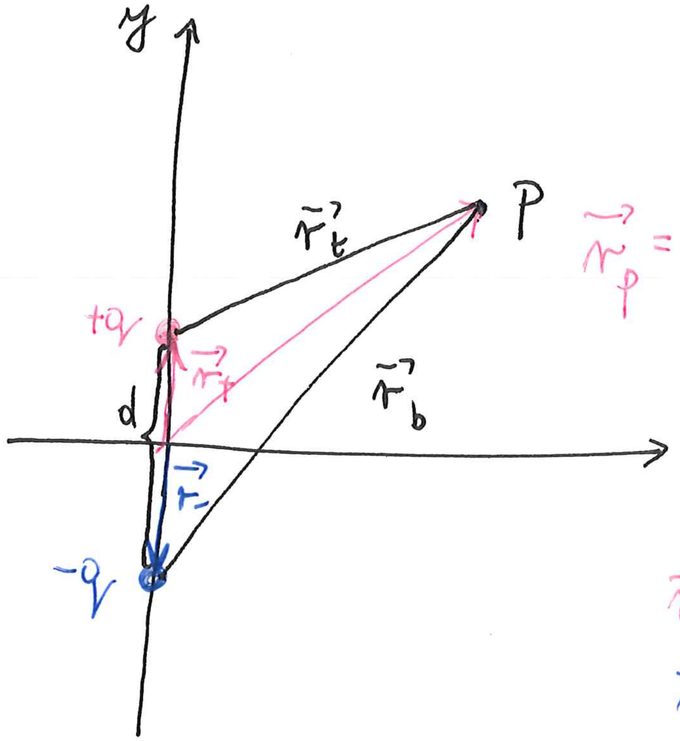
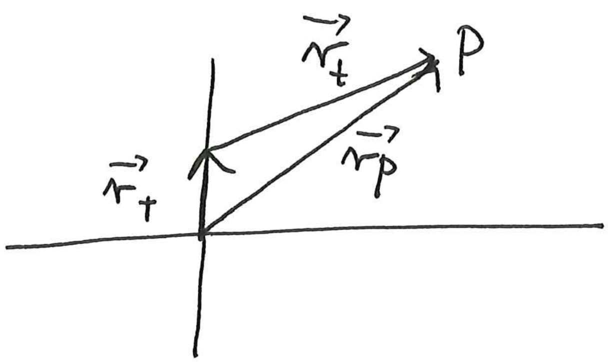
POTENTIAL OF A DIPOLE AT ARBITRARY (x, y)



$\vec{r}_p = (x\hat{i},\ y\hat{j})$

$\vec{r}_+ = (0\hat{i},\ \frac{d}{2}\hat{j})$

$\vec{r}_- = (0\hat{i},\ -\frac{d}{2}\hat{j})$



$$\vec{r}_p = \vec{r}_+ + \underline{\underline{\vec{r}_t}}$$

$$\vec{r}_t = \vec{r}_p - \vec{r}_+$$

$$\vec{r}_b = \vec{r}_p - \vec{r}_-$$

$$V_p = V_{p+} + V_{p-}$$

$$V_{p+} = \frac{1}{4\pi\varepsilon_0}\frac{q}{|r_t|} = \frac{1}{4\pi\varepsilon_0}\frac{q}{|\vec{r}_p - \vec{r}_+|}$$

$$V_{p-} = \frac{1}{4\pi\varepsilon_0}\frac{-q}{|r_b|} = \frac{1}{4\pi\varepsilon_0}\frac{-q}{|\vec{r}_p - \vec{r}_-|}$$

$$V_p = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{\sqrt{x^2+(y-\frac{d}{2})^2}} - \frac{1}{\sqrt{x^2+(y+\frac{d}{2})^2}}\right]$$

$$V_p = \frac{p \cdot y}{4\pi\varepsilon_0}(x^2+y^2)^{-3/2}$$

IDEAL DIPOLE

p – dipole moment

$p = q \cdot d$

$\lim_{d \to 0}$

Finding $\vec{E}$ of a ideal dipole at arbitrary (x, y)

$$\boxed{\vec{E} = E_x\hat{i} + E_y\hat{j} + E_z\hat{k}}$$

$$\boxed{\vec{E} = \underbrace{-\frac{\partial V}{\partial x}}_{E_x}\hat{i} - \underbrace{\frac{\partial V}{\partial y}}_{E_y}\hat{j} - \underbrace{\frac{\partial V}{\partial z}}_{E_z}\hat{k}}$$

$$E_x = -\frac{\partial V_p}{\partial x}$$

$$E_x = -\frac{py}{4\pi\varepsilon_0}\left(-\frac{3}{\cancel{2}}\right)(x^2+y^2)^{-5/2}\,\cancel{2}x$$

$$E_x = \frac{3pxy}{4\pi\varepsilon_0}(x^2+y^2)^{-5/2}$$

$$E_y = -\frac{\partial V_p}{\partial y}\hat{j}$$

$$E_y = -\frac{p}{4\pi\varepsilon_0}(x^2+y^2)^{-3/2} - \frac{py}{4\pi\varepsilon_0}\left(-\frac{3}{2}\right)(x^2+y^2)^{-5/2}2y$$

common denominator

$$E_y = -\frac{p}{4\pi\varepsilon_0}\left(\frac{(x^2+y^2) - 3y^2}{(x^2+y^2)^{5/2}}\right)$$

$$E_y = -\frac{p}{4\pi\varepsilon_0}\left(\frac{-2y^2+x^2}{(x^2+y^2)^{5/2}}\right)$$

$$E_y = \frac{p}{4\pi\varepsilon_0}\left(\frac{2y^2-x^2}{(x^2+y^2)^{5/2}}\right)$$