Electricity and Magnetism

- Physics 259 – L02
- Lecture 43

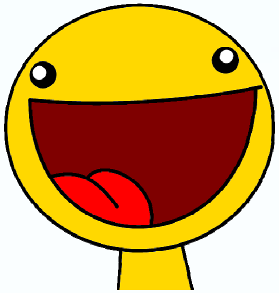
Chapter 29: Magnetic field due to current

Last time:

- Biot-Savart Law (like Coulomb's Law for magnetism)
- B-field of a line of current
- Magnetic force between parallel current-carrying wires
- Ampere's law

Today:

- Applications of Ampere's law



For a single charge $\rightarrow$

$$\boxed{\vec{F}_B = q\,\vec{v}_d \times \vec{B}}$$

For N charges moving through the wire (current carrying wire) $\rightarrow$

$$\boxed{\vec{F}_B = i\vec{\ell} \times \vec{B}}$$

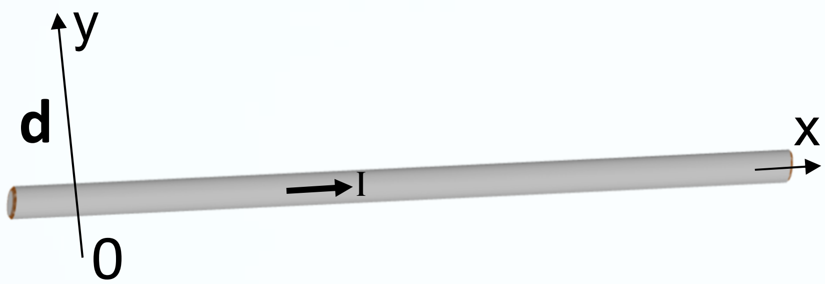
The Biot-Savart Law $\rightarrow$

$$\boxed{\vec{B}_{\text{point charge}} = \frac{\mu_0}{4\pi}\frac{q\vec{v} \times \hat{r}}{r^2}}$$

For an electric current $\rightarrow$

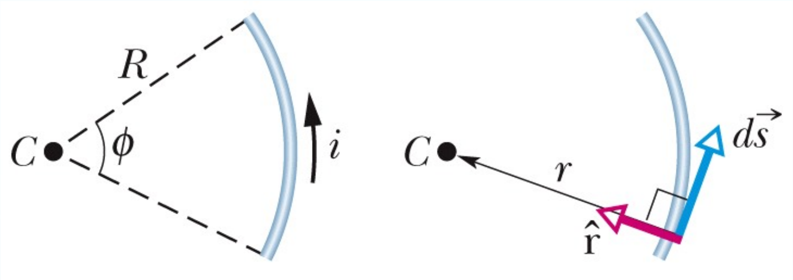
$$\boxed{\vec{B}_{\text{current segment}} = \frac{\mu_0}{4\pi}\frac{I\,\Delta\vec{s} \times \hat{r}}{r^2}}$$

Magnetic field due to current in long straight wire

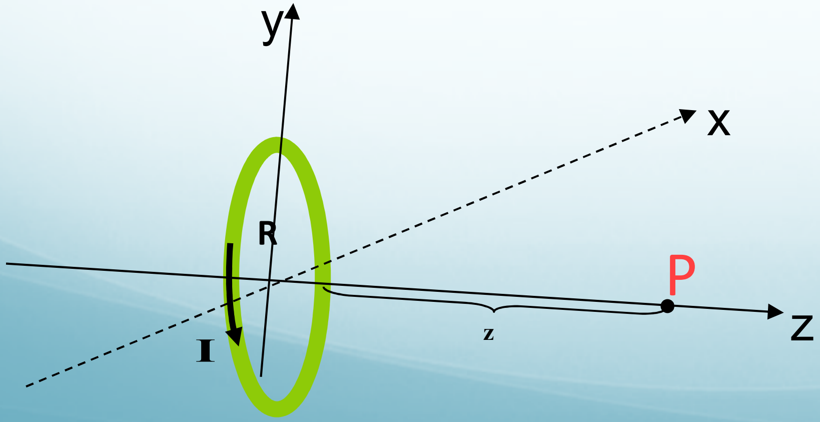


$$B_z = \frac{\mu_0}{2\pi}\frac{I}{d}$$

Non-infinite straight wire→ Appendix 1-chapter 22



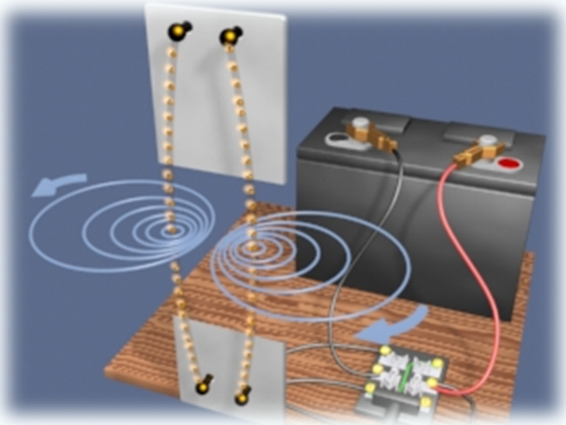
$$B = \frac{\mu_0 i\phi}{4\pi R}$$



$$\vec{B} = \frac{\mu_0}{2}\frac{IR^2}{\left(z^2 + R^2\right)^{3/2}}\hat{k}$$

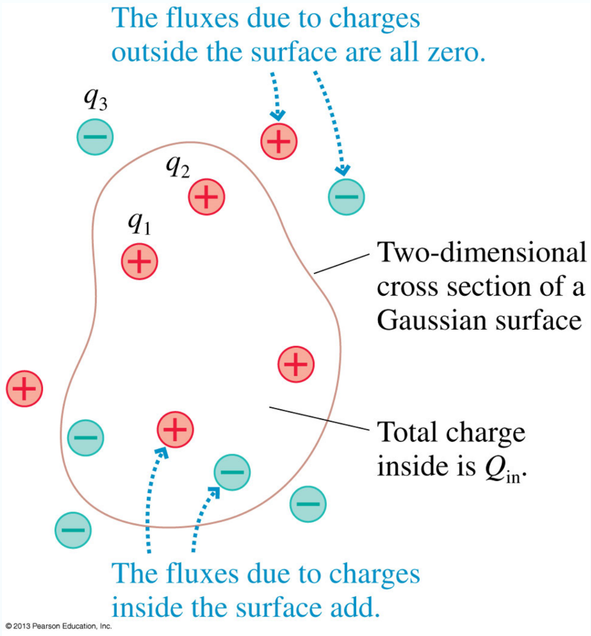
if $\quad z = 0 \quad \vec{B}_{center} = \frac{\mu_0}{2}\frac{I}{R}\hat{k}$

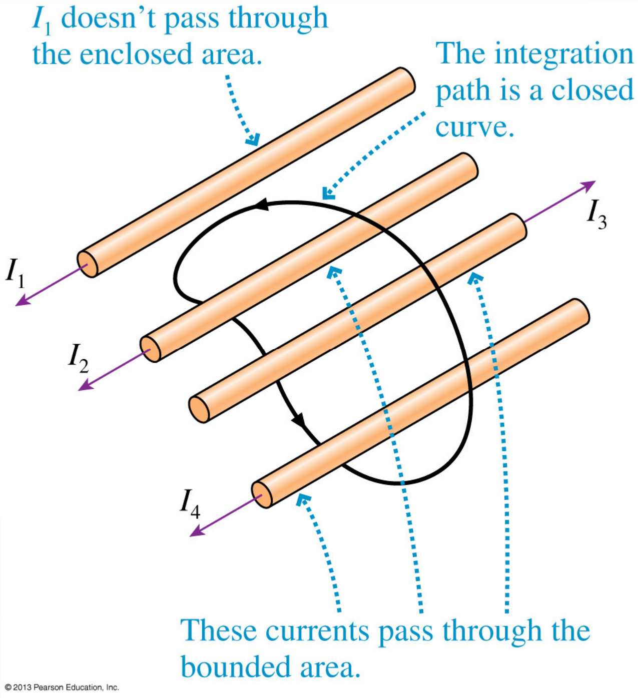
Force between two antiparallel currents



Ampère's law

i.e. $\boxed{\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}}$





$$\oint \vec{E}.d\vec{A} = \frac{Q_{in}}{\varepsilon_0}$$

For a closed surface enclosing total Charge Q

$$\oint \vec{B}.d\vec{l} = \mu_0 I_{enclosing}$$

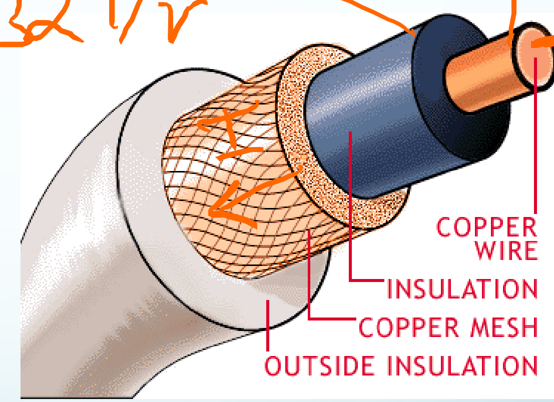
Current I passes through an area bounded by a closed curve

A coaxial cable consists of a wire (radius R_1) surrounded by an insulating sleeve and another cylindrical conducting shell (inner radius R_2) and finally another insulating sleeve. **The wire and the shell carry the same current I but in opposite directions**.

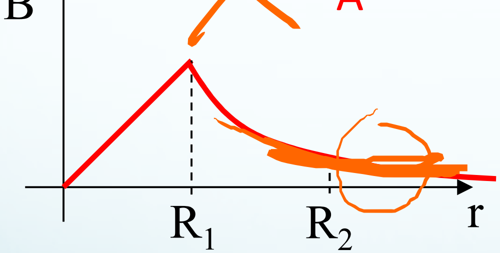
Which diagram best represents the **magnetic field** as a function of radial distance from the cable's axis?

$B \propto r$

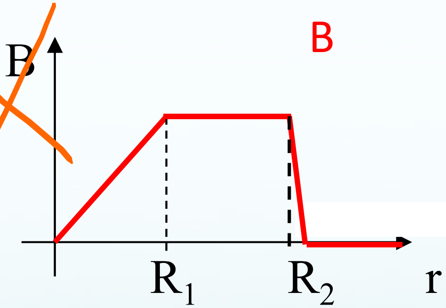
$B \propto 1/r$



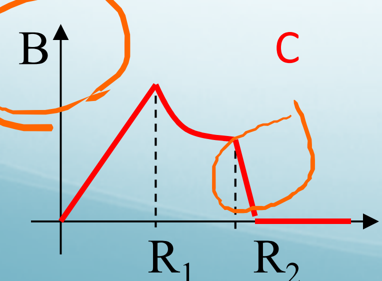
A



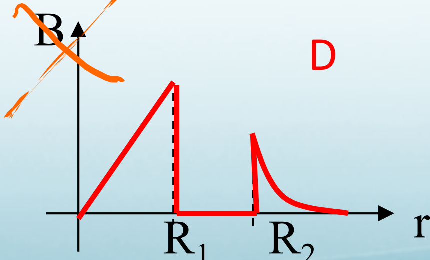
B



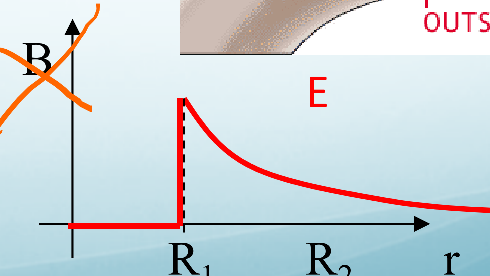
C



D



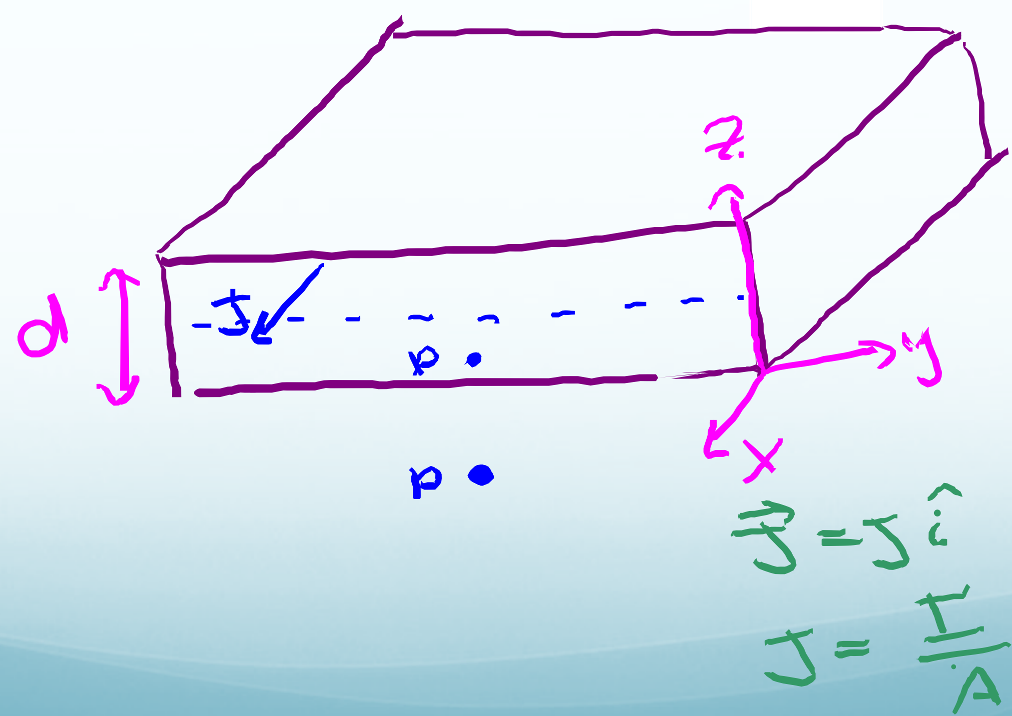
E



Ampère's law: application (3)

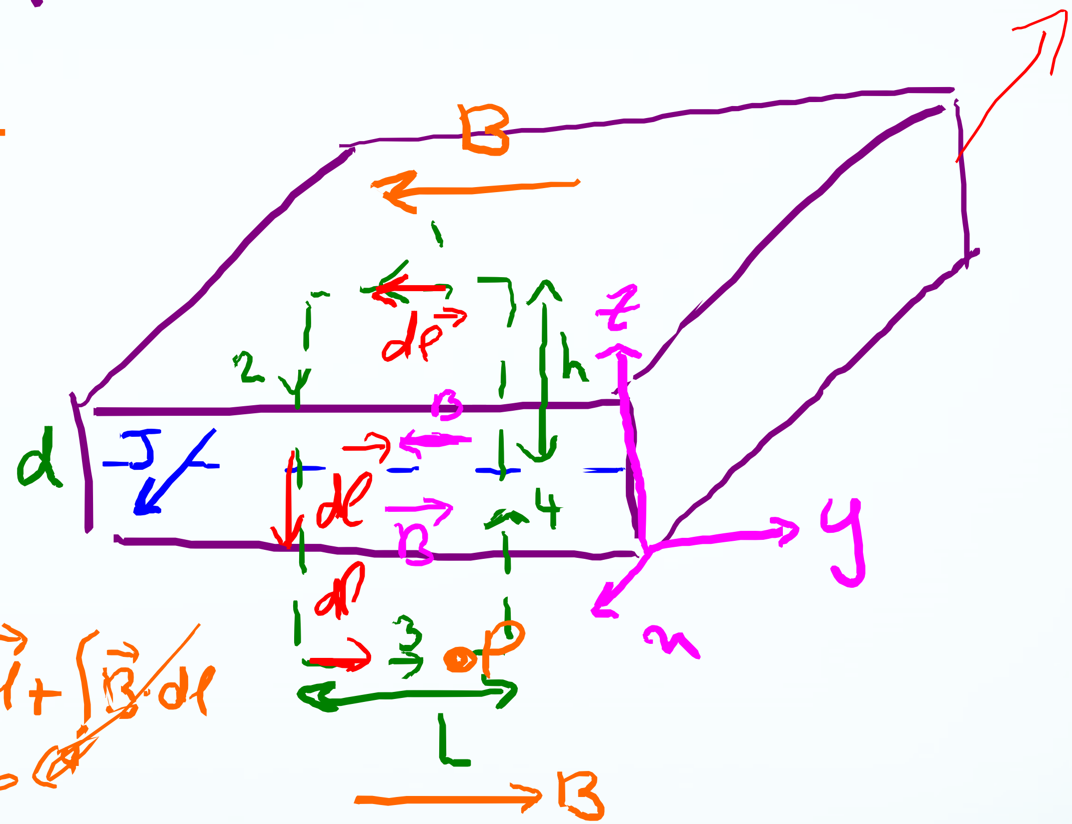
(a) Using Ampère's law, calculate the magnetic field above the current carrying slab

b) Calculate the magnetic field inside the current carrying slab



$\vec{B}$ above or below the slab $\Rightarrow$

$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc}$



$\oint \vec{B}\cdot d\vec{l} \Rightarrow$

$\to \int_1 \vec{B}\cdot d\vec{l} + \cancel{\int_2 \vec{B}\cdot d\vec{l}} + \int_3 \vec{B}\cdot d\vec{l} + \cancel{\int_4 \vec{B}\cdot d\vec{l}}$

$\to 2\int \vec{B}\cdot d\vec{l} = \mu_0 I_{enc}$

$\to 2B\int dl = \mu_0 I_{enc} = \mu_0 J A = \mu_0 J L d$

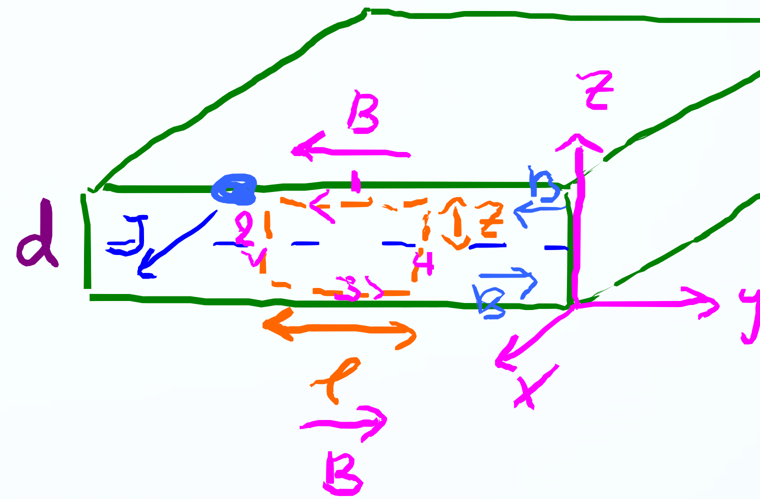
$2B\int dl = \mu_0 J L d \to 2B\cancel{L} = \mu_0 J \cancel{L} d \Rightarrow$

$\to B = \frac{\mu_0 J d}{2}$ $\quad \to B = \frac{\mu_0 J d}{2}$

$\vec{B}$ inside the slab $\Rightarrow$

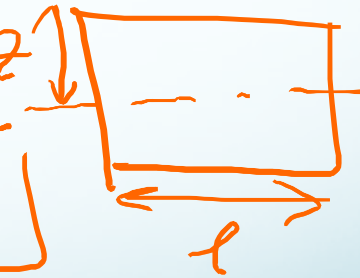
$$\oint \vec{B}\cdot\vec{dl} = \mu_0 I_{enc}$$

$$\rightarrow \int_1 \vec{B}\cdot\vec{dl} + \cancelto{0}{\int_2 \vec{B}\cdot\vec{dl}} + \int_3 + \cancelto{0}{\int_4}$$



$$\rightarrow 2\int B\,dl\cos\theta = \mu_0 I_{enc}$$

$$\rightarrow 2Bl = \mu_0 I_{enc} = \mu_0 J A_{in} = \mu_0 J\, 2zl$$



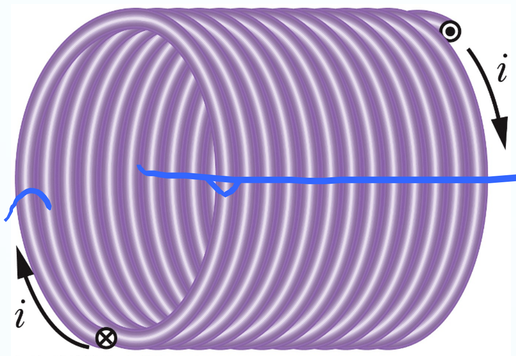
$$\rightarrow \cancel{2}B\cancel{l} = \mu_0 J \cancel{2} z \cancel{l} \rightarrow \boxed{B = \mu_0 J z}$$

$\rightarrow B = \mu_0 J z$ & if $z = \frac{d}{2} \rightarrow B = \frac{\mu_0 J d}{2}$

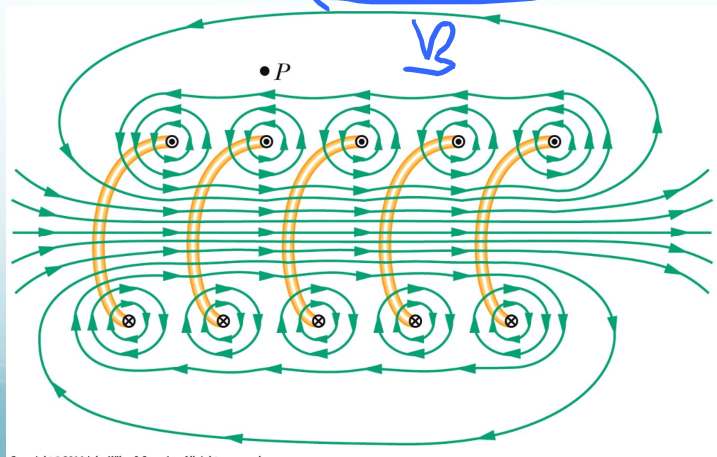
Ampère's law: application(2)

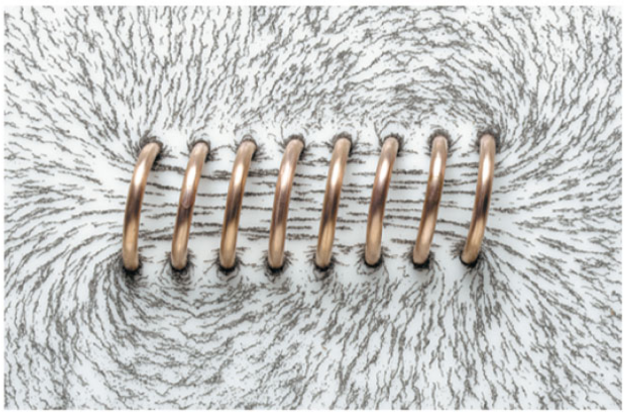
29.3: Solenoids and Toroids

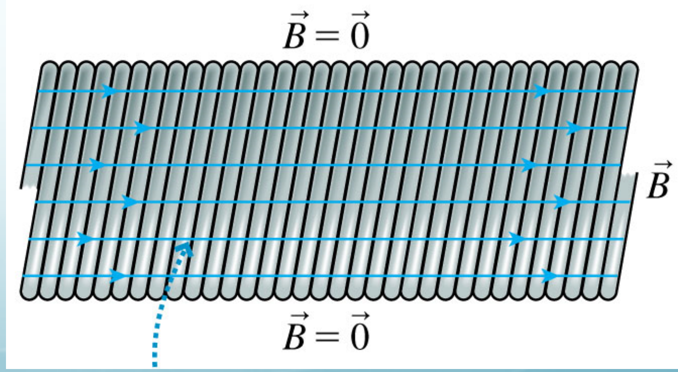
B



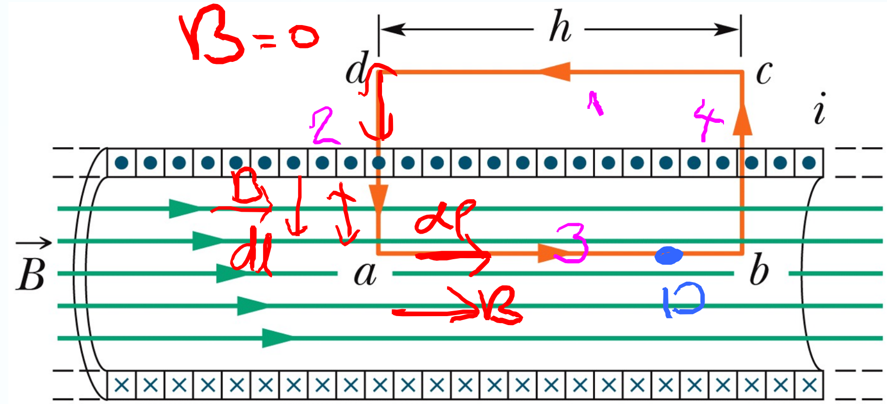
B







$$\oint \vec{B}.d\vec{s} = \mu_0 i_{enc}$$



$$n = \frac{N}{L}$$ **number of turns per unit length**

$$\cancel{\int_1 \vec{B}\cdot\vec{dl}} + \cancel{\int_2} + \int_3 + \cancel{\int_4}$$

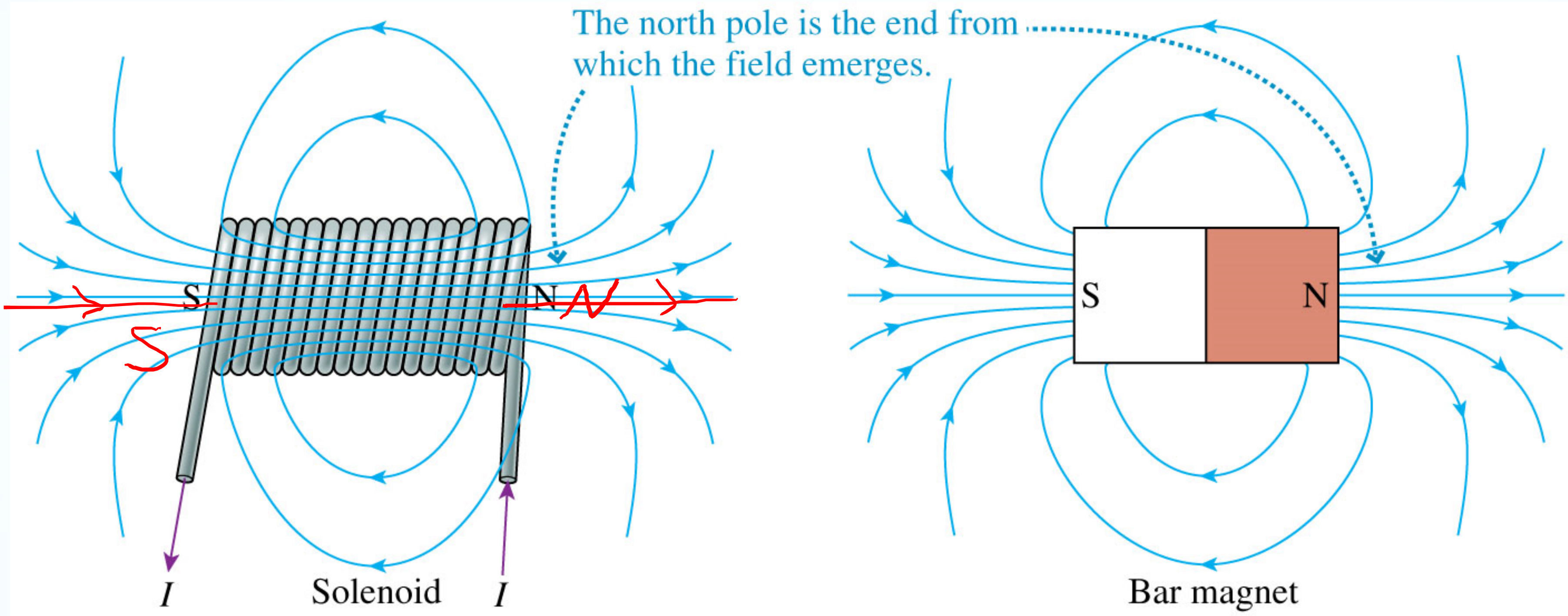
$$i_{enc} = n i h$$

$$\rightarrow \int_3 B\,dl\cos(0) = \mu_0 i_{enc}$$

$$\rightarrow B\cancel{h} = \mu_0 i_{enc} = \mu_0 n i \cancel{h}$$

$$\Rightarrow \boxed{B = \mu_0 n i}$$

$$B_{Solenoid} = \mu_0 n i$$

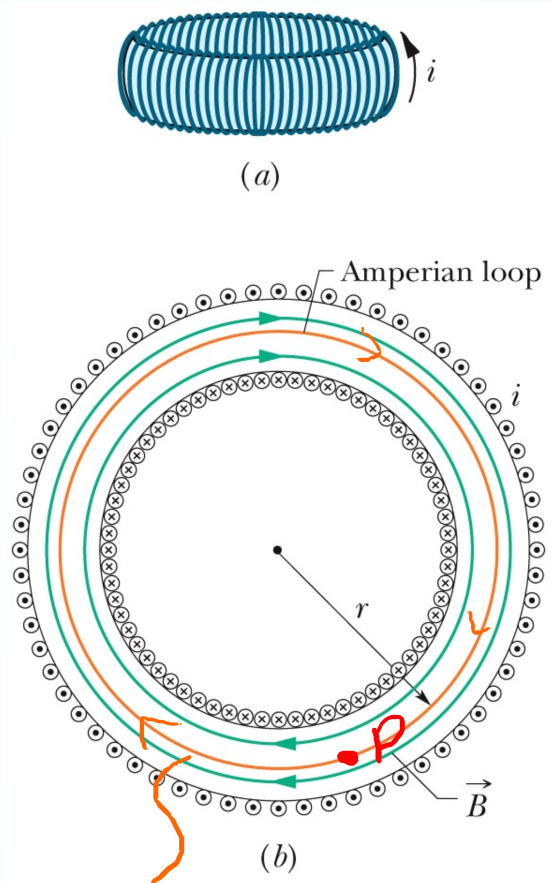


$$B_{Solenoid} = \mu_0 ni$$

magnetic field inside toroid

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \, i_{enc}$$

$$\rightarrow BL = \mu_0 \, i_{enc} \quad \& \quad i_{enc} = Ni$$



Amperian loop

$$\rightarrow B = \frac{\mu_0 i N}{2\pi} \frac{1}{r}$$

This section we talked about:

Chapter 29

See you on Friday