Wednesday March 29, 2017

Last time:

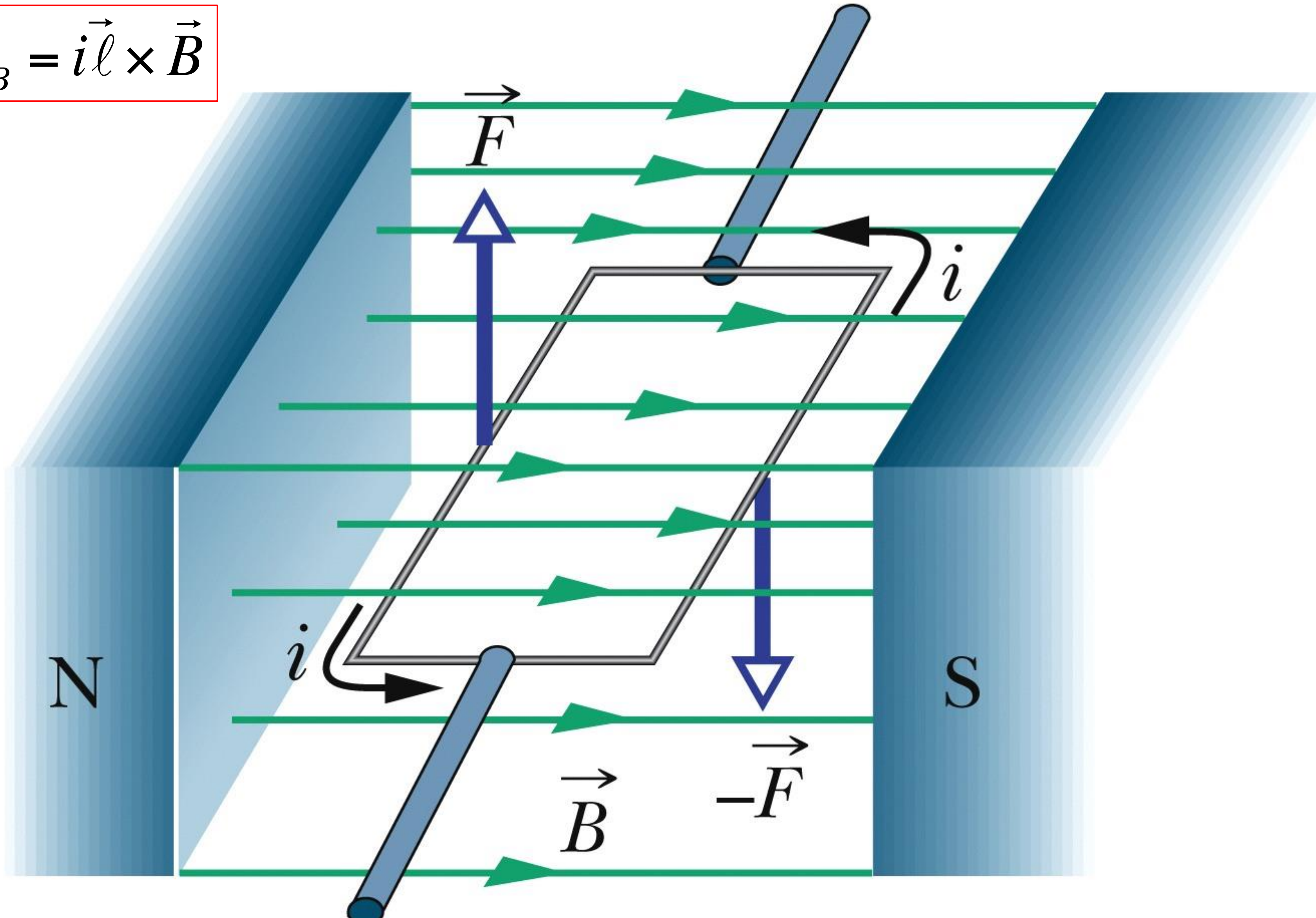
- Magnetic force between parallel current-carrying wires
- Torque on a current loop

Today:

- Biot-Savart Law (like Coulomb’s Law for magnetism)
- B-field of a line of current
- Magnetic force between parallel current-carrying wires
- Applying the Biot-Savart Law:
  - Circular arc of current
- Ampère’s Law: Like Gauss’ Law, but named after Ampère
- Magnetic field of a long wire (inside and outside)

Torque on a current loop

$$\vec{F}_B = i\vec{\ell} \times \vec{B}$$



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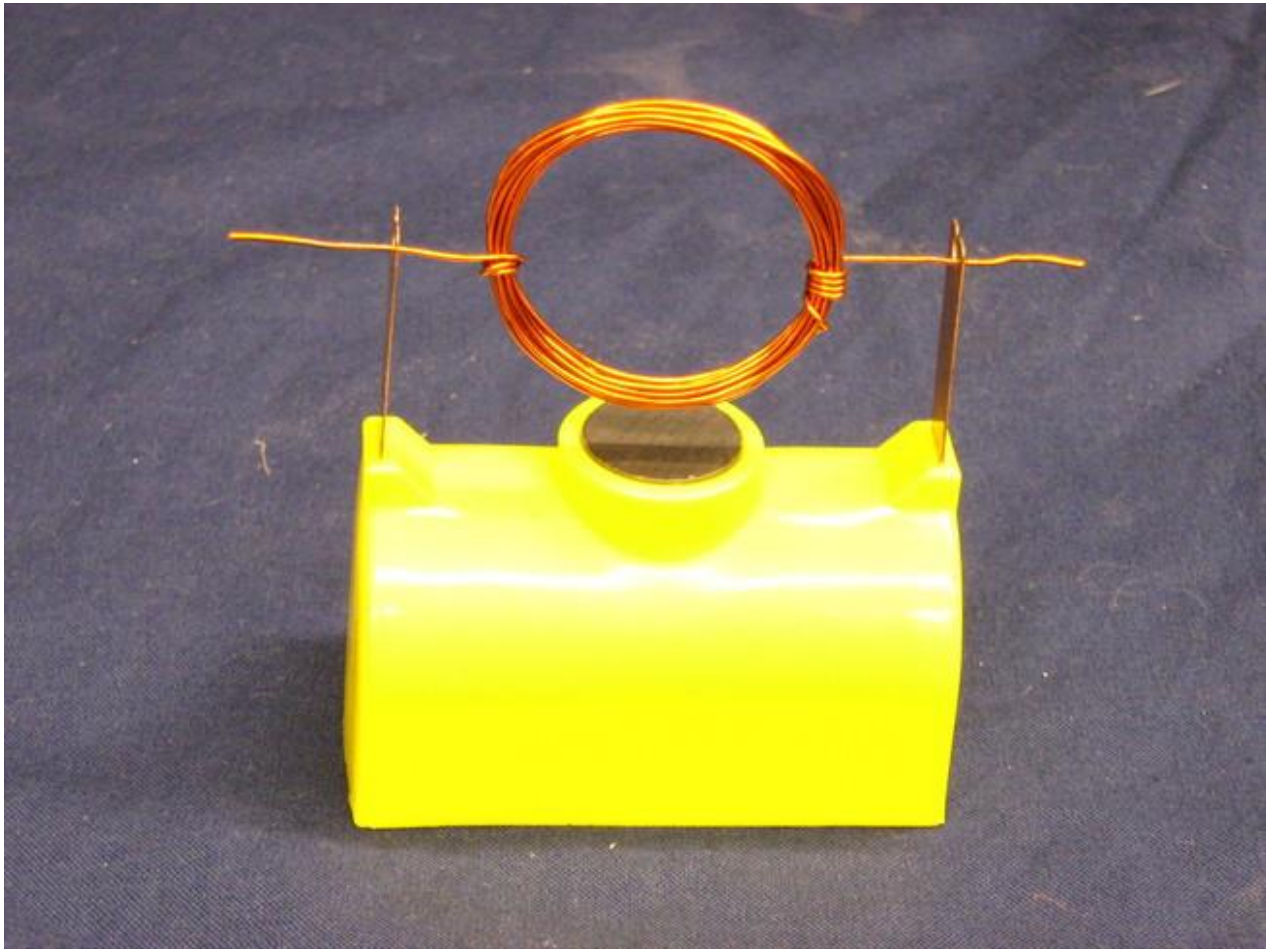
Demonstration more info



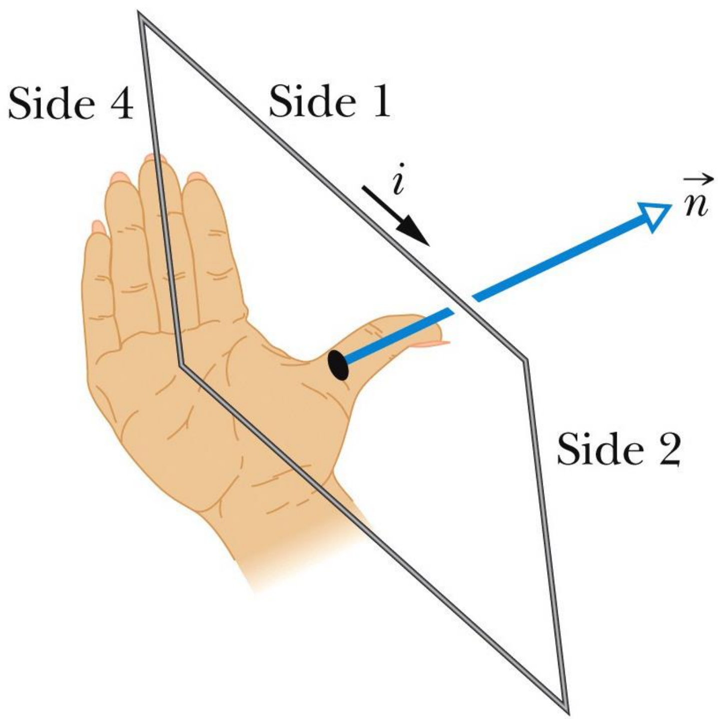
http://www.phas.ucalgary.ca/files/phas/5k40_05_worlds_simplest_motor_pic1.jpg

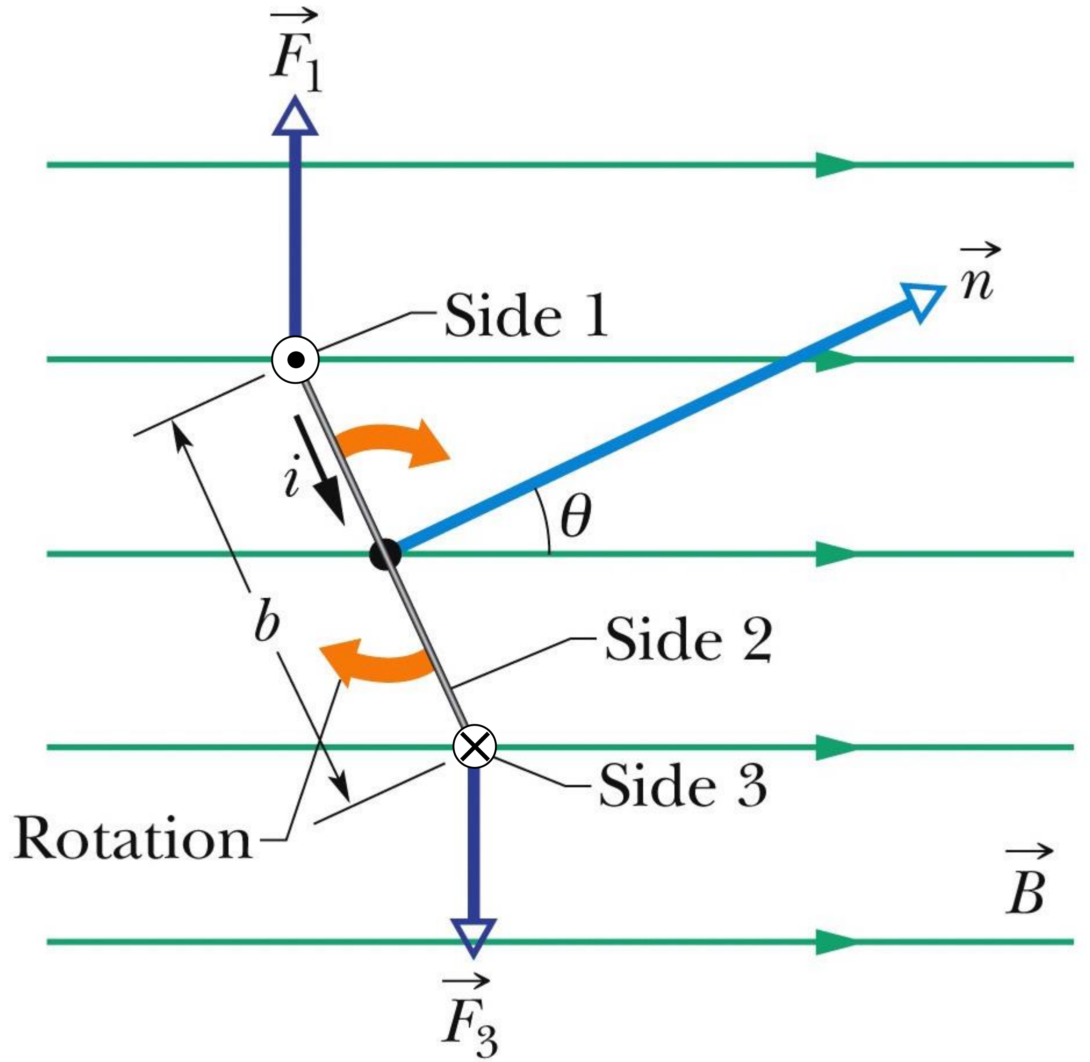
https://www.youtube.com/watch?v=LAprCA8QaXA

http://scitoys.com/scitoys/scitoys/electro/electro.html

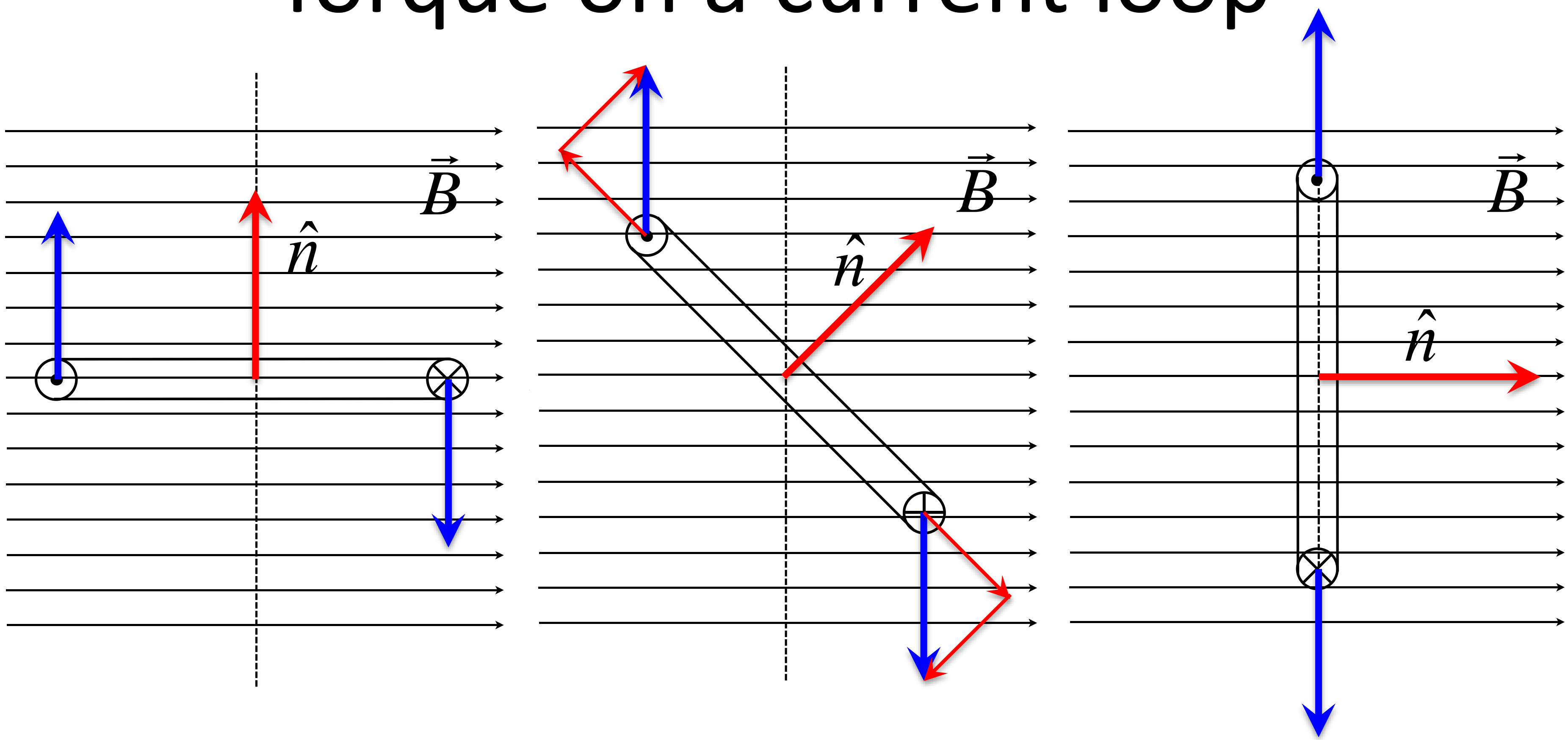
Torque on a current loop

Pick the normal vector to the loop area by RHR: curl your fingers in the direction of i, thumb points in direction of n





Torque on a current loop



The normal vector is at right angles to the B-field: all magnetic force causes rotation of the loop

The normal vector is at some angle to the B-field: some of the magnetic force causes rotation of the loop

The normal vector is parallel to the B-field: none of the magnetic force causes rotation of the loop

Conclusion: components of magnetic force (anti)parallel to normal vector that cause torque

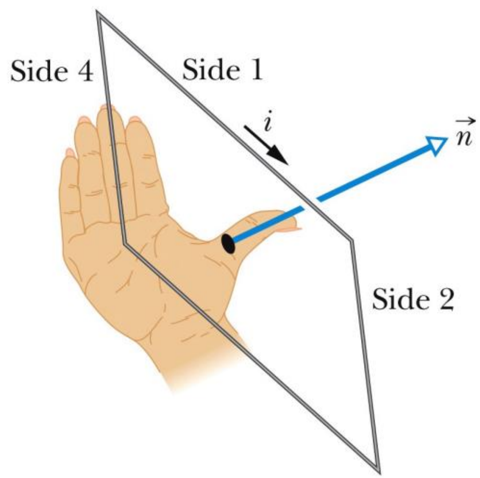
Magnetic moment

$$\vec{\mu} = I\vec{A}$$

- I is the current, $\vec{A}$ is area vector (magnitude equal to the surface are, direction same as $\vec{n}$)

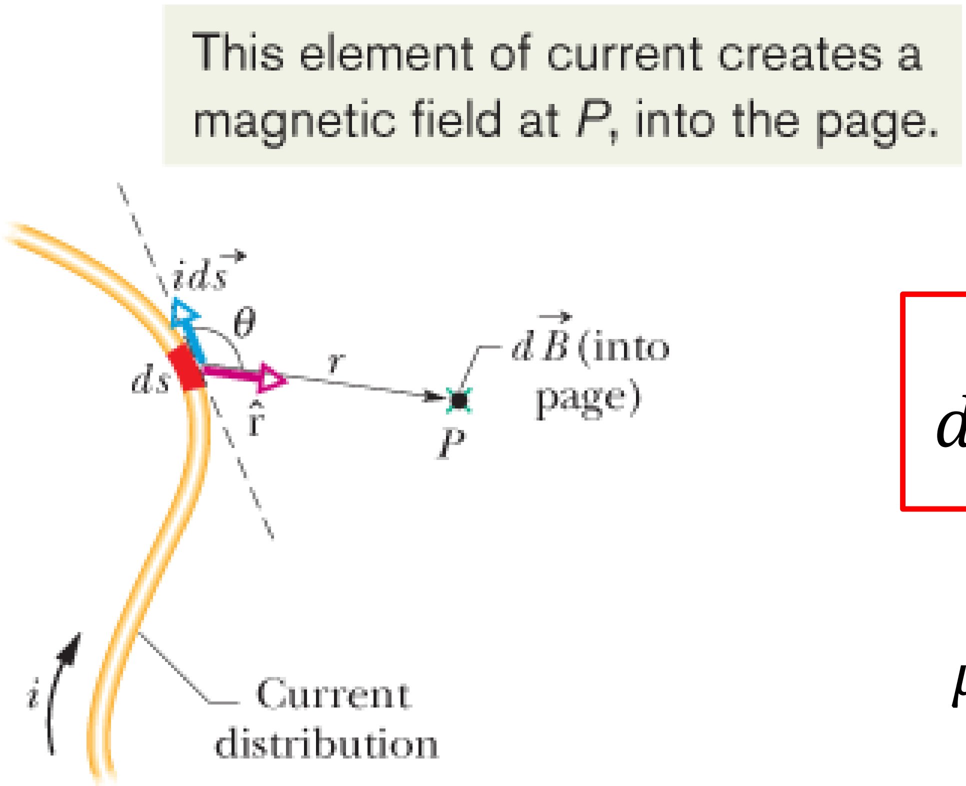
$$\vec{\tau} = \vec{r} \times \vec{F} \qquad \vec{\tau} = \vec{\mu} \times \vec{B}$$

- Labatorial 9



The Biot-Savart Law

(Bee-oh Sah-var)



$$\boxed{d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{l} \times \hat{r}}{r^2}}$$

$$\mu_0 = 4\pi \cdot 10^{-7}\frac{\mathrm{N}s^2}{C^2}$$

“Permeability of free space”

Constants of nature

"Permittivity of free space"

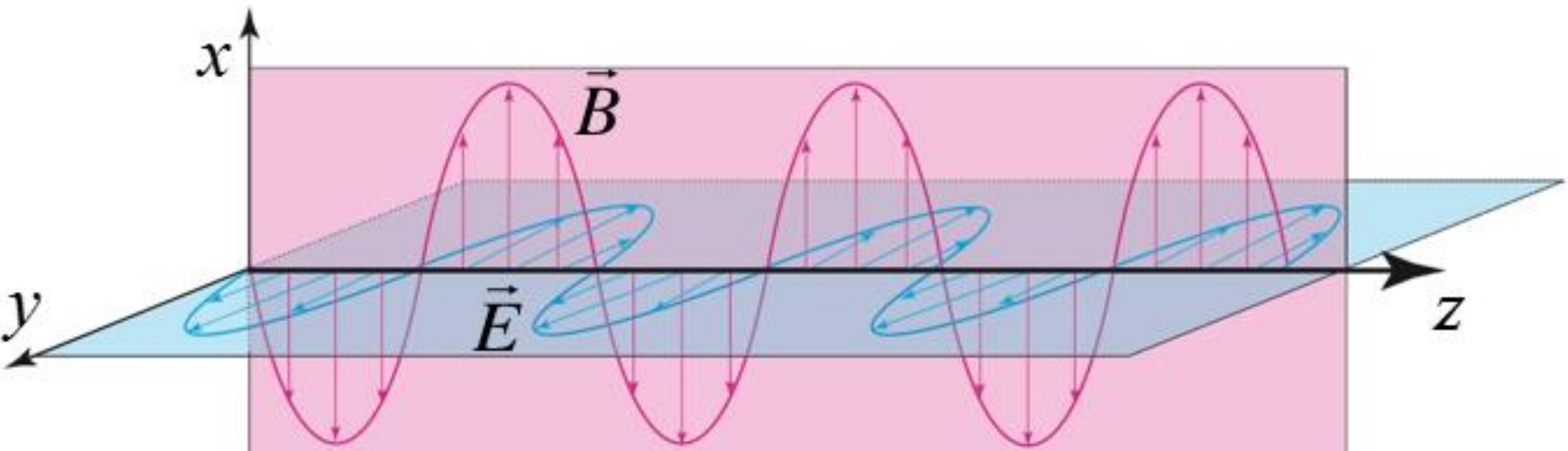
$$\varepsilon_0 = 8.854187781719 \times 10^{-12} \frac{C^2}{N \cdot m^2}$$

"Permeability of free space"

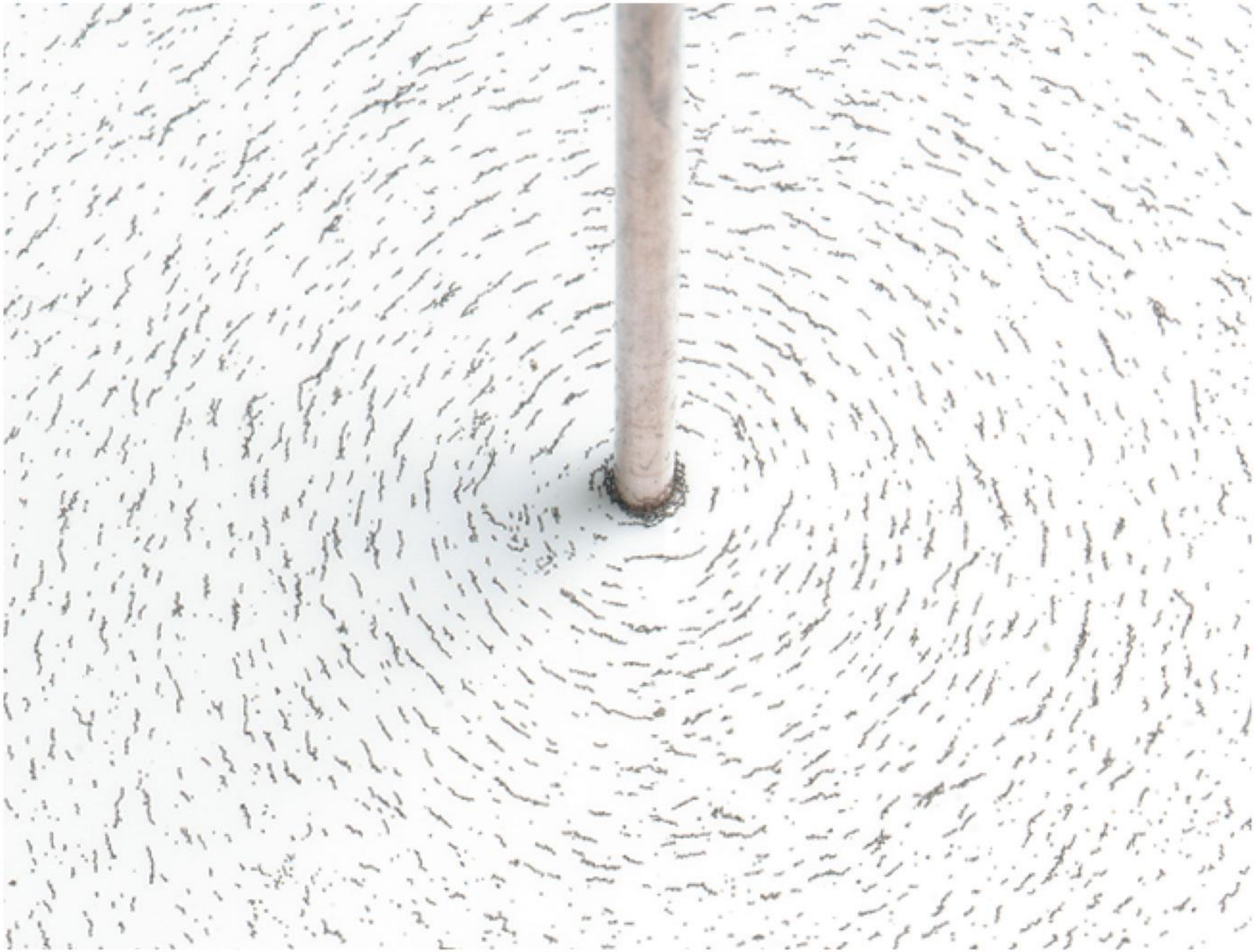
$$\mu_0 = 4\pi \times 10^{-7} \frac{N \cdot s^2}{C^2}$$

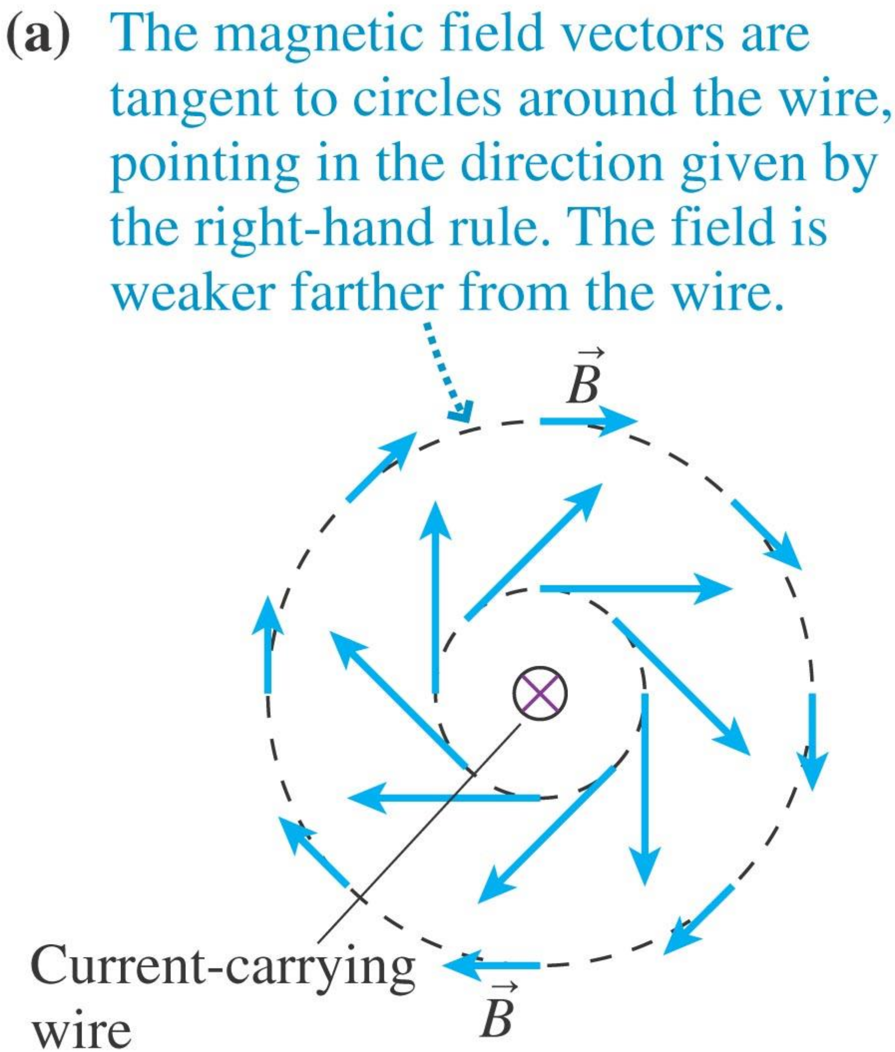
$$\frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 299,792,458 \text{ m/s}$$

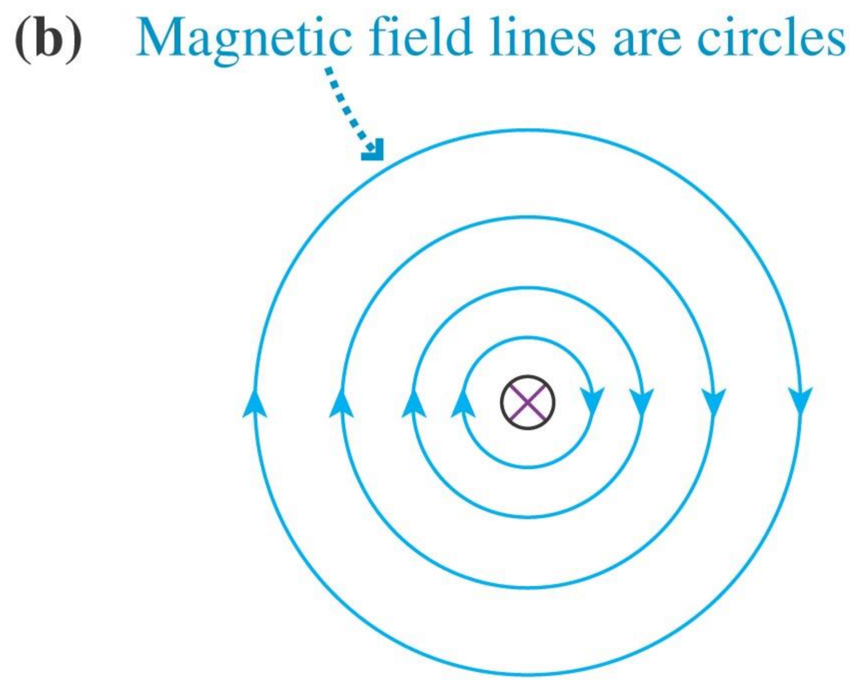
Speed of light!

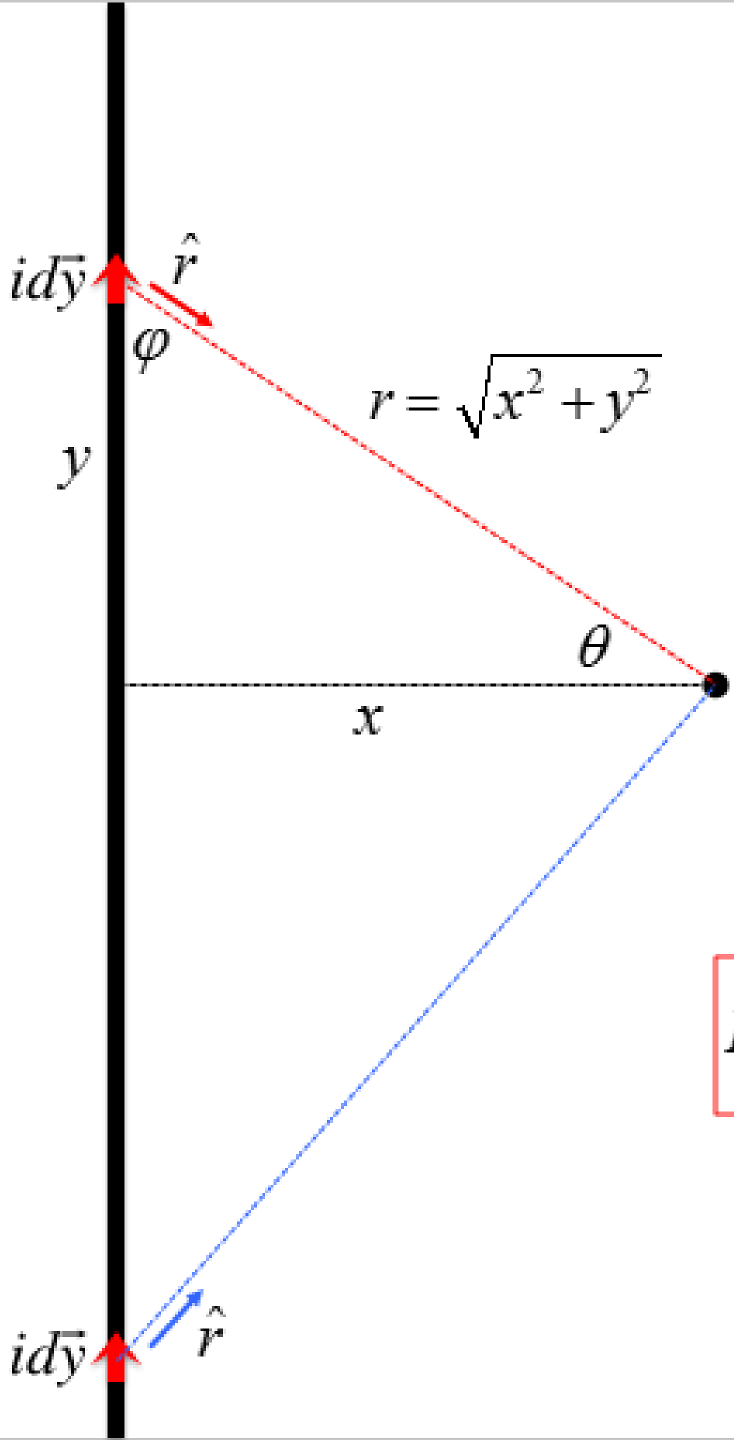


Magnetic Field of a Long, Straight Wire




(a) The magnetic field vectors are tangent to circles around the wire, pointing in the direction given by the right-hand rule. The field is weaker farther from the wire.
$\vec{B}$
Current-carrying wire
$\vec{B}$


(b) Magnetic field lines are circles.



$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{i d\vec{y} \times \hat{r}}{r^2}$$

$$i d\vec{y} \times \hat{r} = i dy \sin\varphi\left(-\hat{k}\right) = -i dy \frac{x}{\sqrt{x^2 + y^2}}\hat{k}$$

$$d\vec{B} = -\frac{\mu_0}{4\pi}\frac{ixdy}{\left(x^2 + y^2\right)^{3/2}}\hat{k}$$

All contributions are in the same direction

$$B = \int_{-\infty}^{\infty}\frac{\mu_0}{4\pi}\frac{ixdy}{\left(x^2 + y^2\right)^{3/2}}$$

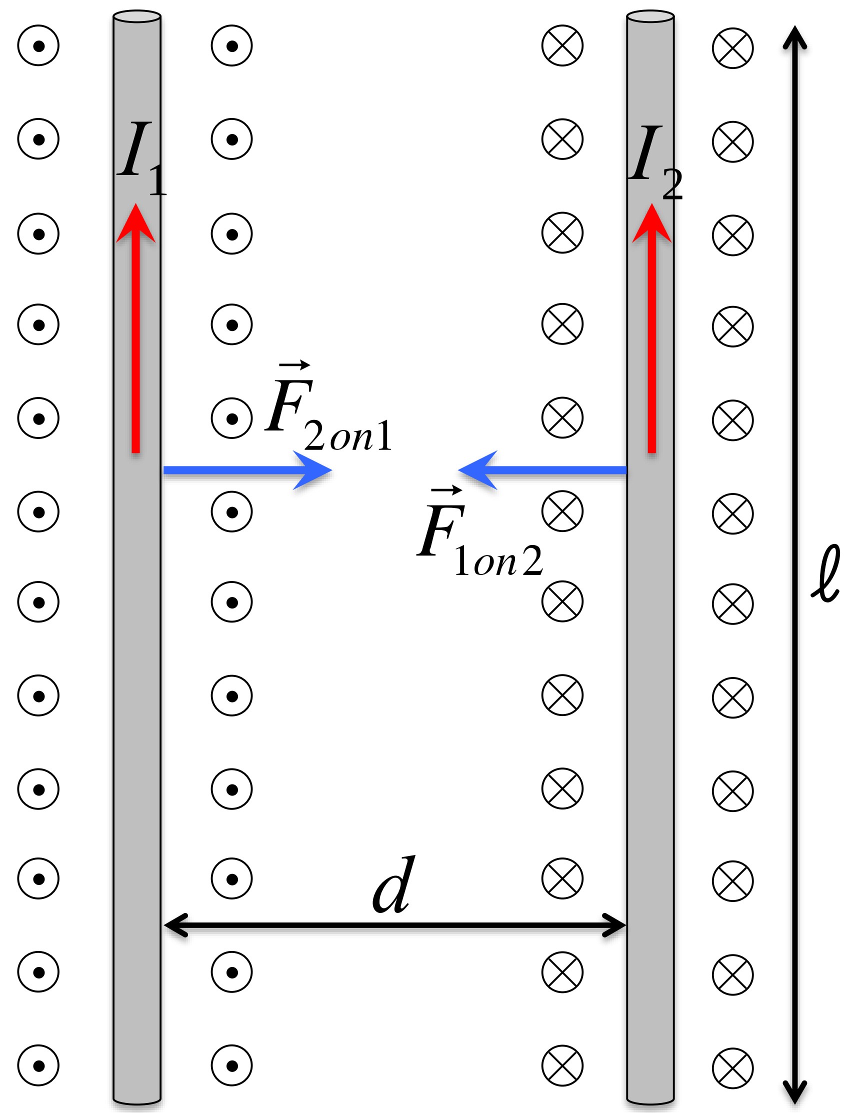
Can just worry about the magnitude

$$\boxed{B_{wire} = \frac{\mu_0 i}{2\pi x}}$$

Magnetic field strength of a long straight wire. Direction from RHR

TopHat Question

$$\left|\vec{B}_2\right| = \frac{\mu_0 I_2}{2\pi d} \qquad \left|\vec{B}_1\right| = \frac{\mu_0 I_1}{2\pi d}$$



Wire 2 exerts a force on wire 1

$$\vec{F}_{2on1} = I_1 \vec{\ell} \times \vec{B}_2$$

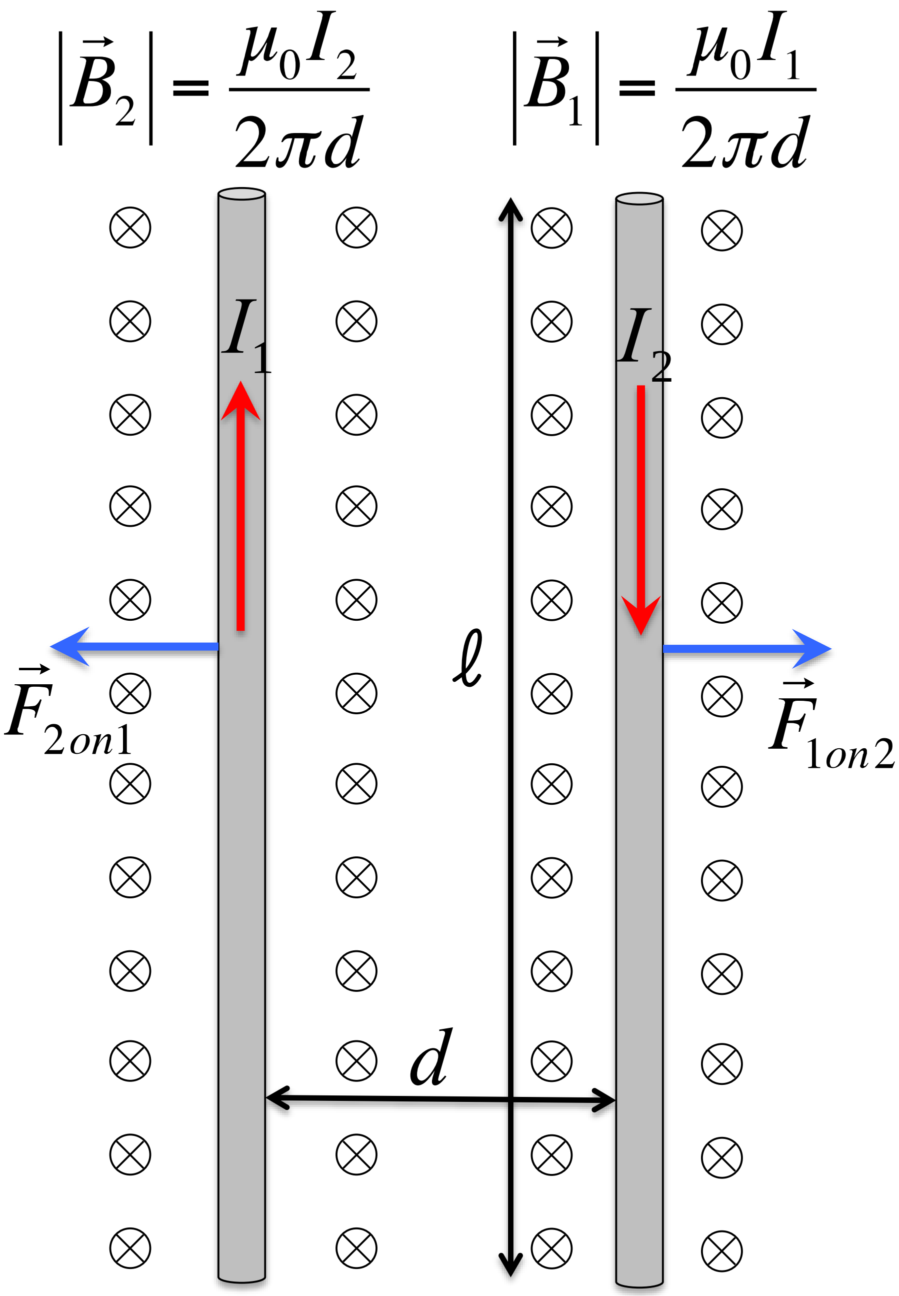
$$\left|\vec{F}_{2on1}\right| = I_1 \ell \frac{\mu_0 I_2}{2\pi d} = \boxed{\frac{\mu_0 \ell I_1 I_2}{2\pi d}}$$

Wire 1 exerts a force on wire 2

$$\vec{F}_{1on2} = I_2 \vec{\ell} \times \vec{B}_1$$

$$\left|\vec{F}_{1on2}\right| = I_2 \ell \frac{\mu_0 I_1}{2\pi d} = \boxed{\frac{\mu_0 \ell I_1 I_2}{2\pi d}}$$

Newton's third law!



Wire 2 exerts a force on wire 1

$$\vec{F}_{2on1} = I_1 \vec{\ell} \times \vec{B}_2$$

$$\left|\vec{F}_{2on1}\right| = I_1 \ell \frac{\mu_0 I_2}{2\pi d} = \boxed{\frac{\mu_0 \ell I_1 I_2}{2\pi d}}$$

Wire 1 exerts a force on wire 2

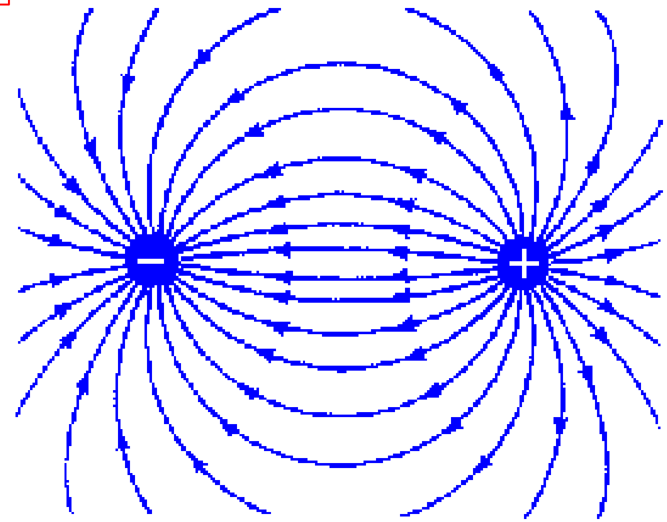
$$\vec{F}_{1on2} = I_2 \vec{\ell} \times \vec{B}_1$$

$$\left|\vec{F}_{1on2}\right| = I_2 \ell \frac{\mu_0 I_1}{2\pi d} = \boxed{\frac{\mu_0 \ell I_1 I_2}{2\pi d}}$$

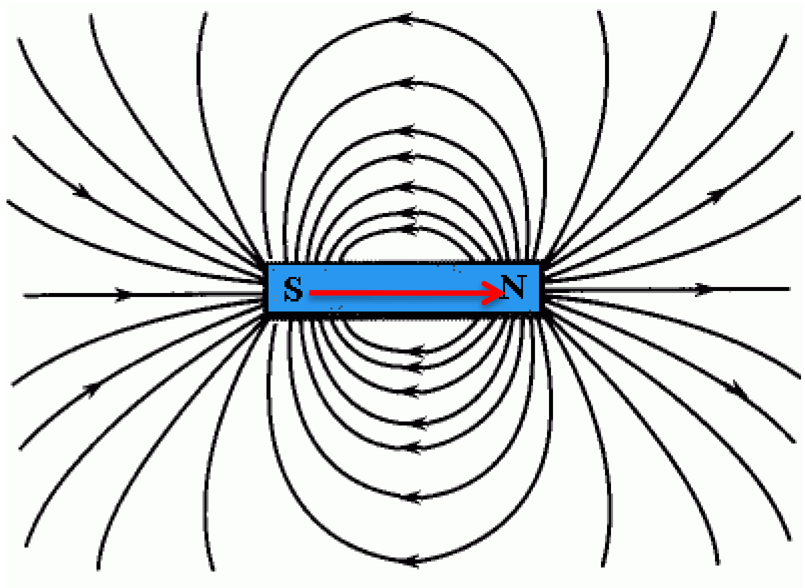
Newton's third law!

Dipole Fields

Electric field from an electric dipole



Magnetic field from a magnetic dipole. Note that the magnetic field lines are continuous – they do NOT stop at the poles!



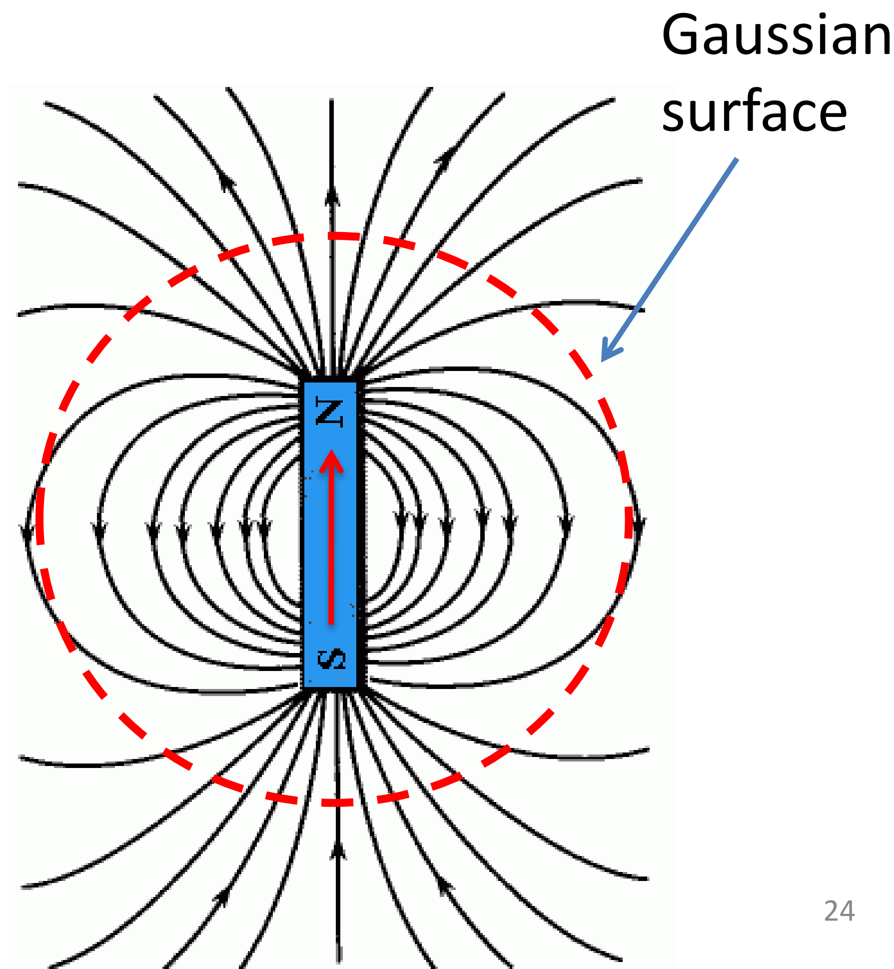
Both fields have the same shape!

Not a Top Hat Question

The magnetic field lines from a magnet point out of the North pole and point into the South pole.

What can you say about the magnetic flux passing through this Gaussian surface?

$$\Phi_B = \oint \vec{B} \cdot d\vec{a}$$



A. Magnetic flux is zero
B. Magnetic flux is greater than zero
C. Magnetic flux is smaller than zero
D. Can't tell without computing the integral

By symmetry the same number of flux lines enter and leave the spherical Gaussian surface

Gauss' Law for Magnetism

The magnetic flux through a closed surface is ALWAYS zero.

$$\Phi_B = \oint \vec{B} \cdot d\vec{a} = 0$$

no enclosed magnetic charges

There is no way to isolate a North or South magnetic pole

The simplest E-field is from a point charge, while the simplest B-field is from a magnetic dipole (e.g. Bar Magnet)

Maxwell's equations

Essentially all of Electricity & Magnetism can be described by a set of 4 equations, referred to as Maxwell's equations.

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\boxed{c = \frac{1}{\sqrt{m_0 e_0}}}$$

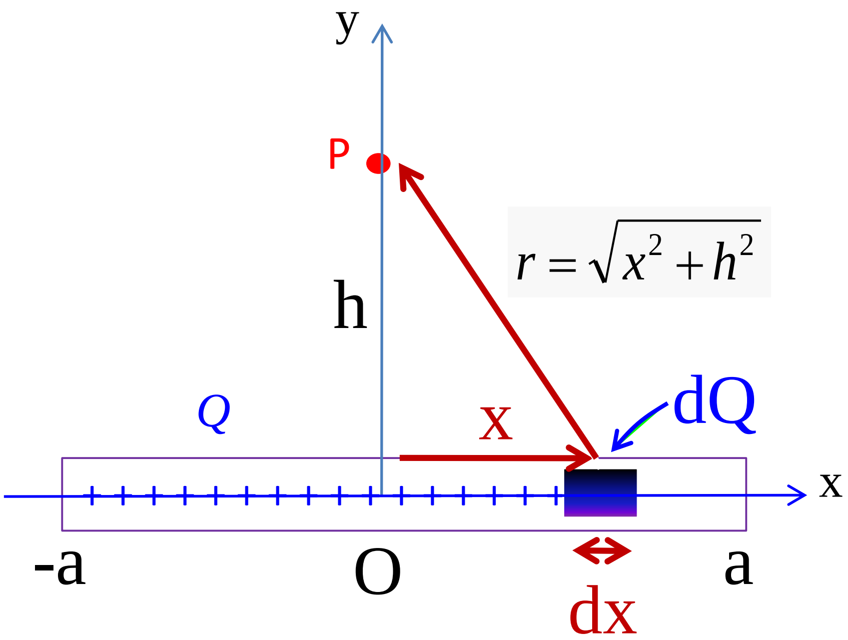
We now have two of them!

$$\Phi_E = \oint \vec{E} \cdot d\vec{a} = \frac{Q_{encl}}{\epsilon_0} \qquad \Phi_B = \oint \vec{B} \cdot d\vec{a} = 0$$

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt} \qquad \oint \vec{B} \cdot d\vec{l} = \mu_0 i_{encl} + \frac{1}{c^2}\frac{d\Phi_E}{dt}$$

We will learn about these other two Maxwell equations today and next week

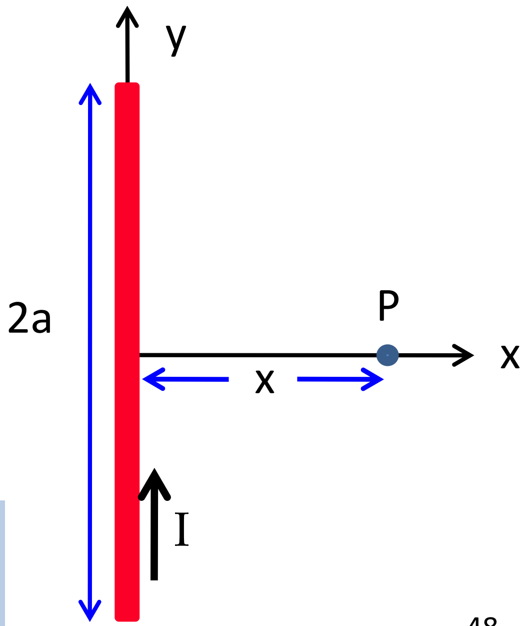
Remember this activity?
Solving for E_P for an infinitely long line of charge (i.e. a >> h) using Coulomb's Law was harder than using



GAUSS'S LAW

$$\oint \vec{E} \cdot d\vec{a} = \frac{Q_{encl}}{\epsilon_0}$$

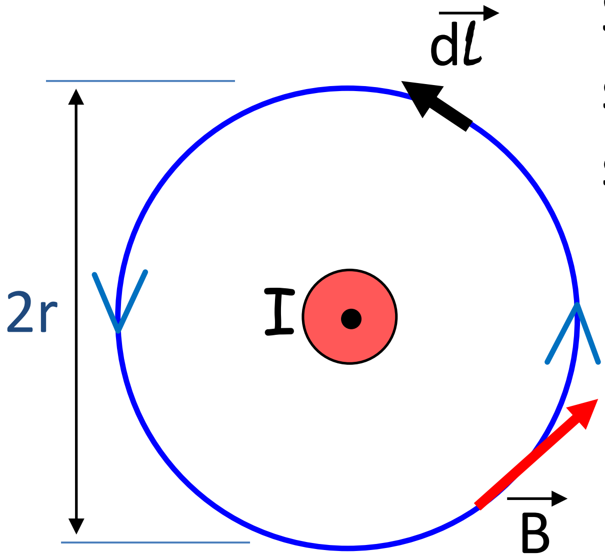
Solving for B_P for an infinitely long current carrying wire (i.e. a >> x) using Biot-Savart's Law was also hard, but there is a MUCH easier alternative!



AMPERE'S LAW

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 i_{encl}$$

Ampère's Law



Suppose we calculate å $\vec{B} \cdot d\vec{l}$ around path shown for the simple case of an infinitely long straight line of current

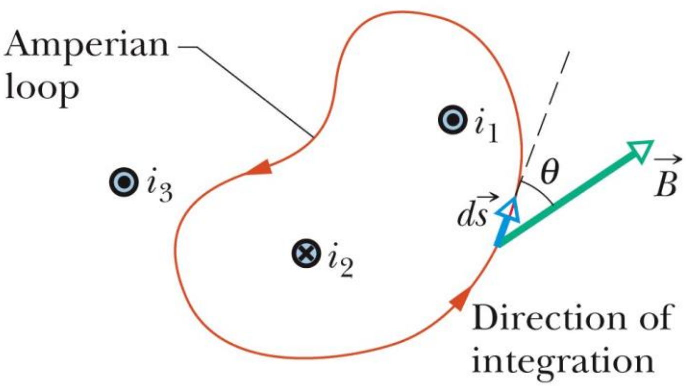
Using our previous result we obtain: $B = \dfrac{\mu_0 I}{2\pi r}$

$$\oint \vec{B} \cdot d\vec{l} = (2\pi r)\left(\frac{\mu_0 I}{2\pi r}\right)$$

i.e. $\boxed{\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}}$

Ampère's *Law is true for any* ***shape of path*** *and any current distribution*

Only the currents encircled by the loop are used in Ampere's law.



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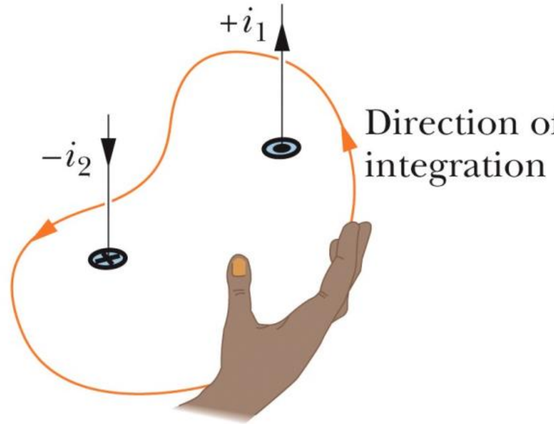
This is how to assign a sign to a current used in Ampere's law.



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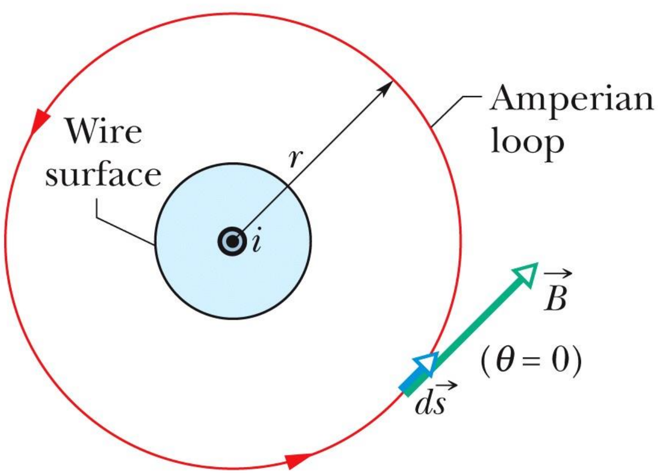
All of the current is encircled and thus all is used in Ampere's law.



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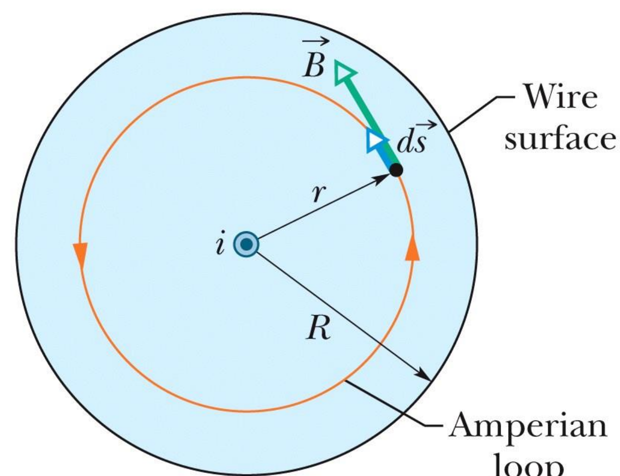
Only the current encircled by the loop is used in Ampere's law.



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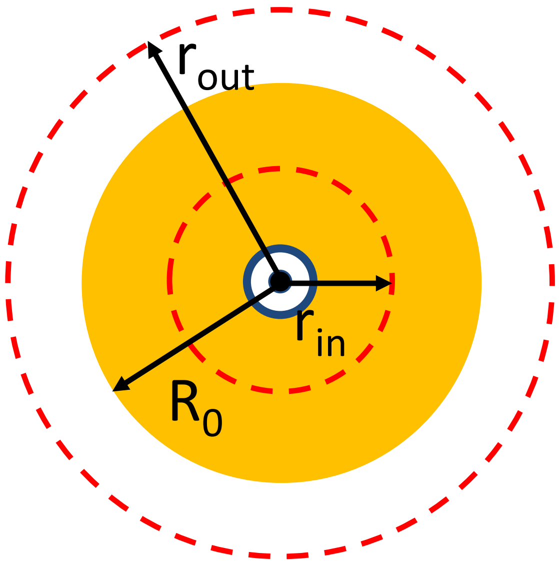
Ampère’s law: application

(a) Using Ampère’s law, calculate the magnetic field inside a solid current carrying wire a distance r_{in} from its axis.

(The length of the solid wire is infinite and the current I is uniformly distributed throughout the solid wire)



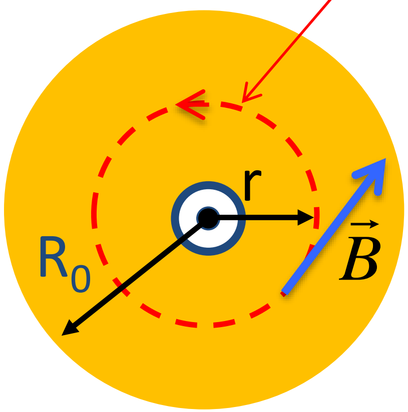
End view:
Wire with radius R
and current I

b) Calculate the magnetic field outside a solid current carrying wire a distance r_{out} from its axis.

Ampère's law: application

(a) B-field inside

We want to know the B-field a distance r, so we choose an Amperian circular loop with radius r < R_0.



Ampère's Law: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$

Left hand side: $\oint \vec{B} \cdot d\vec{l} = BL = B2\pi r$

Right hand side: $\mu_0 I_{enc} = \mu_0 JA = \mu_0 \frac{I}{pR_0^2} pr^2$

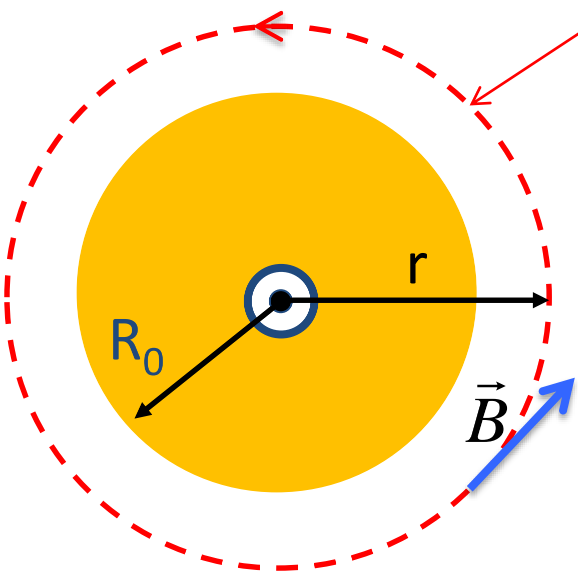
Combine together: $B2pr = \mu_0 \frac{I}{R_0^2} r^2$

$$\boxed{B = \frac{\mu_0 I\, r}{2pR_0^2}}$$

Ampère’s law: application

(a) B-field outside

We want to know the B-field a distance r, so we choose an Ampèrian loop with radius r > R_0.



Ampère’s Law: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$

Left hand side: $\oint \vec{B} \cdot d\vec{l} = BL = B2\pi r$

Right hand side: $\mu_0 I_{enc} = \mu_0 I$

Combine together: $B2pr = \mu_0 I$

$$\boxed{B = \frac{\mu_0 I}{2pr}}$$