transition system $\mathcal{T} = (S, Act, \longrightarrow, S_0, AP, L)$

$\Big\downarrow$ abstraction from actions

state graph $G_{\mathcal{T}}$

- set of nodes $=$ state space S
- edges $=$ transitions without action label

Act for modeling interactions/communication and specifying fairness assumptions

AP, L for specifying properties

State-based view of TS

transition system $\mathcal{T} = (S, Act, \longrightarrow, S_0, AP, L)$

abstraction from actions

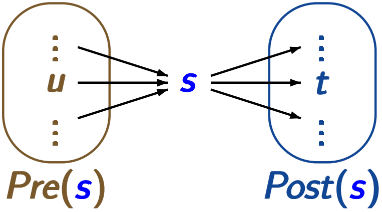
state graph $G_{\mathcal{T}}$

- set of nodes = state space S
- edges = transitions without action label

use standard notations for graphs, e.g.,

$$Post(s) = \{t \in S : s \to t\}$$

$$Pre(s) = \{u \in S : u \to s\}$$



Execution and path fragments

execution fragment: sequence of consecutive transitions

$s_0 \xrightarrow{\alpha_0} s_1 \xrightarrow{\alpha_1} \ldots$ infinite or

$s_0 \xrightarrow{\alpha_0} s_1 \xrightarrow{\alpha_1} \ldots \xrightarrow{\alpha_{n-1}} s_n$ finite

path fragment: sequence of states arising from the projection of an execution fragment to the states

$\pi = s_0\, s_1\, s_2 \ldots$ infinite or $\pi = s_0\, s_1 \ldots s_n$ finite

such that $s_{i+1} \in Post(s_i)$ for all $i < |\pi|$

initial: if $s_0 \in S_0$ = set of initial states

maximal: if infinite or ending in a terminal state

Notations for paths

path fragment: sequence of states

$\pi = s_0\, s_1\, s_2 \ldots$ infinite or $\pi = s_0\, s_1 \ldots s_n$ finite

s.t. $s_{i+1} \in Post(s_i)$ for all $i < |\pi|$

initial: if $s_0 \in S_0$ = set of initial states

maximal: if infinite or ending in terminal state

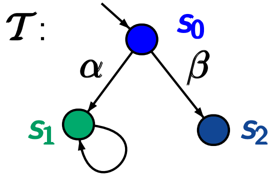
path of TS $\mathcal{T}$ $\widehat{=}$ initial, maximal path fragment

path of state s $\widehat{=}$ maximal path fragment starting in state s

$Paths(\mathcal{T})$ = set of all initial, maximal path fragments

$Paths(s)$ = set of all maximal path fragments starting in state s

Paths of a TS and its states



How many paths are there in $\mathcal{T}$?

answer: 2, namely $s_0\, s_1\, s_1\, s_1 \ldots$ and $s_0\, s_2$

$$Paths(s_1) = \text{set of all maximal paths fragments starting in } s_1 = \{s_1^\omega\} \text{ where } s_1^\omega = s_1\, s_1\, s_1\, s_1\, \ldots$$

$$Paths_{fin}(s_1) = \text{set of all finite path fragments starting in } s_1 = \{s_1^n : n \in \mathbb{N}, n \geq 1\}$$

Linear-time vs branching-time

transition system
$\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$

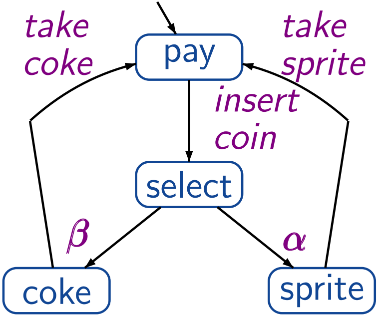
abstraction from actions

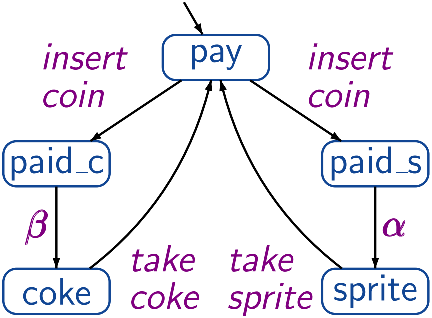
state graph
$+$ labeling

linear-time view
path-based
state sequences
branching structure irrelevant

branching-time view
nondeterministic branches
state & branches

Example: vending machine

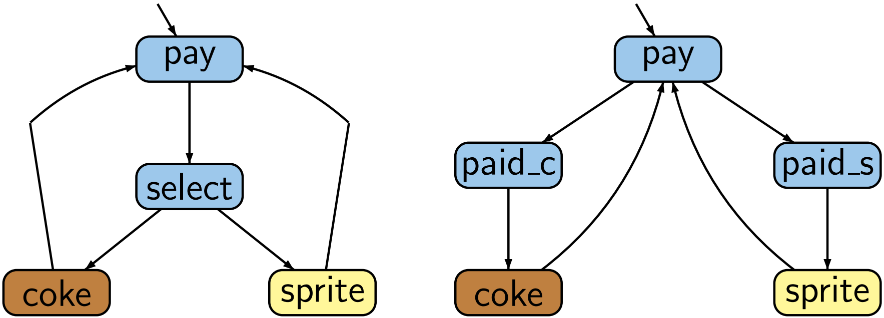




vending machine with
1 coin deposit
select drink after having paid

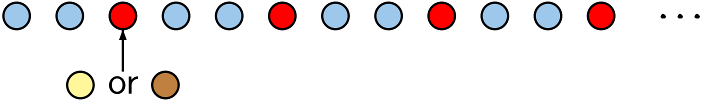
vending machine with
2 coin deposits
select drink by inserting the coin

Example: vending machine

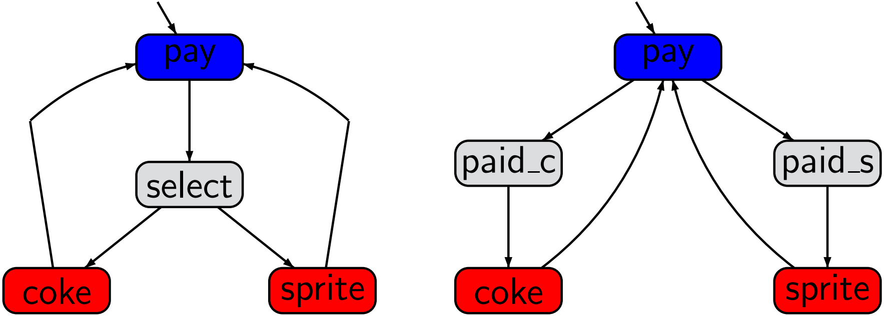


state based view: abstracts from actions and projects onto atomic propositions, e.g. $AP = \{coke, sprite\}$

linear time: all observable behaviors are of the form



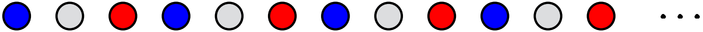
Example: vending machine



state based view: abstracts from actions and projects on atomc propositions, e.g., $AP = \{pay, drink\}$

linear & branching time:

all observable behaviors have the form



transition system

$\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$

abstraction from actions

state graph

$+$ labeling

projection on AP

linear-time view

state sequences

$\Downarrow$

traces

branching-time view

state & branches

$\Downarrow$

computation tree

Traces

for TS with labeling function $L : S \to 2^{AP}$

execution: states + actions

$s_0 \xrightarrow{\alpha_1} s_1 \xrightarrow{\alpha_2} s_2 \xrightarrow{\alpha_3} \ldots$ infinite or ~~finite~~

paths: sequences of states

$s_0\, s_1\, s_2 \ldots$ infinite or ~~$s_0\, s_1 \ldots s_n$ finite~~

traces: sequences of sets of atomic propositions

$L(s_0)\, L(s_1)\, L(s_2)\, \ldots \in (2^{AP})^\omega \cup \cancel{(2^{AP})^+}$

for simplicity: we often assume that the given TS has **no terminal states**

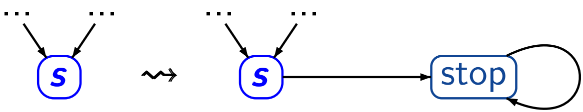
Treatment of terminal states

perform standard graph algorithms to compute the reachable fragment of the given TS

$$\mathit{Reach}(\mathcal{T}) = \left\{ \begin{array}{c} \text{set of states that are reachable} \\ \text{from some initial state} \end{array} \right.$$

for each reachable terminal state s:

- if s stands for an intended halting configuration then add a transition from s to a trap state:



- if s stands for system fault, e.g., deadlock then correct the design before checking further properties

Let $\mathcal{T}$ be a TS $\longleftarrow$ $\boxed{\textit{without} \text{ terminal states}}$

$$Traces(\mathcal{T}) \stackrel{\text{def}}{=} \{trace(\pi) : \pi \in Paths(\mathcal{T})\} \subseteq (2^{AP})^{\omega}$$

$\uparrow$ initial, infinite path fragment

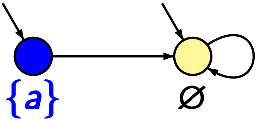
$$Traces_{fin}(\mathcal{T}) \stackrel{\text{def}}{=} \{trace(\widehat{\pi}) : \widehat{\pi} \in Paths_{fin}(\mathcal{T})\} \subseteq (2^{AP})^{*}$$

$\uparrow$ initial, finite path fragment

Let $\mathcal{T}$ be a TS without terminal states.

$$Traces(\mathcal{T}) \stackrel{\text{def}}{=} \{trace(\pi) : \pi \in Paths(\mathcal{T})\} \subseteq (2^{AP})^{\omega}$$

$$Traces_{fin}(\mathcal{T}) \stackrel{\text{def}}{=} \{trace(\widehat{\pi}) : \widehat{\pi} \in Paths_{fin}(\mathcal{T})\} \subseteq (2^{AP})^{*}$$

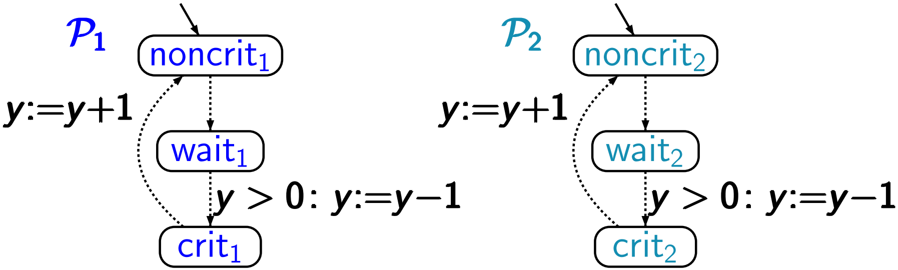


TS $\mathcal{T}$ with a single atomic proposition a

$$Traces(\mathcal{T}) = \{\, \{a\}\varnothing^{\omega}, \varnothing^{\omega} \,\}$$

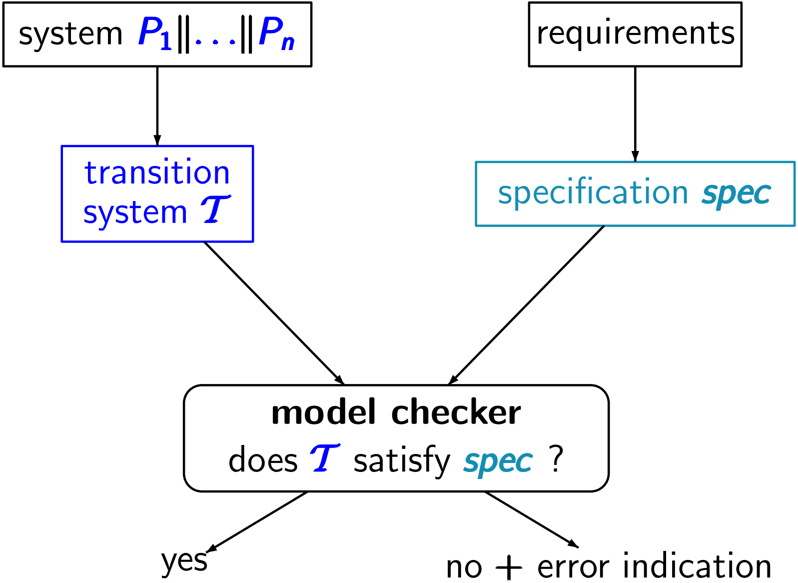
$$Traces_{fin}(\mathcal{T}) = \{\{a\}\varnothing^{n} : n \geq 0\} \cup \{\varnothing^{m} : m \geq 1\}$$

Mutual exclusion with semaphore

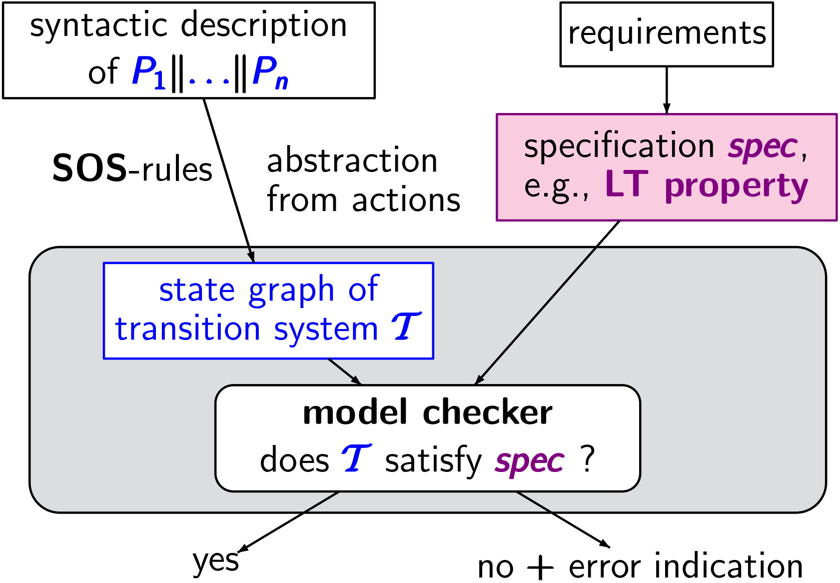


transition system $\mathcal{T}_{\mathcal{P}_1 ||| \mathcal{P}_2}$ arises by unfolding the composite program graph $\mathcal{P}_1 \; ||| \; \mathcal{P}_2$

Model checking



Model checking



Linear-time properties (LT properties)

for TS over AP without terminal states

An LT property over AP is a language E of infinite words over the alphabet $\Sigma = 2^{AP}$, i.e., $E \subseteq \left(2^{AP}\right)^{\omega}$.

E.g., for mutual exclusion problems and
$AP = \{crit_1, crit_2, \ldots\}$

safety:

$$MUTEX = \begin{array}{l}\text{set of all infinite words } A_0\, A_1\, A_2 \ldots \\ \text{over } 2^{AP} \text{ such that for all } i \in \mathbb{N}\text{:} \\ \quad crit_1 \notin A_i \ \text{ or } \ crit_2 \notin A_i\end{array}$$

LT properties for mutual exclusion protocols

$AP = \{wait_1, crit_1, wait_2, crit_2\}$

safety:

$MUTEX$ = set of all infinite words $A_0\, A_1\, A_2 \ldots$ over 2^{AP} such that for all $i \in \mathbb{N}$: $crit_1 \notin A_i$ or $crit_2 \notin A_i$

$\varnothing\ \{wait_1\}\ \{crit_1\}\ \varnothing\ \{wait_1\}\ \{crit_1\}\ \ldots \in MUTEX$

$\varnothing\ \{wait_1\}\ \{crit_1\}\ \{crit_1, wait_2\}\ \{crit_1, crit_2\}\ \ldots \notin MUTEX$

$\varnothing\ \varnothing\ \{wait_1, crit_1, crit_2\}\ \ldots \notin MUTEX$

LT properties for mutual exclusion protocols

$AP = \{wait_1, crit_1, wait_2, crit_2\}$

safety:

$$MUTEX = \begin{array}{l} \text{set of all infinite words } A_0\, A_1\, A_2 \ldots \\ \text{over } 2^{AP} \text{ such that for all } i \in \mathbb{N}: \\ \quad crit_1 \notin A_i \ \text{ or } \ crit_2 \notin A_i \end{array}$$

liveness (starvation freedom):

set of all infinite words $A_0\, A_1\, A_2 \ldots$ s.t.

$$\begin{aligned} LIVE = \quad & \overset{\infty}{\exists}\, i \in \mathbb{N}.wait_1 \in A_i \implies \overset{\infty}{\exists}\, i \in \mathbb{N}.crit_1 \in A_i \\ \wedge \quad & \overset{\infty}{\exists}\, i \in \mathbb{N}.wait_2 \in A_i \implies \overset{\infty}{\exists}\, i \in \mathbb{N}.crit_2 \in A_i \end{aligned}$$

Satisfaction relation for LT properties

An LT property over AP is a language E of infinite words over the alphabet $\Sigma = 2^{AP}$, i.e., $E \subseteq \left(2^{AP}\right)^{\omega}$.

Satisfaction relation $\models$ for TS and states:

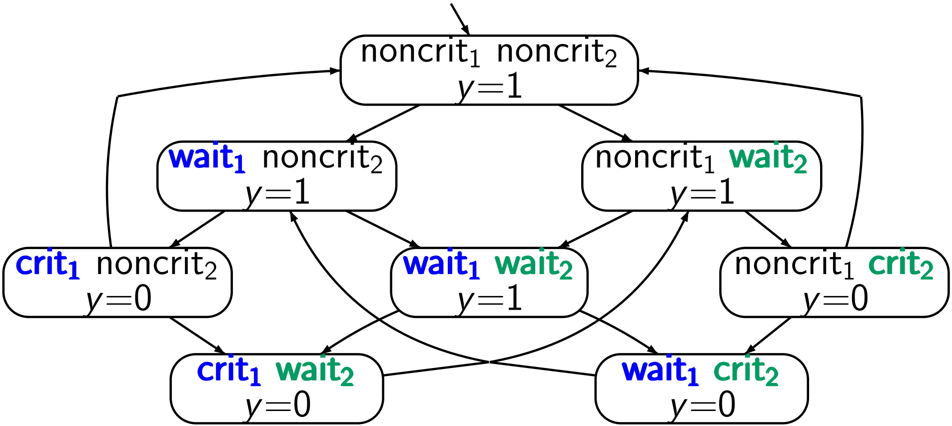
If $\mathcal{T}$ is a TS (without terminal states) over AP and E an LT property over AP then

$$\mathcal{T} \models E \quad \text{iff} \quad \mathit{Traces}(\mathcal{T}) \subseteq E$$

If s is a state in $\mathcal{T}$ then

$$s \models E \quad \text{iff} \quad \mathit{Traces}(s) \subseteq E$$

Mutual exclusion with semaphore



$$\mathcal{T}_{Sem} \models \mathit{MUTEX}, \quad \mathcal{T}_{Sem} \not\models \mathit{LIVE}$$

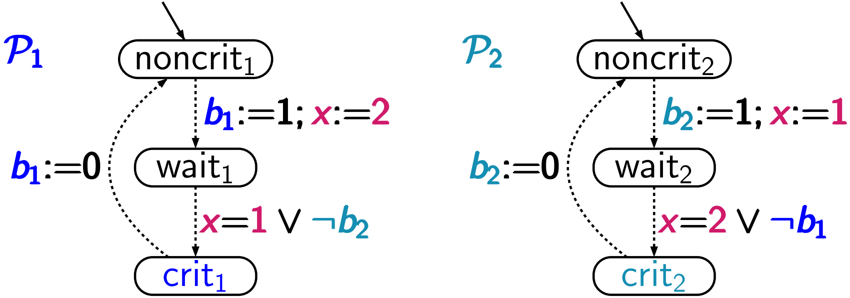
$$\varnothing\ \{\mathsf{wait}_1\}\ \big(\{\mathsf{wait}_1, \mathsf{wait}_2\}\ \{\mathsf{crit}_1, \mathsf{wait}_2\}\{\mathsf{wait}_2\}\big)^\omega \notin \mathit{LIVE}$$

Peterson's mutual exclusion algorithm

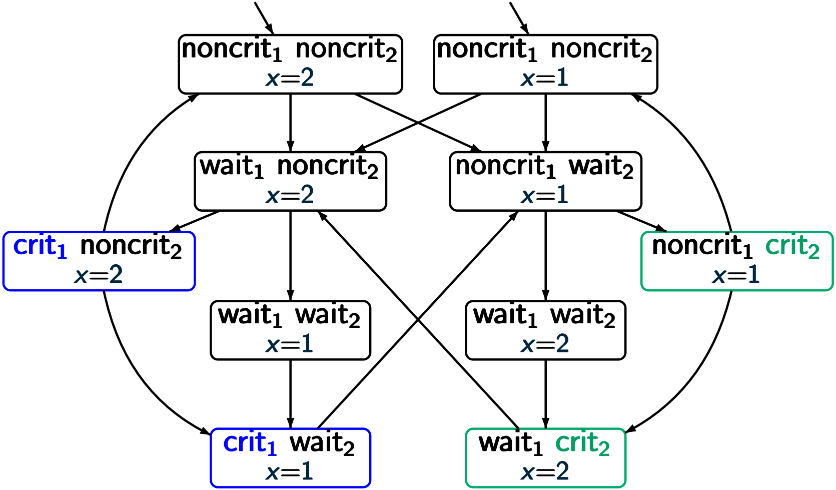
for competing processes $\mathcal{P}_1$ and $\mathcal{P}_2$,

using three additional shared variables

$b_1, b_2 \in \{0,1\}$, $x \in \{1,2\}$



Peterson's mutual exclusion algorithm



$\mathcal{T}_{Pet} \models \mathit{MUTEX}$ and $\mathcal{T}_{Pet} \models \mathit{LIVE}$

LT properties and trace inclusion

An LT property over AP is a language E of infinite words over the alphabet $\Sigma = 2^{AP}$, i.e., $E \subseteq \left(2^{AP}\right)^{\omega}$.

If $\mathcal{T}$ is a TS over AP then $\mathcal{T} \models E$ iff $Traces(\mathcal{T}) \subseteq E$.

Consequence of these definitions:

If $\mathcal{T}_1$ and $\mathcal{T}_2$ are TS over AP then for all LT properties E over AP:

$$Traces(\mathcal{T}_1) \subseteq Traces(\mathcal{T}_2) \wedge \mathcal{T}_2 \models E \Longrightarrow \mathcal{T}_1 \models E$$

note: $Traces(\mathcal{T}_1) \subseteq Traces(\mathcal{T}_2) \subseteq E$

An LT property over AP is a language E of infinite words over the alphabet $\Sigma = 2^{AP}$, i.e., $E \subseteq (2^{AP})^{\omega}$.

If $\mathcal{T}$ is a TS over AP then $\mathcal{T} \models E$ iff $Traces(\mathcal{T}) \subseteq E$.

> If $\mathcal{T}_1$ and $\mathcal{T}_2$ are TS over AP then the following statements are equivalent:
>
> (1) $Traces(\mathcal{T}_1) \subseteq Traces(\mathcal{T}_2)$
>
> (2) for all LT-properties E over AP: whenever $\mathcal{T}_2 \models E$ then $\mathcal{T}_1 \models E$

$(1) \Longrightarrow (2)$: $\checkmark$

LT properties and trace inclusion

An LT property over AP is a language E of infinite words over the alphabet $\Sigma = 2^{AP}$, i.e., $E \subseteq \left(2^{AP}\right)^{\omega}$.

If $\mathcal{T}$ is a TS over AP then $\mathcal{T} \models E$ iff $Traces(\mathcal{T}) \subseteq E$.

> If $\mathcal{T}_1$ and $\mathcal{T}_2$ are TS over AP then the following statements are equivalent:
>
> (1) $Traces(\mathcal{T}_1) \subseteq Traces(\mathcal{T}_2)$
>
> (2) for all LT-properties E over AP: whenever $\mathcal{T}_2 \models E$ then $\mathcal{T}_1 \models E$

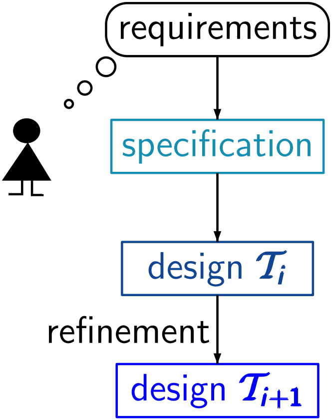
$(2) \Longrightarrow (1)$: consider $E = Traces(\mathcal{T}_2)$

Relevance of trace inclusion

Trace inclusion appears naturally

- as an implementation/refinement relation
- when resolving nondeterminism
- in the context of abstractions

Trace inclusion as an implementation relation



trace inclusion

$\mathcal{T}_{i+1} \sqsubseteq \mathcal{T}_i$ iff

$Traces(\mathcal{T}_{i+1}) \subseteq Traces(\mathcal{T}_i)$

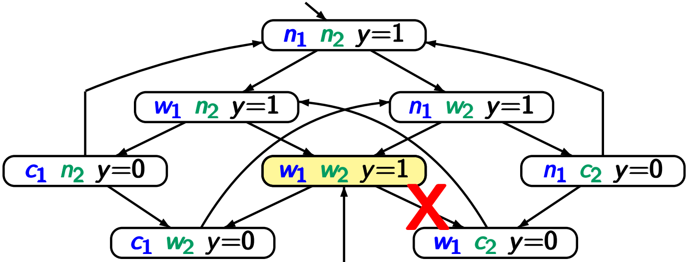
$\longleftarrow \mathcal{T}_i \models E$

$\longleftarrow \mathcal{T}_{i+1} \sqsubseteq \mathcal{T}_i$ implies $\mathcal{T}_{i+1} \models E$

implementation/refinement relation $\sqsubseteq$:

$\mathcal{T}_{i+1} \sqsubseteq \mathcal{T}_i$ iff "$\mathcal{T}_{i+1}$ correctly implements $\mathcal{T}_i$"

Mutual exclusion with semaphore

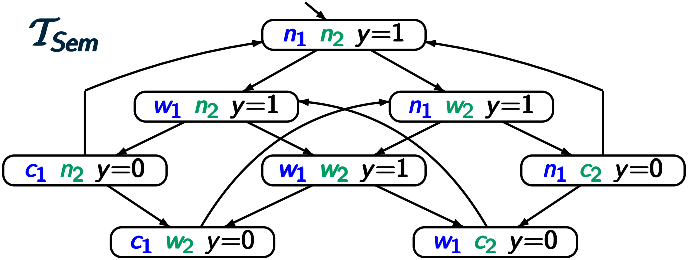


competition in state
$\langle wait_1 \ wait_2 \ y{=}1 \rangle$

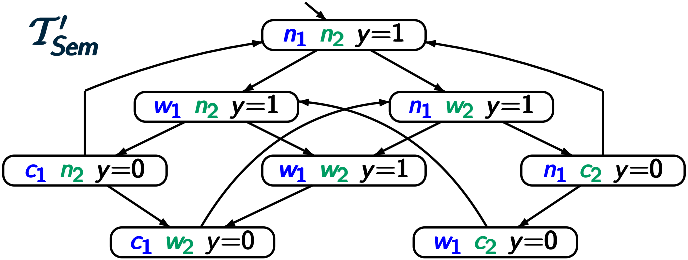
resolve the nondeterminism by giving priority to process P_1

Mutual exclusion with semaphore

$\mathcal{T}_{Sem}$



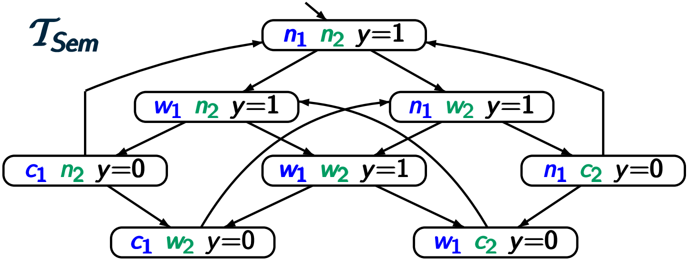
$\mathcal{T}'_{Sem}$



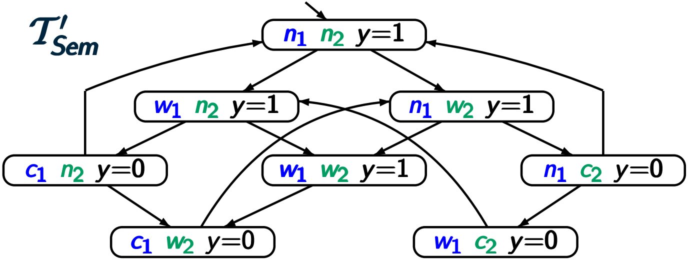
$$Paths(\mathcal{T}'_{Sem}) \subseteq Paths(\mathcal{T}_{Sem})$$

Mutual exclusion with semaphore

$\mathcal{T}_{Sem}$



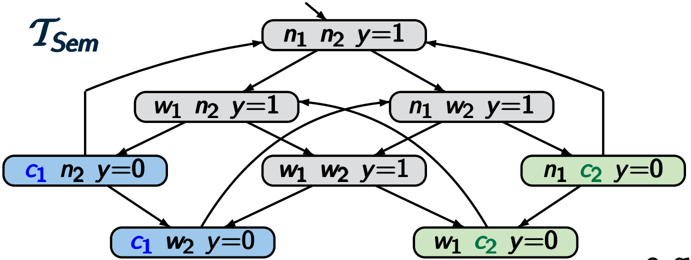
$\mathcal{T}'_{Sem}$



$Traces(\mathcal{T}'_{Sem}) \subseteq Traces(\mathcal{T}_{Sem})$ for any AP

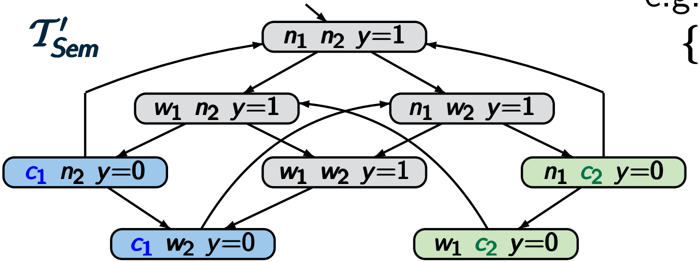
Mutual exclusion with semaphore

$\mathcal{T}_{Sem}$



e.g., for $AP = \{crit_1, crit_2\}$

$\mathcal{T}'_{Sem}$



$Traces(\mathcal{T}_{Sem}) \models E$ implies $Traces(\mathcal{T}'_{Sem}) \models E$ for any E

Relevance of trace inclusion

Trace inclusion appears naturally

- as an implementation/refinement relation
- when resolving nondeterminism

> whenever $\mathcal{T}'$ results from $\mathcal{T}$ by a scheduling policy for resolving nondeterministic choices in $\mathcal{T}$ then
>
> $$Traces(\mathcal{T}') \subseteq Traces(\mathcal{T})$$

- in the context of abstractions

Relevance of trace inclusion

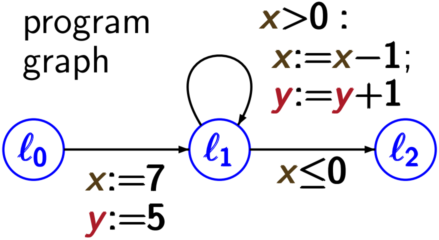
Trace inclusion appears naturally

- as an implementation/refinement relation
- when resolving nondeterminism
- in the context of abstractions $\longleftarrow$

Trace inclusion and data abstraction

$$
\begin{array}{ll}
 & \vdots \\
\ell_0 & x{:=}7;\ y{:=}5; \\
\ell_1 & \texttt{WHILE}\ x>0\ \texttt{DO} \\
 & \qquad x{:=}x-1; \\
 & \qquad y{:=}y+1 \\
 & \texttt{OD} \\
\ell_2 & \vdots
\end{array}
$$

program graph

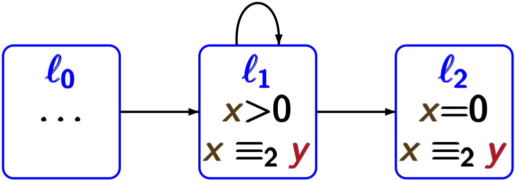


let $\mathcal{T}$ be the associated TS

does $\ell_2 \wedge odd(y)$ never hold ?

data abstraction w.r.t. the predicates

$x>0$, $x=0$, $x \equiv_2 y$

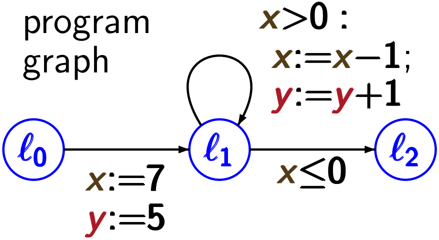


abstract transition system $\mathcal{T}'$

Trace inclusion and data abstraction


```
        ⋮
ℓ0   x:=7;  y:=5;
ℓ1   WHILE x>0 DO
            x:=x−1;
            y:=y+1
     OD
ℓ2      ⋮
```

program graph

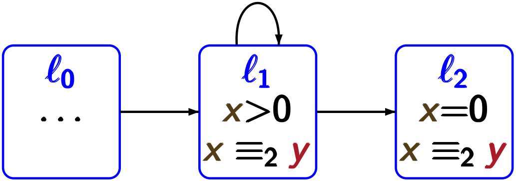


let $\mathcal{T}$ be the associated TS

does $\ell_2 \wedge odd(y)$ never hold ?

data abstraction w.r.t. the predicates

$x>0$, $x=0$, $x \equiv_2 y$



$\mathcal{T}' \models$ "never $\ell_2 \wedge odd(y)$"

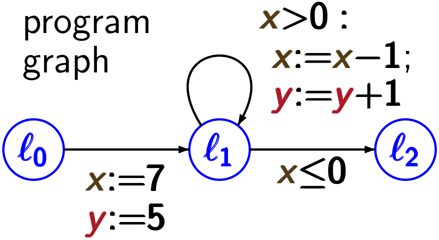
$Traces(\mathcal{T}) \subseteq Traces(\mathcal{T}')$

Trace inclusion and data abstraction

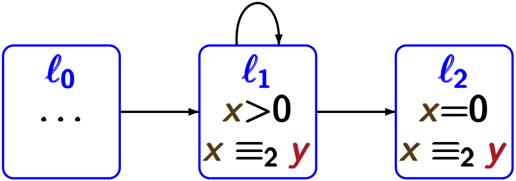

```
       ⋮
ℓ0   x:=7; y:=5;
ℓ1   WHILE x>0 DO
           x:=x−1;
           y:=y+1
     OD
ℓ2     ⋮
```

does $\ell_2 \wedge odd(y)$ never hold ?

program graph



let $\mathcal{T}$ be the associated TS



$\mathcal{T} \models$ "never $\ell_2 \wedge odd(y)$" $\begin{cases} \mathcal{T}' \models \text{"never } \ell_2 \wedge odd(y)\text{"} \\ Traces(\mathcal{T}) \subseteq Traces(\mathcal{T}') \end{cases}$

Trace equivalence

Transition systems $\mathcal{T}_1$ and $\mathcal{T}_2$ over the same set AP of atomic propositions are called trace equivalent iff

$$Traces(\mathcal{T}_1) = Traces(\mathcal{T}_2)$$

i.e., trace equivalence requires trace inclusion in both directions

Trace equivalent TS satisfy the **same LT properties**

LT properties and trace relations

Let $\mathcal{T}_1$ and $\mathcal{T}_2$ be TS over AP.

The following statements are equivalent:

(1) $Traces(\mathcal{T}_1) \subseteq Traces(\mathcal{T}_2)$

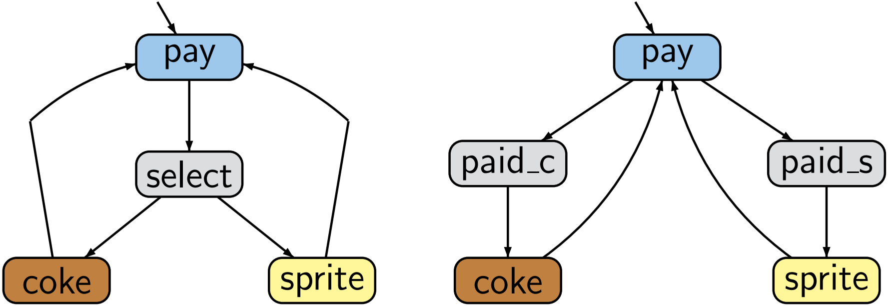
(2) for all LT-properties E: $\mathcal{T}_2 \models E \Longrightarrow \mathcal{T}_1 \models E$

The following statements are equivalent:

(1) $Traces(\mathcal{T}_1) = Traces(\mathcal{T}_2)$

(2) for all LT-properties E: $\mathcal{T}_1 \models E$ iff $\mathcal{T}_2 \models E$

Trace equivalent beverage machines



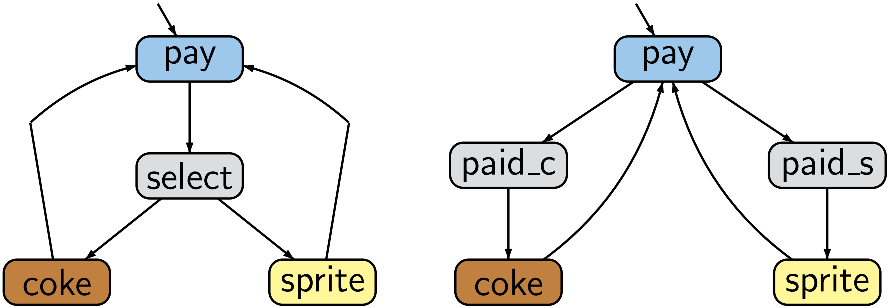
set of atomic propositions $AP = \{pay, coke, sprite\}$

$Traces(\mathcal{T}_1) = Traces(\mathcal{T}_2) =$ set of all infinite words

$\{pay\}\,\varnothing\,\{drink_1\}\,\{pay\}\,\varnothing\,\{drink_2\}\,\ldots$

where $drink_1, drink_2, \ldots \in \{coke, sprite\}$

Trace equivalent beverage machines



set of atomic propositions $AP = \{pay, coke, sprite\}$

$Traces(\mathcal{T}_1) = Traces(\mathcal{T}_2) =$ set of all infinite words

$\{pay\}\,\varnothing\,\{drink_1\}\,\{pay\}\,\varnothing\,\{drink_2\}\,\ldots$

$\mathcal{T}_1$ and $\mathcal{T}_2$ satisfy the same LT-properties over AP

Classification of LT-properties

safety properties *"nothing bad will happen"*

examples:

- mutual exclusion
- deadlock freedom

special case: **invariants**
"no bad state will be reached"

- "every red phase is preceded by a yellow phase"

liveness properties *"something good will happen"*

examples:

- "each waiting process will eventually enter its critical section"
- "each philosopher will eat infinitely often"

$$\Phi \;::=\; \textit{true} \;\Big|\; a \;\Big|\; \Phi_1 \wedge \Phi_2 \;\Big|\; \neg\Phi \;\Big|\; \Phi_1 \vee \Phi_2 \;\Big|\; \Phi_1 \rightarrow \Phi_2 \;\Big|\; \ldots$$

atomic proposition, i.e., $a \in AP$

semantics: Let $A \subseteq AP$

$$\begin{array}{lll} A \models \textit{true} & & \\ A \models a & \text{iff} & a \in A \\ A \models \Phi_1 \wedge \Phi_2 & \text{iff} & A \models \Phi_1 \text{ and } A \models \Phi_2 \\ A \models \neg\Phi & \text{iff} & A \not\models \Phi \end{array}$$

for state s of a TS over AP: $s \models \Phi$ iff $L(s) \models \Phi$

Invariant

Let E be an LT property over AP.

E is called an invariant if there exists a propositional formula Φ over AP such that

$$E = \left\{ A_0 A_1 A_2 \ldots \in \left(2^{AP}\right)^{\omega} : \forall i \geq 0.\, A_i \models \Phi \right\}$$

Φ is called the invariant condition of E.

Examples for invariants

mutual exclusion (safety):

$$MUTEX = \begin{array}{c}\text{set of all infinite words } A_0\, A_1\, A_2 \ldots \text{ s.t.} \\ \forall i \in \mathbb{N}.\ \ \mathsf{crit}_1 \notin A_i \ \text{ or } \ \mathsf{crit}_2 \notin A_i\end{array}$$

invariant condition: $\Phi = \neg\mathsf{crit}_1 \vee \neg\mathsf{crit}_2$

deadlock freedom for 5 dining philosophers:

$$DF = \begin{array}{c}\text{set of all infinite words } A_0\, A_1\, A_2 \ldots \text{ s.t.} \\ \forall i \in \mathbb{N}\ \exists \mathrm{j} \in \{0,1,2,3,4\}.\ \mathsf{wait}_j \notin A_i\end{array}$$

invariant condition:

$$\Phi = \neg\mathsf{wait}_0 \vee \neg\mathsf{wait}_1 \vee \neg\mathsf{wait}_2 \vee \neg\mathsf{wait}_3 \vee \neg\mathsf{wait}_4$$

here: $AP = \{\mathsf{wait}_j : 0 \leq j \leq 4\} \cup \{\ldots\}$

Let E be an LT property over AP. E is called an invariant if there exists a propositional formula Φ s.t.

$$E = \left\{ A_0 A_1 A_2 \ldots \in \left(2^{AP}\right)^\omega : \forall i \geq 0.\, A_i \models \Phi \right\}$$

Let $\mathcal{T}$ be a TS over AP without terminal states. Then:

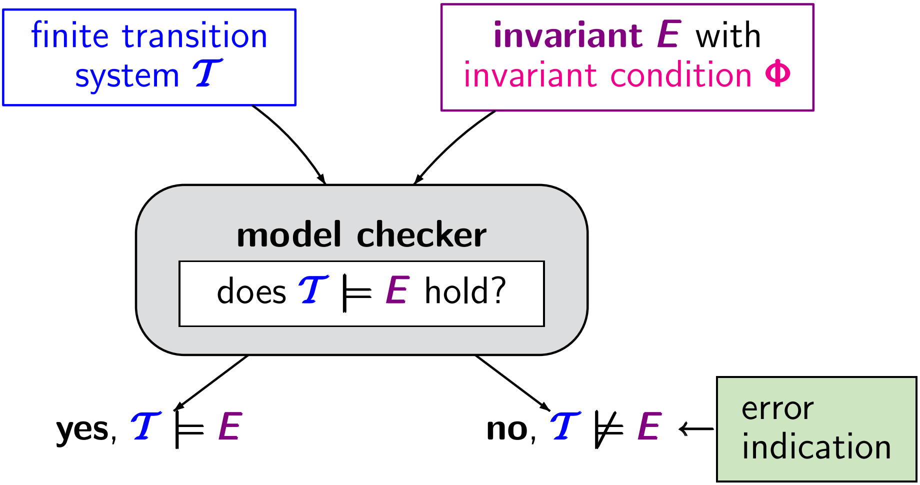
$\mathcal{T} \models E$ iff $trace(\pi) \in E$ for all $\pi \in Paths(\mathcal{T})$

iff $s \models \Phi$ for all states s on a path of $\mathcal{T}$

iff $s \models \Phi$ for all states $s \in Reach(\mathcal{T})$

i.e., Φ holds in all initial states and is invariant under all transitions

Invariant checking



perform a graph analysis (**DFS** or **BFS**) to check whether $s \models \Phi$ for all $s \in \mathit{Reach}(\mathcal{T})$

Invariant checking

finite transition system $\mathcal{T}$

invariant E with invariant condition Φ

model checker

does $\mathcal{T} \models E$ hold?

yes, $\mathcal{T} \models E$

no, $\mathcal{T} \not\models E$ $\longleftarrow$ error indication

error indication: initial path fragment $s_0\, s_1 \ldots s_{n-1} s_n$ such that $s_i \models \Phi$ for $0 \leq i < n$ and $s_n \not\models \Phi$

DFS-based invariant checking

input: finite transition system $\mathcal{T}$, invariant condition Φ

$U := \varnothing \longleftarrow$ stores the "processed" states

$\pi := \varnothing \longleftarrow$ stack for error indication

```
FOR ALL s_0 ∈ S_0 DO
      IF DFS(s_0, Φ) THEN
            return "no" and reverse(π)
      FI
OD
return "yes"
```

$$\begin{array}{|c|}\hline s_n \\ \hline \vdots \\ \hline s_1 \\ \hline s_0 \\ \hline \end{array} \quad s_n = t$$

$DFS(s_0, \Phi)$ returns "true" iff depth-first search from state s_0 leads to some state t with $t \not\models \Phi$

Recursive algorithm $DFS(s, \Phi)$

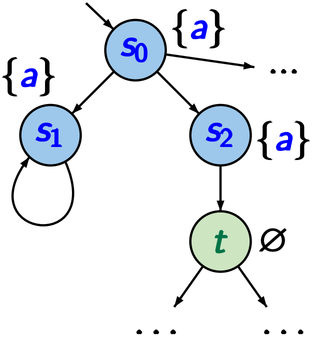
"searches" for a path fragment $s \ldots s' \ldots t$ with $t \not\models \Phi$

$Push(\pi, s)$;
IF $s \notin U$ THEN
    IF $s \not\models \Phi$ THEN return "true" FI
    IF $s \models \Phi$ THEN
        insert s in U;
        FOR ALL $s' \in Post(s)$ DO
            IF $\boxed{DFS(s', \Phi)}$ THEN
                return "true" FI
        OD
    FI
FI
$Pop(\pi)$; return "false"

t	$t \not\models \Phi$
$\vdots$	
s'	$s' \models \Phi$
s	
$\vdots$	
s_0	

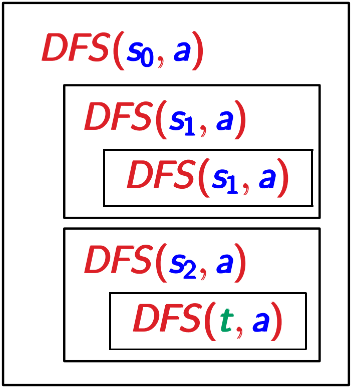
initial state

Example: invariant checking

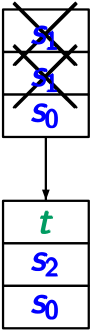


invariant condition a

$$s_0, s_1, s_2 \models a$$
$$t \not\models a$$



stack π



$s_0 \not\models$ "always a" $\longleftarrow$ error indication: $s_0\, s_2\, t$