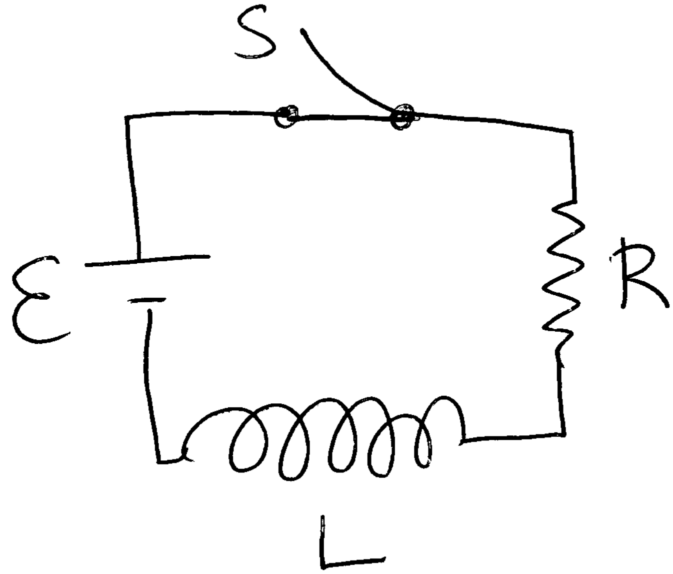
# LR Circuits: current growth and decay.




At $t=0$ S is closed.

$i$ is initially zero.

$i(0) = 0$

loop rule: $\frac{R\mathcal{E}}{R} - iR - L\frac{di}{dt} = 0$

$$-\frac{L}{\cancel{R}}\frac{di}{dt} = \cancel{R}\left(i - \frac{\mathcal{E}}{R}\right) \qquad I = i - \frac{\mathcal{E}}{R}$$

$$\frac{dI}{dt} = \frac{di}{dt}$$

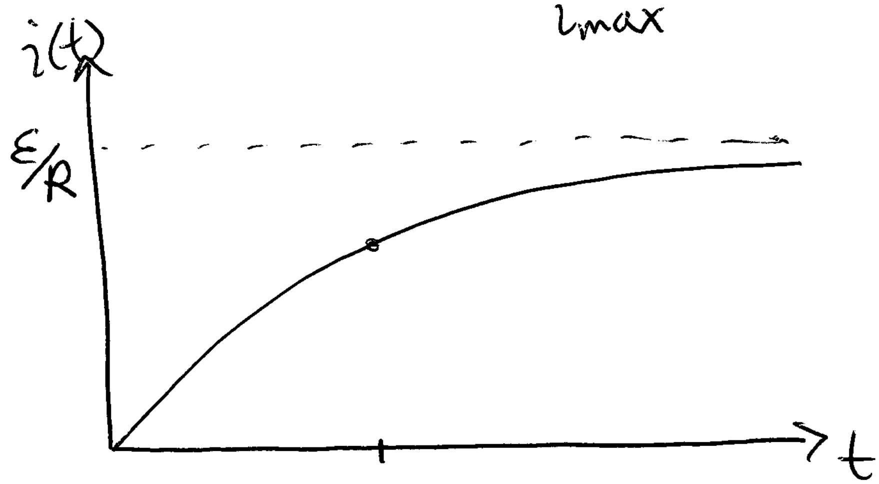
$$+\frac{\cancel{L}}{\cancel{L}}\frac{dI}{\cancel{dt}\,I} = -\frac{R}{L}\cancel{I}\,dt \rightarrow \int_{-\mathcal{E}/R}^{I_f}\frac{dI}{I} = \int_0^t -\frac{R}{L}dt$$

$$\ln\left(\frac{I_f}{-\mathcal{E}/R}\right) = -\frac{R}{L}t$$

$$\frac{I_f}{-\mathcal{E}/R} = e^{-\frac{Rt}{L}}$$

$$i(t) - \mathcal{E}/R = -\frac{\mathcal{E}}{R} e^{-\frac{Rt}{L}}$$

$$i(t) = \underbrace{\frac{\mathcal{E}}{R}}_{i_{max}} \left(1 - e^{-\frac{Rt}{L}}\right) \qquad \tau_{RL} = \frac{L}{R}$$




How much current is there after some time T?

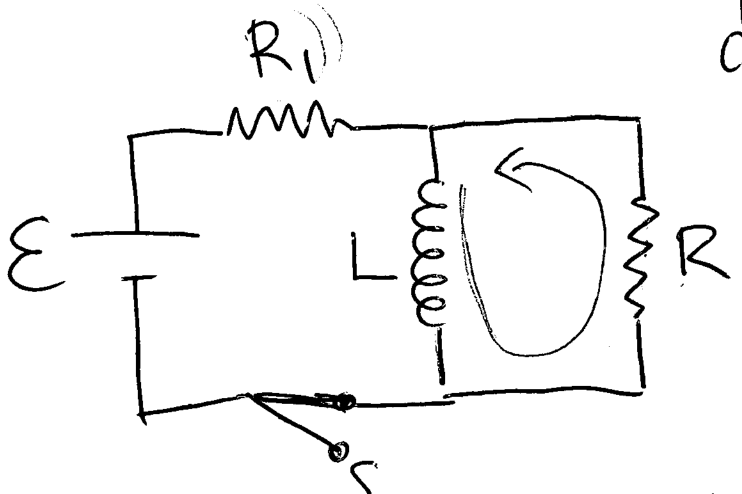
$$i(T) = \frac{\mathcal{E}}{R}\left(1 - e^{-RT/L}\right)$$

at any given time $\mathcal{E} - iR - L\frac{di}{dt} = 0$

$$i = \frac{\mathcal{E}}{R}\left(1 - e^{-Rt/L}\right) \qquad \frac{di}{dt} = \frac{\mathcal{E}}{\cancel{R}}\left(+\left(+\frac{\cancel{R}}{L}\right)e^{-Rt/L}\right)$$

$$\mathcal{E} - \cancel{R}\left(\frac{\mathcal{E}}{\cancel{R}}\left(1 - e^{-Rt/L}\right)\right) - \cancel{L}\left(\frac{\mathcal{E}}{\cancel{L}} e^{-Rt/L}\right) \stackrel{?}{=} 0$$

decay of current.




when switch is closed

$i = \dfrac{\mathcal{E}}{R_1}$ (initial current)

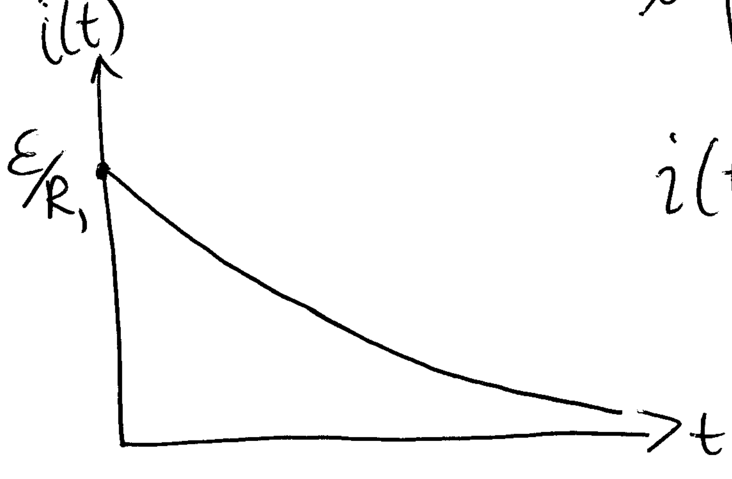
open switch at $t=0$: $i(0) = \dfrac{\mathcal{E}}{R_1}$

$$-L\frac{di}{dt} - iR = 0$$ Kirchhoff's loop rule.

$$-iR = L\frac{di}{dt} \Longrightarrow \int_{\mathcal{E}/R_1}^{i(t)} \frac{di}{i} = \int_0^t -\frac{R}{L}\,dt$$

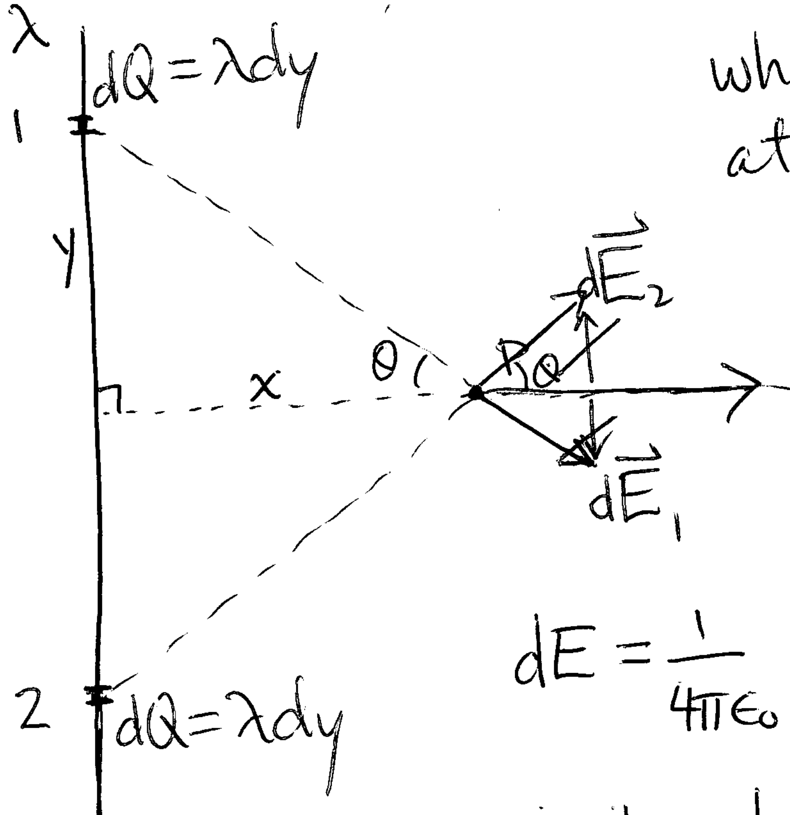
$$\ln\left(\frac{i(t)}{\mathcal{E}/R_1}\right) = -\frac{R}{L}t$$

$$i(t) = \frac{\mathcal{E}}{R_1} e^{-\frac{Rt}{L}}$$




like a discharging capacitor.

# Setting up integrals.




what is E-field at point P?

vertical cancel
horizontal add

$$dE = \frac{1}{4\pi\epsilon_0}\frac{dQ}{(y^2+x^2)}\cos\theta$$

similar triangles: $\cos\theta = \frac{x}{\sqrt{x^2+y^2}}$

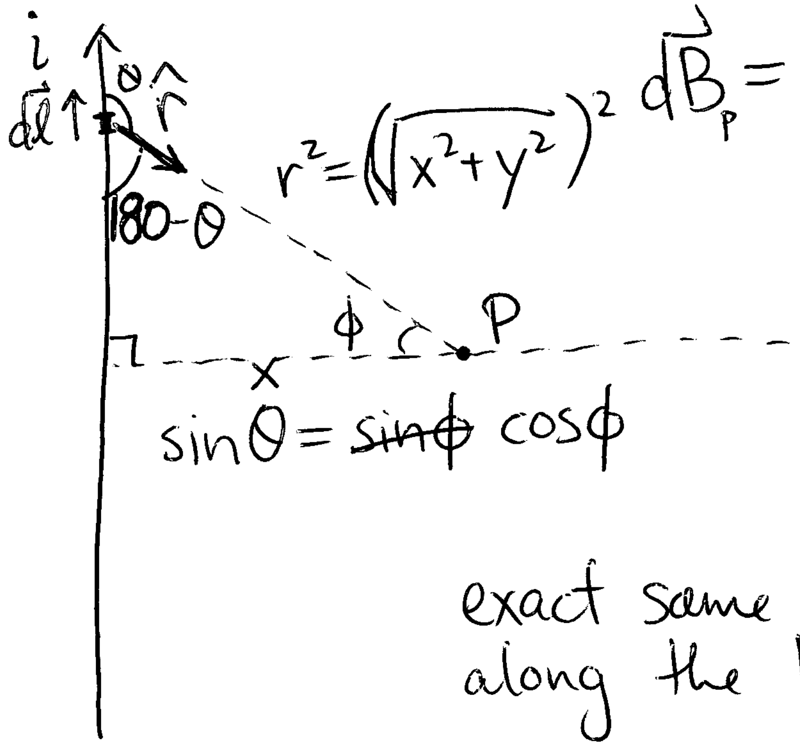
$$dE = \frac{1}{4\pi\epsilon_0}\frac{\lambda dy\, x}{(x^2+y^2)^{3/2}}$$

What about $V_P$? $V$ is a scalar.

$$dV = \frac{1}{4\pi\epsilon_0}\frac{dQ}{(x^2+y^2)^{1/2}}$$ (finite line)

$$dV = \frac{1}{4\pi\epsilon_0}\frac{\lambda\, dy}{(x^2+y^2)^{1/2}}$$ now integrate from $-L/2$ to $+L/2$

Magnetic field due to line of current:




$r^2 = \left(\sqrt{x^2+y^2}\right)^2$

$$d\vec{B}_P = \frac{\mu_0}{4\pi} \frac{i\, d\vec{l} \times \hat{r}}{r^2}$$

$d\vec{l} \times \hat{r} = ?$

$|d\vec{l} \times \hat{r}| = dl \sin\theta$

direction: $\otimes$

exact same direction everywhere along the line.

$\sin\theta = \cancel{\sin\phi} \cos\phi$

$\cancel{180} - \theta + \phi + 90 = \cancel{180}$

$\theta = 90 + \phi$

$\sin\theta = \sin(90+\phi)$

$= \sin 90 \cos\phi + \cancel{\cos 90 \sin\phi}$

$= \cos\phi$

$= \dfrac{x}{(x^2+y^2)^{1/2}}$

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{i\, \overset{dy}{dl}\, x}{(x^2+y^2)^{3/2}} \otimes$$