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Introduction to Model Checking (Summer Term 2018)

— Exercise Sheet 5 (due 4th June) —

General Remarks

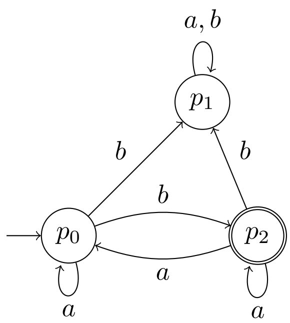
- The exercises are to be solved in groups of *three* students.
- You may hand in your solutions for the exercises just before the exercise class starts at 12:15 or by dropping them into the "Introduction to Model Checking" box at our chair *before 12:00*. Do *not* hand in your solutions via L2P or via e-mail.

Exercise 1★ (2+2+3 Points)

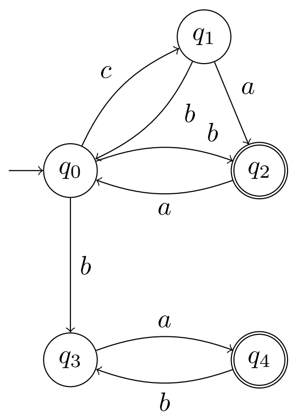
In the following we have $\Sigma = \{a, b, c\}$.

(a) Consider the following NBA $\mathcal{A}_1, \mathcal{A}_2$.

$\mathcal{A}_1$:



$\mathcal{A}_2$:



For each NBA $\mathcal{A}_i$ give an ω-regular expression α_i which characterizes the language accepted by the NBA, i.e., $\mathcal{L}_\omega(\alpha_i) = \mathcal{L}_\omega(\mathcal{A}_i)$.

(b) Consider the following descriptions of ω-regular languages $\mathcal{L}^i_\omega$.

(i) $\mathcal{L}^1_\omega$: a occurs infinitely many times. In between two successive a either

- an odd number of b and no c, or
- an even number of c and no b

has to occur.

(ii) $\mathcal{L}^2_\omega$:

- If c occurs only finitely many times then a and b occur infinitely many times.
- If c occurs infinitely many times then a and b occur only finitely many times.

For each language $\mathcal{L}^i_\omega$ give an NBA $\mathcal{B}_i$ which accepts the language.

(c) Consider again the languages from (b). For each language $\mathcal{L}^i_\omega$ give a DBA $\mathcal{D}_i$ which accepts the language. If you can not find a DBA, justify why there exist no DBA accepting the language.

Exercise 2 (3 Points)

Provide an example for a liveness property P_{live} that is *not* ω-regular. Show that P_{live} is indeed a liveness property and prove that P_{live} is not ω-regular.

Hint: Think about words of the form $\{a\}\,\{b\}\,\{a\}\,\{a\}\,\{b\}\,\{a\}\,\{a\}\,\{a\}\,\{b\}\ldots$.

Exercise 3 (2+2+2+1 Points)

(a) Provide NBA $\mathcal{A}_1$ and $\mathcal{A}_2$ for the languages given by the ω-regular expressions $\alpha_1 = (AC + B)^* B^\omega$ and $\alpha_2 = (B^* AC)^\omega$.

(b) Apply the product construction to obtain a GNBA $\mathcal{G}$ with $\mathcal{L}_\omega(\mathcal{G}) = \mathcal{L}_\omega(\mathcal{A}_1) \cap \mathcal{L}_\omega(\mathcal{A}_2)$.

(c) Transform the GNBA $\mathcal{G}$ into an NBA $\mathcal{A}$ with $\mathcal{L}_\omega(\mathcal{A}) = \mathcal{L}_\omega(\mathcal{G})$.

(d) Justify, why $\mathcal{L}_\omega(\mathcal{G}) = \emptyset$ on the level of the GNBA $\mathcal{G}$.

Hint: For a GNBA $\mathcal{G} = (Q, \Sigma, \delta, Q_0, \mathcal{F})$ with at least one element in $\mathcal{F} = \{F_1, \ldots, F_k\}$. Let $\mathcal{A} = (Q', \Sigma, \delta', Q'_0, F')$ be an NBA with

- $Q' = Q \times \{1, \ldots, k\}$,
- $Q'_0 = Q_0 \times \{1\}$,
- $F' = F_1 \times \{1\}$, and

for all $A \in \Sigma$, it is

$$\delta'(\langle q, i\rangle, A) = \begin{cases} \{\langle q', i\rangle \mid q' \in \delta(q, A)\} & \text{if } q \notin F_i \\ \{\langle q', (i \mod k) + 1\rangle \mid q' \in \delta(q, A)\} & \text{if } q \in F_i. \end{cases}$$

Then $\mathcal{L}_\omega(\mathcal{G}) = \mathcal{L}_\omega(\mathcal{A})$.

Exercise 4 (1+2 Points)

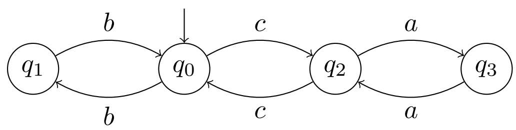
A nondeterministic Muller automaton is a quintuple $\mathcal{A} = (Q, \Sigma, \delta, Q_0, \mathcal{F})$ where Q, Σ, δ and Q_0 are as for NBA and $\mathcal{F} \subseteq 2^Q$. For an infinite run $\rho = q_0 q_1 q_2 \ldots$ of $\mathcal{A}$, let

$$\mathit{inf}(\rho) := \{q \in Q \mid \exists^\infty i \geq 0.\ q_i = q\}\,.$$

Let $\alpha \in \Sigma^\omega$.

$$\mathcal{A} \text{ accepts } \alpha \iff \text{exists infinite run } \rho \text{ of } \mathcal{A} \text{ on } \alpha \text{ s.t. } \mathit{inf}(\rho) \in \mathcal{F}.$$

(a) Consider the following Muller automaton $\mathcal{A}$ with $\mathcal{F} = \{\{q_2, q_3\}, \{q_1, q_3\}, \{q_0, q_2\}\}$:



Give the language accepted by $\mathcal{A}$ by means of an ω-regular expression.

(b) Show that every GNBA $\mathcal{G}$ can be transformed into a nondeterministic Muller automaton $\mathcal{A}$ such that $\mathcal{L}_\omega(\mathcal{A}) = \mathcal{L}_\omega(\mathcal{G})$ by defining the corresponding transformation.