Last time

- More on the electric properties of conductors and Gauss's law

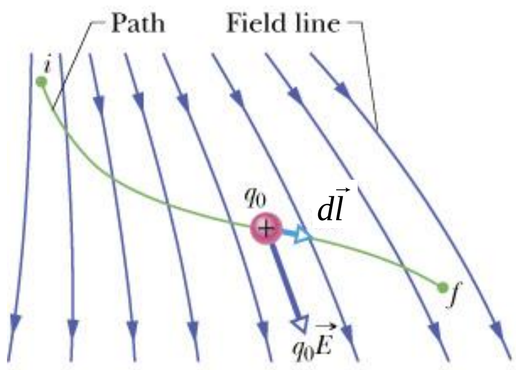
This time

- Potential energy
- Electric potential

Electric potential

Like gravitational potential where we move a mass in the gravitational field and ask for the amount of work required, we can also ask how much work is required for a positive test charge $q_{0,}$ in an external electric field to move from point *i* to point *f*.

The work for an infinitesimal displacement $d\vec{l}$ is given by



$$dW = -\vec{F} \bullet d\vec{l}$$

$$dW = -q_0 \vec{E} \bullet d\vec{l}$$

$$W = -q_0 \int_i^f \vec{E} \bullet d\vec{l}$$

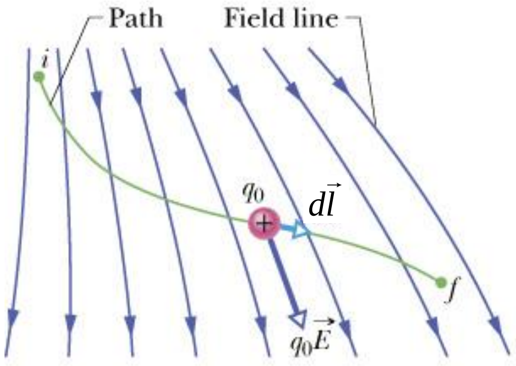
This is a line integral.

The minus sign is only for book keeping purposes to indicate that the work is done by external field rather than by the charge.

Electric potential

The work done by the charge has the opposite sign

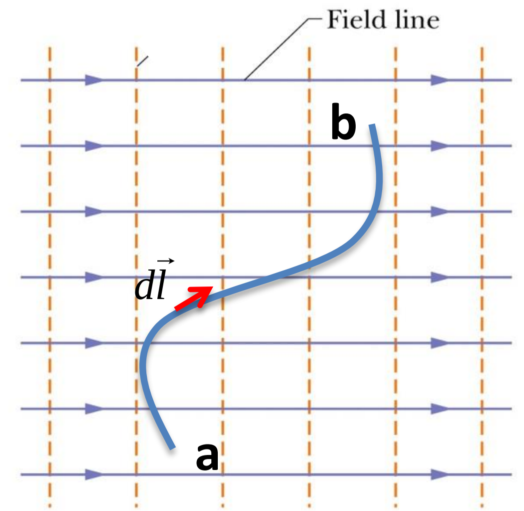
$$W = +q_0 \int_i^f \vec{E} \bullet d\vec{l}$$



The simplest possible case:

$$\vec{E} = \text{constant}$$

Find the work done by the field to move q_0 from point **a** to point **b**.



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$$dW = -q_0 \vec{E} \bullet d\vec{l}$$

$$W = -q_0 \int_a^b \vec{E} \bullet d\vec{l}$$

$$W = -q_0 \vec{E} \bullet \int_a^b d\vec{l}$$

What is the geometrical meaning of $\int_a^b d\vec{l}$?

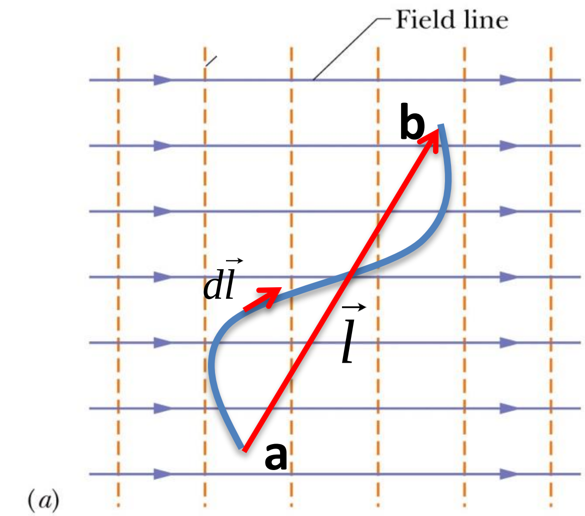
It is a straight vector which starts at **a** and ends at **b**.

The simplest possible case:

$$\vec{E} = \text{constant}$$

$$W = -q_0 \vec{E} \bullet \int_a^b d\vec{l}$$

$$W = -q_0 \vec{E} \bullet \vec{l}$$



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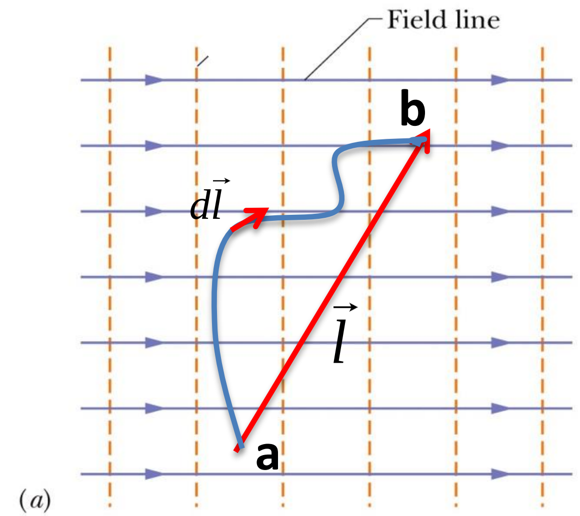
How about a different path?

The simplest possible case:

$$\vec{E} = \text{constant}$$

$$W = -q_0 \vec{E} \bullet \int_a^b d\vec{l}$$

$$W = -q_0 \vec{E} \bullet \vec{l}$$



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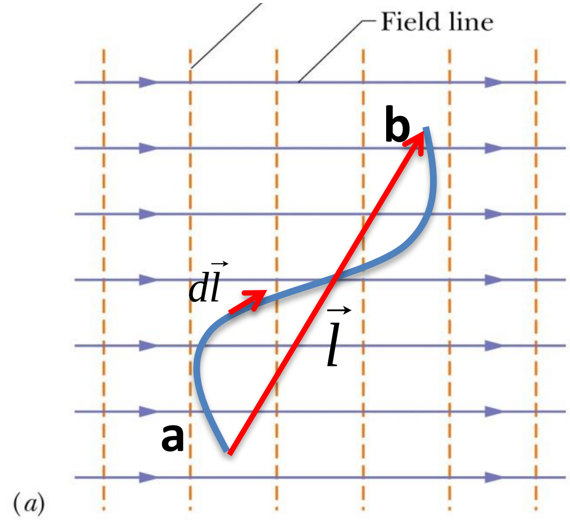
Work done is the same!

What conclusion should we draw from these observations?

$\vec{l}$ is a vector which connects **a** to **b**.

$$W = -q_0 \vec{E} \bullet \vec{l}$$

This implies that the work done is independent of the path chosen as long as start and end points are the same.



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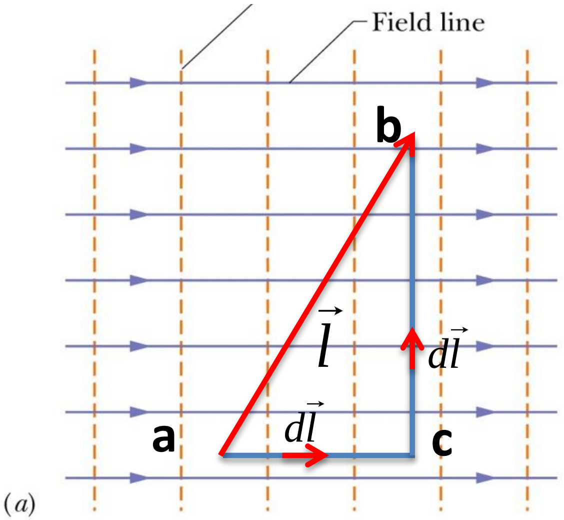
What if we chose the path from **a** to **c** and then from **c** to **b**? Work done is the same as any other path connecting a to b.

$$W = -q_0 \int_{abc} \vec{E} \bullet d\vec{l}$$

$$W = -q_0 \int_{ac} \vec{E} \bullet d\vec{l} - q_0 \int_{cb} \vec{E} \bullet d\vec{l}$$

For the first integral electric field is parallel to the displacement vector.



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For the second integral electric field is perpendicular to the displacement vector and does not contribute to the overall work.

Only the component of $\vec{l}$ which is parallel to the electric field direction will result in non-zero work. The component which is perpendicular to electric field results in no work.