The stability of a unicycle and a bicycle

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Agenda

1. Introduction
2. Representation of a unicycle/bicycle
3. Physics, Lagrangian, kinetic, rotational and potential energies
4. Interpretation and further research

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Why bikes?

- The number of cyclists suffering major injuries in a traffic accident has risen by 30% over the past 10 years
- Most bike accidents do not involve cars or other bikes and in 44% of all incidents, the cyclist's own behaviour was to blame

Dutchies cycling

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Goals

1. Equations of motion of a unicycle/bicycle;
2. Linearization & stability analysis;
3. Numerical solution of the full nonlinear system.

Unicycle description

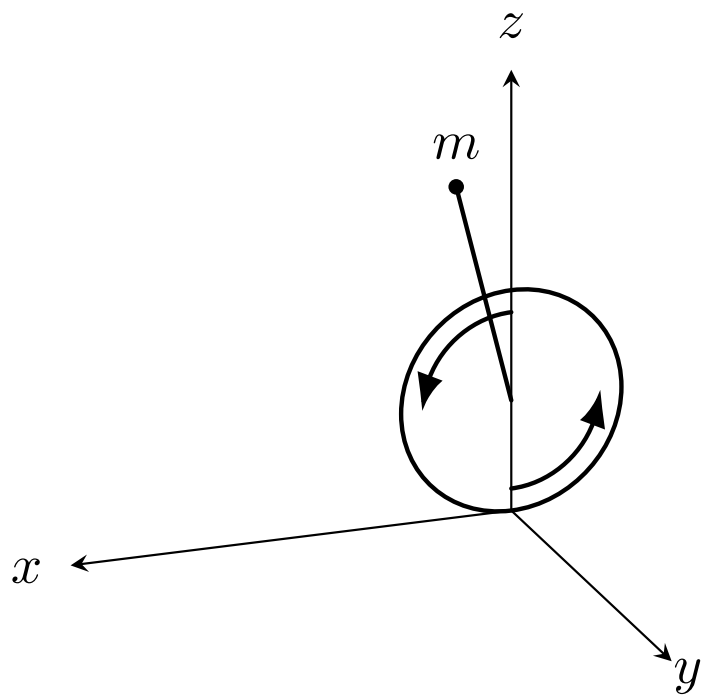

Figure: Static position of the unicycle

Unicycle description

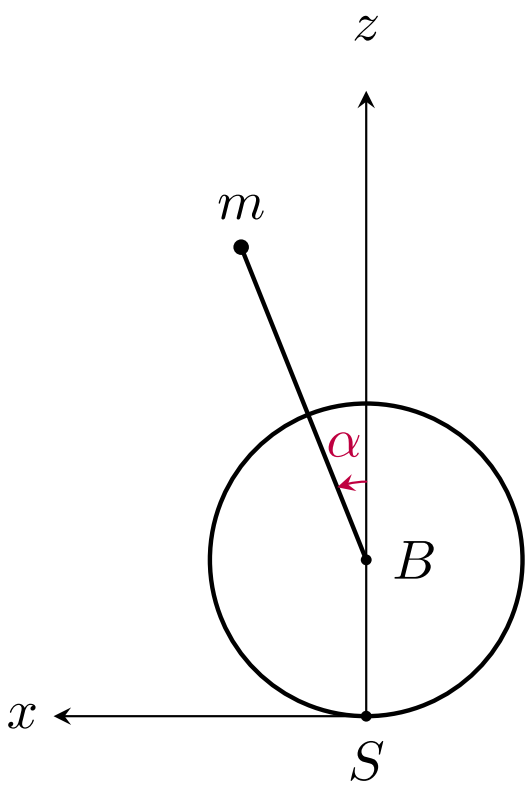

Figure: Position of the unicycle in xz plane

Unicycle description

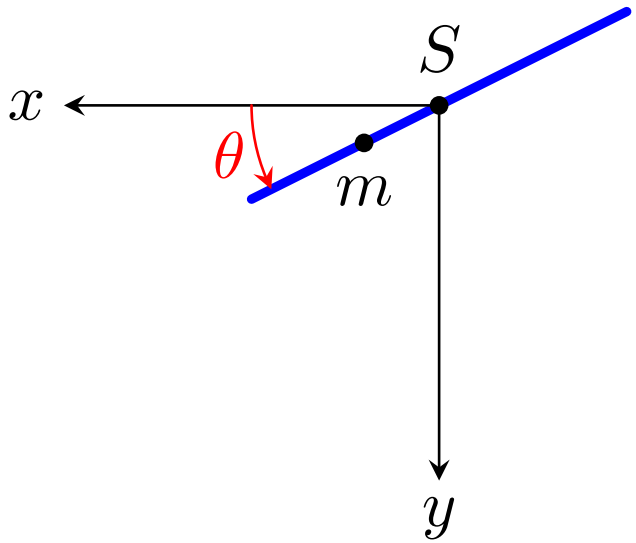

Figure: Position of the unicycle in xy plane

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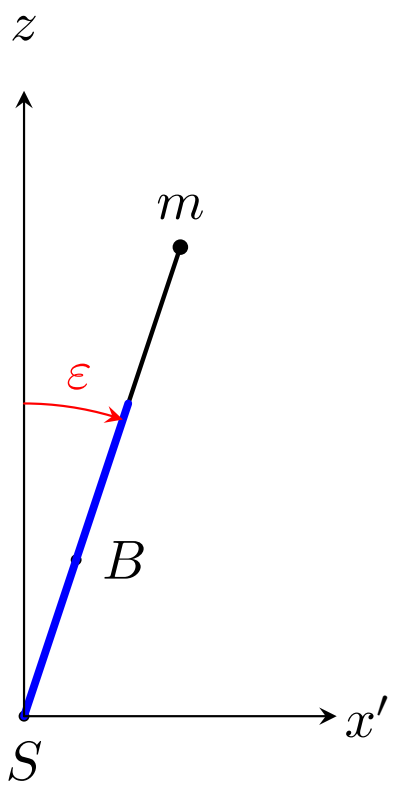

Figure: Position of the unicycle in zx' plane

Physics behind the scene

Lagrangian of the system:

$$L = E_{tr} + E_{rot} - E_{pot}, \tag{1}$$

the velocity v is assumed to be constant.

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = Q_i \tag{2}$$

$$q_1 = x,\ q_2 = y,\ q_3 = \alpha,\ q_4 = \theta,\ q_5 = \varepsilon. \tag{3}$$

Gyroscopic effect

Further research

1. Linearize and compare
2. Solve numerically
3. Interpret results
4. Modify model for bicycle

Interpretation