

Electricity and Magnetism

- Physics 259 – L02
- Lecture 49



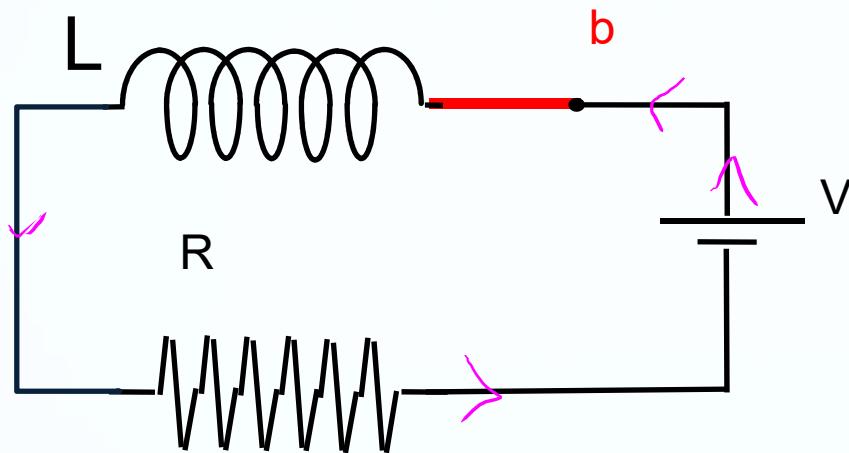
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~~Chapter 29: Magnetic field due to current~~

chapter 30



30.6: R-L Circuit



$$V - L \frac{di}{dt} - iR = 0$$

$$\underbrace{V = -L \frac{di}{dt}}_{v = -iR}$$

If the switch is moved to position **b**, to initiate the current flow, what happens?

$$C = \frac{q}{V} \quad V = \frac{q}{C}$$

Faraday's law applies and so the change in the Magnetic Field in the inductor L means there is a back EMF induced in L.

Discharging \Rightarrow

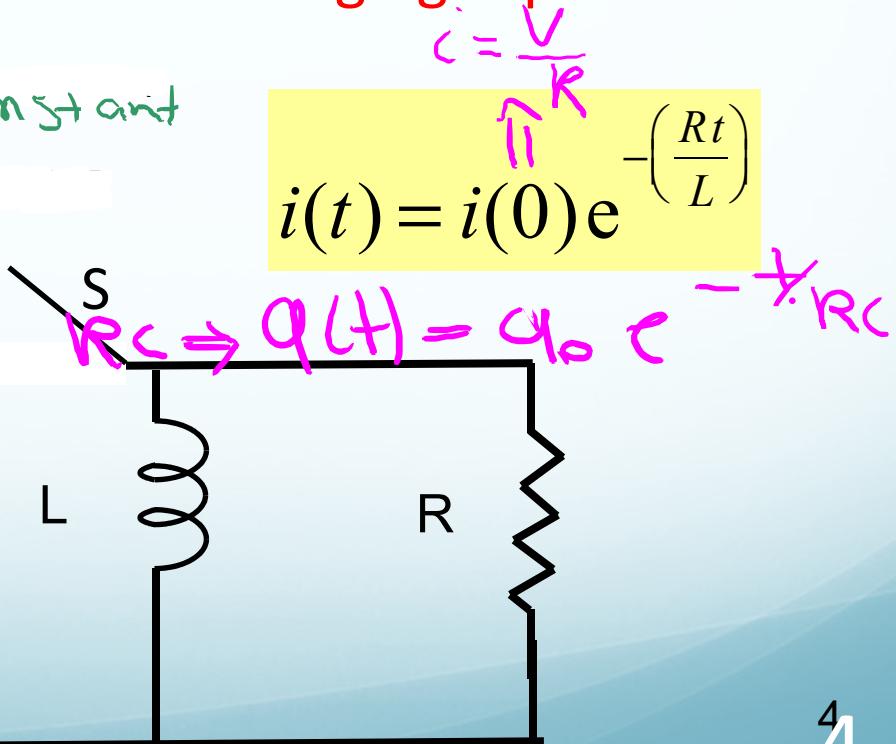
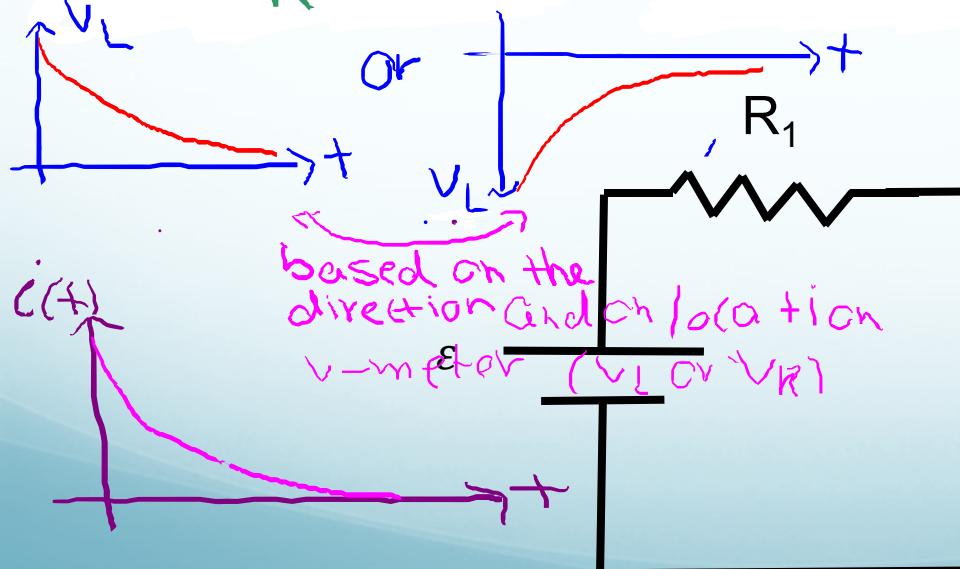
The components have all been connected for a very long time.
At $t=0$ the switch S is **opened**. The current through R_1 and R are 0 and ε/R

Using the loop rules

$$-L \frac{di}{dt} - iR = 0$$

Solving with the method we used for a **discharging capacitor**

$$\tau_L = \frac{L}{R} \quad \text{inductive time constant}$$



Charging

At $t=0$ the switch S is closed.

Using the loop rules

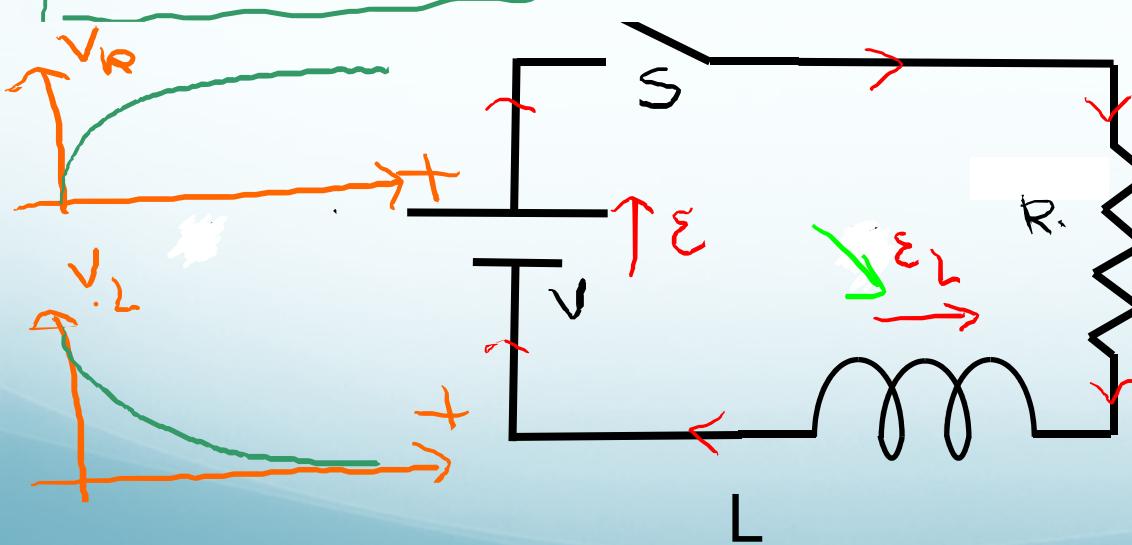
$$V - iR - L \frac{di}{dt} = 0$$

Solving using the method we used for the **charging capacitor**

So in this case at $t = 0$, $i(0) = 0$. Inductor acts like a BATTERY

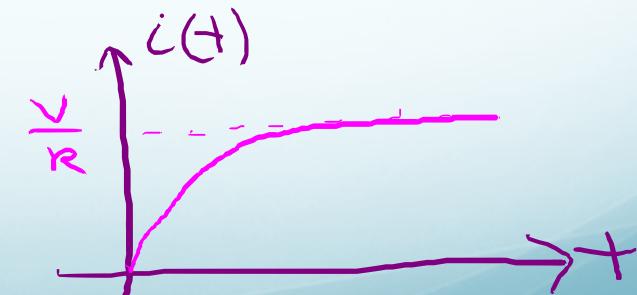
After a long time, $i = V/R$

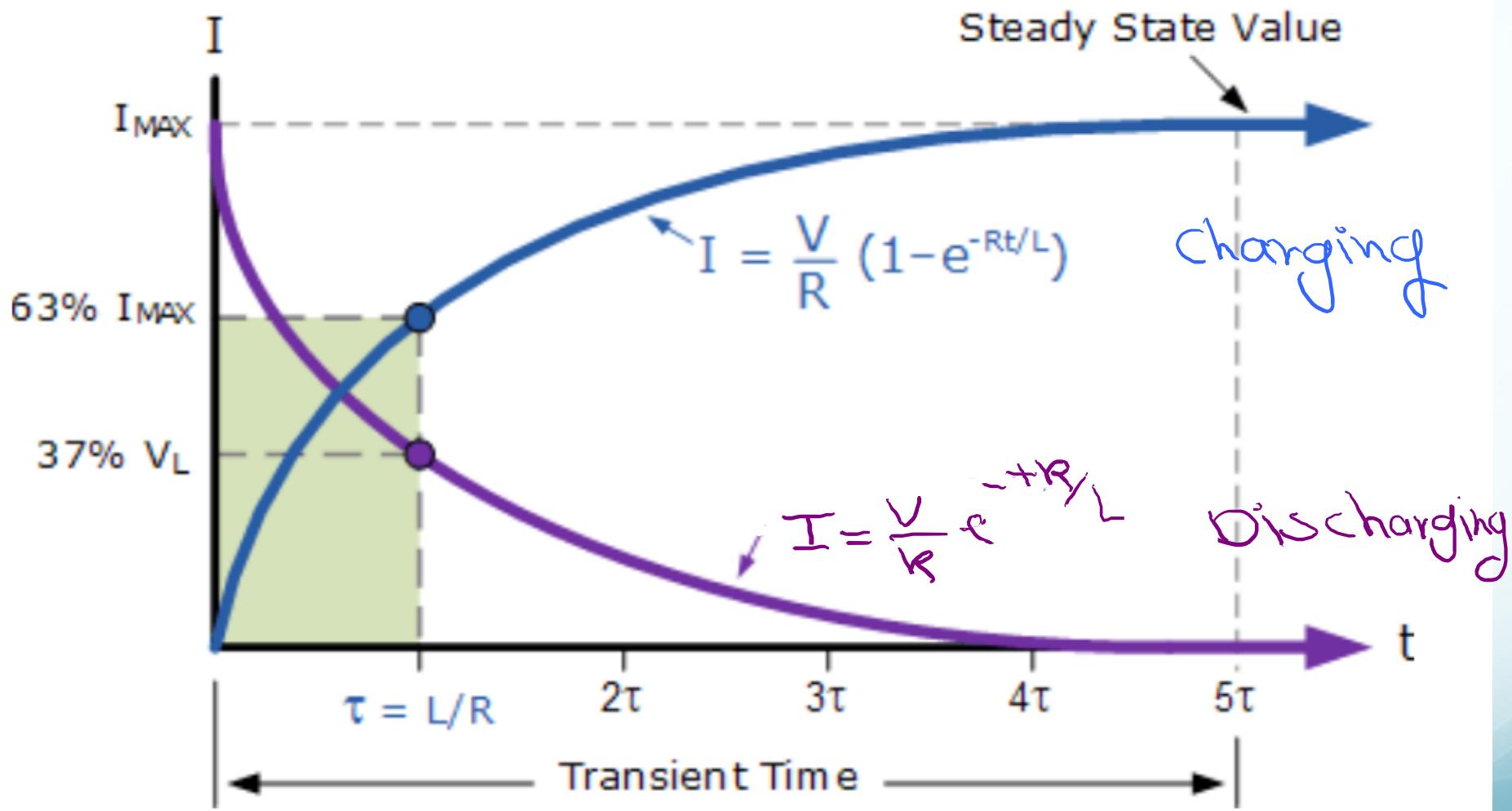
Inductor acts like a WIRE



↓ ↓

$$i(t) = i_{\max} \left(1 - e^{-\left(\frac{Rt}{L}\right)} \right)$$





Top Hat Question

The switch in the series circuit below is closed at $t=0$.

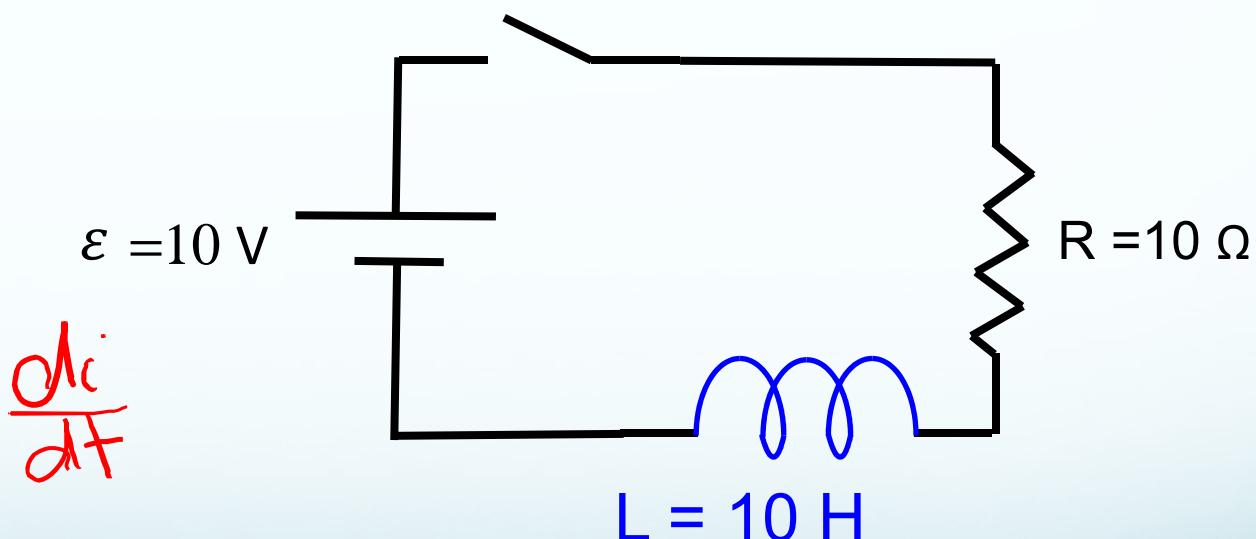
What is the **initial rate of change of current** di/dt in the **inductor**, immediately after the switch is closed (time = $0+$) ?

A. 0 A/s

B. 0.5 A/s

C. 1 A/s

D. 10 A/s



$$\frac{di}{dt}$$

$$V_R = 0 \quad , \quad V_L = L \frac{di}{dt}$$

Top Hat Question

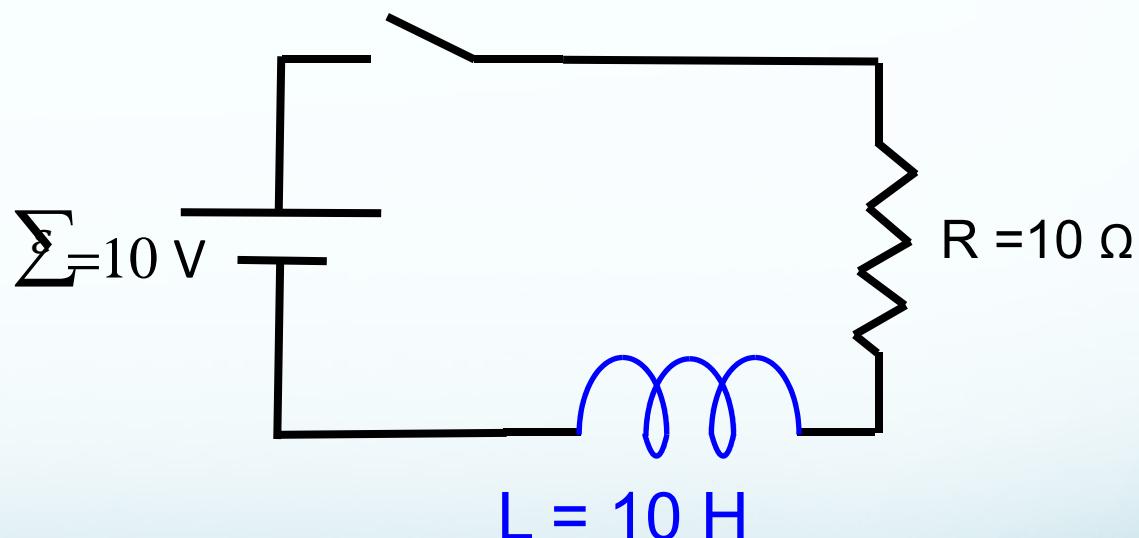
The switch in the series circuit below is closed at $t=0$.

What is the **initial rate of change of current** di/dt in the **inductor**, immediately after the switch is closed (time = $0+$) ?

- A. 0 A/s
- B. 0.5 A/s

- C. 1 A/s

- D. 10 A/s $i = 0$ at $t = 0$ so $V_R(0) = 0$ which means



$$10 \text{ V} = V_L = L \frac{di}{dt} \text{ so } \frac{di}{dt} = 10 \text{ V} / 10 \text{ H} = 1 \text{ A/s}$$

Top Hat Question

The switch in the series circuit below is closed at $t=0$.

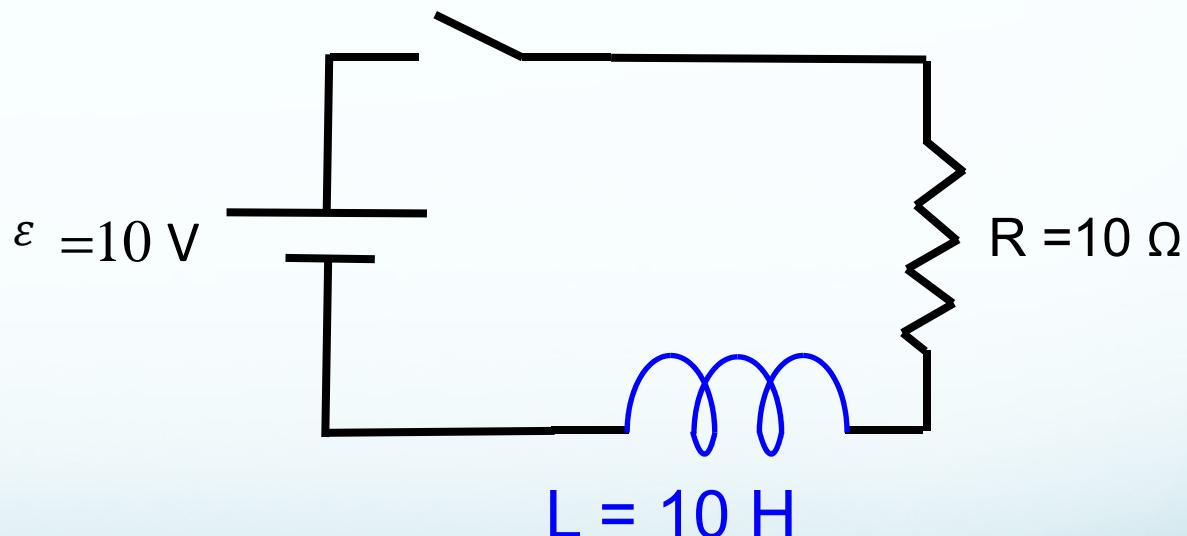
What is the current in the circuit after a time $t = 3.0 \text{ s}$?

A. 0 A

B. 0.63 A

C. 0.86 A

D. 0.95 A



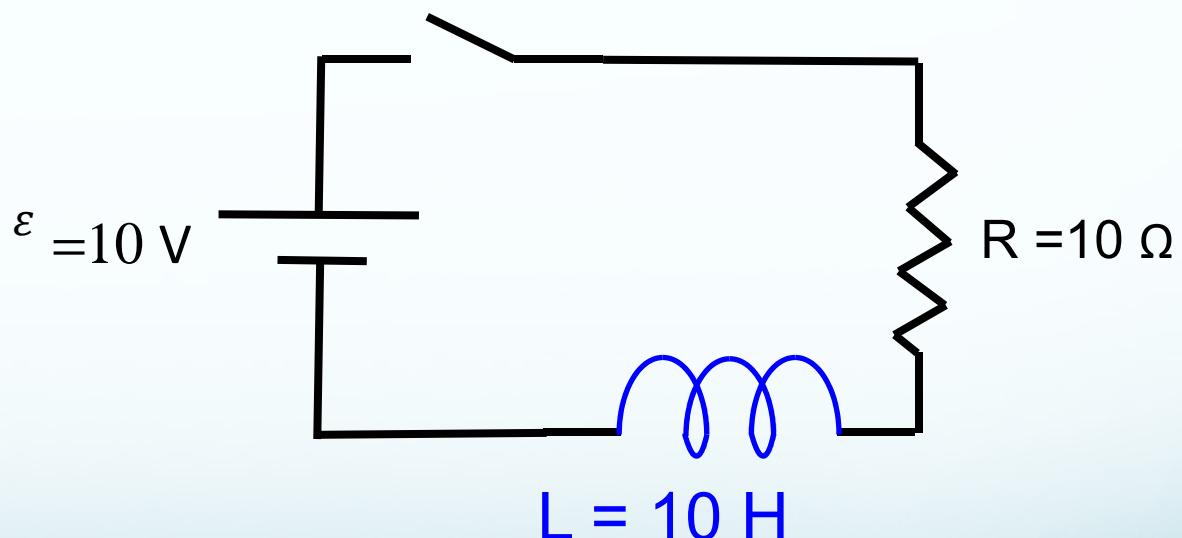
$$i(t) = i_0(1 - e^{-\frac{t}{\tau_L}})$$

Top Hat Question

The switch in the series circuit below is closed at t=0.

What is the current in the circuit after a time t = 3.0 s?

- A. 0 A
- B. 0.63 A
- C. 0.86 A
- D. 0.95 A



$$i(3\text{s}) = \frac{10V}{10\Omega} \left(1 - e^{-3}\right)$$

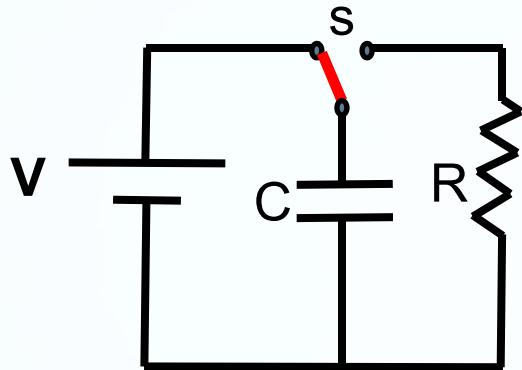


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Final review

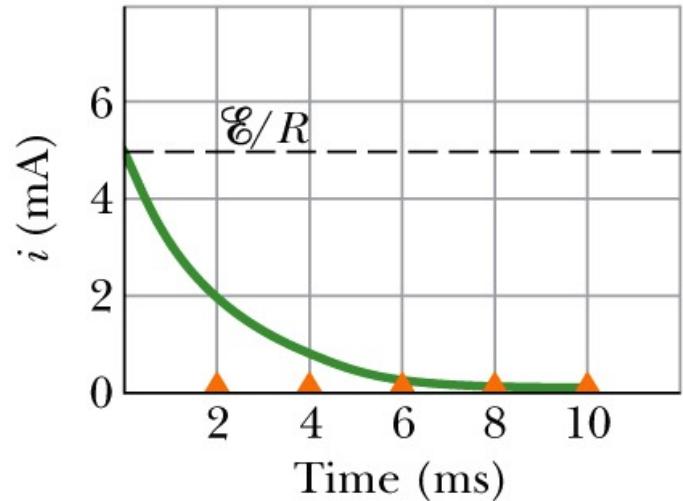


RC circuit: Charging a capacitor

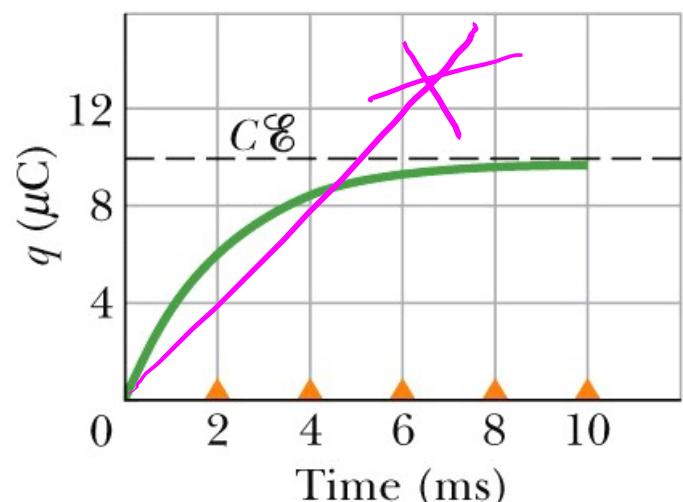


$$i = i_0 e^{-t/RC}$$

$$q = \epsilon C \left(1 - e^{-t/RC}\right) = Q_f \left(1 - e^{-t/RC}\right)$$



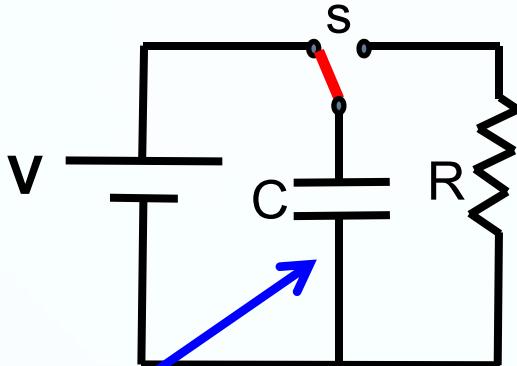
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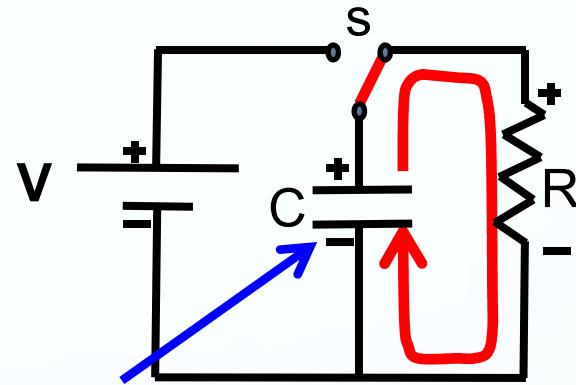
RC circuit: Discharging a capacitor

Switch is connected to the left for a long time until $t=0-$



Capacitor charges up to voltage V

Switch is suddenly flipped to the right at $t=0+$

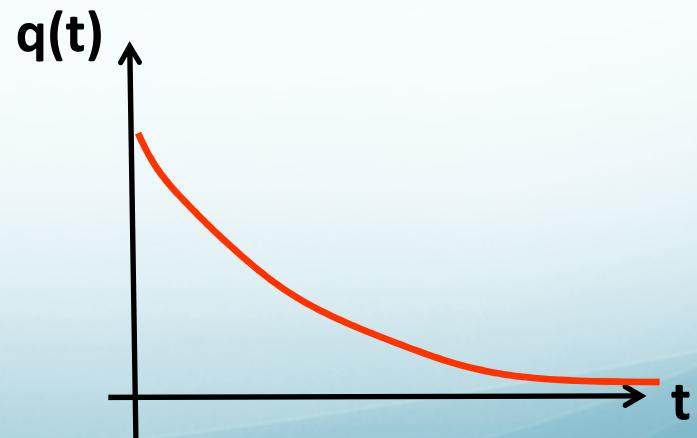


Capacitor discharges

$$q(t) = q_0 e^{-t/RC}$$

$$i(t) = i_0 e^{-t/RC}$$

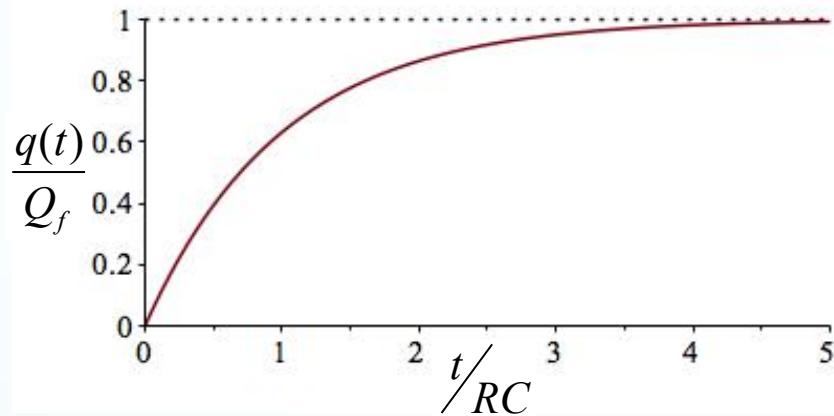
$$q_0 = CV$$



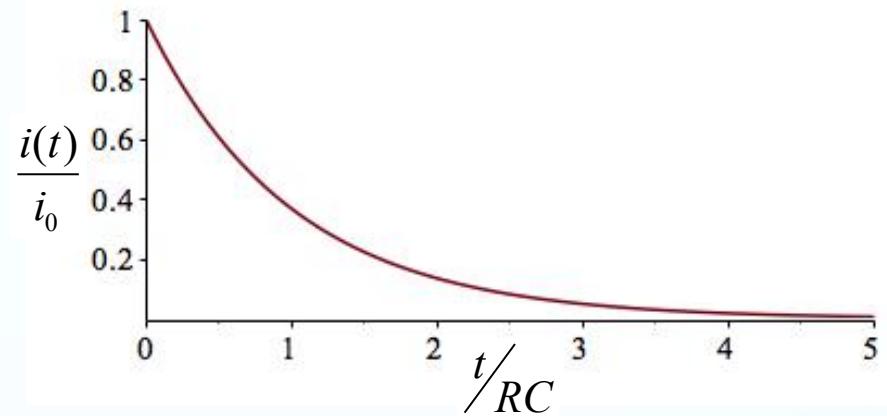
Charging/Discharging Capacitors

Charging:

$$q(t) = Q_f \left(1 - e^{-\frac{t}{RC}} \right)$$

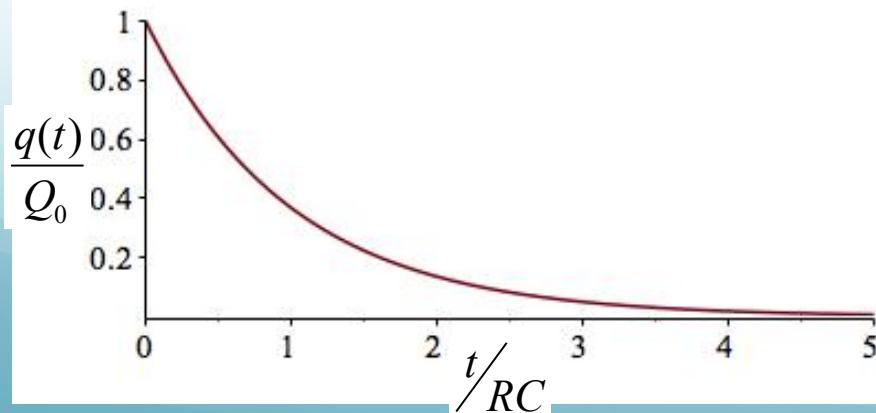


$$i(t) = i_0 e^{-\frac{t}{RC}}$$

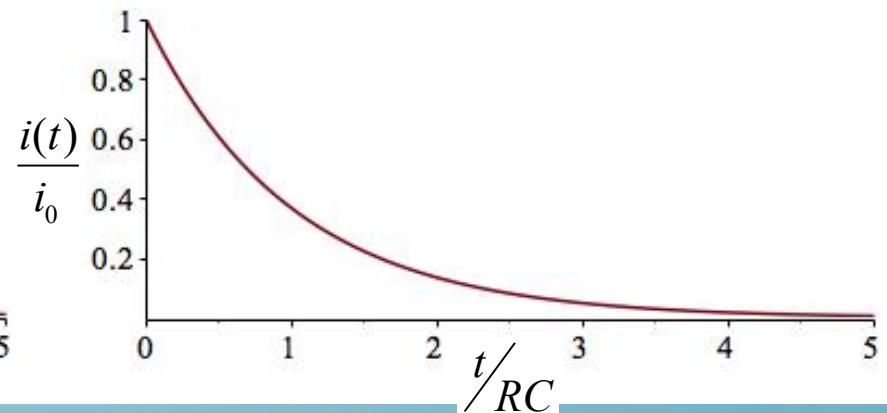


Discharging:

$$q(t) = Q_0 e^{-\frac{t}{RC}}$$



$$i(t) = i_0 e^{-\frac{t}{RC}}$$



The RC time constant

The constant RC pops up in the exponential factor for both charging and discharging capacitors. What does it represent?

The units of RC is seconds: $[RC] = \frac{V}{A} \frac{C}{V} = \frac{C}{\cancel{V}} = s$

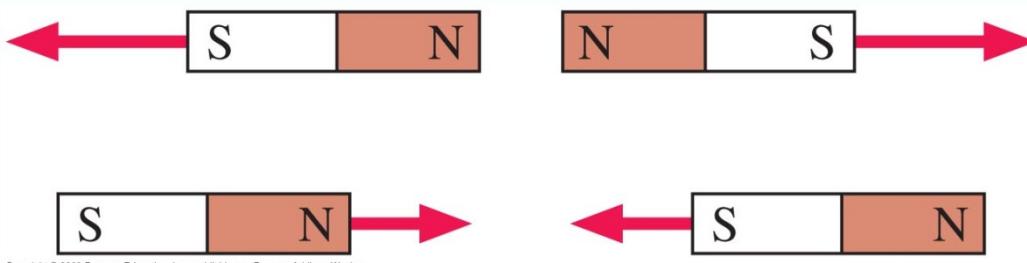
We call RC the “**RC time constant**” and it tells us how quickly a capacitor can charge or discharge.

$$RC \equiv \tau$$

After a time τ , the charge on a discharging capacitor is reduced by a factor of $1/e$. After a time $N\tau$, it is reduced by a factor of $1/e^N$

$$q(t) = Q_0 e^{-\frac{t}{\tau}}$$

28.1: Magnetic fields

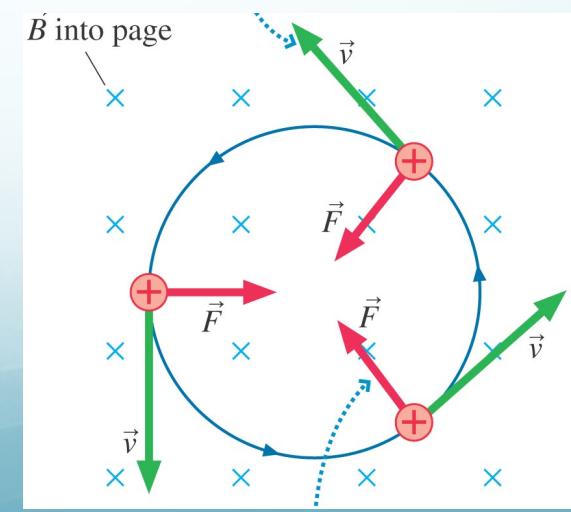


Magnetic force acts only on a moving charge.

$$\vec{F}_B = q \vec{v} \times \vec{B}$$

The Tesla
The Gauss

28.4: A circulating charged particle

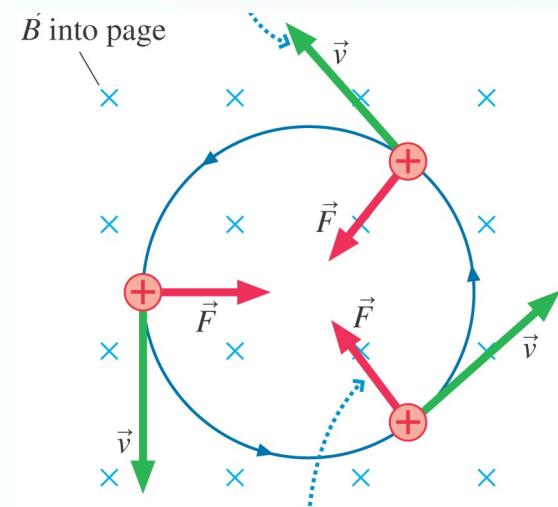


28.4: A circulating charged particle

$$R = \frac{mv}{|q|B}$$

$$T_{cyc} = \frac{2\pi m}{|q|B}$$

$$f_{cyc} = \frac{|q|B}{2\pi m}$$

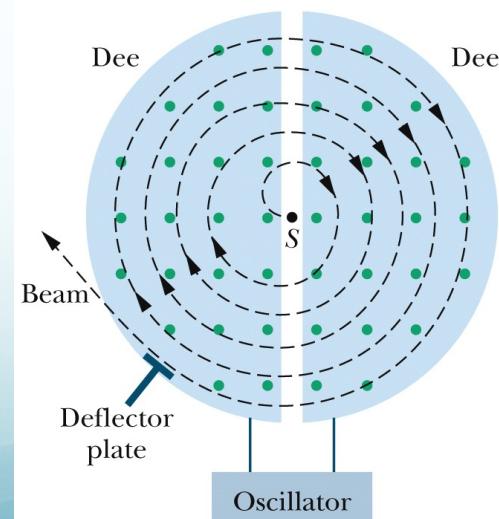


28.5: Cyclotrons and Synchrotrons

Application: Mass Spectrometer

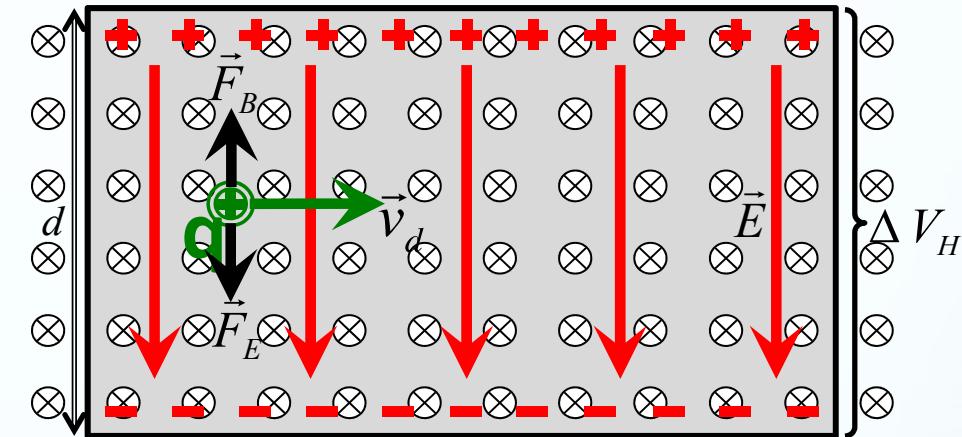
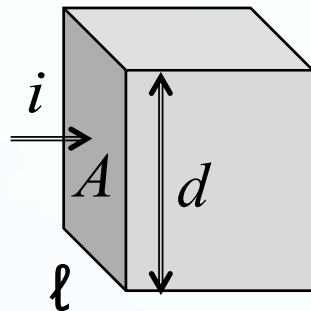
$$R = \frac{mv}{|q|B}$$

The protons spiral outward in a cyclotron, picking up energy in the gap.



28-2 Crossed Fields: Discovery of The Electron

28-3 Crossed Fields: The Hall Effect



$$F_B = q v_d B$$

$$v_d = \frac{i}{neA} \quad A = \ell d$$

$$\Delta V_H = v_d B d$$

$$B = \frac{ne\ell}{i} \Delta V_H$$

n is a material property

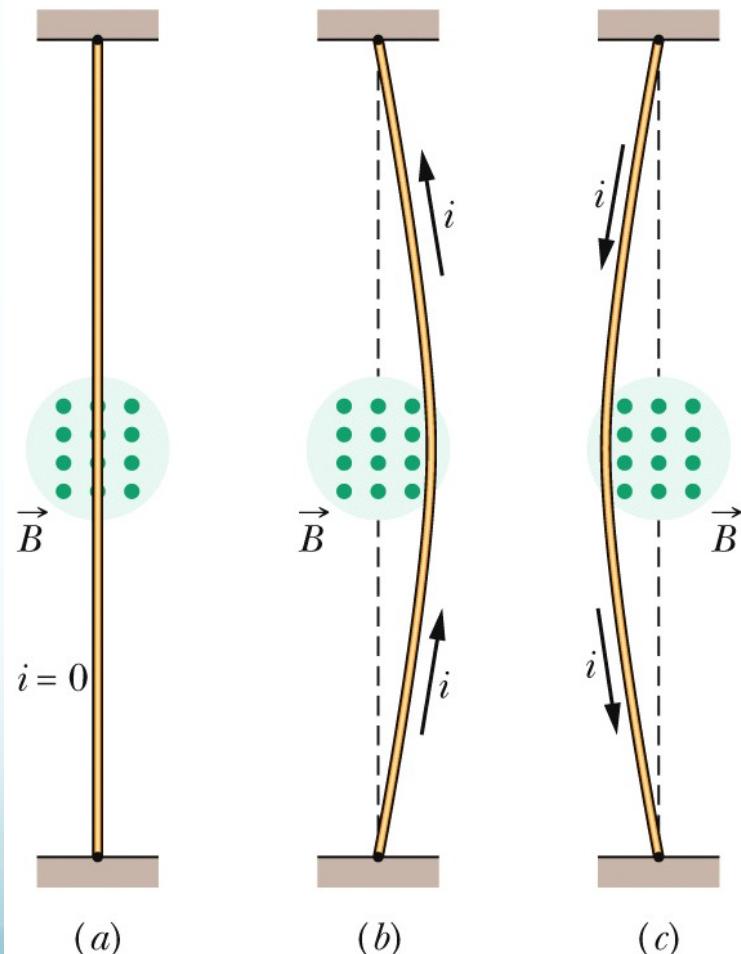
28-6 Magnetic Force on a Current-Carrying Wire

A straight wire carrying a current i in a uniform magnetic field experiences a sideways force

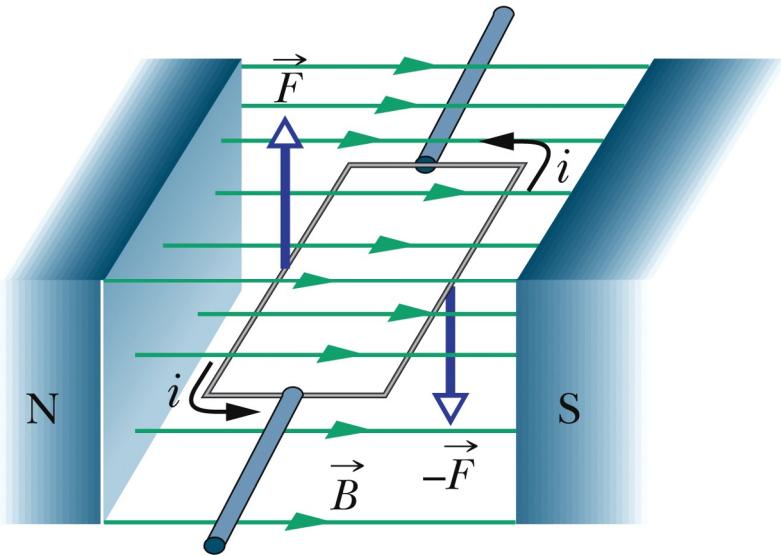
$$\vec{F}_B = i\vec{L} \times \vec{B} \quad (\text{force on a current}).$$

Here \vec{L} is a length vector that has magnitude L and is directed along the wire segment in the direction of the (conventional) current.

A force acts on a current through a B field.

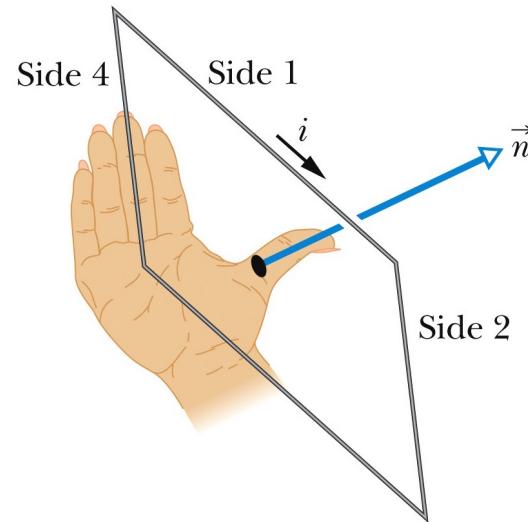


28-7 Torque on a Current Loop



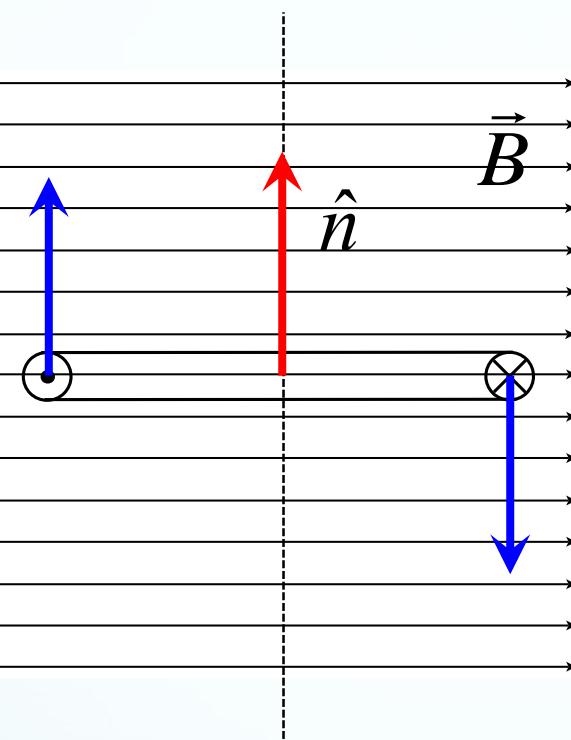
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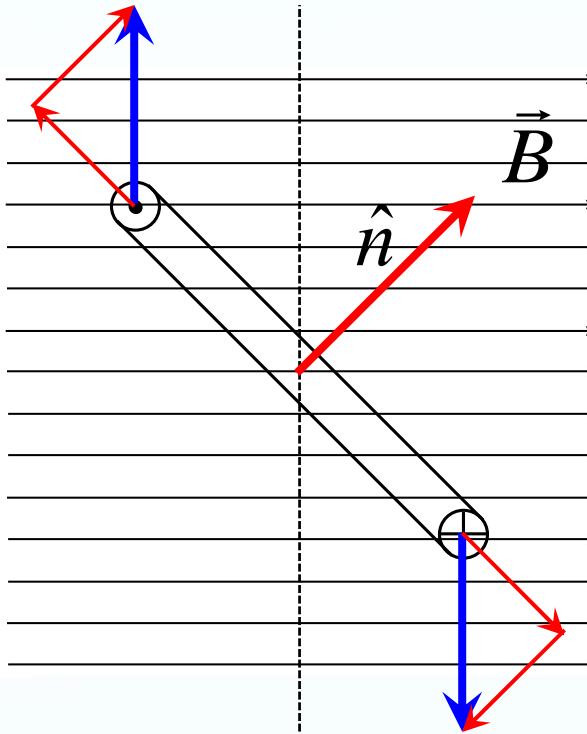


$$\vec{F}_B = i\vec{L} \times \vec{B} \quad (\text{force on a current}).$$

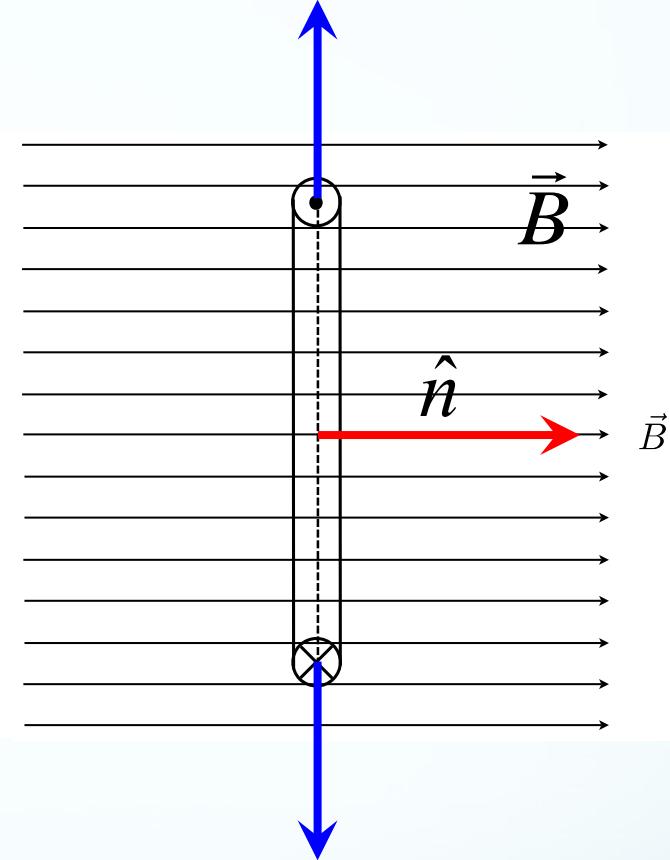
$$\tau = NiAB \sin \theta,$$



The normal vector is at right angles to the B-field: all magnetic force causes rotation of the loop

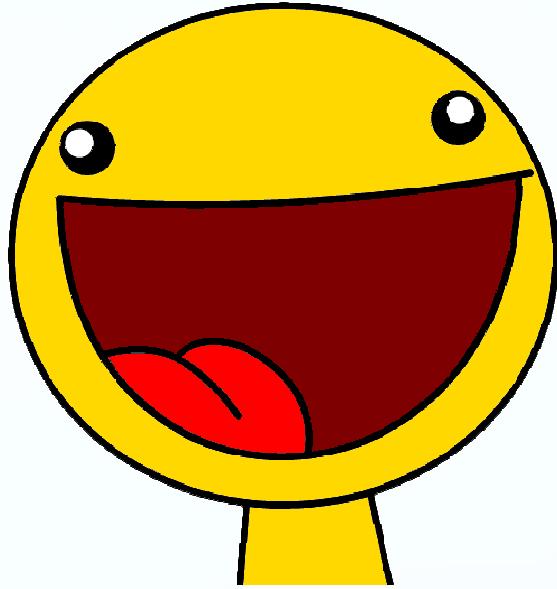


The normal vector is at some angle to the B-field: some of the magnetic force causes rotation of the loop



The normal vector is parallel to the B-field: none of the magnetic force causes rotation of the loop

Conclusion: components of magnetic force (anti)parallel to normal vector cause torque



For a single charge →

$$\vec{F}_B = q \vec{v}_d \times \vec{B}$$

For N charges moving through the wire
(current carrying wire) →

$$\vec{F}_B = i \vec{\ell} \times \vec{B}$$

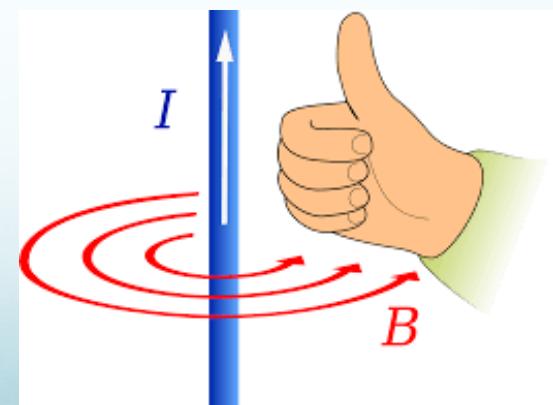
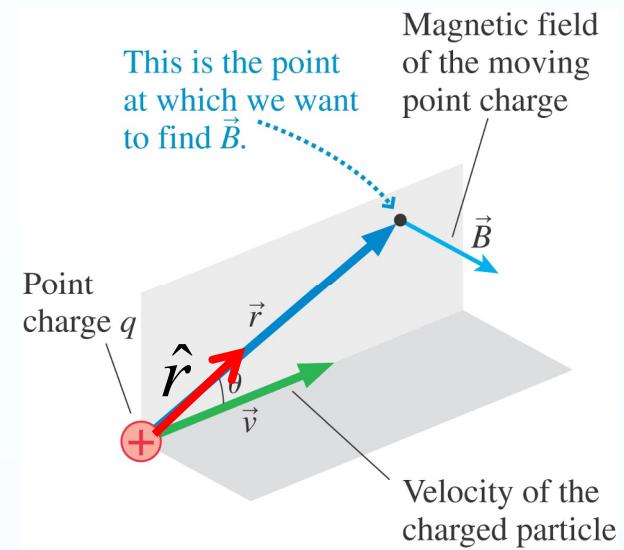
The Biot-Savart Law

Magnetic fields are caused by
moving charges.*

$$\vec{B}_{\text{point charge}} = \frac{\mu_0}{4\pi} \frac{q\vec{v} \times \hat{r}}{r^2}$$

$$\vec{B}_{\text{point charge}} = \frac{\mu_0}{4\pi} \frac{q\vec{v} \times \vec{r}}{r^3}$$

$$\vec{B}_{\text{current segment}} = \frac{\mu_0}{4\pi} \frac{I \Delta \vec{s} \times \hat{r}}{r^2}$$



This section we talked about:

Chapter 30

See you on Monday

