Overview

Introduction

Modelling parallel systems

Linear Time Properties

Regular Properties

- regular safety properties
- ω-regular properties
- model checking with Büchi automata $\longleftarrow$

Linear Temporal Logic

Computation-Tree Logic

Equivalences and Abstraction

Verifying ω-regular properties

given: finite transition system $\mathcal{T}$
ω-regular property E

question: does $\mathcal{T} \models E$ hold **?**

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requires techniques for checking **persistence properties** in finite TS

Persistence property

Let E be an LT-property, i.e., $E \subseteq (2^{AP})^{\omega}$

> E is called a persistence property if there exists a propositional formula Φ over AP such that
>
> $$E = \left\{ \begin{array}{l} \text{set of all infinite words } A_0\,A_1\,A_2\,\ldots \in (2^{AP})^{\omega} \\ \qquad\qquad\qquad \text{s.t.} \quad \overset{\infty}{\forall}\, i \geq 0.\ A_i \models \Phi \end{array} \right.$$

$\overset{\infty}{\forall}\, i{\geq}0.\ \ldots\ =\ \exists j{\geq}0\ \forall i{\geq}j.\ \ldots$ "for all but finitely many"

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"from some moment on Φ"

"eventually forever Φ"

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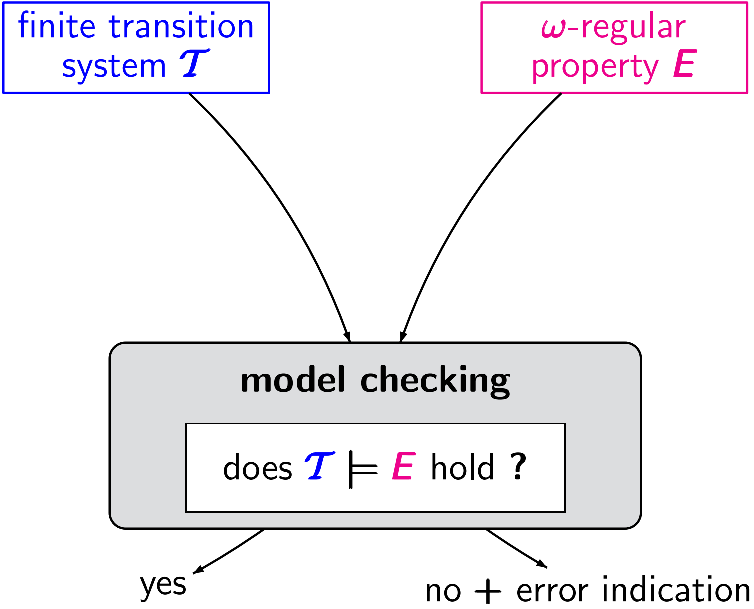
Checking ω-regular properties



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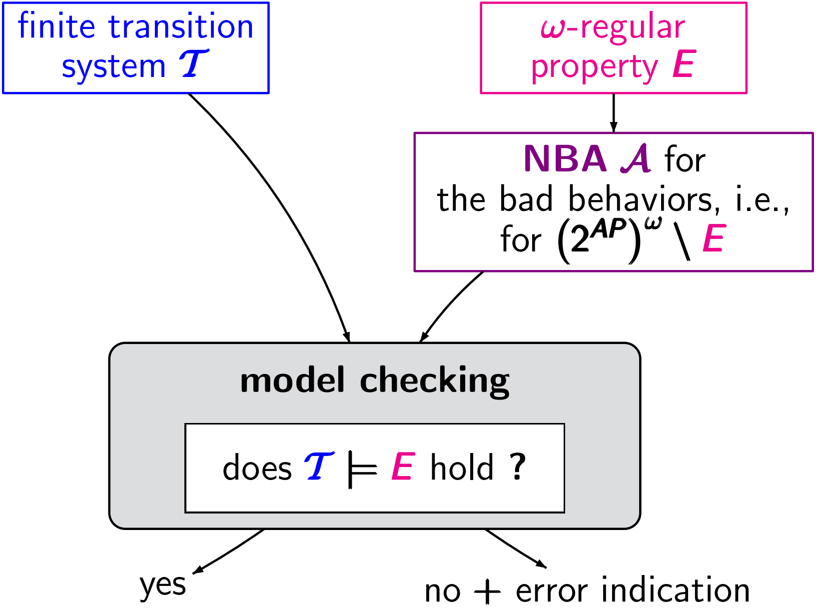

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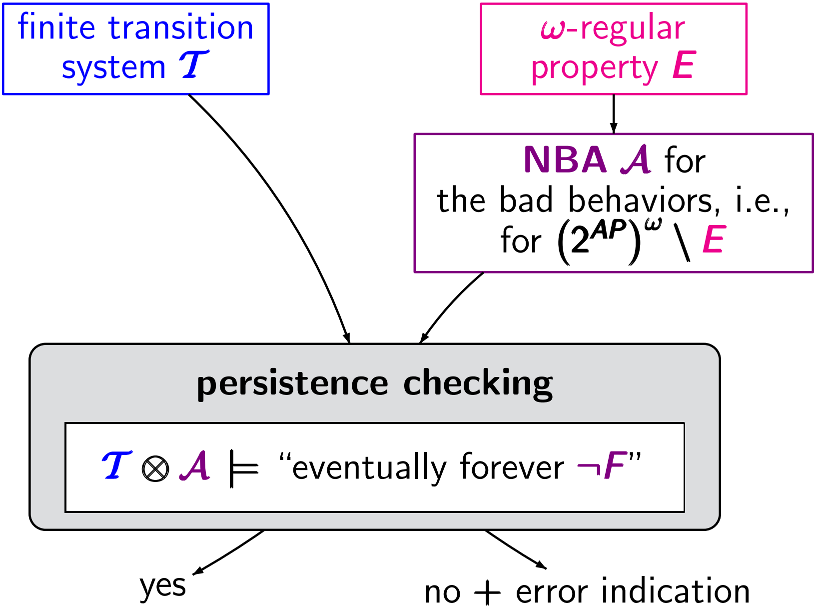

Recall: product of a TS and an NFA

finite transition system
$\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$

NFA for bad prefixes
$\mathcal{A} = (Q, 2^{AP}, \delta, Q_0, F)$

$$\begin{array}{c} s_0 \\ \downarrow \\ s_1 \\ \downarrow \\ s_2 \\ \downarrow \\ \vdots \\ \downarrow \\ s_n \end{array}$$

path
fragment $\widehat{\pi}$

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$$\begin{array}{ccc}
s_0 & & L(s_0)=A_0 \\
\downarrow & & \\
s_1 & & L(s_1)=A_1 \\
\downarrow & & \\
s_2 & & L(s_2)=A_2 \\
\downarrow & & \vdots \\
\vdots & & \\
\downarrow & & \\
s_n & & L(s_n)=A_n
\end{array}$$

path fragment $\widehat{\pi}$ trace

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\vdots & & \vdots \\
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\end{array}$$

path fragment $\widehat{\pi}$ trace

$$\begin{array}{c}
q_0 \in Q_0 \\
\downarrow A_0 \\
q_1 \\
\downarrow A_1 \\
q_2 \\
\downarrow A_2 \\
\vdots \\
\downarrow \\
q_n \\
\downarrow A_n \\
q_{n+1}
\end{array}$$

run for $trace(\widehat{\pi})$

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path fragment $\widehat{\pi}$	trace	path fragm. in product	run for $trace(\widehat{\pi})$
			$q_0 \in Q_0$
s_0	$L(s_0)=A_0$	$\langle s_0, q_1 \rangle$	$q_0 \xrightarrow{A_0} q_1$
s_1	$L(s_1)=A_1$	$\langle s_1, q_2 \rangle$	$q_1 \xrightarrow{A_1} q_2$
s_2	$L(s_2)=A_2$	$\langle s_2, q_3 \rangle$	$q_2 \xrightarrow{A_2} \vdots$
$\vdots$	$\vdots$	$\vdots$	q_n
s_n	$L(s_n)=A_n$	$\langle s_n, q_{n+1} \rangle$	$q_n \xrightarrow{A_n} q_{n+1}$

Product transition system

recall: definition of the product of a **TS** and **NFA**

Product transition system

$\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$ transition system

$\mathcal{A} = (Q, 2^{AP}, \delta, Q_0, F)$ NFA

product-TS $\mathcal{T} \otimes \mathcal{A} \stackrel{\text{def}}{=} (S \times Q, Act, \longrightarrow', S_0', AP', L')$

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$$\frac{s \xrightarrow{\alpha} s' \quad \wedge \quad q' \in \delta(q, L(s'))}{\langle s, q \rangle \xrightarrow{\alpha}{}' \langle s', q' \rangle}$$

initial states: $S_0' = \{ \langle s_0, q \rangle : s_0 \in S_0, q \in \delta\big(Q_0, L(s_0)\big) \}$

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set of atomic propositions: $AP' = Q$

labeling function: $L'(\langle s, q \rangle) = \{q\}$

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Product of a TS and NBA

$$\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L) \quad \text{transition system}$$
$$\mathcal{A} = (Q, 2^{AP}, \delta, Q_0, F) \quad \textbf{NFA} \text{ or } \textbf{NBA}$$

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relies on a reduction to the persistence checking problem

$\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$ finite transition system without terminal states

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representing the bad behaviors of an ω-regular LT-property E

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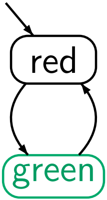
Example: ω-regular model checking

TS $\mathcal{T}$



LT property: "infinitely often green"

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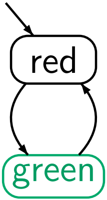
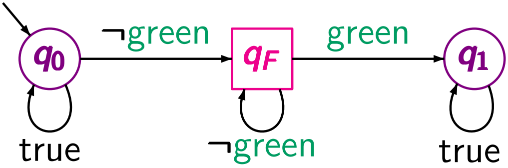


LT property: "infinitely often green"

NBA $\mathcal{A}$ for the complement

"from some moment on $\neg$green"



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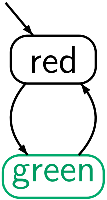

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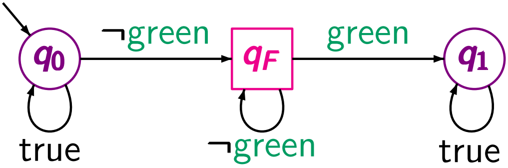

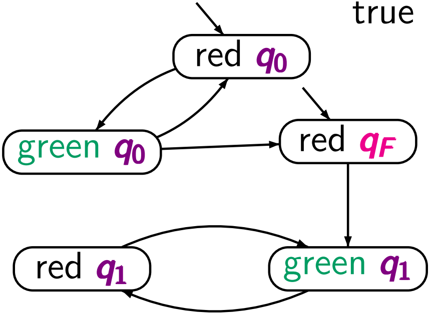

reachable fragment of the product TS $\mathcal{T} \otimes \mathcal{A}$

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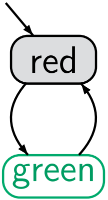

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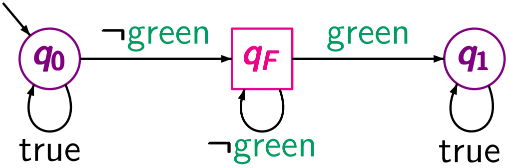
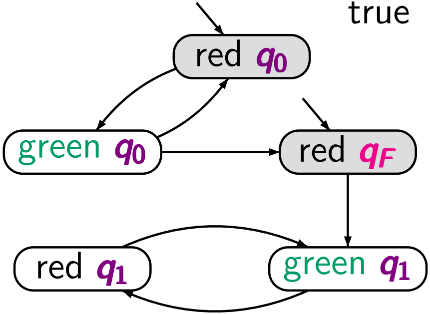


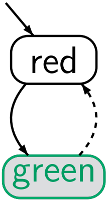


initial states:

$\langle \text{red}, q \rangle$ where

$$q \in \delta(q_0, L(\text{red})) = \delta(q_0, \varnothing) = \{q_0, q_F\}$$

TS $\mathcal{T}$



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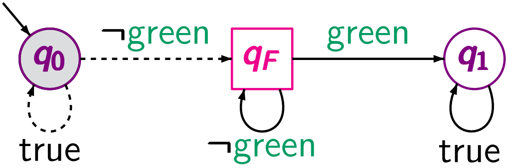

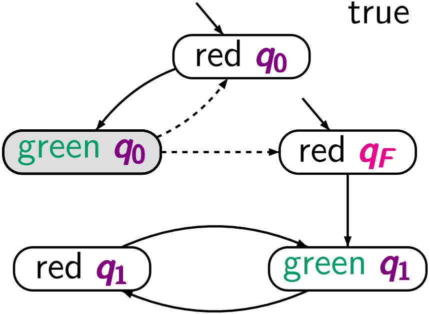

transition

$\langle \text{green}, q_0 \rangle \rightarrow \langle \text{red}, q \rangle$

$$q \in \delta(q_0, L(\text{red})) = \delta(q_0, \varnothing) = \{q_0, q_F\}$$

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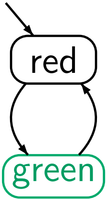

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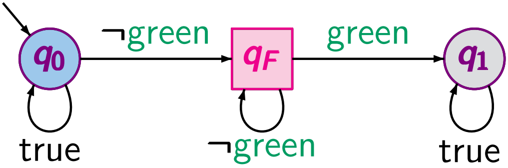

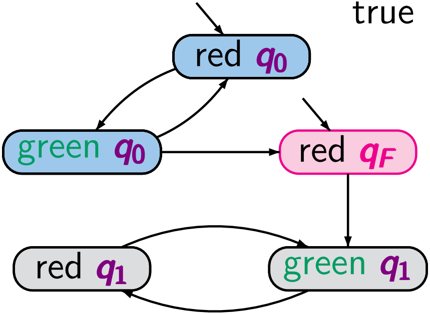

atomic propositions

$AP' = \{q_0, q_F, q_1\}$

obvious labeling function

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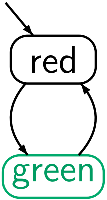

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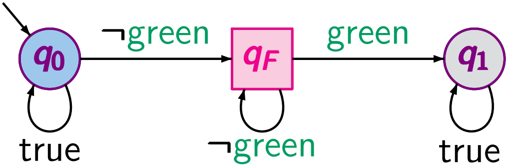

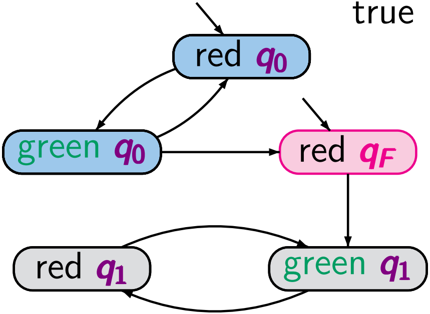

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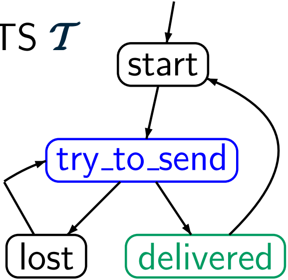

ω-regular LT property E:

"each (repeatedly) sent message will eventually be delivered"

Example: ω-regular model checking

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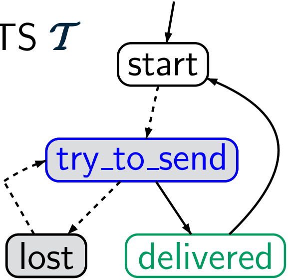

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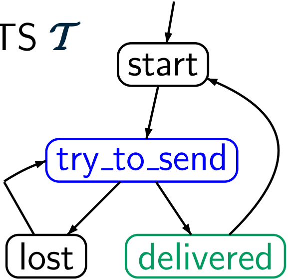
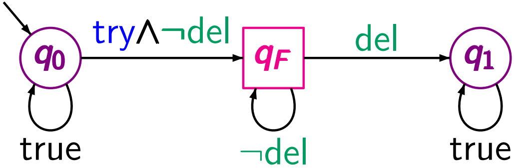
$\mathcal{T} \not\models E$

Example: ω-regular model checking

TS $\mathcal{T}$



NBA $\mathcal{A}$ for the bad behaviors



ω-regular LT property E:

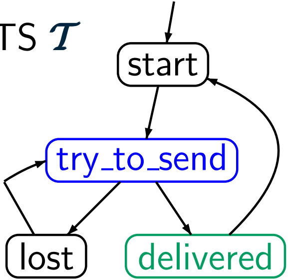
"each (repeatedly) sent message will eventually be delivered"

complement of E, i.e., LT property for the bad behaviors:

"never delivered after some trial"

Example: ω-regular model checking

TS $\mathcal{T}$



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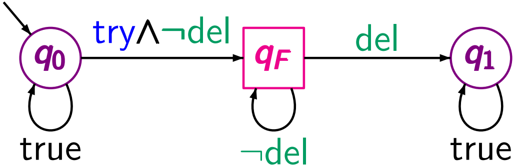

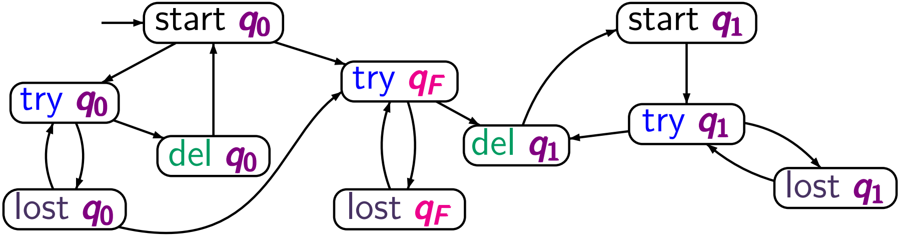

reachable fragment of the product-TS

Example: ω-regular model checking

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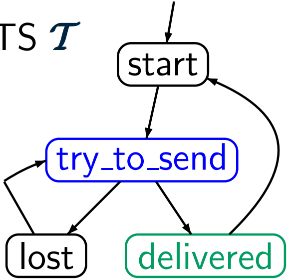

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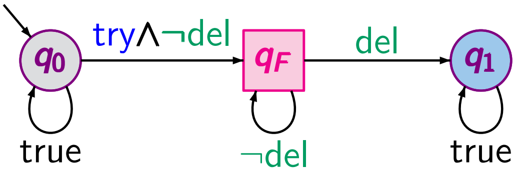

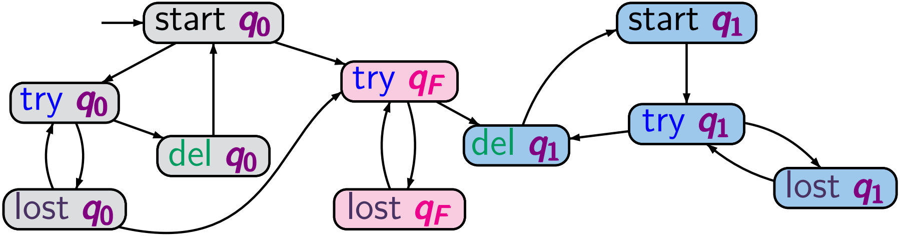

set of atomic propositions $AP' = \{q_0, q_1, q_F\}$

Example: ω-regular model checking

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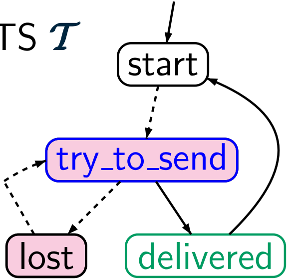

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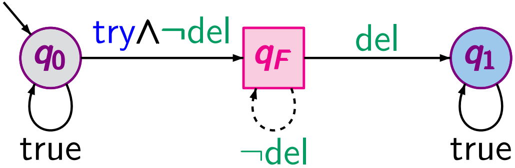

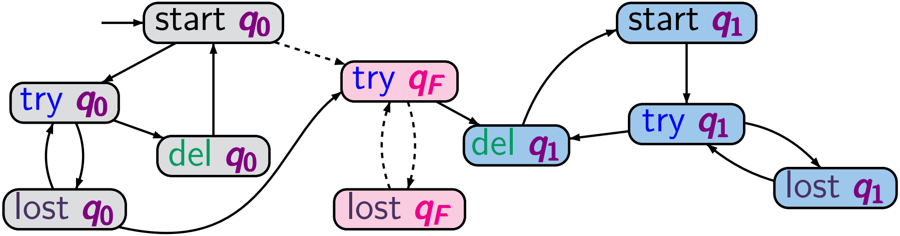

$\mathcal{T} \otimes \mathcal{A} \not\models$ "eventually forever $\neg F$"

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SCC: strongly connected component, i.e., maximal set of states that are reachable from each other

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A SCC is called non-trivial if it has at least one edge.
"either 1 state with a self-loop or 2 or more states"

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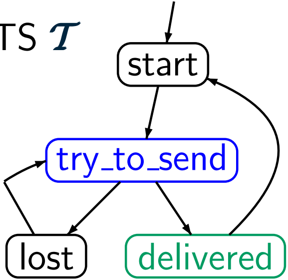
method: calculate and analyze the **SCCs**

TS $\mathcal{T}$



ω-regular LT property E:

"each (repeatedly) sent message will eventually be delivered"

Example: ω-regular model checking

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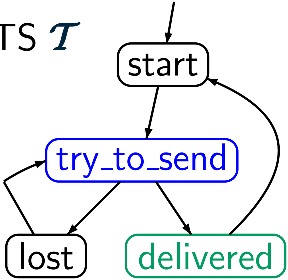

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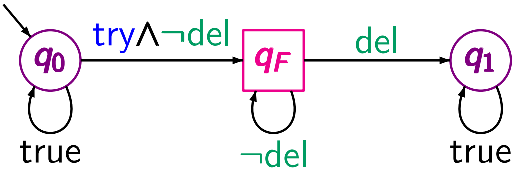

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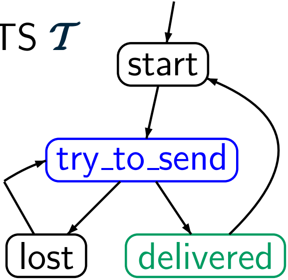

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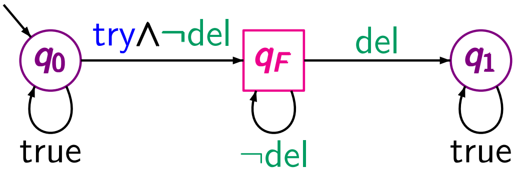

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... analysis of the **SCCs** in product $\mathcal{T} \otimes \mathcal{A}$...

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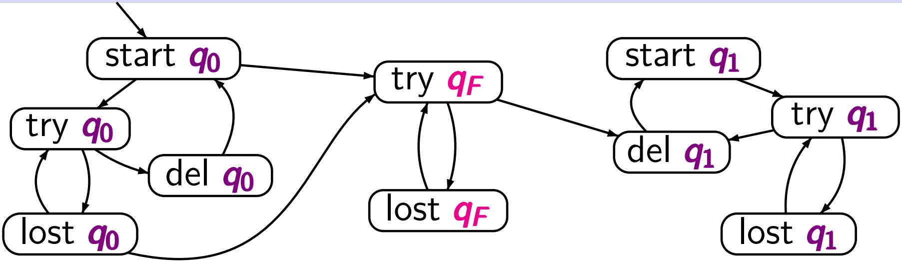

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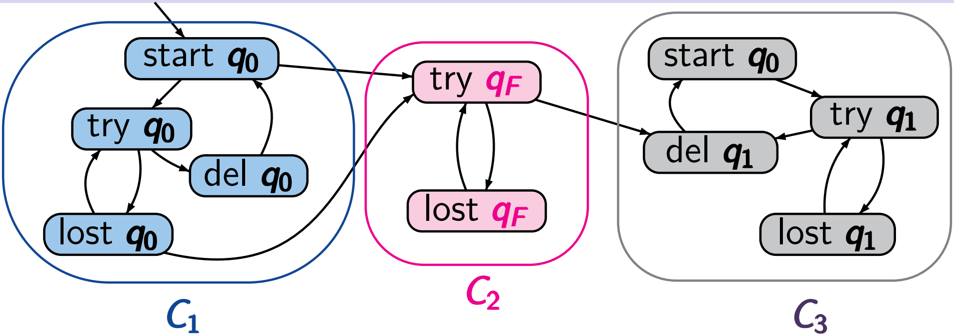

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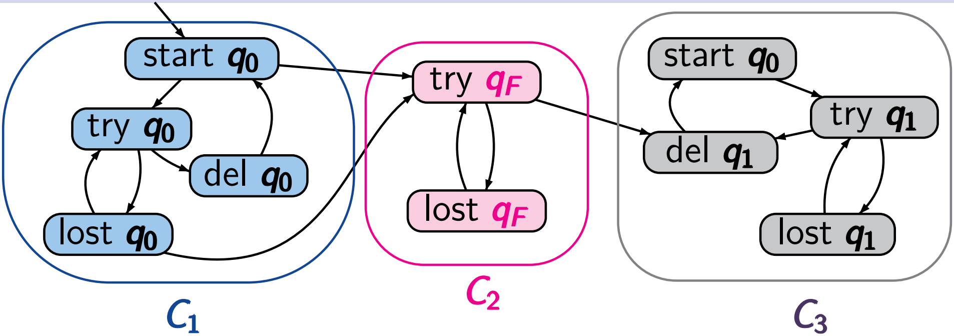

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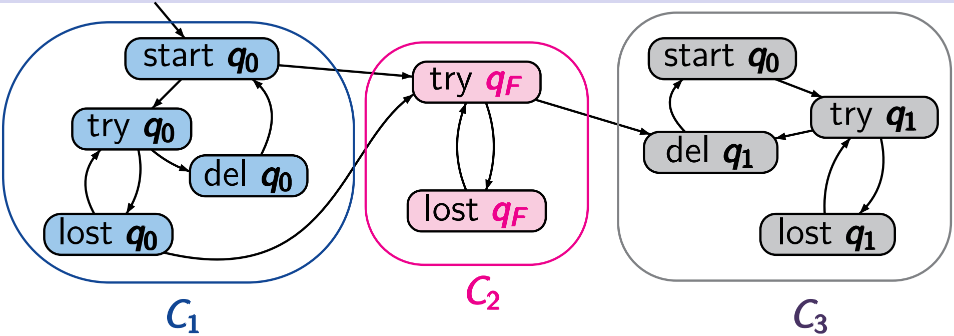

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Persistence checking algorithms based on ...

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method 2: **DFS**-based search for **backward edges**

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Let G be a finite directed graph and s a node in G.

The following statements are equivalent:

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(1) s belongs to a cycle $s\, s_1\, s_2 \ldots s_k\, s$

(2) The DFS started with s finds a backward edge $s' \longrightarrow s$.

Cycle check for fixed node: "does s belong to a cycle?"

- perform by a DFS with starting node s
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```
initially all states are unmarked

REPEAT

  choose an unmarked reachable state s with s ⊭ a;
  mark s;
  IF  CYCLE_CHECK(s) THEN
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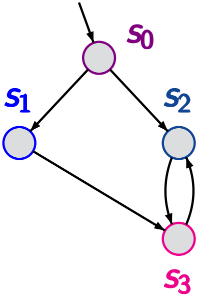
- uses a global visiting set V of states that have been visited so far in the **2. DFS**



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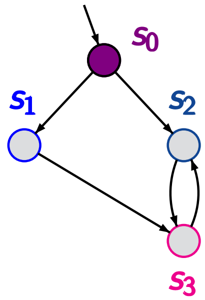

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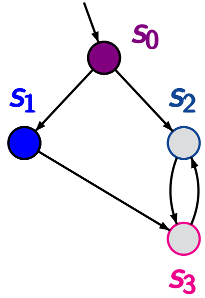

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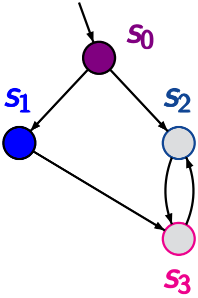

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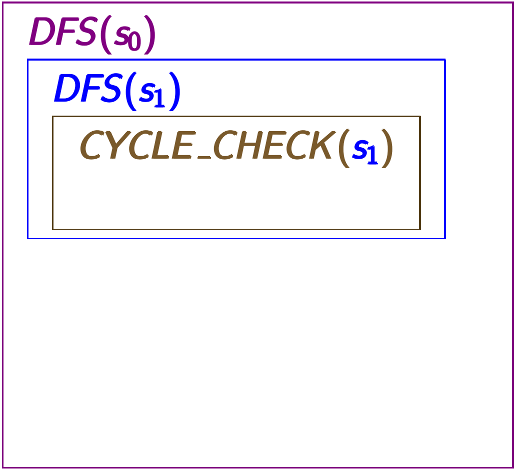

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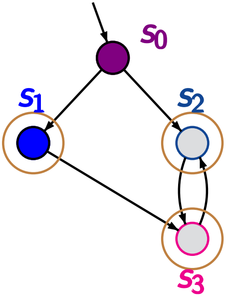

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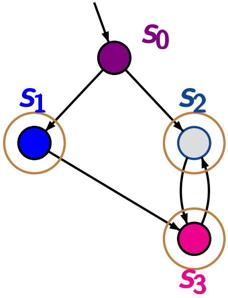

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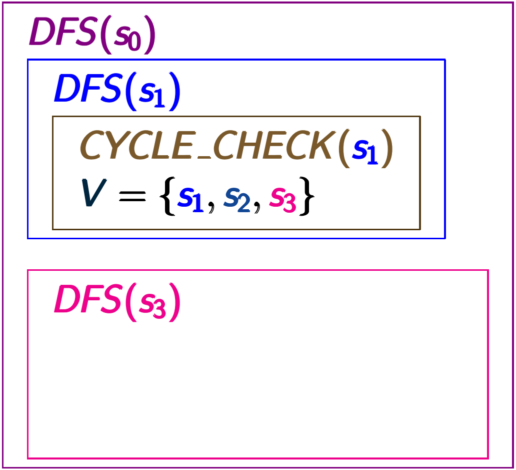

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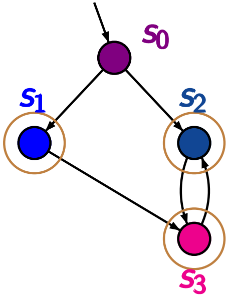

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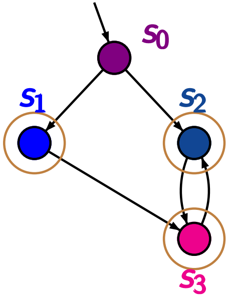

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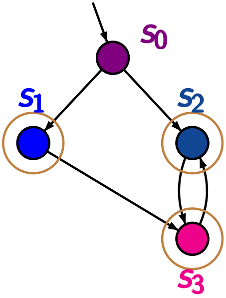

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$CYCLE\_CHECK(s_2)$

↑

returns wrong answer "**yes**"

Nested DFS

- serves to check whether $\mathcal{T} \models$ "eventually forever a"
- relies on two DFSs running in an interleaved way

1. DFS: visits all reachable states

2. DFS: $\mathit{CYCLE\_CHECK}(s)$ for states s with $s \not\models a$

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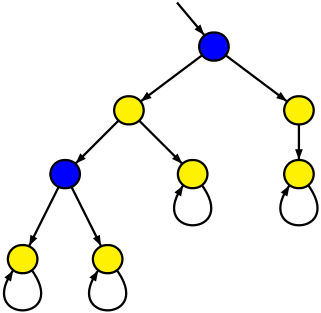
- checks whether s belongs to a cycle
  by searching a backward edge $s' \to s$
- ignores states that have been visited in previous calls of *CYCLE_CHECK* by using a global visiting set V for the **2. DFS**
- is called for state s after s is **fully expanded** in the **1. DFS**

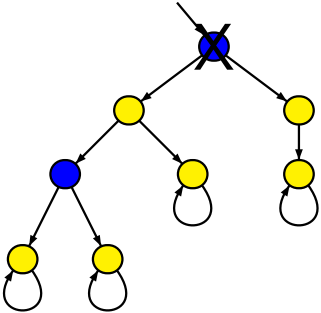
Nested DFS



$\mathcal{T} \models$ "eventually forever $\neg$***blue***"

1. DFS: visits all reachable states

2. DFS: *CYCLE_CHECK*(s) for $s \models$ ***blue***
checks whether s belongs to a cycle
by searching a backward-edge $s' \to s$



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1. **DFS:** visits all reachable states
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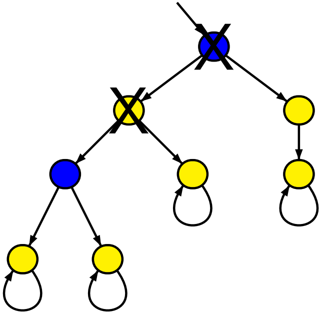

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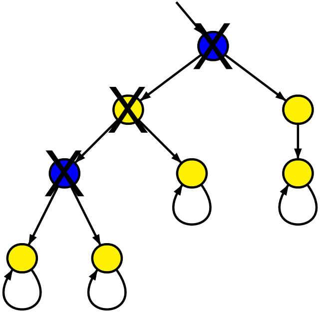

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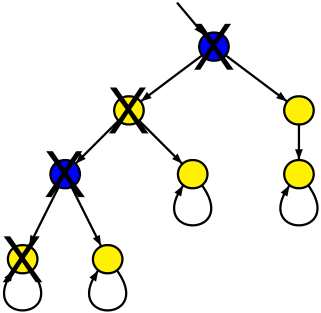

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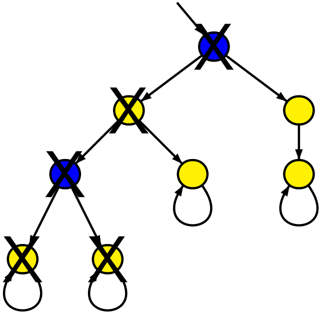

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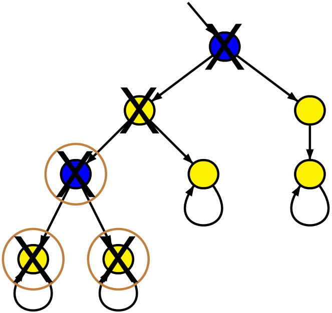

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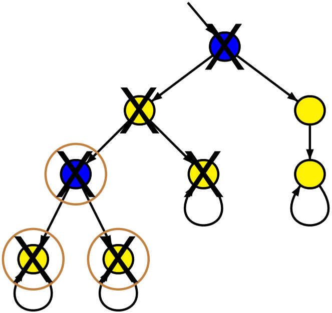

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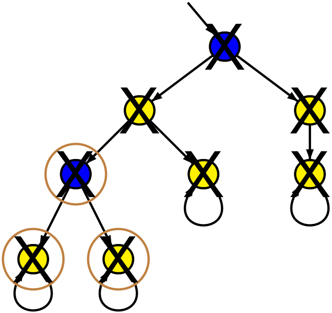

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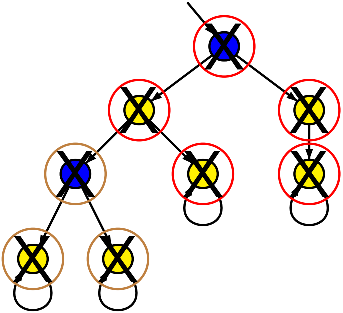

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Nested DFS (pseudo code)

$U := \varnothing$;

FOR ALL $s_0 \in S_0$ DO $DFS(s_0)$ OD ;

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IF $s \notin U$ THEN
    insert s in U;

pseudo code for $DFS(s)$

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pseudo code for $DFS(s)$

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$U := \varnothing$; $\longleftarrow$ visiting set of 1. DFS

```
FOR ALL s_0 ∈ S_0 DO DFS(s_0) OD ;
```

pseudo code for $DFS(s)$

```
IF s ∉ U THEN
    insert s in U;
    FOR ALL s' ∈ Post(s) DO DFS(s') OD ;
    IF s ⊭ a THEN
        IF CYCLE_CHECK(s)
    FI
```

Nested DFS (pseudo code)

$U := \varnothing$; $\longleftarrow$ visiting set of 1. DFS

$V := \varnothing$ $\longleftarrow$ global visiting set of 2. DFS

FOR ALL $s_0 \in S_0$ DO $DFS(s_0)$ OD ;

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IF s ∉ U THEN
     insert s in U;
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IF $s \notin U$ THEN
insert s in U;

FOR ALL $s' \in Post(s)$ DO $DFS(s')$ OD ;

IF $s \not\models a$ THEN
IF $CYCLE\_CHECK(s)$ THEN return "**no**" FI
FI

Nested DFS (pseudo code)

$U := \varnothing;$ ⟵ visiting set of 1. DFS

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$U := \varnothing$; $\longleftarrow$ visiting set of 1. DFS

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```
FOR ALL s_0 ∈ S_0 DO DFS(s_0) OD ;
return "yes"
```

$\longleftarrow$ $\mathcal{T} \models$ "eventually forever a"

```
IF s ∉ U THEN
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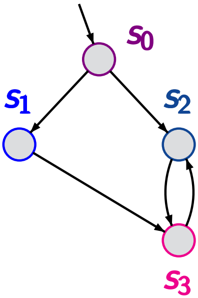
    FI
FI
```

pseudo code for $DFS(s)$

Example: nested DFS

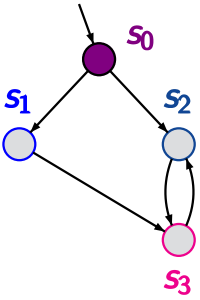


$s_1, s_2 \not\models a$

$s_0, s_3 \models a$

$\mathcal{T} \not\models$ "eventually forever a"

Example: nested DFS



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$DFS(s_0)$

$\mathcal{T} \not\models$ "eventually forever a"

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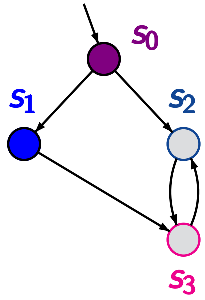

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$s_0, s_3 \models a$

$DFS(s_0)$

$DFS(s_1)$

$\mathcal{T} \not\models$ "eventually forever a"

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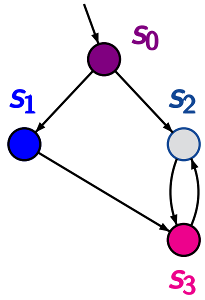

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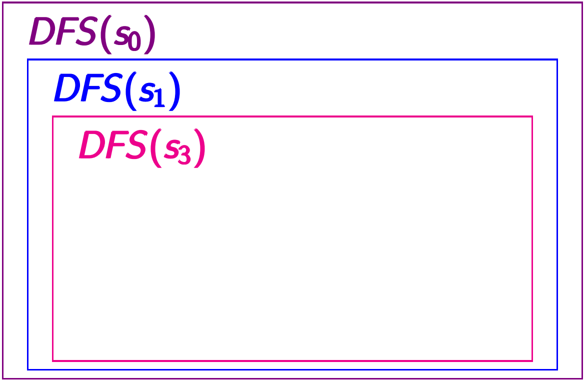

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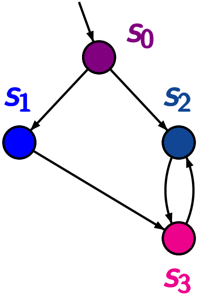

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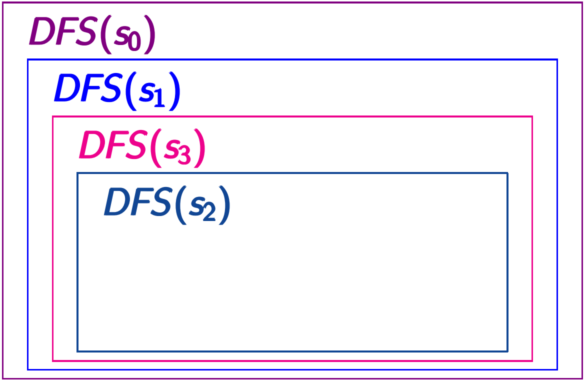

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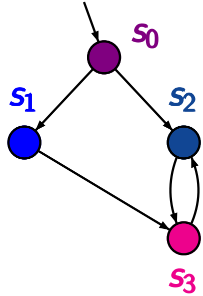

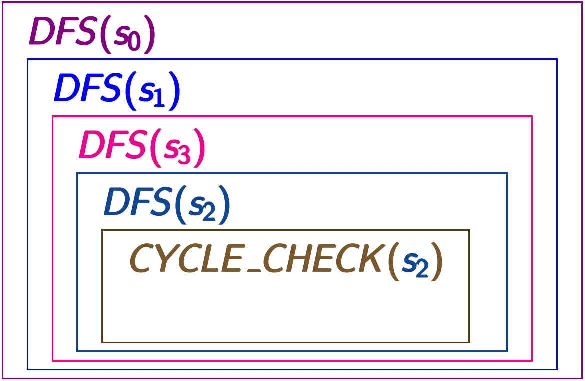

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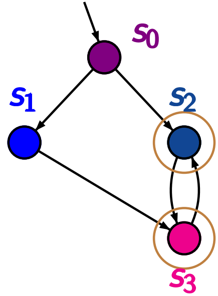


$s_1, s_2 \not\models a$

$s_0, s_3 \models a$

$DFS(s_0)$

$DFS(s_1)$

$DFS(s_3)$

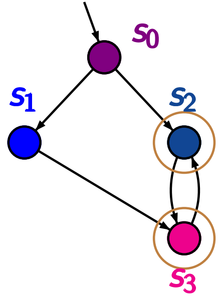
$DFS(s_2)$

$CYCLE\_CHECK(s_2)$

$\rightsquigarrow$ cycle $s_2\ s_3\ s_2$

$\mathcal{T} \not\models$ "eventually forever a"

Example: nested DFS



$s_1, s_2 \not\models a$

$s_0, s_3 \models a$

$\mathcal{T} \not\models$ "eventually forever a"

$DFS(s_0)$

$DFS(s_1)$

$DFS(s_3)$

$DFS(s_2)$

$CYCLE\_CHECK(s_2)$

$\rightsquigarrow$ cycle $s_2\ s_3\ s_2$

$\uparrow$

returns correct answer "**no**"

Nested DFS with counterexample generation

Nested DFS with counterexample generation

input: finite TS $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$
persistence condition $a \in AP$

output: "yes" if $\mathcal{T} \models$ "eventually forever a"
"no" $+$ counterexample otherwise

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$\uparrow$

initial path fragment of the form

$s_0 \ldots s_{n-1}\, s_n\, s_{n+1} \ldots s_{n+m-1}\, s_n$

where $s_n \not\models a$

Nested DFS with counterexample generation

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$\uparrow$

initial path fragment of the form

$s_0 \ldots s_{n-1}\, s_n\, s_{n+1} \ldots s_{n+m-1}\, s_n$

where $s_n \not\models a$

... iterative formulation with 2 stacks ...

$U := \varnothing;\ \pi := \varnothing;$

$U := \varnothing$; $\pi := \varnothing$; $\longleftarrow$ $\boxed{\text{visiting set and stack for 1. DFS}}$

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$V := \varnothing$; $\xi := \varnothing$; $\leftarrow$ $\boxed{\text{visiting set and stack for 2. DFS}}$

WHILE $S_0 \not\subseteq U$ DO

OD

$U := \varnothing$; $\pi := \varnothing$; $\longleftarrow$ $\boxed{\text{visiting set and stack for 1. DFS}}$

$V := \varnothing$; $\xi := \varnothing$; $\longleftarrow$ $\boxed{\text{visiting set and stack for 2. DFS}}$

```
WHILE S_0 ⊈ U DO
    choose s_0 ∈ S_0 \ U; insert s_0 in U;
OD
```

$U := \varnothing;\ \pi := \varnothing;\quad \longleftarrow$ $\boxed{\text{visiting set and stack for 1. DFS}}$

$V := \varnothing;\ \xi := \varnothing;\quad \longleftarrow$ $\boxed{\text{visiting set and stack for 2. DFS}}$

WHILE $S_0 \not\subseteq U$ DO

    choose $s_0 \in S_0 \setminus U$; insert s_0 in U; $Push(\pi, s_0)$;

OD

$U := \varnothing$; $\pi := \varnothing$; $\longleftarrow$ $\boxed{\text{visiting set and stack for 1. DFS}}$

$V := \varnothing$; $\xi := \varnothing$; $\longleftarrow$ $\boxed{\text{visiting set and stack for 2. DFS}}$

```
WHILE S_0 ⊈ U DO
    choose s_0 ∈ S_0 \ U; insert s_0 in U; Push(π, s_0);
    WHILE π ≠ ∅ DO
        s := Top(π);



    OD
OD
```

$U := \varnothing;\ \pi := \varnothing;$ $\longleftarrow$ $\boxed{\text{visiting set and stack for 1. DFS}}$

$V := \varnothing;\ \xi := \varnothing;$ $\longleftarrow$ $\boxed{\text{visiting set and stack for 2. DFS}}$

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WHILE S_0 ⊄ U DO
    choose s_0 ∈ S_0 \ U; insert s_0 in U; Push(π, s_0);
    WHILE π ≠ ∅ DO
        s := Top(π);
        IF Post(s) ⊄ U









        FI
    OD
OD
```

$U := \emptyset$; $\pi := \emptyset$; $\leftarrow$ $\boxed{\text{visiting set and stack for 1. DFS}}$

$V := \emptyset$; $\xi := \emptyset$; $\leftarrow$ $\boxed{\text{visiting set and stack for 2. DFS}}$

```
WHILE S_0 ⊈ U DO
    choose s_0 ∈ S_0 \ U; insert s_0 in U; Push(π, s_0);
    WHILE π ≠ ∅ DO
        s := Top(π);
        IF Post(s) ⊈ U
          THEN choose s' ∈ Post(s) \ U;
               insert s' in U; Push(π, s')
        FI
    OD
OD
```

$U := \varnothing$; $\pi := \varnothing$; $\longleftarrow$ $\boxed{\text{visiting set and stack for 1. DFS}}$

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WHILE S_0 ⊄ U DO
    choose s_0 ∈ S_0 \ U; insert s_0 in U; Push(π, s_0);
    WHILE π ≠ ∅ DO
        s := Top(π);
        IF Post(s) ⊄ U
          THEN choose s' ∈ Post(s) \ U;
               insert s' in U; Push(π, s')
          ELSE Pop(π);


        FI
    OD
OD
```

$U := \varnothing$; $\pi := \varnothing$; $\longleftarrow$ $\boxed{\text{visiting set and stack for 1. DFS}}$

$V := \varnothing$; $\xi := \varnothing$; $\longleftarrow$ $\boxed{\text{visiting set and stack for 2. DFS}}$

```
WHILE S_0 ⊈ U DO
    choose s_0 ∈ S_0 \ U; insert s_0 in U; Push(π, s_0);
    WHILE π ≠ ∅ DO
        s := Top(π);
        IF Post(s) ⊈ U
          THEN choose s' ∈ Post(s) \ U;
               insert s' in U; Push(π, s')
          ELSE Pop(π);
               IF s ⊭ a and CYCLE_CHECK(s)
                  THEN return "no"                      FI
        FI
    OD
OD
```

```
U := ∅; π := ∅;   ← visiting set and stack for 1. DFS
V := ∅; ξ := ∅;   ← visiting set and stack for 2. DFS
WHILE S₀ ⊈ U DO
    choose s₀ ∈ S₀ \ U; insert s₀ in U; Push(π, s₀);
    WHILE π ≠ ∅ DO
        s := Top(π);
        IF Post(s) ⊈ U
          THEN choose s' ∈ Post(s) \ U;
               insert s' in U; Push(π, s')
          ELSE Pop(π);
               IF s ⊭ a and CYCLE_CHECK(s)
                 THEN return "no" + reverse(π, ξ) FI
        FI
    OD
OD
```

$U := \varnothing$; $\pi := \varnothing$; $\longleftarrow$ $\boxed{\text{visiting set and stack for 1. DFS}}$

$V := \varnothing$; $\xi := \varnothing$; $\longleftarrow$ $\boxed{\text{visiting set and stack for 2. DFS}}$

```
WHILE S_0 ⊈ U DO
    choose s_0 ∈ S_0 \ U; insert s_0 in U; Push(π, s_0);
    WHILE π ≠ ∅ DO
        s := Top(π);
        IF Post(s) ⊈ U
         THEN choose s' ∈ Post(s) \ U;
              insert s' in U; Push(π, s')
         ELSE Pop(π);
              IF s ⊭ a and CYCLE_CHECK(s)
                THEN return "no" + reverse(π, ξ) FI
        FI
    OD
OD
return "yes"
```

Algorithm $CYCLE\_CHECK(s)$

- is called for $s \not\models a$
- checks whether s belongs to a cycle
- uses global visiting set V and stack ξ

Algorithm $CYCLE\_CHECK(s)$

$Push(\xi, s)$; insert s in V;

Algorithm $CYCLE\_CHECK(s)$

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WHILE $\xi \neq \varnothing$ DO

Algorithm $CYCLE\_CHECK(s)$

$Push(\xi, s)$; insert s in V;

WHILE $\xi \neq \varnothing$ DO

$\quad s' := Top(\xi)$;

$\quad$ IF $s \in Post(s')$

Algorithm $CYCLE\_CHECK(s)$

$Push(\xi, s)$; insert s in V;

WHILE $\xi \neq \varnothing$ DO

$s' := Top(\xi)$;

IF $s \in Post(s')$

THEN $\cancel{Push(\xi, s)}$; return "true"

Algorithm $CYCLE\_CHECK(s)$

$Push(\xi, s)$; insert s in V;

WHILE $\xi \neq \varnothing$ DO

$\quad s' := Top(\xi)$;

$\quad$ IF $s \in Post(s')$

$\quad\quad$ THEN $Push(\xi, s)$; return "true"

$\quad\quad$ ELSE IF $Post(s') \not\subseteq V$

Algorithm $CYCLE\_CHECK(s)$

$Push(\xi, s)$; insert s in V;

WHILE $\xi \neq \varnothing$ DO

  $s' := Top(\xi)$;

  IF $s \in Post(s')$

    THEN $Push(\xi, s)$; return "true"

    ELSE IF $Post(s') \not\subseteq V$

      THEN choose $s'' \in Post(s') \setminus V$;

        insert s'' in V; $Push(\xi, s'')$;

Algorithm $CYCLE\_CHECK(s)$

$Push(\xi, s)$; insert s in V;

```
WHILE ξ ≠ ∅ DO
    s' := Top(ξ);
    IF s ∈ Post(s')
        THEN  Push(ξ, s); return "true"
        ELSE  IF Post(s') ⊈ V
                 THEN  choose s'' ∈ Post(s') \ V;
                       insert s'' in V; Push(ξ, s'');
                 ELSE  Pop(ξ)
              FI
```

Algorithm $CYCLE\_CHECK(s)$


```
Push(ξ, s); insert s in V;
WHILE ξ ≠ ∅ DO
    s' := Top(ξ);
    IF s ∈ Post(s')
        THEN  Push(ξ, s); return "true"
        ELSE  IF Post(s') ⊈ V
                  THEN  choose s'' ∈ Post(s') \ V;
                        insert s'' in V; Push(ξ, s'');
                  ELSE  Pop(ξ)
              FI
    FI
OD
return "false"
```

Algorithm $CYCLE\_CHECK(s)$

```
Push(ξ, s); insert s in V;
WHILE ξ ≠ ∅ DO
    s' := Top(ξ);
    IF  s ∈ Post(s')
        THEN  Push(ξ, s);  return "true"
        ELSE IF Post(s') ⊈ V
                THEN choose s'' ∈ Post(s') \ V;
                     insert s'' in V; Push(ξ, s'');
                ELSE Pop(ξ)
        FI
    FI
OD
return "false"
```

Algorithm $CYCLE\_CHECK(s)$

$Push(\xi, s)$; insert s in V;

WHILE $\xi \neq \varnothing$ DO

  $s' := Top(\xi)$;

  IF $\boxed{s \in Post(s')}$

    THEN $Push(\xi, s)$; return "true"

    ELSE IF $Post(s') \not\subseteq V$

      THEN choose $s'' \in Post(s') \setminus V$;

        insert s'' in V; $Push(\xi, s'')$;

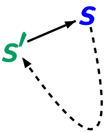
      ELSE $Pop(\xi)$

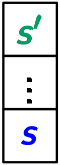
    FI

  FI

OD

return "false"





stack ξ

Algorithm $CYCLE\_CHECK(s)$

$Push(\xi, s)$; insert s in V;

WHILE $\xi \neq \varnothing$ DO

$\quad s' := Top(\xi)$;

$\quad$ IF $\boxed{s \in Post(s')}$

$\quad\quad$ THEN $\boxed{Push(\xi, s)}$ return "true"

$\quad\quad$ ELSE IF $Post(s') \not\subseteq V$

$\quad\quad\quad$ THEN choose $s'' \in Post(s') \setminus V$;

$\quad\quad\quad\quad$ insert s'' in V; $Push(\xi, s'')$;

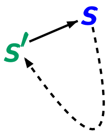
$\quad\quad\quad$ ELSE $Pop(\xi)$

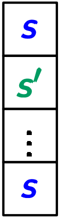
$\quad\quad$ FI

$\quad$ FI

OD

return "false"





stack ξ

```
U := ∅; π := ∅; V := ∅; ξ := ∅;
WHILE S0 ⊈ U DO
    choose s0 ∈ S0 \ U; insert s0 in U; Push(π, s0);
    WHILE π ≠ ∅ DO
        s := Top(π);
        IF Post(s) ⊈ U
          THEN choose s' ∈ Post(s) \ U;
               insert s' in U; Push(π, s')
          ELSE Pop(π);
               IF s ⊭ a and CYCLE_CHECK(s)
                 THEN return "no" + reverse(π, ξ) FI
        FI
    OD
OD
return "yes"
```

```
U := ∅; π := ∅; V := ∅; ξ := ∅;
WHILE S₀ ⊄ U DO
      choose s₀ ∈ S₀ \ U; insert s₀ in U; Push(π, s₀);
      WHILE π ≠ ∅ DO
            s := Top(π);
            IF Post(s) ⊄ U
             THEN choose s′ ∈ Post(s) \ U;
                  insert s′ in U; Push(π, s′)
             ELSE Pop(π);                     ⟵ DFS(s) ends
                  IF s ⊭ a and CYCLE_CHECK(s)
                    THEN return "no" + reverse(π, ξ) FI
            FI
      OD
OD
return "yes"
```

DFS(s) starts when s inserted in U

Soundness of the nested DFS

outer DFS: visits all reachable states s

inner DFS: algorithm $CYCLE\_CHECK(s)$

- is called for $s \not\models a$ when $DFS(s)$ is finished
- uses global data structures V and ξ

Soundness of the nested DFS

outer DFS: visits all reachable states s

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- is called for $s \not\models a$ when $\mathit{DFS}(s)$ is finished
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V:	organizes all states that have been visited in the current and all previous calls of $\mathit{CYCLE\_CHECK}(\cdot)$
ξ:	stack for counterexample

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1. termination
2. partial correctness

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ξ: stack for counterexample

$\mathcal{T} \models$ "eventually forever a"

iff the nested DFS returns "**yes**"

Soundness of the nested DFS

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Soundness of the nested DFS

$\mathcal{T} \not\models$ "eventually forever a"

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Proof of "$\Longleftarrow$":

Soundness of the nested DFS

	$\mathcal{T} \not\models$ "eventually forever a"
iff	the nested DFS returns "**no**"

Proof of "$\Longleftarrow$":

If the nested DFS returns "**no**" then there is a reachable state s such that $s \not\models a$ and *CYCLE_CHECK*(s) finds a backward edge $t \rightarrow s$.

Soundness of the nested DFS

$\mathcal{T} \not\models$ "eventually forever a"
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Proof of "$\Longleftarrow$":

If the nested DFS returns "**no**" then there is a reachable state s such that $s \not\models a$ and *CYCLE_CHECK*(s) finds a backward edge $t \rightarrow s$.

Hence: s belongs to a cycle

Soundness of the nested DFS

$\mathcal{T} \not\models$ "eventually forever a"
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Proof of "$\Longleftarrow$":

If the nested DFS returns "**no**" then there is a reachable state s such that $s \not\models a$ and *CYCLE_CHECK*(s) finds a backward edge $t \longrightarrow s$.

Hence: s belongs to a cycle and there is an ultimatively periodic path $\pi = s_0 \dots s_{n-1}(\, s\, t_1 \dots .\, t_k\,)^\omega$

$\mathcal{T} \not\models$ "eventually forever a"

iff the nested DFS returns "**no**"

Proof of "$\Longleftarrow$":

If the nested DFS returns "**no**" then there is a reachable state s such that $s \not\models a$ and *CYCLE_CHECK*(s) finds a backward edge $t \longrightarrow s$.

Hence: s belongs to a cycle and there is an ultimately periodic path $\pi = s_0 \ldots s_{n-1}(\, s\, t_1 \ldots . \; t_k\,)^\omega$ in $\mathcal{T}$ s.t. $\mathit{trace}(\pi) \notin$ "eventually forever a"

$\mathcal{T} \not\models$ "eventually forever a"
iff the nested DFS returns "**no**"

Proof of "$\Longleftarrow$":

If the nested DFS returns "**no**" then there is a reachable state s such that $s \not\models a$ and *CYCLE_CHECK*(s) finds a backward edge $t \rightarrow s$.

Hence: s belongs to a cycle and there is an ultimately periodic path $\pi = s_0 \ldots s_{n-1} (\, s\, t_1 \ldots .\, t_k\,)^\omega$ in $\mathcal{T}$ s.t. $trace(\pi) \notin$ "eventually forever a"

This yields $\mathcal{T} \not\models$ "eventually forever a".

Soundness of the nested DFS

$\mathcal{T} \not\models$ "eventually forever a"

iff the nested DFS returns "**no**"

Proof of "$\Longrightarrow$":

Soundness of the nested DFS

$\mathcal{T} \not\models$ "eventually forever a"
iff the nested DFS returns "**no**"

Proof of "$\Longrightarrow$": show that:

When $CYCLE\_CHECK(s)$ is called then there is no cycle $t_0\, t_1 \, \ldots \, t_k$ in $\mathcal{T}$ s.t. $s = t_0 = t_k$ and $t_i \in V$ for some $i \in \{1, ..., k\}$

global visiting set of the inner DFS

	$\mathcal{T} \not\models$ "eventually forever a"
iff	the nested DFS returns "**no**"

Proof of "$\Longrightarrow$": show that:

When $CYCLE\_CHECK(s)$ is called then there is no cycle $t_0\, t_1 \ldots t_k$ in $\mathcal{T}$ s.t. $s = t_0 = t_k$ and $t_i \in V$ for some $i \in \{1, \ldots, k\}$

global visiting set of the inner DFS

Hence: if s belongs to a cycle then $CYCLE\_CHECK(s)$ will find a backward edge $t \to s$

Further improvements of the nested DFS

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```
U := ∅; π := ∅;
V := ∅; ξ := ∅;
WHILE S_0 ⊄ U DO
    choose s_0 ∈ S_0 \ U;
        ⋮
    WHILE π ≠ ∅ DO
        s := Top(π);
        IF Post(s) ⊄ U
            THEN ...
            ELSE Pop(π);
             IF s ⊭ a and CYCLE_CHECK(s) THEN ...
      FI
   OD
OD
```

Further improvements of the nested DFS

$U := \varnothing$; $\pi := \varnothing$;
$V := \varnothing$; $\xi := \varnothing$;
WHILE $S_0 \not\subseteq U$ DO
  choose $s_0 \in S_0 \setminus U$;
  ⋮
  WHILE $\pi \neq \varnothing$ DO
    $s := Top(\pi)$;
    IF $Post(s) \not\subseteq U$
      THEN ...
      ELSE $Pop(\pi)$;
       IF $s \not\models a$ and $CYCLE\_CHECK(s)$ THEN ...
    FI
  OD
OD

on the fly construction

Further improvements of the nested DFS

$U := \varnothing$; $\pi := \varnothing$;
$V := \varnothing$; $\xi := \varnothing$;
WHILE $S_0 \not\subseteq U$ DO
    choose $s_0 \in S_0 \setminus U$;
        ⋮
    WHILE $\pi \neq \varnothing$ DO
        $s := Top(\pi)$;
        IF $Post(s) \not\subseteq U$
            THEN ...
            ELSE $Pop(\pi)$;
             IF $s \not\models a$ and $CYCLE\_CHECK(s)$ THEN ...
        FI
    OD
OD

on the fly construction
hash techniques for U and V

Further improvements of the nested DFS


```
U := ∅; π := ∅;
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WHILE S0 ⊈ U DO
    choose s0 ∈ S0 \ U;
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    WHILE π ≠ ∅ DO
        s := Top(π);
        IF Post(s) ⊈ U
            THEN ...
            ELSE Pop(π);
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        FI
    OD
OD
```

on the fly construction

hash techniques for U and V

$$s \in \underbrace{U \setminus V}_{\langle s, 1\rangle} \qquad \underbrace{s \in V}_{\langle s, 0\rangle}$$

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        IF $Post(s) \not\subseteq U$
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             IF $s \not\models a$ and $CYCLE\_CHECK(s)$ THEN . . .
        FI
    OD
OD

on the fly construction

hash techniques for U and V

$$\underbrace{s \in U \setminus V}_{\langle s, 1\rangle} \qquad \underbrace{s \in V}_{\langle s, 0\rangle}$$

early termination of $CYCLE\_CHECK$, e.g., if a state in π is visited