SEMINAR 2. Sequences and series of real numbers

Exercises

1. Use the Cauchy's general criterion for convergence to show the convergence of the sequences:

   (a) $x_n = \frac{\sin x}{2} + \frac{\sin 2x}{2^2} + ... + \frac{\sin nx}{2^n}, \quad n \geq 1$

   (b) $y_n = 1 + \frac{1}{2^2} + \frac{1}{3^2} + ... + \frac{1}{n^2}$

2. Determine the limit of the sequence

$$x_n = \frac{1^p + 2^p + ... + n^p}{n^{p+1}}, \quad p \in \mathbb{N}.$$

3. Study the convergence of the series:

   (a) $\sum_{n\geq 1} \frac{1}{n(n+1)}$

   (b) $\sum_{n\geq 1} \ln(1 + \frac{1}{n})$

   (c) $\sum_{n\geq 1} \frac{n}{n^4+4}$

4. Study the convergence of the series:

   (a) $\sum_{n\geq 1} \frac{1}{n^2}$

   (b) $\sum_{n\geq 1} \frac{\cos nx}{2^n}, \quad x \in \mathbb{R}$