Electricity and Magnetism

- Physics 259 – L02
- Lecture 47

Chapter 30: Induction and inductance

Review

Faraday discovered that there is an induced EMF in the secondary circuit given by

$$\varepsilon = -\frac{d\Phi_M}{dt}$$

This is a new generalized law called Faraday's Law.

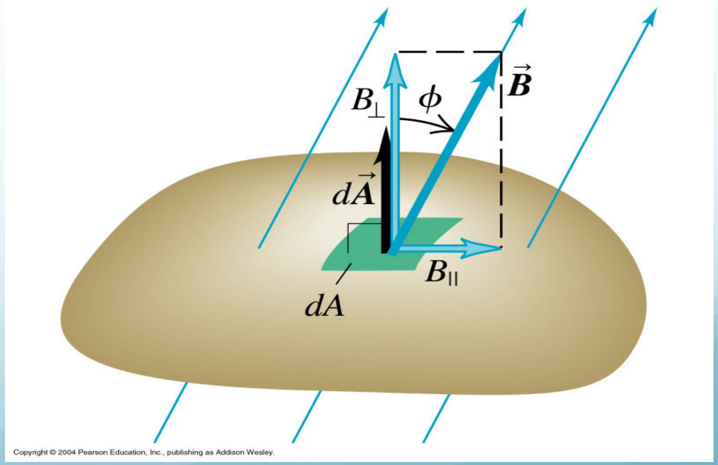
For a coil of N turns→

$$\varepsilon = -N\frac{d\Phi_M}{dt}$$

Recall the definition of magnetic flux:

$$\Phi_M = \int \vec{B} \cdot d\vec{A}$$

Not a closed surface!

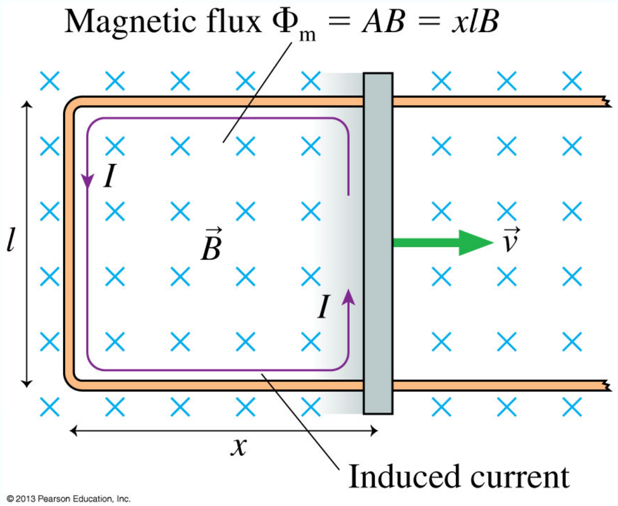


Review

Example of using Faraday's law→

$$\Phi_m = A.B \qquad \varepsilon = \left|\frac{d\Phi_m}{dt}\right| = \frac{d}{dt} xlB = vlB$$

induced current→ $$I = \frac{\mathcal{E}}{R} = \frac{vlB}{R}$$

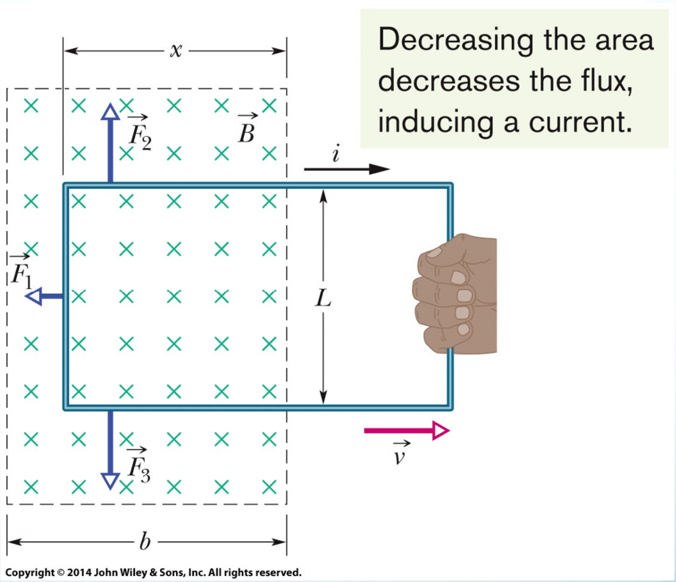


Now→

$$F = iL \times B = iLb\sin(90)$$

Rate of Work: rate at which you do work on the loop as you pull it from the magnetic field:

$$F = \frac{B^2L^2v}{R}.$$



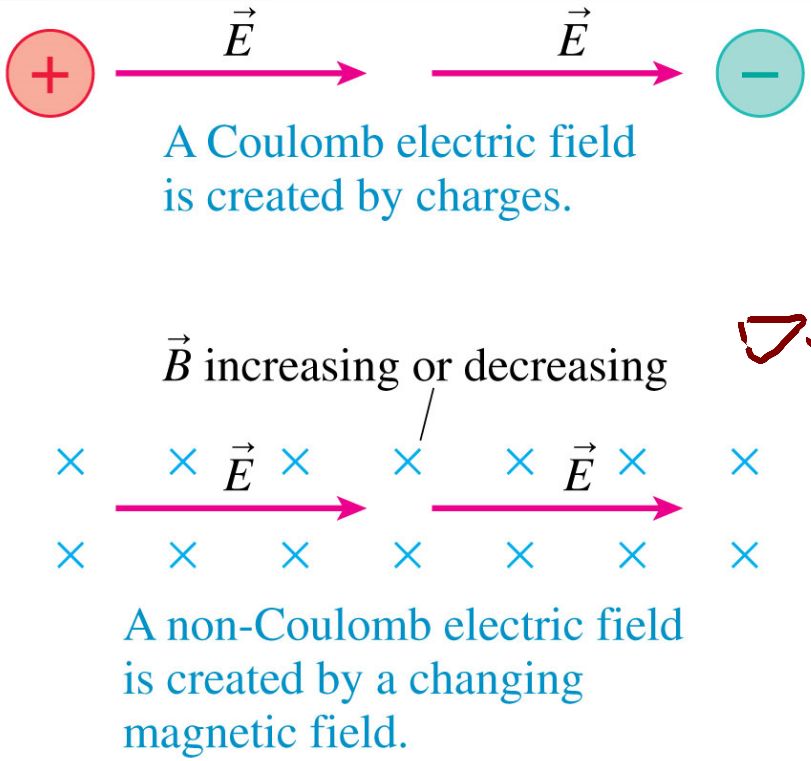
NOTE: The work that you do in pulling the loop through the magnetic field appears as thermal energy in the loop→ $P = Ri^2$

Review 30.3

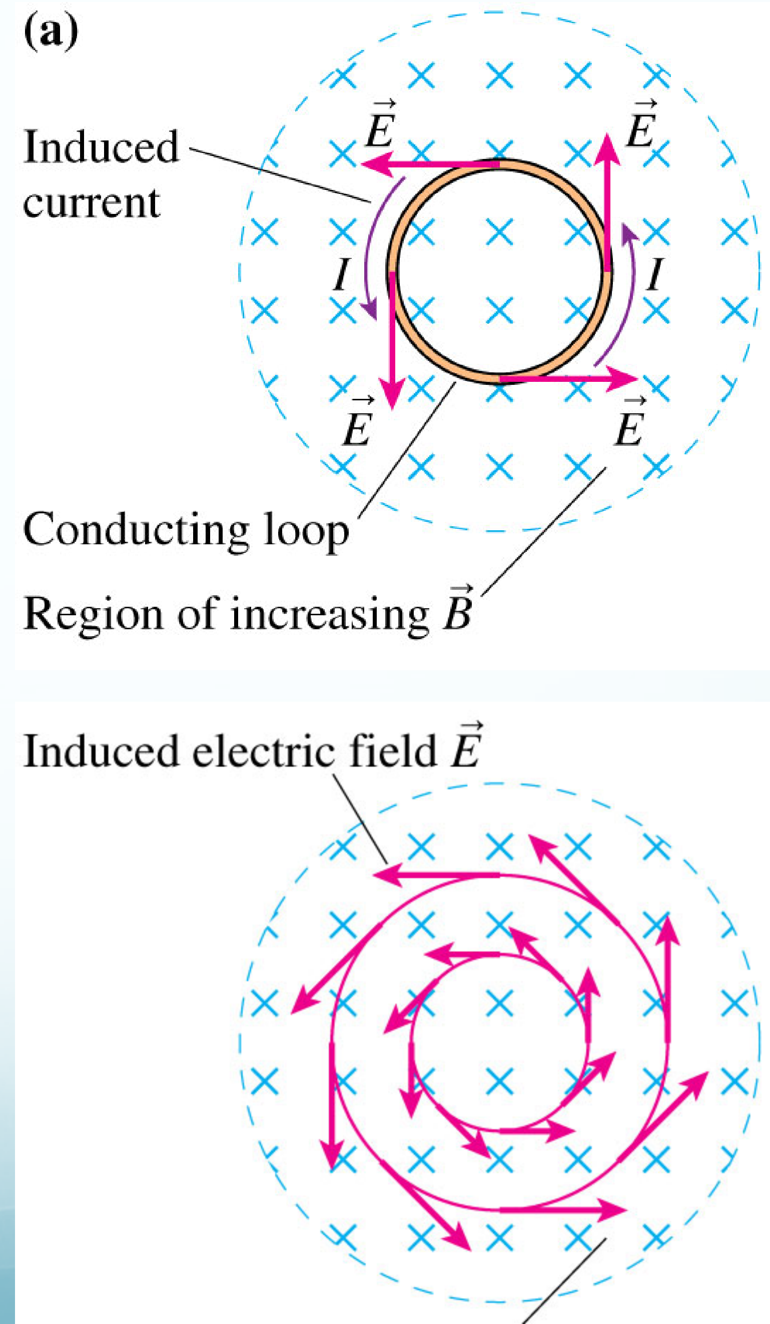
Faraday's law→ strength of induced current

What cause the current?

There is an electric field
caused by changing magnetic field
→ **Induced electric field**



$$\nabla \times E = -\frac{\partial B}{\partial t}$$



Top Hat Question

$$I = \frac{\mathcal{E}}{R} = \frac{vlB}{R}$$

A U-shaped conductor with side length l = 1.0 m is sitting in a uniform magnetic field of field strength 1.0x10^{-2} T. A conducting cross bar is moving with a constant velocity of 1.0 m/s and has a resistance of R = 0.10 ohms. What is the **induced current** in the loop?

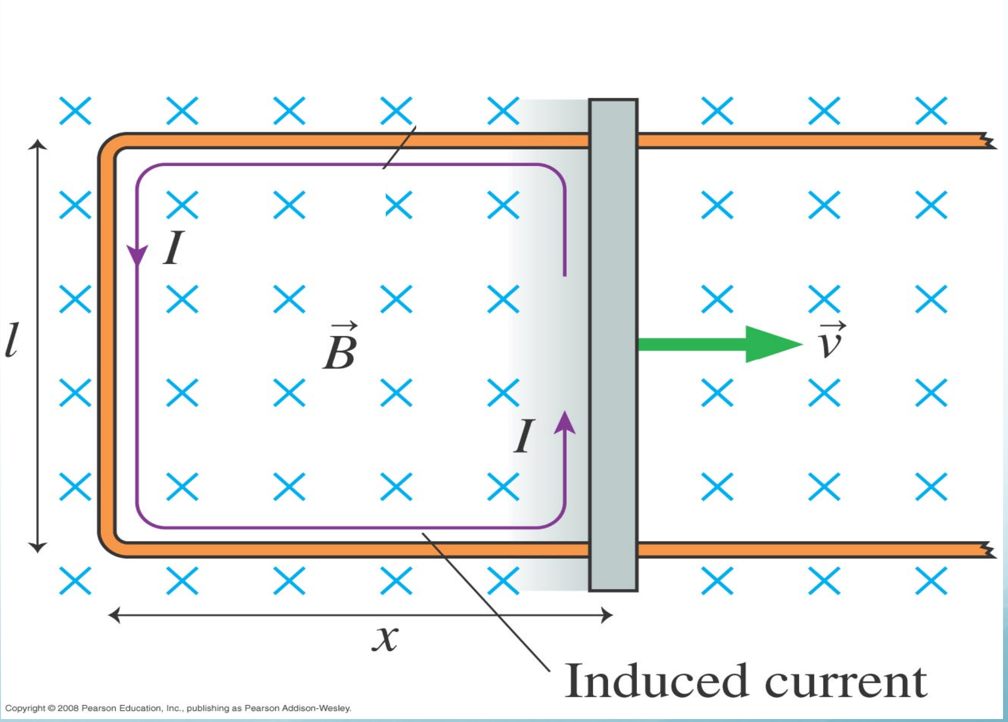
A. 0.0 A

B. 0.010 A

C. 0.10 A

D. 1.0 A

95% Correct.



Top Hat Question

$$P = Fv = \frac{B^2L^2v^2}{R}$$

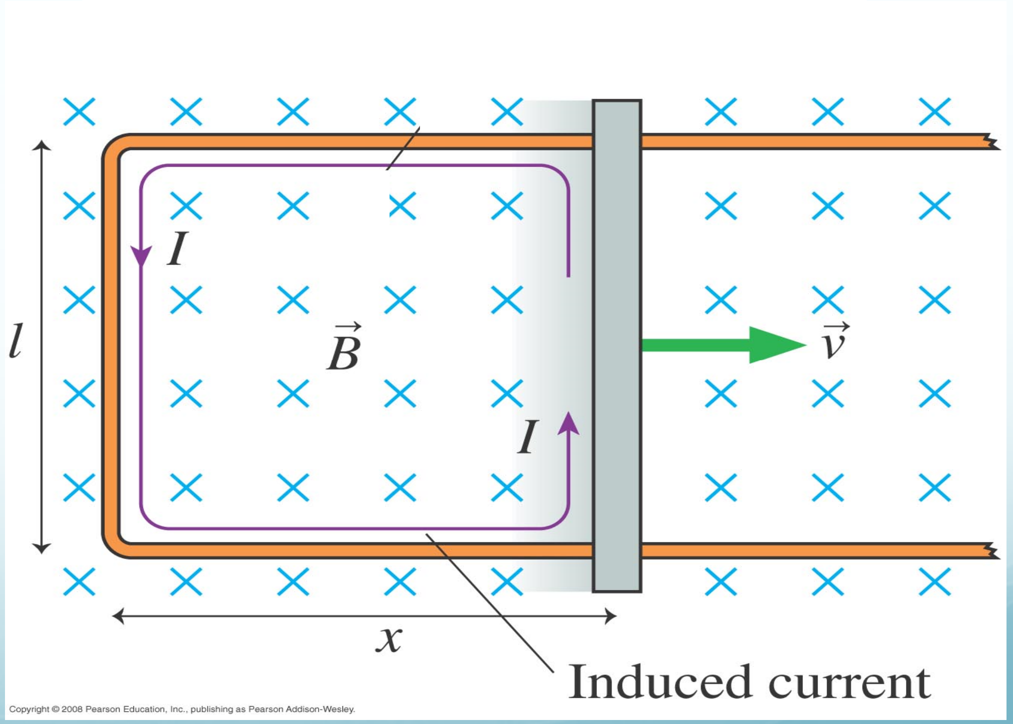
A U-shaped conductor with side length l = 1.0 m is sitting in a uniform magnetic field of field strength 1.0x10^{-2} T. A conducting cross bar is moving with a constant velocity of 1.0 m/s and has a resistance of R = 0.10 ohms. What is the power dissipated by the bar's resistance?

A. 0.0010 W

B. 0.010 W

C. 0.10 W

D. 1.0 W



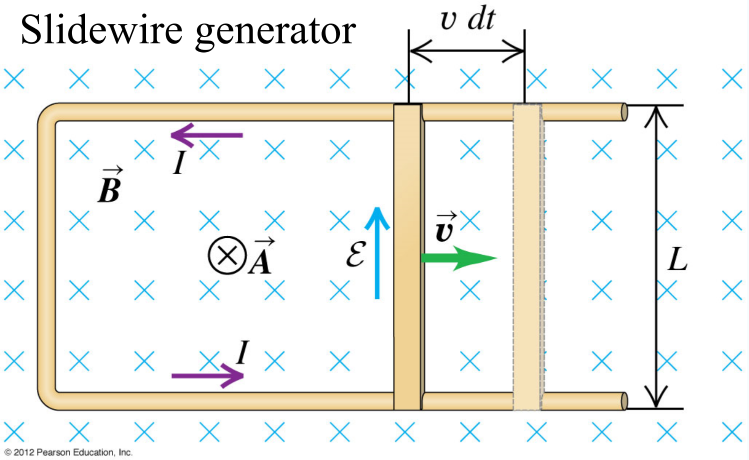
30.2 (continue):Eddy currents

- So far we have considered induction in circuits, where the induced current is confined to wires
- Induction also happens if the magnetic flux through extended metallic objects changes
- As with wires, the induced currents attempt to keep the flux stable: *eddy* currents

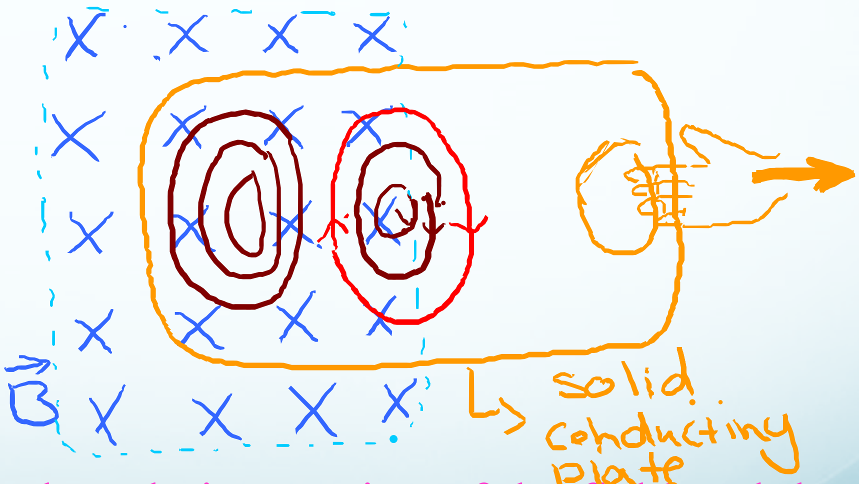
$$I = -\frac{1}{R}\frac{d\phi_B}{dt}$$

F=i LxB

The direction of the currents can be found using Lenz's law



Suppose we replace conducting loop with a Solid conducting plate→

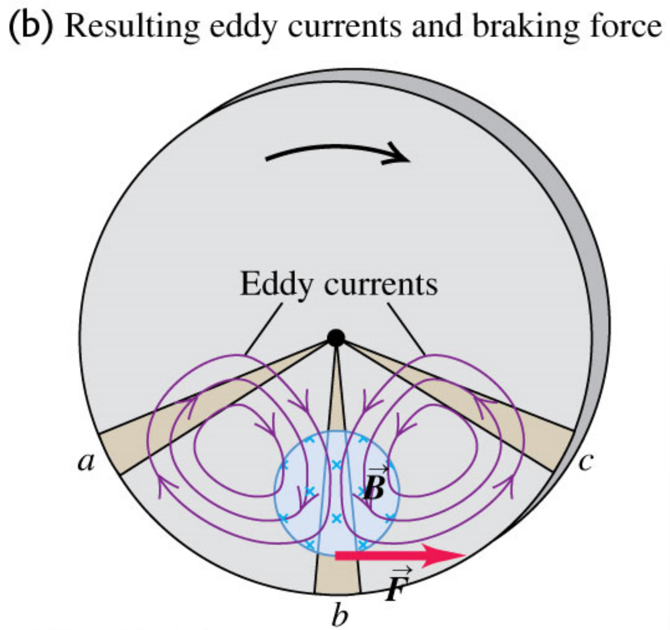


The relative motion of the field and the conductor again induces a current in the conductor→ again an opposing force

lets look at a video fine

Example: Braking system

- Without eddy currents, the magnetic flux at the leading (trailing) edge decreases (increases)
- The induced Eddy currents circulate in a sense that prevents this from happening
- Result: transformation of mechanical energy into heat!



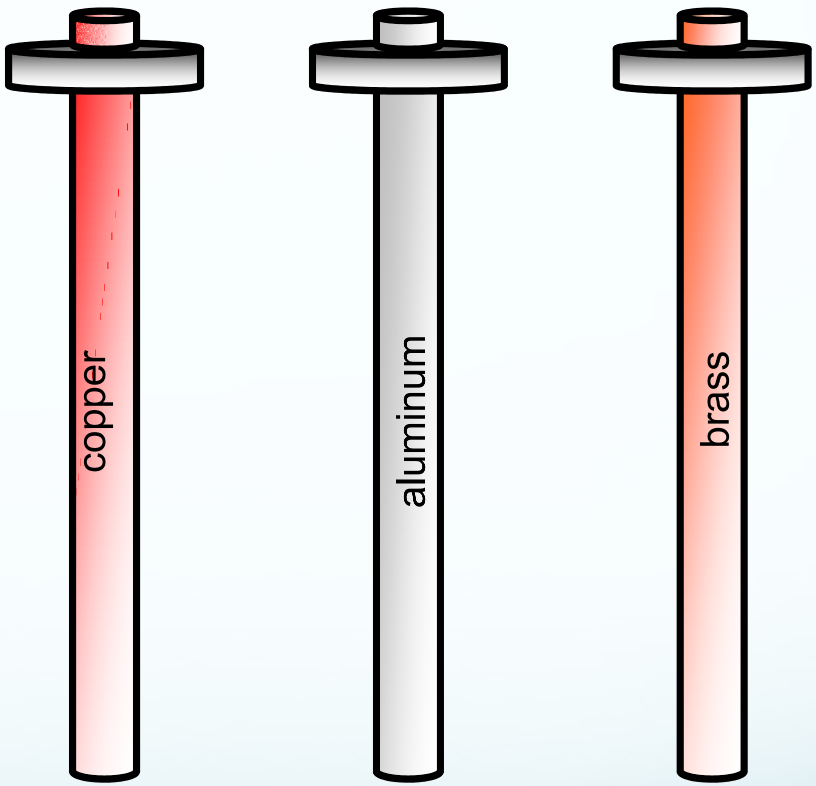
- What changes if the wheel is slotted?
- Slots inhibit the generation of eddy currents, and the braking force is reduced

Question

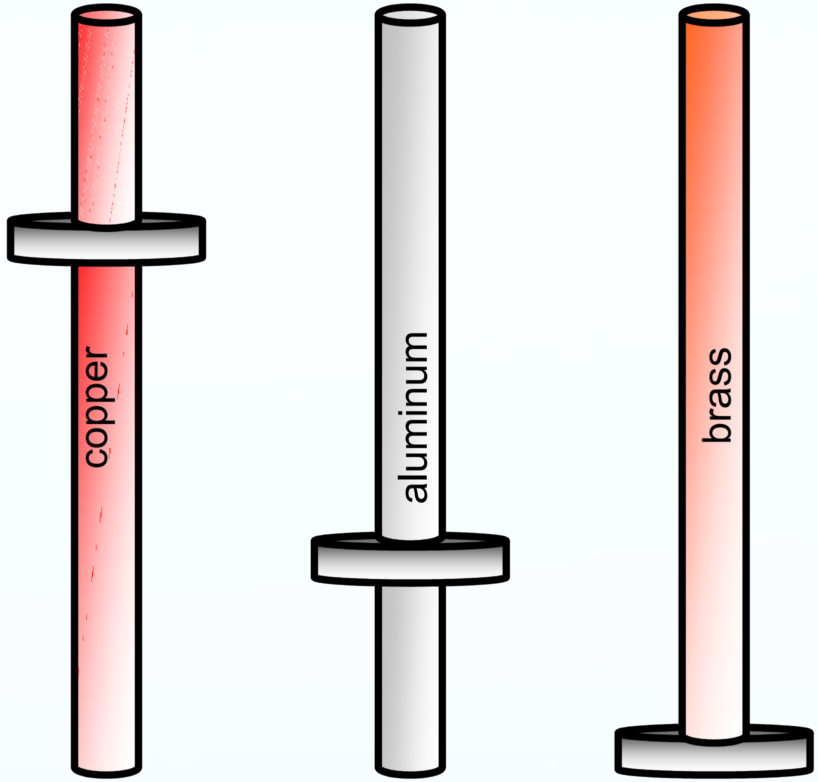
- Three metal rods (brass, aluminum, copper) hold three ring magnets. The three magnets are dropped at the same time, and then slide (fall) down, guided by the rods

- Which magnet (if any) will reach the bottom first?

- *Note*: copper has the least resistivity, followed by aluminum and brass



Let's do the demo



- The *magnet on the brass rod will fall fastest*, as the magnitude of the eddy currents, and hence their capability to slow the magnets down, depends on the material's resistivity.

Recall there are 3 possible terms:

$$\mathcal{E} = \underbrace{-\frac{dB}{dt}A\cos\phi}_{\text{Maxwell Equation}} \underbrace{-\frac{dA}{dt}B\cos\phi + \frac{d\phi}{dt}BA\sin\phi}_{\text{Magnetic Force on free charges}}$$

Maxwell Equation

$$-\frac{d\vec{B}}{dt} = \nabla \times \vec{E}$$

Magnetic Force on free charges

$$F = q\vec{v} \times \vec{B}$$

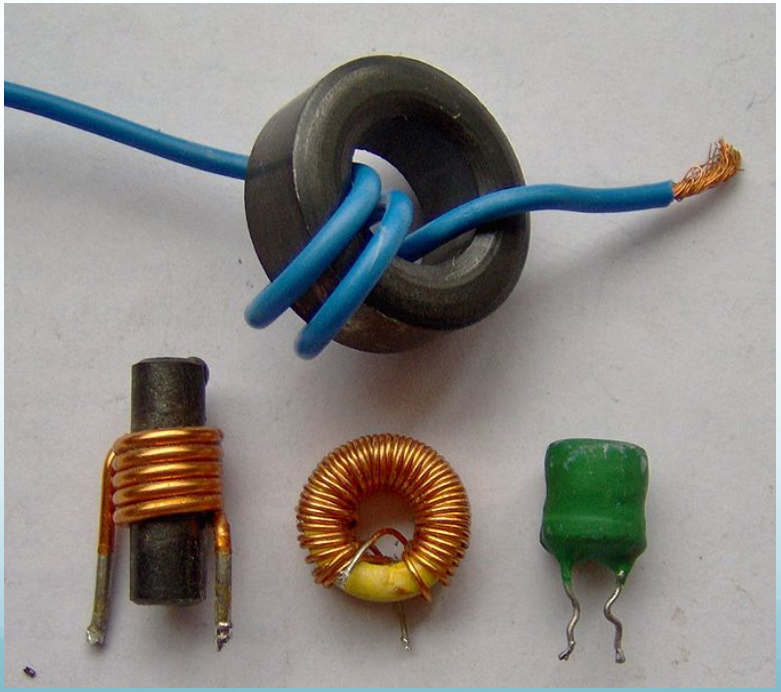
This makes Faraday's Law a particularly powerful tool from a practical engineering standpoint!

Applications of Faraday's Law:

30.4: Inductors and Inductance

An inductor is a device that can be used to produce a known magnetic field in a specified region.

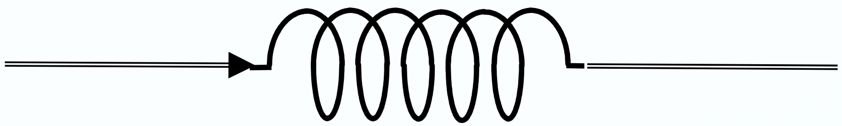
An inductor is a passive electrical component that can store energy in a magnetic field.



Inductance

Note that a changing Magnetic flux produces an induced EMF in a direction which “tries to oppose the change”

$$\frac{di}{dt}$$



$$\varepsilon = -\frac{d\phi}{dt}$$

Changing the current changes the flux through the inductor, which creates a back-emf.

If a current *i* is established through each of the *N* windings of an inductor, a magnetic flux Φ_B links those windings. The inductance *L* of the inductor is

$$\boxed{L = \frac{N\Phi_B}{i}}$$

The SI unit of inductance is the *henry (H)*, where *1 henry = 1H=1T•m²/A*

Energy in a Capacitor is stored in the Electric Field
Energy in an Inductor is stored in the Magnetic Field.

Inductance of a solenoid

The inductance per unit length near the middle of a long solenoid of cross-sectional area *A* and *n* turns per unit length is

$$L = \frac{N\Phi_B}{i}$$

$\rightarrow n = \frac{N}{l} \rightarrow N = nl$

$\Phi = \vec{B}\cdot\vec{A} = BA \Rightarrow L = \frac{nlBA}{i}$

$B = \mu_0 n i$

$\Rightarrow L = \frac{nl\mu_0 n \cancel{i} A}{\cancel{i}} = \mu_0 l n^2 A$

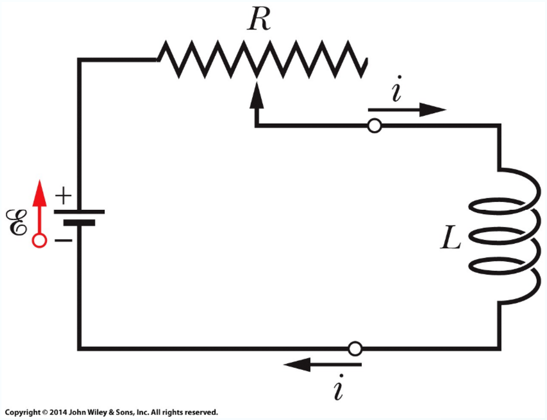
$$\Rightarrow \frac{L}{l} = \mu_0 n^2 A$$

Potential difference across an inductor→ $\boxed{\Delta V = -L\frac{di}{dt}}$

If two coils — which we can now call inductors — are near each other, a current i in one coil produces a magnetic flux Φ_B through the second coil. We have seen that if we change this flux by changing the current, an induced *emf* appears in the second coil according to Faraday's law. An induced *emf* appears in the first coil as well.

30-5 Self-Induction

An induced emf $\mathscr{E}_L$ appears in any coil in which the current is changing.



This process is called self-induction, and the *emf* that appears is called a self-induced *emf*.

It obeys Faraday's law of induction just as other induced emfs do. For any inductor,

$$L = \frac{N\Phi}{i}$$

$$N\Phi_B = Li.$$

Faraday's law tells us that

$$\mathscr{E}_L = -\frac{d(N\Phi_B)}{dt}.$$

By combining these equations, we can write

$$\mathscr{E}_L = -L\frac{di}{dt} \quad \text{(self-induced emf).}$$

Note: a self-induced *emf* appears whenever the current changes with time. The magnitude of the current has no influence on the magnitude of the induced *emf*; only the rate of change of the current counts.

30.7: Energy storage in Inductors

If we build up the current, starting from $\mathbf{I}_0$ = 0 (initial) → $\mathbf{I}_f$,

at the time t when we have achieved a current $\mathbf{I}$, we have to work against an opposing EMF = $Ld\mathbf{I}/dt$ in order to achieve a further increase in current, so our energy source is doing work per unit time

$$dP = IV = IL\frac{dI}{dt}$$

Work → $$W = \int P\,dt = \int IL\frac{dI}{\cancel{dt}}\cancel{dt}$$

total work done:
ie energy stored in system: $$U = \int_0^{I_f} LI\,dI$$

$$\boxed{U = \frac{1}{2}LI^2}$$

& for solenoid→ $$\frac{L}{l} = \mu_0 n^2 A$$

$$\boxed{U=\frac{1}{2}Li^2}$$

$$\boxed{\frac{L}{l}=\mu_0 n^2 A}$$ ← for solenoid

Energy density→

$$u=\frac{U}{V}=\frac{U}{Al}=\frac{\frac{1}{2}Li^2}{Al}=\frac{1}{2}\frac{\mu_0 n^2 A i^2 \cancel{l}}{\cancel{Al}}$$

$$\Rightarrow u=\frac{1}{2}\mu_0 n^2 i^2$$

Capacitor

$$\Rightarrow u=\frac{1}{2}\frac{B^2}{\mu_0}$$

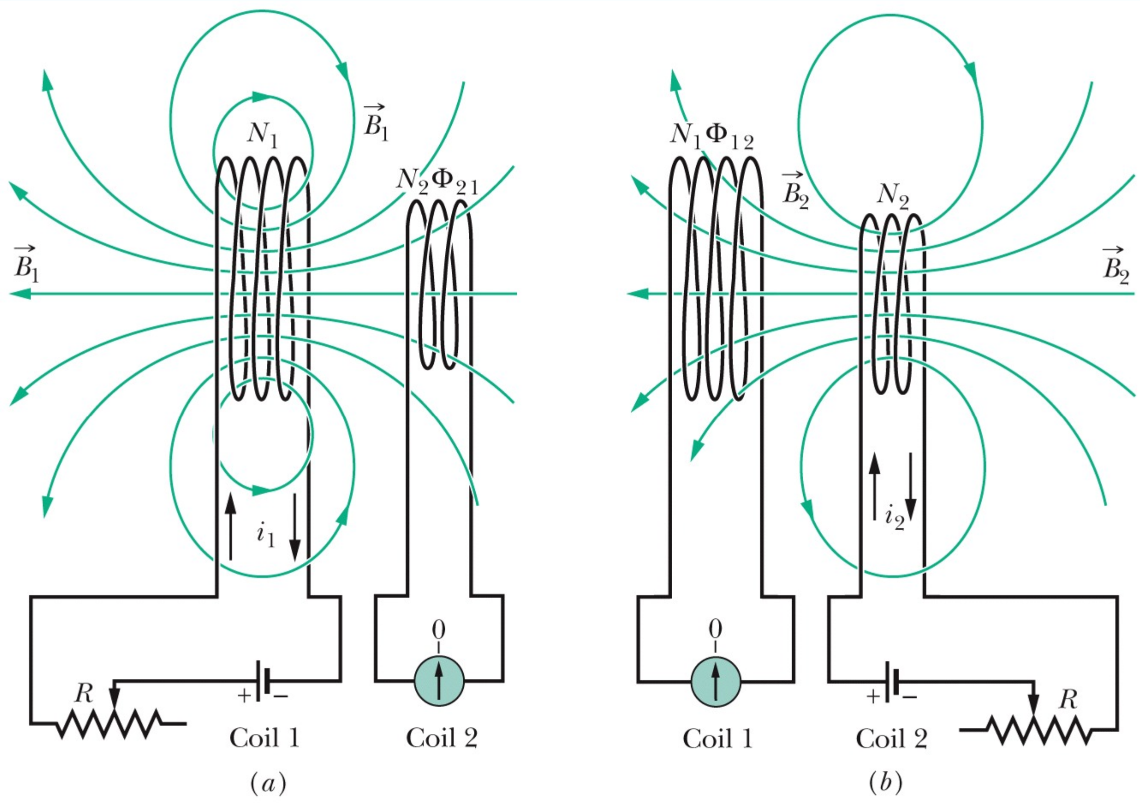
$$\boxed{u=\frac{1}{2}\varepsilon_0 E^2}$$

&

$$B=\mu_0 n i$$

$$\Rightarrow \boxed{=\frac{1}{2\mu_0}B^2}$$ allways true

30-8 Mutual Induction



Mutual induction. (a) The magnetic field B_1 produced by current i_1 in coil 1 extends through coil 2. If i_1 is varied (by varying resistance R), an *emf* is induced in coil 2 and current registers on the meter connected to coil 2. (b) The roles of the coils interchanged.

If coils 1 and 2 are near each other, a changing current in either coil can induce an emf in the other. This mutual induction is described by

$$\mathscr{E}_2 = -M\frac{di_1}{dt}$$

and

$$\mathscr{E}_1 = -M\frac{di_2}{dt}.$$

Mutual inductance M_{21} of coil 2 with respect to coil 1 =>

$$M_{21} = \frac{N_2 \varphi_{21}}{i_1} \rightarrow M_{21} i_1 = N_2 \varphi_{21} \Rightarrow M_{21} \frac{di_1}{dt} = \underbrace{N_2 \frac{d\varphi_{21}}{dt}}$$

we had: $\mathcal{E}_2 = -N_2 \dfrac{d\varphi}{dt}$

$$\rightarrow M_{21} \frac{di_1}{dt} = -\mathcal{E}_2 \rightarrow \mathcal{E}_2 = -M_{21} \frac{di_1}{dt}$$

and the same for $\mathcal{E}_1$ -> $\mathcal{E}_1 = -M_{12} \dfrac{di_2}{dt}$

& $M_{12} = M_{21} = M$ -> we can not prove here :)

$$\Rightarrow \quad \mathcal{E}_2 = -M \frac{di_1}{dt}$$

$$\mathcal{E}_1 = -M \frac{di_2}{dt}$$

This section we talked about:

Chapter 30

See you on Friday