# Last time

- Mutual force between two long straight current carrying conductor
- Magnetic field of a circular current loop on the axis of the loop
- Introduction of Maxwell's Equations

# This time

- Ampere's Law

# Sources of magnetic field

# The wonderful Maxwell's Equations

$$\oint \vec{E} \bullet d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_0}$$

$$\oint \vec{B} \bullet d\vec{A} = 0$$

$$\oint \vec{B} \bullet d\vec{l} = \mu_0 \left( i_c + \varepsilon_0 \frac{d\Phi_E}{dt} \right)$$

$$\oint \vec{E} \bullet d\vec{l} = -\frac{d\Phi_B}{dt}$$

Among other things, they explain the behaviour of light.

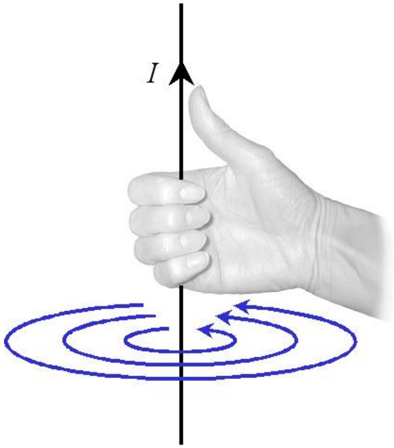
# Ampere's Law

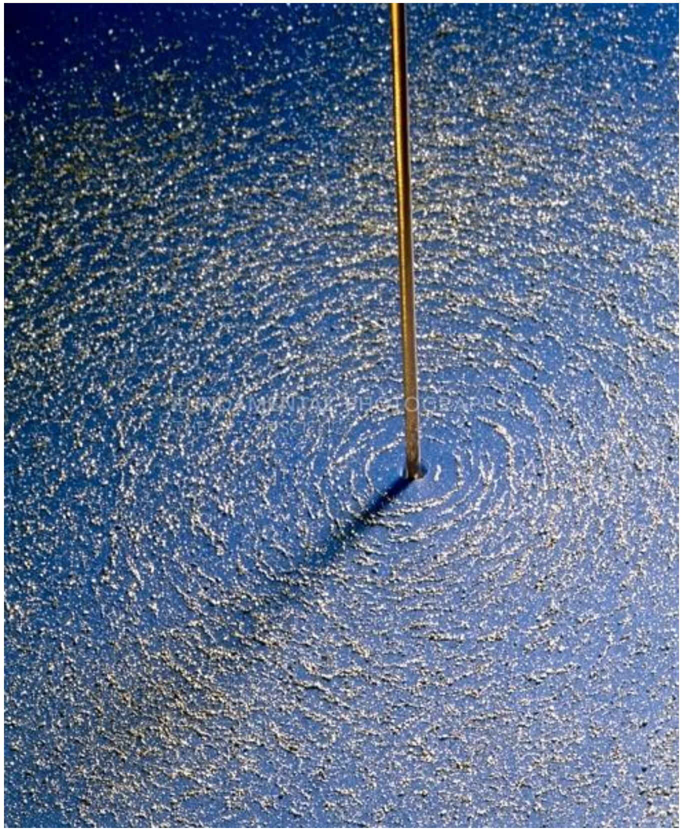
$$\oint \vec{B} \bullet d\vec{l} = \mu_0 i_{enclosed}$$

The path integral of the static magnetic field around any closed path, defining an open surface, is independent of the shape of the path and only a function of the total current enclosed by the path, the total current crossing the surface defined by the closed path.

# Magnetic field of a infinitely long straight current carrying conductor

$$B = \frac{\mu_0 I}{2\pi r}$$



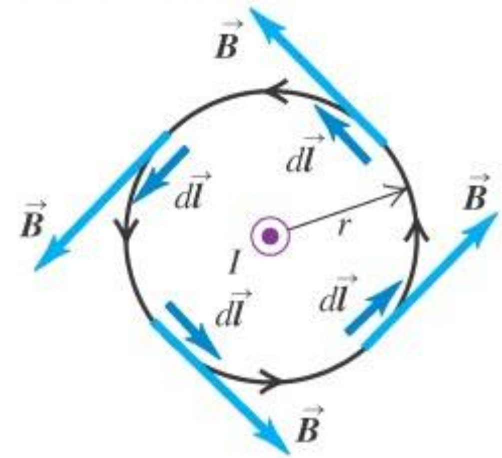
Magnetic field lines are circular loops. Magnetic field vector is tangent to the loop.

# Ampere's Law —specific

$$B = \frac{\mu_0 I}{2\pi r}$$

(a) Integration path is a circle centered on the conductor; integration goes around the circle counterclockwise.

Result: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I$




$d\vec{l}$ and $\vec{B}$ are parallel and in the same direction.

$$\vec{B} \bullet d\vec{l} = Bdl\cos 0 = Bdl$$

$\oint \vec{B} \bullet d\vec{l} = ?$ Line integral (integral around the circle)

$$\oint \vec{B} \bullet d\vec{l} = \oint Bdl = \oint \frac{\mu_0 I}{2\pi r} dl = \frac{\mu_0 I}{2\pi r} \underbrace{\oint dl}_{2\pi r} = \mu_0 I$$

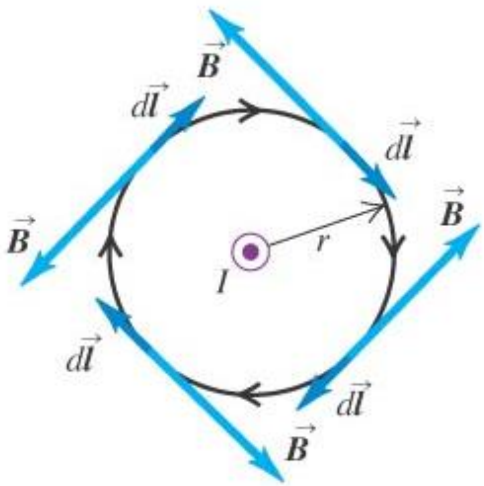
$$\oint \vec{B} \bullet d\vec{l} = \mu_0 I$$

# Ampere's Law —specific

$$B = \frac{\mu_0 I}{2\pi r}$$

(b) Same integration path as in (a), but integration goes around the circle clockwise.

Result: $\oint \vec{B} \cdot d\vec{l} = -\mu_0 I$




$d\vec{l}$ and $\vec{B}$ are parallel and in the opposite directions.

$$\vec{B} \bullet d\vec{l} = Bdl\cos 180 = -Bdl$$

$$\oint \vec{B} \bullet d\vec{l} = -\oint Bdl = -\oint \frac{\mu_0 I}{2\pi r} dl = -\frac{\mu_0 I}{2\pi r} \oint_{2\pi r} dl = -\mu_0 I$$

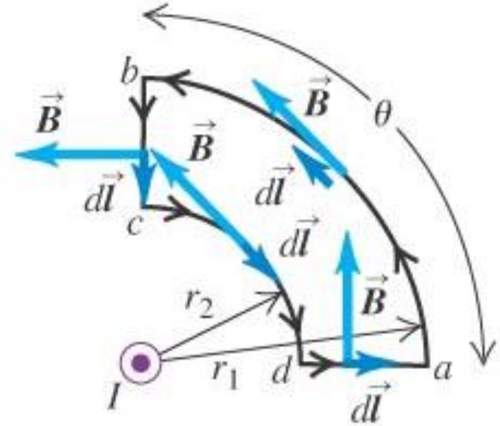
$$\oint \vec{B} \bullet d\vec{l} = -\mu_0 I$$

# Ampere's Law —specific

$$B=\frac{\mu_0 I}{2\pi r}$$

(c) An integration path that does not enclose the conductor.

Result: $\oint \boldsymbol{B}\cdot d\boldsymbol{l}=0$




$$\oint \vec{B}\bullet d\vec{l}=\underset{\uparrow\uparrow}{\int_a^b \vec{B}\bullet d\vec{l}}+\underset{\perp}{\int_b^c \vec{B}\bullet d\vec{l}}+\underset{\uparrow\downarrow}{\int_c^d \vec{B}\bullet d\vec{l}}+\underset{\perp}{\int_d^a \vec{B}\bullet d\vec{l}}$$

$$\oint \vec{B}\bullet d\vec{l}=\int_a^b B dl+0-\int_c^d B dl+0$$

$$\oint \vec{B}\bullet d\vec{l}=\int_a^b \frac{\mu_0 I}{2\pi r_1}dl-\int_c^d \frac{\mu_0 I}{2\pi r_2}dl$$

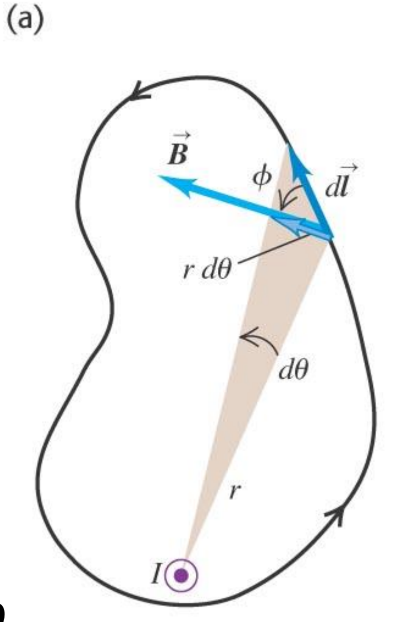
$$\oint \vec{B}\bullet d\vec{l}=\frac{\mu_0 I}{2\pi r_1}\int_a^b dl-\frac{\mu_0 I}{2\pi r_2}\int_c^d dl=\frac{\mu_0 I}{2\pi r_1}r_1\theta-\frac{\mu_0 I}{2\pi r_2}r_2\theta=0$$

$$\oint \vec{B}\bullet d\vec{l}=0$$

An integration that does not enclose the conductor.

# Ampere's Law — general

Integration encloses the conductor.




$$\oint \vec{B}\bullet d\vec{l} = \oint B dl \cos\phi = \oint B r d\theta = \oint \frac{\mu_0 I}{2\pi r} r d\theta = \frac{\mu_0 I}{2\pi} \oint d\theta$$

$2\pi$

$$\oint \vec{B}\bullet d\vec{l} = \mu_0 I$$

Integration is carried out in the same direction as the magnetic field.

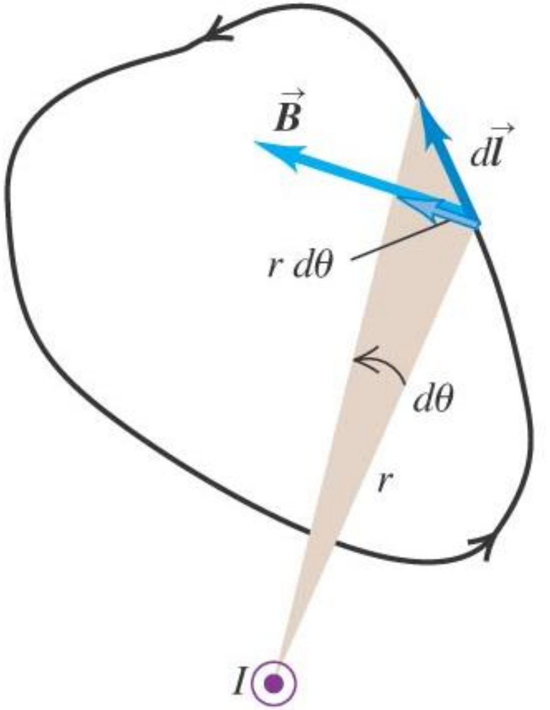
$$\oint \vec{B}\bullet d\vec{l} = -\mu_0 I$$

Integration is carried out in the opposite direction as the magnetic field.

# Ampere's Law — general

Integration that does not enclose the conductor.

(b)




$$\oint \vec{B} \bullet d\vec{l} = ?$$

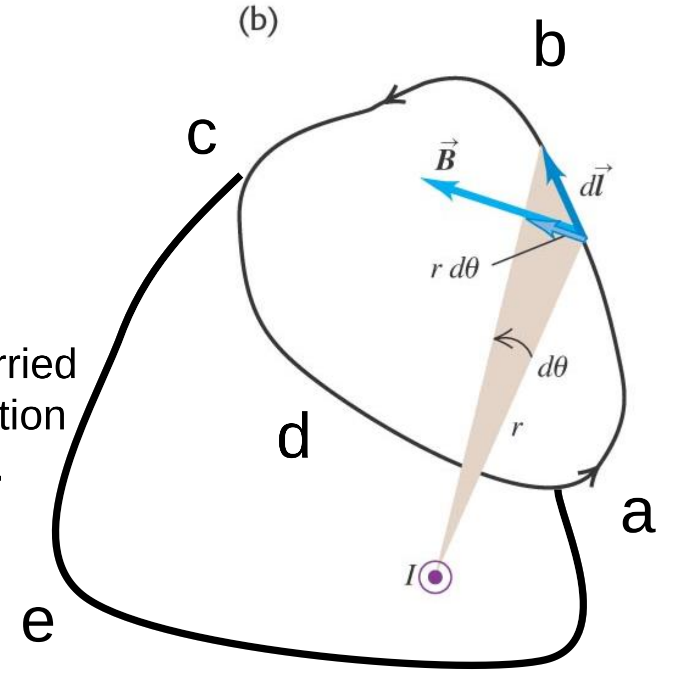
# Ampere's Law — general

Integration that does not enclose the conductor.

$$\oint \vec{B}\bullet d\vec{l} = \int_{\text{Path 1}} \vec{B}\bullet d\vec{l} = \int_{abcea} \vec{B}\bullet d\vec{l} = \mu_0 I$$

$$\oint \vec{B}\bullet d\vec{l} = \int_{abc} \vec{B}\bullet d\vec{l} + \int_{cea} \vec{B}\bullet d\vec{l} = \mu_0 I$$

This integration is carried out in the same direction as the magnetic field.




$$\oint \vec{B}\bullet d\vec{l} = \int_{\text{Path 2}} \vec{B}\bullet d\vec{l} = \int_{adcea} \vec{B}\bullet d\vec{l} = \mu_0 I$$

This integration is also carried out in the same direction as the magnetic field.

$$\oint \vec{B}\bullet d\vec{l} = \int_{adc} \vec{B}\bullet d\vec{l} + \int_{cea} \vec{B}\bullet d\vec{l} = \mu_0 I$$

$$\int_{cda} \vec{B}\bullet d\vec{l} + \int_{aec} \vec{B}\bullet d\vec{l} = -\mu_0 I$$

# Ampere's Law — general

Integration that does not enclose the conductor.

$$\int_{abc} \vec{B}\bullet d\vec{l} + \int_{cea} \vec{B}\bullet d\vec{l} = \mu_0 I$$

add

$$\int_{cda} \vec{B}\bullet d\vec{l} + \int_{aec} \vec{B}\bullet d\vec{l} = -\mu_0 I$$

$$\int_{abc} \vec{B}\bullet d\vec{l} + \int_{cea} \vec{B}\bullet d\vec{l} + \int_{cda} \vec{B}\bullet d\vec{l} + \int_{aec} \vec{B}\bullet d\vec{l} = 0$$

$$\int_{abc} \vec{B}\bullet d\vec{l} + \int_{cda} \vec{B}\bullet d\vec{l} + \int_{cea} \vec{B}\bullet d\vec{l} - \int_{cea} \vec{B}\bullet d\vec{l} = 0$$

$$\int_{abcda} \vec{B}\bullet d\vec{l} = 0$$