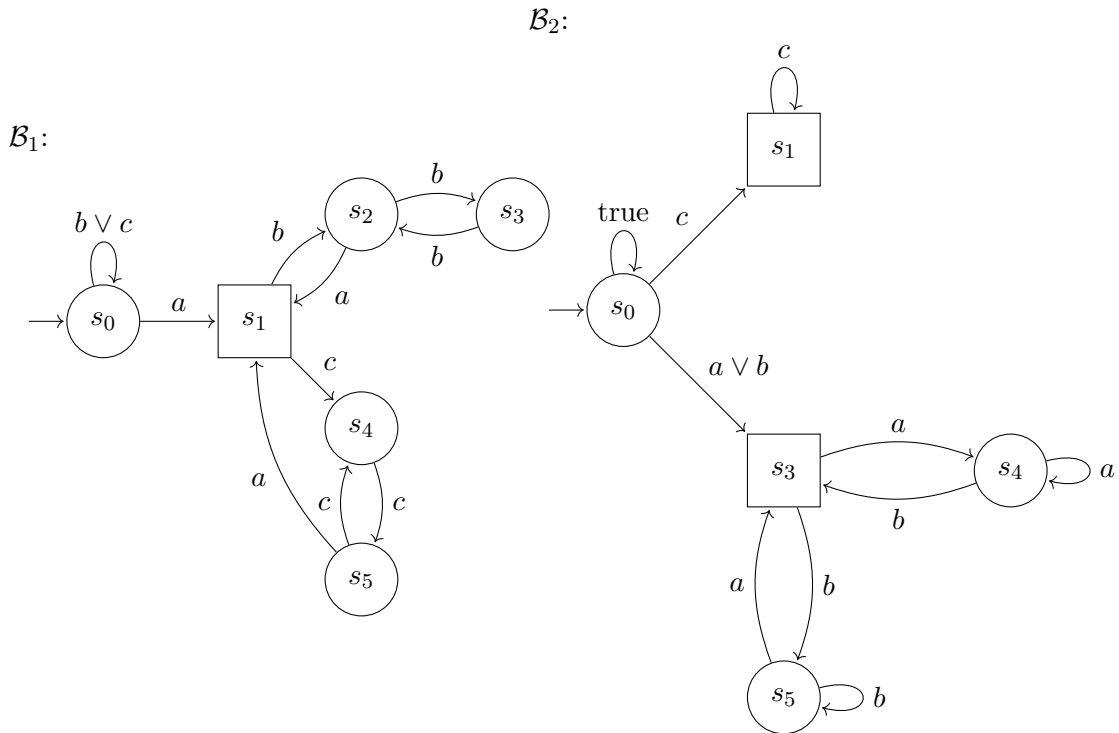


## Exercise 1

a)

- $\alpha_1 = a^*. (b.a^+)^{\omega} + a^*. (b.a^+)^*. b.a^{\omega}$
- $\alpha_2 = ((c.(b.c)^*.a + b).a)^{\omega}$

b)



c)

$\mathcal{L}_{\omega}^1$ :

Since the stated NBA is already a DBA there obviously exists one.

$\mathcal{L}_{\omega}^2$ :

The proof follows the principle of the proof of  $(A + B)^*. A^{\omega}$ . In this case  $A$  corresponds to  $c$  and  $B$  to  $b$ .

Assuming there exists a DBA  $\mathcal{A}$  with  $\mathcal{L}_{\omega}(\mathcal{A}) = \mathcal{L}_{\omega}^2 =: L$ .

Note that  $\mathcal{L}_\omega((c+b)^*.c^\omega) \subseteq L$ , since  $c$  occurs infinitely many times and  $a$  and  $b$  only occur finitely many times.

Following the proof yields a sequence  $n_1, n_2, \dots$  of natural numbers and a sequence  $q_1, q_2, \dots$  of accepting states such that  $\delta(q_0, c^{n_1}bc^{n_2} \dots c^{n_{i-1}}bc^{n_i}) = \{q_i\}$ .

Since  $Q$  is finite, there ex.  $i < j$  such that  $\delta(q_0, c^{n_1}b \dots bc^{n_i}) = \delta(q_0, c^{n_1}b \dots bc^{n_j})$ .

Thus,  $\mathcal{A}$  has an accepting run on  $c^{n_1}b \dots bc^{n_i}(bc^{n_{i+1}} \dots bc^{n_j})^\omega \notin L$ , since  $c$  and  $b$  occur infinitely many times. This contradicts  $\mathcal{L}_\omega(\mathcal{A}) = L$ .

Question:

I guess we cannot just apply the theorem from slide 176 of lec9+10 with  $A = \{c\}$  and  $B = \{a, b\}$ , because the union with  $(a + b + c)^*. (a + b)^\omega$  could theoretically make it DBA-realizable again?

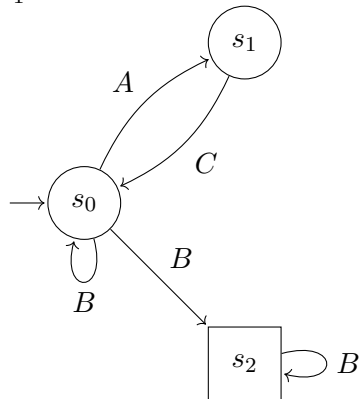
## Exercise 2

Counting?

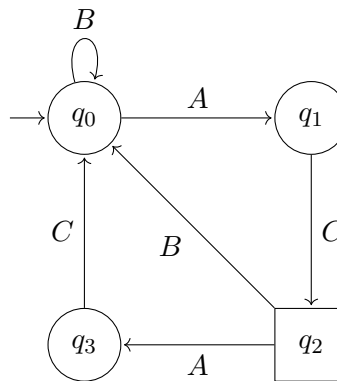
## Exercise 3

a)

$\mathcal{A}_1$ :



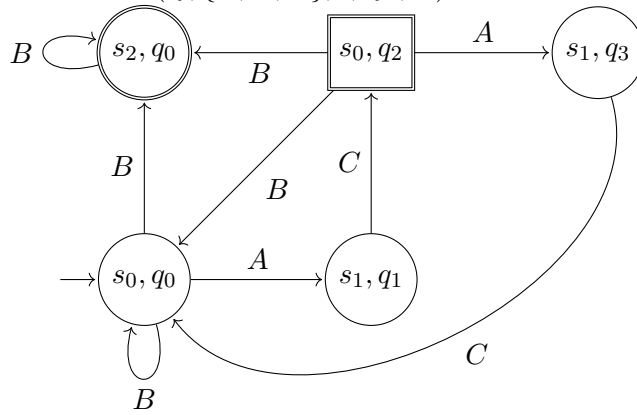
$\mathcal{A}_2$ :



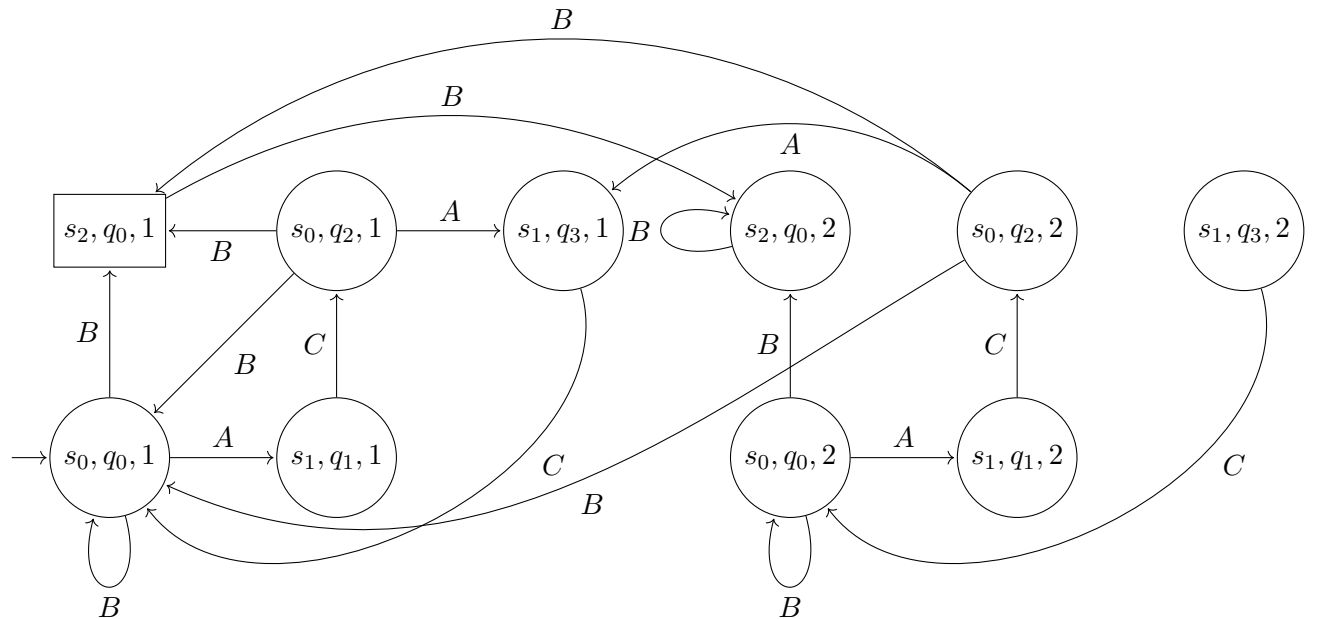
Model Checking Exercise Sheet 5

b)

GNBA  $\mathcal{G} = (Q, \{A, B, C\}, \delta, Q_0, \mathcal{F})$



c)



d)

Based on the picture in Exercise 3 b), we can see that once we enter the state  $(s_2, q_0)$  we will never leave it. Therefore we can never be infinitely often in the final state  $(s_0, q_2) \in F_2$ .

So we cannot fulfill the acceptance criteria that we are infinitely often in a state of every set in  $\mathcal{F} = \{F_1, F_2\}$ . So there is no word  $w \in \mathcal{L}_\omega(\mathcal{G}) \Rightarrow \mathcal{L}_\omega(\mathcal{G}) = \emptyset$

## Exercise 4

a)

$$\alpha_{(q_2, q_3)} = c.a^\omega$$

$$\alpha_{(q_1, q_3)} = false = \text{no combination}$$

$$\alpha_{(q_0, q_2)} = c^\omega$$

$$\alpha = \alpha_{(q_2, q_3)} + \alpha_{(q_1, q_3)} + \alpha_{(q_0, q_2)} = c.a^\omega + c^\omega$$

b)

Let GNBA  $\mathcal{G} = (Q, \Sigma, \Delta, Q_0, \mathcal{F})$  then we can construct the equivalent nondeterministic Muller automaton  $\mathcal{A} = (Q, \Sigma, \delta, Q_0, \mathcal{F}')$  by just taking for every set  $F_i \in \mathcal{F}, 1 \leq i \leq |\mathcal{F}|$  and build the cross product of all these  $F_i$ 's:

$$\mathcal{F}' = F_1 \times F_2 \times \dots \times F_{|\mathcal{F}|}$$

In  $\mathcal{G}$ , in order to be accepting, we had to visit at-least one element of every  $F_i$  infinitely often. In The Muller automaton we have to reach one of every possible combination.

Let  $\rho$  be an arbitrary accepting run of  $\mathcal{G}$ . Then  $\rho$  visits at-least one state of every set  $F_i$  infinitely often, since all those  $F_i$  are of finite length.

Therefore we can state a  $f_i \in F_i$  that is visited infinitely often.

By construction  $\{f_1, f_2, \dots, f_{|\mathcal{F}|}\} \in \mathcal{F}'$  and therefore  $\mathcal{A}$  accepts.