

Introduction

Modelling parallel systems

Linear Time Properties

Regular Properties

Linear Temporal Logic (LTL)

syntax and semantics of LTL

automata-based LTL model checking ←

complexity of LTL model checking

Computation-Tree Logic

Equivalences and Abstraction

given: finite transition system \mathcal{T} over AP
 (without terminal states)
 LTL-formula φ over AP

question: does $\mathcal{T} \models \varphi$ hold ?

given: finite transition system \mathcal{T} over AP
 (without terminal states)
 LTL-formula φ over AP

question: does $\mathcal{T} \models \varphi$ hold ?

basic idea: try to refute $\mathcal{T} \models \varphi$

given: finite transition system \mathcal{T} over AP
 (without terminal states)
 LTL-formula φ over AP

question: does $\mathcal{T} \models \varphi$ hold ?

basic idea: try to refute $\mathcal{T} \models \varphi$ by searching
 for a path π in \mathcal{T} s.t.

$$\pi \not\models \varphi$$

given: finite transition system \mathcal{T} over AP
(without terminal states)
LTL-formula φ over AP

question: does $\mathcal{T} \models \varphi$ hold ?

basic idea: try to refute $\mathcal{T} \models \varphi$ by searching
for a path π in \mathcal{T} s.t.

$$\pi \not\models \varphi, \text{ i.e., } \pi \models \neg \varphi$$

given: finite transition system \mathcal{T} over AP
 LTL-formula φ over AP

question: does $\mathcal{T} \models \varphi$ hold ?

1. construct an **NBA** \mathcal{A} for $Words(\neg\varphi)$

given: finite transition system \mathcal{T} over AP
 LTL-formula φ over AP

question: does $\mathcal{T} \models \varphi$ hold ?

1. construct an **NBA** \mathcal{A} for $Words(\neg\varphi)$
2. search a path π in \mathcal{T} with
 $trace(\pi) \in Words(\neg\varphi)$

given: finite transition system \mathcal{T} over AP
 LTL-formula φ over AP

question: does $\mathcal{T} \models \varphi$ hold ?

1. construct an **NBA** \mathcal{A} for $Words(\neg\varphi)$
2. search a path π in \mathcal{T} with
 $trace(\pi) \in Words(\neg\varphi) = \mathcal{L}_w(\mathcal{A})$

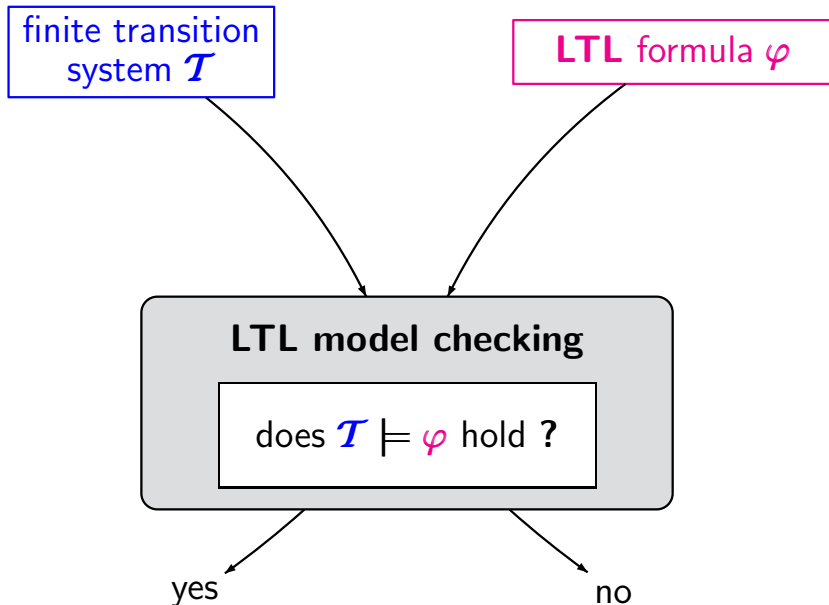
given: finite transition system \mathcal{T} over AP
 LTL-formula φ over AP

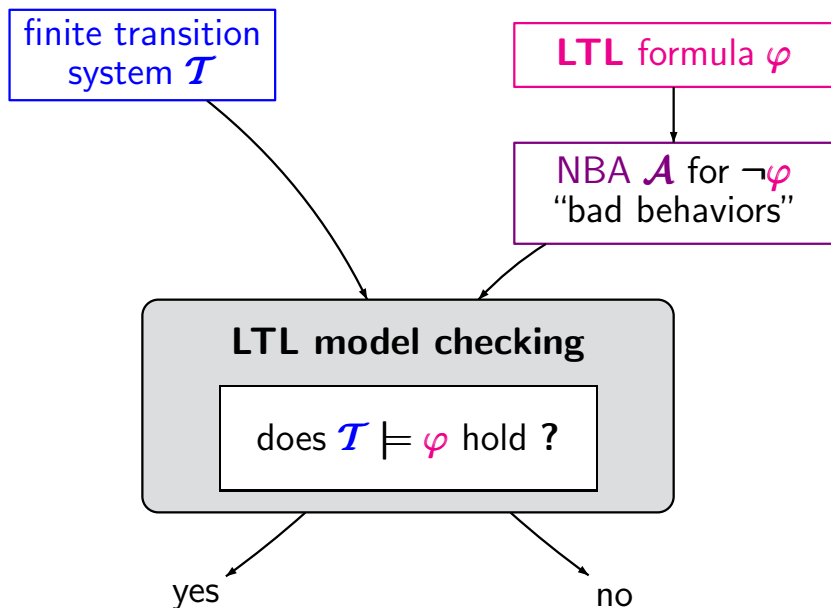
question: does $\mathcal{T} \models \varphi$ hold ?

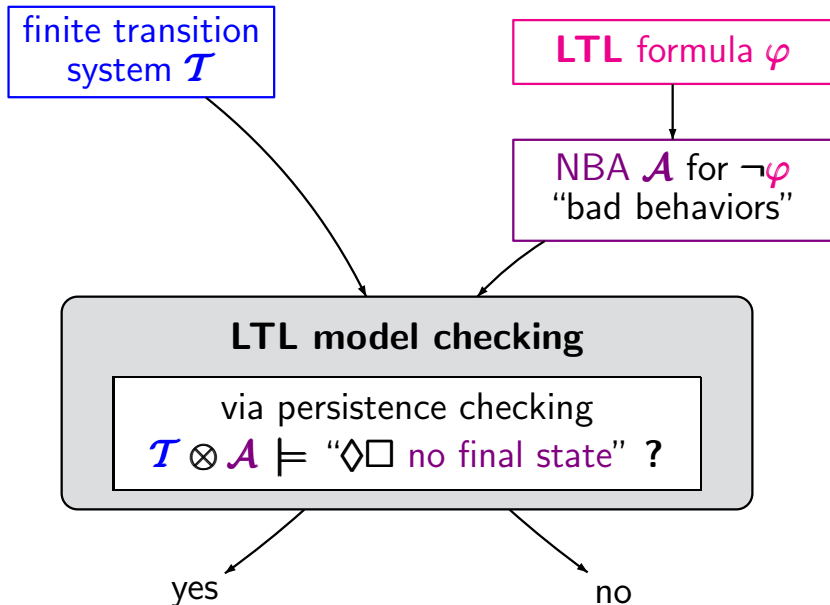
1. construct an **NBA** \mathcal{A} for $Words(\neg\varphi)$
2. **search** a path π in \mathcal{T} with
 $trace(\pi) \in Words(\neg\varphi) = \mathcal{L}_\omega(\mathcal{A})$

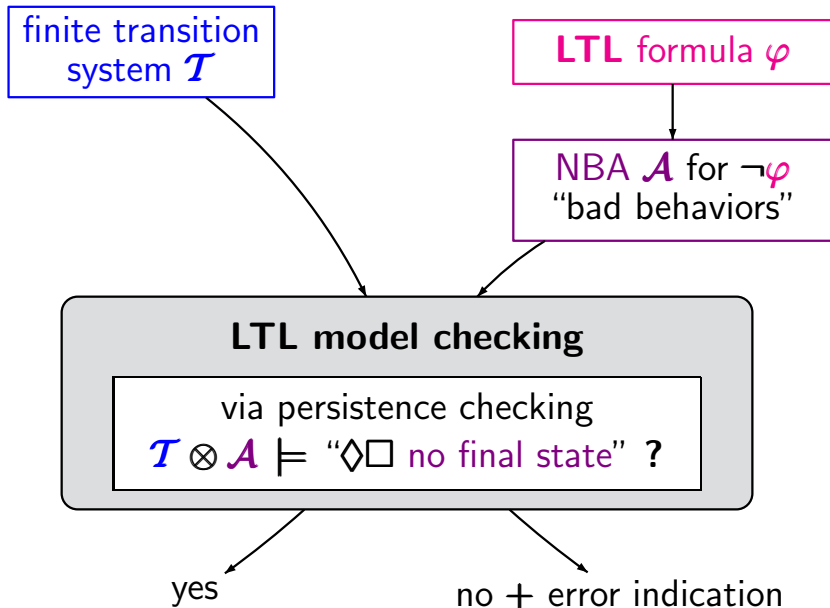


construct the product-TS $\mathcal{T} \otimes \mathcal{A}$
search a path in the product that meets
the acceptance condition of \mathcal{A}









safety property E

LTL-formula φ

safety property E

LTL-formula φ

NFA for the
bad prefixes for E
 $\mathcal{L}(\mathcal{A}) \subseteq (2^{AP})^+$

safety property E

LTL-formula φ

NFA for the
bad prefixes for E
 $\mathcal{L}(\mathcal{A}) \subseteq (2^{AP})^+$

NBA for the
“bad behaviors”
 $\mathcal{L}_w(\mathcal{A}) = \text{Words}(\neg\varphi)$

safety property E

LTL-formula φ

NFA for the
bad prefixes for E
 $\mathcal{L}(\mathcal{A}) \subseteq (2^{AP})^+$

NBA for the
“bad behaviors”
 $\mathcal{L}_w(\mathcal{A}) = \text{Words}(\neg\varphi)$

$$\text{Traces}_{fin}(T) \cap \mathcal{L}(\mathcal{A}) = \emptyset$$

safety property E

LTL-formula φ

NFA for the
bad prefixes for E
 $\mathcal{L}(\mathcal{A}) \subseteq (2^{AP})^+$

NBA for the
“bad behaviors”
 $\mathcal{L}_w(\mathcal{A}) = \text{Words}(\neg\varphi)$

$$\text{Traces}_{fin}(T) \cap \mathcal{L}(\mathcal{A}) = \emptyset$$

$$\text{Traces}(T) \cap \mathcal{L}_w(\mathcal{A}) = \emptyset$$

safety property E

LTL-formula φ

NFA for the
bad prefixes for E
 $\mathcal{L}(\mathcal{A}) \subseteq (2^{AP})^+$

NBA for the
“bad behaviors”
 $\mathcal{L}_w(\mathcal{A}) = \text{Words}(\neg\varphi)$

$$\text{Traces}_{fin}(T) \cap \mathcal{L}(\mathcal{A}) = \emptyset$$

$$\text{Traces}(T) \cap \mathcal{L}_w(\mathcal{A}) = \emptyset$$

invariant checking
in the product

$$T \otimes \mathcal{A} \models \Box \neg F ?$$

safety property E	LTL-formula φ
<p>NFA for the bad prefixes for E</p> $\mathcal{L}(\mathcal{A}) \subseteq (2^{AP})^+$	<p>NBA for the “bad behaviors”</p> $\mathcal{L}_w(\mathcal{A}) = \text{Words}(\neg\varphi)$
$\text{Traces}_{fin}(T) \cap \mathcal{L}(\mathcal{A}) = \emptyset$	$\text{Traces}(T) \cap \mathcal{L}_w(\mathcal{A}) = \emptyset$
<p>invariant checking in the product</p> $T \otimes \mathcal{A} \models \Box \neg F ?$	<p>persistence checking in the product</p> $T \otimes \mathcal{A} \models \Diamond \Box \neg F ?$

safety property E	LTL-formula φ
NFA for the bad prefixes for E $\mathcal{L}(\mathcal{A}) \subseteq (2^{AP})^+$	NBA for the “bad behaviors” $\mathcal{L}_w(\mathcal{A}) = \text{Words}(\neg\varphi)$
$\text{Traces}_{fin}(T) \cap \mathcal{L}(\mathcal{A}) = \emptyset$	$\text{Traces}(T) \cap \mathcal{L}_w(\mathcal{A}) = \emptyset$
invariant checking in the product $T \otimes \mathcal{A} \models \Box \neg F$?	persistence checking in the product $T \otimes \mathcal{A} \models \Diamond \Box \neg F$?
error indication: $\hat{\pi} \in \text{Paths}_{fin}(T)$ s.t. $\text{trace}(\hat{\pi}) \in \mathcal{L}(\mathcal{A})$	

safety property E

LTL-formula φ

NFA for the
bad prefixes for E
 $\mathcal{L}(\mathcal{A}) \subseteq (2^{AP})^+$

NBA for the
“bad behaviors”
 $\mathcal{L}_w(\mathcal{A}) = \text{Words}(\neg\varphi)$

$$\text{Traces}_{fin}(T) \cap \mathcal{L}(\mathcal{A}) = \emptyset$$

$$\text{Traces}(T) \cap \mathcal{L}_w(\mathcal{A}) = \emptyset$$

invariant checking
in the product

$$T \otimes \mathcal{A} \models \Box \neg F ?$$

persistence checking
in the product

$$T \otimes \mathcal{A} \models \Diamond \Box \neg F ?$$

error indication:

$$\hat{\pi} \in \text{Paths}_{fin}(T)$$

$$\text{s.t. } \text{trace}(\hat{\pi}) \in \mathcal{L}(\mathcal{A})$$

error indication:

prefix of a path π

$$\text{s.t. } \text{trace}(\pi) \in \mathcal{L}_w(\mathcal{A})$$

$\mathcal{T} \models$ safety property E

iff $Traces_{fin}(\mathcal{T}) \cap \mathcal{L}(\mathcal{A}) = \emptyset$

where \mathcal{A} is an NFA for the bad prefixes

$\mathcal{T} \models$ LTL-formula φ

iff $Traces(\mathcal{T}) \cap \mathcal{L}_\omega(\mathcal{A}) = \emptyset$

where \mathcal{A} is an NBA for $\neg\varphi$

$\mathcal{T} \models$ safety property E

iff $Traces_{fin}(\mathcal{T}) \cap \mathcal{L}(\mathcal{A}) = \emptyset$

iff there is no path fragment $\langle s_0, q_0 \rangle \langle s_1, q_1 \rangle \dots \langle s_n, q_n \rangle$
in $\mathcal{T} \otimes \mathcal{A}$ s. t. $q_n \in F$

$\mathcal{T} \models$ LTL-formula φ

iff $Traces(\mathcal{T}) \cap \mathcal{L}_\omega(\mathcal{A}) = \emptyset$

iff there is no path $\langle s_0, q_0 \rangle \langle s_1, q_1 \rangle \langle s_2, q_2 \rangle \dots$
in $\mathcal{T} \otimes \mathcal{A}$ s.t. $q_i \in F$ for infinitely many $i \in \mathbb{N}$

$\mathcal{T} \models$ safety property E

iff $Traces_{fin}(\mathcal{T}) \cap \mathcal{L}(\mathcal{A}) = \emptyset$

iff there is no path fragment $\langle s_0, q_0 \rangle \langle s_1, q_1 \rangle \dots \langle s_n, q_n \rangle$
in $\mathcal{T} \otimes \mathcal{A}$ s. t. $q_n \in F$

iff $\mathcal{T} \otimes \mathcal{A} \models \Box \neg F$

$\mathcal{T} \models$ LTL-formula φ

iff $Traces(\mathcal{T}) \cap \mathcal{L}_\omega(\mathcal{A}) = \emptyset$

iff there is no path $\langle s_0, q_0 \rangle \langle s_1, q_1 \rangle \langle s_2, q_2 \rangle \dots$
in $\mathcal{T} \otimes \mathcal{A}$ s.t. $q_i \in F$ for infinitely many $i \in \mathbb{N}$

iff $\mathcal{T} \otimes \mathcal{A} \models \Diamond \Box \neg F$

$\mathcal{T} \models$ safety property E

iff $Traces_{fin}(\mathcal{T}) \cap \mathcal{L}(\mathcal{A}) = \emptyset$

iff there is no path fragment $\langle s_0, q_0 \rangle \langle s_1, q_1 \rangle \dots \langle s_n, q_n \rangle$
in $\mathcal{T} \otimes \mathcal{A}$ s. t. $q_n \in F$

iff $\mathcal{T} \otimes \mathcal{A} \models \Box \neg F \longleftarrow$ invariant checking

$\mathcal{T} \models$ LTL-formula φ

iff $Traces(\mathcal{T}) \cap \mathcal{L}_\omega(\mathcal{A}) = \emptyset$

iff there is no path $\langle s_0, q_0 \rangle \langle s_1, q_1 \rangle \langle s_2, q_2 \rangle \dots$
in $\mathcal{T} \otimes \mathcal{A}$ s.t. $q_i \in F$ for infinitely many $i \in \mathbb{N}$

iff $\mathcal{T} \otimes \mathcal{A} \models \Diamond \Box \neg F \longleftarrow$ persistence checking

From LTL to NBA

LTLMC3.2-THM-LTL-2-NBA

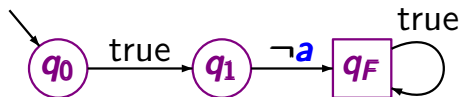
For each **LTL** formula φ over AP there is an **NBA** \mathcal{A} over the alphabet 2^{AP} such that

- $Words(\varphi) = \mathcal{L}_w(\mathcal{A})$
- $size(\mathcal{A}) = \mathcal{O}(\exp(|\varphi|))$

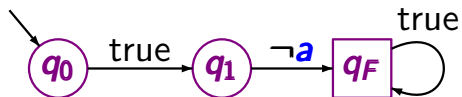
For each **LTL** formula φ over AP there is an **NBA** \mathcal{A} over the alphabet 2^{AP} such that

- $Words(\varphi) = \mathcal{L}_w(\mathcal{A})$
- $size(\mathcal{A}) = \mathcal{O}(\exp(|\varphi|))$

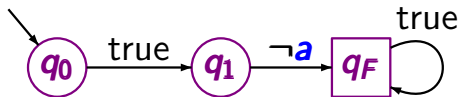
proof: ... later ...



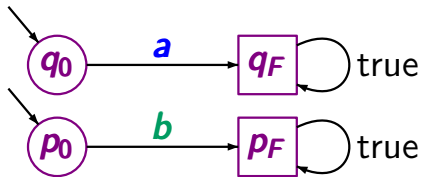
$$\mathcal{L}_\omega(\mathcal{A}) = ?$$



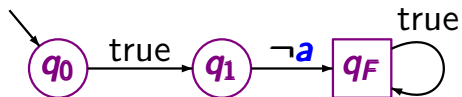
$$\mathcal{L}_\omega(\mathcal{A}) = \text{Words}(\bigcirc \neg a)$$



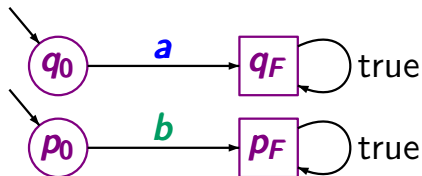
$$\mathcal{L}_w(\mathcal{A}) = \text{Words}(\bigcirc \neg a)$$



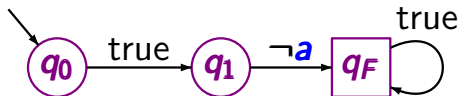
$$\mathcal{L}_w(\mathcal{A}) = ?$$



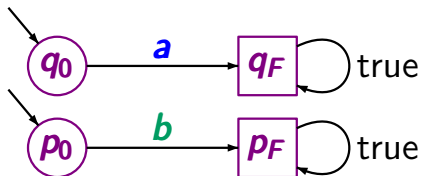
$$\mathcal{L}_w(\mathcal{A}) = \text{Words}(\bigcirc \neg a)$$



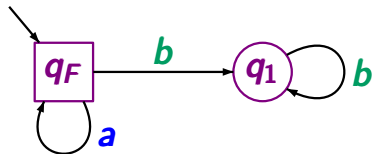
$$\mathcal{L}_w(\mathcal{A}) = \text{Words}(a \vee b)$$



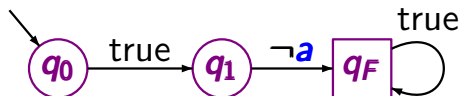
$$\mathcal{L}_\omega(\mathcal{A}) = \text{Words}(\Box \neg a)$$



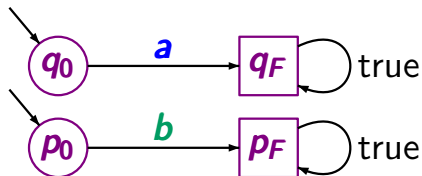
$$\mathcal{L}_\omega(\mathcal{A}) = \text{Words}(a \vee b)$$



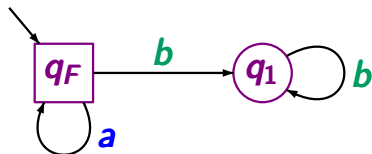
$$\mathcal{L}_\omega(\mathcal{A}) = ?$$



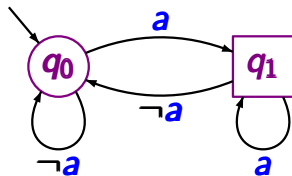
$$\mathcal{L}_\omega(\mathcal{A}) = \text{Words}(\bigcirc \neg a)$$



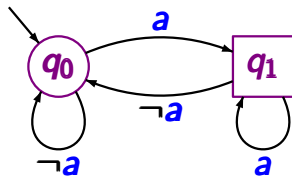
$$\mathcal{L}_\omega(\mathcal{A}) = \text{Words}(a \vee b)$$



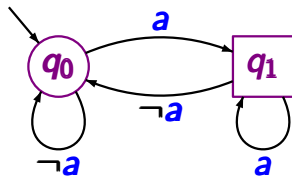
$$\mathcal{L}_\omega(\mathcal{A}) = \text{Words}(\Box a)$$



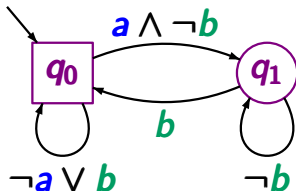
$$\mathcal{L}_\omega(\mathcal{A}) = ?$$



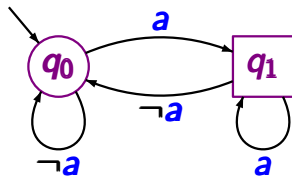
$$\mathcal{L}_\omega(\mathcal{A}) = \text{Words}(\Box \Diamond a)$$



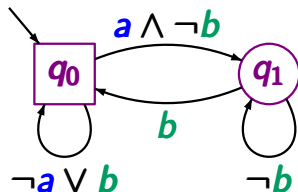
$$\mathcal{L}_\omega(\mathcal{A}) = \text{Words}(\Box \Diamond a)$$



$$\mathcal{L}_\omega(\mathcal{A}) = ?$$

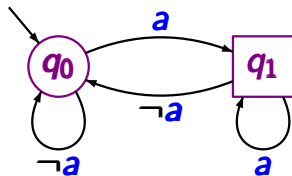


$$\mathcal{L}_\omega(\mathcal{A}) = \text{Words}(\Box\Diamond a)$$

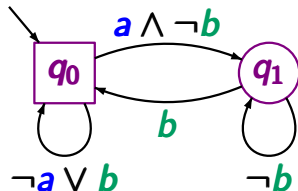


$$\mathcal{L}_\omega(\mathcal{A}) = ?$$

e.g., $\emptyset\emptyset\emptyset\emptyset\dots = \emptyset^\omega$
 $(\{a\}\{b\})^\omega$ } are accepted by \mathcal{A}

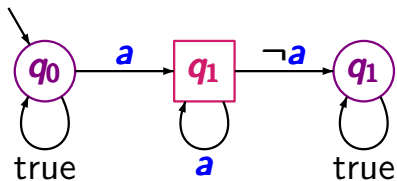


$$\mathcal{L}_\omega(\mathcal{A}) = \text{Words}(\Box\Diamond a)$$

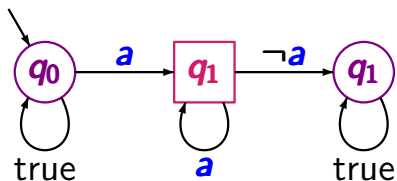


$$\mathcal{L}_\omega(\mathcal{A}) = \text{Words}(\Box(a \rightarrow \Diamond b))$$

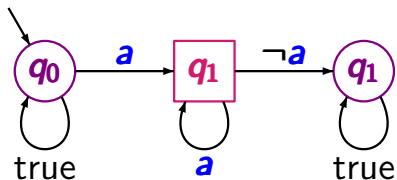
e.g., $\emptyset\emptyset\emptyset\emptyset\dots = \emptyset^\omega$
 $\left. \begin{array}{l} \{\{a\}\{b\}\}^\omega \end{array} \right\}$ are accepted by \mathcal{A}



$$\mathcal{L}_\omega(\mathcal{A}) = ?$$



$$\mathcal{L}_\omega(\mathcal{A}) = \text{Words}(\diamond \Box a)$$



$$\mathcal{L}_\omega(\mathcal{A}) = \text{Words}(\Diamond \Box a)$$

possible runs for $\{a\}^\omega$

$q_0 \ q_0 \ q_0 \ q_0 \ q_0 \ q_0 \ \dots$

$q_0 \ q_1 \ q_1 \ q_1 \ q_1 \ q_1 \ \dots$

$q_0 \ q_0 \ q_1 \ q_1 \ q_1 \ q_1 \ \dots$

$q_0 \ q_0 \ q_0 \ q_1 \ q_1 \ q_1 \ \dots$

\vdots

not accepting

accepting

accepting

accepting

Let \mathcal{A} be an **NFA** for the language of all **bad prefixes** for a safety property E .

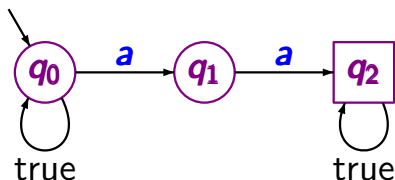
Let \mathcal{A} be an **NFA** for the language of all **bad prefixes** for a safety property E . Then:

$$\mathcal{L}_\omega(\mathcal{A}) = \overline{E} = (2^{AP})^\omega \setminus E$$

Let \mathcal{A} be an **NFA** for the language of all **bad prefixes** for a safety property E . Then:

$$\mathcal{L}_\omega(\mathcal{A}) = \overline{E} = (2^{AP})^\omega \setminus E$$

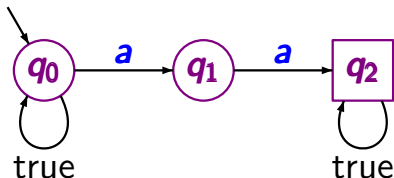
Example: $E \triangleq$ “never a twice in a row”



Let \mathcal{A} be an **NFA** for the language of all **bad prefixes** for a safety property E . Then:

$$\mathcal{L}_\omega(\mathcal{A}) = \overline{E} = (2^{AP})^\omega \setminus E = \text{Words}(\neg\varphi)$$

Example: $E \hat{=} \text{“never } a \text{ twice in a row”}$



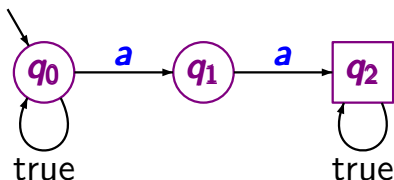
$$\varphi = \Box(a \rightarrow \bigcirc \neg a)$$

Let \mathcal{A} be an **NFA** for the language of all **bad prefixes** for a safety property E . Then:

$$\mathcal{L}_\omega(\mathcal{A}) = \overline{E} = (2^{AP})^\omega \setminus E = \text{Words}(\neg\varphi)$$

wrong, if $\mathcal{L}(\mathcal{A})$ = language of **minimal bad prefixes**

Example: $E \hat{=}$ “never **a** twice in a row”



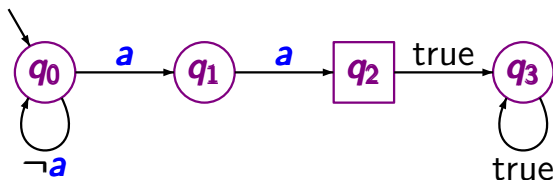
$$\varphi = \Box(a \rightarrow \bigcirc \neg a)$$

Let \mathcal{A} be an **NFA** for the language of all **bad prefixes** for a safety property E . Then:

$$\mathcal{L}_\omega(\mathcal{A}) = \overline{E} = (2^{AP})^\omega \setminus E = \text{Words}(\neg\varphi)$$

wrong, if $\mathcal{L}(\mathcal{A})$ = language of **minimal bad prefixes**

Example: $E \hat{=}$ “never a twice in a row”



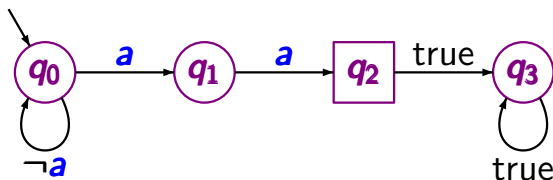
$$\mathcal{L}_\omega(\mathcal{A}) = \emptyset$$

Let \mathcal{A} be an **NFA** for the language of all **bad prefixes** for a safety property E . Then:

$$\mathcal{L}_\omega(\mathcal{A}) = \overline{E} = (2^{AP})^\omega \setminus E = \text{Words}(\neg\varphi)$$

wrong, if $\mathcal{L}(\mathcal{A})$ = language of **minimal bad prefixes** even if \mathcal{A} is a non-blocking DFA

Example: $E \hat{=}$ “never **a** twice in a row”



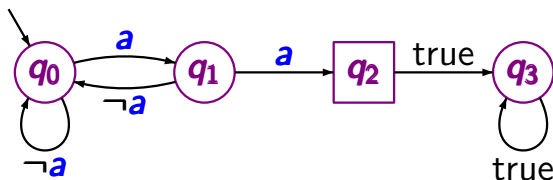
$$\mathcal{L}_\omega(\mathcal{A}) = \emptyset$$

Let \mathcal{A} be an **NFA** for the language of all **bad prefixes** for a safety property E . Then:

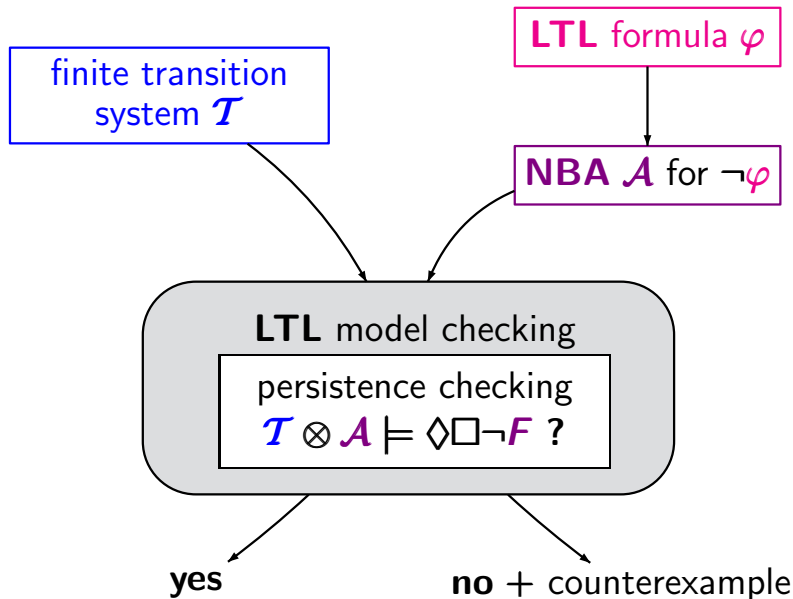
$$\mathcal{L}_\omega(\mathcal{A}) = \overline{E} = (2^{AP})^\omega \setminus E = \text{Words}(\neg\varphi)$$

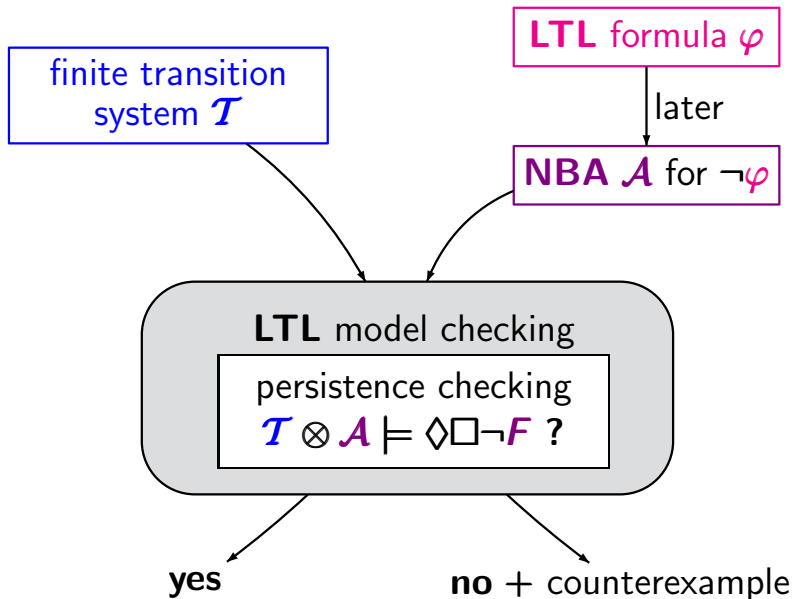
wrong, if $\mathcal{L}(\mathcal{A})$ = language of **minimal bad prefixes** even if \mathcal{A} is a non-blocking DFA

Example: $E \hat{=}$ “never **a** twice in a row”



$$\mathcal{L}_\omega(\mathcal{A}) = \emptyset$$





given: finite TS \mathcal{T} , LTL-formula φ

question: does $\mathcal{T} \models \varphi$ hold ?

given: finite TS \mathcal{T} , LTL-formula φ

question: does $\mathcal{T} \models \varphi$ hold ?

construct an NBA \mathcal{A} for $\neg\varphi$ and the product $\mathcal{T} \otimes \mathcal{A}$

check whether $\mathcal{T} \otimes \mathcal{A} \models \Diamond\Box\neg F \leftarrow$

persistence
checking
nested **DFS**

given: finite TS \mathcal{T} , LTL-formula φ

question: does $\mathcal{T} \models \varphi$ hold ?

construct an NBA \mathcal{A} for $\neg\varphi$ and the product $\mathcal{T} \otimes \mathcal{A}$

check whether $\mathcal{T} \otimes \mathcal{A} \models \Diamond\Box\neg F$ ←

persistence
checking
nested **DFS**

IF $\mathcal{T} \otimes \mathcal{A} \models \Diamond\Box\neg F$

THEN return “yes”

ELSE compute a counterexample

$\langle s_0, p_0 \rangle \dots \langle s_n, p_n \rangle \dots \langle s_n, p_n \rangle$

for $\mathcal{T} \otimes \mathcal{A}$ and $\Diamond\Box\neg F$

return “no” and $s_0 \dots s_n \dots s_n$

given: finite TS \mathcal{T} , LTL-formula φ

question: does $\mathcal{T} \models \varphi$ hold ?

~~construct an NBA \mathcal{A} for $\neg\varphi$ and the product $\mathcal{T} \otimes \mathcal{A}$~~

~~check whether $\mathcal{T} \otimes \mathcal{A} \models \Diamond\Box\neg F$~~ ←

persistence
checking
nested **DFS**

IF $\mathcal{T} \otimes \mathcal{A} \models \Diamond\Box\neg F$

THEN return “yes”

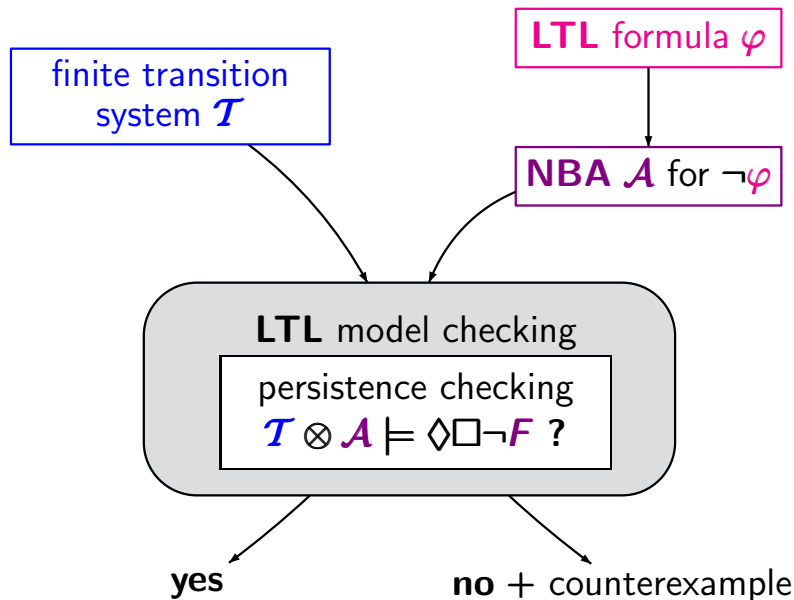
ELSE compute a counterexample

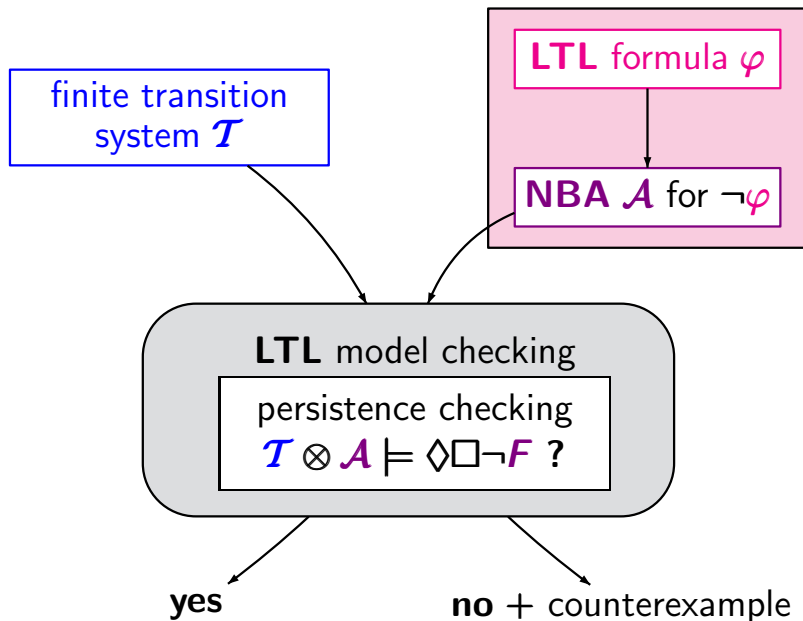
$\langle s_0, p_0 \rangle \dots \langle s_n, p_n \rangle \dots \langle s_n, p_n \rangle$

for $\mathcal{T} \otimes \mathcal{A}$ and $\Diamond\Box\neg F$

return “no” and $s_0 \dots s_n \dots s_n$

time complexity: $\mathcal{O}(\text{size}(\mathcal{T}) \cdot \text{size}(\mathcal{A}))$





For each **LTL** formula φ there is an **NBA** \mathcal{A} s.t.

$$\mathcal{L}_\omega(\mathcal{A}) = \text{Words}(\varphi)$$

For each **LTL** formula φ there is an **NBA** \mathcal{A} s.t.
 $\mathcal{L}_\omega(\mathcal{A}) = \text{Words}(\varphi)$

LTL formula φ



NBA \mathcal{A} s.t.
 $\mathcal{L}_\omega(\mathcal{A}) = \text{Words}(\varphi)$

nondeterministic
Büchi automaton

For each **LTL** formula φ there is an **NBA** \mathcal{A} s.t.
 $\mathcal{L}_\omega(\mathcal{A}) = \text{Words}(\varphi)$

LTL formula φ

GNBA \mathcal{G} s.t.
 $\mathcal{L}_\omega(\mathcal{G}) = \text{Words}(\varphi)$

generalized NBA
several acceptance sets

NBA \mathcal{A} s.t.
 $\mathcal{L}_\omega(\mathcal{A}) = \mathcal{L}_\omega(\mathcal{G})$

nondeterministic
Büchi automaton
1 acceptance set

For each **LTL** formula φ there is an **NBA** \mathcal{A} s.t.
 $\mathcal{L}_\omega(\mathcal{A}) = \text{Words}(\varphi)$

LTL formula φ

GNBA \mathcal{G} s.t.
 $\mathcal{L}_\omega(\mathcal{G}) = \text{Words}(\varphi)$

NBA \mathcal{A} s.t.
 $\mathcal{L}_\omega(\mathcal{A}) = \mathcal{L}_\omega(\mathcal{G})$

generalized NBA
 k acceptance sets

k copies of \mathcal{G}

nondeterministic
Büchi automaton
1 acceptance set

idea: encode the semantics of the operators appearing in φ by appropriate components of the GNBA \mathcal{G}

idea: encode the semantics of the operators appearing in φ by appropriate components of the GNBA \mathcal{G}

semantics of ...	encoding
propositional logic <i>true</i> , \neg , \wedge	
next \bigcirc	
until \mathbf{U}	

idea: encode the semantics of the operators appearing in φ by appropriate components of the GNBA \mathcal{G}

semantics of ...	encoding
propositional logic <i>true</i> , \neg , \wedge	in the <i>states</i>
next \bigcirc	in the <i>transition relation</i>
until \mathbf{U}	via <i>expansion law</i>

idea: encode the semantics of the operators appearing in φ by appropriate components of the GNBA \mathcal{G}

semantics of ...	encoding
propositional logic $\text{true}, \neg, \wedge$	in the states
next \bigcirc	in the transition relation
until \mathbf{U}	via expansion law

$$\psi_1 \mathbf{U} \psi_2 \equiv \psi_2 \vee (\psi_1 \wedge \bigcirc(\psi_1 \mathbf{U} \psi_2))$$

encoded in
the **states**

encoded in the
transition relation

idea: encode the semantics of the operators appearing in φ by appropriate components of the GNBA \mathcal{G}

semantics of ...	encoding
propositional logic <i>true</i> , \neg , \wedge	in the <i>states</i>
next \bigcirc	in the <i>transition relation</i>
until \mathbf{U}	expansion law, <i>least fixed point</i>

$$\psi_1 \mathbf{U} \psi_2 \equiv \psi_2 \vee (\psi_1 \wedge \bigcirc(\psi_1 \mathbf{U} \psi_2))$$

encoded in
the *states*

encoded in the
transition relation

*acceptance
condition*



LTL formula $\varphi \rightsquigarrow$ GNBA \mathcal{G} for $Words(\varphi)$

LTL formula $\varphi \rightsquigarrow$ GNBA \mathcal{G} for $\text{Words}(\varphi)$

states of $\mathcal{G} \hat{=} (\text{certain})$ sets of subformulas of φ

LTL formula $\varphi \rightsquigarrow$ GNBA \mathcal{G} for $Words(\varphi)$

states of $\mathcal{G} \hat{=}$ (certain) sets of subformulas of φ
s.t. each word $\sigma = A_0 A_1 A_2 \dots \in Words(\varphi)$ can be
extended to an accepting run $B_0 B_1 B_2 \dots$ in \mathcal{G}

LTL formula $\varphi \rightsquigarrow$ GNBA \mathcal{G} for $Words(\varphi)$

states of $\mathcal{G} \hat{=}$ (certain) sets of subformulas of φ
s.t. each word $\sigma = A_0 A_1 A_2 \dots \in Words(\varphi)$ can be
extended to an accepting run $B_0 B_1 B_2 \dots$ in \mathcal{G}

$A_0 A_1 A_2 A_3 \dots \in Words(\varphi)$

LTL formula $\varphi \rightsquigarrow$ GNBA \mathcal{G} for $Words(\varphi)$

states of $\mathcal{G} \hat{=}$ (certain) sets of subformulas of φ
 s.t. each word $\sigma = A_0 A_1 A_2 \dots \in Words(\varphi)$ can be
 extended to an accepting run $B_0 B_1 B_2 \dots$ in \mathcal{G}

$$A_0 \ A_1 \ A_2 \ A_3 \ \dots \in Words(\varphi)$$

$$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$$

$$B_0 \ B_1 \ B_2 \ B_3 \ \dots \text{ accepting run}$$

where $B_i = \{ \psi \in cl(\varphi) : A_i A_{i+1} A_{i+2} \dots \models \psi \}$

LTL formula $\varphi \rightsquigarrow$ GNBA \mathcal{G} for $Words(\varphi)$

states of $\mathcal{G} \hat{=}$ (certain) sets of subformulas of φ
 s.t. each word $\sigma = A_0 A_1 A_2 \dots \in Words(\varphi)$ can be
 extended to an accepting run $B_0 B_1 B_2 \dots$ in \mathcal{G}

$A_0 \ A_1 \ A_2 \ A_3 \ \dots \in Words(\varphi)$

$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$

$B_0 \ B_1 \ B_2 \ B_3 \ \dots$ accepting run

where $B_i = \{ \psi \in cl(\varphi) : A_i A_{i+1} A_{i+2} \dots \models \psi \}$

set of subformulas of φ and their negations

LTL formula $\varphi \rightsquigarrow$ GNBA \mathcal{G} for $Words(\varphi)$

states of $\mathcal{G} \hat{=}$ (certain) sets of subformulas of φ
s.t. each word $\sigma = A_0 A_1 A_2 \dots \in Words(\varphi)$ can be
extended to an accepting run $B_0 B_1 B_2 \dots$ in \mathcal{G}

Example: $\varphi = a \text{ U } (\neg a \wedge b)$

LTL formula φ \rightsquigarrow GNBA \mathcal{G} for $Words(\varphi)$

states of $\mathcal{G} \hat{=}$ (certain) sets of subformulas of φ
 s.t. each word $\sigma = A_0 A_1 A_2 \dots \in Words(\varphi)$ can be
 extended to an accepting run $B_0 B_1 B_2 \dots$ in \mathcal{G}

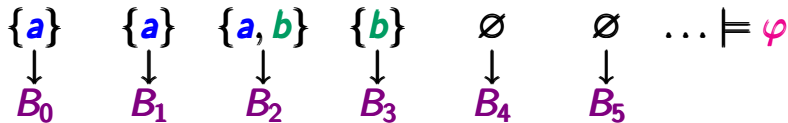
Example: $\varphi = a \text{ U } (\neg a \wedge b)$

$\{a\}$ $\{a\}$ $\{a, b\}$ $\{b\}$ \emptyset \emptyset $\dots \models \varphi$

LTL formula $\varphi \rightsquigarrow$ GNBA \mathcal{G} for $Words(\varphi)$

states of $\mathcal{G} \hat{=}$ (certain) sets of subformulas of φ
 s.t. each word $\sigma = A_0 A_1 A_2 \dots \in Words(\varphi)$ can be
 extended to an accepting run $B_0 B_1 B_2 \dots$ in \mathcal{G}

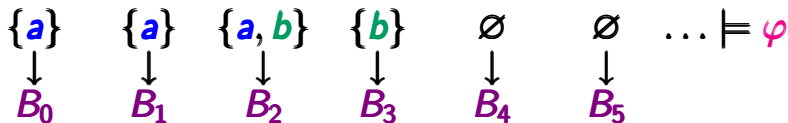
Example: $\varphi = a \mathbf{U}(\neg a \wedge b)$



LTL formula $\varphi \rightsquigarrow$ GNBA \mathcal{G} for $Words(\varphi)$

states of $\mathcal{G} \hat{=}$ (certain) sets of subformulas of φ
 s.t. each word $\sigma = A_0 A_1 A_2 \dots \in Words(\varphi)$ can be
 extended to an accepting run $B_0 B_1 B_2 \dots$ in \mathcal{G}

Example: $\varphi = a \mathbf{U}(\neg a \wedge b)$ $\psi = \neg a \wedge b$



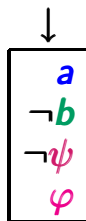
where the B_i 's are subsets of
 $\{a, \neg a, b, \neg b, \psi, \neg \psi, \varphi, \neg \varphi\}$

LTL formula $\varphi \rightsquigarrow$ GNBA \mathcal{G} for $Words(\varphi)$

states of $\mathcal{G} \hat{=}$ (certain) sets of subformulas of φ
 s.t. each word $\sigma = A_0 A_1 A_2 \dots \in Words(\varphi)$ can be
 extended to an accepting run $B_0 B_1 B_2 \dots$ in \mathcal{G}

Example: $\varphi = a \mathbf{U}(\neg a \wedge b)$ $\psi = \neg a \wedge b$

$\{a\}$ $\{a\}$ $\{a, b\}$ $\{b\}$ \emptyset \emptyset $\dots \models \varphi$

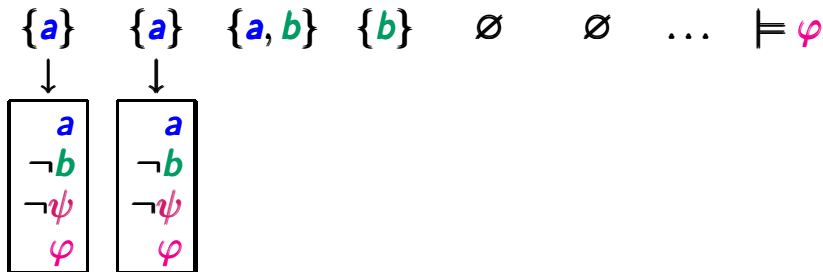


just for better readability:
 tuple rather than set notation

LTL formula $\varphi \rightsquigarrow$ GNBA \mathcal{G} for $Words(\varphi)$

states of $\mathcal{G} \hat{=}$ (certain) sets of subformulas of φ
 s.t. each word $\sigma = A_0 A_1 A_2 \dots \in Words(\varphi)$ can be
 extended to an accepting run $B_0 B_1 B_2 \dots$ in \mathcal{G}

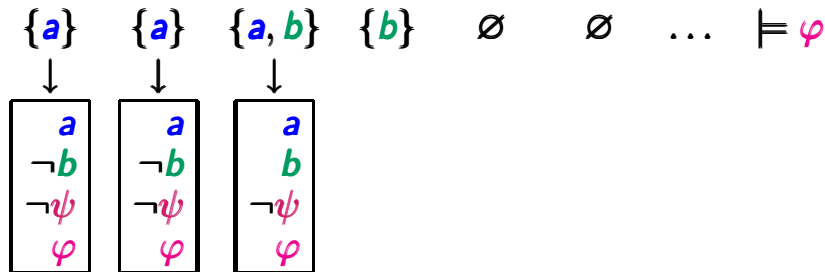
Example: $\varphi = a \mathbf{U}(\neg a \wedge b)$ $\psi = \neg a \wedge b$



LTL formula $\varphi \rightsquigarrow$ GNBA \mathcal{G} for $Words(\varphi)$

states of $\mathcal{G} \hat{=}$ (certain) sets of subformulas of φ
 s.t. each word $\sigma = A_0 A_1 A_2 \dots \in Words(\varphi)$ can be
 extended to an accepting run $B_0 B_1 B_2 \dots$ in \mathcal{G}

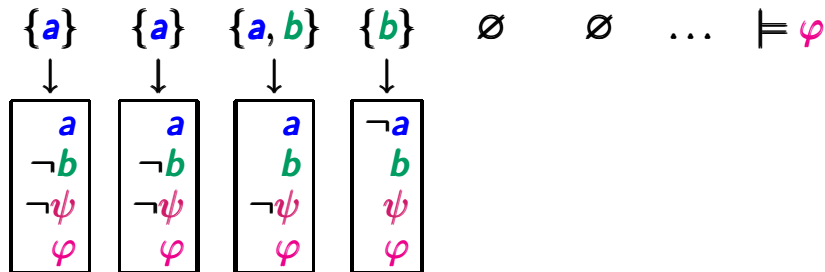
Example: $\varphi = a \text{ U } (\neg a \wedge b)$ $\psi = \neg a \wedge b$



LTL formula $\varphi \rightsquigarrow$ GNBA \mathcal{G} for $Words(\varphi)$

states of $\mathcal{G} \hat{=}$ (certain) sets of subformulas of φ
 s.t. each word $\sigma = A_0 A_1 A_2 \dots \in Words(\varphi)$ can be
 extended to an accepting run $B_0 B_1 B_2 \dots$ in \mathcal{G}

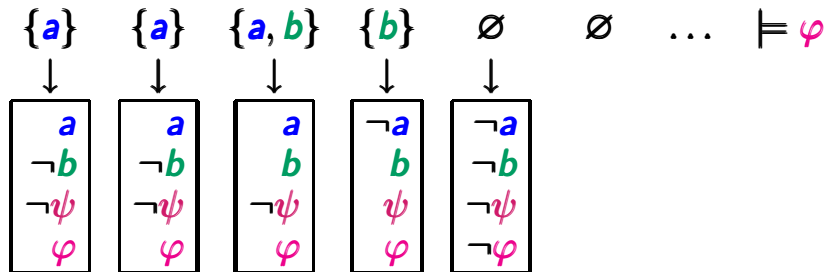
Example: $\varphi = a \text{ U } (\neg a \wedge b)$ $\psi = \neg a \wedge b$



LTL formula $\varphi \rightsquigarrow$ GNBA \mathcal{G} for $Words(\varphi)$

states of $\mathcal{G} \hat{=}$ (certain) sets of subformulas of φ
 s.t. each word $\sigma = A_0 A_1 A_2 \dots \in Words(\varphi)$ can be
 extended to an accepting run $B_0 B_1 B_2 \dots$ in \mathcal{G}

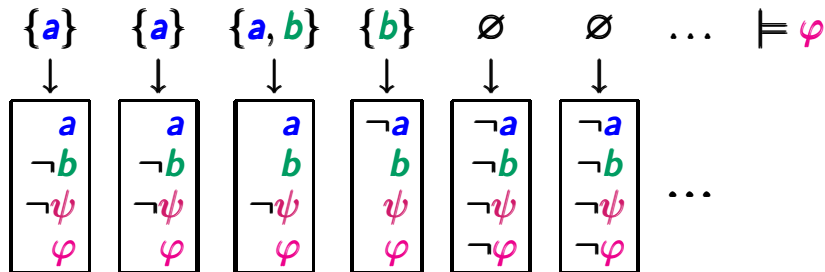
Example: $\varphi = a \text{ U } (\neg a \wedge b)$ $\psi = \neg a \wedge b$



LTL formula $\varphi \rightsquigarrow$ GNBA \mathcal{G} for $Words(\varphi)$

states of $\mathcal{G} \hat{=}$ (certain) sets of subformulas of φ
 s.t. each word $\sigma = A_0 A_1 A_2 \dots \in Words(\varphi)$ can be
 extended to an accepting run $B_0 B_1 B_2 \dots$ in \mathcal{G}

Example: $\varphi = a \mathbf{U}(\neg a \wedge b)$ $\psi = \neg a \wedge b$



Let φ be an LTL formula. Then:

$\text{subf}(\varphi) \stackrel{\text{def}}{=} \text{set of all subformulas of } \varphi$

Let φ be an LTL formula. Then:

$subf(\varphi) \stackrel{\text{def}}{=} \text{set of all subformulas of } \varphi$

$cl(\varphi) \stackrel{\text{def}}{=} subf(\varphi) \cup \{\neg\psi : \psi \in subf(\varphi)\}$

where ψ and $\neg\neg\psi$ are identified

Let φ be an LTL formula. Then:

$subf(\varphi) \stackrel{\text{def}}{=} \text{set of all subformulas of } \varphi$

$cl(\varphi) \stackrel{\text{def}}{=} subf(\varphi) \cup \{\neg\psi : \psi \in subf(\varphi)\}$

where ψ and $\neg\neg\psi$ are identified

Example: if $\varphi = a \mathbf{U} (\neg a \wedge b)$ then

$$cl(\varphi) = \{a, b, \neg a \wedge b, \varphi\} \cup \{\neg a, \neg b, \neg(\neg a \wedge b), \neg\varphi\}$$

Let φ be an LTL formula. Then:

$subf(\varphi) \stackrel{\text{def}}{=} \text{set of all subformulas of } \varphi$

$cl(\varphi) \stackrel{\text{def}}{=} subf(\varphi) \cup \{\neg\psi : \psi \in subf(\varphi)\}$

where ψ and $\neg\neg\psi$ are identified

Example: if $\varphi = a \cup (\neg a \wedge b)$ then

$$cl(\varphi) = \{a, b, \neg a \wedge b, \varphi\} \cup \{\neg a, \neg b, \neg(\neg a \wedge b), \neg\varphi\}$$

Example: if $\varphi' = \Box a$

Let φ be an LTL formula. Then:

$subf(\varphi) \stackrel{\text{def}}{=} \text{set of all subformulas of } \varphi$

$cl(\varphi) \stackrel{\text{def}}{=} subf(\varphi) \cup \{\neg\psi : \psi \in subf(\varphi)\}$

where ψ and $\neg\neg\psi$ are identified

Example: if $\varphi = a \cup (\neg a \wedge b)$ then

$$cl(\varphi) = \{a, b, \neg a \wedge b, \varphi\} \cup \{\neg a, \neg b, \neg(\neg a \wedge b), \neg\varphi\}$$

Example: if $\varphi' = \Box a = \neg\Diamond\neg a = \neg(true \cup \neg a)$

Let φ be an LTL formula. Then:

$subf(\varphi) \stackrel{\text{def}}{=} \text{set of all subformulas of } \varphi$

$cl(\varphi) \stackrel{\text{def}}{=} subf(\varphi) \cup \{\neg\psi : \psi \in subf(\varphi)\}$

where ψ and $\neg\neg\psi$ are identified

Example: if $\varphi = a \cup (\neg a \wedge b)$ then

$$cl(\varphi) = \{a, b, \neg a \wedge b, \varphi\} \cup \{\neg a, \neg b, \neg(\neg a \wedge b), \neg\varphi\}$$

Example: if $\varphi' = \Box a = \neg\Diamond\neg a = \neg(true \cup \neg a)$ then

$$cl(\varphi') = \{a, \neg a, true, \neg true, \Box a, \neg\Box a\}$$

$B \subseteq cl(\varphi)$ is elementary iff:

- (i) B is maximal consistent w.r.t. prop. logic, i.e., if $\psi, \psi_1 \wedge \psi_2 \in cl(\varphi)$ then:

$$\begin{array}{ll} \psi \notin B & \text{iff} \quad \neg\psi \in B \\ \psi_1 \wedge \psi_2 \in B & \text{iff} \quad \psi_1 \in B \text{ and } \psi_2 \in B \\ \text{true} \in cl(\varphi) & \text{implies} \quad \text{true} \in B \end{array}$$

- (ii) B is locally consistent with respect to until \mathbf{U} , i.e., if $\psi_1 \mathbf{U} \psi_2 \in cl(\varphi)$ then:

$$\begin{array}{l} \text{if } \psi_1 \mathbf{U} \psi_2 \in B \text{ and } \psi_2 \notin B \text{ then } \psi_1 \in B \\ \text{if } \psi_2 \in B \text{ then } \psi_1 \mathbf{U} \psi_2 \in B \end{array}$$

Elementary or not?

LTLMC3.2-49

Let $\varphi = a \mathbf{U} (\neg a \wedge b)$.

$$B_1 = \{a, b, \neg a \wedge b, \varphi\}$$

Elementary or not?

LTLMC3.2-49

Let $\varphi = a \mathbf{U}(\neg a \wedge b)$.

$B_1 = \{a, b, \neg a \wedge b, \varphi\}$

not elementary
propositional inconsistent

Elementary or not?

LTLMC3.2-49

Let $\varphi = a \mathbf{U}(\neg a \wedge b)$.

$$B_1 = \{a, b, \neg a \wedge b, \varphi\}$$

not elementary
propositional inconsistent

$$B_2 = \{\neg a, b, \varphi\}$$

Elementary or not?

LTLMC3.2-49

Let $\varphi = a \mathbf{U} (\neg a \wedge b)$.

$$B_1 = \{a, b, \neg a \wedge b, \varphi\}$$

not elementary
propositional inconsistent

$$B_2 = \{\neg a, b, \varphi\}$$

not elementary, not maximal

as $\neg a \wedge b \notin B_2$

$\neg(\neg a \wedge b) \notin B_2$

Elementary or not?

LTLMC3.2-49

Let $\varphi = a \vee (\neg a \wedge b)$.

$$B_1 = \{a, b, \neg a \wedge b, \varphi\}$$

not elementary
propositional inconsistent

$$B_2 = \{\neg a, b, \varphi\}$$

not elementary, not maximal

as $\neg a \wedge b \notin B_2$

$\neg(\neg a \wedge b) \notin B_2$

$$B_3 = \{\neg a, b, \neg a \wedge b, \neg \varphi\}$$

Elementary or not?

LTLMC3.2-49

Let $\varphi = a \mathbf{U} (\neg a \wedge b)$.

$$B_1 = \{a, b, \neg a \wedge b, \varphi\}$$

not elementary
propositional inconsistent

$$B_2 = \{\neg a, b, \varphi\}$$

not elementary, not maximal

as $\neg a \wedge b \notin B_2$

$\neg(\neg a \wedge b) \notin B_2$

$$B_3 = \{\neg a, b, \neg a \wedge b, \neg \varphi\}$$

not elementary
not locally consistent for \mathbf{U}

Let $\varphi = a \mathbf{U} (\neg a \wedge b)$.

$$B_1 = \{a, b, \neg a \wedge b, \varphi\}$$

not elementary
propositional inconsistent

$$B_2 = \{\neg a, b, \varphi\}$$

not elementary, not maximal

$$\text{as } \neg a \wedge b \notin B_2$$

$$\neg(\neg a \wedge b) \notin B_2$$

$$B_3 = \{\neg a, b, \neg a \wedge b, \neg \varphi\}$$

not elementary

not locally consistent for \mathbf{U}

$$B_4 = \{\neg a, \neg b, \neg(\neg a \wedge b), \neg \varphi\}$$

Let $\varphi = a \mathbf{U} (\neg a \wedge b)$.

$$B_1 = \{a, b, \neg a \wedge b, \varphi\}$$

not elementary
propositional inconsistent

$$B_2 = \{\neg a, b, \varphi\}$$

not elementary, not maximal

$$\text{as } \neg a \wedge b \notin B_2$$

$$\neg(\neg a \wedge b) \notin B_2$$

$$B_3 = \{\neg a, b, \neg a \wedge b, \neg \varphi\}$$

not elementary

not locally consistent for \mathbf{U}

$$B_4 = \{\neg a, \neg b, \neg(\neg a \wedge b), \neg \varphi\} \quad \text{elementary}$$

closure $cl(\varphi)$:

- set of all subformulas of φ and their negations
- ψ and $\neg\neg\psi$ are identified

elementary formula-sets: subsets B of $cl(\varphi)$

- maximal consistent w.r.t. propositional logic
- locally consistent w.r.t. \mathbf{U}

For $\varphi = a \mathbf{U}(\neg a \wedge b)$, the elementary sets are:

$\{ a, b, \neg(\neg a \wedge b), \varphi \}$	$\{ a, b, \neg(\neg a \wedge b), \neg\varphi \}$
$\{ a, \neg b, \neg(\neg a \wedge b), \varphi \}$	$\{ a, \neg b, \neg(\neg a \wedge b), \neg\varphi \}$
$\{ \neg a, b, \neg a \wedge b, \varphi \}$	$\{ \neg a, \neg b, \neg(\neg a \wedge b), \neg\varphi \}$

idea: encode the semantics of the operators appearing in φ by appropriate components of the GNBA \mathcal{G} :

semantics of ...	encoding
propositional logic <i>true</i> , \neg , \wedge	in the <i>states</i>
next \bigcirc	in the <i>transition relation</i>
until \mathbf{U}	expansion law, least fixed point

$$\psi_1 \mathbf{U} \psi_2 \equiv \psi_2 \vee (\psi_1 \wedge \bigcirc(\psi_1 \mathbf{U} \psi_2))$$



encoded in
the *states*

encoded in the
transition relation

*acceptance
condition*

idea: encode the semantics of the operators appearing in φ by appropriate components of the GNBA \mathcal{G} :

semantics of ...	encoding
propositional logic <i>true</i> , \neg , \wedge	in the <i>states</i>
next \bigcirc	in the <i>transition relation</i>
until \mathbf{U}	expansion law, least fixed point

$\psi_1 \mathbf{U} \psi_2 \equiv \psi_2 \vee (\psi_1 \wedge \bigcirc(\psi_1 \mathbf{U} \psi_2))$

elementary
formula sets

encoded in the
transition relation

*acceptance
condition*

$$\mathcal{G} = (Q, 2^{AP}, \delta, Q_0, \mathcal{F})$$

$$\mathcal{G} = (Q, 2^{AP}, \delta, Q_0, \mathcal{F})$$

state space: $Q = \{B \subseteq cl(\varphi) : B \text{ is elementary} \}$

$$\mathcal{G} = (Q, 2^{AP}, \delta, Q_0, \mathcal{F})$$

state space: $Q = \{B \subseteq cl(\varphi) : B \text{ is elementary} \}$

initial states: $Q_0 = \{B \in Q : \varphi \in B\}$

$$\mathcal{G} = (\mathcal{Q}, 2^{AP}, \delta, \mathcal{Q}_0, \mathcal{F})$$

state space: $\mathcal{Q} = \{B \subseteq cl(\varphi) : B \text{ is elementary} \}$

initial states: $\mathcal{Q}_0 = \{B \in \mathcal{Q} : \varphi \in B\}$

transition relation: for $B \in \mathcal{Q}$ and $A \in 2^{AP}$:

if $A \neq B \cap AP$ then $\delta(B, A) = \emptyset$

$$\mathcal{G} = (\mathcal{Q}, 2^{AP}, \delta, \mathcal{Q}_0, \mathcal{F})$$

state space: $\mathcal{Q} = \{B \subseteq cl(\varphi) : B \text{ is elementary}\}$

initial states: $\mathcal{Q}_0 = \{B \in \mathcal{Q} : \varphi \in B\}$

transition relation: for $B \in \mathcal{Q}$ and $A \in 2^{AP}$:

if $A \neq B \cap AP$ then $\delta(B, A) = \emptyset$

if $A = B \cap AP$ then $\delta(B, A) = \text{set of all } B' \in \mathcal{Q} \text{ s.t.}$

$$\bigcirc \psi \in B \text{ iff } \psi \in B'$$

$$\psi_1 \mathbf{U} \psi_2 \in B \text{ iff } (\psi_2 \in B) \vee (\psi_1 \in B \wedge \psi_1 \mathbf{U} \psi_2 \in B')$$

$$\mathcal{G} = (Q, 2^{AP}, \delta, Q_0, \mathcal{F})$$

state space: $Q = \{B \subseteq cl(\varphi) : B \text{ is elementary}\}$

initial states: $Q_0 = \{B \in Q : \varphi \in B\}$

transition relation: for $B \in Q$ and $A \in 2^{AP}$:

if $A \neq B \cap AP$ then $\delta(B, A) = \emptyset$

if $A = B \cap AP$ then $\delta(B, A) = \text{set of all } B' \in Q \text{ s.t.}$

$$\bigcirc \psi \in B \text{ iff } \psi \in B'$$

$$\psi_1 \mathbf{U} \psi_2 \in B \text{ iff } (\psi_2 \in B) \vee (\psi_1 \in B \wedge \psi_1 \mathbf{U} \psi_2 \in B')$$

acceptance set $\mathcal{F} = \{F_{\psi_1 \mathbf{U} \psi_2} : \psi_1 \mathbf{U} \psi_2 \in cl(\varphi)\}$

$$\mathcal{G} = (\mathcal{Q}, 2^{AP}, \delta, \mathcal{Q}_0, \mathcal{F})$$

state space: $\mathcal{Q} = \{B \subseteq cl(\varphi) : B \text{ is elementary}\}$

initial states: $\mathcal{Q}_0 = \{B \in \mathcal{Q} : \varphi \in B\}$

transition relation: for $B \in \mathcal{Q}$ and $A \in 2^{AP}$:

if $A \neq B \cap AP$ then $\delta(B, A) = \emptyset$

if $A = B \cap AP$ then $\delta(B, A) = \text{set of all } B' \in \mathcal{Q} \text{ s.t.}$

$$\bigcirc \psi \in B \text{ iff } \psi \in B'$$

$$\psi_1 \mathbf{U} \psi_2 \in B \text{ iff } (\psi_2 \in B) \vee (\psi_1 \in B \wedge \psi_1 \mathbf{U} \psi_2 \in B')$$

acceptance set $\mathcal{F} = \{F_{\psi_1 \mathbf{U} \psi_2} : \psi_1 \mathbf{U} \psi_2 \in cl(\varphi)\}$

where $F_{\psi_1 \mathbf{U} \psi_2} = \{B \in \mathcal{Q} : \psi_1 \mathbf{U} \psi_2 \notin B \vee \psi_2 \in B\}$

Example: GNBA for $\varphi = \bigcirc a$

LTLMC3.2-52

Example: GNBA for $\varphi = \bigcirc a$

LTLMC3.2-52

$a, \bigcirc a$

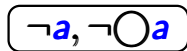
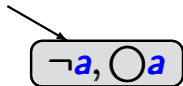
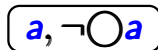
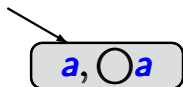
$a, \neg \bigcirc a$

$\neg a, \bigcirc a$

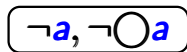
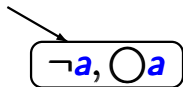
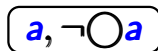
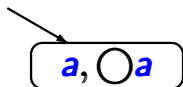
$\neg a, \neg \bigcirc a$

Example: GNBA for $\varphi = \bigcirc a$

LTLMC3.2-52



initial states: formula-sets B with $\bigcirc a \in B$



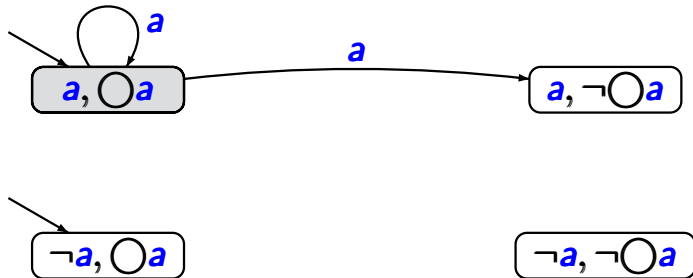
initial states: formula-sets B with $\bigcirc a \in B$

transition relation:

if $\bigcirc a \in B$ then $\delta(B, B \cap \{a\}) = \{B' : a \in B'\}$

Example: GNBA for $\varphi = \bigcirc a$

LTLMC3.2-52



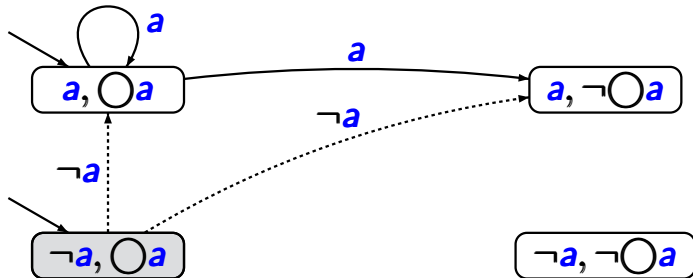
initial states: formula-sets B with $\bigcirc a \in B$

transition relation:

if $\bigcirc a \in B$ then $\delta(B, B \cap \{a\}) = \{B' : a \in B'\}$

Example: GNBA for $\varphi = \bigcirc a$

LTLMC3.2-52



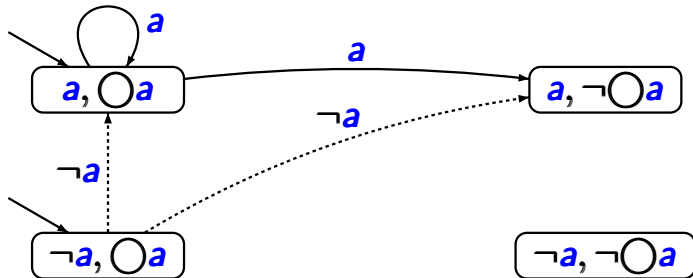
initial states: formula-sets B with $\bigcirc a \in B$

transition relation:

$$\text{if } \bigcirc a \in B \text{ then } \delta(B, B \cap \{a\}) = \{B' : a \in B'\}$$

Example: GNBA for $\varphi = \bigcirc a$

LTLMC3.2-52



initial states: formula-sets B with $\bigcirc a \in B$

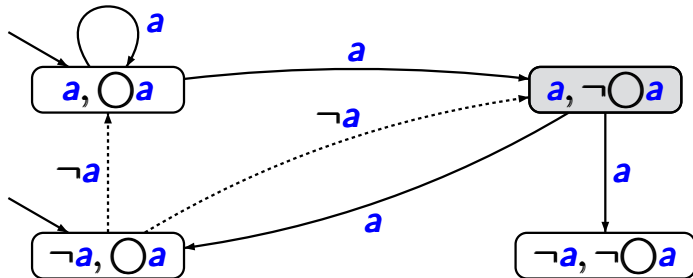
transition relation:

if $\bigcirc a \in B$ then $\delta(B, B \cap \{a\}) = \{B' : a \in B'\}$

if $\bigcirc a \notin B$ then $\delta(B, B \cap \{a\}) = \{B' : a \notin B'\}$

Example: GNBA for $\varphi = \bigcirc a$

LTLMC3.2-52



initial states: formula-sets B with $\bigcirc a \in B$

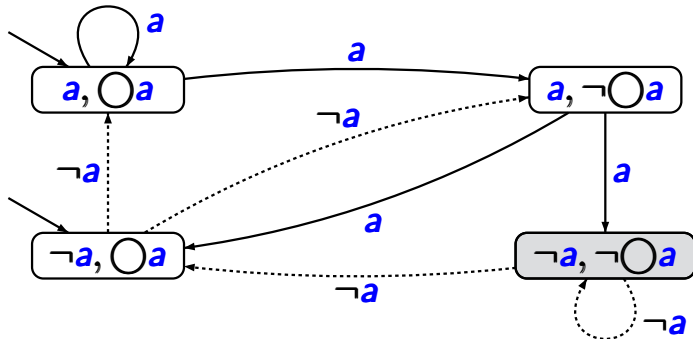
transition relation:

if $\bigcirc a \in B$ then $\delta(B, B \cap \{a\}) = \{B' : a \in B'\}$

if $\bigcirc a \notin B$ then $\delta(B, B \cap \{a\}) = \{B' : a \notin B'\}$

Example: GNBA for $\varphi = \bigcirc a$

LTLMC3.2-52



initial states: formula-sets B with $\bigcirc a \in B$

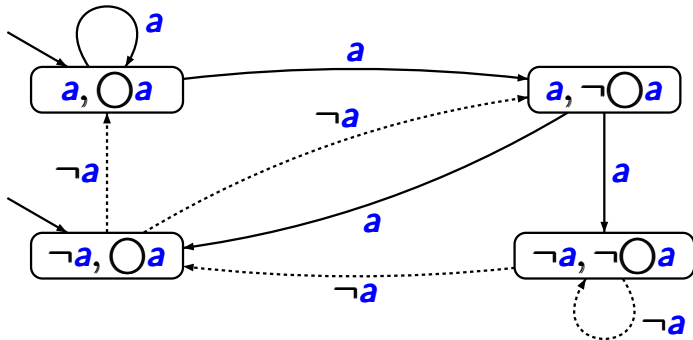
transition relation:

if $\bigcirc a \in B$ then $\delta(B, B \cap \{a\}) = \{B' : a \in B'\}$

if $\bigcirc a \notin B$ then $\delta(B, B \cap \{a\}) = \{B' : a \notin B'\}$

Example: GNBA for $\varphi = \bigcirc a$

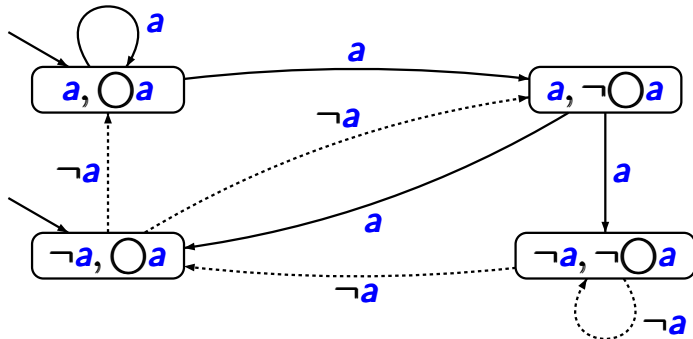
LTLMC3.2-53



set of acceptance sets:

Example: GNBA for $\varphi = \bigcirc a$

LTLMC3.2-53

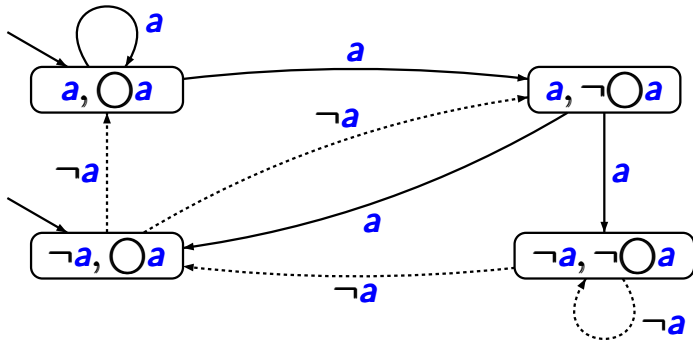


set of acceptance sets: $\mathcal{F} = \emptyset$

hence: all words having an **infinite run** are accepted

Example: GNBA for $\varphi = \bigcirc a$

LTLMC3.2-53

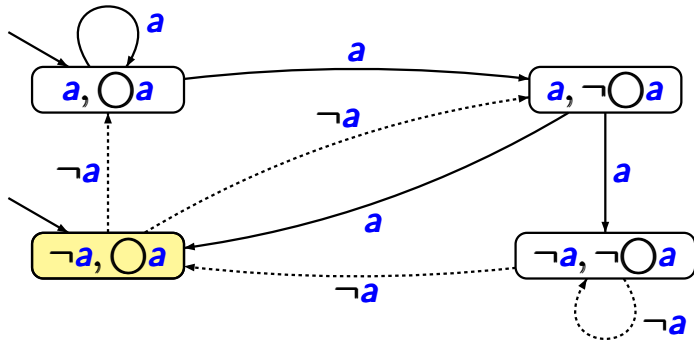


set of acceptance sets: $\mathcal{F} = \emptyset$

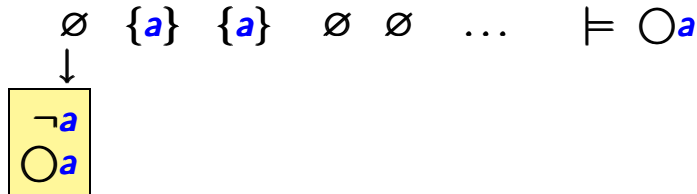
$\emptyset \quad \{a\} \quad \{a\} \quad \emptyset \quad \emptyset \quad \dots \models \bigcirc a$

Example: GNBA for $\varphi = \bigcirc a$

LTLMC3.2-53

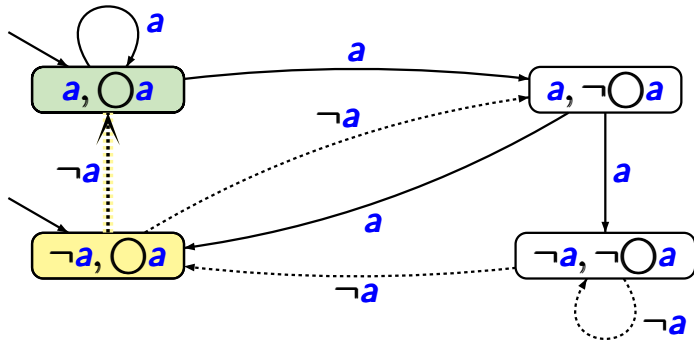


set of acceptance sets: $\mathcal{F} = \emptyset$

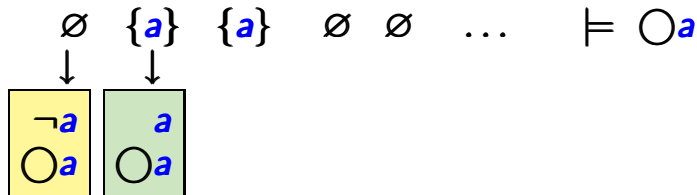


Example: GNBA for $\varphi = \bigcirc a$

LTLMC3.2-53

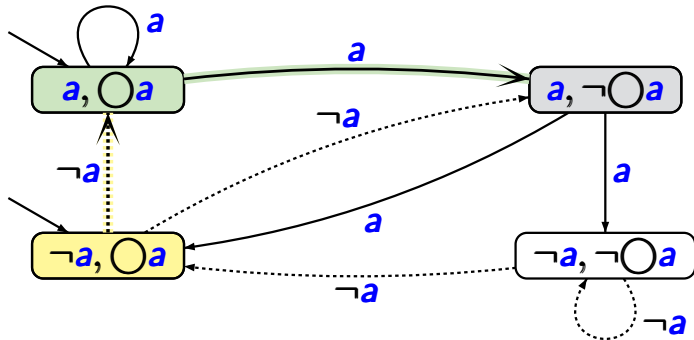


set of acceptance sets: $\mathcal{F} = \emptyset$

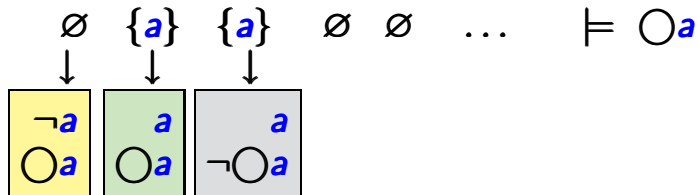


Example: GNBA for $\varphi = \bigcirc a$

LTLMC3.2-53

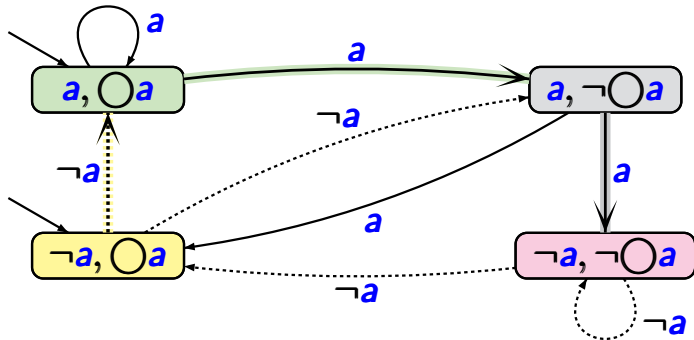


set of acceptance sets: $\mathcal{F} = \emptyset$

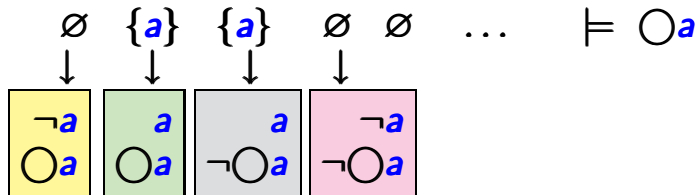


Example: GNBA for $\varphi = \bigcirc a$

LTLMC3.2-53

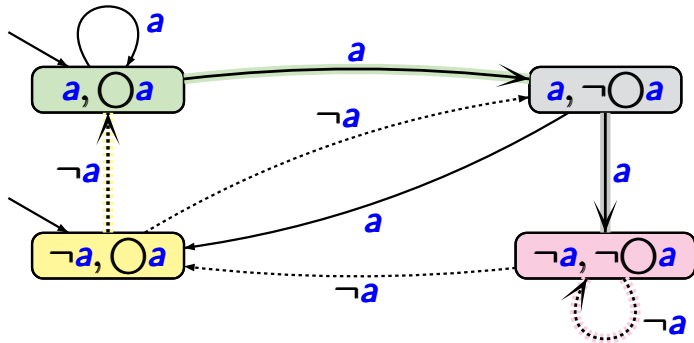


set of acceptance sets: $\mathcal{F} = \emptyset$

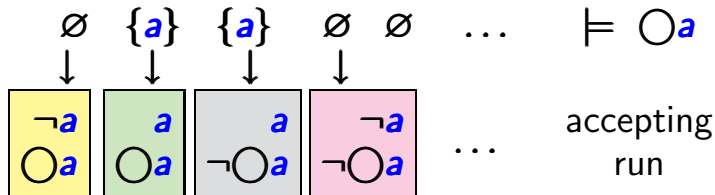


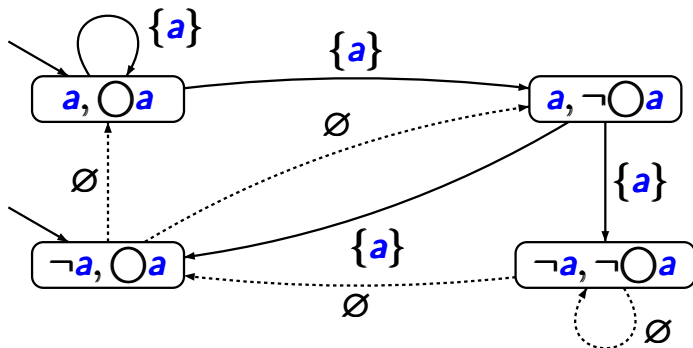
Example: GNBA for $\varphi = \bigcirc a$

LTLMC3.2-53

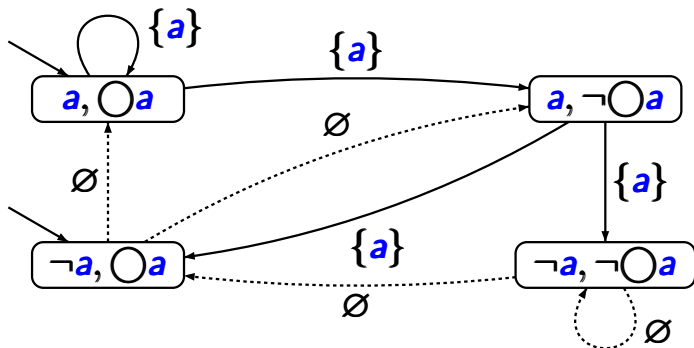


set of acceptance sets: $\mathcal{F} = \emptyset$



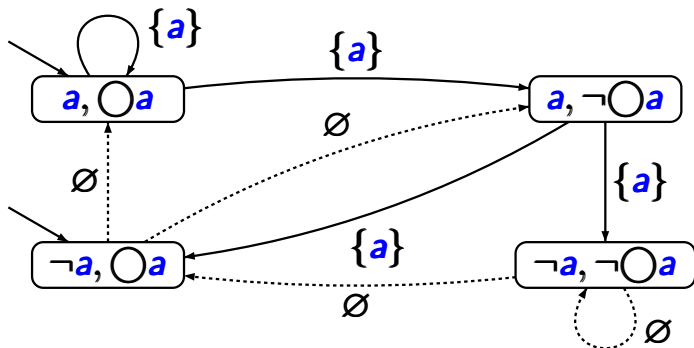


for all words $\sigma = A_0 A_1 A_2 A_3 \dots \in \mathcal{L}_\omega(\mathcal{G})$: $A_1 = \{a\}$



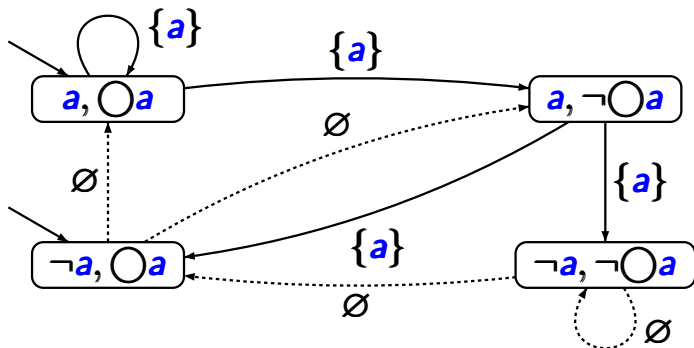
for all words $\sigma = A_0 A_1 A_2 A_3 \dots \in \mathcal{L}_\omega(\mathcal{G})$: $A_1 = \{a\}$

proof:



for all words $\sigma = A_0 A_1 A_2 A_3 \dots \in \mathcal{L}_\omega(\mathcal{G})$: $A_1 = \{a\}$

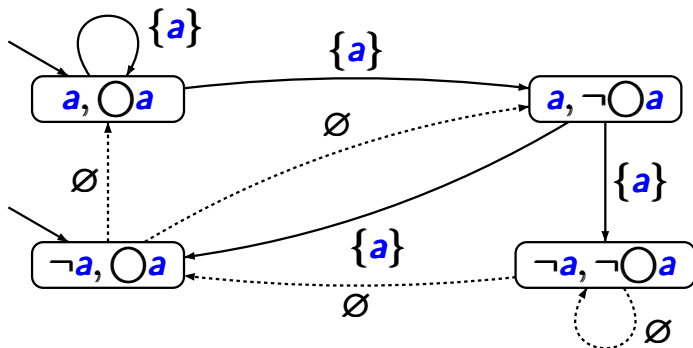
proof: Let $B_0 B_1 B_2 \dots$ be an accepting run for σ .



for all words $\sigma = A_0 A_1 A_2 A_3 \dots \in \mathcal{L}_\omega(\mathcal{G})$: $A_1 = \{a\}$

proof: Let $B_0 B_1 B_2 \dots$ be an accepting run for σ .

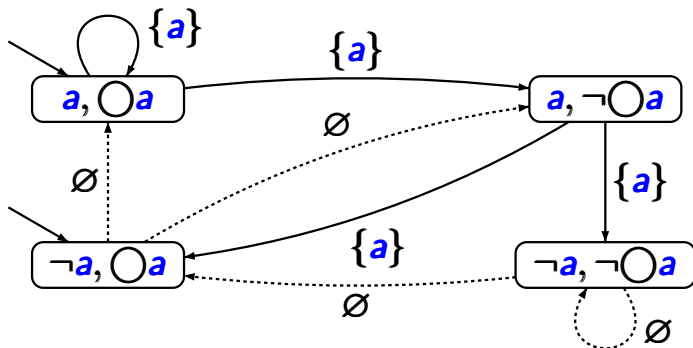
$\Rightarrow \bigcirc a \in B_0$



for all words $\sigma = A_0 A_1 A_2 A_3 \dots \in \mathcal{L}_\omega(\mathcal{G})$: $A_1 = \{a\}$

proof: Let $B_0 B_1 B_2 \dots$ be an accepting run for σ .

$\Rightarrow \bigcirc a \in B_0$ and therefore $a \in B_1$

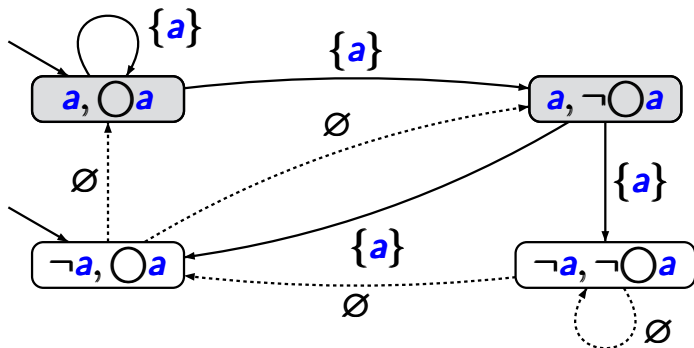


for all words $\sigma = A_0 A_1 A_2 A_3 \dots \in \mathcal{L}_\omega(\mathcal{G})$: $A_1 = \{a\}$

proof: Let $B_0 B_1 B_2 \dots$ be an accepting run for σ .

$\Rightarrow \bigcirc a \in B_0$ and therefore $a \in B_1$

\Rightarrow the outgoing edges of B_1 have label $\{a\}$



for all words $\sigma = A_0 A_1 A_2 A_3 \dots \in \mathcal{L}_\omega(\mathcal{G})$: $A_1 = \{a\}$

proof: Let $B_0 B_1 B_2 \dots$ be an accepting run for σ .

$\Rightarrow \bigcirc a \in B_0$ and therefore $a \in B_1$

\Rightarrow the outgoing edges of B_1 have label $\{a\}$

$\Rightarrow \{a\} = B_1 \cap AP = A_1$

Example: GNBA for $\varphi = aU b$

LTLMC3.2-54

$a, b, a \mathbf{U} b$

$\neg a, \neg b, \neg(a \mathbf{U} b)$

$a, \neg b, a \mathbf{U} b$

$a, \neg b, \neg(a \mathbf{U} b)$

$\neg a, b, a \mathbf{U} b$

locally inconsistent: $\{a, b, \neg(a \mathbf{U} b)\}$

$\{\neg a, b, \neg(a \mathbf{U} b)\}$

$\{\neg a, \neg b, a \mathbf{U} b\}$

$a, b, a \mathbf{U} b$

$\neg a, \neg b, \neg(a \mathbf{U} b)$

$a, \neg b, a \mathbf{U} b$

$a, \neg b, \neg(a \mathbf{U} b)$

$\neg a, b, a \mathbf{U} b$

initial states:

B with $\varphi = a \mathbf{U} b \in B$

$\longrightarrow a, b, a \mathbf{U} b$

$\neg a, \neg b, \neg(a \mathbf{U} b)$

$\longrightarrow a, \neg b, a \mathbf{U} b$

$a, \neg b, \neg(a \mathbf{U} b)$

$\longrightarrow \neg a, b, a \mathbf{U} b$

initial states:

B with $\varphi = a \mathbf{U} b \in B$

→ $a, b, a \mathbf{U} b$

$\neg a, \neg b, \neg(a \mathbf{U} b)$

→ $a, \neg b, a \mathbf{U} b$

$a, \neg b, \neg(a \mathbf{U} b)$

→ $\neg a, b, a \mathbf{U} b$

initial states:

B with $\varphi = a \mathbf{U} b \in B$

acceptance condition: just one set of accept states

$F =$ set of all B with $\varphi \notin B$ or $b \in B$

$\longrightarrow a, b, a \mathbf{U} b$

$\neg a, \neg b, \neg(a \mathbf{U} b)$

$\longrightarrow a, \neg b, a \mathbf{U} b$

$a, \neg b, \neg(a \mathbf{U} b)$

$\longrightarrow \neg a, b, a \mathbf{U} b$

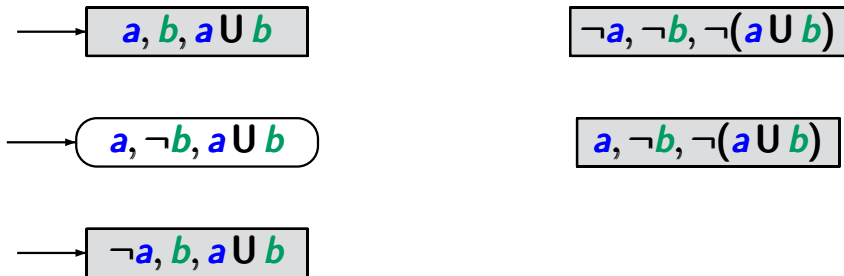
initial states:

B with $\varphi = a \mathbf{U} b \in B$

acceptance condition:

just one set of accept states

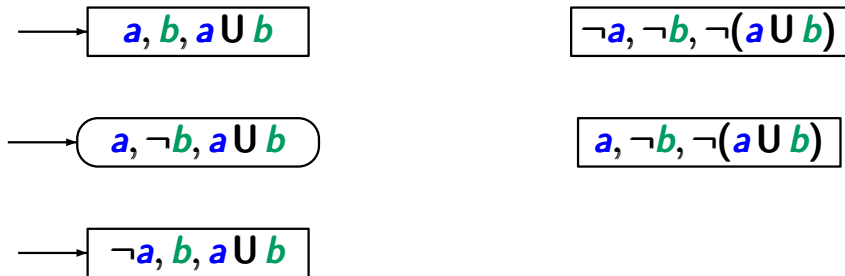
$F =$ set of all B with $\varphi \notin B$ or $b \in B$



initial states: B with $\varphi = a \mathbf{U} b \in B$

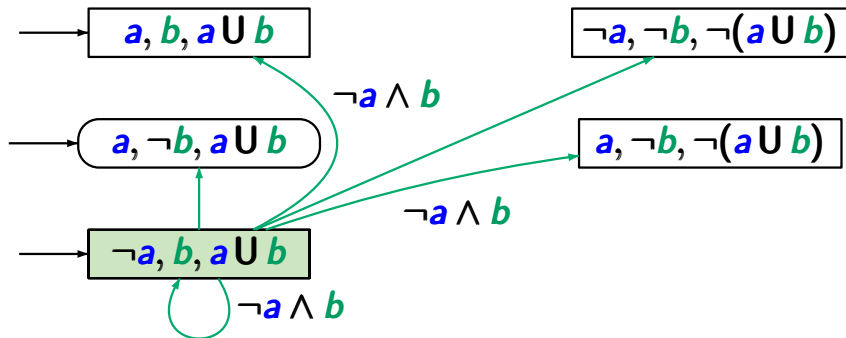
acceptance condition: just one set of accept states

$F =$ set of all B with $\varphi \notin B$ or $b \in B$



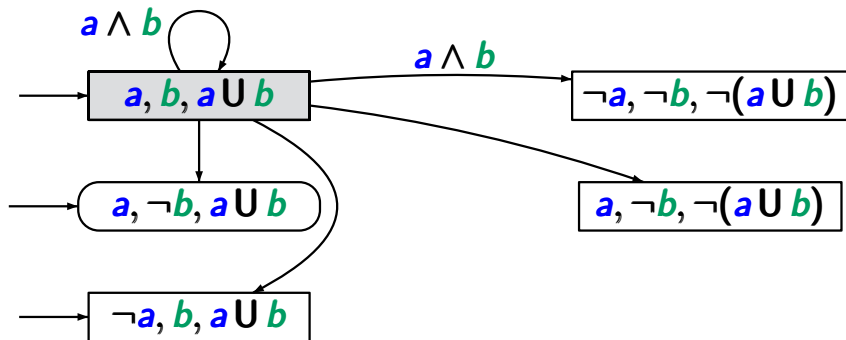
transition relation: $B' \in \delta(B, B \cap AP)$ iff

$$a \cup b \in B \iff (b \in B \vee (a \in B \wedge a \cup b \in B'))$$



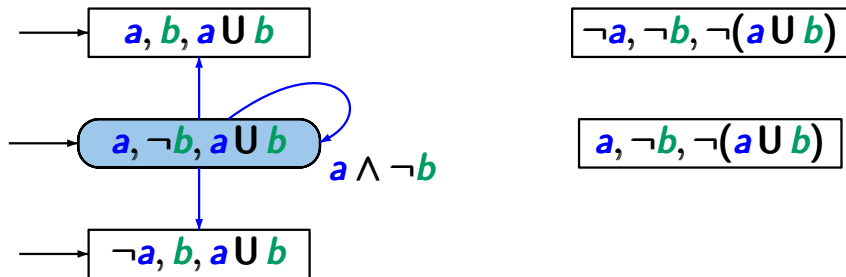
transition relation: $B' \in \delta(B, B \cap AP)$ iff

$$a \cup b \in B \iff (b \in B \vee (a \in B \wedge a \cup b \in B'))$$



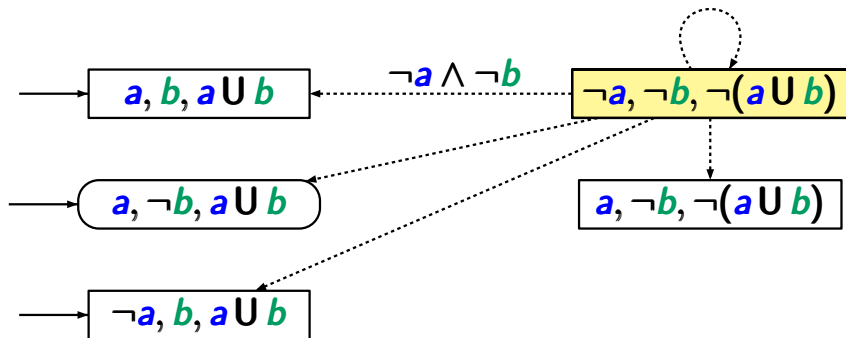
transition relation: $B' \in \delta(B, B \cap AP)$ iff

$$a \mathbf{U} b \in B \iff (b \in B \vee (a \in B \wedge a \mathbf{U} b \in B'))$$



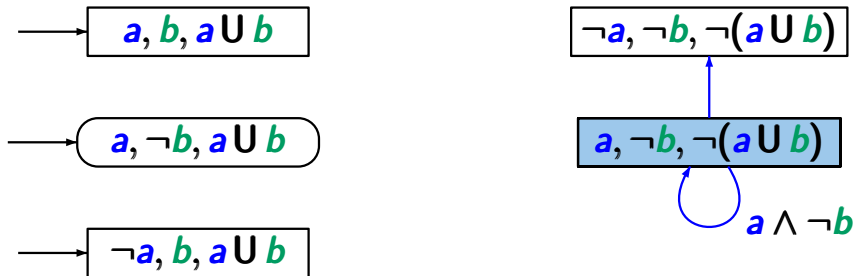
transition relation: $B' \in \delta(B, B \cap AP)$ iff

$$a \cup b \in B \iff (b \in B \vee (a \in B \wedge a \cup b \in B'))$$



transition relation: $B' \in \delta(B, B \cap AP)$ iff

$$a \mathbf{U} b \in B \iff (b \in B \vee (a \in B \wedge a \mathbf{U} b \in B'))$$

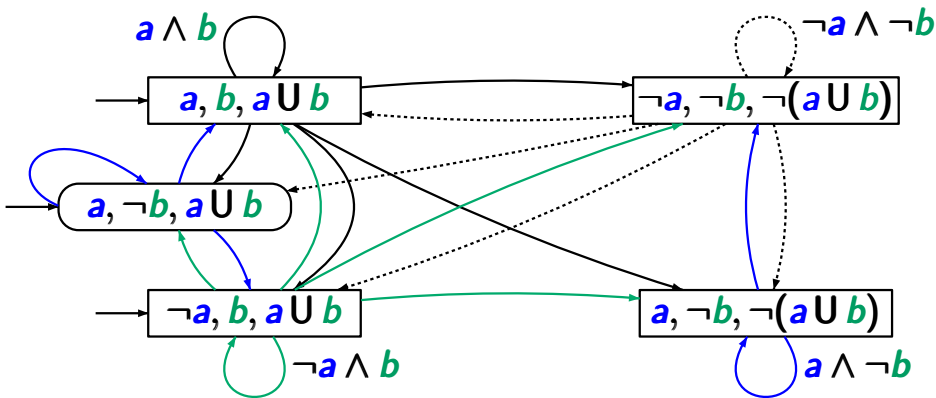


transition relation: $B' \in \delta(B, B \cap AP)$ iff

$$a \cup b \in B \iff (b \in B \vee (a \in B \wedge a \cup b \in B'))$$

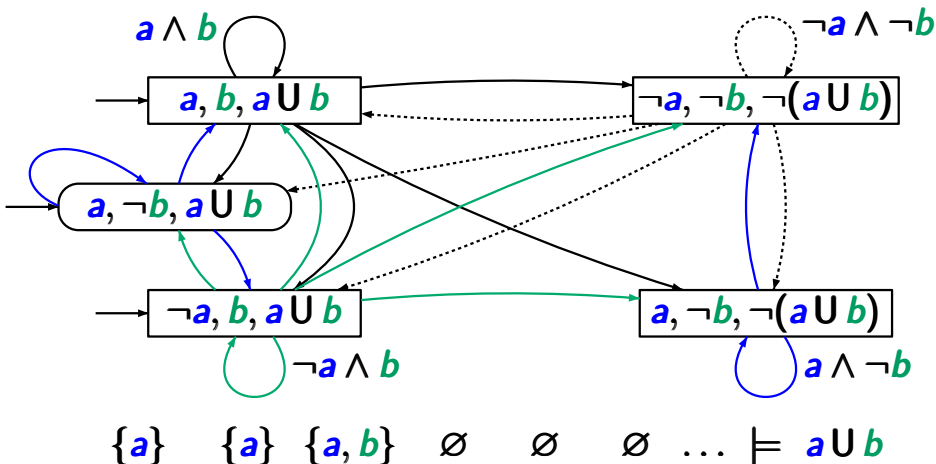
Example: (G)NBA for $\varphi = a \cup b$

LTLMC3.2-55



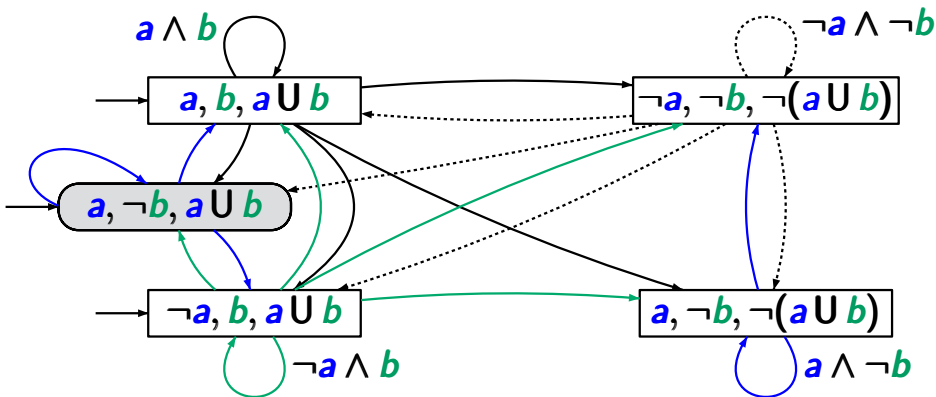
Example: (G)NBA for $\varphi = a \cup b$

LTLMC3.2-55

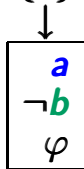


Example: (G)NBA for $\varphi = a \cup b$

LTLMC3.2-55

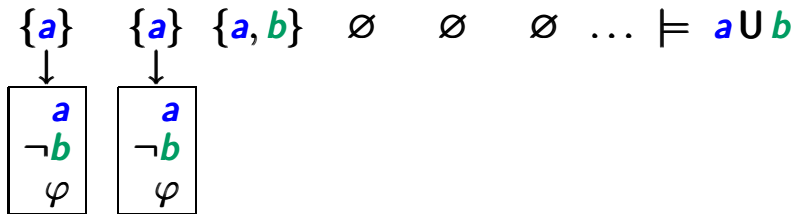
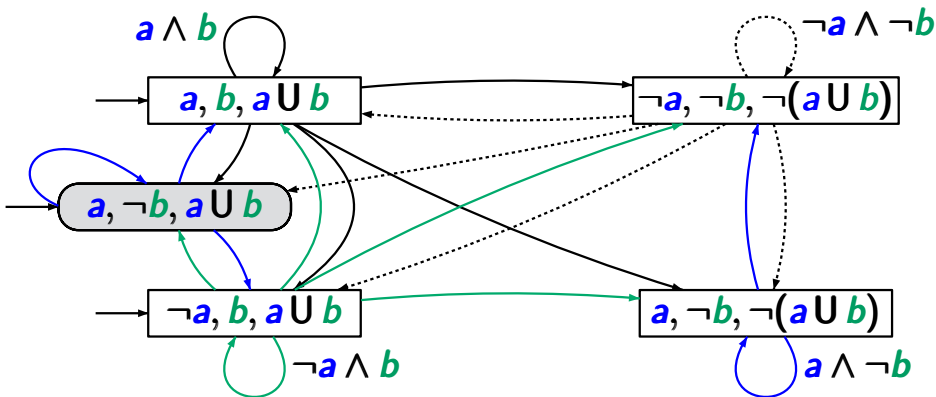


$\{a\} \quad \{a\} \quad \{a, b\} \quad \emptyset \quad \emptyset \quad \emptyset \quad \dots \models a \cup b$

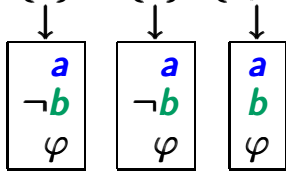


Example: (G)NBA for $\varphi = a \cup b$

LTLMC3.2-55

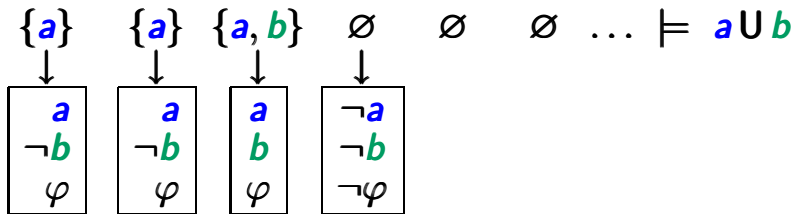
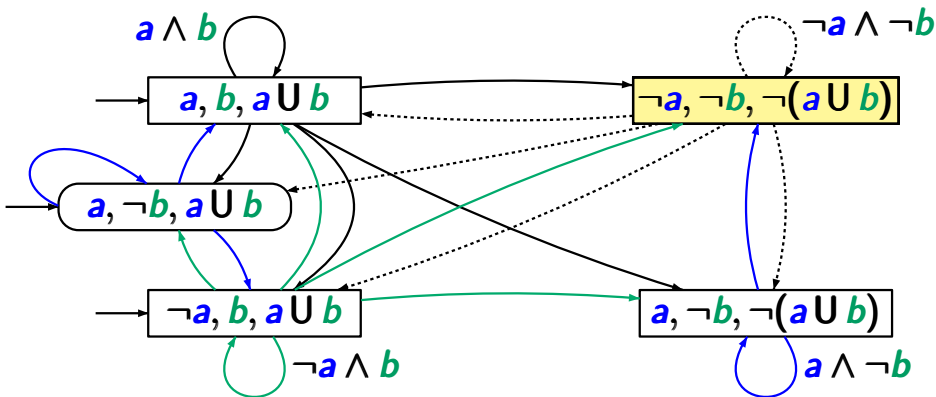


LTLMC3.2-55



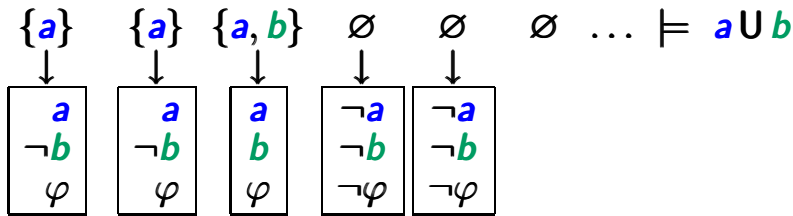
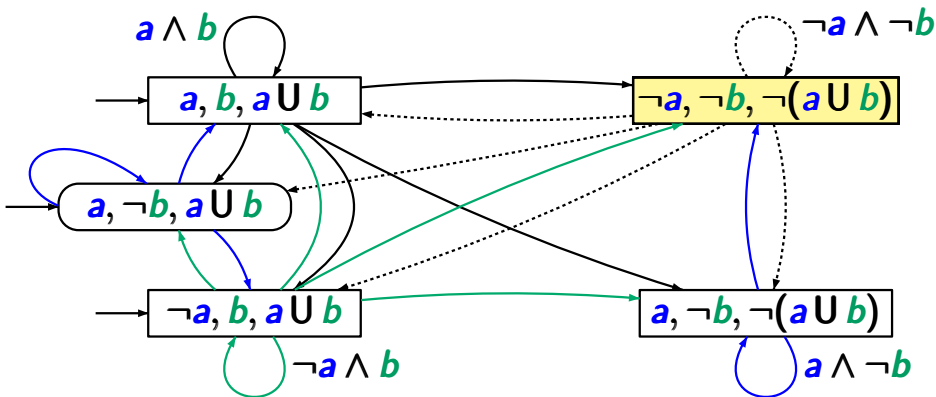
Example: (G)NBA for $\varphi = a \cup b$

LTLMC3.2-55



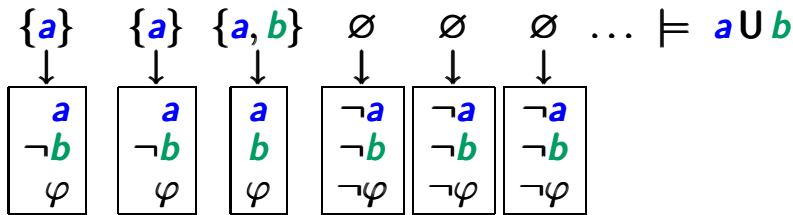
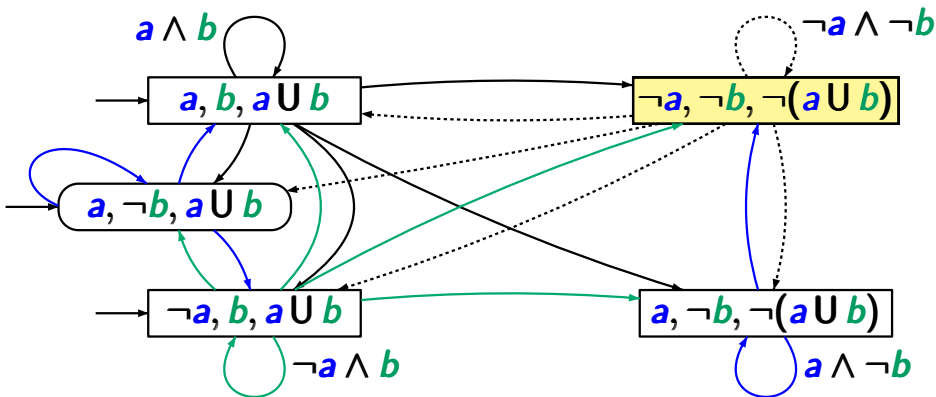
Example: (G)NBA for $\varphi = a \cup b$

LTLMC3.2-55



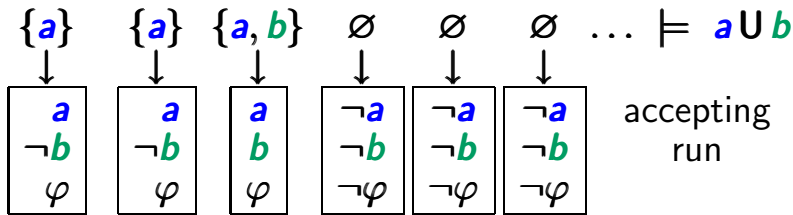
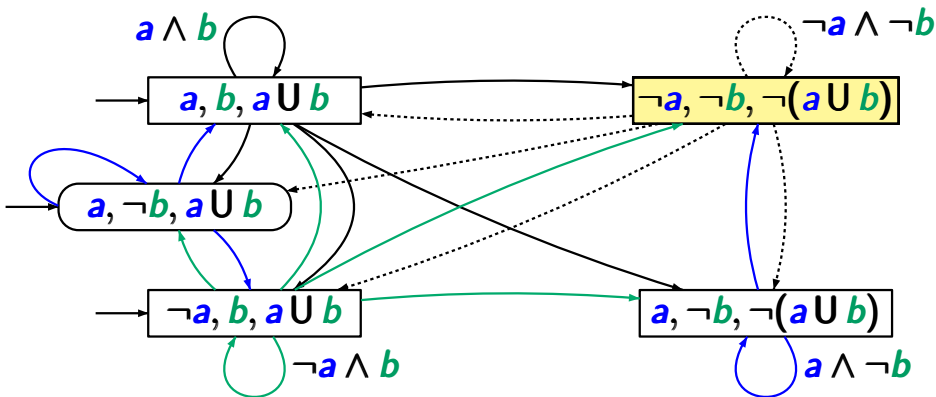
Example: (G)NBA for $\varphi = a \cup b$

LTLMC3.2-55



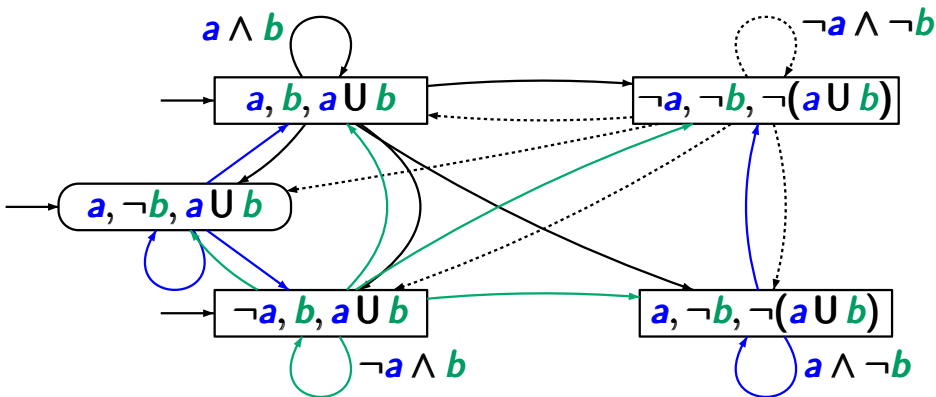
Example: (G)NBA for $\varphi = a \cup b$

LTLMC3.2-55



Example: (G)NBA for $\varphi = a \cup b$

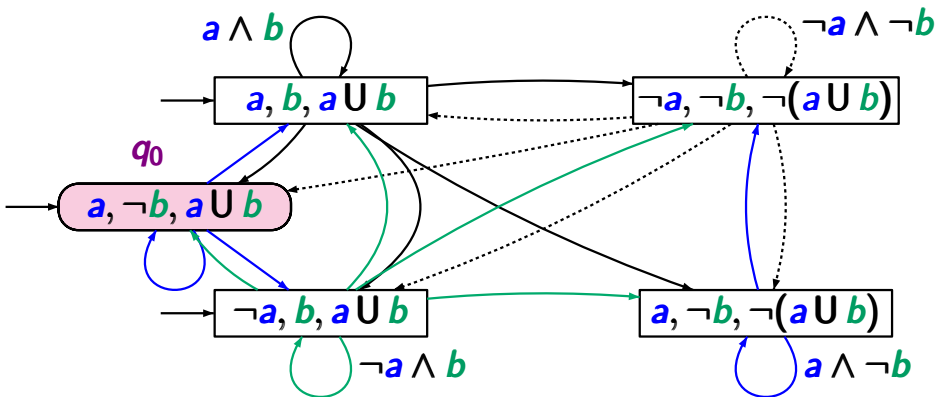
LTLMC3.2-56



$\{a\} \{a\} \{a\} \{a\} \dots \not\models \varphi$

Example: (G)NBA for $\varphi = a \cup b$

LTLMC3.2-56

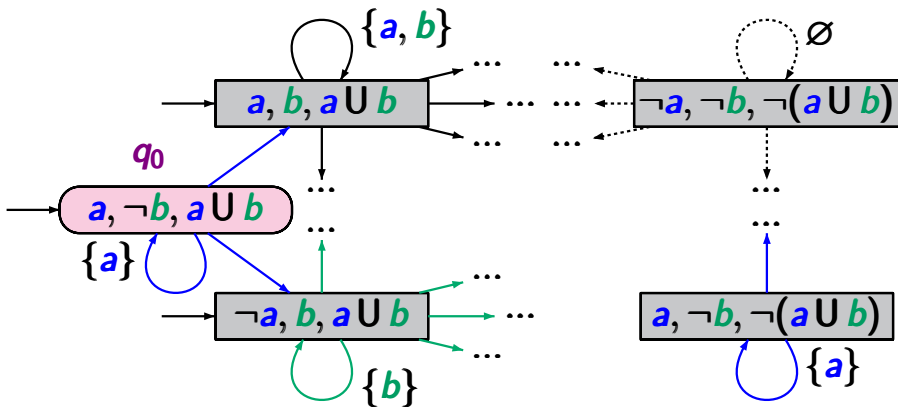


$\{a\} \{a\} \{a\} \{a\} \dots \not\models \varphi$

only 1 infinite run: $q_0 q_0 q_0 \dots$

Example: (G)NBA for $\varphi = a \cup b$

LTLMC3.2-56

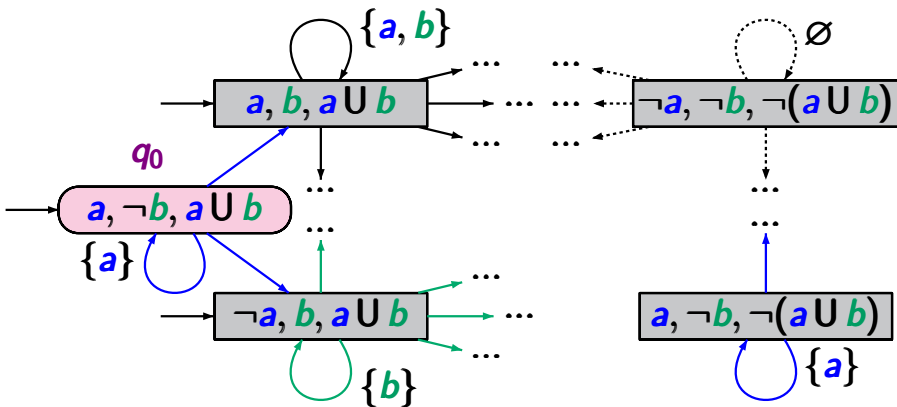


$\{a\} \{a\} \{a\} \{a\} \dots \not\models \varphi$

only 1 infinite run: $q_0 q_0 q_0 \dots$

Example: (G)NBA for $\varphi = a \cup b$

LTLMC3.2-56



$\{a\} \{a\} \{a\} \{a\} \dots \not\models \varphi$

only 1 infinite run: $q_0 q_0 q_0 \dots$ not accepting

$$\mathcal{G} = (Q, 2^{AP}, \delta, Q_0, \mathcal{F})$$

state space: $Q = \{B \subseteq cl(\varphi) : B \text{ is elementary}\}$

initial states: $Q_0 = \{B \in Q : \varphi \in B\}$

transition relation: for $B \in Q$ and $A \in 2^{AP}$:

if $A \neq B \cap AP$ then $\delta(B, A) = \emptyset$

if $A = B \cap AP$ then $\delta(B, A) = \text{set of all } B' \in Q \text{ s.t.}$

$$\bigcirc \psi \in B \text{ iff } \psi \in B'$$

$$\psi_1 \mathbf{U} \psi_2 \in B \text{ iff } (\psi_2 \in B) \vee (\psi_1 \in B \wedge \psi_1 \mathbf{U} \psi_2 \in B')$$

acceptance set $\mathcal{F} = \{F_{\psi_1 \mathbf{U} \psi_2} : \psi_1 \mathbf{U} \psi_2 \in cl(\varphi)\}$

where $F_{\psi_1 \mathbf{U} \psi_2} = \{B \in Q : \psi_1 \mathbf{U} \psi_2 \notin B \vee \psi_2 \in B\}$

.... of the construction LTL formula $\varphi \rightsquigarrow$ GNBA \mathcal{G}

Let φ be an LTL-formula and $\mathcal{G} = (Q, 2^{AP}, \delta, Q_0, \mathcal{F})$ be the constructed GNBA.

Claim: $Words(\varphi) = \mathcal{L}_\omega(\mathcal{G})$

Let φ be an LTL-formula and $\mathcal{G} = (Q, 2^{AP}, \delta, Q_0, \mathcal{F})$ be the constructed GNBA.

Claim: $Words(\varphi) = \mathcal{L}_\omega(\mathcal{G})$

“ \subseteq ” show: each infinite word $A_0 A_1 A_2 \dots \in (2^{AP})^\omega$

with $A_0 A_1 A_2 \dots \models \varphi$

has an accepting run in \mathcal{G}

Let φ be an LTL-formula and $\mathcal{G} = (Q, 2^{AP}, \delta, Q_0, \mathcal{F})$ be the constructed GNBA.

Claim: $Words(\varphi) = \mathcal{L}_\omega(\mathcal{G})$

“ \subseteq ” show: each infinite word $A_0 A_1 A_2 \dots \in (2^{AP})^\omega$

with $A_0 A_1 A_2 \dots \models \varphi$

has an accepting run in \mathcal{G}

“ \supseteq ” show: for all infinite words $A_0 A_1 A_2 \dots \in \mathcal{L}_\omega(\mathcal{G})$:

$A_0 A_1 A_2 \dots \models \varphi$

Let φ be an LTL-formula and $\mathcal{G} = (Q, 2^{AP}, \delta, Q_0, \mathcal{F})$ be the constructed GNBA.

Claim: $Words(\varphi) = \mathcal{L}_\omega(\mathcal{G})$

“ \subseteq ” show: each infinite word $A_0 A_1 A_2 \dots \in (2^{AP})^\omega$
with $A_0 A_1 A_2 \dots \models \varphi$
has an accepting run in \mathcal{G}

“ \supseteq ” show: for all infinite words $A_0 A_1 A_2 \dots \in \mathcal{L}_\omega(\mathcal{G})$:
 $A_0 A_1 A_2 \dots \models \varphi$

Accepting runs for the elements of $Words(\varphi)$

LTLMC3.2-47-COPY

LTL formula $\varphi \rightsquigarrow$ GNBA \mathcal{G} for $Words(\varphi)$

states of $\mathcal{G} \hat{=} \text{elementary formula-sets } B \subseteq cl(\varphi)$

Accepting runs for the elements of $Words(\varphi)$

LTLMC3.2-47-COPY

LTL formula $\varphi \rightsquigarrow$ GNBA \mathcal{G} for $Words(\varphi)$

states of $\mathcal{G} \hat{=} \text{elementary formula-sets } B \subseteq cl(\varphi)$

s.t. each word $\sigma = A_0 A_1 A_2 \dots \in Words(\varphi)$ can be extended to an accepting run $B_0 B_1 B_2 \dots$ in \mathcal{G}

Accepting runs for the elements of $Words(\varphi)$

LTLMC3.2-47-COPY

LTL formula $\varphi \rightsquigarrow$ GNBA \mathcal{G} for $Words(\varphi)$

states of $\mathcal{G} \hat{=} \text{elementary formula-sets } B \subseteq cl(\varphi)$

s.t. each word $\sigma = A_0 A_1 A_2 \dots \in Words(\varphi)$ can be extended to an accepting run $B_0 B_1 B_2 \dots$ in \mathcal{G}

Example: $\varphi = a \text{ U } (\neg a \wedge b)$

Accepting runs for the elements of $Words(\varphi)$

LTLMC3.2-47-COPY

LTL formula $\varphi \rightsquigarrow$ GNBA \mathcal{G} for $Words(\varphi)$

states of $\mathcal{G} \hat{=} \text{elementary formula-sets } B \subseteq cl(\varphi)$

s.t. each word $\sigma = A_0 A_1 A_2 \dots \in Words(\varphi)$ can be extended to an accepting run $B_0 B_1 B_2 \dots$ in \mathcal{G}

Example: $\varphi = a \mathbf{U}(\neg a \wedge b)$

$\{a\} \quad \{a\} \quad \{a, b\} \quad \{b\} \quad \emptyset \quad \emptyset \quad \dots \models \varphi$

Accepting runs for the elements of $Words(\varphi)$

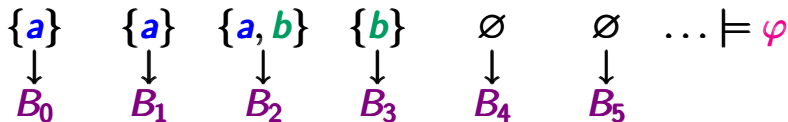
LTLMC3.2-47-COPY

LTL formula $\varphi \rightsquigarrow$ GNBA \mathcal{G} for $Words(\varphi)$

states of $\mathcal{G} \hat{=} \text{elementary formula-sets } B \subseteq cl(\varphi)$

s.t. each word $\sigma = A_0 A_1 A_2 \dots \in Words(\varphi)$ can be extended to an accepting run $B_0 B_1 B_2 \dots$ in \mathcal{G}

Example: $\varphi = a \mathbf{U}(\neg a \wedge b)$



Accepting runs for the elements of $Words(\varphi)$

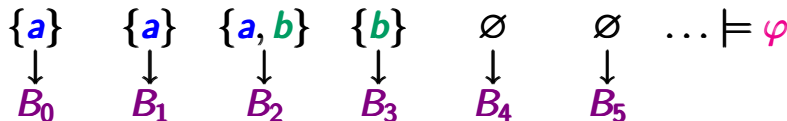
LTLMC3.2-47-COPY

LTL formula $\varphi \rightsquigarrow$ GNBA \mathcal{G} for $Words(\varphi)$

states of $\mathcal{G} \hat{=} \text{elementary formula-sets } B \subseteq cl(\varphi)$

s.t. each word $\sigma = A_0 A_1 A_2 \dots \in Words(\varphi)$ can be extended to an accepting run $B_0 B_1 B_2 \dots$ in \mathcal{G}

Example: $\varphi = a \mathbf{U}(\neg a \wedge b)$ $\psi = \neg a \wedge b$



where the B_i 's are states in \mathcal{G} , i.e., elementary subsets of $\{a, \neg a, b, \neg b, \psi, \neg \psi, \varphi, \neg \varphi\}$

Accepting runs for the elements of $Words(\varphi)$

LTLMC3.2-47-COPY

LTL formula $\varphi \rightsquigarrow$ GNBA \mathcal{G} for $Words(\varphi)$

states of $\mathcal{G} \hat{=} \text{elementary formula-sets } B \subseteq cl(\varphi)$

s.t. each word $\sigma = A_0 A_1 A_2 \dots \in Words(\varphi)$ can be extended to an accepting run $B_0 B_1 B_2 \dots$ in \mathcal{G}

Example: $\varphi = a \mathbf{U}(\neg a \wedge b)$ $\psi = \neg a \wedge b$

$\{a\}$ $\{a\}$ $\{a, b\}$ $\{b\}$ \emptyset \emptyset $\dots \models \varphi$



Accepting runs for the elements of $Words(\varphi)$

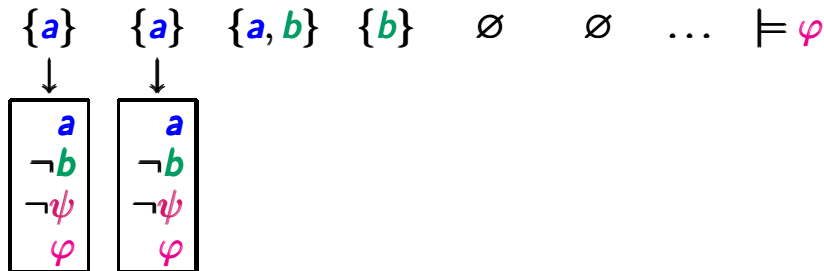
LTLMC3.2-47-COPY

LTL formula $\varphi \rightsquigarrow$ GNBA \mathcal{G} for $Words(\varphi)$

states of $\mathcal{G} \hat{=} \text{elementary formula-sets } B \subseteq cl(\varphi)$

s.t. each word $\sigma = A_0 A_1 A_2 \dots \in Words(\varphi)$ can be extended to an accepting run $B_0 B_1 B_2 \dots$ in \mathcal{G}

Example: $\varphi = a \mathbf{U}(\neg a \wedge b)$ $\psi = \neg a \wedge b$



Accepting runs for the elements of $Words(\varphi)$

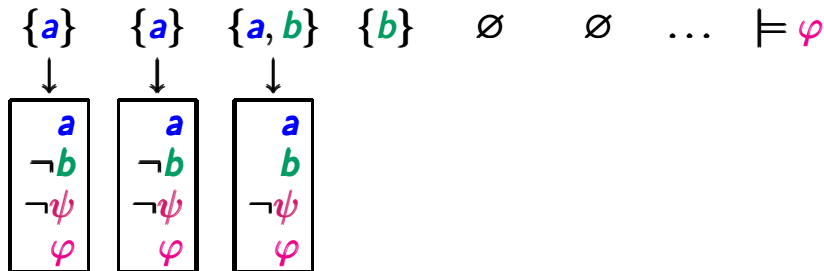
LTLMC3.2-47-COPY

LTL formula $\varphi \rightsquigarrow$ GNBA \mathcal{G} for $Words(\varphi)$

states of $\mathcal{G} \hat{=} \text{elementary formula-sets } B \subseteq cl(\varphi)$

s.t. each word $\sigma = A_0 A_1 A_2 \dots \in Words(\varphi)$ can be extended to an accepting run $B_0 B_1 B_2 \dots$ in \mathcal{G}

Example: $\varphi = a \mathbf{U}(\neg a \wedge b)$ $\psi = \neg a \wedge b$



Accepting runs for the elements of $Words(\varphi)$

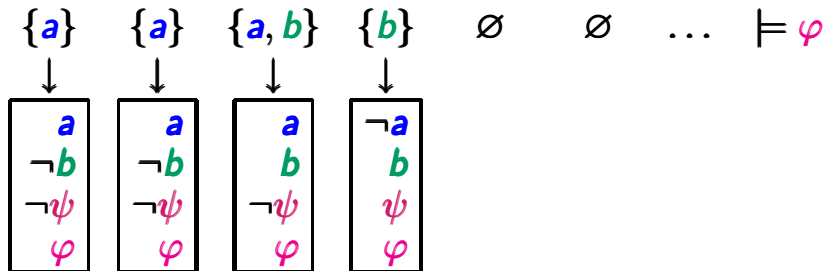
LTLMC3.2-47-COPY

LTL formula $\varphi \rightsquigarrow$ GNBA \mathcal{G} for $Words(\varphi)$

states of $\mathcal{G} \hat{=} \text{elementary formula-sets } B \subseteq cl(\varphi)$

s.t. each word $\sigma = A_0 A_1 A_2 \dots \in Words(\varphi)$ can be extended to an accepting run $B_0 B_1 B_2 \dots$ in \mathcal{G}

Example: $\varphi = a \mathbf{U}(\neg a \wedge b)$ $\psi = \neg a \wedge b$



Accepting runs for the elements of $Words(\varphi)$

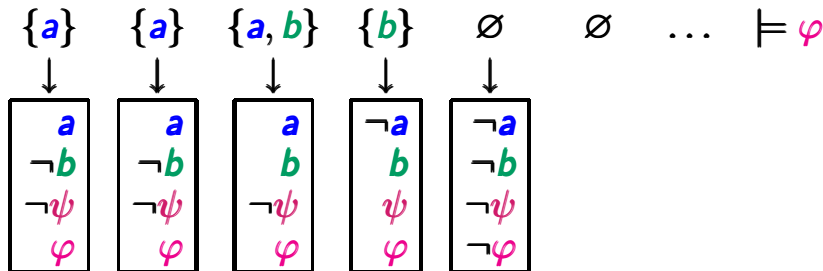
LTLMC3.2-47-COPY

LTL formula $\varphi \rightsquigarrow$ GNBA \mathcal{G} for $Words(\varphi)$

states of $\mathcal{G} \hat{=} \text{elementary formula-sets } B \subseteq cl(\varphi)$

s.t. each word $\sigma = A_0 A_1 A_2 \dots \in Words(\varphi)$ can be extended to an accepting run $B_0 B_1 B_2 \dots$ in \mathcal{G}

Example: $\varphi = a \mathbf{U}(\neg a \wedge b)$ $\psi = \neg a \wedge b$



Accepting runs for the elements of $Words(\varphi)$

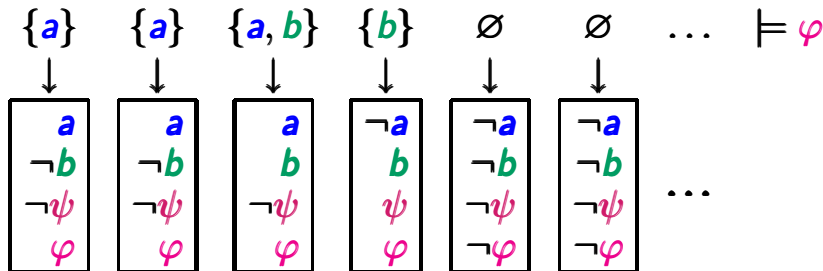
LTLMC3.2-47-COPY

LTL formula $\varphi \rightsquigarrow$ GNBA \mathcal{G} for $Words(\varphi)$

states of $\mathcal{G} \hat{=} \text{elementary formula-sets } B \subseteq cl(\varphi)$

s.t. each word $\sigma = A_0 A_1 A_2 \dots \in Words(\varphi)$ can be extended to an accepting run $B_0 B_1 B_2 \dots$ in \mathcal{G}

Example: $\varphi = a \mathbf{U}(\neg a \wedge b)$ $\psi = \neg a \wedge b$



$$\mathcal{G} = (Q, 2^{AP}, \delta, Q_0, \mathcal{F})$$

state space: $Q = \{B \subseteq cl(\varphi) : B \text{ is elementary}\}$

initial states: $Q_0 = \{B \in Q : \varphi \in B\}$

transition relation: for $B \in Q$ and $A \in 2^{AP}$:

if $A \neq B \cap AP$ then $\delta(B, A) = \emptyset$

if $A = B \cap AP$ then $\delta(B, A) = \text{set of all } B' \in Q \text{ s.t.}$

$$\bigcirc \psi \in B \text{ iff } \psi \in B'$$

$$\psi_1 \mathbf{U} \psi_2 \in B \text{ iff } (\psi_2 \in B) \vee (\psi_1 \in B \wedge \psi_1 \mathbf{U} \psi_2 \in B')$$

acceptance set $\mathcal{F} = \{F_{\psi_1 \mathbf{U} \psi_2} : \psi_1 \mathbf{U} \psi_2 \in cl(\varphi)\}$

where $F_{\psi_1 \mathbf{U} \psi_2} = \{B \in Q : \psi_1 \mathbf{U} \psi_2 \notin B \vee \psi_2 \in B\}$

$B \subseteq cl(\varphi)$ is elementary iff:

- (i) B is maximal consistent w.r.t. prop. logic, i.e., if $\psi, \psi_1 \wedge \psi_2 \in cl(\varphi)$ then:

$$\begin{array}{ll} \psi \notin B & \text{iff} \quad \neg\psi \in B \\ \psi_1 \wedge \psi_2 \in B & \text{iff} \quad \psi_1 \in B \text{ and } \psi_2 \in B \\ \text{true} \in cl(\varphi) & \text{implies} \quad \text{true} \in B \end{array}$$

- (ii) B is locally consistent with respect to until \mathbf{U} , i.e., if $\psi_1 \mathbf{U} \psi_2 \in cl(\varphi)$ then:

$$\begin{array}{l} \text{if } \psi_1 \mathbf{U} \psi_2 \in B \text{ and } \psi_2 \notin B \text{ then } \psi_1 \in B \\ \text{if } \psi_2 \in B \text{ then } \psi_1 \mathbf{U} \psi_2 \in B \end{array}$$

Let φ be an LTL-formula and $\mathcal{G} = (Q, 2^{AP}, \delta, Q_0, \mathcal{F})$ be the constructed GNBA.

Claim: $Words(\varphi) = \mathcal{L}_\omega(\mathcal{G})$

“ \subseteq ” show: each infinite word $A_0 A_1 A_2 \dots \in (2^{AP})^\omega$

with $A_0 A_1 A_2 \dots \models \varphi$

has an accepting run in \mathcal{G}

“ \supseteq ” show: for all infinite words $A_0 A_1 A_2 \dots \in \mathcal{L}_\omega(\mathcal{G})$:

$A_0 A_1 A_2 \dots \models \varphi$

Let φ be an LTL-formula and $\mathcal{G} = (Q, 2^{AP}, \delta, Q_0, \mathcal{F})$ be the constructed GNBA.

Claim: $Words(\varphi) = \mathcal{L}_\omega(\mathcal{G})$

“ \subseteq ” show: each infinite word $A_0 A_1 A_2 \dots \in (2^{AP})^\omega$

with $A_0 A_1 A_2 \dots \models \varphi$

has an accepting run in \mathcal{G}

“ \supseteq ” show: for all infinite words $A_0 A_1 A_2 \dots \in \mathcal{L}_\omega(\mathcal{G})$:

$A_0 A_1 A_2 \dots \models \varphi$

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \dots$ is a path in \mathcal{G} s.t.

$$\forall F \in \mathcal{F} \quad \exists j \geq 0. B_j \in F$$

then for all formulas $\psi \in \text{cl}(\varphi)$:

$$\psi \in B_0 \quad \text{iff} \quad A_0 A_1 A_2 \dots \models \psi$$

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \dots$ is a path in \mathcal{G} s.t.

$$\forall F \in \mathcal{F} \quad \exists j \geq 0. B_j \in F$$

then for all formulas $\psi \in \text{cl}(\varphi)$:

$$\psi \in B_0 \quad \text{iff} \quad A_0 A_1 A_2 \dots \models \psi$$

The claim yields that for each $\sigma = A_0 A_1 A_2 \dots \in \mathcal{L}_\omega(\mathcal{G})$:

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \dots$ is a path in \mathcal{G} s.t.

$$\forall F \in \mathcal{F} \quad \exists j \geq 0. B_j \in F$$

then for all formulas $\psi \in \text{cl}(\varphi)$:

$$\psi \in B_0 \quad \text{iff} \quad A_0 A_1 A_2 \dots \models \psi$$

The claim yields that for each $\sigma = A_0 A_1 A_2 \dots \in \mathcal{L}_\omega(\mathcal{G})$:
 \implies there is an accepting run $B_0 B_1 B_2 \dots$ for σ

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \dots$ is a path in \mathcal{G} s.t.

$$\forall F \in \mathcal{F} \quad \exists j \geq 0. B_j \in F$$

then for all formulas $\psi \in \text{cl}(\varphi)$:

$$\psi \in B_0 \quad \text{iff} \quad A_0 A_1 A_2 \dots \models \psi$$

The claim yields that for each $\sigma = A_0 A_1 A_2 \dots \in \mathcal{L}_\omega(\mathcal{G})$:

\implies there is an accepting run $B_0 B_1 B_2 \dots$ for σ

$\implies B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \dots$ is a path in \mathcal{G}

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \dots$ is a path in \mathcal{G} s.t.

$$\forall F \in \mathcal{F} \quad \exists j \geq 0. B_j \in F$$

then for all formulas $\psi \in \text{cl}(\varphi)$:

$$\psi \in B_0 \quad \text{iff} \quad A_0 A_1 A_2 \dots \models \psi$$

The claim yields that for each $\sigma = A_0 A_1 A_2 \dots \in \mathcal{L}_\omega(\mathcal{G})$:

\implies there is an accepting run $B_0 B_1 B_2 \dots$ for σ

$\implies B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \dots$ is a path in \mathcal{G} s.t. $\varphi \in B_0$

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \dots$ is a path in \mathcal{G} s.t.

$$\forall F \in \mathcal{F} \exists j \geq 0. B_j \in F$$

then for all formulas $\psi \in \text{cl}(\varphi)$:

$$\psi \in B_0 \quad \text{iff} \quad A_0 A_1 A_2 \dots \models \psi$$

The claim yields that for each $\sigma = A_0 A_1 A_2 \dots \in \mathcal{L}_\omega(\mathcal{G})$:

\implies there is an accepting run $B_0 B_1 B_2 \dots$ for σ

$\implies B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \dots$ is a path in \mathcal{G} s.t. $\varphi \in B_0$

as $B_0 \in Q_0$

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \dots$ is a path in \mathcal{G} s.t.

$$\forall F \in \mathcal{F} \quad \exists j \geq 0. B_j \in F \quad (*)$$

then for all formulas $\psi \in \text{cl}(\varphi)$:

$$\psi \in B_0 \quad \text{iff} \quad A_0 A_1 A_2 \dots \models \psi$$

The claim yields that for each $\sigma = A_0 A_1 A_2 \dots \in \mathcal{L}_\omega(\mathcal{G})$:

\implies there is an accepting run $B_0 B_1 B_2 \dots$ for σ

$\implies B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \dots$ is a path in \mathcal{G} s.t. $\varphi \in B_0$
and $(*)$ holds

as $B_0 \in Q_0$

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \dots$ is a path in \mathcal{G} s.t.

$$\forall F \in \mathcal{F} \quad \exists j \geq 0. B_j \in F \quad (*)$$

then for all formulas $\psi \in \text{cl}(\varphi)$:

$$\psi \in B_0 \quad \text{iff} \quad A_0 A_1 A_2 \dots \models \psi$$

The claim yields that for each $\sigma = A_0 A_1 A_2 \dots \in \mathcal{L}_\omega(\mathcal{G})$:

\implies there is an accepting run $B_0 B_1 B_2 \dots$ for σ

$\implies B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \dots$ is a path in \mathcal{G} s.t. $\varphi \in B_0$
and $(*)$ holds

as $B_0 \in Q_0$

$\implies \sigma = A_0 A_1 A_2 \dots \models \varphi$

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \dots$ is a path in \mathcal{G} s.t.

$$\forall F \in \mathcal{F} \quad \exists j \geq 0. B_j \in F \quad (*)$$

then for all formulas $\psi \in \text{cl}(\varphi)$:

$$\psi \in B_0 \quad \text{iff} \quad A_0 A_1 A_2 \dots \models \psi$$

The claim yields that for each $\sigma = A_0 A_1 A_2 \dots \in \mathcal{L}_\omega(\mathcal{G})$:

\implies there is an accepting run $B_0 B_1 B_2 \dots$ for σ

$\implies B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \dots$ is a path in \mathcal{G} s.t. $\varphi \in B_0$

and $(*)$ holds

as $B_0 \in Q_0$

$\implies \sigma = A_0 A_1 A_2 \dots \models \varphi$

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \dots$ is a path in \mathcal{G} s.t.

$$\forall F \in \mathcal{F} \quad \exists^{\infty} j \geq 0. B_j \in F \quad (*)$$

then for all formulas $\psi \in \text{cl}(\varphi)$:

$$\psi \in B_0 \quad \text{iff} \quad A_0 A_1 A_2 \dots \models \psi$$

Proof by structural induction on ψ

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \dots$ is a path in \mathcal{G} s.t.

$$\forall F \in \mathcal{F} \quad \exists^\infty j \geq 0. B_j \in F \quad (*)$$

then for all formulas $\psi \in \text{cl}(\varphi)$:

$$\psi \in B_0 \quad \text{iff} \quad A_0 A_1 A_2 \dots \models \psi$$

Proof by structural induction on ψ

base of induction:

$$\psi = \text{true}$$

$$\psi = a \in AP$$

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \dots$ is a path in \mathcal{G} s.t.

$$\forall F \in \mathcal{F} \quad \exists j \geq 0. B_j \in F \quad (*)$$

then for all formulas $\psi \in \text{cl}(\varphi)$:

$$\psi \in B_0 \quad \text{iff} \quad A_0 A_1 A_2 \dots \models \psi$$

Proof by structural induction on ψ

base of induction:

$$\psi = \text{true}$$

$$\psi = a \in AP$$

induction step:

$$\psi = \neg \psi'$$

$$\psi = \psi_1 \wedge \psi_2$$

$$\psi = \bigcirc \psi'$$

$$\psi = \psi_1 \cup \psi_2$$

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \dots$ is a path in \mathcal{G} s.t.

$$\forall F \in \mathcal{F} \quad \exists j \geq 0. B_j \in F$$

then for all formulas $\psi \in cl(\varphi)$:

$$\psi \in B_0 \quad \text{iff} \quad A_0 A_1 A_2 \dots \models \psi$$

Base of induction:

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \dots$ is a path in \mathcal{G} s.t.

$$\forall F \in \mathcal{F} \quad \exists j \geq 0. B_j \in F$$

then for all formulas $\psi \in cl(\varphi)$:

$$\psi \in B_0 \quad \text{iff} \quad A_0 A_1 A_2 \dots \models \psi$$

Base of induction:

Suppose $\psi = \text{true} \in cl(\varphi)$.

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \dots$ is a path in \mathcal{G} s.t.

$$\forall F \in \mathcal{F} \quad \exists j \geq 0. B_j \in F$$

then for all formulas $\psi \in cl(\varphi)$:

$$\psi \in B_0 \quad \text{iff} \quad A_0 A_1 A_2 \dots \models \psi$$

Base of induction:

Suppose $\psi = \mathbf{true} \in cl(\varphi)$. Then $\mathbf{true} \in B_0$

note: \mathbf{true} is contained in all elementary formula-sets

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \dots$ is a path in \mathcal{G} s.t.

$$\forall F \in \mathcal{F} \quad \exists j \geq 0. B_j \in F$$

then for all formulas $\psi \in cl(\varphi)$:

$$\psi \in B_0 \quad \text{iff} \quad A_0 A_1 A_2 \dots \models \psi$$

Base of induction:

Suppose $\psi = \text{true} \in cl(\varphi)$. Then $\text{true} \in B_0$ and

$$A_0 A_1 A_2 \dots \models \text{true}$$

note: **true** is contained in all elementary formula-sets
true holds for all paths/traces

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \dots$ is a path in \mathcal{G} s.t.

$$\forall F \in \mathcal{F} \quad \exists j \geq 0. B_j \in F$$

then for all formulas $\psi \in cl(\varphi)$:

$$\psi \in B_0 \quad \text{iff} \quad A_0 A_1 A_2 \dots \models \psi$$

Base of induction:

Suppose $\psi = \text{true} \in cl(\varphi)$. Then $\text{true} \in B_0$ and

$$A_0 A_1 A_2 \dots \models \text{true}$$

Let $\psi = a \in AP$.

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \dots$ is a path in \mathcal{G} s.t.

$$\forall F \in \mathcal{F} \quad \exists j \geq 0. B_j \in F$$

then for all formulas $\psi \in cl(\varphi)$:

$$\psi \in B_0 \quad \text{iff} \quad A_0 A_1 A_2 \dots \models \psi$$

Base of induction:

Suppose $\psi = \text{true} \in cl(\varphi)$. Then $\text{true} \in B_0$ and

$$A_0 A_1 A_2 \dots \models \text{true}$$

Let $\psi = a \in AP$. Then:

$$a \in B_0$$

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \dots$ is a path in \mathcal{G} s.t.

$$\forall F \in \mathcal{F} \quad \exists j \geq 0. B_j \in F$$

then for all formulas $\psi \in cl(\varphi)$:

$$\psi \in B_0 \quad \text{iff} \quad A_0 A_1 A_2 \dots \models \psi$$

Base of induction:

Suppose $\psi = \text{true} \in cl(\varphi)$. Then $\text{true} \in B_0$ and
 $A_0 A_1 A_2 \dots \models \text{true}$

Let $\psi = a \in AP$. Then:

$$a \in B_0 \iff a \in A_0$$

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \dots$ is a path in \mathcal{G} s.t.

$$\forall F \in \mathcal{F} \quad \exists j \geq 0. B_j \in F$$

then for all formulas $\psi \in cl(\varphi)$:

$$\psi \in B_0 \quad \text{iff} \quad A_0 A_1 A_2 \dots \models \psi$$

Base of induction:

Suppose $\psi = \text{true} \in cl(\varphi)$. Then $\text{true} \in B_0$ and

$$A_0 A_1 A_2 \dots \models \text{true}$$

Let $\psi = a \in AP$. Then:

$$a \in B_0 \iff a \in A_0$$

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \dots$ is a path in \mathcal{G} s.t.

$$\forall F \in \mathcal{F} \quad \exists j \geq 0. B_j \in F \quad A_0 = B_0 \cap AP$$

then for all formulas $\psi \in cl(\varphi)$:

$$\psi \in B_0 \quad \text{iff} \quad A_0 A_1 A_2 \dots \models \psi$$

Base of induction:

Suppose $\psi = \text{true} \in cl(\varphi)$. Then $\text{true} \in B_0$ and $A_0 A_1 A_2 \dots \models \text{true}$

Let $\psi = a \in AP$. Then:

$$a \in B_0 \iff a \in A_0$$

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \dots$ is a path in \mathcal{G} s.t.

$$\forall F \in \mathcal{F} \quad \exists j \geq 0. B_j \in F \quad A_0 = B_0 \cap AP$$

then for all formulas $\psi \in cl(\varphi)$:

$$\psi \in B_0 \quad \text{iff} \quad A_0 A_1 A_2 \dots \models \psi$$

Base of induction:

Suppose $\psi = \text{true} \in cl(\varphi)$. Then $\text{true} \in B_0$ and

$$A_0 A_1 A_2 \dots \models \text{true}$$

Let $\psi = a \in AP$. Then:

$$a \in B_0 \iff a \in A_0 \iff A_0 A_1 A_2 \dots \models a$$

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \dots$ is a path in \mathcal{G} s.t.

$$\forall F \in \mathcal{F} \quad \exists j \geq 0. B_j \in F$$

then for all formulas $\psi \in cl(\varphi)$:

$$\psi \in B_0 \quad \text{iff} \quad A_0 A_1 A_2 \dots \models \psi$$

Induction step: for $\psi = \neg\psi'$:

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \dots$ is a path in \mathcal{G} s.t.

$$\forall F \in \mathcal{F} \quad \exists j \geq 0. B_j \in F$$

then for all formulas $\psi \in cl(\varphi)$:

$$\psi \in B_0 \quad \text{iff} \quad A_0 A_1 A_2 \dots \models \psi$$

Induction step: for $\psi = \neg\psi'$:

$$\psi \in B_0$$

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \dots$ is a path in \mathcal{G} s.t.

$$\forall F \in \mathcal{F} \quad \exists j \geq 0. B_j \in F$$

then for all formulas $\psi \in cl(\varphi)$:

$$\psi \in B_0 \quad \text{iff} \quad A_0 A_1 A_2 \dots \models \psi$$

Induction step: for $\psi = \neg\psi'$:

$$\psi \in B_0$$

$$\text{iff } \psi' \notin B_0 \quad (\text{maximal consistency})$$

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \dots$ is a path in \mathcal{G} s.t.

$$\forall F \in \mathcal{F} \quad \exists j \geq 0. B_j \in F$$

then for all formulas $\psi \in cl(\varphi)$:

$$\psi \in B_0 \quad \text{iff} \quad A_0 A_1 A_2 \dots \models \psi$$

Induction step: for $\psi = \neg\psi'$:

$$\psi \in B_0$$

iff $\psi' \notin B_0$ (maximal consistency)

iff $A_0 A_1 A_2 \dots \not\models \psi'$ (induction hypothesis)

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \dots$ is a path in \mathcal{G} s.t.

$$\forall F \in \mathcal{F} \quad \exists j \geq 0. B_j \in F$$

then for all formulas $\psi \in cl(\varphi)$:

$$\psi \in B_0 \quad \text{iff} \quad A_0 A_1 A_2 \dots \models \psi$$

Induction step: for $\psi = \neg\psi'$:

$$\psi \in B_0$$

iff $\psi' \notin B_0$ (maximal consistency)

iff $A_0 A_1 A_2 \dots \not\models \psi'$ (induction hypothesis)

iff $A_0 A_1 A_2 \dots \models \psi$ (semantics of \neg)

$B \subseteq cl(\varphi)$ is elementary iff:

- (i) B is maximal consistent w.r.t. prop. logic, i.e., if $\psi, \psi_1 \wedge \psi_2 \in cl(\varphi)$ then:

$$\begin{aligned}\psi \notin B & \text{ iff } \neg\psi \in B \\ \psi_1 \wedge \psi_2 \in B & \text{ iff } \psi_1 \in B \text{ and } \psi_2 \in B \\ \text{true} \in cl(\varphi) & \text{ implies } \text{true} \in B\end{aligned}$$

- (ii) B is locally consistent with respect to until U , i.e., if $\psi_1 U \psi_2 \in cl(\varphi)$ then:

$$\begin{aligned}\text{if } \psi_1 U \psi_2 \in B \text{ and } \psi_2 \notin B & \text{ then } \psi_1 \in B \\ \text{if } \psi_2 \in B & \text{ then } \psi_1 U \psi_2 \in B\end{aligned}$$

$B \subseteq cl(\varphi)$ is elementary iff:

- (i) B is maximal consistent w.r.t. prop. logic, i.e., if $\psi, \psi_1 \wedge \psi_2 \in cl(\varphi)$ then:

$$\psi \notin B \text{ iff } \neg\psi \in B$$

$$\psi_1 \wedge \psi_2 \in B \text{ iff } \psi_1 \in B \text{ and } \psi_2 \in B$$

$$true \in cl(\varphi) \text{ implies } true \in B$$

- (ii) B is locally consistent with respect to until U , i.e., if $\psi_1 U \psi_2 \in cl(\varphi)$ then:

$$\text{if } \psi_1 U \psi_2 \in B \text{ and } \psi_2 \notin B \text{ then } \psi_1 \in B$$

$$\text{if } \psi_2 \in B \text{ then } \psi_1 U \psi_2 \in B$$

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \dots$ is a path in \mathcal{G} s.t.

$$\forall F \in \mathcal{F} \quad \exists j \geq 0. B_j \in F$$

then for all formulas $\psi \in cl(\varphi)$:

$$\psi \in B_0 \quad \text{iff} \quad A_0 A_1 A_2 \dots \models \psi$$

Induction step: for $\psi = \psi_1 \wedge \psi_2$

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \dots$ is a path in \mathcal{G} s.t.

$$\forall F \in \mathcal{F} \quad \exists j \geq 0. B_j \in F$$

then for all formulas $\psi \in cl(\varphi)$:

$$\psi \in B_0 \quad \text{iff} \quad A_0 A_1 A_2 \dots \models \psi$$

Induction step: for $\psi = \psi_1 \wedge \psi_2$

$$\psi \in B_0$$

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \dots$ is a path in \mathcal{G} s.t.

$$\forall F \in \mathcal{F} \quad \exists j \geq 0. B_j \in F$$

then for all formulas $\psi \in cl(\varphi)$:

$$\psi \in B_0 \quad \text{iff} \quad A_0 A_1 A_2 \dots \models \psi$$

Induction step: for $\psi = \psi_1 \wedge \psi_2$

$$\psi \in B_0$$

$$\text{iff } \psi_1, \psi_2 \in B_0 \quad (\text{maximal consistency})$$

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \dots$ is a path in \mathcal{G} s.t.

$$\forall F \in \mathcal{F} \quad \exists j \geq 0. B_j \in F$$

then for all formulas $\psi \in cl(\varphi)$:

$$\psi \in B_0 \quad \text{iff} \quad A_0 A_1 A_2 \dots \models \psi$$

Induction step: for $\psi = \psi_1 \wedge \psi_2$

$$\psi \in B_0$$

iff $\psi_1, \psi_2 \in B_0$ (maximal consistency)

iff $A_0 A_1 A_2 \dots \models \psi_1$ and $A_0 A_1 A_2 \dots \models \psi_2$ (IH)

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \dots$ is a path in \mathcal{G} s.t.

$$\forall F \in \mathcal{F} \quad \exists j \geq 0. B_j \in F$$

then for all formulas $\psi \in cl(\varphi)$:

$$\psi \in B_0 \quad \text{iff} \quad A_0 A_1 A_2 \dots \models \psi$$

Induction step: for $\psi = \psi_1 \wedge \psi_2$

$$\psi \in B_0$$

iff $\psi_1, \psi_2 \in B_0$ (maximal consistency)

iff $A_0 A_1 A_2 \dots \models \psi_1$ and $A_0 A_1 A_2 \dots \models \psi_2$ (IH)

iff $A_0 A_1 A_2 \dots \models \psi$ (semantics of \wedge)

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \dots$ is a path in \mathcal{G} s.t.

$$\forall F \in \mathcal{F} \quad \exists j \geq 0. B_j \in F$$

then for all formulas $\psi \in cl(\varphi)$:

$$\psi \in B_0 \quad \text{iff} \quad A_0 A_1 A_2 \dots \models \psi$$

Induction step: for $\psi = \bigcirc \psi'$:

$$\mathcal{G} = (Q, 2^{AP}, \delta, Q_0, \mathcal{F})$$

state space: $Q = \{B \subseteq cl(\varphi) : B \text{ is elementary}\}$

initial states: $Q_0 = \{B \in Q : \varphi \in B\}$

transition relation: for $B \in Q$ and $A \in 2^{AP}$:

if $A \neq B \cap AP$ then $\delta(B, A) = \emptyset$

if $A = B \cap AP$ then $\delta(B, A) = \text{set of all } B' \in Q \text{ s.t.}$

$$\bigcirc \psi \in B \text{ iff } \psi \in B'$$

$$\psi_1 \mathbf{U} \psi_2 \in B \text{ iff } (\psi_2 \in B) \vee (\psi_1 \in B \wedge \psi_1 \mathbf{U} \psi_2 \in B')$$

acceptance set $\mathcal{F} = \{F_{\psi_1 \mathbf{U} \psi_2} : \psi_1 \mathbf{U} \psi_2 \in cl(\varphi)\}$

where $F_{\psi_1 \mathbf{U} \psi_2} = \{B \in Q : \psi_1 \mathbf{U} \psi_2 \notin B \vee \psi_2 \in B\}$

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \dots$ is a path in \mathcal{G} s.t.

$$\forall F \in \mathcal{F} \quad \exists j \geq 0. B_j \in F$$

then for all formulas $\psi \in cl(\varphi)$:

$$\psi \in B_0 \quad \text{iff} \quad A_0 A_1 A_2 \dots \models \psi$$

Induction step: for $\psi = \bigcirc \psi'$:

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \dots$ is a path in \mathcal{G} s.t.

$$\forall F \in \mathcal{F} \quad \exists j \geq 0. B_j \in F$$

then for all formulas $\psi \in cl(\varphi)$:

$$\psi \in B_0 \quad \text{iff} \quad A_0 A_1 A_2 \dots \models \psi$$

Induction step: for $\psi = \bigcirc \psi'$:

$$\psi \in B_0$$

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \dots$ is a path in \mathcal{G} s.t.

$$\forall F \in \mathcal{F} \quad \exists j \geq 0. B_j \in F \quad B_1 \in \delta(B_0, A_0)$$

then for all formulas $\psi \in cl(\varphi)$:

$$\psi \in B_0 \quad \text{iff} \quad A_0 A_1 A_2 \dots \models \psi$$

Induction step: for $\psi = \bigcirc \psi'$:

$$\psi \in B_0$$

$$\text{iff } \psi' \in B_1 \quad (\text{definition of } \delta)$$

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \dots$ is a path in \mathcal{G} s.t.

$$\forall F \in \mathcal{F} \quad \exists^\infty j \geq 0. B_j \in F \quad B_1 \in \delta(B_0, A_0)$$

then for all formulas $\psi \in cl(\varphi)$:

$$\psi \in B_0 \quad \text{iff} \quad A_0 A_1 A_2 \dots \models \psi$$

Induction step: for $\psi = \bigcirc \psi'$:

$$\psi \in B_0$$

$$\text{iff } \psi' \in B_1 \quad (\text{definition of } \delta)$$

$$\text{iff } A_1 A_2 A_3 \dots \models \psi' \quad (\text{induction hypothesis})$$

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \dots$ is a path in \mathcal{G} s.t.

$$\forall F \in \mathcal{F} \quad \exists j \geq 0. B_j \in F \quad B_1 \in \delta(B_0, A_0)$$

then for all formulas $\psi \in cl(\varphi)$:

$$\psi \in B_0 \quad \text{iff} \quad A_0 A_1 A_2 \dots \models \psi$$

Induction step: for $\psi = \bigcirc \psi'$:

$$\psi \in B_0$$

$$\text{iff } \psi' \in B_1 \quad (\text{definition of } \delta)$$

$$\text{iff } A_1 A_2 A_3 \dots \models \psi' \quad (\text{induction hypothesis})$$

$$\text{iff } A_0 A_1 A_2 A_3 \dots \models \psi \quad (\text{semantics of } \bigcirc)$$

Induction step: until

LTLMC3.2-63

$B \subseteq cl(\varphi)$ is elementary iff:

- (i) B is maximal consistent w.r.t. prop. logic,
i.e., if $\psi, \psi_1 \wedge \psi_2 \in cl(\varphi)$ then:

$$\begin{array}{ll} \psi \notin B & \text{iff} \quad \neg\psi \in B \\ \psi_1 \wedge \psi_2 \in B & \text{iff} \quad \psi_1 \in B \text{ and } \psi_2 \in B \\ \text{true} \in cl(\varphi) & \text{implies } \text{true} \in B \end{array}$$

- (ii) B is locally consistent with respect to until \mathbf{U} ,
i.e., if $\psi_1 \mathbf{U} \psi_2 \in cl(\varphi)$ then:

$$\begin{array}{l} \text{if } \psi_1 \mathbf{U} \psi_2 \in B \text{ and } \psi_2 \notin B \text{ then } \psi_1 \in B \\ \text{if } \psi_2 \in B \text{ then } \psi_1 \mathbf{U} \psi_2 \in B \end{array}$$

$$\mathcal{G} = (Q, 2^{AP}, \delta, Q_0, \mathcal{F})$$

state space: $Q = \{B \subseteq cl(\varphi) : B \text{ is elementary}\}$

initial states: $Q_0 = \{B \in Q : \varphi \in B\}$

transition relation: for $B \in Q$ and $A \in 2^{AP}$:

if $A \neq B \cap AP$ then $\delta(B, A) = \emptyset$

if $A = B \cap AP$ then $\delta(B, A) = \text{set of all } B' \in Q \text{ s.t.}$

$$\bigcirc \psi \in B \text{ iff } \psi \in B'$$

$$\psi_1 \mathbf{U} \psi_2 \in B \text{ iff } (\psi_2 \in B) \vee (\psi_1 \in B \wedge \psi_1 \mathbf{U} \psi_2 \in B')$$

acceptance set $\mathcal{F} = \{F_{\psi_1 \mathbf{U} \psi_2} : \psi_1 \mathbf{U} \psi_2 \in cl(\varphi)\}$

where $F_{\psi_1 \mathbf{U} \psi_2} = \{B \in Q : \psi_1 \mathbf{U} \psi_2 \notin B \vee \psi_2 \in B\}$

$$\mathcal{G} = (Q, 2^{AP}, \delta, Q_0, \mathcal{F})$$

state space: $Q = \{B \subseteq cl(\varphi) : B \text{ is elementary}\}$

initial states: $Q_0 = \{B \in Q : \varphi \in B\}$

transition relation: for $B \in Q$ and $A \in 2^{AP}$:

if $A \neq B \cap AP$ then $\delta(B, A) = \emptyset$

if $A = B \cap AP$ then $\delta(B, A) = \text{set of all } B' \in Q \text{ s.t.}$

$$\bigcirc \psi \in B \text{ iff } \psi \in B'$$

$$\psi_1 \mathbf{U} \psi_2 \in B \text{ iff } (\psi_2 \in B) \vee (\psi_1 \in B \wedge \psi_1 \mathbf{U} \psi_2 \in B')$$

acceptance set $\mathcal{F} = \{F_{\psi_1 \mathbf{U} \psi_2} : \psi_1 \mathbf{U} \psi_2 \in cl(\varphi)\}$

where $F_{\psi_1 \mathbf{U} \psi_2} = \{B \in Q : \psi_1 \mathbf{U} \psi_2 \notin B \vee \psi_2 \in B\}$

Induction step: until

LTLMC3.2-63

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \dots$ is a path in \mathcal{G} s.t.

$$\forall F \in \mathcal{F} \quad \exists^{\infty} j \geq 0. B_j \in F$$

then for all $\psi \in cl(\varphi)$: $\psi \in B_0$ iff $A_0 A_1 A_2 \dots \models \psi$

Induction step for $\psi = \psi_1 \mathbf{U} \psi_2$:

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \dots$ is a path in \mathcal{G} s.t.

$$\forall F \in \mathcal{F} \quad \exists^\infty j \geq 0. B_j \in F$$

then for all $\psi \in cl(\varphi)$: $\psi \in B_0$ iff $A_0 A_1 A_2 \dots \models \psi$

Induction step for $\psi = \psi_1 \mathbf{U} \psi_2$:

“ \Leftarrow ”: Suppose $A_0 A_1 A_2 \dots \models \psi$.

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \dots$ is a path in \mathcal{G} s.t.

$$\forall F \in \mathcal{F} \quad \exists j \geq 0. B_j \in F$$

then for all $\psi \in cl(\varphi)$: $\psi \in B_0$ iff $A_0 A_1 A_2 \dots \models \psi$

Induction step for $\psi = \psi_1 \mathbf{U} \psi_2$:

“ \Leftarrow ”: Suppose $A_0 A_1 A_2 \dots \models \psi$. Let $j \geq 0$ s.t.

$$A_j A_{j+1} A_{j+2} \dots \models \psi_2$$

$$A_{j-1} A_j A_{j-1} \dots \models \psi_1$$

$$A_{j-2} A_{j-1} A_j \dots \models \psi_1$$

$$\vdots$$

$$A_0 A_1 A_2 A_3 \dots \models \psi_1$$

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \dots$ is a path in \mathcal{G} s.t.

$$\forall F \in \mathcal{F} \quad \exists j \geq 0. B_j \in F$$

then for all $\psi \in cl(\varphi)$: $\psi \in B_0$ iff $A_0 A_1 A_2 \dots \models \psi$

Induction step for $\psi = \psi_1 \mathbf{U} \psi_2$:

“ \Leftarrow ”: Suppose $A_0 A_1 A_2 \dots \models \psi$. Let $j \geq 0$ s.t.

$$\begin{array}{lll} A_j A_{j+1} A_{j+2} \dots & \models \psi_2 & \stackrel{\text{IH}}{\Rightarrow} \psi_2 \in B_j \\ A_{j-1} A_j A_{j-1} \dots & \models \psi_1 & \Rightarrow \psi_1 \in B_{j-1} \\ A_{j-2} A_{j-1} A_j \dots & \models \psi_1 & \Rightarrow \psi_1 \in B_{j-2} \\ \vdots & & \vdots \\ A_0 A_1 A_2 A_3 \dots & \models \psi_1 & \Rightarrow \psi_1 \in B_0 \end{array}$$

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \dots$ is a path in \mathcal{G} s.t.

$\forall F \in \mathcal{F} \exists j \geq 0. B_j \in F$ B_j is elementary

then for all $\psi \in cl(\varphi)$: $\psi \in B_0$ iff $A_0 A_1 A_2 \dots \models \psi$

Induction step for $\psi = \psi_1 \mathbf{U} \psi_2$:

“ \Leftarrow ”: Suppose $A_0 A_1 A_2 \dots \models \psi$. Let $j \geq 0$ s.t.

$$\begin{array}{llll}
 A_j A_{j+1} A_{j+2} \dots & \models \psi_2 & \xRightarrow{\text{IH}} & \psi_2 \in B_j \Rightarrow \psi \in B_j \\
 A_{j-1} A_j A_{j-1} \dots & \models \psi_1 & \Rightarrow & \psi_1 \in B_{j-1} \\
 A_{j-2} A_{j-1} A_j \dots & \models \psi_1 & \Rightarrow & \psi_1 \in B_{j-2} \\
 \vdots & & & \vdots \\
 A_0 A_1 A_2 A_3 \dots & \models \psi_1 & \Rightarrow & \psi_1 \in B_0
 \end{array}$$

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \dots$ is a path in \mathcal{G} s.t.

$$\forall F \in \mathcal{F} \quad \exists j \geq 0. B_j \in F \quad B_j \in \delta(B_{j-1}, A_{j-1})$$

then for all $\psi \in cl(\varphi)$: $\psi \in B_0$ iff $A_0 A_1 A_2 \dots \models \psi$

Induction step for $\psi = \psi_1 \mathbf{U} \psi_2$:

“ \Leftarrow ”: Suppose $A_0 A_1 A_2 \dots \models \psi$. Let $j \geq 0$ s.t.

$$\begin{array}{llll} A_j A_{j+1} A_{j+2} \dots & \models \psi_2 & \xRightarrow{\text{IH}} & \psi_2 \in B_j \Rightarrow \psi \in B_j \\ A_{j-1} A_j A_{j-1} \dots & \models \psi_1 & \Rightarrow & \psi_1 \in B_{j-1} \wedge \psi \in B_{j-1} \\ A_{j-2} A_{j-1} A_j \dots & \models \psi_1 & \Rightarrow & \psi_1 \in B_{j-2} \\ \vdots & & & \vdots \\ A_0 A_1 A_2 A_3 \dots & \models \psi_1 & \Rightarrow & \psi_1 \in B_0 \end{array}$$

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \dots$ is a path in \mathcal{G} s.t.

$$\forall F \in \mathcal{F} \quad \exists j \geq 0. B_j \in F \quad B_{j-1} \in \delta(B_{j-2}, A_{j-2})$$

then for all $\psi \in cl(\varphi)$: $\psi \in B_0$ iff $A_0 A_1 A_2 \dots \models \psi$

Induction step for $\psi = \psi_1 \mathbf{U} \psi_2$:

“ \Leftarrow ”: Suppose $A_0 A_1 A_2 \dots \models \psi$. Let $j \geq 0$ s.t.

$$\begin{array}{llll} A_j A_{j+1} A_{j+2} \dots & \models \psi_2 & \xRightarrow{\text{IH}} & \psi_2 \in B_j \Rightarrow \psi \in B_j \\ A_{j-1} A_j A_{j-1} \dots & \models \psi_1 & \Rightarrow & \psi_1 \in B_{j-1} \wedge \psi \in B_{j-1} \\ A_{j-2} A_{j-1} A_j \dots & \models \psi_1 & \Rightarrow & \psi_1 \in B_{j-2} \wedge \psi \in B_{j-2} \\ \vdots & & & \vdots \\ A_0 A_1 A_2 A_3 \dots & \models \psi_1 & \Rightarrow & \psi_1 \in B_0 \end{array}$$

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \dots$ is a path in \mathcal{G} s.t.

$$\forall F \in \mathcal{F} \quad \exists j \geq 0. B_j \in F \quad B_1 \in \delta(B_0, A_0)$$

then for all $\psi \in cl(\varphi)$: $\psi \in B_0$ iff $A_0 A_1 A_2 \dots \models \psi$

Induction step for $\psi = \psi_1 \mathbf{U} \psi_2$:

“ \Leftarrow ”: Suppose $A_0 A_1 A_2 \dots \models \psi$. Let $j \geq 0$ s.t.

$$\begin{array}{llll} A_j A_{j+1} A_{j+2} \dots & \models \psi_2 & \xRightarrow{\text{IH}} & \psi_2 \in B_j \Rightarrow \psi \in B_j \\ A_{j-1} A_j A_{j-1} \dots & \models \psi_1 & \Rightarrow & \psi_1 \in B_{j-1} \wedge \psi \in B_{j-1} \\ A_{j-2} A_{j-1} A_j \dots & \models \psi_1 & \Rightarrow & \psi_1 \in B_{j-2} \wedge \psi \in B_{j-2} \\ \vdots & & & \vdots \\ A_0 A_1 A_2 A_3 \dots & \models \psi_1 & \Rightarrow & \psi_1 \in B_0 \wedge \psi \in B_0 \end{array}$$

Induction step: until (part “ \Rightarrow ”)

LTLMC3.2-64

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \dots$ is a path in \mathcal{G} s.t.

$$\forall F \in \mathcal{F} \quad \exists^{\infty} j \geq 0. B_j \in F$$

then for all $\psi \in cl(\varphi)$: $\psi \in B_0$ iff $A_0 A_1 A_2 \dots \models \psi$

Induction step for $\psi = \psi_1 \mathbf{U} \psi_2$:

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \dots$ is a path in \mathcal{G} s.t.

$$\forall F \in \mathcal{F} \quad \exists j \geq 0. B_j \in F$$

then for all $\psi \in cl(\varphi)$: $\psi \in B_0$ iff $A_0 A_1 A_2 \dots \models \psi$

Induction step for $\psi = \psi_1 \mathbf{U} \psi_2$:

“ \implies ” Suppose $\psi \in B_0$.

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \dots$ is a path in \mathcal{G} s.t.

$$\forall F \in \mathcal{F} \quad \exists j \geq 0. B_j \in F$$

then for all $\psi \in cl(\varphi)$: $\psi \in B_0$ iff $A_0 A_1 A_2 \dots \models \psi$

Induction step for $\psi = \psi_1 \mathbf{U} \psi_2$:

“ \implies ” Suppose $\psi \in B_0$. There exists $j \geq 0$ with $\psi_2 \in B_j$,

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \dots$ is a path in \mathcal{G} s.t.

$$\forall F \in \mathcal{F} \quad \exists j \geq 0. B_j \in F$$

then for all $\psi \in cl(\varphi)$: $\psi \in B_0$ iff $A_0 A_1 A_2 \dots \models \psi$

Induction step for $\psi = \psi_1 \mathbf{U} \psi_2$:

“ \implies ” Suppose $\psi \in B_0$. There exists $j \geq 0$ with $\psi_2 \in B_j$, since otherwise $\forall j \geq 0. \psi_2 \notin B_j$

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \dots$ is a path in \mathcal{G} s.t.

$$\forall F \in \mathcal{F} \quad \exists j \geq 0. B_j \in F$$

then for all $\psi \in cl(\varphi)$: $\psi \in B_0$ iff $A_0 A_1 A_2 \dots \models \psi$

Induction step for $\psi = \psi_1 \mathbf{U} \psi_2$:

“ \implies ” Suppose $\psi \in B_0$. There exists $j \geq 0$ with $\psi_2 \in B_j$, since otherwise $\forall j \geq 0. \psi_2 \notin B_j$ and therefore:

$$\psi \in B_0 \wedge \psi_2 \notin B_0$$

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \dots$ is a path in \mathcal{G} s.t.

$$\forall F \in \mathcal{F} \quad \exists j \geq 0. B_j \in F \quad B_{i+1} \in \delta(B_i, A_i)$$

then for all $\psi \in cl(\varphi)$: $\psi \in B_0$ iff $A_0 A_1 A_2 \dots \models \psi$

Induction step for $\psi = \psi_1 \mathbf{U} \psi_2$:

“ \implies ” Suppose $\psi \in B_0$. There exists $j \geq 0$ with $\psi_2 \in B_j$, since otherwise $\forall j \geq 0. \psi_2 \notin B_j$ and therefore:

$$\psi \in B_0 \wedge \psi_2 \notin B_0$$

$$\Rightarrow \psi \in B_1$$

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \dots$ is a path in \mathcal{G} s.t.

$$\forall F \in \mathcal{F} \quad \exists j \geq 0. B_j \in F \quad \boxed{B_{i+1} \in \delta(B_i, A_i)}$$

then for all $\psi \in cl(\varphi)$: $\psi \in B_0$ iff $A_0 A_1 A_2 \dots \models \psi$

Induction step for $\psi = \psi_1 \mathbf{U} \psi_2$:

“ \implies ” Suppose $\psi \in B_0$. There exists $j \geq 0$ with $\psi_2 \in B_j$, since otherwise $\forall j \geq 0. \psi_2 \notin B_j$ and therefore:

$$\psi \in B_0 \wedge \psi_2 \notin B_0$$

$$\Rightarrow \psi \in B_1 \wedge \psi_2 \notin B_1$$

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \dots$ is a path in \mathcal{G} s.t.

$$\forall F \in \mathcal{F} \quad \exists j \geq 0. B_j \in F \quad B_{i+1} \in \delta(B_i, A_i)$$

then for all $\psi \in cl(\varphi)$: $\psi \in B_0$ iff $A_0 A_1 A_2 \dots \models \psi$

Induction step for $\psi = \psi_1 \mathbf{U} \psi_2$:

“ \implies ” Suppose $\psi \in B_0$. There exists $j \geq 0$ with $\psi_2 \in B_j$, since otherwise $\forall j \geq 0. \psi_2 \notin B_j$ and therefore:

$$\psi \in B_0 \wedge \psi_2 \notin B_0$$

$$\Rightarrow \psi \in B_1 \wedge \psi_2 \notin B_1$$

$$\Rightarrow \psi \in B_2$$

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \dots$ is a path in \mathcal{G} s.t.

$$\forall F \in \mathcal{F} \quad \exists j \geq 0. B_j \in F \quad B_{i+1} \in \delta(B_i, A_i)$$

then for all $\psi \in cl(\varphi)$: $\psi \in B_0$ iff $A_0 A_1 A_2 \dots \models \psi$

Induction step for $\psi = \psi_1 \mathbf{U} \psi_2$:

“ \implies ” Suppose $\psi \in B_0$. There exists $j \geq 0$ with $\psi_2 \in B_j$, since otherwise $\forall j \geq 0. \psi_2 \notin B_j$ and therefore:

$$\begin{aligned} & \psi \in B_0 \wedge \psi_2 \notin B_0 \\ \Rightarrow & \psi \in B_1 \wedge \psi_2 \notin B_1 \\ \Rightarrow & \psi \in B_2 \wedge \psi_2 \notin B_2 \\ & \vdots \end{aligned}$$

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \dots$ is a path in \mathcal{G} s.t.

$$\forall F \in \mathcal{F} \quad \exists j \geq 0. B_j \in F$$

then for all $\psi \in cl(\varphi)$: $\psi \in B_0$ iff $A_0 A_1 A_2 \dots \models \psi$

Induction step for $\psi = \psi_1 \mathbf{U} \psi_2$:

“ \implies ” Suppose $\psi \in B_0$. There exists $j \geq 0$ with $\psi_2 \in B_j$, since otherwise $\forall j \geq 0. \psi_2 \notin B_j$ and therefore:

$$\left. \begin{array}{l} \psi \in B_0 \wedge \psi_2 \notin B_0 \\ \implies \psi \in B_1 \wedge \psi_2 \notin B_1 \\ \implies \psi \in B_2 \wedge \psi_2 \notin B_2 \\ \vdots \end{array} \right\} \implies \forall j \geq 0. B_j \notin F_\psi \text{ where } F_\psi = \{B : \psi \notin B \text{ or } \psi_2 \in B\}$$

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \dots$ is a path in \mathcal{G} s.t.

$$\forall F \in \mathcal{F} \quad \exists j \geq 0. B_j \in F$$

then for all $\psi \in cl(\varphi)$: $\psi \in B_0$ iff $A_0 A_1 A_2 \dots \models \psi$

Induction step for $\psi = \psi_1 \mathbf{U} \psi_2$:

“ \implies ” Suppose $\psi \in B_0$. There exists $j \geq 0$ with $\psi_2 \in B_j$, since otherwise $\forall j \geq 0. \psi_2 \notin B_j$ and therefore:

$$\left. \begin{array}{l} \psi \in B_0 \wedge \psi_2 \notin B_0 \\ \Rightarrow \psi \in B_1 \wedge \psi_2 \notin B_1 \\ \Rightarrow \psi \in B_2 \wedge \psi_2 \notin B_2 \\ \vdots \end{array} \right\} \implies \forall j \geq 0. B_j \notin F_\psi \text{ where } F_\psi = \{B : \psi \notin B \text{ or } \psi_2 \in B\}$$

Contradiction!

Induction step: until (part “ \Rightarrow ”)

LTLMC3.2-65

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \dots$ is a path in \mathcal{G} s.t.

$$\forall F \in \mathcal{F} \quad \exists j \geq 0. B_j \in F$$

then for all $\psi \in cl(\varphi)$: $\psi \in B_0$ iff $A_0 A_1 A_2 \dots \models \psi$

Induction step for $\psi = \psi_1 \mathbf{U} \psi_2$:

Let $\psi \in B_0$ and $j \geq 0$ minimal s.t. $\psi_2 \in B_j$

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \dots$ is a path in \mathcal{G} s.t.

$$\forall F \in \mathcal{F} \quad \exists j \geq 0. B_j \in F$$

then for all $\psi \in cl(\varphi)$: $\psi \in B_0$ iff $A_0 A_1 A_2 \dots \models \psi$

Induction step for $\psi = \psi_1 \mathbf{U} \psi_2$:

Let $\psi \in B_0$ and $j \geq 0$ minimal s.t. $\psi_2 \in B_j$

$$\xRightarrow{\text{IH}} A_j A_{j+1} \dots \models \psi_2$$

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \dots$ is a path in \mathcal{G} s.t.

$$\forall F \in \mathcal{F} \quad \exists j \geq 0. B_j \in F$$

then for all $\psi \in cl(\varphi)$: $\psi \in B_0$ iff $A_0 A_1 A_2 \dots \models \psi$

Induction step for $\psi = \psi_1 \mathbf{U} \psi_2$:

Let $\psi \in B_0$ and $j \geq 0$ minimal s.t. $\psi_2 \in B_j$

$$\stackrel{\text{IH}}{\implies} A_j A_{j+1} \dots \models \psi_2$$

$$\neg \psi_2 \in B_{j-1}$$

$$\neg \psi_2 \in B_{j-2}$$

$$\vdots$$

$$\neg \psi_2 \in B_1$$

$$\neg \psi_2 \in B_0$$

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \dots$ is a path in \mathcal{G} s.t.

$$\forall F \in \mathcal{F} \quad \exists j \geq 0. B_j \in F$$

then for all $\psi \in cl(\varphi)$: $\psi \in B_0$ iff $A_0 A_1 A_2 \dots \models \psi$

Induction step for $\psi = \psi_1 \mathbf{U} \psi_2$:

Let $\psi \in B_0$ and $j \geq 0$ minimal s.t. $\psi_2 \in B_j$

$$\stackrel{\text{IH}}{\implies} A_j A_{j+1} \dots \models \psi_2$$

$$\neg \psi_2 \in B_{j-1}$$

$$\neg \psi_2 \in B_{j-2}$$

$$\vdots$$

$$\neg \psi_2 \in B_1$$

$$\neg \psi_2, \psi \in B_0 \quad \longleftarrow \text{by assumption}$$

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \dots$ is a path in \mathcal{G} s.t.

$$\forall F \in \mathcal{F} \quad \exists j \geq 0. B_j \in F$$

then for all $\psi \in cl(\varphi)$: $\psi \in B_0$ iff $A_0 A_1 A_2 \dots \models \psi$

Induction step for $\psi = \psi_1 \mathbf{U} \psi_2$:

Let $\psi \in B_0$ and $j \geq 0$ minimal s.t. $\psi_2 \in B_j$

$$\stackrel{\text{IH}}{\implies} A_j A_{j+1} \dots \models \psi_2$$

$$\neg \psi_2 \in B_{j-1}$$

$$\neg \psi_2 \in B_{j-2}$$

$$\vdots$$

$$\neg \psi_2 \in B_1$$

$$\neg \psi_2, \psi_1, \psi \in B_0$$

\leftarrow local consistency w.r.t. \mathbf{U}

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \dots$ is a path in \mathcal{G} s.t.

$$\forall F \in \mathcal{F} \quad \exists j \geq 0. B_j \in F$$

$$B_{i+1} \in \delta(B_i, A_i)$$

then for all $\psi \in cl(\varphi)$: $\psi \in B_0$ iff $A_0 A_1 A_2 \dots \models \psi$

Induction step for $\psi = \psi_1 \mathbf{U} \psi_2$:

Let $\psi \in B_0$ and $j \geq 0$ minimal s.t. $\psi_2 \in B_j$

$$\xRightarrow{\text{IH}} A_j A_{j+1} \dots \models \psi_2$$

$$\neg \psi_2 \in B_{j-1}$$

$$\neg \psi_2 \in B_{j-2}$$

$$\vdots$$

$$\neg \psi_2, \psi_1, \psi \in B_1$$

$$\neg \psi_2, \psi_1, \psi \in B_0$$

\leftarrow local consistency w.r.t. \mathbf{U}

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \dots$ is a path in \mathcal{G} s.t.

$$\forall F \in \mathcal{F} \quad \exists j \geq 0. B_j \in F$$

$$B_{i+1} \in \delta(B_i, A_i)$$

then for all $\psi \in cl(\varphi)$: $\psi \in B_0$ iff $A_0 A_1 A_2 \dots \models \psi$

Induction step for $\psi = \psi_1 \mathbf{U} \psi_2$:

Let $\psi \in B_0$ and $j \geq 0$ minimal s.t. $\psi_2 \in B_j$

$$\stackrel{\text{IH}}{\implies} A_j A_{j+1} \dots \models \psi_2$$

$$\neg \psi_2, \psi_1, \psi \in B_{j-1}$$

$$\neg \psi_2, \psi_1, \psi \in B_{j-2}$$

$$\vdots$$

$$\neg \psi_2, \psi_1, \psi \in B_1$$

$$\neg \psi_2, \psi_1, \psi \in B_0$$

← local consistency w.r.t. \mathbf{U}

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \dots$ is a path in \mathcal{G} s.t.

$$\forall F \in \mathcal{F} \quad \exists j \geq 0. B_j \in F$$

then for all $\psi \in cl(\varphi)$: $\psi \in B_0$ iff $A_0 A_1 A_2 \dots \models \psi$

Induction step for $\psi = \psi_1 \mathbf{U} \psi_2$:

Let $\psi \in B_0$ and $j \geq 0$ minimal s.t. $\psi_2 \in B_j$

$$\xRightarrow{\text{IH}} A_j A_{j+1} \dots \models \psi_2$$

$$\neg \psi_2, \psi_1, \psi \in B_{j-1} \implies A_{j-1} A_j \dots \models \psi_1$$

$$\neg \psi_2, \psi_1, \psi \in B_{j-2}$$

$$\vdots$$

$$\neg \psi_2, \psi_1, \psi \in B_1$$

$$\neg \psi_2, \psi_1, \psi \in B_0$$

\leftarrow local consistency w.r.t. \mathbf{U}

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \dots$ is a path in \mathcal{G} s.t.

$$\forall F \in \mathcal{F} \quad \exists j \geq 0. B_j \in F$$

then for all $\psi \in cl(\varphi)$: $\psi \in B_0$ iff $A_0 A_1 A_2 \dots \models \psi$

Induction step for $\psi = \psi_1 \mathbf{U} \psi_2$:

Let $\psi \in B_0$ and $j \geq 0$ minimal s.t. $\psi_2 \in B_j$

$$\xRightarrow{\text{IH}} A_j A_{j+1} \dots \models \psi_2$$

$$\neg \psi_2, \psi_1, \psi \in B_{j-1} \implies A_{j-1} A_j \dots \models \psi_1$$

$$\neg \psi_2, \psi_1, \psi \in B_{j-2} \implies A_{j-2} A_{j-1} \dots \models \psi_1$$

\vdots

$$\neg \psi_2, \psi_1, \psi \in B_1$$

$$\neg \psi_2, \psi_1, \psi \in B_0$$

\leftarrow local consistency w.r.t. \mathbf{U}

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \dots$ is a path in \mathcal{G} s.t.

$$\forall F \in \mathcal{F} \exists j \geq 0. B_j \in F$$

then for all $\psi \in cl(\varphi)$: $\psi \in B_0$ iff $A_0 A_1 A_2 \dots \models \psi$

Induction step for $\psi = \psi_1 \mathbf{U} \psi_2$:

Let $\psi \in B_0$ and $j \geq 0$ minimal s.t. $\psi_2 \in B_j$

$$\begin{array}{lll}
 & \xRightarrow{\text{IH}} & A_j A_{j+1} \dots \models \psi_2 \\
 \neg \psi_2, \psi_1, \psi \in B_{j-1} & \implies & A_{j-1} A_j \dots \models \psi_1 \\
 \neg \psi_2, \psi_1, \psi \in B_{j-2} & \implies & A_{j-2} A_{j-1} \dots \models \psi_1 \\
 \vdots & & \vdots \\
 \neg \psi_2, \psi_1, \psi \in B_1 & \implies & A_1 A_2 A_3 \dots \models \psi_1 \\
 \neg \psi_2, \psi_1, \psi \in B_0 & &
 \end{array}$$

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \dots$ is a path in \mathcal{G} s.t.

$$\forall F \in \mathcal{F} \exists j \geq 0. B_j \in F$$

then for all $\psi \in cl(\varphi)$: $\psi \in B_0$ iff $A_0 A_1 A_2 \dots \models \psi$

Induction step for $\psi = \psi_1 \mathbf{U} \psi_2$:

Let $\psi \in B_0$ and $j \geq 0$ minimal s.t. $\psi_2 \in B_j$

$$\begin{array}{lll}
 & \xRightarrow{\text{IH}} & A_j A_{j+1} \dots \models \psi_2 \\
 \neg \psi_2, \psi_1, \psi \in B_{j-1} & \implies & A_{j-1} A_j \dots \models \psi_1 \\
 \neg \psi_2, \psi_1, \psi \in B_{j-2} & \implies & A_{j-2} A_{j-1} \dots \models \psi_1 \\
 \vdots & & \vdots \\
 \neg \psi_2, \psi_1, \psi \in B_1 & \implies & A_1 A_2 A_3 \dots \models \psi_1 \\
 \neg \psi_2, \psi_1, \psi \in B_0 & \implies & A_0 A_1 A_2 \dots \models \psi_1
 \end{array}$$

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \dots$ is a path in \mathcal{G} s.t.

$$\forall F \in \mathcal{F} \exists j \geq 0. B_j \in F$$

then for all $\psi \in cl(\varphi)$: $\psi \in B_0$ iff $A_0 A_1 A_2 \dots \models \psi$

Induction step for $\psi = \psi_1 \mathbf{U} \psi_2$:

Let $\psi \in B_0$ and $j \geq 0$ minimal s.t. $\psi_2 \in B_j$

$$\begin{array}{ccc} & \xRightarrow{\text{IH}} & A_j A_{j+1} \dots \models \psi_2 \\ \neg \psi_2, \psi_1, \psi \in B_{j-1} & \implies & A_{j-1} A_j \dots \models \psi_1 \\ \vdots & & \vdots \\ \neg \psi_2, \psi_1, \psi \in B_0 & \implies & A_0 A_1 A_2 \dots \models \psi_1 \end{array}$$

\Downarrow

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \dots$ is a path in \mathcal{G} s.t.

$$\forall F \in \mathcal{F} \quad \exists j \geq 0. B_j \in F$$

then for all $\psi \in cl(\varphi)$: $\psi \in B_0$ iff $A_0 A_1 A_2 \dots \models \psi$

Induction step for $\psi = \psi_1 \mathbf{U} \psi_2$:

Let $\psi \in B_0$ and $j \geq 0$ minimal s.t. $\psi_2 \in B_j$

$$\begin{array}{ccc} & \xRightarrow{\text{IH}} & A_j A_{j+1} \dots \models \psi_2 \\ \neg \psi_2, \psi_1, \psi \in B_{j-1} & \implies & A_{j-1} A_j \dots \models \psi_1 \\ \vdots & & \vdots \\ \neg \psi_2, \psi_1, \psi \in B_0 & \implies & A_0 A_1 A_2 \dots \models \psi_1 \end{array}$$

\Downarrow

$$A_0 A_1 A_2 \dots \models \psi = \psi_1 \mathbf{U} \psi_2$$

Complexity: LTL \rightsquigarrow NBA

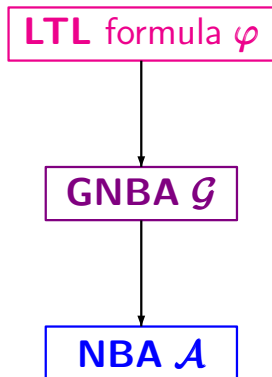
LTLMC3.2-67

For each **LTL** formula φ , there is an **NBA** \mathcal{A} s.t.

$$\mathcal{L}_w(\mathcal{A}) = \text{Words}(\varphi)$$

For each **LTL** formula φ , there is an **NBA** \mathcal{A} s.t.

$$\mathcal{L}_w(\mathcal{A}) = \text{Words}(\varphi)$$



For each **LTL** formula φ , there is an **NBA** \mathcal{A} s.t.

$$\mathcal{L}_w(\mathcal{A}) = \text{Words}(\varphi)$$

LTL formula φ



GNBA \mathcal{G}



NBA \mathcal{A}

size: $\text{size}(\mathcal{G}) \cdot |\mathcal{F}|$

For each **LTL** formula φ , there is an **NBA** \mathcal{A} s.t.

$$\mathcal{L}_w(\mathcal{A}) = \text{Words}(\varphi)$$

LTL formula φ



GNBA \mathcal{G}



NBA \mathcal{A}

size: $\text{size}(\mathcal{G}) \cdot |\mathcal{F}|$

$|\mathcal{F}|$ = number of
acceptance
sets in \mathcal{G}

For each **LTL** formula φ , there is an **NBA** \mathcal{A} s.t.

$$\mathcal{L}_w(\mathcal{A}) = \text{Words}(\varphi)$$

LTL formula φ

GNBA \mathcal{G}

NBA \mathcal{A}

size: $\text{size}(\mathcal{G}) \cdot |\mathcal{F}|$

$|\mathcal{F}|$ = number of
acceptance
sets in \mathcal{G}
 $\leq |\varphi|$

For each **LTL** formula φ , there is an **NBA** \mathcal{A} s.t.

$$\mathcal{L}_w(\mathcal{A}) = \text{Words}(\varphi)$$

LTL formula φ

GNBA \mathcal{G}

size: $2^{|\text{cl}(\varphi)|}$

NBA \mathcal{A}

size: $\text{size}(\mathcal{G}) \cdot |\mathcal{F}|$

$|\mathcal{F}|$ = number of
acceptance
sets in \mathcal{G}
 $\leq |\varphi|$

Complexity: LTL \rightsquigarrow NBA

LTLMC3.2-67

For each **LTL** formula φ , there is an **NBA** \mathcal{A} s.t.

$$\mathcal{L}_w(\mathcal{A}) = \text{Words}(\varphi) \text{ and}$$

$$\text{size}(\mathcal{A}) \leq 2^{|\text{cl}(\varphi)|} \cdot |\varphi|$$

LTL formula φ

GNBA \mathcal{G}

size: $2^{|\text{cl}(\varphi)|}$

NBA \mathcal{A}

size: $\text{size}(\mathcal{G}) \cdot |\mathcal{F}|$

$|\mathcal{F}|$ = number of
acceptance
sets in \mathcal{G}
 $\leq |\varphi|$

Complexity: LTL \rightsquigarrow NBA

LTLMC3.2-67

For each **LTL** formula φ , there is an **NBA** \mathcal{A} s.t.

$$\mathcal{L}_w(\mathcal{A}) = \text{Words}(\varphi) \text{ and}$$

$$\text{size}(\mathcal{A}) \leq 2^{|\text{cl}(\varphi)|} \cdot |\varphi| = 2^{\mathcal{O}(|\varphi|)}$$

LTL formula φ

GNBA \mathcal{G}

size: $2^{|\text{cl}(\varphi)|}$

NBA \mathcal{A}

size: $\text{size}(\mathcal{G}) \cdot |\mathcal{F}|$

$|\mathcal{F}|$ = number of
acceptance
sets in \mathcal{G}
 $\leq |\varphi|$

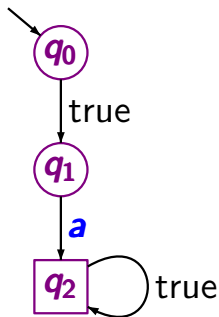
For the proposed transformation **LTL** \rightsquigarrow **NBA**:

The constructed NBA for LTL formulas are often
unnecessarily complicated

For the proposed transformation **LTL** \rightsquigarrow **NBA**:

The constructed NBA for LTL formulas are often
unnecessarily complicated

NBA for $\bigcirc a$

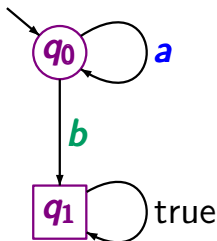


constructed GNBA has
4 states and **8** edges

For the proposed transformation **LTL** \rightsquigarrow **NBA**:

The constructed NBA for LTL formulas are often
unnecessarily complicated

NBA for ***a*** **U** ***b***



constructed (G)NBA has
5 states and **20** edges

For the proposed transformation **LTL** \rightsquigarrow **NBA**:

The constructed NBA for LTL formulas are often
unnecessarily complicated

... but there exists LTL formulas φ_n such that

- $|\varphi_n| = \mathcal{O}(\text{poly}(n))$
- each NBA for φ_n has at least 2^n states

LT-properties that have no “small” NBA

LTLMC3.2-69

consider the following family of LT-properties $(E_n)_{n \geq 1}$:

$$E_n = \left\{ \begin{array}{l} \text{set of all infinite words over } 2^{AP} \text{ of the form} \\ A_1 A_2 A_3 \dots A_n A_1 A_2 A_3 \dots A_n B_1 B_2 B_3 B_4 \dots \end{array} \right.$$

LT-properties that have no “small” NBA

consider the following family of LT-properties $(E_n)_{n \geq 1}$:

$$E_n = \left\{ \begin{array}{l} \text{set of all infinite words over } 2^{AP} \text{ of the form} \\ \underbrace{A_1 A_2 A_3 \dots A_n A_1 A_2 A_3 \dots A_n}_{= \textcolor{blue}{xx}} \underbrace{B_1 B_2 B_3 B_4 \dots}_{\in (2^{AP})^\omega} \\ \text{for some } \textcolor{blue}{x} \in (2^{AP})^* \text{ of length } \textcolor{blue}{n} \quad \text{arbitrary} \end{array} \right.$$

consider the following family of LT-properties $(E_n)_{n \geq 1}$:

$$E_n = \left\{ \underbrace{A_1 A_2 A_3 \dots A_n A_1 A_2 A_3 \dots A_n}_{= xx \text{ for some } x \in (2^{AP})^* \text{ of length } n} \underbrace{B_1 B_2 B_3 B_4 \dots}_{\in (2^{AP})^\omega \text{ arbitrary}} \right\}$$

LTL formula φ_n with $Words(\varphi_n) = E_n$

consider the following family of LT-properties $(E_n)_{n \geq 1}$:

$$E_n = \left\{ \begin{array}{l} \text{set of all infinite words over } 2^{AP} \text{ of the form} \\ \underbrace{A_1 A_2 A_3 \dots A_n A_1 A_2 A_3 \dots A_n}_{= xx} \underbrace{B_1 B_2 B_3 B_4 \dots}_{\in (2^{AP})^\omega} \end{array} \right.$$

for some $x \in (2^{AP})^*$
of length n arbitrary

LTL formula φ_n with $Words(\varphi_n) = E_n$

$$\varphi_n = \bigwedge_{a \in AP} \bigwedge_{0 \leq i < n} (\bigcirc^i a \leftrightarrow \bigcirc^{i+n} a)$$

LT-properties that have no “small” NBA

consider the following family of LT-properties $(E_n)_{n \geq 1}$:

$$E_n = \left\{ \underbrace{A_1 A_2 A_3 \dots A_n A_1 A_2 A_3 \dots A_n}_{= \textcolor{blue}{xx} \text{ for some } \textcolor{blue}{x} \in (2^{AP})^* \text{ of length } n} \underbrace{B_1 B_2 B_3 B_4 \dots}_{\in (2^{AP})^\omega \text{ arbitrary}} \right\}$$

LTL formula φ_n with $\text{Words}(\varphi_n) = E_n$

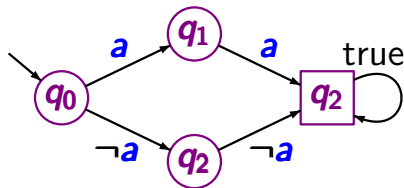
$$\varphi_n = \bigwedge_{a \in AP} \bigwedge_{0 \leq i < n} (\bigcirc^i \textcolor{blue}{a} \leftrightarrow \bigcirc^{i+n} \textcolor{blue}{a})$$

length
 $\mathcal{O}(\textcolor{purple}{poly}(n))$

$$E_1 = \left\{ \begin{array}{l} \text{set of all infinite words over } 2^{AP} \text{ of the form} \\ \textcolor{brown}{A} \textcolor{brown}{A} B_1 B_2 B_3 B_4 \dots \text{ where } \textcolor{brown}{A}, B_j \subseteq \textcolor{blue}{AP} \text{ for } j \geq 0 \end{array} \right.$$

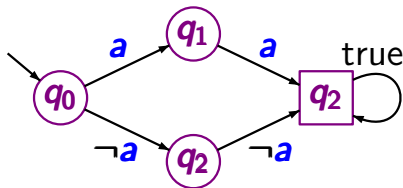
$$E_1 = \left\{ \begin{array}{l} \text{set of all infinite words over } 2^{AP} \text{ of the form} \\ \textcolor{brown}{A} \textcolor{brown}{A} B_1 B_2 B_3 B_4 \dots \text{ where } \textcolor{brown}{A}, B_j \subseteq \textcolor{blue}{AP} \text{ for } j \geq 0 \end{array} \right.$$

NBA for E_1 if $AP = \{a\}$:



$$E_1 = \left\{ \begin{array}{l} \text{set of all infinite words over } 2^{AP} \text{ of the form} \\ \textcolor{brown}{A} \textcolor{brown}{A} B_1 B_2 B_3 B_4 \dots \text{ where } \textcolor{brown}{A}, B_j \subseteq \textcolor{blue}{AP} \text{ for } j \geq 0 \end{array} \right.$$

NBA for E_1 if $AP = \{a\}$:



LTL-formula:

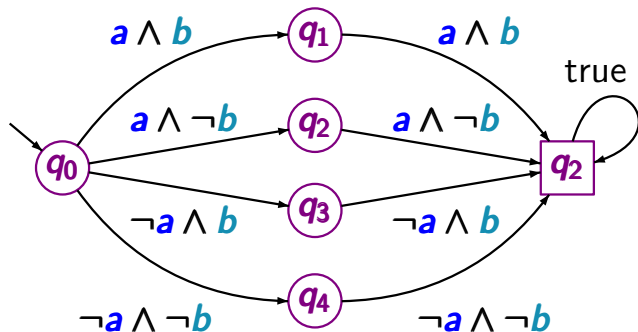
$$\textcolor{blue}{a} \leftrightarrow \bigcirc \textcolor{blue}{a}$$

LT-property E_n for $n=1$

LTLMC3.2-69A

$$E_1 = \left\{ \begin{array}{l} \text{set of all infinite words over } 2^{AP} \text{ of the form} \\ \mathbf{A} \mathbf{A} B_1 B_2 B_3 B_4 \dots \text{ where } \mathbf{A}, B_j \subseteq AP \text{ for } j \geq 0 \end{array} \right.$$

NBA for E_1 if $AP = \{a, b\}$:

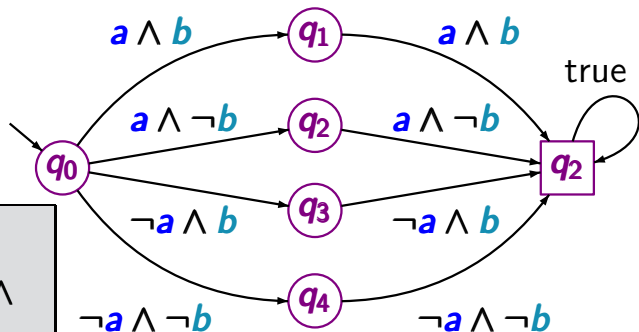


LT-property E_n for $n=1$

LTLMC3.2-69A

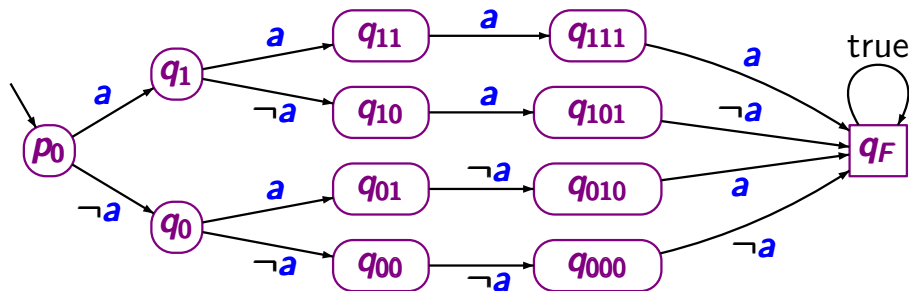
$$E_1 = \left\{ \begin{array}{l} \text{set of all infinite words over } 2^{AP} \text{ of the form} \\ \textcolor{brown}{A} \textcolor{brown}{A} B_1 B_2 B_3 B_4 \dots \text{ where } \textcolor{brown}{A}, B_j \subseteq \textcolor{blue}{AP} \text{ for } j \geq 0 \end{array} \right.$$

NBA for E_1 if $AP = \{a, b\}$:

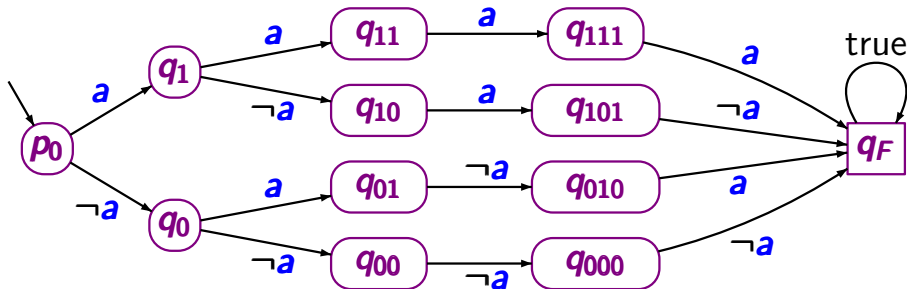


LTL-formula:

$$(a \leftrightarrow \bigcirc a) \wedge (b \leftrightarrow \bigcirc b)$$

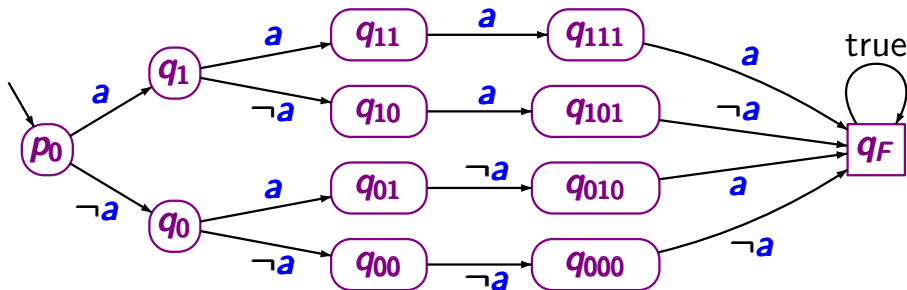


$$E_2 = \{A_1 A_2 A_1 A_2 \sigma : A_1, A_2 \subseteq AP, \sigma \in (2^{AP})^\omega\}$$

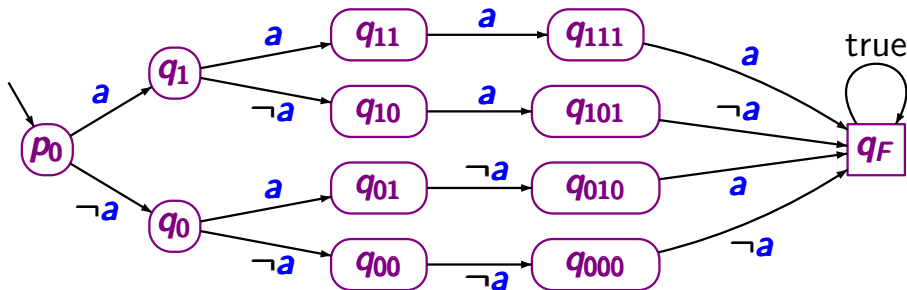


$$E_2 = \{A_1 A_2 A_1 A_2 \sigma : A_1, A_2 \subseteq AP, \sigma \in (2^{AP})^\omega\}$$

LTL-formula: $(a \leftrightarrow \bigcirc \bigcirc a) \wedge (\bigcirc a \leftrightarrow \bigcirc \bigcirc \bigcirc a)$

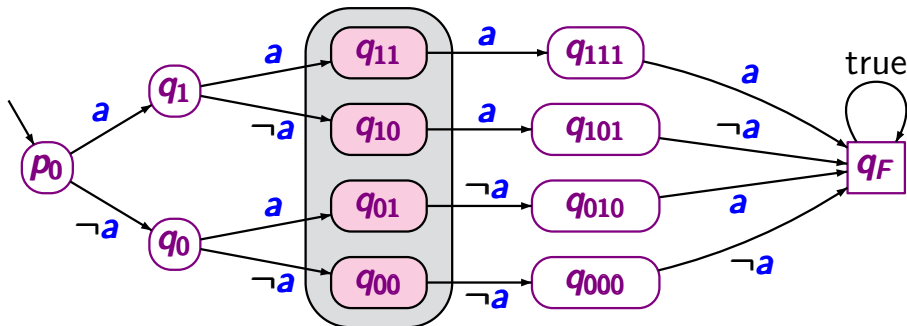


general case: each **NBA** for E_n has $\geq 2^n$ states



general case: each **NBA** for E_n has $\geq 2^n$ states

$$E_n = \text{Words}(\varphi_n) \text{ where } \varphi_n = \bigwedge_{a \in AP} \bigwedge_{0 \leq i < n} (\bigcirc^i a \leftrightarrow \bigcirc^{n+i} a)$$



general case: each **NBA** for E_n has $\geq 2^n$ states

$$E_n = \text{Words}(\varphi_n) \text{ where } \varphi_n = \bigwedge_{a \in AP} \bigwedge_{0 \leq i < n} (\bigcirc^i a \leftrightarrow \bigcirc^{n+i} a)$$