Monday Feb 13, 2017

Announcements

- Midterm tomorrow:
  - Please bring pen (dark), pencil, calculator, student ID
  - Eating well and sleeping properly are important
  - Solve the questions before looking at the answers

- There are NO LABATORIALS this week

- Assignment 5 is due this Wednesday March 1

Midterm Review

What we've done so far: Electrostatics

- Coulomb's Law: force between two charged particles
- Electric field of a point charge
- Principle of superposition: electric field of distributions of charges
- Electric field lines to visualize the electric field
- Uniform electric field & kinematics
- Electric flux through any surface (open or closed)
- Gauss' Law: relating electric flux to charge enclosed
- Properties of conductors in electrostatic equilibrium (no moving charges)
- Calculating electric field in scenarios with high symmetry
- Superposition with Gauss' Law

Maxwell's equations

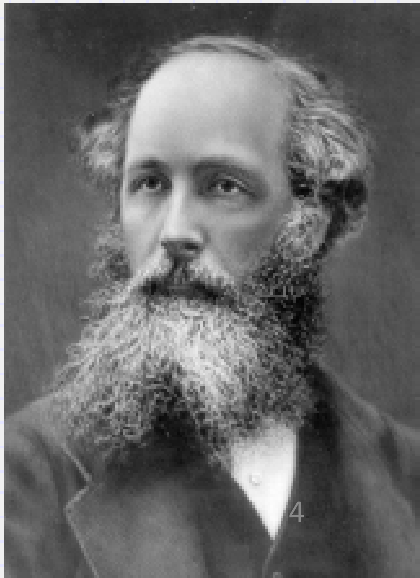
$$\oiint \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\epsilon_0}$$ Gauss's law for electricity

$$\oiint \vec{B} \cdot d\vec{A} = 0$$ Gauss's law for magnetism

$$\oint \vec{B} \cdot d\vec{l} = \mu_0\left(i_c + \epsilon_0 \frac{d\Phi_E}{dt}\right)$$ Ampere's law

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$ Faraday's law

James Clerk Maxwell
1831-1879



Electric Charges and Forces

- There are positive and negative charges
- Like charges repel each other
- Opposite charges attract each other
- The force between charged objects varies with distance
- The force between charged objects depends on the amount of charge

This can be quantified by Coulomb's Law

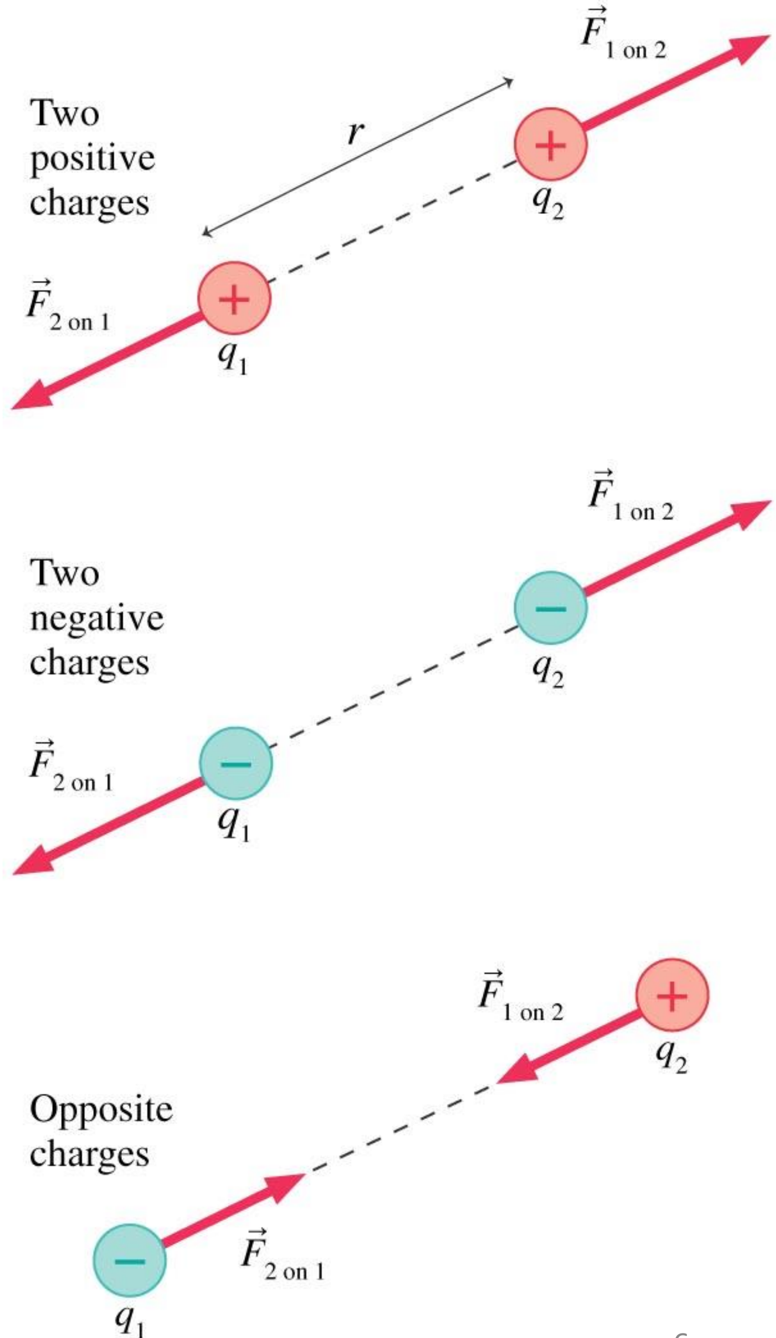
Coulomb's Law

There are only two kinds of charges:

positive and **negative**.

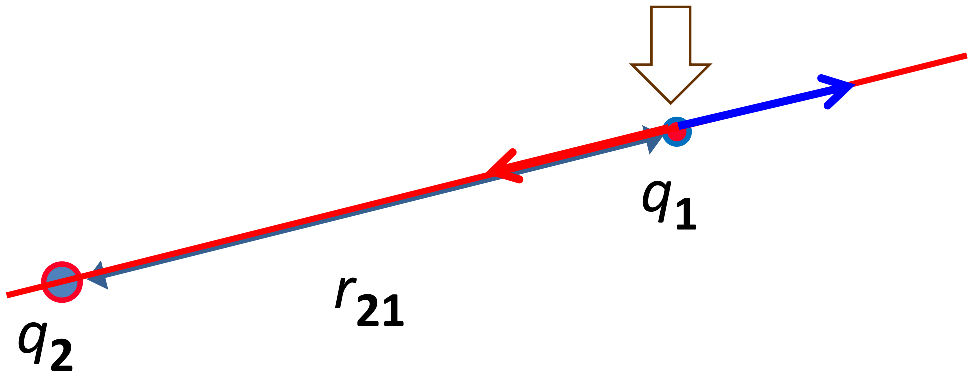
Charges of the **same** sign **repel** each other.

Charges of **opposite** sign **attract** each other.



Coulomb's Law

How to compute the magnitude and direction properly.



$$\left|\vec{F}_{21}\right| = \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_2|}{r_{21}^2}$$

1) Find the distance between the charges.

2) Draw a line passing through the two charges.

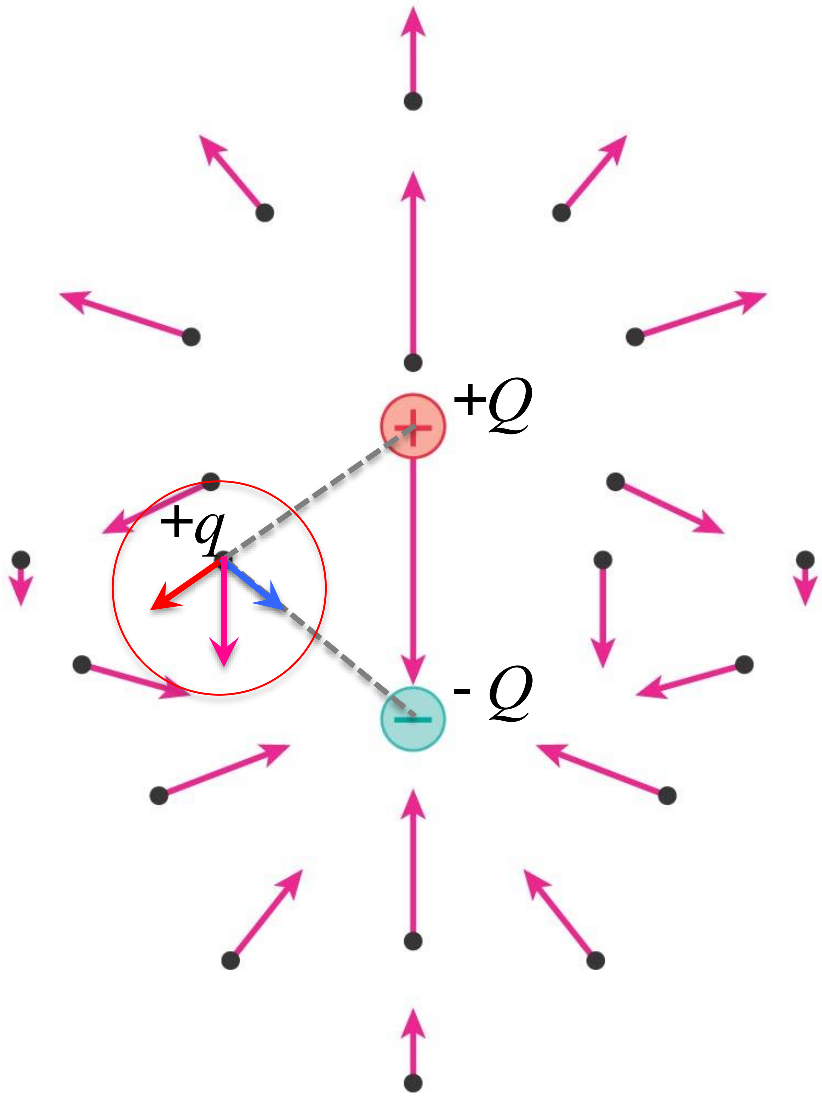
3) The force on q_1 due to q_2 has its tail at location 1 and points either towards q_2 or away from q_2.

4) Pick the direction according to basic rule of charges:

Like charges repel, Opposite charges attract

Superposition with Building Blocks

The vector represents the magnitude and direction of the electric force on the charge q **at that point**. It comes from superposition of the individual forces from +Q and –Q.



Step 1: draw the lines connecting the charge pairs
Step 2: draw the force vector for each charge pair
Step 3: sum all forces to find net force

Electric Force vs Electric Field

If we think there is an electric field somewhere in space, then we can measure it by placing a charge q in the field. If q feels an electric force, then

$$\vec{E} = \frac{\vec{F}_{on\,q}}{q}$$

Or, if we know the electric field, then the electric force on any charge q placed in this field is

$$\vec{F}_{on\,q} = q\vec{E}$$

(How nature really works)

We'll come back to this

Full force on charged particle due to electromagnetic field:

$$\boxed{\vec{F}_{on\,q} = q\vec{E} + q\vec{v} \times \vec{B}}$$

Facts about the Electric Field

Fact 1: The electric field is a property of the space surrounding charges. It exists whether other charges are present to feel a force from it or not.

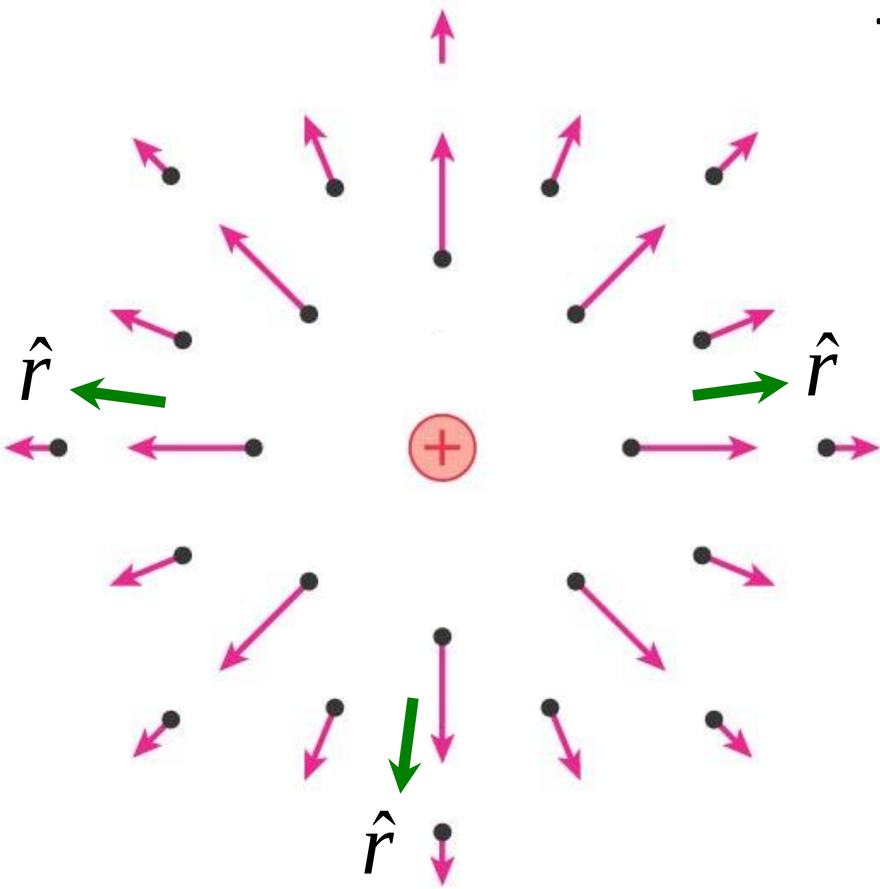
Fact 2: The electric field is a vector at every point in space whose direction is the same as the force on a positive test charge.

Fact 3: Negative charges feel a force in the direction opposite the direction of the electric field.

Fact 4: The electric field of a collection of charges is obtained by the superposition principle.

Fact 5: The electric force felt by a charge q in an electric field **E** is given by **F** = q**E** (boldface to denote vectors)

Electric Field building blocks



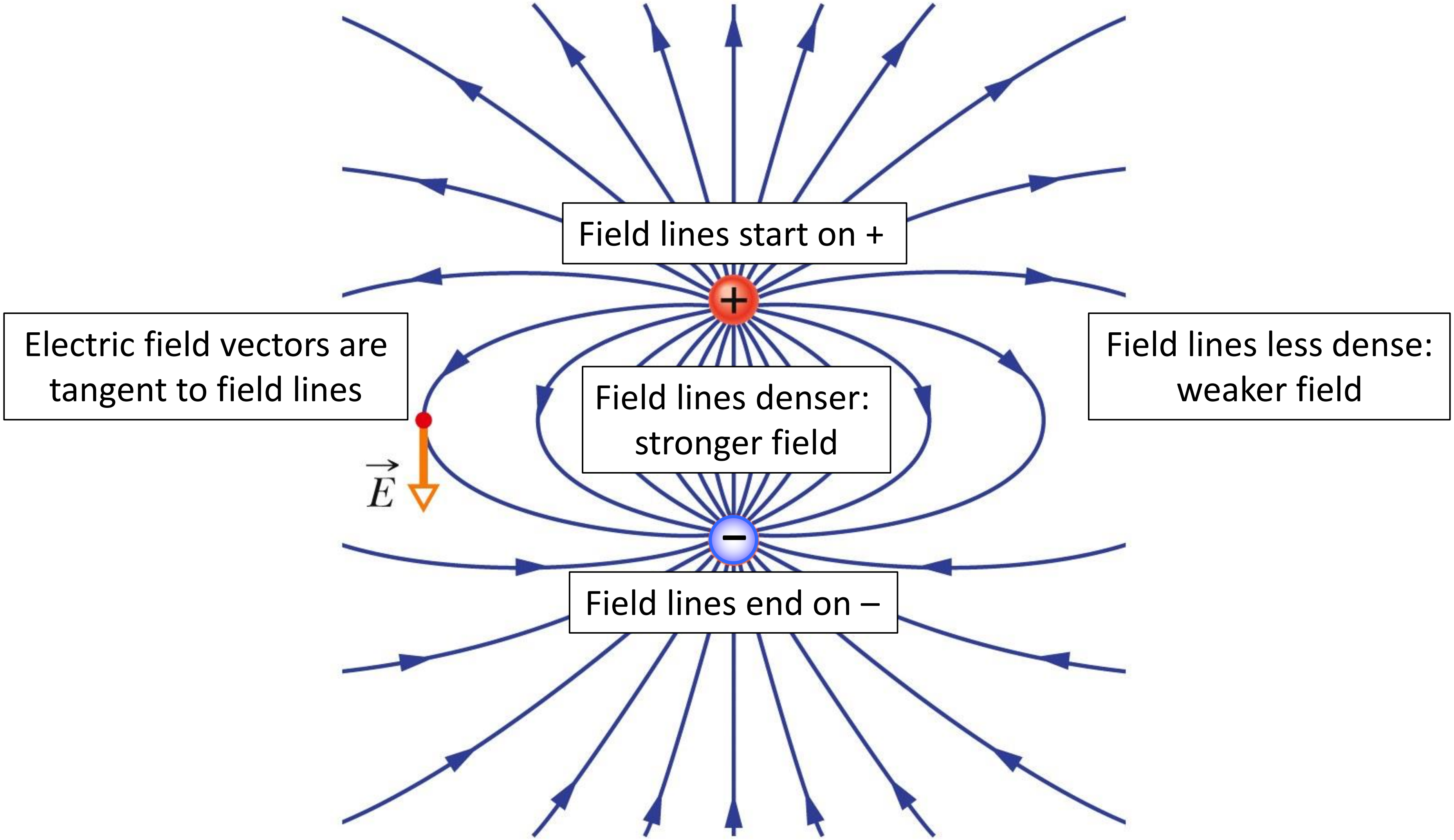
The electric field around a **point charge, q,** is given by

$$\boxed{\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}}$$

$\hat{r}$ is a **unit vector** that always points **away from** q.

We can use this with superposition to find the electric field of more complicated objects.

Electric Field Lines



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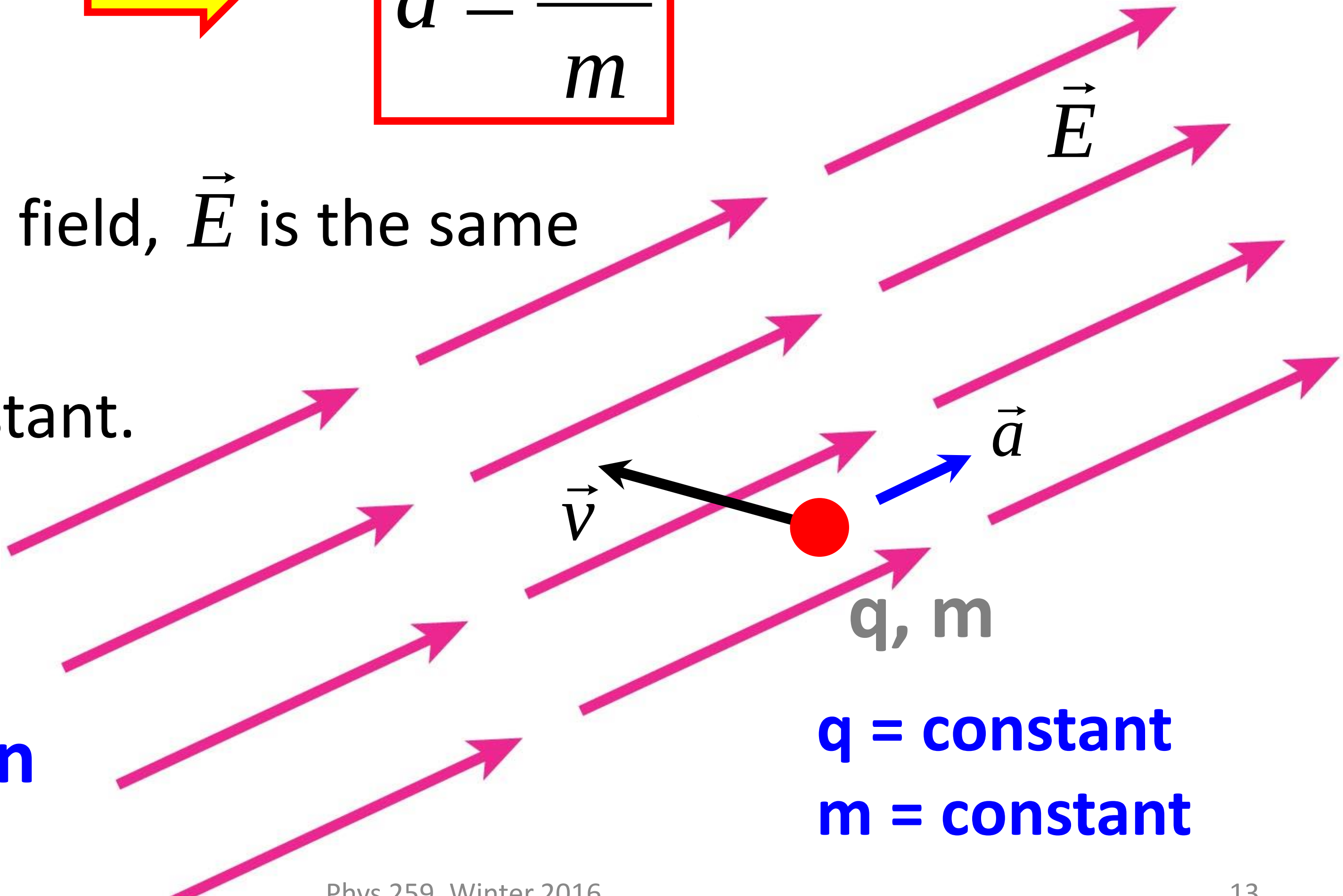
Uniform Electric Field

$$q\vec{E} = m\vec{a} \Rightarrow \boxed{\vec{a} = \frac{q\vec{E}}{m}}$$

In a **uniform** field, $\vec{E}$ is the same everywhere.

So $\vec{a}$ is constant.



q = constant

m = constant

Constant acceleration motion!

Uniform E-field: projectile motion

$$\vec{F}_{net} = q\vec{E}$$

Take E to point along +x-direction

$$a_x = \frac{qE}{m}$$

If q is +, a_x is +
If q is −, a_x is −

$$a_y = 0$$

$$x_f = x_i + v_{ix}\Delta t + \frac{1}{2}\frac{qE}{m}\Delta t^2$$

$$y_f = y_i + v_{iy}\Delta t$$

$$v_{fx} = v_{ix} + \frac{qE}{m}\Delta t$$

$$v_{fy} = v_{iy}$$

Rules for finding field due to continuous charge distribution

Step 1: cut up the distribution into a bunch of tiny pieces each with charge dQ.

Step 2: look for a symmetry that will make your calculations easier. If one exists, argue why you can use it and what it does.

Step 3: Calculate the magnitude of the force due to an ARBITRARY piece of charge dQ. Use geometric information to find the appropriate distance of dQ to the point of interest.

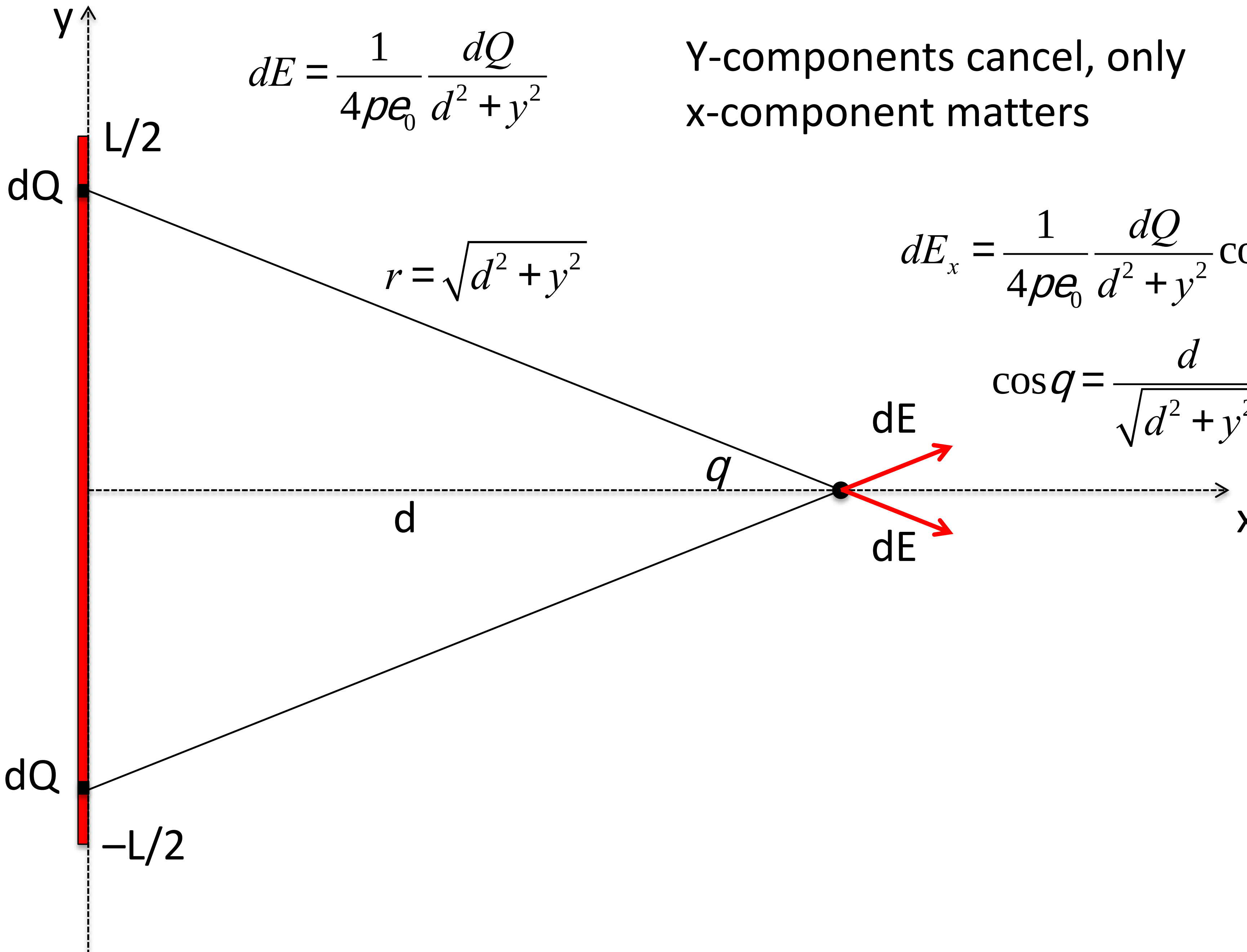
Step 4: pick the direction of the field based on the sign of the charge, then decompose the field into components.

$$dE = \frac{1}{4\pi\epsilon_0}\frac{dQ}{d^2 + y^2}$$

Y-components cancel, only x-component matters

$$dE_x = \frac{1}{4\pi\epsilon_0}\frac{dQ}{d^2 + y^2}\cos\theta$$

$$\cos\theta = \frac{d}{\sqrt{d^2 + y^2}}$$



Rules for finding field due to continuous charge distribution

Step 1: cut up the distribution into a bunch of tiny pieces each with charge dQ.

Step 2: look for a symmetry that will make your calculations easier. If one exists, argue why you can use it and what it does.

Step 3: Calculate the magnitude of the force due to an ARBITRARY piece of charge dQ. Use geometric information to find the appropriate distance of dQ to the point of interest.

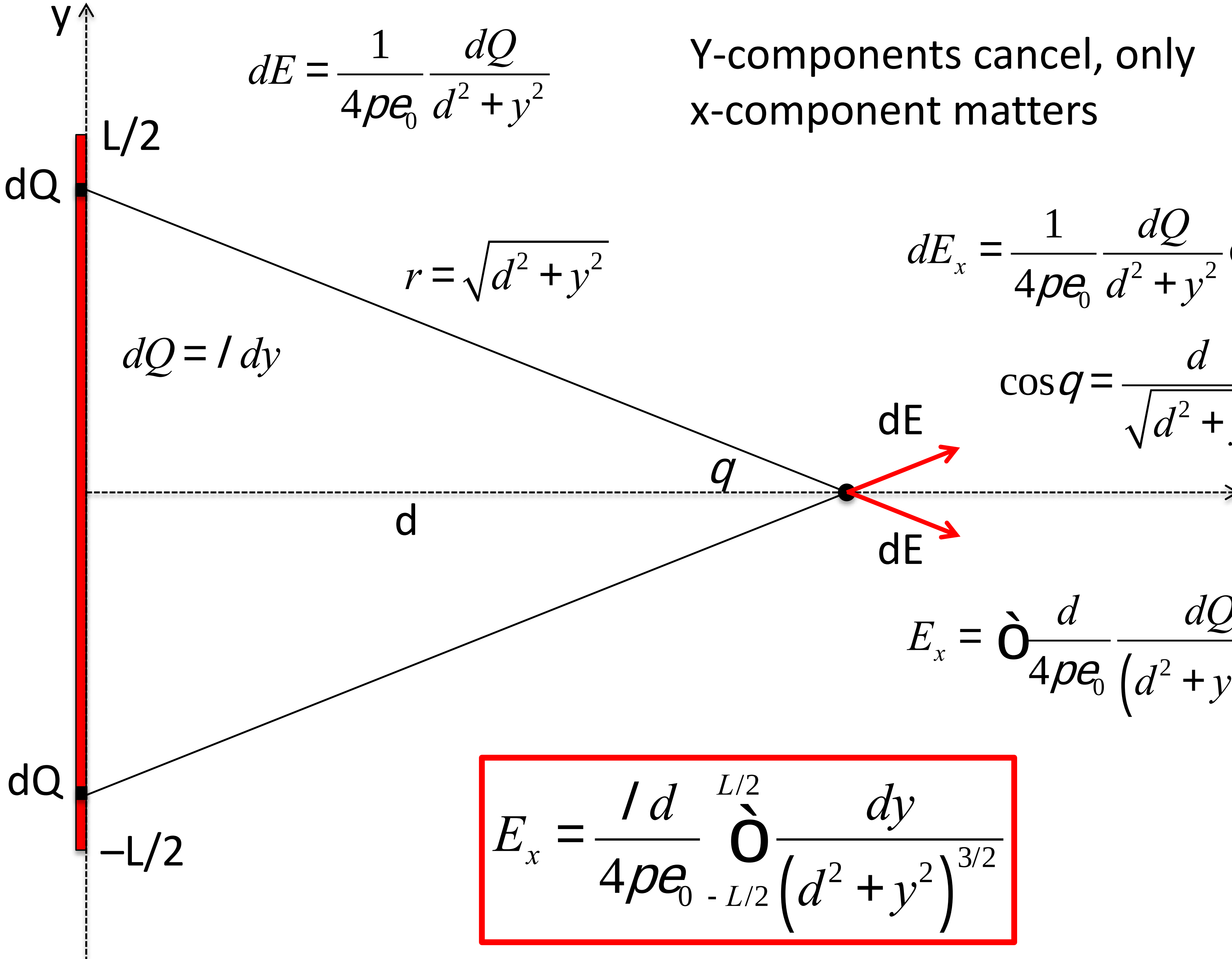
Step 4: pick the direction of the field based on the sign of the charge, then decompose the field into components.

Step 5: For each non-zero component, sum up all pieces dQ by integrating over the whole charge distribution.

Step 6: Express dQ in terms of a variable to be integrated over using linear/surface/volume charge density.

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dQ}{d^2+y^2}$$

Y-components cancel, only x-component matters



$$r = \sqrt{d^2+y^2}$$

$$dQ = \lambda\, dy$$

$$dE_x = \frac{1}{4\pi\varepsilon_0}\frac{dQ}{d^2+y^2}\cos\theta$$

$$\cos\theta = \frac{d}{\sqrt{d^2+y^2}}$$

$$E_x = \int \frac{d}{4\pi\varepsilon_0}\frac{dQ}{\left(d^2+y^2\right)^{3/2}}$$

$$\boxed{E_x = \frac{\lambda d}{4\pi\varepsilon_0}\int_{-L/2}^{L/2}\frac{dy}{\left(d^2+y^2\right)^{3/2}}}$$

Electric Flux; Gauss' Law

Gauss' Law is equivalent to Coulomb's law. It will provide us:

(i) an **easier way to calculate the electric field** in specific circumstances (especially situations with a high degree of symmetry)

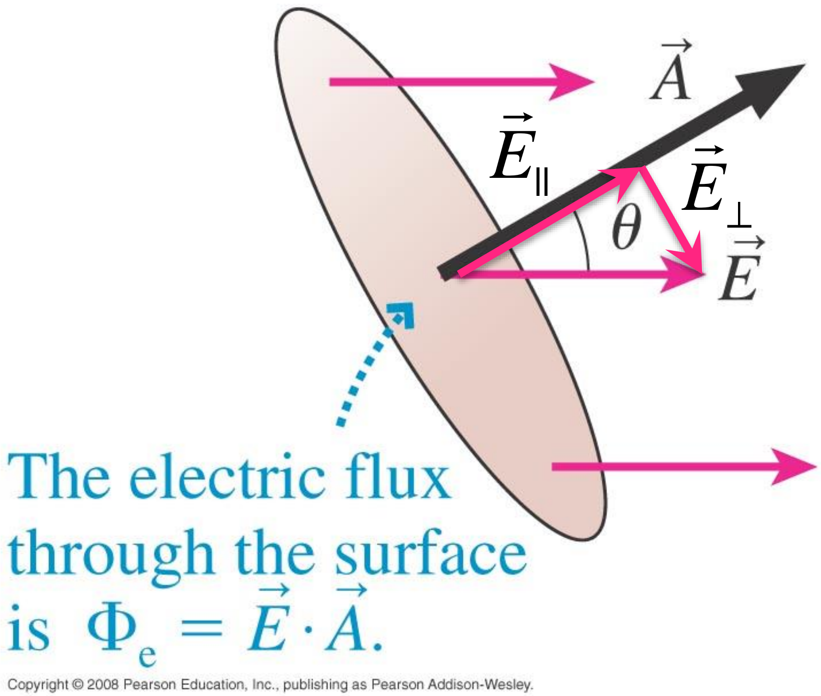
(ii) a better understanding of the properties of conductors in electrostatic equilibrium (more on this as we go)

(iii) It is valid for moving charges – not limited to electrostatics.

Electric flux, passing through a **closed** Gaussian surface

$$\Phi_E = \oiint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$$

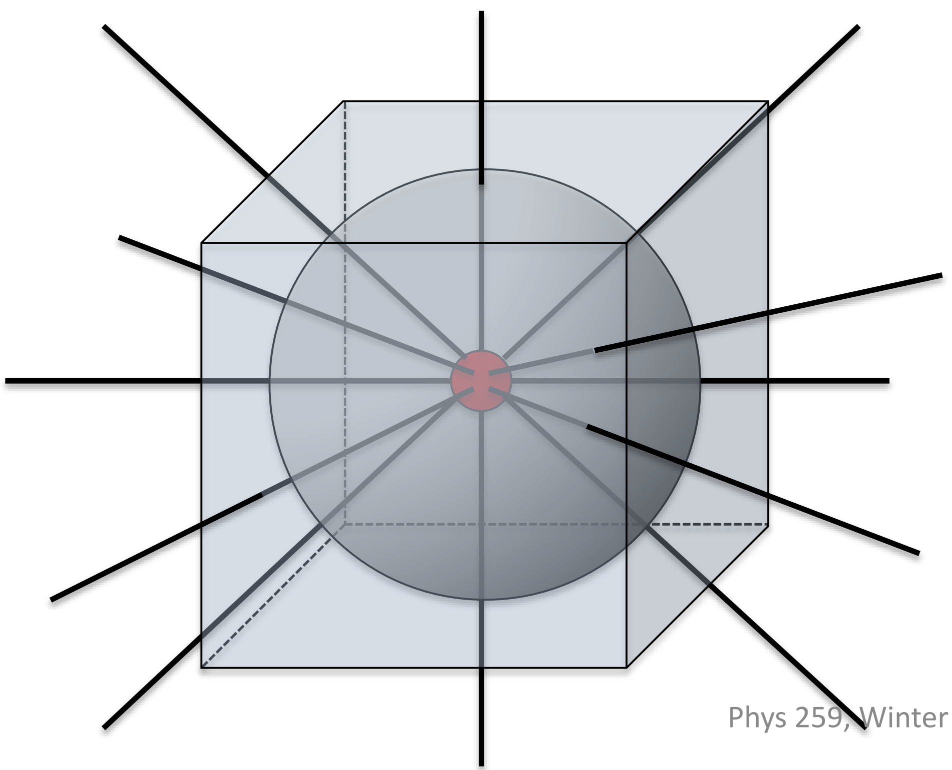
Electric flux through a surface with area A



$$F_E = EA\cos q = E_{\parallel} A$$

The **electric flux** through a closed surface does not depend on the **SHAPE** of the surface, it only depends on the **charges enclosed** by the surface.

This gives a nice interpretation of the flux as the **number of electric field lines passing through the surface**. Take the example of a point charge surrounded by a sphere, surrounded by a cube



The number of field lines passing through the sphere is the same as the number of field lines passing through the cube.

The electric flux through each surface is the same.

How to evaluate $\Phi_E = \oiint \vec{E} \cdot d\vec{A}$

- If the electric field is tangent to the surface:

$$\Phi = 0$$

- If the electric field is normal to the surface and is constant at every point:

$$\Phi = EA$$

Rules for using Gauss' Law to find **E**

Step 1: figure out the symmetry of the charge distribution: spherical, cylindrical, or planar, & the centre of symmetry.

Step 2: argue which direction the E-field must be pointing based on that symmetry

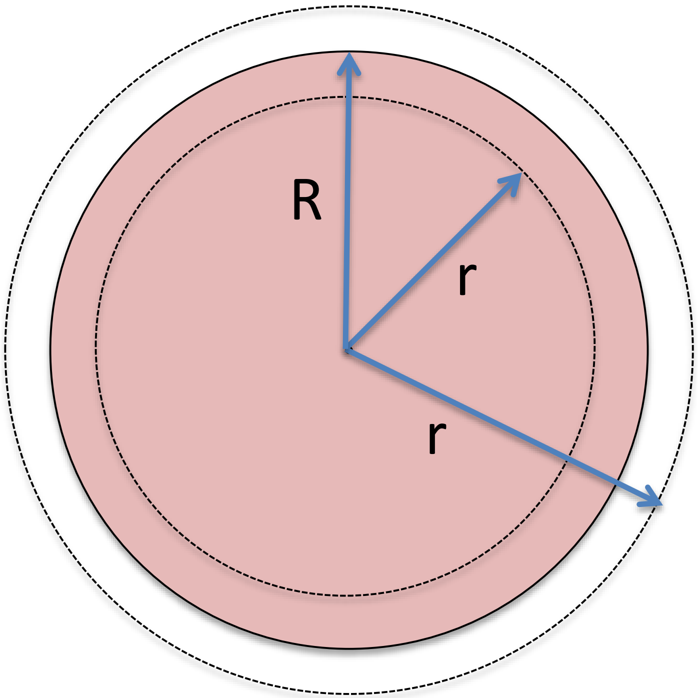
Step 3: argue where the E-field must have constant magnitude based on the symmetry

Step 4: choose a Gaussian surface such that over part of it, **E** has constant magnitude and points in the direction of the normal vector and such that everywhere else the flux is zero.

Step 5: everywhere **E** has constant magnitude, the flux is EA.

Step 6: determine the net charge enclosed by the Gaussian surface. Use this to find an expression for E.

1. Solid ball of uniform charge density (charge Q)



Outside the ball: $q_{enc} = Q$

$$\boxed{E = \frac{Q}{4\pi\varepsilon_0 r^2} \qquad \text{for } \; r > R}$$

Task

Use Gauss’ law to compute the E field inside and outside a uniformly charged ball

Symmetry argument:

1. Electric field must point in the **radial direction** only.
2. The electric field must be the **same magnitude** at constant radius.

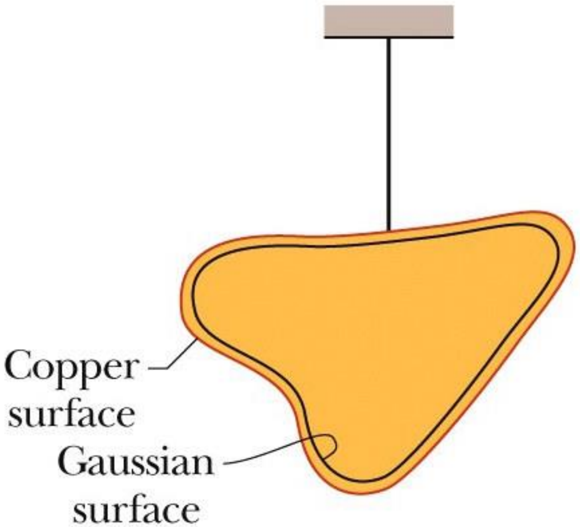
$$\Phi_E = \oint \vec{E} \cdot (d\vec{A}) = EA_{\text{sphere}} = \frac{q_{enc}}{\varepsilon_0}$$

Inside the ball:

$$q_{enc} = \rho V_{ball} = \left(\frac{Q}{\cancel{\tfrac{4}{3}\pi} R^3}\right)\cancel{\tfrac{4}{3}\pi} r^3 = Q\frac{r^3}{R^3}$$

$$E = \frac{Qr^3}{\varepsilon_0 (4\pi r^2) R^3} = \boxed{\frac{Qr}{4\pi\varepsilon_0 R^3}} \qquad \text{for } \; r < R$$

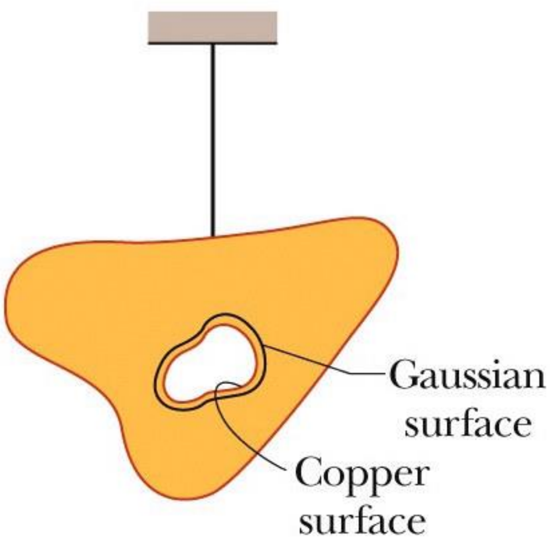
Hollow Conductors



(a)

The electric field inside a conductor is zero. This immediately implies that conductors are electrically neutral in their interiors.

$$\oint \cancel{\vec{E}} \cdot d\vec{A} = 0 = \frac{q_{enc}}{\varepsilon_0}$$



(b)

This also means that the surface of a hollow cavity inside a conductor cannot carry any excess charge. All excess charge must reside on the outside surface only.

Summary of Conductors and Electric Fields