Electricity and Magnetism

- Physics 259 – L02
- Lecture 31

Chapter 25: Capacitance

Last time

- Cylindrical capacitors
- Capacitors in parallel and series
- Energy in Capacitors

This time

- Energy in Capacitors
- Capacitors with a dielectric

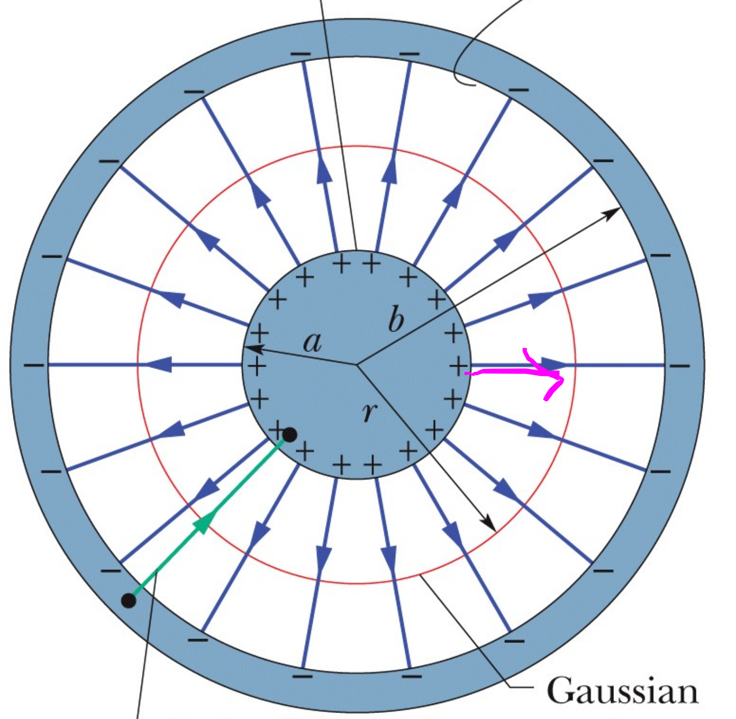
25-2 Calculating the Capacitance: Cylindrical Capacitor

1. Use Gauss's law

$$q = \varepsilon_0 EA = \varepsilon_0 E(2\pi rL)$$

2. Find potential

$$V = \int_-^+ E\,ds = -\frac{q}{2\pi\varepsilon_0 L}\int_b^a \frac{dr}{r} = \frac{q}{2\pi\varepsilon_0 L}\ln\left(\frac{b}{a}\right)$$



$V = -\int_a^b \vec{E}\cdot d\vec{s} = -\int_-^+ \vec{E}\cdot d\vec{s} = -\int_-^+ E\,ds\cos 180$

$\rightarrow V = \int_-^+ E\,ds = \int_-^+ \frac{q}{\varepsilon_0 2\pi L}\frac{ds}{r}$

$ds = -dr \Rightarrow V = -\int \frac{q}{''}\frac{dr}{r}$

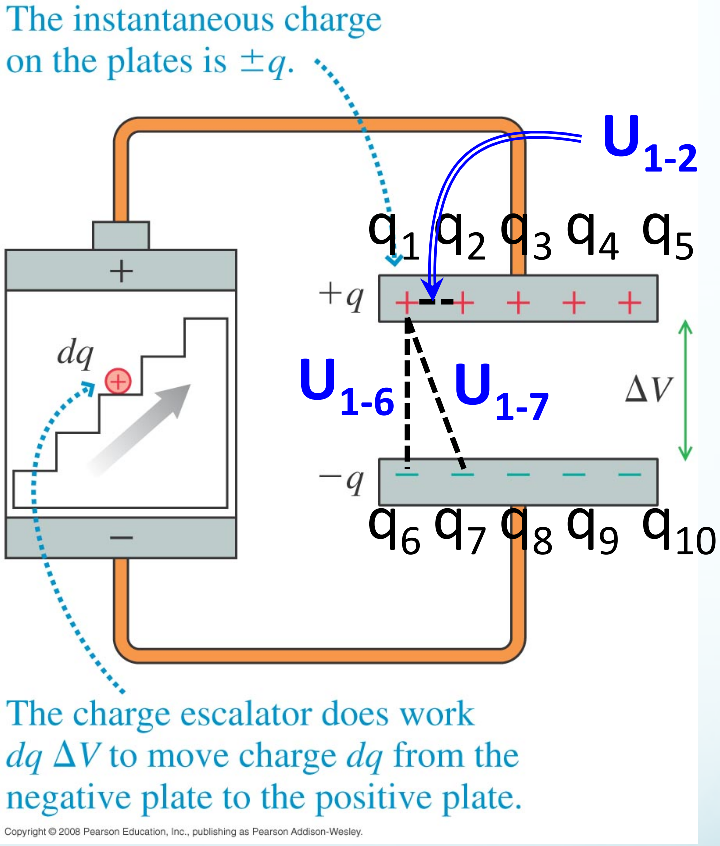
$$C = 2\pi\varepsilon_0 \frac{L}{\ln(b/a)} \quad \text{(cylindrical capacitor).}$$

25-4 Energy Stored in an Electric Field

Energy Storage in Capacitors

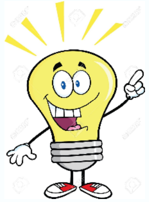
We want to calculate **potential energy** stored in the capacitor

VERYYYYY hard



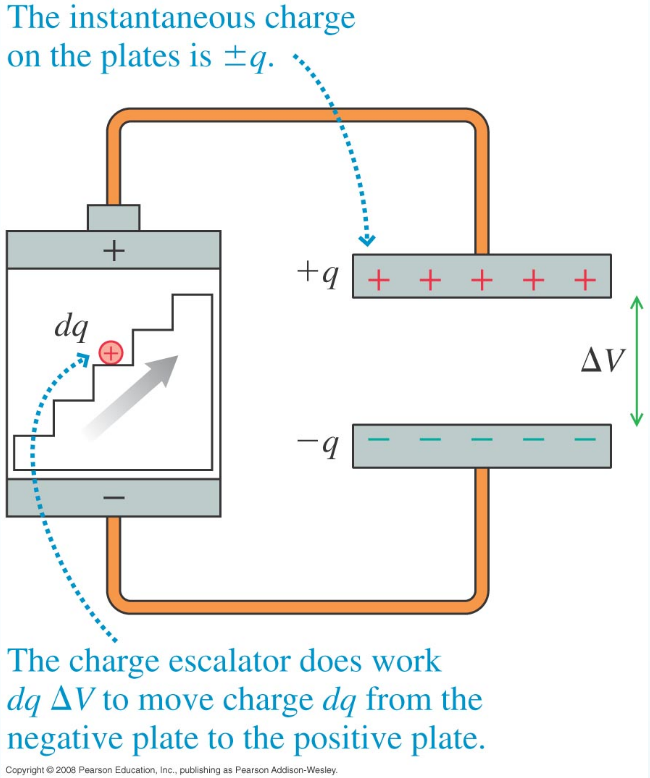
$$U = U_{1-2} + U_{1-3} + \ldots + U_{1-10} + U_{2-1} + U_{i\text{-}j} \text{ of every other pair}$$

Easier way!



Move a tiny charge, dq, from negative plate to positive plate→

It moves through a potential difference ΔV→ its potential energy increases by an amount



$$\rightarrow dU = dq\Delta V_C$$

$$\& \Delta V_C = \frac{q}{C}$$

$$\rightarrow dU = dq\,\frac{q}{C}$$

$$\rightarrow dU = \frac{q\,dq}{C}$$

$$dU = \frac{q\,dq}{C}$$

$$U = \int dU$$

$$\boxed{U = \frac{1}{C}\int_0^Q q\,dq = \frac{1}{2}\frac{Q^2}{C}}$$

✓ **Energy** storage in terms of the charge on the plates: $\boxed{U = \frac{1}{2}\frac{Q^2}{C}}$

✓ **Energy** storage in terms of the voltage across the plates:

$Q = CV$ $\quad U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\frac{C^2V^2}{C} = \frac{1}{2}CV^2 \Rightarrow$ $\boxed{U = \frac{1}{2}CV^2}$

✓ **Energy density:**

The potential energy of a charged capacitor may be viewed as being stored in the electric field between its plates.

Energy density→ Potential energy per unit volume between the plates

u

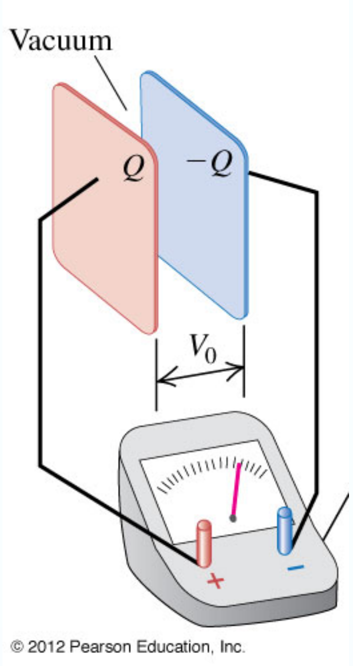
For parallel-plate capacitor→ $\quad U = \frac{1}{2}CV^2 \, , \; C = \varepsilon_0 \frac{A}{d}$

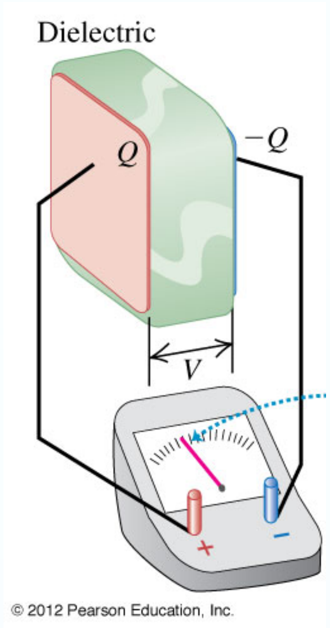
$$u = \frac{U}{Ad} = \frac{\frac{1}{2}CV^2}{Ad} = \frac{\frac{1}{2}V^2\varepsilon_0\frac{\cancel{A}}{d}}{\cancel{A}d}$$

$$\rightarrow u = \frac{1}{2}\varepsilon_0\frac{V^2}{d^2} \longrightarrow \boxed{\rightarrow u = \frac{1}{2}\varepsilon_0\left(\frac{V}{d}\right)^2 = \frac{1}{2}\varepsilon_0 E^2}$$

25-5 Capacitor with a Dielectric

25-5 Capacitor with a Dielectric

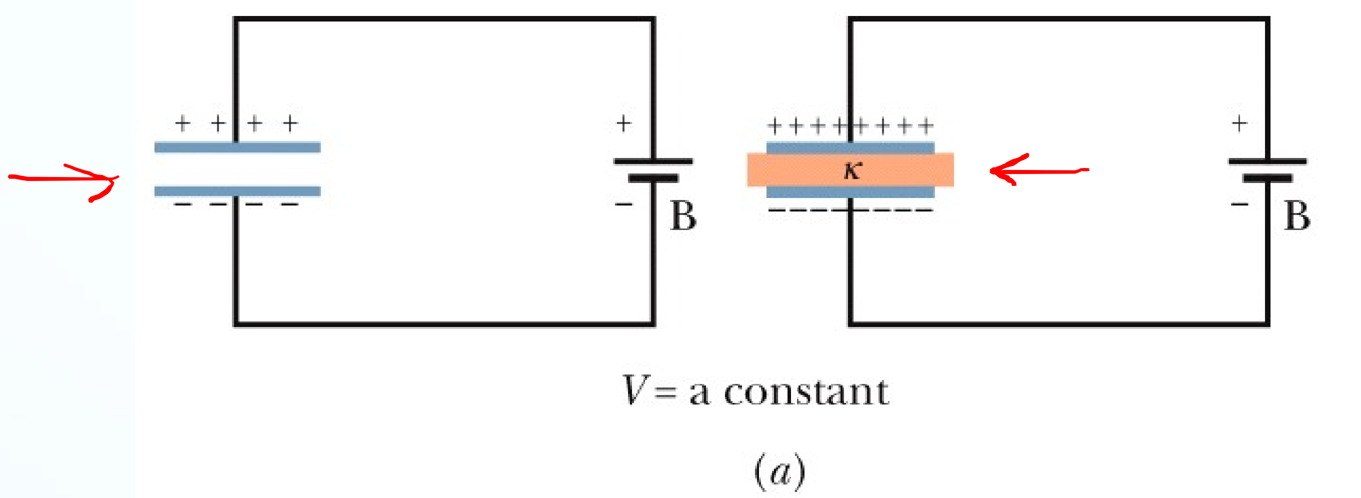




If the space between the plates of a capacitor is completely filled with a **dielectric material**, the capacitance C in vacuum (or, effectively, in air) is multiplied by the material's **dielectric constant** *κ*, which is a number greater than 1.

$$\boxed{C = \kappa C_{air}} \rightarrow C \uparrow$$

If the **potential difference** between the plates of a capacitor is maintained, as by the presence of battery B→

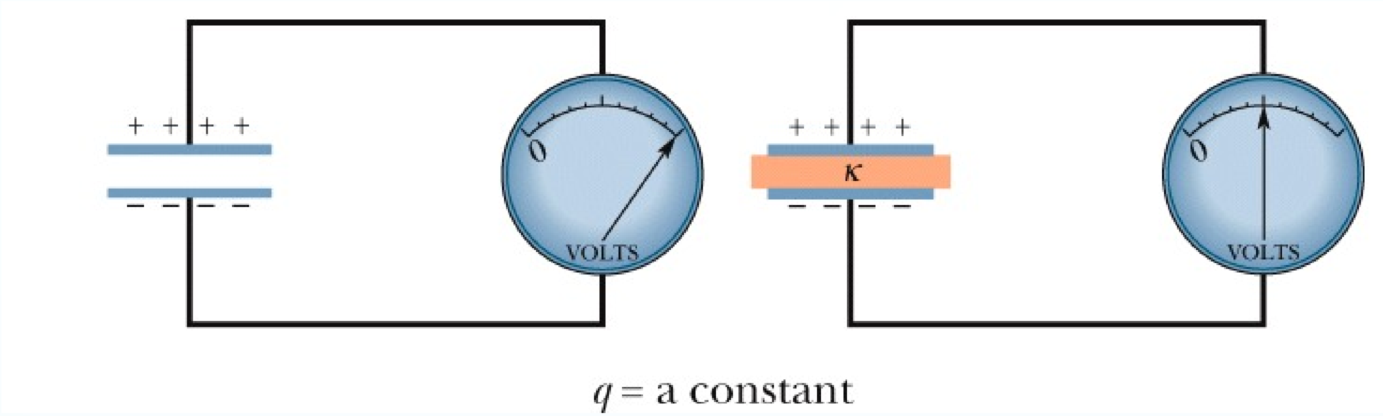


The effect of a dielectric is to increase the charge on the plates.

$$\boxed{q = CV = \kappa C_{air} V}$$

$$\boxed{q = \kappa q_{air}}$$

If the charge on the capacitor plates is maintained, as in this case by isolating the capacitor→



The effect of a dielectric is to reduce the potential difference between the plates.

$$\boxed{V = \frac{q}{C} = \frac{q}{\kappa C_{air}} \rightarrow} \qquad \boxed{V = \frac{V_{air}}{\kappa}}$$

The scale shown is that of a potentiometer, a device used to measure potential difference (here, between the plates). A capacitor cannot discharge through a potentiometer.

In a region completely filled by a dielectric material of dielectric constant κ, all electrostatic equations containing the permittivity constant ε_0 are to be modified by replacing ε_0 with $\kappa\varepsilon_0$.

Examples:

The magnitude of electric field produced by a point charge inside a dielectric→

$\varepsilon_0 \rightarrow \kappa\,\varepsilon_0$

$$\boxed{E = \frac{1}{4\pi\kappa\varepsilon_0}\frac{q}{r^2}}$$

The magnitude of electric field outside an isolated conductor immersed inside a dielectric→

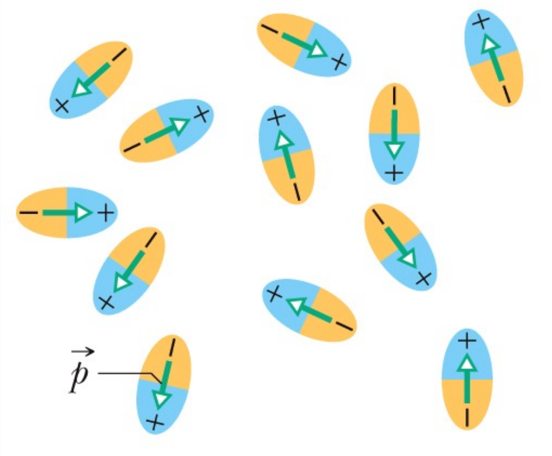
$$\boxed{E = \frac{\sigma}{\kappa\varepsilon_0}}$$

Dielectrics: An Atomic View

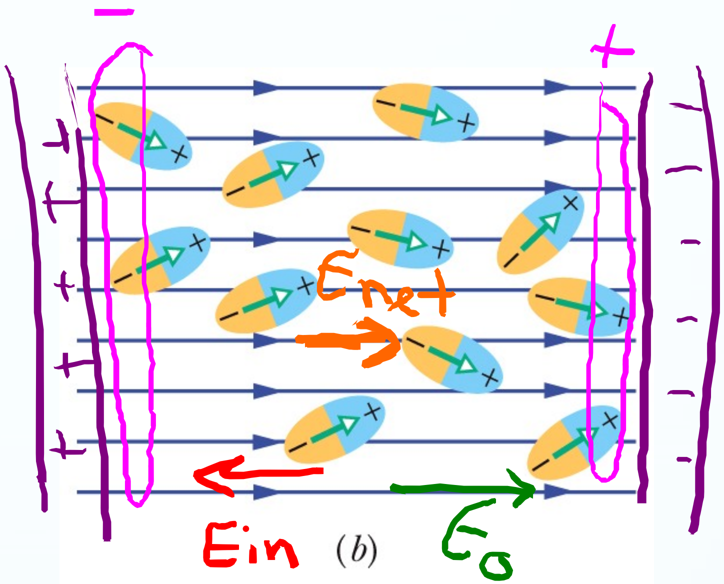
What happens in atomic view when we put a dielectric in an electric field?

1. Polar dielectric
2. Nonpolar dielectric

1. Polar dielectrics



(a)



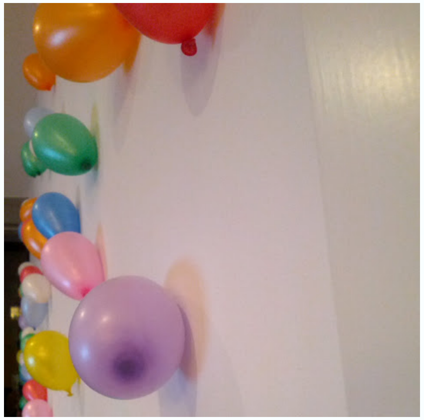
(b)

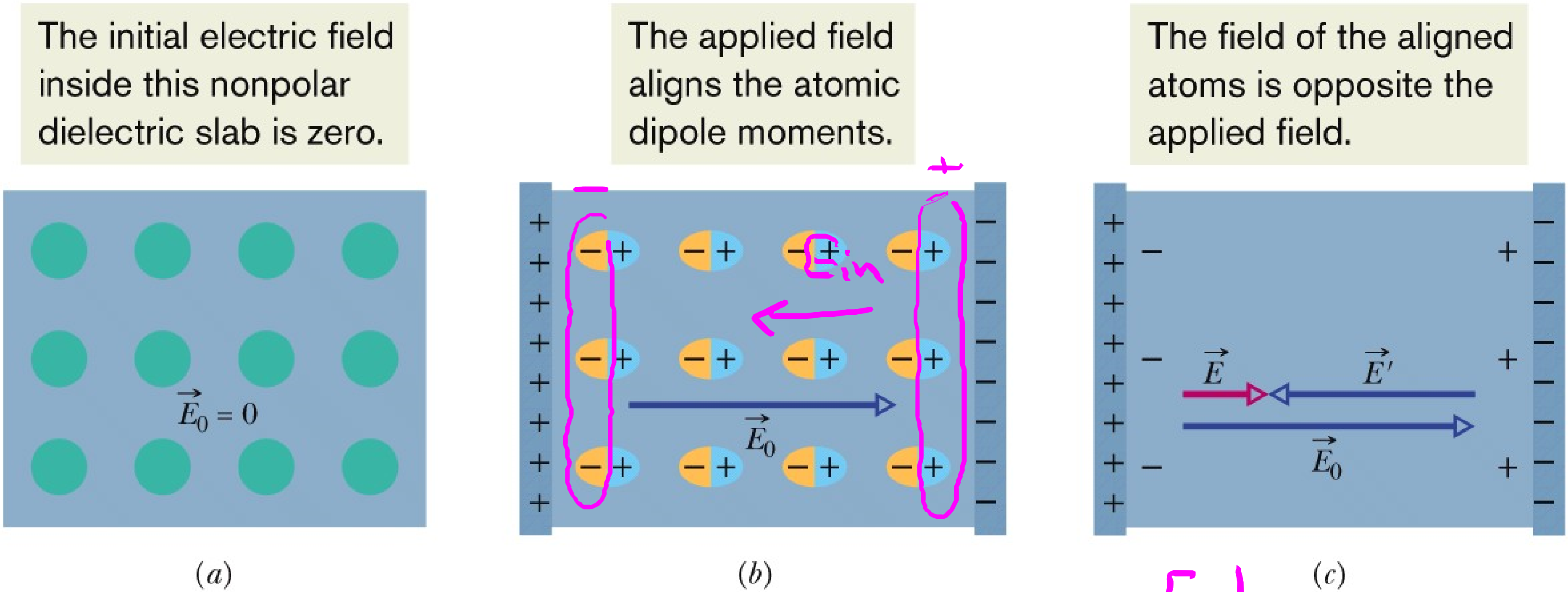
Molecules with a permanent electric dipole moment, showing their random orientation in the absence of an external electric field.

An electric field is applied, producing partial alignment of the dipoles. Thermal agitation prevents complete alignment.

E ↓

2. Nonpolar dielectrics

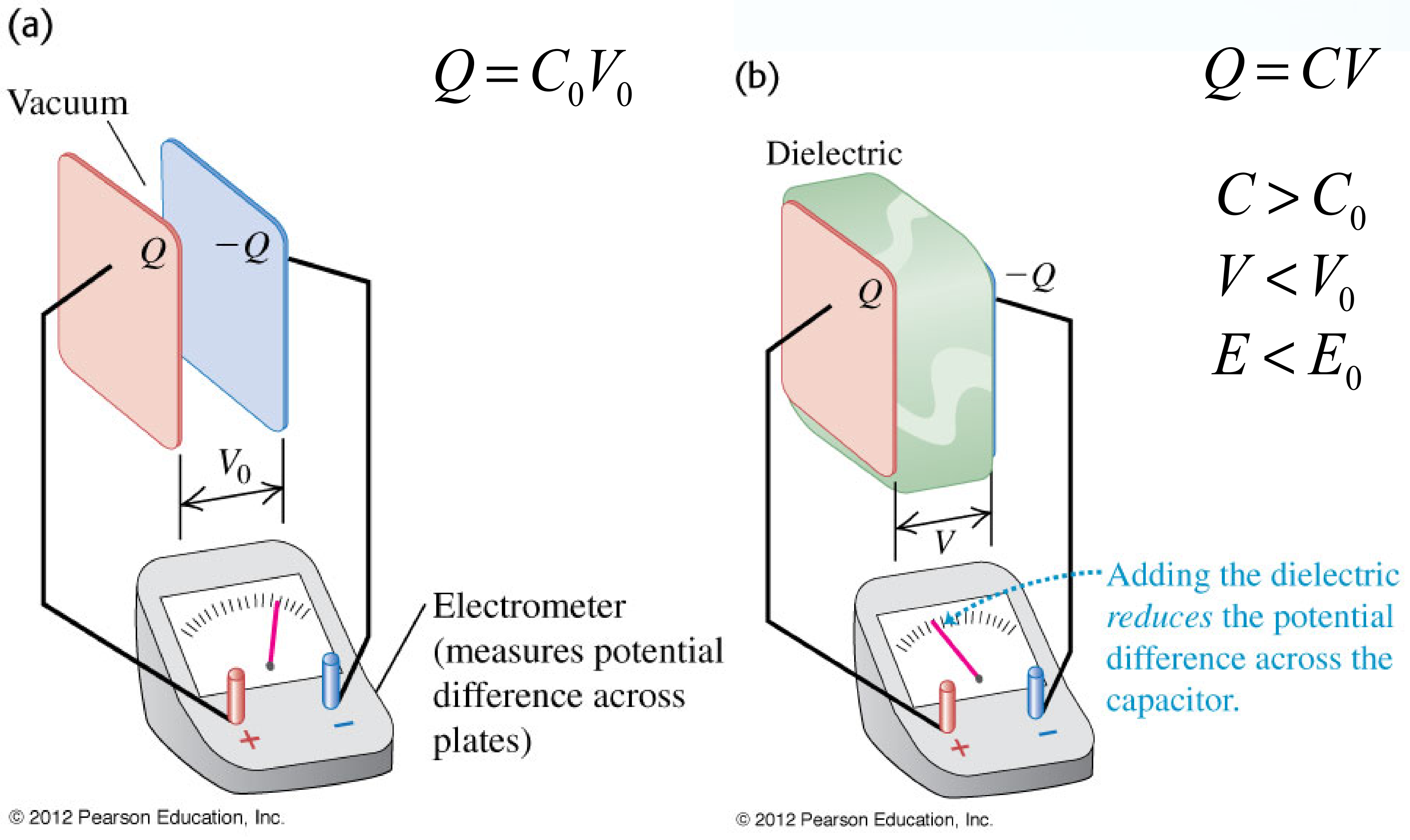




Dielectrics: An Atomic View

The effect of both polar and nonpolar dielectrics is to weaken any applied field within them.

To sum up →



$$Q = C_0 V_0$$

$$Q = CV$$

$$C > C_0$$

$$V < V_0$$

$$E < E_0$$

TopHat Question

A capacitor without a dielectric is charged up so that it stores potential energy U_0, and it is then disconnected so that **its charge remains the same**. A dielectric with constant $\kappa = 2$ is then inserted between the plates. What is the new potential energy stored in the capacitor **with the dielectric**?

$$U_C = \frac{Q^2}{2C} = \frac{V_C^2 C}{2}$$

A. $4U_0$

A. $2U_0$

C. $\frac{1}{2}U_0$

D. $\frac{1}{4}U_0$

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$C \rightarrow \kappa C$

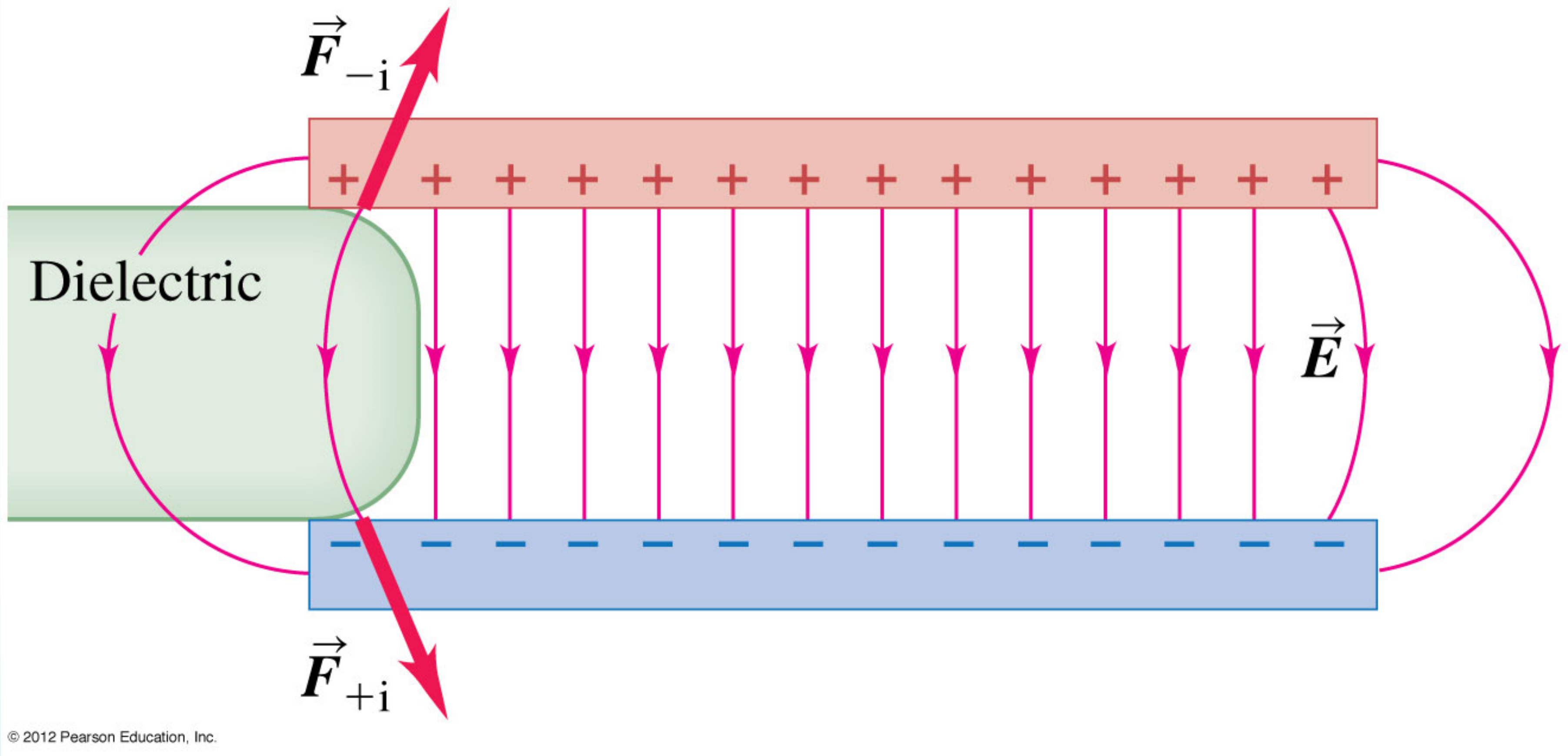
$C \rightarrow 2C$

A. $2U_0$

D. $\frac{1}{4}U_0$

The **potential energy lowers** when the dielectric is added, so it will feel a **force sucking it into the gap** between the plates.

Can find the force using $F_x = -dU/dx$ and



Fringe Electric Field pulls the dielectric into the gap

TopHat Question

A capacitor without a dielectric is charged up so that it stores potential energy U_0, and is kept connected **at constant voltage**. A dielectric with constant $\kappa = 2$ is then inserted between the plates. What is the new potential energy stored in the capacitor **with the dielectric**?

$$U_C = \frac{Q^2}{2C} = \frac{V_C^2 C}{2}$$

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$$U_C = \frac{Q^2}{2C} = \frac{V_C^2 C}{2}$$

A. $4U_0$

C. $\frac{1}{2}U_0$

A. $2U_0$

D. $\frac{1}{4}U_0$

The **potential energy raises** when the dielectric is added, so it will feel a **force pushing it out of the gap** between the plates.

This section we talked about:

Chapter 25

See you on Friday