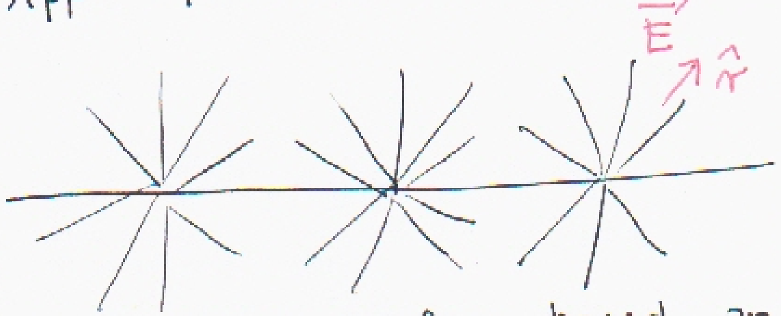
Electric field of a charged wire

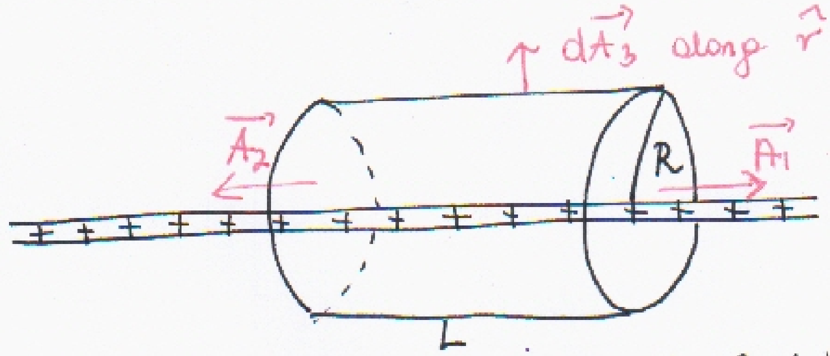
– using Gauss' Law

Consider a thin plastic rod uniformly charged ($+\lambda$)

1. Look at the distribution of E-field lines (App: falstad.com/vector3de/ → charged line)



2. Choose Gaussian surface based on the symmetry ⇒ cylinder

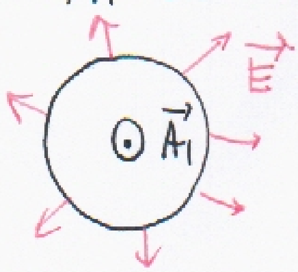


⇒ for a given radius E-field is constant
IT IS ∞-LONG ROD → no end effects. UNIFORM FIELD

3. Find the electric field flux through the surface of the cylinder
   → determine the direction of E-field – $\hat{r}$
   → determine the direction of surface vectors for each side

$$\oiint \vec{E}\cdot d\vec{A} = \int_{\text{cap } 1} \overset{=0}{\vec{E}\cdot\vec{A_1}} + \int_{\text{cap } 2} \overset{=0}{\vec{E}\cdot\vec{A_2}} + \int_{\text{core}} E\cdot d\vec{A_3}$$

For A_1 and A_2



$\vec{E} \perp \vec{A_1} \Rightarrow \vec{E}\cdot\vec{A_1} = 0$

$\Rightarrow \cos 90° \Rightarrow 0$ $\qquad \vec{E}\cdot\vec{A_2} = 0$

DOT PRODUCT

For the core:

$\vec{E}$ along $\hat{r}$, $d\vec{A_3}$ along $\hat{r}$

$$\Rightarrow \vec{E} \parallel d\vec{A_3} \Rightarrow \int_{core} \vec{E} \cdot d\vec{A_3} = \int_{core} E\, dA \cdot \overset{=1}{\cos 0^\circ}$$

E - field is constant

$$\Phi_3 = E \int_{core} dA = E \cdot \underbrace{2\pi R \cdot L}_{\text{area of the core}}$$

$$\oiint \vec{E} \circ d\vec{A} = E \cdot 2\pi R \cdot L$$

4. Find charge enclosed within the cylinder:

$$q_{enc} = \lambda \cdot L$$

5. Apply Gauss' law

$$\oiint \vec{E} \circ d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$E \cdot 2\pi R \cancel{L} = \frac{\lambda \cdot \cancel{L}}{\varepsilon_0}$$

$$E = \frac{1}{2\pi \varepsilon_0} \frac{\lambda}{R}$$

$$\vec{E} = \frac{1}{2\pi \varepsilon_0} \frac{\lambda}{R} \hat{r}$$