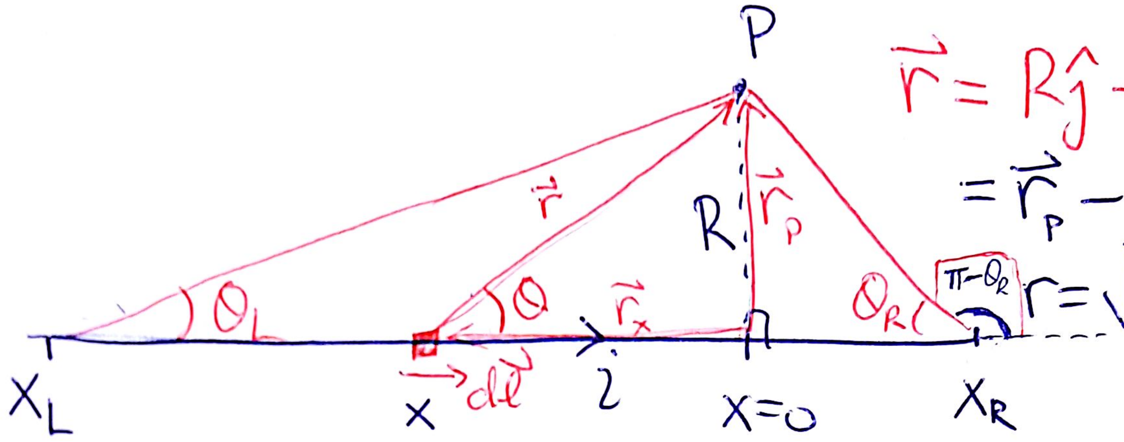

$$\vec{r} = R\hat{j} - x\hat{i}$$
$$= \vec{r}_p - \vec{r}_x$$
$$r = \sqrt{R^2+x^2}$$

$$\hat{r} = \frac{\vec{r}}{r}$$

$$\vec{dl} = dl\,\hat{i}$$
$$= dx\,\hat{i}$$

$$d\vec{B}_p = \frac{\mu_0}{4\pi}\frac{i\,\vec{dl}\times}{r^2}\frac{\vec{r}}{r}$$

$$d\vec{B}_p = \frac{\mu_0}{4\pi}\frac{i\,dl\,\hat{i}}{r^3}\times\left(R\hat{j}\cancel{-x\hat{i}}\right)$$

$$d\vec{B}_p = \frac{\mu_0 i\,dl\,R}{4\pi r^3}\hat{k}$$

$$\vec{B}_p = \left(\frac{\mu_0 i R}{4\pi}\int\frac{dx}{(x^2+R^2)^{3/2}}\right)\hat{k}$$

slight mistake initially. The way θ is defined here, $\cot\theta > 0$ but x is negative. Need $x = -R\cot\theta$ instead.

$$x = -R\cot\theta$$
$$dx = +R\csc^2\theta\,d\theta$$
$$x^2+R^2 = R^2\csc^2\theta$$

$$\vec{B}_p = \frac{\mu_0 i R}{4\pi}\int_{\theta_L}^{\pi-\theta_R}\frac{+\sin\theta\,d\theta}{R^2}$$

Note minor correction made

$$\boxed{\vec{B}_p = \frac{\mu_0 i}{4\pi R}\left(\cos\theta_R + \cos\theta_L\right)\hat{k}}$$

$\theta_R \to 0$ $\theta_L \to 0$: ∞ line.