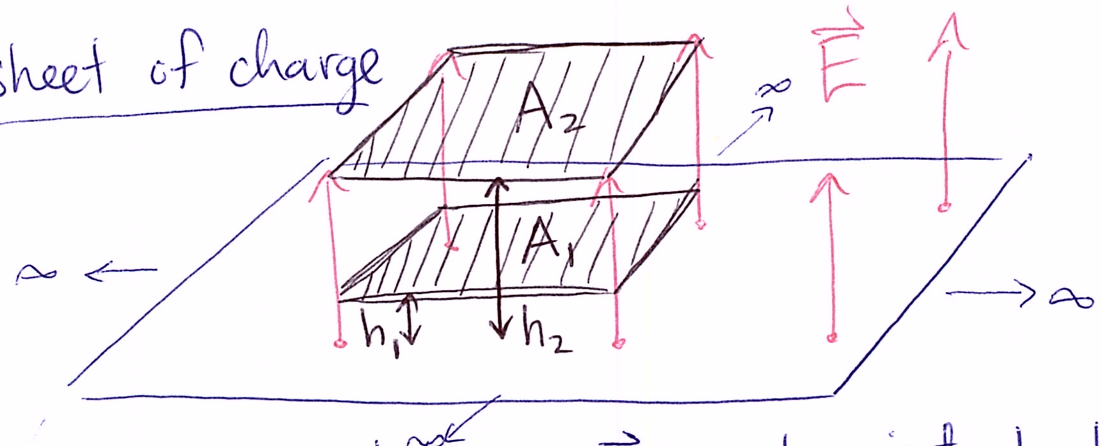
Predicting E-field behaviour

sheet of charge



use symmetry to say $\vec{E}$ must point $\perp$ to surface

The flux is the number of field lines passing through a surface, so A_1 and A_2 must have the same flux through them. The field lines are not spreading out, so $A_1 = A_2$

$$\Phi_{E1} = \int \vec{E}_1 \cdot d\vec{A}_1 = E_1 A_1 \quad (E \text{ is constant along } A_1)$$

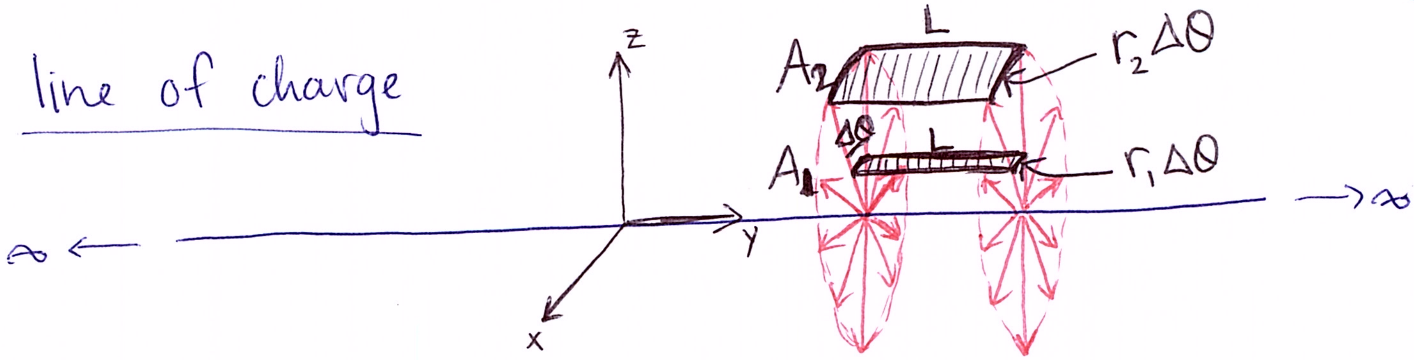
$$\Phi_{E2} = \int \vec{E}_2 \cdot d\vec{A}_2 = E_2 A_2 \quad (E \text{ is constant along } A_2)$$

$$\Phi_{E1} = \Phi_{E2} \quad \text{so} \quad E_1 \cancel{A_1} = E_2 \cancel{A_2} \quad (A_1 = A_2)$$

This implies the field is uniform: $E_1 = E_2$

Just based on symmetry we were able to predict $\vec{E}$ is uniform for infinite plane!

line of charge



Use symmetry to say the E-field only points radially away from the wire and that $|\vec{E}|$ is constant at constant radius. The flux through A_1 must be equal to the flux through A_2

$$\Phi_{E1} = \int \vec{E}_1 \cdot d\vec{A}_1 = E_1 A_1 \qquad A_1 = L r_1 \Delta\theta$$

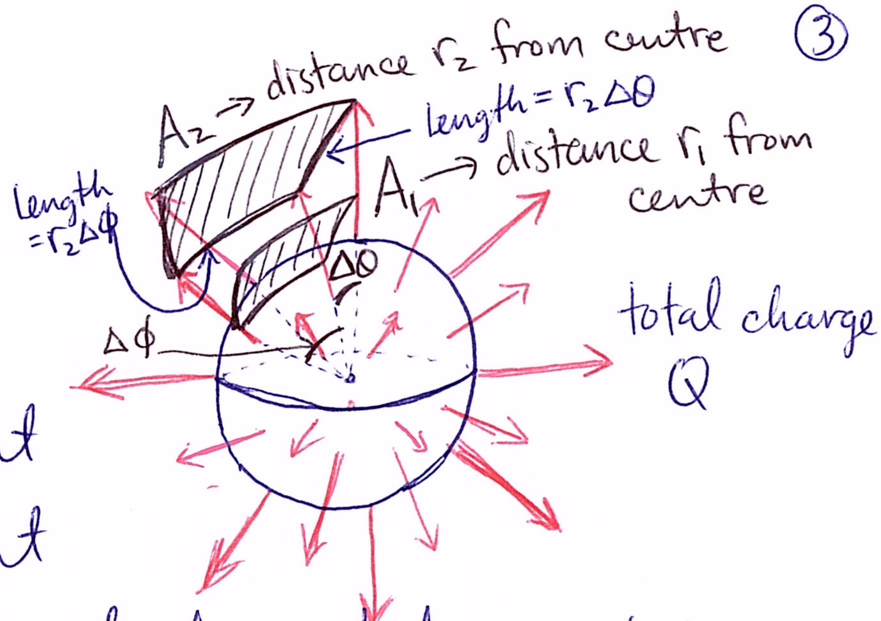
$$\Phi_{E2} = \int \vec{E}_2 \cdot d\vec{A}_2 = E_2 A_2 \qquad A_2 = L r_2 \Delta\theta$$

Area is increasing linearly in r, so to maintain constant flux, E must be decreasing like $\frac{1}{r}$

$$\boxed{E_{line} \sim \frac{1}{r}}$$

This result is again based solely on the symmetry of the line of charge and how constant flux areas increase with distance.

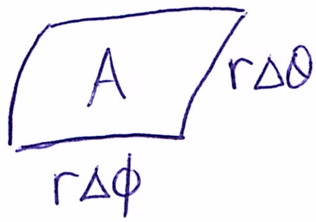
Sphere of charge



use symmetry to say that E points radial only and has constant magnitude at constant radius. The flux through A_1 and A_2 must be equal:

$$\Phi_{E_1} = \int \vec{E}_1 \cdot d\vec{A}_1 = E_1 A_1$$

$$\Phi_{E_2} = \int \vec{E}_2 \cdot d\vec{A}_2 = E_2 A_2$$

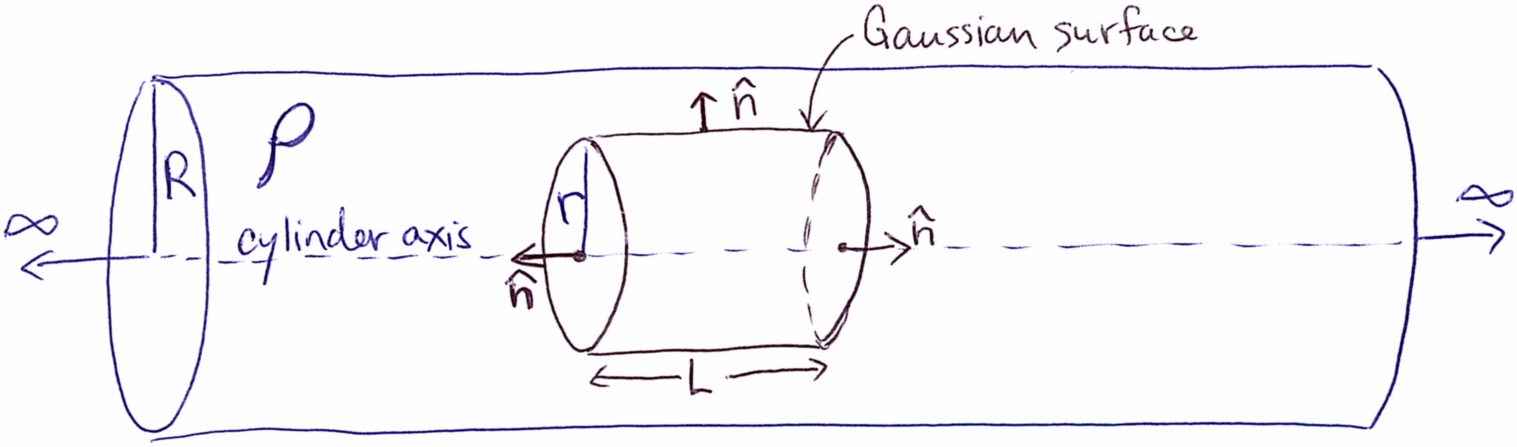


both areas are $r^2 \Delta\theta \Delta\phi$, so area is increasing like r^2.

To maintain constant flux, the E-field must be decreasing like $\frac{1}{r^2}$.

Again, just based on symmetry we have found that a spherical mass distribution must have a $\frac{1}{r^2}$ dependence for the E-field.

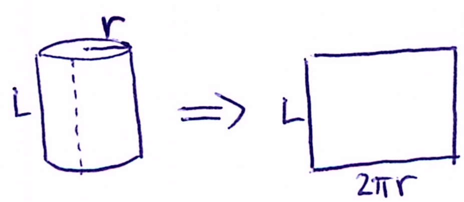
Solid, uniformly charged, infinite rod



The rod is invariant (looks the same) when moved side to side and when rotated around its axis. The E-field must also look the same under these two operations, so E must point radially away from the centre and be constant magnitude at constant radius. Choose a cylindrical Gaussian surface co-axial with the rod. On the ends of the cylinder $\hat{n} \cdot \vec{E} = 0$ because $\hat{n} \perp \vec{E}$ so the ends contribute zero flux. Along the "tube" of the cylinder, $\hat{n} = \hat{r}$, so $\hat{n} \cdot \vec{E} = E$

$$\oint \vec{E} \cdot d\vec{A} = \int_{tube} E\, dA = E A_{tube} = \frac{q_{enc}}{\epsilon_0}$$

$A_{tube} = 2\pi r L$



cut along line, flatten out to rectangle.

What is q_{enc}? $\quad q_{enc} = \rho V = \rho(\pi r^2)(L)$

area × length

Put this all together:

$$E(2\cancel{\pi} \cancel{r} \cancel{L}) = \frac{\rho \cancel{\pi} r^{\cancel{2}} \cancel{L}}{\epsilon_0}$$

$$\boxed{E = \frac{\rho r}{2\epsilon_0}}$$ valid for $r < R$

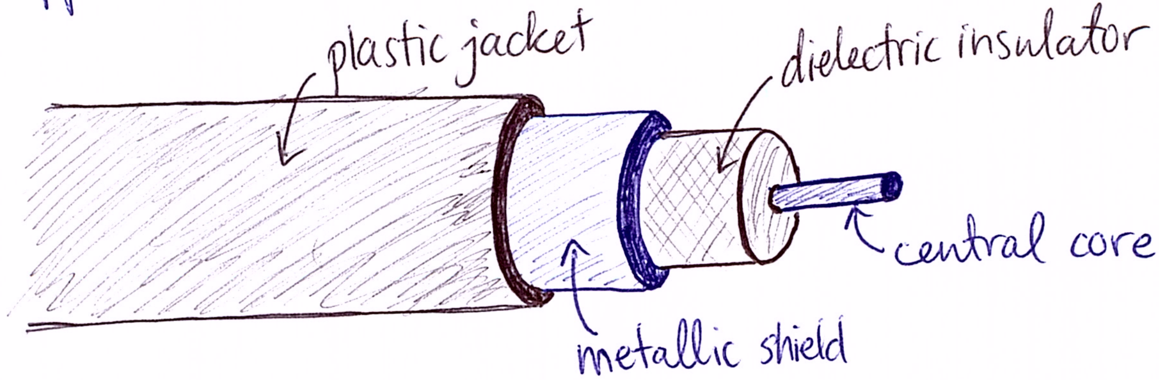
At the surface when $r = R$: $E = \dfrac{\rho R}{2\epsilon_0}$

but $\rho = \dfrac{Q}{\pi R^2 L} = \dfrac{\lambda}{\pi R^2}$ (defining an equivalent linear charge density)

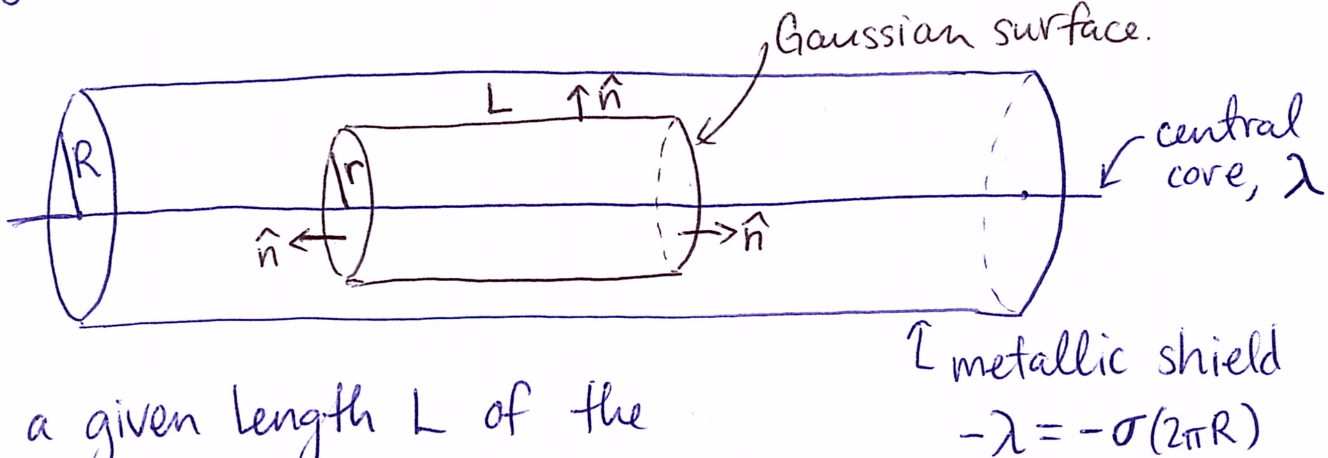
So $E = \left(\dfrac{\lambda}{\pi R^{\cancel{2}}}\right)\left(\dfrac{\cancel{R}}{2\epsilon_0}\right) = \dfrac{\lambda}{2\pi\epsilon_0 R}$

This agrees with the result of a line of charge $E = \dfrac{\lambda}{2\pi\epsilon_0 r}$. Outside the charged rod, it looks just like a line of charge with linear charge density λ.

Application: Co-axial cable



Ignore the dielectric insulator and plastic jacket.



In a given length L of the coax cable, the central core carries $+Q = \lambda L$ and the metallic shield carries $-Q = -\lambda L = -\sigma(2\pi R L)$

By symmetry, the E-field can only point radially (assuming cable is infinite) and must be constant magnitude at constant radius. Choose cylindrical Gaussian surface. On the two ends $\hat{n} \perp \vec{E}$ so they contribute zero flux. On the "tube" part, $\hat{n} = \hat{r}$ so $\vec{E} \cdot d\vec{A} = E\,dA$

$$\oint \vec{E}\cdot d\vec{A} = \int_{tube} E\,dA = E\int_{tube} dA = E(2\pi r L)$$

Therefore $E(2\pi r L) = \dfrac{q_{enc}}{\epsilon_0}$

q_{enc} is only from the central core: $q_{enc} = \lambda L$

$$E(2\pi r \cancel{L}) = \frac{\lambda \cancel{L}}{\epsilon_0} \Rightarrow \boxed{E = \frac{\lambda}{2\pi\epsilon_0 r}} \quad \text{for } r<R$$

Inside the cable, the field is that of a line of charge λ from the central core.

Outside the cable the field must be zero because by symmetry E must still be radial and constant magnitude at constant radius, so

$$\oint \vec{E}\cdot d\vec{A} = \int_{tube} E\,dA = E A_{tube} = \frac{q_{enc}}{\epsilon_0}$$

but $q_{enc} = 0$.

This is why coaxial cables are used! They shield the outside from the fields inside. We will see these again when we talk about magnetic fields.