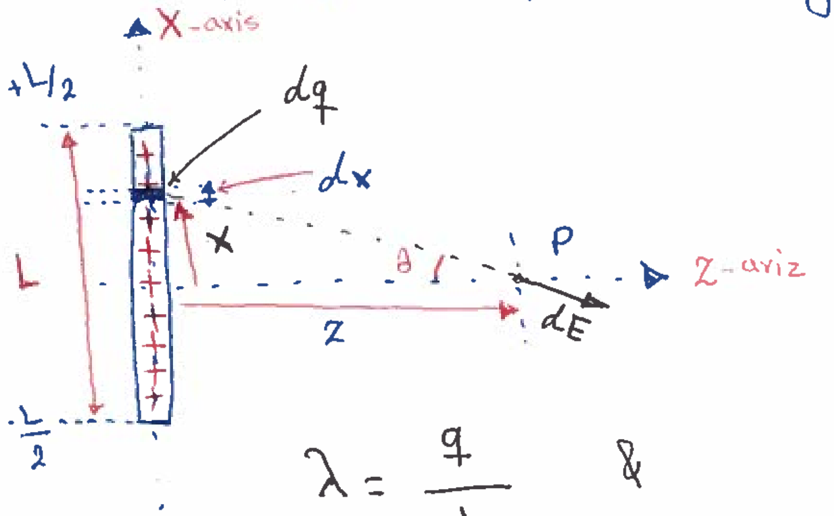
Electric Field of a uniformly charged rod.



$$\lambda = \frac{q}{L} \quad \& \quad dq = \lambda\, dx \quad \text{(I)}$$

$$dE = \frac{k\, dq}{x^2 + z^2}$$

we know the x component of dE would cancel out in the calculation of the net Electric Field due to the symmetry. Therefore, we only calculate the z component.

$$dE_z = \frac{k\, dq}{x^2+z^2} \cos\theta = \frac{k\, dq}{x^2+z^2} \times \frac{z}{\sqrt{x^2+z^2}} = \frac{k z\, dq}{(x^2+z^2)^{3/2}} \overset{\text{(I)}}{=} \frac{k z \lambda\, dx}{(x^2+z^2)^{3/2}}$$

$$E_{Net} = E_{znet} = \int_{-L/2}^{+L/2} dE_z = \int_{-L/2}^{+L/2} \frac{k z \lambda\, dx}{(x^2+z^2)^{3/2}} = k z \lambda \int_{-\frac{L}{2}}^{+L/2} \frac{dx}{(x^2+z^2)^{3/2}}$$

Here I used table of Integrals! $\rightsquigarrow$

$$= k z \lambda \left. \frac{x}{z^2 \sqrt{x^2+z^2}} \right|_{-L/2}^{+L/2}$$

$$= \frac{1}{4\pi\epsilon_0} \lambda \frac{\cancel{z}\, L}{z^{\cancel{2}} \sqrt{(L/2)^2 + z^2}} = \frac{1}{4\pi\epsilon_0} \frac{q}{\cancel{L}} \frac{\cancel{L}}{z \sqrt{\left(\frac{L}{2}\right)^2 + z^2}} = \boxed{\frac{q}{2\pi\epsilon_0 z} \frac{1}{\sqrt{L^2 + 4z^2}}}$$

if $z \gg L$ it will act like a point charge ← prove it yourself.

For an infite line with a uniform charge: $\boxed{E = \frac{\lambda}{2\pi\epsilon_0 z}}$