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Fill in these boxes with your LAST NAME followed by your FIRST INITIAL.

University of Calgary

Faculty of Science

Midterm Test Solutions

PHYSICS 259 ALL LECTURE SECTIONS

February 13, 2014, 7:00–8:30 p.m.

Time: 90 minutes.

DO NOT TEAR OFF THIS PAGE! You may, however, tear off the last page, which has an equation sheet and table of integrals.

This is a closed-book exam worth a total of 30 points. Please answer all questions. Use of the Schulich calculator or equivalent is allowed.

**Write your Last Name and First Initial on this top sheet in the grid above.** (Do not write your ID number on this page.)

Make sure this question paper booklet contains 14 pages. If you are missing any pages, get a new booklet from the exam supervisor.

You should also have a **separate set of Answer Sheets**. This is where you enter Multiple Choice answers of Part I and also detailed solutions to the problems of Part II. Only work entered in the indicated spaces on the Answer Sheets will be marked.

**IMPORTANT: YOUR ID NUMBER MUST BE ENTERED AT THE TOP OF EACH AND EVERY ONE OF THE ANSWER SHEETS. DO THIS NOW.**

Begin working on the examination when instructed to do so by the supervisor.

# Part I: Multiple-Choice Questions (Total: 13 marks)

Enter answers to multiple choice questions on the first Answer Sheet using space provided in the upper right of the page. Each question in Part I is worth one point. You should try to complete Part I in about 35 minutes or less.

- 1) A point charge  $q$  is placed at the center of a cube with sides of length  $L$ . The net electric flux  $\Phi_E$  through the surface will change if
- the cube expands in volume
  - the cube rotates about the center
  - the point charge moves off centre (but still inside the cube)
  - the point charge moves to just outside the cube  $\Leftarrow \checkmark$
  - a second point charge is placed just outside the cube

- 2) When a piece of paper is held with the area vector  $\vec{A}$  parallel to a uniform electric field  $\vec{E}$ , the flux through the paper is  $25.0 \text{ Nm}^2/\text{C}$ . If the paper is tilted by some angle  $\alpha$  then the flux is reduced to  $19.0 \text{ Nm}^2/\text{C}$ . What is the angle  $\alpha$ ? (choose the closest answer)

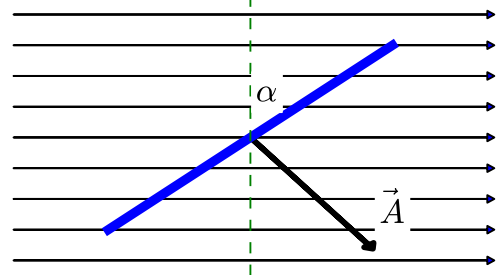


Figure 2: Tilted sheet in uniform electric field

- 3) An electron is released from rest at the surface of a large metal plate held at  $-10.0$  volts. A second, parallel metal plate at potential  $0 \text{ V}$  is placed  $3.00 \text{ cm}$  away. What is the electron speed at a point halfway between the two plates?
- $1.3 \times 10^6 \text{ m/s}$   $\Leftarrow \checkmark$
  - $2.6 \times 10^6 \text{ m/s}$
  - $4.9 \times 10^6 \text{ m/s}$
  - $1.1 \times 10^5 \text{ m/s}$
  - The electron does not move to the second plate.

- 4) Figure 4 shows the electric potential  $V(x)$  measured as a function of position along the  $x$ -axis. At which of the indicated locations does the  $x$ -component of the electric field have its largest positive value?

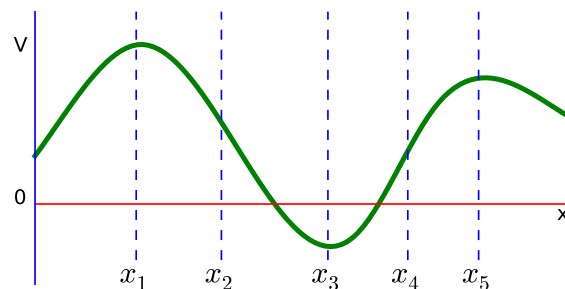
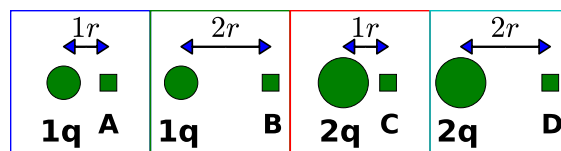


Figure 4: Potential profile along the  $x$ -axis.

- $x_1$
- $x_2$   $\Leftarrow \checkmark$
- $x_3$
- $x_4$
- not enough information provided

- 5) In Figure 5 the electric field is measured at two different distances ( $r$  and  $2r$ ) from two different charges ( $q$  and  $2q$ ). Rank from largest to smallest the magnitudes  $E_A$ ,  $E_B$ ,  $E_C$ , and  $E_D$  of the electric field at points A, B, C and D, respectively. ( $E_A = E_D$  means  $E_A$  and  $E_D$  are equal.)



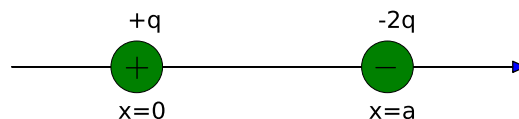
**Figure 5:** Electric field at two different distances from two different point charges.

- $E_B > E_A = E_D > E_C$
- $E_D = E_C > E_B = E_A$
- $E_B > E_D > E_A > E_C$
- $E_C > E_A > E_D > E_B \quad \Leftarrow \checkmark$
- $E_C > E_D > E_A > E_B$

- 6) A point charge  $q$  is placed at the center of a cube with sides of length  $L$ . If  $q = 3\mu C$  and  $L = 7cm$  then what is the flux through a single face of the cube? (choose the closest answer)

- $0 Nm^2/C$
- $3.4 \times 10^5 Nm^2/C$
- $5.6 \times 10^4 Nm^2/C \quad \Leftarrow \checkmark$
- $3.3 \times 10^2 Nm^2/C$
- $1.1 \times 10^7 Nm^2/C$

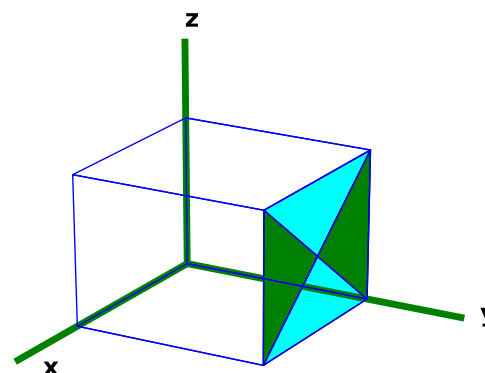
- 7) Where in Figure 7 is the electric field zero (other than at infinity)?



**Figure 7:** Two charges

- Somewhere in each of the regions  $x < 0$  and  $0 < x < a$
- Somewhere in the region  $x > a$ , only
- Somewhere in the region  $0 < x < a$ , only
- Somewhere in each of the regions  $0 < x < a$  and  $x > a$
- Somewhere in the region  $x < 0$ , only  $\Leftarrow \checkmark$

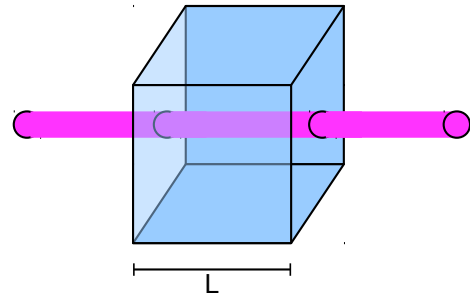
- 8) Figure 8 shows a cube with edges of length 2.00 m, and one corner at the origin. If there is a uniform electric field given by  $\vec{E} = 4.25\hat{i} - 2.75\hat{j}$  V/m in this region of space, what is the electric flux through the right-hand shaded face shown in the figure?



**Figure 8:** Gaussian cube

- $-17 Nm^2/C$
- $-11 Nm^2/C \quad \Leftarrow \checkmark$
- $+6 Nm^2/C$
- $+11 Nm^2/C$
- $+17 Nm^2/C$

- 9) In Figure 9 a horizontal, insulating rod carries a uniform, positive linear charge density (charge per unit length)  $\lambda$ . A student has drawn a Gaussian surface in the shape of a cube with edges of length  $L$ , centered on the rod. The net outward flux through this closed Gaussian surface is

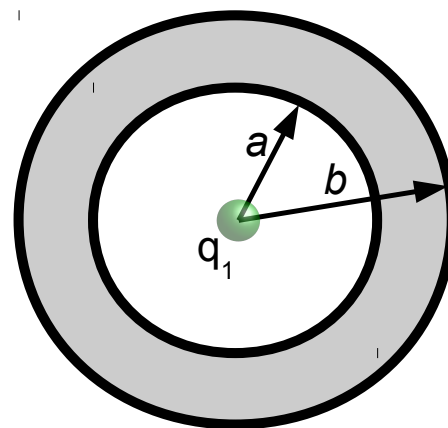


**Figure 9:** Rod of charge inside a Gaussian cube.

- a)  $2L\lambda/\epsilon_0$
  - b) impossible to calculate.
  - c)  $L\lambda/\epsilon_0$        $\Leftarrow \checkmark$
  - d)  $6L^2\lambda/\epsilon_0$
  - e)  $4L^2\lambda/\epsilon_0$
- 10) A test charge  $q$  initially feels a force,  $F$ , due to a point charge  $Q$  located a distance  $d$  away. Then the distance between charges is increased to 10 times the initial separation and the charge  $Q$  is changed so that the electrostatic force,  $F$ , remains the same ( $q$  does not change). How much must the charge  $Q$  increase or decrease?
- a) Increase by a factor of 10.
  - b) None of the other statements is correct.
  - c) Increase by a factor of 100.       $\Leftarrow \checkmark$
  - d) Decrease by a factor of 10.
  - e) Decrease by a factor of 100.
- 
- 11) A solid metal sphere has been given an excess charge. As a consequence, the electric potential at the surface of the sphere is 450 V, relative to  $V = 0$  at infinity. What is the electric potential at the centre of the sphere (and what is the reason)?
- a) 900 V (the electric potential at the surface of a sphere is half that at the centre, if  $V = 0$  at infinity)
  - b) 0 V (the electric potential is zero in a conductor)
  - c) Indeterminate (the charge distribution inside the sphere has not been specified)
  - d) 450 V (the electric potential gradient is zero in a conductor)       $\Leftarrow \checkmark$
  - e) Infinite (the radius is zero at the centre)
-

- 12) In Figure 12 the central point charge is  $q_1 = +2Q$  and there is a net negative charge  $-Q$  on the spherical conducting shell. Distance from the center of the sphere is  $r$ . Which of the following statements about the electric field magnitude  $E$  is true?

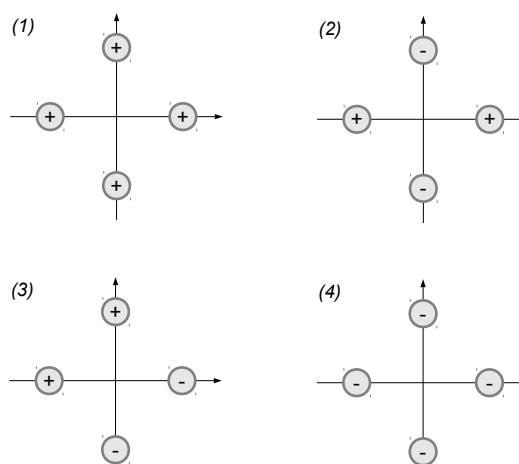
- a)  $E$  is  $\frac{Q}{4\pi\epsilon_0 r^2}$  for all  $r > b$ .  $\Leftarrow \checkmark$
- b)  $E$  is zero for all  $r < b$ .
- c)  $E$  is  $\frac{Q}{4\pi\epsilon_0 r^2}$  for all  $r < a$ .
- d)  $E$  is  $\frac{2Q}{4\pi\epsilon_0 r^2}$  for all  $r > b$ .
- e) none of the above



**Figure 12:** Spherical conducting shell with inner radius  $a$  and outer radius  $b$ . Point charge  $q_1$  is located at the centre of the hollow shell.

- 13) In Figure 13, which charge configuration(s) have a non-zero electric field at the origin?

- a) Configuration 2.
- b) Configuration 1.
- c) Configuration 3.  $\Leftarrow \checkmark$
- d) Configurations 2 and 3.
- e) Configuration 4.



**Figure 13:** Four sets of four charges.

- 14) This is **version @V@** of the exam. Please select **@V@** in the row marked "Version" in the Multiple Choice Answers area on the first Answer Sheet. **Be sure to do this now, before moving on to other questions on the paper.**

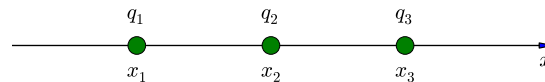
Solutions for each randomized version

- A : eecba bcdad cbe
- B : ceae addad cac
- C : deebe eedde edd

## Part II: Written Answer Questions (Total: 17 marks)

**IMPORTANT:** Write your answers to the problems in Part II in the corresponding boxes on the Answer Sheets. Work must be shown for full marks. Rough work can be done on the back of this question paper, but only the work appearing on the Answer Sheets will be marked.

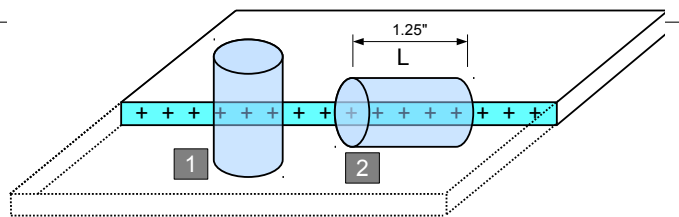
- 15) [6 marks] Three point charges are located on the x-axis as shown in Figure 15. The amounts of charge are  $q_1 = -1 \mu\text{C}$ ,  $q_2 = +3 \mu\text{C}$ ,  $q_3 = -1 \mu\text{C}$  and the positions are  $x_1 = 1\text{cm}$ ,  $x_2 = 3\text{cm}$ ,  $x_3 = 5\text{cm}$ .



**Figure 15:** Three charges equally spaced along the x-axis.

- Calculate the electric force on the right-most charge  $q_3$  due to the other two charges. [3 marks]
- Calculate the change in electrostatic potential energy of the system if the middle charge ( $q_2$ ) were moved out to infinity. [3 marks]

- 16) [6 marks] Figure 16 shows a very large thin sheet with uniform positive charge density  $+\sigma$ . Two cylindrical Gaussian surfaces both have identical radii ( $r$ ) and lengths ( $L$ ) but are oriented vertically (#1) and horizontally (#2). The volume of each cylinder is equally split, with half above and half below the charge sheet.



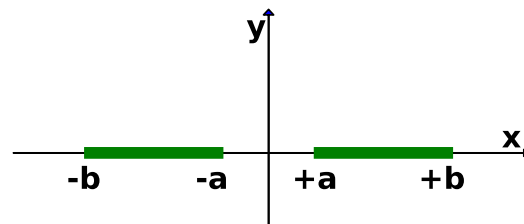
**Figure 16:** Two cylindrical Gaussian surfaces centered on a very large uniform sheet of charge. (Cut-away of charge sheet is for display only).

- Using only symmetry arguments, what can be said about the electric field  $\vec{E}(x, y, z)$  above the sheet ( $z > 0$ )? [1 mark]
- Calculate the electric flux through each surface (top, bottom, and sides) of the vertical cylinder (#1). [3 marks]
- Assume that both cylinders have the same total electric flux passing through them. If this is true then the quantities  $L$  and  $r$  must be related. Derive that relationship. [2 marks]

- 17) [5 marks] Two charged rods are located on the x-axis as shown in Figure 17. Both rods have the same length  $L$  and are located the same distance from the origin.

Assume both rods have the same positive charge density  $+\lambda$ .

- Write down an approximate formula for electric potential  $V(x)$  at large distances along the x-axis ( $x \gg +b$ ). Briefly explain the idea behind this approximation. [1 mark]
- Calculate the electric potential at the origin  $V(0)$ . [3 marks]



**Figure 17:** Two charged rods.

Assume the two rods have opposite charge density:  $-\lambda$  on the left and  $+\lambda$  on the right.

- Calculate the electric potential at the origin  $V(0)$ . [1 mark]

# CONSTANTS AND USEFUL EQUATIONS

$$k = \text{Coulomb constant} = 8.99 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

$$\epsilon_0 = \text{permittivity of free space} = 8.85 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$$

$$e = \text{fundamental charge} = 1.602 \times 10^{-19} \text{ C}$$

$$m_p = \text{mass of proton} = 1.67 \times 10^{-27} \text{ kg}$$

$$m_e = \text{mass of electron} = 9.11 \times 10^{-31} \text{ kg}$$

$$m = 10^{-3}$$

$$\mu = 10^{-6}$$

$$n = 10^{-9}$$

$$p = 10^{-12}$$

$$\text{Surface area of a sphere: } A = 4\pi r^2$$

$$\text{Volume of a sphere: } A = \frac{4}{3}\pi r^3$$

$$\text{Area of a circle: } A = \pi r^2$$

$$\text{Circumference of a circle: } C = 2\pi r$$

$$x = x_0 + v_{x0}t + \frac{1}{2}a_x t^2 \quad v_x = v_{x0} + a_x t \quad v_x^2 = v_{x0}^2 + 2a_x x \quad \vec{F} = m\vec{a}$$

$$\vec{F} = k \frac{q_1 q_2}{r^2} \hat{r} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \hat{r} \quad \vec{E} = \frac{\vec{F}}{q} \quad \vec{E} = k \frac{q}{r^2} \hat{r} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r}$$

$$\Phi_E = \oint_A \vec{E} \cdot d\vec{A} = \oint_A E dA \cos\theta = \frac{Q_{encl}}{\epsilon_0} \quad V = \frac{U}{q} \quad U = k \frac{q_1 q_2}{r} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r}$$

$$V = k \frac{q}{r} = \frac{1}{4\pi\epsilon_0} \frac{q}{r} \quad W = qV_{ab} \quad V_{ab} = V_a - V_b = \int_a^b \vec{E} \cdot d\vec{l}$$

$$\vec{E} = -\vec{\nabla}V = -\left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)V \quad C = \frac{Q}{V_{ab}} \quad C = \frac{\epsilon_0 A}{d}$$

$$U = \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} QV = \frac{1}{2} CV^2 \quad \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \quad C = C_1 + C_2 + C_3$$

$$u = \frac{1}{2}\epsilon_0 E^2$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} \quad \int \frac{dx}{\sqrt{x^2 + a^2}} = \ln \left( x + \sqrt{x^2 + a^2} \right)$$

$$\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \arctan \frac{x}{a} \quad \int \frac{dx}{(x^2 + a^2)^{3/2}} = \frac{1}{a^2} \frac{x}{\sqrt{x^2 + a^2}}$$

$$\int \frac{x dx}{(x^2 + a^2)^{3/2}} = -\frac{1}{\sqrt{x^2 + a^2}}$$

## Written Answer Sheets

Phys 259 Midterm Exam (Winter 2014): Instructions to Markers

You are responsible for marking the Problems, Part II. The multiple choice answers are scored automatically. The "Version" number applies only to the multiple-choice answers; there is only one version of Part II. You should also have these general instructions, plus a set of solutions with detailed instructions, and a copy of an exam question paper. The solutions are presented in Answer Page format for ease of reference.

Print out these Instructions, and read them before starting to mark. You might be tempted to bypass these, but you will save us much time and trouble if you follow instructions.

On the following pages, I've suggested a marking scheme for each question. However, not all solutions will follow the one I've given, especially for the 2-mark and 3-mark questions, so use your judgment. The following broad guidelines may help:

- In 3-mark questions, the first half mark should be easy to get: if a student shows any reasonable insight at all, give a half mark. In an integration problem, if the student simply makes a valiant attempt at integrating, this might be enough.
- Give half of the allotted marks (e.g., 1.5 marks out of 3) if you feel that the student has got half way to the right answer. For example, if a student makes five mistakes but still has the solution half right, give 1.5; don't deduct five half-marks.
- In numerical questions, the final answer must have correct units. Deduct 0.5 mark if the units on the final answer are not shown or are incorrect. Don't worry about units for intermediate calculations.
- Only subtract 1/2 mark once for each class of mistake in a given question. For example, missing vectors in one equation would be the same as missing them in two or three equations.
- Be lenient on significant figures. Final answer can have anything between 2 and 5 digits, take off 1/2 mark if 6 or more. 1 sig-fig can sometimes be okay depending on context. In general keep 2 or 3 eg.  $2.53 - 2.49 = 0.04$ .
- If a lot of students are making the same "mistake", check my answer - I could be the one who is wrong! If you discover any errors in my solutions, please let me know as soon as possible (physics259@ucalgary.ca) so I can alert all other markers.



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**Q15a** [3.0 marks]

The net force on charge  $q_3$  is the vector sum of the forces from each of the other charges (superposition). **1/2 mark for superposition in words or equation or free body diagram. Vectors not required for 1D problem if signs are correct.**

$$\vec{F}_1 = \vec{F}_{13} + \vec{F}_{23}$$

The force  $F_{13}$  of charge  $q_1$  acting on  $q_3$  is given by Coulomb's law

$$\vec{F}_{13} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_3}{r_{13}^2} \hat{r} = 8.99 \times 10^9 \text{ Nm}^2/\text{C}^2 \frac{(-1 \times 10^{-6} \text{ C})(-1 \times 10^{-6} \text{ C})}{(5 \text{ cm} - 1 \text{ cm})^2} \hat{x} = \boxed{5.619 \text{ N } \hat{x}}$$

The force  $F_{23}$  of charge  $q_2$  acting on  $q_3$  is given by Coulomb's law

$$\vec{F}_{23} = \frac{1}{4\pi\epsilon_0} \frac{q_2 q_3}{r_{23}^2} \hat{r} = 8.99 \times 10^9 \text{ Nm}^2/\text{C}^2 \frac{(+3 \times 10^{-6} \text{ C})(-1 \times 10^{-6} \text{ C})}{(5 \text{ cm} - 3 \text{ cm})^2} \hat{i} = \boxed{-67.425 \text{ N } \hat{i}}$$

**2 marks for these two calculations. -1/2 if either (or both) has missing or wrong units, -1/2 for scalar=vector, -1/2 for numerical error.** The total force is

$$\vec{F}_1 = \vec{F}_{13} + \vec{F}_{23} = 5.619 \text{ N } \hat{x} - 67.425 \text{ N } \hat{i} = \boxed{-61.81 \text{ N } \hat{i}}$$

**1/2 mark for correct numerical value including negative sign.**

**Q15b -approach#1** [3.0 marks]

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The net change in electrostatic potential energy is **1/2 mark for basic idea in words or equation.**

$$\Delta U = \Delta U_f - \Delta U_i$$

The initial energy between  $q_1$  and  $q_2$  and  $q_3$  is given by

$$U_f = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_{12}} + \frac{1}{4\pi\epsilon_0} \frac{q_1 q_3}{r_{13}} + \frac{1}{4\pi\epsilon_0} \frac{q_3 q_2}{r_{32}}$$

The final energy between  $q_1$  and  $q_3$  is given by

$$U_f = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_3}{r_{13}}$$

so the difference is

$$\begin{aligned} \Delta U_f &= -\frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_{12}} - \frac{1}{4\pi\epsilon_0} \frac{q_3 q_2}{r_{32}} \\ &= -8.99 \times 10^9 \frac{(-1 \times 10^{-6} C)(+3 \times 10^{-6} C)}{0.02 m} - 8.99 \times 10^9 \frac{(-1 \times 10^{-6} C)(+3 \times 10^{-6} C)}{0.02 cm} \\ &= +1.349 J + 1.349 J = \boxed{+2.698 J} \end{aligned} \quad (1)$$

**1/2 mark for correct numerical value including sign. Calculating  $U_i$  and  $U_f$  individually then subtracting is ok (but more work).**

*Continued on next page*

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**Q15b -alternate** [3.0 marks]

The net change in electrostatic potential energy is the scalar sum of change in potential energy between  $q_2$  and each of the other charges. **1/2 mark for scalar superposition in words or equation.**

$$\Delta U_2 = \Delta U_{12} + \Delta U_{32}$$

The potential energy between  $q_1$  and  $q_2$  is given by

$$U_{12} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_{12}} = 8.99 \times 10^9 \frac{(-1 \times 10^{-6} C)(+3 \times 10^{-6} C)}{|3cm - 1cm|} = \boxed{-1.349 \text{ J}}$$

The potential energy between  $q_3$  and  $q_2$  is the same because charges and distances are the same

$$U_{32} = \frac{1}{4\pi\epsilon_0} \frac{q_3 q_2}{r_{23}} = 8.99 \times 10^9 \frac{(-1 \times 10^{-6} C)(+3 \times 10^{-6} C)}{|5cm - 3cm|} = \boxed{-1.349 \text{ J}}$$

**-1/2 if either has missing or wrong units, -1 if vectors appear, -1/2 for numerical error. Allow skipping of second calculation with brief statement that result will be the same.** The total change in potential energy is

$$\Delta U_2 = U_f - U_i = 0 - (U_{12} + U_{32}) = 0 - 1.349 \text{ J} - 1.349 \text{ J} = \boxed{+2.698 \text{ J}}$$

**1/2 mark for correct numerical value including sign.**

*Continued on next page*

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**Q16a** [1.0 mark]

In the region above a very large sheet of positive charge the electric field direction will point away from the sheet

$$\vec{E}(x, y, z > 0) = E_z(z)\hat{z}$$

and will not depend on x and y (staying away from the edges). **1 mark for correct direction. Give 1/2 if appears confused about direction and didn't clearly define what they assumed. That is, the figure on the test didn't explicitly give x,y,z axes (although the z-direction is implicit in this question).**

**Q16b** [3.0 marks]

**1 mark for each of 3 surfaces, subtract 1/2 mark for each sign error. For at least one surface need to define  $\Phi_E$  in terms of integral, subtract 1/2 mark if not done. Details of integrals (ie. limits) not important but unit vectors are. Give at least half marks (1.5) for no math but vectors clearly drawn or explained. -1/2 for inappropriate closed integrals  $\oint$**

Cylinder#1 is composed of 3 surfaces: top, bottom, and sides. Using the symmetry result from Q16a tells us that there will be no flux through the sides

$$\Phi_E = \iint \vec{E} \cdot d\vec{a} = \iint (\pm E_z \hat{z}) \cdot (r d\phi dz \hat{r}) = \pm E_z \iint (\hat{z} \cdot \hat{r}) r d\phi dz = \boxed{0}$$

because the electric field is at  $90^\circ$  to the surface normal vector.

For the top cap

$$\Phi_E = \iint \vec{E} \cdot d\vec{a} = \iint (+E_z \hat{z}) \cdot (r dr d\phi \hat{z}) = +E_z \int_{r=0}^{r=R} \int_{\phi=0}^{\phi=2\pi} (\hat{z} \cdot \hat{z}) r dr d\phi = \boxed{E_z \pi R^2}$$

it's just the area of the cap (a circle) times a constant electric field (at some fixed z).

For the bottom cap the surface normal vector is  $-\hat{z}$  (down) but so is the electric field direction below the sheet

$$\Phi_E = \iint \vec{E} \cdot d\vec{a} = \iint (-E_z \hat{z}) \cdot r dr d\phi (-\hat{z}) = +E_z \int_{r=0}^{r=R} \int_{\phi=0}^{\phi=2\pi} (-\hat{z}) \cdot (-\hat{z}) r dr d\phi = \boxed{E_z \pi R^2}$$

so the flux is the same as for the top cap. **Subtract 1 mark for not stating  $\vec{E} = -E_z \hat{z}$  (or word/figure equivalent) below the sheet. Q16a was a pretty big hint. Alternative is using Gauss's law enclosed charge divided by two for top & bottom. Fine if clear about what's being done, -1 otherwise.**

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**Q16c** [2.0 marks]

If the flux  $\Phi_E$  is the same through both cylinders, then by Gauss's law there must be equal amounts of charge  $Q_{enc}$  inside each cylinder. This requires equal areas of the same charge density.

$$\frac{Q_1}{\epsilon_0} = \Phi_1 = \Phi_2 = \frac{Q_2}{\epsilon_0}$$

1/2 mark for basic idea, either words or equation

The charge enclosed inside cylinder #1 is a circular disk  $Q_1 = \sigma \pi r^2$ . 1/2 mark for correct result

The charge enclosed inside cylinder #2 is a rectangle  $Q_2 = \sigma 2r L$  1/2 mark for correct result

Set  $Q_1 = Q_2$  and solve for  $L$  in terms of  $r$

$$L = \frac{\pi}{2} r$$

or vice versa

$$r = \frac{2}{\pi} L$$

1/2 mark for either result

**Q17a** [1.0 mark]

At large distances non-neutral objects look like a point charge.

$$V(x) \approx \frac{1}{4\pi\epsilon_0} \frac{Q}{r} \quad x \rightarrow +\infty$$

1/2 mark for basic idea in words or equation

The charge on each rod is  $\lambda L$  or  $\lambda(b-a)$  so the net charge is twice that.

$$V(x) \approx \frac{1}{4\pi\epsilon_0} \frac{2\lambda L}{r} = \frac{1}{4\pi\epsilon_0} \frac{2\lambda(b-a)}{r} \quad x \rightarrow +\infty$$

1/2 mark for correct expression

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**Q17b** [3.0 marks]

The electrostatic potential  $dV$  produced by a small amount of charge  $dq$  at distance  $r$  is

$$dV = \frac{1}{4\pi\epsilon_0} \frac{dq}{r}$$

The charge on a small segment  $dx$  of the rod is  $dq = \lambda dx$  so the potential at the origin due to the entire rod is

$$V = \int dV = \int \frac{1}{4\pi\epsilon_0} \frac{dq}{r} = \frac{1}{4\pi\epsilon_0} \int \frac{\lambda dx}{|x-0|} = \frac{\lambda}{4\pi\epsilon_0} \int \frac{dx}{|x|}$$

1/2 mark for V as an integral involving  $\lambda$ .

For the  $x > 0$  rod the integral is from one end of the rod to the other

$$V = \frac{\lambda}{4\pi\epsilon_0} \int_a^b \frac{dx}{x} = \frac{\lambda}{4\pi\epsilon_0} \ln|x|_a^b = \frac{\lambda}{4\pi\epsilon_0} (\ln b - \ln a) = \frac{\lambda}{4\pi\epsilon_0} \ln\left(\frac{b}{a}\right)$$

1 mark for correct answer showing work.

Repeating the calculation for the other rod will give the same result, because it has the same sign of charge at the same distances

$$V = \frac{\lambda}{4\pi\epsilon_0} \int_{-b}^{-a} \frac{dx}{-x} = \frac{\lambda}{4\pi\epsilon_0} \ln|x|_{-a}^{-b} = \frac{\lambda}{4\pi\epsilon_0} (\ln b - \ln a) = \frac{\lambda}{4\pi\epsilon_0} \ln\left(\frac{b}{a}\right)$$

1 mark for correct answer, either with calculation or simple statement that same as first rod. Sign errors are likely, only take off 1/2 mark once.

$$V = V_1 + V_2 = \boxed{\frac{2\lambda}{4\pi\epsilon_0} \ln\left(\frac{b}{a}\right)}$$

1/2 mark for correct answer.

**Q17c** [1.0 mark]

This is just Q17b: add two segments, but flip the sign of one. Either repeat part b with the appropriate signs, or just note symmetry: every bit of positive charge has a matching bit of negative charge at the same distance. The result will cancel at the origin

$$\boxed{V(0) = 0}$$

1/2 mark for correct answer, 1/2 for calculation or very brief explanation.