

FedDCSR: Federated Cross-domain Sequential Recommendation via Disentangled Representation Learning

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Abstract

Cross-domain Sequential Recommendation (CSR) which leverages user sequence data from multiple domains has received extensive attention in recent years. However, the existing CSR methods require sharing origin user data across domains, which violates the General Data Protection Regulation (GDPR). Thus, it is necessary to combine federated learning (FL) and CSR to fully utilize knowledge from different domains while preserving data privacy. Nonetheless, the sequence feature heterogeneity across different domains significantly impacts the overall performance of FL. In this paper, we propose **FedDCSR**, a novel federated cross-domain sequential recommendation framework via disentangled representation learning. Specifically, to address the sequence feature heterogeneity across domains, we introduce an approach called inter-intra domain sequence representation disentanglement (SRD) to disentangle the user sequence features into domain-shared and domain-exclusive features. In addition, we design an intra domain contrastive infomax (CIM) strategy to learn richer domain-exclusive features of users by performing data augmentation on user sequences. Extensive experiments on three real-world scenarios demonstrate that FedDCSR achieves significant improvements over existing baselines¹.

Keywords: federated learning; recommendation system; sequential recommendation; cross-domain recommendation; disentangled representation learning.

1 Introduction

In recent years, cross-domain sequential recommendation (CSR) has found wide applications in various scenarios such as e-commerce, social media, and video-sharing platforms. Based on the assumption that users have similar preferences across domains, CSR can significantly promote the next-item recommendation performance for users by leveraging user sequence data

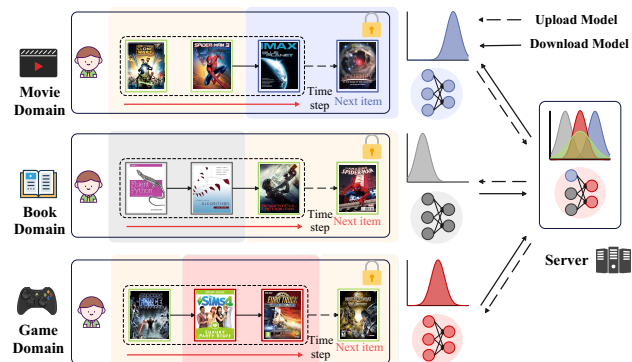


Figure 1: Sequence feature heterogeneity across domains in the FedCSR scenario.

from multiple domains. However, existing works around CSR [1–3] require sharing origin data between domains, which violates the General Data Protection Regulation (GDPR). How to provide high-quality cross-domain sequential recommendations while preserving data privacy has become an urgent issue.

In this paper, we focus on a new paradigm called federated cross-domain sequential recommendation (FedCSR). In this context, user sequences are considered private information, which cannot be directly shared between domains. Although FedCSR can effectively solve the privacy issue in CSR, it also faces the challenge of sequence feature heterogeneity across domains, that is, user sequences in different domains contain domain-exclusive interaction information.

It is impractical to directly apply classical federated learning methods such as FedAvg [4] in the FedCSR scenario due to the sequence feature heterogeneity across domains. Specifically, Figure 1 gives a toy example of the sequence feature heterogeneity across different domains. As shown in Figure 1, a user watches action movies and documentary movies in the Movie domain, reads professional books and action books in the Book domain, and plays action games and simulation games in the Game domain. As mentioned above, documentary movies in the Movie domain, professional books in the Book domain, and simulation games in the Game

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¹Code available at <https://github.com/orion-orion/FedDCSR>

domain can all be regarded as domain-exclusive interaction information. Therefore, due to the presence of sequence feature heterogeneity, we can only obtain sub-optimal results if all domains share the same model.

To address the challenge above, we propose a novel federated cross-domain sequential recommendation framework via disentangled representation learning (FedDCSR), which allows different domains to train better-performing CSR models collaboratively without sharing origin user data. Specifically, inspired by the disentangled representation learning [5, 6], we introduce an inter-intra domain sequence representation disentanglement method called SRD to address the sequence feature heterogeneity across domains. Following this method, the model of each domain is split into a local branch and a global branch, which are responsible for extracting domain-shared and domain-exclusive sequence representations separately. In addition, we design a contrastive infomax strategy CIM to learn richer domain-exclusive features of users by performing data augmentation on user sequences. We conduct the evaluation on Amazon datasets under the FL setting. The experimental results show that our FedDCSR can improve the performance of recommendations in three different CSR scenarios.

To summarize, our contributions are as follows:

- We propose a novel federated cross-domain sequential framework FedDCSR, which allows different domains to collaboratively train better performing CSR models, while effectively preserving data privacy.
- We introduce SRD, an inter-intra domain sequence representation disentanglement method, which disentangles the user sequence features into domain-shared and domain-exclusive features to solve the sequence feature heterogeneity problem across domains.
- We design an intra domain contrastive infomax strategy CIM, to learn richer domain-exclusive sequence features of users by performing data augmentation on user sequences.

2 Related Work.

Sequential Recommendation SASRec [7] first uses the self-attention network to model user sequences. VSAN [8] introduces variational inference into self-attention networks for sequential recommendation. ContrastVAE [9] proposes to use contrastive learning to solve the posterior collapse problem in VAE. CL4SRec [10] and DuoRec [11] introduce the contrastive learning framework to extract self-supervised signals from user

sequences. However, the above methods only focus on a single domain and cannot fully utilize user data from multiple domains.

Cross-domain Recommendation DDTCDR [12] proposes a dual transfer learning based model that improves recommendation performance across domains. DisAlign [13] proposes Stein path alignment to align the distributions of embeddings across domains. CDRIB [14] proposes information bottleneck regularizers to build user-item correlations across domains. However, the above methods should access the whole user-item interactions across domains and are not feasible in the FL setting.

Cross-domain Sequential Recommendation As a pioneer work, π -net [1] utilizes a gating mechanism to enhance the domain knowledge transfer. Based on this, MIFN [2] further introduces knowledge graphs to build bridges between different domains. C²DSR [3] proposes a contrastive infomax objective for modeling the relationship between domains. While these methods still face the same problems as above.

Federated Cross-domain Recommendation FedCT [15] employs variational autoencoders to generate shared encoding latent vectors across domains. FedCDR [16] sets up the user personalization model on the client side and uploads other models to the server during aggregation. FedCTR [17] designs a framework to train a CTR prediction model across multiple platforms under privacy protection. However, none of them have considered the sequential data and addressed the sequence feature heterogeneity problem across domains.

3 Methodology.

3.1 Problem Definition. Assume there are K local clients and a central server. The k -th client maintains its own user sequence dataset denoted as $\mathcal{D}_k = \{\mathbf{S}_k^{(u)}\}_{u \in \mathcal{U}_k}$, which forms a distinct domain, where \mathcal{U}_k is the user set in client k and $\mathbf{S}_k^{(u)}$ is the user sequence for a certain user u . The sequence $\mathbf{S}_k^{(u)} = (s_1, s_2, \dots, s_T)$ contains the interactions between user u and items within the past T time steps.

For client k , we disentangle the user sequence feature into domain-shared representations \mathbf{Z}_k^s and domain-exclusive representations \mathbf{Z}_k^e . After the local model update is completed, the central server aggregates $\{\mathbf{Z}_k^s\}_{k=1}^K$ to obtain the global representations \mathbf{Z}^g used in the next training round. The local augmented user representations is denoted as $\tilde{\mathbf{Z}}_k^e$. Each client's local model is split into a global branch including a domain-shared encoder (parameterized by ϕ_k^s), and a local branch including a domain-exclusive encoder (parameterized by ϕ_k^e) and a prediction layer (parameter-

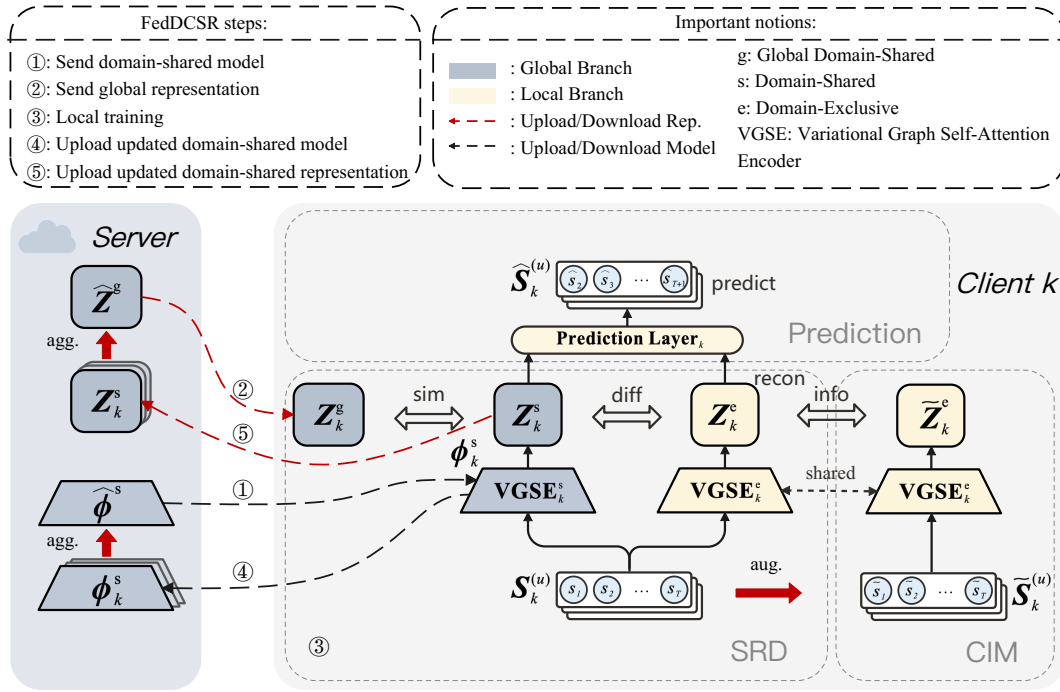


Figure 2: An overview of FedDCSR.

ized by θ_k). After each round of training is completed, the server aggregates $\{\phi_k^s\}_{k=1}^K$ to get ϕ^g which will be shared among clients in the next training round.

3.2 Overview of FedDCSR. As shown in Figure2, our proposed FedDCSR adopts client-server federated learning architecture. The model of each client is split into a local branch (in yellow) and a global branch (in purple). Only domain-shared representations and model parameters are aggregated in each training round. During the test stage, domain-shared and domain-exclusive representations are jointly employed for local prediction.

3.3 Variational Graph Self-Attention Encoder (VGSE). In this section, we introduce our proposed VGSE, which is used to model the representation distribution of local user sequences, taking into account the relationships between items.

Assume that user sequences in the local dataset $\mathcal{D}_k = \{S_k^{(u)}\}_{u \in \mathcal{U}_k}$ are drawn from the following distribution:

$$p_{\theta_k}(S_k) = \int p_{\theta_k}(S_k | Z_k^s, Z_k^e) p(Z_k^s) p(Z_k^e) dZ_k^s dZ_k^e, \quad (3.1)$$

where we assume the prior distributions $p(Z_k^s)$, $p(Z_k^e)$ following normal Gaussian distribution $\mathcal{N}(\mathbf{0}, \mathbf{I})$, and the

conditional distribution of S_k is defined as follows:

$$\begin{aligned} p_{\theta_k}(S_k | Z_k^s, Z_k^e) &= \prod_{t=2}^T p_{\theta_k}(s_t | s_1, \dots, s_{t-1}, Z_k^s, Z_k^e) \\ &= \prod_{t=2}^T p_{\theta_k}(s_t | z_k^{s,1}, \dots, z_k^{s,t-1}, z_k^{e,1}, \dots, z_k^{e,t-1}), \end{aligned} \quad (3.2)$$

where $Z_k^s, Z_k^e \in \mathbb{R}^{T \times d}$, and $z_k^{s,t}, z_k^{e,t} \in \mathbb{R}^d, t = [1, 2, \dots, T]$ denote the representation of the user sequence at the t -th time step.

Because the integral of the marginal likelihood in (3.1) is intractable and the true posterior $p_{\theta_k}(Z_k^s, Z_k^e | S_k)$ is intractable as well, we use the approximated posterior $q_{\phi_k}(Z_k^s, Z_k^e | S_k)$, which is coefficientized according to the probabilistic graphical model in Figure4 as follows:

$$q_{\phi_k}(Z_k^s, Z_k^e | S_k) = q_{\phi_k^s}(Z_k^s | S_k) q_{\phi_k^e}(Z_k^e | S_k), \quad (3.3)$$

where $q_{\phi_k^s}(Z_k^s | S_k)$ and $q_{\phi_k^e}(Z_k^e | S_k)$ are domain-shared and domain-exclusive VGSE respectively.

Next, we will illustrate how VGSE is designed. As shown in Figure 3, we first process the item relationship matrix A_k through two GNNs to obtain the domain-shared and domain-exclusive embeddings of items. The update formula of the l -th GNN layer (totally L layer)

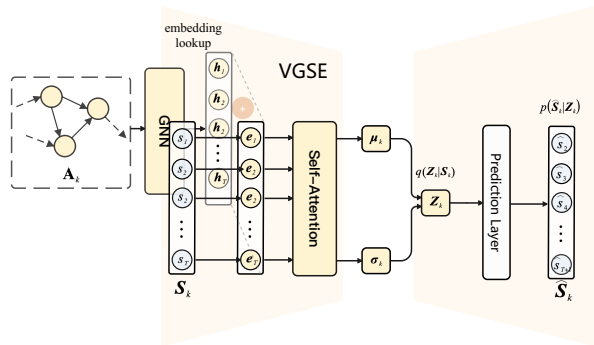


Figure 3: The architecture of the variational graph self-attention encoder.

is defined as follows:

$$(3.4) \quad \mathbf{H}_k^{s,l} = \text{Norm}(\mathbf{A}_k) \mathbf{H}_k^{s,l-1}, \quad \mathbf{H}_k^{e,l} = \text{Norm}(\mathbf{A}_k) \mathbf{H}_k^{e,l-1},$$

where $l = 1, 2, \dots, L$. Let $\mathbf{H}_k^s, \mathbf{H}_k^e$ denote the average embeddings of the items obtained in all layers of GNN. Then for input sequence \mathbf{S}_k , we obtain its corresponding domain-shared and domain-exclusive relationship embeddings $\mathbf{H}_{k,s_k}^s, \mathbf{H}_{k,s_k}^e$ through the EmbeddingLookup operation. Let $\mathbf{E}_k^s, \mathbf{E}_k^e$ denote the sequential embeddings of the sequence, and $\mathbf{P}_k^s, \mathbf{P}_k^e$ denote the positional embeddings of the sequence. Then we add up $\mathbf{H}_{k,s_k}^s, \mathbf{P}_k^s, \mathbf{E}_k^s$ and $\mathbf{H}_{k,s_k}^e, \mathbf{P}_k^e, \mathbf{E}_k^e$ separately, then feed these sums into self-attention networks to get attention aggregation results μ_k^s, σ_k^s and μ_k^e, σ_k^e :

$$(3.5) \quad \begin{aligned} \mu_k^s, \sigma_k^s &= \text{Self-Attention}_k^s(\mathbf{E}_k^s + \mathbf{P}_k^s + \mathbf{H}_{k,s_k}^s), \\ \mu_k^e, \sigma_k^e &= \text{Self-Attention}_k^e(\mathbf{E}_k^e + \mathbf{P}_k^e + \mathbf{H}_{k,s_k}^e). \end{aligned}$$

Next, we use the reparameterization trick to sample the domain-shared and the domain-exclusive representations $\mathbf{Z}_k^s, \mathbf{Z}_k^e$ from $q(\mathbf{Z}_k^s | \mathbf{S}_k)$ and $q(\mathbf{Z}_k^e | \mathbf{S}_k)$:

$$(3.6) \quad \mathbf{Z}_k^s = \mu_k^s + \sigma_k^s \odot \epsilon, \quad \mathbf{Z}_k^e = \mu_k^e + \sigma_k^e \odot \epsilon,$$

where $\epsilon \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$ is Gaussian noise.

3.4 Intra-Inter Domain Sequence Representation Disentanglement (SRD). In this section, we introduce SRD, which disentangles user sequence representations into domain-shared and domain-exclusive representations. Specifically, SRD includes three components: difference loss, similarity loss, and reconstruction loss.

3.4.1 Difference Loss The difference loss is applied to each domain and encourages the domain-shared and domain-exclusive encoders to encode distinct information of the inputs. The structured probabilistic relationship between $\mathbf{Z}_k^s, \mathbf{Z}_k^e$ and \mathbf{S}_k is shown in Figure 3.

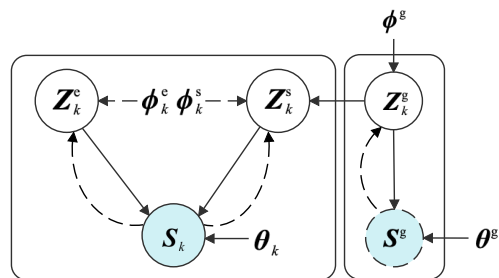


Figure 4: The graphical model illustrating the relationship between $\mathbf{Z}_k^s, \mathbf{Z}_k^e, \mathbf{Z}_k^g, \mathbf{S}_k$ and the dummy variable \mathbf{S}^g , which follows the global sequence distribution.

Then the difference loss $\mathcal{L}_k^{\text{diff}}$ during local training can be formulated as:

$$(3.7) \quad \mathcal{L}_k^{\text{diff}} = I(\mathbf{Z}_k^s; \mathbf{Z}_k^e),$$

Here $I(\cdot, \cdot)$ denotes the mutual information of two random variables. Intuitively, The above formula aims to amplify the difference between domain-shared representations and domain-exclusive representations.

To minimize $I(\mathbf{Z}_k^s; \mathbf{Z}_k^e)$, we need to make a further equivalent transformation. Following the definition of interaction information [18], we have:

$$(3.8) \quad I(\mathbf{Z}_k^s; \mathbf{Z}_k^e) = I(\mathbf{Z}_k^s; \mathbf{Z}_k^e; \mathbf{S}_k) + I(\mathbf{Z}_k^s; \mathbf{Z}_k^e | \mathbf{S}_k).$$

According to the structural assumption in Figure 4, we have $q(\mathbf{Z}_k^s | \mathbf{S}_k) = q(\mathbf{Z}_k^s | \mathbf{Z}_k^e, \mathbf{S}_k)$, thus the last term $I(\mathbf{Z}_k^s; \mathbf{Z}_k^e | \mathbf{S}_k) = H(\mathbf{Z}_k^s | \mathbf{S}_k) - H(\mathbf{Z}_k^s | \mathbf{Z}_k^e, \mathbf{S}_k) = 0$, where $H(\cdot)$ denotes the information entropy. Then we have

$$(3.9) \quad \begin{aligned} I(\mathbf{Z}_k^s; \mathbf{Z}_k^e) &= I(\mathbf{Z}_k^s; \mathbf{Z}_k^e; \mathbf{S}_k) \\ &= I(\mathbf{S}_k; \mathbf{Z}_k^s) - I(\mathbf{S}_k; \mathbf{Z}_k^s | \mathbf{Z}_k^e) \\ &= I(\mathbf{S}_k; \mathbf{Z}_k^s) + I(\mathbf{S}_k; \mathbf{Z}_k^e) - I(\mathbf{S}_k; \mathbf{Z}_k^s, \mathbf{Z}_k^e). \end{aligned}$$

However, directly minimizing the above formula is still intractable, so we need to minimize its variational upper bound [6] instead:

$$(3.10) \quad \begin{aligned} I(\mathbf{Z}_k^s; \mathbf{Z}_k^e) &\leq D_{\text{KL}}(q(\mathbf{Z}_k^s | \mathbf{S}_k) \| p(\mathbf{Z}_k^s)) \\ &\quad + D_{\text{KL}}(q(\mathbf{Z}_k^e | \mathbf{S}_k) \| p(\mathbf{Z}_k^e)) \\ &\quad - \mathbb{E}_{q(\mathbf{Z}_k^s | \mathbf{S}_k)q(\mathbf{Z}_k^e | \mathbf{S}_k)}[\log p(\mathbf{S}_k | \mathbf{Z}_k^s, \mathbf{Z}_k^e)]. \end{aligned}$$

Intuitively, the above formula is to align the posterior distribution $q(\mathbf{Z}_k^s | \mathbf{S}_k)$ and $q(\mathbf{Z}_k^e | \mathbf{S}_k)$ with the Gaussian distribution, and enable \mathbf{Z}_k^s and \mathbf{Z}_k^e can jointly reconstruct \mathbf{S}_k .

3.4.2 Similarity Loss The similarity loss of each domain encourages the domain-shared encoders to encode the information shared between domains. For client k , The structured probabilistic relationship between \mathbf{Z}_k^s and \mathbf{Z}_k^g is shown in Figure 4. Then the similarity loss $\mathcal{L}_k^{\text{sim}}$ during local training can be defined as follows:

$$(3.11) \quad \mathcal{L}_k^{\text{sim}} = -I(\mathbf{Z}_k^s; \mathbf{Z}_k^g).$$

Intuitively, the purpose of the above formula is to make the domain-shared representations and global representations similar to facilitate knowledge transfer.

As mentioned above, directly maximizing $I(\mathbf{Z}_k^s; \mathbf{Z}_k^g)$ is intractable. We adopt the idea of Deep Infomax [19, 20] to use Jensen-Shannon Divergence (JSD) as its variational lower bound:

$$(3.12) \quad \begin{aligned} I(\mathbf{Z}_k^s; \mathbf{Z}_k^g) &\geq \hat{I}^{\text{JSD}}(\mathbf{Z}_k^s; \mathbf{Z}_k^g) \\ &= \mathbb{E}_{q(\mathbf{Z}_k^s, \mathbf{Z}_k^g | \mathbf{S}_k, \mathbf{S}^g)} [-\text{sp}(-T_k(\mathbf{Z}_k^s, \mathbf{Z}_k^g))] \\ &\quad - \mathbb{E}_{q(\mathbf{Z}_k^s | \mathbf{S}_k)q(\mathbf{Z}_k^g | \mathbf{S}^g)} [\text{sp}(T_k(\mathbf{Z}_k^{s'}, \mathbf{Z}_k^g))]. \end{aligned}$$

In the above formula, the discriminator T_k differentiates between a paired sample from the joint (positive pair) and a paired sample from the product of marginals (negative pair), $\text{sp}(z) = \log(1 + e^z)$ is the softplus function, and $\mathbf{Z}_k^{s'}$ denotes negative sample latent vectors, here we set $\mathbf{Z}_k^{s'} = \mathbf{Z}_k^e$ for each user. This bound enables the estimation of mutual information with only a single negative example for each positive example.

3.4.3 Reconstruction Loss Furthermore, it is crucial to empower domain-exclusive representations with the capability to reconstruct sequential data in their respective domains in order to minimize the dilution of domain-exclusive information. The structural probabilistic relationship between \mathbf{Z}_k^e and \mathbf{S}_k is shown in Figure 3. Then the reconstruction loss $\mathcal{L}_k^{\text{recon}}$ during training can be formulated as:

$$(3.13) \quad \mathcal{L}_k^{\text{recon}} = -I(\mathbf{Z}_k^e; \mathbf{S}_k).$$

Intuitively, the above formula correlates the domain-exclusive representations \mathbf{Z}_k^e with the user sequence \mathbf{S}_k to complete the reconstruction of intra domain information.

As before, to maximize $I(\mathbf{Z}_k^e; \mathbf{S}_k)$, we need to maximize its variational lower bound [21]:

$$(3.14) \quad \begin{aligned} I(\mathbf{Z}_k^e; \mathbf{S}_k) &= \mathbb{E}_{p(\mathbf{S}_k)q(\mathbf{Z}_k^e | \mathbf{S}_k)} \left[\log \frac{p(\mathbf{S}_k | \mathbf{Z}_k^e)}{p(\mathbf{S}_k)} \right] \\ &\geq \mathbb{E}_{p(\mathbf{S}_k)q(\mathbf{Z}_k^e | \mathbf{S}_k)} [\log p(\mathbf{S}_k | \mathbf{Z}_k^e)] + H(\mathbf{S}_k). \end{aligned}$$

Afterward, adding up the difference loss, the similarity loss, and the reconstruction loss, the intra-inter

domain representation disentanglement loss can be derived as:

$$(3.15) \quad \begin{aligned} \mathcal{L}_k^{\text{disen}} &= \alpha \mathcal{L}_k^{\text{diff}} + \beta \mathcal{L}_k^{\text{sim}} + \gamma \mathcal{L}_k^{\text{recon}} \\ &= \alpha I(\mathbf{Z}_k^s; \mathbf{Z}_k^e) - \beta I(\mathbf{Z}_k^s; \mathbf{Z}_k^g) - \gamma I(\mathbf{Z}_k^e; \mathbf{S}_k) \\ &\leq \alpha D_{\text{KL}}(q(\mathbf{Z}_k^s | \mathbf{S}_k) \| p(\mathbf{Z}_k^s)) \\ &\quad + \alpha D_{\text{KL}}(q(\mathbf{Z}_k^e | \mathbf{S}_k) \| p(\mathbf{Z}_k^e)) \\ &\quad - \alpha \mathbb{E}_{q(\mathbf{Z}_k^s | \mathbf{S}_k)q(\mathbf{Z}_k^e | \mathbf{S}_k)} [\log p(\mathbf{S}_k | \mathbf{Z}_k^s, \mathbf{Z}_k^e)] \\ &\quad - \beta \mathbb{E}_{q(\mathbf{Z}_k^s, \mathbf{Z}_k^g | \mathbf{S}_k, \mathbf{S}^g)} [-\text{sp}(-T_k(\mathbf{Z}_k^s, \mathbf{Z}_k^g))] \\ &\quad + \beta \mathbb{E}_{q(\mathbf{Z}_k^s | \mathbf{S}_k)q(\mathbf{Z}_k^g | \mathbf{S}^g)} [\text{sp}(T_k(\mathbf{Z}_k^{s'}, \mathbf{Z}_k^g))] \\ &\quad - \gamma \mathbb{E}_{q(\mathbf{Z}_k^e | \mathbf{S}_k)} [\log p(\mathbf{S}_k | \mathbf{Z}_k^e)], \end{aligned}$$

where α , β , and γ are hyperparameters that control the degree of intra-inter domain representation disentanglement.

3.5 Intra Domain Contrastive Infomax (CIM).

In this section, we introduce CIM, which uses contrastive information maximization [22, 23] to enhance the user preference information contained in the domain-exclusive representations \mathbf{Z}_k^e .

In client k , let us denote the user sequence after data augmentation as $\tilde{\mathbf{S}}_k$. The data augmentation method employed in this study is randomly shuffling the sequence of user sequences, i.e., $\tilde{\mathbf{S}}_k = (\tilde{s}_1, \tilde{s}_2, \dots, \tilde{s}_T)$. Then let $\tilde{\mathbf{Z}}_k^e$ denote the augmented domain-exclusive representations, and $\mathbf{z}_k^u = (\mathbf{z}_k^e)_T$ denote the user representation, where T is the last position of the sequence. Assuming that the batch size is N , the augmentation operation yields $2N$ sequences $\{\mathbf{z}_k^{(1)}, \tilde{\mathbf{z}}_k^{(1)}, \dots, \mathbf{z}_k^{(N)}, \tilde{\mathbf{z}}_k^{(N)}\}$. Therefore, for each positive pair in the batch, the negative set \mathcal{Z}_k^- comprises $2(N-1)$ negative pairs. For example, consider the augmented pair of sequence representations $\mathbf{z}_k^{(1)}$ and $\tilde{\mathbf{z}}_k^{(1)}$, the corresponding negative set \mathcal{Z}_k^- consists of $\{\mathbf{z}_k^{(2)}, \tilde{\mathbf{z}}_k^{(2)}, \dots, \mathbf{z}_k^{(N)}, \tilde{\mathbf{z}}_k^{(N)}\}$. Thus, for domain k , the intra domain contrastive infomax loss can be formulated as:

$$(3.16) \quad \begin{aligned} \mathcal{L}_k^{\text{info}} &= -I(\mathbf{z}_k^u, \tilde{\mathbf{z}}_k^u) \\ &\leq -\mathbb{E}_{\mathbf{z}_k^e} \log \frac{e^{(\text{sim}(\mathbf{z}_k^u, \tilde{\mathbf{z}}_k^u)/\tau)}}{e^{(\text{sim}(\mathbf{z}_k^u, \tilde{\mathbf{z}}_k^u)/\tau)} + \sum_{\mathbf{z}_k^-} e^{(\text{sim}(\mathbf{z}_k^u, \mathbf{z}_k^-)/\tau)}} \\ &\quad + \log \frac{e^{(\text{sim}(\tilde{\mathbf{z}}_k^u, \mathbf{z}_k^u)/\tau)}}{e^{(\text{sim}(\tilde{\mathbf{z}}_k^u, \mathbf{z}_k^u)/\tau)} + \sum_{\mathbf{z}_k^-} e^{(\text{sim}(\tilde{\mathbf{z}}_k^u, \mathbf{z}_k^-)/\tau)}}. \end{aligned}$$

The objective of the formula above is to minimize the distance between the positive sample pair $\mathbf{z}_k^u, \tilde{\mathbf{z}}_k^u$, and maximize the distance between the negative sample pair $\mathbf{z}_k^u, \mathbf{z}_k^-$ and $\tilde{\mathbf{z}}_k^u, \mathbf{z}_k^-$, where $\mathbf{z}_k^- \in \mathcal{Z}_k^-$.

3.6 Federated Training and Evaluation The overall federated learning algorithm is shown in Algorithm 1. In each round, the server sends the current global model $\phi^{s,t}$ and global user representations $\mathbf{Z}^{g,t}$ to clients, receives the updated local models $\{\phi_k^{s,t+1}\}_{k=1}^K$ and local domain-shared representations $\{\mathbf{Z}_k^{s,t+1}\}_{k=1}^K$ from clients, and updates the global model using weighted averaging.

In the local training stage, after receiving the global model ϕ^s and the global representations \mathbf{Z}^g , each client updates its local model by the training objectives mentioned above. The overall objective is defined as follows:

$$(3.17) \quad \begin{aligned} \mathcal{L}_k^{\text{total}} &= \mathcal{L}_k^{\text{disen}} + \lambda \mathcal{L}_k^{\text{info}} \\ &= \alpha \mathcal{L}_k^{\text{diff}} + \beta \mathcal{L}_k^{\text{sim}} + \gamma \mathcal{L}_k^{\text{recon}} + \lambda \mathcal{L}_k^{\text{info}}, \end{aligned}$$

In the local test stage, we fuse the representations \mathbf{Z}_k^s and representations \mathbf{Z}_k^e , and then we have $\mathbf{h}_{k,T} = f_{\theta_k}(\mathbf{Z}_k^s + \mathbf{Z}_k^e)_T$, where $f_{\theta_k}(\cdot)$ is the prediction layer. Then the probability of being interacted next for item i can be formulated as follows:

$$(3.18) \quad p_{\theta_k}(s_{k,T+1} = i \mid \mathbf{z}_{k,1}, \dots, \mathbf{z}_{k,T}) = \text{softmax}(\mathbf{h}_{k,T})_i,$$

where $\mathbf{z}_{k,T}$ is the representation corresponding to the T -th time step of the sequence, and $\mathbf{h}_{k,T}$ is the output of the T -th position of the prediction layer $f_{\theta_k}(\cdot)$.

4 Experiments.

4.1 Datasets As used in many cross-domain recommendation methods, we utilize the publicly available datasets from Amazon² (an e-commerce platform) to construct the federated CSR scenarios. We select ten domains to generate three cross-domain scenarios: Food-Kitchen-Cloth-Beauty (FKCB), Movie-Book-Game (MBG), and Sports-Garden-Home (SGH). Following the approach of previous studies [1–3], we remove users and items with fewer than 10 interactions. Additionally, we only preserve sequences that contain a minimum of 4 items, and a maximum of 16 items. For the dataset split, we select the latest 20% of each user’s interactions as the validation/test set, and the remaining 80% interaction sequences as the training set. We summarize the statistics of the federated CSR scenarios in Table 1.

4.2 Experimental Setting

4.2.1 Evaluation Protocol. Following previous sequential recommendation works [7, 10], we utilize the leave-one-out method to evaluate the recommendation performance. To ensure unbiased evaluation, we

Algorithm 1: FedDCSR

Input: Local datasets $\mathcal{D} = \{\mathcal{D}_k\}_{k=1}^K$, local epochs E , learning rate η

Output: The optimal encoder parameters $\{\phi_k\}_{k=1}^K = \{(\phi_k^s, \phi_k^e)\}_{k=1}^K$

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1 Server executes:
2 initialize  $\phi^{s,0}$ ;
3 for round  $t = 0, 1, \dots, T-1$  do
4   for each client  $k \in K$  in parallel do
5     send  $\phi^{s,t}$  and  $\mathbf{Z}^{g,t}$  to client  $k$ ;
6      $\phi_k^{s,t+1}, \mathbf{Z}_k^{s,t+1} = \text{ClientUpdate}(k, \phi^{s,t}, \mathbf{Z}^{g,t})$ ;
7   end
8    $\phi^{s,t+1} = \sum_{k=1}^K \frac{|\mathcal{D}_k|}{|\mathcal{D}|} \phi_k^{s,t+1}$ ;
9    $\mathbf{Z}^{g,t+1} = \sum_{k=1}^K \frac{|\mathcal{D}_k|}{|\mathcal{D}|} \mathbf{Z}_k^{s,t+1}$ ;
10 end
11 ClientUpdate ( $k, \phi^s, \mathbf{Z}^g$ ):
12   save  $\phi^s, \mathbf{Z}^g$  as  $\phi_k^s, \mathbf{Z}_k^g$ ;
13   for local epoch  $i = 1, 2, \dots, E$  do
14     for sequence batch  $\mathbf{S}_k^{(b)}$  from  $\mathcal{D}_k$  do
15        $\tilde{\mathbf{S}}_k^{(b)} = \text{aug}(\mathbf{S}_k^{(b)})$ ;
16        $\mathbf{Z}_k^s = \text{VGSE}_k^s(\mathbf{S}_k^{(b)}; \phi_k^s)$ ;
17        $\mathbf{Z}_k^e, \tilde{\mathbf{Z}}_k^e = \text{VGSE}_k^e(\mathbf{S}_k^{(b)}, \tilde{\mathbf{S}}_k^{(b)}; \phi_k^e)$ ;
18        $\phi_k^s \leftarrow \phi_k^s - \eta \nabla_{\phi_k^s} \left( \alpha \mathcal{L}^{\text{diff}}(\mathbf{Z}_k^s, \mathbf{Z}_k^e) \right.$ 
19          $\left. - \beta \mathcal{L}^{\text{sim}}(\mathbf{Z}_k^s, \mathbf{Z}_k^g) \right)$ ;
20        $\phi_k^e \leftarrow \phi_k^e - \eta \nabla_{\phi_k^e} \left( \alpha \mathcal{L}^{\text{diff}}(\mathbf{Z}_k^s, \mathbf{Z}_k^e) \right.$ 
21          $\left. - \gamma \mathcal{L}^{\text{recon}}(\mathbf{Z}_k^e, \mathbf{S}_k^{(b)}) \right.$ 
22          $\left. - \lambda \mathcal{L}^{\text{info}}(\mathbf{Z}_k^e, \tilde{\mathbf{Z}}_k^e) \right)$ ;
23     end
24   end
25   return  $\phi_k^s, \mathbf{Z}_k^s$ ;

```

Table 1: Statistics of Three Federated CSR scenarios.

| Domain | #Users | #Items | #Train | #Valid | #Test | Avg.Len |
|----------|--------|--------|--------|--------|-------|---------|
| Food | 4658 | 13564 | 4977 | 1307 | 1332 | 8.79 |
| Kitchen | 13382 | 32918 | 11100 | 2172 | 2254 | 8.60 |
| Clothing | 9240 | 34909 | 5720 | 818 | 866 | 9.30 |
| Beauty | 5902 | 17780 | 4668 | 836 | 855 | 10.29 |
| Movie | 34792 | 44464 | 57405 | 10944 | 11654 | 7.97 |
| Book | 19419 | 72246 | 63157 | 11168 | 12149 | 7.20 |
| Game | 5588 | 10336 | 6631 | 1374 | 1444 | 6.49 |
| Sport | 28139 | 88992 | 51477 | 13720 | 14214 | 10.65 |
| Garden | 6852 | 21604 | 10479 | 3074 | 3113 | 9.48 |
| Home | 20784 | 62499 | 37361 | 10058 | 10421 | 10.41 |

²<https://jmcauley.ucsd.edu/data/amazon/>

Table 2: **Federated experimental results(%) on the FKCB scenario. Avg denotes the average result calculated from all domains. The best results are boldfaced.**

| Method | Food | | | Kitchen | | | Clothing | | | Beauty | | | Avg | | |
|----------------------|--------------|--------------|--------------|--------------|--------------|--------------|-------------|-------------|-------------|--------------|--------------|--------------|--------------|--------------|--------------|
| | MRR | HR | NDCG | MRR | HR | NDCG | MRR | HR | NDCG | MRR | HR | NDCG | MRR | HR | NDCG |
| | | @10 | @10 | | @10 | @10 | | @10 | @10 | | @10 | @10 | | @10 | @10 |
| FedSASRec | 6.84 | 14.41 | 8.03 | 0.92 | 1.64 | 0.95 | 0.32 | 0.46 | 0.33 | 3.66 | 7.02 | 4.14 | 2.94 | 5.88 | 3.36 |
| FedVSAN | 21.31 | 35.21 | 23.92 | 6.46 | 12.38 | 7.06 | 1.60 | 2.89 | 1.49 | 11.58 | 20.70 | 13.01 | 10.24 | 17.79 | 11.37 |
| FedContrastVAE | 23.38 | 39.19 | 26.53 | 7.15 | 12.20 | 7.63 | 1.74 | 3.58 | 1.71 | 15.08 | 25.73 | 16.88 | 11.84 | 20.18 | 13.19 |
| FedCL4SRec | 21.89 | 34.53 | 24.32 | 5.56 | 9.80 | 5.93 | 1.49 | 2.31 | 1.29 | 12.79 | 21.52 | 14.27 | 10.43 | 17.04 | 11.45 |
| FedDuoRec | 21.60 | 33.63 | 23.93 | 5.45 | 9.23 | 5.74 | 1.61 | 2.42 | 1.38 | 13.15 | 21.05 | 14.45 | 10.45 | 16.58 | 11.38 |
| FedDCSR(Ours) | 28.87 | 45.65 | 32.30 | 11.37 | 21.96 | 13.07 | 1.99 | 3.23 | 1.87 | 18.73 | 33.45 | 21.66 | 15.24 | 26.07 | 17.23 |

Table 3: **Federated experimental results(%) on the MBG scenario. Avg denotes the average result calculated from all domains. The best results are boldfaced.**

| Method | Movie | | | Book | | | Game | | | Avg | | |
|----------------------|--------------|--------------|--------------|--------------|--------------|--------------|-------------|--------------|-------------|--------------|--------------|--------------|
| | MRR | HR | NDCG | MRR | HR | NDCG | MRR | HR | NDCG | MRR | HR | NDCG |
| | | @10 | @10 | | @10 | @10 | | @10 | @10 | | @10 | @10 |
| FedSASRec | 10.36 | 17.79 | 11.26 | 6.91 | 10.91 | 7.19 | 5.20 | 6.44 | 5.15 | 7.49 | 11.71 | 7.87 |
| FedVSAN | 5.93 | 12.72 | 6.43 | 6.54 | 13.09 | 7.27 | 1.83 | 3.20 | 1.74 | 4.77 | 9.67 | 5.15 |
| FedContrastVAE | 11.25 | 19.14 | 12.22 | 7.63 | 12.76 | 8.18 | 4.95 | 6.72 | 4.96 | 7.94 | 12.87 | 8.45 |
| FedCL4SRec | 10.39 | 17.68 | 11.24 | 6.86 | 10.94 | 7.18 | 5.21 | 6.44 | 5.17 | 7.49 | 11.69 | 7.87 |
| FedDuoRec | 10.42 | 17.83 | 11.55 | 6.79 | 10.80 | 7.12 | 5.16 | 6.42 | 5.13 | 7.46 | 11.68 | 7.93 |
| FedDCSR(Ours) | 16.11 | 28.32 | 18.09 | 10.38 | 18.68 | 11.56 | 7.65 | 10.60 | 7.83 | 11.38 | 19.20 | 12.49 |

Table 4: **Federated experimental results(%) on the SGH scenario. Avg denotes the average result calculated from all domains. The best results are boldfaced.**

| Method | Sport | | | Garden | | | Home | | | Avg | | |
|----------------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| | MRR | HR | NDCG | MRR | HR | NDCG | MRR | HR | NDCG | MRR | HR | NDCG |
| | | @10 | @10 | | @10 | @10 | | @10 | @10 | | @10 | @10 |
| FedSASRec | 3.59 | 4.34 | 3.43 | 4.74 | 5.36 | 4.56 | 3.13 | 3.77 | 2.93 | 3.82 | 4.49 | 3.64 |
| FedVSAN | 1.16 | 2.05 | 1.00 | 1.18 | 2.02 | 1.05 | 1.27 | 2.26 | 1.10 | 1.21 | 2.11 | 1.05 |
| FedContrastVAE | 3.93 | 4.95 | 3.81 | 5.07 | 5.97 | 4.91 | 3.54 | 4.37 | 3.39 | 4.18 | 5.10 | 4.03 |
| FedCL4SRec | 3.60 | 4.26 | 3.41 | 4.74 | 5.27 | 4.55 | 3.11 | 3.74 | 2.91 | 3.82 | 4.42 | 3.62 |
| FedDuoRec | 3.75 | 4.43 | 3.64 | 4.86 | 5.47 | 4.79 | 3.07 | 3.60 | 2.83 | 3.89 | 4.50 | 3.75 |
| FedDCSR(Ours) | 5.29 | 6.70 | 5.22 | 6.41 | 8.35 | 6.52 | 4.52 | 5.84 | 4.45 | 5.41 | 6.96 | 5.40 |

adopt the approach described in Rendle’s literature [24]. Specifically, for each validation or test sample, we calculate its score along with 999 negative items. Subsequently, we evaluate the performance of the Top-K recommendation by analyzing the 1,000 ranked list using metrics like MRR (Mean Reciprocal Rank) [25], NDCG@10 (Normalized Discounted Cumulative Gain) [26], and HR@10 (Hit Ratio).

4.2.2 Compared Baselines. We compare our methods with three types of representative sequential recommendation models: (1) attention-based methods, like SASRec [7]. (2) VAE-based methods, such as VSAN [8] and ContrastVAE [9]. (3) CL-based methods, including CL4SRec [10] and DuoRec [11]. We integrate these methods with FedAvg [4] to form baselines.

4.2.3 Implementation and Hyperparameter Setting. For all methods, the common hyperparameters are listed as follows: the training round is fixed as 40, the local epoch per client is fixed as 3, the

early stopping patience is fixed as 5, the mini-batch size is fixed as 256, the learning rate is fixed as 0.001, the dropout rate is fixed as 0.3.

4.3 Performance Comparisons Table 2, 3, 4 show the performance of compared methods on the Food-Kitchen-Clothing-Beauty, Movie-Book-Game and Sports-Garden-Home CSR scenarios.

From the experimental results, we have several insightful observations: (1) For the variational baselines, FedContrastVAE shows better performance than FedSASRec, which validates that modeling the uncertainty of user behaviors could be helpful for better representations in the federated CSR scenario. (2) For the CL baselines, FedCL4SRec and FedDuoRec both show better performance than FedSASRec, which indicates that the contrastive learning has promising advantages in learning representations in the federated CSR scenario. However, we find that CL-based federated CSR sometimes performs equally or worse than FedSASRec, we believe that this is because the CL could intensify the se-

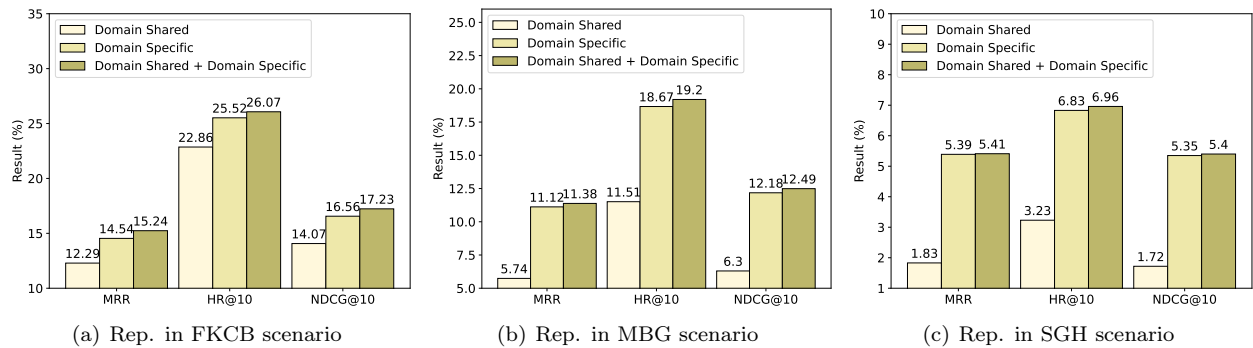


Figure 5: The predictive results of representations in FKCB, MBG, and SGH scenario.

Table 5: Ablation study on FKCB, MBG, and SGH scenarios.

| Method | FKCB | | MBG | | SGH | |
|--------------------------|--------------|--------------|--------------|--------------|-------------|-------------|
| | MRR | NDCG@10 | MRR | NDCG@10 | MRR | NDCG@10 |
| LocalVSAN | 11.57 | 13.16 | 5.63 | 5.87 | 2.06 | 1.96 |
| FedVSAN | 10.24 | 11.37 | 4.77 | 5.15 | 1.21 | 1.05 |
| LocalContrastVAE | 12.11 | 13.38 | 8.67 | 9.24 | 4.10 | 4.02 |
| FedContrastVAE | 11.84 | 13.19 | 8.82 | 9.75 | 4.18 | 4.03 |
| LocalVGSE | 13.49 | 15.30 | 10.63 | 11.63 | 5.24 | 5.20 |
| FedDCSR - w/o (SRD, CIM) | 12.24 | 14.01 | 10.31 | 11.14 | 5.37 | 5.32 |
| FedDCSR - w/o CIM | 14.17 | 16.20 | 10.78 | 11.67 | 5.40 | 5.36 |
| FedDCSR(Ours) | 15.24 | 17.23 | 11.38 | 12.49 | 5.41 | 5.40 |

quence feature heterogeneity and ultimately have a negative impact on the prediction accuracy of the recommendation model. (3) Our FedDCSR significantly outperforms all baselines in many metrics, highlighting the crucial role of disentangled representation learning and contrastive infomax strategy in capturing intra-domain and inter-domain user preferences.

4.4 Ablation Study We perform an ablation study on FedDCSR by showing how the SRD and CIM affect its performance. Table 5 displays the performance of several model variants in three CSR scenarios. LocalVGSE is the VGSE model without federated aggregation, FedDCSR-w/o (SRD, CIM) is FedDCSR without SRD and CIM, and FedDCSR-w/o CIM is FedDCSR without CIM. We observe that FedDCSR-w/o (SRD, CIM) sometimes performs worse than LocalVGSE, which indicates that the sequence feature heterogeneity is very significant. Additionally, We notice that FedDCSR-w/o CIM performs better than LocalVGSE and FedDCSR-w/o (SRD, CIM), demonstrating the effectiveness of SRD in addressing the feature heterogeneity problem across domains. Finally, CIM allows the model performance to be further improved.

4.5 Discussion of the user representations In this section, to further validate that our SRD can learn domain-shared and domain-exclusive representations for users, we conduct a comparative analysis between

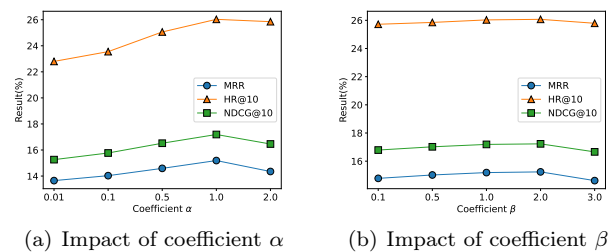


Figure 6: Impact of coefficient α and β . the predictive performance of three types of representations: domain-shared, domain-exclusive, and domain-shared+domain-exclusive representations. As shown in Figure 5, our analysis reveals several interesting observations: (1) The predictive results of three types of representations are different, indicating the efficiency of SRD. (2) The domain-shared+domain-exclusive representation outperforms both the domain-shared and domain-exclusive representations, which demonstrates that both leveraging the domain-shared and domain-exclusive features can effectively integrate information from multiple domains.

4.6 Influence of hyperparameters Figure 6 shows the MRR, @HR10, @NDCG@10 performance with the increase of coefficient α and β . We can observe that: (1) The overall performance of FedDCSR increases first and then decreases with the increase of α , and reaches its peak at 1.0. It indicates that the α coefficient of 1.0 is optimal for feature disentanglement. (2) The overall performance of FedDCSR increases first and then decreases with the increase of coefficient β , and reaches its peak at 2.0. It indicates that a β coefficient of 2.0 is optimal for sharing intra-domain information, and shows that there is a trade-off between inter domain and intra domain information.

5 Conclusion

In this paper, we propose a novel federated cross-domain sequential recommendation framework FedD-

CSR, which allows domains to train better performing CSR models collaboratively under privacy protection. Specifically, we propose a sequence representation disentanglement method SRD, which disentangles the user sequence features into domain-shared and domain-exclusive features to address the feature heterogeneity. Besides, we design a contrastive infomax strategy CIM to learn richer domain-exclusive features of users by performing data augmentation on user sequences.

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