Electricity and Magnetism

- Physics 259 – L02
- Lecture 42

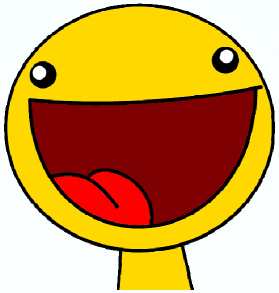
Chapter 29: Magnetic field due to current

Last time:

- Biot-Savart Law (like Coulomb's Law for magnetism)

Today:

- B-field of a line of current
- Magnetic force between parallel current-carrying wires
- Ampere's law



For a single charge $\rightarrow$

$$\boxed{\vec{F}_B = q\vec{v}_d \times \vec{B}}$$

For N charges moving through the wire (current carrying wire) $\rightarrow$

$$\boxed{\vec{F}_B = i\vec{\ell} \times \vec{B}}$$

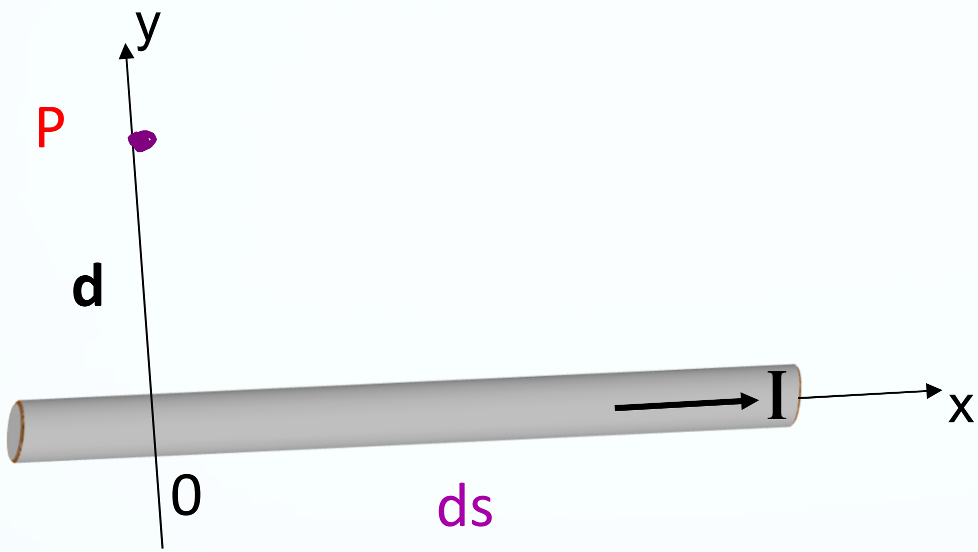
The Biot-Savart Law $\rightarrow$

$$\boxed{\vec{B}_{\text{point charge}} = \frac{\mu_0}{4\pi}\frac{q\vec{v} \times \hat{r}}{r^2}}$$

For an electric current $\rightarrow$

$$\boxed{\vec{B}_{\text{current segment}} = \frac{\mu_0}{4\pi}\frac{I\,\Delta\vec{s} \times \hat{r}}{r^2}}$$

Magnetic field due to current in long straight wire



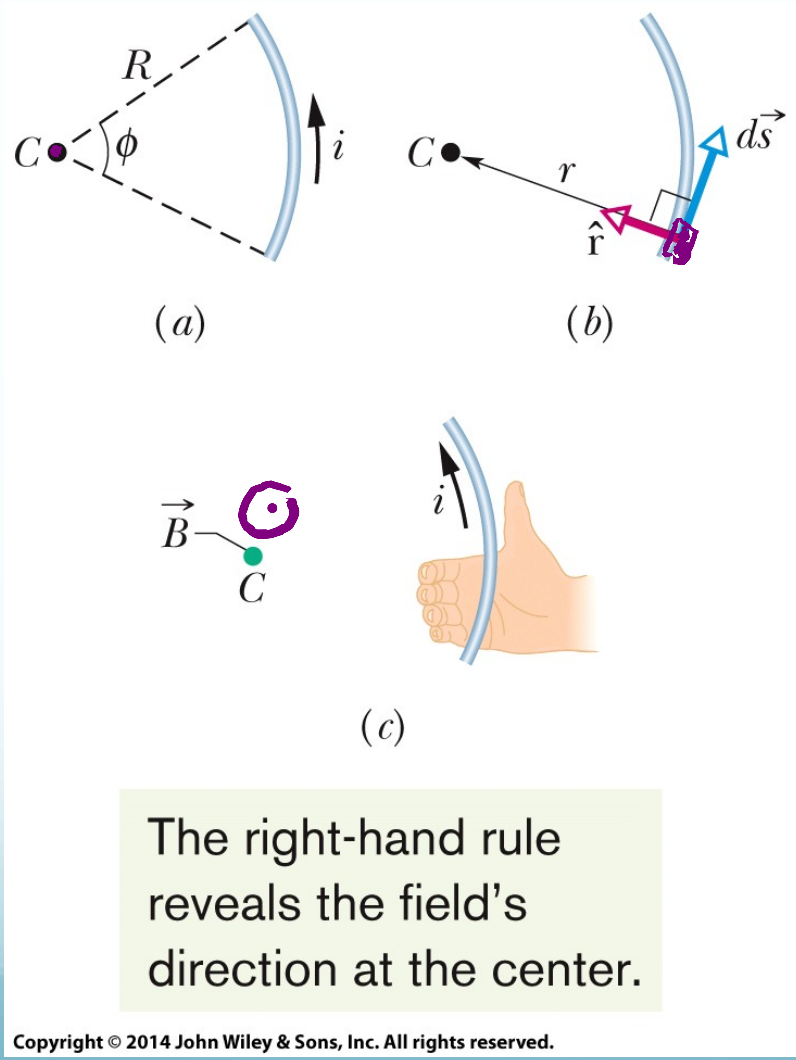
$$\vec{B} = \frac{\mu_0}{4\pi} \frac{I\Delta\vec{s} \times \hat{r}}{r^2}$$

$B_z = \dfrac{\mu_0}{2\pi} \dfrac{I}{d}$, tangent to a circle around the wire in the right-hand direction

Non-infinite straight wire→ Appendix 1-chapter 22

Magnetic field due to a current in a circular arc of wire

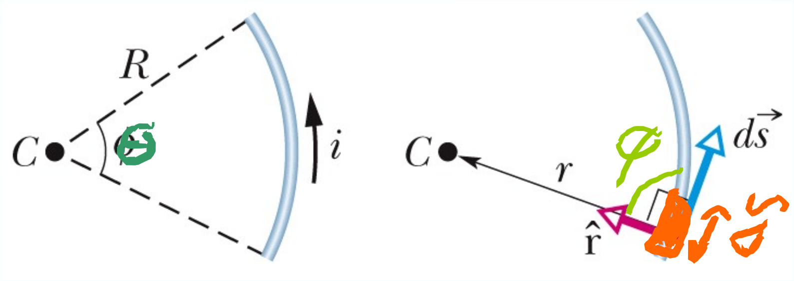
The magnitude of the **magnetic field at the center of a circular arc**, of radius R and central angle ϕ (in radians), carrying current I →



$\phi = 90^\circ$

$$dB = \frac{\mu_0}{4\pi} \frac{I\, d\vec{s} \times \hat{r}}{r^2} \rightarrow$$

$$\rightarrow dB = \frac{\mu_0}{4\pi} \frac{I\, ds\, |\hat{r}| \sin\phi}{R^2}$$



$\phi = 90^\circ$

ϕ between $\hat{r}$ and ds

$\sin\phi = 1$

$|\hat{r}| = 1$

$ds = R\,d\theta$

$$\rightarrow dB = \frac{\mu_0}{4\pi} \frac{I\, ds}{R^2}$$

$$\rightarrow dB = \frac{\mu_0}{4\pi} \frac{I\, R\, d\phi}{R^2} \rightarrow$$

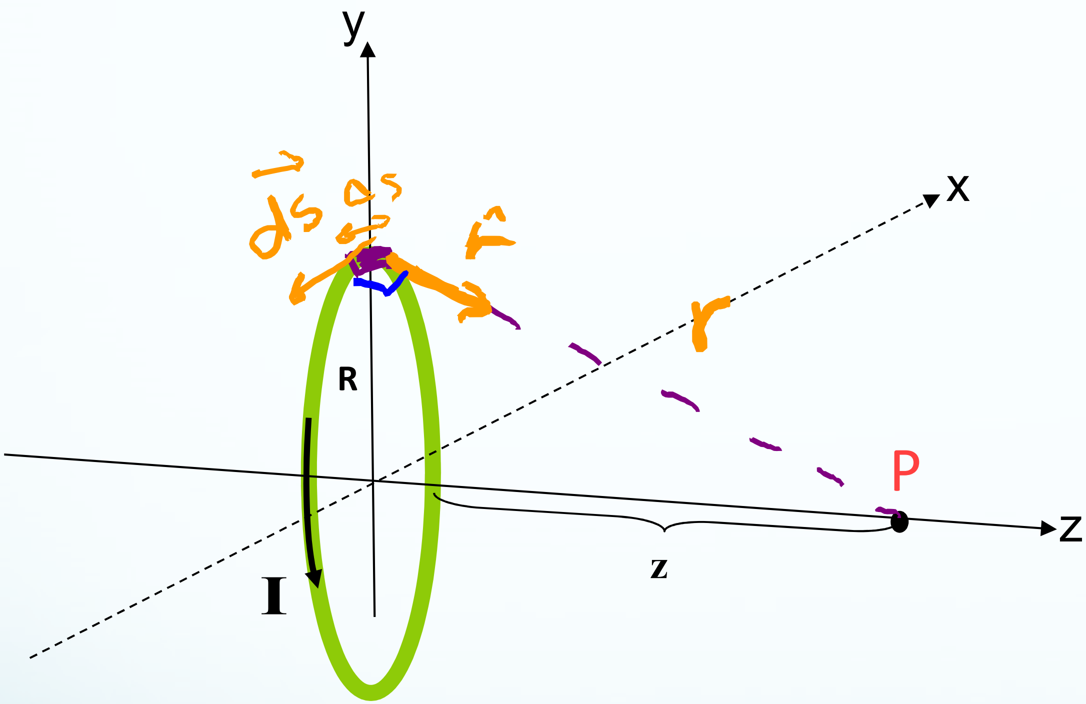
$$B = \int dB = \int_0^{\phi} \frac{\mu_0}{4\pi} \frac{I \cancel{R}\, d\theta}{R^{\cancel{2}}}$$

$I \rightarrow i$

$$\rightarrow B = \frac{\mu_0}{4\pi} \frac{I}{R} \int_0^{\theta} d\theta = \frac{\mu_0}{4\pi} \frac{I}{R} \theta =$$

$$B = \frac{\mu_0 i \theta}{4\pi R}$$

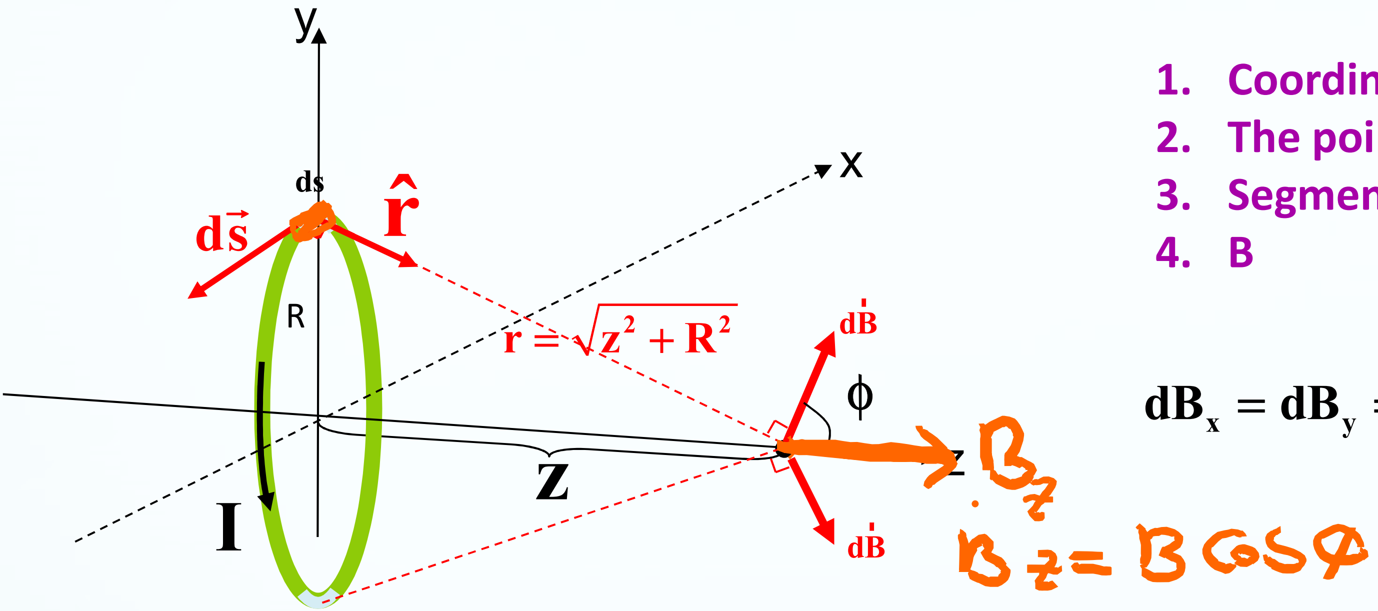
Magnetic field due to a current in a circular loop (at distance z from the loop)



1. Coordinate system
2. The point to calculate field
3. Segments
4. B

$$\vec{B}_{\text{current segment}} = \frac{\mu_o}{4\pi}\frac{I\Delta\vec{s}\times\hat{r}}{r^2} = \frac{\mu_0}{4\pi}\frac{I\,ds\,|\hat{r}|\sin\phi}{r^2}, \quad \phi = 90^\circ$$

Magnetic field of a circular loop



1. Coordinate system
2. The point to calculate field
3. Segments
4. B

$$dB_x = dB_y = 0 \qquad dB_z = dB\cos\phi$$

$$\cos\phi = \frac{R}{\sqrt{z^2 + R^2}}$$

$$dB = \frac{\mu_0}{4\pi}\frac{Ids\sin 90^\circ}{r^2} = \frac{\mu_0}{4\pi}\frac{Ids}{r^2} \rightarrow dB_z = \frac{\mu_0}{4\pi}\frac{Ids}{r^2}\cos\phi$$

$$\rightarrow dB_z = \frac{\mu_0}{4\pi}\frac{Ids}{z^2+R^2}\frac{R}{\sqrt{z^2+R^2}}$$

$$\longrightarrow dB_z = \frac{\mu_0}{4\pi}\frac{IRds}{\left(z^2+R^2\right)^{3/2}}$$

Integrate!

$$dB_z = \frac{\mu_0}{4\pi}\frac{IRds}{(z^2+R^2)^{3/2}}$$

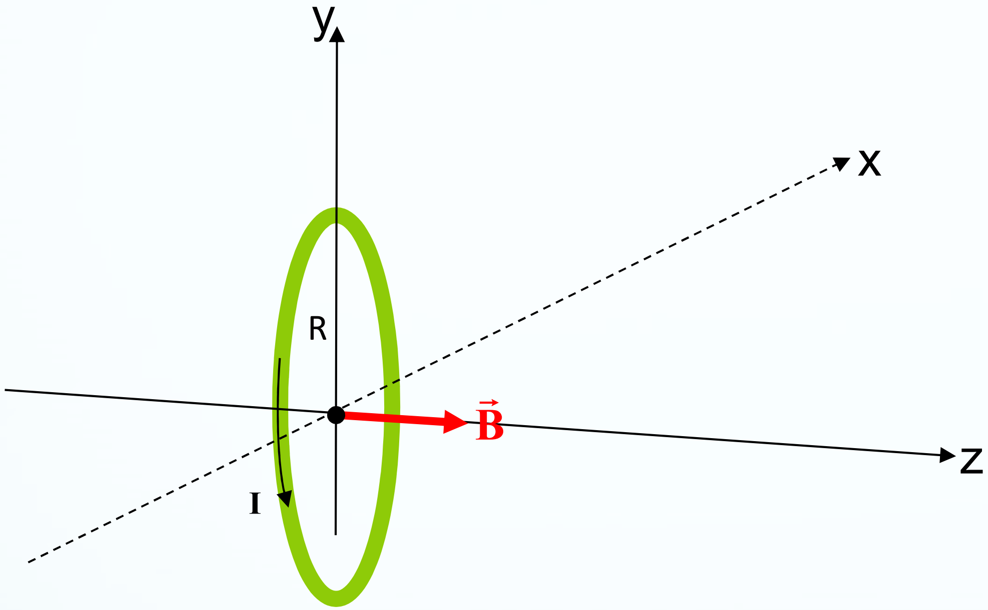
$$B_z = \int_{\text{circle}} dB_z = \int_{\text{circle}} \frac{\mu_0}{4\pi}\frac{IRds}{(z^2+R^2)^{3/2}}$$

$$\rightarrow B_z = \frac{\mu_0}{4\pi}\frac{IR}{(z^2+R^2)^{3/2}} \underbrace{\int_{\text{circle}} ds}_{2\pi R}$$

$$\rightarrow B_z = \frac{\mu_0}{\underset{2}{\cancel{4\pi}}}\frac{IR\,\cancel{2\pi}R}{(z^2+R^2)^{3/2}} \rightarrow B_z = \frac{\mu_0}{2}\frac{IR^2}{(z^2+R^2)^{3/2}}$$

$$\rightarrow \vec{B} = \frac{\mu_0}{2}\frac{IR^2}{(z^2+R^2)^{3/2}}\hat{k}$$

Magnetic field of a current loop at the center (z=0)

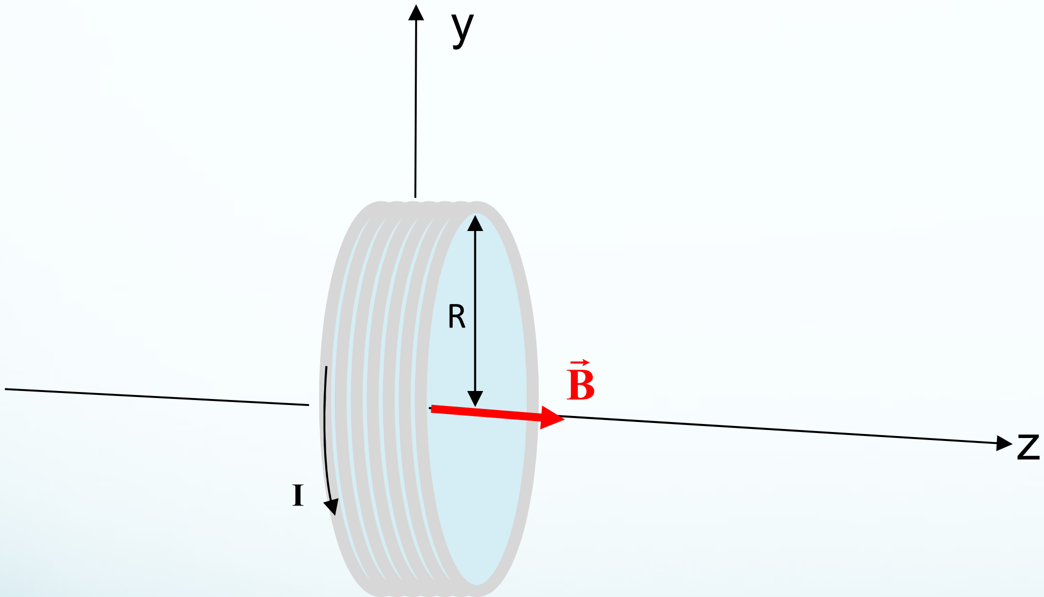


$$\vec{B} = \frac{\mu_0}{2} \frac{IR^2}{\left(z^2 + R^2\right)^{3/2}} \hat{k}$$

if $z = 0$ $\rightarrow$ $B = \frac{\mu_0}{2} \frac{IR^2}{(R^2)^{3/2}} = \frac{\mu_0}{2} \frac{I}{R}$

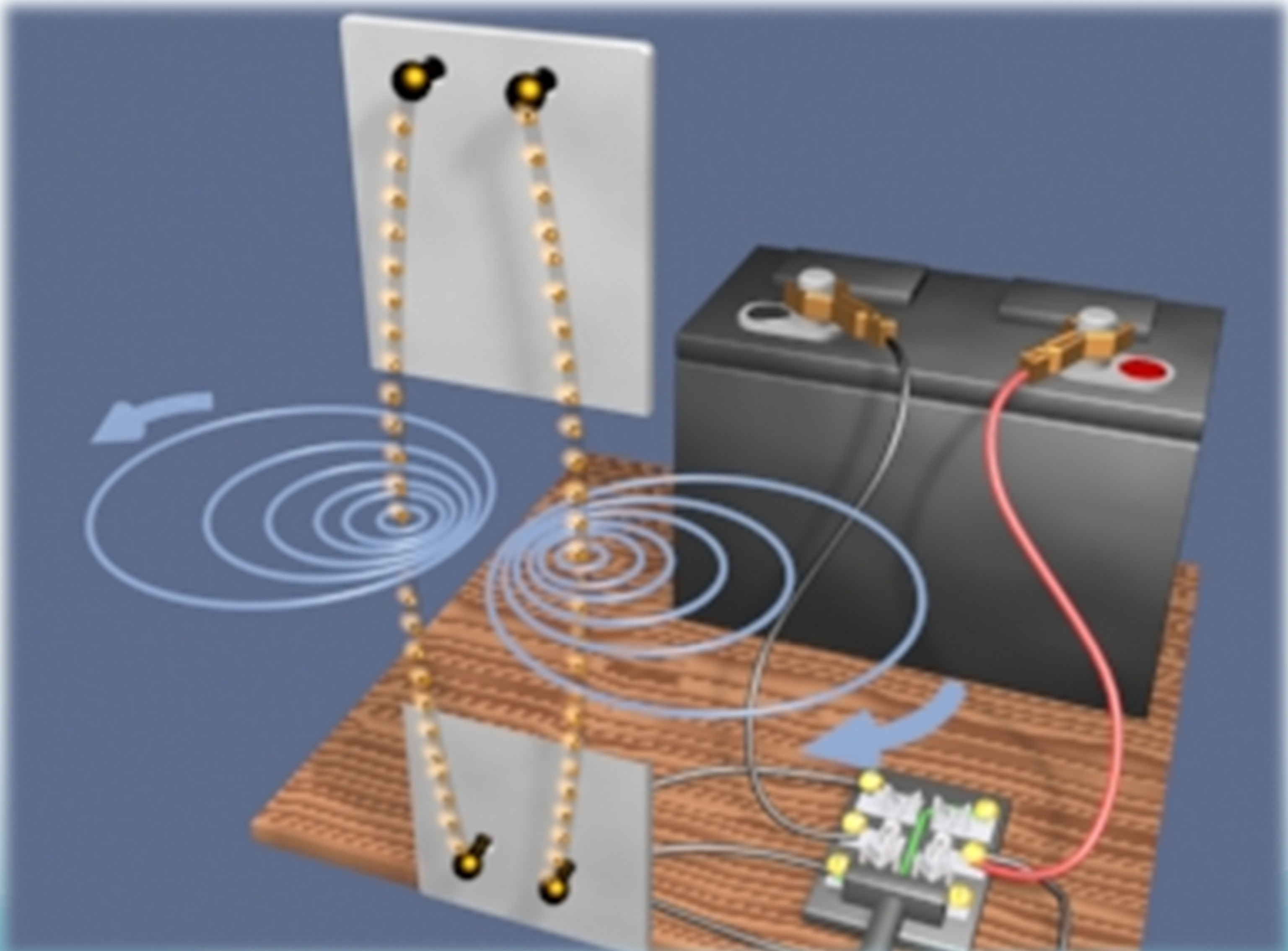
$$\vec{B}_{center} = \frac{\mu_0}{2} \frac{I}{R} \hat{k}$$

Magnetic field of a coil consists of N current loop (with the radius R) at the center of the coil:



$$\vec{B}_{coil \quad center} = \frac{\mu_0}{2}\frac{NI}{R}\hat{k}$$

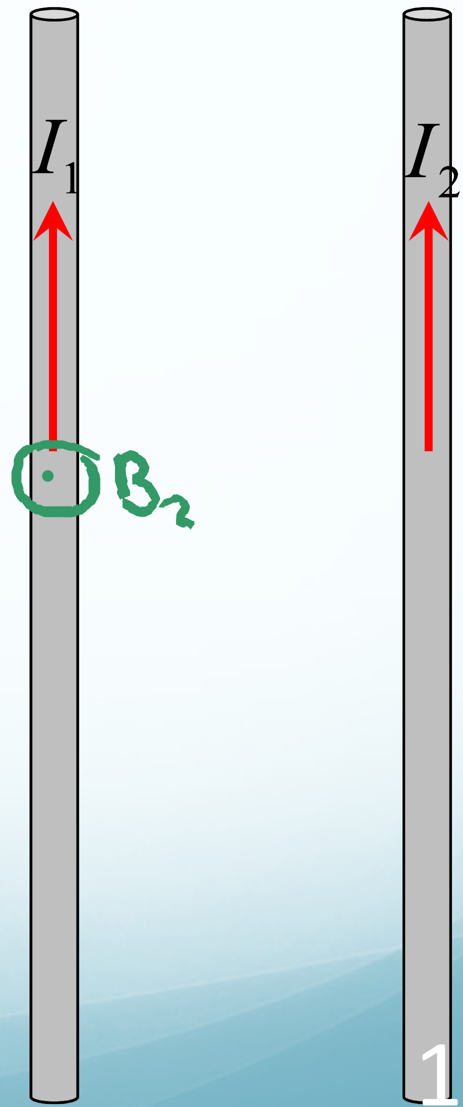
29.2: Force between two antiparallel currents



https://tinyurl.com/lghkxt8

TopHat Question

Two wires carry currents I_1 and I_2 as shown. What direction is the magnetic field produced by wire 2 at the location of wire 1?



A. Downward

B. Upward

C. Into the page

D. Out of the page

TopHat Question

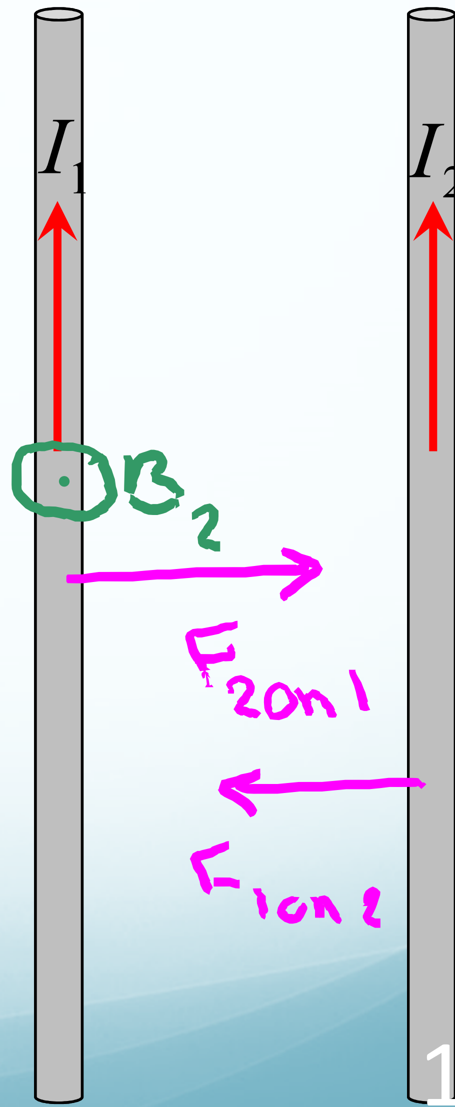
Two wires carry currents I_1 and I_2 as shown. What direction is the force of wire 2 on wire 1?

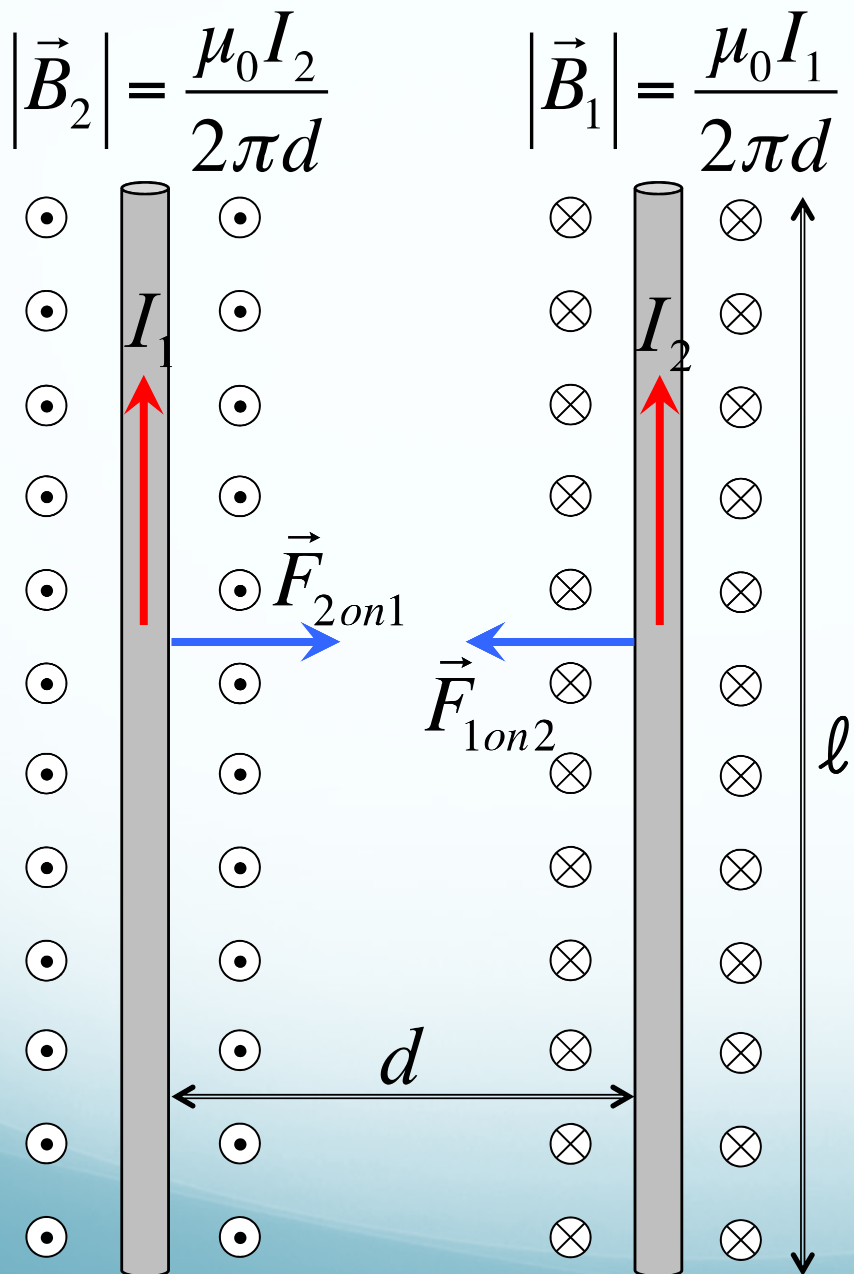
A. Left

B. Right

C. Up

D. Down





Wire 2 exerts a force on wire 1

$$\vec{F}_{2on1} = I_1 \vec{\ell} \times \vec{B}_2$$

$$\left|\vec{F}_{2on1}\right| = I_1 \ell \frac{\mu_0 I_2}{2\pi d} = \boxed{\frac{\mu_0 \ell I_1 I_2}{2\pi d}}$$

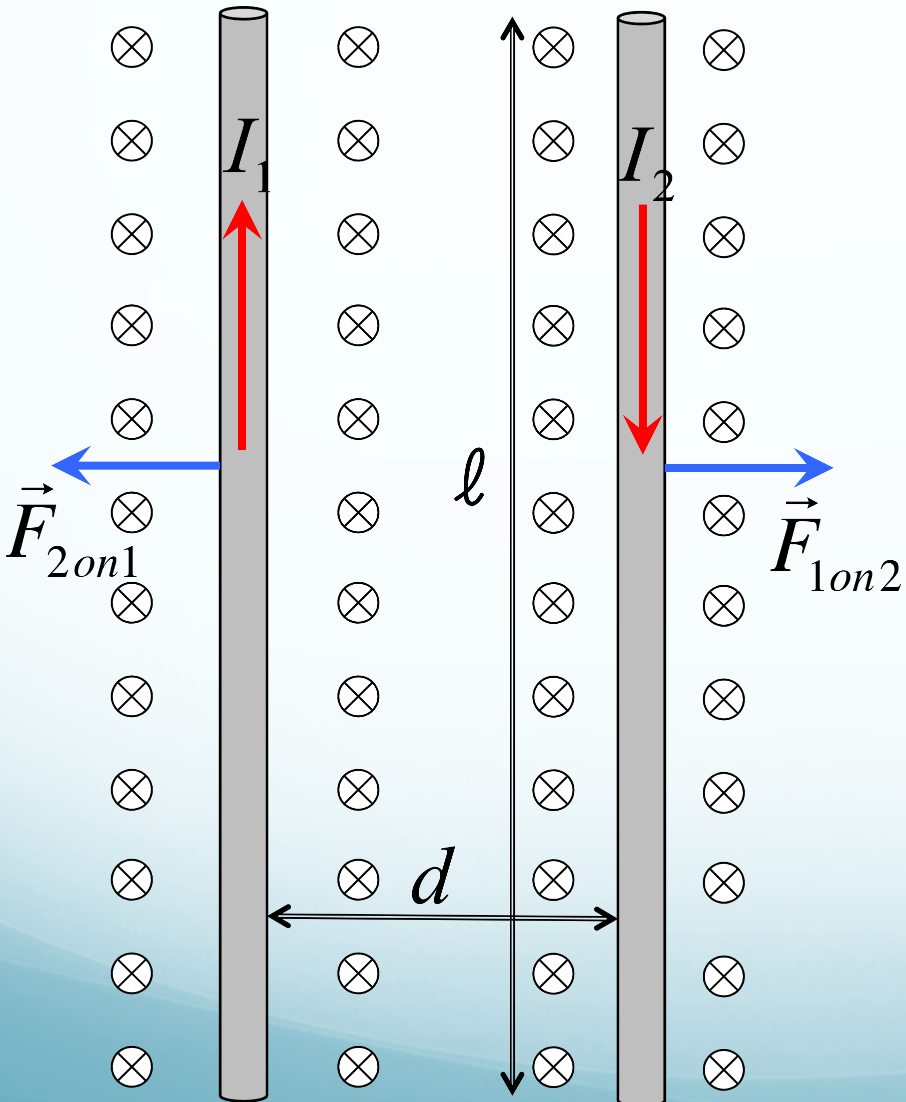
Wire 1 exerts a force on wire 2

$$\vec{F}_{1on2} = I_2 \vec{\ell} \times \vec{B}_1$$

$$\left|\vec{F}_{1on2}\right| = I_2 \ell \frac{\mu_0 I_1}{2\pi d} = \boxed{\frac{\mu_0 \ell I_1 I_2}{2\pi d}}$$

Newton's third law!

$$\left|\vec{B}_2\right| = \frac{\mu_0 I_2}{2\pi d} \qquad \left|\vec{B}_1\right| = \frac{\mu_0 I_1}{2\pi d}$$



Wire 2 exerts a force on wire 1

$$\vec{F}_{2on1} = I_1 \vec{\ell} \times \vec{B}_2$$

$$\left|\vec{F}_{2on1}\right| = I_1 \ell \frac{\mu_0 I_2}{2\pi d} = \boxed{\frac{\mu_0 \ell I_1 I_2}{2\pi d}}$$

Wire 1 exerts a force on wire 2

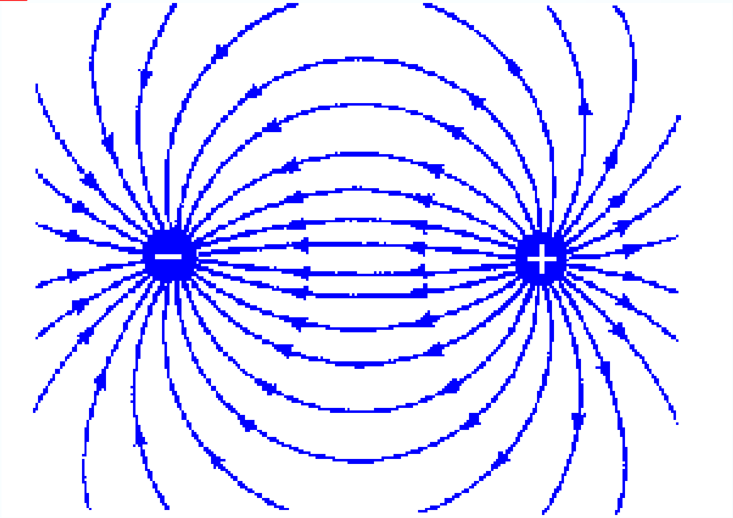
$$\vec{F}_{1on2} = I_2 \vec{\ell} \times \vec{B}_1$$

$$\left|\vec{F}_{1on2}\right| = I_2 \ell \frac{\mu_0 I_1}{2\pi d} = \boxed{\frac{\mu_0 \ell I_1 I_2}{2\pi d}}$$

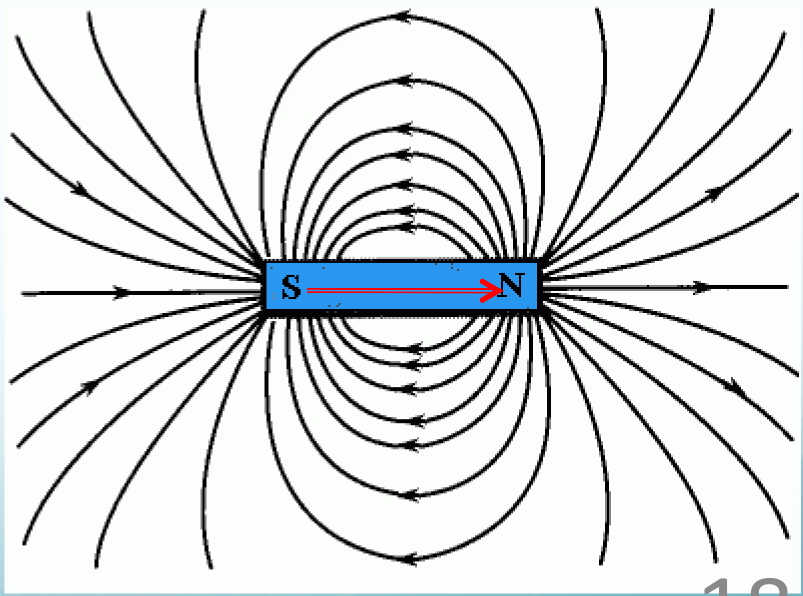
Newton's third law!

Dipole Fields

Electric field from an electric dipole



Magnetic field from a magnetic dipole. Note that the magnetic field lines are continuous – they do NOT stop at the poles!



Both fields have the same shape!

Not a Top Hat Question

The magnetic field lines from a magnet point out of the North pole and point into the South pole.

What can you say about the magnetic flux passing through this Gaussian surface?

$\Phi_E = \oint E \cdot da = \frac{q_{enc}}{\varepsilon_0}$

$$\Phi_B = \oint \vec{B} \cdot d\vec{a}$$

$\Phi_B = 0$

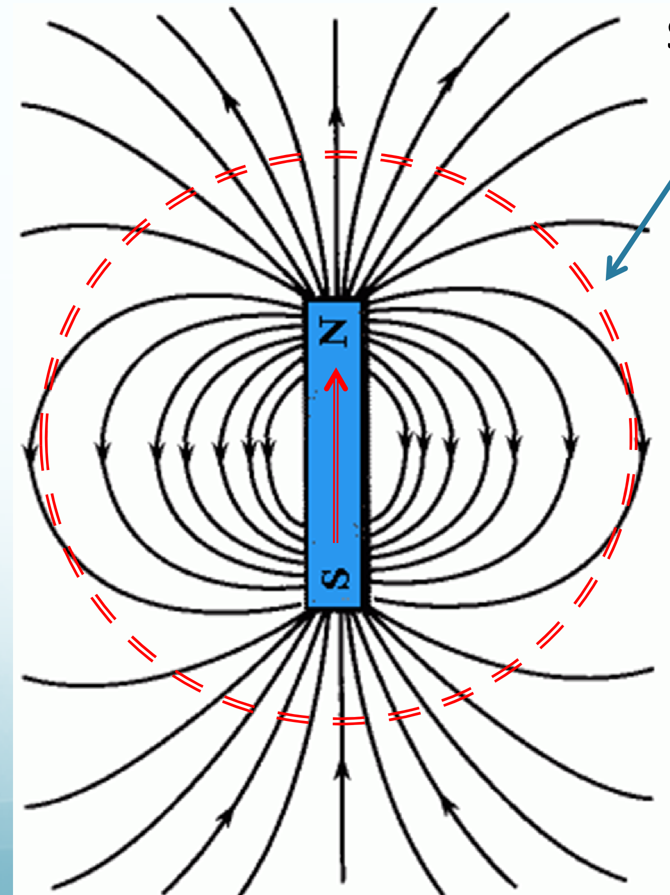
A. Magnetic flux is zero

B. Magnetic flux is greater than zero

C. Magnetic flux is smaller than zero

D. Can't tell without computing the integral

Gaussian surface



Gauss' Law for Magnetism

The magnetic flux through a closed surface is ALWAYS zero.

$$\Phi_B = \oint \vec{B} \cdot d\vec{a} = 0$$

no enclosed
magnetic charges

There is no way to isolate a North or South magnetic pole

The simplest E-field is from a point charge, while the simplest B-field is from a magnetic dipole (e.g. Bar Magnet)

Maxwell's equations

Essentially all of Electricity & Magnetism can be described by a set of 4 equations, referred to as Maxwell's equations.

$\oint E \cdot dA$

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

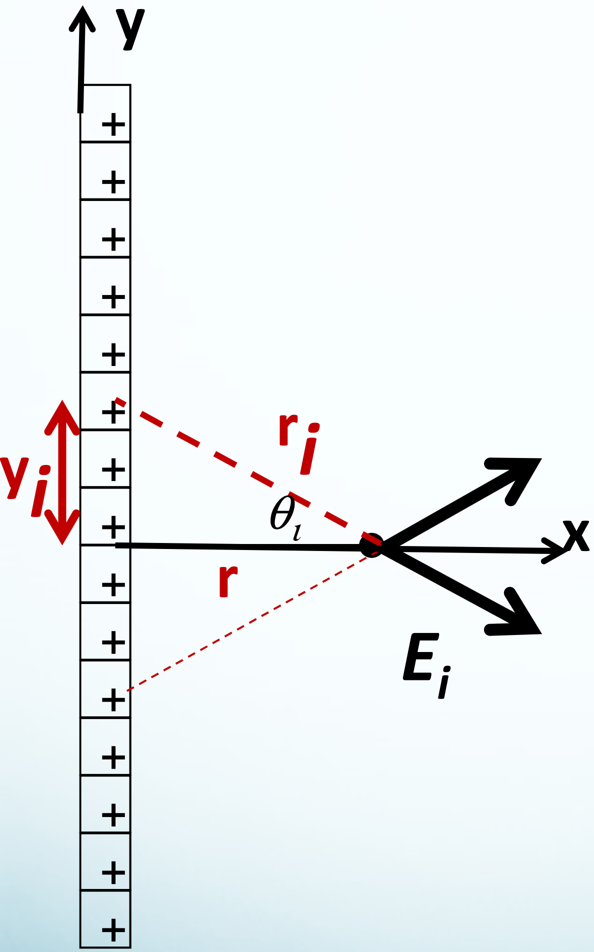
$\oint B \cdot dA = 0$

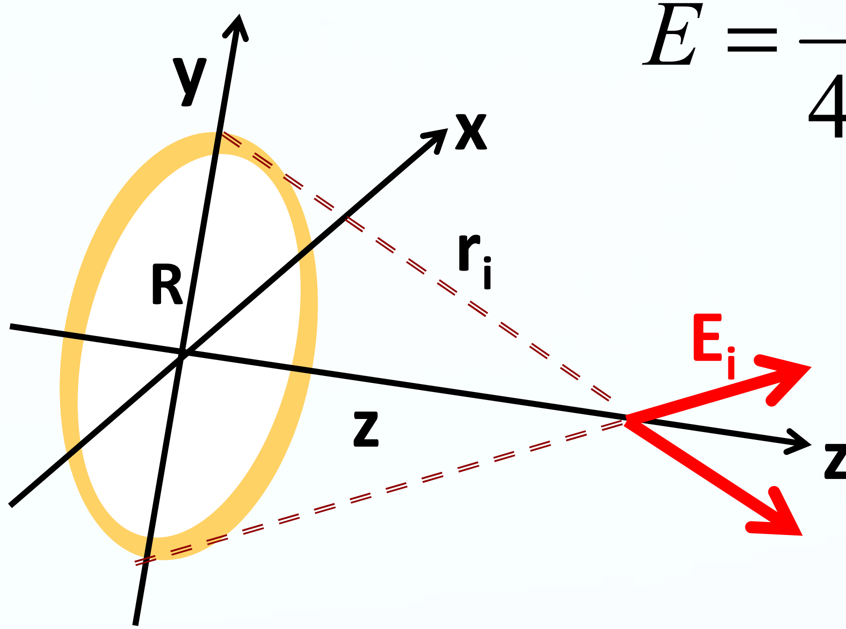
→ We now have two of them!

$$\Phi_E = \oint \vec{E} \cdot d\vec{a} = \frac{Q_{encl}}{\epsilon_0} \qquad \Phi_B = \oint \vec{B} \cdot d\vec{a} = 0$$

Electrostatics

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$$





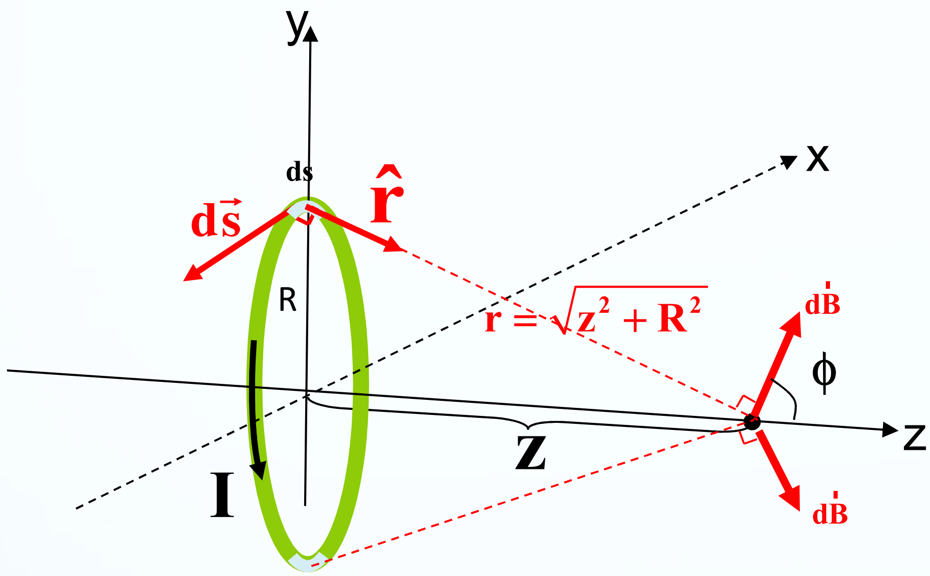
Savior:

Gausses' law

$$\oint \vec{E}.d\vec{A} = \frac{Q_{in}}{\varepsilon_0}$$

Magnetostatics

$$\vec{B}_{\text{current segment}} = \frac{\mu_o}{4\pi}\frac{I\Delta\vec{s}\times\hat{r}}{r^2}$$



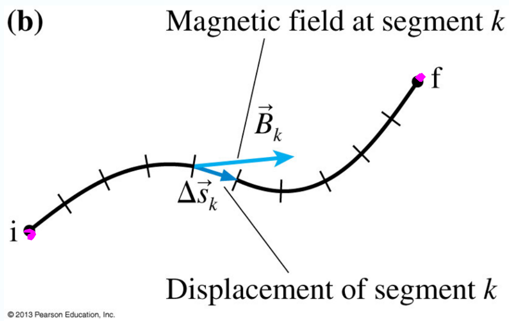
Savior:

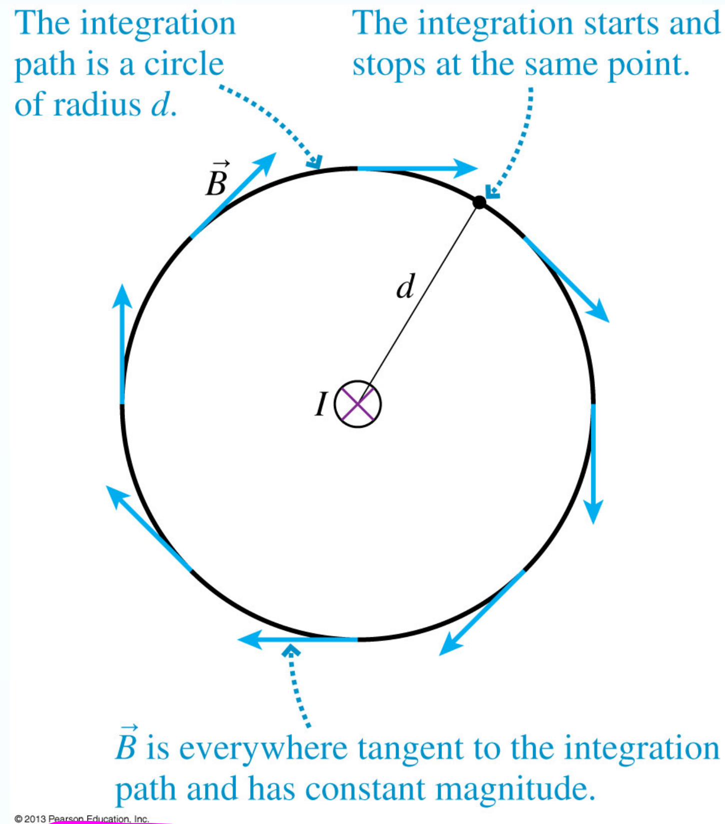
Ampere's law

Expression?

Ampère's law

The line integral of **B** along the path:





$$\int_i^f \vec{B}.d\vec{l}$$

$$\oint \vec{B} \cdot d\vec{l} = (2\pi r)\left(\frac{\mu_0 I}{2\pi r}\right)$$

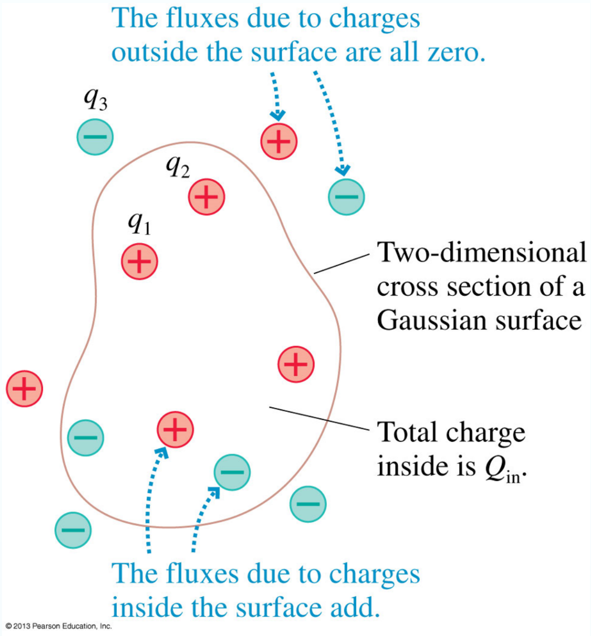
$= B \ell$

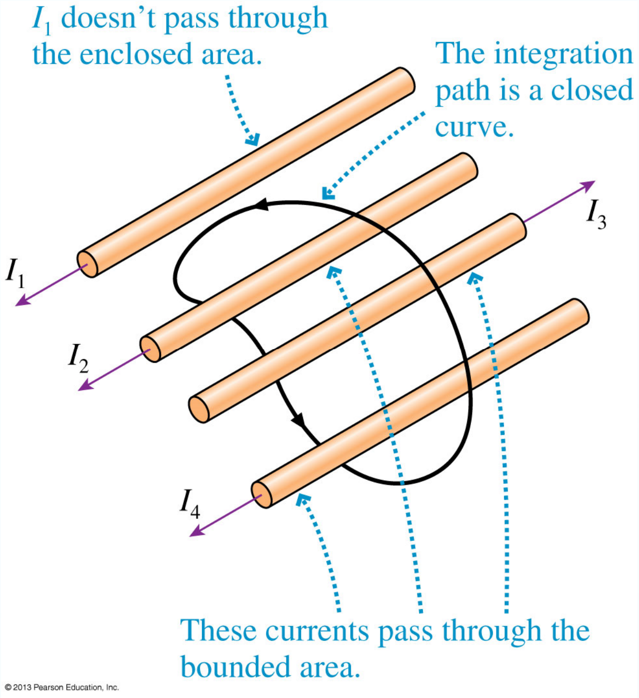
Infinite wire→ $B = \dfrac{\mu_0 I}{2\pi r}$

$\Rightarrow$ $\boxed{\text{i.e.} \oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}}$

$\oint E \cdot dA = \dfrac{q_{enc}}{\varepsilon_0}$

Ampère's *Law is true for any* ***shape of path*** *and any current distribution*



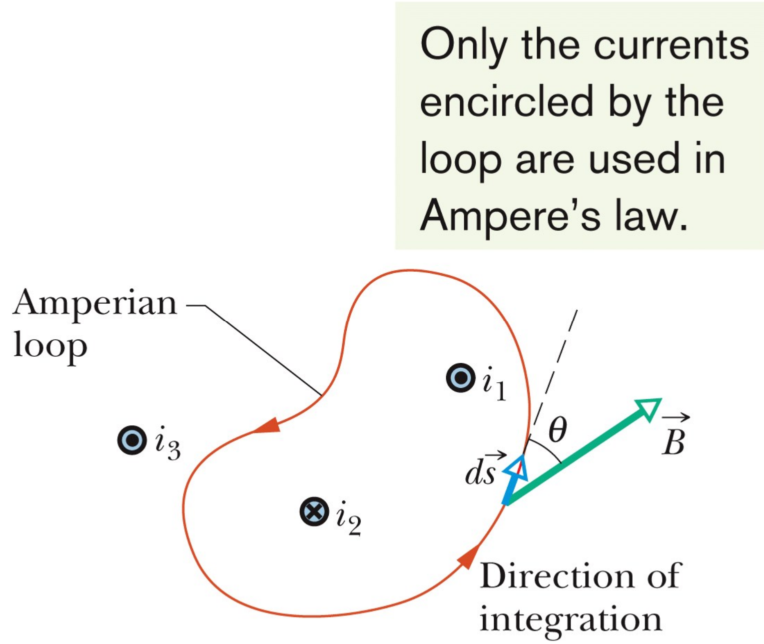


$$\oint \vec{E}.d\vec{A} = \frac{Q_{in}}{\varepsilon_0}$$

For a closed surface enclosing total Charge Q

$$\oint \vec{B}.d\vec{l} = \mu_0 I_{enclosing}$$

Current I passes through an area bounded by a closed curve



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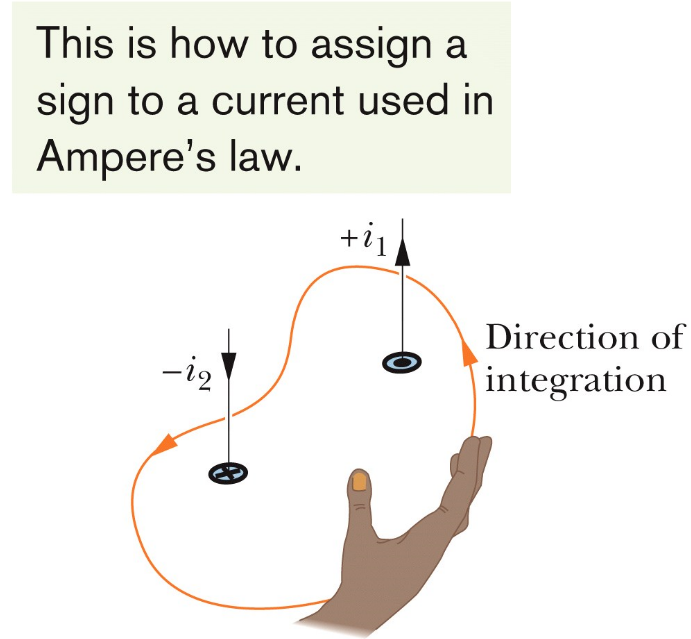

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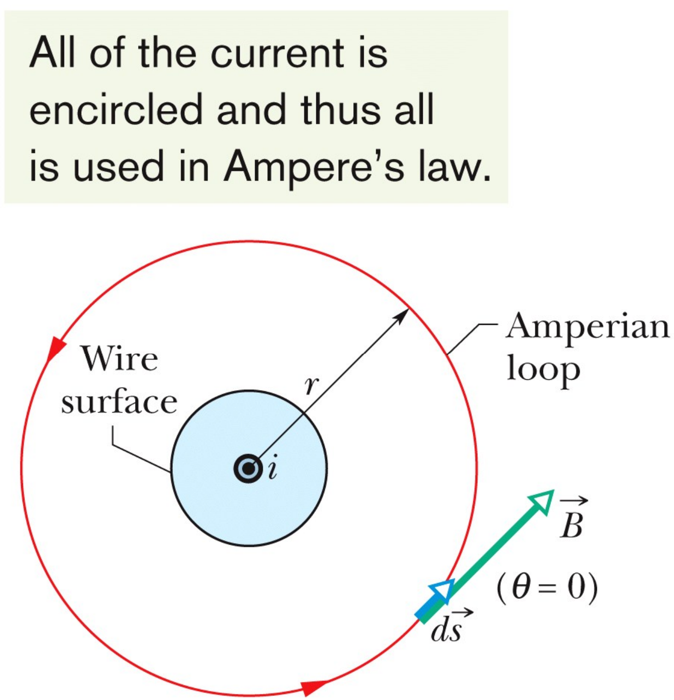

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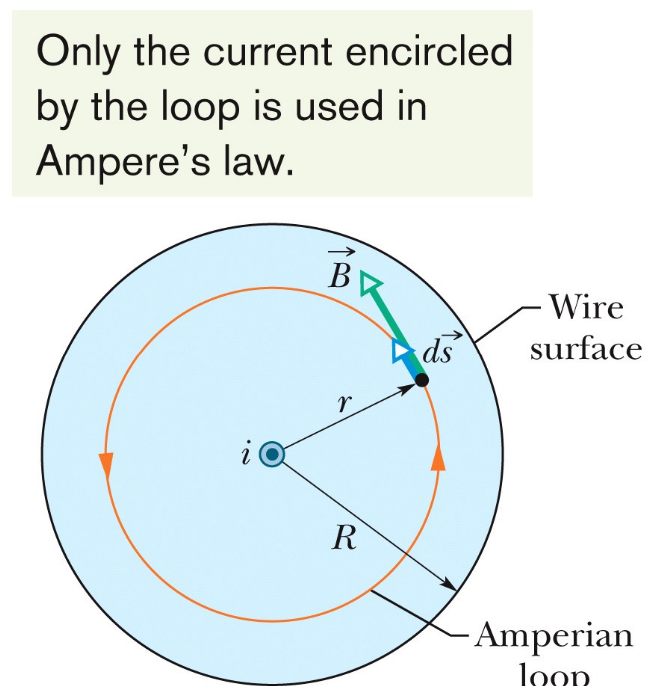

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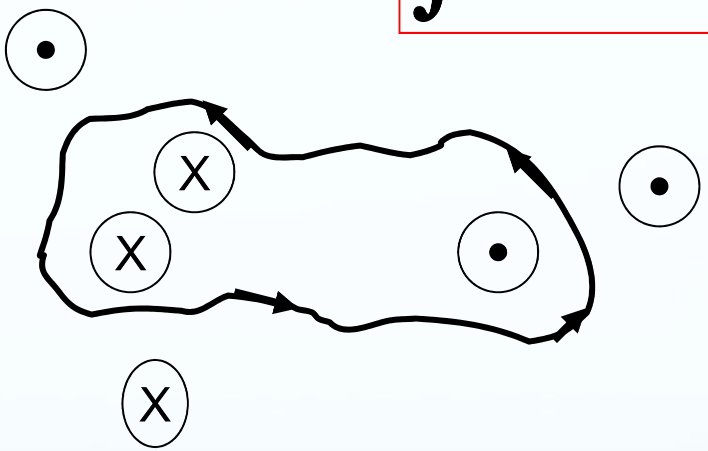
TopHat Question

What is I_{encl} here, where all three wires have 5 A?

$$\oint \vec{B} \cdot d\vec{L} = \mu_0 I_{through}$$



A) -5 A

B) 5A

C) 15 A

D) -15 A

E) other

This section we talked about:

Chapter 29

See you on Thursday