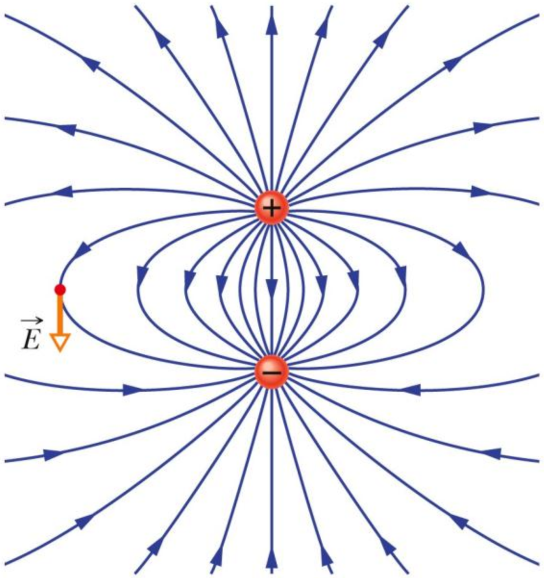
Important Notes from Prev. Sessions

- Electric Fields is stronger in the regions that lines are closer.
- Electric force is tangent to the direction of eclectic field
- Electric Field lines cannot cross!
- A uniformly spherical charge distribution act OUTSIDE the sphere as if the charge is concentrated at the center (point charge)!
- Calculation for the Electric Field of a uniform line of a charge is now available online. Answer to TOPHAT Q8.



 halliday_10e_fig_22_08

So far

- Motion of charges in electric fields
- Using superposition with extended objects (annulus)
- Scaling argument for arc of charge (+ full calculation)

To be continued!

- Electric field visualization applet
- Introduction to Gauss' Law: the first of the four Maxwell equations of electromagnetism
- Electric Flux, calculating flux

Electric Fields

www.falstad.com/vector3de

shows electric field vectors, electric field lines, equipotential surfaces, etc. for a large number of objects we have been considering

Electric Flux; Gauss' Law

Gauss' Law is equivalent to Coulomb's law. It will provide us:

(i) an **easier way to calculate the electric field** in specific circumstances (especially situations with a high degree of symmetry)

(ii) a better understanding of the properties of conductors in electrostatic equilibrium (more on this as we go)

(iii) It is valid for moving charges – not limited to electrostatics.

Electric flux passing through a **closed** Gaussian surface

$$\oint \vec{E}.\vec{n}\, dA = \frac{Q_{enc}}{\varepsilon_0} = 4\pi k\, Q_{enc}$$

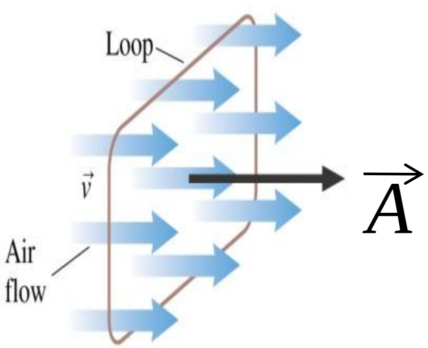
Electric flux passing through a **closed** Gaussian surface

$$\oint \vec{E} \cdot \overrightarrow{da} = \frac{Q_{enclosed}}{\epsilon_0} = 4pk\, Q_{enc}$$

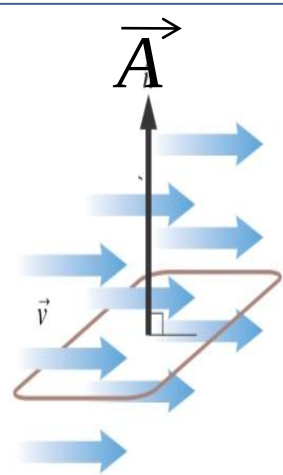
$$\boxed{F_E = Q_{encl} / e_0}$$

Gauss's law: the **net flux** passing through a closed surface (Gaussian surface) is **proportional to the net charge inside** the surface. **It does NOT depend on the shape of the surface.**

Flux : amount of 'something' (air, water……) flowing through an area

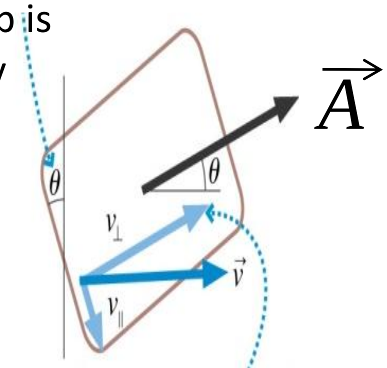


The air flowing through the loop is maximum when $\theta = 0^o$



No air flows through the loop when $\theta = 90^o$

The loop is tilted by angle θ

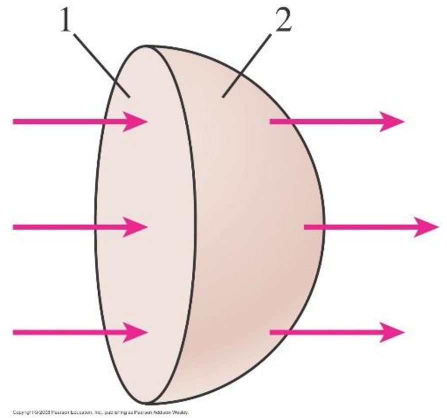


$v_\perp = v \cos\theta$ is the component of the air velocity perpendicular to the loop.

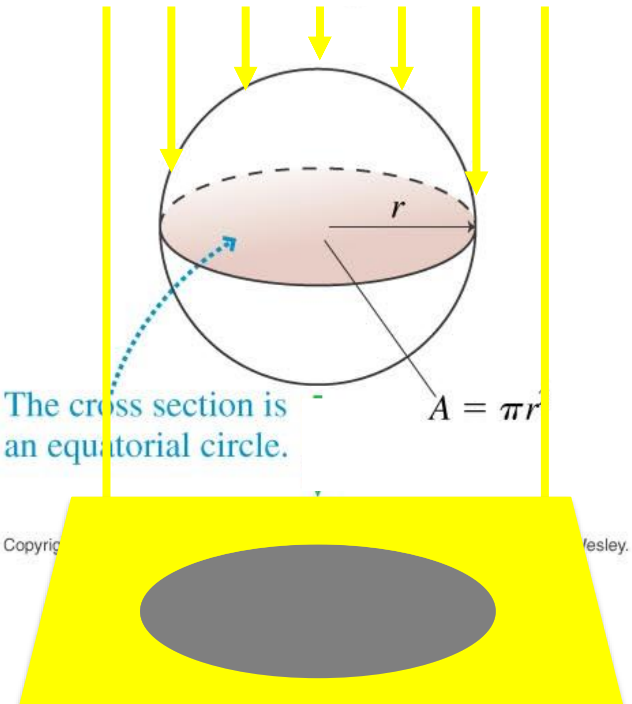
Cross-sectional area

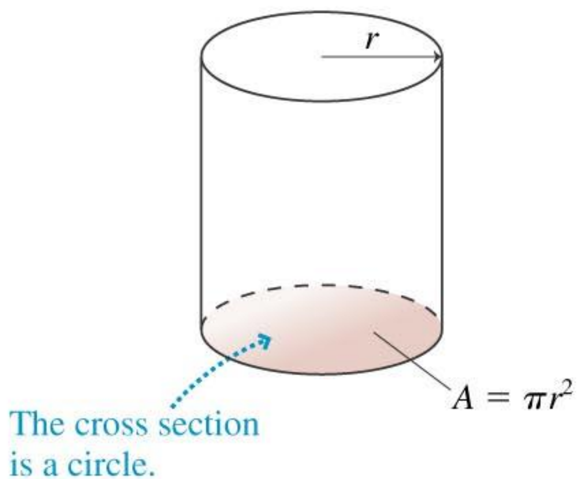
Area measured in a plane ⊥ to the direction of flow.

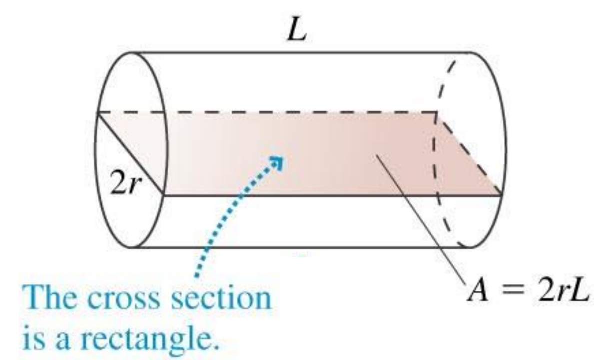
Area of shadow cast by || light rays



Flux through 1 = Flux through 2





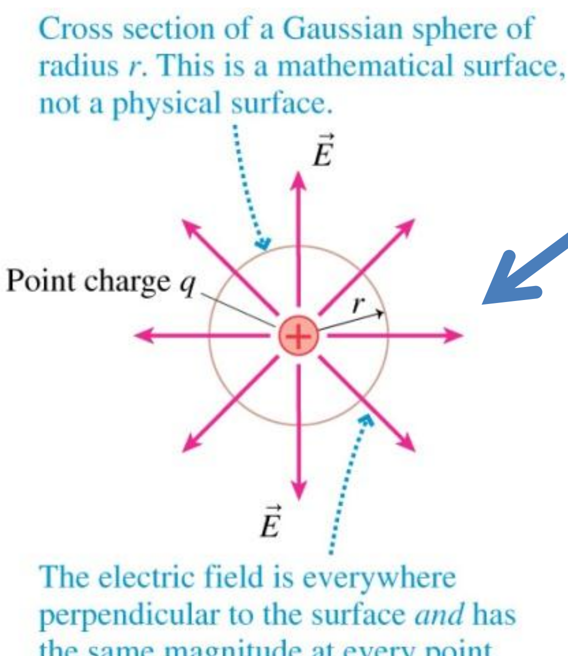


Electric flux passing through a **closed** Gaussian surface

$$\oint \vec{E} \cdot \overrightarrow{da} = \frac{Q_{enclosed}}{\epsilon_0}$$

$$\boxed{\Phi_E = Q_{encl} / \epsilon_0}$$

Flux : amount of ‘something’ (air, water……) flowing through an area

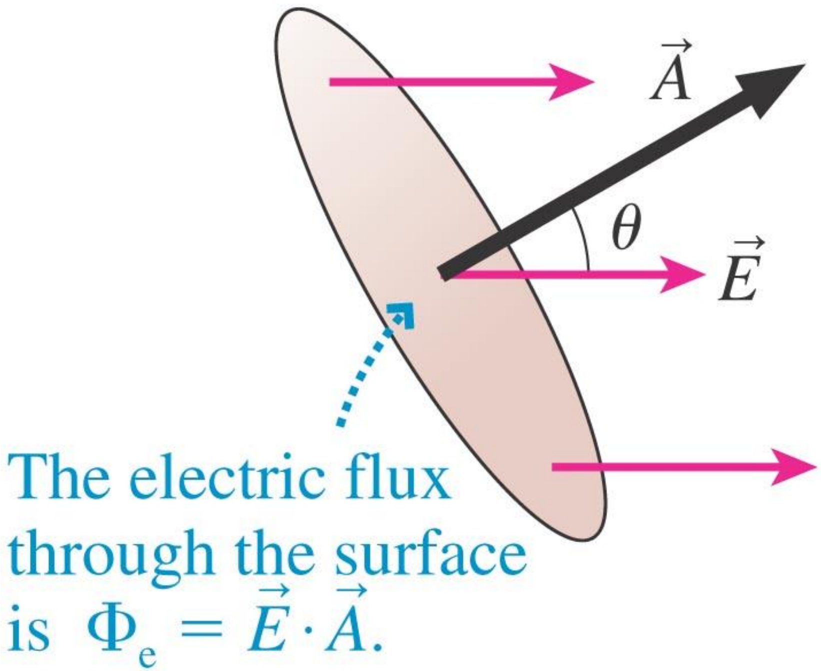


The flux passing **out** of the closed Gaussian surface is **positive**. The flux is: $\boxed{\Phi_E = +q / \epsilon_0}$

If the positive charge is replaced by a negative charge, the flux would be:

$$\boxed{\Phi_E = -q / \epsilon_0}$$

<u>Electric flux</u> through a surface with area A



$$\Phi_E = EA\cos\theta$$

How to evaluate $\Phi_E = \oint \vec{E}.\vec{n}\, dA$

- If the electric field is tangent to the surface:

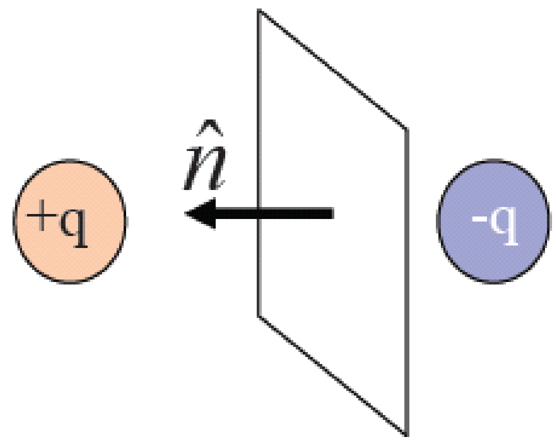
$$\Phi = 0$$

- If the electric field is normal to the surface and is constant at every point:

$$\Phi = EA$$

TopHat Question

The flux through the planar surface below (positive unit normal to left)



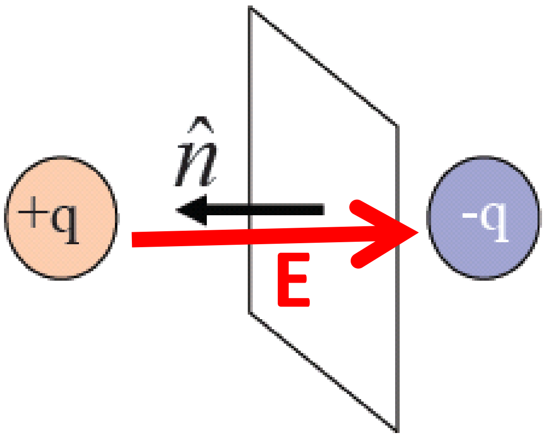
A: is positive.

B: is negative.

C: is zero.

D: not enough information

The flux through the planar surface below (positive unit normal to left)



A: is positive.

B: is negative.

C: is zero.

D: not enough info

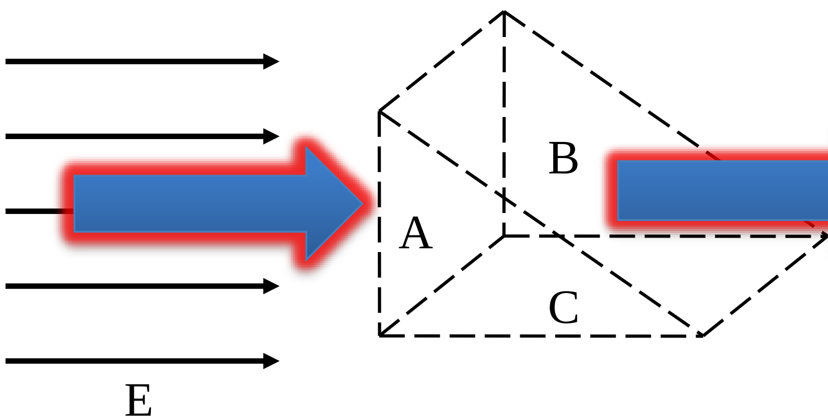
Here we choose the outward direction to be to the left. That is completely arbitrary. Since the E field is flowing inward (negative), the flux is negative.

E always points from + to - charges

Gaussian Surfaces

TopHat Question

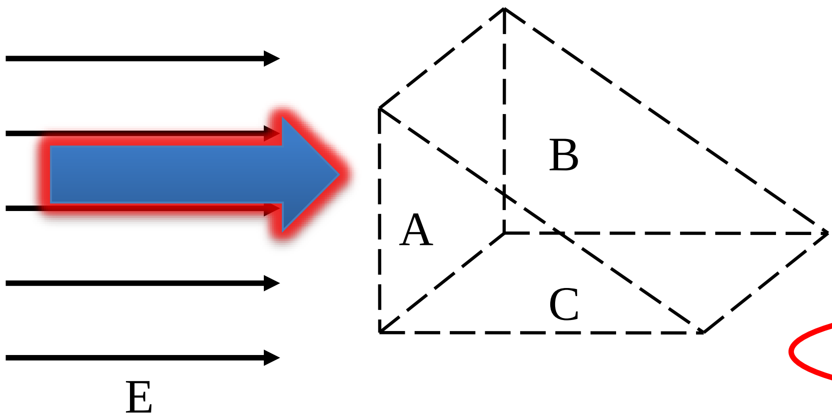
A prism-shaped closed surface is in a constant, uniform electric field **E**, filling all space, pointing to the right. The 3 rectangular faces of the prism are labeled A, B, and C. Face A is perpendicular to the E-field. The bottom face C is parallel to **E**. Face B is the leaning face.



Which face has the largest magnitude electric flux passing through it?

A) side A
B) side B
C) side C
D) sides A and B have the same Electric flux

A prism-shaped closed surface is in a constant, uniform electric field **E**, filling all space, pointing to the right. The 3 rectangular faces of the prism are labeled A, B, and C. Face A is perpendicular to the E-field. The bottom face C is parallel to **E**. Face B is the leaning face.



Which face has the largest magnitude electric flux passing through it?

A) A

B) B

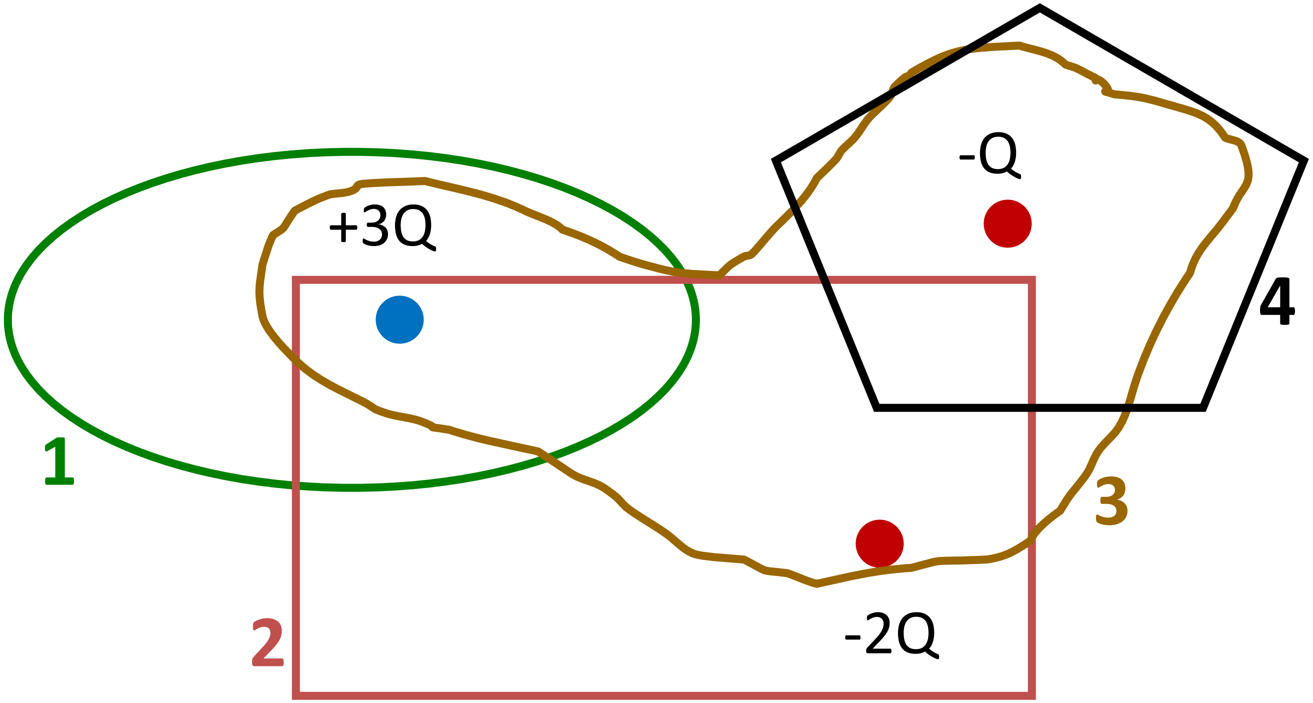
C) C

D) A and B have the same E flux

The same amount of E flux flows through the perpendicular face A and the slanted face B. The amount of **E**-flux flowing through the bottom face C is 0. Remember, although the slanted face B has larger surface area, the cos(θ) term keeps the flux equal to the flux through the perpendicular face A. Recall, θ is the angle between E and the normal to the slanted surface B.

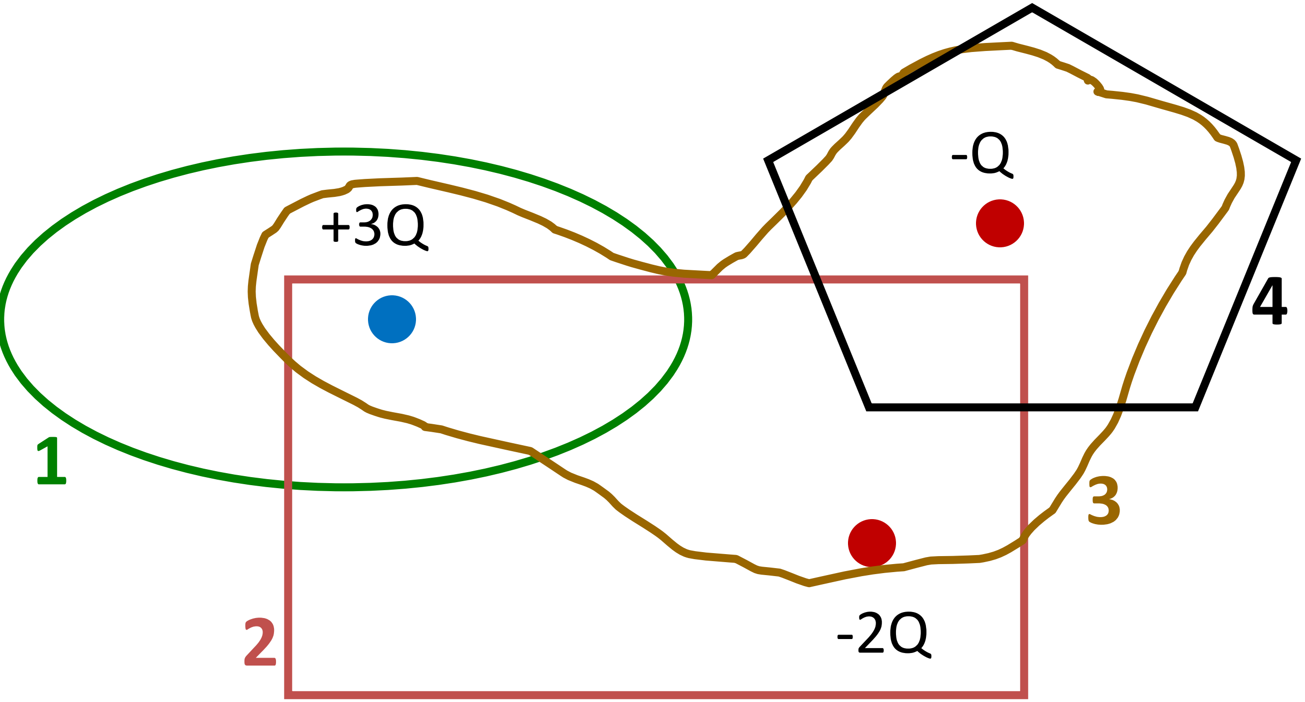
TopHat Question

Which surface has the largest **magnitude** electric flux?



A) 1 = 2 = 3 = 4
B) 1 > 2 = 4 > 3
C) 3 > 2 > 1 > 4
D) 3 > 2 > 1 = 4
E) None of the above

Which surface has the largest <u>magnitude</u> electric flux?



A) 1 = 2 = 3 = 4

B) 1 > 2 = 4 > 3

C) 3 > 2 > 1 > 4

D) 3 > 2 > 1 = 4

E) None of the above

Region 1: Net charge is +3Q

Region 2: Net charge is +Q

Region 3: Net charge is 0

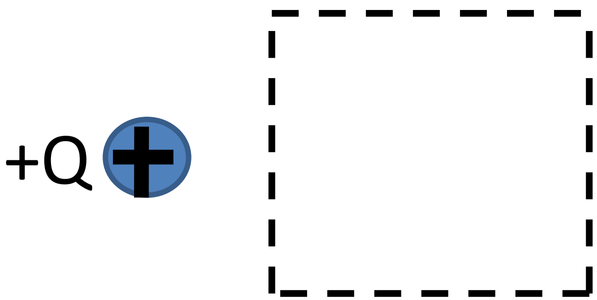
Region 4: Net charge is -Q

Here, we are interested in the **magnitude** of the flux, so the flux through regions 2 & 4 is the same. The flux through region 2 is outward (**+**), while the flux through region 4 is inward (**-**).

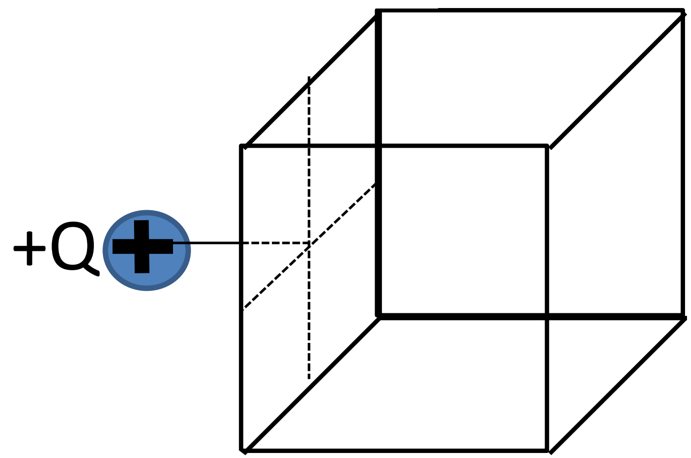
Again, electric flux is equal to "$Q_{enclosed}/\varepsilon_0$"

TopHat Question

What is the **sign** (positive or negative) of the NET electric flux passing through the **4 side surfaces**? (To be clear, the cube has 6 sides. It has a TOP, BOTTOM and 4 SIDE surfaces)



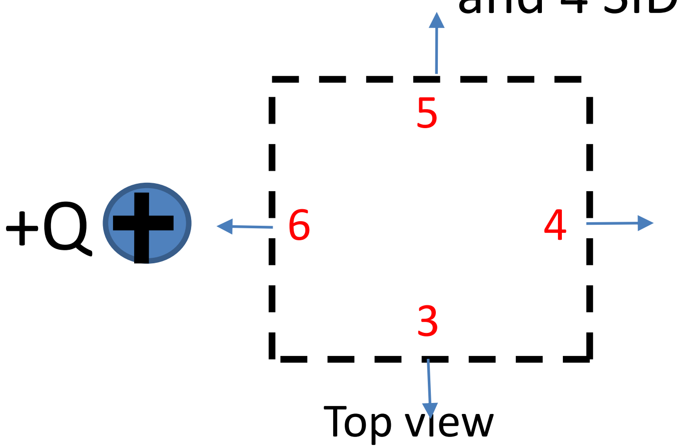
Side view

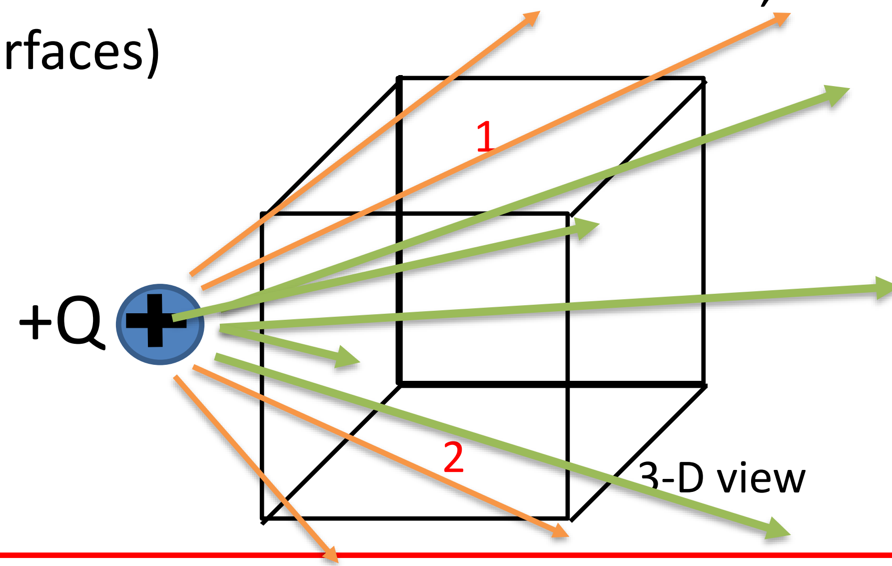


3-D view

(A) Positive
(B) Negative
(C) Zero
(D) Not enough information

What is the **sign** (positive or negative) of the NET electric flux passing through the **4 side surfaces**?
(To be clear, the cube has 6 sides. It has a TOP, BOTTOM and 4 SIDE surfaces)





ANSWER: The **net** flux going thru the 4 sides **must be negative**

Reasoning: (1) Since no charge is enclosed in the Gaussian cube, **E Flux = 0 through the whole closed surface.**
(2) The flux going through the TOP(1) and BOTTOM(2) is positive. (the E field at the top and bottom surfaces is flowing **outward**, so the flux is positive).
(3) Therefore, the flux through the 4 sides MUST be a net negative
Basically, the flux going through the nearest side surface to the charge +Q is very large & negative because the E field at the nearest surface is strongest at that surface.

Last time

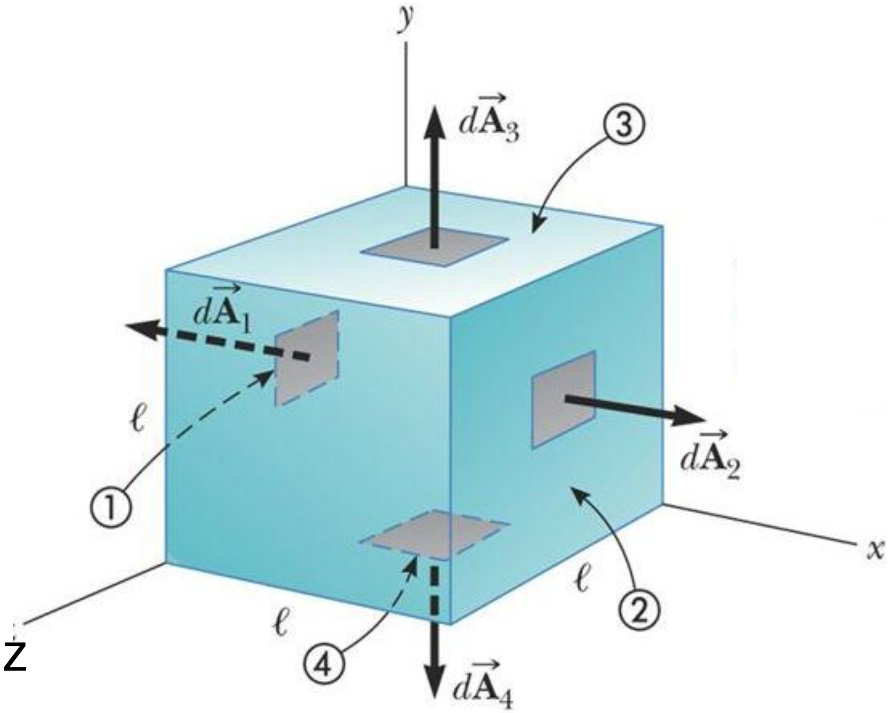
- Introduction to Gauss' Law: the first of the four Maxwell equations of electromagnetism
- Electric Flux, calculating flux

This time

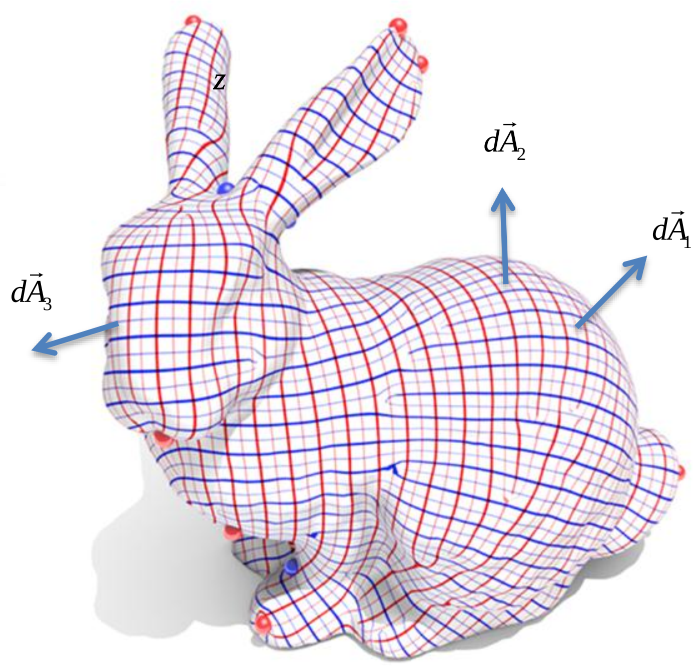
- Applying Gauss' Law: start simple, verify what we expect
  - Electric field of a point charge
  - Electric field of a spherical shell of charge
  - Electric field of a uniformly charged ball

How do we define the area vector?

Flat closed surface



Arbitrary closed surface



A closed surface defines a volume.

Electric Flux; Gauss' Law

Gauss' Law is equivalent to Coulomb's law. It will provide us:

(i) an **easier way to calculate the electric field** in specific circumstances (especially situations with a high degree of symmetry)

(ii) a better understanding of the properties of conductors in electrostatic equilibrium (more on this as we go)

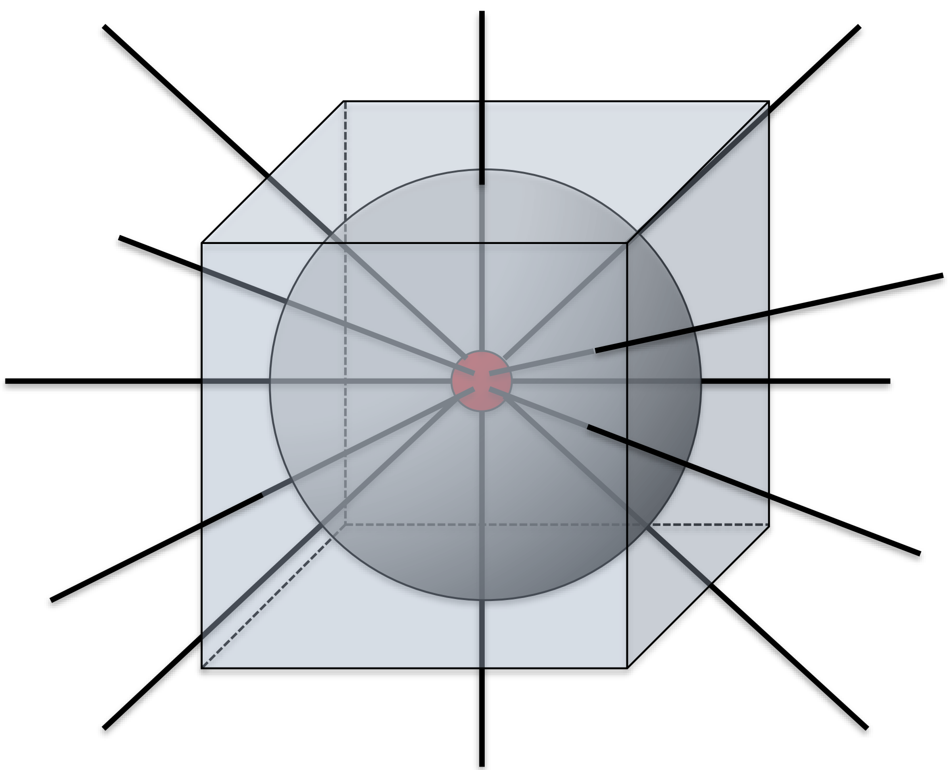
(iii) It is valid for moving charges – not limited to electrostatics.

Electric flux passing through a **closed** Gaussian surface

$$\oint \vec{E}.\vec{n}\, dA = \frac{Q_{enc}}{\varepsilon_0} = 4\pi k\, Q_{enc}$$

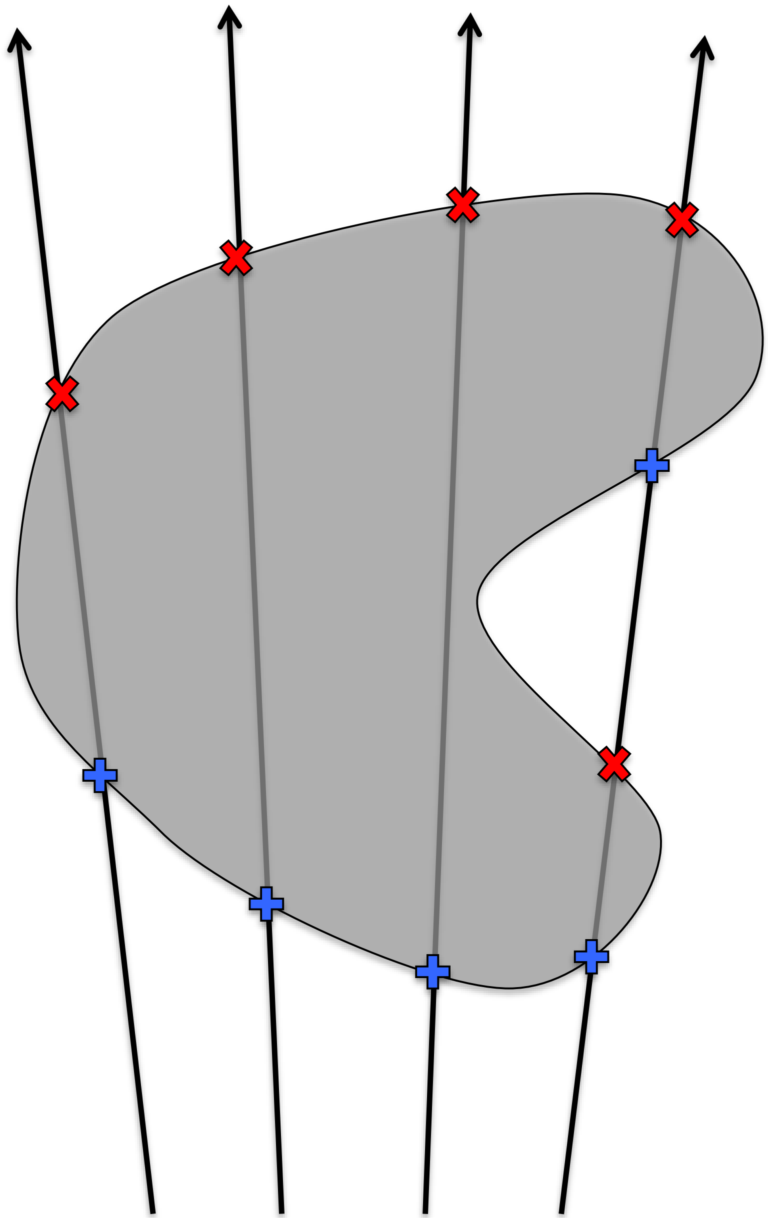
The **electric flux** through a closed surface does not depend on the **SHAPE** of the surface, it only depends on the **charges enclosed** by the surface.

This gives a nice interpretation of the flux as the **number of electric field lines passing through the surface**. Take the example of a point charge surrounded by a sphere, surrounded by a cube



The number of field lines passing through the sphere is the same as the number of field lines passing through the cube.

The electric flux through each surface is the same.

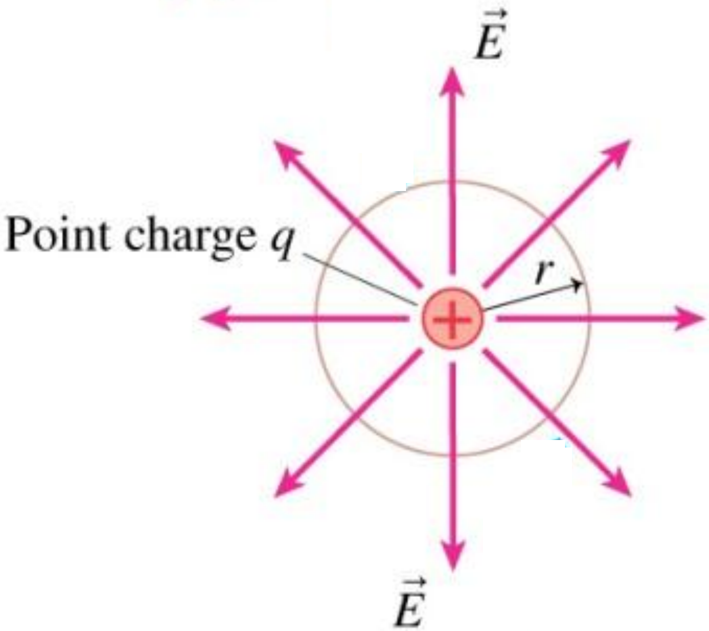


Using the idea that electric flux "counts the number of field lines" passing through a surface, we immediately see why external electric field lines contribute nothing to the net flux.

Any external field line that enters later has to leave.

Something to think about: what is the flux through a surface surrounding an electric dipole? How can you think about it in terms of this idea?

1. Point source



Task

Use Gauss' law to compute the E field at a distance r from the positive charge

We will not assume anything about the E-field of a point charge but we can be guided by symmetry.

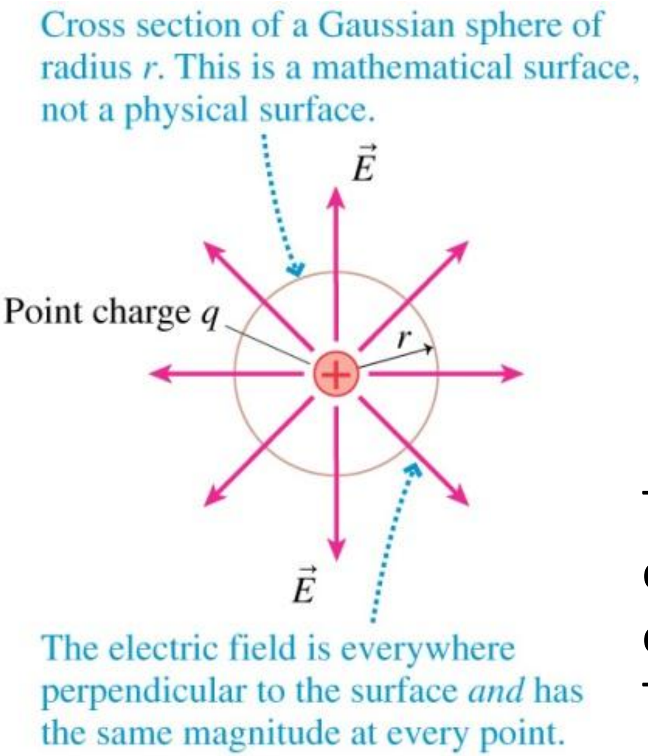
Symmetry argument:

1. Electric field must point in the **radial direction** only.
2. The electric field must be the **same magnitude** at constant radius.

1. Point source

(spherical symmetry → choose a sphere for Gaussian surface)

Use Gauss's law to compute the E field at a distance r from the positive charge



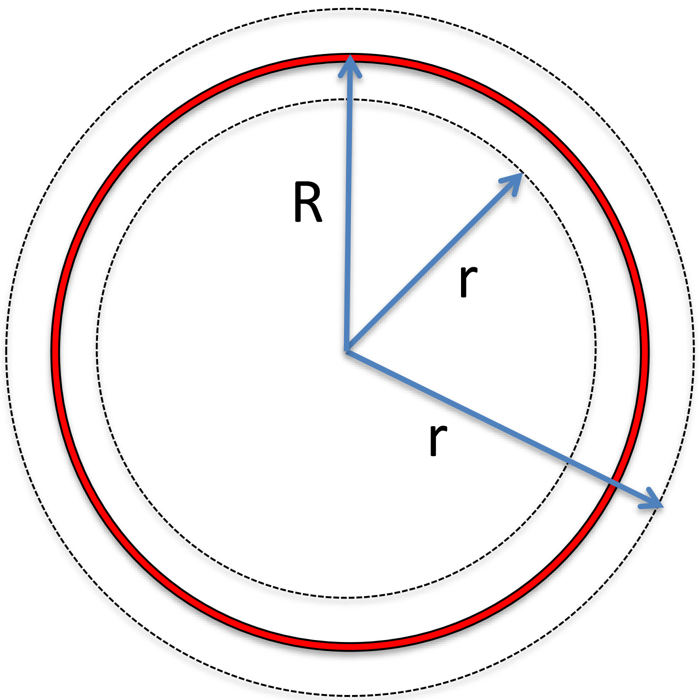
$$\Phi_E = \oint \vec{E} \bullet (d\vec{A}) = \mathrm{EA_{sphere}} = q / \varepsilon_0$$

$$\boxed{E = \frac{q}{\varepsilon_0 \mathrm{A_{sphere}}} = \frac{q}{\varepsilon_0 4\pi r^2}}$$

This is Coulomb Law!

The integral is very simple because the E field has constant magnitude everywhere on the surface and can therefore be factored outside of the integral. The area of a sphere is $4\pi r^2$, so the flux integral is equal to $E*4\pi r^2 = Q/\varepsilon_0$.

1. Shell of charge



Task

Use Gauss' law to compute the E field inside and outside a spherical shell of charge

Symmetry argument:

1. Electric field must point in the **radial direction** only.

2. The electric field must be the **same magnitude** at constant radius.

$$\Phi_E = \oint \vec{E} \cdot (d\vec{A}) = EA_{\text{sphere}} = \frac{q_{enc}}{\varepsilon_0}$$

Inside the shell: $q_{enc} = 0$

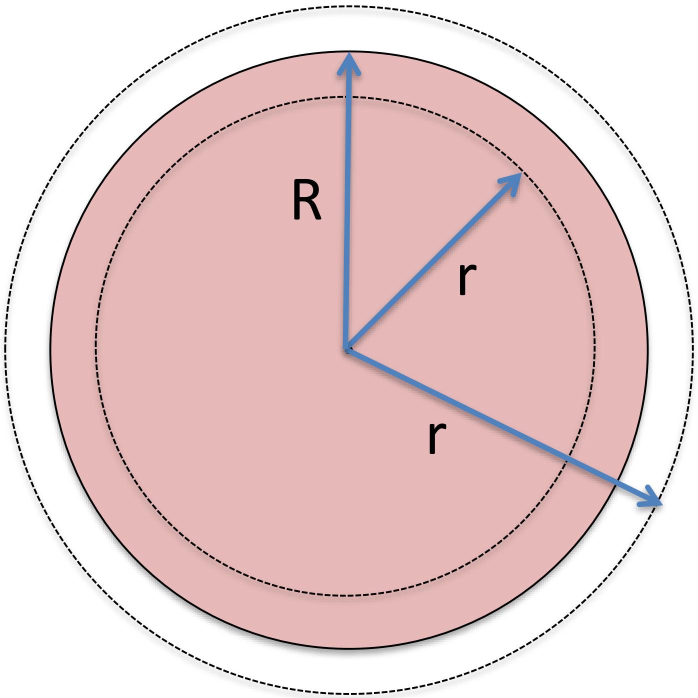
$$\boxed{E = 0 \quad \text{for } r < R}$$

Outside the shell: $q_{enc} = Q$

$$\boxed{E = \frac{Q}{4\pi\varepsilon_0 r^2} \quad \text{for } r > R}$$

Using Gauss' Law

1. Solid ball of uniform charge density (charge Q)



Outside the ball: $q_{enc} = Q$

$$\boxed{E = \frac{Q}{4\pi\epsilon_0 r^2} \qquad \text{for} \quad r > R}$$

Task

Use Gauss' law to compute the E field inside and outside a uniformly charged ball

Symmetry argument:

1. Electric field must point in the **radial direction** only.

2. The electric field must be the **same magnitude** at constant radius.

$$\Phi_E = \oint \vec{E} \cdot (d\vec{A}) = EA_{\text{sphere}} = \frac{q_{enc}}{\varepsilon_0}$$

Inside the ball:

$$q_{enc} = \rho V_{ball} = \left(\frac{Q}{\cancel{\tfrac{4}{3}\pi} R^3} \right) \cancel{\tfrac{4}{3}\pi} r^3 = Q \frac{r^3}{R^3}$$

$$E = \frac{Qr^3}{\epsilon_0 (4\pi r^2) R^3} = \boxed{\frac{Qr}{4\pi\epsilon_0 R^3}} \qquad \text{for} \quad r < R$$

KEEP
CALM
AND
STUDY
ON

Last time

- Gauss’ Law and superposition
- Properties of conductors
- Gauss’ Law applied to conductors

This time

- Re-examine the charged ball
- Gauss’ Law and superposition (powerful!)
- Properties of conductors
- Gauss’ Law applied to conductors (also powerful!)

Superposition

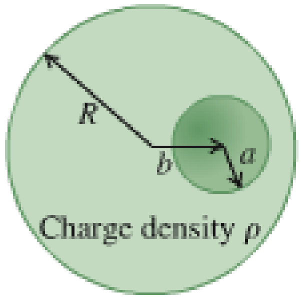
$E_{total} = E_1 + E_2$

Can you solve this problem?

1. Solve it by Monday
2. Present it in class
   → bonus on TopHat
3. For entire class

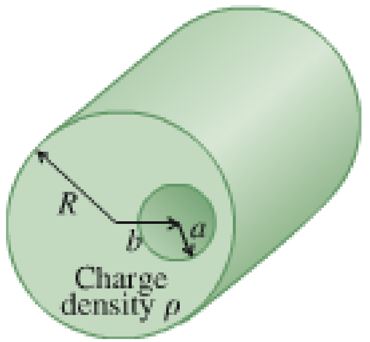
$\vec{E} = \rho(\vec{r} - \vec{b})/3\epsilon_0$. (b) An insulating sphere of radius R has a spherical hole of radius a located within its volume and centered a distance b from the center of the sphere, where $a < b < R$ (a cross section of the sphere is shown in Fig. P22.61). The solid part of the sphere has a uniform volume charge density ρ. Find the magnitude and direction of the electric field $\vec{E}$ inside the hole, and show that $\vec{E}$ is uniform over the entire hole. [*Hint:* Use the principle of superposition and the result of part (a).]

Figure **P22.61**



22.62 • A very long, solid insulating cylinder with radius R has a cylindrical hole with radius a bored along its entire length. The axis of the hole is a distance b from the axis of the cylinder, where $a < b < R$ (Fig. P22.62). The solid material of the cylinder has a uniform volume charge density ρ. Find the magnitude and direction of the electric field $\vec{E}$ inside the hole, and show that $\vec{E}$ is uniform over the entire hole. (*Hint:* See Problem 22.61.)

Figure **P22.62**



Conductors

A conductor is a material in which the charges are free to move.

This means that two things are true:

1. There is zero net charge inside a conductor. ($Q_{net} = 0$)

2. There is zero electric field inside a conductor. ($E_{in} = 0$)

Conductors -- Explanations

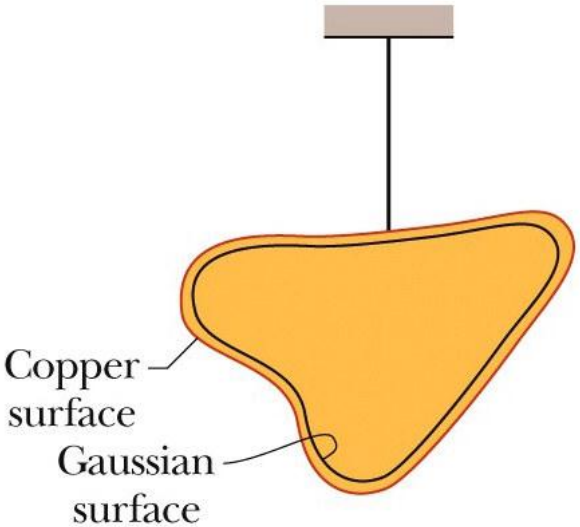
1. There is zero net charge inside a conductor. ($Q_{net} = 0$)

If there are 2 (or more) like charges inside a conductor then they will repel each other and push each other far away. (ie --to the surface)

2. There is zero electric field inside a conductor. ($E_{in} = 0$)

If there is a non-zero E field then F = qE implies there is a net force which means charges would move until the force on them is zero – we have a STATIC situation. (Equilibrium)

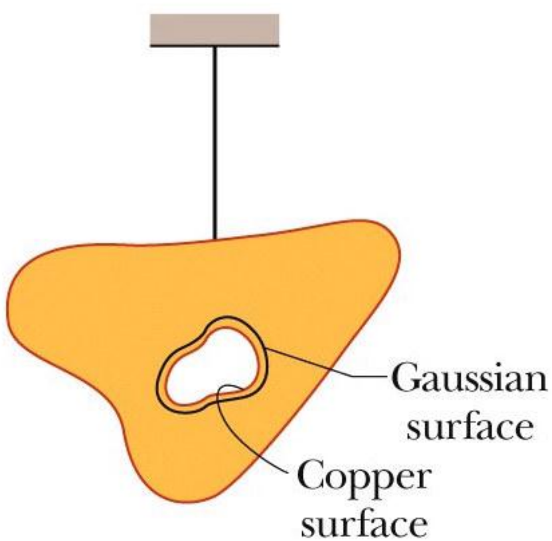
Hollow Conductors



(a)

The electric field inside a conductor is zero. This immediately implies that conductors are electrically neutral in their interiors.

$$\oint \cancel{\vec{E}} \cdot d\vec{A} = 0 = \frac{q_{enc}}{\varepsilon_0}$$



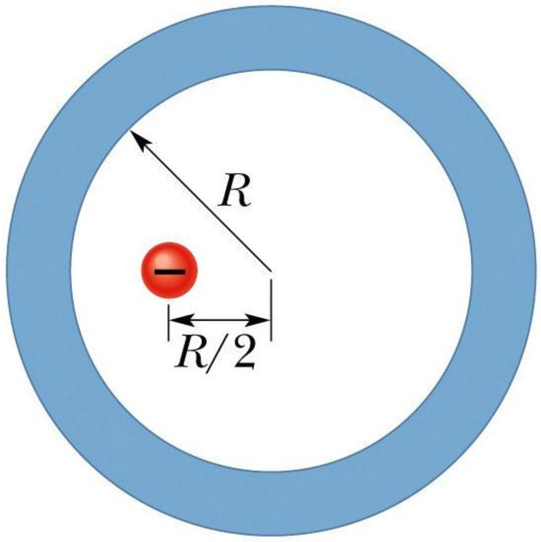
(b)

This also means that the surface of a hollow cavity inside a conductor cannot carry any excess charge. All excess charge must reside on the outside surface only.

TopHat Question

Consider a spherical metal shell of inner radius R.
A point charge of -5.0 μC is located at a distance R/2 from the centre of the shell. If the shell is neutral – -- what is the induced charge on its **outer surface**?

A) Zero
B) +5.0 μC
C) - 5.0 μC

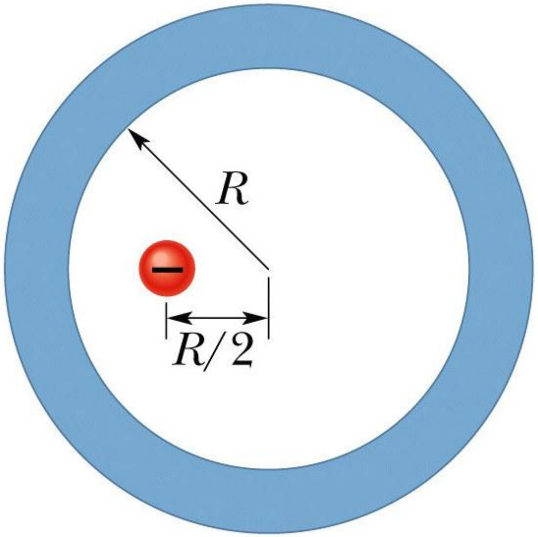


TopHat Question

Consider a spherical metal shell of inner radius R.

A point charge of - 5.0 μC is located at a distance R/2 from the centre of the shell.

If the shell is neutral, are these charges uniformly distributed on the **inner surface**?



A. Yes
B. No
C. Perhaps

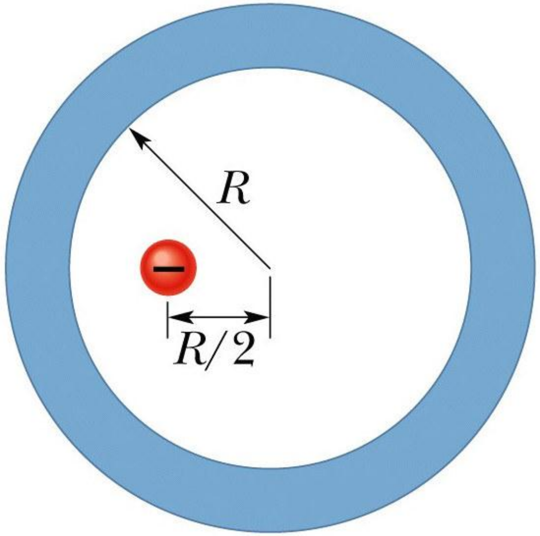
Hint: What must the electric field be inside the shell?

Charge in a hollow conductor

Consider a spherical metal shell of inner radius R.

A point charge of -5.0 μC is located at a distance R/2 from the centre of the shell. If the shell is neutral --

1. Calculate the induced charge on its outer surface ?
2. Sketch the E field lines inside and outside the metal shell

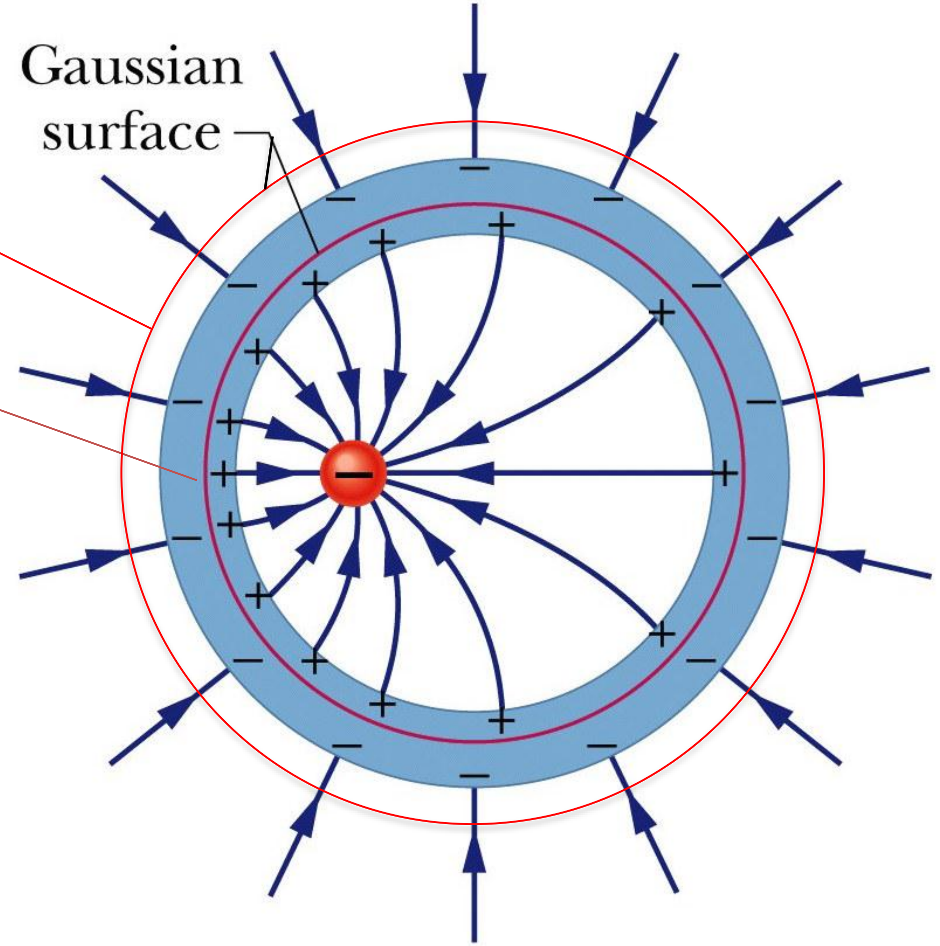


Hint: Use Gauss's law.

Solution

- Point Charge inside is $-5\mu C$
- Inner surface has $+5\ \mu C$ non-uniformly distributed
- Outer surface has $-5\ \mu C$ uniformly distributed

In this Gaussian surface, the **total charge enclosed** is –5 μC.
E = 0 inside the metal:
The **net charge** enclosed in this Gaussian surface must be **zero**.
There must be +5 μC on the inside of the shell and –5 μC on the outside.
The charges on the outside "don't know" about the inside since E = 0 inside the conductor.



So Far

- Gauss' Law and superposition
- Properties of conductors
- Gauss' Law applied to conductors

To be continued!

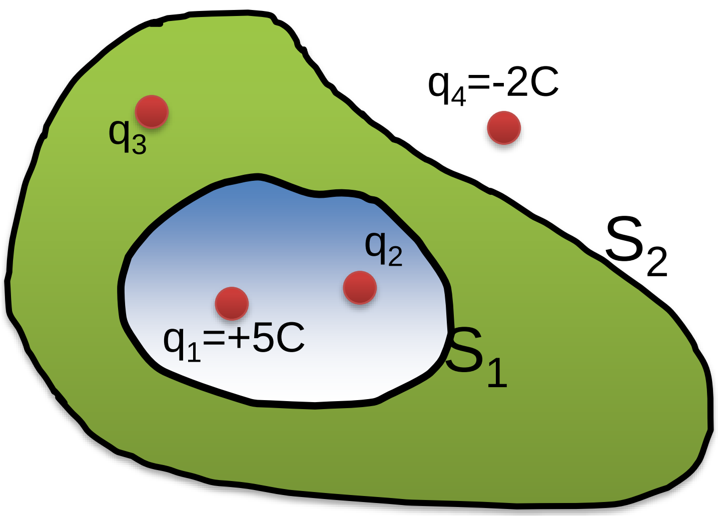
- TopHat

Applications of Gauss's Law

Provides a relation between the electric field on a *closed surface (the total flux)* and the charge inside.

$$\Phi_E = \oint E\,dA = \oint E\,dA\cos\phi = \frac{Q_{encl}}{\epsilon_0}$$

The total flux through S_1 is $+10\ C/\varepsilon_0$, and the total flux through S_2 is $0\ C/\varepsilon_0$. Which of the following statements is true?



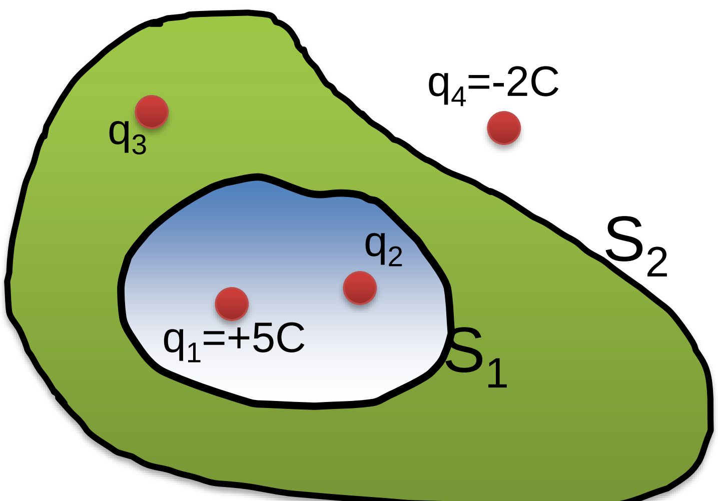
a) q_2=+5C and q_3=+2C
b) q_2= +5C and q_3=0
c) q_2=-5C and q_3=-10C
d) q_2=+5C and q_3=-10C
e) The value of q_4 is relevant

Applications of Gauss's Law

Provides a relation between the electric field on a *closed surface (the total flux)* and the charge inside.

$$\Phi_E = \oint E\,dA = \oint E\,dA\cos\phi = \frac{Q_{encl}}{\epsilon_0}$$

The total flux through S_1 is $+10\ C/\varepsilon_0$, and the total flux through S_2 is $0\ C/\varepsilon_0$. Which of the following statements is true?



a) q_2=+5C and q_3=+2C
b) q_2= +5C and q_3=0
c) q_2=-5C and q_3=-10C
d) q_2=+5C and q_3=-10C
e) The value of q_4 is relevant

✓

Last Time!

This Time!

& Forever!

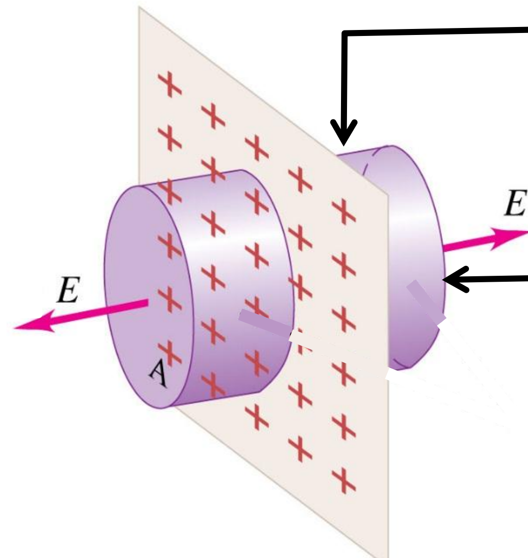
$$\Phi = \oint \vec{E}.\vec{n}\, dA = \frac{Q_{enc}}{\varepsilon_0} = 4\pi k\, Q_{enc}$$

Last Lecture: Gaussian surfaces

- Gauss’ law allows calculating the electric field due to an extended object if we can select a convenient *Gaussian surface* that makes calculating

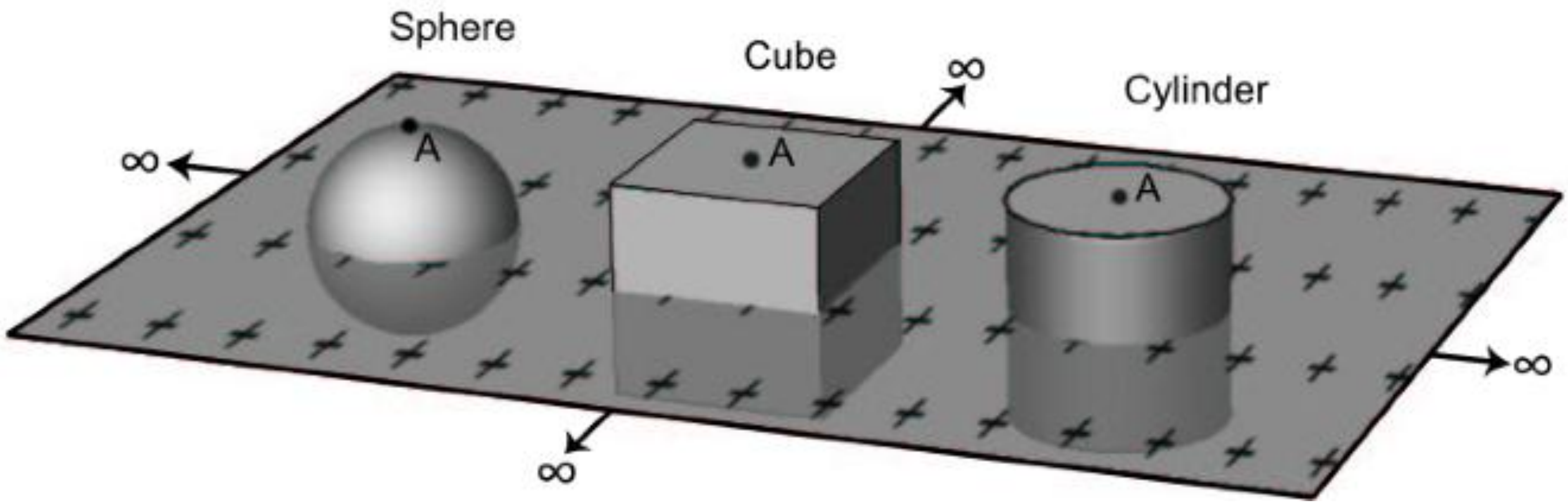
  $\Phi_{total} = \oint E dA$ easy.
- We always try to use surfaces for which one of the three cases applies:
  - $\vec{\boldsymbol{E}}.\overrightarrow{\boldsymbol{dA}}$=0 $\overrightarrow{\boldsymbol{dA}}$ = 0 zero field at any point of the surface
  - $\vec{\boldsymbol{E}}.\overrightarrow{\boldsymbol{dA}}$ =EdA $\boldsymbol{E} \parallel \boldsymbol{A}$ at any point of the surface
  - $\vec{\boldsymbol{E}}.\overrightarrow{\boldsymbol{dA}}$ =0 $\boldsymbol{E} \perp \boldsymbol{A}$ at any point of the surface

- Make sure the point for which you want to calculate E lies on the Gaussian surface (and E is constant over a part of it)
- The Gaussian surface does not exist in reality – it is just an imaginary geometric surface



Applications of Gauss' s Law

For which of these Gaussian surfaces will Gauss' law help us to calculate E at point A due to the sheet of charge? *Point A is at the top center of each surface*.

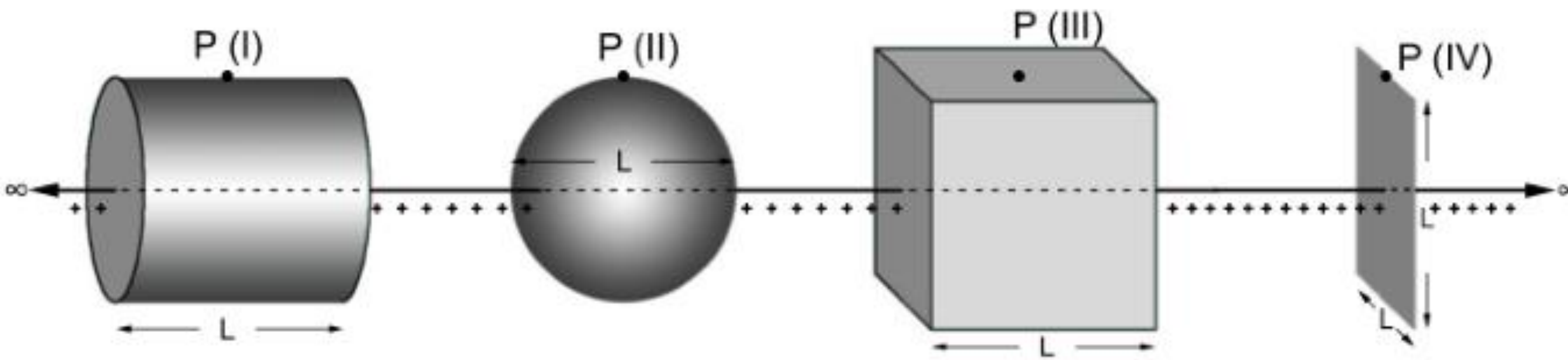


A) Only the sphere
B) Only the cylinder
C) Only the cylinder and the cube
D) Only the sphere and the cylinder
E) All surfaces will work

Applications of Gauss's Law

4 surfaces are coaxial with an infinitely long line of charge with a uniform linear charge density = λ. Choose all the surfaces through which

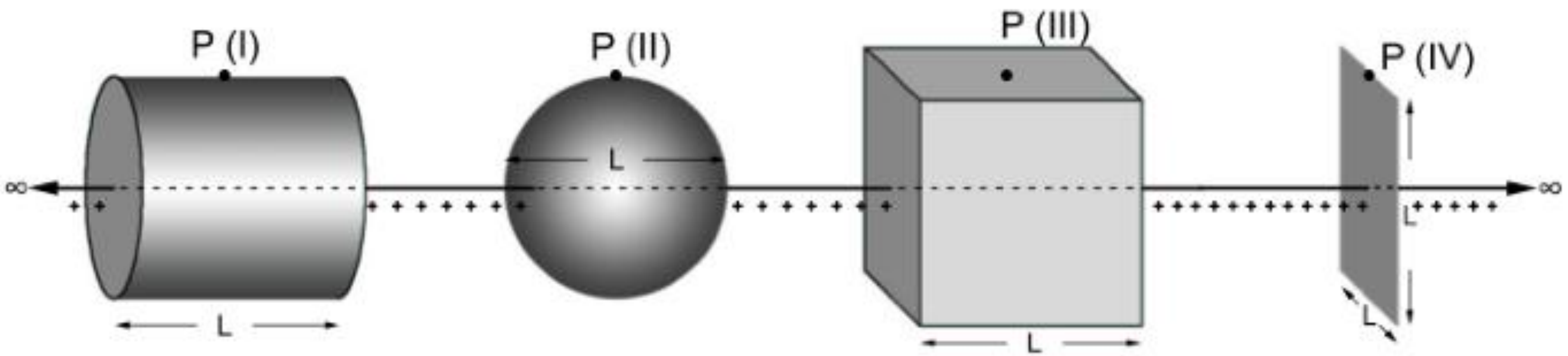
$$F_E = / L / e_0$$



A) I only
B) I and II only
C) I and III only
D) I, II, and III only
E) All four.

Applications of Gauss’ s Law

4 surfaces are coaxial with an infinitely long line of charge with a uniform linear charge density = λ. Choose all the surfaces that can be used to calculate E at point P.



A) I only
B) I and II only
C) I and III only
D) I, II, and III only
E) All four.

Last lecture: Field of a line

$$\Phi_E = \oint E\,dA = \frac{Q_{encl}}{\varepsilon_0}$$

– Calculate the electric field due to a positively charged line of linear charge density λ.

We note that the direction of E is radially away from the rod.

$$\Phi_{total} = \oint_{cylinder} E\,dA = \int_{tube} E\,dA + \int_{top} E\,dA + \int_{bottom} E\,dA$$

Angle between E and dA is always 0!

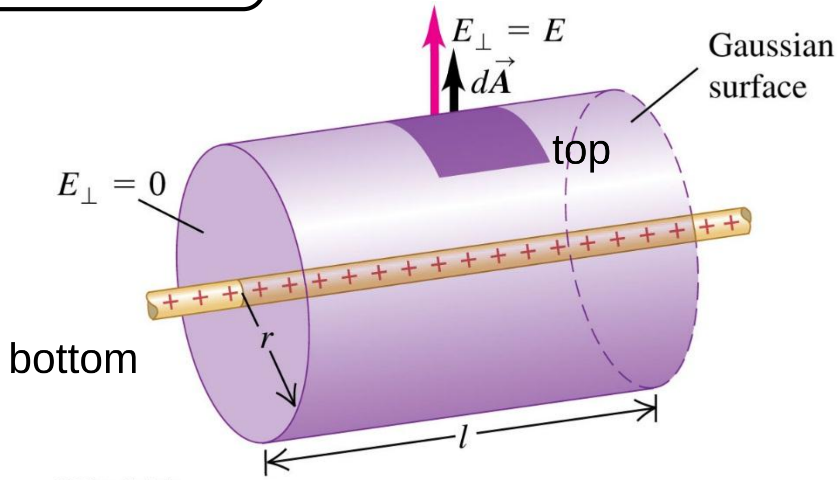
E and dA are always perpendicular!

$$= E \oint_{tube} dA$$

$$= 2\pi r l E$$

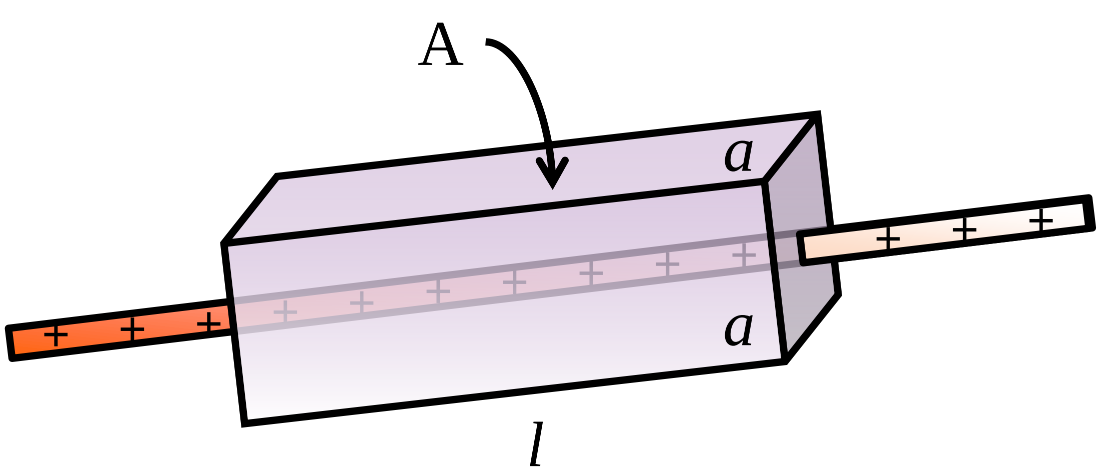
$$= \frac{\lambda l}{\varepsilon_0}$$

-> $$E = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{r}$$



Field of a line charge

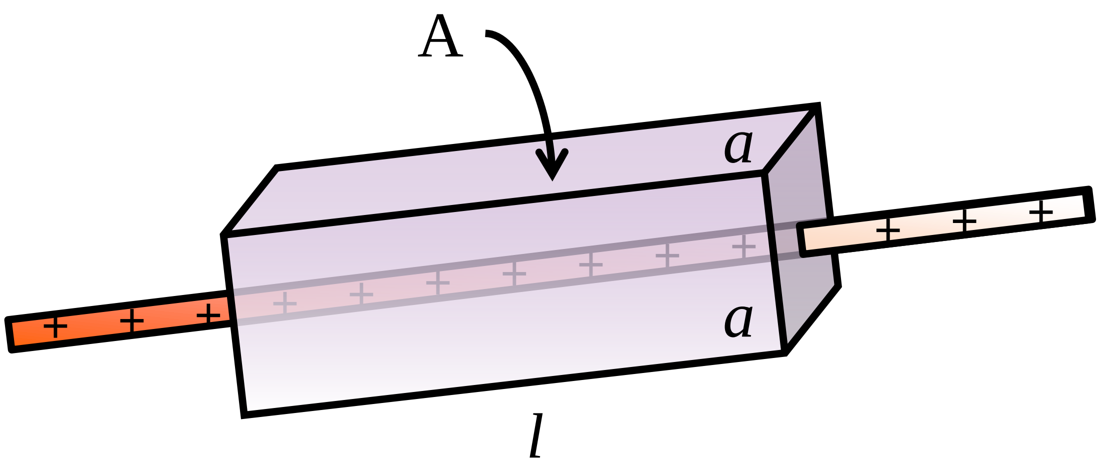
- Consider an infinitely long, positively charged rod of linear charge density λ. How large is the flux through side A of the box? Suppose the values for l, a and λ are given.



Field of a line charge

- Consider an infinitely long, positively charged rod of linear charge density λ. How large is the flux through side A of the box? Suppose the values for l, a and λ are given.
- Gauss' law tells us that the total electric flux only depends on the enclosed charge – not the shape of the (closed) Gaussian surface:

$$\Phi_{tot} = Q_{encl}/\varepsilon_0 = \lambda l/\varepsilon_0$$



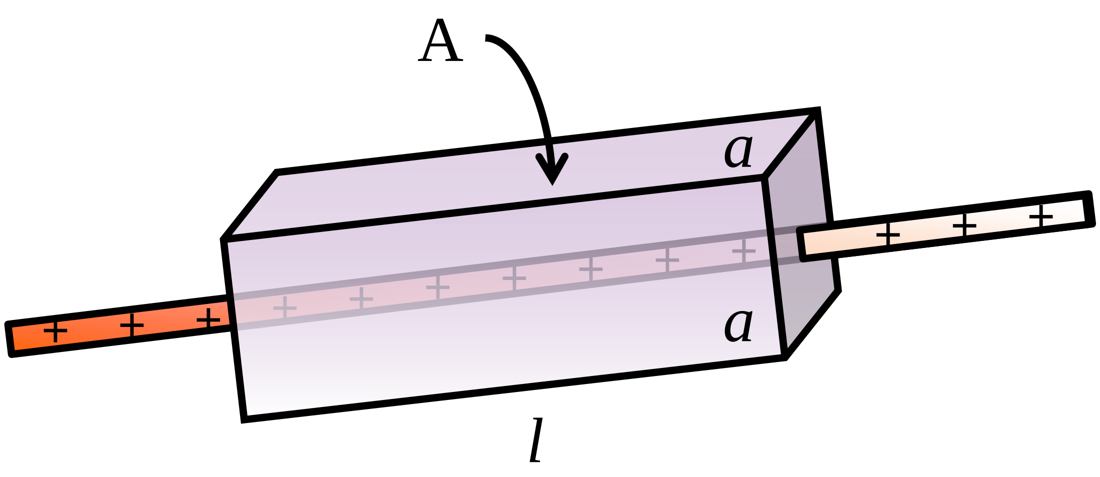
Field of a line charge

- Consider an infinitely long, positively charged rod of linear charge density λ. How large is the flux through side A of the box? Suppose the values for l, a and λ are given.
- Gauss' law tells us that the total electric flux only depends on the enclosed charge – not the shape of the (closed) Gaussian surface:

$$\Phi_{tot} = Q_{encl}/\varepsilon_0 = \lambda l/\varepsilon_0$$

- The total flux must be equally partitioned into flux through the four surfaces whose area vectors are parallel to the electric field.

Hence, $\Phi_A = \lambda l/4\varepsilon_0$



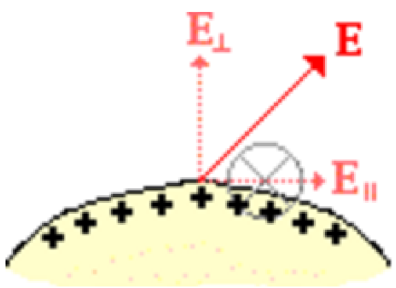
So far

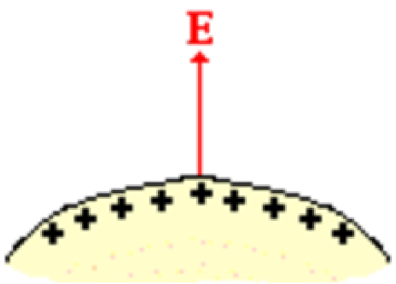
- Gauss' Law and superposition
- Properties of conductors
- Gauss' Law applied to conductors
- Choosing a right symmetry

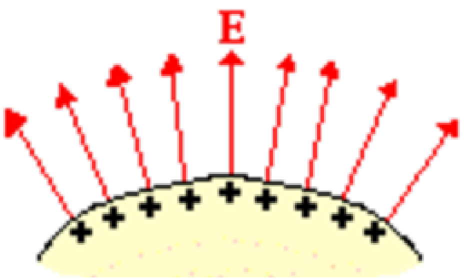
To be continued!

- Finish off discussion about conductors
- Gauss' Law applied to conductors: local surface charge density
- Electric field of a uniformly charged line
- Exercise: electric field of a coaxial cable

Conductor







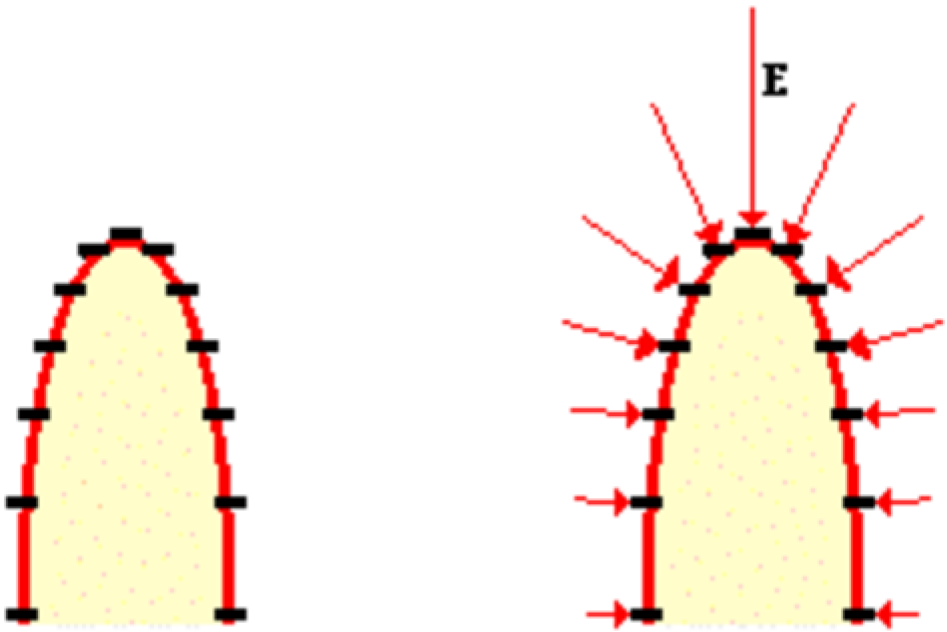
If a component of E exist ($E_\parallel$)
The charge would move

Therefore, ($E_\parallel$) =0
And only E_T exist.

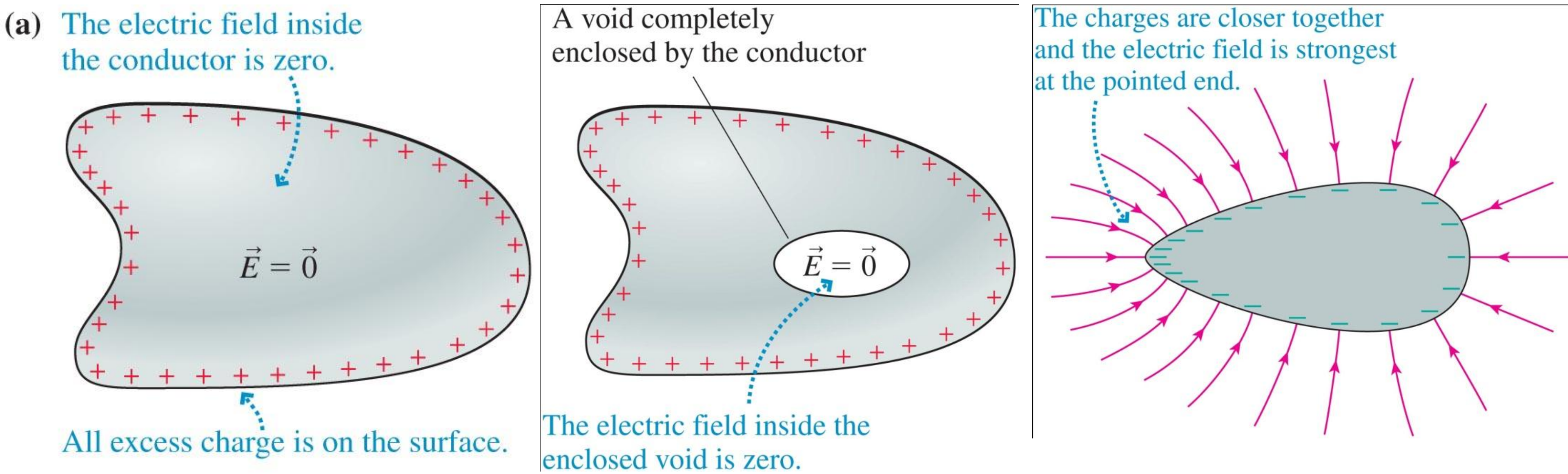
This means the electric field on a conductor has to be perpendicular to the surface!

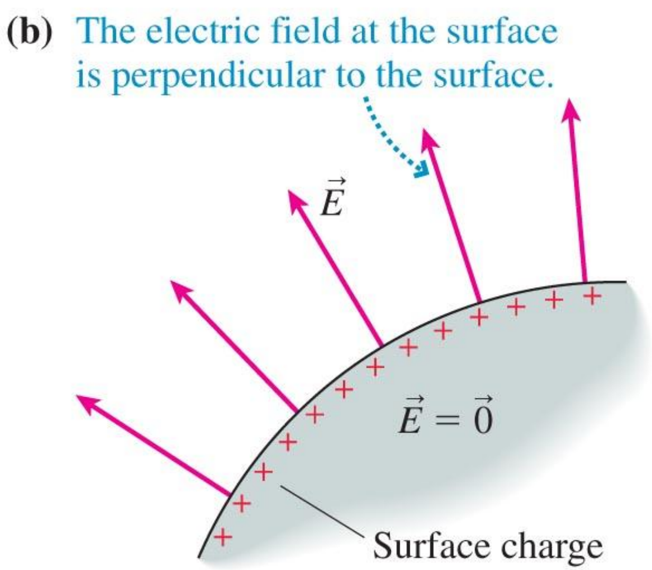
Conductor Sharp Edges!

- What will happen?



Summary of Conductors and Electric Fields

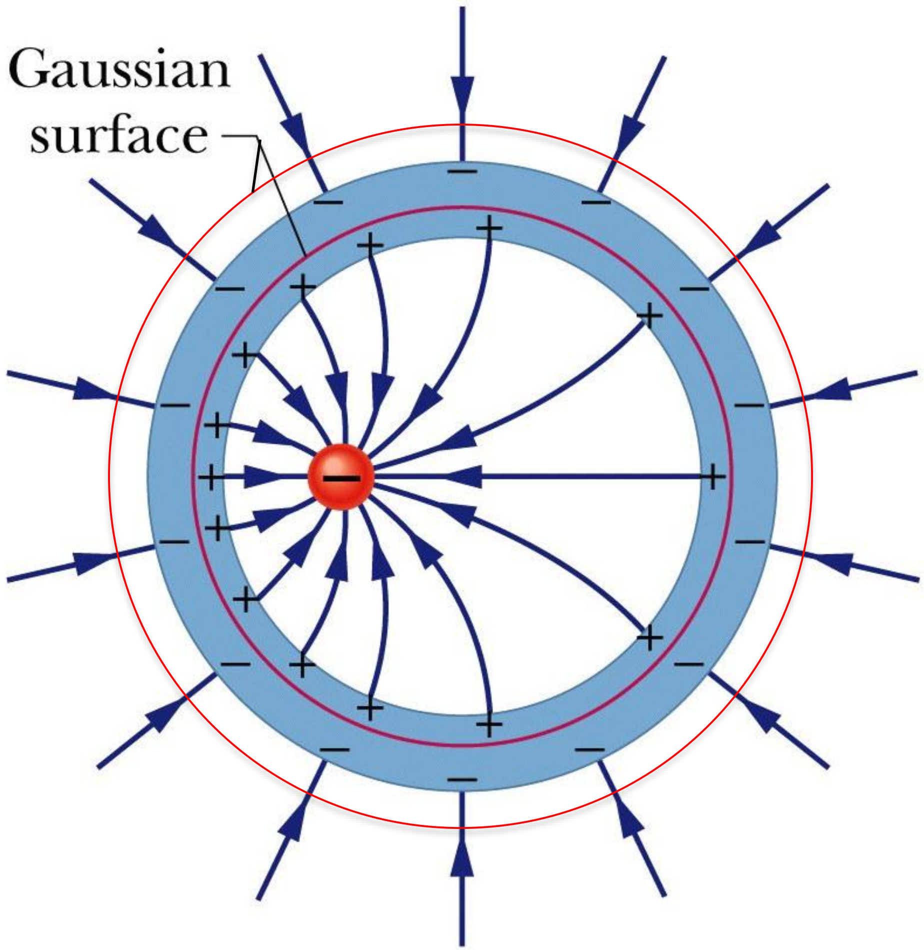




https://www.youtube.com/watch?v=x7uCAvEhP1E

Recall from last time

Faraday cages also shield the outside from fields on the inside. If the charge in the cavity were moved around, the field inside would be complicated and time-varying, but that information would not make it outside.



Application: Airplane Wifi

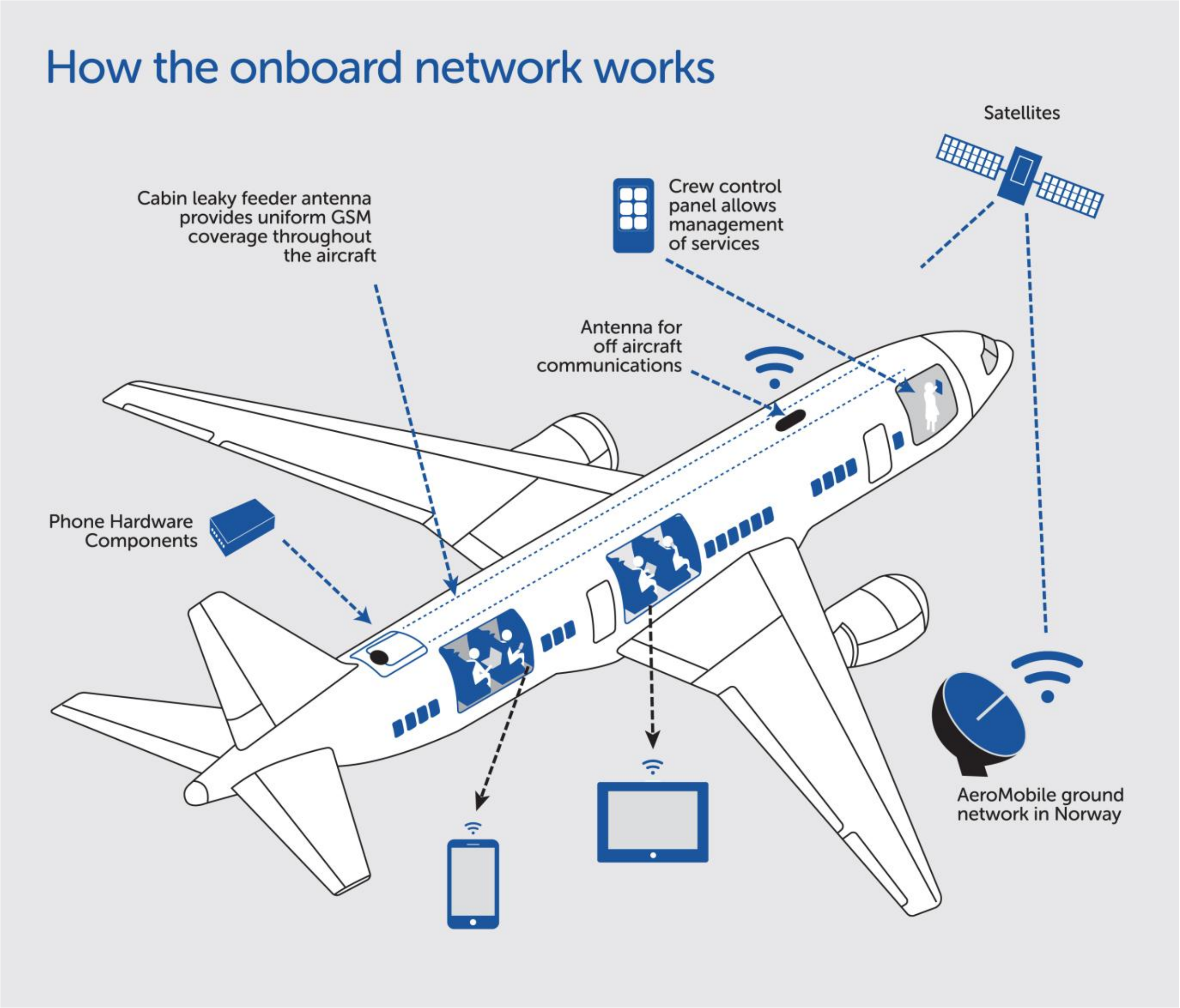


Image from http://www.mobileindustryreview.com/media/AeroMobile-How-it-Works.jpg

Application: Microwaves

Properties of Conductors

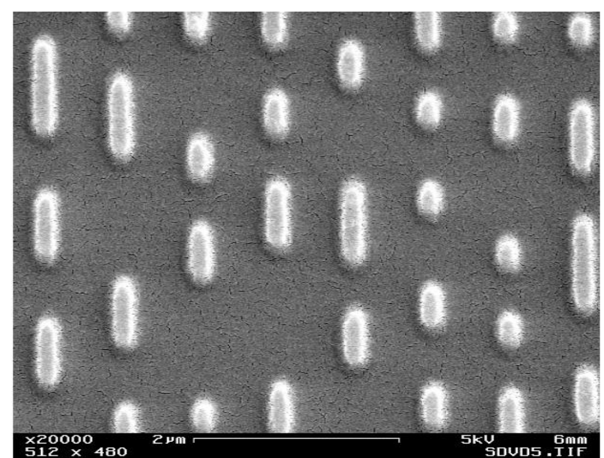
Summary:

- 1. The electric field is zero inside a conductor. – Static Case
- 2. All excess charge is distributed over the outside surface.

  Inside, a conductor is neutral.
- 3. The electric field outside a conductor is parallel to the area vector (perpendicular to the surface) at each point and has a magnitude $\mathbf{E} = \sigma/\varepsilon_0$
- 4. The charge density is greatest where the radius of curvature is smallest.

Example: CD in the microwave.

Image: close-up of the surface of a CD



CD in the microwave

https://www.youtube.com/watch?v=0JkCIfLE_-M

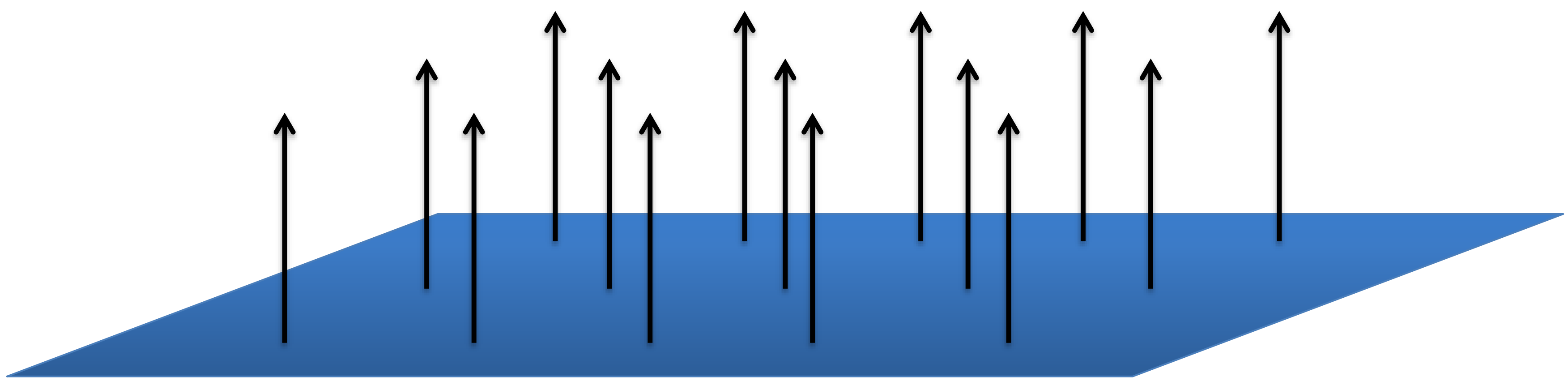
More in MicroWave

https://www.youtube.com/watch?v=c2ivYqToCLQ

Planar Symmetry

An infinite plane is perfectly symmetric under side to side translations, and rotations in the x,y plane. Consequences:

1) E-field must point in same direction everywhere (translations)
2) E-field must point perpendicular to the surface (rotations)



This alone is enough to predict E-field is uniform for infinite plane!

- LOOK at my notes called:
- Feb_Appendix1_plannar symmetry

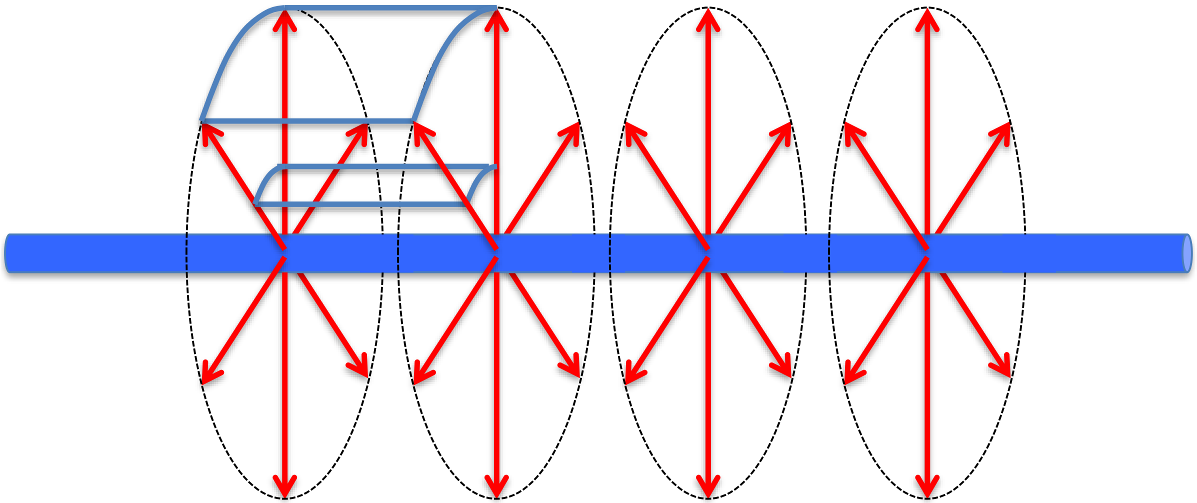
&

- Feb_Appendix2_Cylinderical symmetry

&

- Feb_Appendix3

Predicting E-field behaviour



L

$r_2\Delta\theta$

$r_1\Delta\theta$

L

Same flux through both
Area ~ r, so E ~ 1/r

Point charge: much harder to draw but a similar method shows the area is increasing like r^2, so the E-field must behave like $1/r^2$

Exercise: Coaxial Cable

Assume there is a charge **+Q** on the centre core and **-Q** on the metallic shield. (Ignore the dielectric insulator and plastic jacket.)

Find the electric field outside the metallic shield (**E_2**) and just outside the central core (**E_1**).