Electricity and Magnetism

- Physics 259 – L02
- Lecture 45

Chapter 30: Induction and inductance

Last Time:

- Chapter 29

Today:

- Faraday’s Law of Induction
- Non-conservative electric fields
- Lenz’s Law

Section 3.1: Faraday's law and Lens's law

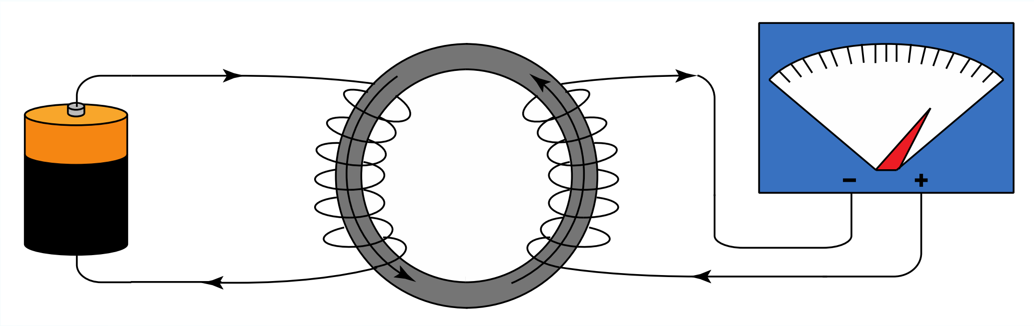
30-1

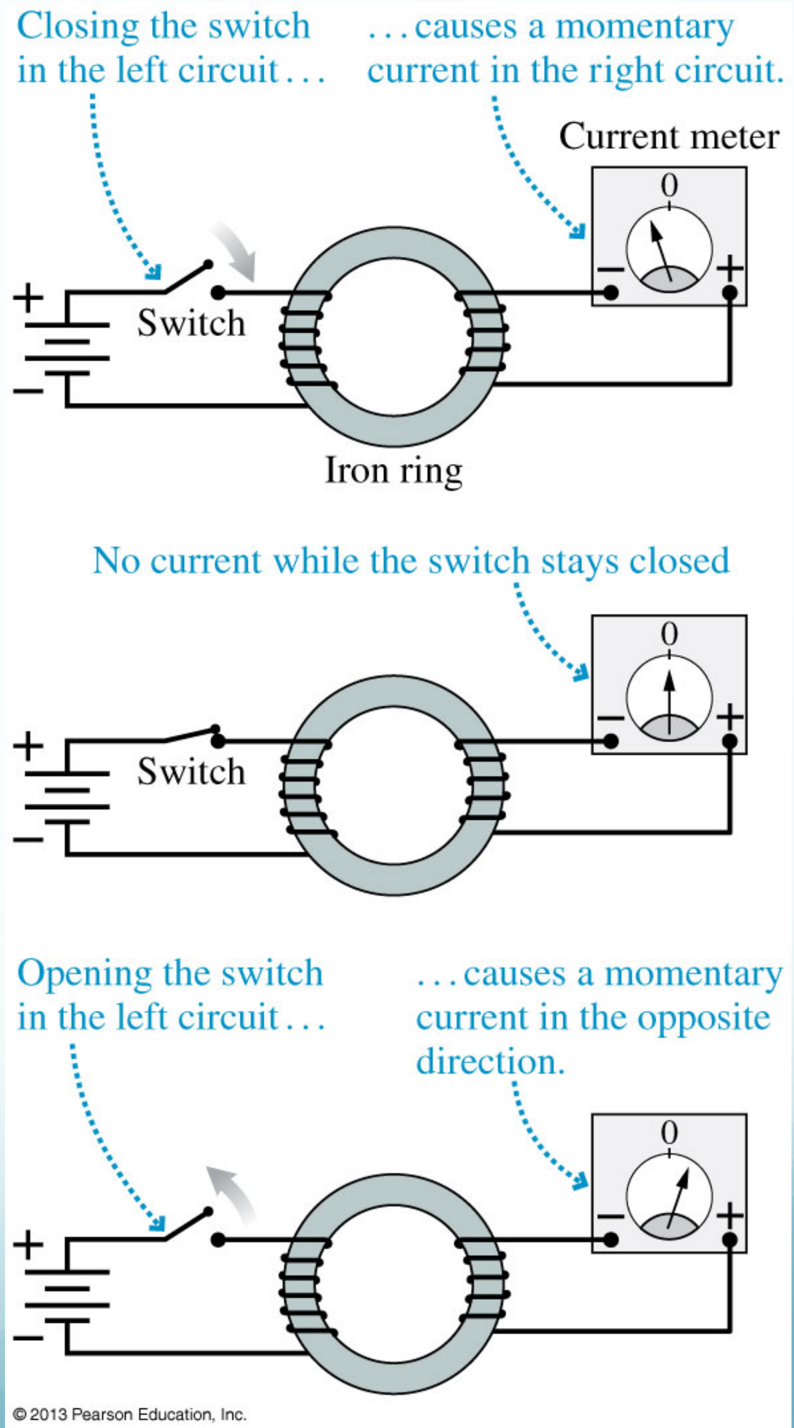
Electric currents cause magnetic fields

- Much of chapter 29 was about magnetic fields being caused by moving electrical charges

The reverse is true

- Magnetic fields can cause electrical currents
- Only changing magnetic fields can cause currents
- Which makes sense since only moving electrical charges can cause magnetic fields




Closing the switch
in the left circuit . . .
. . . causes a momentary
current in the right circuit.
Current meter
0
Switch
Iron ring
No current while the switch stays closed
0
Switch
Opening the switch
in the left circuit . . .
. . . causes a momentary
current in the opposite
direction.
0
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Changing magnetic field

In order for the magnetic field to change

A) The strength of the field would change

B) The direction of the field would change

C) The size of the loop of wire would change

D) The direction of the loop of wire would change

$\boxed{?} \propto$ E, $\cos\theta$, A

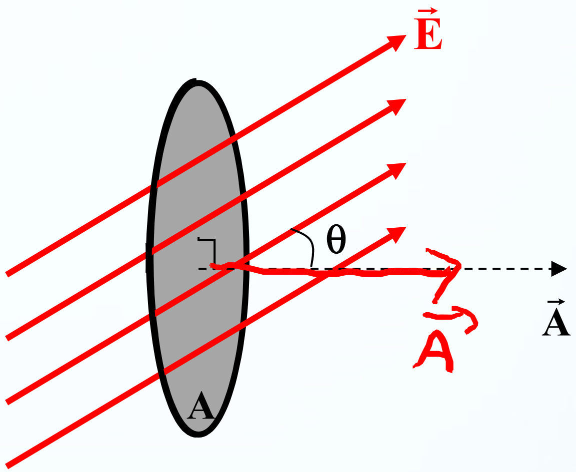
$\Phi_E = \vec{E} \cdot \vec{A} = EA\cos\theta$

Electric flux

Electric flux versus Magnetic flux

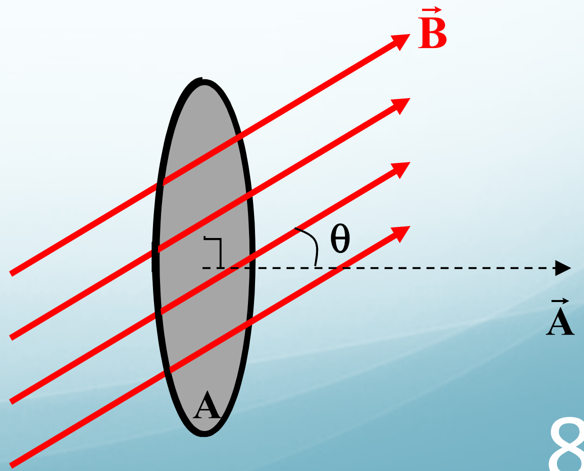
Electric flux of a constant electric field:

$$\Phi_e = \vec{E} \cdot \vec{A} = EA\cos\theta$$



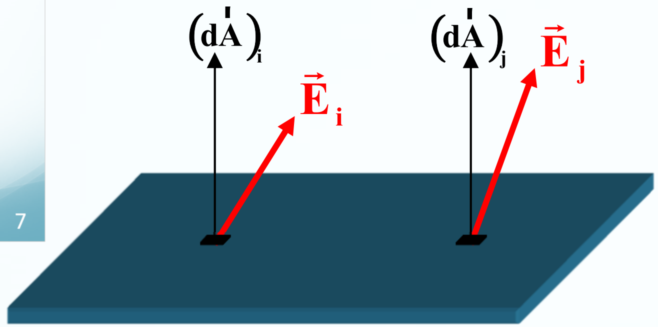
Magnetic flux of a uniform magnetic field:

$$\Phi_B = \vec{B} \cdot \vec{A} = BA\cos\theta$$

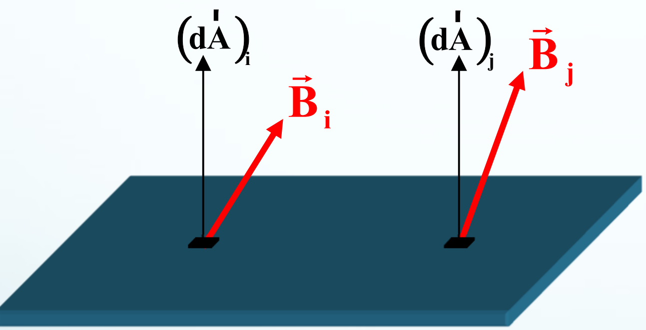


Unit: $\mathrm{T \cdot m^2 = Wb}$

More generally



$$\Phi_e = \oint \vec{E} \cdot d\vec{A}$$



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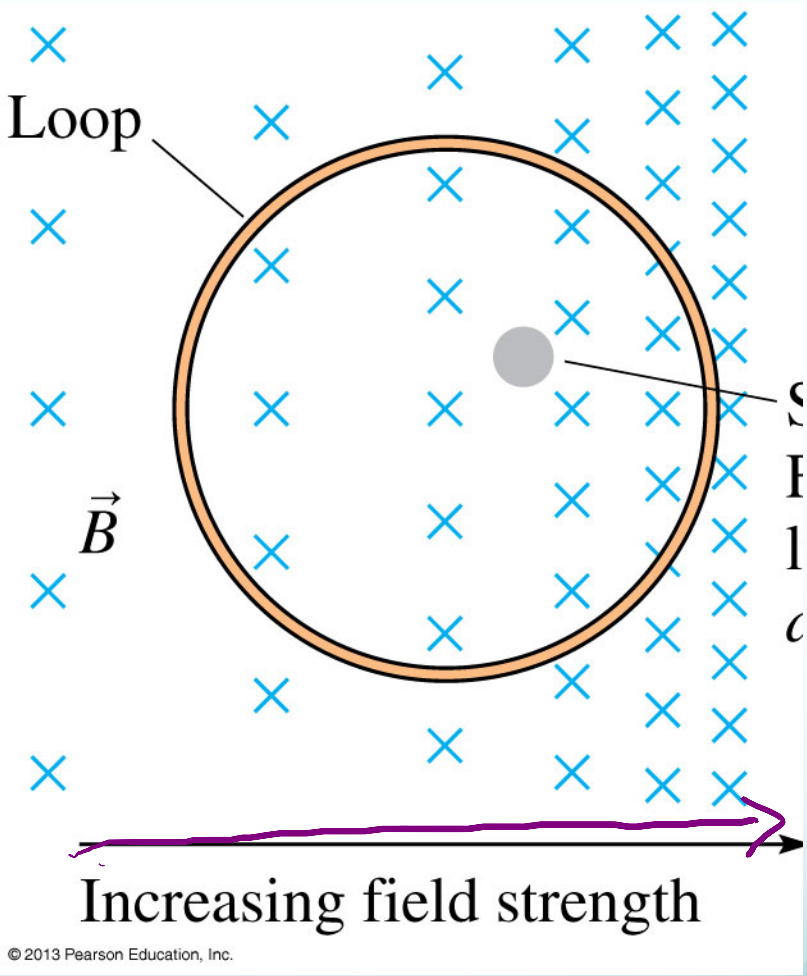


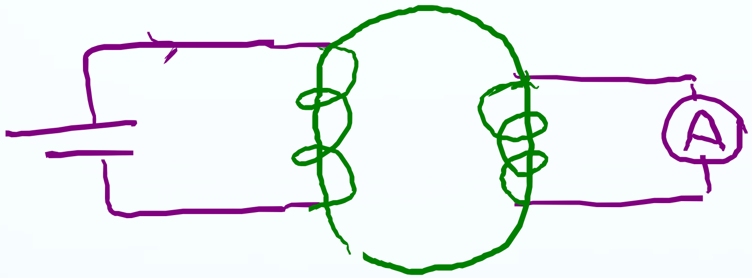
Magnetic flux can be thought of us counting the field lines

Charges don’t start moving spontaneously!

A current requires an emf to exist

$$I_{induced} = \frac{\varepsilon}{R}$$

induced emf



emf associated with a changing magnetic flux→induced emf

The induced emf is the rate of change of magnetic flux through the loop.

Faraday discovered that there is an induced EMF in the secondary circuit given by

$$\varepsilon = -\frac{d\Phi_M}{dt}$$

This is a new generalized law called **Faraday's Law**.

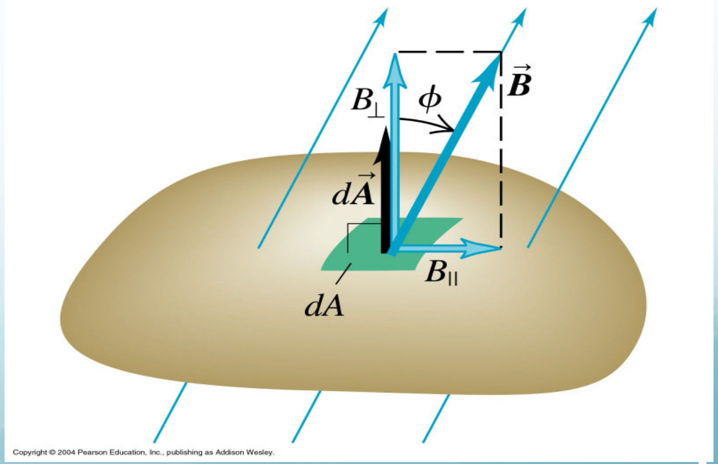
For a coil of N turns→

$$\varepsilon = -N\frac{d\Phi_M}{dt}$$

Recall the definition of magnetic flux:

$$\Phi_M = \int \vec{B} \cdot d\vec{A}$$

Not a closed surface!



This is valid even if Φ_M changes because of a time dependent A or angle φ (without changing the magnetic field)!

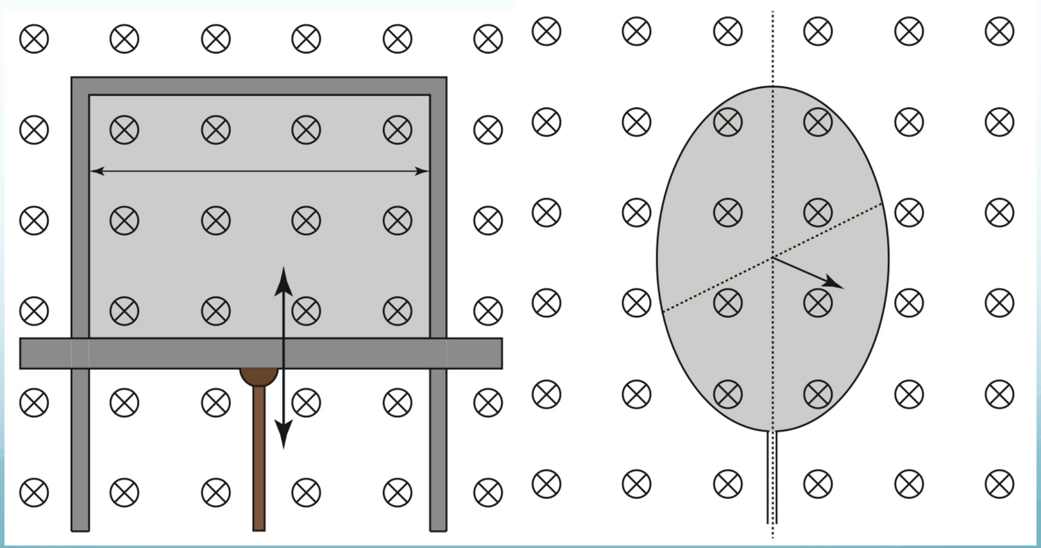
$$\varepsilon = -\frac{d\Phi_M}{dt}$$

$$\rightarrow \quad \varepsilon = -\frac{d}{dt}\left(BA\cos\phi\right)$$

→ 3 possible terms

$$\varepsilon = -\frac{dB}{dt}A\cos\phi - \frac{dA}{dt}B\cos\phi + \frac{d\phi}{dt}BA\sin\phi$$

$$\frac{d(\cos\phi)}{dt}\,BA$$



Example of using Faraday's law

$$\Phi_m = A.B = AB\cos\varphi$$

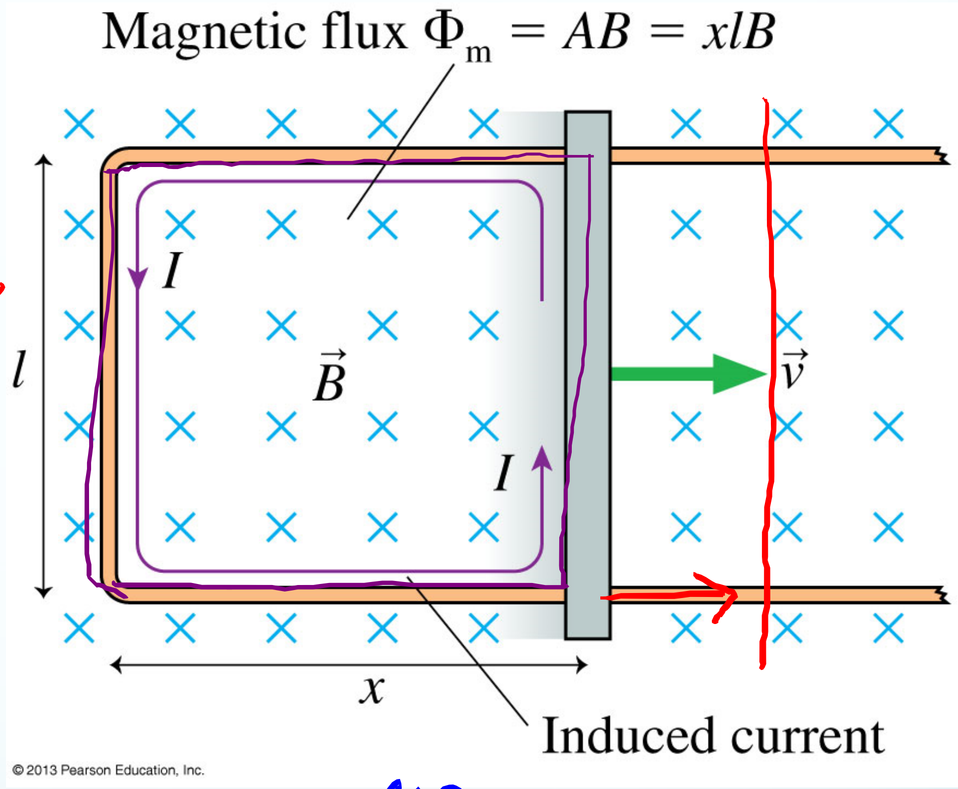
$$\mathcal{E} = -\frac{d\Phi_m}{dt} = \frac{dA}{dt} B\cos\varphi$$

$$A = x\ell$$

$$\mathcal{E} = -\frac{d\Phi_m}{dt} = -\frac{d}{dt}(BA)$$

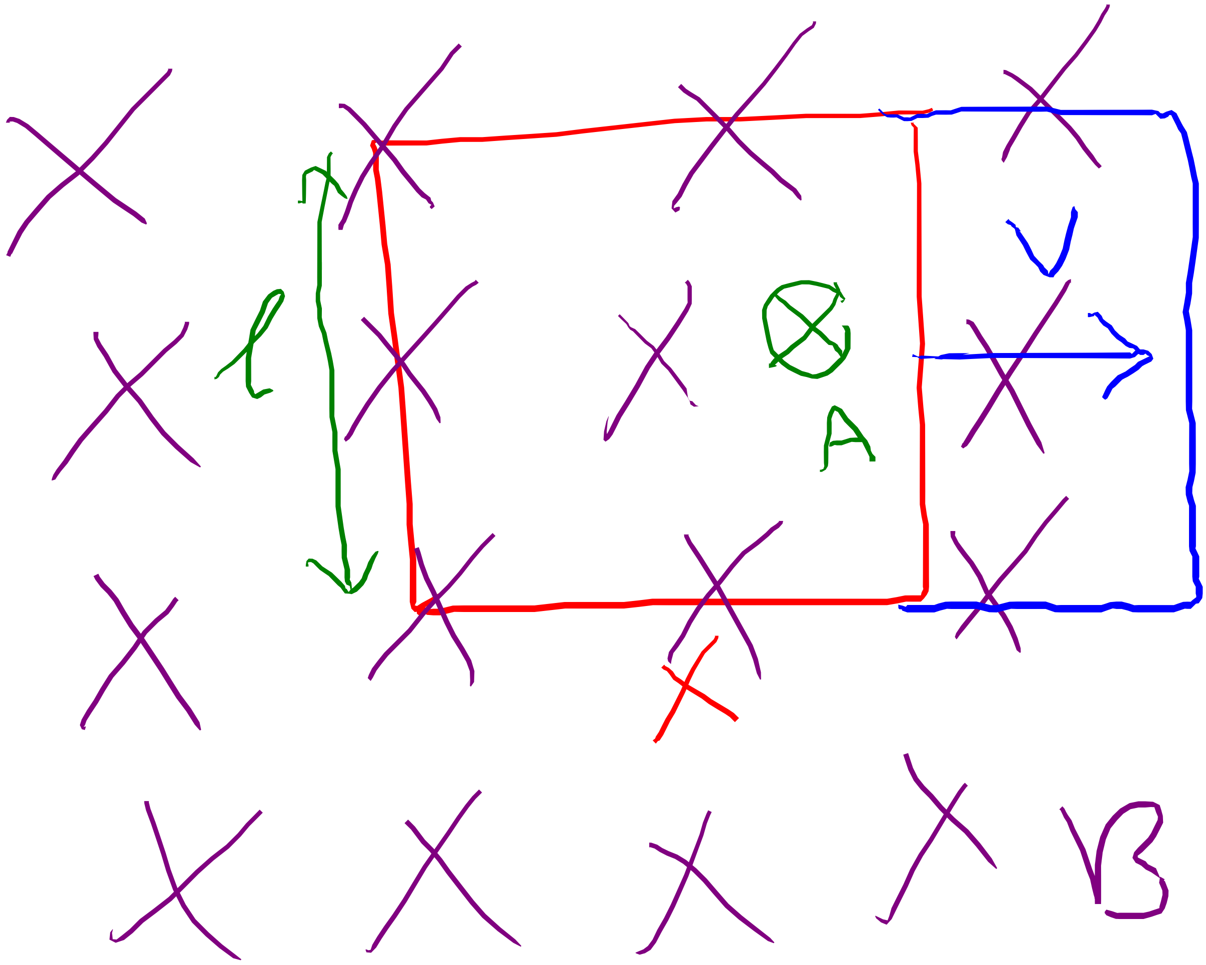
$$\rightarrow |\varepsilon| = \left|\frac{d\Phi_m}{dt}\right| = \left(\frac{d}{dt}\, xl\right)B = vlB$$

$$\rightarrow |\mathcal{E}| = v\ell B \Rightarrow I_{ind} = \frac{\mathcal{E}}{R} = \frac{v\ell B}{R}$$



Use Ohm's law to find induced current

$$I = \frac{\mathcal{E}}{R} = \frac{vlB}{R}$$

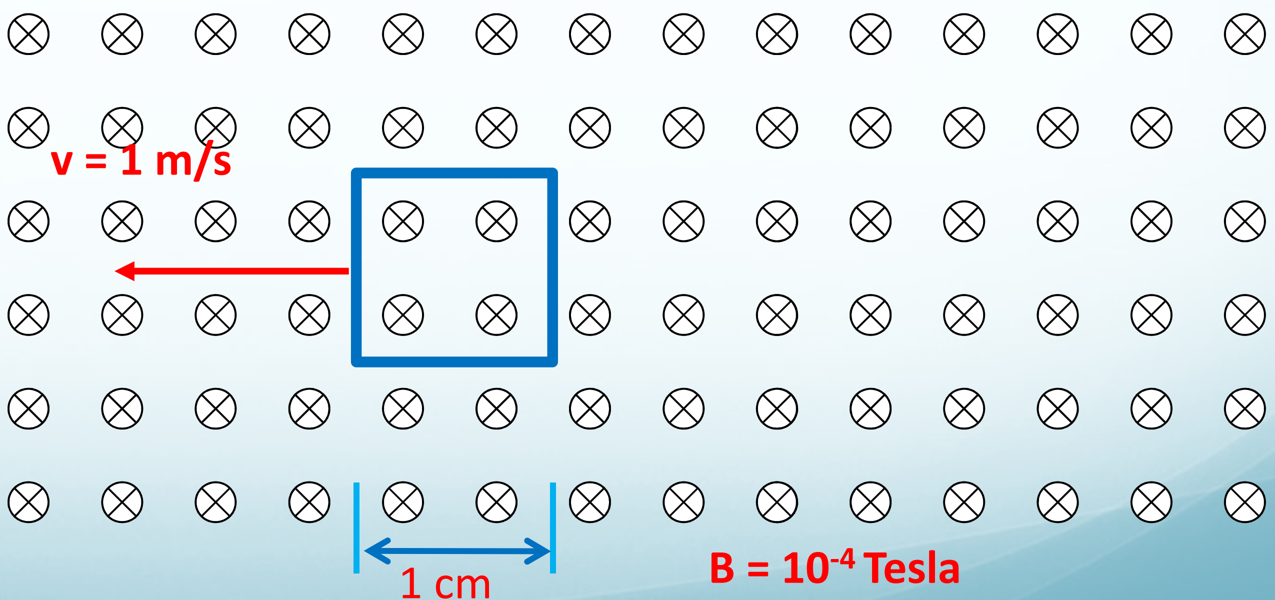


$$\varphi_m = \vec{B}\cdot\vec{A} = BA\cos\phi$$
$$= BA$$

Top Hat Question

A square loop of wire with a resistance of 1 Ω is moving with a constant velocity of 1 m/s through a uniform magnetic field as shown. What is the current induced in the loop? Pick the closest answer
(Note: 1 Ampere = 1 Coulomb/sec)

A. 0 A
B. 0.001 A
C. 0.01 A
D. 0.1 A
E. 1 A



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$\Phi_M = AB\cos\theta$ & $\mathcal{E}_{ind} = -\frac{d\Phi_M}{dt}$

$\mathcal{E}_{ind} = 0 \rightarrow I_{ind} = 0$

The magnetic flux through the loop is not changing, so there is no induced emf and hence no induced current

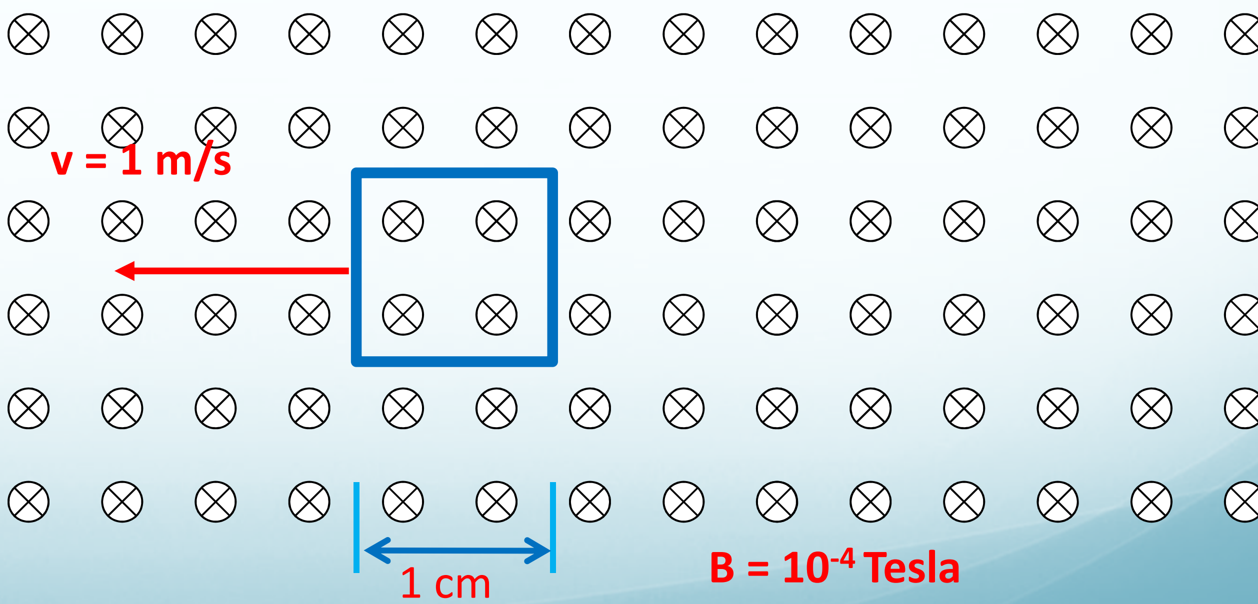
A. 0 A

B. 0.001 A

C. 0.01 A

D. 0.1 A

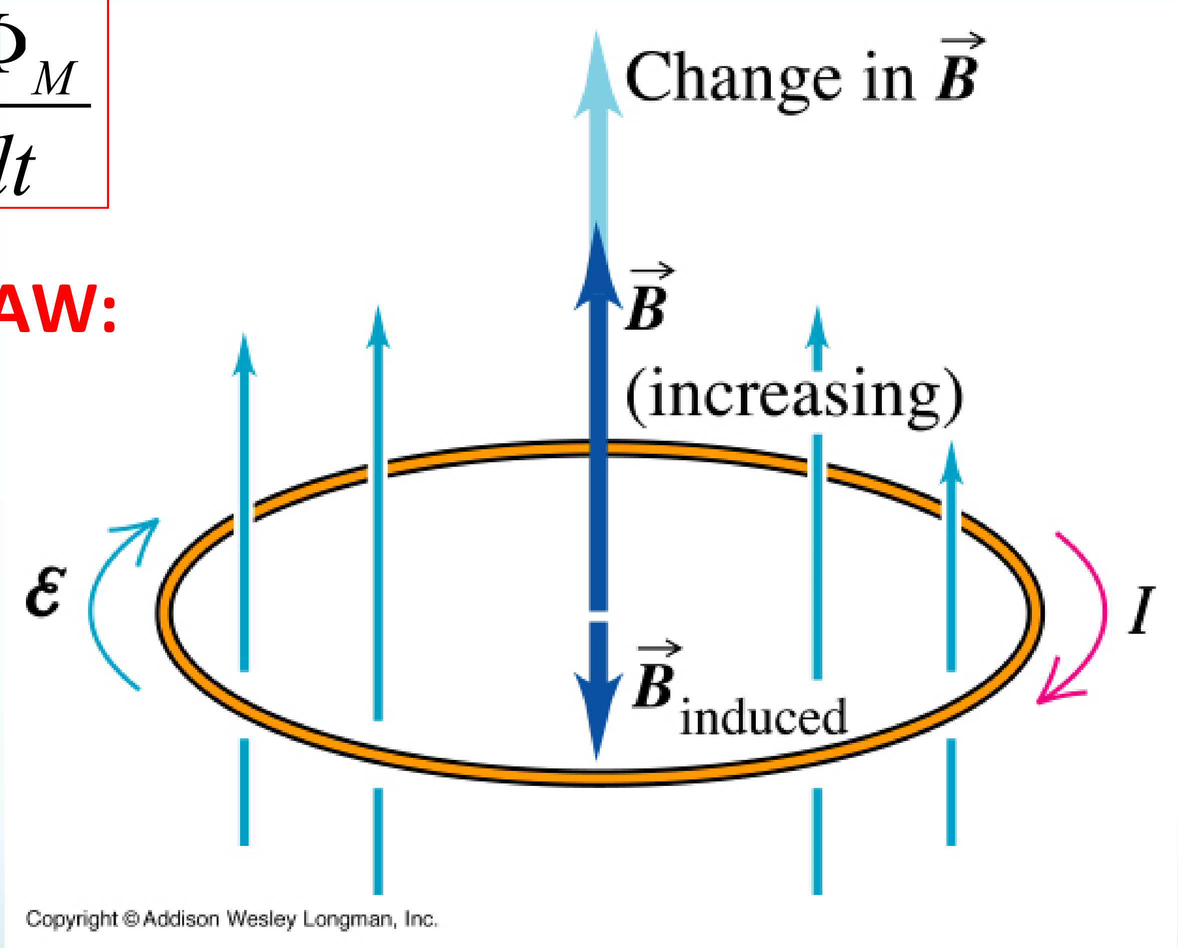
E. 1 A



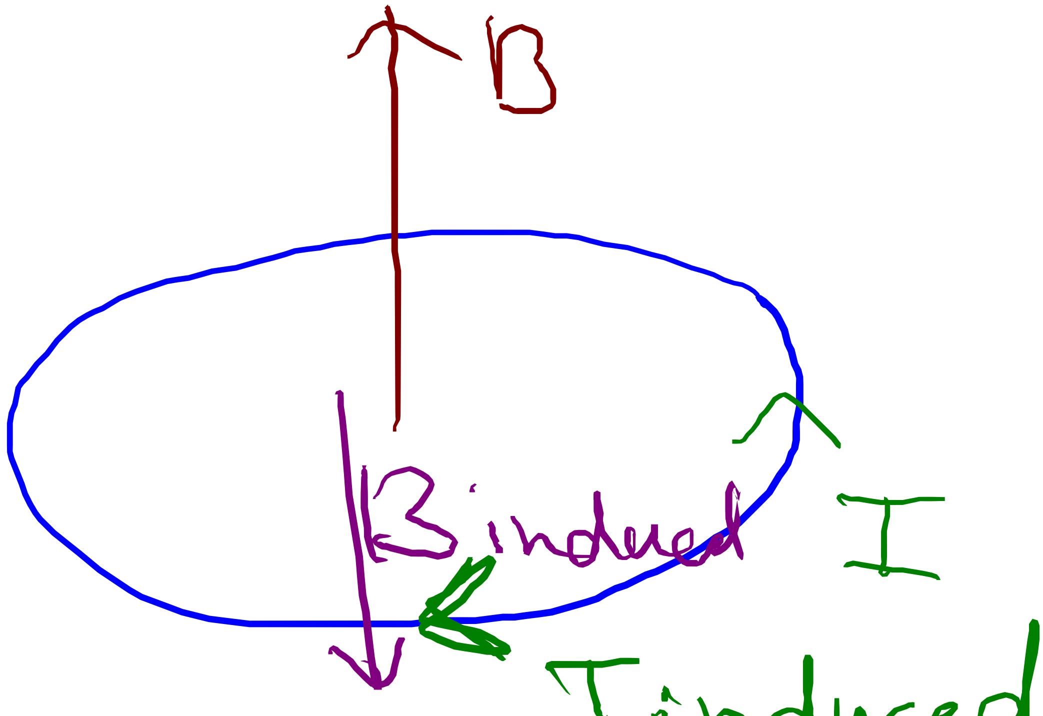
What about the **minus sign** in Faraday's law?

$$\boxed{\varepsilon = -\frac{d\Phi_M}{dt}}$$

LENZ'S LAW:



The changing magnetic flux generates an induced current which creates an induced magnetic field which, in turn, resists the change in magnetic flux.

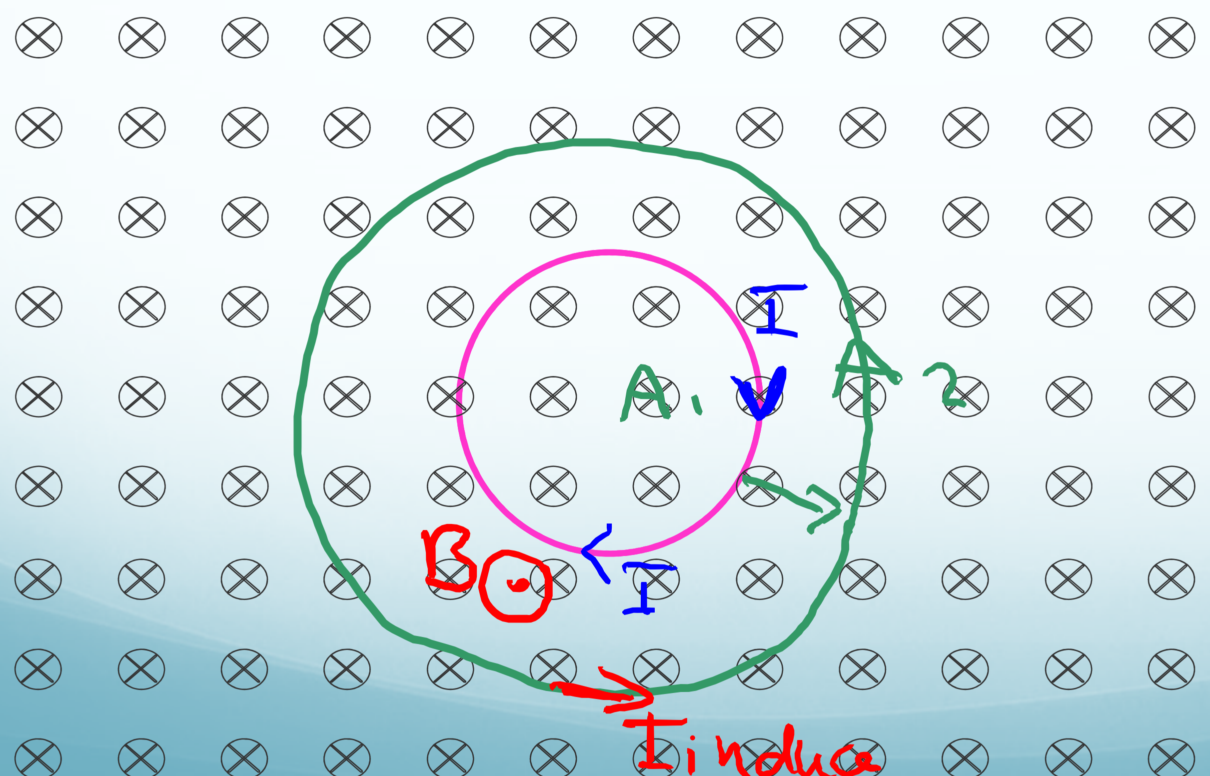


B ↑ by time

Lenz's Law

The induced current from Faraday's Law is always in a direction such that the induced magnetic field from the induced current opposes the change in the magnetic flux through the loop.

$$\Phi_M = BA\cos\phi \rightarrow \varepsilon = -\frac{d\Phi_m}{dt}$$



More B-field lines inside the loop: induced B-field from induced current must be out of the page to compensate.
Induced current is CCW

A↑

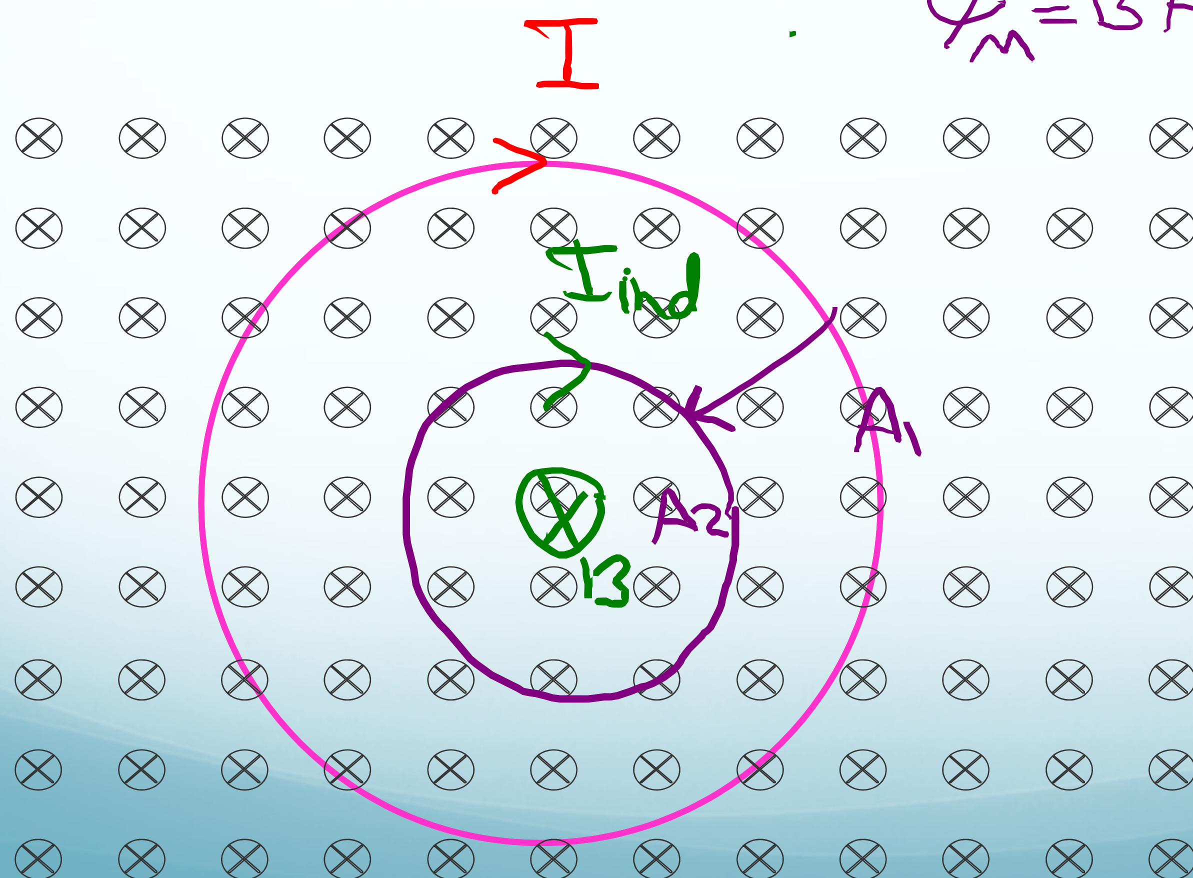
I_{ind}

B_{ind}

Lenz's Law

The induced current from Faraday's Law is always in a direction such that the induced magnetic field from the induced current opposes the change in the magnetic flux through the loop.

$\Phi_M = B A \cos\theta$



Fewer B-field lines inside the loop: induced B-field from induced current must be into the page to compensate.

Induced current is CW

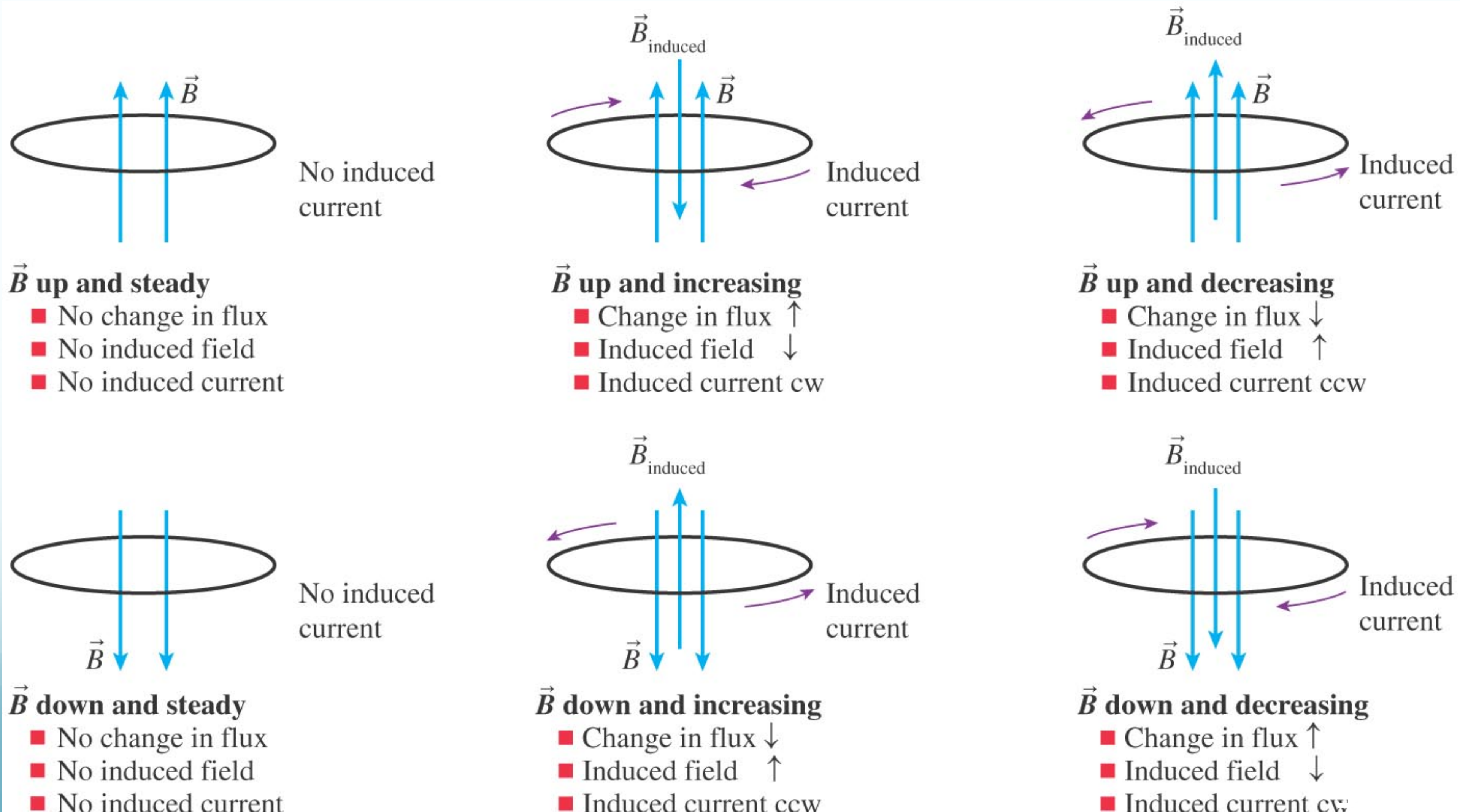
A↓

I_{ind}

B_{ind}

Lenz's Law

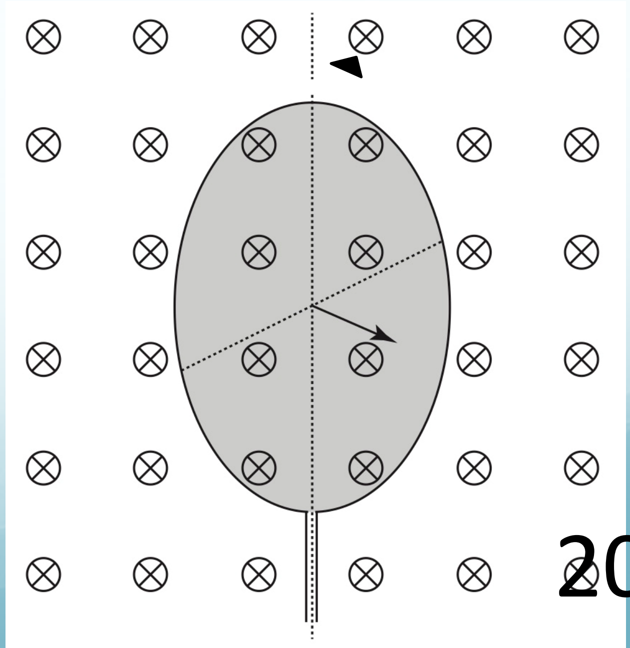
The currents always generate a field to oppose the change in flux.



Top Hat Question

A loop of wire is spinning rapidly about a stationary vertical axis in a uniform B-field. Is there a current (or EMF) induced in the loop?

A. Yes, a DC current is induced
B. Yes, an AC (time varying) current is induced
C. The B-field is not changing, so no currents are induced



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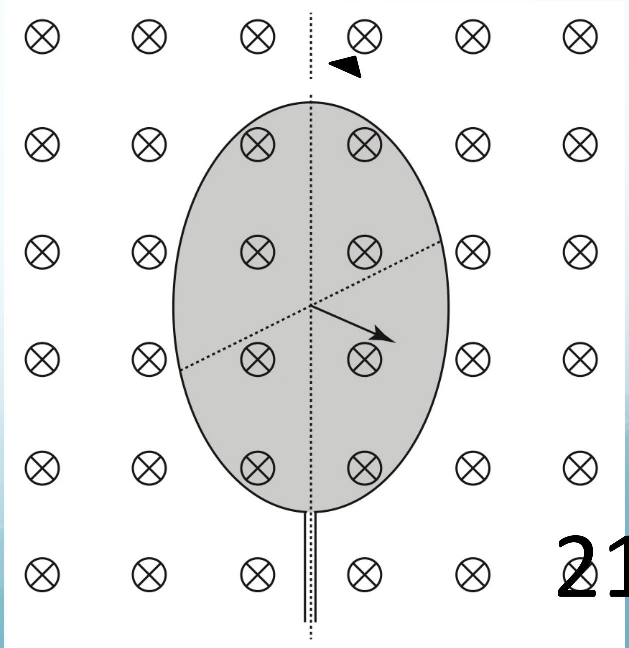
In this case, the flux through the loop is changing with time because of the B•A term, so there will be an induced current (or emf) in the loop. The normal vector is changing direction so half the time the flux is positive and half the time it is negative: i.e. an AC current is induced.

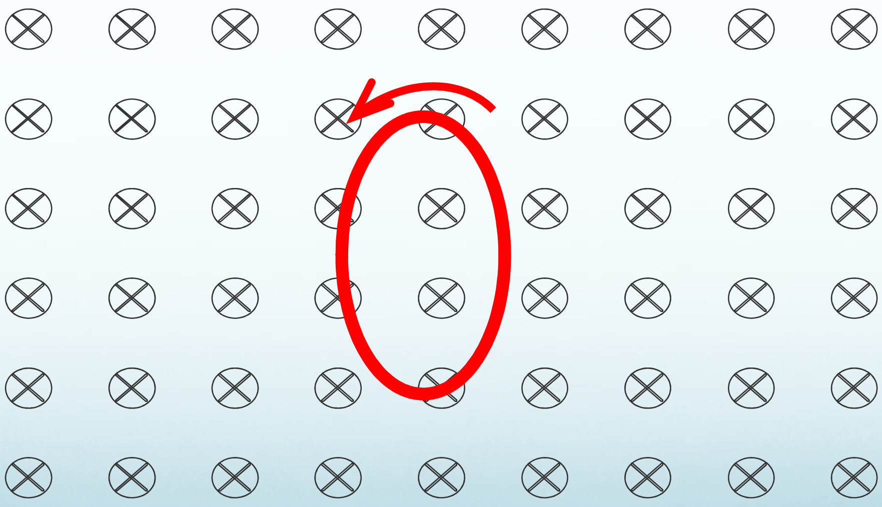


Top Hat Question

An oval shaped loop is spun around an axis pointing out of the page passing through the center of the loop. Is there a current (or EMF) induced in the loop?

A: Yes, there is

B: No, there is not



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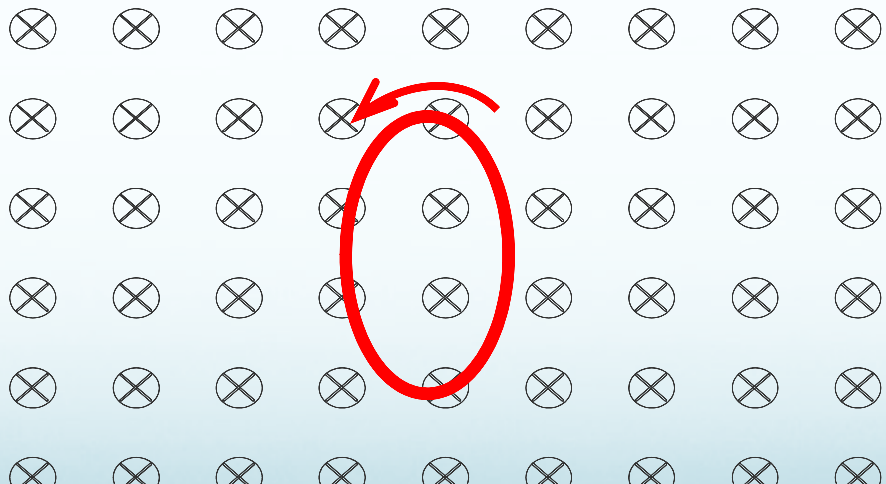
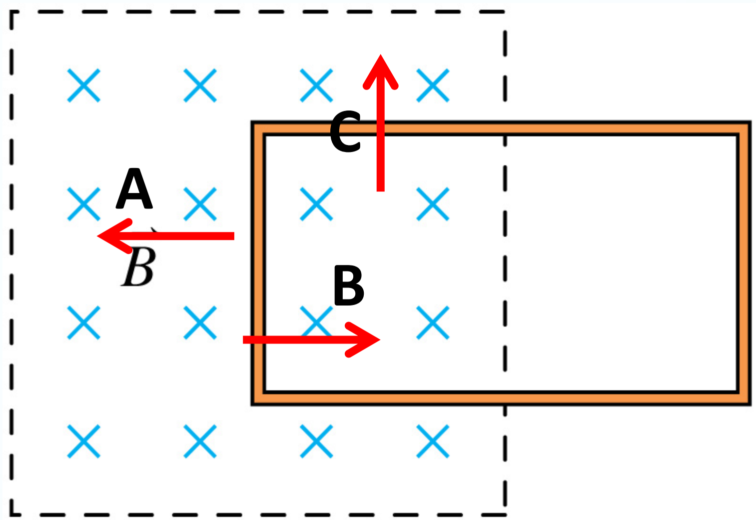


Top Hat Question

A conducting loop is halfway inside a magnetic field. Suppose the magnetic field begins to increase rapidly in strength. What happens to the loop?



A. The loop is pulled to the left, into the magnetic field.
B. The loop is pushed to the right, out of the magnetic field.
C. The loop is pushed upward, out of the magnetic field
D. The tension in the wire increases but the loop does not move.

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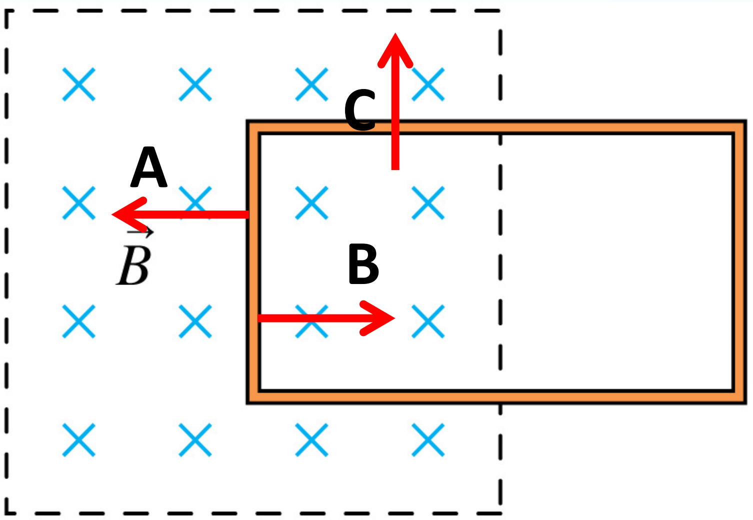

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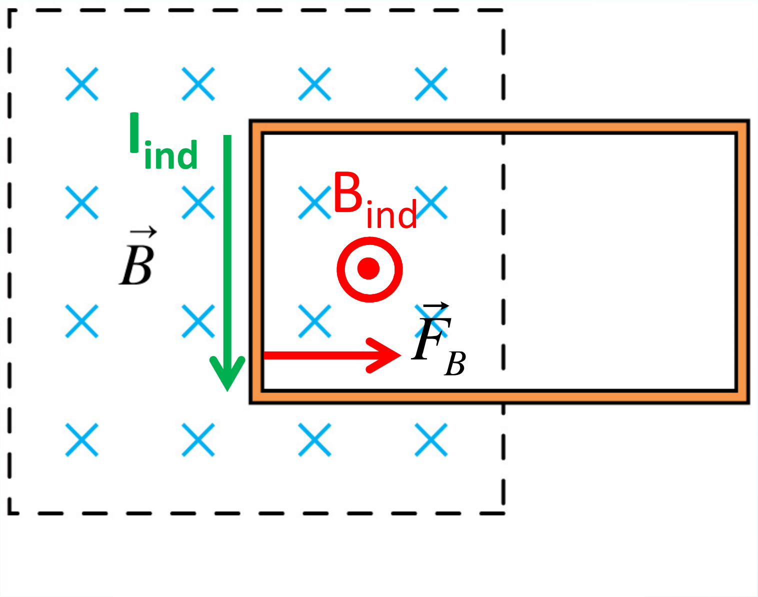
D. The tension in the wire increases but the loop does not move.

Lenz's Law: whatever happens must be such that it maintains the "amount of B-field" inside the loop. Since the strength of B is increasing, the loop must be pushed outside so that there are fewer B-field lines inside the loop.

Top Hat Question Feedback

A conducting loop is halfway inside a magnetic field. Suppose the magnetic field begins to increase rapidly in strength. What happens to the loop?



More rigorous argument

Lenz's Law→ B_{ind} must point out, so I_{ind} is CCW

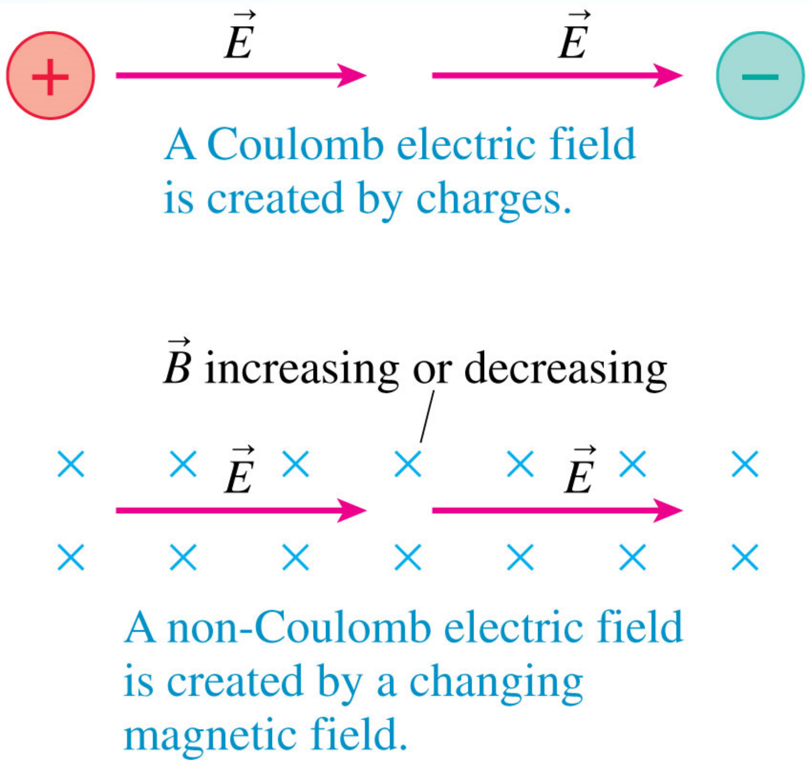
Recall→ the Lorentz force on a current carrying wire

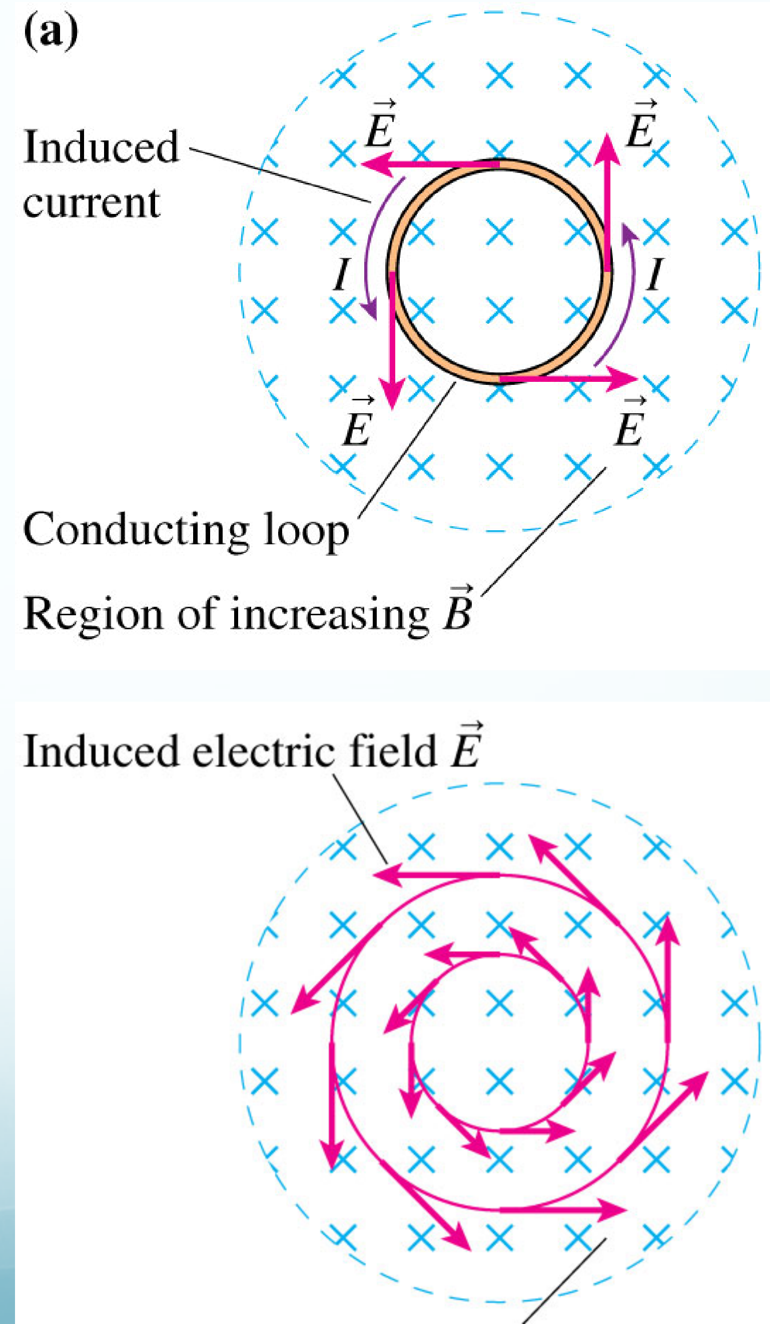
$$\vec{F}_B = I\vec{\ell} \times \vec{B}$$ ➔ points to the RIGHT

Faraday's law→ strength of induced current

What cause the current?

There is an electric field
caused by changing magnetic field
→ **Induced electric field**

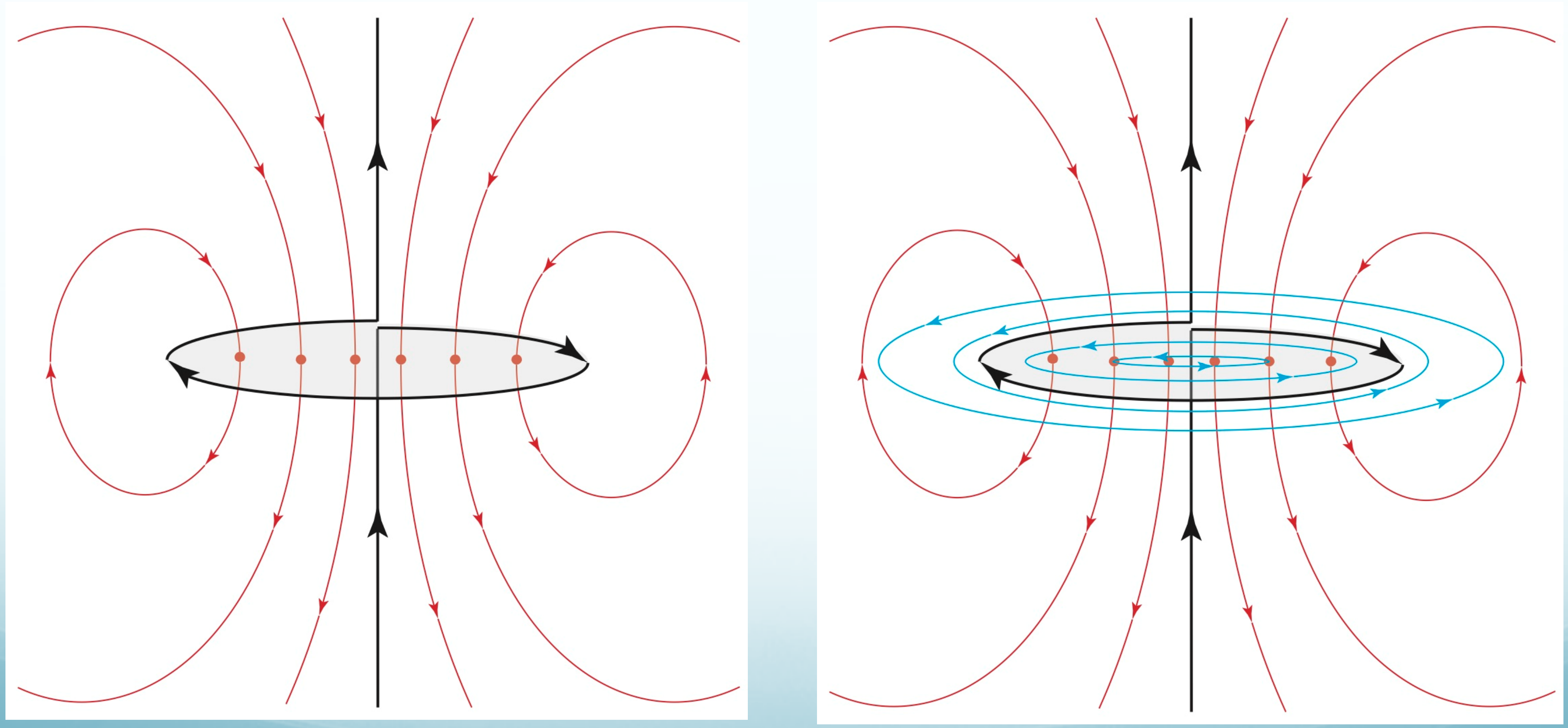




Imagine a loop in a wire carrying a current I_1. The current is then increased to $I_2 > I_1$, increasing the magnetic flux.

Changing B-fields induce non-conservative E-fields.

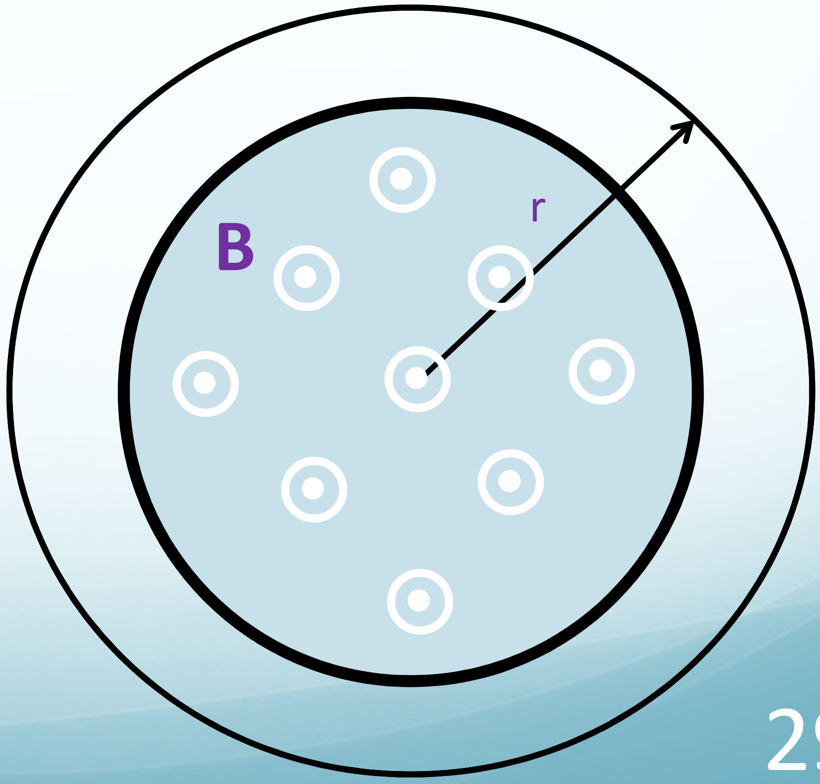
$$-\frac{d\vec{B}}{dt} = \nabla \times \vec{E}$$



Top Hat Question

The current in an infinitely long solenoid with uniform magnetic field B inside is increasing so that the magnitude B increases in time as $B=B_0+kt$. A circular loop of radius r is placed coaxially outside the solenoid as shown. In what direction is the induced E-field around the loop ?

A. CW
B. CCW
C. The induced E is zero
D. Not enough information



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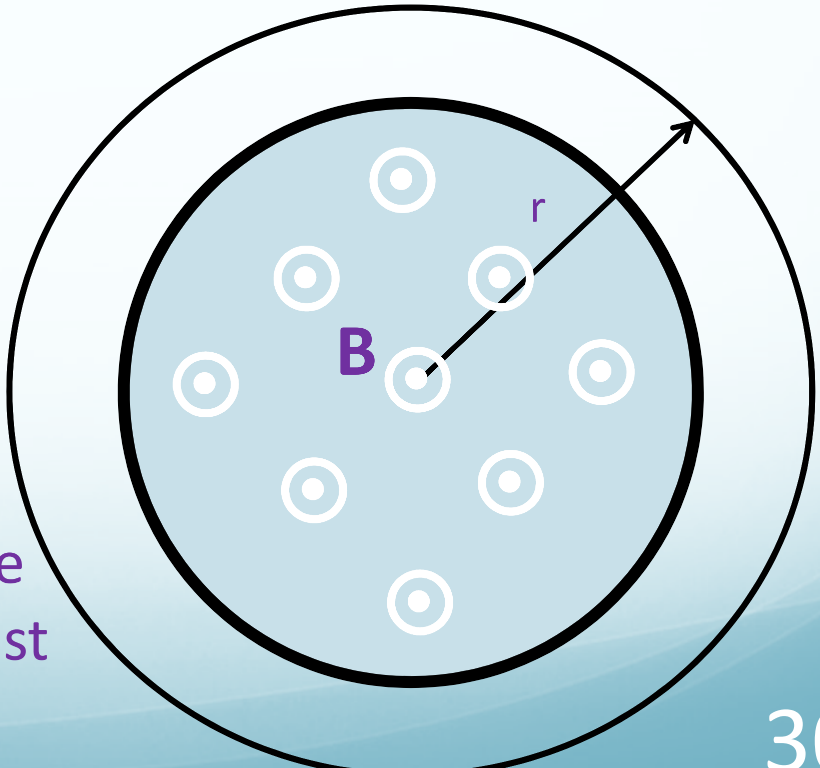

Lenz' law:
induced EMF around the loop is in the CW direction. The induced E-field must therefore be in the CW direction

This section we talked about:

Chapter 30

See you on Wednesday