(Section -5) (Matrices)

* Diagonal Matrix

	1	2	3	4	5
1	3	0	0	0	0
2	0	7	0	0	0
3	0	0	4	0	0
4	0	0	0	9	0
5	0	0	0	0	6

M [i,j] = 0 if i ≠ j

Representing in 1-D form

A	3	7	4	9	6
	0	1	2	3	4

M [i,j]
if (i == j) A [i-1]; //OR A [j-1]

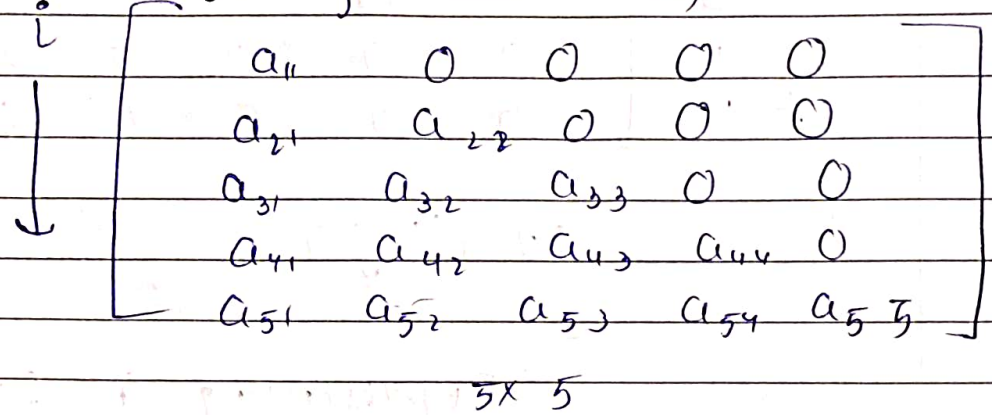
Source's code :-

```
int A[5];

void set (int A[], int i,
          int j, int x)
{
  if (i == j)
  {
    A [i-1] = x;
  }
}
```

```
int get (int A[], int i, int j)
{
    if (i == j) return A[i-1];
    else return 0;
}
```

* **Lower Triangular Matrix**



5 × 5

M[i, j] = 0 if i < j
M[i, j] = non-zero if i >= j

non-zero = 1 + 2 + 3 + 4 + 5
= 1 + 2 + 3 + .. n = $\frac{n(n+1)}{2}$

zero = $n^2 - \frac{n(n+1)}{2} \Rightarrow \frac{n(n-1)}{2}$

Row-Major Method :-

{ Storing Row-by-Row }

__/__/__

A	a_{11}	a_{21}	a_{22}	a_{31}	a_{32}	a_{33}	a_{41}	a_{42}	a_{43}	a_{44}	a_{51}	a_{52}	a_{53}	a_{54}	a_{55}
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
	row 1		row 2		row 3		row 4				row 5				

Index (A [4] [3]) = [1+2+3] + 2 = 8

Index (A [5] [4]) = [1+2+3+4] +3

= 13

$$\text{Index } [A[i][j]) = \left[\frac{i(i-1)}{2}\right] + (j-1)$$

Column - Major Method :-

a_{11}	a_{21}	a_{31}	a_{41}	a_{31}	a_{22}	a_{32}	a_{42}	a_5	a_{33}	a_{43}	a_{33}	a_{44}	a_{54}	a_{5}
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
column 1					column 2				column 3			column 4		(5

Index (A [4] [4]) = [5+4+3] + 0 = 12

Index (A [5] [4]) = [5+4+3] + 1 = 13

Index (A [5] [3]) = [5+4] + 2 = 11

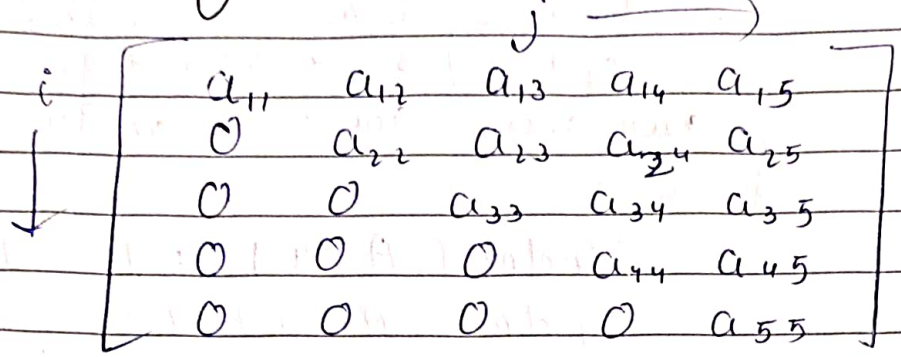
Index (A [i] [j]) = [n + (n-1) + (n-2) + ... n-(j-2)]

+ (i-j)

$\Rightarrow$ n (j-1) - [1+2+3+ .. j-2]] + (i-j)

$$\text{Index } (A[i][j]) = \left[n(j-1) - \frac{(j-2)(j-1)}{2}\right] + (i-j)$$

* **Upper Triangular Matrix**



3 X 5

M [i, j] = 0 if i > j

M [i, j] = non-zero if i <= j

No of elements.

non-zero = 5 + 4 + 3 + 2 + 1

= n + (n-1) + 3 + 2 + 1

= n (n+1) / 2

zero = $n^2 - n(n+1)/2 = n(n-1)/2$

Row-Major Method

rows 5

a_{55}

a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	a_{22}	a_{23}	a_{24}	a_{25}	a_{33}	a_{34}	a_{35}	a_{44}	a_{45}	a_{55}
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
row 1					row 2				row 3			row 4		

Index (A[4][5]) = [5 + 4 + 3] + 1

= 13

Index (A[i][j]) = [n + (n-1) + (n-2) + ... + n - (i-2)] + (j - i)

$$\Rightarrow \left[(i-1)n - \frac{(i-2)(i-1)}{2}\right] + (j-i)$$

$$\text{Index } (A[i][j]) = \left[(i-1)n - \frac{(i-2)(i-1)}{2}\right] + (j-i)$$

Column - Major Method

a_{11}	a_{12}	a_{22}	a_{13}	a_{23}	a_{33}	a_{14}	a_{24}	a_{34}	a_{44}	a_{15}	a_{25}	a_{35}	a_{45}	a_{55}
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14

(1) (2) (3) (4) (5)

Index (A[4][5]) = [(1 + 2 + 3 + 4) + 3]

$\Rightarrow$ 13

$$\text{Index } (A[i][j]) = [1 + 2 + 3 + \ldots + j - 1] + i - 1$$

$$\Rightarrow \frac{j(j-1)}{2} + i - 1$$

Symmetric Matrix

A square Matrix where the elements are mirrored across the main diagonal

j ——→

i ↓

2	2	2	2	2
2	3	3	3	3
2	3	4	4	4
2	3	4	5	5
2	3	4	5	6

5X5

if (M[i,j] = [M[j,i])

can be represent → ① lower triangular
② upper triangular

* Tridiagonal Matrix

j ——→

i ↓

$$M = \begin{bmatrix} a_{11} & a_{12} & 0 & 0 & 0 \\ a_{21} & a_{22} & a_{23} & 0 & 0 \\ a_{31} & a_{32} & a_{33} & a_{34} & 0 \\ 0 & 0 & a_{43} & a_{44} & a_{45} \\ 0 & 0 & 0 & a_{54} & a_{55} \end{bmatrix}$$

5X5

Main diagonal $i-j=0$
Lower " $i-j=1$
upper " $i-j=-1$
$|i-j| \leq 1$

M[i,j] = non zero if $|i-j| \leq 1$
M[i,j] = 0 if $|i-j| > 1$

No of elements

non-zero = 3 + 4 + 4
= n + (n-1) + (n-1)
= 3n - 2

{ Note :- Rows and columns don't have uniform no of elements }

A	a_{21}	a_{32}	a_{43}	a_{54}	a_{11}	a_{22}	a_{33}	a_{44}	a_{55}	a_{12}	a_{23}	a_{34}	a_{45}
	0	1	2	3	4	5	6	7	8	9	10	11	12
	lower Diag.				main diagonal					upper Diag			

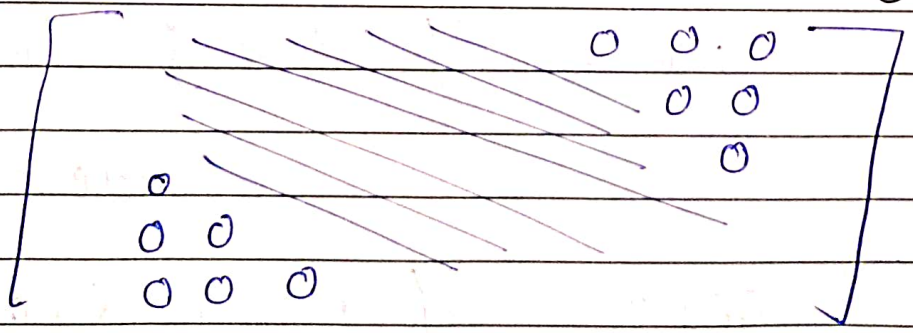
Index (A[i][j])

case 1 :- if $i-j=1$ index = $i-2$

case 2 :- if $i-j=0$ index = $n-1+i-1$

case 3 : if $i-j=-1$ index = $2n-1+i-1$

* Square Band {Diagonal Band formation}



← Example

{ we can store here diagonal-by-diagonal too }

* **Toeplitz Matrix**

$$M = \begin{bmatrix} 2 & 3 & 4 & 5 & 6 \\ 7 & 2 & 3 & 4 & 5 \\ 8 & 7 & 2 & 3 & 4 \\ 9 & 8 & 7 & 2 & 3 \\ 10 & 9 & 8 & 7 & 2 \end{bmatrix}$$

5 × 5

$$M[i,j] = M[i-1, j-1]$$

no of elements → $n + (n-1)$

A	2	3	4	5	6	7	8	9	10
	0	1	2	3	4	5	6	7	8

Row | Column

Index (A[i][j])

Case 1 if $i \leq j$ (upper triangle)

index = $j - i$

Case 2 if $i >= j$ (lower triangle)

__/__/____

index = n + i - j - 1