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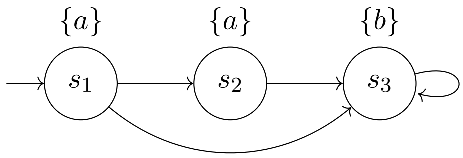
Exercise 1

a)

TS :



Then we have, that $Traces(s_2) = \{ab^\omega\} \subset \{aab^\omega, ab^\omega\} = Traces(s_1)$ So there is a formula $\psi = a \wedge \bigcirc a$, with $s_1 \models \psi$ and $s_2 \not\models \psi$, but there is no formula φ, s.t. $s_2 \models \psi$ and $s_1 \not\models \psi$. Therefore $s_1 \models \varphi$ iff $s_2 \models \varphi$ is not true and therefore $s_1 \not\equiv_{LTL} s_2$

b)

Let TS_1 and TS_2 be transition systems without terminal states and $TS_1 \not\equiv_{CTL} TS_2$. By definition on page 475 in the book, we know that there is a formula Φ, s.t. one of the following cases hold:

1. $TS_1 \models \Phi$ and $TS_2 \not\models \Phi$
   This is what is up to be proven and therefore already done.
2. $TS_1 \not\models \Phi$ and $TS_2 \models \Phi$
   For this we can take $\Psi' = \neg\Psi$ and then have through the semantics of CTL again a CTL formula for which $TS_1 \models \Phi'$ and $TS_2 \not\models \Phi'$

In both cases we are able to find a formula, s.t. the statement holds.

Exercise 2

Subformulas in $\Phi = \{\underbrace{\exists\Box b}_{c}, \underbrace{\exists\bigcirc(a \text{ U } c)}_{d}\} \Rightarrow \Phi = \forall\Diamond\Box d$

<u>Algorithm for $Sat(\exists\Box b)$:</u>

- $T = Sat(b) = \{s_0, s_2, s_4, s_6, s_7\}$
- $E = S \setminus Sat(b) = \{s_1, s_3, s_5\}$
- $c[s_0] = 1; c[s_2] = 2; c[s_4] = 2; c[s_6] = 3; c[s_7] = 1$
- 1. Iteration:
  - Pick s_1

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  - set $c[s_0] = 0$ and remove it from T
  - set $c[s_2] = 1$

  $\Rightarrow$ $T = \{s_2, s_4, s_6, s_7\}; E = \{s_0, s_3, s_5\}$ and
  $c[s_0] = 0; c[s_2] = 1; c[s_4] = 2; c[s_6] = 2; c[s_7] = 1$

- 2. Iteration:
  - Pick s_0
  - set $c[s_2] = 0$ and remove it from T

  $\Rightarrow$ $T = \{s_4, s_6, s_7\}; E = \{s_2, s_3, s_5\}$ and
  $c[s_0] = 0; c[s_2] = 0; c[s_4] = 1; c[s_6] = 2; c[s_7] = 1$

- 3. Iteration:
  - Pick s_2, where $Pre(s_2) \cap T = \emptyset$

  $\Rightarrow$ $T = \{s_4, s_6, s_7\}; E = \{s_3, s_5\}$ and
  $c[s_0] = 0; c[s_2] = 0; c[s_4] = 2; c[s_6] = 3; c[s_7] = 1$

- 4. Iteration:
  - Pick s_3
  - set $c[s_6] = 2$

  $\Rightarrow$ $T = \{s_4, s_6, s_7\}; E = \{s_5\}$ and
  $c[s_0] = 0; c[s_2] = 0; c[s_4] = 2; c[s_6] = 2; c[s_7] = 1$

- 5. Iteration:
  - Pick s_5, where $Pre(s_5) \cap T = \emptyset$

  $\Rightarrow$ $T = \{s_4, s_6, s_7\}; E = \emptyset$ and
  $c[s_0] = 0; c[s_2] = 0; c[s_4] = 2; c[s_6] = 2; c[s_7] = 1$

Therefore $Sat(\exists\Box b) = \{s_4, s_6, s_7\}$ and we label them with c

Per definition on slide 57: $Sat(\exists \bigcirc (a \text{ U } c)) = \{s_1, s_3, s_4, s_5, s_6, s_7\}$ and we label those states with d.

Now through slide 50: $Sat(\Phi) = Sat(\forall\Diamond\Box d) = Sat(\Diamond\Box d) = \{s_0, s_1, s_3, s_4, s_6, s_7\}$ (Note: not s_5 and s_2 since there is the path $(s_5 s_2)^\omega$)

So since $\{s_0, s_5\} = S_0 \not\subseteq Sat(\Phi)$ if follows that $TS \not\models \Phi$

Exercise 3

a)

Counterexample used:

- $t_1 = \{a\}\{a,b\}\{a\}\{b\}^\omega$

For TS_1:

- TS_1 and TS_2 are not equivalent since, t_1 is a trace in TS_2 but not in TS_1.
- TS_1 and TS_3 are not equivalent since, t_1 is a trace in TS_3 but not in TS_1.
- TS_1 and TS_4 are not equivalent since, t_1 is a trace in TS_4 but not in TS_1.

For TS_2:

- TS_2 and TS_3 are equivalent with the set
  $Traces(TS_2) = \{\{a\}\{a,b\}^\omega, (\{a\}\{a,b\}^*)^\omega, (\{a\}\{a,b\}^*)^*\{a\}\{b\}^\omega(\{a\}\{a,b\}^*)^*\emptyset^\omega\}$
- TS_2 and TS_4 are equivalent with the same set as above

For TS_3:

- TS_3 and TS_4 are, since they are both equivalent to TS_2 and because of transitivity, also equivalent with the same set as above

b)

For TS_1 and any other $TS_i, i \in \{2,3,4\}$ we can use the formula $\phi = \exists \bigcirc (a \wedge b \wedge \bigcirc a)$
For TS_2 and TS_3 we can give $\mathcal{R}_1 = \{(s_0,s_0),(s_1,s_1),(s_2,s_2),(s_3,s_2),(s_4,s_3),(s_5,s_3)\}$
For TS_3 and TS_4 we can give $\mathcal{R}_2 = \{(s_0,s_0),(s_0,s_4),(s_1,s_1),(s_2,s_2),(s_2,s_3),(s_3,s_5)\}$
For TS_2 and TS_4 we can build the concatenation as described on slide 80 and receive $\mathcal{R}_3 = \{(s_0,s_0),(s_0,s_4),(s_1,s_1),(s_2,s_2),(s_2,s_3),(s_3,s_2),(s_3,s_2),(s_4,s_5),(s_5,s_5)\})$