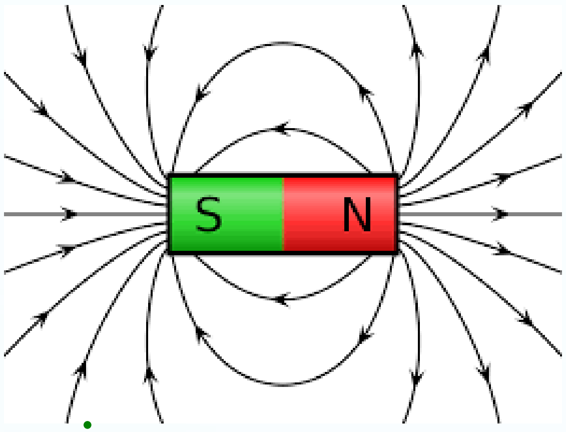
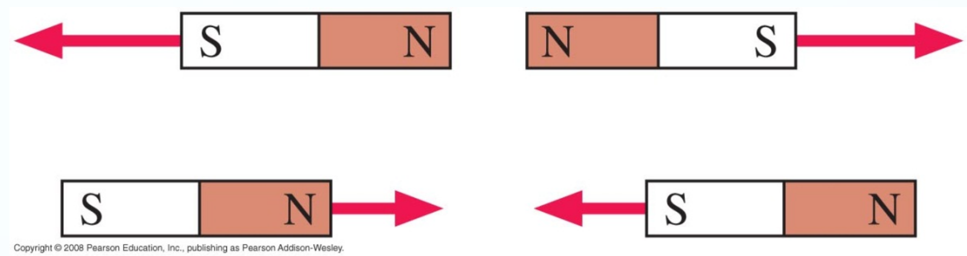
Electricity and Magnetism

- Physics 259 – L02
- Lecture 38

Chapter 28: Magnetic fields

28.1: Magnetic fields

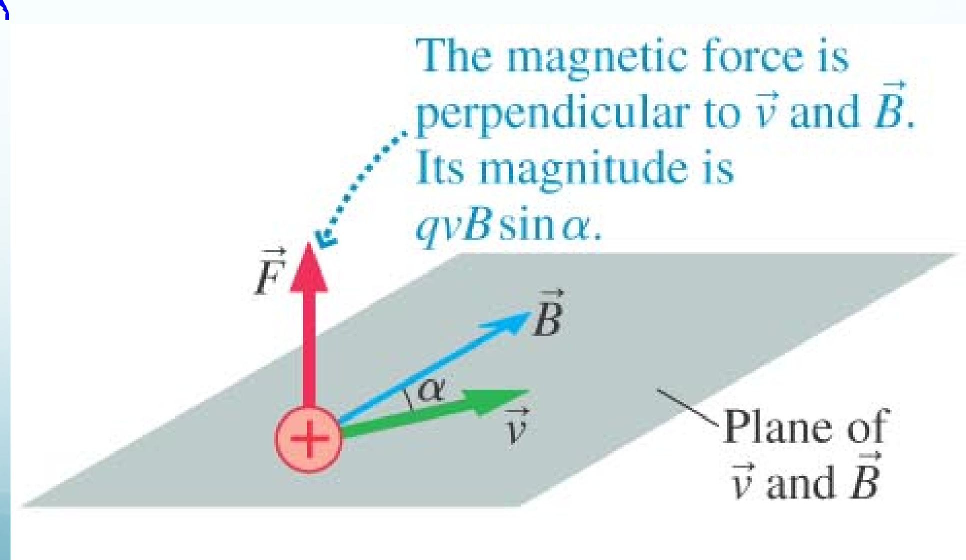




electric magnetic

$$\vec{F} = \underbrace{q\vec{E}}_{\text{electric}} + \underbrace{q\vec{v}\times\vec{B}}_{\text{magnetic}}$$

$$\vec{F}_B = q\,\vec{v}\times\vec{B}$$

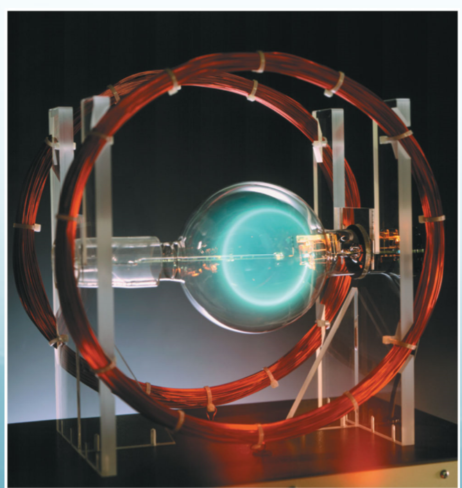


28.4: A circulating charged particle

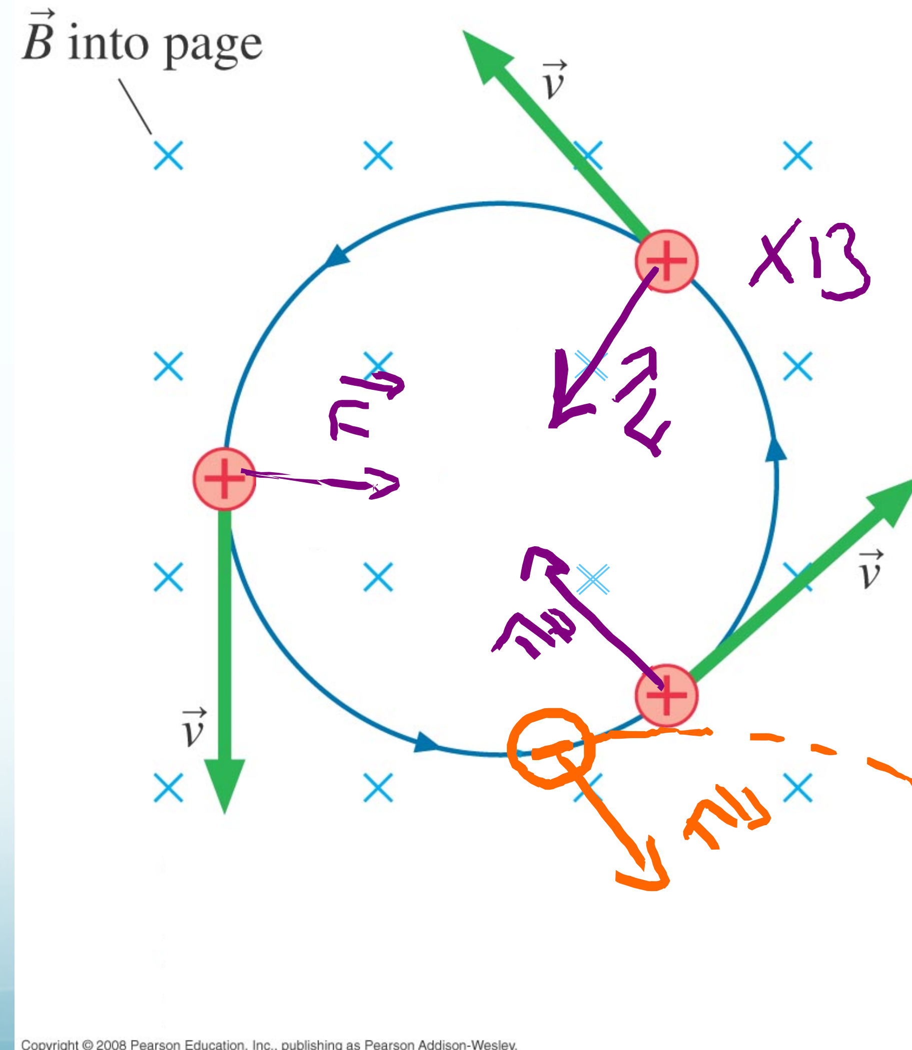
Charged particles in uniform magnetic fields undergo **uniform circular motion**.

The radius of the circle depends on how fast the particle is moving:

$$\boxed{R = \frac{mv}{|q|B}}$$



$$\vec{F}_B = q\,\vec{v} \times \vec{B}$$

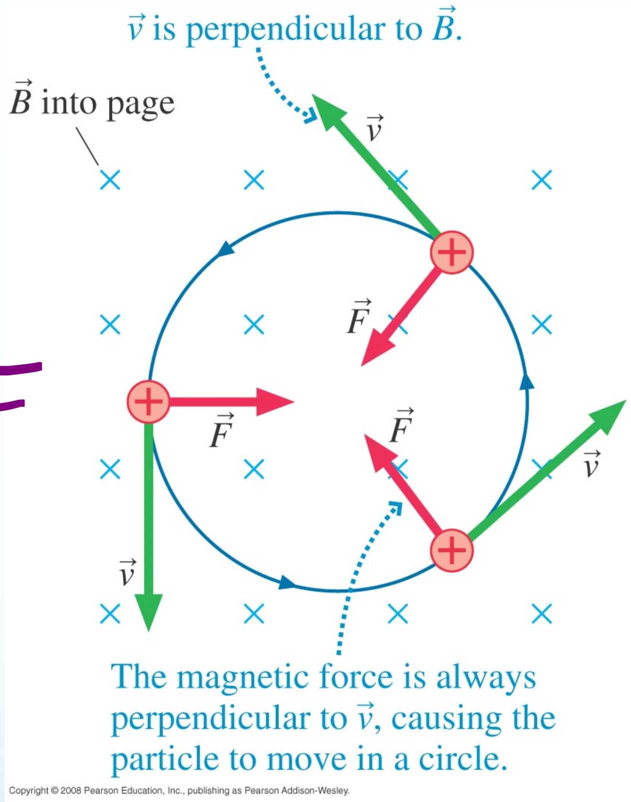


$$v = \frac{2\pi R}{T_{cyc}}$$

T_{cyc} is period (time it takes to make one cycle)

$$R = \frac{mv}{|q|B} \rightarrow \cancel{R} = \frac{m}{|q|B}\,\frac{2\pi \cancel{R}}{T_{cyc}}$$

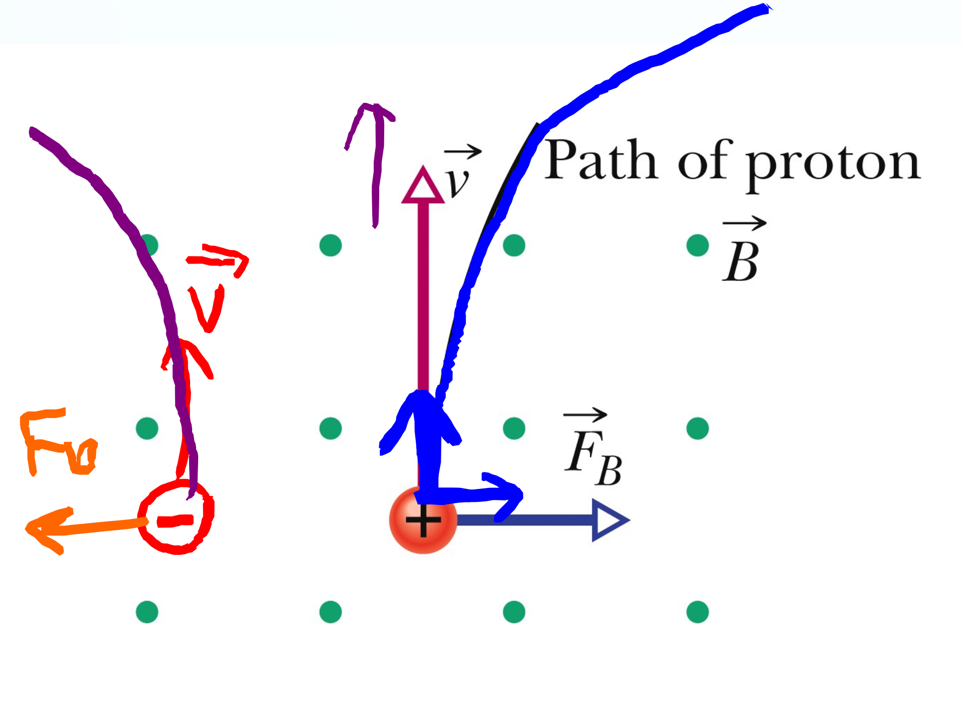
$$\rightarrow T_{cyc} = \frac{2\pi m}{|q|B} \quad \& \quad f_{cyc} = \frac{1}{T}$$



$$\boxed{T_{cyc} = \frac{2\pi m}{|q|B}} \qquad \boxed{f_{cyc} = \frac{|q|B}{2\pi m}}$$

The period (and also the frequency of the circular motion) depend on the B-field strength and the charge-to-mass ratio *q/m*

Motion of charges in B-field



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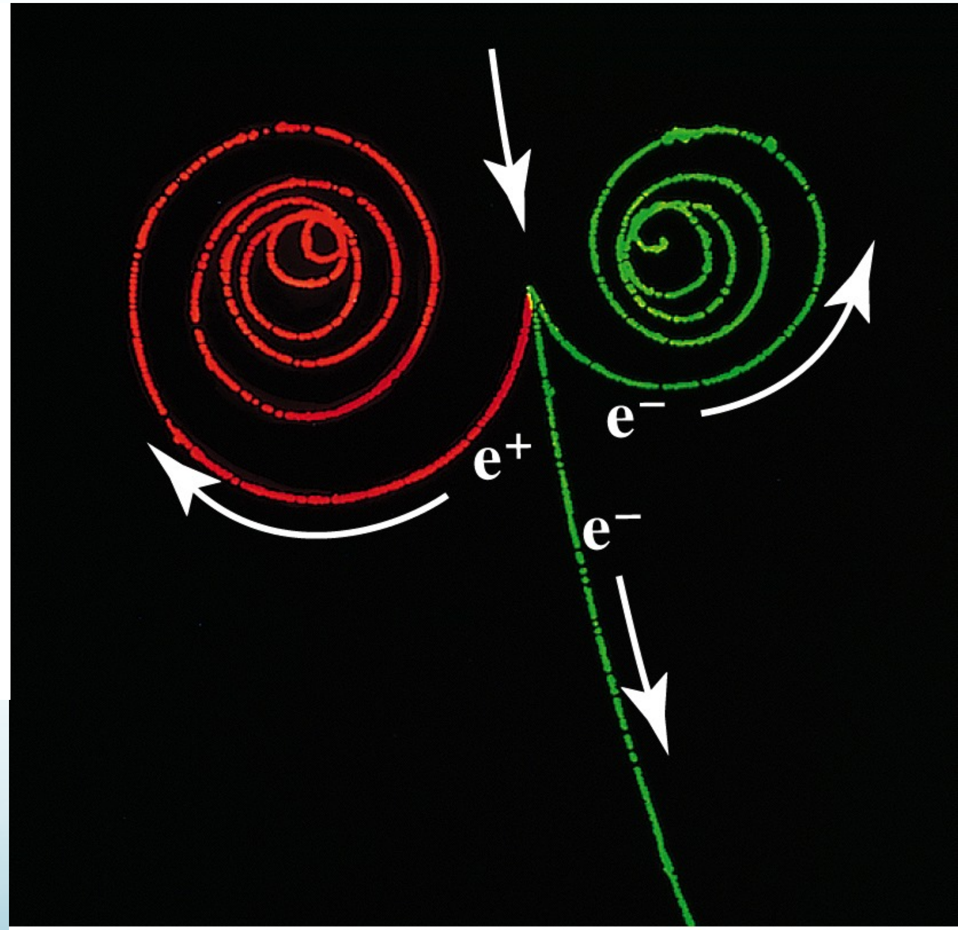
kinetic energy remains constant



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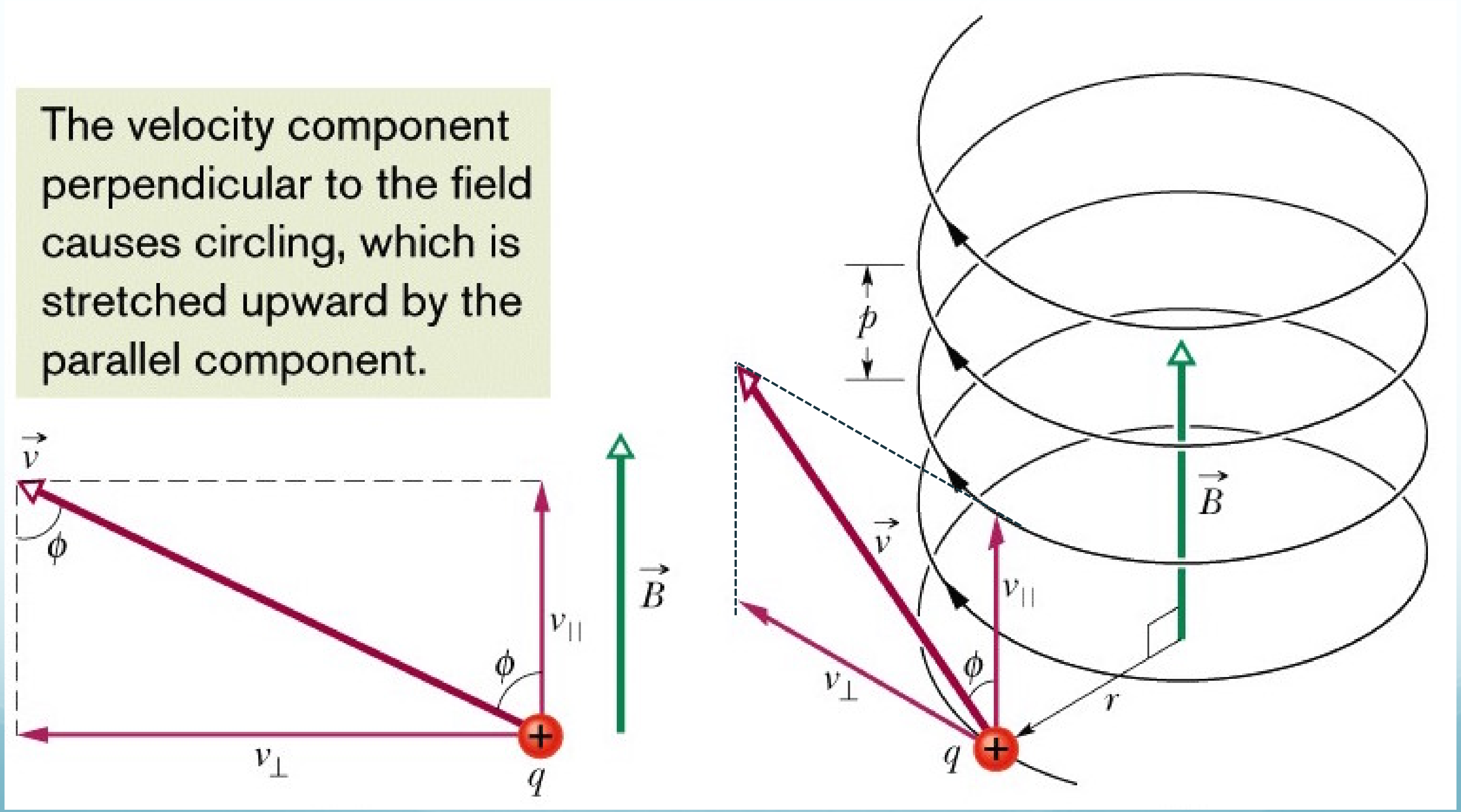
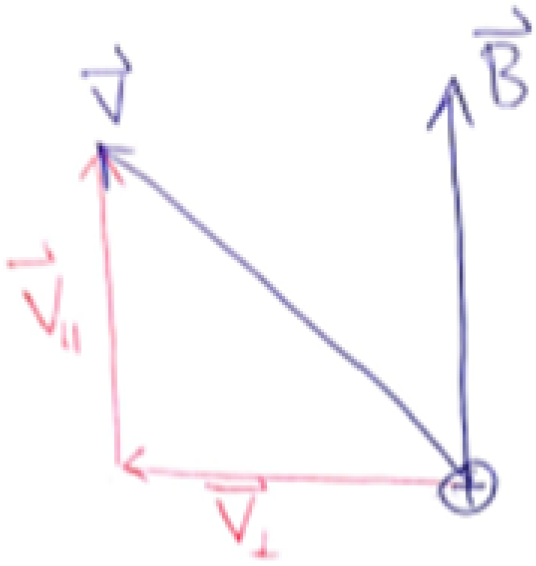
Helical Paths Through a B-field

Splitting up the velocity into a component parallel to B-field and a component perpendicular to B-field immediately leads to helical motion



Charges on Helical Paths.

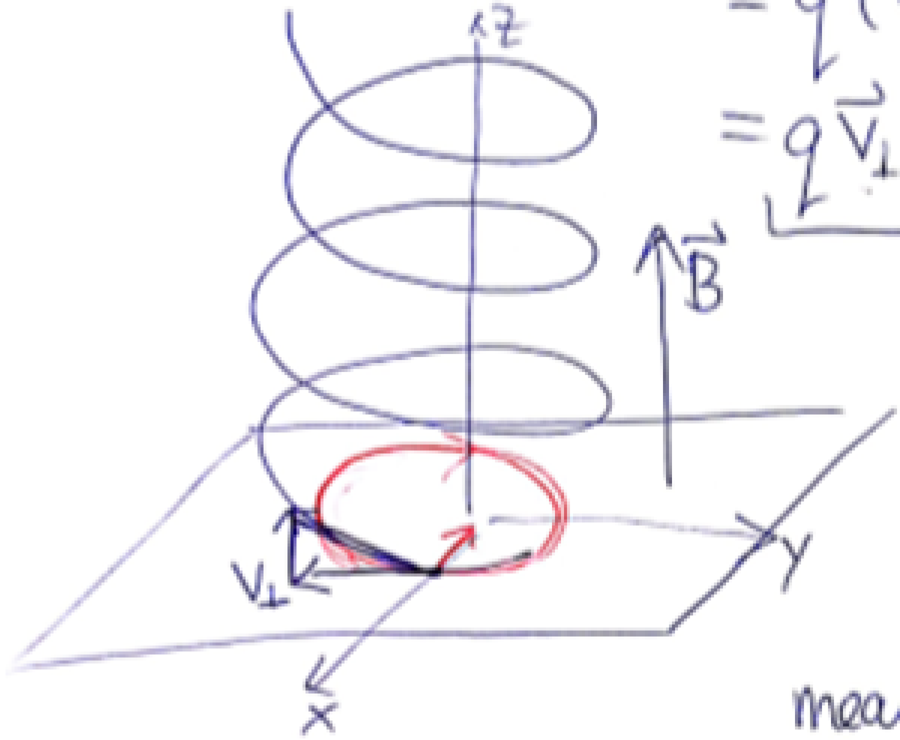


$$\vec{v} = \vec{v}_\perp + \vec{v}_\parallel$$

$$\vec{F}_B = q\vec{v} \times \vec{B}$$

$$= q(\vec{v}_\perp + \vec{v}_\parallel) \times \vec{B}$$

$$= q\vec{v}_\perp \times \vec{B} + \underbrace{q\vec{v}_\parallel \times \vec{B}}_{=0}$$



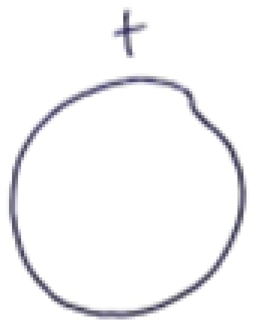
$$\vec{F}_B = q\vec{v}_\perp \times \vec{B}$$

never in z-direction

means: $\vec{v}_\parallel$ is constant

↳ only in z.

steady velocity upward (z)



circular motion in x,y.

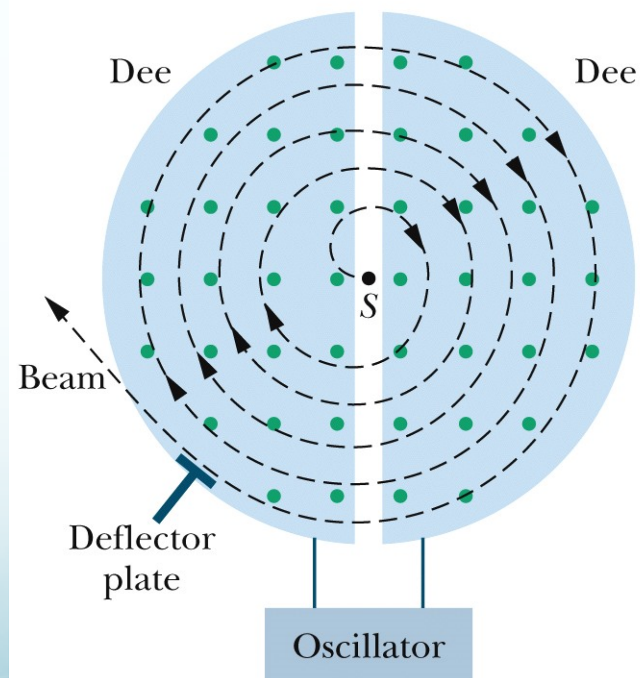
28.5: Cyclotrons and Synchrotrons

We need beams of high-energy particles→
Two accelerators that employ a magnetic field to repeatedly bring particles back to an accelerating region are:

- ✓ The Cyclotron
- ✓ The proton Synchrotron

watch the video file

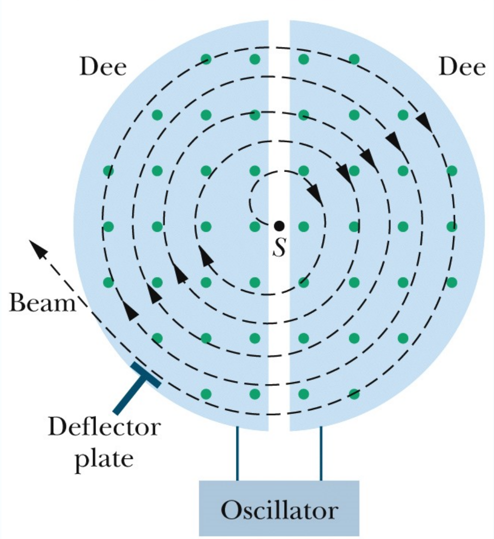
The protons spiral outward in a cyclotron, picking up energy in the gap.



✓ The Cyclotron

The key to the operation of the cyclotron is that the frequency f at which the proton circulates in the magnetic field (and that does not depend on its speed) must be equal to the fixed frequency f_{osc} of the electrical oscillator, or

$$f = f_{osc} \quad \text{(resonance condition).}$$



✓ The proton Synchrotron

Magnetic field ***B*** and the oscillator frequency f_{osc}, instead of having fixed values as in the conventional cyclotron, are made to vary with time during the accelerating cycle→

(1) the frequency of the circulating protons remains in step with the oscillator at all times

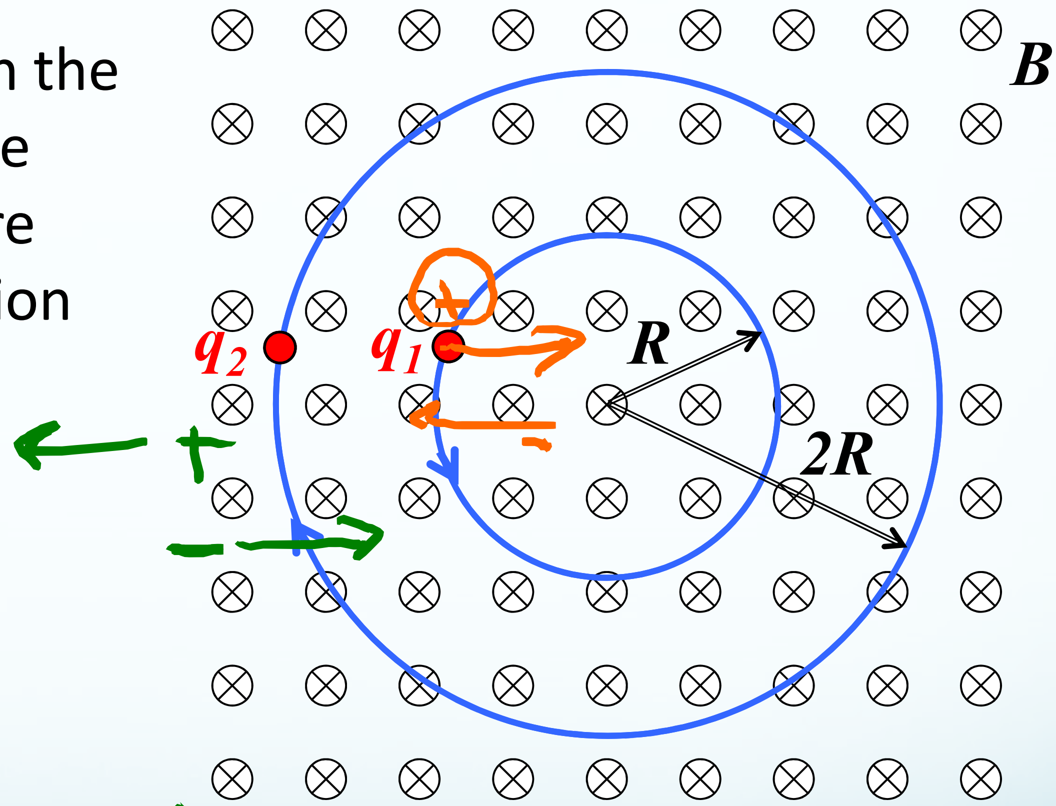
(2) the protons follow a circular — not a spiral — path. Thus, the magnet need extend only along that circular path, not over some 4×10^6 m^2.

$$\boxed{R = \frac{mv}{|q|B}} \quad \boxed{T_{cyc} = \frac{2\pi m}{|q|B}} \quad \boxed{f_{cyc} = \frac{|q|B}{2\pi m}}$$

Top Hat Question

Two charges q_1 and q_2 with the same mass m and the same magnitude of charge $|q|$ are undergoing cyclotron motion in a uniform B-field.

What are the **signs of the charges**?



A. Both positive

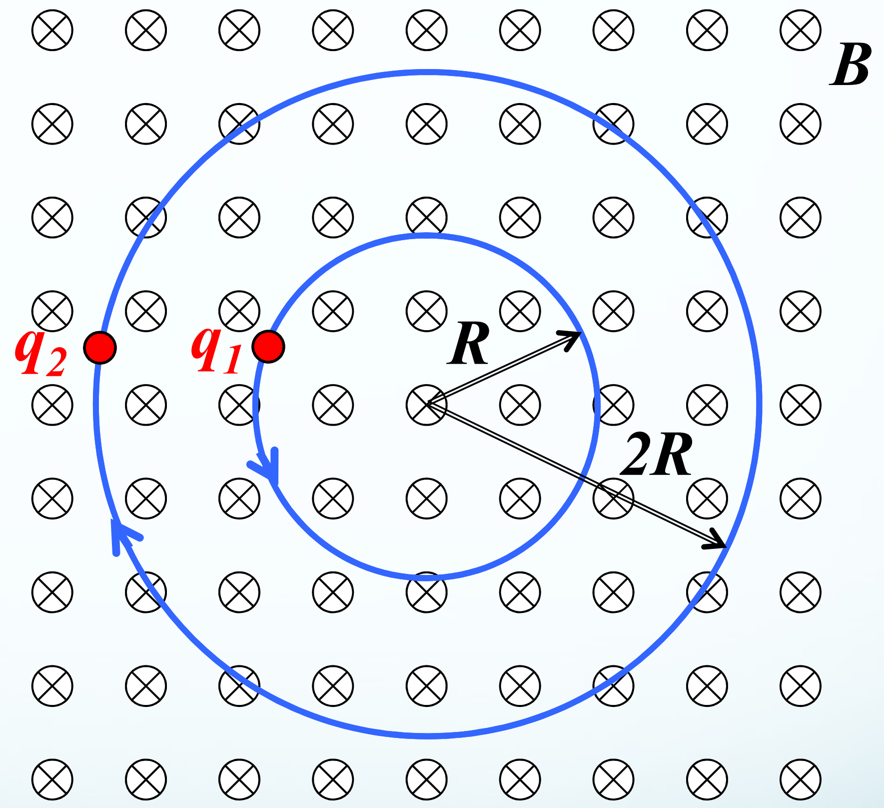
B. Both negative

C. q_1 positive, q_2 negativ

D. q_1 negative, q_2 positive

Top Hat Question

Two charges q_1 and q_2 with the same mass m and the same magnitude of charge $|q|$ are undergoing cyclotron motion in a uniform B-field.



If the speed of q_1 is v, what is the speed of q_2?

A. v

B. $2v$ ✓

C. $\frac{1}{2}v$

D. $4v$

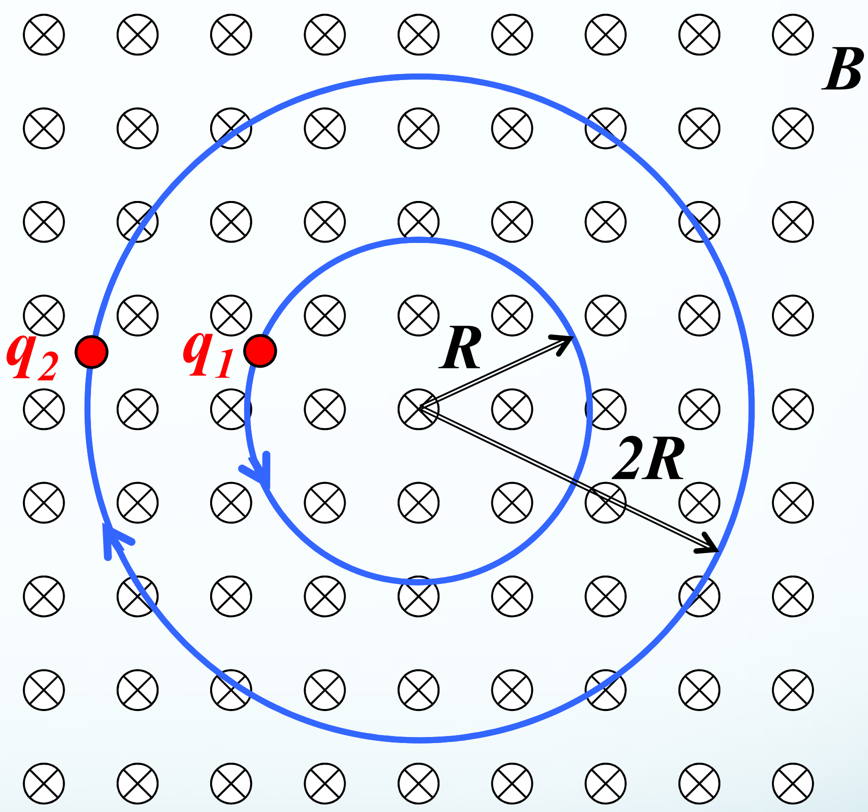
$$v = \frac{2\pi R}{T_{cyc}}$$

Top Hat Question

$$R = \frac{mv}{|q|B} \qquad T_{cyc} = \frac{2\pi m}{|q|B} \qquad f_{cyc} = \frac{|q|B}{2\pi m}$$

Two charges q_1 and q_2 with the same mass m and the same magnitude of charge $|q|$ are undergoing cyclotron motion in a uniform B-field.

If the period of rotation of q_1 is T, what is the period of rotation of q_2?



A. T

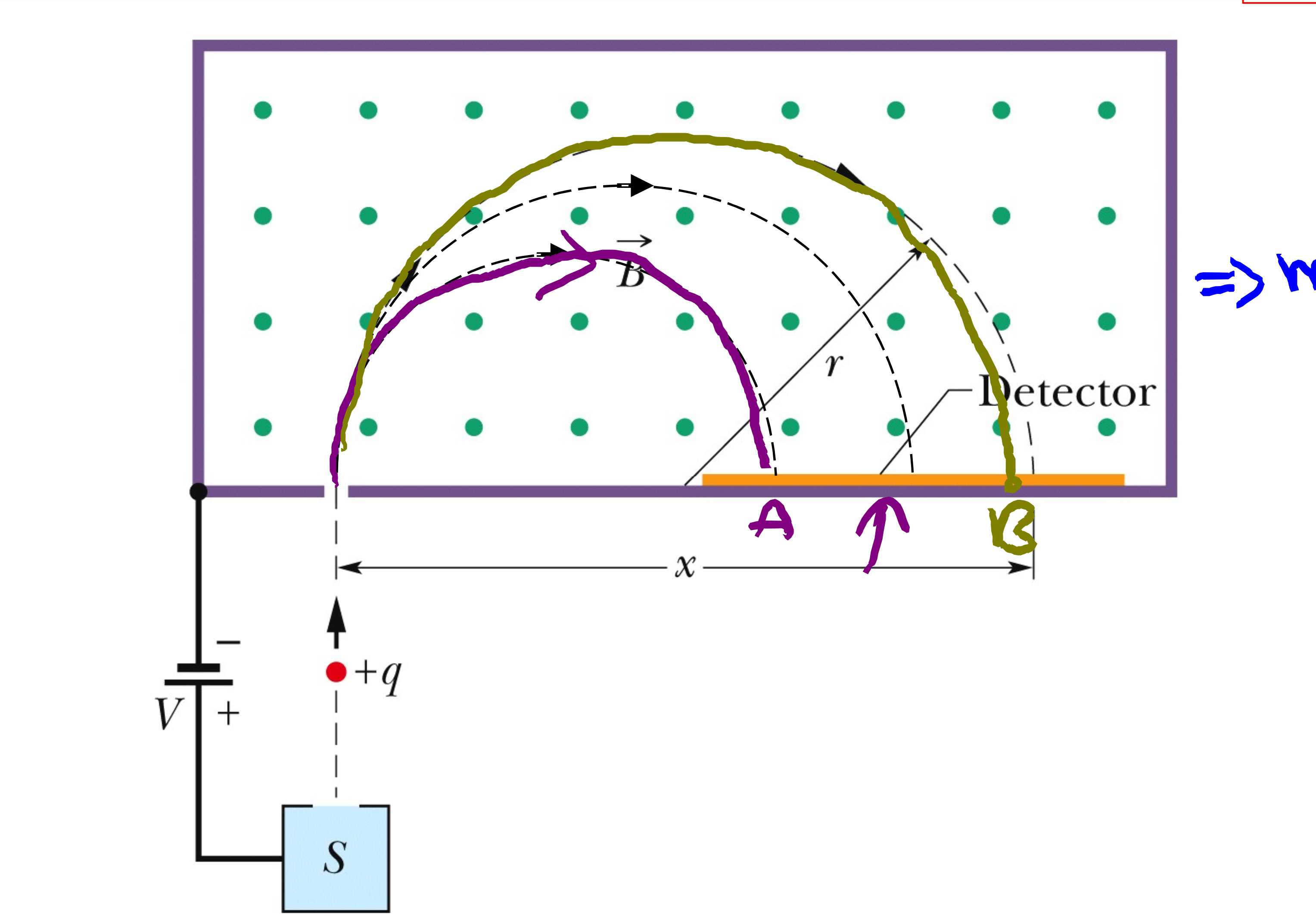
B. $2T$

C. ½ T

D. $4T$

Application: Mass Spectrometer

$$\boxed{R = \frac{mv}{|q|B}}$$



$\Rightarrow m_B > m_A$

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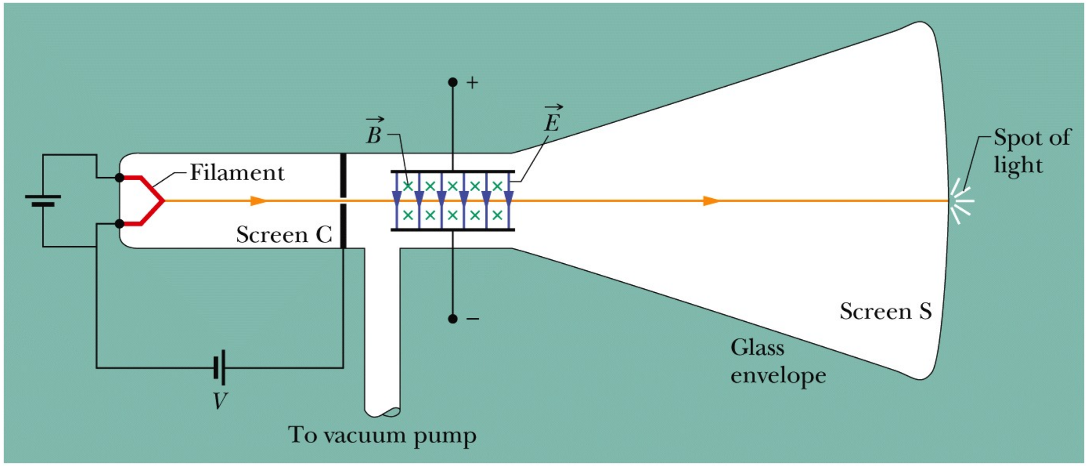
28-2 Crossed Fields: Discovery of The Electron

A modern version of J.J. Thomson's apparatus for measuring the ratio of mass to charge for the electron



If a charged particle moves through a region containing both an electric field and a magnetic field, it can be affected by both an electric force and a magnetic force.

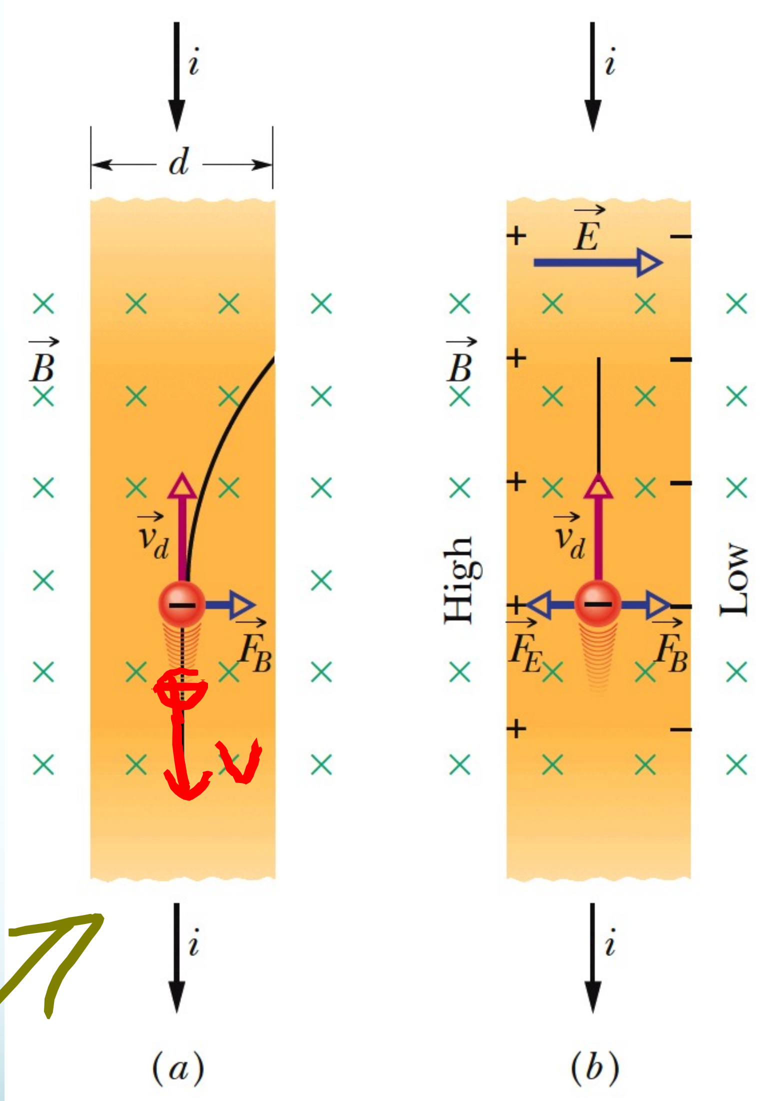
When the two fields are perpendicular to each other, they are said to be **crossed fields**.

28-3 Crossed Fields: The Hall Effect

A beam of electrons in a vacuum can be deflected by ad magnetic field.

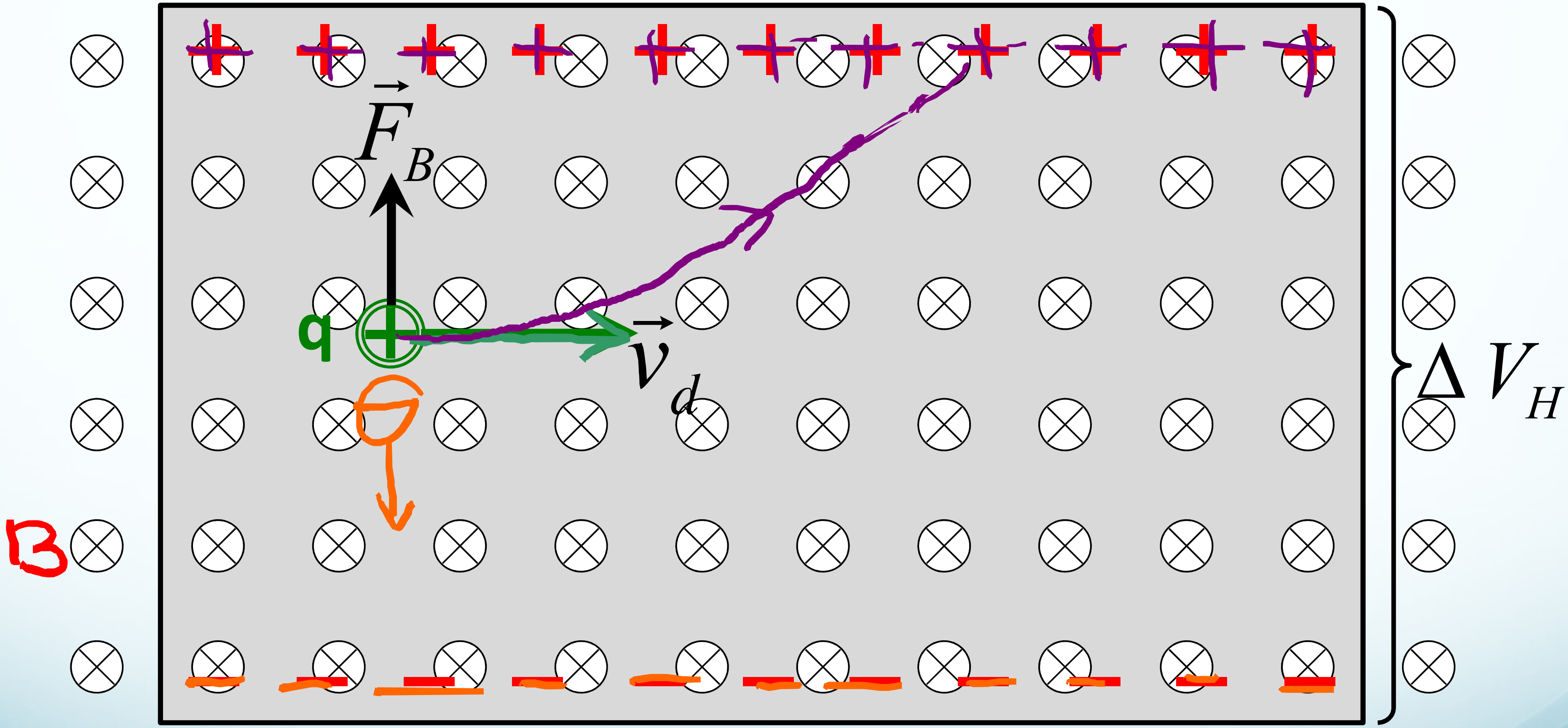
Can the drifting conduction electrons in a copper wire also be deflected by a magnetic field?

In 1879, Edwin H. Hall, then a 24-year-old graduate student at the Johns Hopkins University, showed that they can.

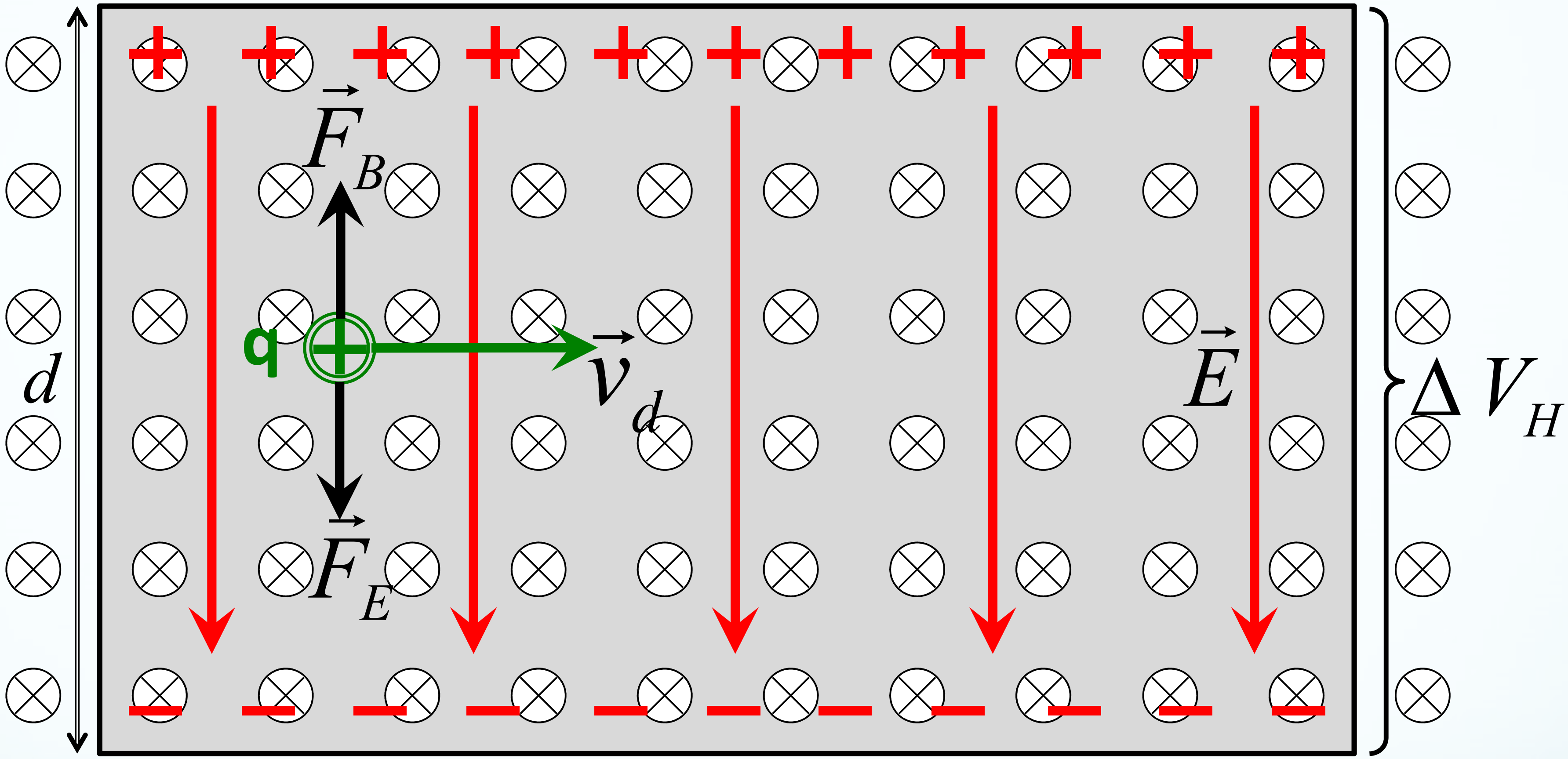


Explanation: Let's talk about positive charge

Due to the B-field, net charge build up on the edges.



In equilibrium, current still flows. Need to balance the magnetic and electric forces on the charge carriers.



$$F_B = q\, v_d B$$

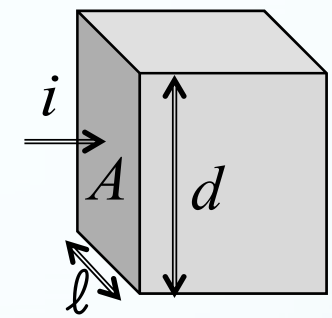
$$F_E = q \underbrace{\frac{\Delta V_H}{d}}_{\vec{E}}$$

$$\Longrightarrow \quad \cancel{q}\frac{\Delta V_H}{d} = \cancel{q} v_d B \rightarrow \boxed{\Delta V_H = v_d B d}$$

→ Voltage established across a conductor carrying a current in a magnetic field →

$$\Delta V_H = v_d B d$$

We previously related the drift speed to the current via



$$v_d = \frac{i}{neA}$$ where $A = \ell d$ and n is a material property

We can then relate the Hall voltage to known quantities:

$$\Delta V_H = \frac{i}{ne\ell \cancel{d}} B\cancel{d} = \frac{iB}{ne\ell}$$

In practical applications, you measure ΔV_H to find B:

$$\boxed{B = \frac{ne\ell}{i} \Delta V_H}$$

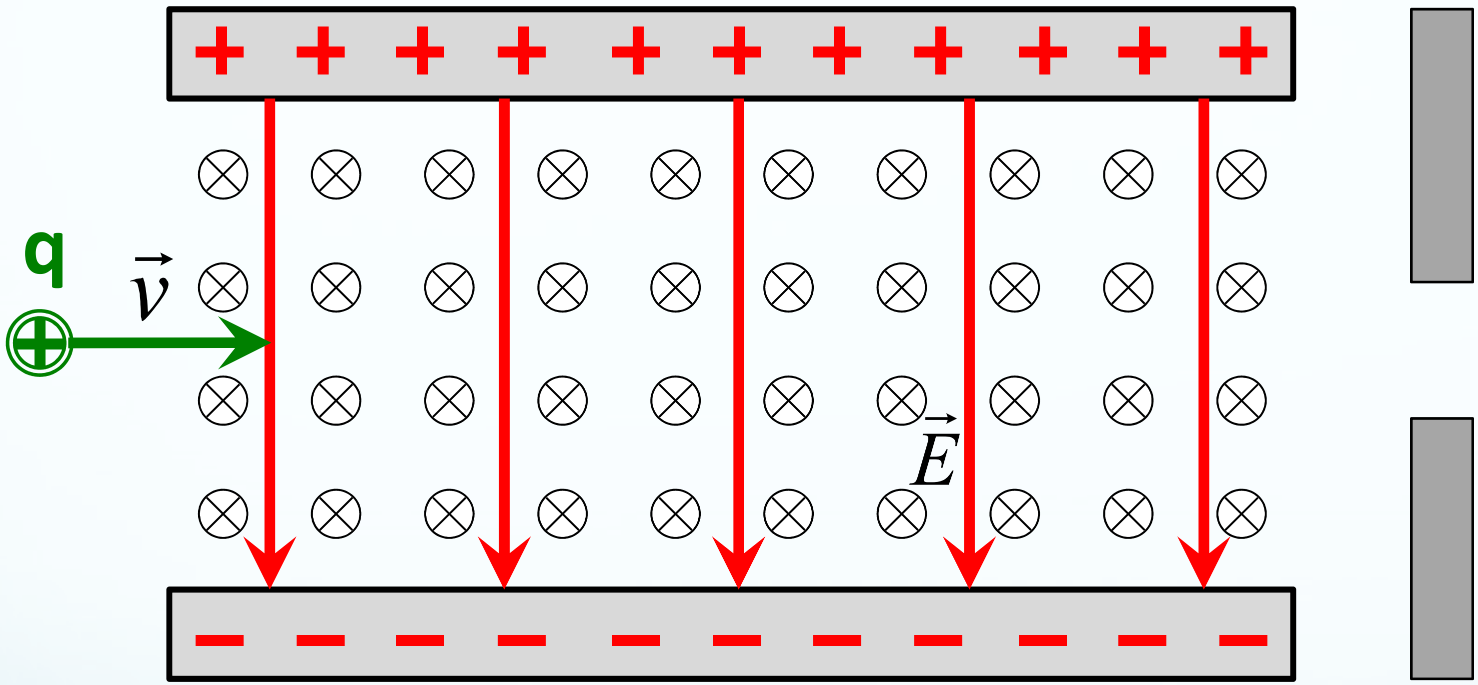
How the B-field probe used in the next lab works

This section we talked about:

Chapter 28.2, 28.4, 28.3, 28.5

See you on Thursday

Similar concept: velocity selector

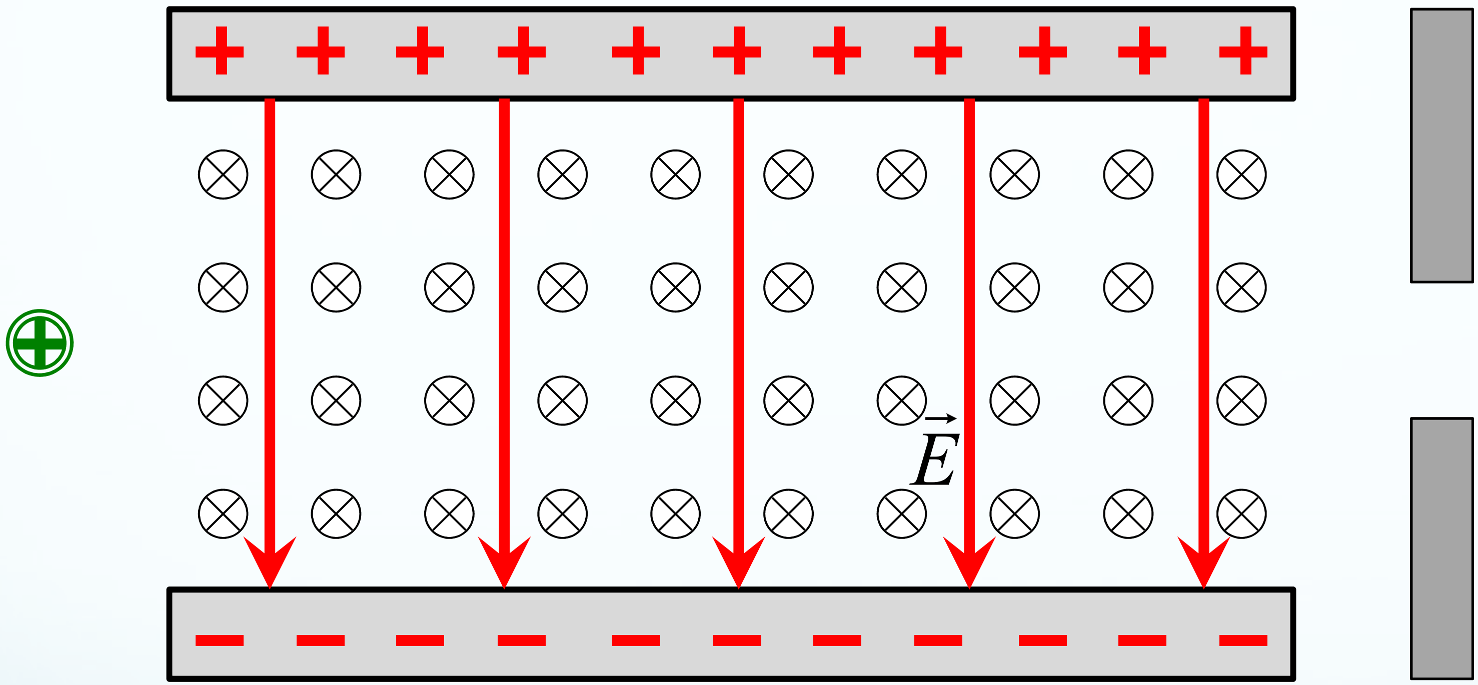


In a velocity selector, you send a charge through a region with crossed E and B fields, which leads to electric and magnetic forces:

$$\vec{F}_e = q\vec{E} \qquad \vec{F}_B = q\vec{v} \times \vec{B} \qquad qE = qvB \qquad v = \frac{E}{B}$$

If the forces balance ($F_{net} = 0$) the charge makes it through the slit

Similar concept: velocity selector



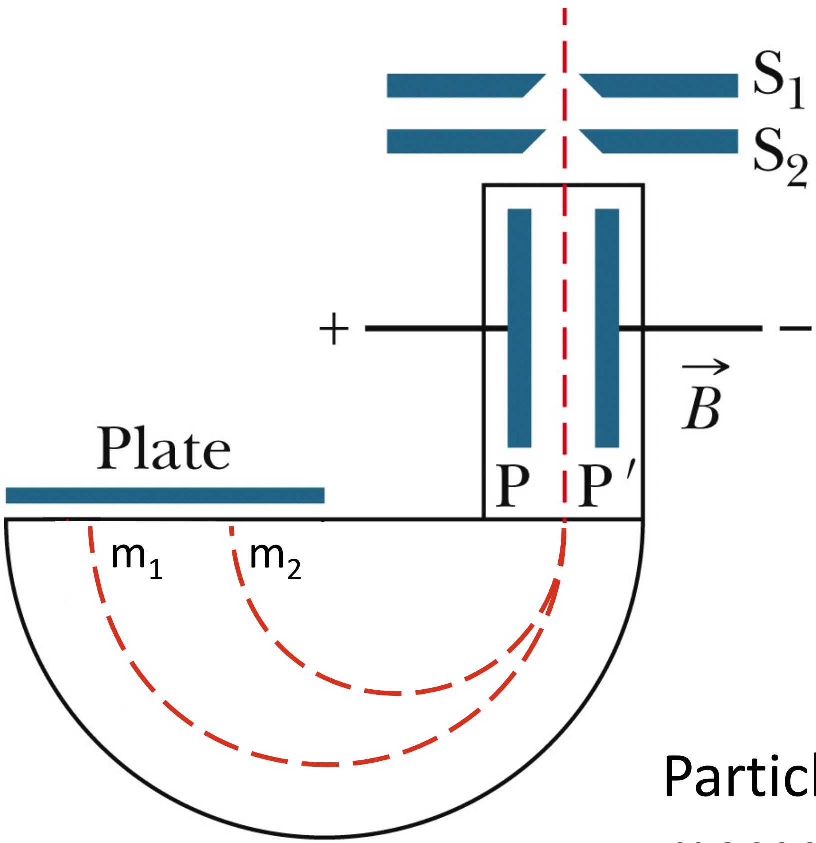
If the forces don't balance the charge hits the wall

$$qE - qvB = ma$$

We pick the E and B magnitudes to select the speeds we want

Bainbridge Mass Spectrometer

Accelerate charges through ΔV so they all have same Kinetic Energy



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The slits S_1 and S_2 ensure the beam of particles is collimated.

The beam enters a region of crossed E and B-fields

A narrow slit ensures only particles with a specific speed enter

Particles with same KE but different masses and charges will have different radius in B field