2015

1. spectral radius of adjacency matrix

$$E[D]\sqrt{1+\frac{Var[D]}{E^2[D]}} \leq \lambda_1 \leq d_{\max}$$

2. Head of the line (HoL) with non-preemptive mode

3. Erdos-Renyi, Betweenness

$E(B_l) = \frac{1}{L}\cdot\frac{N(N-1)}{2}E(H_N) = \frac{1}{L}\cdot\frac{N(N-1)}{2}\cdot(\Pr[H_N=1]+2\Pr[H_N=2])$

$= \frac{1}{\frac{N(N-2)}{2}\rho}\cdot\frac{N(N-1)}{2}\cdot(\rho(1-\rho)+2(1-\rho)(1-(1-\rho^2)^{N-2}))$

$\lim_{N\to\infty} = \frac{1}{\rho}(\rho+2(1-\rho)) = \frac{2-\rho}{\rho}$

4. diameter, clustering coefficient, connectivity

diameter = the longest shortest path
clustering coefficient = $\frac{2y}{d_v(d_v-1)}$

higher connectivity $\sim$ higher μ_{N-1}

5. R-model/R-value: $R=\sum_{k=1}^{m} s_t t_k$

R is between [0,1]

R model is linear

All the m metrics should be orthogonal to each other (Ideally)

6. K-hop walk, closed walk

k-hop walk: $N_k = \sum_i\sum_j (A^k)_{ij}$

closed k-hop walk: $W_k = \sum_j (A^k)_{jj} = \sum_i diag(A^k)_i$

7. token bucket, burstiness
   - if σ is small, bursty traffic is delayed or discard

8. union Erdos-Renyi

9. CAC = Connection Admission Control

10. electrical graph, effective resistance

11. graph, histogram

12. different between walk and path
    - walk: Vertices may repeat. Edges may repeat (Closed or Open)

- path: Vertices cannot repeat. Edges cannot repeat (Open)

13. complete graph, the spectrum of the Laplacian matrix

    $\mu = [N, N, N, ..., 0]$

14. Tree

2016

1. spectral radius of adjacency matrix

$$E[D]\sqrt{1+\frac{Var[D]}{E^2[D]}} \le \lambda_1 \le d_{\max}$$

regular graph 取下界 (Var=0), complete graph 取上界

2. Erdos-Renyi degree distribution

   Degree distribution: Binomial distribution, when $N \to \infty$ it becomes Gaussian distribution

3. spectrum of adjacency matrix

   the sum of the eigen values of A is 0

4. **complete graph**, spectrum of adjacency matrix

   - number of triangles:

   $$\sum_{j=1}^{N} \lambda_j^3 = N(N-1)(N-2) = 6\#triangle$$

   - number of links:

   $$\sum_{j=1}^{N} \lambda_j^2 = N(N-1) = 2L$$

   - $\sum_{j=2}^{N} \lambda_j = -\lambda_1 = 1 - N$

5. DPR, assortative

6. average eigenvalue $E[\mu]$

   代入法, suppose that $p = 1$, only A is correct

7. PBS

   - Below threshold T: identical to FIFO and sequence order is preserved
   - Above threshold T: only a high priority cell regime until buffer is full

8. clustering coefficient, average clustering coefficient

$$C_G(V) = \frac{2y}{dv(dv-1)}$$

$$\frac{1}{10}(6 * \frac{2 \times 1}{2 \times 1} + 3 * \frac{2 \times 1}{3 \times 2} + 1\frac{2 \times 0}{3 \times 2}) = 0.7$$

9. Laplacian matrix
    - $Q = BB^T = \Delta - A$, $\Delta = diag(d1, d2, ..., d_N)$
    - Q is symmetric
    - $Qu = 0$ u is an eigenvector of Q belonging to eigenvalue $\mu = 0$
10. electrical graph, effective resistance
11. electrical graph, effective resistance
    - compute the effective resistance between node 1 and 3
12. Laplacian matrix
13. Star graph, spectrum of the Laplacian

    $\mu = N, 1, 1, ..., 1, 0$
14. star graph, wheel graph, cycle graph, spectrum
15. Robustness envelopes
16. K-hop walk
17. Erdos-Renyi graph, number of triangles
    - expected number of triangles incident of one vertex is $\binom{N-1}{2} \rho^3$
    - expected number of triangles in the graph is $\binom{N}{3} \rho^3$
18. 看不懂
19. degree, spectrum of the adjacency matrix, boundary
20. Erdos-Renyi graph

2018 (1)

1. disconnected graph, spectral radius, link density
    - If the graph is disconnected, its complementary graph is connected
    - If $d_{\min} = \min_{1 \le k \le N} \lambda_k^2(A)$, the graph is disconnected
    - Increasing assortativity creates more disconnected components
2. remove one link in the graph
    - disconnected graph, $\mu_{N-1}(G) = 0$

3. delay

   $D_{\max} < \frac{\sigma}{\lambda}$

4. Laplacian properties:
   - symmetric
   - sum of any rows or columns equals 0

5. degree, connected graph
   - The number of nodes with odd degree is even
   - At least two nodes in G have the same degree
   - if connected, $2 - \frac{2}{N} \leq E(D) \leq N - 1$

6. robustness
   - $E[S]$ diverges for $1 < \tau < 2$ if $k \to \infty$

7. effective resistance matrix
   - symmetric
   - all diagonal elements are 0s
   - triangular inequality: $w_{ij} \leq w_{ik} + w_{kj}$

8. spectrum, tree/bipartite graph
   - Peaks refer to a specific structure or pattern in the graph
   - A broader, bell-shape form of fl(x) around the origin (x=0) is a fingerprint of randomness
   - A tree/bipartite graph has a symmetric spectrum (Any tree can be represented as a bipartite graph)

9. Envelope definitions

10. line graph: two nodes in l(G) are connected by a link if the corresponding two links in G have a node in common

11. Spectrum of Q:
    - complexity (number of spanning tree) is $\xi(G) = \frac{1}{N} \prod_{k=1}^{N-1} \mu_k$
    - $\prod_{k=1}^{N-1} \mu_k$ is an integer

12. Watts-Strogatz small world graph
    - with the increase of the rewiring probability:
      - the average clustering coefficient decreases
      - the average hop count decreases

13. path, walk, connected graph

14. Pseudoinverse of the Laplacian
    - The best spreader minimizes the sum of potential differences between its own and all other node potentials

- the best spreader is the node k with minimum $Q^{\dagger}_{kk} \leq Q^{\dagger}_{ii}$ for all i

- diagonal elements $Q^{\dagger}_{ii} > 0$, but the off-diagonal elements $Q^{\dagger}_{ij} > 0$ can be negative as well as non-negative, thus, $Q^{\dagger}$ is not always a Laplacian!
- the effective graph resistance $R_G = N \times Trace(Q^{\dagger}) = Nu^T diag(Q^{\dagger})$

15. edge (node) connectivity: the minimum number of links (or nodes) whose removal disconnects G

16. degree

17. power law (let k=2, A incorrect)

18. $L(u,t) \leq max_{\tau \in [u,t]} \lambda(\tau)(t-u)$

19. R-model: difficult to build a general theory

20. assortativity

- only numerator changes when DPR

$$\rho_D = 1 - \frac{\sum_{i\sim j}(d_i - d_j)^2}{\sum_j d_j^3 - \frac{1}{2L}(\sum_j d_j^2)^2}$$

2018(2)

1. line graph of the complete graph
   - node: $\frac{N(N-1)}{2}$
   - degree: $2(N-2)$
   - λ: n-2
   - μ: 4
2. real-world complex network
   - small-world property
   - scale-free degree distribution
   - clustering and community structure (peaks in the spectrum of adjacency matrix)
   - robustness to random node failure
   - vulnerability to targeted attacks
3. complement graph
   - if the graph $G is disconnected, then its complement G^C is connected
4. power law, moment
   - $E[D^m]$ only exist for $\tau > m+1$ when $N \to \infty$
5. spectrum
6. subgraph, spectral radius
   - The spectral radius of a graph G is larger than or equal to the spectral radius of any subgraph of G
7. electrical network, effective resistance
8. adjacency matrix of bipartite graph
   - diagonal entries of A^{2n+1} is 0 for bipartite graph without self-loops
9. degree
   - every degree $\leq$ N-1
   - The number of nodes with odd degree is even

  - At least two nodes in G have the same degree
10. tree graph
11. Erdos-Renyi
12.
13. Robustness envelope
    - Low sensitivity, High energy, most desirable
14. effective graph resistance:
    - $R_G = \frac{1}{2}u^T \omega u$
15. loss-less multiplexing
    - stability requires: $\sum \lambda_k < \mu$, μ is the constant link rate
    - $Q_t \leq G$
16. token bucket
17. effective graph resistance of complete graph
    - $R_G \geq N - 1$, with equality for the complete graph
18. the spectrum of the adjacency matrix
19. electrical network
20. FIFO POB: the first entered low priority cell is discarded

2019(1)

1. degree
2. FIFO, PBS
3. Burstiness constraint

$$\lim_{t\to\infty} \frac{L(u,t)}{t-u} = \lambda + \lim_{t\to\infty} \frac{\sigma}{t-u}$$

4. measurement for burstiness $B = \frac{max\ S}{E[S]}$
5. Burstiness constraint
6. Linearity of R-model
7. weighted Laplacian:

$$\tilde{Q} = \tilde{\Delta} - \tilde{A},\ \tilde{a}_{ij} = \frac{1}{r_{ij}} a_{ij}$$

8. spectrum of adjacency matrix, number of triangles $\sum_{j=1}^N \lambda_j^3 = 6\#triangle$
9. spectrum of adjacency matrix
    - $\lambda_1 - \lambda_2 \leq N$ (equals when N=2, $\lambda_1 = 1$ and $\lambda_2 = -1$)
    - $\lambda_1 \leq N - 1$
    - $\sum_k \lambda_k = 0$
10. the effective graph resistance for a complete graph with N nodes is $N - 1$
11. degree

12. closeness, effective resistance matrix
    - $Cl_i^{-1} \geq NQ_{ii}^{\dagger}$
    - A positive semidefinite matrix is a Hermitian matrix all of whose eigenvalues are nonnegative.
    - the effective resistance matrix R is a Euclidean distance matrix: it has zero diagonal, nonnegative entries.

13. Erdos-Renyi

14. R-model: the more independent m metrics are, the better robustness measure R for network is

15. small-world compared with random graph p.73

- Purely random graphs, built according to the Erdős–Rényi (ER) model, exhibit a small average shortest path length (varying typically as the logarithm of the number of nodes) along with a small clustering coefficient.

16. spectrum of adjacency matrix

- $\Sigma_{j=1}^{N}\lambda_j = 0$
- $\Sigma_{j=1}^{N}\lambda_j^2 = 2L$
- $\Sigma_{j=1}^{N}\lambda_j^3 = 6 \times triangles$

17. token bucket
18. Robustness envelope
    - for scenario B, $K_{\text{attack}} = K_{\text{recovery}}$
19. electrical network, current on the link
20. line graph, assortativity, path graph
    - line graph of path graph is still path graph only removing one node from original graph
    - path graph assortativity is $-\frac{1}{N-2}$
    - so, line graph of path graph assortativity is $-\frac{1}{N-3}$

2019(2)

1. trace of the Laplacian: $trace(Q) = total\ degree = 2L$

2. Adjacency matrix

3. average Laplacian eigenvalue of an Erdos-Renyi graph is $E[\mu] = (N-1)\rho$

4. Laplacian matrix

5. electrical network

6. Laplacian matrix

7. star, wheel, cycle graph

8. k-hop walks

    - 0-hop walks $N_0 = N$

- 1-hop walks $N_1 = 2L$
- 2-hop walks $N_2 = d^T d$

9. complexity, the number of spanning tree, complete graph
   - $\xi(G) = \frac{1}{N}\prod_{j=1}^{N-1} \mu_j$
10. number of triangle
    - $\Sigma_{j=1}^{N}\lambda_j^3 = 6 \times triangles$
11. power-law
12. all eigenvalue of Q are always non-negative
13. LIFO POB
14. effective graph resistance
15. adjacency matrix
16. incidence matrix
17. R-model, linearity
18. algebraic connectivity μ_{N-1} is zero when graph is disconnected
19. QoS
20. On-off source