# Downside risk

*Capturing what's at stake in investment situations.*

*Frank A. Sortino and Robert van der Meer*

A growing number of academics and practitioners are claiming that standard deviation and beta are not relevant measures of risk for many investment situations because they do not capture what's at stake. *Business Week* in its survey of mutual funds proffers an ad hoc risk measure that "doesn't penalize a fund on the upside," because "few investors gripe about risk when it turns up handsome gains" (February 23, 1987, p. 65).

According to Arnott and Bernstein [1988, p. 98], "Elimination of risk does not refer to variability as such, but to the risk of having insufficient assets to meet the obligations as they come due." Hagigi and Kluger [1987] claim the investment objectives on which traditional CAPM measures are based might not accurately describe the case for a defined benefit plan and offer a "safety-first" rule that emphasizes the avoidance of downside risk. Leibowitz [1986] cites the need for a new risk measure that specifically takes into consideration the liability characteristics of investment decisions.

This article examines the problem of measuring risk in general and three downside risk measures in particular. We conclude that Downside Variance is the superior risk measure for many investment situations.

#### WHAT IS RISK?

Hazard, peril, danger, jeopardy ... these synonyms for risk have to do with the "chance of bad consequences," the definition of risk given by the Oxford Dictionary. We will argue that risk is situation-specific and that none of these terms is adequate for describing investment risk. The body of knowledge that has developed for making investment decisions under conditions of uncertainty, and for which Harry Markowitz, William Sharpe, and Merton Miller received the Nobel Prize, requires a clear distinction between uncertainty and risk.

Risk and return are inseparable components of the concept of uncertainty, and the way we describe uncertainty in the financial markets is in terms of a range of possible returns and their chances of occurring. This is called a probability distribution, which describes the shape of uncertainty. The shape most often used is a bell shape, or normal distribution.

There is a great deal of research to indicate that, in reality, distributions are anything but normal. The shape is instead skewed, not symmetric, the high and low returns occur much more often than would be indicated by a bell shape, and the distribution is more "pointy."

We have chosen a triangle (Figure 1) to illustrate our basic points. This triangle is positively skewed, and the area within the triangle describes the uncertainty associated with achieving returns between -100% and +1000%. Some of these returns incur risk, others do not.



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FIGURE 1

PROBABILITY DISTRIBUTION

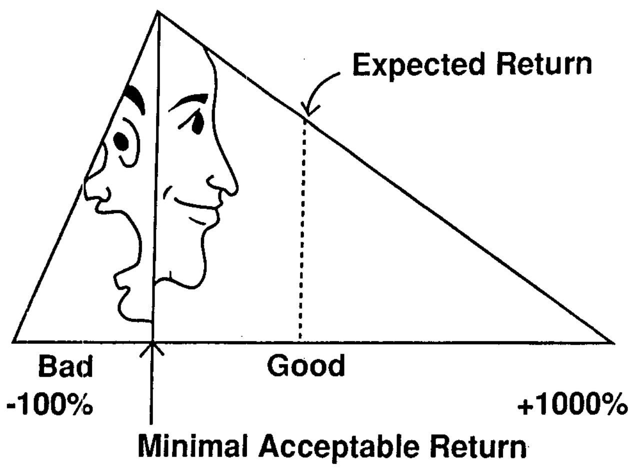
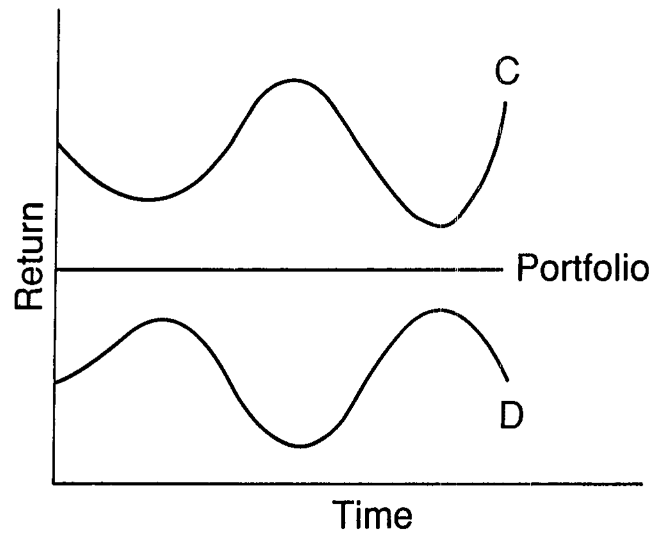



FIGURE 2

RETURNS RELATIVE TO THE MEAN




**THE FACE REVEALS ALL**[1]

For many investment decisions, there is some return that must be earned at minimum in order to accomplish some established goals. This could be the minimum return necessary to fund a plan or project within some cost constraints, or it might be the returns necessary to maintain the surplus at a desired level.

From a negative point of view, it is the return that must be earned at minimum in order to prevent some bad outcome from occurring. Returns to the right of the Minimal Acceptable Return (MAR) ensure accomplishment of the goals and therefore incur no risk. The farther the returns are to the right, the less risk there is of a bad outcome, and the happier we are.

One of the good returns in this diagram is the expected return or mean of this distribution of returns. Standard deviation considers returns that fall in the area between the MAR and a 1000% gain to be risky because they create dispersion about the mean.

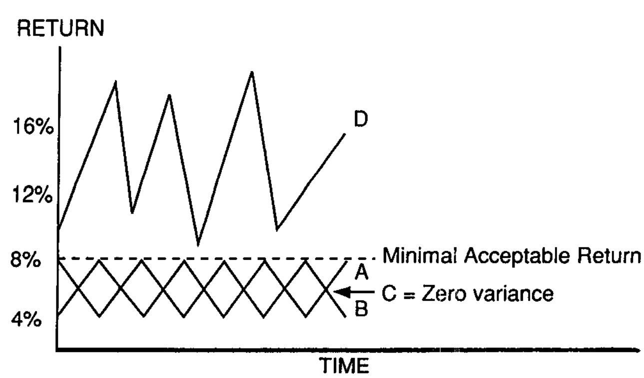
It could be said that standard deviation captures the risk associated with achieving the mean, but that can be totally unrelated to bad outcomes that make us unhappy. Only those returns that fall below the MAR incur risk and the farther they fall below, the greater the risk, and the greater our unhappiness. Ironically, standard deviation considers the best possible return (+1000%) to be the most risky.

The problem of measuring risk relative to the mean is illustrated in Figure 2. Here we have a picture of how returns on Assets C and D are expected to vary over time. An investor who measures risk in terms of standard deviation supposedly would want to invest equal amounts in C and D to create a portfolio that was riskless, i.e., had no variance about the mean. If the Figure is an accurate description of the uncertainty associated with these returns, however, then every investor should want to invest entirely in Asset C regardless of risk tolerance or desired MAR, because the returns with Asset C are always expected to exceed the returns on Asset D.[2]

Figure 3 illustrates the problem of identifying riskless assets in terms of volatility about the mean and the confusion caused by equating uncertainty with risk. Assets A and B are perfectly negatively correlated and therefore can be combined into Portfolio C with zero variance. Mean-variance rules view Asset C as a riskless asset and D as a very risky asset.

FIGURE 3

GOOD OR BAD VOLATILITY




If the minimum return necessary to accomplish the goals is 8%, however, how can constantly earning 200 basis points less than required to accomplish the goals be riskless? Uncertainty of achieving the mean may have been eliminated, but failure to achieve the goal is now a certainty.

On the other hand, Asset D has a great deal of volatility, but all of it above the MAR. There is virtually no risk of failing to achieve the goal if one invests in the most volatile asset, D.

So, in situations where there is an identifiable MAR that is economically significant, one must distinguish between good volatility and bad volatility. Bad volatility is dispersion below the MAR; the farther below the MAR, the worse it is. Good volatility is dispersion above the MAR; the farther above the MAR, the better it is.

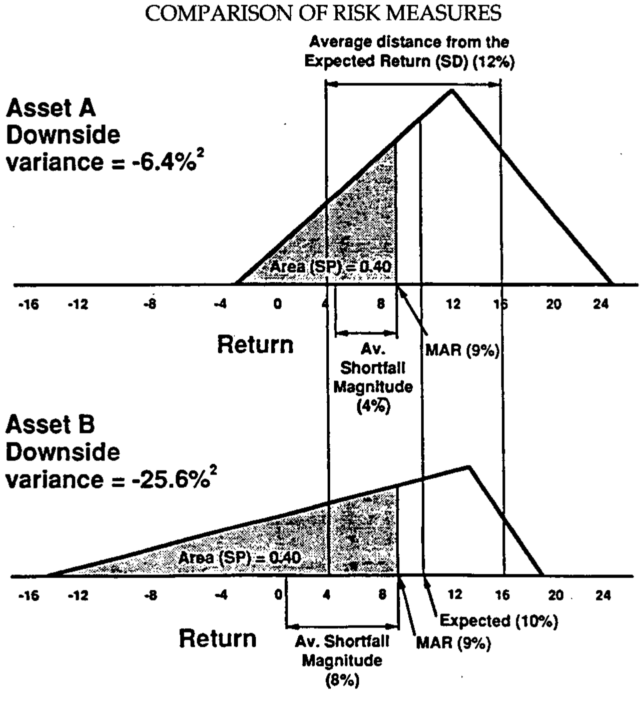
### DOWNSIDE RISK

Now the question is how best to measure this risk of falling below the MAR. The earliest suggestion was to consider the shortfall probability or chances of falling below the MAR. A measure that considers the magnitude as well as the chance of shortfall was proposed by Baumol [1963] and has been recently adopted by Salomon Brothers [1989]. In 1977 Fishburn offered a measure of risk that overcomes the weaknesses academicians have noted with the mean-variance model while remaining within the framework of so-called modern portfolio theory (MPT). We will refer to Fishburn's measure as Downside Variance.

Figure 4 allows us to compare all these risk measures. Assets A and B both have the same mean of 10% and standard deviation of 12%, but they are clearly not equivalent. Asset B has a great deal more downside risk; the question is how to measure it. Shortfall probability fails to distinguish between the comparative risk of A and B because each has the same probability of falling below the MAR of 9% (40% probability of failure).

FIGURE 4

COMPARISON OF RISK MEASURES




Applying shortfall magnitude in this example, we assume the investor wants to be 90% confident the returns will be above 9%. As 10% of the distribution lies below a return of 5% in Asset A, the shortfall magnitude is 4% (9% - 5%). Similarly, the shortfall magnitude of Asset B is 8%. This gives the impression that Asset B is twice as risky as Asset A. Unfortunately, this measure is not consistent with expected utility theory or MPT.

Downside Variance indicates that Asset B in fact is four times riskier than Asset A, and this is the correct perspective. Furthermore, Van Harlow and Rao [1989] have shown that this measure can be incorporated in the CAPM framework without losing any of the important concepts developed by Markowitz and Sharpe. In an earlier paper, Bawa and Lindenberg [1977] show that an efficient frontier of portfolios can be constructed that provides the highest expected return for a given level of Downside Variance. This model will always do at least as well as the mean-variance model.[3]

We now test a mean-downside variance optimizer against a mean-variance optimizer to see if the claim of Bawa and Lindenberg holds. Because active allocation with any optimizer requires some degree of forecasting skill to beat a passive market mix (60% equity, 35% bonds, 5% cash), we give investors the ability to forecast the economic scenario that will occur in each quarter, but they must accept the average historical risk-return characteristics for each asset given that scenario.[4] We call this "near perfect" forecasting ability, admittedly a Herculean assumption, but less so than assuming an ability to forecast the risk-return characteristics of each asset.

The purpose here is to test two models that are the same in every feature, except how they measure risk, in an attempt to determine which risk measure produces the best results if one could forecast. Absent some ability to forecast, one should passively settle for the return on the market mix. To compare these active strategies with the passive, we subtract the return on the market mix from the realized returns on the active asset allocation strategies (i.e., values above the zero line beat the market mix).

The results are shown for a very risk-averse investor who wants to minimize the risk of achieving 10%, and a less risk-averse investor who is willing to take three units of risk to get one unit of return. It is clear that how one measures risk before the fact dramatically affects the performance results after the fact.

### PERFORMANCE RESULTS

The results for the Netherlands (Figure 5) show that the Downside Variance model performed better

FIGURE 5

DOWNSIDE VARIANCE VERSUS VARIANCE RESULTS

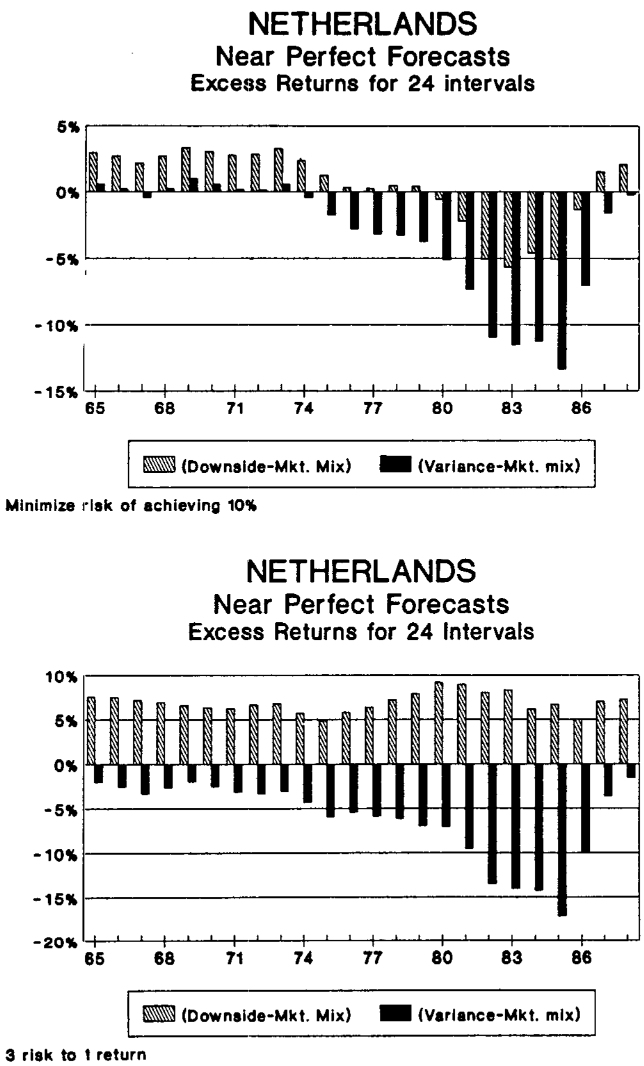
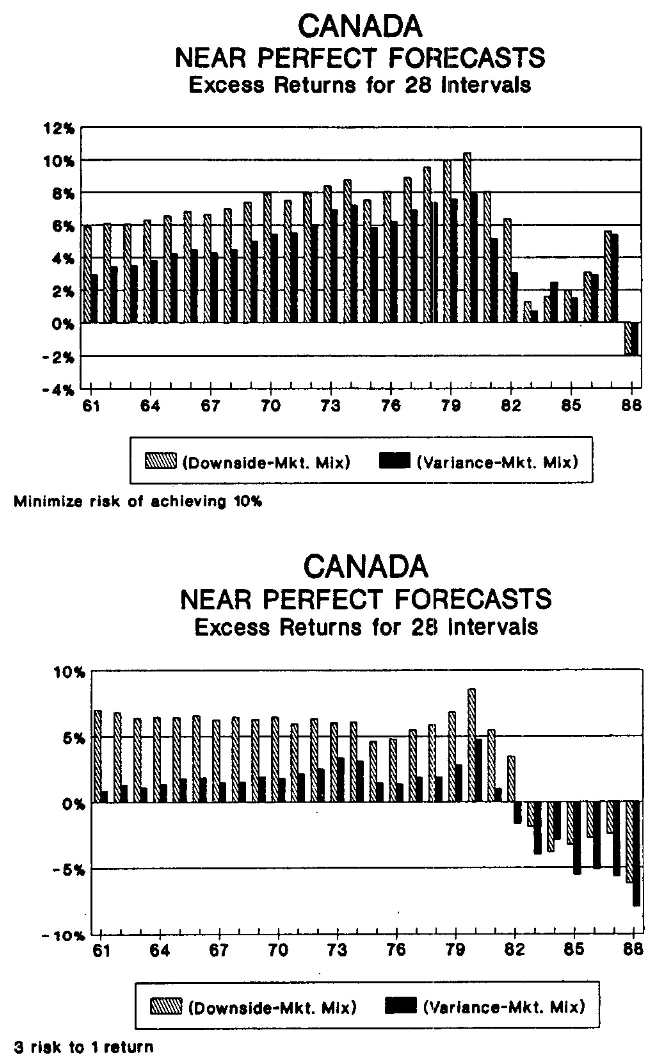



FIGURE 6

DOWNSIDE VARIANCE VERSUS VARIANCE RESULTS




than the variance model for every interval[5] for a wide range of risk-averse investors and always beats the market mix for the less risk-averse investor. The reason is that returns during growth scenarios in Holland exhibited a great deal of volatility in stocks, but most of it above the MAR of 10%. Therefore, the Downside Variance model takes stocks as having very little risk of falling below the MAR, while the variance model sees stocks as having a great deal of risk associated with achieving the mean.

The market mix, however, often performs better than the Downside Variance strategy for the very risk-averse investor. This is not surprising because the market mix took more risk than the active strategies (e.g., in periods of growth the market mix took 48% more risk than the very risk-averse investor).

The results for Canada (Figure 6) are markedly different from the Dutch experience. Both active strategies beat the market mix for every interval (except for 1987 to 1988) for the very risk-averse investor, and the Downside Variance model always beats the variance model. The less risk-averse investor beats the market mix with both active strategies until 1983, but once again the Downside Variance model beats the variance model in every interval.

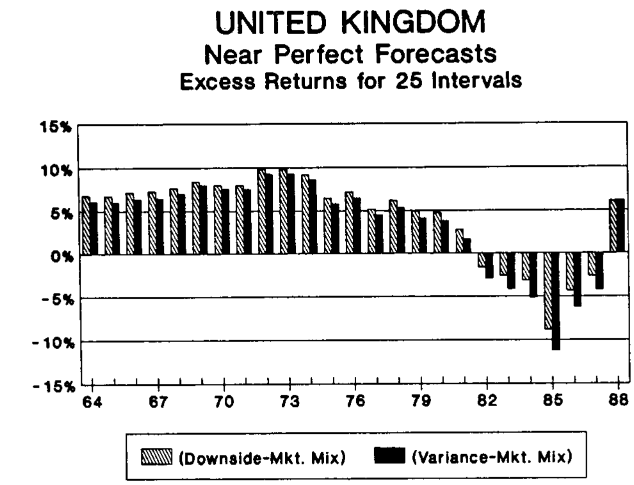
Results for the U.K. are different still (Figure 7). While the Downside Variance model still beats the variance model for every interval, the results are close for the very risk-averse investor.

These results support the Bawa and Lindenberg claim that downside variance will do as well as or better than variance. It is important to note that results vary greatly depending on the country, the interval of time, and the investor's degree of risk aversion.

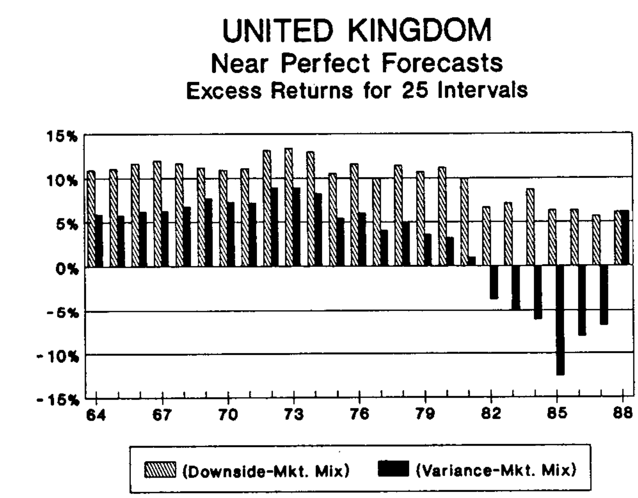
While these results are admittedly of a preliminary nature, they do tend to confirm tests conducted at the Royal Dutch Shell pension fund and Aegon

FIGURE 7

DOWNSIDE VARIANCE VERSUS VARIANCE RESULTS




Minimize risk of achieving 10%




3 risk to 1 return

Insurance Company in the Netherlands over the last three years. These results support the call for a new set of criteria for defining risk that:

1. Explicitly consider the investor's goals.
2. Distinguish between true risk and uncertainty.
3. Consider both the chance and the magnitude of adverse outcomes.
4. Are intuitively appealing.

## CONCLUSIONS

There are many circumstances where a rate of return must be earned at minimum in order to prevent bad outcomes. For corporate planning purposes, the minimum return may be the cost of capital; for a defined benefit plan it may be the return that will minimize negative impacts on earnings. Optimization models have been criticized for not specifically taking into consideration the liability characteristics of a plan. A return can be estimated that discounts the promised benefits to the present value of the contribution stream plus the portfolio assets, thus providing a link between assets and liabilities.

Downside Variance meets these criteria and captures what's at stake in many investment situations. It appears moreover to resolve the shortcomings of variance and beta while retaining the important risk-return framework of modern portfolio theory. This makes Downside Variance worthy of consideration by researchers and practitioners.

[1] Old Dutch saying.

[2] In terms of stochastic dominance rules, Asset C is said to dominate Asset D by first-degree stochastic dominance.

[3] To incorporate Downside Variance into a non-linear programming algorithm, the combined distribution must first be estimated. Then the integral of the downside risk equation

$$DR = \int_{-\infty}^{m} (m-r)^2 f(r)dr$$

where r is the return on some asset, and M is the MAR, can be evaluated by analytical methods. Now the optimization procedure can begin to generate the mean-downside variance efficient frontier.

[4] For a detailed description of this procedure, see Sortino [1990].

[5] The first set of bars is for the 1965 to 1989 interval. The last set of bars is for the 1988 to 1989 interval. The results vary depending on the starting date for performance measurement.

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