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Exercise 1

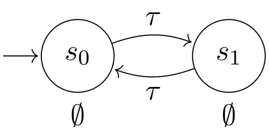
a)

Exercise 2

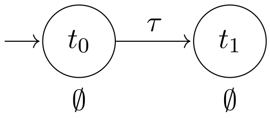
a)

The statement does not hold.
Counterexample:

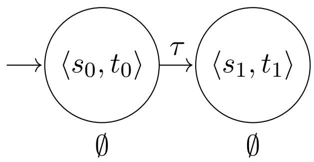
TS_1:



TS_2:



$TS_1||TS_2$:



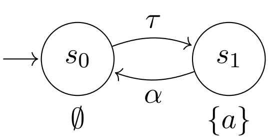
Then:

- $trace(TS_1) = \{\emptyset^\omega\}$
- $trace(TS_1) = \{\emptyset\emptyset\}$
- $trace(TS_1||TS_2) = \{\emptyset\emptyset\}$
- $\Rightarrow \emptyset^\omega \in Traces(TS_1)$, but $\emptyset^\omega \notin Traces(TS_1||TS_2)$

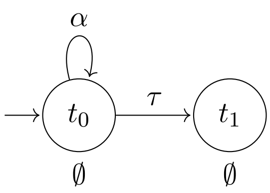
b)

The statement does not hold.
Counterexample:

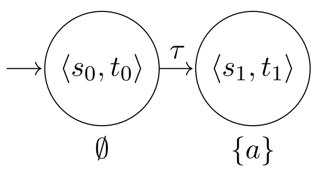
TS_1:



TS_2:



$TS_1||TS_2$:



Then:

- $trace(TS_1) = \{(\emptyset\{a\})^\omega\}$
- $trace(TS_1) = \{\emptyset\emptyset\}$

- $trace(TS_1||TS_2) = \{\emptyset\{a\}\}$
- $\Rightarrow \emptyset\{a\} \in Traces(TS_1||TS_2)$, but $\emptyset\{a\} \notin Traces(TS_1)$

c)

The statement holds.
Let $Traces(TS_1) \subseteq Traces(TS_2)$. Then $Traces(TS_1) \setminus Traces(TS_2) = \emptyset$.
Further let t be an arbitrary trace, with $t \in Traces(TS_1) \cap Traces(TS_2)$:
If t is a $\mathcal{F}$-fair execution then$t \in FairTraces_{\mathcal{F}}(TS_1)$ and $t \in FairTraces_{\mathcal{F}}(TS_2)$
If t is not a $\mathcal{F}$-fair execution then $t \notin FairTraces_{\mathcal{F}}(TS_1)$ and $t \notin FairTraces_{\mathcal{F}}(TS_2)$
So $FairTraces(TS_1) \subseteq FairTraces(TS_2)$.

d)

Let $Traces(TS_1) \subseteq Traces(TS_2)$,$E$ =eventually a will occur, with $a \in Act_2$, but $a \notin Act_1$, and $TS_2 \models_F E$.

Through the lecture we know, that liveness properties are not effected by fairness assumptions.
So $TS \models_{\mathcal{F}} E \Leftrightarrow TS \models E$ for any liveness property E.

So $TS_1 \models_{\mathcal{F}} E \Leftrightarrow TS_1 \models E$. Let a finite word σ be given, not containing an a. Since $a \notin Act_1$ we cannot find a σ' s.t. $\sigma\sigma' \in E$. So $TS_1 \not\models E$ and therefore $TS_1 \not\models_{\mathcal{F}} E$.

Exercise 3

a)

E_a: $TS \not\models E_a$, since $\sigma = (\emptyset\{b\})^\omega \in Traces(TS)$, but σ obviously contains no a.

E_b: $TS \not\models E_b$, since $\sigma' = (\emptyset\{a\})^\omega \in Traces(TS)$, but σ' obviously contains no b.

E': $TS \not\models E'$, since $\sigma'' = (\emptyset\{a\}\{a,b\}\emptyset)^\omega \in Traces(TS)$, but there ex. a $i = 1$, such that $A_i = \{a\}, A_{i+1} = \{a,b\}$ and $A_{i+2} = \emptyset$.

b)

3.2.1 E_a

$B = B_1$: $TS \models_{\mathcal{F}^1_{strong}} E_a$
There is no path from an initial state s.t. never an α-transition is enabled. Therefore we have to take an α infinitely many times. So we either go the loop $s_1s_2s_4$ or the loop $s_1s_3s_4$ both fulfilling the premise and conclusion of strong fairness. But then we

infinitely often visit state s_4 containing an a in its label. So we have have a infinitely often.

$B = B_2$: $TS \not\models \mathcal{F}^2_{strong} E_a$
One path that fulfils the strong fairness condition is $\pi = s_1 s_3 s_1 s_3 \ldots$.
But $trace(\pi) = (\emptyset\{b\})^\omega$ does not have infinitely many a's.

$B = B_3$: $TS \not\models \mathcal{F}^3_{strong} E_a$
One path that fulfils the strong fairness condition is $\pi = s_1 s_3 s_1 s_3 \ldots$.
But $trace(\pi) = (\emptyset\{b\})^\omega$ does not have infinitely many a's.

3.2.2 E_b

$B = B_1$: $TS \not\models \mathcal{F}^1_{strong} E_b$
One path that fulfils the strong fairness condition is $\pi = s_1 s_2 s_1 s_2 \ldots$
But $trace(\pi) = (\emptyset\{a\})^\omega$ does not have infinitely many b's.

$B = B_2$: $TS \not\models \mathcal{F}^2_{strong} E_b$
One path that fulfils the strong fairness condition is $\pi = s_1 s_2 s_1 s_2 \ldots$
But $trace(\pi) = (\emptyset\{a\})^\omega$ does not have infinitely many b's.

$B = B_3$: $TS \models \mathcal{F}^3_{strong} E_b$
There is just one path fulfilling the strong fairness condition: $\pi = s_1 s_3 s_4 s_1 s_3 s_4 \ldots$
Then $trace(\pi) = (\emptyset\{b\}\{a,b\})^\omega$ has infinitely many b's.

3.2.3 E'

$B = B_1$: $TS \not\models \mathcal{F}^1_{strong} E'$
One path that fulfils the strong fairness condition is $\pi = s_1 s_2 s_4 s_1 s_2 s_4 \ldots$
But $trace(\pi) = (\emptyset\{a\}\{a,b\})^\omega \not\models E'$.

$B = B_2$: $TS \not\models \mathcal{F}^2_{strong} E'$
One path that fulfils the strong fairness condition is $\pi = s_1 s_2 s_4 s_1 s_2 s_4 \ldots$
But $trace(\pi) = (\emptyset\{a\}\{a,b\})^\omega \not\models E'$.

$B = B_3$: $TS \not\models \mathcal{F}^2_{strong} E'$
One path that fulfils the strong fairness condition is $\pi = s_1 s_3 s_4 s_1 s_2 s_4 s_1 s_3 s_4 s_1 s_2 s_4 \ldots$
But $traces(\pi) = (\emptyset\{b\}\{a,b\}\emptyset\{a\}\{a,b\})^\omega \not\models E'$

c)

3.3.1 E_a

$B = B_1$: $TS \models_{\mathcal{F}^1_{weak}} E_a$
The only path not containing infinitely many a's is the path $\pi = s_1 s_3 s_1 s_3 \ldots$. Then

π fulfils the conditions of weak fairness, but not the conclusion and therefore is so eligible path. So we have to take $s_3 \xrightarrow{\alpha} a_4$ or $s_1 \xrightarrow{\alpha} s_2$ infinitely often leading to a state having a in the label. So we have infinitely many a's.

$B = B_2$: $TS \not\models \mathcal{F}^2_{weak} E_a$
One path that fulfils the weak fairness condition is $\pi = s_1 s_3 s_1 s_3 \ldots$. We fulfil the premise, but by taking β every second time we also fulfil the conclusion.
But $trace(\pi) = (\emptyset\{b\})^\omega$ does not have infinitely many a's.

$B = B_3$: $TS \not\models \mathcal{F}^3_{weak} E_a$
There is no path in TS, such that the premise holds. So all paths are fair with respect to the stated fairness condition. But again we can take the path $\pi = s_1 s_3 s_1 s_3 \ldots$ and observe, that $trace(\sigma) = (\emptyset\{b\})^\omega$ does not contain infinitely many a's.

3.3.2 E_b

$B = B_1$: $TS \not\models \mathcal{F}^1_{weak} E_b$
One path that does not fulfil the weak fairness premise is $\pi = s_1 s_2 s_1 s_2 \ldots$. Since it does not satisfy the premise is satisfies the whole condition.
But $trace(\pi) = (\emptyset\{a\})^\omega$ does not have infinitely many b's.

$B = B_2$: $TS \not\models \mathcal{F}^2_{weak} E_b$
One path that fulfils the weak fairness premise and conclusion is $\pi = s_1 s_2 s_1 s_2 \ldots$
But $trace(\pi) = (\emptyset\{a\})^\omega$ does not have infinitely many b's.

$B = B_3$: $TS \not\models \mathcal{F}^3_{weak} E_b$
There is not path s.t. contentiously β transitions are enabled. So the premise does not hold and therefore the weak fairness is fulfilled. But obviously there are paths not fulfilling the property, like $\pi = s_1 s_2 s_1 s_2 \ldots$, with $trace(\pi) = (\emptyset\{a\})^\omega$ does not have infinitely many b's.

3.3.3 E'

$B = B_1$: $TS \not\models \mathcal{F}^1_{strong} E'$
One path that does not fulfil the weak fairness premise is $\pi = s_1 s_2 s_4 s_1 s_2 s_4 \ldots$. Since it does not satisfy the premise is satisfies the whole condition.
But $trace(\pi) = (\emptyset\{a\}\{a,b\})^\omega$ does not satisfy the property.

$B = B_2$: $TS \not\models \mathcal{F}^2_{strong} E'$
One path that does not fulfil the weak fairness premise is $\pi = s_1 s_2 s_4 s_1 s_2 s_4 \ldots$. Since it does not satisfy the premise is satisfies the whole condition.
But $trace(\pi) = (\emptyset\{a\}\{a,b\})^\omega$ does not satisfy the property.

$B = B_3$: $TS \not\models \mathcal{F}^2_{strong} E'$
There is not path s.t. contentiously β transitions are enabled. So the premise does

not hold and therefore the weak fairness is fulfilled.
But for the path $\pi = s_1 s_2 s_4 s_1 s_2 s_4 \ldots$ the trace looks like the following $trace(\pi) = (\emptyset\{a\}\{a,b\})^\omega$ and obviously does not satisfy the property.