Fill in these spaces with your SURNAME and INITIAL.

University of Calgary

Faculty of Science

Solutions to the Midterm Test

PHYSICS 259 ALL LECTURE SECTIONS

February 28, 2012, 6:45-8:15 p.m. **Time: 90 minutes.**

DO NOT TEAR OFF THIS PAGE! You may, however, tear off the last page, which has an equation sheet and table of integrals on the back.

This is a closed-book exam worth a total of 30 points. Please answer all questions. Use of the Schulich calculator or equivalent is allowed.

Write your Last Name and Initial on this top sheet in the grid above. (Do not write your ID number on this page.) **Also write your ID in the grid at top right of Page 2 of the Question paper. DO THIS NOW**.

Make sure this question paper booklet contains 12 pages, with page 12 being the Formula Sheet. (Pages 10 and 11 are blank pages for rough work.) If you are missing any pages, get a new booklet from the exam supervisor.

You should also have a **separate set of Answer Sheets**. This is where you enter Multiple Choice answers of Part I and also detailed solutions to the problems of Part II. Only work entered in the indicated spaces on the Answer Sheets will be marked.

IMPORTANT: YOUR ID NUMBER IS TO BE ENTERED AT THE TOP OF EACH AND EVERY ONE OF THE ANSWER SHEETS. DO THIS NOW.

Begin working on the examination when instructed to do so by the supervisor.

ID:							

Part I

Multiple-Choice Questions

Enter answers to multiple choice questions on the first Answer Sheet using space provided in the upper right of the page. Each question in Part I is worth one point. You should complete Part I in about 30 minutes or less.

1. Two point charges are 0.300 metres apart. One of the charges is +3.00 mC, and it feels an attractive force of 600 newtons from the other charge. The value of the other charge must be (*select the closest answer*)

*(a) $-2\ \mu C$ (b) $+1\ mC$ (c) $-2\ mC$ (d) $+2\ \mu C$ (e) $+2\ mC$

Reasoning: *Label the +3.00 mC charge q_1 and the unknown charge q_2. The force between them is attractive, so q_2 must be negative. Its magnitude can be found from Coulomb's law:*

$$F_E = \frac{1}{4\pi\epsilon_0}\frac{|q_1|\,|q_2|}{r^2}$$

$$|q_2| = \frac{4\pi\epsilon_0 F_E r^2}{|q_1|} = \frac{4\pi(8.85\times10^{-12}\,\frac{C^2}{N\,m^2})(600\,N)(0.3\,m)^2}{3\times10^{-3}\,C} = 2.00\times10^{-6}\,C = 2.00\,\mu C$$

2. A test charge q initially feels a force, F, due to a point charge Q located a distance d away. Then the distance between charges is increased to 10 times the initial separation and the charge Q is changed so that the electrostatic force, F, remains the same (q does not change). How much must the charge Q increase or decrease?

a) Increase by a factor of 10.

b) None of statements a, c, d or e is correct.

*c) Increase by a factor of 100.

d) Decrease by a factor of 10.

e) Decrease by a factor of 100.

Reasoning: *The force is inversely proportional to r^2, so increasing the distance weakens the force by a factor or 100. The charge therefore has to be increased by a factor of 100 to maintain the same force.*

3. Consider two lightweight metal spheres, each hanging from its own insulating nylon thread. One of the spheres has been negatively charged, while the other sphere is neutral. If the spheres are close together,

$*$a) they attract each other.

b) they repel each other.

c) the negatively charged sphere attracts the neutral one, but the neutral sphere does not attract the negatively charged one.

d) they do not exert an electrostatic force on each other.

e) the neutral sphere attracts the negatively charged one, but the negatively charged sphere does not attract the neutral one.

Reasoning: *Call the negatively-charged sphere "sphere N" and the uncharged sphere "sphere U". Then sphere N attracts the positive charges and repels the negative charges in sphere U. The positive charges on sphere U are now closer to sphere N, where the field is stronger, and the negative charges on sphere U are further away from sphere N, where the field is weaker. Therefore, the positive charges are attracted to sphere N more strongly than the negative charges are repelled, and there is a net attraction.*

4. **Figure 1** shows a cylindrical surface, 0.800 m long and 0.200 m in diameter, whose axis is aligned parallel to the x-axis. A nonuniform electric field is directed parallel to the x-axis at all points in space. The electric fields E_1 and E_2 at the ends of the cylindrical surface have magnitudes of 8,000 N/C and 5,000 N/C, respectively, and are directed as shown. The charge enclosed by the cylindrical surface in **Figure 1** is closest to

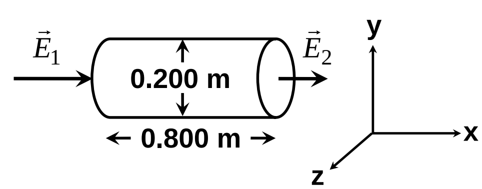

Figure 1

$*$(a) -0.83 nC (b) -3.6 nC (c) -1.8 nC (d) 0.83 nC (e) 3.6 nC

Reasoning: *$\vec{E}$ is parallel to the long axis of the cylinder, so only the ends of the cylinder contribute to the electric flux through the surface of the cylinder. $\vec{E}_1$ is inward, so the flux there is negative, and $\vec{E}_2$ is outward, so the flux there is positive. Then from Gauss's law,*

$$-E_1\,A + E_2\,A = \frac{Q_{encl}}{\epsilon_0}$$

$$Q_{encl} = (-E_1 + E_2)(\pi\,r^2)\epsilon_0 = (-8000\,N/C + 5000\,N/C)\pi(0.1\,m)^2\epsilon_0 = -0.83\,nC$$

5. The outer cylinder in **Figure 2** is part of an infinitely-long cylindrical distribution of charge with constant charge density ρ and radius R. In attempting to find the electric field at a point inside the cylinder ($r < R$), someone has drawn a cylindrical Gaussian surface of radius r inside the charged cylinder, and then has written down the following four statements:

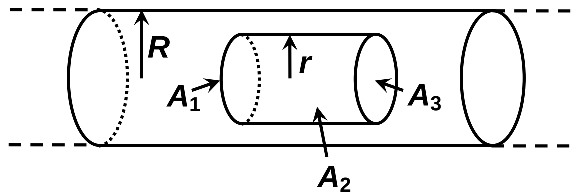

Figure 2

$$\oint_A \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0} \tag{1}$$

where $$\oint_A \vec{E} \cdot d\vec{A} = \int_{A_1} \vec{E} \cdot d\vec{A} + \int_{A_2} \vec{E} \cdot d\vec{A} + \int_{A_3} \vec{E} \cdot d\vec{A} \tag{2}$$

$$= \int_{A_2} \vec{E} \cdot d\vec{A} \tag{3}$$

$$= \vec{E} \cdot \int_{A_2} d\vec{A} \tag{4}$$

Referring to **Figure 2**, we are able to go from equation (2) to equation (3) because

a) there is no charge on surfaces A_1 or A_3.

b) E is zero on A_1 and A_3.

c) the contributions from the integrals over A_1 and A_3 have equal magnitude but opposite sign, and thus add to zero.

* d) $\vec{E}$ is perpendicular to $d\vec{A}$ over A_1 and A_3.

e) E is constant over the surfaces A_1 and A_3.

Reasoning: *On each end, $d\vec{A}$ is perpendicular to the surface and therefore parallel to the axis, but $\vec{E}$ is perpendicular to the axis, so $\vec{E} \perp d\vec{A}$. The dot product of two mutually perpendicular vectors is zero.*

6. In **Figure 3**, which charge configuration(s) have a non-zero electric field at the origin?

a) Configuration 2.

b) Configuration 1.

*c) Configuration 3.

d) Configurations 2 and 3.

e) Configuration 4.

Reasoning: *In configuration 3, the charges on the x-axis produce a field toward the right at the origin and the charges on the y-axis produce a field to downward. Then the total field at the origin is toward the lower right. For the other three configurations, a similar analysis gives a field of zero at the origin.*

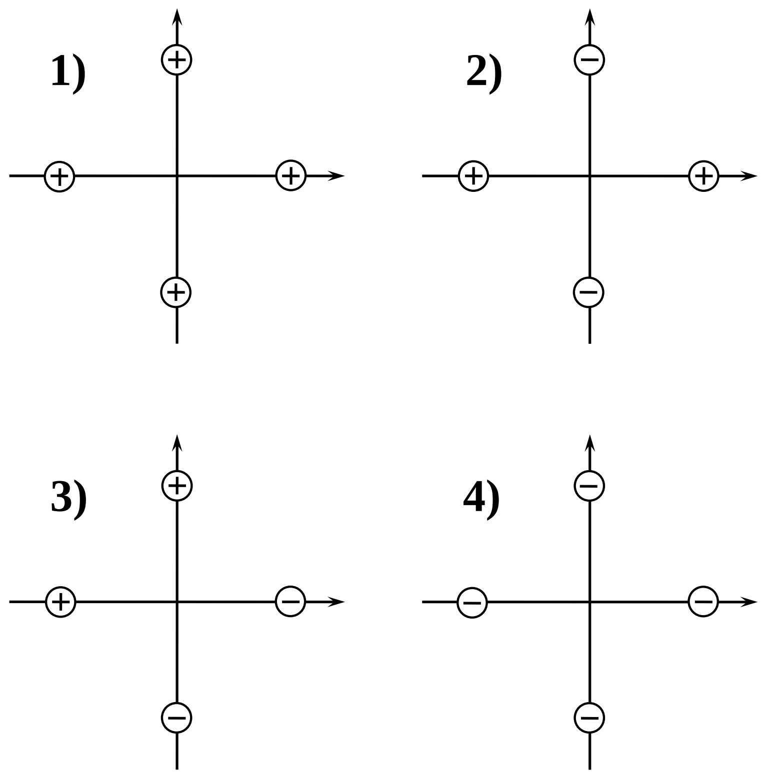

Figure 3

7. A point charge $q_1 = +10.0$ nC is fixed at the origin. A second point charge $q_2 = +5.00$ nC is located at $x = 3.00$ cm, $y = 0.00$ cm, $z = 0.00$ cm. Calculate the work done by the electric force if q_2 is brought to the location $x = 2.00$ cm, $y = 2.00$ cm, $z = 2.00$ cm. *Select the closest answer.*

(a) -2.01 μJ (b) 1.50 μJ (c) 12.5 mJ *(d) 2.01 μJ (e) -12.5 mJ

Reasoning: *Define the first position to be a and the second to be b. Then the work done on q_2 by the electric force is $W^{ELEC}_{a\to b} = q_1 V_{ab}$.*

$$V_a = \frac{1}{4\pi\epsilon_0}\frac{q_1}{r_a} = \frac{1}{4\pi\epsilon_0}\frac{10\times 10^{-6}\,C}{0.03\,m} = 2997\,V$$

$$V_b = \frac{1}{4\pi\epsilon_0}\frac{q_1}{r_b} = \frac{1}{4\pi\epsilon_0}\frac{10\times 10^{-6}\,C}{\sqrt{(0.02\,m)^2+(0.02\,m)^2+(0.02\,m)^2}}\frac{1}{4\pi\epsilon_0}\frac{10\times 10^{-6}\,C}{0.0346\,m} = 2596\,V$$

$$V_{ab} = V_a - V_b = 2997\,V - 2596\,V = 401\,V$$

$$W^{ELEC}_{a\to b} = q_2 V_{ab} = (5\times 10^{-9}\,C)(401\,V) = 2.01\,\mu J$$

8. A solid metal sphere has been given an excess charge. As a consequence, the electric potential at the surface of the sphere is 450 V, relative to V = 0 at infinity. What is the electric potential at the centre of the sphere (and what is the reason)?

a) 900 V (the electric potential at the surface of a sphere is half that at the centre, if V = 0 at infinity).

b) 0 (the electric potential is zero in a conductor).

c) Indeterminate (the charge distribution inside the sphere has not been specified).

$*$d) 450 V (the electric potential gradient is zero in a conductor).

e) Infinite (the radius is zero at the centre).

Reasoning: *In a static situation, the electric field inside a conducting sphere is zero. The electric field equals the negative of the potential gradient, so the potential gradient between the surface and the centre is zero; i.e, the potential does not change between the surface and the centre. Another way of looking at it is to write* $V_{ab} = \int_a^b \vec{E} \cdot d\vec{l} = \int_a^b 0 \cdot d\vec{l} = 0$*; i.e., the potential difference between the surface and the centre is zero.*

9. An electron is released from rest between the plates of a parallel plate capacitor. As it moves,

(a) its potential energy increases.

(b) its potential energy may increase or decrease, depending on the direction of the electric field.

(c) its potential energy stays constant.

(d) its potential energy cannot be determined, because its charge is negative.

$*$(e) its potential energy decreases.

Reasoning: *Any object, released from rest and acted upon only by a conservative force, always "falls" toward lower potential energy. Looking at it in more detail, when released, the electric force on the electron causes it to move in the opposite direction to the electric field, and therefore toward higher potential. Then* V_b *is greater than* V_a*. But* q *is negative, so* $U_b = qV_b$ *is less than (i.e., more negative than)* $U_a = qV_a$*; i.e., its potential energy has decreased.*

10. In **Figure 4**, two parallel, metal plates in vacuum have been connected to a battery of potential difference 300 V. Assume that the plates are large enough that the electric field between them is uniform. A small particle of mass 1.00 g and charge 2.00 μC has been injected next to the right-hand plate with an initial velocity of 1.00 m/s toward the opposite plate. Does the particle reach the other plate? If so, what is its speed when it arrives? If not, how far does it get? *Select the closest answer.*

(a) Yes, with a speed of 2.2 m/s.

(b) Yes, just barely; its speed becomes zero just as it reaches the other plate.

(c) No, it turns back about 1/3 of the way across.

(d) Yes, with a speed of 1.5 m/s.

∗(e) No, it turns back about 5/6 of the way across.

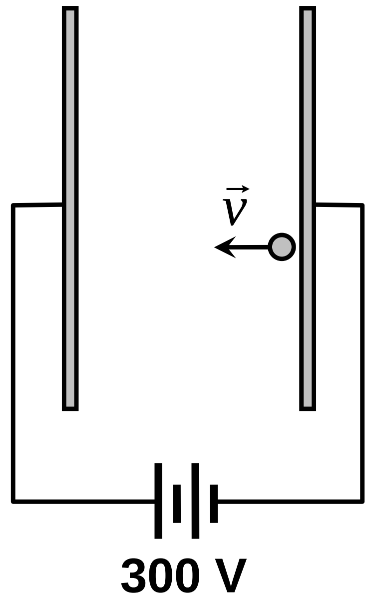

Figure 4

Reasoning: *Find the potential difference, V_{ab}, needed to stop the particle, and then compare V_{ab} to the potential difference between the plates. From conservation of mechanical energy,*

$$K_f + U_f = K_i + U_i$$

$$K_f - K_i = U_i - U_f$$

$$\Delta K = qV_a - qV_b = q(V_a - V_b) = qV_{ab}$$

$$V_{ab} = \frac{\Delta K}{q} = \frac{\frac{1}{2}mv^2 - 0}{q} = \frac{\frac{1}{2}(10^{-3}\,kg)(1\,m/s)^2}{2 \times 10^{-6}\,C} = 250\,V$$

There is a potential difference of 300V between the plates, and only 250 V are needed to stop the particle, so the particle stops before reaching the other plate. In a uniform electric field, the electric potential decreases linearly along a field line, so the particle travels 250/300 = 5/6 of the distance to the other plate before turning around.

11. This is **version A** of the exam. Please select (A) in the row marked "Version" in the Multiple Choice Answers area on the first Answer Sheet. **Be sure to do this now, before moving on to other questions on the paper**.

Part II: Written-answer Problems (Total: 20 marks)

IMPORTANT: Write your answers to the five problems in Part II in the corresponding boxes on the Answer Sheets. Work must be shown for full marks. Rough work can be done on the blank spaces provided in this question paper, but only the work appearing on the Answer sheets will be marked.

12. [**5.5 marks**] In **Figure 5**, a total charge of $+Q$ is distributed uniformly over a circular ring of radius a.

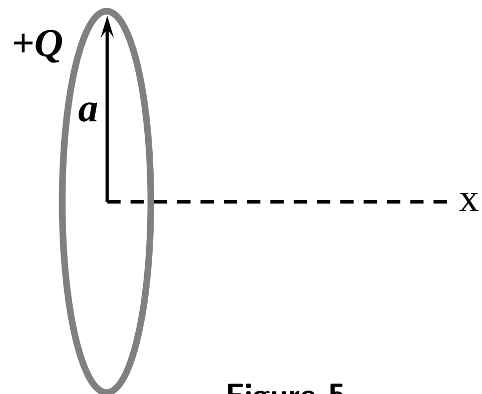

Figure 5

(a) What is the linear charge density, λ? [**1 mark**]

(b) Derive an expression for the magnitude of the electric field on the axis of the ring as a function of x. Use a diagram to illustrate your method. *Express your answer in terms of the quantities Q, a, x, and constants.* [**3 marks**]

(c) What is the electric field at the center of the ring ($x = 0$)? (Show your work or justify your answer clearly.) [**1 mark**]

(d) What would you expect the electric field to look like at large distances from the ring ($x >> a$)? (Write down a mathematical expression or give a short description; you do not need to derive anything.) [**0.5 mark**]

Pages 10 and 11 and blank spaces anywhere else in this Question Booklet can be used for ROUGH WORK which will not be marked. Enter your finished solutions in the ANSWER PAGES. Be sure to show your steps for possible part marks.

13. [**7.5 marks**] **Figure 6** shows a small conducting spherical shell with inner radius a and outer radius b, concentric with a larger conducting spherical shell with inner radius c and outer radius d.

The inner shell has charge $+2q$ and the outer shell has charge $-4q$.

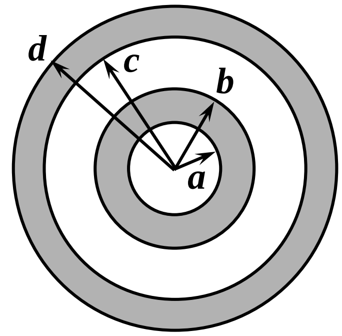

Figure 6

(a) Write down Gauss's law and use it to obtain an expression for the magnitude of the electric field in the region $b < r < c$ (the region between the two spheres) in terms of q and the distance r from the centre of the two shells. Show all steps in applying Gauss's law, and explain your reasoning. [**3 marks**]

(b) What is the direction of the electric field in the region $b < r < c$? [**0.5 mark**]

(c) For each of the following regions, give an expression for the magnitude of the electric field and describe its direction. *Just the answers are required for this part; you do not need to show your work.*

(i) $r < a$. [**1 mark**]

(ii) $a < r < b$. [**1 mark**]

(iii) $c < r < d$. [**1 mark**]

(iv) $r > d$. [**1 mark**]

Pages 10 and 11 and blank spaces anywhere else in this Question Booklet can be used for ROUGH WORK which will not be marked. Enter your finished solutions in the ANSWER PAGES. Be sure to show your steps for possible part marks.

14. [**1 mark**] In **Figure 7**, four point charges are fixed in a square of side 5.00 cm, and a fifth point charge $+q$ (not shown) is located at infinity. What is the change in the electric potential energy of this system of five point charges if the fifth point charge $+q$ is now brought from infinity and placed at the centre of the square? Justify your answer. [**1 mark**]

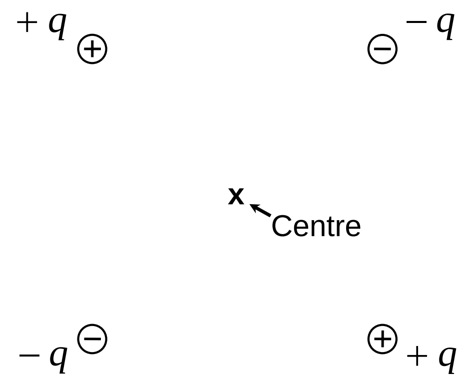

Figure 7

15. [**3 marks**] A thin rod with length a and uniform linear charge density λ is oriented along the x-axis between $-a$ and the origin, as shown in **Figure 8**. Find an expression for the electric potential at point P, at arbitrary position x on the $+x$ axis. *Express your answer in terms of a, x, λ and constants.* [**3 marks**]

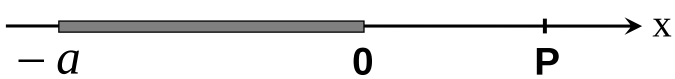

Figure 8

Pages 10 and 11 and blank spaces anywhere else in this Question Booklet can be used for ROUGH WORK which will not be marked. Enter your finished solutions in the ANSWER PAGES. Be sure to show your steps for possible part marks.

16. [**3 marks**] Three identical point charges of charge q are initially held at rest 1.00 cm apart on the x-axis, as shown in **Figure 9**. Each point charge has a mass of 10.0 grams and a charge of 5.0 μC. At time $t = 0$ s, the two outer charges are released simultaneously while the central charge remains fixed in place. If friction can be neglected, what is the speed of each outer charge a very long time after it is released? [**3 marks**]

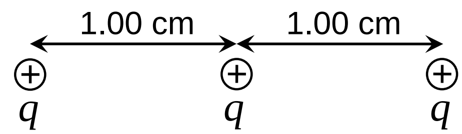

Figure 9

Pages 10 and 11 and blank spaces anywhere else in this Question Booklet can be used for ROUGH WORK which will not be marked. Enter your finished solutions in the ANSWER PAGES. Be sure to show your steps for possible part marks.

This page is for ROUGH WORK. Enter your finished solutions, showing steps, in the ANSWER PAGES.

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CONSTANTS AND USEFUL EQUATIONS

k = Coulomb constant = 8.99×10^{9} N m^2 C^{-2}
ϵ_0 = permittivity of free space = 8.85×10^{-12} C^2 N^{-1} m^{-2}
e = fundamental charge = 1.602×10^{-19} C
m_e = mass of electron = 9.11×10^{-31} kg

m = 10^{-3} $\quad$ $\mu = 10^{-6}$ $\quad$ n = 10^{-9} $\quad$ p = 10^{-12}

Surface area of a sphere: $A = 4\pi r^2$
Volume of a sphere: $A = \frac{4}{3}\pi r^3$
Area of a circle: $A = \pi r^2$
Circumference of a circle: $C = 2\pi r$

$$x = x_0 + v_{x0}t + \frac{1}{2}a_x t^2 \qquad v_x = v_{x0} + a_x t \qquad v_x^2 = v_{x0}^2 + 2a_x x \qquad \vec{F} = m\vec{a}$$

$$\vec{F} = k\frac{q_1 q_2}{r^2}\,\hat{r} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}\,\hat{r} \qquad \vec{E} = \frac{\vec{F}}{q} \qquad \vec{E} = k\frac{q}{r^2}\,\hat{r} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\,\hat{r}$$

$$\Phi_E = \oint_A \vec{E}\cdot d\vec{A} = \oint_A E dA cos\theta = \frac{Q_{encl}}{\epsilon_0} \qquad V = \frac{U}{q} \qquad U = k\frac{q_1 q_2}{r} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r}$$

$$V = k\frac{q}{r} = \frac{1}{4\pi\epsilon_0}\frac{q}{r} \qquad W = qV_{ab} \qquad V_{ab} = V_a - V_b = \int_a^b \vec{E}\cdot d\vec{l}$$

$$\vec{E} = -\vec{\nabla}V = -\left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)V \qquad C = \frac{Q}{V_{ab}} \qquad C = \frac{\epsilon_0 A}{d}$$

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}QV = \frac{1}{2}CV^2 \qquad \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \qquad C = C_1 + C_2 + C_3$$

$$u = \frac{1}{2}\epsilon_0 E^2$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = arcsin\frac{x}{a} \qquad \int \frac{dx}{\sqrt{x^2 + a^2}} = ln\left(x + \sqrt{x^2 + a^2}\right)$$

$$\int \frac{dx}{x^2 + a^2} = \frac{1}{a}arctan\frac{x}{a} \qquad \int \frac{dx}{(x^2 + a^2)^{3/2}} = \frac{1}{a^2}\frac{x}{\sqrt{x^2 + a^2}}$$

$$\int \frac{xdx}{(x^2 + a^2)^{3/2}} = -\frac{1}{\sqrt{x^2 + a^2}}$$