Wednesday March 15, 2017

Last time:

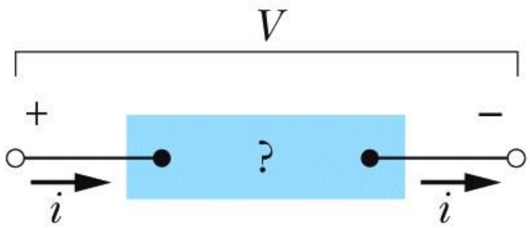
- Electric current: a microscopic picture
- Current density (a vector) vs current (a scalar)
- Electric fields in conductors and electron drift speed
- Resistance as geometric quantity
- Resistors in series
- Resistors in parallel

Today:

- Ohmic vs non-ohmic materials
- Microscopic view of resistivity
- Temperature dependence of resistivity/resistance
- Ideal vs non-ideal batteries
- RC circuits (charging/discharging capacitors)

TopHat questions

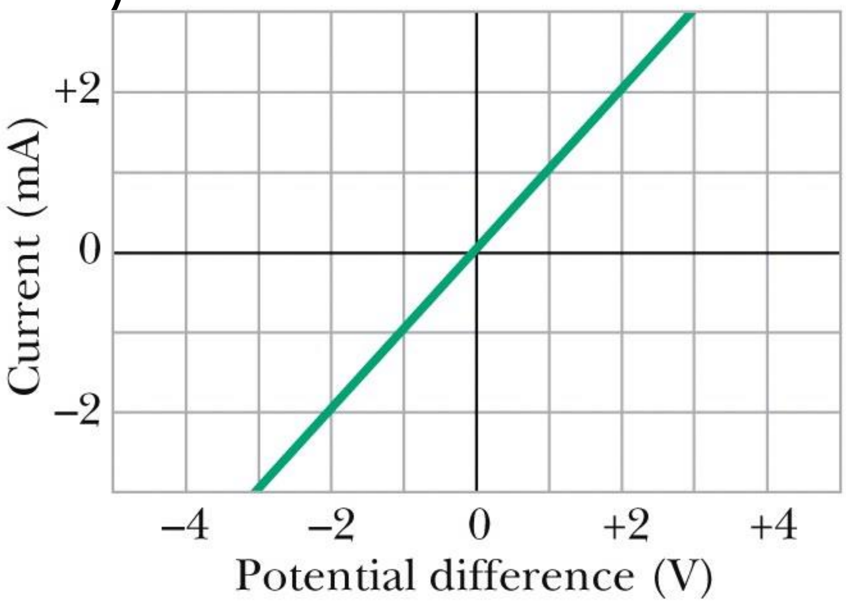
Ohmic vs non-Ohmic devices



(a)

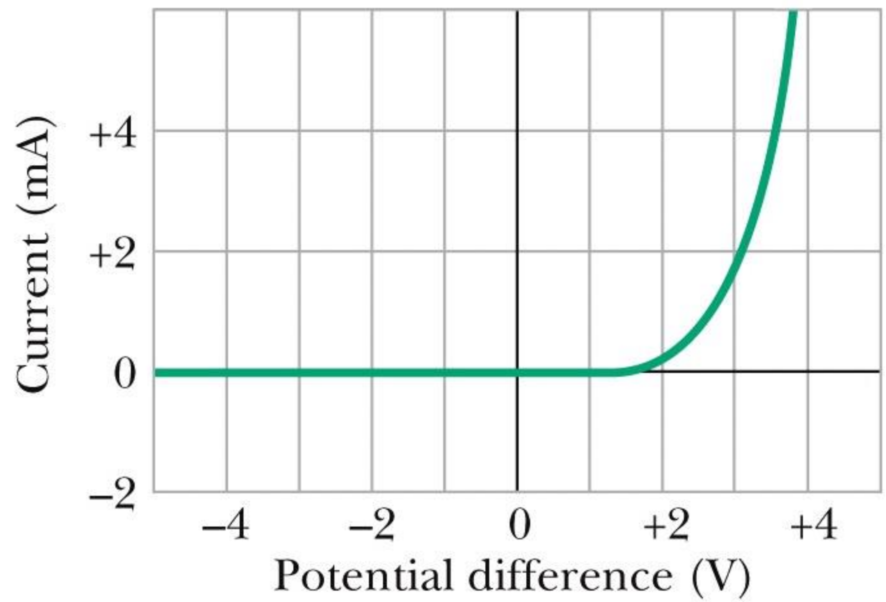
Materials with isotropic electrical properties

$\rho = E / J = const$



(b)

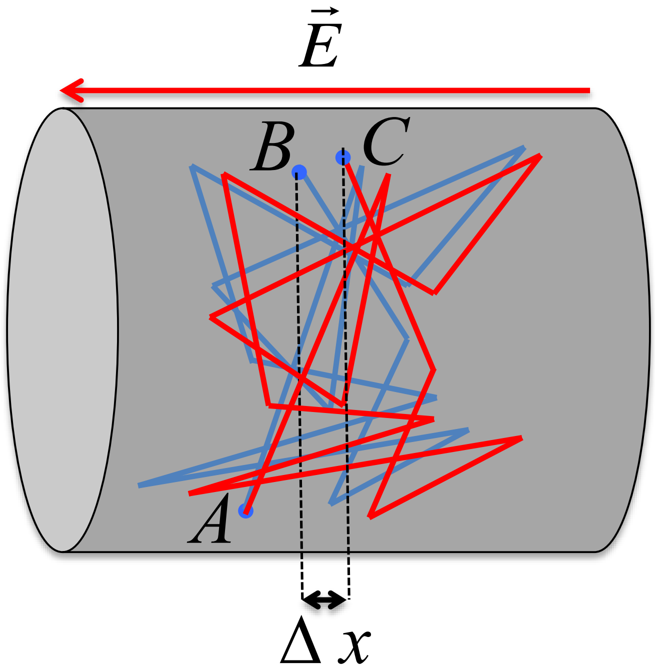
Materials with anisotropic electrical properties (pn junction diode)



(c)

 halliday_10e_fig_26_11

Microscopic view of resistivity



Electrons bounce around inside the metal at speeds very high speeds on the order of 0.5% light speed.

When an electric field is applied in the conductor, there is a net force on the electrons, leading to an average "drift speed" of $v_d = 0.5\ \mu\text{m/s}$

The acceleration felt by the electrons from the E-field is

$$a_x = \frac{eE}{m}$$

So the average drift speed of the electrons will be given by

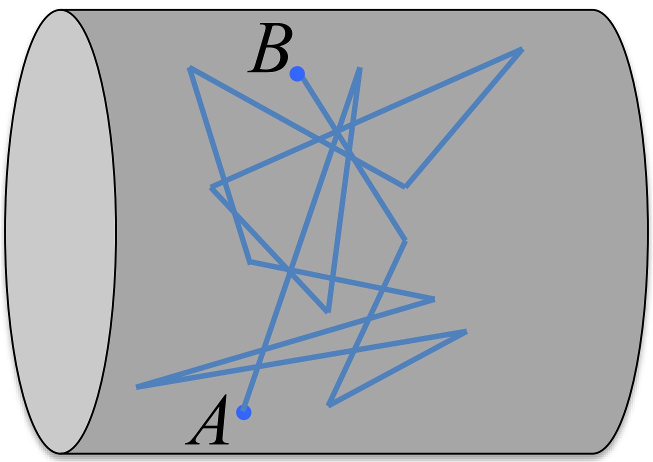
$$v_d = at = \frac{eEt}{m}$$

but we found before:

$$v_d = \frac{J}{ne}$$

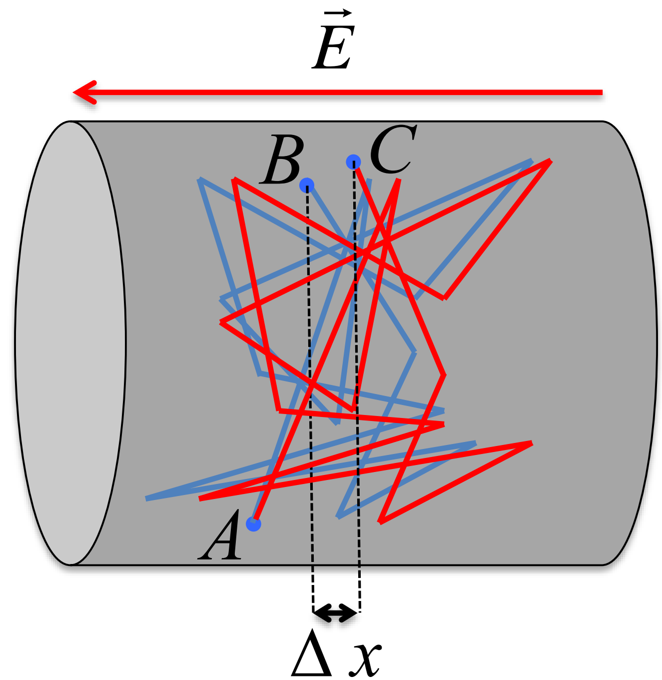
Microscopic view of resistivity

No E-field applied



$$v_d = \frac{eE\tau}{m}$$

$$v_d = \frac{J}{ne}$$



The average time between collisions is τ and is called the *mean free time*. Equating the two expressions for the drift speed, we get:

$$\frac{eE\tau}{m} = \frac{J}{ne}$$ Rearrange this to find $$E = \left(\frac{m}{ne^2\tau} \right) J$$

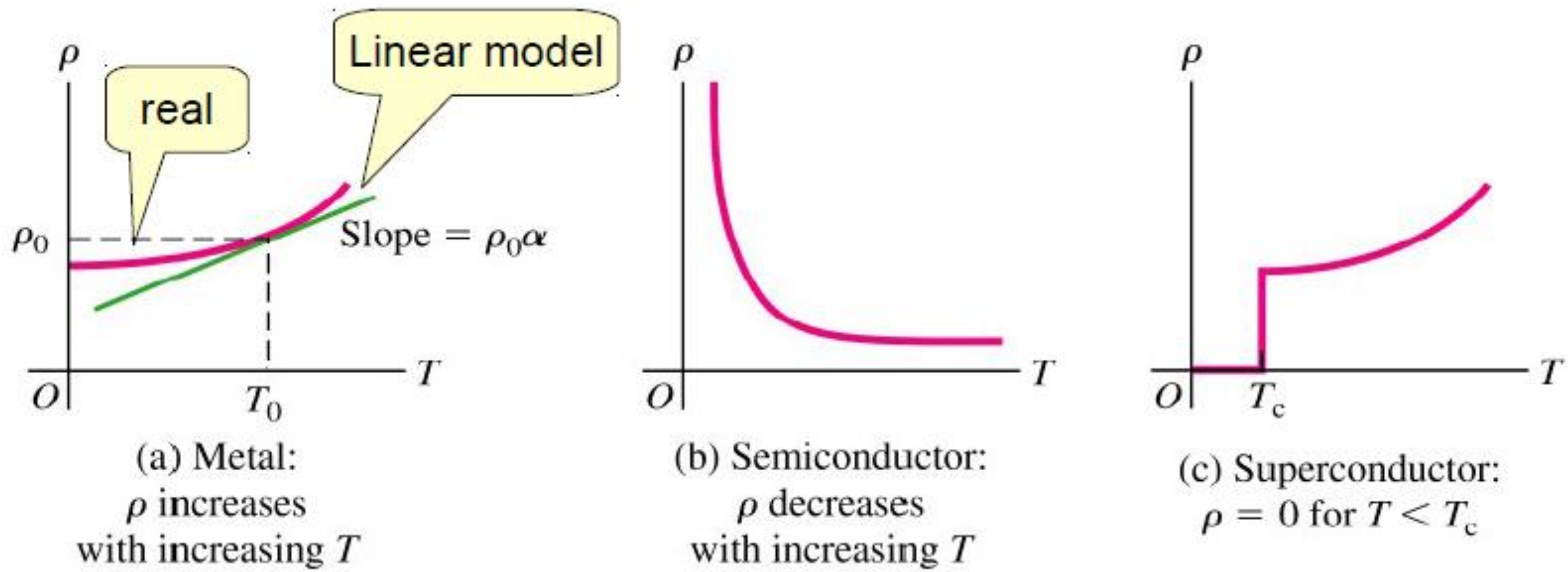
This gives a microscopic picture of resistivity: $$\boxed{\rho = \frac{m}{ne^2\tau}}$$

Resistivity and temperature

- As temperature increases, the ions in the conductor vibrate more, and electrons can only travel shorter distances before they have a collision.
- The resistivity of a metallic conductor tends to increase as it gets warmer.
- A simple linear model of this effect on resistivity is approximately valid over a small (100 C) temperature range.

$$r - r_0 = r_0 a\left(T - T_0\right)$$

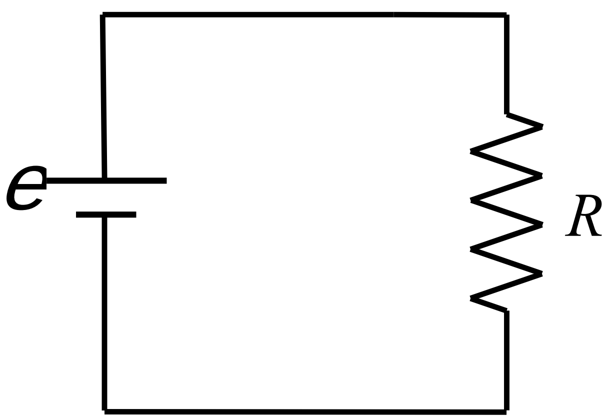
Resistivity and temperature

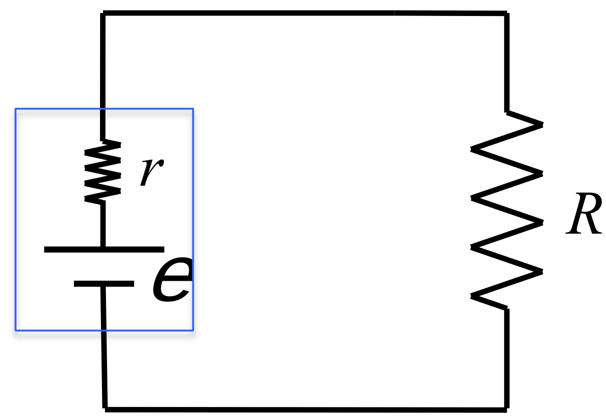


https://www.youtube.com/watch?v=2xOAb68agy0

Non-ideal Batteries: internal resistance

Every voltage source has **some** internal resistance to it. Usually this can be ignored but not always





The internal resistance simply acts as a resistor in series with the rest of the circuit.

$$e - Ir - IR = 0$$

$$I = \frac{e}{(r+R)}$$

Power in circuits

Recall that **POWER** is the **rate at which work is done**.

$$P = \frac{W}{\Delta t}$$

A battery with voltage ΔV raises the **potential energy** of a single charge q by an amount $q\Delta V$. This is the **work done** by the battery. For N charges

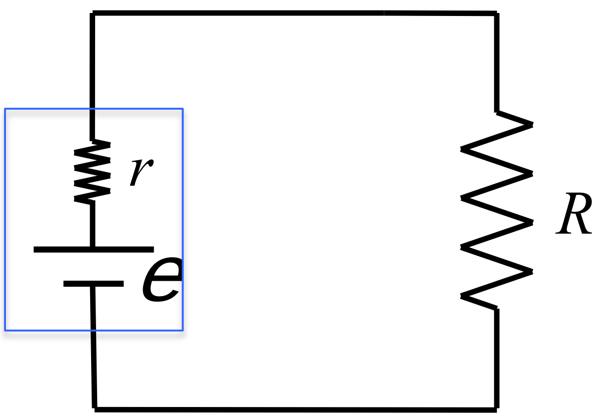
$$P = \frac{Nq\Delta V}{\Delta t} = \left(\frac{Nq}{\Delta t}\right)\Delta V$$

$Nq/\Delta t$ is the number of **charges passing through the battery** in time Δt, i.e. it is the **current**

$$\boxed{P = I\Delta V}$$

Power Non-ideal Batteries

$$I = \frac{e}{(r+R)}$$



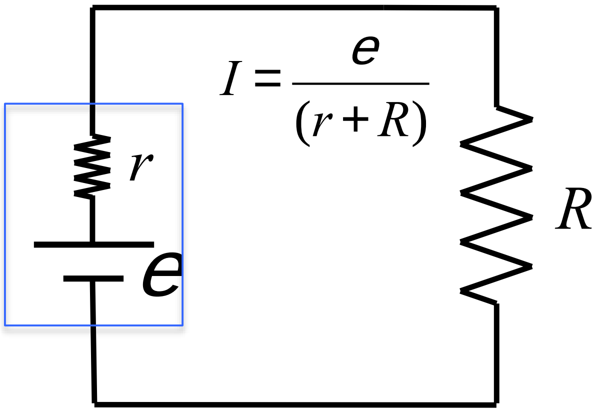
$$P_e = Ie = \frac{e^2}{R+r}$$

Power output required by the emf source

$$P_R = I^2 R = \frac{e^2 R}{(R+r)^2}$$

Power dissipated by the resistive load

Non-ideal Batteries: internal resistance



Conservation of energy requires power in must be equal to power out, but power by emf is not power dissipated by R.

$$P_e = Ie = \frac{e^2}{R+r} \qquad P_r = I^2 R = \frac{e^2 R}{(R+r)^2}$$

Resolution: power dissipated by emf

$$P_r = I^2 r = \frac{e^2 r}{(R+r)^2}$$

The emf must do more work because it fights against its own internal resistance

Now we can verify that power in = power out

$$P_e = P_r + P_R = \frac{e^2 r}{(R+r)^2} + \frac{e^2 R}{(R+r)^2} = \frac{e^2 (R+r)}{(R+r)^2} = \frac{e^2}{R+r}$$

Wednesday March 15, 2017
lecture 2

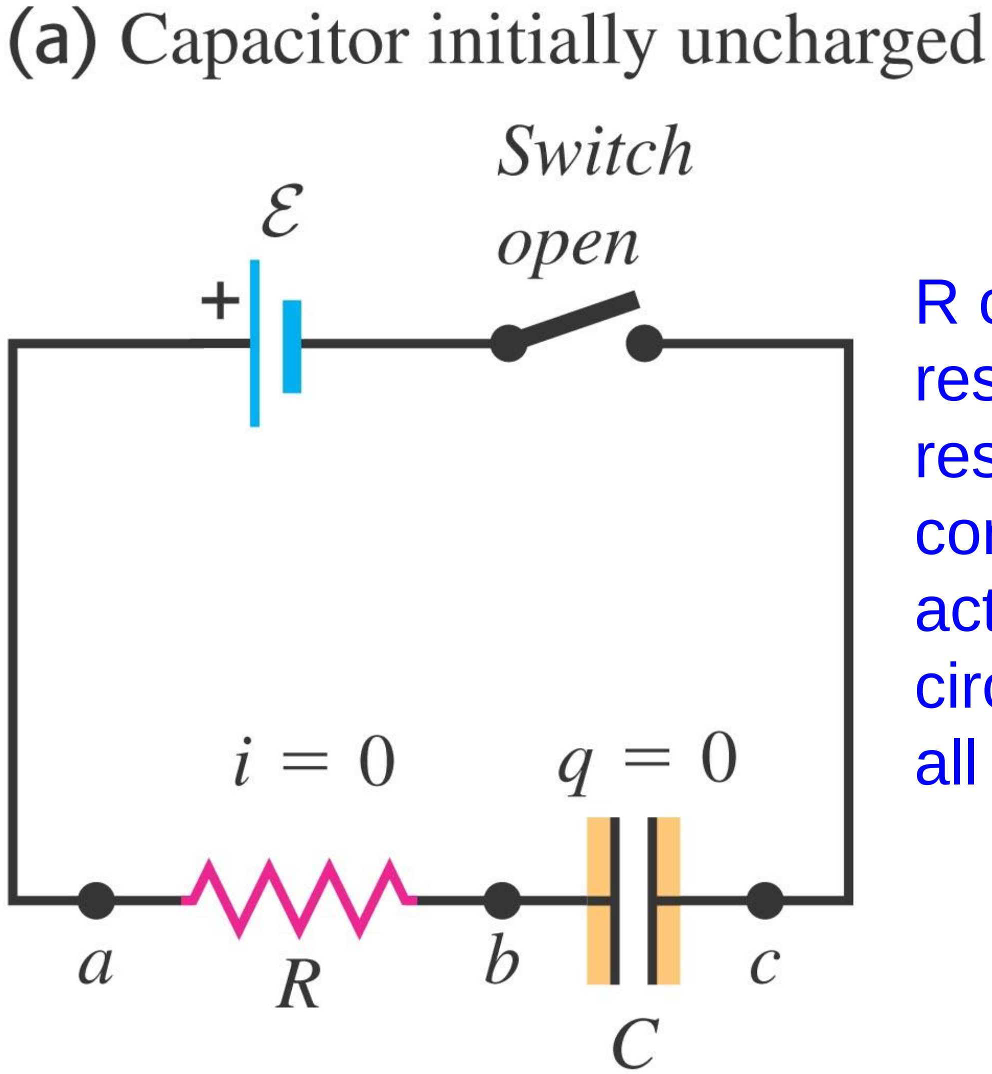
Last time:

- Ohmic vs non-ohmic materials
- Microscopic view of resistivity
- Temperature dependence of resistivity/resistance
- Ideal vs non-ideal batteries
- RC circuits (charging/discharging capacitors)

Today:

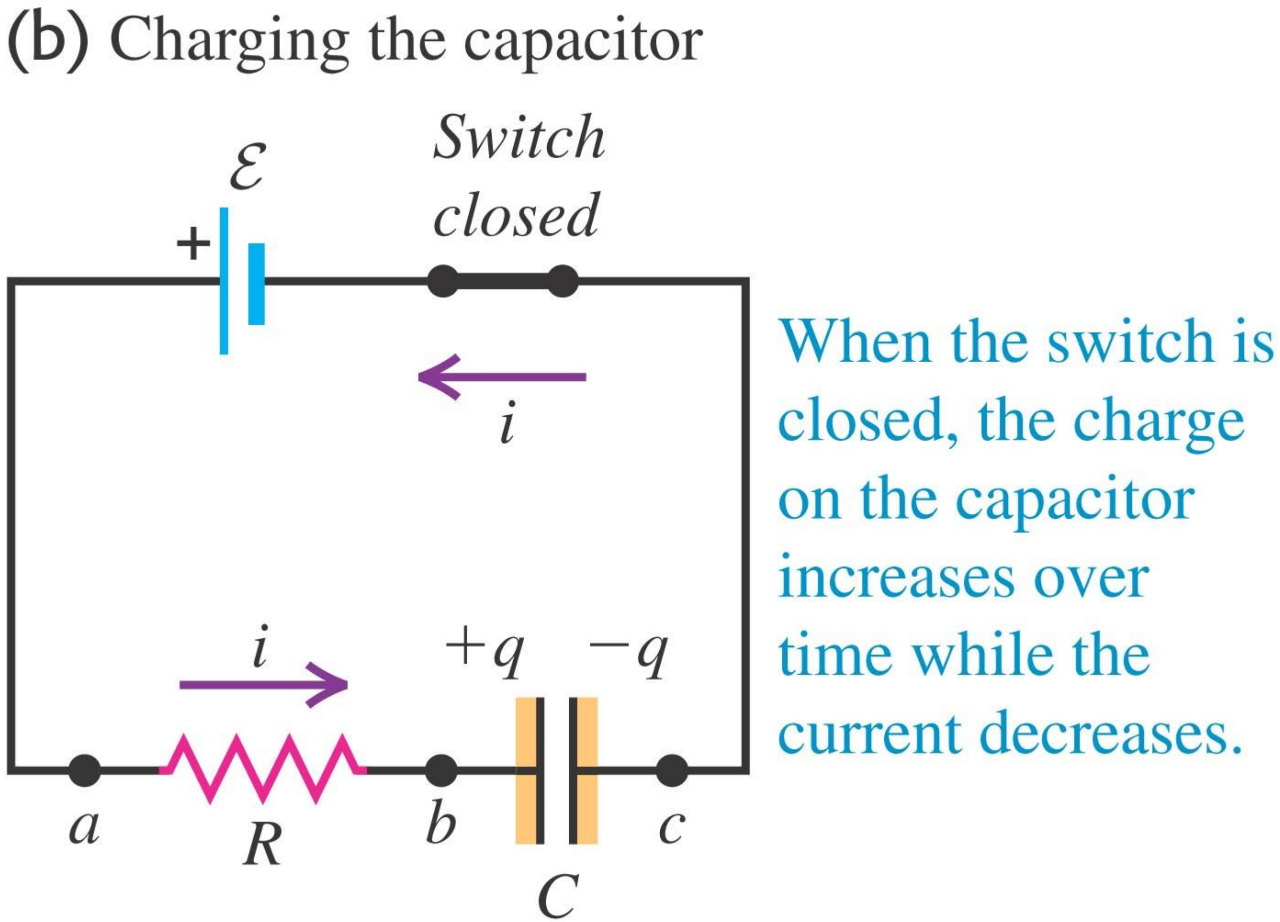
- RC circuits (charging/discharging capacitors)
- RC time constant and its meaning
- Early and late time behaviour of RC circuits

Charging a capacitor

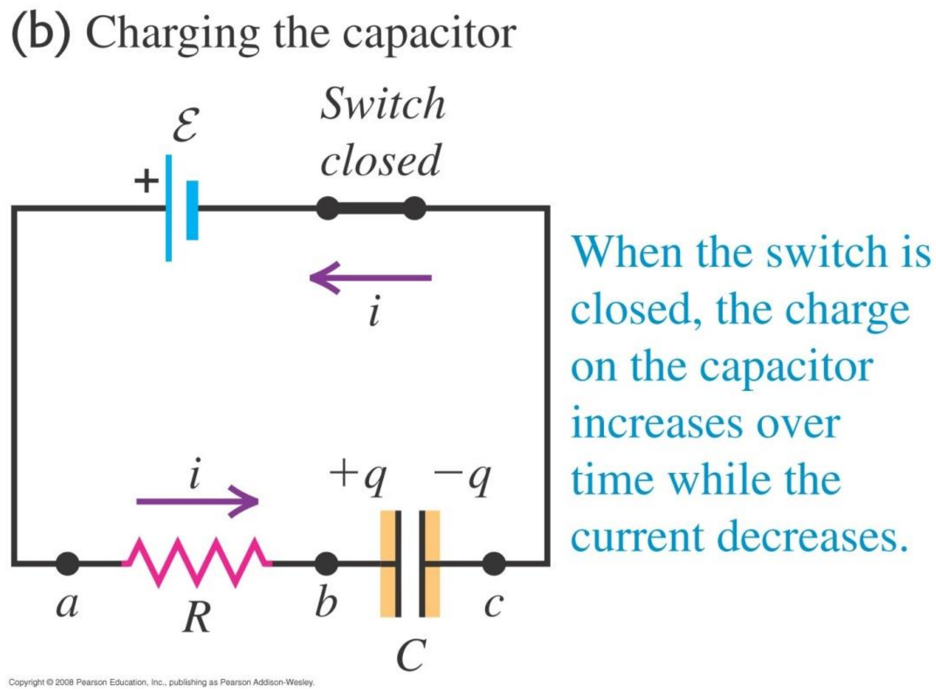


R could be the internal resistance of the battery, resistance of the connecting wires, an actual resistance in the circuit or combination of all the above.

Charging a capacitor



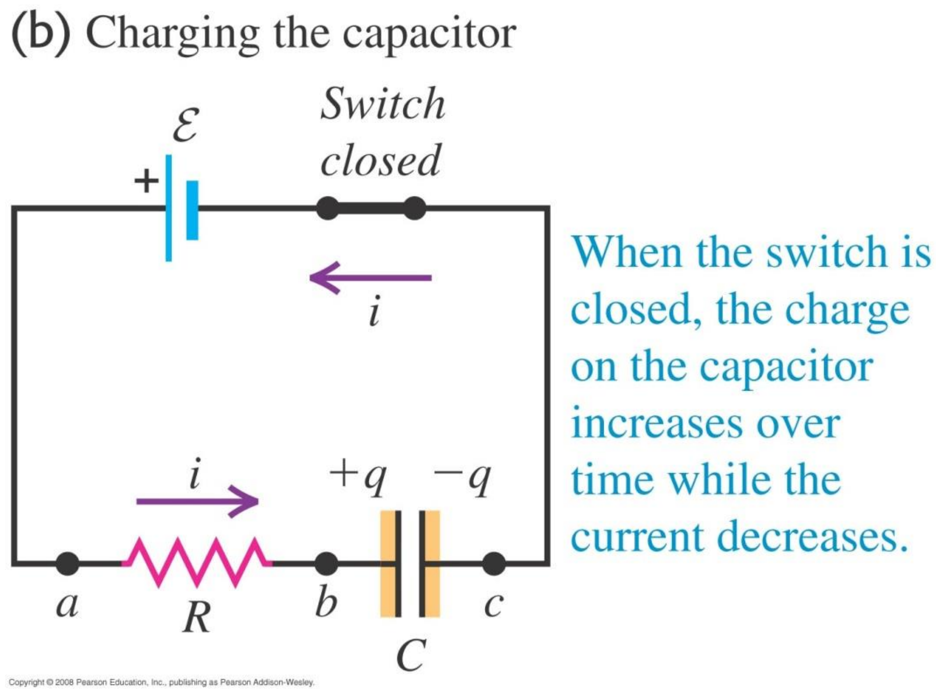
Coulomb repulsion doesn't allow for q to increase on the + plate indefinitely. As more charge is stored on the plates the coulomb repulsion gets larger resulting in smaller current in the circuit.



$t = 0$ at the time swithed is closed. $q = 0$ $V_C = \dfrac{q}{C} = 0$

Potential drop across the capacitor is zero. Capacitor acts momentarily as a short (not in the circuit). This means that the potential drop is entirely across the resistor. This is when the current in the circuit assumes its maximum value.

$$i = i_0 = \frac{\varepsilon}{R}$$

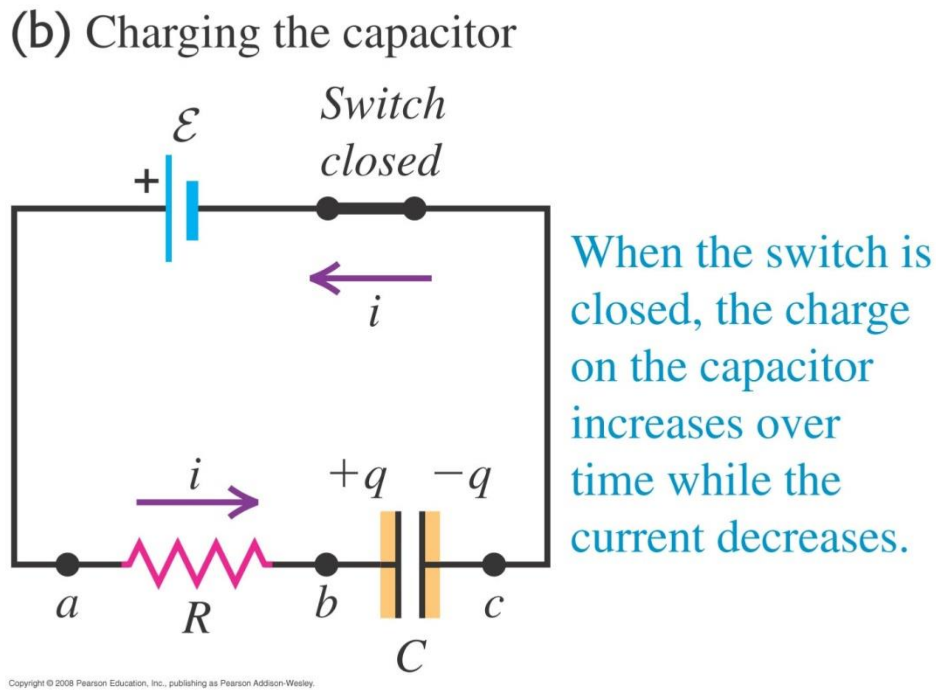


What happens after a long time? $t = \infty$

Capacitor is fully charged. No more charge can be displaced by the battery. Capacitor acts as an open switch. The current in the circuit is zero. Potential drop across the resistor is also zero.

$$q = Q_f \qquad V_C = \varepsilon = \frac{Q_f}{C} \Rightarrow Q_f = C\varepsilon$$

$$i_f = 0$$



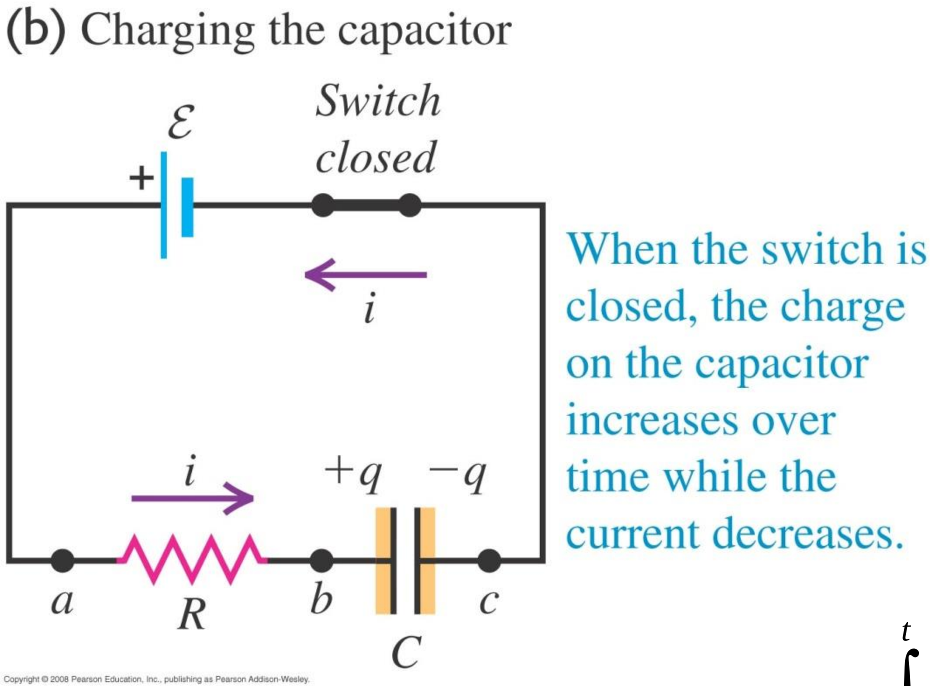
$$t = \text{intermediate}$$

$$\varepsilon - iR - \frac{q}{C} = 0$$

Emf is shared by voltage drop across the resistor and voltage drop across the capacitor.

$$\frac{d\varepsilon}{dt} - R\frac{di}{dt} - \frac{1}{C}\frac{dq}{dt} = 0 \Rightarrow -R\frac{di}{dt} - \frac{1}{C}\frac{dq}{dt} = 0 \Rightarrow \frac{di}{dt} = -\frac{i}{RC}$$

Current as a function of time



$$\frac{di}{i} = -\frac{dt}{RC}$$

$$\int_0^t \frac{di}{i} = -\frac{1}{RC}\int_0^t dt$$

$$\ln i\Big|_0^t = -t/RC \Rightarrow \ln i - \ln i_0 = -t/RC \Rightarrow \ln\frac{i}{i_0} = -t/RC$$

$$i = i_0 e^{-t/RC} = \frac{\mathcal{E}}{R} e^{-t/RC}$$

Charge as a function of time

$$i = \frac{\varepsilon}{R} e^{-t/RC}$$

$$\frac{dq}{dt} = \frac{\varepsilon}{R} e^{-t/RC}$$

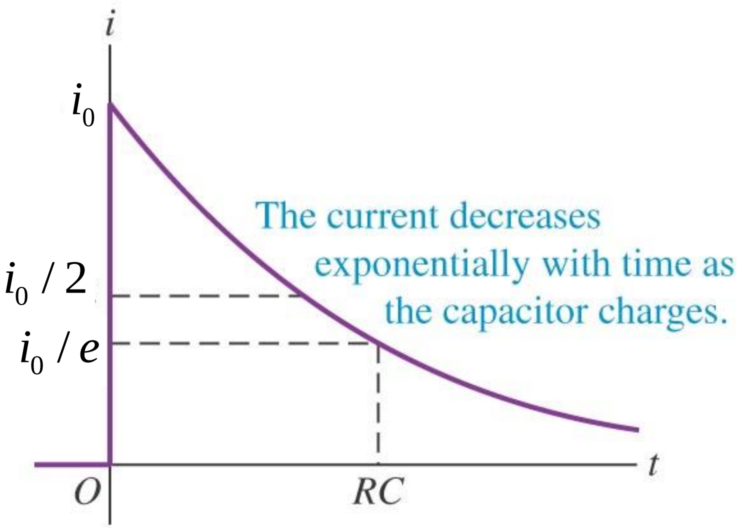
$$dq = \frac{\varepsilon}{R} e^{-t/RC} dt$$

$$\int_0^q dq = \frac{\varepsilon}{R} \int_0^t e^{-t/RC} dt$$

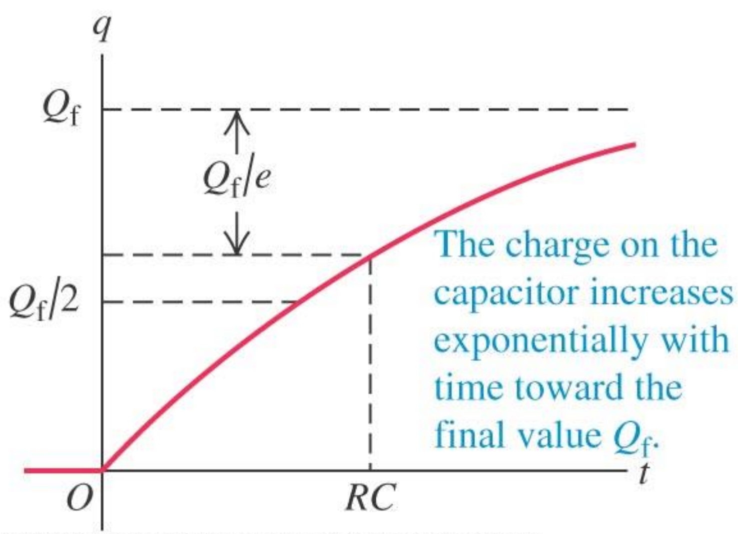
$$q = -\frac{\varepsilon RC}{R} e^{-t/RC} \Big|_0^t$$

$$q = \varepsilon C \left(1 - e^{-t/RC}\right) = Q_f \left(1 - e^{-t/RC}\right)$$

(a) Graph of current versus time for a charging capacitor



(b) Graph of capacitor charge versus time for a charging capacitor

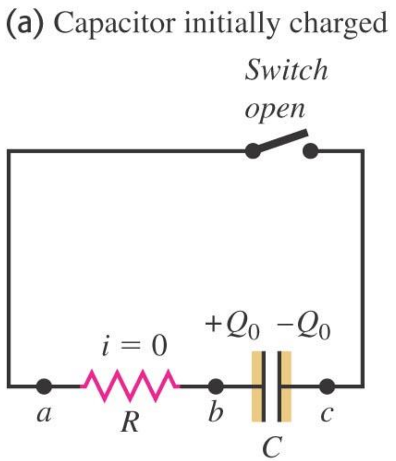


$$i = i_0 e^{-t/RC}$$

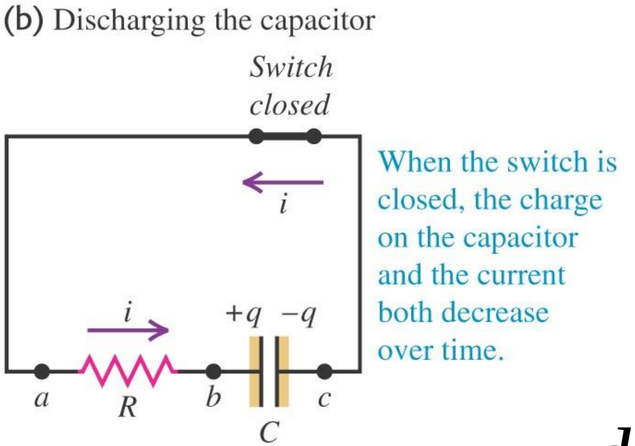
time	q	i
RC	$0.63Q_f$	$0.36i_0$
$2RC$	$0.86Q_f$	$0.13i_0$
$3RC$	$0.95Q_f$	$0.05i_0$
$4RC$	$0.98Q_f$	$0.02i_0$
$5RC$	$0.99Q_f$	$0.01i_0$

$$q = \varepsilon C\left(1 - e^{-t/RC}\right) = Q_f\left(1 - e^{-t/RC}\right)$$

$$t = 0 \text{ at the time switch is closed.}$$



$$q = Q_0, i = I_0 = \frac{\varepsilon C}{R}$$



$$t = \text{intermediate} \qquad -R\frac{dq}{dt} + \frac{q}{C} = 0$$

$$\frac{dq}{dt} = -\frac{q}{RC} \qquad \frac{dq}{q} = -\frac{dt}{RC}$$

$$\frac{dq}{q} = -\frac{dt}{RC}$$

$$\int_{Q_0}^{q} \frac{dq}{q} = -\int_{0}^{t} \frac{dt}{RC}$$

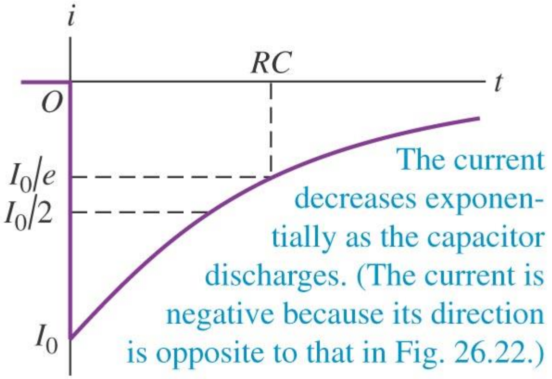
$$\ln\frac{q}{Q_0} = -\frac{t}{RC}$$

$$\frac{q}{Q_0} = e^{-\frac{t}{RC}} \Rightarrow q = Q_0 e^{-\frac{t}{RC}}$$

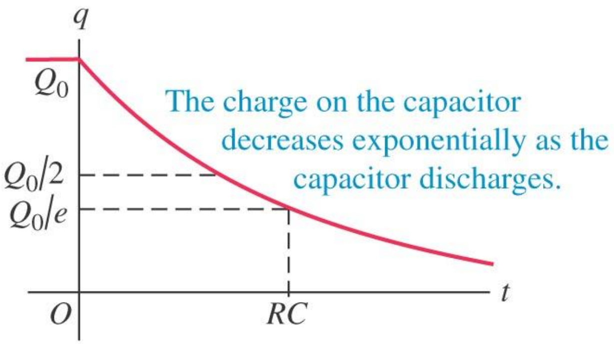
$$\frac{dq}{dt} = \frac{Q_0}{RC} e^{-\frac{t}{RC}}$$

$$i = I_0 e^{-\frac{t}{RC}}$$

(a) Graph of current versus time for a discharging capacitor



(b) Graph of capacitor charge versus time for a discharging capacitor



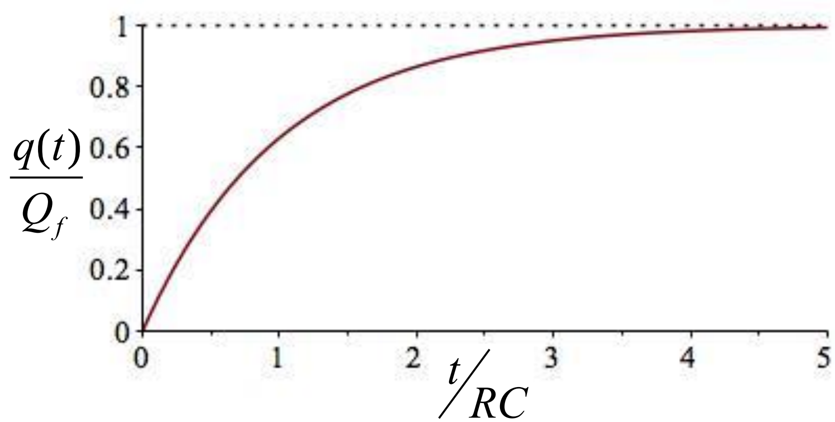
$$q = Q_0 e^{-\frac{t}{RC}}$$

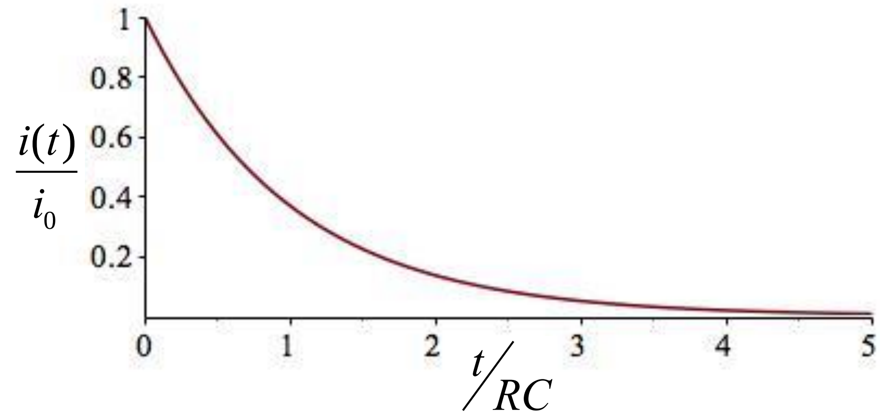
$$i = I_0 e^{-\frac{t}{RC}}$$

$$I_0 = \frac{Q_0}{RC}$$

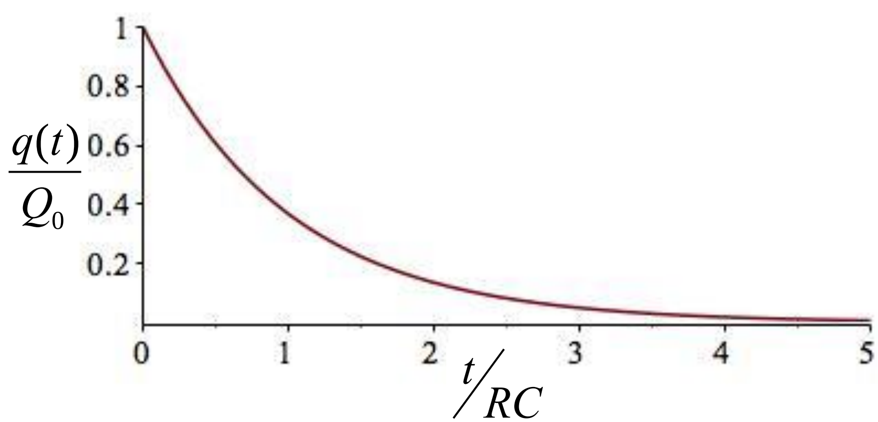
Charging/Discharging Capacitors

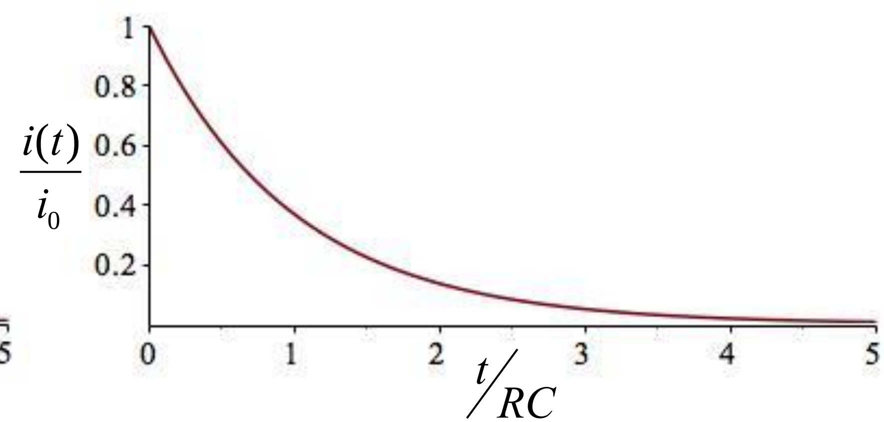
Charging: $q(t) = Q_f\left(1 - e^{-\frac{t}{RC}}\right)$ $i(t) = i_0 e^{-\frac{t}{RC}}$





Discharging: $q(t) = Q_0 e^{-\frac{t}{RC}}$ $i(t) = i_0 e^{-\frac{t}{RC}}$





The RC time constant

The constant RC pops up in the exponential factor for both charging and discharging capacitors. What does it represent?

The units of RC is seconds: $$[RC] = \frac{V}{A}\frac{C}{V} = \frac{C}{C/s} = s$$

We call RC the "RC time constant" and it tells us how quickly a capacitor can charge or discharge.

$$RC \circ t$$

After a time τ, the charge on a discharging capacitor is reduced by a factor of 1/e. After a time $N\tau$, it is reduced by a factor of $1/e^N$

$$q(t) = Q_0 e^{-\frac{t}{t}}$$

Document Camera Calculation

An RC circuit is shown below. Initially the switch is open and the capacitor is uncharged. At time t = 0 s, the switch is thrown to the left, connecting the capacitor to the battery. At time t = 15 ms the switch is thrown to the right, connecting the capacitor to the resistor.

1) How much charge builds up on the capacitor while it is connected to the battery?
2) What is the voltage across the resistor as a function of time as the capacitor discharges?
3) What is the ratio of the charging time to discharging time?