IPSA

Au511 - Modélisation d'un aéronef - Pilote automatique

Aéro 5

Practical Work - Control of MIRAGE III

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1 Introduction

1.1 In flight operating point

For this study, we were asked to choose a group number from the following table, which would then correspond to a mach and altitude for our aircraft in this project :

Alt (ft) \ Mach	0.76	0.91	1.18	1.29	1.49	1.68	1.88
575	11	12	13	14	15	16	17
3015	21	22	23	24	25	26	27
6115	31	32	33	34	35	36	37
10085	41	42	43	44	45	46	47
13105	51	52	53	54	55	56	57
16335	61	62	63	64	65	66	67
19365	71	72	73	74	75	76	77
22265	81	82	83	84	85	86	87

Figure 1: Mach and altitude table

We have chosen number 42, which gave us the following parameters for our study: **Mach = 1.49 and Altitude (ft) = 10085**.

1.2 Aircraft characteristics

For this study, we will consider that our aircraft will be a MIRAGE III class with the following characteristics:

- Total lenght : $L_t = \frac{3}{2} L_e$
- Mass : $m = 8400kg$
- Aircraft centering (center of gravity position) $c = 52\%$ (as percentage of total length)
- Reference surface (Wings) $S = 34m^2$
- Radius of gyration $r_g = 2,65m$
- Reference length $L_{ref} = 5,24m = \frac{2}{3} L_t$

For the calculus of air density and speed of sound as a function of altitude, we will use the US 76 standard atmosphere model.

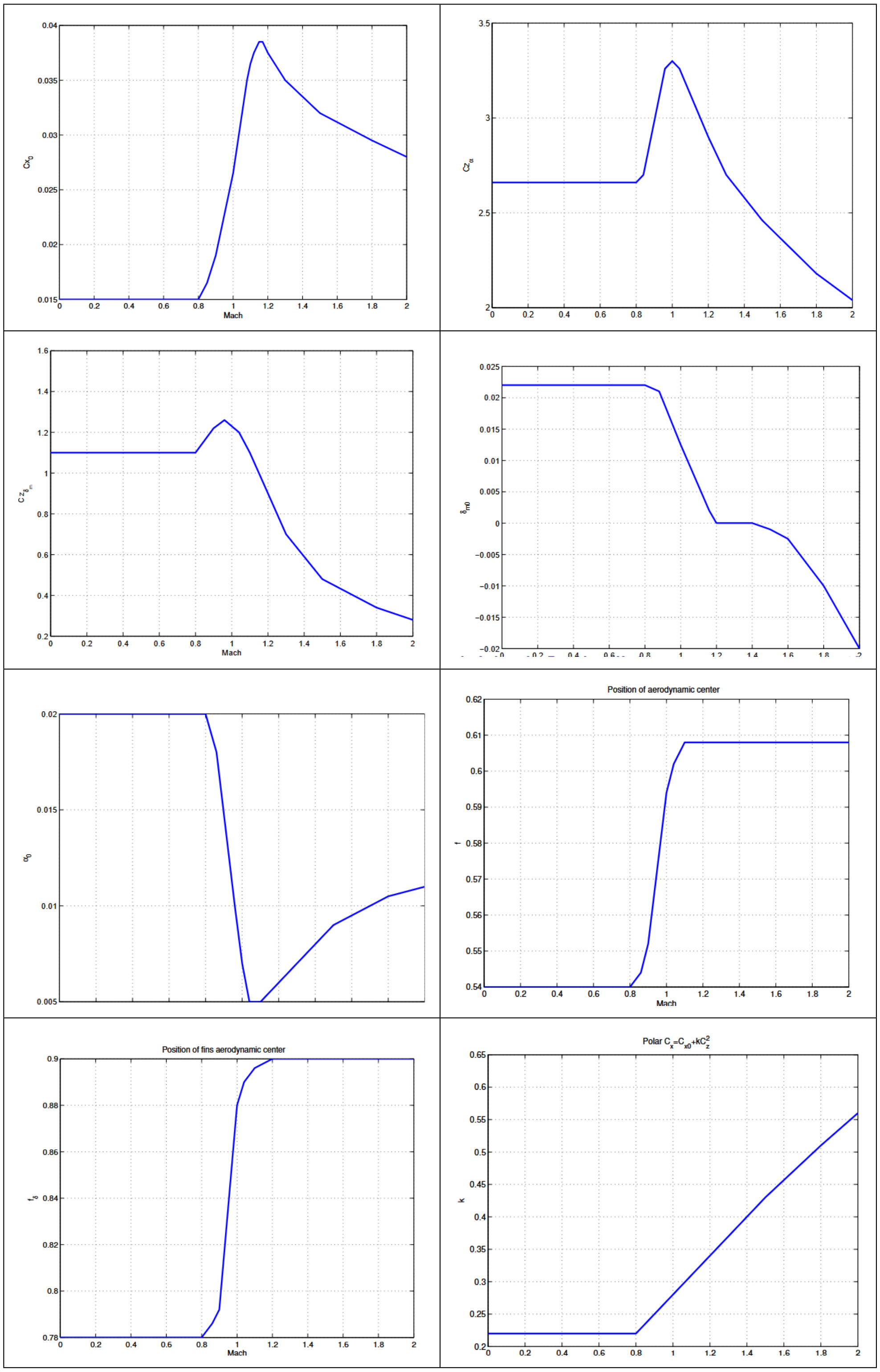
1.3 Hypothesis

For the study, we will also consider the following hypothesis for the flight and the modeling:

- Symmetrical flight, in the vertical plane (null sideslip and roll)
- Thrust axis merged with aircraft longitudinal axis
- Inertia principal axis = aircraft transverse axis (diagonal inertia matrix)
- Fin control loop: its dynamics will be neglected for the controller synthesis
- The altitude sensor is modeled by a 1st order transfer function with a time constant $\tau = 0.75s$

2 Aircraft aerodynamic model

For the study, we will also consider the following hypothesis for the flight and the modeling:



3 Study of the uncontrolled aircraft

3.1 Equilibrium point

First, we need to find the aircraft's equilibrium point for our chosen conditions. To find this equilibrium point, we will use the following algorithm, coded in Python:

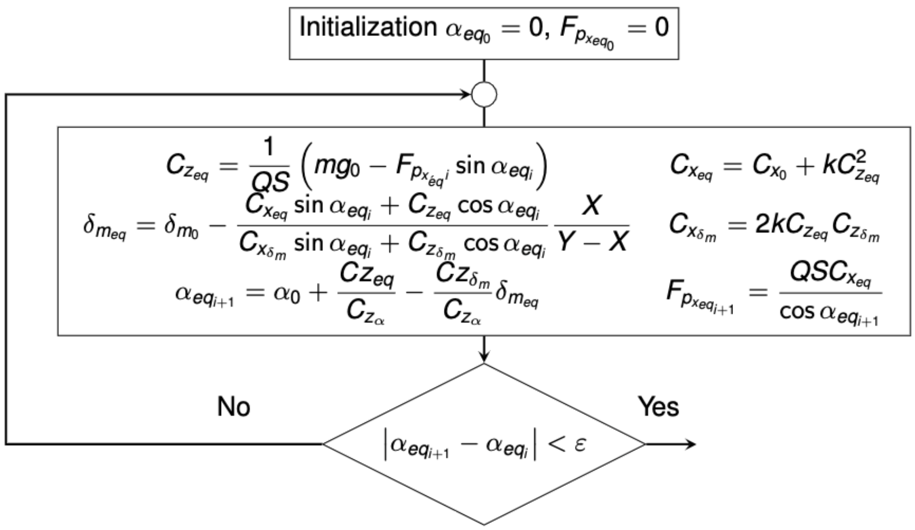


Figure 2: Algorithm to find the equilibrium point

We obtain the following results for the equilibrium point :

- $\alpha_{eq} = 0.0202615458$ rad. A low angle of incidence is typical for stable cruising. Here, the angle is small enough to guarantee a good compromise between lift and drag.
- $F_{pxeq} = 120522.663752933$ N. This order of magnitude seems correct for our type of aircraft.
- $C_{Xeq} = 0.0326968071$. This drag coefficient is slightly higher than C_{X_0}. This means that drag is slightly increased due to the angle of incidence.
- $C_{Zeq} = 0.0216469004$. This coefficient is relatively low, which is typical for cruise flight, where lift exactly balances the aircraft's weight. It indicates that the aircraft is in a state of equilibrium, with no excess or lack of lift.
- $C_{X\delta m} = 0.0087280302$

Now, let's give the state space representation (A, B, C, D) around this equilibrium point, considering the following state vector, with 6 variables: $X = \begin{pmatrix} V & \gamma & \alpha & q & \theta & z \end{pmatrix}^T$ and as the command vector, only U = (δ_m). The state space model is $\dot{X} = AX + BU$ with (X) being the state vector, (A) the dynamic matrix, (B) the command matrix, and (U) the command.
If we express this model in matrix form, we get the following result:

$$\begin{pmatrix} \dot{V} \\ \dot{\gamma} \\ \dot{\alpha} \\ \dot{q} \\ \dot{\theta} \\ \dot{z} \end{pmatrix} = \begin{pmatrix} -X_V & -X_\gamma & -X_\alpha & 0 & 0 & 0 \\ Z_V & 0 & Z_\alpha & 0 & 0 & 0 \\ -Z_V & 0 & -Z_\alpha & 1 & 0 & 0 \\ 0 & 0 & m_\alpha & m_q & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & V_{eq} & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} V \\ \gamma \\ \alpha \\ q \\ \theta \\ z \end{pmatrix} + \begin{pmatrix} 0 \\ Z_{\delta_m} \\ -Z_{\delta_m} \\ m_{\delta_m} \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} \delta_m \end{pmatrix}$$

To calculate the coefficients, we will need the formulas below:

$$X_V = \frac{2QSC_{x_{eq}}}{mV_{eq}} \qquad m_V = 0 \qquad Z_V = \frac{2QSC_{z_{eq}}}{mV_{eq}} \approx \frac{2g_0}{V_{eq}}$$

$$X_\alpha = \frac{F_{eq}}{mV_{eq}} \sin\alpha_{eq} + \frac{QSC_{x_\alpha}}{mV_{eq}} \qquad m_\alpha = \frac{QS\ell_{\mathrm{ref}}C_{m_\alpha}}{I_{YY}} \qquad Z_\alpha = \frac{F_{eq}}{mV_{eq}} \cos\alpha_{eq} + \frac{QSC_{z_\alpha}}{mV_{eq}}$$

$$X_\gamma = \frac{g_0 \cos\gamma_{eq}}{V_{eq}} \qquad m_q = \frac{QS\ell_{\mathrm{ref}}^2 C_{m_q}}{V_{eq}I_{YY}} \qquad Z_\gamma = \frac{g_0 \sin\gamma_{eq}}{V_{eq}}$$

$$X_{\delta_m} = \frac{QS\ell_{\mathrm{ref}}C_{x_{\delta m}}}{mV_{eq}} \qquad m_{\delta_m} = \frac{QS\ell_{\mathrm{ref}}C_{m_{\delta m}}}{I_{YY}} \qquad Z_{\delta_m} = \frac{QSC_{z_{\delta m}}}{mV_{eq}}$$

$$X_\tau = -\frac{F_\tau \cos\alpha_{eq}}{mV_{eq}} \qquad m_\tau = \frac{F_\tau \sin\alpha_{eq}}{mV_{eq}} \qquad Z_\tau = \frac{F_\tau \sin\alpha_{eq}}{mV_{eq}}$$

We get the following matrices with Python:

$$A = \begin{pmatrix} -0.0587 & -0.02 & -0.0397 & 0 & 0 & 0 \\ 0.0388 & 0 & 2.1821 & 0 & 0 & 0 \\ -0.0388 & 0 & -2.1821 & 1 & 0 & 0 \\ 0 & 0 & -104.9054 & -1.1223 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 489.1036 & 0 & 0 & 0 & 0 \end{pmatrix}$$
$$B = \begin{pmatrix} 0 \\ 0.4306 \\ -0.4306 \\ -89.5821 \\ 0 \\ 0 \end{pmatrix}$$
$$C = I_6$$
$$D = \mathbf{0}_{6\times 6}$$

Using the damp function, we can get the values of the modes, the damping ratio, and the proper pulsation of the open-loop modes:

Eigenvalue (pole)	**Damping**	**Frequency**
0	1	0
0	1	0
$-1.652 + 10.23j$	0.1595	10.36
$-1.652 - 10.23j$	0.1595	10.36
-0.01932	1	0.01932
-0.03931	1	0.03931

3.2 Short period mode

In the short period mode, the conjugate eigenvalues of the double fast pole are associated mainly with q and α. In this mode, we neglect the coupling between $\{V, \gamma\}$. We get the following data in short period mode:

- Pole : -1.652 + 10.23j
- Damping ratio : 0.1595
- Proper pulsation : 10.36

- State-space representation :

$$\begin{pmatrix} \dot{V} \\ \dot{\gamma} \\ \dot{\alpha} \\ \dot{q} \\ \dot{\theta} \\ \dot{z} \end{pmatrix} = \begin{pmatrix} -X_V & -X_\gamma & -X_\alpha & 0 & 0 & 0 \\ Z_V & 0 & Z_\alpha & 0 & 0 & 0 \\ 0 & 0 & -Z_\alpha & 1 & 0 & 0 \\ 0 & 0 & m_\alpha & m_q & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & V_{eq} & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} V \\ \gamma \\ \alpha \\ q \\ \theta \\ z \end{pmatrix} + \begin{pmatrix} 0 \\ Z_{\delta_m} \\ -Z_{\delta_m} \\ m_{\delta_m} \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} \delta_m \end{pmatrix}$$

- Transfer function: $\frac{\alpha}{\delta_m}$:

$$\frac{-0.4306s - 90.07}{s^2 + 3.304s + 107.4}$$

- Transfer function: $\frac{q}{\delta_m}$:

$$\frac{-89.58s - 150.3}{s^2 + 3.304s + 107.4}$$

- Step response for each variable:

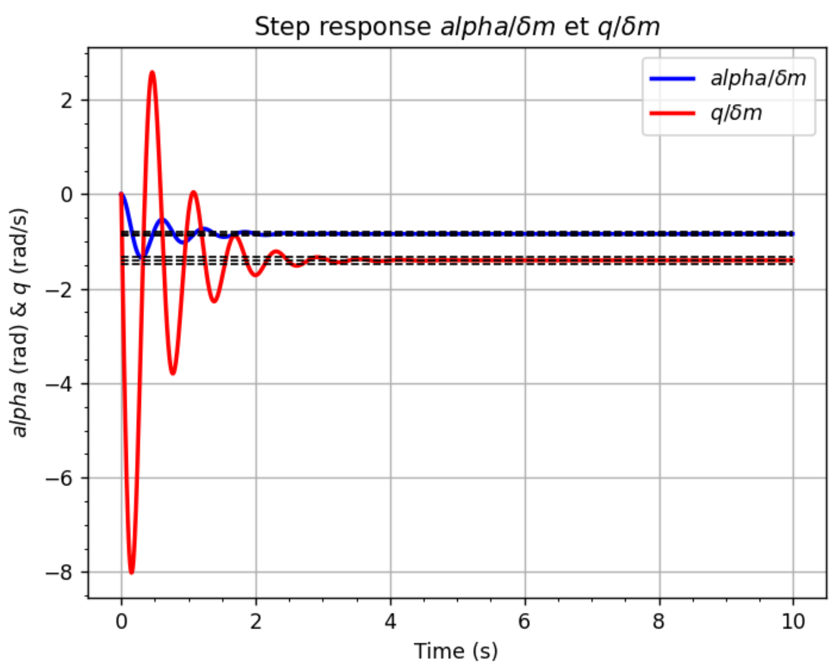


Figure 3: Step response for each variable in short period mode

In short period mode, we can see that the step response is stable with a 5% settling time of **2.69 s for q and 1.61 s for α.**

3.3 Phugoid oscillation mode

In the phugoid oscillation mode, the conjugate eigenvalues of the double slow pole are associated mainly with (V) and (γ). In this mode, we neglect the coupling between $\{\alpha, q\}$. We get the following data in short phugoid oscillation mode:

- Pole : -0.03844 and -0.02022
- Damping ratio : 1 and 1
- Proper pulsation : 0.03844 and 0.02022
- State-space representation :

$$\begin{pmatrix} \dot{V} \\ \dot{\gamma} \\ \dot{\alpha} \\ \dot{q} \\ \dot{\theta} \\ \dot{z} \end{pmatrix} = \begin{pmatrix} -X_V & -X_\gamma & 0 & 0 & 0 & 0 \\ Z_V & 0 & 0 & 0 & 0 & 0 \\ -Z_V & 0 & -Z_\alpha & 1 & 0 & 0 \\ 0 & 0 & m_\alpha & m_q & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & V_{eq} & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} V \\ \gamma \\ \alpha \\ q \\ \theta \\ z \end{pmatrix} + \begin{pmatrix} 0 \\ Z_{\delta_m} \\ -Z_{\delta_m} \\ m_{\delta_m} \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} \delta_m \end{pmatrix}$$

- Transfer function: $\frac{V}{\delta_m}$:

$$\frac{-0.008616}{s^2 + 0.05866s + 0.0007772}$$

- Transfer function: $\frac{\gamma}{\delta_m}$:

$$\frac{0.4306s + 0.02526}{s^2 + 0.05866s + 0.0007772}$$

- Step response for each variable:

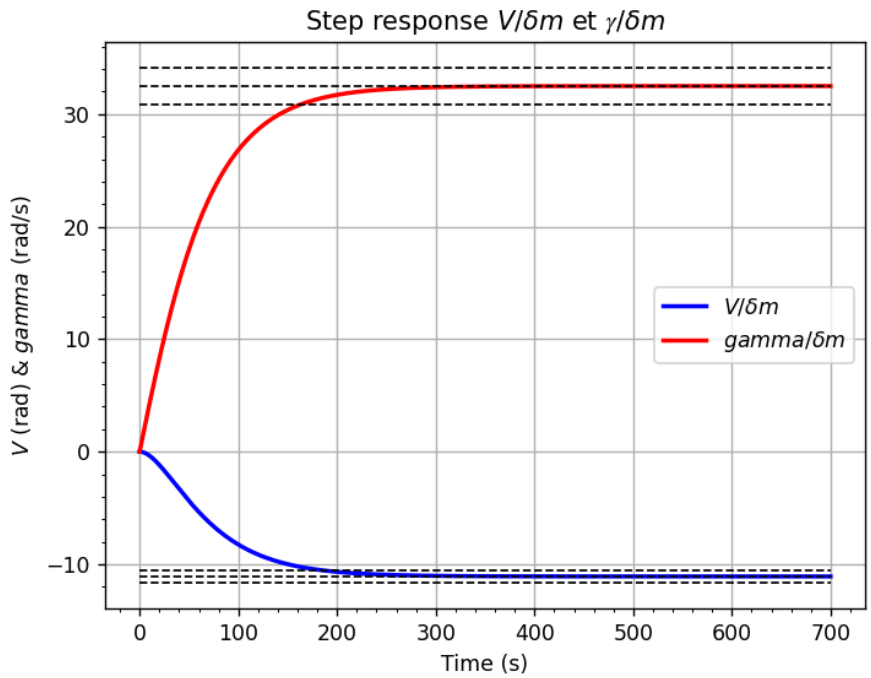


Figure 4: Step response for each variable in phugoid oscillation mode

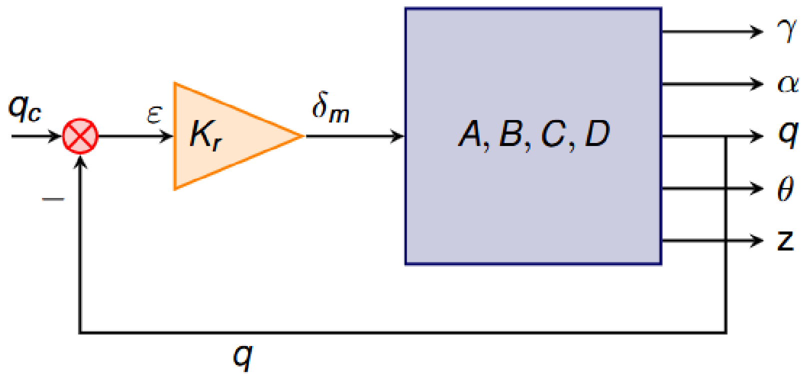
In phugoid oscillation mode, we can see that the step response is stable with a very high 5% settling time of **184 s for V and 163.3 s for γ.**

To build an autopilot, we're going to add a control feedback loop to our model.
Moreover, we will now consider that the speed is controlled with an auto-throttle which is perfect (with an instantaneous response). Consequently, the speed (V) can be removed from the state vector. We will now consider the following (5×1) state vector ($\mathbf{X} = (\gamma\ \alpha\ q\ \theta\ z)^T$) and ($\mathbf{U} = (\delta_m)$) as the command vector.

4 Controllers synthesis

4.1 q feedback loop

We are beginning to build an autopilot by adding a gyrometric feedback loop (with q as the measured variable).



a

Figure 5: System with a gyrometric feedback loop

Using the sisopy31VS script, and more specifically its sisotool function, we can determine the kain K_r of q feedback loop. To do this, we'll look in particular at the 'Root locus' diagram displayed using the sisotool function. In order for the closed-loop damping ratio to be 0.7, the closed-loop poles represented by the red stars need to be superimposed on the black dotted line representing the xi-line. This gives us a gain Kr = -0.13225

With q as the output and the gain K_r, we will use the following formulas to get the state space representation (A_k, B_k, C_k, D_k) of the closed-loop system :

$$
\begin{aligned}
A_k &= AK_rBC_q \\
B_k &= K_rB \\
C_k &= C_{\text{out}} = C_q \\
D_k &= K_rD
\end{aligned}
$$

Thanks to thoose formulas, we will get the following matrices for the state space representation :

$$A_k = \begin{pmatrix} 0 & 2.1821 & 0.0569 & 0 & 0 \\ 0 & -2.1821 & 0.9431 & 0 & 0 \\ 0 & -104.9054 & -12.9696 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 489.1036 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$B_k = \begin{pmatrix} -0.0569 \\ 0.0569 \\ 11.8472 \\ 0 \\ 0 \end{pmatrix}$$

$$C_k = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

$$D_k = \begin{pmatrix} 0 \end{pmatrix}$$

We can also give the transfer function of the closed-loop system :

$$\frac{11.85s + 19.88}{s^2 + 15.15s + 127.2}$$

Using the damp function, we can get the values of the modes, the damping ratio, and the proper pulsation of the closed-loop system:

Eigenvalue (pole)	**Damping**	**Frequency**
0	1	0
0	1	0
0	1	0
$-7.576 + 8.357j$	0.6716	11.28
$-7.576 - 8.357j$	0.6716	11.28

Now let's plot the step response of the closed-loop system :

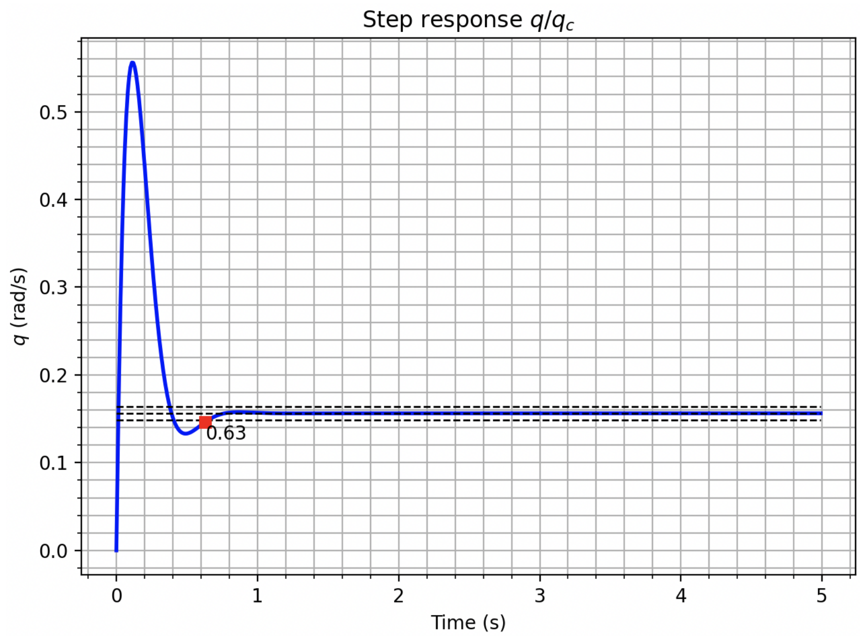


Figure 6: Step response of the closed-loop system

The step response is stable with a pretty low 5% settling time of **0.63 s.** However, there is an important overshoot.

We suppose that we use the q feedback loop as an assistance for a human pilot. In case of damper failure (q loop controller), the human pilot must act smoothly to counter the incidence oscillations and pilot the aircraft with modified efficiency commands (different open loop and closed loop static gain). We need to cancel the effects of the damper at low frequency by adding high-pass filter named washout filter: $\frac{\tau s}{1+\tau s}$.

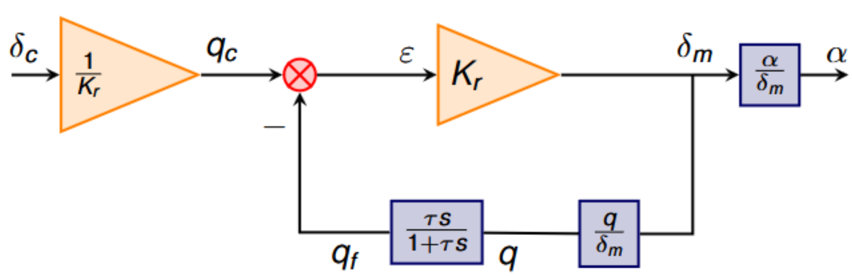


Figure 7: System with the washout filter

To do so, we will choose a value of $\tau = 0.7\,\mathrm{s}$
Using the "feedback" and "series" commands of the control toolbox, we can plot the open-loop response, the closed-loop response without filter, and the closed-loop response with the washout filter :

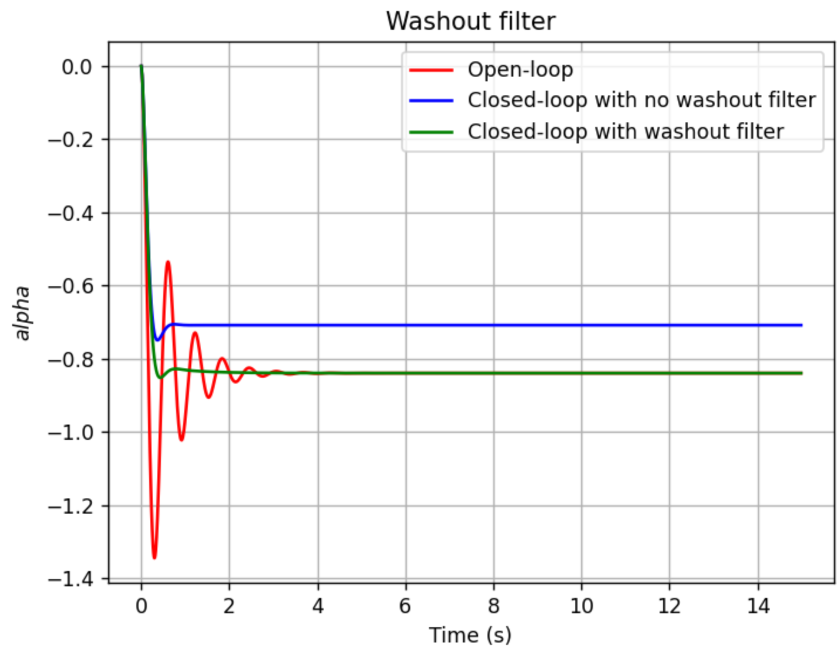


Figure 8: Step responses for different configurations

With the washout filter, the steady-state response is the same as the open-loop response.

4.2 γ feedback loop

We consider that the auto-throttle perfectly ensures that the speed is constant, so that $\dot{v} = \frac{dv}{dt} = 0$. A flight path angle feedback loop is added to the preceding controlled system (with the q feedback loop, keeping the preceding K_r tuning).

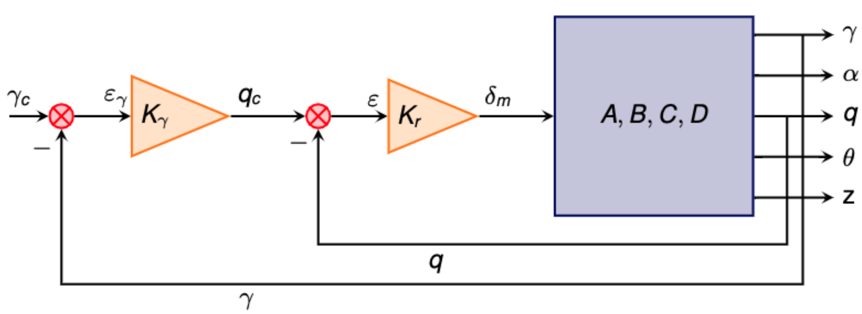


Figure 9: System with a gyrometric and a flight path angle feedback loop

We first want to propose a gain allowing a gain margin $\geq 8\,\text{dB}$ and a phase margin $\geq 35°$ and an optimized settling time (to within a 5 % threshold).
In the above configuration, we find the first gain $K_\gamma = 8.61376$. We have a gain margin of 8.039dB, a phase margin of 80.456 deg, and a 5 % settling time of 1.927 s. The conditions are therefore all well met, but the 5 % settling time could be improved by reducing the gain and phase margins and adding a slight overshoot.

It's the reason why in a second time we will choose a tuning (that will be kept for going on with the study), with the following requirements:

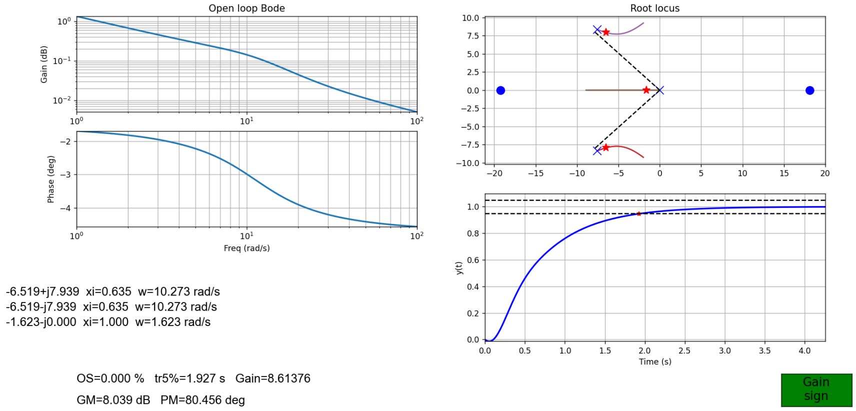


Figure 10: First tuning for K_γ

- An overshoot $D_1 \leq 5\%$
- A settling time to within 5%, for a step response that must be minimized
- The pseudo-periodic modes must be correctly damped ($\xi \geq 0.5$).

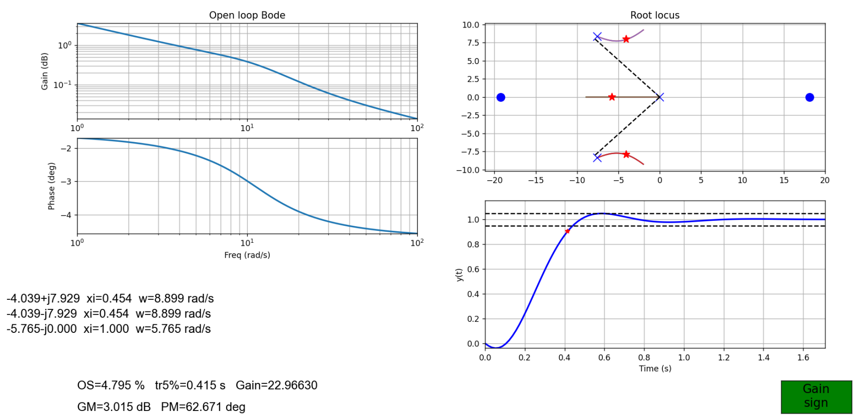


Figure 11: Second tuning for K_γ

In this configuration, we find the second gain $K_\gamma = 22.96630$. We have an overshoot of 4.795 % , a damping ratio higher than 0.5 and a 5 % settling time of 0.415 s. The conditions are therefore all well met, and the 5 % settling time is minimized as wanted.

State space representation with γ output :

$$
A_\gamma = \begin{pmatrix} 1.3077 & 2.1821 & 0.0569 & 0 & 0 \\ -1.3077 & -2.1821 & 0.9431 & 0 & 0 \\ -272.0872 & -104.9054 & -12.9696 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 489.1036 & 0 & 0 & 0 & 0 \end{pmatrix}
$$

$$
B_\gamma = \begin{pmatrix} -1.3077 \\ 1.3077 \\ 272.0872 \\ 0 \\ 0 \end{pmatrix}
$$

$$
C_\gamma = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \end{pmatrix}
$$

$$
D_\gamma = \begin{pmatrix} 0 \end{pmatrix}
$$

Transfer function in closed loop :

$$
\frac{-1.308s^2 - 1.468s + 456.5}{s^3 + 13.84s^2 + 125.8s + 456.5}
$$

Using the damp function, we can get the values of the modes, the damping ratio, and the proper pulsation of the closed-loop modes:

Eigenvalue (pole)	**Damping**	**Frequency**
0	1	0
0	1	0
$-4.039 + 7.929j$	0.4539	8.899
$-4.039 - 7.929j$	0.4539	8.899
-5.765	1	5.765

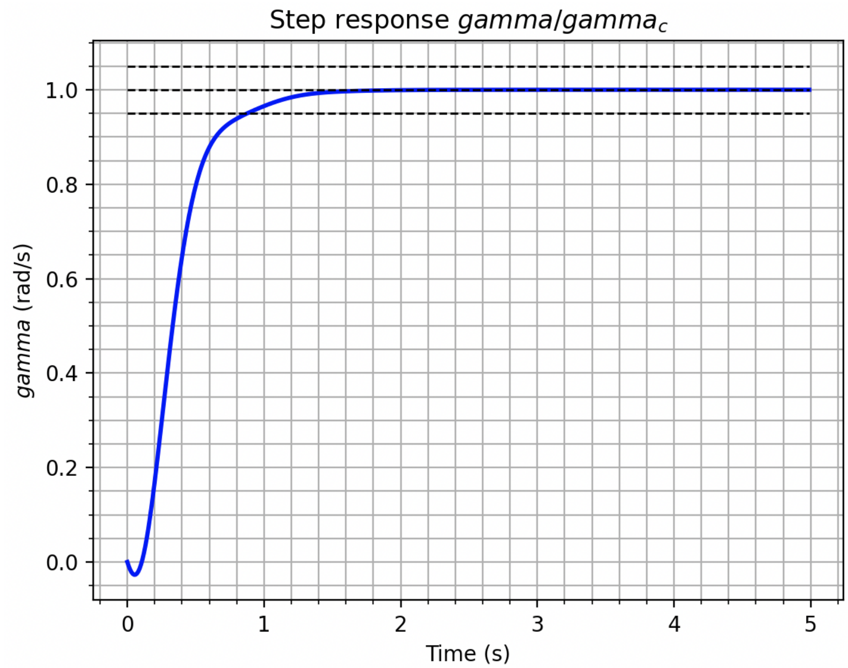


Figure 12: Step response of the closed loop

The step response is stable with a slightly higher 5% settling time of **0.8 s** but no overshoot.

4.3 z feedback loop

We add another control loop, using the measurement of the altitude z to the previous controlled system (aircraft + q feedback loop + γ feedback loop, while keeping the K_r and K_γ tuning).

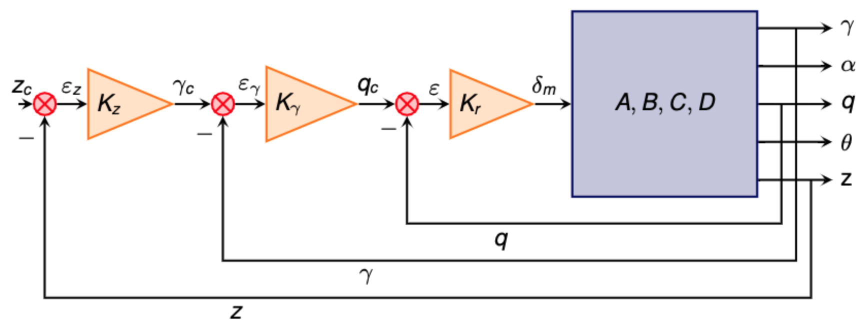


Figure 13: System with a gyrometric, a flight path angle and an altitude feedback loop

We want to tune the gain K_z to respect the conditions

- An overshoot $D_1 \leq 5\%$
- A settling time to within 5%, for a step response that must be minimized
- The pseudo-periodic modes must be correctly damped ($\xi \geq 0.5$).

Unfortunately, the root locus curve for this example seems to be wrong, as there is no way of satisfying the stated conditions, as can be seen from the graph below:

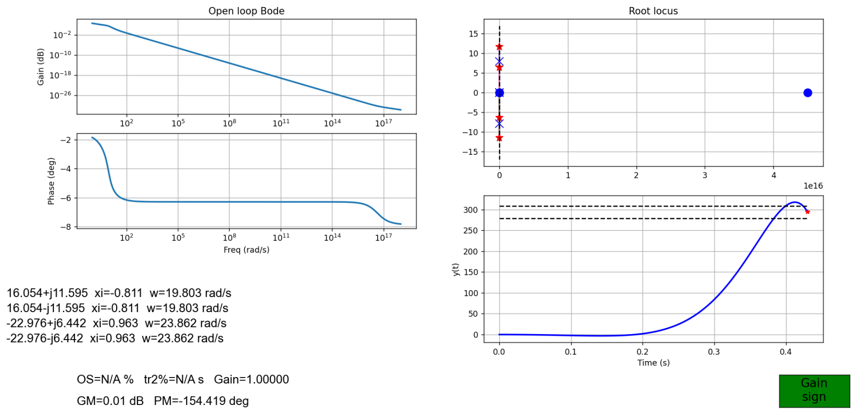


Figure 14: Sisotool function for z

We will then arbitrarily choose a kain $K_z = 0.00010$
State space representation with z output :

$$A_z = \begin{pmatrix} 1.3077 & 2.1821 & 0.0569 & 0 & 0.0001 \\ -1.3077 & -2.1821 & 0.9431 & 0 & -0.0001 \\ -272.0872 & -104.9054 & -12.9696 & 0 & -0.0272 \\ 0 & 0 & 1 & 0 & 0 \\ 489.1036 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$B_z = \begin{pmatrix} -0.0001 \\ 0.0001 \\ 0.0272 \\ 0 \\ 0 \end{pmatrix}$$

$$C_z = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$D_z = \begin{pmatrix} 0 \end{pmatrix}$$

Transfer function in closed loop

$$\frac{-0.06396s^2 - 0.07179s + 22.33}{s^4 + 13.84s^3 + 125.7s^2 + 456.5s + 22.33}$$

Using the damp function, we can get the values of the modes, the damping ratio, and the proper pulsation of the closed-loop modes

Eigenvalue (pole)	**Damping**	**Frequency**
0	1	0
$-4.042 + 7.907j$	0.4552	8.88
$-4.042 - 7.907j$	0.4552	8.88
-5.71	1	5.71
-0.04959	1	0.04959

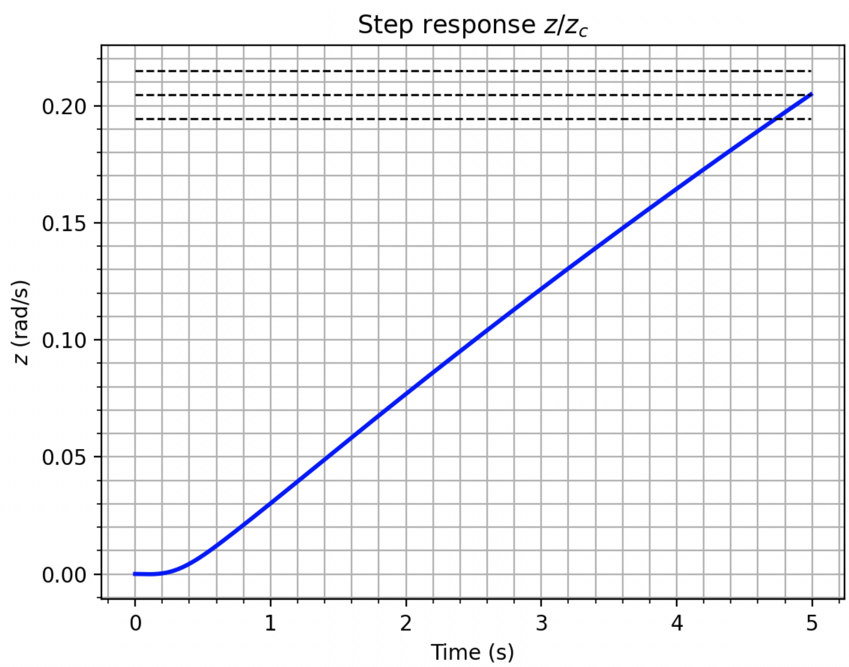


Figure 15: Step response of the closed loop

4.4 Addition of a saturation in the γ control loop

A saturation is added at the input of the γ feedback loop. In this question, we are going to determine the value of γ_{csat}, but we will not implement the non-linear simulation of the saturated autopilot.

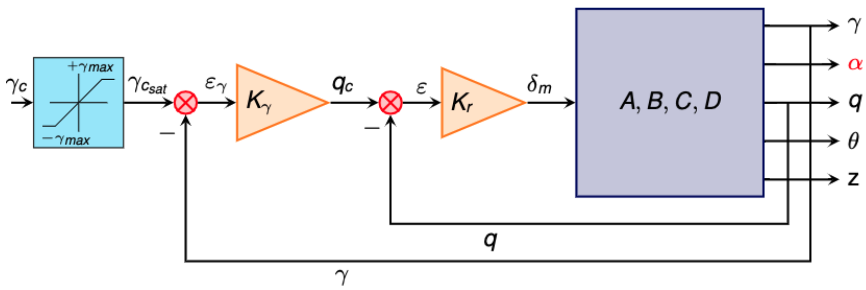


Figure 16: System with a saturation

State-space representation :

$$A_{sat} = \begin{pmatrix} 1.3077 & 2.1821 & 0.0569 & 0 & 0 \\ -1.3077 & -2.1821 & 0.9431 & 0 & 0 \\ -272.0872 & -104.9054 & -12.9696 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 489.1036 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$B_{sat} = \begin{pmatrix} -0.0411 \\ 0.0411 \\ 8.5513 \\ 0 \\ 0 \end{pmatrix}$$

$$C_{sat} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

$$D_{sat} = \begin{pmatrix} 0 \end{pmatrix}$$

We want to evaluate α_{max} knowing that $\Delta n_z = 3.2\,g$. We can deduce $\alpha_{\max}$ with the following formula :

$$\alpha_{\max} = \alpha'_{\text{eq}} + (\alpha'_{\text{eq}} - \alpha_0)\Delta n_z$$

We obtain $\alpha_{\max} = 0.059498 rad$

We now want to obtain the value of γ_{max} knowing that the maximum incidence α equals $\alpha_{\max}$. To do this, we'll use a first method (bisection method) which uses a coded function f which associates γ_{csat} to the difference $\max(\alpha(t)) - \alpha_{\max}$, $\alpha(t)$ being the response of the transfer between γ_{csat} and α to a step of a value of γ_{csat}, then a second method (scalling method) to check our result. We obtain the following results:

	Bissection Method	**Scaling Method**
γ_{max}	0.042533	0.042533

The results are the same for both methods, which seems to give a correct value.

4.5 Change gravitity center

Finally, we modified the center of gravity of our aircraft by applying a factor $f = 1.1$ to its original value. This adjustment reflects a hypothetical scenario where the aircraft's center of gravity shifts due to changes in load distribution or design modifications. Such a shift impacts critical aerodynamic parameters, including the moments and forces acting on the aircraft, which in turn alter the state-space representation of the system. Recomputing the state-space matrices A, B, C, D allows us to accurately model the dynamics of the aircraft with the new center of gravity. This recalibration is essential for analyzing how the control system responds to the updated dynamics and ensuring that the stability, performance, and control objectives are still met under these altered conditions.

Our new state space :

$$A_{new} = \begin{pmatrix} -0.0587 & -0.02 & -0.0398 & 0. & 0. & 0. \\ 0.0389 & 0. & 2.1821 & 0. & 0. & 0. \\ -0.0389 & 0. & -2.1821 & 1. & 0. & 0. \\ 0. & 0. & -43.6125 & -1.1223 & 0. & 0. \\ 0. & 0. & 0. & 1. & 0. & 0. \\ 0. & 489.1036 & 0. & 0. & 0. & 0. \end{pmatrix}$$

$$B_{new} = \begin{pmatrix} 0 \\ 0.4306 \\ -0.4306 \\ -77.3238 \\ 0 \\ 0 \end{pmatrix}$$

$$C_{new} = I_6$$

$$D_{new} = \mathbf{0}_{6\times 6}$$

Using the "damp" command, we can now give the poles of the system, their damping ratio and their pulsation :

Eigenvalue (pole)	**Damping**	**Frequency (Hz)**
0	1	0
0	1	0
$-1.652 + 6.584j$	0.2434	6.788
$-1.652 - 6.584j$	0.2434	6.788
-0.01796	1	0.01796
-0.04107	1	0.04107

The system exhibits a mix of fast oscillatory dynamics and slower decaying modes. The two poles at the origin indicate the presence of integrators, which could lead to steady-state drift or instability if not properly controlled. The complex-conjugate poles at $-1.652 \pm 10.23j$ suggest oscillatory behavior with a low damping ratio of 0.1595, implying significant overshoot and sustained oscillations in the system's response. These oscillations occur at a natural frequency of 10.36 Hz, indicating relatively fast dynamics. In contrast, the real poles at -0.01932 and -0.03931 have small magnitudes, introducing slower decay modes that may not significantly affect the system's transient response. Overall, the system's dynamic behavior is dominated by the complex-conjugate poles, making them critical for stability and performance analysis. Proper tuning of the controller will be necessary to manage the oscillatory nature and ensure robust system stability.

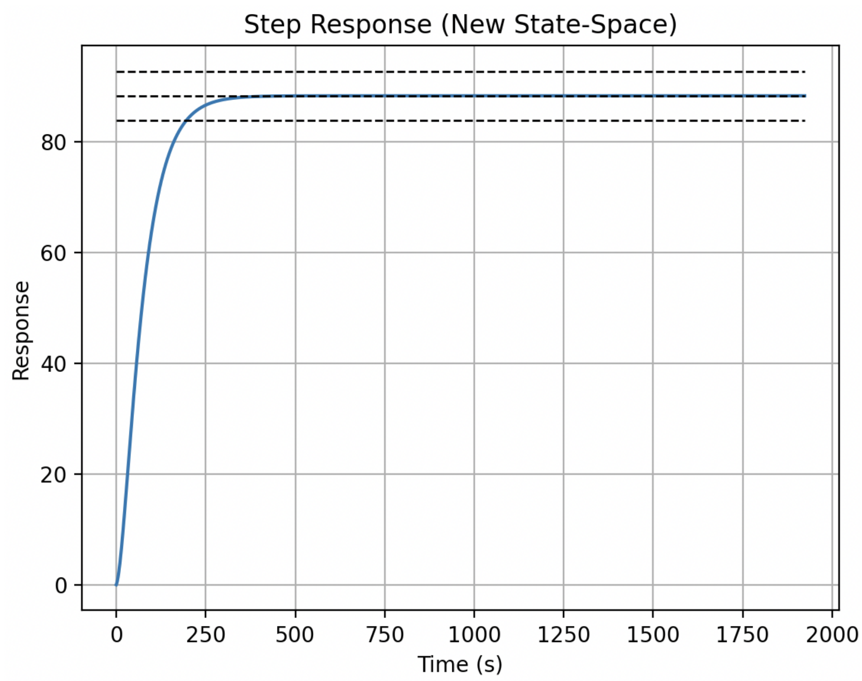


Figure 17: Step response for the new State-Space

4.6 LQ Controller

The Linear Quadratic (LQ) method is an optimal control technique used to regulate the behavior of dynamic systems. It is based on minimizing a cost function that balances the performance of the system and the control effort. The cost function typically takes the form:

$$J = \int_0^\infty \left(x^\top Q x + u^\top R u\right) dt$$

where x is the state vector, u is the control input, Q is a positive semi-definite matrix penalizing deviations in the states, and R is a positive definite matrix penalizing the control effort. By solving this optimization problem, the LQ method computes a feedback gain matrix K such that the control law:

$$u = -Kx$$

minimizes the cost function. This ensures the system achieves desired performance characteristics, such as stability, rapid response

The selection of the matrices Q and R in the Linear Quadratic Regulator (LQR) design process is critical to achieving the desired system behavior. The matrix Q is used to penalize deviations in the system states, while R is used to penalize the control effort. The goal is to balance system performance with the required control energy.

In our case, Q was chosen as a diagonal matrix to assign specific weights to the states. The weights were selected based on the relative importance of each state for the system's performance. For example, a high weight was assigned to the altitude state to prioritize its precision. Similarly, R was selected to limit the control effort. A smaller value for R allows for more aggressive control actions, but may result in higher control energy. In our case, R was set as a scalar value, leading to the following matrix:

$$Q = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 100 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \quad R = \begin{pmatrix} 0.1 \end{pmatrix}$$

The following figure give us the step response with LQ Controller :

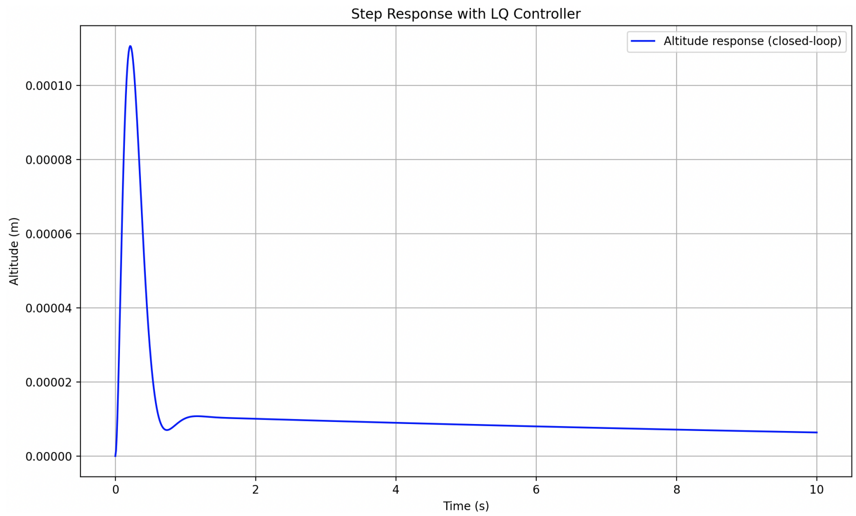


Figure 18: Step Response with LQ Controller

The figure shows a **well-damped step response**, with a moderate transient overshoot and a rapid settling time. This reflects good performance of the LQ controller, balancing dynamic response and control effort. The system demonstrates stability, as the response converges without divergence or prolonged oscillations. The very small response amplitude (10^{-4}) suggests either a very small input command or that the observed variable (altitude) has limited influence in the model. If this low amplitude is intentional, it indicates that the altitude reacts minimally to the input, which could be desirable if the goal is to minimize disturbances to the system. However, it may be worth verifying the units and scaling of the simulated variables to ensure consistency with the physical expectations of the system.

5 Conclusion

This report has presented a thorough study of an aircraft's stability and control, from its aerodynamic model to the synthesis of various control loops. We analyzed the uncontrolled aircraft dynamics, focusing on equilibrium points, the short-period mode, and phugoid oscillations, and then designed several controllers to ensure stability and performance under different scenarios.

Throughout the project, we encountered challenges in certain stages of the implementation. One significant issue arose during the use of the SISO Tool for tuning the control gains. Despite our efforts to achieve precise values, the tool introduced complications that made it difficult to refine the gains to the desired accuracy. As a result, we opted for approximate gain values in some cases to proceed with the analysis and ensure the program's operability.

While these approximations were essential for maintaining the continuity of our work, they might explain some of the anomalies observed in the response curves, such as unexpected oscillations or slight discrepancies from expected behaviors. Nevertheless, these gains played a critical role in demonstrating the overall functionality of the designed controllers and validating the control strategies applied to the system.