Electricity and Magnetism

- Physics 259 – L02
- Lecture 19

Chapter 23.3-4

Last time

- Chapter 23.2 

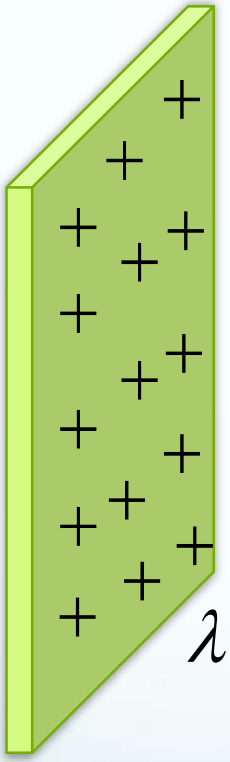
This time

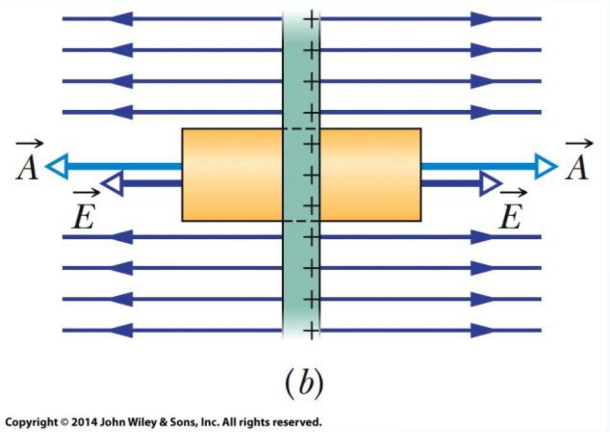
- Chapter 23. and 23.4 

23-5: Electric field of a plane of charge

Nonconduction infinite sheet

$$\Phi_e = \oint \vec{E}.d\vec{A} = \frac{Q_{in}}{\varepsilon_0}$$





$$E_{plane} = \frac{\sigma}{2\varepsilon_0}$$

Q2) Two very thin infinite sheets are uniformly charged with surface charge densities -2η and $+5\ \eta$ as indicated in the figure. What is the magnitude and direction of the electric field at point P located between the sheets? (note the direction of +x in the figure)

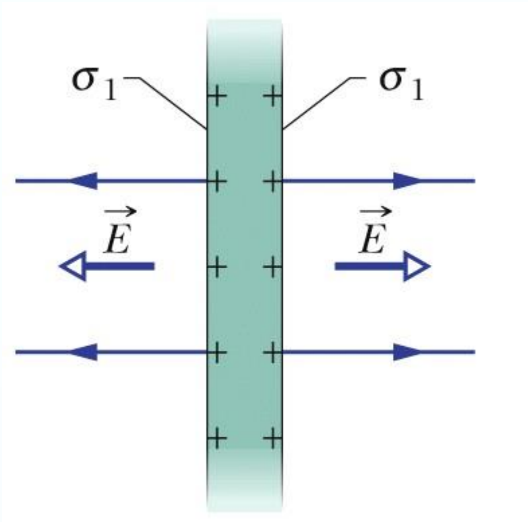
a) $-3\eta/2\varepsilon_o$

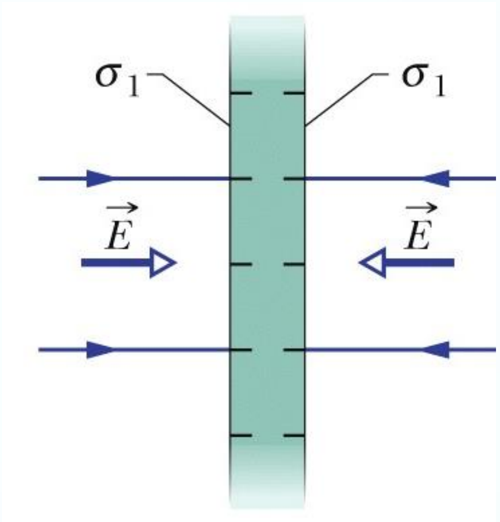
b) $+3\eta/2\varepsilon_o$

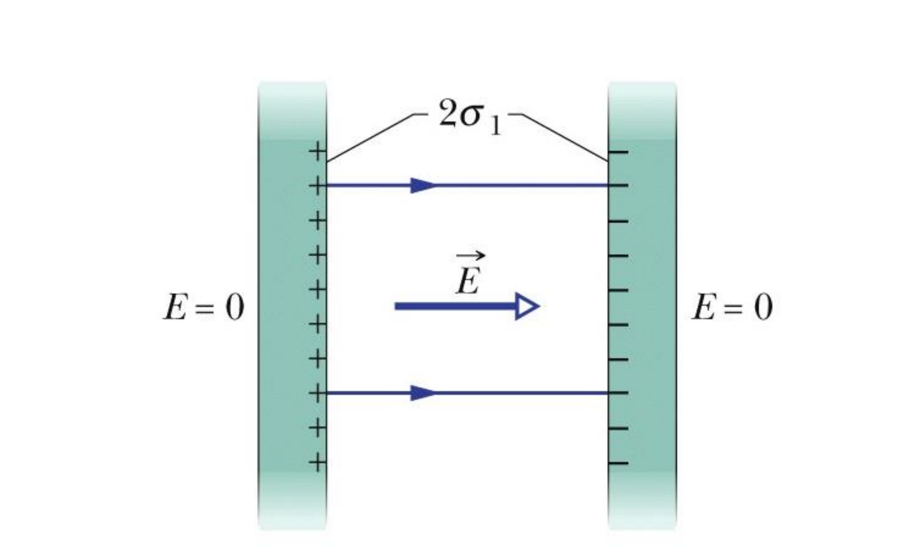
c) $-7\eta/2\varepsilon_o$

d) $+7\eta/2\varepsilon_o$

Two conducting Plates



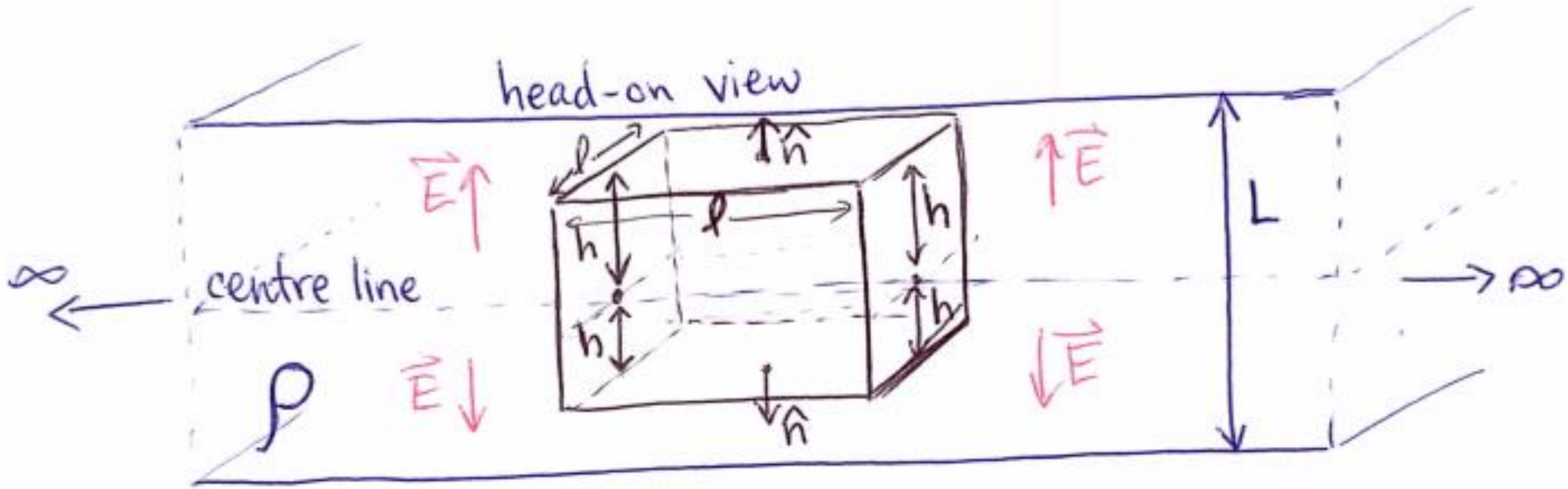




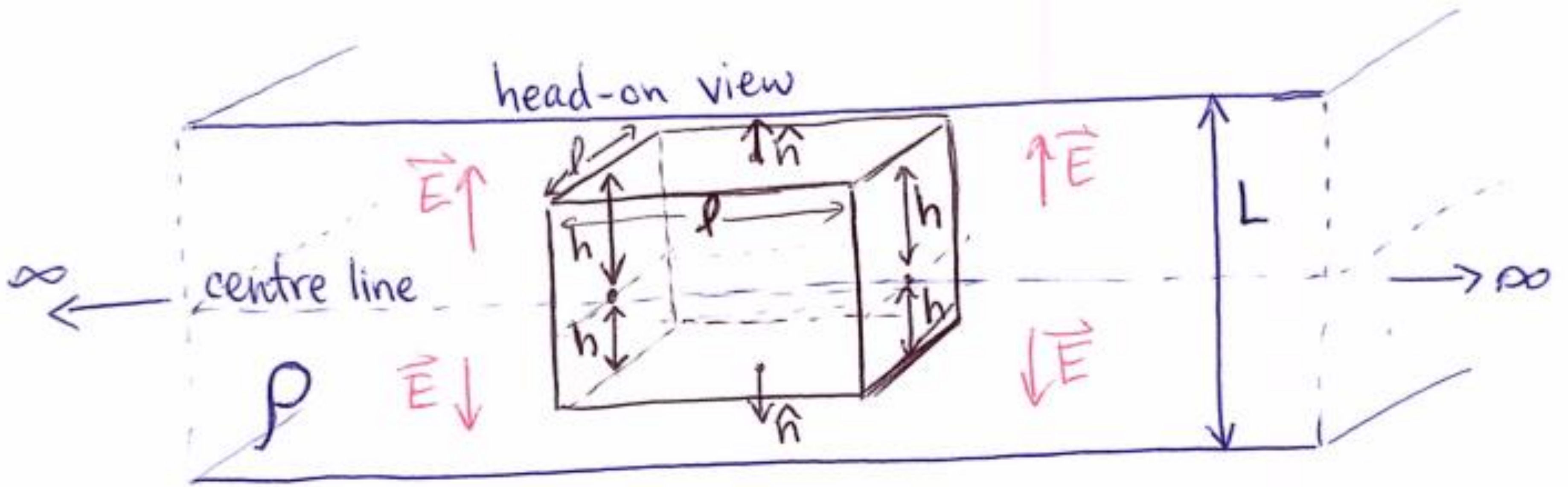
(c)

$$E = \frac{2\sigma_1}{\varepsilon_0} = \frac{\sigma}{\varepsilon_0}.$$

What is the field inside the slab?



The slab has thickness L, we have to choose a Gaussian surface with the same symmetries as the slab: choose a box whose centre coincides with the centre of the slab.



$$\oint \vec{E}\cdot d\vec{A} = \int_{top} \vec{E}\cdot dA + \int_{bottom} \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0}$$

$$E\int_{top} dA + E\int_{bottom} dA = \frac{q_{enc}}{\epsilon_0}$$

$$2EA = \frac{q_{enc}}{\epsilon_0} \qquad q_{enc} = \rho V = \rho A(2h)$$

$$\cancel{2}E\cancel{A} = \frac{\rho \cancel{A}(\cancel{2}h)}{\epsilon_0}$$

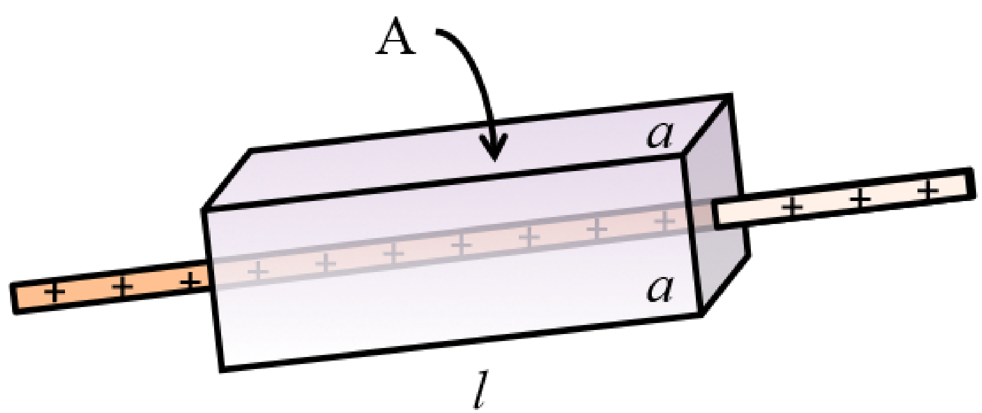
$$\boxed{E = \frac{\rho h}{\epsilon_0}}$$

What about cylinder?

Read appendix 1-chapter 23 posted on D2l.

Field of a line charge

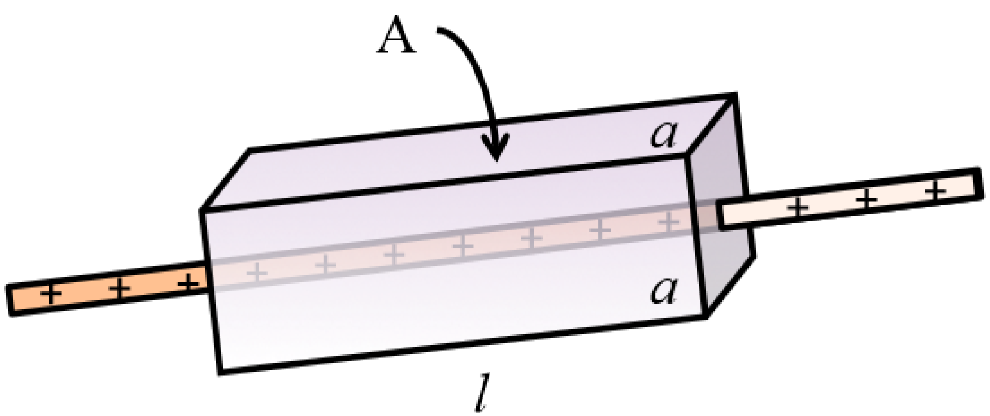
Consider an infinitely long, positively charged rod of linear charge density λ. How large is the flux through side A of the box? Suppose the values for l, a and λ are given.



Field of a line charge

- Consider an infinitely long, positively charged rod of linear charge density λ. How large is the flux through side A of the box? Suppose the values for l, a and λ are given.
- Gauss' law tells us that the total electric flux only depends on the enclosed charge – not the shape of the (closed) Gaussian surface:

  $\Phi_{tot} = Q_{encl}/\varepsilon_0 = \lambda l/\varepsilon_0$

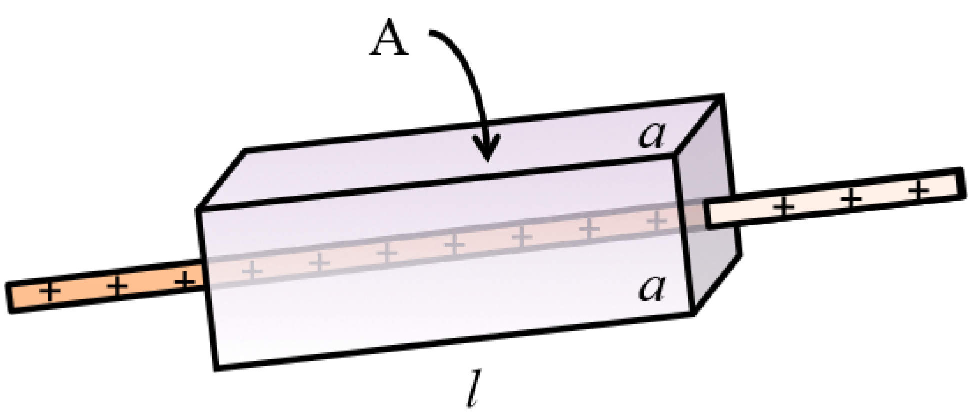


Field of a line charge

- Consider an infinitely long, positively charged rod of linear charge density λ. How large is the flux through side A of the box? Suppose the values for l, a and λ are given.
- Gauss' law tells us that the total electric flux only depends on the enclosed charge – not the shape of the (closed) Gaussian surface:

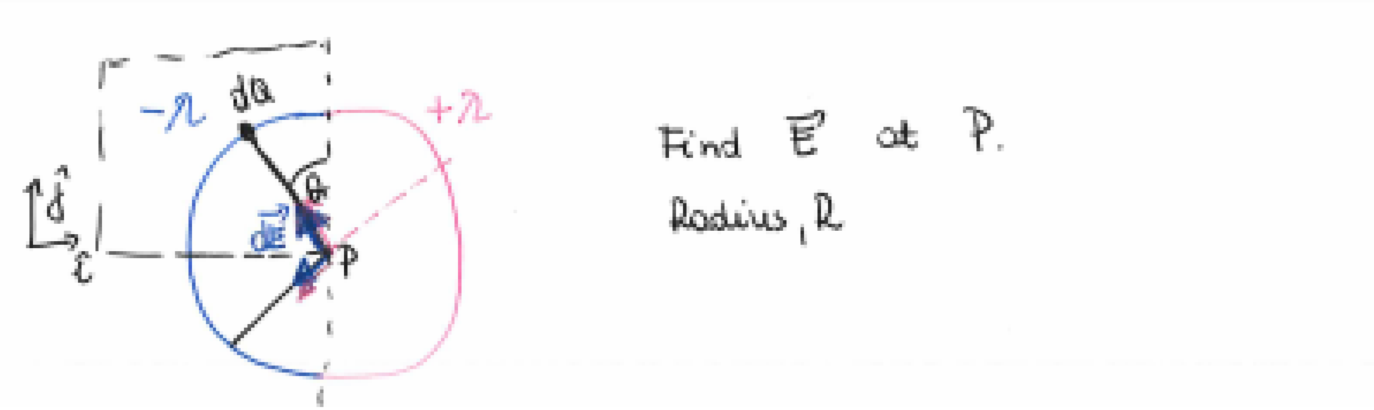
  $\Phi_{tot} = Q_{encl}/\varepsilon_0 = \lambda l/\varepsilon_0$
- The total flux must be equally partitioned into flux through the four surfaces whose area vectors are parallel to the electric field.

  Hence, $\Phi_A = \lambda l/4\varepsilon_0$

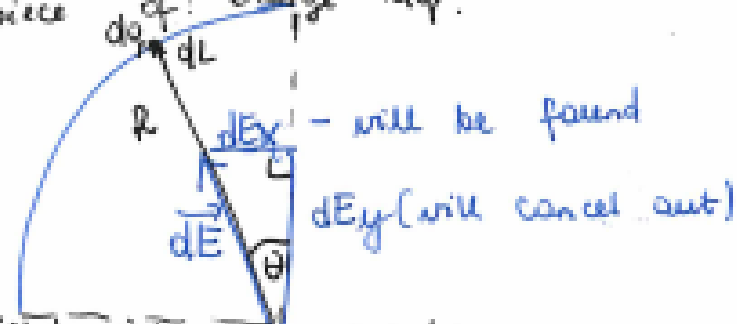


Practice question:

Find the field at point p at the centre of a ring of charge composed of two oppositely charged half rings.



1. Cut the distribution into a bunch of tiny pieces each with charge dQ
2. Look for a symetry → can use 1/4 circle (arc) Find $\vec{E}$ and multiply it by 4.
3. Calculate the magnitude of E-field due to ARBITRARY piece of charge dq.



4. Decompose field into components

$$\frac{dE_x}{dE} = \sin\theta \qquad \frac{dE_y}{dE} = \cos\theta$$

$$dE_x = dE \sin\theta$$
$$= \frac{1}{4\pi\varepsilon_0} \frac{dq}{R^2} \cdot \sin\theta$$

$$dE_y = \frac{1}{4\pi\varepsilon_0} \frac{dq}{R^2} \cos\theta$$

⟹ not needed due to symetry

5. For each non-zero component, sum up all pieces dQ by integrating over the whole charge distribution

$$\int dE_x = \frac{1}{4\pi\varepsilon_0} \int \frac{dq}{R^2} \sin\theta$$

6. Express dQ in terms of a variable to be integrated over using linear / surface / volume density

$$\int dE_x = \frac{1}{4\pi\varepsilon_0}\int \frac{\lambda\, dL}{R^2}\cdot \sin\theta = \frac{1}{4\pi\varepsilon_0}\int \frac{\lambda \cdot \cancel{R} \sin\theta \, d\theta}{R^{\cancel{2}}}$$

$$dL = R\, d\theta$$

$\pi/2$ - quarter of the circle

$$E_x = \frac{\lambda}{4\pi\varepsilon_0 R}\int_0^{\pi/2} \sin\theta \, d\theta \qquad \int \sin x \, dx = -\cos x$$

$$E_x = \frac{\lambda}{4\pi\varepsilon_0 R}\left[-\cos\theta\right]_0^{\pi/2} = \frac{\lambda}{4\pi\varepsilon_0 R}\left[-\underbrace{\cos\left(\tfrac{\pi}{2}\right)}_{=0} - \left(-\underbrace{\cos 0^\circ}_{=1}\right)\right]$$

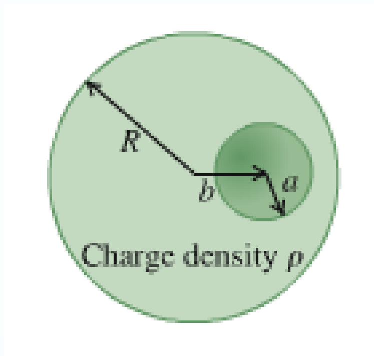
$$= \frac{\lambda}{4\pi\varepsilon_0 R}$$

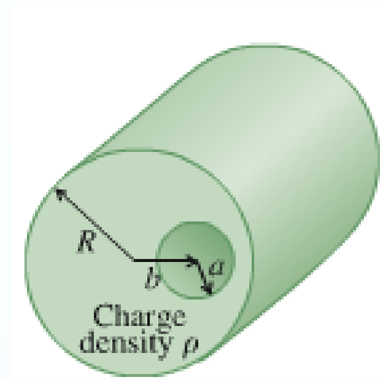
Total electric field

$$E_{net} = 4E_x = \frac{\cancel{4}\lambda}{\cancel{4}\pi\varepsilon_0 R} = \frac{\lambda}{4\pi\varepsilon_0 R^2}$$

$$\vec{E}_{net} = \frac{\lambda}{4\pi\varepsilon_0 R}\,(-\hat{i})$$

Superposition





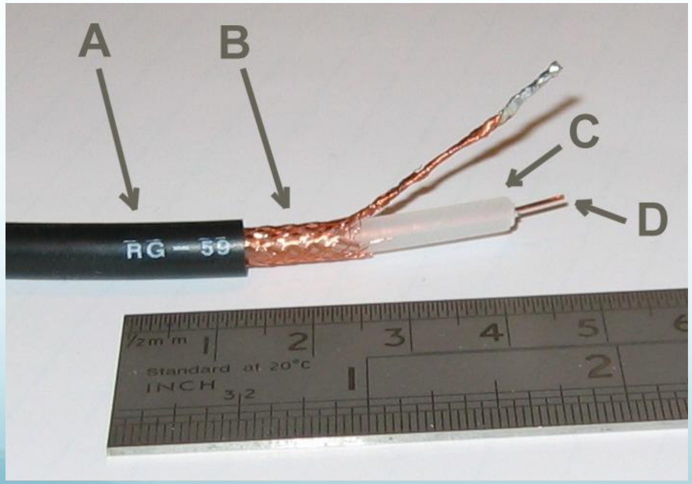
$$E_{total} = E_1 + E_2$$

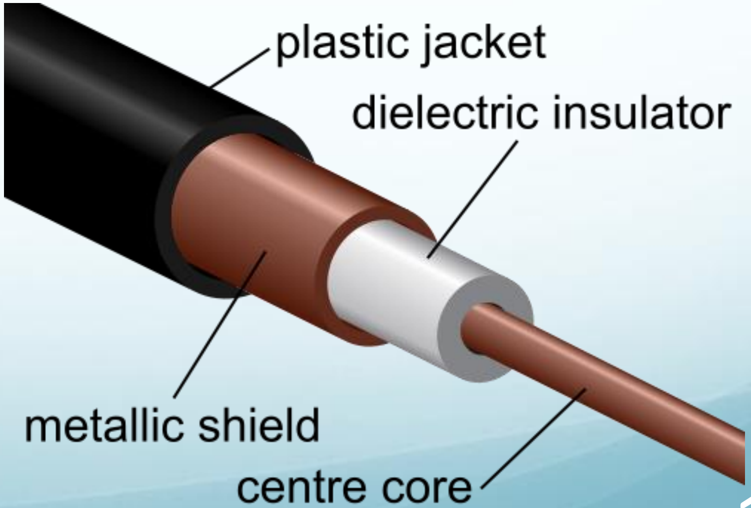
Exercise: Coaxial Cable

Study appendix 1-Chapter 23

Assume there is a charge **+Q** on the centre core and **-Q** on the metallic shield. (Ignore the dielectric insulator and plastic jacket.)

Find the electric field outside the metallic shield ($\mathbf{E_2}$) and just outside the central core ($\mathbf{E_1}$).





This section we talked about:

Chapter 23 & Midterm Review

See you on Friday