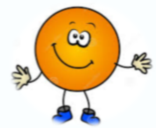
# Electricity and Magnetism

- Physics 259 – L02
- Lecture 13

# Chapter 23

**(please read chapter 22 of the textbook)**

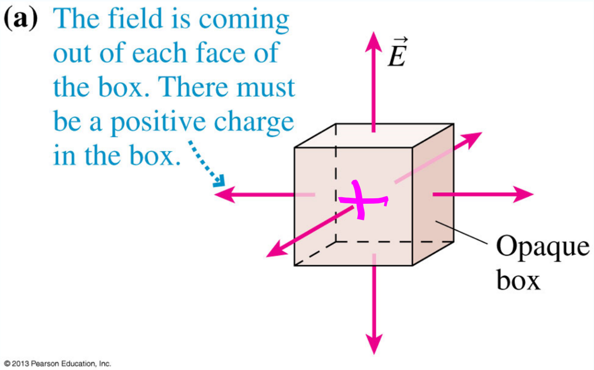
# Last time

- Chapter 22 and 23.1 

# This time

- Chapter 23.1 

# 23-1: The Electric Flux


(a) The field is coming out of each face of the box. There must be a positive charge in the box.
$\vec{E}$
Opaque box
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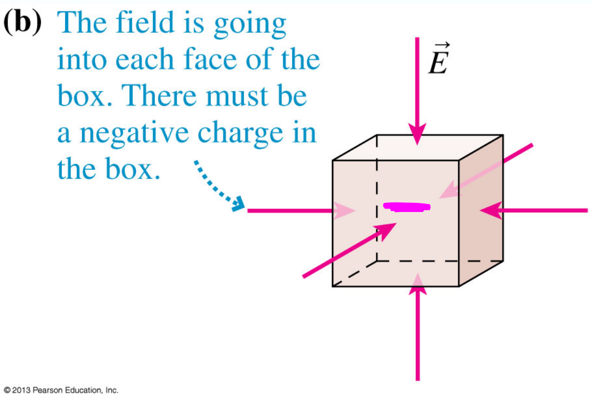
(b) The field is going into each face of the box. There must be a negative charge in the box.
$\vec{E}$
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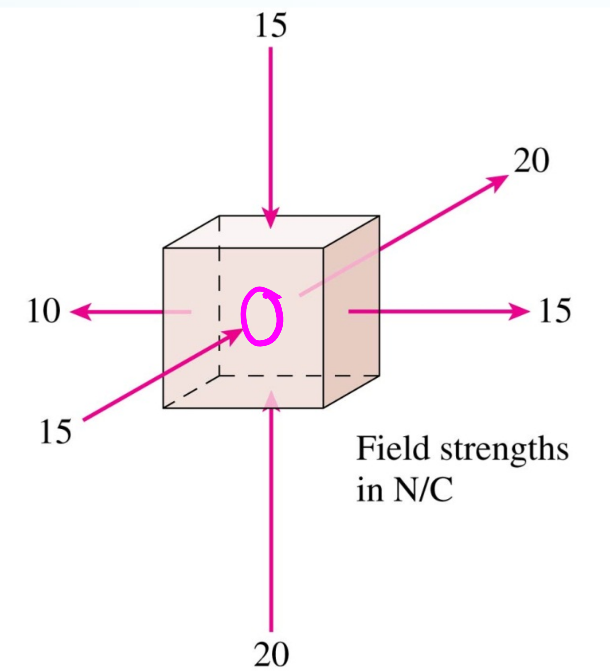
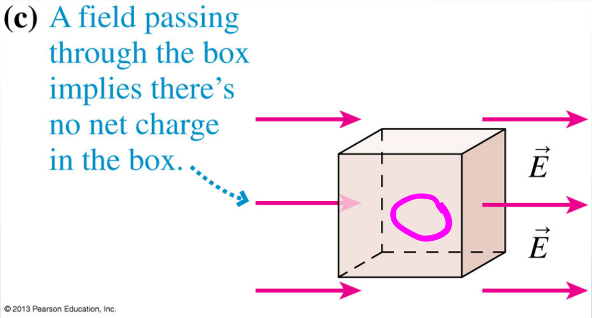

15
20
10
15
15
20
Field strengths in N/C


(c) A field passing through the box implies there's no net charge in the box.
$\vec{E}$
$\vec{E}$
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# Electric Flux; Gauss' Law

Gauss' Law is equivalent to Coulomb's law. It will provide us:

(i) an **easier way to calculate the electric field** in specific circumstances (especially situations with a high degree of symmetry)

(ii) a better understanding of the properties of conductors in electrostatic equilibrium (more on this as we go)

(iii) It is valid for moving charges – not limited to electrostatics.

# A closed surface through which an electric field passes is called **Gaussian surface**

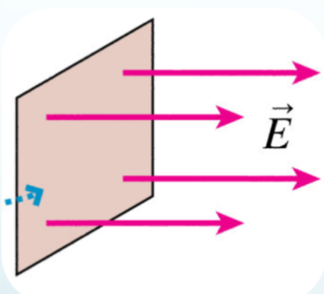
An imaginary mathematical surface

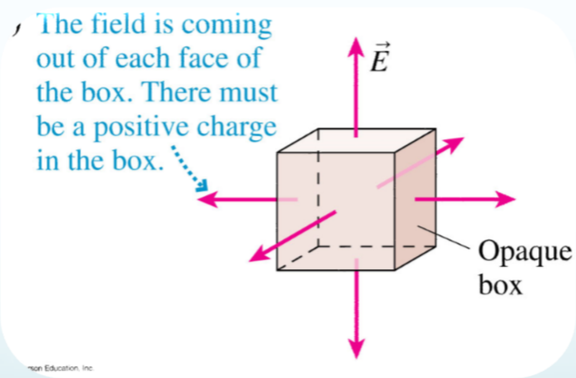
**The Gaussian surface is most useful when it matches the shape of the field**

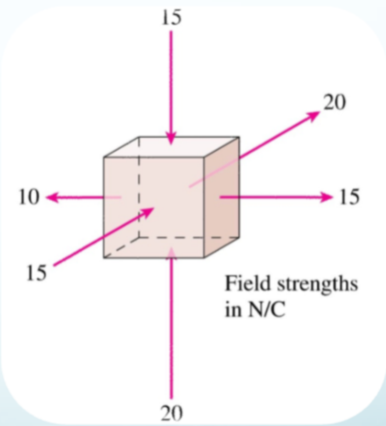
**Gauss's law relates the electric field at points on a closed Gaussian surface to the net charge enclosed by that surface**

# Electric Flux ($\Phi_e$)

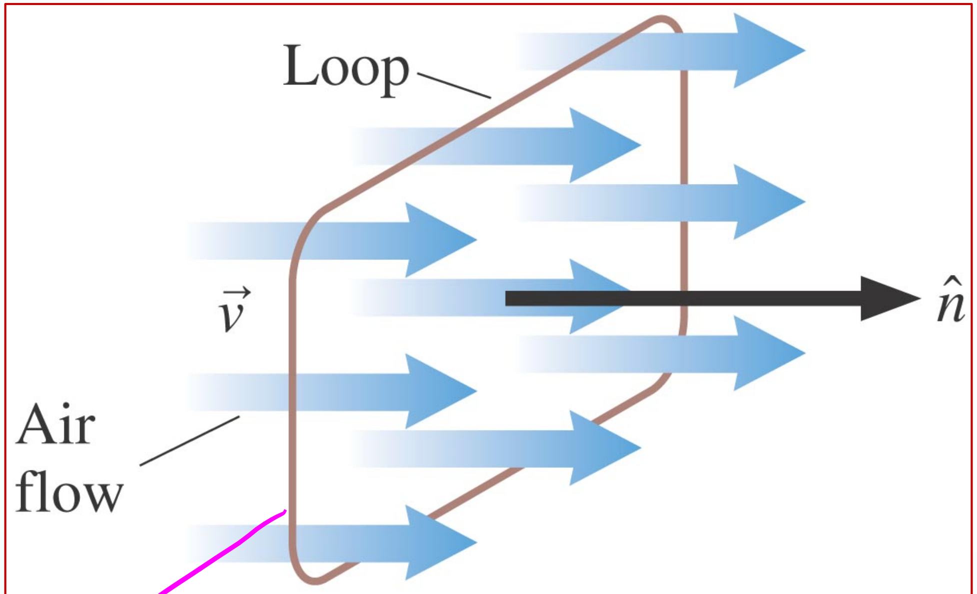
- **Amount of electric field going through a surface**
- **The number of field lines coming through a surface**



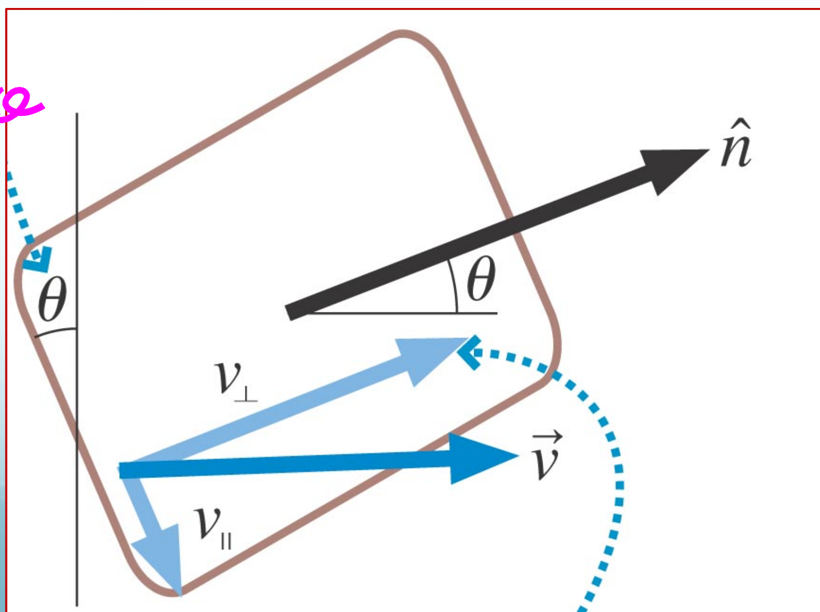



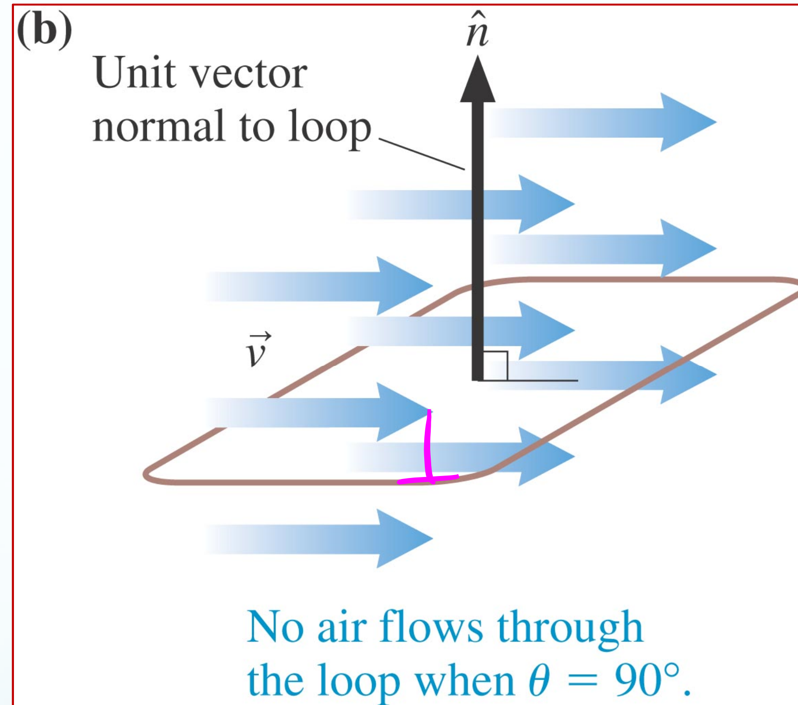



# Wind going through a loop




surface






$W \propto A$

$W \propto \theta$

$W \propto W$

$\Phi_e = \vec{E} \cdot \vec{A}$

# The Electric Flux

**Amount of electric field going through a surface**

$\rightarrow \Phi_e \propto \vec{E}$

$\rightarrow \Phi_e \propto \vec{A}$

$\rightarrow \Phi_e \propto \theta$

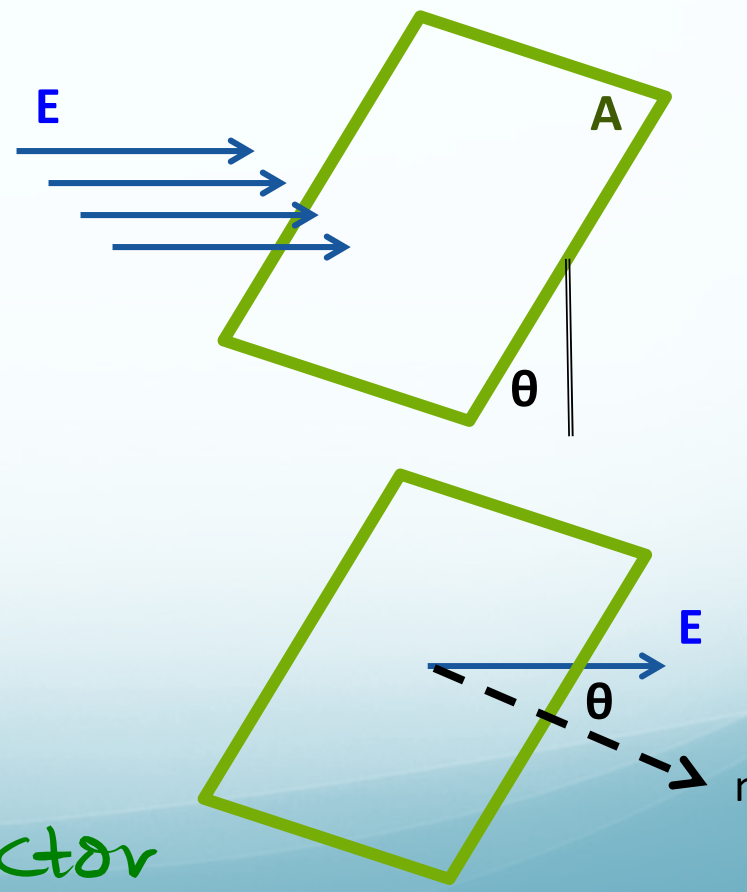
$\rightarrow$ $$\boxed{\Phi_e = E_{\perp} A = EA\cos\theta}$$

$\vec{E}.\vec{A} = |E||A|\cos\theta$

$$\rightarrow \Phi_e = \vec{E}.\vec{A}$$

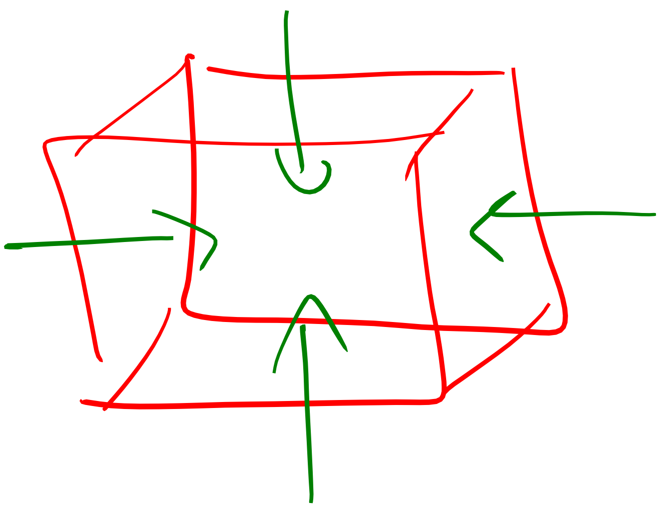
Scalar

vector. vector

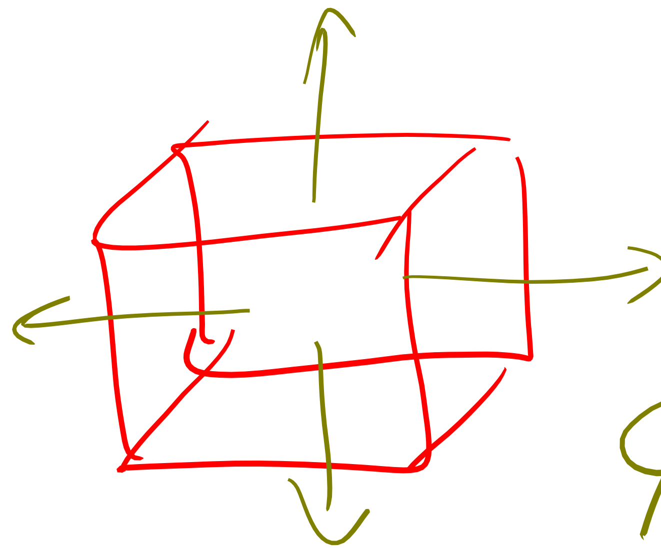


$$C = \vec{A} \cdot \vec{B} = |A||B| \cos\theta$$

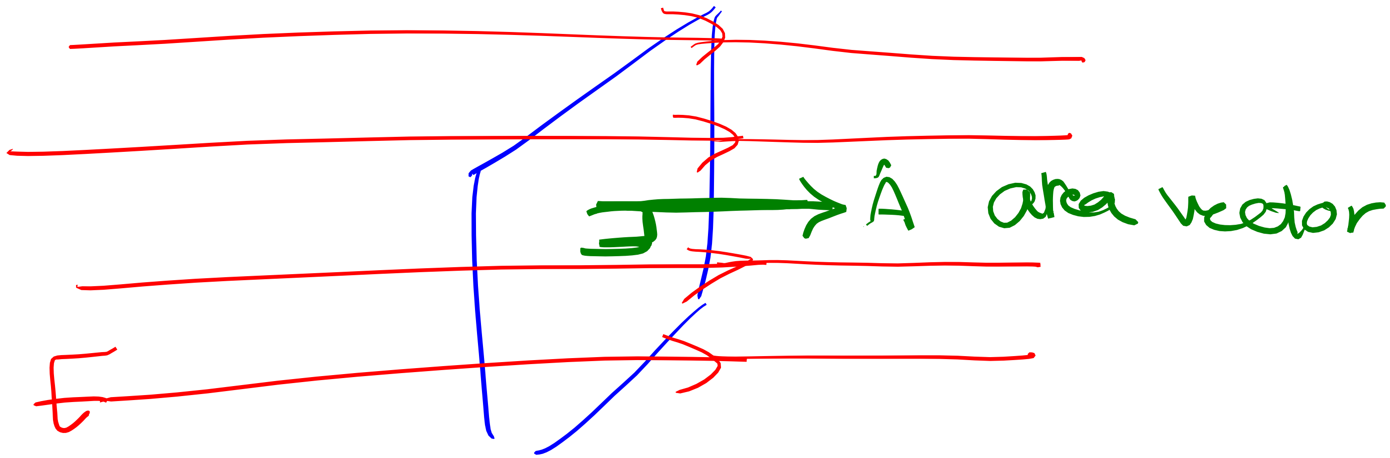
Scalar

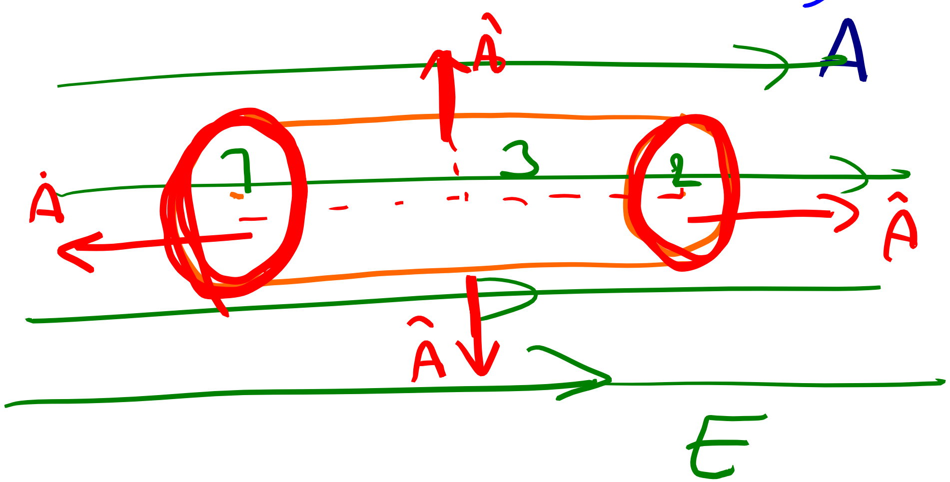


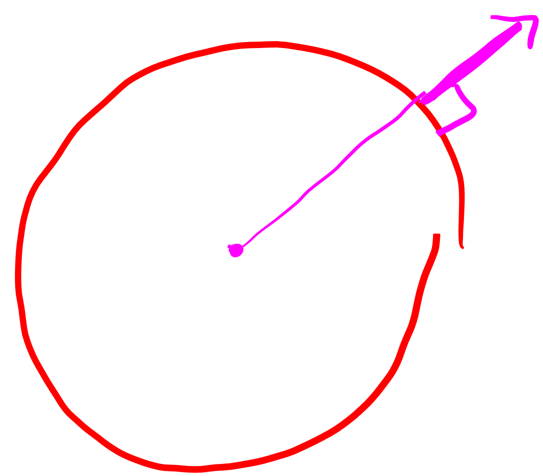
$\phi_e \to -$



$\phi_e \to +$



$1 \rightarrow \theta = 180^\circ \qquad 2 \rightarrow \theta = 0^\circ , \quad 3 \rightarrow 90^\circ$

# QuickCheck 27.2

$$\Phi_e = \vec{E}\cdot\vec{A} = EA\cos\theta$$

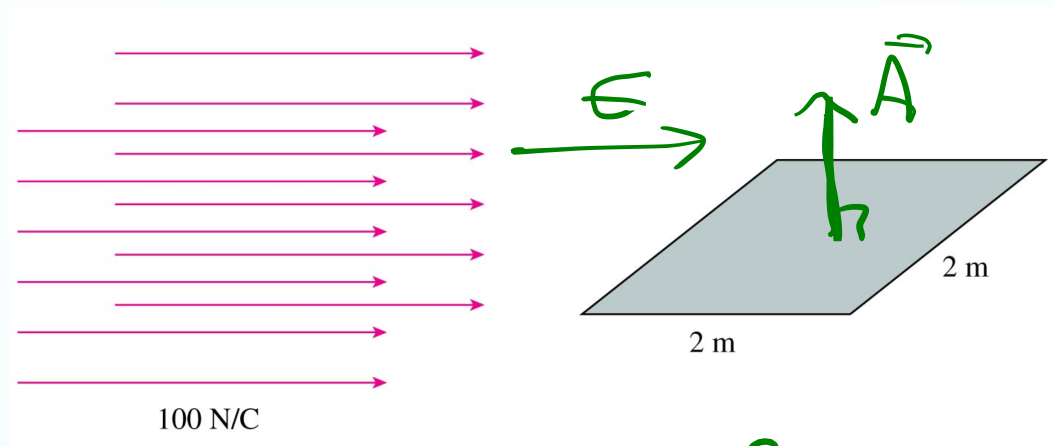
The electric flux through the shaded surface is

A. 0.

B. 200 N m/C.

C. 400 N $m^2$/C.

D. Some other value.




$\theta = 90^\circ$

$\cos\theta = 0$

# QuickCheck 27.1

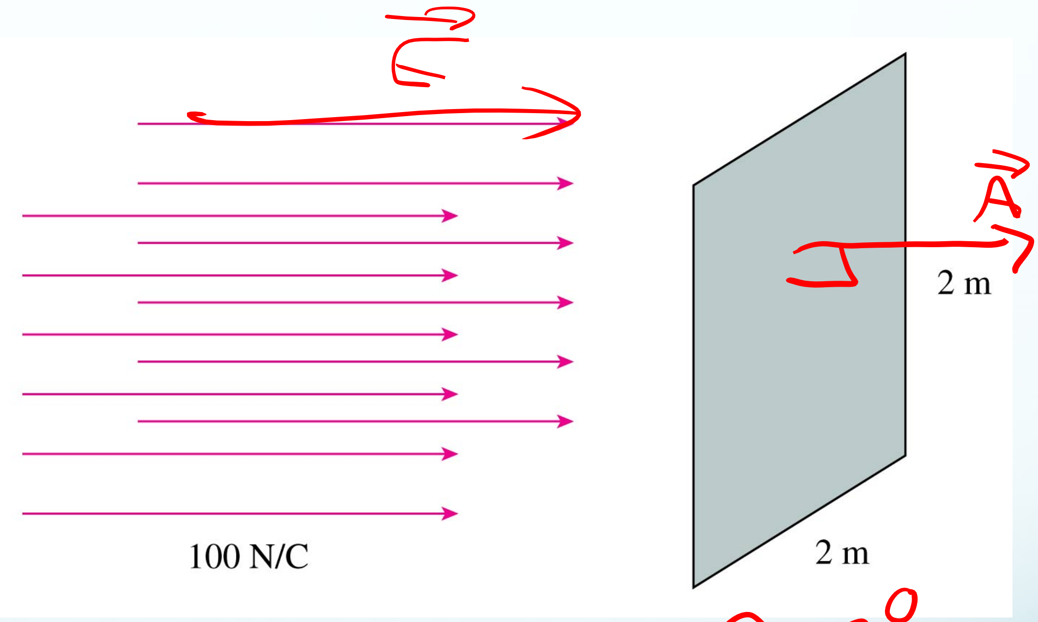
The electric flux through the shaded surface is

A. 0.

B. 200 N m/C.

C. 400 N $m^2$/C.

D. Flux isn't defined for an open surface.




$\theta = 0^\circ$

$$\Phi_e = \vec{E} \cdot \vec{A} = EA\cos\theta$$

$$= 100 \times 4 = 400 \frac{Nm^2}{C}$$

# QuickCheck 27.3

**Top Hat Question**

The electric flux through the shaded surface is

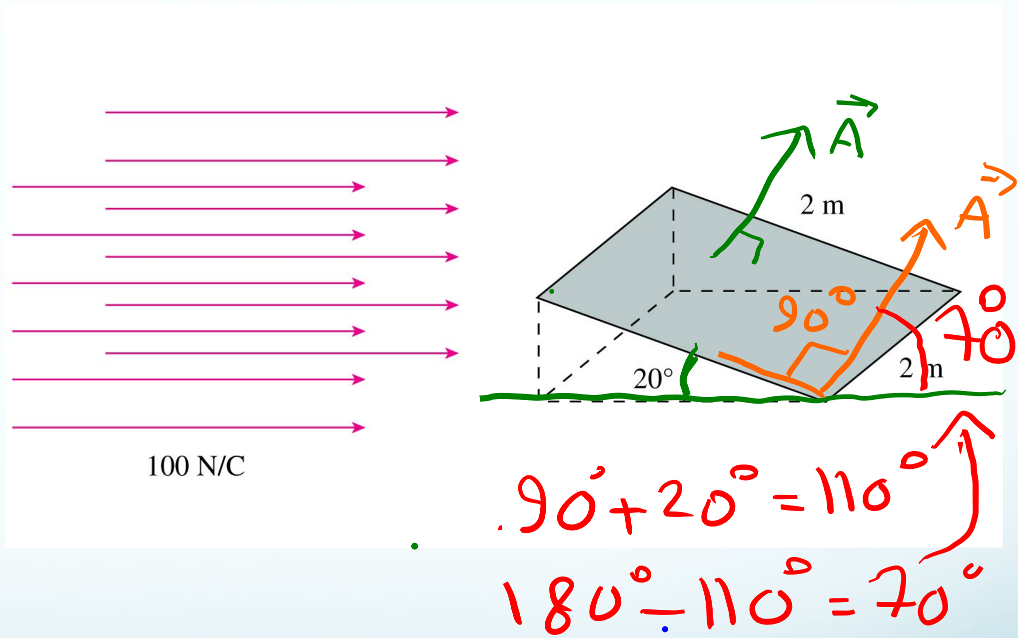
A. 0.

B. 400cos20° N m$^2$/C.

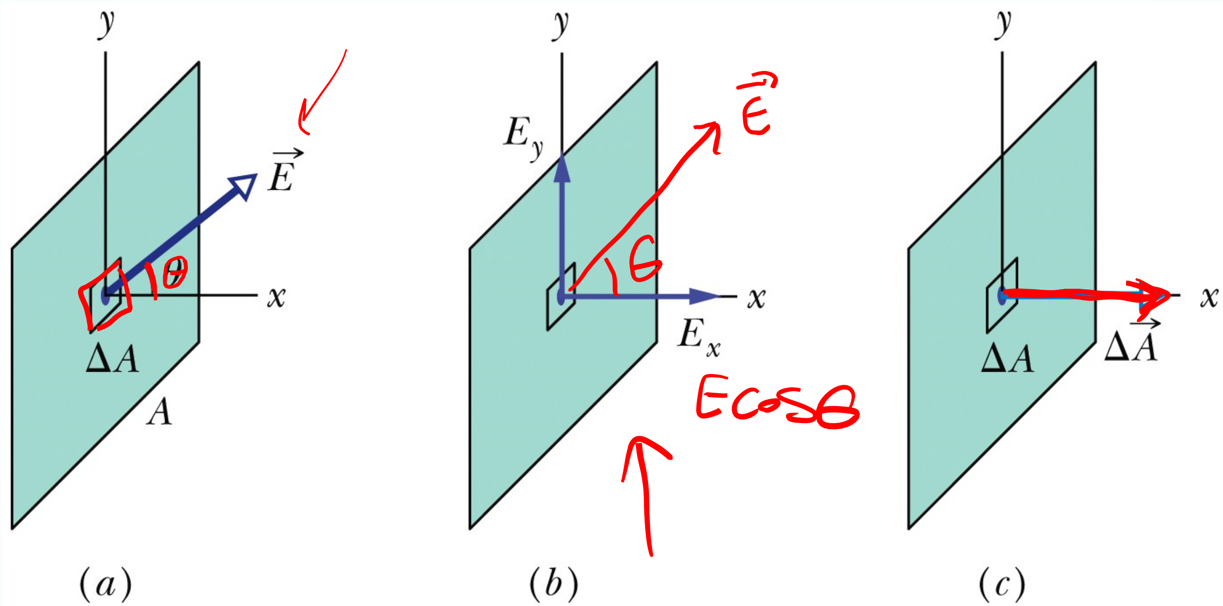
C. 400cos70° N m$^2$/C.

D. 400 N m$^2$/C.

E. Some other value.



# Electric flux: Flat surface, uniform field






$$\Phi_e = \vec{E}\cdot\vec{A} = EA\cos\theta$$

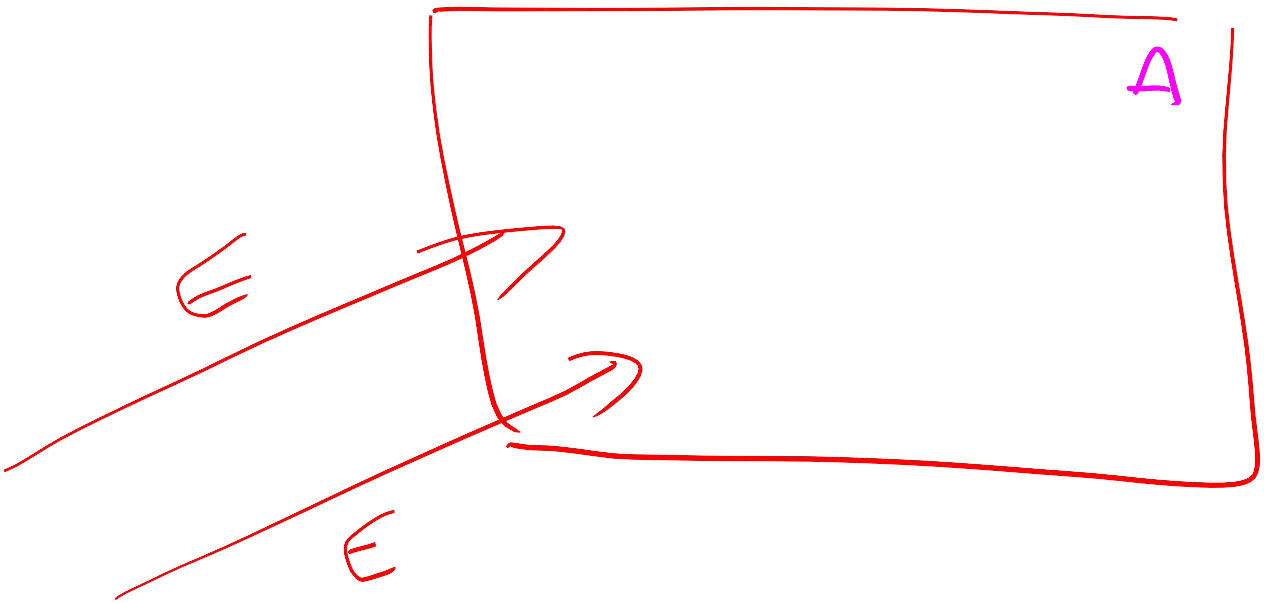
$$\rightarrow \Delta A \rightarrow \Delta\Phi = E\cos(\theta)\Delta A \rightarrow \Delta\Phi = E.\Delta A$$

SUM over $\Delta\phi$

$$\int \rightarrow \boxed{\Phi_e = \int \vec{E}.d\vec{A}} = \vec{E}.\vec{A}$$

$$\Phi_e = \int \vec{E}\cdot\vec{dA} = \vec{E}\cdot\int \vec{dA} = E\cos\theta \underbrace{\int dA}_{A}$$

$E \rightarrow$ uniform



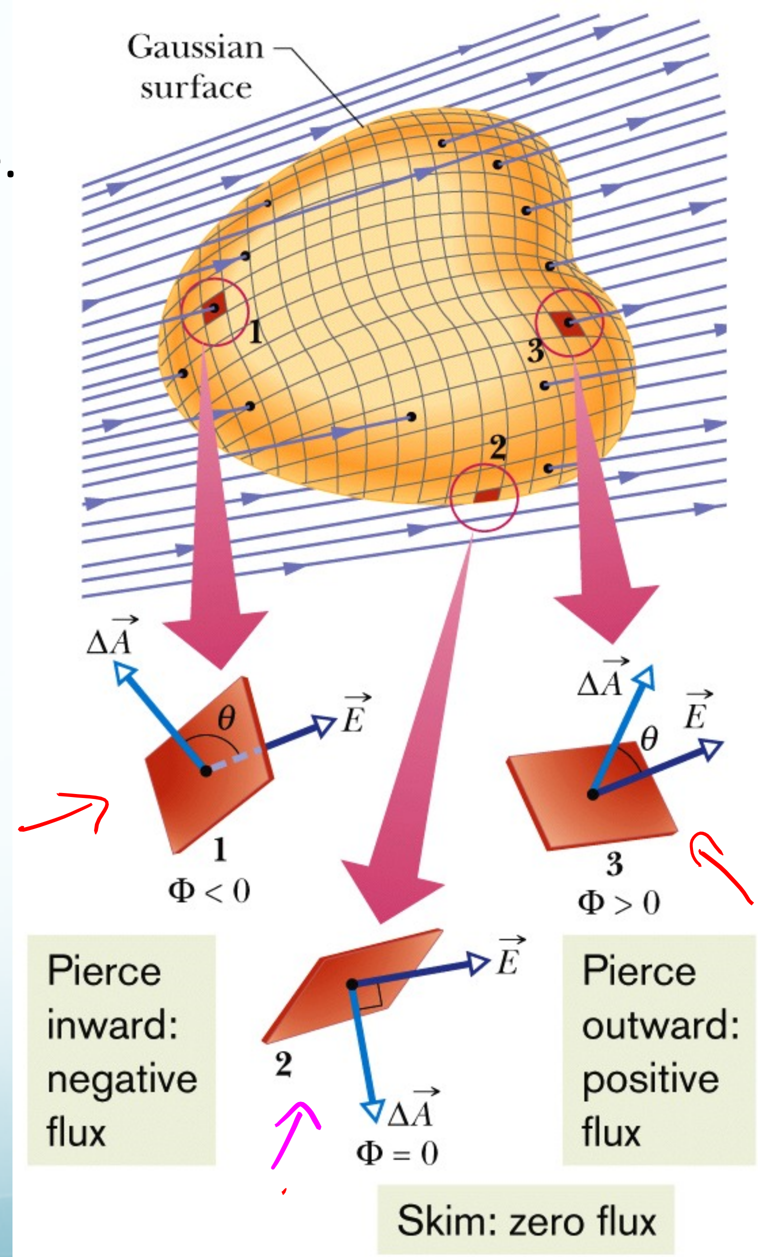
# Electric flux: Closed surface, uniform field

Total flux through a surface →
Integrating the dot product over the full surface.

$$\Phi = \int \vec{E} \cdot d\vec{A} \quad \text{(total flux).}$$

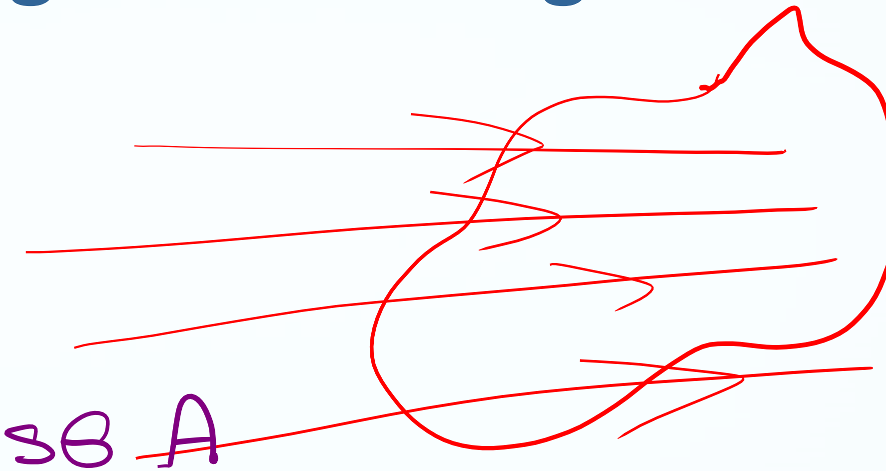
The **net flux** through a closed surface (which is used in Gauss' law) →

$$\Phi = \oint \vec{E} \cdot d\vec{A} \quad \text{(net flux).}$$



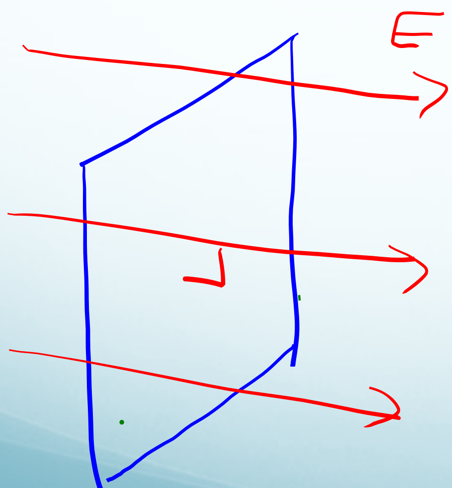
# Tactics: Evaluating surface integrals

For uniform E:

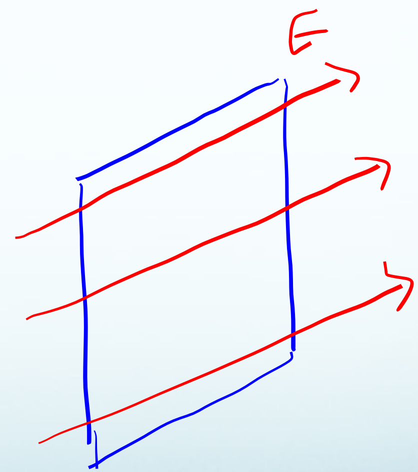


$$\Phi_e = \oint \vec{E}.d\vec{A} = E \cdot \oint d\vec{A}$$

$$= E\cos\theta \oint dA = E\cos\theta\, \underline{\underline{A}}$$




$E \parallel dA$

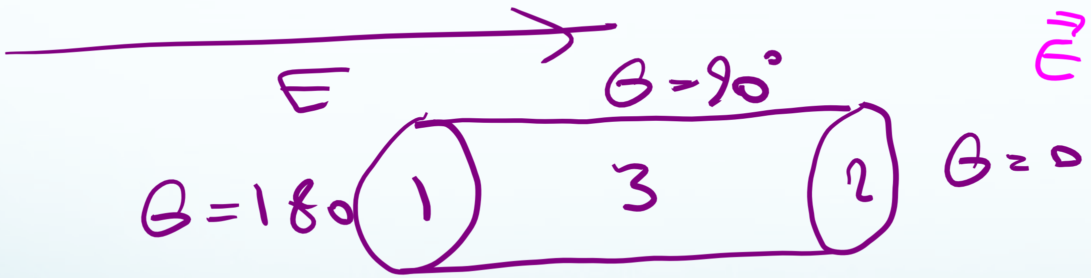



$E \perp dA$

# Finding the flux through a closed surface

1. Divide the closed surface into pieces that are tangent to the electric field, perpendicular to the electric field, or with a specific angle
2. Evaluate the surface integral

$\theta \rightarrow$ angle between $\vec{E}$ and $\vec{dA}$

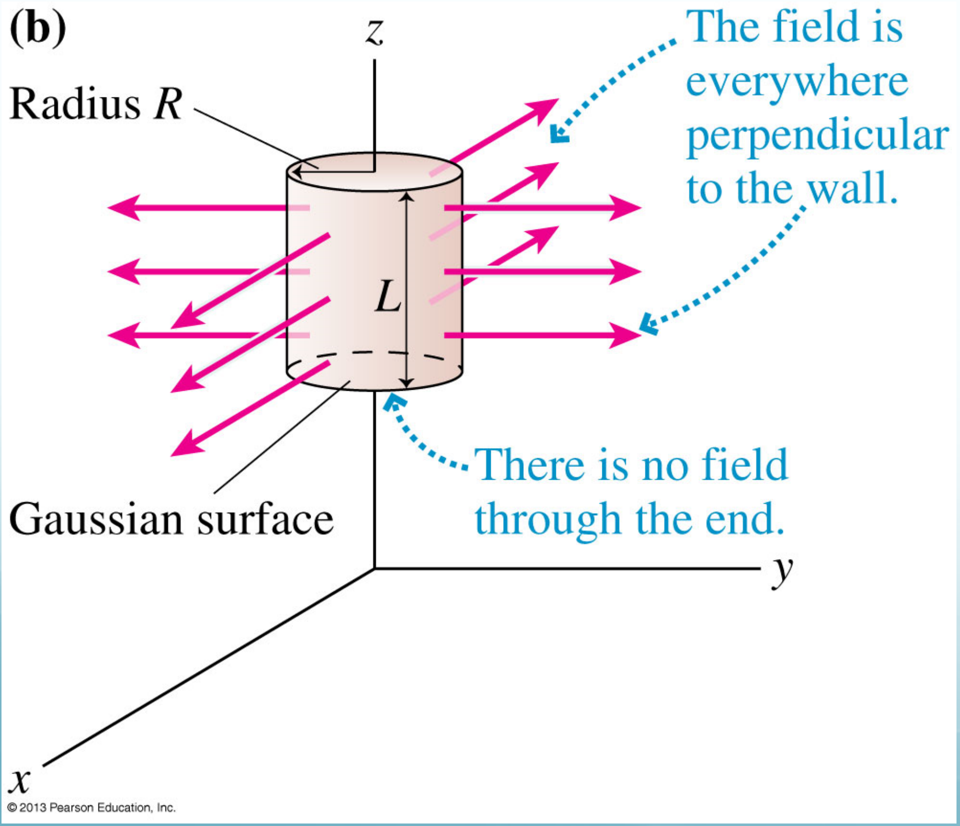



$$\Phi = \oint \vec{E} \cdot \vec{dA}$$

# The Electric Flux through a Closed Surface

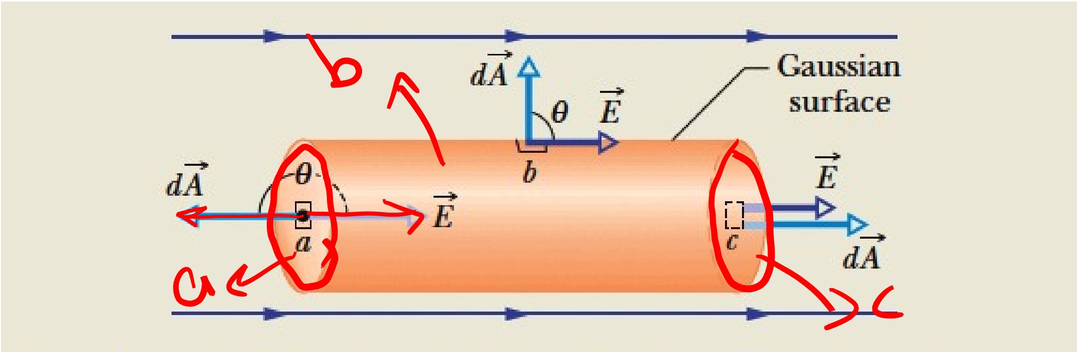
$$\Phi_e = \oint \vec{E} \cdot d\vec{A}$$

The area vector dA of a closed surface is always defined to point toward the outside



# Electric Flux: Example

Figure 23-6 shows a Gaussian surface in the form of a closed cylinder (a Gaussian cylinder or G-cylinder) of radius $R$. It lies in a uniform electric field $\vec{E}$ with the cylinder's central axis (along the length of the cylinder) parallel to the field. What is the net flux $\Phi$ of the electric field through the cylinder?



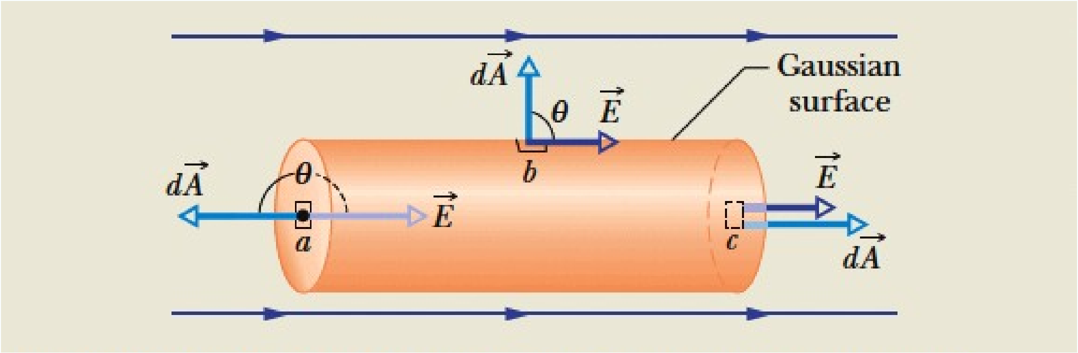

$$\Phi_e = \oint \vec{E} \cdot d\vec{A}$$

$$= \oint E\, dA \cos\theta$$

a: $\theta = 180^\circ$

b: $\theta = 90^\circ$

c: $\theta = 0^\circ$




$$\Phi_e = \oint \vec{E}\cdot d\vec{A} = \oint_a \vec{E}\cdot dA + \oint_b \vec{E}\cdot d\vec{A} + \oint_c E\cdot d\vec{A}$$

$$E\cos\theta \oint d\vec{A} = -E\oint dA \qquad E\cos\theta \oint dA = 0 \qquad E\cos\theta \int d\vec{A} = E\oint dA$$

$$\rightarrow \Phi_e = -E\oint dA + 0 + E\oint dA = 0$$

This section we talked about:

Chapter 23.1

*See you on Wednesday*