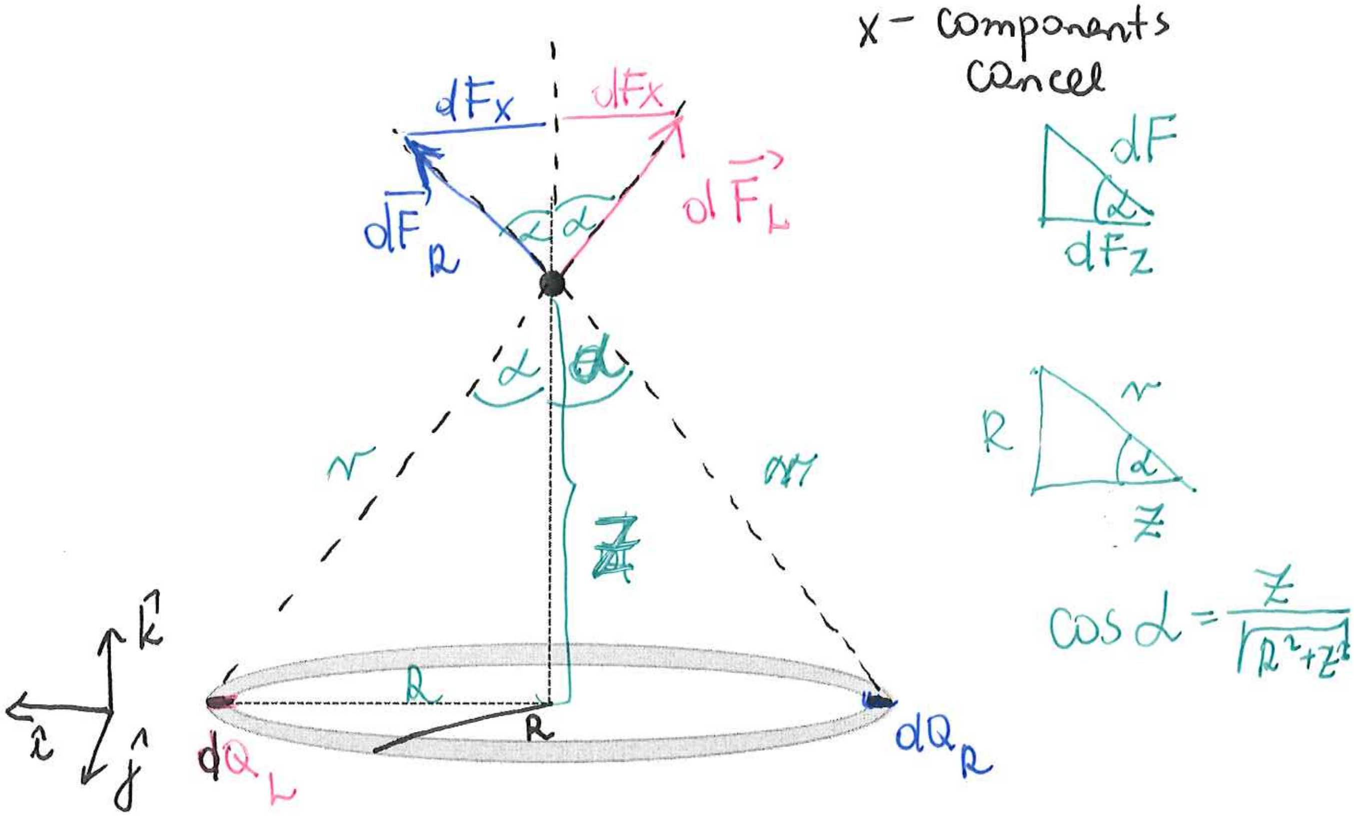
Force from a ring of charge



Step 1: Choose coordinate system

Step 2: Break the ring into small charges

Step 3. Draw line connecting dQ with q

Determine direction of the force

Step 4: Use symmetry

For each dQ there is a diametrically opposite dQ whose force on q cancels x - component (AND Y COMPONENT)

Step 5: Calculate z - component for a single charge element dQ

$$dF_z = \underbrace{dF}_{\text{from Coulomb's law}} \underbrace{\cos\alpha}_{\text{geometry}} = \frac{kq\,dQ}{(z^2+R^2)} \cdot \frac{z}{\sqrt{z^2+R^2}} = \frac{kq\,z\,dQ}{(z^2+R^2)^{3/2}}$$

Two ways to proceed:

1. $$\int dF_{net,z} = \int_0^Q \frac{k q z \, dQ}{(z^2+R^2)^{3/2}}$$

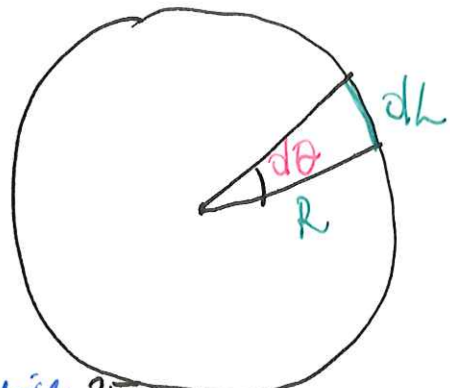
For each element dQ, z and R are the same
→ they can be treated as constant.
Can integrate dQ directly

$$F_{net,z} = \frac{k q z}{(z^2+R^2)^{3/2}} \int_0^Q dQ$$

$$F_{net,z} = \frac{k z Q q}{(z^2+R^2)^{3/2}}$$

2. Use the linear charge density, λ (systematic)

$\lambda = \frac{Q}{L}$ $\quad Q = \lambda \cdot L$



$dL = R \, d\theta$

arc length of circle

change in angle

$$dQ = \lambda \, dL = \lambda \cdot R \, d\theta$$

circle

$$\int dF_{net,z} = \int_0^{2\pi} \boxed{\frac{k q z \lambda \cdot R \, d\theta}{(z^2+R^2)^{3/2}}} = \frac{k q z \lambda R}{(z^2+R^2)^{3/2}} \int_0^{2\pi} d\theta$$

constant for each dQ

$$F_{net,z} = \frac{k q z \, \lambda R \cdot 2\pi}{(z^2+R^2)^{3/2}}$$

Q

$$= \frac{k q z Q}{(z^2+R^2)^{3/2}}$$

$\lambda = \frac{Q}{L}$

$\lambda = \frac{Q}{2\pi R}$

$\Rightarrow Q = \lambda \cdot 2\pi R$

Same as before

END OF FORCE CALCULATION.

LIMITS (SPECIAL CASES)

CASE 1: $z >> R^2$ CHARGE LOCATAED FAR FROM THE RING

$$z^2 + R^2 \approx z^2$$

$$F_{net,z} = \frac{kzqQ}{(z^2)^{3/2}} = \frac{\cancel{z} kqQ}{z^{3/2}} = \frac{kqQ}{z^2}$$ – point charge

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CASE 2: $z << R^2$ CHARGE LOCATED near the center of the ring

$$z^2 + R^2 \approx R^2$$

constant

$$F_{net,z} = \frac{kqzQ}{(R^2)^{3/2}} = \frac{kqQ}{R^3} \cdot z$$

linear function of z

NOW CONSIDER A NEGATIVE CHARGE:

Consider a force on a charge $-q$ for

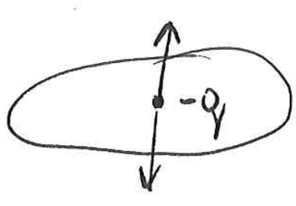
$z << R^2$

Constant

$$F_z = -\underbrace{\boxed{\frac{kqQ}{R^3}}}_{ks} \cdot z$$

$F_z = -ks \cdot z$ → this is Hook's law

A negative charge acts like it is attached to the spring



oscillates back and forth around the center of the ring

→ simple harmonic motion

SIMPLE HARMONIC MOTION

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