Mixed integer program for allocating university tutors to course tutorials/workshops.

Liam Timms
liam.timms@uq.edu.au

Sets

I	Set of tutors, i.
S	Set of supertutors, $S \subseteq I$.
GID	Set of gender identities that tutors in I identify with.
W	Set of workshops, w, over all courses.
T	Set of time slots that workshops in W occupy (in 24-hour time), e.g. $\{[0800, 1000], [0900, 1100], \ldots, [1600, 1800]\}$.
D	Set of days in a week.
C	Set of courses offered in the semester. $C = \{\text{SCIE1000}, \text{SCIE1100}\}$, or $C = \{\text{SCIE1000}\}$.

Data

Assume that all workshops are 2 hours long, and they begin on the hour (i.e. 8am, 9am, etc.).

P_{iw}	Tutor i's availability to teach workshop w, for $i \in I$, $w \in W$. $P_{iw} \in \{0, 1, A\}$, corresponding to "Unavailable", "If needed", and "Available".
A	Weighting for "Available" status, represents tutors' preferences for specific workshops. $A > 1$ means that "Available" allocations will be preferenced over "If needed" in the objective function.
Exp_i	1 if tutor $i \in I$ is experienced, 0 otherwise.
G_i	Gender identity of tutor $i \in I$. $G_i \in \text{GID}\, \forall i \in I$.
Div_{ij}	1 if $G_i = G_j$, 0 otherwise. Indicates if tutor pair (i, j) is "diverse", for $i, j \in I$.
Conflict_{ij}	1 if tutor i and tutor j cannot tutor together, $i, j \in I$ and, 0 otherwise.
N_w	Number of tutors required for workshop $w \in W$.
M_{ic}	Number of workshops in course $c \in C$ assigned to tutor $i \in I$.
Start_w, End_w	Times when workshop $w \in W$ begins and ends. $[\text{Start}_w, \text{End}_w] \in T$, $\forall w \in W$.
Day_w	Day of workshop $w \in W$.
DayOne	First day of the week with a workshop.
Overlap_w	The set of workshops that overlap with workshop $w \in W$. These are the workshops that start within an hour of workshop w and are on the same day: $\text{Overlap}_w = \left\{v \in W \text{ s.t. } \lvert\text{Start}_v - \text{Start}_w\rvert \leq 100 \wedge \text{Day}_v = \text{Day}_w\right\}$.
α	Weighting for the diversity contribution to the objective function. $\alpha \in [0, \infty)$. Represents how important diverse tutor teams are relative to fulfilling tutors' preferences.

Variables

Assume that the only options for tutor team sizes are 1, 2, or 3, i.e. $N_w \in \{1, 2, 3\}$.

x_{iw}	1 if tutor $i \in I$ is allocated to workshop $w \in W$, 0 otherwise.
y_{ijw}	1 if tutors $i, j \in I$, $i \neq j$ are both allocated to workshop $w \in \{W \mid N_w = 2\}$, 0 otherwise.
z_{ijkw}	1 if tutors $i, j, k \in I$, $i \neq j \neq k$ are all allocated to workshop $w \in \{W \mid N_w = 3\}$, 0 otherwise.

Objective

Define the total number of tutors required to staff all workshops, $M_{\text{total}} = \sum_{i \in I} M_i$, the number of workshops that require 2 tutors, $N^2 = |\{w \in W \mid N_w = 2\}|$, and the number of workshops requiring 3 tutors, $N^3 = |\{w \in W \mid N_w = 3\}|$.

The total tutor preferences is

$$\text{TutorPref} = \sum_{i\in I,\, w\in W} P_{iw} x_{iw}.$$

The total gender diversity over all tutoring teams is given by

$$\text{GenderDiv} = \sum_{\substack{i,j\in I,\, w\in W\\ i\neq j}} y_{ijw}\text{Div}_{ij} + \sum_{\substack{i,j,k\in I,\, w\in W\\ i\neq j\neq k}} z_{ijkw}(\text{Div}_{ij} + \text{Div}_{ik} + \text{Div}_{jk})$$

The objective is then maximise tutors' preferences for workshop allocations, with a bonus for increased average tutor team diversity over all workshops. This is given by the sum of the normalised tutor preferences and the normalised average gender diversity, weighted appropriately:

$$\max \frac{1}{AM_{\text{total}}}\text{TutorPref} + \frac{\alpha}{N^2 + 3N^3}\text{GenderDiv}$$

Constraints

$$\sum_{i\in I} x_{iw} = N_w, \quad \forall w \in W, \tag{1}$$

$$\sum_{w\in W} x_{iw} = M_{ic}, \quad \forall i \in I,\, c \in C, \tag{2}$$

$$\sum_{i\in I,\, v\in \text{Overlap}_w} x_{iw} \leq 1, \quad \forall i \in I,\, w \in W, \tag{3}$$

$$\sum_{\substack{i\in I\\ |\text{Exp}_i=1}} x_{iw} \geq 1, \quad \forall w \in W, \tag{4}$$

$$x_{iw} + xjw \leq 1, \quad \forall i, j \in I,\, w \in W, \text{ if Conflict}_{ij} = 1, \tag{5}$$

$$\sum_{\substack{w\in W,\\ |\text{Day}_w=\text{DayOne}}} x_{iw} \geq 1, \quad \forall i \in S, \tag{6}$$

$$\sum_{i\in S} x_{iw} \leq 1, \quad \forall w \in W, \tag{7}$$

$$y_{ijw} \leq x_{iw} \quad \forall i, j \in I,\, i \neq j,\, w \in W, \tag{8}$$

$$y_{ijw} \leq x_{jw} \quad \forall i, j \in I,\, i \neq j,\, w \in W, \tag{9}$$

$$y_{ijw} \geq x_{iw} + x_{jw} - 1 \quad \forall i, j \in I,\, i \neq j,\, w \in W, \tag{10}$$

$$z_{ijkw} \leq x_{iw} \quad \forall i, j, k \in I,\, i \neq j \neq k,\, w \in W, \tag{11}$$

$$z_{ijkw} \leq x_{jw} \quad \forall i, j, k \in I,\, i \neq j \neq k,\, w \in W, \tag{12}$$

$$z_{ijkw} \leq x_{kw} \quad \forall i, j, k \in I,\, i \neq j \neq k,\, w \in W, \tag{13}$$

$$z_{ijkw} \geq x_{iw} + x_{jw} + x_{kw} - 2 \quad \forall i, j, k \in I,\, i \neq j \neq k,\, w \in W, \tag{14}$$

$$x_{iw}, y_{ijw}, z_{ijkw} \in \{0, 1\}, \quad \forall i, j, k \in I,\, w \in W. \tag{15}$$

Constraints 1 ensure that each workshop is staffed with the required number of tutors. Constraints 2 make sure that each tutor is allocated the correct number of workshops. Each tutor can only be in one workshop at a given time on a given day, enforced by Constraints 3. Some tutors are unable to teach together for various reasons, e.g. being in a relationship, personal conflict, etc. These tutors cannot be allocated to the same workshop, shown in Constraints 5.

Tutors are divided into experienced and inexperienced tutors. Constraints 4 ensure that there is at least one experienced tutor in each workshop. Additionally, one or more tutors are "supertutors" who manage the tutoring team. These supertutors should be allocated to a workshop on the first day of workshops, enforced by Constraints 6, so that they discover any issues with the workshop as soon as possible. However, allocating multiple supertutors to the same workshop is a suboptimal use of resources, as supertutors are among the most experienced tutors. Constraints 7 prevent this from happening.

Constraints 8 - 10 ensure that $y_{ijw} = 1$ if and only if tutors i and j are assigned to workshop w, and is 0 otherwise. Constraints 11 - 14 perform a similar role for the z_{ijkw} variables.

Finally, Constraints 15 prescribe the domain of the decision variables.

Comments

Firstly, I have approximated the gender spectrum with the set GID. Elements of GID can be any gender identity that a tutor identifies with, however GID requires some level of discretisation. For example, if two tutors identify as male, then their G_i values should be the same, even though their ideas of what male means may be different.

I use a limited definition for "diverse" that only accounts for differences in gender identity.

There are many options for incorporating tutor team diversity in the objective, one of which is maximising the minimum tutor team diversity. One issue with this approach is that tutor availabilities can be fairly inflexible, so there may be at least one workshop where all of the tutors have the same gender identity - the only possible tutor team diversity value is 0. In this case, the minimum tutor team diversity will always be 0, and so diversity no longer contributes to the objective function (since max-min only considers the minimum diversity). To solve this, I have used the average tutor team diversity instead of max-min.

To properly weight the preference and diversity terms in the objective, I scaled them both to be between 0 and 1.

$$\max \sum_{i \in I,\, w \in W} P_{iw} x_{iw} = A \sum_{i \in I} M_i.$$

We can then normalise this term by dividing it by $A \sum_{i \in I} M_i$. For diversity,

$$\max \sum_{\substack{i,j \in I,\, w \in W \\ i \neq j}} y_{ijw} \mathrm{Div}_{ij} = N^2,$$

and

$$\max \sum_{\substack{i,j,k \in I,\, w \in W \\ i \neq j \neq k}} z_{ijkw} (\mathrm{Div}_{ij} + \mathrm{Div}_{ik} + \mathrm{Div}_{jk}) = 3N^3.$$

The average divergence is given by

$$\frac{\text{total Div.}}{N^2 + N^3} = \frac{1}{N^2 + N^3} \left(\sum_{\substack{i,j \in I,\, w \in W \\ i \neq j}} y_{ijw} + \sum_{\substack{i,j,k \in I,\, w \in W \\ i \neq j \neq k}} z_{ijkw} (\mathrm{Div}_{ij} + \mathrm{Div}_{ik} + \mathrm{Div}_{jk}) \right)$$

This can be normalised by dividing by $(N^2 + 3N^3)/(N^2 + N^3)$, which leaves us with

$$\frac{\text{total Div.}}{N^2 + 3N^3}.$$