4. Exercise

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Exercise 4.1

1.

T_1 and T_2

$$T_1 \subseteq T_2$$
?

Find mapping
$$h:T_2\to T_1,x\rightarrowtail \begin{cases} b_3&x=b_1\\5&x=b_2\\b_4&x=b_3\\b_3&x=b_4\\a_1&x=a_1\\a_2&x=a_2 \end{cases} \Rightarrow T_1\subseteq T_2$$

$$T_2 \subseteq T_1$$
?

Impossible since we would have to map the constant 5 to another constant and T_2 does not contain a constant. So $T_2 \not\subseteq T_1$

Overall

$$\Rightarrow T_1 \subset T_2$$
, but $T_2 \not\subseteq T_1$ and therefore $T_1 \not\equiv T_2$

T_2 and T_3

$$T_2 \subseteq T_3$$
?

When we map T_3 to T_2 :

$$a1 \rightarrow a1$$

$$a2 \rightarrow a2$$

$$b3 \rightarrow b4$$

$$b2 \rightarrow b3$$

 $b1 \rightarrow a1$, but then for (b4, b1) in T_3 we cannot find a matched instance in T_1

So there cannot exist a function $h: T_3 \to T_2$. $\Rightarrow T_2 \not\subseteq T_3$

 $T_3 \subseteq T_2$?

When we map T_2 to T_3 :

- $a1 \rightarrow a1$
- $a2 \rightarrow a2$
- $b4 \rightarrow b3$
- $b1 \rightarrow b4$
- $b3 \rightarrow b2$
- $b2 \rightarrow b2$ (or b1)

but then for (b3,b2) in T_2 we cannot find a matched instance in T_3

So there cannot exist a function $h: T_2 \to T_3$. $\Rightarrow T_3 \not\subseteq T_2$

Overall

 $\Rightarrow T_2 \not\subseteq T_3, T_3 \not\subseteq T_2$ and therefore especially $T_2 \not\equiv T_3$

T_1 and T_3

$$T_1 \subseteq T_3$$
?

When we map T_3 to T_1 :

- $a1 \rightarrow a1$
- $a2 \rightarrow a2$
- $b3 \rightarrow b3$
- $b2 \rightarrow b4$
- $b1 \rightarrow a1$

 $b2 \rightarrow 5$, because of (b2, b3) in T_3 and b3 \rightarrow b3, so conflict with b2 \rightarrow b4

So there cannot exist a function $h:T_3\to T_1.\Rightarrow T_1\not\subseteq T_3$

$$T_3 \subseteq T_3$$
?

Analogously to $T_2 \subseteq T_1$ we would have to map the constant 5 to an other constant and T_3 does not contain a constant. So $T_3 \not\subseteq T_1$

Overall

 \Rightarrow $T_1 \not\subseteq T_3, T_3 \not\subseteq T_1$ and therefore especially $T_1 \not\equiv T_3$

2.

 T_1 :

 T_1 is minimal, because if we delete any row and so create T' we can not have a function $h:T\to T'$ such that $T'\subseteq T$

2

 T_2 :

1. delete row (b_4, a_2) , since we have a function h_1 , with $h_1(x) = \begin{cases} b_1 & \text{if } x = b_4 \\ x & \text{if } x \neq b_4 \end{cases}$

New Tableau 7		au T_2^\prime	
	a_1	a_2	
	b_1	a_2	(R)
	a_1	b_3	(R)
	b_2	b_1	(R)
	b_3	b_2	(R)

2. we can't minimize $T_2^{\prime\prime}$ any further, so $T_2^{\prime\prime}$ is the minimal tableau

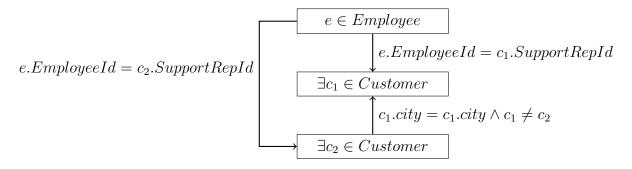
T_3 :

 T_3 is minimal, because if we delete any row and so create T' we can not have a function $h:T\to T'$ such that $T'\subseteq T$

Exercise 4.2

1.a)

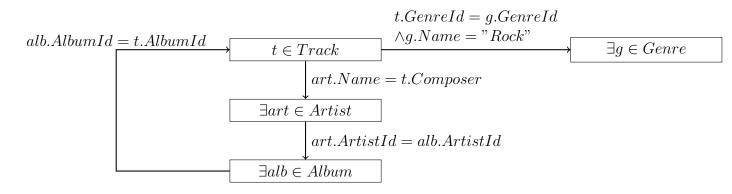
 $\{ \langle e.EmployeeId, e.LastName \rangle | e \in Employee \land \exists c_1 \in Customer \land c_1.SupportedRepId = e.EmployeeId \land \exists c_2 \in Customer \land c_2.SupportRepId = e.EmployeeId \land c_1.City = c_2.City \land c_1 \neq c_2 \}$



This graph is cycle free, which means the query is optimizable using semi-joins.

1.b)

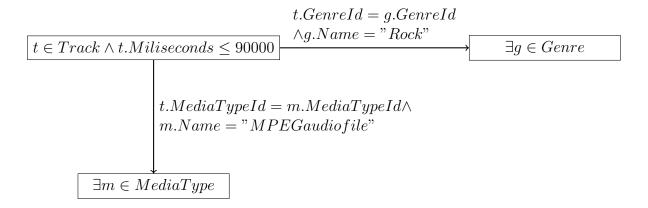
 $\{ < t.name, t.composer > | \ t \in Track \land \exists g \in Genre \land g.GenreId = t.GenreId \land g.Name = "Rock" \land \exists art \in Artist \land \exists alb \in Album \land alb.ArtistId = art.ArtistId \land t.AlbumId = alb.AlbumId \land t.Composer = art.Name \}$



This Graph has a cycle. This means the query is not optimizable using semi-joins.

2.

 $\{ \langle t.Name \rangle | \ t \in Track \land t.Miliseconds \leq 90000 \land \exists g \in Genre \land t.GenreId = t.GenreId \land g.Name = "Rock" \land \exists m \in MediaType \land t.MediaTypeId = m.MediaTypeid \land m.Name = "MPEGaudiofile" \}$



The graph is cycle free, which means it can be optimized using semi-joins.

Exercise 4.3

1.

B = # buffer pages in memory = 4 N = # pages in the input file = $\lceil \frac{7500 \cdot 36}{1024 - 64} \rceil = 282$ After the initial pass: # runs (subfiles) = $\lceil \frac{N}{B} \rceil = \lceil \frac{282}{4} \rceil = 71$, each subfiles: 4-pages runs

2.

passes =
$$1 + \lceil \log_{B-1} \lceil \frac{N}{B} \rceil \rceil = 1 + \lceil \log_{4-1} 71 \rceil = 1 + 4 = 5$$

3.

Total costs = $2N \cdot (\#passes) = 2 \cdot 282 \cdot 5 = 2820$ I/Os

4.

If B = 4:
$$1 + \lceil \log_3 \lceil \frac{N}{B} \rceil \rceil = 2 \Rightarrow (\log_3 \lceil \frac{N}{4} \rceil)_{max} = 1 \Rightarrow (\lceil \frac{N}{4} \rceil)_{max} = 3 \Rightarrow N_{max} = 12$$
 # records in the file = $\frac{12 \cdot (1024 - 64)}{36} = 320$ records

If B = 42: $1 + \lceil \log_{41} \lceil \frac{N}{B} \rceil \rceil = 2 \Rightarrow (\log_4 1 \lceil \frac{N}{42} \rceil)_{max} = 1 \Rightarrow (\lceil \frac{N}{42} \rceil)_{max} = 41 \Rightarrow N_{max} = 41 \cdot 42 = 1722$ # records in the file = $\frac{1722 \cdot (1024 - 64)}{36} = 45920$ records