Announcements

- Complete Assignment #2 before **11:59 pm, Wednesday, January 25**.
- Go on D2L to see solutions and feedback on your solution for activity #1.
- Laboratorials start next week.

Last time

- Coulomb’s force due to a line charge, making approximations
- Coulomb’s force due to a line charge, exact solution

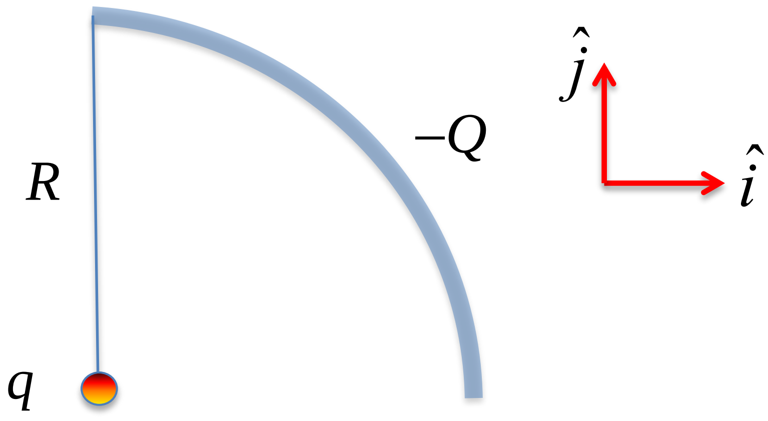
This time

- Coulomb’s force due to a 90° arc of radius R at its center
- Coulomb’s force due to a 180° arc of radius R at its center.
- Activity #2

Electric force due to an arc at its center

Consider a arc of radius *R* with the total charge of -*Q* and a uniform charge distribution.

Compute the force due to the arc on a point charge *q* located at the center of the arc.



solution

Divide the arc into an infinite number of pieces each with a small amount of charge, then calculate the force for each and add up all the forces.

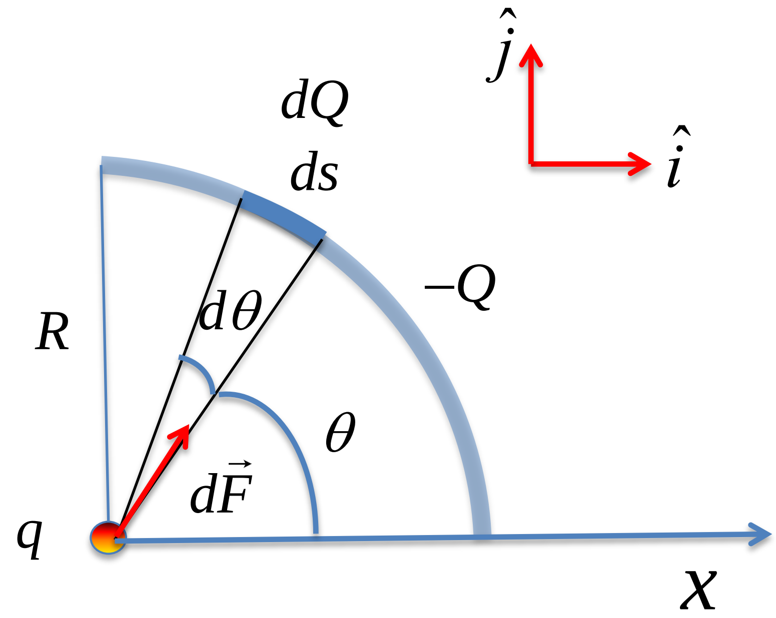
Electric force due to an arc at its center

Charge per unit length: $\lambda = \dfrac{|-Q|}{\pi R/2}$

ds Infinitesimal arc length

$d\theta$ Infinitesimal angle subtended by ds

dQ Infinitesimal charge on ds

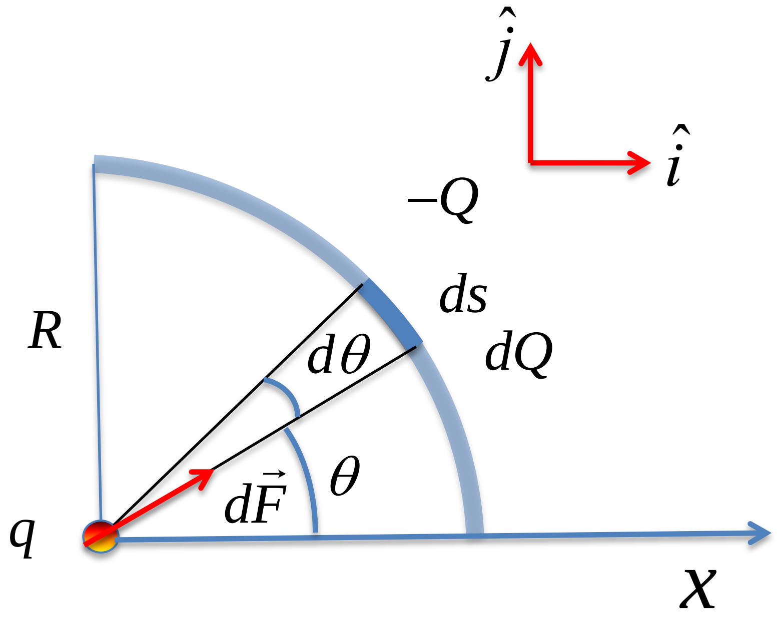


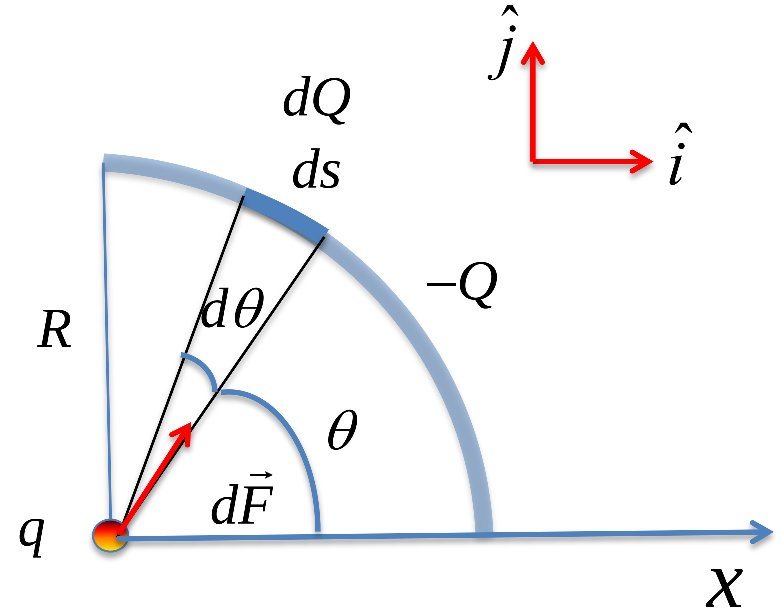
If ds is infinitely small then dQ is infinitely small and can be considered a point charge.

$$dQ = \lambda ds$$

$$d\vec{F} = \frac{k_e q dQ}{R^2}\hat{r} = \frac{k_e q \lambda R d\theta}{R^2}\hat{r}$$

Electric force due to an arc at its center





$$dQ = \lambda ds$$

$$d\vec{F} = \frac{k_e q dQ}{R^2}\hat{r} = \frac{k_e q \lambda R d\theta}{R^2}\hat{r}$$

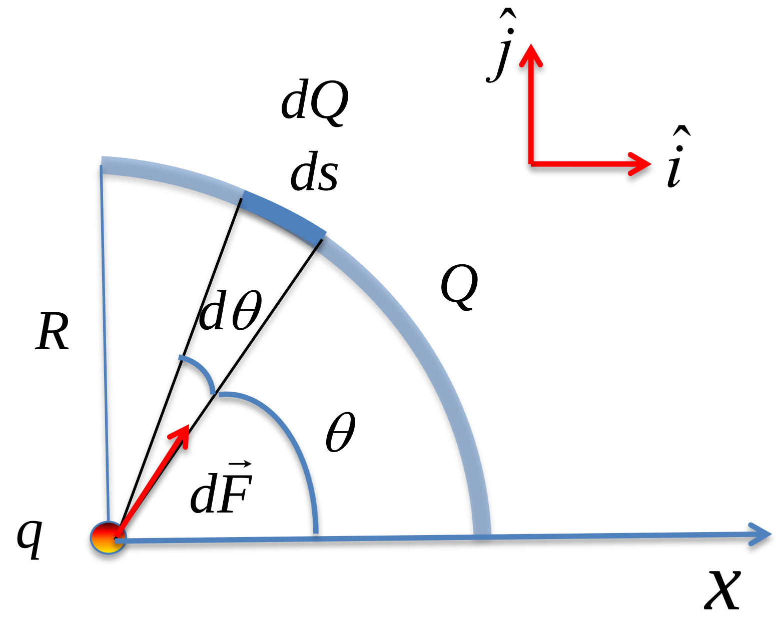
Electric force due to 45° arc at its center

Charge per unit length: $\lambda = \dfrac{|-Q|}{\pi R/2}$

$$d\vec{F} = \frac{k_e q dQ}{R^2}\hat{r} = \frac{k_e q \lambda R d\theta}{R^2}\hat{r}$$

$$dF_x = \frac{k_e q \lambda R d\theta}{R^2}\cos\theta$$

$$dF_y = \frac{k_e q \lambda R d\theta}{R^2}\sin\theta$$



$$F_x = \int dF_x = \int_0^{\pi/2} \frac{k_e q \lambda R d\theta}{R^2}\cos\theta = \frac{k_e q \lambda}{R}\int_0^{\pi/2}\cos\theta d\theta = \frac{k_e q\lambda}{R}\sin\theta\,\Big|_0^{\pi/2} = \frac{k_e q \lambda}{R} = \left(\frac{2}{\pi}\right)\frac{k_e q Q}{R^2}$$

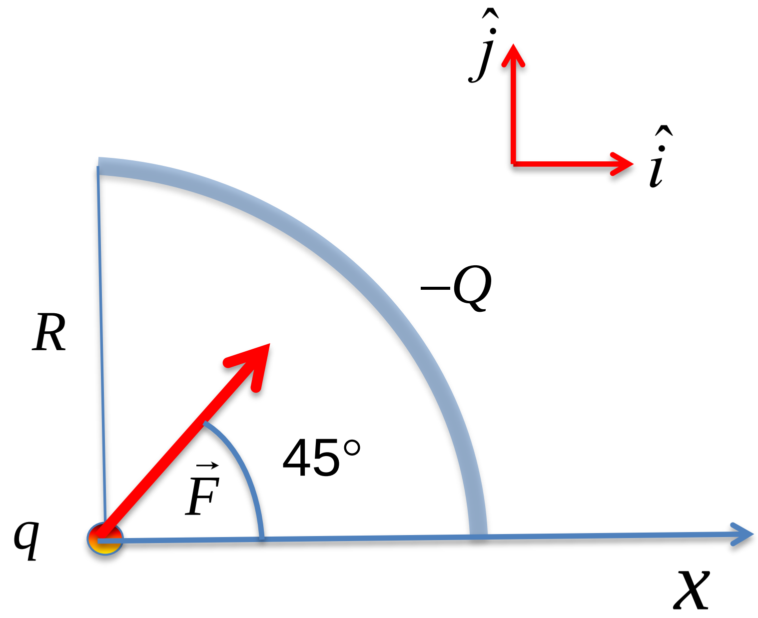
$$F_y = \int dF_y = \int_0^{\pi/2} \frac{k_e q \lambda R d\theta}{R^2}\sin\theta = \frac{k_e q \lambda}{R}\int_0^{\pi/2}\sin\theta d\theta = -\frac{k_e q\lambda}{R}\cos\theta\,\Big|_0^{\pi/2} = \frac{k_e q \lambda}{R} = \left(\frac{2}{\pi}\right)\frac{k_e q Q}{R^2}$$

Electric force due to 45° arc at its center

$$F_x = \left(\frac{2}{\pi}\right)\frac{k_e qQ}{R^2}$$

$$F_y = \left(\frac{2}{\pi}\right)\frac{k_e qQ}{R^2}$$

$$\vec{F} = \frac{2}{\pi}\frac{k_e qQ}{R^2}\left(\hat{i} + \hat{j}\right)$$



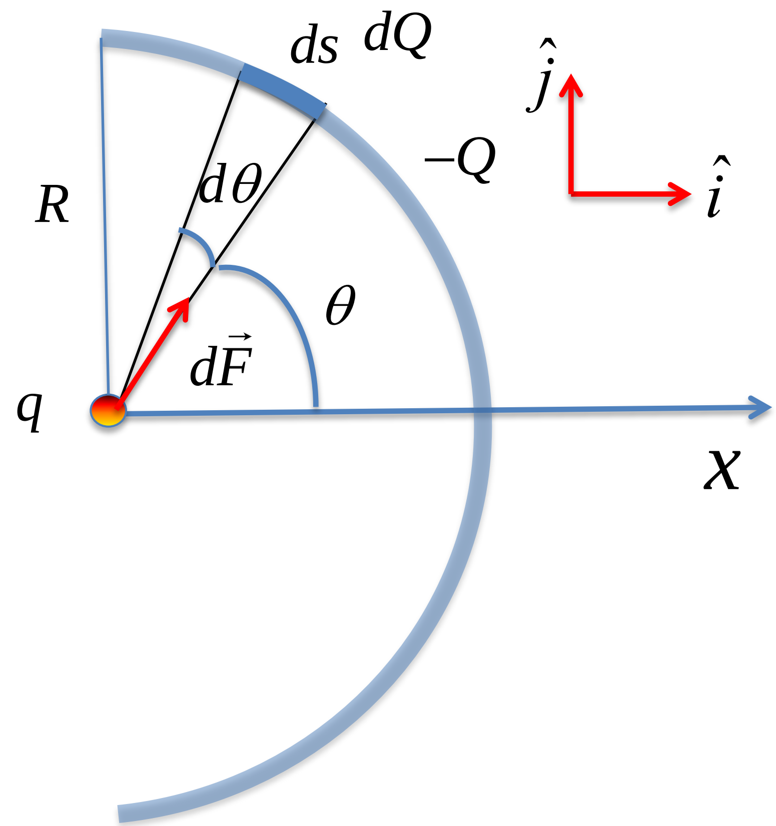
Electric force due to 90° arc at its center

$$\lambda = \frac{|-Q|}{\pi R}$$

$$d\vec{F} = \frac{k_e q dQ}{R^2}\hat{r} = \frac{k_e q \lambda R d\theta}{R^2}\hat{r}$$

$$dF_x = \frac{k_e q \lambda R d\theta}{R^2}\cos\theta$$

$$dF_y = \frac{k_e q \lambda R d\theta}{R^2}\sin\theta$$



$$F_x = \int dF_x = \int_{-\pi/2}^{\pi/2}\frac{k_e q \lambda R d\theta}{R^2}\cos\theta = \frac{k_e q \lambda}{R}\int_{-\pi/2}^{\pi/2}\cos\theta d\theta = \frac{k_e q \lambda}{R}\sin\theta\,\Big|_{-\pi/2}^{\pi/2} = \frac{2k_e q \lambda}{R} = \left(\frac{2}{\pi}\right)\frac{k_e q Q}{R^2}$$

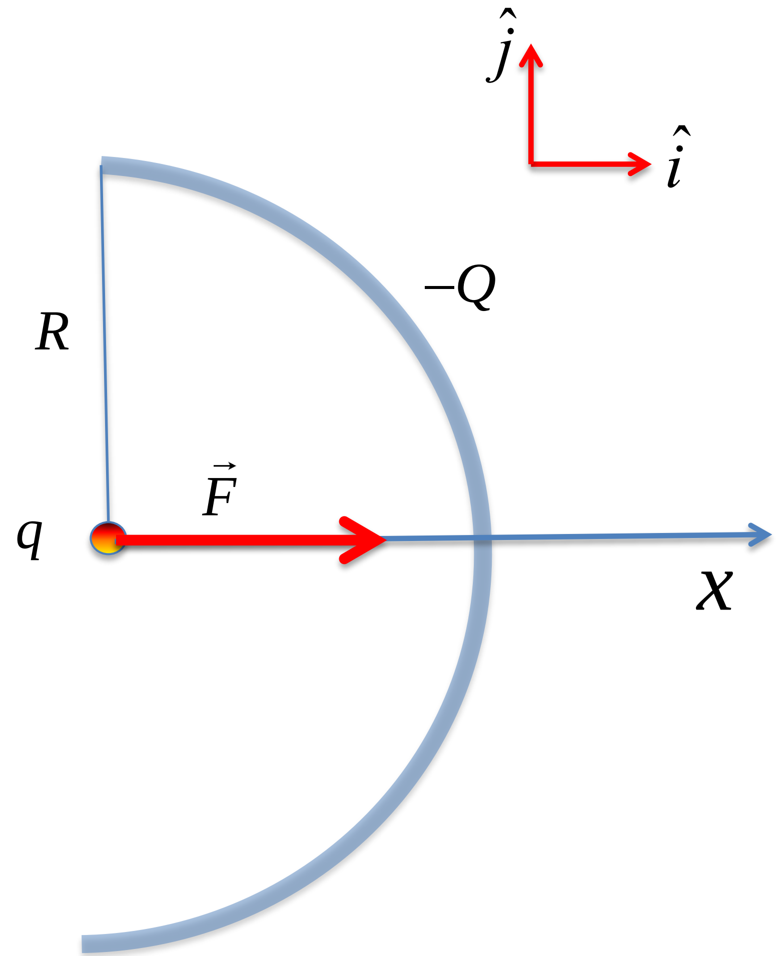
$$F_y = \int dF_y = \int_{-\pi/2}^{\pi/2}\frac{k_e q \lambda R d\theta}{R^2}\sin\theta = \frac{k_e q \lambda}{R}\int_{-\pi/2}^{\pi/2}\sin\theta d\theta = -\frac{k_e q \lambda}{R}\cos\theta\,\Big|_{-\pi/2}^{\pi/2} = 0$$

Electric force due to 90° arc at its center

$$F_x = \left(\frac{2}{\pi}\right)\frac{k_e qQ}{R^2}$$

$$F_y = 0$$

$$\vec{F} = \frac{2}{\pi}\frac{k_e qQ}{R^2}\hat{i}$$

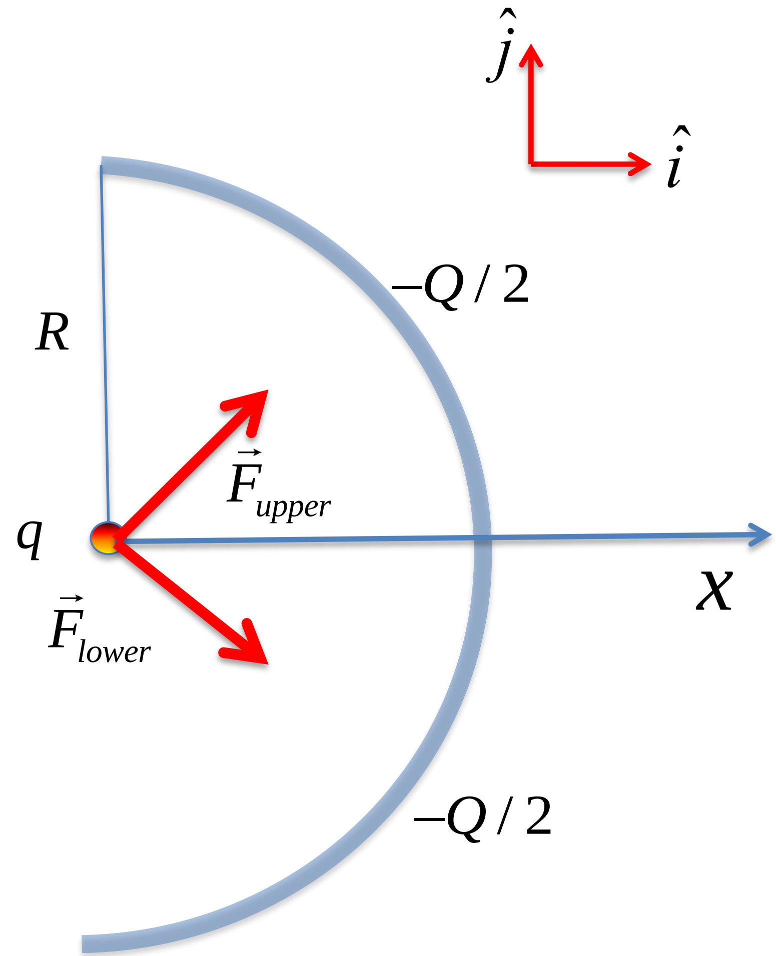


Electric force due to 90° arc at its center

Or divide the arc into two 45° arcs and use the superposition principle.

$$\vec{F}_{upper} = \left(\frac{2}{\pi}\right)\frac{k_e qQ/2}{R^2}(\hat{i}+\hat{j})$$

$$\vec{F}_{lower} = \left(\frac{2}{\pi}\right)\frac{k_e qQ/2}{R^2}(\hat{i}-\hat{j})$$



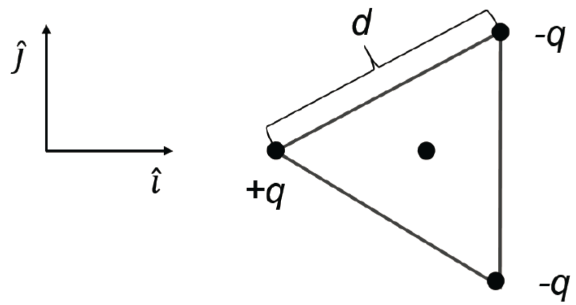
SYMMERTY ABOUT X-AXIS!

$$F_x = \frac{2}{\pi}\frac{k_e qQ}{R^2} \qquad F_y = 0$$

$$\vec{F} = \frac{2}{\pi}\frac{k_e qQ}{R^2}\hat{i}$$

Group #	Student	Last Name	First Name
	1		
	2		
	3		
	4		

(10 marks) Three charges are placed in the corners of the equilateral triangle with side-length d as shown on the diagram below. All charges are equal in magnitude, and their sign is indicated on the diagram below. What is the net force (magnitude and direction) on the positive charge of the same magnitude located in the center of the triangle due to the other three charges? Note: the outer charges are placed equidistant from the center charge.



The parts below walk you through related questions, and the steps with which to solve this problem. Please show all work in the boxes provided and then choose the correct answer at the bottom

1. **(1.5 marks)** On the diagram above draw the distances r for the different Coulomb's law pairs that are needed to solve this question (0.5 mark). From properties of 30-60-90 triangles find the value of the distances as a function of d (1 mark).

2. **(2 marks)** On the diagram above, draw and label the forces on the central charge due to each surrounding charge (2 marks). Make sure that the correct magnitude and direction are accurately portrayed (0.5 mark). Indicate any angles needed for find the components of the forces (0.5 marks).

3. **(1.5 mark)** Will any of the force vector components cancel? (0.5) Explain (1 mark).

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4. **(1 mark)** Write a mathematical expression for the magnitude of the force from each of the charges on the central charge.

5. **(3 marks)** Using the principle of superposition, find the net force (magnitude and direction) on the central charge due to the surrounding charges (1 mark). Remember to show your work (2 marks).

(1 mark for the correct answer) Circle answer below:

A. $3\frac{kq^2}{d^2}\hat{\imath}$ B. $-3\frac{kq^2}{d^2}\hat{\jmath}$ C. $6\frac{kq^2}{d^2}\hat{\imath}$ D. $6\sqrt{3}\frac{kq^2}{d^2}\hat{\imath}$