Consider the following linear system:

$$3x_1 + 4x_2 + 3x_3 = 10$$
$$x_1 + 5x_2 - x_3 = 7$$
$$6x_1 + 3x_2 + 7x_3 = 15$$

Exercise 1: Solve the system using Naive Gaussian Elimination.

$$\left[\begin{array}{ccc|c} ③ & 4 & 3 & 10 \\ ① & 5 & -1 & 7 \\ 6 & 3 & 7 & 15 \end{array}\right] \quad \tfrac{1}{3}$$

$$R_2 \mathrel{-{=}} \tfrac{1}{3}(R_1)$$

$$\left[\begin{array}{ccc|c} ③ & 4 & 3 & 10 \\ 0 & \tfrac{11}{3} & -2 & \tfrac{11}{3} \\ ⑥ & 3 & 7 & 15 \end{array}\right] \quad \tfrac{6}{3}$$

$$R_3 \mathrel{-{=}} 2(R_1)$$

$$\left[\begin{array}{ccc|c} 3 & 4 & 3 & 10 \\ 0 & \tfrac{11}{3} & -2 & \tfrac{11}{3} \\ 0 & -5 & 1 & -5 \end{array}\right] \quad \tfrac{-5}{\tfrac{11}{3}}$$

$$R_3 \mathrel{+{=}} \tfrac{15}{11}(R_1)$$

$$\left[\begin{array}{ccc|c} 3 & 4 & 3 & 10 \\ 0 & \tfrac{11}{3} & -2 & \tfrac{11}{3} \\ 0 & 0 & \tfrac{-19}{11} & 0 \end{array}\right]$$

$$x_3 = \frac{0}{-\tfrac{19}{11}}$$
$$= 0$$
$$x_2 = \frac{(\tfrac{11}{3}) - (-2 \cdot 0)}{\tfrac{11}{3}}$$
$$= 1$$
$$x_1 = \frac{(10) - (4 \cdot 1)}{3}$$
$$= 2$$

$$x_1 = 2, \quad x_2 = 1, \quad x_3 = 0 \quad \checkmark$$

Exercise 2: Solve the system using Gaussian Elimination w/ Scaled Partial Pivoting

$$\begin{bmatrix} 3 & ④ & 3 & | & 10 \\ 1 & ⑤ & -1 & | & 7 \\ 6 & 3 & ⑦ & | & 15 \end{bmatrix}$$ scaling

$R_1 \leftrightarrow R_3$

$$\begin{bmatrix} ⑥ & 3 & 7 & | & 15 \\ ① & 5 & -1 & | & 7 \\ 3 & 4 & 3 & | & 10 \end{bmatrix}$$ $1/6$

$R_2 \mathrel{-}= 1/6(R_1)$

$$\begin{bmatrix} ⑥ & 3 & 7 & | & 15 \\ 0 & \frac{9}{2} & \frac{-13}{6} & | & \frac{27}{6} \\ ③ & 4 & 3 & | & 10 \end{bmatrix}$$ $3/6$

$R_3 \mathrel{-}= 1/2(R_1)$

$$\begin{bmatrix} 6 & 3 & 7 & | & 15 \\ 0 & \frac{9}{2} & \frac{-13}{6} & | & \frac{9}{2} \\ 0 & \frac{5}{2} & -\frac{1}{2} & | & \frac{5}{2} \end{bmatrix}$$ $5/9$

$R_3 \mathrel{-}= \frac{5}{9}(R_2)$

$$\begin{bmatrix} 6 & 3 & 7 & | & 15 \\ 0 & \frac{9}{2} & \frac{-13}{6} & | & \frac{9}{2} \\ 0 & 0 & \frac{92}{54} & | & 0 \end{bmatrix}$$

i	j	s
1	1	3
1	2	4
1	3	4
2	1	1
2	2	5
2	3	5
3	1	6
3	2	6
3	3	7

scaling = [4, 5, 7]

k	i	max	ind	r	mult	
1	1	3/4	1	3/4	nan	
1	2	3/4	1	1/5	nan	
1	3	6/7	3	6/7	nan	← swapped
1	2	nan	nan	nan	1/6	
1	3	nan	nan	nan	1/2	
2	2	45/2	2	45/2	nan	
2	3	45/2	2	20/2	nan	← no swap
2	3	nan	nan	nan	5/9	

$x_3 = 0/\frac{92}{54}$
$\quad = 0$

$x_2 = \dfrac{(9/2) - (-13/6 * 0)}{9/2}$
$\quad = 1$

$x_1 = \dfrac{(15) - (3*1)}{6}$
$\quad = 2$

$x_1 = 2, \quad x_2 = 1, \quad x_3 = 0$