Electricity and Magnetism

- Physics 259 – L02
- Lecture 26

Chapter 24.4 and 24.5:

Potential due to an electric dipole

Potential due to a continuous charge distribution

Last time

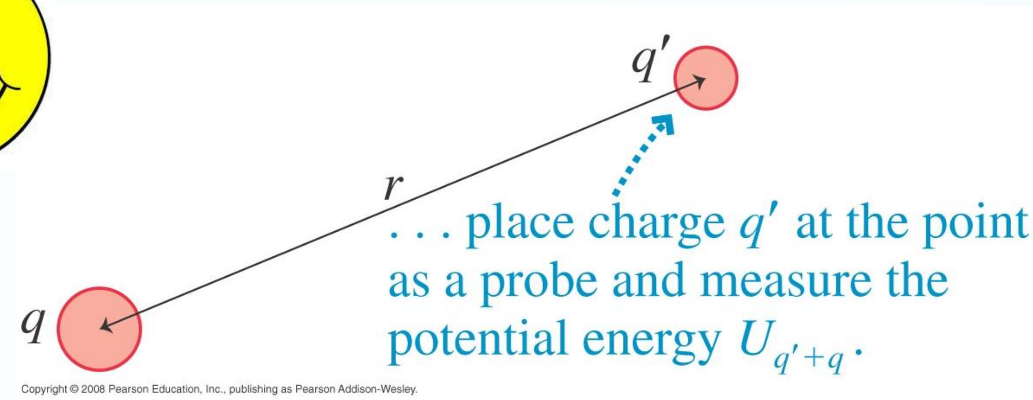
- Electric potential energy of a collection of charges
- Interpreting equipotential surfaces
- Equipotential surfaces: visualizing electric potential

This time

- Potential due to an electric dipole
- Potential due to a continuous charge distribution

Starting from the end

The whole story is:



Electric force on q' from q

$$\vec{F}_{qq'} = \frac{1}{4\pi\varepsilon_0}\frac{qq'}{r^2}\hat{r}$$

Then the electric field of q is

$$\vec{E} = \frac{\vec{F}_{qq'}}{q'} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$$

Potential energy of q and q'

$$U_{q'+q} = \frac{1}{4pe_0}\frac{qq'}{r}$$

Then the potential of q is

$$V = \frac{U_{q'+q}}{q'} = \frac{1}{4pe_0}\frac{q}{r}$$

Electric Potential

$$\boxed{V \equiv \frac{U_{q+sources}}{q}}$$

$$\boxed{U_{q+sources} = qV}$$

Potential Gradient -- E and V

Note: E is always $\perp$ equipotential lines

$$\vec{E} = -\vec{\nabla} V = -\frac{\partial V}{\partial x}\hat{i} - \frac{\partial V}{\partial y}\hat{j} - \frac{\partial V}{\partial z}\hat{k}$$

In 3 dimensions we must take 3 derivatives, then add them **VECTORIALLY**

Alternatively, the potential is found from the electric field integrated along any path connecting points A and B

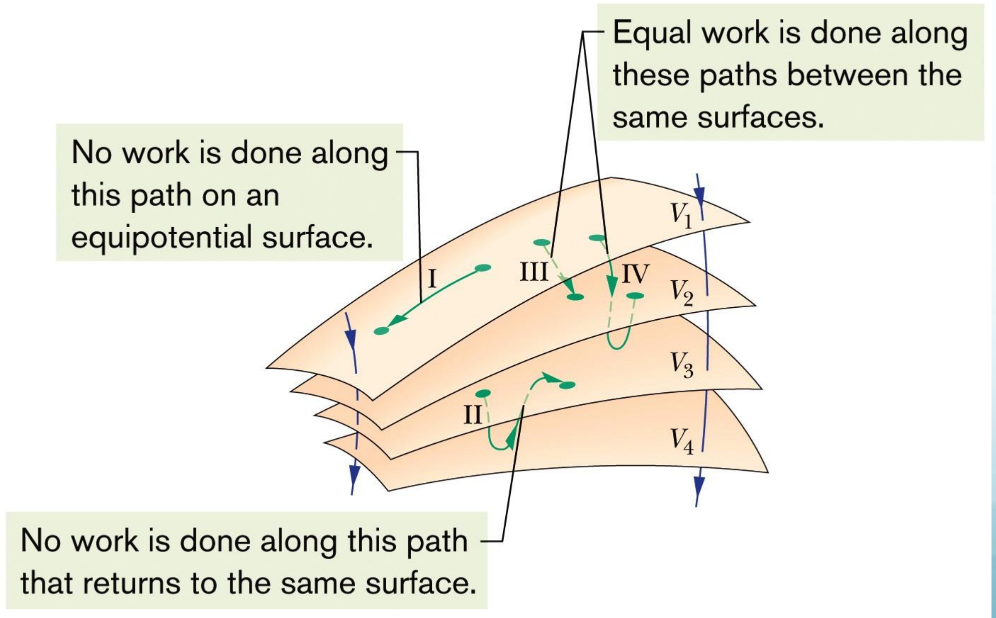
$$V_{AB} = \int_A^B \vec{E} \cdot ds$$

Equipotential Surfaces

- ✓ **Adjacent points with the same electric potential from an equipotential surface**

- ✓ No net work W is done on a charged particle by an electric field when particle moves on the same equipotential surface

$$W = -\Delta U = -q\,\Delta V = -q(V_f - V_i).$$



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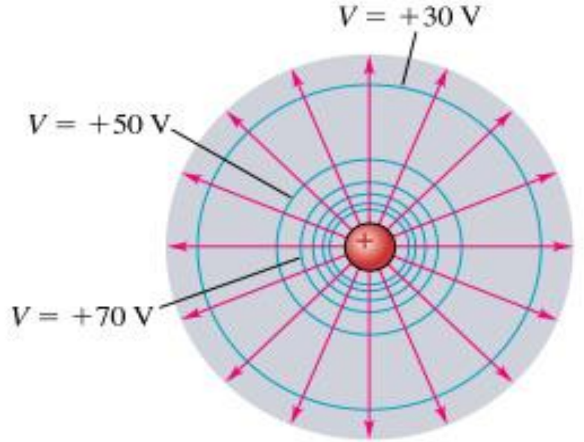
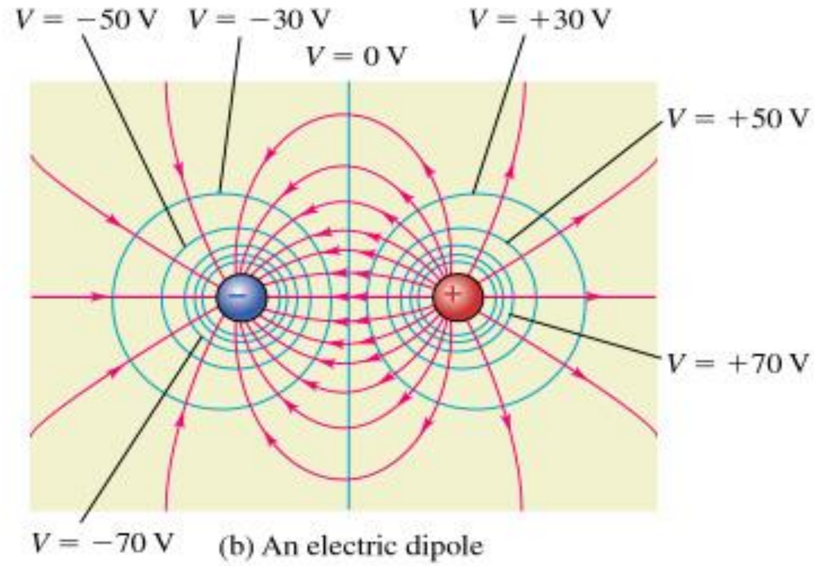
Equipotential Surfaces



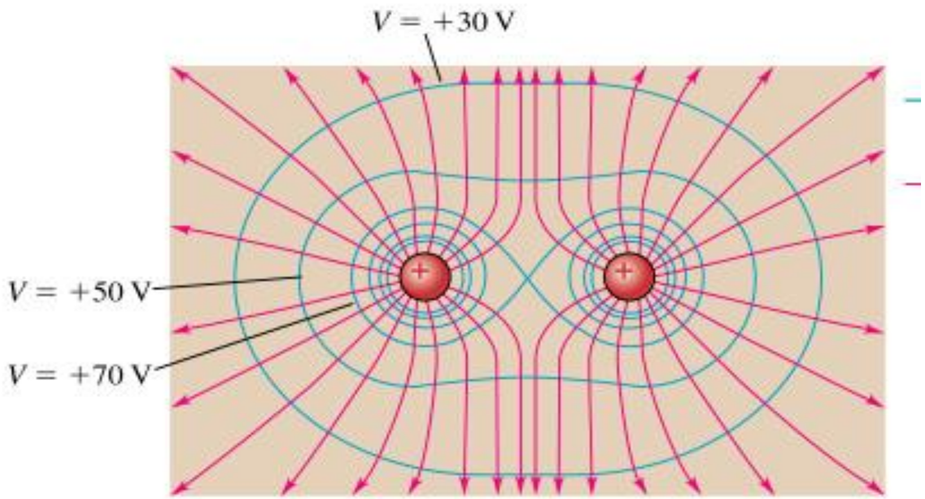
(a) A single positive charge



(b) An electric dipole

Note – E is always ⊥ V !!

Conducting sphere
+ sheet



(c) Two equal positive charges

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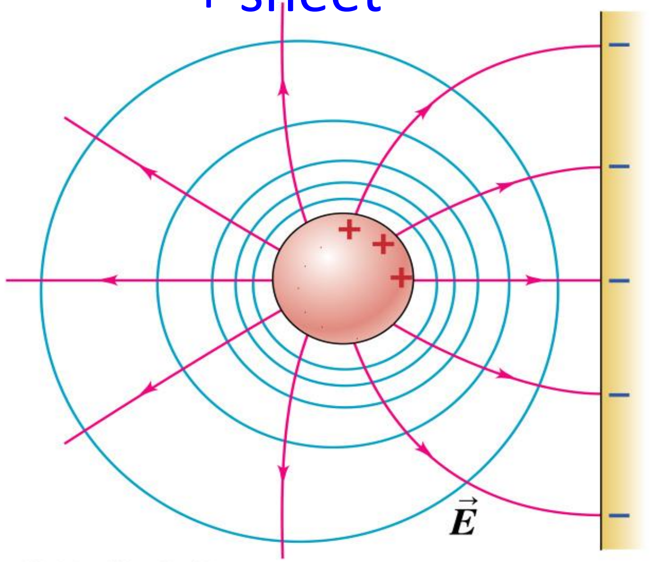

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Calculating Potential from the Field

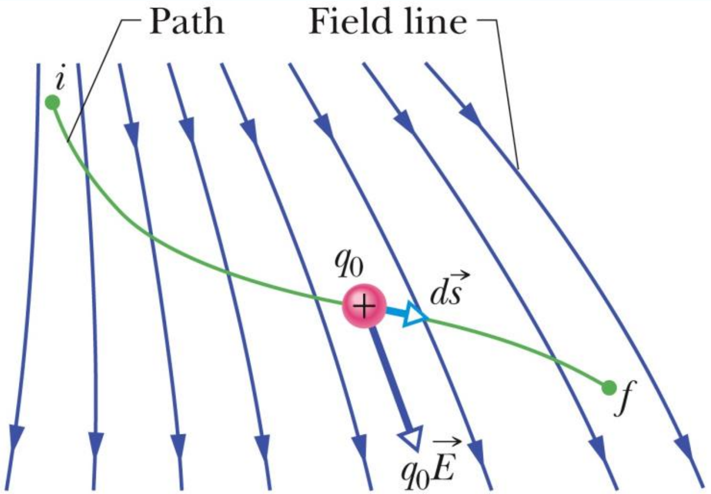
Differential work dW done on a particle by force F→

$$dW = \vec{F}.d\vec{s} = q\vec{E}.d\vec{s} \rightarrow W = q\int_i^f \vec{E}.d\vec{s}$$

& $$W = -\Delta U = -q\,\Delta V = -q(V_f - V_i).$$

→ $$\boxed{V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s},}$$

$$\boxed{V = -\int_i^f \vec{E} \cdot d\vec{s}.}$$



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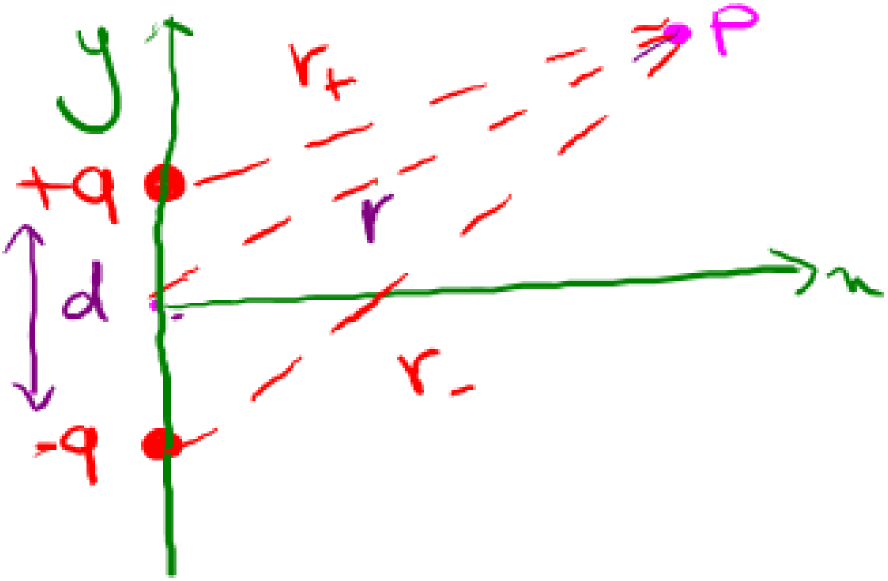
For uniform electric field→ $$\boxed{\Delta V = -E\,\Delta x.}$$

Problem 24.02 of textbook:
Finding potential change from the electric field

Find the potential difference $v_f - v_i$ by moving a positive test charge q from I to f along the path shown.

Electric potential of a dipole at arbitrary point p



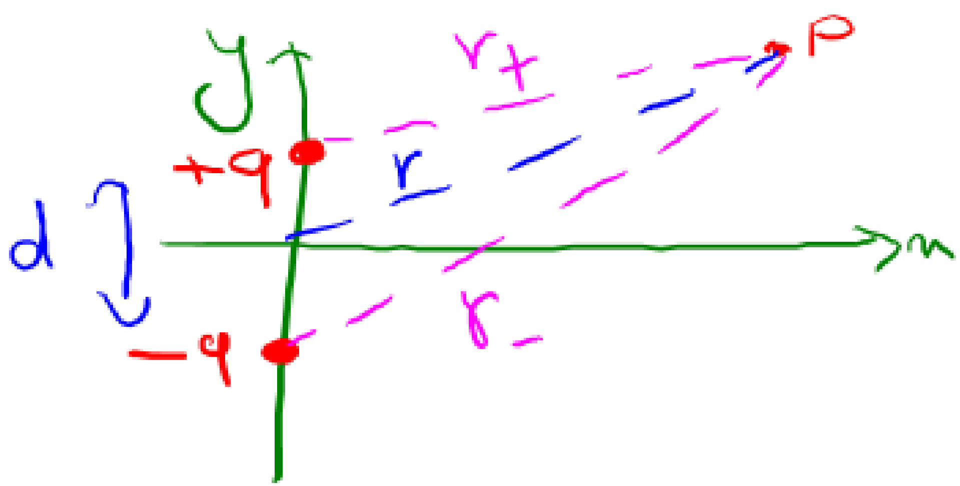
$$V = V_+ + V_-$$

$$\begin{cases} V_+ = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r_+} \\ V_- = \dfrac{-1}{4\pi\varepsilon_0}\dfrac{q}{r_-} \end{cases}$$

$$\rightarrow V = V_+ + V_- = \frac{1}{4\pi\varepsilon_0}\frac{q}{r_+} + \frac{-1}{4\pi\varepsilon_0}\frac{q}{r_-}$$

$$V =$$

$$\Rightarrow V = \frac{q}{4\pi\varepsilon_0}\frac{r_- - r_+}{r_- r_+}$$



natural dipoles $\rightarrow$

$r \gg d \Rightarrow r_- \simeq r_+ \simeq r$

$\Rightarrow r_- r_+ \simeq r^2$ & $r_- - r_+ \rightarrow$

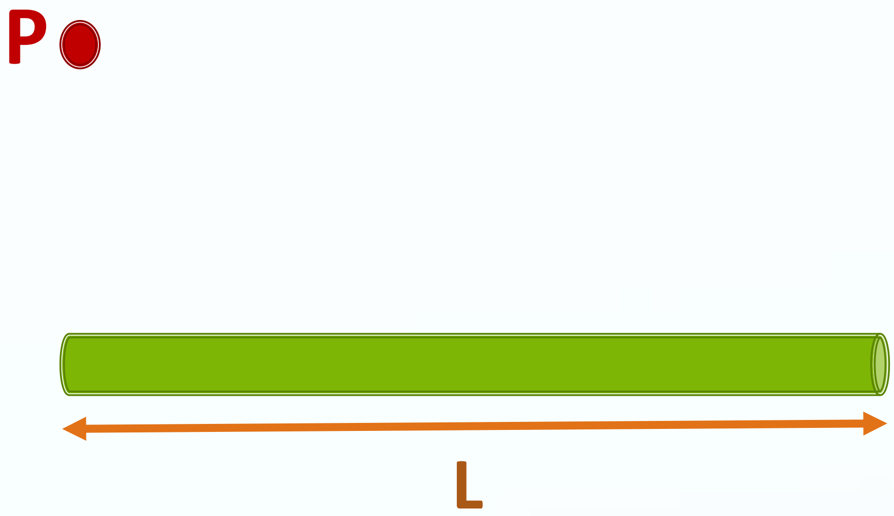
$\rightarrow r_- - r_+ = d\cos\theta$

$\Rightarrow V =$

$$\rightarrow V = \frac{q}{4\pi\varepsilon_0}\frac{d\cos\theta}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}$$

- **Go through “Appendix 1-chapter 24” in D2L (different approach)**

Electric potential of a line of charge at point p



Thin nonconducting rod of length L with uniform positive charge with charge density λ.

Find electric potential V due to the rod at p, a perpendicular distance d from the left end of the rod.


P
L

This section we talked about:

Chapter 24.4 and 24.5

See you on Wednesday