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Exercise 1

Φ_1)

$Sat(\Phi_1) = \{s_1, s_2, s_3, s_4\}$, since $s_0 \notin Sat(\Phi_1) \Rightarrow TS \not\models \Phi_1$

Φ_2)

$Sat(\Phi_2) = \{s_4\}$, since $s_0 \notin Sat(\Phi_2) \Rightarrow TS \not\models \Phi_2$

Φ_3)

$Sat(\Phi_3) = \{s_0, s_1, s_3, s_4\}$, since $\{s_0, s_3\} \subset Sat(\Phi_3) \Rightarrow TS \models \Phi_3$

Φ_4)

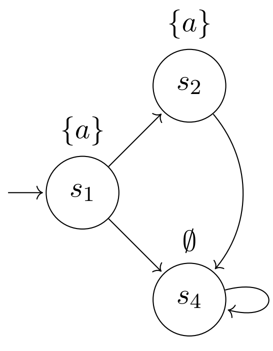
$Sat(\Phi_4) = \emptyset$, since $s_0 \notin Sat(\Phi_4) \Rightarrow TS \not\models \Phi_4$

Exercise 2

Exercise 3

a)

Using the theorem of slide 27 of lec18-2-1. If a CTL formula Φ has an equivalent LTL formula φ, it can be obtained by removing the quantifiers.
Therefore we obtain the formula $\varphi = \Diamond(a \wedge \bigcirc a)$. Now either they are equivalent or there ex. no LTL-formula, which is equivalent to Φ_1.
Now consider the following transition system TS:



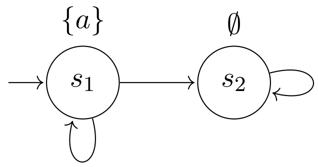
Then $TS \not\models \Diamond(a \wedge \bigcirc a)$, because there is $trace(s_1 s_4^\omega) = \{a\}\emptyset^\omega$.
But, since $Sat(a \wedge \exists \bigcirc a) = \{s_1\}$ and for all paths π it holds that $s_1 \in Reach_{TS}(\pi)$ it follows that $TS \models \forall\Diamond(a \wedge \exists \bigcirc a)$.

Since through TS it is proven that Φ_1 is not equivalent to φ we can conclude that there is no LTL-formula that is equivalent to Φ.

b)

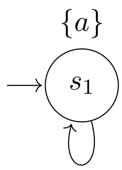
Suppose we have an LTL-formula φ, s.t. $\varphi \equiv \forall\Diamond\exists\bigcirc\forall\Diamond\neg a$.

Consider now TS $\mathcal{T}_1$:



Since $\mathcal{T}_1 \models \forall\Diamond\exists\bigcirc\forall\Diamond\neg a \Rightarrow \mathcal{T}_1 \models \varphi$.

Also consider TS $\mathcal{T}_2$:



Now $Traces(\mathcal{T}_2) = \{\{a\}^\omega\} \subset Traces(\mathcal{T}_1) \subset Words(\varphi)$, but $\mathcal{T}_2 \not\models \forall\Diamond\exists\bigcirc\forall\Diamond\neg a$.
Contradiction

Exercise 4

a)

1 $\Rightarrow$ 2)

Let $s \models_{LTL} \Box a$.
Then for every trace $\pi = s_1 s_2 s_3 \cdots \in Traces(s)$, $s_1 = s$, we have that every $s_i \models a$, $i \geq 0$.
Therefore especially every path $\pi' \in Paths(s)$ rooted in s, fulfills $\pi' \models \Box a$ and therefore $s \models_{CTL} \forall\Box a$

2 $\Rightarrow$ 3)

Let $s \models_{CTL} \Box a$.
Then for every path $\pi \in Paths(s)$ it holds that $\pi \models \Box a$.

So every node s' within π has to fulfill $s' \models a$
This especially means that every $s' \in Reach_{TS}(s)$ it holds that $s' \models a$.
So $\forall s' \in Reach_{TS}(s) \,.\, s' \models a$

3 ⇒ 4)

Let $\forall s' \in Reach_{TS}(s) \,.\, s' \models a$.
Through the given hint, we can infer that for every $s'' \in Reach_{TS}(s')$ it also holds, that $s'' \in Reach_{TS}(s)$ and therefore $s'' \models a$.
Now since for all $\pi' = s's'' \cdots \in Paths(s')$ it holds that $s'' \models a$, by definition of slide 66(73) we can rewrite it as $s' \models_{CTL} \forall \Box a$.
And therefore infer $\forall s' \in Reach_{TS}(s) \,.\, s' \models_{CTL} \forall \Box a$.

4 ⇒ 1)

Let $\forall s' \in Reach_{TS}(s) \,.\, s' \models_{CTL} \forall \Box a$.
For every possible state s' it holds that $s' \models_{CTL} \forall \Box a$. Taking definition on slide 66(73) into account, also every descendant state s'' of s' has to fulfill $s'' \models a$. Since $s \in Reach_{TS}(s)$ this means every state of every path has to model a, this concludes to $s \models_{LTL} \Box a$

b)

We use the theorem of slide 27 of lec18-2-1. If a CTL formula Φ has an equivalent LTL formula φ, it can be obtained by removing the quantifiers. Since the exercise is to prove the equivalence we can assume that an equivalent LTL formula exists.

$$\forall\big(a \text{ U } (b \land \forall \Box a)\big) \rightsquigarrow a \text{ U } (b \land \Box a)$$

($\star$): Now we can see that the Until-formula holds if we have consecutive a's until we encounter a b and have $\Box a$ = always a. So we have to always have a's. In order to fulfill any side. This can be stated separately by using simply $\Box a$. Then the formula can be simplified as follows:

$$\begin{aligned}
& a \text{ U } (b \land \Box a) && \text{with: } (\star) \\
& \equiv \Box a \land (a \text{ U } (b \land \Box a)) && \\
& \equiv \Box a \land (\mathit{true} \text{ U } b) && \text{def. of } \Diamond \\
& \equiv \Box a \land \Diamond b &&
\end{aligned}$$

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Exercise 5

a)

$$\begin{aligned}
\Phi_1 &= \forall \bigcirc (\exists(\neg a \text{ U } (b \wedge \neg c)) \vee \exists\Box\forall \bigcirc a) && \text{with: } \exists\Box\Phi = \neg\forall\neg\Phi \\
&\Leftrightarrow \forall \bigcirc (\exists(\neg a \text{ U } (b \wedge \neg c)) \vee \neg\forall\Diamond\neg\forall \bigcirc a) && \text{with: } \forall\Diamond\Phi = \forall(true \text{ U } \Phi) \\
&\Leftrightarrow \forall \bigcirc (\exists(\neg a \text{ U } (b \wedge \neg c)) \vee \neg\forall(true \text{ U } \neg\forall \bigcirc a)) && \text{rewrite: } \neg\forall \\
&\Leftrightarrow \forall \bigcirc (\exists(\neg a \text{ U } (b \wedge \neg c)) \vee \exists(\forall \bigcirc a \text{ W } (\neg true \wedge \forall \bigcirc a))) && \text{rewrite } \neg true \\
&\Leftrightarrow \forall \bigcirc (\exists(\neg a \text{ U } (b \wedge \neg c)) \vee \exists(\forall \bigcirc a \text{ W } (false \wedge \forall \bigcirc a))) && \text{rewrite: } false \wedge \Phi' = false \\
&\Leftrightarrow \forall \bigcirc (\exists(\neg a \text{ U } (b \wedge \neg c)) \vee \exists(\forall \bigcirc a \text{ W } false))
\end{aligned}$$

b)

$$\begin{aligned}
\Phi_1 &= \forall \bigcirc (\exists(\neg a \text{ U } (b \wedge \neg c)) \vee \exists\Box\forall \bigcirc a) && \text{with: } \forall \bigcirc \Phi = \neg\exists \bigcirc \neg\Phi \\
&\Leftrightarrow \neg\exists \bigcirc \neg(\exists(\neg a \text{ U } (b \wedge \neg c)) \vee \exists\Box\forall \bigcirc a) && \text{with: } \forall \bigcirc \Phi = \neg\exists \bigcirc \neg\Phi \\
&\Leftrightarrow \neg\exists \bigcirc \neg(\exists(\neg a \text{ U } (b \wedge \neg c)) \vee \exists\Box\neg\exists \bigcirc \neg a) \\
&\Leftrightarrow \neg\exists \bigcirc (\neg\exists(\neg a \text{ U } (b \wedge \neg c)) \wedge \neg\exists\Box\neg\exists \bigcirc \neg a)
\end{aligned}$$

c)

Let $(\star)$ be : $\forall(\Phi \text{ W } \Psi) = \neg\exists((\Phi \wedge \neg\Psi) \text{ U } (\neg\Phi \wedge \neg\Psi)$

$$\begin{aligned}
\Phi_2 &= \forall(\neg a \text{ W } (b \rightarrow \forall \bigcirc c)) && \text{with: } (star) \\
&\Leftrightarrow \neg\exists((\neg a \wedge \neg(b \rightarrow \forall \bigcirc c)) \text{ U } (\neg\neg a \wedge \neg(b \rightarrow \forall \bigcirc c)) && \text{simplify and de Morgan} \\
&\Leftrightarrow \neg\exists((\neg a \wedge \neg(\neg b \vee \forall \bigcirc c)) \text{ U } (a \wedge \neg(\neg b \vee \forall \bigcirc c)) && \text{simplify} \\
&\Leftrightarrow \neg\exists((\neg a \wedge b \wedge \neg\forall \bigcirc c) \text{ U } (a \wedge b \wedge \neg\forall \bigcirc c)) && \text{with: } \neg\forall \bigcirc \Phi = \exists \bigcirc \neg\Phi \\
&\Leftrightarrow \neg\exists((\neg a \wedge b \wedge \exists \bigcirc \neg c) \text{ U } (a \wedge b \wedge \exists \bigcirc \neg c))
\end{aligned}$$