LUND UNIVERSITY
SCHOOL OF ECONOMICS AND MANAGEMENT

Fixed Income Securities as a Hedge against Equity Market Downside

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September 2022

Bachelor's Programme in Business and Economics

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Abstract

Investors are constantly searching for ways to protect their wealth, ideally without loosing out on long-term returns. To achieve this, some investors chose to diversify their equity portfolios with fixed income assets, believing they can protect their portfolios from drawdowns. However, the current literature is divided on the effectiveness of fixed income assets as a shock-absorber during severe equity drawdowns. This paper analyzes sub-classes of fixed income assets to understand if they mitigate downside risk as part of a multi-asset equity portfolio. The results of this thesis shows that investors cannot rely on simple heuristics to decide weather to include fixed income or not for this purpose. While some findings support the effectiveness of fixed income, the results are divided. Overall, this paper supports the claim that fixed income can act as a shock-absorber for equity portfolios, in particular government bonds and some investment grade corporate bonds.

This thesis was written in collaboration with Citroneer AB as the concluding part of an bachelor's degree at Lund University School of Economics and Management.

Keywords: Fixed Income, Financial Markets, Hedge, Portfolio, Downside Risk, Value-at-Risk, Expected Shortfall, Extreme Value Theory, Kernel Density Estimation.

Acknowledgements

Erik Nordenskjöld at Citroneer AB deserves a special recognition for his contribution to this thesis. He has always been engaged, eager to help and has offered fresh perspectives and ideas. During the initial phases of the thesis, Erik was the one who suggested that it might be an idea to take a closer look at the assumptions around fixed income. With that said, thank you Erik! For anyone about to write a bachelors thesis in economics, Erik has the best of my recommendations.

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Introduction

The literature has shown that investors generally avoid positions where they would be vulnerable to exceptional losses, despite the low likelihood of these scenarios. Such a "disaster avoidance attitude" suggests that investors are worried about worst-case situations and are wary of the possibility of rapid price drops (Menezes et al., 1980). Such downside risk is important to take into consideration when constructing portfolios due to the fact that price drops can have a significant real-world impact and extreme losses occur far more frequently than would be predicted by commonly assumed return distributions (Huang et al., 2012). In the words of a senior European bank executive: "Normal distribution is man-made. Life is negatively skewed." (Hünseler, 2013).

Despite this risk-averse profile of investors, most choose to build their portfolios around equities. Understandably so, since long-term returns have been alluring, leading to substantial capital growth. It seems that equities' high levels of exposure to macroeconomic growth risk and subsequently repeated large losses are not enough to dissuade investors. To balance the prospects of great long-term return with these downside risks, investors are encouraged to look for asset classes and investing techniques that provide enticing conditional correlations. Safe-haven assets like gold, government bonds, and derivative-based hedging strategies are a few potential choices. Finding solutions to mitigate the tail risk that equities bring while continuing to take on as much risk as tolerable is essential in order to increase the likelihood of great long-term performance (Githaiga et al., 2021).

A common approach to mitigate the tail risk of equities is to include fixed income assets such as government bonds or corporate bonds in the portfolio. The main thesis is that these assets are negatively correlated with equities and will therefore perform well during times of turmoil in the equity market. If so, this would result in fewer severe drawdowns for the portfolio as a whole since the fixed income assets will increase in value when equities are experiencing losses. Especially when equities are experiencing severe drawdowns, fixed income hopefully maintains this negative correlation to equities, acting like a shock-absorber.

In recent years, however, investor sentiment has shifted, and some recommend leaving the conventional "safe haven" of government bonds. Mainly due to the historically low bond yields and the lack of downside protection obtained. More specifically, two statements are frequently made: that investment-grade corporate bonds can serve as a viable alternative to government bonds and offer equivalent downside risk protection; and that government bonds have lost their diversification benefits as equity shock absorbers (Renzi-Ricci and Baynes, 2021).

The objective of this thesis is to investigate if fixed income still acts as a hedge to equity market downside and if fixed income assets have protective characteristics relative to equities. That is, does fixed income continue to act as a shock absorber, limiting downside risk in multi-asset equity portfolios? Investigating this is accomplished by examining changes in the correlation between fixed income sub-asset classes and equity indices over time, as well as changes in their accumulated returns over the same time frames. Thereafter, the distributions of the historical returns in combination with the downside risk measurements are analyzed. In addition to examining the returns derived from the data as a whole, extreme value theory will also be used to evaluate a subset of the returns separately. Finally, the thesis evaluates the assertions that investment-grade corporate bonds can be a reliable alternative to government bonds in particular. Evaluated are the US, European, UK, and German markets. They are evaluated using historical time series and the focus will be on approximately the last 30 years.

The results of this thesis indicate that fixed income acts as a hedge against equity drawdowns, that fixed income does have protective qualities as compared to equities, and that investment-grade bonds are generally not a reliable replacement for government bonds.

2

Technical Background

This section covers the theoretical and technical concepts pervading this thesis. First, concepts within the realm of economics, such as fixed income securities and portfolio theory, are covered. Thereafter, the most important statistical theory underlying the analysis will be outlined.

2.1 Fixed Income

Fixed income securities are investment products with fixed payments. This typically includes a principal payment on the bond's maturity date as well as periodic installments known as coupons. The coupons are specified by a yearly coupon rate, which is a percentage of the principal amount, and the regularity (normally quarterly, semi-annually, or annually) with which they are paid until maturity. A zero-coupon bond is one that does not pay coupons. Bonds are first sold to the primary market, meaning that the issuer sells to a counter party and, in return, receives funds that can be used to finance spending. After the first issuance, the bonds can be traded on a secondary market where the bonds already issued are bought and sold. The price of bonds in the secondary market is what, over time, creates the time series underlying the indices analyzed in this thesis.

A subtype of fixed income is government bonds; this is fixed income issued by governments to finance the government's spending. Since these assets are backed by the government, they are typically seen as lower risk investments, given that the government's default risk is low. Bonds sold by the United States Treasury are considered among the safest in the world, together with government bonds issued by other developed nations. The yields are often low on government bonds due to their relative lack of risk. However, this is not always true, as some government-backed bonds, particularly those issued in emerging countries, can have substantially higher yields. Another subtype of fixed income is corporate bonds. These are instead backed by corporations. An investor who purchases a corporate bond on the primary market effectively lends money to the company in exchange for a series of interest payments. Corporate bonds are frequently viewed as riskier than government bonds issued by stable governments, something that is reflected in the higher yield.

The assets have credit ratings assigned by credit rating companies based on the issuer's financial health. These agencies assess the creditworthiness of corporate and government bonds, as well as the ability of the issuers to repay these loans. The credit grading scale has about 20 levels; in the case of Standard & Poor's scale, the range goes from "AAA" to "D", with "AAA" being the highest rating. On these scales, government bonds are typically rated higher than investment-grade corporate bonds, resulting in lower yields. The lower yields are due to the fact that governments are seen as more reliable counterparties with a minimal chance of default, hence the higher credit rating. In some sense, investors are questioning the yield resulting from this credit rating with statements that investment grade corporate bonds can serve as a viable alternative to government bonds. The difference in yields, the credit spread, between investment-grade corporate bonds and government bonds is seen as potentially too large, resulting in favorable yields for the investment-grade corporate bonds. There are also non-investment grade bonds, also known as junk bonds or high-yield bonds, which have even lower credit ratings and higher yields due to the higher likelihood of default.

2.2 Portfolio Theory

Diversification is the concept of decreasing risk by investing in a diverse set of financial instruments, industries, and asset classes. It reduces idiosyncratic risk but does not affect systematic risk, and the aim is to achieve higher risk-adjusted returns. However, if there is contagion, diversification benefits might fade as correlations between asset returns tend to increase. In extreme market situations, for instance, correlations between assets tend to converge to one, a negative consequence for portfolio managers. Thus, it is essential to comprehend one's risk exposure and the variability of this exposure over time, regardless of diversification. By observing the correlation between sub-types of fixed income and equities over time, it is possible to, in part, understand this risk exposure.

Finding the best portfolio that balances risk and returns while outperforming the market portfolio is one of the primary objectives for portfolio managers, and diversification plays an essential part in being successful at this. A measure portfolio managers sometimes opt to maximize is the Sharpe ratio, a risk-adjusted return metric developed by the Nobel laureate Sharpe (1964). Over the years, several approaches to building a portfolio by giving the proper weights to the right assets have been established and advocated in an effort to address the portfolio management problem and identify the ideal portfolio. Modern portfolio theory (MPT), first proposed by Markowitz (1952), is a common approach that provides an uncomplicated technique for addressing these concerns. When investors are confronted with a vast universe of risky securities and given estimates of their anticipated returns, standard deviations, and pair-wise correlations, they can compute the set of efficient portfolios. Markowitz method is centered on maximizing the risk-return trade-off in a diversified portfolio, which results in a portfolio that is less volatile than the sum of its parts. The methodology has nevertheless drawn criticism because it is based on assumptions that are not supported by empirical data. Additionally, investors often choose more heuristic weighting strategies because they are simpler to apply, such as giving each asset an identical weight in an attempt to reduce the portfolio's volatility. This thesis will leave volatility as a measure of risk and instead focus on maximal drawdowns, enabling us to comprehend negative performance and what constitutes an extreme event for an asset, rather than just its variability over time.

To even construct an optimal portfolio, the assumption must be made that it is possible to do so. The efficient market hypothesis (EMH), created by Fama (1970), is a theory that asserts that asset prices reflect all available information. As a result, an investor cannot buy or sell assets below their fair market value, making it impossible to outperform the market using various stock picking strategies. Proponents of the EMH often deploy an investment strategy tilted toward passive ownership of market capitalization-weighted portfolios that follow stock indices like the Swedish stock market's OMXS30 or the American S&P 500. Some of the world's largest asset managers, such as BlackRock, Vanguard, and State Street, are labeled as passive index fund managers (Fichtner et al., 2017). Results indicating that there are benefits for or against using any particular sub-type of fixed income as a hedge to equities would be considered to be a deviation away from EHM.

2.3 Non-Parametric Models

Non-parametric models are those in which no parametric assumptions are made regarding the distribution of the underlying data. This is in contrast to parametric statistics, which make such assumptions. Histograms and kernel density estimation are two examples of statistical methods that are typically considered to be non-parametric.

2.3.1 Histograms

To approximate the distribution of numerical data, one can construct a histogram representation of the data. Histograms provide a basic idea of the density of the underlying data and are a common tool used in density estimation. To find the probability density histogram, one normalizes the density histogram so that its total area sums to one. For a time series of returns, this will give an understanding of how the returns were distributed historically, which potentially indicates something about the likelihood of observing the same returns in the future.

2.3.2 Kernel density estimation

Kernel Density Estimation (KDE) is a way of estimating an unknown probability density function (PDF) given some data. Opting to use a kernel density estimate instead of a histogram is usually motivated by the belief that it produces a more accurate representation of the unknown probability density function.

For every data point x_i, we center a kernel function K over it. The kernel density estimate is

$$\hat{f}(x) = \frac{1}{N} \sum_{i=1}^{N} K\left(x - x_i\right).$$

The Kernel function K usually fulfills that it is:

1. everywhere non-negative: $K(x) \geq 0, \ \forall x$.
2. symmetric: $K(x) = K(-x), \ \forall x$.
3. decreasing: $K'(x) < 0, \ \forall x > 0$.

A common choice of Kernel according to Fan and Yao (2003) is the Gaussian Kernel

$$K(u) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{u^2}{2}\right), \quad u \in R.$$

The kernel function K does however not normally have a great influence over the outcome of the estimation, that is, the KDE will be very similar for different kernels, and increasingly so as the number of data points increase (Odland, 2018).

To control the bandwidth, h, of the kernel we introduce

$$\hat{f}(x) = \frac{1}{Nh} \sum_{i=1}^{N} K\left(\frac{x - x_i}{h}\right).$$

The kernel bandwidth does usually have a significant impact on the outcome of the estimation (Odland, 2018), despite this there is no straightforward way to decide on what bandwidth to choose. The predicted PDF will closely match the histogram if one chooses a too small bandwidth. For an excessively large bandwidth, the estimated PDF will instead be too smoothed out, potentially losing properties of the data. Density peaks might be underestimated and tail probabilities overestimated.

2.4 Descriptive Statistics and Measures

It is difficult, if not impossible, to construct and carry out rigorous statistical tests when there is no naturally objective and systematic way to construct the test and when there is a limited number of historical events available for analysis. When it comes to bear markets in the past, this is precisely the situation. According to Myers (2022), the technical definition of a bear market is a decline of 20% or more from recent highs. However, this definition is arbitrary, and depending on what basket of assets one is observing (indices, for example), we could either be in a bear market or we could not be in a bear market. The assets that are typically used to indicate bear markets are also subject to change over time. That is to say, indices such as the S&P500, the Dax, or the FTSE are not static; rather, they are constructed out of different assets over time. As a consequence of this, the scope of the analysis is sometimes limited to a more informal comparison of the results.

2.4.1 Correlation

An estimation of the linear relationship between two variables, in this case, stocks and bonds, can be provided by correlation. With a series of n readings from (X_i, Y_i) indexed $i = 1, \ldots, n$, the population Pearson correlation $\rho_{X,Y}$ between X and Y can be estimated by using the sample correlation coefficient. The sample correlation coefficient is defined as

$$r_{xy} = \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n} (x_i - \bar{x})^2 \sum_{i=1}^{n} (y_i - \bar{y})^2}}$$

where $\bar{x}$ and $\bar{y}$ are the sample means of X and Y (Blom et al., 2005).

A negative correlation between two indexed price series (e.g. the equity indices) indicates that there is a protective relationship between the two, that is, they act as hedges to each other. The inverse is true for the correlation between an indexed price series and a yield series, a positive correlation indicates that the two series will act as hedges to each other. A lower yield implies a higher asset price. Therefore, a yield decrease during an indexed price decrease results in a value loss for the indexed price asset but a value increase for the yield asset.

For the purpose of this thesis, it is important to remember that just because there is a correlation, it does not necessarily mean that there is some intrinsic connection between the variables; in this case, intrinsic protection between the government bonds and equity indices. Instead, there could be a confounding variable that affects both.

2.4.2 Sample standard deviation

The standard deviation is a statistic that expresses how much variance or dispersion there is in a group of numbers. While a high standard deviation suggests that the values are dispersed over a wider range, a low standard deviation suggests that the values tend to be close to the expected value of the collection.

The sample standard deviation denoted by s is defined as

$$s = \left(\frac{1}{N-1} \sum_{i=1}^{N} (x_i - \bar{x})^2 \right)^{1/2}$$

2.4.3 Value at risk and expected shortfall

Value-at-Risk (VaR) and Expected Shortfall (ES) are risk management tools that calculate the amount of financial risk that exists connected to an asset or a portfolio over a given time frame. More precisely, for a given time horizon t and confidence level α, the VaR is the loss in market value over the time horizon t that is exceeded with probability $1 - \alpha$. Organizations with substantial assets and/or liabilities frequently employ these tools to understand and control their overall exposure to risk. The general protective qualities of an asset, like fixed income, as compared to another asset, like equities, could in part be deduced from comparing the VaR and ES results of these assets. If the tail risk of equities is larger than that of fixed income, then fixed income should in general act as a shock-absorber to equities. The reasoning behind this is that if downside events do not occur in close proximity to each other, then great, a downside event in one asset is not seen in the other, meaning that we have a somewhat protective relationship. If the downside events do occur in proximity to each other, then, if assumed that the events are of the same percentile magnitude, a lesser magnitude for the fixed income assets means that there is still some protection. Although these are the least desirable dynamics, and the downside risk measure would probably need to be significantly smaller than the equities equivalents to justify labeling that asset as one that provides downside protection.

Let L be a loss distribution. The VaR at level $\alpha \in (0, 1)$ is the smallest number l such that the probability that l does not exceed L is at least $1-\alpha$. That is, $\mathrm{VaR}_\alpha(L)$ is the $(1-\alpha)$ quantile of L.

$$VaR_\alpha(L) = \min\{l : \Pr(L > l) \leq 1 - \alpha\}$$

However, VaR as a risk measurement has significant flaws. For example, the measurement simply indicates the likelihood that a portfolio may experience a loss greater than VaR. It does not impose an upper bound on the loss itself, making it very dependent on the behavior of the tail once VaR has been calculated. ES is an enhanced measure that deals with some of these problems. The predicted loss determined by ES is the expected loss when exceeding the VaR quantile. As a result, it provides a more informative estimate of the loss one should expect in the occasion of an extreme market event.

Hence, Expected shortfall is the expected value of all returns in the distribution that are worse than the VaR at a given level of confidence. For example, at a 95% confidence level, the expected shortfall is found by taking the average of the returns in the worst 5% of cases.

$$\mathrm{ES}_\alpha(L) = \mathrm{E}\left[L \mid L \geq \mathrm{VaR}_\alpha(L)\right] = \frac{1}{1-\alpha}\int_\alpha^1 \mathrm{VaR}_u(L)du$$

2.5 Extreme Value Theory

To improve VaR and ES estimates and to increase the confidence level in them, Extreme Value Theory (EVT) can be used. EVT smoothens the tails of a probability distribution. By doing this, the computational uncertainties normally associated with deriving VaR and ES are avoided. As a result, estimates of VaR and ES are produced that take into account the entire shape of the distribution's tail rather than simply a small number of loss locations (Hull, 2012). In essence, EVT tries to more accurately characterize tail losses with a distribution of its own, and in risk management, we are ultimately only interested in the tail of the distribution. Therefore, EVT might produce better estimates of the tail losses than traditional VaR or ES methods when considering high confidence levels, such as 99% or 99.9% (Hull, 2012).

There are two EVT strategies:

1. Block Maximia: Analyzing the largest value with a chunk of observations (e.g. within a certain fixed time period)

2. Peaks Over Threshold: Analyzing values that exceed a predetermined threshold (often the dataset's percentile).

Both strategies rely on their respective Extreme Value Theorem. In finance, and for most practical purposes, Peaks Over Threshold (POT) is more appropriate and simpler to implement. Of interest is therefore the distribution of losses

$$P(L \leq y + \gamma \mid L > \gamma)$$

where L is the loss distribution and γ is some percentile, usually the 95th, of a dataset $\mathcal{D}\{r_i\}_{i=1}^{n}$ of returns. With the help of Bayes' theorem one obtains

$$P(L \leq y + \gamma \mid L > \gamma) = \frac{P(\{L \leq y + \gamma\} \cap \{L > \gamma\})}{P(L > \gamma)}.$$

A central finding in EVT for the POT approach says that (Gomes and Guillou, 2015)

$$P(L \leq y + \gamma \mid L > \gamma) \approx G_{\xi,\beta}(y)$$

where $G_{\xi,\beta}(y)$ is a Generalized Pareto Distribution (GPD). The GPD is expressed as (Gomes and Guillou, 2015)

$$G_{\xi,\beta}(y) = \begin{cases} 1 - \left[1 + \xi \frac{y}{\beta}\right]^{-\frac{1}{\xi}}, & \xi \neq 0. \\ 1 - \exp\left\{-\frac{y}{\beta}\right\}, & \xi = 0. \end{cases}$$

and their respective probability density function are (Gomes and Guillou, 2015)

$$g_{\xi,\beta}(y) = \begin{cases} \frac{1}{\beta}\left(1 + \xi \frac{y}{\beta}\right)^{-\frac{1}{\xi}-1}, & \xi \neq 0. \\ \frac{1}{\beta} \exp\left\{-\frac{y}{\beta}\right\}, & \xi = 0. \end{cases}$$

The distribution of losses exceeding γ, $F_{L_\gamma}(x)$, can now be derived (Nilsson, 2022) as

$$F_{L_\gamma}(l) = G_{\xi,\beta}(l - \gamma)\left(1 - \hat{F}_L(\gamma)\right) + \hat{F}_L(\gamma) \tag{2.1}$$

where $\frac{n_\gamma}{N} = 1 - \hat{F}_L(\gamma)$, in which $\frac{n_\gamma}{N}$ is the fraction of losses above our threshold γ, and $\hat{F}_L(\gamma)$ is the empirical cumulative density function of the observations smaller than γ.

3

Methodology

This section aims to explain and motivate details regarding data collection, implementation, and analysis methods. The theory was implemented by code using 16 assets. The assets consist of stock indices, government bonds, and credit indices. The data was handled in Microsoft Excel before being loaded into Python for analysis. Access to high quality time series data of transaction prices spanning decades back prompted the decision to use this approach. The markets of the three largest economies in the Occident are assessed.

3.1 Software

The data was extracted and concatenated using Microsoft Excel. The analyses were performed in Python3 on the Jupyter Note-Book platform within the code editor Visual Studio Code. A number of packages were imported. The packages used most extensively were Pandas, NumPy, PyPlot and Matplotlib.

3.2 Data Collection

Historical time series data for weekly closing prices on three equity indices, four corporate bond indices, and nine generalized government bond yield contracts were collected from the Bloomberg Terminal. The datasets vary in length, with the longest starting in 1959. All datasets end 2022-08-31. Information about the data can be found in table 3.1.

The choice to use the data specified in table 3.1 was influenced by whether large amounts of historical data could be accessed; if the quality of that data was satisfactory; and finally, if the format of the data was such that analysis could be performed after a reasonable amount of data manipulation.

Table 3.1: Data obtained from the Bloomberg terminal.

Series name	Ticker	Frequency	Unit	Start date
US Corporate High Yield	LF98TRUU	Weekly	Indexed	1998-07-31
US Corporate Bond Index (IG)	LUACTRUU	Weekly	Indexed	1988-12-30
US Generic Govt 3y Index	USGG3YR	Weekly	Yield	1962-01-05
US Generic Govt 5y Index	USGG5YR	Weekly	Yield	1962-01-05
US Generic Govt 10y Index	USGG10YR	Weekly	Yield	1962-01-05
US Govt Bond Index (BB)	LUAGTRUU	Weekly	Indexed	1990-01-05
S&P 500 Total Return Index	SPXT	Weekly	Indexed	1988-01-08
Sterling Corporate Bonds	LC61TRGU	Weekly	Indexed	1992-01-03
3Y UK Govt Bond	GTGBP3YR	Weekly	Yield	1992-01-03
5Y UK Govt Bond	GTGBP5YR	Weekly	Yield	1992-01-03
10Y UK Govt Bond	GTGBP10YR	Weekly	Yield	1992-01-03
Sterling Gilt Index (BB)	LSG1TRGU	Weekly	Indexed	1999-01-29
FTSE All-Share Index TR	ASXTR	Weekly	Indexed	1986-01-03
Bloomberg Global Credit - Ger TR	I04140EU	Weekly	Indexed	2001-06-29
GER Govt Bond 3Y	GTDEM3YR	Weekly	Yield	1990-08-10
GER Govt Bond 5Y	GTDEM5YR	Weekly	Yield	1990-08-10
GER Govt Bond 10Y	GTDEM10YR	Weekly	Yield	1990-08-10
DAX Index	DAX INDEX	Weekly	Indexed	1959-10-02
Euro-Aggregate Treasury Index (BB)	LEATTREU	Weekly	Indexed	1998-07-03
Euro-Aggregate Corpporate Index. IG (BB)	LECPTREU	Weekly	Indexed	1998-07-03
Euro Stoxx Gross Return Index (BB)	SXXGT Index	Weekly	Indexed	2001-01-05

3.3 Time-Varying Measurements

Correlation and standard deviation are calculated using three-year lags. Returns are calculated with a one-year lag. The choice of time-lags is of an arbitrary nature. Consequently, to avoid choosing an arbitrary time-lag, it would be preferable to add a "time-lag" axis with a continuous spectrum. However, the scope of this analysis is limited to one time-lag for each rolling series. Notably, the chosen time lags do not significantly impact the analysis since different time lags would produce graphs indicating the same or similar conclusions. The main difference would be in the visualization of the results.

3.3.1 Rolling correlation

Fix a variable l, take the l first readings, $i = 1, ..., l$, from a series of $n > l$ readings of (X_i, Y_i). With these readings, calculate the correlation r_{xy_1} between X and Y. Increment the starting position one step forward so that $i = 2, ..., l + 1$ and repeat the calculations to obtain r_{xy_2}. Continue this process until $l + k = n$ to calculate the *rolling correlation* estimate series $r_{xy_1}, ..., r_{xy_{k+1}}$ between X and Y.

3.3.2 Rolling return

Fix a variable l, from a series of prices S_i where $i = 1, ..., n$, take the first and l:*th* reading of S_i. Calculate the return $R_1 = \frac{S_l - S_1}{S_1}$. Increment one step forward and calculate R_2 in the same manner using the second and $l + 1$:*th* readings. Proceed to increment forward and calculate the return until $l + k = n$ to obtain the *rolling* *return* series $R_1, ..., R_{k+1}$ of S_i with interval l.

3.3.3 Rolling standard deviation

Fix a variable l, from a series of prices S_i where $i = 1, ..., n$, take the $l < n$ first readings, $i = 1, ..., l$. Calculate the sample standard deviation s_1 from these readings. Increment the starting position one step forward so that $i = 2, ..., l + 1$ and repeat the calculations to calculate s_2 in the same manner. Proceed to increment forward and calculate the sample standard deviations until $l + k = n$ to obtain the *rolling standard deviation* series $s_1, ..., s_{k+1}$ of S_i with length l.

3.4 Distributions

3.4.1 Histograms

To construct a histogram, first "bin" the range of values, that is, divide them into a series of intervals, and then count the number of values that fall into each interval. The bins are defined as a series of discrete intervals that do not overlap, are next to each other, and are of the same width.

Some of the time periods in the histograms differ. If it is assumed that the returns are the fallout of a specific stochastic variable, then cutting short the time series would be of no benefit; it would only leave one with less data. If it is not assumed that the returns are the fallout of a stochastic variable, then modelling the data as such is of questionable meaning. It is therefore not a straight-forward task to decide on what is better. For this thesis, we have chosen to plot histograms for both the full time series and, when appropriate for comparison, a time series cut short.

3.4.2 Kernel density estimation

Define a kernel function, K. Center the kernel function on each data point. That is, over every data point in the sample, place the kernel function. Sum the kernel functions to arrive at a kernel density estimate. This estimate is normalized by the number of data points, N, since the integral of the kernel must evaluate to one for it to be a probability density function. The Gaussian kernel has been used for all kernel density estimations in this thesis. By using a heuristic approach, the bandwidth h used for the KDE's was set to $h = 0.4$ and the number of bins N in the histogram to $N = 300$.

3.5 Value at risk and expected shortfall

To better understand how an asset might contribute to the risk exposure as part of a portfolio, VaR and ES are calculated for the asset. Since VaR and ES do not tell us when extreme scenarios will happen, the results will only state something about the asset in general. Even though it is not ideal, it will give us a general idea of how much downside protection we can anticipate in extreme cases. When combined with time-varying correlation, this is helpful. For instance, if the correlation is high, it is expected that a significant decline in equity prices will also result in a significant decline in fixed income prices, in the sense that both will experience, say, a 99th percentile event. Having a number on the magnitude of such a decline from the ES and VaR estimates, it is possible to somewhat anticipate how much protection one can expect.

3.5.1 Historical method

The historical VaR methodology is a conventional way to compute VaR. Using a historical time period of usually daily returns, VaR is derived from the return distribution as a percentile of the distribution multiplied with the value of the portfolio. To find the corresponding ES of the same distribution, the expected value of the tail beyond the same percentile is calculated. In this case, that is just an average of the losses above our threshold.

VaR and ES are sometimes stated as a percentage and sometimes as an asset value. We will not assume any portfolio size in this thesis and therefore keep the number as a percentage. If one wants to find the equivalent asset value loss, this is simply done by multiplying the percentage VaR or ES number with the total portfolio value.

3.5.2 Kernel method

Instead of using the historical returns directly, we calculate a kernel density estimate for the probability density function that represents these historical returns. Starting from the left-most tail value of this PDF, we step to the right along the x-axis and evaluate the area under the curve to the left of the current position. This is done until we are one step prior to the position where the area under the curve will evaluate to the percentile we have chosen, or larger. The x-value at which we stop before passing the threshold will be the VaR percentage. To find the ES, we, as before, take the average of the losses larger than this VaR threshold.

3.5.3 Extreme value theory method

From equation 2.1 we can derive VaR and ES estimates. To find the VaR estimate we solve the equation $F_{L_\gamma}(\mathrm{VaR}_c) = c$, where c is the percentile chosen. By using this result we can then find an estimate for ES. As stated in (Nilsson, 2022) the estimates will evaluate to;

$$\mathrm{VaR}_c = \gamma + \frac{\beta}{\xi}\left(\left(\frac{N}{n_\gamma}(1-c)\right)^{-\xi} - 1\right)$$

$$\mathrm{ES}_c = \frac{\mathrm{VaR}_c}{1-\xi} + \frac{\beta - \gamma\xi}{1-\xi}.$$

The parameters used in the Generalized Pareto Distribution are found by maximum likelihood estimation. An example of this can be found in (Nilsson, 2022).

3.6 Limitations

No matter how sophisticated the method is, it is critical to remember that the results we derive are outcomes of calculations that require inputs. The starting point in this case is a historical time series of prices. This implies that the results reached are merely forecasts of future behavior based on some earlier behavior. Precisely, the earlier behavior during the time period of the data series that was used as an input. In essence, this means that it is expected that the dynamics of price changes in the future will largely follow those of the past. If the dynamics do not match, which they most likely won't, the results will not match reality going forward. Hopefully, the mismatch is not too large, and the results will be somewhat accurate, at least for some future time period. In practice, this would, for example, imply that one would like to base the model on just the last few weeks or even days' worth of data. If so, the results would capture and include only the most recent market dynamics, assuming that the dynamics of an asset or market change over time but are similar to those that have just been observed.

4

Results

In the following chapter, the results of the study are presented along with some remarks. First, the three time-varying measurements are presented. Second, we combine rolling correlation with rolling standard deviation to highlight a visually appealing pattern. Third, histogram plots together with their respective kernel density estimates are displayed. At last, value-at-risk and expected shortfall estimates are presented.

4.1 Time-Varying Measurements

Time-varying measurements for each asset in each region are displayed in this segment.

4.1.1 Returns

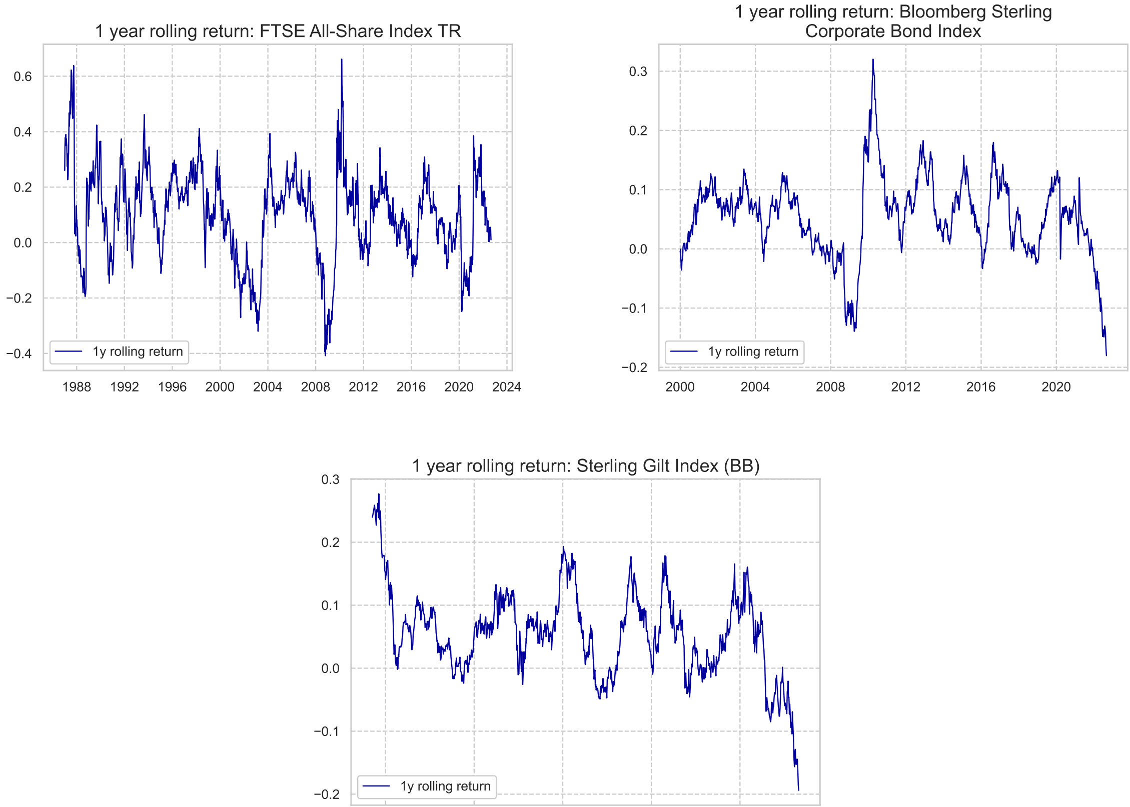

Figure 4.1: One-year rolling return, UK assets. It is worth noting that the scaling on the x and y axes differs.

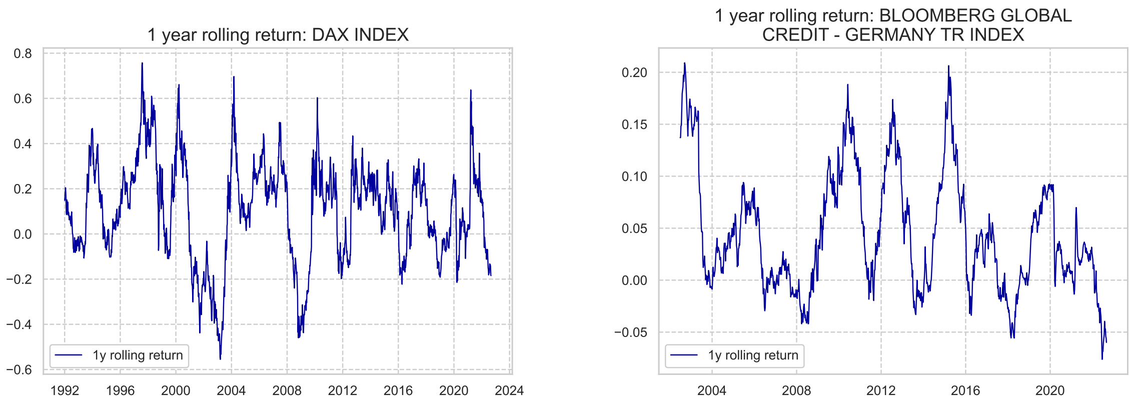

Figure 4.2: One-year rolling return, German assets. It is worth noting that the scaling on the x and y axes differs.

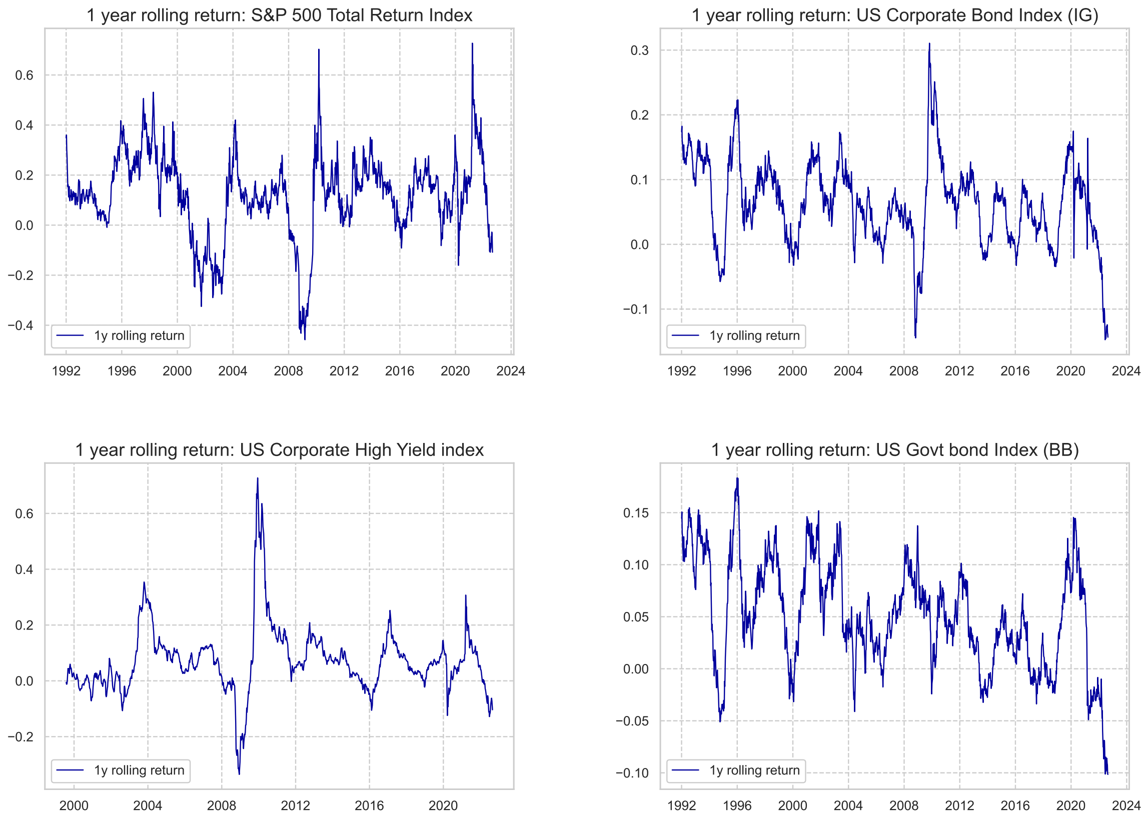

Figure 4.3: One-year rolling return, US assets. It is worth noting that the scaling on the x and y axes differs.

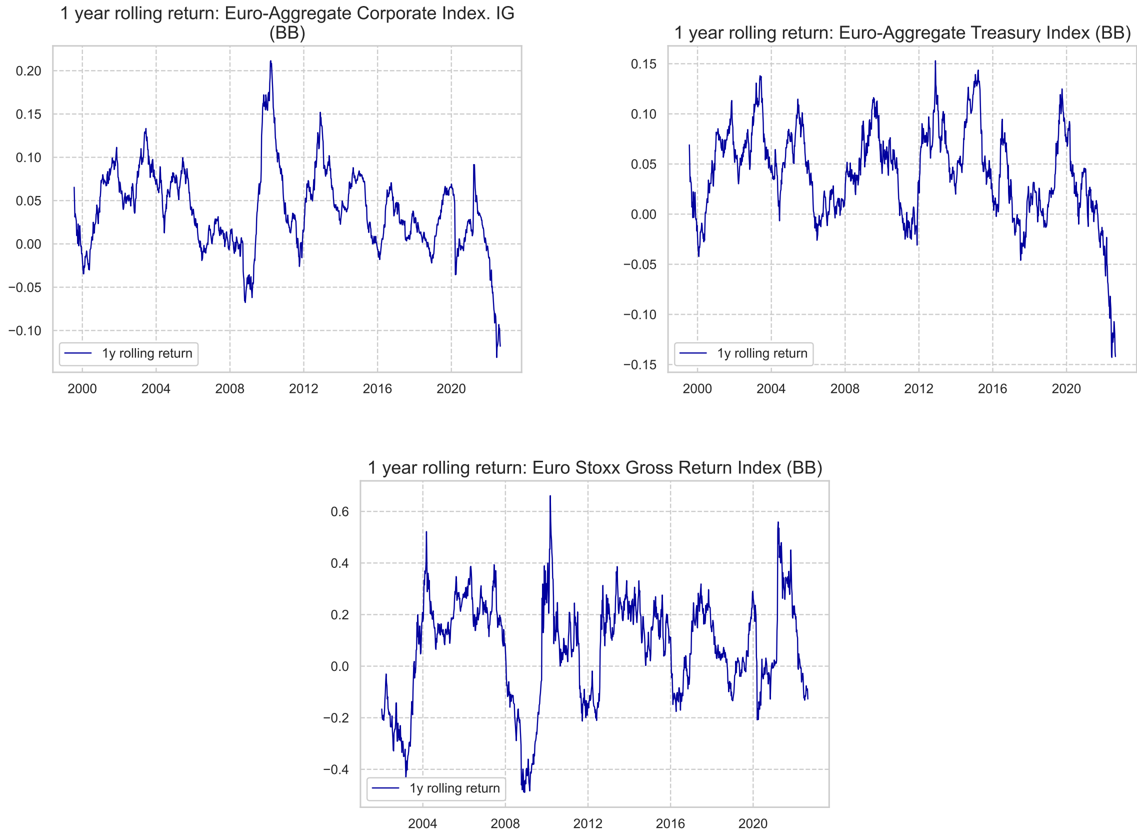

Figure 4.4: One-year rolling return, European assets. It is worth noting that the scaling on the x and y axes differs.

As seen in figure 4.1, 4.2, 4.3 and 4.8, the equity indices' rolling returns have higher variability than both the government bond indices and the corporate credit indices. Note that the lowest reading in 2012, 2016, 2020 and 2022 is almost the same for the US Corporate High Yield Index and S&P500 while the highest readings are generally higher for the S&P500 Index during the same time period ($2012 - 2022$). That is to say, we did not see any benefits associated with downside protection, but we did miss out on the benefits associated with upside.

It appears that the German credit index has performed well in comparison to the US and UK credit indices. German credits have consistently had one year rolling returns above -5% except for during two short periods around 2018 and 2022, where it barely dips below -5%. This is in contrast to the two other investment-grade indices that both returned less than -10% on a rolling basis after the financial crisis and in 2022.

4.1.2 Correlation

For a correct interpretation of the figures, see the second paragraph of subsection 2.4.1.

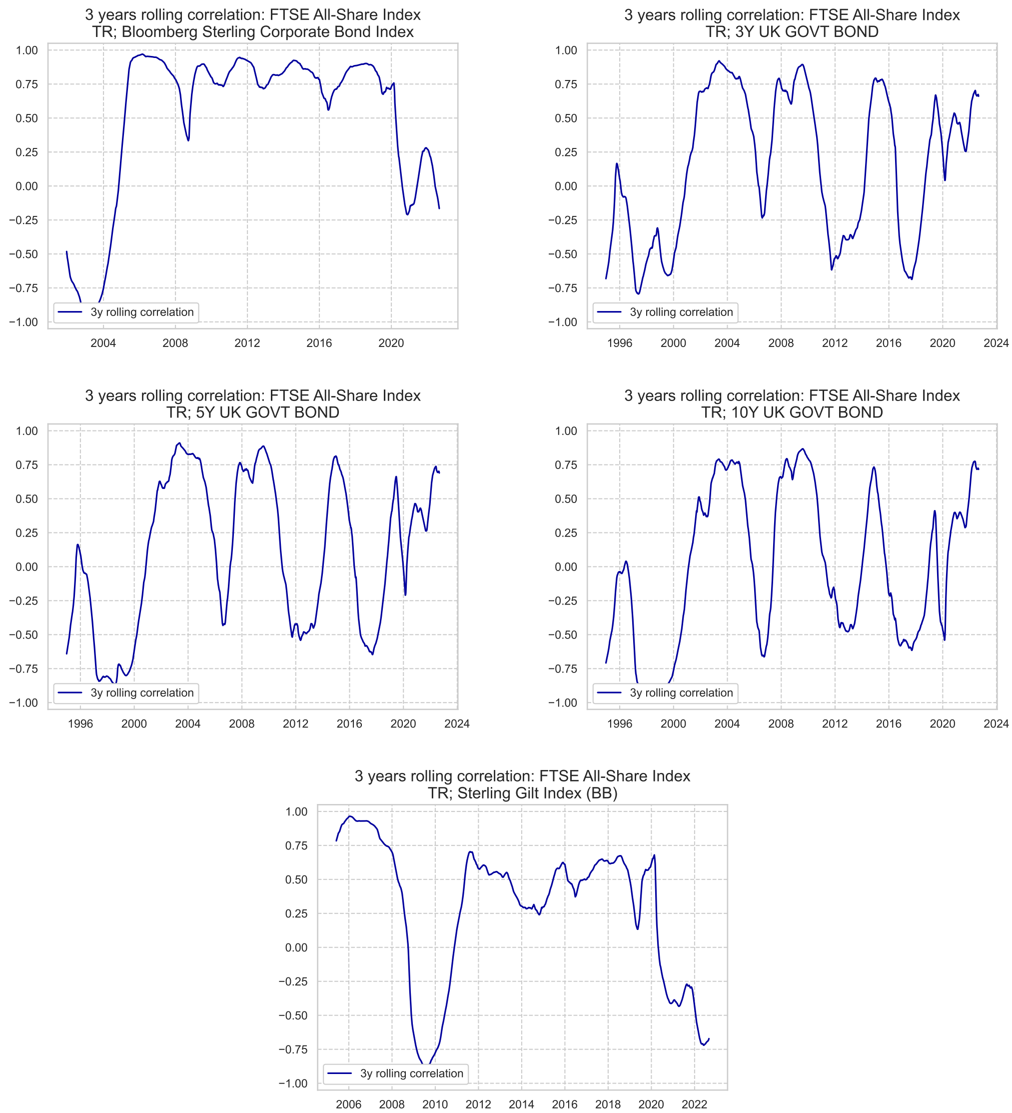

Figure 4.5: Three-year rolling correlations, UK assets. It is worth noting that the scaling on the x and y axes differs in (row, col) = (1,1).

UK stocks exhibit a high and positive correlation with UK corporate bonds. The correlation between UK stocks and UK government bonds has often been more negative when considering the correlation to UK bond asset prices (inverse to yield correlation). This implies that UK government bonds exhibit more favorable protection against equity downside in general than UK corporate bonds. The Sterling gilt index also suggests that government bonds provide better protection than UK corporate bonds. In contrast to the UK corporate bond index, there is a significant decline in correlation for the gilt index during 2008, indicating that government bonds give better protection than credits. The same goes for the period after 2020. However, when observing the returns in figure 4.1, it seems that this decline in correlation is the result of a significant downfall in the sterling index that is not matched by the stock index, indicating worse protection. Highlighting how crucial it is to examine the correlation in conjunction with the rolling returns for a better understanding of the dynamics.

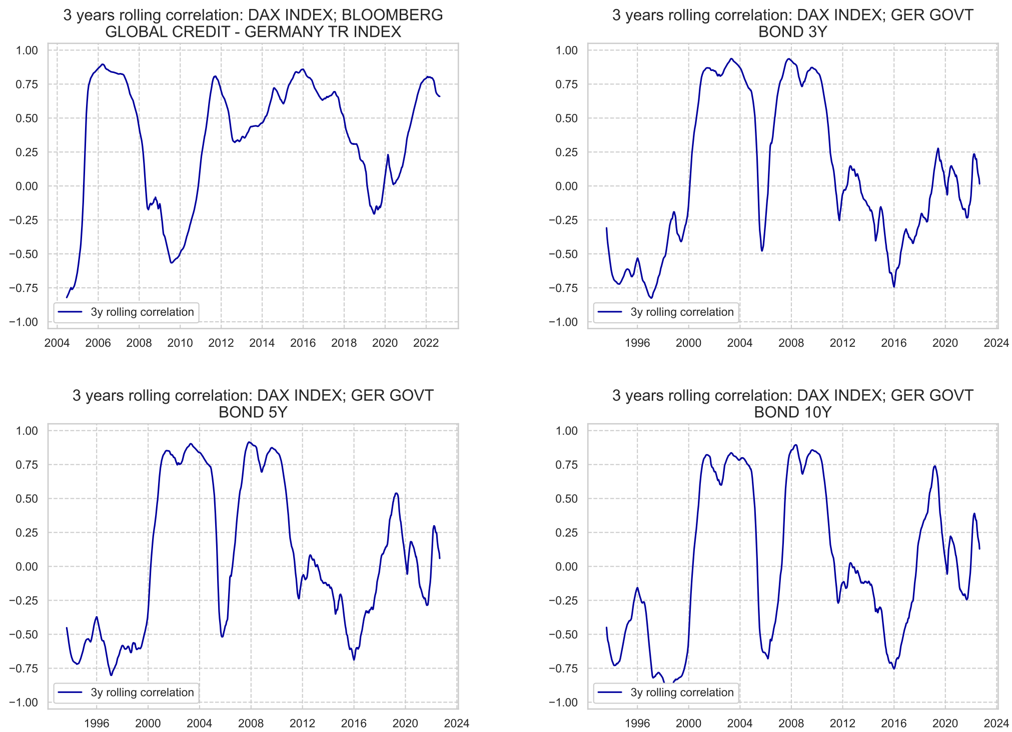

Figure 4.6: Three-year rolling correlations, German assets. It is worth noting that the scaling on the x and y axes differs in (row, col) = (1,1).

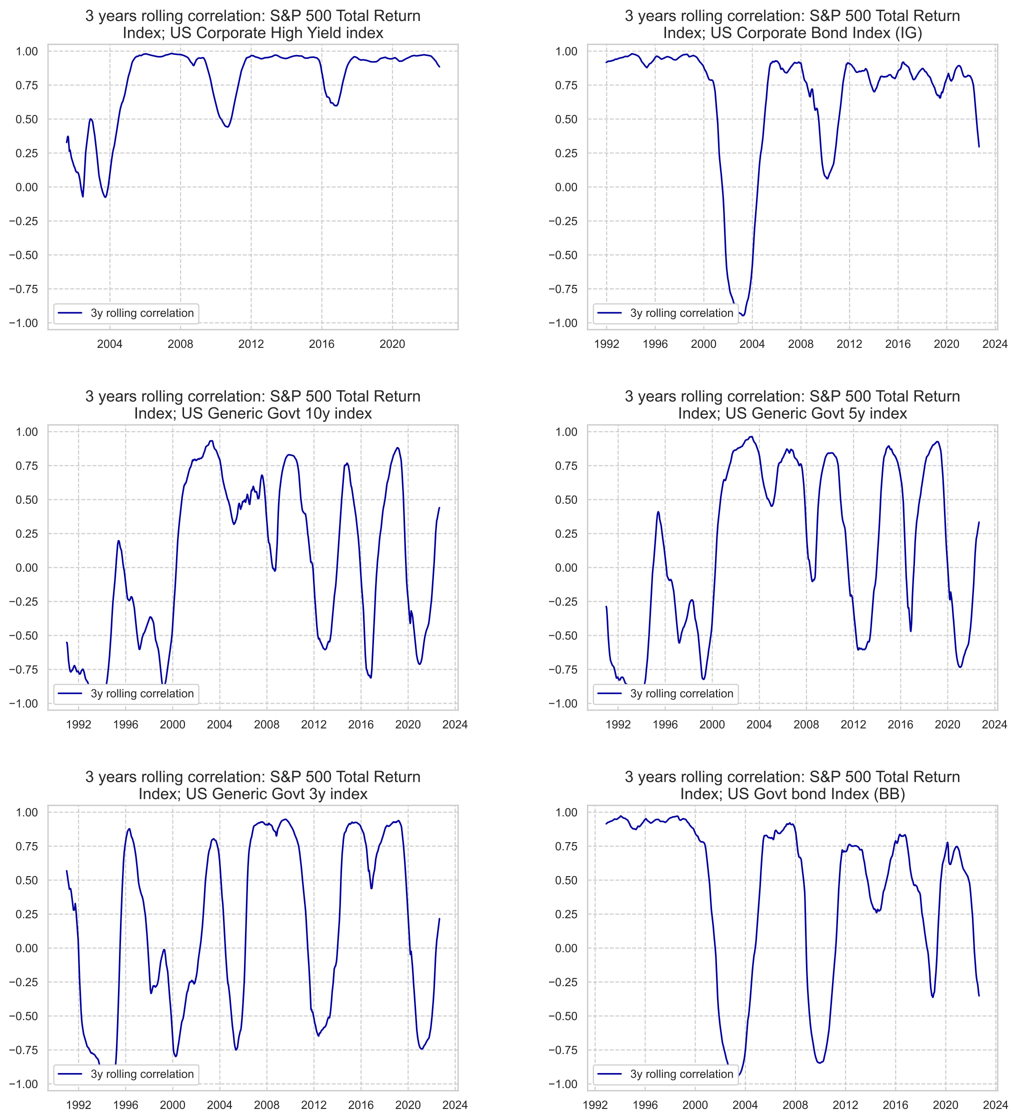

Figure 4.7: Three-year rolling correlations, US assets. It is worth noting that the scaling on the x and y axes differs in (row, col) = (1,1).

The correlation between US stocks and US IG corporate bonds, US stocks and US government bonds, as well as German stocks and German investment grade credit, indicates some protection against equity downfall in contrast to UK corporate bonds. Downfalls in the correlation can be observed before 2004 and around 2010 in the US. In Germany, downfalls can be observed before 2010 and before 2020. This is what prompts us to suspect that investment grade corporate bonds do offer some protection against equity downside in the US and German markets. US government bonds have a similar structure to US IG corporate bonds, differing in that the protection appears better for the government bonds. The rolling returns of US Government bonds and US/German IG corporate bonds in figure 4.3 and 4.2 confirm that the downfalls where not as severe for the fixed income as for the equity indices around 2002 and 2008.

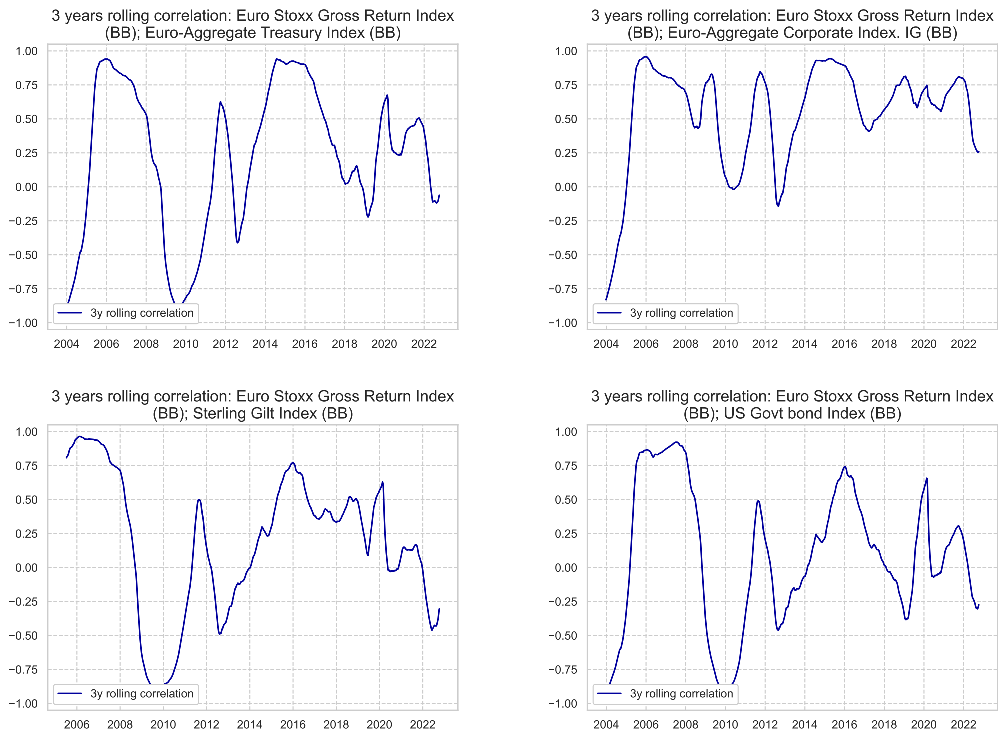

Figure 4.8: Three-year rolling correlations, Stoxx 600. It is worth noting that the scaling on the x and y axes differs.

There is a strong similarity in the correlation between European stocks and all three types of government bonds: US, UK, and European. Notably, they are all similar to the correlation seen between German stocks and German credits. Thus, the same analysis can be performed. That is, there is a protective relationship between EU stocks and each one of the assets. It is noteworthy that the patterns across the assets are strikingly similar when the bonds are compared to the same stock index, the Stoxx 600, implying that while government bond indices do not fluctuate significantly across nations, stock indices do. With the asterisk that, starting in 2017, UK government bonds began to depart from this. While EU stocks experienced greater rolling return declines in 2000 and 2008 than did EU IG corporate credit and EU government bonds, all three experienced the same decline in 2022, resulting in the lowest one-year rolling return readings of the bonds since their creation. Along with the UK's two credit indices, the US government bond index also registered a historically low reading in 2022.

4.2 Volatility and Correlation

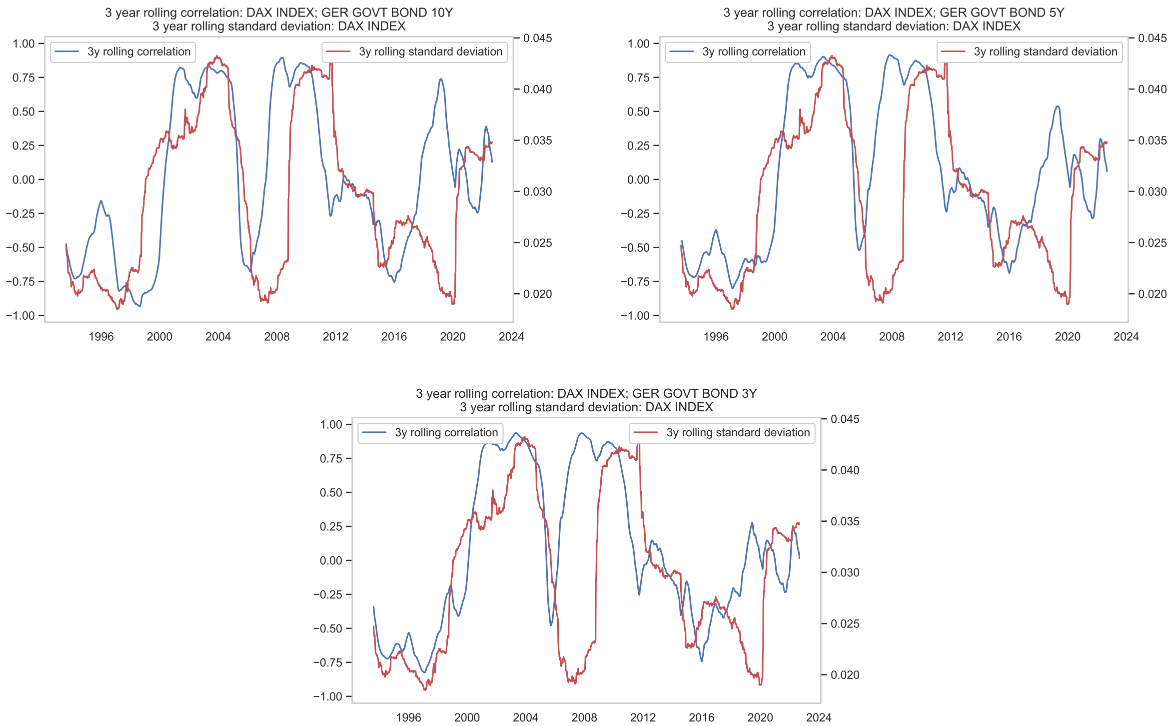

Figure 4.9: Three-year rolling correlation and three-year rolling return, German assets. Note the two-sided y-axis.

From figure 4.9 one could deduce that there is some overlap between the environments of correlation and volatility. In particular, a more negative stock-bond correlation appears to be linked with lower equity market volatility and vice versa.

Mortensen (2021) states that unlike correlation regimes, volatility regimes appear to be less consistent and shift more frequently. This is in contrast to the apparent covariation in figure 4.9. Meanwhile, Ilmanen (2003) found that the correlation between stocks and bonds tends to be low near the peak of the business cycle and during the tightening of monetary policy. In figure 4.9 the correlation bottoms just before 2000, 2008 and 2016.

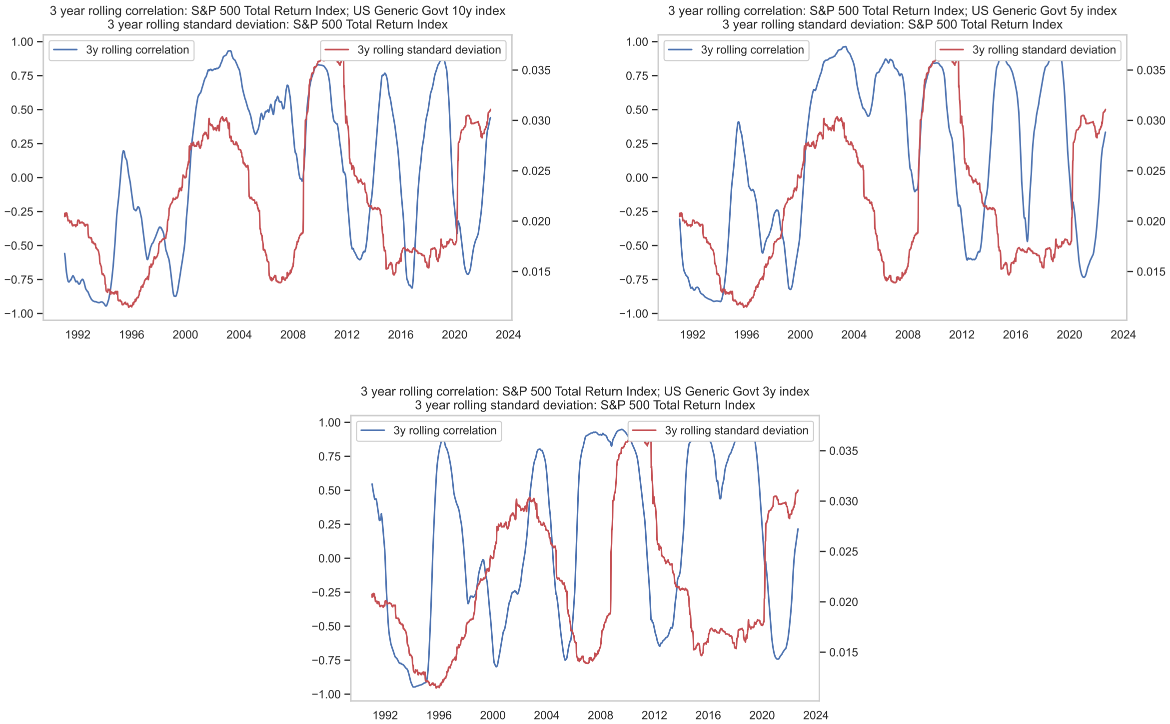

Figure 4.10: Three-year rolling correlation and three-year rolling return, US assets. Note the two-sided y-axis.

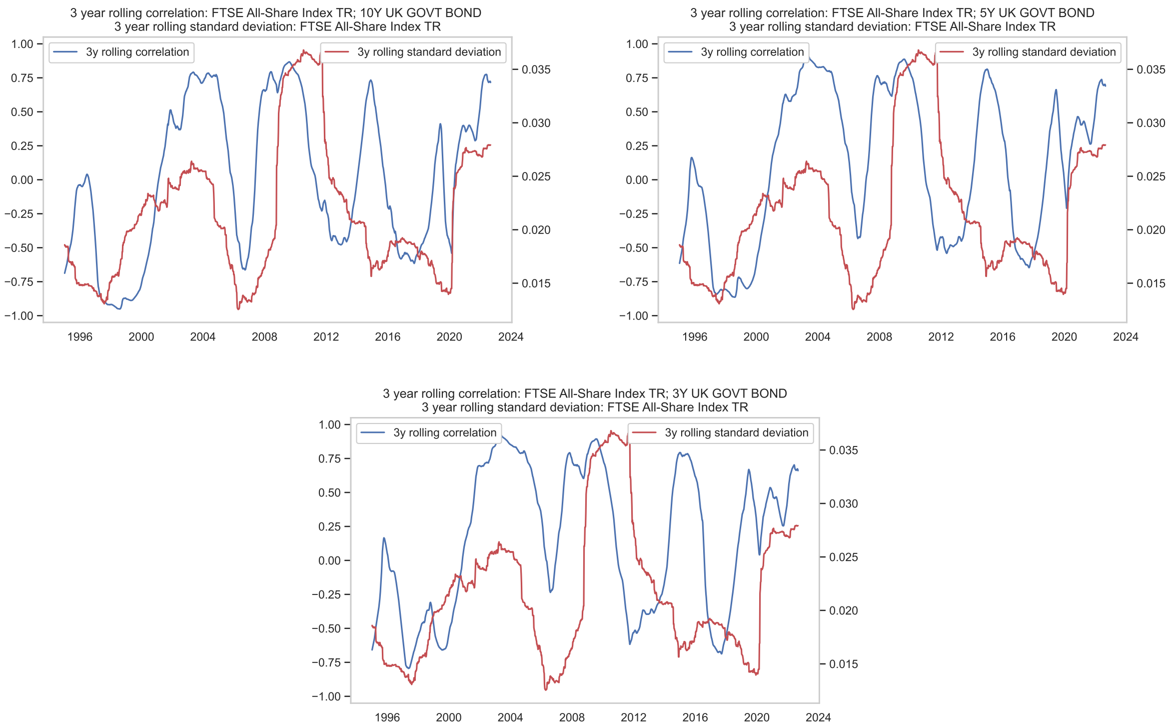

Figure 4.11: Three-year rolling correlation and three-year rolling return, UK assets. Note the two-sided y-axis.

The strong covariation observed in figure 4.9 is not seen in figures 4.11 and 4.10.

4.3 Return Distributions

It is vital to keep in mind that the time period varies while looking at the histograms below. The likelihood of an outlier measurement increases with the length of the time period. We limit the comparison to time series plotted using the same time frames.

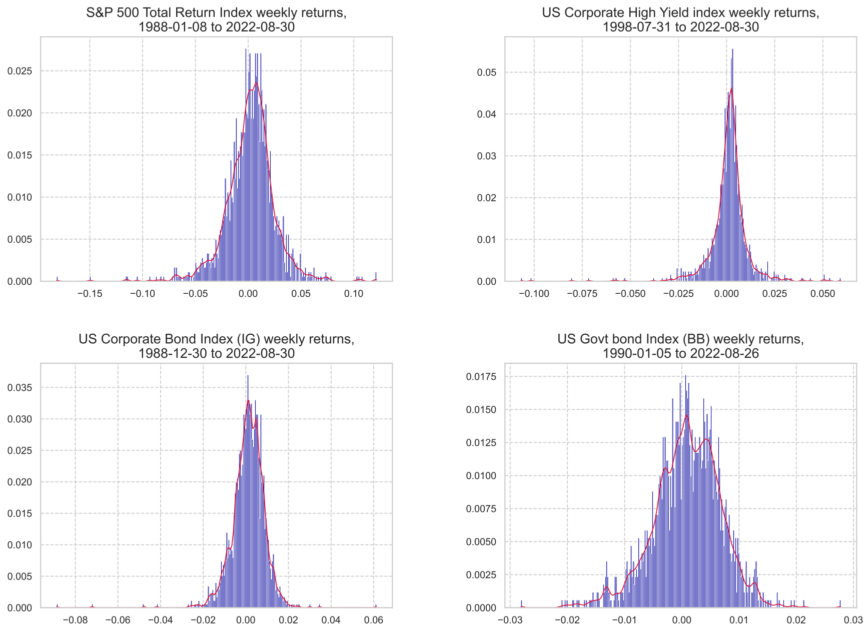

Figure 4.12: Histogram of returns with a kernel density estimate, US assets. Note that the scaling on both the x and y-axis differs.

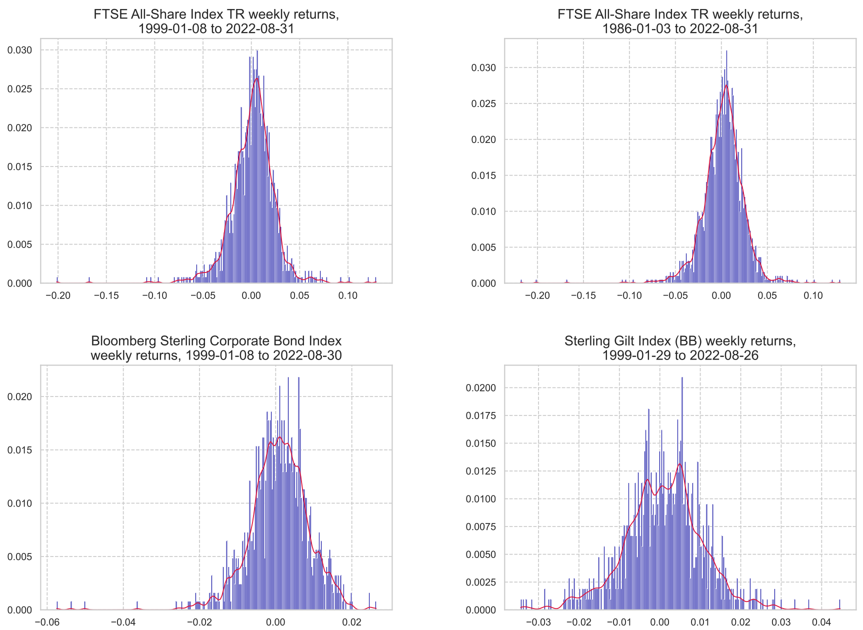

Figure 4.13: Histogram of returns with a kernel density estimate, German assets. Note that the scaling on both the x and y-axis differs.

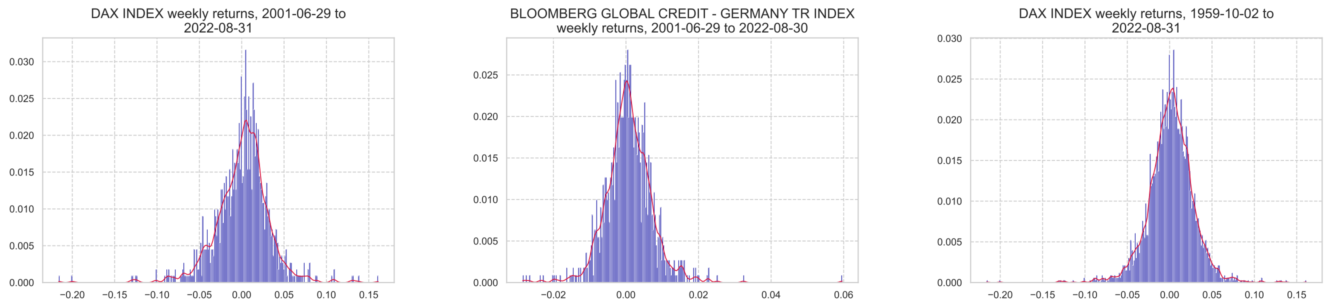

Figure 4.14: Histogram of returns with a kernel density estimate, UK assets. Note that the scaling on both the x and y-axis differs.

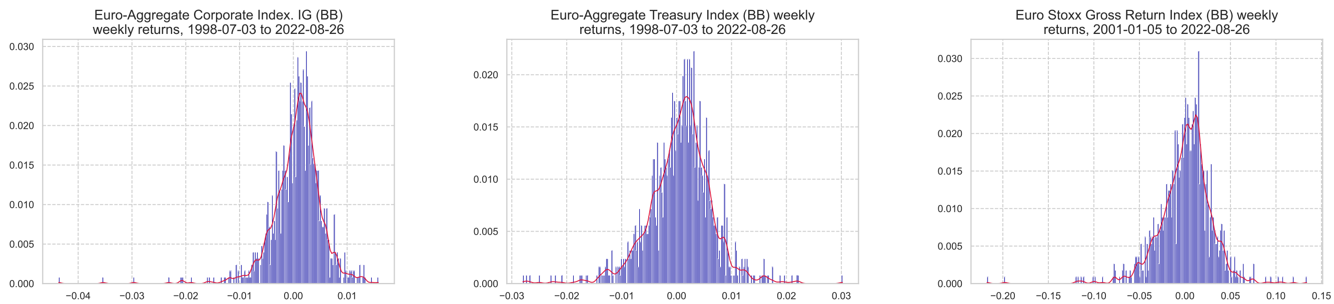

Figure 4.15: Histogram of returns with a kernel density estimate, European assets. Note that the scaling on both the x and y-axis differs.

US equity returns have greater dispersion than US investment grade (IG) returns. More specifically, the spread around the mean for the S&P500 is wider, and the most extreme measures are more excessive. When comparing UK equities and UK IG returns, as well as German equities and German IG returns, the same differences are observed. These observations align with the VaR and ES results in section 4.4.

4.4 Value at Risk and Expected Shortfall

The VaR estimates for the various approaches are the same or very similar in table 4.1. The predicted ES values deviate more between methods, particularly the KDE estimates (2) which are substantially larger than the estimates provided by both the vanilla estimates (1) and the EVT estimates (3).

Table 4.1: 99th percentile Value at Risk and Expected Shortfall estimates.
(1) Derived from the historical return distribution.
(2) Derived from the kernel density estimates seen plotted in figure 4.12, 4.15 and 4.14.
(3) Derived using EVT.

The 99th Percentile	Value at Risk (%)	Expected Shortfall (%)
1. Quantile of the return distribution		
US Assets		
US Corporate High Yield Index	-2.7	-5.7
US Corporate Bond Index (IG)	-1.8	-3.1
US Government Bond Index	-2.1	-2.4
S&P 500 Total Return Index	-6.7	-9.4
UK Assets		
Bloomberg Sterling Corporate Bond Index	-1.6	-2.1
UK Government Bond Index	-2.4	-3.4
FTSE All-Share Index TR	-5.9	-9.8
German Assets		
Bloomberg Global Credit - GER TR index	-1.6	-2.1
DAX Index	-7.3	-10.5
European Assets		
Euro-Aggregate Treasury Index	-1.6	-2.2
Euro-Aggregate Corpporate Index, IG	-1.3	-2.2
Euro Stoxx Gross Return Index	-8.1	-12.5
2. Kernel Density Estimation Estimate		
US Assets		
US Corporate High Yield Index	-2.9	-7.3
US Corporate Bond Index (IG)	-1.8	-5.7
US Government Bond Index	-1.5	-2.3
S&P 500 Total Return Index	-6.8	-13.7
UK Assets		
Bloomberg Sterling Corporate Bond Index	-2.0	-4.1
UK Government Bond Index	-2.5	-4.4
FTSE All-Share Index TR	-6.1	-15.6
German Assets		
Bloomberg Global Credit - GER TR index	-1.6	-2.4
DAX Index	-7.4	-16.2
European Assets		
Euro-Aggregate Treasury Index	-1.6	-2.3
Euro-Aggregate Corpporate Index, IG	-1.4	-3.0
Euro Stoxx Gross Return Index	-8.2	-16.7
3. Extreme Value Theory Estimate		
US Assets		
US Corporate High Yield Index	-2.9	-6.7
US Corporate Bond Index (IG)	-1.8	-2.4
US Government Bond Index	-2.4	-3.5
S&P 500 Total Return Index	-6.8	-8.0
UK Assets		
Bloomberg Sterling Corporate Bond Index	-1.7	-3.2
UK Government Bond Index	-2.4	-3.5
FTSE All-Share Index TR	-6.0	-8.0
German Assets		
Bloomberg Global Credit - GER TR index	-1.7	-3.2
DAX Index	-7.3	-9.9
European Assets		
Euro-Aggregate Treasury Index	-1.6	-2.9
Euro-Aggregate Corpporate Index, IG	-1.3	-2.4
Euro Stoxx Gross Return Index	-8.2	-11.7

Moving further out on the tail of the distribution, it seems that the estimates differ less between the methods used. Instead, they all generate quite similar results for both the VaR and ES estimates. Notable is the extremely low estimate across methods for German credits and EU treasuries.

Notice the expected shortfall estimates for US investment grade bonds and US high yield bonds in particular. The statistically most reliable EVT estimate indicates that the average fallout in extreme cases, specifically the 99.9th percentile cases, is almost the same. If one were to observe the ES estimates for the 99th percentile case, it would be reasonable to assume that US high yield does have heavier tails than US investment grade. However, when we travel farther out on the tail, this ceases to be the case; instead, US investment grade and US high yield appear to have equally fat tails.

Table 4.2: 99.9th percentile Value at Risk and Expected Shortfall estimates.
(1) Derived from the historical return distribution.
(2) Derived from the kernel density estimates seen plotted in figure 4.12, 4.15 and 4.14.
(3) Derived using EVT.

The 99.9th Percentile	Value at Risk (%)	Expected Shortfall (%)
1. Quantile of the return distribution		
US Assets		
US Corporate High Yield Index	-10.1	-11.0
US Corporate Bond Index (IG)	-5.5	-8.4
US Government Bond Index	-2.4	-3.4
S&P 500 Total Return Index	-13.0	-18.1
UK Assets		
Bloomberg Sterling Corporate Bond Index	-5.4	-5.7
UK Government Bond Index	-3.7	-4.8
FTSE All-Share Index TR	-18.7	-23.6
German Assets		
Bloomberg Global Credit - GER TR index	-2.8	-2.8
DAX Index	-13.9	-18.7
European Assets		
Euro-Aggregate Treasury Index	-2.8	-2.8
Euro-Aggregate Corpporate Index, IG	-3.4	-4.0
Euro Stoxx Gross Return Index	-20.8	-23.3
2. Kernel Density Estimation Estimate		
US Assets		
US Corporate High Yield Index	-10.8	-11.2
US Corporate Bond Index (IG)	-7.5	-8.5
US Government Bond Index	-2.1	-2.6
S&P 500 Total Return Index	-16.0	-18.3
UK Assets		
Bloomberg Sterling Corporate Bond Index	-5.6	-5.9
UK Government Bond Index	-3.9	-5.1
FTSE All-Share Index TR	-22.3	-23.8
German Assets		
Bloomberg Global Credit - GER TR index	-2.9	-3.0
DAX Index	-14.1	-19.5
European Assets		
Euro-Aggregate Treasury Index	-2.8	-2.9
Euro-Aggregate Corpporate Index, IG	-3.6	-4.1
Euro Stoxx Gross Return Index	-22.4	-23.9
3. Extreme Value Theory Estimate		
US Assets		
US Corporate High Yield Index	-10.7	-12.0
US Corporate Bond Index (IG)	-6.2	-11.7
US Government Bond Index	-2.5	-5.6
S&P 500 Total Return Index	-13.8	-21.8
UK Assets		
Bloomberg Sterling Corporate Bond Index	-5.7	-6.2
UK Government Bond Index	-3.7	-3.7
FTSE All-Share Index TR	-19.1	-26.8
German Assets		
Bloomberg Global Credit - GER TR index	-2.9	-3.0
DAX Index	-13.9	-14.2
European Assets		
Euro-Aggregate Treasury Index	-2.8	-2.9
Euro-Aggregate Corpporate Index, IG	-4.1	-5.5
Euro Stoxx Gross Return Index	-24.3	-30.9

5

Discussion

Stocks and government bonds are said to have historically had a negative correlation with each other, suggesting that when equity returns were historically lower than average, returns on government bonds were typically higher than normal, and vice versa. Our findings resonate with this. Correlation, however, has two limitations: it only provides a general idea of the relationship, whereas investors are typically more concerned with how bonds respond to falling stock prices; and it provides no indication of the extent and magnitude of the relationship. But, a rolling correlation shows the trend of the correlation over time, allowing for a relative comparison. The relative comparison in this thesis indicates that corporate credits are less protective against equity downside as compared to government bonds. However, it is challenging to draw any decisive conclusions since the relationship is not constant but rather unpredictable, and the magnitude of the link varies between marketplaces. In the UK and US, IG bonds do not appear to be a reliable substitute for government bonds, but in Germany, there does indeed seem to be some truth to the statements, and the potential effectiveness of German IG bonds is hinted at from all three perspectives taken in this thesis.

The most conveying example of the contrary, that is, the effectiveness of government bonds over IG corporate bonds, is the comparison between the correlations of US stocks to US IG corporate bonds and US stocks to the US government bonds index, upper and lower right of figure 4.7. The two graphs are almost identical, differing mostly in the magnitude of the downfalls during 2008, 2019 and 2022, all of which were times of major equity market drawdowns. Another compelling illustration of this is the comparison between the correlation of EU IG corporate bonds and EU treasuries, respectively, to EU stocks. Compared to EU treasuries, EU IG corporate bonds seem to be a less effective hedge. So, for the assertions that IG bonds could act as a replacement for government bonds, these examples refute that. However, the results are conflicting, and thus, conclusions from particular markets may serve as the foundation for generalizations about IG bonds' serving as an adequate replacement for government bonds. Therefore, while it might be true to some extent in Germany and perhaps in other unexplored markets, it is not necessarily true elsewhere, highlighting the need for caution when making generalizations or extrapolating findings. Another notable finding is that there are no apparent advantages to using either EU treasuries or US government bonds to hedge against EU stocks. Something that goes against the intuition that geographic diversification should make US assets a better hedge than EU assets. However, in contrast to US government bonds, EU treasuries consist of government bonds from multiple countries, an argument for EU treasuries being the better hedge.

The general protective qualities of fixed income as compared to equities are apparent from the VaR and ES results. The tail risk of equities is, across all regions and for all models, larger. This means that fixed income should, in general, act as a shock-absorber for these portfolios. In this sense, some of the fixed income assets provide what seems to be very enticing downside protection in the most extreme cases, where the 99.9th percentile drawdowns are just slightly larger than the 99th percentile ones. This is especially true for EU treasuries and German credits. But EU corporate bonds and US treasuries could also be on this list, depending on where the line is drawn between what is slightly bigger and what is not.

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