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Problem ①:-

Let p: Neeraan is brave

q: Neeraan is kind

i) $(p \vee q) \wedge (p \to \neg q)$

→ (Neeraan is brave or Neeraan is kind) and (if Neeraan is brave then Neeraan is not kind).

ii) $(p \to q) \wedge (q \to p)$

$p \to q$: if Neeraan is brave, then Neeraan is kind

$q \to p$: if Neeraan is kind, then Neeraan is brave.

Therefore both are True or both are false, it is $p \leftrightarrow q$ (biimplication).

→ (Neeraan is kind if and only if Neeraan is brave).

Problem ②:-

A: The project is funded

B: The team expands

C: Productivity increases.

1. If the project is funded, then team will expand: $A \to B$
2. If team expands, productivity increases: $B \to C$
3. Productivity did not increase: $\neg C$

CONCLUSION: $\neg A$

Using rules of inference:-

$A \to B$
$B \to C$

then $A \to C$

. Now we have $A \to C$ and $\neg C$ Then $\neg A$

Therefore conclusion matches, argument is valid ✓.

Problem ③ :-

(i) Some vegetarians do not like bitter gourd.

$V(x)$: x is a vegetarian

$L(x)$: x likes bittergourd.

There exists atleast one, who is vegetarian and don't like bittergourd.

predicate form $\exists x (V(x) \land \neg L(x))$

(ii) Either some people like bitter gourd or some people are vegetarians.

→ There exists someone who like bittergourd. OR

→ There exists someone who is vegetarian.

predicate form : $(\exists x L(x)) \lor (\exists x V(x))$

Problem ④ :-

$B(x)$: x is a bird , $W(x)$: x is worm , $E(x,y)$: x eats y .

(i) $\forall x \forall y \, (B(x) \wedge W(y) \rightarrow E(x,y))$

for any x and y, if x is a bird and y is a worm, then x eats y .

English Sentence :- "Every bird eats every worm".

(ii) $\forall x \forall y \, (E(x,y) \rightarrow (B(x) \wedge W(y)))$

English :- Whenever something eats something, the eater is a bird and the thing eaten is a worm .

(iii) $\exists x \, (B(x) \wedge \forall y \, (B(y) \rightarrow E(x,y)))$

C there is atleast one bird x for every y such that if y is a bird x eats y.

English :- There exists a bird that eats every bird .

Problem ⑤ :-

$$\neg p \rightarrow (q \rightarrow r) \equiv q \rightarrow (p \vee r)$$

Solving with Truth Table :-

p	q	r	$\neg p$	$q \rightarrow r$	$\neg p \rightarrow (q \rightarrow r)$	$p \vee r$	$q \rightarrow (p \vee r)$
T	T	T	F	T	T	T	T
T	T	F	F	F	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	F	F
F	F	T	T	T	T	T	T
F	F	F	T	T	T	F	T

Therefore :- $\neg p \to (q \to r) \equiv q \to (p \vee r)$

Solving without Truth Table :

$L.H.S = \neg p \to (q \to r)$

Step 1 :- Remove the inner implication

$$q \to r \equiv \neg q \vee r$$

so $\neg p \to (\neg q \vee r)$

Step 2 :- Remove the outer implication.

if $a \to b \equiv \neg a \vee b$, then.

$$\neg p \to (\neg q \vee r) \equiv \neg(\neg p) \vee (\neg q \vee r) \equiv p \vee (\neg q \vee r)$$

Step 3 :- Associative law.

$$p \vee (\neg q \vee r) \equiv \neg q \vee (p \vee r)$$

as $\neg q \vee X \equiv q \to X$

$\therefore$ $\neg q \vee (p \vee r) \equiv q \to (p \vee r)$

Hence proved $\neg p \to (q \to r) \equiv q \to (p \vee r)$.

Problem ⑥ :- $(p \to q) \wedge (r \to s)$; $(q \to t) \wedge (s \to u)$; $\neg(t \wedge u)$; $(p \to r) \Rightarrow \neg p$

1) Direct Method (Two Column proof)

Statements		Reasons
1.	$(p \to q) \wedge (r \to s)$	premise.
2.	$(q \to t) \wedge (s \to u)$	premise
3.	$\neg (t \wedge u)$	premise.
4.	$p \to r$	premise.
5.	$p \to q$	from (1)
6.	$r \to s$	from (1)
7.	$q \to t$	from (2)
8.	$s \to u$	from (2)
9.	$p \to t$	from (5, 7)
10.	$p \to s$	from (4, 6).
11.	$p \to u$	from (10, 8)
12.	$p \to (t \wedge u)$	from (9, 11) by conjunction.
13.	$\neg p$	Modus Tollens (12, 3).

Therefore $\neg p$.

11) Indirect Method (Proof by Contradiction).

assume p.

Statements	Reasons
1. $(p \Rightarrow q) \wedge (r \Rightarrow s)$	Simplification premise
2. $(q \Rightarrow t) \wedge (s \Rightarrow u)$	premise
3. $\neg (t \wedge u)$	premise
4. $p \rightarrow r$	premise
5. Assume p	Assumption.
6. $p \Rightarrow q$	from (1)
7. $r \rightarrow s$	from (1)
8. $q \rightarrow t$	from (2)
9. $s \rightarrow u$.	from (2)
10. r	Modus Ponens (4, 5)
11. s	Modus Ponens (7, 10)
12. u	Modus Ponens (9, 11)
13. q	Modus Ponens (6, 5)
14. t	Modus Ponens (8, 13)
15. $t \wedge u$	Conjunction (14, 12)
16. Contradiction.	from (3, 15)

$\therefore$ p leads to contradiction $\neg p$.

Problem (7) :-

Show $\forall_x (P(x) \to (Q(y) \land R(x)))$ and $\exists_x P(x) \Rightarrow Q(y) \land \exists_x (P(x) \land R(x))$

Proof :-

1. $\forall_x (P(x) \to (Q(y) \land R(x)))$ (premise / given)
2. $\exists_x P(x)$ (premise / given)
3. $P(a)$ (Existential Instantiation)
4. $P(a) \to (Q(y) \land R(a))$
5. $Q(y) \land R(a)$ (Modus Ponens) (from 3, 4).
6. $Q(y)$ (from 5)
7. $R(a)$ (from 5)
8. $P(a) \land R(a)$ (Conjunction)
9. $\exists_x (P(x) \land R(x))$ (from 8)
10. $Q(y) \land \exists_x (P(x) \land R(x))$ (Conjunction) (combine 6, 9)

Hence proved.