Wednesday March 15, 2017

Last time:

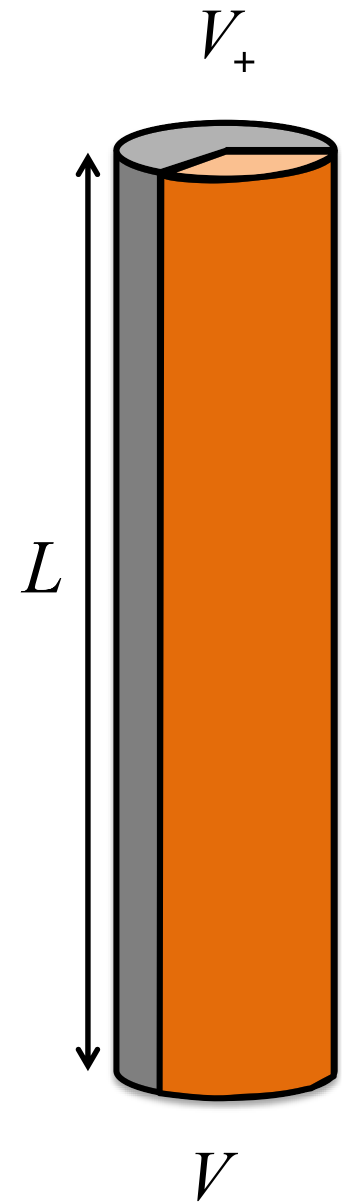
- Electric current: a microscopic picture
- Current density (a vector) vs current (a scalar)
- Electric fields in conductors and electron drift speed
- Resistance as geometric quantity
- Resistors in series
- Resistors in parallel

Today:

- Ohmic vs non-ohmic materials
- Microscopic view of resistivity
- Temperature dependence of resistivity/resistance
- Ideal vs non-ideal batteries
- RC circuits (charging/discharging capacitors)

TopHat Question

A cylindrical resistor is composed of 1/3 copper and 2/3 Tungsten as shown (ρ_{Cu} = 1.69x10^{-8} Ωm, and ρ_W = 5.25x10^{-8} Ωm). The radius of the cylinder is R = 0.105 mm, and it is L = 5.26 cm long.

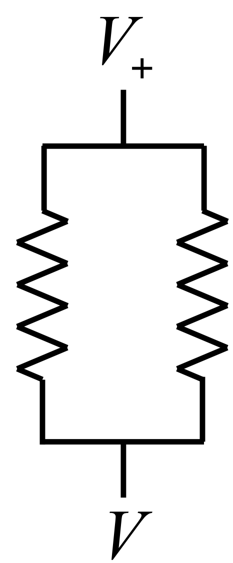


Each piece has a resistance. Are these resistors in series or in parallel?

A. Series

B. Parallel

C. Neither



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What is the total resistance of this device?

A. 0.0468 Ω

B. 0.197 Ω

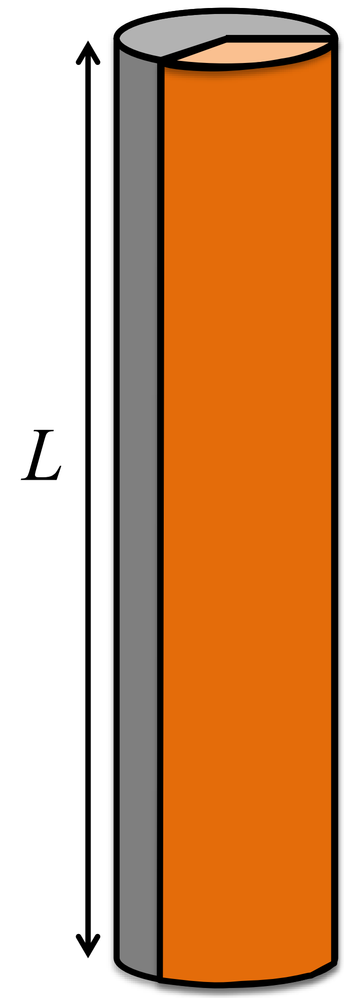
C. 0.105 Ω

D. 0.0729 Ω

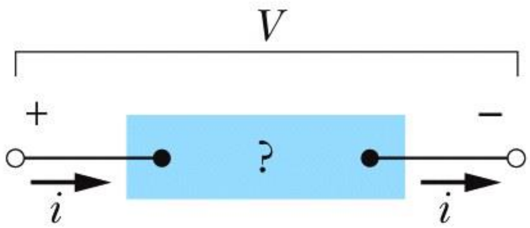
$$R = \left(\frac{1}{R_{Cu}} + \frac{1}{R_W} \right)^{-1}$$

$$R_{Cu} = \rho_{Cu} \frac{L}{\frac{1}{3}\pi R^2}$$

$$R_W = \rho_W \frac{L}{\frac{2}{3}\pi R^2}$$



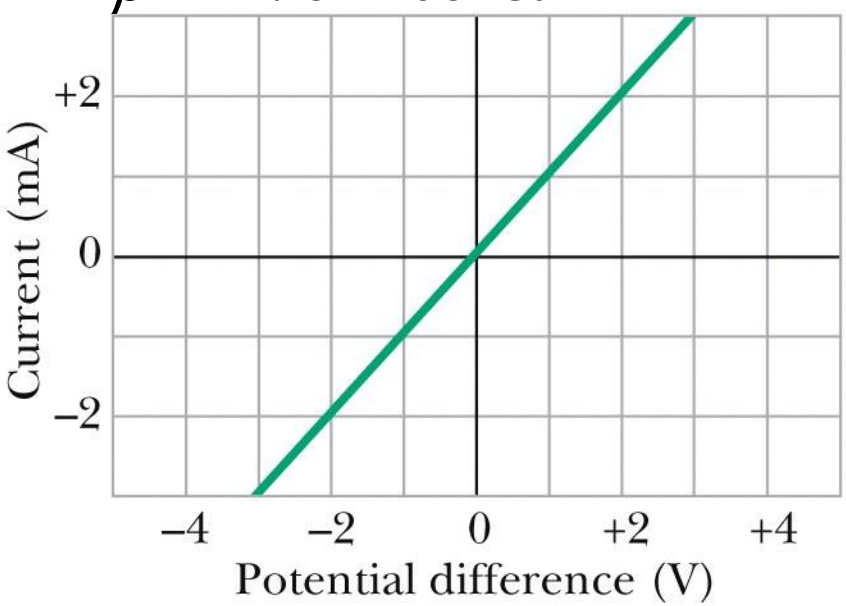
Ohmic vs non-Ohmic devices



(a)

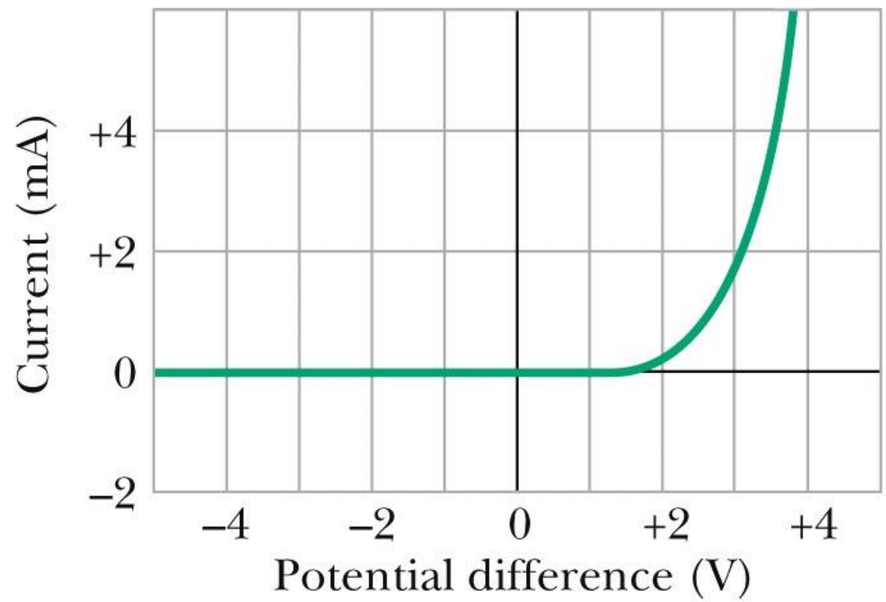
$$\rho = E / J = const$$

Materials with isotropic electrical properties



(b)

Materials with anisotropic electrical properties (pn junction diode)



(c)

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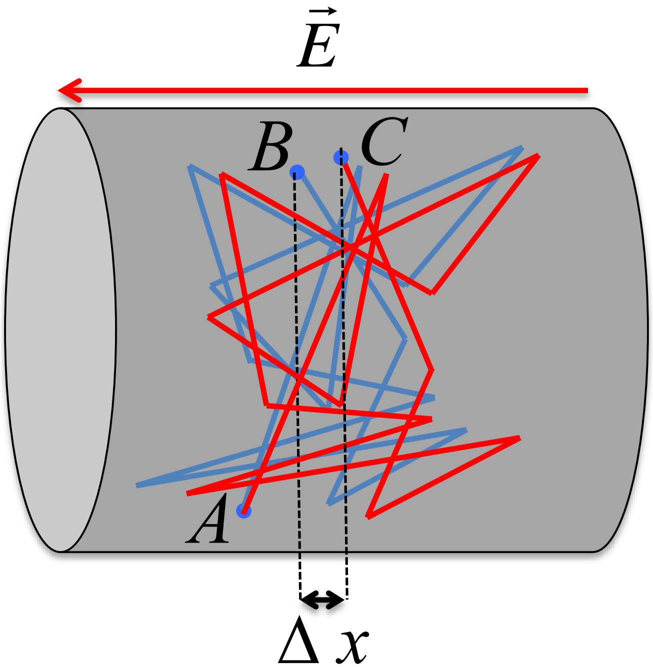
Microscopic view of resistivity



Electrons bounce around inside the metal at speeds very high speeds on the order of 0.5% light speed.

When an electric field is applied in the conductor, there is a net force on the electrons, leading to an average "drift speed" of $v_d = 0.5\ \mu\text{m/s}$

The acceleration felt by the electrons from the E-field is

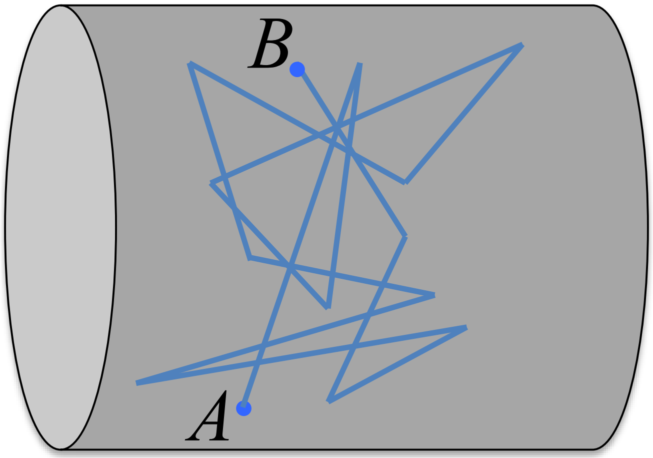
$$a_x = \frac{eE}{m}$$

So the average drift speed of the electrons will be given by

$$v_d = at = \frac{eEt}{m}$$ but we found before: $$v_d = \frac{J}{ne}$$

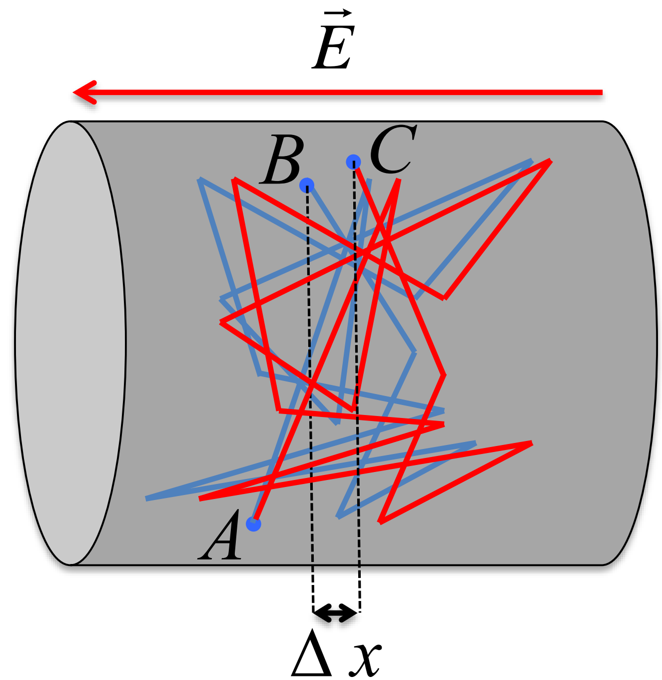
Microscopic view of resistivity

No E-field applied



$$v_d = \frac{eE\tau}{m}$$

$$v_d = \frac{J}{ne}$$



The average time between collisions is τ and is called the *mean free time*. Equating the two expressions for the drift speed, we get:

$$\frac{eE\tau}{m} = \frac{J}{ne}$$

Rearrange this to find $E = \left(\frac{m}{ne^2\tau}\right)J$

This gives a microscopic picture of resistivity:

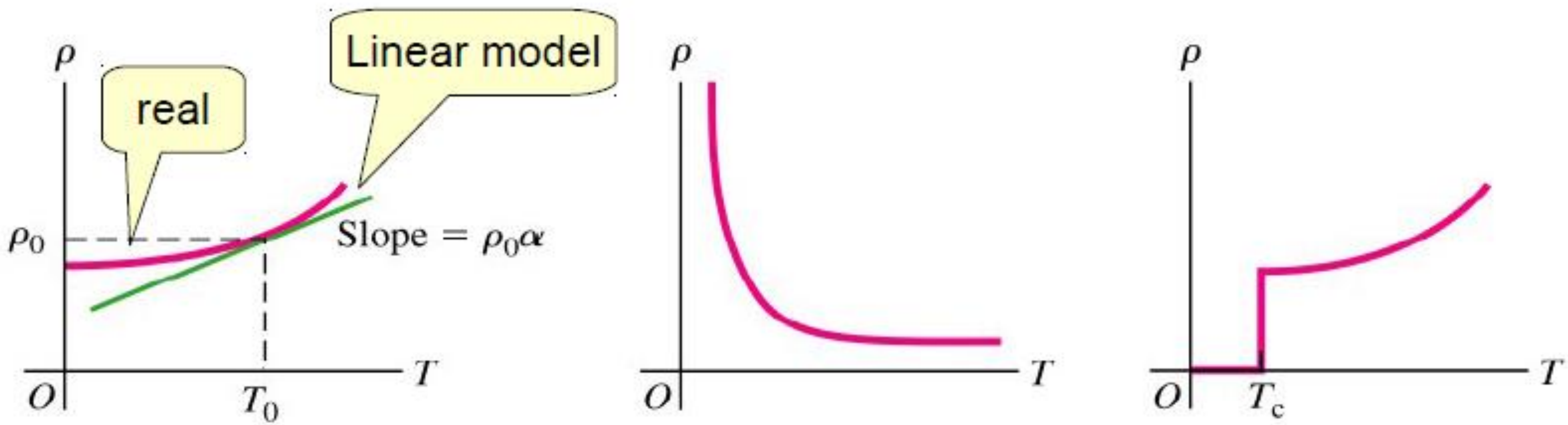
$$\boxed{\rho = \frac{m}{ne^2\tau}}$$

Resistivity and temperature

- As temperature increases, the ions in the conductor vibrate more, and electrons can only travel shorter distances before they have a collision.
- The resistivity of a metallic conductor tends to increase as it gets warmer.
- A simple linear model of this effect on resistivity is approximately valid over a small (100 C) temperature range.

$$r - r_0 = r_0 a \left(T - T_0\right)$$

Resistivity and temperature



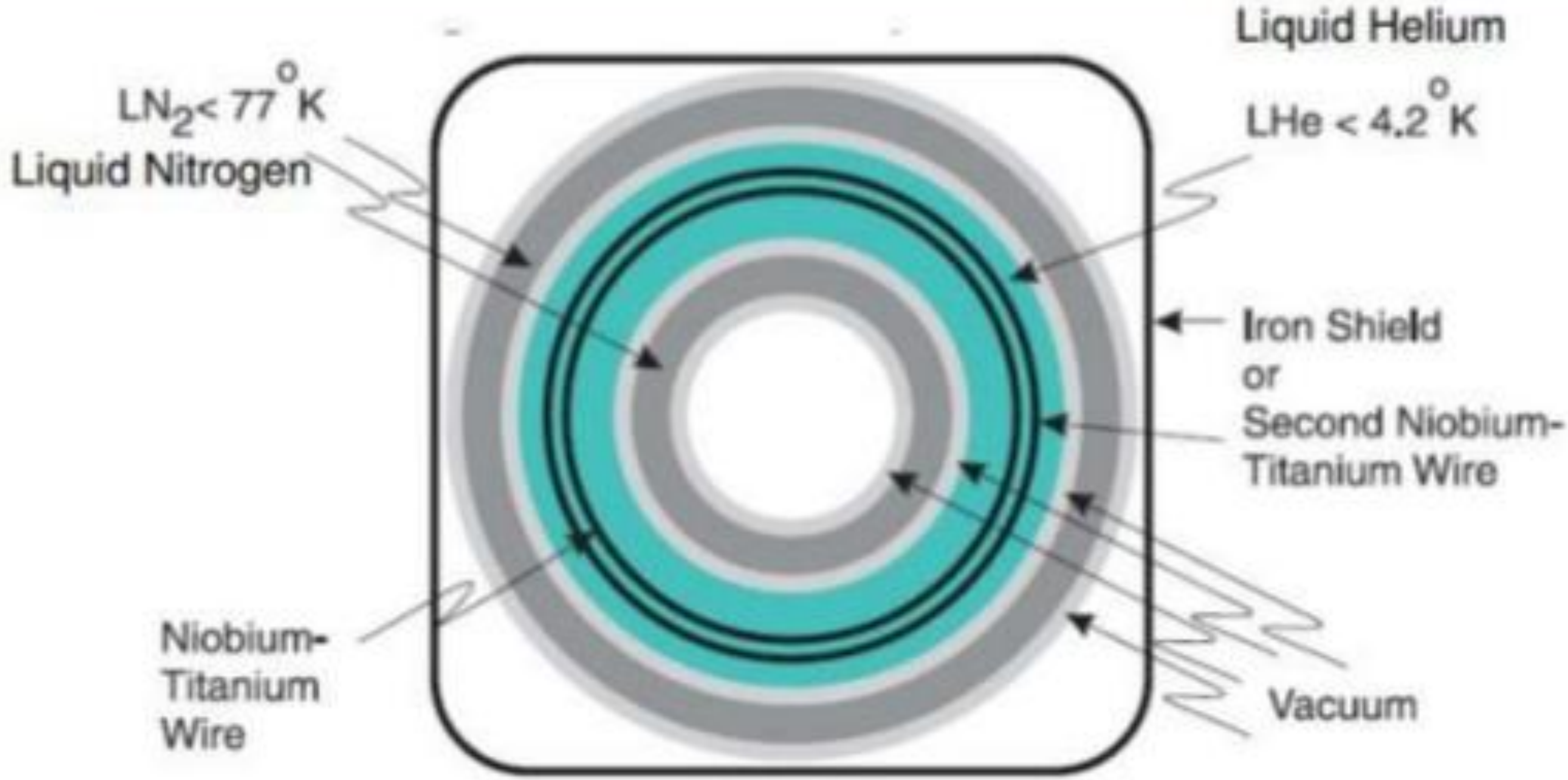
(a) Metal:
ρ increases
with increasing T

(b) Semiconductor:
ρ decreases
with increasing T

(c) Superconductor:
$\rho = 0$ for $T < T_c$

https://www.youtube.com/watch?v=2xOAb68agy0

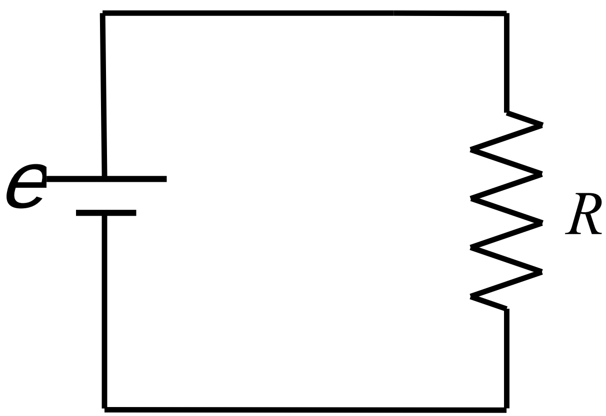
Superconducting Magnet

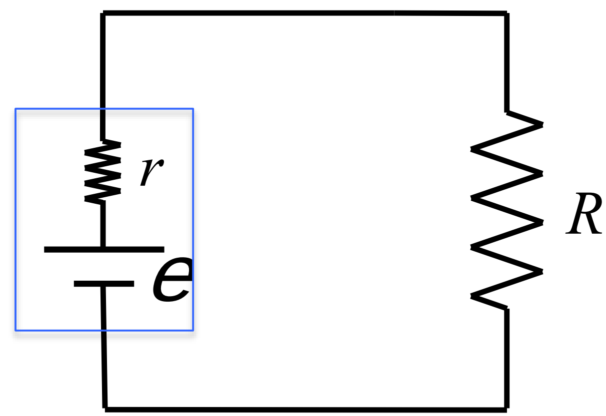


The length of superconducting wire in the magnet is typically several miles. The coil of wire is kept at a temperature of 4.2K by immersing it in liquid helium. The coil and liquid helium is kept in a large dewar. The typical volume of liquid Helium in an MRI magnet is 1700 liters. This dewar is surrounded by a liquid nitrogen (77.4K) dewar which acts as a thermal buffer between the room temperature (293K) and the liquid helium.

Non-ideal Batteries: internal resistance

Every voltage source has **some** internal resistance to it. Usually this can be ignored but not always





The internal resistance simply acts as a resistor in series with the rest of the circuit.

$$e - Ir - IR = 0$$

$$I = \frac{e}{(r+R)}$$

TopHat question

- A real battery with internal resistance r = 5.0 Ω is connected to a circuit with a resistive load of R = 150 Ω as shown in the diagram. If the voltage across the terminals of the battery (labeled A and B) is 12.0 V, what is the emf of the battery?

A. 12.4 V
B. 12.2 V
C. 12.0 V
D. 11.6 V