Monday March 20, 2017

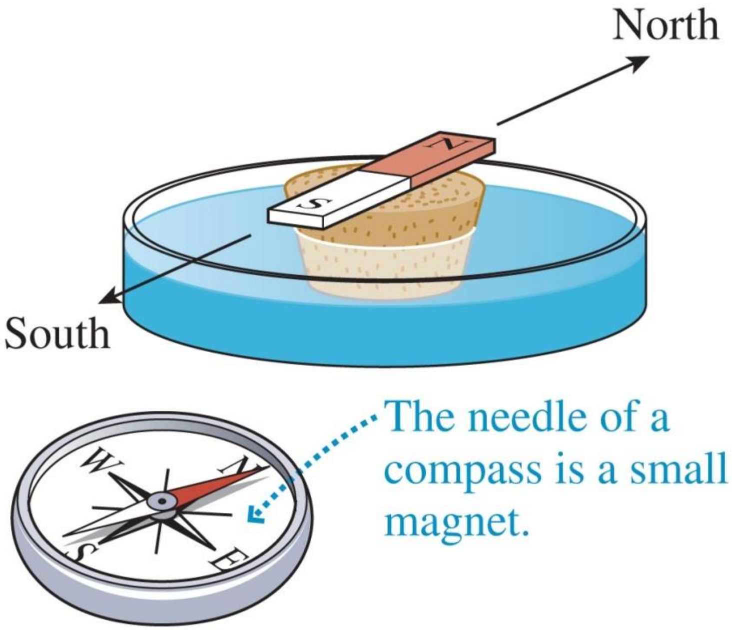
Last time:

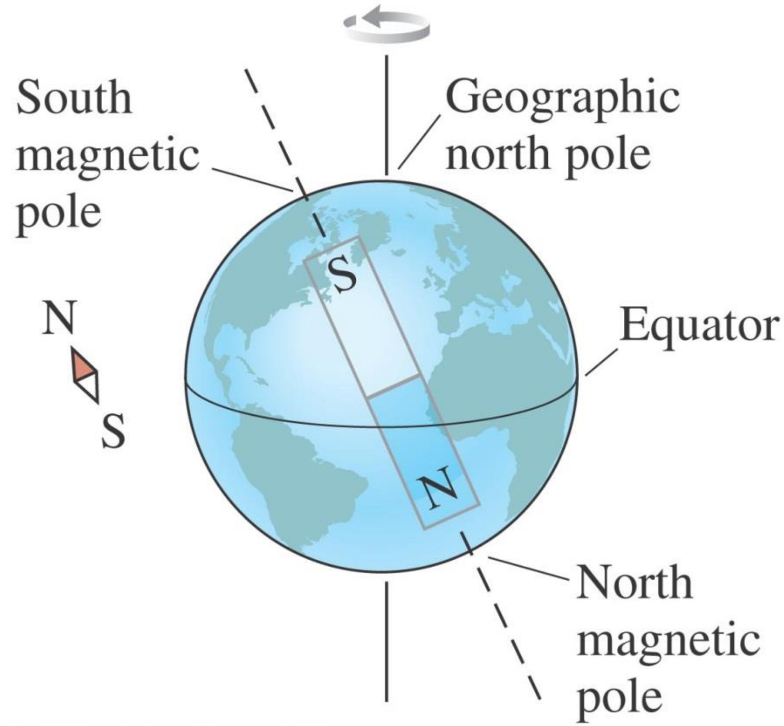
- RC time constant and its meaning
- Charging/discharging capacitors calculation

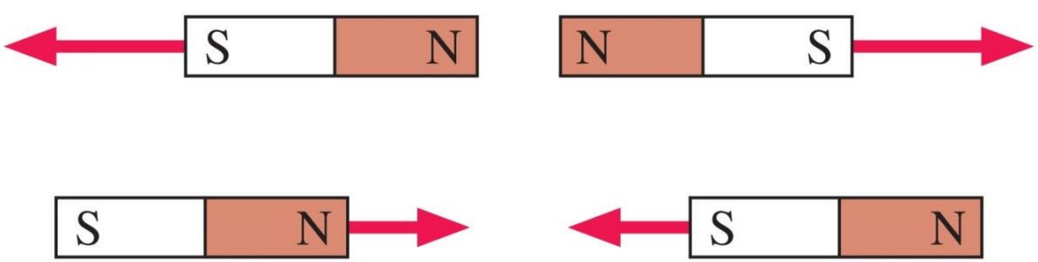
Today:

- Introduction to magnetism
- Electric force vs magnetic force on charges
- Vector cross product
- Consequences of magnetic force

Magnetism



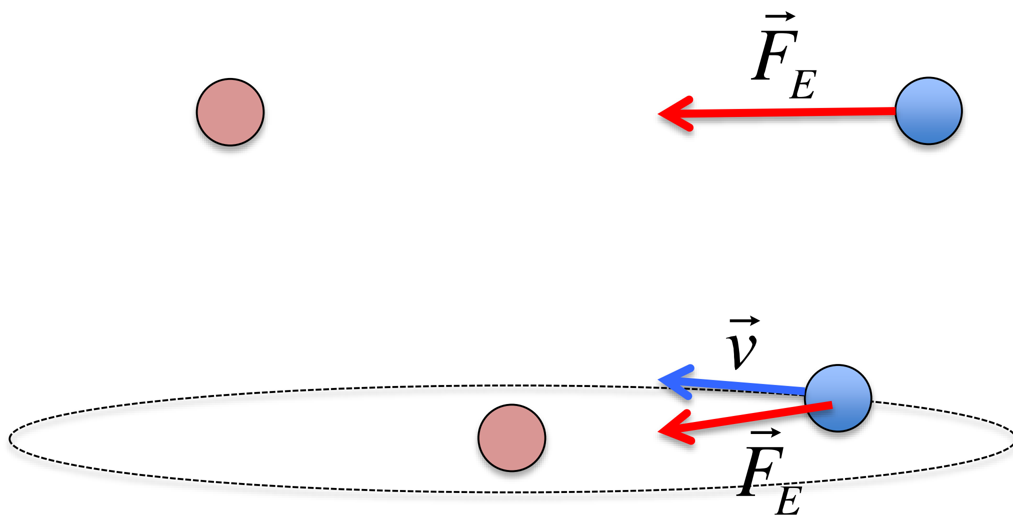




Like poles repel.

Unlike poles attract.

Electric Force on Charges



Electric force acts on a charge regardless of its motion.

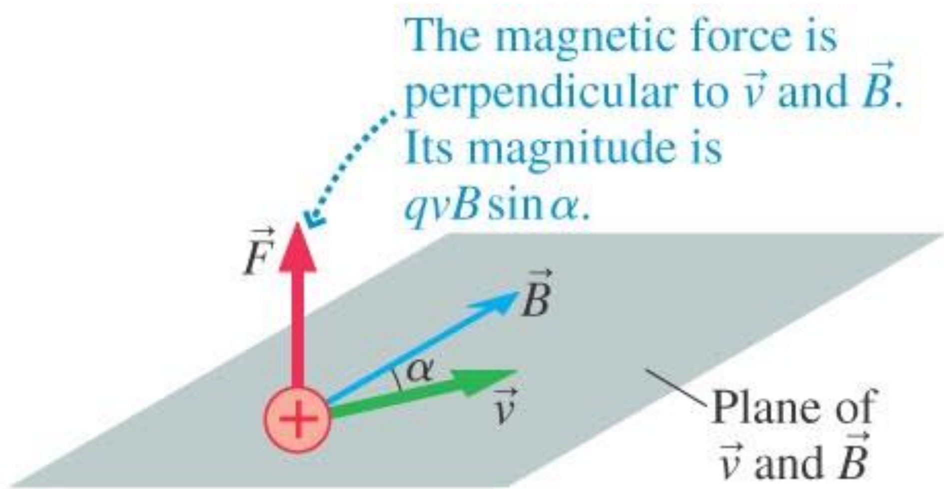
$$\vec{F}_E = q\vec{E}$$

Magnitude: $F_E = qE$

Direction: direction of $\vec{E}$

Magnetic Force on Charges

Magnetic force acts only on a moving charge.

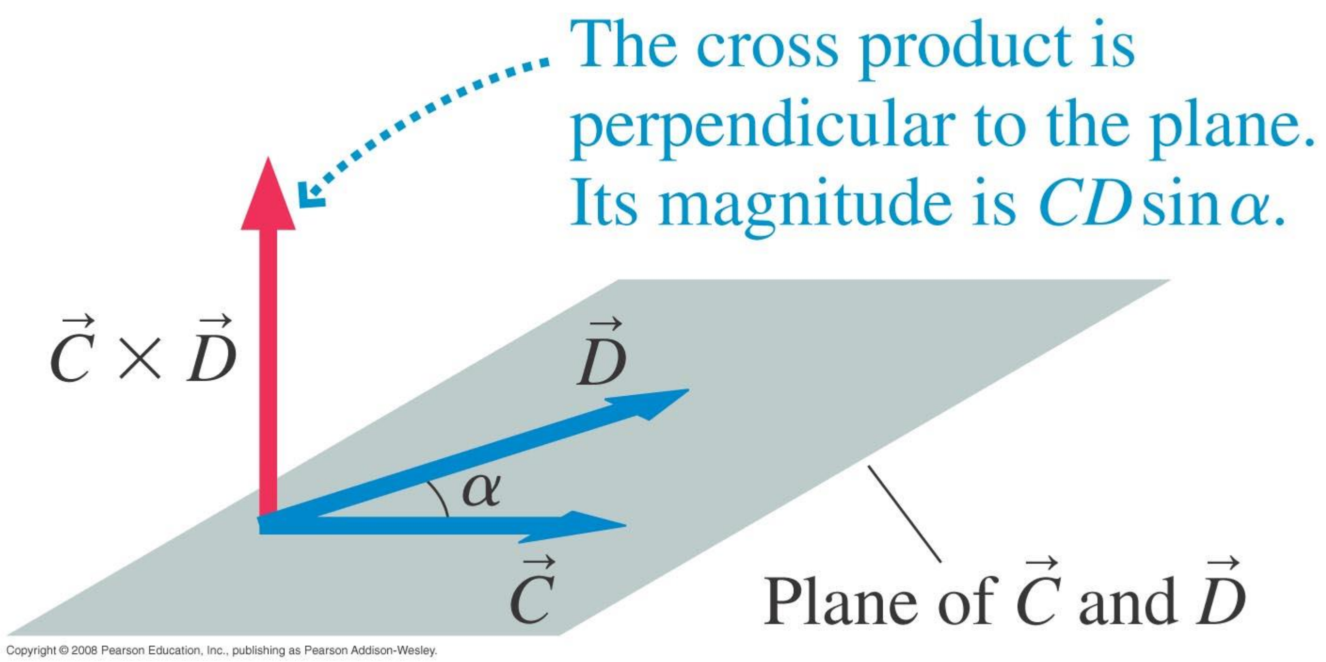


$$\vec{F}_B = q\vec{v}\times\vec{B}$$

Magnitude: $F_B = qvB\sin\alpha$

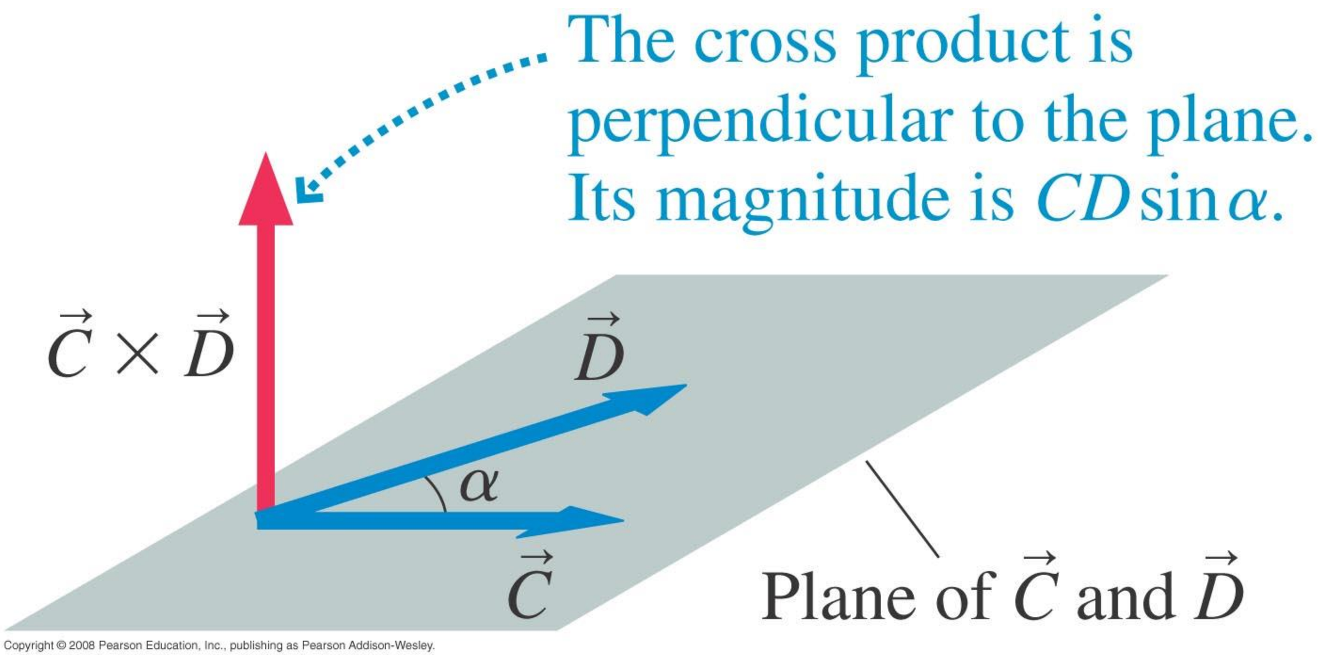
Direction: RH rule

The Vector Cross Product



Point the fingers of your right hand along the first vector in the cross product (vector C), then curl them so they point along the second vector (vector D). Your thumb gives the direction of the cross product.

The Vector Cross Product



So $\vec{C} \times \vec{D}$ points up and $\vec{D} \times \vec{C}$ points down.

$$\left|\vec{C} \times \vec{D}\right| = \left|\vec{C}\right|\left|\vec{D}\right|\sin\alpha$$

Cross product vs regular product

Regular/dot product	**Cross product**
Distributive	Distributive
$\vec{B}\cdot\left(\vec{C}+\vec{D}\right)=\vec{B}\cdot\vec{C}+\vec{B}\cdot\vec{D}$	$\vec{B}\times\left(\vec{C}+\vec{D}\right)=\vec{B}\times\vec{C}+\vec{B}\times\vec{D}$
Commutative	Anticommutative
$CD=DC$ $\vec{C}\cdot\vec{D}=\vec{D}\cdot\vec{C}$	$\vec{C}\times\vec{D}=-\vec{D}\times\vec{C}$
Associative	Non-Associative
$B\left(CD\right)=\left(BC\right)D$	$\vec{B}\times\left(\vec{C}\times\vec{D}\right)\neq\left(\vec{B}\times\vec{C}\right)\times\vec{D}$

Unit vector notation

The cross product becomes easy to deal with when using unit vector notation

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

$$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$$

$$\hat{i} \times \hat{j} = \hat{k}$$

$$\hat{j} \times \hat{k} = \hat{i}$$

$$\hat{k} \times \hat{i} = \hat{j}$$

Now let’s see what the cross product between A and B is:

$$\vec{C} = \vec{A} \times \vec{B}$$

$$\vec{C} = \left(A_x\hat{i} + A_y\hat{j} + A_z\hat{k}\right) \times \left(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}\right)$$

$$\vec{C} = \left(A_yB_z - A_zB_y\right)\hat{i} + \left(A_zB_x - A_xB_z\right)\hat{j} + \left(A_xB_y - A_yB_x\right)\hat{k}$$

Another way to think about it

Start with the two vectors in component form

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k} \qquad\qquad \vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$$

The cross product is given by the determinant of the following matrix:

$$\vec{C} = \vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$\vec{C} = \left(A_yB_z - A_zB_y\right)\hat{i} + \left(A_zB_x - A_xB_z\right)\hat{j} + \left(A_xB_y - A_yB_x\right)\hat{k}$$

Parallel and Perpendicular vectors

For parallel vectors

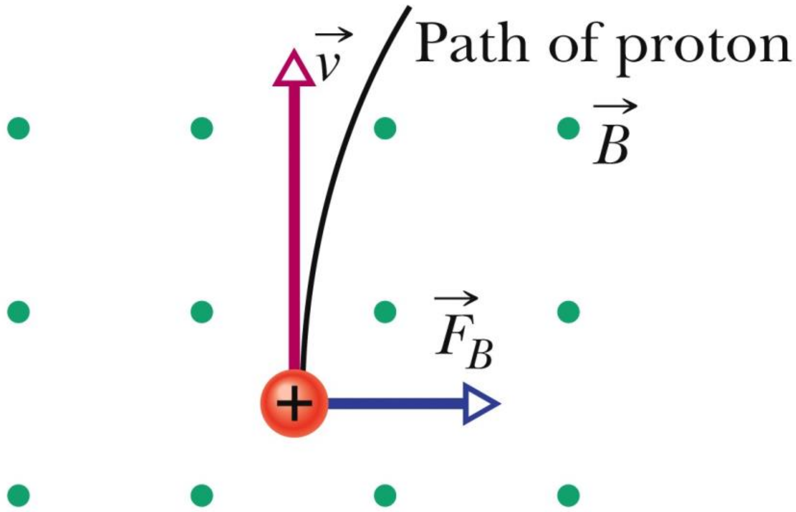
$$\vec{A} = A\hat{i} \qquad \vec{B} = B\hat{i} \qquad \vec{C} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A & 0 & 0 \\ B & 0 & 0 \end{vmatrix} = \vec{0}$$

For perpendicular vectors

$$\vec{A} = A\hat{i} \qquad \vec{B} = B\hat{j} \qquad \vec{C} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A & 0 & 0 \\ 0 & B & 0 \end{vmatrix} = AB\hat{k}$$

TopHat questions

Motion of charges in B-field



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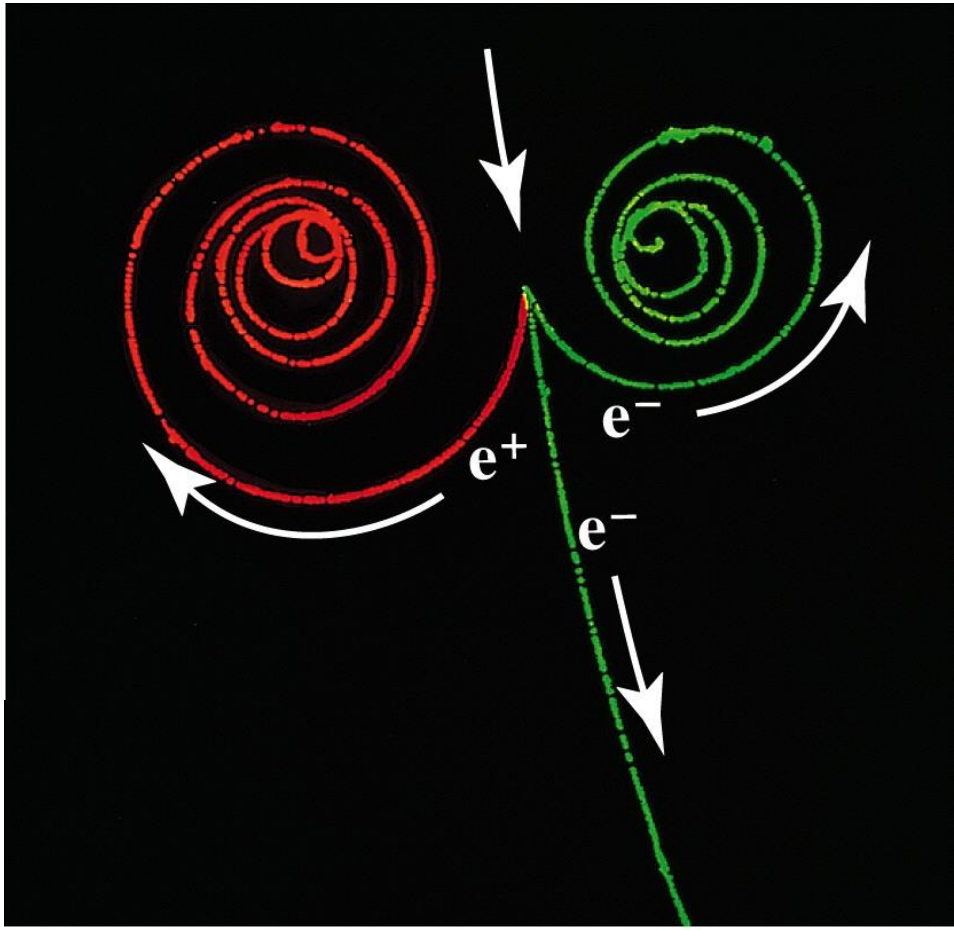

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Cyclotron Motion

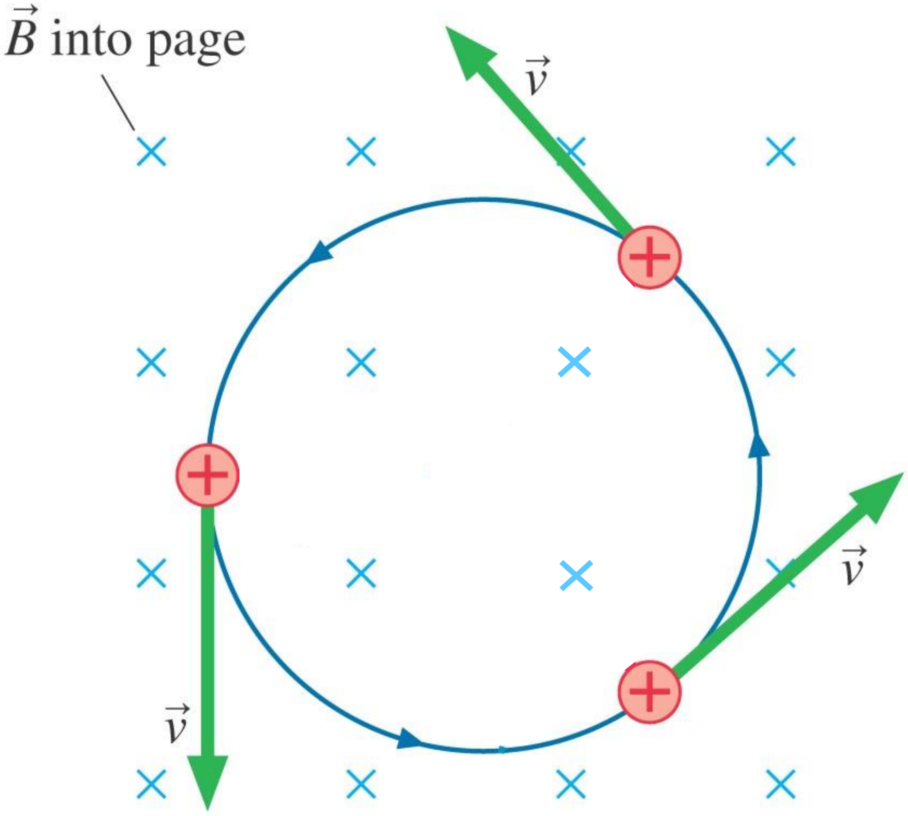
Charged particles in uniform magnetic fields undergo **uniform circular motion**.
The radius of the circle depends on how fast the particle is moving:

$$\vec{F}_B = q\vec{v} \times \vec{B}$$

$$\left|\vec{F}_B\right| = |q| vB \sin\alpha = |q| vB$$

The magnetic force is the **net force**

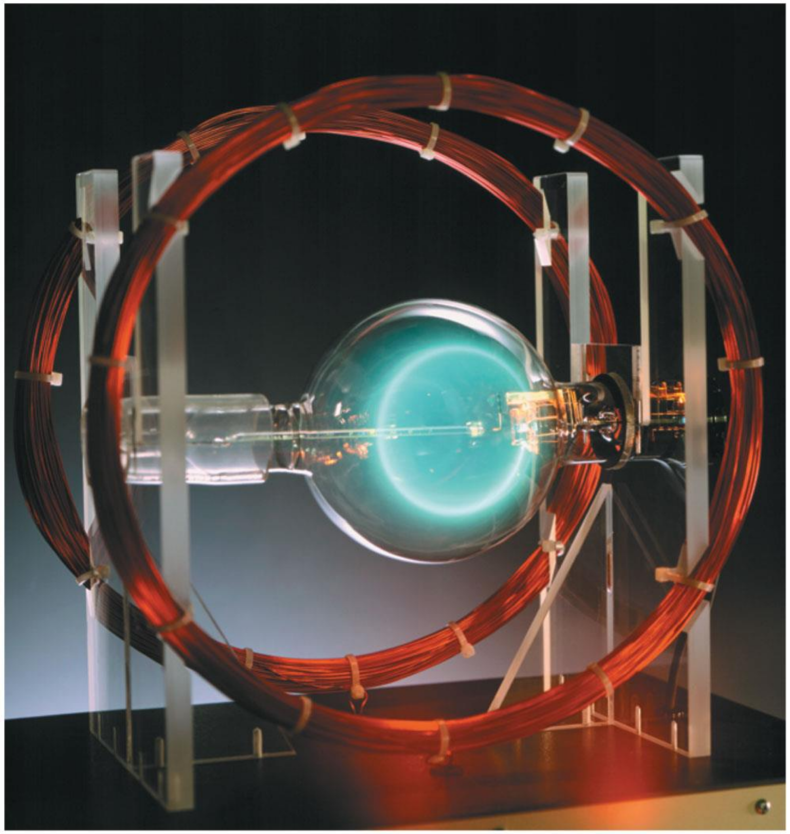
$$\left|\vec{F}_B\right| = m\frac{v^2}{R}$$

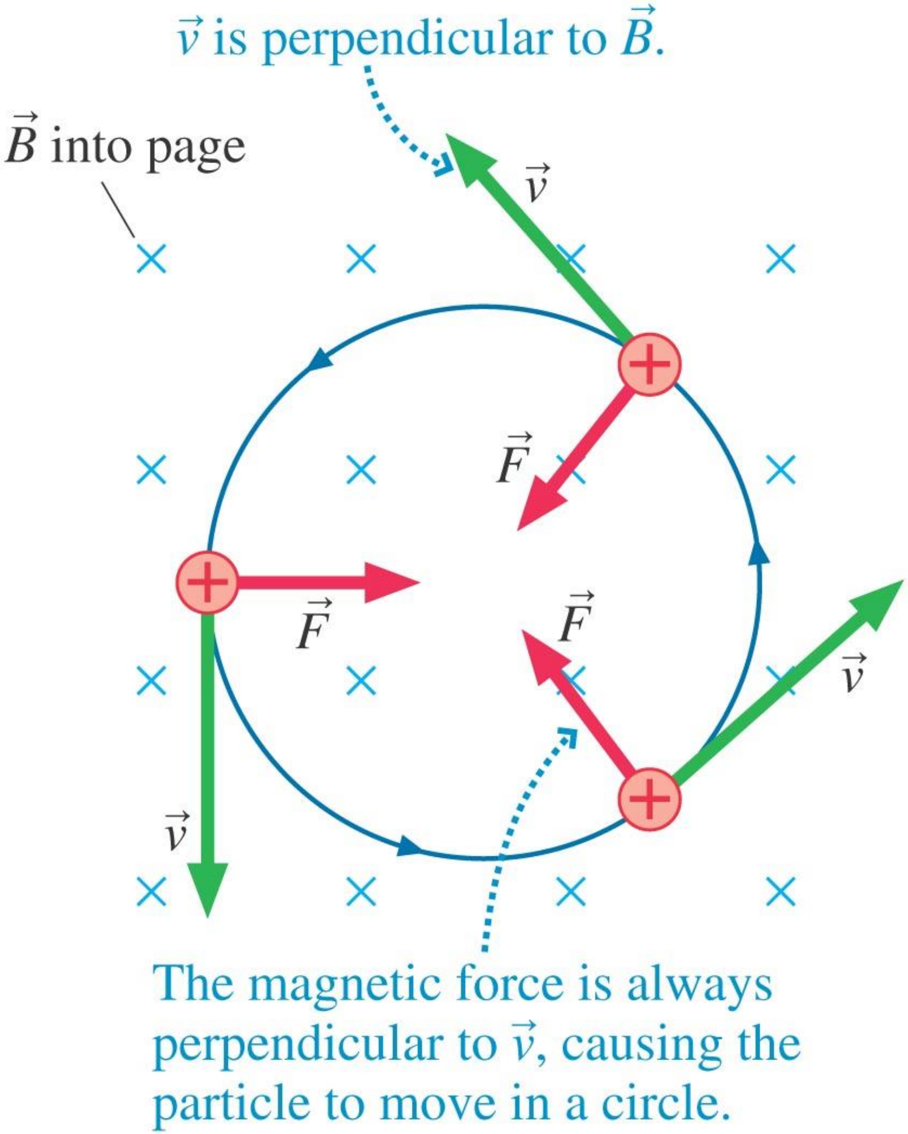


Cyclotron Motion

$$\left|\vec{F}_B\right| = |q|\cancel{v}B = m\frac{v^{\cancel{2}}}{R}$$

$$\boxed{R = \frac{mv}{|q|B}}$$





Cyclotron Motion

$$v = \frac{2pR}{T_{cyc}}$$

T_{cyc} is the cyclotron period (time it takes to make one cycle)

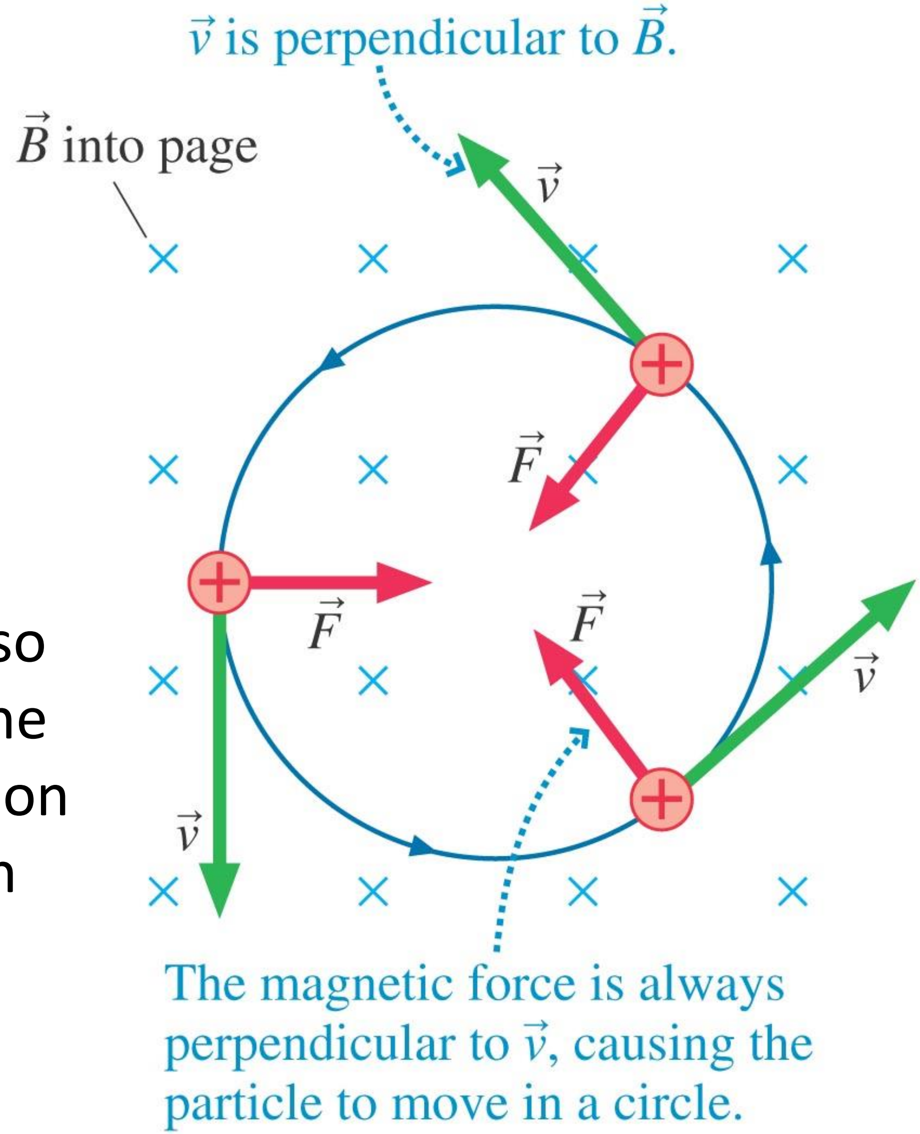
$$R = \frac{mv}{|q|B}$$

$$\cancel{R} = \frac{m}{|q|B}\frac{2p\cancel{R}}{T}$$

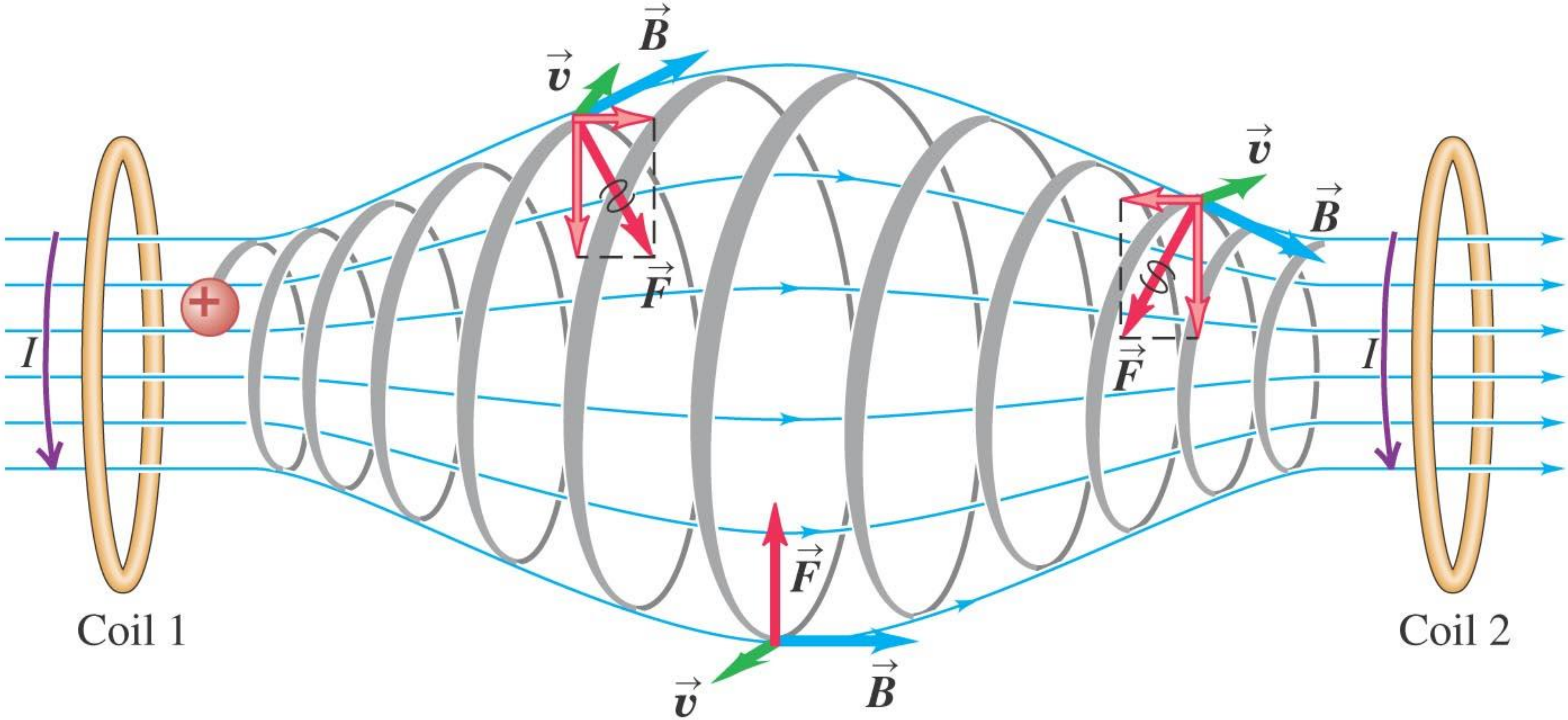
$$\boxed{T_{cyc} = \frac{2pm}{|q|B}}$$

$$\boxed{f_{cyc} = \frac{|q|B}{2pm}}$$

The period (and also the frequency of the cyclotron) depend on the B-field strength and the charge-to-mass ratio *q/m*

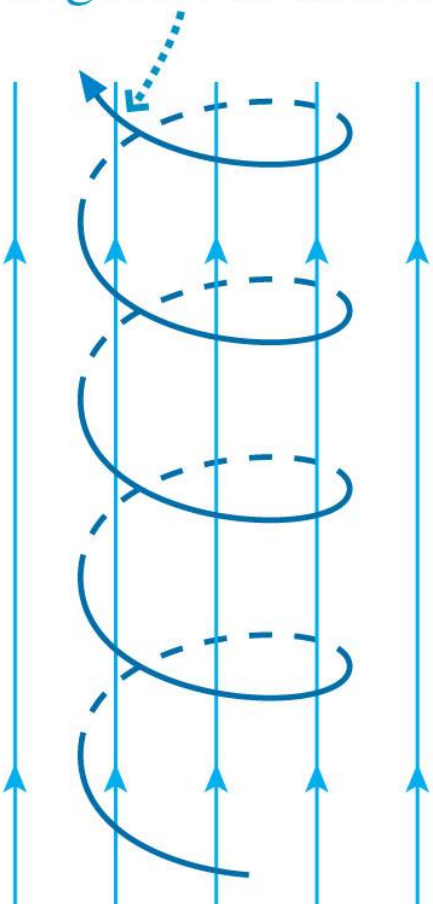


Magnetic Ion Trap

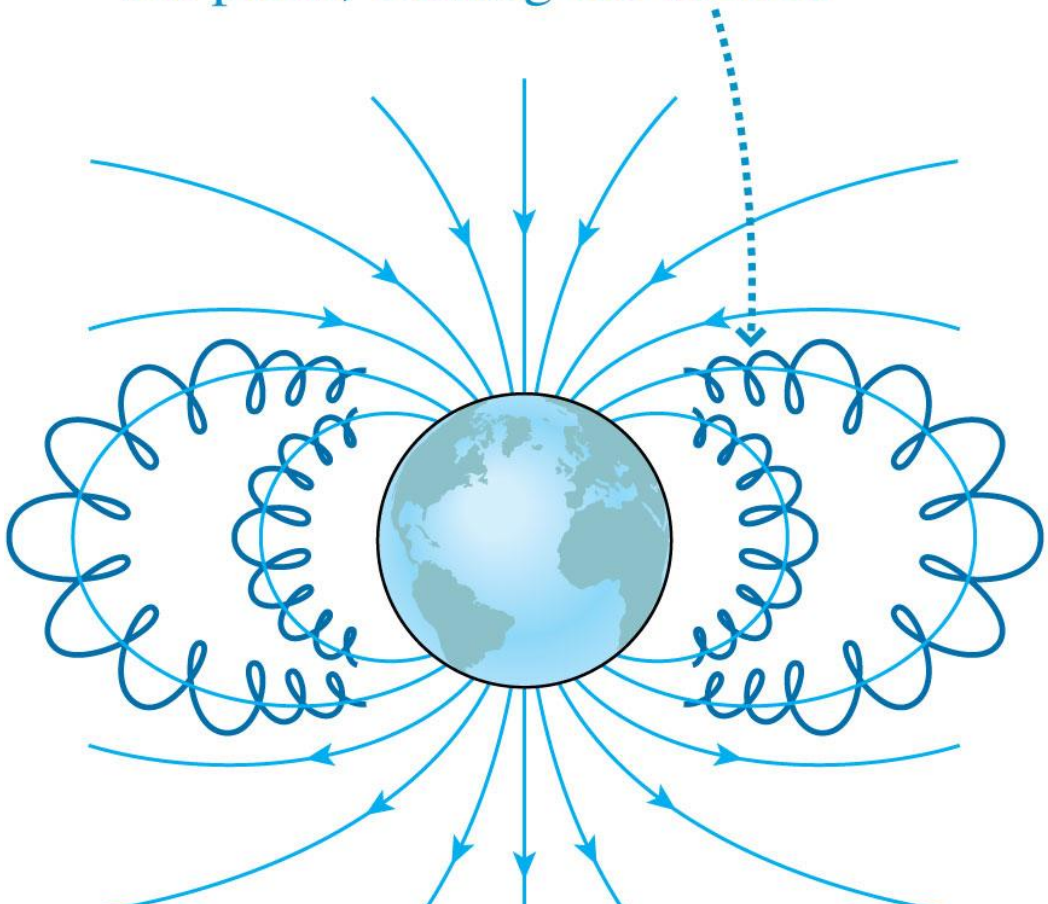


Earth's Van Allen belt
(aurora borealis/australis)

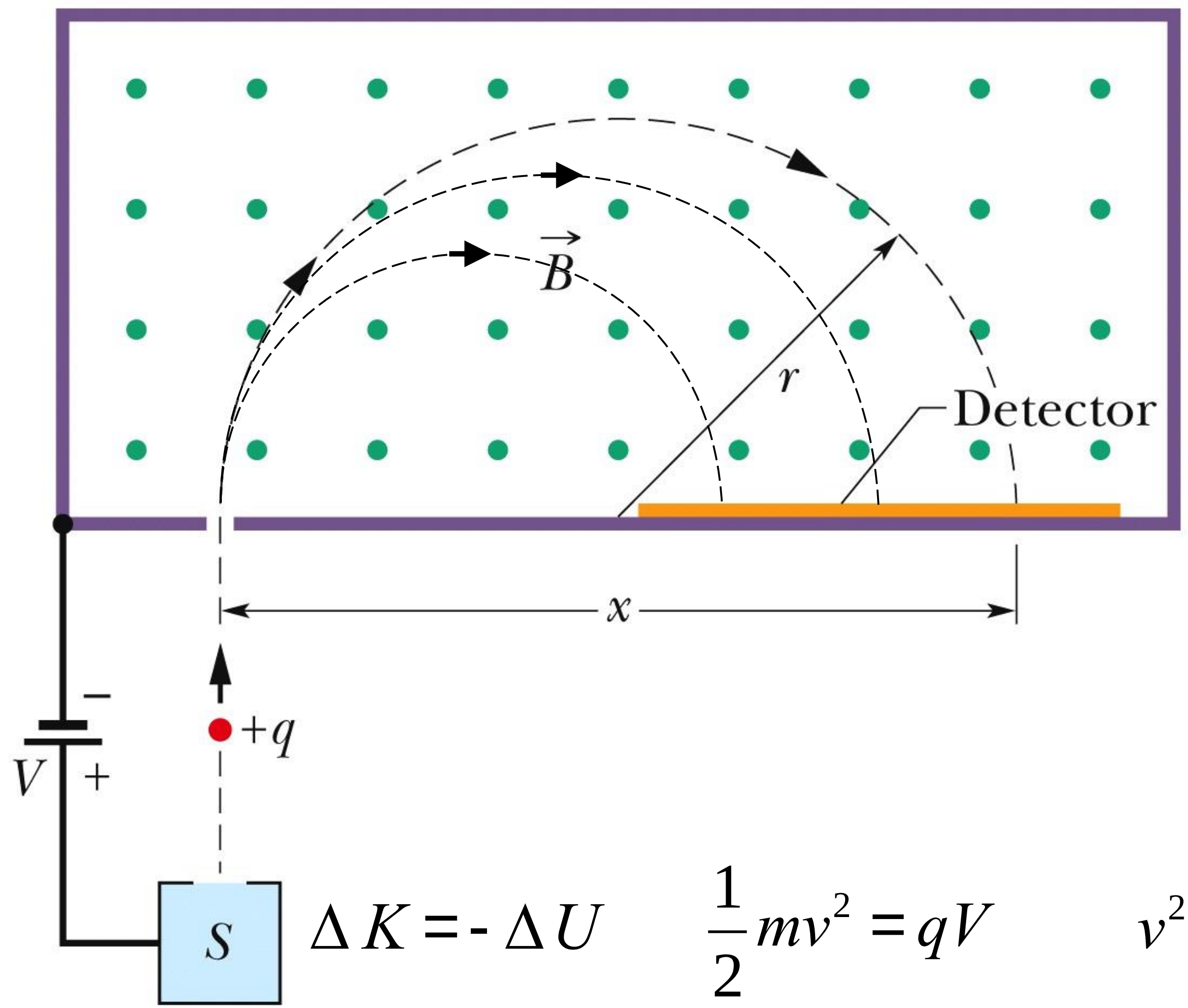
(a) Charged particles spiral around the magnetic field lines.



(b) The earth's magnetic field leads particles into the atmosphere near the poles, causing the aurora.



Application: Mass Spectrometer



$$r = \frac{mv}{qB} = \frac{x}{2}$$

$$m^2 = \frac{q^2B^2x^2}{4v^2}$$

$$m^2 = \frac{q^2B^2x^2}{4}\frac{m}{2qV}$$

$$\boxed{m = \frac{qB^2x^2}{8V}}$$

$$\Delta K = -\Delta U \qquad \frac{1}{2}mv^2 = qV \qquad v^2 = \frac{2qV}{m}$$

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Application: Cyclotron

In the gap between the dees, charges are accelerated by E-field:

$$\Delta K_{gap} = -\Delta U_{gap} = q\Delta V$$

After N times through the gap:

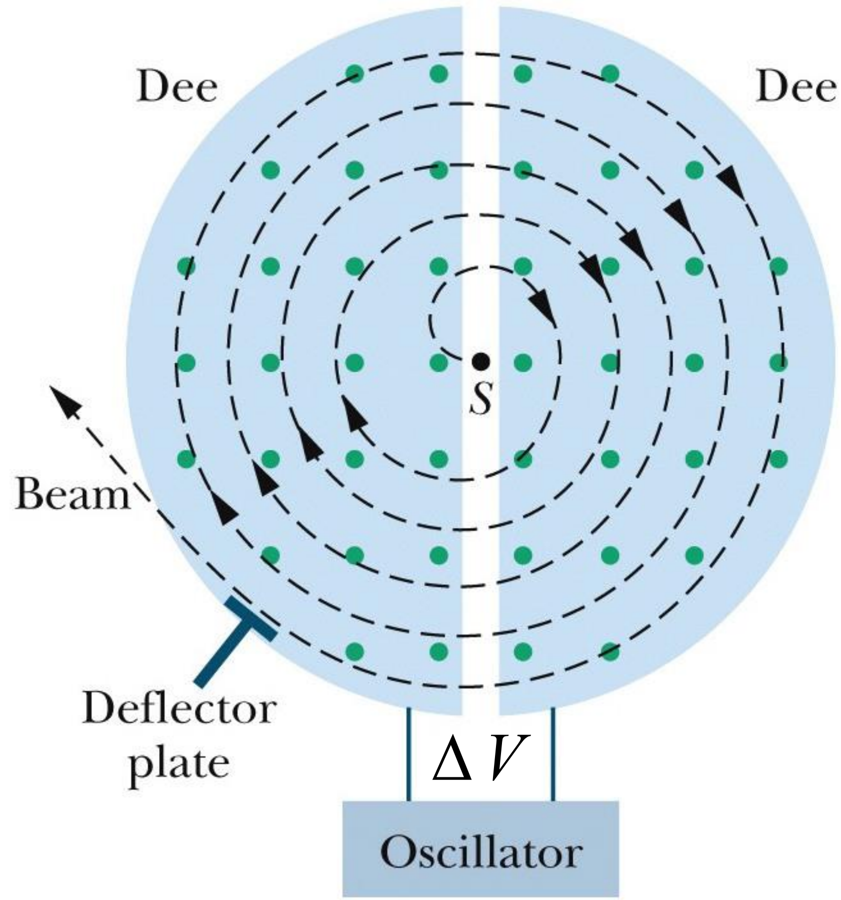
$$\frac{1}{2}mv^2 = Nq\Delta V$$

$$v = \sqrt{\frac{2Nq\Delta V}{m}}$$

$$r = \frac{mv}{qB} = \frac{m}{qB}\sqrt{\frac{2Nq\Delta V}{m}}$$

$$\boxed{= \sqrt{\frac{2Nm\Delta V}{qB^2}}}$$

The protons spiral outward in a cyclotron, picking up energy in the gap.



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