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Image Processing 06
Histograms
Part 1

SS 2020

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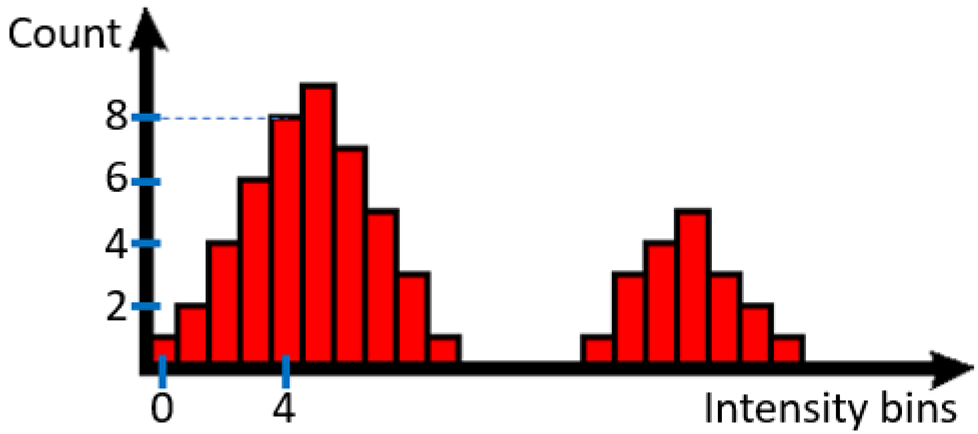
Outline

- Part 1: Histograms: Motivation and unnormalized h.
- Part 2: Normalized & cumulative histograms
- Part 3: Mean and Variance
- Part 4: Histogram Equalization

Histogram

An *image histogram* is a graphical representation of the distribution of values (like intensity or color) in a digital image (it models a *probability distribution* if normalized)



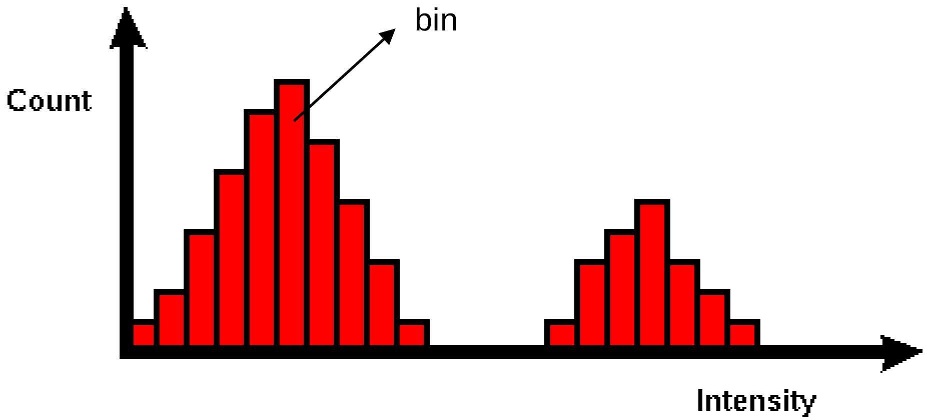
[Image: http://homepages.inf.ed.ac.uk/rbf/HIPR2]

Histogram

Definition: Let r_k, $k = 0, 1, ..., L-1$ denote the intensities of a digital image $f(x, y)$. The unnormalized *gray value histogram* is defined as

$$h(r_k) = n_k$$

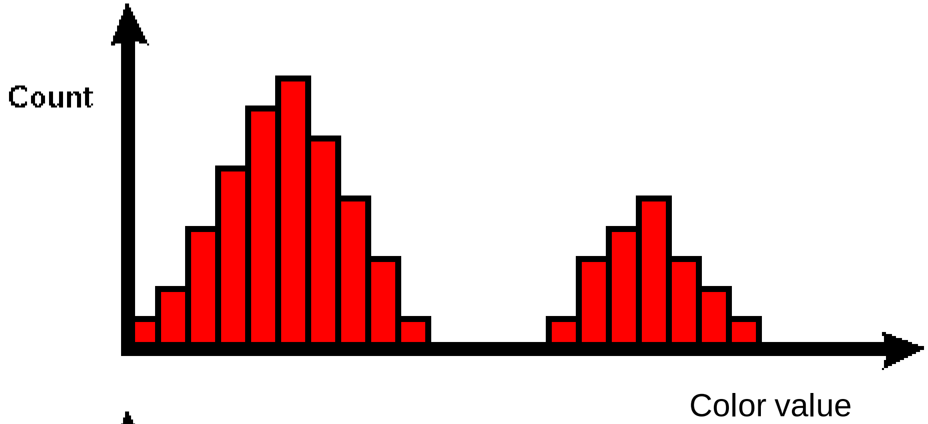
where r_k is the k-th intensity value and n_k is the number of pixels in f with intensity r_k. The subdivisions of the intensity scale are called *bins*.

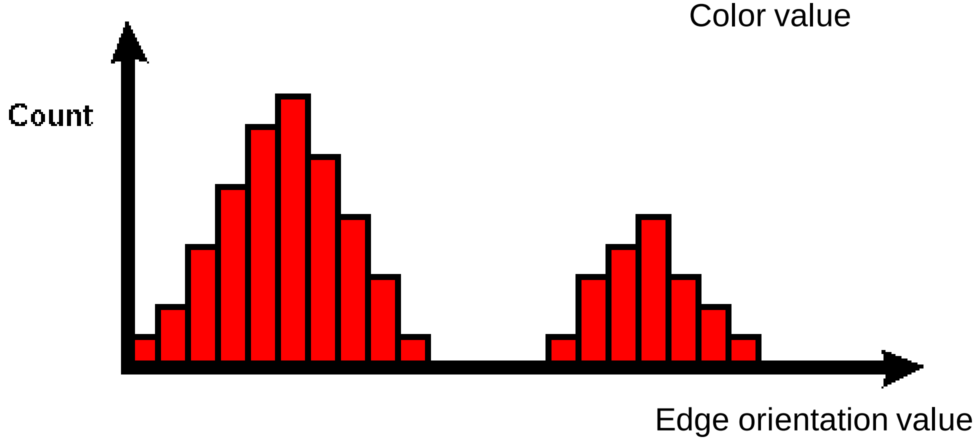


[Image: http://homepages.inf.ed.ac.uk/rbf/HIPR2]

Histogram

Side note: There are also color histograms, orientation histograms, etc.

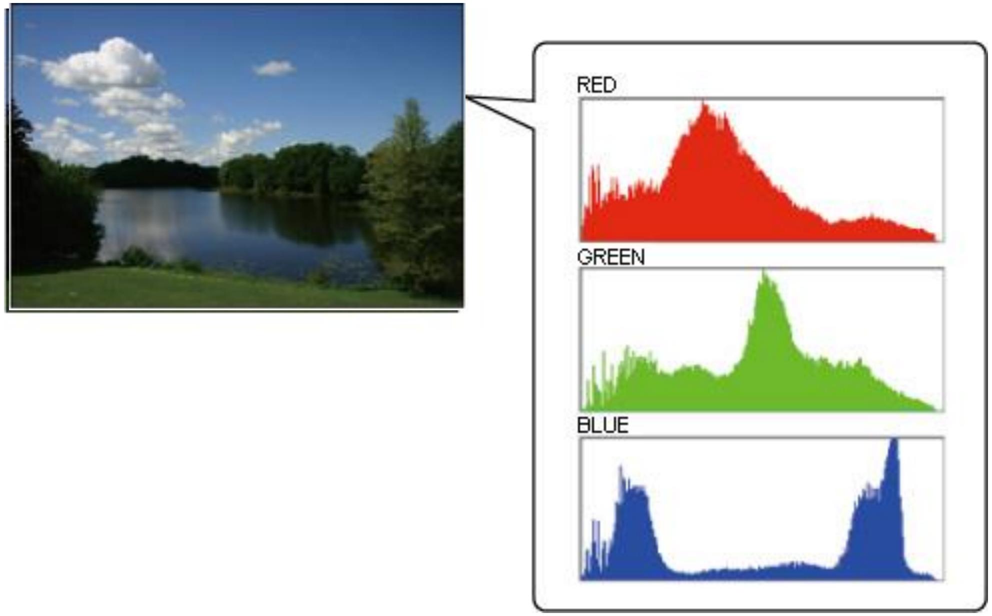




Color Histograms

Color histograms, simple solution:

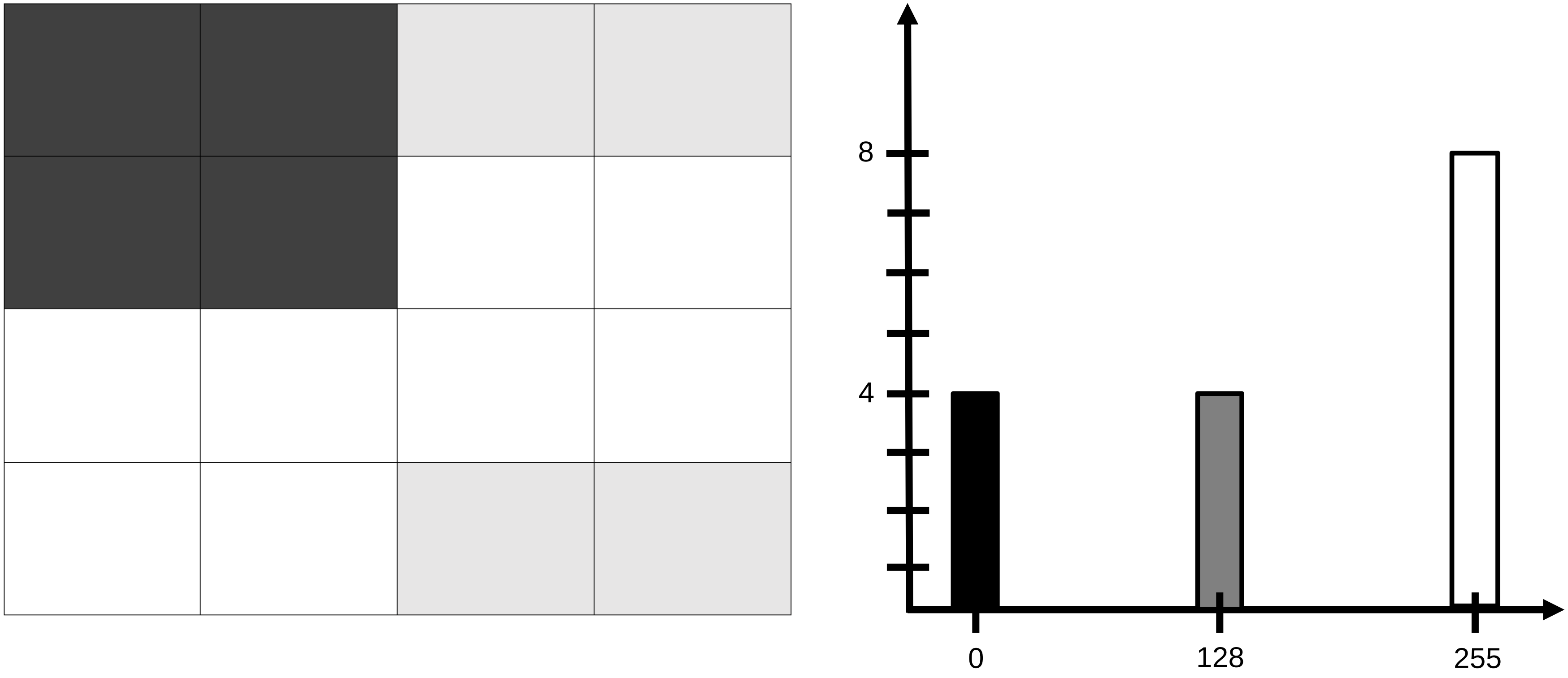
- Create a histogram for each channel of the color image
- An example for the RGB space:



- The same is possible for arbitrary color spaces

[Image: Canon, Canada]

Histograms



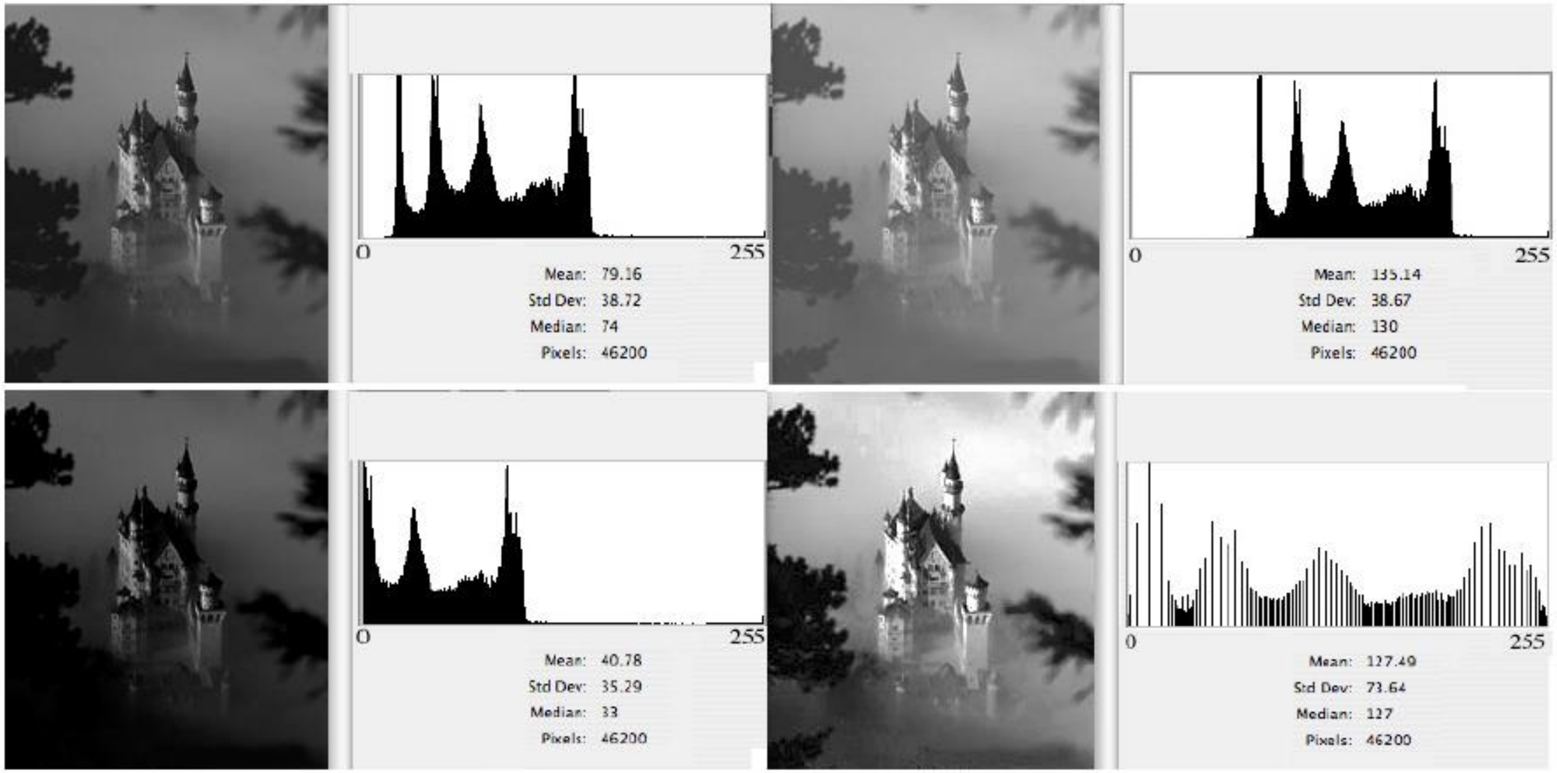
[Gonzales/Woods Fig. 2.49]

Histograms for Two Image Windows

Two 104 × 98 windows in image Yan and corresponding histograms

[Klette 2014]

Histograms



Histograms for a 200×231 image Neuschwanstein

Upper left: Original image. *Upper right*: Brighter version. *Lower left*: Darker version. *Lower right*: After histogram equalization

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Part 2

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Histogram

A *normalized histogram* is a histogram in which the value n_k in each bin is divided by the number of pixels in the image:

Definition: Let r_k, $k = 0, 1, ..., L-1$ denote the intensities of a digital image $f(x, y)$. The *normalized histogram* of f is defined as

$$p(r_k) = \frac{n_k}{MN}$$

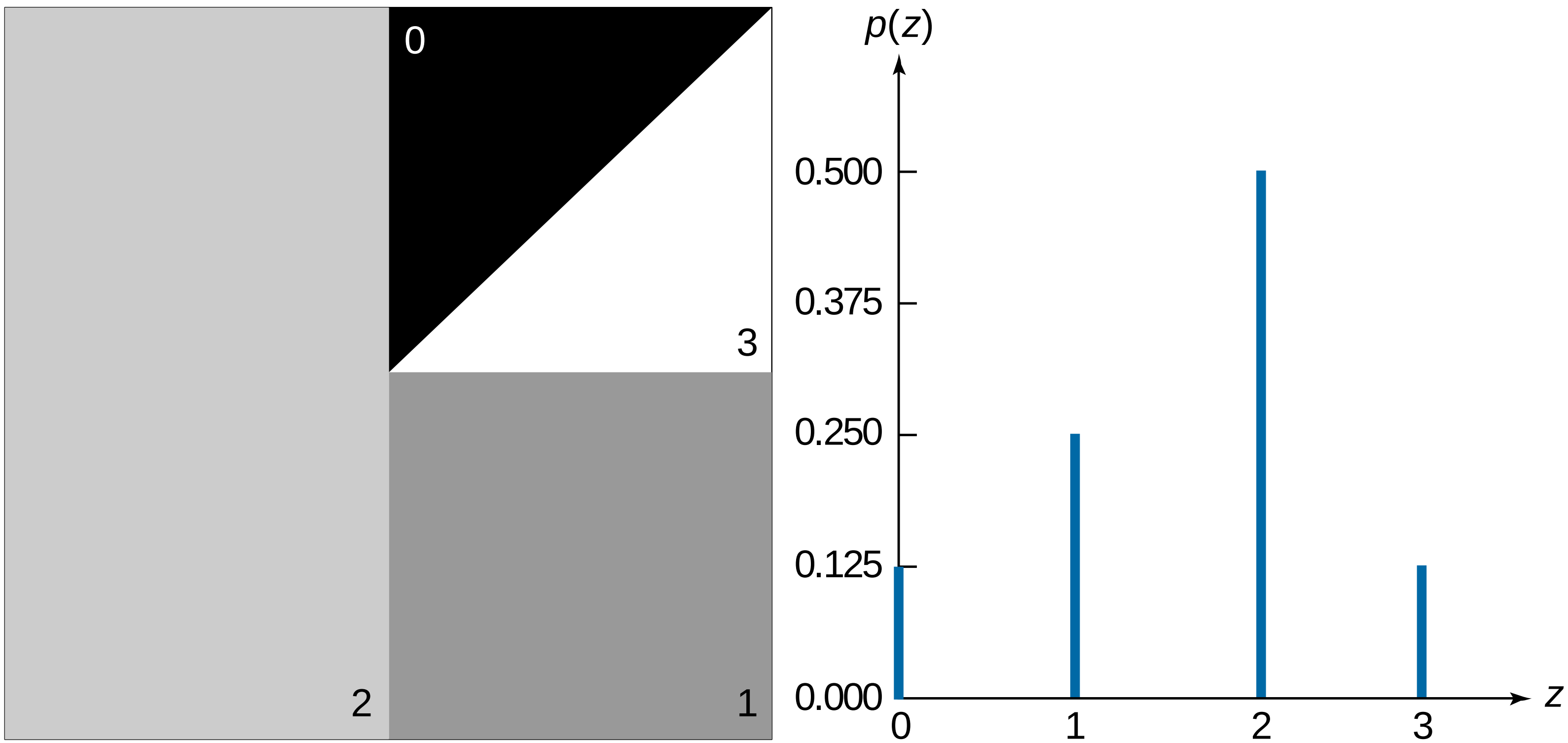
where n_k is the number of pixels in f with intensity r_k and M and N are the number of rows and columns respectively.

http://homepages.inf.ed.ac.uk/rbf/HIPR2

Histograms

a b
c d

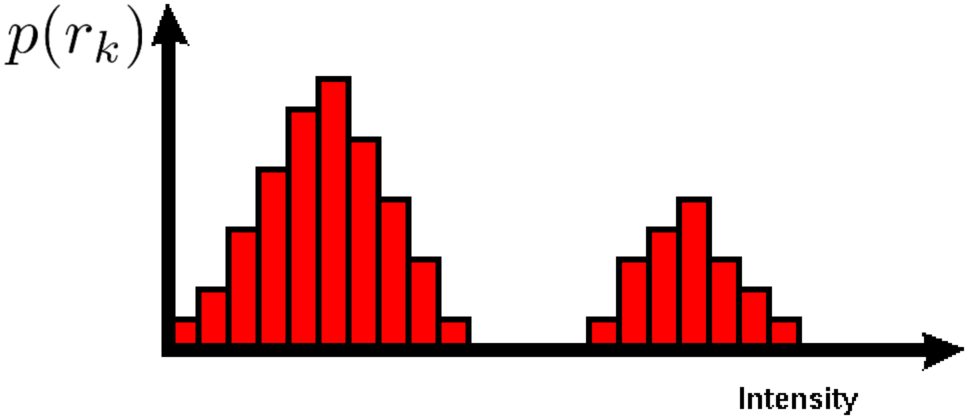
FIGURE 2.49
(a) A synthetic image, and (b) its histogram. (c) A natural image, and (d) its histogram.



[Gonzales/Woods Fig. 2.49]

Histogram

- We work mostly with normalized histograms: if nothing is mentioned explicitly, the histogram is normalized
- $p(r_k)$ is an estimate of the probability distribution of the intensity levels in image f (it tells us how intensity values are distributed in the image).



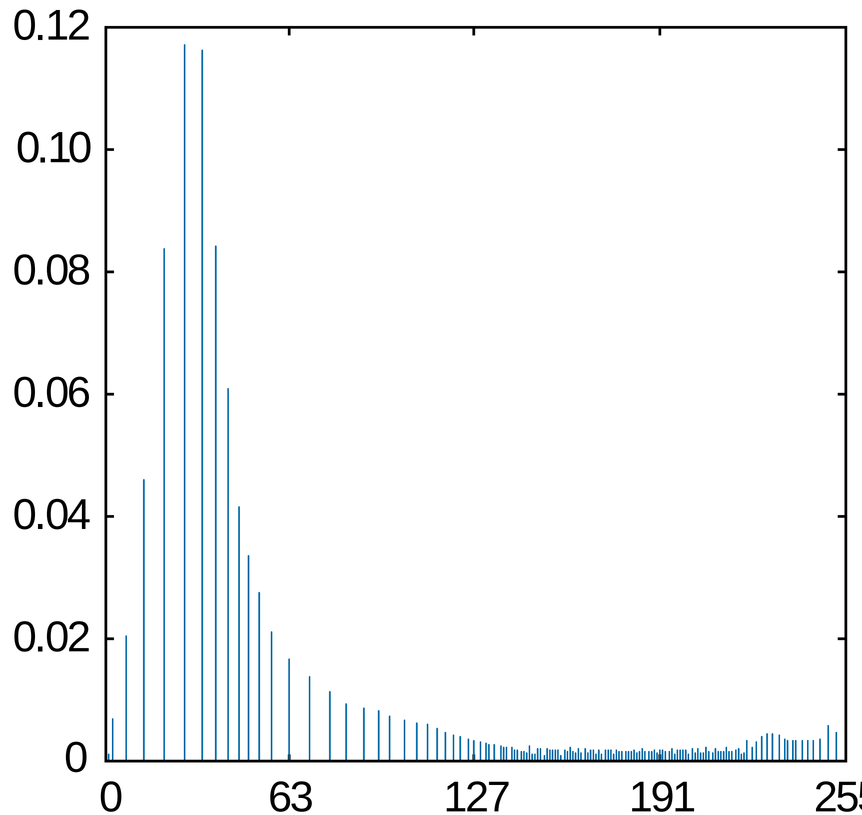
- What is the sum of all values in a normalized histogram?
- Answer: 1

http://homepages.inf.ed.ac.uk/rbf/HIPR2

Histograms

a b
c d

FIGURE 2.49
(a) A synthetic image, and (b) its histogram. (c) A natural image, and (d) its histogram.



[Gonzales/Woods Fig. 2.49]

Histograms

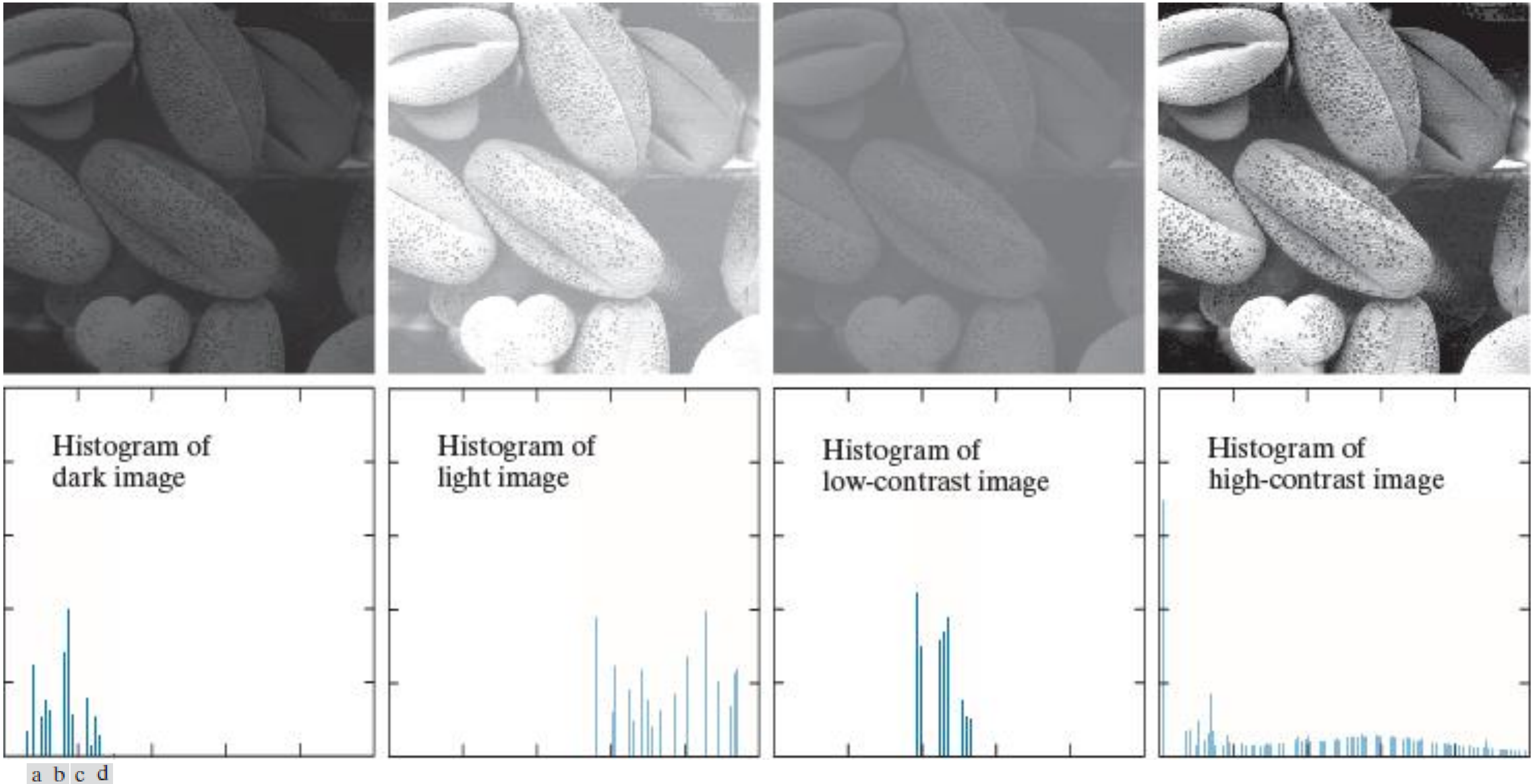
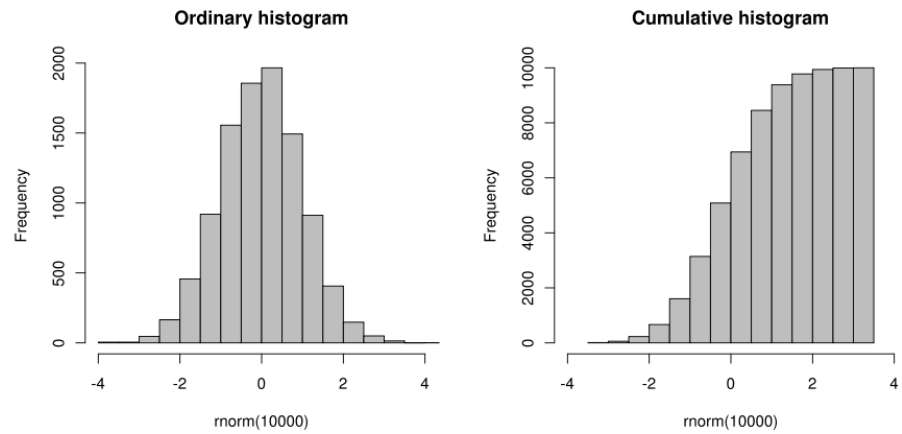


FIGURE 3.16 Four image types and their corresponding histograms. (a) dark; (b) light; (c) low contrast; (d) high contrast. The horizontal axis of the histograms are values of r_k and the vertical axis are values of $p(r_k)$.

Histogram

Definition: a *cumulative histogram* c for histogram h is a histogram that counts in each bin $c(i)$ the number of occurrences in all bins $j \leq i$:

$$c(i) = \sum_{j=1}^{i} h(i)$$



[Image: by Kierano, Wikimedia Commons]

Histogram

- We work mostly with normalized histograms: if nothing is mentioned explicitly, the histogram is normalized
- $p(r_k)$ is an estimate of the probability distribution of the intensity levels in image f (it tells us how intensity values are distributed in the image).
- Example: A binary image with two bins 0 (black) and 1 (white), has the histogram values $p(0) = 0.25$ and $p(1) = 0.75$.
  This means: 25% of the pixels are black and 75% of the pixels are white

http://homepages.inf.ed.ac.uk/rbf/HIPR2

Purpose of histograms

Histograms are very useful representations of images:

1. They can be manipulated to modify (usually to improve) the image appearance
2. They can be compared to determine the similarity of image regions (e.g. for object recognition etc.)

Here we will mainly cover the 1st aspect. The 2nd aspect will be treated in the master lecture „Computer Vision 1“

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- Part 1: Histograms: Motivation and unnormalized h.
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- Part 4: Histogram Equalization

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Part 3

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Mean and Variance

Some basic statistics in images:

Mean = average of all pixels of an image I (or image region):

$$\mu_I = \frac{1}{N_{cols} \cdot N_{rows}} \sum_{x=1}^{N_{cols}} \sum_{y=1}^{N_{rows}} I(x,y)$$

Variance = average of squared deviation of all pixels from mean

$$\sigma^2 = \frac{1}{N_{cols} \cdot N_{rows}} \sum_{x=1}^{N_{cols}} \sum_{y=1}^{N_{rows}} [I(x,y) - \mu_I]^2$$

Both can be computed in one run over the image (see exercise).

Mean

We can compute *image statistics* (e.g. mean and variance) of an image region also based on its histogram:

Definition (mean):

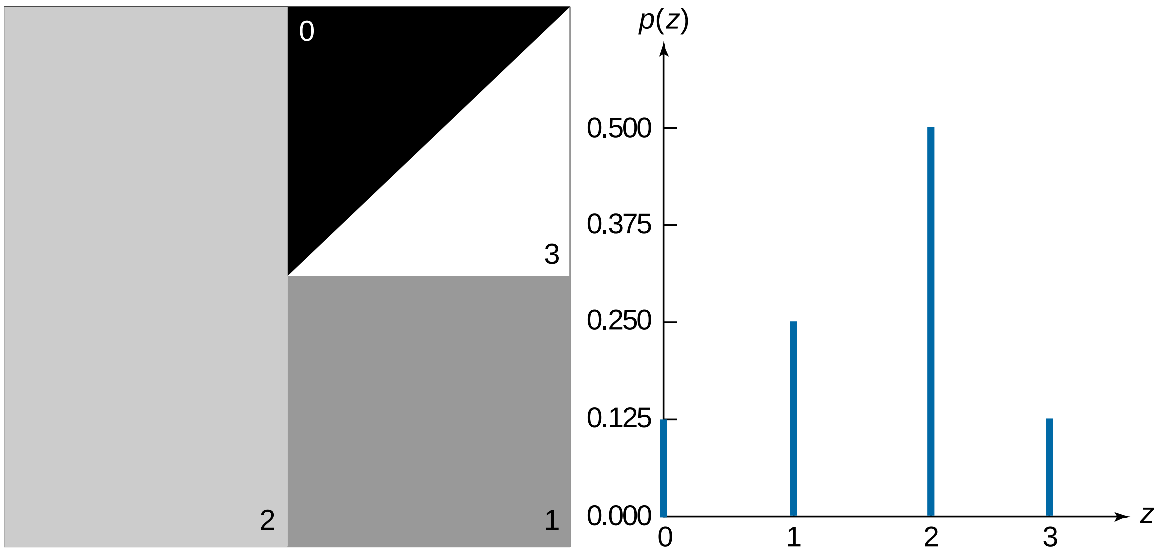
The mean m (average intensity value) of a digital image f with intensity levels r_k and the normalized histogram $p(r_k)$ is defined as:

$$m = \sum_{i=0}^{L-1} r_i p(r_i)$$

Mean

a b
c d

FIGURE 2.49
(a) A synthetic image, and (b) its histogram. (c) A natural image, and (d) its histogram.



$$\bar{z} = E[z] = \sum_{z \in \{0,1,2,3\}} z p(z) = (0)p(0) + (1)p(1) + (2)p(2) + 3p(3)$$

$$= (0)(0.125) + (1)p(0.250) + (2)(0.500) + (3)(0.125) = 1.625$$

[Gonzales/Woods]

Variance

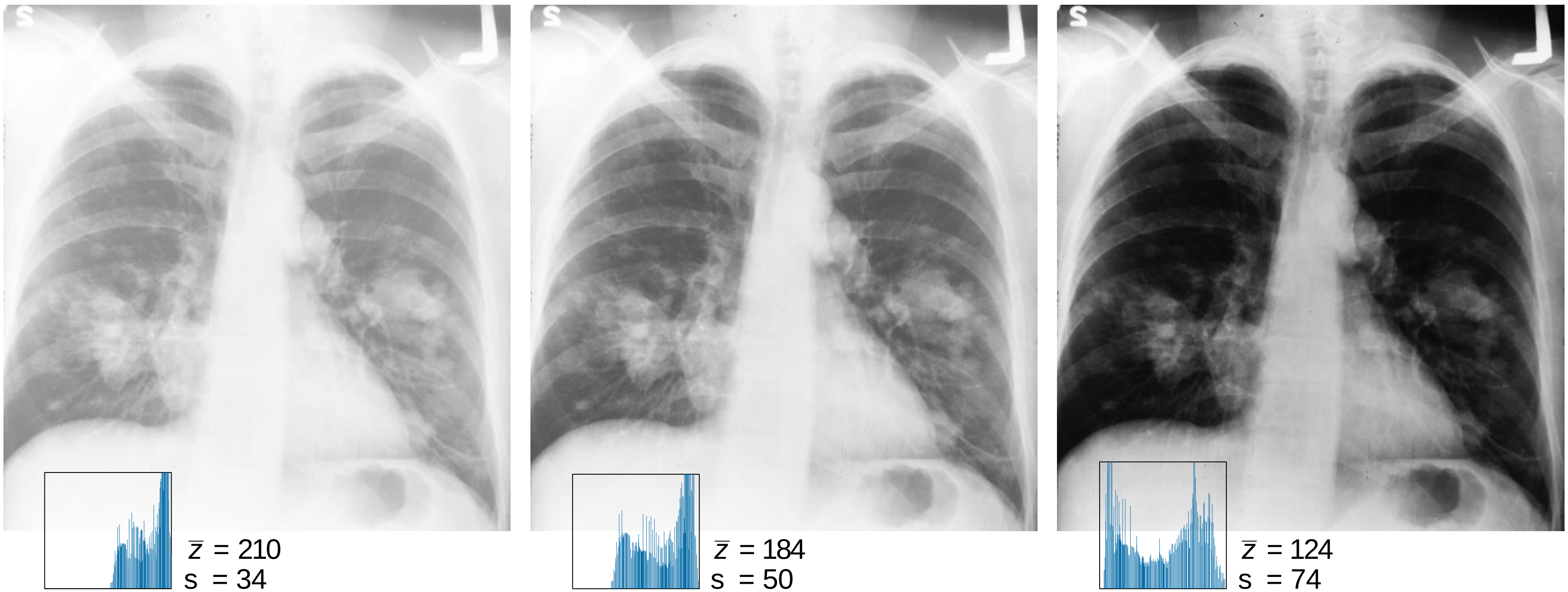
Definition (variance):

The variance of a digital image f with L intensity levels r_k and the normalized histogram $p(r_k)$ and mean m is defined as

$$\sigma^2 = \sum_{i=0}^{L-1} (r_i - m)^2 p(r_i)$$

The variance measures how much pixel intensities vary from the mean. This is a measure of *image contrast*.

Mean and Variance



a b c

FIGURE 2.51 Illustration of the mean and standard deviation as functions of image contrast. (a)-(c) Images with low, medium, and high contrast, respectively. (Original image courtesy of the National Cancer Institute.)

[Gonzales/Woods]

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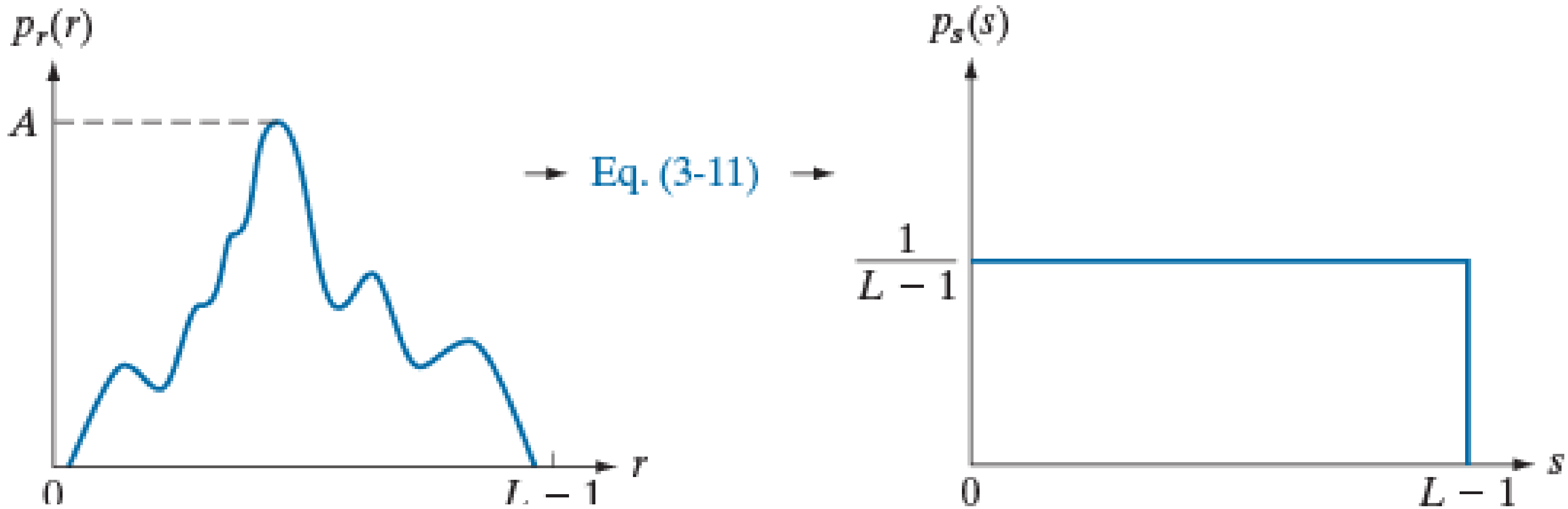
Histogram Equalization

- Idea of histogram equalization: transform the histogram to a uniform distribution



a b

FIGURE 3.18 (a) An arbitrary PDF. (b) Result of applying Eq. (3-11) to the input PDF. The resulting PDF is always uniform, independently of the shape of the input.

- In practice with discrete intensity values, this is not possible in general.
- But we can spread the histogram over all intensity values

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Histogram Equalization

- Discrete transformation for histogram equalization:

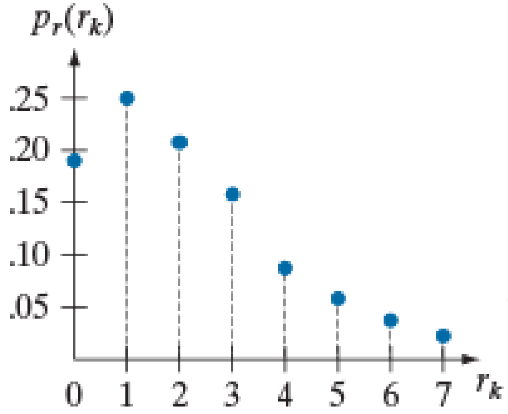
$$s_k = T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j) \quad k = 0, 1, 2, \ \ldots, L-1$$

Histogram Equalization

- Let us look at an example:

TABLE 3.1 Intensity distribution and histogram values for a 3-bit, 64×64 digital image.

r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02



$$s_0 = T(r_0) = 7\sum_{j=0}^{0} p_r(r_j) = 7p_r(r_0) = 1.33$$

$$s_1 = T(r_1) = 3.08,\, s_2 = 4.55,\, s_3 = 5.67,\, s_4 = 6.23,\, s_5 = 6.65,\, s_6 = 6.86 \text{ and } s_7 = 7.00$$

Histogram Equalization

- Let us look at an example:

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r_k	n_k	$p_r(r_k) = n_k/MN$
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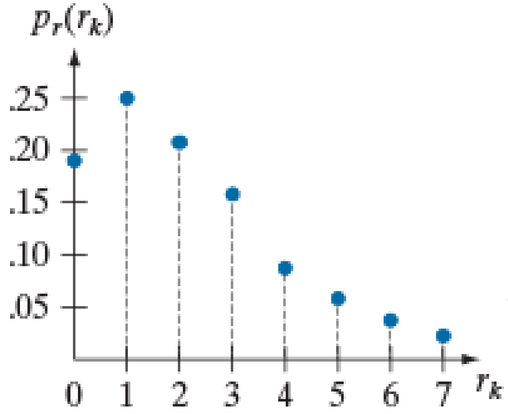
$r_7 = 7$	81	0.02



- We round the values to integers:

$$s_0 = 1.33 \rightarrow 1 \quad s_2 = 4.55 \rightarrow 5 \quad s_4 = 6.23 \rightarrow 6 \quad s_6 = 6.86 \rightarrow 7$$
$$s_1 = 3.08 \rightarrow 3 \quad s_3 = 5.67 \rightarrow 6 \quad s_5 = 6.65 \rightarrow 7 \quad s_7 = 7.00 \rightarrow 7$$

Histogram Equalization

TABLE 3.1
Intensity distribution and histogram values for a 3-bit, 64 × 64 digital image.

r_k	n_k	$p_r(r_k) = n_k/MN$
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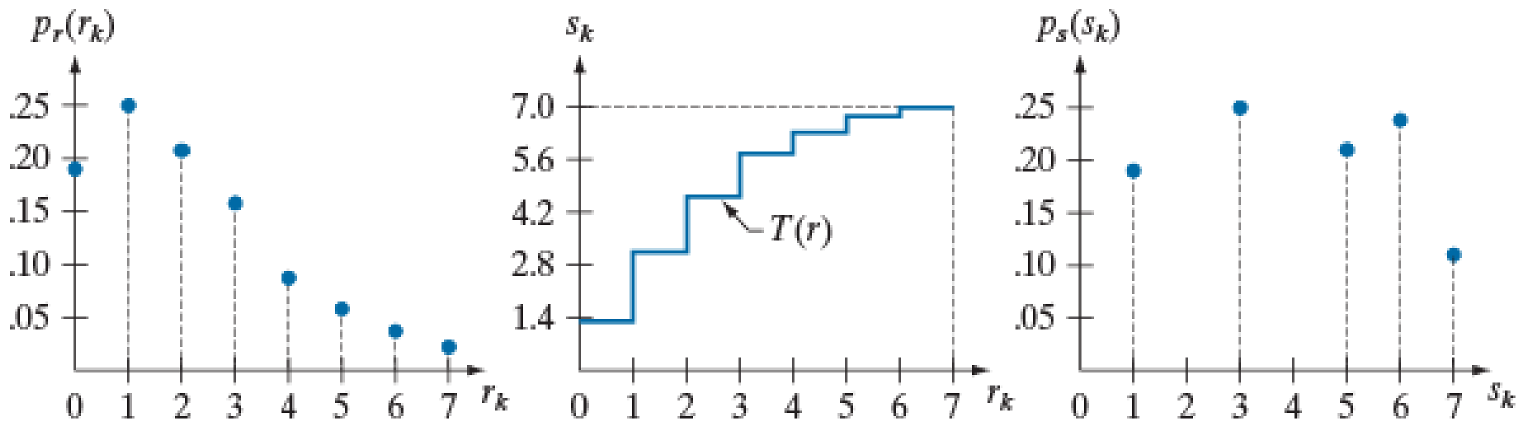


a b c
FIGURE 3.19 Histogram equalization. (a) Original histogram. (b) Transformation function. (c) Equalized histogram.

Histogram

It does (almost) not matter, with which image we start (low contrast, too bright, too dark), after histogram equalization, the result images look very similar

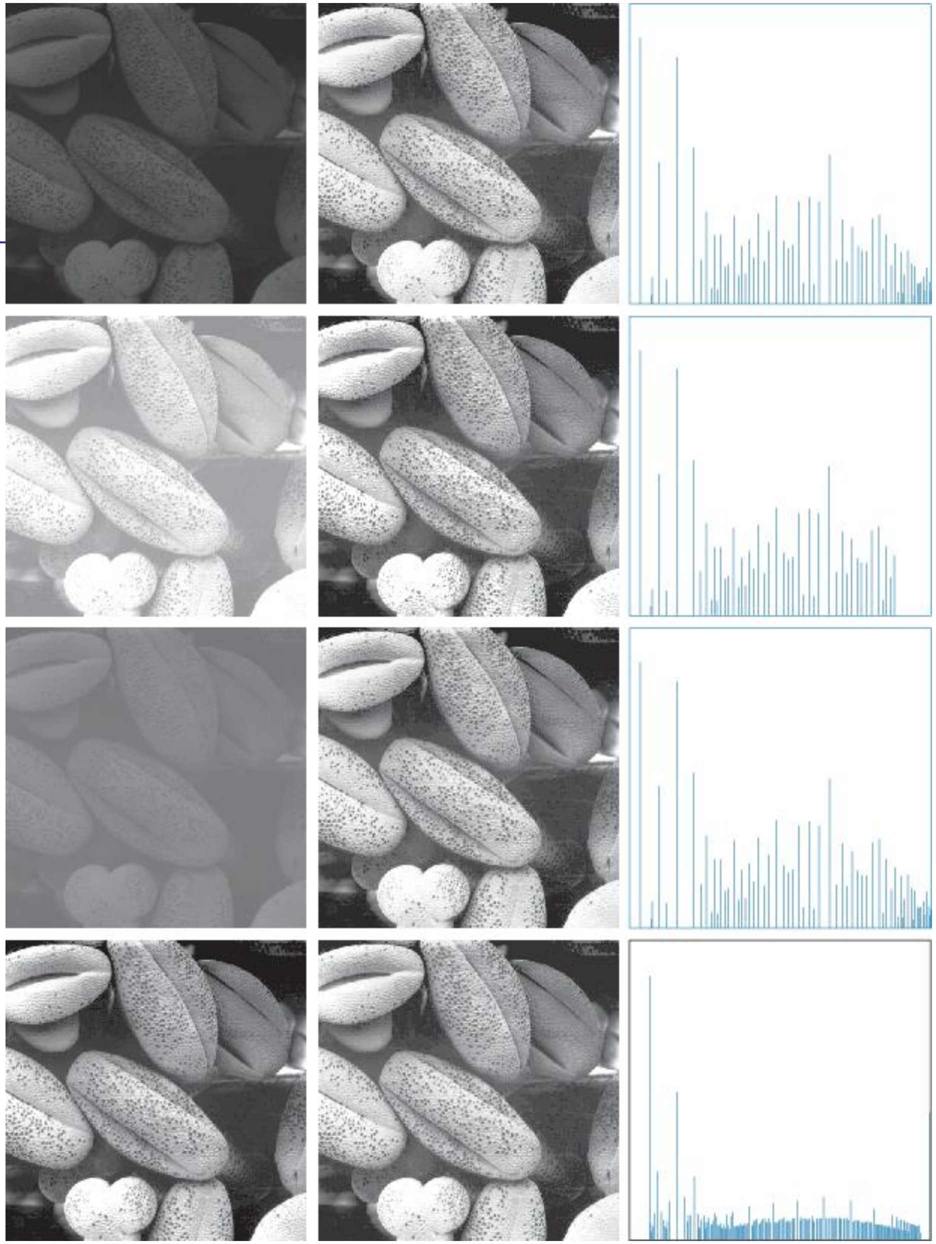
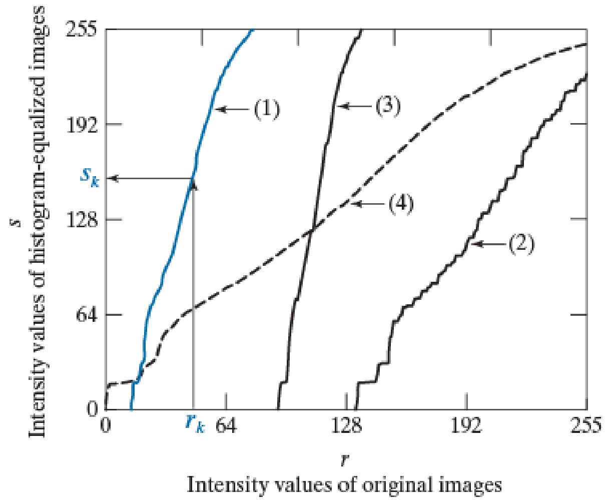


FIGURE 3.20 Left column: Images from Fig. 3.16. Center column: Corresponding histogram-equalized images. Right column: histograms of the images in the center column (compare with the histograms in Fig. 3.16).

Histogram Equalization

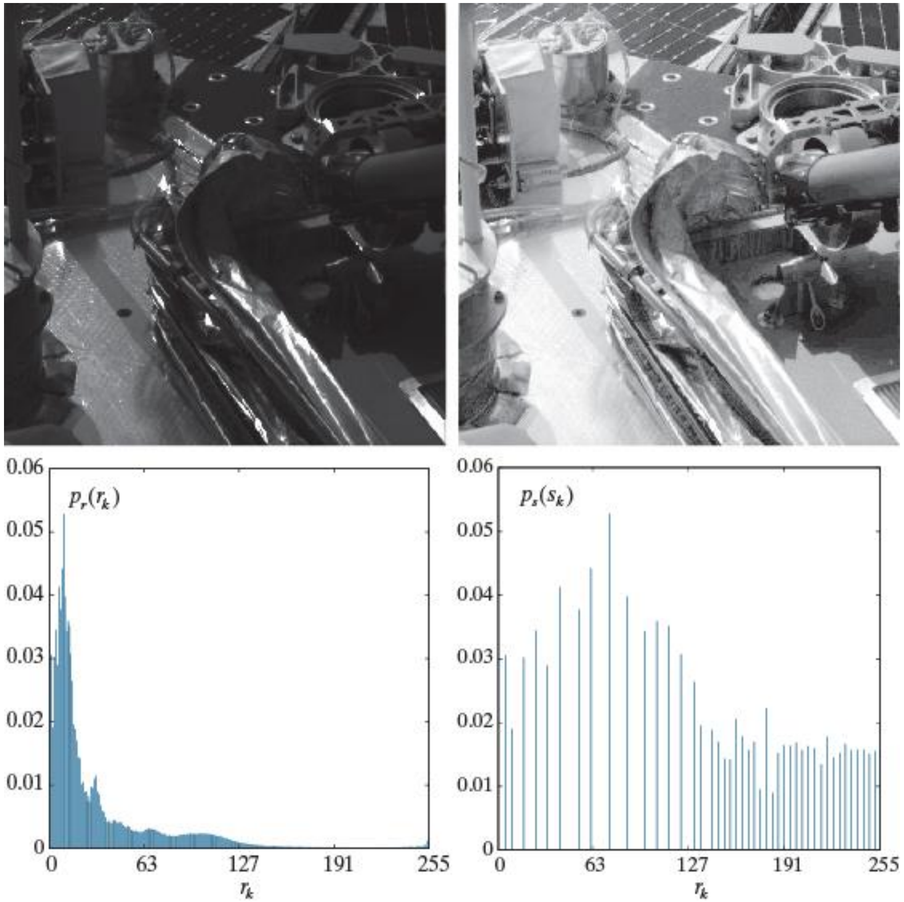
FIGURE 3.21 Transformation functions for histogram equalization. Transformations (1) through (4) were obtained using Eq. (3-15) and the histograms of the images on the left column of Fig. 3.20. Mapping of one intensity value r_k in image 1 to its corresponding value s_k is shown.



Histogram Equalization

a b
c d

FIGURE 3.22
(a) Image from Phoenix Lander. (b) Result of histogram equalization. (c) Histogram of image (a). (d) Histogram of image (b). (Original image courtesy of NASA.)



Histogram Equalization

Summary histogram equalization:

- Histogram equalization is a powerful contrast enhancement method
- The method is parameter-free and can be applied to every image without pre-knowledge
- Images taken under different conditions of illumination or with different cameras will look very similar after histogram equalization

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