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Fixed Income Securities as a Hedge against Equity Market Downside

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Abstract

Investors are constantly searching for ways to protect their wealth, ideally without loosing out on long-term returns. To achieve this, some investors chose to diversify their equity portfolios with fixed income assets, believing they can protect their portfolios from drawdowns. However, the current literature is divided on the effectiveness of fixed income assets as a shock-absorber during severe equity drawdowns. This paper analyzes sub-classes of fixed income assets to understand if they mitigate downside risk as part of a multi-asset equity portfolio. The results of this paper shows that investors cannot rely on simple heuristics to decide weather to include fixed income or not for this purpose. While some findings support the effectiveness of fixed income, the results are divided. Overall, this paper supports the claim that fixed income can act as a shock-absorber for equity portfolios, in particular government bonds and some investment grade corporate bonds.

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Keywords: Fixed Income, Financial Markets, Hedge, Portfolio, Downside Risk, Value-at-Risk, Expected Shortfall, Extreme Value Theory, Kernel Density Estimation.

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Introduction

The literature has shown that investors generally avoid positions where they would be vulnerable to exceptional losses, despite the low likelihood of these scenarios. Such a "disaster avoidance attitude" suggests that investors are worried about the worst-case situations and are wary of the possibility of rapid price drops (Menezes et al., 1980). Such downside risk is important to take into consideration when construction portfolios due to the fact that price drops can have a significant real world impact and extreme losses occur far more frequently than would be predicted by commonly assumed return distributions (Huang et al., 2012). In the words of a senior European bank executive: "Normal distribution is man made. Life is negatively skewed." (Hünseler, 2013).

Despite this risk averse profile of investors, most choose to build their portfolios around equities. Understandably so since long-term returns have been alluring, leading to substantial capital growth. It seems that equities high levels of exposure to macroeconomic growth risk and subsequently repeated large losses are not enough to dissuade investors. To balance the prospects of great long-term return with these downside risks investors are encouraged to look for asset classes and investing techniques that provide enticing conditional correlations. Safe haven assets like gold, government bonds and derivative-based hedging strategies are a few potential choices. Finding solutions to mitigate the tail risk that equities bring while continuing to take on as much risk as tolerable is essential in order to increase the likelihood of great long-term performance (Githaiga et al., 2021).

A common approach to mitigate the tail risk of equities is to include fixed income assets such as government bonds or corporate bonds in the portfolio. The main thesis is that these assets are negatively correlated with equities and will therefore perform well during times of turmoil in the equities market. If so, this would result in less severe drawdowns for the portfolio as a whole since the fixed income assets will increase in value when equities are experiencing losses. Especially, when equities are experiencing severe drawdowns, fixed income hopefully maintains this negative correlation to equities, acting like a shock-absorber.

In recent years however, investor sentiment has shifted and some recommend leaving the conventional "safe haven" of fixed income instruments. Mainly due to the historically low bond yields and the lack of downside protection obtained. In March of 2020 the trading in some Swedish fixed income funds closed down for a couple of days, these same funds where advertised as safer investment options. More specifically, two statements are frequently made: that investment-grade corporate bonds can serve as a viable alternative to government bonds and offer equivalent downside risk protection; and that government bonds have lost their diversification benefits as equities shock absorbers (Renzi-Ricci and Baynes, 2021).

This thesis assess whether fixed income continues to act as a shock absorber, limiting downside risk, in multi-asset equity portfolios. This is accomplished by examining changes in the correlation between fixed income sub-asset classes and equity indices over time, as well as changes in their accumulated returns over the same time frames. The distributions of the historical returns in combination with downside risk measurements are also analyzed. In addition to examining the returns derived from the data as a whole, extreme value theory will also be used to evaluate a subset of the returns separately. Finally, the thesis evaluates the assertions that investment grade corporate bonds can be a reliable alternative to government bonds in particular. Taken into consideration are the US, UK and German markets.

2

Technical Background

This section covers the theoretical and technical concepts pervading this thesis. First, concepts within the realm of economics such as fixed-income securities and portfolio theory are covered. Thereafter, the most important statistical theory underlying the analysis will be outlined.

2.1 Fixed Income

Fixed-Income securities are investment products with fixed payments. This typically includes a principal payment on the bond's maturity date, as well as periodic installments known as coupons. The coupons are specified by a yearly coupon rate, which is a percentage of the principal amount, and the regularity (normally quarterly, semi-annually, or annually) with which they are paid until maturity. A zero-coupon bond is one that does not pay coupons. The bond certificate specifies the terms and circumstances of future payments.

Bonds are first sold to the primary market, meaning that the issuer sells to a counter party and in return receives funds that can be used to finance spending. After the first issuance, the bonds can be traded on a secondary market where the bonds already issued are bought and sold.

Fixed-income assets have credit ratings assigned by credit rating companies based on the issuer's financial health. These agencies assess the creditworthiness of corporate and government bonds, as well as the ability of the issuers to repay these loans. One could separate bonds into two buckets: investment grade and non-investment grade. Investment grade bonds will have lower yields due to the fact that they are issued by reliable counterparties with a minimal chance of default. Non-investment grade bonds, also known as junk bonds or high-yield bonds, have lower credit ratings due to the higher likelihood of default.

A subtype of fixed income is government bonds; this is fixed income issued by governments to finance the governments spending. Since these assets are backed by the government they are typically seen as lower risk investments, given that the governments default risk is low. Bonds sold by the United States Treasury are considered among the safest in the world together with government bonds issued by other developed nations. The yields are often low on government bonds due to their relative lack of risk. However, this is not always true, as some government-backed bonds, particularly those issued in emerging countries, can have substantially higher yields.

Corporate bonds are another subtype of fixed income that are instead backed by corporations. An investor who purchases a corporate bond on the primary market effectively lends money to the company in exchange for a series of interest payments. Corporate bonds are frequently viewed as riskier than government bonds issued by stable governments which is reflected in that the yield usually is higher on corporate bonds.

2.2 Portfolio Theory

Diversification is the concept of decreasing risk by investing in a diverse set of financial instruments, industries, and asset classes. It reduces idiosyncratic risk but does not affect systematic risk, and the aim is to achieve higher risk adjusted returns. However, if there is contagion, diversification benefits might fade as correlations between asset returns tend to increase. In extreme market situations, for instance, correlations between assets tends to converge to one. A negative consequence for portfolio managers. Thus, it is essential to comprehend one's risk exposure, regardless of diversification (Nilsson, 2022).

Finding the best portfolio that balances risk and returns while outperforming the market portfolio is one of the primary objectives for portfolio managers, and diversification plays an essential part in being successful at this. A measure portfolio managers sometimes opt to maximize is the Sharpe ratio, a risk-adjusted return metric developed by the Nobel laureate Sharpe (1964). Over the years, several approaches to building a portfolio by giving the proper weights to the right assets have been established and advocated in an effort to address the portfolio management problem and identify the ideal portfolio. Modern portfolio theory (MPT), first proposed by Markowitz (1952), is an common approach that provides an uncomplicated technique for addressing these concerns. When investors are confronted with a vast universe of risky securities and given estimates of their anticipated returns, standard deviations, and pair-wise correlations, they can compute the set of efficient portfolios.

Markowitz method is centered on maximizing the risk-return trade-off in a diversified portfolio, which results in a portfolio that is less volatile than the sum of its parts. The methodology has nevertheless drawn criticism because it is based on assumptions that are not supported by empirical data. Investors choose more heuristic weighting strategies because they are simpler to apply, such as the naively diversified portfolio that gives each asset an identical weight or the minimum variance portfolio, which attempts to reduce the portfolio's variation.

To even construct an optimal portfolio, the assumption must be made that it is possible to do so. The efficient market hypothesis (EMH), created by Fama (1970), is a theory that asserts that asset prices reflect all available information. As a result, an investor cannot buy or sell assets below their fair market value, making it impossible to outperform the market using various stock picking strategies. Proponents of the

EMH often deploy a investment strategy tilted toward passive ownership of market capitalization-weighted portfolios that follow stock indices like the Swedish stock market's OMXS30 or the American S&P 500. Some of the world's largest asset managers such as BlackRock, Vanguard, and State Street are labeled passive index fund managers (Fichtner et al., 2017).

2.3 Non-Parametric Models

Contrary to parametric statistics, non-parametric models are those in which no parametric assumptions are made regarding the distribution of the underlying data. Kernel density estimation and histograms are normally regarded as non-parametric techniques.

2.3.1 Histograms

To approximate the distribution of numerical data one can construct a histogram representation of the data. Histograms provide a basic idea of the density of the underlying data and it is a common tool used in density estimation. To find the probability density histogram one normalizes the density histogram so that its total area sums to one.

2.3.2 Kernel density estimation

Kernel Density Estimation (KDE) is a way of estimating an unknown probability density function (PDF) given some data. Opting to use a kernel density estimate instead of a histogram is usually motivated by the belief that it produces a more accurate representation of the unknown probability density function.

For every data point x_i, we center a kernel function K over it. The kernel density estimate is

$$\hat{f}(x) = \frac{1}{N} \sum_{i=1}^{N} K\left(x - x_i\right).$$

The Kernel function K usually fulfills that it is:

1. everywhere non-negative: $K(x) \geq 0, \ \forall x$.
2. symmetric: $K(x) = K(-x), \ \forall x$.
3. decreasing: $K'(x) < 0, \ \forall x > 0$.

A common choice of Kernel according to Fan and Yao (2003) is the Gaussian Kernel

$$K(u) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{u^2}{2}\right), \quad u \in R.$$

The kernel function K does however not normally have a great influence over the outcome of the estimation, that is, the KDE will be very similar for different kernels, and increasingly so as the number of data points increase (Odland, 2018).

To control the bandwidth, h, of the kernel we introduce

$$\hat{f}(x) = \frac{1}{Nh}\sum_{i=1}^{N} K\left(\frac{x - x_i}{h}\right).$$

The kernel bandwidth does usually have a significant impact on the outcome of the estimation (Odland, 2018), despite this there is no straightforward way to decide on what bandwidth to choose. The predicted PDF will closely match the histogram if one chooses a too small bandwidth. For an excessively large bandwidth, the estimated PDF will instead be too smoothed out, potentially losing properties of the data. Density peaks might be underestimated and tail probabilities overestimated.

2.4 Descriptive Statistics and Measures

2.4.1 Correlation

An estimation of the linear relationship between two variables, in this case, stocks and bonds, can be provided by correlation. With a series of n readings from (X_i, Y_i) indexed $i = 1, \ldots, n$, the population Pearson correlation $\rho_{X,Y}$ between X and Y can be estimated by using the sample correlation coefficient. The sample correlation coefficient is defined as

$$r_{xy} \stackrel{\text{def}}{=} \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2 \sum_{i=1}^{n}(y_i - \bar{y})^2}}$$

where $\bar{x}$ and $\bar{y}$ are the sample means of X and Y (Blom et al., 2005).

A negative correlation between two indexed price series (e.g. the equity indices) indicates that there is a protective relationship between the two, that is, they act as hedges to each other. The inverse is true for the correlation between an indexed price series and an yield series, a positive correlation indicates that the two series will act as hedges to each other. A lower yield implies a higher asset price. Therefore, a yield decrease during a indexed price decrease results in a value loss for the indexed price asset but a value increase for the yield asset.

For the purpose of this thesis, it is important to remember that just because there is correlation, it does not necessarily mean that there is some intrinsic connection between the variables, in this case intrinsic protection between the government bonds and equity indices; instead, there could be a confounding variable that affects both.

2.4.2 Sample standard deviation

The standard deviation is a statistic that expresses how much variance or dispersion there is in a group of numbers. While a high standard deviation suggests that the values are dispersed over a wider range, a low standard deviation suggests that the values tend to be close to the expected value of the collection.

The sample standard deviation denoted by s is defined as

$$s = \left(\frac{1}{N-1} \sum_{i=1}^{N} (x_i - \bar{x})^2 \right)^{1/2}$$

2.4.3 Value at risk and expected shortfall

Value-at-Risk (VaR) and Expected Shortfall (ES) are risk management tools that calculates the amount of financial risk that exists connected to an asset or a portfolio over a given time frame. Organizations with substantial assets and/or liabilities frequently employ these tools to understand and control their overall exposure to risk. More precisely, for a given time horizon t and confidence level α, the VaR is the loss in market value over the time horizon t that is exceeded with probability $1-\alpha$.

Let L be a loss distribution. The VaR at level $\alpha \in (0,1)$ is the smallest number l such that the probability that l does not exceed L is at least $1-\alpha$. That is, $\mathrm{VaR}_\alpha(L)$ is the $(1-\alpha)$ quantile of L.

$$VaR_\alpha(L) = \min\{l : \Pr(L > l) \leq 1-\alpha\}$$

However, VaR as a risk measurement has significant flaws. For example, the measurement simply indicates the likelihood that a portfolio may experience a loss greater than VaR. It does not impose an upper bound on the loss itself, making it very dependent on the behavior of the tail once VaR has been calculated. ES is an enhanced measure that deals with some of these problems. The predicted loss determined by ES is expected loss when exceeding the VaR quantile. As a result, it provides a more informative estimate of the loss one should expect in the occasion of an extreme market event.

Hence, Expected shortfall is the expected value of all returns in the distribution that are worse than the VaR at a given level of confidence. For example, at a 95% confidence level, the expected shortfall is found by taking the average of the returns in the worst 5% of cases.

$$\mathrm{ES}_\alpha(L) = \mathrm{E}\left[L \mid L \geq \mathrm{VaR}_\alpha(L)\right] = \frac{1}{1-\alpha} \int_\alpha^1 \mathrm{VaR}_u(L) du$$

2.5 Extreme Value Theory

To Improve VaR and ES estimates and to increase the confidence level in them, Extreme Value Theory (EVT) can be used. EVT smoothens the tails of a probability distribution. By doing this, the computational uncertainties normally associated with deriving VaR and ES are avoided. As a result, estimates of VaR and ES are produced that take into account the entire shape of the distribution's tail rather than simply a small number of loss' locations (Hull, 2012). In essence, EVT tries to more accurately characterize tail losses with a distribution of its own, and in risk management, we are ultimately only interested in the tail of the distribution. Therefore, EVT might produce better estimates of the tail losses than traditional VaR or ES methods when considering high confidence levels, such as 99% or 99.9% (Hull, 2012).

There are two EVT strategies:

1. Block Maximia: Analyzing the largest value with a chunk of observations (e.g. within a certain fixed time period)

2. Peaks Over Threshold: Analyzing values that exceed a predetermined threshold (often the dataset's percentile).

Both strategies rely on their respective Extreme Value Theorem. In finance, and for most practical purposes, Peaks Over Threshold (POT) is more appropriate and simpler to implement. Of interest are therefore the distribution of losses

$$P(L \leq y + \gamma \mid L > \gamma)$$

where L is the loss distribution and γ is some percentile, usually the 95th, of a dataset $\mathcal{D}\left\{r_i\right\}_{i=1}^n$ of returns. With the help of Bayes' theorem one obtains

$$P(L \leq y + \gamma \mid L > \gamma) = \frac{P(\{L \leq y + \gamma\} \cap \{L > \gamma\})}{P(L > \gamma)}.$$

A central finding in EVT for the POT approach says that (Gomes and Guillou, 2015)

$$P(L \leq y + \gamma \mid L > \gamma) \approx G_{\xi,\beta}(y)$$

where $G_{\xi,\beta}(y)$ is a Generalized Pareto Distribution (GPD). The GPD is expressed as (Gomes and Guillou, 2015)

$$G_{\xi,\beta}(y) = \begin{cases} 1 - \left[1 + \xi \frac{y}{\beta}\right]^{-\frac{1}{\xi}}, & \xi \neq 0. \\ 1 - \exp\left\{-\frac{y}{\beta}\right\}, & \xi = 0. \end{cases}$$

and their respective probability density function are (Gomes and Guillou, 2015)

$$g_{\xi,\beta}(y) = \begin{cases} \frac{1}{\beta}\left(1 + \xi \frac{y}{\beta}\right)^{-\frac{1}{\xi}-1}, & \xi \neq 0. \\ \frac{1}{\beta}\exp\left\{-\frac{y}{\beta}\right\}, & \xi = 0. \end{cases}$$

The distribution of losses exceeding γ, $F_{L_\gamma}(x)$, can now be derived (Nilsson, 2022) as

$$F_{L_\gamma}(l) = G_{\xi,\beta}(l - \gamma)\left(1 - \hat{F}_L(\gamma)\right) + \hat{F}_L(\gamma) \tag{2.1}$$

where $\frac{n_\gamma}{N} = 1 - \hat{F}_L(\gamma)$, in which $\frac{n_\gamma}{N}$ is the fraction of losses above our threshold γ, and $\hat{F}_L(\gamma)$ is the empirical cumulative density function of the observations smaller than γ.

3

Methodology

This section aims to explain and motivate details regarding data collection, implementation, and analysis methods. The theory was implemented by code using 16 assets. The assets consist of stock indices, government bonds, and credit indices. The data was handled in Microsoft Excel before being loaded into Python for analysis. Access to high quality time series data of transaction prices spanning decades back prompted the decision to use this approach. Markets of the three largest economies in the Occident are assessed.

3.1 Software

The data was extracted and concatenated using Microsoft Excel. The analyses were performed in Python3 on the Jupyter Note-Book platform within the code editor Visual Studio Code. A number of packages was imported, the packages used most extensively was Pandas, NumPy, PyPlot and Matplotlib.

3.2 Data Collection

Historical time series data for weekly closing prices on three equity indices, four corporate bond indices, and nine generalized government bond yield contracts were collected from the Bloomberg Terminal. The datasets vary in length, with the longest starting in 1959. All datasets end 2022-08-31. Information about the data can be found in table 3.1.

The choice to use the data specified in table 3.1 was influenced by; if large amounts of historical data could be accessed; if the quality of that data was satisfactory; and finally, if the format of the data was such that analysis could be performed after a reasonable amount of data manipulation.

Table 3.1: Data obtained from the Bloomberg terminal.

Series name	Ticker	Frequency	Unit	Start date
US Corporate High Yield	LF98TRUU	Weekly	Indexed	1998-07-31
US Corporate Bond Index (IG)	LUACTRUU	Weekly	Indexed	1988-12-30
US Generic Govt 3y Index	USGG3YR	Weekly	Yield	1962-01-05
US Generic Govt 5y Index	USGG5YR	Weekly	Yield	1962-01-05
US Generic Govt 10y Index	USGG10YR	Weekly	Yield	1962-01-05
S&P 500 Total Return Index	SPXT	Weekly	Indexed	1988-01-08
Sterling Corporate Bonds	LC61TRGU	Weekly	Indexed	1992-01-03
3Y UK Govt Bond	GTGBP3YR	Weekly	Yield	1992-01-03
5Y UK Govt Bond	GTGBP5YR	Weekly	Yield	1992-01-03
10Y UK Govt Bond	GTGBP10YR	Weekly	Yield	1992-01-03
FTSE All-Share Index TR	ASXTR	Weekly	Indexed	1986-01-03
Bloomberg Global Credit - Ger TR	I04140EU	Weekly	Indexed	2001-06-29
GER Govt Bond 3Y	GTDEM3YR	Weekly	Yield	1990-08-10
GER Govt Bond 5Y	GTDEM5YR	Weekly	Yield	1990-08-10
GER Govt Bond 10Y	GTDEM10YR	Weekly	Yield	1990-08-10
DAX Index	DAX INDEX	Weekly	Indexed	1959-10-02

3.3 Time-Following Measurements

Correlation and standard deviation are calculated using three-year lags, returns are calculated with a one-year lag. The time-lags decided on are of arbitrary nature. Consequently, to avoid choosing an arbitrary time-lag, it would be preferable to add a "time-lag" axis with a continuous spectrum. However, the scope of this analysis is limited to one time-lag for each rolling series.

3.3.1 Rolling correlation

Fix a variable l, take the l first readings, $i = 1, ..., l$, from a series of $n > l$ readings of (X_i, Y_i). With these readings calculate the correlation r_{xy_1} between X and Y. Increment the starting position one step forward so that $i = 2, ..., l + 1$ and repeat the calculations to obtain r_{xy_2}. Continue this process until $l + k = n$ to calculate the *rolling correlation* estimate series $r_{xy_1}, ..., r_{xy_{k+1}}$ between X and Y.

3.3.2 Rolling return

Fix a variable l, from a series of prices S_i where $i = 1, ..., n$ take the first and l:*th* reading of S_i. Calculate the return $R_1 = \frac{S_l - S_1}{S_1}$. Increment one step forward and calculate R_2 in the same manner using the second and $l + 1$:*th* readings. Proceed to increment forward and calculate the return until $l + k = n$ to obtain the *rolling return* series $R_1, ..., R_{k+1}$ of S_i with interval l.

3.3.3 Rolling standard deviation

Fix a variable l, from a series of prices S_i where $i = 1, ..., n$ take the $l < n$ first readings, $i = 1, ..., l$. Calculate the sample standard deviation s_1 from these readings.

Increment the starting position one step forward so that $i = 2, ..., l + 1$ and repeat the calculations to calculate s_2 in the same manner. Proceed to increment forward and calculate the sample standard deviations until $l + k = n$ to obtain the *rolling standard deviation* series $s_1, ..., s_{k+1}$ of S_i with length l.

3.4 Distributions

3.4.1 Histograms

To construct a histogram, first "bin" the range of values, that is divide it into a series of intervals, and then count the number of values that fall into each interval. The bins are defined as a series of discrete intervals that do not overlap, are next to each other and of the same width.

Some of the time period in the histograms differ. If it is assumed that the returns are the fallout of a specific stochastic variable then cutting short the time series would be of no benefit, it would only leave one with less data. If it is not assumed that the returns are the fallout of a stochastic variable, then modelling the data as such is of questionable meaning. It is therefore not a straight forward task to decide on what is better. In this thesis we have chosen to plot histograms for both the full time series, and when appropriate for comparison, a time series cut short.

3.4.2 Kernel density estimation

Define a kernel function, K. Center the kernel function on each data point. That is, over every data point in the sample, place the kernel function. Sum the kernel functions to arrive at a kernel density estimate. This estimate is normalized by the number of data points, N, since the integral of the kernel must evaluate to one for it to be a probability density function. The Gaussian kernel has been used for all kernel density estimations in this thesis. By using a heuristic approach the bandwidth h used for the KDE's was set to $h = 0.4$ and the number of bins N in the histogram to $N = 300$.

3.5 Value at risk and expected shortfall

3.5.1 Historical method

The historical VaR methodology is a conventional way to compute VaR. Using a historical time period of usually daily returns, VaR is derived from the return distribution as a percentile of the distribution multiplied with the value of the portfolio. To find the corresponding ES of the same distribution, the expected value of the tail beyond the same percentile is calculated. In this case, that is just an average of the losses above our threshold.

VaR and ES are sometimes stated as a percentage and sometimes as an asset value. We will not assume any portfolio size in this thesis and therefore keep the number as a percentage. If one wants to find the equivalent asset value loss this is simply done by multiplying the percentage VaR or ES number with the total portfolio value.

3.5.2 Kernel method

Instead of using the historical returns directly, we calculate a kernel density estimate for the probability density function that represents these historical returns. Starting from the left-most tail value of this PDF, we step to the right along the x-axis and evaluate the area under the curve to the left of the current position. This is done until we are one step prior the position where the area under the curve will evaluate to the percentile we have chosen, or larger. The x-value at which we stop before passing the threshold will be the VaR percentage. To find the ES we as before take the average of the losses larger than this VaR threshold.

3.5.3 Extreme value theory method

From equation 2.1 we can derive VaR and ES estimates. To find the VaR estimate we solve the equation $F_{L_\gamma}(\mathrm{VaR}_c) = c$, where c is the percentile chosen. By using this result we can then find an estimate for ES. As stated in (Nilsson, 2022) the estimates will evaluate to;

$$\mathrm{VaR}_c = \gamma + \frac{\beta}{\xi}\left(\left(\frac{N}{n_\gamma}(1-c)\right)^{-\xi} - 1\right)$$

$$\mathrm{ES}_c = \frac{\mathrm{VaR}_c}{1-\xi} + \frac{\beta - \gamma\xi}{1-\xi}.$$

The parameters used in the Generalized Pareto Distribution is found by maximum likelihood estimation. An example of this can be found in (Nilsson, 2022).

3.6 Limitations

No matter how sophisticated the method, it is critical to remember that the results we derive are outcomes of calculations that require inputs. The starting point in this case is a historical time series of prices. This implies that the results reached are merely forecasts of future behavior based on some earlier behavior. Precisely the earlier behavior during the time period of the data series used as an input. In essence, this means that it is expected that the dynamics of price changes in the future will largely follow those of the past. If the dynamics do not match, which they most likely won't, the results will not match reality going forward. Hopefully, the mismatch is not too large, and the results will be somewhat accurate at least for some future time period. In practice this would, for example, imply that one would like to base the model on just the last few weeks or even days worth of data. If so, the results would capture and include only the most recent market dynamics; assuming that the dynamics of an asset or market changes over time but are similar to those that has just been observed.

4

Results

In the following chapter the results of the study are presented along with some remarks. First, the three time-following measurements are presented. Second, we combine rolling correlation with rolling standard deviation to highlight a visually appealing pattern. Third, histogram plots together with their respective kernel density estimates are displayed. At last, value-at-risk and expected shortfall estimates are presented.

4.1 Time-Following Measurements

Time-following measurements for each asset in each region are displayed in this segment. For a correct interpretation of the figures, see the second paragraph of subsection 2.4.1.

4.1.1 Correlation

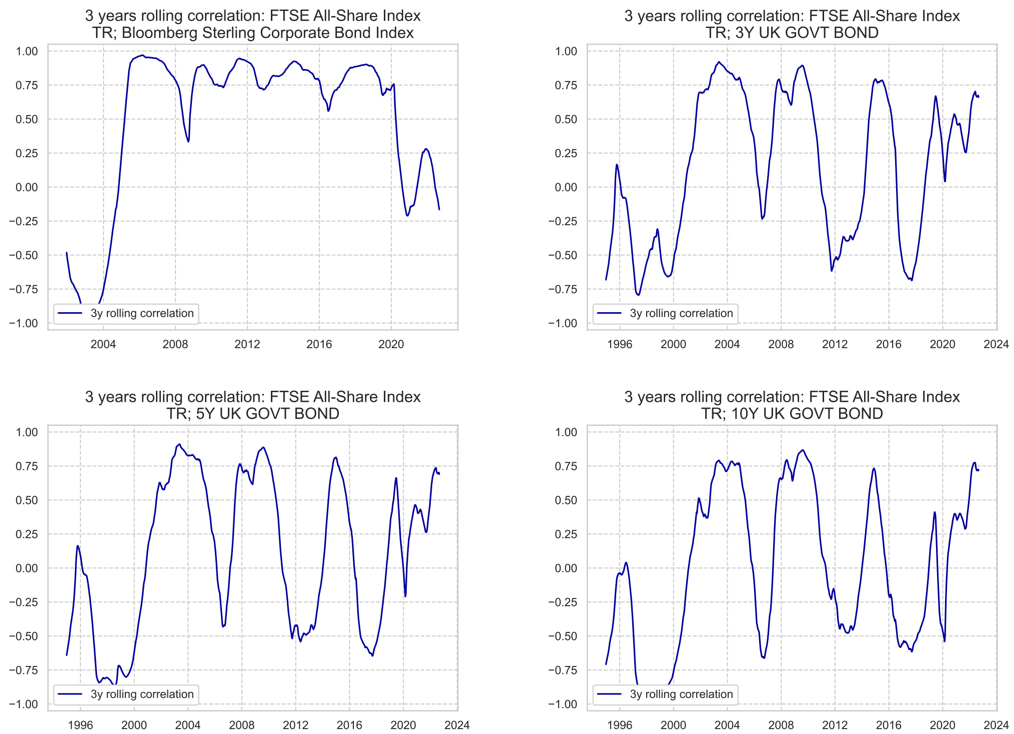

Figure 4.1: Three year rolling correlations, UK assets. Note that the x-axis time frame differs in (row, col) = (1,1).

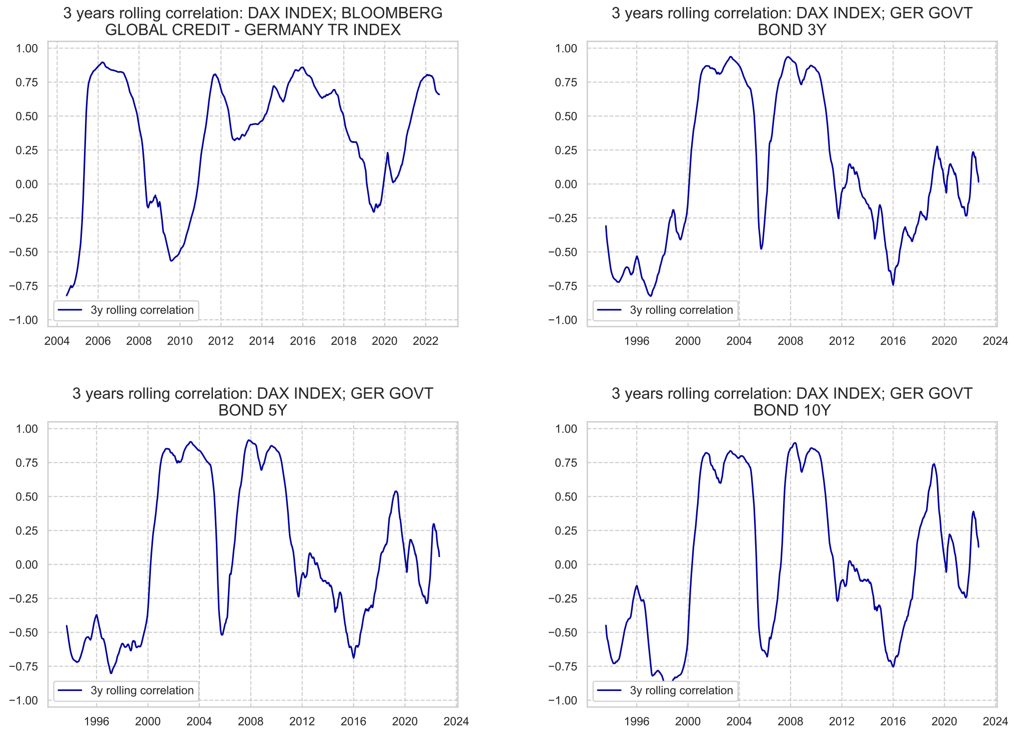

Figure 4.2: Three year rolling correlations, German assets. Note that the x-axis time frame differs in (row, col) = (1,1).

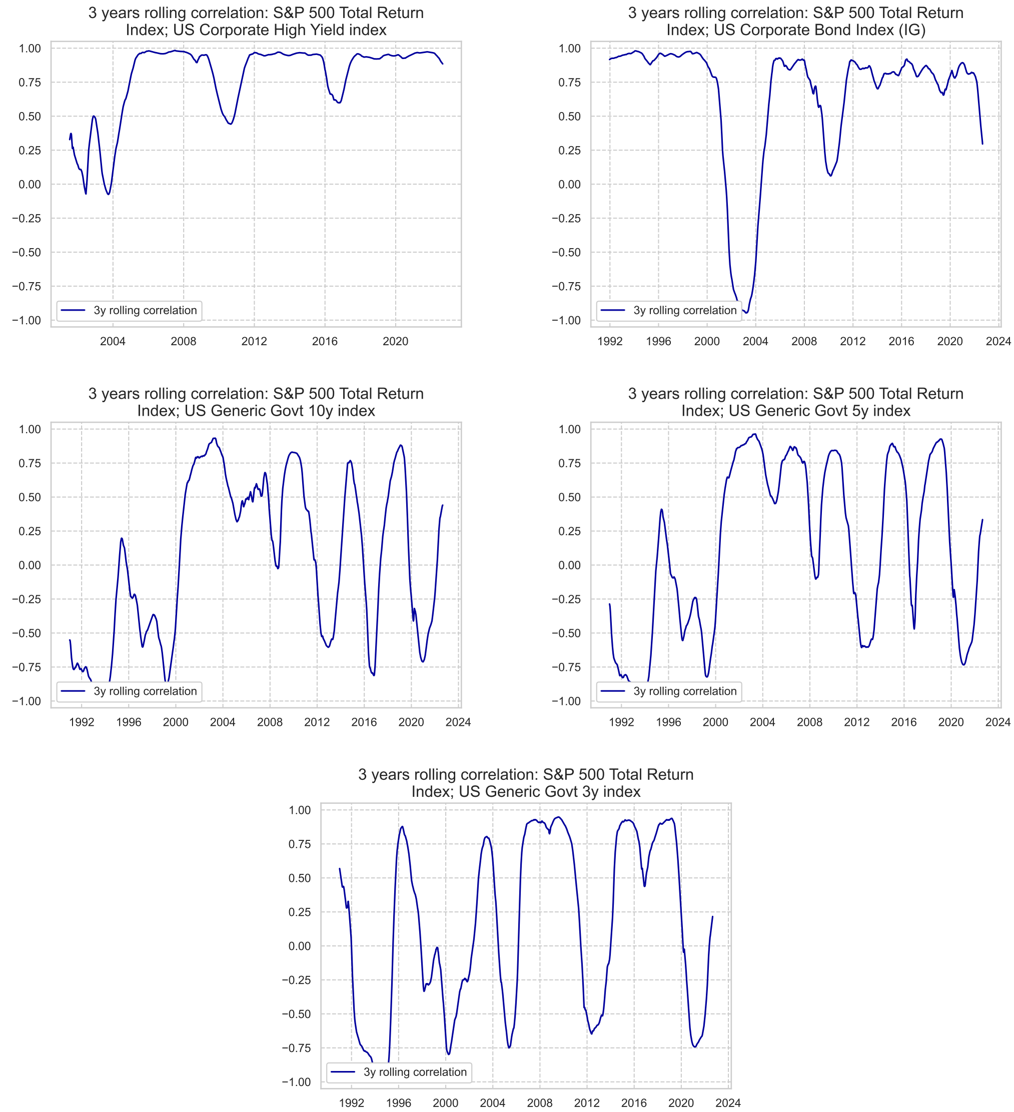

Figure 4.3: Three year rolling correlations, US assets. Note that the x-axis time frame differs in (row, col) = (1,1).

In figure 4.3, 4.1 and 4.2 all assets show variability in the 3-year rolling correlation over time, although the variance for government bonds appears to be more prominent. The correlation of the corporate credit indices appears to be positive in general, implying less protective protective characteristics against equity downside. The government bonds indices variability, particularly to the positive correlation side, indicates that there is a protective relationship.

4.1.2 Returns

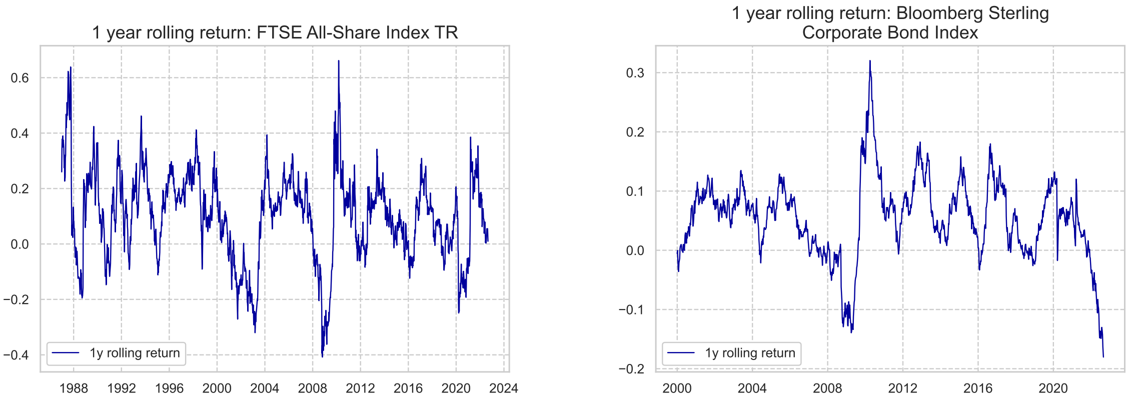

Figure 4.4: One year rolling return, UK assets. Note that the x-axis time frame differs.

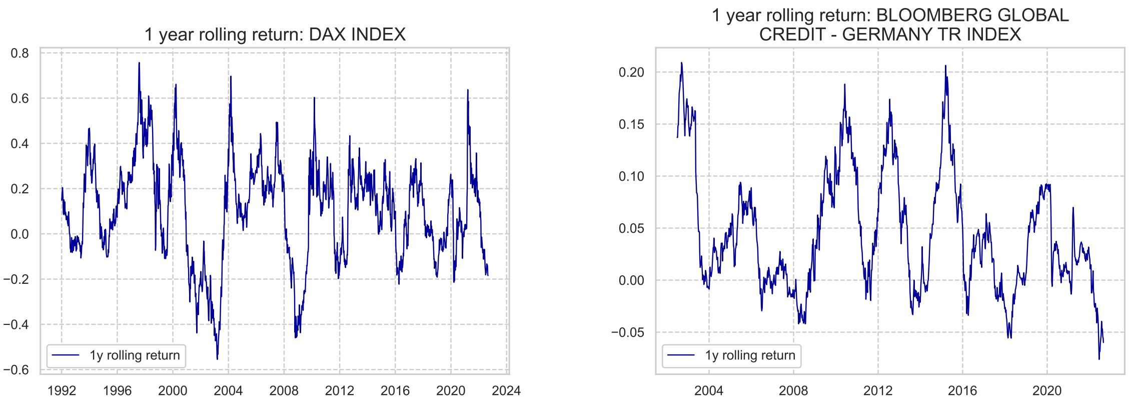

Figure 4.5: One year rolling return, German assets. Note that the x-axis time frame differs.

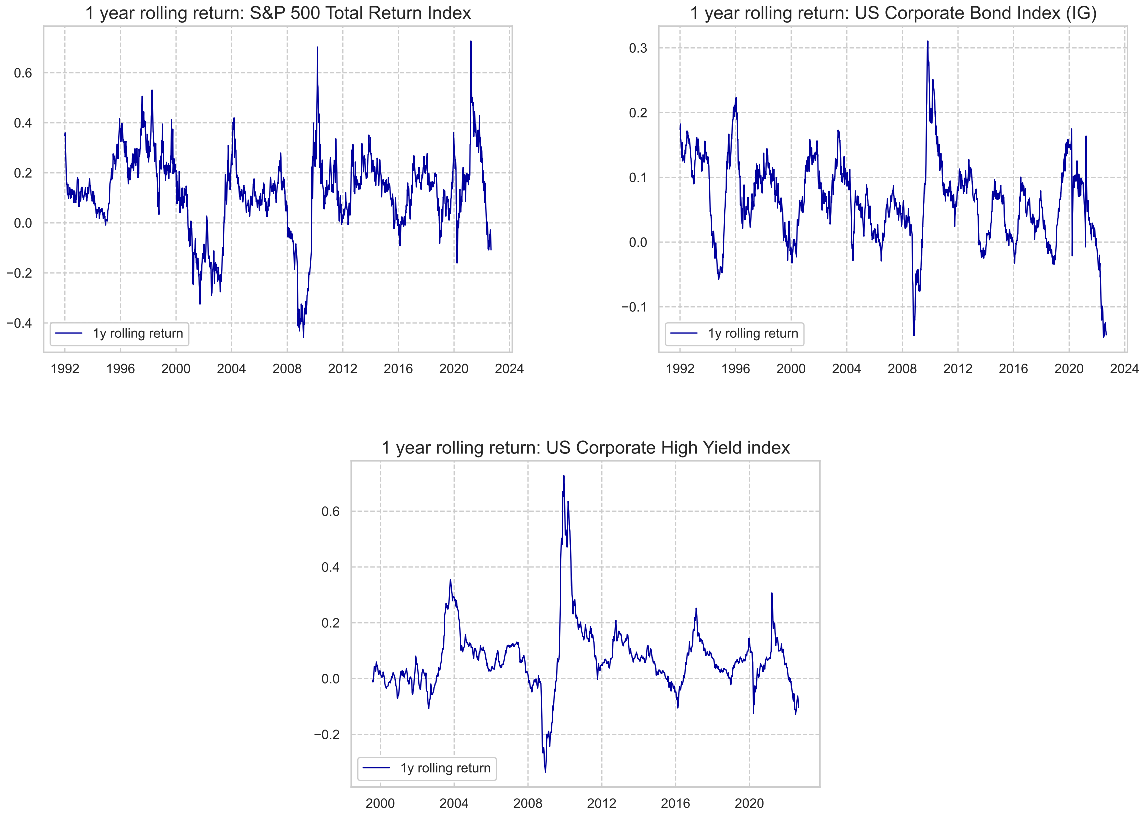

Figure 4.6: One year rolling return, US assets. Note that the x-axis time frame differs.

As seen in figure 4.6, figure 4.4 and figure 4.5 the equity indices rolling returns have higher variability than both the government bond indices and the corporate credit indices. Note that the lowest reading in 2012, 2016, 2020 and 2022 is almost the same for the US Corporate High Yield Index and S&P500 while the highest readings are generally higher for the SP500 Index during the same time period ($2012 - 2022$).

It appears that the German credit index has performed well in comparison to the US and UK credits indices. German credits has consistently had one year rolling return above -5% except for during two short periods around 2018 and 2022, where it barely dips below -5%. This is in contrast to the two other investment-grade indices that both returned less than -10% on a rolling basis after the financial crisis.

4.1.3 Standard deviation

Figure 4.7: Three year rolling standard deviation, US assets. Note that the x-axis time frame differs.

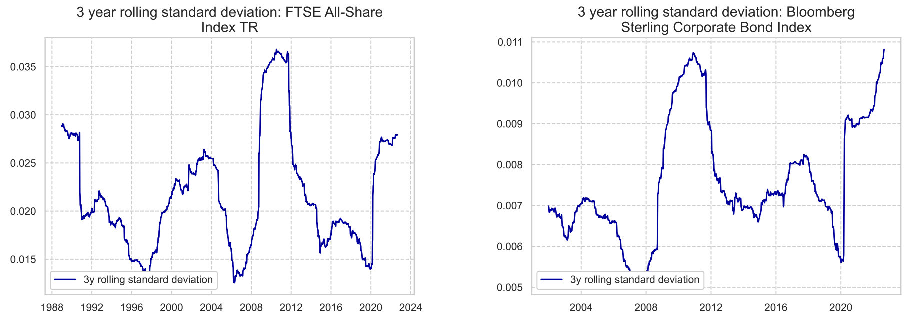

Figure 4.8: Three year rolling standard deviation, UK assets. Note that the x-axis time frame differs.

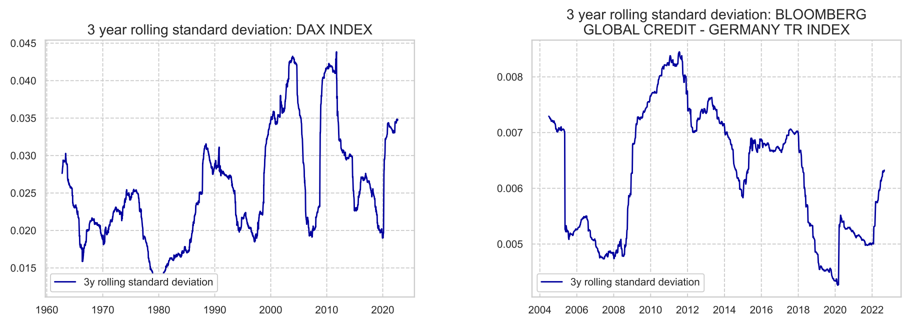

Figure 4.9: Three year rolling standard deviation, German assets.

4.2 Volatility and Correlation

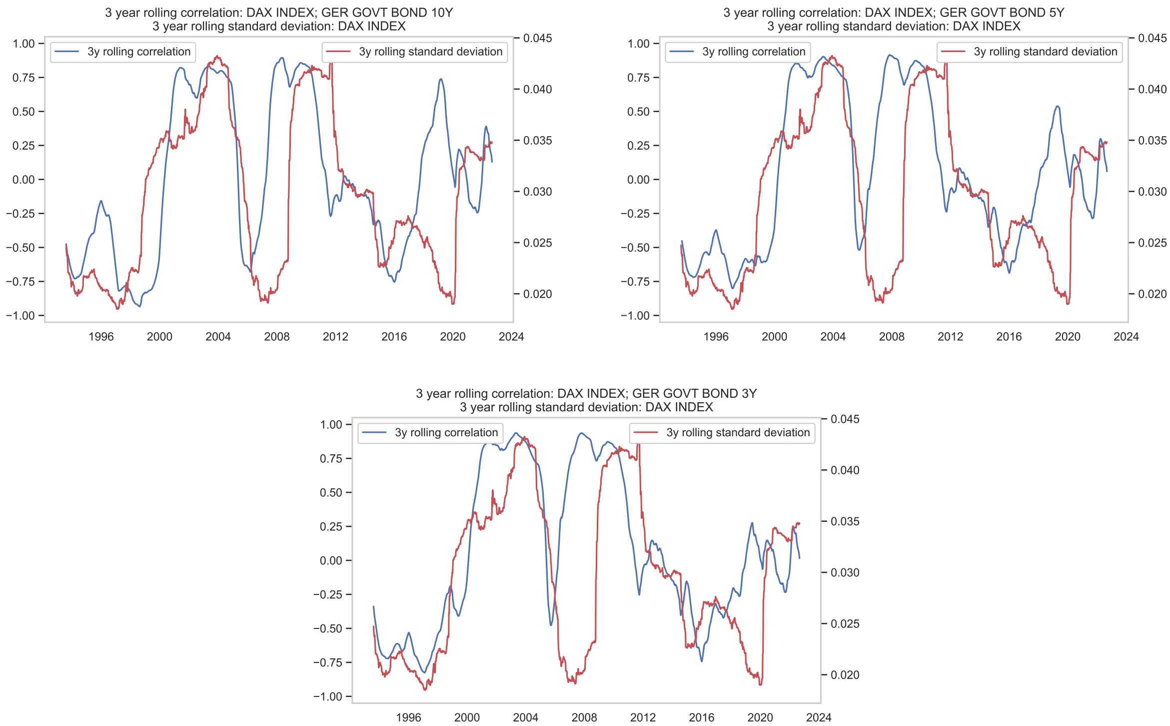

Figure 4.10: Three year rolling correlation and three year rolling return, German assets. Note the two-sided y-axis.

From figure 4.10 one could deduce that there is some overlap between the environments of correlation and volatility. Particularly, a more negative stock-bond correlation appears to be linked with low equity market volatility and vice versa.

Mortensen (2021) states that unlike correlation regimes, volatility regimes appear to be less consistent and shift more frequently. This is in contrast to the apparent covariation in figure 4.10. Meanwhile, Ilmanen (2003) found that the correlation between stocks and bonds tends to be low near the peak of the business cycle and during the tightening of monetary policy. In figure 4.10 the correlation bottoms just before 2000, 2008 and 2016.

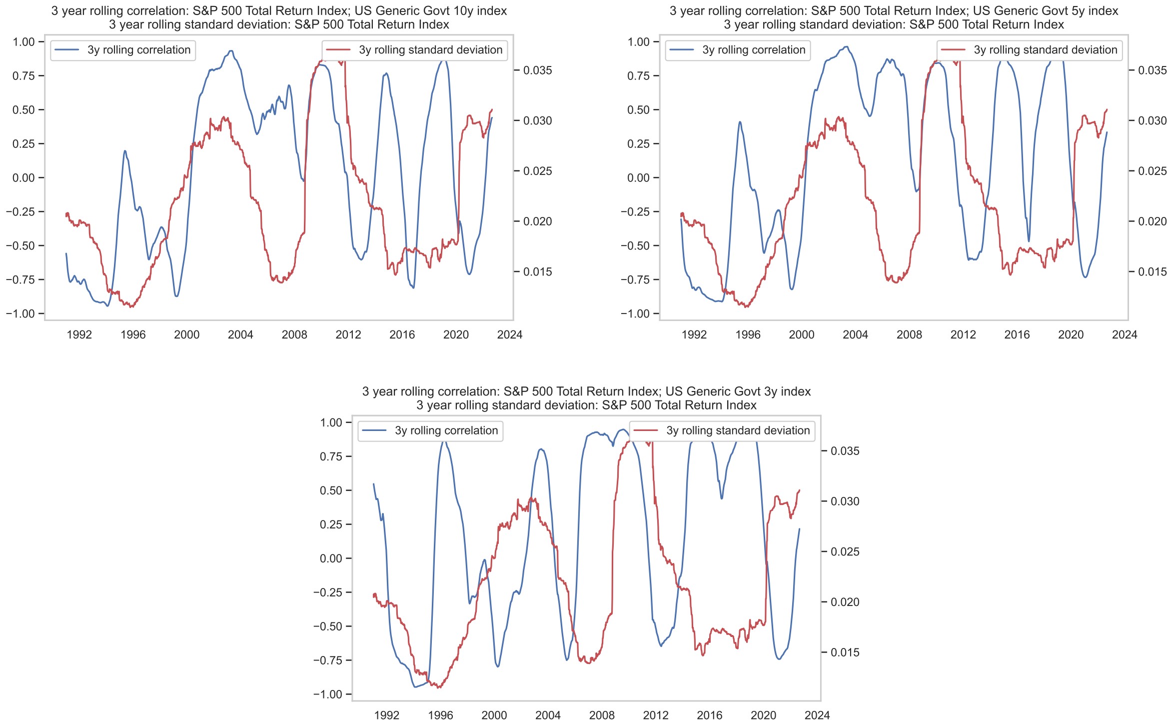

Figure 4.11: Three year rolling correlation and three year rolling return, US assets. Note the two-sided y-axis.

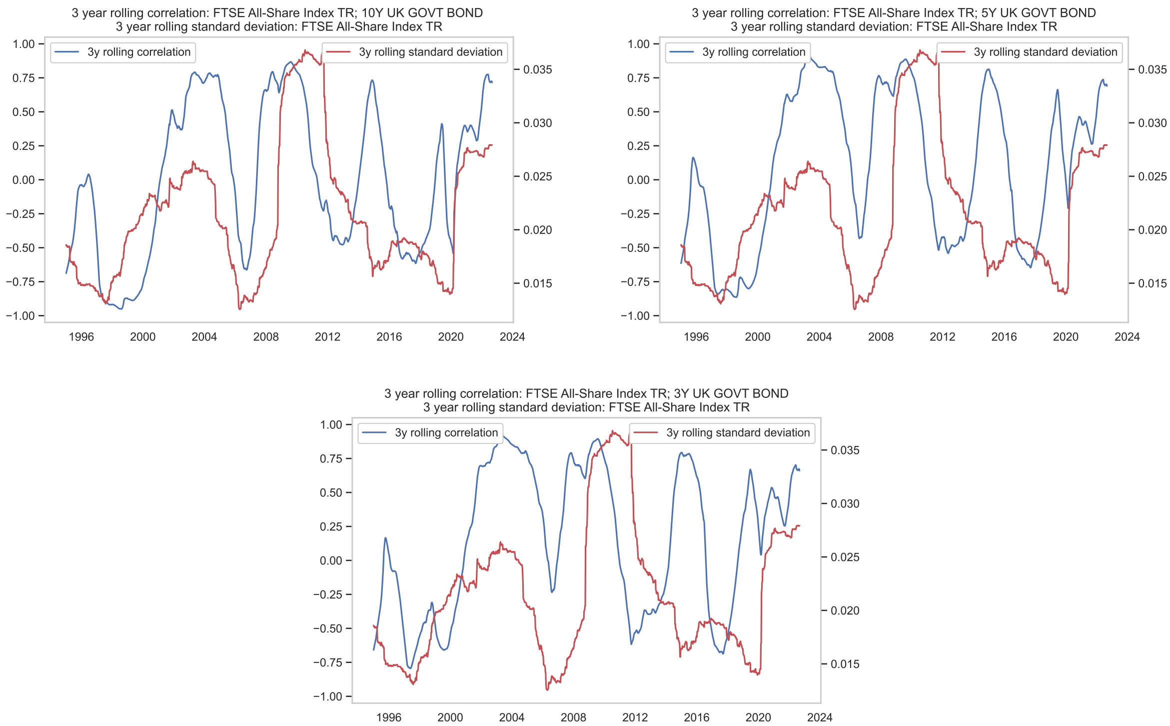

Figure 4.12: Three year rolling correlation and three year rolling return, UK assets. Note the two-sided y-axis.

The strong covariation observed in figure 4.10 is not seen in figures 4.12 and 4.11.

4.3 Return Distributions

It is vital to keep in mind that the time period varies while looking at the histograms below. The likelihood of an outlier measurement increases with the length of the time period. We restrict the comparison to time series that are plotted with the same time frames.

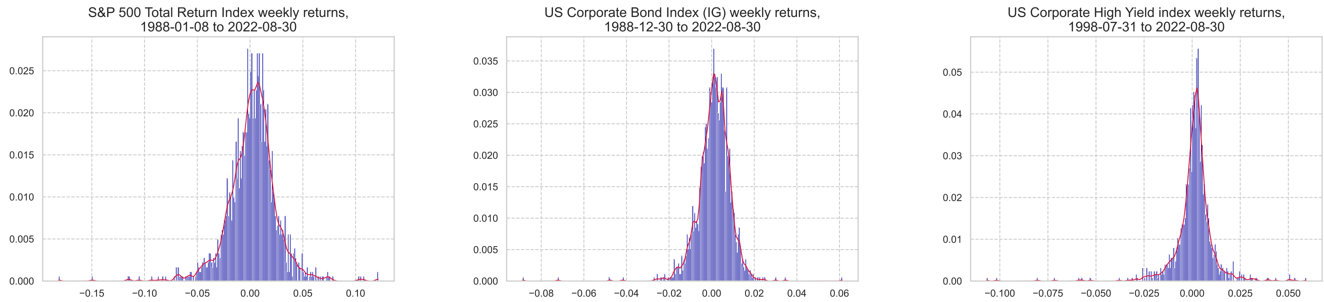

Figure 4.13: Histogram of returns with a kernel density estimate, US assets. Note that the scaling on both the x and y-axis differs.

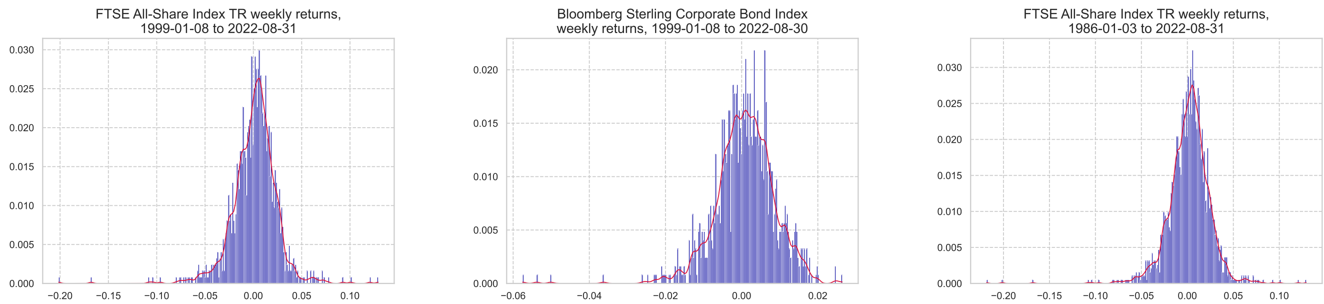

Figure 4.14: Histogram of returns with a kernel density estimate, German assets. Note that the scaling on both the x and y-axis differs.

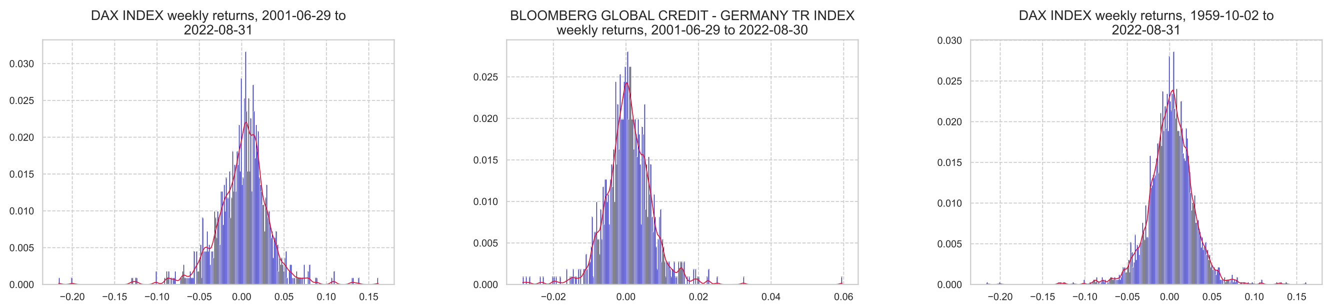

Figure 4.15: Histogram of returns with a kernel density estimate, UK assets. Note that the scaling on both the x and y-axis differs.

US equity returns has greater dispersion than US investment grade (IG) returns. More specifically, for the S&P500, the spread around the mean is greater and the most extreme measures are more excessive. Comparing UK equities and UK IG returns, as well as German equities and German IG returns, the same differences are observed. These observations align with the VaR and ES results in section 4.4.

4.4 Value at Risk and Expected Shortfall

The VaR estimates for the various approaches are the same or very similar in table 4.1. The predicted ES values deviate more between methods, particularly the KDE estimates (2) which are substantially larger than the estimates provided by both the vanilla estimates (1) and the EVT estimates (3).

Table 4.1: 99th percentile Value at Risk and Expected Shortfall estimates.
(1) Derived from the historical return distribution.
(2) Derived from the kernel density estimates seen plotted in figure 4.13, 4.14 and 4.15.
(3) Derived using EVT.

The 99th Percentile	Value at Risk (%)	Expected Shortfall (%)
1. Quantile of the return distribution		
US Assets		
US Corporate High Yield Index	-2.7	-5.7
US Corporate Bond Index (IG)	-1.8	-3.1
S&P 500 Total Return Index	-6.7	-9.4
UK Assets		
Bloomberg Sterling Corporate Bond Index	-1.6	-2.1
FTSE All-Share Index TR	-5.9	-9.8
German Assets		
Bloomberg Global Credit - GER TR index	-1.6	-2.1
DAX Index	-7.3	-10.5
2. Kernel Density Estimation Estimate		
US Assets		
US Corporate High Yield Index	-2.9	-7.3
US Corporate Bond Index (IG)	-1.8	-5.7
S&P 500 Total Return Index	-6.8	-13.7
UK Assets		
Bloomberg Sterling Corporate Bond Index	-2.0	-4.1
FTSE All-Share Index TR	-6.1	-15.6
German Assets		
Bloomberg Global Credit - GER TR index	-1.6	-2.4
DAX Index	-7.4	-16.2
3. Extreme Value Theory Estimate		
US Assets		
US Corporate High Yield Index	-2.9	-6.7
US Corporate Bond Index (IG)	-1.8	-2.4
S&P 500 Total Return Index	-6.8	-8.0
UK Assets		
Bloomberg Sterling Corporate Bond Index	-1.7	-3.2
FTSE All-Share Index TR	-6.0	-8.0
German Assets		
Bloomberg Global Credit - GER TR index	-1.7	-3.2
DAX Index	-7.3	-9.9

Moving further out on the tail of the distribution, it seems that the estimates differ less between the methods used. Instead they are all generate quite similar results for both the VaR and ES estimates. Noticeable is the extremely low estimate across methods for German credits.

Notice the expected shortfall estimates for US investment grade bonds and US high yield bonds in particular. The statistically most reliable EVT estimate indicates that the average fallout in extreme cases, specifically the 99.9th percentile cases, are almost the same. If one were to observe the ES estimates for the 99th percentile case, it would be reasonable to assume that US high yield does have heaver tails that US investment grade. However, when we travel farther out on the tail, this ceases to be the case; instead, US investment grade and US high yield appear to have equally fat tails.

Table 4.2: 99.9th percentile Value at Risk and Expected Shortfall estimates.
(1) Derived from the historical return distribution.
(2) Derived from the kernel density estimates seen plotted in figure 4.13, 4.14 and 4.15.
(3) Derived using EVT.

The 99.9th Percentile	Value at Risk (%)	Expected Shortfall (%)
1. Quantile of the return distribution		
US Assets		
US Corporate High Yield Index	-10.1	-11.0
US Corporate Bond Index (IG)	-5.5	-8.4
S&P 500 Total Return Index	-13.0	-18.1
UK Assets		
Bloomberg Sterling Corporate Bond Index	-5.4	-5.7
FTSE All-Share Index TR	-18.7	-23.6
German Assets		
Bloomberg Global Credit - GER TR index	-2.8	-2.8
DAX Index	-13.9	-18.7
2. Kernel Density Estimation Estimate		
US Assets		
US Corporate High Yield Index	-10.8	-11.2
US Corporate Bond Index (IG)	-7.5	-8.5
S&P 500 Total Return Index	-16.0	18.3
UK Assets		
Bloomberg Sterling Corporate Bond Index	-5.6	-5.9
FTSE All-Share Index TR	-22.3	-23.8
German Assets		
Bloomberg Global Credit - GER TR index	-2.9	-3.0
DAX Index	-14.1	-19.5
3. Extreme Value Theory Estimate		
US Assets		
US Corporate High Yield Index	-10.7	-12.0
US Corporate Bond Index (IG)	-6.2	-11.7
S&P 500 Total Return Index	-13.8	-21.8
UK Assets		
Bloomberg Sterling Corporate Bond Index	-5.7	-6.2
FTSE All-Share Index TR	-19.1	-26.8
German Assets		
Bloomberg Global Credit - GER TR index	-2.9	-3.0
DAX Index	-13.9	-14.2

5

Discussion

Stocks and government bonds are said to historically have had negative correlation with each other, suggesting that when equity returns were historically lower than average, returns on government bonds were typically higher than normal, and vice versa. Our findings resonate with this. Correlation, however, has two limitations: it only provides a general idea of the relationship, whereas investors are typically more concerned in how bonds respond to falling stock prices; and it provides no indication of the extent and magnitude of the relationship. However, a rolling correlation shows the trend of the correlation over time, enabling a relative comparison.

The relative comparison in this thesis indicates that corporate credits are less protective against equity downside as compared to government bonds. Although it is challenging to draw any decisive conclusions since the relationship is not constant, but rather unpredictable, and the magnitude of the link varies between marketplaces. More precisely, the findings demonstrate that UK stocks have exhibited a high and positive correlation with UK corporate bonds. The correlation between UK stocks and UK government bonds has often been more negative when considering the correlation to UK bond asset prices (inverse to yield correlation). This implies that UK government bonds exhibit a more favorable protection against equity downside in general than UK corporate bonds. The same can be said for the relationship between US stocks and US High Yield corporate bonds, see the upper left of figure 4.3. The correlation between US stocks and US investment grade corporate bonds, as well as German stocks and German investment grade credit does not appear to follow the same pattern as clearly. Downfalls in the correlation can be observed before 2004 and around 2010 in US. In Germany downfalls can be observed before 2010 and before 2020. This prompts us to suspect that investment grade corporate bonds do offer some protection against equity downside in the US and German markets. The rolling returns for IG corporate bonds in figure 4.6 and 4.5 confirms that the downfalls where not as severe for the IG indices as the equity indices around 2002 and 2008.

As for the assertion that IG bonds could act as a replacement for government bonds, the results are conflicting. In the UK and US, IG bonds do not appear to be a reliable substitute for government bonds, but in Germany there do indeed seem to be some truth to the statements. The potential effectiveness of German IG bonds are hinted at from all three perspectives taken in this thesis. Generalizations about IG bonds being a sufficient substitute for government bonds may be based on findings from specific markets. Therefore, while it may hold some truth in Germany and possibly in other unexplored markets, it is not universal, emphasizing the importance of exercising caution when making generalizations or extrapolating findings.

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