
Fill in these boxes with your SURNAME, a space, and your INITIALS.

University of Calgary

Faculty of Science

Final Exam Test - Solutions

PHYSICS 259 ALL LECTURE SECTIONS

April 21 2014, 12:00-15:00 **Time: 3 hours.**

DO NOT TEAR OFF THIS PAGE!

Answer all questions. There is an equation sheet and table of integrals on the last two pages. You may tear these two pages off if you wish.

This is a closed-book exam worth a total of 52 points. Use of the Schulich calculator or equivalent is allowed.

Write your Last Name and Initial on this top sheet in the grid above. (Do not write your ID number on this page.) **Also write your ID in the grid at top right of Page 2 of the Question paper. DO THIS NOW**.

Make sure this question paper booklet contains 26 pages. If you are missing any pages, get a new booklet from the exam supervisor.

You should also have a **separate set of Answer Sheets**. This is where you enter Multiple Choice answers of Part I and also detailed solutions to the problems of Part II. Only work entered in the indicated spaces on the Answer Sheets will be marked.

IMPORTANT: YOUR ID NUMBER IS TO BE ENTERED AT THE TOP OF EACH AND EVERY ONE OF THE ANSWER SHEETS. DO THIS NOW.

Begin working on the examination when instructed to do so by the supervisor.

UCID:

Part I: Multiple-Choice Questions (Total: 30 marks)

Enter answers to multiple choice questions on the first Answer Sheet using space provided in the upper right of the page. Each question in Part I is worth one point. You should complete Part I in about 90 minutes.

1) In Figure 1 the central point charge is $q_1 = +2Q$ and there is a net negative charge $-Q$ on the spherical conducting shell. Distance from the center of the sphere is r. Which of the following statements about the electric field magnitude E is true?

a) E is $\dfrac{Q}{4\pi\epsilon_0 r^2}$ for all $r < a$.

b) E is $\dfrac{Q}{4\pi\epsilon_0 r^2}$ for all $r > b$. $\Leftarrow \checkmark$

c) E is zero for all $r < b$.

d) E is $\dfrac{2Q}{4\pi\epsilon_0 r^2}$ for all $r > b$.

e) no other answer is correct

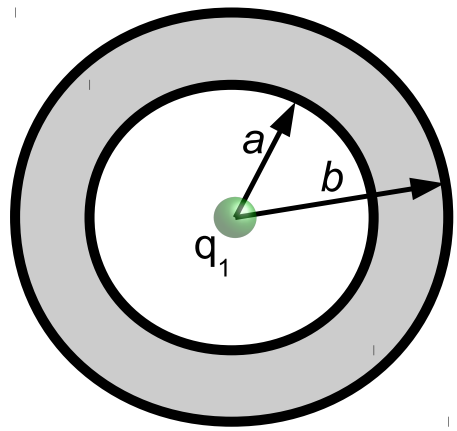

Figure 1: Spherical conducting shell with inner radius a and outer radius b. Point charge q_1 is located at the centre of the hollow shell.

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2) Figure 2 shows the electric potential $V(x)$ measured as a function of position along the x-axis. An electron is placed at x_2 and a proton is placed at x_4. Both charges are initially at rest ($v = 0$), then they are released and can move freely along the x-axis. Assume the only force on each charge is due to the electrostatic potential (the charges do not interact). At what locations will each charge be moving the fastest?

a) electron at x_1, proton at x_3 $\Leftarrow \checkmark$
b) electron at x_3, proton at x_5
c) both electron and proton at x_3
d) electron at x_2, proton at x_4
e) electron at x_1, proton at x_5

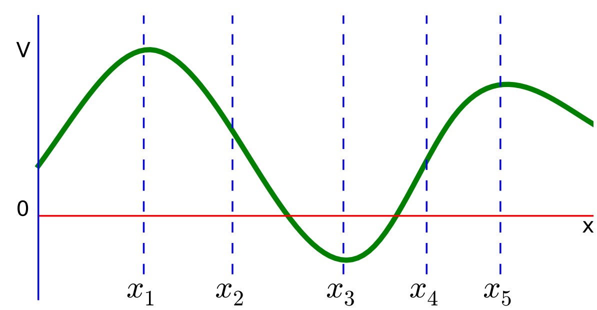

Figure 2: Potential profile along the x-axis.

3) A particle with charge $q = -1.0$ Coulombs and mass $m = 1$ kg is moving in the positive z-direction with a speed of $v = 5$ m/s. If the magnetic field is $\vec{B} = (3\hat{i} - 4\hat{j})$ T, then what is the magnetic force on the particle?

a) $(-20\hat{i} - 15\hat{j})\ N$ $\Leftarrow \checkmark$
b) $(+20\hat{i} + 15\hat{j})\ N$
c) $(+20\hat{i} - 15\hat{j})\ N$
d) $(-20\hat{i} + 15\hat{j})\ N$
e) $0\ N$

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4) A particle with mass m_1, charge q_1, and speed v_1 is moving perpendicular to a uniform magnetic field with magnitude $|\vec{B}|$. A second particle with $m_2 = m_1$, $q_2 = q_1$ and $v_2 = 3v_1$ is also moving perpendicular to $\vec{B}$. Which of the following statements about gyroradius (radius of the orbit) R and gyrofrequency (cyclotron frequency) ω is correct?

a) $R_1 < R_2$ and $\omega_1 = \omega_2$ $\Leftarrow \checkmark$
b) $R_1 = R_2$ and $\omega_1 = \omega_2$
c) $R_1 = R_2$ and $\omega_1 > \omega_2$
d) $R_1 > R_2$ and $\omega_1 = \omega_2$
e) none of the other answers are correct

---

5) A wire segment of length $L = 1.2$ metres carries a current of $I = 3.5$ Amperes towards the origin in the x-y plane at an angle $\theta = 30^\circ$ as shown in Figure 5. A magnetic field of $B = 0.5$ Tesla points out of the page ($+\hat{z}$). The magnetic force on the wire (in Newtons) is closest to

a) $+1.1\hat{i} + 1.8\hat{j}$
b) $-1.1\hat{i} + 1.8\hat{j}$ $\Leftarrow \checkmark$
c) $-1.8\hat{i} + 1.1\hat{j}$
d) $+1.1\hat{i} - 1.8\hat{j}$
e) $+1.8\hat{i} + 1.1\hat{j}$

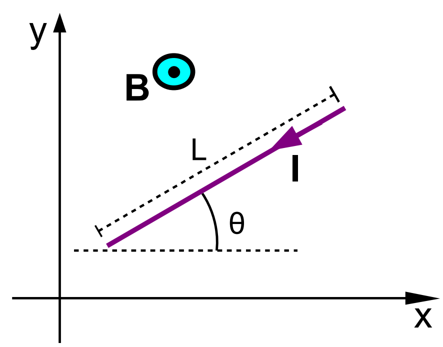

Figure 5: Current segment in a magnetic field.

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6) An infinitely long straight wire lies along the y-axis and carries a current in the positive y-direction. A positive point charge moves along the x-axis in the positive x-direction. What is the direction of the magnetic force acting on the moving charge when it is located at $x = 1$?

a) the positive x-direction.
b) the negative x-direction.
c) the positive y-direction. $\Leftarrow \checkmark$
d) the negative y-direction.
e) none of the above

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7) Three very long straight parallel wires each carry 4.0 Amperes of current directed out of the page as shown in Figure 7. The wires pass through the vertices of a right isoceles triangle with dimensions of H=2.0 cm. What is the magnitude of the magnetic field B produced by these currents at the midpoint of the hypotenuse (point P)?

a) 2.83×10^{-5} Tesla
b) 1.77×10^{-5} Tesla
c) 5.66×10^{-5} Tesla $\Leftarrow \checkmark$
d) 1.70×10^{-4} Tesla
e) 1.77×10^{-6} Tesla

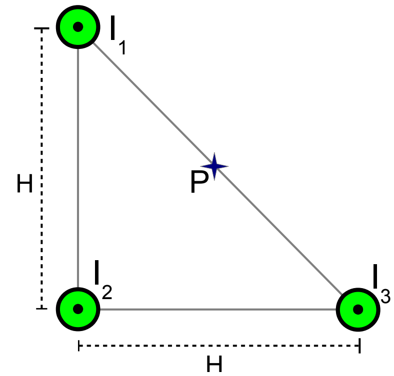

Figure 7: Three parallel currents.

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8) In Figure 8 we define the potential to be zero very far away from the sphere. An amount of positive charge $+2Q$ is placed on the spherical conducting shell. Which of the following is true?

a) the potential is constant for $r > b$.
b) the potential is zero everywhere inside the spherical shell ($r < a$).
c) the potential is zero in the conductor ($a < r < b$).
d) the potential is $\dfrac{Q}{2\pi\epsilon_0 r}$ for $r > b$. $\Leftarrow \checkmark$
e) none of the other answers is correct

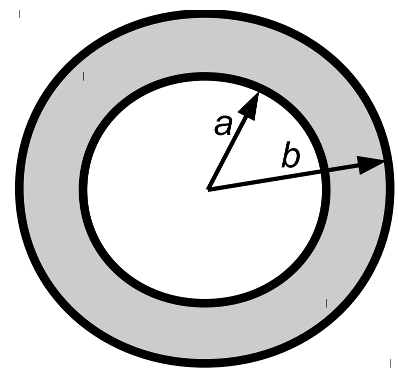

Figure 8: Spherical conducting shell with inner radius a and outer radius b.

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9) What are SI units for the time rate of change of magnetic flux?

a) Tesla metre second
b) (Coulomb metre^2) / second
c) Volt $\Leftarrow \checkmark$
d) Watt
e) Ampere

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10) For the circuit shown in Figure 10, assume that the magnetic field has a magnitude of 4 Tesla, pointing into the page. Use a value of $L = 2$ metre for the length of the sliding bar. If the bar is moving at 3 m/s, what is the EMF around the loop?

a) 12 volts clockwise
b) 12 volts counter-clockwise
c) 24 volts clockwise
d) 24 volts counter-clockwise $\Leftarrow \checkmark$
e) 0 V

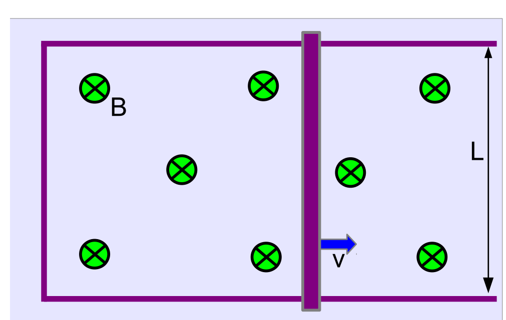

Figure 10: U-shaped conducting rails and a conducting bar of length L moving with speed v in the $+x$ direction.

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11) The graph in Figure 11 is a plot of power (in watts) vs voltage (in volts) for a particular material. What is the resistance of the material?

- **a)** $R = 1.0\,\Omega$
- **b)** $R = 0.1\,\Omega$ $\Leftarrow \checkmark$
- **c)** $R = 10\,\Omega$
- **d)** $R = 0.01\,\Omega$
- **e)** $R = 2.0\,\Omega$

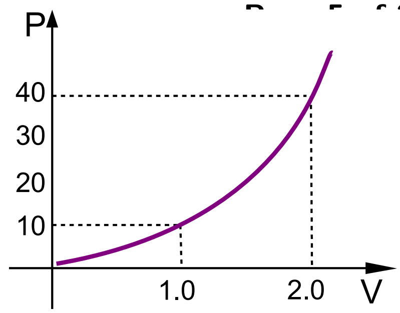

Figure 11: Voltage versus power.

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12) Two equal positive point charges, each of charge q, are separated by distance L. There are no other charges anywhere. The potential at the midpoint of the line joining the charges is defined to be zero. The electrostatic potential at an infinite distance from the two charges is

- **a)** $-\dfrac{1}{4\pi\epsilon_0}\dfrac{2q}{L}$
- **b)** $+\dfrac{1}{4\pi\epsilon_0}\dfrac{2q}{L}$
- **c)** $-\dfrac{1}{4\pi\epsilon_0}\dfrac{4q}{L}$ $\Leftarrow \checkmark$
- **d)** $+\dfrac{1}{4\pi\epsilon_0}\dfrac{4q}{L}$
- **e)** Zero

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13) The switch S in Figure 13 is closed at time t=0. When is the current in the circuit equal to 1.50 A? (Select the closest answer.)

- **a)** 0.231 s
- **b)** 8.32 s $\Leftarrow \checkmark$
- **c)** 1.39 s
- **d)** 1.73 s
- **e)** 2.88 s

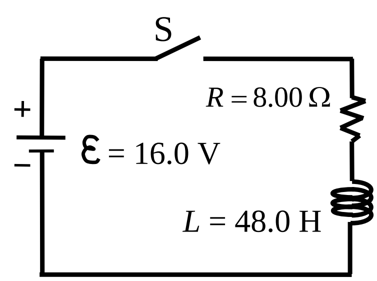

Figure 13: RL circuit.

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14) Figure 14 shows a uniform electric field with a magnitude of 6,000 V/m. Point "a" is at (x,y) = (2,2) and point "b" is at (6,5), measured in centimetres. What is the potential difference, Vab?

- **a)** 180 V $\Leftarrow \checkmark$
- **b)** 200,000 V
- **c)** 240 V
- **d)** 120,000 V
- **e)** 300 V

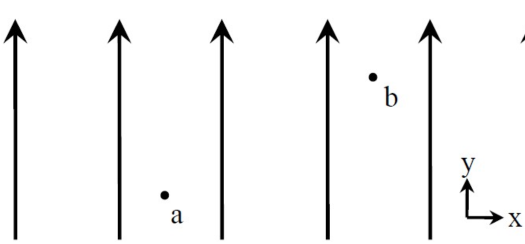

Figure 14: Uniform electric field.

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15) For the capacitor network in Figure 15, use the values $C_1 = 1\mu F$, $C_2 = C_3 = 2\mu F$, and $C_4 = C_5 = 4\mu F$. What is the equivalent (or effective) capacitance between points A and B?

a) $1.00\ \mu F$
b) $1.25\ \mu F$
c) $2.00\ \mu F$ $\Leftarrow \checkmark$
d) $4.00\ \mu F$
e) $0.80\ \mu F$

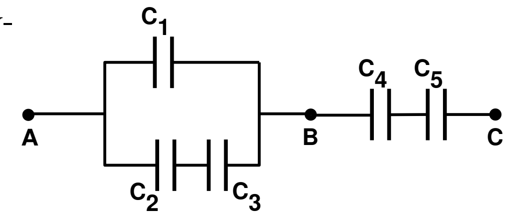

Figure 15: Capacitor network.

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16) Figure 16 shows a hollow sphere of inner radius r_1 and outer radius r_2. The solid part of the sphere carries a uniform volume charge density $\rho\ C/m^3$. What is the electric field strength outside the sphere, at radius $r > r_2$?

a) $\dfrac{\rho(r_2^3 - r_1^3)}{3r^2\,\epsilon_0}$ $\Leftarrow \checkmark$

b) $\dfrac{\rho(r_2 - r_1)^3}{3r^2\,\epsilon_0}$

c) $\dfrac{\rho\, r}{3\epsilon_0}$

d) $\dfrac{\rho\, r}{3\epsilon_0} - \dfrac{\rho\, r_1^3}{3r^2\,\epsilon_0}$

e) $\dfrac{\rho(r_2^3 - r_1^3)}{r^2\,\epsilon_0}$

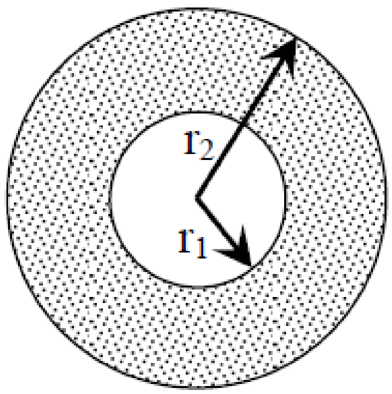

Figure 16: Hollow sphere with uniform volume charge density.

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17) The electric potential in a particular region of space is given by $V = -2xy + 3y^2$. Rounded to the nearest degree, the electric field vector at the point (x,y) = (1,2) points at an angle of

a) 22° above the $-x$ axis
b) 68° above the $+x$ axis
c) 22° below the $-x$ axis
d) 68° above the $-x$ axis
e) 68° below the $+x$ axis $\Leftarrow \checkmark$

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18) The electric field in a particular region of space is given by $\vec{E} = y^2\hat{\imath} + 2x\hat{\jmath}$. What is the electric potential difference, $V_{ab} \equiv V_a - V_b$, between point "a" at $(x_a, y_a) = (1, 2)$ and point "b" at $(x_b, y_b) = (3, 2)$?

a) $V_{ab} = +8V$ $\Leftarrow \checkmark$
b) $V_{ab} = -8V$
c) $V_{ab} = +24V$
d) $V_{ab} = -24V$
e) $V_{ab} = 0V$

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19) In Figure 19, two identical permanent magnets are dropped from equal heights above the ground at the same instant of time. There is a continuous ring of conducting material lying on the ground below magnet A. What happens? (Assume the two magnets are far enough apart that they do not influence each other, and the ground is nonconducting.)

a) Magnet A hits the ground before magnet B

b) Magnet B hits the ground before magnet A $\Leftarrow \checkmark$

c) Both magnets hit the ground at the same time

d) The answer depends on which pole of magnet A faces downwards

e) The answer depends on whether A and B are parallel or anti-parallel

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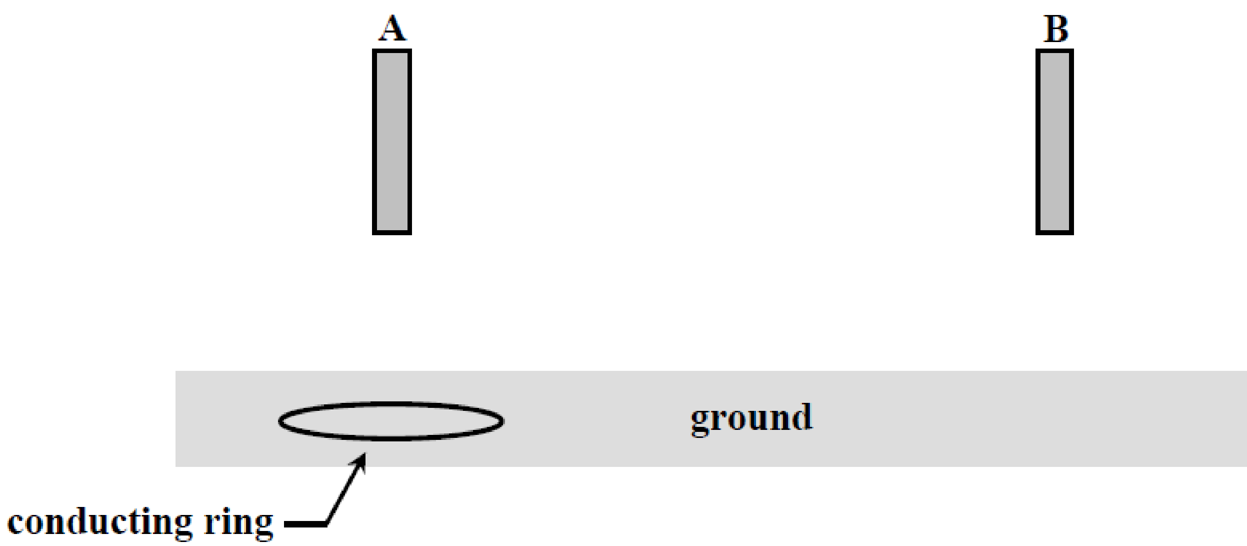

Figure 19: Two bar magnets.

20) In Figure 20, a particle of negative charge q is fired into a region of crossed electric and magnetic fields, with the electric field directed downwards between two charged parallel metal plates, and the magnetic field (denoted by the symbol X) directed into the page. The configuration is called a velocity selector and it allows particles of one specific speed, $v_0 = E/B$, to pass through undeflected (in a straight line). What happens if the particle is traveling faster than the required speed (i.e., $v > v_0$)?

a) It is deflected toward the bottom of the page $\Leftarrow$ $\checkmark$

b) It is deflected toward the top of the page

c) It is deflected out of the page

d) It is deflected into the page

e) It is slowed until its speed equals the required v_0

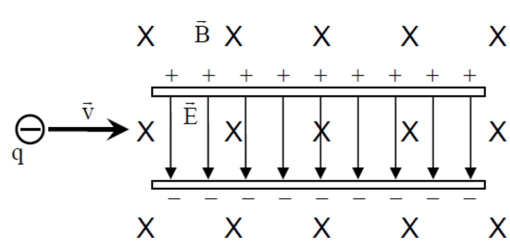

Figure 20: Velocity selector.

21) All capacitors in the circuit shown in Figure 21 are initially uncharged. What total charge flows through the battery after the switch, S, is closed?

a) 10.0 nC
b) 1.90 nC
c) 0.01 nC
d) 4.00 nC $\Leftarrow \checkmark$
e) 0.210 nC

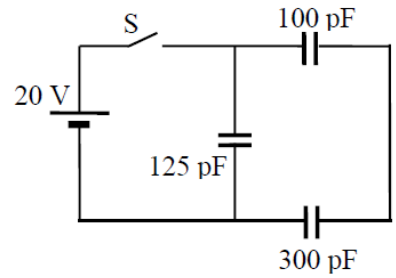

Figure 21: three capacitors.

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22) In Figure 22 the capacitor C is filled with a material of dielectric constant K = 5, and has a capacitance of 50.0 μF with the dielectric present. The switch, S, is closed until the capacitor is fully charged by a V = 9.00 volt battery, and then opened again. If the dielectric is now pulled completely out of the capacitor while S remains open, what will be the final potential difference across the capacitor?

a) 9.00 V
b) 1.80 V
c) 45.0 V $\Leftarrow \checkmark$
d) 0 V
e) 4.00 V

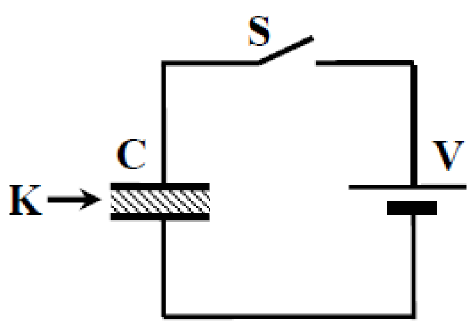

Figure 22

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23) What is the current in resistor R_3 in the circuit shown in Figure 23? (Choose the closest answer.)

a) 2.0 A $\Leftarrow \checkmark$
b) 4.0 A
c) 15 A
d) 11 A
e) 6.0 A

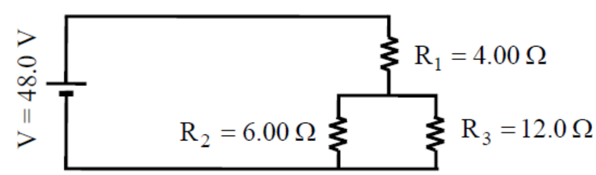

Figure 23: three resistors.

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24) The electric field in some region of space is uniform in magnitude and direction. Which one of the following five statements best describes the volume charge density (ρ) in this region of space?

a) $\rho = 0$ $\Leftarrow \checkmark$
b) ρ decreases linearly in the direction of the electric field
c) ρ increases linearly in the direction of the electric field
d) It is not possible to know anything about ρ from the information given
e) ρ has a uniform value throughout the region

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25) Equation 25 gives the electric field strength (E) on the axis of a thin ring of charge Q and radius r at a distance x from the centre of the ring.

$$E = \frac{1}{4\pi\epsilon_0}\frac{Q\,x}{(x^2+r^2)^{3/2}} \qquad (25)$$

Which of the following integrals correctly gives the electric field strength, E, on the axis of a uniformly-charged disk of radius R and surface charge density σ, at a distance x from the centre of the disk? [HINT: Consider the disk to be divided into thin rings, as illustrated in Figure 25]

a) $E = \int\limits_{r=0}^{R} \frac{4x\,\sigma\,r^2\,dr}{\epsilon_0(x^2+r^2)^{3/2}}$

b) $E = \int\limits_{r=0}^{R} \frac{x\,\sigma\,r^2\,dr}{4\epsilon_0(x^2+r^2)^{3/2}}$

c) $E = \int\limits_{r=0}^{R} \frac{2x\,\sigma\,r\,dr}{\epsilon_0(x^2+r^2)^{3/2}}$

d) $E = \int\limits_{r=0}^{R} \frac{x\,\sigma\,r\,dr}{2\epsilon_0(x^2+r^2)^{3/2}}$ $\Leftarrow \checkmark$

e) $E = \int\limits_{r=0}^{R} \frac{\sigma\,r^2\,dx}{4\epsilon_0(x^2+r^2)^{3/2}}$

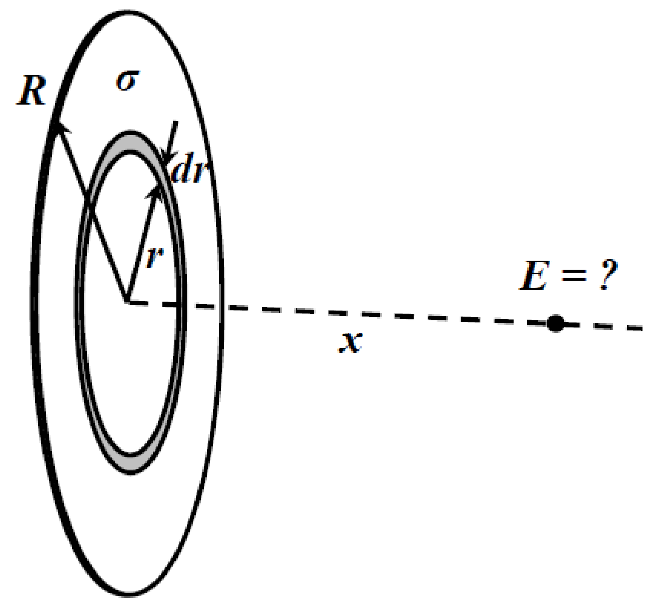

Figure 25

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26) When resistor R_1 and capacitor C_1 are used in an RC series circuit, the charging curve is given by the solid line in Figure 26. After R_1 and C_1 are replaced by R_2 and C_2, the charging curve is given by the dashed line. What can you say about R_1, R_2, C_1 and C_2?

a) $R_2 > R_1,\ C_1 = C_2$ $\Leftarrow \checkmark$
b) $R_1 = R_2,\ C_1 > C_2$
c) $R_1 = R_2,\ C_1 < C_2$
d) $C_1 = C_2,\ Q_1(final) > Q_2(final)$
e) $R_2 < R_1,\ C_1 = C_2$

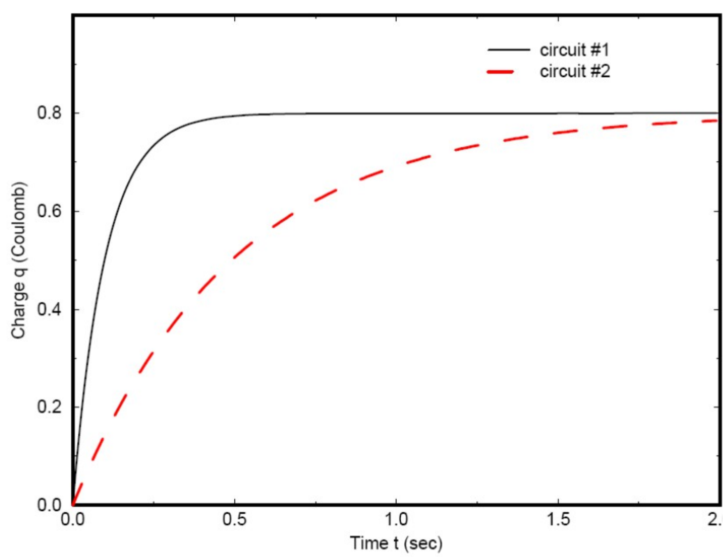

Figure 26

27) A cube of side 3.00 m has edges placed along the x, y, and z axes as shown in Figure 27. The electric field in this region of space is directed toward the right with a magnitude that varies with position, y, according to $E = 4.00y$ where y is in metres and E is in V/m. How much charge is contained within the cube? (Select the closest answer, to three significant figures.)

- **a)** $1.06 \times 10^{-10}\ C$
- **b)** $1.22 \times 10^{13}\ C$
- **c)** $9.56 \times 10^{-10}\ C$ ⇐ ✓
- **d)** $108\ C$
- **e)** $1.36 \times 10^{12}\ C$

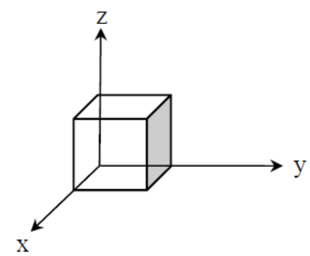

Figure 27

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28) In Figure 28, two particles of the same charge (q) are executing cyclotron motion in a region of uniform magnetic field directed into the page. The upper particle has mass m and travels at speed v. The lower particle has mass 2m and travels at speed 2v. If the period of motion of the upper particle is T, then what is the period of motion of the lower particle?

- **a)** T/2
- **b)** T/4
- **c)** T
- **d)** 2T ⇐ ✓
- **e)** 4T

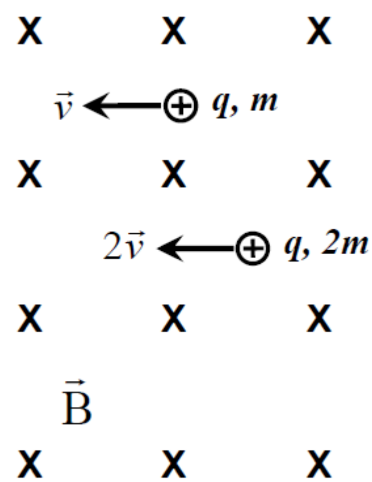

Figure 28

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29) In Figure 29 the long, straight wire carries a current with positive charge flow downwards, and the loop carries a current with positive charge flow counter-clockwise. Both the long wire and the loop are in the plane of the page. The loop experiences

- **a)** a torque trying to rotate side a toward you and side b away from you.
- **b)** a net force toward the right, away from the long wire
- **c)** no net force or torque
- **d)** a torque trying to rotate side a away from you and side b toward you.
- **e)** a net force toward the left, toward the long wire ⇐ ✓

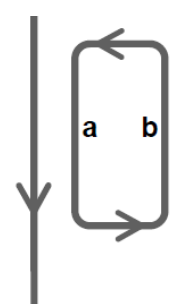

Figure 29

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30) Figure 30 shows two circular coils of wire. The larger coil has N_1 turns and radius a. The smaller coil (radius b) has N_2 turns, and is both coplanar and coaxial with the larger coil. The magnetic field strength at the centre of a circular coil of N turns and radius r is

$$B = \frac{\mu_0 N i}{2r}$$

Assume $b \ll a$, so that the magnetic field of the larger coil is approximately uniform over the area of the smaller coil. The mutual inductance of this combination is given by the expression

a) $\frac{\mu_0 \pi N_1 N_2 \, b}{a}$

b) $\frac{\mu_0 N_1 N_2}{2a}$

c) $\frac{\mu_0 \pi N_2 \, b^2}{2a}$

d) $\frac{\mu_0 N_1 N_2 \, b^2}{2a}$

e) $\frac{\mu_0 \pi N_1 N_2 \, b^2}{2a}$ $\Leftarrow \checkmark$

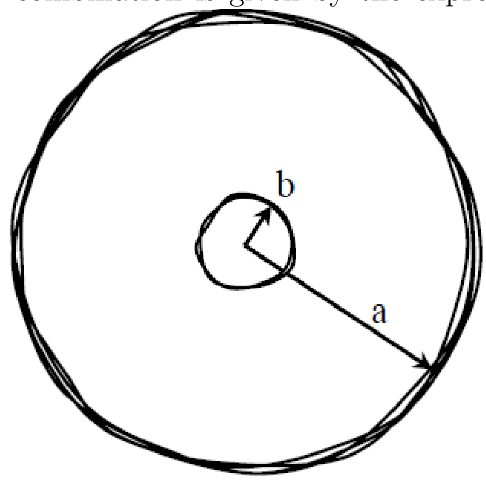


Figure 30: Two current carrying coils.

This is the end of the Multiple Choice part of the exam. You may now proceed to Part II. Remember that your answers to Part II are to be entered on the Answer Sheet pages.

Part II: Written Answer Questions (Total: 22 marks)

IMPORTANT: Write your answers to the problems in Part II in the corresponding boxes on the Answer Sheets. Work must be shown for full marks. Rough work can be done on the back of this question paper, but only the work appearing on the Answer Sheets will be marked.

31) *[7.0 marks]* A loop with dimensions shown in Figure 31a is initially located in a region with no magnetic field. It moves at constant speed v in the positive x-direction, and eventually enters into a different region with constant uniform magnetic field B pointing into the page. There are three important transition times:

$\mathbf{t_0}$ when the right edge of the loop is at $x = 0$ (just entering the magnetic field region) as shown in Figure 31a

$\mathbf{t_1}$ when the right edge is at $x = w_1$, so the wider portion is inside the field region and the narrower portion is outside

$\mathbf{t_2}$ when the right edge is at $x = w_1 + w_2$ and the left edge is at $x = 0$ (entire loop completely inside the field region)

For convenience we can set $t_0 = 0$.

- **a)** *[2.0 mark]* Calculate the magnetic flux Φ_B through the loop at two times: t_1 and t_2.
- **b)** *[1.0 mark]* Draw a graph of Φ_B during the time range starting before t_0 and ending after t_2. Use the axes in Answer 31b and add appropriate labels.
- **c)** *[3.0 mark]* Calculate the EMF induced in the loop during each of these four intervals: $t < 0$, $0 < t < t_1$, $t_1 < t < t_2$, and $t > t_2$
- **d)** *[1.0 mark]* Draw a graph of EMF during the time range starting before t_0 and ending after t_2.

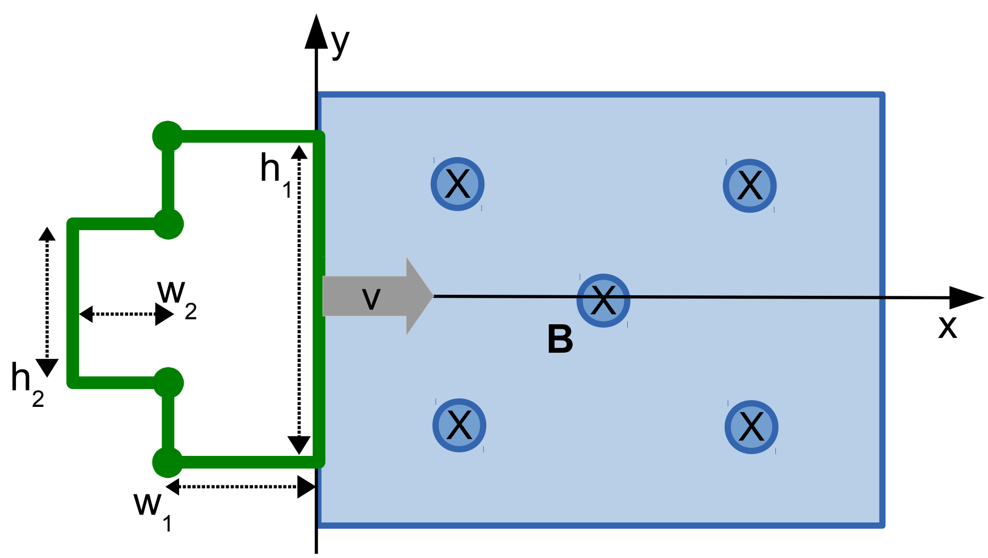


Figure 31a: Loop moving into a uniform magnetic field.

32) *[6.0 marks]* The loop in Figure 32 has a total current $I = 500\ mA$ flowing clockwise in a constant uniform magnetic field of $B = -3\,\hat{i}\ nT$. The curved segment is a half-circle with diameter $w_1 = 10\ cm$ and the rectangular portion has dimensions $w_2 = 5\ cm$ and $h_2 = 3\ cm$.

a) *[1.0 mark]* What is the net force on the entire loop?

b) *[1.0 mark]* Calculate the force on the horizontal linear segment of length w_2 from $(x = -\frac{1}{2}w_2, y = h_2)$ to $(x = +\frac{1}{2}w_2, y = h_2)$.

c) *[1.0 mark]* Calculate the force on the vertical linear segment of length h_2 from $(x = +\frac{1}{2}w_2, y = 0)$ to $(x = +\frac{1}{2}w_2, y = h_2)$.

d) *[1.0 mark]* What is the net force on the semi-circular segment?

e) *[1.0 mark]* Calculate the magnetic moment of the loop.

f) *[1.0 mark]* Calculate the torque on the loop.

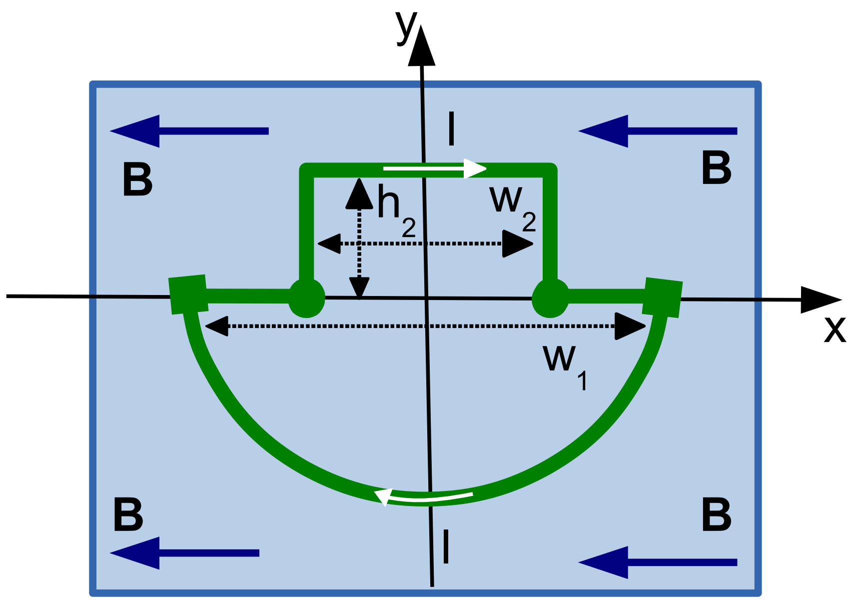


Figure 32: Current loop in a uniform magnetic field.

33) *[5.0 marks]* A 9 V battery is connected across points A and F in Figure 33, causing currents to flow through a network of steel wires. The wires have resistivity of $2 \times 10^{-7}\ \Omega\, m$ and circular cross-sections with 2 mm radius. Point "G" is on the y-axis at $y_G = -10\ cm$, all other indicated points are on the x-axis: $x_A = -20\ cm$, $x_B = -10\ cm$, $x_C = -5\ cm$, $x_D = +5\ cm$, $x_E = +10\ cm$, $x_F = +20\ cm$. **What is the current I_1?**

a) *[1.0 mark]* Briefly outline the steps required to answer this question.

b) *[4.0 mark]* Calculate the value of I_1, following the sequence of steps you outlined in part a.

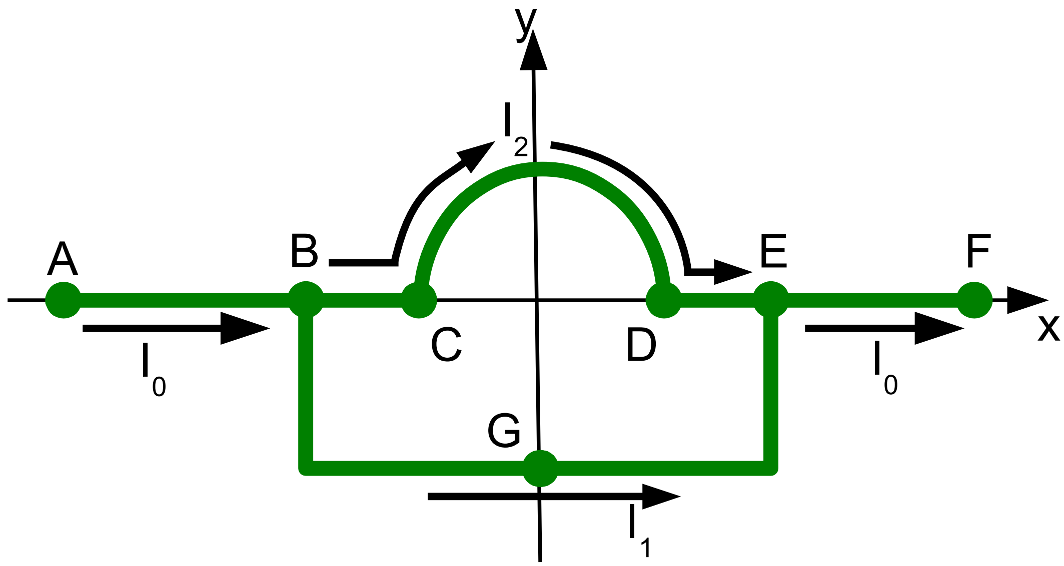


Figure 33: Current flowing along a system of conducting wires. All wire segments are straight lines except for the curve between C and D which is a half-circle.

34) *[4.0 marks]* Figure 34 shows a portion of a very long solid insulating cylinder of radius R that carries a positive charge density $\rho\ C/m^3$ distributed uniformly throughout its volume.

a) *[0.5 mark]* What is the direction of the electric field in the region $r < R$?

b) *[3.5 mark]* Write down Gauss's law and use it to obtain an expression for the magnitude of the electric field in the region $r < R$ (inside the cylinder) in terms of ρ and the distance r from the axis of the cylinder. Draw a diagram to illustrate how you are applying Gauss's law, show all steps and explain your reasoning.

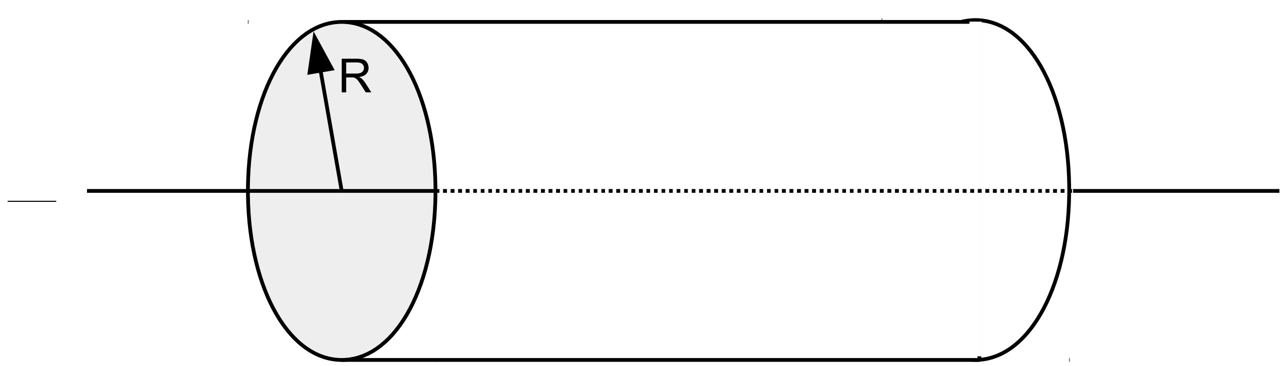


Figure 34: Cylinder of charge.

CONSTANTS AND USEFUL EQUATIONS

k = Coulomb constant = 8.99×10^{9} N m^2 C^{-2}
ϵ_0 = permittivity of free space = 8.85×10^{-12} C^2 N^{-1} m^{-2}
μ_0 = permeability of free space = $4\pi \times 10^{-7}$ Wb A^{-1} m^{-1}
e = fundamental charge = 1.602×10^{-19} C
m_e = mass of electron = 9.11×10^{-31} kg
m_p = mass of proton = 1.67×10^{-27} kg
K_{air} = dielectric constant of air = 1.00059

m = 10^{-3} $\mu = 10^{-6}$ n = 10^{-9} p = 10^{-12}

Area of a circle: $A = \pi r^2$
Surface area of a sphere: $A = 4\pi r^2$
Surface area of a cylinder: $A = 2\pi r L$
Circumference of a circle: $C = 2\pi r$
Volume of a sphere: $V = \frac{4}{3}\pi r^3$
Volume of a cylinder: $V = \pi r^2 L$

$$x = x_0 + v_{x0}t + \frac{1}{2}a_x t^2 \qquad v_x = v_{x0} + a_x t \qquad v_x^2 = v_{x0}^2 + 2a_x x \qquad \vec{F} = m\vec{a}$$

$$\vec{F} = k\frac{q_1 q_2}{r^2}\,\hat{r} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}\,\hat{r} \qquad \vec{E} = \frac{\vec{F}}{q} \qquad \vec{E} = k\frac{q}{r^2}\,\hat{r} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\,\hat{r}$$

$$\Phi_E = \oint_A \vec{E}\cdot d\vec{A} = \oint_A E dA cos\theta = \frac{Q_{encl}}{\epsilon_0} \qquad V = \frac{U}{q} \qquad U = k\frac{q_1 q_2}{r} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r}$$

$$V = k\frac{q}{r} = \frac{1}{4\pi\epsilon_0}\frac{q}{r} \qquad W = qV_{ab} \qquad V_{ab} = V_a - V_b = \int_a^b \vec{E}\cdot d\vec{l}$$

$$\vec{E} = -\vec{\nabla}V = -\left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)V \qquad C = \frac{Q}{V_{ab}} \qquad C = \frac{\epsilon_0 A}{d}$$

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}QV = \frac{1}{2}CV^2 \qquad \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \qquad C = C_1 + C_2 + C_3$$

$$C = KC_0 = K\epsilon_0\frac{A}{d} = \epsilon\frac{A}{d} \qquad u = \frac{1}{2}\epsilon_0 E^2 \qquad P = I^2 R = \frac{V^2}{R}$$

More Equations on Next Page

CONSTANTS AND USEFUL EQUATIONS (Continued)

$$V = IR \qquad P = \mathcal{E}I \qquad \frac{1}{R} = \frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3} \qquad R = R_1+R_2+R_3$$

$$R = \frac{\rho L}{A} \qquad \sum i = 0 \qquad \sum (\mathcal{E} + iR) = 0 \qquad nq = -\frac{J_x B_y}{E_z} \qquad \vec{J} = \sum_i n_i q_i \vec{v}_i$$

$$\vec{F} = q\vec{v}\times\vec{B} \qquad \Phi_B = \int \vec{B}\cdot d\vec{A} \qquad r = \frac{mv}{qB} \qquad \vec{F} = I\vec{l}\times\vec{B} \qquad \vec{\mu} = I\vec{A}$$

$$\vec{\tau} = \vec{\mu}\times\vec{B} \qquad \vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2} \qquad d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{l}\times\hat{r}}{r^2} \qquad U = -\vec{\mu}\cdot\vec{B}$$

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{encl} \qquad \mathcal{E}_2 = -N_2\frac{d\Phi_{B2}}{dt} \qquad \mathcal{E}_2 = -M\frac{di_1}{dt} \qquad \mathcal{E} = -L\frac{di}{dt}$$

$$\mathcal{E} = \int_a^b \left(\vec{v}\times\vec{B}\right)\cdot d\vec{l} \qquad M = \frac{N_2\Phi_2}{i_1} = \frac{N_1\Phi_1}{i_2} \qquad L = \frac{N\Phi}{i} \qquad u = \frac{B^2}{2\mu_0}$$

$$\tau = RC \qquad \tau = \frac{L}{R} \qquad x = x_0 e^{-\frac{t}{\tau}} \qquad x = x_0\left(1 - e^{-\frac{t}{\tau}}\right) \qquad U = \frac{1}{2}LI^2$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin\frac{x}{a} \qquad \int \frac{dx}{\sqrt{x^2+a^2}} = ln\left(x + \sqrt{x^2+a^2}\right)$$

$$\int \frac{dx}{x^2+a^2} = \frac{1}{a}\arctan\frac{x}{a} \qquad \int \frac{dx}{\left(x^2+a^2\right)^{3/2}} = \frac{1}{a^2}\frac{x}{\sqrt{x^2+a^2}}$$

$$\int \frac{xdx}{\left(x^2+a^2\right)^{3/2}} = -\frac{1}{\sqrt{x^2+a^2}}$$

Multiple Choice Answer Sheet - Version A

#1 (b)
#2 (d)
#3 (a)
#4 (c)
#5 (d)
#6 (c)
#7 (d)
#8 (d)
#9 (b)
#10 (b)

#11 (e)
#12 (a)
#13 (b)
#14 (a)
#15 (e)
#16 (d)
#17 (e)
#18 (d) Paul's bubble sheet goes here
#19 (e)
#20 (b)

#21 (e)
#22 (a)
#23 (b)
#24 (a)
#25 (e)
#26 (d)
#27 (e)
#28 (d)
#29 (e)
#30 (b)

Written Answer Sheets

Phys 259 Final Exam (Winter, 2014): Instructions to Markers

You are responsible for marking the Problems, Part II. The multiple choice answers are scored automatically. The "Version" number applies only to the multiple-choice answers; there is only one version of Part II. You should also have these general instructions, plus a set of solutions with detailed instructions, and a copy of an exam question paper. The solutions are presented in Answer Page format for ease of reference.

Print out these Instructions, and read them before starting to mark. You might be tempted to bypass these, but you will save us much time and trouble if you follow instructions.

On the following pages, I've given a suggested detailed marking scheme for each question. However, not all solutions will follow the one I've given, especially for the 2-mark and 3-mark questions, so use your judgment. The following broad guidelines may help:

- In 3-mark questions, the first half mark should be easy to get: if a student shows any reasonable insight at all, give a half mark. In an integration problem, if the student simply makes a valiant attempt at integrating, this might be enough.
- Give half of the allotted marks (e.g., 1.5 marks out of 3) if you feel that the student has got half way to the right answer. For example, if a student makes five mistakes but still has the solution half right, give 1.5; don't deduct five half-marks.
- In numerical questions, the final answer must have correct units. Deduct 0.5 mark if the units on the final answer are not shown or are incorrect.
- Be lenient on significant figures.
- vectors $\neq$ scalars, subtract 0.5 (once for any problem) eg. $\vec{E} = kqr$ or $E = kq\hat{r}$
- If a lot of students are making the same "mistake", check my answer - I could be the one who is wrong! If you discover any errors in my solutions, please let me know as soon as possible (physics259@ucalgary.ca) so I can alert all other markers.

Q31a *[2.0 mark]*

Magnetic flux through a surface is defined as $\Phi_B = \int \vec{B} \cdot d\vec{A}$. NOT $\oint \vec{B} \cdot d\vec{A}$ unless it is a closed surface. For a flat loop and a constant magnetic field the flux is simply $\Phi_B = \int BdA\ cos\theta = B\cos\theta \int dA = BA\cos\theta$ where A is the total area and $\cos\theta$ is the angle between $\vec{B}$ and the surface normal vector $\vec{A}$. +0.5 for writing correct formula

A flat loop is not a closed surface, so we must choose either of two directions $\pm\hat{k}$. Here I pick $\hat{A} = -\hat{k}$ to be parallel to $\vec{B}$ so $\Phi_B = AB$ is positive, but the other choice is also valid. -0.5 if no explicit choice, which is a very common mistake.

At t_1 the wider potion is inside the field region and the narrower part is outside so

$$\Phi(t_1) = A_1 B + A_2\, 0 = h_1 \times w_1 \times B$$

At t_2 the entire loop is in the constant magnetic field so

$$\Phi(t_2) = A_1 B + A_2 B = h_1 \times w_1 \times B + h_2 \times w_2 \times B$$

+0.5 for each answer. Note that flux is a scalar, not a vector.

Q31b *[1.0 mark]*

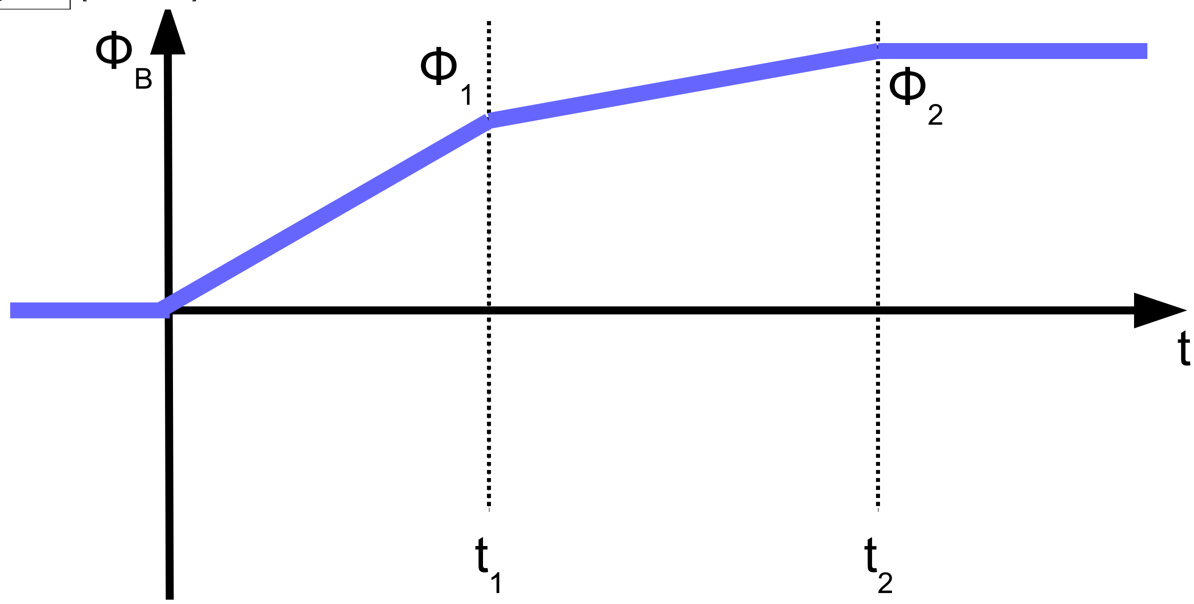


Figure 31b: Magnetic flux through a moving loop.
Existing labels are sufficient, additional $\Phi_{1,2}$ nice but not required.
+0.5 for different slopes between 0,t1 and t1,t2
+0.5 for constant (non-zero) flux for $t > t_2$
should have flux=0 for $t < 0$, but hard to see so ignore

Continued on next page

Q31c *[3.0 mark]* two possible approaches: Faraday (this page) & motional EMF (next page)

Electromotive force (EMF) induced by changing magnetic flux through a loop is given by $\mathcal{E} = -\frac{d\Phi_B}{dt}$. In general both the field and the loop can change $\frac{d\Phi_B}{dt} = \frac{d}{dt}(AB\cos\theta)$, but for a constant $\vec{B}$ and flat loop oriented as given we get a simple result $\mathcal{E} = -\frac{dA}{dt}B$

When the loop is entirely outside ($t < 0$) the magnetic field region then flux through the loop is zero and $\mathcal{E} = 0$. 0.5 for correct answer with even a few words, 0 for just "zero"

When the loop is entirely inside ($t > t_2$) the magnetic field region then flux through the loop is constant ($\Phi_B = AB$) and $\mathcal{E} = 0$. 0.5 for correct answer

When the loop is first entering the magnetic field region ($0 < t < t_1$) then the area inside increases with time as $dA = h_1\,dw = h_1\,v\,dt$ so $dA/dt = h_1\,v$ and the EMF is [0.5 for correct magnitude] (see below for direction)

$$\mathcal{E} = -\frac{dA}{dt}B = -h_1\,v\,B \text{ clockwise}$$

When the loop is almost entirely inside the magnetic field region ($t_1 < t < t_2$) then the area inside increases with time as $dA = h_2\,dw = h_2\,v\,dt$ so $dA/dt = h_2\,v$ and the EMF is [0.5 for correct magnitude]

$$\mathcal{E} = -\frac{dA}{dt}B = -h_2\,v\,B \text{ clockwise}$$

[0.5 for correct directions, 0.5 for justification] The direction of positive EMF comes from our previous definition of $\vec{A}$ pointing <u>into</u> the page, so by the right-hand rule clockwise (viewed from above) will be positive. To get the actual direction we must include the minus sign, so **EMF will be counter-clockwise**. This will drive ccw currents in the loop , which will produce $\vec{B}$ inside the loop pointing out of the page. This opposes the initial field as required by Lenz's law.

Q31d *[1.0 mark]*

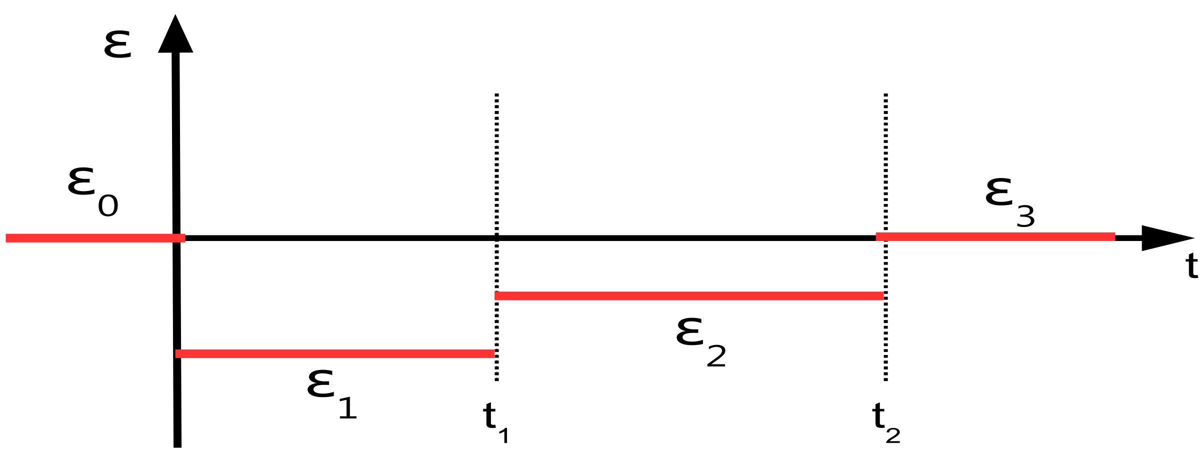


Figure 31d: EMF induced in a moving loop.

Axis labels ($\mathcal{E}, t$) and times t_1, t_2 required [+0.5] ; all others optional

+0.5 for $|\mathcal{E}_1| > |\mathcal{E}_2|$, $\mathcal{E}_3 = 0$

Continued on next page

Q31c *[3.0 mark]* two possible approaches: Faraday (previous page) & motional EMF (this page)

The EMF induced around a conducting loop moving in a magnetic field is

$$\mathcal{E} = \oint (\vec{v} \times \vec{B}) \cdot d\vec{l}$$

where we can break the loop down into vertical (y) & horizontal (x) segments. For the horizontal segments the EMF is zero

$$\mathcal{E}_x = \int \left[v(\hat{i}) \times B(-\hat{k})\right] \cdot (dl\hat{i}) = vB(\hat{j} \cdot \hat{i})dl = 0$$

For the vertical segments the EMF is

$$\mathcal{E}_y = \int \left[v(\hat{i}) \times B(-\hat{k})\right] \cdot (dl\hat{j}) = vB(\hat{j} \cdot \hat{j})dl = vBL$$

When the loop is entirely outside ($t < 0$) the magnetic field region, EMF is zero on all segments and net $\mathcal{E} = 0$. 0.5 for correct answer with even a few words, 0 for just "zero"

When the loop is first entering the magnetic field region ($0 < t < t_1$) then the only EMF comes from the long vertical segment

$$\mathcal{E} = h_1\, v\, B$$

but we need to get the direction (up or down?). Lenz's law "wants" flux inside the loop not to change, which requires current moving up. Thinking about force on a single proton (positive charge carrier) gives the same result. **EMF around the entire loop will be counterclockwise.** +1.0 for correct direction with some justification, +0.5 for right answer with no reason,+0.5 for wrong answer with right reason.

When $t > t_1$ there will be two more short vertical segments, each of length $\frac{h_1 - h_2}{2}$ contributing EMF that also points up (+y), so the net EMF around the loop is reduced

$$\mathcal{E} = h_1\, v\, B - 2\left[\frac{h_1 - h_2}{2}\right] v\, B = h_2 v B$$

When the loop is entirely inside ($t > t_2$) the magnetic field region then the trailing edge cancels out everything and $\mathcal{E} = 0$.

Continued on next page

Q32a *[1.0 mark]*

The net force on a current loop in a uniform magnetic field is always zero. Invoking symmetry (each element balanced by another) is acceptable.

only 0.5 for just writing "0"

Q32b *[1.0 mark]*

The force on a straight current in a uniform magnetic field is $\vec{F} = I\vec{L} \times \vec{B}$. For this case $\vec{B} = B(-\hat{i})$ and $\vec{L} = w_2\hat{i}$ so the force will be zero

$$\vec{F} = I(w_2\hat{i}) \times (-B\hat{i}) = -Iw_2B(\hat{i} \times \hat{i}) = 0$$

simply noting that $\vec{F}||\vec{v}$ or $\vec{F}||\vec{dl}$ is okay

Q32c *[1.0 mark]*

The force on a straight current in a uniform magnetic field is $\vec{F} = I\vec{L} \times \vec{B}$. For this case $\vec{B} = B(-\hat{i})$ and $\vec{L} = h_2(-\hat{j})$ so

$$\vec{F} = I(-h_2\hat{j}) \times (-B\hat{i}) = Ih_2B(\hat{j} \times \hat{i}) = Ih_2B(-\hat{k}) = -Ih_2B\hat{k}$$

-0.5 if numbers are calculated incorrectly

$$\vec{F} = (0.5\ A)\ (0.03\ m)\ (3 \times 10^{-9}\ T)\ (-\hat{k}) = 4.5 \times 10^{-11}\ N\ (-\hat{k})$$

result is vector with units

Continued on next page

Q32d *[1.0 mark]*

Short answer: net force is zero (part a), force on horizontal segments is zero (part b), force on vertical segments will cancel (part c revisited), so force on curved segment must be zero. Symmetry argument is acceptable, so is breaking down into small horizontal & vertical segments.

Actually calculating force requires expressing the length vector orientation as a function of path eg. $d\vec{L} = R(-\sin\theta\hat{i} - \cos\theta\hat{j})d\theta$ where θ sweeps out an angle from 0 to 180° (π radians). The force on a small portion will always point in the z-direction

$$d\vec{F} = I\vec{L} \times \vec{B} = Iw_1/2(-\sin\theta\hat{i} - \cos\theta\hat{j}) \times (-B\hat{i}) = Iw_1/2B\cos\theta d\theta(-\hat{k})$$

and ends up cancelling over the entire semicircle

$$\int d\vec{F} = Iw_1/2B\int_0^\pi \cos\theta d\theta = Iw_1/2B\sin\theta|_0^\pi = 0$$

Q32e *[1.0 mark]*

The magnetic moment of a current loop is defined as $\vec{\mu} = I\vec{A}$ where $|\vec{A}|$ is the loop area and I is the current.

For this loop

$$A = \frac{1}{2}\pi R^2 + hw = \frac{\pi w_1^2}{8} + h_2w_2 = 3.93 \times 10^{-3} + 1.5 \times 10^{-3} = 5.43 \times 10^{-3}\ m^2$$

so the magnetic moment is

$$\vec{\mu} = -I\left(\frac{\pi w_1^2}{8} + h_2w_2\right)\hat{k} = 2.71 \times 10^{-3}\ A \cdot m^2\,(-\hat{k})$$

where the direction is defined in terms of clockwise current direction.

Q32f *[1.0 mark]*

Torque on a current loop in a magnetic field is defined as $\vec{\tau} = \vec{\mu} \times \vec{B}$

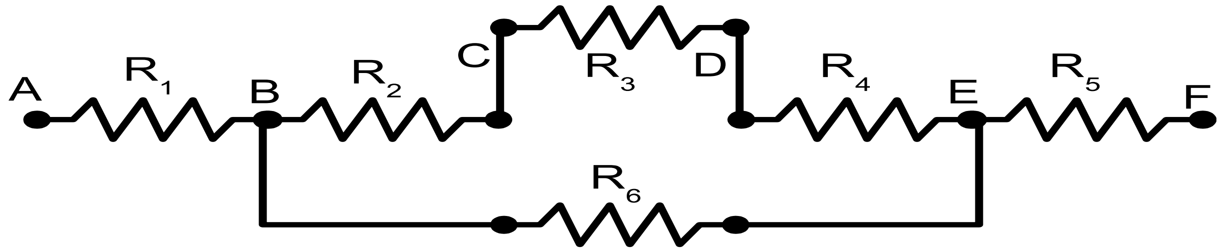
For this case

$$\vec{\tau} = I\left(\frac{\pi w_1^2}{8} + h_2w_2\right)(-\hat{k}) \times (-B\hat{i}) = IB\left(\frac{\pi w_1^2}{8} + h_2w_2\right)\hat{j} = 8.15 \times 10^{-12}\ N \cdot m\,\hat{j}$$

Continued on next page

Q33a *[1.0 mark]*
This is a simple resistor network, so we can replace it with a single equivalent resistor, use Ohm's law to find total current I_0, and then solve for the branch with I_1. First step is to use resistivity to calculate resistance of each segment. Second step is to merge segments together in series and parallel. Third step is to find I_0. Fourth step is to find potential drop V_{BE}. Final step is to find current I_1 flowing between B and E. 0 for nothing, 1/2 for plausible but unclear, 1 for complete (may use current bridge approach).

Q33b *[4.0 mark]*
-1 if solution does not follow the general pattern from part "a". -1 if total effective resistance R_{AF} not given. If final answer is wrong, but clearly presented, then please make an effort to identify error(s). If final answer is wrong and presentation is confusing (ie. no intermediate check-points) then assume the problem is major and mark accordingly.



Resistance of a wire is related to resistivity of the conductor, as well as length and cross-sectional area, according to the formula $R = \rho L/A$. All segments have the same area $A = \pi r^2 = \pi(0.002\ m)^2 = 1.26 \times 10^{-5}\ m^2$ so $\rho/A = 0.0159\ \Omega/m$. +0.5 for the correct formula, +0.5 for correct numerical value(s).

		L [cm]	R [$m\Omega$]
R_1	AB	10	1.59
R_2	BC	5	0.80
R_3	CD	5π	2.50
R_4	DE	5	0.80
R_5	EF	10	1.59
R_6	BGE	40	6.37

I'm showing every detail, students can skip obvious steps, but then lose benefit of the doubt in case of errors.

Three resistors in series can be added: $R_{BCDE} = R_2 + R_3 + R_4 = 4.10\ m\Omega$.
Two resistors in parallel: $R_{BE}^{-1} = R_6^{-1} + R_{BCDE}^{-1} = 40.09\ \Omega^{-1}$ so $R_{BE} = 2.49\ m\Omega$.
Three resistors in series: $R_{AF} = R_1 + R_{BE} + R_5 = 5.67\ m\Omega$
+1.0 for correct parallel & series combinations, -0.5 for either error.

Ohm's law gives $I_0 = V/R_0 = 9.0/5.67 \times 10^{-3} = 1587\ Amperes$, which gives the potential drop

$$V_{AB} = V_{EF} = IR = 1587\ A \times 1.59\ m\Omega = 2.52\ volts$$

so $V_{BE} = 9V - 2.52V - 2.52V = 3.95\ volts$ and the current through the lower branch is

$$I_1 = V/R = 3.95\ V/6.37 \times 10^{-3}\ \Omega = 620\ Amperes.$$

Accept "current bridge" approach (from circuits course)
Accept 2 significant figures (final result may be 10 A different)

Continued on next page

$\boxed{\textbf{Q34a}}$ *[0.5 mark]*

Inside the cylinder the electric field will be directed radially (symmetry). We can also say that it will be pointed outward because of the positive charge density.

$\boxed{\textbf{Q34b}}$ *[3.5 mark]*

Start with Gauss's law

$$\frac{Q_{enc}}{\epsilon_0} = \Phi_E = \oint \vec{E} \cdot d\vec{A}$$

where the total charge is just the density multiplied by the volume of a cylinder

$$Q_{enc} = \pi r^2 L \rho$$

Use the result from part "a" to calculate the (non-zero) flux through the sides of the cylinder

$$\Phi_{E_{tube}} = \int (E\hat{r}) \cdot (\hat{r}) dA = E \int dA = E 2\pi r L$$

Need to show that there is no flux through the end-caps

$$\Phi_{E_{end}} = \int (E\hat{r}) \cdot (\pm\hat{z}) da = E\ 0 = 0$$

-1 if no mention of endcaps

Pull everything together

$$\frac{\pi r^2 L \rho}{\epsilon_0} = E 2\pi r L \qquad \rightarrow \qquad E = \frac{\rho r}{2\epsilon_0}$$

and write a complete expression for the electric field inside the cylinder

$$E(r) = \frac{\rho r}{2\epsilon_0} \qquad r < R$$

Question explicitly asks for electric field magnitude, so vectors not required.

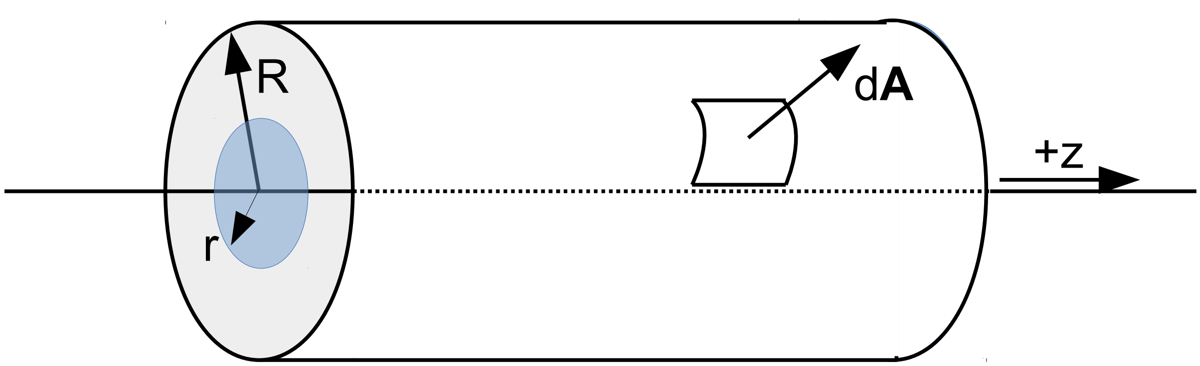


Figure 34b: Cylinder of charge.

-1 for no figure