Overview

- Introduction
- Modelling parallel systems
- **Linear Time Properties**
  - state-based and linear time view
  - definition of linear time properties
  - invariants and safety
  - liveness and fairness ⟵
- Regular Properties
- Linear Temporal Logic
- Computation-Tree Logic
- Equivalences and Abstraction

“liveness: something good will happen.”

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"event a will occur eventually"

Liveness

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e.g., termination for sequential programs

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LF2.6-1

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"event ***a*** will occur infinitely many times"

e.g., starvation freedom for dining philosophers

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"whenever event b occurs then event a
will occur sometimes in the future"

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"event ***a*** will occur eventually"

e.g., termination for sequential programs

"event ***a*** will occur infinitely many times"

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"whenever event ***b*** occurs then event ***a***
will occur sometimes in the future"

e.g., every waiting process enters eventually
its critical section

which property type?

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- Whenever a philosopher eats then he has been thinking at some time before.

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- Whenever a philosopher eats then he will think some time afterwards.

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Liveness

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here: one just example for a formal definition
of liveness

Definition of liveness properties

LF2.6-DEF-LIVENESS

Definition of liveness properties

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recall: $pref(E) =$ set of all finite, nonempty prefixes of words in E

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Examples:

- each process will eventually enter its critical section
- each process will enter its critical section infinitely often
- whenever a process has requested its critical section then it will eventually enter its critical section

Examples for liveness properties

An LT property E over AP is called a liveness property if $pref(E) = \left(2^{AP}\right)^+$

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E = set of all infinite words $A_0\, A_1\, A_2 \ldots$ s.t.

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$$\forall i \in \{1, \ldots, n\} \; \forall j \geq 0. \, wait_i \in A_j \longrightarrow \exists k > j. \, crit_i \in A_k$$

Recall: safety properties, prefix closure

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iff $\forall \sigma \in (2^{AP})^\omega \setminus E \;\; \exists A_0 A_1 \ldots A_n \in pref(\sigma)$ s.t.

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iff $\mathit{cl}(E) = E$

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Show that:

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Show that:

- $E = SAFE \cap LIVE$ $\surd$
- $SAFE$ is a safety property as $cl(SAFE) = SAFE$
- $LIVE$ is a liveness property, i.e., $pref(LIVE) = (2^{AP})^{+}$

Being safe and live

Which LT properties are both a safety and a liveness property?

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- If E is a liveness property then

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If E is a safety property too, then $cl(E) = E$.

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- If E is a liveness property then

$$pref(E) = (2^{AP})^+$$

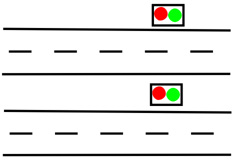
$$\Longrightarrow \quad cl(E) = (2^{AP})^\omega$$

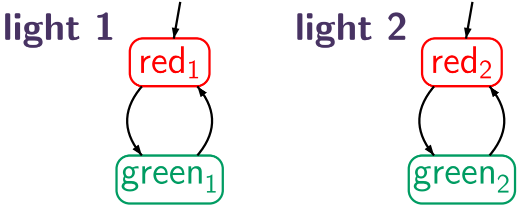
If E is a safety property too, then $cl(E) = E$.
Hence $E = cl(E) = (2^{AP})^\omega$.

Observation

liveness properties are often violated
although we expect them to hold

Two independent traffic lights





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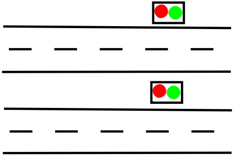
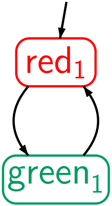
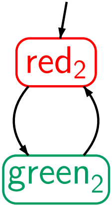


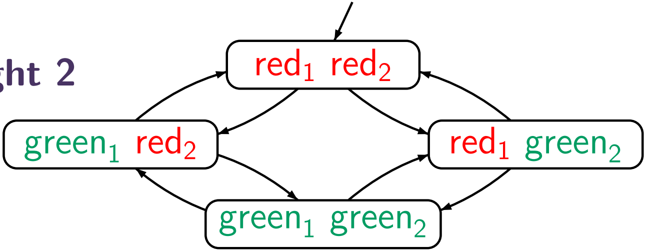
light 1



light 2



light 1 ||| light 2



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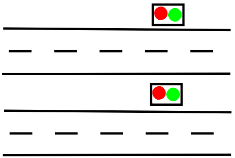

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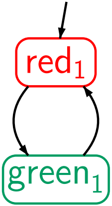

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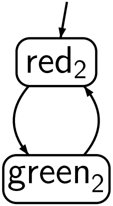

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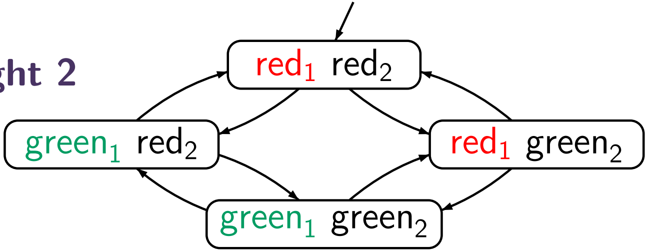
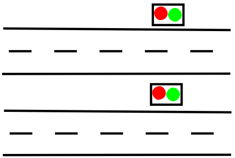


light 1 ||| light 2 $\not\models$ "infinitely often ***green$_1$***"

Two independent traffic lights



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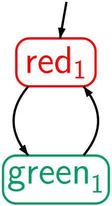

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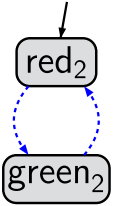

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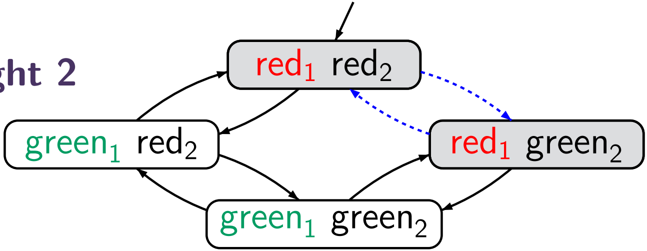

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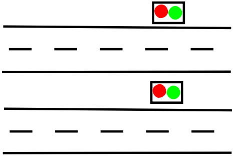

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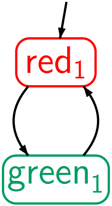

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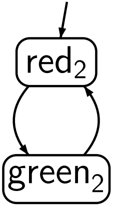

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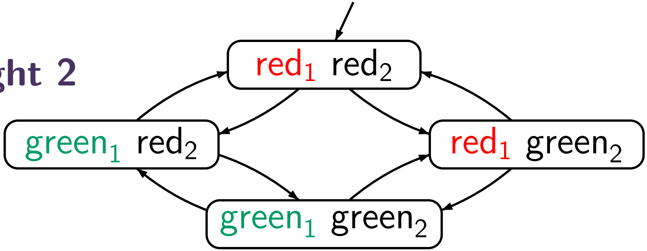
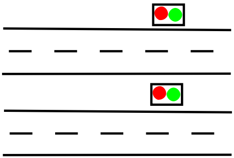


light 1 ||| light 2 $\not\models$ "infinitely often ***green$_1$***"

although **light 1** $\models$ "infinitely often ***green$_1$***"

Two independent traffic lights



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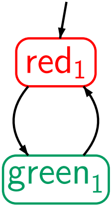

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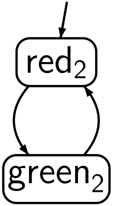

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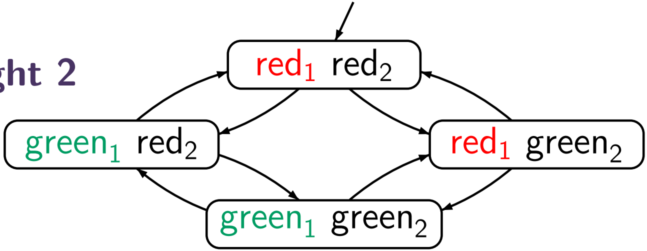

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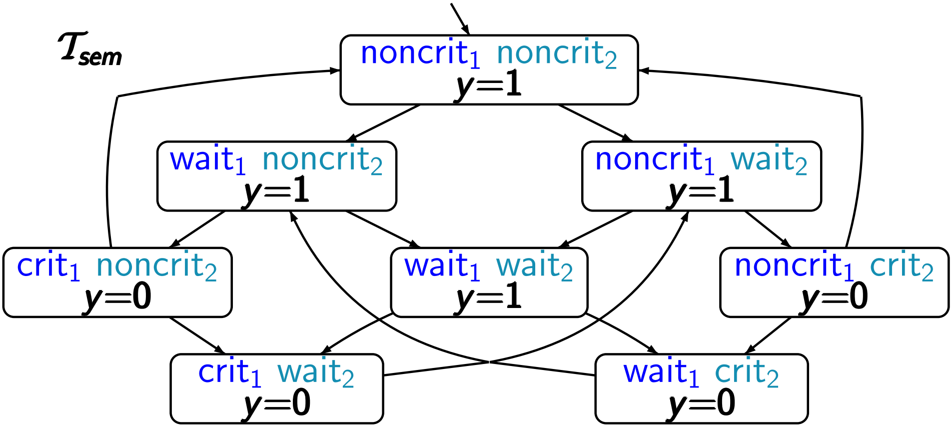
interleaving is completely time abstract **!**

Mutual exclusion (semaphore)



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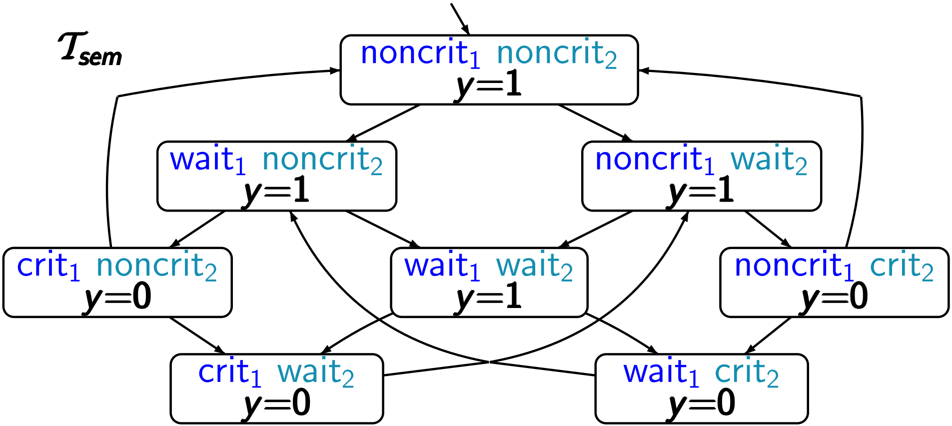

liveness property $\widehat{=}$ "each waiting process will eventually enter its critical section"

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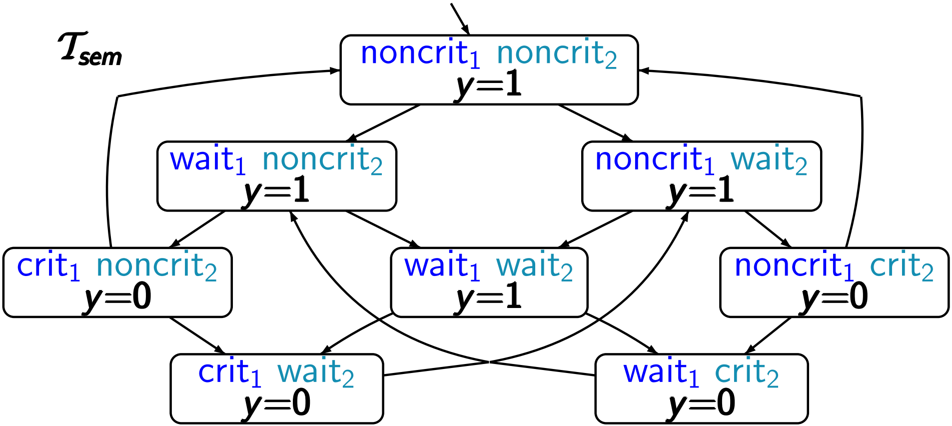

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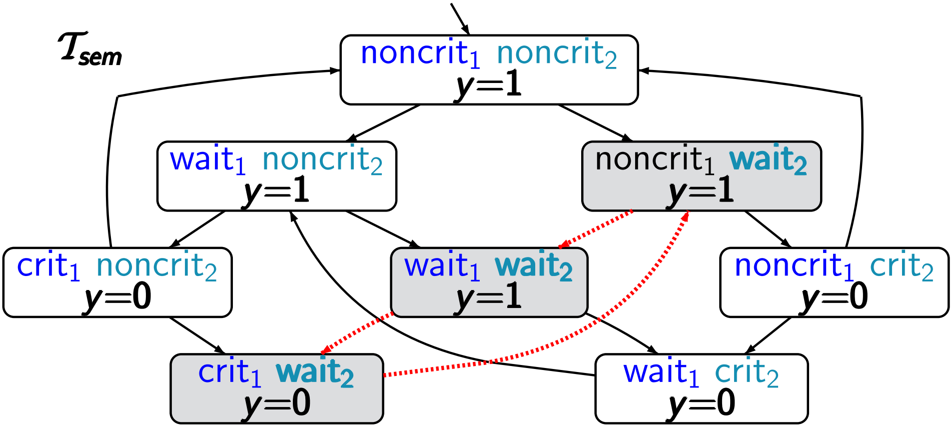

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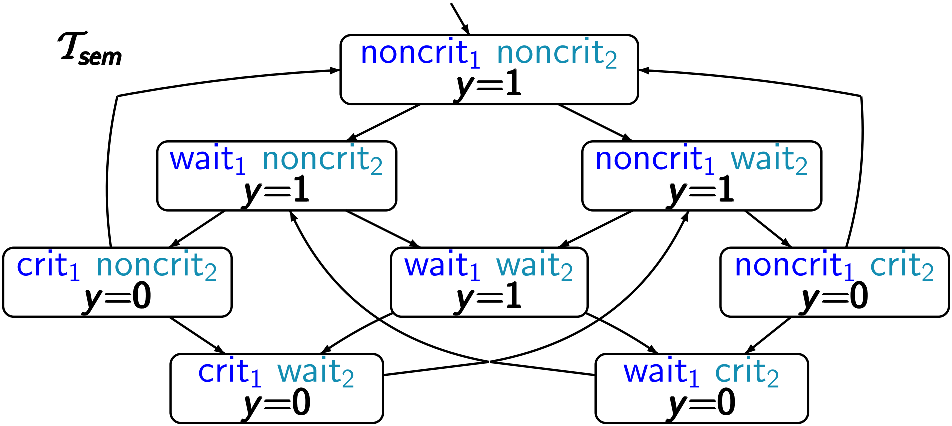

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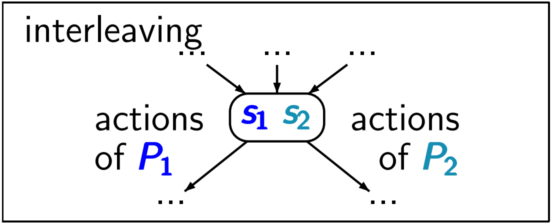
level of abstraction is too coarse !

Process fairness

LF2.6-5

two independent non-communicating processes $P_1 \mid\mid\mid P_2$



possible interleavings:

$P_1\ P_2\ P_2\ P_1\ P_1\ P_1\ P_2\ P_1\ P_2\ P_2\ P_2\ P_1\ P_1\ \ldots$

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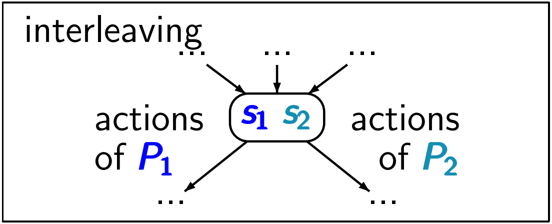
Process fairness

two independent non-communicating processes $P_1 \; ||| \; P_2$



possible interleavings:

$P_1 \; P_2 \; P_2 \; P_1 \; P_1 \; P_1 \; P_2 \; P_1 \; P_2 \; P_2 \; P_2 \; P_1 \; P_1 \; \ldots$
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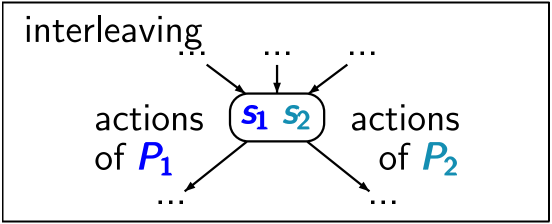
Process fairness

two independent non-communicating processes $P_1 \mid\mid\mid P_2$



possible interleavings:

P_1 P_2 P_2 P_1 P_1 P_1 P_2 P_1 P_2 P_2 P_2 P_1 P_1 ... fair

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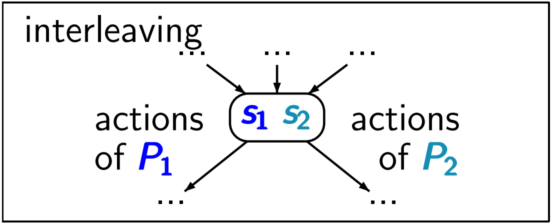
Process fairness

two independent non-communicating processes $P_1 \,|||\, P_2$



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$P_1\ P_1\ P_1\ P_1\ P_1\ P_1\ P_1\ P_1\ P_1\ P_1\ P_1\ P_1\ P_1 \ldots$ unfair

process fairness assumes an appropriate resolution of the nondeterminism resulting from interleaving and competitions

Nuances of fairness

- unconditional fairness

- strong fairness

- weak fairness

Nuances of fairness

- unconditional fairness, e.g.,

  every process enters gets its turn infinitely often.

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- strong fairness, e.g.,

  every process that is enabled infinitely often
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- weak fairness, e.g.,

  every process that is continuously enabled
  from a certain time instance on,
  gets its turn infinitely often.

Fairness for action-set

LF2.6-7

Fairness for action-set

Let $\mathcal{T}$ be a TS with action-set $\textit{Act}$, $A \subseteq \textit{Act}$ and

$\rho = s_0 \xrightarrow{\alpha_0} s_1 \xrightarrow{\alpha_1} s_2 \xrightarrow{\alpha_2} \ldots$ infinite execution fragment

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we will provide conditions for

- unconditional A-fairness of ρ
- strong A-fairness of ρ
- weak A-fairness of ρ

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using the following notations:

$$Act(s_i) \;=\; \{\beta \in Act : \exists s' \text{ s.t. } s_i \xrightarrow{\beta} s'\}$$

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$$\overset{\infty}{\exists} \ \widehat{=} \ \text{"there exists infinitely many ..."}$$

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$$\overset{\infty}{\exists} \;\widehat{=}\; \text{“there exists infinitely many ...”}$$

$$\overset{\infty}{\forall} \;\widehat{=}\; \text{“for all, but finitely many ...”}$$

Fairness for action-set

Let $\mathcal{T}$ be a TS with action-set Act, $A \subseteq Act$ and
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"actions in A will be taken infinitely many times"

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$$\overset{\infty}{\exists}\, i \geq 0.\, A \cap Act(s_i) \neq \varnothing \quad \Longrightarrow \quad \overset{\infty}{\exists}\, i \geq 0.\, \alpha_i \in A$$

"If infinitely many times some action in A is enabled, then actions in A will be taken infinitely many times."

Fairness for action-set

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$$\overset{\infty}{\forall} i \geq 0.\, A \cap Act(s_i) \neq \varnothing \quad \Longrightarrow \quad \overset{\infty}{\exists} i \geq 0.\, \alpha_i \in A$$

"If from some moment, actions in A are enabled, then actions in A will be taken infinitely many times."

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unconditionally A-fair $\Longrightarrow$ strongly A-fair
$\Longrightarrow$ weakly A-fair

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unconditionally A-fair	$\Longrightarrow$	strongly A-fair
	$\Longrightarrow$	weakly A-fair

Strong and weak action fairness

strong A-fairness is *violated* if

$$s_0 \longrightarrow s_1 \dashrightarrow s_2 \longrightarrow s_3 \longrightarrow s_4 \longrightarrow s_5 \longrightarrow s_6 \longrightarrow s_7 \longrightarrow s_8 \longrightarrow s_9 \longrightarrow \ldots$$

- no A-actions are executed from a certain moment
- A-actions are enabled infinitely many times

Strong and weak action fairness

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- no A-actions are executed from a certain moment
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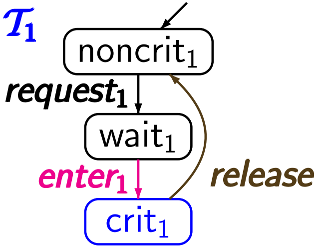
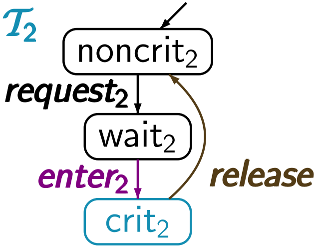
weak A-fairness is *violated* if

$$s_0 \longrightarrow s_1 \dashrightarrow s_2 \longrightarrow s_3 \longrightarrow s_4 \longrightarrow s_5 \longrightarrow s_6 \longrightarrow s_7 \longrightarrow s_8 \longrightarrow s_9 \longrightarrow \cdots$$

- no A-actions are executed from a certain moment
- A-actions are continuously enabled from some moment on

Mutual exclusion with arbiter





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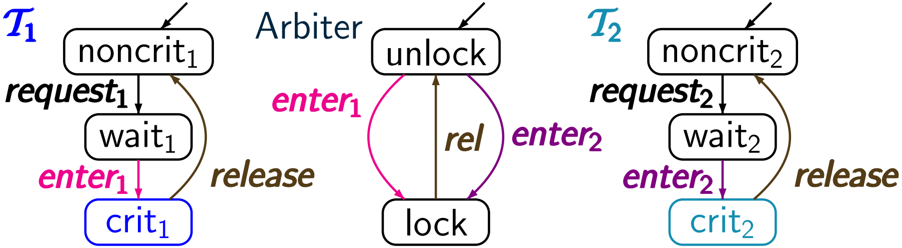

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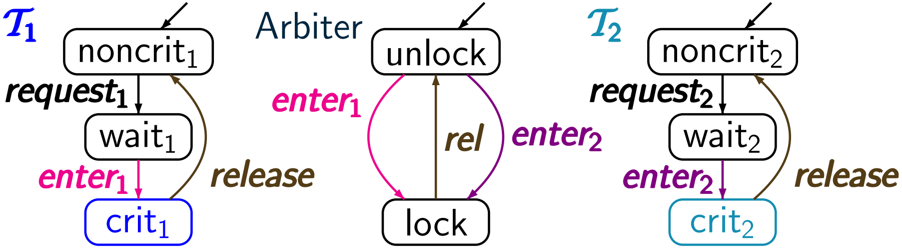
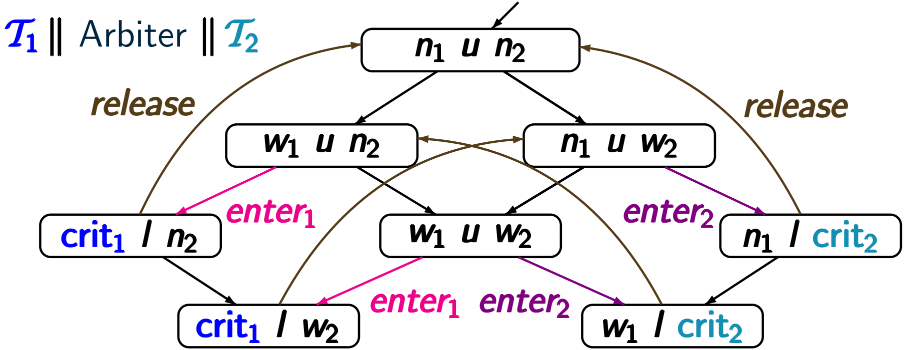
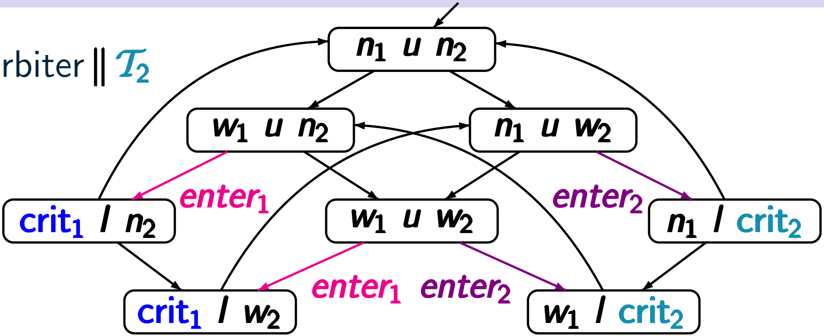




Unconditional, strongly or weakly fair?

$\mathcal{T}_1 \parallel \text{Arbiter} \parallel \mathcal{T}_2$



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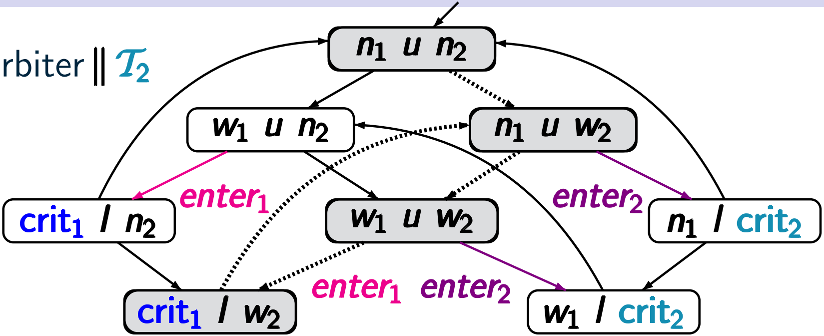

fairness for action set $A = \{enter_1\}$:

$$\langle n_1, u, n_2\rangle \longrightarrow \Big(\langle n_1, u, w_2\rangle \longrightarrow \langle w_1, u, w_2\rangle \longrightarrow \langle crit_1, l, w_2\rangle \Big)^{\omega}$$

- unconditional A-fairness:
- strong A-fairness:
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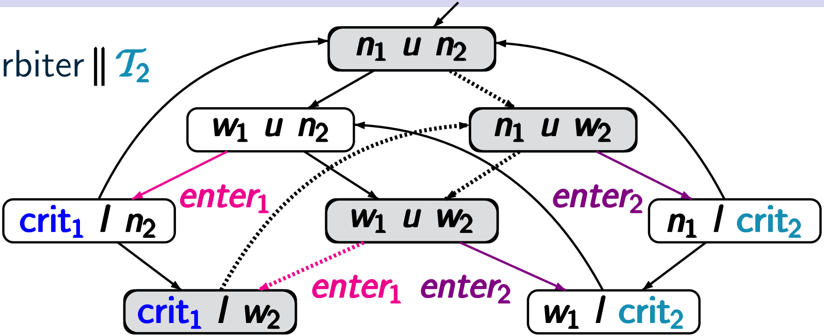

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- unconditional A-fairness: **yes**
- strong A-fairness: **yes** $\leftarrow$ unconditionally fair
- weak A-fairness: **yes** $\leftarrow$ unconditionally fair

Unconditional, strongly or weakly fair?

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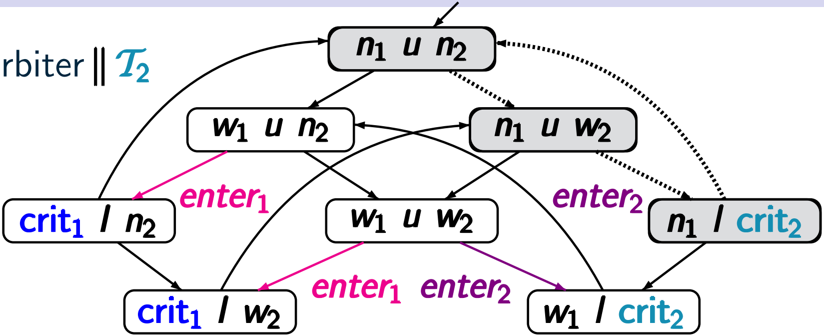

fairness for action-set $A = \{enter_1\}$:

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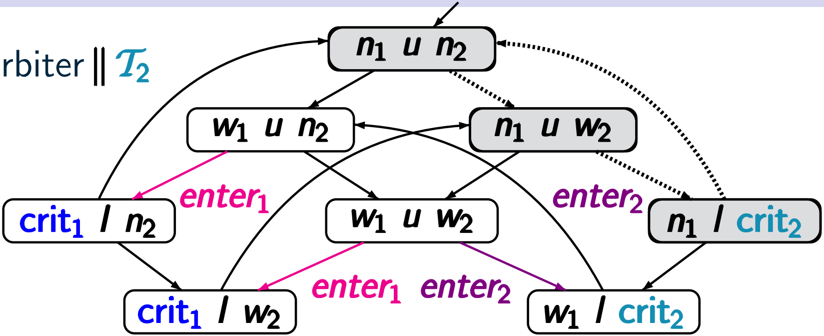

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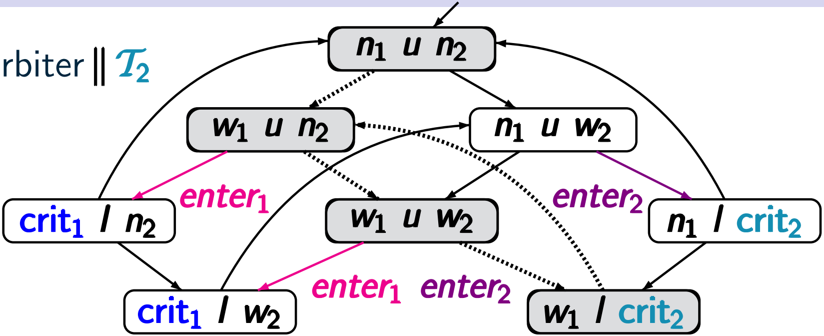
- unconditional A-fairness: **no**
- strong A-fairness: **yes** $\leftarrow$ A never enabled
- weak A-fairness: **yes** $\leftarrow$ strongly A-fair

Unconditional, strongly or weakly fair?

$\mathcal{T}_1 \parallel \text{Arbiter} \parallel \mathcal{T}_2$



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Unconditional, strongly or weakly fair?

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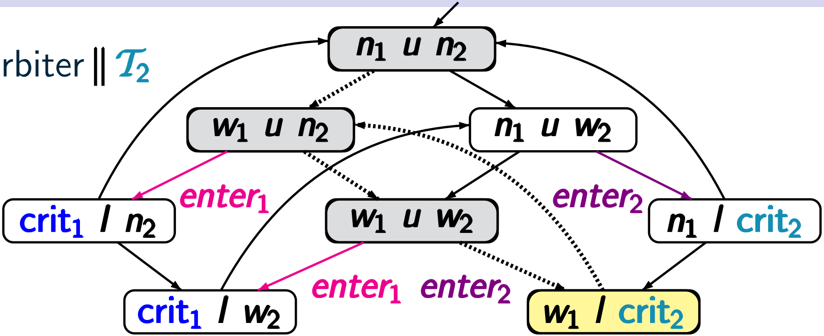

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- unconditional A-fairness: **no**
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Unconditional, strongly or weakly fair?

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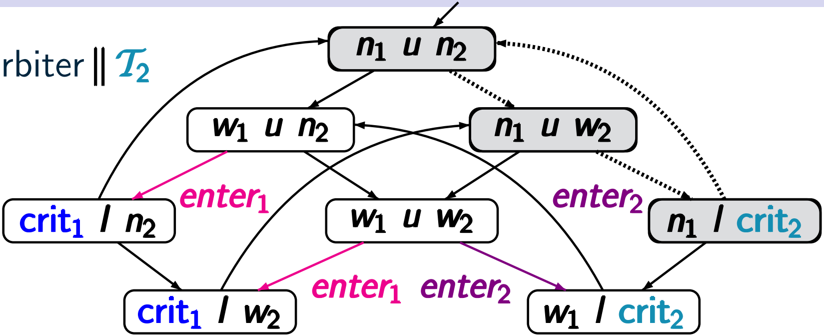

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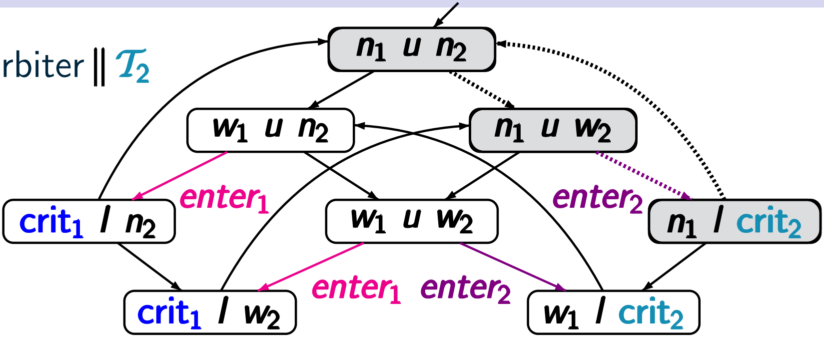

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- unconditional A-fairness: **yes**
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Action-based fairness assumptions

Action-based fairness assumptions

Let $\mathcal{T}$ be a transition system with action-set Act.
A fairness assumption for $\mathcal{T}$ is a triple

$$\mathcal{F} = (\mathcal{F}_{\mathit{ucond}}, \mathcal{F}_{\mathit{strong}}, \mathcal{F}_{\mathit{weak}})$$

where $\mathcal{F}_{\mathit{ucond}}$, $\mathcal{F}_{\mathit{strong}}$, $\mathcal{F}_{\mathit{weak}} \subseteq 2^{\mathit{Act}}$.

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An execution ρ is called $\mathcal{F}$-fair iff

- ρ is unconditionally A-fair for all $A \in \mathcal{F}_{ucond}$
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$$FairTraces_{\mathcal{F}}(\mathcal{T}) \stackrel{\text{def}}{=} \{ trace(\rho) : \rho \text{ is a } \mathcal{F}\text{-fair execution of } \mathcal{T} \}$$

Fair satisfaction relation

Fair satisfaction relation

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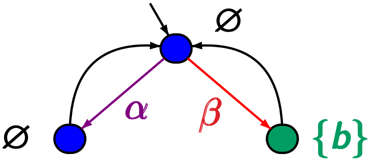
If $\mathcal{T}$ is a TS and E a LT property over AP then:

$$\mathcal{T} \models_{\mathcal{F}} E \quad \overset{\text{def}}{\Longleftrightarrow} \quad FairTraces_{\mathcal{F}}(\mathcal{T}) \subseteq E$$

Example: fair satisfaction relation



fairness assumption $\mathcal{F}$

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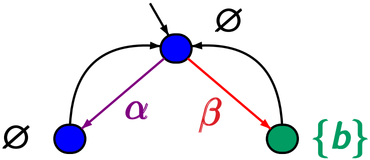

fairness assumption $\mathcal{F}$

- no unconditional fairness condition $\leftarrow \mathcal{F}_{ucond} = \varnothing$
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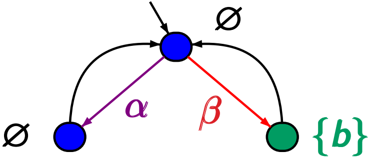
Example: fair satisfaction relation



$\mathcal{T} \models_{\mathcal{F}}$ "infinitely often b" ?

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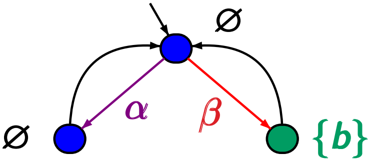

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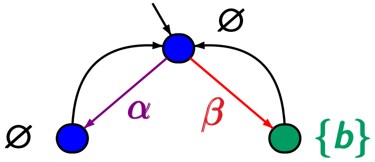

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- strong fairness for $\{\alpha, \beta\}$ $\leftarrow \mathcal{F}_{strong} = \{\{\alpha, \beta\}\}$
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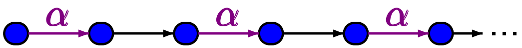
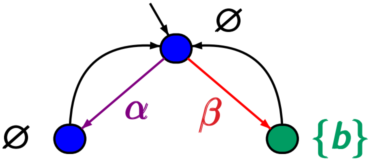


$\mathcal{F}$-fair

actions in $\{\alpha, \beta\}$ are executed infinitely many times

Example: fair satisfaction relation



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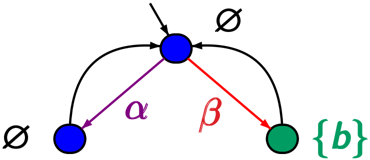

$\mathcal{T} \models_{\mathcal{F}}$ "infinitely often b" ?

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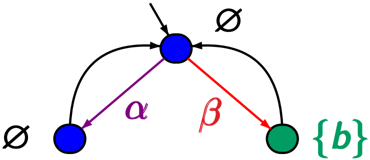

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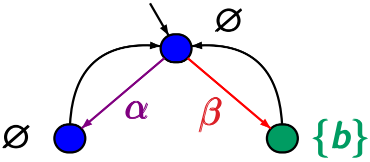

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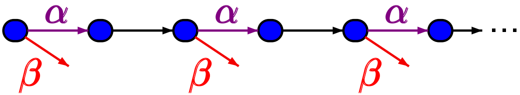

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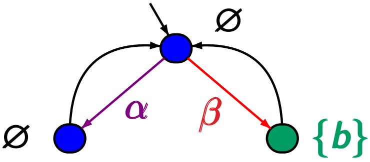

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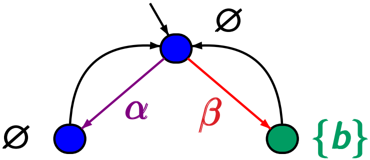

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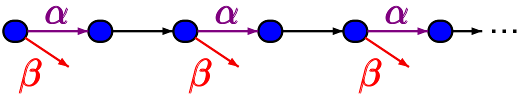

is not $\mathcal{F}$-fair

Which type of fairness?

LF2.6-13A

Which type of fairness?

fairness assumptions should be
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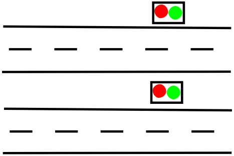
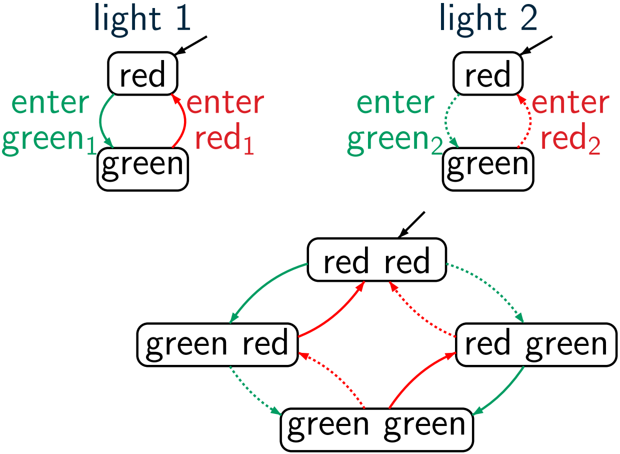
Two independent traffic lights





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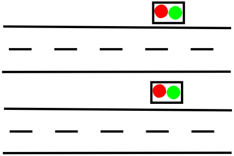

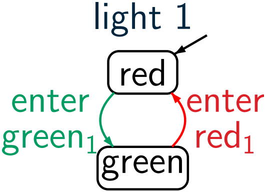

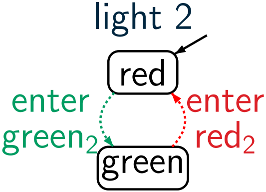

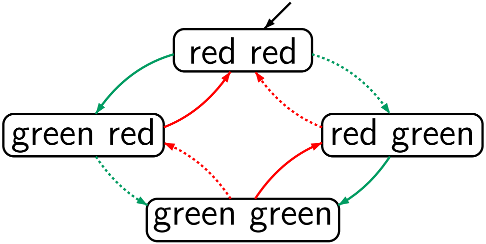

fairness assumption $\mathcal{F}$:

$\mathcal{F}_{ucond} = ?$

$\mathcal{F}_{strong} = ?$

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light 1 $|||$ light 2 $\models_{\mathcal{F}} E$

$E \mathrel{\widehat{=}}$ "both lights are infinitely often green"

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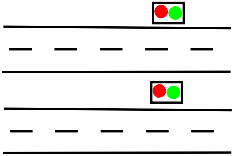
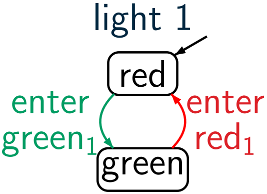
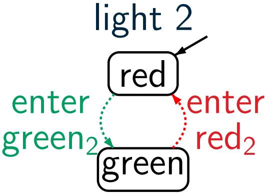
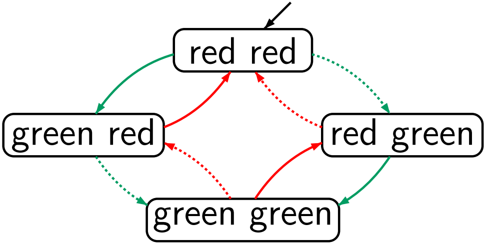






A_1 = actions of light 1
A_2 = actions of light 2



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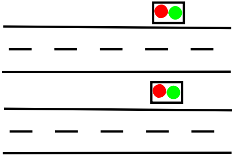

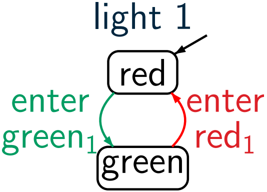

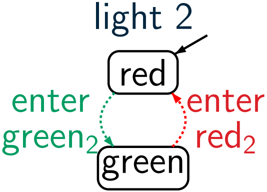

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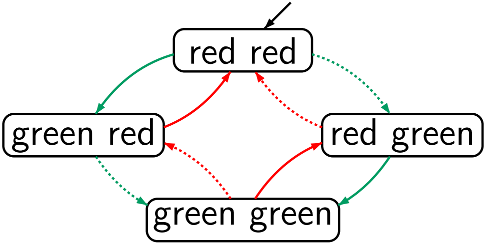

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Example: MUTEX with fair arbiter

$\mathcal{T} = \mathcal{T}_1 \parallel \text{Arbiter} \parallel \mathcal{T}_2$

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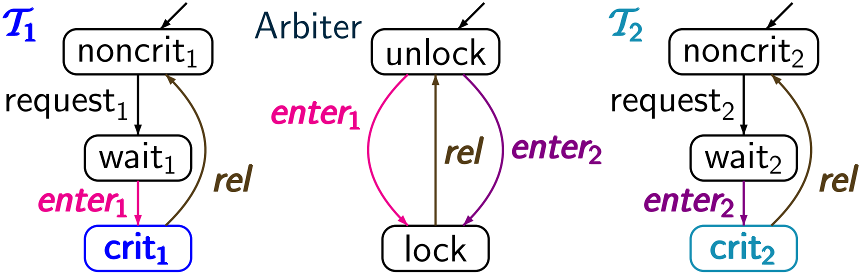

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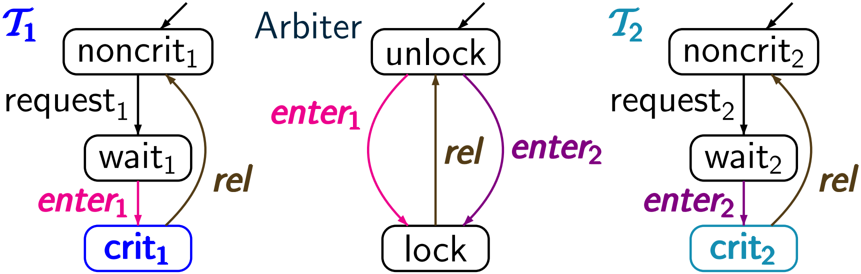
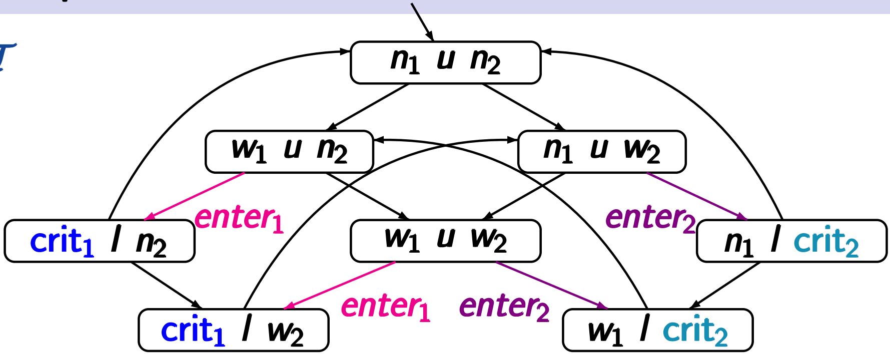


$\mathcal{T}_1$ and $\mathcal{T}_2$ compete to communicate with the arbiter by means of the actions ***enter***$_1$ and ***enter***$_2$, respectively

Example: MUTEX with fair arbiter

$\mathcal{T}$



LT property E: each waiting process eventually enters its critical section

$\mathcal{T} \not\models E$

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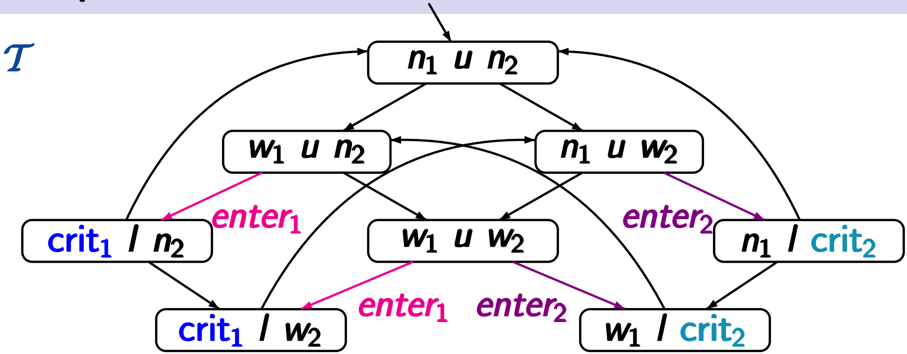

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Example: MUTEX with fair arbiter

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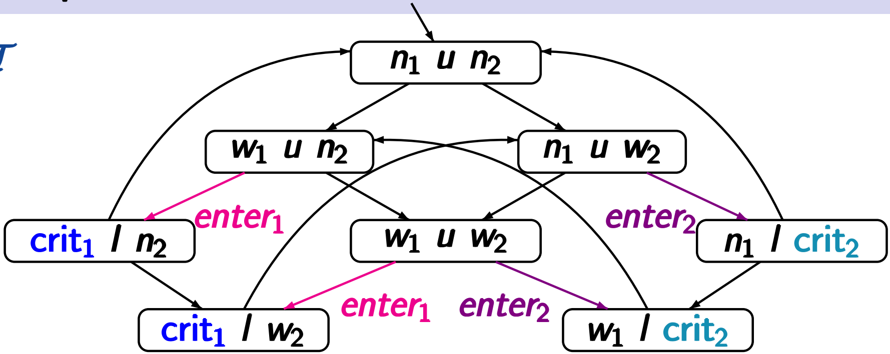

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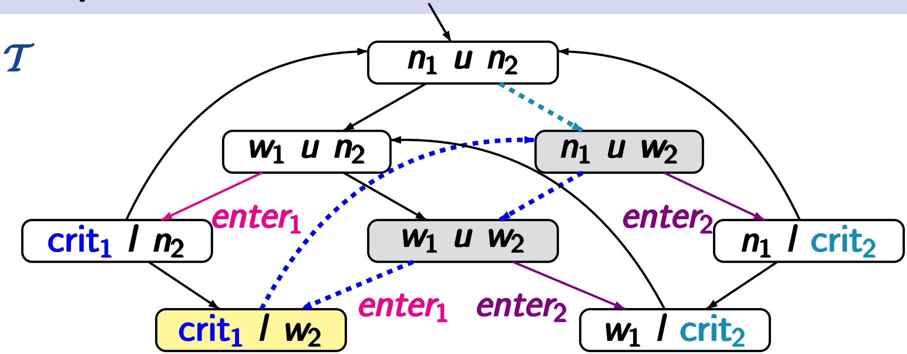

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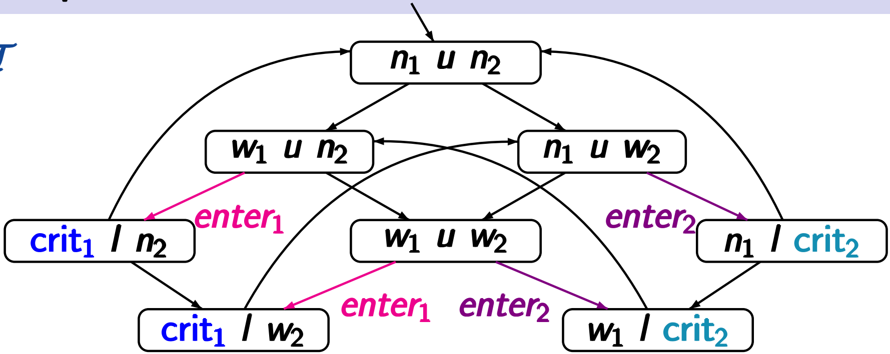
as $enter_2$ is not enabled in $\langle crit_1, l, w_2 \rangle$

Example: MUTEX with fair arbiter

$\mathcal{T}$



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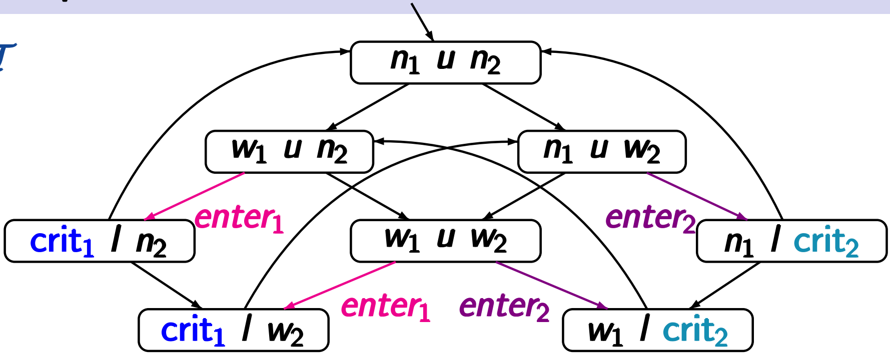

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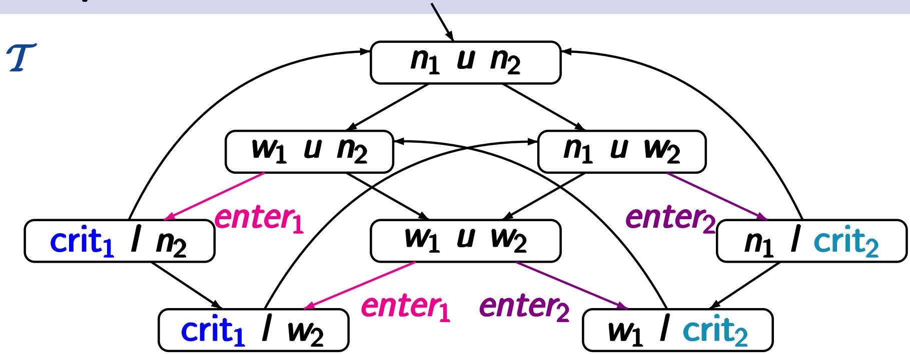

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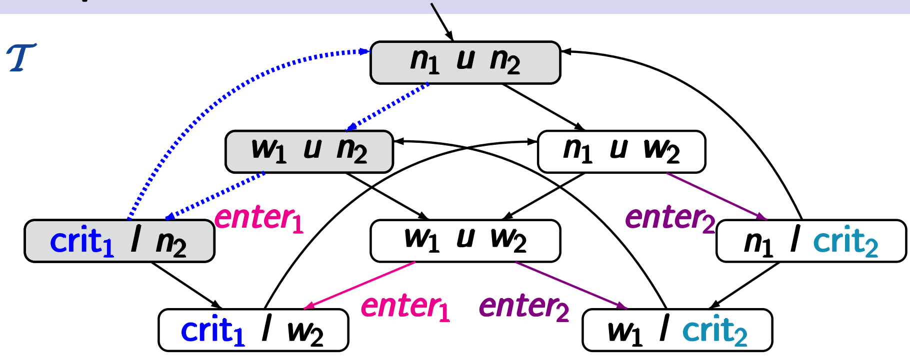

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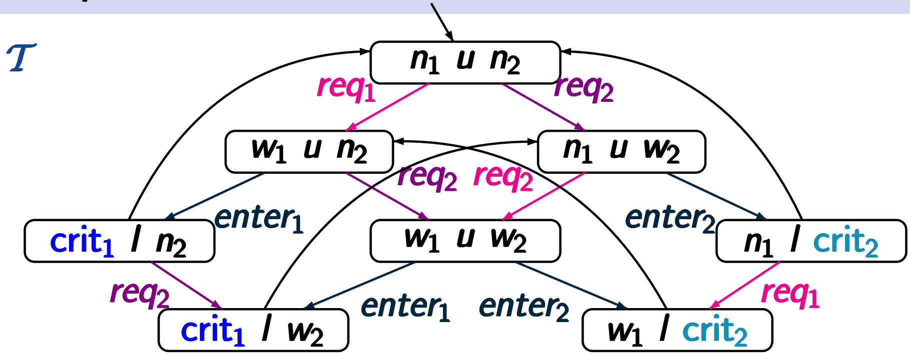

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Process fairness

LF2.6-19

Process fairness

For asynchronous systems:

$$\boxed{\text{parallelism} \;=\; \text{interleaving} + \text{fairness}}$$

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rule of thumb:

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  - ∗ resolution of competitions
- weak fairness for the nondetermism obtained from the interleaving of independent actions
- unconditional fairness: only of theoretical interest

Purpose of fairness conditions

$$\boxed{\text{parallelism} \;=\; \text{interleaving} + \text{fairness}}$$

Process fairness and other fairness conditions

- can compensate information loss due to interleaving
  or rule out other unrealistic pathological cases
- can be requirements for a scheduler
  or requirements for environment
- can be verifiable system properties

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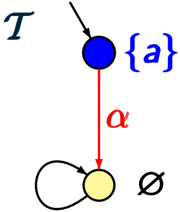
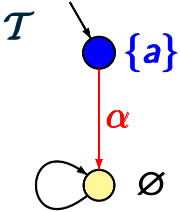
liveness properties:	fairness can be essential
safety properties:	fairness is irrelevant



fairness assumption $\mathcal{F}$:
unconditional fairness for action set $\{\alpha\}$

does $\mathcal{T} \models_{\mathcal{F}}$ "infinitely often a" hold ?



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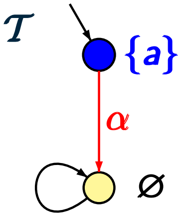

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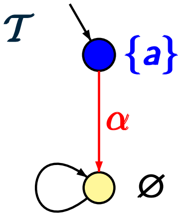
Realizability of fairness assumptions



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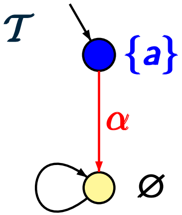
Realizability requires that each initial finite path fragment can be extended to a $\mathcal{F}$-fair path

Realizability of fairness assumptions



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Fairness assumption $\mathcal{F}$ is said to be realizable for a transition system $\mathcal{T}$ if for each reachable state s in $\mathcal{T}$ there exists a $\mathcal{F}$-fair path starting in s

Realizability of fairness assumptions

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fairness assumption $\mathcal{F} = (\mathcal{F}_{\mathit{ucond}}, \mathcal{F}_{\mathit{strong}}, \mathcal{F}_{\mathit{weak}})$ for TS $\mathcal{T}$

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$\uparrow$

can always be guaranteed by a scheduler, i.e.,
an instance that resolves the nondeterminism in $\mathcal{T}$

Safety and realizable fairness

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Realizable fairness assumptions are irrelevant
for safety properties

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If $\mathcal{F}$ is a realizable fairness assumption for TS $\mathcal{T}$ and E a safety property then:

$$\mathcal{T} \models E \quad \text{iff} \quad \mathcal{T} \models_{\mathcal{F}} E$$

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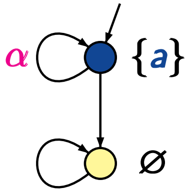

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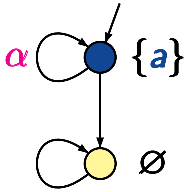

$\mathcal{F}$: unconditional fairness for $\{\alpha\}$

E = invariant "always a"

$\mathcal{T} \not\models E$, but $\mathcal{T} \models_{\mathcal{F}} E$