz}B(z) = -a = -0.022nT/m}$$

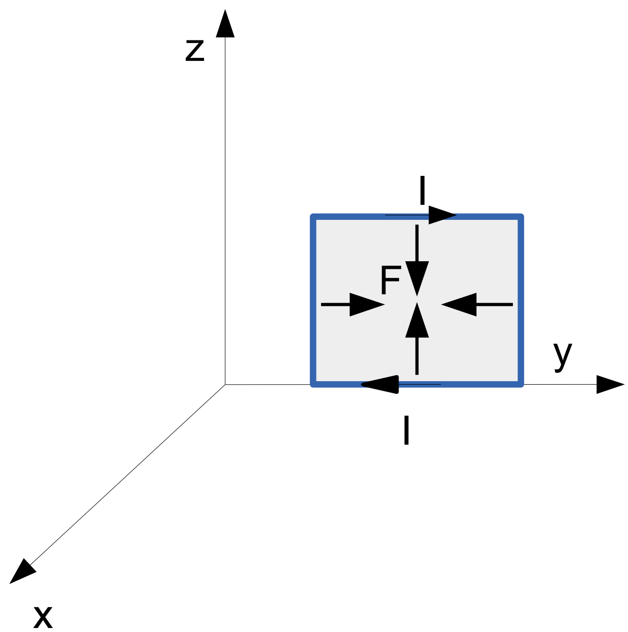
1.0 for either "–a" or the correct numerical value with units. Subtract 0.5 for one or more errors or omissions

Continued on next page

UCID:

Q27b *[2.0 mark]*

0.5 for a right-handed coordinate system, 0.5 for loop in x-z or y-z plane (depending on how student defined North), 0.5 for correct current direction (clockwise as defined above), 0.5 for drawing force vectors on each of four sides. No math is required for this part of the question.



Q27c *[2.0 mark]*

The net magnetic force on the loop is

$$\vec{F} = IL^2\, a\, \hat{k}$$

so lifting an object of mass m against gravity will require

$$F = ma = mg$$

and solving for m

$$m = \frac{F}{g} = \frac{IL^2 a}{g} = \frac{1A \times 1m \times 1m \times 0.022\; nT/m}{9.81 m/s^2} = 2.24 \times 10^{-12} kg$$

so even a very thin wire loop could not lift itself, much less any battery required to produce the rather large 1 Ampere current. *0.5 for net force equation, 0.5 for gravity balance, 1.0 for correct answer with units*

Continued on next page

UCID:

Q27d *[1.0 mark]*

The question does not ask for a calculation, so we're just looking for some general statements about geometry. If the loop perimeter is the same then $2\pi r = 4L$ and the radius is given by

$$r = \frac{2}{\pi} L \approx 0.637L$$

so top and bottom of the circular loop are 1.2 times further apart than the square loop. This will give more force from these segments, but there are also segments that are closer together. Just by inspection it is not obvious what the net force will be, except to say that it will be vertical (horizontal components will still cancel). *1.0 for basically any correct statement with justification, 0.5 for assertion without proof or incorrect proof.*

Q27e *[2.0 mark]*

Start with Faraday's law that relates the change in magnetic flux through a closed loop to the EMF around that loop

$$EMF = -\frac{d\Phi_B}{dt}$$

So how fast is the flux through the loop changing? Use the chain rule to combine change in flux with height (part a) and change in height with time (speed v)

$$\frac{d\Phi_B}{dt} = \frac{d\Phi_B}{dz}\frac{dz}{dt} = av$$

drop in the numbers to get a tiny result

$$EMF = -av = 0.022nT/m\ 90m/s = \boxed{19.8nV/m}$$

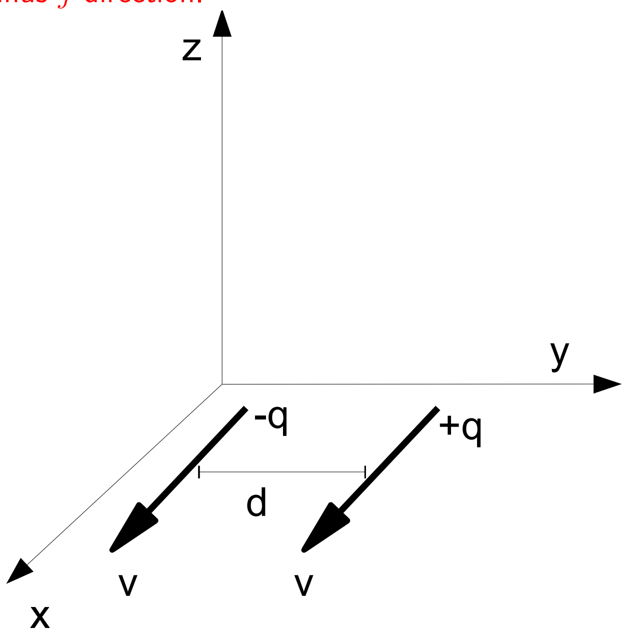
0.5 marks for Faraday's law, 0.5 for the general idea of the chain rule, 1.0 for the correct answer with units (don't worry about the sign).

Continued on next page

UCID:

Q28a *[1.0 mark]*

0.5 for right-handed coordinate system, 0.5 for both charges moving in $\hat{i}$ (x) direction separated by d in plus or minus $\hat{j}$ direction.



Q28b *[1.0 mark]*

The electric field produced by a point charge is given by

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$$

so the field of a proton ($+q$) at $d = 3m$ "above" ($+\hat{j}$) the electron will be

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{d^2}(-\hat{j}) = 8.99 \times 10^9 \frac{1.60 \times 10^{-19}}{(3m)^2}(-\hat{j}) = -1.6 \times 10^{-10}\ \hat{j}\ N/C$$

0.5 marks for correct magnitude (either in terms of d and q or numerical) and 0.5 for correct direction.

Continued on next page

UCID: | | | | | | | |

Q28c *[1.0 mark]*
The magnetic field due to a moving charge is

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2}$$

so for an electron moving in the $+\hat{i}$ direction

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{-qv(+\hat{i})\times(+\hat{j})}{d^2} = \boxed{-\frac{\mu_0}{4\pi}\frac{qv}{d^2}\hat{k}}$$

which can be evaluated as

$$\vec{B} = 10^{-7}\frac{1.60\times10^{-19}(500m/s)}{(3m)^2}(-\hat{k}) = \boxed{-8.9\times10^{-25}\hat{k}\ T}$$

1.0 for correct magnitude (can be symbolic or numerical) and direction

Q28d *[1.0 mark]*
Electric field acting on a charge is

$$\vec{F}_e = q\vec{E}$$

so the E field of the proton acting on the electron is

$$\vec{F}_e = \frac{1}{4\pi\epsilon_0}\frac{q^2}{d^2}(+\hat{j}) = 2.56\times10^{-29}\hat{j}\ N$$

1.0 for correct magnitude (can be symbolic or numerical) and direction

Q28e *[1.0 mark]*
Magnetic field acting on a moving charge is

$$\vec{F}_b = q\vec{v}\times\vec{B}$$

so the B field of an electron acting on the proton is

$$= qv(+\hat{i})\times B(-\hat{k}) = \frac{\mu_0}{4\pi}\frac{q^2v^2}{d^2}(+\hat{j}) = +7.11\times10^{-41}\hat{j}\ N$$

1.0 for correct magnitude (can be symbolic or numerical) and direction

Continued on next page

UCID:

Q29a *[1.0 mark]*

Both conductors have the same total current I flowing in opposite directions. We are told that the current density

$$J = \frac{I}{A}$$

is uniform. For the inner conductor the cross-sectional area is just a circle

$$J = \frac{I}{A} = \frac{I}{\pi R_1^2}$$

while for the outer conductor the area can be found by subtracting two circles of radius $R_2 = 4R_1$ and $R_3 = 5R_1$

$$J = \frac{I}{A} = \frac{I}{\pi R_3^2 - \pi R_2^2} = \frac{I}{\pi(25R_1^2 - 16R_1^2)} = \frac{I}{9\pi R_1^2}$$

0.5 for correct J for each conductor

Q29b *[1.0 mark]*

Ampere's law relates the magnetic field integrated around a closed loop to the total current which passes through the loop

$$\boxed{\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}}$$

For a cylindrically symmetric system of currents, the magnetic field will be directed azimuthally (clockwise or counterclockwise) and should be constant for any given radius r. Consequently, the magnetic field integral is just

$$\oint \vec{B} \cdot d\vec{l} = \oint B dl = B \oint dl = B 2\pi r$$

For the region outside both conductors $r > R_3$ the net current enclosed is I into the page and I out of the page, so

$$I_{enc} = I - I = 0$$

In order for this to be true, the magnetic field must be zero

$$\boxed{B = 0}$$

0.5 for Ampere's Law, 0.5 for the correct answer

Continued on next page

UCID: | | | | | | | |

Q29c *[1.0 mark]*

For the region $R_1 < r < R_2$ the total current I enclosed is constant. The magnetic field integral gives the usual result for cylindrical symmetry

$$\oint \vec{B} \cdot d\vec{l} = \oint B dl = B \oint dl = 2\pi r B$$

so Ampere's law gives

$$\boxed{\vec{B} = \frac{\mu_0 I}{2\pi r}\hat{\phi}}$$

where $\hat{\phi}$ indicates a clockwise orientation (from the right-hand rule or Biot-Savart law). 0.5 for correct magnitude, 0.5 for direction

Q29d *[2.0 mark]*

For the region $0 < r < R_1$ the Amperean loop will contain some fraction of the total current

$$I_{enc} = \frac{\pi r^2}{\pi R_1^2} I$$

ranging from 0% when $r = 0$ and 100% when $r = R_1$. With cylindrical symmetry the magnetic field is parallel to a circular loop

$$\oint \vec{B} \cdot d\vec{l} = \oint B dl = B \oint dl = 2\pi r B$$

so Ampere's law becomes

$$2\pi r B = \frac{\pi r^2}{\pi R_1^2}\mu_0 I$$

so the magnetic field strength is

$$\boxed{B = \frac{\mu_0 I r}{2\pi R_1^2}}$$

and the direction from the right-hand rule is clockwise if the current I flows into the page. 0.5 for correct magnitude, 0.5 for direction