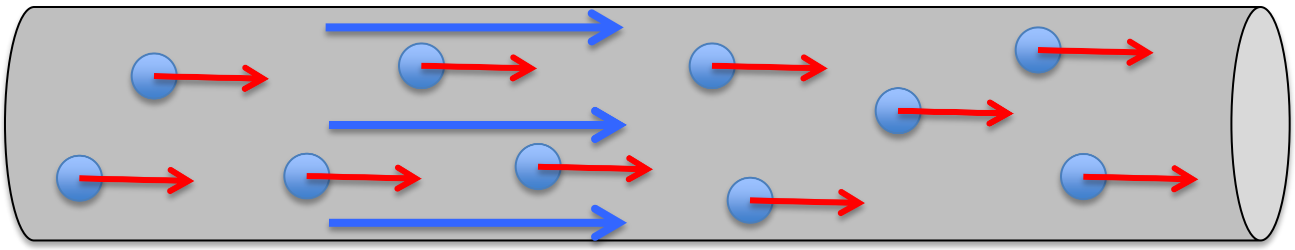
Last time

- Electric current: a microscopic picture
- Current density (a vector) vs current (a scalar)
- Electric fields in conductors and electron drift speed
- Resistance as a geometrical factor
- Resistivity: a microscopic picture

This time

- Review of the last lecture
- Resistivity and temperature
- Semiconductors
- Superconductors
- Measuring temperature
- Introduction of magnetism
- Magnetic force

Current



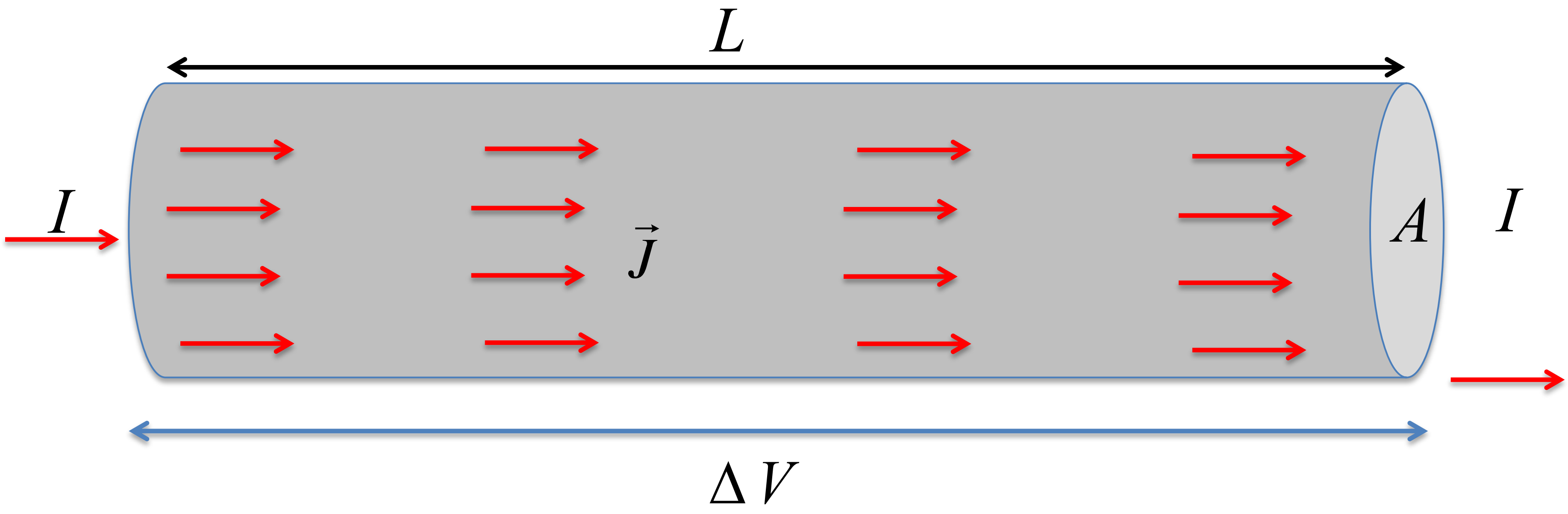
The ordered flow of charges is called the electric current.

In the absence of an electric field the drift velocity (ordered flow) due to random motion is zero.

$$\mathrm{v}_{\mathrm{th}} = \sqrt{\frac{3kT}{m_e}} \approx 10^5 \ \mathrm{m/s} \qquad \bar{\mathrm{v}}_{\mathrm{th}} = 0$$

In the presence of an electric field the drift velocity (ordered flow) is

$$i = n_e A e v_d \qquad \vec{J} = n_e e \vec{v}_d \qquad v_d = 0.1 - 1 \ \mathrm{mm/s} \ll \mathrm{v}_{\mathrm{th}}$$



For Ohmic materials (linear)

$$\vec{J} = \sigma \vec{E}$$

$$\vec{J} = \sigma \vec{E} = \frac{1}{\rho} \vec{E}$$

$$\rho \frac{L}{A} = \frac{\Delta V}{I}$$

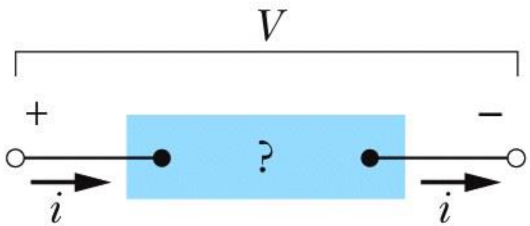
Quantities on the right hand side are function of the type of conductor and the geometry.

$$\rho \frac{L}{A} \equiv R$$

$$\boxed{\Delta V = IR}$$

Quantities on the left hand side can be directly measured.

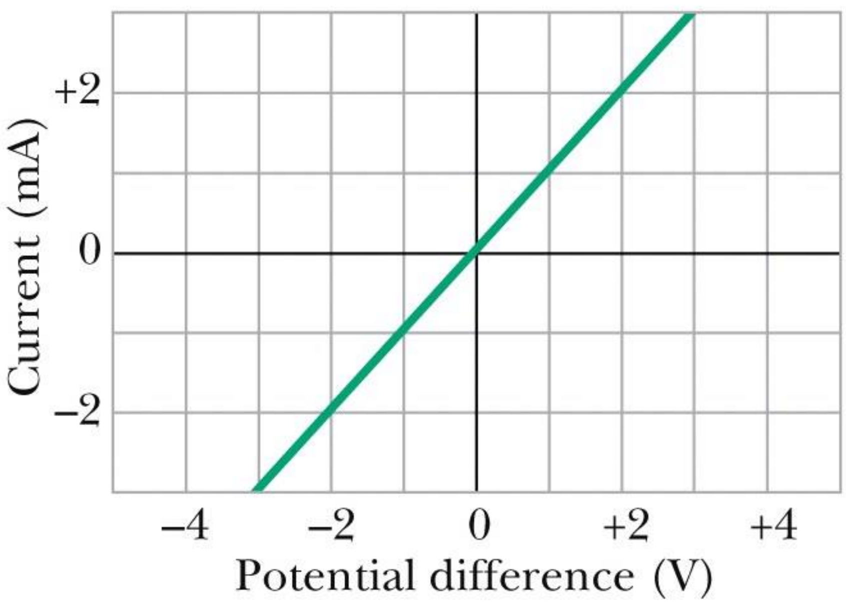
Ohmic vs non-Ohmic devices



(a)

Materials with isotropic electrical properties

Materials with anisotropic electrical properties (pn junction diode)



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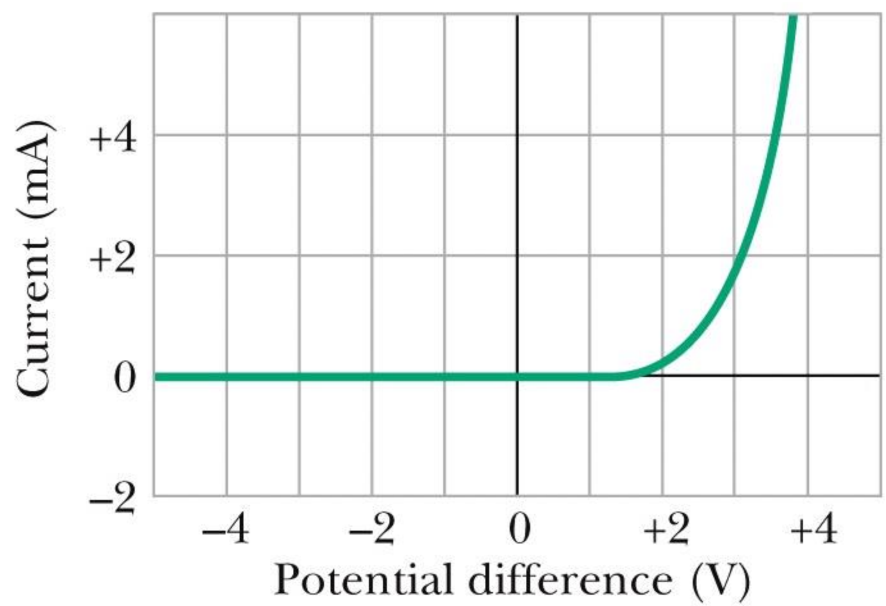

(c)

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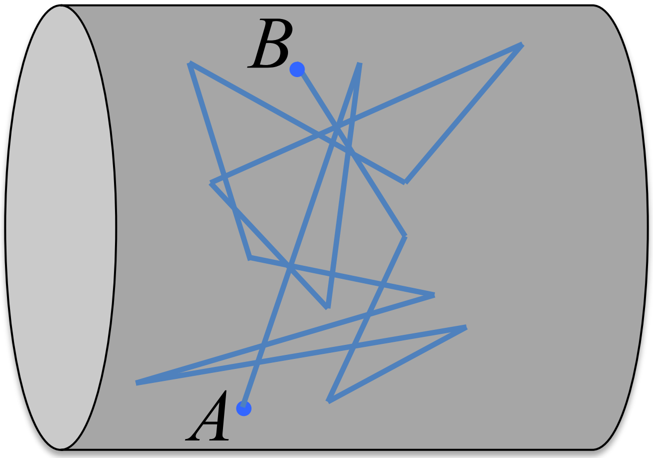
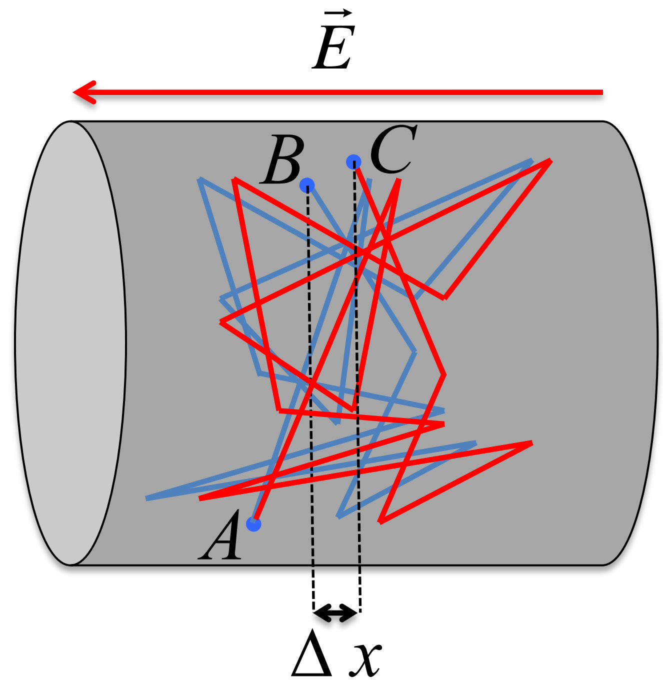
Microscopic view of resistivity

No E-field applied



$$v_d = \frac{eEt}{m}$$

$$v_d = \frac{J}{n_e e}$$



The average time between collisions is τ and is called the *mean free time*. Equating the two expressions for the drift speed, we get: Microscopic picture of resistivity:

$$\boxed{\rho = \frac{m}{n_e e^2 \tau}}$$

Consequence of this microscopic view

$\tau = 2.4 \times 10^{-14}$ s Averaged time between collisions

$\frac{1}{\tau} = 4 \times 10^{13}$ 1/s Number of collisions per second (collision frequency)

For linear materials and in the lowest order

$$\rho - \rho_0 = \rho_0 \alpha \left(T - T_0 \right)$$

This is why the resistance of a device depends on temperature. We can use this property to measure temperature.

Resistivity is intrinsic to a metal sample (like density is)

Table 25.1 Resistivities at Room Temperature (20 °C)

Substance		$\rho\,(\Omega \cdot m)$	Substance	$\rho\,(\Omega \cdot m)$
Conductors			**Semiconductors**	
Metals	Silver	1.47×10^{-8}	Pure carbon (graphite)	3.5×10^{-5}
	Copper	1.72×10^{-8}	Pure germanium	0.60
	Gold	2.44×10^{-8}	Pure silicon	2300
	Aluminum	2.75×10^{-8}	**Insulators**	
	Tungsten	5.25×10^{-8}	Amber	5×10^{14}
	Steel	20×10^{-8}	Glass	10^{10}–10^{14}
	Lead	22×10^{-8}	Lucite	$>10^{13}$
	Mercury	95×10^{-8}	Mica	10^{11}–10^{15}
Alloys	Manganin (Cu 84%, Mn 12%, Ni 4%)	44×10^{-8}	Quartz (fused)	75×10^{16}
	Constantan (Cu 60%, Ni 40%)	49×10^{-8}	Sulfur	10^{15}
	Nichrome	100×10^{-8}	Teflon	$>10^{13}$
			Wood	10^{8}–10^{11}

Resistivity and temperature

In general resistivity is a function of temperature. It increases with increasing temperature.

So resistors can be used to measure temperature (thermocouple).

For a limited range of temperature resistivity may be considered to be a linear function of temperature. If so,

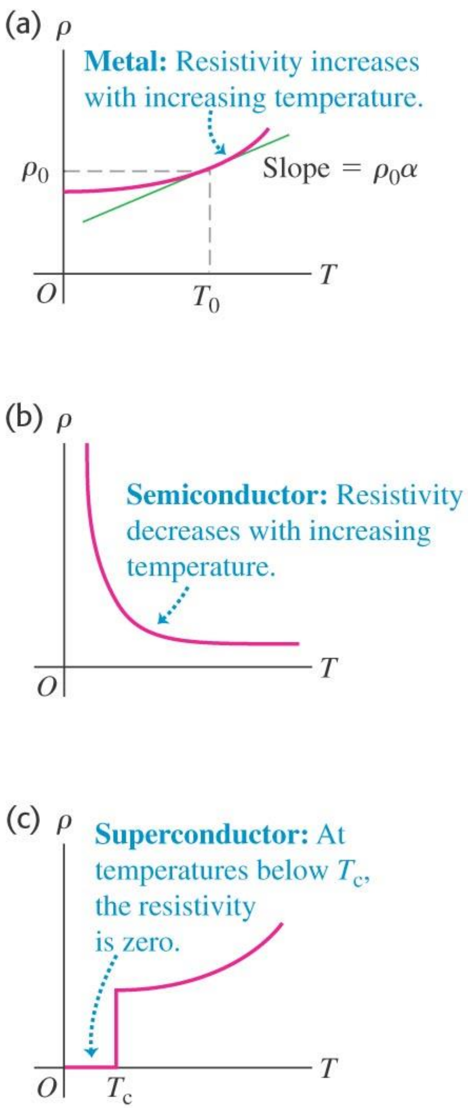
$$\rho = \rho_0 \left[1 + \alpha \left(T - T_0 \right) \right]$$

$$\rho = \rho_0\left[1+\alpha\left(T-T_0\right)\right]$$

$$\rho = \rho_0 + \rho_0\alpha\left(T-T_0\right)$$

$$\rho - \rho_0 = \rho_0\alpha\left(T-T_0\right)$$

$$\frac{\Delta\rho}{\Delta T} = \rho_0\alpha$$



This property can be used to measure temperature.

Note that temperature is not an easy quantity to measure.

Resistivity and temperature

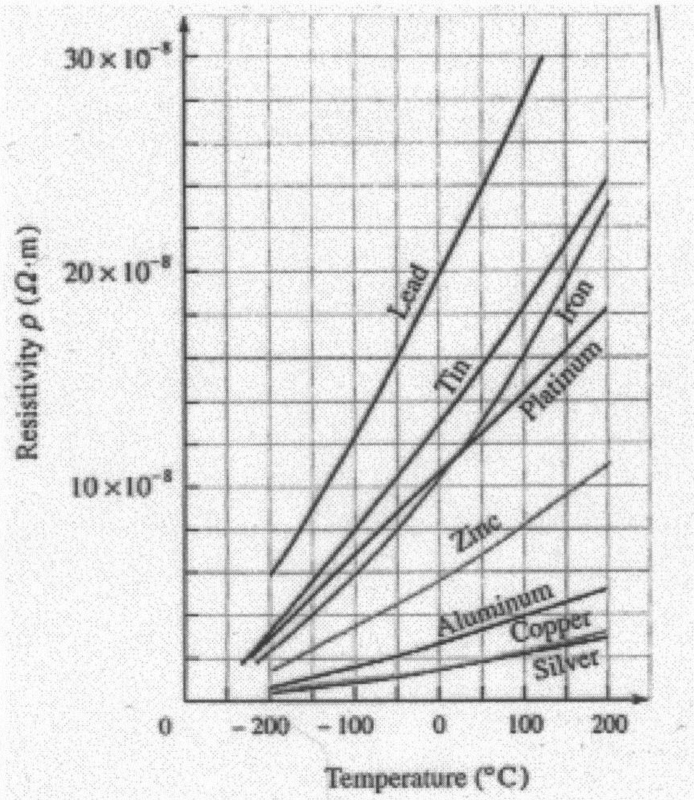
Table 25.2 Temperature Coefficients of Resistivity (Approximate Values Near Room Temperature)

Material	$\alpha\,[(°C)^{-1}]$	Material	$\alpha\,[(°C)^{-1}]$
Aluminum	0.0039	Lead	0.0043
Brass	0.0020	Manganin	0.00000
Carbon (graphite)	−0.0005	Mercury	0.00088
Constantan	0.00001	Nichrome	0.0004
Copper	0.00393	Silver	0.0038
Iron	0.0050	Tungsten	0.0045

For negative coefficient of resistivity the gain in density of the conduction electrons is larger than increase in resistivity because of random collisions with vibrating atomic sites and other electrons.

Thermocouples

$$\frac{\Delta\rho}{\Delta T} = \rho_0\alpha$$



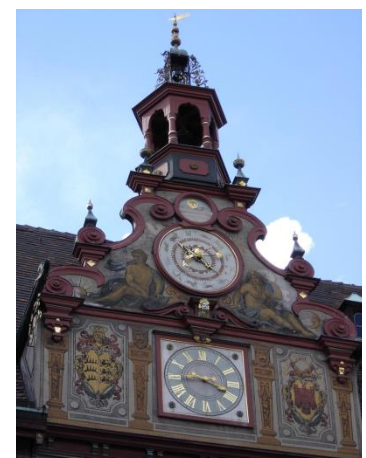
Slope of the resistivity curve near the temperature range of interest

Critical temperatures for some high T superconductors

Formula	Notation	T_c (K)	No. of Cu-O planes in unit cell	Crystal structure
$YBa_2Cu_3O_7$	123	92	2	Orthorhombic
$Bi_2Sr_2CuO_6$	Bi-2201	20	1	Tetragonal
$Bi_2Sr_2CaCu_2O_8$	Bi-2212	85	2	Tetragonal
$Bi_2Sr_2Ca_2Cu_3O_6$	Bi-2223	110	3	Tetragonal
$Tl_2Ba_2CuO_6$	Tl-2201	80	1	Tetragonal
$Tl_2Ba_2CaCu_2O_8$	Tl-2212	108	2	Tetragonal
$Tl_2Ba_2Ca_2Cu_3O_{10}$	Tl-2223	125	3	Tetragonal
$TlBa_2Ca_3Cu_4O_{11}$	Tl-1234	122	4	Tetragonal
$HgBa_2CuO_4$	Hg-1201	94	1	Tetragonal
$HgBa_2CaCu_2O_6$	Hg-1212	128	2	Tetragonal
$HgBa_2Ca_2Cu_3O_8$	Hg-1223	134	3	Tetragonal

Measuring mass

Measuring length



The NIST F-1 atomic clock is accurate to within one second every thirty million years.

Measuring time

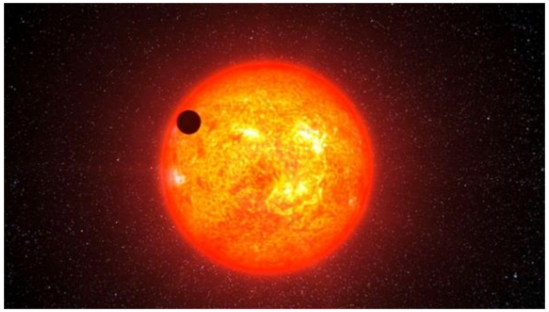
Measuring temperature

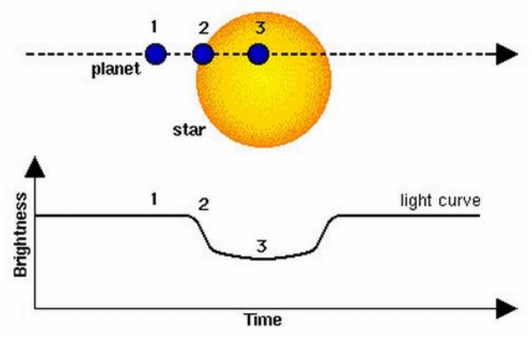
Ear thermometer

Infrared thermometer

Uses coefficient of volume expansion

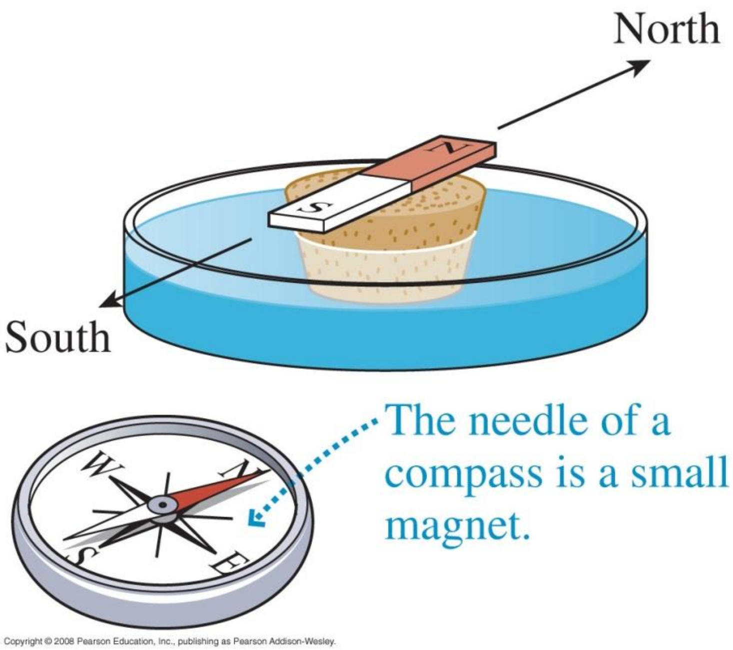
Detecting exo-planets

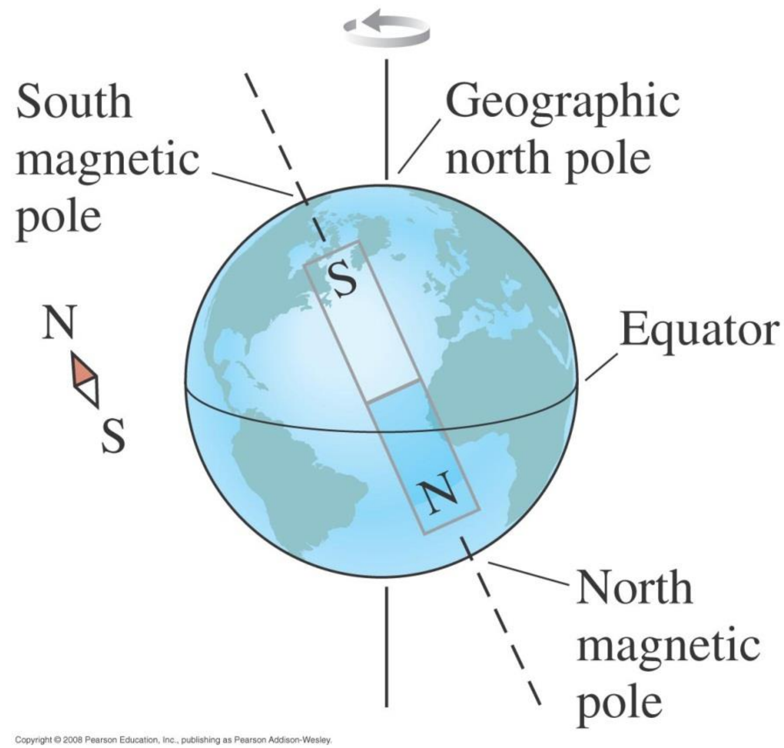


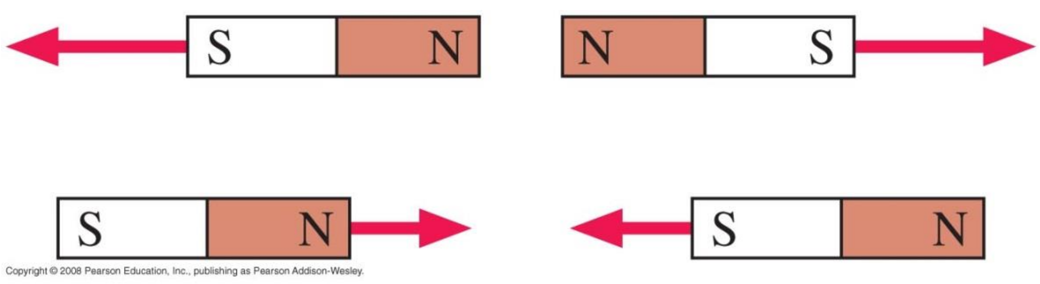


Magnetism

Magnetism



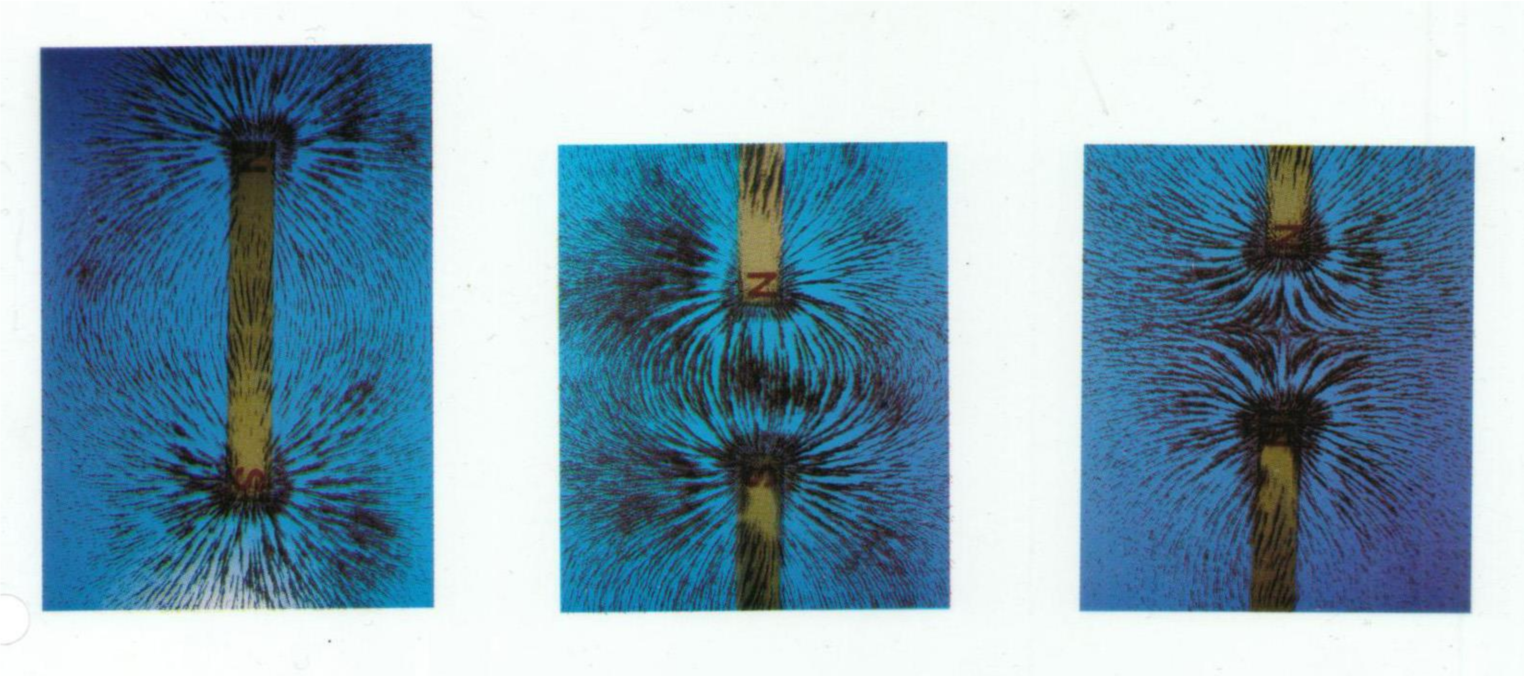




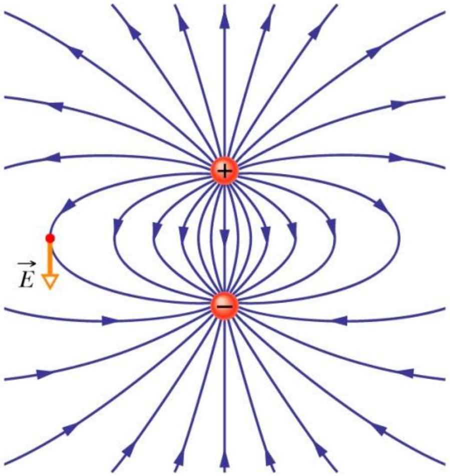
Like poles repel.

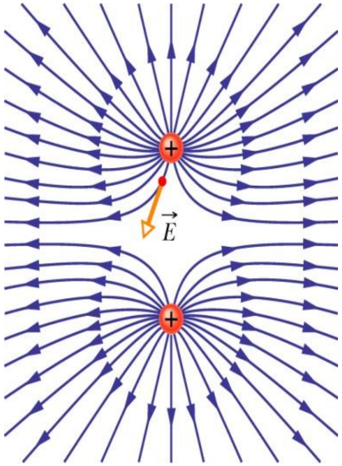
Unlike poles attract.

Bar magnets and their magnetic fields



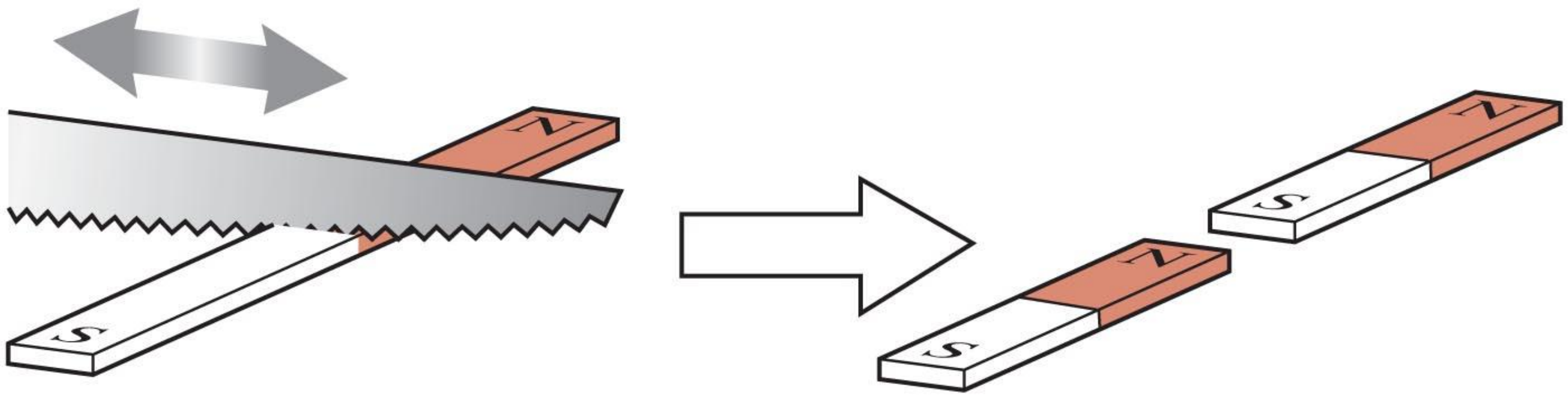
Electric charges and their electric fields





Magnetism is not the same as electricity!!

For example, cutting a magnet does not create one north-pole piece and one south-pole piece.

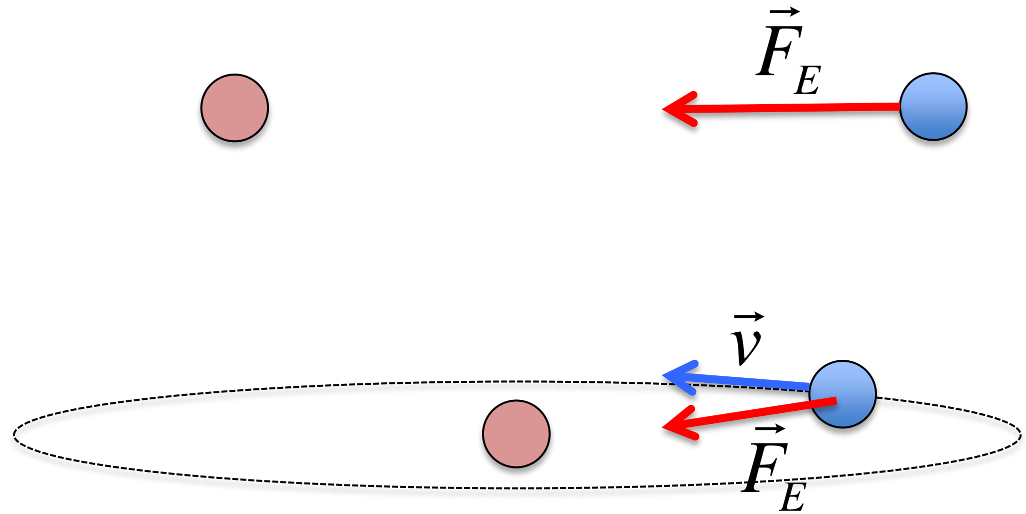


Magnetic monopoles do not seem to exist:

We cannot have a north pole without a south pole.

Electric Force on Charges

Electric force acts on a charge regardless of its motion.



$$\vec{F}_E = q\vec{E}$$

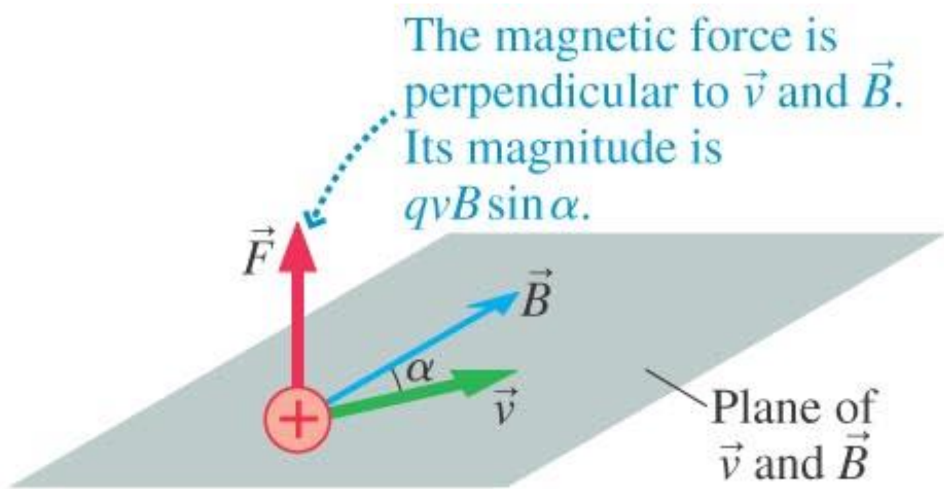
Coulomb's force

Magnitude: $F_E = qE$

Direction: Parallel to $\vec{E}$

Magnetic Force on Charges

Magnetic force acts only on a moving charge.



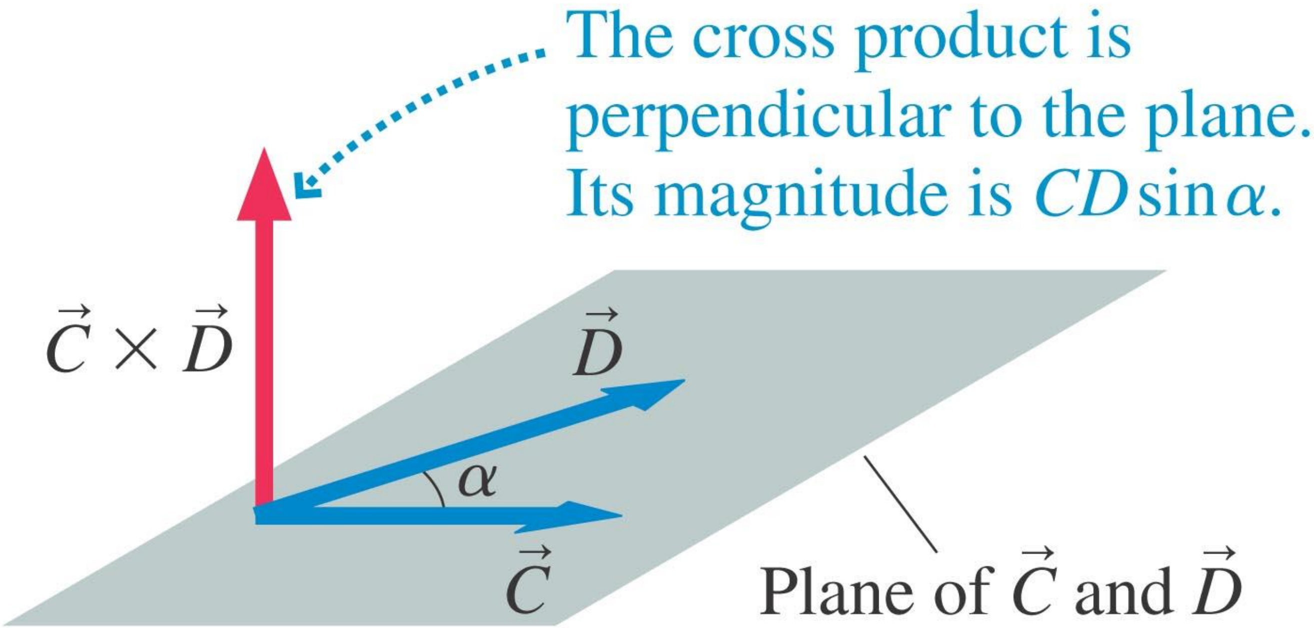
$$\vec{F}_B = q\vec{v} \times \vec{B}$$

Magnetic force

Magnitude: $F_B = qvB\sin\alpha$

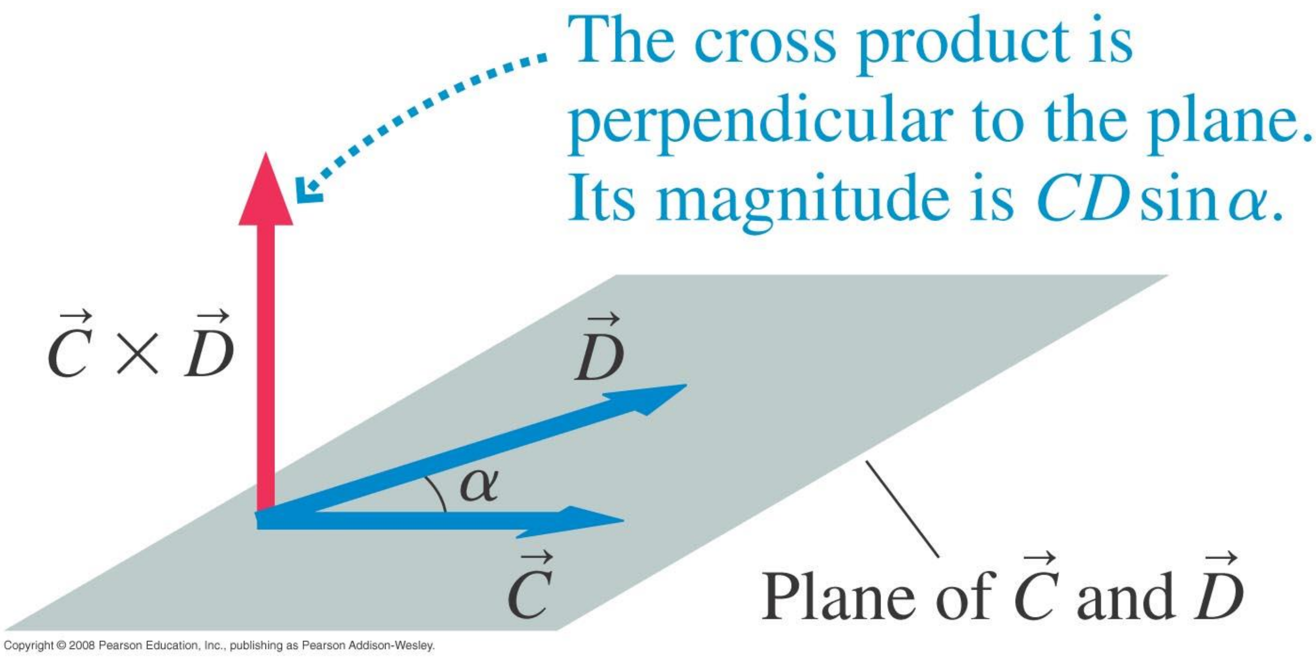
Direction: Right hand rule

The Vector Cross Product



Point the fingers of your right hand along the first vector in the cross product (vector C), then curl them so they point along the second vector (vector D). Your thumb gives the direction of the cross product.

The Vector Cross Product



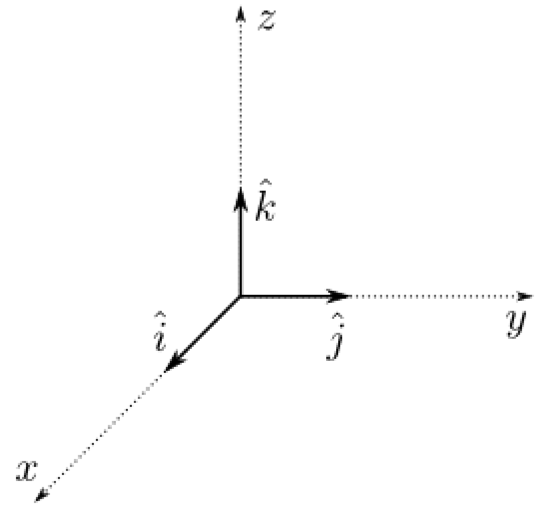
So $\vec{C}\times\vec{D}$ points up and $\vec{D}\times\vec{C}$ points down.

$$\left|\vec{C}\times\vec{D}\right| = \left|\vec{C}\right|\left|\vec{D}\right|\sin\alpha$$

Cross product vs regular product

Regular/dot product	**Cross product**
Distributive	Distributive
$\vec{B}\cdot\left(\vec{C}+\vec{D}\right)=\vec{B}\cdot\vec{C}+\vec{B}\cdot\vec{D}$	$\vec{B}\times\left(\vec{C}+\vec{D}\right)=\vec{B}\times\vec{C}+\vec{B}\times\vec{D}$
Commutative	Anticommutative
$CD=DC$ $\vec{C}\cdot\vec{D}=\vec{D}\cdot\vec{C}$	$\vec{C}\times\vec{D}=-\vec{D}\times\vec{C}$
Associative	Non-Associative
$B\left(CD\right)=\left(BC\right)D$	$\vec{B}\times\left(\vec{C}\times\vec{D}\right)\neq\left(\vec{B}\times\vec{C}\right)\times\vec{D}$

$$\hat{i} \times \hat{j} = \hat{k}$$



$$\hat{j} \times \hat{k} = \hat{i}$$

$$\hat{k} \times \hat{i} = \hat{j}$$

$$\hat{i} \times \hat{i} = 0 \qquad \hat{j} \times \hat{j} = 0 \qquad \hat{k} \times \hat{k} = 0$$

Unit vector notation

The cross product becomes easy to deal with when using unit vector notation

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

$$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$$

$$\hat{i}\times\hat{j} = \hat{k}$$

$$\hat{j}\times\hat{k} = \hat{i}$$

$$\hat{k}\times\hat{i} = \hat{j}$$

Now let's see what the cross product between A and B is:

$$\vec{C} = \vec{A}\times\vec{B}$$

$$\vec{C} = \left(A_x\hat{i} + A_y\hat{j} + A_z\hat{k}\right)\times\left(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}\right)$$

$$\vec{C} = \left(A_yB_z - A_zB_y\right)\hat{i} + \left(A_zB_x - A_xB_z\right)\hat{j} + \left(A_xB_y - A_yB_x\right)\hat{k}$$

Another way to think about it

Start with the two vectors in component form

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k} \qquad \vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$$

The cross product is given by the determinant of the following matrix:

$$\vec{C} = \vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$\vec{C} = \left(A_yB_z - A_zB_y\right)\hat{i} + \left(A_zB_x - A_xB_z\right)\hat{j} + \left(A_xB_y - A_yB_x\right)\hat{k}$$

Parallel and Perpendicular vectors

For parallel vectors

$$\vec{A} = A\hat{i} \qquad \vec{B} = B\hat{i}$$

$$\left|\vec{C}\right| = \left|\vec{A}\times\vec{B}\right| = AB\sin 0 = 0$$

$$\vec{C} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A & 0 & 0 \\ B & 0 & 0 \end{vmatrix} = \vec{0}$$

For perpendicular vectors

$$\vec{A} = A\hat{i} \qquad \vec{B} = B\hat{j}$$

$$\left|\vec{C}\right| = \left|\vec{A}\times\vec{B}\right| = AB\sin \pi/2 = AB$$

$$\vec{C} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A & 0 & 0 \\ 0 & B & 0 \end{vmatrix} = AB\hat{k}$$

Top Hat Question

A charged particle q enters a region with a constant B-field pointing into the page as shown. If the particle follows the path from **a** to **b** as shown

$$\boxed{\vec{F} = q\vec{v} \times \vec{B}}$$

What is the sign of q?

A. Positive

B. Negative

C. Not enough info