Announcements

- Complete Assignment #1 **(Math Review)** before **11:59 pm, Wednesday, January 18**.

- Assignment #2 went online **Wednesday Jan. 18 at 8:00 a.m.**

- No laboratorial this week.

Last time

- Polarization
- More on unit vectors
- More on Coulomb's law
- Calculation of Coulomb's force between two point charges
- Superposition principle

This time

- More on superposition principle
- An example involving four point charges
- Define electric dipole and force due to a dipole
- Line, surface and volume charge density

Superposition principle

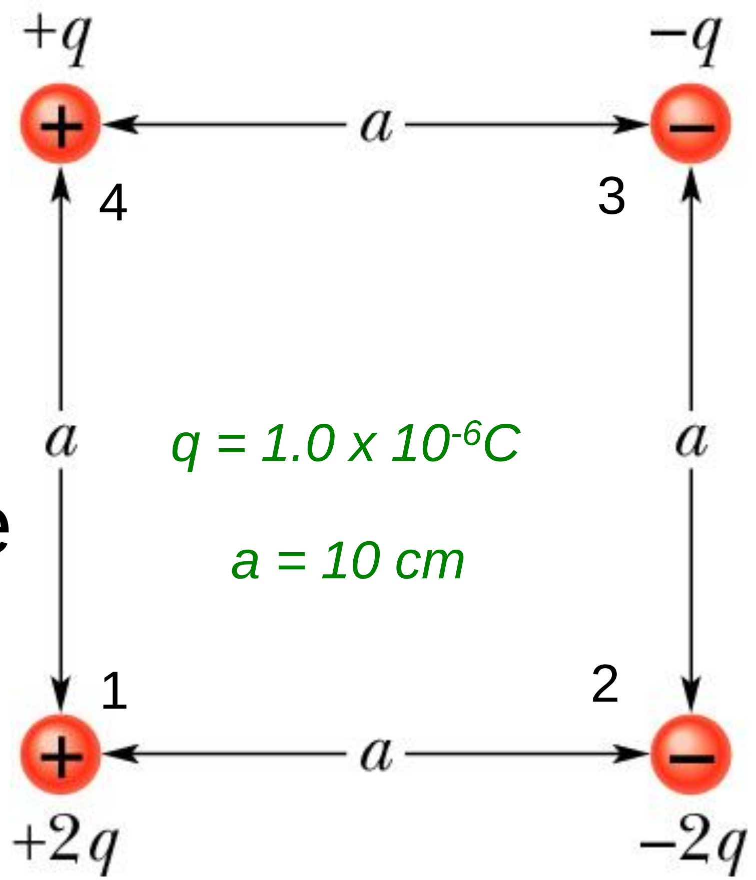
Principle of superposition states that **the total force on a particle in is simply the vector sum of the individual forces.**

$$\vec{F}_{1,net} = \vec{F}_{2\ on\ 1} + \vec{F}_{3\ on\ 1} + \vec{F}_{4\ on\ 1} + \vec{F}_{5\ on\ 1} + \cdots$$

$$\vec{F}_{4,net} = \vec{F}_{1\ on\ 4} + \vec{F}_{2\ on\ 4} + \vec{F}_{3\ on\ 4} + \vec{F}_{5\ on\ 4} + \cdots$$

Example

Calculate the net force on particle 1.



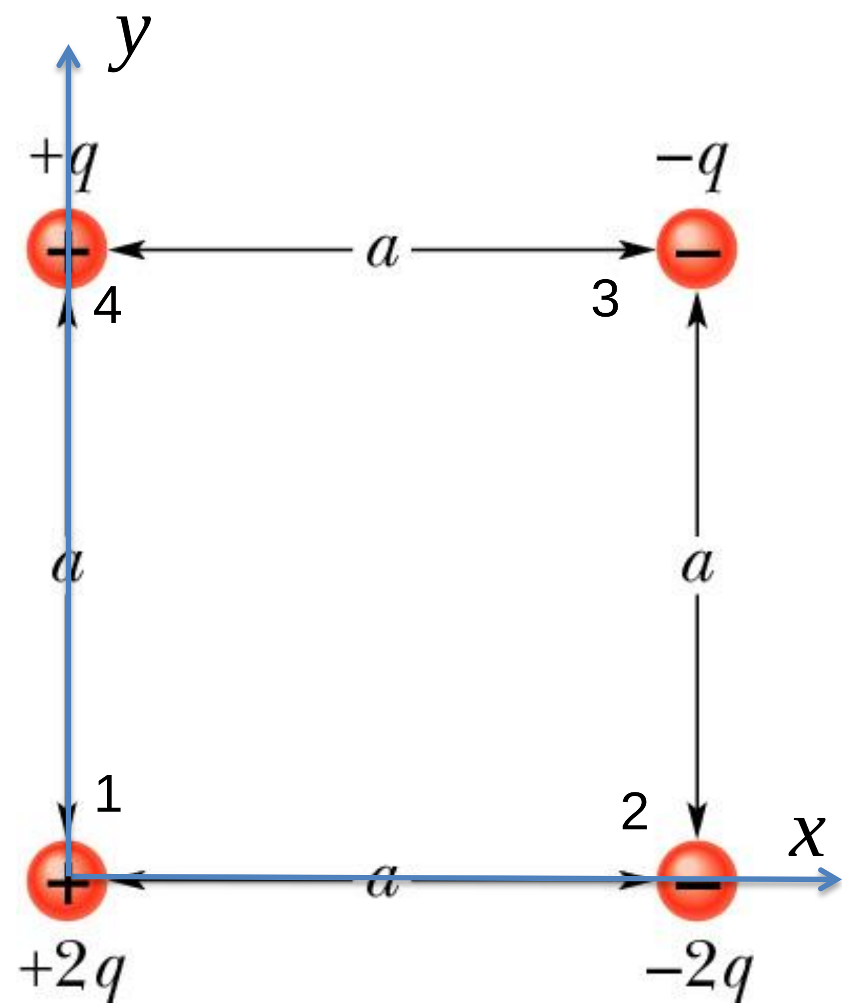
Use superposition principle

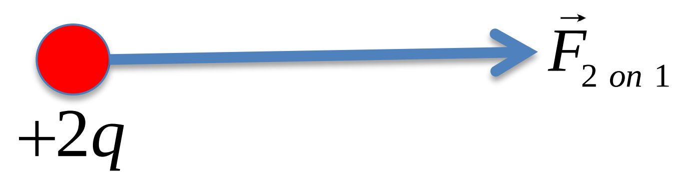
$$\vec{F}_{1,net} = \vec{F}_{2\ on\ 1} + \vec{F}_{3\ on\ 1} + \vec{F}_{4\ on\ 1}$$

Force from 2 on 1 is attractive. It lies along the line connecting charges 1 and 2 and pointing in the positive x-direction.

$$\vec{F}_{2\ on\ 1} = k_e \frac{|q_1||q_2|}{r^2}\hat{r}_{21}$$

$$\begin{aligned}\vec{F}_{2\ on\ 1} &= k_e \frac{|q_1||q_2|}{r^2}\hat{r}_{21} \\ &= k_e \frac{(2q)(2q)}{a^2}\hat{i} \\ &= 4k_e \frac{q^2}{a^2}\hat{i}\end{aligned}$$

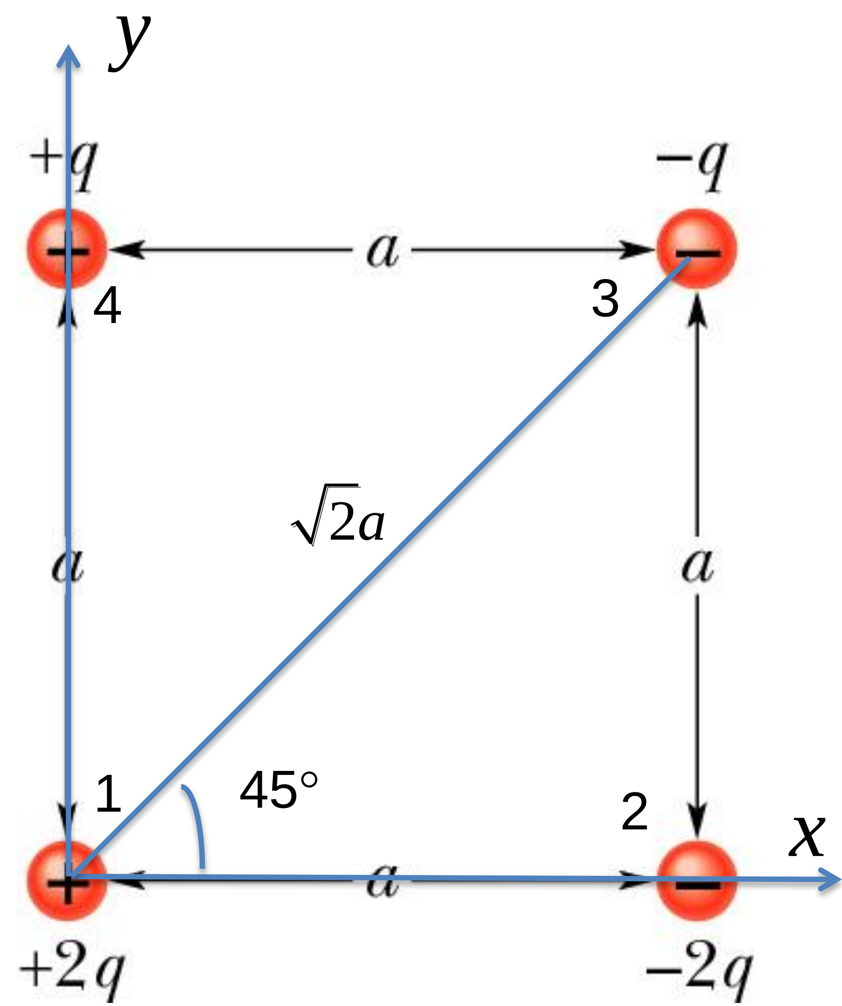


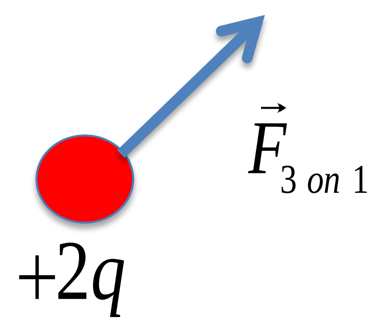


Force from 3 on 1 is attractive. It lies along the line connecting charges 1 and 3 and pointing 45° north of east.

$$\vec{F}_{3\ on\ 1} = k_e \frac{|q_1||q_3|}{r^2}\hat{r}_{31}$$

$$\begin{aligned}\vec{F}_{3\ on\ 1} &= k_e \frac{|q_1||q_3|}{r^2}\hat{r}_{31} \\ &= k_e \frac{(2q)(q)}{\left(\sqrt{2}a\right)^2}\hat{r}_{31} \\ &= k_e \frac{q^2}{a^2}\hat{r}_{31} = k_e \frac{q^2}{a^2}\left(\cos 45\ \hat{i} + \sin 45\ \hat{j}\right)\end{aligned}$$

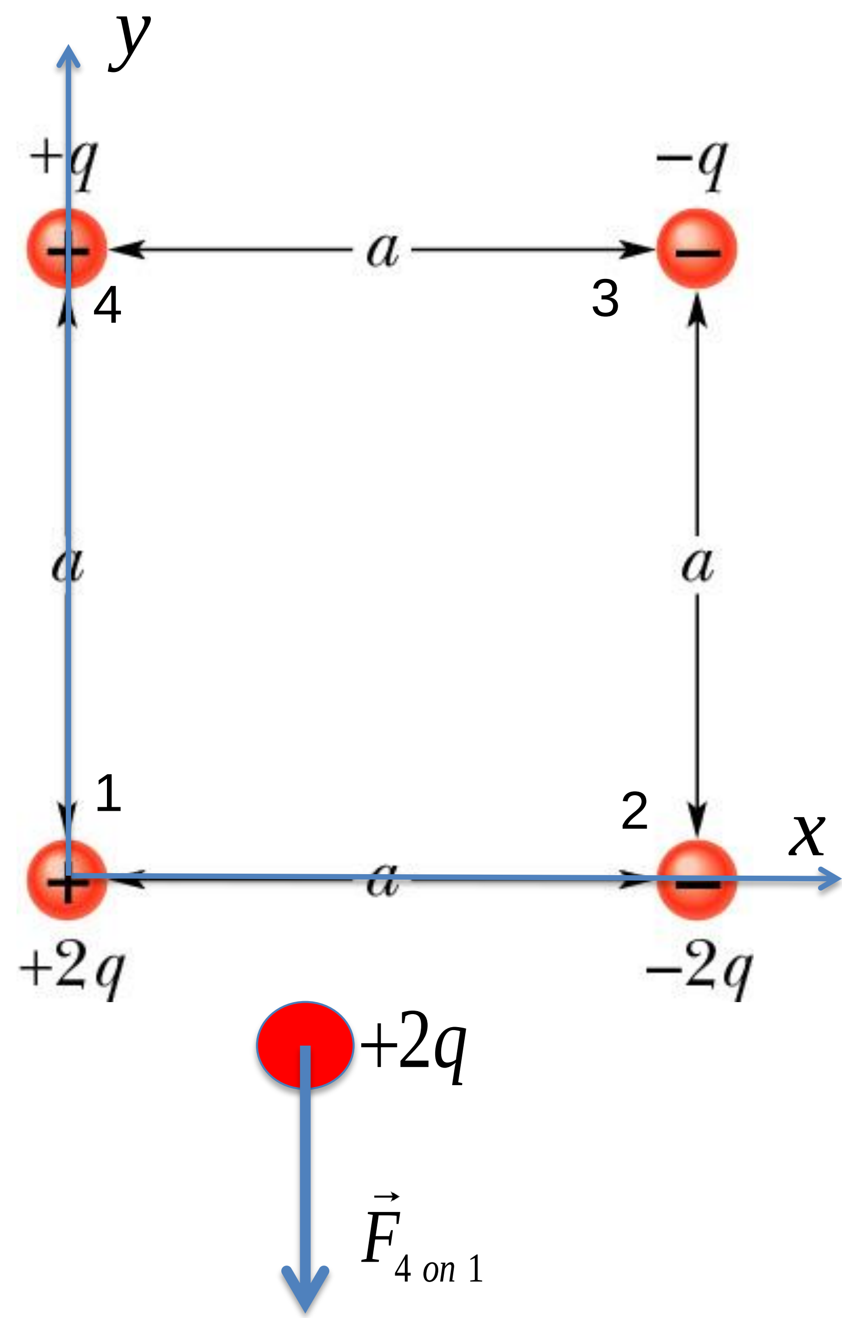




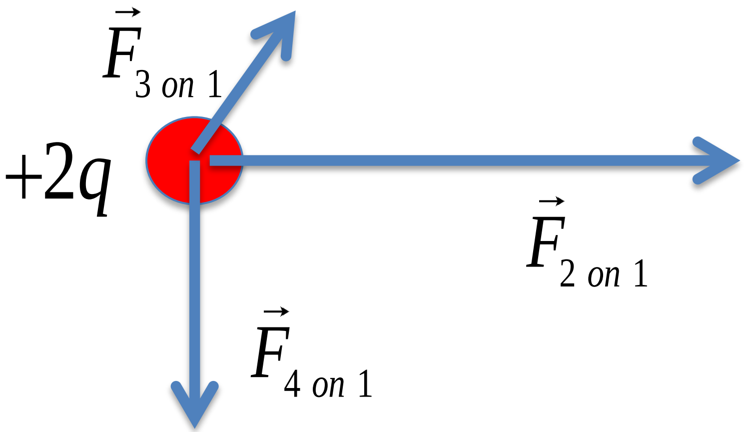
Force from 4 on 1 is repulsive. It lies along the line connecting charges 1 and 4 and pointing south.

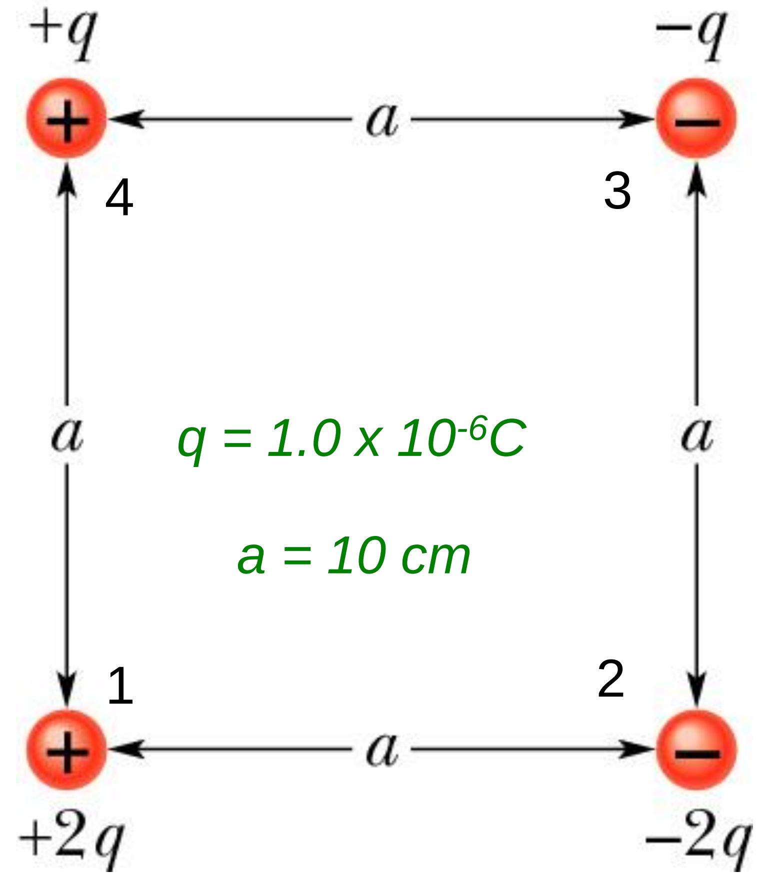
$$\vec{F}_{4\ on\ 1} = k_e \frac{|q_1||q_4|}{r^2}\hat{r}_{41}$$

$$\begin{aligned}\vec{F}_{4\ on\ 1} &= k_e \frac{|q_1||q_4|}{r^2}\hat{r}_{41} \\ &= k_e \frac{(2q)(q)}{a^2}\hat{r}_{41} \\ &= 2k_e \frac{q^2}{a^2}\hat{r}_{41} = -2k_e \frac{q^2}{a^2}\hat{j}\end{aligned}$$



Putting it all together.





$$\vec{F}_{on\ 1} = 4k_e \frac{q^2}{a^2}\hat{i} + k_e \frac{q^2}{a^2}\left(\cos 45\ \hat{i} + \sin 45\ \hat{j}\right) - 2k_e \frac{q^2}{a^2}\hat{j}$$

$$= k_e \frac{q^2}{a^2}\left[\left(4 + \cos 45\right)\ \hat{i} + \left(-2 + \sin 45\right)\ \hat{j}\right]$$

$$\vec{F}_{on\ 1} = k_e \frac{q^2}{a^2}\left[\left(4+\cos 45\right)\ \hat{i} + \left(-2+\sin 45\right)\ \hat{j}\right]$$

$$= \left(8.99\times10^{9}\ \text{Nm}^2/\text{C}^2\right)\frac{\left(10^{-12}\ \text{C}^2\right)}{\left(0.10\ \text{m}\right)^2}\left(4.71\hat{i} - 1.29\,\hat{j}\right)$$

$$= \left(4.23\hat{i} - 1.16\,\hat{j}\right)\text{N}$$

$$F_x = +4.23\ \text{N}$$

$$F_y = -1.16\ \text{N}$$

TopHat Question 1: **JOIN CODE:** 419305

What is the dimension of a unit vector showing the direction of a force on a point charge?

(A) Newton

(B) Dimensionless

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We can obtain a unit vector from a vector by dividing the vector by

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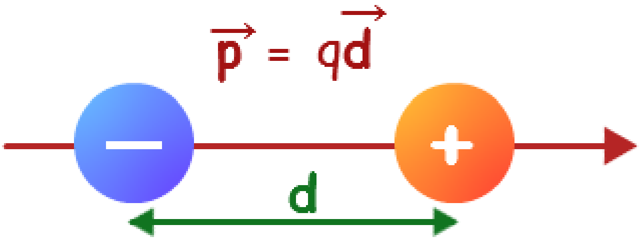
Definition of electric dipole moment

A pair of opposite charges of equal magnitude q separated by a distance d form an electric dipole.



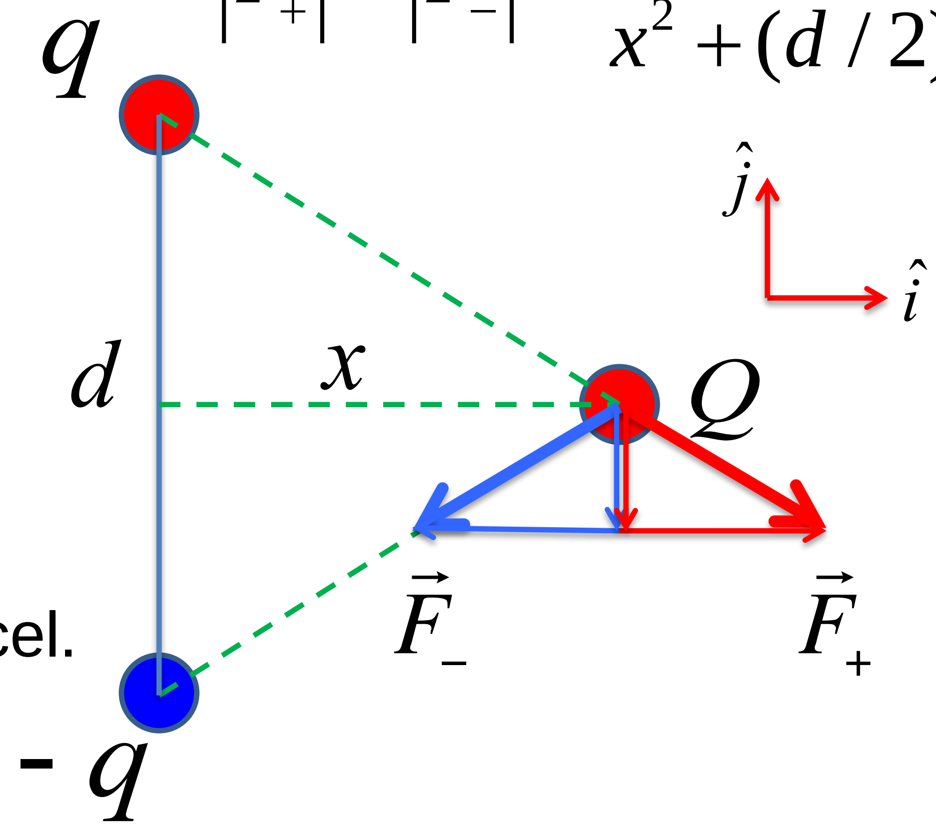
Electric dipole moment is a vector quantity. The defined direction is from the negative charge to the positive charge and the magnitude is given by $p = qd$.

Example: force due to a dipole

A charge *Q* sits at a distance *x* on the axis perpendicular to the dipole. What is the force (magnitude and direction) it experiences?

$$\left|\vec{F}_+\right| = \left|\vec{F}_-\right| = \frac{k_e qQ}{x^2 + (d/2)^2}$$

Free Body Digram:



Horizontal components cancel.
Vertical components add.
SYMMETRY!

Example: force due to a dipole

$$\left|\vec{F}_{net}\right| = 2\left(\frac{k_e qQ}{x^2+(d/2)^2}\right)\sin\theta$$

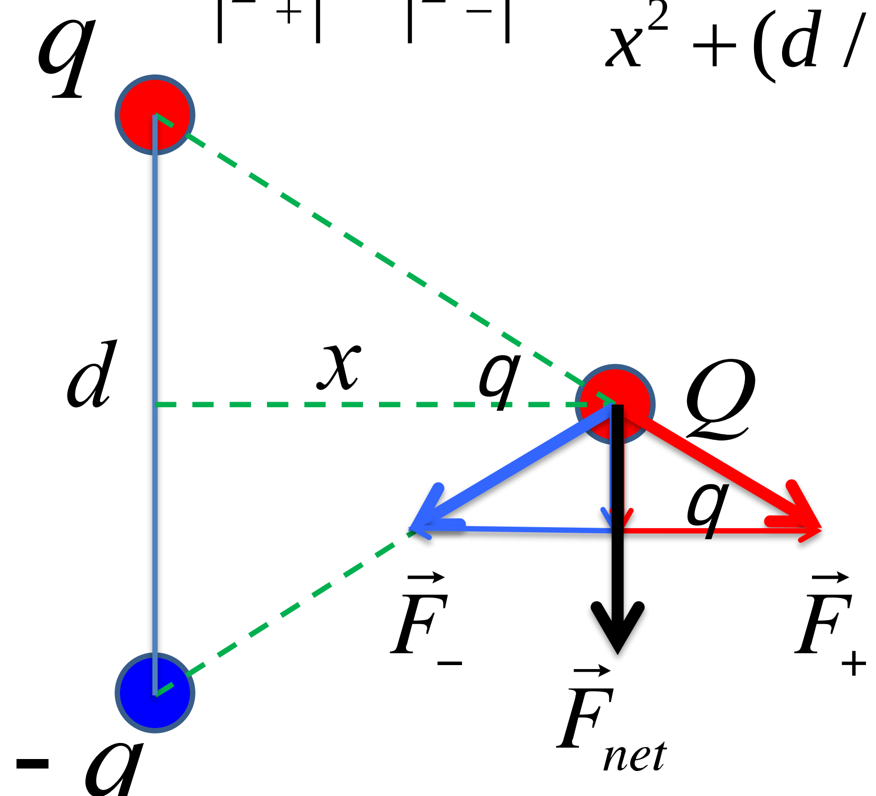
$$\sin q = \frac{d/2}{\sqrt{x^2+(d/2)^2}}$$

$$\left|\vec{F}_{net}\right| = \frac{k_e qQd}{\left(x^2+(d/2)^2\right)^{3/2}}$$

Direction: **downward**

$$\vec{F}_{net} = -\frac{k_e qQd}{\left(x^2+(d/2)^2\right)^{3/2}}\,\hat{j}$$

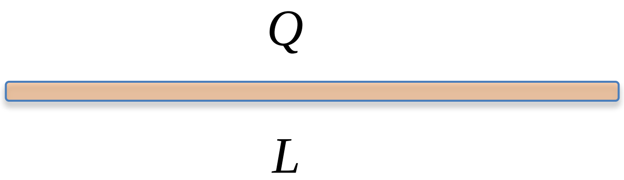
$$\left|\vec{F}_{+}\right| = \left|\vec{F}_{-}\right| = \frac{k_e qQ}{x^2+(d/2)^2}$$



How to compute Coulomb's force for a charge distribution?

Line charge density:

A total charge of Q is uniformly distributed over an infinitely thin rod of length L. The other dimensions are infinitely thin.

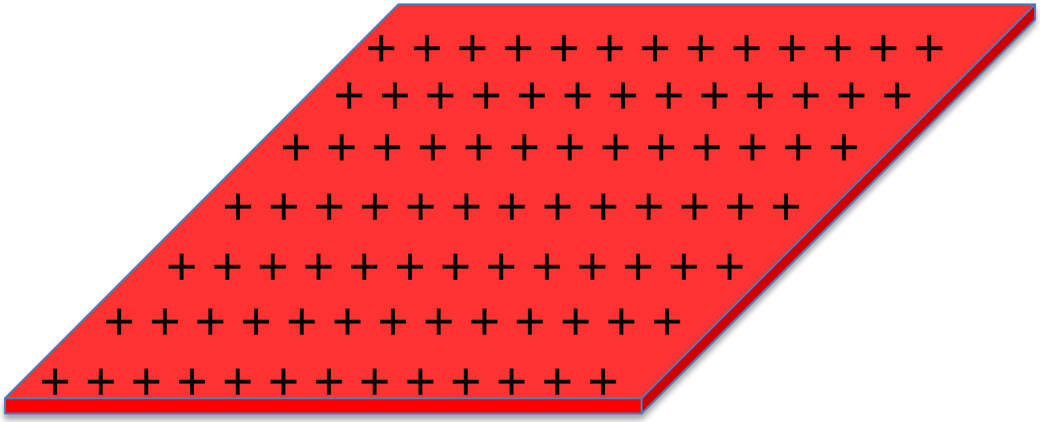


Charge per unit length:

$$\lambda = \frac{Q}{L}$$

Surface charge density:

A total charge of Q is uniformly distributed over an infinitely thin sheet of area A.



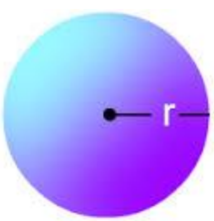
Charge per unit area:

$$\sigma = \frac{Q}{A}$$

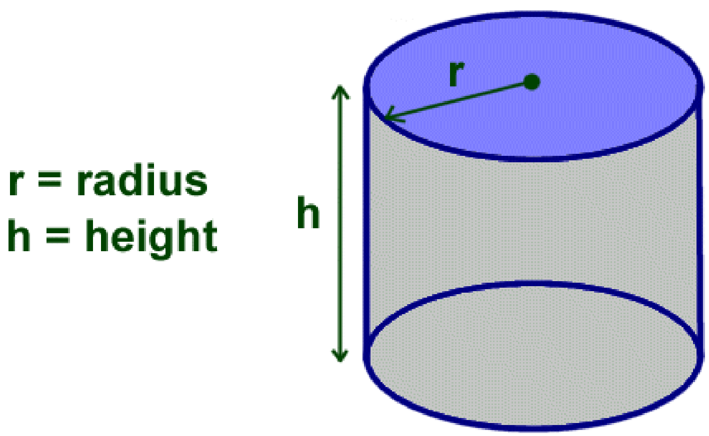
Volume charge density:

A total charge of Q is uniformly distributed in a volume V.

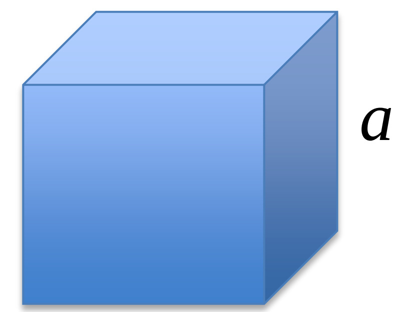
Charge per unit volume: $\rho = \dfrac{Q}{V}$



$$\rho = \frac{Q}{\frac{4}{3}\pi r^3}$$



$$\rho = \frac{Q}{\pi r^2 h}$$



$$\rho = \frac{Q}{a^3}$$