Prof. Dr. Ir. Dr. h. c. Joost-Pieter Katoen
Christian Hensel, Matthias Volk

Introduction to Model Checking (Summer Term 2018)

— Exercise Sheet 9 (due 2nd July) —

General Remarks

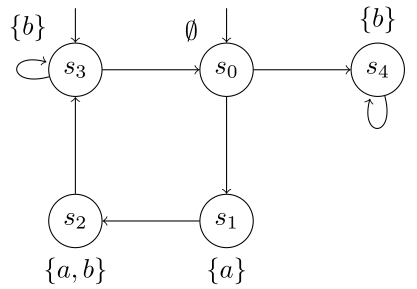
- The exercises are to be solved in groups of *three* students.
- You may hand in your solutions for the exercises just before the exercise class starts at 12:15 or by dropping them into the "Introduction to Model Checking" box at our chair *before 12:00*. Do *not* hand in your solutions via L2P or via e-mail.
- If a task asks you to justify your answer, an explanation of your reasoning is sufficient. If you are required to prove a statement, you need to give a *formal* proof.

Exercise 1 (4 Points)

Consider the following CTL formulae and the transition system TS outlined on the right:

$$\Phi_1 = \forall(a \mathsf{U} b) \vee \exists \bigcirc \forall\Box b$$
$$\Phi_2 = \forall\Box \forall(a \mathsf{U} b)$$
$$\Phi_3 = (a \wedge b) \rightarrow \exists\Box \exists \bigcirc \forall(b \mathsf{W} a)$$
$$\Phi_4 = \forall\Box \exists\Diamond \neg(a \vee b)$$

TS :



Give the satisfaction sets $Sat(\Phi_i)$ for each CTL formula Φ_i, $1 \leq i \leq 4$.
Does $\text{TS} \models \Phi_i$ hold?

Exercise 2 (5 Points)

Prove that $Sat(\exists(\Phi \mathsf{W} \Psi))$ is the largest set T such that

$$T \subseteq Sat(\Psi) \cup \{s \in Sat(\Phi) \mid \text{Post}(s) \cap T \neq \emptyset\}. \tag{9.1}$$

Exercise 3 (2 + 2 Points)

(a) Using an appropriate theorem from the lecture, prove that there does not exist an equivalent LTL-formula for the CTL-formula $\Phi_1 = \forall\Diamond\,(a \wedge \exists \bigcirc a)$.

(b) Now prove directly (i.e. without the theorem from the lecture) that there does not exist an equivalent LTL-formula for the CTL-formula $\Phi_2 = \forall\Diamond \exists \bigcirc \forall\Diamond \neg a$.
Hint: Argue by contraposition. In particular, think about trace inclusion versus CTL-equivalence.

Exercise 4★ (4 + 7* Points)

(a) For an arbitrary transition system $TS = (S, \mathrm{Act}, \rightarrow, S_0, \mathrm{AP}, L)$ without terminal states, let $\mathrm{Reach}_{\mathrm{TS}}(s)$ denote the states reachable from $s \in S$ in TS. In other words $s' \in \mathrm{Reach}_{\mathrm{TS}}(s)$ if and only if there exists a path $\pi = s_0 s_1 \ldots \in \mathit{Paths}(s)$ such that there exists $i \geq 0$ with $s_i = s'$. Show that for all $s \in S$ the following is equivalent:

1. $s \models_{\mathrm{LTL}} \Box a$,
2. $s \models_{\mathrm{CTL}} \forall \Box a$,
3. $\forall s' \in \mathrm{Reach}_{\mathrm{TS}}(s) \,.\, s' \models a$, and
4. $\forall s' \in \mathrm{Reach}_{\mathrm{TS}}(s) \,.\, s' \models_{\mathrm{CTL}} \forall \Box a$.

Hint: You may use the fact that for all $s, s', s'' \in S$ we have $s' \in \mathrm{Reach}_{\mathrm{TS}}(s) \wedge s'' \in \mathrm{Reach}_{\mathrm{TS}}(s') \implies s'' \in \mathrm{Reach}_{\mathrm{TS}}(s)$ or, equivalently, $s' \in \mathrm{Reach}_{\mathrm{TS}}(s) \implies \mathrm{Reach}_{\mathrm{TS}}(s') \subseteq \mathrm{Reach}_{\mathrm{TS}}(s)$.

(b) ***This subexercise does not count towards the total number of points that you can achieve. Not solving it does not decrease the percentage of points you achieved while solving it may increase it.**

Prove
$$\forall \Big(a \,\mathsf{U}\, (b \wedge \forall \Box a) \Big) \equiv \Box a \wedge \Diamond b.$$

Exercise 5★ (1+1+1 Points)

Consider the following CTL formulae:

$$\Phi_1 = \forall \bigcirc \Big(\exists(\neg a \,\mathsf{U}\, (b \wedge \neg c)) \vee \exists \Box \forall \bigcirc a \Big)$$
$$\Phi_2 = \forall(\neg a \,\mathsf{W}\, (b \rightarrow \forall \bigcirc c))$$

(a) Transform Φ_1 into PNF.

(b) Transform Φ_1 into ENF.

(c) Transform Φ_2 into ENF.