Introduction

Modelling parallel systems

Linear Time Properties

Regular Properties

Linear Temporal Logic (LTL)

Computation-Tree Logic

Equivalences and Abstraction

Expansion laws

LTLSF3.1-28

Expansion law for U

until: $\varphi \, \mathsf{U} \, \psi \;\equiv\; \psi \;\vee\; (\varphi \wedge \bigcirc(\varphi \, \mathsf{U} \, \psi))$

Expansion laws for U and $\Diamond$

until: $\varphi \,\mathsf{U}\, \psi \;\equiv\; \psi \;\vee\; (\varphi \wedge \bigcirc(\varphi \,\mathsf{U}\, \psi))$

eventually: $\Diamond\psi \;\equiv\; \psi \;\vee\; \bigcirc\Diamond\psi$

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note: $\Diamond\psi = \mathit{true} \,\mathsf{U}\, \psi$

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note: $\Diamond\psi = \mathit{true} \mathbin{\mathsf{U}} \psi$

$$\equiv \psi \vee (\mathit{true} \wedge \bigcirc(\underbrace{\mathit{true} \mathbin{\mathsf{U}} \psi}_{=\Diamond\psi}))$$

Expansion laws for U and $\Diamond$

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note:

$$\begin{aligned}\Diamond\psi &= \mathit{true}\,\mathsf{U}\,\psi \\ &\equiv \psi \vee (\mathit{true} \wedge \bigcirc(\underbrace{\mathit{true}\,\mathsf{U}\,\psi}_{=\Diamond\psi})) \\ &\equiv \psi \vee \bigcirc\Diamond\psi\end{aligned}$$

Expansion laws for U, $\Diamond$ and $\Box$

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eventually: $\Diamond\psi \equiv \psi \vee \bigcirc\Diamond\psi$

always: $\Box\psi \equiv$ **?**

Expansion laws for U, $\Diamond$ and $\Box$

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$\Box\psi = \neg\Diamond\neg\psi$

Expansion laws for U, $\Diamond$ and $\Box$

until: $\varphi \,\mathsf{U}\, \psi \equiv \psi \vee (\varphi \wedge \bigcirc(\varphi \,\mathsf{U}\, \psi))$

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$$\begin{aligned} \Box\psi &= \neg\Diamond\neg\psi \\ &\equiv \neg(\neg\psi \vee \bigcirc\Diamond\neg\psi) \leftarrow \boxed{\text{expansion law for } \Diamond} \end{aligned}$$

Expansion laws for U, $\Diamond$ and $\Box$

until: $\varphi \,\mathsf{U}\, \psi \equiv \psi \lor (\varphi \land \bigcirc(\varphi \,\mathsf{U}\, \psi))$

eventually: $\Diamond\psi \equiv \psi \lor \bigcirc\Diamond\psi$

always: $\Box\psi \equiv \psi \land \bigcirc\Box\psi$

$$\begin{aligned}
\Box\psi &= \neg\Diamond\neg\psi \\
&\equiv \neg(\neg\psi \lor \bigcirc\Diamond\neg\psi) \\
&\equiv \neg\neg\psi \land \neg\bigcirc\Diamond\neg\psi \quad \leftarrow \boxed{\text{de Morgan}}
\end{aligned}$$

Expansion laws for U, $\Diamond$ and $\Box$

until: $\varphi \,\mathsf{U}\, \psi \;\equiv\; \psi \vee (\varphi \wedge \bigcirc(\varphi \,\mathsf{U}\, \psi))$

eventually: $\Diamond\psi \;\equiv\; \psi \vee \bigcirc\Diamond\psi$

always: $\Box\psi \;\equiv\; \psi \wedge \bigcirc\Box\psi$

$$
\begin{aligned}
\Box\psi &= \neg\Diamond\neg\psi \\
&\equiv \neg(\neg\psi \vee \bigcirc\Diamond\neg\psi) \\
&\equiv \neg\neg\psi \wedge \neg\bigcirc\Diamond\neg\psi \\
&\equiv \psi \wedge \neg\bigcirc\Diamond\neg\psi \quad \leftarrow \boxed{\text{double negation}}
\end{aligned}
$$

Expansion laws for U, $\Diamond$ and $\Box$

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eventually: $\Diamond \psi \equiv \psi \vee \bigcirc \Diamond \psi$

always: $\Box \psi \equiv \psi \wedge \bigcirc \Box \psi$

$$\begin{aligned}
\Box \psi &= \neg \Diamond \neg \psi \\
&\equiv \neg(\neg \psi \vee \bigcirc \Diamond \neg \psi) \\
&\equiv \neg\neg \psi \wedge \neg \bigcirc \Diamond \neg \psi \\
&\equiv \psi \wedge \bigcirc \neg \Diamond \neg \psi \quad \leftarrow \boxed{\text{self duality of } \bigcirc}
\end{aligned}$$

Expansion laws for U, $\Diamond$ and $\Box$

until: $\varphi \, \mathsf{U} \, \psi \equiv \psi \vee (\varphi \wedge \bigcirc(\varphi \, \mathsf{U} \, \psi))$

eventually: $\Diamond\psi \equiv \psi \vee \bigcirc\Diamond\psi$

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$$
\begin{aligned}
\Box\psi &= \neg\Diamond\neg\psi \\
&\equiv \neg(\neg\psi \vee \bigcirc\Diamond\neg\psi) \\
&\equiv \neg\neg\psi \wedge \neg\bigcirc\Diamond\neg\psi \\
&\equiv \psi \wedge \bigcirc\neg\Diamond\neg\psi \\
&\equiv \psi \wedge \bigcirc\Box\psi \qquad \longleftarrow \boxed{\text{definition of } \Box}
\end{aligned}
$$

Expansion laws are fixed point equations

until: $\varphi \,\mathsf{U}\, \psi \equiv \psi \vee (\varphi \wedge \bigcirc(\varphi \,\mathsf{U}\, \psi))$

eventually: $\Diamond\psi \equiv \psi \vee \bigcirc\Diamond\psi$

always: $\Box\psi \equiv \psi \wedge \bigcirc\Box\psi$

Expansion laws are fixed point equations

until: $\boxed{\varphi \, \mathsf{U} \, \psi} \equiv \psi \lor (\varphi \land \bigcirc \boxed{\varphi \, \mathsf{U} \, \psi})$

eventually: $\boxed{\Diamond \psi} \equiv \psi \lor \bigcirc \boxed{\Diamond \psi}$

always: $\boxed{\Box \psi} \equiv \psi \land \bigcirc \boxed{\Box \psi}$

Expansion laws are fixed point equations

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always: $\boxed{\Box \psi} \equiv \psi \wedge \bigcirc \boxed{\Box \psi}$

...don't yield a complete characterization, e.g.,

$$\boxed{\begin{array}{lcl} \mathit{false} & \equiv & a \wedge \bigcirc \mathit{false} \\ \Box a & \equiv & a \wedge \bigcirc \Box a \end{array}}$$

consider $\psi = a$

Expansion laws are fixed point equations

until: $\boxed{\varphi \, \mathsf{U} \, \psi} \equiv \psi \vee (\varphi \wedge \bigcirc \boxed{\varphi \, \mathsf{U} \, \psi})$

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although
$\Box a \not\equiv \mathit{false}$

Expansion laws are fixed point equations

until:	$\varphi \mathsf{U} \psi$	$\equiv$	$\psi \vee \underbrace{(\varphi \wedge \bigcirc(\varphi \mathsf{U} \psi))}_{\boxed{\text{least fixed point}}}$
eventually:	$\Diamond\psi$	$\equiv$	$\psi \vee \underbrace{\bigcirc\Diamond\psi}_{\boxed{\text{least fixed point}}}$
always:	$\Box\psi$	$\equiv$	$\psi \wedge \bigcirc\Box\psi$

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although $\Box a \not\equiv \mathit{false}$

Expansion laws are fixed point equations

until: $\varphi \, \mathsf{U} \, \psi \equiv \psi \vee (\varphi \wedge \bigcirc(\varphi \, \mathsf{U} \, \psi))$

least fixed point

eventually: $\Diamond\psi \equiv \psi \vee \bigcirc\Diamond\psi$

least fixed point

always: $\Box\psi \equiv \psi \wedge \bigcirc\Box\psi$

greatest fixed point

. . . don't yield a complete characterization, e.g.,

$$\boxed{\begin{array}{lcl} \mathit{false} & \equiv & a \wedge \bigcirc \mathit{false} \\ \Box a & \equiv & a \wedge \bigcirc\Box a \end{array}}$$

although $\Box a \not\equiv \mathit{false}$

Expansion law for U

The LTL formula $\chi = \varphi \, \mathsf{U} \, \psi$ is the least solution of

$$\chi \equiv \psi \vee (\varphi \wedge \bigcirc \chi)$$

Expansion law for U

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i.e., $\mathit{Words}(\varphi \, \mathsf{U} \, \psi)$ least LT-property E s.t.

$$E = \mathit{Words}(\psi) \cup \{ A_0 A_1 A_2 \ldots \in \mathit{Words}(\varphi) : A_1 A_2 \ldots \in E \}$$

Expansion law for U

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$$E = \mathit{Words}(\psi) \cup \left\{ A_0 A_1 A_2 \ldots \in \mathit{Words}(\varphi) : A_1 A_2 \ldots \in E \right\}$$

It even holds that *Words*$(\varphi \,\mathbf{U}\, \psi)$ least LT-property E s.t.

(1) $\mathit{Words}(\psi) \subseteq E$

(2) $\left\{ A_0 A_1 A_2 \ldots \in \mathit{Words}(\varphi) : \; A_1 A_2 \ldots \in E \right\} \subseteq E$

The weak until operator W

The weak until operator W

$$\varphi \, \mathsf{W} \, \psi \;\stackrel{\text{def}}{=}\; (\varphi \, \mathsf{U} \, \psi) \vee \Box \varphi$$

The weak until operator W

$$\boxed{\varphi \, \mathsf{W} \, \psi \ \overset{\text{def}}{=} \ (\varphi \, \mathsf{U} \, \psi) \lor \Box\varphi}$$

deriving "always" and "until" from "weak until":

$$\Box\varphi \quad \equiv \quad ?$$

The weak until operator W

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deriving "always" and "until" from "weak until":

$$\Box\varphi \quad \equiv \quad \varphi \,\mathsf{W}\, \mathit{false}$$

The weak until operator W

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deriving "always" and "until" from "weak until":

$$\Box\varphi \quad \equiv \quad \varphi \, \mathsf{W} \, \mathit{false}$$

$$\varphi \, \mathsf{U} \, \psi \quad \equiv \quad ?$$

The weak until operator W

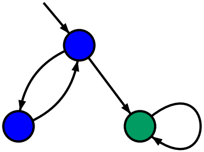
$$\boxed{\varphi \, \mathsf{W} \, \psi \;\stackrel{\text{def}}{=}\; (\varphi \, \mathsf{U} \, \psi) \vee \Box\varphi}$$

deriving "always" and "until" from "weak until":

$$\Box\varphi \equiv \varphi \, \mathsf{W} \, \mathit{false}$$

$$\varphi \, \mathsf{U} \, \psi \equiv (\varphi \, \mathsf{W} \, \psi) \wedge \Diamond\psi$$

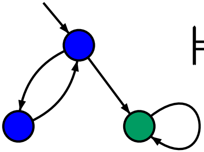
Does $\mathcal{T} \models a \,\mathsf{W}\, b$ hold?



 $\widehat{=}$ $\{a\}$

 $\widehat{=}$ $\{b\}$

Does $\mathcal{T} \models a \mathsf{W}\, b$ hold?

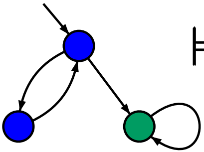


$\models a \mathsf{W}\, b$

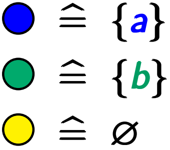
$\widehat{=}$ $\{a\}$

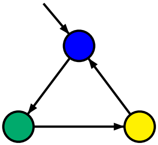
$\widehat{=}$ $\{b\}$

Does $\mathcal{T} \models a \, \mathsf{W} \, b$ hold?

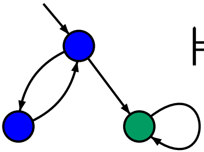


$\models a \, \mathsf{W} \, b$





Does $\mathcal{T} \models a \mathsf{W}\, b$ hold?

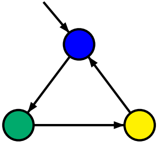


$\models a \mathsf{W}\, b$

$\widehat{=} \; \{a\}$

$\widehat{=} \; \{b\}$

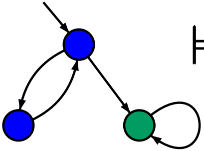
$\widehat{=} \; \varnothing$



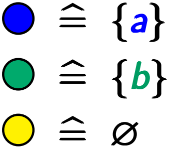
$\models \; a \mathsf{W}\, b \quad$ (even $a \mathsf{U}\, b$)

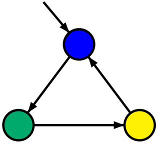
Does $\mathcal{T} \models a \, \mathsf{W} \, b$ hold?

LTLSF3.1-32

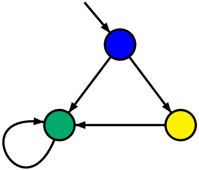


$\models a \, \mathsf{W} \, b$

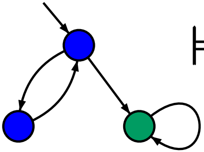




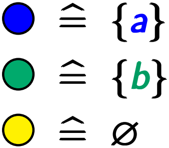
$\models \; a \, \mathsf{W} \, b \qquad$ (even $a \, \mathsf{U} \, b$)

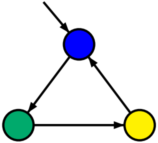


Does $\mathcal{T} \models a \,\mathsf{W}\, b$ hold?

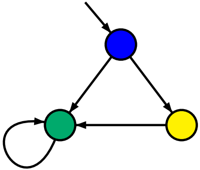


$\models a \,\mathsf{W}\, b$



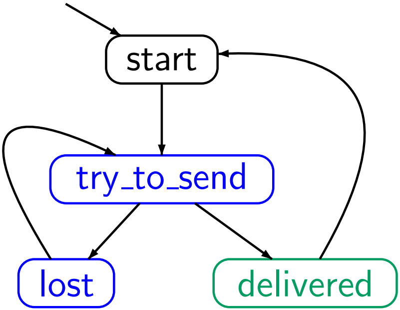


$\models a \,\mathsf{W}\, b$ (even $a \,\mathsf{U}\, b$)

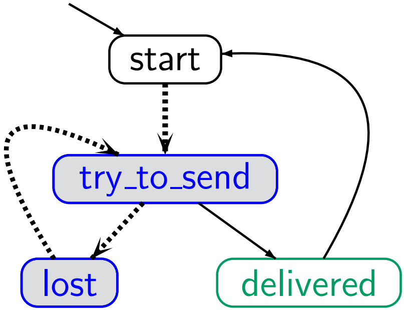


$\not\models a \,\mathsf{W}\, b$

Example: simple communication protocol

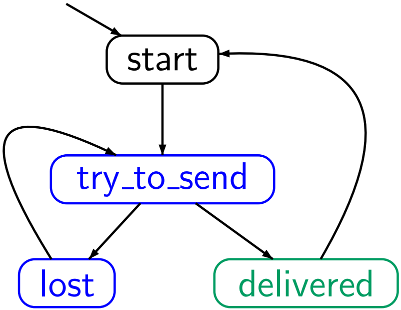


Example: simple communication protocol



$$\mathcal{T} \not\models \Box(\textit{blue} \longrightarrow \textit{blue}\ \mathsf{U}\ \textit{delivered})$$

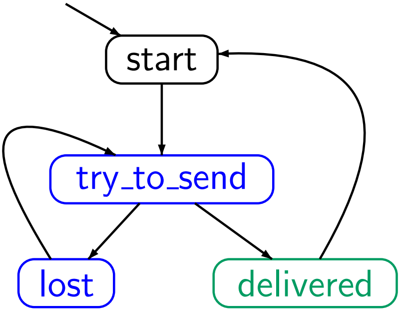
Example: until versus weak until



$$\mathcal{T} \not\models \Box(\mathit{blue} \longrightarrow \mathit{blue}\ \mathsf{U}\ \mathit{delivered})$$

$$\mathcal{T} \models \Box(\mathit{blue} \longrightarrow \mathit{blue}\ \mathsf{W}\ \mathit{delivered})$$

Example: until versus weak until



constrained liveness:

$$\mathcal{T} \not\models \Box(\mathit{blue} \longrightarrow \mathit{blue}\,\mathsf{U}\,\mathit{delivered})$$

safety:

$$\mathcal{T} \models \Box(\mathit{blue} \longrightarrow \mathit{blue}\,\mathsf{W}\,\mathit{delivered})$$

Duality of U and W

$$\boxed{\varphi \, \mathsf{W} \, \psi \ \stackrel{\text{def}}{=} \ (\varphi \, \mathsf{U} \, \psi) \ \vee \ \Box\varphi}$$

goal: express $\neg(\varphi \, \mathsf{U} \, \psi)$ via **W**, and vice versa

Duality of U and W

$$\boxed{\varphi \, \mathsf{W} \, \psi \;\stackrel{\text{def}}{=}\; (\varphi \, \mathsf{U} \, \psi) \lor \Box\varphi}$$

$$\begin{aligned} & \neg(\varphi \, \mathsf{U} \, \psi) \\ \equiv \;& \big((\varphi \land \neg\psi) \; \mathsf{U} \; (\neg\varphi \land \neg\psi)\big) \lor \Box(\varphi \land \neg\psi) \end{aligned}$$

Duality of U and W

$$\boxed{\varphi \,\mathsf{W}\, \psi \;\stackrel{\text{def}}{=}\; (\varphi \,\mathsf{U}\, \psi) \;\vee\; \Box\varphi}$$

$$\begin{aligned} & \neg(\varphi \,\mathsf{U}\, \psi) \\ \equiv \;& \big((\varphi \wedge \neg\psi) \;\mathsf{U}\; (\neg\varphi \wedge \neg\psi)\big) \;\vee\; \Box(\varphi \wedge \neg\psi) \\ \equiv \;& (\varphi \wedge \neg\psi) \,\mathsf{W}\, (\neg\varphi \wedge \neg\psi) \end{aligned}$$

Duality of U and W

$$\boxed{\varphi \,\mathsf{W}\, \psi \;\stackrel{\text{def}}{=}\; (\varphi \,\mathsf{U}\, \psi) \;\vee\; \Box\varphi}$$

$$\begin{aligned}
& \neg(\varphi \,\mathsf{U}\, \psi) \\
\equiv\; & ((\varphi \wedge \neg\psi) \;\mathsf{U}\; (\neg\varphi \wedge \neg\psi)) \;\vee\; \Box(\varphi \wedge \neg\psi) \\
\equiv\; & (\varphi \wedge \neg\psi) \;\mathsf{W}\; (\neg\varphi \wedge \neg\psi) \\
\equiv\; & (\neg\psi) \;\mathsf{W}\; (\neg\varphi \wedge \neg\psi)
\end{aligned}$$

Duality of U and W

$$\varphi \mathsf{W} \psi \overset{\text{def}}{=} (\varphi \mathsf{U} \psi) \vee \Box\varphi$$

$$\begin{aligned}
& \neg(\varphi \mathsf{U} \psi) \\
\equiv\ & ((\varphi \wedge \neg\psi) \ \mathsf{U} \ (\neg\varphi \wedge \neg\psi)) \vee \Box(\varphi \wedge \neg\psi) \\
\equiv\ & (\varphi \wedge \neg\psi) \mathsf{W} (\neg\varphi \wedge \neg\psi) \\
\equiv\ & (\neg\psi) \mathsf{W} (\neg\varphi \wedge \neg\psi)
\end{aligned}$$

$$\neg(\varphi \mathsf{U} \psi) \equiv (\neg\psi) \mathsf{W} (\neg\varphi \wedge \neg\psi)$$

$$\neg(\varphi \mathsf{W} \psi) \equiv ?$$

Duality of U and W

$$\varphi \,\mathsf{W}\, \psi \;\stackrel{\text{def}}{=}\; (\varphi \,\mathsf{U}\, \psi) \vee \Box\varphi$$

$$\begin{aligned}
& \neg(\varphi \,\mathsf{U}\, \psi) \\
\equiv\ & ((\varphi \wedge \neg\psi) \,\mathsf{U}\, (\neg\varphi \wedge \neg\psi)) \vee \Box(\varphi \wedge \neg\psi) \\
\equiv\ & (\varphi \wedge \neg\psi) \,\mathsf{W}\, (\neg\varphi \wedge \neg\psi) \\
\equiv\ & (\neg\psi) \,\mathsf{W}\, (\neg\varphi \wedge \neg\psi)
\end{aligned}$$

$$\neg(\varphi \,\mathsf{U}\, \psi) \equiv (\neg\psi) \,\mathsf{W}\, (\neg\varphi \wedge \neg\psi)$$

$$\neg(\varphi \,\mathsf{W}\, \psi) \equiv (\neg\psi) \,\mathsf{U}\, (\neg\varphi \wedge \neg\psi)$$

Expansion laws for U and W

Expansion laws for U and W

$$\varphi \,\mathsf{U}\, \psi \quad \equiv \quad \psi \vee (\varphi \wedge \bigcirc(\varphi \,\mathsf{U}\, \psi))$$

$$\varphi \,\mathsf{W}\, \psi \quad \equiv \quad ?$$

Expansion laws for U and W

$$\varphi \; \mathsf{U} \; \psi \quad \equiv \quad \psi \; \vee \; (\varphi \wedge \bigcirc(\varphi \, \mathsf{U} \, \psi))$$

$$\varphi \; \mathsf{W} \; \psi \quad \equiv \quad \psi \; \vee \; (\varphi \wedge \bigcirc(\varphi \, \mathsf{W} \, \psi))$$

Expansion laws for U and W

$$\varphi \ \mathsf{U} \ \psi \quad \equiv \quad \psi \ \vee \ (\varphi \wedge \bigcirc(\varphi \, \mathsf{U} \, \psi))$$

smallest solution

$$\varphi \ \mathsf{W} \ \psi \quad \equiv \quad \psi \ \vee \ (\varphi \wedge \bigcirc(\varphi \, \mathsf{W} \, \psi))$$

Expansion laws for U and W

$\varphi \,\mathsf{U}\, \psi$	$\equiv$	$\psi \vee (\varphi \wedge \bigcirc(\varphi \,\mathsf{U}\, \psi))$	smallest solution
$\varphi \,\mathsf{W}\, \psi$	$\equiv$	$\psi \vee (\varphi \wedge \bigcirc(\varphi \,\mathsf{W}\, \psi))$	largest solution

Expansion laws for U and W

$\varphi \, \mathsf{U} \, \psi$	$\equiv$	$\psi \vee (\varphi \wedge \bigcirc(\varphi \, \mathsf{U} \, \psi))$	smallest solution
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$\mathit{Words}(\varphi \, \mathsf{U} \, \psi)$ smallest LT-property E s.t.

Expansion laws for U and W

$\varphi \, \mathsf{U} \, \psi$	$\equiv$	$\psi \vee (\varphi \wedge \bigcirc(\varphi \, \mathsf{U} \, \psi))$	smallest solution
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$Words(\varphi \, \mathsf{U} \, \psi)$ smallest LT-property E s.t.

(1) $Words(\psi) \subseteq E$

(2) $\{A_0 A_1 A_2 \ldots \in Words(\varphi) : A_1 A_2 \ldots \in E\} \subseteq E$

Expansion laws for U and W

$\varphi \,\mathsf{U}\, \psi$	$\equiv$	$\psi \vee (\varphi \wedge \bigcirc(\varphi \,\mathsf{U}\, \psi))$	smallest solution
$\varphi \,\mathsf{W}\, \psi$	$\equiv$	$\psi \vee (\varphi \wedge \bigcirc(\varphi \,\mathsf{W}\, \psi))$	largest solution

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(1) $Words(\psi) \subseteq E$

(2) $\{A_0A_1A_2\ldots \in Words(\varphi) : A_1A_2\ldots \in E\} \subseteq E$

$$\Updownarrow$$

$$Words(\psi) \cup \{A_0A_1A_2\ldots \in Words(\varphi) : A_1A_2\ldots \in E\} \subseteq E$$

Expansion laws for U and W

$\varphi \, \mathsf{U} \, \psi$	$\equiv$	$\psi \vee (\varphi \wedge \bigcirc(\varphi \, \mathsf{U} \, \psi))$	smallest solution
$\varphi \, \mathsf{W} \, \psi$	$\equiv$	$\psi \vee (\varphi \wedge \bigcirc(\varphi \, \mathsf{W} \, \psi))$	largest solution

$\mathit{Words}(\varphi \, \mathsf{U} \, \psi)$ smallest LT-property E s.t.

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Expansion laws for U and W

$\varphi \,\mathsf{U}\, \psi$	$\equiv$	$\psi \vee (\varphi \wedge \bigcirc(\varphi \,\mathsf{U}\, \psi))$	smallest solution
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$\mathit{Words}(\varphi \,\mathsf{U}\, \psi)$ smallest LT-property E s.t.

$$\mathit{Words}(\psi) \cup \{A_0 A_1 A_2 \ldots \in \mathit{Words}(\varphi) : A_1 A_2 \ldots \in E\} \subseteq E$$

$\mathit{Words}(\varphi \,\mathsf{W}\, \psi)$ largest LT-property E s.t.

Expansion laws for U and W

$\varphi \,\mathsf{U}\, \psi$	$\equiv$	$\psi \vee (\varphi \wedge \bigcirc(\varphi \,\mathsf{U}\, \psi))$	smallest solution
$\varphi \,\mathsf{W}\, \psi$	$\equiv$	$\psi \vee (\varphi \wedge \bigcirc(\varphi \,\mathsf{W}\, \psi))$	largest solution

$Words(\varphi \,\mathsf{U}\, \psi)$ smallest LT-property E s.t.

$$Words(\psi) \cup \{A_0A_1A_2\ldots \in Words(\varphi) : A_1A_2\ldots \in E\} \subseteq E$$

$Words(\varphi \,\mathsf{W}\, \psi)$ largest LT-property E s.t.

$$Words(\psi) \cup \{A_0A_1A_2\ldots \in Words(\varphi) : A_1A_2\ldots \in E\} \supseteq E$$

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$\varphi \,\mathsf{U}\, \psi$	$\equiv$	$\psi \vee (\varphi \wedge \bigcirc(\varphi \,\mathsf{U}\, \psi))$	smallest solution
$\varphi \,\mathsf{W}\, \psi$	$\equiv$	$\psi \vee (\varphi \wedge \bigcirc(\varphi \,\mathsf{W}\, \psi))$	largest solution

$Words(\varphi \,\mathsf{U}\, \psi)$ smallest LT-property E s.t.

$$Words(\psi) \cup \{A_0A_1A_2\ldots \in Words(\varphi) : A_1A_2\ldots \in E\} \subseteq E$$

$Words(\varphi \,\mathsf{W}\, \psi)$ largest LT-property E s.t.

$$E \subseteq Words(\psi) \cup \{A_0A_1A_2\ldots \in Words(\varphi) : A_1A_2\ldots \in E\}$$

Expansion laws for U and W

$$\varphi \, \mathsf{U} \, \psi \quad \equiv \quad \psi \; \vee \; (\varphi \wedge \bigcirc(\varphi \, \mathsf{U} \, \psi))$$

smallest solution

$$\varphi \, \mathsf{W} \, \psi \quad \equiv \quad \psi \; \vee \; (\varphi \wedge \bigcirc(\varphi \, \mathsf{W} \, \psi))$$

largest solution

Expansion laws for U, W, $\Diamond$, and $\Box$

$$\varphi \, \mathsf{U} \, \psi \quad \equiv \quad \psi \ \vee \ (\varphi \wedge \bigcirc(\varphi \, \mathsf{U} \, \psi))$$

smallest solution

$$\Diamond\psi \quad \equiv \quad \psi \ \vee \ \bigcirc\Diamond\psi$$

smallest solution

$$\varphi \, \mathsf{W} \, \psi \quad \equiv \quad \psi \ \vee \ (\varphi \wedge \bigcirc(\varphi \, \mathsf{W} \, \psi))$$

largest solution

$$\Box\varphi \quad \equiv \quad \varphi \wedge \bigcirc\Box\varphi$$

largest solution

---

remind: $\Diamond\psi \ = \ \mathit{true} \, \mathsf{U} \, \psi, \quad \Box\varphi \ \equiv \ \varphi \, \mathsf{W} \, \mathit{false}$

Positive normal form (PNF)

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- negation only on the level of literals
- uses for each operator its dual

PNF for propositional logic

- negation only on the level of literals
- uses for each operator its dual

syntax of propositional formulas in PNF:

$$\varphi \ ::= \ \mathit{true} \mid \mathit{false} \mid a \mid \neg a \mid \varphi_1 \wedge \varphi_2 \mid \varphi_1 \vee \varphi_2$$

PNF for propositional logic

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syntax of propositional formulas in PNF:

$$\varphi \ ::= \ \mathit{true} \mid \mathit{false} \mid a \mid \neg a \mid \varphi_1 \wedge \varphi_2 \mid \varphi_1 \vee \varphi_2$$

$\neg \mathit{true} \ \equiv \ \mathit{false}$	duality of the constant truth values
$\neg(\varphi_1 \wedge \varphi_2) \ \equiv \ \neg\varphi_1 \vee \neg\varphi_2$	duality of $\vee$ and $\wedge$ (de Morgan's law)

LTL in positive normal form (PNF)

- negation only on the level of literals
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$$\varphi \quad ::= \quad \mathit{true} \;\big|\; \mathit{false} \;\big|\; a \;\big|\; \neg a \;\big|\; \varphi_1 \wedge \varphi_2 \;\big|\; \varphi_1 \vee \varphi_2$$

using duality of constants and duality of $\vee$ and $\wedge$

LTL in positive normal form (PNF)

- negation only on the level of literals
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$$\varphi \ ::= \ \mathit{true} \mid \mathit{false} \mid a \mid \neg a \mid \varphi_1 \wedge \varphi_2 \mid \varphi_1 \vee \varphi_2 \mid$$

$\bigcirc\varphi$ + dual operator for $\bigcirc$

using duality of constants and duality of $\vee$ and $\wedge$

LTL in positive normal form (PNF)

- negation only on the level of literals
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$$\varphi \quad ::= \quad \mathit{true} \mid \mathit{false} \mid a \mid \neg a \mid \varphi_1 \wedge \varphi_2 \mid \varphi_1 \vee \varphi_2 \mid$$

$$\bigcirc \varphi \longleftarrow$$ no new operator needed for $\neg\bigcirc$

using duality of constants and duality of $\vee$ and $\wedge$

$\neg \bigcirc \varphi \;\equiv\; \bigcirc \neg \varphi$ self-duality of the next operator

LTL in positive normal form (PNF)

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$$\varphi \;::=\; \mathit{true} \mid \mathit{false} \mid a \mid \neg a \mid \varphi_1 \wedge \varphi_2 \mid \varphi_1 \vee \varphi_2 \mid \bigcirc\varphi \mid \varphi_1 \, \mathsf{U} \, \varphi_2 \; + \text{dual operator for } \mathsf{U}$$

using duality of constants and duality of $\vee$ and $\wedge$

$\neg \bigcirc \varphi \equiv \bigcirc \neg \varphi$ self-duality of the next operator

LTL in positive normal form (PNF)

- negation only on the level of literals
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$$\varphi \ ::= \ \mathit{true} \mid \mathit{false} \mid a \mid \neg a \mid \varphi_1 \wedge \varphi_2 \mid \varphi_1 \vee \varphi_2 \mid \bigcirc\varphi \mid \varphi_1 \,\mathsf{U}\, \varphi_2 \mid \varphi_1 \,\mathsf{W}\, \varphi_2$$

using duality of constants and duality of $\vee$ and $\wedge$

$\neg \bigcirc \varphi \ \equiv \ \bigcirc\neg\varphi$ self-duality of the next operator

$\neg(\varphi_1 \,\mathsf{U}\, \varphi_2) \ \equiv \ (\neg\varphi_2)\,\mathsf{W}(\neg\varphi_1 \wedge \neg\varphi_2)$

duality of **U** and **W**

Derivation of $\Diamond$ and $\Box$ in LTL-PNF

$$\varphi ::= \mathit{true} \mid \mathit{false} \mid a \mid \neg a \mid \varphi_1 \wedge \varphi_2 \mid \varphi_1 \vee \varphi_2 \mid \bigcirc\varphi \mid \varphi_1 \,\mathsf{U}\, \varphi_2 \mid \varphi_1 \,\mathsf{W}\, \varphi_2$$

Derivation of $\Diamond$ and $\Box$ in LTL-PNF

$$\varphi \quad ::= \quad \mathit{true} \mid \mathit{false} \mid a \mid \neg a \mid \varphi_1 \wedge \varphi_2 \mid \varphi_1 \vee \varphi_2 \mid \bigcirc\varphi \mid \varphi_1 \,\mathsf{U}\, \varphi_2 \mid \varphi_1 \,\mathsf{W}\, \varphi_2 \mid \Diamond\varphi \mid \Box\varphi$$

$\Diamond$ and $\Box$ can (still) be derived:

$$\Diamond\varphi \quad \stackrel{\text{def}}{=} \quad \mathit{true} \,\mathsf{U}\, \varphi$$

$$\Box\varphi \quad \stackrel{\text{def}}{=} \quad \varphi \,\mathsf{W}\, \mathit{false}$$

Universality of LTL-PNF

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Each LTL formula can be transformed into an equivalent LTL formula in **PNF**

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LTL formula $\varphi \rightsquigarrow$ LTL formula in PNF φ'
by successive application of the following rules:

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by successive application of the following rules:

$$\begin{array}{lcl}
\neg \mathit{true} & \rightsquigarrow & \mathit{false} \\
\neg\neg\varphi & \rightsquigarrow & \varphi \\
\neg(\varphi_1 \wedge \varphi_2) & \rightsquigarrow & \neg\varphi_1 \vee \neg\varphi_2 \\
\neg \bigcirc \varphi & \rightsquigarrow & \bigcirc\neg\varphi \\
\neg(\varphi_1 \,\mathsf{U}\, \varphi_2) & \rightsquigarrow & (\neg\varphi_2)\,\mathsf{W}(\neg\varphi_1 \wedge \neg\varphi_2)
\end{array}$$

Universality of LTL-PNF

Each LTL formula can be transformed into an equivalent LTL formula in **PNF**

LTL formula $\varphi \leadsto$ LTL formula in PNF φ'
by successive application of the following rules:

$$
\begin{array}{lcl}
\neg \mathit{true} & \leadsto & \mathit{false} \\
\neg\neg\varphi & \leadsto & \varphi \\
\neg(\varphi_1 \wedge \varphi_2) & \leadsto & \neg\varphi_1 \vee \neg\varphi_2 \\
\neg \bigcirc \varphi & \leadsto & \bigcirc\neg\varphi \\
\neg(\varphi_1 \,\mathsf{U}\, \varphi_2) & \leadsto & (\neg\varphi_2)\,\mathsf{W}(\neg\varphi_1 \wedge \neg\varphi_2)
\end{array}
$$

exponential-blow up is possible

Example: LTL $\rightsquigarrow$ LTL-PNF

$$
\begin{array}{lll}
\neg \mathit{true} & \rightsquigarrow & \mathit{false} \\
\neg\neg\varphi & \rightsquigarrow & \varphi \\
\neg(\varphi_1 \wedge \varphi_2) & \rightsquigarrow & \neg\varphi_1 \vee \neg\varphi_2 \\
\neg \bigcirc \varphi & \rightsquigarrow & \bigcirc\neg\varphi \\
\neg(\varphi_1 \,\mathsf{U}\, \varphi_2) & \rightsquigarrow & (\neg\varphi_2)\,\mathsf{W}(\neg\varphi_1 \wedge \neg\varphi_2)
\end{array}
$$

Example: LTL $\rightsquigarrow$ LTL-PNF

$\neg \mathit{true}$	$\rightsquigarrow$	false	+ analogue rule for $\neg \mathit{false}$
$\neg\neg\varphi$	$\rightsquigarrow$	φ	
$\neg(\varphi_1 \wedge \varphi_2)$	$\rightsquigarrow$	$\neg\varphi_1 \vee \neg\varphi_2$	+ analogue rule for $\neg\vee$
$\neg \bigcirc \varphi$	$\rightsquigarrow$	$\bigcirc\neg\varphi$	
$\neg(\varphi_1 \,\mathsf{U}\, \varphi_2)$	$\rightsquigarrow$	$(\neg\varphi_2)\,\mathsf{W}(\neg\varphi_1 \wedge \neg\varphi_2)$	

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$\neg(\varphi_1 \wedge \varphi_2)$	$\rightsquigarrow$	$\neg\varphi_1 \vee \neg\varphi_2$	+ analogue rule for $\neg\vee$
$\neg \bigcirc \varphi$	$\rightsquigarrow$	$\bigcirc\neg\varphi$	
$\neg(\varphi_1 \,\mathsf{U}\, \varphi_2)$	$\rightsquigarrow$	$(\neg\varphi_2)\,\mathsf{W}(\neg\varphi_1 \wedge \neg\varphi_2)$	
$\neg\Diamond\varphi$	$\rightsquigarrow$	$\Box\neg\varphi$	$\neg\Box\varphi \rightsquigarrow \Diamond\neg\varphi$

Example: LTL $\rightsquigarrow$ LTL-PNF

$$\neg \mathit{true} \rightsquigarrow \mathit{false} \quad + \text{ analogue rule for } \neg \mathit{false}$$

$$\neg\neg\varphi \rightsquigarrow \varphi$$

$$\neg(\varphi_1 \wedge \varphi_2) \rightsquigarrow \neg\varphi_1 \vee \neg\varphi_2 \quad + \text{ analogue rule for } \neg\vee$$

$$\neg \bigcirc \varphi \rightsquigarrow \bigcirc\neg\varphi$$

$$\neg(\varphi_1 \,\mathsf{U}\, \varphi_2) \rightsquigarrow (\neg\varphi_2)\,\mathsf{W}(\neg\varphi_1 \wedge \neg\varphi_2)$$

$$\neg\Diamond\varphi \rightsquigarrow \Box\neg\varphi \qquad \neg\Box\varphi \rightsquigarrow \Diamond\neg\varphi$$

$$\neg\Box((a \,\mathsf{U}\, b) \vee \bigcirc c)$$

Example: LTL $\rightsquigarrow$ LTL-PNF

$$
\begin{array}{lll}
\neg \mathit{true} & \rightsquigarrow & \mathit{false} \quad + \text{ analogue rule for } \neg \mathit{false} \\
\neg\neg\varphi & \rightsquigarrow & \varphi \\
\neg(\varphi_1 \wedge \varphi_2) & \rightsquigarrow & \neg\varphi_1 \vee \neg\varphi_2 \quad + \text{ analogue rule for } \neg\vee \\
\neg \bigcirc \varphi & \rightsquigarrow & \bigcirc \neg\varphi \\
\neg(\varphi_1 \,\mathsf{U}\, \varphi_2) & \rightsquigarrow & (\neg\varphi_2)\,\mathsf{W}(\neg\varphi_1 \wedge \neg\varphi_2) \\
\neg\Diamond\varphi & \rightsquigarrow & \Box\neg\varphi \qquad \neg\Box\varphi \rightsquigarrow \Diamond\neg\varphi
\end{array}
$$

$$\neg\Box((a \,\mathsf{U}\, b) \vee \bigcirc c)$$

$$\equiv \Diamond\neg((a \,\mathsf{U}\, b) \vee \bigcirc c) \qquad \longleftarrow \boxed{\text{duality of } \Diamond \text{ and } \Box}$$

Example: LTL $\rightsquigarrow$ LTL-PNF

$$
\begin{array}{lll}
\neg \mathit{true} & \rightsquigarrow & \mathit{false} \quad + \text{ analogue rule for } \neg \mathit{false} \\
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\neg(\varphi_1 \wedge \varphi_2) & \rightsquigarrow & \neg\varphi_1 \vee \neg\varphi_2 \quad + \text{ analogue rule for } \neg\vee \\
\neg \bigcirc \varphi & \rightsquigarrow & \bigcirc\neg\varphi \\
\neg(\varphi_1 \, \mathsf{U} \, \varphi_2) & \rightsquigarrow & (\neg\varphi_2)\, \mathsf{W} (\neg\varphi_1 \wedge \neg\varphi_2) \\
\neg\Diamond\varphi & \rightsquigarrow & \Box\neg\varphi \qquad \neg\Box\varphi \rightsquigarrow \Diamond\neg\varphi
\end{array}
$$

$$\neg\Box((a \, \mathsf{U} \, b) \vee \bigcirc c)$$

$$\equiv \Diamond\neg((a \, \mathsf{U} \, b) \vee \bigcirc c) \qquad \longleftarrow \boxed{\text{duality of } \Diamond \text{ and } \Box}$$

$$\equiv \Diamond(\neg(a \, \mathsf{U} \, b) \wedge \neg\bigcirc c) \qquad \longleftarrow \boxed{\text{duality of } \wedge \text{ and } \vee}$$

Example: LTL $\rightsquigarrow$ LTL-PNF

$\neg \mathit{true}$	$\rightsquigarrow$	false	+ analogue rule for $\neg \mathit{false}$
$\neg\neg\varphi$	$\rightsquigarrow$	φ	
$\neg(\varphi_1 \wedge \varphi_2)$	$\rightsquigarrow$	$\neg\varphi_1 \vee \neg\varphi_2$	+ analogue rule for $\neg\vee$
$\neg \bigcirc \varphi$	$\rightsquigarrow$	$\bigcirc\neg\varphi$	
$\neg(\varphi_1 \mathsf{U} \varphi_2)$	$\rightsquigarrow$	$(\neg\varphi_2) \mathsf{W} (\neg\varphi_1 \wedge \neg\varphi_2)$	
$\neg\Diamond\varphi$	$\rightsquigarrow$	$\Box\neg\varphi$	$\neg\Box\varphi \rightsquigarrow \Diamond\neg\varphi$

$$\neg\Box((a \mathsf{U} b) \vee \bigcirc c)$$

$$\equiv \Diamond\neg((a \mathsf{U} b) \vee \bigcirc c) \quad \leftarrow \boxed{\text{duality of } \Diamond \text{ and } \Box}$$

$$\equiv \Diamond(\neg(a \mathsf{U} b) \wedge \neg\bigcirc c) \quad \leftarrow \boxed{\text{duality of } \wedge \text{ and } \vee}$$

$$\equiv \Diamond(\neg(a \mathsf{U} b) \wedge \bigcirc\neg c) \quad \leftarrow \boxed{\text{self-duality of } \bigcirc}$$

Example: LTL $\rightsquigarrow$ LTL-PNF

$\neg \mathit{true}$	$\rightsquigarrow$	false	+ analogue rule for $\neg \mathit{false}$
$\neg\neg\varphi$	$\rightsquigarrow$	φ	
$\neg(\varphi_1 \wedge \varphi_2)$	$\rightsquigarrow$	$\neg\varphi_1 \vee \neg\varphi_2$	+ analogue rule for $\neg\vee$
$\neg \bigcirc \varphi$	$\rightsquigarrow$	$\bigcirc\neg\varphi$	
$\neg(\varphi_1 \mathsf{U} \varphi_2)$	$\rightsquigarrow$	$(\neg\varphi_2)\mathsf{W}(\neg\varphi_1 \wedge \neg\varphi_2)$	
$\neg\Diamond\varphi$	$\rightsquigarrow$	$\Box\neg\varphi$	$\neg\Box\varphi \rightsquigarrow \Diamond\neg\varphi$

$$\neg\Box((a \mathsf{U} b) \vee \bigcirc c)$$

$$\equiv \Diamond\neg((a \mathsf{U} b) \vee \bigcirc c) \quad \longleftarrow \boxed{\text{duality of } \Diamond \text{ and } \Box}$$

$$\equiv \Diamond(\neg(a \mathsf{U} b) \wedge \neg\bigcirc c) \quad \longleftarrow \boxed{\text{duality of } \wedge \text{ and } \vee}$$

$$\equiv \Diamond((\neg b)\mathsf{W}(\neg a \wedge \neg b) \wedge \bigcirc\neg c) \longleftarrow \boxed{\text{duality of } \mathsf{U} \text{ and } \mathsf{W}}$$

Example: LTL $\rightsquigarrow$ LTL-PNF

$$\begin{array}{lll}
\neg \mathit{true} & \rightsquigarrow & \mathit{false} \quad + \text{ analogue rule for } \neg \mathit{false} \\
\neg\neg\varphi & \rightsquigarrow & \varphi \\
\neg(\varphi_1 \wedge \varphi_2) & \rightsquigarrow & \neg\varphi_1 \vee \neg\varphi_2 \quad + \text{ analogue rule for } \neg\vee \\
\neg \bigcirc \varphi & \rightsquigarrow & \bigcirc \neg\varphi \\
\neg(\varphi_1 \, \mathsf{U} \, \varphi_2) & \rightsquigarrow & (\neg\varphi_2) \, \mathsf{W} (\neg\varphi_1 \wedge \neg\varphi_2) \\
\neg\Diamond\varphi & \rightsquigarrow & \Box\neg\varphi \qquad \neg\Box\varphi \rightsquigarrow \Diamond\neg\varphi
\end{array}$$

$$\begin{array}{ll}
 & \neg\Box((a \, \mathsf{U} \, b) \ \vee \ \bigcirc c) \\
\equiv & \Diamond\neg((a \, \mathsf{U} \, b) \ \vee \ \bigcirc c) \\
\equiv & \Diamond(\neg(a \, \mathsf{U} \, b) \ \wedge \ \neg\bigcirc c) \\
\equiv & \Diamond((\neg b) \, \mathsf{W} (\neg a \wedge \neg b) \ \wedge \ \bigcirc\neg c) \longleftarrow \boxed{\textbf{PNF}}
\end{array}$$

Fairness in LTL

Recall: action-based fairness

Recall: action-based fairness

fairness assumption for TS $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$:

$$\mathcal{F} = (\mathcal{F}_{ucond}, \mathcal{F}_{strong}, \mathcal{F}_{weak})$$

where $\mathcal{F}_{ucond}, \mathcal{F}_{strong}, \mathcal{F}_{weak} \subseteq 2^{Act}$

- $\mathcal{F}_{ucond}$ unconditional fairness assumption
- $\mathcal{F}_{strong}$ strong fairness assumption
- $\mathcal{F}_{weak}$ weak fairness assumption

fairness assumption for TS $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$:

$$\mathcal{F} \; = \; (\mathcal{F}_{ucond}, \mathcal{F}_{strong}, \mathcal{F}_{weak})$$

where $\mathcal{F}_{ucond}$, $\mathcal{F}_{strong}$, $\mathcal{F}_{weak} \subseteq 2^{Act}$

execution $s_0 \xrightarrow{\alpha_1} s_1 \xrightarrow{\alpha_2} s_2 \xrightarrow{\alpha_3} \ldots$ $\mathcal{F}$-fair if

Recall: action-based fairness

fairness assumption for TS $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$:

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execution $s_0 \xrightarrow{\alpha_1} s_1 \xrightarrow{\alpha_2} s_2 \xrightarrow{\alpha_3} \ldots$ $\mathcal{F}$-fair if

- for all $A \in \mathcal{F}_{ucond}$: $\overset{\infty}{\exists} i \geq 1. \alpha_i \in A$

Recall: action-based fairness

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$$\mathcal{F} = (\mathcal{F}_{ucond}, \mathcal{F}_{strong}, \mathcal{F}_{weak})$$

where $\mathcal{F}_{ucond}$, $\mathcal{F}_{strong}$, $\mathcal{F}_{weak} \subseteq 2^{Act}$

execution $s_0 \xrightarrow{\alpha_1} s_1 \xrightarrow{\alpha_2} s_2 \xrightarrow{\alpha_3} \ldots$ $\mathcal{F}$-fair if

- for all $A \in \mathcal{F}_{ucond}$: $\overset{\infty}{\exists}\, i \geq 1.\, \alpha_i \in A$
- for all $A \in \mathcal{F}_{strong}$:

$$\overset{\infty}{\exists}\, i \geq 1.\, A \cap Act(s_i) \neq \varnothing \implies \overset{\infty}{\exists}\, i \geq 1.\, \alpha_i \in A$$

Recall: action-based fairness

fairness assumption for TS $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$:

$$\mathcal{F} \;=\; (\mathcal{F}_{ucond}, \mathcal{F}_{strong}, \mathcal{F}_{weak})$$

where $\mathcal{F}_{ucond}$, $\mathcal{F}_{strong}$, $\mathcal{F}_{weak} \subseteq 2^{Act}$

execution $s_0 \xrightarrow{\alpha_1} s_1 \xrightarrow{\alpha_2} s_2 \xrightarrow{\alpha_3} \ldots$ $\mathcal{F}$-fair if

- for all $A \in \mathcal{F}_{ucond}$: $\overset{\infty}{\exists}\, i \geq 1.\, \alpha_i \in A$
- for all $A \in \mathcal{F}_{strong}$:

  $$\overset{\infty}{\exists}\, i \geq 1.\, A \cap Act(s_i) \neq \varnothing \implies \overset{\infty}{\exists}\, i \geq 1.\, \alpha_i \in A$$
- for all $A \in \mathcal{F}_{weak}$:

  $$\overset{\infty}{\forall}\, i \geq 1.\, A \cap Act(s_i) \neq \varnothing \implies \overset{\infty}{\exists}\, i \geq 1.\, \alpha_i \in A$$

Recall: action-based fairness

fairness assumption for TS $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$:

$$\mathcal{F} = (\mathcal{F}_{ucond}, \mathcal{F}_{strong}, \mathcal{F}_{weak})$$

where $\mathcal{F}_{ucond}$, $\mathcal{F}_{strong}$, $\mathcal{F}_{weak} \subseteq 2^{Act}$

satisfaction relation for LT-properties under fairness:

$\mathcal{T} \models_{\mathcal{F}} E$ iff for all $\mathcal{F}$-fair paths π of $\mathcal{T}$: $trace(\pi) \in E$

Process fairness is LTL-definable

Process fairness is LTL-definable

$$\varphi \ ::= \ \mathit{true} \ \big| \ a \ \big| \ \varphi_1 \wedge \varphi_2 \ \big| \ \neg\varphi \ \big| \ \bigcirc \varphi \ \big| \ \varphi_1 \mathsf{U} \varphi_2$$

eventually $\Diamond\varphi \stackrel{\text{def}}{=} \mathit{true}\, \mathsf{U}\, \varphi$

always $\Box\varphi \stackrel{\text{def}}{=} \neg\Diamond\neg\varphi$

infinitely often $\Box\Diamond\varphi$

eventually forever $\Diamond\Box\varphi$

Process fairness is LTL-definable

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eventually $\Diamond\varphi \overset{\text{def}}{=} \mathit{true}\,\mathsf{U}\,\varphi$

always $\Box\varphi \overset{\text{def}}{=} \neg\Diamond\neg\varphi$

infinitely often $\Box\Diamond\varphi$

eventually forever $\Diamond\Box\varphi$

e.g., unconditional fairness $\Box\Diamond \mathit{crit}_i$

strong fairness $\Box\Diamond \mathit{wait}_i \rightarrow \Box\Diamond \mathit{crit}_i$

Process fairness is LTL-definable

$$\varphi \ ::= \ \mathit{true} \mid a \mid \varphi_1 \wedge \varphi_2 \mid \neg\varphi \mid \bigcirc \varphi \mid \varphi_1 \,\mathsf{U}\, \varphi_2$$

eventually	$\Diamond\varphi$	$\stackrel{\text{def}}{=}$	$\mathit{true}\,\mathsf{U}\,\varphi$
always	$\Box\varphi$	$\stackrel{\text{def}}{=}$	$\neg\Diamond\neg\varphi$
infinitely often	$\Box\Diamond\varphi$		
eventually forever	$\Diamond\Box\varphi$		

e.g.,	unconditional fairness	$\Box\Diamond \mathit{crit}_i$
	strong fairness	$\Box\Diamond \mathit{wait}_i \rightarrow \Box\Diamond \mathit{crit}_i$
	weak fairness	$\Diamond\Box \mathit{wait}_i \rightarrow \Box\Diamond \mathit{crit}_i$

LTL fairness assumptions

LTL fairness assumptions

. . . are conjunctions of LTL formulas of the form:

- unconditional fairness $\Box\Diamond\phi$
- strong fairness $\Box\Diamond\phi_1 \rightarrow \Box\Diamond\phi_2$
- weak fairness $\Diamond\Box\phi_1 \rightarrow \Box\Diamond\phi_2$

where ϕ_1, ϕ_2, ϕ are propositional formulas

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If *fair* is a LTL fairness assumption, s a state in a TS, and φ an LTL formula then

$$s \models_{fair} \varphi \quad \text{iff} \quad \text{for all } \pi \in Paths(s): \text{ if } \pi \models fair \text{ then } \pi \models \varphi$$

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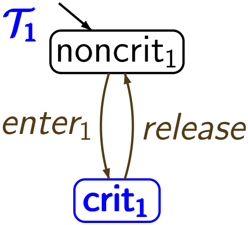
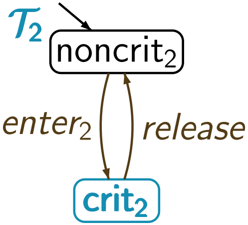
If *fair* is a LTL fairness assumption, s a state in a TS, and φ an LTL formula then

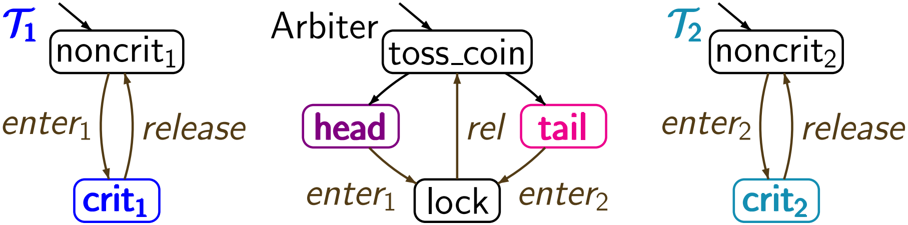
$$\begin{array}{lll} s \models_{fair} \varphi & \text{iff} & \text{for all } \pi \in Paths(s): \\ & & \quad \text{if } \pi \models fair \text{ then } \pi \models \varphi \\ & \text{iff} & s \models fair \rightarrow \varphi \end{array}$$

Randomized arbiter for MUTEX





Randomized arbiter for MUTEX



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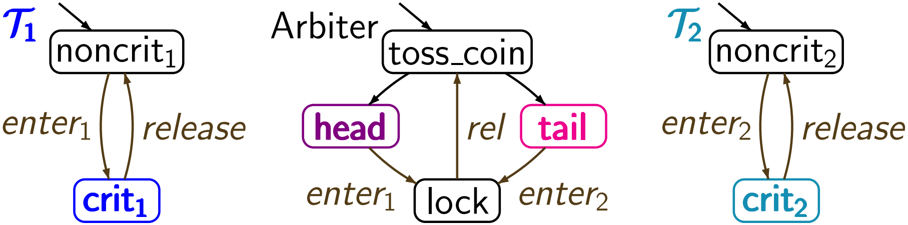
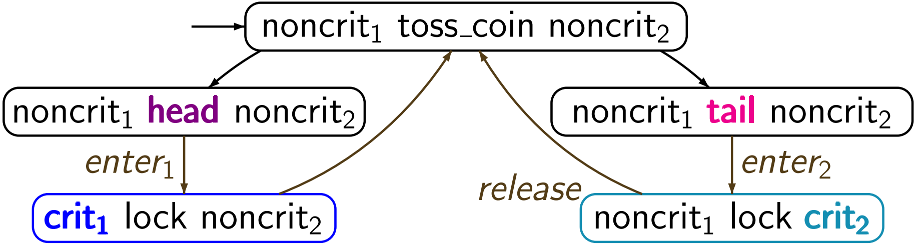
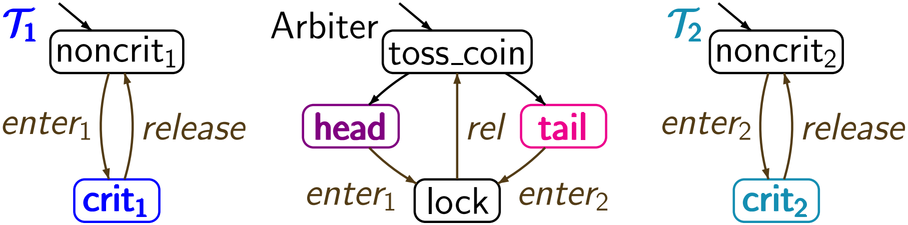


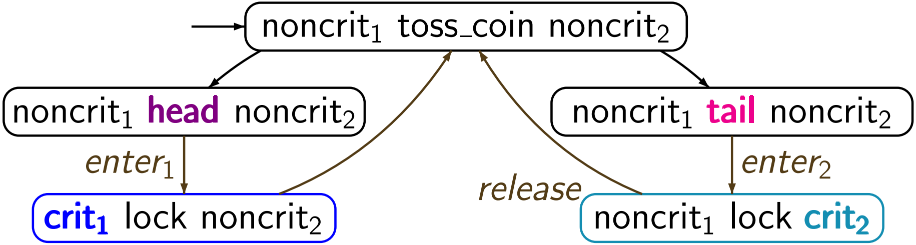
$(\mathcal{T}_1 \mid\mid\mid \mathcal{T}_2) \parallel \textit{Arbiter}$



Randomized arbiter for MUTEX



$$(\mathcal{T}_1 \mid\mid\mid \mathcal{T}_2) \parallel \mathit{Arbiter} \not\models \Box\Diamond \mathsf{crit}_1 \wedge \Box\Diamond \mathsf{crit}_2$$



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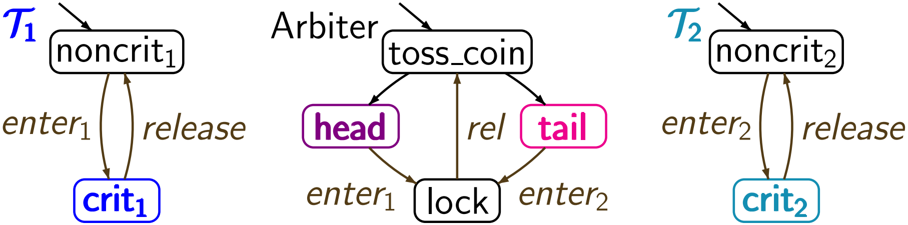
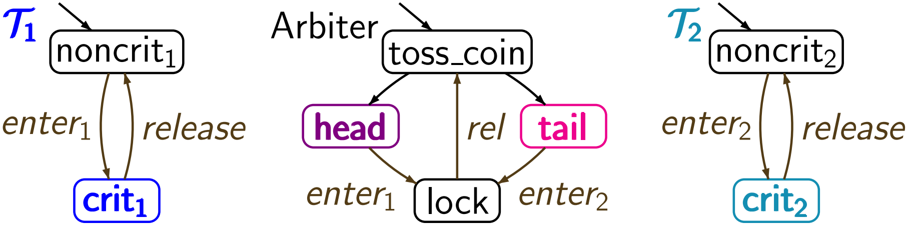


unconditional LTL-fairness:

$$fair \; = \; \Box\Diamond \textbf{head} \; \wedge \; \Box\Diamond \textbf{tail}$$

Randomized arbiter for MUTEX



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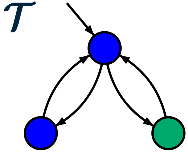
$$fair = \Box\Diamond \mathbf{head} \wedge \Box\Diamond \mathbf{tail}$$

$$(\mathcal{T}_1 \,|||\, \mathcal{T}_2) \,\|\, Arbiter \models_{fair} \Box\Diamond \mathbf{crit_1} \wedge \Box\Diamond \mathbf{crit_2}$$

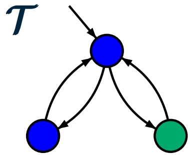
Correct or wrong?



LTL fairness assumption
$fair = \Diamond\Box a \rightarrow \Box\Diamond b$

● $\widehat{=}\{a\}$ ● $\widehat{=}\{b\}$

Correct or wrong?



LTL fairness assumption
$fair = \Diamond\Box a \rightarrow \Box\Diamond b$

$\bullet \;\widehat{=}\; \{a\}$ $\bullet \;\widehat{=}\; \{b\}$

$\mathcal{T} \models_{fair} \bigcirc b$ **?**

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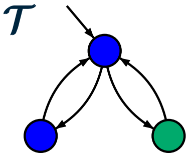
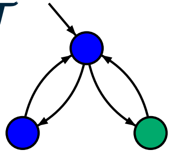


LTL fairness assumption
$fair = \Diamond\Box a \rightarrow \Box\Diamond b$

● $\widehat{=}\{a\}$ ● $\widehat{=}\{b\}$

$\mathcal{T} \not\models_{fair} \bigcirc b$ as ●→●→●→●→●→●→ ... is fair

$\mathcal{T}$



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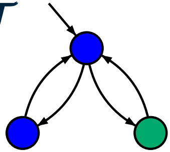
$\mathcal{T} \models_{fair} a \, \mathsf{U} \, b$ **?**

Correct or wrong?

$\mathcal{T}$



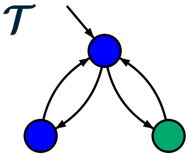
LTL fairness assumption
$fair = \Diamond\Box a \rightarrow \Box\Diamond b$

● (blue) $\widehat{=}\{a\}$ ● (green) $\widehat{=}\{b\}$

$\mathcal{T} \not\models_{fair} \bigcirc b$ as ●→●→●→●→●→●→ . . . is fair

$\mathcal{T} \models_{fair} a \, \mathsf{U} \, b$ $\surd$

Correct or wrong?



LTL fairness assumption

$fair = \Diamond\Box a \to \Box\Diamond b$

$\bullet \mathrel{\widehat{=}} \{a\} \quad \bullet \mathrel{\widehat{=}} \{b\}$

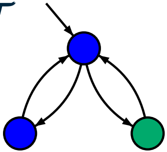
$\mathcal{T} \not\models_{fair} \bigcirc b$ as $\bullet \to \bullet \to \bullet \to \bullet \to \bullet \to \bullet \to \ldots$ is fair

$\mathcal{T} \models_{fair} a \,\mathsf{U}\, b$ $\checkmark$

$\mathcal{T} \models_{fair} a \,\mathsf{U}\, \Box(b \leftrightarrow \bigcirc a)$ **?**

Correct or wrong?

$\mathcal{T}$



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as ● → ● → ● → ● → ● → ● → ... is fair

- can be necessary to prove liveness properties, e.g., mutual exclusion with arbiter/semaphore

$$\mathcal{T}_{sem} \not\models \Box\Diamond crit_1 \wedge \Box\Diamond crit_2$$

$$\mathcal{T}_{sem} \models_{fair} \Box\Diamond crit_1 \wedge \Box\Diamond crit_2$$

for appropriate fairness condition

LTL-fairness assumptions

- can be necessary to prove liveness properties, e.g., mutual exclusion with arbiter/semaphore

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$$\mathcal{T}_{sem} \models_{fair} \Box\Diamond crit_1 \wedge \Box\Diamond crit_2$$

for appropriate fairness condition, e.g.,

$$fair = \bigwedge_{i=1,2} \big((\Box\Diamond wait_i \rightarrow \Box\Diamond crit_i) \wedge (\Diamond\Box noncrit_i \rightarrow \Box\Diamond wait_i) \big)$$

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for appropriate fairness condition

- can be verifiable system properties

e.g., Peterson algorithm guarantees strong fairness

$$\mathcal{T}_{Pet} \models \Box\Diamond wait_1 \rightarrow \Box\Diamond crit_1$$

LTL-fairness assumptions

- can be necessary to prove liveness properties, e.g.,

  $$\mathcal{T}_{sem} \not\models \Box\Diamond crit_1 \wedge \Box\Diamond crit_2$$
  $$\mathcal{T}_{sem} \models_{fair} \Box\Diamond crit_1 \wedge \Box\Diamond crit_2$$

  for appropriate fairness condition

- can be verifiable system properties, e.g.,

  $$\mathcal{T}_{Pet} \models \Box\Diamond wait_1 \rightarrow \Box\Diamond crit_1$$

- are irrelevant for verifying safety properties

> $\mathcal{T} \models \varphi_{safe}$ iff $\mathcal{T} \models_{fair} \varphi_{safe}$
>
> if *fair* is realizable

Correct or wrong?

Each strong **LTL** fairness assumption

$$fair = \Box\Diamond a \rightarrow \Box\Diamond b$$

is realizable for each TS over $AP = \{a, b, \ldots\}$.

Correct or wrong?

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recall: a fairness condition is called realizable
if for each reachable state s there exists
a fair path starting in s

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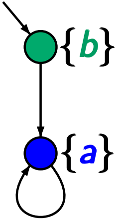
wrong



$fair = \Box\Diamond a \rightarrow \Box\Diamond b$

is <u>not</u> realizable

Action-based fairness $\rightsquigarrow$ LTL-fairness

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idea: use new atomic propositions ***enabled*(*A*)** and ***taken*(*A*)** and extend the labeling function:

$$\mathit{enabled}(A) \in L(s) \quad \text{iff} \quad s \xrightarrow{\alpha} \ldots \text{ for some } \alpha \in A$$

$$\mathit{taken}(A) \in L(s) \quad \text{iff} \quad \text{for all transitions } \ldots \xrightarrow{\alpha} s\text{: } \alpha \in A$$

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- unconditional A-fairness: $\Box\Diamond taken(A)$
- strong A-fairness: $\Box\Diamond enabled(A) \rightarrow \Box\Diamond taken(A)$
- weak A-fairness: $\Diamond\Box enabled(A) \rightarrow \Box\Diamond taken(A)$

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problem: each state s can have several incoming transitions

$$t \xrightarrow{\alpha} s, \quad u \xrightarrow{\beta} s, \; \ldots$$

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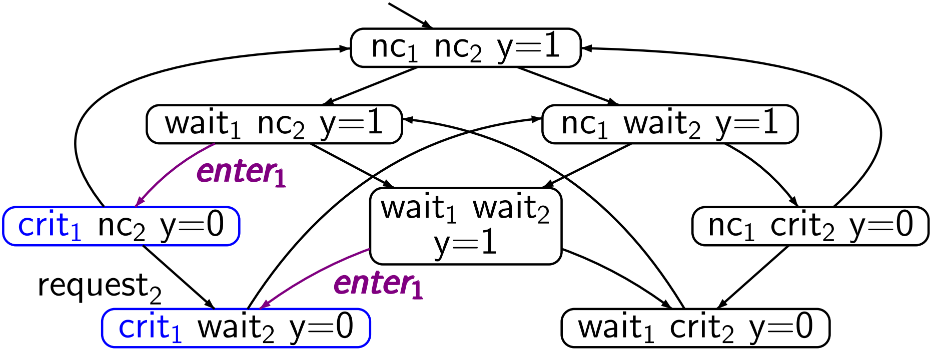
alternative 1: ad-hoc choice of "***taken***-predicate"

alternative 2: modify the given transition system by adding an action component to the states

Ad-hoc: action fairness ⇝ LTL-fairness



TS for mutual exclusion with semaphore

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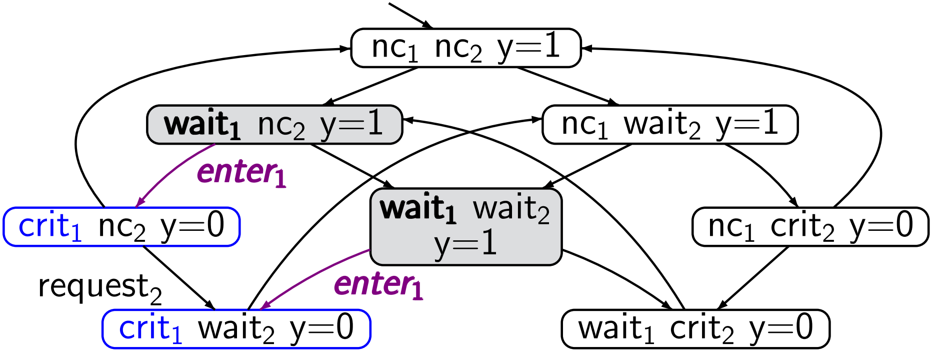

$\widehat{=}$ *enabled*({*enter*$_1$})

TS for mutual exclusion with semaphore

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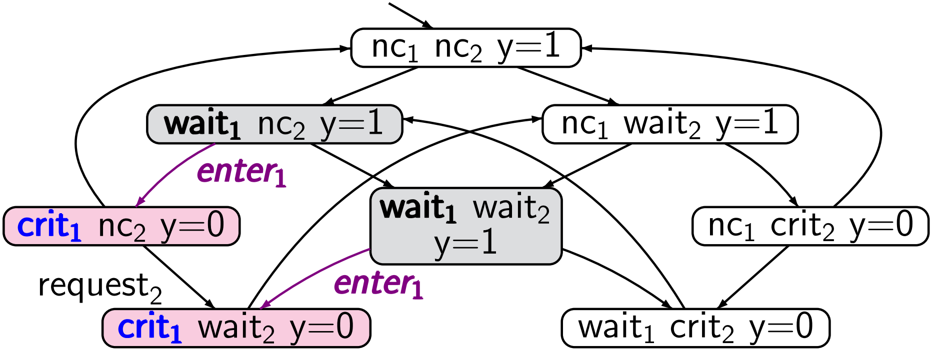

■ $\widehat{=}$ *taken*({*enter*$_1$})

■ $\widehat{=}$ *enabled*({*enter*$_1$})

TS for mutual exclusion with semaphore

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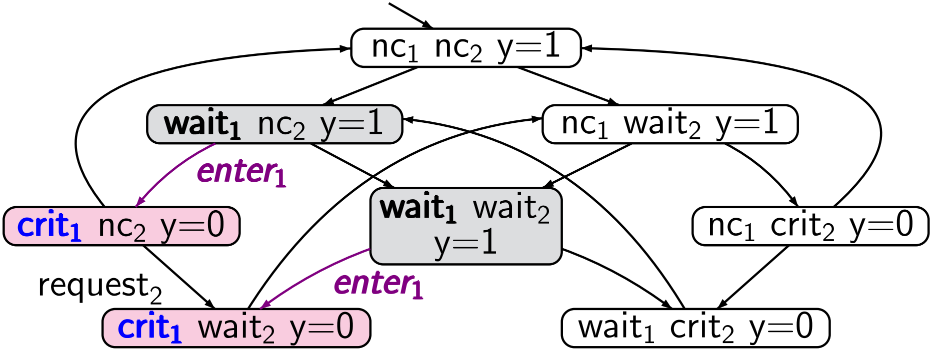
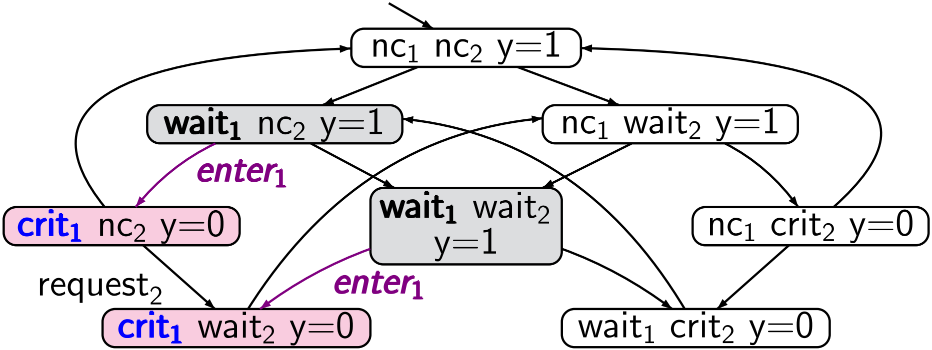


■ (pink)	$\widehat{=}$	*taken*({*enter*$_1$})	$\widehat{=}$	**crit$_1$**
■ (grey)	$\widehat{=}$	*enabled*({*enter*$_1$})	$\widehat{=}$	**wait$_1$** $\wedge \neg$**crit$_2$**

Ad-hoc: action fairness ⇝ LTL-fairness



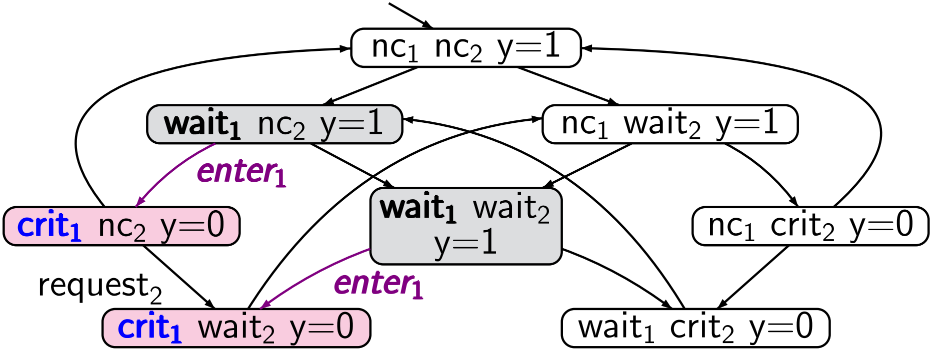
$\widehat{=}$ *taken*({*enter*$_1$}) $\widehat{=}$ **crit$_1$**

$\widehat{=}$ *enabled*({*enter*$_1$}) $\widehat{=}$ **wait$_1$** $\wedge\neg$**crit$_2$**

strong {*enter*$_1$}-fairness: LTL formula

$$\Box\Diamond\ \mathit{enabled}(\{\mathit{enter}_1\}) \rightarrow \Box\Diamond\ \mathit{taken}(\{\mathit{enter}_1\})$$

Ad-hoc: action fairness ⇝ LTL-fairness



$$\text{(pink)} \ \widehat{=} \ \mathit{taken}(\{\mathit{enter}_1\}) \ \widehat{=} \ \mathsf{crit}_1$$

$$\text{(grey)} \ \widehat{=} \ \mathit{enabled}(\{\mathit{enter}_1\}) \ \widehat{=} \ \mathsf{wait}_1 \wedge \neg \mathsf{crit}_2$$

$$\Box\Diamond\, \mathit{enabled}(\{\mathit{enter}_1\}) \rightarrow \Box\Diamond\, \mathit{taken}(\{\mathit{enter}_1\})$$

$$\widehat{=} \ \Box\Diamond\, (\mathsf{wait}_1 \wedge \neg \mathsf{crit}_2) \rightarrow \Box\Diamond\, \mathsf{crit}_1$$

Action-based fairness $\rightsquigarrow$ LTL-fairness

idea: use new atomic propositions $enabled(A)$ and $taken(A)$ and extend the labeling function:

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alternative 1: **ad-hoc choice** of "***taken***-predicate"

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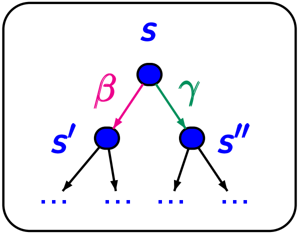
Action-based fairness $\rightsquigarrow$ LTL-fairness

transition system
$\mathcal{T} = (S, Act, \rightarrow, \ldots)$



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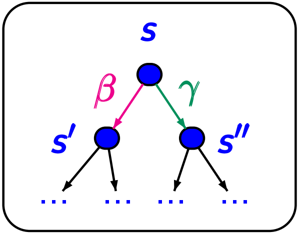
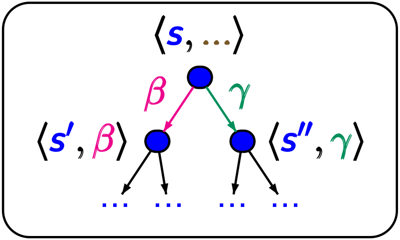


transition system
$\mathcal{T}' = (S \times Act, \ldots, AP', L')$



Action-based fairness $\rightsquigarrow$ LTL-fairness

transition system
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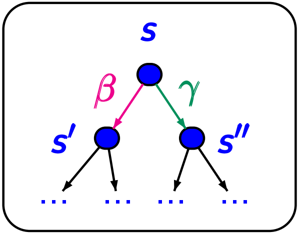

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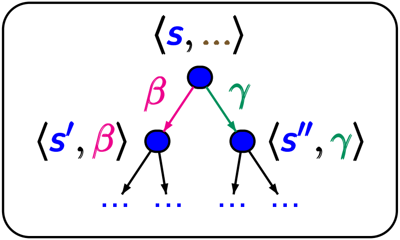
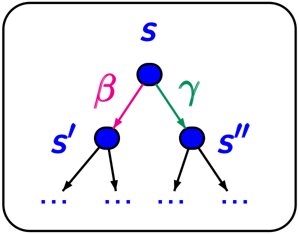


strong A-fairness
for $A \subseteq Act$

strong **LTL**-fairness
$\Box\Diamond enabled(A) \to \Box\Diamond taken(A)$

Action-based fairness $\rightsquigarrow$ LTL-fairness

transition system
$\mathcal{T} = (S, Act, \to, \ldots)$



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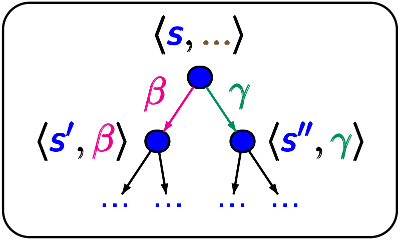


strong A-fairness
for $A \subseteq Act$

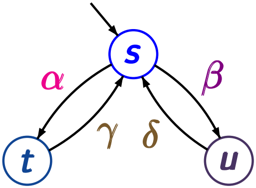
strong **LTL**-fairness
$\Box\Diamond enabled(A) \to \Box\Diamond taken(A)$

$enabled(A) \in L'(\langle s, \alpha \rangle)$ iff $s \xrightarrow{\beta} \ldots$ for some $\beta \in A$

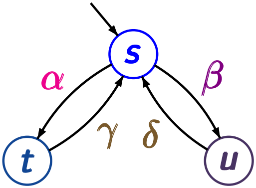
$taken(A) \in L'(\langle s, \alpha \rangle)$ iff $\alpha \in A$

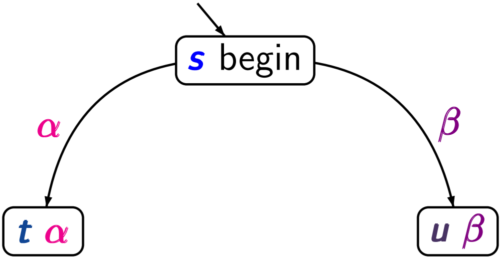
Example: action fairness ⇝ LTL-fairness

action-based fairness ⇝ LTL-fairness



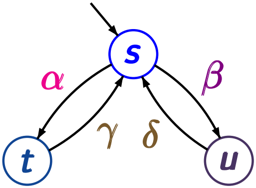
action-based fairness ⇝ LTL-fairness

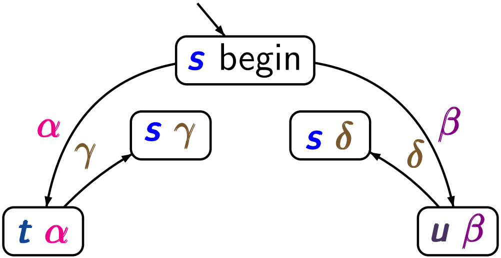




Example: action fairness ⇝ LTL-fairness

action-based fairness ⇝ LTL-fairness





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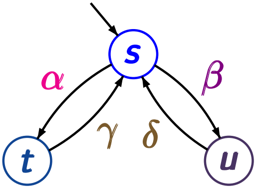

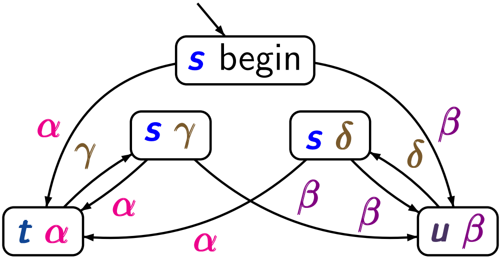

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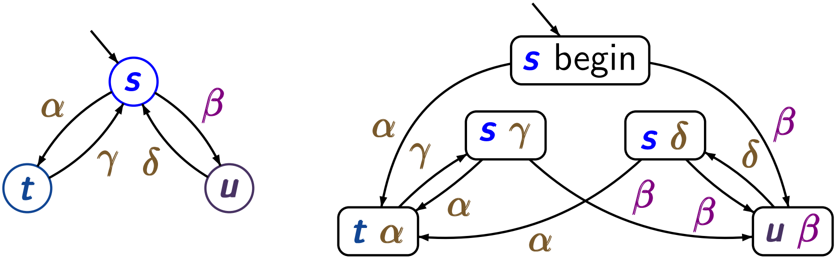
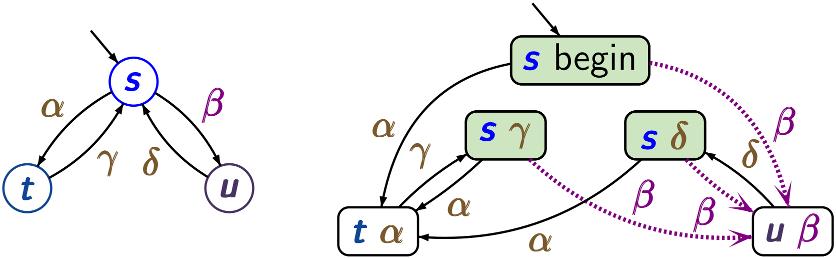


strong fairness for $\{\beta\}$:

$$\Box\Diamond\, enabled(\beta) \rightarrow \Box\Diamond\, taken(\beta)$$

Example: action fairness $\rightsquigarrow$ LTL-fairness

action-based fairness $\rightsquigarrow$ LTL-fairness



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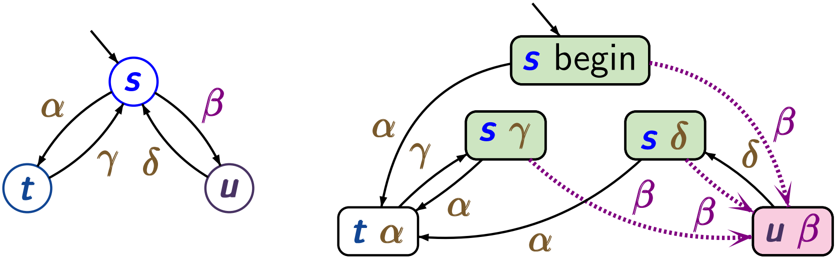
action-based fairness $\rightsquigarrow$ LTL-fairness



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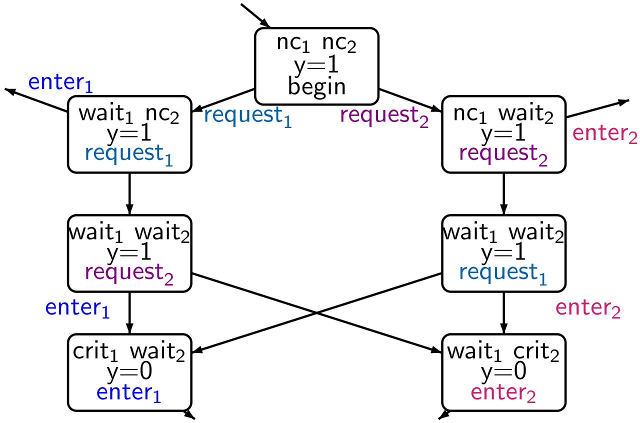
Example: mutual exclusion with semaphore

add additional variable last_action with domain Act $\cup$ {begin}



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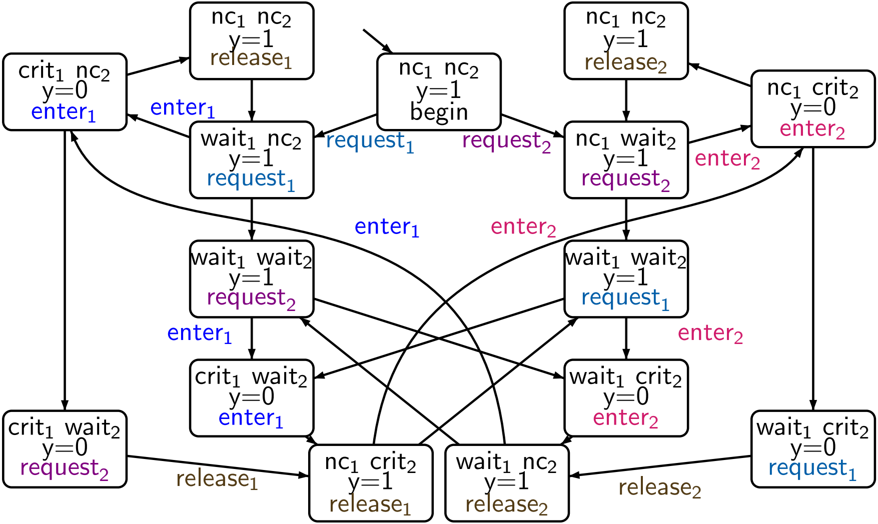

add additional variable last_action with domain $Act \cup \{begin\}$