Z-TRANSFORM

LAB 1

Assigment 1

(4) a) $x(k) = \left(\frac{1}{2}\right)^k$

$$Z(x(k)) = \sum_{k=0}^{\infty} x(k) \cdot z^{-k} = x(0) \cdot z^0 + x(1) z^{-1} + x(2) \cdot z^{-2} + \ldots =$$

$$= 1 + \left(\frac{1}{2}\right)^1 \cdot z^{-1} + \left(\frac{1}{2}\right)^2 \cdot z^{-2} + \left(\frac{1}{2}\right)^3 \cdot z^{-3} + \ldots =$$

$$= 1 + \left(\frac{1}{2} \cdot \frac{1}{z}\right)^1 + \left(\frac{1}{2} \cdot \frac{1}{z}\right)^2 + \ldots = 1 + \left(\frac{1}{2z}\right)^1 + \left(\frac{1}{2z}\right)^2 + \left(\frac{1}{2z}\right)^3 + \ldots$$

Geometric series $k \to \infty, |r| < 1$:

$$a + a \cdot r + a \cdot r^2 + a \cdot r^3 + \ldots = \frac{a}{1-r}$$

$$Z(x(k)) = 1 + \left(\frac{1}{2z}\right)^1 + \left(\frac{1}{2z}\right)^2 + \ldots \quad \Rightarrow \quad a = 1, \quad r = \frac{1}{2z}$$

$$Z(x(k)) = \frac{1}{1 - \frac{1}{2z}} = \frac{1}{\frac{2z-1}{2z}} = \frac{2z}{2z-1}$$

$$\boxed{Z(x(k)) = \frac{2z}{2z-1}}$$

(5) $y(k+1) = (1+r) y(k)$ with $r = 0.1$

$y(0) = 100$

$$Z(y(k+1)) = Z(1.1 \cdot y(k)) \iff$$

$$\iff z(Y(z)) - 100 = 1.1 \cdot Y(z) \iff (z - 1.1)(Y(z)) = 100 \cdot z$$

$$Y(z) = \frac{z}{z - 1.1} \cdot 100$$

$$y(k) = Z^{-1}\left\{\frac{z/z}{z-1.1} \cdot 100\right\} = 100 \cdot Z^{-1}\underbrace{\left\{\frac{1}{1 - 1.1 \cdot z^{-1}}\right\}}_{\Downarrow \text{ according to the table of transf } (1.1)^k} = 100 \cdot (1.1)^k$$

$$\boxed{y(k) = 100 \cdot (1.1)^k}$$