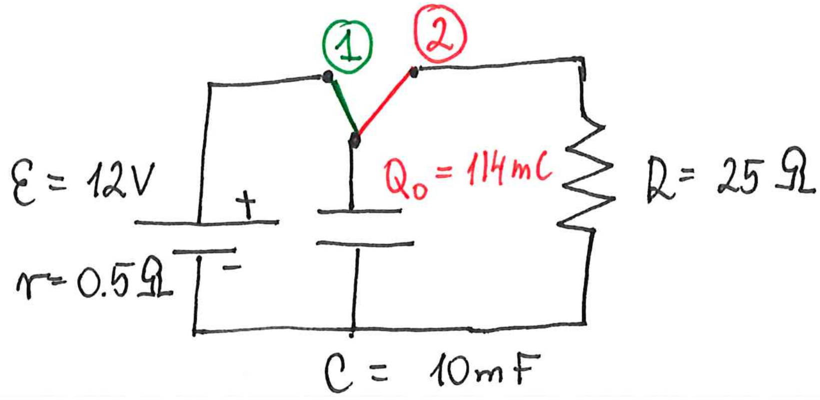


1) Charging the capacitor

$t = 0 \rightarrow t = 15\ ms$

Find $\tau = RC$ $\quad$ $R = r$ (internal resistance of battery)

$\tau = rC \qquad \tau = 0.5\,\Omega \cdot 10mF = 5\,ms$

Total charging time $\quad t = 15\,s$

$t = 3\tau$

Calculating charge on capacitor after a very long time – maximum possible charge Q_f $(t \to \infty)$

$$Q_f = C \cdot \mathcal{E}$$

$$Q_f = 10\,mF \cdot 12\,V = 120\,mC$$

Calculating charge on capacitor after $t = 15\,ms$ (3τ) time.

$$q(t) = Q_f\left(1 - e^{-t/RC}\right) = Q_f\left(1 - e^{-t/\tau}\right)$$

$$q(3\tau) = 120\,mC \cdot \left(1 - e^{-3\tau/\tau}\right)$$

$$q(3\tau) = 120\,mC\,\underbrace{\left(1 - \underbrace{e^{-3}}_{0.049}\right)}_{0.95}$$

$$q(3\tau) = 114\,mC$$

Capacitor is not fully charged after 15 ms.

2) What is the voltage?

We start at $\boxed{Q_0 = 114 \text{ mC}}$

→ this is the charge on capacitor after charging for 15 ms (see part 1)

$$q(t) = Q_0 e^{-t/RC}$$

$$i(t) = -\frac{dq}{dt} = -\left(\frac{Q_0}{(-RC)} e^{-t/RC}\right)$$

discharging current flows in opposite direction

$$i(t) = \frac{Q_0}{RC} e^{-t/RC}$$

$$V(t) = R \cdot i(t)$$

$$V(t) = \cancel{R} \cdot \frac{Q_0}{\cancel{R}C} e^{-t/RC}$$

$$V(t) = \frac{Q_0}{C} e^{-t/RC}$$

$$RC = 25\,\Omega \cdot 10\,\text{mF} = 250\,\text{ms} = 0.25\,\text{s}$$

$$Q_0 = 114\,\text{mC}$$

$$C = 10\,\text{mF}$$

$$V(t) = \frac{114\,\text{mC}}{10\,\text{mF}} \cdot e^{-t/0.25\,\text{s}}$$

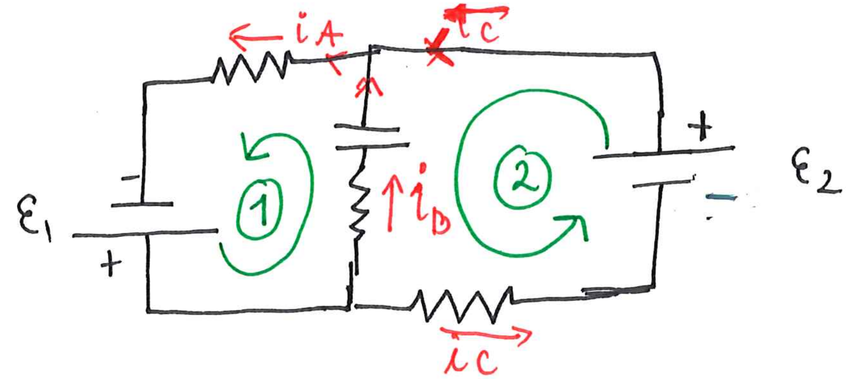
$$\boxed{V(t) = 11.4\,\text{V}\, e^{-t/0.25\text{s}}}$$

3) Calculating the ratio of charging to discharging time

$$\frac{\tau_{\text{charge}}}{\tau_{\text{discharge}}} = \frac{r \cdot \cancel{C}}{R \cdot \cancel{C}} = \frac{0.5\,\Omega}{2.5\,\Omega} = \frac{1}{50}$$

or:

$$\frac{\tau_{\text{charge}}}{\tau_{\text{discharge}}} = \frac{5\,\text{ms}}{250\,\text{ms}} = \frac{1}{50}$$



$$\varepsilon_1 > \varepsilon_2$$

$$i_A - i_C = i_B$$

$$\frac{dq}{dt} = i_B = i_A - i_C$$

1 $$\varepsilon_1 - i_B R - \frac{q(t)}{C} - i_A R = 0$$

2 $$\varepsilon_2 + \frac{q(t)}{C} + i_B R - i_C R = 0 \quad / \ominus$$

$$\varepsilon_1 - \varepsilon_2 - i_B R - \frac{q(t)}{C} - i_A R - \frac{q(t)}{C} - i_B R + i_C R = 0$$

$$\varepsilon_1 - \varepsilon_2 = 2 i_B R + 2\frac{q(t)}{C} + \underbrace{R(i_A - i_C)}_{R \cdot i_B}$$

$$\varepsilon_1 - \varepsilon_2 = 3 i_B R + \frac{2q(t)}{C}$$

$$\varepsilon_1 - \varepsilon_2 = 3\frac{dq}{dt} + \frac{2q(t)}{C} \quad /:2$$

$$\frac{\varepsilon_1 - \varepsilon_2}{2} = \frac{3}{2}\frac{dq}{dt} + \frac{q(t)}{C}$$