Announcements

- Phys 259 tutorial in ST 141 form 2:00-4:00 p.m. on Sunday April 2, 2017.
- Phys 259 tutorial in ST 141 form 2:00-4:00 p.m. on Sunday April 9, 2017.
- Possible Phys 259 tutorial in ST 141 form 2:00-4:00 p.m. on Saturday April 15, 2017.

Last time

- Force on a current carrying conductor
- Force on a current loop
- Torque on a current loop
- Magnetic moment
- Symmetry between electric and magnetic field
- DC motor

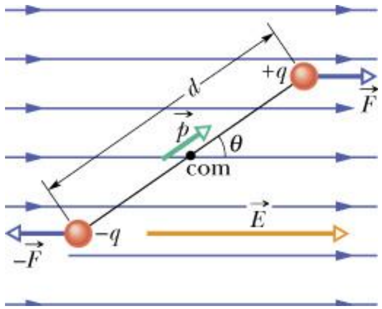
This time

- Sources of magnetic field (Biot-Savart Law)
- Magnetic field of a moving point charge
- Magnetic field of a current carrying conductor
- Magnetic field of an infinitely long straight current carrying conductor

Symmetry between E and B

Electrostatic and electric dipole moment

$$\vec{p} = q\vec{d}$$



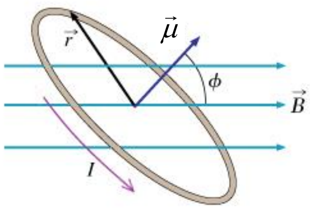
$$\vec{F}_E = 0$$

$$\vec{\tau}_E = \vec{p} \times \vec{E}$$

$$U_E = -\vec{p} \bullet \vec{E} = -pE\cos\theta$$

Magneto-static and magnetic moment

$$\vec{\mu} = I\vec{A}$$



$$\vec{F}_B = 0$$

$$\vec{\tau}_B = \vec{\mu} \times \vec{B}$$

$$U_B = -\vec{\mu} \bullet \vec{B} = -\mu B\cos\theta$$

The minimum energy state of current loop is when its magnetic moment lines up with the magnetic field. The maximum energy state is when its magnetic moment is anti-parallel with the B field. Given the chance a magnetic moment would line up with the B field.

$$\vec{\tau}_B = \vec{\mu} \times \vec{B}$$

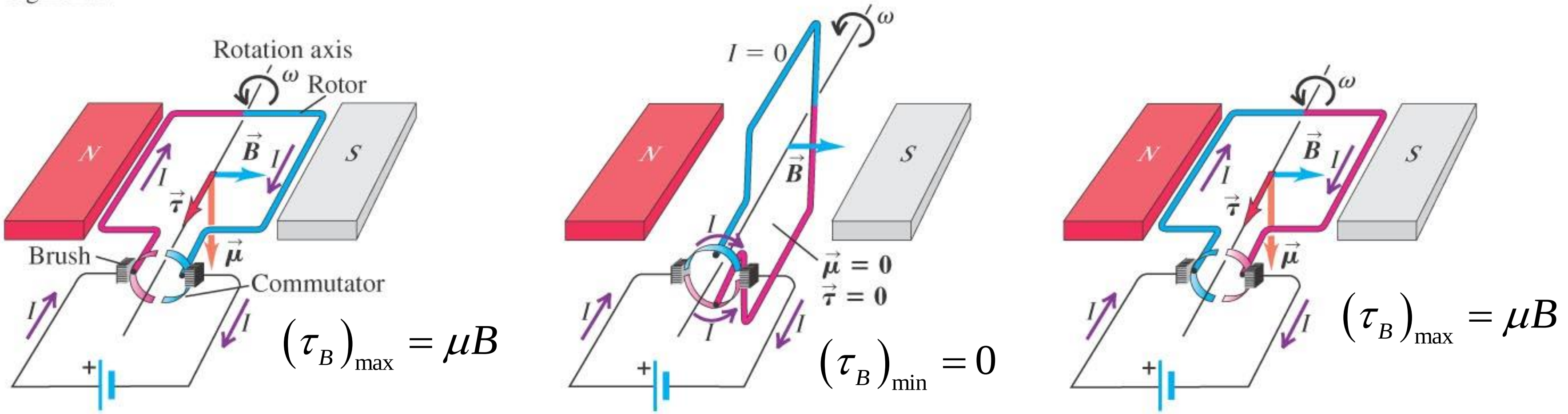
Direct current motor

$$\tau_B = \mu B \sin\phi$$

(a) Brushes are aligned with commutator segments.

(b) Rotor has turned 90°.

(c) Rotor has turned 180°.



$$(\tau_B)_{\max} = \mu B$$

$$(\tau_B)_{\min} = 0$$

$$(\tau_B)_{\max} = \mu B$$

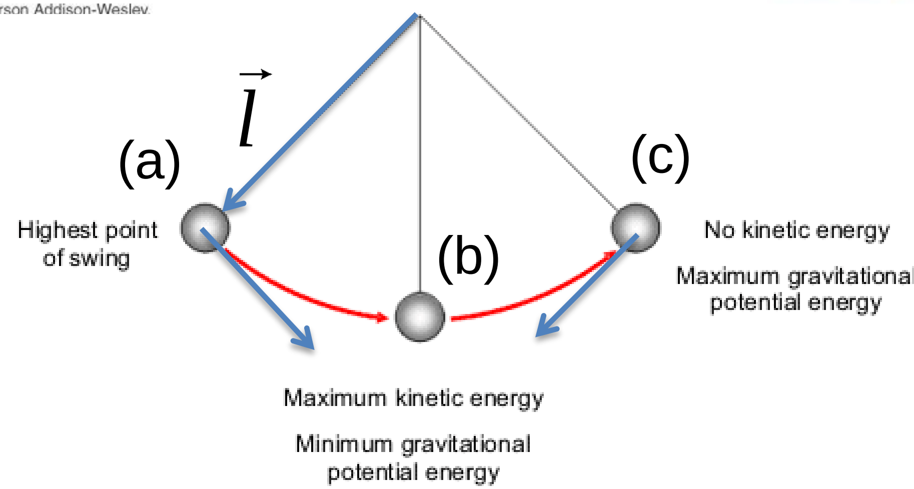
- Current flows into the red side of the rotor and out of the blue side.
- Therefore the magnetic torque causes the rotor to spin counterclockwise.

- Each brush is in contact with both commutator segments, so the current bypasses the rotor altogether.
- No magnetic torque acts on the rotor.

- The brushes are again aligned with commutator segments. This time the current flows into the blue side of the rotor and out of the red side.
- Therefore the magnetic torque again causes the rotor to spin counterclockwise.

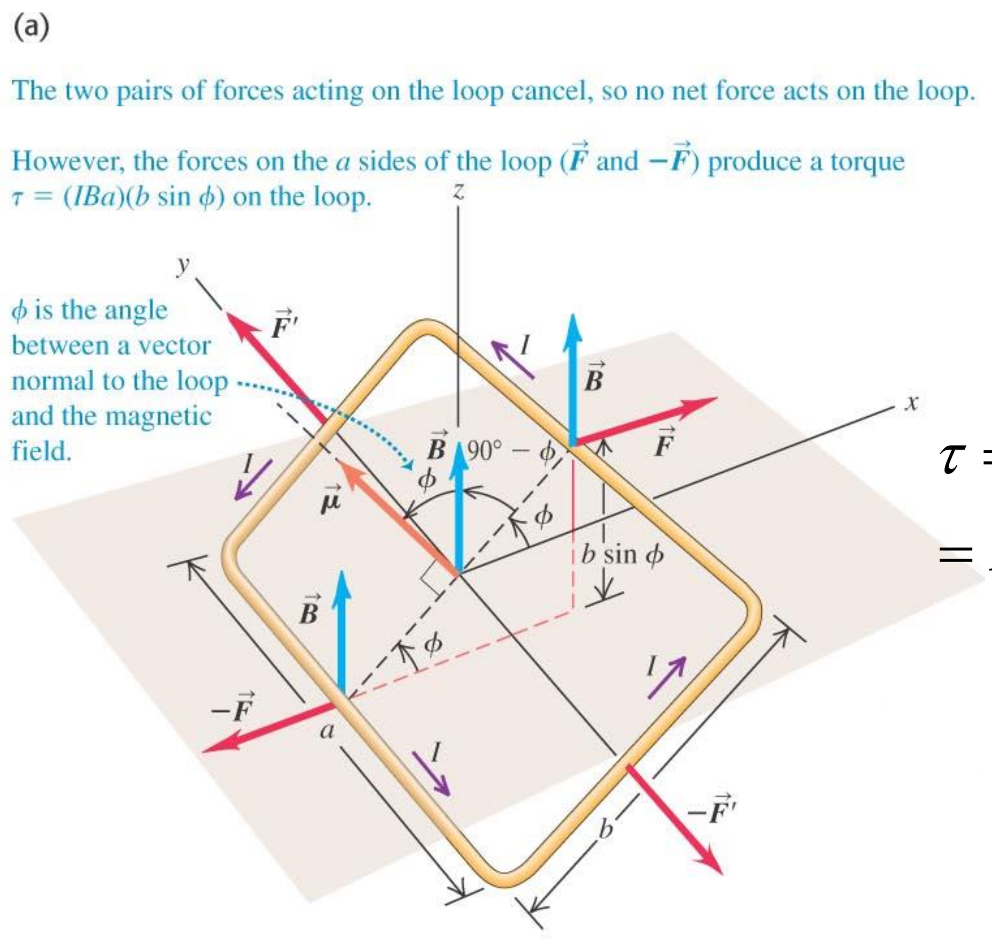
Pendulum

$$\vec{\tau}_g = m\vec{l} \times \vec{g}$$



Force and torque on a current loop

- This basis of electric motors



$$\tau = 2F(b/2)\sin\phi$$
$$= IBab\sin\phi = \left(Iab\right)B\sin\phi = \mu B\sin\phi$$

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

Sources of magnetic field

The Biot-Savart Law

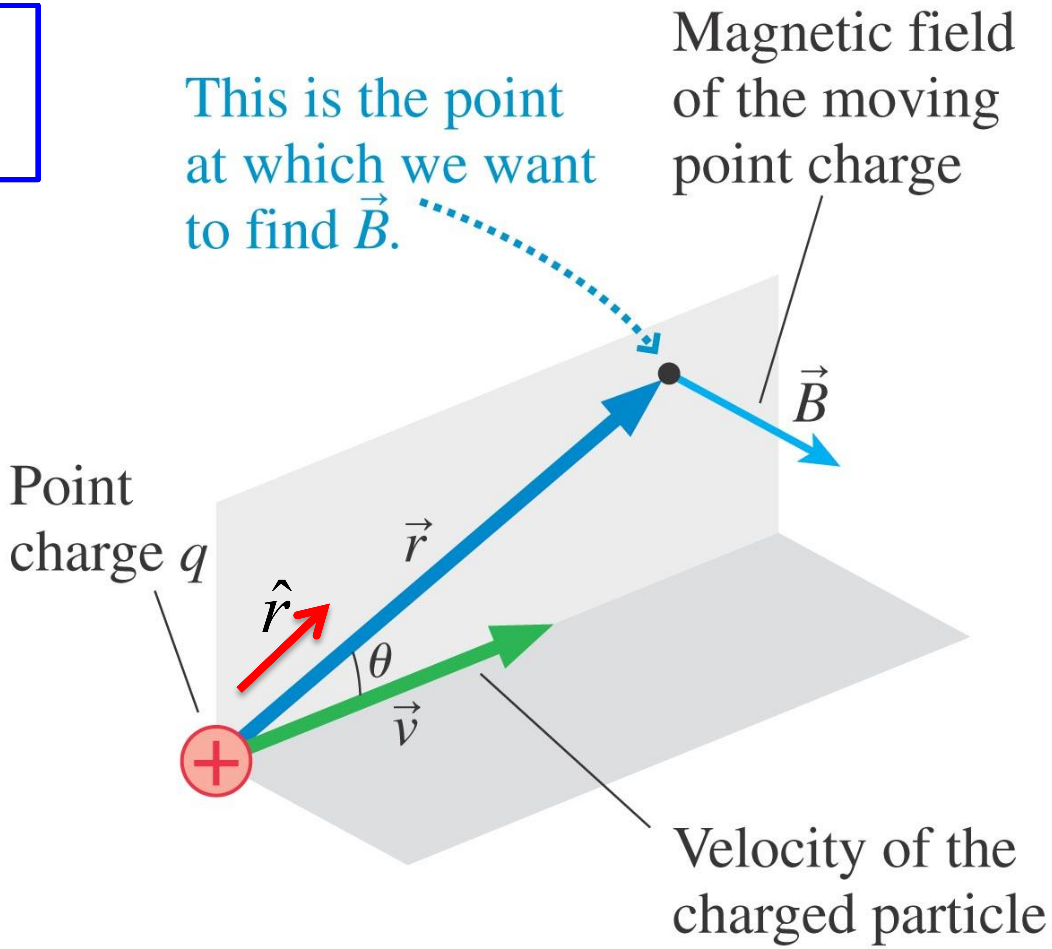
(Bee-oh Sah-var)

Magnetic fields are caused by **moving charges**.*

$$\vec{B}_{\text{point charge}} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2}$$

Or, using the definition $\hat{r} = \dfrac{\vec{r}}{|\vec{r}|}$

$$\vec{B}_{\text{point charge}} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\vec{r}}{r^3}$$



*One exception is due to quantum mechanics: charged particles with "spin" produce B fields

Comparing sources of magnetic field and electric field

Magnetic field of a moving point charge.

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2} \qquad B = \frac{\mu_0}{4\pi}\frac{|q|v\sin\theta}{r^2}$$

μ_0 is the magnetic permeability of free space.

Electric field of a point charge.

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q\hat{r}}{r^2} \qquad E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

ε_0 is the electric permittivity of free space.

Constants of nature

"Permittivity of free space"

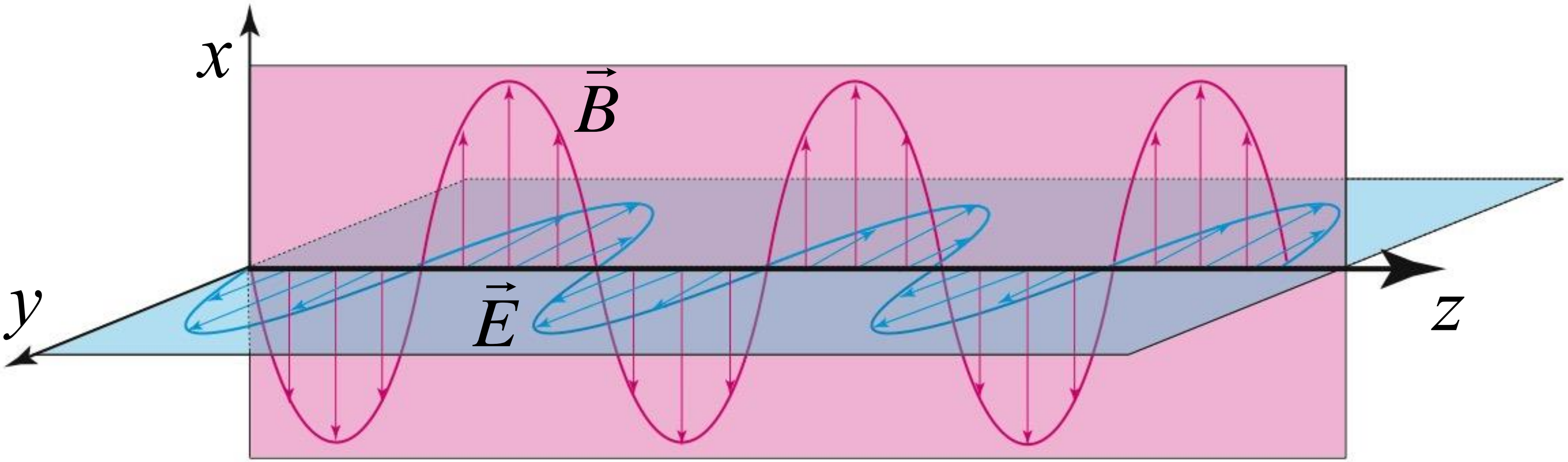
$$\varepsilon_0 = 8.85418781719 \times 10^{-12} \frac{C^2}{N \cdot m^2}$$

"Permeability of free space"

$$\mu_0 = 4\pi \times 10^{-7} \frac{N \cdot s^2}{C^2}$$

$$\frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 299,792,458 \text{ m/s}$$

Speed of light!

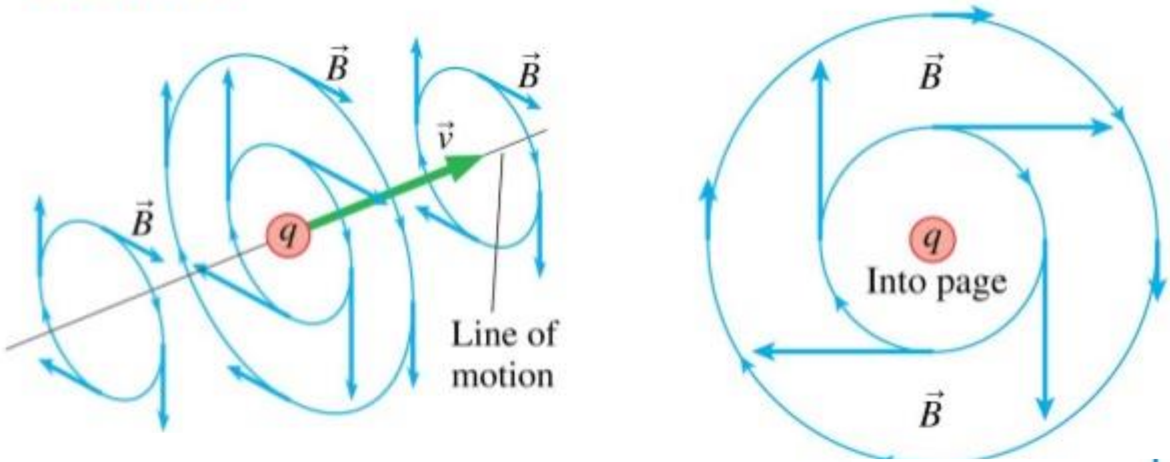


$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$$

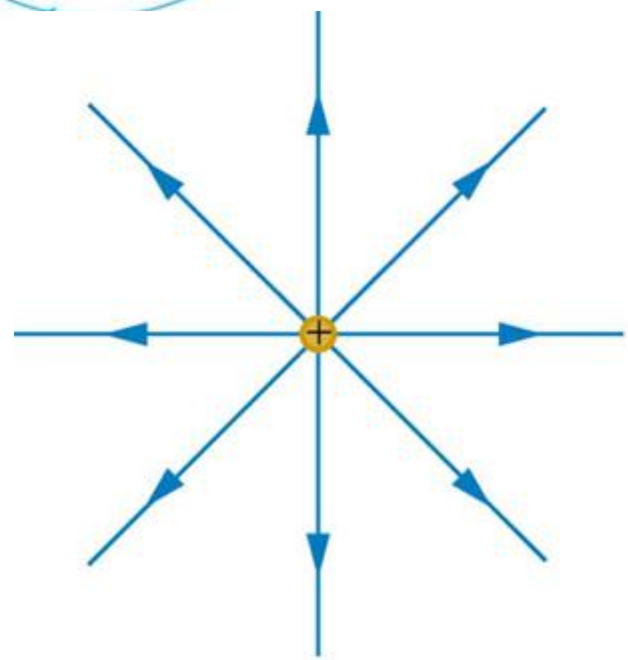
Discovered by Maxwell from his theory of electromagnetic field which led to the notion that electromagnetic field is light and all colors of light have the same speed in vacuum.

Magnetic field pattern generated by a moving charge

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2}$$



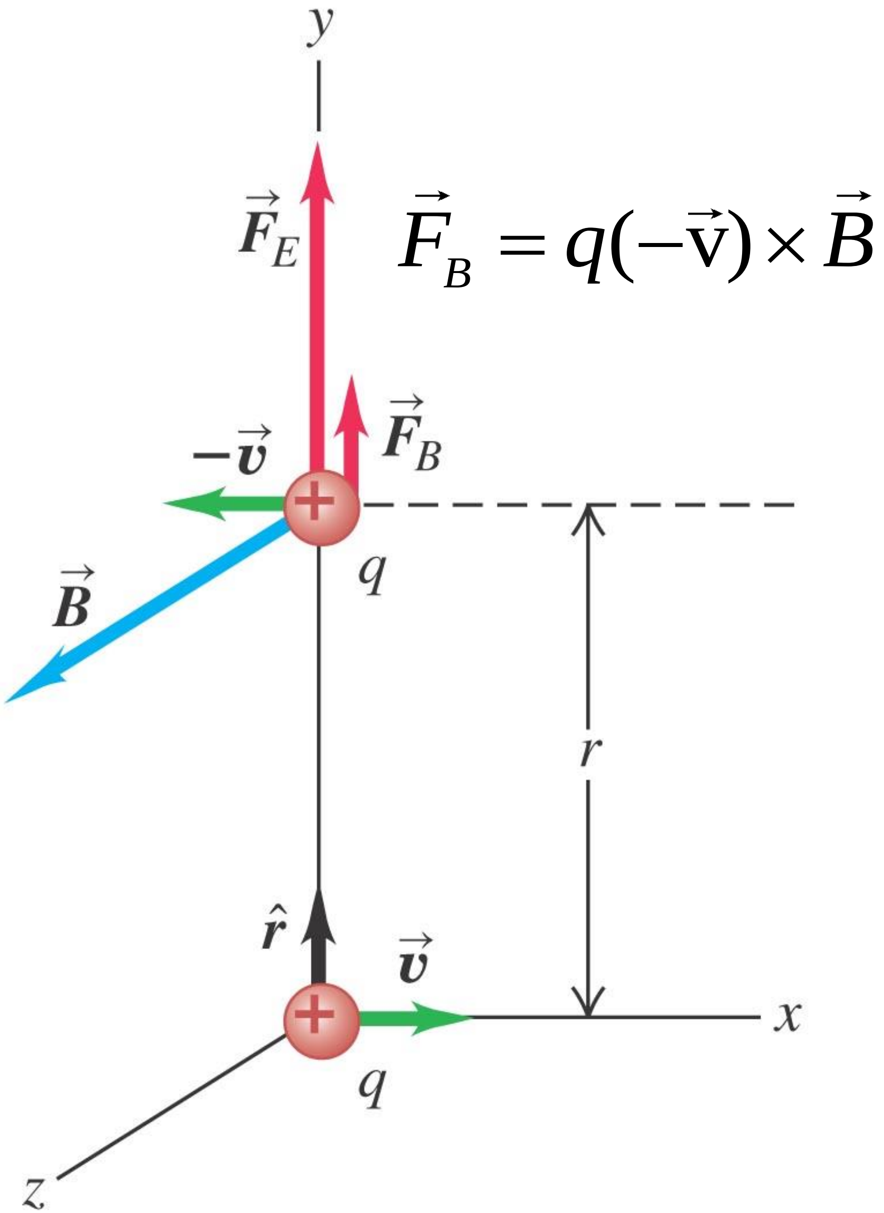
Electric field pattern generated by a point charge



Moving charges—field lines

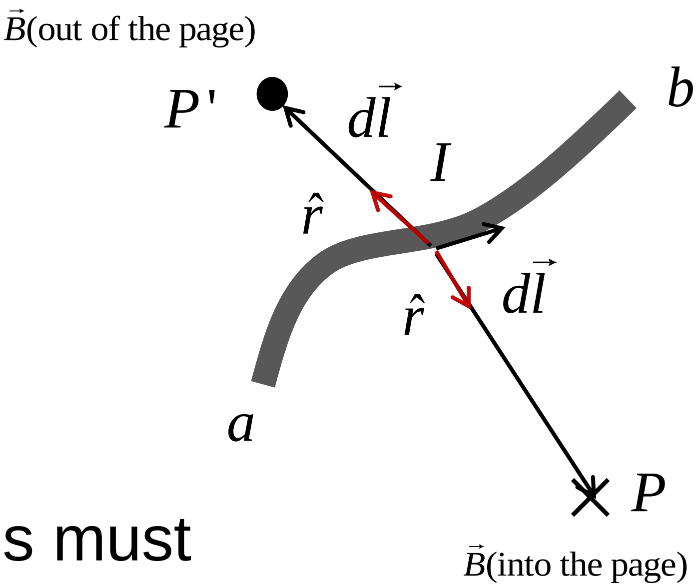
- The moving charge will generate field lines in circles around the charge in planes perpendicular to the line of motion.

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2}$$



Magnetic field of a current carrying conductor

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I d\vec{l} \times \hat{r}}{r^2}$$



Note that magnetic field lines must be closed loops.

$$\vec{B} = \frac{\mu_0}{4\pi}\int_a^b \frac{I d\vec{l} \times \hat{r}}{r^2}$$

Biot-Savart law

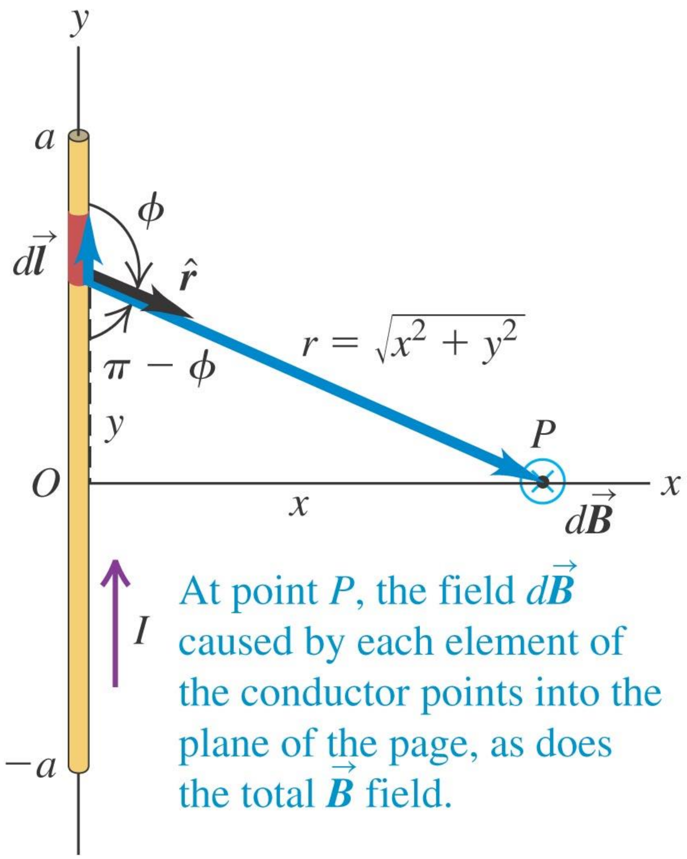
Analogous to

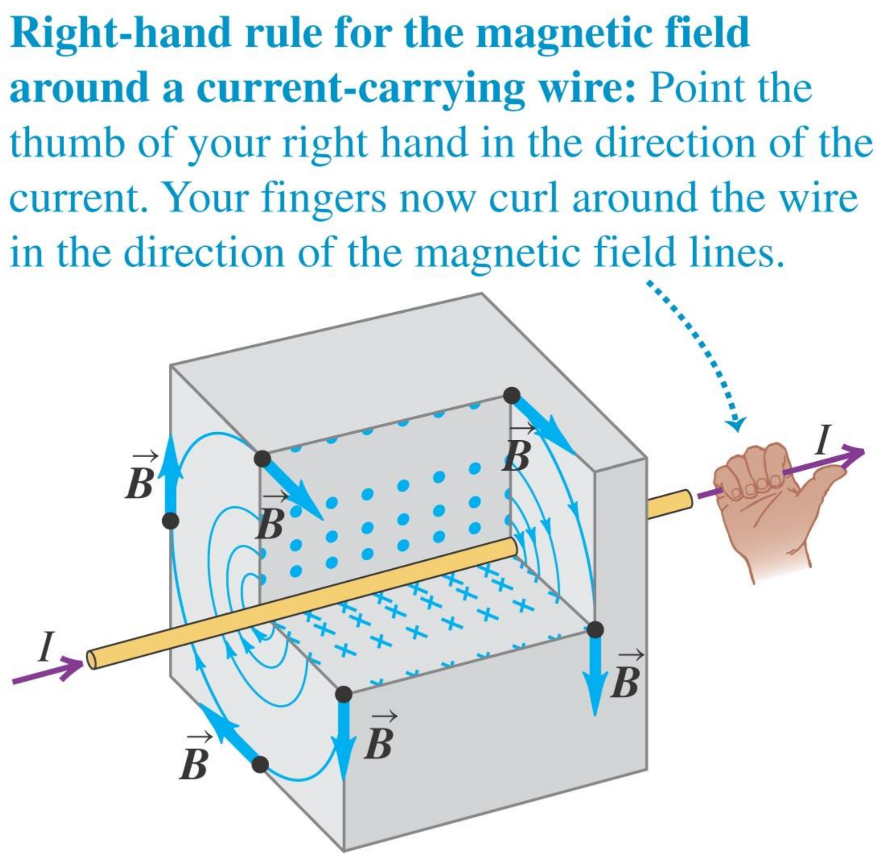
$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int \frac{dq\hat{r}}{r^2}$$

Coulomb law

Magnetic field of a straight current-carrying conductor

- Biot and Savart contributed to finding the magnetic field produced by a single current-carrying conductor.



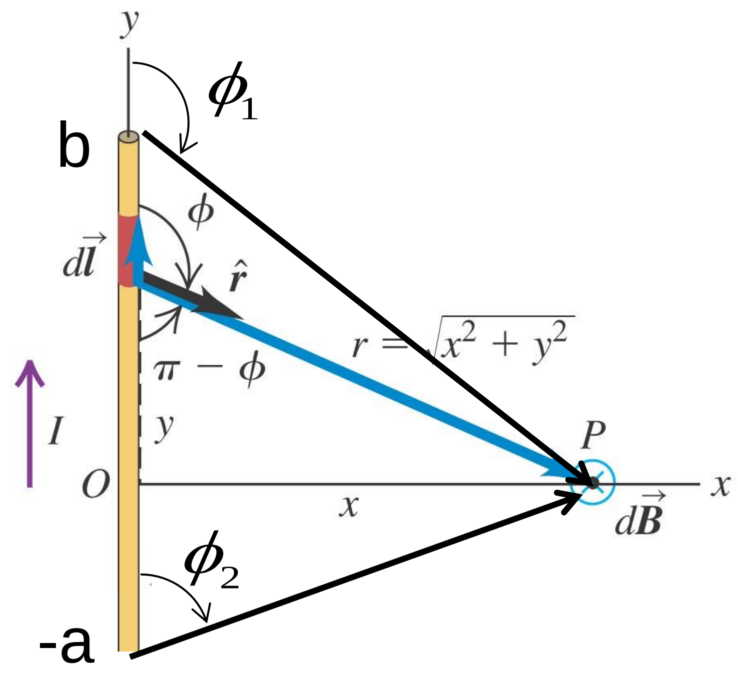


$$\sin(\pi-\phi)=\sin\phi=\frac{x}{r}$$

$$\cos(\pi-\phi)=-\cos\phi=\frac{y}{r}$$

$$\cot\phi=-\frac{y}{x}\Rightarrow y=-x\cot\phi$$

$$dy=-x\left(-\frac{d\phi}{\sin^2\phi}\right)=\frac{xd\phi}{\sin^2\phi}$$



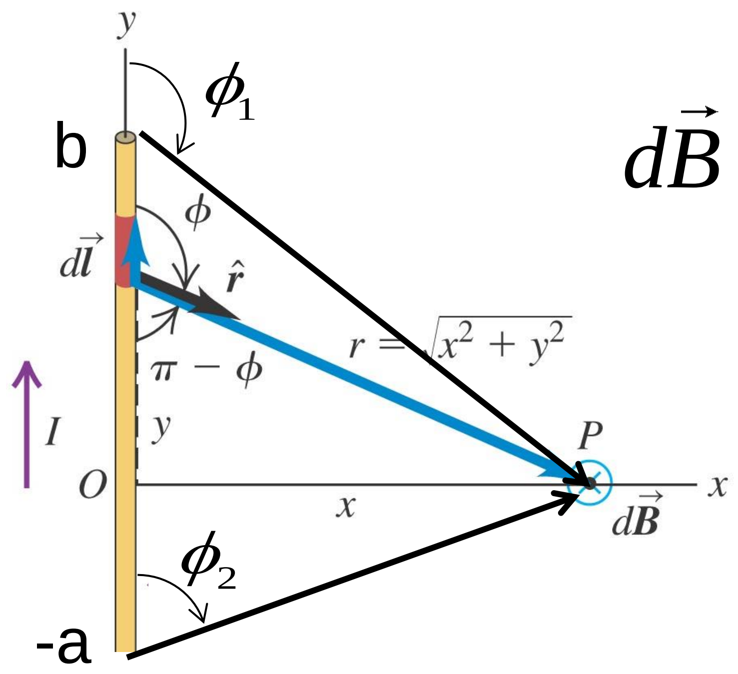
$$d\vec{B}=\frac{\mu_0}{4\pi}\frac{Id\vec{l}\times\hat{r}}{r^2}$$

$$dB=\frac{\mu_0}{4\pi}\frac{Idl\sin\phi}{r^2}=\frac{\mu_0}{4\pi}\frac{Idy\sin\phi}{r^2}$$

$$dB=\frac{\mu_0}{4\pi}\frac{I\sin\phi dy}{x^2/\sin^2\phi}=\frac{\mu_0}{4\pi}\frac{I\sin^3\phi dy}{x^2}=\frac{\mu_0}{4\pi}\frac{Ix\sin^3\phi d\phi}{x^2\sin^2\phi}=\frac{\mu_0}{4\pi}\frac{I\sin\phi d\phi}{x}$$

$$B=\int dB=\frac{\mu_0 I}{4\pi x}\int_{\phi_2}^{\phi_1}\sin\phi d\phi=\frac{\mu_0 I}{4\pi x}\left[-\cos\phi_1+\cos\phi_2\right]$$

Magnetic field of a straight current-carrying conductor



$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I d\vec{l} \times \hat{r}}{r^2}$$

Out of the page to the left of the wire.

Into the page to the right of the wire.

$$B = \frac{\mu_0 I}{4\pi x}\left[\cos\phi_2 - \cos\phi_1\right]$$

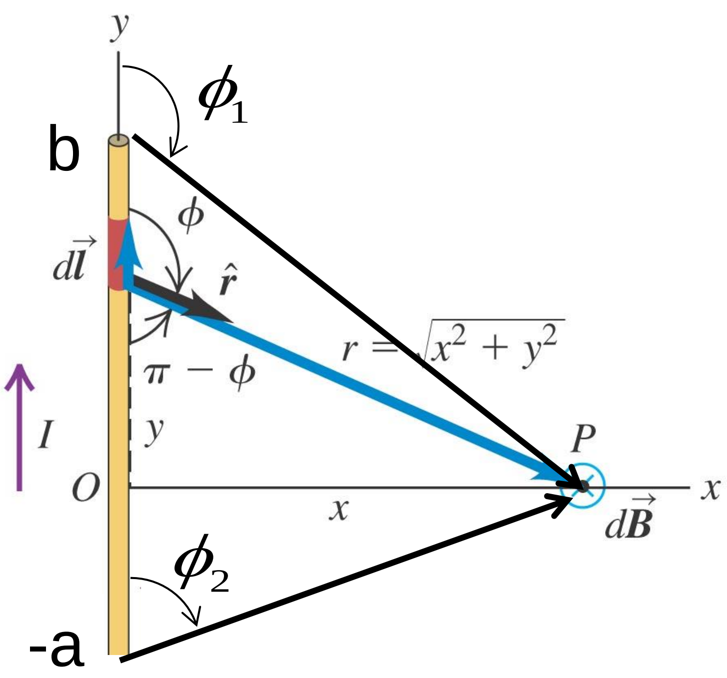
Magnetic field of a infinitely long straight current carrying conductor

$$B = \frac{\mu_0 I}{4\pi x}\left[\cos\phi_2 - \cos\phi_1\right]$$

$$b \to \infty \Rightarrow \phi_1 \to \pi$$

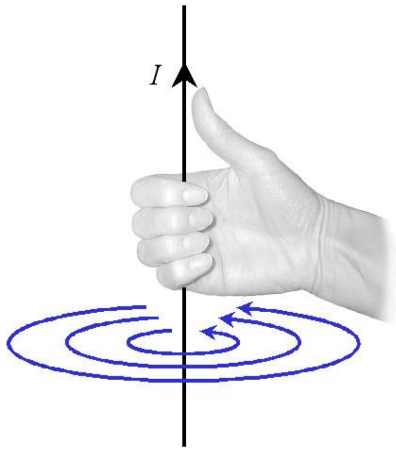
$$-a \to -\infty \Rightarrow \phi_2 \to 0$$

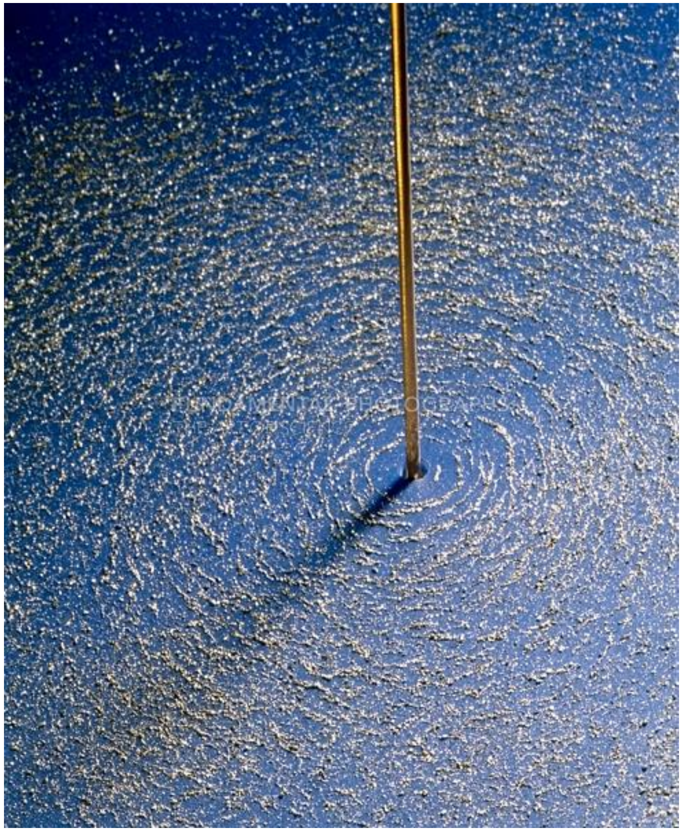
$$B = \frac{\mu_0 I}{4\pi x}\left[1+1\right] = \frac{\mu_0 I}{2\pi x}$$



Magnetic field of a infinitely long straight current carrying conductor

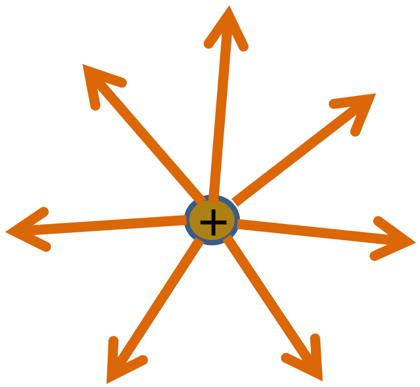
$$B = \frac{\mu_0 I}{2\pi x}$$





Magnetic field lines are circular loops.

The electric field of an infinite line of charge



View from the top

$$E = \frac{\lambda}{2\pi\varepsilon_0 x}$$