Problem Session 108.3 & 4.

Wednesday, November 22, 2023 6:26 PM

Sample Problem

Induced emf in coil due to a solenoid

The long solenoid S shown (in cross section) in Fig. 30-3 has 220 turns/cm and carries a current $i = 1.5$ A; its diameter D is 3.2 cm. At its center we place a 130-turn closely packed coil C of diameter $d = 2.1$ cm. The current in the solenoid is reduced to zero at a steady rate in 25 ms. What is the magnitude of the emf that is induced in coil C while the current in the solenoid is changing?

KEY IDEAS

1. Because it is located in the interior of the solenoid, coil C lies within the magnetic field produced by current i in the solenoid; thus, there is a magnetic flux Φ_B through coil C.
2. Because current i decreases, flux Φ_B also decreases.
3. As Φ_B decreases, emf $\mathscr{E}$ is induced in coil C.

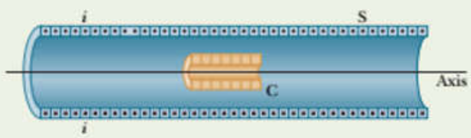


Fig. 30-3 A coil C is located inside a solenoid S, which carries current i.

4. The flux through each turn of coil C depends on the area A and orientation of that turn in the solenoid's magnetic field $\vec{B}$. Because $\vec{B}$ is uniform and directed perpendicular to area A, the flux is given by Eq. 30-2 ($\Phi_B = BA$).
5. The magnitude B of the magnetic field in the interior of a solenoid depends on the solenoid's current i and its number n of turns per unit length, according to Eq. 29-23 ($B = \mu_0 in$).

Calculations: Because coil C consists of more than one turn, we apply Faraday's law in the form of Eq. 30-5 ($\mathscr{E} = -N\, d\Phi_B/dt$), where the number of turns N is 130 and $d\Phi_B/dt$ is the rate at which the flux changes.

Because the current in the solenoid decreases at a steady rate, flux Φ_B also decreases at a steady rate, and so we can write $d\Phi_B/dt$ as $\Delta\Phi_B/\Delta t$. Then, to evaluate $\Delta\Phi_B$, we need the final and initial flux values. The final flux $\Phi_{B,f}$ is zero because the final current in the solenoid is zero. To find the initial flux $\Phi_{B,i}$, we note that area A is $\frac{1}{4}\pi d^2$ ($= 3.464 \times 10^{-4}$ m^2) and the number n is 220 turns/cm, or 22 000 turns/m. Substituting Eq. 29-23 into Eq. 30-2 then leads to

$$\Phi_{B,i} = BA = (\mu_0 in)A$$
$$= (4\pi \times 10^{-7}\,\text{T}\cdot\text{m/A})(1.5\text{ A})(22\,000\text{ turns/m}) \times (3.464 \times 10^{-4}\text{ m}^2)$$
$$= 1.44 \times 10^{-5}\text{ Wb}.$$

Now we can write

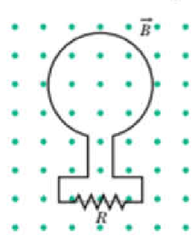
$$\frac{d\Phi_B}{dt} = \frac{\Delta\Phi_B}{\Delta t} = \frac{\Phi_{B,f} - \Phi_{B,i}}{\Delta t} = \frac{(0 - 1.44 \times 10^{-5}\text{ Wb})}{25 \times 10^{-3}\text{ s}}$$
$$= -5.76 \times 10^{-4}\text{ Wb/s} = -5.76 \times 10^{-4}\text{ V}.$$

We are interested only in magnitudes; so we ignore the minus signs here and in Eq. 30-5, writing

$$\mathscr{E} = N\frac{d\Phi_B}{dt} = (130\text{ turns})(5.76 \times 10^{-4}\text{ V})$$
$$= 7.5 \times 10^{-2}\text{ V} = 75\text{ mV}. \qquad \text{(Answer)}$$

tion for the field magnitude.

•7 In Fig. 30-36, the magnetic flux through the loop increases according to the relation $\Phi_B = 6.0t^2 + 7.0t$, where Φ_B is in milliwebers and t is in seconds. (a) What is the magnitude of the emf induced in the loop when $t = 2.0$ s? (b) Is the direction of the current through R to the right or left?



•8 A uniform magnetic field $\vec{B}$ is per-

••11 A rectangular coil of N turns and of length a and width b is rotated at frequency f in a uniform magnetic field $\vec{B}$, as indicated in Fig. 30-38. The coil is connected to co-rotating cylinders, against which metal brushes slide to make contact. (a) Show that the emf induced in the coil is given (as a function of time t) by

$$\mathscr{E} = 2\pi f NabB \sin(2\pi f t) = \mathscr{E}_0 \sin(2\pi f t).$$

This is the principle of the commercial alternating-current generator. (b) What value of Nab gives an emf with $\mathscr{E}_0 = 150$ V when the loop is rotated at 60.0 rev/s in a uniform magnetic field of 0.500 T?

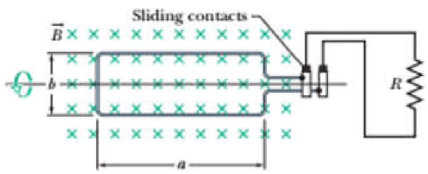


Fig. 30-38 Problem 11.

If a 100-volt emf is applied, what energy is stored in the magnetic field after the current has built up to its maximum value $\mathcal{E}/R$?

The maximum current is given by

$$i = \frac{\mathcal{E}}{R} = \frac{100 \text{ volts}}{20 \text{ ohms}} = 5.0 \text{ amp.}$$

The stored energy is given by Eq. 36–18:

$$U_B = \tfrac{1}{2}Li^2 = \tfrac{1}{2}(5.0 \text{ henrys})(5.0 \text{ amp})^2 = 63 \text{ joules.}$$

Note that the time constant for this coil ($= L/R$) is 0.25 sec. After how many time constants will *half* of this equilibrium energy be stored in the field?

Example 4. A 3.0-henry inductor is placed in series with a 10-ohm resistor, an emf of 3.0 volts being suddenly applied to the combination. At 0.30 sec (which is one inductive time constant) after the contact is made, (*a*) what is the rate at which energy is being delivered by the battery?

The current is given by Eq. 36–12, or

$$i = \frac{\mathcal{E}}{R}(1 - e^{-t/\tau_L}),$$

which at $t = 0.30$ sec ($= \tau_L$) has the value

$$i = \left(\frac{3.0 \text{ volts}}{10 \text{ ohms}}\right)(1 - e^{-1}) = 0.189 \text{ amp.}$$

The rate $P_\mathcal{E}$ at which energy is delivered by the battery is

$$\begin{aligned} P_\mathcal{E} &= \mathcal{E}i \\ &= (3.0 \text{ volts})(0.189 \text{ amp}) \\ &= 0.567 \text{ watt.} \end{aligned}$$

(*b*) At what rate does energy appear as Joule heat in the resistor? This is given by

$$\begin{aligned} P_J &= i^2R \\ &= (0.189 \text{ amp})^2(10 \text{ ohms}) \\ &= 0.357 \text{ watt.} \end{aligned}$$

(*c*) At what rate P_B is energy being stored in the magnetic field? This is given by the last term in Eq. 36–16, which requires that we know di/dt. Differentiating Eq. 36–12 yields

$$\begin{aligned} \frac{di}{dt} &= \left(\frac{\mathcal{E}}{R}\right)\left(\frac{R}{L}\right)e^{-t/\tau_L} \\ &= \frac{\mathcal{E}}{L}e^{-t/\tau_L}. \end{aligned}$$

At $t = \tau_L$ we have

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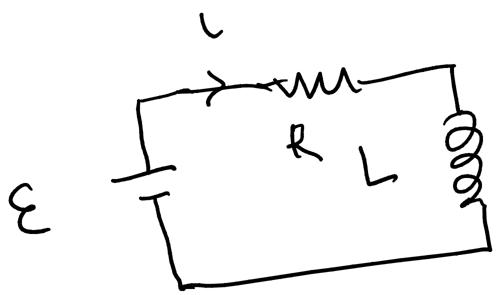
R-L ckt



$L = 3$ H

$R = 10$ ohm

$\mathcal{E} = 3$ Volt

$\tau = L$ $3 = 0 \cdot 3$ sec

$\mathcal{E} = 3$ Volt

$$T_L = \frac{L}{R} = \frac{3}{10} = 0.3 \text{ sec}$$

$$\mathcal{E}i = i^2R + Li\frac{di}{dt}$$

$$P_{\mathcal{E}} = P_R + P_B$$

$$P_{\mathcal{E}} = \mathcal{E}i \; ; \; t = 0.3\,s$$

$$i(t) = \frac{\mathcal{E}}{R}\left(1 - e^{-Rt/L}\right)$$

$$= \frac{3}{10}\left(1 - e^{-\frac{10\times .3}{3}}\right)$$

$$= 0.189\text{ A.}$$

If a 100-volt emf is applied, what energy is stored in the magnetic field after the current has built up to its maximum value $\mathcal{E}/R$?

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At $t = \tau_L$ we have

(a) $P_{\mathcal{E}} = \mathcal{E}i = 3 \times 0.189$

$= 0.567$ W

(b) $P_R = i^2R = (0.189)^2 \times 10$

$= 0.357$ W

(c) $P_B = Li\frac{di}{dt}$

(c) $P_B = L i \frac{di}{dt}$

; $\frac{di}{dt} = \frac{d}{dt}\left(\frac{\mathcal{E}}{R}\left(1 - e^{-Rt/L}\right)\right)$

at $t = 0.3$; $\frac{di}{dt} = \frac{\mathcal{E}}{R}\left(0 - e^{-Rt/L}\cdot\left(-\frac{R}{L}\right)\right)$

$= \frac{\mathcal{E}}{L} e^{-Rt/L}$

$= 0.37$ A/s

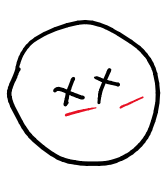
$P_B = L i \frac{di}{dt} = (3)(0.189)(0.37)$

$= 0.21$ W

Farcday's law:

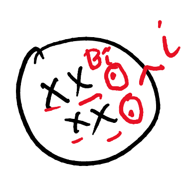
$|\mathcal{E}| = N \frac{d\phi_B}{dt}$

$i = \frac{\mathcal{E}}{R}$



$t_i = 0$

$\phi_{Bi} = 2$



$t_f = 2$ s

$\phi_{Bf} = 4$

$t_i = 0$

$\Phi_{Bi} = 2$

$t_f = 2$

$\Phi_{Bf} = 4$

; $R = 1$ ohm

$d\Phi_B = \Phi_{Bf} - \Phi_{Bi} = 4 - 2 = 2$

$dt = t_f - t_i = 2 - 0 = 2$ s

$\frac{d\Phi_B}{dt} = \frac{2}{2} = 1$ volt

$|\varepsilon| = N \frac{d\Phi_B}{dt}$; $N = 1$

$\varepsilon = 1$ volt

$i = \frac{\varepsilon}{R} = \frac{1}{1} = 1\ A.$

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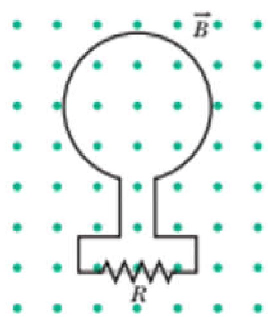

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$\Phi_B = 6t^2 + 7t$

$t = 2$ s

$\varepsilon = \frac{d}{dt}\Phi_B = \frac{d}{dt}(6t^2 + 7t)$

$\mathcal{E} = 6.2t + 7 \quad ; \quad t = 2$

$\mathcal{E} = 12.2 + 7 = 31$ volt

$i = \frac{\mathcal{E}}{R} = \frac{31}{2}$

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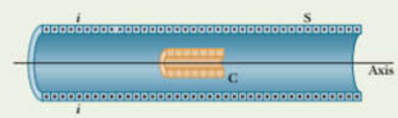

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$$\begin{aligned}\frac{d\Phi_B}{dt} = \frac{\Delta\Phi_B}{\Delta t} &= \frac{\Phi_{B,f} - \Phi_{B,i}}{\Delta t} \\ &= \frac{(0 - 1.44 \times 10^{-5}\,\text{Wb})}{25 \times 10^{-3}\,\text{s}} \\ &= -5.76 \times 10^{-4}\,\text{Wb/s} = -5.76 \times 10^{-4}\,\text{V}.\end{aligned}$$

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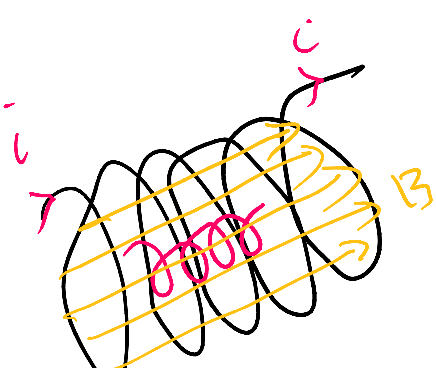
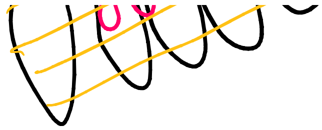
A = x-sectional area of the inner coil :





$B = \mu_0 i n$

$i = 1.5\,A$ $\quad dt = 25\,ms$

$\downarrow$

0

$i \downarrow \rightarrow B \downarrow \rightarrow \phi_B \downarrow 0$

$\mathcal{E} = N \frac{d\phi_B}{dt}$

$= 130 \left[\frac{d\phi_B = \phi_{Bf} - \phi_{Bi}}{dt = 25\,ms} \right]$

••11 A rectangular coil of N turns and of length a and width b is rotated at frequency f in a uniform magnetic field $\vec{B}$, as indicated in Fig. 30-38. The coil is connected to co-rotating cylinders, against which metal brushes slide to make contact. (a) Show that the emf induced in the coil is given (as a function of time t) by

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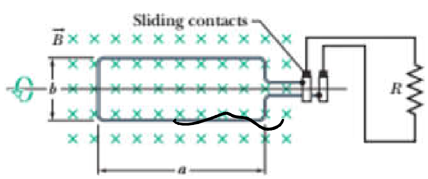


Fig. 30-38 Problem 11.

$\mathcal{E} = \frac{d}{dt}(\phi_B)$

$= \frac{d}{dt}(BA \cos\theta)$

$= BA \frac{d}{dt} \cos\omega t$

$= BA \sin\omega t \cdot (\omega)$

$\omega = 2\pi f$

$\theta = \omega t$

$= BA \frac{}{dt}$

$|\mathcal{E}| = BA \sin\omega t . (\omega)$

$A = ab$

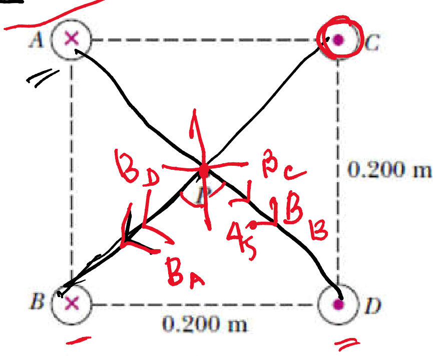
$N = N$

$\mathcal{E} = N B A \underline{\omega} \sin\omega t$

$= \underline{N B (ab) 2\pi f} \sin \underline{2\pi t}$

Q. Four long, parallel conductors carry equal currents of I= 5.00 A. Figure is an end view of the conductors. The direction of the current is into the page at points A and B (indicated by the crosses) and out of the page at C and D (indicated by the dots). Calculate the magnitude and direction of the magnetic field at point P, located at the center of the square with an edge length of 0.200 m

$I = 5A = I_A$



$$\frac{\sqrt{.2^2 + .2^2}}{2}$$

$$B_A = \frac{\mu_0 i}{2\pi r} = B_B = B_C = B_D$$