PURE MATHEMATICS 319 L01 WINTER 2016
Practice Problems for Quiz 3

2. Recall that $\mathcal{I}$ if the group of all isometries of $\mathbb{R}^2$. Determine whether the following statements are true or false.

(a) If a subgroup of $\mathcal{I}$ contains a non-identity translation then it must contain at least two reflections.

(b) If a subgroup of $\mathcal{I}$ contains at least two reflections then it must contain a non-identity translation.

(c) The product of two halfturns with different centres is always a translation.

(d) The product of two rotations with different centres is always a rotation.

(e) The product of a rotation and a translation is always a rotation.

(f) The set of all translations in $\mathcal{I}$ is a subgroup of $\mathcal{I}$.

(g) The set of all halfturns in $\mathcal{I}$ is a subgroup of $\mathcal{I}$.

(h) The set of all rotations in $\mathcal{I}$ is a subgroup of $\mathcal{I}$.

2. Determine whether the following statements are true or false, and briefly explain why.
(a) $\mathcal{F}_2$ can be generated by a translation and a halfturn.

(b) $\mathcal{F}_2$ can be generated by two a halfturns.

(c) $\mathcal{F}_2^1$ can be generated by a halfturn and a reflection.

(d) $\mathcal{F}_2^2$ can be generated by a halfturn and a reflection.

(e) $\mathcal{F}_1^2$ can be generated by two reflections.

(f) Among all frieze group, only $\mathcal{F}_1^1$, $\mathcal{F}_1^3$ and $\mathcal{F}_2^1$ have glide-reflections.

(g) $\mathcal{F}_1 \subseteq \mathcal{F}_1^2 \subseteq \mathcal{F}_2^2$.

(h) $\mathcal{F}_1 \subseteq \mathcal{F}_1^1 \subseteq \mathcal{F}_2^1$.

(i) $\mathcal{F}_1^3 \subseteq \mathcal{F}_1^1$

(j) If a frieze group contains a reflection then it must contain infinitely many reflections.

(k) If a frieze group has a glide reflection then it must have infinitely many glide reflections.