

Mon Jan 30, 2017

# Last time

- The electric field: demonstration
- Calculating electric field: Group Activity

# This time

- Electric field visualization applet - reminder
- Introduction to Gauss' Law: the first of the four Maxwell equations of electromagnetism
- Electric Flux, calculating flux

# Electric Fields

[www.falstad.com/vector3de](http://www.falstad.com/vector3de)

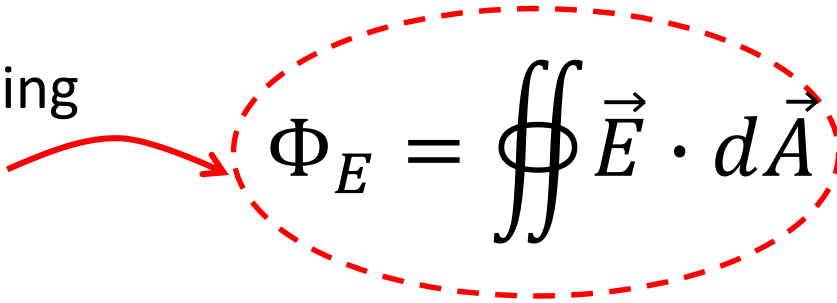
shows electric field vectors, electric field lines, equipotential surfaces, etc. for a large number of objects we have been considering

# Electric Flux; Gauss' Law

Gauss' Law is equivalent to Coulomb's law. It will provide us:

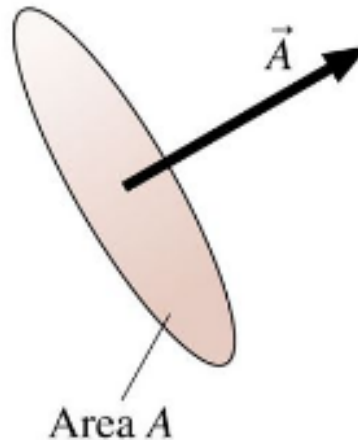
- (i) an **easier way to calculate the electric field** in specific circumstances (especially situations with a **high degree of symmetry**)
- (ii) a better understanding of the properties of conductors in electrostatic equilibrium (more on this as we go)
- (iii) It is valid for moving charges – not limited to electrostatics.

Electric flux, passing  
through a **closed**  
Gaussian surface


$$\Phi_E = \oiint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0}$$

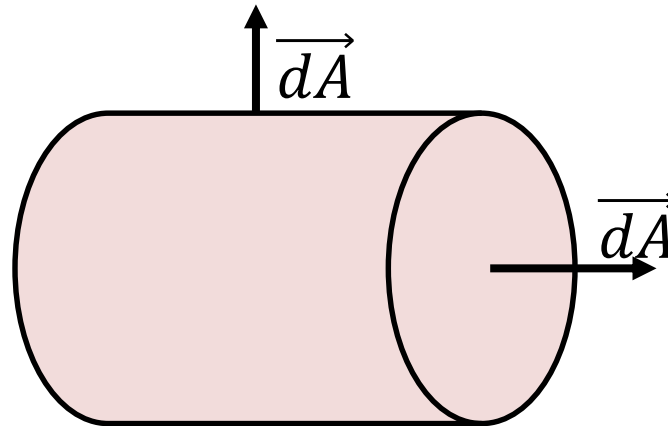
# Area vector

- Area vector,  $\vec{A}$ :
  - magnitude is equal to the surface area
  - direction normal to the plane of the area (unit vector,  $\vec{n}$  )



# Area vector

- Area vector,  $\vec{dA}$ :
  - area vector for infinitesimally small surface segment (small enough to be considered flat)
  - on closed surfaces we choose  $\vec{dA}$  to point outwards



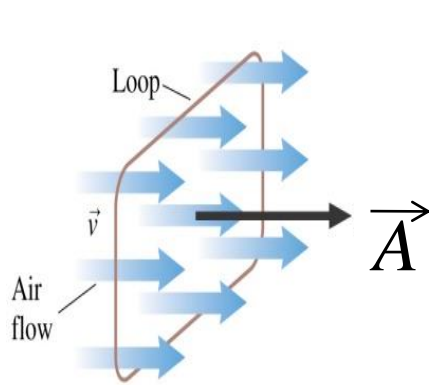
Electric flux passing through a **closed** Gaussian surface

$$\oiint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0}$$

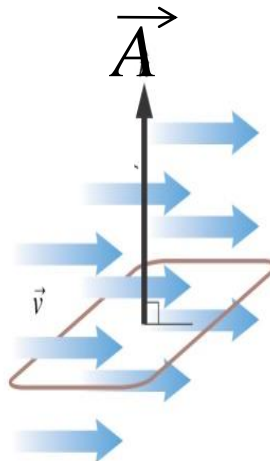
$$\Phi_E = Q_{encl} / \epsilon_0$$

**Gauss's law:** the **net flux** passing through a closed surface (Gaussian surface) is **proportional to the net charge inside** the surface. It **does NOT** depend on the shape of the surface.

**Flux :** amount of 'something' (air, water.....) flowing through an area

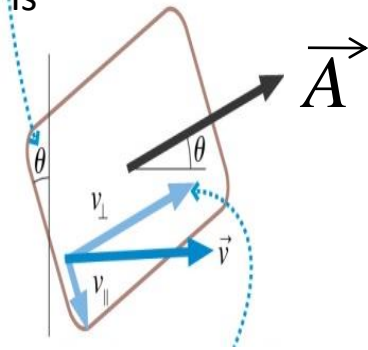


The air flowing through the loop is maximum when  $\theta = 0^\circ$



No air flows through the loop when  $\theta = 90^\circ$

The loop is tilted by angle  $\theta$

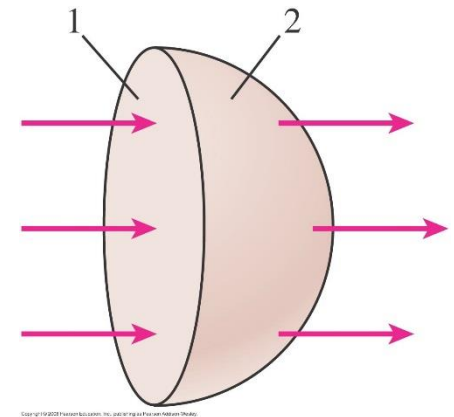


$v_{\perp} = v \cos\theta$  is the component of the air velocity perpendicular to the loop.

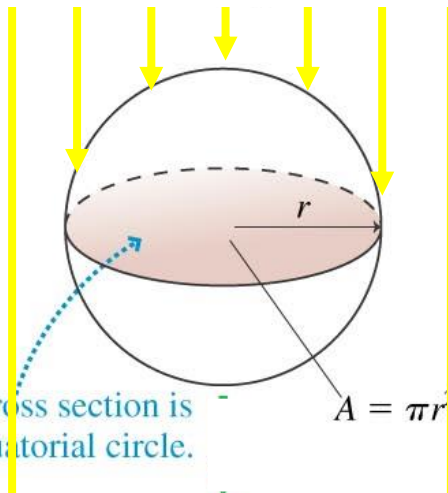
# Cross-sectional area

Area measured in a plane  $\perp$  to the direction of flow.

Area of shadow cast by || light rays

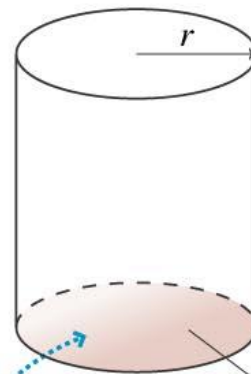


Flux through 1 = Flux through 2



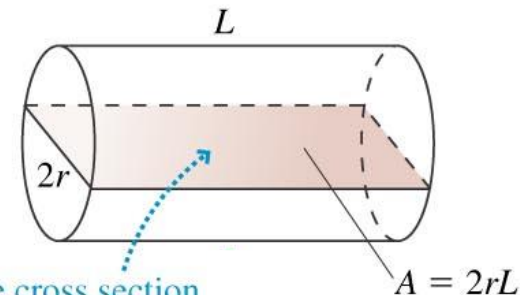
The cross section is an equatorial circle.

$$A = \pi r^2$$



The cross section is a circle.

$$A = \pi r^2$$



The cross section is a rectangle.

$$A = 2rL$$



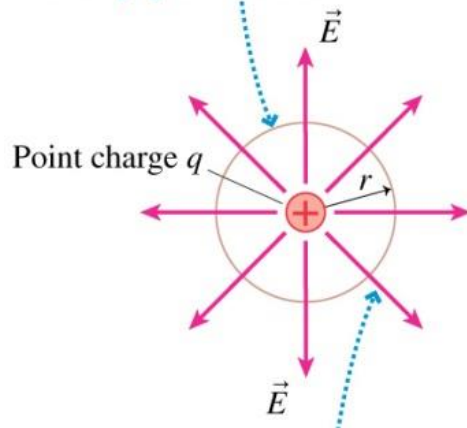
Electric flux passing through a **closed** Gaussian surface

$$\oint \vec{E} \cdot d\vec{a} = \frac{Q_{enclosed}}{\epsilon_0}$$

$$F_E = Q_{encl} / \epsilon_0$$

**Flux** : amount of 'something' (air, water.....) flowing through an area

Cross section of a Gaussian sphere of radius  $r$ . This is a mathematical surface, not a physical surface.



The electric field is everywhere perpendicular to the surface and has the same magnitude at every point

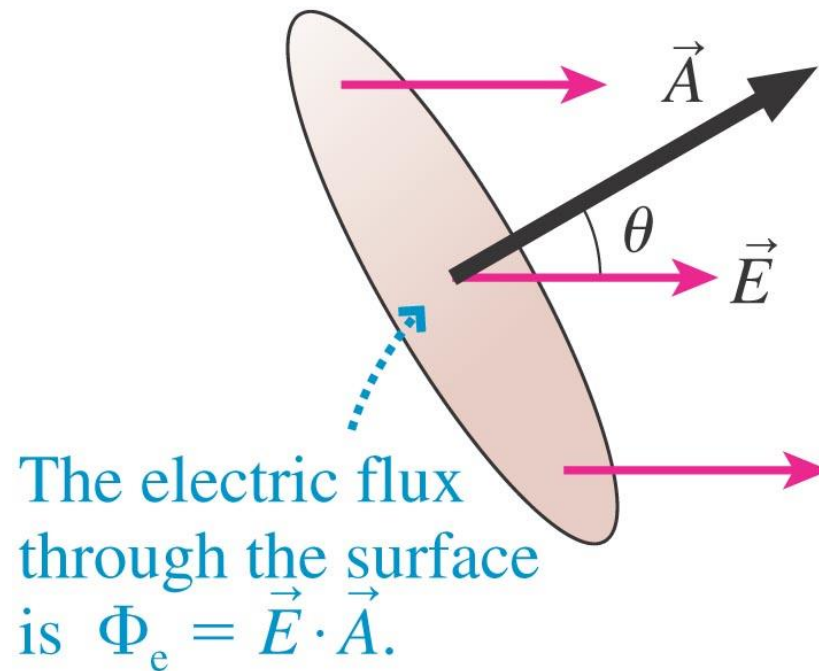
The flux passing **out** of the closed Gaussian surface is **positive**. The flux is:

$$F_E = +q / \epsilon_0$$

If the positive charge is replaced by a **negative** charge, the flux would be:

$$F_E = -q / \epsilon_0$$

## Electric flux through a surface with area $A$



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$$\Phi_E = EA \cos \theta$$

How to evaluate  $\Phi_E = \oiint \vec{E} \cdot d\vec{A}$

- If the electric field is **tangent** to the surface:

$$\Phi = 0$$

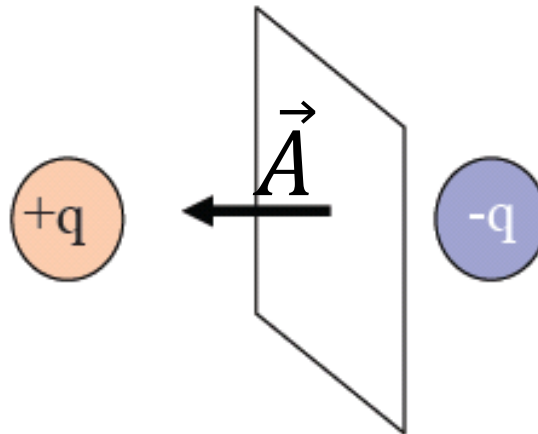
- If the electric field is **normal** to the surface and is **constant** at every point:

$$\Phi = EA$$

# TopHat Questions

# TopHat Question

The flux through the planar surface below  
(surface vector pointing to the left)



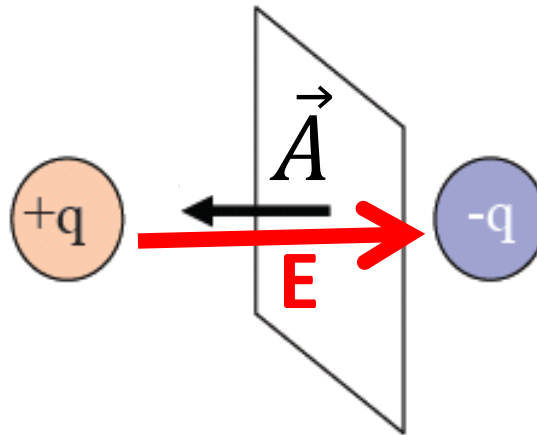
A: is positive.

B: is negative.

C: is zero.

D: not enough information

The flux through the planar surface below  
(surface vector pointing to the left)



A: is positive.

B: is negative.

C: is zero.

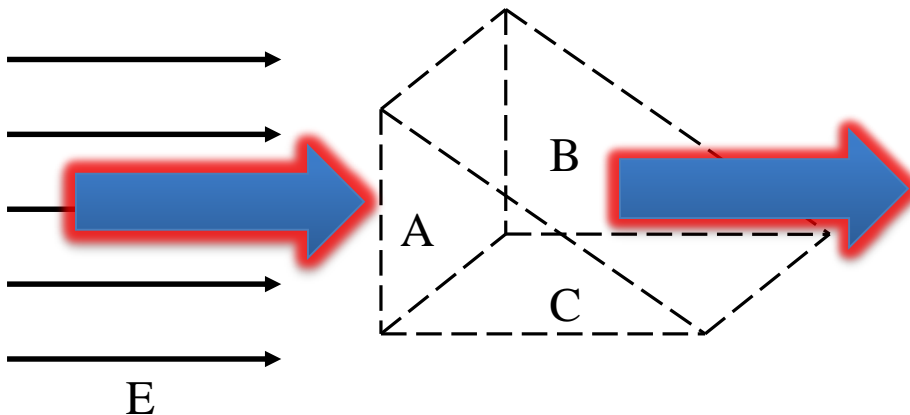
D: not enough info

Here we choose the outward direction to be to the left. That is completely arbitrary. Since the  $\vec{E}$  field is flowing inward (negative), the flux is negative.

$\vec{E}$  always points from  $+$  to  $-$  charges

# TopHat Question

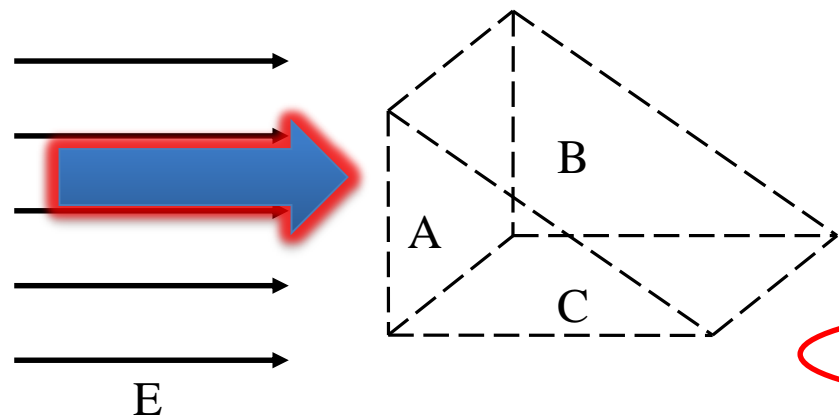
A prism-shaped closed surface is in a constant, uniform electric field  $\mathbf{E}$ , filling all space, pointing to the right. The 3 rectangular faces of the prism are labeled A, B, and C. Face A is perpendicular to the E-field. The bottom face C is parallel to  $\mathbf{E}$ . Face B is the leaning face.



Which face has the largest magnitude electric flux passing through it?

- A) side A
- B) side B
- C) side C
- D) sides A and B have the same Electric flux

A prism-shaped closed surface is in a constant, uniform electric field  $\mathbf{E}$ , filling all space, pointing to the right. The 3 rectangular faces of the prism are labeled A, B, and C. Face A is perpendicular to the E-field. The bottom face C is parallel to  $\mathbf{E}$ . Face B is the leaning face.



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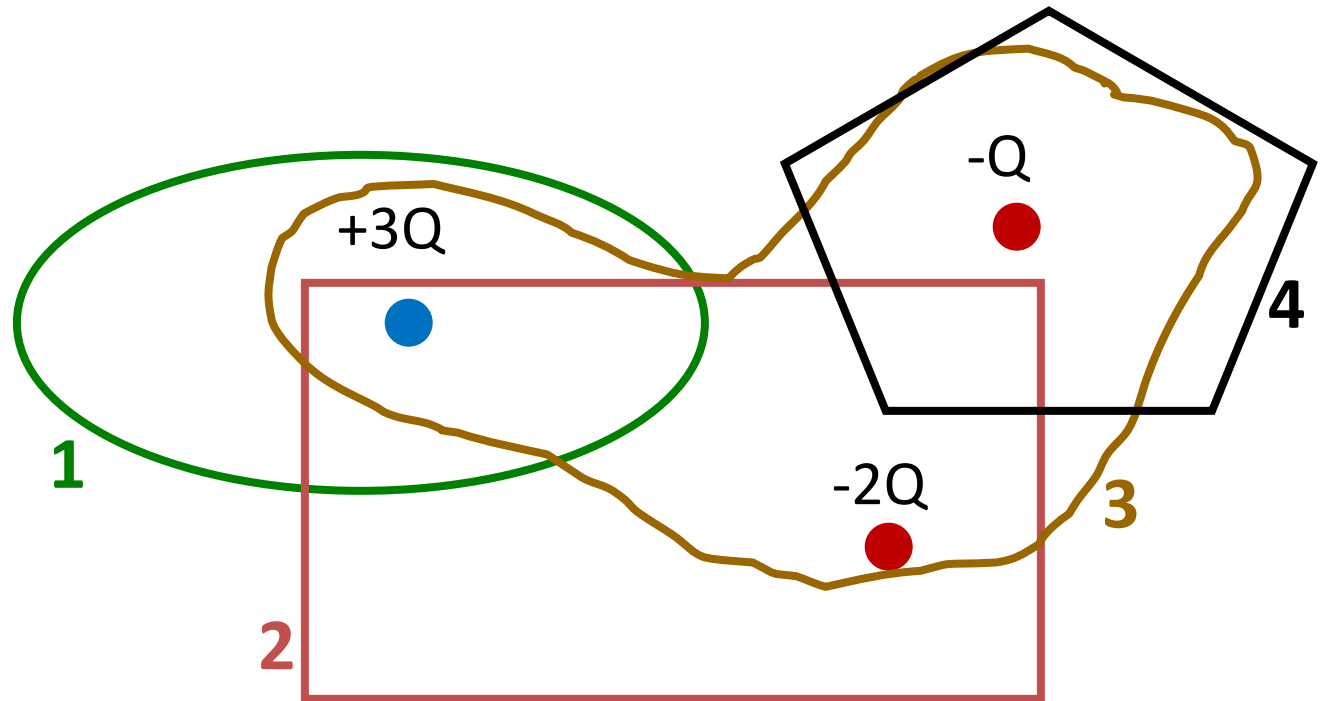
- A) A                      B) B                      C) C  
D) A and B have the same E flux

The same amount of  $\mathbf{E}$  flux flows through the perpendicular face A and the slanted face B. The amount of  $\mathbf{E}$ -flux flowing through the bottom face C is 0. Remember, although the slanted face B has larger surface area, the  $\cos(\theta)$  term keeps the flux equal to the flux through the perpendicular face A. Recall,  $\theta$  is the angle between  $\mathbf{E}$  and the normal to the slanted surface B.



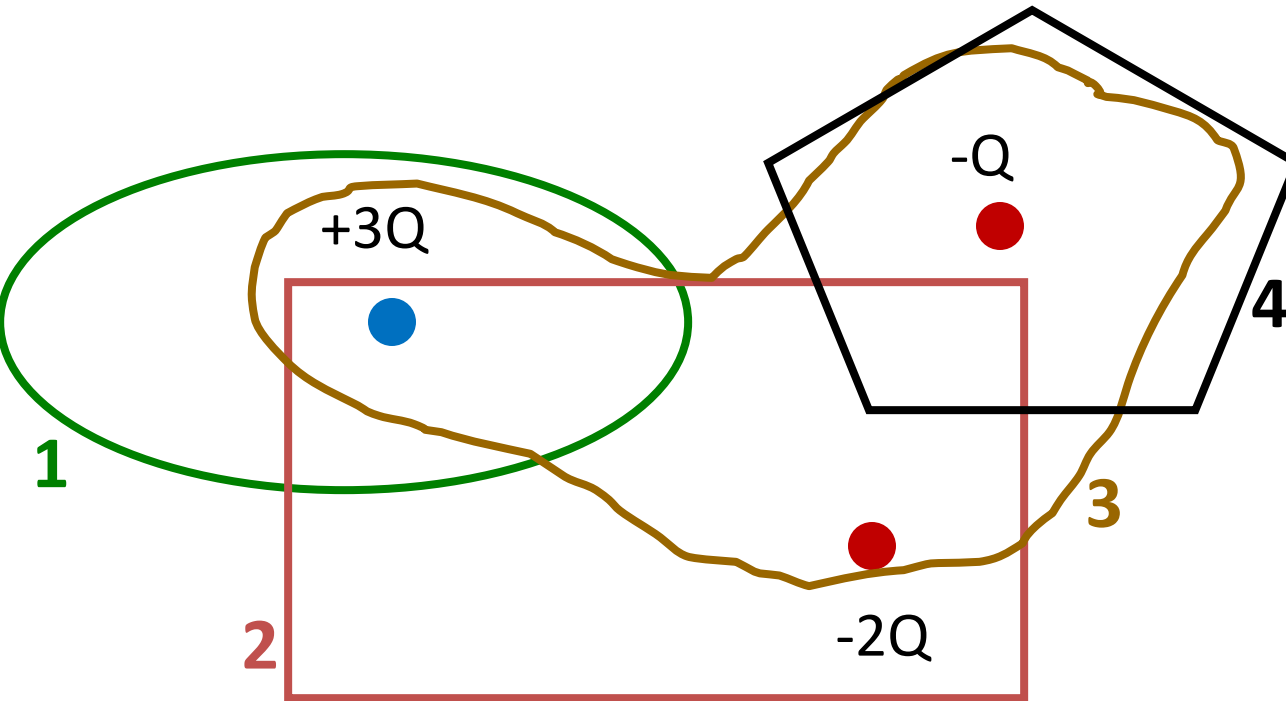
# TopHat Question

Which surface has the largest magnitude electric flux?



- A)  $1 = 2 = 3 = 4$
- B)  $1 > 2 = 4 > 3$
- C)  $3 > 2 > 1 > 4$
- D)  $3 > 2 > 1 = 4$
- E) None of the above

Which surface has the largest magnitude electric flux?



A)  $1 = 2 = 3 = 4$

B)  $1 > 2 = 4 > 3$

C)  $3 > 2 > 1 > 4$

D)  $3 > 2 > 1 = 4$

E) None of the above

**Region 1: Net charge is  $+3Q$**

**Region 2: Net charge is  $+Q$**

**Region 3: Net charge is  $0$**

**Region 4: Net charge is  $-Q$**

Here, we are interested in the **magnitude** of the flux, so the flux through regions 2 & 4 is the same. The flux through region 2 is outward (+), while the flux through region 4 is inward (-).

Again, electric flux is equal to " $Q_{\text{enclosed}}/\epsilon_0$ "