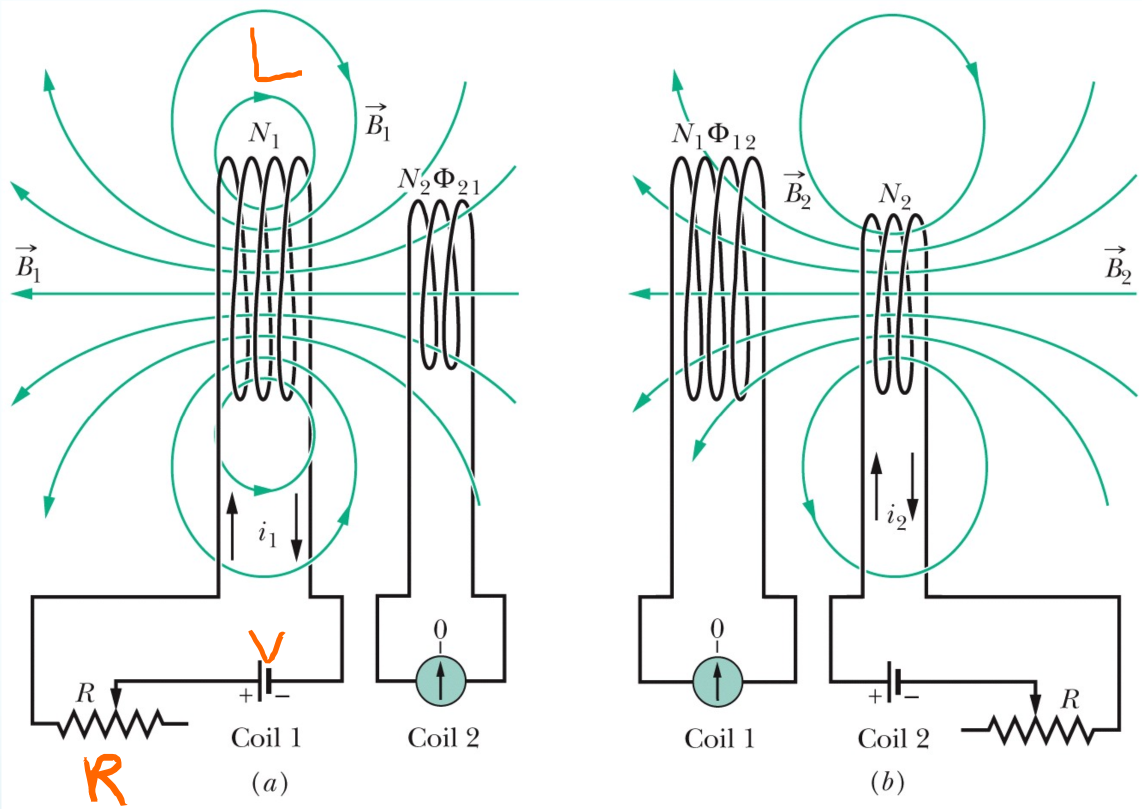
Electricity and Magnetism

- Physics 259 – L02
- Lecture 48

Chapter 29: Magnetic field due to current

30-8 Mutual Induction



Mutual induction. (a) The magnetic field B_1 produced by current i_1 in coil 1 extends through coil 2. If i_1 is varied (by varying resistance R), an *emf* is induced in coil 2 and current registers on the meter connected to coil 2. (b) The roles of the coils interchanged.

If coils 1 and 2 are near each other, a changing current in either coil can induce an emf in the other. This mutual induction is described by

$$\mathcal{E}_{ind} = -\frac{d\Phi_m}{dt} \rightarrow \Phi_m \text{ change}$$

$$\mathcal{E}_2 = -M\frac{di_1}{dt}$$

$$\mathcal{E}_1 = -M\frac{di_2}{dt}.$$

Mutual inductance M_{21} of coil 2 with respect to coil 1 =>

$$M_{21} = \frac{N_2 \Phi_{21}}{i_1} \Rightarrow M_{21}\, i_1 = N_2 \Phi_{21} \Rightarrow M_{21} \frac{di_1}{dt} = N_2 \frac{d\Phi_{21}}{dt} = -\mathcal{E}_2$$

we had: $\mathcal{E}_2 = -N_2 \frac{d\Phi}{dt}$

$$\rightarrow M_{21} \frac{di_1}{dt} = -\mathcal{E}_2 \rightarrow \mathcal{E}_2 = -M_{21} \frac{di_1}{dt}$$

and the same for $\mathcal{E}_1 \rightarrow \mathcal{E}_1 = -M_{12} \frac{di_2}{dt}$

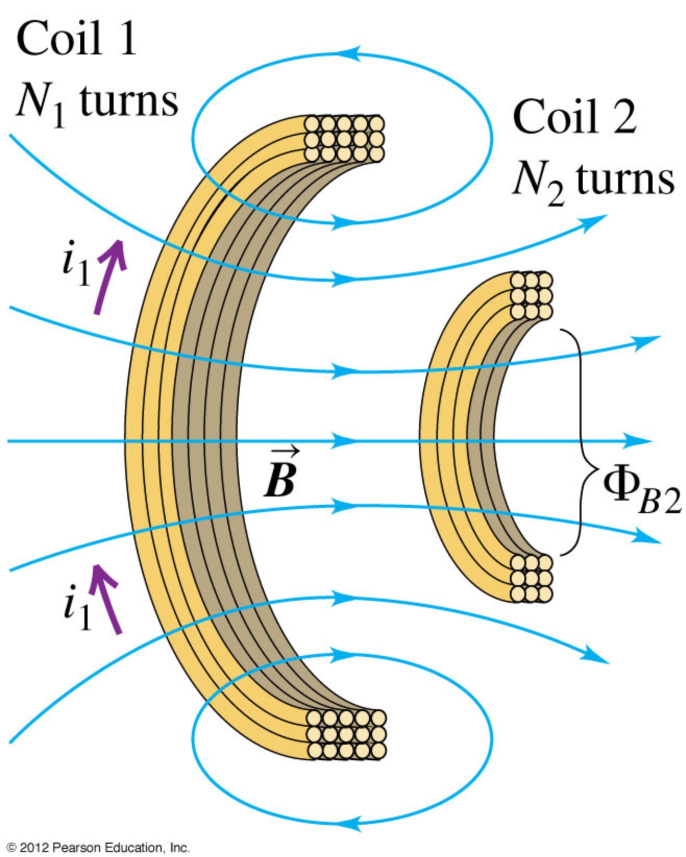
& $M_{12} = M_{21} = M$ → we can not prove here

$$\Rightarrow \quad \mathcal{E}_2 = -M \frac{di_1}{dt} \qquad \mathcal{E}_1 = -M \frac{di_2}{dt}$$

M: *mutual inductance*, depends on the geometry of the two coils

[*M*]=1*H* =1*Wb*/*A*=1*Vs*/*A*=1*Ωs*=1*J*/*A*^{2}. Typical values: *M*=*μH*-*mH*

Mutual inductance: If the current in coil 1 is changing, the changing flux through coil 2 induces an emf in coil 2.



Induced EMF in coil 2:

$$\varepsilon_2 = -N_2 \frac{d\phi_{B2}}{dt} = -\frac{d\left(N_2 \phi_{B2}\right)}{dt}$$

Note: ϕ_{B2} is the magn. flux through a single loop of coil 2. N_2 is the number of loops.

The magnetic field in coil 2 is prop. to the current through coil 1.

Same

$$dB = \frac{\mu_0}{4\pi} \frac{i_1 d\boldsymbol{l} \times \boldsymbol{r}}{r^2}$$

Biot-Savart

Hence, the magnetic flux through coil 2 is proportional to i_1:

$$N_2 \phi_{B2} = M_{21} i_1$$

and

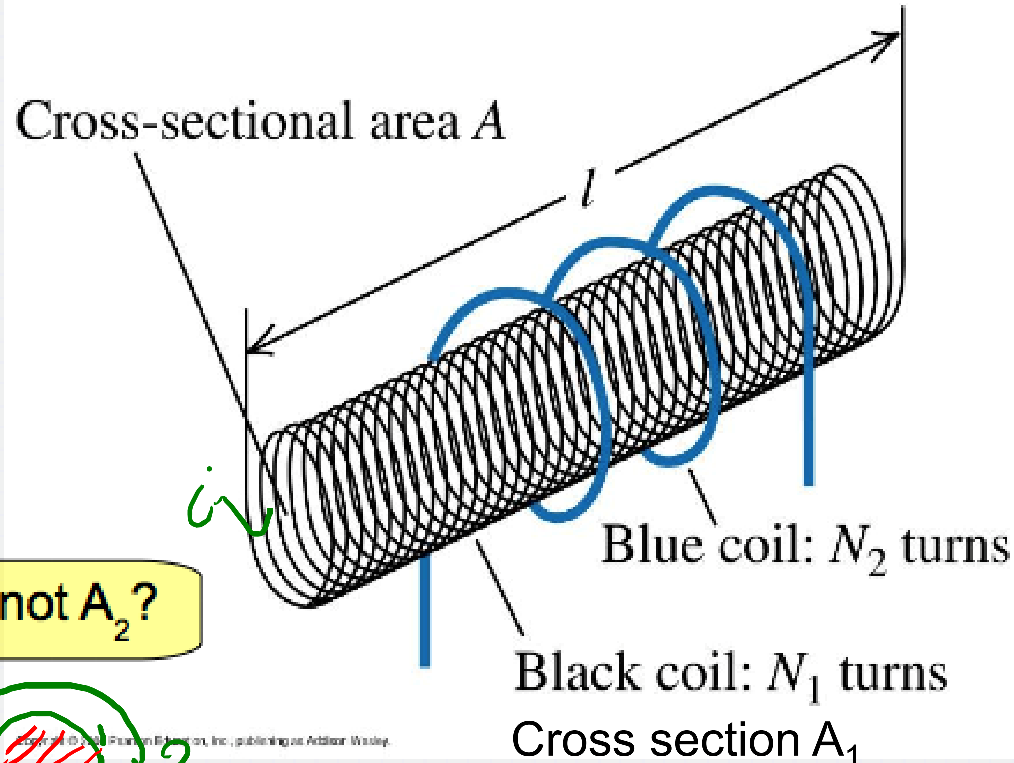
$$\varepsilon_2 = -\frac{d\left(N_2 \phi_{B2}\right)}{dt} = -M_{21} \frac{di_1}{dt}$$

M_{21}: *mutual inductance*, depends on the geometry of the two coils

Question 1 – Mutual inductance

The long solenoid will produce a magnetic field that is proportional to the current I_1 and the number of turns per unit length n_1

$$B_1 = \frac{\mu_0 N_1 I_1}{L} = \mu_0 n_1 I_1$$



and the total flux through each loop of the outer coil is

$$\Phi_{B2} = B_1 A_1$$

Why not A_2?

so the mutual inductance is

$$M = \frac{N_2 \Phi_{B2}}{I_1} = \frac{N_2 (B_1 A_1)}{I_1} = \frac{\mu_0 A_1 N_1 N_2}{L}$$

does not depend on I!

For a 0.5m long coil with 10cm^2 area and N_1=1000, N_2=10 turns

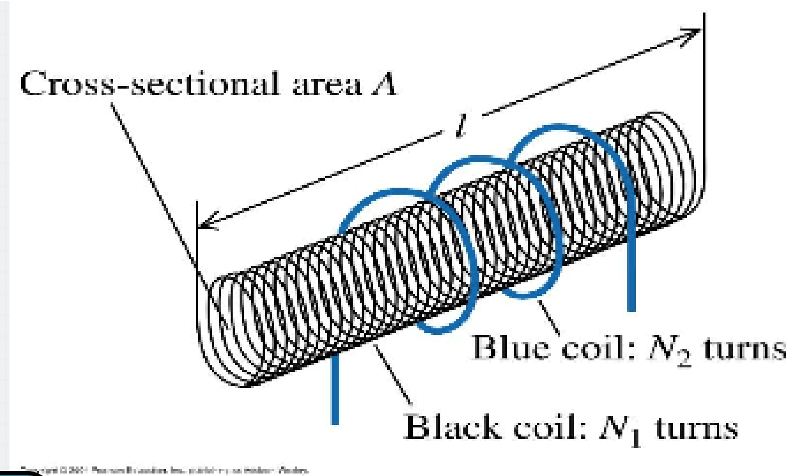
$$M = \frac{(4\pi\times10^{-7}\,T\,m/A)(1.0\times10^{-3}\,m^2)(1000)(10)}{0.5\text{m}} = 2.5\times10^{-6}\,H = 25\,\mu H$$

Question 1 (b)

If a rapidly increasing current is driven through the outer coil

$$i_2(t) = (2.0\times 10^6\, A/s)\, t$$

what EMF will be induced in the inner coil?



Note: M also allows calculating ε_2 if I_1 changes

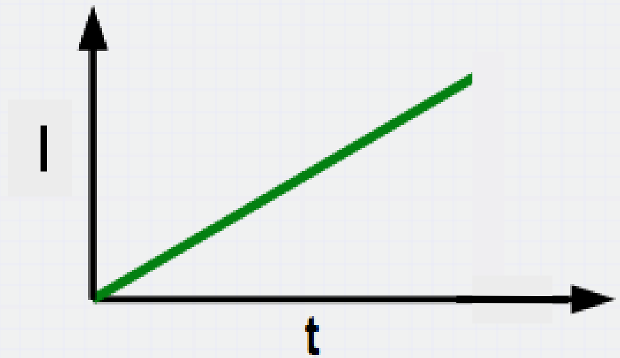
$$\varepsilon_1 = -M\frac{di_2}{dt}$$

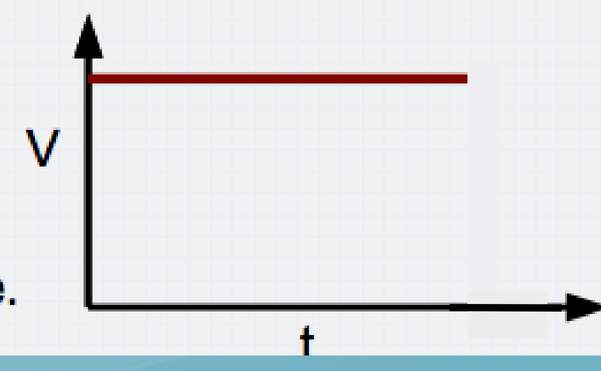
$$= -M\frac{d}{dt}(2\times 10^6)t = -M\, 2\times 10^6$$

$$= -(25\times 10^{-6} H)\frac{d}{dt}\left[(2.0\times 10^{-6}\, A/s)\, t\right]$$

$$= -(25\times 10^{-6} H)(2.0\times 10^{-6}\, A/s)$$

$$= -50\, V$$





This allows electrical energy in one circuit to be converted to electric energy in a separate device.

Question 2

16. The diagram below shows two nested, circular coils of wire. The larger coil has radius a and consists of N_1 turns. The smaller coil (radius b) consists of N_2 turns, and is both coplanar and coaxial with the larger coil. Assume b << a, so that the magnetic field of the larger coil is approximately uniform over the area of the smaller coil. The **mutual inductance** of this combination is given by the expression

$$M = \frac{N_2 \phi_{B2}}{i_1} = \frac{N_1 \phi_{B1}}{i_2}$$

Flux through one loop of coil 2 (area A_2) due to magnetic field generated by current in coil 1

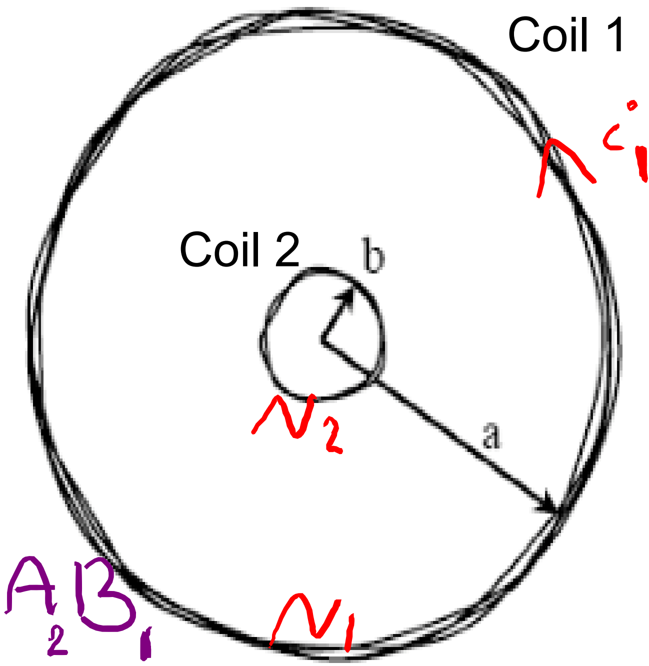
a) $\frac{\mu_0 N_1 N_2}{2a}$.

b) $\frac{\pi \mu_0 N_1 N_2 b}{a}$.

c) $\frac{\pi \mu_0 N_1 N_2 b^2}{2a}$.

d) $\frac{\mu_0 N_1 N_2 b^2}{2a}$.

e) $\frac{\pi \mu_0 N_2 b^2}{2a}$.



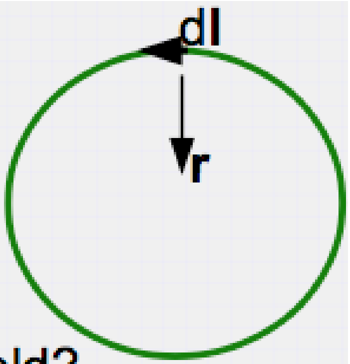
$$M = \frac{N_2 \Phi_2}{i_1} \rightarrow \Phi_2 = A_2 B_1$$

We assume current in the larger coil (coil 1), which generates a roughly uniform field in the area covered by the much smaller coil.

How large is B?

How large is B?

Calculate for one loop!



A circular loop of radius a carries a constant current I.
What is the magnetic field at the center of the loop?

What are the two methods we know for calculating magnetic field?
Biot-Savard law & Ampere's law.

Ampere's law isn't useful for a loop, so use the Biot-Savard law:

$B_1 = ?$

$\Phi_2 = A_2 B_1$

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{l}\times\hat{r}}{r^2} = \frac{\mu_0}{4\pi}\frac{I}{a^2}\,dl\,\hat{z}$$

$$\vec{B} = \int d\vec{B} = \frac{\mu_0}{4\pi}\frac{I}{a^2}\hat{z}\int dl = \frac{\mu_0}{4\pi}\frac{I}{a^2}(2\pi a)\hat{z} = \frac{\mu_0}{2}\frac{I}{a}\hat{z}$$

With the B-field direction directed out of the page, either from **dl** x **r**, or right thumb in direction of current and fingers curl in direction of **B**.

$$M = \frac{N_2 \phi_{B2}}{i_1} = \frac{N_2 BA}{i_1}$$

$$B = N_1 \frac{\mu_0 i_1}{2a}$$

$$A = \pi b^2$$

$$\Phi_2 = B_1 A_2 = N_1 \frac{\mu_0 i_1}{2a} \pi b^2$$

$$M = \frac{N_2}{\cancel{i_1}} N_1 \frac{\mu_0 \cancel{i_1}}{2a} \pi b^2 = \mu_0 \frac{N_1 N_2 \pi b^2}{2a}$$

$$\Longrightarrow \quad \boxed{M = \mu_0 N_1 N_2 \frac{\pi b^2}{2a}}$$

Question

16. The diagram below shows two nested, circular coils of wire. The larger coil has radius a and consists of N_1 turns. The smaller coil (radius b) consists of N_2 turns, and is both coplanar and coaxial with the larger coil. Assume b << a, so that the magnetic field of the larger coil is approximately uniform over the area of the smaller coil. The **mutual inductance** of this combination is given by the expression

a) $\dfrac{\mu_0 N_1 N_2}{2a}$.

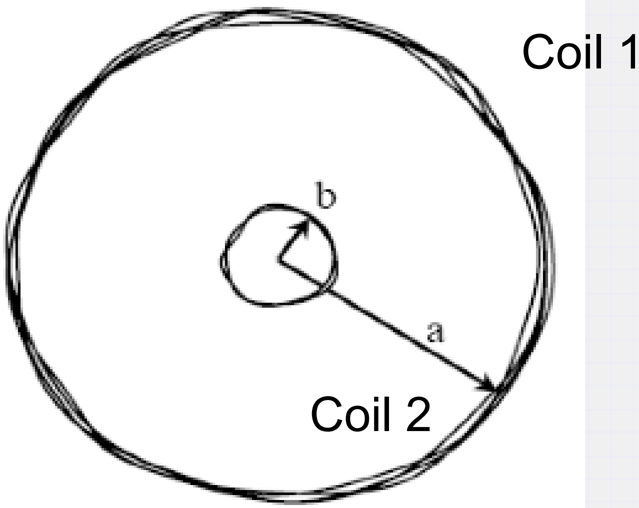
b) $\dfrac{\pi\mu_0 N_1 N_2 b}{a}$.

c) $\dfrac{\pi\mu_0 N_1 N_2 b^2}{2a}$. ✓

d) $\dfrac{\mu_0 N_1 N_2 b^2}{2a}$.

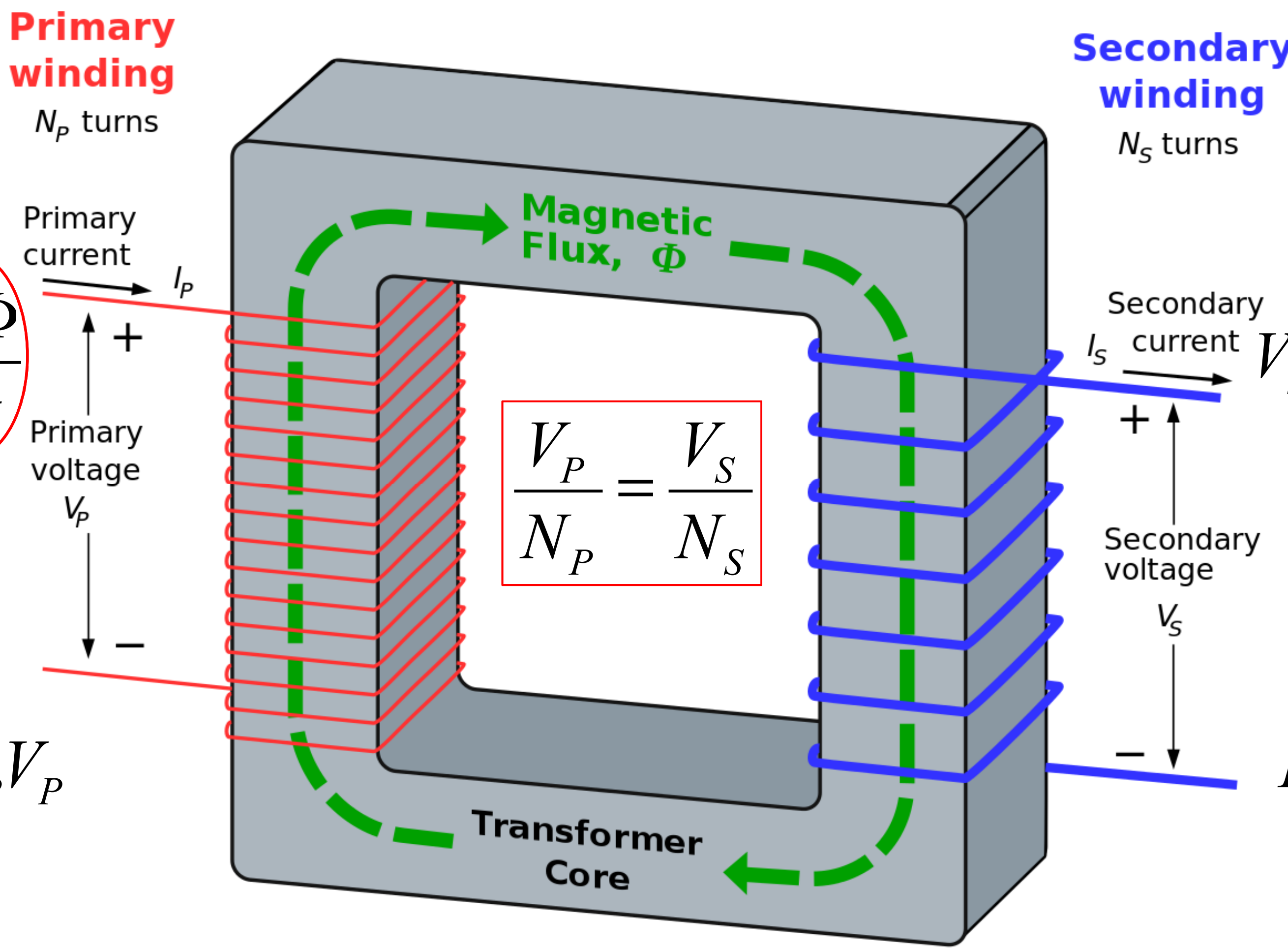
e) $\dfrac{\pi\mu_0 N_2 b^2}{2a}$.

$$M = \frac{N_2 \phi_{B2}}{i_1} = \frac{N_1 \phi_{B1}}{i_2}$$



- ✓ We expect the result to be proportional to the area of the coil that sees the field of the other coil, i.e. πb^2.
- ✓ Furthermore, we expect a dependence on N_1 and N_2: the field depends on N_1, and the flux on N_2.
- ✓ This leaves only answer c).

Transformers



$$V_P = N_P \frac{d\Phi}{dt}$$

$$V_S = N_S \frac{d\Phi}{dt}$$

$$\boxed{\frac{V_P}{N_P} = \frac{V_S}{N_S}}$$

$$P_{in} = I_P V_P$$

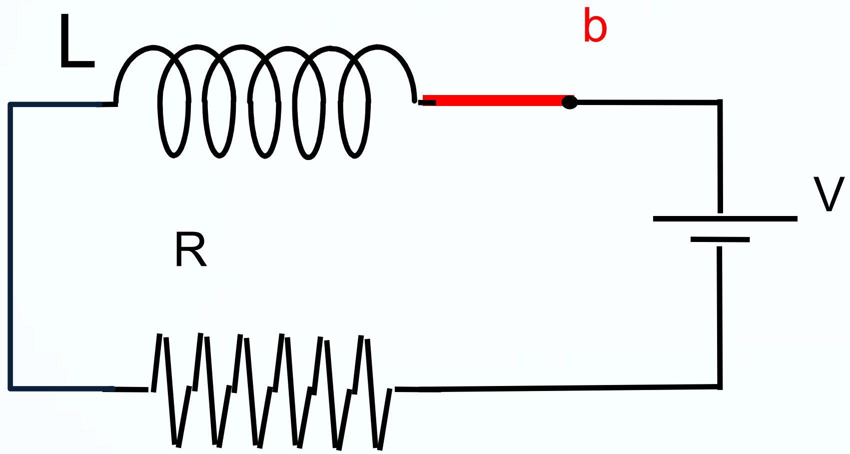
$$P_{out} = I_S V_S$$

$$P_{in} = P_{out}$$

$$I_P V_P = I_P \frac{N_P}{N_S} V_S = I_S V_S$$

$$\boxed{I_P N_P = I_S N_S}$$

30.6: R-L Circuit



$$V - L\frac{di}{dt} - iR = 0$$

If the switch is moved to position b, to initiate the current flow, what happens?

Faraday's law applies and so the change in the Magnetic Field in the inductor L means there is a back EMF induced in L.

So in this case at t = 0, $i(0) = 0$.

Inductor acts like a BATTERY

After a long time, $i = V/R$

Inductor acts like a WIRE

The components have all been connected for a very long time. At t=0 the switch S is opened. The current through R_1 and R are 0 and ${}^{\varepsilon}/_{R}$

Using the loop rules

$$-L\frac{di}{dt} - iR = 0$$

Solving with the method we used for a discharging capacitor

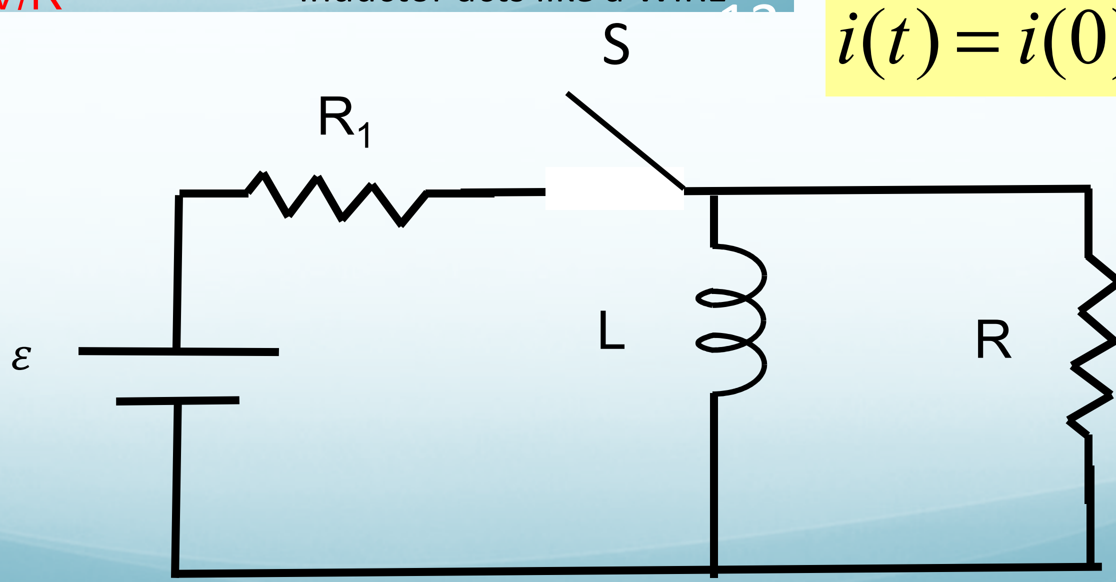
So in this case at t = 0, $i(0) = 0$.

After a long time, i= V/R

Inductor acts like a BATTERY

Inductor acts like a WIRE

$$i(t) = i(0)\,e^{-\left(\frac{Rt}{L}\right)}$$



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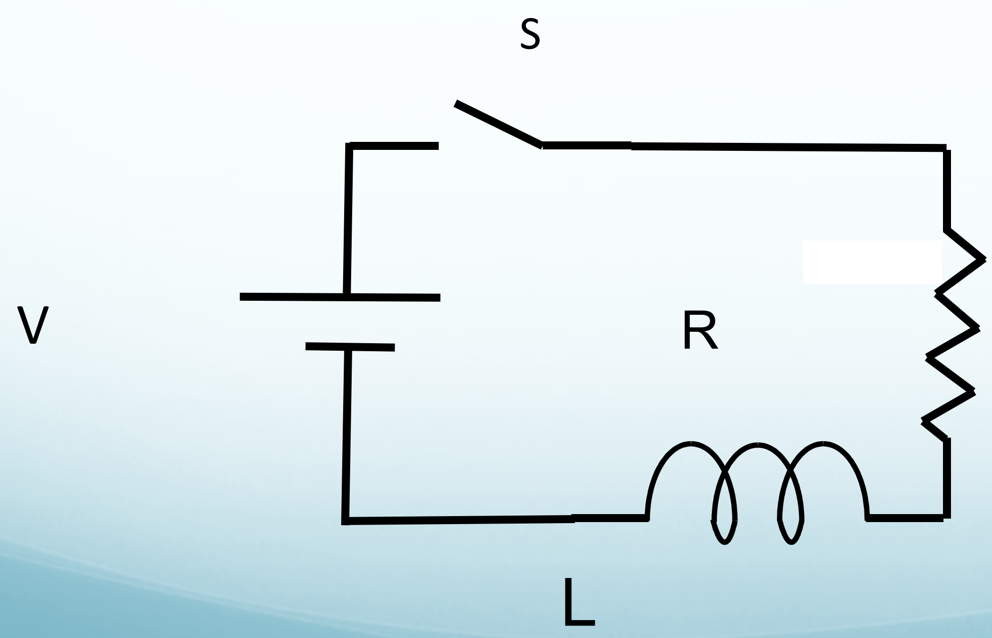
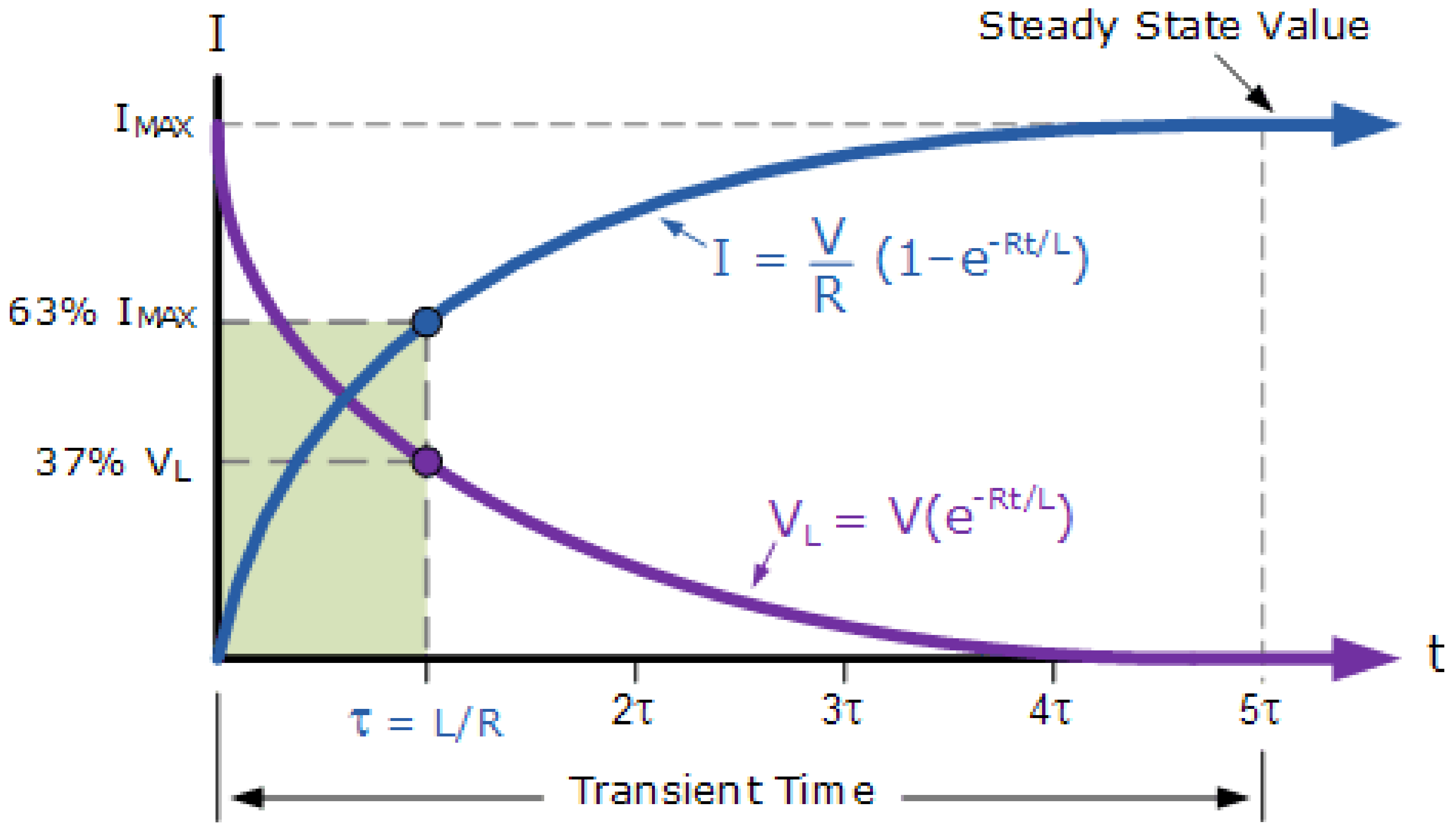
At t=0 the switch S is closed.

Using the loop rules $$V - iR - L\frac{di}{dt} = 0$$

Solving using the method we used for the charging capacitor



$$i(t) = i_{\max}\left(1 - e^{-\left(\frac{Rt}{L}\right)}\right)$$



https://tinyurl.com/msfn7b5

Top Hat Question

The switch in the series circuit below is closed at t=0.

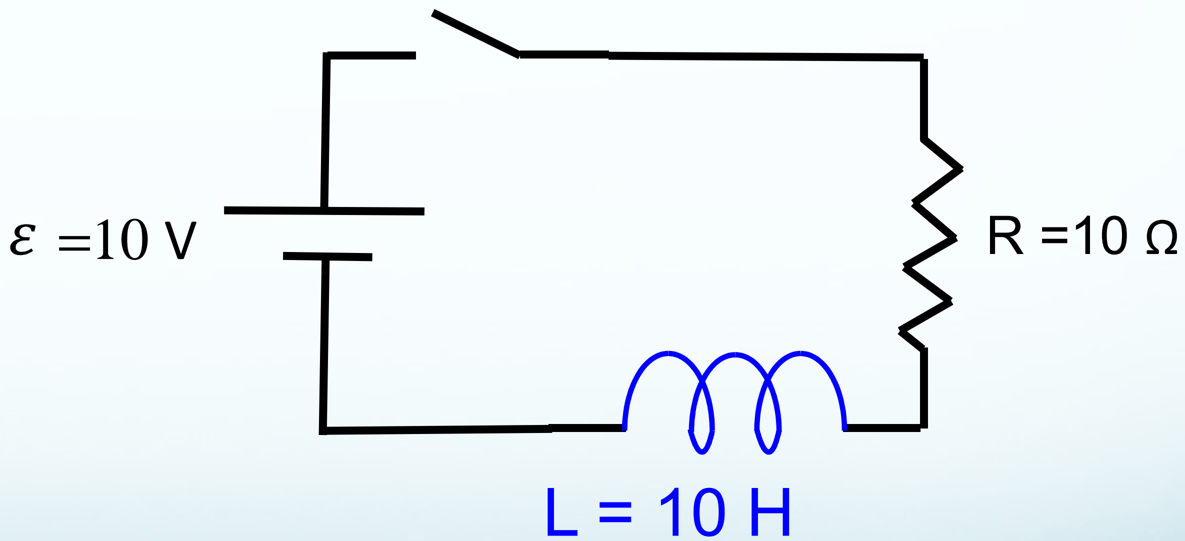
What is the initial rate of change of current di/dt in the inductor, immediately after the switch is closed (time = 0+) ?

A. 0 A/s

B. 0.5 A/s

C. 1 A/s

D. 10 A/s



V_R $\quad$ $V_L = L\,di/dt$

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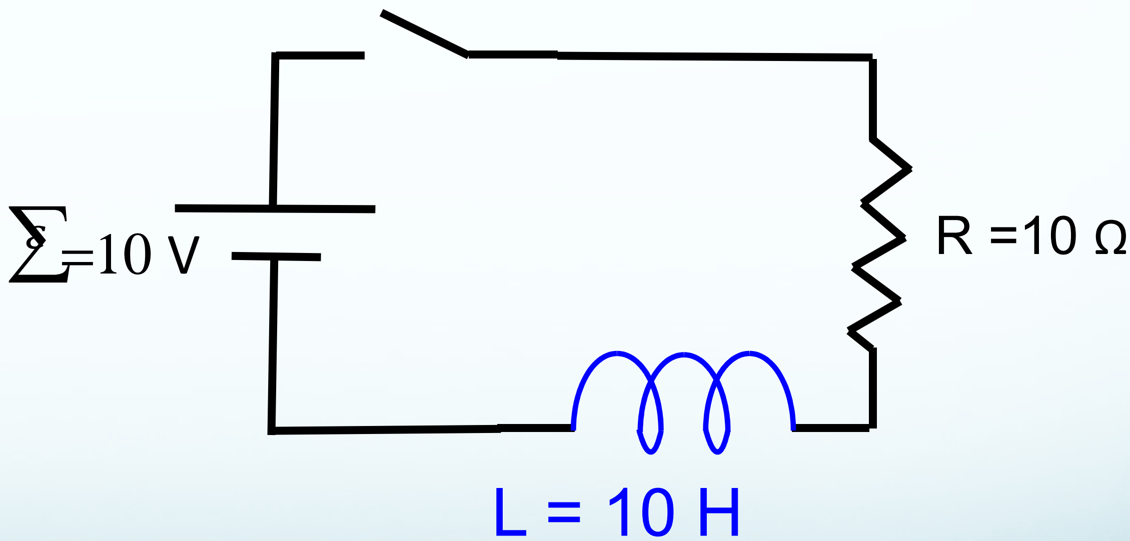
Top Hat Question

The switch in the series circuit below is closed at t=0.

What is the initial rate of change of current di/dt in the inductor, immediately after the switch is closed (time = 0+) ?



A. 0 A/s

B. 0.5 A/s

C. 1 A/s

D. 10 A/s

i = 0 at t = 0 so V_R(0)= 0 which means

10 V = V_L= Ldi/dt so di/dt = 10V/10H = 1 A/s

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Top Hat Question

The switch in the series circuit below is closed at t=0.

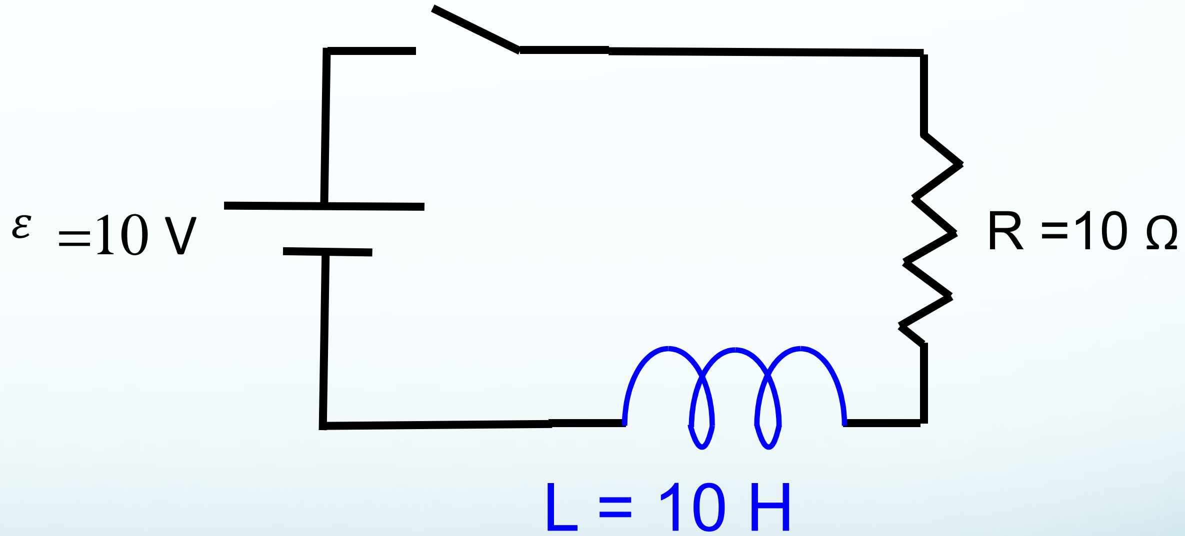
What is the current in the circuit after a time t = 3.0 s?

A. 0 A

B. 0.63 A

C. 0.86 A

D. 0.95 A



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Top Hat Question

The switch in the series circuit below is closed at t=0.

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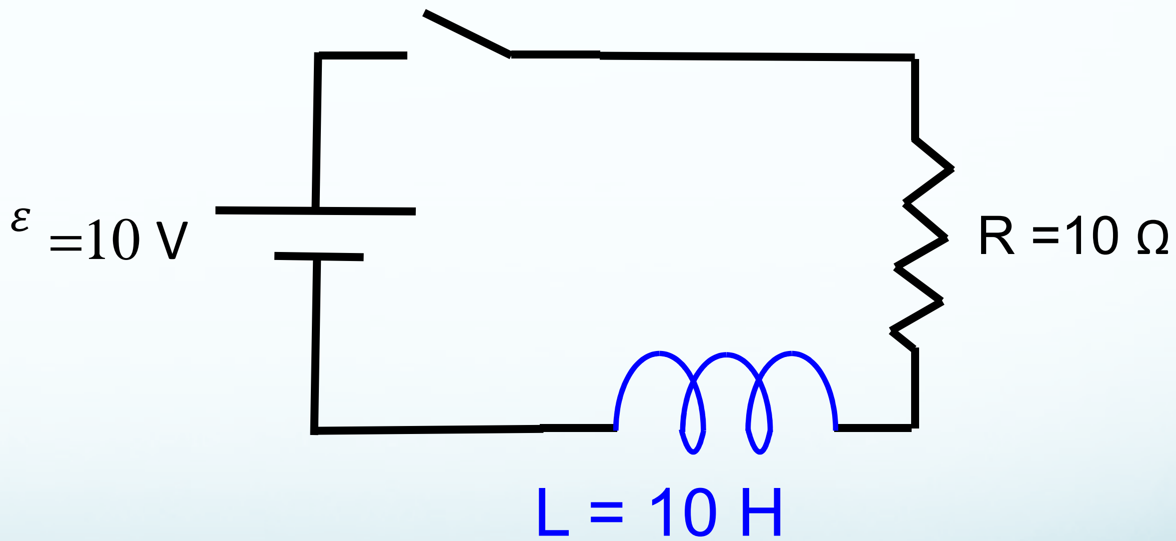

$$i(3\text{s}) = \frac{10V}{10\Omega}\left(1 - e^{-3}\right)$$

This section we talked about:

Chapter 30

See you on Monday