**Course: Discrete Mathematics and Graph Theory**

---

**Problems:**

1. Write an equivalent english sentence for each of the following logical formulas, where $p$: Rama is smart, and $q$: Rama is honest.

   i. $(p \vee q) \wedge (p \rightarrow \neg q)$.

   ii. $(p \rightarrow q) \wedge (q \rightarrow p)$.

2. Express the following two English sentences symbolically:

   (i) Every state of India has a capital.

   (ii) There are states in India which have common capital.

3. Determine the validity of the following argument by using predicte logic:

   all my friends are musicians
   John is my friend
   None of neighbours are musicians

   ---

   John is not my neighbour.

4. Let the domain of the following consists of all people. Formulate the following English statements using predicate logic.

   i. Every comedian is funny.

   ii. At least one of your friends is a comedian.

   iii. Some people are funny but not comedians.

5. Formulate the following English statements using predicate logic.

   i. Some vegetarians do not like Jaffa cakes.

   ii. Either some people like Jaffa cakes or some people are vegetarians.

   iii. Some people like either Jaffa cakes or are vegetarians.

6. Let $B(x)$ mean $x$ is a bird, let $W(x)$ mean $x$ is a worm, and let $E(x, y)$ mean $x$ eats $y$. Find an English sentence to describe each of the following statements.

   i. $\forall x \ \forall y \ \ (B(x) \wedge W(y) \rightarrow E(x, y))$.

   ii. $\forall x \ \forall y \ \ (E(x, y) \rightarrow B(x) \wedge W(y))$.

   iii. $\exists x \ (B(x) \wedge \forall y \ (B(y) \rightarrow e(x, y)))$.

7. With and without constructing the truth tables prove that
   $\neg p \rightarrow (q \rightarrow r) \equiv q \rightarrow (p \vee r)$.

8. Prove the following using both direct and indirect methods ?
   $(p \rightarrow q) \wedge (r \rightarrow s); (q \rightarrow t) \wedge (s \rightarrow u); \neg(t \wedge u); (p \rightarrow r) \implies \neg p$.

9. Show that $\forall x(P(x) \rightarrow (Q(y) \wedge R(x)))$ and $\exists x P(x) \implies Q(y) \wedge \exists x(P(x) \wedge R(x))$.

10. Using truth table, show that $(p \rightarrow q) \wedge (p \rightarrow r) \equiv (p \rightarrow (q \wedge r)$.