Group #	Student	Last Name	First Name
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(10 marks) In the circuit given below, a capacitor has capacitance C and time-dependent charge q(t). The emf of the batteries follow the relation $\mathcal{E}_1 > \mathcal{E}_2$. Find the time constant τ that characterizes the charging of the capacitor after the switch is closed.

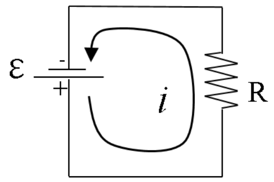

Figure 1: Current convention

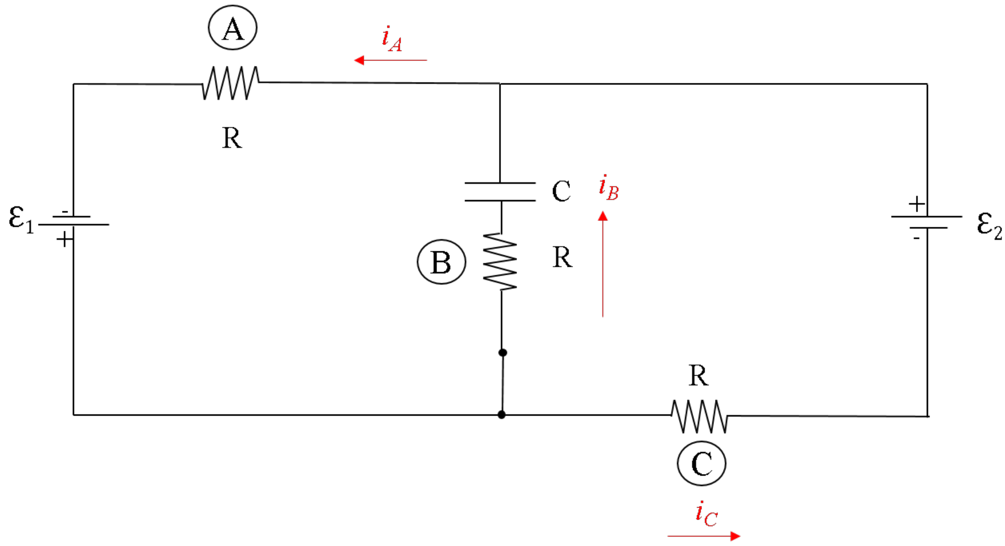

Figure 2: Multi-loop circuit

The parts below walk you through related questions, and the steps with which to solve this problem. Please <u>show all work</u> in the boxes provided and then choose the correct answer at the bottom

1. **(1 mark)** For each of the following situations, explain what happens in the circuit:
Need to have 2/3 situations correct to get full marks here.

• The switch has just been closed for the first time
The capacitor begins to charge from current flowing through the center branch of the circuit. Initially there is no charge on the capacitor so it acts like a wire.

• The switch has been closed for a long time
The capacitor becomes fully charged and acts like a broken wire. Current will flow only along the outside loop

• The switch has been opened. *Nothing will change from the previous situation. The charged capacitor retains its charge but now acts like a broken wire, which is analogous to opening the switch.*

<u>**The switch has recently been closed.** Answer the following questions.</u>

2. **(1 mark)** Let i_A, i_B and i_C represent the currents through resistors A, B and C (Fig. 2), respectively. Draw and label arrows indicating the direction of current through each resistor. *The direction of current MUST follow the convention depicted in Fig. 1).* **See annotations on Fig. 2. Must have directions and labels as requested in question.**

3. **(1 mark)** Write an expression that relates the rate of the capacitor charging, $dq(t)/dt$, to i_A and i_C.

$$\frac{dq}{dt} = i_B = i_A - i_C$$

4. **(2 marks)** Express the electromotive forces $\mathcal{E}_1$ and $\mathcal{E}_2$ in terms of R, C, i_A, i_B, q(t) and i_C.
Note: The circuit in Fig. 2 can be separated into a lefthand loop including $\mathcal{E}_1$ and a righthand loop including $\mathcal{E}_2$. Using Kirchoff's law we get:

$$\begin{aligned}\mathcal{E}_1 &= i_B R + \frac{q(t)}{C} + i_A R \\ \mathcal{E}_2 &= -(\frac{q(t)}{C} + i_B) + i_C R \\ &= -\frac{q(t)}{C} - i_B R + i_C R\end{aligned}$$

Note that the elements in the middle branch act as voltage *gains* in the loop with $\mathcal{E}_2$ (this is because i_B is in the opposite direction of the righthand loop current).

5. **(1 mark)** Use the two equations you obtained in Question 4 to create *one simplified equation* that contains R, C, q(t) and i_B but does **NOT** contain i_A nor i_C.

$$\begin{aligned}\mathcal{E}_1 - \mathcal{E}_2 &= \frac{2q(t)}{C} + 2i_B R + (i_A - i_C)R \\ &= \frac{2q(t)}{C} + 3i_B R\end{aligned}$$

6. **(1 mark)** Re-write the charging equation from Question 5 in terms of R, C, q(t) and $\frac{dq}{dt}$.

$$\begin{aligned}i_B &= \frac{dq(t)}{dt}, so: \\ \mathcal{E}_1 - \mathcal{E}_2 &= \frac{2q(t)}{C} + 3\frac{dq}{dt}R\end{aligned}$$

7. **(2 marks)** Compare your equation from Question 6 to the **single-loop** circuit charging equation:

$$\tilde{\mathcal{E}} = \tilde{R}\frac{d\tilde{q}}{d\tilde{t}} + \frac{\tilde{q}}{\tilde{C}}.$$

In the following table, fill in the substitutions that would allow us to re-write the charging equation from Question 6 in the standard, single-loop form given here. **Note: the $\sim$ above variables here serve only to distinguish them from the multi-loop circuit**

Variable	**Single Loop**	**Fig. 2 Circuit**	**Show your work here:**
Capacitance	$\tilde{C}$	C	
Resistance	$\tilde{R}$	$\frac{3}{2}R$	
Time	$\tilde{t}$	t	$\frac{\mathcal{E}_1 - \mathcal{E}_2}{2} = \frac{q(t)}{C} + \frac{3}{2}\frac{dq}{dt}R$
Charge	$\tilde{q}(\tilde{t})$	$q(t)$	
Electromotive force	$\tilde{\mathcal{E}}$	$(\mathcal{E}_1 - \mathcal{E}_2)/2$	

Note: Substitutions in table only worth 0.5 marks on their own – must either follow from question (6) OR be justified by additional work/comments in "show your work" box. Coefficients of the variables here could also differ, so long as they are consistent with the complete differential equation (e.g. $\tilde{C} \to C/2; \tilde{R} \to 3R; \tilde{\mathcal{E}} \to \mathcal{E}_1 - \mathcal{E}_2$) .

Since we know such an equation can be solved to give time constant $\tau = \widetilde{R}\widetilde{C}$, subbing in we get $\tau = \frac{3}{2}RC$ **(1 mark for the correct answer)** What is the time constant τ that governs charging of the capacitor in Fig. 2?

A. $\tau = 6RC$ B. $\tau = RC$ C. $\boxed{\tau = \dfrac{3}{2}RC}$ D. $\tau = \frac{1}{C} + \frac{3}{2}R$