Time: 105 minutes.

University of Calgary Faculty of Science Midterm Test

PHYSICS 259 ALL LECTURE SECTIONS

February 9, 2016, 7:00-8:45 p.m.

This is a closed-book exam worth a total of 36 points. Please answer all questions. Use of the Schulich calculator or equivalent is allowed.

You need the following:

- 1. THIS QUESTION BOOKLET, which includes multiple choice exam questions and the formula sheet (last page). THIS BOOKLET WILL NOT BE HANDED IN. Make sure this question booklet contains 9 pages (including formula sheet). If you are missing any pages, get a new book from the exam supervisor.
- 2. A BUBBLE SHEET used to answer multiple choice questions. IT WILL BE HANDED IN.

IMPORTANT: START by entering your ID NUMBER, NAME, and COURSE ID on the bubble sheet. Also fill in the corresponding numbers below your ID and name. DO THIS NOW.

All answers to the multiple choice questions of PART I must be entered by marking the appropriate character (one of a, b, c, d, e) beside the question number on the bubble sheet. Make sure that you darken the entire interior of the circle that contains the character.

3. LONG ANSWER BOOKLET WITH ANSWER SHEETS which you will use to write your answers to long answer questions. THIS BOOKLET WILL BE HANDED IN.

Please enter your student ID on the first page, and your last name & initial on the second page. DO THIS NOW.

Your solutions to the long-answer problems of PART II must be written in the spaces defined by the boxes. Be sure to show all important steps in your solution and underline final answers, including units where applicable.

If you are missing anything from the above items, raise your hand and ask an exam supervisor to supply what is missing.

Please wait for the exam supervisor to signal when to start the test.

Part I: Multiple-Choice Questions (Total: 20 marks)

Enter answers to multiple choice questions on the bubble sheet. Each multiple choice question is worth one point.

- 1) A total charge Q is uniformly distributed, with surface charge density σ , over a very thin disk of radius R. The electric field at a distance d along the disk axis is given by $\vec{E} = \frac{\sigma}{2\epsilon_0} \left[1 \frac{d}{\sqrt{d^2 + R^2}} \right] \hat{n}$ where \hat{n} is the normal unit vector perpendicular to the disk. At what distance along the axis will the electric field be closest to half of its peak value $E_{max} = \frac{\sigma}{2\epsilon_0}$ if the radius is R = 40 centimetres?
 - a) d = 34.64 cm
 - **b)** d = 20.00 cm
 - c) d = 40.00 cm
 - **d)** d = 23.09 cm
 - e) none of the above
- 2) In all situations, the electric field lines in a given region of space
 - a) point in the direction that a negative charge will move.
 - b) point in the direction of the force on a positive charge.
 - c) point in the direction of the force on a negative charge.
 - d) point in the direction that a positive charge will move.
 - e) none of the above
- 3) Two very large square insulating sheets of area L^2 are separated by a small distance $d \ll L$ in the z-direction. Total charge -3Q is evenly distributed on the lower sheet A, while +Q is evenly distributed on the upper sheet B. What is the electric field \vec{E} in the region between the charge sheets?
 - $\mathbf{a)} \ \frac{+1Q}{\epsilon_0 L^2} \hat{k}$
 - $\mathbf{b)} \ \frac{+2Q}{\epsilon_0 L^2} \hat{k}$
 - $\mathbf{c)} \ \frac{-2Q}{\epsilon_0 L^2} \hat{k}$
 - $\mathbf{d)} \ \frac{-1Q}{\epsilon_0 L^2} \hat{k}$
 - e) none of the above

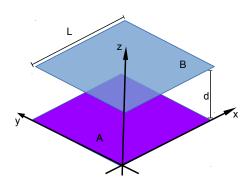


Figure 3:Two large sheets of charge

- 4) Two flat, square, metal plates are parallel to each other and separated by a gap of 2.00 mm. There is vacuum between the plates. An electron released at rest at one plate accelerates across the gap and reaches the other plate with a speed of 4.00×10^6 m/s. The electric field strength between the plates is closest to
 - a) $2.27 \times 10^4 \text{ N/C}$
 - **b)** $4.40 \times 10^7 \text{ N/C}$
 - c) $2.27 \times 10^{-8} \text{ N/C}$
 - **d)** $4.54 \times 10^4 \text{ N/C}$
 - e) $4.40 \times 10^{-5} \text{ N/C}$

5) A half-circular segment radius R has total charge Q distributed uniformly along length L. The electric field magnitude at the origin due to this charge segment is E_0 . What is E_1 from a half-circular segment with twice the total charge at double the radius $R_1 = 2R_0$?



b)
$$E_1 = 2E_0$$

c)
$$E_1 = 1/2E_0$$

d)
$$E_1 = 4E_0$$

e)
$$E_1 = 1/4E_0$$

f) none of the above

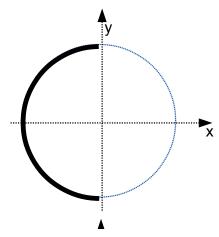


Figure 5: Half circle of charge.

- 6) A point charge +q is placed on the x-axis at (x=R, y=0). An equal amount of charge +q is distributed uniformly along an arc of length $2\theta R$ as shown in Figure 6. What is the direction of the electric field at the origin (x=0,y=0)?
 - a) towards the point charge $(+\hat{i})$
 - **b)** towards the charged arc $(-\hat{i})$
 - c) \vec{E} will be zero at the origin
 - $\mathbf{d)} + \hat{j}$
 - $e) -\hat{j}$

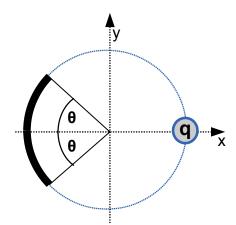


Figure 6: Point charge and circular charge segment.

- 7) A conducting sphere of radius 0.01 m has a charge of 1.0×10^{-11} C deposited on it. What is the magnitude of the electric field in N/C just outside the surface of the sphere? (choose the closest answer)
 - **a)** 90,000
 - **b)** 4,500
 - **c**) 900
 - d) zero
 - **e**) 450
- 8) Figure 8 shows a cube with edges of length 2.00 m, and one corner at the origin. If there is a uniform electric field given by $\vec{E} = 4.25\hat{i} 2.75\hat{j}$ N/C in this region of space, what is the electric flux through the right-hand face shown in the figure?
 - a) $-17 Nm^2/C$
 - **b)** $-11 \ Nm^2/C$
 - c) $6 Nm^2/C$
 - **d**) $11 \ Nm^2/C$
 - e) $17 \ Nm^2/C$

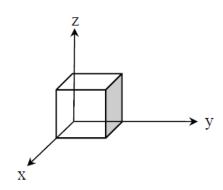


Figure 8: Cube with corner at origin.

- 9) Figure 9 portrays a square, plane insulating sheet carrying uniform positive charge density σ (charge per unit area of sheet). A spherical Gaussian surface of radius R is constructed so that it intersects the sheet with its centre exactly coinciding with the sheet of charge. The net outward flux through the closed Gaussian sphere in this situation is
 - a) zero
 - b) $\pi R^2 \sigma / \epsilon_0$
 - c) $2\pi R^2 \sigma / \epsilon_0$
 - d) $4\pi R^2 \sigma / \epsilon_0$
 - e) none of the above

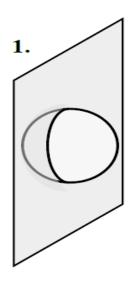


Figure 9: Sheet of charge with spherical Gaussian surface.

- 10) A solid material carries a static excess charge distributed uniformly throughout its volume (i.e., the material has a volume charge density ρ). Which one of the following statements correctly describes this situation?
 - a) The material is a conductor.
 - b) The material is an insulator.
 - c) The material can be either a conductor or an insulator, but it is not possible to tell which from the information given.
 - d) The situation described is impossible.
 - e) none of the above
- 11) In Figure 11, $q_1 = 4q$ and $q_2 = -q$, and a=1cm. Where on the x-axis is the electric field equal to zero?



- **b)** x=1/3 cm
- \mathbf{c}) $\mathbf{x}=0$ cm
- **d)** x=-1/3 cm
- e) x=3 cm

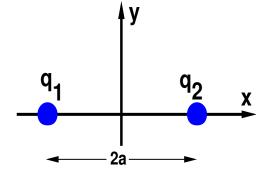


Figure 11: Two point charges.

- 12) Which of the following statements about Gauss's law are correct?
 - a) Only charge enclosed within a Gaussian surface can produce an electric field at points on that surface.
 - b) Gauss's law is valid only for symmetric charge distributions, such as spheres and cylinders.
 - c) The normal to a closed surface points inward for negatively charged objects.
 - d) Gauss's law requires that the electric field be exactly perpendicular or parallel to the surface normal.
 - e) none of the above
- 13) In Figure 13 there are 5 charges with $Q_1 = 5$ Coulombs and $Q_2 = -5$ Coulombs. The electric flux through closed surface B is $10/\epsilon_0$ Coulombs and the flux through closed surface A is zero. Which of the following statements is true?
 - a) $Q_3 = +5$ Coulombs and $Q_5 = +5$ Coulombs
 - b) $Q_3 = -5$ Coulombs and $Q_5 = -5$ Coulombs
 - c) $Q_3 = -5$ Coulombs and $Q_5 = +5$ Coulombs
 - d) $Q_3 = +5$ Coulombs and $Q_5 = -5$ Coulombs
 - e) none of the above

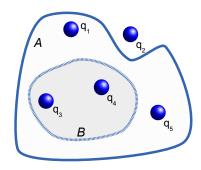


Figure 13: five point charges and two closed surfaces

- 14) A point charge $Q_1=+2Q$ is located at the centre of the cavity inside a spherical conducting shell of inner radius a and outer radius b (see Figure 14). There is a net negative charge -Q on the spherical conductor. Which of the following is true?
 - a) the magnitude of the electric field is zero for all r < b.
 - **b)** the magnitude of the electric field is $\frac{Q}{4\pi\epsilon_0 r^2}$ for all r > b.
 - c) the magnitude of the electric field is $\frac{Q}{4\pi\epsilon_0 r^2}$ for all r < a.
 - d) the magnitude of the electric field is $\frac{2Q}{4\pi\epsilon_0 r^2}$ for all r > b.
 - e) none of the above

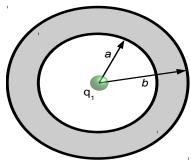


Figure 14: point charge inside a conducting shell.

- 15) Consider two lightweight metal spheres, each hanging from its own insulating nylon thread. One of the spheres has been negatively charged, while the other sphere is neutral. If the spheres are close together then which of the following statements will be true?
 - a) they attract each other.
 - b) they repel each other.
 - c) the negatively charged sphere attracts the neutral one, but the neutral sphere does not attract the negatively charged one.
 - d) they do not exert an electrostatic force on each other.
 - e) the neutral sphere attracts the negatively charged one, but the negatively charged sphere does not attract the neutral one.
- 16) A test charge q feels a force F when located a distance d away from a point charge Q. The amount of charge Q now changes and at the same time the test charge is moved 10 times further away, while keeping q constant. At this new location the test charge still feels the same force F. How much has the charge Q increased/decreased?
 - a) It has increased by a factor of 10
 - b) It has decreased by a factor of 10
 - c) It has increased by a factor of 100
 - d) It has decreased by a factor of 100
 - e) none of the above

- 17) The SI units for electric flux are
 - a) Nm/C
 - \mathbf{b}) Nm²/C
 - c) Nm^2/C^2
 - \mathbf{d}) Nm/C²
 - e) V/C
- 18) A total charge Q is uniformly distributed, with surface charge density σ , over a very thin disk of radius R. The electric field at a distance d along the disk axis is given by $\vec{E} = \frac{\sigma}{2\epsilon_0} \left[1 \frac{d}{\sqrt{d^2 + R^2}} \right] \hat{n}$ where \hat{n} is a normal unit vector perpendicular to the disk. What is the best approximation for the electric field magnitude E at large distances from the disk?
 - a) $\frac{\sigma}{2\epsilon_0}$
 - $\mathbf{b)} \ \frac{QR}{\epsilon_0 d^2}$
 - $\mathbf{c)} \ \frac{\sigma R}{4\epsilon_0 d}$
 - $\mathbf{d)} \ \frac{\sigma}{\epsilon_0} \left(\frac{R}{2d}\right)^2$
 - e) none of the above
- 19) A solid insulating sphere of radius R carries a positive charge density ρ (C/m^3) distributed uniformly throughout its volume. What is the electric field strength E as a function of radius r inside the sphere?
 - $\mathbf{a)} \ E = \frac{\rho R}{3\epsilon_0 r}$
 - $\mathbf{b)} \ E = \frac{1}{4\pi\epsilon_0} \frac{\rho}{r^2}$
 - c) $E = \frac{4\pi\rho r^3}{3\epsilon_0 R^2}$
 - $\mathbf{d)} \ E = \frac{\rho r^2}{3\epsilon_0}$
 - e) $E = \frac{\rho r}{3\epsilon_0}$

20) For the charged rod shown in Figure 20, assume that the total charge in the rod is positive $(Q \ge 0)$ and uniformly distributed along the rod. Which of the following statements is true for the electric field on the x-axis?

a) for
$$x \gg L$$
 $E_x \approx \frac{Q}{4\pi\epsilon_0 x^2}$

b) for
$$x \gg L$$
 $E_x \approx \frac{Q}{2\pi\epsilon_0 x}$

c) for
$$x \gg L$$
 $E_x \approx \frac{Q}{2\pi\epsilon_0 x^3}$

d) for all
$$x > 0$$
 $E_x = \frac{Q}{4\pi\epsilon_0} \frac{L}{x^3}$

e) none of the above

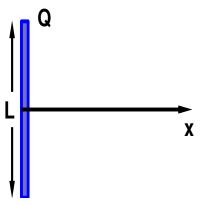


Figure 20: Charged rod of length L.

This is the end of the Multiple Choice part of the exam. Make sure that you have entered all your answers from this section on the Answer Sheet "bubble" page.

You may now proceed to Part II. All your answers for Part II must be written in the appropriate box on the Answer Sheet pages.

CONSTANTS AND USEFUL EQUATIONS

$$k = Coulomb constant = 8.99 \times 10^9 N m^2 C^{-2}$$

$$\epsilon_0 = \text{permittivity of free space} = 8.85 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$$

$$e = fundamental charge = 1.602 \times 10^{-19} C$$

$$m_p = \text{mass of proton} = 1.67 \times 10^{-27} \text{ kg}$$

$$m_e = \text{mass of electron} = 9.11 \times 10^{-31} \text{ kg}$$

$$m = 10^{-3}$$

$$\mu = 10^{-6}$$

$$n = 10^{-9}$$

$$p = 10^{-12}$$

Surface area of a sphere: $A = 4\pi r^2$

Area of a circle: $A = \pi r^2$

Volume of a sphere: $A = \frac{4}{3}\pi r^3$

Circumference of a circle: $C = 2\pi r$

$$x = x_0 + v_{x0}t + \frac{1}{2}a_xt^2$$
 $v_x = v_{x0} + a_xt$ $v_x^2 = v_{x0}^2 + 2a_xx$ $\vec{F} = m\vec{a}$

$$v_x = v_{x0} + a_x$$

$$v_x^2 = v_{x0}^2 + 2a_x x$$

$$\vec{F} = m\vec{a}$$

$$\vec{F} = k \frac{q_1 q_2}{r^2} \, \hat{r} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \, \hat{r} \qquad \qquad \vec{E} = \frac{\vec{F}}{q} \qquad \qquad \vec{E} = k \frac{q}{r^2} \, \hat{r} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \, \hat{r}$$

$$\vec{E} = \frac{\vec{F}}{q}$$

$$\vec{E} = k \frac{q}{r^2} \, \hat{r} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \, \hat{r}$$

$$\Phi_E = \oint_A \vec{E} \cdot d\vec{A} = \oint_A E \, dA \cos\theta = \frac{Q_{encl}}{\epsilon_0} \qquad V = \frac{U}{q} \qquad U = k \frac{q_1 q_2}{r} = \frac{1}{4\pi \epsilon_0} \frac{q_1 q_2}{r}$$

$$V = \frac{U}{a}$$

$$U = k \frac{q_1 q_2}{r} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r}$$

$$V = k \frac{q}{r} = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$$

$$W = qV_{ab}$$

$$W = qV_{ab}$$

$$V_{ab} = V_a - V_b = \int_a^b \vec{E} \cdot d\vec{l}$$

$$\vec{E} = -\vec{\nabla}V = -\left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)V$$

$$C = \frac{Q}{V_{ab}} \qquad \qquad C = \frac{\epsilon_0 A}{d}$$

$$C = \frac{\epsilon_0 A}{d}$$

$$u = \frac{1}{2}\epsilon_0 E^2$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a}$$

$$\int \frac{dx}{\sqrt{x^2 + a^2}} = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

$$\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \arctan \frac{x}{a}$$

$$\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \arctan \frac{x}{a} \qquad \int \frac{dx}{(x^2 + a^2)^{3/2}} = \frac{1}{a^2} \frac{x}{\sqrt{x^2 + a^2}}$$

$$\int \frac{x \, dx}{\left(x^2 + a^2\right)^{3/2}} = -\frac{1}{\sqrt{x^2 + a^2}}$$