Last time

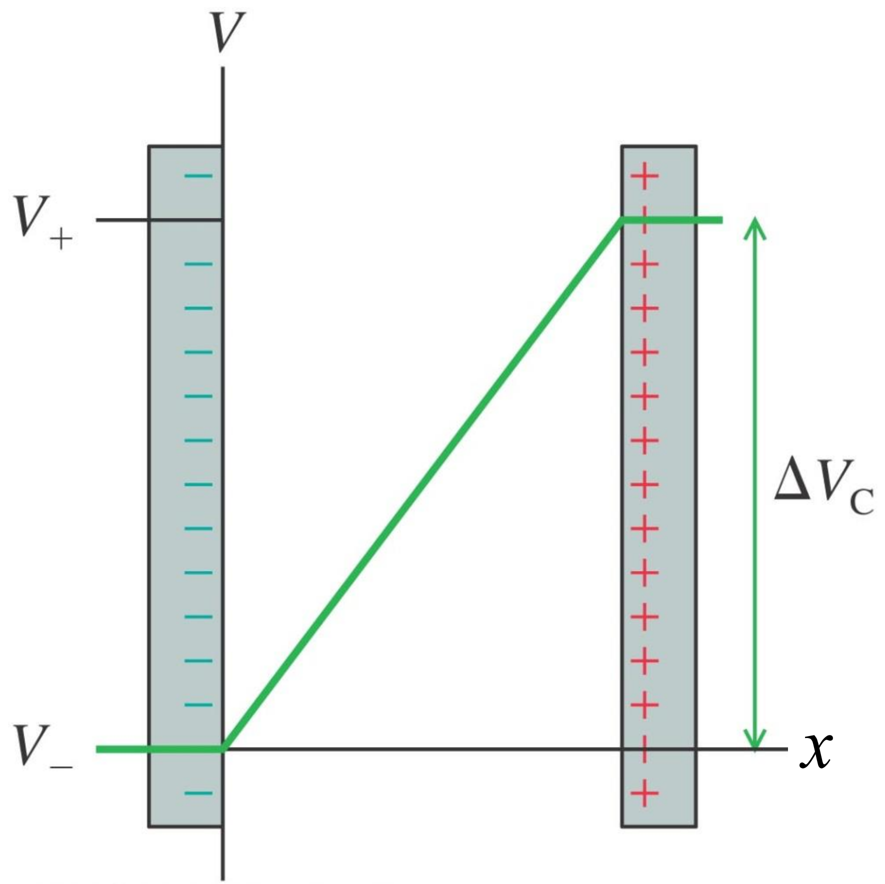
- Insulating spherical shell and a solid spherical conductor
- Potential between two parallel charged plates

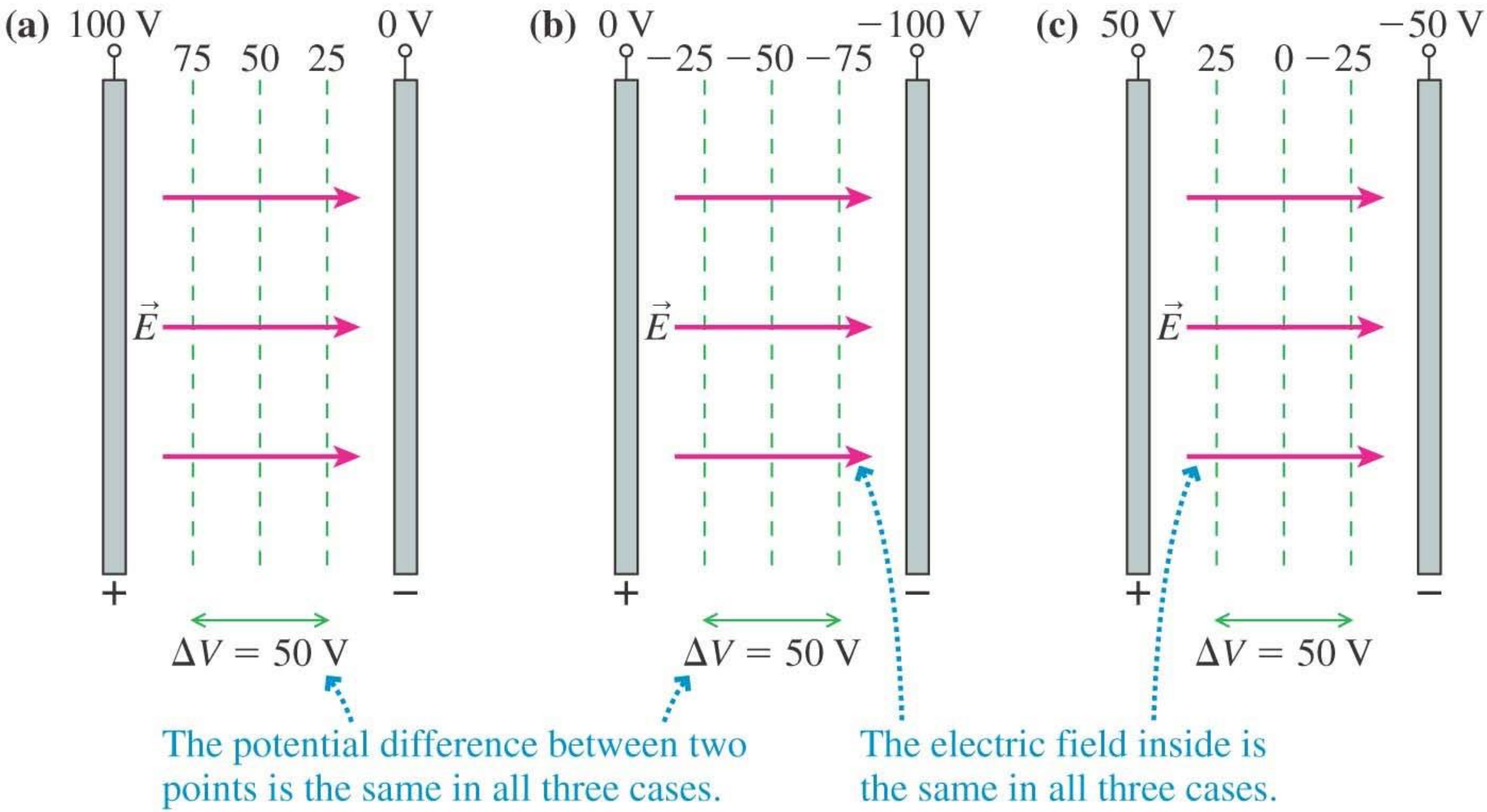
This time

- Capacitance as a geometric quantity
- Capacitance for a parallel plate capacitor
- Capacitance for a solid spherical conductor
- Energy stored in a parallel plate capacitor

$$\Delta V_C = \frac{\sigma}{\varepsilon_0} d = \left(\frac{Q}{A\varepsilon_0} \right) d$$

The electric potential inside a charged capacitor increases linearly from the negative to the positive plate.





We can define V = 0 anywhere we want. Our choice of V = 0 does not affect any potential differences or the electric field.

Capacitance

Capacitance of a conductor is defined as

$$C = \frac{Q}{\Delta V}$$

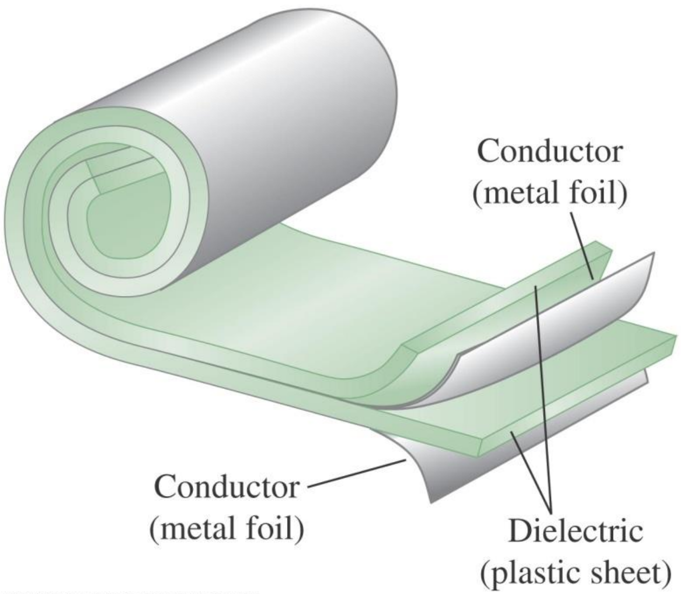
For a parallel plate capacitor

$$\Delta V_C = \frac{\sigma}{\varepsilon_0} d = \left(\frac{d}{A\varepsilon_0} \right) Q$$

$$C = \frac{Q}{\Delta V} = \frac{\varepsilon_0 A}{d}$$

C is a geometric factor. Roughly speaking capacitance is the ability of a geometrical shape to store charge.

In practice, a capacitor is made of two conductor with a dialectic material filling the gap between the two. An example of a parallel plate capacitor is shown below.



$$C = \frac{Q}{\Delta V} = \frac{\varepsilon_0 A}{d}$$

$$\varepsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$$

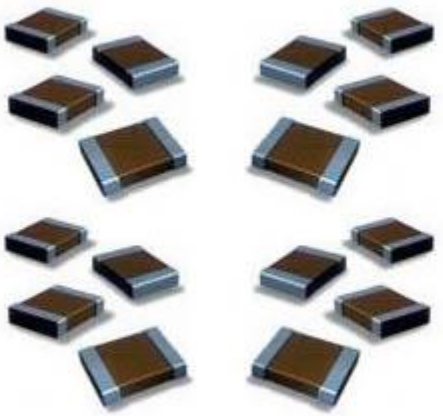
$$A = 10 \text{ cm}^2$$

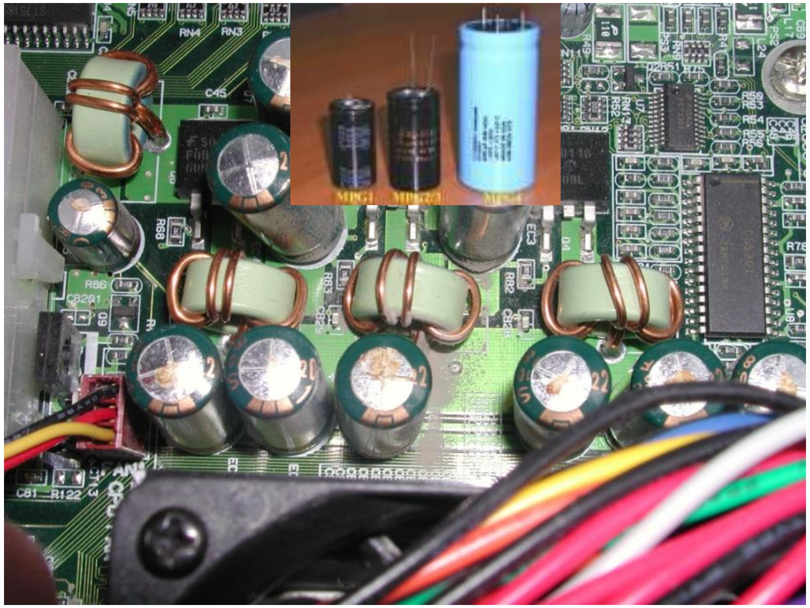
$$d = 1 \text{ mm}$$

$$C = 8.85 \text{ pF}$$

Capacitors

Capacitors come in all kind of shapes and sizes.





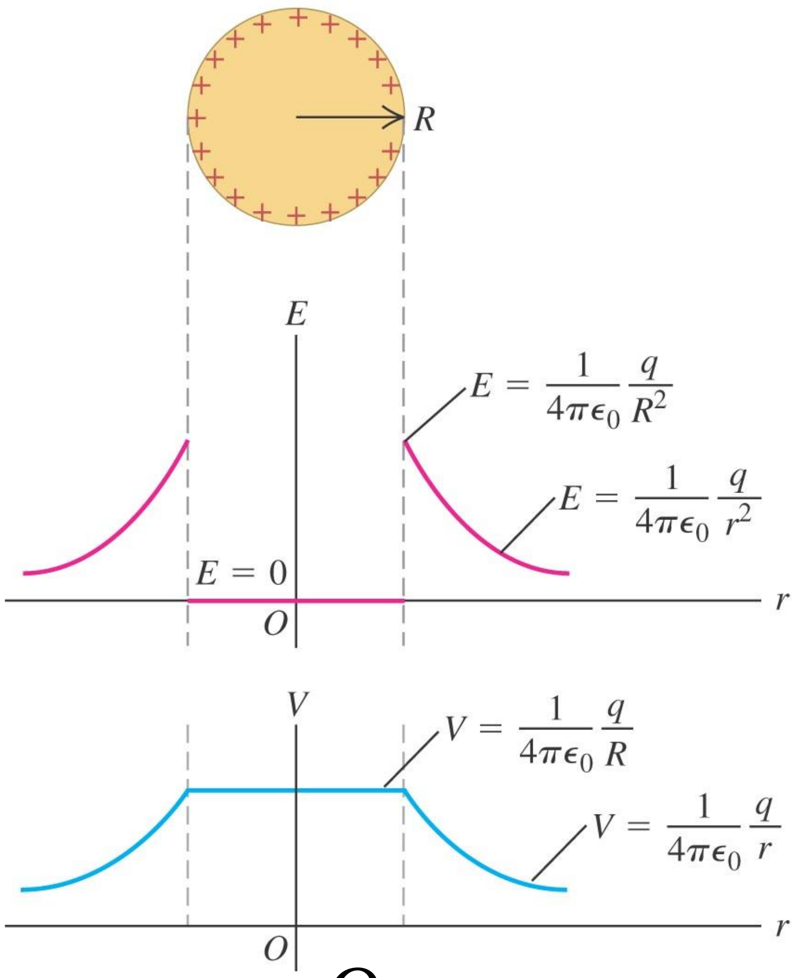
Spherical conductor

$$V(R) = \frac{Q}{4\pi\varepsilon_0 R}$$

$$C = \frac{Q}{V} = 4\pi\varepsilon_0 R$$

In this case the shape factor is $4\pi R$.

$$C = 4\pi \times \left(8.85\times10^{-12}\ \mathrm{F/m}\right)\times\left(0.1\ \mathrm{m}\right) = 11\ \mathrm{pF}$$



$$V(\vec{r}) = \frac{Q}{4\pi\varepsilon_0 R} \quad \text{For } r \le R$$

As we will see shortly, this is essentially a two-plate capacitor. The second spherical surface is at infinity with V=0.

Energy for a parallel plate capacitor

Calculate the work done to accumulate an additional charge dq on the plates if the plates are already charged.

The plates have an initial charge of q. Then

$$V = \frac{q}{C}$$

The work done to transfer an additional charge dq is given by

$$dW = Vdq$$

The work required to charge a capacitor from zero to Q is

$$W = \int_0^Q Vdq = \int_0^Q \frac{q}{C} dq = \frac{Q^2}{2C}$$

The energy stored in the capacitor is then

$$U = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV \qquad V = \frac{Q}{C}$$

Energy density for a parallel plate capacitor is given by

$$u = \frac{U}{\text{volume}} = \frac{CV^2}{2Ad} = \frac{\varepsilon_0 \frac{A}{d}\left(Ed\right)^2}{2Ad} = \frac{1}{2}\varepsilon_0 E^2$$

$$\boxed{u = \frac{1}{2}\varepsilon_0 E^2}$$

This is a general result!