Electricity and Magnetism

- Physics 259 – L02
- Lecture 7

Section 21.1-3

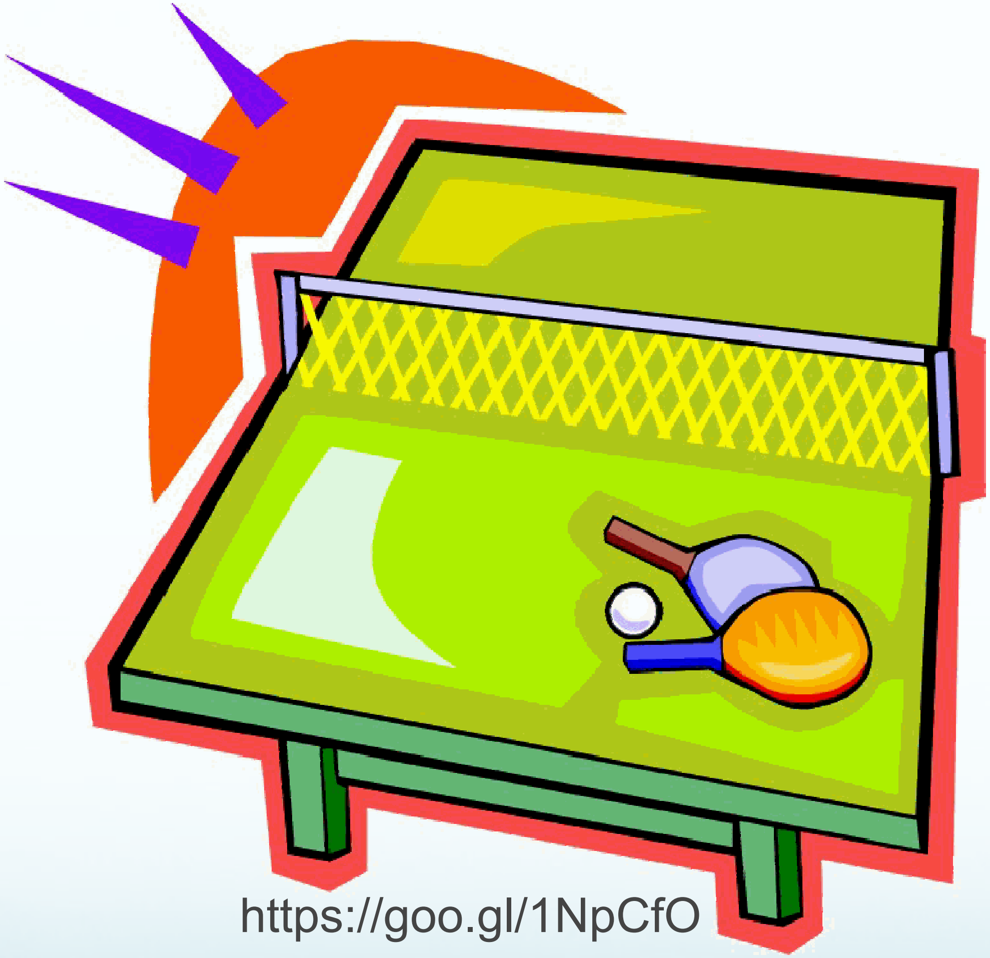
Last time

- Charges and Force Between Charges
- Conductors and Insulators
- Van De Graaff Generator Experiment
- Solve Class Activity Question
- Coulomb's Law

This time

- Examples for Coulomb's law

Let's play Ping Pong



https://goo.gl/1NpCfO

Let's play electric ping pong

physics works

Coulomb's Law

$$\boxed{F_{1\,on\,2} = F_{2\,on\,1} = K\frac{|q_1||q_2|}{r^2}}$$

K = electrostatic constant

$$K = 8.99\times10^{9}\,\frac{N\cdot m^2}{C^2}$$

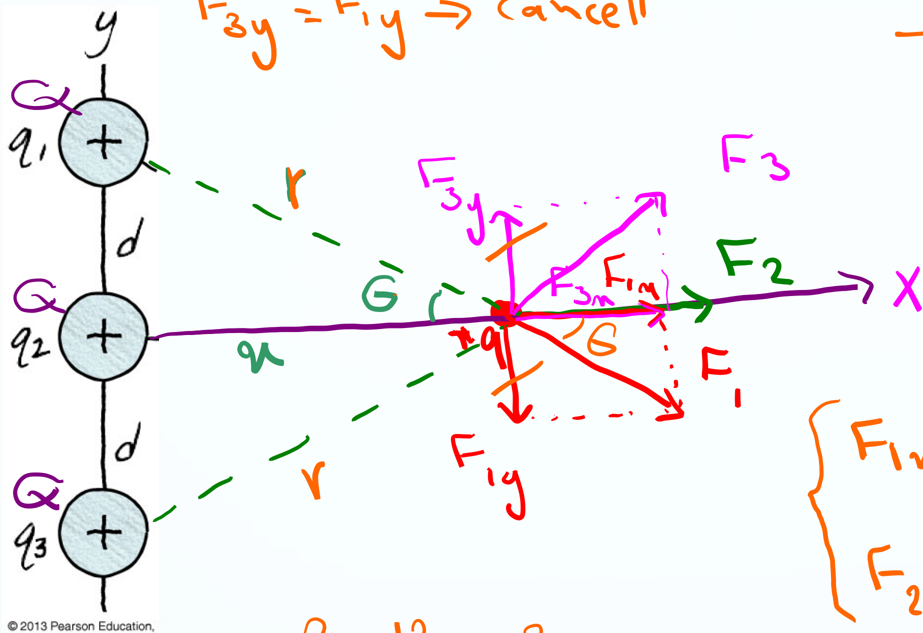
$$\boxed{F_{1\,on\,2} = F_{2\,on\,1} = \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_2|}{r^2}}$$

ε_0 = permittivity of free space

$$\varepsilon_0 = \frac{1}{4\pi K} = 8.85\times10^{12}\,\frac{C^2}{N\cdot m^2}$$

Example #1: Three point charges

$F_{3y} = F_{1y} \Rightarrow$ cancell

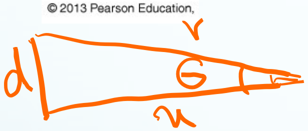


$\vec{F}_{net} = F_x \hat{i} + \cancel{F_y \hat{j}}^{\,0}$

$F_x = F_{1x} + F_{2x} + F_{3x} = 2F_{1x} + F_{2x}$

$F_y = 0$

$$\begin{cases} F_{1x} = F_{3x} = k \dfrac{Qq}{r^2} \cos\theta \\ F_{2x} = F_2 = k \dfrac{Qq}{x^2} \end{cases}$$



$r^2 = d^2 + x^2$

$\cos\theta = \dfrac{x}{r} = \dfrac{x}{\sqrt{d^2+x^2}}$

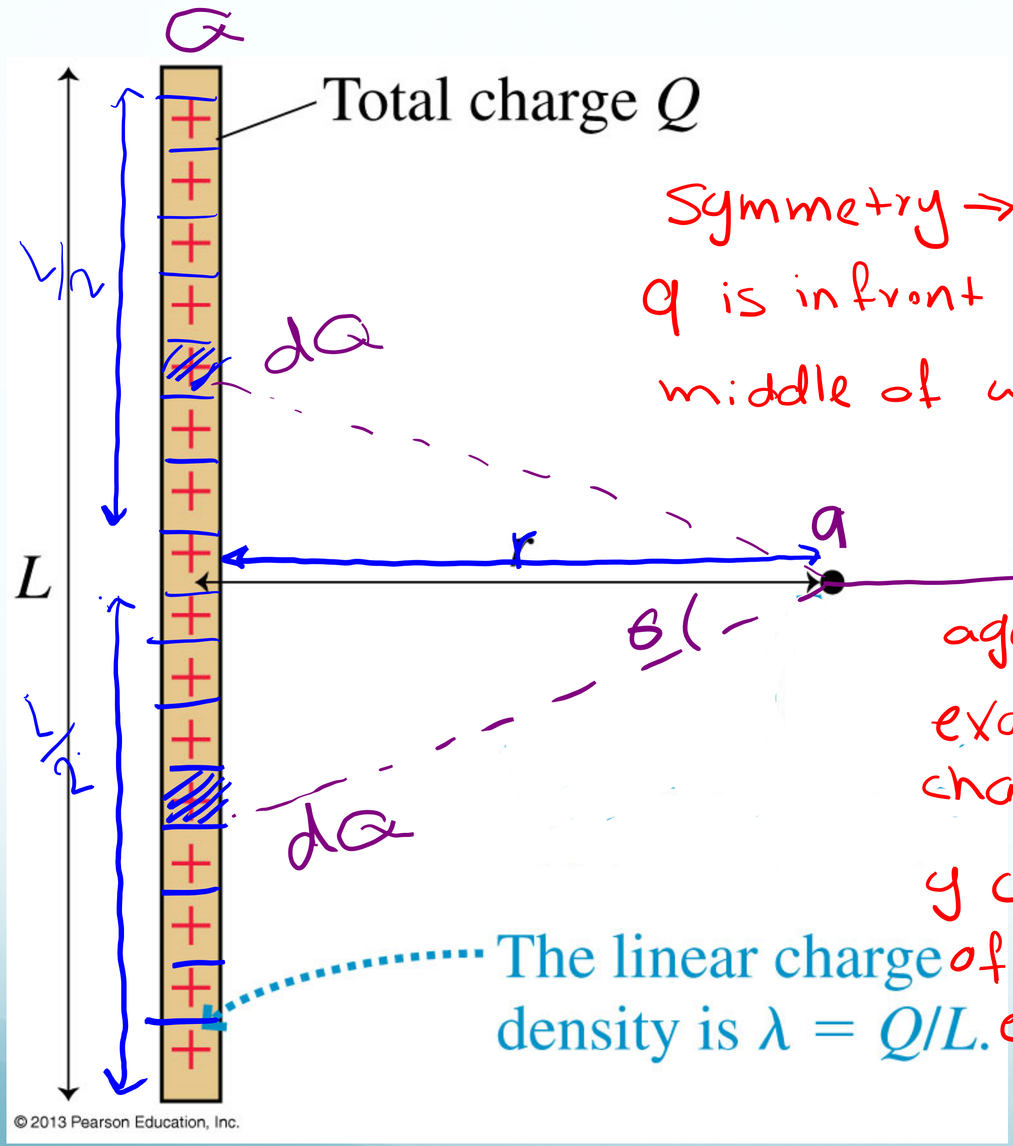
$$F_{3x} = F_{1x} = \underbrace{k}_{\frac{1}{4\pi\varepsilon_0}} \frac{Qq}{d^2+x^2} \cdot \frac{x}{\sqrt{d^2+x^2}} = \frac{kQqx}{(d^2+x^2)^{3/2}}$$

$$(F_{net})_x = 2(F_1)_x + (F_2)_x = \frac{qQ}{4\pi\varepsilon_o}[\overbrace{\frac{1}{x^2}}^{F_{2x}} + \overbrace{\frac{2x}{(x^2+d^2)^{3/2}}}^{2 \times F_{1x}}]$$

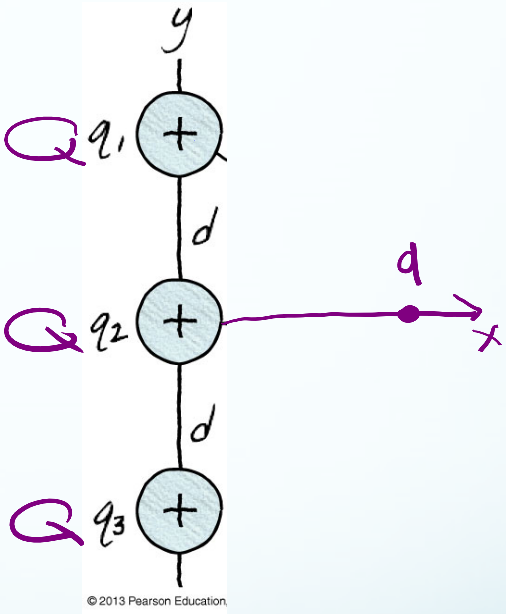
$$\boxed{\vec{F}_{net} = \frac{qQ}{4\pi\varepsilon_o}[\frac{1}{x^2} + \frac{2x}{(x^2+d^2)^{3/2}}]\hat{i}}$$

Example #2: Force from a line of charge

wire with
total charge Q
total length L



Symmetry →
q is in front of the
middle of wire
↓
again, like the
example of 3
charges, the
y components
of forces cancel
eachother



$\vec{F}_{net} = F_x \hat{i} + F_y \hat{j}$, $F_x = F_{1x} + F_{2x} + F_{3x} + \dots$, $F_y = 0$

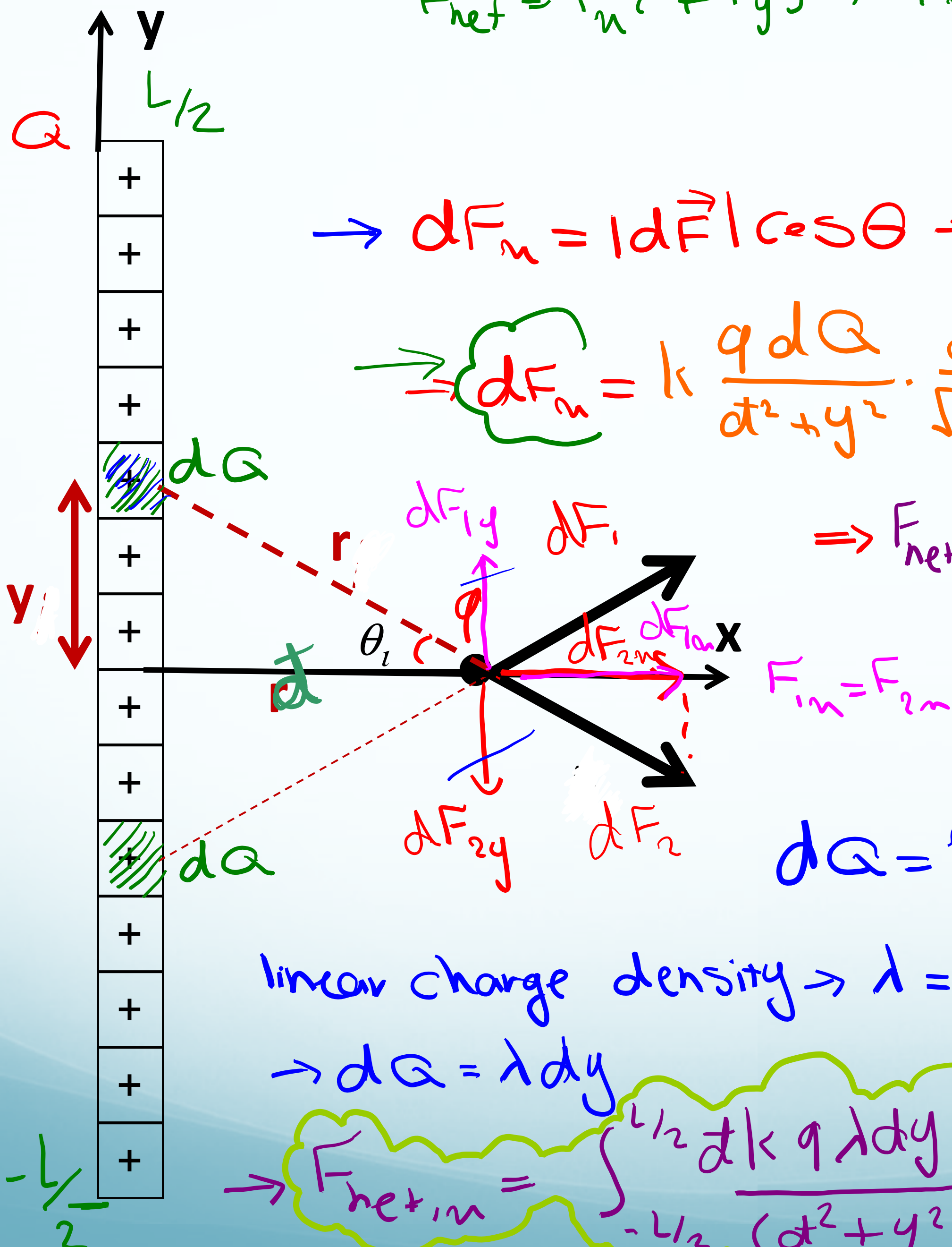
$\downarrow$ again symmetry

$\cos\theta = \frac{d}{r} = \frac{d}{\sqrt{d^2+y^2}}$

$\rightarrow dF_x = |d\vec{F}|\cos\theta \rightarrow |d\vec{F}| = k\frac{q\,dQ}{r^2}$

$r = \sqrt{d^2+y^2}$

$\rightarrow dF_x = k\frac{q\,dQ}{d^2+y^2}\cdot\frac{d}{\sqrt{d^2+y^2}} = k\frac{d\,q\,dQ}{(d^2+y^2)^{3/2}}$



$\Rightarrow F_{net_x} = \int_{-L/2}^{L/2} dF_x = \int k\frac{d\,q\,dQ}{(d^2+y^2)^{3/2}}$

$= k\,d\,q\int\frac{dQ}{(d^2+y^2)^{3/2}}$ ✩

$dQ = ?$

linear charge density $\Rightarrow \lambda = \frac{Q}{L} \rightarrow Q = \lambda L \Rightarrow dQ = \lambda\, \underbrace{dL}_{dy}$

$\rightarrow dQ = \lambda dy$

$dQ = \lambda dy$ ✩

$\rightarrow F_{net,x} = \int_{-L/2}^{L/2}\frac{d\,k\,q\,\lambda\,dy}{(d^2+y^2)^{3/2}}$

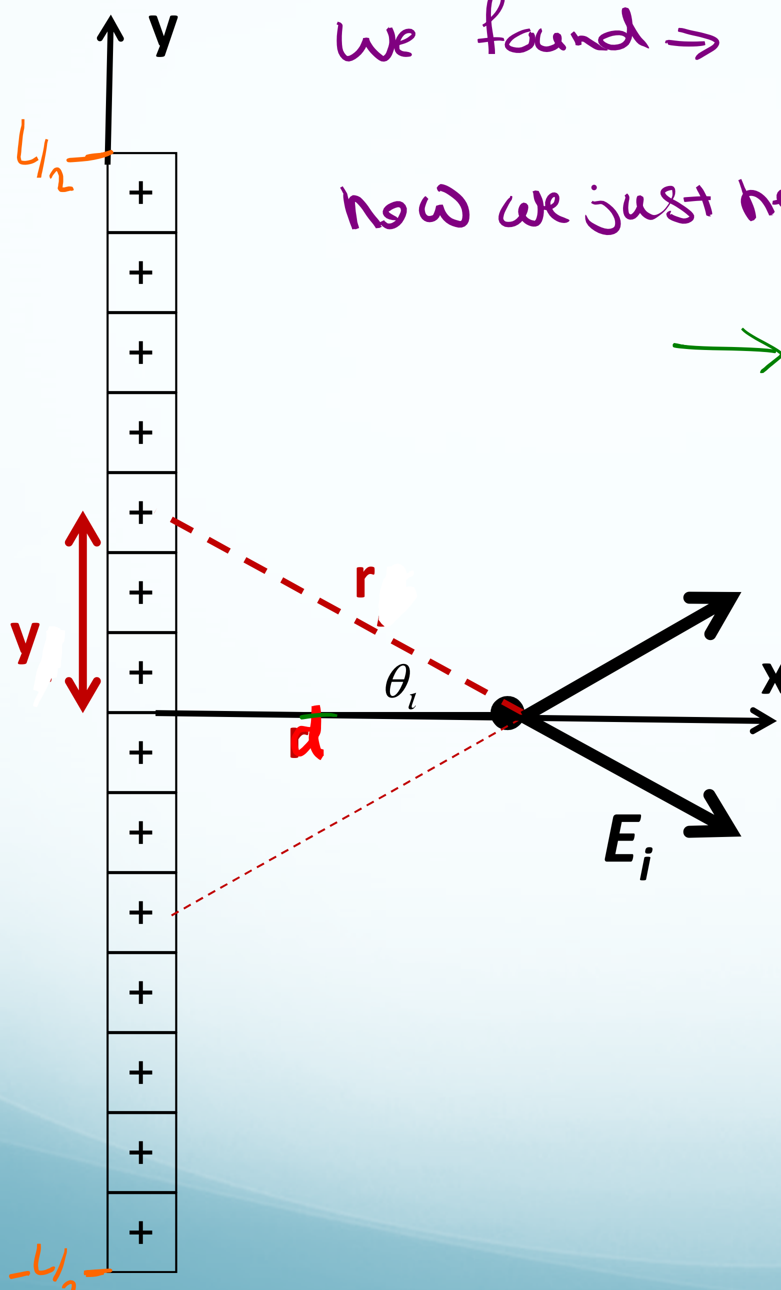
Integration is Just Continous Summation

$a_1 + a_2 + a_3 + \cdots$

Sum $(a_1 + a_2 + \cdots)$ Sum

$$\sum_i a_i = a_1 + a_2 + a_3 \cdots \qquad \Sigma$$

$S \rightarrow \int \qquad \int$



we found => $F_{net,x} = \int_{-L/2}^{L/2} \frac{d\,k\,q\,\lambda\,dy}{(d^2+y^2)^{3/2}}$

now we just need to solve the integral =>

$$\longrightarrow \vec{F}_{net} = \frac{kQq}{d\sqrt{(L/2)^2+d^2}}\,\hat{i}$$

limiting cases =>

1 $d \gg L \Rightarrow d^2 + \cancel{(L/2)^2} \simeq d^2$ 0

$\vec{F}_{net} = \vec{F}_{net} = k\,\frac{Qq}{d^2}$

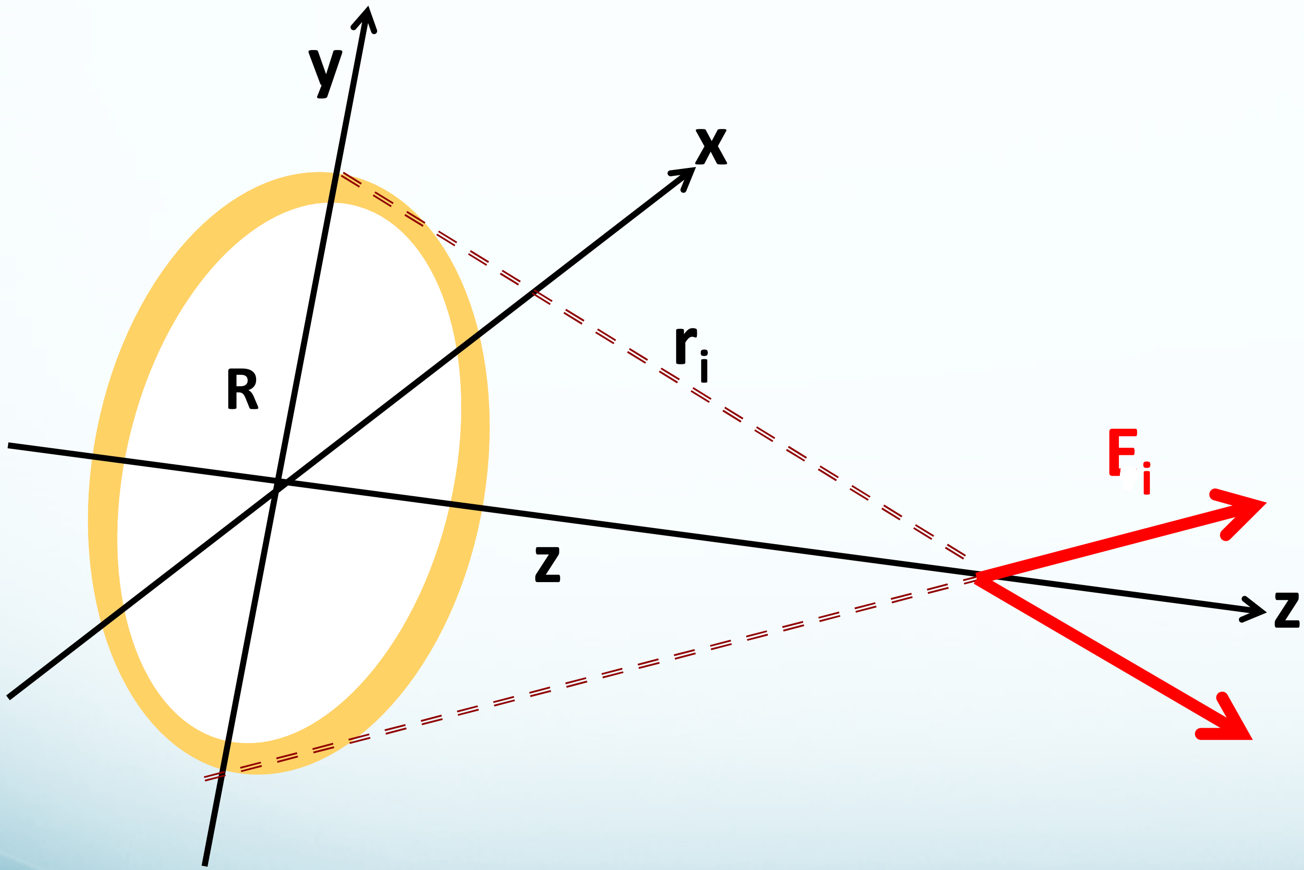
2 $d \ll L \Rightarrow \cancel{d^2} + (\frac{L}{2})^2 \simeq (\frac{L}{2})^2$ 0

$\vec{F}_{net} = k\,\frac{qQ}{L/2} = k\,\frac{2qQ}{L}$

infinite long wire

Example #3: Force from a ring of charge

next week



Why should we care? Applications:

Ring antenna (very directional)

Photo taken from https://en.wikipedia.org/wiki/Loop_antenna

This section we talked about:

Chapter 21.1-3: Examples

See you on Friday