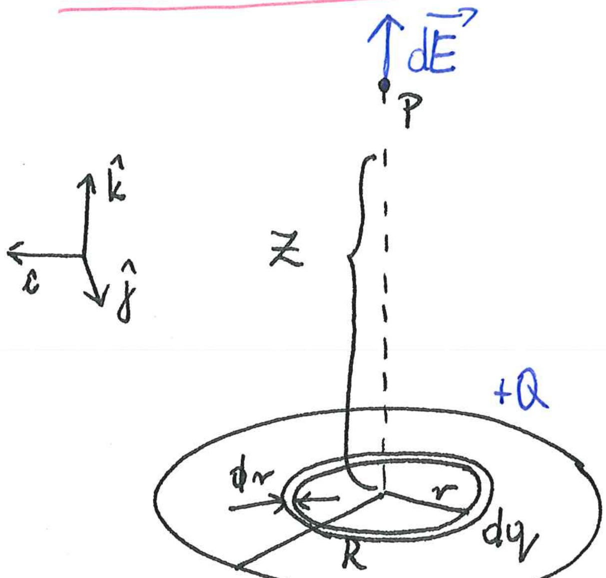
CHARGED DISK (SURFACE OF CHARGE)

surface density σ



Approach:

- $\vec{E}$ due to ring of charge R, Q

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{Q z}{(z^2 + R^2)^{3/2}} \hat{k}$$

$$d\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{z\, dq}{(z^2 + r^2)^{3/2}} \hat{k}$$

- Consider concentric rings, each with radius r and thickness dr, $r < R$
- Use principle of superposition – add up the rings starting from the center
  → integration variable should be r, integration limits: $r = 0 \to r = R$

$$\sigma = \frac{Q}{A}$$

- Find the surface area of the element

$$dA = \text{circumference} * \text{thickness}$$

$$dA = 2\pi r \cdot dr$$

- Find dq located on the ring

$$Q = \sigma \cdot A$$

$$dq = \sigma \cdot 2\pi r \cdot dr$$

$$dE = \frac{1}{4\pi\varepsilon_0} \frac{z\,\sigma\,2\pi r\,dr}{(z^2+r^2)^{3/2}}$$

$$\int dE = \int_0^R \frac{1}{\cancel{4\pi}\varepsilon_0 \; 2} \frac{r\,dr\; z\cdot\sigma\cancel{\cdot 2\pi}}{(z^2+r^2)^{3/2}}$$

$$E = \frac{z\cdot\sigma}{2\varepsilon_0} \int_0^R \frac{r\,dr}{(z^2+r^2)^{3/2}}$$

$$\int \frac{r\,dr}{(z^2+r^2)^{3/2}} = \frac{-1}{(z^2+r^2)^{1/2}}$$

$$E = \frac{z\cdot\sigma}{2\varepsilon_0}\left[\frac{-1}{(z^2+R^2)^{1/2}} - \left(-\frac{1}{(z^2+0^2)^{1/2}}\right)\right]$$

$$E = \frac{z\sigma}{2\varepsilon_0}\left[\frac{1}{z} - \frac{1}{(z^2+R^2)^{1/2}}\right]$$

$$E = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{z}{(z^2+R^2)^{1/2}}\right]$$

- TWO EXTREMES

$z \ll R$

$z^2 + R^2 \approx R^2$

small

$$E = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{z}{R}\right] = \frac{\sigma}{2\varepsilon_0}$$

electric field of a charged inifite plane

CHARGED SHEET (INFINITE)

- FAR AWAY FROM THE DISK

$R \ll z$

Should look like a point charge, but the approximation of $z^2 + R^2 \approx z^2$ would not work

$$E = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{z}{(z^2)^{1/2}}\right] = \frac{\sigma}{2\varepsilon_0}[1-1] = 0 \quad \text{NOT TRUE}$$

Use binomial expension

For $f(x) = (1+x)^n$

$$f(x) \approx 1 - nx + \frac{n(n-1)}{2}x^2 + \dots$$

If $x \ll 1$ then

$$f(x) = 1 + nx \ , \quad \text{take} \quad x = \frac{R^2}{z^2} \quad \text{and} \quad n = -\frac{1}{2}$$

$$E = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{z}{(z^2+R^2)^{1/2}}\right] = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{z/z}{\left(\frac{z^2}{z^2} + \frac{R^2}{z^2}\right)^{1/2}}\right]$$

$$E = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{1}{\left(1+\frac{R^2}{z^2}\right)^{1/2}}\right] = \frac{\sigma}{2\varepsilon_0}\left[1 - \left(1+\frac{R^2}{z^2}\right)^{-1/2}\right]$$

$$x = \frac{R^2}{z^2} \quad \text{and} \quad n = -\frac{1}{2}$$

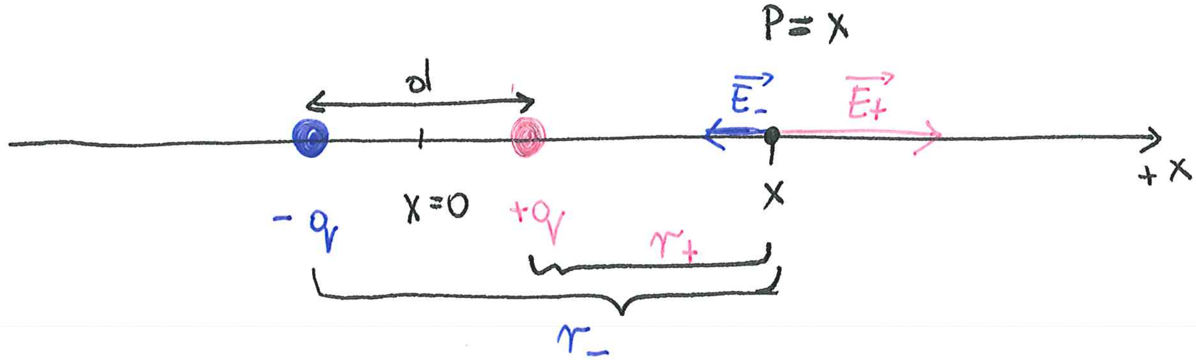
For $R \ll z$ x is very small $\rightarrow$ binomial expension

$$E = \frac{\sigma}{2\varepsilon_0}\left[1 - \left(1 + \overset{n}{\left(-\frac{1}{2}\right)} \cdot \overset{x}{\frac{R^2}{z^2}}\right)\right] = \frac{\sigma}{2\varepsilon_0}\left[\cancel{1} - \cancel{1} + \frac{1}{2}\frac{R^2}{z^2}\right]$$

$$E = \frac{\sigma}{2\varepsilon_0} \cdot \frac{1}{2}\frac{R^2}{z^2} \qquad \sigma = \frac{Q}{\pi R^2}$$

$$E = \frac{Q}{\pi \cancel{R^2}} \cdot \frac{1}{4\varepsilon_0}\frac{\cancel{R^2}}{z^2} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{z^2} \qquad \text{POINT CHARGE}$$

ELECTRIC FIELD OF A DIPOLE ALONG AXIS



$$E_+ = \frac{1}{4\pi\varepsilon_0}\frac{q}{r_+^2} = \frac{1}{4\pi\varepsilon_0}\frac{q}{\left(x-\frac{d}{2}\right)^2}$$

$$E_- = \frac{1}{4\pi\varepsilon_0}\frac{q}{r_-^2} = \frac{1}{4\pi\varepsilon_0}\frac{q}{\left(x+\frac{d}{2}\right)^2}$$

$$E_x = E_+ - E_- = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{\left(x-\frac{d}{2}\right)^2} - \frac{1}{\left(x+\frac{d}{2}\right)^2}\right)$$

Superposition

$$E_x = \frac{q}{4\pi\varepsilon_0}\left(\frac{(x+d/2)^2 - \left(x-\frac{d}{2}\right)^2}{\left(x-\frac{d}{2}\right)^2\left(x-\frac{d}{2}\right)^2}\right)$$ Get a common denominator

$$E_x = \frac{q}{4\pi\varepsilon_0}\left(\frac{(\cancel{x^2} + xd + \cancel{\tfrac{d^2}{4}}) - (\cancel{x^2} - xd + \cancel{d^2/4})}{(x^2 - d^2/4)^2}\right)$$

expend and cancel, use $(a+b)(a-b) = (a^2-b^2)$

$$E_x = \frac{q}{4\pi\varepsilon_0}\left(\frac{2xd}{(x^2-d^2/4)^2}\right)$$

$$E_x = \frac{1}{4\pi\varepsilon_0}\frac{2qxd}{(x^2-d^2/4)^2}$$ DIPOLE MOMENT $p = qd$

Perfect dipole: kepp p fixed, but let $d \to 0$
(or equivalent $x \gg d$) $\quad \underline{x^2 - d^2/4 \approx x^2}$

$$E_x = \frac{1}{4\pi\varepsilon_0}\frac{2p\cancel{x}}{x^{\cancel{4}\,3}}$$

$$E_x = \frac{1}{4\pi\varepsilon_0}\frac{2p}{x^3}$$

Monopole (single charge) $\sim \frac{1}{r^2}$ falloff
Dipole $\left((+q)\underset{d}{\text{—}}(-q)\right) \sim \frac{1}{r^3}$ falloff