Monday April 3, 2017

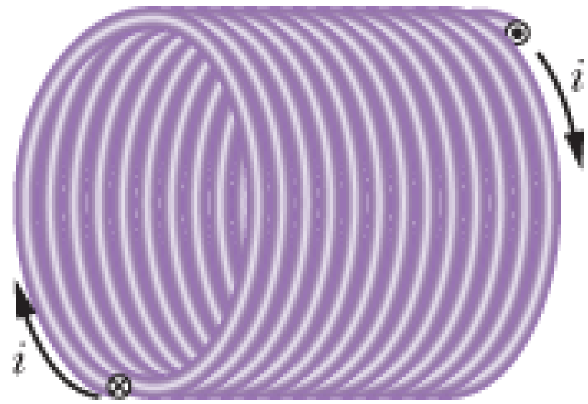
Last time:

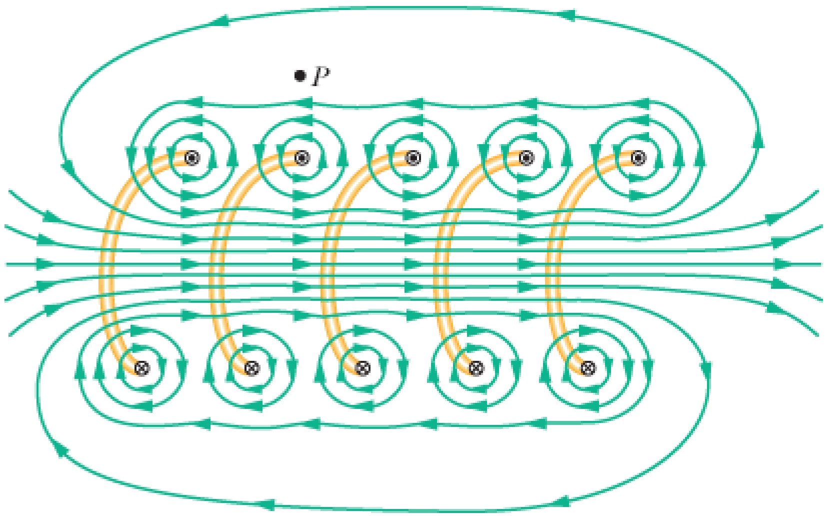
- Applying Ampère’s Law:
  - ➢ Magnetic field of a long wire (inside and outside)
  - ➢ Magnetic field of solenoid
- Applying the Biot-Savart Law: Circular arc of current (take-home example)

Today:

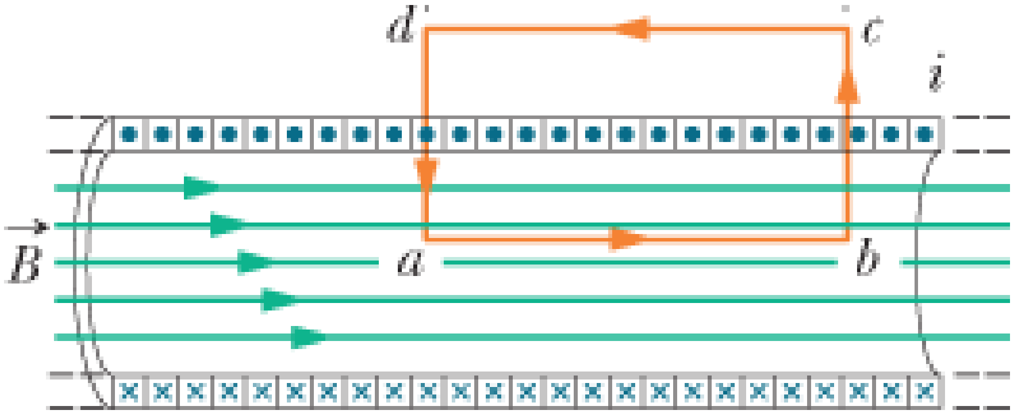
- Applying Ampère’s Law:
  - ➢ Magnetic field of solenoid and toroid
- Faraday’s Law of Induction
- Non-conservative electric fields
- Motional emf

Magnetic field of solenoid

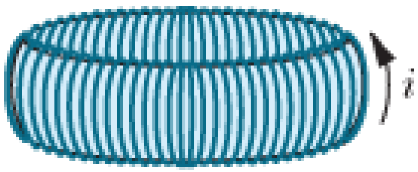




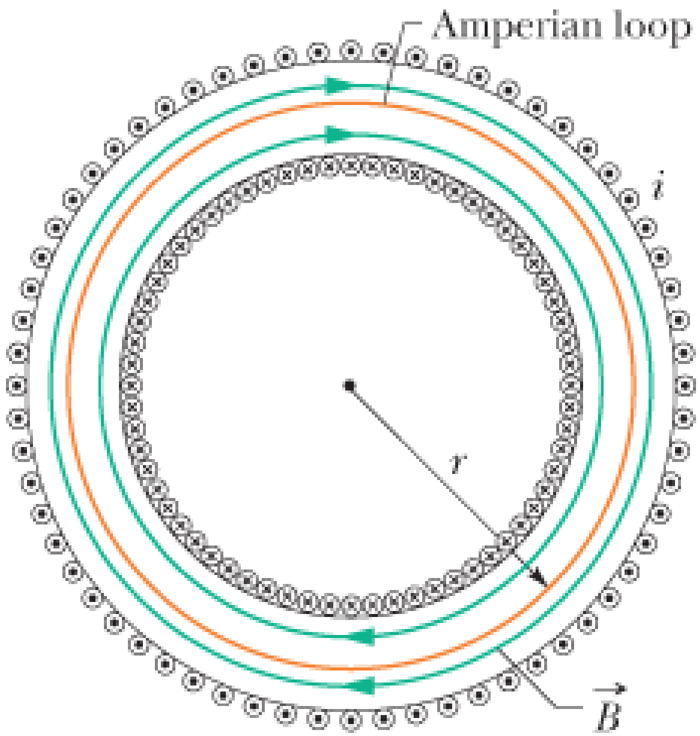
$$B = \mu_0 i n \quad \text{(ideal solenoid)}$$



Magnetic field of toroid



(a)



(b)

$$B = \frac{\mu_0 i N}{2\pi}\frac{1}{r} \quad \text{(toroid)}$$

Faraday's Law of Induction

Electrostatics: E-field from motionless charges
Magnetostatics: B-field from charges in motion

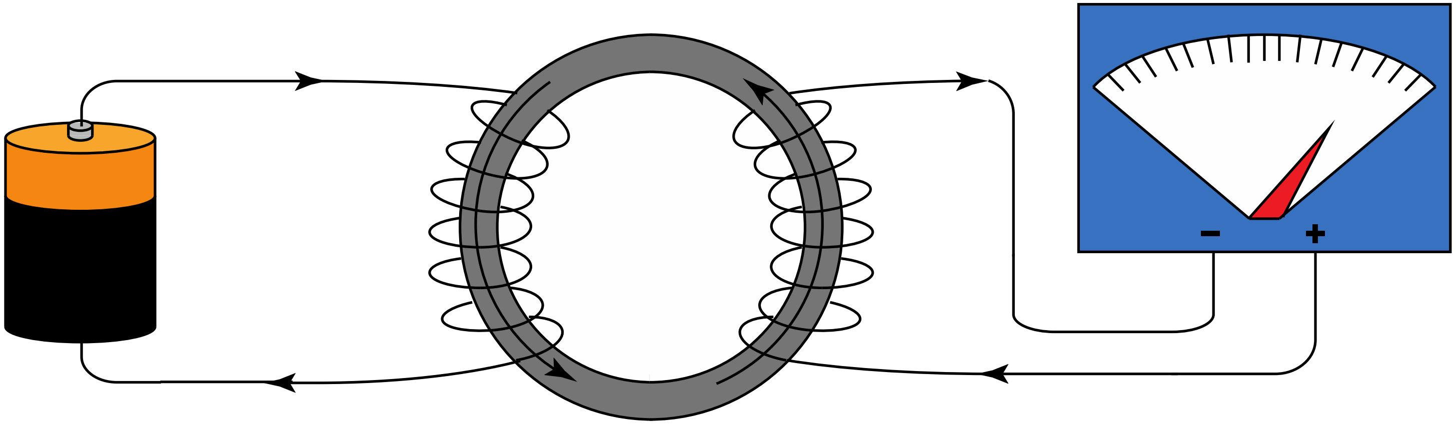
Changing electric fields (moving charges) create magnetic fields. Is the opposite true?

YES!

$$\varepsilon = -\frac{d\Phi_B}{dt}$$

i.e., A changing magnetic flux creates an induced EMF.

Faraday's Initial Experiment



Faraday discovered that there is an **induced EMF** in the secondary circuit given by

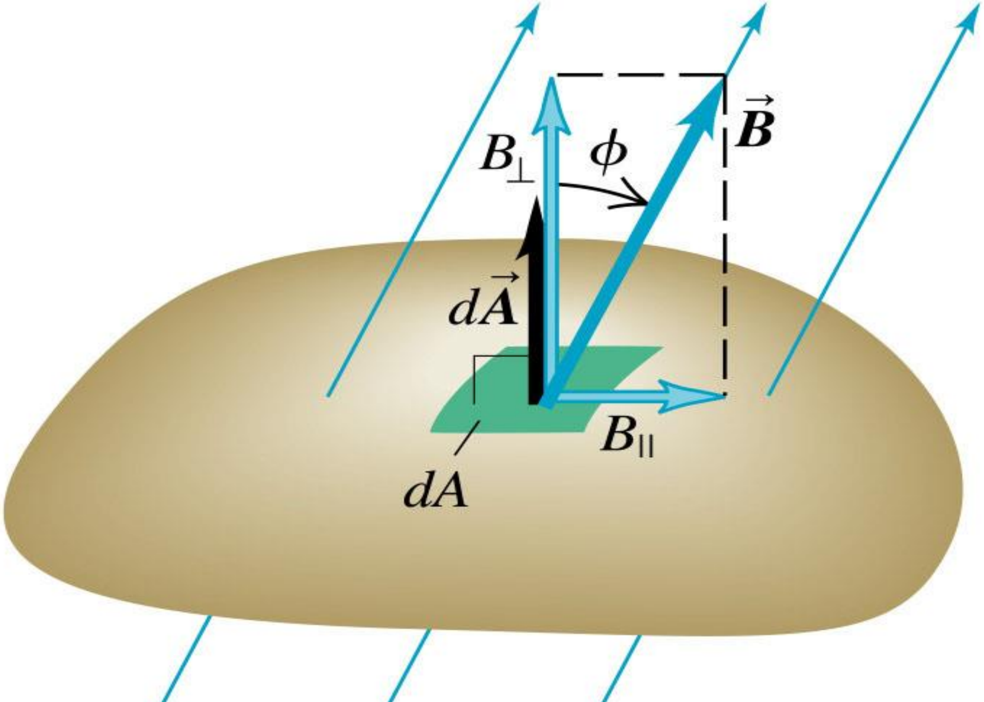
$$\varepsilon = -\frac{d\Phi_B}{dt}$$

This is a new generalized law called **Faraday's Law**.

Recall the definition of magnetic flux:

$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$

Not a closed surface!



This is valid even if Φ_B changes because of a time dependent A or angle ϕ (without changing the magnetic field)!

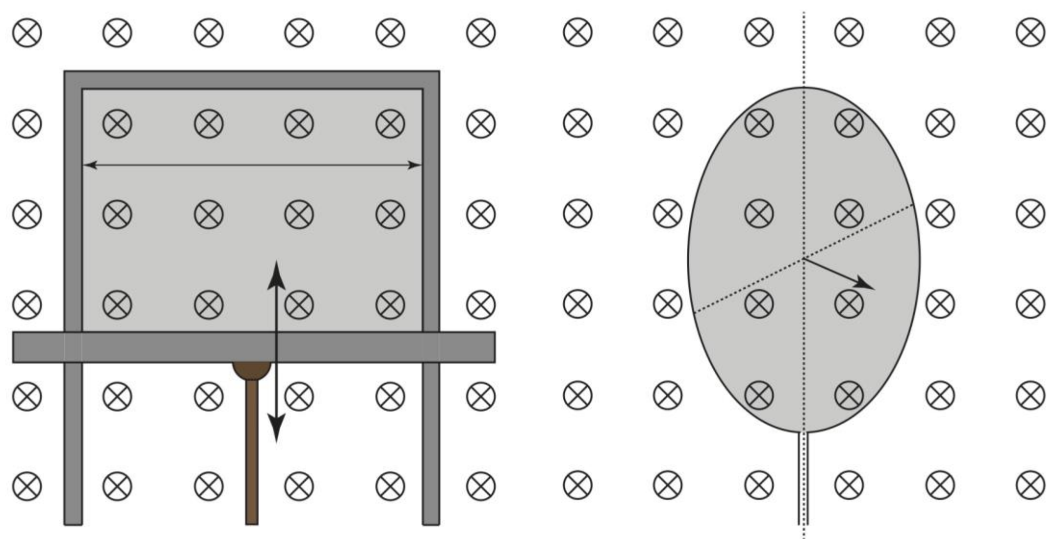
$$\boxed{e = -\frac{d}{dt}\left(BA\cos f\right)}$$

→ 3 possible terms

$$e = -\frac{dB}{dt}A\cos f - \frac{dA}{dt}B\cos f + \frac{df}{dt}BA\sin f$$

From Maxwell Eq.

$$-\frac{d\vec{B}}{dt} = \nabla \times \vec{E}$$

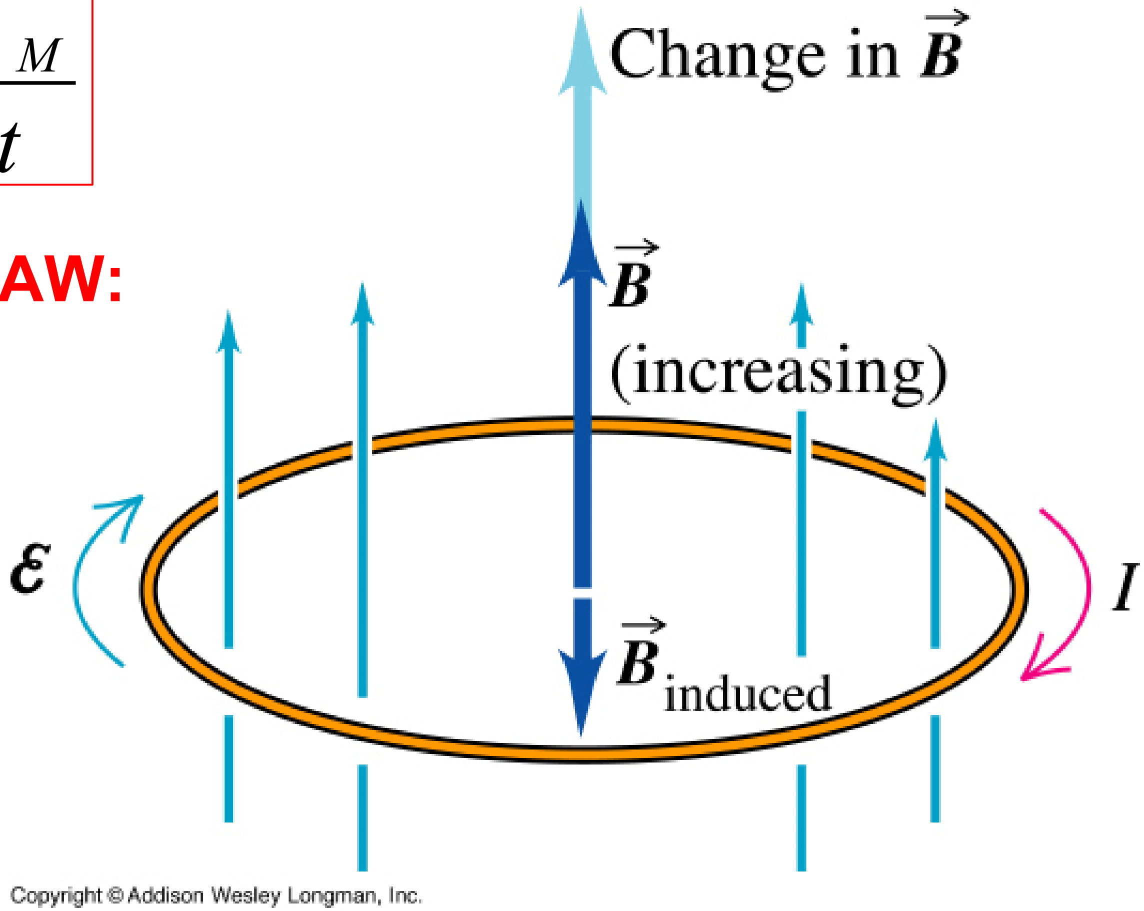


Top Hat Question

What about the minus sign in Faraday's law?

$$\boxed{e = -\frac{dF_M}{dt}}$$

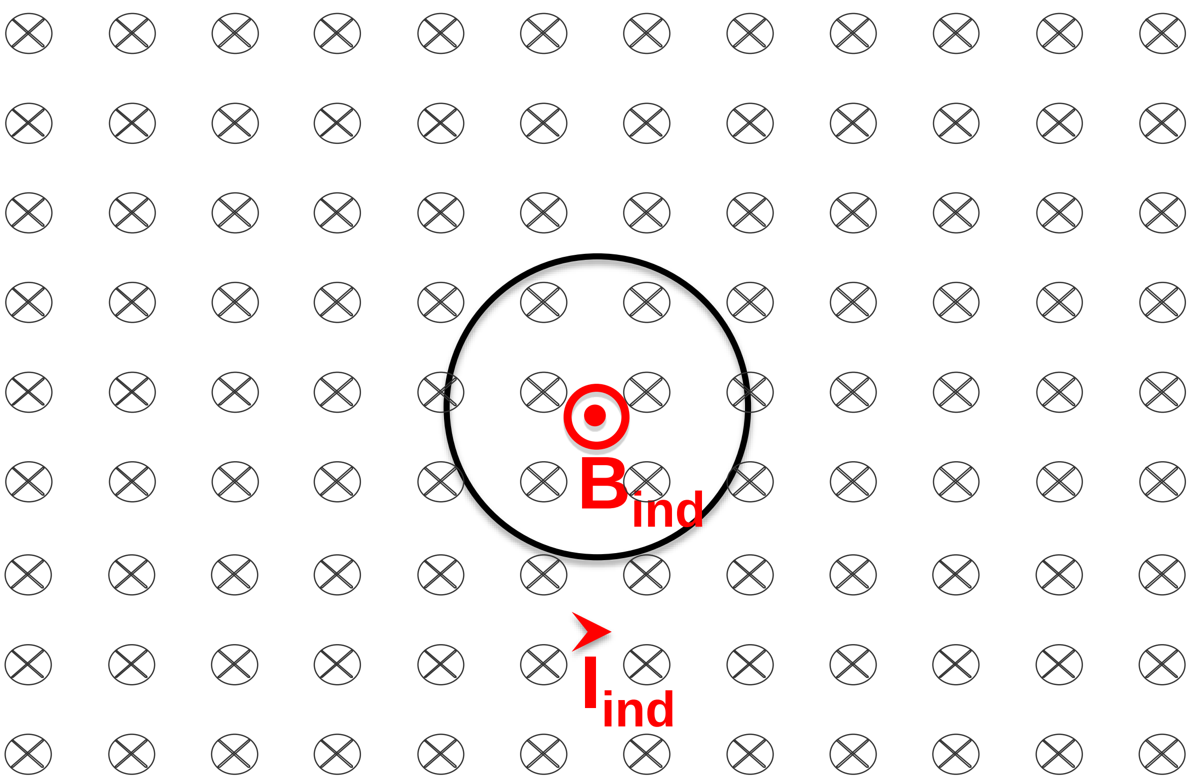
LENZ'S LAW:



The changing magnetic flux generates an induced current which creates an induced magnetic field which, in turn, resists the change in magnetic flux.

Lenz's Law

The induced current from Faraday's Law is always in a direction such that the induced magnetic field from the induced current opposes the change in the magnetic flux through the loop.



More B-field lines inside the loop: induced B-field from induced current must be out of the page to compensate.
Induced current is CCW

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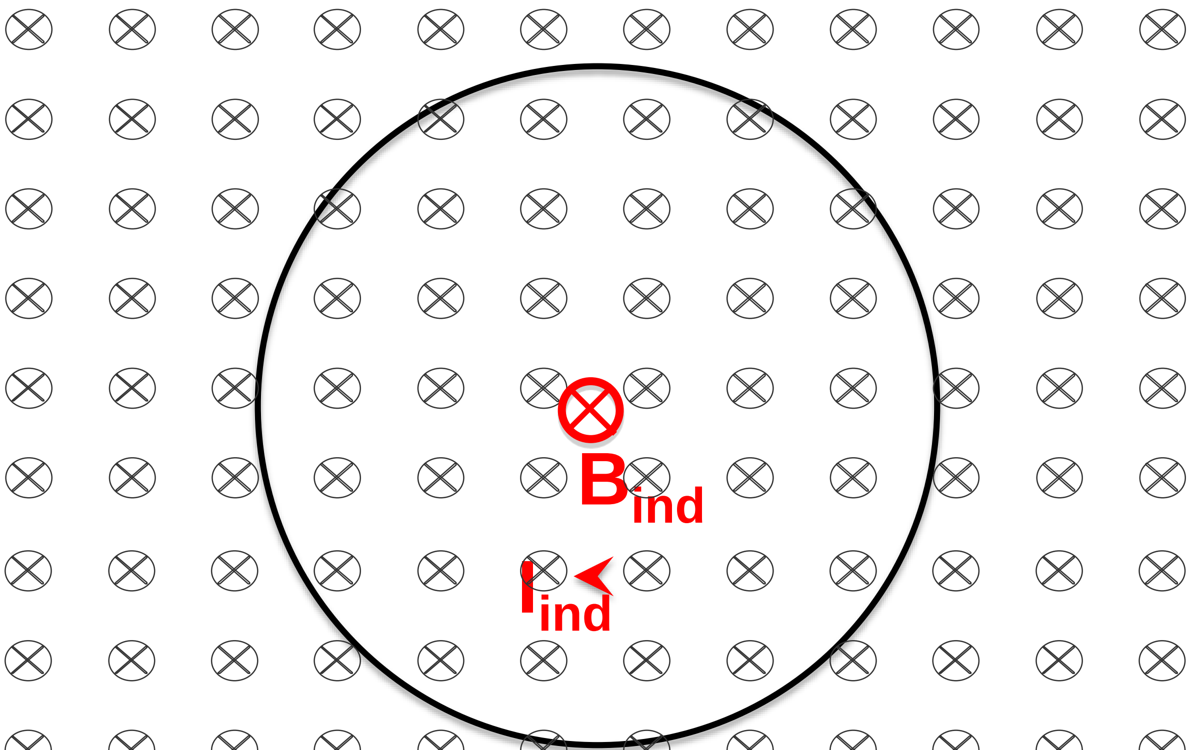

Fewer B-field lines inside the loop: induced B-field from induced current must be into the page to compensate. Induced current is CW