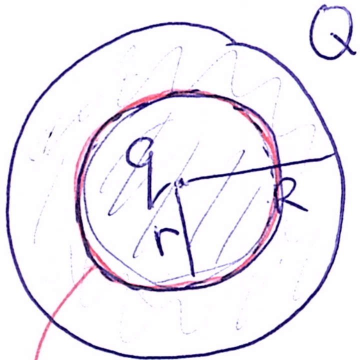
Charged ball (treating it as solid ball + shell)

uniform charge density $\rho = \dfrac{Q}{V}$ (total charge) / (total volume)



Gaussian surface

$$V = \frac{4}{3}\pi R^3$$

result from last time using Gauss' Law directly

$$E = \frac{Q r}{4\pi\epsilon_0 R^3} = \boxed{\frac{\rho r}{3\epsilon_0}}$$

Outside a solid ball: $E_B = \boxed{\dfrac{1}{4\pi\epsilon_0}\dfrac{q}{r^2}}$ ← q of solid ball of radius r.

inside a spherical shell: $E_S = 0$

input value of q

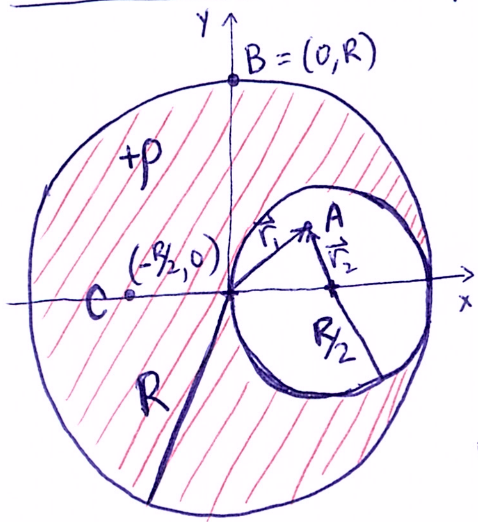
volume of solid ball of radius r

$$q = V_r \rho = \frac{4}{3}\pi r^3 \rho$$

Electric field $E = \boxed{\dfrac{1}{\cancel{4\pi}\epsilon_0}\dfrac{\cancel{4\pi} r^{\cancel{3}}\rho}{3\,\cancel{r^2}}} = \boxed{\dfrac{\rho r}{3\epsilon_0}}$

$E_B + E_S$ (superposition)

Gauss' Law + superposition: offset spherical hole



Calculate E-field at points A, B, C.

Use superposition of large ball with charge density $+\rho$ and small ball with charge density $-\rho$

At point A: field inside large ball at radius $r_1 = |\vec{r}_1|$

$$\vec{E}_+ = \frac{+\rho \vec{r}_1}{3\epsilon_0}$$

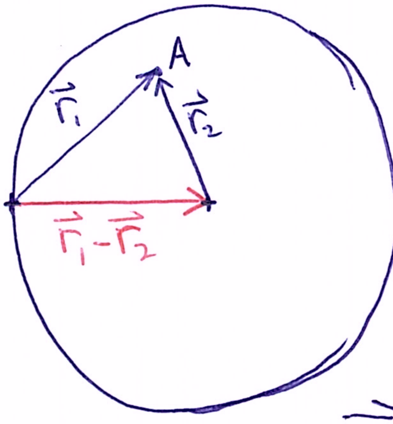
need to take the direction explicitly into account.

field inside small ball at radius $r_2 = |\vec{r}_2|$

$$\vec{E}_- = \frac{-\rho \vec{r}_2}{3\epsilon_0}$$

Net field at point A: $\vec{E}_A = \vec{E}_+ + \vec{E}_-$

$$\vec{E}_A = \frac{\rho}{3\epsilon_0}(\vec{r}_1 - \vec{r}_2)$$



wherever we put point A inside the hole, we get $\vec{r}_1 - \vec{r}_2$, which is a constant vector: $\vec{r}_1 - \vec{r}_2 = \frac{R}{2}\hat{\imath}$

$\Rightarrow$ uniform E-field $\boxed{\vec{E}_A = \frac{\rho R}{6\epsilon_0}\hat{\imath}}$

At point B: field outside large ball at radius R

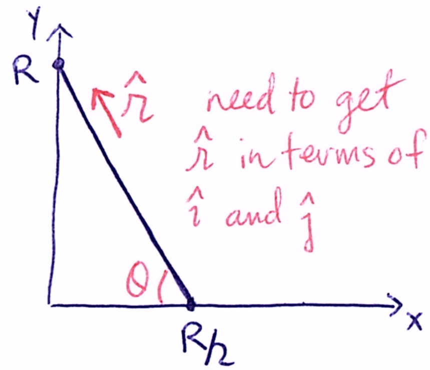
$$\vec{E}_+ = \frac{1}{3\cdot\frac{4\pi\epsilon_0}{3}}\frac{Q_+}{R^2}\frac{R}{R}\hat{j} = \frac{\rho R}{3\epsilon_0}\hat{j}$$

field outside small ball at radius $\sqrt{R^2+(R/2)^2} = r$

unit vector pointing away from small ball.

$$\vec{E}_- = \frac{1}{4\pi\epsilon_0}\frac{Q_-}{r^2}\hat{r} = \frac{1}{4\pi\epsilon_0}\frac{Q_-\hat{r}}{\underbrace{R^2(5/4)}_{R^2+(R/2)^2}} = \frac{1}{3\cdot\frac{4\pi\epsilon_0}{3}}\frac{Q_-}{(R/2)^2 5}\frac{R/2}{R/2}\hat{r}$$

$$\vec{E}_- = \frac{-\rho(R/2)}{5\cdot 3\cdot\epsilon_0}\hat{r} = -\frac{\rho R}{30\epsilon_0}\hat{r}$$



need to get $\hat{r}$ in terms of $\hat{i}$ and $\hat{j}$

$$\hat{r} = \underbrace{\frac{-R/2}{\sqrt{(R/2)^2+R^2}}}_{-\cos\theta}\hat{i} + \underbrace{\frac{R}{\sqrt{(R/2)^2+R^2}}}_{\sin\theta}\hat{j}$$

$$\hat{r} = \frac{-\cancel{R}/\cancel{2}}{\cancel{R}\sqrt{5}/\cancel{2}}\hat{i} + \frac{R}{R\sqrt{5}/2}\hat{j} = -\frac{1}{\sqrt{5}}\hat{i} + \frac{2}{\sqrt{5}}\hat{j}$$

$$\vec{E}_- = \frac{-\rho R}{30\epsilon_0}\left[-\frac{1}{\sqrt{5}}\hat{i} + \frac{2}{\sqrt{5}}\hat{j}\right]$$

Net field at point B: $\vec{E}_B = \vec{E}_+ + \vec{E}_-$

$$\vec{E}_B = \frac{\rho R}{3\epsilon_0}\left[\frac{10\sqrt{5}}{10\sqrt{5}}\hat{j} - \frac{1}{10}\left(-\frac{1}{\sqrt{5}}\hat{i} + \frac{2}{\sqrt{5}}\hat{j}\right)\right]$$

$$\boxed{\vec{E}_B = \frac{\rho R}{30\sqrt{5}\epsilon_0}\left[\hat{i} + (10\sqrt{5}-2)\hat{j}\right]}$$

At point C: field inside large ball at radius $R/2$

$$\vec{E}_+ = \frac{\rho(R/2)}{3\epsilon_0}(-\hat{\imath}) = -\frac{\rho R}{6\epsilon_0}\hat{\imath}$$

field due to $+\rho$ is pointing in $-x$ direction

field outside small ball at radius R

$$\vec{E}_- = \frac{1}{3\cdot\frac{4}{3}\pi\epsilon_0}\frac{Q(R/2)}{R^2\left(\frac{4(R/2)}{4}\right)}\hat{\imath} = \frac{Q}{\frac{4}{3}\pi(R/2)^3}\frac{(R/2)}{3\epsilon_0 4}\hat{\imath} = \frac{\rho R}{24\epsilon_0}\hat{\imath}$$

Net field at point C: $\vec{E}_c = \vec{E}_+ + \vec{E}_-$

$$\vec{E}_c = -\frac{\rho R}{6\epsilon_0}\hat{\imath} + \frac{\rho R}{24\epsilon_0}\hat{\imath} = \frac{\rho R}{24\epsilon_0}(-4+1)\hat{\imath}$$

$$\boxed{\vec{E}_c = -\frac{\rho R}{8\epsilon_0}\hat{\imath}}$$

Is there a position where $\vec{E} = \vec{0}$? (Extra Question)

Due to symmetry, it would lie along the x-axis: it would have to be somewhere in the range $-R < x < 0$. Let $x = -x_0$

field inside large ball at radius x_0

$$\vec{E}_+ = \frac{\rho x_0}{3\epsilon_0}(-\hat{\imath}) = -\frac{\rho x_0}{3\epsilon_0}\hat{\imath}$$

field outside small ball at radius $R/2 + x_0$

$$\vec{E}_- = \frac{\left(\rho\frac{4}{3}\pi\left(\frac{R}{2}\right)^3\right)}{4\pi\epsilon_0\left(\frac{R}{2}+\frac{2x_0}{2}\right)^2}\hat{\imath} = \frac{\rho R^3}{6\epsilon_0(R+2x_0)^2}\hat{\imath}$$

For $\vec{E}=\vec{0}$ we need $|\vec{E}_+|=|\vec{E}_-|$

$$\frac{\cancel{\rho} x_0}{\cancel{3\epsilon_0}} = \frac{\cancel{\rho} R^3}{2\cancel{3\epsilon_0}(R+2x_0)^2}$$

$$2x_0(R+2x_0)^2 = R^3$$

$$2x_0(R^2+4Rx_0+4x_0^2) = R^3$$

$$8x_0^3 + 8Rx_0^2 + 2R^2x_0 - R^3 = 0$$ need to solve a cubic

Solving with Mathematica yields:

$$x_0 = \left[-\frac{1}{3} + \frac{(928-96\sqrt{93})^{1/3}}{24} + \frac{2^{2/3}(29+3\sqrt{93})^{1/3}}{12}\right]R$$

or $\boxed{x_0 = 0.232786\,R}$

The electric field is zero at the position $(-0.232786\mathbf{R}, 0)$