PMAT 319 Winter 2016.
Chapter 2: Transformations.

A *transformation* of $\mathbb{R}^2$ is a bijection from $\mathbb{R}^2$ to $\mathbb{R}^2$. Let $\mathcal{G}$ be the set of all transformations of $\mathbb{R}^2$. We define the binary operation $\circ$ on $\mathcal{G}$ as follows:

$$\text{for } \alpha, \beta \in, \ \alpha \circ \beta\,(P) = \alpha\,(\beta\,(P)) \text{ for each } P \in \mathbb{R}^2.$$

Theorem 2.1: $(\mathcal{G}, \circ)$ is a group.
Proof:

For simplicity, when we write $\alpha\beta$, we means $\alpha \circ \beta$.
Let $\mathcal{C}$ be the set of all collineations from $\mathbb{R}^2$ to $\mathbb{R}^2$.
Theorem 2.2: $(\mathcal{C}, \circ)$ is a group.
Proof: We prove that $(\mathcal{C}, \circ)$ is a subgroup of $(\mathcal{G}, \circ)$ using Theorm 0.1.

We note that $\mathcal{G}$ and $\mathcal{C}$ are not abelian.
An element $\alpha \in \mathcal{G}$ is called an *involution* if and only if $\alpha^2 = i$ and $\alpha \neq i$.