Electricity and Magnetism

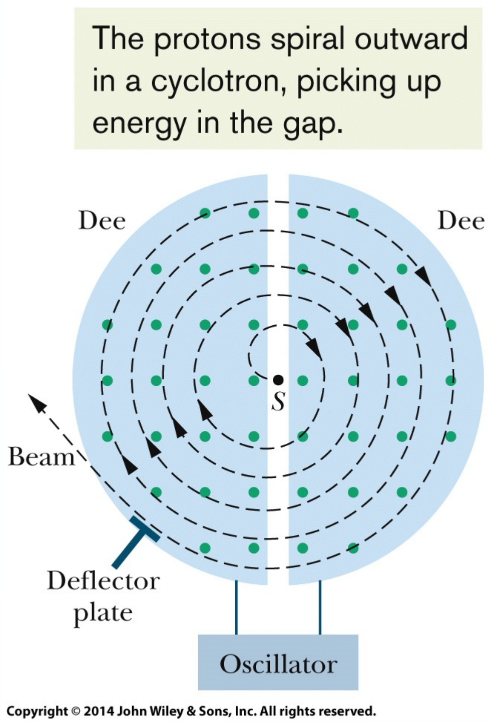
- Physics 259 – L02
- Lecture 39

Chapter 28: Magnetic fields

Last Section:

✓ The Cyclotron

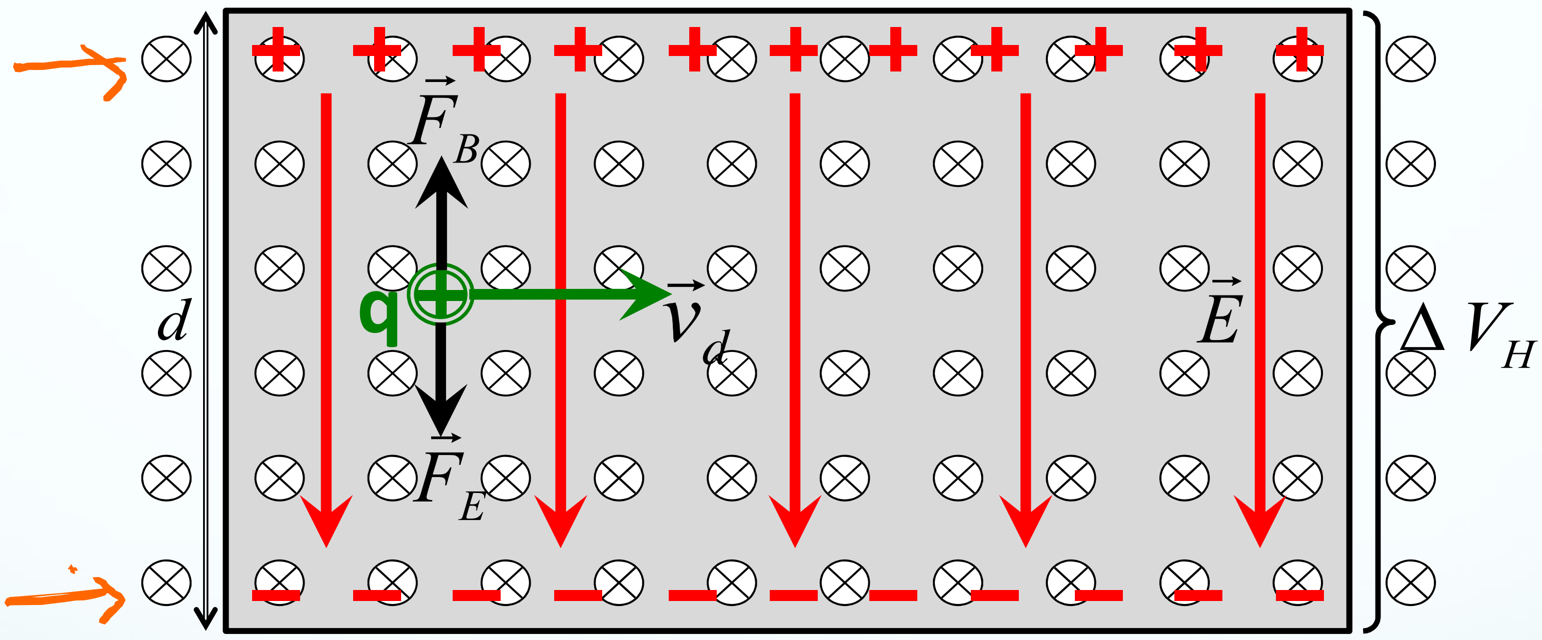
✓ The proton Synchrotron



$$\boxed{R = \frac{mv}{|q|B}} \quad \boxed{T_{cyc} = \frac{2\pi m}{|q|B}} \quad \boxed{f_{cyc} = \frac{|q|B}{2\pi m}}$$

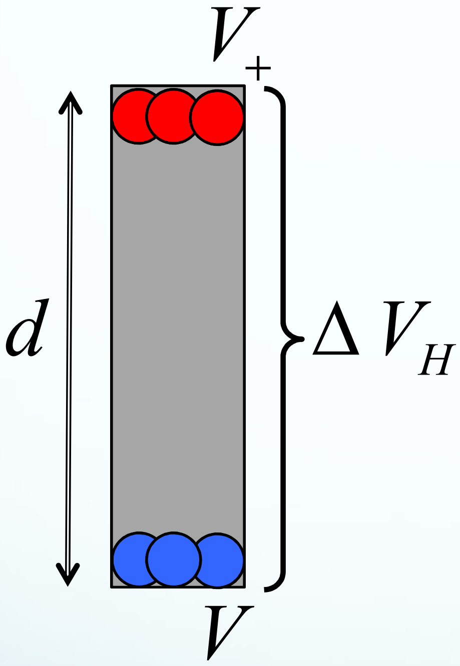
Last Section:

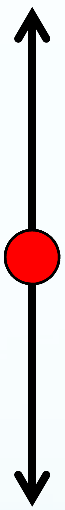
Hall effect



$$\boxed{\Delta V_H = v_d B d}$$

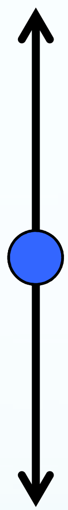
Conductors moving in B-fields





$$F_B = qvB$$

$$F_E = q\frac{\Delta V_H}{d}$$



$$F_E = q\frac{\Delta V_H}{d}$$

$$F_B = qvB$$

$qE = qv \times B$

In equilibrium, forces balance, leading to a constant voltage

$$q\frac{\Delta V_H}{d} = qvB$$

$$\boxed{\Delta V_H = vBd}$$

v of moving conductor

28-6 Magnetic Force on a Current-Carrying Wire

Conductors in B-fields

Free charges moving in a B-field feel→

$$\vec{F}_B = q\,\vec{v} \times \vec{B}$$

Conductors are full of charges that are free to move around→

If a conductor moves in a magnetic field, these charges also feel a magnetic force

28-6 Magnetic Force on a Current-Carrying Wire

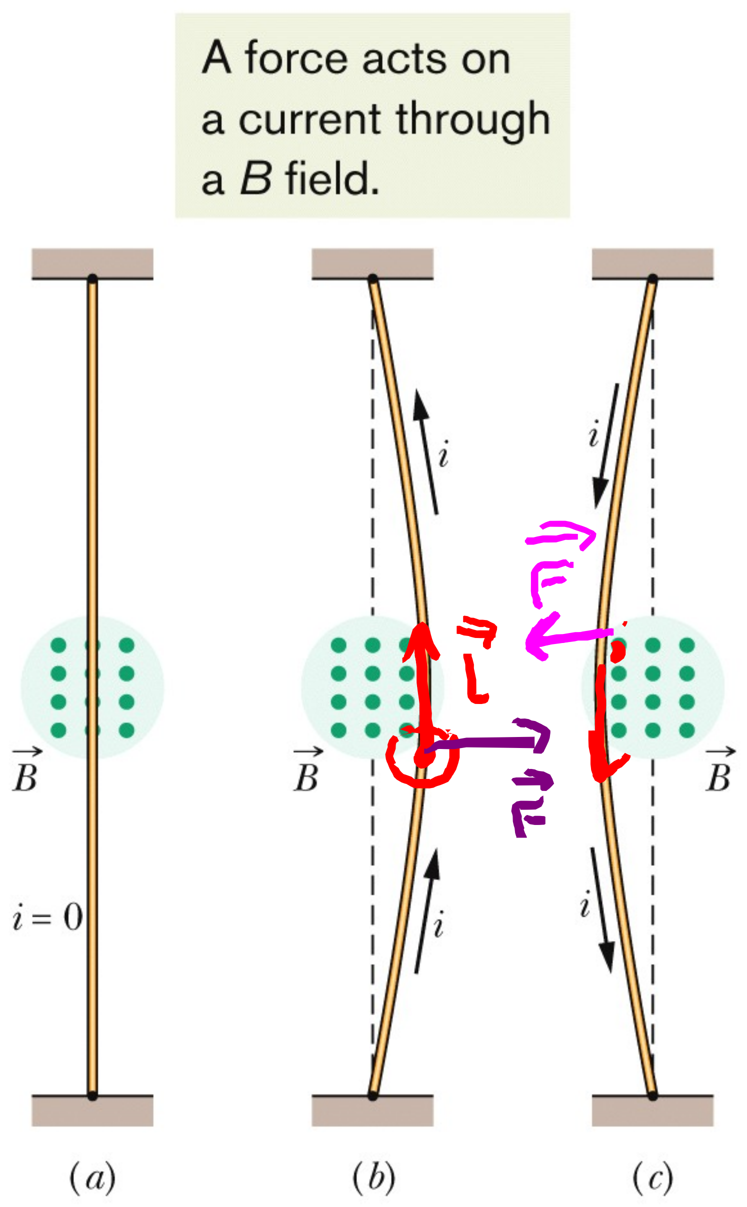
A straight wire carrying a current *i* in a uniform magnetic field experiences a sideways force

$$\vec{F}_B = i\vec{L} \times \vec{B} \quad \text{(force on a current).}$$

Here ***L*** is a length vector that has magnitude *L* and is directed along the wire segment in the direction of the (conventional) current.

$i \rightarrow$ current

$\vec{L} \rightarrow$ length vector



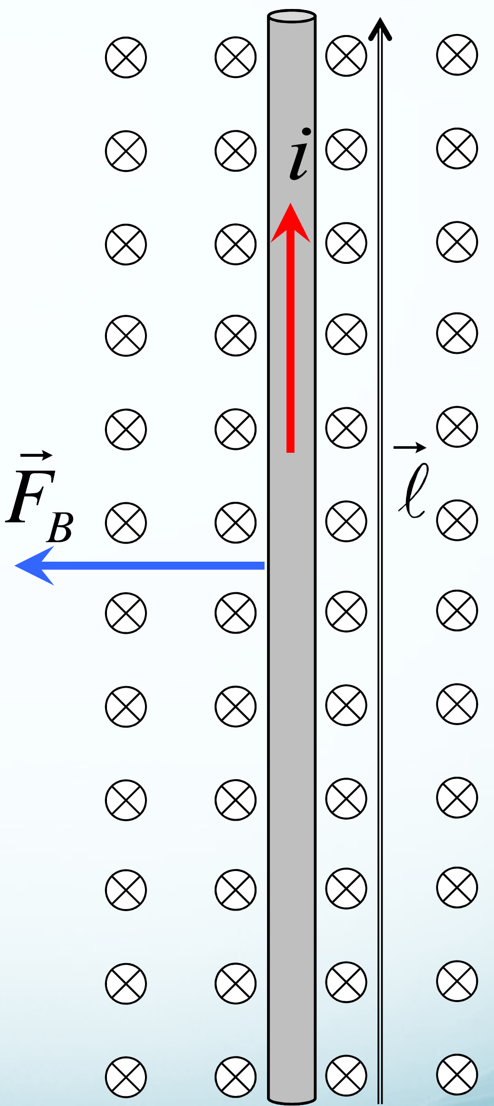
Proof

Current in wires is nothing more than charges in motion.

It doesn't matter if we consider $-q$ moving opposite i or $+q$ moving in the same direction as i

For a single charge→

$$\vec{F}_B = q\vec{v}_d \times \vec{B}$$



For N charges moving through the wire:

$q = it = i\frac{L}{v_d} \Rightarrow \vec{F}_B = q\vec{v}_d \times \vec{B}$

$\rightarrow \vec{F}_B = i\frac{L}{\cancel{v_d}}\cancel{v_d} \times B = iLB\sin\theta$

$\vec{l}$

$\rightarrow \vec{F}_B = i\vec{l} \times \vec{B}$

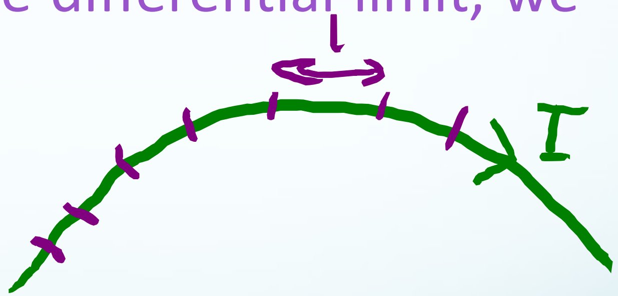
$$\boxed{\vec{F}_B = i\vec{\ell} \times \vec{B}}$$

Crooked Wire

If a wire **is not straight** or the **field is not uniform**→
we can imagine the wire broken up into small straight segments.

The force on the wire as a whole is then the vector sum of all the forces on the segments that make it up. In the differential limit, we can write

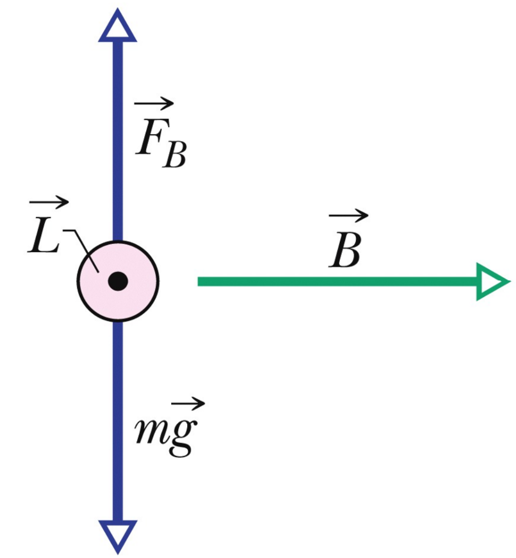
$$\boxed{d\vec{F}_B = i\, d\vec{L} \times \vec{B}.}$$



and the direction of length vector ***L*** or *d**L*** is in the direction of *i.*

TopHat Question

A wire of length $50\ \mathrm{cm}$ is carrying a current i out of the page and is sitting in a uniform magnetic field of $500\ \mathrm{mT}$ pointing to the right. If the wire has a mass of $25\ \mathrm{g}$, what current i is needed to support its weight?



halliday_10e_fig_28_17

$$\vec{F}_B = i\vec{\ell} \times \vec{B}$$

$$\vec{F}_B = iLB\sin\theta = mg \rightarrow i = \frac{mg}{LB\sin\theta} \ \& \ \theta = 90^\circ$$

A. 9.81×10^{-3} A

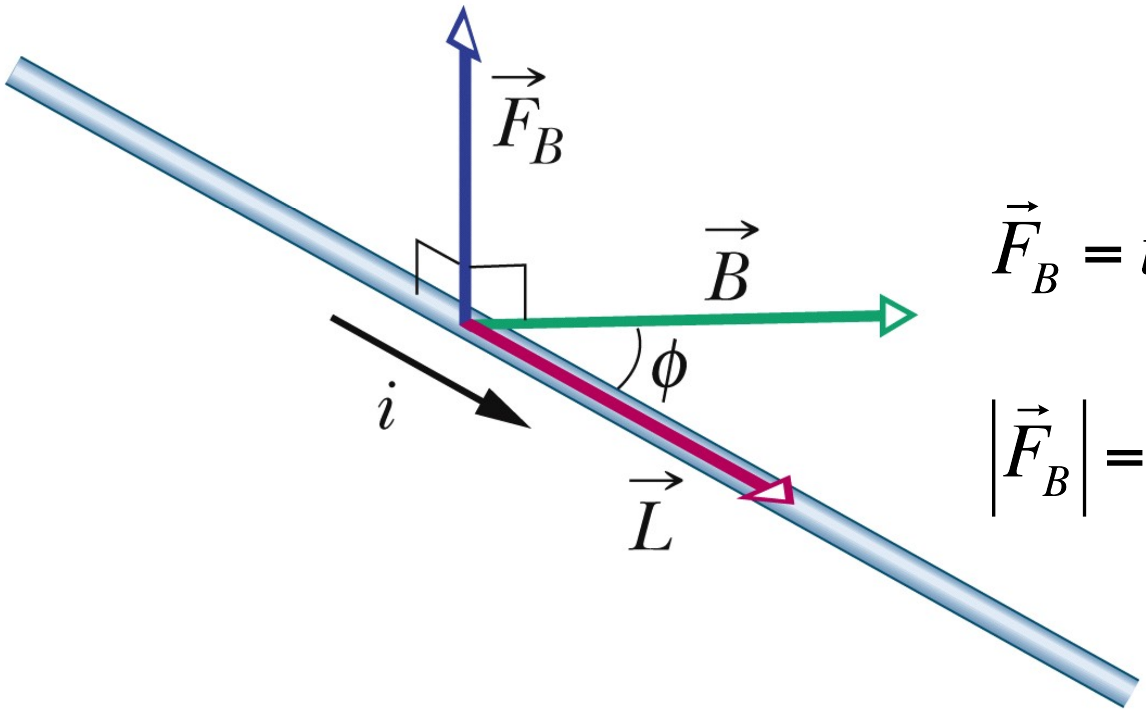
C. 0.981 A ✓

A. 1.02 A

D. 1.02×10^{-3}

Forces on Current-Carrying Wires: B and L not perpendicular

The force is perpendicular to both the field and the length.



$$\vec{F}_B = i\vec{L} \times \vec{B}$$

$$\left|\vec{F}_B\right| = \left|i\vec{L} \times \vec{B}\right| = iLB\sin\phi$$

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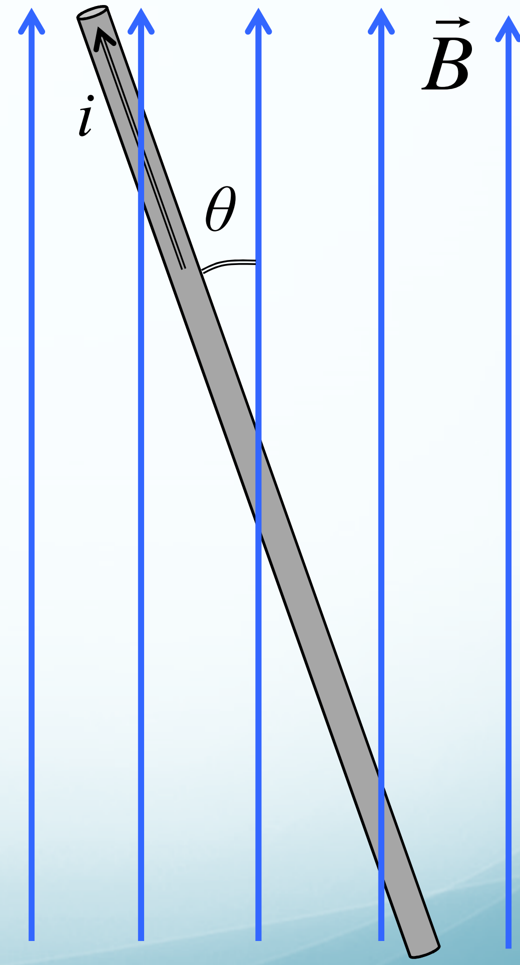
TopHat Question

A wire of length $50\ \mathrm{cm}$ is carrying a current i and is sitting in a uniform magnetic field B as shown. What is the magnitude and direction of the magnetic force on the wire?



$$\vec{F}_B = i\vec{\ell} \times \vec{B}$$

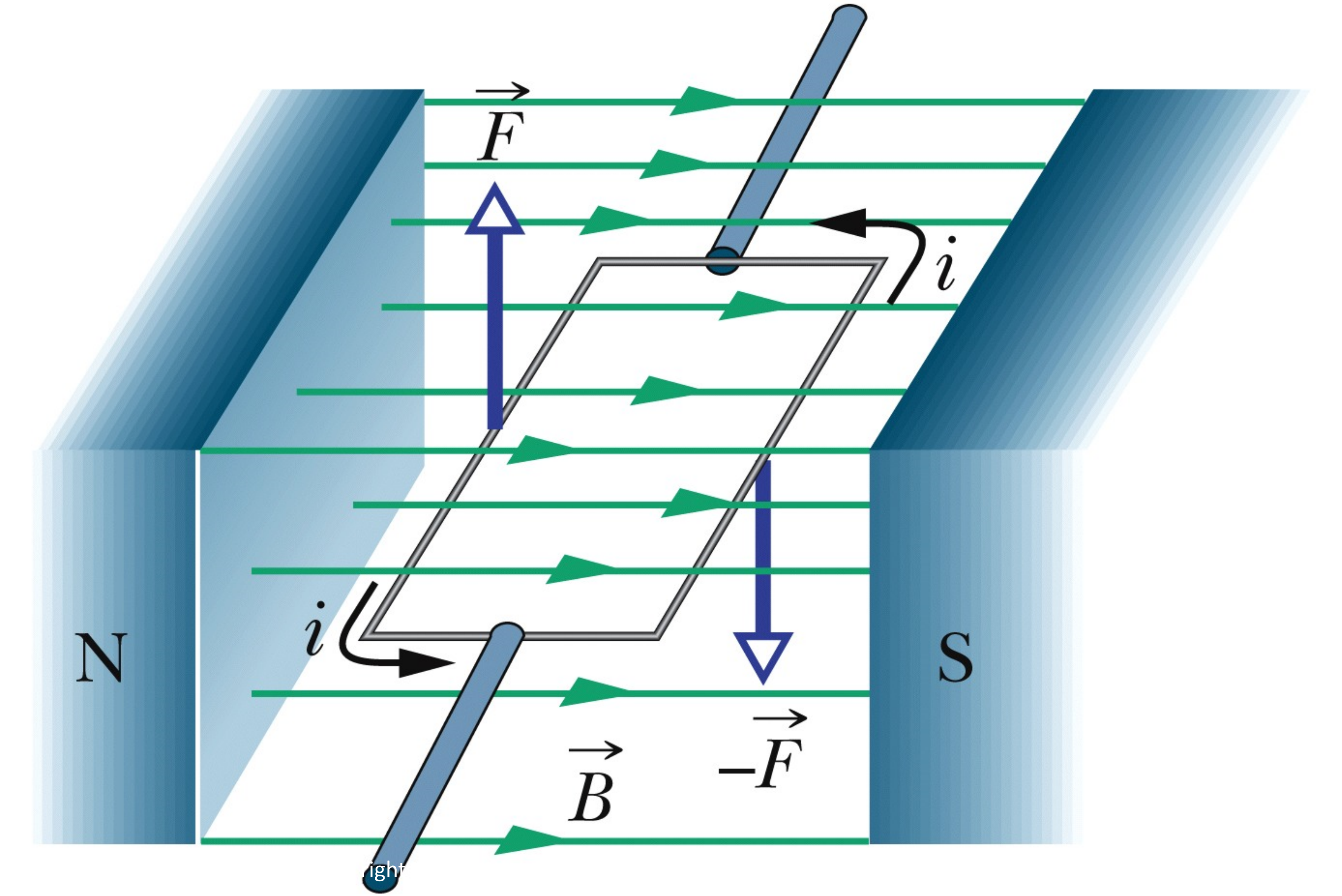
A. ilB $\odot$

B $ilB\sin\theta$ $\otimes$

C. $ilB\sin\theta$ $\swarrow$

D. ilB $\nearrow$

28-7 Torque on a Current Loop



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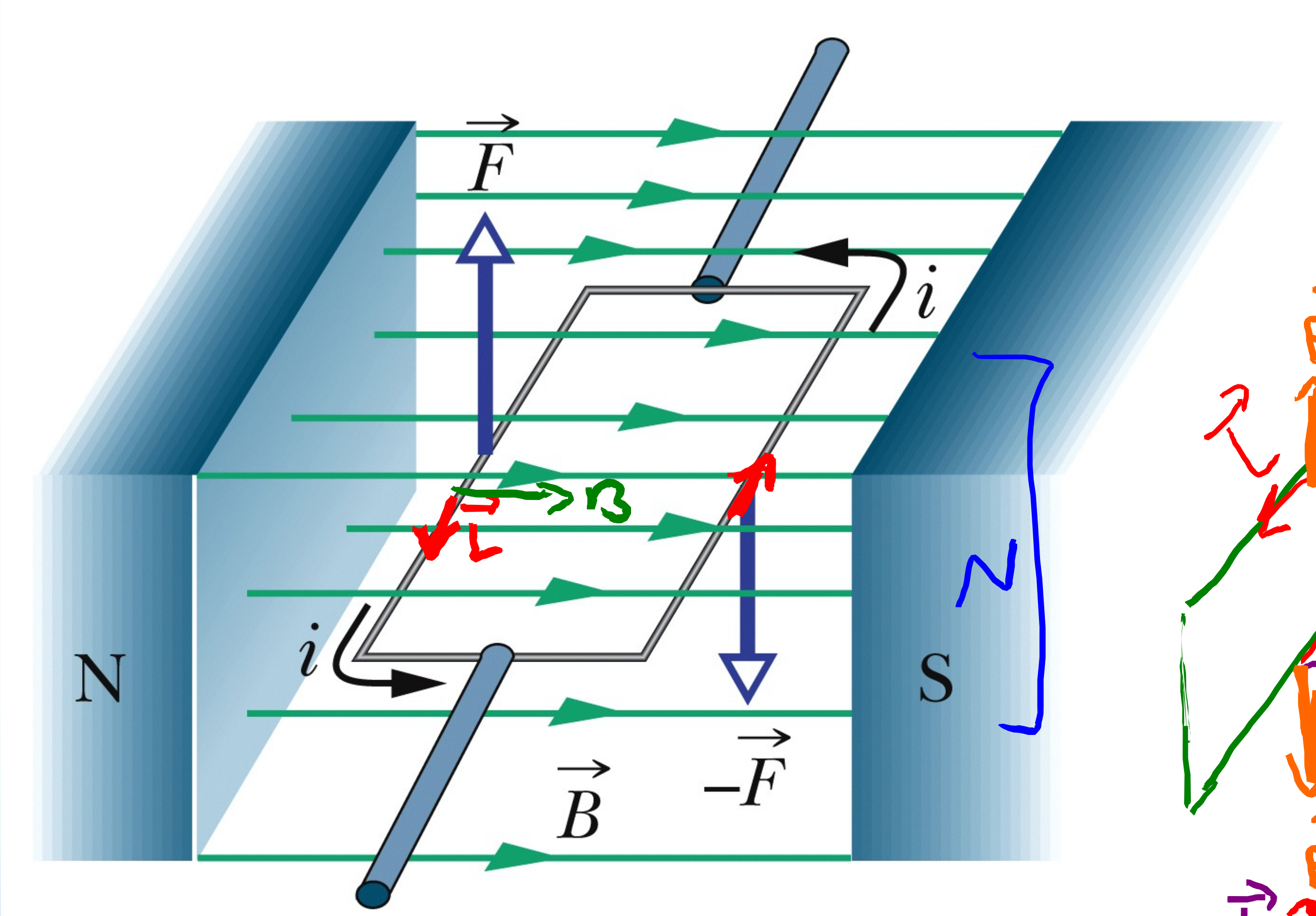
$$\vec{F}_B = i\vec{L} \times \vec{B} \quad \text{(force on a current).}$$



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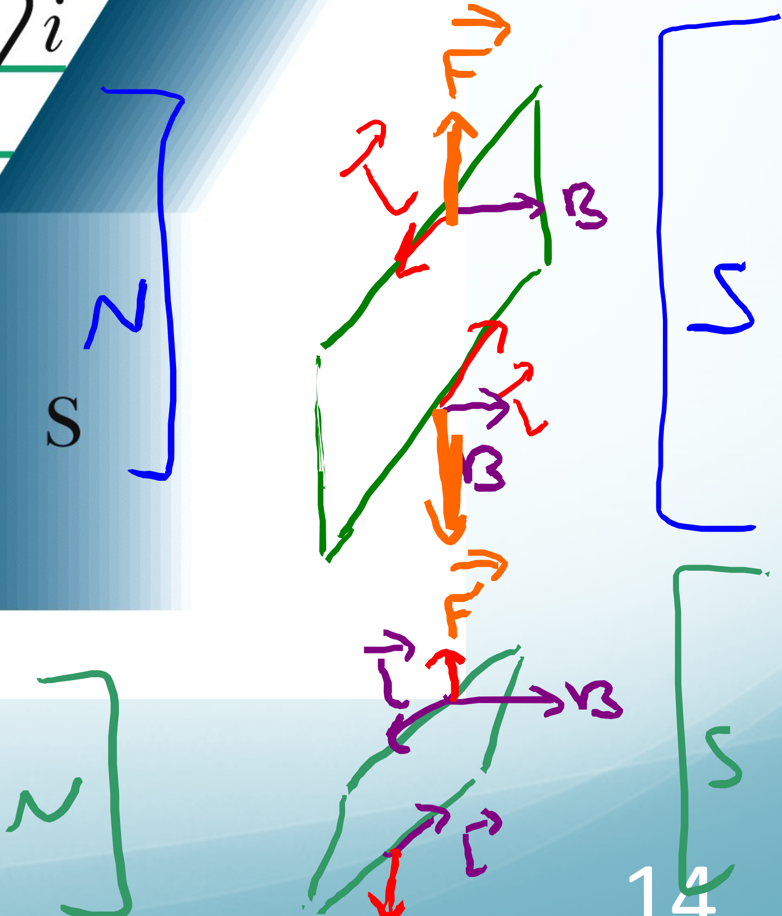
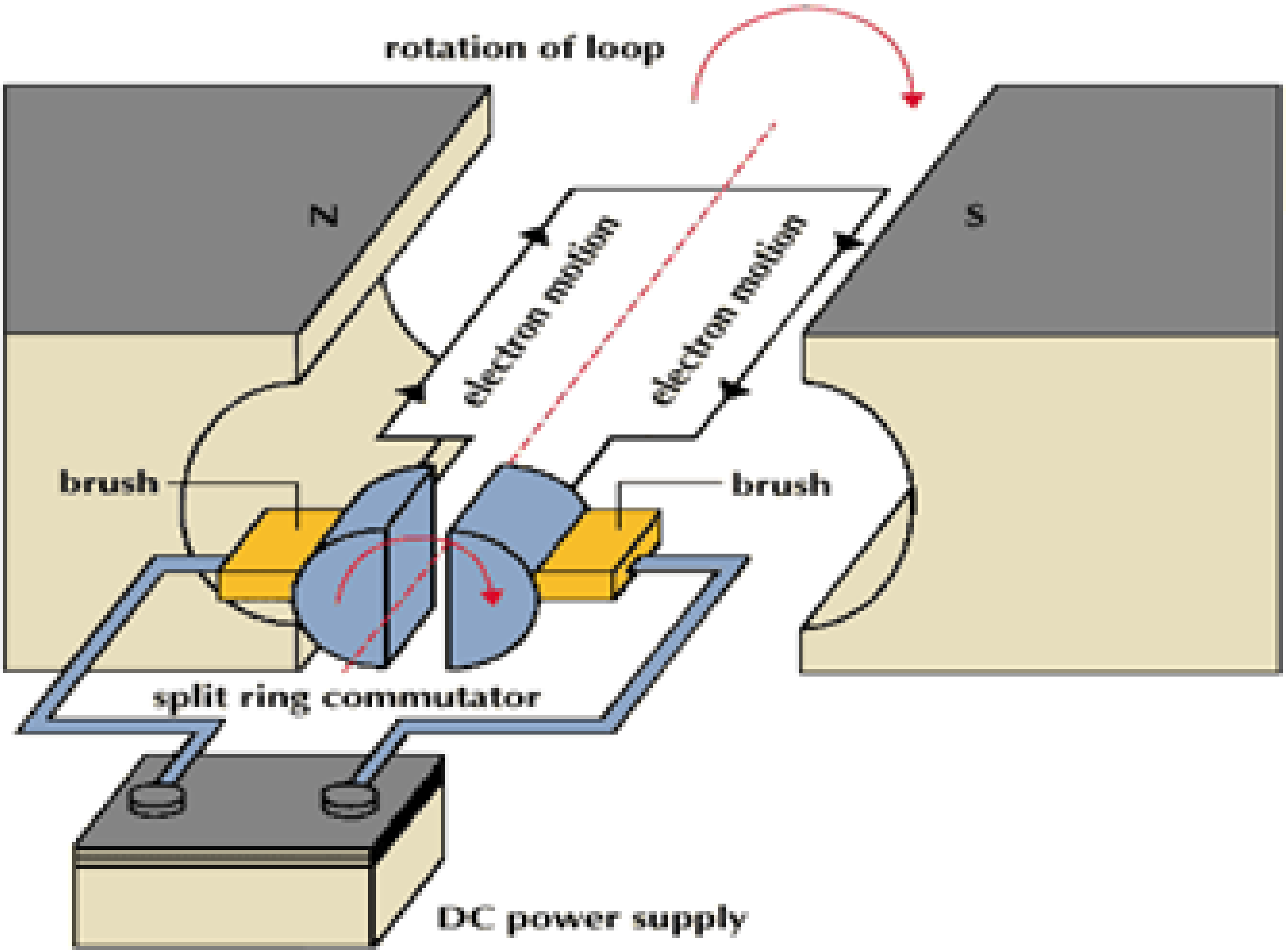
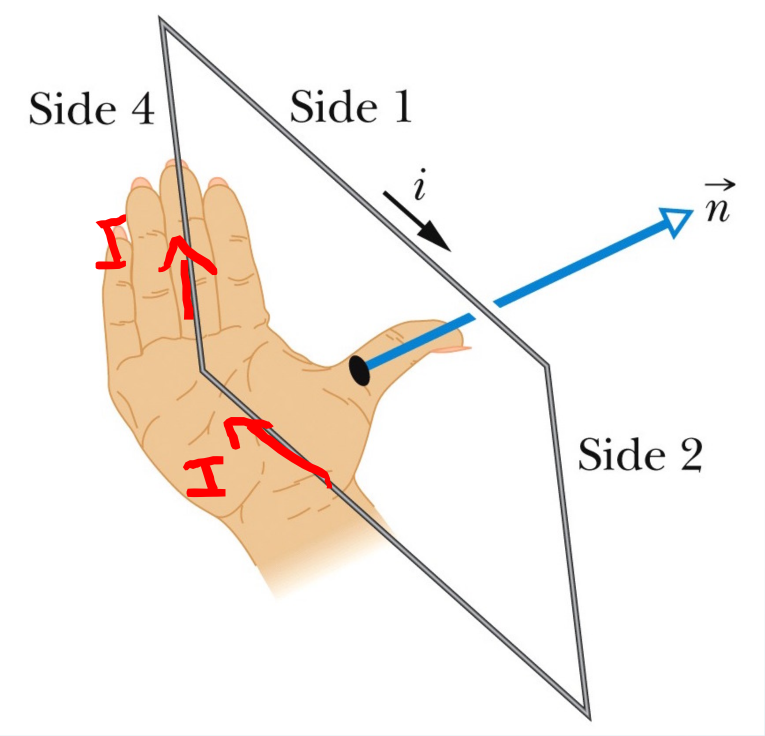
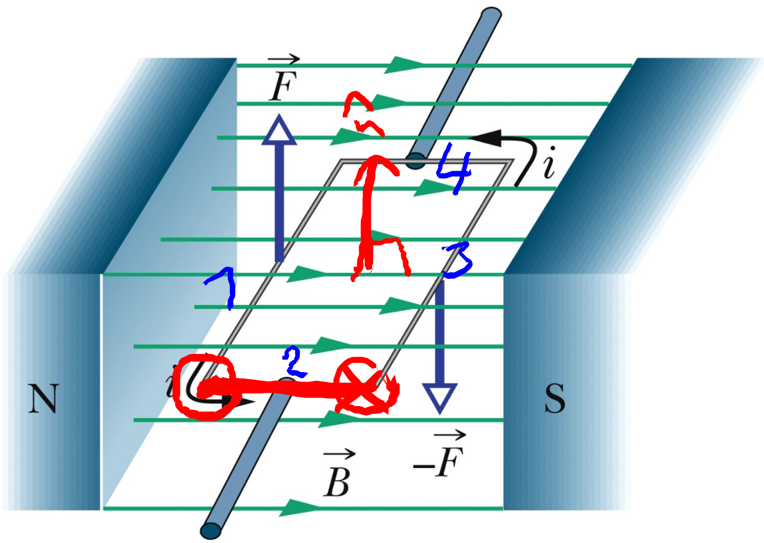



rotation of loop
N
S
electron motion
electron motion
brush
brush
split ring commutator
DC power supply

Pick the normal vector to the loop area by RHR: curl your fingers in the direction of i, thumb points in direction of n





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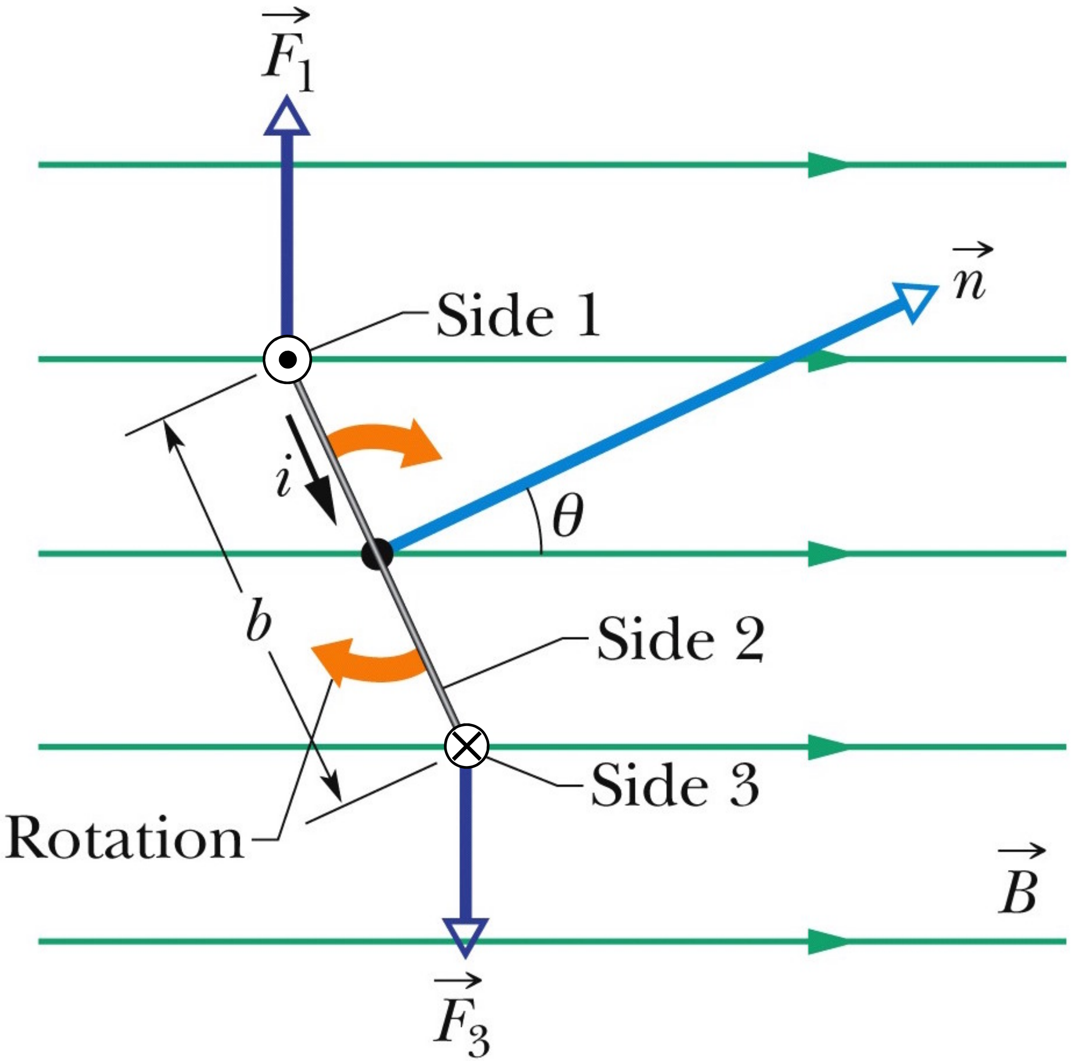
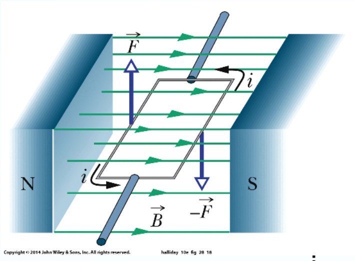
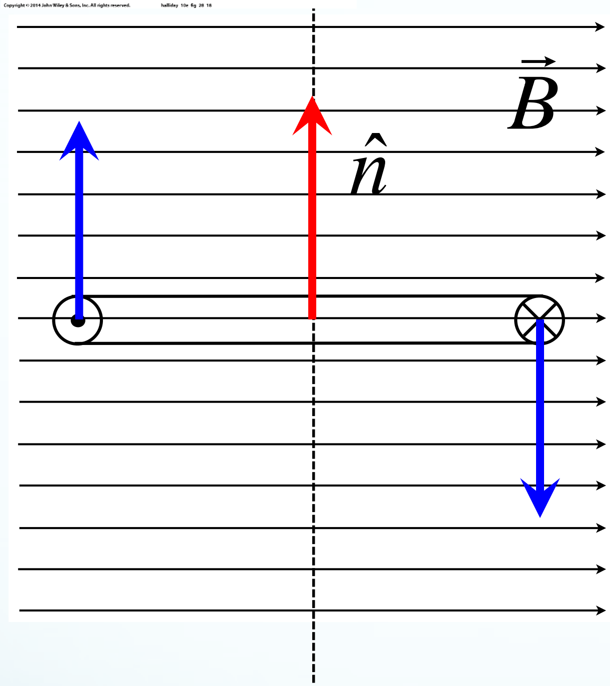
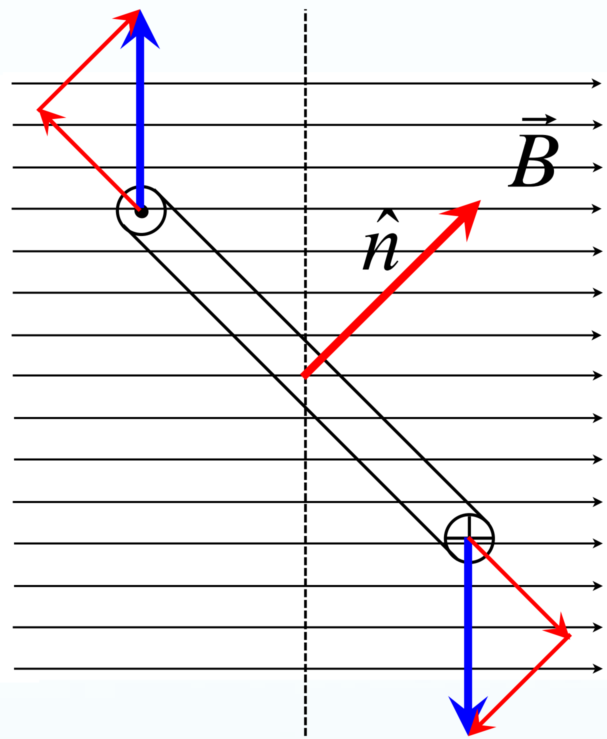


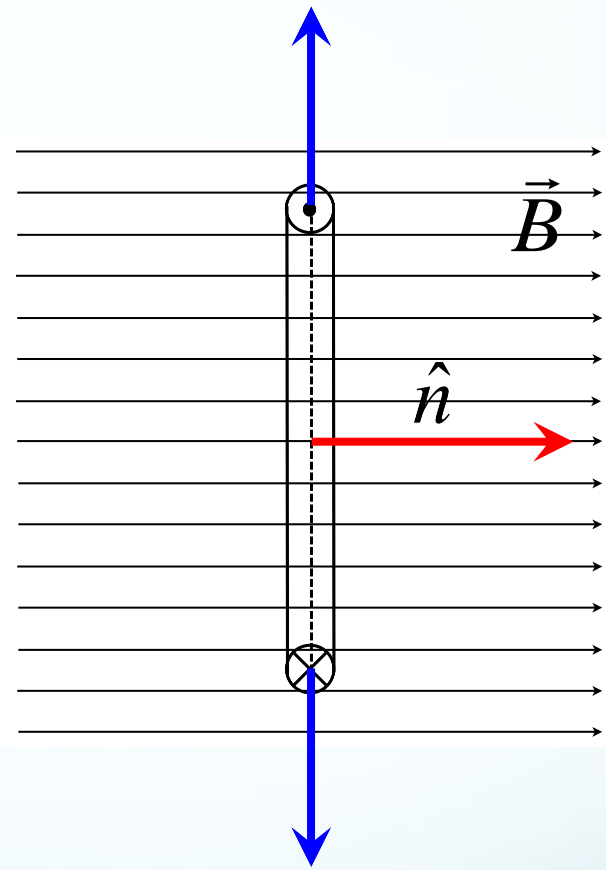




The normal vector is at right angles to the B-field: all magnetic force causes rotation of the loop



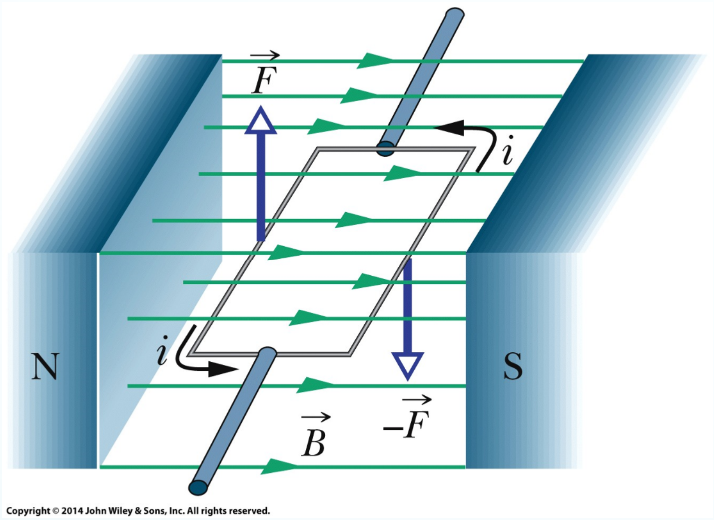
The normal vector is at some angle to the B-field: some of the magnetic force causes rotation of the loop



The normal vector is parallel to the B-field: none of the magnetic force causes rotation of the loop

Conclusion: components of magnetic force (anti)parallel to normal vector cause torque

Conclusion: Torque on a Current Loop



Net force on the loop is the vector sum of the forces acting on its four sides and comes out to be zero. The net torque acting on the coil has a magnitude given by

$$\tau = NiAB \sin\theta,$$

where *N* is the number of turns in the coil, *A* is the area of each turn, *i* is the current, *B* is the field magnitude, and θ is the angle between the magnetic field ***B*** and the normal vector to the coil ***n***.

Sample problem: 28.05

Suppose a cyclotron is operated at an oscillator frequency of 12 MHz and has a dee radius of R=53 cm.

(a) What is the magnitude of magnetic field needed for deuterons to be accelerated in the cyclotron? (mass of deuteron is m=3.34 e-27 kg)

(b) What is the resulting kinetic energy of the deuterons?

a) $f_{osc} = \frac{|q|B}{2\pi m} \rightarrow B = ?$ $\rightarrow B = \frac{2\pi m f}{|q|} =$

$q = 1.6 \times 10^{-19}$ C

$B = 1.57 \times 10^{-6}$ T

1.57 T

b) $K = \frac{1}{2}mv^2$ and $v = \frac{R|q|B}{m}$

$K =$

a) $f_{osc} = \frac{|q|B}{2\pi m} \rightarrow B = ?$

$q = 1.6 \times 10^{-19}\ C$

$\rightarrow B = 1.57\ T$

b) $K = \frac{1}{2} m v^2$ and $v = \frac{R|q|B}{}$

$\rightarrow K = 2.7 \times 10^{-12}\ J$

This section we talked about:

Chapter 28

See you on Friday