

Root Finding Methods for Polynomials

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Setup

- **Systems:** 3 polynomial functions tested
- **Methods:** Bisection, Newton's, Secant, Hybrid (Bisection \rightarrow Newton after 5 iterations)
- **Convergence:** IEEE 754 single precision (successive approximations equal at float precision)
- **Maximum iterations:** 10,000 per run

Results

Performance Summary

Method	Avg. Iterations	Success Rate	Performance vs Bisection
Bisection	23.75	100% (4/4)	Baseline
Newton	4.00	100% (4/4)	$5.94\times$ faster
Secant	5.50	100% (4/4)	$4.32\times$ faster
Hybrid	8.25	100% (4/4)	$2.88\times$ faster

Detailed Results by Polynomial

Polynomial	Bisection	Newton	Secant	Hybrid
$x^3 + 3x - 1$ (Ex 1a)	25	4	7	8
$x^3 + 2x^2 + 10x - 20$ (Ex 2)	23	4	5	8
$3x^3 + 5x^2 - 7$	24	4	5	9

Table 1: Iterations to convergence

Polynomial	Bisection	Newton	Secant	Hybrid
$x^3 + 3x - 1$	0.322185	0.322185	0.322185	0.322185
$x^3 + 2x^2 + 10x - 20$ (Ex 2)	1.36881	1.36881	1.36881	1.36881
$3x^3 + 5x^2 - 7$	0.94518	0.94518	0.94518	0.94518

Table 2: All methods converged to identical roots

Findings

Convergence Speed

- **Newton's method** is the clear winner at 4 iterations consistently. This demonstrates quadratic convergence—each iteration approximately doubles the number of correct digits.
- **Secant method** averaged 5.5 iterations, nearly matching Newton's speed without requiring derivative calculations. Shows super-linear convergence.
- **Hybrid method** converged in 8 iterations, balancing the reliability of bisection's early iterations with Newton's rapid final convergence.
- **Bisection** required the most iterations (24 average) due to linear convergence, but never failed.