Monday April 3, 2017

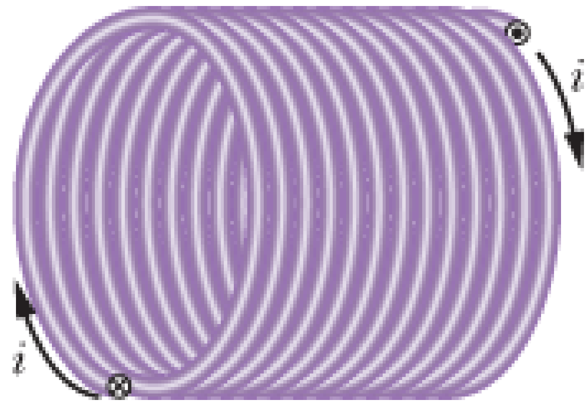
Last time:

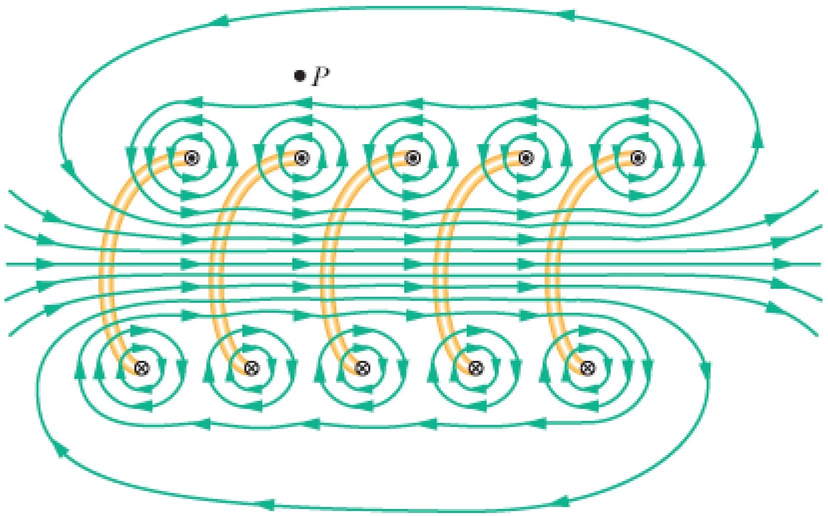
- Applying Ampère’s Law:
  - Magnetic field of a long wire (inside and outside)
  - Magnetic field of solenoid
- Applying the Biot-Savart Law: Circular arc of current (take-home example)

Today:

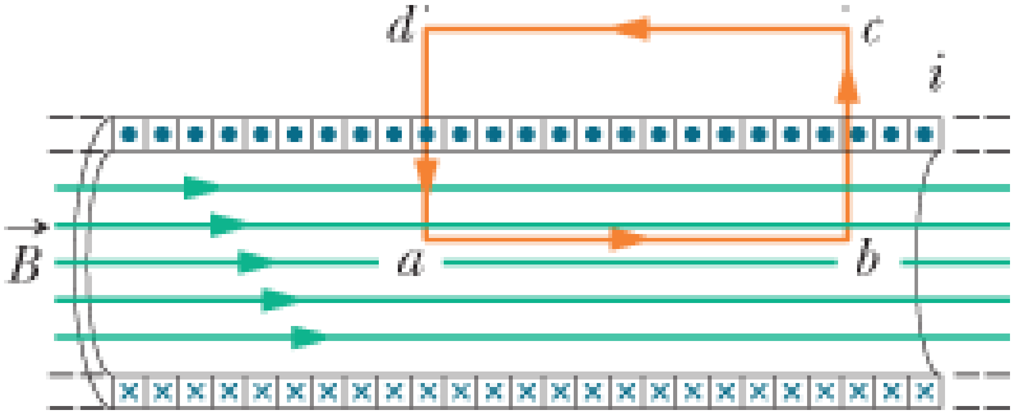
- Applying Ampère’s Law:
  - Magnetic field of solenoid and toroid
- Faraday’s Law of Induction
- Non-conservative electric fields
- Motional emf

Magnetic field of solenoid

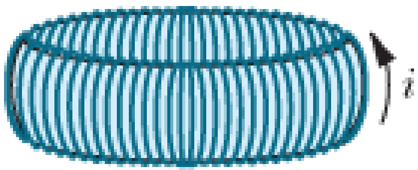




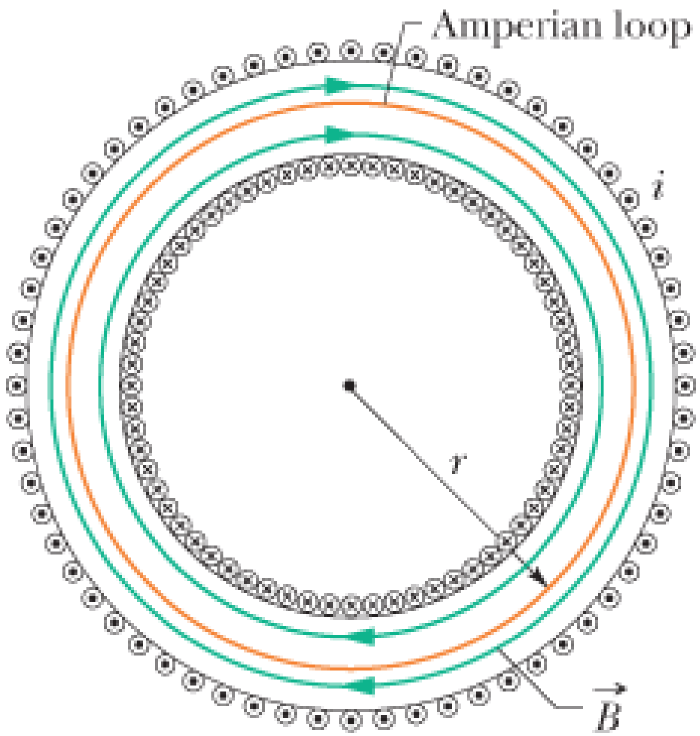
$$B = \mu_0 i n \quad \text{(ideal solenoid)}$$



Magnetic field of toroid



(a)



(b)

$$B = \frac{\mu_0 i N}{2\pi}\frac{1}{r} \quad \text{(toroid)}$$

Faraday's Law of Induction

Electrostatics: E-field from motionless charges
Magnetostatics: B-field from charges in motion

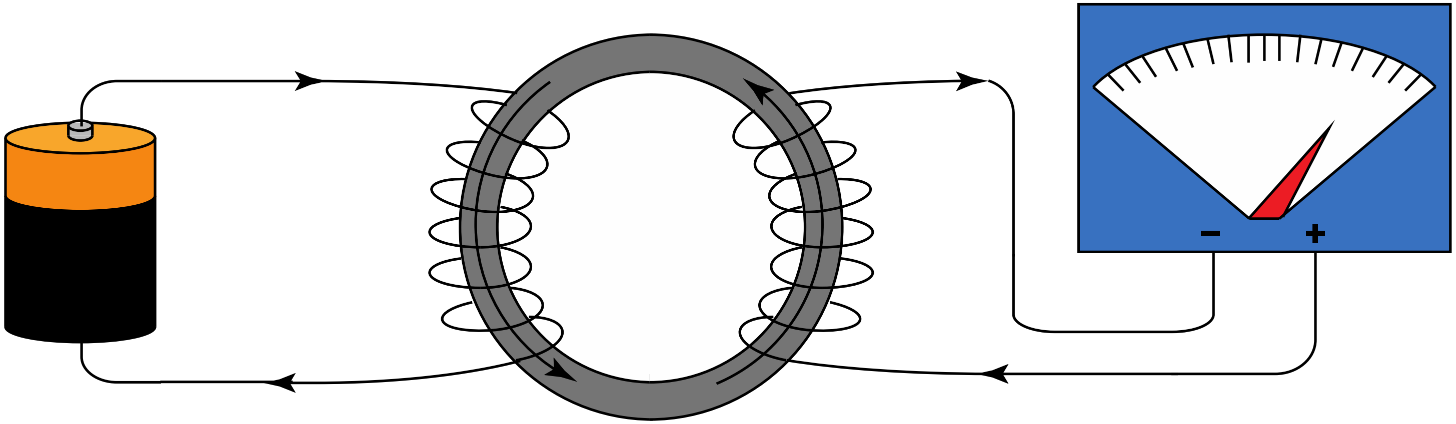
Changing electric fields (moving charges) create magnetic fields. Is the opposite true?

YES!

$$\boxed{\varepsilon = -\frac{d\Phi_B}{dt}}$$

i.e., A changing magnetic flux creates an induced EMF.

Faraday’s Initial Experiment



Faraday discovered that there is an **induced EMF** in the secondary circuit given by

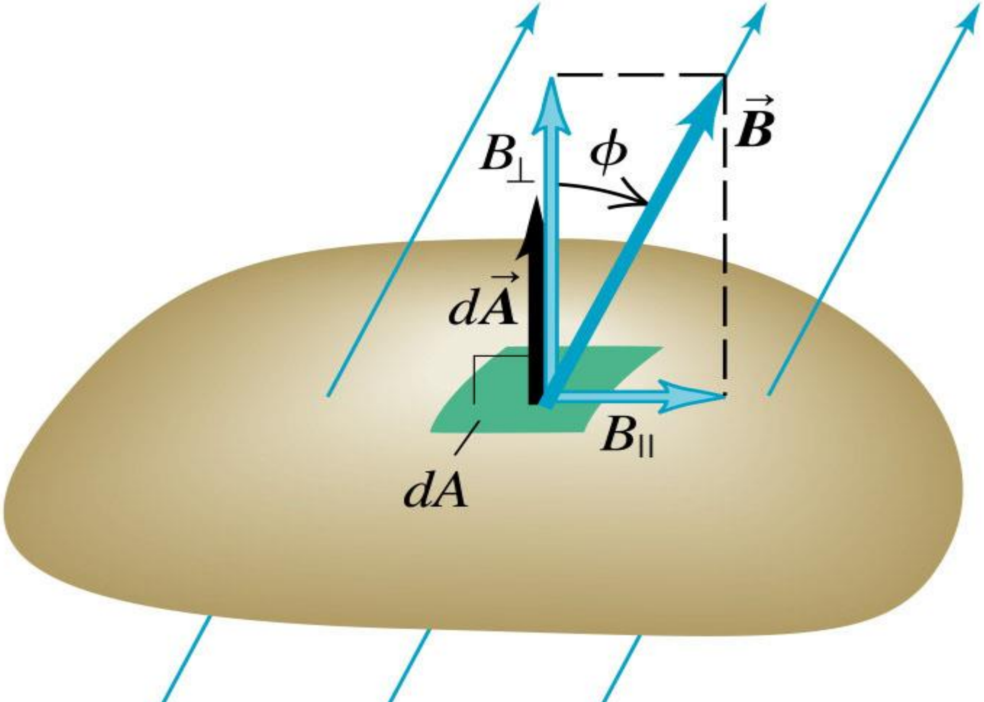
$$\varepsilon = -\frac{d\Phi_B}{dt}$$

This is a new generalized law called **Faraday's Law**.

Recall the definition of magnetic flux:

$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$

Not a closed surface!



This is valid even if Φ_B changes because of a time dependent A or angle ϕ (without changing the magnetic field)!

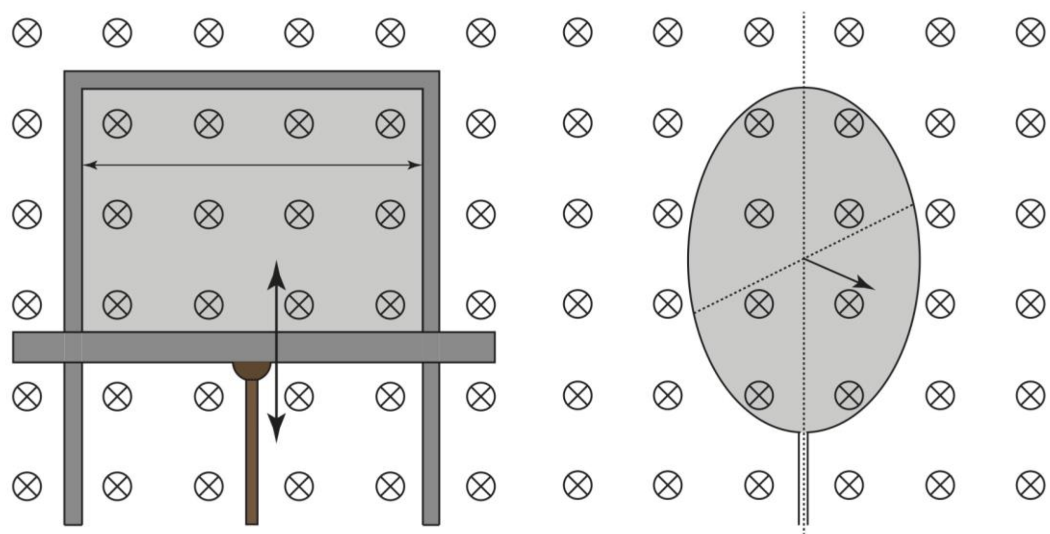
$$e = -\frac{d}{dt}\left(BA\cos f\right)$$

→ 3 possible terms

$$e = -\frac{dB}{dt}A\cos f - \frac{dA}{dt}B\cos f + \frac{df}{dt}BA\sin f$$

From Maxwell Eq.

$$-\frac{d\vec{B}}{dt} = \nabla \times \vec{E}$$



Top Hat Question

Top Hat Question

A square loop of wire with a resistance of 1 Ω is moving with a constant velocity of 1 m/s through a uniform magnetic field as shown. What is the current induced in the loop? Pick the closest answer
(Note: 1 Ampere = 1 Coulomb/sec)

The magnetic flux through the loop is not changing, so there is no induced emf and hence no induced current

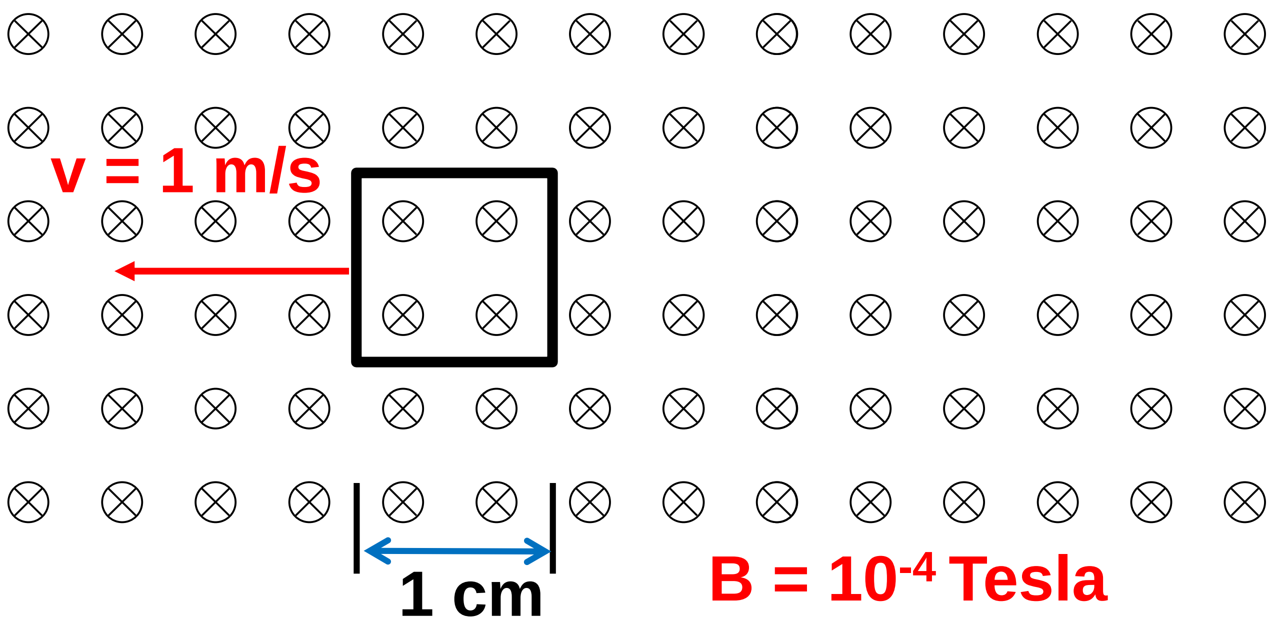
A. 0 A

B. 0.001 A

C. 0.01 A

D. 0.1 A

E. 1 A

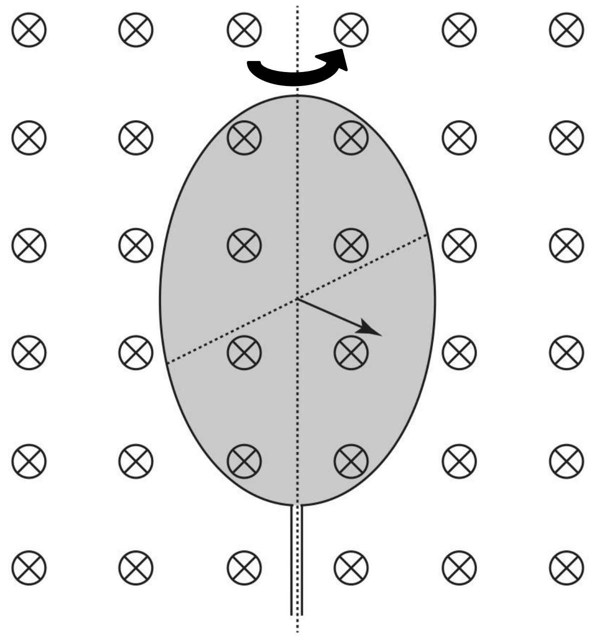


Top Hat Question

A loop of wire is spinning rapidly about a stationary vertical axis in a uniform B-field. Is there a current (or EMF) induced in the loop?

A. Yes, a DC current is induced
B. Yes, an AC (time varying) current is induced
C. The B-field is not changing, so no currents are induced

In this case, the flux through the loop is changing with time because of the **B•A** term, so there will be an induced current (or emf) in the loop. The normal vector is changing direction so half the time the flux is positive and half the time it is negative: i.e. an AC current is induced.



Top Hat Question

An oval shaped loop is spun around an axis pointing out of the page passing through the center of the loop. Is there a current (or EMF) induced in the loop?

A: Yes, there is

B: No, there is not