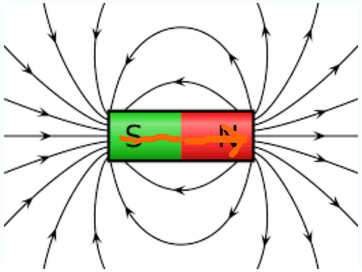
Electricity and Magnetism

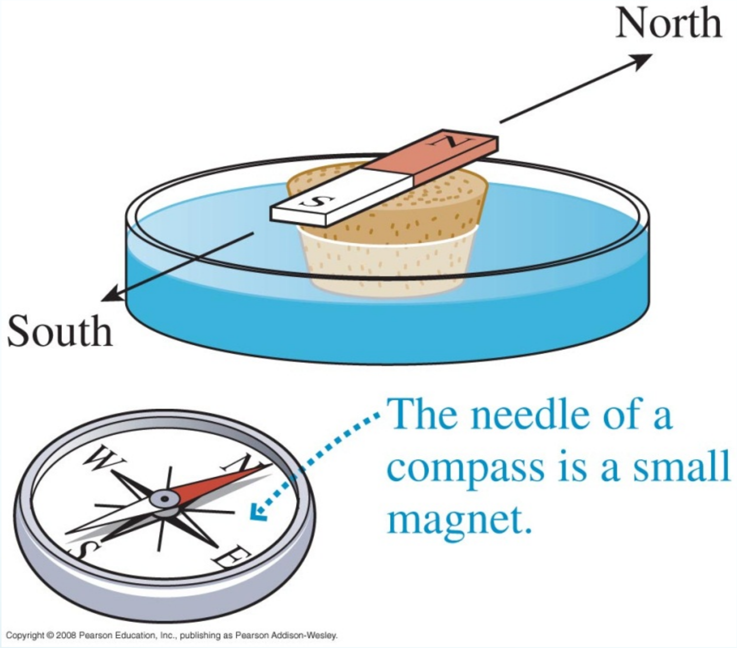
- Physics 259 – L02
- Lecture 37

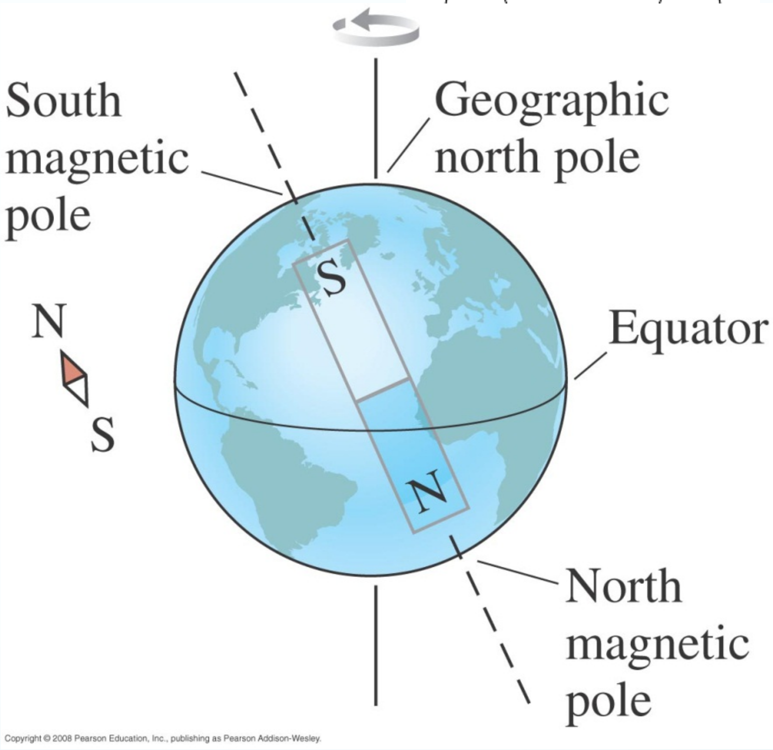
Chapter 28: Magnetic fields

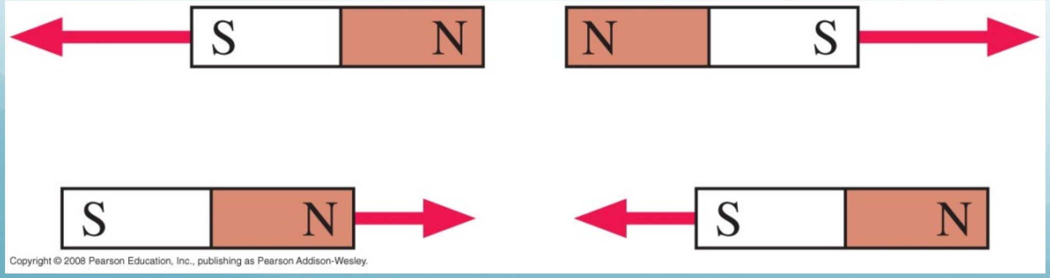
28.1: Magnetic fields

Magnetism







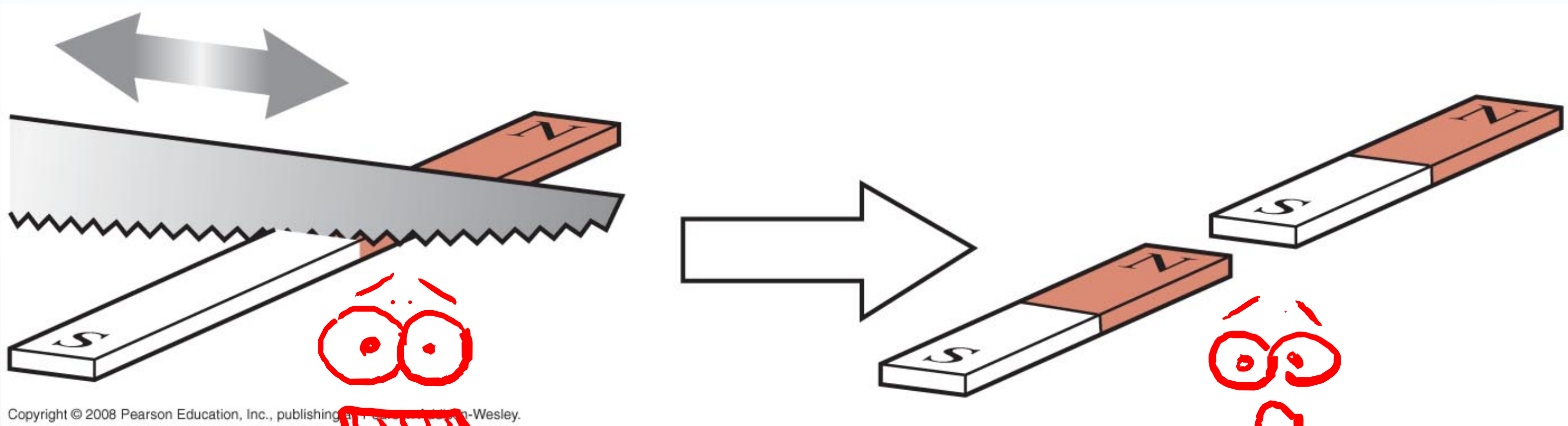


Like poles repel.

Unlike poles attract.

Magnetism is not the same as electricity!!

For example, cutting a magnet does not create one north-pole piece and one south-pole piece.



again $\boxed{N|S}$

Magnetic monopoles do not seem to exist: ~~N~~ ~~S~~

We cannot have a north pole without a south pole. till now!

Except…

M. W. Ray et al., Nature **505**, 657 (2014); doi:10.1038/nature12954

Observation of Dirac Monopoles in a Synthetic Magnetic Field

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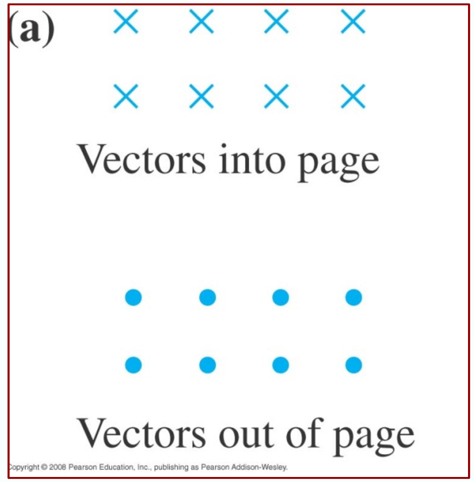
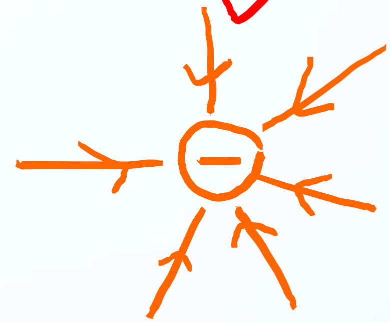
Abstract

Magnetic monopoles — particles that behave as isolated north or south magnetic poles — have been the subject of speculation since the first detailed observations of magnetism several hundred years ago[1]. Numerous theoretical investigations and hitherto unsuccessful experimental searches[2] have followed Dirac's 1931 development of a theory of monopoles consistent with both quantum mechanics and the gauge invariance of the electromagnetic field[3]. The existence of even a single Dirac

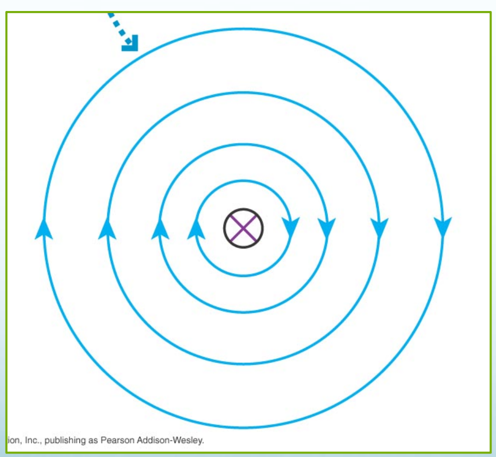
Magnetic fields are necessarily 3 dimensional

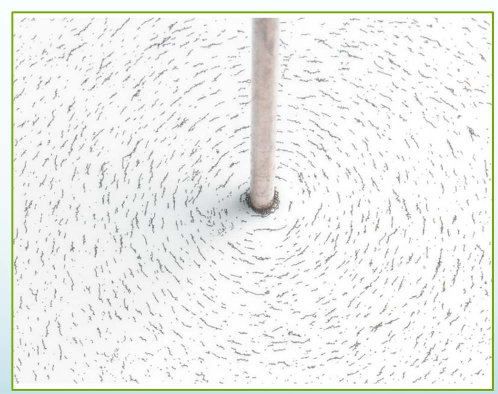


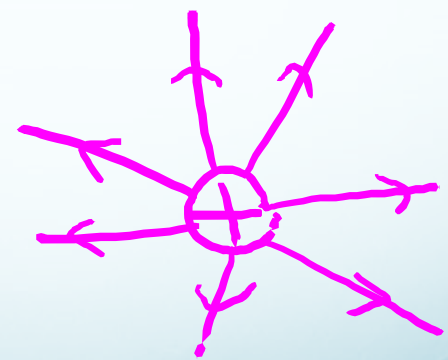
electric charge



Magnetic field lines never start or stop anywhere



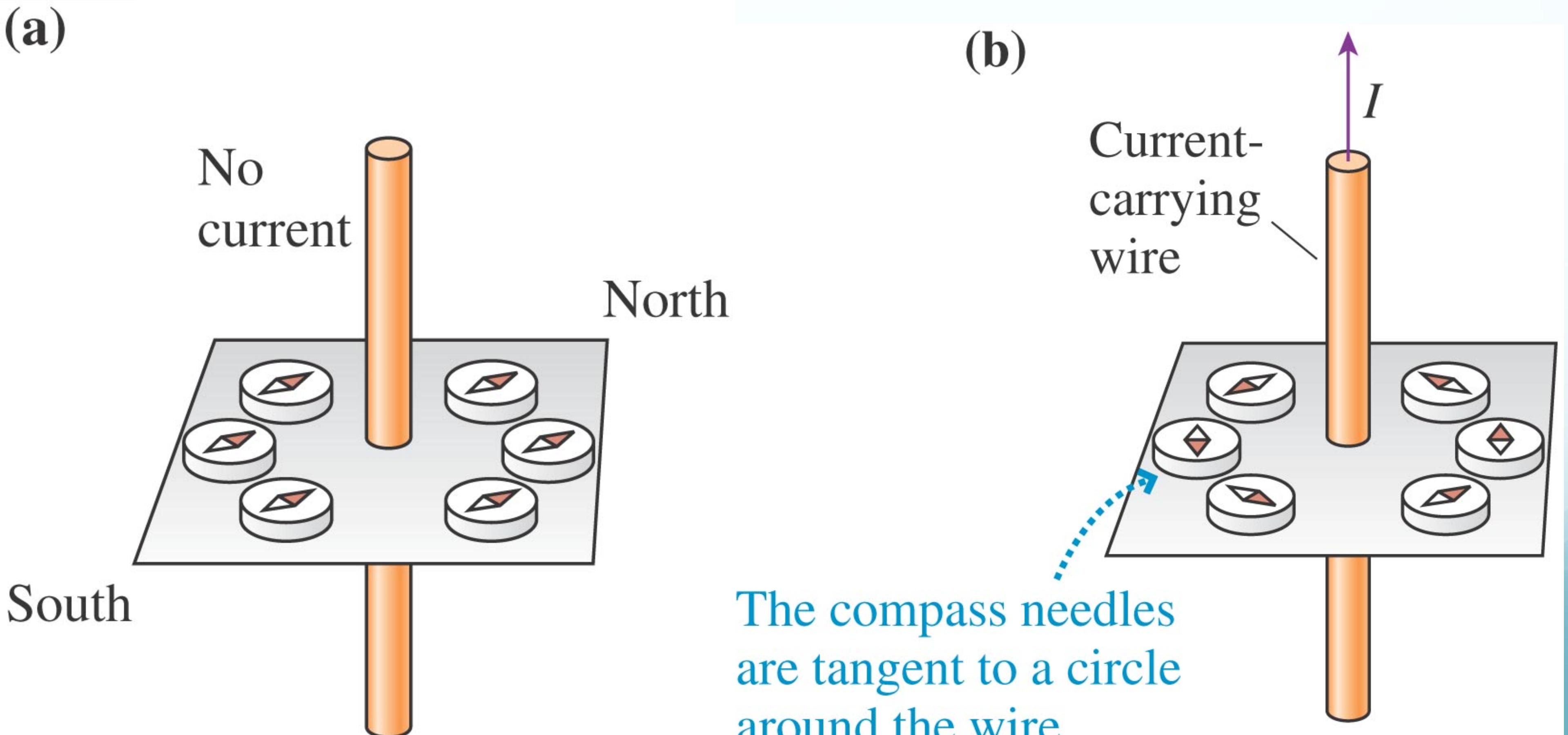




No magnetic charge till now!

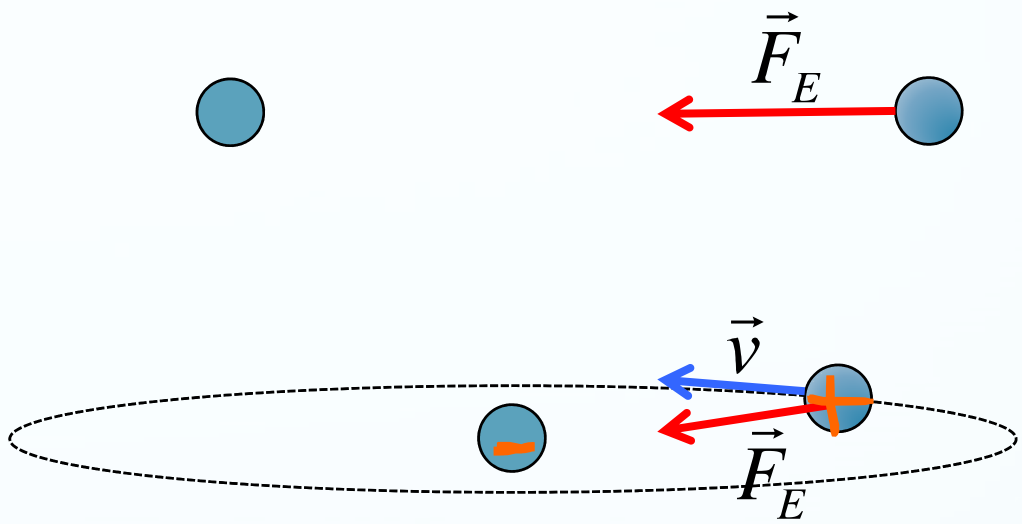
Oersted was doing a physics lecture in 1819

Response of compass needle to a current in a straight wire



Electric Force on Charges

Electric force acts on a charge regardless of its motion.



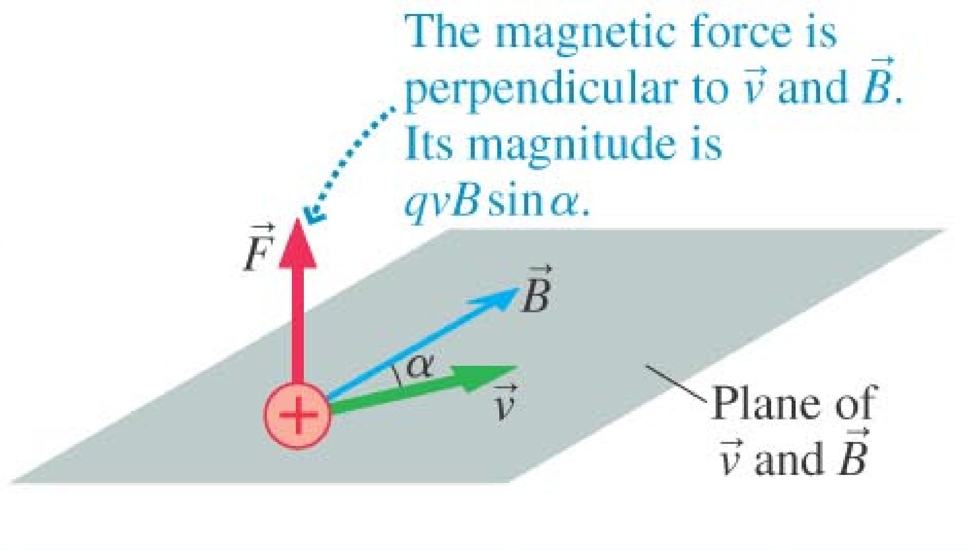
$$\vec{F}_E = q\vec{E}$$

Magnitude: $F_E = qE$

Direction: direction of $\vec{E}$

Magnetic Force on Charges

Magnetic force acts only on a moving charge.



$$\vec{F}_E = q\vec{E}$$

$$\vec{F}_B = q\,\vec{v}\times\vec{B}$$

charge moving in B

Magnitude: $F_B = qvB\sin\alpha$

Direction: RH rule

The Tesla

- The SI unit of magnetic field is the Tesla.
- 1 tesla=1 N/A m

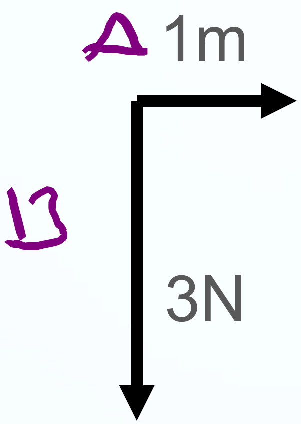
Field location	Field strength (T)
Surface of the earth	5×10^{-5}
Refrigerator magnet	5×10^{-3}
Laboratory magnet	0.1 to 1

The Gauss

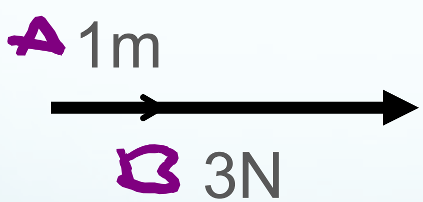
- More useful size: 10000 G = 1 T
- Earth's magnetic field ~ 0.5 G

Vector

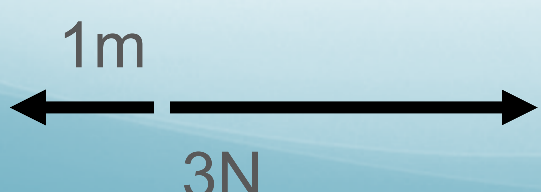
The magnitude $$\vec{A}\times\vec{B} = \left|\vec{A}\right|\left|\vec{B}\right|\sin\theta$$



$|\vec{A}\times\vec{B}| = ? = |3||1|\sin(90) = 3$



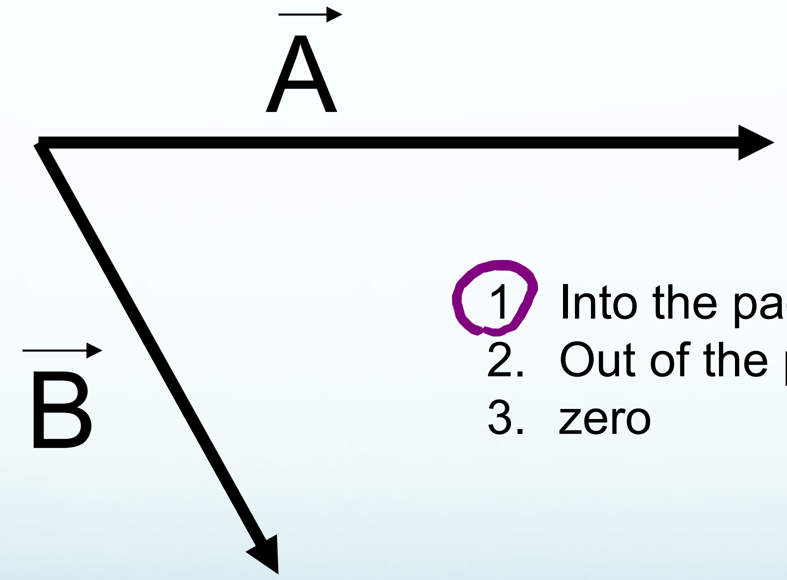
$|\vec{A}\times\vec{B}| = |A||B|\sin(0) = 0$



$|\vec{A}\times\vec{B}| = 0$

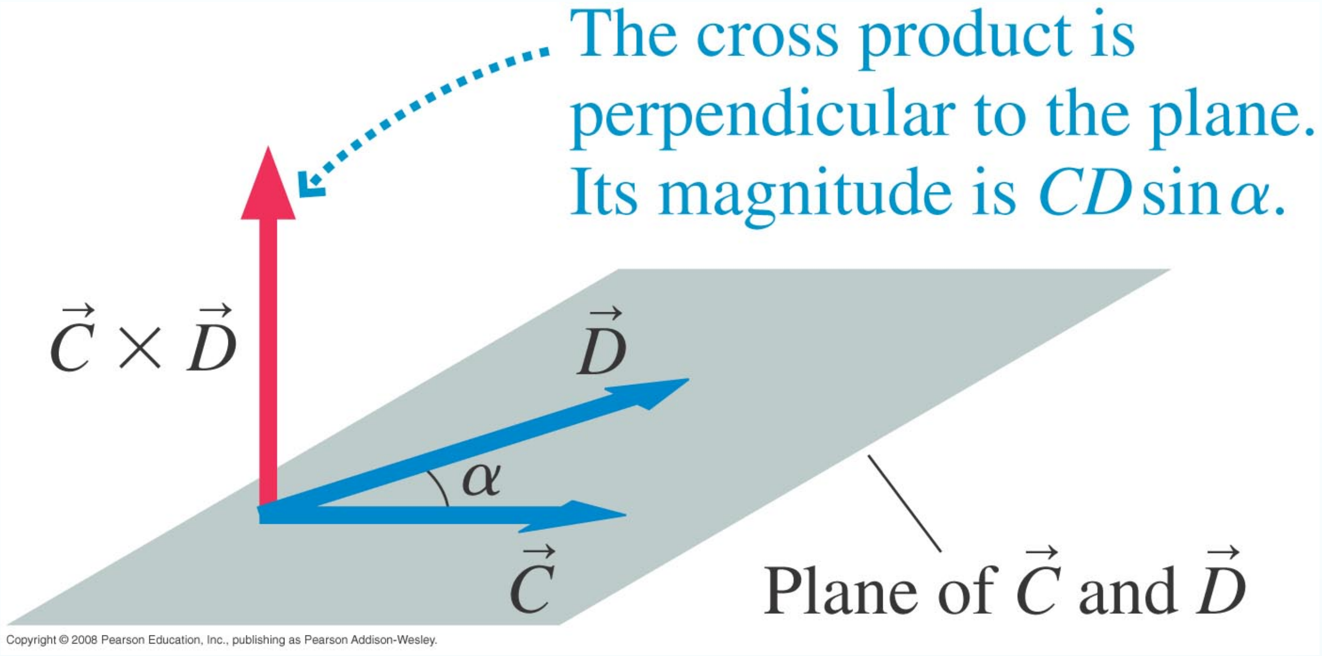
The direction?

$$\vec{A} \times \vec{B} = \vec{C}$$



1. Into the page.
2. Out of the page
3. zero

The Vector Cross Product



Point the fingers of your right hand along the first vector in the cross product (vector C), then curl them so they point along the second vector (vector D). Your thumb gives the direction of the cross product.

Cross product vs regular product

Regular/dot product	**Cross product**
Distributive	Distributive
$\vec{B}\cdot\left(\vec{C}+\vec{D}\right)=\vec{B}\cdot\vec{C}+\vec{B}\cdot\vec{D}$	$\vec{B}\times\left(\vec{C}+\vec{D}\right)=\vec{B}\times\vec{C}+\vec{B}\times\vec{D}$
Commutative	Anticommutative
$CD=DC$ $\vec{C}\cdot\vec{D}=\vec{D}\cdot\vec{C}$	$\vec{C}\times\vec{D}=-\vec{D}\times\vec{C}$
Associative	Non-Associative
$B\left(CD\right)=\left(BC\right)D$	$\vec{B}\times\left(\vec{C}\times\vec{D}\right)\neq\left(\vec{B}\times\vec{C}\right)\times\vec{D}$

Appendix: Unit vector notation

The cross product becomes easy to deal with when using unit vector notation

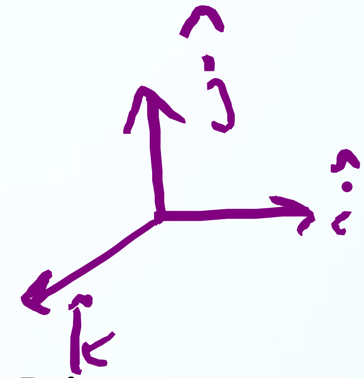
$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

$$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$$

$$\hat{i}\times\hat{j} = \hat{k}$$

$$\hat{j}\times\hat{k} = \hat{i}$$

$$\hat{k}\times\hat{i} = \hat{j}$$



Now let's see what the cross product between A and B is:

$$\vec{C} = \vec{A}\times\vec{B}$$

$$\vec{C} = \left(A_x\hat{i} + A_y\hat{j} + A_z\hat{k}\right)\times\left(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}\right)$$

$$\vec{C} = \left(A_yB_z - A_zB_y\right)\hat{i} + \left(A_zB_x - A_xB_z\right)\hat{j} + \left(A_xB_y - A_yB_x\right)\hat{k}$$

Appendix: Another way to think about it

Start with the two vectors in component form

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k} \qquad \vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$$

The cross product is given by the determinant of the following matrix:

$$\vec{C} = \vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$\vec{C} = \left(A_yB_z - A_zB_y\right)\hat{i} + \left(A_zB_x - A_xB_z\right)\hat{j} + \left(A_xB_y - A_yB_x\right)\hat{k}$$

Top Hat Question

A charged particle q enters a region with a constant B-field pointing into the page as shown. If the particle follows the path from **a** to **b** as shown

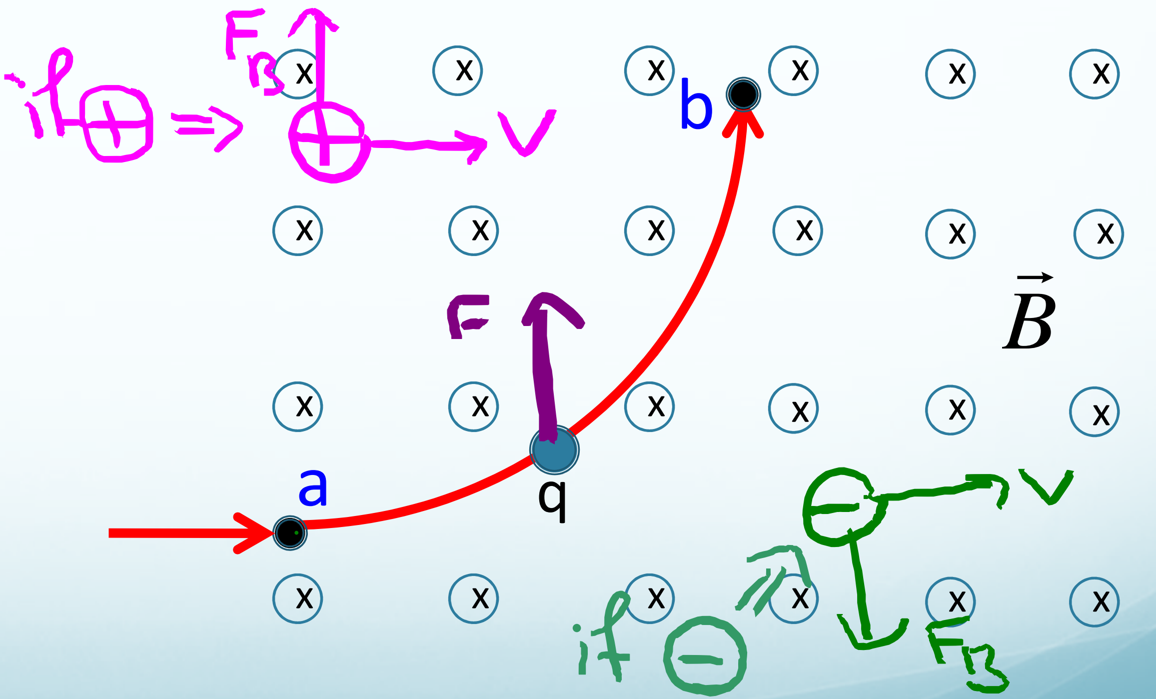
$$\boxed{\vec{F} = q\vec{v} \times \vec{B}}$$

What is the sign of q?

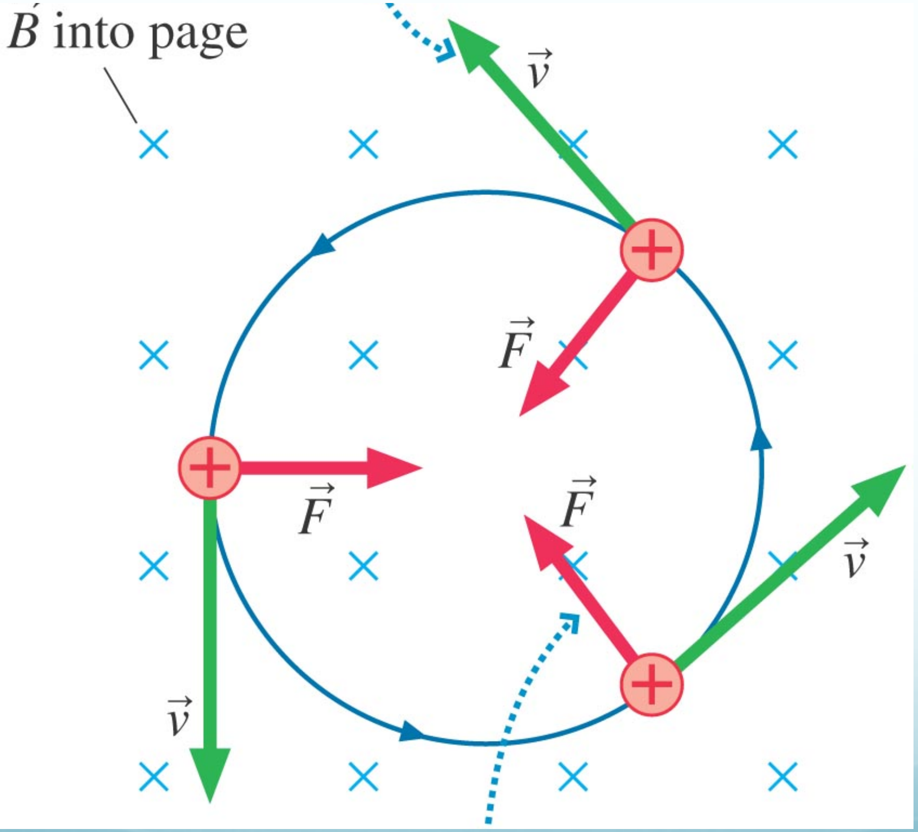
A. Positive

A. Negative

A. Not enough info



28.4: A circulating charged particle



Charged particles in uniform magnetic fields undergo **uniform circular motion**.

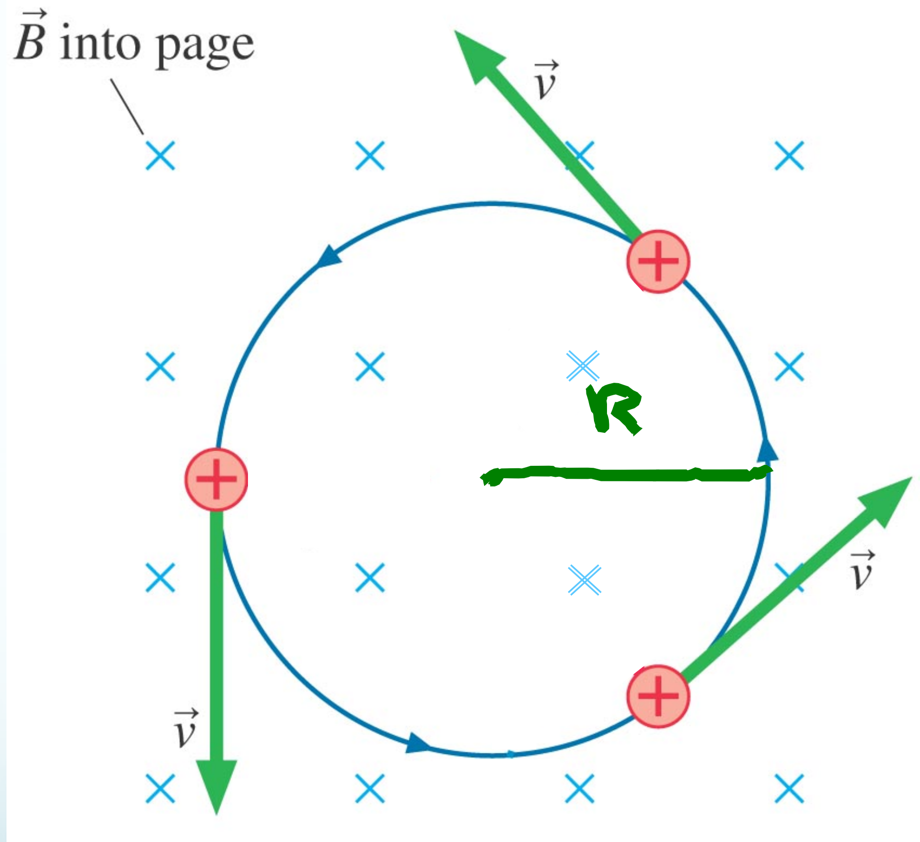
The radius of the circle depends on how fast the particle is moving:

$$\vec{F}_B = q\vec{v} \times \vec{B}$$

$$\left|\vec{F}_B\right| = |q| vB \sin\alpha = |q| vB$$

The magnetic force is the **net force**

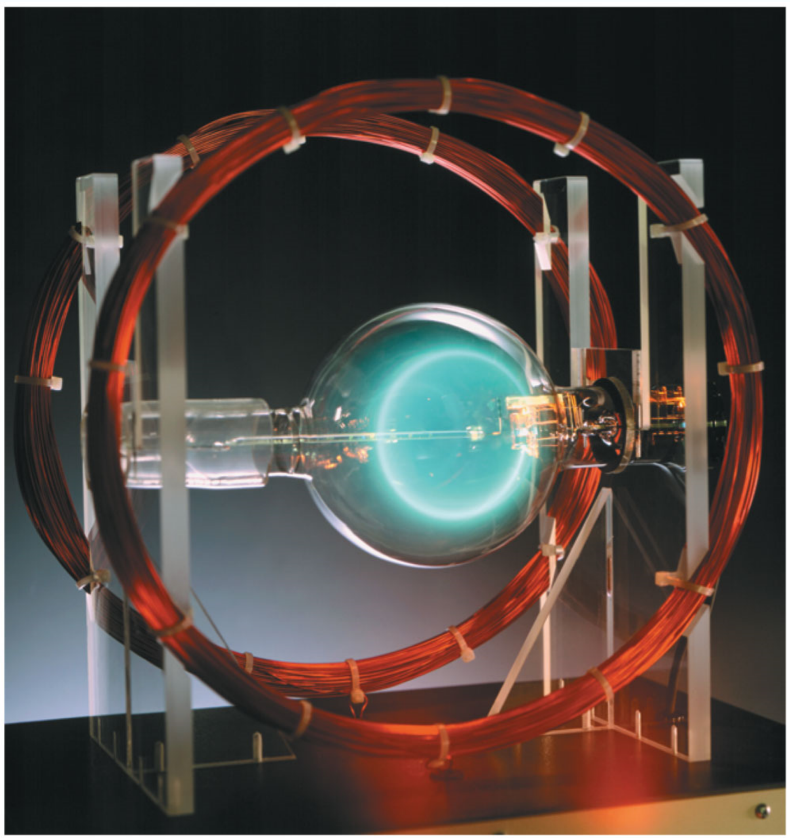
$$\left|\vec{F}_B\right| = m\frac{v^2}{R}$$

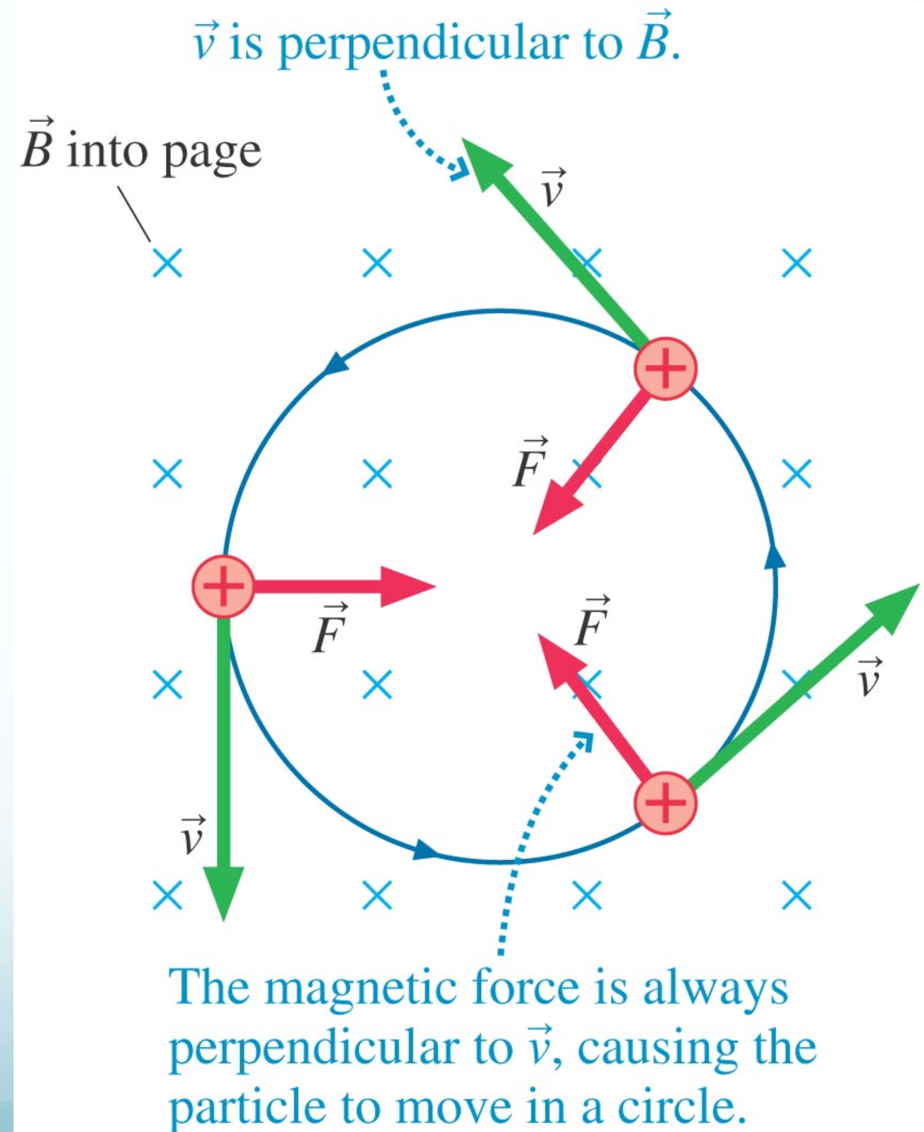


Cyclotron Motion

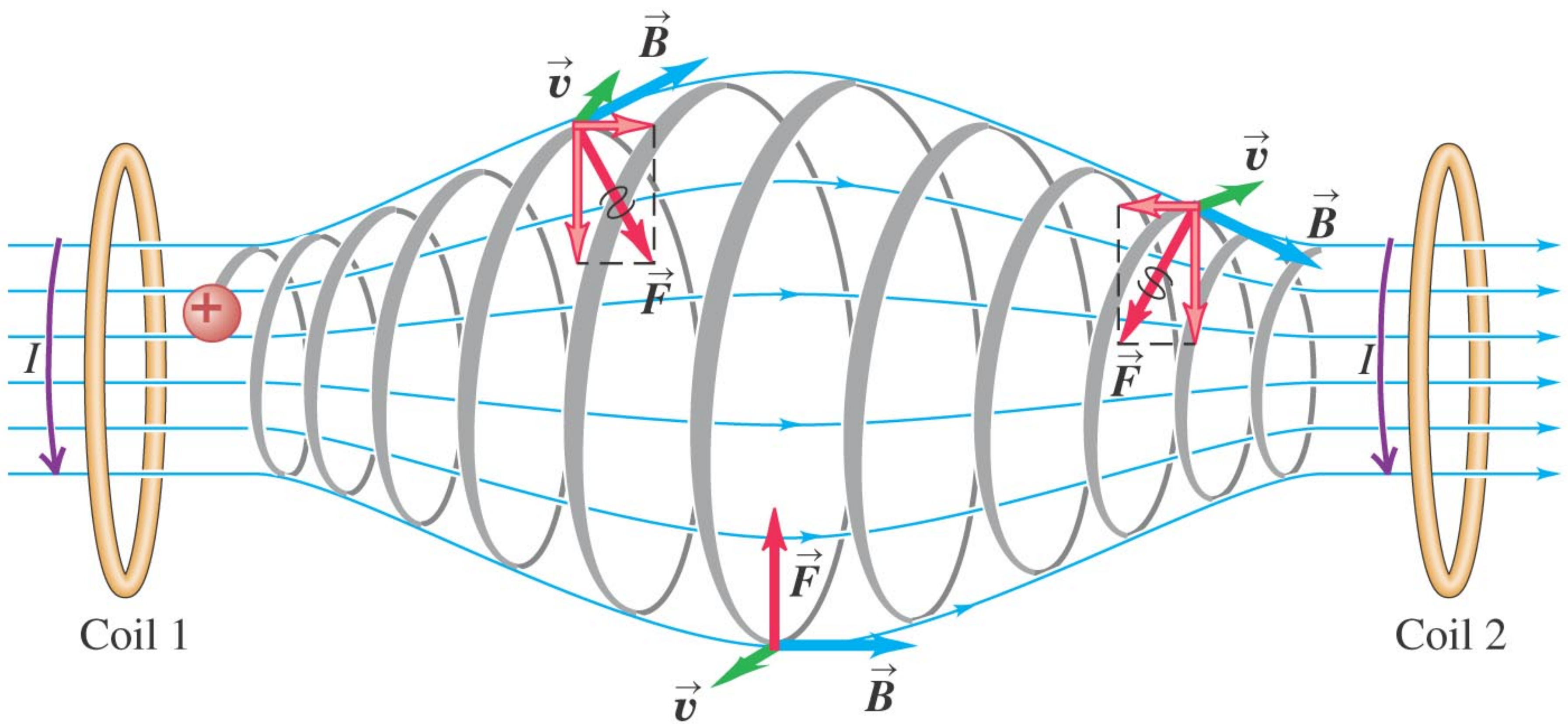
$$\left|\vec{F}_B\right| = |q|\cancel{v}B = m\frac{v^{\cancel{2}}}{R}$$

$$\boxed{R = \frac{mv}{|q|B}}$$



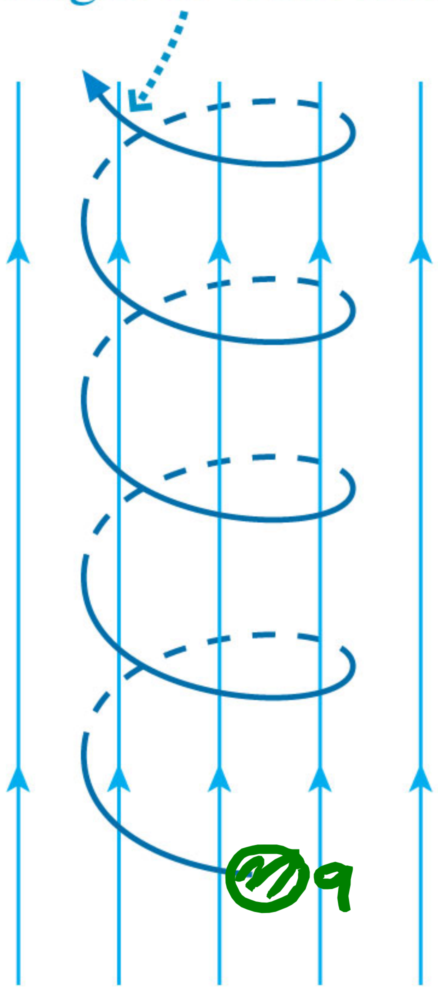


Magnetic Ion Trap

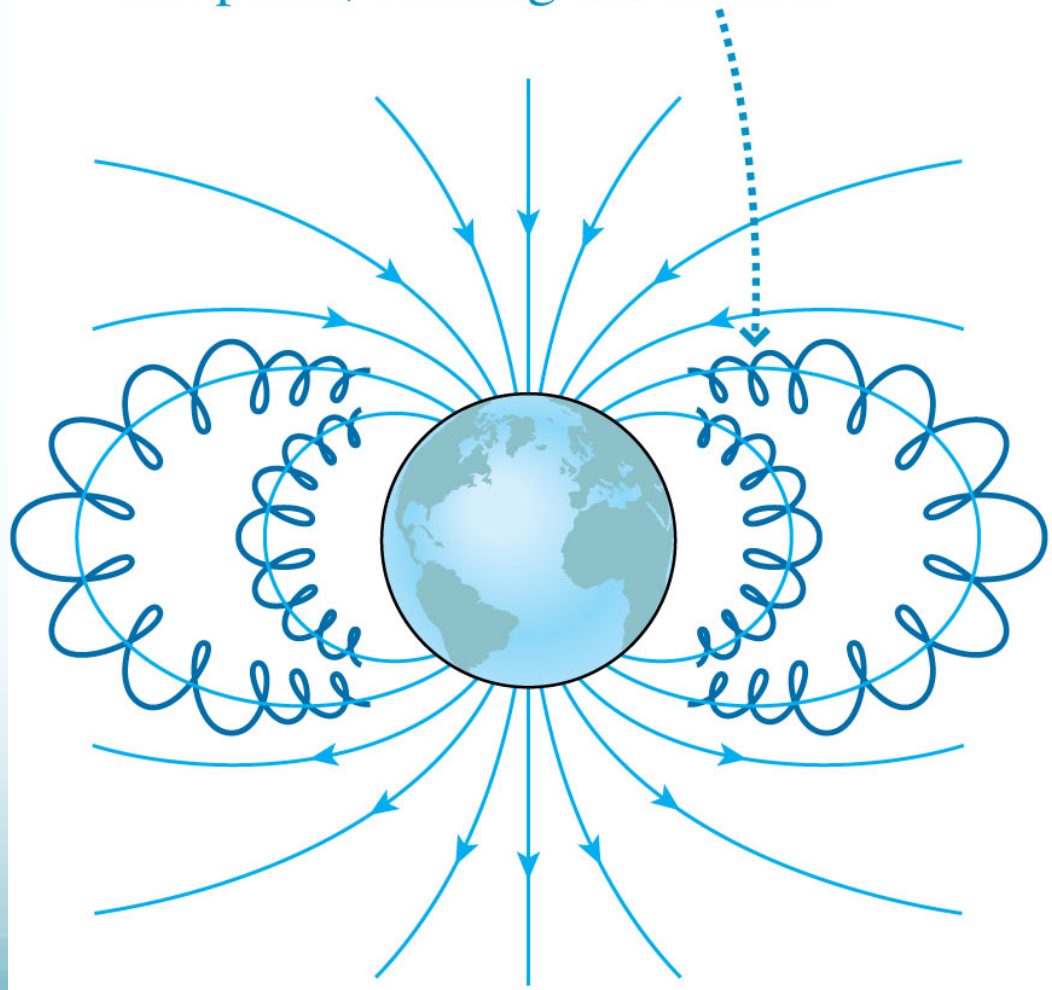


Earth's Van Allen belt
(aurora borealis/australis)

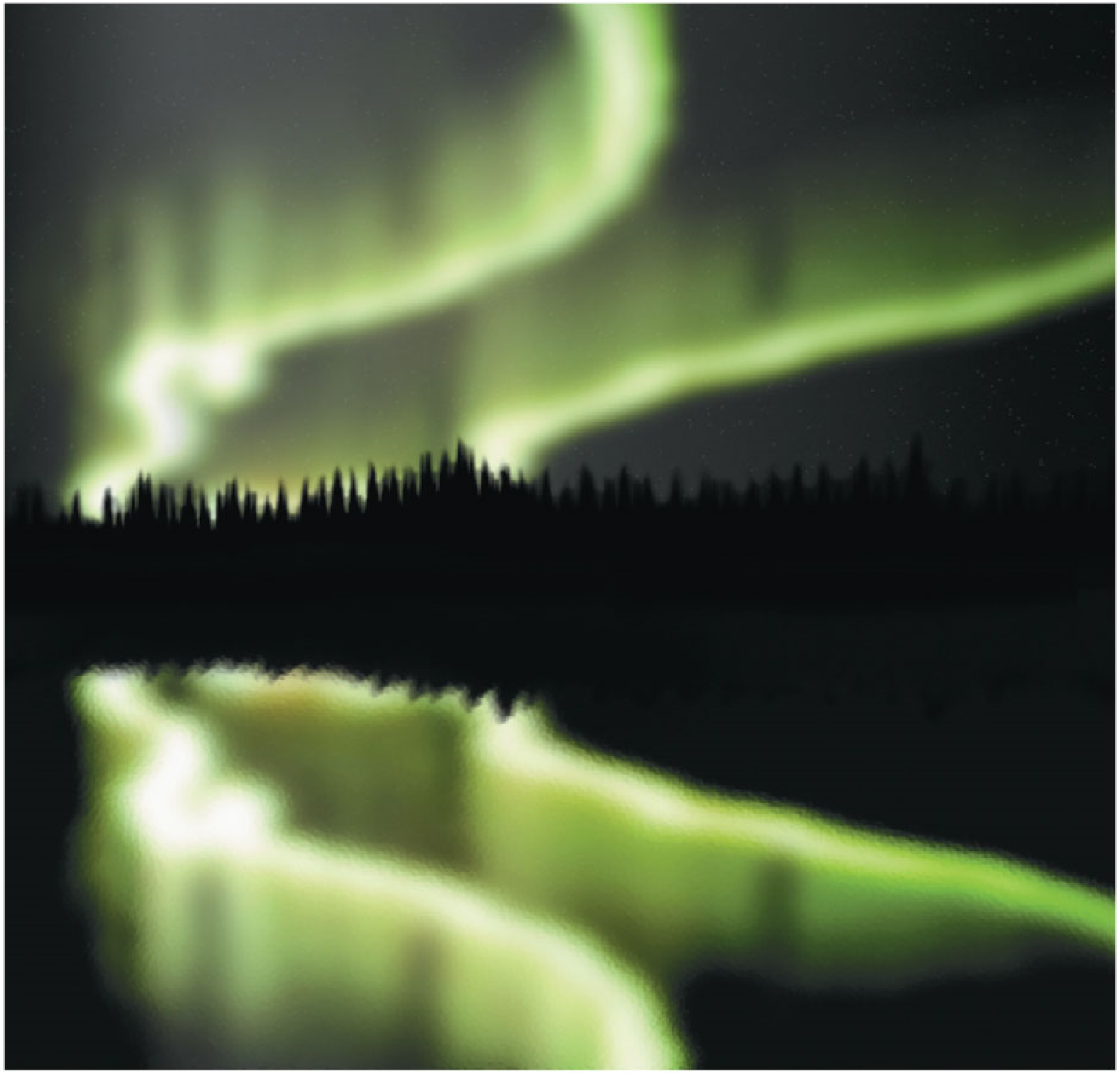
(a) Charged particles spiral around the magnetic field lines.



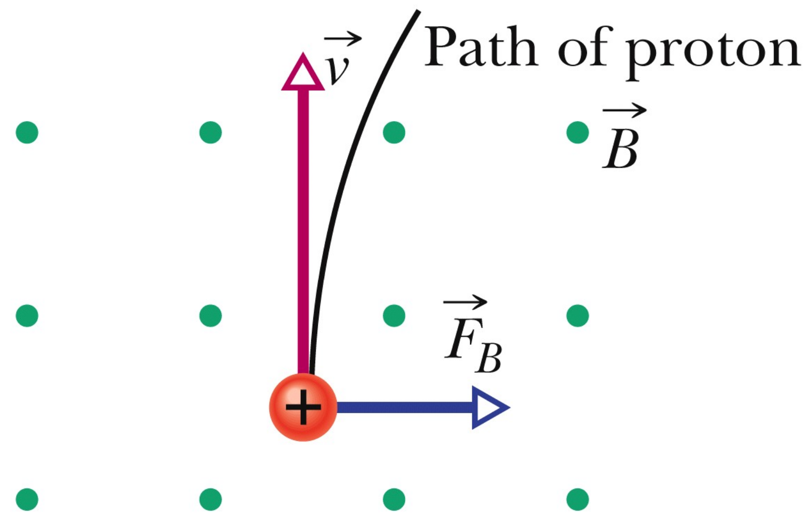
(b) The earth's magnetic field leads particles into the atmosphere near the poles, causing the aurora.



(c) The aurora



Motion of charges in B-field



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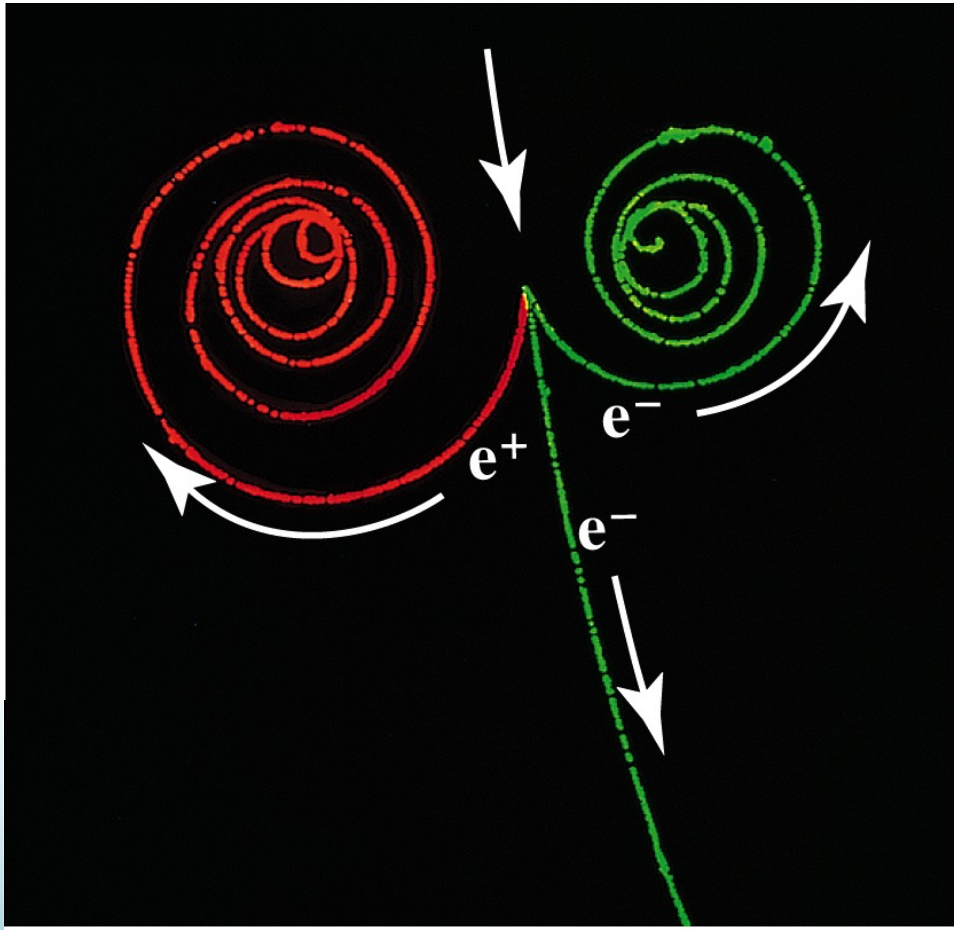

Lawrence Berkeley Laboratory/Photo Researchers, Inc.

This section we talked about:

Chapter 28.1

See you on Wednesday