Group #	Student	Last Name	First Name
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(10 marks) The figure below shows a ring of charge with total charge dq (Figure.1) and a solid disk of constant charge density σ (Figure.2).The points P are located a distance z above the center of both the ring and disk. Find the electrical potential at a point P above the center of the disk.

Useful formulas: $\int \frac{xdx}{\sqrt{x^2+a^2}} = \sqrt{x^2+a^2}$ $\qquad E_z^{disk} = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{z}{\sqrt{z^2+R^2}}\right]$

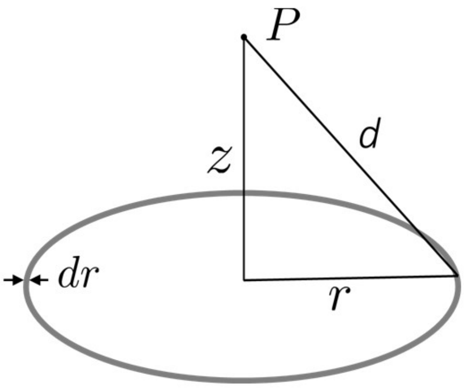

Figure 1. Ring

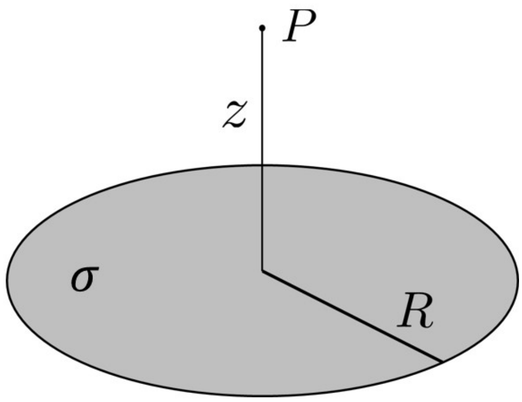

Figure 2. Disk

The parts below walk you through related questions, and the steps with which to solve this problem. Please show all work in the boxes provided and then choose the correct answer at the bottom

1. **(1 mark)** What is the distance d from some point on the ring of radius r to point P a distance z above the ring?

Using the pythagorean theorem we can compute this to be $d^2 = z^2 + r^2$. Then solving for d we have $d = \sqrt{r^2 + z^2}$

2. **(2 marks)** If you knew the potential at point P for a ring of thickness dr and charge dQ, how would go about calculating the potential at point P for a disk?

We can use the same approach that we did for the electric field due to a disk of charge. Start from the potential contributed due to a ring of charge. From this expression we can integrate over rings from radius 0 to R to sum all of these contributions and find a final expression for potential.

3. **(1 mark)** Considering the fact that all points on the ring are at the same distance from point P, write the expression for the small contribution to the potential at point P due to the ring of radius r and thickness dr shown in Figure 1?

Considering the fact that all points on the ring are at the same distance from point P, we can consider a disk to be be a point charge. Similar to the electric field, we know that an infinitesimal bit of charge dq due to a ring of charge is $dq = 2\pi\sigma r dr$. Inserting this definition for charge, and the distance calculated above, we can find the potential contributed by a ring of charge.

$$dV = \frac{1}{4\pi\epsilon_o}\frac{dq}{d} = \frac{1}{4\pi\epsilon_o}\frac{2\pi\sigma r dr}{\sqrt{r^2+z^2}} = \frac{\sigma}{2\epsilon_o}\frac{r dr}{\sqrt{r^2+z^2}} \tag{1}$$

4. **(2 marks)**What is the total potential at point P due to the disk (Figure.2)? State explicitly what the limits of integration are and evaluate the integral.

Since we are summing all of the rings of radii from 0 to R that form the disk we have the expression

$$V = \int_0^R \frac{\sigma}{2\epsilon_o} \frac{rdr}{\sqrt{r^2 + z^2}}. \tag{2}$$

Conveniently, this matched with an integral given on the activity(;P) so we can easily compute the potential.

$$V = \frac{\sigma}{2\epsilon_o}(\sqrt{R^2 + z^2} - z). \tag{3}$$

5. **(1 mark)** Is there a direction associated with the electric potential in question 4? Why or why not?

The electric potential has no direction because it is a scalar field. For every point in space the potential can be represented by a number not a vector.

6. **(2 marks)** Verify your expression for the potential of the disk (question 4) by calculating $E_z = -\frac{\partial V}{\partial z}$. Does this correspond with the electric field produced by a disk that you would expect?

As stated above $E = -\nabla V$. We can use this to check that we have the right answer. Lets compute $E_z = -\frac{\partial V}{\partial z}$ since we know that partials of our potential in the x, and y directions would be zero at point P.

$$E_z = -\frac{\sigma}{2\sigma\epsilon_o}\frac{\partial}{\partial z}(\sqrt{R^2 + z^2} - z) = \frac{\sigma}{2\epsilon_o}(1 - \frac{z}{\sqrt{R^2 + z^2}}). \tag{4}$$

This matches the expression given earlier in the activity for the electric field from a charged disk, so our expression for potential must be correct. That means that the correct answer below must be D.

(1 mark for the correct answer) What is the electrical potential at a point P due to the disk?

A. $V = \frac{\sigma}{2\epsilon_o}\left[1 - \frac{z}{\sqrt{z^2+R^2}}\right]$ B. $V = \frac{\sigma}{2\epsilon_o}\left[1 - \frac{z}{\sqrt{z^2+R^2}}\right]\hat{k}$ C. $V = \frac{\sigma}{2\epsilon_o}\left[\sqrt{z^2 + R^2} - z\right]\hat{k}$ D. $V = \frac{\sigma}{2\epsilon_o}\left[\sqrt{z^2 + R^2} - z\right]$

D is correct.