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Exercise 1

a)

$$\begin{aligned}
\varphi &= (a \rightarrow \bigcirc\neg b) \text{ W } (a \wedge b) \\
\varphi' = \neg\varphi &= \neg\big[(a \rightarrow \bigcirc\neg b) \text{ W } (a \wedge b)\big] \\
&\Leftrightarrow ((a \rightarrow \bigcirc\neg b) \wedge \neg(a \wedge b)) \text{ U } (\neg(a \rightarrow \bigcirc\neg b) \wedge \neg(a \wedge b)) \\
&\Leftrightarrow ((\neg a \vee \bigcirc\neg b) \wedge \neg a \vee \neg b) \text{ U } (\neg(a \rightarrow \bigcirc\neg b) \wedge \neg(a \wedge b)) \\
&\Leftrightarrow ((\neg a \vee \bigcirc\neg b) \wedge \neg a \vee \neg b) \text{ U } (\neg(\neg a \vee \bigcirc\neg b) \wedge \neg(a \wedge b)) \\
&\Leftrightarrow ((\neg a \vee \bigcirc\neg b) \wedge \neg a \vee \neg b) \text{ U } ((a \wedge \bigcirc b) \wedge \neg(a \wedge b)) \\
&\Leftrightarrow ((\neg a \vee \bigcirc\neg b) \wedge \neg a \vee \neg b) \text{ U } ((a \wedge \bigcirc b) \wedge (\neg a \vee \neg b)) \\
&\Leftrightarrow ((\neg a \vee \bigcirc\neg b) \wedge \neg a \vee \neg b) \text{ U } ((a \wedge \bigcirc b) \wedge (\neg a \vee \neg b)) \\
&\Leftrightarrow ((\neg a \vee \bigcirc\neg b) \wedge \neg a \vee \neg b) \text{ U } (a \wedge \bigcirc b \wedge \neg b) \\
&\Leftrightarrow (\neg a \vee (\neg b \wedge \bigcirc\neg b)) \text{ U } (a \wedge \neg b \wedge \bigcirc b) \\
&\Leftrightarrow \neg(a \wedge \neg(\neg b \wedge \bigcirc\neg b)) \text{ U } (a \wedge \neg b \wedge \bigcirc b)
\end{aligned}$$

b)

$$\begin{aligned}
cl(\varphi') = \{ \quad & \underline{a}, \neg a, \\
& \underline{b}, \neg b, \\
& \underline{\bigcirc b}, \neg \bigcirc b, \\
& \underline{\neg b \wedge \bigcirc\neg b}, \neg(\neg b \wedge \bigcirc\neg b), \\
& \underline{a \wedge \neg(\neg b \wedge \bigcirc\neg b)}, \neg(a \wedge \neg(\neg b \wedge \bigcirc\neg b)) \\
& \underline{a \wedge \neg b}, \neg(a \wedge \neg b), \\
& \underline{\neg b \wedge \bigcirc b}, \neg(\neg b \wedge \bigcirc b), \\
& \underline{a \wedge \bigcirc b}, \neg(a \wedge \bigcirc b), \\
& \underline{a \wedge \neg b \wedge \bigcirc b}, \neg(a \wedge \neg b \wedge \bigcirc b), \\
& \underline{\varphi'}, \neg\varphi' \}
\end{aligned}$$

(Note: $\neg \bigcirc b \equiv \bigcirc\neg b$)

	a	b	$\bigcirc b$	$\overbrace{\neg b \land \bigcirc \neg b}^{\eta}$	$a \land \neg \eta$	$a \land \neg b$	$\neg b \land \bigcirc b$	$a \land \bigcirc b$	$a \land \neg b \land \bigcirc b$	φ'
B_0	0	0	0	1	0	0	0	0	0	1
B_0'	0	0	0	1	0	0	0	0	0	0
B_1	0	0	1	0	0	0	1	0	0	1
B_1'	0	0	1	0	0	0	1	0	0	0
B_2	0	1	0	0	0	0	0	0	0	1
B_2'	0	1	0	0	0	0	0	0	0	0
B_3	0	1	1	0	0	0	0	0	0	1
B_3'	0	1	1	0	0	0	0	0	0	0
B_4	1	0	0	1	0	1	0	0	0	1
B_4'	1	0	0	1	0	1	0	0	0	0
B_5	1	0	1	0	1	1	1	1	1	1
B_6	1	1	0	0	1	0	0	0	0	0
B_7	1	1	1	0	1	0	0	1	0	0

(Note: Every B_i' is the row representing the possibility but not necessity of rule (ii) first item on slide 138)

c)

We construct the GNBA $\mathcal{G} = (Q, 2^{AP}, \delta, Q_0, \mathcal{F})$, where

- $Q = \{B_0, B_0', B_1, B_1', B_2, B_2', B_3, B_3', B_4, B_4', B_5, B_6, B_7\}$
- $Q_0 = \{B_0, B_1, B_2, B_3, B_4, B_5\}$
- $\mathcal{F} = \{\{B_0', B_1', B_2', B_3', B_4', B_5, B_6, B_7\}\}$
- and δ as follows:

	B_0	B_0'	B_1	B_1'	B_2	B_2'	B_3	B_3'	B_4	B_4'	B_5	B_6	B_7
B_0	$\emptyset$		$\emptyset$						$\emptyset$		$\emptyset$		
B_0'		$\emptyset$		$\emptyset$						$\emptyset$			
B_1					$\emptyset$		$\emptyset$						
B_1'						$\emptyset$		$\emptyset$				$\emptyset$	$\emptyset$
B_2	$\{b\}$		$\{b\}$						$\{b\}$		$\{b\}$		
B_2'		$\{b\}$		$\{b\}$		$\{b\}$				$\{b\}$			
B_3					$\{b\}$								
B_3'						$\{b\}$							
B_4	$\{a\}$		$\{a\}$								$\{a\}$		
B_4'		$\{a\}$		$\{a\}$									
B_5					$\{a\}$		$\{a\}$						
B_6'		$\{a,b\}$		$\{a,b\}$						$\{a,b\}$			
B_7						$\{a,b\}$		$\{a,b\}$					

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Exercise 2

a)

$$\begin{aligned}
\varphi &= \Box(a \to ((\neg b) \text{ U } (a \wedge b))) \\
\varphi' = \neg\varphi &= \neg\big[\Box(a \to ((\neg b) \text{ U } (a \wedge b)))\big] \\
&\Leftrightarrow \neg\big[\neg\Diamond\neg(a \to ((\neg b) \text{ U } (a \wedge b)))\big] \\
&\Leftrightarrow \Diamond\neg(a \to ((\neg b) \text{ U } (a \wedge b))) \\
&\Leftrightarrow \mathit{true} \text{ U } \neg(a \to ((\neg b) \text{ U } (a \wedge b))) \\
&\Leftrightarrow \mathit{true} \text{ U } \neg(\neg a \vee ((\neg b) \text{ U } (a \wedge b))) \\
&\Leftrightarrow \mathit{true} \text{ U } (a \wedge \neg((\neg b) \text{ U } (a \wedge b)))
\end{aligned}$$

So $closure(\varphi) = \{\underline{\mathit{true}}, \neg\mathit{true}$
$\underline{a}, \neg a,$
$\underline{b}, \neg b,$
$\underline{a \wedge b}, \neg(a \wedge b),$
$\underline{(\neg b) \text{ U } (a \wedge b)}, \neg((\neg b) \text{ U } (a \wedge b)),$
$\underline{\varphi'}, \neg\varphi'\}$

b)

	true	a	b	$a \wedge b$	$\overbrace{(\neg b) \text{ U } (a \wedge b)}^{\eta}$	$a \wedge \eta$	φ'
B_0	1	0	0	0	1	0	1
B_0'	1	0	0	0	1	0	0
B_0''	1	0	0	0	0	0	0
B_0'''	1	0	0	0	0	0	1
B_1	1	0	1	0	0	0	1
B_1'	1	0	1	0	0	0	0
B_2	1	1	0	0	1	0	1
B_2'	1	1	0	0	1	0	0
B_2''	1	1	0	0	0	0	0
B_2'''	1	1	0	0	0	0	1
B_3	1	1	1	1	1	1	1

Exercise 3

$$\begin{aligned} \neg \bigcirc \psi \in B &\Leftrightarrow \bigcirc\psi \notin B \\ &\overset{*}{\Leftrightarrow} \psi \notin B' \end{aligned}$$

where $*$ holds, because of $B' \in \delta(B, B \cap AP)$