Electricity and Magnetism

- Physics 259 – L02
- Lecture 30

UNIVERSITY OF CALGARY

Chapter 25: Capacitance

Last time

- Chapter 25-1 and 25-2

This time

- Cylindrical capacitors
- Capacitors in parallel and series
- Energy in Capacitors

25-2 Calculating the Capacitance

https://goo.gl/p3vvfm

TopHat Question

If we double the amount of charge on a capacitor, how does the capacitance change?

$C = \frac{q}{V}$

a. Capacitance increases.
b. Capacitance decreases.
c. Capacitance remains the same.
d. More information is needed.

Review: Calculating electric field and potential difference

1. To relate the electric field ***E*** between the plates of a capacitor to the charge *q* on either plate→ use Gauss' law:

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q.$$

2. the potential difference between the plates of a capacitor is related to the field ***E*** by

$$V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s},$$

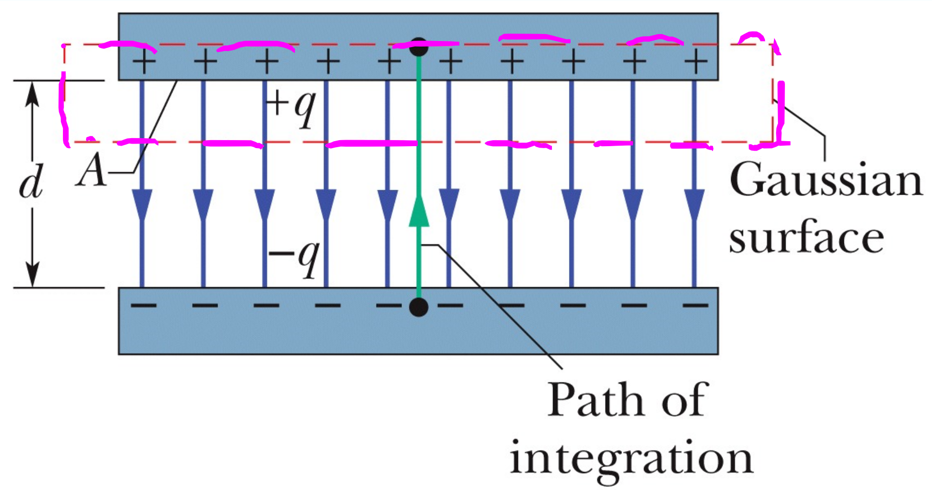
Letting V represent the difference $V_f = V_i$, we can then recast the above equation as:

$$V = \int_-^+ E\, ds$$

3. Find Capacitance

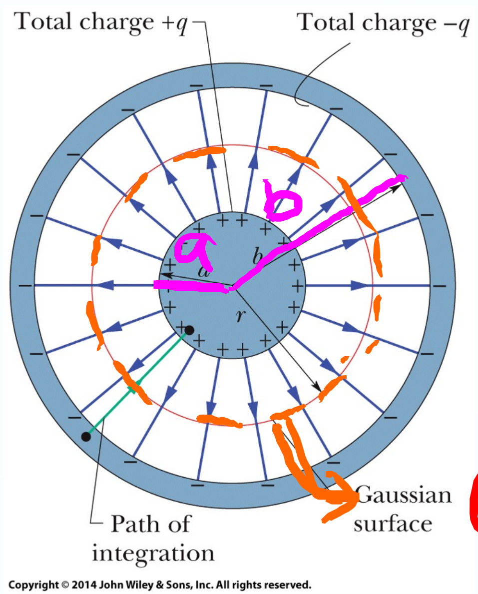
$$q = CV.$$

Review: 25-2 Calculating the Capacitance: Parallel-Plate Capacitor



$$C = \frac{\varepsilon_0 A}{d} \quad \text{(parallel-plate capacitor).}$$

25-2 Calculating the Capacitance: Cylindrical Capacitor



Gaussion surface

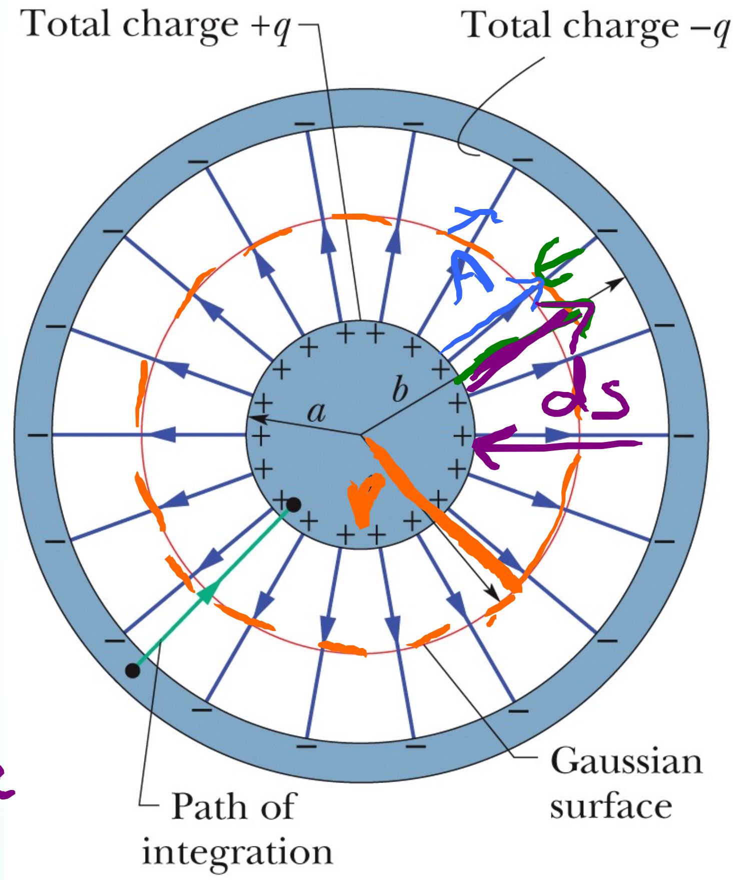
- ✓ **cylindrical capacitor of length *L* formed by two coaxial cylinders of radii *a* and *b*.**
- ✓ ***L* >> *b* →neglect fringing of electric field that occurs at ends of the cylinders.**
- ✓ **Each plate contains a charge of magnitude *q*.**

1. Use Gauss's law E

2. Find potential V

3. Find Capacitance $C = \frac{q}{V}$

25-2 Calculating the Capacitance: Cylindrical Capacitor



1. Use Gauss's law

$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0} = \frac{q}{\varepsilon_0}$

$\rightarrow EA = \frac{q}{\varepsilon_0} \Rightarrow$

$$\boxed{q = \varepsilon_0 EA = \varepsilon_0 E(2\pi rL)}$$

$\rightarrow E = \frac{q}{\varepsilon_0 2\pi r L}$

2. Find potential

$$\boxed{V = \int_{-}^{+} E\,ds = -\frac{q}{2\pi\varepsilon_0 L}\int_b^a \frac{dr}{r} = \frac{q}{2\pi\varepsilon_0 L}\ln\left(\frac{b}{a}\right)}$$

$\rightarrow$ * next page

3. Find Capacitance

$C = \frac{q}{V} = \frac{\cancel{q}}{\frac{\cancel{q}}{2\pi\varepsilon_0 L}\ln\frac{b}{a}}$

$\rightarrow C = 2\pi\varepsilon_0 L \frac{1}{\ln b/a} \Rightarrow$

$$C = 2\pi\varepsilon_0 \frac{L}{\ln(b/a)} \quad \text{(cylindrical capacitor).}$$

* explanation of $V = \int_{-}^{+} E\,ds$

the expression for potential →

$$V = -\int_a^b \vec{E}\cdot\vec{ds} = -\int_a^b E\,ds\cos\theta$$

because we consider $\int_{-}^{+}$ ⇒ θ is allways $\theta = 180°$

$$\Rightarrow V = -\int_{-}^{+} E\,ds\cos(180) \Rightarrow V = \int_{-}^{+} E\,ds$$

$$\Rightarrow V = \int_{-}^{+} E\,ds = \int_{-}^{+} \frac{q}{2\pi\varepsilon_0 L}\frac{ds}{r} \quad \& \quad ds = -dr$$

$$\Rightarrow V = \frac{-q}{2\pi\varepsilon_0 L}\int_{-}^{+}\frac{dr}{r} = \frac{-q}{2\pi\varepsilon_0 L}\int_b^a \frac{dr}{r} = \frac{-q}{2\pi\varepsilon_0 L}\ln\left(\frac{a}{b}\right)$$

25-2 Calculating the Capacitance

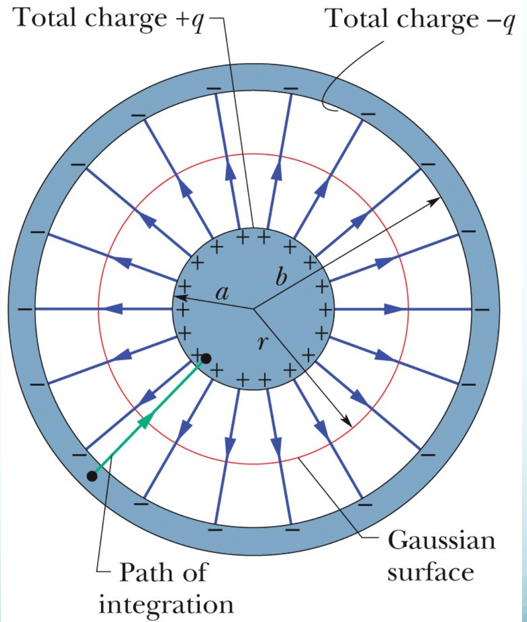
Others...

For **spherical capacitor** the capacitance is:

$$C = 4\pi\varepsilon_0 \frac{ab}{b-a} \quad \text{(spherical capacitor).}$$

Capacitance of an **isolated sphere:**

$$C = 4\pi\varepsilon_0 R \quad \text{(isolated sphere).}$$



TopHat Question

How big does the radius of an isolated sphere have to be in order for it to have a capacitance of 1 F? Choose the closest answer.

A. 10^9 km

B. 10^8 km

C. 10^7 km

D. 10^6 km

$$R = \frac{C}{4\pi\varepsilon_0} = \frac{1}{4\pi\varepsilon_0} = 8.99\times10^6$$

$$\simeq 10\times10^6 = 10^7$$

$$C = 4\pi\varepsilon_0 R \quad \text{(isolated sphere).}$$

$$\varepsilon_0 = 8.85\times10^{-12}$$

TopHat Question

How big does the radius of an isolated sphere have to be in order for it to have a capacitance of 1 F? Choose the closest answer.

A. 10^9 km

B. 10^8 km

C. 10^7 km

D. 10^6 km

For comparison, the orbit of Mercury around the sun is 5×10^7 km! veryyyyy large

Spheres make bad capacitors; a 1 F capacitor used in circuits can fit in your hand. → with new technology

→ because their construction is hard

Capacitors

General relationship: $$Q = C\Delta V_C$$

Parallel plate capacitor: $$Q = \left(\frac{\varepsilon_o A}{d}\right)\Delta V_C$$

Spherical capacitor: $$Q = \left(\frac{4\pi\varepsilon_0 r_b r_a}{r_b - r_a}\right)\Delta V_C$$

Isolated sphere: $$Q = \left(4\pi\varepsilon_0 R\right)\Delta V_C$$

Cylindrical capacitor: $$Q = \left(\frac{2\pi\varepsilon_0 L}{\ln\left(\frac{r_B}{r_A}\right)}\right)\Delta V_C$$

25-3 Capacitors in parallel and series

Capacitors in Series

https://tinyurl.com/j6cb8sr

$$1/C=1/C_1+1/C_2$$

& q is the same

Capacitors in Series

When a potential difference V is applied across several capacitors connected in series, the capacitors have identical charge q. The sum of the potential differences across all the capacitors is equal to the applied potential difference V.

$$V_1 = \frac{q}{C_1}, \quad V_2 = \frac{q}{C_2}, \quad \text{and} \quad V_3 = \frac{q}{C_3}.$$

The total potential difference V due to the battery is the sum

$$V = V_1 + V_2 + V_3 = q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right).$$

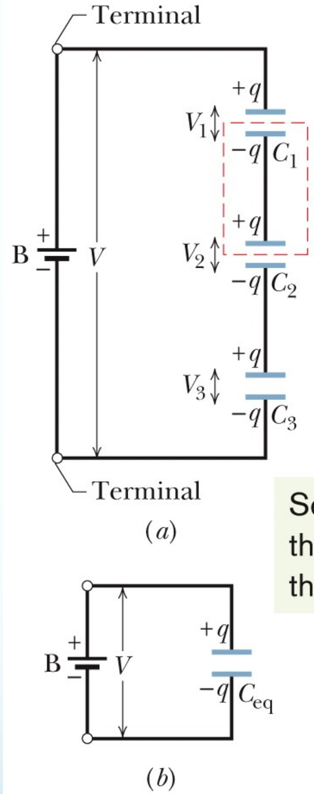
The equivalent capacitance is then

$$C_{eq} = \frac{q}{V} = \frac{1}{1/C_1 + 1/C_2 + 1/C_3},$$

or

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}.$$

$$\frac{1}{C_{eq}} = \sum_{j=1}^{n} \frac{1}{C_j} \quad (n \text{ capacitors in series}).$$



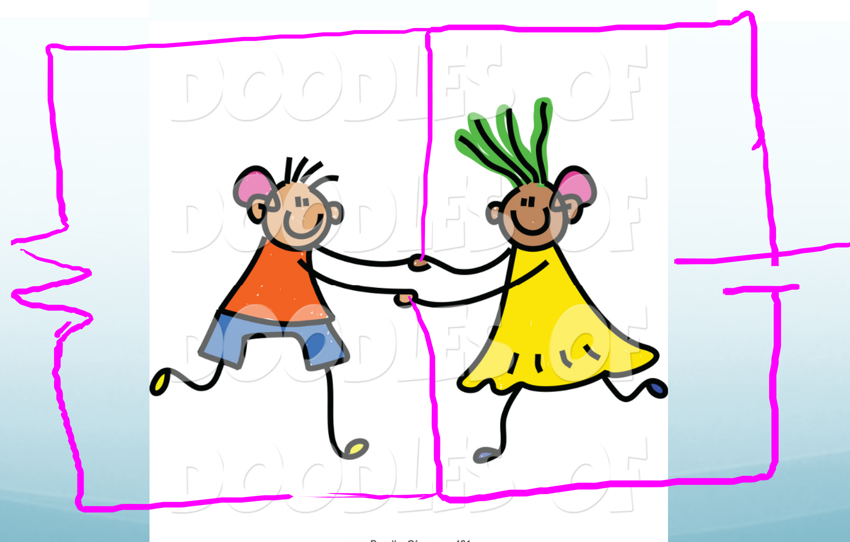
Series capacitors and their equivalent have the same q ("seri-q").

Capacitors that are connected in series can be replaced with an equivalent capacitor that has the same charge q and the same *total* potential difference V as the actual series capacitors.

Capacitors in Parallel

$$C = C_1 + C_2$$

& V is the same



Capacitors in Parallel

When a potential difference V is applied across several capacitors connected in parallel, that potential difference V is applied across each capacitor. The total charge q stored on the capacitors is the sum of the charges stored on all the capacitors.

$$q_1 = C_1V, \quad q_2 = C_2V, \quad \text{and} \quad q_3 = C_3V.$$

The total charge on the parallel combination of Fig. 25-8*a* is then

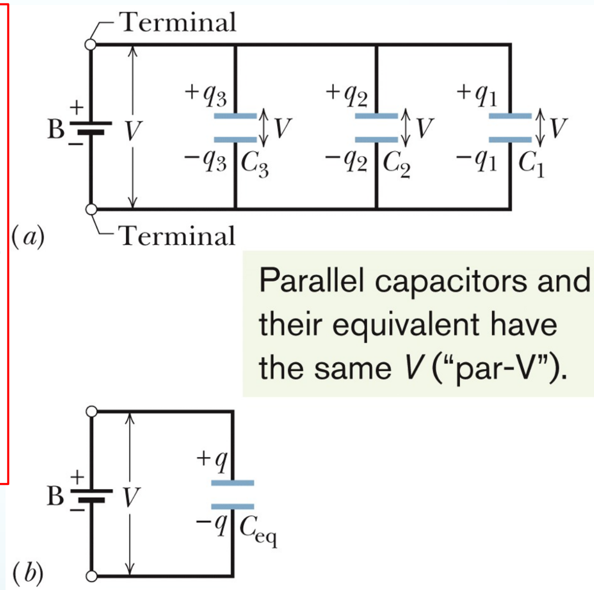
$$q = q_1 + q_2 + q_3 = (C_1 + C_2 + C_3)V.$$

The equivalent capacitance, with the same total charge q and applied potential difference V as the combination, is then

$$C_{eq} = \frac{q}{V} = C_1 + C_2 + C_3,$$

a result that we can easily extend to any number n of capacitors, as

$$C_{eq} = \sum_{j=1}^{n} C_j \quad (n \text{ capacitors in parallel}).$$



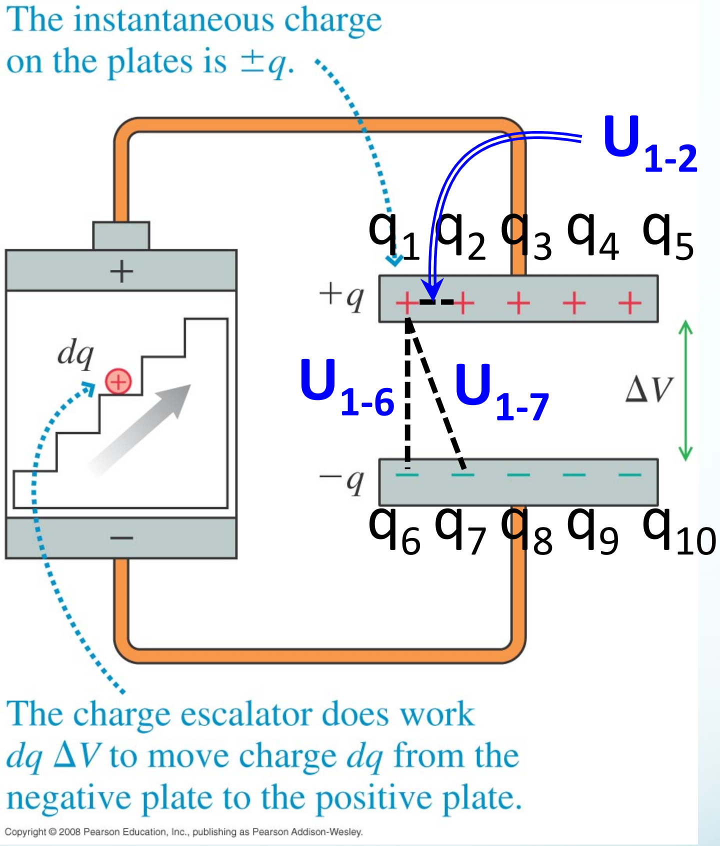
Capacitors connected in parallel can be replaced with an equivalent capacitor that has the same *total* charge q and the same potential difference V as the actual capacitors.

25-4 Energy Stored in an Electric Field

Energy Storage in Capacitors

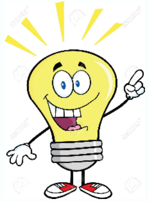
We want to calculate **potential energy** stored in the capacitor

VERYYYYY hard



$$U = U_{1-2} + U_{1-3} + \ldots + U_{1-10} + U_{2-1} + U_{i\text{-}j} \text{ of every other pair}$$

Easier way!

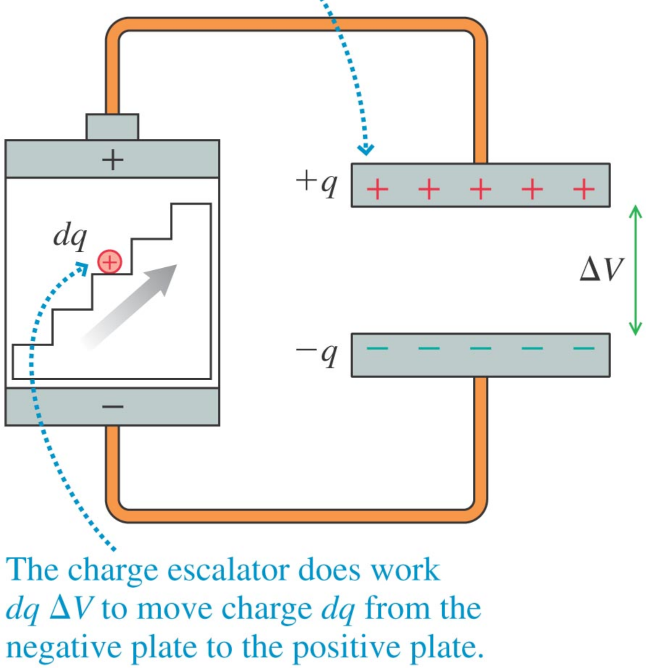


Move a tiny charge, dq, from negative plate to positive plate→

It moves through a potential difference ΔV→ its potential energy increases by an amount

$$dU = dq\Delta V_C$$

$$\& \Delta V_C = \frac{q}{C}$$



$$dU = \frac{qdq}{C}$$

$$\boxed{U = \frac{1}{C}\int_0^Q q\,dq = \frac{1}{2}\frac{Q^2}{C}}$$

✓ **Energy** storage in terms of the charge on the plates:

$$\boxed{U = \frac{1}{2}\frac{Q^2}{C}}$$

✓ **Energy** storage in terms of the voltage across the plates:

$$Q = CV$$

$$\boxed{U = \frac{1}{2}CV^2}$$

✓ **Energy density:**

The potential energy of a charged capacitor may be viewed as being stored in the electric field between its plates.

Energy density→ Potential energy per unit volume between the plates

For parallel-plate capacitor→ $U = \frac{1}{2}CV^2$, $C = \varepsilon_0 \frac{A}{d}$

$$u = \frac{U}{Ad} =$$

$$\boxed{\rightarrow u = \frac{1}{2}\varepsilon_0\left(\frac{V}{d}\right)^2 = \frac{1}{2}\varepsilon_0 E^2}$$

The following two slides you do **NOT** need to know how to reproduce for this course. They simply illustrate that the result from the previous slide applies more generally than for just a parallel plate capacitor.

Spherical Capacitor

Start with integrating $dU = udV$ over the volume between the plates

$$U = \iiint_{Vol} u\,dV = \iiint_{Vol} \frac{\varepsilon_0}{2} E^2\,dV \quad \text{where} \quad \vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$$

In spherical coordinates, $dV = r^2 sin\theta drd\theta d\phi$. Integrals over angles give.

$$\int_0^{\pi} \sin\theta\,d\theta \int_0^{2\pi} d\phi = 4\pi$$

Then U becomes

$$U = \frac{\varepsilon_0}{2}\int_{r_a}^{r_b} \frac{Q^2}{16\pi^2\varepsilon_0^2 r^4} 4\pi r^2\,dr = \frac{1}{2}\frac{Q^2}{4\pi\varepsilon_0}\int_{r_a}^{r_b}\frac{dr}{r^2}$$

Performing the integral and rewriting, we indeed get U

$$U = \frac{1}{2}\frac{Q^2}{4\pi\varepsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right) = \frac{1}{2}\frac{Q^2}{4\pi\varepsilon_0\left(\frac{r_b r_a}{r_b - r_a}\right)} = \boxed{\frac{1}{2}\frac{Q^2}{C}}$$

Cylindrical Capacitor

Start with integrating $dU = udV$ over the volume between the plates

$$U = \iiint_{Vol} u\,dV = \iiint_{Vol} \frac{\varepsilon_0}{2} E^2\,dV \qquad \text{where} \qquad \vec{E} = \frac{\lambda}{2\pi\varepsilon_0 r}\hat{r}$$

In cylindrical coordinates, $dV = rdrd\theta dl$.
Integrals over angle and l give

$$\int_0^{2\pi} d\theta \int_0^L d\ell = 2\pi L$$

Then U becomes

$$U = \frac{\varepsilon_0}{2}\int_{r_a}^{r_b} \frac{\lambda^2}{4\pi^2\varepsilon_0^2 r^2} 2\pi L r\,dr = \frac{1}{2}\frac{\lambda^2 L}{2\pi\varepsilon_0}\int_{r_a}^{r_b}\frac{dr}{r}$$

Performing the integral and rewriting, we indeed get U

$$U = \frac{1}{2}\frac{\lambda^2 L}{2\pi\varepsilon_0}\ln\left(\frac{r_b}{r_a}\right) = \frac{1}{2}\frac{\ln\left(\frac{r_b}{r_a}\right)}{2\pi\varepsilon_0 L}Q^2 \boxed{= \frac{1}{2}\frac{Q^2}{C}}$$

This section we talked about:

Chapter 25

See you on next Thursday