Last time

- Top hat questions
- Electric field pattern due to point charges
- Electric field pattern for a dipole
- Electric field due to a charged 90° arc of radius R at its center, constant charge density
- Electric field due to a charged semi-circle of radius R at its center using superposition principle
- Electric field due to a charged circle of radius R at its center using superposition principle

This time

- Electric field of a thin ring on a point along the axis of the ring
- Activity #3

Infinitely thin ring of charge

$$\lambda = \frac{|Q|}{2\pi R}$$ Uniformly charged ring

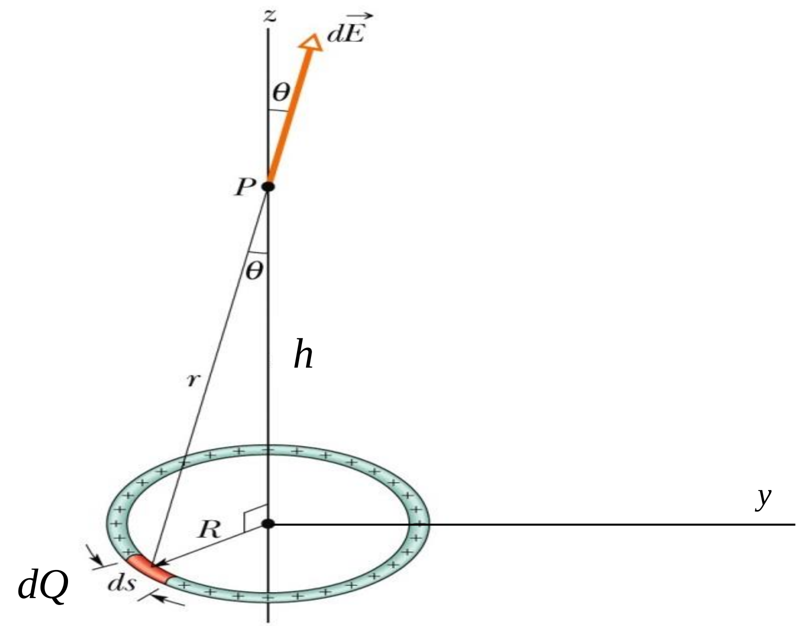
ds Infinitesimal arc length

dQ Infinitesimal charge on ds

$$dQ = \lambda ds$$

$$r^2 = \left(R^2 + h^2\right)$$

$$\cos\theta = \frac{h}{r} = \frac{h}{\left(R^2 + h^2\right)^{1/2}}$$



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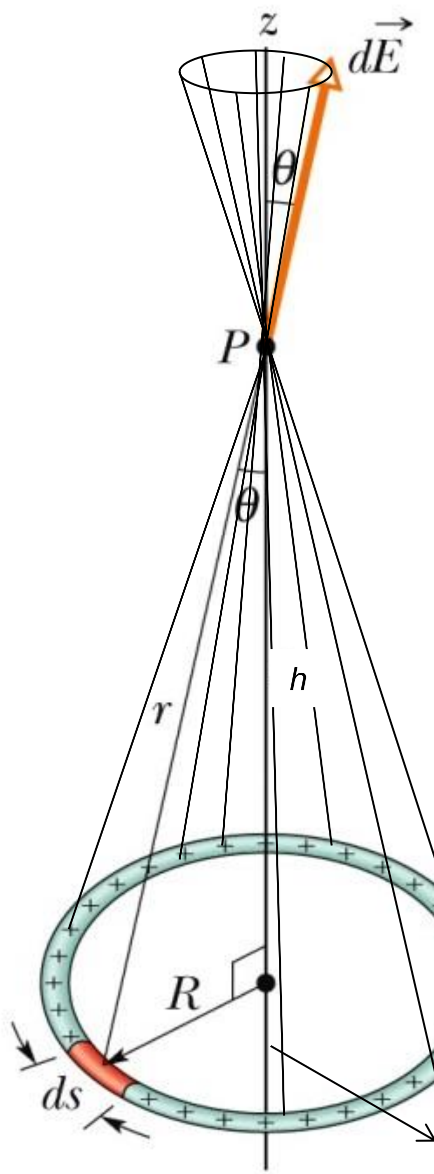
$d\vec{E}$ is the electric field generated by dQ at point P.

Do this for a large number pieces around the ring.

$d\vec{E}$ traces a cone.



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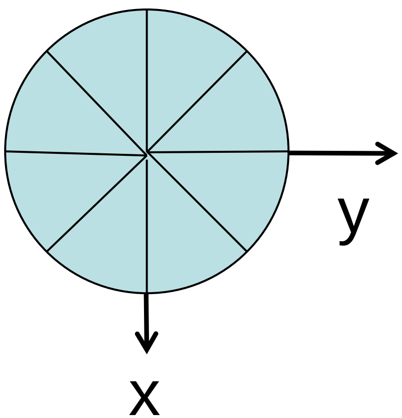
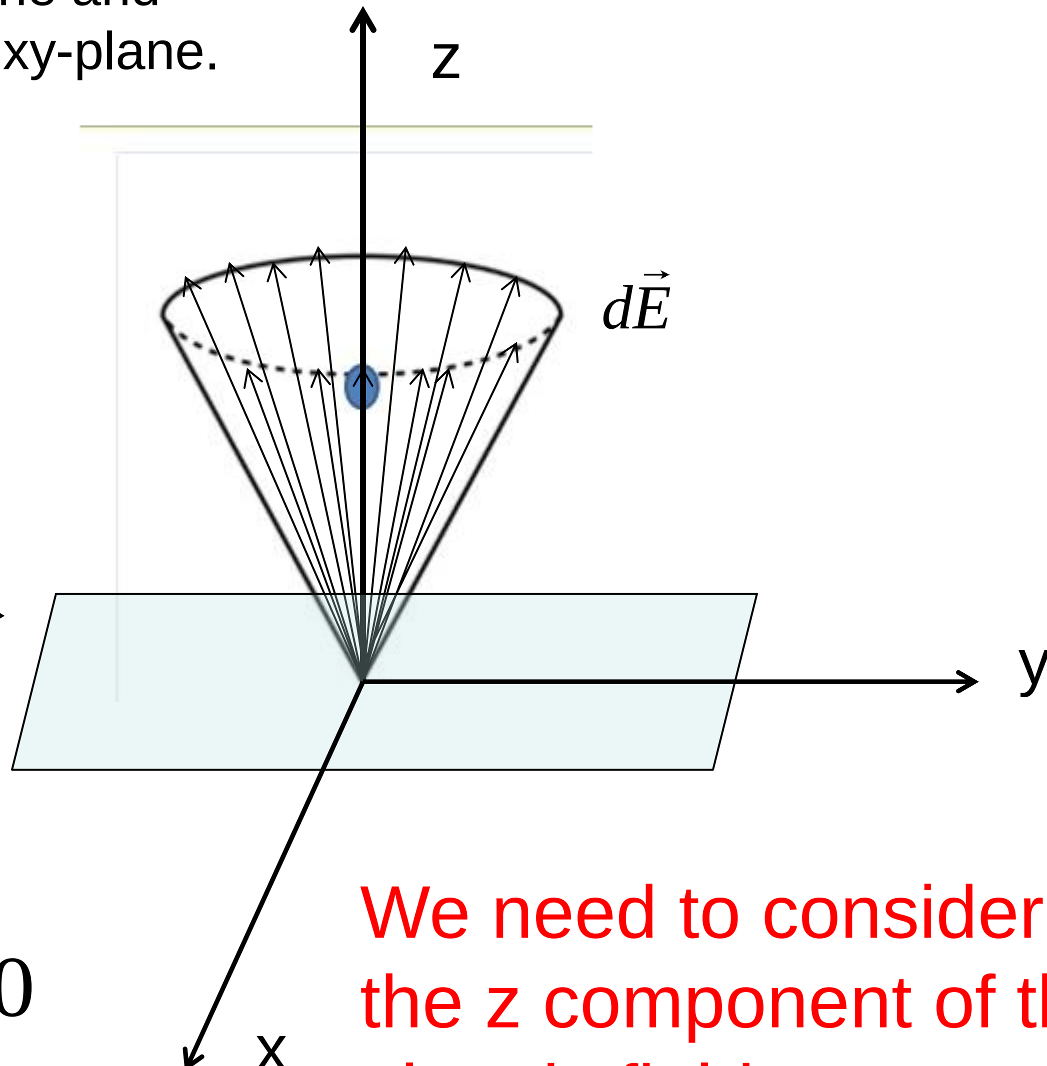
Now consider the cone and project out $d\vec{E}$ in the xy-plane.





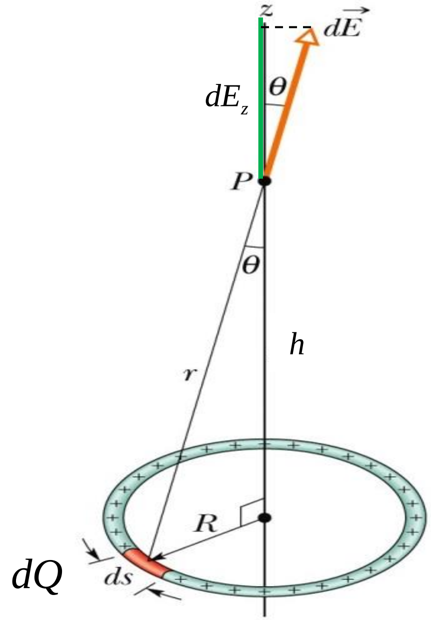
$$E_x = 0, E_y = 0$$

We need to consider only the z component of the electric field.

$$\lambda = \frac{Q}{2\pi R}, Q = 2\pi\lambda R$$

$$r^2 = \left(R^2 + h^2\right)$$

$$\cos\theta = \frac{h}{\left(R^2 + h^2\right)^{1/2}}$$



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$$dE_z = dE\cos\theta$$

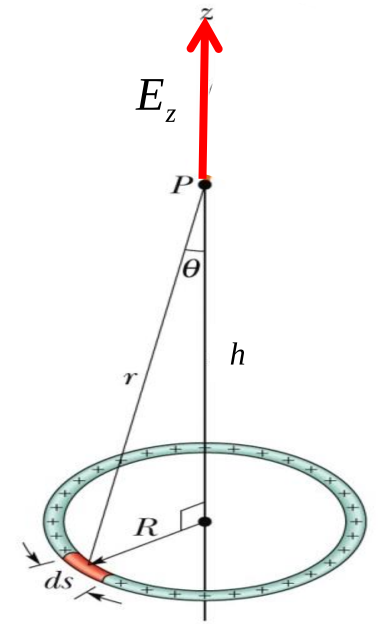
$$dE_z = dE\cos\theta = k_e\frac{dQ}{r^2}\cos\theta = k_e\frac{dQ}{\left(R^2+h^2\right)}\frac{h}{\left(R^2+h^2\right)^{1/2}} = k_e\frac{h\lambda ds}{\left(R^2+h^2\right)^{3/2}}$$

$$E_z = \int_{\text{Around the circle}} k_e\frac{h\lambda ds}{\left(R^2+h^2\right)^{3/2}} = k_e\frac{h\lambda}{\left(R^2+h^2\right)^{3/2}}\int_{\text{Around the circle}} ds$$

$$E_z = k_e\frac{h\lambda}{\left(R^2+h^2\right)^{3/2}}2\pi R = k_e\frac{hQ}{\left(R^2+h^2\right)^{3/2}}$$

$$\vec{E} = k_e\frac{hQ}{\left(R^2+h^2\right)^{3/2}}\hat{k}$$

$$\vec{E} = k_e \frac{hQ}{\left(R^2 + h^2\right)^{3/2}} \hat{k}$$



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Move down to the center of the ring.

$$h \to 0? \Rightarrow E_z = 0$$

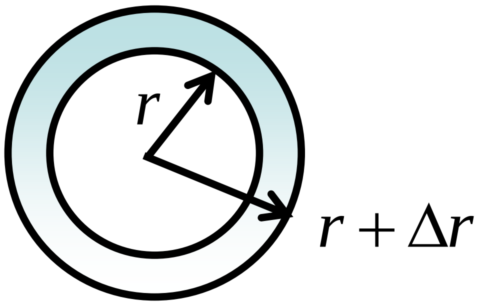
Move very far away from the ring.

$$h \gg R? \quad (R^2 + h^2) \approx h^2$$

$$E_z = k_e \frac{hQ}{\left(R^2 + h^2\right)^{3/2}} \approx k \frac{hQ}{h^3} = k \frac{Q}{h^2}$$

Appears as a point charge Q located at the center of the disk.

What is the change in the area of a circle if its radius *r* is increased by an infinitesimal amount (*dr*)?



$$\Delta A = \pi(r + \Delta r)^2 - \pi r^2 = \pi r^2 + 2\pi r \Delta r + \Delta r^2 - \pi r^2$$

$$\Delta A = 2\pi r \Delta r + \Delta r^2 \approx 2\pi r \Delta r$$

When can we make the ≈ sign =?

$$\lim_{\Delta r \to 0} \Delta r = dr$$

$$\lim_{\Delta A \to 0} \Delta A = dA$$

$$dA = 2\pi r dr$$