Last time

- Non-ideal batteries and electromotive force
- Kirchhoff's loop rule with capacitors
- Capacitors in parallel
- More complicated capacitor circuit (tutorial)

This time

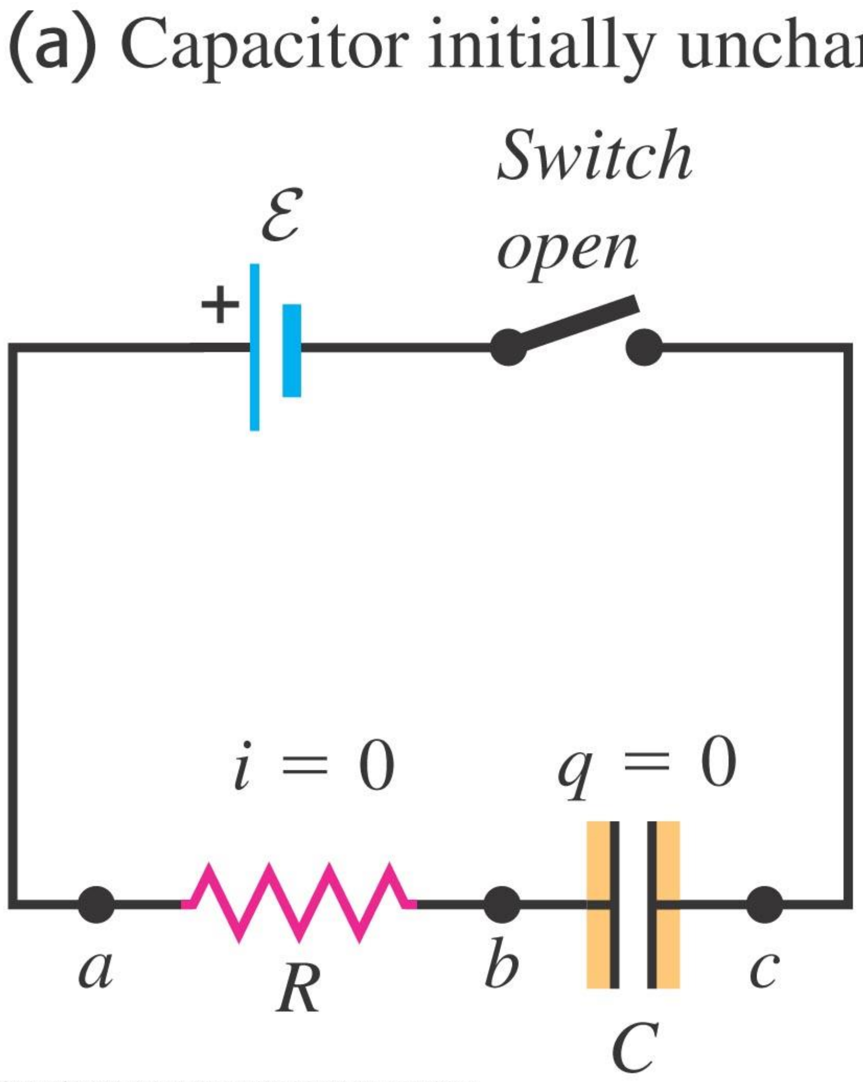
- Non-steady state cases (time dependent processes)
- Charging a capacitor
- Time constant of a capacitor
- Discharging a capacitor

Non-steady state cases

R-C circuits

https://docs.google.com/forms/d/1V90A-uxudcBxZAW4BMw6sZ2H9lKxFYATGpQcbFF_KOw/viewform

Charging a capacitor



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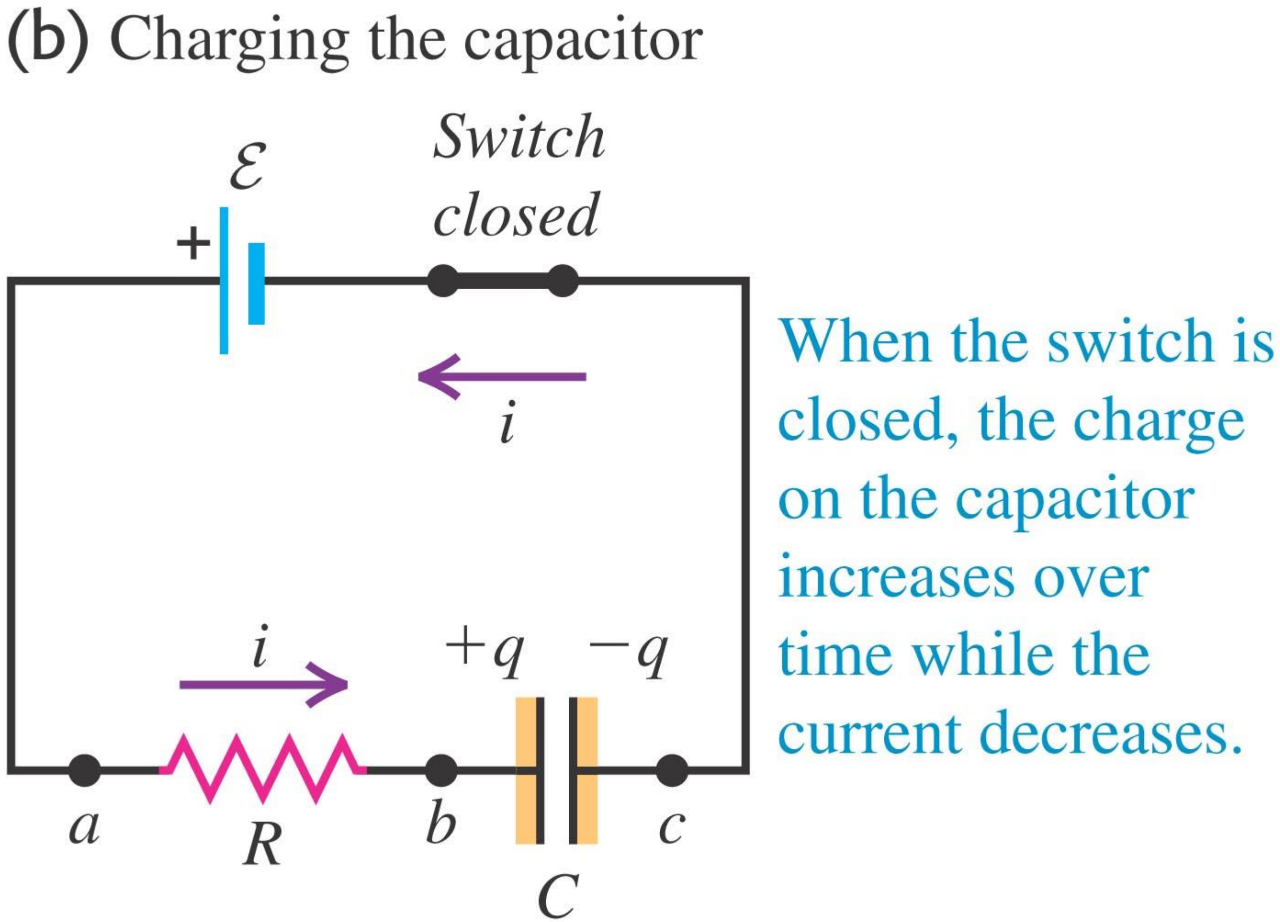
R could be the internal resistance of the battery, resistance of the connecting wires, an actual resistance in the circuit or combination of all the above.

Charging a capacitor



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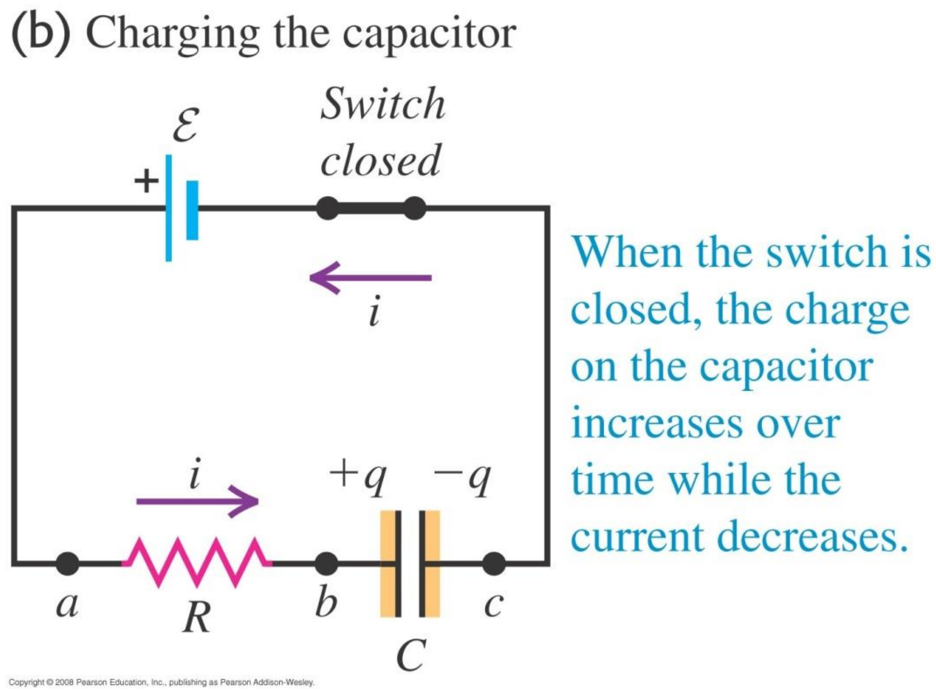
Coulomb repulsion doesn't allow for q to increase on the + plate indefinitely. As more charge is stored on the plates the coulomb repulsion gets larger resulting in smaller current in the circuit.



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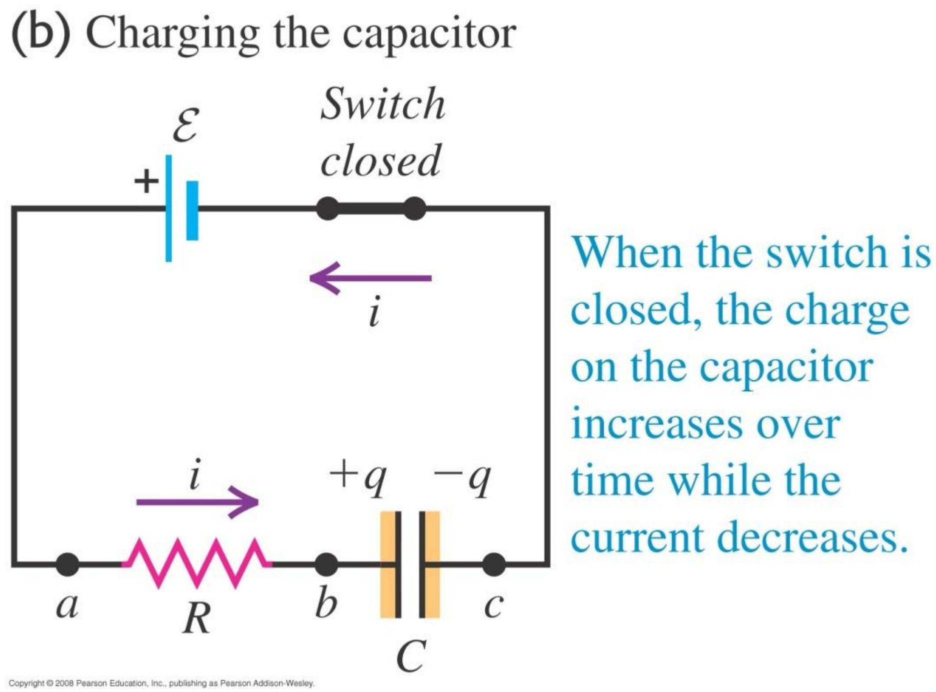
$t = 0$ at the time swithed is closed. $q = 0$ $V_C = \dfrac{q}{C} = 0$

Potential drop across the capacitor is zero. Capacitor acts momentarily as a short (not in the circuit). This means that the potential drop is entirely across the resistor. This is when the current in the circuit assumes its maximum value.

$$i = i_0 = \frac{\varepsilon}{R}$$

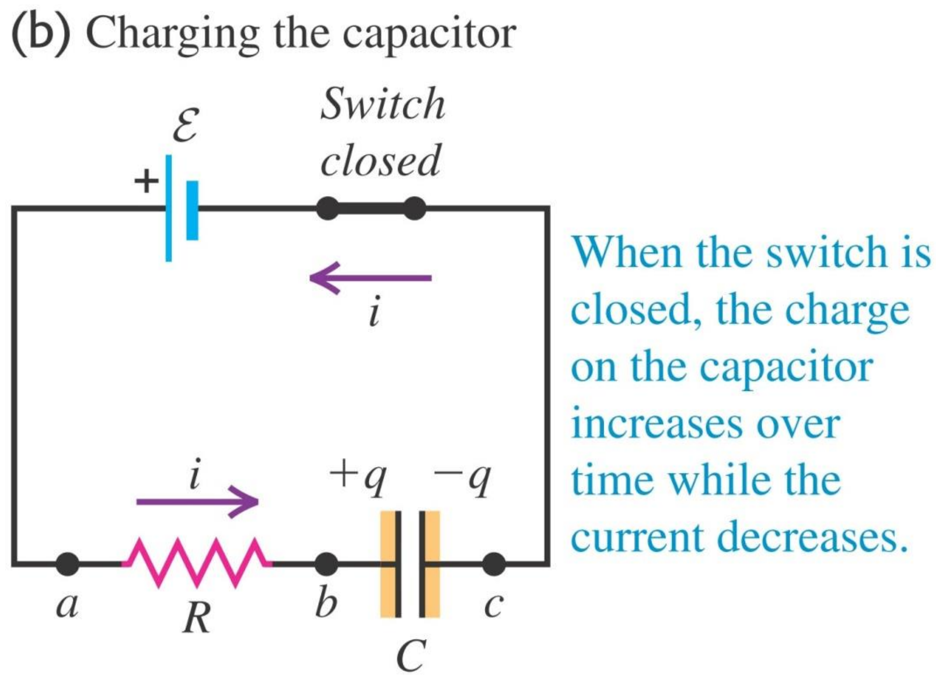


What happens after a long time? $t = \infty$

Capacitor is fully charged. No more charge can be displaced by the battery. Capacitor acts as an open switch. The current in the circuit is zero. Potential drop across the resistor is also zero.

$$q = Q_f \qquad V_C = \varepsilon = \frac{Q_f}{C} \Rightarrow Q_f = C\varepsilon$$

$$i_f = 0$$



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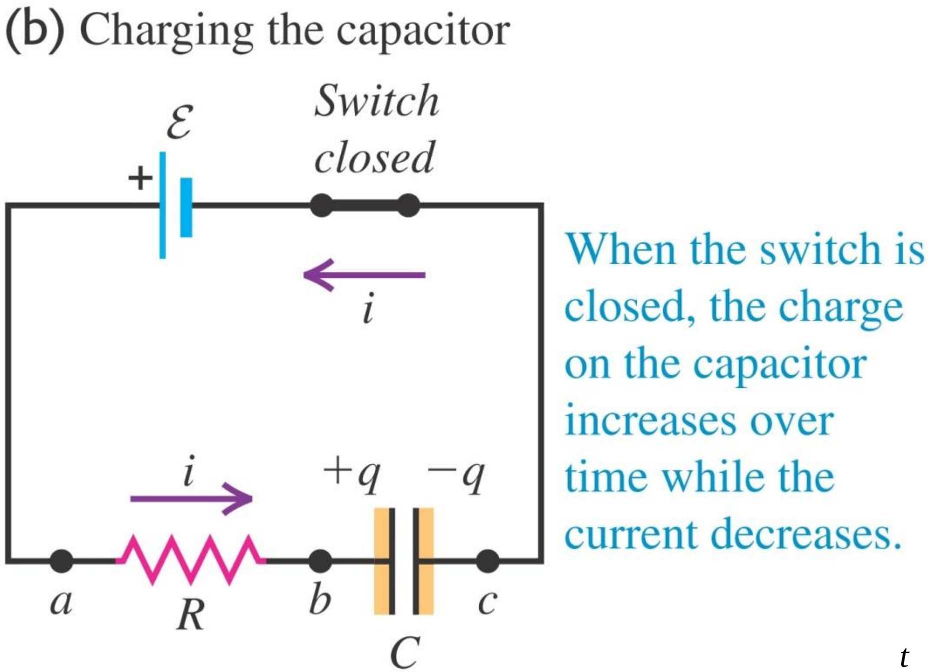
$$t = \text{intermediate}$$

$$\varepsilon - iR - \frac{q}{C} = 0$$

Emf is shared by voltage drop across the resistor and voltage drop across the capacitor.

$$\frac{d\varepsilon}{dt} - R\frac{di}{dt} - \frac{1}{C}\frac{dq}{dt} = 0 \Rightarrow -R\frac{di}{dt} - \frac{1}{C}\frac{dq}{dt} = 0 \Rightarrow \frac{di}{dt} = -\frac{i}{RC}$$

Current as a function of time



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$$\frac{di}{i} = -\frac{dt}{RC}$$

$$\int_0^t \frac{di}{i} = -\frac{1}{RC}\int_0^t dt$$

$$\ln i\Big|_0^t = -t/RC \Rightarrow \ln i - \ln i_0 = -t/RC \Rightarrow \ln\frac{i}{i_0} = -t/RC$$

$$i = i_0 e^{-t/RC} = \frac{\mathcal{E}}{R} e^{-t/RC}$$

Charge as a function of time

$$i = \frac{\varepsilon}{R} e^{-t/RC}$$

$$\frac{dq}{dt} = \frac{\varepsilon}{R} e^{-t/RC}$$

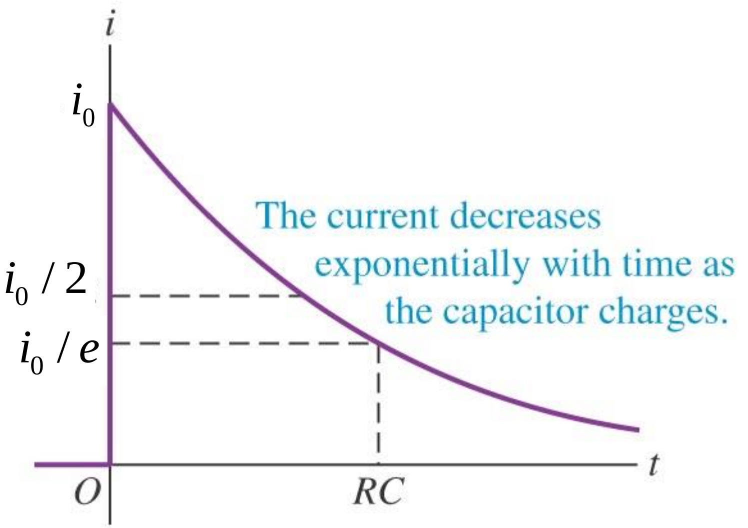
$$dq = \frac{\varepsilon}{R} e^{-t/RC} dt$$

$$\int_0^q dq = \frac{\varepsilon}{R} \int_0^t e^{-t/RC} dt$$

$$q = -\frac{\varepsilon RC}{R} e^{-t/RC} \Big|_0^t$$

$$q = \varepsilon C\left(1 - e^{-t/RC}\right) = Q_f\left(1 - e^{-t/RC}\right)$$

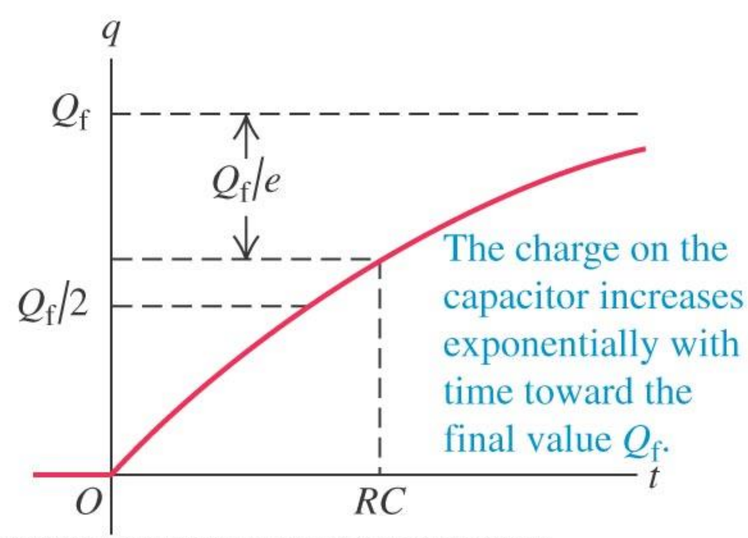
(a) Graph of current versus time for a charging capacitor



$$i = i_0 e^{-t/RC}$$

time	q	i
RC	$0.63Q_f$	$0.36i_0$
$2RC$	$0.86Q_f$	$0.13i_0$
$3RC$	$0.95Q_f$	$0.05i_0$
$4RC$	$0.98Q_f$	$0.02i_0$
$5RC$	$0.99Q_f$	$0.01i_0$

(b) Graph of capacitor charge versus time for a charging capacitor



$$q = \varepsilon C\left(1 - e^{-t/RC}\right) = Q_f\left(1 - e^{-t/RC}\right)$$

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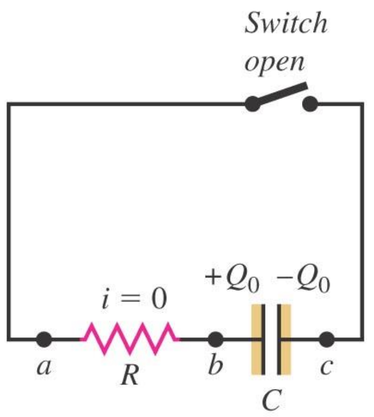
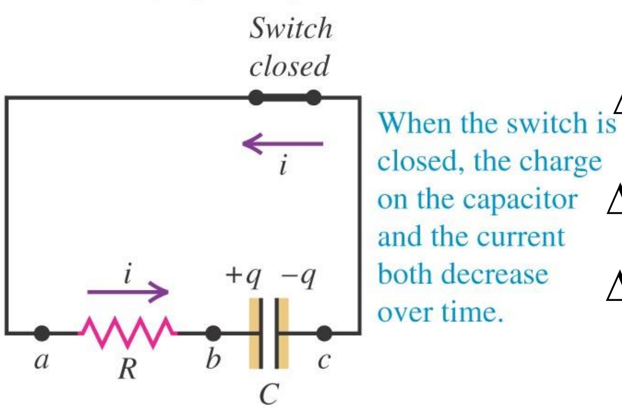
Discharging a capacitor

(a) Capacitor initially charged



(b) Discharging the capacitor



$t = 0$ at the time switch is closed.

$$q = Q_0 = \varepsilon C$$

$$t = \text{intermediate}$$

$$\Delta V_{ab} + \Delta V_{bc} + \Delta V_{ca} = 0$$

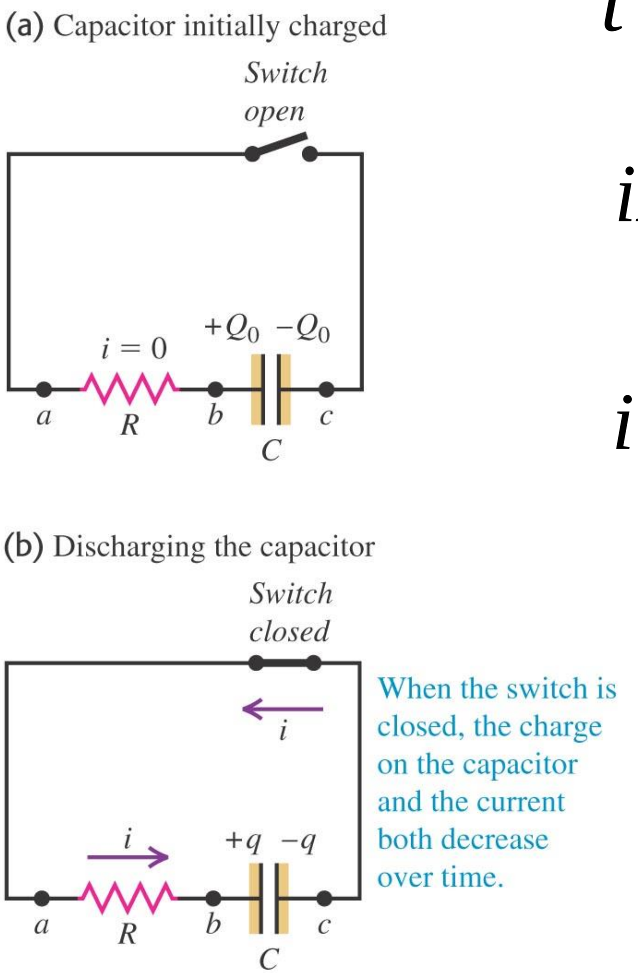
$\Delta V_{ab} = V_b - V_a = iR > 0$ Since b is connetcted to the plate with +q

$\Delta V_{bc} = V_c - V_b = -\dfrac{q}{C} < 0$ Since c is connetcted to the plate with -q

$\Delta V_{ca} = V_a - V_c = 0$ Since c is connetec to a by conneting wires

$$iR - \frac{q}{C} = 0$$

Discharging a capacitor



$$t = \text{intermediate}$$

$$iR - \frac{q}{C} = 0$$

$$i = -\frac{dq}{dt}$$

Negative sign is due to the fact that current is decreasing as a function of time.

$$-R\frac{dq}{dt} - \frac{q}{C} = 0$$

$$\frac{dq}{q} = -\frac{dt}{RC}$$

Discharging a capacitor

$$\frac{dq}{q} = -\frac{dt}{RC}$$

$$\int_{Q_0}^{q} \frac{dq}{q} = -\int_{0}^{t} \frac{dt}{RC}$$

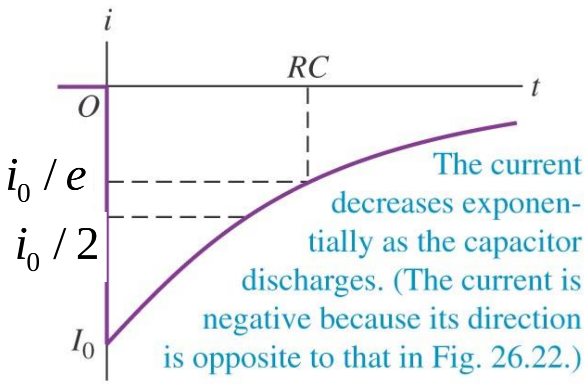
$$\ln\frac{q}{Q_0} = -\frac{t}{RC}$$

$$\frac{q}{Q_0} = e^{-\frac{t}{RC}} \Rightarrow q = Q_0 e^{-\frac{t}{RC}}$$

$$\frac{dq}{dt} = \frac{Q_0}{RC} e^{-\frac{t}{RC}} \qquad -i = i_0 e^{-\frac{t}{RC}} \qquad i = -i_0 e^{-\frac{t}{RC}}$$

Discharging a capacitor

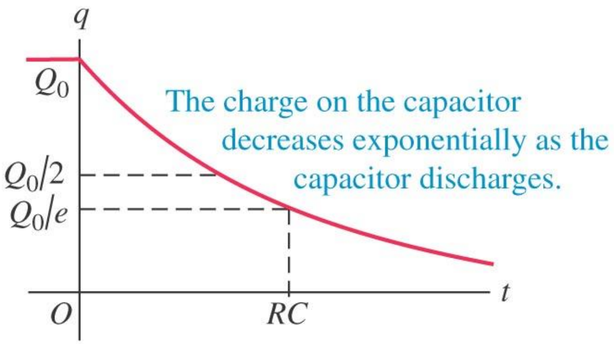
(a) Graph of current versus time for a discharging capacitor



$$q = Q_0 e^{-\frac{t}{RC}}$$

$$Q_0 = \varepsilon C$$

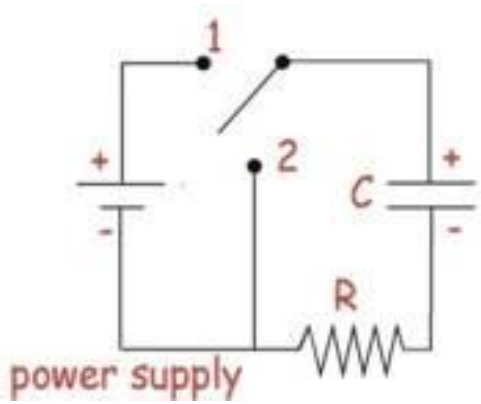
(b) Graph of capacitor charge versus time for a discharging capacitor



$$i = -i_0 e^{-\frac{t}{RC}}$$

$$i_0 = \frac{Q_0}{RC}$$

Charging and discharging a capacitor

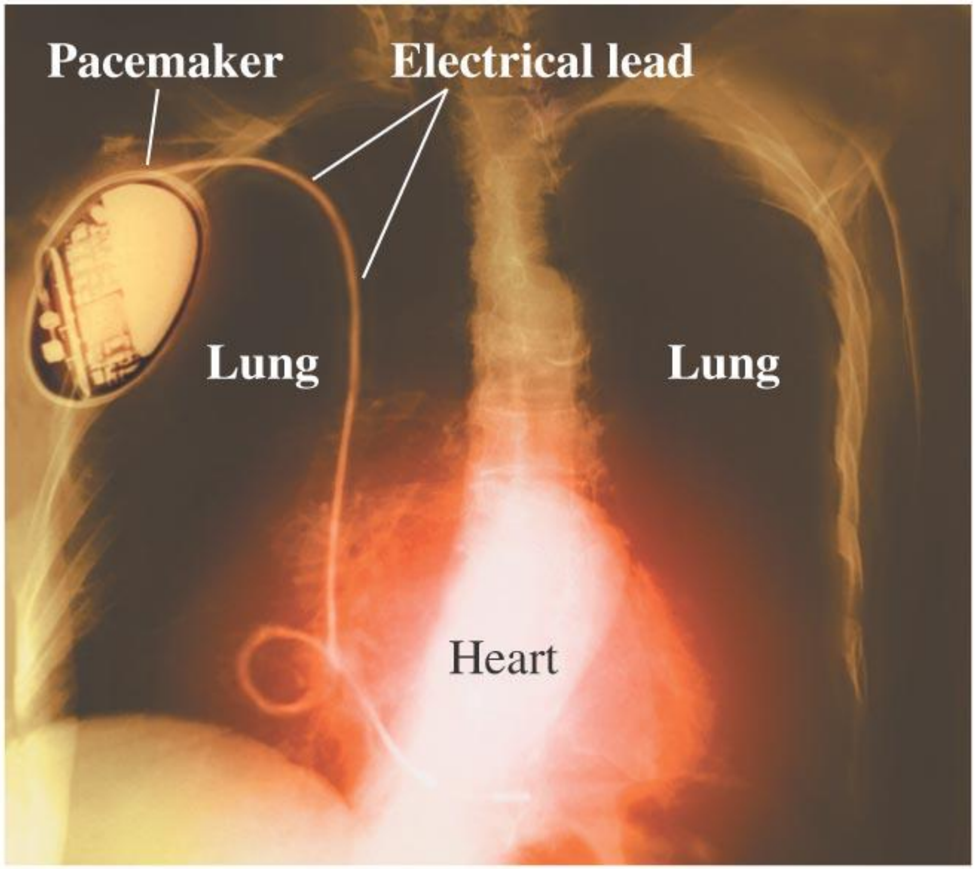


Move the switch to position 1 and wait for five time constants to charge the capacitor fully.

Move the switch to position 2 and wait for five time constants to discharge the capacitor fully.

Repeat the steps above.

The physiology of a heartbeat, the medical intervention of a pacemaker, and charging a capacitor to take a flash picture.



..helmet flasher
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