RUHR-UNIVERSITÄT BOCHUM

Z3 - Tutorial

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Introduction

1. Smartphone company OnePluZ3 is about to launch their new flagship phone
2. You are facing several issues that need to be solved ASAP

Problem 1 - What to produce?

- You can produce 3 different items
  - Phone cases, chargers, and smartphones
- Each take different amounts of resources to produce and generate a different amount of profit
- You have limited labor hours, machine hours and material available

Problem 1 - What to produce?

Resources available:

- 500 labor hours
- 800 machine hours
- 600 units of material

Name	Profit	Labor Hours	Machine Time	Raw Materials
Phone Case	10	3	3	4
Phone Charger	30	5	3	2
Smartphone	50	4	5	6

Problem 1 - Formalization

This can be expressed as a linear programming problem.

$$\max f(x) = 10 * A + 30 * B + 50 * C \tag{1}$$

$$\text{with contraints} \tag{2}$$

$$3 * A + 5 * B + 4 * C <= 500 \tag{3}$$

$$3 * A + 3 * B + 5 * C <= 800 \tag{4}$$

$$4 * A + 2 * B + 6 * C <= 600 \tag{5}$$

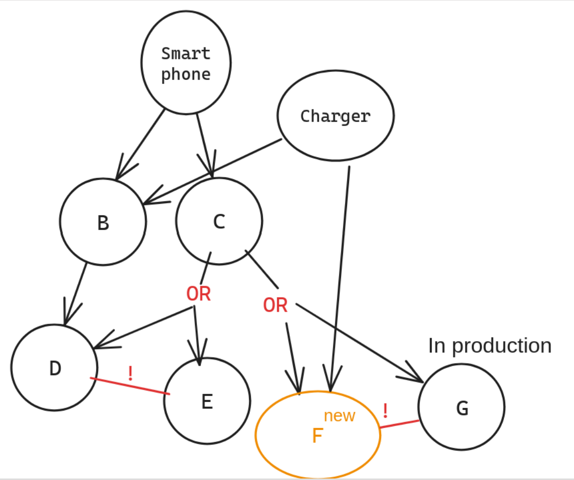
$$A >= 0 \tag{6}$$

$$B >= 0 \tag{7}$$

$$C >= 0 \tag{8}$$

Problem 2 - Dependency Chaos

As part of the production line, you need to manage different parts and chips that are used in different devices.



Problem 2 - Formalization

- Each part is represented by a boolean variable
  - True if in production
  - False if not in production
- A depends on B : $A \implies B$
- A conflicts with B : $\neg A \vee \neg B$

Problem 3 - Code Verification

The day 1 patch is currently in code review. You notice a strange function written by a coworker.

```
// Magic function
uint32_t f(int32_t v) {
    int32_t const mask = v >> 31;
    uint32_t r = (v + mask) ^ mask;
    return r;
}
```

Z3

- SMT solver
- open-source
- from Microsoft Research

Z3 - Usage

4 Step pattern:

1. Create Solver
2. Define variables
3. Add constraints
4. Check

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```python
s = Solver()
x = Real("x")

s.add(3*x + 6 == 0)

if s.check() == sat:
        m = s.model()
        x = m.evaluate(x)

        print(f"{x=}") # x=-2
else:
        print(s.check())
```

Variables

```
# Bool
x = Bool('a')

a = BoolVal(True)
b = BoolVal(False)

# Integers
x = Int('x')
y = Int('y')

a = IntVal(5)
```

```
# Real numbers
x = Real('x')
y,z = Reals("y z")

a = RealVal(3.141)
b = Q(1,3) # 1/3

# Bit-vector
x = BitVec('x', 16) # 16 Bits
y = BitVec('y', 32) # 32 Bits

a = BitVecVal(16, 32) # 32 Bits
```

Arrays

Arrays in Z3 map from one datatype to another. They support Store and Select operations.

```
# A is an array mapping from integer to integer
A = Array("A", IntSort(), IntSort())
```

Custom Datatypes - Enum

```python
Color, (red, green, blue) = EnumSort('Color',
          ('red', 'green', 'blue'))


print (simplify(green == blue)) # False

c = Const('c', Color)
solve(c != green, c != blue) # [c = red]
```

Custom Datatypes - List

```
def DeclareList(sort):
    List = Datatype('List_of_%s' % sort.name())
    List.declare('cons', ('head', sort), ('tail', List))
    List.declare('nil')
    return List.create()

IntList     = DeclareList(IntSort())
RealList    = DeclareList(RealSort())
IntListList = DeclareList(IntList)

l1 = IntList.cons(10, IntList.nil)
l2 = RealList.cons("1/3", RealList.nil)
```

Conclusion

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- Z3 is versatile and can be applied to a lot of problems, such as
  - Software Verification / Static analysis
  - Optimization problems
  - Mathematical proofs
  - ...
- Z3 uses formal methods like first-order logic and SMT to guarantee the correctness and consistency of solutions.

References

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