
Fill in these boxes with your LAST NAME and FIRST INITIAL.

University of Calgary

Faculty of Science

Midterm Test

PHYSICS 259 ALL LECTURE SECTIONS

February 12, 2015, 7:00–8:30 p.m. **Time: 90 minutes.**

DO NOT TEAR OFF THIS PAGE! You may, however, tear off the last page, which has an equation sheet and table of integrals.

This is a closed-book exam worth a total of 35 points. Please answer all questions. Use of the Schulich calculator or equivalent is allowed.

Write your Last Name and First Initial on this top sheet in the grid above. (Do not write your ID number on this page.)

Make sure this question paper booklet contains 8 pages. If you are missing any pages, get a new booklet from the exam supervisor.

You should also have a **separate set of Answer Sheets**. This is where you enter Multiple Choice answers of Part I and also detailed solutions to the problems of Part II. Only work entered in the indicated spaces on the Answer Sheets will be marked.

IMPORTANT: YOUR ID NUMBER IS TO BE ENTERED AT THE TOP OF EACH AND EVERY ONE OF THE ANSWER SHEETS. DO THIS NOW.

Begin working on the examination when instructed to do so by the supervisor.

Part I: Multiple-Choice Questions (Total: 15 marks)

Enter answers to multiple choice questions on the first Answer Sheet using space provided in the upper right of the page. Each question in Part I is worth one point. You should complete Part I in about 40 minutes or less.

1) In all situations, the electric field lines in a given region of space

a) point in the direction that a negative charge will move.
b) point in the direction of the force on a positive charge.
c) point in the direction of the force on a negative charge.
d) point in the direction that a positive charge will move.
e) none of the above

---

2) A conducting sphere of radius 0.01 m has a charge of 1.0×10^{-11} C deposited on it. The magnitude of the electric field in V/m just outside the surface of the sphere is closest to

a) 90,000
b) 900
c) 4,500
d) zero
e) 450

---

3) A solid insulating sphere of radius R carries a positive charge density ρ (C/m^3) distributed uniformly throughout its volume. What is the electric field strength E as a function of radius r inside the sphere?

a) $E = \dfrac{\rho R}{3\epsilon_0 r}$

b) $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{\rho}{r^2}$

c) $E = \dfrac{4\pi\rho r^3}{3\epsilon_0 R^2}$

d) $E = \dfrac{\rho r^2}{3\epsilon_0}$

e) $E = \dfrac{\rho r}{3\epsilon_0}$

---

4) Two very large thin parallel charge sheets of area A are separated by a small distance $d \ll \sqrt{A}$. A total charge $-3Q$ is evenly distributed over sheet #1, while $+Q$ is evenly distributed over sheet #2. What is the electric field magnitude E at a point halfway between the charge sheets?

a) $\dfrac{2Q}{\epsilon_0 A}$

b) $\dfrac{4Q}{\epsilon_0 A}$

c) $\dfrac{3Q}{\epsilon_0 A}$

d) $\dfrac{5Q}{\epsilon_0 A}$

e) none of the above

---

5) A solid material carries a static excess charge distributed throughout its volume (i.e., the material has a volume charge density ρ). Which one of the following statements correctly describes this situation?

a) The material is a conductor.
b) The material is an insulator.
c) The material can be either a conductor or an insulator, but it is not possible to tell which from the information given.
d) The situation described is impossible.
e) none of the above

---

6) A total charge Q is uniformly distributed over a very thin disk of radius R. The electric field at a distance d along the disk axis is given by $\vec{E} = \dfrac{\sigma}{2\epsilon_0}\left[1 - \dfrac{1}{\sqrt{(R^2/d^2)+1}}\right]\hat{n}$ where $\hat{n}$ points away from the disk. At what distance along the axis will the electric field be closest to half of its peak value $E_{max} = \dfrac{\sigma}{2\epsilon_0}$ if the radius is $R = 40$ centimetres?

a) $d = 23.09$ cm
b) $d = 34.64$ cm
c) $d = 20.00$ cm
d) $d = 40.00$ cm
e) none of the above

---

7) Which of the following statements about Gauss's law are correct?

- **a)** Only charge enclosed within a Gaussian surface can produce an electric field at points on that surface.
- **b)** Gauss's law is valid only for symmetric charge distributions, such as spheres and cylinders.
- **c)** The normal to a closed surface points inward for negatively charged objects.
- **d)** Gauss's law requires that the electric field be exactly perpendicular or parallel to the surface normal.
- **e)** none of the above

---

8) Consider two lightweight metal spheres, each hanging from its own insulating nylon thread. One of the spheres has been negatively charged, while the other sphere is neutral. If the spheres are close together then which of the following statements will be true?

- **a)** they attract each other.
- **b)** they repel each other.
- **c)** the negatively charged sphere attracts the neutral one, but the neutral sphere does not attract the negatively charged one.
- **d)** they do not exert an electrostatic force on each other.
- **e)** the neutral sphere attracts the negatively charged one, but the negatively charged sphere does not attract the neutral one.

---

9) A test charge q feels a force F when located a distance d away from a point charge Q. The amount of charge Q now changes and at the same time the test charge is moved 10 times further away, while keeping q constant. At this new location the test charge still feels the same force F. How much has the charge Q increased/decreased?

- **a)** It has increased by a factor of 10
- **b)** It has decreased by a factor of 10
- **c)** It has increased by a factor of 100
- **d)** It has decreased by a factor of 100
- **e)** none of the above

---

10) Three point charges q_1, q_2, and q_3 are located on the x-axis at x_1 = -3.0 cm, $x_2 = 0$, and x_3 = +5.0 cm. The force on q_1 equals

- **a)** The charge of q_1 multiplied by the scalar sum of the electric fields of q_1, q_2, and q_3.
- **b)** The vector sum of the forces exerted by q_1 on q_2 and q_3.
- **c)** The charge of q_1 multiplied by the vector sum of the electric fields of q_1, q_2, and q_3.
- **d)** The vector sum of the forces exerted by q_2 and q_3.
- **e)** The sum of the charges q_1, q_2, and q_3 divided by ϵ_0.

---

11) Figure 1 shows a thin insulating rod of length L that carries a uniform linear charge density λ. Which of the following integrals do we need to evaluate to find the x-component of the electric field at point P due to this rod?

a) $E_x = \int_{y=0}^{L} \frac{1}{4\pi\epsilon_0} \frac{\lambda}{x^2 + y^2} dy$

b) $E_x = \int_{y=0}^{L} \frac{1}{4\pi\epsilon_0} \frac{\lambda x}{(x^2 + y^2)^{3/2}} dy$

c) $E_x = \int_{y=0}^{L} \frac{1}{4\pi\epsilon_0} \frac{\lambda y}{(x^2 + y^2)^{3/2}} dy$

d) $E_x = \int_{y=0}^{L} \frac{1}{4\pi\epsilon_0} \frac{\lambda xy}{(x^2 + y^2)^{3/2}} dy$

e) $E_x = \int_{y=0}^{L} \frac{1}{4\pi\epsilon_0} \frac{\lambda L}{x^2 + y^2} dy$

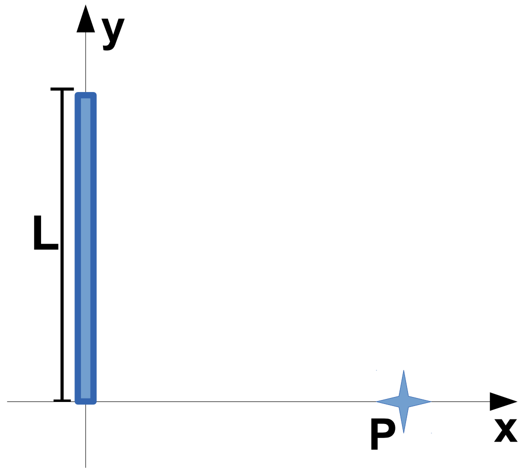

Figure 1: Line of charge along positive y-axis.

---

12) In Figure 2 the electric potential is zero at infinity. Where else is the electric potential zero?

a) Somewhere in the region $0 < x < a$, only
b) Somewhere in each of the regions $x < 0$ and $0 < x < a$
c) Somewhere in the region $x < 0$, only
d) Somewhere in each of the regions $0 < x < a$ and $x > a$
e) Somewhere in the region $x > a$, only

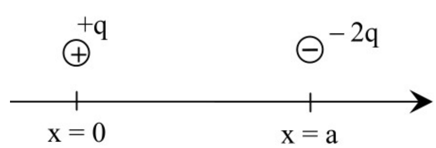

Figure 2: Two charges on a line.

---

13) The SI units for electric flux Φ_E are

a) Nm/C
b) $\mathrm{Nm^2/C}$
c) $\mathrm{Nm^2/C^2}$
d) $\mathrm{Nm/C^2}$
e) V/C

---

14) A half-circular segment with radius R has total charge Q distributed uniformly along length L. The total electric field at the origin due to this charge segment is $E_0\hat{i}$. What is the electric field $\vec{E}$ at the origin produced by a quarter-circular segment with the same radius R and total charge Q?

a) $-E_0\hat{i} + E_0\hat{j}$
b) $-\sqrt{2}E_0\hat{i} + \sqrt{2}E_0\hat{j}$
c) $-2E_0\hat{i} + 2E_0\hat{j}$
d) $+E_0\hat{i} - E_0\hat{j}$
e) none of the above

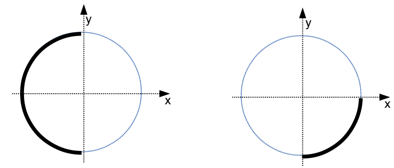

Figure 3: Half circle and quarter circle of charge.

---

15) An electric dipole consists of two opposite charges $+q$ and $-q$ separated by a fixed distance d. An electric quadrupole is a combination of two dipoles locked at right angles to each other. An electric dipole and quadrupole are initially located as shown in Figure 4. The electric quadrupole is then released so that it can move freely in the plane of the page. What statement best describes what will happen?

a) The quadrupole will rotate by 90° and will move toward the dipole.
b) The quadrupole will rotate by 90° and will move away from the dipole.
c) The quadrupole will rotate by 45° and will move toward the dipole.
d) The quadrupole will rotate by 45° and will move away from the dipole.
e) none of the above

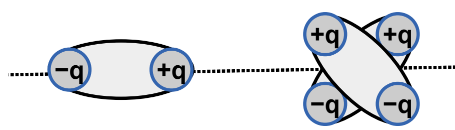

Figure 4: Dipole and quadrupole.

---

16) This is **version @V@** of the exam. Please select @V@ in the row marked "Version" in the Multiple Choice Answers area on the first Answer Sheet. **Be sure to do this now, before moving on to other questions on the paper**.

This is the end of the Multiple Choice part of the exam. Make sure that you have entered all your answers from this section on the Answer Sheet "bubble" page.

You may now proceed to Part II. All your answers for Part II must be written in the appropriate box on the Answer Sheet pages.

Part II: Written Answer Questions (Total: 20 marks)

IMPORTANT: Write your answers to the problems in Part II in the corresponding boxes on the Answer Sheets. Work must be shown for full marks. Rough work can be done on the back of this question paper, but only the work appearing on the Answer Sheets will be marked.

17) *[6 marks]* Eight point charges are evenly spaced around a circle of radius $R = 1$ mm as shown in Figure 5. All of the even-numbered charges (2,4,6,8) are protons and all of the odd-numbered charges (1,3,5,7) are electrons.

- **a)** Calculate the force that charge #1 applies on charge #5.
- **b)** Calculate the force on charge #1 due to charges #3 and #7.
- **c)** What is the total electric field $\vec{E}$ at the center of the circle due to all eight charges?
- **d)** If charge #8 is removed, what is the total electric field $\vec{E}$ at the center of the circle?

---

18) *[7 marks]* Total charge $+Q_a$ is uniformly distributed on a half-circle of radius R_a and total charge $+Q_b$ is uniformly distributed on a smaller half-circle of radius R_b as shown in Figure 6.

- **a)** What is the linear charge density λ_a on the larger segment?
- **b)** Determine the electric potential V at the center of the circle.
- **c)** Determine the ratio of charge Q_b/Q_a required to produce zero electric field $\vec{E}$ at the center of the circle.

---

19) *[7 marks]* For an infinite line of charge density λ we normally use a "Gaussian cylinder" to find the electric field $\vec{E} = \dfrac{\lambda}{2\pi\epsilon_0 r}\hat{r}$. What happens if we use a cube of volume $L{\times}L{\times}L$? Assume that the line charge enters exactly through the center of one face, passes through the center of the cube, and exits through the center of the opposite side (see Figure 7). Note that the electric field magnitude E will <u>not</u> be constant everywhere on the surface of the cube. All of the following questions refer to the top face with normal $\hat{n}$ pointing in the $+\hat{k}$ direction.

- **a)** Consider just the top face: what is the maximum electric field magnitude E and where does it occur? Provide an equation for E with a brief explanation and a quick sketch including all the essential details.
- **b)** Consider just the top face: what is the maximum electric field component E_z parallel to the surface normal $\hat{n}$ and where does it occur? Provide an equation, a brief explanation, and a quick sketch.
- **c)** What is the minimum value of E_z on the top face? Where is it?
- **d)** Determine the total electric flux Φ_E through the top surface.
- **e)** Determine the average of E_z at the top surface.

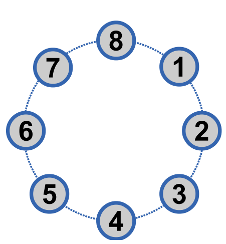

Figure 5: Eight charges on a circle.

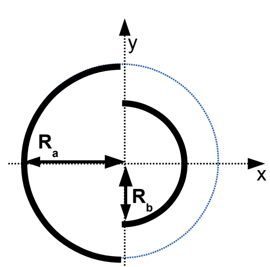

Figure 6: Two half-circles of charge.

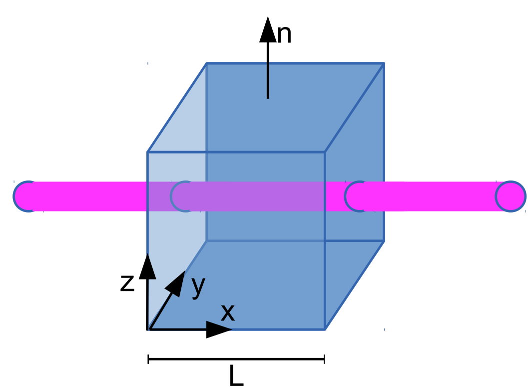

Figure 7: Line charge and Gaussian cube.

CONSTANTS AND USEFUL EQUATIONS

k = Coulomb constant = 8.99×10^{9} N m^2 C^{-2}
ϵ_0 = permittivity of free space = 8.85×10^{-12} C^2 N^{-1} m^{-2}
e = fundamental charge = 1.602×10^{-19} C
m_p = mass of proton = 1.67×10^{-27} kg
m_e = mass of electron = 9.11×10^{-31} kg

m = 10^{-3} $\mu = 10^{-6}$ n = 10^{-9} p = 10^{-12}

Surface area of a sphere: $A = 4\pi r^2$
Volume of a sphere: $A = \frac{4}{3}\pi r^3$
Area of a circle: $A = \pi r^2$
Circumference of a circle: $C = 2\pi r$

$$x = x_0 + v_{x0}t + \frac{1}{2}a_x t^2 \qquad v_x = v_{x0} + a_x t \qquad v_x^2 = v_{x0}^2 + 2a_x x \qquad \vec{F} = m\vec{a}$$

$$\vec{F} = k\frac{q_1 q_2}{r^2}\,\hat{r} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}\,\hat{r} \qquad \vec{E} = \frac{\vec{F}}{q} \qquad \vec{E} = k\frac{q}{r^2}\,\hat{r} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\,\hat{r}$$

$$\Phi_E = \oint_A \vec{E}\cdot d\vec{A} = \oint_A E\,dA\,cos\theta = \frac{Q_{encl}}{\epsilon_0} \qquad V = \frac{U}{q} \qquad U = k\frac{q_1 q_2}{r} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r}$$

$$V = k\frac{q}{r} = \frac{1}{4\pi\epsilon_0}\frac{q}{r} \qquad W = qV_{ab} \qquad V_{ab} = V_a - V_b = \int_a^b \vec{E}\cdot d\vec{l}$$

$$\vec{E} = -\vec{\nabla}V = -\left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)V \qquad C = \frac{Q}{V_{ab}} \qquad C = \frac{\epsilon_0 A}{d}$$

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}QV = \frac{1}{2}CV^2 \qquad \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \qquad C = C_1 + C_2 + C_3$$

$$u = \frac{1}{2}\epsilon_0 E^2$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin\frac{x}{a} \qquad \int \frac{dx}{\sqrt{x^2 + a^2}} = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

$$\int \frac{dx}{x^2 + a^2} = \frac{1}{a}\arctan\frac{x}{a} \qquad \int \frac{dx}{(x^2 + a^2)^{3/2}} = \frac{1}{a^2}\frac{x}{\sqrt{x^2 + a^2}}$$

$$\int \frac{x\,dx}{(x^2 + a^2)^{3/2}} = -\frac{1}{\sqrt{x^2 + a^2}}$$