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Exercise 1

a)

$Traces(TS) = \{\emptyset(\emptyset^+[\{a\}])^*\emptyset\{b\}\} \cup \{\emptyset^\omega\} \cup \{\emptyset(\emptyset^+\{a\})^\omega\} \cup \{\emptyset(\emptyset^+(\{a\}\emptyset)^*)^+\emptyset^\omega\}$
We have always enter the "second" state, which is the state reached by the initial state using τ. Now we have several options. We could either take the self loop infinitely times. If we take it only finitely many times (including 0) we could also go to the state labelled with $\{a\}$ and back. This could also be an infinite circle. Otherwise we again could take the self loop as often as we like. This process of selfloop-circle-selfloop can also be repeated infinitely often. We could also stop after having performed the desired amount of these loops and enter the state labelled with $\{b\}$ and have a finite trace.
So we denote, like in regular expressions, the ability to take the loops finitely or infinitely often with the Kleene-star.

b)

No, since $t = \emptyset\emptyset\emptyset\{b\} \in Traces(TS_1)$ but $t \notin Traces(TS_2)$

Exercise 2

a)

(i) $LT_{(i)} = \{A_0A_1A_2\cdots \in (2^AP)^\omega : \exists i \in \mathbb{N} \text{ s.t. } winter \in A_i\}$

(ii) $LT_{(ii)} = \{A_0A_1A_2\cdots \in (2^AP)^\omega : \forall i \in \mathbb{N} : awesome \in A_i\}$

(iii) $LT_{(iii)} = \{A_0A_1A_2\cdots \in (2^AP)^\omega : A_0 = \{here\} \wedge \exists i,j \in \mathbb{N}, 1 \leq i < j, A_i = \emptyset, A_j = \{here\}\}$

(iv) $LT_{(iv)} = \{A_0A_1A_2\cdots \in (2^AP)^\omega : A_0 = \{live, hero\} \wedge \exists i \in \mathbb{N} \text{ s.t. } A_i \in \{\{hero\},\{live\}\} \wedge \forall j \in \mathbb{N}, i \leq j A_i = A_j\}$

(v) $LT_{(v)} = \{A_0A_1A_2\cdots \in (2^AP)^\omega : A_0 = \{day, form\_1\} \wedge \forall i \in \mathbb{N} \setminus \{0\} :$
$(A_{i-1} = \{day, form\_1\} \Rightarrow A_i = \{night, form\_2\} \vee A_i = \{kiss\}) \wedge$
$(A_{i-1} = \{night, form\_2\} \Rightarrow A_i = \{day, form\_1\} \vee A_i = \{kiss\}) \wedge$
$(A_{i-1} = \{kiss\} \Rightarrow A_i = \{true\_form\}) \wedge$
$(A_{i-1} = \{true\_form\} \Rightarrow A_i = \{true\_form\})\}$

(vi) $LT_{(vi)} = \{A_0A_1A_2\cdots \in (2^AP)^\omega : (\exists i \in \mathbb{N}, A_i = \{in\_debt\}) \Rightarrow (\exists j \in \mathbb{N}, i < j, A_j = \emptyset)\}$

(vii) it's a valid statement. It contains every possible word $A \in 2^{AP^\omega}$, since anything and therefore every combination of $ap_1, \ldots, ap_n$ is possible

(viii) $LT_{(viii)} = \{A_0A_1A_2\cdots \in (2^AP)^\omega : A_0 = \{legen\} \wedge \exists n \in \mathbb{N}(A_n = \{dary\} \wedge \forall i \in \mathbb{N}, 0 < i < n, A_i = \{wait\_for\_it\})\}$

b)

(i)

is saftey	is liveness?
no	yes

Justification: every given word can be expanded such that *winter* is contained, by simply adding if it is not originally in the word.

(ii)

is saftey	is liveness?
yes	no

Justification: Once having a prefix of a word that in one particular state does not contain the AP *awesome* then we cannot satisfy this property any longer

(iii)

is saftey	is liveness?
no	no

Justification: There are two forms in which this LT can be violated

1. We dont start with *here*
2. We start with here, but never come back

We therefore cannot define one single prefix s.t. it holds for every word violating $LT_{(iii)}$. Also we can not find a suffix σ' to the word $\sigma = \emptyset$ s.t. $\sigma\sigma' \models LT_{(iii)}$. Therefore it is neither.

(iv)

is saftey	is liveness?
no	no

Justification: for $\omega = \emptyset$ we cannot find a continuation ω' s.t. $\omega\omega' \models LT_{(vi)}$.
But we can not find a prefix s.t. if we start with $\{live, here\}\{hero\}\{hero\}\ldots$ and continue with $\{hero\}$ finitely many times that we go back to a state $\{live, hero\}$. For a safety property such a prefix has to exists for every word not in $LT_{(vi)}$. So it's neither.

(v)

is saftey	is liveness?
yes	no

Justification: for $\omega = day \in BadPref_{(v)}$ we cannot find a continuation ω' s.t. $\omega\omega' \models LT_{(v)}$

(vi)

is saftey	is liveness?
no	yes

Justification: every finite word not containing *in_dept* already satisfies $LT_{(vi)}$. Every finite word containing ...

(vii)

is saftey	is liveness?
yes	yes

Justification: per definition in lecture 6 on slide 162.

(viii)

is saftey	is liveness?
yes	no

Justification: if the word has the prefix $\omega = dary$ we cannot find a continuation s.t. $\omega\omega' \models LT_{(viii)}$