Regular LT properties

Idea: define regular LT properties to be those languages of infinite words over the alphabet 2^{AP} that have a representation by a finite automata

- regular safety properties:
  NFA-representation for the bad prefixes
- other regular LT properties:
  representation by ω-automata, i.e., acceptors for infinite words

Regular safety properties

Let $E \subseteq (2^{AP})^\omega$ be a safety property.

E is called regular iff the language

$BadPref$ = set of all bad prefixes for E $\subseteq (2^{AP})^+$

$BadPref = \mathcal{L}(\mathcal{A})$ for some NFA $\mathcal{A}$ over the alphabet 2^{AP}

is regular.

Nondeterministic finite automata (NFA)

NFA $\mathcal{A} = (Q, \Sigma, \delta, Q_0, F)$

- Q finite set of states
- Σ alphabet
- $\delta : Q \times \Sigma \to 2^Q$ transition relation
- $Q_0 \subseteq Q$ set of initial states
- $F \subseteq Q$ set of final states, also called accept states

run for a word $A_0 A_1 \ldots A_{n-1} \in \Sigma^*$:

state sequence $\pi = q_0\, q_1 \ldots q_n$ where $q_0 \in Q_0$
and $q_{i+1} \in \delta(q_i, A_i)$ for $0 \leq i < n$

run π is called accepting if $q_n \in F$

Nondeterministic finite automata (NFA)

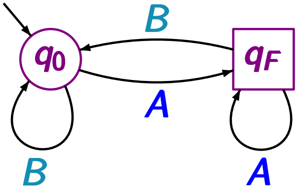
NFA $\mathcal{A} = (Q, \Sigma, \delta, Q_0, F)$

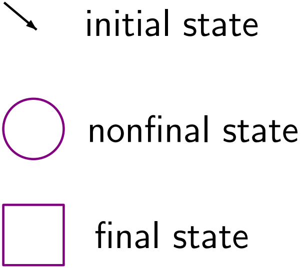
- Q finite set of states
- Σ alphabet $\longleftarrow$ $\boxed{\text{here: } \Sigma = 2^{AP}}$
- $\delta : Q \times \Sigma \to 2^Q$ transition relation
- $Q_0 \subseteq Q$ set of initial states
- $F \subseteq Q$ set of final states, also called accept states

accepted language $\mathcal{L}(\mathcal{A}) \subseteq \Sigma^*$ is given by:

$\mathcal{L}(\mathcal{A})$ = set of finite words over Σ that have an accepting run in $\mathcal{A}$

Notations in pictures for NFA





accepted language $\mathcal{L}(\mathcal{A})$:

set of all finite words over $\{A, B\}$
ending with letter A

Symbolic notations

NFA $\mathcal{A} = (Q, \Sigma, \delta, Q_0, F)$ over the alphabet $\Sigma = 2^{AP}$

symbolic notation for the labels of transitions:

> If Φ is a propositional formula over AP then
> $q \xrightarrow{\Phi} p$ stands for the set of transitions $q \xrightarrow{A} p$
> where $A \subseteq AP$ such that $A \models \Phi$

Example: if $AP = \{a, b, c\}$ then

$$q \xrightarrow{a \wedge \neg b} p \;\widehat{=}\; \left\{ q \xrightarrow{A} p : A = \{a, c\} \text{ or } A = \{a\} \right\}$$

$$q \xrightarrow{true} p \;\widehat{=}\; \left\{ q \xrightarrow{A} p : A \subseteq AP \right\}$$

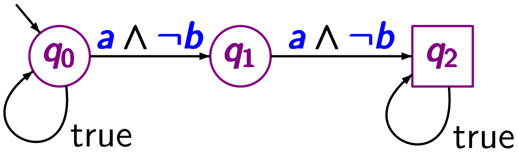
Regular safety properties

A safety property $E \subseteq (2^{AP})^\omega$ is called regular iff

$BadPref$ = set of all bad prefixes for $E \subseteq (2^{AP})^+$

$BadPref = \mathcal{L}(\mathcal{A})$ for some NFA $\mathcal{A}$ over the alphabet 2^{AP}

is regular.



$AP = \{a, b\}$

symbolic notation:

$a \land \neg b \mathrel{\widehat{=}} \{a\}$

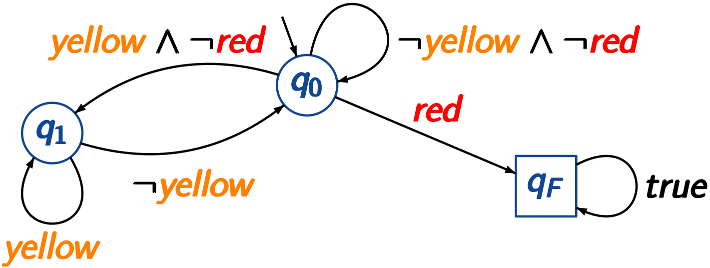
safety property E: "$a \land \neg b$ never holds twice in a row"

"Every red phase is preceded by a yellow phase"

set of all infinite words $A_0\, A_1\, A_2 \ldots$ s.t. for all $i \geq 0$:

$$red \in A_i \quad \Longrightarrow \quad i \geq 1 \text{ and } yellow \in A_{i-1}$$

DFA for all (possibly non-minimal) bad prefixes



Bad prefixes vs minimal bad prefixes

Let $E \subseteq (2^{AP})^\omega$ be a safety property.

$$\begin{array}{lcl} \mathit{BadPref} & = & \text{set of all bad prefixes for } E \\ \mathit{MinBadPref} & = & \text{set of minimal bad prefixes for } E \end{array}$$

Claim: $\mathit{BadPref}$ is regular $\Longleftrightarrow$ $\mathit{MinBadPref}$ is regular

"$\Longleftarrow$": Let $\mathcal{A}$ be an NFA for $\mathit{MinBadPref}$.

An NFA $\mathcal{A}'$ for $\mathit{BadPref}$ is obtained from $\mathcal{A}$ by adding self-loops $p \xrightarrow{\mathit{true}} p$ to all final states p.

"$\Longrightarrow$": Let $\mathcal{A}$ be a DFA for $\mathit{BadPref}$.

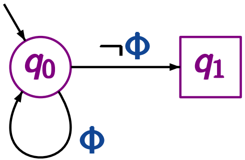
A DFA $\mathcal{A}'$ for $\mathit{MinBadPref}$ is obtained from $\mathcal{A}$ by removing all outgoing transitions of final states.

Correct or wrong?

Every invariant is regular.

correct.

Let E be an invariant with invariant condition Φ

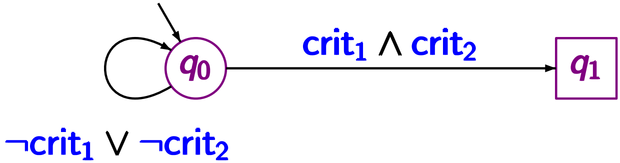


is a DFA for the language of all minimal bad prefixes

Example: DFA for *MUTEX*

"The two processes are never simultaneously in their critical sections"

DFA for minimal bad prefixes over the alphabet 2^{AP} where $AP = \{\mathsf{crit}_1, \mathsf{crit}_2\}$



Every safety property is regular.

wrong. e.g., $AP = \{pay, drink\}$

E = set of alle infinite words $A_0\, A_1\, A_2 \ldots \in (2^{AP})^\omega$
such that for all $j \in \mathbb{N}$:
$$\big|\{i \le j : pay \in A_i\}\big| \ge \big|\{i \le j : drink \in A_i\}\big|$$

- E is a safety property, but
- the language of (minimal) bad prefixes is *not* regular

Verifying regular safety properties

given: finite TS $\mathcal{T}$
regular safety property E
(represented by an **NFA** for its bad prefixes)

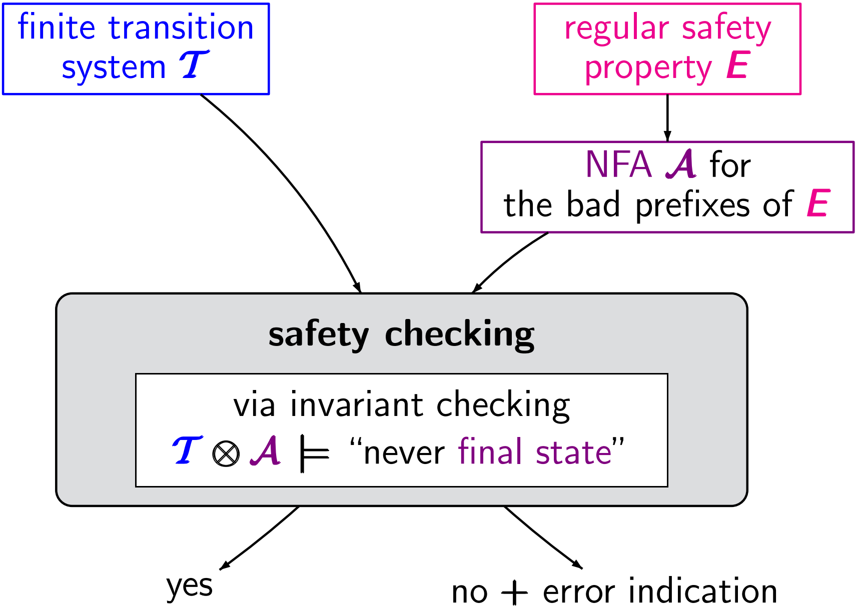
question: does $\mathcal{T} \models E$ hold **?**

method: relies on an analogy between the tasks:

- checking language inclusion for **NFA**
- model checking regular safety properties

language inclusion for NFA	verification of regular safety properties
$\mathcal{L}(\mathcal{A}_1) \subseteq \mathcal{L}(\mathcal{A}_2)$ **?**	$\mathit{Traces}(\mathcal{T}) \subseteq E$ **?**
check whether $\mathcal{L}(\mathcal{A}_1) \cap (\Sigma^* \setminus \mathcal{L}(\mathcal{A}_2))$ is empty	check whether $\mathit{Traces}_{\mathit{fin}}(\mathcal{T}) \cap \mathit{BadPref}$ is empty
1. complement $\mathcal{A}_2$, i.e., construct NFA $\overline{\mathcal{A}_2}$ with $\mathcal{L}(\overline{\mathcal{A}_2}) = \Sigma^* \setminus \mathcal{L}(\mathcal{A}_2)$ 2. construct NFA $\mathcal{A}$ with $\mathcal{L}(\mathcal{A}) = \mathcal{L}(\mathcal{A}_1) \cap \mathcal{L}(\overline{\mathcal{A}_2})$ 3. check if $\mathcal{L}(\mathcal{A}) = \varnothing$	1. construct NFA $\mathcal{A}$ for the bad prefixes $\mathcal{L}(\overline{\mathcal{A}}) = \mathit{BadPref}$ 2. construct TS $\mathcal{T}'$ with $\mathit{Traces}_{\mathit{fin}}(\mathcal{T}') = \ldots$ 3. invariant checking for $\mathcal{T}'$

Checking regular safety properties



Product of a TS and an NFA

finite transition system
$\mathcal{T} = (S, Act, \to, S_0, AP, L)$

NFA for bad prefixes
$\mathcal{A} = (Q, 2^{AP}, \delta, Q_0, F)$

path fragment $\widehat{\pi}$	trace	path fragm. in product	run for $trace(\widehat{\pi})$
			$q_0 \in Q_0$
s_0	$L(s_0) = A_0$	$\langle s_0, q_1 \rangle$	$q_0 \xrightarrow{A_0} q_1$
$\downarrow$		$\downarrow$	
s_1	$L(s_1) = A_1$	$\langle s_1, q_2 \rangle$	$q_1 \xrightarrow{A_1} q_2$
$\downarrow$		$\downarrow$	
s_2	$L(s_2) = A_2$	$\langle s_2, q_3 \rangle$	$q_2 \xrightarrow{A_2} \ldots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\downarrow$		$\downarrow$	$\downarrow$
s_n	$L(s_n) = A_n$	$\langle s_n, q_{n+1} \rangle$	$q_n \xrightarrow{A_n} q_{n+1}$

Product transition system

$$\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L) \quad \text{transition system}$$

$$\mathcal{A} = (Q, 2^{AP}, \delta, Q_0, F) \quad \text{NFA}$$

product-TS $\mathcal{T} \otimes \mathcal{A} \stackrel{\text{def}}{=} (S \times Q, Act, \longrightarrow', S_0', AP', L')$

$$\frac{s \xrightarrow{\alpha} s' \quad \wedge \quad q' \in \delta(q, L(s'))}{\langle s, q \rangle \xrightarrow{\alpha}{}' \langle s', q' \rangle}$$

initial states: $S_0' = \{ \langle s_0, q \rangle : s_0 \in S_0, q \in \delta\big(Q_0, L(s_0)\big) \}$

for $P \subseteq Q$ and $A \subseteq AP$: $\quad \delta(P, A) = \bigcup_{p \in P} \delta(p, A)$

Product transition system

$\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$ transition system

$\mathcal{A} = (Q, 2^{AP}, \delta, Q_0, F)$ NFA

product-TS $\mathcal{T} \otimes \mathcal{A} \stackrel{\text{def}}{=} (S \times Q, Act, \longrightarrow', S_0', AP', L')$

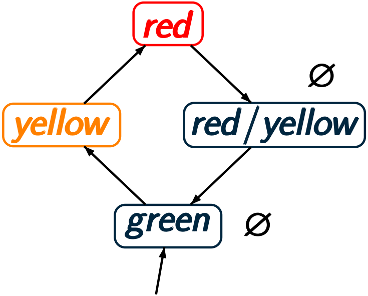
$$\frac{s \xrightarrow{\alpha} s' \quad \wedge \quad q' \in \delta(q, L(s'))}{\langle s, q \rangle \xrightarrow{\alpha}{}' \langle s', q' \rangle}$$

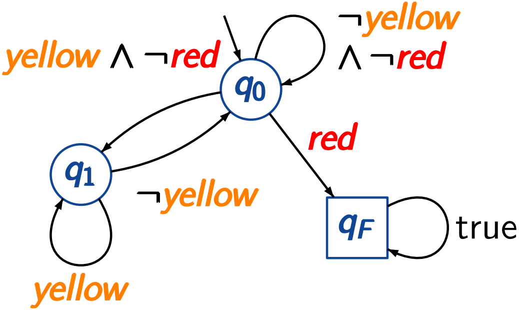
initial states: $S_0' = \{ \langle s_0, q \rangle : s_0 \in S_0, q \in \delta(Q_0, L(s_0)) \}$

set of atomic propositions: $AP' = Q$

labeling function: $L'(\langle s, q \rangle) = \{q\}$

Example: product-TS



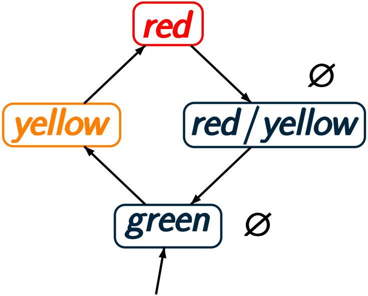


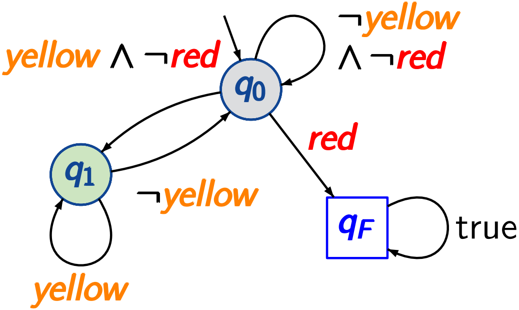
transition system $\mathcal{T}$ over $AP = \{red, yellow\}$

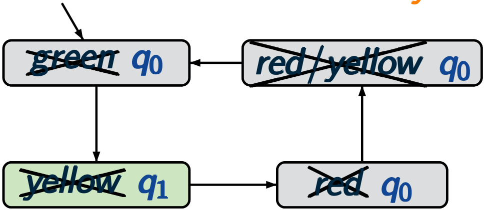
DFA $\mathcal{A}$ for the bad prefixes for E

$\mathcal{T}$ satisfies the safety property E
"every red phase is preceded by a yellow phase"

Example: product-TS







set of propositions

$AP' = \{q_0, q_1, q_F\}$

invariant condition $\neg q_F$ holds
for all reachable states

Technical remark on the product-TS

definition of the product of

- a transition system $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$

without terminal states

- an NFA $\mathcal{A} = (Q, 2^{AP}, \delta, Q_0, F)$

then the product $\mathcal{T} \otimes \mathcal{A} = (S \times Q, Act, \rightarrow', \ldots)$ is a TS

without terminal states

assumptions on the NFA $\mathcal{A}$:

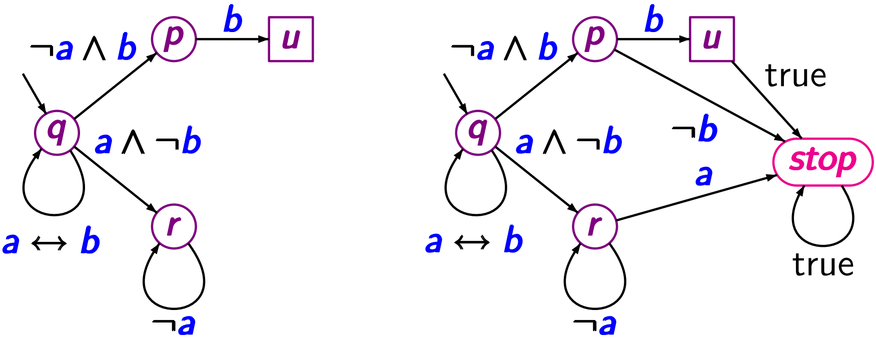
- $\mathcal{A}$ is non-blocking, i.e.,

  $$Q_0 \neq \varnothing \ \wedge \ \forall q \in Q \, \forall A \in 2^{AP}. \, \delta(q, A) \neq \varnothing$$

- no initial state of $\mathcal{A}$ is final, i.e., $Q_0 \cap F = \varnothing$

Non-blocking NFA

NFA $\mathcal{A}$ $\rightsquigarrow$ equivalent NFA $\mathcal{A}'$

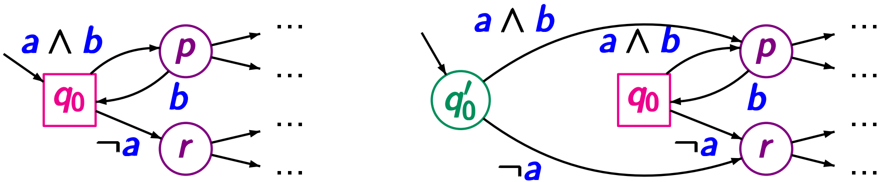


blocks for input
$\{a\}\,\varnothing\,\{a\}$

non-blocking

NFA where no initial state is final

NFA $\mathcal{A}$ with $Q_0 \cap F \neq \varnothing$ $\rightsquigarrow$ NFA $\mathcal{A}'$ with $Q_0 \cap F = \varnothing$



$$\boxed{\mathcal{L}(\mathcal{A}') = \mathcal{L}(\mathcal{A}) \setminus \{\varepsilon\}}$$

note: if $\mathcal{A}$ is an NFA for the bad prefixes of a safety property then

$$\varepsilon \notin \mathcal{L}(\mathcal{A}) = \mathit{BadPref}$$

Model checking regular safety properties

Let $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$ be a transition system (without terminal states)

$\mathcal{A} = (Q, 2^{AP}, \delta, Q_0, F)$ be an NFA for the bad prefixes of a regular safety property E (non-blocking and $Q_0 \cap F = \varnothing$)

The following statements are equivalent:

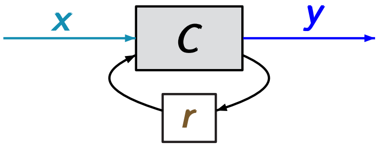
(1) $\mathcal{T} \models E$

(2) $Traces_{fin}(\mathcal{T}) \cap \mathcal{L}(\mathcal{A}) = \varnothing$

(3) $\mathcal{T} \otimes \mathcal{A} \models$ invariant "always $\neg F$"

where "$\neg F$" denotes $\bigwedge_{q \in F} \neg q$

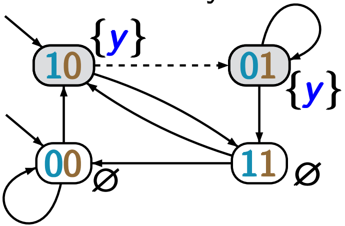
Example: sequential circuit



$\lambda_y = \delta_r = x \oplus r$

initially $r = 0$

transition system $\mathcal{T}$

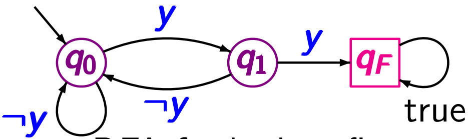


$\mathcal{T} \not\models E$

error indication, e.g.,

$\langle 10 \rangle \langle 01 \rangle$

bad prefix: $\{y\}\{y\}$



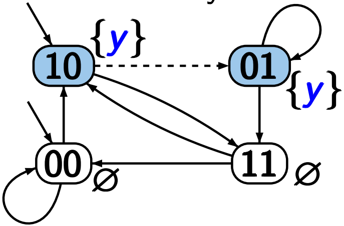
DFA for bad prefixes

safety property E

The circuit will never ouput two ones after each other

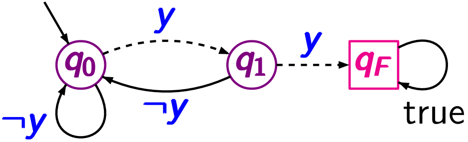
Example: product-TS

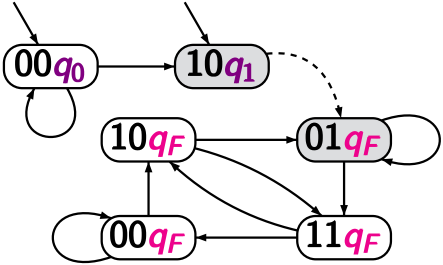
transition system $\mathcal{T}$



safety property E

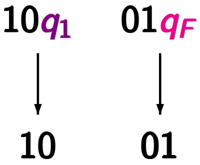
... never two ones in a row ...





error indication for

$\mathcal{T} \otimes \mathcal{A} \not\models$ "never q_F"



error indication for $\mathcal{T} \not\models E$

Model checking regular safety properties

input: finite TS $\mathcal{T}$,
NFA $\mathcal{A}$ for the bad prefixes of E

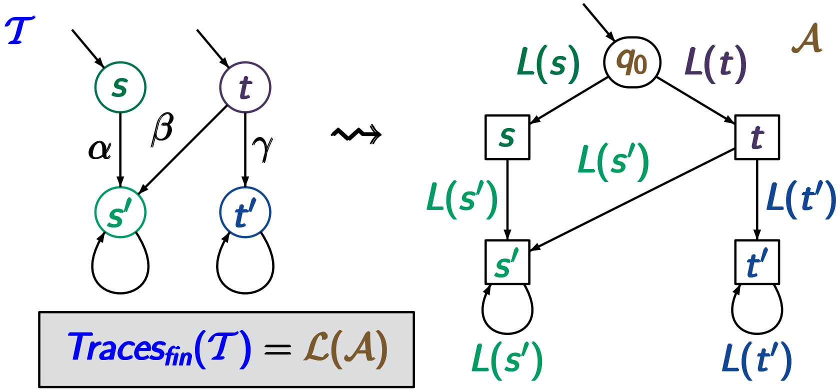
output: "yes" if $\mathcal{T} \models E$
otherwise "no" + error indication

construct product transition system $\mathcal{T} \otimes \mathcal{A}$
check whether $\mathcal{T} \otimes \mathcal{A} \models$ "always $\neg F$"
if so, then return "yes"
if not, then return "no" $\longleftarrow$ and an error indication

where F = set of final states in $\mathcal{A}$

> If $\mathcal{T}$ is a finite transition system then $\mathit{Traces}_{fin}(\mathcal{T})$ is regular.

correct. $\mathcal{T}$ can be transformed into an **NFA**.



$$\boxed{\mathit{Traces}_{fin}(\mathcal{T}) = \mathcal{L}(\mathcal{A})}$$