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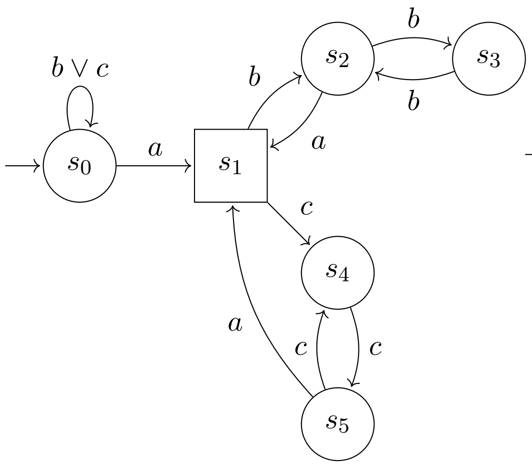
Exercise 1

a)

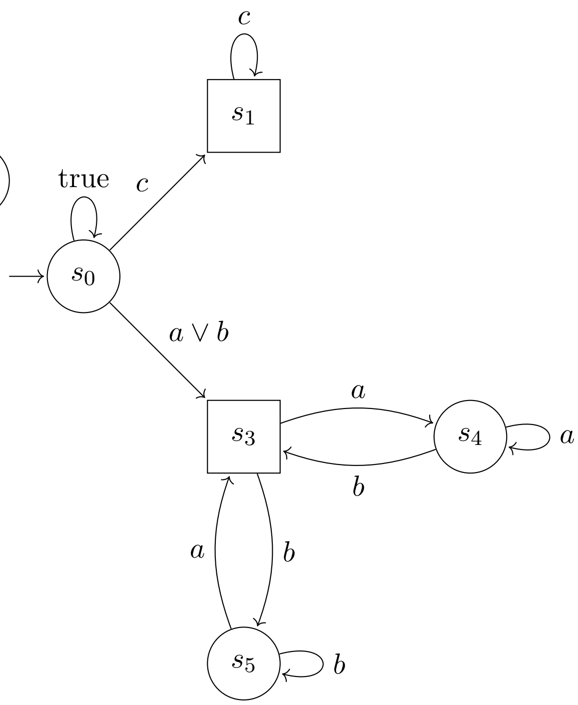
- $\alpha_1 = a^*.(b.a^+)^\omega + a^*.(b.a^+)^*.b.a^\omega$
- $\alpha_2 = ((c.(b.c)^*.a + b).a)^\omega$

b)

$\mathcal{B}_1$:



$\mathcal{B}_2$:



c)

$\mathcal{L}_\omega^1$:

Since the stated NBA is already a DBA there obviously exists one.

$\mathcal{L}_\omega^2$:

The proof follows the principle of the proof of $(A+B)^*.A^\omega$. In this case A corresponds to c and B to b.
Assuming there exists a DBA $\mathcal{A}$ with $\mathcal{L}_\omega(\mathcal{A}) = \mathcal{L}_\omega^2 =: L$.

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Note that $\mathcal{L}_\omega((c+b)^*.c^\omega) \subseteq L$, since c occurs infinitely many times and a and b only occur finitely many times.
Following the proof yields a sequence $n_1, n_2, \ldots$ of natural numbers and a sequence $q_1, q_2, \ldots$ of accepting states such that $\delta(q_0, c^{n_1}bc^{n_2}\ldots c^{n_{i-1}}bc^{n_i}) = \{q_i\}$.
Since Q is finite, there ex. $i < j$ such that $\delta(q_0, c^{n_1}b\ldots bc^{n_i}) = \delta(q_0, c^{n_1}b\ldots bc^{n_j})$.
Thus, $\mathcal{A}$ has an accepting run on $c^{n_1}b\ldots bc^{n_i}(bc^{n_{i+1}}\ldots bc^{n_j})^\omega \notin L$, since c and b occur infinitely many times. This contradicts $\mathcal{L}_\omega(\mathcal{A}) = L$.

Question:
I guess we cannot just apply the theorem from slide 176 of lec9+10 with $A = \{c\}$ and $B = \{a, b\}$, because the union with $(a+b+c)^*.(a+b)^\omega$ could theoretically make it DBA-realizable again?

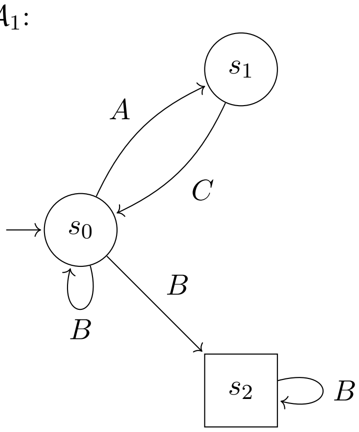
Exercise 2

Counting?

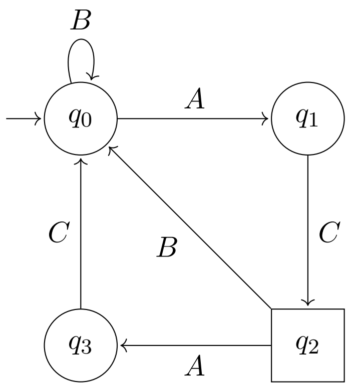
Exercise 3

a)

$\mathcal{A}_1$:

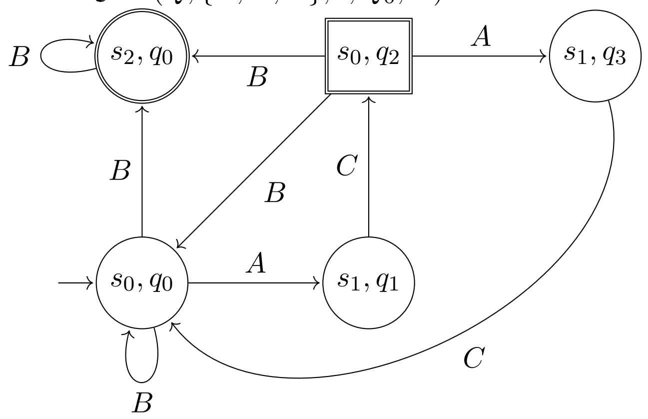


$\mathcal{A}_2$:

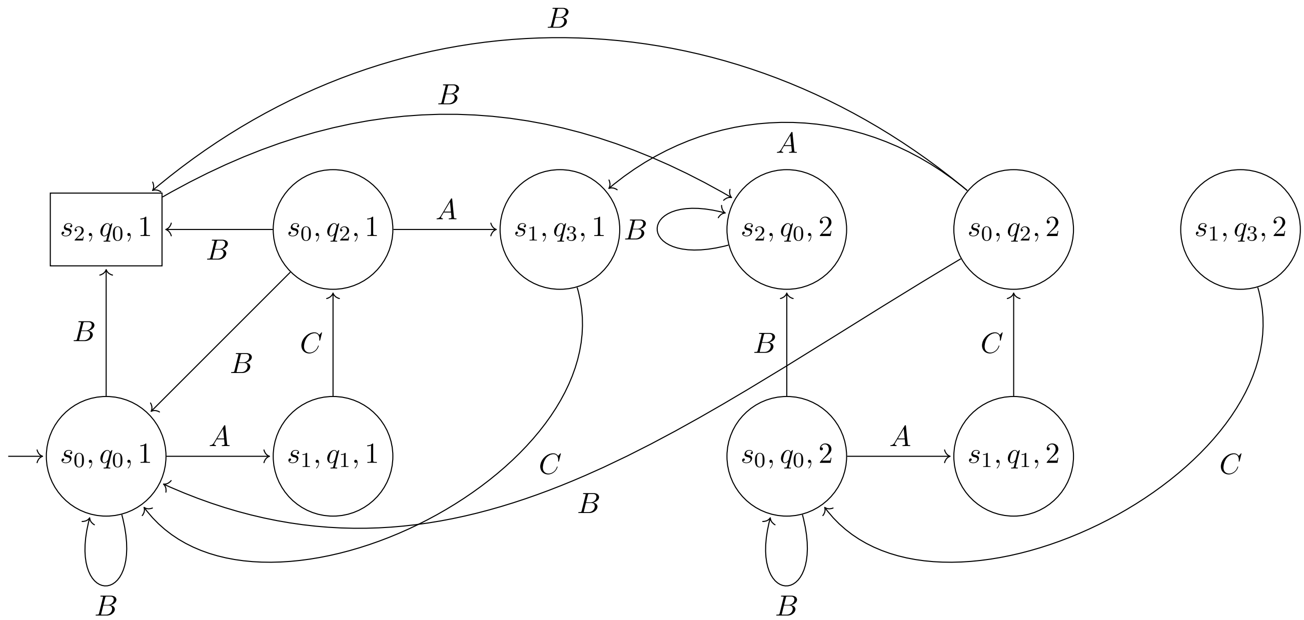


b)

GNBA $\mathcal{G} = (Q, \{A, B, C\}, \delta, Q_0, \mathcal{F})$



c)



d)

Based on the picture in Exercise 3 b), we can see that once we enter the state (s_2, q_0) we will never leave it. Therefore we can never be infinitely often in the final state $(s_0, q_2) \in F_2$.

So we cannot fulfill the acceptance criteria that we are infinitely often in a state of every set in $\mathcal{F} = \{F_1, F_2\}$. So there is no word $w \in \mathcal{L}_\omega(\mathcal{G}) \Rightarrow \mathcal{L}_\omega(\mathcal{G}) = \emptyset$

Exercise 4

a)

$\alpha_{pre} = ((b.b)^* + (c.(a.a)^*.c)^*)^*$
$\alpha_{(q_2,q_3)} = c.a^\omega$
$\alpha_{(q_1,q_3)} = false =$ no combination
$\alpha_{(q_0,q_2)} = c^\omega$
$\alpha = \alpha_{pre}.(\alpha_{(q_2,q_3)} + \alpha_{(q_1,q_3)} + \alpha_{(q_0,q_2)}) = ((b.b)^* + (c.(a.a)^*.c)^*)^* + (c.a^\omega + c^\omega)$

b)

Let GNBA $\mathcal{G} = (Q, \Sigma, \delta, Q_0, \mathcal{F})$ then we can construct the equivalent nondeterministic Muller automaton $\mathcal{A} = (Q, \Sigma, \delta, Q_0, \mathcal{F}')$ by just taking for every set $F_i \in \mathcal{F}, 1 \leq i \leq |\mathcal{F}|$ and build the cross product of all these F_i's:

$$\mathcal{F}' = F_1 \times F_2 \times \cdots \times F_{|\mathcal{F}|}$$

In $\mathcal{G}$, in order to be accepting, we had to visit at-least one element of every F_i infinitely often. In The Muller automaton we have to reach one of every possible combination.

Let ρ be an arbitrary accepting run of $\mathcal{G}$. Then ρ visits at-least one state of every set F_i infinitely often, since all those F_i are of finite length.
Therefore we can state a $f_i \in F_i$ that is visited infinitely often.
By construction $\{f_1, f_2, \ldots, f_{|\mathcal{F}|}\} \in \mathcal{F}'$ and therefore $\mathcal{A}$ accepts.
Let ρ' be an accepting run of $\mathcal{A}$. Then there is a $F' \in \mathcal{F}'$, s.t. $\exists^\infty i \geq 0.q_i = q$ or in words: every element of F' is visited infinitely often. By construction this set F' is a combination of elements, s.t.

- $|F'| = |\mathcal{F}$
- $\forall 0 \leq i \leq |F'|.$for $f_i \in F'$ the i-th element also $f_i \in F_i \in \mathcal{F}$

  or in words: every i-th element of F' is an element of the i-th element of $\mathcal{F}$

So for every set in $\mathcal{F}$ there is an element that is visited infinitely often. And so it is also an accepting run in $\mathcal{G}$.
Since every accepting run of $\mathcal{G}$ is an accepting run of $\mathcal{A}$ and vice versa those two are equivalent.