# Announcements

- Complete Assignment #2 before **11:59 pm, Wednesday, January 25**.

- Laboratorials start this week.

- Midterm and the final will consist entirely of MCQ

- To get a passing grade letter you must obtain a weighted average of 50% or better for the midterm and the final.

# Last time

- Coulomb's force due to a 90° arc of radius R at its center
- Coulomb's force due to a 180° arc of radius R at its center.
- Activity #2

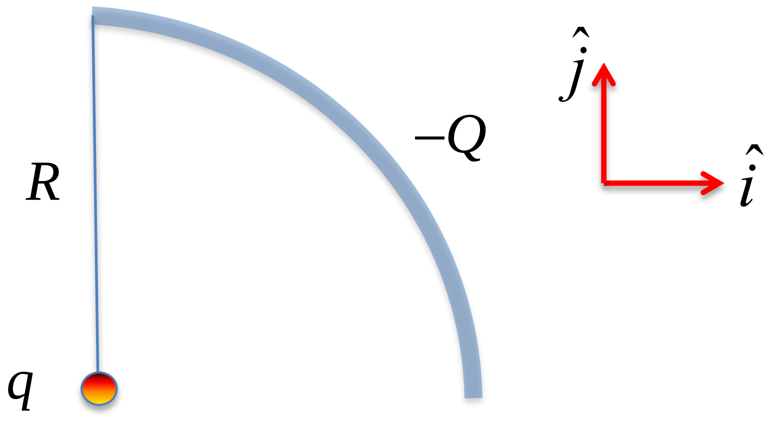
# This time

- Review of Coulomb's force due to a 90° arc of radius R at its center, constant charge density
- Coulomb's force due to a semi-circle of radius R at its center using superposition principle
- Coulomb's force due to a  circle of radius R at its center using superposition principle
- Coulomb's force due to a semi-circle of radius R at its center with a twist using superposition principle
- Electric field

# Electric force due to an arc at its center

Consider a arc of radius *R* with the total charge of -*Q* and a uniform charge distribution.

Compute the force due to the arc on a point charge *q* located at the center of the arc.



# Electric force due to 90° arc at its center

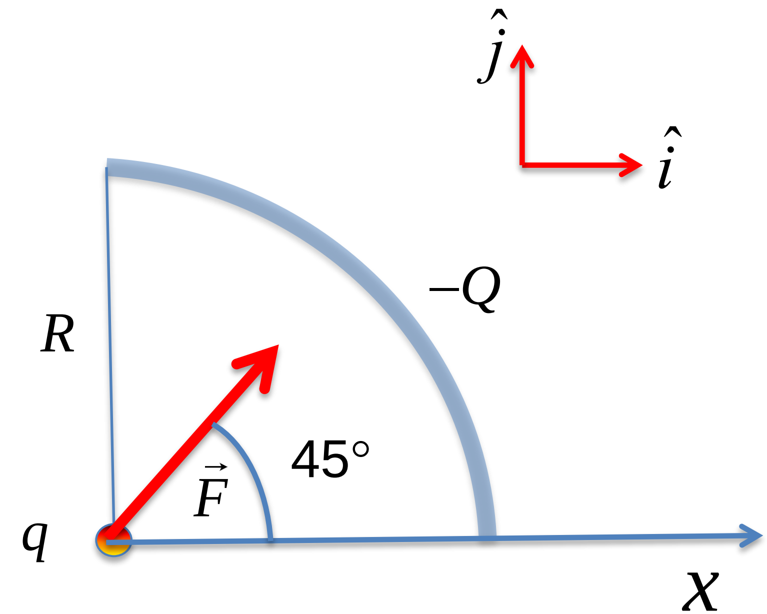
$$F_x = \int dF_x = \int_0^{\pi/2} \frac{k_e q \lambda R d\theta}{R^2} \cos\theta = \frac{k_e q \lambda}{R} \sin\theta \Big|_0^{\pi/2} = \frac{k_e q \lambda}{R} = \left(\frac{2}{\pi}\right) \frac{k_e q Q}{R^2}$$

$$F_y = \int dF_y = \frac{k_e q \lambda}{R} \int_0^{\pi/2} \sin\theta d\theta = -\frac{k_e q \lambda}{R} \cos\theta \Big|_0^{\pi/2} = \frac{k_e q \lambda}{R} = \left(\frac{2}{\pi}\right) \frac{k_e q Q}{R^2}$$

$$F_x = \left(\frac{2}{\pi}\right) \frac{k_e q Q}{R^2}$$

$$F_y = \left(\frac{2}{\pi}\right) \frac{k_e q Q}{R^2}$$

$$\vec{F} = \frac{2}{\pi} \frac{k_e q Q}{R^2} \left(\hat{i} + \hat{j}\right)$$



# Electric force due to 90° arc at its center

$$F_x = \int dF_x = \frac{k_e q\lambda}{R}\sin\theta\,|_{-\pi/2}^{\pi/2} = \frac{2k_e q\lambda}{R} = \left(\frac{2}{\pi}\right)\frac{k_e qQ}{R^2}$$

$$F_y = \int dF_y\,\frac{k_e q\lambda}{R}\int_{-\pi/2}^{\pi/2}\sin\theta d\theta = -\frac{k_e q\lambda}{R}\cos\theta\,|_{-\pi/2}^{\pi/2} = 0$$

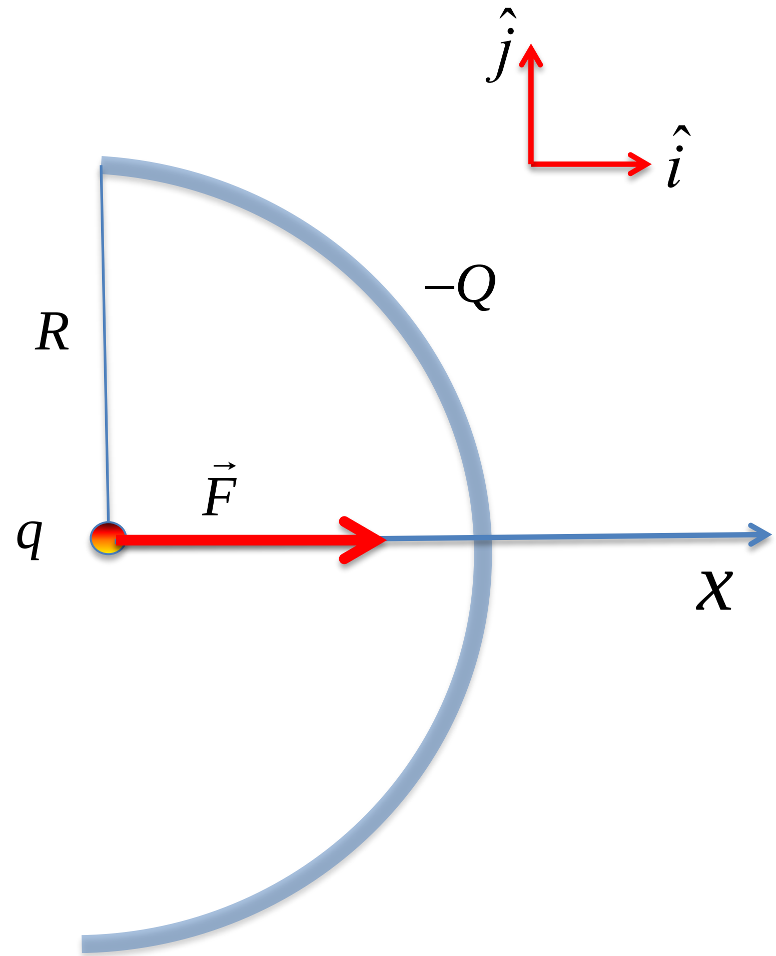
$$F_x = \left(\frac{2}{\pi}\right)\frac{k_e qQ}{R^2}$$

$$F_y = 0$$

$$\vec{F} = \frac{2}{\pi}\frac{k_e qQ}{R^2}\hat{i}$$

Remember that charge density in this case is

$$\lambda = \frac{|-Q|}{\pi R}$$

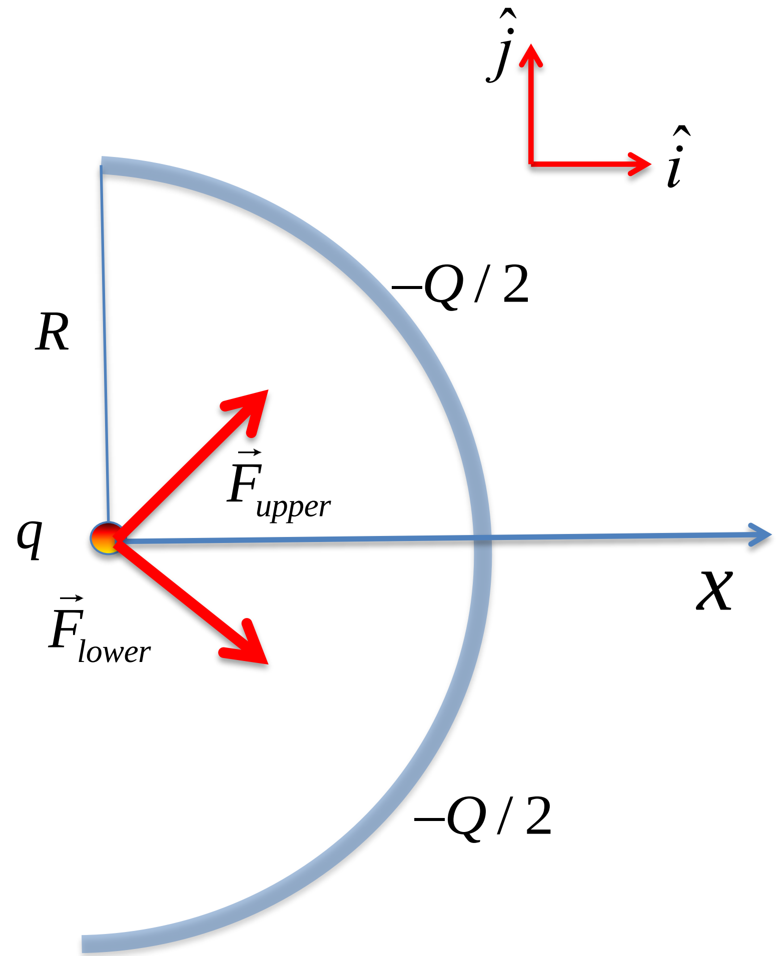


# Electric force due to 180° arc at its center

Or divide the semi-circle into two 90° arcs and use the superposition principle.

$$\vec{F}_{upper} = \left(\frac{2}{\pi}\right)\frac{k_e qQ/2}{R^2}(\hat{i}+\hat{j})$$

$$\vec{F}_{lower} = \left(\frac{2}{\pi}\right)\frac{k_e qQ/2}{R^2}(\hat{i}-\hat{j})$$



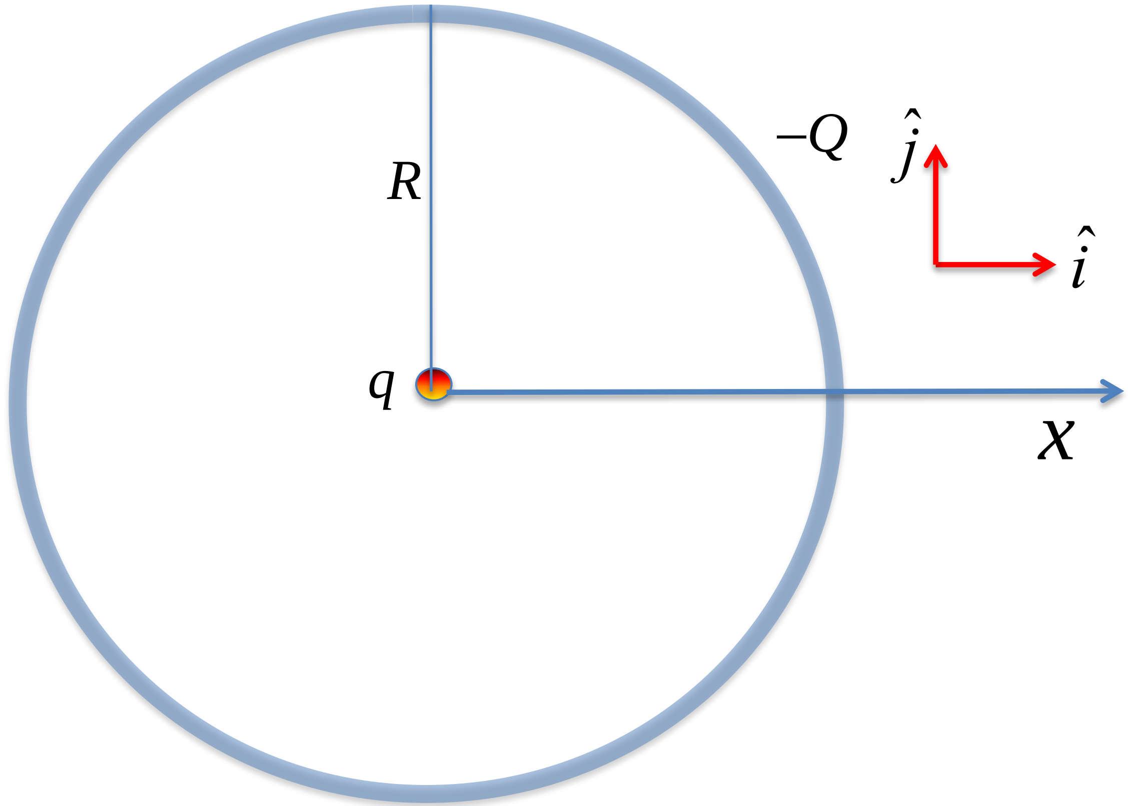

SYMMERTY ABOUT X-AXIS!

$$F_x = \frac{2}{\pi}\frac{k_e qQ}{R^2} \qquad F_y = 0$$

$$\vec{F} = \frac{2}{\pi}\frac{k_e qQ}{R^2}\hat{i}$$

# Electric force due to circular ring at its center

$$\lambda = \frac{|-Q|}{2\pi R}$$




$$F_x = \int dF_x = \frac{k_e q\lambda}{R}\int_0^{2\pi}\cos\theta d\theta = \frac{k_e q\lambda}{R}\sin\theta\,|_0^{2\pi} = 0$$

$$F_y = \int dF_y = \frac{k_e q\lambda}{R}\int_0^{2\pi}\sin\theta d\theta = -\frac{k_e q\lambda}{R}\cos\theta\,|_0^{2\pi} = 0$$

$$F_x = 0$$

$$F_y = 0$$

# Electric force due to circular ring at its center

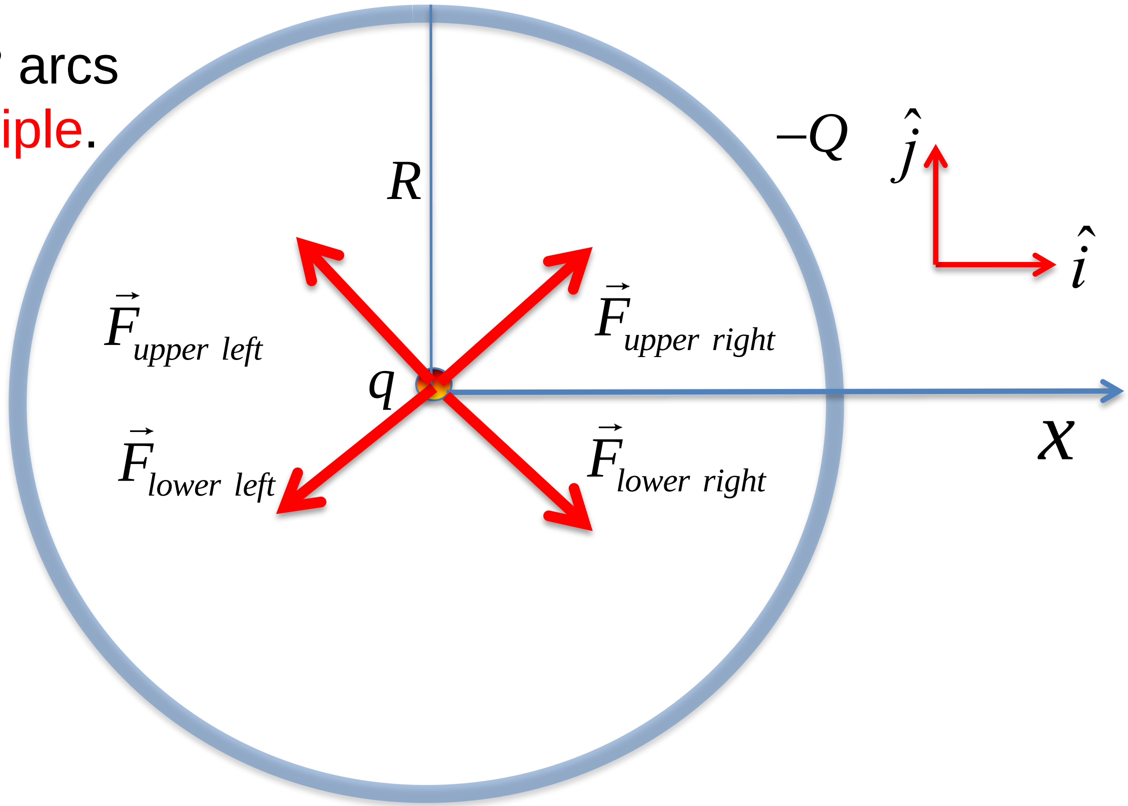
Or divide the circle into four 90° arcs and use the superposition principle.

$$\vec{F}_{upper\ right} = \left(\frac{2}{\pi}\right)\frac{k_e qQ/4}{R^2}(\hat{i}+\hat{j})$$

$$\vec{F}_{lower\ \ right} = \left(\frac{2}{\pi}\right)\frac{k_e qQ/4}{R^2}(\hat{i}-\hat{j})$$

$$\vec{F}_{lower\ \ left} = \left(\frac{2}{\pi}\right)\frac{k_e qQ/4}{R^2}(-\hat{i}-\hat{j})$$

$$\vec{F}_{upper\ left} = \left(\frac{2}{\pi}\right)\frac{k_e qQ/4}{R^2}(-\hat{i}+\hat{j})$$




$$F_x = 0$$

$$F_y = 0$$

# Electric force due to 180° arc at its center with a twist

Upper half with the total charge –Q and the lower half with the total charge +Q

## Use the superposition principle

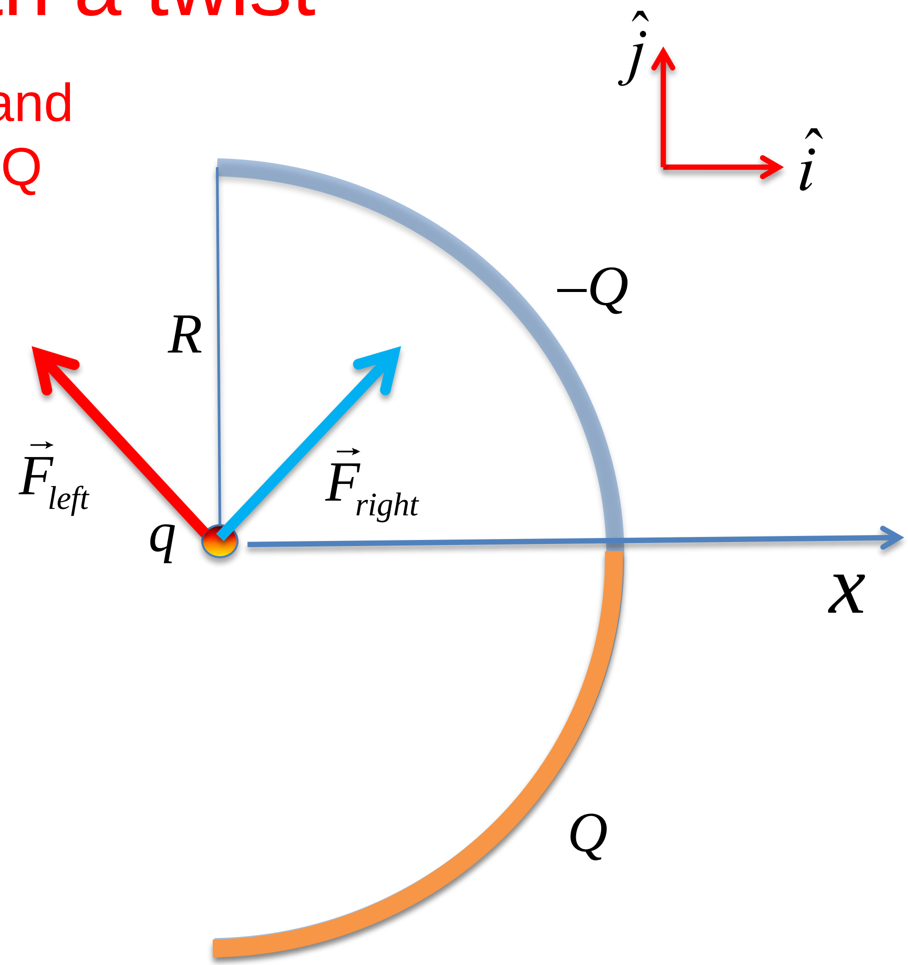
Consider the upper and lower half separately

$$\lambda_{top} = \frac{|-Q|}{\pi R/2} \qquad \lambda_{bottom} = \frac{|Q|}{\pi R/2}$$

$$\vec{F}_{right} = \left(\frac{2}{\pi}\right)\frac{k_e q Q}{R^2}\left(\hat{i} + \hat{j}\right)$$

$$\vec{F}_{left} = \left(\frac{2}{\pi}\right)\frac{k_e q Q}{R^2}\left(-\hat{i} + \hat{j}\right)$$

$$F_x = 0 \qquad F_y = \frac{4}{\pi}\frac{k_e q Q}{R^2}$$




**SYMMETRY ABOUT THE X-AXIS**

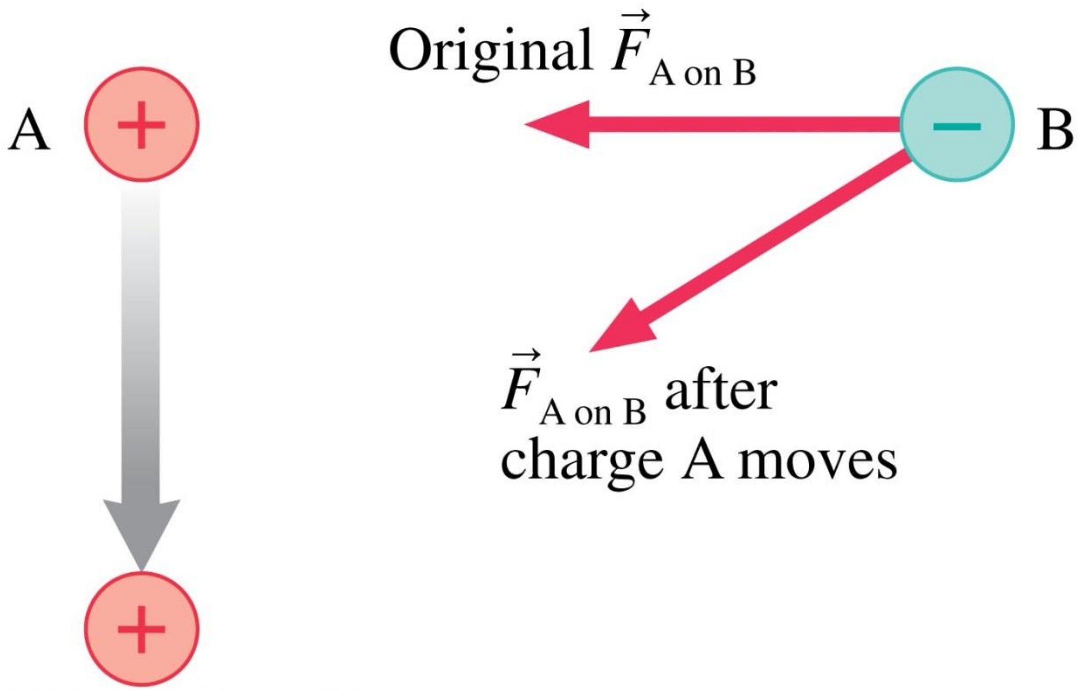
# Action-at-a-Distance Forces
## (Electric field)

**A** exerts a force on **B** through empty space.

- No contact.
- No apparent **mechanism**.

# Action-at-a-Distance Forces
## (Electric field)

If **A** suddenly moves to a new position, the force on **B** varies to match this new position. **How?**



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A hipster is allowed to look to the right but not to the left.

(a) If a charge B is placed at the position shown and is free to move, what will he observe? Assume tha he does not affect the outcome of any observations he makes.


(b) What will he observe when charge A is placed behind him? He sees that charge B will move to the right or to the left.

We conclude that the space surrounding charge A is modified even if charge B is not present. We say charge A creates an electric field.

# What if B wasn't there?

If B is not there, charge A still "does something" to the **surrounding space**. We can quantify this by using the concept of an **electric field**. Start with a single positive charge:

Coulomb's Law rewritten:

$$\vec{F}_e = \left(\frac{k_e q}{r^2}\hat{r}\right)q' = \vec{E}q'$$

**Electric field**

A

Universal gravity rewritten:

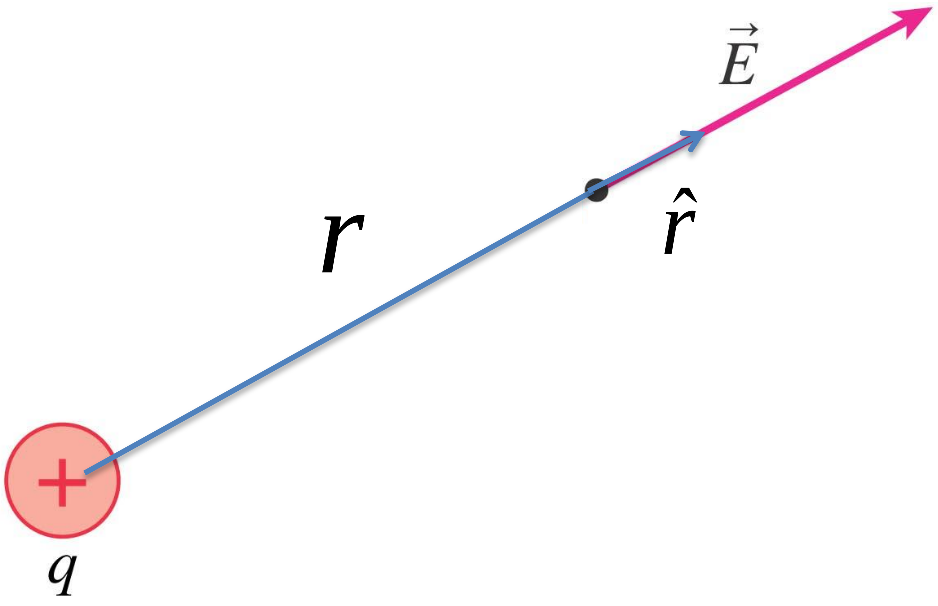
$$\vec{F}_G = \left(\frac{-GM}{r^2}\hat{r}\right)m = \vec{g}m$$

**Gravitational field**

Electric field is the mediator between q and q'!

# Electric field of a point charge at a distance r from the charge

$$\vec{E} = \frac{k_e q}{r^2}\hat{r}$$

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