Last time

- Spherical capacitor
- Energy density stored in a spherical capacitor
- Cylindrical capacitor and co-axial cable

This time

- Dielectrics
- Activity #7

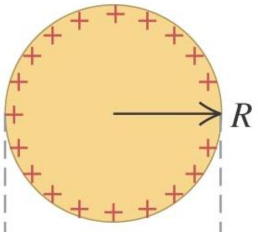
General result

$$u = \frac{1}{2}\varepsilon_0 E^2$$

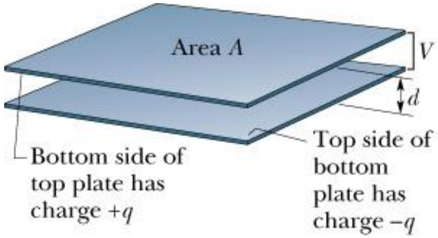
$$U = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

Capacitance

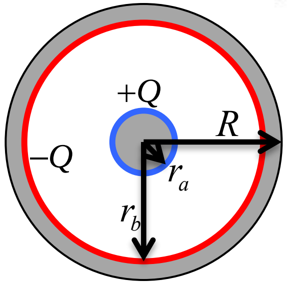
$$C = \frac{Q}{V} = 4\pi\varepsilon_0 R$$



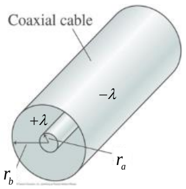
$$C = \frac{\varepsilon_0 A}{d}$$



$$C = \frac{4\pi\varepsilon_0 ab}{(b-a)}$$

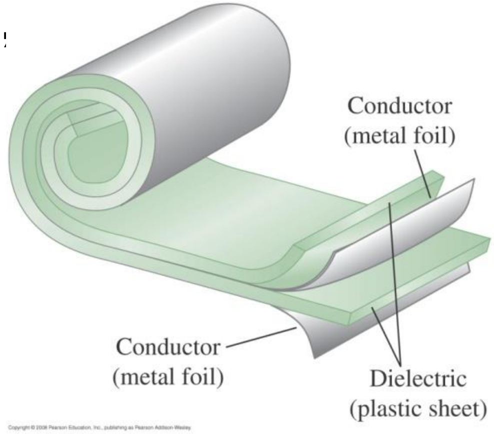


$$\frac{C}{L} = \frac{2\pi\varepsilon_0}{\ln(r_b / r_a)}$$



DIELECTRICS

Most capacitors have a non-conducting material, or a dielectric between their plates.



Placing a solid dielectric between the plates of a capacitor serves three functions:

1. It solves the mechanical problem of maintaining two large metal sheets at a very small separation without actual contact.

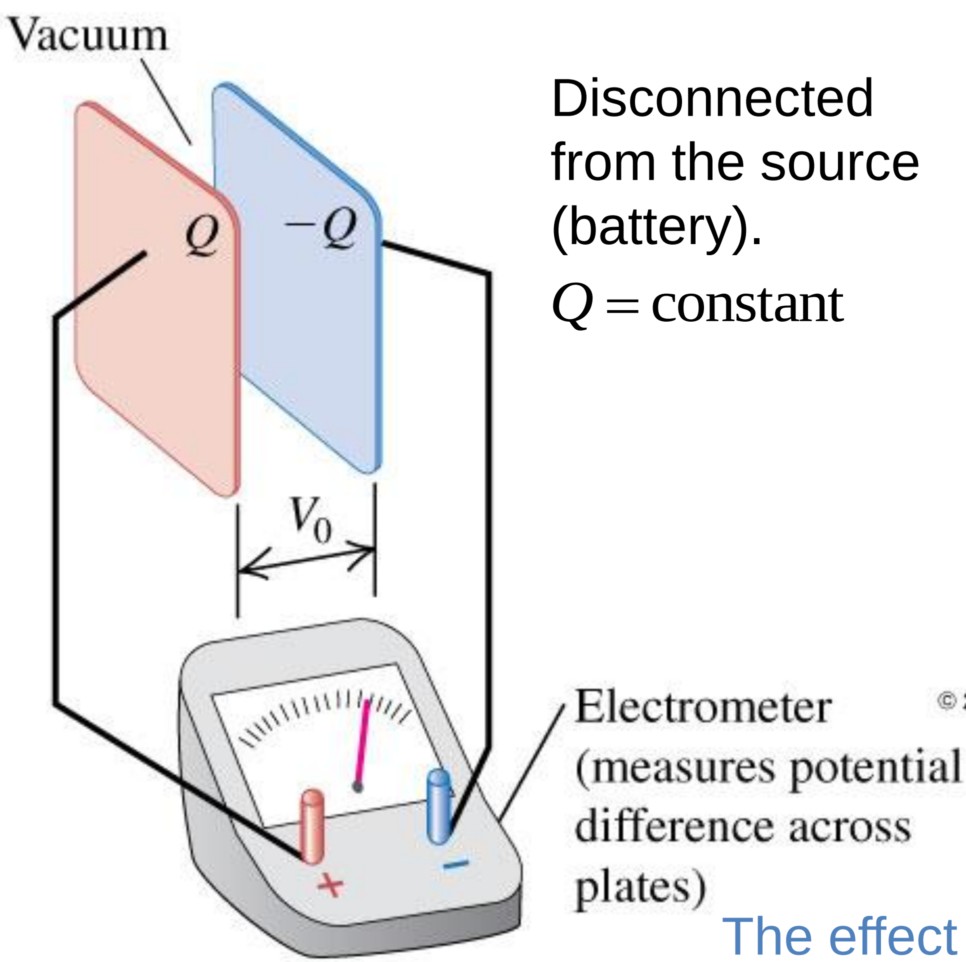
2. It increases the maximum potential difference between the capacitor plates.

3. It increases the capacitance of a capacitor.

$$Q = C_0 V_0$$

$$V_0 = E_0 d$$

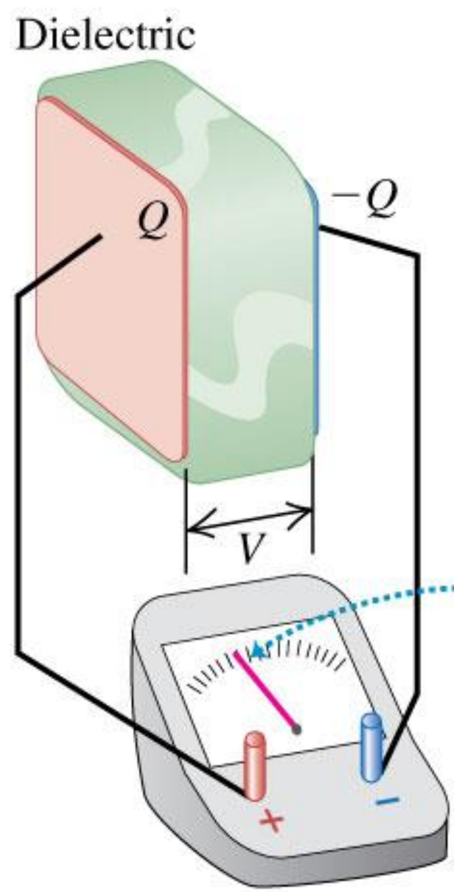
(a)



Disconnected from the source (battery).

$Q = \text{constant}$

(b)



$$Q = CV$$

$$V < V_0$$

$$C > C_0$$

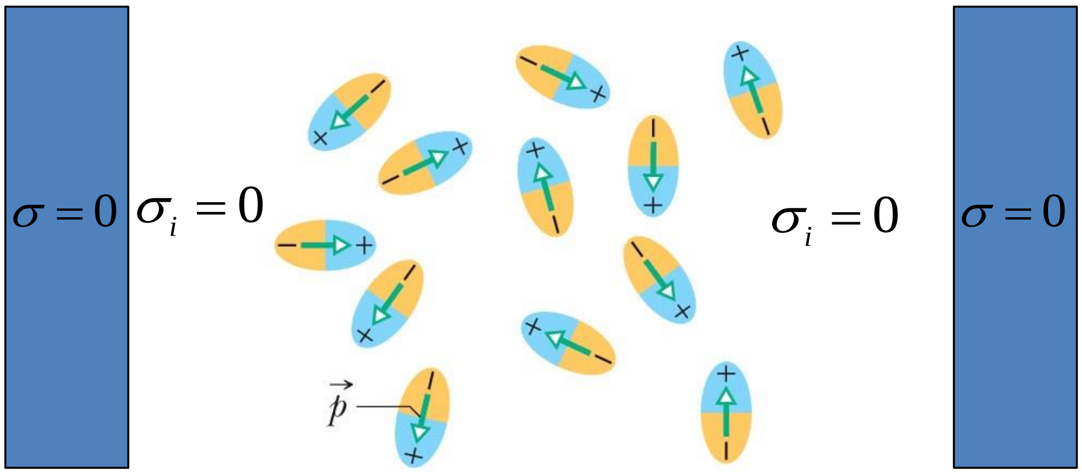
$$V = Ed$$

$$E < E_0$$

The effect of the dielectric is to reduce the electric field magnitude from its initial value E_0.

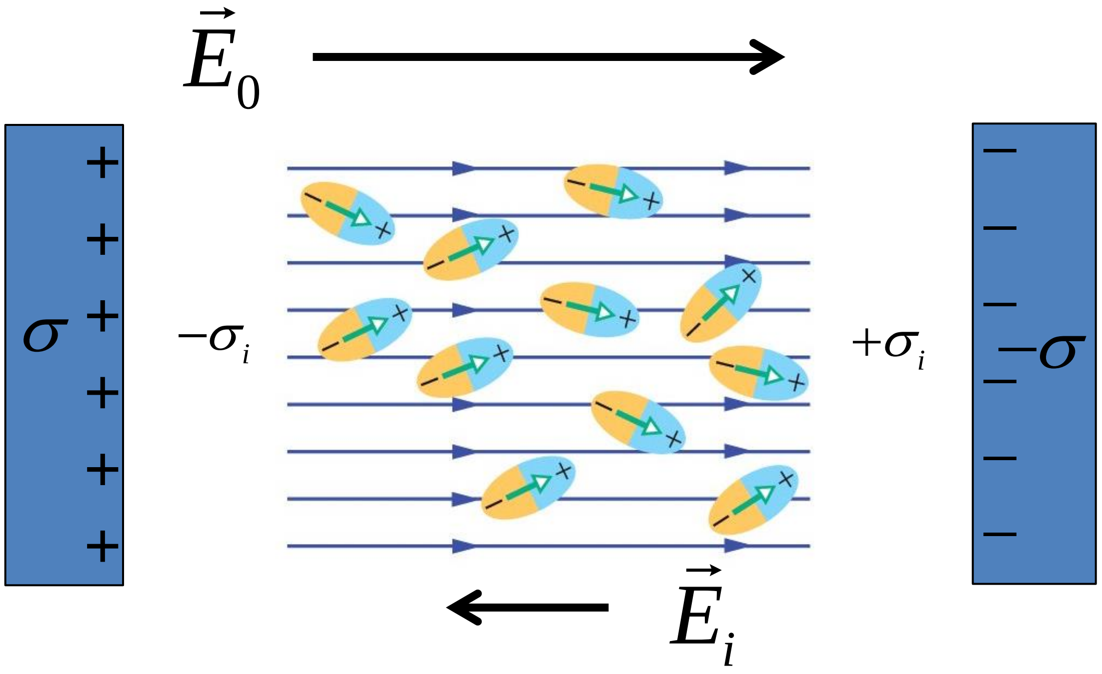
The microscopic picture

Polar molecules have random orientations when there is no applied electric field.



$$\vec{E}_0 = 0$$

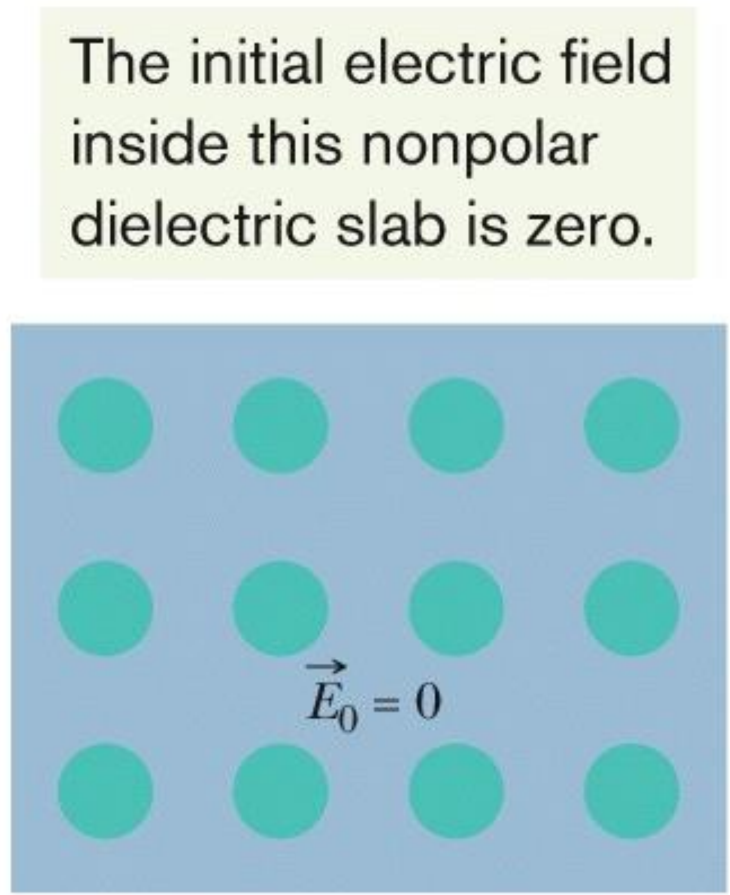
Molecular alignment of a polar dielectric in an external electric field gives rise to an induced surface charge density



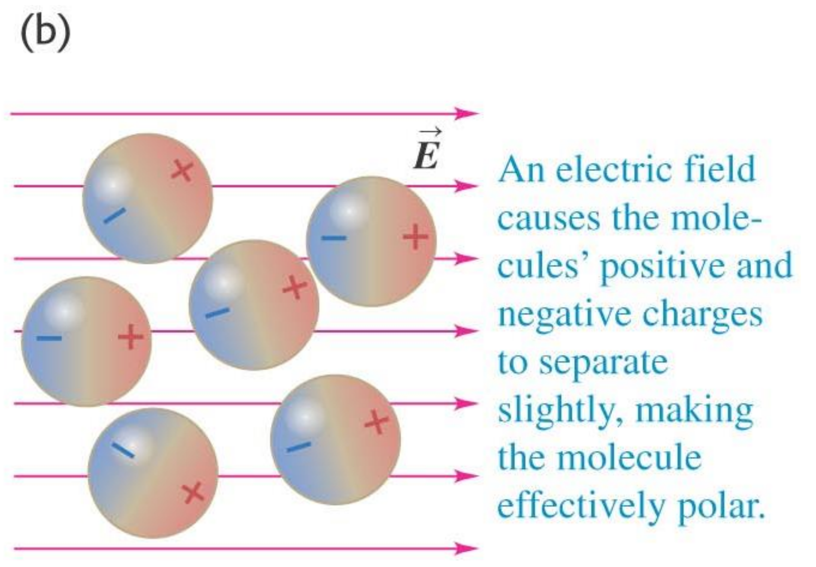
Induced charge density by the applied field σ_i .
Charge density on the plates σ.

$$\vec{E}_{\text{Total}} = \vec{E}_0 - \vec{E}_i$$

Nonpolar molecules have their positive and negative charge centers at the same point.

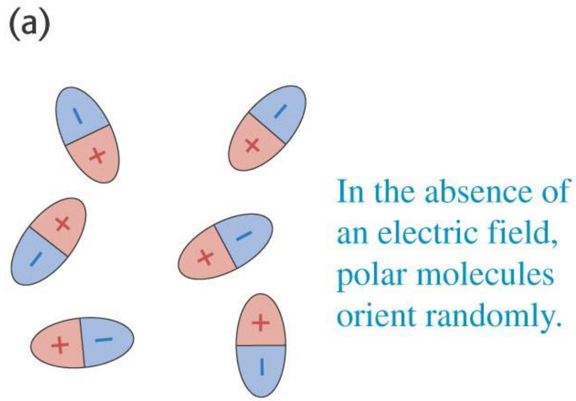


Molecular alignment of a non-polar dielectric in an external electric field also gives rise to surface charge density.
The positive and negative charge centers become separated slightly by the applied electric field.

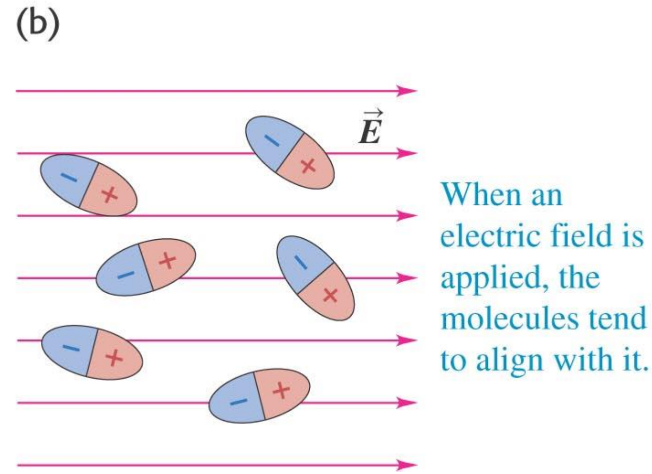


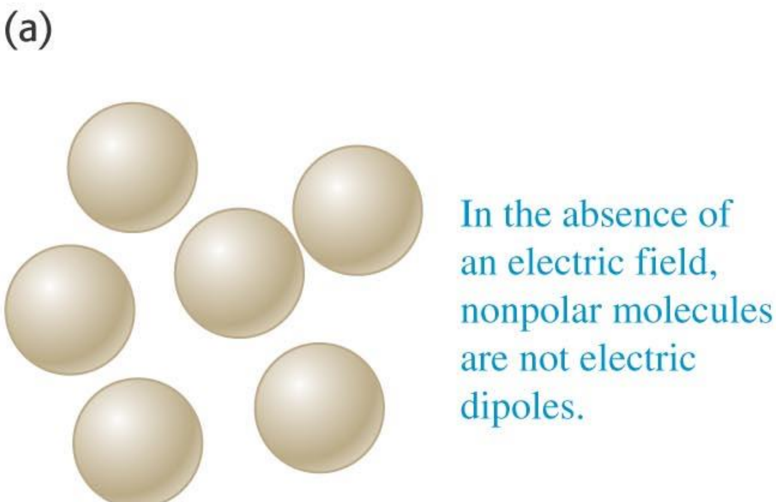
Molecular models

- the effect of an applied field on individual molecules.

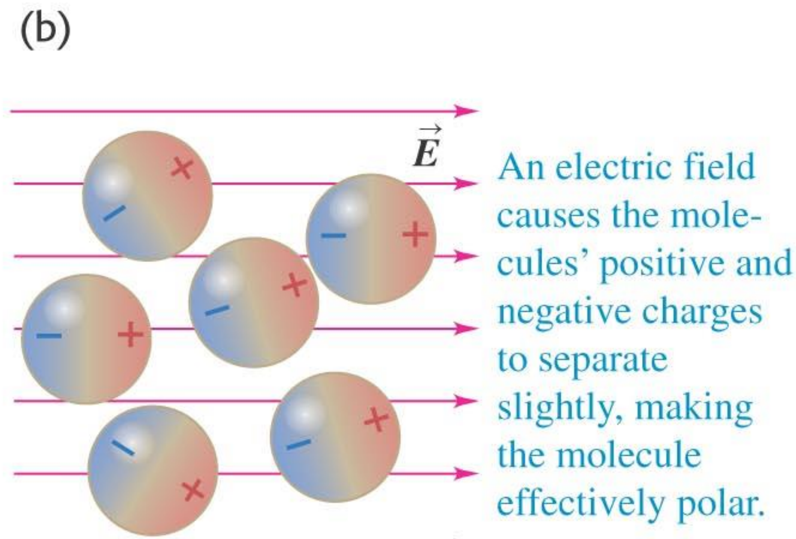


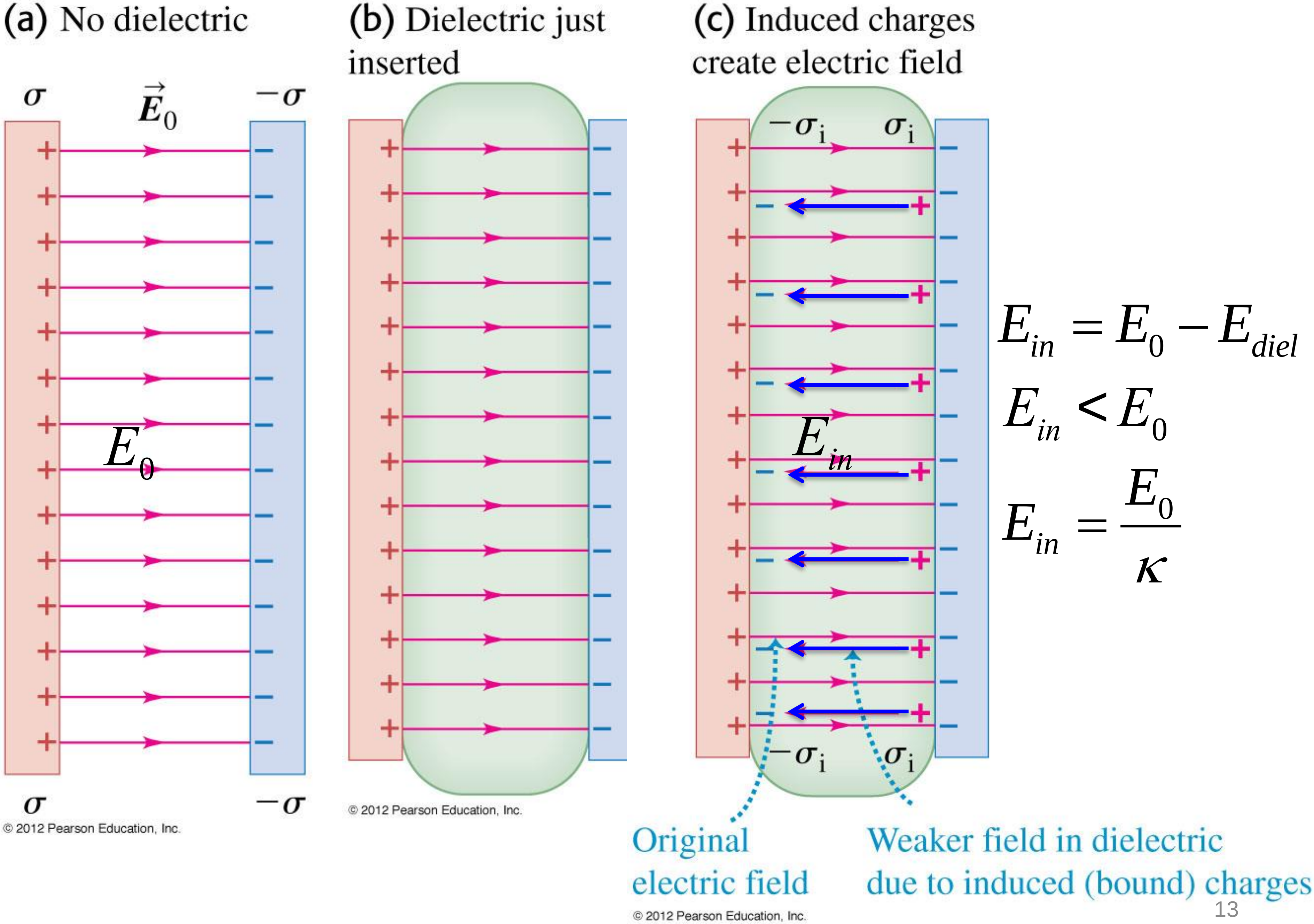
$$\vec{E}_0 = 0$$





$$\vec{E}_0 = 0$$





$$E_{in} = E_0 - E_{diel}$$

$$E_{in} < E_0$$

$$E_{in} = \frac{E_0}{\kappa}$$

The resultant field

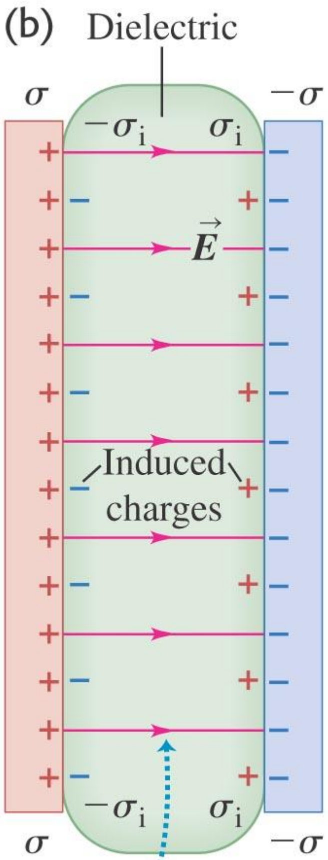
$$\kappa = \frac{V_0}{V}$$

For a parallel plate capacitor

$$E = \frac{V}{d}$$

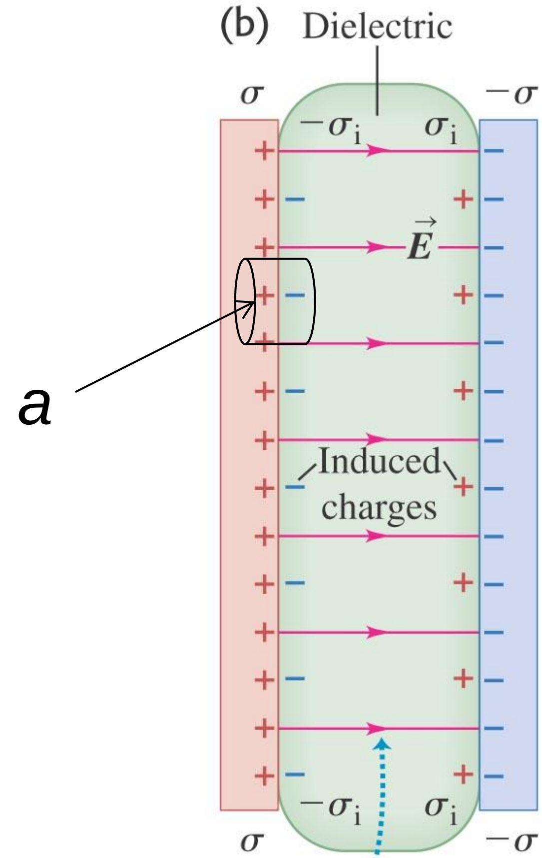
$$E_0 = \frac{V_0}{d}$$

$$E = \frac{E_0}{\kappa}$$ When Q is constant.



For a given charge density σ, the induced charges on the dielectric's surfaces reduce the electric field between the plates.

Using Guass's law:



For a given charge density σ, the induced charges on the dielectric's surfaces reduce the electric field between the plates.

Using Guass's law:

$$\oint \vec{E} \bullet d\vec{A} = \frac{Q_{\text{Enclosed}}}{\varepsilon_0}$$

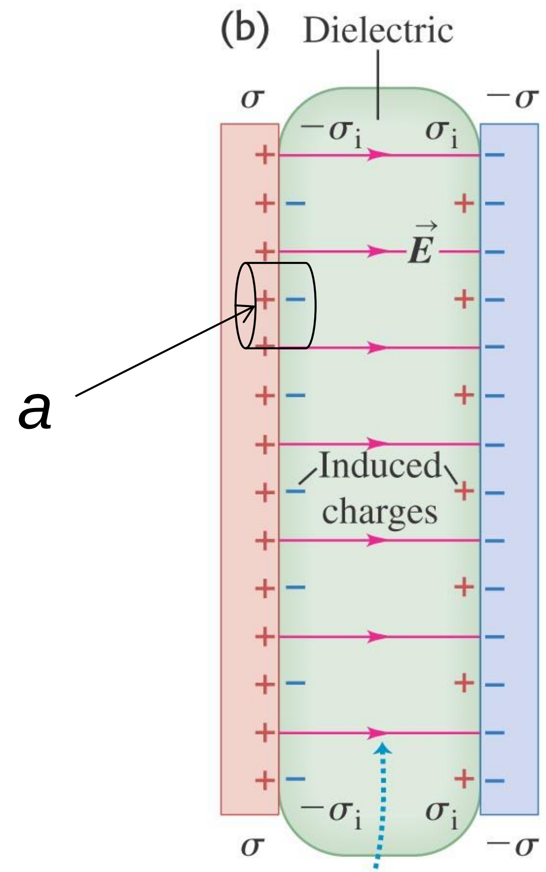
$$Ea = \frac{\sigma_{\text{net}} a}{\varepsilon_0}$$

$$Ea = \frac{(\sigma - \sigma_i) a}{\varepsilon_0}$$

$$E = \frac{(\sigma - \sigma_i)}{\varepsilon_0}$$

Also

$$E = \frac{E_0}{\kappa} = \frac{\dfrac{\sigma}{\varepsilon_0}}{\kappa} = \frac{\sigma}{\varepsilon_0 \kappa}$$



For a given charge density σ, the induced charges on the dielectric's surfaces reduce the electric field between the plates.

$$E = \frac{(\sigma - \sigma_i)}{\varepsilon_0}$$

$$E = \frac{\sigma}{\varepsilon_0 \kappa}$$

$$\frac{(\sigma - \sigma_i)}{\varepsilon_0} = \frac{\sigma}{\varepsilon_0 \kappa}$$

$$\sigma_i = \sigma\left(1 - \frac{1}{\kappa}\right)$$

Induced surface charge density

The electric permittivity of the dielectric is defined to be $\varepsilon = \varepsilon_0 \kappa$

Hence

$$E = \frac{\sigma}{\varepsilon}$$

Hence

With the dielectric

$$E = \frac{\sigma}{\varepsilon}$$

$$C = \kappa C_0$$

$$C = \varepsilon \frac{A}{d}$$

$$u = \frac{1}{2}\varepsilon E^2$$

Without the dielectric

$$E_0 = \frac{\sigma}{\varepsilon_0}$$

$$C_0 = \varepsilon_0 \frac{A}{d}$$

$$u = \frac{1}{2}\varepsilon_0 E_0^2$$

In all equations replace ε_0 by ε.

Table 24.1—Dielectric constants

Table 24.1 Values of Dielectric Constant K at 20°C

Material	K	Material	K
Vacuum	1	Polyvinyl chloride	3.18
Air (1 atm)	1.00059	Plexiglas	3.40
Air (100 atm)	1.0548	Glass	5–10
Teflon	2.1	Neoprene	6.70
Polyethylene	2.25	Germanium	16
Benzene	2.28	Glycerin	42.5
Mica	3–6	Water	80.4
Mylar	3.1	Strontium titanate	310

Dielectric breakdown

- A very strong electrical field can exceed the strength of the dielectric to contain it. Table 24.2 at the bottom of the page lists some limits.

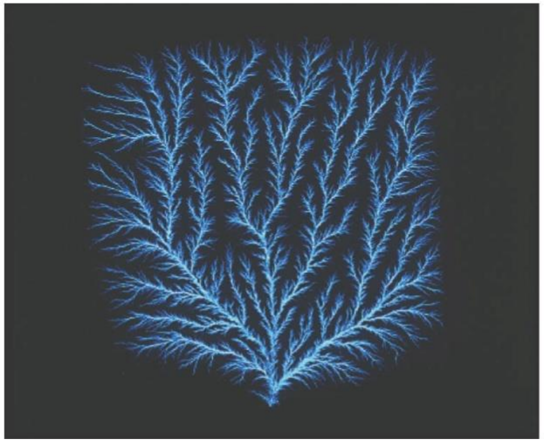


Table 24.2 Dielectric Constant and Dielectric Strength of Some Insulating Materials

Material	Constant, K	E_m (V/m)
Polycarbonate	2.8	3×10^7
Polyester	3.3	6×10^7
Polypropylene	2.2	7×10^7
Polystyrene	2.6	2×10^7
Pyrex glass	4.7	1×10^7
air		3×10^6

Introduction of a slab with a dielectric constant κ between the plates