Course: Discrete Mathematics and Graph Theory

Problems:

1. Write an equivalent english sentence for each of the following logical formulas, where p: Newan is Brave, and q: Newan is Kind.

   i. $(p \vee q) \wedge (p \rightarrow \neg q)$.

   ii. $(p \rightarrow q) \wedge (q \rightarrow p)$.

2. Determine the validity of the following argument by using predicate logic:

   If the project is funded, then the team will expand.
   If the team expands, productivity increases.
   Productivity did not increase.

   Therefore, the project was not funded.

3. Formulate the following English statements using predicate logic.

   i. Some vegetarians do not like bitter gourd.

   ii. Either some people like bitter gourd or some people are vegetarians.

   iii. Some people like either bitter gourd or are vegetarians.

4. Let $B(x)$ mean x is a bird, let $W(x)$ mean x is a worm, and let $E(x, y)$ mean x eats y. Find an English sentence to describe each of the following statements.

   i. $\forall x \ \forall y \ \ (B(x) \wedge W(y) \rightarrow E(x, y))$.

   ii. $\forall x \ \forall y \ \ (E(x, y) \rightarrow B(x) \wedge W(y))$.

   iii. $\exists x \ (B(x) \wedge \forall y \ (B(y) \rightarrow e(x, y)))$.

5. With and without constructing the truth tables prove that
   $\neg p \rightarrow (q \rightarrow r) \equiv q \rightarrow (p \vee r)$.

6. Prove the following using both direct and indirect methods?
   $(p \rightarrow q) \wedge (r \rightarrow s); (q \rightarrow t) \wedge (s \rightarrow u); \neg(t \wedge u); (p \rightarrow r) \implies \neg p$.

7. Show that $\forall x(P(x) \rightarrow (Q(y) \wedge R(x)))$ and $\exists x P(x) \implies Q(y) \wedge \exists x(P(x) \wedge R(x))$.