

**A Tutorial on Tailored Simulation-Based Sample Size Planning for Experimental Designs  
with Generalized Linear Mixed Models**

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This is version 2.5 of our preprint published at <https://doi.org/10.31234/osf.io/rpjem>. All materials (reproducible manuscript, including R code and all simulation results) are available in the project's repository on the Open Science Framework (OSF) at <https://osf.io/dhwf4/>. A Quarto Manuscripts website is hosted at [https://Timo-Ko.github.io/glmm\\_simulation\\_tutorial/](https://Timo-Ko.github.io/glmm_simulation_tutorial/). The authors declare that there were no conflicts of interest with respect to the authorship or the publication of this article. This research was funded by a grant from the Volkswagen Foundation (Grant No. 98525). Author roles were classified using the Contributor Role Taxonomy (CRediT; <https://credit.niso.org/>) as follows: Florian Pargent: formal analysis, methodology, visualization, writing, editing; Timo K. Koch: conceptualization, formal analysis, writing, editing; Anne-Kathrin Kleine: formal analysis, editing; Eva Lermer: conceptualization, funding acquisition, editing; Susanne Gaube: conceptualization, supervision, editing

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### **Abstract**

When planning experimental research, determining an appropriate sample size and using suitable statistical models are crucial for robust and informative results. The recent replication crisis underlines the need for more rigorous statistical methodology and adequately powered designs. Generalized linear mixed models (GLMMs) offer a flexible statistical framework to analyze experimental data with complex (e.g., dependent and hierarchical) data structures. However, available methods and software for a priori sample size planning for GLMMs are often limited to specific designs. Tailored data simulation approaches are a more flexible alternative. Based on a practical case study, the current tutorial equips researchers with a step-by-step guide and corresponding code for conducting tailored a priori sample size planning with GLMMs. We not only focus on power analysis but also explain how to use the precision of parameter estimates to determine appropriate sample sizes. We conclude with an outlook on the increasing importance of simulation-based sample size planning.

*Keywords:* tutorial, sample size planning, generalized linear mixed model, power analysis, precision, data simulation

## **A Tutorial on Tailored Simulation-Based Sample Size Planning for Experimental Designs with Generalized Linear Mixed Models**

### **Introduction**

When planning experimental research, it is essential to determine an appropriate sample size and to use appropriate statistical models to analyze the data to ensure that the results obtained are both robust and informative ([Lakens, 2022a](#)). The recent replication crisis has illustrated many challenges surrounding the reproducibility and reliability of study findings ([Yarkoni, 2022](#)). As a result, there is a growing need for more rigorous statistical methodology and the adoption of adequately powered experimental designs. Multiple easy-to-use software solutions exist for simple statistical models and experimental designs. However, many researchers lack the skills and tools to conduct “a priori” (i.e., before data collection) sample size planning for more complex research designs such as flexible generalized linear mixed models (GLMM) framework. In the present work, we provide a tutorial on how to determine adequate sample sizes by performing tailored simulation-based sample size planning for GLMMs. After introducing some theoretical background on sample size planning, we review existing software solutions in R and discuss under which circumstances tailored data simulations are necessary. Then we describe general steps and decisions involved in tailored data simulation. To illustrate the details of these steps, we finish with a hypothetical case study from the field of human-AI (artificial intelligence) interaction research.

To benefit most of this tutorial paper, we recommend readers to familiarise themselves with basic statistical concepts like hypothesis tests (HTs) and their statistical power as well as confidence intervals (CIs) and their precision ([Kumle et al., 2021](#); [Lakens, 2022b](#); [Riesthuis, 2024](#)). Some knowledge of causal inference is useful but not necessary ([Deffner et al., 2022](#); [Lundberg et al., 2021](#)). In addition, readers should have a rough understanding of R ([Wickham et al., 2023](#)) and how to simulate data. For data simulation, we use functions from the tidyverse ([Wickham et al., 2019](#)) and the faux package ([DeBruine, 2023](#)). Finally, readers should be familiar with regression modeling in general and GLMMs in particular. In this tutorial, we simulate data by manually specifying the model equation of a GLMM that represents our assumed

data generating process ([DeBruine & Barr, 2021](#)). It is not necessary to understand the technical details of how GLMMs are estimated. However it is crucial to understand the structure of a basic GLMM (e.g., logistic regression with random intercepts) and how the model assumes that the values in the dependent variable are determined by the predictor variables and the random effects.

### **Theoretical background**

#### **Planning for statistical power or precision**

Conducting research with insufficiently large sample sizes can have many negative consequences. First, experiments may yield inconclusive or misleading results, hindering the accumulation of knowledge. Second, studies that are doomed to never finding a postulated effect waste resources by consuming time, effort, and funding without delivering meaningful results. For these reasons, many journals and funding bodies now require that a justification of sample size is included in study protocols and grant proposals, recognizing its significance in ensuring robust and meaningful findings. Although scientists often do not justify sample size or use general heuristics from the literature, resource constraints often play an important role ([Lakens, 2022a](#)). But ideally, a suitable sample size should be determined a priori (i.e., before the study is conducted) based on some meaningful computation to ensure that the study will be able to fulfill its purpose.

The majority of empirical studies in psychology and other social sciences apply hypothesis testing. As a consequence, the dominant approach for determining an adequate sample size is based on power analysis (i.e., planning for power) ([Lakens, 2022a](#); [Maxwell et al., 2008](#)). Statistical power is defined as the probability that a HT has a significant p-value when analyzing repeated samples from a population with a true effect of some pre-specified size ([Cohen, 1992](#)). Less formally, power is described as the probability that a HT correctly rejects the null hypothesis when the alternative hypothesis is true. If the sample size (i.e., the number of participants and/or stimuli) used for data collection is insufficient to detect the effects or relationships being investigated with high probability, the study is considered “underpowered”. When planning for power, a target is set for the statistical power of a HT of interest. Assuming some effect size of

interest and a desired significance level, a minimum sample size can be determined that, on average, would guarantee reaching this target. Although it is recommended to justify the desired significance level and power ([Lakens, Adolphi, et al., 2018](#)), most empirical studies adopt the heuristic of  $\alpha = 0.05$  and  $1 - \beta = 0.80$ .

In contrast to power analysis, sample size planning can also be based on the precision of parameter estimates (i.e., planning for precision or planning for accuracy) ([Lakens, 2022a](#); [Maxwell et al., 2008](#)). Not all research questions are best answered using hypothesis testing. It has been argued that basic research rarely requires making discrete decisions on whether some effect has been “discovered” and should thus shift from a hypothesis testing towards an estimation framework ([Cumming, 2014](#); [Kruschke & Liddell, 2018](#); [McElreath, 2020](#)). Although this view is not without critique ([Uygun Tunç et al., 2023](#)), at least for exploratory or pilot studies where little previous research has been conducted, more scientists seem to agree that simply estimating the effects of interest and making the estimation uncertainty transparent by reporting CIs is more useful. Assuming that no HTs are conducted for a planned study, power analysis is not relevant for sample size planning. Nonetheless, the sample size still has a crucial effect on how informative the planned study will be, because an effect of interest is estimated more precisely with bigger samples. In the precision framework, the target quantity commonly used for sample size planning is the expected width of a CI ([Kelley & Rausch, 2006](#); [Lakens, 2022a](#); [Maxwell et al., 2008](#)). The values inside a CI are often interpreted as plausible values for the quantity of interest it is supposed to estimate. More formally, a CI with confidence level 0.95 provides the smallest interval with the property that upon repeated sampling, 95% of individual CIs would include the true quantity of interest. Thus, a narrow CI is more informative about the size of the true effect than a wide interval. Apart from the desired confidence level, the width of a CI depends strongly on the sample size. Because bigger samples carry more information, they lead to smaller CIs. When planning for precision, a target can be set for the expected width of a CI of interest. Assuming some effect size of interest and a desired confidence level, a minimum sample size can be determined that, on average, would guarantee reaching this target. Because planning for

precision is still rare, there are no common heuristics on how to choose the desired width of the CI ([Lakens, 2022a](#)).

### **Generalized linear mixed models (GLMMs)**

As study designs become more complex, psychological researchers require more sophisticated statistical models to capture the nuanced relationships and grouping structures introduced by them ([Yarkoni, 2022](#)). GLMMs (also called multilevel models) are gaining increasing popularity because they offer great flexibility ([Fahrmeir et al., 2021](#)). GLMMs are an extension of LMMs (Linear Mixed Models), which are, in turn, extensions of linear regression models that account for correlated data, including hierarchical structures ([Fahrmeir et al., 2021](#)). In this context, correlated data means that the value in the outcome variable for one observation may be related to the value of another observation in a systematic way that is not already accounted for by the usual (fixed) predictor variables (e.g., age of participants). This correlation can arise for various reasons: For instance, responses to some stimuli from some participants might be more similar because the same person was measured twice (repeated measurements), participants come from the same neighborhood (clustering), or participants responded to the same stimulus (stimulus effects). Thus, modeling such correlations is important whenever the data has a clear structure, while the grouping variables can be hierarchically nested (e.g., grouping variables students and schools: each student belongs to exactly one school) or cross-classified (e.g., grouping variables students and math exercises: each student is presented with several math exercises). LMMs are used when the outcome variable is continuous and follows a normal distribution. They allow for the modeling of fixed effects, which capture the relationships between the usual predictors and the outcome, as well as random effects, which account for the different types of correlation structure and grouping effects. Random effects are typically assumed to follow a normal distribution with a mean of zero and a variance that quantifies the heterogeneity across groups. GLMMs extend the LMM framework to accommodate non-normally distributed continuous and categorical outcome variables. GLMMs involve a link function that connects the linear combination of predictor variables to the expected value of the outcome variable. The link

function allows for modeling the relationship between predictors and the outcome in a non-linear way that is appropriate for the specific distribution family of the outcome variable. For example, think of an experiment with different design factors (e.g., picture positions, headline aesthetics) impacting the likelihood of users clicking on an online advertisement. The participants' behavior is measured repeatedly over several sessions. The click patterns of participants in one session are likely to be correlated with their previous sessions and the outcome variable is binary (click/no click) for each session, which follows a binomial distribution.

### **Simulation-based sample size planning with GLMMs**

To our knowledge, existing approaches for sample size planning for GLMMs have exclusively focused on planning for power. Power analysis methods for multilevel models can be categorized into formula-based methods and simulation-based methods ([Murayama et al., 2022](#)). Formula-based methods rely on formulas to calculate power directly while simulation-based methods rely on repeatedly simulating data with a known true effect size and estimating power empirically (i.e., what percentage of simulated datasets produces a significant p-value). Available formula-based software packages for multilevel models often do not include GLMMs or are limited to very simple designs ([Murayama et al., 2022](#); [Westfall et al., 2014](#)), making it necessary to build data simulations tailored specifically to the study design. A number of tutorials have been published describing how to perform such simulation-based power analysis for multilevel models ([Arend & Schäfer, 2019](#); [Brysbaert & Stevens, 2018](#); [DeBruine & Barr, 2021](#); [Green & MacLeod, 2016](#); [Kain et al., 2015](#); [Kumle et al., 2021](#); [Lafit et al., 2021](#); [Zimmer et al., 2022](#)). However, many of these tutorials focus on linear mixed models (LMMs) and the common study designs (but see [Kumle et al., 2021](#), for a tutorial that also covers more advanced settings). This narrow focus provides limited guidance for researchers faced with more complex study designs, especially when little prior knowledge about plausible effect sizes is available (see the discussion in [Kumle et al., 2021](#)). Simulation-based power analysis with GLMMs requires making a range of assumptions: The (conditional) distribution assumption specifies the distributional family for the outcome variable. Assumptions about the random effects include the assumption of normality



(i.e., that the random effects follow a normal distribution) and the covariance structure among the random effects (i.e., if and how they are correlated). Making these decisions requires understanding the underlying assumptions of the model and ensuring they align with the characteristics of the data being analyzed. Existing tutorials often rely on heuristics for specifying variance components (e.g., the standard deviation of random intercepts) or assume that results from meta-analyses or data from pilot studies are available to determine plausible values for all model parameters. However, in practice, knowledge about those parameters from prior studies is often limited, which makes specifying assumptions a practical challenge (see the discussion in (Maxwell et al., 2008) and (Kumle et al., 2021)).

INSERT TABLE 1 HERE!

In Table 1, we give a short review of existing R packages that can be used for power analysis for GLMMs.

### **When to use tailored data simulation?**

Performing tailored simulation-based sample size planning is more complicated and time-consuming than using the existing software tools outlined in TABLE 1. The most important circumstances under which tailored simulation-based sample size planning is necessary are 1) complex study designs, 2) complex statistical hypotheses, 3) planning for precision, 4) no available prior studies or pilot data.

Requirements of real-world studies are often more complex than the simplified designs assumed by many user-friendly software packages for sample size planning. One frequent issue in applied data analysis is missing data, and there can be various reasons for this (Little & Rubin, 2014). For example, data can be missing completely at random (e.g., because an electronic measurement device randomly failed for some technical reasons). Alternatively, subjects might systematically drop out or produce missing data, but this dropout can be explained by some attributes also measured in the dataset (e.g., older subjects have a higher probability to refuse answering a question on income). In a more complicated scenario, missing data in some variable is caused by the measured attribute itself (e.g., wealthy people are more likely to refuse reporting

their income). Moreover, many experimental designs contain conditions in which values of the predictor variables are missing by design. This can make data analysis more complicated because predictors have to be coded in specific ways that prevent the estimated GLMM from becoming unidentified. Whether missing data has an effect on the sample size planning depends on our theoretical assumptions on how the missingness is caused. However, it is often challenging to decide whether missing data can be safely ignored in the data analysis and sample size planning process based on a merely theoretical approach ([Gomila & Clark, 2022](#)). Tailored simulation-based approaches offer the possibility to include the assumed process of how data become missing in the data simulation, thereby determining the required sample based on simulated datasets that contain missing values (for an example, see [Lane & Hennes, 2018](#)). As a byproduct, the simulated datasets can also be used to test whether the intended data analysis provides the expected (unbiased) results, despite the missing data. Although GLMMs can handle a large variety of outcome variables, researchers are becoming increasingly aware that many datasets might profit from even more sophisticated models. Common examples are zero-inflated outcomes, censoring, and nonlinear predictor effects that can be modeled with the R packages `glmmTMB` ([Brooks et al., 2017](#)) or `brms` ([Bürkner, 2018](#)). Tailored simulation-based approaches do not share the same limitation than the existing software solutions for power analysis that focus exclusively on GLMMs. As long as there is a software package available to estimate the model of interest, it is always possible to perform tailored simulation-based sample size planning.

The most common hypotheses tested in psychological research are of the type  $H_0 : \beta = 0$ , where  $\beta$  is a slope or intercept of a regression model. However, many research questions in psychology actually require testing more complex statistical hypotheses. In the new era of preregistration and registered reports ([Chambers & Tzavella, 2022](#)), most research questions should be tested with directed hypotheses because good theories at least postulate whether some psychological effect of interest is positive or negative. Even better theories should be able to specify the smallest effect sizes of interest (SESOI) that must be exceeded if the effect has any practical relevance ([Lakens, Scheel, et al., 2018](#)). In combination, this might require a test such as

$H_0 : \beta \leq 0.1$ . More elaborate research questions often require testing hypotheses that consist of a combination of model parameters, for example testing simple slopes (Preacher et al., 2006) with a hypothesis such as  $H_0 : \beta_0 + \beta_1 \leq 0$ . If the research question consists only of a single hypothesis of this sort, it might be possible to reduce the hypothesis to a single regression coefficient by clever coding and/or centering of predictor variables. However, interesting research questions often consist of combined hypotheses that consist of more than one separate statistical hypothesis (for a tutorial on contrast analysis in GLMMs, see Schad et al., 2020). For example, a combined null hypothesis  $H_0$  might consist of two single null hypotheses  $H_{01} : \beta_1 \leq 0$  and  $H_{02} : \beta_0 + \beta_1 \leq 0$ . For some research questions, the combined null hypothesis  $H_0$  would be rejected if both  $H_{01}$  AND  $H_{02}$  are rejected. For other research questions, the combined null hypothesis  $H_0$  would be rejected if  $H_{01}$  OR  $H_{02}$  OR both are rejected. If the global hypothesis  $H_0$  is combined with OR, the p-values of the single hypotheses must be corrected for multiple testing to avoid  $\alpha$ -inflation for the global hypothesis (Dmitrienko & D'Agostino, 2013). However, if the global hypothesis  $H_0$  is combined with AND, a correction for multiple testing is not necessary but rather a mistake that unnecessarily reduces the power of the global HT. None of the software packages for sample size planning in table X can handle combined hypotheses as discussed here and only some can handle directed hypotheses. In contrast, our case study will demonstrate how we can test directed combined hypotheses with tailored simulation-based sample size planning.

Although planning for precision (Cumming, 2014; Lakens, 2022a; Maxwell et al., 2008) has been increasingly discussed as a useful strategy for empirical research, all available software packages for sample size planning with GLMMs are based on power analysis. Therefore, researchers that want to want to apply an estimation strategy in their studies, without testing any statistical hypotheses, currently cannot use the software packages outlined in TABLE 1. However, tailored simulation-based sample size planning can easily handle the planning for precision approach (Maxwell et al., 2008). The only change in procedure is that instead of computing HTs for each simulated dataset and estimating statistical power across repetitions, CIs are computed for each simulated dataset, and the expected width is estimated.

All frameworks for sample size planning require the user to make assumptions about the expected effect size. Assuming the true effect is of this size (or greater), one can compute the (minimum) power of a HT or the (maximum) expected width of a CI. Existing software packages for sample size planning for GLMMs usually require to provide the assumed effect in the unit of some standardized measure of effect size. When the researcher has access to similar studies or pilot data, providing such standardized effect sizes is feasible. However, note that choosing effect sizes based on small pilot studies is generally not recommended, as those estimates can be heavily biased (Albers & Lakens, 2018; Lakens, 2022a). Providing an informed standardized effect size can be an almost impossible challenge when no prior studies or pilot data are available. This problem is further exacerbated by the fact that GLMMs are so flexible that general heuristics of what should be considered a small effect do not exist or are difficult to defend. In our experience, using domain knowledge to construct a tailored data simulation is the only solution to determine plausible effect sizes in the absence of prior evidence. It would be possible to use these tailored simulations to extract plausible values for standardized effect sizes that could then be inserted in existing software packages for sample size planning. However, we would argue that when tailored data simulations necessary to determine effect sizes anyway, performing the whole sample size planning in a customized way is preferred over using the existing software packages.

### **General steps in tailored simulation-based sample size planning**

Although the details differ depending on the specific study characteristics, each tailored simulation-based sample size planning requires a series of steps and decisions. We will introduce each step in a theoretical section, followed by the practical application based on a hypothetical case study. All code in this manuscript and simulation results are available in the project's repository on the Open Science Framework (<https://osf.io/dhwf4/>).

#### **Step 1: Define the estimand**

##### ***THEORY***

The first step in every research process should be a clear definition of the theoretical *estimand* (Lundberg et al., 2021), i.e. the theoretical quantity which is necessary to answer a

specific research question. The estimand consists of a quantity, that can be described for each unit under investigation and a clear definition of the target population, for which the quantity is of interest. For example, an estimand might be the probability that a clinical psychologist makes the correct diagnosis for a psychiatric patient with major depression, averaged across all clinical psychologists and depressed patients in psychiatric institutions in Germany.

The estimand should always be defined outside of any statistical model, because there are usually a range of statistical methods that can be used to estimate the same estimand, depending on the study design (e.g., a randomized experiment) that will produce the observed data in the planned study. For many common research questions in psychology, the estimand can be expressed as a statistical quantity that can be estimated with a regression model, for example a single  $\beta$  coefficient. However, this is not possible for all estimands, which is why the literature discusses many estimation strategies beyond regression (Deffner et al., 2022; Lundberg et al., 2021).

## **PRACTICE**

In the present hypothetical case study, we consider the effectiveness of feedback provided by an artificial intelligence (AI) embedded in a diagnostic decision support system. The context is a clinical setting, where expert radiologists and students under training must detect bleeding based on head scans from computer tomography (CT). In the investigated AI-enabled diagnostic decision support system, an AI model can provide initial diagnostic advice, which can be used as guidance by the humans who are required to make the final diagnostic decision. The research goal is to validate the effectiveness of the AI-enabled advice. We consider the AI-enabled advice as effective, if the following pattern holds, which we will first describe verbally:

*We expect that for BOTH expert radiologists and medical students, correct AI-advice leads to a higher probability of accurately diagnosing a CT scan compared to no AI-advice presented, AND, we expect that for BOTH experts and non-experts, incorrect advice leads to a lower probability of accurately diagnosing a CT scan compared to no advice presented.*

It becomes clear that our estimand consists of four comparisons between experimental

conditions (Lundberg et al., 2021). However, the verbal description is still somewhat vague, which is why we try to give a more precise expression for each comparison:

$$P(\text{correct diagnosis}|\text{correct advice, average expert, average scan})$$

$$- P(\text{correct diagnosis}|\text{no advice, average expert, average scan})$$

$$P(\text{correct diagnosis}|\text{no advice, average expert, average scan})$$

$$- P(\text{correct diagnosis}|\text{wrong advice, average expert, average scan})$$

$$P(\text{correct diagnosis}|\text{correct advice, average student, average scan})$$

$$- P(\text{correct diagnosis}|\text{no advice, average student, average scan})$$

$$P(\text{correct diagnosis}|\text{no advice, average student, average scan})$$

$$- P(\text{correct diagnosis}|\text{wrong advice, average student, average scan})$$

For example, the first expression is concerned with the difference between the probability that a correct diagnosis is made if correct AI-advice is presented and the probability that a correct diagnosis is made if no AI-advice is presented. This contrast is quantified for a hypothetical *typical* expert and a *typical* scan, where *typical* is usually defined as an average score on all attributes of the expert or scan.<sup>1</sup> To complete our definition of the estimand, we have to define our target population that consists of persons, stimuli, and AI-advice: With respect to persons, we are only interested in expert radiologists and medical students at German universities. With respect to stimuli, we are only interested in the head CT scans made from subjects that do or do not suffer from intracerebral hemorrhage. Lastly, we are only interested in AI-advice given by a specific AI-enabled diagnostic decision support system.

Although the estimand is initially defined outside of any statistical model, it is only useful if we find a way to estimate it based on observed data. For our exemplary research question, it is

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<sup>1</sup> Note that a different estimand would be the so-called average treatment effect (ATE). For the ATE, the probability contrast is defined for each combination of expert and scan, and then these contrasts are averaged across all experts and scans from the target population (Lundberg et al., 2021).

possible to construct an experimental study, where all participants are confronted with the same set of head CT scans, but the kind of AI-advice given for each scan is randomly assigned within participants. This random intervention allows us to produce an empirical estimate of our estimand, although, in reality, each person receives only one kind of AI-advice (correct advice, wrong advice, no advice) for each scan. We will see later how each of the probability expressions in our estimand can be modeled with the same GLMM. Estimating this GLMM based on the data observed in our planned study will produce an estimate for each probability, and these estimates can then be combined to compute an estimate for each of the four probability contrasts. For pedagogical reasons, we will skip the concrete definition of our estimate until we have discussed how to simulate data based on a concrete GLMM in the next section.

## **Step 2: Simulate the data generating process**

**THEORY..** When the estimand has been defined, the next step in the research process is to write code that simulates the (assumed) data generating process of the planned study. This requires specifying a generative process for all predictor variables used in the final data analysis. While such assumptions can be quite challenging for observational studies or continuous predictor variables, this is less of a problem for experimental studies with only categorical predictor variables. When all predictor variables have been simulated, one can use the structure of a suitable GLMM to simulate the dependent variable. To simulate the GLMM, one requires plausible values for all model parameters. We will discuss strategies on how these values can be obtained later. Because we have full control over the data generating process in a tailored simulation, it is possible to model specific aspects of the planned study, like data missing by design or assuming that some subjects might drop out. The quality of the results of the sample size planning crucially depends on the plausibility of the simulated data generating process. However, we would argue that even a strongly simplified data generating process (e.g. only a small number of interaction effects; only random intercepts and no random slopes; assuming that data is missing completely at random) can yield informative results.

**PRACTICE..** In our case study, we simulate data for an experiment where the diagnostic performance of users of an AI-enabled diagnostic decision support system will be evaluated. Radiologists (task experts) and students/interns (non experts) review a series of head computer tomography (CT) scans to assess the presence of a bleeding. An AI model provides initial diagnostic advice to assist their decision-making. In the control condition, no AI advice is presented. When AI advice is given, this advice can be either correct or incorrect. The type of advice (no advice, wrong advice, correct advice) is randomized within subjects across brain scans. After reviewing a CT scan, participants deliver a medical diagnosis (bleeding or no bleeding), which may be either accurate or inaccurate. This experimental design introduces some missing values by design since the advice is neither correct nor incorrect when no advice is present, which must be taken into account when simulating and analyzing the data. In this example, recruiting task experts (i.e., radiologists) is more challenging due to their limited availability, while non-experts (i.e., students/interns) are more readily accessible. The goal of simulation-based sample size planning is to determine how many task experts and non-experts must be recruited to achieve sufficient statistical power or precision in the planned experiment.

**Our specific GLMM..** In a GLMM, the expected value of the dependent variable  $Y$  conditioned on the vector of predictor variables  $\mathbf{X}$  and random effects  $\mathbf{U}$ , transformed by a link function  $g()$  is modeled as a linear combination  $\eta$  of the predictor variables  $\mathbf{X}$ , the random effects  $\mathbf{U}$  and the model parameters  $\beta$  ([Fahrmeir et al., 2021](#)):

$$g(E(Y|\mathbf{X} = \mathbf{x}, \mathbf{U} = \mathbf{u})) = \eta$$

Equivalently, the conditional expected value is modeled as the linear combination  $\eta$ , transformed by the inverse link function  $g^{-1}()$ :

$$E(Y|\mathbf{X} = \mathbf{x}, \mathbf{U} = \mathbf{u}) = g^{-1}(\eta)$$

If the dependent variable (i.e., diagnostic decision)  $Y$  is a binary variable with values 0 (i.e., inaccurate), or 1 (i.e., accurate), the conditional expected value is equivalent to the probability:

$$P_{si} := P(Y = 1|\mathbf{X} = \mathbf{x}, \mathbf{U} = \mathbf{u})$$



In our case study,  $P_{si}$  is the conditional probability that subject  $s$  gives the correct response to item (i.e., CT scan)  $i$ .

In such a setting, we model this probability as

$$P_{si} = \text{inverse\_logit}(\eta_{si})$$

with the inverse-logit link  $g^{-1}(\eta_{si}) = \text{inverse\_logit}(\eta_{si}) = \frac{\exp(\eta_{si})}{1+\exp(\eta_{si})}$  or equivalently

$$\text{logit}(P_{si}) = \eta_{si}$$

with the logit link  $g(P_{si}) = \text{logit}(P_{si}) = \ln\left(\frac{P_{si}}{1-P_{si}}\right)$ .

In our case study, the probability of making an accurate diagnostic decision is assumed to depend on the predictors:

- $\text{advice\_present}_{si}$ : whether subject  $s$  was presented with AI advice (1) or not (0) when asked to assess item  $i$
- $\text{advice\_correct}_{si}$ : whether this advice was correct (1) or not (0)
- $\text{expert}_s$ : whether subject  $s$  was a task expert (1) or not (0)

and the random effects:

- $u_{0s}$ : the deviation of subject  $s$  from the average ability to solve an item (i.e., CT scan) with average difficulty; assumed to be distributed as  $u_{0s} \sim N(0, \sigma_S^2)$
- $u_{0i}$ : the deviation of item (i.e., CT scan)  $i$  from the average difficulty to be solved by a person with average ability; assumed to be distributed as  $u_{0i} \sim N(0, \sigma_I^2)$

In total, we assume the model

$$\begin{aligned} \text{logit}[P_{si}] = & (\beta_0 + u_{0s} + u_{0i}) + \\ & \beta_a \cdot \text{advice\_present}_{si} + \beta_c \cdot \text{advice\_correct}_{si} + \beta_e \cdot \text{expert}_s + \\ & \beta_{ea} \cdot \text{expert}_s \cdot \text{advice\_present}_{si} + \beta_{ec} \cdot \text{expert}_s \cdot \text{advice\_correct}_{si} \end{aligned}$$

or equivalently

$$P_{si} = \text{inverse\_logit}[(\beta_0 + u_{0s} + u_{0i}) + \beta_a \cdot \text{advice\_present}_{si} + \beta_c \cdot \text{advice\_correct}_{si} + \beta_e \cdot \text{expert}_s + \beta_{ea} \cdot \text{expert}_s \cdot \text{advice\_present}_{si} + \beta_{ec} \cdot \text{expert}_s \cdot \text{advice\_correct}_{si}]$$

with model parameters  $\beta_0, \beta_e, \beta_a, \beta_c, \beta_{ea}, \beta_{ec}, \sigma_S$ , and  $\sigma_I$ .

In the GLMM literature, this would be called a binomial GLMM with two random intercepts (for subjects and items), two level-1 predictors (*advice\_present*, *advice\_correct*), one level-2 predictor (*expert*) and two cross-level interactions (*expert · advice\_present*, *expert · advice\_correct*). To limit complexity, we do not consider random slopes, additional predictors or higher-level interactions.

***Simulation function in R.*** The following R function simulates a full dataset structured according to the design of our case study.

```
simulate <- function(n_subjects = 100, n_items = 50,
  b_0 = 0.847, b_e = 1.350, b_a = -1.253, b_c = 2.603,
  b_ea = 0.790, b_ec = -1.393,
  sd_u0s = 0.5, sd_u0i = 0.5, ...){
  require(dplyr)
  require(faux)
  # simulate design
  dat <- add_random(subject = n_subjects, item = n_items) |>
    add_between("subject", expert = c(1, 0), .prob = c(0.25, 0.75)) |>
    mutate(advice_present = rbinom(n(), 1, prob = 2/3)) |>
    mutate(advice_correct = if_else(advice_present == 1L,
                                   rbinom(n(), 1L, prob = 0.8), 0L)) |>
  # add random effects
  add_ranef("subject", u0s = sd_u0s) |>
```

```

add_ranef("item", u0i = sd_u0i) |>
# compute dependent variable
mutate(linpred = b_0 + u0i + u0s +
          b_e * expert + b_a * advice_present + b_c * advice_correct +
          b_ea * expert * advice_present + b_ec * expert * advice_correct) |>
mutate(y_prob = plogis(linpred)) |>
mutate(y_bin = rbinom(n = n(), size = 1, prob = y_prob))

dat
}

```

In the first four lines of the function definition, we set some default parameter values (which we will explain in the next section) and load the packages we use to manipulate and simulate data.<sup>2</sup> In our case study, each subject (`n_subjects` in total) is assumed to respond to each item (i.e., CT scan; `n_items` in total). Thus, the `add_random` command creates a fully-crossed `data.frame` with `n_subjects × n_items` rows. We add a between-subject effect with the `add_between` command, simulating that about 25% of subjects are experts. The next two lines simulate that in  $\frac{2}{3}$  of trials, subjects will be presented with AI advice, and if advice is presented, the advice will be correct in about 80% of cases (the variable `advice_correct` is always 0 when no advice is presented). Next, we simulate one random effect for each subject (`u0s`) and for each item (`u0i`). As assumed by standard GLMMs, the `add_ranef` function draws the random effects from a normal distribution with a mean 0 and a standard deviation specified by the user. With all design variables done, we are ready to simulate our model equation outlined in the last section. The linear predictor variable `linpred` ( $\eta$  in the GLMM model equations) combines the predictor variables, random effects, and model parameters as assumed by our model. We then transform the linear predictor with the inverse-link function to compute `y_prob`,

---

<sup>2</sup> The `faux` package (DeBruine, 2023) contains useful functions when simulating factorial designs, including random effects.

the probability that the subject correctly solved the item (in R, the inverse-logit link is computed with `plogis` and the logit link with `qlogis`). In the final step, we simulate the binary dependent variable `y_bin` (i.e., whether the subject makes an accurate diagnostic decision for the CT scan) by – for each trial – drawing from a Bernoulli distribution with success probability `y_prob`.

### **Step 3: Specify the population parameters**

#### ***THEORY***

In the absence of previous studies with the same design or pilot data, researchers require strategies on how to specify the population parameters used in their data simulation. Population parameters are all model parameters estimated in a GLMM, in particular the regression coefficients of the fixed effects and the standard deviation of the random effects (and the correlation between random effects in more complicated models). In contrast to non-hierarchical linear regression, common heuristics based on standardized effect sizes are less useful or not even available for GLMMs. Our strategies to specify population parameters will require access to domain knowledge from domain experts. Because most domain knowledge can only be expressed in unstandardized measurement units of a specific application, we argue that unstandardized effect sizes are usually preferable over standardized effect sizes for tailored simulation-based sample size planning. The basic idea of all strategies is how the data generating process implied by certain values of population parameters can be quantified or visualized in an intuitive way that enables a calibration of population parameters based on the available knowledge of domain experts.

Although we use frequentist model estimation in our tutorial, many strategies described in this chapter are inspired by research on how to monitor the plausibility of model assumptions in applied Bayesian statistics ([Gelman et al., 2020](#)).

#### ***PRACTICE***

When introducing the simulation function for our case study, we have used theoretically plausible values as defaults for all model parameters ( $\beta_0$ ,  $\beta_e$ ,  $\beta_a$ ,  $\beta_c$ ,  $\beta_{ea}$ ,  $\beta_{ec}$ ,  $\sigma_S$ , and  $\sigma_I$ ), but have not talked about where these numbers came from.

The starting point for all parameter values in our present case study were based on results

from distantly related study designs in the literature. Additionally, we had repeated discussions with our affiliated domain experts in radiology to check whether our assumptions seem plausible.

We now outline our main strategy to determine plausible parameter values for the fixed effects ( $\beta$  parameters): Unfortunately, the model parameters in a binomial GLMM are hard to interpret in isolation because 1) the  $\beta$  parameters are connected to the modeled probability via the non-linear inverse-logit link, and 2) we also have to consider the random effects. The most simple interpretation, that allows us to ignore the random effects for now, works by imagining a subject with average ability ( $u_{0s} = 0$ ) responding to an item (i.e., CT scan) with average difficulty ( $u_{0i} = 0$ ). Then the model implied probability that such a person solves such an item accurately is given by:

$$\begin{aligned} P(Y = 1 | \mathbf{X} = \mathbf{x}, \mathbf{U} = \mathbf{0}) = \\ = \text{inverse\_logit}[\beta_0 + \beta_a \cdot \text{advice\_present}_{si} + \beta_c \cdot \text{advice\_correct}_{si} + \beta_e \cdot \text{expert}_s + \\ \beta_{ea} \cdot \text{expert}_s \cdot \text{advice\_present}_{si} + \beta_{ec} \cdot \text{expert}_s \cdot \text{advice\_correct}_{si}] \end{aligned}$$

In fact, we would only need the full equation if the subject is an expert and correct advice is presented. In all other experimental conditions, some terms drop from the equation because they are multiplied by 0. For example, the probability that a non-expert with average ability solves an item with average difficulty when no advice is presented, only requires the intercept:

$$\begin{aligned} P(Y = 1 | \text{advice\_present} = 0, \text{advice\_correct} = 0, \text{expert} = 0, u_{0s} = 0, u_{0i} = 0) = \\ = \text{inverse\_logit}[\beta_0] \end{aligned}$$

We can revert this perspective by choosing plausible probability values based on domain knowledge and deriving the parameter values implied by these probabilities for each experimental condition.

Table 1 shows our set of assumptions concerning the probability that an average subject solves an average item for each experimental condition, as well as the corresponding equations implied by the model. The table can be used to compute the implied values for the  $\beta$  parameters,

**Table 1**

*Assumed probabilities that an average subject solves an average item in each experimental condition.*

Experimental		
condition	$P(Y = 1 \mid \mathbf{X} = \mathbf{x}, \mathbf{U} = \mathbf{0})$	Implied equation
no advice, no expert	0.70	$\text{logit}(0.70) = \beta_0$
no advice, expert	0.90	$\text{logit}(0.90) = \beta_0 + \beta_e$
false advice, no expert	0.40	$\text{logit}(0.40) = \beta_0 + \beta_a$
false advice, expert	0.85	$\text{logit}(0.85) = \beta_0 + \beta_e + \beta_a + \beta_{ea}$
correct advice, no expert	0.90	$\text{logit}(0.90) = \beta_0 + \beta_a + \beta_c$
correct advice, expert	0.95	$\text{logit}(0.95) = \beta_0 + \beta_e + \beta_a + \beta_c + \beta_{ea} + \beta_{ec}$

*Note.* Implied equations are derived based on the model equations and setting all random intercept terms to 0.

starting with the first equation and reinserting the computed  $\beta$  values in all following equations (b\_0 stands for the intercept  $\beta_0$ , b\_e for the slope  $\beta_e$ , and so on):

```

b_0 <- qlogis(0.7)
b_e <- qlogis(0.9) - b_0
b_a <- qlogis(0.4) - b_0
b_ea <- qlogis(0.85) - b_0 - b_e - b_a
b_c <- qlogis(0.9) - b_0 - b_a

```

```
b_ec <- qlogis(0.95) - b_0 - b_e - b_a - b_c - b_ea
c(b_0 = b_0, b_e = b_e, b_a = b_a, b_c = b_c, b_ea = b_ea, b_ec = b_ec)
```

b_0	b_e	b_a	b_c	b_ea	b_ec
0.8472979	1.3499267	-1.2527630	2.6026897	0.7901394	-1.3928518

It is always possible to double-check these computations by transforming the parameter values back to probabilities, e.g.

$$P(Y = 1 | expert = 1, advice\_present = 1, advice\_correct = 1, u_{0s} = 0, u_{0i} = 0) = \\ = inverse\_logit[\beta_0 + \beta_e + \beta_a + \beta_c + \beta_{ea} + \beta_{ec}]$$

which we compute in R as:

```
plogis(b_0 + b_e + b_a + b_c + b_ea + b_ec)
```

```
[1] 0.95
```

This leaves us with the question on how to determine plausible values for the two remaining model parameters ( $\sigma_S$ , and  $\sigma_I$ ) that are the standard deviations for the random intercepts. For this, we introduce two more strategies.

### *Insightful descriptive statistics*

**THEORY..** The mathematical structure of GLMMs determines which patterns in data would be produced by the model, if a specific set of values for the population parameters would be specified. The knowledge of how to simulate from a GLMM enables us to compute insightful descriptive statistics that can be compared to available domain knowledge much more easily than the opaque values of model parameters. For example, domain experts might not be able to directly choose plausible values for the coefficients in a logistic regression model (which are measured on the log-odds scale). However, they should be able to reason about the expected ratio of the binary dependent variable in different experimental conditions, i.e. which relative frequency they expect

to observe (on average). The job of the analyst who is familiar with the mathematical structure of the GLMM is to produce the model implied value of the insightful descriptive statistic expected by the domain expert. Although insightful descriptive statistics usually depend on the model parameters in a non-linear way, it is not necessary to solve the exact relationship mathematically. Instead, one can simply adjust the population parameters by trial and error until the model implied quantities produce the desired result.

**PRACTICE..** In the last section, we showed how we can derive the model implied probability that a subject with average ability solves an item with average difficulty for each experimental condition. Although these derivations are straightforward, it is important not to misinterpret their implications: In binomial GLMMs, the average probability to solve an item (averaged across persons of varying ability and items of varying difficulty) is **not** equal to the probability that a person with average ability solves an item with average difficulty ([Fahrmeir et al., 2021](#)). The first perspective implies a so-called marginal interpretation, while the second one implies a conditional interpretation.

For example, we determined the  $\beta$  parameters in a way that corresponds to a desired conditional probability of 0.95, that an expert with average ability solves an item with average difficulty when presented with correct advice (the conditional perspective). However, even if the model were true, we would not observe that 95% of experts responding to items presented with correct advice from a big sample of subjects drawn from their natural distribution of ability, and items drawn from their natural distribution of difficulty (the marginal perspective). How much the two probabilities differ depends on the standard deviations of the random intercepts (the two probabilities are only equal if both standard deviations would be zero). We want to use the model implied observed proportion of correct diagnoses in each experimental condition as an insightful descriptive statistics to determine plausible values for the random effect standard deviations.

We will simulate a large dataset (for which the observed values of the descriptive statistic will be close to their model implied true values) and simply compute the relative frequency of correct diagnoses for each experimental condition.



```

library(tidyverse)
set.seed(1)
dat <- simulate(n_subjects = 3000, n_items = 3000,
  sd_u0s = 0.5, sd_u0i = 0.5)
dat |>
  mutate(condition = fct_cross(
    factor(expert), factor(advice_present), factor(advice_correct))) |>
  mutate(condition = fct_recode(condition,
    "no expert, no advice" = "0:0:0", "expert, no advice" = "1:0:0",
    "no expert, wrong advice" = "0:1:0", "expert, wrong advice" = "1:1:0",
    "no expert, correct advice" = "0:1:1", "expert, correct advice" = "1:1:1")) |>
  group_by(condition) |>
  summarize(relative_frequency = sum(y_bin) / n())

```

# A tibble: 6 x 2

condition	relative_frequency
<fct>	<dbl>
1 no expert, no advice	0.683
2 expert, no advice	0.881
3 no expert, wrong advice	0.409
4 expert, wrong advice	0.828
5 no expert, correct advice	0.883
6 expert, correct advice	0.938

We tried using these descriptive statistics to judge together with domain experts whether our chosen values for the random effect standard deviations would produce data that align with out domain expertise. However, although the result was deemed plausible, these statistics were not informative enough to determine a final set of plausible parameter values. For this reason, we will

additionally look at insightful model based quantities.

### *Insightful model based quantities*

**THEORY..** Because GLMMs are complicated models, descriptive statistics alone are usually not enough to specify plausible values for all model parameters. This is especially true for the standard deviation of random effects that have non-linear (and often unexpected) effects on the model-implied results. An important advantage of data simulation (where one has full control over parameter values and sample sizes) is that one can produce insightful model based quantities that can never be directly observed in an actual empirical dataset. For example, in a logistic model with random intercepts for participants, one can produce a visualization of the implied distribution of the probability that a participant on average solves a cognitive task. Although domain knowledge will probably not suffice to specify this distribution completely, it should be possible to rule out implausible boundary conditions. For example, the domain expert might deem it implausible that the 5% most able participants have a probability of more than 0.99 to solve the difficult cognitive task.

**PRACTICE..** The discussed inequality of conditional and marginal effects in GLMMs ([Fahrmeir et al., 2021](#)) makes their interpretation more difficult. One must be careful when specifying parameter values based on previous studies or pilot data that use the marginal interpretation (e.g., a pilot study providing an estimate of how often radiologists make an accurate diagnosis based on brain scans). However, this does not mean that we cannot use the marginal interpretation (average probability across persons and items) to inform plausible parameter values: When parameter values have been selected, we can compute the implied marginal distributions and compare this information to our domain knowledge. Then, we can iteratively adjust the parameter values until we are satisfied with the implied distributions. In the last section, we simulated a large dataset and computed descriptive statistics, the relative frequencies of correct diagnoses, for each experimental condition. We will now use the model implied probability of each simulated data point (stored in the variable `y_prob`) to visualize the whole model implied marginal distribution of correct diagnoses for each experimental condition.

```
library(ggdist)

dat |>
  mutate(condition = fct_cross(
    factor(expert), factor(advice_present), factor(advice_correct))) |>
  mutate(condition = fct_recode(condition,
    "no expert, no advice" = "0:0:0", "expert, no advice" = "1:0:0",
    "no expert, wrong advice" = "0:1:0", "expert, wrong advice" = "1:1:0",
    "no expert, correct advice" = "0:1:1", "expert, correct advice" = "1:1:1")) |>
  ggplot(aes(x = y_prob, y = condition)) +
  stat_histinterval(point_interval = "mean_qi", slab_color = "gray45",
    breaks = "Sturges") +
  scale_x_continuous(breaks = seq(0, 1, 0.1), limits = c(0, 1))
```

**Figure 1**

*Marginal distributions including means, 66% and 95% confidence intervals for all experimental conditions.*

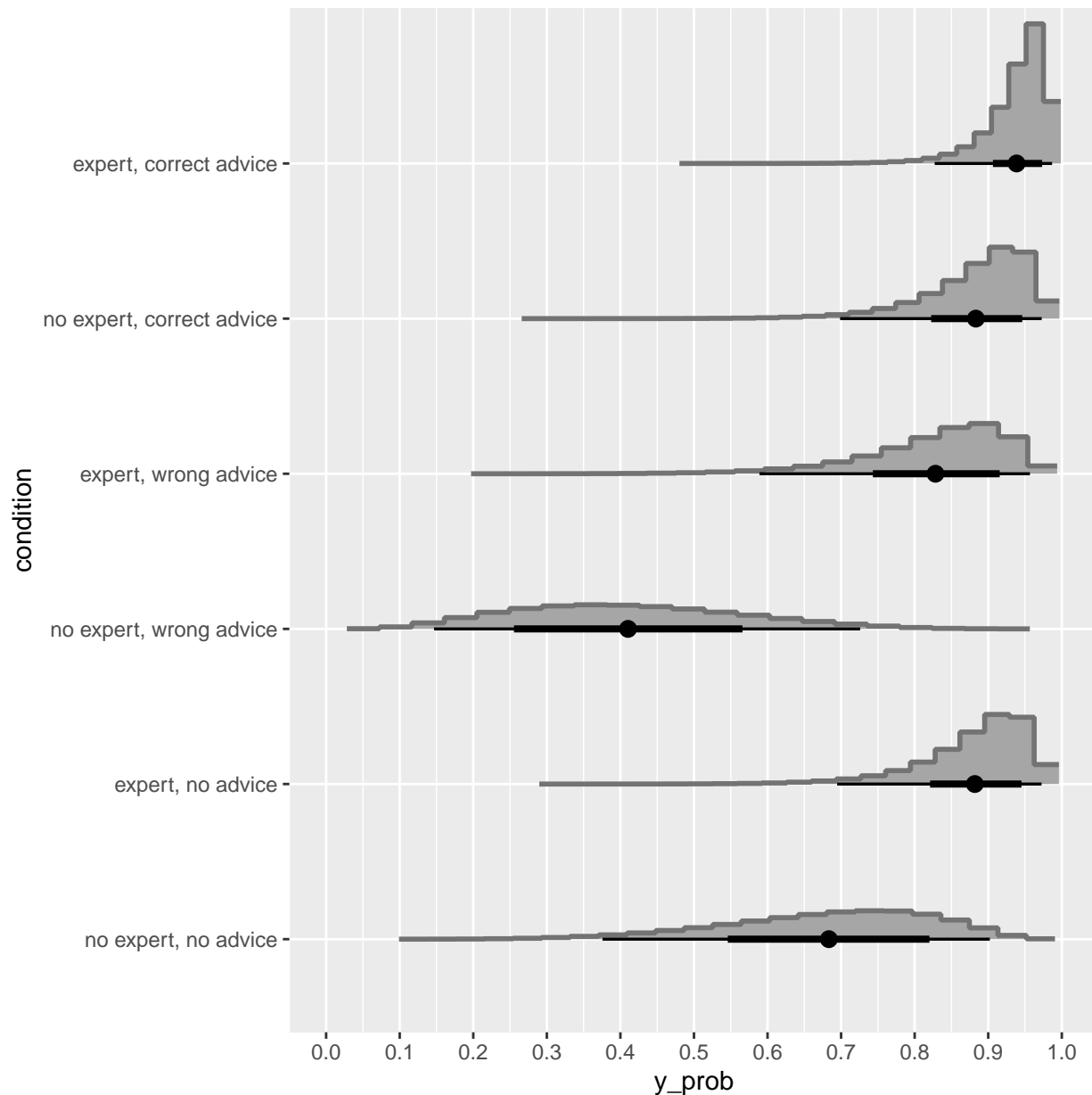
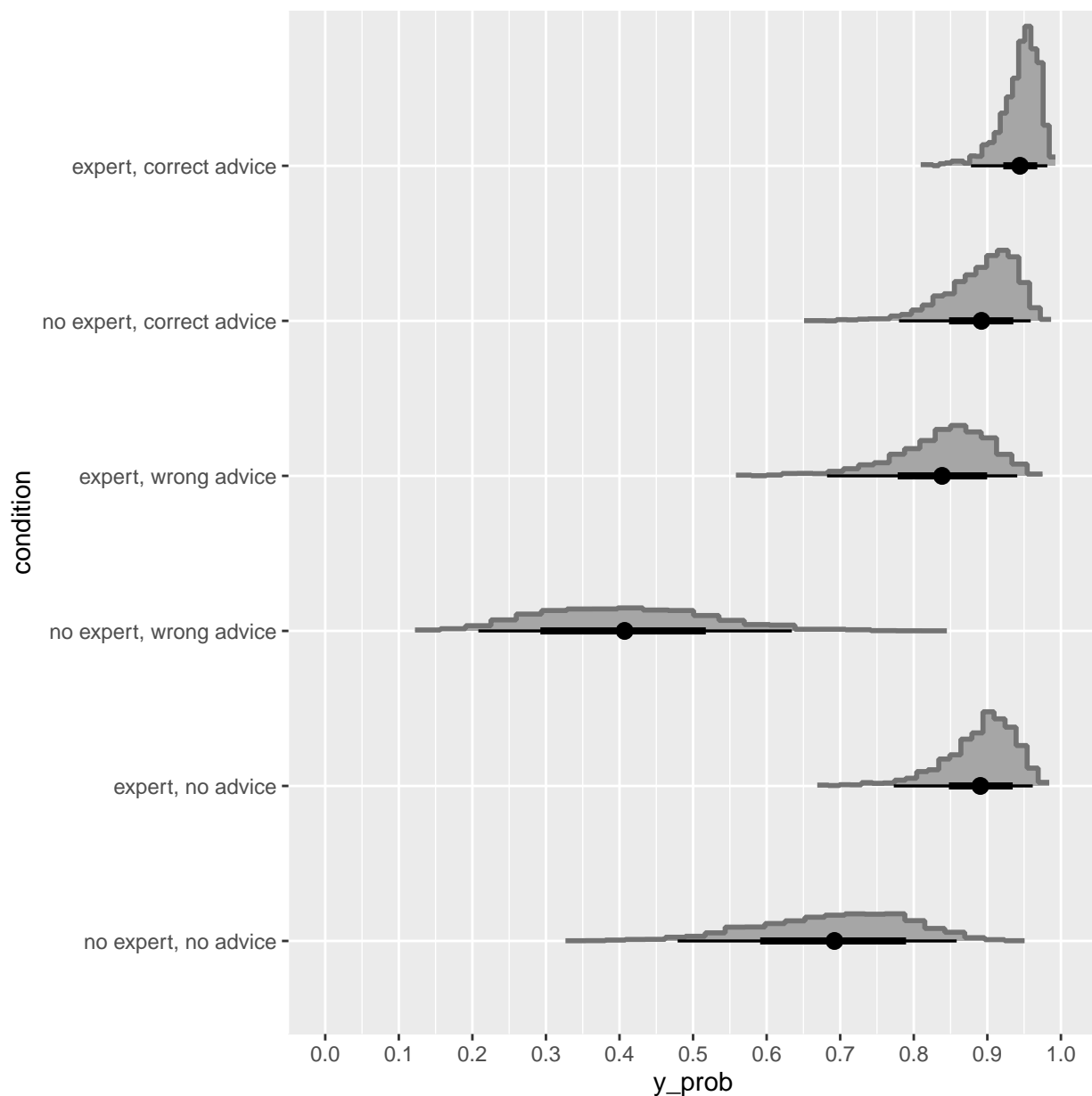


Figure 1 shows the model implied marginal distributions, including the mean, 66% and 95% intervals. We can see that, indeed, the average probabilities (black dots) slightly differ from the probabilities of average subjects and items considered in the previous section. This difference increases with the variability of the random effects. We can use plots like the one above as a

useful tool to decide whether the specified standard deviations of the subject and item random intercepts ( $\sigma_S$  and  $\sigma_I$ ) are reasonable by comparing the ranges and overlap between conditions to domain knowledge.

**Figure 2**

*Marginal distributions including means, 66% and 95% confidence intervals for all experimental conditions while setting the standard deviation of item random intercepts to 0.01.*

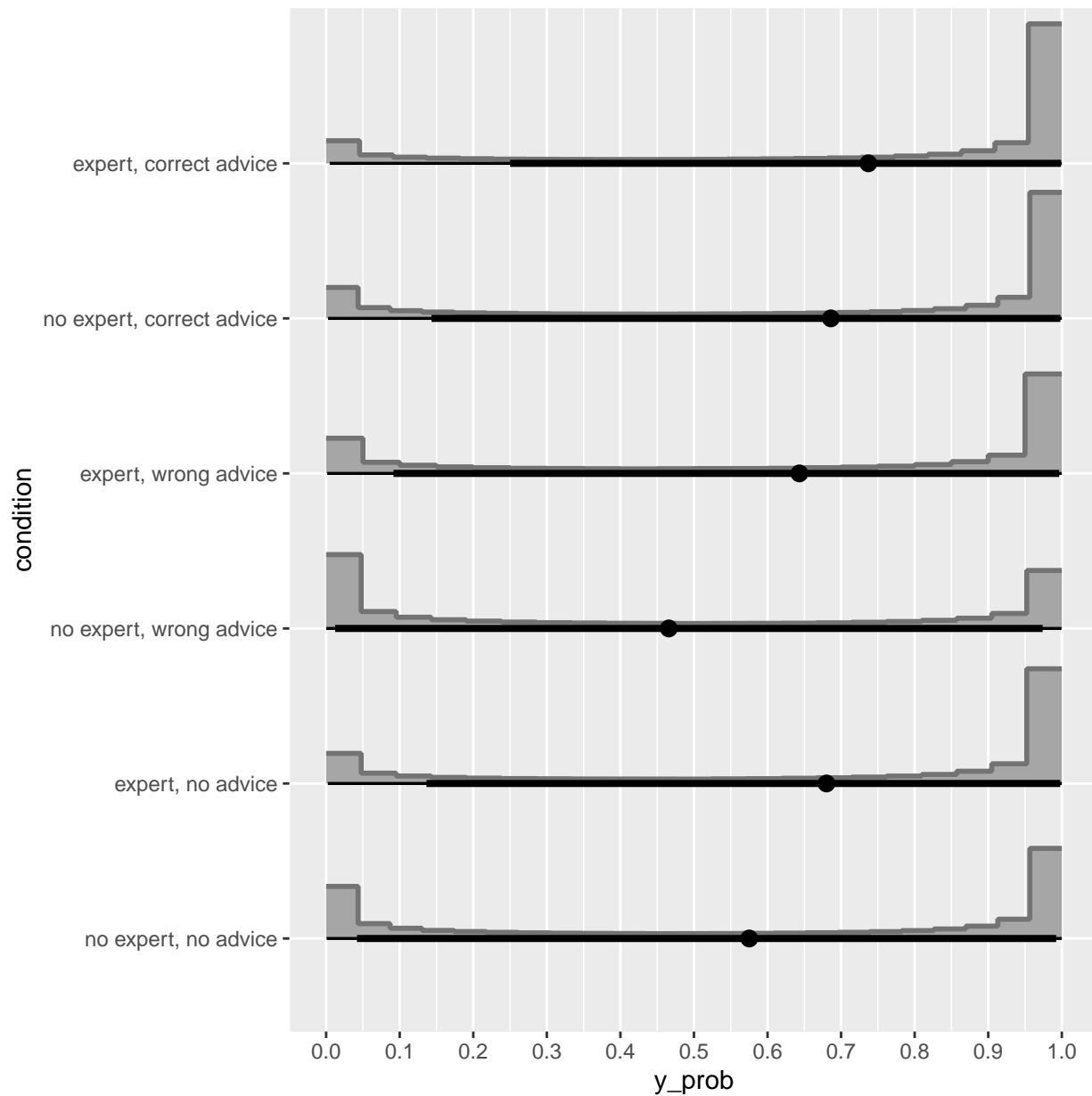


In the next plot, we have set the item standard deviation to almost zero ( $\sigma_I = 0.01$ ). This gives us a better way to see the variability between persons. As an example, Figure 2 reveals a number of implicit assumptions about the comparison between experts and non-experts: With wrong advice, virtually all experts have a higher probability of making a correct diagnosis compared to non-experts when considering only items with average difficulty. In contrast, there is considerable overlap in probability between experts and non-experts with no advice and even higher overlap with correct advice. Patterns like these should be considered carefully and discussed with the domain experts. Parameter values ( $\beta$  parameters, and  $\sigma_S$ ) should be adjusted if the implications do not seem reasonable. We could also have a closer look at variability between items by setting the subject standard deviation to almost zero ( $\sigma_S = 0.01$ ).

The final plot demonstrates that these plots are also useful for spotting standard deviations that are specified too high. For Figure 3 we have set  $\sigma_S = 3$  and  $\sigma_I = 3$ . This implies that in each experimental condition, the probabilities that a subject solves an item are usually close to either 0 or 1, which is not a plausible assumption. These high standard deviations do not account for the inherent variability and complexity of human performance. For example, we would expect that a participant with low ability compared to other task experts to solve a difficult item with a probability substantially larger than zero even when presented with wrong advice.

**Figure 3**

*Marginal distributions including means, 66% and 95% confidence intervals for all experimental conditions while setting the standard deviation of subject and item random intercepts to 3.*



### *Iterative process with domain experts*

**THEORY..** In our experience, the gathering of domain knowledge by domain experts and the consecutive specification of population parameter values used in data simulation is not a

one-time event but rather an iterative process. In a first step, domain experts can be interviewed to “elicit” their domain knowledge about how the future data of the planned study is expected to look like. As most domain experts are no experts in statistical modeling and GLMMs, they often struggle without further guidance to communicate their knowledge in a way that is useful when specifying the parameters for data simulation. For this reason, we suggest that after an initial unstructured interview of domain experts, the analyst who is familiar with the structure of the GLMM under study selects an initial set of insightful descriptive statistics and model based quantities. Then they reenter into an iterative discussion where some set of population values are selected and the plausibility of resulting implied quantities are discussed with the domain experts. Then, the population parameters are updated based on this discussion until the domain experts are satisfied with the final result. During this process, the monitored model based quantities and descriptive statistics can be updated or extended to capture as much available domain knowledge as possible.

***PRACTICE..*** All parameter values in our present case study have been determined based on repeated discussions with our affiliated domain experts in Radiology to validate our assumptions. Initially, we reviewed the literature to establish a reasonable baseline performance rate for examining head CT scans for intracranial hemorrhage. Existing studies indicate that radiologists typically demonstrate high accuracies, often exceeding or hovering around 90%, while interns have been shown to perform below 80%, and medical students fall even shorter. For simplicity, we assumed plausible probability values of .90 for experts and .70 for non-experts, respectively. Our experts confirmed that these values are realistic baselines for reviewing diverse head CT images without AI assistance. Subsequently, we consulted several published papers investigating the effect of correct and incorrect advice on decision-making performance in other settings. From their findings, we inferred that both experts and non-experts should benefit from correct and suffer losses from incorrect advice. However, the magnitude of these effects should be substantially greater for non-experts, given their demonstrated reliance on advice compared to experts. We further validated the plausibility of our estimated gains and losses with the



collaborating radiologists. For our simulation, we used the probabilities of average participants to solve an average case, as shown in Table 1.

#### **Step 4: Estimate the statistical model**

##### ***THEORY***

At this point, the researcher is capable of producing a simulated dataset similar to the actual dataset that will later be collected in the planned study. The next step is to specify how the statistical model shall be estimated in the actual study collected later. This usually includes the selection of 1) a statistical framework (e.g., frequentist statistics), 2) a software package that is capable of estimating the model class of interest (e.g., the lme4 R package), 3) an estimation algorithm (e.g., the default optimizer “bobyqa”), and 4) the specific model structure including all fixed effects, random effects, and the model family of the dependent variable.

Note that this does not always mean that one will specify the same GLMM that was used when specifying the data generating process. On the one hand, using a simpler model for data simulation than for model estimation can be a useful strategy in scenarios where making plausible assumptions for complicated random effect structures and interactions is not feasible. On the other hand, using a more complex model for data simulation than for model estimation can be a useful strategy in scenarios where one has specific domain knowledge about aspects of the data generating process that are still difficult to estimate with the current state-of-the-art in multilevel modeling.

##### ***PRACTICE***

In our case study, we use the lme4 R package ([Bates et al., 2015](#)), which is a state-of-the-art tool for fitting frequentist GLMMs.<sup>3</sup> For the current example, we simulate data according to our model, in which 100 subjects respond to 50 items (we use `set.seed` to make the simulation reproducible). However, for the sake of the exercise, we can imagine that this would be real data resulting from our future experiment and think about how we would analyze this data.

---

<sup>3</sup> For Bayesian GLMMs, the brms R package is currently the most prominent option ([Bürkner, 2017](#)).

```
library(tidyverse)
set.seed(1)
dat <- simulate(n_subjects = 100, n_items = 50)
```

The lme4 package uses a special syntax for model specification. Our specific GLMM is represented by the formula:

```
library(lme4)
f <- y_bin ~ 1 + expert + advice_present + advice_correct +
  expert:advice_present + expert:advice_correct +
  (1|subject) + (1|item)
```

The first two lines look similar to any linear model in R (general intercept indicated by 1; main effects indicated by variable names in the dataset; interactions indicated by variable1:variable2). The third line specifies a random intercept for each subject (1|subject) and for each item (1|item). The complete set of rules for the syntax is outlined in Bates et al. (2015) and in the documentation of the lme4 package.

In lme4, a GLMM is fitted with the glmer function. By setting family = "binomial", we request a binomial GLMM appropriate for our binary dependent variable y\_bin (the binomial GLMM uses the canonical logit link by default), which is defined as an accurate (1) vs. inaccurate (0) diagnosis.

```
fit <- glmer(f, data = dat, family = "binomial")
```

We can inspect the estimates for all model parameters with the summary command:

```
summary(fit)
```

Generalized linear mixed model fit by maximum likelihood (Laplace  
Approximation) [glmerMod]

Family: binomial ( logit )

Formula:

y\_bin ~ 1 + expert + advice\_present + advice\_correct + expert:advice\_present +  
expert:advice\_correct + (1 | subject) + (1 | item)

Data: dat

AIC	BIC	logLik	deviance	df.resid
4149.4	4201.6	-2066.7	4133.4	4992

Scaled residuals:

Min	1Q	Median	3Q	Max
-5.7669	0.2125	0.3046	0.4317	2.1056

Random effects:

Groups	Name	Variance	Std.Dev.
subject	(Intercept)	0.3148	0.5611
item	(Intercept)	0.1624	0.4029

Number of obs: 5000, groups: subject, 100; item, 50

Fixed effects:

	Estimate	Std. Error	z value	Pr(> z )
(Intercept)	1.0339	0.1103	9.374	< 2e-16 ***
expert	1.1849	0.2096	5.654	1.56e-08 ***
advice_present	-1.3436	0.1206	-11.143	< 2e-16 ***
advice_correct	2.6154	0.1273	20.540	< 2e-16 ***
expert:advice_present	1.0589	0.2940	3.601	0.000317 ***
expert:advice_correct	-1.8104	0.2915	-6.210	5.29e-10 ***

```

---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Correlation of Fixed Effects:
              (Intr) expert advc_p advc_c exprt:dvc_p
expert          -0.377
advic_prsnt     -0.349  0.176
advic_crrct     0.023  0.001 -0.668
exprt:dvc_p     0.143 -0.448 -0.412  0.276
exprt:dvc_c    -0.008  0.004  0.292 -0.435 -0.686

```

In the lme4 output, the Estimate column in the Fixed effects table contains the estimates for the  $\beta$  parameters, while the Std.Dev. column in the Random effects table contains the estimates for  $\sigma_S$  and  $\sigma_I$ .

### Step 5: Compute the estimate

#### *THEORY*

In previous steps, we have defined the theoretical estimand, written a data simulation function and specified how to estimate a GLMM using simulated data. The next step is to specify how to compute a concrete point estimate of the theoretical estimand within the framework of the fitted GLMM. For some research question, the estimate corresponds with a single regression coefficient. In more complicated scenarios, the estimate is computed from a combination of coefficients. Beyond computing the point estimate, we have already discussed that both hypothesis testing and interval estimation can be used to answer the research question. The decision on testing or estimating is then followed by selecting the specific statistical method that shall be applied to compute the HT(s) or CI(s) (e.g., compute HTs and CIs with the `marginalEffects` R package using the delta method).

**PRACTICE**

In the estimand section, we have translated a verbal description of our research question into four probability statements that are specified outside of any specific statistical model. For a concrete estimate within the context of our specified GLMM, we must compute the following probability contrasts:

$$\begin{aligned}
 &P(Y = 1 | \text{advice\_present} = 1, \text{advice\_correct} = 1, \text{expert} = 1, u_{0s} = 0, u_{0i} = 0) \\
 &\quad - P(Y = 1 | \text{advice\_present} = 0, \text{advice\_correct} = 0, \text{expert} = 1, u_{0s} = 0, u_{0i} = 0) \\
 &P(Y = 1 | \text{advice\_present} = 0, \text{advice\_correct} = 0, \text{expert} = 1, u_{0s} = 0, u_{0i} = 0) \\
 &\quad - P(Y = 1 | \text{advice\_present} = 1, \text{advice\_correct} = 0, \text{expert} = 1, u_{0s} = 0, u_{0i} = 0) \\
 &P(Y = 1 | \text{advice\_present} = 1, \text{advice\_correct} = 1, \text{expert} = 0, u_{0s} = 0, u_{0i} = 0) \\
 &\quad - P(Y = 1 | \text{advice\_present} = 0, \text{advice\_correct} = 0, \text{expert} = 0, u_{0s} = 0, u_{0i} = 0) \\
 &P(Y = 1 | \text{advice\_present} = 0, \text{advice\_correct} = 0, \text{expert} = 0, u_{0s} = 0, u_{0i} = 0) \\
 &\quad - P(Y = 1 | \text{advice\_present} = 1, \text{advice\_correct} = 0, \text{expert} = 0, u_{0s} = 0, u_{0i} = 0)
 \end{aligned}$$

We have already discussed how to compute the involved probabilities in the section on specifying population parameters. Plugging in the model equation of the GLMM produces an equation on how to compute each contrast if all model parameters were known. When we want to estimate the above contrasts based on *observed* data, the only difference is that model parameters are not known and we instead use the corresponding parameter *estimates*.

We could use our knowledge of the structure of the GLMM to determine the exact formula needed to compute the contrasts of interest and then plug in the parameter estimates manually from the `summary(fit)` output. However, this would be tedious and we can use R to compute this contrast without doing the math. Using the first contrast (*correct advice, expert*) - (*no advice, expert*) as our example, we could apply the `predict` function of the `lme4` package to compute the

predicted probability for a correct diagnosis based on our fitted model, plug in the two sets of predictor values, and compute the difference between the two estimated probabilities.

```
grid1 <- data.frame(advice_present = c(1, 0), advice_correct = c(1, 0),
  expert = c(1, 1))
```

```
grid1
```

	advice_present	advice_correct	expert
1	1	1	1
2	0	0	1

```
pred <- predict(fit, newdata = grid1, type = "response", re.form = NA)
```

```
pred
```

	1	2
	0.939292	0.901923

```
pred[1] - pred[2]
```

	1
	0.03736901

The argument `type = "response"` specifies that predictions are made on the probability scale (instead of the log-odds scale of the  $\beta$  parameters), while `re.form = NA` sets all random effects to 0. We could use this method to compute point estimates for all four contrasts that are part of our estimand. However, depending on whether we are interested in hypothesis testing or parameter estimation, we also need a method to compute HTs or CI. The `marginalEffects` package ([Arel-Bundock, 2024](#)) is a very flexible, increasingly popular package to compute HTs and CIs for contrasts with a variety of statistical models, including GLMMs estimated with `lme4`. First, we specify a grid of all combinations of predictor variable and then compute estimated probabilities for all experimental conditions in our experiment with the `predictions` function:

```
library(tidyverse)
library(marginaleffects)
library(tinytable)
grid2 <- expand_grid(advice_present = 0:1,
  advice_correct = 0:1, expert = 0:1)
grid2
```

```
# A tibble: 8 x 3
```

	advice_present	advice_correct	expert
	<int>	<int>	<int>
1	0	0	0
2	0	0	1
3	0	1	0
4	0	1	1
5	1	0	0
6	1	0	1
7	1	1	0
8	1	1	1

```
preds <- predictions(fit, newdata = grid2,
  type = "response", re.form = NA)
print(preds, style = "tinytable") |> theme_tt(theme = "resize")
```

The point estimates for all experimental conditions are reported in the Estimate column. Note that the output also contains the two missing by design conditions that will never be observed in the actual study (*advice\_present* = 0, *advice\_correct* = 1, *expert* = 1 and *advice\_present* = 0, *advice\_correct* = 1, *expert* = 0). This is no problem as long as we never interpret those estimates. Next, we use the estimated probabilities to compute the four specific

Estimate	Std. Error	z	Pr(> z )	S	2.5 %	97.5 %	advice_present	advice_correct	expert
0.738	0.02134	34.6	<0.001	867.2	0.696	0.779	0	0	0
0.902	0.01739	51.9	<0.001	Inf	0.868	0.936	0	0	1
0.975	0.00421	231.6	<0.001	Inf	0.966	0.983	0	1	0
0.954	0.01454	65.6	<0.001	Inf	0.925	0.982	0	1	1
0.423	0.03221	13.1	<0.001	128.6	0.360	0.486	1	0	0
0.874	0.02793	31.3	<0.001	711.4	0.819	0.928	1	0	1
0.909	0.00967	94.1	<0.001	Inf	0.890	0.928	1	1	0
0.939	0.01091	86.1	<0.001	Inf	0.918	0.961	1	1	1

Type: response

Columns: rowid, estimate, std.error, statistic, p.value, s.value, conf.low, conf.high, advice\_present, advice\_correct, expert, y\_bin

contrasts that are part of our estimand. For this we must specify which rows in probs have to be subtracted from each other. We will use the `hypotheses` function to compute our four contrasts of interest together with HTs and CIs. We use the default inference options of the `marginalEffects` package that compute HTs and CIs based on the approximate delta method.

```
contrasts <- preds |>
  hypotheses(hypothesis = c(
    "b8 = b2", # (correct advice, expert) - (no advice, expert)
    "b2 = b6", # (no advice, expert) - (wrong advice, expert)
    "b7 = b1", # (correct advice, no expert) - (no advice, no expert)
    "b1 = b5"), # (no advice, no expert) - (wrong advice, no expert)
  equivalence = c(0, 0))
print(contrasts, style = "tinytable") |> theme_tt(theme = "resize")
```

The expression `"b8 = b2"` is special syntax to subtract the estimate in row number 8 from the estimate in row number 2 in the probs-output. The argument `equivalence = c(0, 0)` can



Term	Estimate	Std. Error	z	Pr(> z )	S	2.5 %	97.5 %	p (NonSup)	p (NonInf)	p (Equiv)
b8=b2	0.0374	0.0162	2.31	0.021	5.6	0.00563	0.0691	0.989	0.0105	0.989
b2=b6	0.0282	0.0279	1.01	0.312	1.7	-0.02653	0.0830	0.844	0.1562	0.844
b7=b1	0.1717	0.0173	9.93	<0.001	74.8	0.13780	0.2056	1.000	<0.001	1.000
b1=b5	0.3145	0.0280	11.24	<0.001	95.0	0.25965	0.3693	1.000	<0.001	1.000

Type: response

Columns: term, estimate, std.error, statistic, p.value, s.value, conf.low, conf.high, statistic.noninf, statistic.nonsup, p.value.noninf, p.value.nonsup, p.value.equiv

be used to compute one-sided p-values, testing whether the contrast in the population is smaller than 0 (p (NonSub) column) or greater than 0 (p (NonInf) column). The point estimates for four contrasts are reported in the Estimate column. Note that to facilitate interpretation, we arranged the contrasts in a way that we theoretically expect positive values for all four of them.

**Hypothesis testing.** If we chose hypothesis testing for our case study, we would test a combined null hypothesis  $H_0$  that consists of four separate null hypotheses:

$$H_{01} : P(Y = 1 | advice\_present = 1, advice\_correct = 1, expert = 1, u_{0s} = 0, u_{0i} = 0) \leq$$

$$P(Y = 1 | advice\_present = 0, advice\_correct = 0, expert = 1, u_{0s} = 0, u_{0i} = 0)$$

$$H_{02} : P(Y = 1 | advice\_present = 0, advice\_correct = 0, expert = 1, u_{0s} = 0, u_{0i} = 0) \leq$$

$$P(Y = 1 | advice\_present = 1, advice\_correct = 0, expert = 1, u_{0s} = 0, u_{0i} = 0)$$

$$H_{03} : P(Y = 1 | advice\_present = 1, advice\_correct = 1, expert = 0, u_{0s} = 0, u_{0i} = 0) \leq$$

$$P(Y = 1 | advice\_present = 0, advice\_correct = 0, expert = 0, u_{0s} = 0, u_{0i} = 0)$$

$$H_{04} : P(Y = 1 | advice\_present = 0, advice\_correct = 0, expert = 0, u_{0s} = 0, u_{0i} = 0) \leq$$

$$P(Y = 1 | advice\_present = 1, advice\_correct = 0, expert = 0, u_{0s} = 0, u_{0i} = 0)$$

The combined null hypothesis  $H_0$  should only be rejected if **all** individual null hypotheses are rejected (i.e., intersection-union setting; [Dmitrienko & D'Agostino, 2013](#)). In such cases, the error probabilities do not accumulate, and we would waste power when correcting for multiple tests.

With a standard significance level of  $\alpha = 0.05$ , we would not reject all four null hypotheses

(the p-value in the p (NonInf) column for the second hypothesis is not significant) and therefore also not reject the combined null hypothesis for this particular (simulated) dataset. Note that this decision would be wrong because we have simulated the data such that the combined alternative hypothesis  $H_1$  is actually true in the population.

**Interval estimation.** If we chose parameter estimation for our case study, we would focus on the two-sided CIs of the four contrasts of interest. With a standard confidence level of  $1 - \alpha = 0.95$ , plausible values are clearly in the positive range for the first, third and fourth contrast, while both negative and positive values seem plausible for the second contrast. Note that due to the constrained range of the probability scale, the width of the CI differs between the four contrasts (which is the expected behavior for binomial GLMMs). The smallest width is observed for the first contrast (expert with correct advice vs. expert without advice) where both underlying probabilities are close to 1. The largest width is observed for the fourth contrast (non-expert with wrong advice vs. non-expert without advice), where both underlying probabilities are closer to 0.5.

## Step 6: Perform repeated simulations

### *THEORY*

Conducting all previous steps enables the analyst to 1) simulate a dataset, 2) estimate a GLMM, and 3) compute HTs or CIs for estimands of interest, mirroring the analysis that will later be performed for the actual dataset of the planned study. The last missing piece is to write code to perform the above steps repeatedly and allow for a setting using different sample sizes. On a conceptual level, we first require a function that takes as input the sample size and the full set of population parameter values. When planning for power, the function should return the p-value(s) of the HT(s) of interest when conducted on the simulated dataset. When planning for precision, the function should return the width of the CI(s) of interest. Secondly, we must run this function repeatedly with the same sample size and population parameters. Because even fitting GLMMs with frequentist methods can quickly become time-consuming, it is recommended to use parallel computing, that is running simulations on multiple cores of the computer at the same time to reduce total run time. Thirdly, the results of the repeated simulation must be collected and

aggregated. When planning for power, we compute the relative frequency of (a) significant p-value(s) across repeated simulations. When planning for precision, we compute the average width of the CI(s). Lastly, we have to repeat the complete simulation for different sample sizes, to determine how big the sample must be in order to achieve the targeted power or precision.

### ***PRACTICE***

We are finally ready to run our simulation-based sample size planning analyses to plan for power and for precision. Wrapping the `simulate` function already constructed earlier, the helper function `sim_and_analyse` performs all previous steps (simulate a dataset, fit a GLMM, compute p-values and CIs) in a single command.

```
sim_and_analyse <- function(
  formula_chr = "y_bin ~ 1 + expert + advice_present + advice_correct +
    expert:advice_present + expert:advice_correct + (1|subject) + (1|item)",
  contrasts = c("b8 = b2", "b2 = b6", "b7 = b1", "b1 = b5"), ...) {
  require(lme4)
  require(marginaleffects)
  require(tidyr)
  # simulate data
  dat <- simulate(...)
  # fit model
  model <- glmer(as.formula(formula_chr), data = dat, family = "binomial")
  # compute contrasts
  contr_df <- expand_grid(advice_present = 0:1, advice_correct = 0:1,
    expert = 0:1)
  predictions(model, newdata = contr_df, type = "response", re.form = NA) |>
    hypotheses(hypothesis = contrasts, equivalence = c(0, 0)) |>
    data.frame()
```

```
}
```

Simulation-based sample size planning can quickly become computationally intensive when we repeatedly simulate data and fit models for different parameter combinations or sample sizes. Thus, we use the `future` (Bengtsson, 2021) and `furrr` (Vaughan & Dancho, 2022) packages to perform computations in parallel. First, we enable parallelization with the `plan` function and specify how many parallel cores (“workers”) of our computer to use (users can find out the maximum number of cores on their computer with the command `parallel::detectCores()`), and set a seed to make the simulation reproducible.

```
library(future)
plan("multisession", workers = 6)
set.seed(2)
```

The next code chunk specifies a simulation grid with different settings for both the number of subjects (`n_subjects`) and the number of items (`n_items`), each combination being repeated `rep` times. We chose 300 repetitions for the data simulation at hand as it strikes a balance between achieving a robust statistical estimate and remaining computationally feasible. With the current settings, this simulation takes about one hour on a MacBook Pro from 2020 with M1 chip and 16 GB working memory. If you want to quickly experiment with the code yourself, a setting with `workers = 4` and `rep = 5` should finish in less than 5 minutes, even on smaller machines.

```
library(furrr)
sim_result <- crossing(
  rep = 1:300,
  n_subjects = c(100, 150, 200, 250),
  n_items = c(10, 30, 50, 70)
) |>
mutate(res = future_pmap(., sim_and_analyse,
```

```
.options = furrr_options(seed = TRUE))) |>
unnest(col = res)
```

The result of this computation is a data frame that contains the p-values and CIs of all specified contrasts for each simulated dataset. In some iterations (predominantly in conditions with small sample sizes), model estimation did not converge with the lme4 package. When the model fails to converge, it means that the statistical model being fitted to the data failed to reach a stable or valid solution during the estimation process. We do not remove these results because non-convergence can also happen when analyzing the real data we plan to collect, thus, we want to factor in this possibility to keep our simulation more realistic.

**Power results.** For our exemplary combined hypothesis, power is defined as the (long-run) percentage of simulations in which all four p-values of our individual hypotheses are significant at the  $\alpha = 0.05$  level. Based on our simulation outcomes, we compute a power estimate for each combination of  $n\_subjects \times n\_items$  (including 95% CIs) and visualize the results with the following code.<sup>4</sup>

```
library(binom)
alpha <- 0.05
power <- sim_result |>
  pivot_wider(names_from = term, names_sep = "_",
    values_from = estimate:p.value.equiv) |>
  group_by(n_subjects, n_items) |>
  summarise(
    power = mean(`p.value.noninf_b1=b5` < alpha &
      `p.value.noninf_b8=b2` < alpha & `p.value.noninf_b2=b6` < alpha &
      `p.value.noninf_b7=b1` < alpha),
```

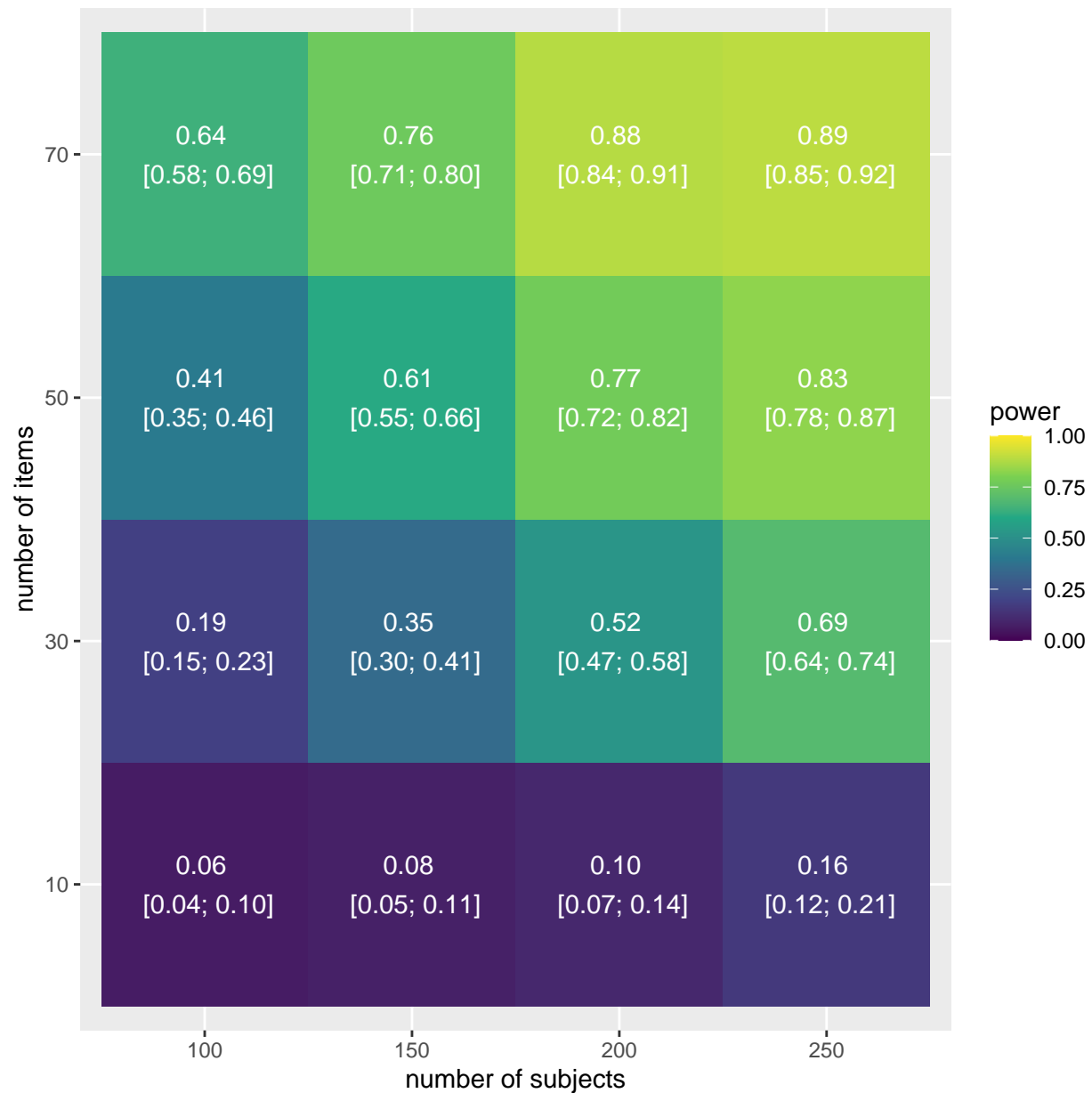
---

<sup>4</sup> This code was inspired by the “Mixed Design Simulation” vignette of the faux package at [https://debruine.github.io/faux/articles/sim\\_mixed.html](https://debruine.github.io/faux/articles/sim_mixed.html).

```
n_sig = sum(`p.value.noninf_b1=b5` < alpha &
            `p.value.noninf_b8=b2` < alpha & `p.value.noninf_b2=b6` < alpha &
            `p.value.noninf_b7=b1` < alpha),
n = n(),
ci.lwr = binom.confint(n_sig, n, method = "wilson")$lower,
ci.upr = binom.confint(n_sig, n, method = "wilson")$upper,
.groups = "drop")
power |>
mutate(across(c(n_subjects, n_items), factor)) |>
ggplot(aes(n_subjects, n_items, fill = power)) +
geom_tile() +
geom_text(aes(label = sprintf("%.2f \n [%.2f; %.2f]",
                             power, ci.lwr, ci.upr)),
          color = "white", size = 4) +
scale_fill_viridis_c(limits = c(0, 1)) +
xlab("number of subjects") + ylab("number of items")
```

**Figure 4**

*Simulation-based power estimates including 95% confidence interval of the case study for different numbers of subjects and items, based on a significance level of 0.05.*



As should be the case, power estimates in Figure 4 increase with both the number of subjects and the number of items. The CIs reported here indicate how precisely power was estimated by our simulation. Higher precision (which would be reflected in narrower CIs) could be obtained by increasing the number of repetitions (rep) in the simulation. In practice, data

simulations are often run multiple times with adjusted combinations of sample sizes. When running for the first time, it might be revealed that power is way too low (or much higher than required) for some combinations of `n_subjects` and `n_items`. When narrowing down the best combination that achieves sufficient power while at the same time striking a good balance of how many subjects and items are practically feasible, later rounds of data simulation will typically include a smaller grid of sample sizes combined with a higher number of repetitions. This will assure high precision for the final power estimates, which are then used for the sample size justification of the future study.

Much has been written on the optimal amount of power to target in empirical research. The most prominent heuristic is to target a power of 0.8 (when combined with a type I error rate of  $\alpha = 0.05$ ), but depending on the research goals of the study, there are often good reasons to move away from this standard depending on the research goals and resource constraints ([Lakens, Adolphi, et al., 2018](#); [Lakens, 2022a](#)). When target power has been specified, the number of subjects and the number of items in our study design can be traded against each other based on practical considerations. For the sake of the example, let the targeted power be indeed about 0.8, using an  $\alpha$  of 0.05 to detect an effect of the expected size implied by our data simulation. This could be achieved by collecting data from 200 subjects (about 25% of which will be experts), each completing the same 50 items (with advice present in about 67% of cases, which is correct in about 80% of cases with present advice). If collecting data from 200 subjects is not feasible, an alternative would be to recruit 150 subjects but increase the length of the experiment to over 70 items. However, 70 items might take too long to complete for the radiologists participating in the study, who have a busy schedule. The simulation suggests that it might also be possible to plan a shorter experiment with only 30 items if it is feasible to recruit an even higher number of subjects (> 250, to be determined by additional rounds of power analysis). Design parameters that also affect power, and which could be investigated in the simulation to find a more optimal trade-off, are the ratio of experts, the frequency of whether advice is presented and whether it is correct.



**Precision results.** When planning for precision, one could monitor the width of all four CIs at the same time. However, because the CIs of the four contrasts strongly differ in width, it is not trivial to decide which width one should target when deciding on the appropriate sample size. In contrast to planning for power, there are no common standards on how to specify the targeted precision. For our example, we use a simple heuristic but we strongly encourage readers to think about better alternatives that are appropriate in their own applications. Our simulations show that the smallest CI can be expected for the first contrast (expert with correct advice vs. expert without advice). The true contrast in probability for an average expert and an average item in this condition is  $\text{plogis}(b_0 + b_e + b_a + b_c + b_{ea} + b_{ec}) - \text{plogis}(b_0 + b_e) = 0.05$ . We want the width of this CI to be smaller than 0.1. This would mean that if the point estimate happens to be close to the true value, the plausible values inside of a 95% CI would all be positive.

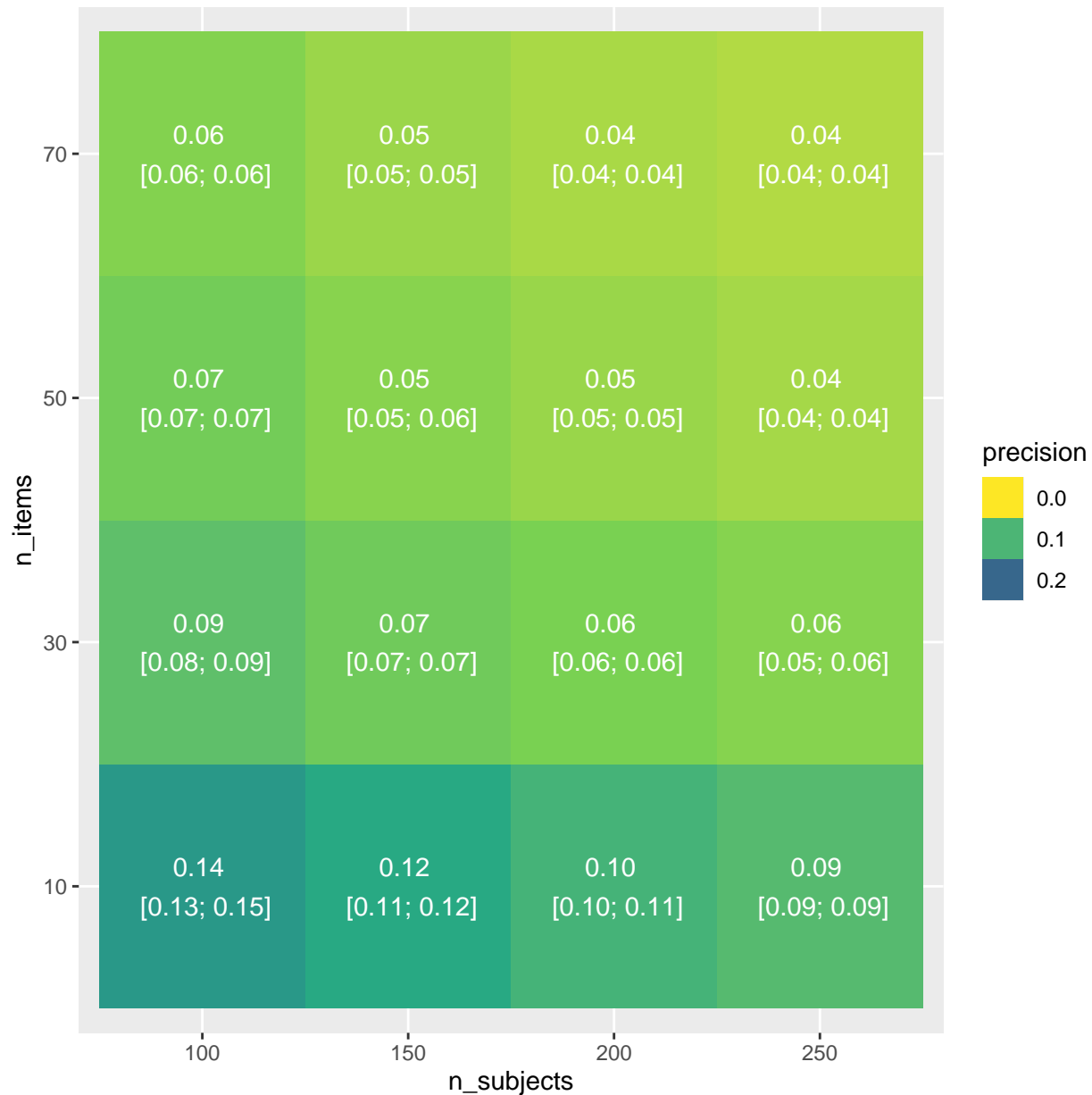
Thus in our example, precision is defined as the (long-run) average width of a 95% CI for the probability contrast between experts with correct advice and experts without advice. Of course, lower width implies better precision. Based on our simulation outcomes, we compute the precision estimate for each combination of  $n_{\text{subjects}} \times n_{\text{items}}$  (including 95% CIs) and visualize the results with the following code.

```
precision <- sim_result |>
  pivot_wider(names_from = term, names_sep = "_",
    values_from = estimate:p.value.equiv) |>
  group_by(n_subjects, n_items) |>
  mutate(width = `conf.high_b8=b2` - `conf.low_b8=b2`) |>
  summarise(precision = mean(width),
    ci.lwr = t.test(width)$conf.int[1],
    ci.upr = t.test(width)$conf.int[2],
    .groups = "drop")
precision |>
  mutate(across(c(n_subjects, n_items), factor)) |>
```

```
ggplot(aes(n_subjects, n_items, fill = precision)) +  
  geom_tile() +  
  geom_text(aes(label = sprintf("%.2f \n [%.2f; %.2f]",  
                                precision, ci.lwr, ci.upr)),  
            color = "white", size = 4) +  
  scale_fill_viridis_c(limits = c(0, 0.3), direction = -1) +  
  guides(fill = guide_legend(reverse=FALSE))
```

**Figure 5**

*Simulation-based precision estimates (expected width of confidence intervals) including 95% confidence interval of the case study for different numbers of subjects and items, based on a confidence level of 0.95.*



As should be the case, precision estimates in Figure 5 increase (i.e., average width of CI decreases) with both the number of subjects and the number of items. The CIs reported here indicate how precisely the expected width of the CI for our focal contrast was estimated by our

simulation. Applying our simple heuristic of targeting an expected width smaller than 0.1, we see the same trade-off between the number of subjects and the number of items as with planning for power. We could either choose 100 subjects and 30 items or 200 subjects and 10 items. Note that our simple heuristic for determining sample size in the planning for precision scenario was quite liberal. This is reflected by the result that we would need a smaller sample size than in the planning for power scenario. With a more conservative precision target, the result is generally the opposite: As a rule, precise parameter estimates usually require bigger samples than null hypothesis testing.

### **Sensitivity analysis**

In our case study, we have performed simulation-based sample size planning from a single set of parameter values that reflect our assumptions of an expected effect size. Instead of extracting this expected effect size from meta-analyses or pilot data, which has been the main focus of previous tutorials (e.g., [Kumle et al., 2021](#)), we have demonstrated some strategies to determine plausible parameter values in GLMMs based on domain knowledge. Domain knowledge can be considered a vague theoretical model about the data-generating process that is less formal and can only be accessed by a back-and-forth exchange in which domain experts assess the plausibility of simulated data. When sample sizes are chosen based on the results of our simulation-based power analysis, a future study will be informative to reject the null hypothesis if an effect of our *expected size* is present (or estimate the effect with satisfying precision). However, if the true effect is indeed smaller, power (or precision) will be lower, and the study might not be sufficiently informative. A common, more conservative strategy for sample size justification is to perform sample size planning for the smallest effect size of interest (SESOI). An effect smaller than the SESOI would be considered too small to be interesting or practically meaningful, even if the effect is not actually zero ([King, 2011](#); [Lakens, Scheel, et al., 2018](#)). For strategies on the even more difficult task of specifying a plausible SESOI, as well as a thorough discussion of various topics concerning power analysis, see ([Lakens, 2022b](#)). When domain knowledge or formal theories about the research topic of interest are too vague to specify a meaningful SESOI, it is still

recommended to demonstrate power or precision for different effect sizes in what is called *sensitivity power analysis*. By simulating power (or precision) for different effect sizes (in addition to the different number of subjects and items), one can make sure that power (or precision) would still be sufficient to detect smaller effect sizes than our expected effect or at least get an impression of how strongly power (or precision) depends on the size of the true effect. For our case study that investigates combined hypotheses in a GLMM modeling framework, sensitivity analysis would require manually specifying additional sets of plausible parameter values that reflect scenarios with smaller or larger differences between groups with respect to our specific research question. Power (or precision) could then be simulated for several of these scenarios (across different numbers of subjects and items, as considered earlier).

### **Conclusion and outlook**

The goal of this tutorial was to teach researchers how to perform tailored simulation-based sample size planning for GLMMs. Beyond the specifics of our concrete case study, we want to outline six developments regarding the future role of simulation-based sample size planning in experimental research:

In light of the ongoing replication crisis and an existing literature full of underpowered studies, there is a growing need for simulation-based sample size planning in experimental research: In order to conduct informative research, GLMMs offer a flexible statistical framework to analyze complex experimental study designs. However, existing formula-based heuristics and user-friendly software tools for a priori power analysis are often not sufficient. Therefore, simulation-based power analysis is becoming increasingly needed since it provides experimental researchers with a tailored approach to estimating required sample sizes before data collection.

Managing data simulations more easily with discrete predictor variables:  
Simulation-based sample size planning becomes more manageable when all predictor variables are discrete (like in the presented case study) and fixed by the study design. This allows researchers to focus on simulating outcome variables while avoiding the need for complex simulations of predictor values, which would introduce additional assumptions. By simplifying

the simulation process, researchers can obtain reliable estimates for power or precision without compromising realistic assumptions about the data-generating process implied by the study design.

**Teaching data simulation skills:** The ability to conduct simulation-based sample size planning is a valuable skill that should be taught to experimental researchers. By incorporating such training into research methods courses and workshops, researchers can gain a deeper understanding of statistical power or precision, and improve the quality of their experimental designs. Equipping researchers with the knowledge and tools to perform simulation-based sample size planning enables them to make informed decisions and enhance the rigor of their studies. The need to reason about how to simulate plausible data that is in line with the research hypothesis, while not violating domain expertise on how plausible data should look, might also contribute to planning more insightful studies that can answer more precise research questions ([Yarkoni, 2022](#)).

**Addressing the mismatch in effort perception:** There is often a significant disconnect between the amount of effort required to perform tailored simulation-based sample size planning and the perceived effort estimated by researchers and collaborators in experimental research. Many researchers request simulation-based power analyses from statisticians or methodological experts without fully comprehending the complexity and time-consuming nature of these tailored simulations. It is crucial to raise awareness about the effort involved to ensure realistic expectations and effective collaboration between researchers and methodological experts.

**Recognizing the value of simulation-based design analysis:** Tailored data simulations and power analyses are not mere technicalities; they are valuable research contributions that deserve recognition in experimental research. They offer insights into the robustness and sensitivity of experimental designs, helping researchers make informed decisions about sample sizes, effect sizes, and statistical power or precision. Their importance can be reflected by allocating them a separate publication or incorporating them as a significant component of stage 1 preregistered reports ([Chambers & Tzavella, 2022](#)).

**Integration with Open Science and preregistration practices:** Simulation-based sample

size planning aligns well with the principles of Open Science and preregistration in experimental research. When researchers have access to simulated data based on their pre-specified model, analyzing the collected dataset becomes straightforward and unambiguous. By preregistering their simulation-based sample size plan, researchers enhance the transparency and accountability of their experimental procedures, contributing to the credibility and reproducibility of research.

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