
Fill in these boxes with your SURNAME and INITIAL.

University of Calgary

Faculty of Science

Final Exam Test

PHYSICS 259 ALL LECTURE SECTIONS

April 19 2013, 8:00-11:00 a.m. **Time: 3 hours.**

DO NOT TEAR OFF THIS PAGE!

Answer all questions. There is an equation sheet and table of integrals on the last two pages. You may tear these two pages off if you wish.

This is a closed-book exam worth a total of 52 points. Use of the Schulich calculator or equivalent is allowed.

Write your Last Name and Initial on this top sheet in the grid above. (Do not write your ID number on this page.) **Also write your ID in the grid at top right of Page 2 of the Question paper. DO THIS NOW**.

Make sure this question paper booklet contains 20 pages. If you are missing any pages, get a new booklet from the exam supervisor.

You should also have a **separate set of Answer Sheets**. This is where you enter Multiple Choice answers of Part I and also detailed solutions to the problems of Part II. Only work entered in the indicated spaces on the Answer Sheets will be marked.

IMPORTANT: YOUR ID NUMBER IS TO BE ENTERED AT THE TOP OF EACH AND EVERY ONE OF THE ANSWER SHEETS. DO THIS NOW.

Begin working on the examination when instructed to do so by the supervisor.

UCID: | | | | | | | | |

Part I: Multiple-Choice Questions (Total: 20 marks)

Enter answers to multiple choice questions on the first Answer Sheet using space provided in the upper right of the page. Each question in Part I is worth one point. You should complete Part I in about 1 hour.

1) A particle with charge $q = -1.0$ Coulombs and mass $m = 1$ kg is moving in the positive z-direction with a speed of $v = 5$ m/s. If the magnetic field is $\vec{B} = (3\hat{\imath} - 4\hat{\jmath})$ T, then what is the magnetic force on the particle?

a) $(-20\hat{\imath} - 15\hat{\jmath})N$ $\Leftarrow \checkmark$
b) $(+20\hat{\imath} + 15\hat{\jmath})N$
c) $(+20\hat{\imath} - 15\hat{\jmath})N$
d) $(-20\hat{\imath} + 15\hat{\jmath})N$
e) none of the above

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2) A particle with mass m_1, charge q_1, and speed v_1 is moving perpendicular to a uniform magnetic field with magnitude $|\vec{B}|$. A second particle with $m_2 = m_1$, $q_2 = q_1$ and $v_2 = 3v_1$ is also moving perpendicular to $\vec{B}$. Which of the following statements about gyroradius (radius of the orbit) R and gyrofrequency (cyclotron frequency) ω is correct?

a) $R_1 < R_2$ and $\omega_1 = \omega_2$ $\Leftarrow \checkmark$
b) $R_1 = R_2$ and $\omega_1 = \omega_2$
c) $R_1 = R_2$ and $\omega_1 > \omega_2$
d) $R_1 > R_2$ and $\omega_1 = \omega_2$
e) none of the above

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3) A wire segment of length $L = 1.2$ metres carries a current of $I = 3.5$ Amperes in the x-y plane at an angle $\theta = 30°$ as shown in Figure 1. A magnetic field of $B = 0.5$ Tesla points out of the page. The magnetic force on the wire (in Newtons) is closest to

a) $+1.1\hat{\imath} + 1.8\hat{\jmath}$
b) $-1.1\hat{\imath} + 1.8\hat{\jmath}$ $\Leftarrow \checkmark$
c) $-1.8\hat{\imath} + 1.1\hat{\jmath}$
d) $+1.1\hat{\imath} - 1.8\hat{\jmath}$
e) $+1.8\hat{\imath} + 1.1\hat{\jmath}$

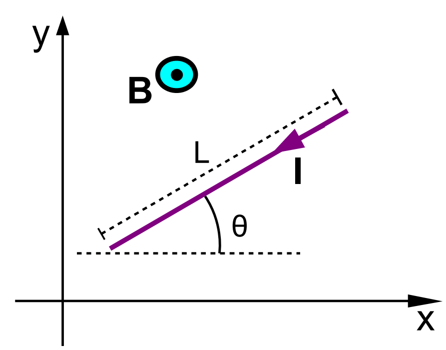

Figure 1: Current segment in a magnetic field.

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4) Three very long straight parallel wires each carry 3.0 Amperes of current directed out of the page as shown in Figure 2. The dimensions are H=2.0 cm and L=3.0 cm. What is the magnitude of the magnetic field B produced by these currents at the origin?

a) 7.0×10^{-5} Tesla
b) 1.0×10^{-5} Tesla
c) 3.0×10^{-5} Tesla $\Leftarrow \checkmark$
d) zero
e) none of the above

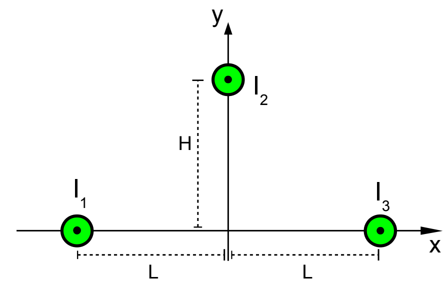

Figure 2: Three parallel currents.

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5) In Figure 3 the wires carry currents of $I_1 = I_5 = 0.5$ Amperes, $I_2 = I_4 = 1.0$ A and $I_3 = 0$. Which of the following statements is true?

a) The magnitude of $\oint \vec{B} \cdot \vec{dl}$ around loop A is $2\mu_0$ Tesla-metres.
b) The quantity $\oint \vec{B} \cdot \vec{dl}$ around loop A is zero $\Leftarrow \checkmark$
c) The quantity $\oint \vec{B} \cdot \vec{dl}$ around loop B is zero.
d) The quantity $\oint \vec{B} \cdot \vec{dl}$ clockwise around loop B is positive, and negative going counterclockwise.
e) none of the above

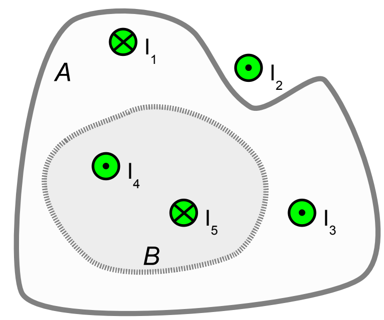

Figure 3: Five infinitely long wires and two closed loops.

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6) A proton with a velocity $\vec{v} = v_0\hat{i}$ moves in a magnetic field $\vec{B} = B_0\hat{j}$. The sum of forces due to electric and magnetic fields is zero if:

a) $\vec{E} = -B_0 v_0 \hat{j}$
b) $\vec{E} = B_0 v_0 \hat{j}$
c) $\vec{E} = -B_0 v_0 \hat{k}$ $\Leftarrow \checkmark$
d) $\vec{E} = B_0 v_0 \hat{k}$
e) none of the above

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7) In Figure 4 we define the potential very far away to be zero. An amount of positive charge $+2Q$ is placed on the spherical conducting shell. Which of the following is true?

a) the potential is constant for $r > b$.
b) the potential is zero everywhere inside the spherical shell ($r < a$).
c) the potential is zero in the conductor ($a < r < b$).
d) the potential is $\dfrac{Q}{2\pi\epsilon_0 r}$ for $r > b$. $\Leftarrow \checkmark$
e) none of the above

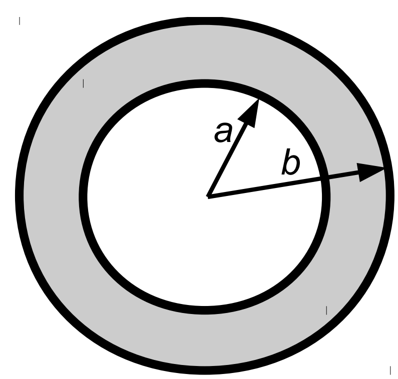

Figure 4: Spherical conducting shell with inner radius a and outer radius b.

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8) Two protons are both moving along the x-axis with equal speeds v and are separated by a constant distance d. There are no external electric or magnetic fields. The strength of the magnetic force between the two charges is

a) $\frac{\mu_0}{4\pi}\frac{q^2v^2}{d^2}$
b) $\frac{\mu_0}{4\pi}\frac{qv}{d^2}$
c) v^2/c^2
d) zero $\Leftarrow \checkmark$
e) none of the above

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9) For the capacitor network in Figure 5, use the values $C_1 = 1\mu F$, $C_2 = C_3 = 2\mu F$, and $C_4 = C_5 = 4\mu F$. What is the equivalent (or effective) capacitance?

a) $\frac{1}{2}\mu F$
b) $1\mu F$ $\Leftarrow \checkmark$
c) $2\mu F$
d) $4\mu F$
e) $8.8\mu F$

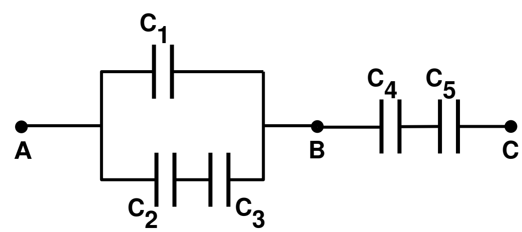

Figure 5: Capacitor network.

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10) The capacitor in Figure 6 is charged up by the battery to a voltage V. The battery is then disconnected without changing the voltage across the capacitor. After this, the separation between plates is increased to $d' = 2d$. Which of the following is true?

a) the charge doubles
b) the voltage doubles $\Leftarrow \checkmark$
c) the voltage halves
d) the charge halves
e) none of the above

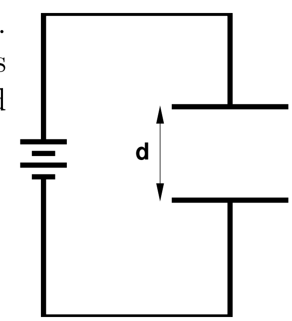

Figure 6: A parallel plate capacitor and battery.

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11) For ideal current and voltage measuring devices (Ammeters and Voltmeters) which of the following statements is true?

a) An ammeter should have zero resistance and be placed in parallel with the part of the circuit being measured.
b) A voltmeter should have zero resistance and be placed in parallel with the part of the circuit being measured.
c) A voltmeter should have an infinite resistance and be placed in series with the part of the circuit being measured.
d) An ammeter should have an infinite resistance and be placed in series with the part of the circuit being measured.
e) none of the above. $\Leftarrow \checkmark$

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12) For the circuit shown in Figure 7, assume that the magnetic field has a magnitude of 4 Tesla, pointing into the page. Use a value of $L = 1$ metre for the length of the sliding bar. If the bar is moving at $3m/s$, what is the magnitude of the EMF around the loop?

a) 12 volts $\Leftarrow \checkmark$
b) 20 volts
c) 16 volts
d) 0.2 volts
e) none of the above

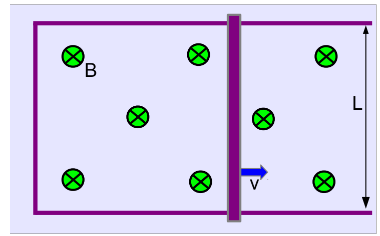

Figure 7: U-shaped conducting rails and a conducting bar of length L moving with speed v in the $+x$ direction.

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13) For the circuit shown in Figure 8 assume that the normal to the loop is pointing into the page ($\hat{n} = -\hat{k}$). If the current through the infinite wire is constant in the direction shown ($+\hat{i}$), and the loop is moving away from the wire in the $+\hat{j}$ direction, which the the following statements are true?

a) $\Phi_B > 0$ and $\dfrac{d\Phi_B}{dt} < 0$

b) $\Phi_B < 0$ and $\dfrac{d\Phi_B}{dt} > 0$ $\Leftarrow \checkmark$

c) $\Phi_B < 0$ and $\dfrac{d\Phi_B}{dt} < 0$

d) $\Phi_B > 0$ and $\dfrac{d\Phi_B}{dt} > 0$

e) none of the above

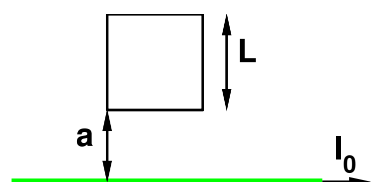

Figure 8: Long straight current I_0 and square loop.

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14) In Figure 8 imagine that you are able to move the square loop anywhere in the plane of the page at a constant velocity. If a constant current I_0 is flowing through the wire, what direction of motion of the loop would generate the largest EMF in the loop?

a) Towards the wire $\Leftarrow \checkmark$
b) Away from the wire
c) To the left
d) To the right
e) The direction does not matter, all directions would generate the same EMF.

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15) The graph in Figure 9 is a plot of power (in watts) vs voltage (in volts) for a particular material. What is the resistance of the material?

a) $R = 1.0\,\Omega$
b) $R = 0.1\,\Omega$ $\Leftarrow \checkmark$
c) $R = 10\,\Omega$
d) $R = 0.01\,\Omega$
e) $R = 2.0\,\Omega$

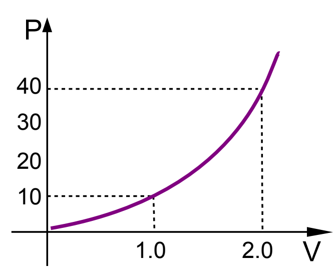

Figure 9: Voltage versus power.

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16) Two equal positive point charges, each of charge q, are separated by distance L. There are no other charges anywhere. The potential at the midpoint of the line joining the charges is defined to be zero. The electrostatic potential at an infinite distance from the two charges is

a) $-\frac{1}{4\pi\epsilon_0}\frac{2q}{L}$

b) $+\frac{1}{4\pi\epsilon_0}\frac{2q}{L}$

c) $-\frac{1}{4\pi\epsilon_0}\frac{4q}{L}$ $\Leftarrow \checkmark$

d) $+\frac{1}{4\pi\epsilon_0}\frac{4q}{L}$

e) Zero

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17) Figure 10 shows a Hall probe 5.0 cm in length, 1.8 cm in width and 0.12 cm in thickness in a 0.40 T magnetic field directed upward.The drift speed of the electrons in the Hall probe is 0.82 mm/s. The strength of the electrostatic field generated in the Hall probe is (select the closest answer)

a) $5.2 \times 10^{-23}\, V/m$
b) $6.7 \times 10^{-6}\, V/m$
c) $3.3 \times 10^{-4}\, V/m$ $\Leftarrow \checkmark$
d) Zero
e) Need more information.

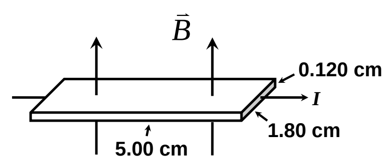

Figure 10: Hall effect probe.

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18) The switch S in Figure 11 is closed at time t=0. At what time is the current in the circuit equal to 1.50 A? (Select the closest answer.)

a) 0.231 s
b) 8.32 s $\Leftarrow \checkmark$
c) 1.39 s
d) 1.73 s
e) 2.88 s

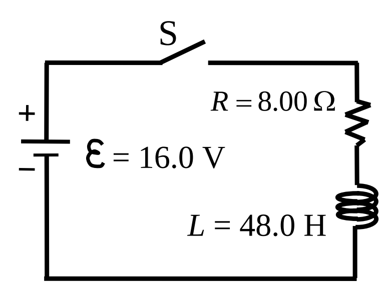

Figure 11: RL circuit.

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19) A capacitor has two parallel plates of area A separated by distance d. A slab of material with dielectric coefficient κ completely fills the space between the plates. Which of the following expressions for the total energy stored in the capacitor is correct?

a) $\frac{\kappa}{2}\frac{Q^2 d}{A}$

b) $\frac{\kappa\epsilon_0}{2}\frac{Q^2 d}{A}$

c) $\frac{\kappa\epsilon_0}{2}\frac{V^2 d}{A}$

d) $\frac{\kappa}{2}\frac{V^2 d}{A}$

e) none of the above $\Leftarrow \checkmark$

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20) Material with resistivity ρ is used to construct a resistor of length L and cross-sectional area A. Which of the following choices correctly expresses the relationship between potential drop V across the resistor and current I flowing through the resistor?

a) $V = \frac{\rho L}{A} I$ $\Leftarrow \checkmark$

b) $\vec{J} = \sigma \vec{E}$

c) $V = \frac{\rho A}{L} I$

d) $V = \frac{L}{\rho A} I$

e) none of the above

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21) This is **version @V@** of the exam. Please select @V@ in the row marked "Version" in the Multiple Choice Answers area on the first Answer Sheet. **Be sure to do this now, before moving on to other questions on the paper**.

This is the end of the Multiple Choice part of the exam. You may now proceed to Part II. Remember that your answers to Part II are to be entered on the Answer Sheet pages.

Part II: Written Answer Questions (Total: 32 marks)

IMPORTANT: Write your answers to the problems in Part II in the corresponding boxes on the Answer Sheets. Work must be shown for full marks. Rough work can be done on the back of this question paper, but only the work appearing on the Answer Sheets will be marked.

22) *[5.0 marks]* For each of the concepts listed below, write down the defining equation.

- **a)** Coulomb's law *[1.0 marks]*
- **b)** Gauss's law *[1.0 marks]*
- **c)** Biot-Savart law *[1.0 marks]*
- **d)** Ampere's law *[1.0 mark]*
- **e)** Faraday's law *[1.0 mark]*

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23) *[5.0 marks]* The electric potential in some region of space is given by $V = 3xy + 7yz - 11x^2$ (x,y,z have units of metres). A proton starts at a point $P_1 = (x=1, y=2, z=4)$ with an initial speed of $v_1 = 2.5\,m/s$ and moves to $P_2 = (x=1, y=1, z=2)$.

- **a)** What is the electric field at point P_1? *[3.0 marks]*
- **b)** Assume that the only force acting on the proton is due to the electric field $\vec{E}$. How fast will the proton be moving at point P_2? *[2.0 marks]*

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24) *[5.0 marks]* A current I flows through a conductor as shown in Figure 12. The two linear segments are each of length $L_1 = L_2 = L$ and run along the y and x axes respectively. The quarter-circle segment has radius R centered at the origin.

- **a)** What is the magnetic field $\vec{B}$ produced at the origin (x=0,y=0) by each of the two linear current segments L_1 and L_2? *[1.0 marks]*
- **b)** What is the magnetic field $\vec{B}$ produced at the origin (x=0,y=0) by the quarter-circular current segment of radius R? *[3.0 marks]*
- **c)** Consider an additional current I_2 coming out of the page at the origin. What is the force produced by this current on the quarter-circular current segment of radius R? *[1.0 marks]*

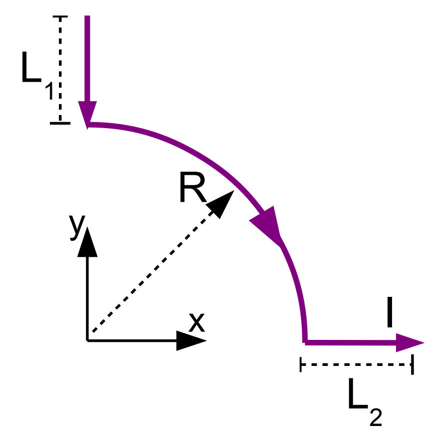

Figure 12: Quarter-circle current loop in the x-y plane.

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25) *[6.0 marks]* A very thin spherical conducting shell with radius r_1 has charge Q_1 uniformly distributed over the surface. A second very thin spherical conducting shell with radius $r_2 > r_1$ has charge Q_2 uniformly distributed over the surface. Both spheres are centered at the origin $(x = 0, y = 0, z = 0)$.

- **a)** Assume that $Q_1 = 0$ and $Q_2 = +Q$. What is the electric potential outside the larger sphere $(r > r_2)$? *[1.0 marks]*

b) Assume that $Q_1 = 0$ and $Q_2 = +Q$. What is the electric potential inside the larger sphere ($r < r_2$)? *[1.0 marks]*

c) Assume that the two spheres carry equal and opposite charges $Q_1 = -Q$ and $Q_2 = +Q$. What is the capacitance of this system of conductors? (show your work) *[3.0 marks]*

d) Assume that the two spheres are separated by a distance $d = r_2 - r_1$ which is very small compared to either radius ($d << r_1$). What is a useful approximation for the capacitance? *[1.0 marks]*

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26) *[6.0 marks]* A circuit is built with two capacitors $C_1 = 3.0\,mF$ and $C_2 = 5\,mF$ and an ideal battery $V = 9$ volts as shown in Figure 13. Initially the circuit behaves as expected, fully charging the two capacitors. However, at time $t = 0$ the battery fails and starts acting like a resistor $R = 3\,k\Omega$.

a) What is the total charge on the capacitors at $t = 0$? *[1.0 marks]*

b) How long will it take for the charge on C_1 to drop to 50% of its initial value? *[2.0 marks]*

c) What will be the current I through the resistor (formerly a battery) at $t = 60s$? *[2.0 marks]*

d) What is the total energy dissipated in the resistor after a long time ($t \to \infty$)? *[1.0 marks]*

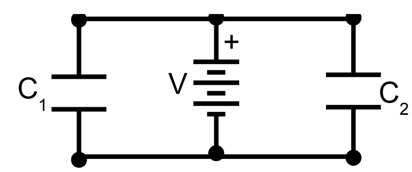

Figure 13: Circuit with two capacitors and one battery.

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27) *[5.0 marks]* The core of a toroidal solenoid has a rectangular cross-section as shown in Figure 14 with inner radius r_1, outer radius r_2, and thickness H. A steady current I flows through N evenly spaced loops wound around the core.

a) Assume that the magnetic field inside the solenoid ($r_1 < r < r_2$) is a constant B_0. What is the self inductance of this system? *[2.0 marks]*

b) Use Ampere's law to determine the magnetic field for $r_1 < r < r_2$. *[3.0 marks]*

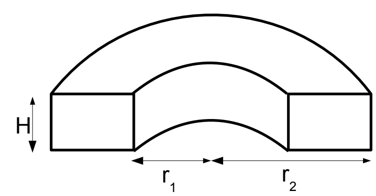

Figure 14: Toroidal solenoid.

CONSTANTS AND USEFUL EQUATIONS

k = Coulomb constant = 8.99×10^{9} N m^2 C^{-2}
ϵ_0 = permittivity of free space = 8.85×10^{-12} C^2 N^{-1} m^{-2}
μ_0 = permeability of free space = $4\pi \times 10^{-7}$ Wb A^{-1} m^{-1}
e = fundamental charge = 1.602×10^{-19} C
m_e = mass of electron = 9.11×10^{-31} kg
m_p = mass of proton = 1.67×10^{-27} kg
K_{air} = dielectric constant of air = 1.00059

m = 10^{-3} $\mu = 10^{-6}$ n = 10^{-9} p = 10^{-12}

Area of a circle: $A = \pi r^2$
Circumference of a circle: $C = 2\pi r$
Surface area of a sphere: $A = 4\pi r^2$
Volume of a sphere: $V = \frac{4}{3}\pi r^3$
Surface area of a cylinder: $A = 2\pi r L$
Volume of a cylinder: $V = \pi r^2 L$

$$x = x_0 + v_{x0}t + \frac{1}{2}a_x t^2 \qquad v_x = v_{x0} + a_x t \qquad v_x^2 = v_{x0}^2 + 2a_x x \qquad \vec{F} = m\vec{a}$$

$$\vec{F} = k\frac{q_1 q_2}{r^2}\,\hat{r} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}\,\hat{r} \qquad \vec{E} = \frac{\vec{F}}{q} \qquad \vec{E} = k\frac{q}{r^2}\,\hat{r} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\,\hat{r}$$

$$\Phi_E = \oint_A \vec{E}\cdot d\vec{A} = \oint_A E dA cos\theta = \frac{Q_{encl}}{\epsilon_0} \qquad V = \frac{U}{q} \qquad U = k\frac{q_1 q_2}{r} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r}$$

$$V = k\frac{q}{r} = \frac{1}{4\pi\epsilon_0}\frac{q}{r} \qquad W = qV_{ab} \qquad V_{ab} = V_a - V_b = \int_a^b \vec{E}\cdot d\vec{l}$$

$$\vec{E} = -\vec{\nabla}V = -\left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)V \qquad C = \frac{Q}{V_{ab}} \qquad C = \frac{\epsilon_0 A}{d}$$

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}QV = \frac{1}{2}CV^2 \qquad \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \qquad C = C_1 + C_2 + C_3$$

$$C = KC_0 = K\epsilon_0\frac{A}{d} = \epsilon\frac{A}{d} \qquad u = \frac{1}{2}\epsilon_0 E^2 \qquad P = I^2 R = \frac{V^2}{R}$$

More Equations on Next Page

CONSTANTS AND USEFUL EQUATIONS (Continued)

$$V = IR \qquad P = \mathcal{E}I \qquad \frac{1}{R} = \frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3} \qquad R = R_1+R_2+R_3$$

$$\sum i = 0 \qquad \sum (\mathcal{E} + iR) = 0 \qquad nq = -\frac{J_x B_y}{E_z} \qquad \vec{J} = \sum_i n_i q_i \vec{v}_i$$

$$\vec{F} = q\vec{v}\times\vec{B} \qquad \Phi_B = \int \vec{B}\cdot d\vec{A} \qquad r = \frac{mv}{qB} \qquad \vec{F} = I\vec{l}\times\vec{B} \qquad \vec{\mu} = I\vec{A}$$

$$\vec{\tau} = \vec{\mu}\times\vec{B} \qquad \vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2} \qquad d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\vec{dl}\times\hat{r}}{r^2} \qquad U = -\vec{\mu}\cdot\vec{B}$$

$$\oint \vec{B}\cdot\vec{dl} = \mu_0 I_{encl} \qquad \mathcal{E}_2 = -N_2\frac{d\Phi_{B2}}{dt} \qquad \mathcal{E}_2 = -M\frac{di_1}{dt} \qquad \mathcal{E} = -L\frac{di}{dt}$$

$$\mathcal{E} = \int_a^b \left(\vec{v}\times\vec{B}\right)\cdot\vec{dl} \qquad M = \frac{N_2\Phi_2}{i_1} = \frac{N_1\Phi_1}{i_2} \qquad L = \frac{N\Phi}{i} \qquad u = \frac{B^2}{2\mu_0}$$

$$\tau = RC \qquad \tau = \frac{L}{R} \qquad x = x_0 e^{-\frac{t}{\tau}} \qquad x = x_0\left(1 - e^{-\frac{t}{\tau}}\right) \qquad U = \frac{1}{2}LI^2$$

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} \qquad \int \frac{dx}{\sqrt{x^2+a^2}} = ln\left(x+\sqrt{x^2+a^2}\right)$$

$$\int \frac{dx}{x^2+a^2} = \frac{1}{a}\arctan\frac{x}{a} \qquad \int \frac{dx}{\left(x^2+a^2\right)^{3/2}} = \frac{1}{a^2}\frac{x}{\sqrt{x^2+a^2}}$$

$$\int \frac{xdx}{\left(x^2+a^2\right)^{3/2}} = -\frac{1}{\sqrt{x^2+a^2}}$$

Multiple Choice Answer Sheet - Version A

#1 b
#2 d
#3 a

#4 c
#5 d
#6 c
#7 d

#8 d
#9 b
#10 b
#11 e

#12 a
#13 b
#14 a
#15 e Paul's bubble sheet goes here

#16 d
#17 e
#18 d

#19 e
#20 b

Written Answer Sheets

Phys 259 Final Exam (Winter, 2013): Instructions to Markers

You are responsible for marking the Problems, Part II. The multiple choice answers are scored automatically. The "Version" number applies only to the multiple-choice answers; there is only one version of Part II. You should also have these general instructions, plus a set of solutions with detailed instructions, and a copy of an exam question paper. The solutions are presented in Answer Page format for ease of reference.

Print out these Instructions, and read them before starting to mark. You might be tempted to bypass these, but you will save us much time and trouble if you follow instructions.

On the following pages, I've given a suggested detailed marking scheme for each question. However, not all solutions will follow the one I've given, especially for the 2-mark and 3-mark questions, so use your judgment. The following broad guidelines may help:

- In 3-mark questions, the first half mark should be easy to get: if a student shows any reasonable insight at all, give a half mark. In an integration problem, If the student simply makes a valiant attempt at integrating, this might be enough.
- Give half of the allotted marks (e.g., 1.5 marks out of 3) if you feel that the student has got half way to the right answer. For example, if a student makes five mistakes but still has the solution half right, give 1.5; don't deduct five half-marks.
- In numerical questions, the final answer must have correct units. Deduct 0.5 mark if the units on the final answer are not shown or are incorrect.
- Be lenient on significant figures.
- If a lot of students are making the same "mistake", check my answer - I could be the one who is wrong! If you discover any errors in my solutions, please let me know as soon as possible (physics259@ucalgary.ca) so I can alert all other markers.

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Q22 *[5.0 marks]*
For each of five laws, write down the corresponding equation. (Descriptive sentence not required)

The correct answer for each is worth 1.0 marks.

Subtract 0.5 if there are one or more minor errors such as missing vector signs, dot/cross symbols, or circles for closed integrals. Give 0.5 even if there are multiple minor errors.

Give 0 if there are any major errors such as $\vec{B}$ instead of $\vec{E}$ or $\cdot d\vec{l}$ instead of $\cdot d\vec{a}$

Q22a *[1.0 marks]*

Coulomb's law $\boxed{\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}}$

Q22b *[1.0 marks]*

Gauss's law $\boxed{\oint \vec{E}\cdot d\vec{a} = \frac{Q_{enc}}{\epsilon_0}}$ Accept k for $\frac{1}{4\pi\epsilon_0}$

Q22c *[1.0 marks]*

Biot-Savart law $\boxed{d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{l}\times\hat{r}}{r^2}}$

Q22d *[1.0 marks]*

Ampere's law $\boxed{\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc}}$

Q22e *[1.0 marks]*

Faraday's law $\boxed{\mathcal{E} = -\frac{d\Phi_B}{dt}}$

Q23a *[3.0 marks]*
The electric field vector is the negative gradient of potential 0.5 for correct equation

$$\vec{E} = -\nabla V = -\frac{\partial V}{\partial x}\hat{i} - \frac{\partial V}{\partial y}\hat{j} - \frac{\partial V}{\partial z}\hat{k}$$

with components evaluated at location $P_1 = (1, 2, 4)$ 1.0 for correct derivatives & 0.5 for substituting location correctly

$$E_x = -(3y + 0 - 22x) = 22x - 3y = 22 - 6 = 16V/m$$

$$E_y = -(3x + 7z + 0) = -3x - 7z = -3 - 28 = -31V/m$$

$$E_z = -(0 + 7y + 0) = -7y = -14V/m$$

The electric field at P_1 is $\boxed{\vec{E} = 16\hat{i} - 31\hat{j} - 14\hat{k}\, V/m}$ 0.5 for numerical result & 0.5 for units

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Q23b *[2.0 marks]*

0.5 for either of these equations If there are no other forces then we can use conservation of energy

$$K_i + U_i = K_f + U_f$$

for initial and final kinetic and electrostatic potential energies

$$\frac{1}{2}mv_i^2 + qV_i = \frac{1}{2}mv_f^2 + qV_f$$

We'll need to determine the initial and final potentials

$$V_i = 3xy + 7yz - 11x^2 = 3(1)(2) + 7(2)(4) + 11(1)^2 = 6 + 56 + 11 = 73V$$

$$V_f = 3xy + 7yz - 11x^2 = 3(1)(1) + 7(1)(2) + 11(1)^2 = 3 + 14 + 11 = 28V$$

and then re-arrange to isolate the final kinetic energy 0.5 for general idea

$$\begin{aligned}
\frac{1}{2}mv_f^2 &= \frac{1}{2}mv_i^2 + q(V_i - V_f) \\
&= \frac{1}{2}(1.67 \times 10^{-27}kg)(2.5m/s)^2 + (+1.6 \times 10^{-19}C)(73 - 28)V \\
&= 5.22 \times 10^{-27}J + 7.20 \times 10^{-18}J \approx 7.20 \times 10^{-18}J
\end{aligned}$$

from which we can solve for the final speed 0.5 for numerical result & 0.5 for units

$$v_f = \sqrt{\frac{2K_f}{m}} = \sqrt{\frac{2(7.20 \times 10^{-18}J)}{1.67 \times 10^{-27}kg}} = \boxed{9.29 \times 10^4\ m/s}$$

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Q24a *[1.0 marks]*

The magnetic field produced by a current element is given by the Biot-Savart law

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{l} \times \hat{r}}{r^2}$$

For segment #1 current is flowing down so $d\vec{l} = -\hat{j}dy$. The vector pointing from the current to the observation point at the origin

$$\vec{r} = (0\hat{i} + 0\hat{j}) - (0\hat{i} + y\hat{j}) = -y\hat{j}$$

is parallel to the current element, so the cross product gives

$$d\vec{l} \times \hat{r} = (-\hat{j}) \times (-\hat{j}) = 0$$

and the total magnetic field from the upper linear segment is

$$\vec{B} = \int d\vec{B} = \int 0 = \boxed{0 \text{ Tesla}}$$

For segment #2

$$d\vec{l} = +\hat{i} \qquad \hat{r} = -\hat{i}$$

so the cross product is zero, and so is the total magnetic field is

$$\vec{B} = \int d\vec{B} = \int 0 = \boxed{0 \text{ Tesla}}$$

0.5 for correct answer (units not required) & 0.5 for mathematical proof or plausible explanation ie. right-hand-rule

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Q24b *[3.0 marks]*

The magnetic field produced by a current element is given by the Biot-Savart law

0.5 for the correct equation

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I d\vec{l} \times \hat{r}}{r^2}$$

For the quarter-circular current we can use the right-hand rule to determine that all the field contributions will be directed into the page 0.5 for this key property

$$d\vec{l} \times \hat{r} = -\hat{k}$$

so the total magnetic field is 0.5 for simplifying equation

$$\vec{B} = \int d\vec{B} = \int \frac{\mu_0}{4\pi}\frac{I}{r^2}(-\hat{k})dl = \frac{\mu_0}{4\pi}\frac{I}{R^2}(-\hat{k})\int dl$$

because all points on the quarter circle are at a constant distance R from the origin. The circumference of a circle is $2\pi R$ so the length of a quarter-circle is 0.5 for correct length

$$\int dl = \frac{\pi R}{2}$$

and the total magnetic field will be 0.5 for answer & 0.5 for units

$$\vec{B} = \boxed{-\frac{\mu_0 I}{8R}\,\hat{k}\ \text{Tesla}}$$

Q24c *[1.0 marks]*

The force on a current segment in a magnetic field is 0.5 for equation

$$\vec{F} = I\vec{L} \times \vec{B}$$

A current I_2 coming out of the page $(+\hat{k})$ at the origin will produce a magnetic field directed counter-clockwise with magnitude (don't require this equation)

$$\vec{B} = \frac{\mu_0 I}{4\pi r}\hat{\phi}$$

that is constant everywhere along the quarter-circle. Since the quarter-circular current and magnetic field are anti-parallel everywhere

$$\hat{L} \cdot \hat{B} = -1 \qquad\qquad \vec{L} \times \vec{B} = 0$$

and the net force on the quarter-circular current is zero. 0.5 for answer (ignore units)

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Q25a *[1.0 marks]*

Outside a spherically symmetric charge distribution the field and potential are indistinguishable from a point charge at the center so

$$\boxed{V(r > r_2) = \frac{1}{4\pi\epsilon_0}\frac{Q_2}{r} = \frac{+Q}{4\pi\epsilon_0 r}}$$

0.5 for answer (units not required) & 0.5 for proof or plausible agument

Q25b *[1.0 marks]*

Inside a spherically symmetric charge distribution the electric field is zero and the potential is constant. The potential is continuous, so it must match the surface of the sphere

$$\boxed{V(r < r_2) = \frac{1}{4\pi\epsilon_0}\frac{Q}{r_2}}$$

0.5 for answer (units not required) & 0.5 for proof or plausible agument

Q25c *[3.0 marks]*

Between the two conductors the electric potential is 0.5 for superposition

$$V(r_1 < r < r_2) = \frac{1}{4\pi\epsilon_0}\frac{Q_1}{r} + \frac{1}{4\pi\epsilon_0}\frac{Q_2}{r_2} = \frac{1}{4\pi\epsilon_0}\left(\frac{-Q}{r} + \frac{+Q}{r_2}\right)$$

so the potential difference between the shells is 0.5 for for ΔV

$$V(r_2) - V(r_1) = \frac{1}{4\pi\epsilon_0}\left(\frac{-Q}{r_2} - \frac{-Q}{r_1}\right) = \frac{Q}{4\pi\epsilon_0}\frac{r_2 - r_1}{r_1 r_2}$$

From the definition of capacitance 1.0 for definition

$$C = Q/V = \boxed{4\pi\epsilon_0 \frac{r_1 r_2}{r_2 - r_1}}$$

1.0 for correct answer (units not required), 0.5 for minor error

Q25d *[1.0 marks]*

For large plates very close together, all the cases we have studied will reduce to a parallel plate capacitor 1.0 for correct answer, 0.5 for minor error ie. reciprocal or missing ϵ_0

$$\boxed{C \approx \epsilon_0 \frac{A}{d}}$$

which we could also write as $C \approx \frac{4\pi\epsilon_0 r^2}{r_2 - r_1}$ not required

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Q26a *[1.0 marks]*
The two capacitors in this circuit are in parallel, so the effective capacitance is

$$C = C_1 + C_2 = 3.0mF + 5.0mF = 8mF$$

The total charge can be found using the definition of capacitance 0.5 for numerical answer & 0.5 for units

$$Q = CV = (8.0 \times 10^{-3}F)(9V) = \boxed{7.2 \times 10^{-2}\text{ Coulombs}}$$

Q26b *[2.0 marks]*
The time constant for this resistor-capacitor circuit is 0.5 for time constant

$$\tau = RC = (3.0 \times 10^3\Omega)(8.0 \times 10^{-3}F) = 24.0\,s$$

The charge on both capacitors will decay as $C(t) = C_0 \exp(-t/\tau)$ so the charge will be at 50% when 0.5 for exponential decay

$$\frac{1}{2}C_0 = C_0 e^{-t/\tau} \quad \Rightarrow \quad \ln(0.5) = -t/\tau \quad \Rightarrow \quad \boxed{t = \tau \ln 2 = 16.6\,s}$$

0.5 for numerical answer & 0.5 for units

Q26c *[2.0 marks]*
The initial "spike" of current will be 0.5 for correct initial current

$$I_0 = V_0/R = 9.0V/3k\Omega = 3 \times 10^{-3}A = 3\,mA$$

and then decay with time as given by $I(t) = I_0 \exp(-t/\tau)$ so that at $t = 60\,s$
0.5 for exponential decay

$$I(60) = (3.0\,mA)\ \exp(-60.0/24.0) = 2.464 \times 10^{-4}A$$

0.5 for numerical answer & 0.5 for units

Q26d *[1.0 marks]*
The total energy dissipated in the resistor could be found by integrating $P = I^2R$ over time from zero to infinity. It is easier to note that the initial energy stored in the capacitors

$$U = \frac{1}{2}CV^2 = \frac{1}{2}(8.0 \times 10^{-3}F)(9V)^2 = \boxed{0.324\text{ Joules}}$$

will all end up as heat in the resistor. 0.5 for numerical answer & 0.5 for units

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Q27a *[2.0 marks]*

The rectangular cross-sectional area of the toroid is $A = (r_2 - r_1)L$ so the total flux through each loop is 0.5 for idea of flux through one loop

$$\Phi_B = \int \vec{B} \cdot d\vec{a} = \int B_0 \, da = B_0 \int da = B_0 A$$

The total inductance is 0.5 for definition of L

$$L = \frac{N\Phi}{i} = \boxed{\frac{NB_0L(r_2 - r_1)}{I}}$$

1.0 for final result

Q27b *[3.0 marks]*

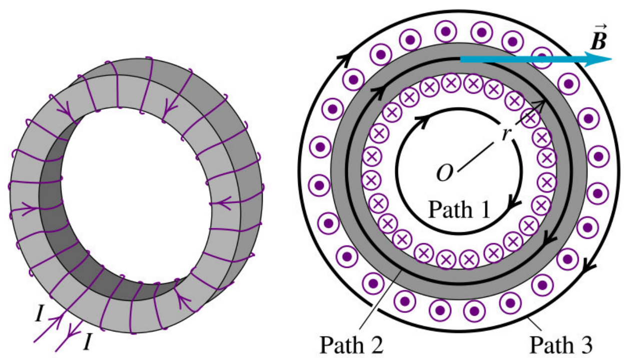
1.0 for figure showing path 2 with direction of current, integration, and $\vec{B}$

Ampere's law $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$

0.5 for equation

By symmetry, we know that $\vec{B}$ inside the toroid will be purely azimuthal in direction, and have constant magnitude at a given radius r. (Could also use RHR argument or cross-product in Biot-Savart law.) Choose a closed loop integral which is also circular so that $\vec{B}$ and $d\vec{l}$ are parallel

0.5 for path integral result



(a) (b)

$$\oint \vec{B} \cdot d\vec{l} = \oint B \, dl = B \oint dl = B 2\pi r$$

This path encloses N coils, each carrying current I into the page (positive for clockwise path integral). 0.5 for enclosed current result

The magnetic field inside the toroid is 0.5 for final result

$$B = \boxed{\frac{\mu_0 N I}{2\pi r}}$$