Last time

- Potential energy of a collection of point charges
- Electric potential due to a point charge
- Electric potential due to a collection of point charges
- Electric potential due to an arbitrary charge distribution

This time

- Review of electrostatic equations
- Obtaining electric field from electric potential
- Equipotential surface

Vector quantities

$$\vec{F}_{qq'} = \frac{1}{4\pi\varepsilon_0}\frac{qq'}{r^2}\hat{r}$$

$$\vec{E} = \frac{\vec{F}_{qq'}}{q'} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$$

$$\vec{F} = q\vec{E}$$

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r^2}\hat{r}$$

Scalar quantities

$$U_{q'+q} = \frac{1}{4\pi\varepsilon_0}\frac{qq'}{r}$$

$$V = \frac{U_{q'+q}}{q'} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

$$U = qV$$

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r}$$

$$U_b - U_a = -q_0\int_a^b \vec{E}\bullet d\vec{l}$$

$$V_b - V_a = -\int_a^b \vec{E}\bullet d\vec{l} = \int_b^a \vec{E}\bullet d\vec{l}$$

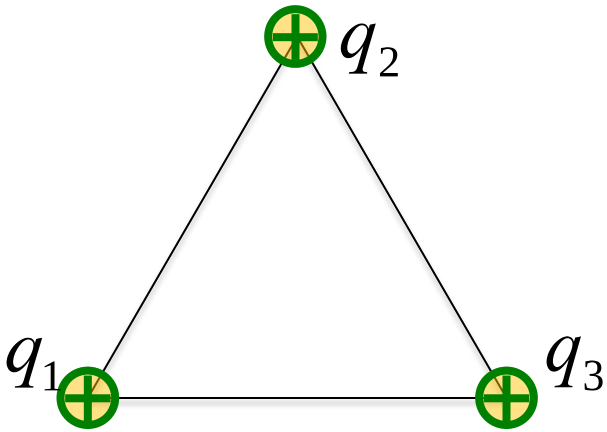
Example 1

Three charges q_1 = 1.0 nC, q_2 = –2.0 nC, and q_3 = 3.0 nC are fixed in an equilateral triangle of side length d = 5.0 cm. What is the electric potential energy of this configuration?

$$\boxed{\varepsilon_0 = 8.85 \times 10^{-12}\ \mathrm{C^2 / N\ m^2}}$$

$$\boxed{k_e = 8.99 \times 10^{+9}\ \mathrm{N\ m^2 / C^2}}$$

$$U_{ij} = \frac{1}{4\pi\varepsilon_0} \frac{q_i q_j}{r_{ij}}$$



$$U_{12} = -3.596 \times 10^{-7}\ \mathrm{J}$$

$$U_{23} = -10.79 \times 10^{-7}\ \mathrm{J}$$

$$U_{13} = +5.394 \times 10^{-7}\ \mathrm{J}$$

$$U = U_{12} + U_{13} + U_{23}$$

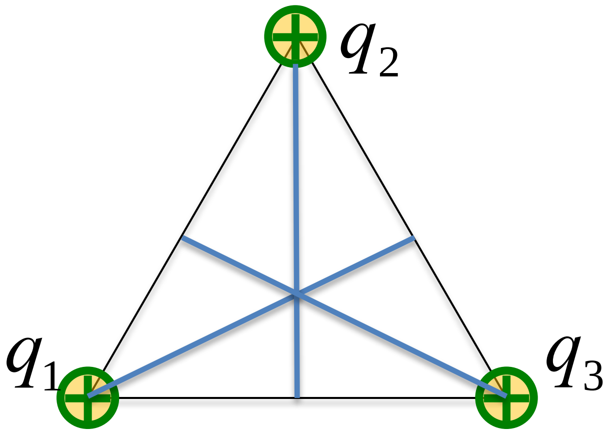
Answer: U = –9.0 x 10^{-7} J

Example 2

Three charges q_1 = 1.0 nC, q_2 = –2.0 nC, and q_3 = 3.0 nC are fixed in an equilateral triangle of side length d = 5.0 cm. What is the electric potential at the center of the triangle?

$$\frac{1}{4\pi\varepsilon_0} = 8.99\times10^9 \text{ Nm}^2/\text{C}^2$$

$$V_i = \frac{1}{4\pi\varepsilon_0}\frac{q_i}{r_i}$$



$$V_1 = +310 \text{ V}$$
$$V_2 = -620 \text{ V}$$
$$V_3 = +930 \text{ V}$$

$$r_1 = r_2 = r_3 = \frac{.05}{\sqrt{3}}\text{m} = .029 \text{ m}$$

$$V = V_1 + V_2 + V_3$$

Answer: V = 620 V

Obtaining $\vec{E}$ from V

$$V_f - V_i = -\int_i^f \vec{E} \bullet d\vec{l} = \int_i^f -\vec{E} \bullet d\vec{l} = \int_i^f dV$$

$$dV = -\vec{E} \bullet d\vec{l}$$
$$= -\left(E_x dx + E_y dy + E_z dz\right)$$

$$\frac{\partial V}{\partial x} = -E_x \qquad \left.\frac{\partial V(x,y,z)}{\partial x}\right|_{y\&z=\text{constants}} = -E_x$$

$$\frac{\partial V}{\partial y} = -E_y \qquad \left.\frac{\partial V(x,y,z)}{\partial y}\right|_{x\&z=\text{constants}} = -E_y$$

$$\frac{\partial V}{\partial z} = -E_z \qquad \left.\frac{\partial V(x,y,z)}{\partial z}\right|_{x\&y=\text{constants}} = -E_z$$

Moving in the direction of the electrical field

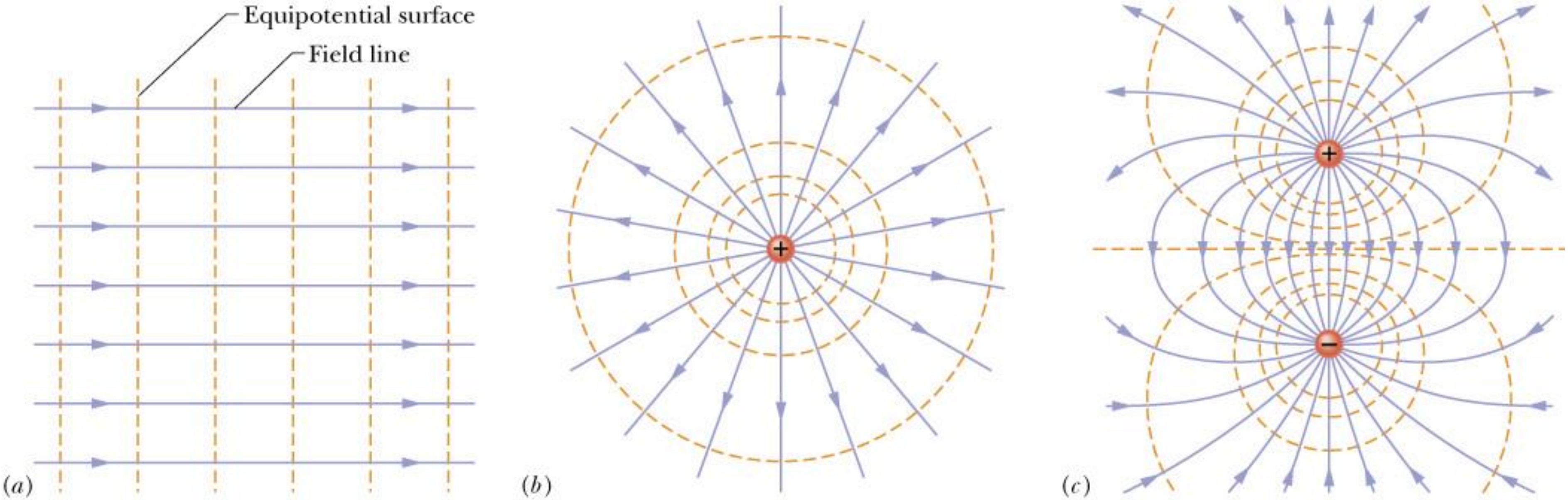
$$dV = -\vec{E} \bullet d\vec{l} = -Edl\cos 0 = -Edl < 0$$

Moving in the opposite direction of the electrical field

$$dV = -\vec{E} \bullet d\vec{l} = -Edl\cos 180 = Edl > 0$$

Moving with the electrical field decreases the electrical potential. Moving against the field increases it.

What happens to the electric potential when the displacement vector is perpendicular to the electric field?

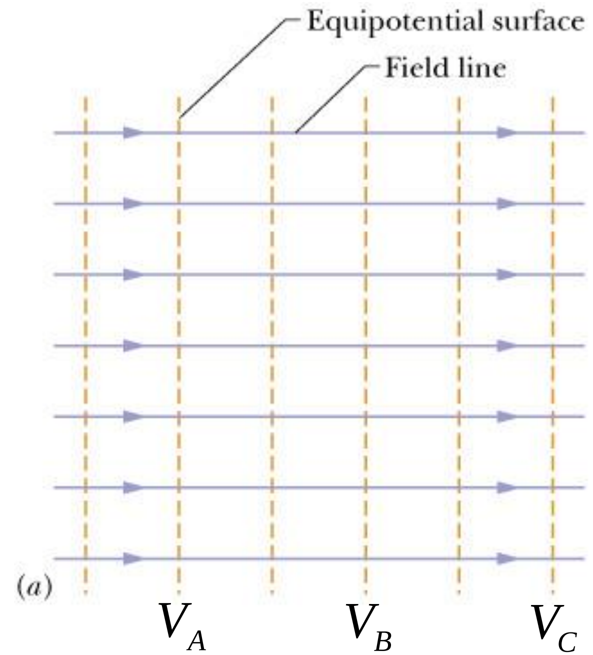


$$V_f - V_i = -\int_i^f \vec{E} \bullet d\vec{l} = -\int_i^f Edl\cos 90 = 0$$

Electric potential doesn't change. A surface with this property is called an **equipotential surface**.

TopHat Question

How do the values of electric potential compare at the three surfaces shown on the figure?



A. $V_A=V_B=V_C$

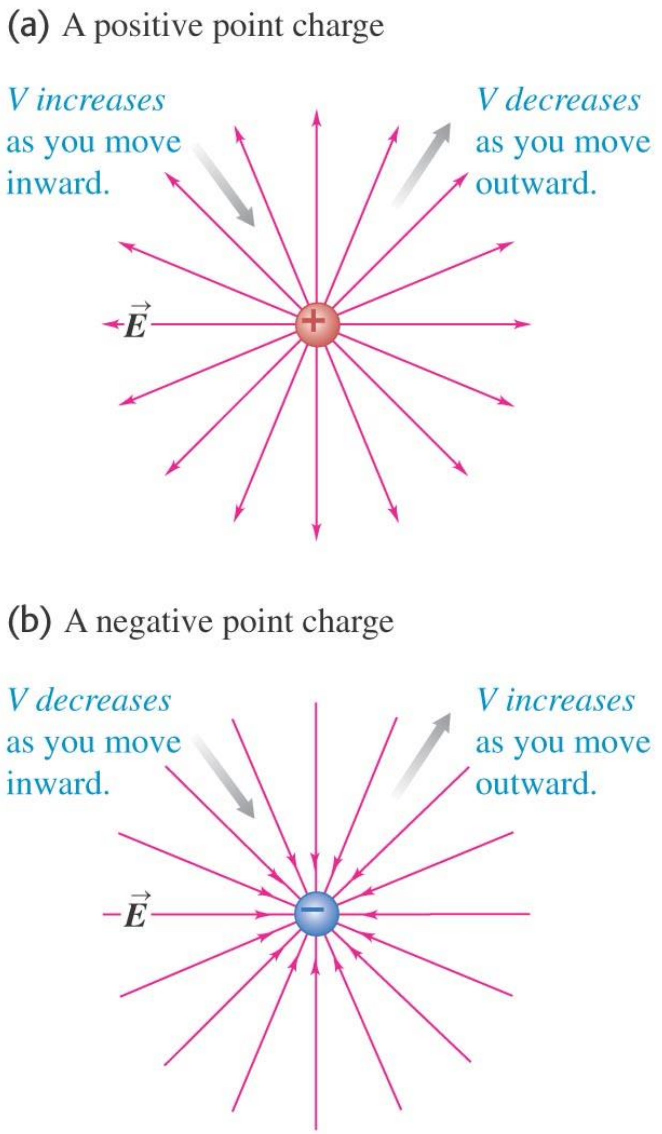
B. $V_A<V_B<V_C$

C. $V_A>V_B>V_C$

D. Need More information

The electrical potential

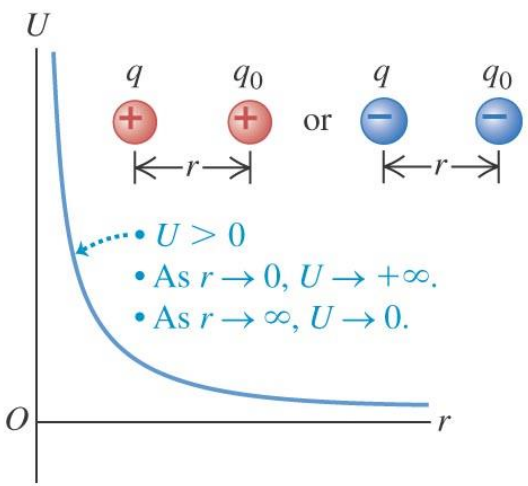
Moving with the electrical field decreases the electrical potential. Moving against the field increases it.



Potential energy curves—PE versus r

- Graphically, the potential energy between like charges increases sharply to positive (repulsive) values as the charges become close.

(a) q and q_0 have the same sign.



- Unlike charges have potential energy becoming sharply negative as they become close (attractive).

(b) q and q_0 have opposite signs.