PURE MATHEMATICS 319 L01 WINTER 2016
Practice Problems 6

1. For each of the following statements, determine whether the statement is TRUE or FALSE and give a brief explanation.
(a) $\mathcal{F}_2$ is generated by a halfturn and a non-identity translation.

(b) Both $\mathcal{W}_1^1$ and $\mathcal{W}_1^2$ have glide reflections.

(c) If a point A is a 4-centre of a wallpaper group $\mathcal{W}$ then σ_A is an element of $\mathcal{W}$.

(d) If α and β are isometries so that $\alpha^2 = \beta^2$ then $\alpha = \beta$.

(e) If a frieze group contains a glide reflection γ then γ^2 generates all translations in the frieze group.

(f) $\mathcal{W}_2^1$ can be generated by two halfturns and a reflection.

(g) $\mathcal{W}_1^1$ and $\mathcal{W}_1^2$ can be generated by two translations and a reflection.

(h) $\mathcal{W}_3^1$ and $\mathcal{W}_3^2$ can be generated by two rotations of 120° and a reflection.

(i) $\mathcal{W}_1^3$ and $\mathcal{W}_2^4$ are the only wallpaper groups which has glides reflections but does not have any reflections.

(j) The product of two rotations is always a rotation.

(k) The product of two glide-reflections with parallel axes is a translation.

(l) The product of a translation and a reflection is always a glide-reflection.

(m) The product of two glide-reflections with non-parallel axes is a translation.

(n) $\mathcal{W}_2$ is a subgroup of $\mathcal{W}_4$.

(o) $\mathcal{W}_2$ is a subgroup of $\mathcal{W}_3$.

(p) $\mathcal{W}_2$ is a subgroup of $\mathcal{W}_6$.

(q) $\mathcal{W}_1^3$ is a subgroup of $\mathcal{W}_2^4$.

(r) $\mathcal{W}_2$ does not have a glide-reflection.

(r) $\mathcal{W}_2^1$ does not have a glide-reflection.