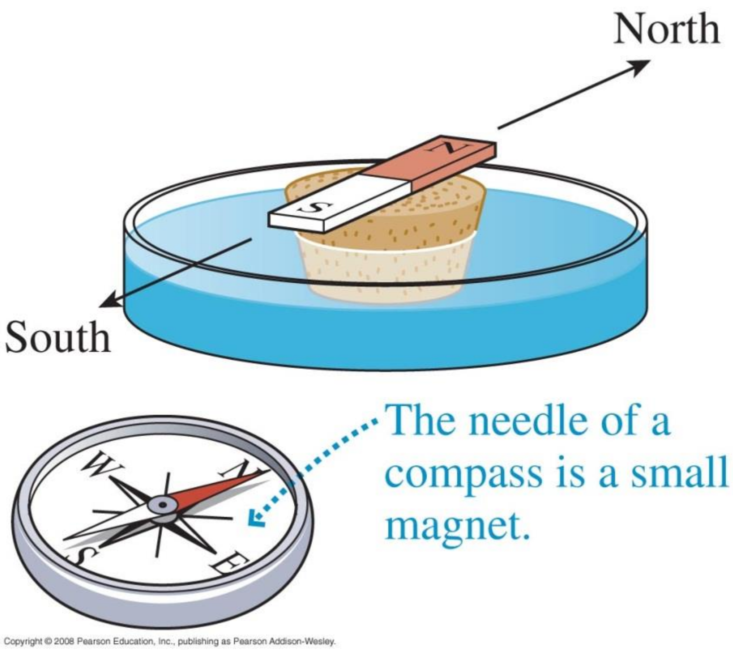
Last time:

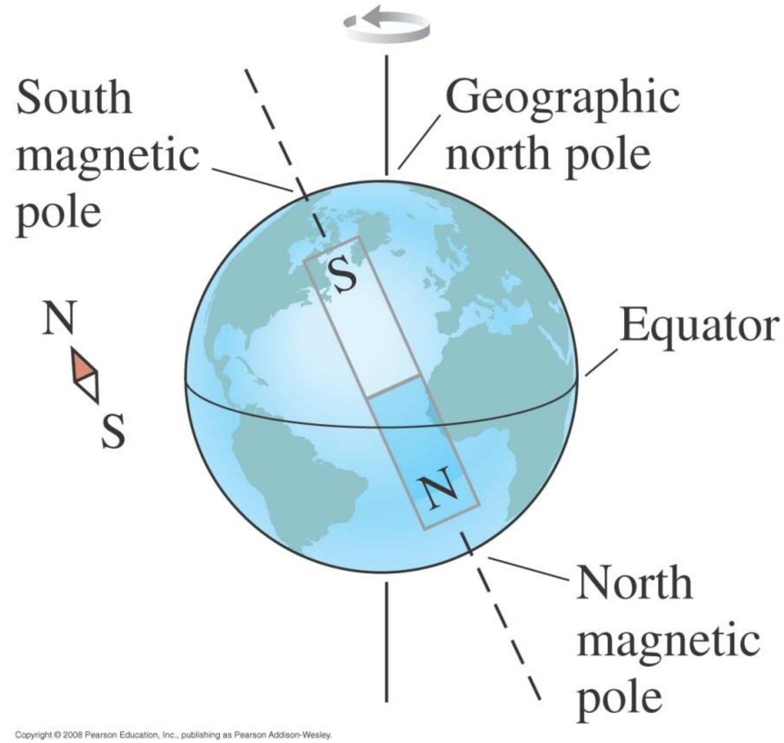
- RC time constant and its meaning
- Charging/discharging capacitors calculation

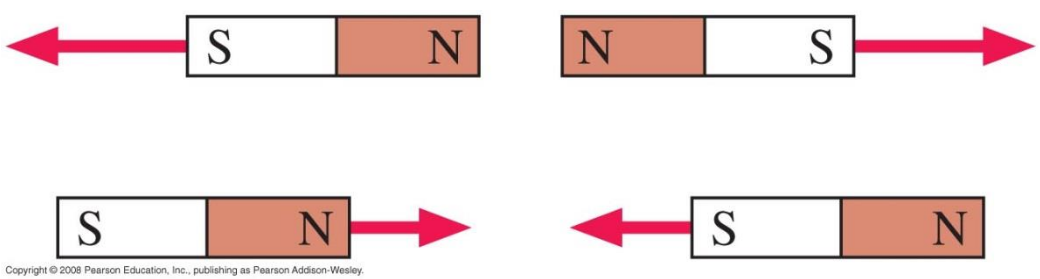
Today: Happy Vernal Equinox

- Introduction to magnetism
- Electric force vs magnetic force on charges
- Vector cross product
- Consequences of magnetic force

Magnetism





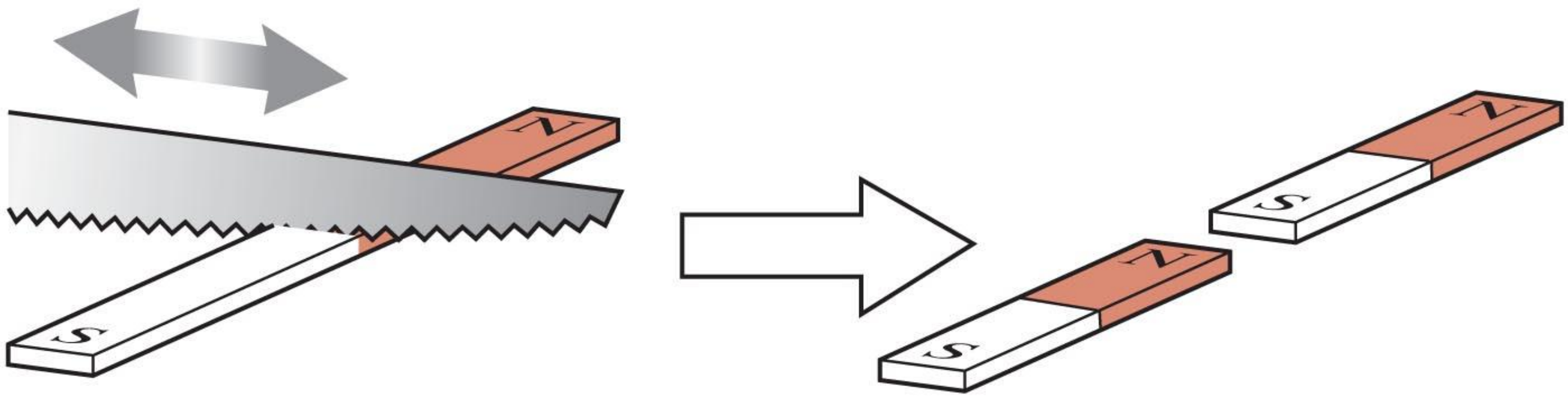


Like poles repel.

Unlike poles attract.

Magnetism is not the same as electricity!!

For example, cutting a magnet does not create one north-pole piece and one south-pole piece.



Magnetic monopoles do not seem to exist:

We cannot have a north pole without a south pole.

Except…

M. W. Ray et al., Nature **505**, 657 (2014); doi:10.1038/nature12954

Observation of Dirac Monopoles in a Synthetic Magnetic Field

M. W. Ray,[1] E. Ruokokoski,[2] S. Kandel,[1,*] M. Möttönen,[2,3] and D. S. Hall[1]

[1]*Department of Physics, Amherst College,*
Amherst, Massachusetts 01002–5000, USA

[2]*QCD Labs, COMP Centre of Excellence,*
Department of Applied Physics, Aalto University,
P.O. Box 13500, FI–00076 Aalto, Finland

[3]*Low Temperature Laboratory (OVLL), Aalto University,*
P.O. Box 13500, FI-00076 Aalto, Finland

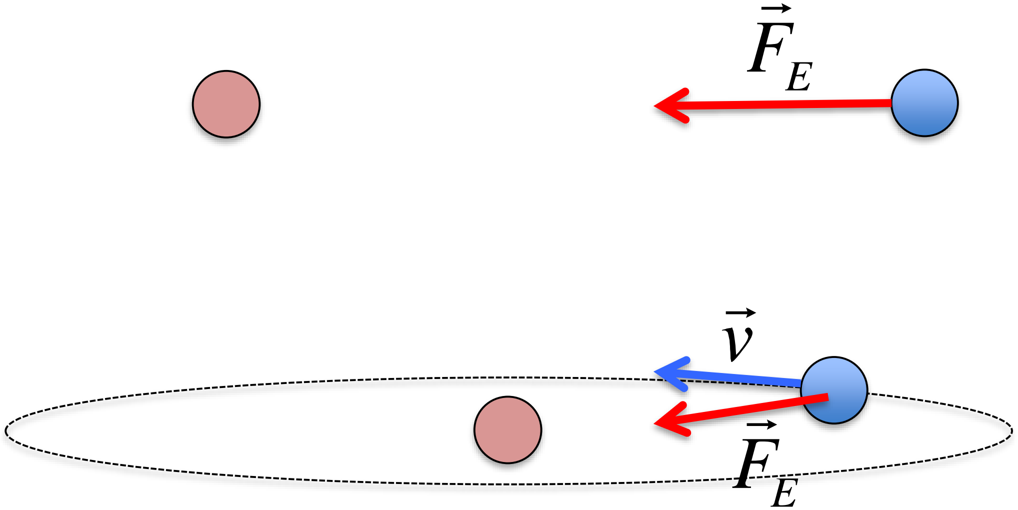
(Dated: 20 September 2013; accepted 4 December 2013)

Abstract

Magnetic monopoles — particles that behave as isolated north or south magnetic poles — have been the subject of speculation since the first detailed observations of magnetism several hundred years ago[1]. Numerous theoretical investigations and hitherto unsuccessful experimental searches[2] have followed Dirac's 1931 development of a theory of monopoles consistent with both quantum mechanics and the gauge invariance of the electromagnetic field[3]. The existence of even a single Dirac

Electric Force on Charges

Electric force acts on a charge regardless of its motion.



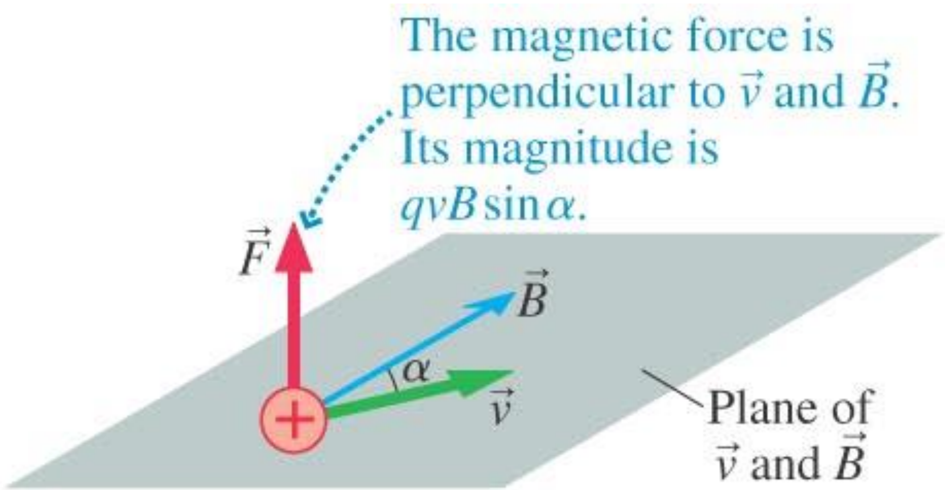
$$\vec{F}_E = q\vec{E}$$

Magnitude: $F_E = qE$

Direction: direction of $\vec{E}$

Magnetic Force on Charges

Magnetic force acts only on a moving charge.

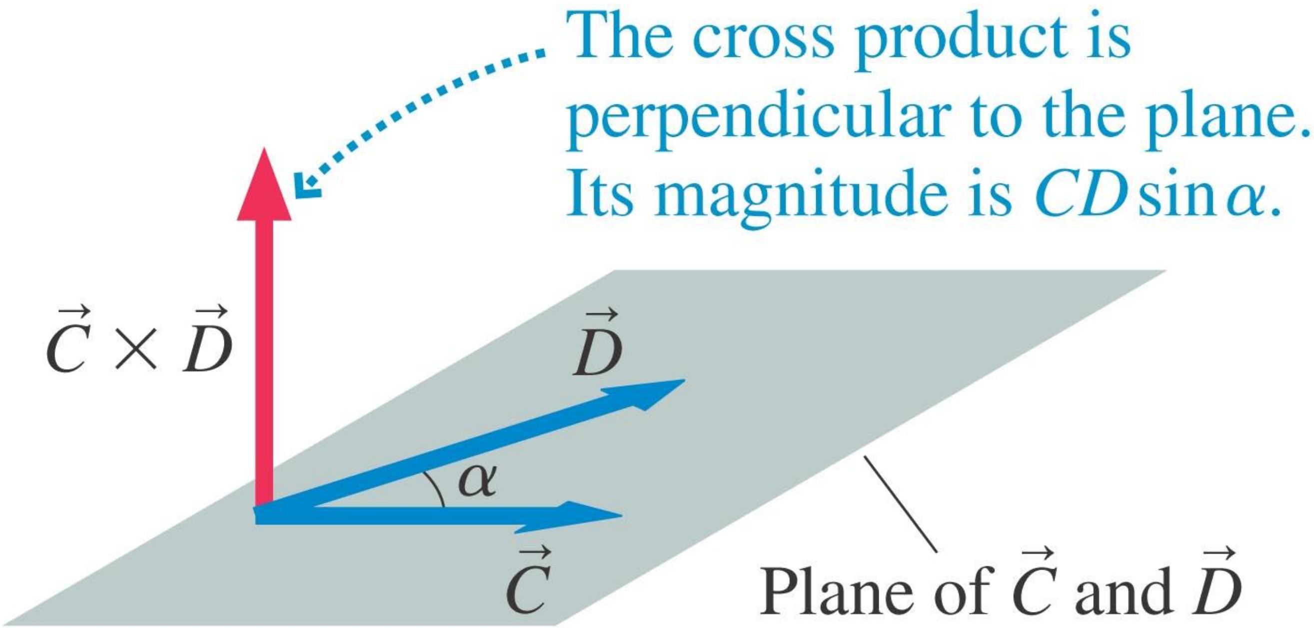


$$\vec{F}_B = q\vec{v}\times\vec{B}$$

Magnitude: $F_B = qvB\sin\alpha$

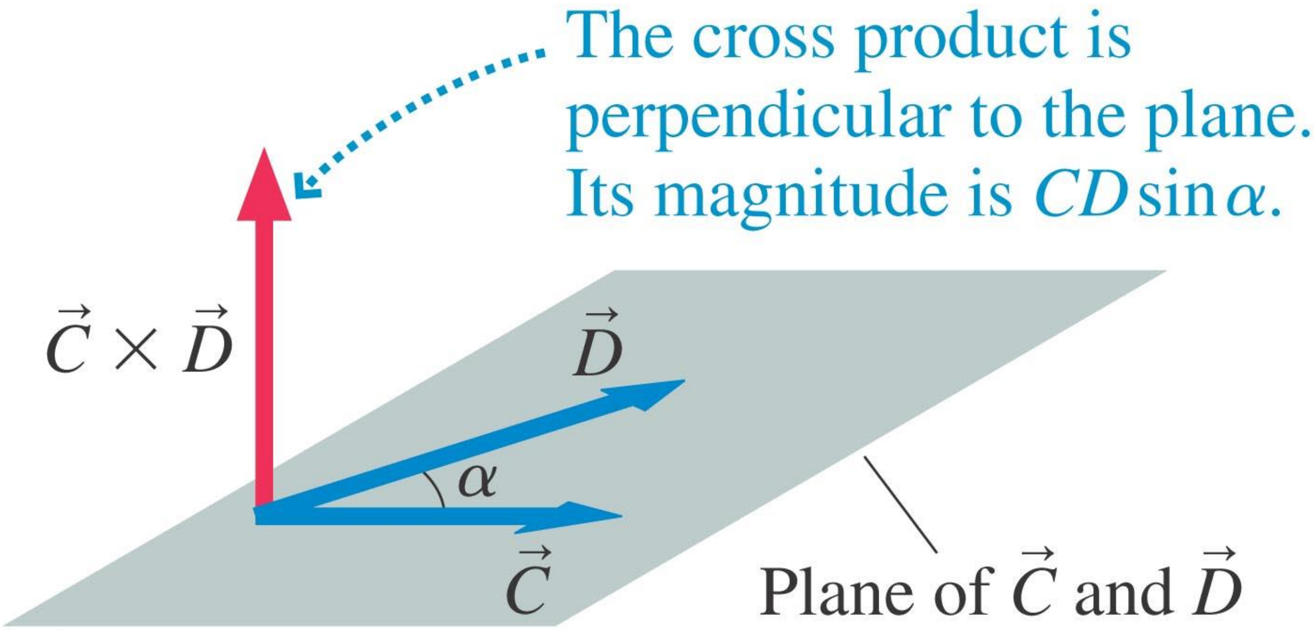
Direction: RH rule

The Vector Cross Product



Point the fingers of your right hand along the first vector in the cross product (vector C), then curl them so they point along the second vector (vector D). Your thumb gives the direction of the cross product.

The Vector Cross Product



So $\vec{C}\times\vec{D}$ points up and $\vec{D}\times\vec{C}$ points down.

$$\left|\vec{C}\times\vec{D}\right| = \left|\vec{C}\right|\left|\vec{D}\right|\sin\alpha$$

Cross product vs regular product

Regular/dot product	**Cross product**
Distributive	Distributive
$\vec{B}\cdot\left(\vec{C}+\vec{D}\right)=\vec{B}\cdot\vec{C}+\vec{B}\cdot\vec{D}$	$\vec{B}\times\left(\vec{C}+\vec{D}\right)=\vec{B}\times\vec{C}+\vec{B}\times\vec{D}$
Commutative	Anticommutative
$CD=DC$ $\vec{C}\cdot\vec{D}=\vec{D}\cdot\vec{C}$	$\vec{C}\times\vec{D}=-\vec{D}\times\vec{C}$
Associative	Non-Associative
$B\left(CD\right)=\left(BC\right)D$	$\vec{B}\times\left(\vec{C}\times\vec{D}\right)\neq\left(\vec{B}\times\vec{C}\right)\times\vec{D}$

Unit vector notation

The cross product becomes easy to deal with when using unit vector notation

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

$$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$$

$$\hat{i} \times \hat{j} = \hat{k}$$

$$\hat{j} \times \hat{k} = \hat{i}$$

$$\hat{k} \times \hat{i} = \hat{j}$$

Now let's see what the cross product between A and B is:

$$\vec{C} = \vec{A} \times \vec{B}$$

$$\vec{C} = \left(A_x\hat{i} + A_y\hat{j} + A_z\hat{k}\right) \times \left(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}\right)$$

$$\vec{C} = \left(A_yB_z - A_zB_y\right)\hat{i} + \left(A_zB_x - A_xB_z\right)\hat{j} + \left(A_xB_y - A_yB_x\right)\hat{k}$$

Unit vector notation

The cross product becomes easy to deal with when using unit vector notation

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

$$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$$

$$\hat{i}\times\hat{j} = \hat{k}$$

$$\hat{j}\times\hat{k} = \hat{i}$$

$$\hat{k}\times\hat{i} = \hat{j}$$

Now let's see what the cross product between A and B is:

$$\vec{C} = \vec{A}\times\vec{B}$$

$$\vec{C} = \left(A_x\hat{i} + A_y\hat{j} + A_z\hat{k}\right)\times\left(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}\right)$$

$$\vec{C} = \left(A_yB_z - A_zB_y\right)\hat{i} + \left(A_zB_x - A_xB_z\right)\hat{j} + \left(A_xB_y - A_yB_x\right)\hat{k}$$

Another way to think about it

Start with the two vectors in component form

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k} \qquad \vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$$

The cross product is given by the determinant of the following matrix:

$$\vec{C} = \vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$\vec{C} = \left(A_yB_z - A_zB_y\right)\hat{i} + \left(A_zB_x - A_xB_z\right)\hat{j} + \left(A_xB_y - A_yB_x\right)\hat{k}$$

Parallel and Perpendicular vectors

For parallel vectors

$$\vec{A} = A\hat{i} \qquad \vec{B} = B\hat{i} \qquad \vec{C} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A & 0 & 0 \\ B & 0 & 0 \end{vmatrix} = \vec{0}$$

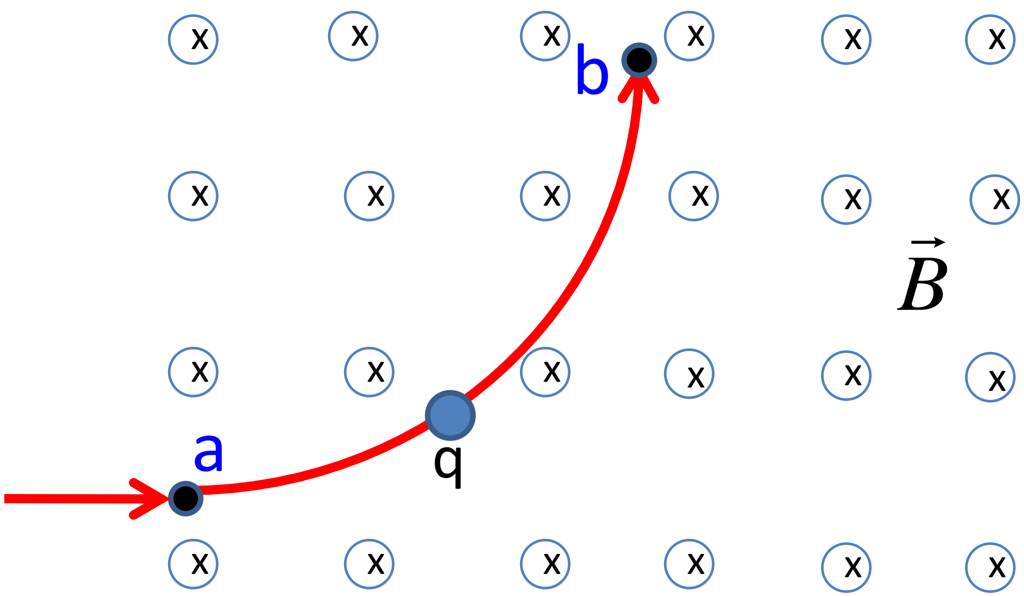
For perpendicular vectors

$$\vec{A} = A\hat{i} \qquad \vec{B} = B\hat{j} \qquad \vec{C} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A & 0 & 0 \\ 0 & B & 0 \end{vmatrix} = AB\hat{k}$$

Top Hat Question

A charged particle q enters a region with a constant B-field pointing into the page as shown. If the particle follows the path from **a** to **b** as shown

$$\boxed{\vec{F} = q\vec{v} \times \vec{B}}$$



What is the sign of q?

A. Positive

B. Negative

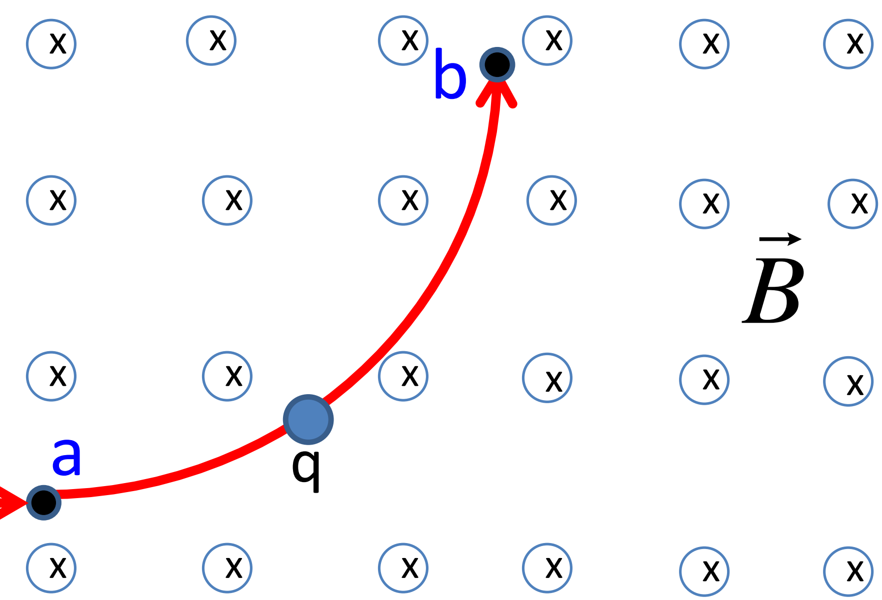
C. Not enough info

Top Hat Question

A charged particle q enters a region with a constant B-field pointing into the page. The force on the charged particle is

$$\boxed{\vec{F} = q\vec{v} \times \vec{B}}$$

As the particle travels from point **a** to point **b**, its kinetic energy:



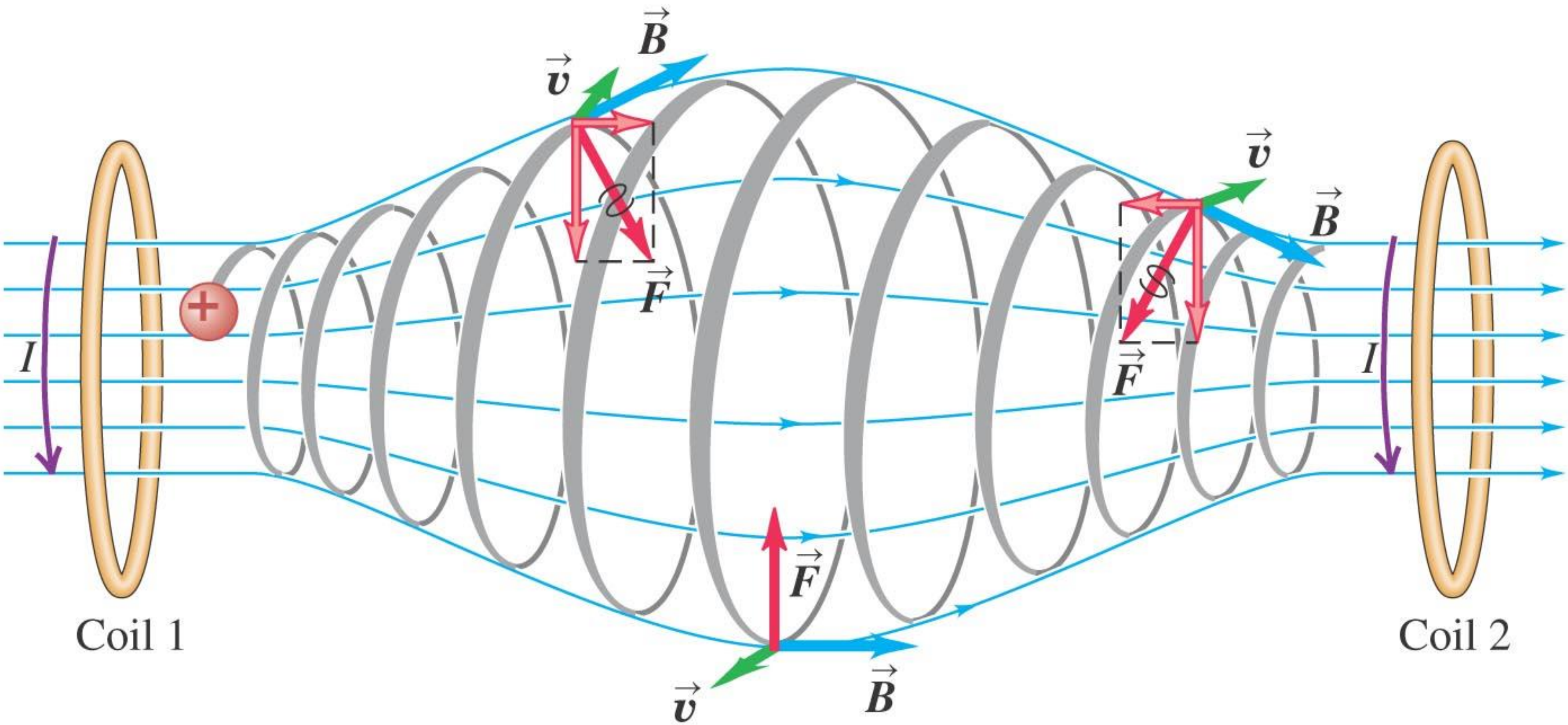
A. Should increase

B. Should decrease

C. Should stay the same

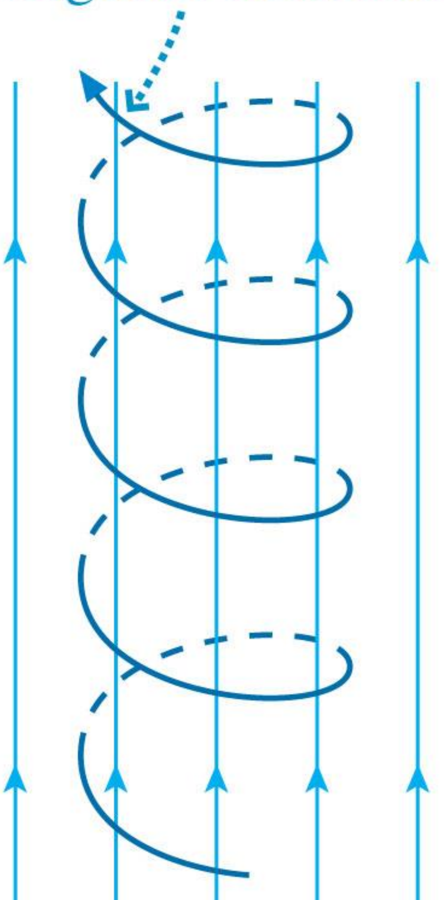
D. Not enough info

Magnetic Ion Trap

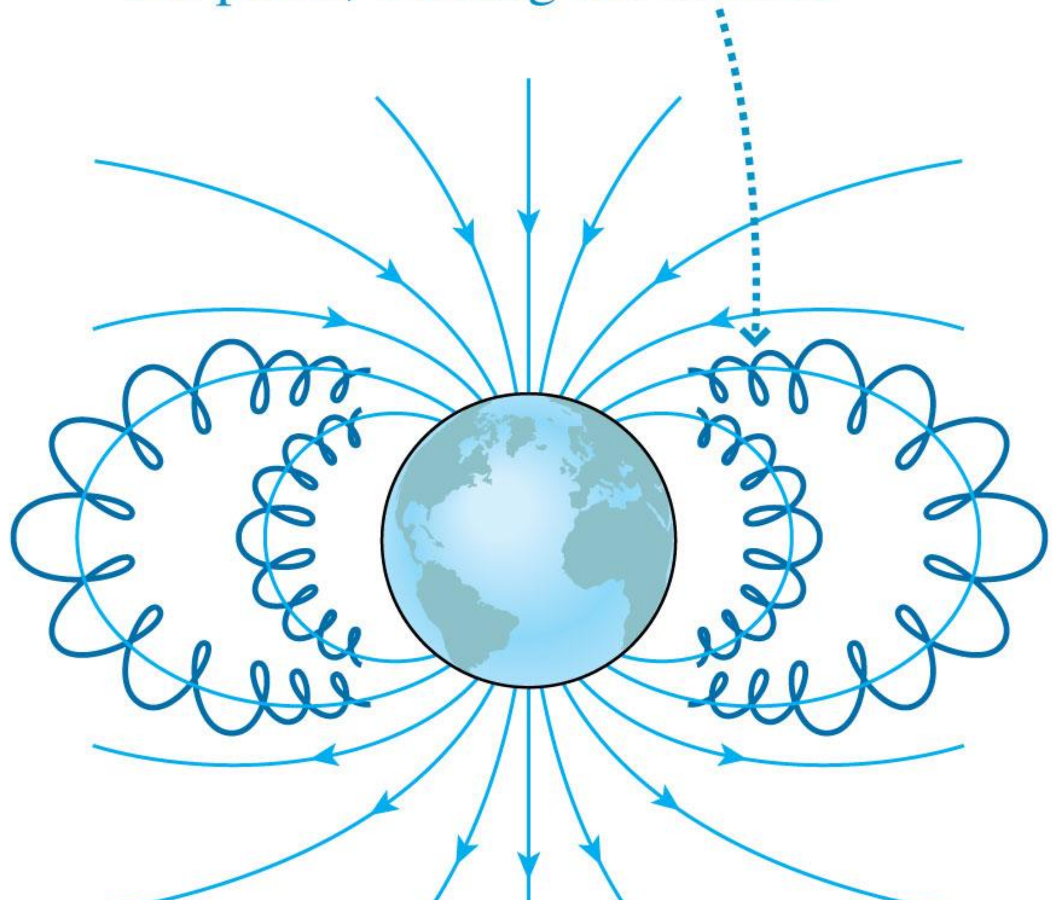


Earth's Van Allen belt
(aurora borealis/australis)

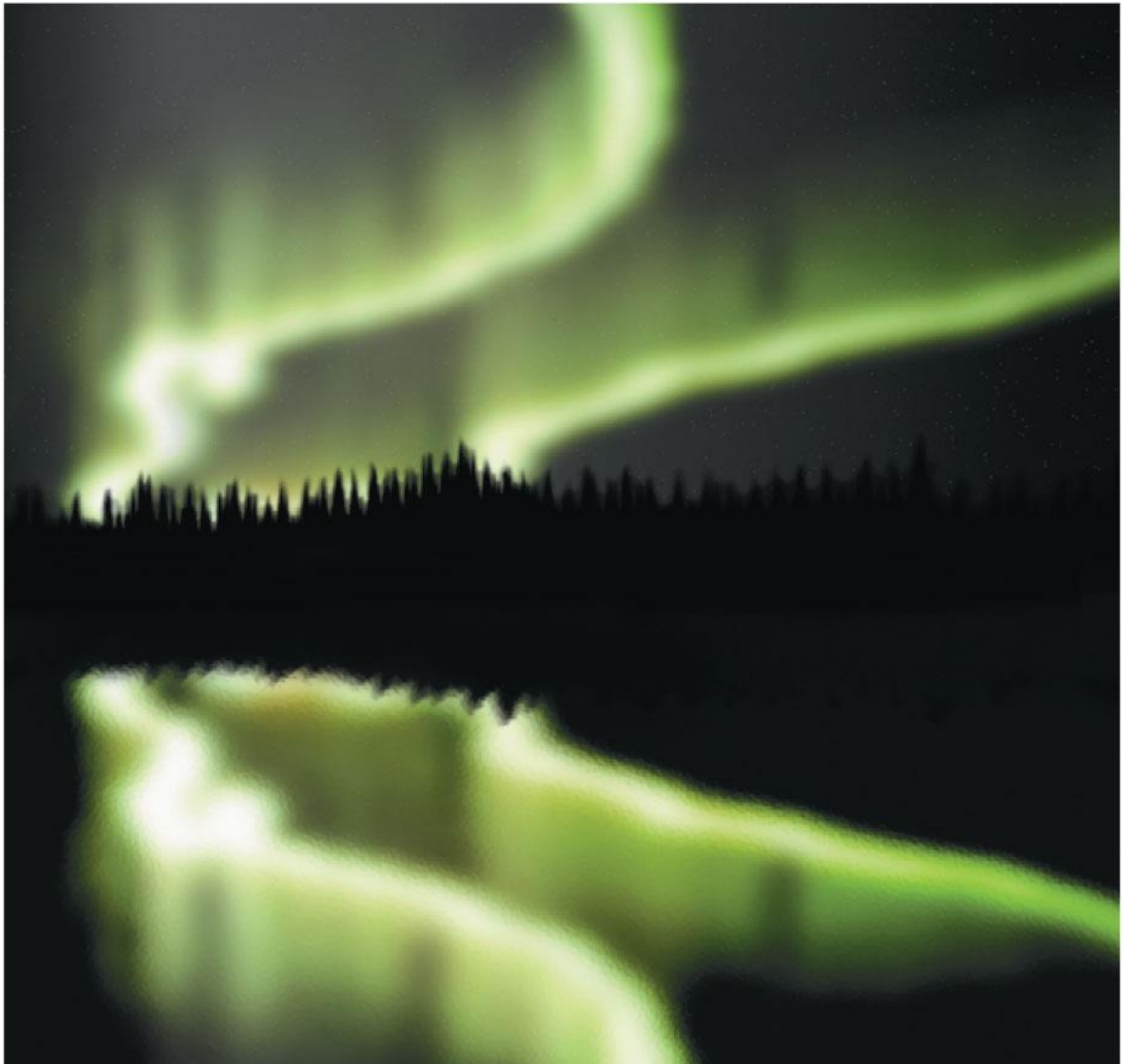
(a) Charged particles spiral around the magnetic field lines.



(b) The earth's magnetic field leads particles into the atmosphere near the poles, causing the aurora.



(c) The aurora



Last time:

- Introduction to magnetism
- Electric force vs magnetic force on charges
- Vector cross product
- Consequences of magnetic force

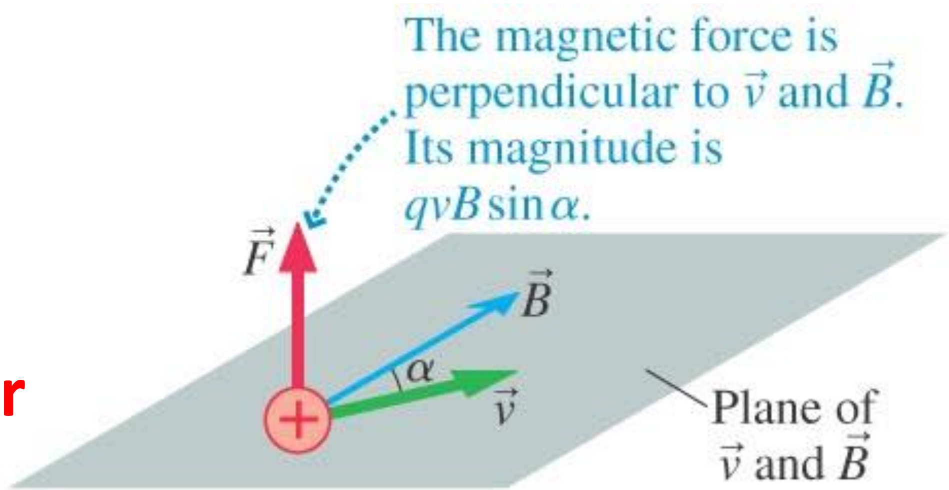
Today:

- Motion of charges in magnetic fields
- Cyclotron motion, cyclotron frequency, q/m
- Mass spectrometers
- Cyclotron as a particle accelerator
- Charges on helical paths in B-field (aurora)

Magnetic Force on Charges

Magnetic force acts only on a moving charge.

It is perpendicular to both B and v.

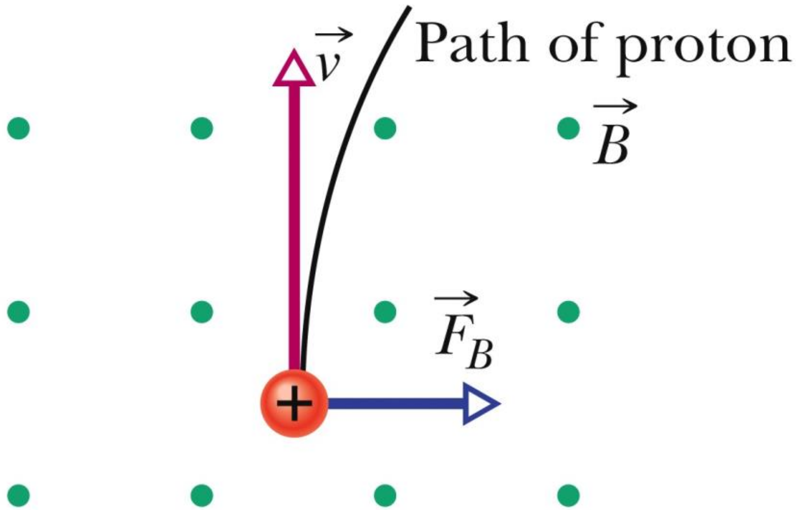


$$\vec{F}_B = q\,\vec{v} \times \vec{B}$$

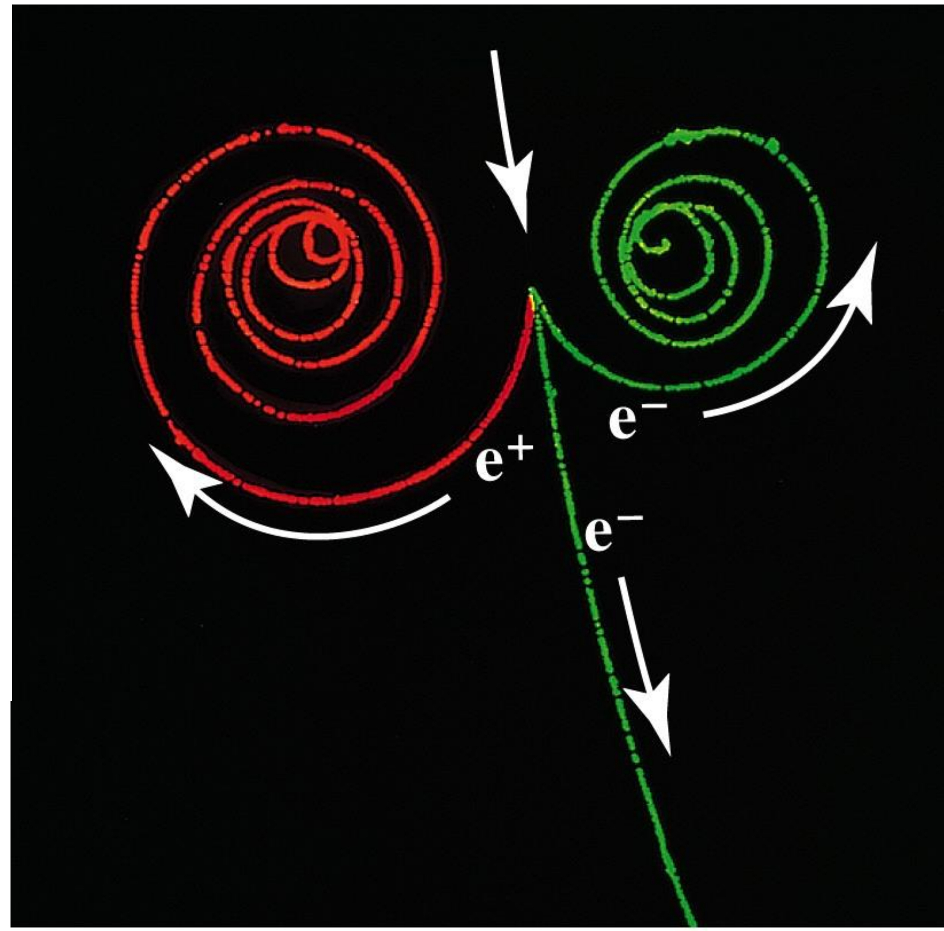
Magnitude: $F_B = qvB\sin\alpha$

Direction: RH rule

Motion of charges in B-field



 halliday_10e_fig_28_06



Lawrence Berkeley Laboratory/Photo Researchers, Inc.

Particle Tracks (Dr Stotyn's Desktop Picture)

Cyclotron Motion

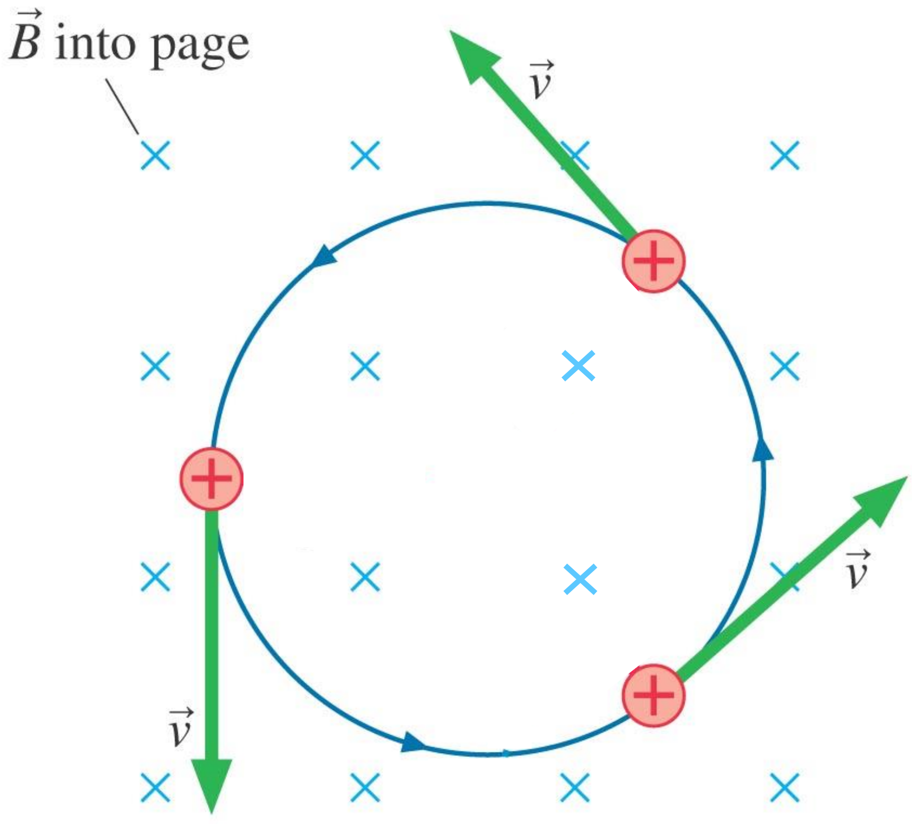
Charged particles in uniform magnetic fields undergo **uniform circular motion**.
The radius of the circle depends on how fast the particle is moving:

$$\vec{F}_B = q\vec{v} \times \vec{B}$$

$$\left|\vec{F}_B\right| = |q| vB \sin\alpha = |q| vB$$

The magnetic force is the **net force**

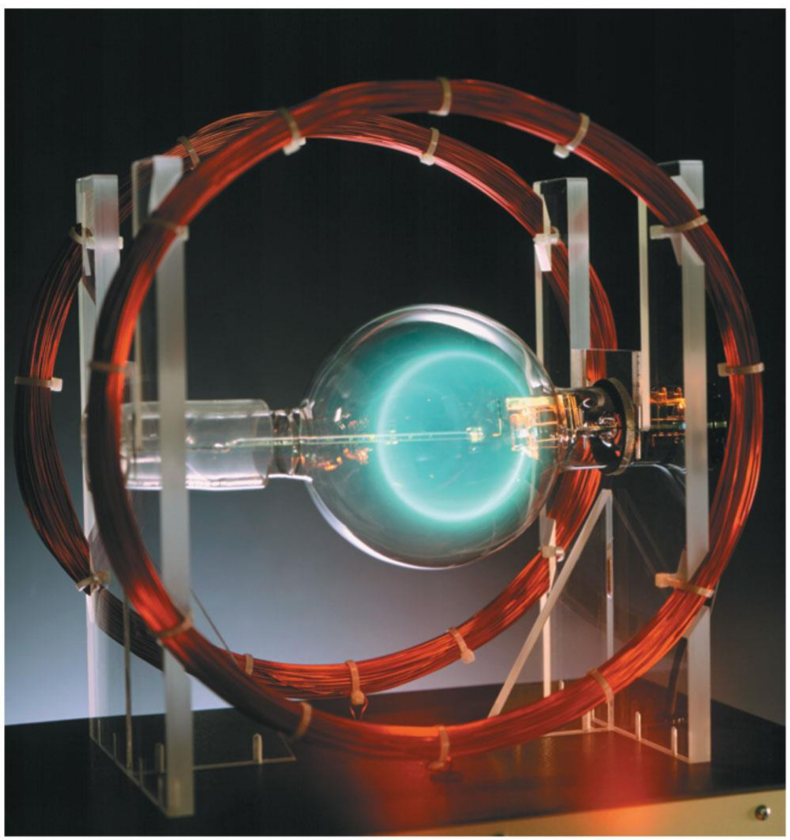
$$\left|\vec{F}_B\right| = m\frac{v^2}{R}$$

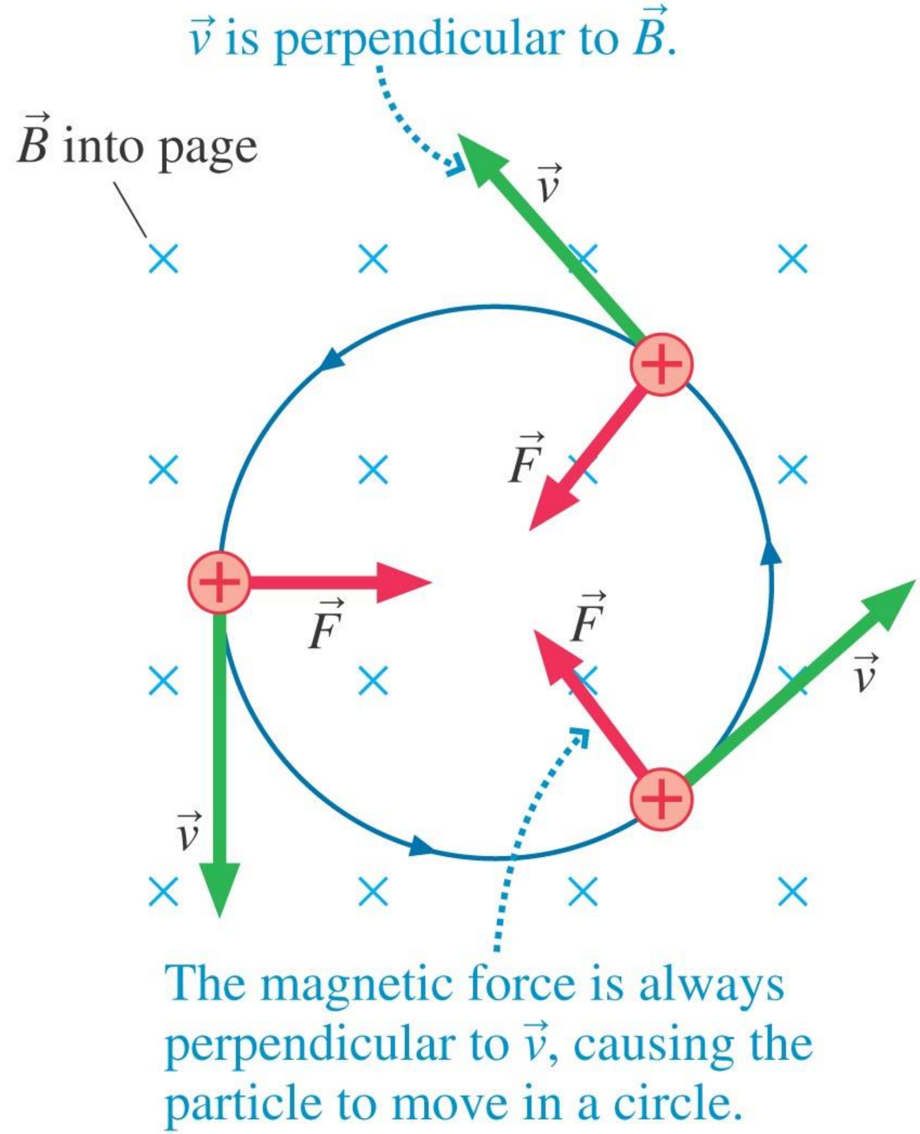


Cyclotron Motion

$$\left|\vec{F}_B\right| = |q|\cancel{v}B = m\frac{v^{\cancel{2}}}{R}$$

$$\boxed{R = \frac{mv}{|q|B}}$$





Cyclotron Motion

$$v = \frac{2pR}{T_{cyc}}$$

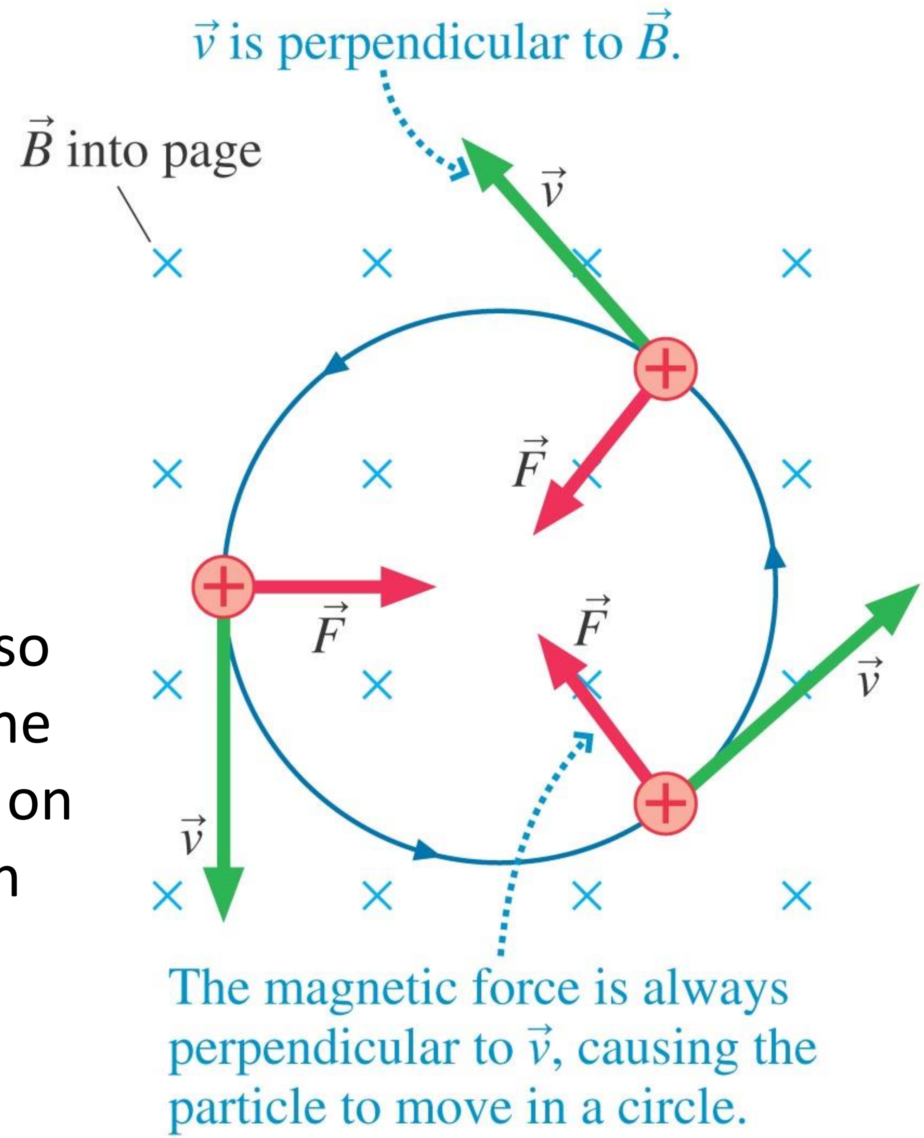
T_{cyc} is the cyclotron period (time it takes to make one cycle)

$$R = \frac{mv}{|q|B} \qquad \cancel{R} = \frac{m}{|q|B}\frac{2p\cancel{R}}{T}$$

$$\boxed{T_{cyc} = \frac{2pm}{|q|B}}$$

$$\boxed{f_{cyc} = \frac{|q|B}{2pm}}$$

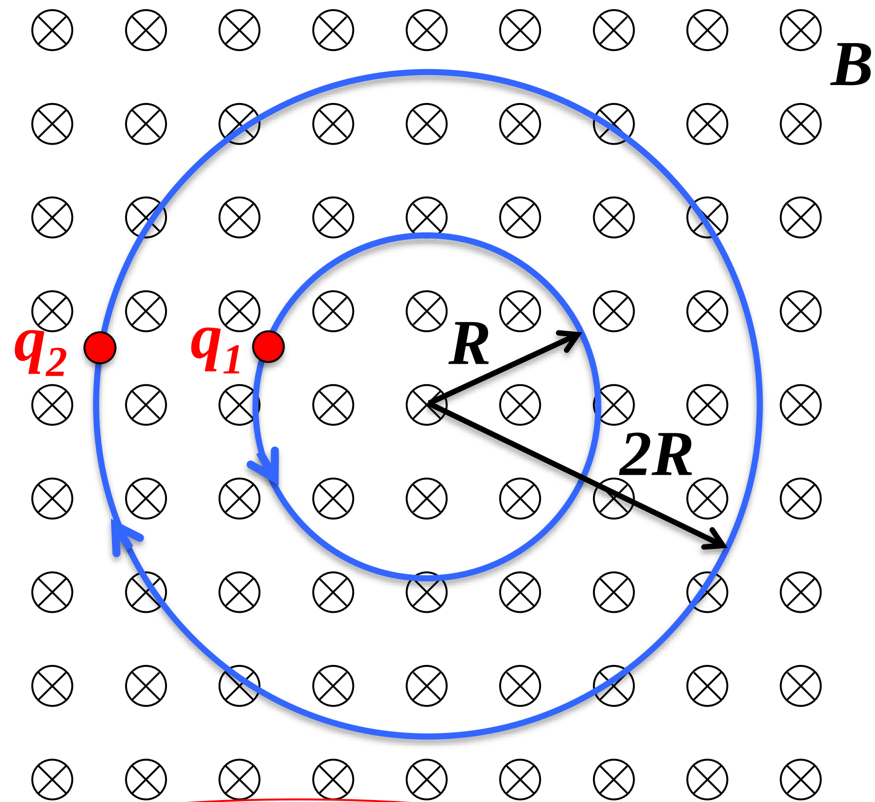
The period (and also the frequency of the cyclotron) depend on the B-field strength and the charge-to-mass ratio q/m



Top Hat Question

Two charges q_1 and q_2 with the same mass m and the same magnitude of charge $|q|$ are undergoing cyclotron motion in a uniform B-field.

What are the **signs of the charges**?



A. Both positive

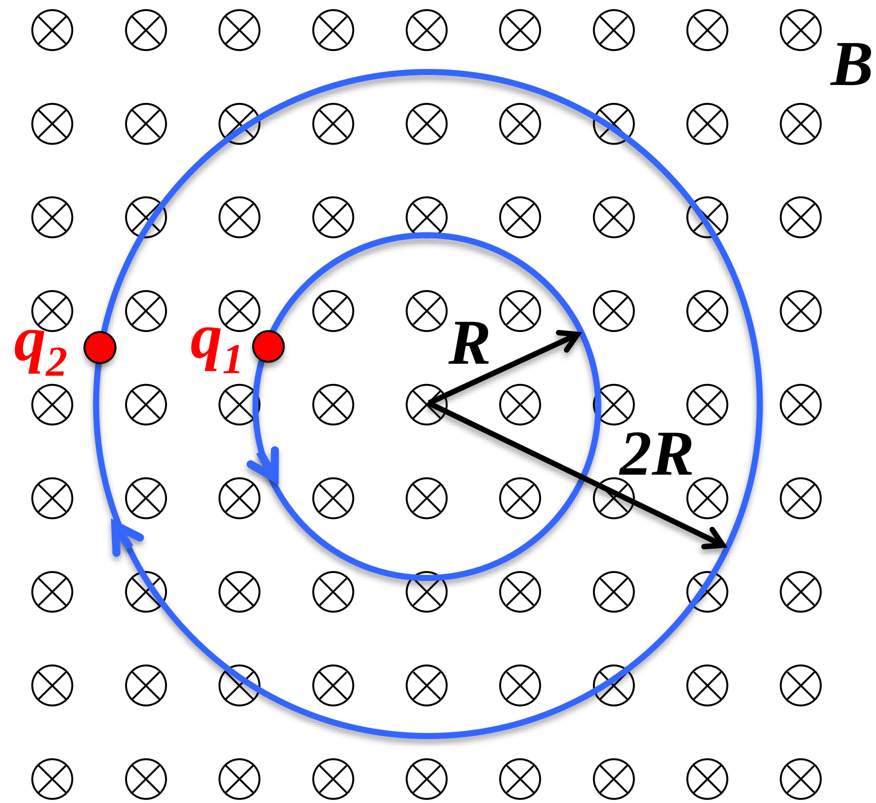
B. Both negative

C. q_1 positive, q_2 negative

D. q_1 negative, q_2 positive

Top Hat Question

Two charges q_1 and q_2 with the same mass m and the same magnitude of charge $|q|$ are undergoing cyclotron motion in a uniform B-field.



If the speed of q_1 is v, what is the speed of q_2?

$$\boxed{R = \frac{mv}{|q|B}}$$

A. v

B. $2v$

C. $\frac{1}{2}v$

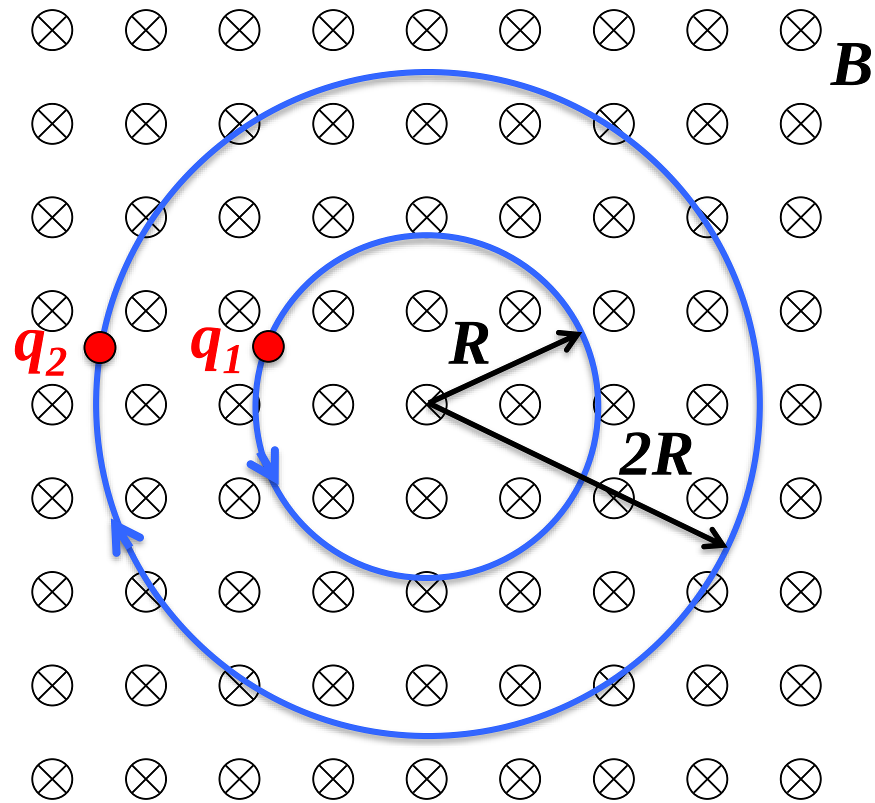
D. $4v$

Top Hat Question

Two charges q_1 and q_2 with the same mass m and the same magnitude of charge $|q|$ are undergoing cyclotron motion in a uniform B-field.

If the period of rotation of q_1 is T, what is the period of rotation of q_2?

$$\boxed{T_{cyc} = \frac{2\pi m}{|q|B}}$$



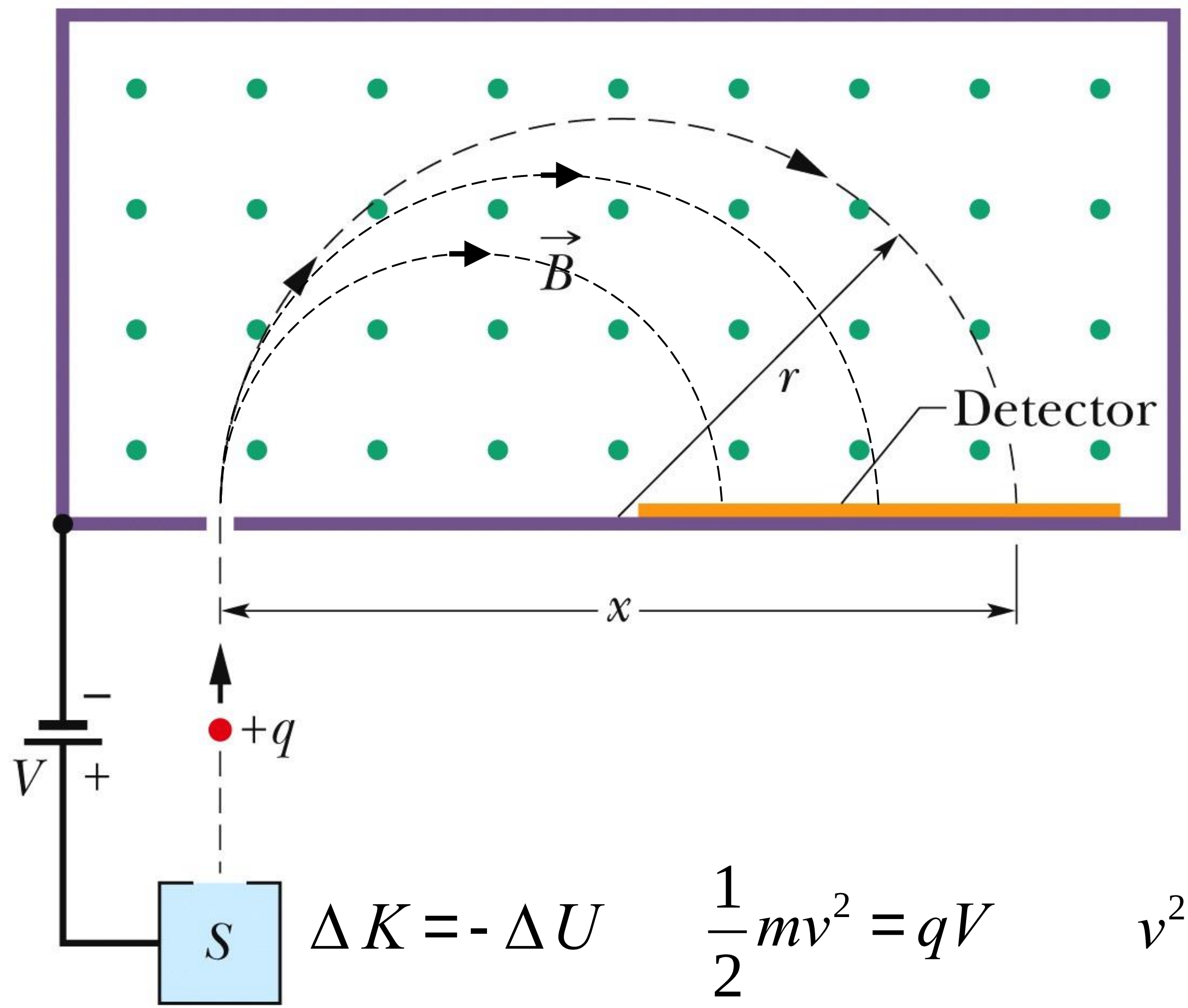
A. T

B. $2T$

C. $\frac{1}{2}T$

D. $4T$

Application: Mass Spectrometer



halliday_10e_fig_28_12

$$r = \frac{mv}{qB} = \frac{x}{2}$$

$$m^2 = \frac{q^2B^2x^2}{4v^2}$$

$$m^2 = \frac{q^2B^2x^2}{4}\frac{m}{2qV}$$

$$\boxed{m = \frac{qB^2x^2}{8V}}$$

$$\Delta K = -\Delta U \qquad \frac{1}{2}mv^2 = qV \qquad v^2 = \frac{2qV}{m}$$

Application: Cyclotron

In the gap between the dees, charges are accelerated by E-field:

$$\Delta K_{gap} = -\Delta U_{gap} = q\Delta V$$

After N times through the gap:

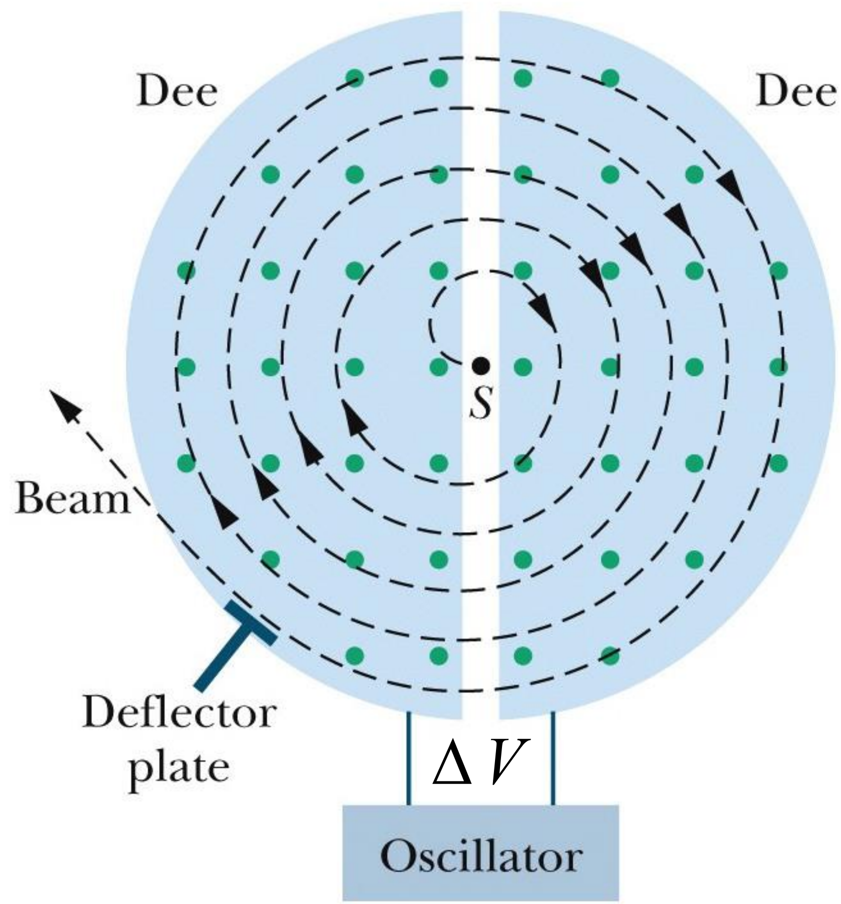
$$\frac{1}{2}mv^2 = Nq\Delta V$$

$$v = \sqrt{\frac{2Nq\Delta V}{m}}$$

$$r = \frac{mv}{qB} = \frac{m}{qB}\sqrt{\frac{2Nq\Delta V}{m}}$$

$$\boxed{= \sqrt{\frac{2Nm\Delta V}{qB^2}}}$$

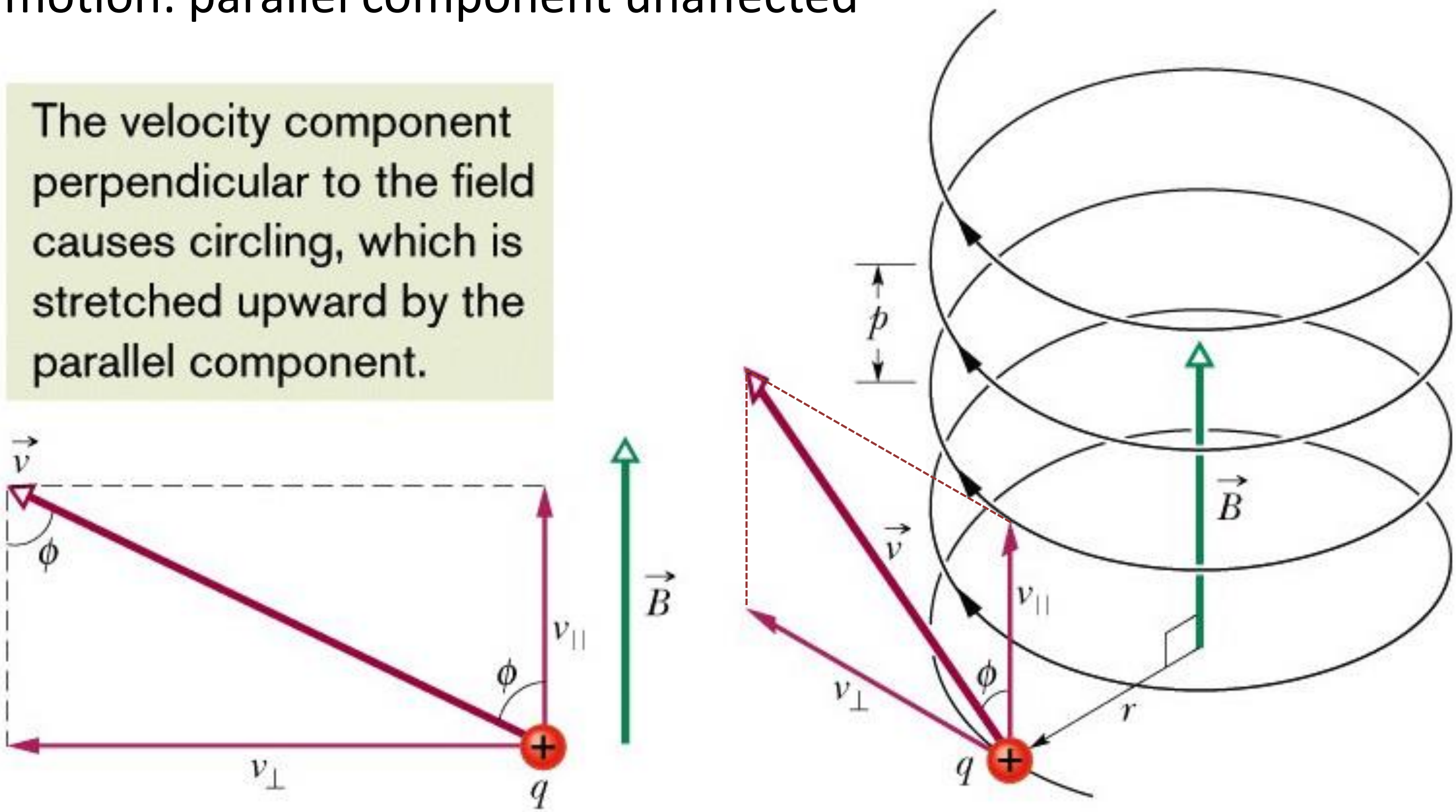
The protons spiral outward in a cyclotron, picking up energy in the gap.



halliday_10e_fig_28_13

Helical Paths Through a B-field

Splitting up the velocity into a component parallel to B-field and a component perpendicular to B-field immediately leads to helical motion: parallel component unaffected



Helical Paths: document camera

We can analyze and specify the motion exactly as the charge moves in a helix.

Last time:

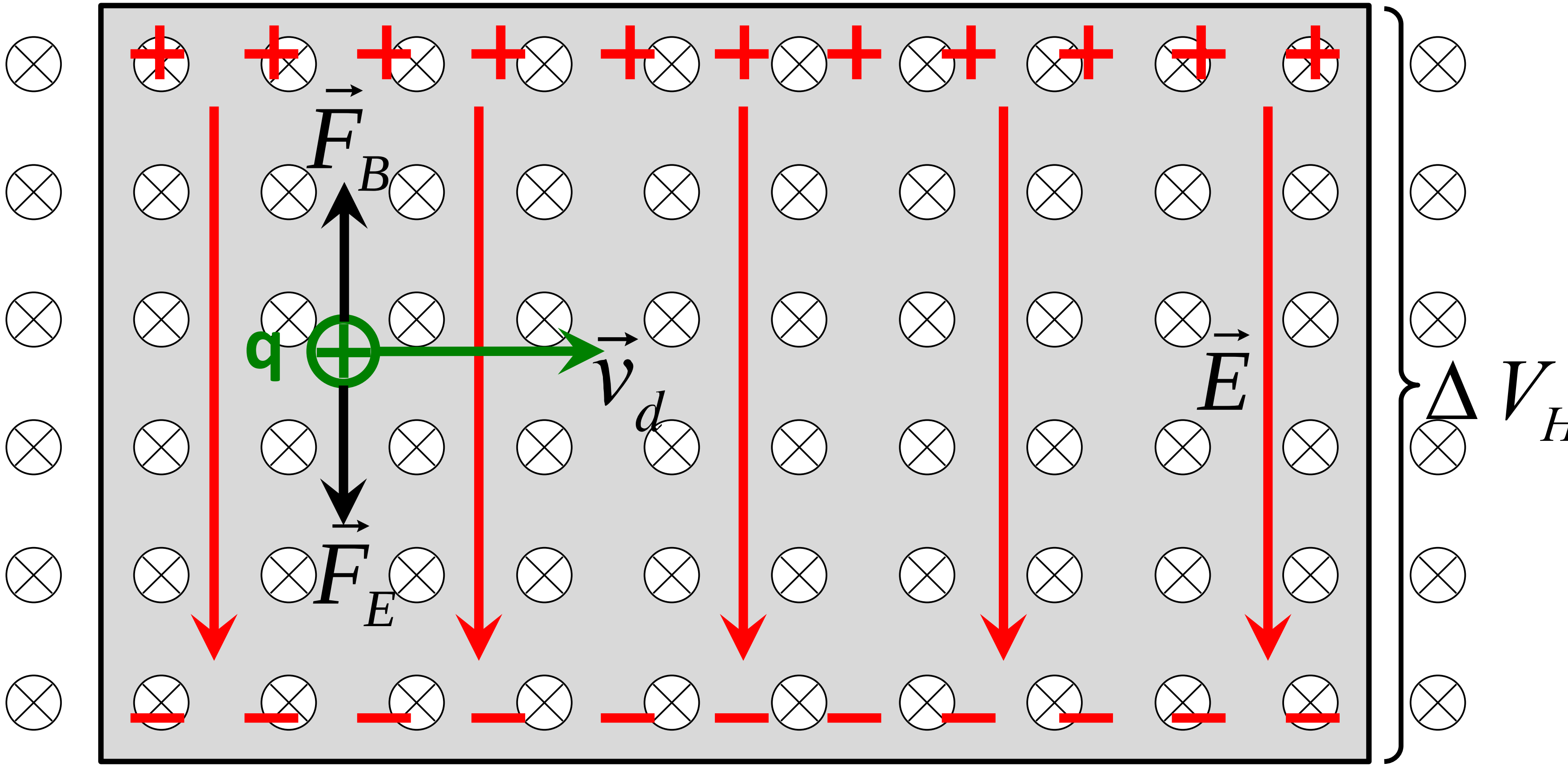
- Motion of charges in magnetic fields
- Cyclotron motion, cyclotron frequency, q/m
- Mass spectrometers
- Cyclotron as a particle accelerator

Today:

- Conductors moving through B-fields: Hall(ish) Effect
- Magnetic force on current carrying wires
- Torque on a current loop

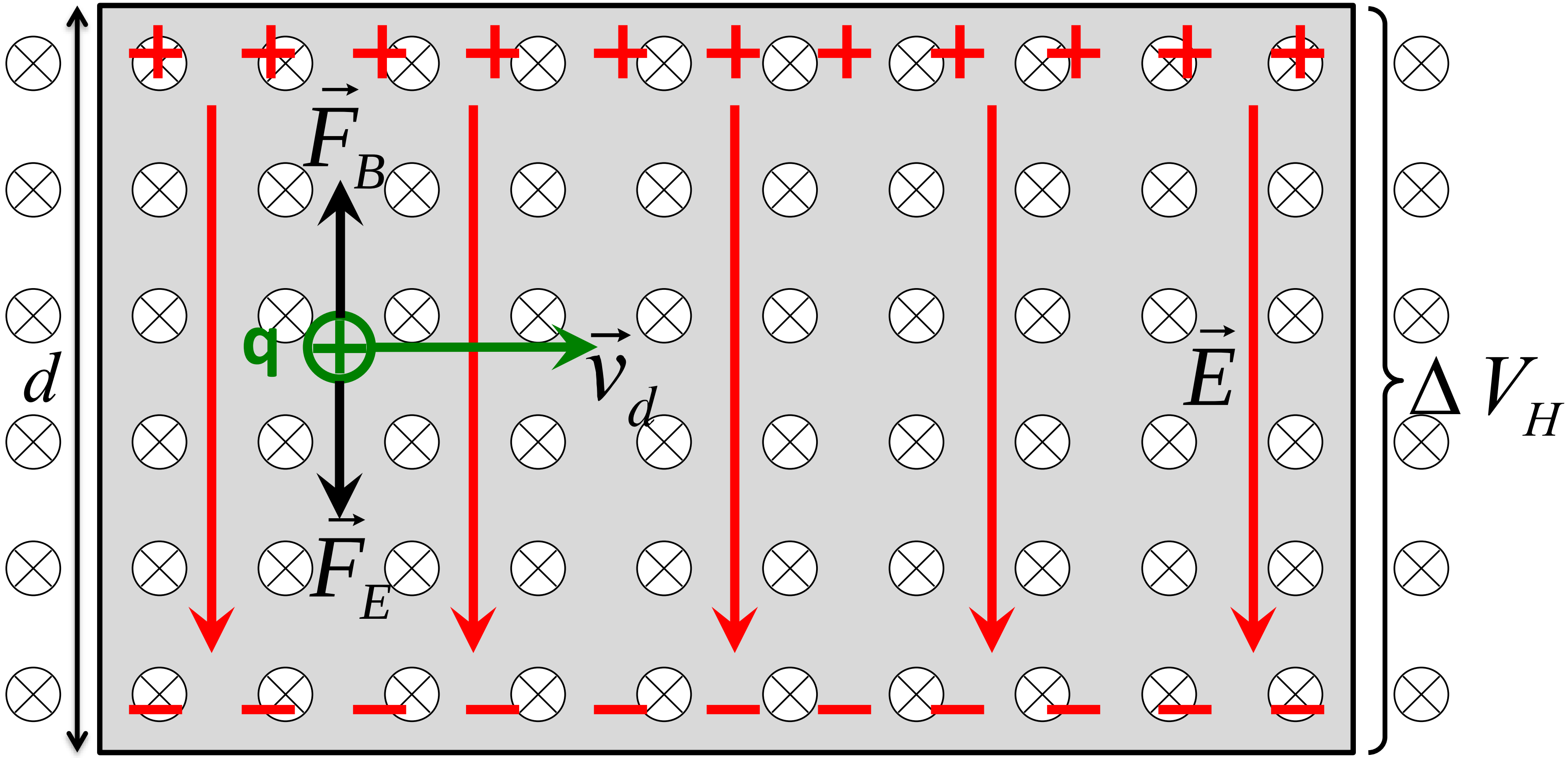
The Hall Effect

Due to the B-field, net charge build up on the edges.



In equilibrium, current still flows. Need to balance the magnetic and electric forces on the charge carriers.

The Hall Effect



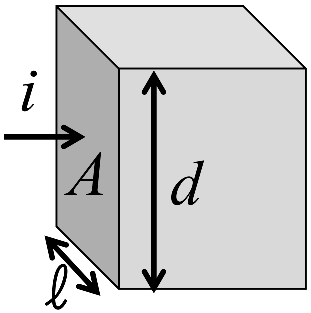
$$F_B = q v_d B \qquad F_E = q\frac{\Delta V_H}{d} \qquad q\frac{\Delta V_H}{d} = q v_d B \qquad \boxed{\Delta V_H = v_d B d}$$

The Hall Effect

We have just found that the voltage established across a conductor carrying a current in a magnetic field is

$$\Delta V_H = v_d B d$$

We previously related the drift speed to the current via



$$v_d = \frac{i}{neA}$$ where $A = \ell d$ and n is a material property

We can then relate the Hall voltage to known quantities:

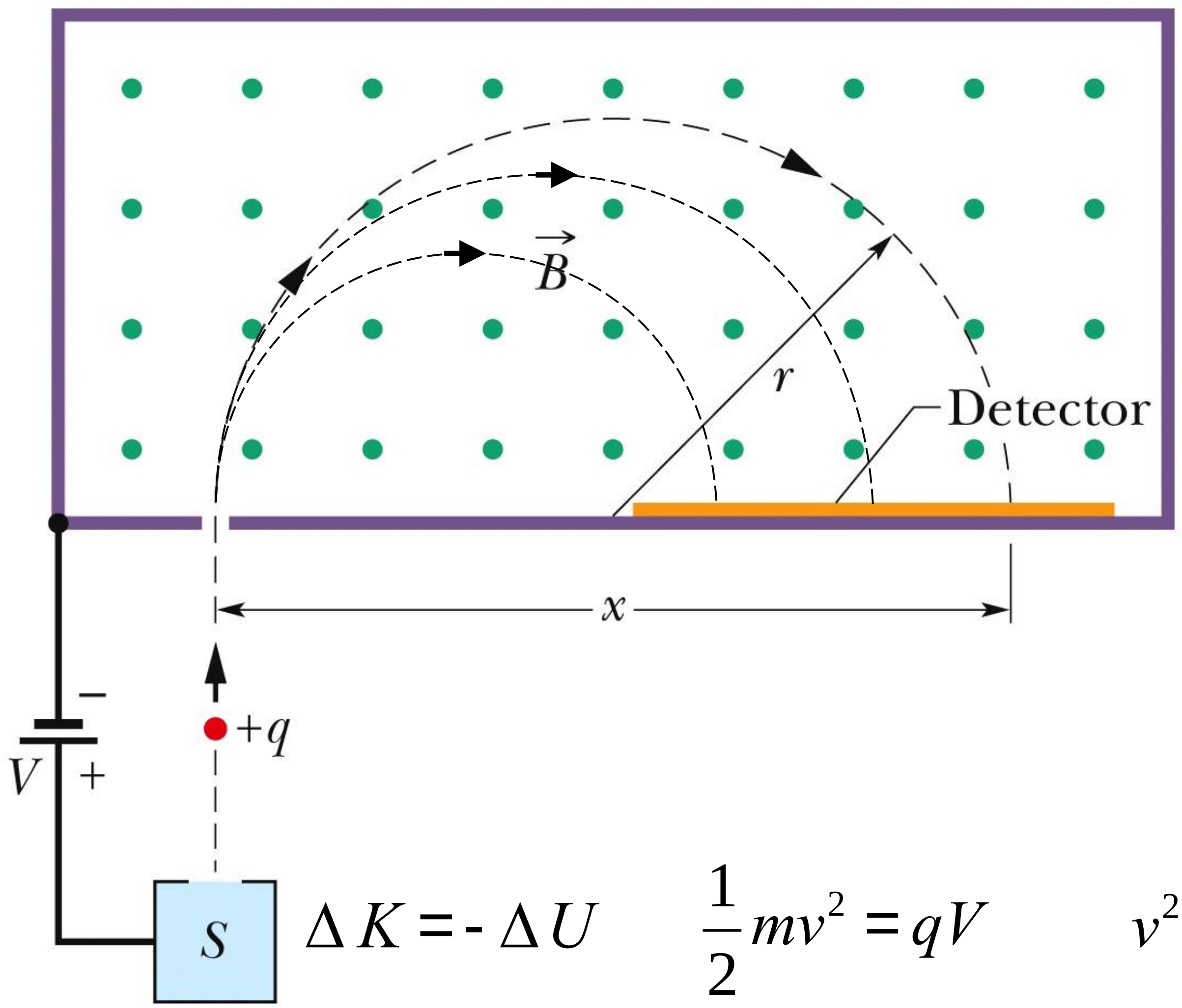
$$\Delta V_H = \frac{i}{ne\ell \cancel{d}} B\cancel{d} = \frac{iB}{ne\ell}$$

In practical applications, you measure ΔV_H to find B:

$$\boxed{B = \frac{ne\ell}{i} \Delta V_H}$$

How the B-field probe used in the next lab works

Reminder: Mass Spectrometer



$$r = \frac{mv}{qB} = \frac{x}{2}$$

$$m^2 = \frac{q^2B^2x^2}{4v^2}$$

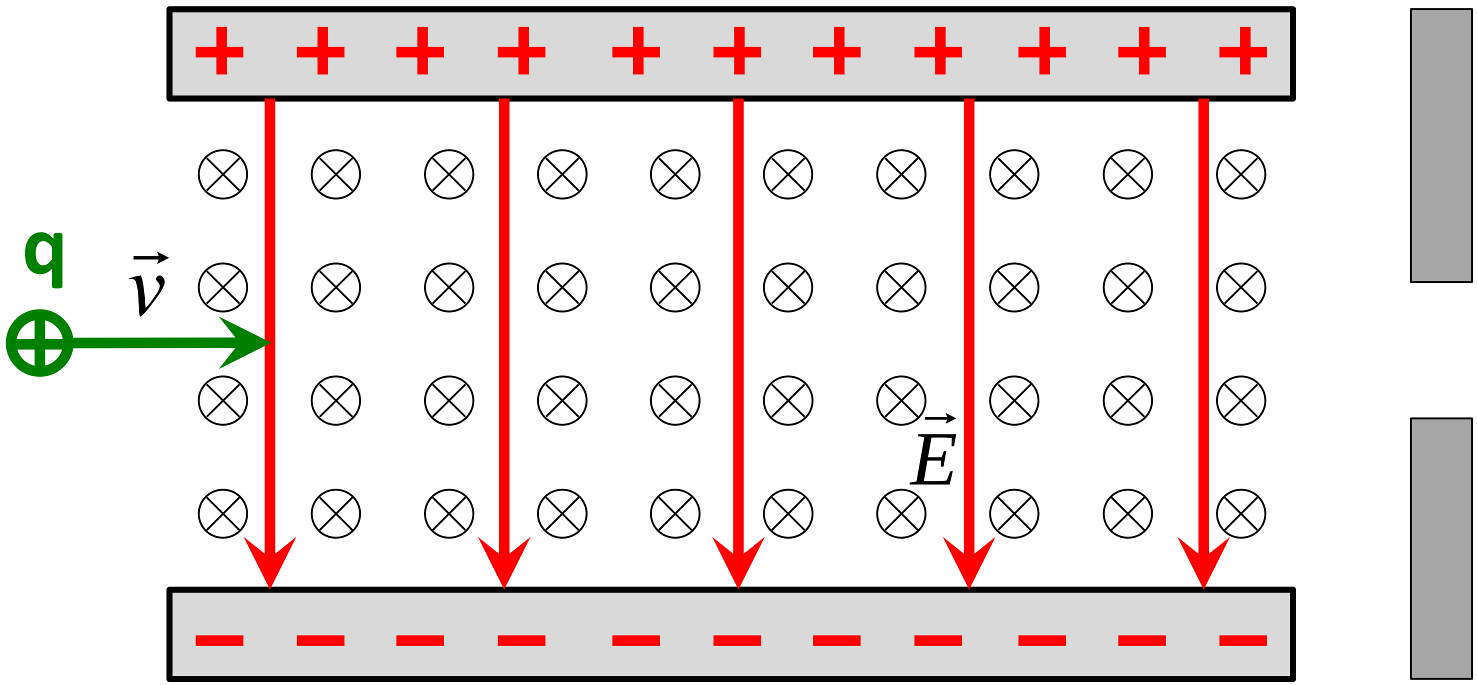
$$m^2 = \frac{q^2B^2x^2}{4}\frac{m}{2qV}$$

$$\boxed{m = \frac{qB^2x^2}{8V}}$$

$$\Delta K = -\Delta U \qquad \frac{1}{2}mv^2 = qV \qquad v^2 = \frac{2qV}{m}$$

 halliday_10e_fig_28_12

Similar concept: velocity selector

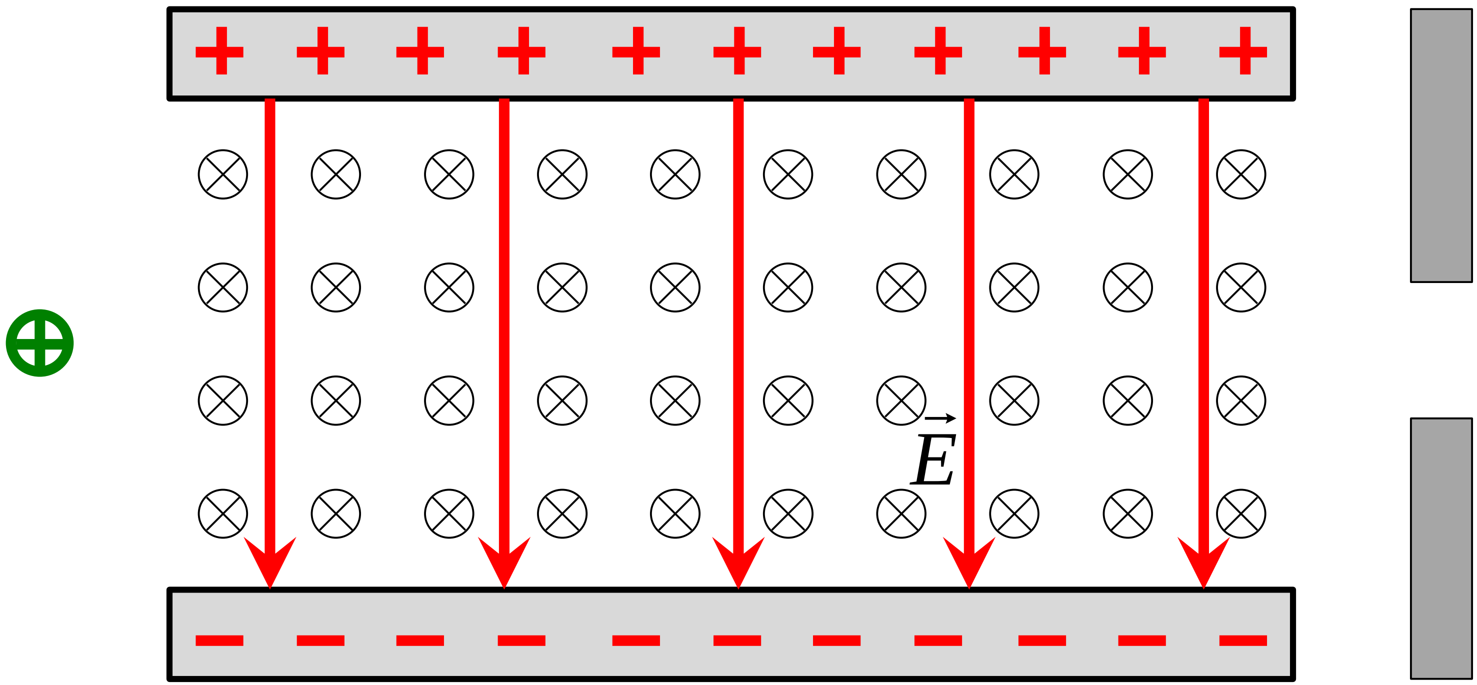


In a velocity selector, you send a charge through a region with crossed E and B fields, which leads to electric and magnetic forces:

$$\vec{F}_e = q\vec{E} \qquad \vec{F}_B = q\vec{v} \times \vec{B} \qquad qE = qvB \qquad v = \frac{E}{B}$$

If the forces balance ($F_{net} = 0$) the charge makes it through the slit

Similar concept: velocity selector



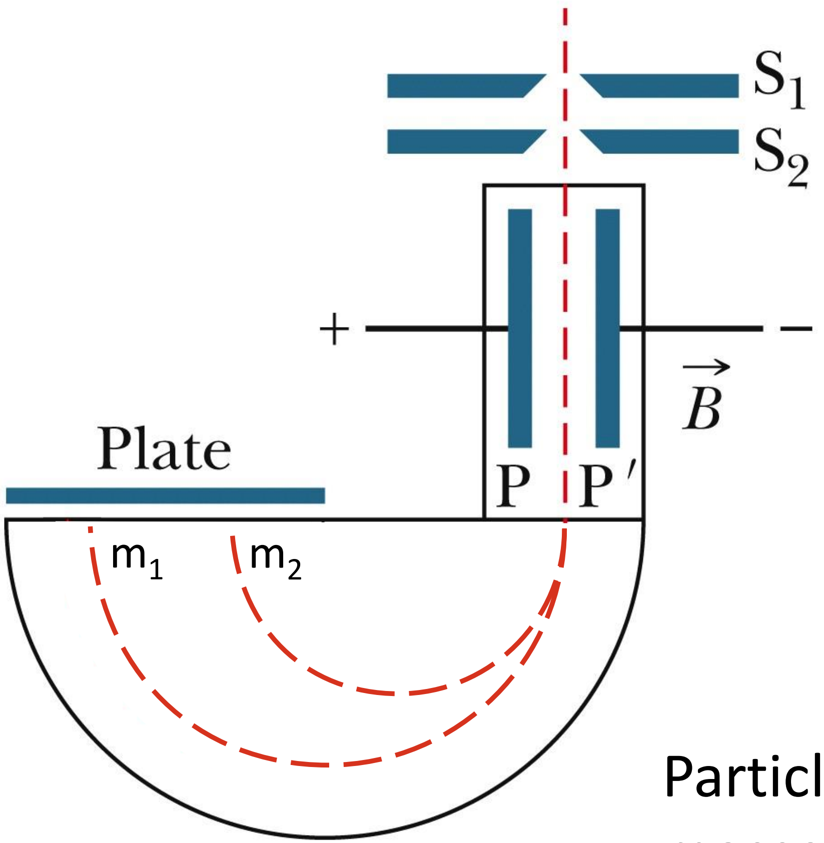
If the forces don't balance the charge hits the wall

$$qE - qvB = ma$$

We pick the E and B magnitudes to select the speeds we want

Bainbridge Mass Spectrometer

Accelerate charges through ΔV so they all have same Kinetic Energy



 halliday_10e_fig_28_54

The slits S_1 and S_2 ensure the beam of particles is collimated.

The beam enters a region of crossed E and B-fields

A narrow slit ensures only particles with a specific speed enter

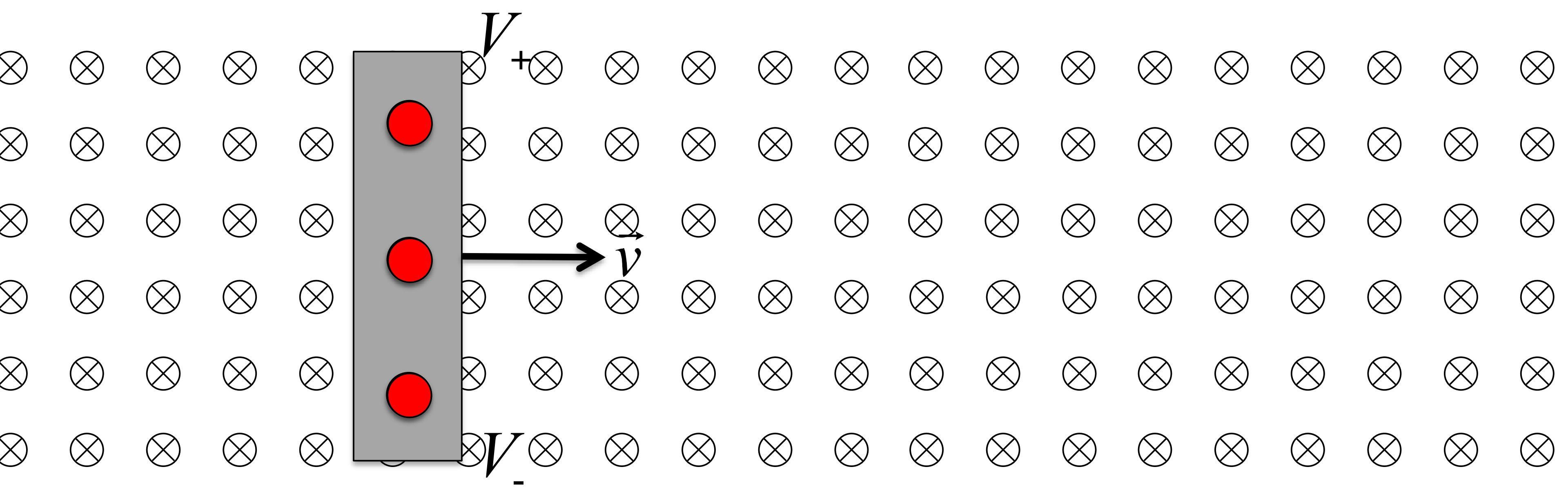
Particles with same KE but different masses and charges will have different radius in B field

Conductors moving in B-fields

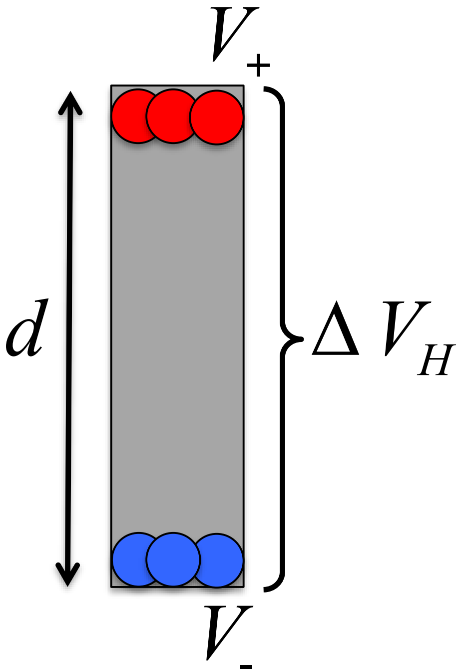
We've seen that free charges moving in a B-field feel a force perpendicular to the field and the charge's velocity:

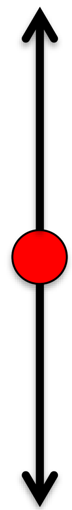
$$\vec{F}_B = q\vec{v}\times\vec{B}$$

Conductors are full of charges that are free to move around (yet they have to stay confined to the conductor itself). If a conductor moves in a magnetic field, these charges also feel a magnetic force



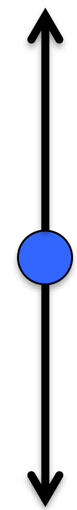
Conductors moving in B-fields





$$F_B = q v B$$

$$F_E = q \frac{\Delta V_H}{d}$$



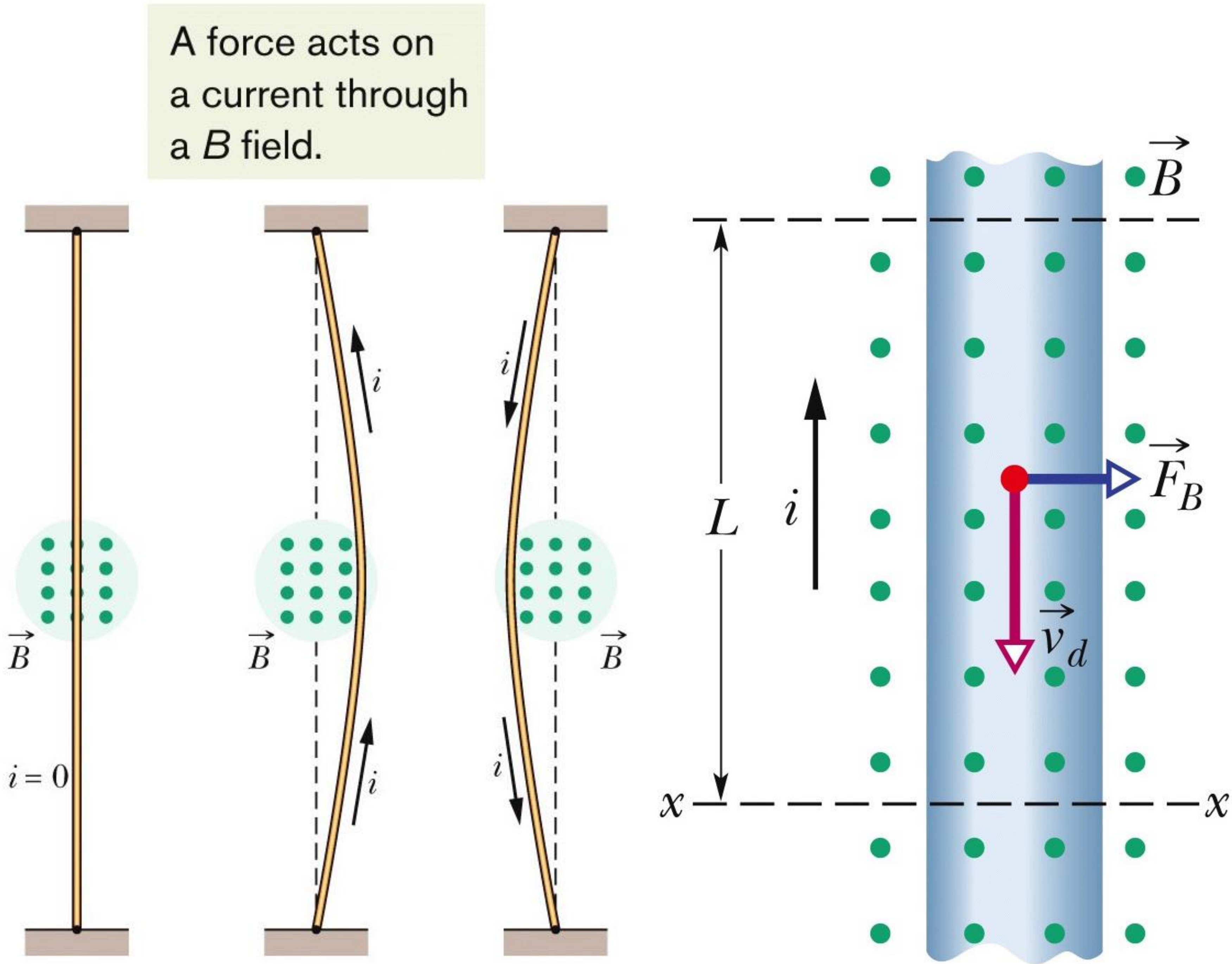
$$F_E = q \frac{\Delta V_H}{d}$$

$$F_B = q v B$$

In equilibrium, forces balance, leading to a constant voltage

$$q \frac{\Delta V_H}{d} = q v B$$

$$\boxed{\Delta V_H = vBd}$$


A force acts on
a current through
a B field.
B
i = 0
i
B
i
i
B
L
i
F_B
v_d
x
x

Forces on Current-Carrying Wires

Current in wires is nothing more than charges in motion. It doesn't matter if we consider $-q$ moving opposite i or $+q$ moving in the same direction as i

In a magnetic field, these charges feel a force and get deflected from their normal straight path. For a single charge:

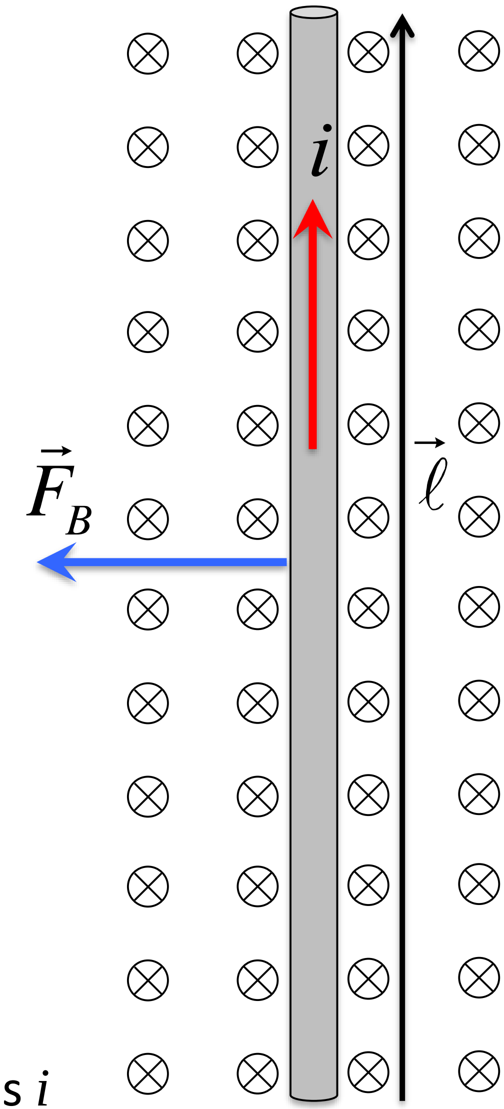
$$\vec{F}_B = q\vec{v}_d \times \vec{B}$$

For N charges moving through the wire:

$$Nq\vec{v}_d = \left(nAq\vec{v}_d\right)\ell = i\vec{\ell}$$

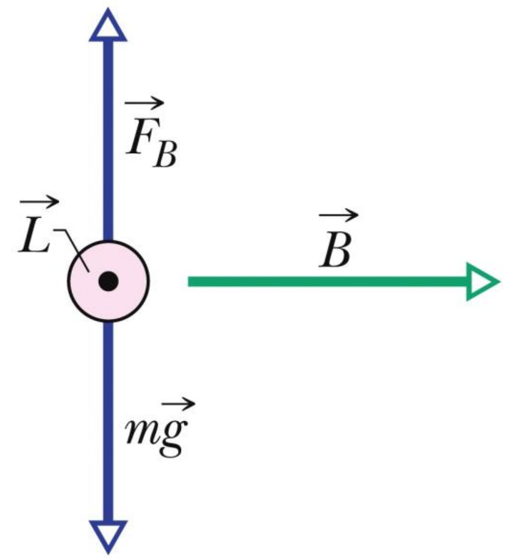
$$\boxed{\vec{F}_B = i\vec{\ell} \times \vec{B}}$$

Length of wire, direction same as i



TopHat Question

A wire of length 50 cm is carrying a current i out of the page and is sitting in a uniform magnetic field of 500 mT pointing to the right. If the wire has a mass of 25 g, what current i is needed to support its weight?



halliday_10e_fig_28_17

$$\vec{F}_B = i\vec{\ell} \times \vec{B}$$

A. 9.81×10^{-3} A

C. 0.981 A

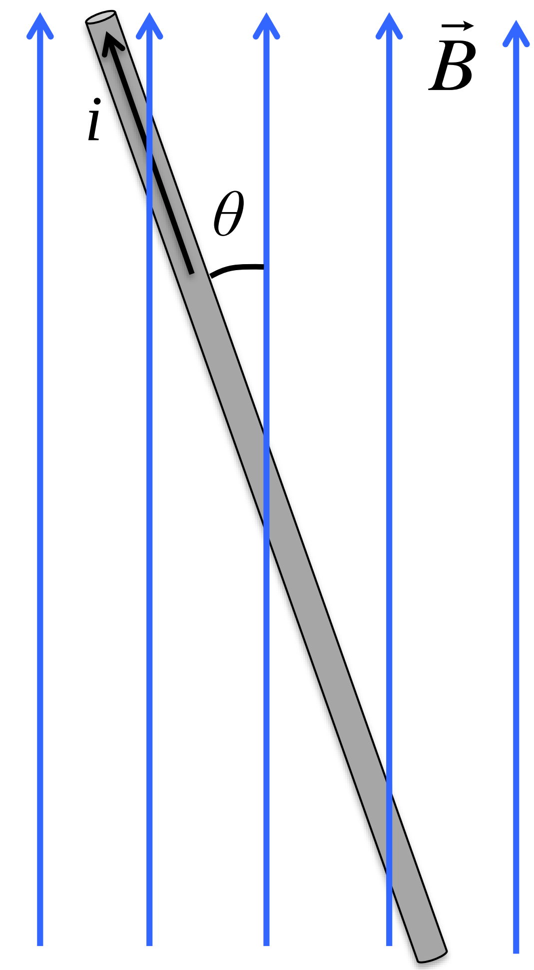
A. 1.02 A

D. 1.02×10^{-3}

TopHat Question

A wire of length 50 cm is carrying a current i and is sitting in a uniform magnetic field B as shown. What is the magnitude and direction of the magnetic force on the wire?

$$\vec{F}_B = i\vec{\ell} \times \vec{B}$$



A. ilB $\odot$

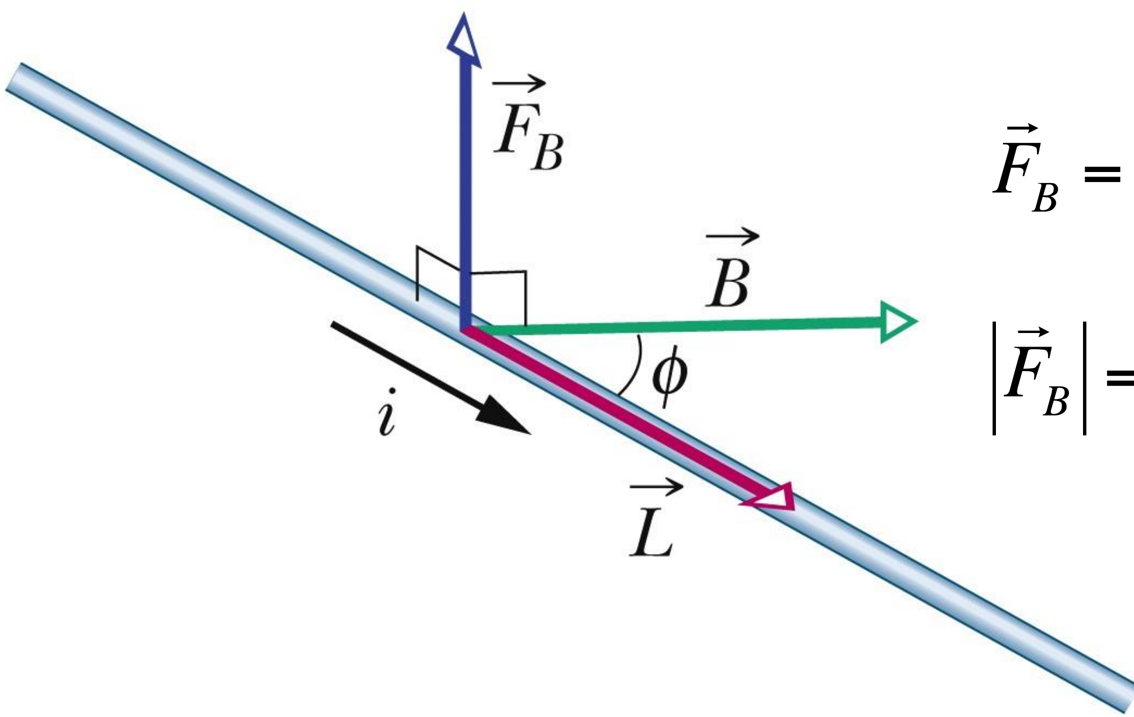
B. $ilB\sin\theta$ $\otimes$

C. $ilB\sin\theta$ $\swarrow$

D. ilB $\nearrow$

Forces on Current-Carrying Wires: B and L not perpendicular

The force is perpendicular to both the field and the length.

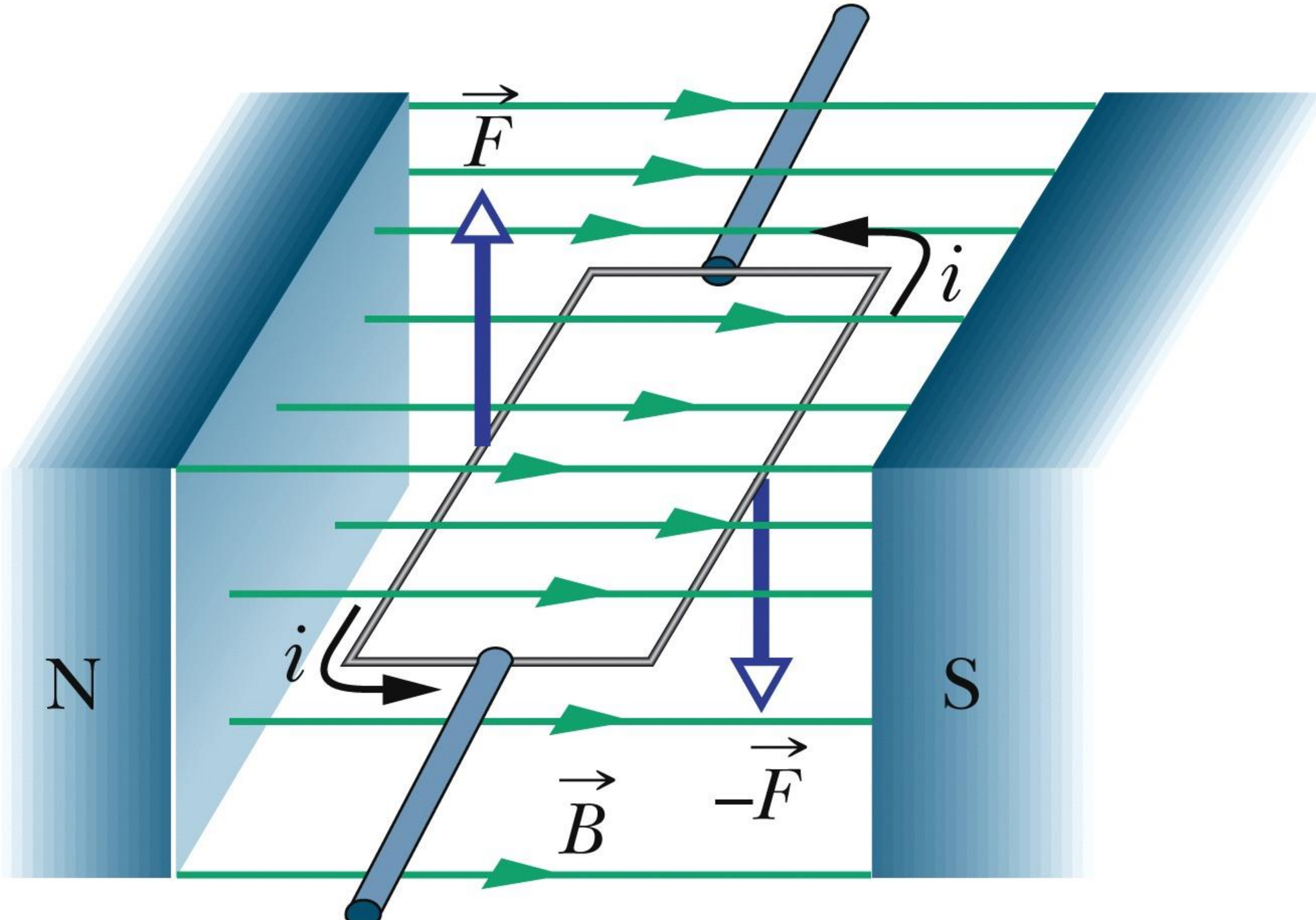


$$\vec{F}_B = i\vec{L} \times \vec{B}$$

$$\left|\vec{F}_B\right| = \left|i\vec{L} \times \vec{B}\right| = iLB\sin\phi$$

halliday_10e_fig_28_16

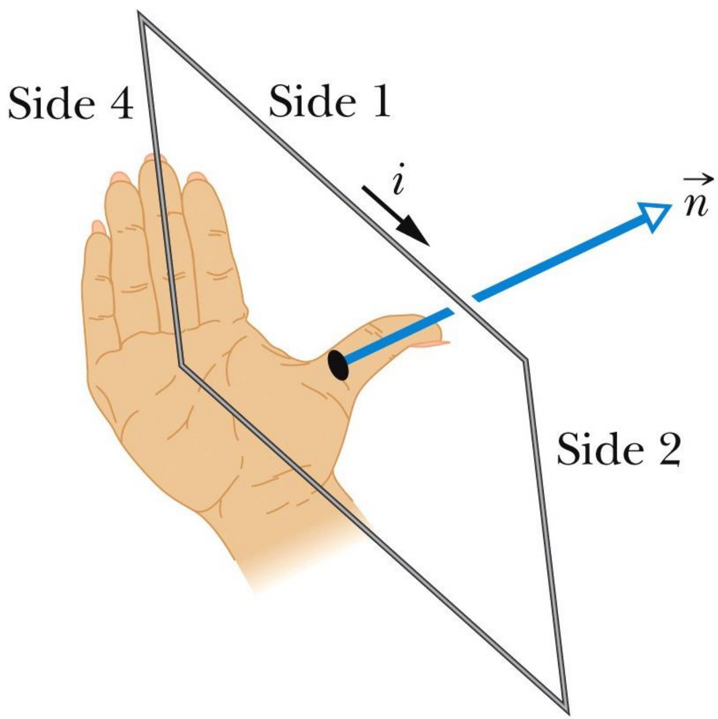
Torque on a current loop

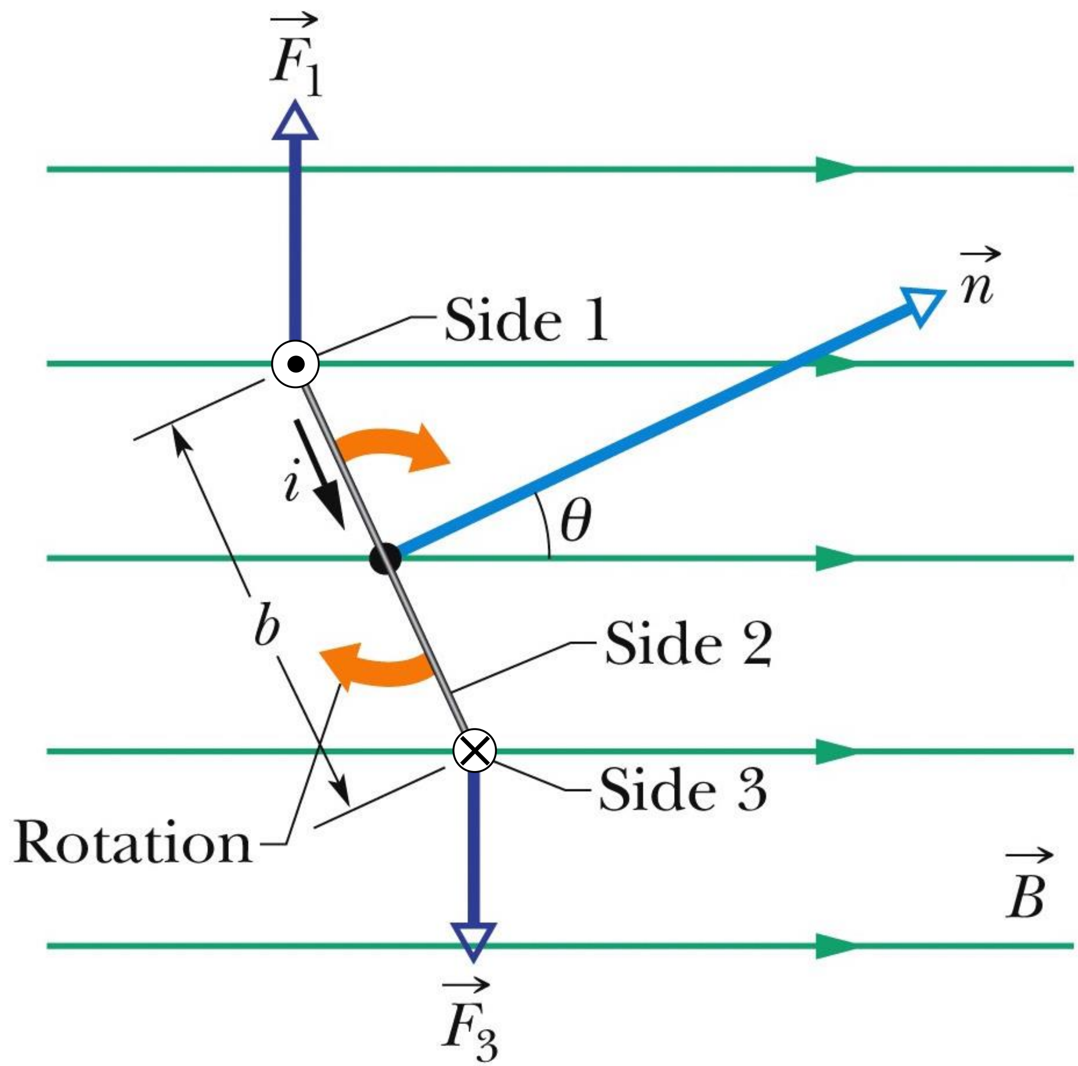


halliday_10e_fig_28_18

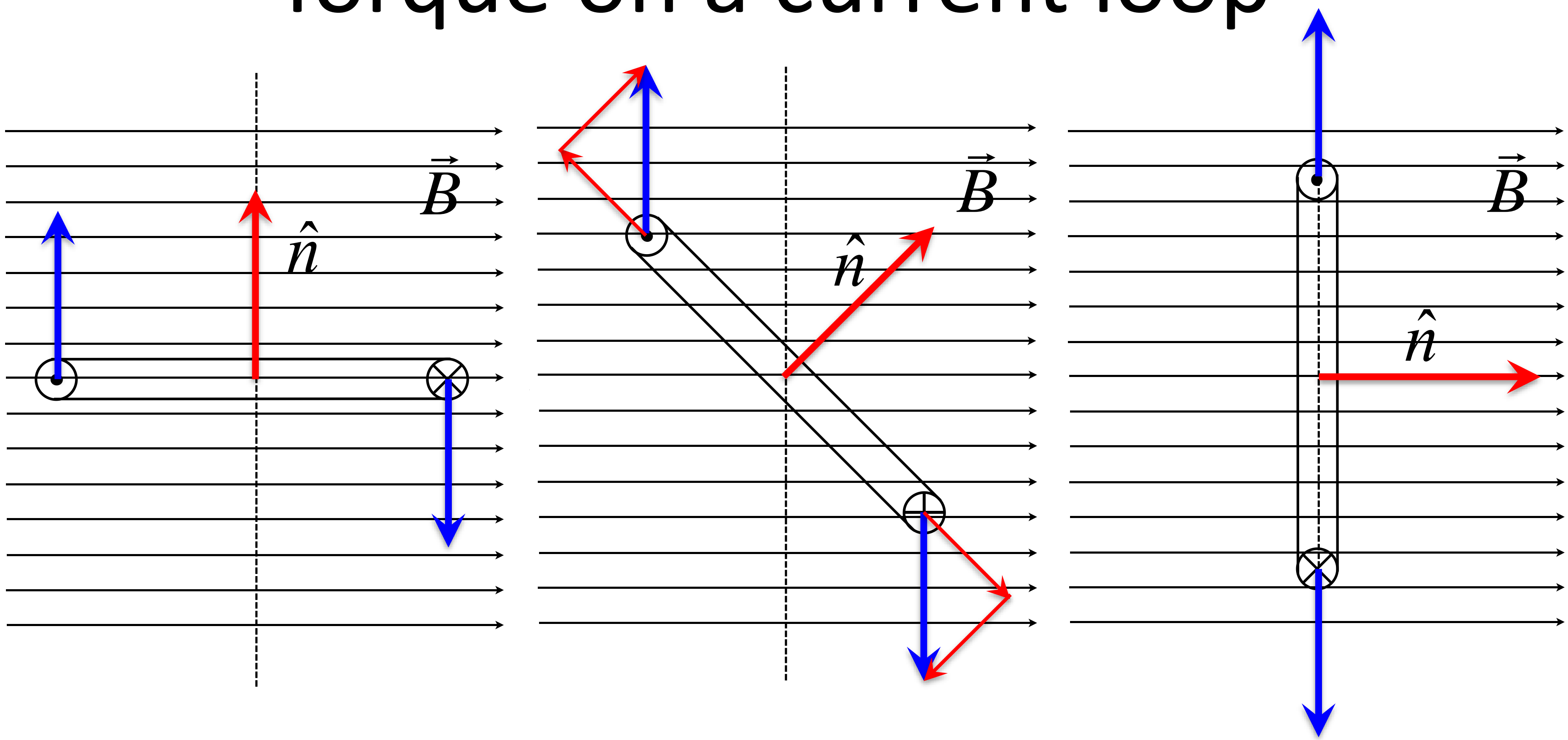
Torque on a current loop

Pick the normal vector to the loop area by RHR: curl your fingers in the direction of i, thumb points in direction of n





Torque on a current loop



The normal vector is at right angles to the B-field: all magnetic force causes rotation of the loop

The normal vector is at some angle to the B-field: some of the magnetic force causes rotation of the loop

The normal vector is parallel to the B-field: none of the magnetic force causes rotation of the loop

Conclusion: components of magnetic force (anti)parallel to normal vector that cause torque