Overview

Introduction

Modelling parallel systems

Linear Time Properties

Regular Properties

Linear Temporal Logic

Computation-Tree Logic

Equivalences and Abstraction

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Linear Time Properties

- state-based and linear time view ⟵
- definition of linear time properties
- invariants and safety
- liveness and fairness

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State-based view of TS

transition system $\mathcal{T} = (S, Act, \longrightarrow, S_0, AP, L)$

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Act for modeling interactions/communication

AP, L for specifying properties

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$\downarrow$ abstraction from actions

state graph $G_{\mathcal{T}}$

- set of nodes = state space S
- edges = transitions without action label

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State-based view of TS

transition system $\mathcal{T} = (S, Act, \longrightarrow, S_0, AP, L)$

$\downarrow$ abstraction from actions

state graph $G_{\mathcal{T}}$

- set of nodes = state space S
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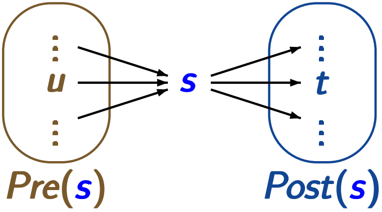
use standard notations for graphs, e.g.,

$$Post(s) = \{t \in S : s \rightarrow t\}$$
$$Pre(s) = \{u \in S : u \rightarrow s\}$$



Execution fragments

execution fragment: sequence of consecutive transitions

$s_0 \xrightarrow{\alpha_0} s_1 \xrightarrow{\alpha_1} \ldots$ infinite or

$s_0 \xrightarrow{\alpha_0} s_1 \xrightarrow{\alpha_1} \ldots \xrightarrow{\alpha_{n-1}} s_n$ finite

Execution and path fragments

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$\pi = s_0\, s_1\, s_2 \ldots$ infinite or $\pi = s_0\, s_1 \ldots s_n$ finite

such that $s_{i+1} \in Post(s_i)$ for all $i < |\pi|$

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Notations for paths

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s.t. $s_{i+1} \in Post(s_i)$ for all $i < |\pi|$

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path of TS $\mathcal{T}$ $\widehat{=}$ initial, maximal path fragment

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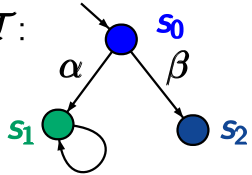
$Paths(\mathcal{T})$ = set of all initial, maximal path fragments

$Paths(s)$ = set of all maximal path fragments starting in state s

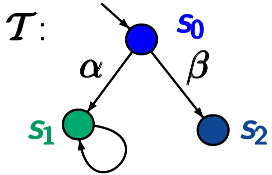
Paths of a TS

$\mathcal{T}$:



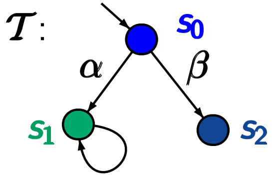
How many paths are there in $\mathcal{T}$?

Paths of a TS



How many paths are there in $\mathcal{T}$?

answer: 2, namely $s_0\, s_1\, s_1\, s_1 \ldots$ and $s_0\, s_2$



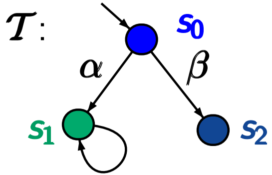
How many paths are there in $\mathcal{T}$?

answer: 2, namely $s_0\, s_1\, s_1\, s_1 \ldots$ and $s_0\, s_2$

$\mathit{Paths}(s_1)$ = set of all maximal paths fragments starting in s_1

$= \{s_1^\omega\}$ where $s_1^\omega = s_1\, s_1\, s_1\, s_1\, \ldots$

Paths of a TS and its states



How many paths are there in $\mathcal{T}$?

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$$\begin{aligned} Paths(s_1) &= \text{set of all maximal paths fragments starting in } s_1 \\ &= \{s_1^{\omega}\} \text{ where } s_1^{\omega} = s_1\, s_1\, s_1\, s_1\, \ldots \end{aligned}$$

$$\begin{aligned} Paths_{fin}(s_1) &= \text{set of all finite path fragments starting in } s_1 \\ &= \{s_1^{n} : n \in \mathbb{N}, n \geq 1\} \end{aligned}$$

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Linear-time vs branching-time

Linear-time vs branching-time

transition system

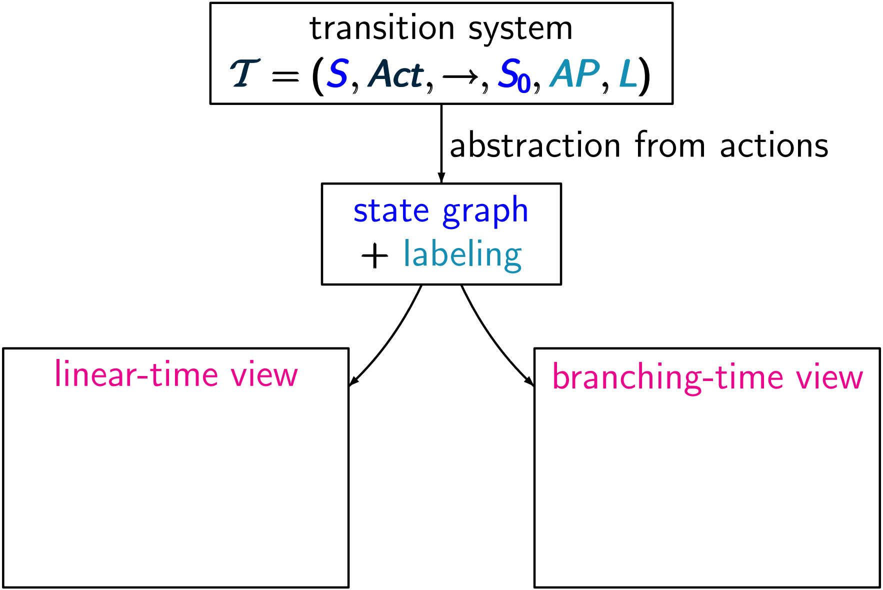
$$\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$$

$$\begin{array}{|c|}\hline \text{transition system} \\ \mathcal{T} = (S, Act, \rightarrow, S_0, AP, L) \\ \hline \end{array}$$

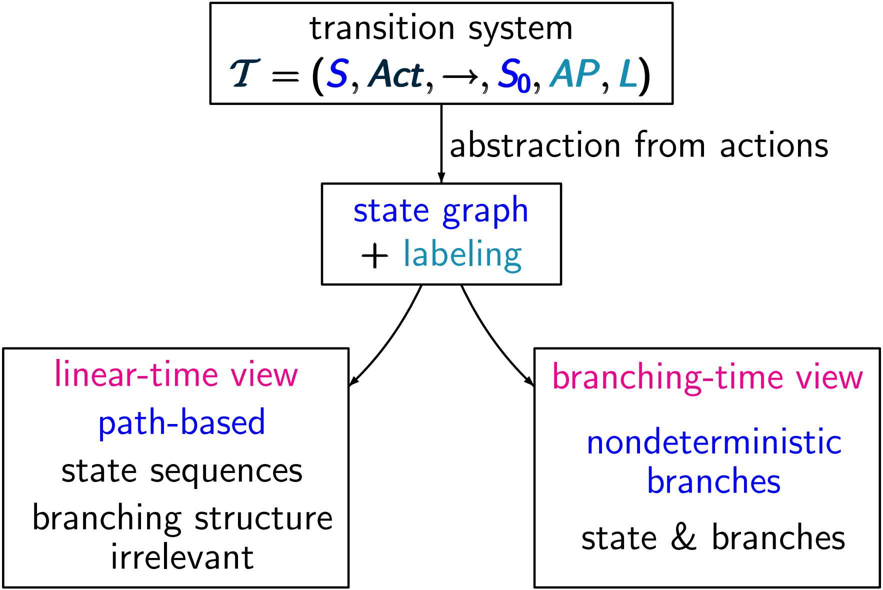
$$\downarrow \text{abstraction from actions}$$

$$\begin{array}{|c|}\hline \text{state graph} \\ + \text{ labeling} \\ \hline \end{array}$$

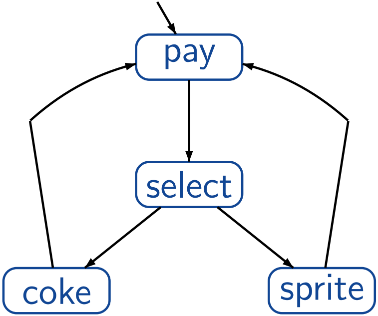
Linear-time vs branching-time



Linear-time vs branching-time

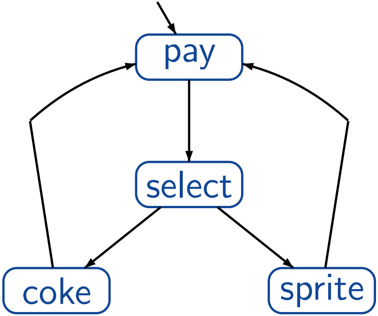


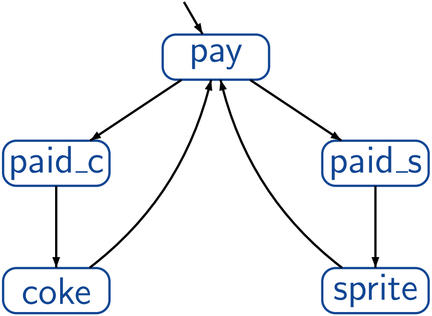
Example: vending machine



vending machine with
1 coin deposit
select drink after
having paid

Example: vending machine





vending machine with
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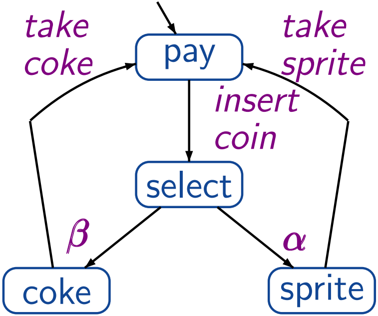
vending machine with
2 coin deposits
select drink by inserting the coin

Example: vending machine



vending machine with
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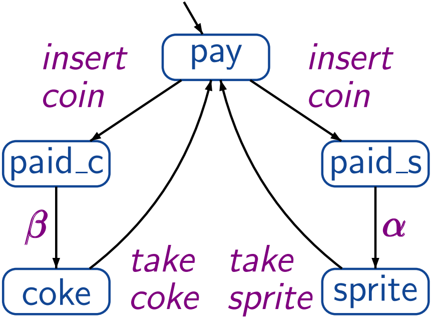

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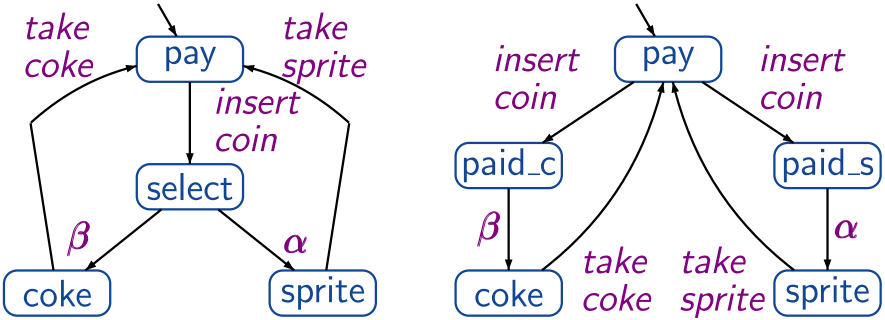
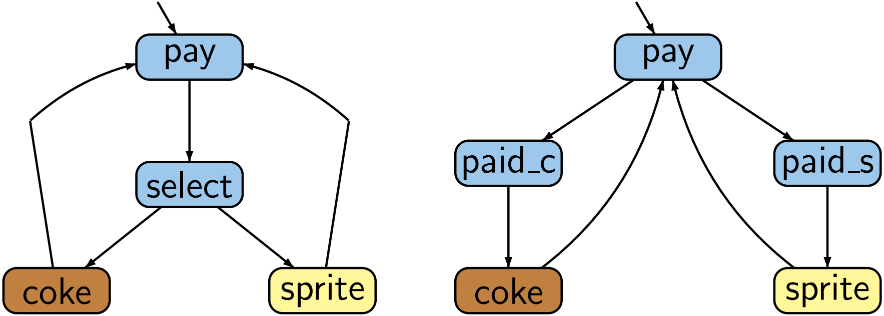
Example: vending machine



state based view: abstracts from actions and projects onto atomic propositions, e.g. $AP = \{coke, sprite\}$

Example: vending machine



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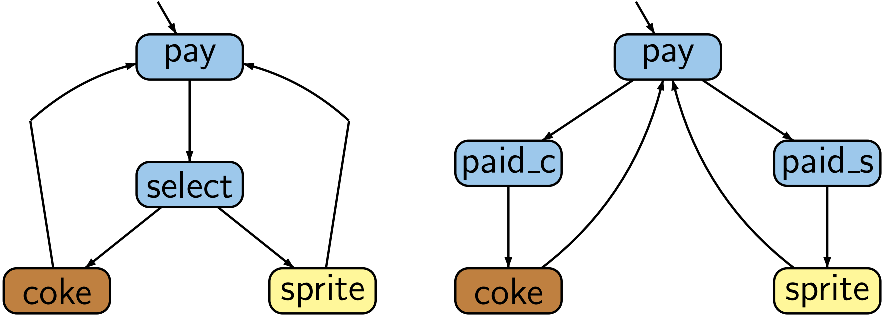
e.g., $L(\text{coke}) = \{coke\}$, $L(\text{pay}) = \varnothing$

Example: vending machine



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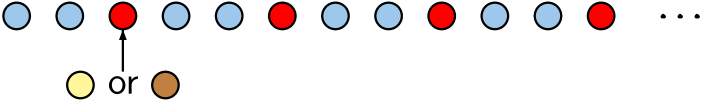
linear time: all observable behaviors are of the form



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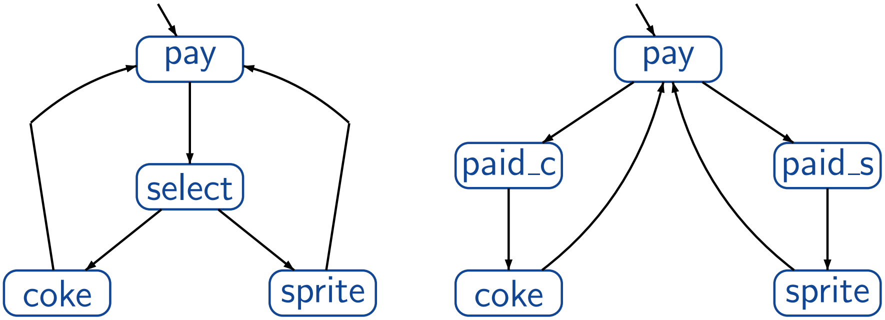

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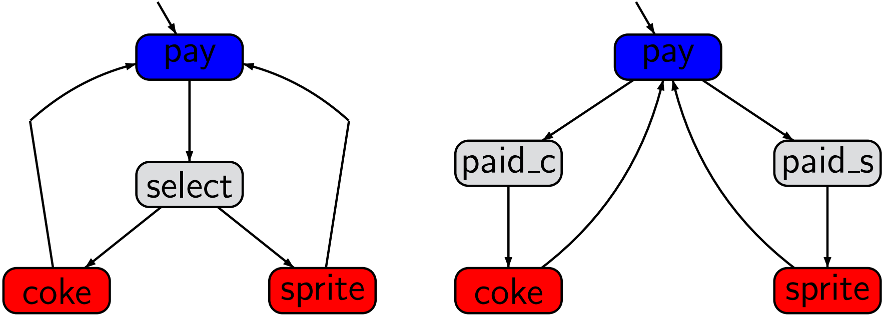

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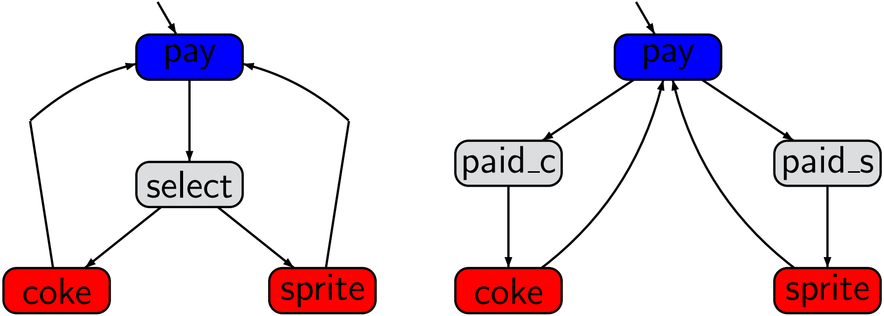

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linear & branching time:

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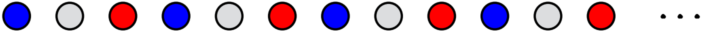
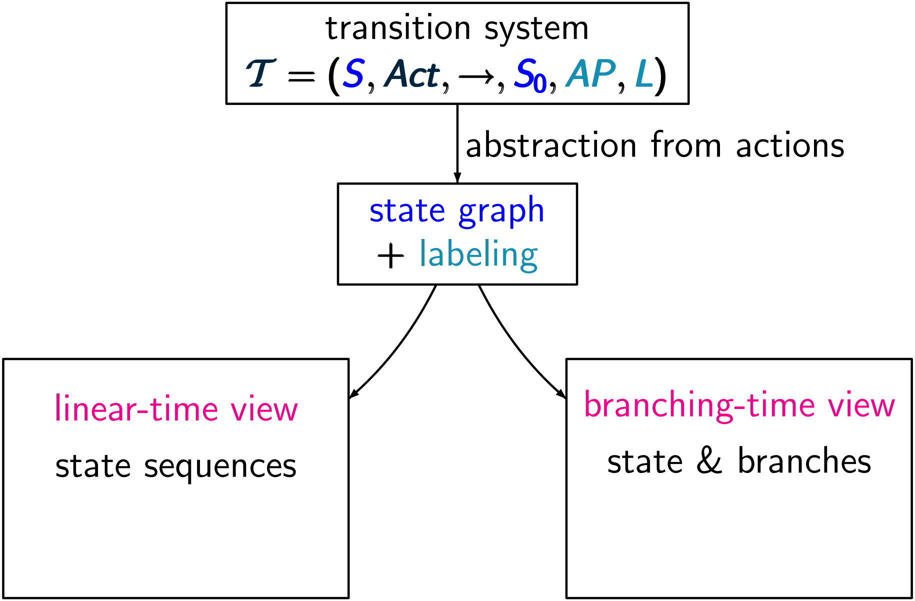


Linear-time vs branching-time



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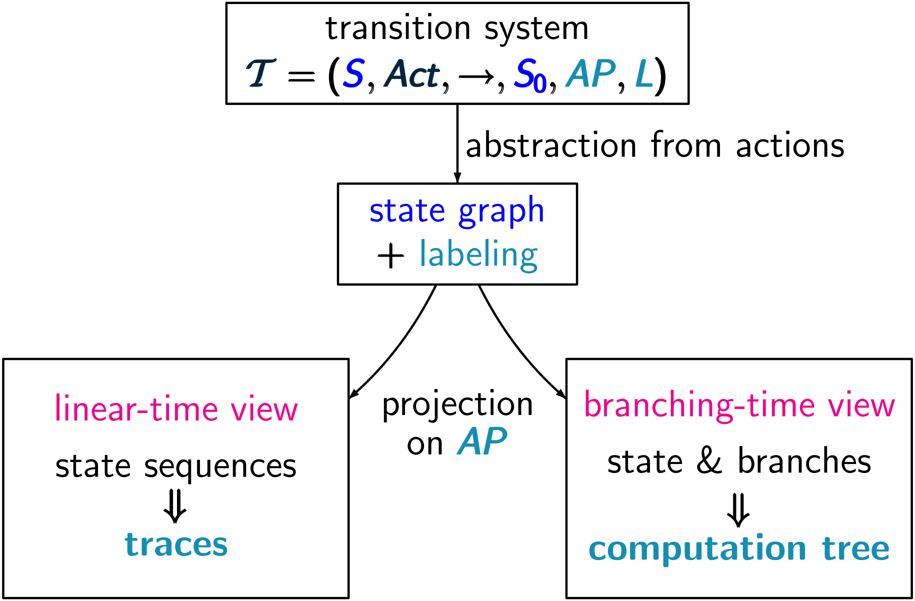

Traces

LTB2.4-4

Traces

for TS with labeling function $L : S \rightarrow 2^{AP}$

execution: states + actions

$s_0 \xrightarrow{\alpha_1} s_1 \xrightarrow{\alpha_2} s_2 \xrightarrow{\alpha_3} \ldots$ infinite or finite

$\downarrow$

paths: sequences of states

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↓

traces: sequences of sets of atomic propositions

$L(s_0)\, L(s_1)\, L(s_2)\, \ldots$

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traces: sequences of sets of atomic propositions

$L(s_0)\, L(s_1)\, L(s_2) \ldots \quad \in (2^{AP})^\omega \cup (2^{AP})^+$

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for simplicity: we often assume that the given TS has **no terminal states**

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Treatment of terminal states

perform standard graph algorithms to compute the reachable fragment of the given TS

$$Reach(\mathcal{T}) = \left\{ \begin{array}{c} \text{set of states that are reachable} \\ \text{from some initial state} \end{array} \right.$$

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for each reachable terminal state s:

- if s stands for an intended halting configuration then add a transition from s to a trap state:

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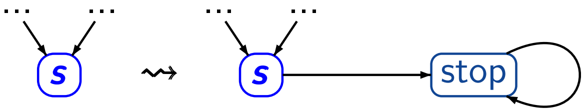

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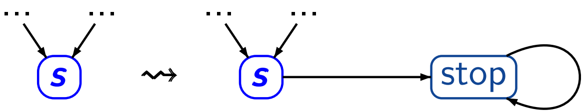

- if s stands for system fault, e.g., deadlock then correct the design before checking further properties

Traces of a transition system

Let $\mathcal{T}$ be a TS

$$Traces(\mathcal{T}) \quad \stackrel{\text{def}}{=} \left\{ trace(\pi) : \pi \in Paths(\mathcal{T}) \right\}$$

$$Traces_{fin}(\mathcal{T}) \stackrel{\text{def}}{=} \left\{ trace(\widehat{\pi}) : \widehat{\pi} \in Paths_{fin}(\mathcal{T}) \right\}$$

Traces of a transition system

Let $\mathcal{T}$ be a TS

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$$\mathit{Traces}_{\mathit{fin}}(\mathcal{T}) \stackrel{\text{def}}{=} \{\mathit{trace}(\underset{\substack{\uparrow\\ \text{initial, finite path fragment}}}{\widehat{\pi}}) : \widehat{\pi} \in \mathit{Paths}_{\mathit{fin}}(\mathcal{T})\}$$

Traces of a transition system

Let $\mathcal{T}$ be a TS $\longleftarrow$ $\boxed{\textit{without}\text{ terminal states}}$

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$$\mathit{Traces}_{\mathit{fin}}(\mathcal{T}) \overset{\text{def}}{=} \{\underset{\substack{\uparrow \\ \text{initial, finite path fragment}}}{\mathit{trace}(\widehat{\pi})} : \widehat{\pi} \in \mathit{Paths}_{\mathit{fin}}(\mathcal{T})\}$$

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$$\mathit{Traces}(\mathcal{T}) \stackrel{\text{def}}{=} \{\mathit{trace}(\underset{\uparrow}{\pi}) : \pi \in \mathit{Paths}(\mathcal{T})\} \subseteq (2^{AP})^{\omega}$$

initial, infinite path fragment

$$\mathit{Traces}_{\mathit{fin}}(\mathcal{T}) \stackrel{\text{def}}{=} \{\mathit{trace}(\underset{\uparrow}{\widehat{\pi}}) : \widehat{\pi} \in \mathit{Paths}_{\mathit{fin}}(\mathcal{T})\} \subseteq (2^{AP})^{*}$$

initial, finite path fragment

Example: traces

Let $\mathcal{T}$ be a TS without terminal states.

$$Traces(\mathcal{T}) \stackrel{\text{def}}{=} \{trace(\pi) : \pi \in Paths(\mathcal{T})\} \subseteq (2^{AP})^{\omega}$$

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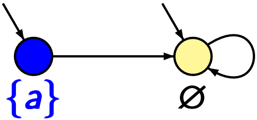


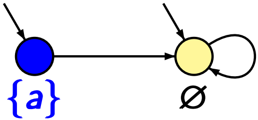
TS $\mathcal{T}$ with a single atomic proposition a

Example: traces

Let $\mathcal{T}$ be a TS without terminal states.

$$Traces(\mathcal{T}) \stackrel{\text{def}}{=} \{trace(\pi) : \pi \in Paths(\mathcal{T})\} \subseteq (2^{AP})^\omega$$

$$Traces_{fin}(\mathcal{T}) \stackrel{\text{def}}{=} \{trace(\widehat{\pi}) : \widehat{\pi} \in Paths_{fin}(\mathcal{T})\} \subseteq (2^{AP})^*$$

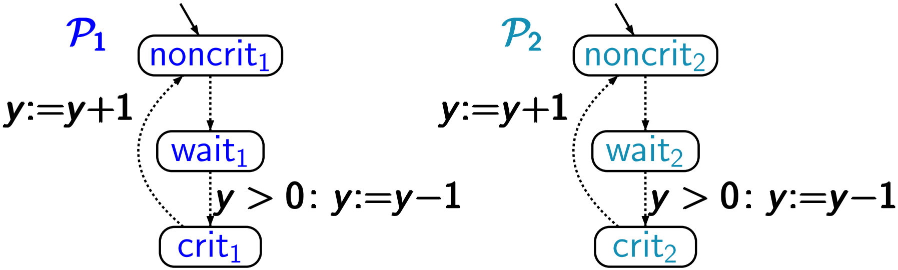


TS $\mathcal{T}$ with a single atomic proposition a

$$Traces(\mathcal{T}) = \{ \{a\}\varnothing^\omega, \varnothing^\omega \}$$

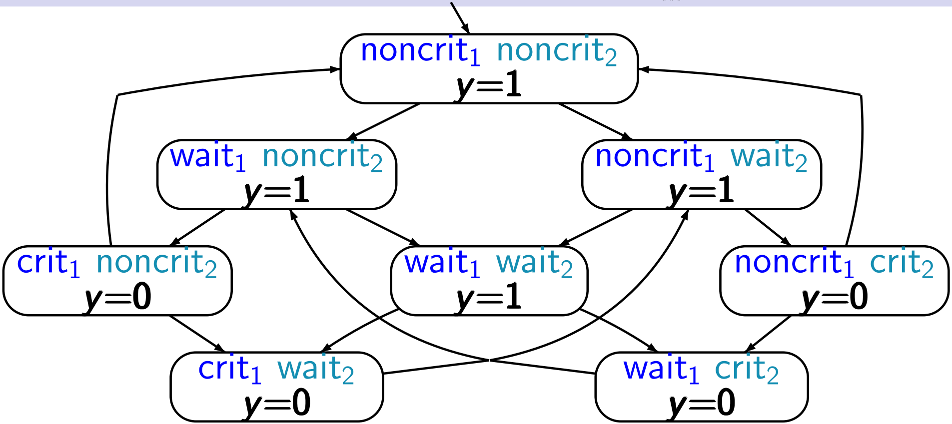
$$Traces_{fin}(\mathcal{T}) = \{\{a\}\varnothing^n : n \geq 0\} \cup \{\varnothing^m : m \geq 1\}$$

Mutual exclusion with semaphore



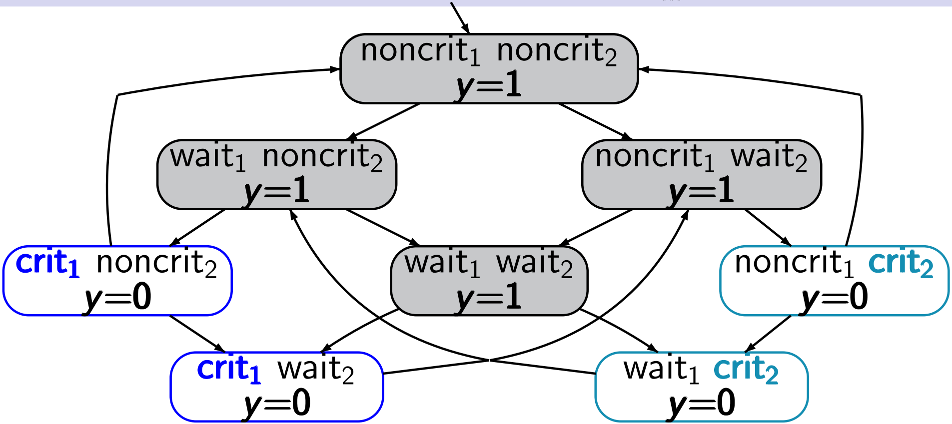
transition system $\mathcal{T}_{\mathcal{P}_1 ||| \mathcal{P}_2}$ arises by unfolding the composite program graph $\mathcal{P}_1 \ ||| \ \mathcal{P}_2$

Mutual exclusion with semaphore $\mathcal{T}_{\mathcal{P}_1 ||| \mathcal{P}_2}$



set of atomic propositions $AP = \{\mathbf{crit_1}, \mathbf{crit_2}\}$

Mutual exclusion with semaphore $\mathcal{T}_{\mathcal{P}_1 ||| \mathcal{P}_2}$



set of atomic propositions $AP = \{\text{crit}_1, \text{crit}_2\}$

e.g., $L(\langle \text{noncrit}_1, \text{noncrit}_2, y{=}1 \rangle) =$
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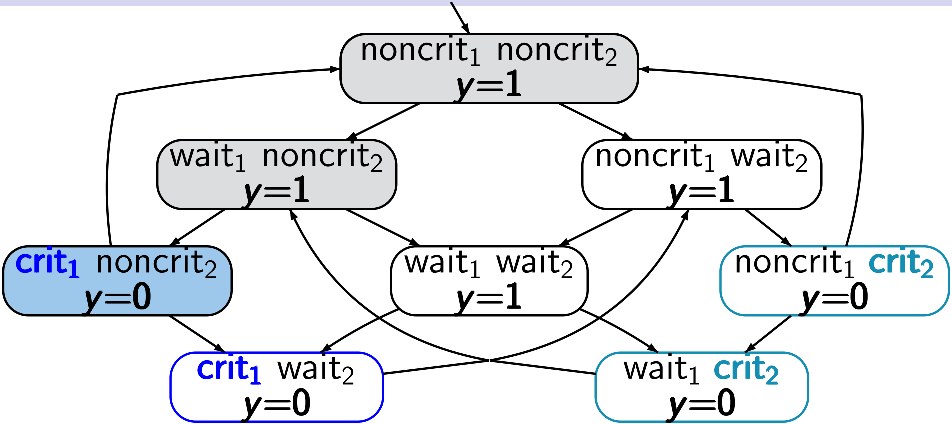
Mutual exclusion with semaphore $\mathcal{T}_{\mathcal{P}_1 ||| \mathcal{P}_2}$



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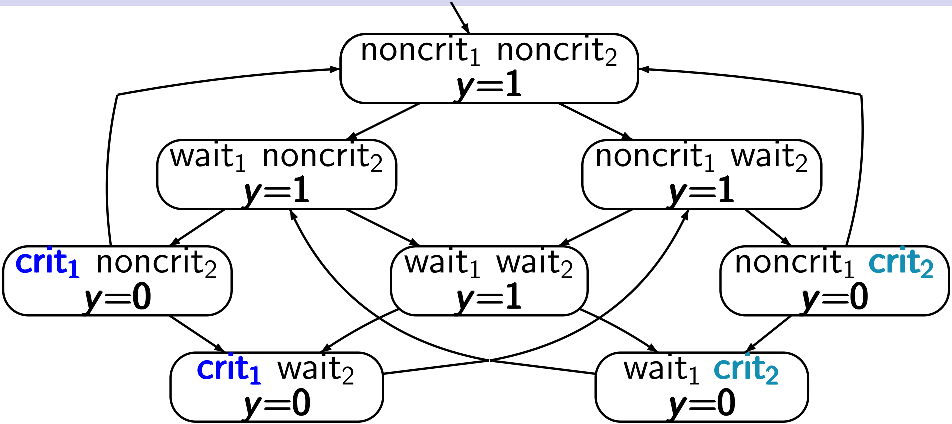

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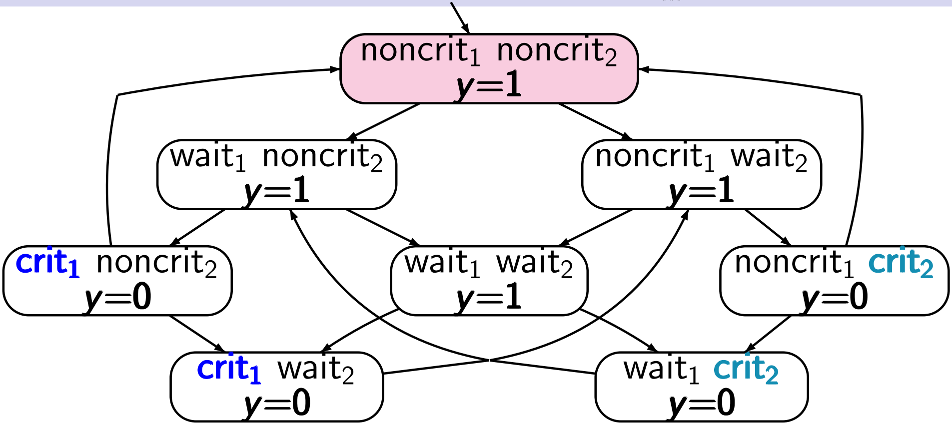

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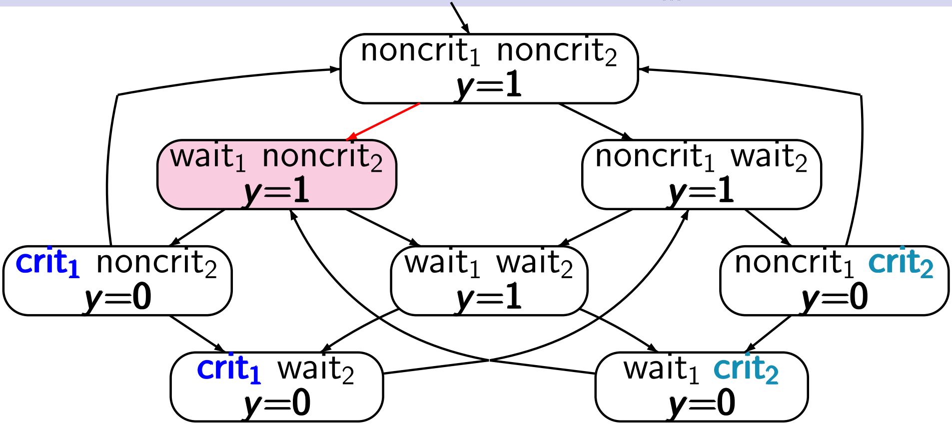

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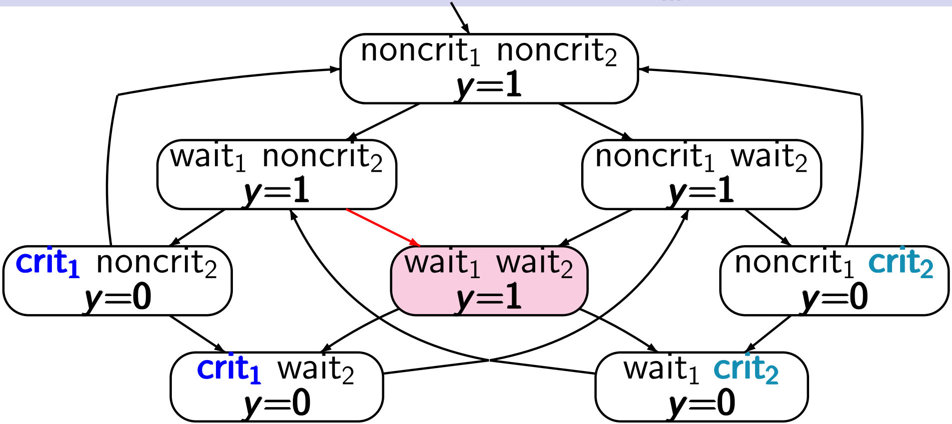

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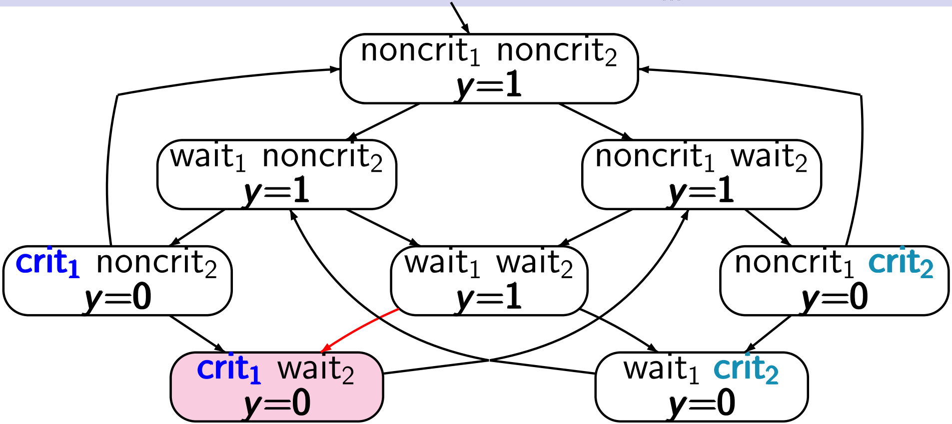

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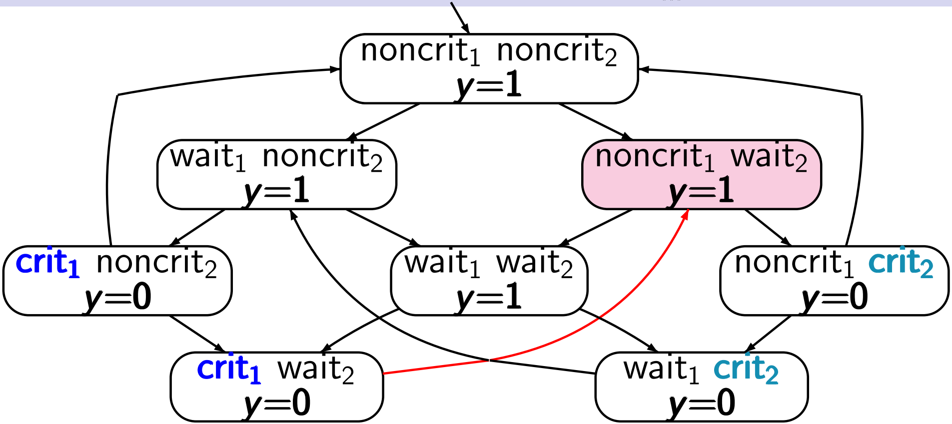

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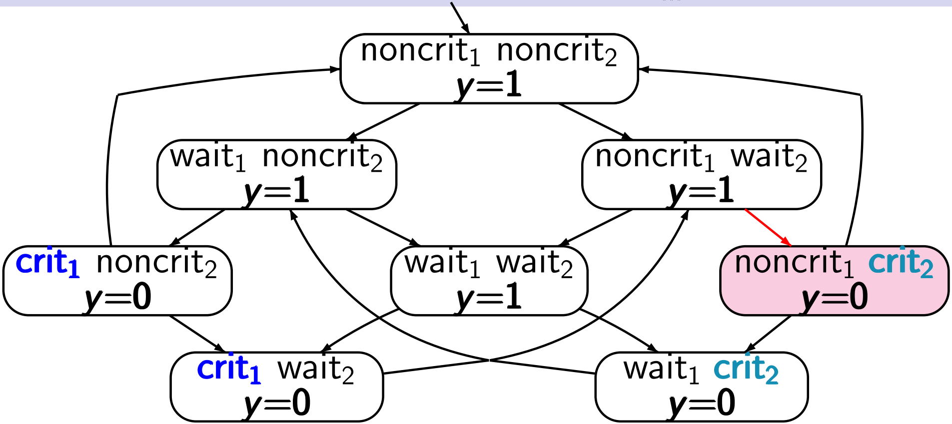

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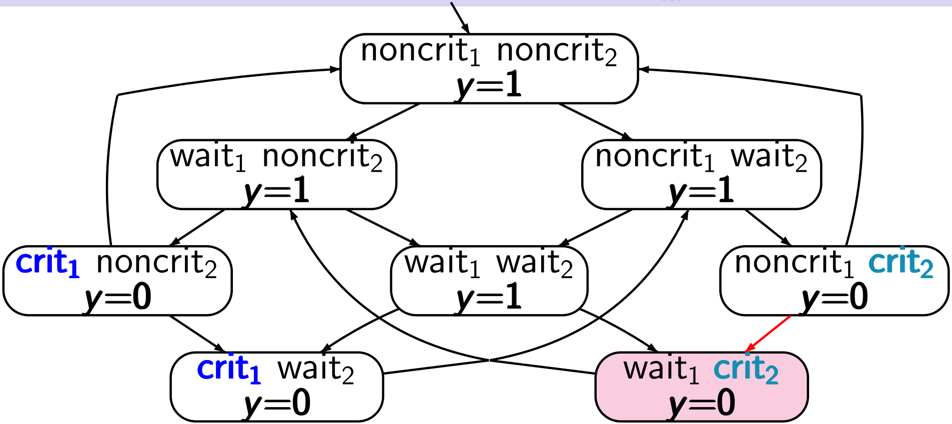

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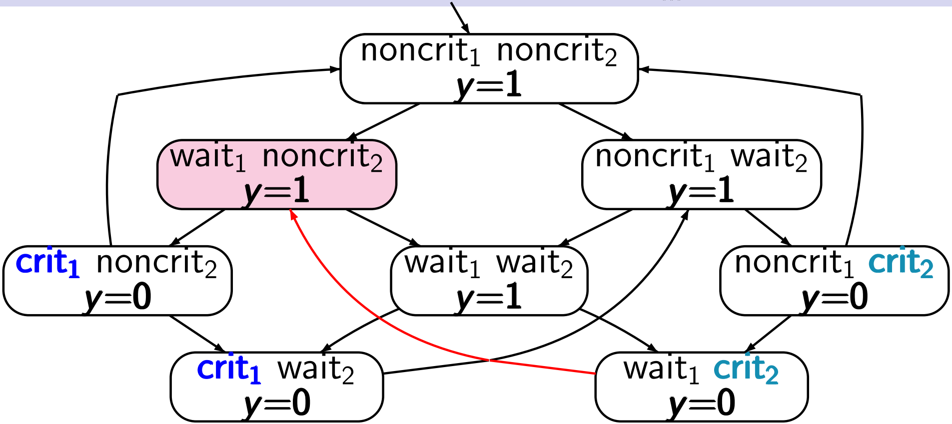

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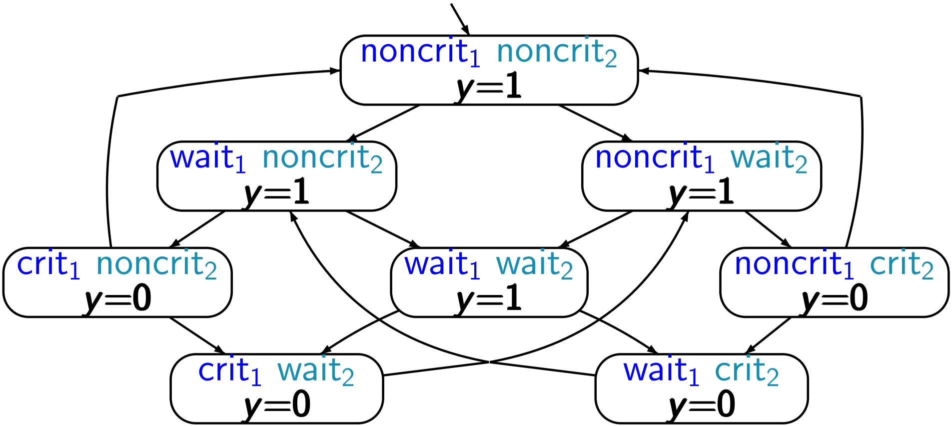

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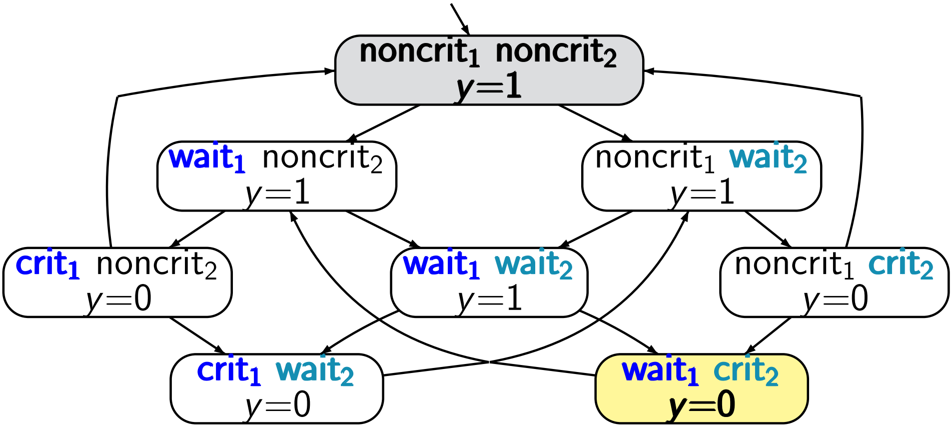

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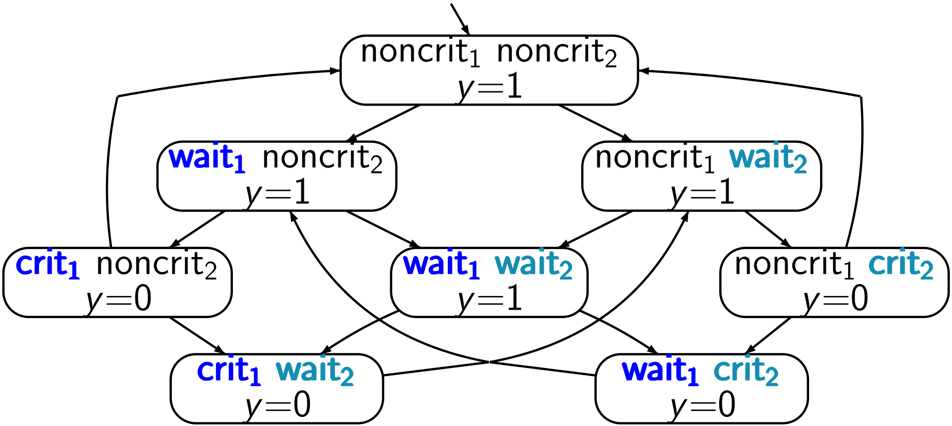

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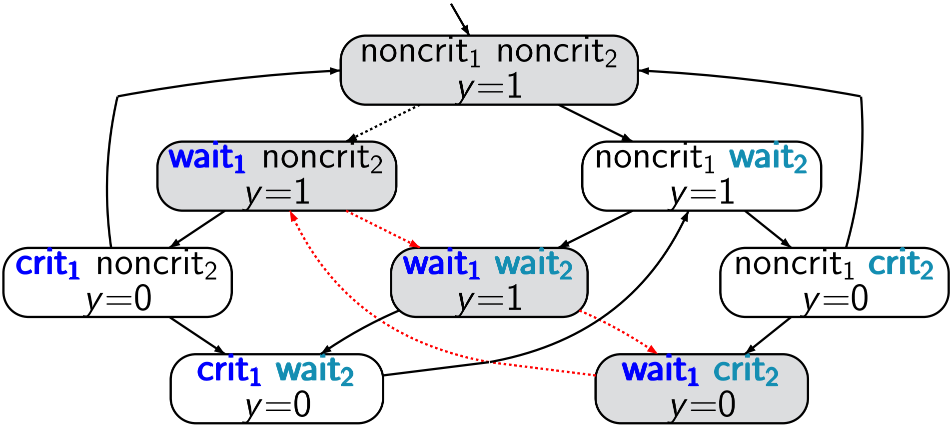

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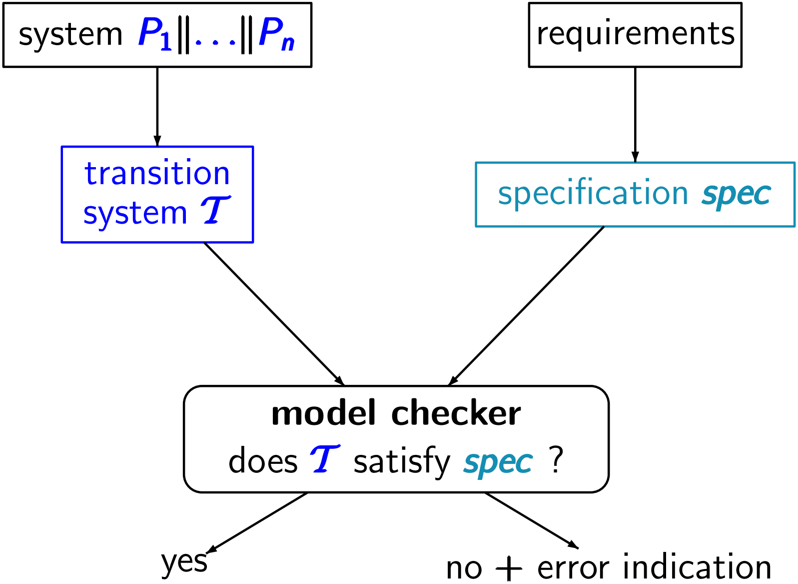
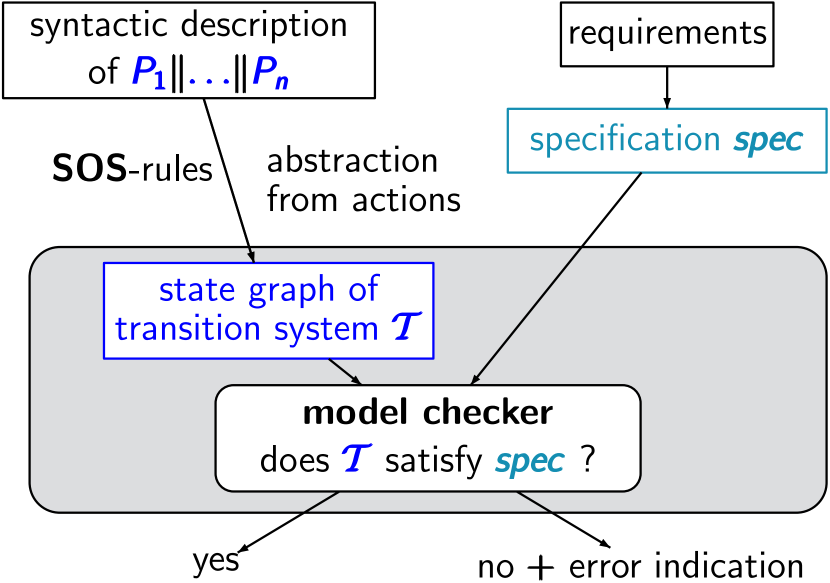
Overview

Model checking



Model checking



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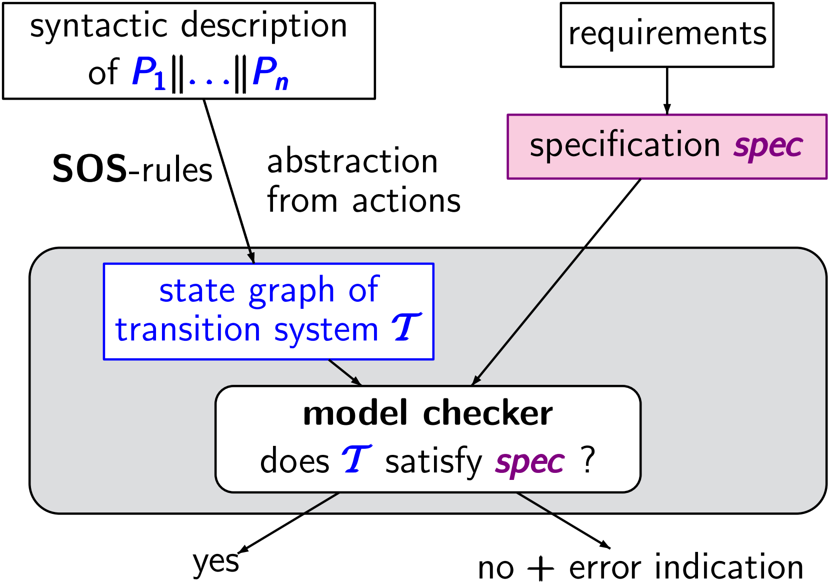

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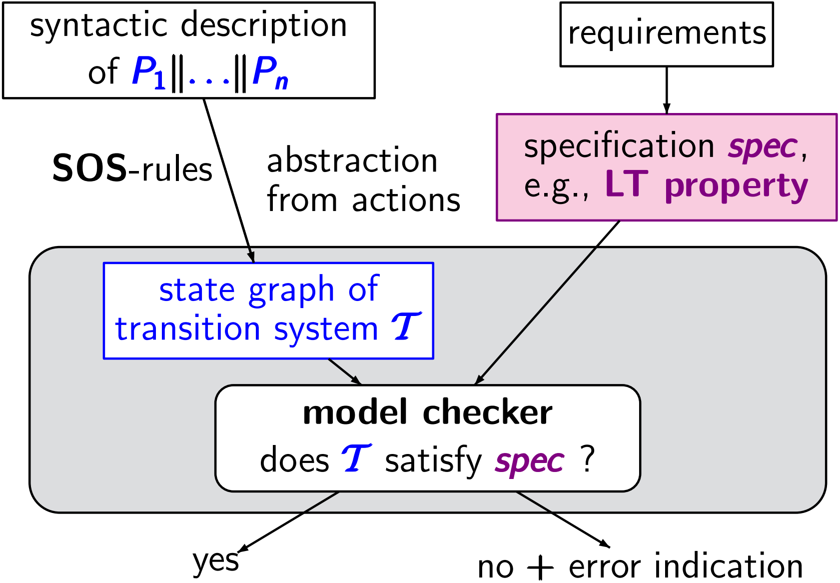

Linear-time properties (LT properties)

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for TS over AP without terminal states

An LT property over AP is a language E of infinite words over the alphabet $\Sigma = 2^{AP}$,

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E.g., for mutual exclusion problems and
$AP = \{crit_1, crit_2, \ldots\}$

safety:

$$MUTEX = \begin{array}{l}\text{set of all infinite words } A_0\, A_1\, A_2 \ldots \\ \text{over } 2^{AP} \text{ such that for all } i \in \mathbb{N}\text{:} \\ \quad crit_1 \notin A_i \ \text{ or } \ crit_2 \notin A_i\end{array}$$

LT properties for mutual exclusion protocols

$AP = \{\text{wait}_1, \text{crit}_1, \text{wait}_2, \text{crit}_2\}$

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$\varnothing\ \{\text{wait}_1\}\,\{\text{crit}_1\}\,\varnothing\ \{\text{wait}_1\}\,\{\text{crit}_1\}\ \ldots \qquad \in MUTEX$

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LT properties for mutual exclusion protocols

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liveness (starvation freedom):

$LIVE$ = set of all infinite words $A_0\, A_1\, A_2 \ldots$ s.t.

$$\overset{\infty}{\exists}\, i \in \mathbb{N}.\mathsf{wait}_1 \in A_i \implies \overset{\infty}{\exists}\, i \in \mathbb{N}.\mathsf{crit}_1 \in A_i$$

$$\wedge \;\; \overset{\infty}{\exists}\, i \in \mathbb{N}.\mathsf{wait}_2 \in A_i \implies \overset{\infty}{\exists}\, i \in \mathbb{N}.\mathsf{crit}_2 \in A_i$$

Satisfaction relation for LT properties

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Satisfaction relation $\models$ for TS:

If $\mathcal{T}$ is a TS (without terminal states) over AP and E an LT property over AP then

$$\mathcal{T} \models E \quad \text{iff} \quad Traces(\mathcal{T}) \subseteq E$$

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Satisfaction relation $\models$ for TS and states:

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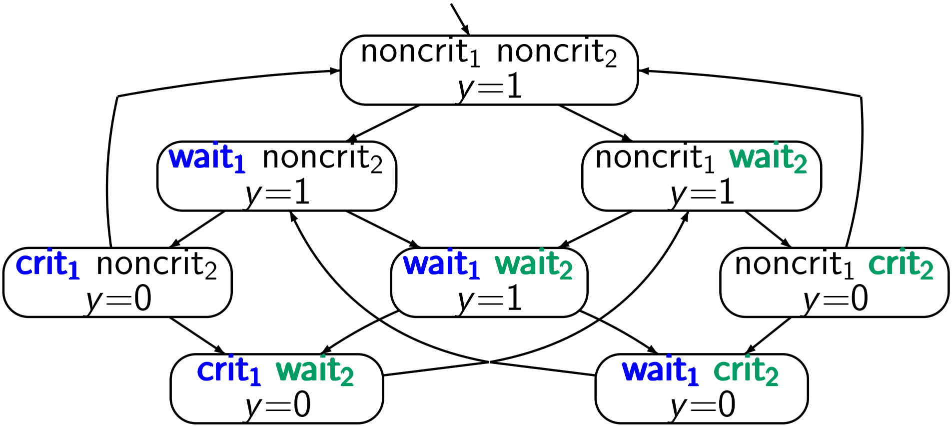
If s is a state in $\mathcal{T}$ then

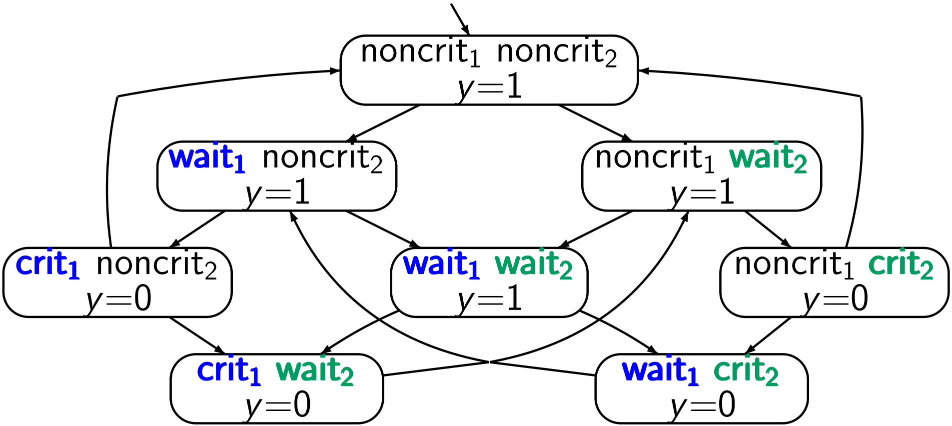
$$s \models E \quad \text{iff} \quad Traces(s) \subseteq E$$

Mutual exclusion with semaphore



$\mathcal{T}_{Sem} \models \mathit{MUTEX}$

Mutual exclusion with semaphore



$\mathcal{T}_{Sem} \models \mathit{MUTEX}, \quad \mathcal{T}_{Sem} \models \mathit{LIVE}$?

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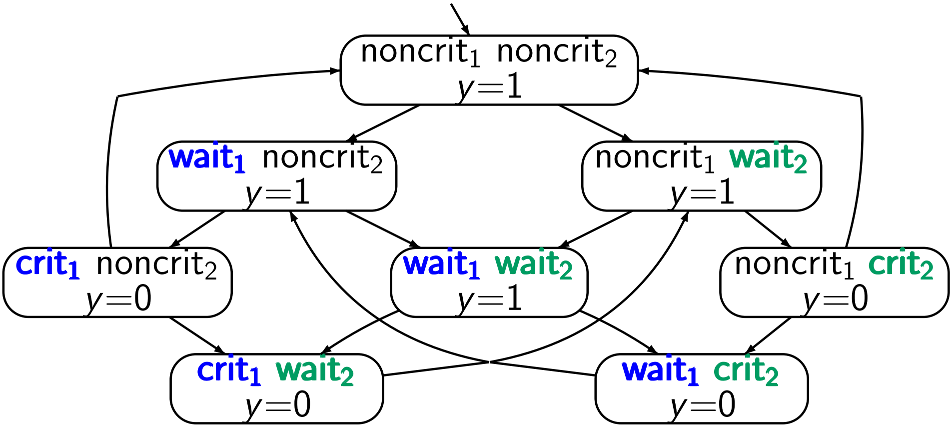

$$\mathcal{T}_{Sem} \models \mathit{MUTEX}, \quad \mathcal{T}_{Sem} \not\models \mathit{LIVE}$$

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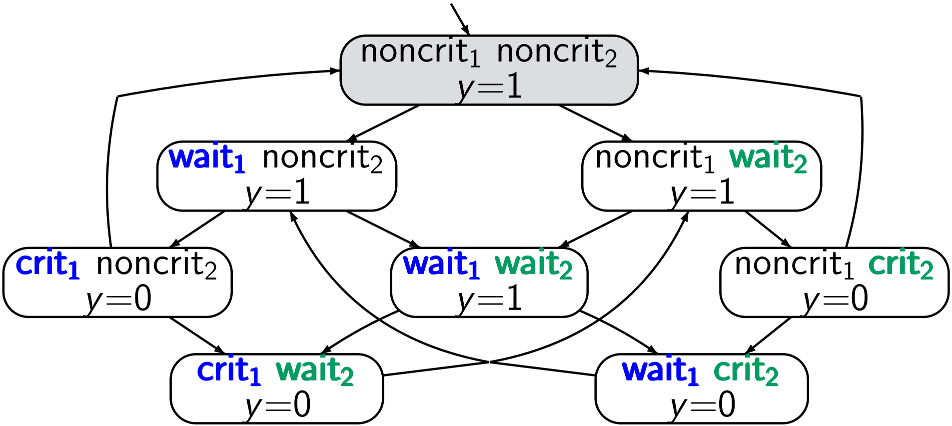

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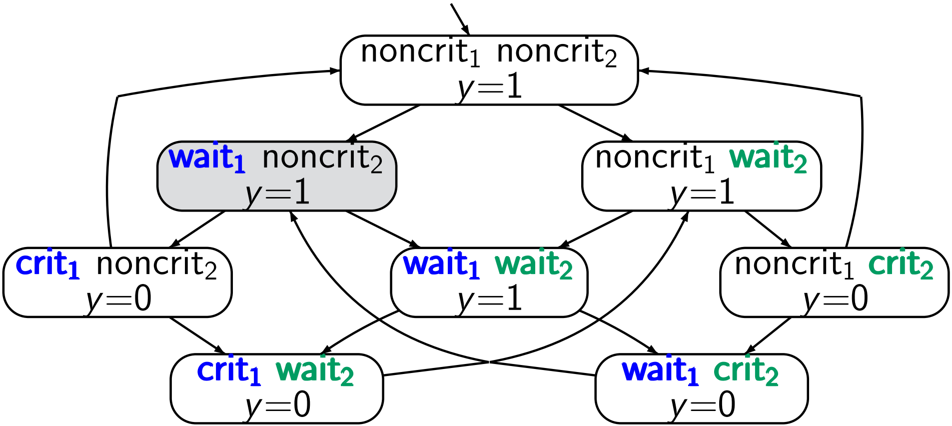

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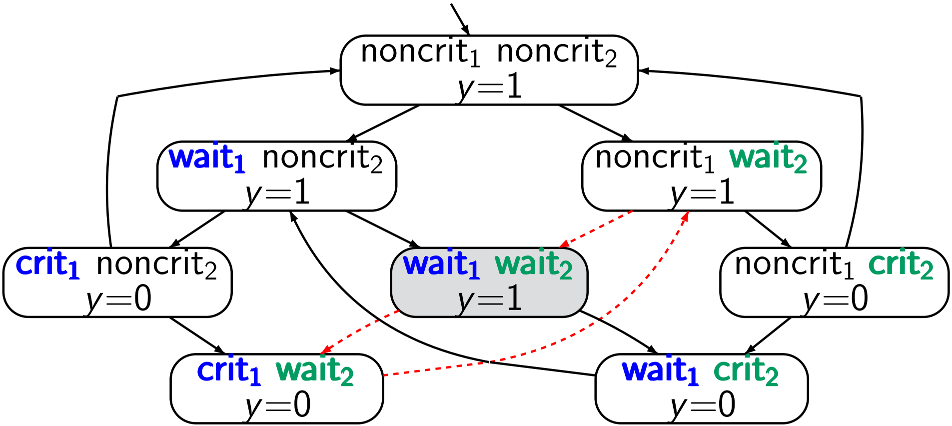

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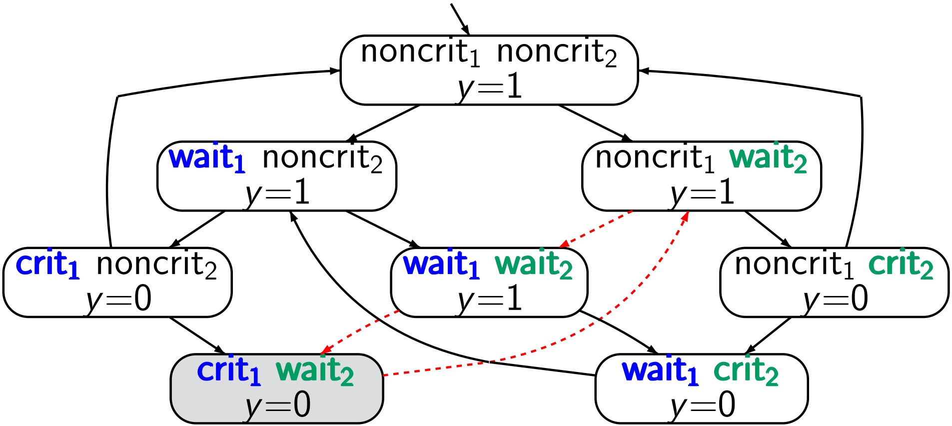

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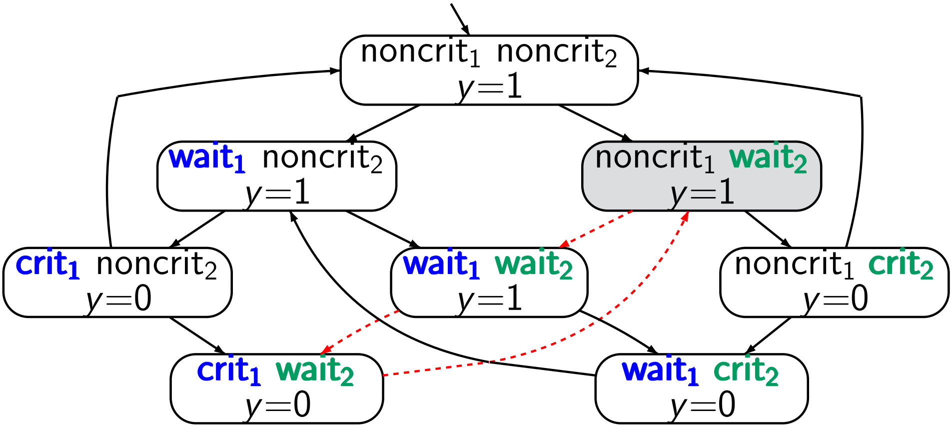

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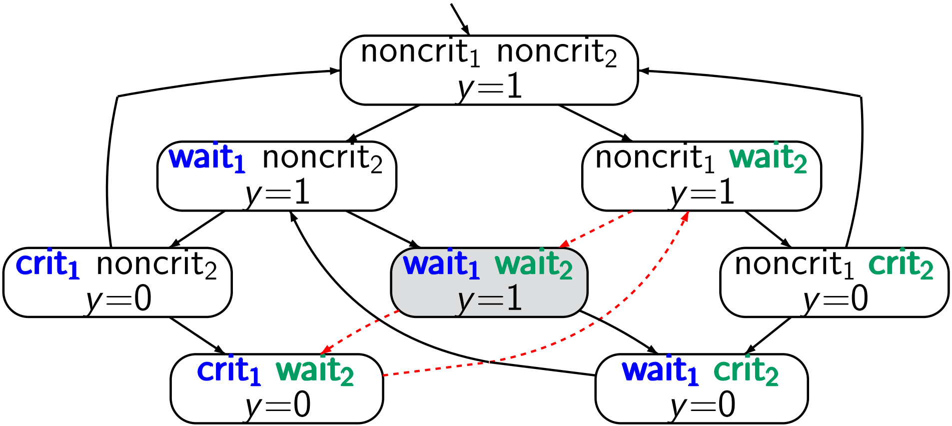

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Peterson's mutual exclusion algorithm

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for competing processes $\mathcal{P}_1$ and $\mathcal{P}_2$,

using three additional shared variables

$b_1, b_2 \in \{0,1\}$, $x \in \{1,2\}$

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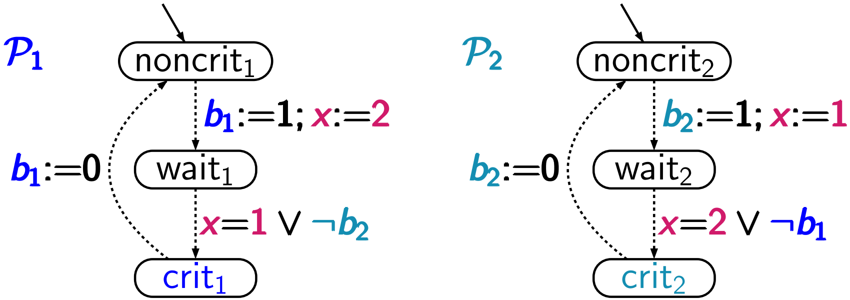

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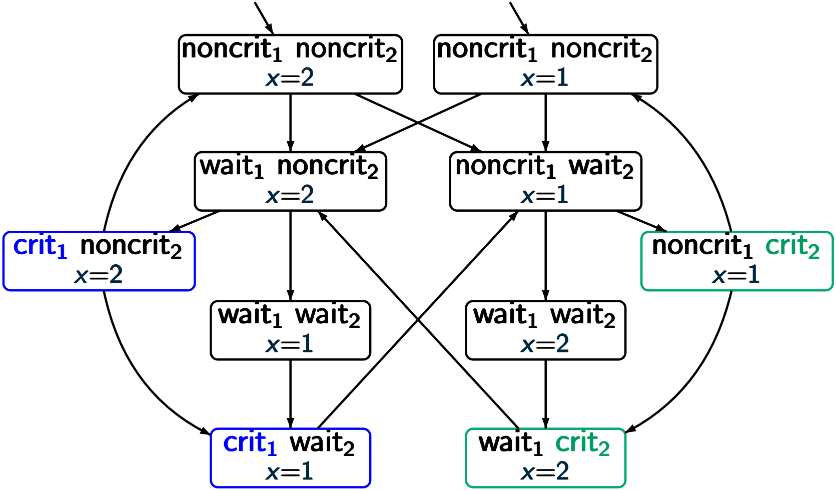
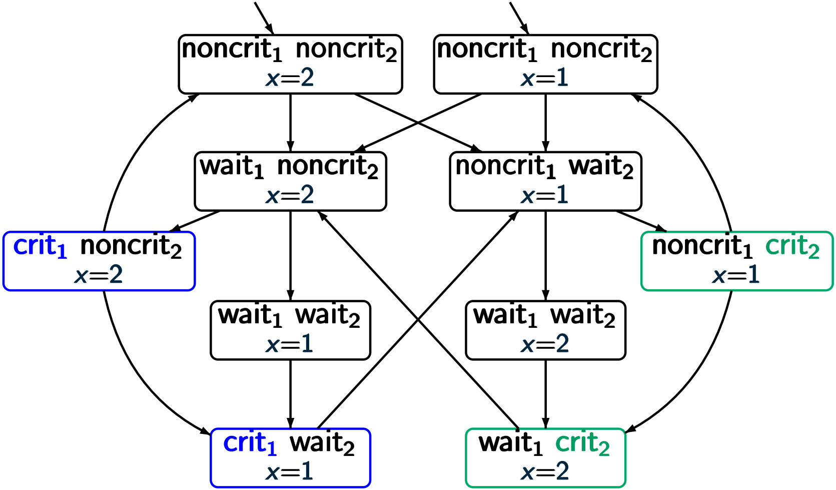


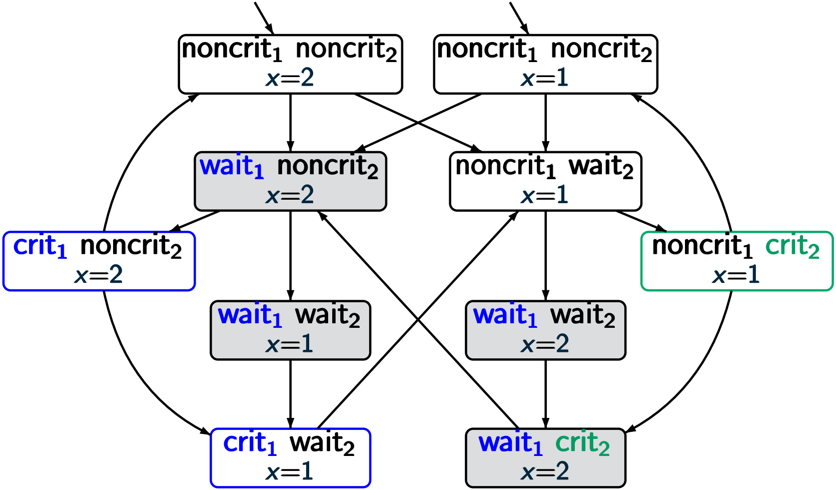
$\mathcal{T}_{Pet} \models \mathit{MUTEX}$

Peterson's mutual exclusion algorithm



$\mathcal{T}_{Pet} \models \mathit{MUTEX}$ and $\mathcal{T}_{Pet} \models \mathit{LIVE}$

Peterson's mutual exclusion algorithm



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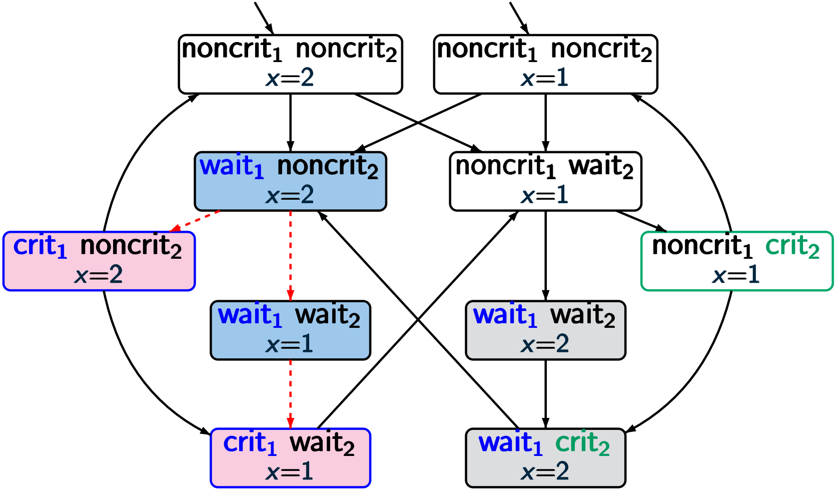

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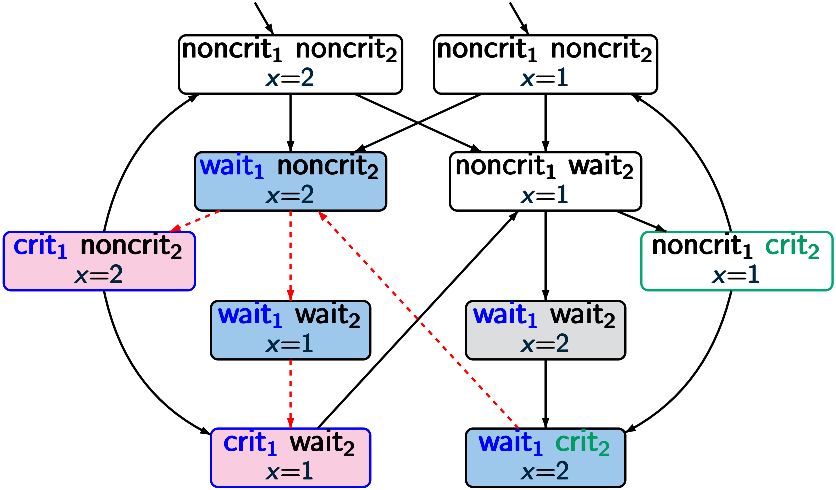

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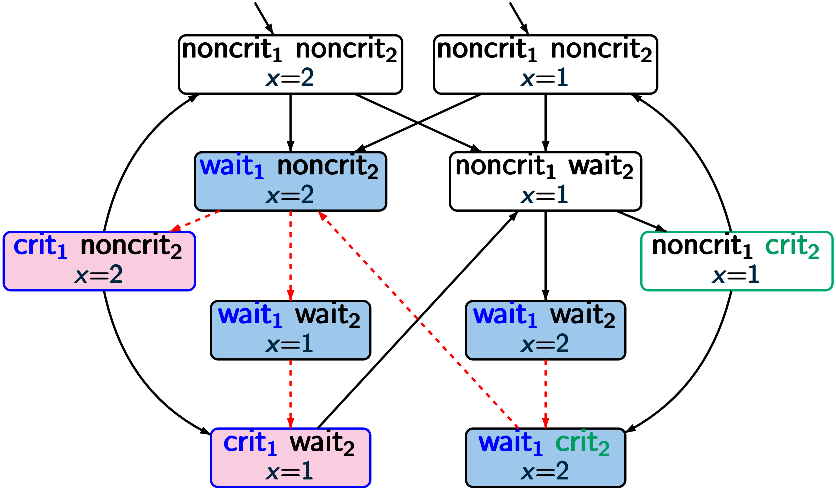
Peterson's mutual exclusion algorithm



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LT properties and trace inclusion

An LT property over AP is a language E of infinite words over the alphabet $\Sigma = 2^{AP}$, i.e., $E \subseteq \left(2^{AP}\right)^{\omega}$.

If $\mathcal{T}$ is a TS over AP then $\mathcal{T} \models E$ iff $Traces(\mathcal{T}) \subseteq E$.

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Consequence of these definitions:

If $\mathcal{T}_1$ and $\mathcal{T}_2$ are TS over AP then for all LT properties E over AP:

$$Traces(\mathcal{T}_1) \subseteq Traces(\mathcal{T}_2) \wedge \mathcal{T}_2 \models E \implies \mathcal{T}_1 \models E$$

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note: $Traces(\mathcal{T}_1) \subseteq Traces(\mathcal{T}_2) \subseteq E$

LT properties and trace inclusion

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If $\mathcal{T}$ is a TS over AP then $\mathcal{T} \models E$ iff $Traces(\mathcal{T}) \subseteq E$.

If $\mathcal{T}_1$ and $\mathcal{T}_2$ are TS over AP then the following statements are equivalent:

(1) $Traces(\mathcal{T}_1) \subseteq Traces(\mathcal{T}_2)$

(2) for all LT-properties E over AP: whenever $\mathcal{T}_2 \models E$ then $\mathcal{T}_1 \models E$

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$(1) \Longrightarrow (2)$: $\checkmark$

LT properties and trace inclusion

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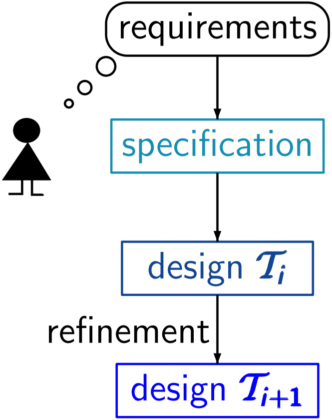
$(2) \Longrightarrow (1)$: consider $E = Traces(\mathcal{T}_2)$

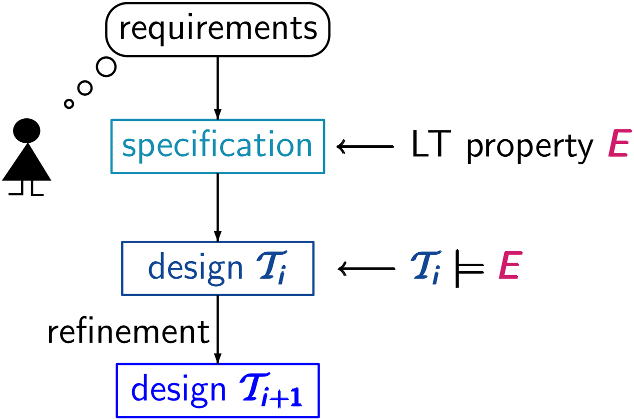
Relevance of trace inclusion

Trace inclusion appears naturally

- as an implementation/refinement relation
- when resolving nondeterminism
- in the context of abstractions

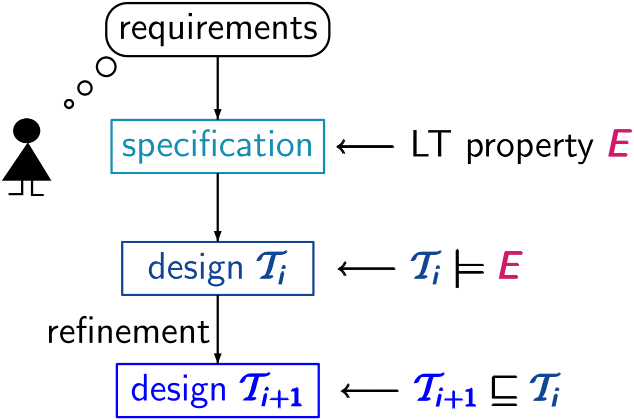
Software design cycle




requirements
specification
⟵ LT property E
design $\mathcal{T}_i$
⟵ $\mathcal{T}_i \models E$
refinement
design $\mathcal{T}_{i+1}$

Software design cycle

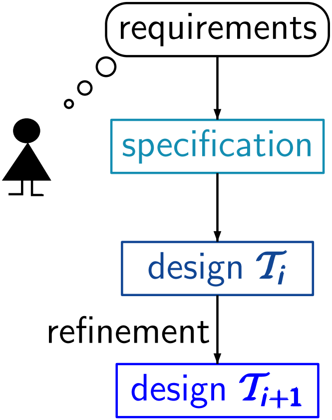
LTB2.4-19



implementation/refinement relation $\sqsubseteq$:

$\mathcal{T}_{i+1} \sqsubseteq \mathcal{T}_i$ iff "$\mathcal{T}_{i+1}$ correctly implements $\mathcal{T}_i$"

Trace inclusion as an implementation relation



trace inclusion

$\mathcal{T}_{i+1} \sqsubseteq \mathcal{T}_i$ iff

$\mathit{Traces}(\mathcal{T}_{i+1}) \subseteq \mathit{Traces}(\mathcal{T}_i)$

$\longleftarrow \mathcal{T}_i \models E$

$\longleftarrow \mathcal{T}_{i+1} \sqsubseteq \mathcal{T}_i$

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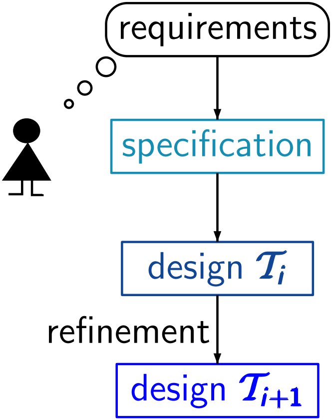

$\longleftarrow \mathcal{T}_i \models E$

$\longleftarrow \mathcal{T}_{i+1} \sqsubseteq \mathcal{T}_i$ implies $\mathcal{T}_{i+1} \models E$

trace inclusion

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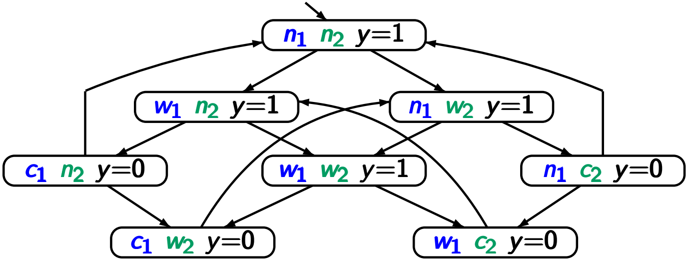
Mutual exclusion with semaphore



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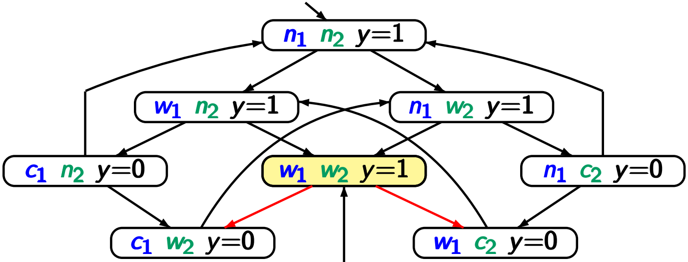
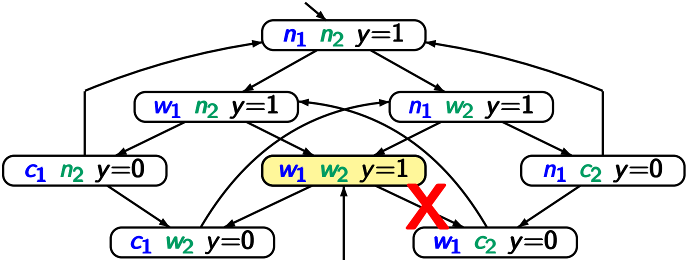


competition in state
$\langle wait_1 \; wait_2 \; y{=}1 \rangle$

Mutual exclusion with semaphore

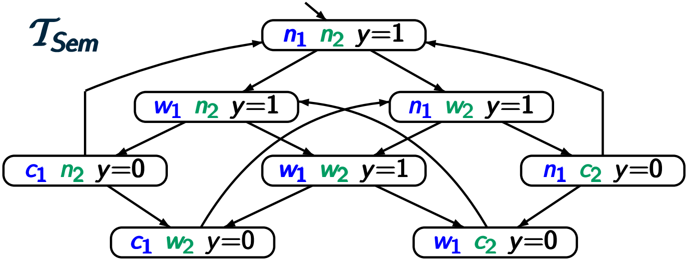


competition in state
$\langle \mathit{wait}_1\ \mathit{wait}_2\ y{=}1 \rangle$

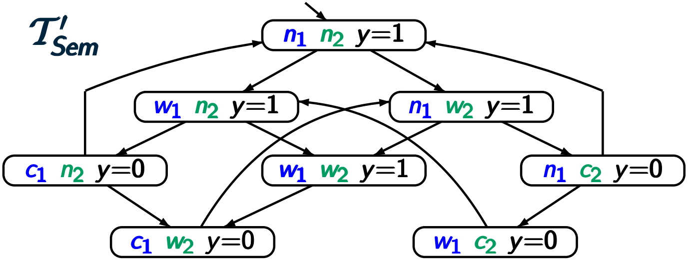
resolve the nondeterminism by giving priority to process P_1

Mutual exclusion with semaphore

$\mathcal{T}_{Sem}$



$\mathcal{T}'_{Sem}$



Mutual exclusion with semaphore

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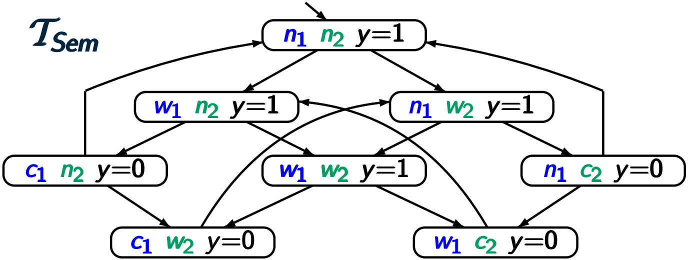

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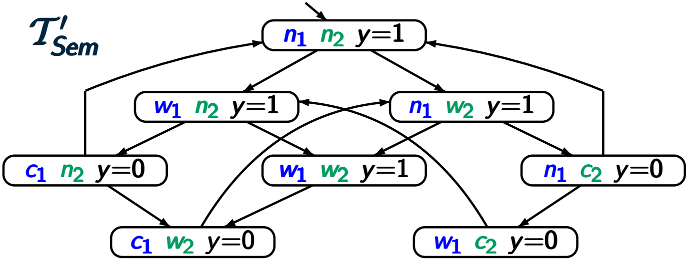
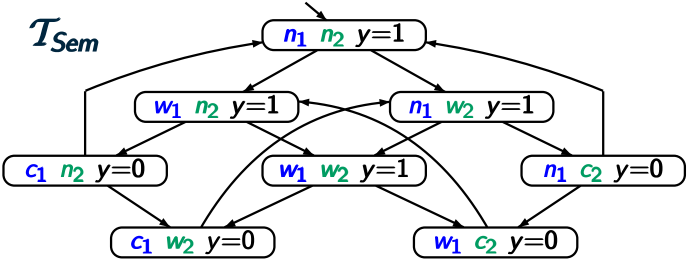


$$Paths(\mathcal{T}'_{Sem}) \subseteq Paths(\mathcal{T}_{Sem})$$

Mutual exclusion with semaphore

$\mathcal{T}_{Sem}$



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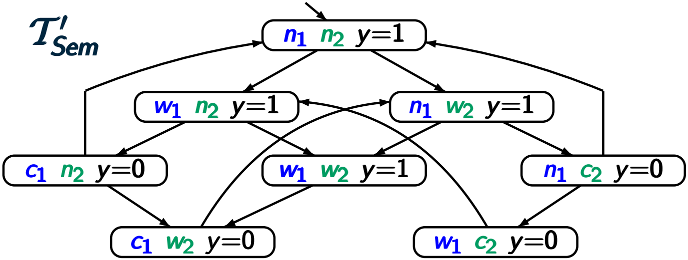
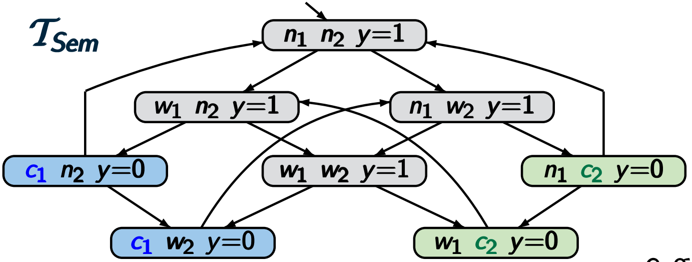
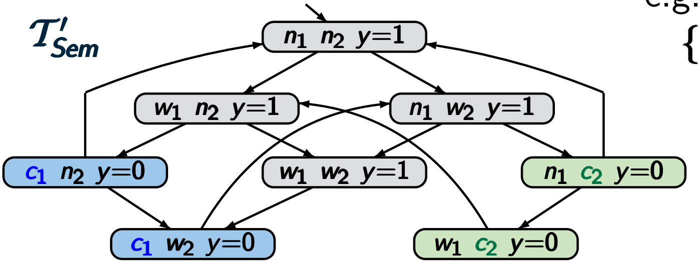


$Traces(\mathcal{T}'_{Sem}) \subseteq Traces(\mathcal{T}_{Sem})$ for any AP

Mutual exclusion with semaphore



e.g., for $AP = \{crit_1, crit_2\}$



$Traces(\mathcal{T}_{Sem}) \models E$ implies $Traces(\mathcal{T}'_{Sem}) \models E$ for any E

Relevance of trace inclusion

Trace inclusion appears naturally

- as an implementation/refinement relation
- when resolving nondeterminism ⟵

  e.g., $\mathit{Traces}(\mathcal{T}'_{Sem}) \subseteq \mathit{Traces}(\mathcal{T}_{Sem})$
- in the context of abstractions

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Trace inclusion appears naturally

- as an implementation/refinement relation
- when resolving nondeterminism

whenever $\mathcal{T}'$ results from $\mathcal{T}$ by a scheduling policy for resolving nondeterministic choices in $\mathcal{T}$ then

$$\mathit{Traces}(\mathcal{T}') \subseteq \mathit{Traces}(\mathcal{T})$$

- in the context of abstractions

Relevance of trace inclusion

Trace inclusion appears naturally

- as an implementation/refinement relation
- when resolving nondeterminism
- in the context of abstractions $\longleftarrow$

Trace inclusion and data abstraction

$$\begin{array}{l} \quad\vdots \\ x{:=}7;\ \ y{:=}5; \\ \texttt{WHILE}\ x>0\ \texttt{DO} \\ \qquad x{:=}x-1; \\ \qquad y{:=}y+1 \\ \texttt{OD} \\ \quad\vdots \end{array}$$

Trace inclusion and data abstraction


```
        ⋮
ℓ0   x:=7;  y:=5;
ℓ1   WHILE x>0 DO
           x:=x−1;
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     OD
ℓ2      ⋮
```

does $\ell_2 \wedge odd(y)$ never hold **?**

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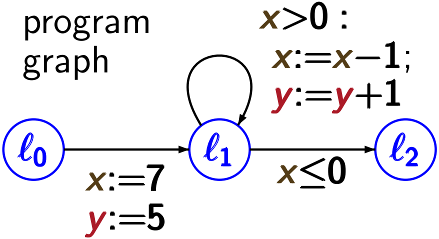

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Trace inclusion and data abstraction

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\begin{array}{ll}
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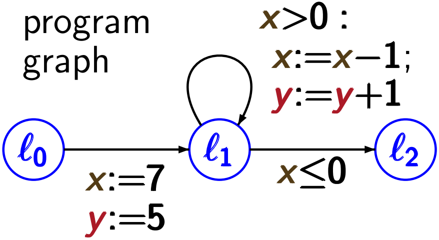
 & \texttt{OD} \\
\ell_2 & \vdots
\end{array}
$$

program graph



let $\mathcal{T}$ be the associated TS

does $\ell_2 \wedge odd(y)$ never hold ? $\longleftarrow$ $\mathcal{T} \models$ "never $\ell_2 \wedge odd(y)$" ?

Trace inclusion and data abstraction

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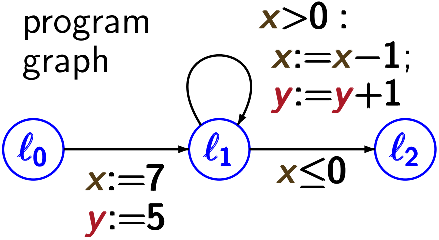

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data abstraction w.r.t. the predicates

$x>0$, $x=0$, $x \equiv_2 y$

Trace inclusion and data abstraction

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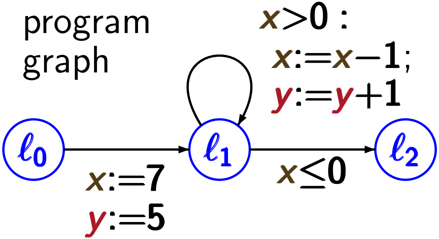

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data abstraction w.r.t. the predicates

$x>0$, $x=0$, $x \equiv_2 y$ $\longleftarrow$ i.e., $x-y$ is even

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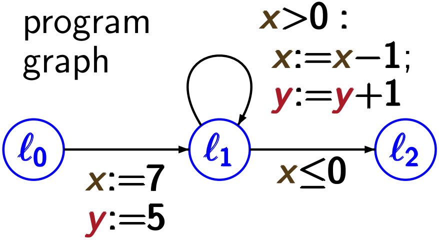

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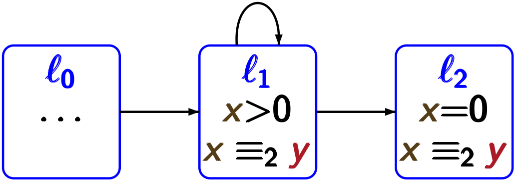

abstract transition system $\mathcal{T}'$

$\vdots$

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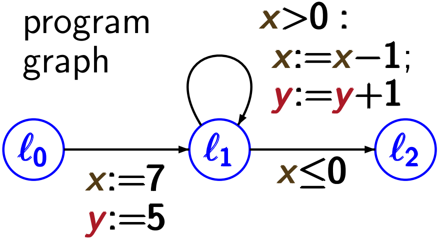
OD

ℓ_2 $\;\vdots$

program graph



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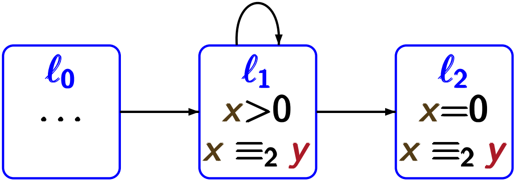

$\mathcal{T}' \models$ "never $\ell_2 \wedge odd(y)$"

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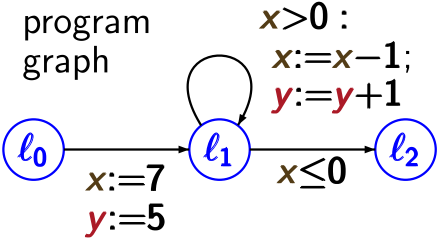

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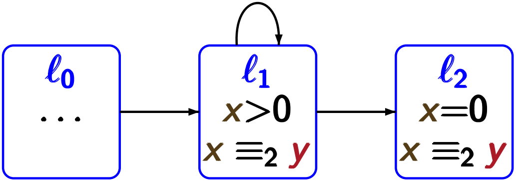

$\mathcal{T}' \models$ "never $\ell_2 \wedge \mathit{odd}(y)$"

$\mathit{Traces}(\mathcal{T}) \subseteq \mathit{Traces}(\mathcal{T}')$

Trace inclusion and data abstraction

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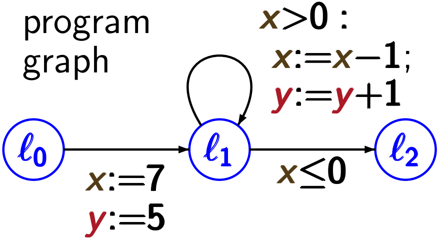

does $\ell_2 \wedge odd(y)$ never hold ?

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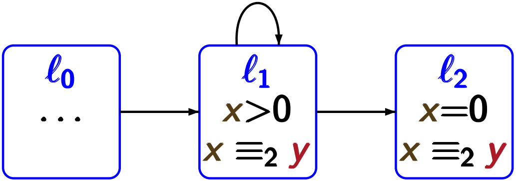

$$\mathcal{T} \models \text{“never } \ell_2 \wedge odd(y)\text{”} \quad \begin{cases} \mathcal{T}' \models \text{“never } \ell_2 \wedge odd(y)\text{”} \\ Traces(\mathcal{T}) \subseteq Traces(\mathcal{T}') \end{cases}$$

Trace equivalence

Trace equivalence

Transition systems $\mathcal{T}_1$ and $\mathcal{T}_2$ over the same set AP of atomic propositions are called trace equivalent iff

$$Traces(\mathcal{T}_1) \; = \; Traces(\mathcal{T}_2)$$

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i.e., trace equivalence requires trace inclusion in both directions

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i.e., trace equivalence requires trace inclusion in both directions

> Trace equivalent TS satisfy the **same LT properties**

LT properties and trace relations

Let $\mathcal{T}_1$ and $\mathcal{T}_2$ be TS over AP.

The following statements are equivalent:

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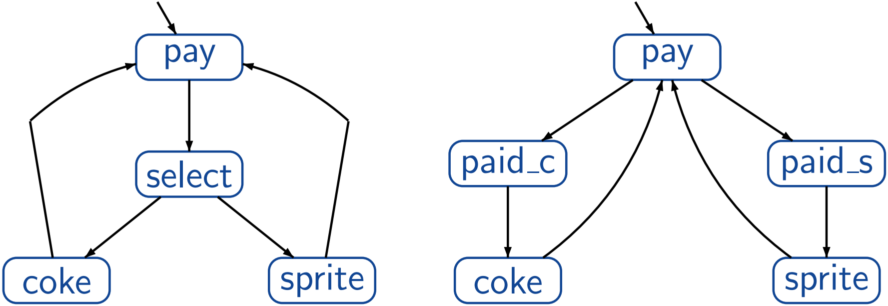
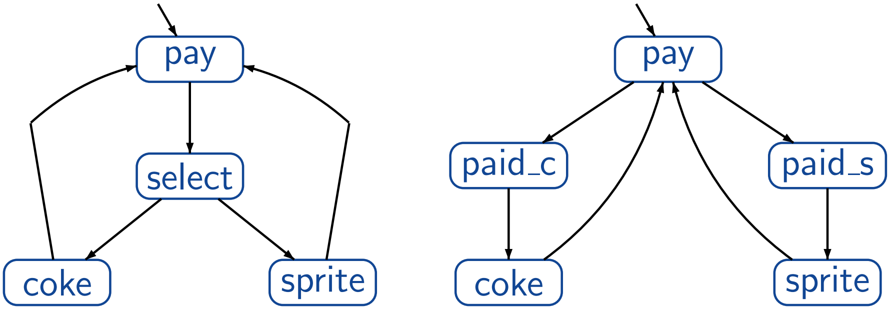
Trace equivalent beverage machines

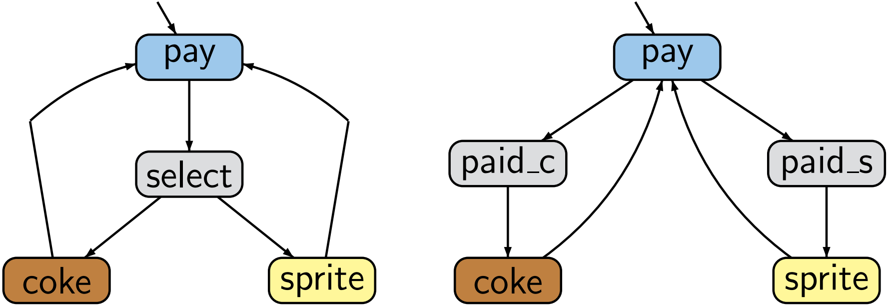


Trace equivalent beverage machines



set of atomic propositions $AP = \{pay, coke, sprite\}$

Trace equivalent beverage machines



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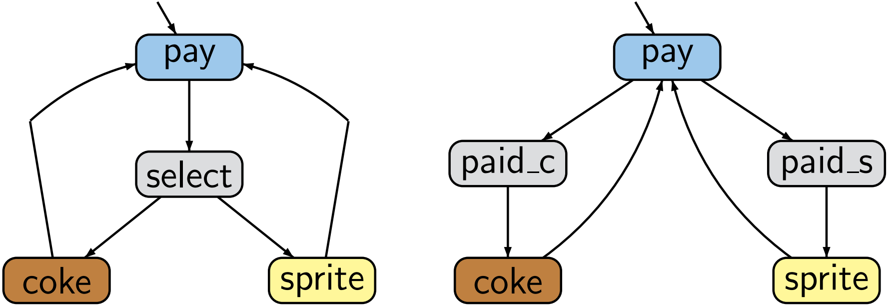


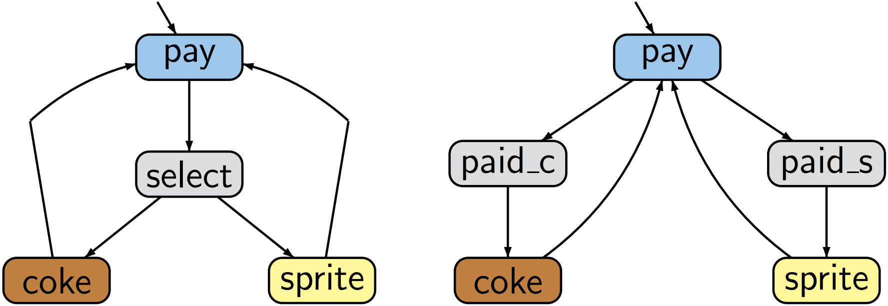
set of atomic propositions $AP = \{pay, coke, sprite\}$

$Traces(\mathcal{T}_1) = Traces(\mathcal{T}_2) =$ set of all infinite words

$\{pay\} \varnothing \{drink_1\} \{pay\} \varnothing \{drink_2\} \ldots$

where $drink_1, drink_2, \ldots \in \{coke, sprite\}$

Trace equivalent beverage machines



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$\{pay\} \varnothing \{drink_1\} \{pay\} \varnothing \{drink_2\} \ldots$

$\mathcal{T}_1$ and $\mathcal{T}_2$ satisfy the same LT-properties over AP

Introduction

Modelling parallel systems

Linear Time Properties

- state-based and linear time view
- definition of linear time properties
- invariants and safety $\longleftarrow$
- liveness and fairness

Regular Properties

Linear Temporal Logic

Computation-Tree Logic

Equivalences and Abstraction

Classification of LT-properties

safety properties *"nothing bad will happen"*

liveness properties *"something good will happen"*

Classification of LT-properties

safety properties *"nothing bad will happen"*

examples:

- mutual exclusion
- deadlock freedom
- "every red phase is preceded by a yellow phase"

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Classification of LT-properties

safety properties *"nothing bad will happen"*

examples:

- mutual exclusion
- deadlock freedom
- "every red phase is preceded by a yellow phase"

liveness properties *"something good will happen"*

examples:

- "each waiting process will eventually enter its critical section"
- "each philosopher will eat infinitely often"

Classification of LT-properties

safety properties *"nothing bad will happen"*

examples:

- mutual exclusion
- deadlock freedom

$\Big\}$ special case: **invariants** *"no bad state will be reached"*

- "every red phase is preceded by a yellow phase"

liveness properties *"something good will happen"*

examples:

- "each waiting process will eventually enter its critical section"
- "each philosopher will eat infinitely often"

Propositional logic

$$\Phi \; ::= \; \textit{true} \;\Big|\; a \;\Big|\; \Phi_1 \wedge \Phi_2 \;\Big|\; \neg\Phi$$

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semantics: interpretation over a subsets of AP

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e.g., $\{a, b\} \not\models (a \rightarrow \neg b) \vee c \qquad \{a, b\} \models a \vee c$

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for state s of a TS over AP: $s \models \Phi$ iff $L(s) \models \Phi$

Invariant

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Let E be an LT property over AP.

E is called an invariant if there exists a propositional formula Φ over AP such that

$$E = \left\{ A_0 A_1 A_2 \ldots \in \left(2^{AP}\right)^{\omega} : \forall i \geq 0.\, A_i \models \Phi \right\}$$

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Φ is called the invariant condition of E.

Examples for invariants

mutual exclusion (safety):

$$\mathit{MUTEX} = \begin{array}{c} \text{set of all infinite words } A_0\, A_1\, A_2 \ldots \text{ s.t.} \\ \forall i \in \mathbb{N}.\ \ \mathsf{crit}_1 \notin A_i \ \text{ or } \ \mathsf{crit}_2 \notin A_i \end{array}$$

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deadlock freedom for 5 dining philosophers:

$$DF = \begin{array}{l}\text{set of all infinite words } A_0 A_1 A_2 \ldots \text{ s.t.} \\ \forall i \in \mathbb{N}\ \exists \mathrm{j} \in \{0,1,2,3,4\}.\ \mathit{wait}_j \notin A_i\end{array}$$

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invariant condition:

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Satisfaction of invariants

Let E be an LT property over AP. E is called an invariant if there exists a propositional formula Φ s.t.

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set of reachable states in $\mathcal{T}$

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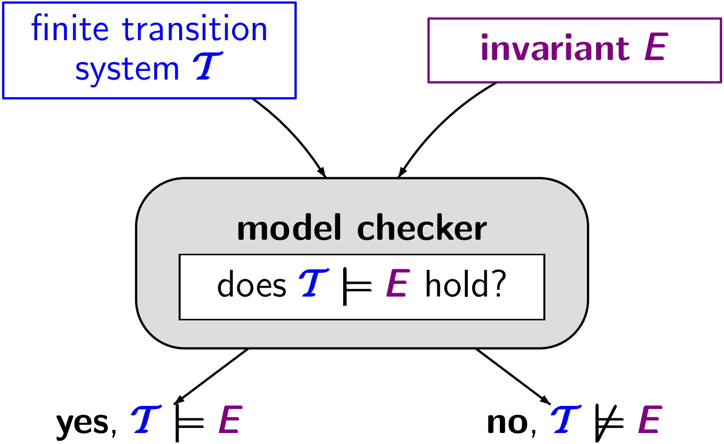
i.e., Φ holds in all initial states and is invariant under all transitions

Invariant checking



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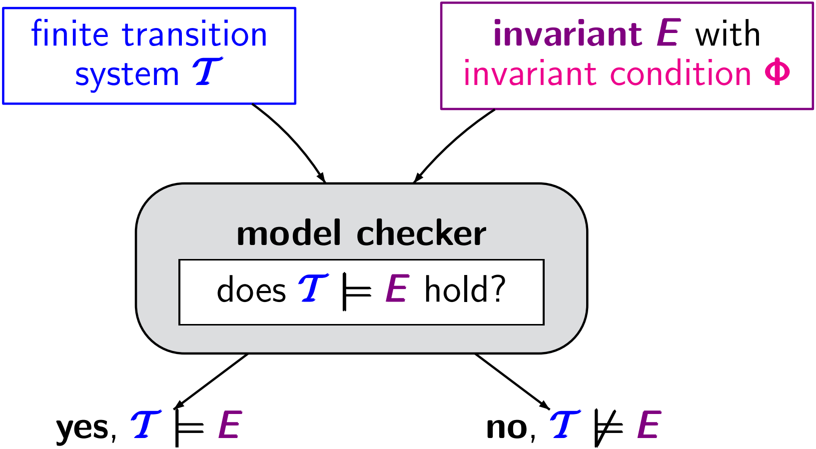

perform a graph analysis (**DFS** or **BFS**) to check whether $s \models \Phi$ for all $s \in Reach(\mathcal{T})$

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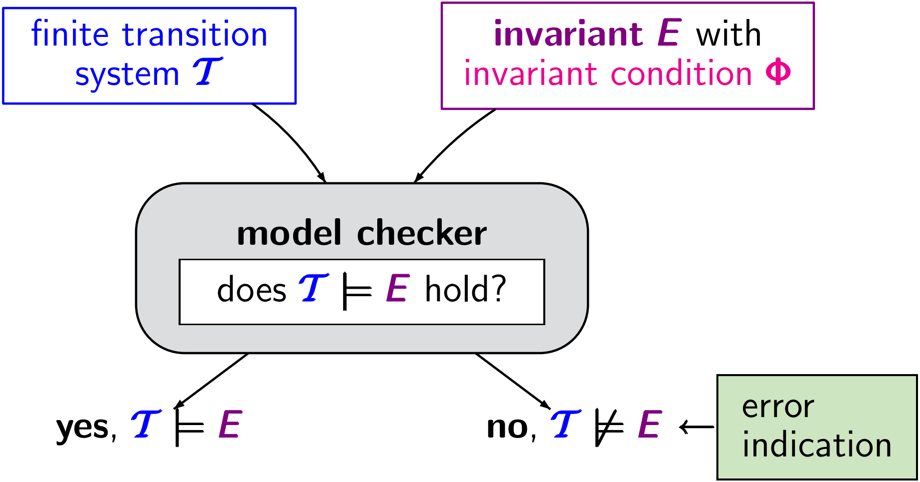

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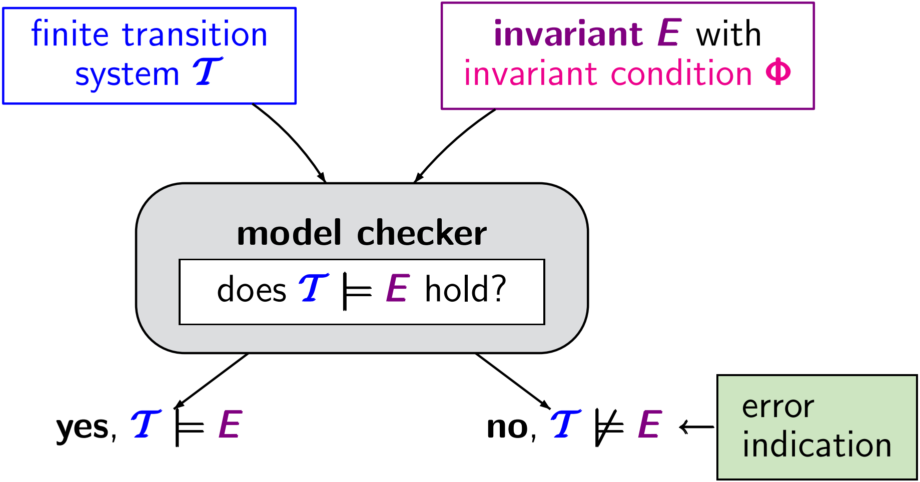

error indication: initial path fragment $s_0\, s_1 \ldots s_{n-1} s_n$
such that $s_i \models \Phi$ for $0 \le i < n$ and $s_n \not\models \Phi$

DFS-based invariant checking

input: finite transition system $\mathcal{T}$, invariant condition Φ

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```
FOR ALL s_0 ∈ S_0 DO
      IF DFS(s_0, Φ) THEN
          return "no"
      FI
OD
return "yes"
```

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$DFS(s_0, \Phi)$ returns "true" iff depth-first search from state s_0 leads to some state t with $t \not\models \Phi$

DFS-based invariant checking

input: finite transition system $\mathcal{T}$, invariant condition Φ

```
π := ∅ ⟵ stack for error indication
FOR ALL s₀ ∈ S₀ DO
      IF DFS(s₀, Φ) THEN
            return "no" and reverse(π)
      FI
OD
return "yes"
```

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FOR ALL s0 ∈ S0 DO
      IF DFS(s0, Φ) THEN
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      FI
OD
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```

s_n
$\vdots$
s_1
s_0

$s_n = t$

$DFS(s_0, \Phi)$ returns "true" iff depth-first search from state s_0 leads to some state t with $t \not\models \Phi$

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input: finite transition system $\mathcal{T}$, invariant condition Φ

$U := \varnothing \longleftarrow$ stores the "processed" states

$\pi := \varnothing \longleftarrow$ stack for error indication

```
FOR ALL s_0 ∈ S_0 DO
    IF DFS(s_0, Φ) THEN
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```

$$\begin{array}{|c|}\hline s_n \\ \hline \vdots \\ \hline s_1 \\ \hline s_0 \\ \hline \end{array} \quad s_n = t$$

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Recursive algorithm $DFS(s, \Phi)$

"searches" for a path fragment $s \ldots t$ with $t \not\models \Phi$

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```
IF s ∉ U THEN
    IF s ⊭ Φ THEN return "true" FI
    IF s ⊨ Φ THEN

            ⋮

    FI
FI
return "false"
```

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```
IF s ∉ U THEN
    IF s ⊭ Φ THEN return "true" FI
    IF s ⊨ Φ THEN
        insert s in U;



    FI
FI
return "false"
```

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```
IF s ∉ U THEN
     IF s ⊭ Φ THEN return "true" FI
     IF s ⊨ Φ THEN
          insert s in U;
          FOR ALL s′ ∈ Post(s) DO
               IF DFS(s′, Φ) THEN
                    return "true" FI
          OD
     FI
FI
return "false"
```

Recursive algorithm $DFS(s, \Phi)$

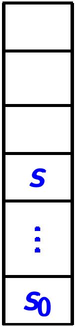
"searches" for a path fragment $s \ldots t$ with $t \not\models \Phi$

```
Push(π, s);
IF s ∉ U THEN
     IF s ⊭ Φ THEN return "true" FI
     IF s ⊨ Φ THEN
          insert s in U;
          FOR ALL s' ∈ Post(s) DO
               IF DFS(s', Φ) THEN
                    return "true" FI
          OD
     FI
FI
Pop(π); return "false"
```

Recursive algorithm $DFS(s, \Phi)$

"searches" for a path fragment $s \dots t$ with $t \not\models \Phi$

$Push(\pi, s)$;
IF $s \notin U$ THEN
    IF $s \not\models \Phi$ THEN return "true" FI
    IF $s \models \Phi$ THEN
        insert s in U;
        FOR ALL $s' \in Post(s)$ DO
            IF $DFS(s', \Phi)$ THEN
                return "true" FI
        OD
    FI
FI
$Pop(\pi)$; return "false"



initial state

Recursive algorithm $DFS(s, \Phi)$

"searches" for a path fragment $s \ldots t$ with $t \not\models \Phi$

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IF s ∉ U THEN
     IF s ⊭ Φ THEN return "true" FI
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          OD
     FI
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Pop(π); return "false"
```

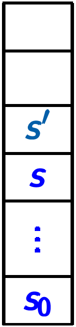

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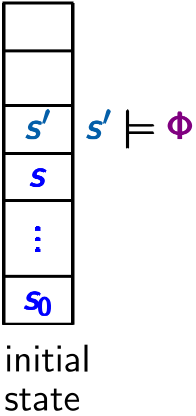

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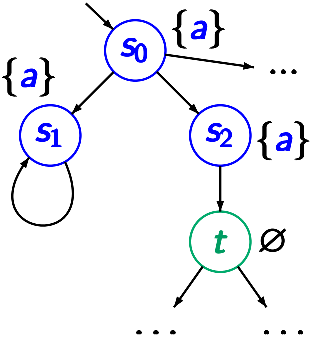
t	$t \not\models \Phi$
$\vdots$	
s'	$s' \models \Phi$
s	
$\vdots$	
s_0	

initial state

Example: invariant checking



invariant
condition a

$$s_0, s_1, s_2 \models a$$
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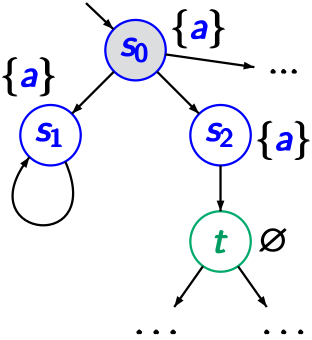

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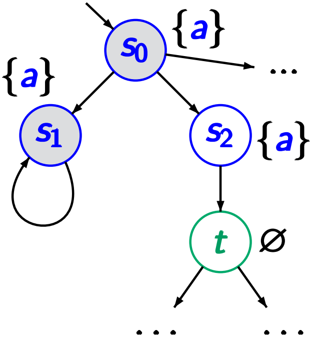
$DFS(s_0, a)$

stack π

$\boxed{s_0}$

Example: invariant checking



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s_1
s_0

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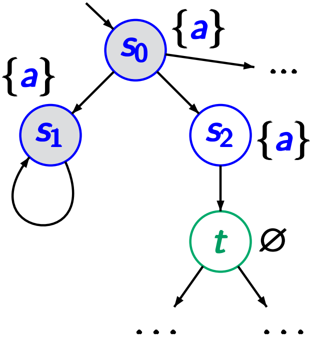

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$\boxed{DFS(s_1, a)}$

stack π

~~s_1~~
s_1
s_0

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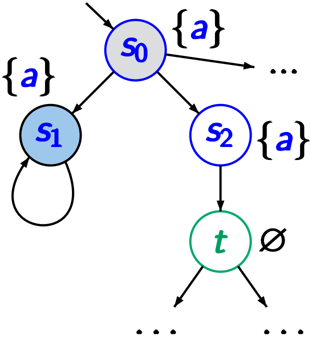

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$DFS(s_1, a)$

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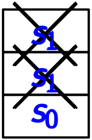
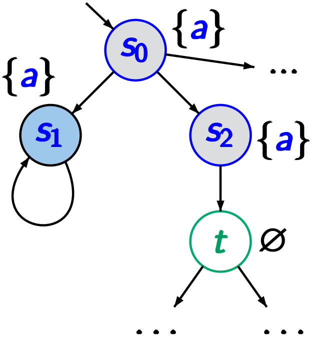


invariant condition a

$$\begin{array}{rl} s_0, s_1, s_2 & \models a \\ t & \not\models a \end{array}$$

Example: invariant checking

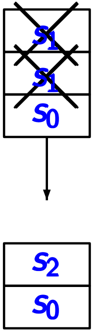


$DFS(s_0, a)$

$\boxed{DFS(s_1, a) \quad \boxed{DFS(s_1, a)}}$

$DFS(s_2, a)$

stack π



invariant condition a

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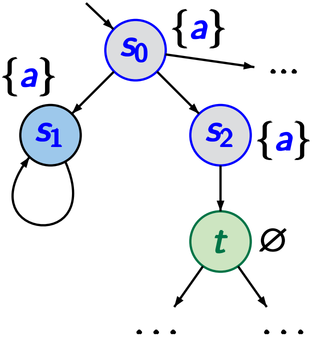

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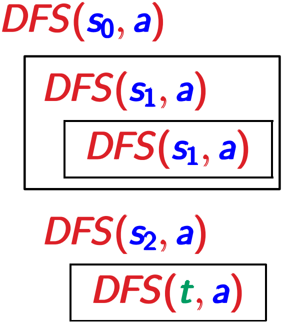

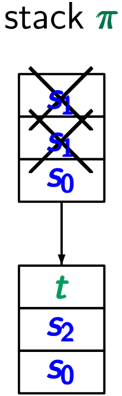

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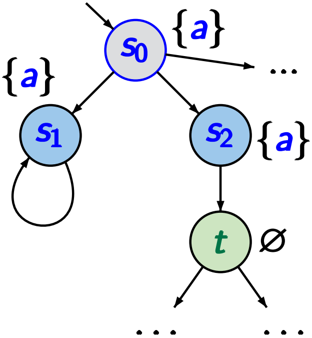

invariant condition a

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$DFS(s_0, a)$

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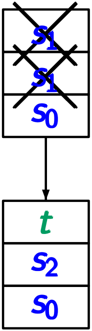
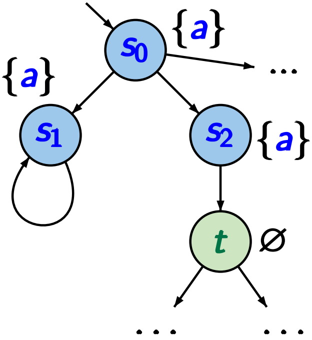
$\boxed{DFS(s_2, a) \quad \boxed{DFS(t, a)}}$

stack π



Example: invariant checking

IS2.5-9



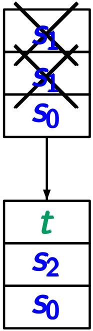
invariant condition a

$$s_0, s_1, s_2 \models a$$
$$t \not\models a$$

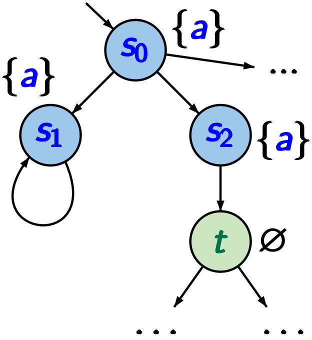
$DFS(s_0, a)$

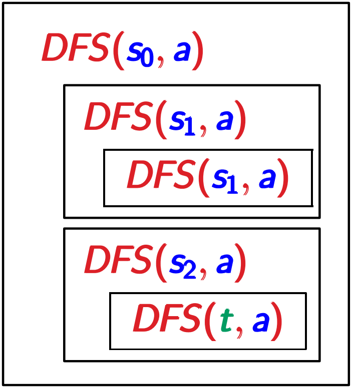
- $DFS(s_1, a)$
  - $DFS(s_1, a)$
- $DFS(s_2, a)$
  - $DFS(t, a)$

stack π

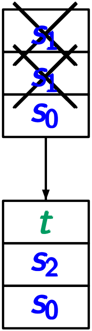


Example: invariant checking





stack π

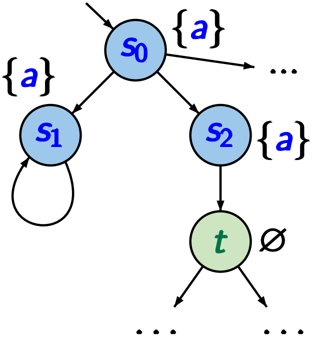


invariant
condition a

$$\begin{array}{r|l} s_0, s_1, s_2 & \models a \\ t & \not\models a \end{array}$$

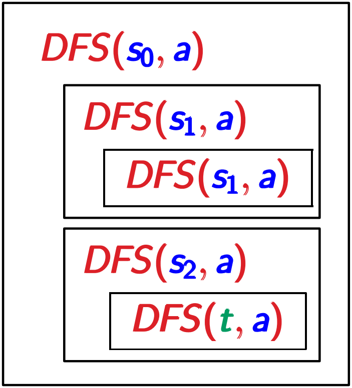
$s_0 \not\models$ “always a”

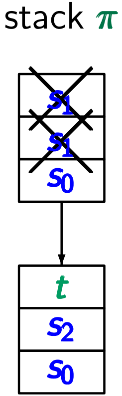
Example: invariant checking



invariant condition a

$$s_0, s_1, s_2 \models a$$
$$t \not\models a$$





$s_0 \not\models$ "always a" $\longleftarrow$

error indication: $s_0\, s_2\, t$