# Overview

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- regular safety properties:
  **NFA**-representation for the bad prefixes
- other regular LT properties:
  representation by $\omega$-automata, i.e.,
  acceptors for infinite words

# Overview

# Recall: definition of safety properties



Let $E$ be a LT property over $AP$, i.e., $E \subseteq (2^{AP})^\omega$.

$E$ is called a safety property if for all words

$$\sigma = A_0 A_1 A_2 \ldots \in (2^{AP})^\omega \setminus E$$

there exists a finite prefix $A_0 A_1 \ldots A_n$ of $\sigma$ such that *none* of the words $A_0 A_1 \ldots A_n B_{n+1} B_{n+2} B_{n+3} \ldots$ belongs to $E$, i.e.,

$$E \cap \{\sigma' \in (2^{AP})^\omega : A_0 \ldots A_n \text{ is a prefix of } \sigma'\} = \varnothing$$

Such words $A_0 A_1 \ldots A_n$ are called bad prefixes for $E$.

$$\mathit{BadPref} \stackrel{\text{def}}{=} \text{set of bad prefixes for } E \subseteq (2^{AP})^+$$

# Regular safety properties

IS2.5-REG-SAFE

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Let $E \subseteq \left(2^{AP}\right)^{\omega}$ be a safety property.

$E$ is called regular iff the language

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is regular.

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$$\mathit{BadPref} = \text{set of all bad prefixes for } E \subseteq \left(2^{AP}\right)^{+}$$

$$\mathit{BadPref} = \mathcal{L}(\mathcal{A}) \text{ for some NFA } \mathcal{A} \text{ over the alphabet } 2^{AP}$$

is regular.

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NFA $\mathcal{A} = (Q, \Sigma, \delta, Q_0, F)$

- $Q$ finite set of states
- $\Sigma$ alphabet
- $\delta : Q \times \Sigma \to 2^Q$ transition relation
- $Q_0 \subseteq Q$ set of initial states
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run for a word $A_0 A_1 \ldots A_{n-1} \in \Sigma^*$:

state sequence $\pi = q_0\, q_1 \ldots q_n$ where $q_0 \in Q_0$ and $q_{i+1} \in \delta(q_i, A_i)$ for $0 \leq i < n$

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run $\pi$ is called accepting if $q_n \in F$

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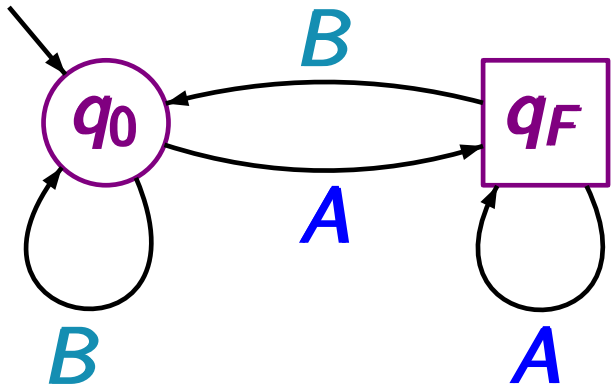
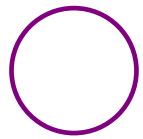
# Notations in pictures for NFA

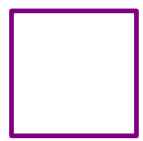





initial state



nonfinal state



final state

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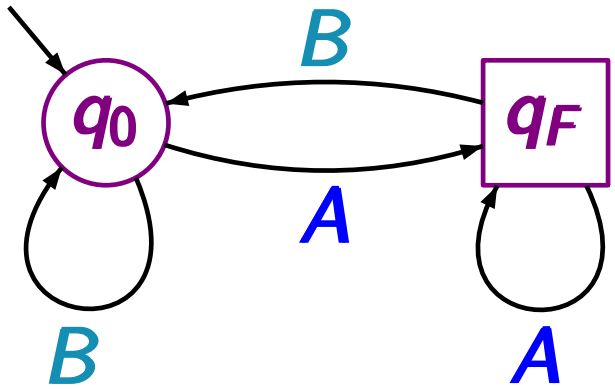
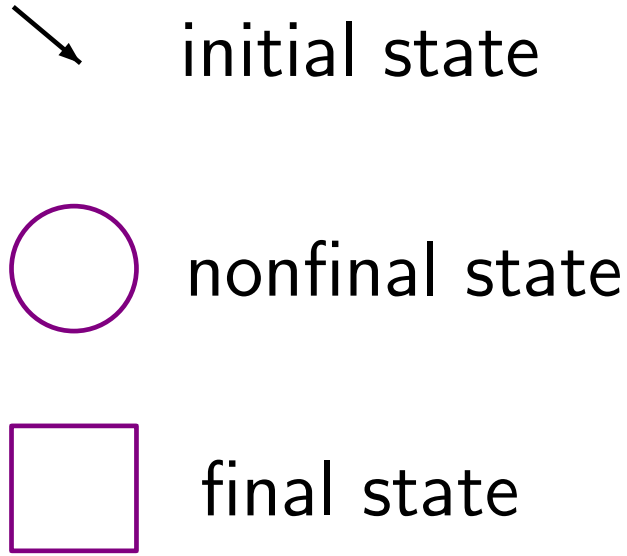



NFA $\mathcal{A}$ with state space $\{q_0, q_F\}$

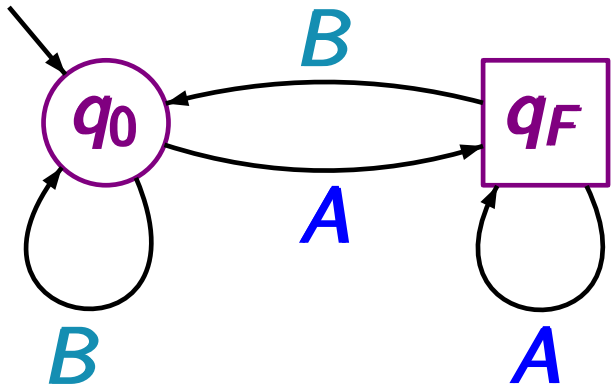
$q_0$ initial state

$q_F$ final state

alphabet $\Sigma = \{A, B\}$

# Notations in pictures for NFA



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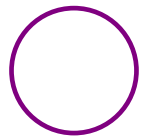

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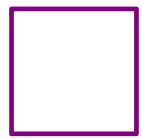

final state

accepted language $\mathcal{L}(\mathcal{A})$:

set of all finite words over $\{A, B\}$
ending with letter $A$

# Symbolic notations



for transitions in **NFA** over the alphabet $\Sigma = 2^{AP}$

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NFA $\mathcal{A} = (Q, \Sigma, \delta, Q_0, F)$ over the alphabet $\Sigma = 2^{AP}$

*symbolic notation* for the labels of transitions:

If $\Phi$ is a propositional formula over $AP$ then

$q \xrightarrow{\Phi} p$ stands for the set of transitions $q \xrightarrow{A} p$

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$$q \xrightarrow{true} p \;\widehat{=}\; \{q \xrightarrow{A} p : A \subseteq AP\}$$

# Regular safety properties



A safety property $E \subseteq (2^{AP})^{\omega}$ is called regular iff

$BadPref$ = set of all bad prefixes for $E$ $\subseteq (2^{AP})^{+}$

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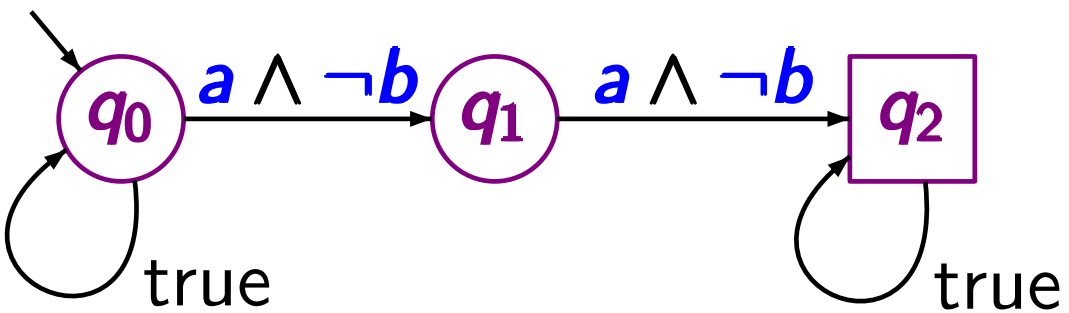


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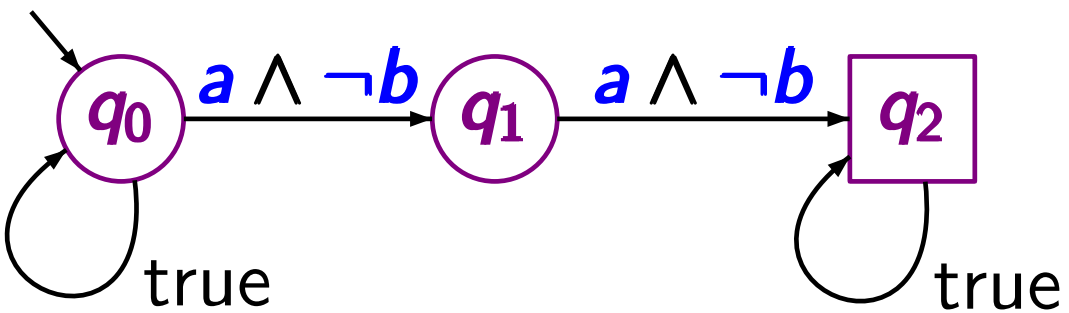


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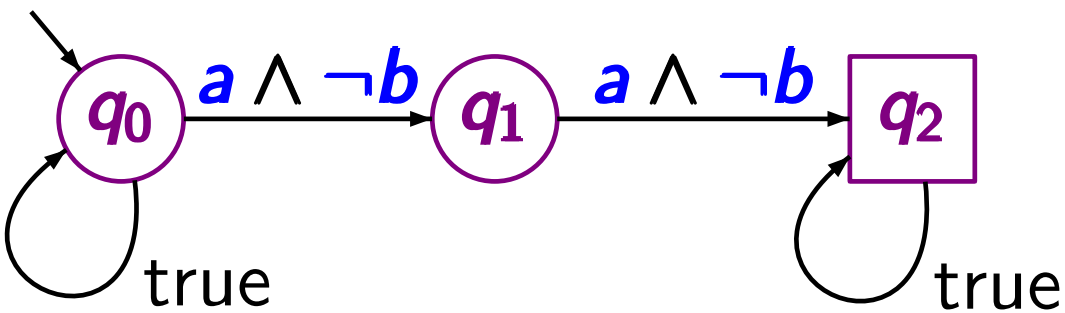


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safety property $E$: "$a \wedge \neg b$ never holds twice in a row"

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set of all infinite words $A_0\, A_1\, A_2 \ldots$ s.t. for all $i \geq 0$:

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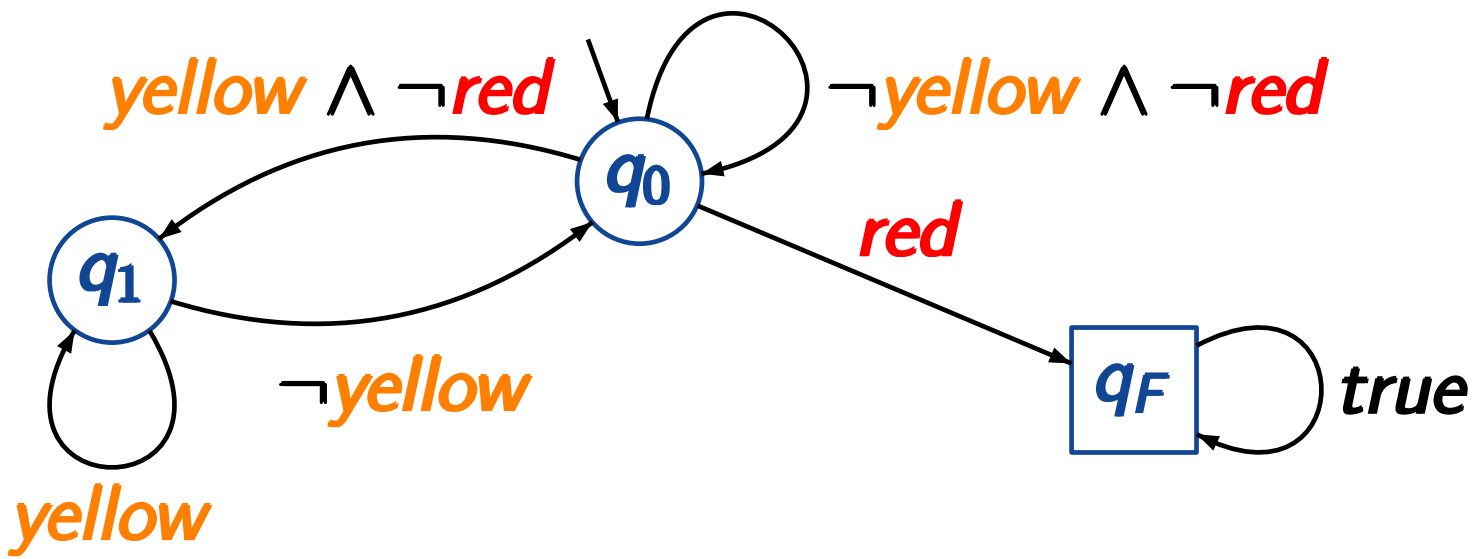
**DFA** for all (possibly non-minimal) bad prefixes



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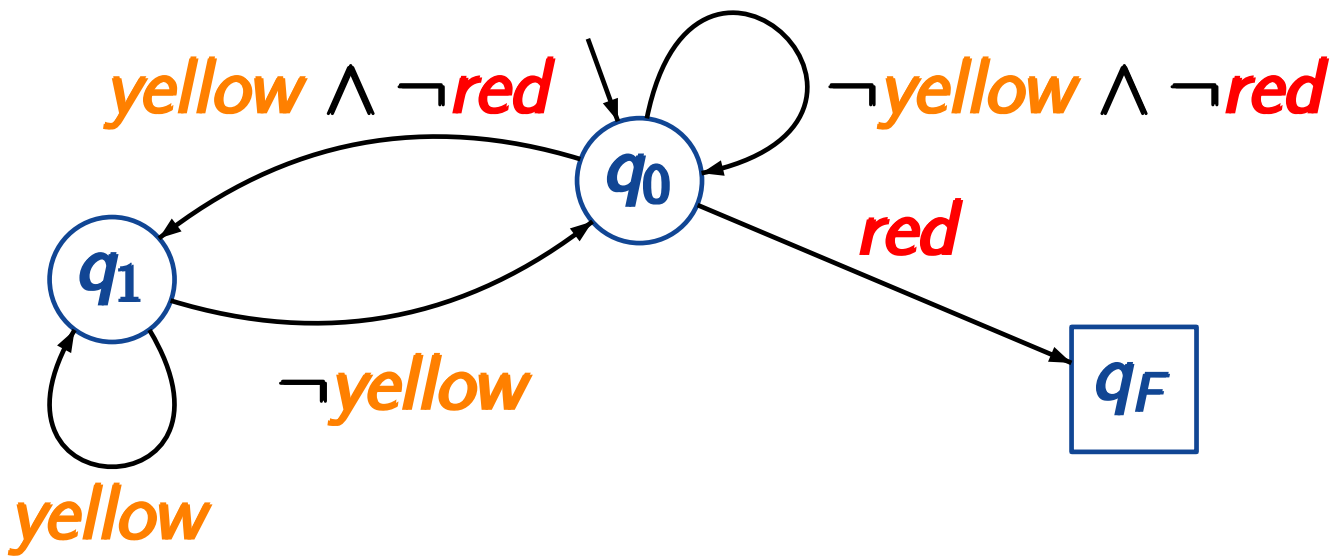
**DFA** for minimal bad prefixes



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A DFA $\mathcal{A}'$ for $MinBadPref$ is obtained from $\mathcal{A}$ by removing all outgoing transitions of final states.

# Correct or wrong?



Every invariant is regular.

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**correct.**

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Let $E$ be an invariant with invariant condition $\Phi$

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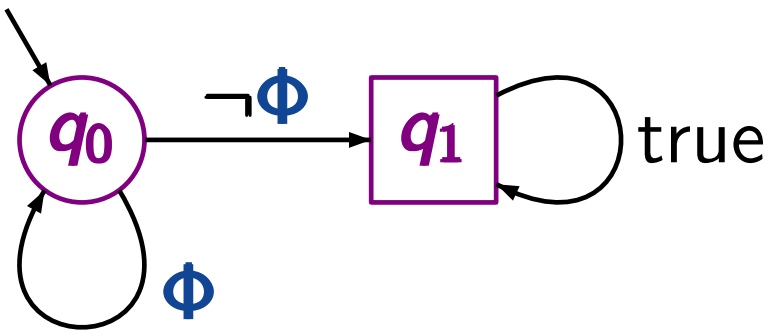


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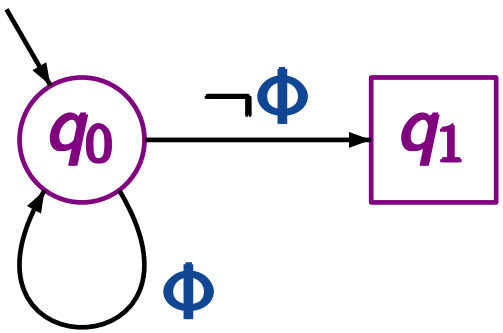


is a DFA for the language of all minimal bad prefixes

# Example: DFA for *MUTEX*



"The two processes are never simultaneously in their critical sections"

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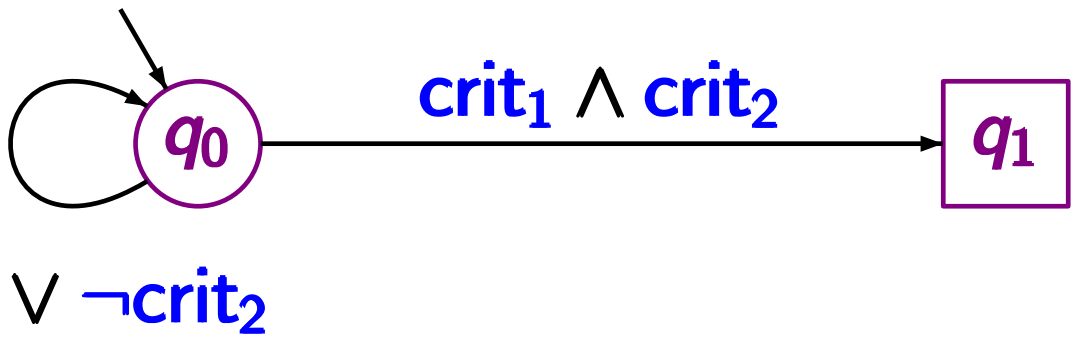
**DFA** for minimal bad prefixes over the alphabet $2^{AP}$ where $AP = \{\mathbf{crit_1}, \mathbf{crit_2}\}$



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- $E$ is a safety property, but
- the language of (minimal) bad prefixes is *not* regular

# Verifying regular safety properties

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*given:* finite TS $\mathcal{T}$
regular safety property $E$
(represented by an **NFA** for its bad prefixes)

*question:* does $\mathcal{T} \models E$ hold **?**

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*given:* finite TS $\mathcal{T}$
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*question:* does $\mathcal{T} \models E$ hold **?**

*method:* relies on an analogy between the tasks:

- checking language inclusion for **NFA**
- model checking regular safety properties

| language inclusion for NFA | verification of regular safety properties |
| --- | --- |
| $\mathcal{L}(\mathcal{A}_1) \subseteq \mathcal{L}(\mathcal{A}_2)$ ? | $\mathit{Traces}(\mathcal{T}) \subseteq E$ ? |
| | |
| | |

| language inclusion for NFA | verification of regular safety properties |
| --- | --- |
| $\mathcal{L}(\mathcal{A}_1) \subseteq \mathcal{L}(\mathcal{A}_2)$ ? | $\mathit{Traces}(\mathcal{T}) \subseteq E$ ? |
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| $\mathcal{L}(\mathcal{A}_1) \subseteq \mathcal{L}(\mathcal{A}_2)$ ? | $\mathit{Traces}(\mathcal{T}) \subseteq E$ ? |
| check whether $\mathcal{L}(\mathcal{A}_1) \cap (\Sigma^* \setminus \mathcal{L}(\mathcal{A}_2))$ is empty | check whether $\mathit{Traces}_{\mathit{fin}}(\mathcal{T}) \cap \mathit{BadPref}$ is empty |
| 1. complement $\mathcal{A}_2$, i.e., construct NFA $\overline{\mathcal{A}_2}$ with $\mathcal{L}(\overline{\mathcal{A}_2}) = \Sigma^* \setminus \mathcal{L}(\mathcal{A}_2)$<br>2. construct NFA $\mathcal{A}$ with $\mathcal{L}(\mathcal{A}) = \mathcal{L}(\mathcal{A}_1) \cap \mathcal{L}(\overline{\mathcal{A}_2})$<br>3. check if $\mathcal{L}(\mathcal{A}) = \varnothing$ | |

| language inclusion for NFA | verification of regular safety properties |
|---|---|
| $\mathcal{L}(\mathcal{A}_1) \subseteq \mathcal{L}(\mathcal{A}_2)$ ? | $\mathit{Traces}(\mathcal{T}) \subseteq E$ ? |
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| $\mathcal{L}(\mathcal{A}_1) \subseteq \mathcal{L}(\mathcal{A}_2)$ ? | $\mathit{Traces}(\mathcal{T}) \subseteq E$ ? |
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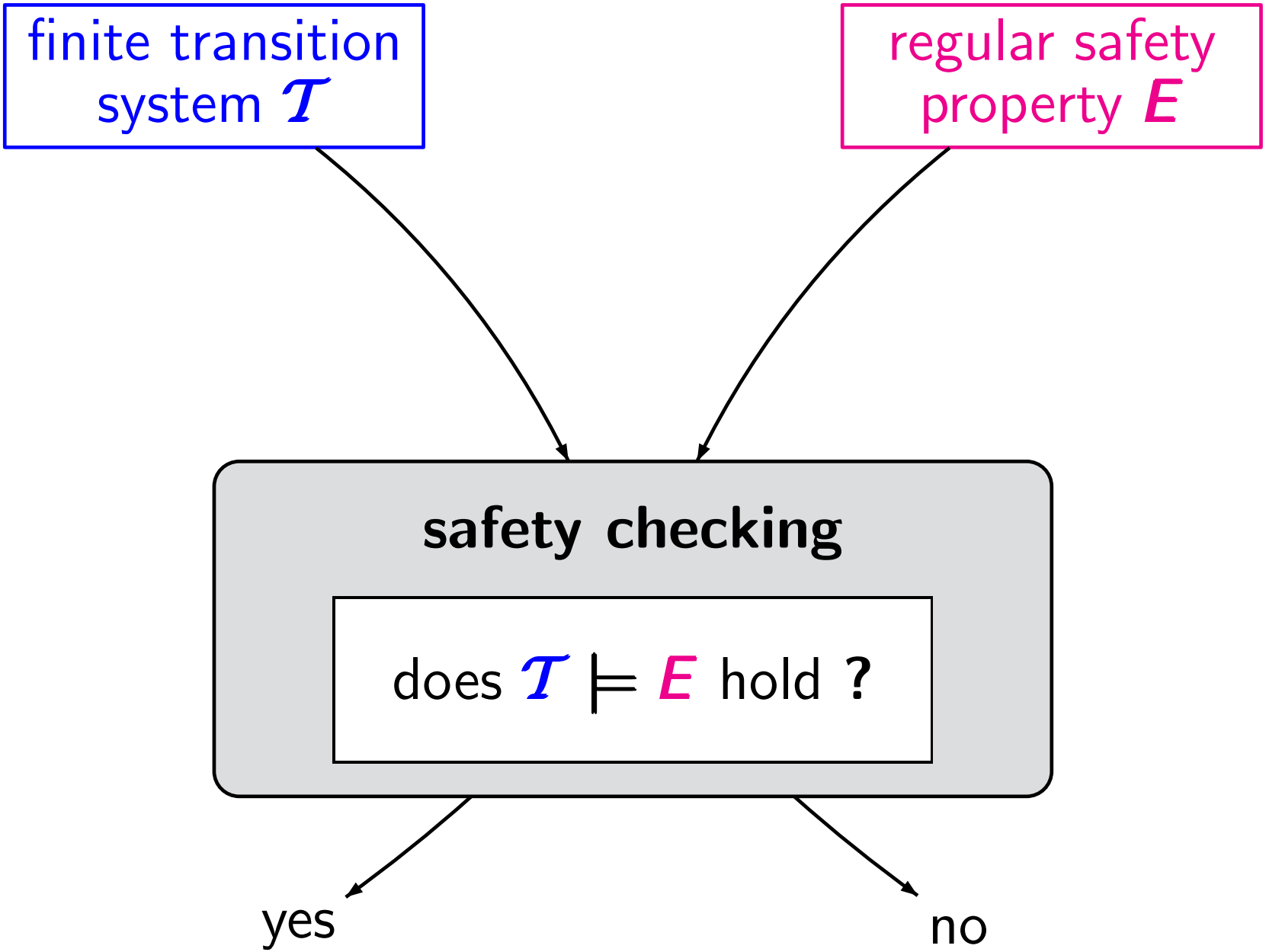
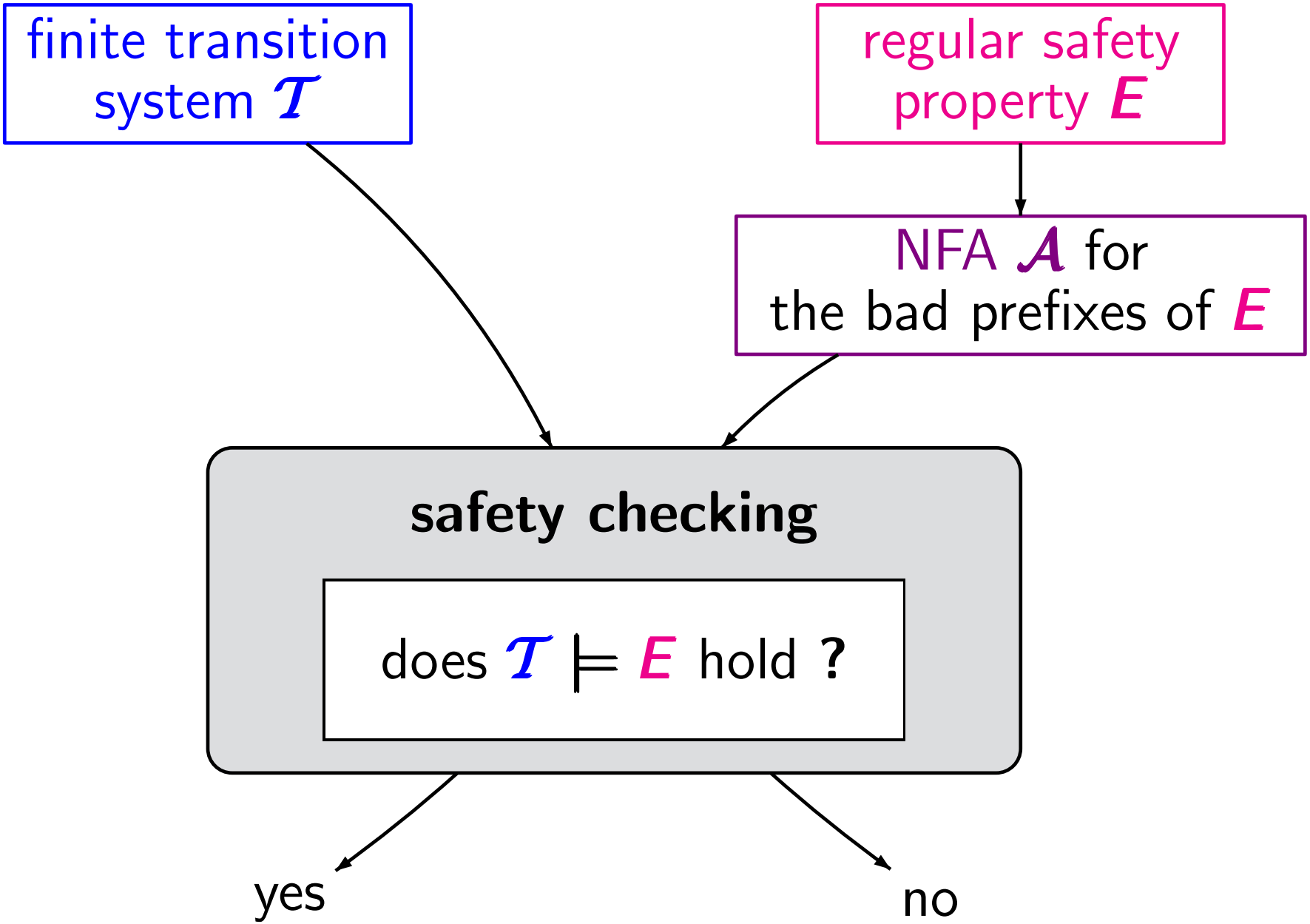
# Checking regular safety properties





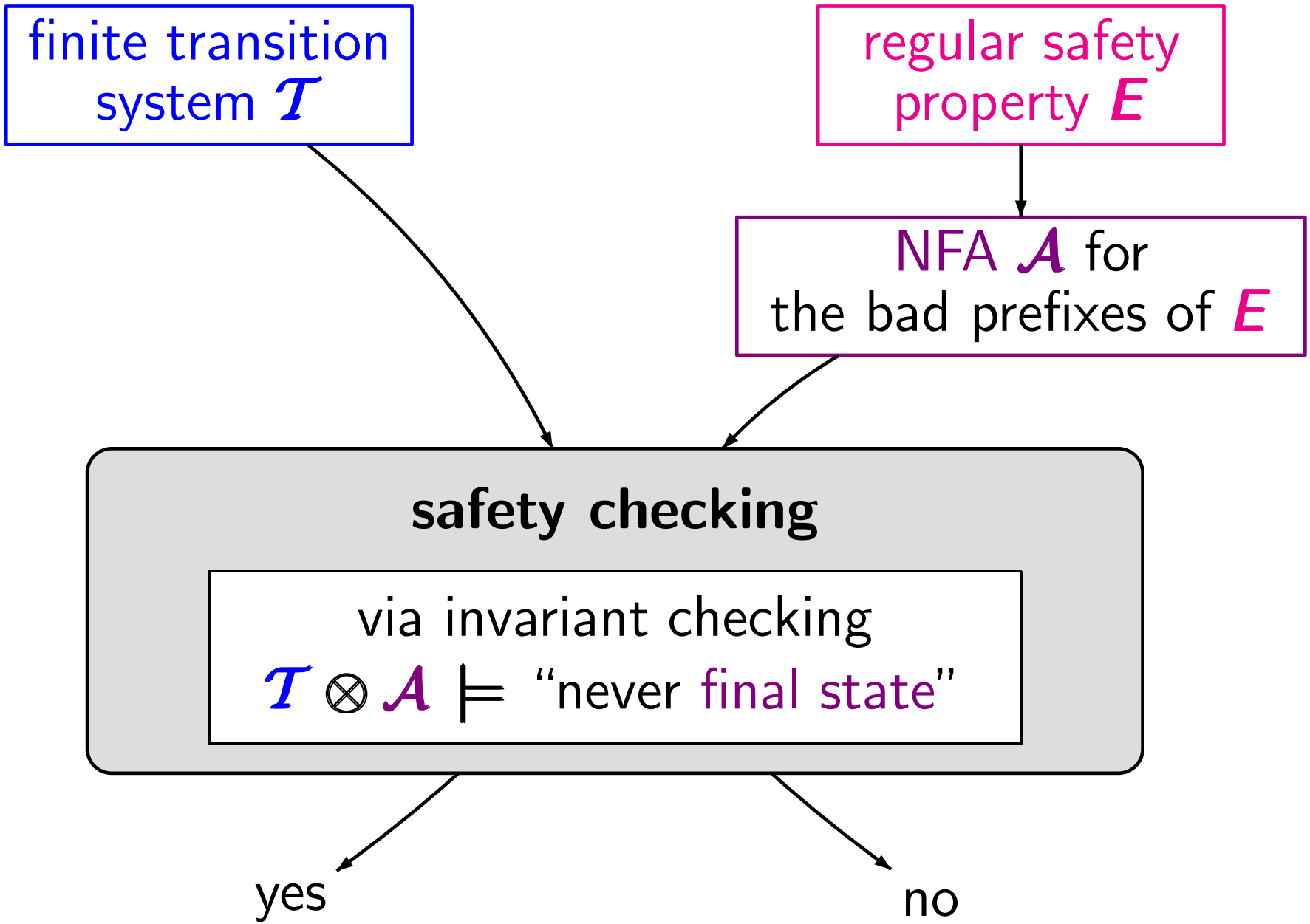
# Checking regular safety properties





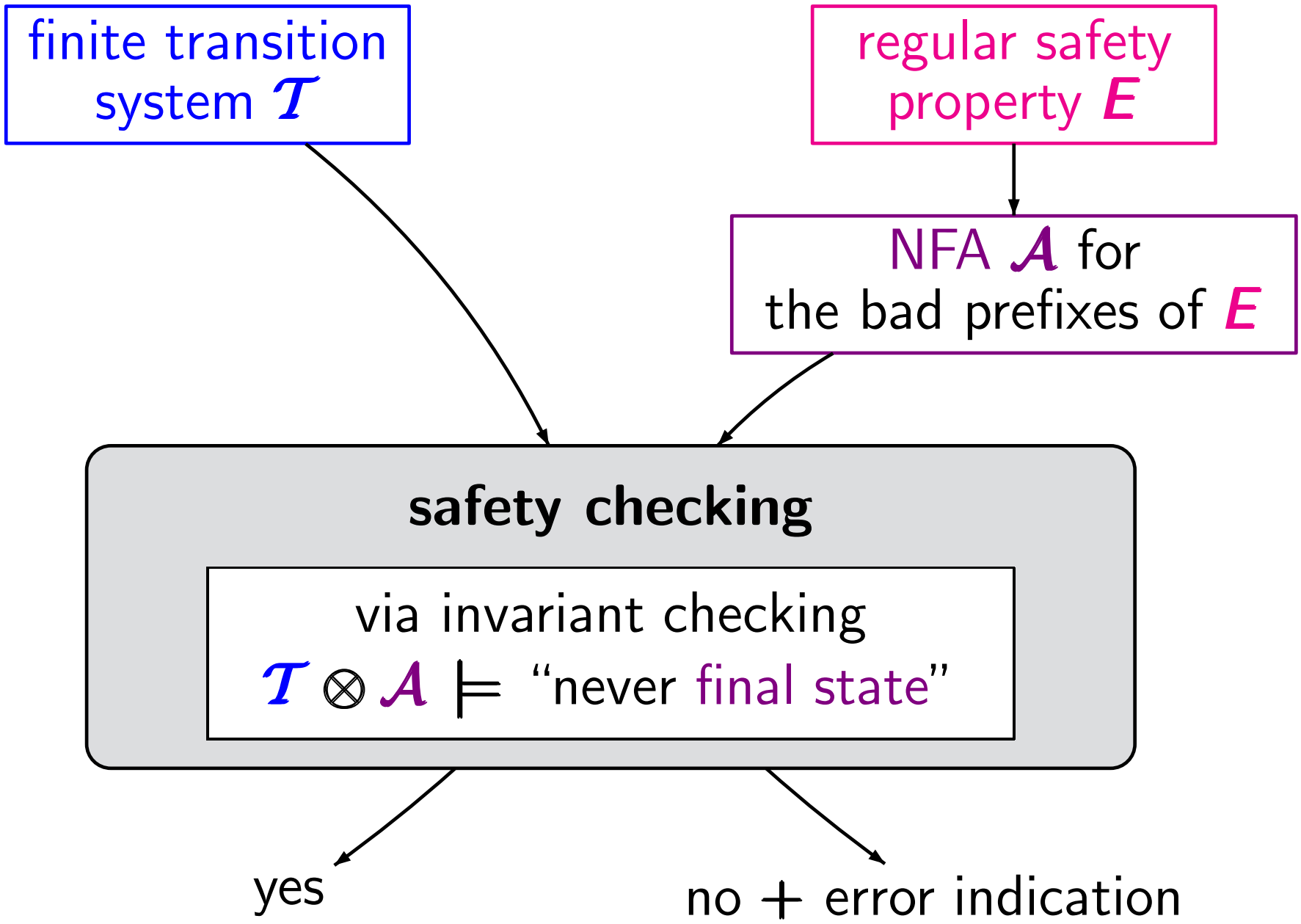
# Checking regular safety properties





# Checking regular safety properties





# Product of a TS and an NFA

# Product of a TS and an NFA



finite transition system
$\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$

NFA for bad prefixes
$\mathcal{A} = (Q, 2^{AP}, \delta, Q_0, F)$


path fragment $\widehat{\pi}$

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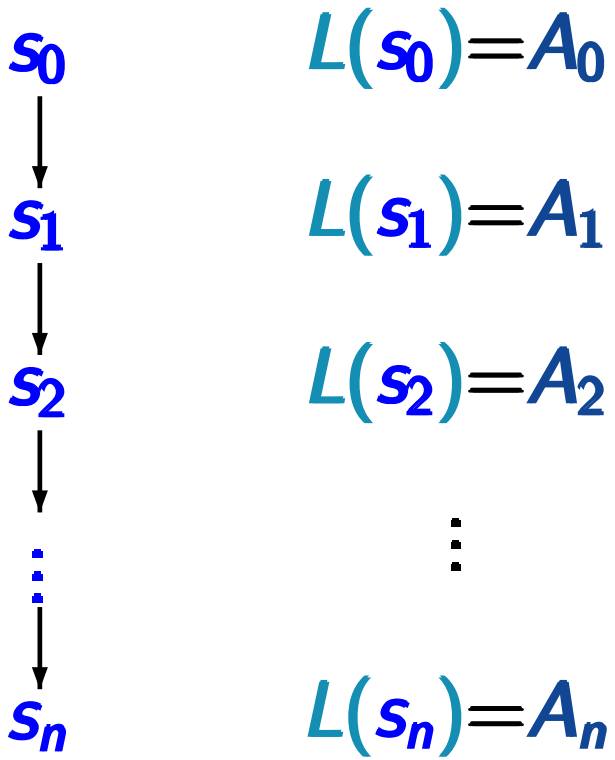


path fragment $\widehat{\pi}$

trace

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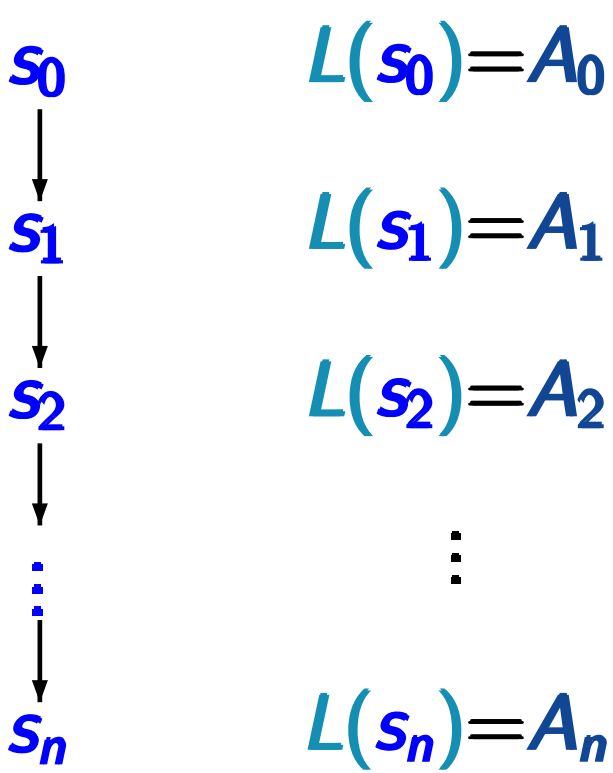
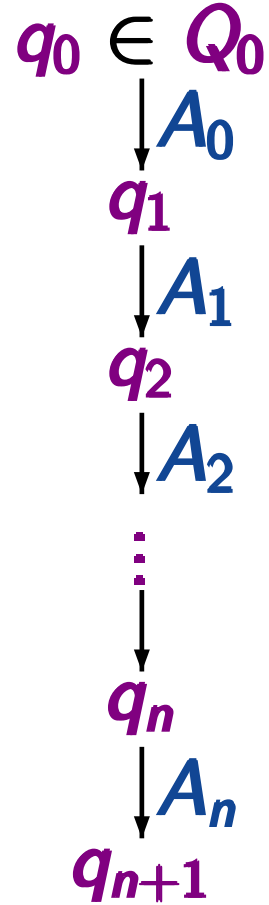



path fragment $\widehat{\pi}$

trace




run for $trace(\widehat{\pi})$

# Product of a TS and an NFA



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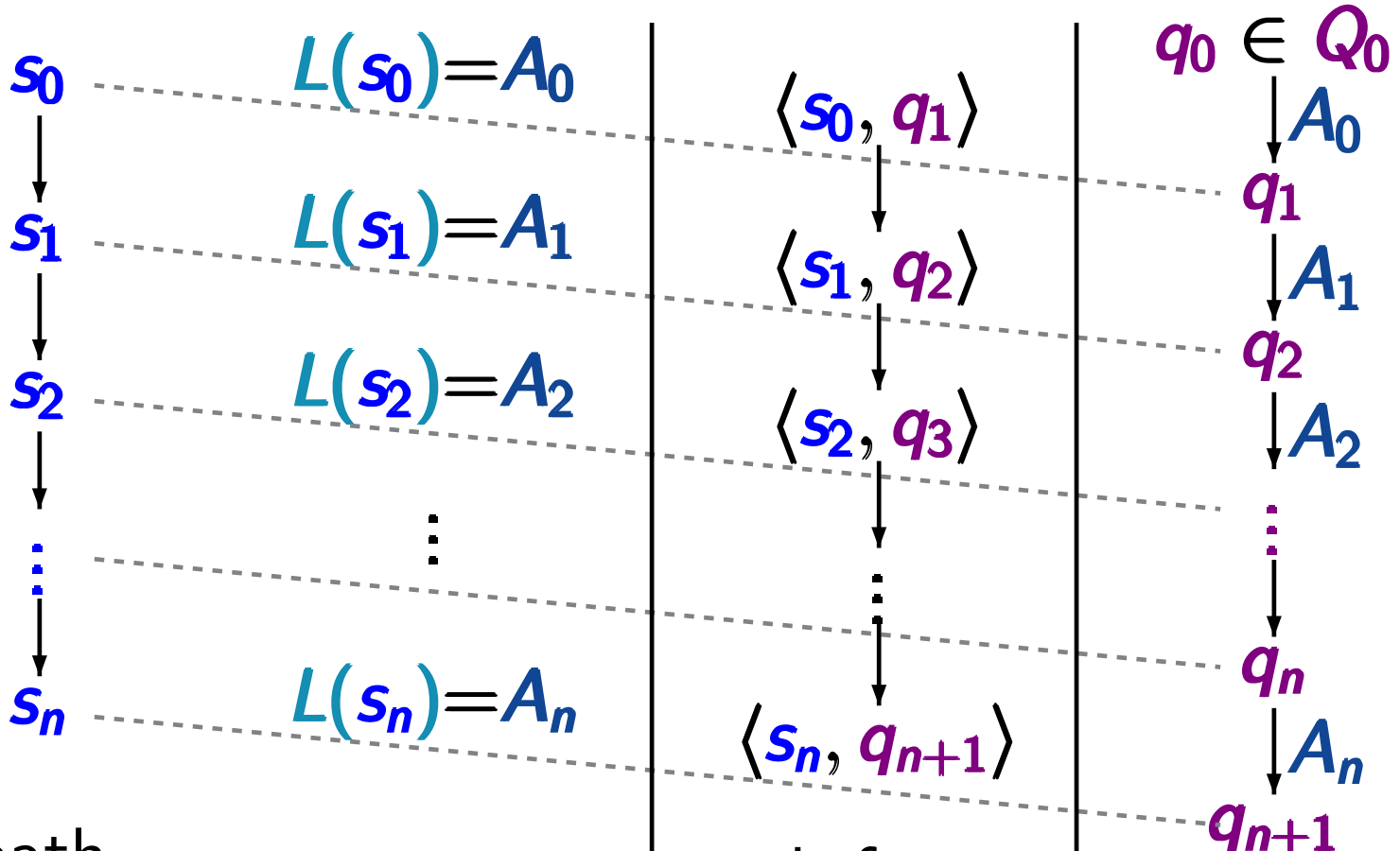


path fragment $\widehat{\pi}$

trace

path fragm. in product

run for $trace(\widehat{\pi})$

# Product transition system

# Product transition system



$$\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L) \quad \text{transition system}$$

$$\mathcal{A} = (Q, 2^{AP}, \delta, Q_0, F) \quad \text{NFA}$$

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# Product transition system



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$$\frac{s \xrightarrow{\alpha} s' \quad \wedge \quad q' \in \delta(q, L(s'))}{\langle s, q \rangle \xrightarrow{\alpha}' \langle s', q' \rangle}$$

# Product transition system



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initial states: $S_0' = \{ \langle s_0, q \rangle : s_0 \in S_0, q \in \delta(Q_0, L(s_0)) \}$

# Product transition system



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initial states: $S_0' = \{ \langle s_0, q \rangle : s_0 \in S_0, q \in \delta(Q_0, L(s_0)) \}$

for $P \subseteq Q$ and $A \subseteq AP$: $\delta(P, A) = \bigcup_{p \in P} \delta(p, A)$

# Product transition system



$\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$ transition system

$\mathcal{A} = (Q, 2^{AP}, \delta, Q_0, F)$ NFA

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set of atomic propositions: $AP' = Q$

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$$\frac{s \xrightarrow{\alpha} s' \quad \wedge \quad q' \in \delta(q, L(s'))}{\langle s, q \rangle \xrightarrow{\alpha}' \langle s', q' \rangle}$$

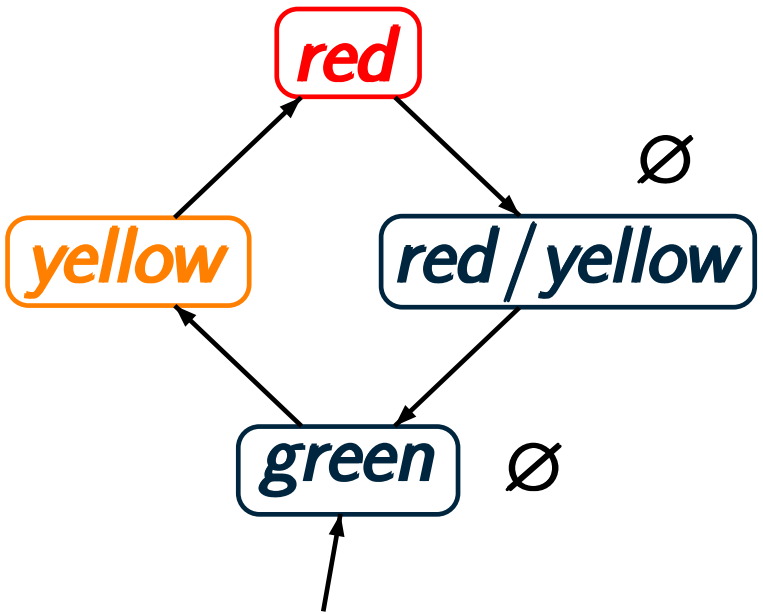
initial states: $S'_0 = \{ \langle s_0, q \rangle : s_0 \in S_0, q \in \delta(Q_0, L(s_0)) \}$

set of atomic propositions: $AP' = Q$

labeling function: $L'(\langle s, q \rangle) = \{q\}$

# Example: product-TS

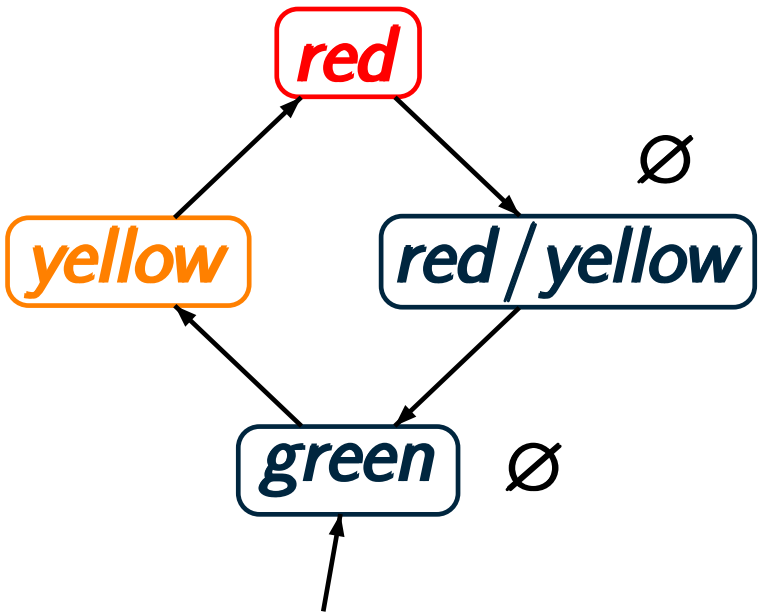





transition system $\mathcal{T}$ over
$AP = \{red, yellow\}$

# Example: product-TS



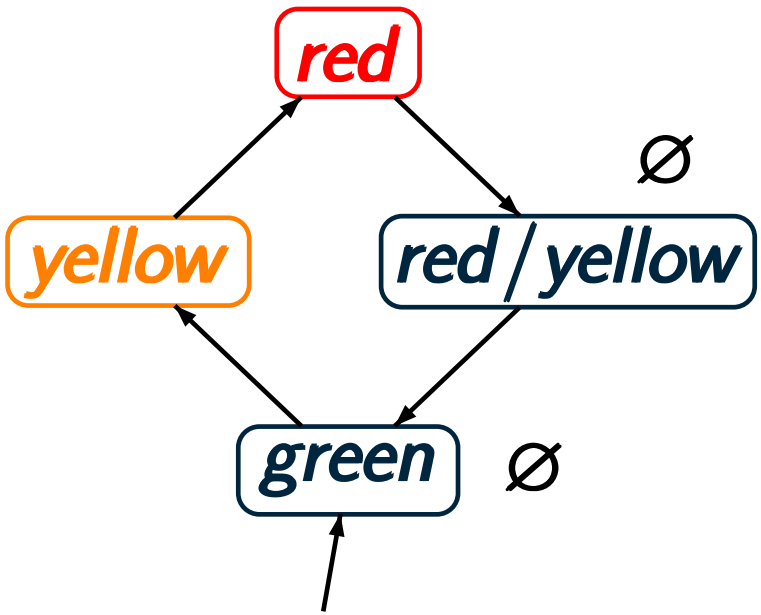


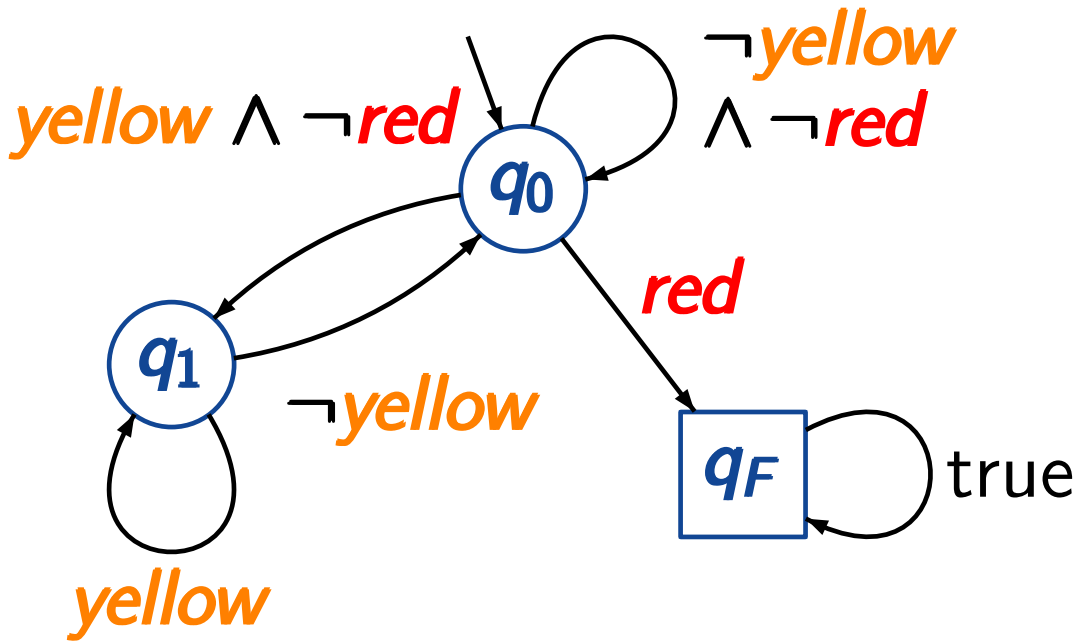

transition system $\mathcal{T}$ over
$AP = \{red, yellow\}$

$\mathcal{T}$ satisfies the safety property $E$
*"every red phase is preceded by a yellow phase"*

# Example: product-TS





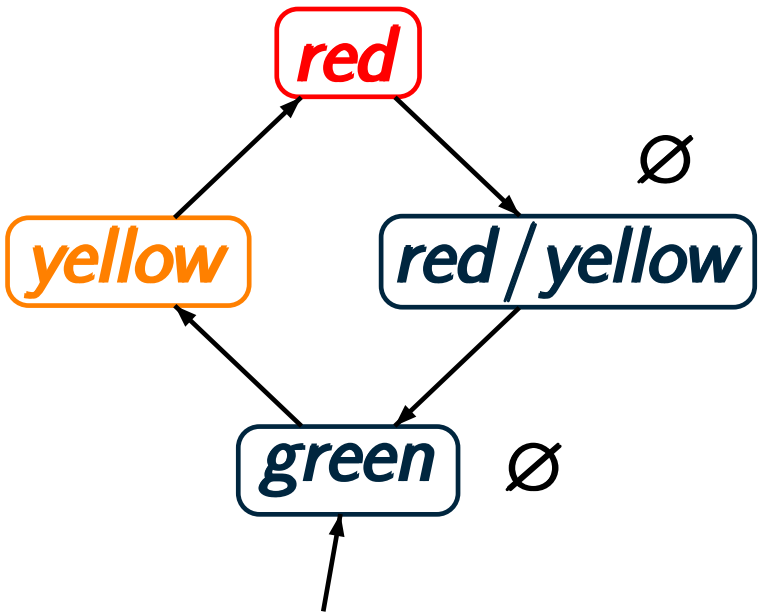

transition system $\mathcal{T}$ over
$AP = \{red, yellow\}$

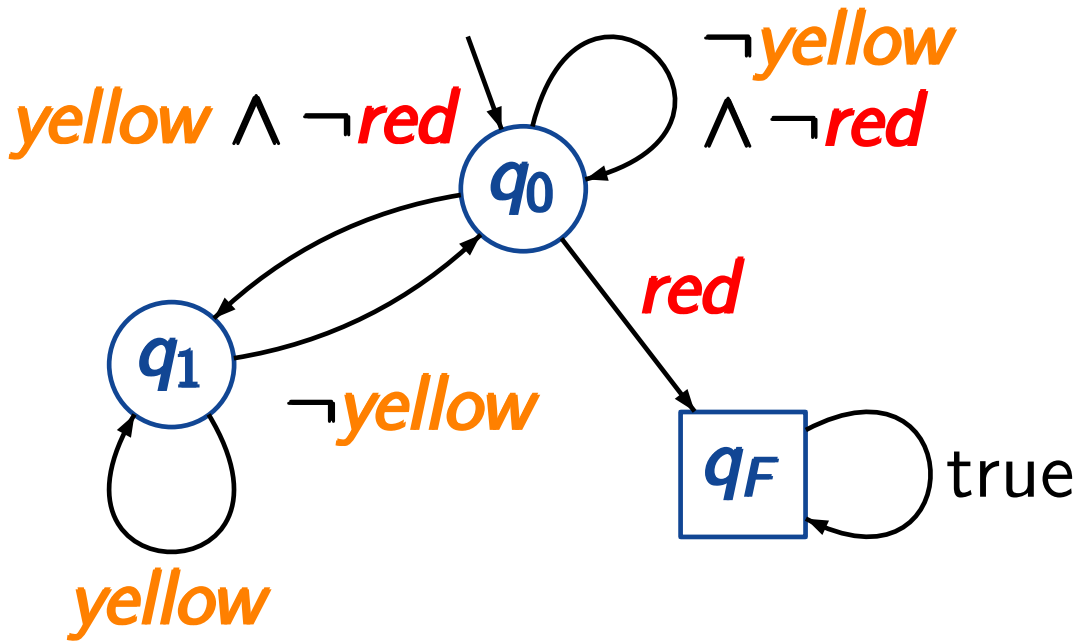
DFA $\mathcal{A}$ for the
bad prefixes for $E$

$\mathcal{T}$ satisfies the safety property $E$
*"every red phase is preceded by a yellow phase"*

# Example: product-TS






green $q_0$

red / yellow $q_0$

yellow $q_1$

red $q_0$

...

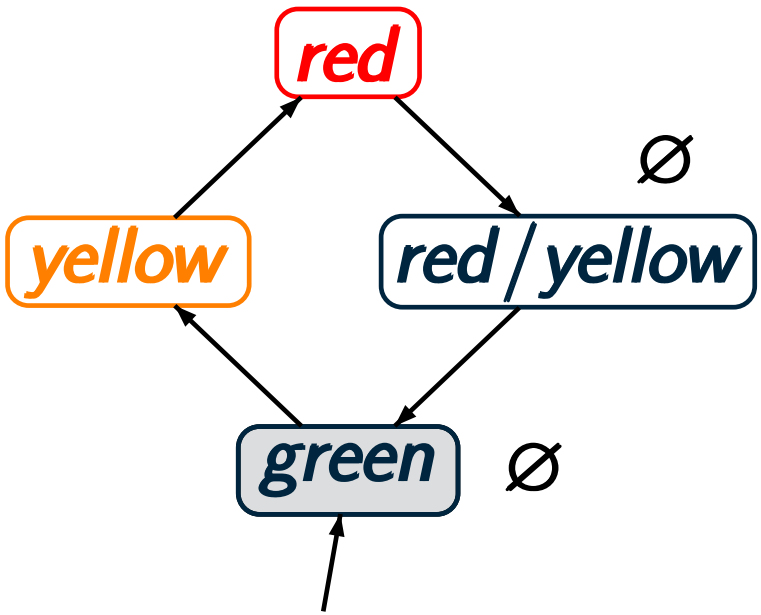
product-TS

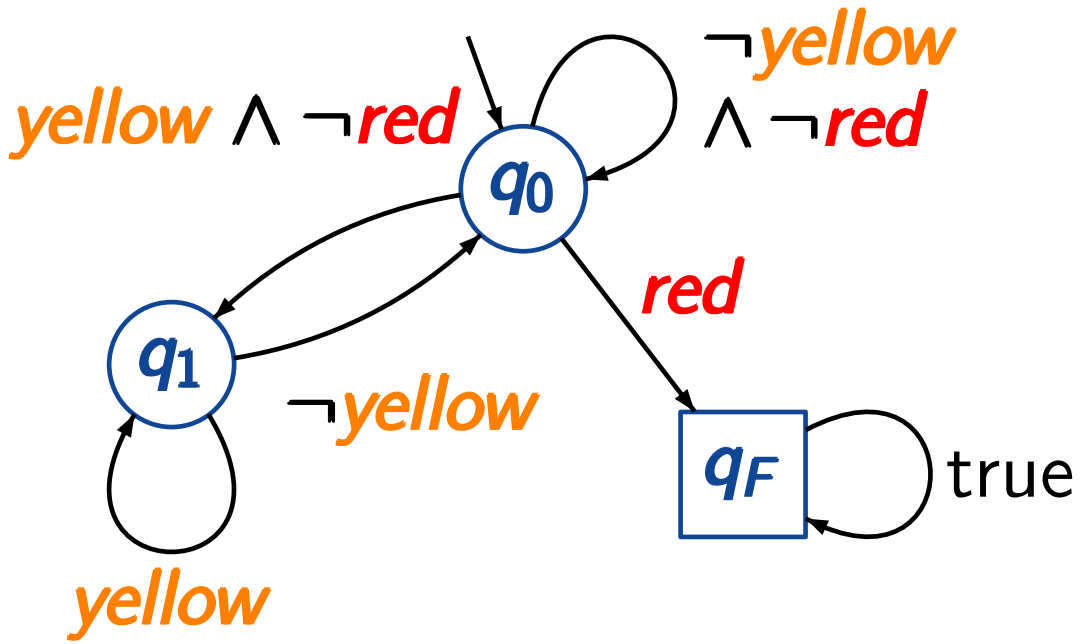
$\mathcal{T} \otimes \mathcal{A}$

$(4 * 3 = 12$ states$)$

# Example: product-TS






green $q_0$

red / yellow $q_0$

yellow $q_1$

red $q_0$

...

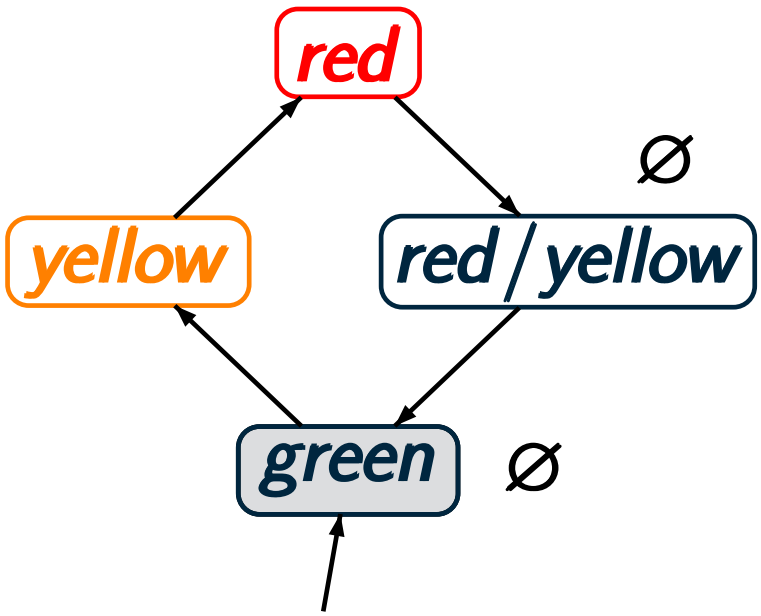
initial state

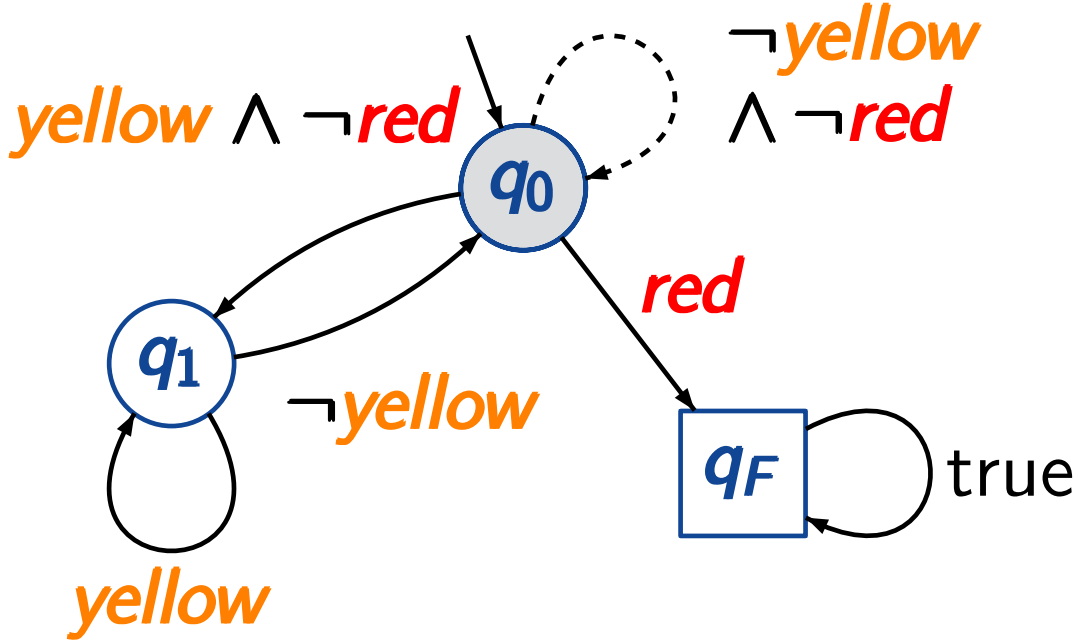
$\langle green, \delta(q_0, \varnothing) \rangle$

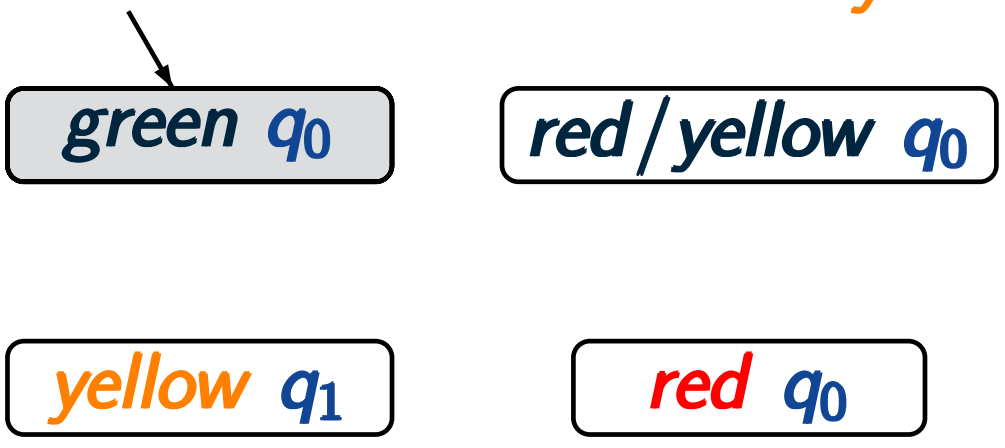
$L(green) = \varnothing$

# Example: product-TS





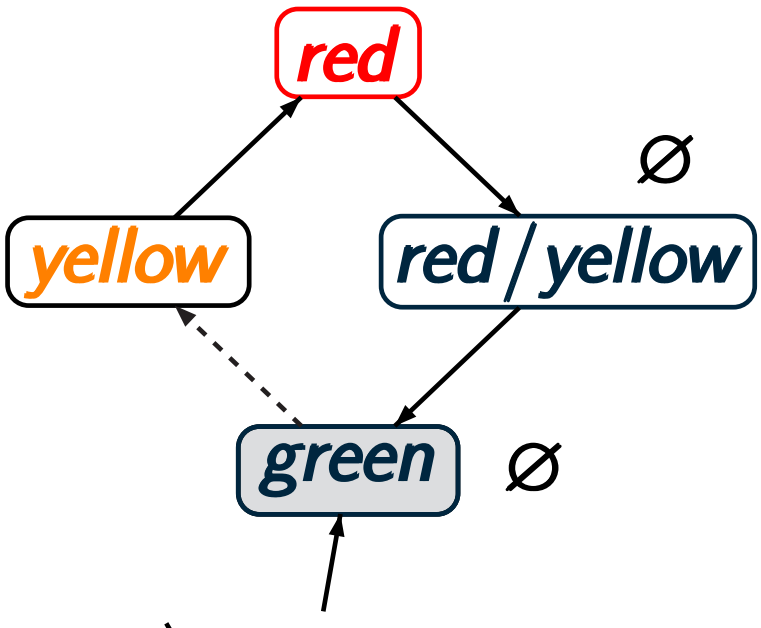



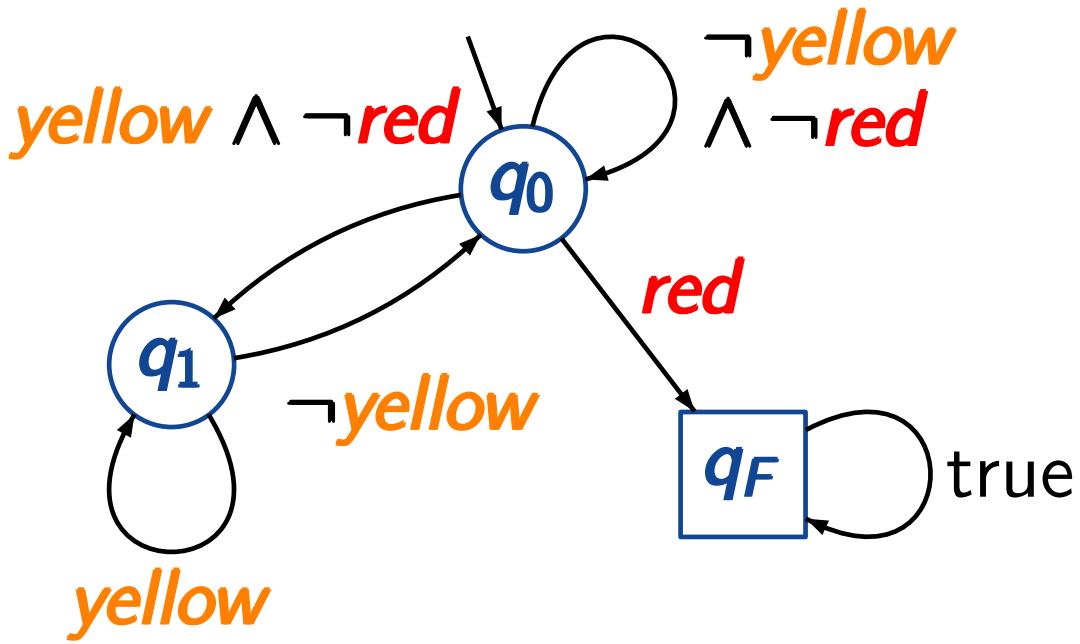

initial state

$$\langle \textit{green}, \underbrace{\delta(q_0, \varnothing)}_{= q_0} \rangle$$

# Example: product-TS






green $q_0$

red / yellow $q_0$

yellow $q_1$

red $q_0$

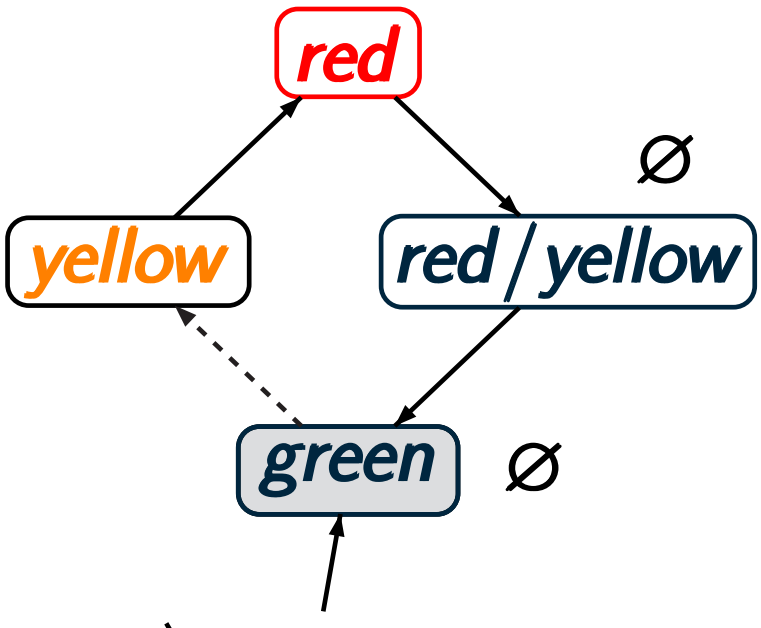
...

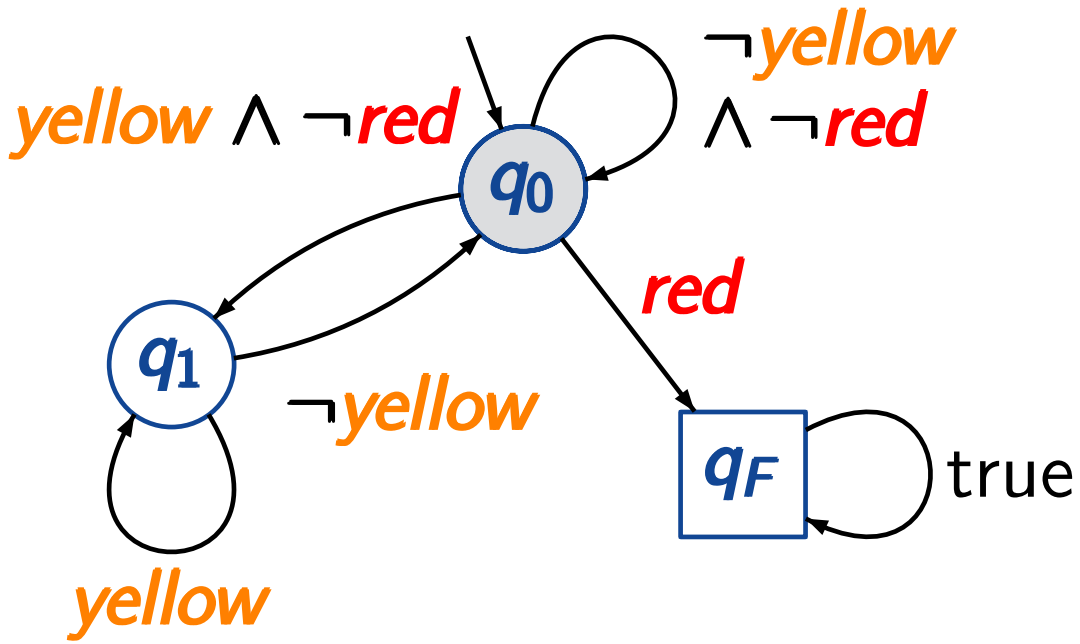
lifting the transition

*green* ⟶ *yellow*

# Example: product-TS






green $q_0$

red / yellow $q_0$

yellow $q_1$

red $q_0$

...

lifting the transition

*green* ⟶ *yellow*

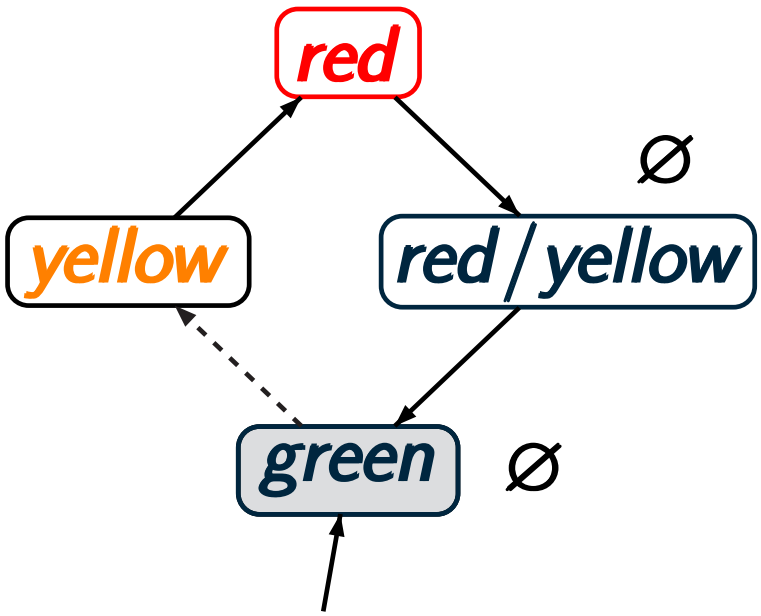
$\langle green, q_0 \rangle$

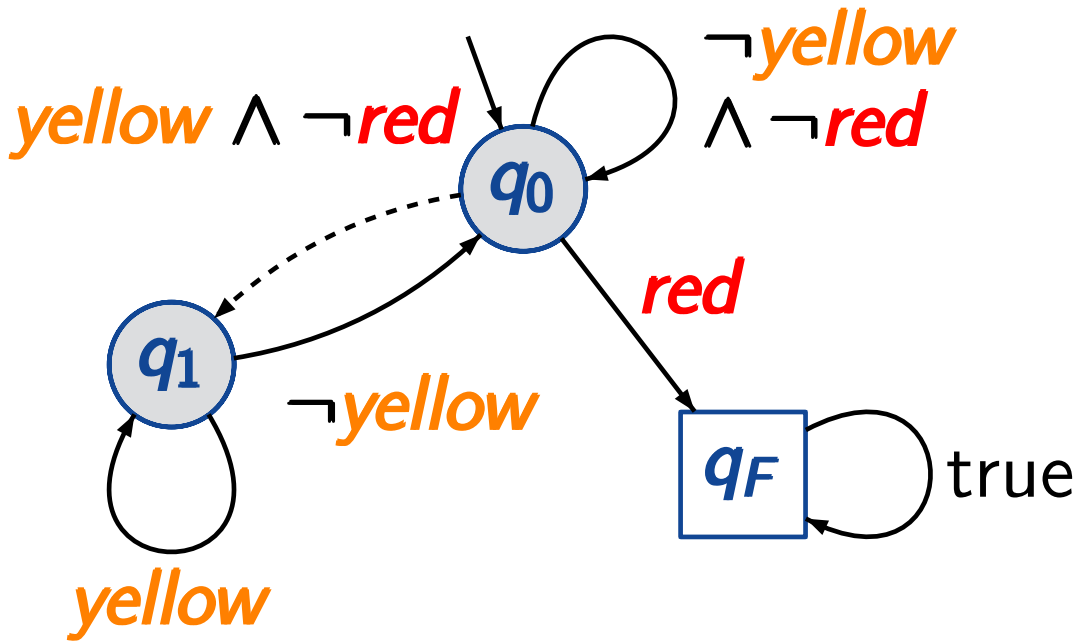
↓

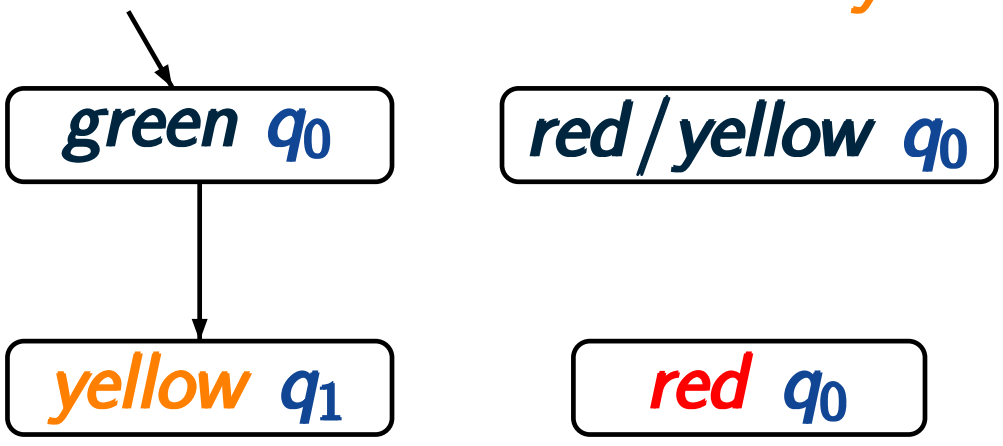
$\langle yellow, ? \rangle$

# Example: product-TS






lifting the transition

$$green \longrightarrow yellow$$

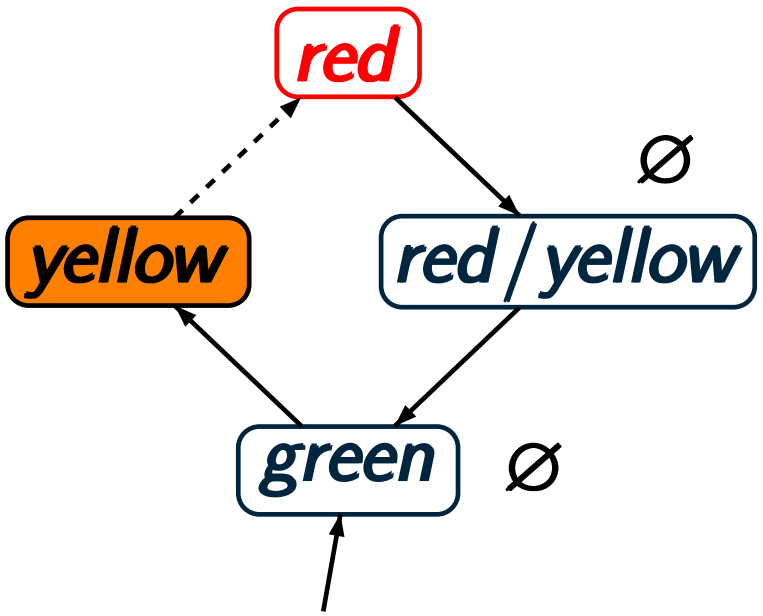
$$\langle green, q_0 \rangle$$

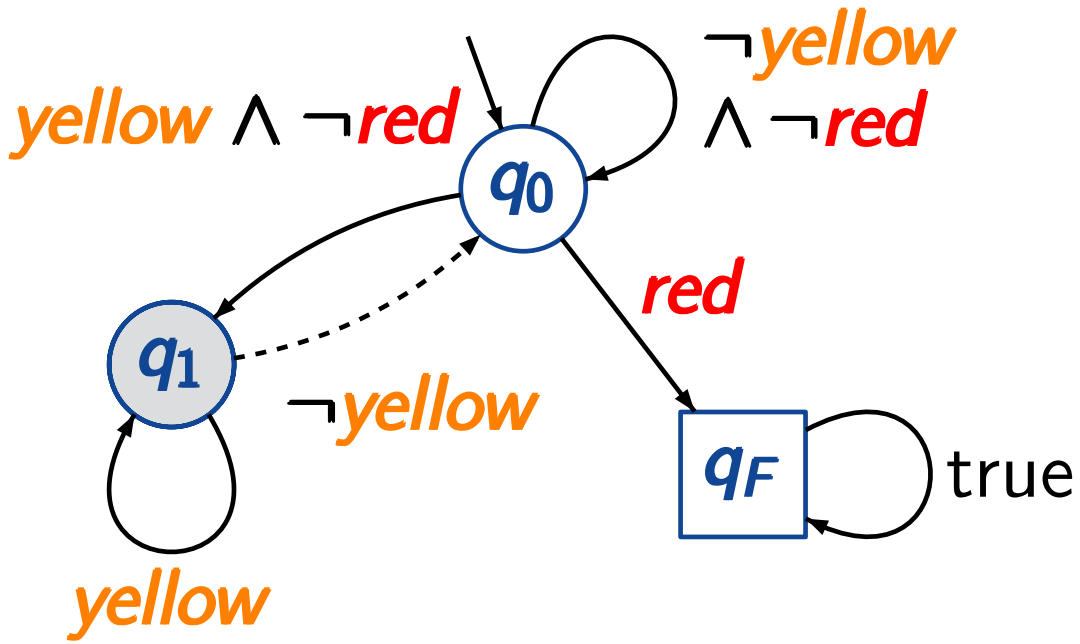
$$\downarrow$$

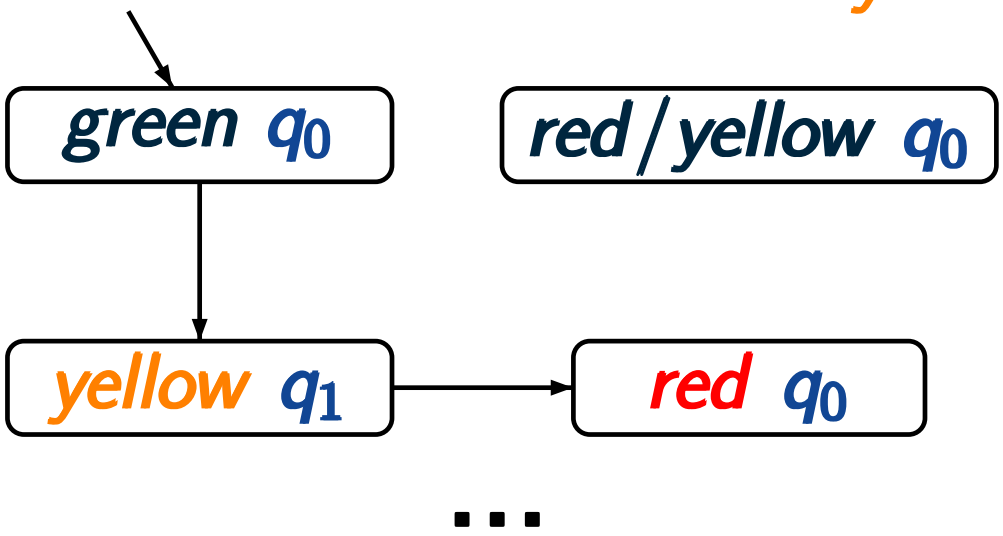
$$\langle yellow, \underbrace{\delta(q_0, \{yellow\})}_{= q_1} \rangle$$

# Example: product-TS






lifting the transition

$$\textit{yellow} \longrightarrow \textit{red}$$

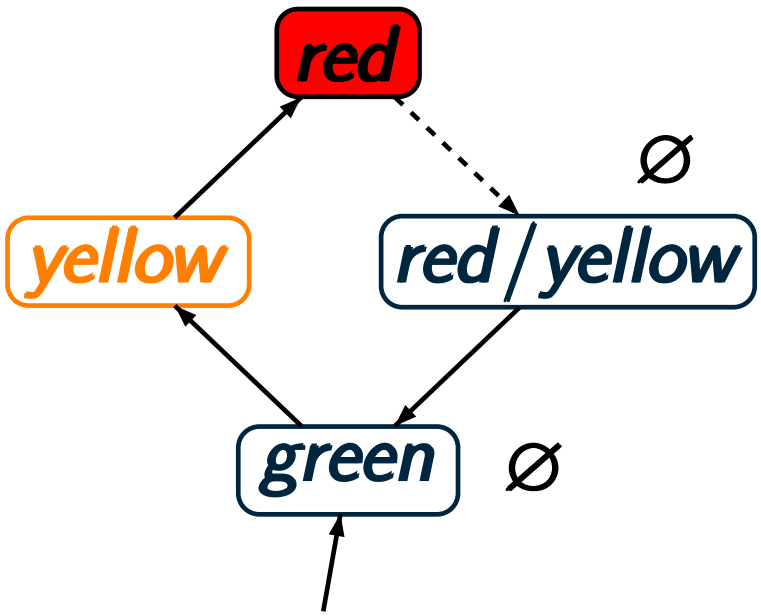
$$\langle \textit{yellow}, q_1 \rangle$$

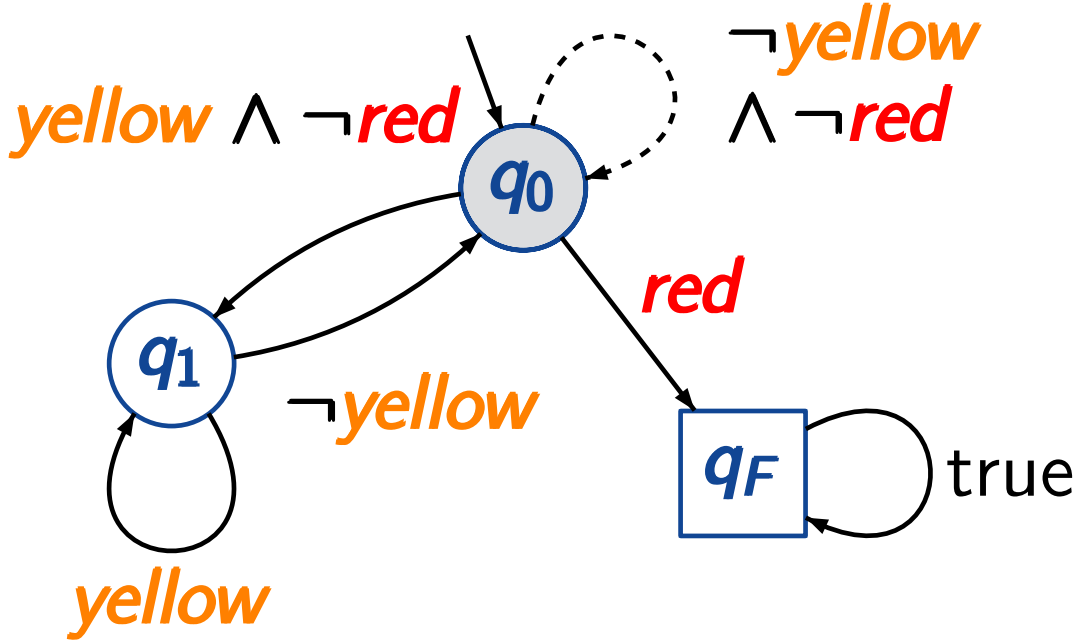
$$\downarrow$$

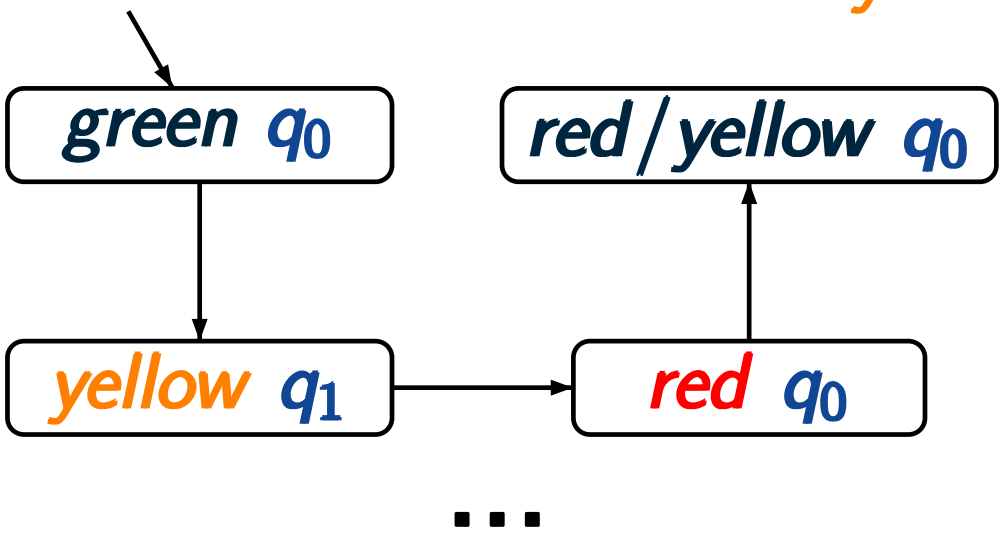
$$\langle \textit{red}, \underbrace{\delta(q_1, \{\textit{red}\})}_{= q_0} \rangle$$

# Example: product-TS






lifting the transition

$red \longrightarrow red/yellow$

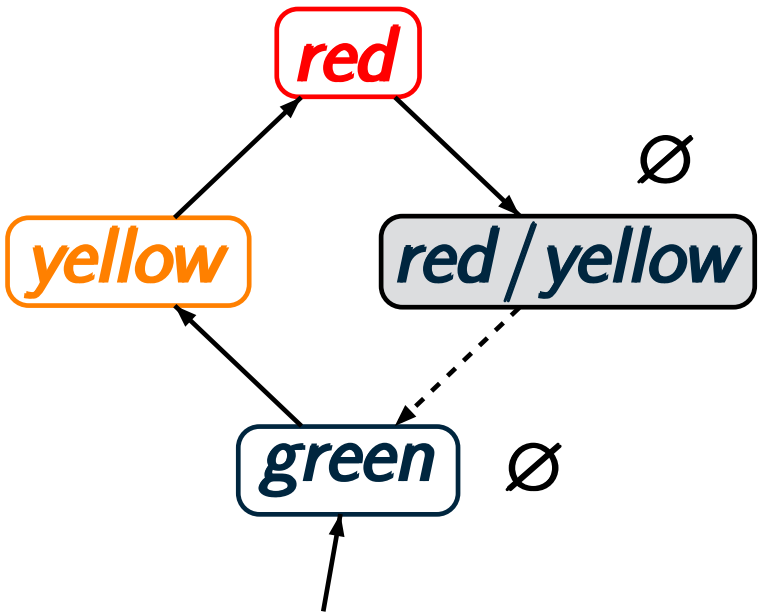
$\langle red, q_0 \rangle$

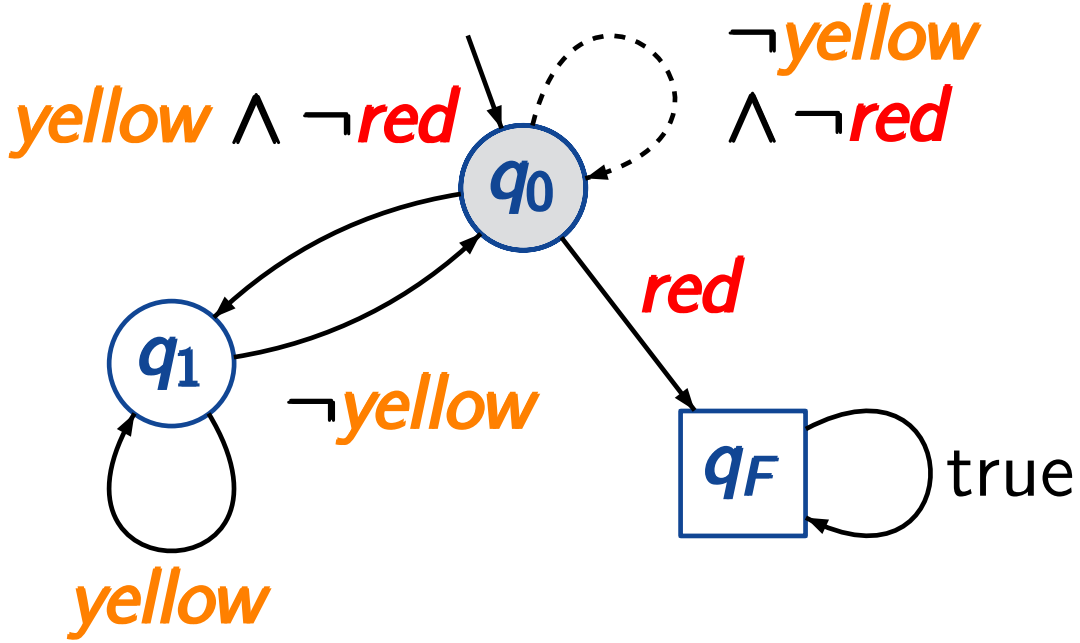
$\downarrow$

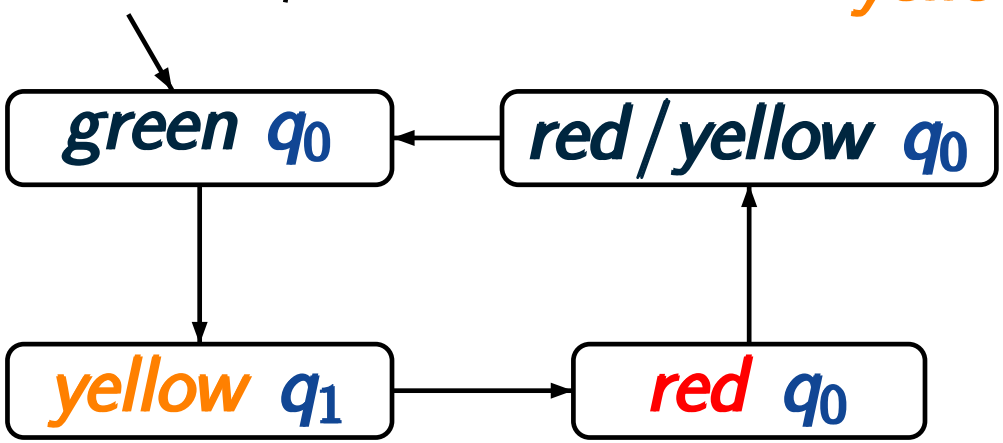
$\langle red/yellow, \underbrace{\delta(q_0, \varnothing)}_{= q_0} \rangle$

# Example: product-TS






lifting the transition

$$\textit{red}/\textit{yellow} \longrightarrow \textit{green}$$

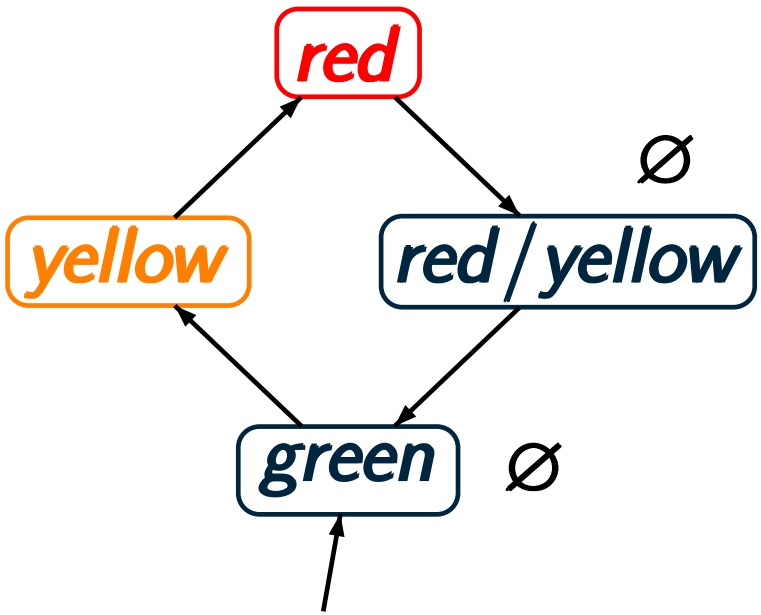
$$\langle \textit{red}/\textit{yellow}, q_0 \rangle$$

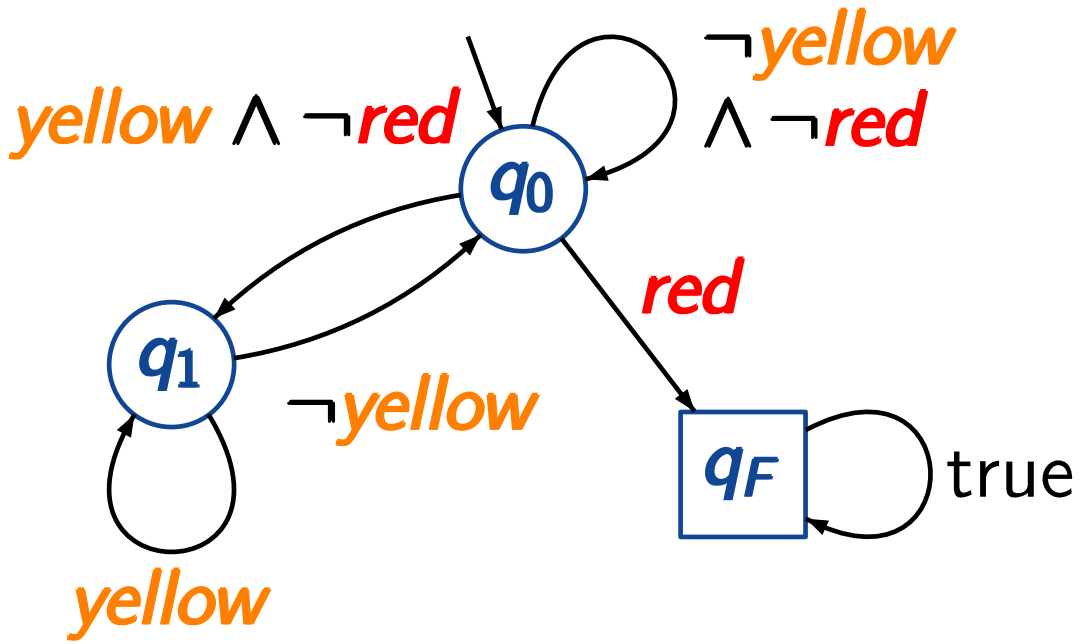
$$\downarrow$$

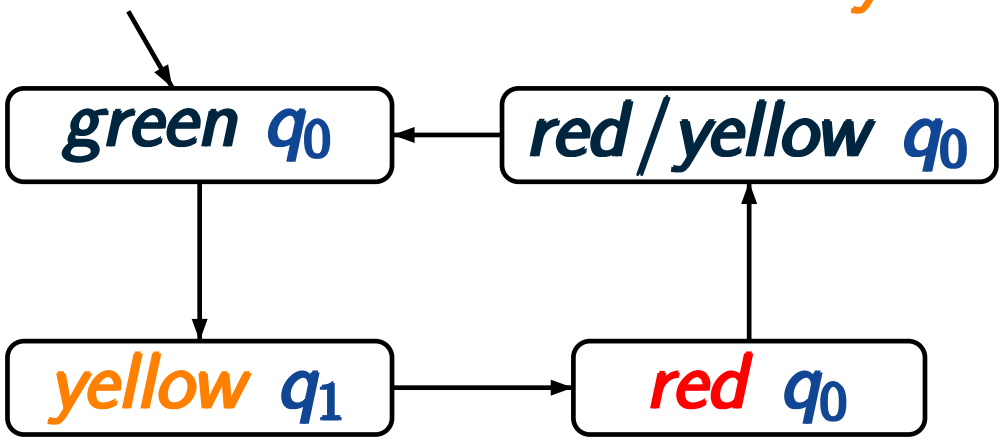
$$\langle \textit{green}, \underbrace{\delta(q_0, \varnothing)}_{= q_0} \rangle$$

# Example: product-TS






product-TS

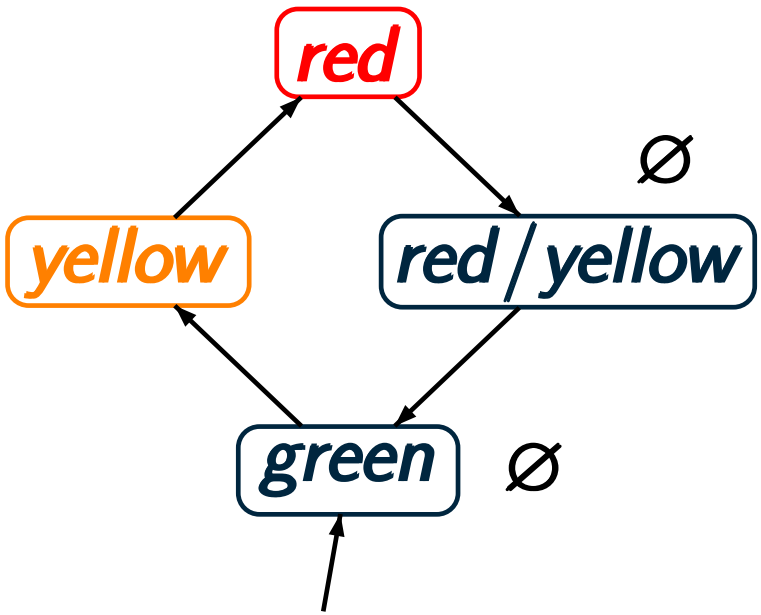
$\mathcal{T} \otimes \mathcal{A}$

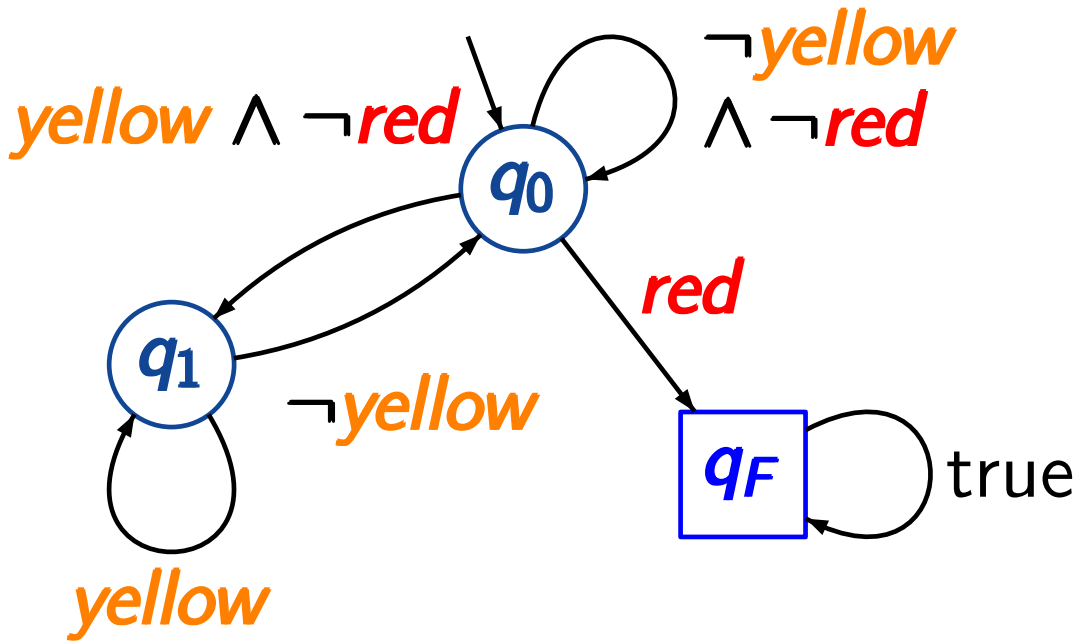
$4 * 3 = 12$ states, but

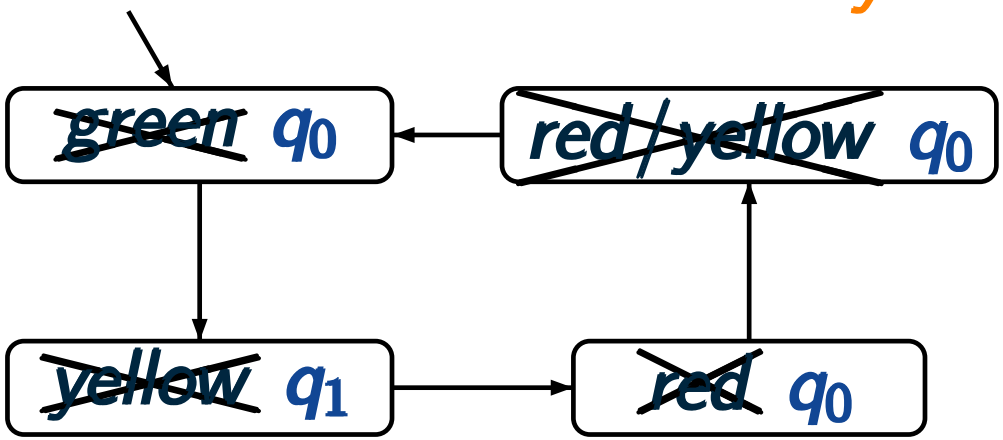
just 4 reachable states

# Example: product-TS





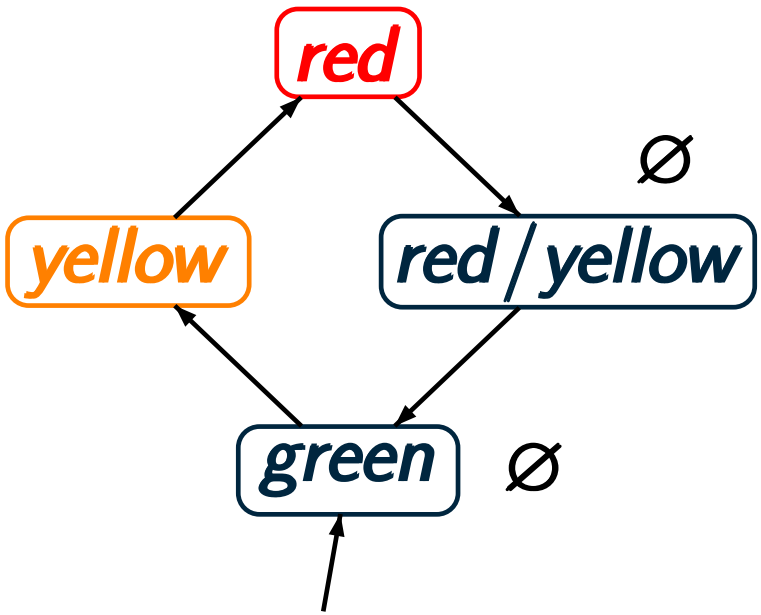



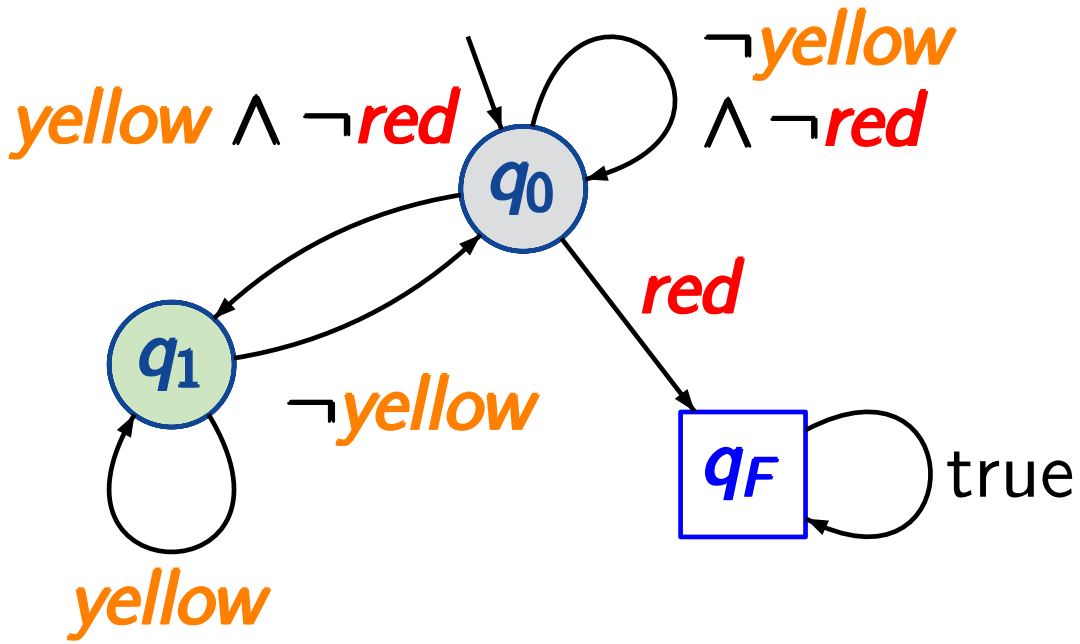

set of propositions

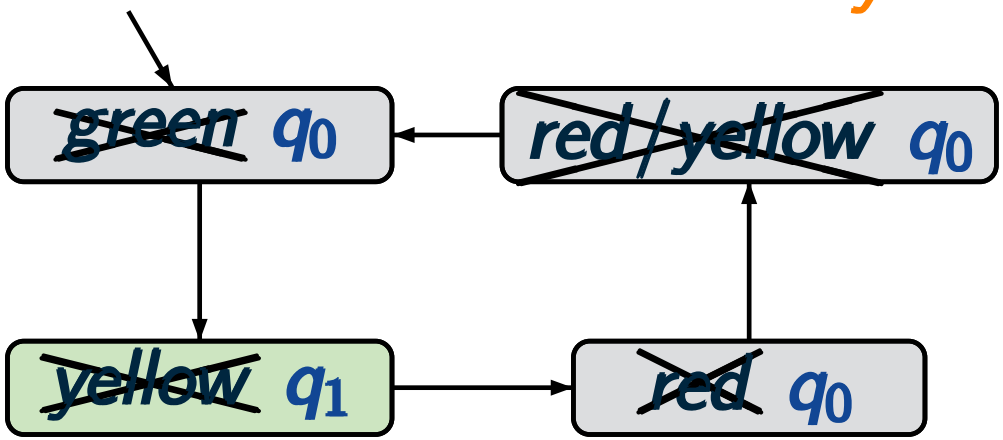
$AP' = \{q_0, q_1, q_F\}$

# Example: product-TS






set of propositions

$AP' = \{q_0, q_1, q_F\}$

invariant condition $\neg q_F$ holds
for all reachable states

# Technical remark on the product-TS



definition of the product of

- a transition system $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$

- an NFA $\mathcal{A} = (Q, 2^{AP}, \delta, Q_0, F)$

then the product $\mathcal{T} \otimes \mathcal{A} = (S \times Q, Act, \rightarrow', \ldots)$ is a TS

# Technical remark on the product-TS



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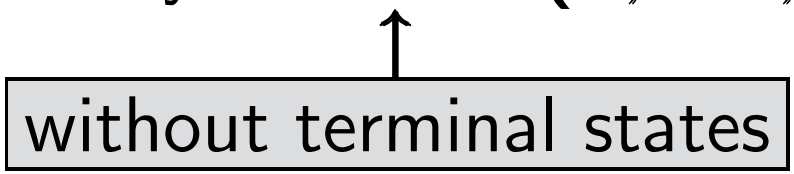


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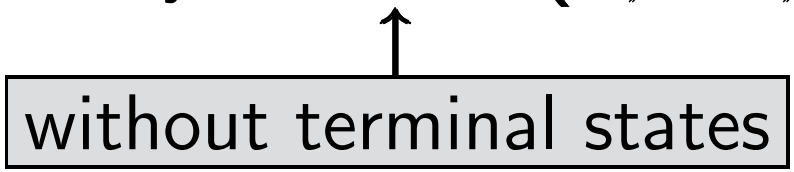


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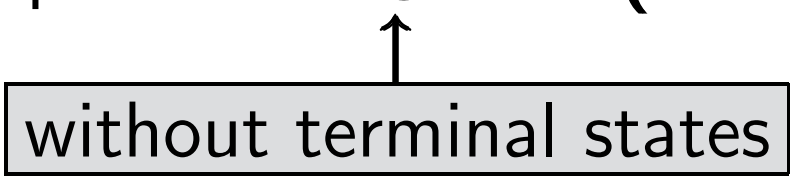

# Technical remark on the product-TS



definition of the product of

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  without terminal states

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assumptions on the NFA $\mathcal{A}$:

# Technical remark on the product-TS



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↑ without terminal states

assumptions on the NFA $\mathcal{A}$:

- $\mathcal{A}$ is non-blocking, i.e.,

$$Q_0 \neq \varnothing \ \wedge \ \forall q \in Q \, \forall A \in 2^{AP}. \ \delta(q, A) \neq \varnothing$$

# Technical remark on the product-TS



definition of the product of

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then the product $\mathcal{T} \otimes \mathcal{A} = (S \times Q, Act, \rightarrow', \ldots)$ is a TS

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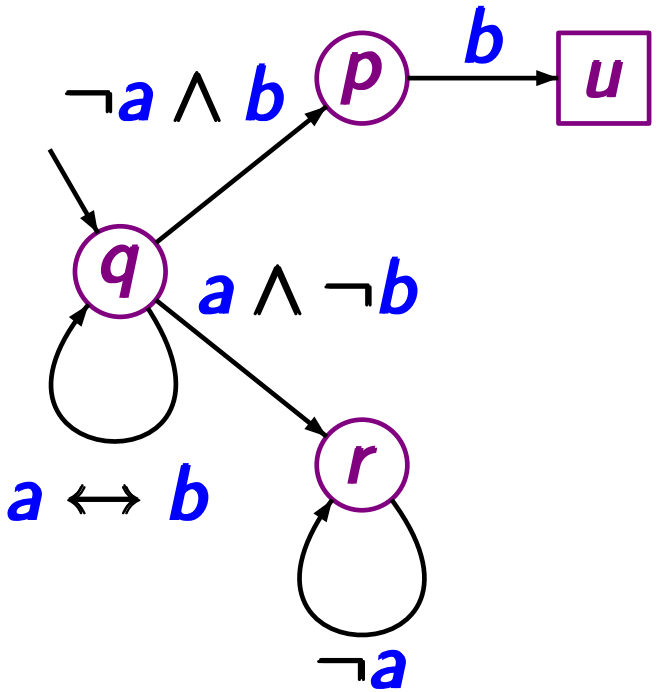
$$Q_0 \neq \varnothing \ \wedge \ \forall q \in Q \, \forall A \in 2^{AP}. \ \delta(q, A) \neq \varnothing$$

- no initial state of $\mathcal{A}$ is final, i.e., $Q_0 \cap F = \varnothing$

# Non-blocking NFA



NFA $\mathcal{A}$

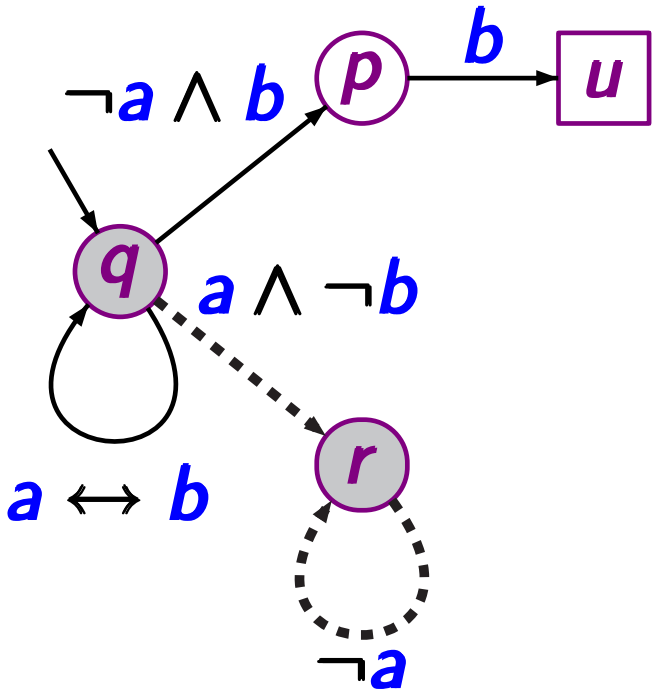



alphabet $\Sigma = 2^{AP}$ where $AP = \{a, b\}$

# Non-blocking NFA



NFA $\mathcal{A}$



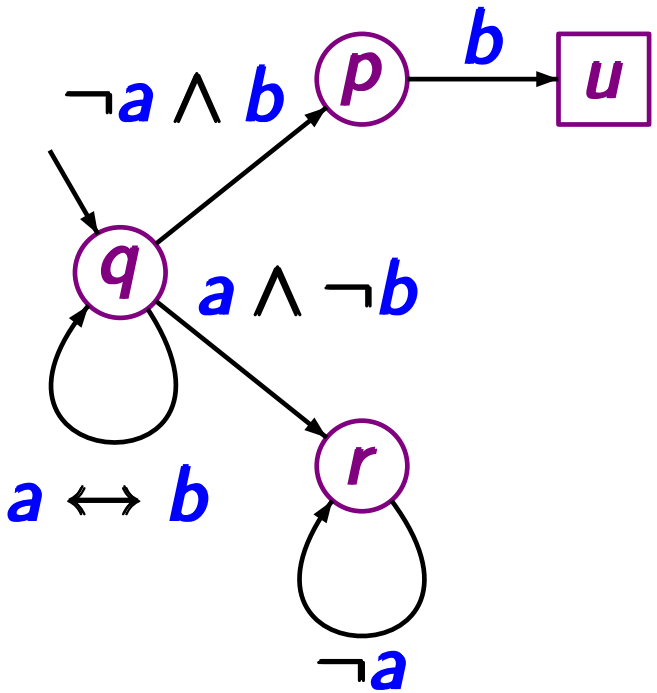

blocks for input
$\{a\}\varnothing\{a\}$

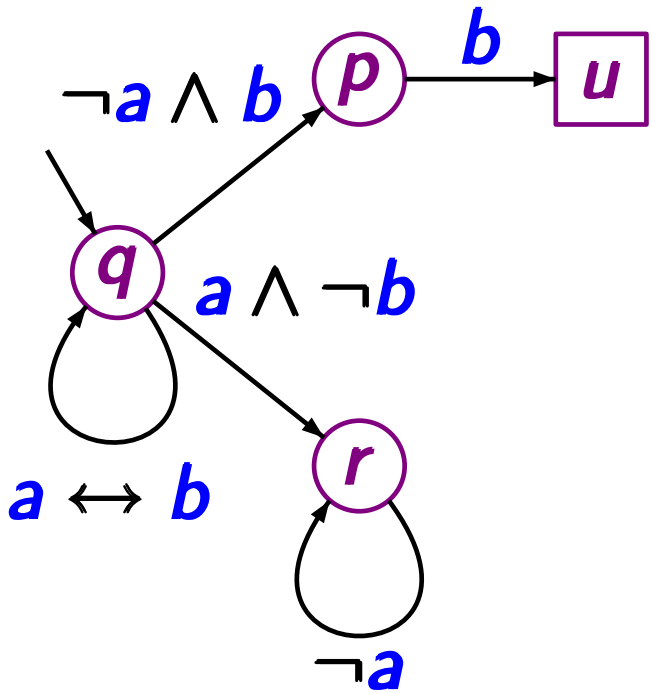
alphabet $\Sigma = 2^{AP}$ where $AP = \{a, b\}$

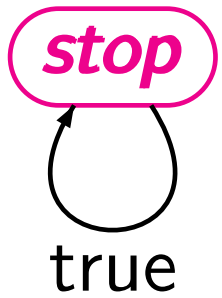
# Non-blocking NFA



NFA $\mathcal{A}$ $\rightsquigarrow$ equivalent NFA $\mathcal{A}'$



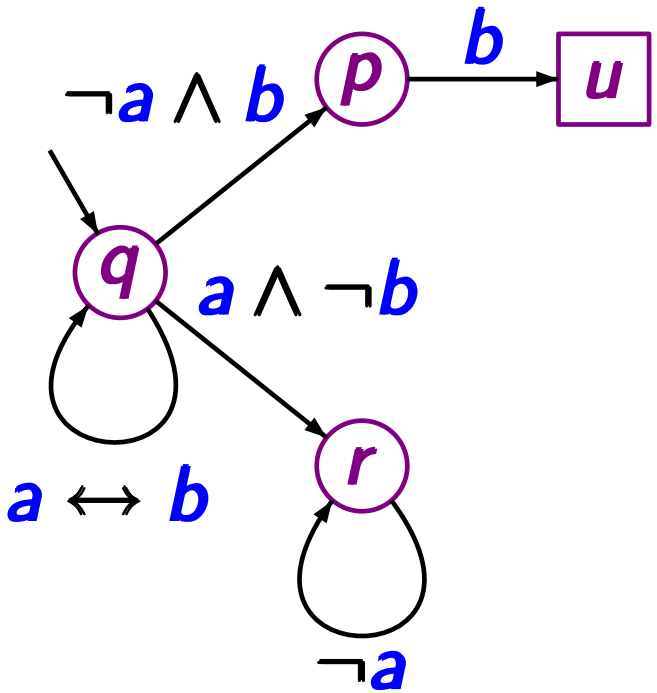

blocks for input
$\{a\}\,\varnothing\,\{a\}$

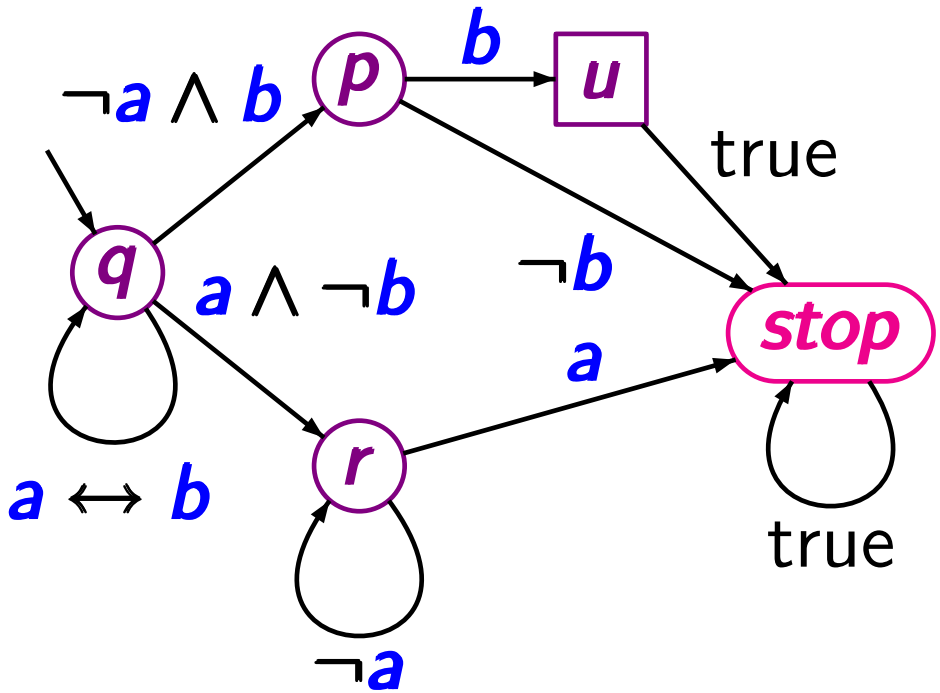
add a trap state *stop*

# Non-blocking NFA



NFA $\mathcal{A}$ $\rightsquigarrow$ equivalent NFA $\mathcal{A}'$



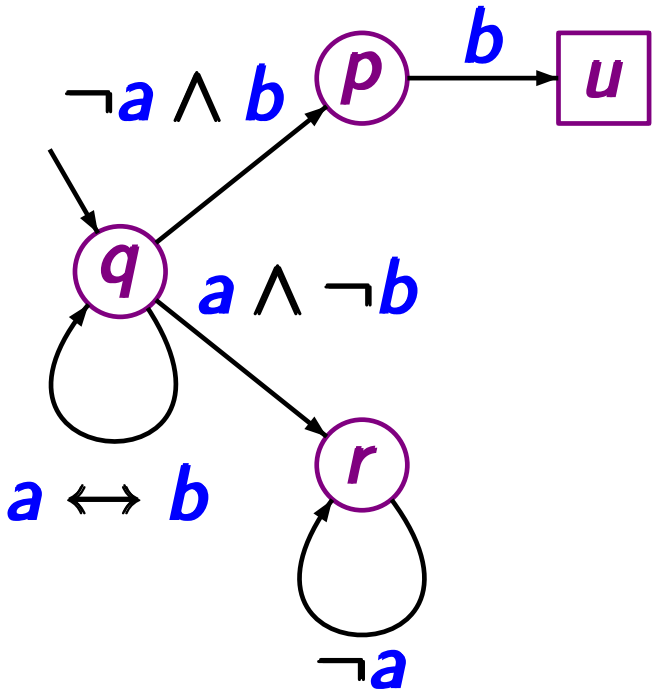

blocks for input
$\{a\}\,\varnothing\,\{a\}$

add a trap state *stop*

# Non-blocking NFA



NFA $\mathcal{A}$ $\rightsquigarrow$ equivalent NFA $\mathcal{A}'$




blocks for input
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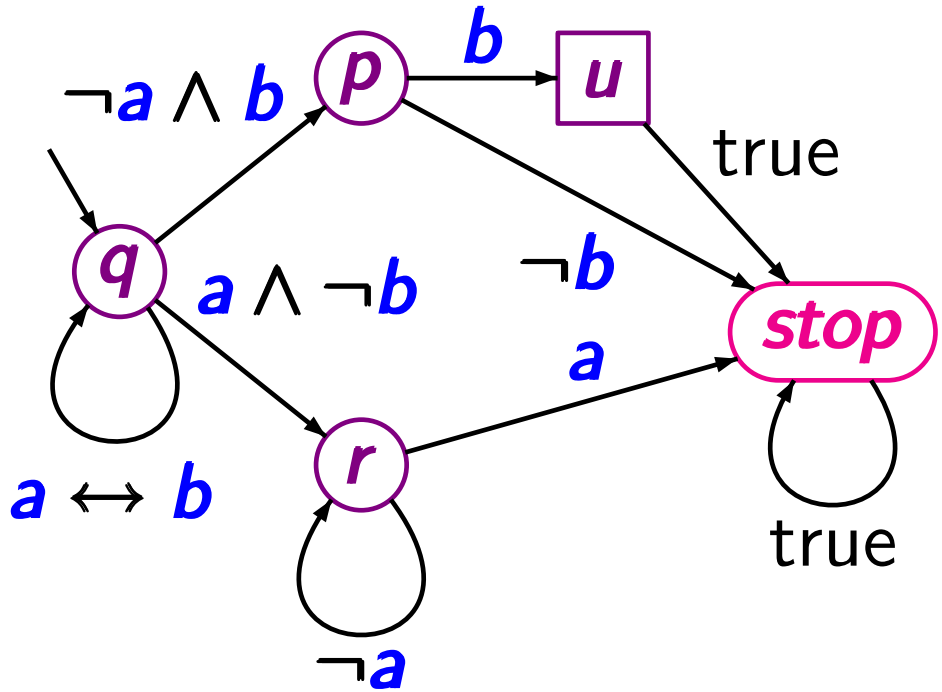
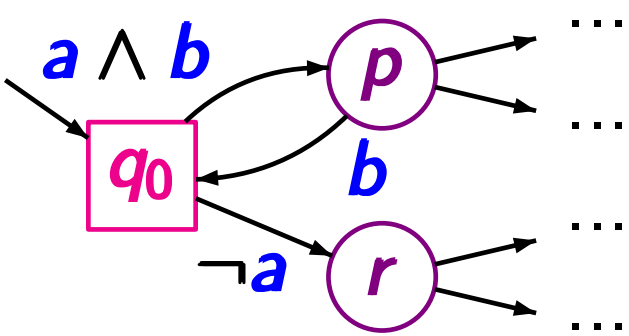



**non-blocking**

# NFA where no initial state is final



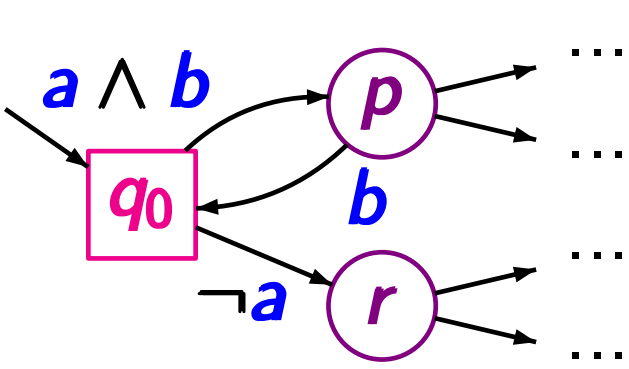
NFA $\mathcal{A}$ with $Q_0 \cap F \neq \varnothing$

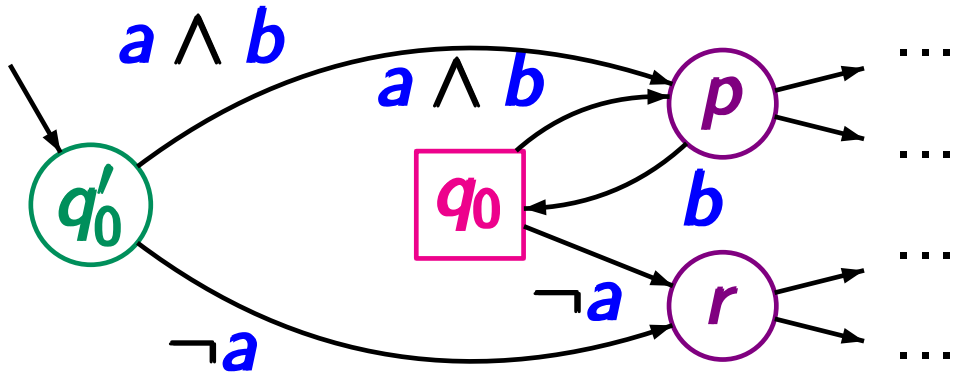


# NFA where no initial state is final



NFA $\mathcal{A}$ with $Q_0 \cap F \neq \varnothing$ $\rightsquigarrow$ NFA $\mathcal{A}'$ with $Q_0 \cap F = \varnothing$



# NFA where no initial state is final



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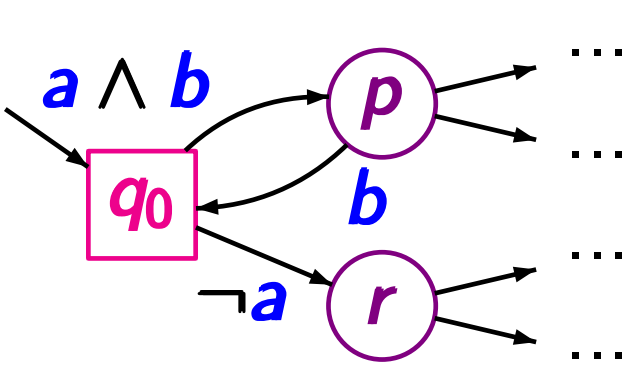
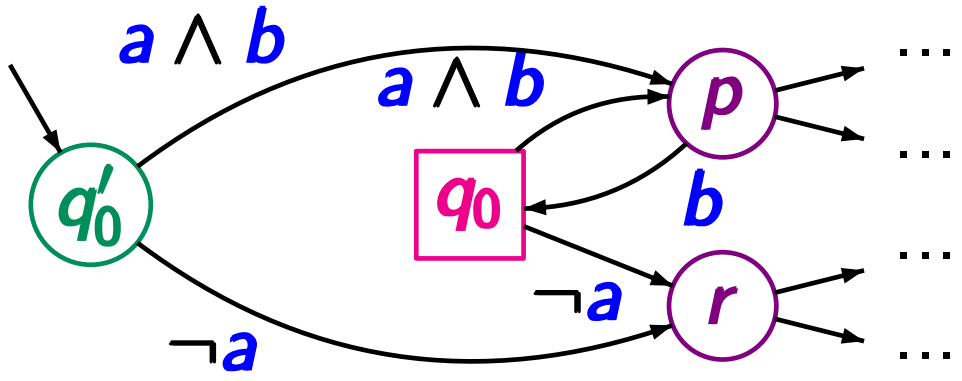


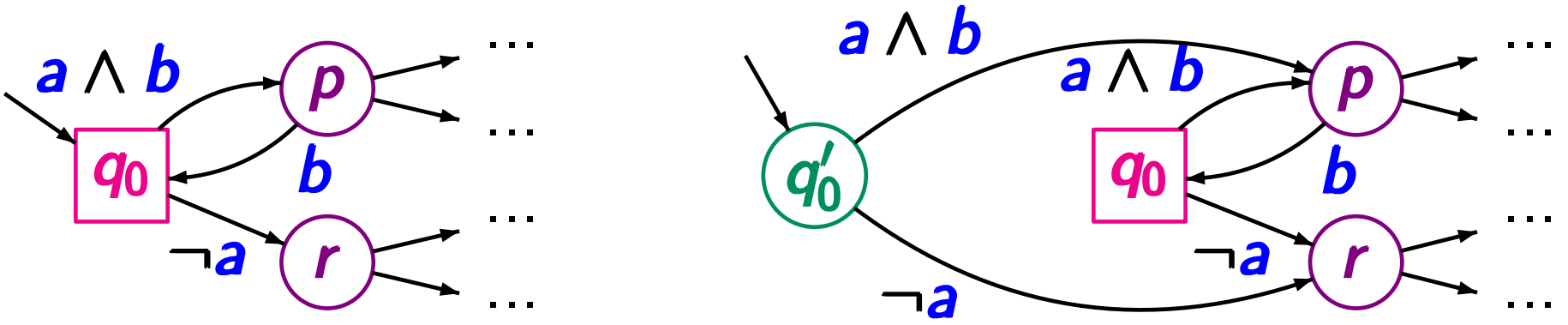




$$\mathcal{L}(\mathcal{A}') = \mathcal{L}(\mathcal{A}) \setminus \{\varepsilon\}$$

# NFA where no initial state is final



NFA $\mathcal{A}$ with $Q_0 \cap F \neq \emptyset$ $\rightsquigarrow$ NFA $\mathcal{A}'$ with $Q_0 \cap F = \emptyset$




$$\mathcal{L}(\mathcal{A}') = \mathcal{L}(\mathcal{A}) \setminus \{\varepsilon\}$$

*note:* if $\mathcal{A}$ is an NFA for the bad prefixes of a safety property then

$$\varepsilon \notin \mathcal{L}(\mathcal{A}) = \mathit{BadPref}$$

# Model checking regular safety properties

# Model checking regular safety properties



... via a reduction to invariant checking .....

# Model checking regular safety properties



Let $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$ be a transition system

$\mathcal{A} = (Q, 2^{AP}, \delta, Q_0, F)$ be an NFA
for the bad prefixes of a regular safety property $E$

# Model checking regular safety properties



Let $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$ be a transition system (without terminal states)

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The following statements are equivalent:

(1) $\mathcal{T} \models E$

(2) $Traces_{fin}(\mathcal{T}) \cap \mathcal{L}(\mathcal{A}) = \varnothing$

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(3) $\mathcal{T} \otimes \mathcal{A} \models$ invariant "always $\neg F$"

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Let $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$ be a transition system (without terminal states)

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The following statements are equivalent:

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(3) $\mathcal{T} \otimes \mathcal{A} \models$ invariant "always $\neg F$"

where "$\neg F$" denotes $\bigwedge_{q \in F} \neg q$

# Product transition system



$\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$ transition system

$\mathcal{A} = (Q, 2^{AP}, \delta, Q_0, F)$ NFA

product-TS $\mathcal{T} \otimes \mathcal{A} \stackrel{\text{def}}{=} (S \times Q, Act, \longrightarrow', S'_0, AP', L')$

$$\frac{s \xrightarrow{\alpha} s' \quad \wedge \quad q' \in \delta(q, L(s'))}{\langle s, q \rangle \xrightarrow{\alpha}' \langle s', q' \rangle}$$

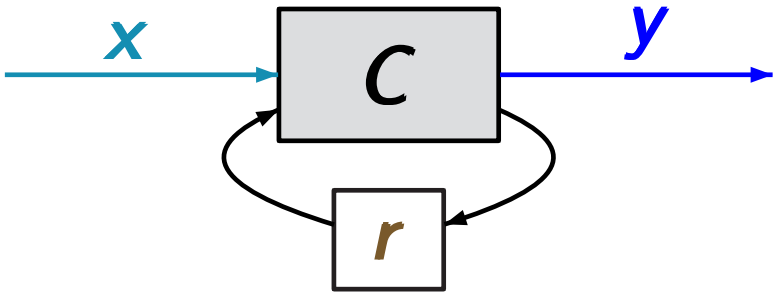
initial states: $S'_0 = \{ \langle s_0, q \rangle : s_0 \in S_0, q \in \delta(Q_0, L(s_0)) \}$

set of atomic propositions: $AP' = Q$

labeling function: $L'(\langle s, q \rangle) = \{q\}$

# Example: sequential circuit






$$\lambda_y = \delta_r = x \oplus r$$

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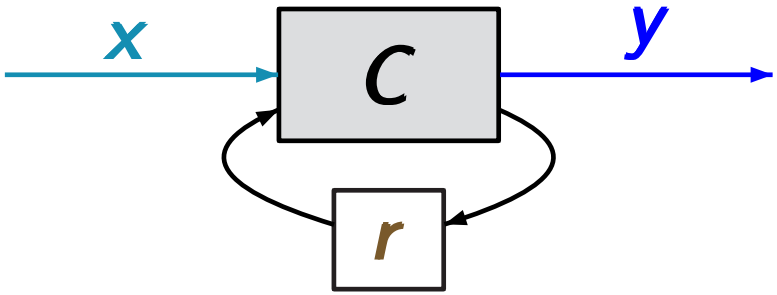


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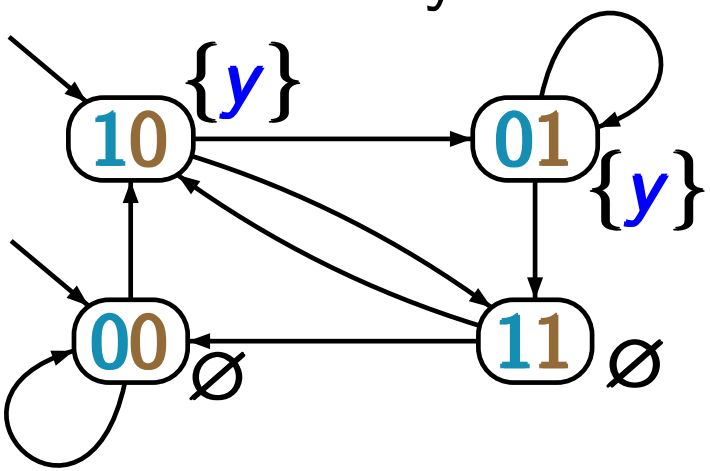
initially $r = 0$

transition system $\mathcal{T}$

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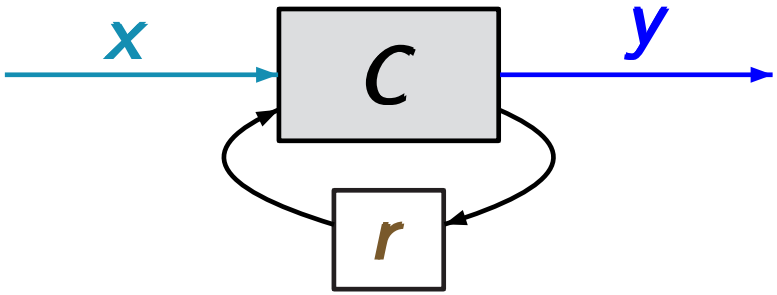


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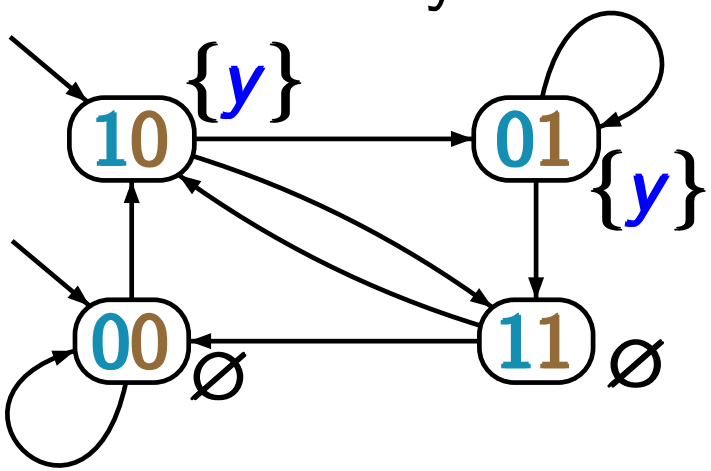


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safety property $E$

*The circuit will never ouput two ones after each other*

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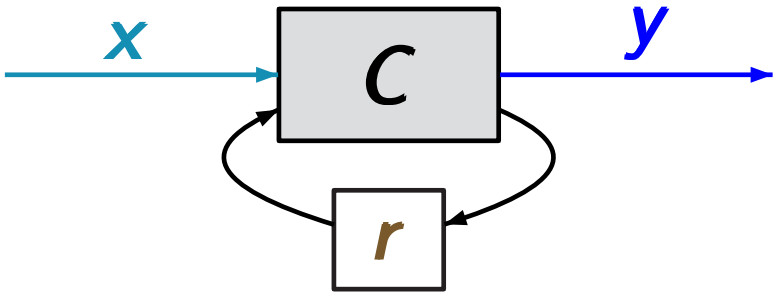


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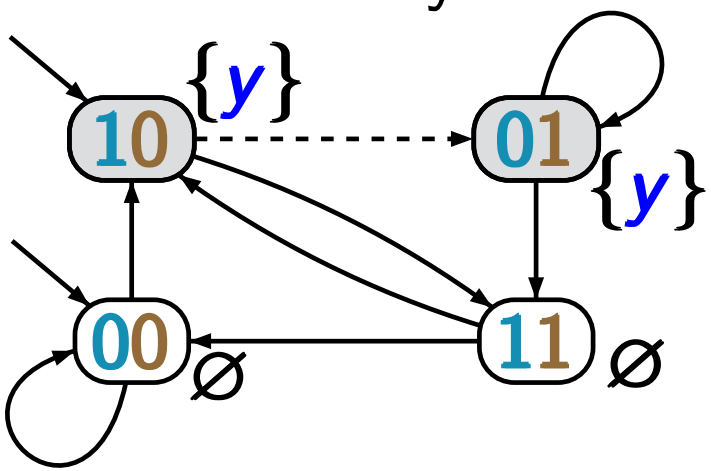


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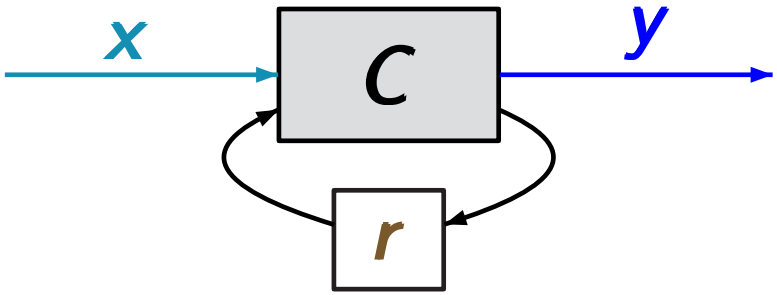


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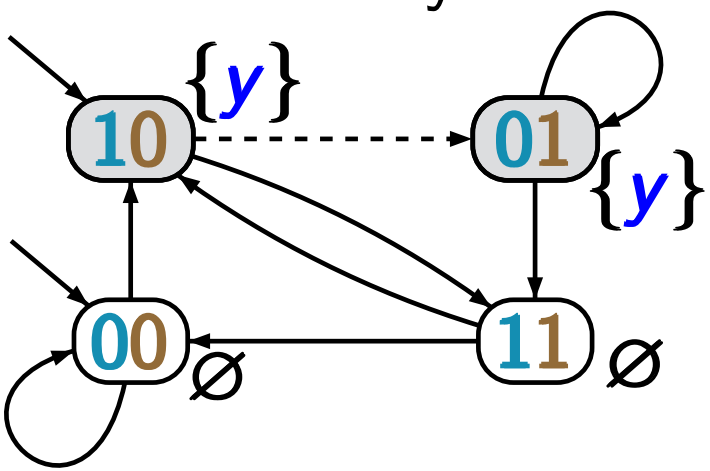


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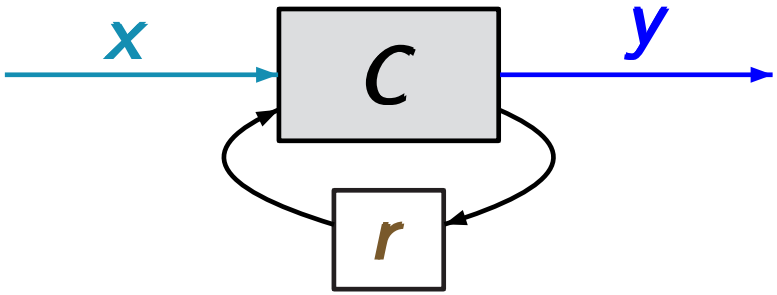


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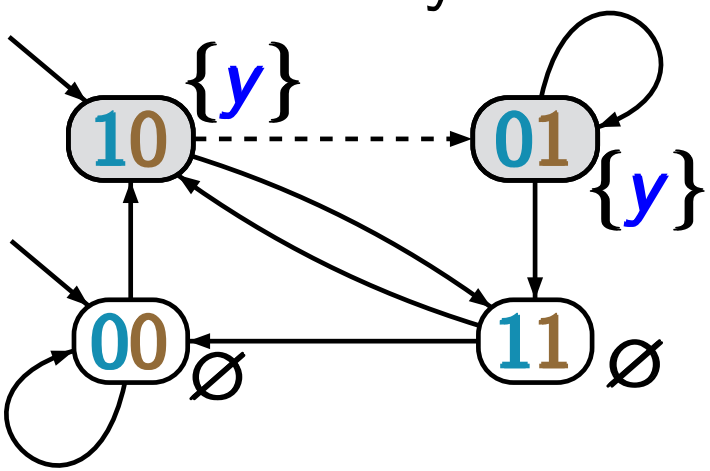


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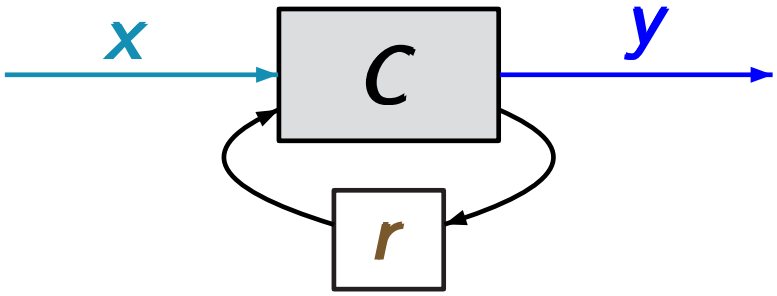


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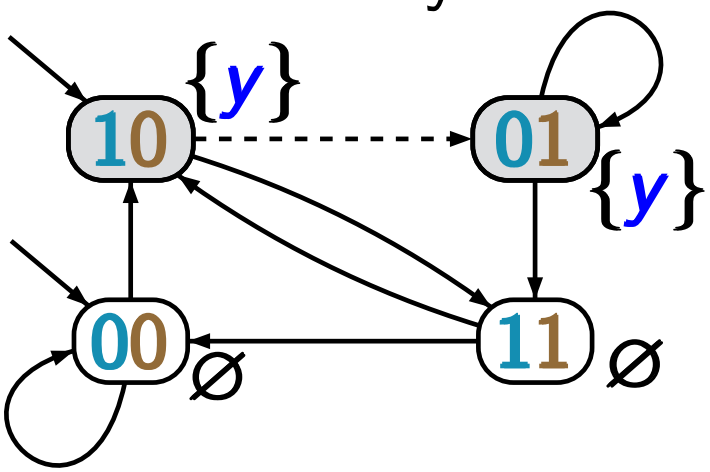


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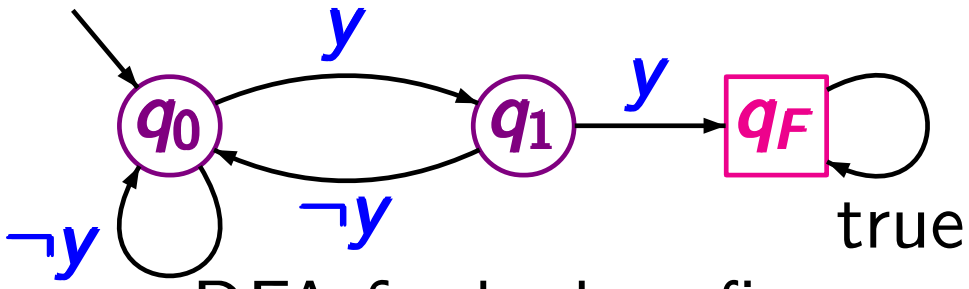


DFA for bad prefixes

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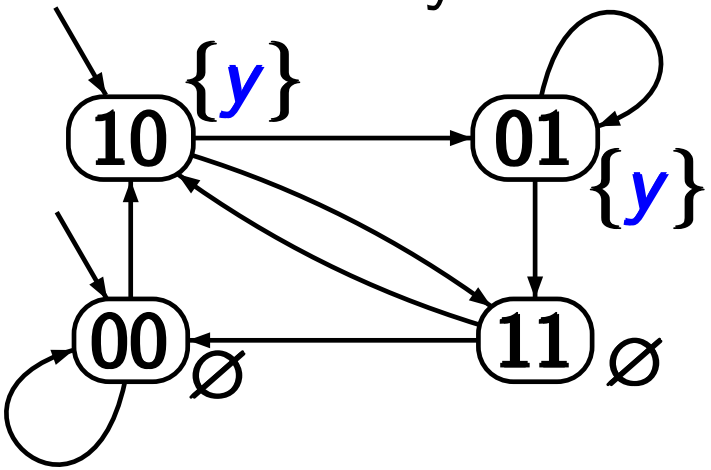
# Example: product-TS

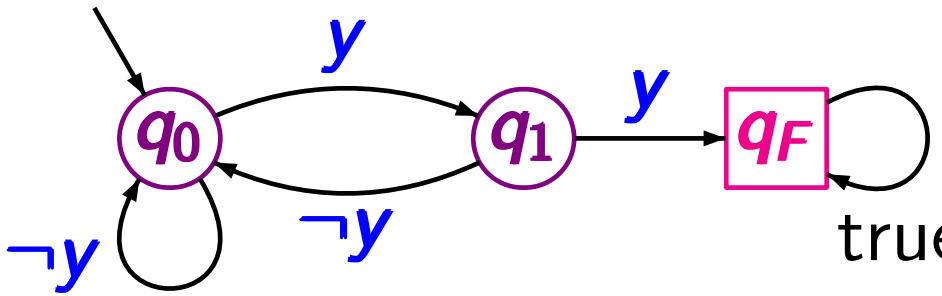


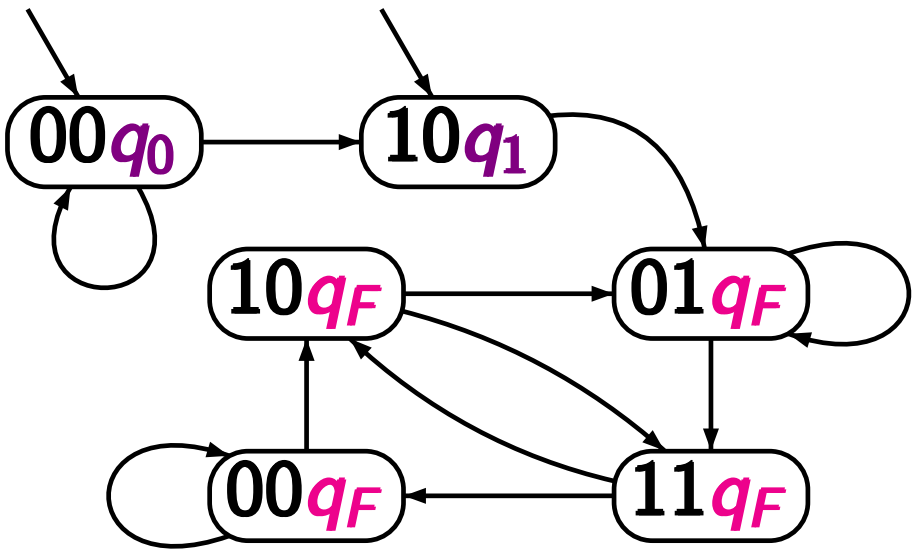
transition system $\mathcal{T}$




safety property $E$

*... never two ones in a row ...*




product-TS $\mathcal{T} \otimes \mathcal{A}$

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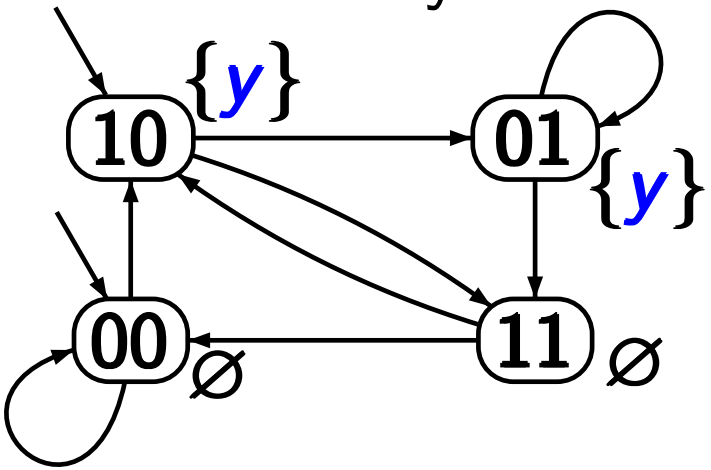


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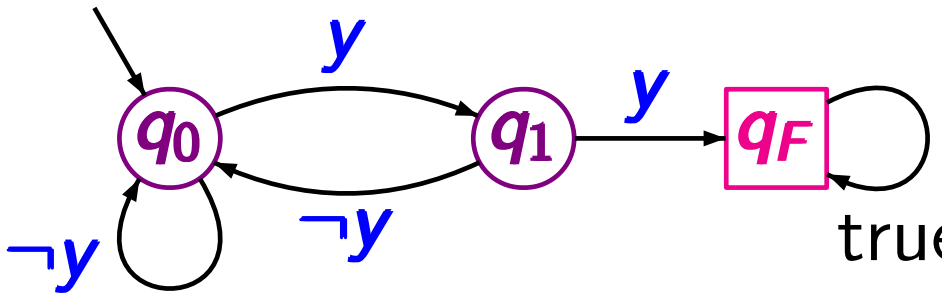


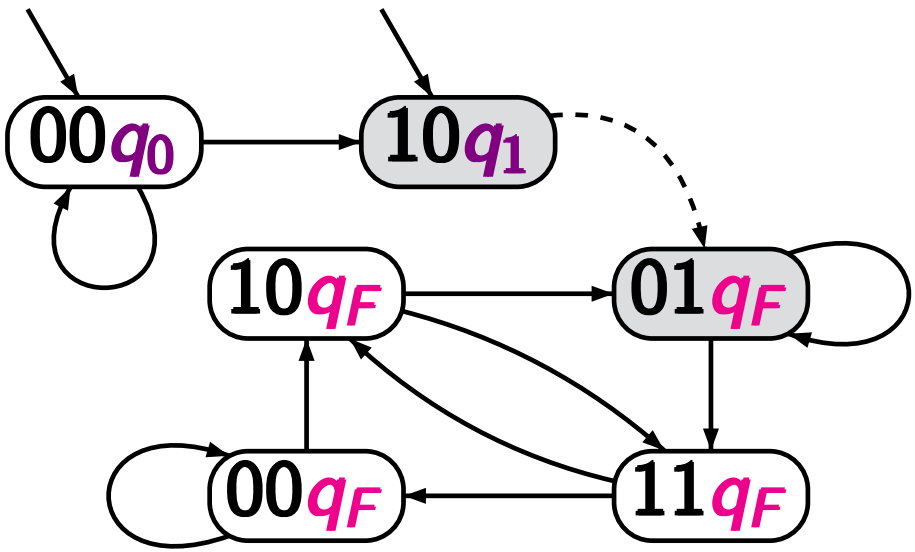


$\mathcal{T} \otimes \mathcal{A} \not\models$ "never $q_F$"

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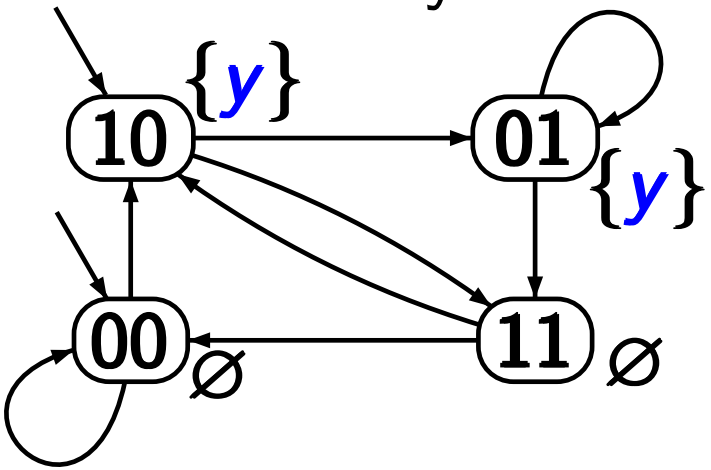


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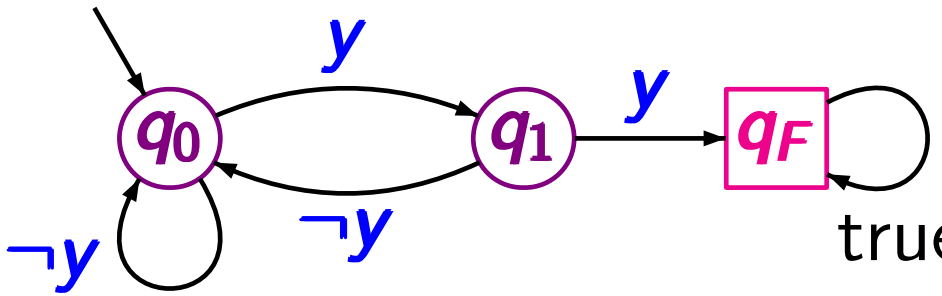


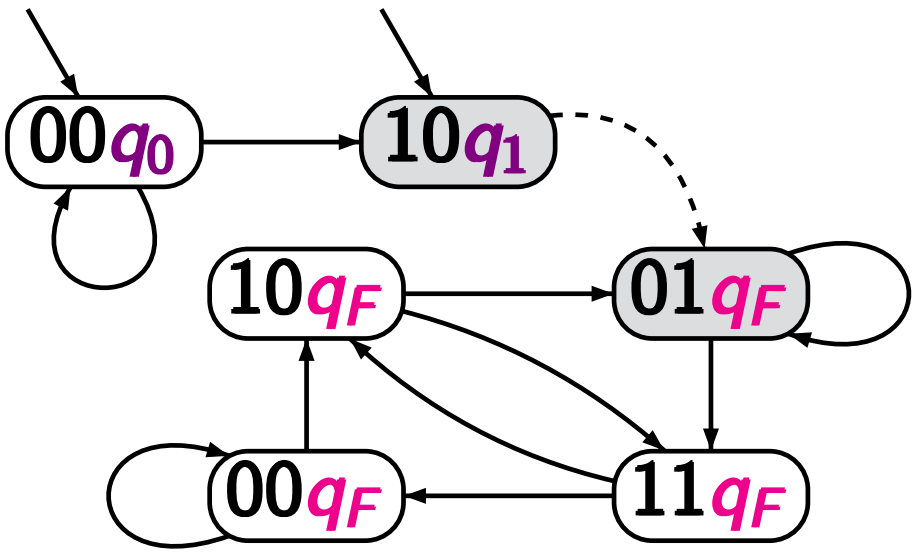


error indication for

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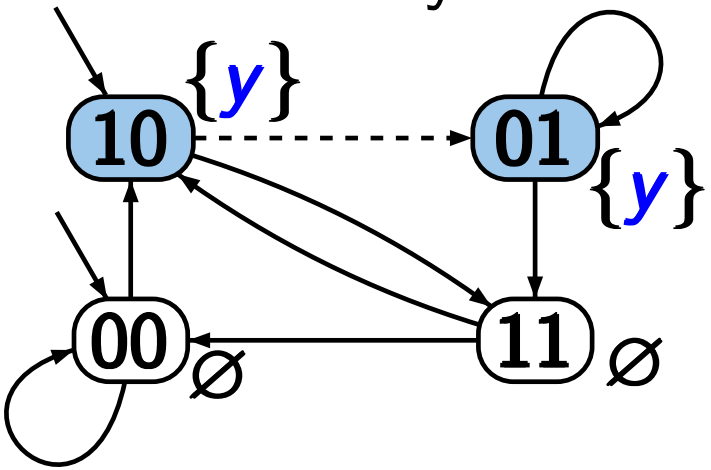


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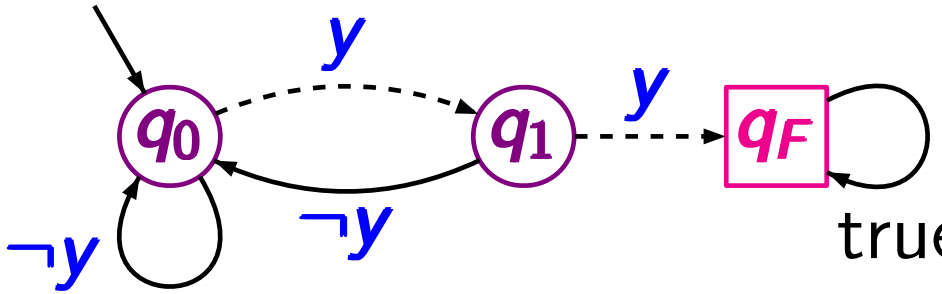


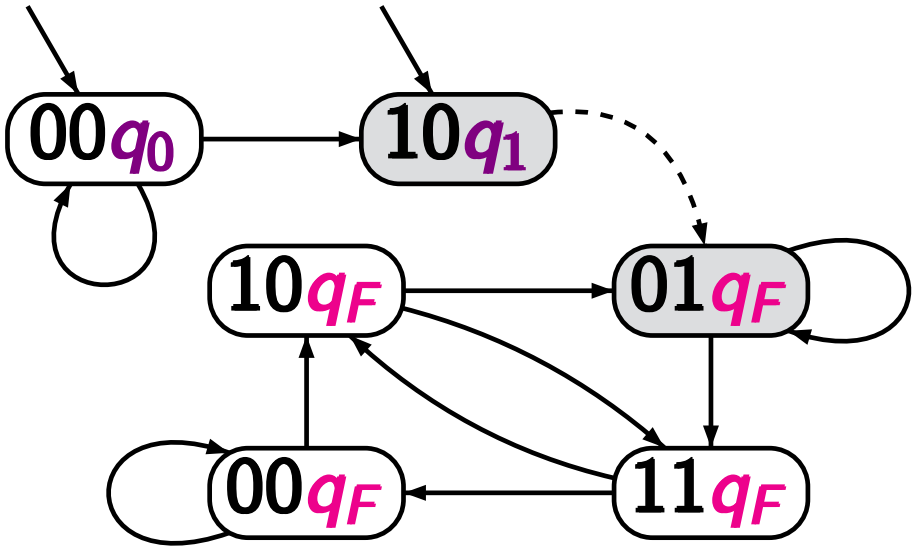


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↓ ↓

**10** **01**

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*output*: "yes" if $\mathcal{T} \not\models E$
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**time complexity**: $\mathcal{O}\big(size(\mathcal{T}) \cdot size(\mathcal{A})\big)$

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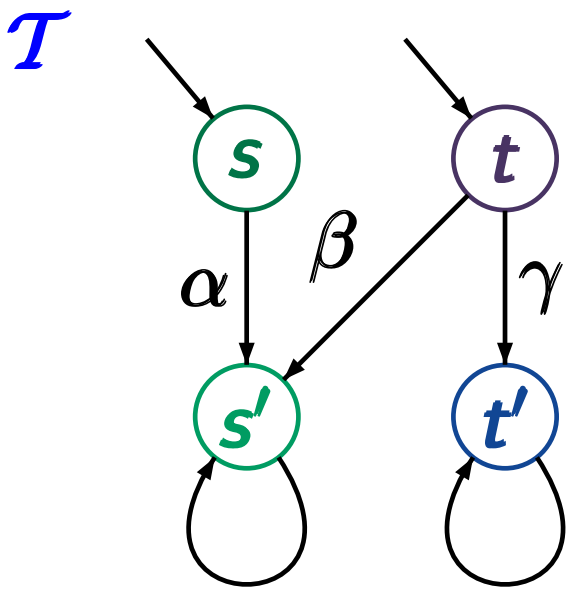

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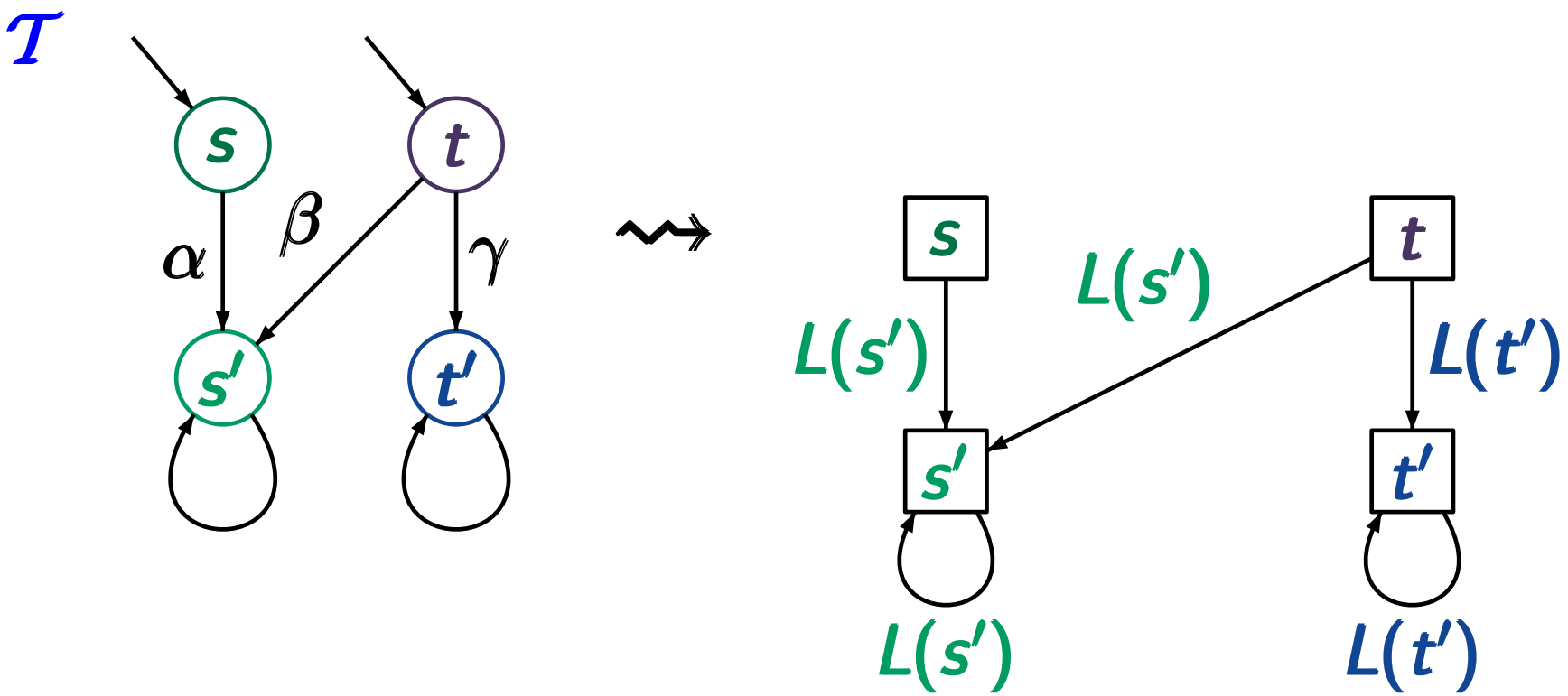

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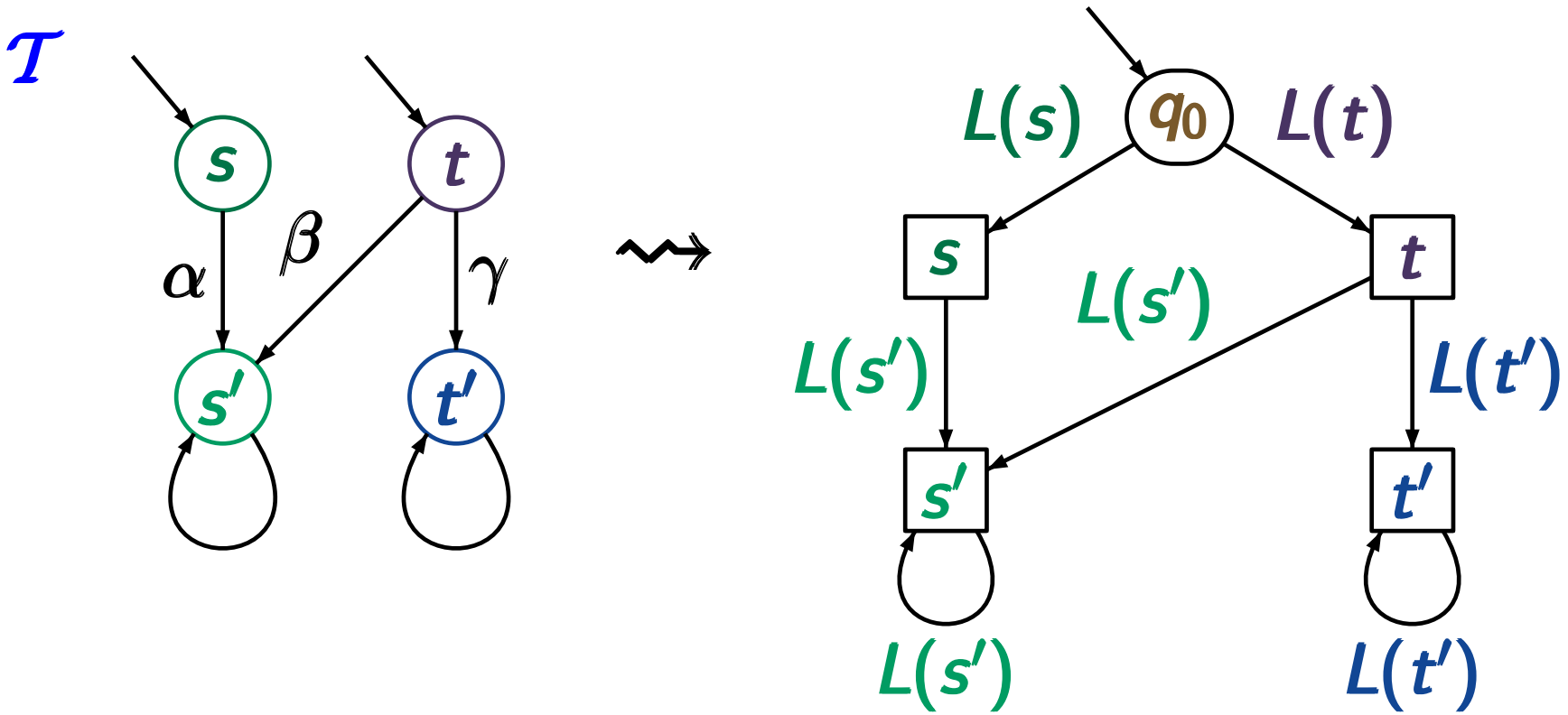

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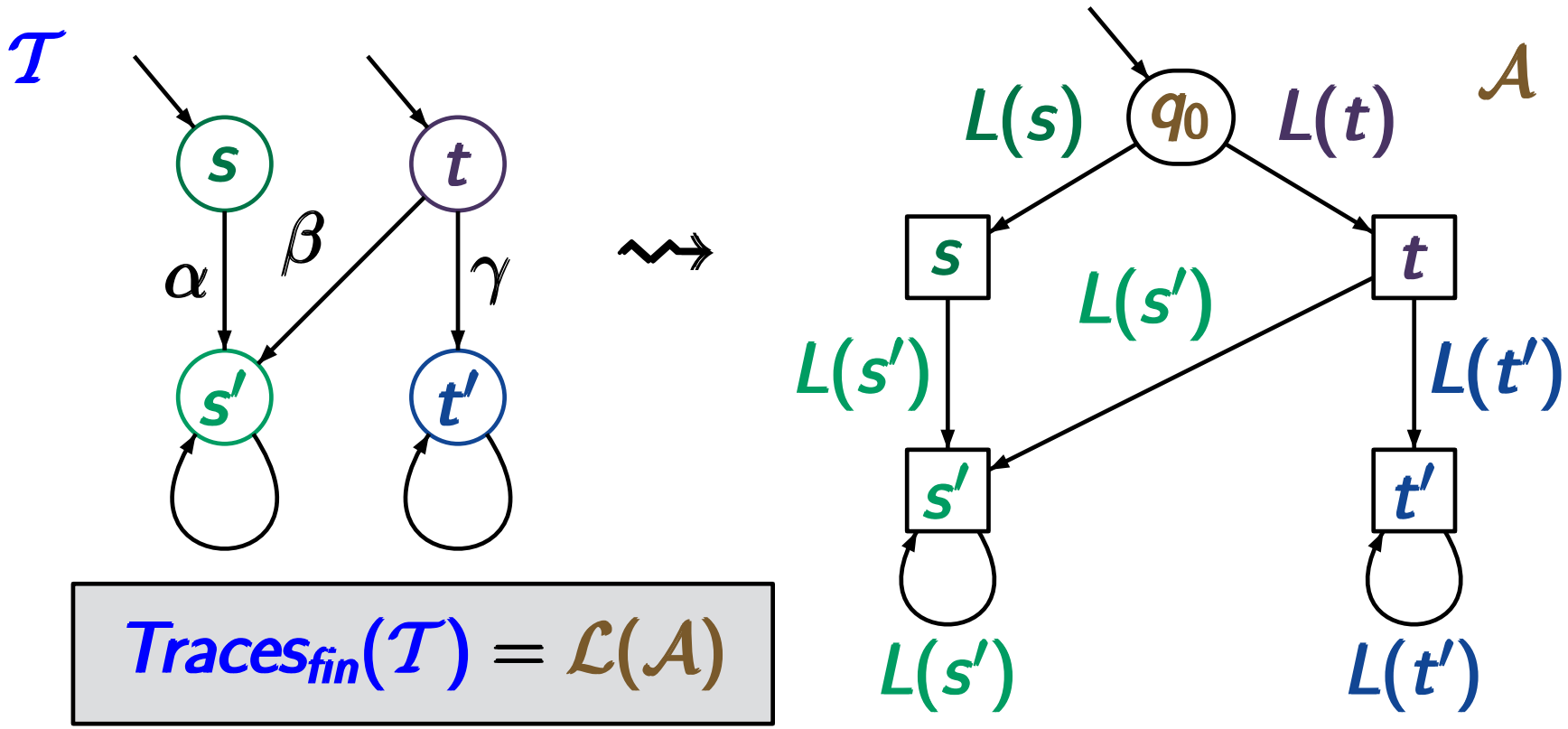


$$\mathit{Traces}_{\mathit{fin}}(\mathcal{T}) = \mathcal{L}(\mathcal{A})$$