Friday March 17, 2017

Last time:

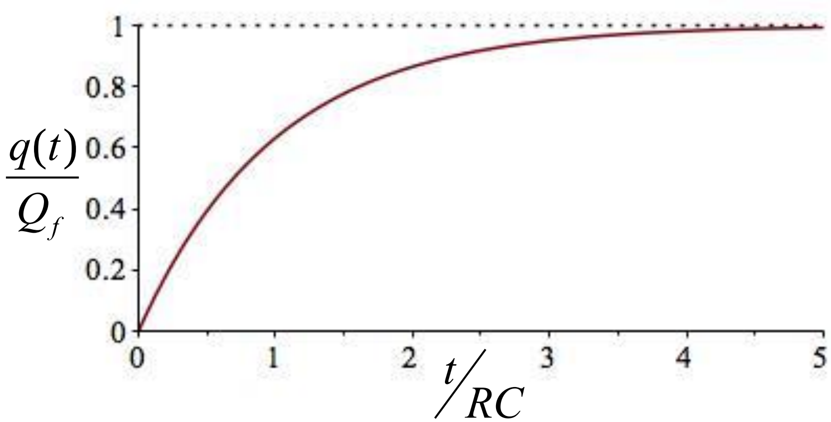
- RC circuits (charging/discharging capacitors)
- RC time constant and its meaning
- Early and late time behaviour of RC circuits

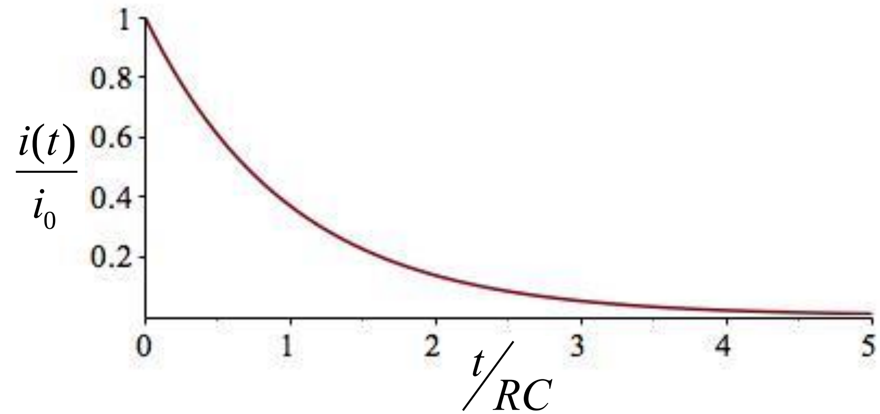
Today:

- RC circuits example
- Power in circuits
- Group activity

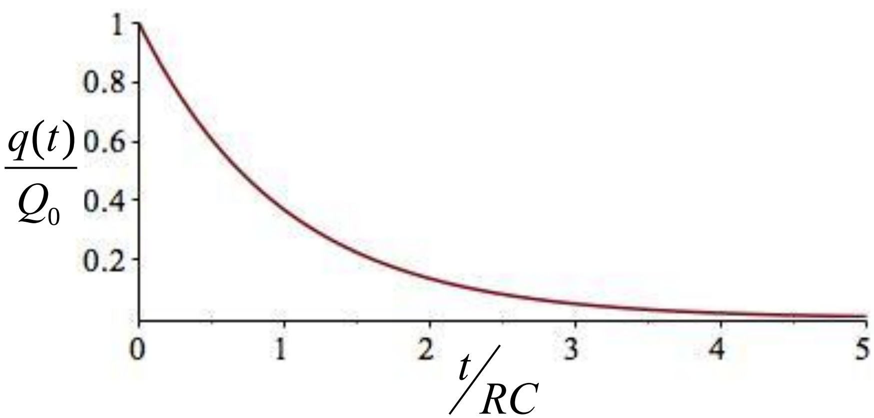
Charging/Discharging Capacitors

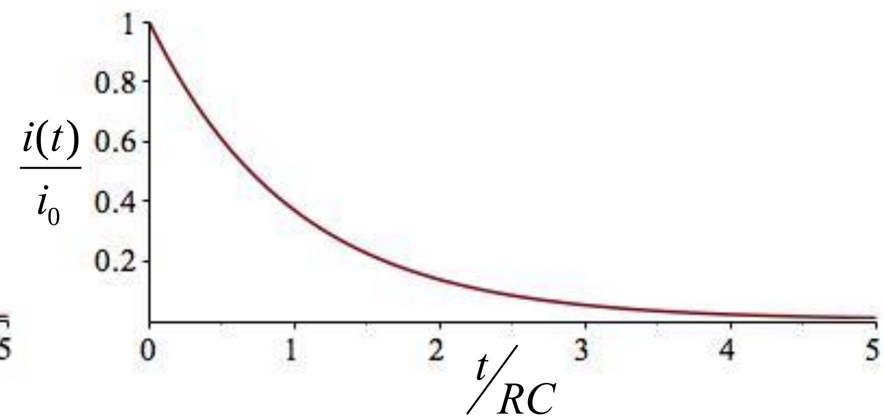
Charging: $q(t) = Q_f\left(1 - e^{-\frac{t}{RC}}\right)$ $i(t) = i_0 e^{-\frac{t}{RC}}$





Discharging: $q(t) = Q_0 e^{-\frac{t}{RC}}$ $i(t) = i_0 e^{-\frac{t}{RC}}$





The RC time constant

The constant RC pops up in the exponential factor for both charging and discharging capacitors. What does it represent?

The units of RC is seconds: $$[RC] = \frac{V}{A}\frac{C}{V} = \frac{C}{C/s} = s$$

We call RC the "RC time constant" and it tells us how quickly a capacitor can charge or discharge.

$$RC \equiv \tau$$

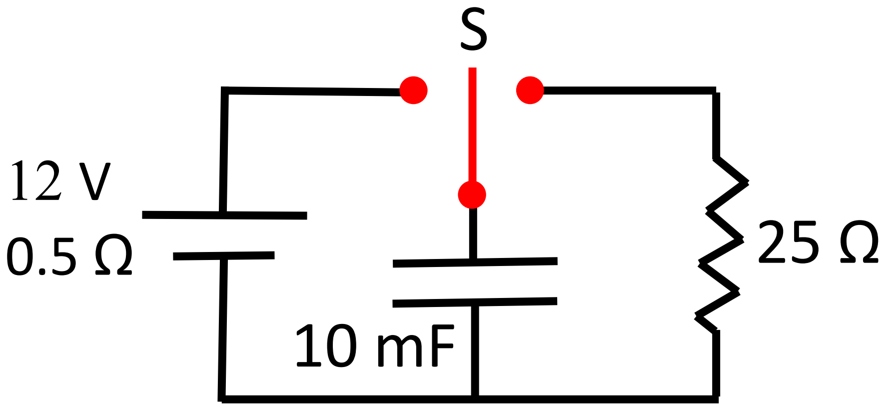
After a time τ, the charge on a discharging capacitor is reduced by a factor of 1/e. After a time $N\tau$, it is reduced by a factor of $1/e^N$

$$q(t) = Q_0 e^{-\frac{t}{t}}$$

Document Camera Calculation

An RC circuit is shown below. Initially the switch is open and the capacitor is uncharged. At time t = 0 s, the switch is thrown to the left, connecting the capacitor to the battery. At time t = 15 ms the switch is thrown to the right, connecting the capacitor to the resistor.

1) How much charge builds up on the capacitor while it is connected to the battery?
2) What is the voltage across the resistor as a function of time as the capacitor discharges?
3) What is the ratio of the charging time to discharging time?



Power in circuits

Recall that **POWER** is the **rate at which work is done**.

$$P = \frac{W}{\Delta t}$$

A battery with voltage ΔV raises the **potential energy** of a single charge q by an amount $q\Delta V$. This is the **work done** by the battery. For N charges

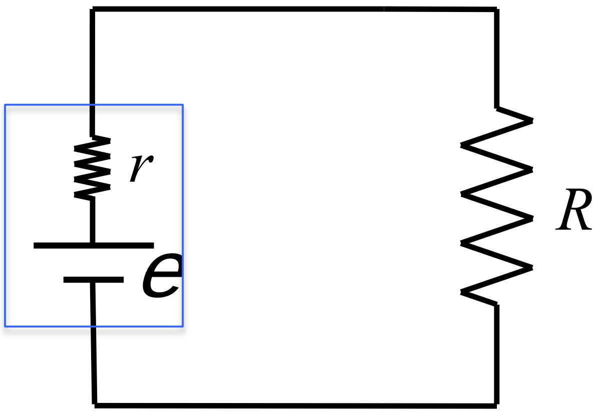
$$P = \frac{Nq \quad V}{t} = \left(\frac{Nq}{t} \right) \quad V$$

$Nq/\Delta t$ is the number of **charges passing through the battery** in time Δt, i.e. it is the **current**

$$\boxed{P = I \Delta V}$$

Power Non-ideal Batteries

$$I = \frac{e}{(r+R)}$$



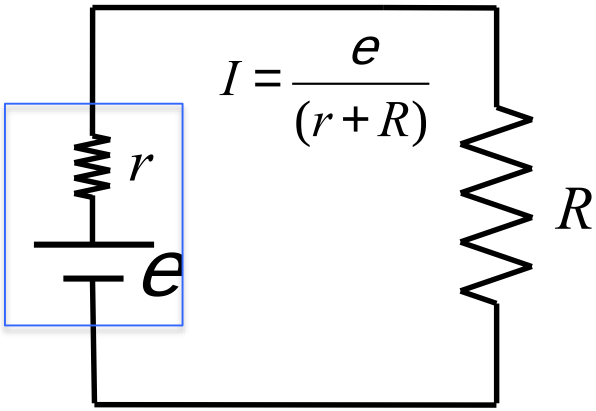
$$P_e = Ie = \frac{e^2}{R+r}$$

Power output required by the emf source

$$P_R = I^2 R = \frac{e^2 R}{(R+r)^2}$$

Power dissipated by the resistive load

Non-ideal Batteries: internal resistance



Conservation of energy requires power in must be equal to power out, but power by emf is not power dissipated by R.

$$P_e = Ie = \frac{e^2}{R+r} \qquad P_r = I^2 R = \frac{e^2 R}{(R+r)^2}$$

Resolution: power dissipated by emf

$$P_r = I^2 r = \frac{e^2 r}{(R+r)^2}$$

The emf must do more work because it fights against its own internal resistance

Now we can verify that power in = power out

$$P_e = P_r + P_R = \frac{e^2 r}{(R+r)^2} + \frac{e^2 R}{(R+r)^2} = \frac{e^2 (R+r)}{(R+r)^2} = \frac{e^2}{R+r}$$