Friday April 7, 2017

Last time:

- Lenz’s Law
- Non-conservative electric fields
- Motional emf
- Examples
- RL circuits

Today:

- Eddie currents demo
- Energy stored in a capacitor (reminder)
- Energy stored in an inductor
- Mutual inductance
- Transformers

Potential Energy in a Capacitor

Energy storage in terms of the charge on the plates:

$$\boxed{U = \frac{1}{2}\frac{Q^2}{C}}$$

Use the general relation for a capacitor to swap charge for voltage

$$Q = C\mathrm{D}V_C$$

Energy storage in terms of the voltage across the plates:

$$U = \frac{1}{2}\frac{\left(C\mathrm{D}V_C\right)^2}{C}$$

$$\boxed{= \frac{1}{2}C\left(\mathrm{D}V_C\right)^2}$$

Energy density

$$U = \frac{1}{2} C \left(\Delta V_C \right)^2$$

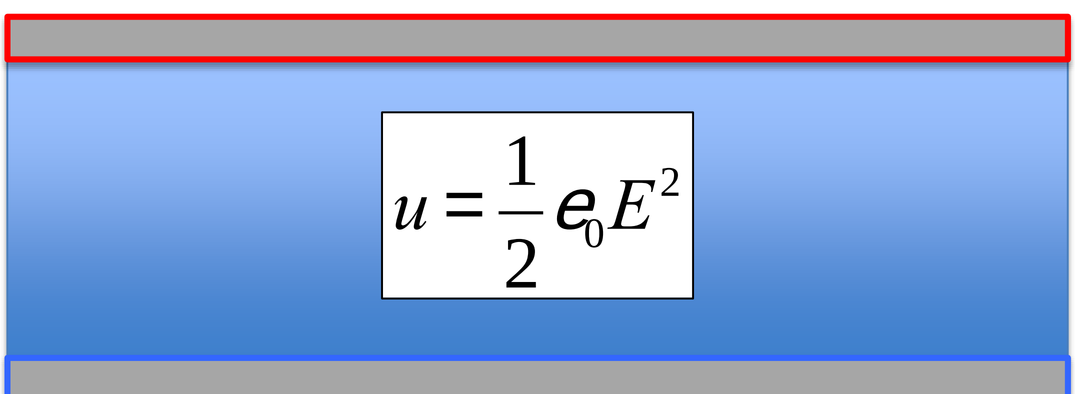
$$= \frac{1}{2} C E^2 d^2$$

$$= \frac{1}{2} \frac{\epsilon_0 A}{d} E^2 d^2 = \frac{1}{2} \epsilon_0 E^2 (Ad)$$

$$\Delta V = Ed$$

$$C = \frac{\epsilon_0 A}{d}$$

$$u = \frac{U}{Ad}$$



$$\boxed{u = \frac{1}{2} \epsilon_0 E^2}$$

The capacitor's energy is stored in the electric field between the plates!

Potential Energy density

- Potential energy

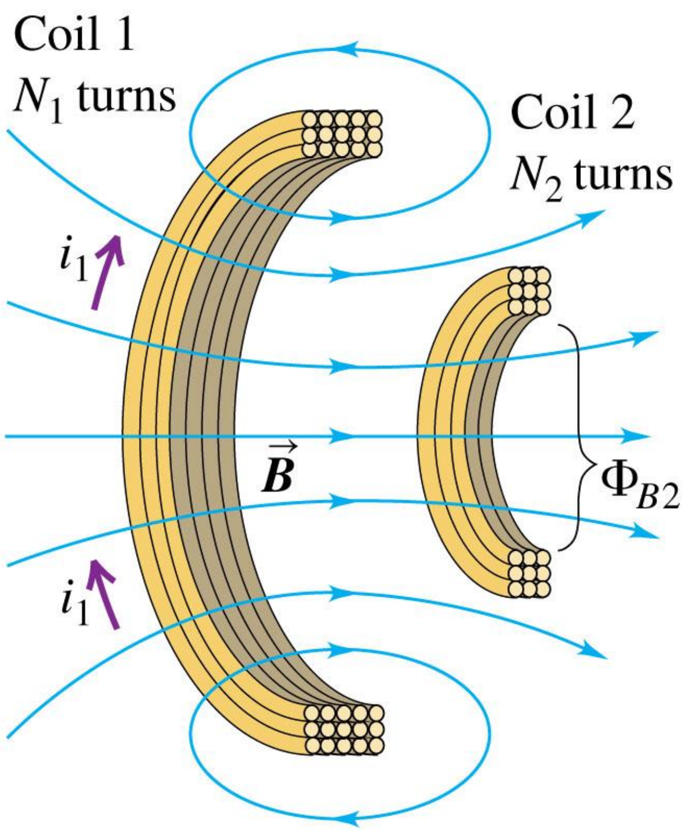
$$\boxed{U = \frac{1}{2} L I^2}$$

- For a solenoid: $\boxed{L = m_0 \frac{N^2}{\ell} A}$ $\quad N = nl$

$$u = \frac{U}{V} = \frac{1}{2V}\left(m_0 n N A\right) I^2 = \frac{1}{2m_0}\left(m_0^2 n^2 I^2\right)\frac{A\ell}{V} = \boxed{\frac{1}{2m_0} B^2}$$

Mutual Inductance

Mutual inductance: If the current in coil 1 is changing, the changing flux through coil 2 induces an emf in coil 2.



Induced EMF in coil 2:

$$\varepsilon_2 = -N_2 \frac{d\phi_{B2}}{dt} = -\frac{d\left(N_2 \phi_{B2}\right)}{dt}$$

Note: ϕ_{B2} is the magn. flux through a single loop of coil 2. N_2 is the number of loops.

The magnetic field in coil 2 is prop. to the current through coil 1:

$$d\mathbf{B} = \frac{\mu_0}{4\pi} \frac{i_1 d\mathbf{l} \times \mathbf{r}}{r^2}$$

Biot-Savart

Hence, the magnetic flux through coil 2 is proportional to i_1:

$$N_2 \phi_{B2} = M_{21} i_1$$

and

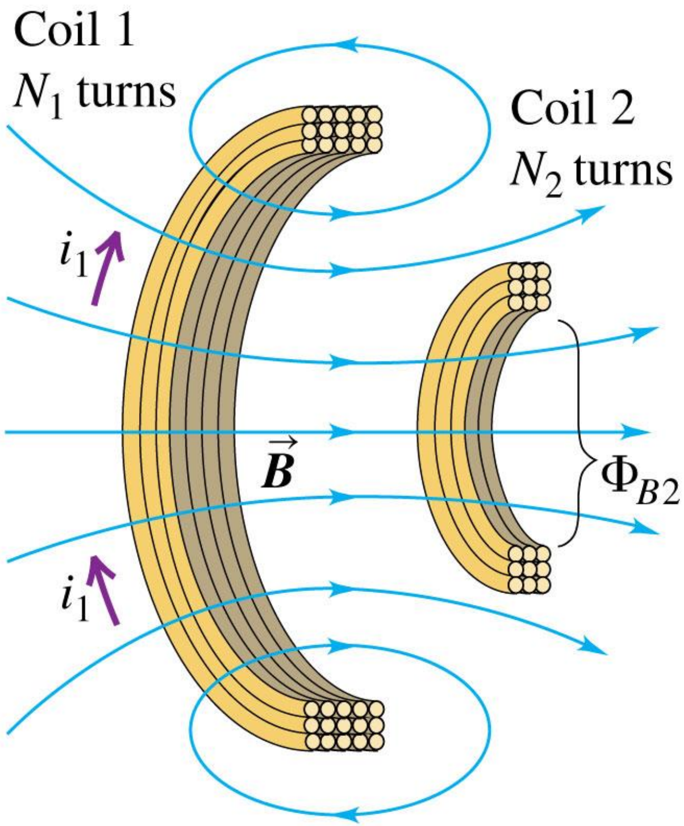
$$\varepsilon_2 = -\frac{d\left(N_2 \phi_{B2}\right)}{dt} = -M_{21} \frac{di_1}{dt}$$

M_{21}: *mutual inductance*, depends on the geometry of the two coils

Mutual Inductance

$$N_2 f_{B2} = M_{21} i_1$$

Mutual inductance: If the current in coil 1 is changing, the changing flux through coil 2 induces an emf in coil 2.



$$e_2 = -\frac{d(N_2 f_{B2})}{dt} = -M_{21}\frac{di_1}{dt}$$

Similarly, if current flows through coil 2:

$$e_1 = -\frac{d(N_1 f_{B1})}{dt} = -M_{12}\frac{di_2}{dt}$$

One can show that M_{21}=M_{12}, hence

$$e_2 = -M\frac{di_1}{dt} \qquad e_1 = -M\frac{di_2}{dt}$$

(mutually induced EMF)

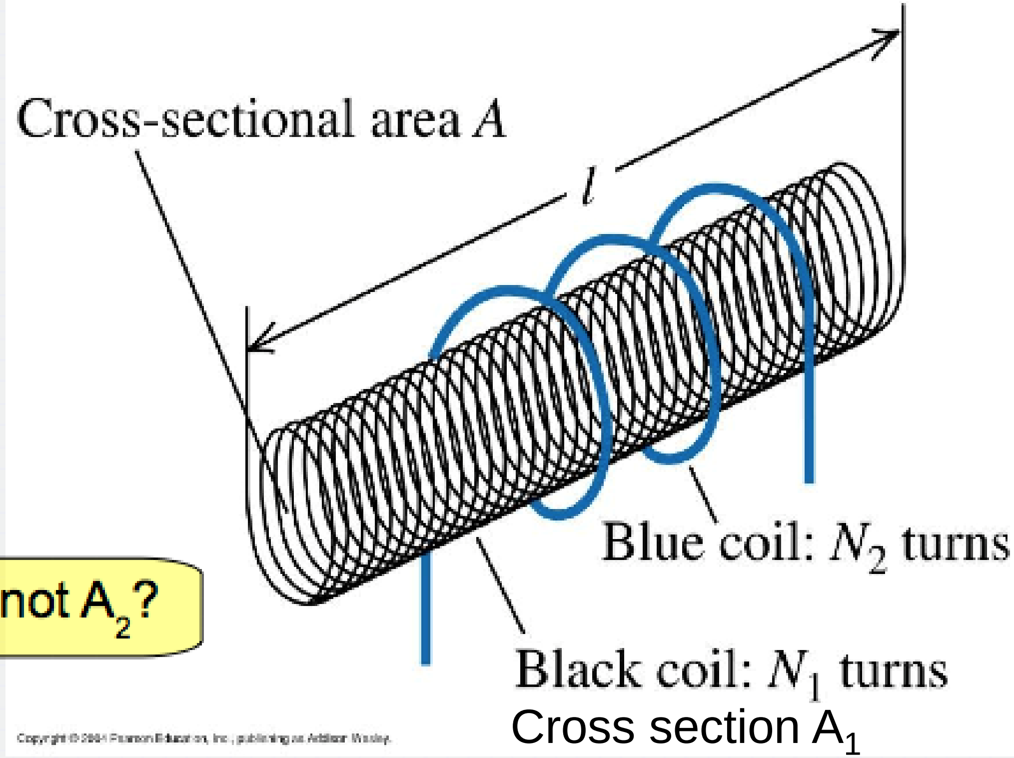
$$M = \frac{N_2 f_{B2}}{i_1} = \frac{N_1 f_{B1}}{i_2}$$

(mutual inductance, <u>can be calculated either way</u>)

[M]=1H =1Wb/A=1Vs/A=1Ωs=1J/A^2. Typical values: M=μH-mH

Example – Mutual inductance

The long solenoid will produce a magnetic field that is proportional to the current I_1 and the number of turns per unit length n_1



$$B_1 = \frac{\mu_0 N_1 I_1}{L} = \mu_0 n_1 I_1$$

and the total flux through each loop of the outer coil is

$$\Phi_{B2} = B_1 A_1$$

Why not A_2?

so the mutual inductance is

$$M = \frac{N_2 \Phi_{B2}}{I_1} = \frac{N_2 (B_1 A_1)}{I_1} = \frac{\mu_0 A_1 N_1 N_2}{L}$$

does not depend on I!

For a 0.5m long coil with 10cm^2 area and N_1=1000, N_2=10 turns

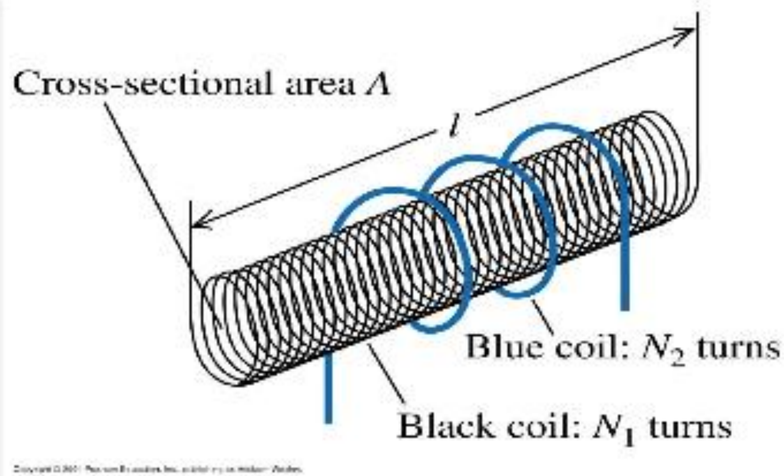
$$M = \frac{(4\pi\times10^{-7}\,T\,m/A)(1.0\times10^{-3}\,m^2)(1000)(10)}{0.5\mathrm{m}} = 2.5\times10^{-6}\,H = 25\,\mu H$$

Example

If a rapidly increasing current is driven through the outer coil

$$i_2(t) = (2.0\times10^{6}\,A/s)\,t$$

what EMF will be induced in the inner coil?



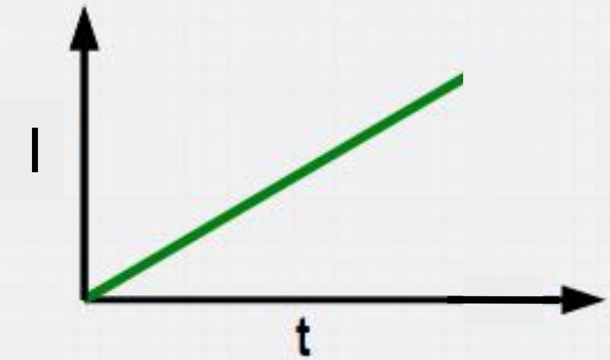
$$\mathcal{E}_1 = -M\frac{di_2}{dt}$$

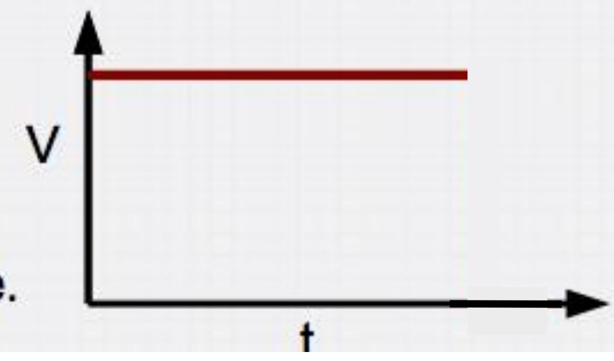
$$= -(25\times10^{-6}\,H)\,\frac{d}{dt}\left[(2.0\times10^{-6}\,A/s)\,t\right]$$

$$= -(25\times10^{-6}\,H)(2.0\times10^{-6}\,A/s)$$

$$= -50\,V$$

Note: M also allows cal-culating ε_2 if I_1 changes





This allows electrical energy in one circuit to be converted to electric energy in a separate device.

Question

16. The diagram below shows two nested, circular coils of wire. The larger coil has radius a and consists of N_1 turns. The smaller coil (radius b) consists of N_2 turns, and is both coplanar and coaxial with the larger coil. Assume b << a, so that the magnetic field of the larger coil is approximately uniform over the area of the smaller coil. The **mutual inductance** of this combination is given by the expression

a) $\dfrac{\mu_0 N_1 N_2}{2a}$.

b) $\dfrac{\pi\mu_0 N_1 N_2 b}{a}$.

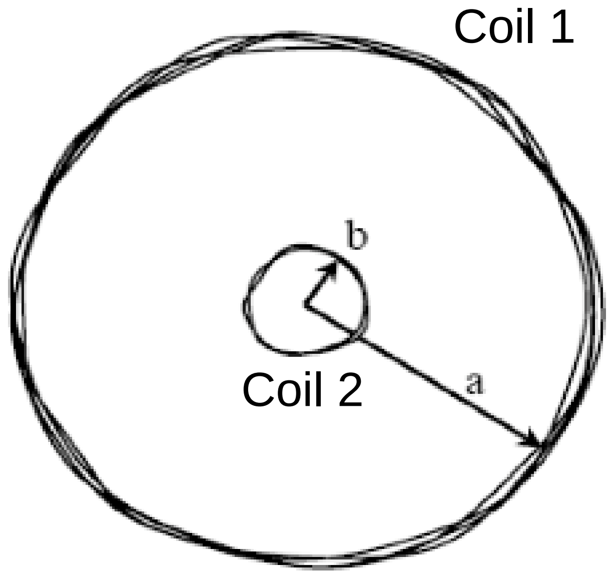
c) $\dfrac{\pi\mu_0 N_1 N_2 b^2}{2a}$.

d) $\dfrac{\mu_0 N_1 N_2 b^2}{2a}$.

e) $\dfrac{\pi\mu_0 N_2 b^2}{2a}$.

$$M = \frac{N_2 \Phi_{B2}}{i_1} = \frac{N_1 \Phi_{B1}}{i_2}$$

Flux through one loop of coil 2 (area A_2) due to magnetic field generated by current in coil 1



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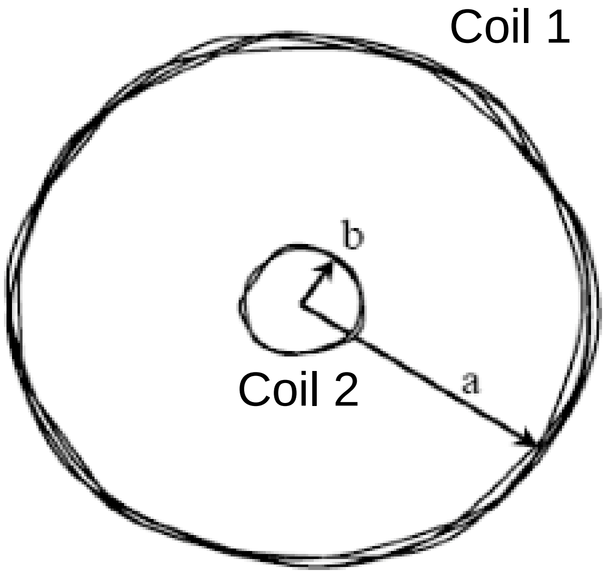
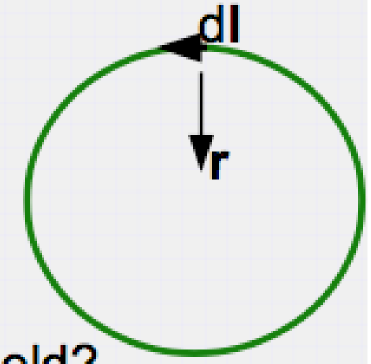


We assume current in the larger coil (coil 1), which generates a roughly uniform field in the area covered by the much smaller coil.

But how large is B?

Question

Calculate for one loop!



A circular loop of radius a carries a constant current I.
What is the magnetic field at the center of the loop?

What are the two methods we know for calculating magnetic field?
Biot-Savard law & Ampere's law.

Ampere's law isn't useful for a loop, so use the Biot-Savard law:

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{l}\times\hat{r}}{r^2} = \frac{\mu_0}{4\pi}\frac{I}{a^2}\,dl\,\hat{z}$$

$$\vec{B} = \int d\vec{B} = \frac{\mu_0}{4\pi}\frac{I}{a^2}\hat{z}\int dl = \frac{\mu_0}{4\pi}\frac{I}{a^2}(2\pi a)\hat{z} = \frac{\mu_0}{2}\frac{I}{a}\hat{z}$$

With the B-field direction directed out of the page, either from dl x **r**, or right thumb in direction of current and fingers curl in direction of **B**.

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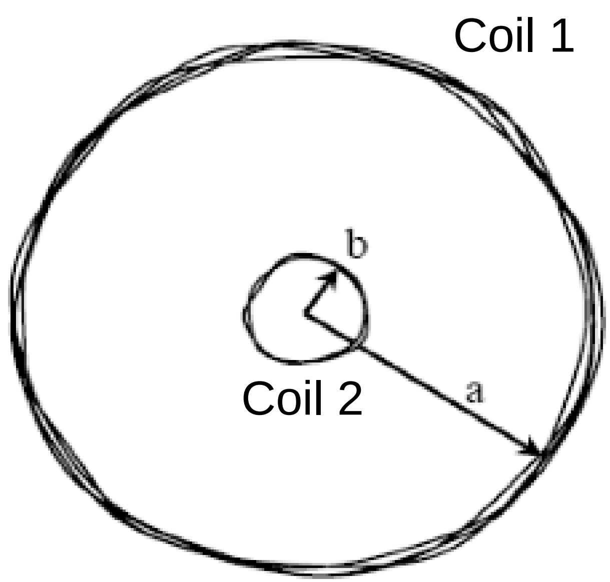

We assume current in the larger coil (coil 1), which generates a roughly uniform field in the area covered by the much smaller coil. Hence,

$$M = \frac{N_2 \Phi_{B2}}{i_1} = \frac{N_2}{i_1} \underbrace{N_1 \frac{\mu_0 i_1}{2a}}_{B_1} \underbrace{\pi b^2}_{A_2} = \mu_0 N_1 N_2 \frac{\pi b^2}{2a}$$

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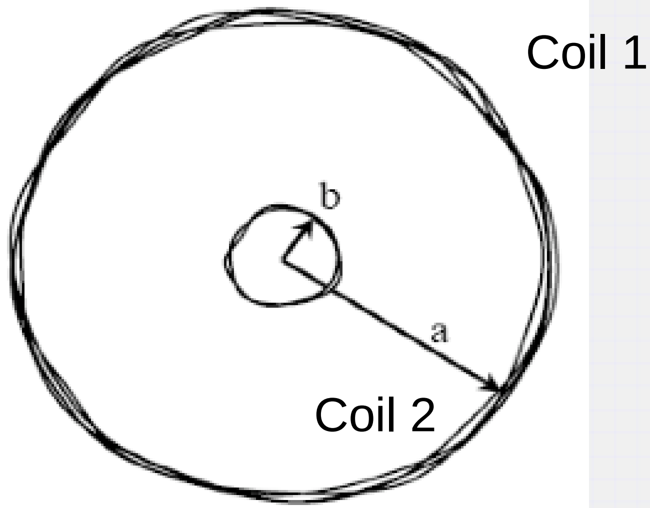
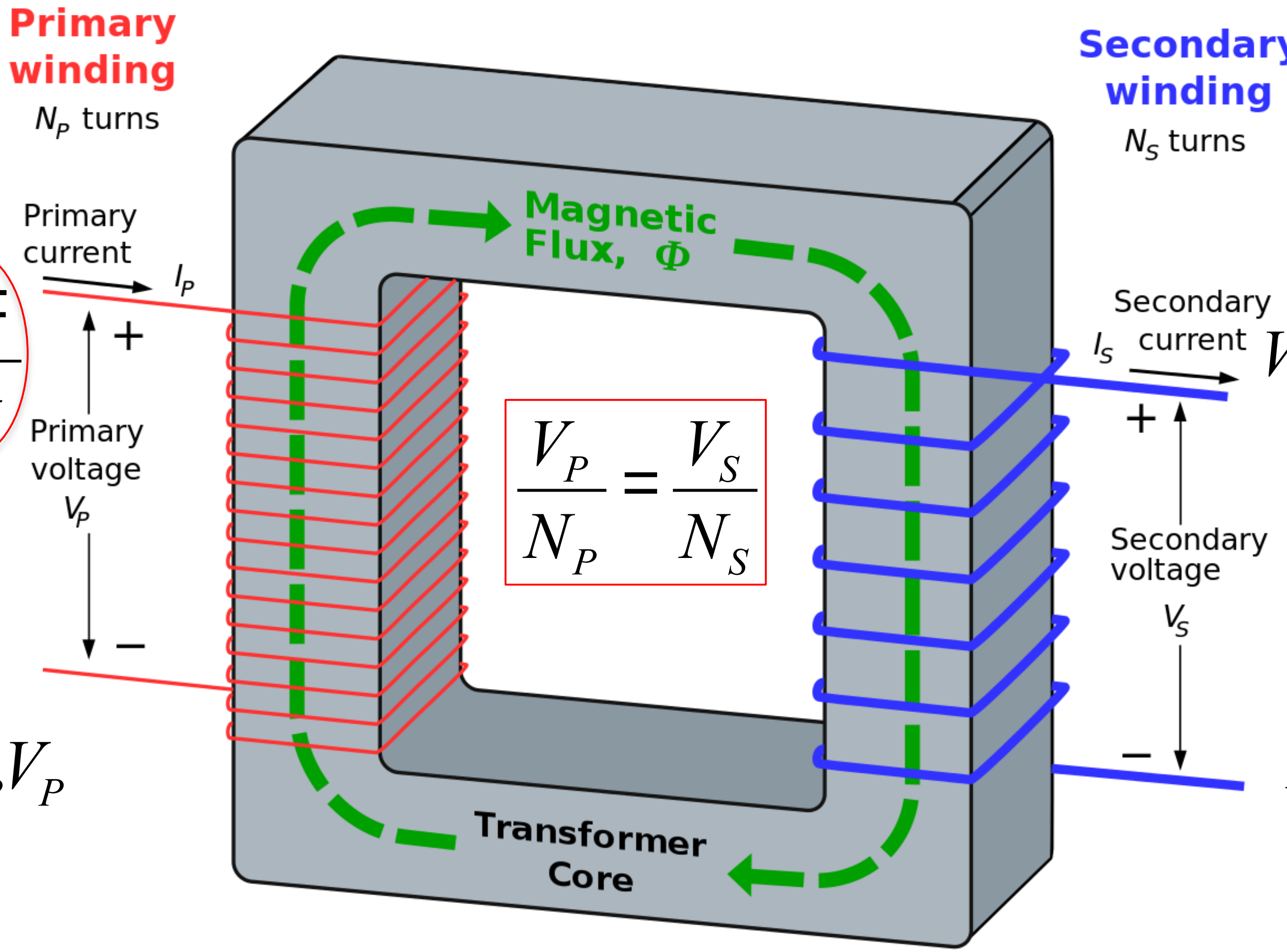


We expect the result to be proportional to the area of the coil that sees the field of the other coil, i.e. πb^2. Furthermore, we expect a dependence on N_1 and N_2: the field depends on N_1, and the flux on N_2. This leaves only answer c).

Transformers

$$V_P = N_P \frac{d\mathrm{F}}{dt}$$



$$\boxed{\frac{V_P}{N_P} = \frac{V_S}{N_S}}$$

$$V_S = N_S \frac{d\mathrm{F}}{dt}$$

$$P_{in} = I_P V_P$$

$$P_{out} = I_S V_S$$

$$P_{in} = P_{out}$$

$$I_P V_P = I_P \frac{N_P}{N_S} V_S = I_S V_S$$

$$\boxed{I_P N_P = I_S N_S}$$

Top Hat Question

Top Hat Question

The transformer for your laptop (the adaptor) has an output voltage of 18.5V. Your laptop uses about 85W of energy. The adaptor uses a step down transformer – what is the ratio of turns, primary to seconday, N_P/N_S ?

a) 0.065

b) 0.65

c) 6.5

d) 65

That’s all for content!

Monday’s class: Review