Electricity and Magnetism

- Physics 259 – L02
- Lecture 43

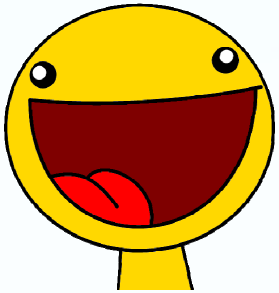
Chapter 29: Magnetic field due to current

Last time:

- Biot-Savart Law (like Coulomb’s Law for magnetism)
- B-field of a line of current
- Magnetic force between parallel current-carrying wires

Today:

- Ampere’s law
- Applications of Ampere’s law



For a single charge →

$$\boxed{\vec{F}_B = q\,\vec{v}_d \times \vec{B}}$$

For N charges moving through the wire (current carrying wire) →

$$\boxed{\vec{F}_B = i\vec{\ell}\times\vec{B}}$$

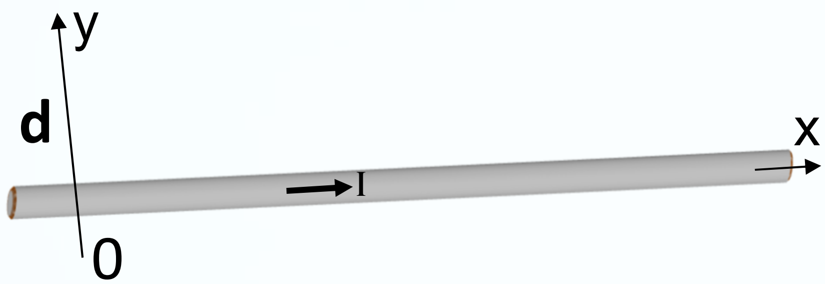
The Biot-Savart Law →

$$\boxed{\vec{B}_{\text{point charge}} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2}}$$

For an electric current →

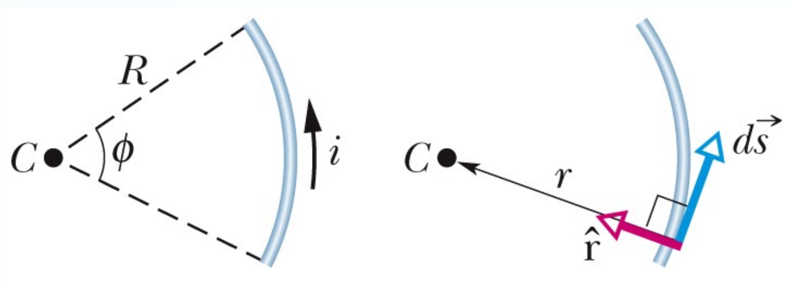
$$\boxed{\vec{B}_{\text{current segment}} = \frac{\mu_0}{4\pi}\frac{I\,\Delta\vec{s}\times\hat{r}}{r^2}}$$

Magnetic field due to current in long straight wire

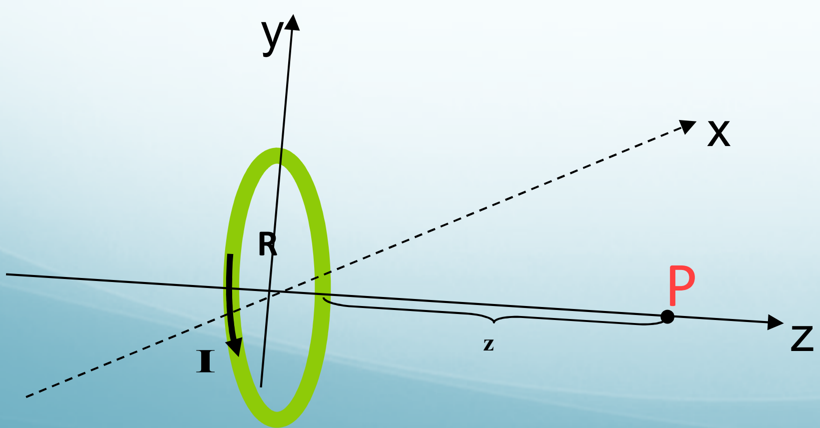


$$B_z = \frac{\mu_0}{2\pi}\frac{I}{d}$$

Non-infinite straight wire→ Appendix 1-chapter 22



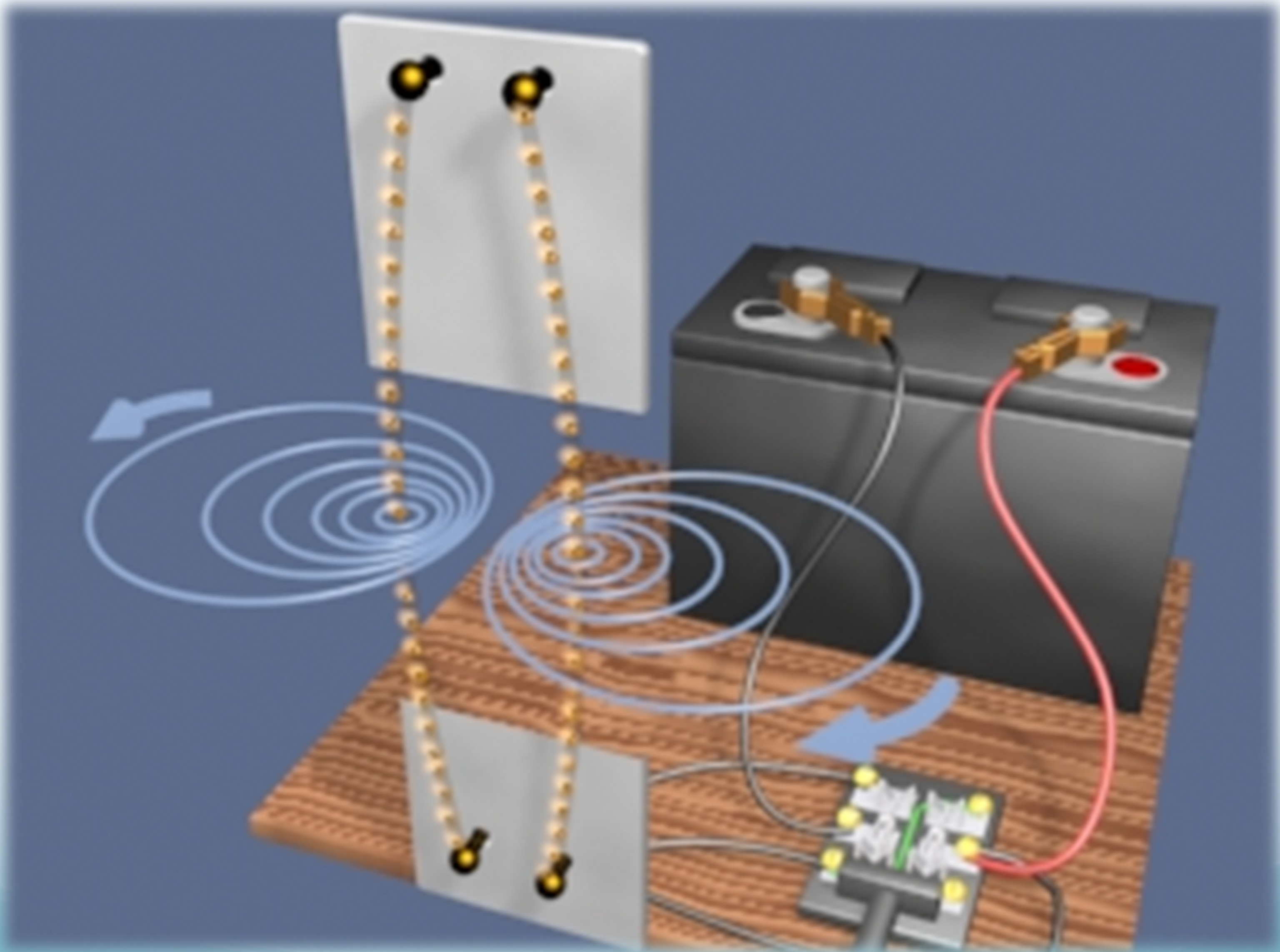
$$B = \frac{\mu_0 i \phi}{4\pi R}$$



$$\vec{B} = \frac{\mu_0}{2}\frac{IR^2}{\left(z^2 + R^2\right)^{3/2}}\hat{k}$$

if $\quad z = 0 \quad \vec{B}_{center} = \frac{\mu_0}{2}\frac{I}{R}\hat{k}$

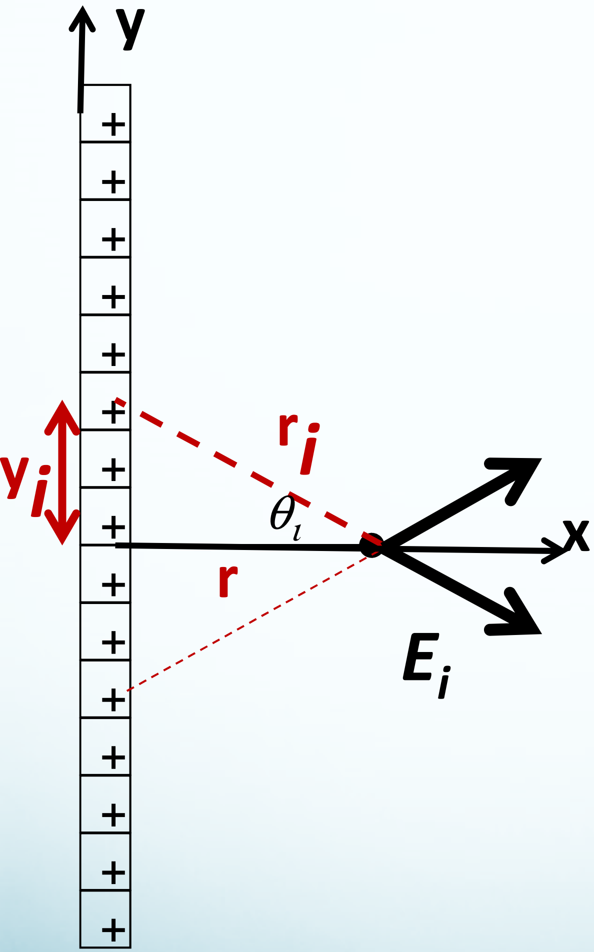
29.2: Force between two antiparallel currents

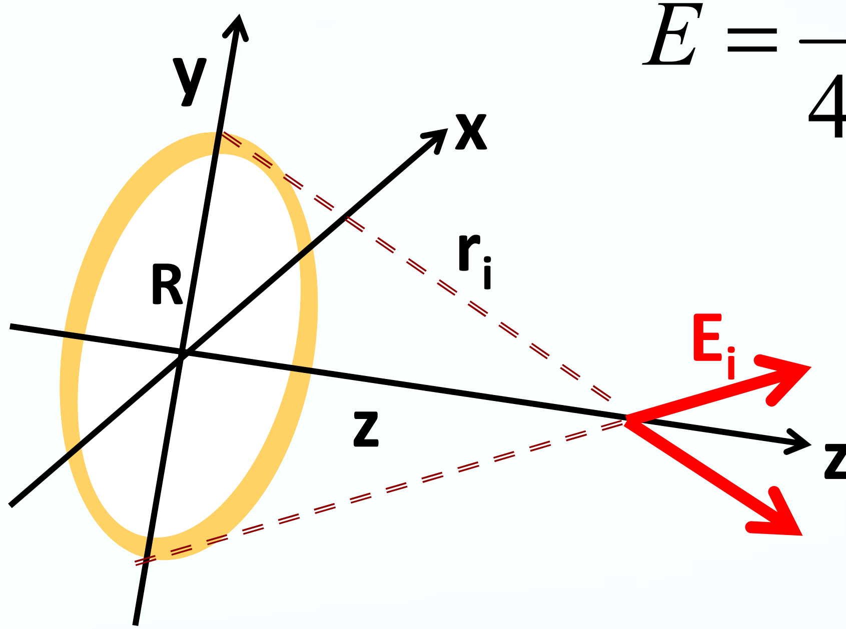


https://tinyurl.com/lghkxt8

Electrostatics

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$$





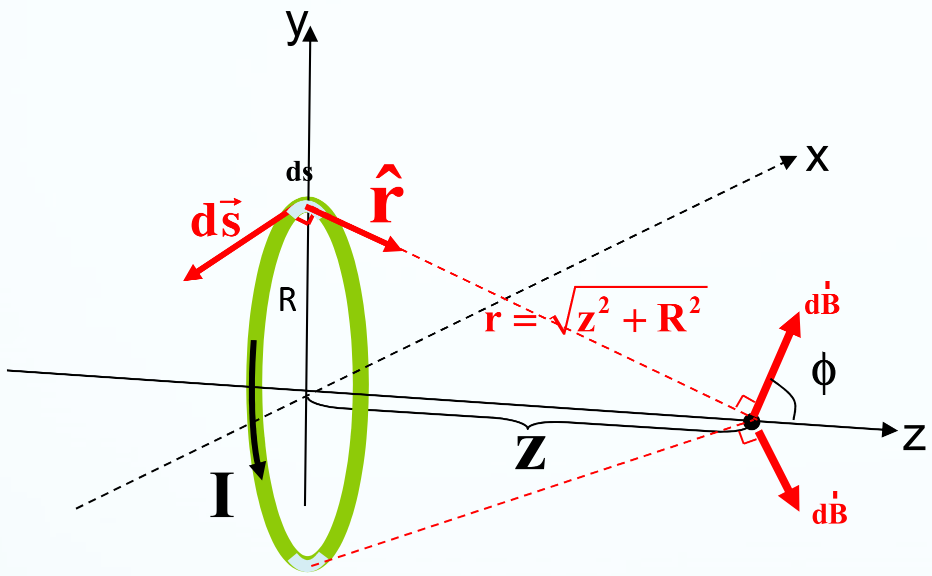
Savior:

Gausses' law

$$\oint \vec{E}.d\vec{A} = \frac{Q_{in}}{\varepsilon_0}$$

Magnetostatics

$$\vec{B}_{\text{current segment}} = \frac{\mu_o}{4\pi}\frac{I\Delta\vec{s}\times\hat{r}}{r^2}$$



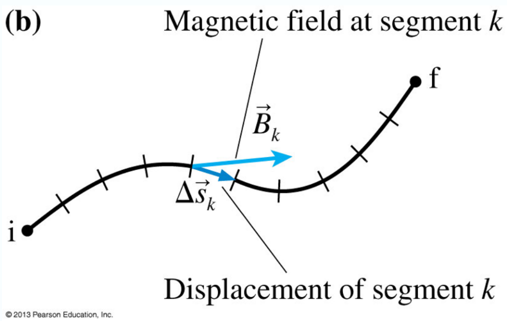
Savior:

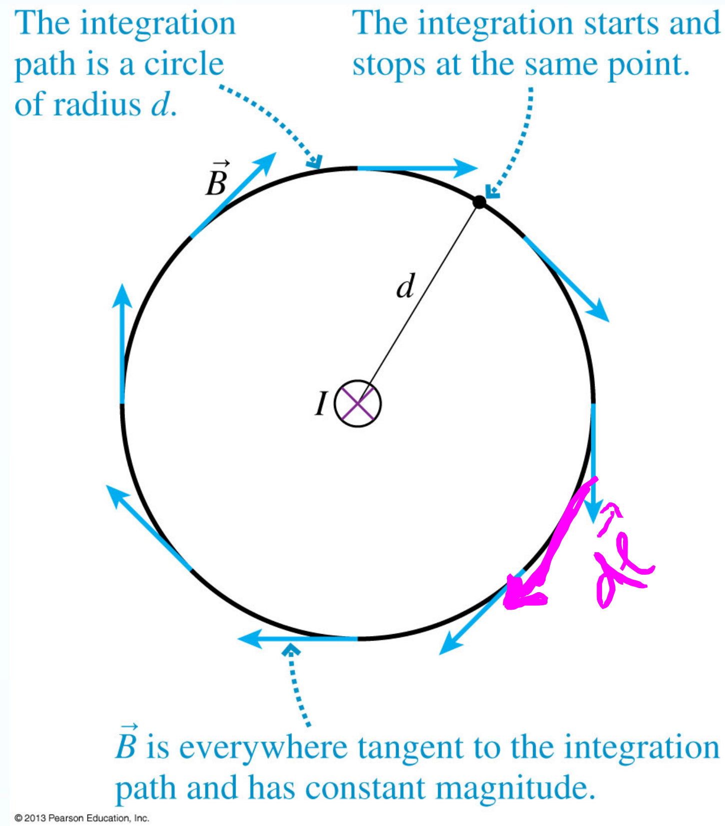
Ampere’s law

Expression?

Ampère's law

The line integral of **B** along the path:



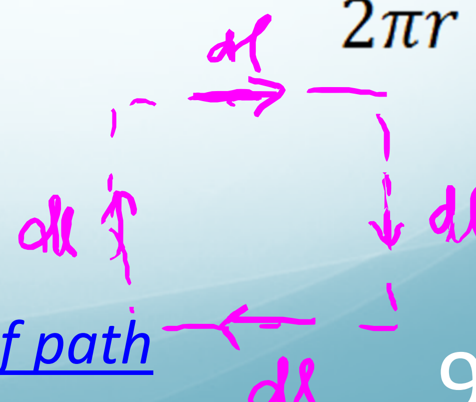


$$\int_i^f \vec{B}.d\vec{l}$$

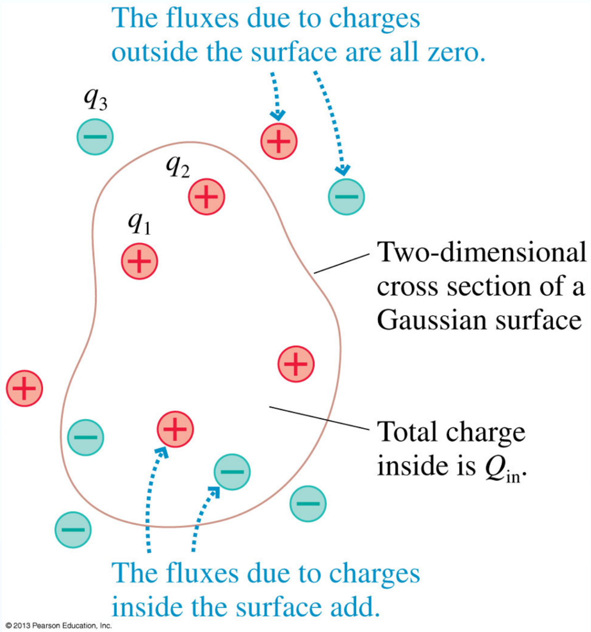
$$\oint \vec{B}\cdot d\vec{l} = (2\pi r)\left(\frac{\mu_0 I}{2\pi r}\right)$$

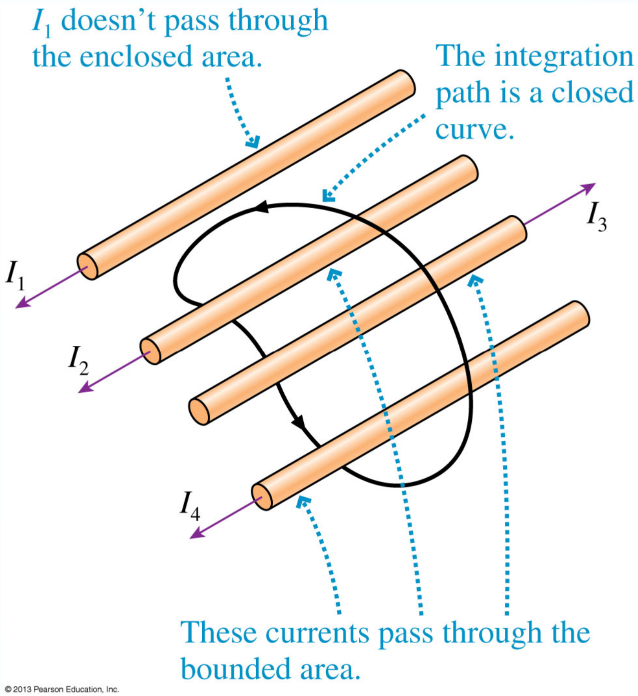
Infinite wire→ $B = \dfrac{\mu_0 I}{2\pi r}$

⟹ $\boxed{\text{i.e.} \oint \vec{B}\cdot d\vec{l} = \mu_0 I_{encl}}$



Ampère's Law is true for any ***shape of path*** *and any current distribution*





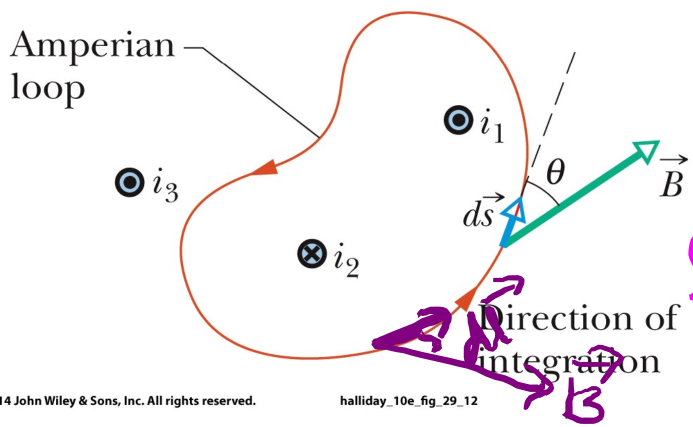
$$\oint \vec{E}.d\vec{A} = \frac{Q_{in}}{\varepsilon_0}$$

For a closed surface enclosing total Charge Q

$$\oint \vec{B}.d\vec{l} = \mu_0 I_{enclosing}$$

Current I passes through an area bounded by a closed curve

Only the currents encircled by the loop are used in Ampere's law.



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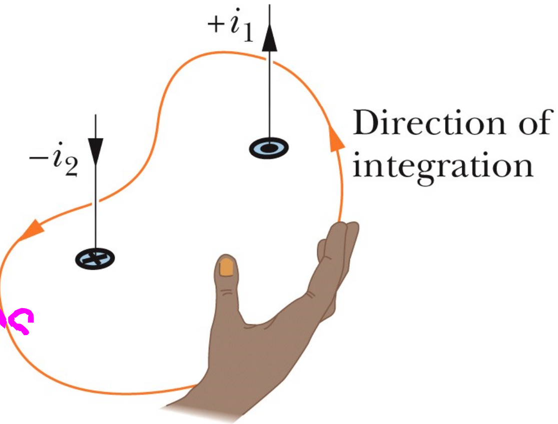
This is how to assign a sign to a current used in Ampere's law.

$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$



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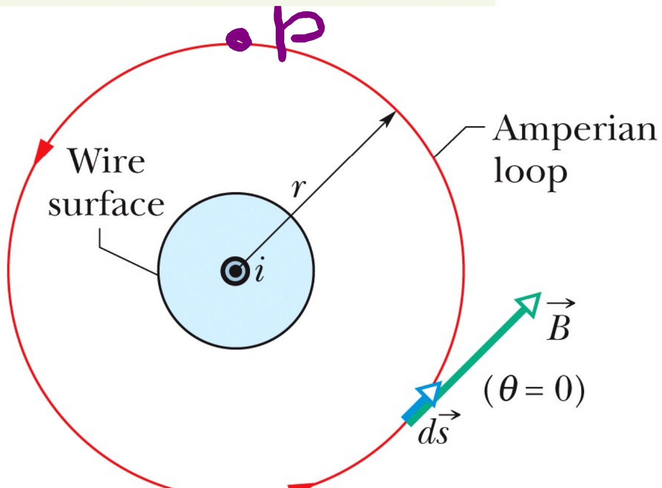
All of the current is encircled and thus all is used in Ampere's law.



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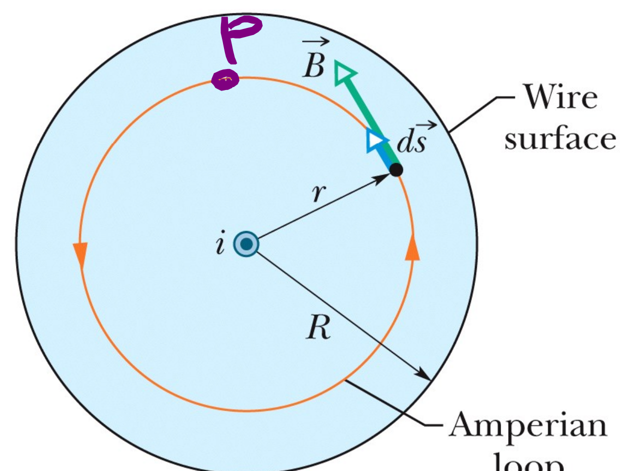
Only the current encircled by the loop is used in Ampere's law.



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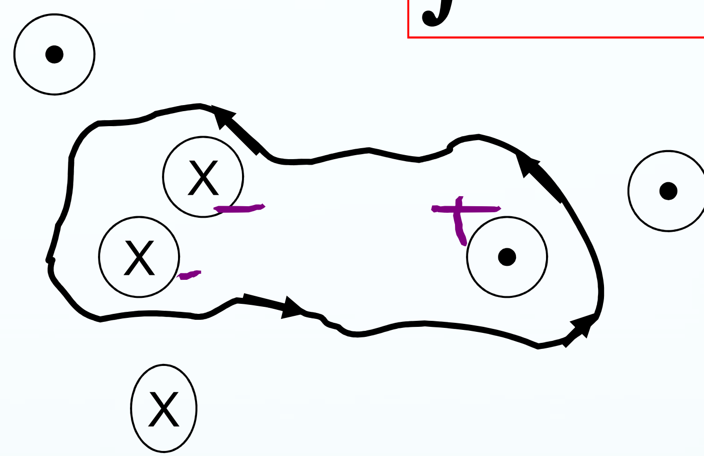
TopHat Question

What is I_{encl} here, where all three wires have 5 A?

$$\oint \vec{B} \cdot d\vec{L} = \mu_0 I_{through}$$



A) -5 A

B) 5A

C) 15 A

D) -15 A

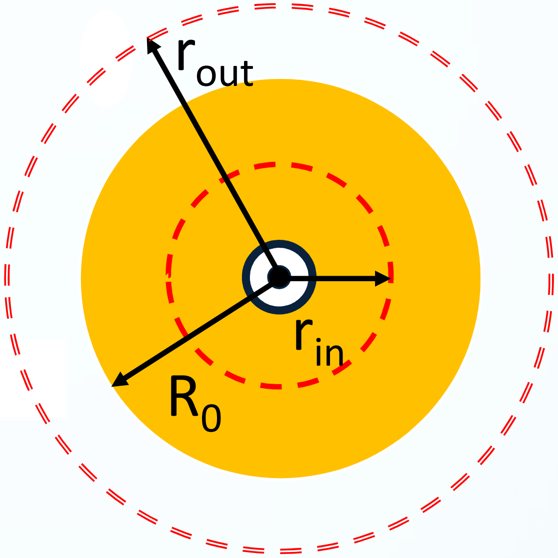
E) other

Ampère's law: application

(a) Using Ampère's law, calculate the magnetic field outside a solid current carrying wire a distance r_{out} from its axis

(The length of the solid wire is infinite and the current I is uniformly distributed throughout the solid wire)

b) Calculate the magnetic field inside a solid current carrying wire a distance r_{in} from its axis.



End view:
Wire with radius R and current I

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

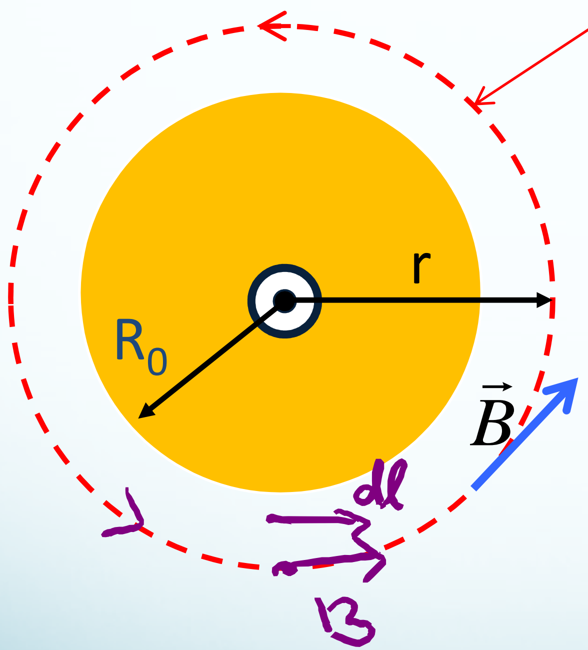
$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{enc}$$

$$J = \frac{I}{A}$$

Ampère's law: application(1)

(a) B-field outside

We want to know the B-field a distance r, so we choose an Ampèrian loop with radius r > R_0.



Ampère's Law: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$

Left hand side: $\oint \vec{B} \cdot d\vec{l} = BL = B2\pi r$

Right hand side: $\mu_0 I_{enc} = \mu_0 I$

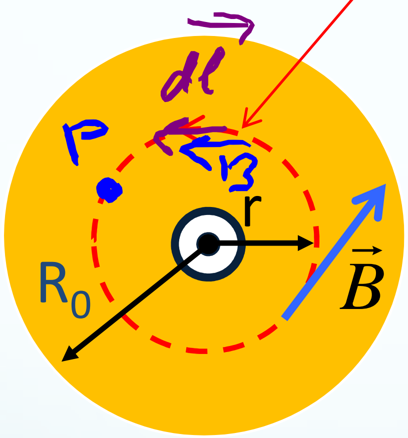
Combine together: $B2\pi r = \mu_0 I$

$$\boxed{B = \frac{\mu_0 I}{2\pi r}}$$

Ampère's law: application(1)

(b) B-field inside

We want to know the B-field a distance r, so we choose an Amperian circular loop with radius $r < R_0$.



Ampère's Law: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$

Left hand side: $\oint \vec{B} \cdot d\vec{l} = BL = \boxed{B 2\pi r}$

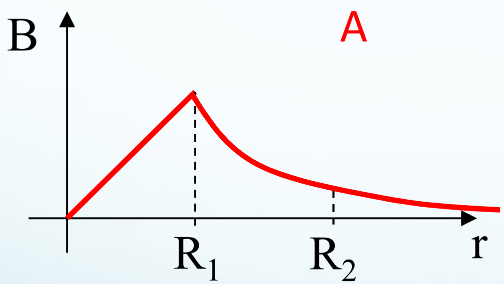
Right hand side: $\mu_0 I_{enc} = \mu_0 JA = \mu_0 \dfrac{I}{\pi R_0^2} \pi r^2$

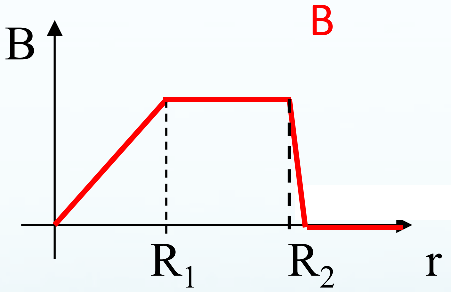
$\rightarrow \mu_0 I_{enc} = \mu_0 JA = \mu_0 J \pi r^2 = \mu_0 \dfrac{I}{A} \pi r^2 = \mu_0 \dfrac{I}{\pi R_0^2} \pi r^2$

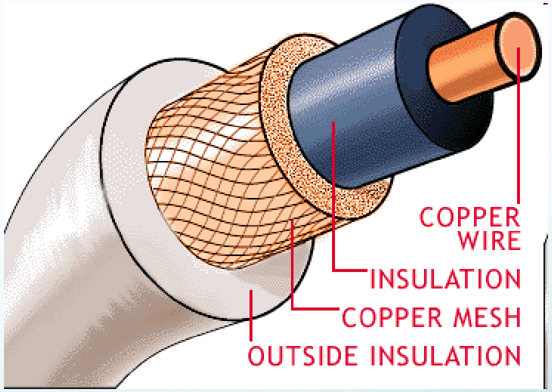
Combine together: $B 2\pi r = \mu_0 \dfrac{I}{R_0^2} r^2$ $\qquad \boxed{B = \dfrac{\mu_0 I r}{2\pi R_0^2}}$

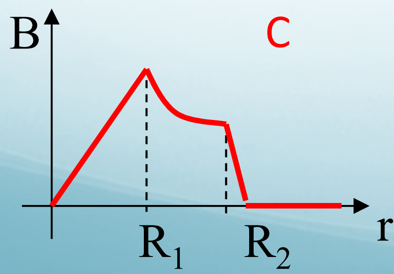
A coaxial cable consists of a wire (radius R_1) surrounded by an insulating sleeve and another cylindrical conducting shell (inner radius R_2) and finally another insulating sleeve. **The wire and the shell carry the same current I but in opposite directions**.

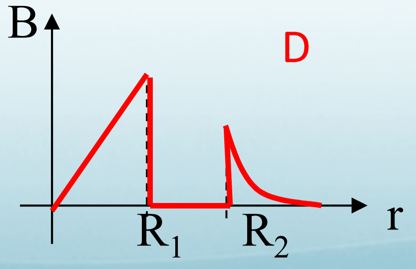
Which diagram best represents the **magnetic field** as a function of radial distance from the cable's axis?

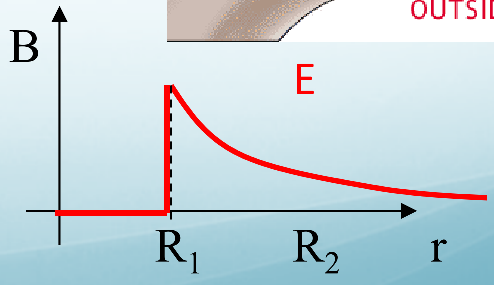








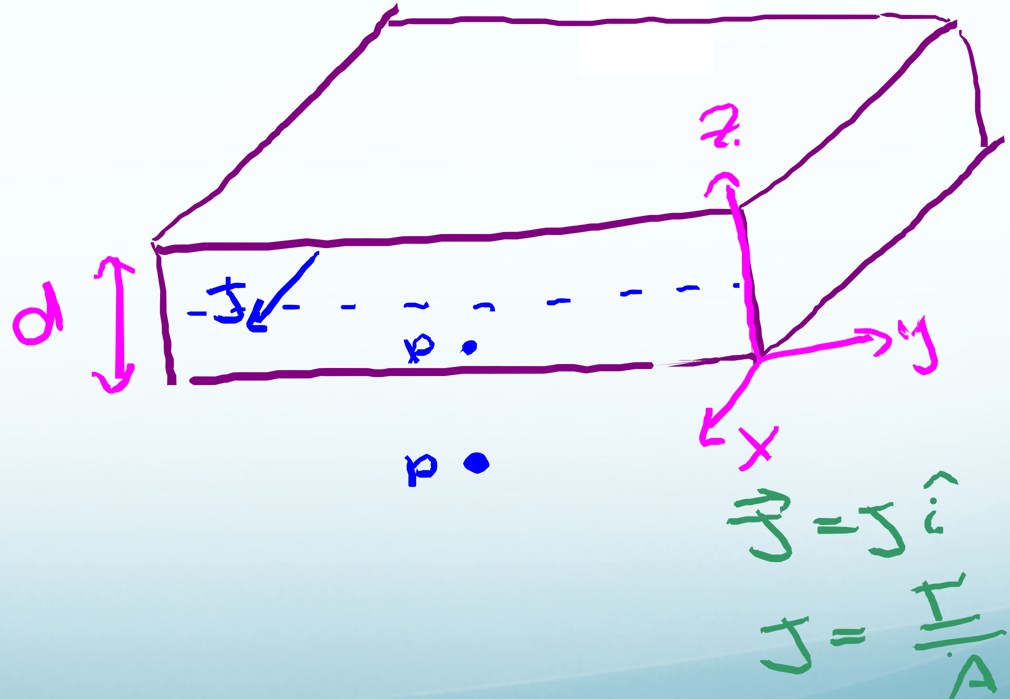




Ampère's law: application (3)

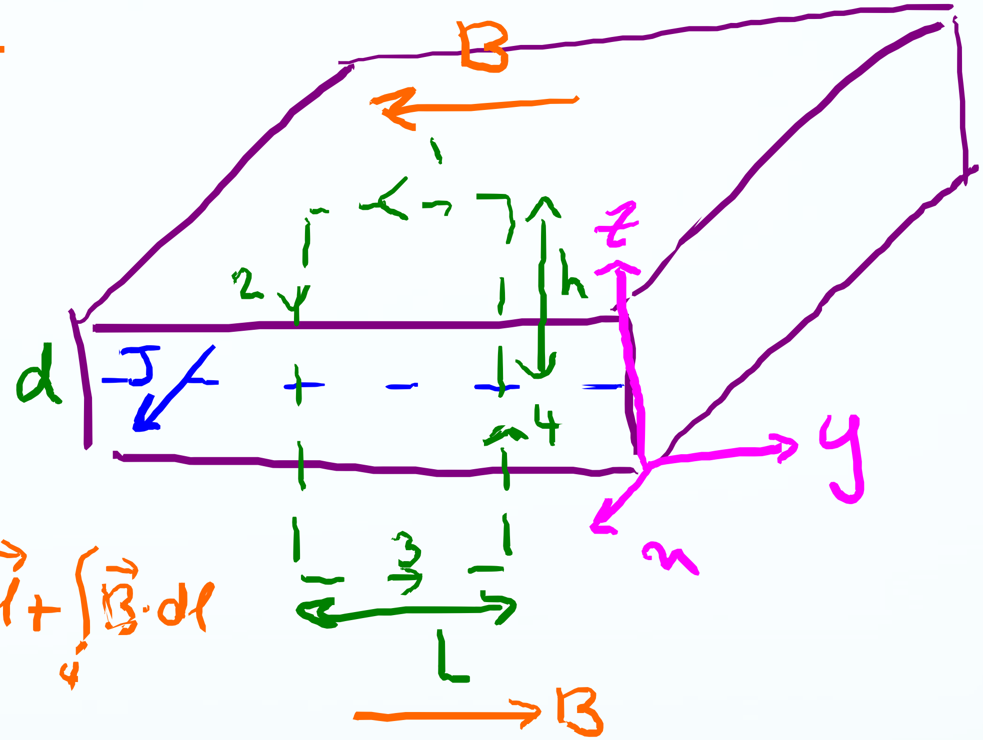
(a) Using Ampère's law, calculate the magnetic field above the current carrying slab

b) Calculate the magnetic field inside the current carrying slab



$\vec{B}$ above or below the slab ⇒

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$$



$$\oint \vec{B} \cdot d\vec{l} \Rightarrow$$

$$\rightarrow \int_1 \vec{B} \cdot d\vec{l} + \int_2 \vec{B} \cdot d\vec{l} + \int_3 \vec{B} \cdot d\vec{l} + \int_4 \vec{B} \cdot d\vec{l}$$

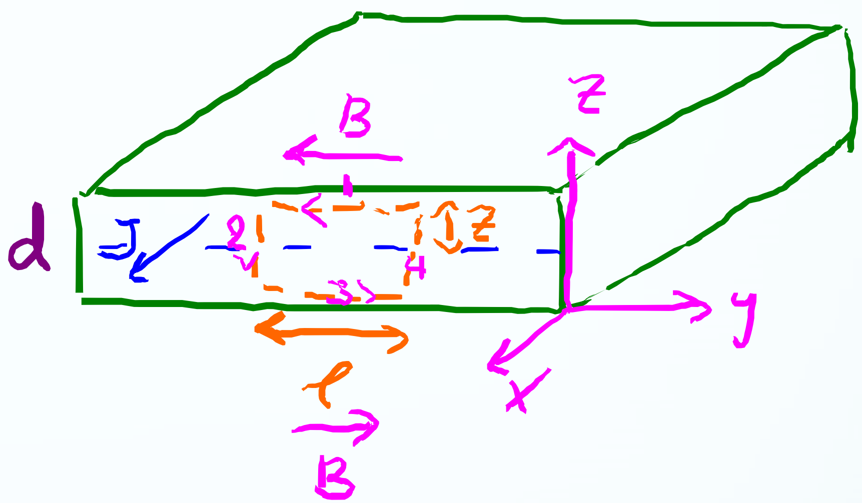
$$\rightarrow 2B \int dl = \mu_0 I_{enc} = \mu_0 J A = \mu_0 J l d$$

$$\rightarrow B = \frac{\mu_0 J d}{2}$$

$\vec{B}$ inside the slab $\Rightarrow$

$$\oint \vec{B}\cdot\vec{dl} = \mu_0 I_{enc}$$

$$\rightarrow \int_1 \vec{B}\cdot\vec{dl} + \int_2 \vec{B}\cdot\vec{dl} + \int_3 + \int_4$$

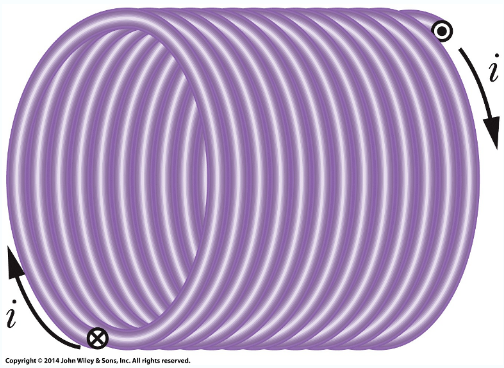


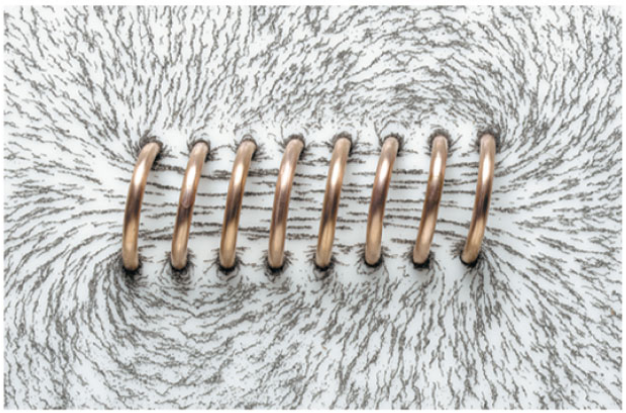
$$\rightarrow 2Bl = \mu_0 I_{enc} = \mu_0 J A_{in} = \mu_0 J\, 2zl$$

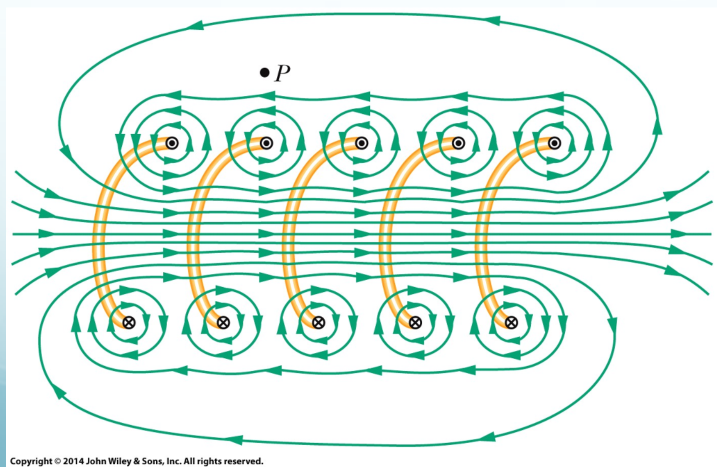
$$\rightarrow B = \mu_0 J z$$ & if $z = \frac{d}{2} \rightarrow B = \frac{\mu_0 J d}{2}$

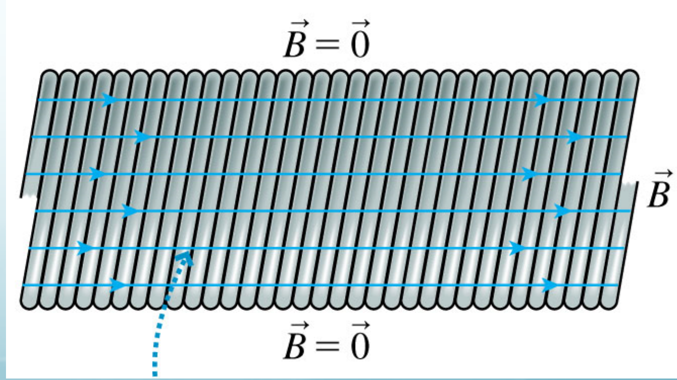
Ampère’s law: application(2)

29.3: Solenoids and Toroids

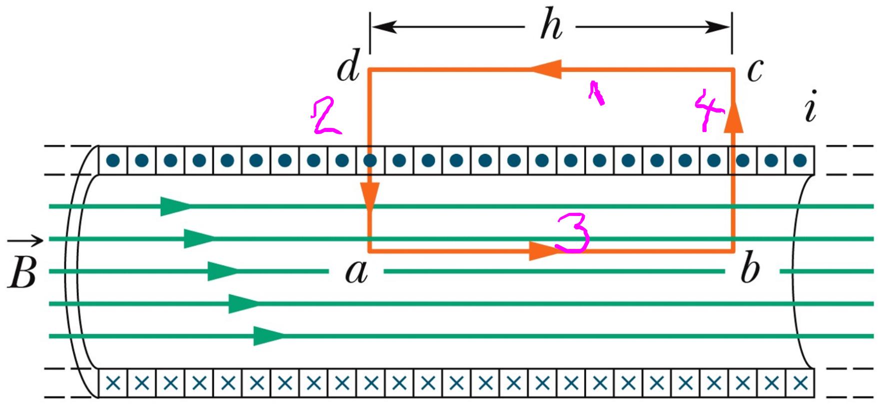








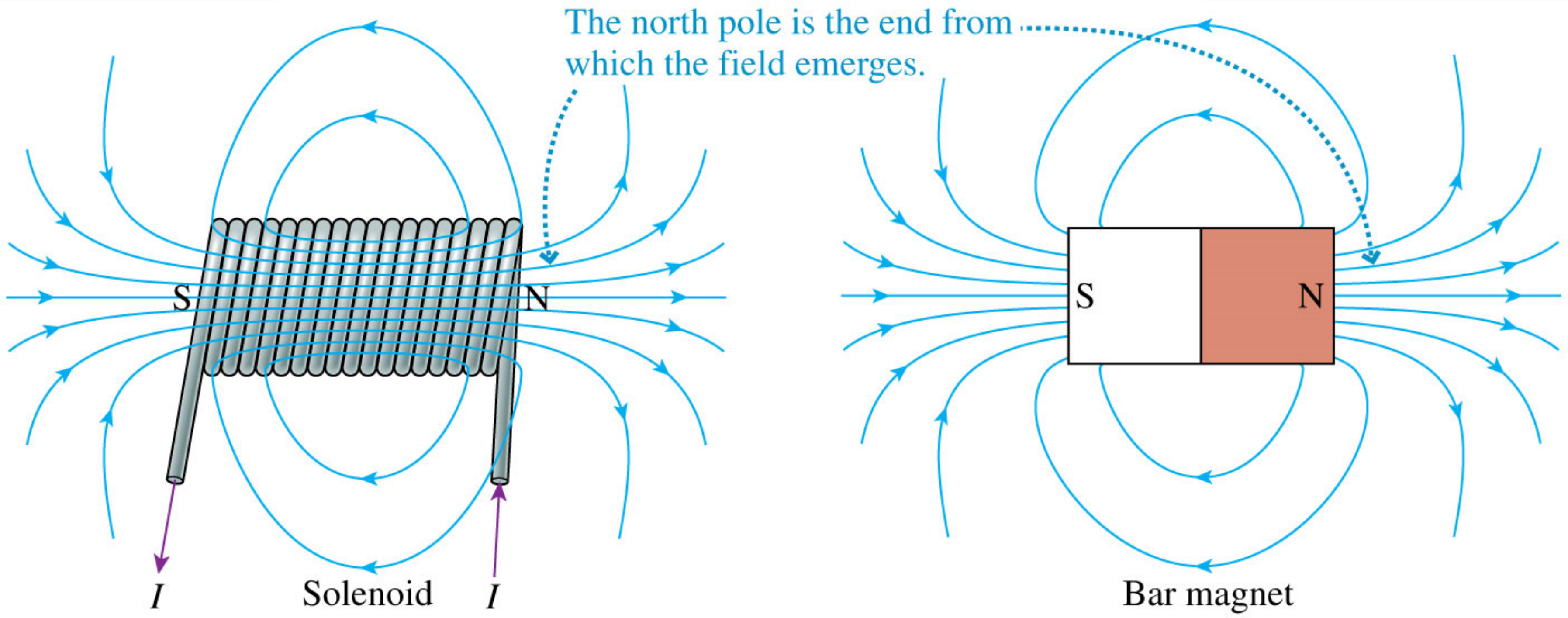
$$\oint \vec{B}.d\vec{s} = \mu_0 i_{enc}$$



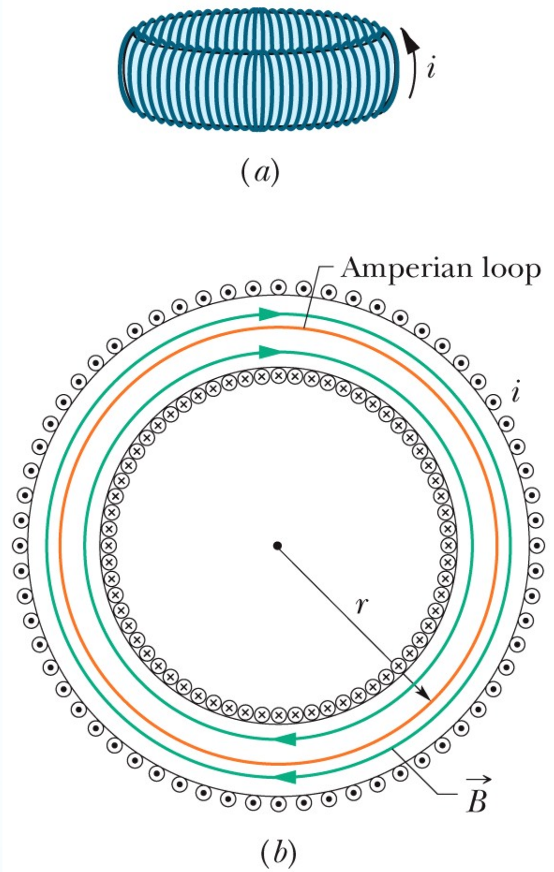
$\mathbf{n} = \dfrac{\mathbf{N}}{\mathbf{L}}$ **number of turns per unit length**

$i_{enc} = nih$

$$B_{Solenoid} = \mu_0 ni$$



$$B_{Solenoid} = \mu_0 n i$$


i
(a)
Amperian loop
i
r
$\vec{B}$
(b)

This section we talked about:

Chapter 29

See you on Friday