# EE3-25: Deep Learning

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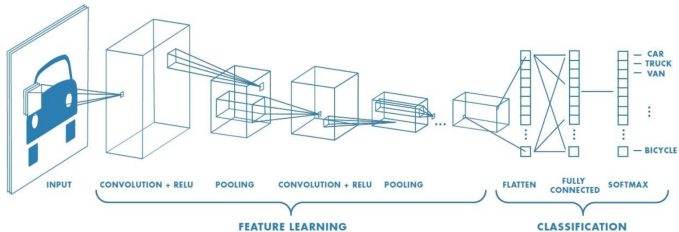
# Convolutional Neural Networks

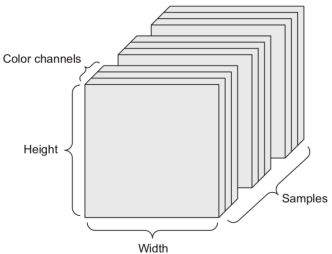
Type of neural network with a special architecture

- Convolution
- Filters, strides, padding
- Pooling
- FC layer
- Loss layer
- Practicals

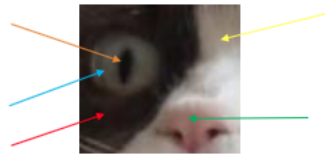


# CNN Input Data

- Neural network architecture optimised for k-D data e.g. images
  - RGB, hyperspectral, etc.
- Images are represented in 4D tensors: (samples, height, width, channels)
- Neighbouring variables are locally correlated

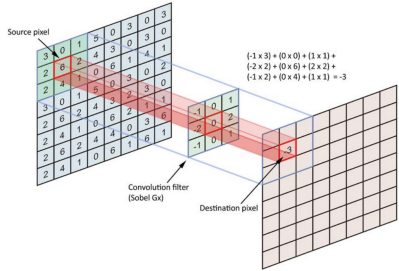




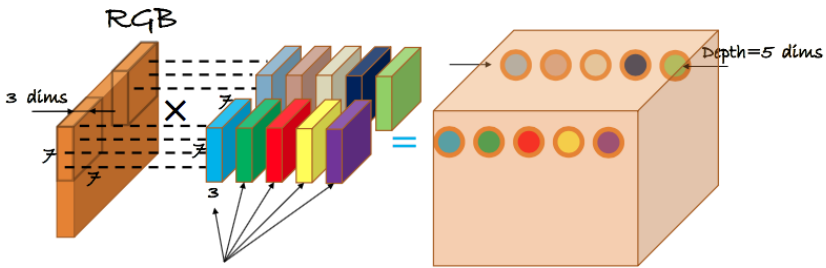


# CNN Neurons (filters, kernels, tensors)

- Spatially large input data
  - Many neurons distributed spatialy
  - Many weights per neurons



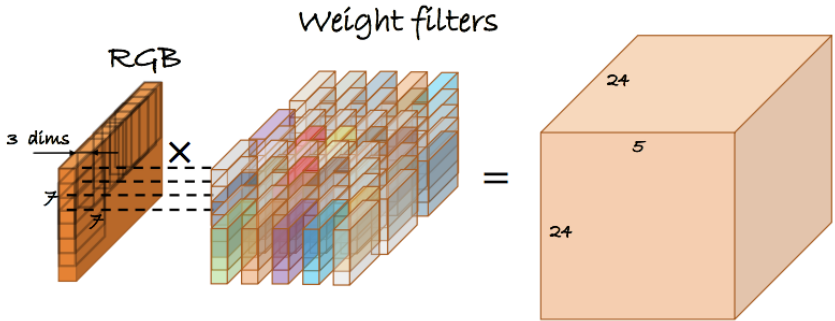

The activations of a hidden layer form a volume of neurons, not a 1-d "chain"
This volume has a depth 5, as we have 5 filters




How many weights for these 5 neurons?

# CNN Neurons

- Covering whole 2D input with different filters




Assume the image is 30x30x3.
1 filter every pixel (stride =1)
How many parameters in total?

24 filters along the $x$ axis
24 filters along the $y$ axis
Depth of 5
$\times$ $7 * 7 * 3$ parameters per filter

$423K$ parameters in total

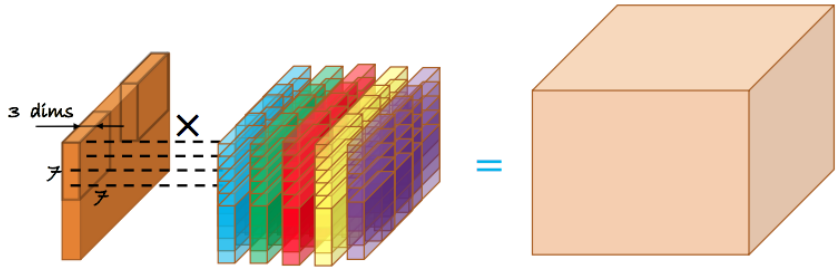
# CNN Neurons

- Many parameters to train
  - With an image of only $30 \times 30$ pixels and a single hidden layer of depth $5$ we need $85k$ parameters
  - With a $256 \times 256$ image we need $46 \times 10^6$ parameters
- Problems
  - How to fit a model with that many parameters ?
  - Where to find the data for training such a model ?

Are all these weights necessary?

# CNN shared filters

- If the data repeats, why not repeat filters?
  - Sliding window (filter)



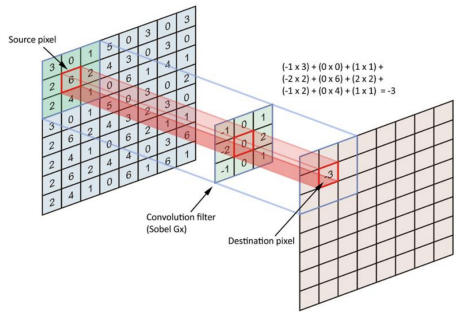

Assume the image is 30x30x3.
1 column of filters common across the image.
How many parameters in total?

Depth of 5
× 7 ∗ 7 ∗ 3 parameters per filter

735 parameters in total

# Convolution

- Convolution operator in continuous space
  $(f * w)(t) = \int_{-\inf}^{+\inf} f(\tau) w(t - \tau) d\tau$
  - a weighted average of the input $f$ according to the weighting (filter, kernel, tensor) $w$ at each point in time $t$ or space
- Discrete convolution
  $(f * w)(t) = \sum_{-\inf}^{+\inf} f(\tau) w(t - \tau)$
- In practice both signal and kernel are finite
  1D $(\mathbf{x} * F)(i) = \sum_m \mathbf{x}(m) F(i - m)$
  2D $(\mathbf{x} * F)(i, i) = \sum_m \sum_n \mathbf{x}(m, n) F(i - m, j - n)$
  or $\sum_m \sum_n \mathbf{x}(i + m, j + n) F(m, n)$




| | | |
|---|---|---|
| 16 * 0 | 24*-1 | 32 |
| 47*1 | 18*0 | 26 |
| 68 | 12 | 9 |

| | |
|---|---|
| 23 | 0 |
| 0 | 0 |

| | | |
|---|---|---|
| 16 | 24*0 | 32*-1 |
| 47 | 18*1 | 26*0 |
| 68 | 12 | 9 |

| | |
|---|---|
| 23 | -14 |
| 50 | 0 |

| | | |
|---|---|---|
| 16 | 24 | 32 |
| 47*0 | 18*-1 | 26 |
| 68*1 | 12*0 | 9 |

| | |
|---|---|
| 23 | 0 |
| 50 | 0 |

| | | |
|---|---|---|
| 16 | 24 | 32 |
| 47 | 18*0 | 26*-1 |
| 68 | 12*1 | 9*0 |

| | |
|---|---|
| 23 | -14 |
| 50 | -14 |

# Convolutional Layer

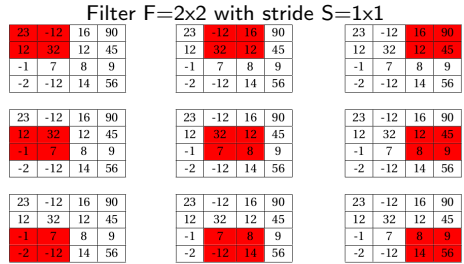
## Convolution

- The size of filter $F$ is its receptive field
- The result of a convolution is a feature map
- A layer can be formed from several channels (feature maps)
  - Typically, there are many filters and feature maps for the same input
  - RGB image has 3 channels
- Filters (kernels) are therefore multi dimensional tensors (3 for input image)
- The convolution is applied across all channels in the input layer

  $\sum_m \sum_n \sum_c \mathbf{x}(i+m, j+n, c) F(m, n, c)$
- The convolution is usually combined with a shared bias (one per channel) and input into an activation function

# Strides

- Step of the shift when applying the filter (kernel, tensor)
- Stride 1x1 shifts by 1 pixel at a time
  - Most frequently used
  - Patches are heavily overlapping
- Stride 2x2 skips one patch horizontally and vertically
  - Convolution with downsampling



# Padding

- VALID - no padding
- SAME - adds $P = F - 1$ zeros to the input to constrain the output size to be the same for unit strides, $H_o = H_i$.
  - For odd-sized kernels, add $P = \lfloor F/2 \rfloor$ zeros on both sides of the input
- FULL - adds $P = 2(F - 1)$ zeros to the input ($F - 1$ zeros on both sides), resulting in an output height of $H_i + 2(F - 1)$

X

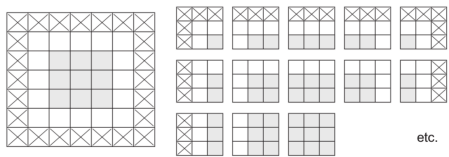
| | | | |
|---|---|---|---|
| -12 | 16 | 90 | 34 |
| 32 | 12 | 45 | -8 |
| 7 | 8 | 9 | 12 |
| -12 | 14 | 56 | 18 |

X with 1-padding of zeros

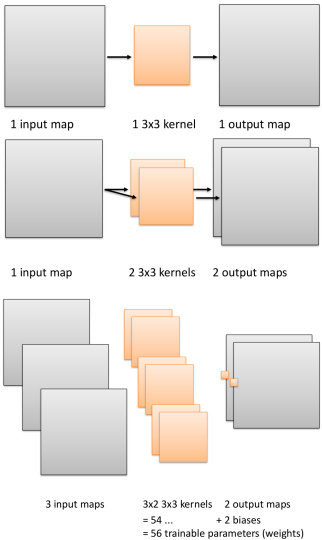
| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | -12 | 16 | 90 | 34 | 0 |
| 0 | 32 | 12 | 45 | -8 | 0 |
| 0 | 7 | 8 | 9 | 12 | 0 |
| 0 | -12 | 14 | 56 | 18 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |

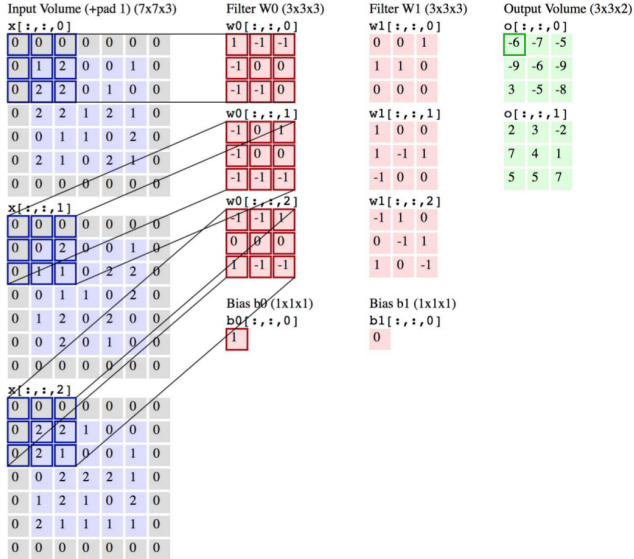
X with 2-padding of zeros

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | -12 | 16 | 90 | 34 | 0 | 0 |
| 0 | 0 | 32 | 12 | 45 | -8 | 0 | 0 |
| 0 | 0 | 7 | 8 | 9 | 12 | 0 | 0 |
| 0 | 0 | -12 | 14 | 56 | 18 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |



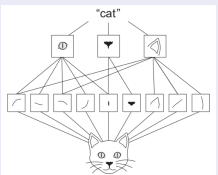
# Convolutional tensors and kernels (filters)

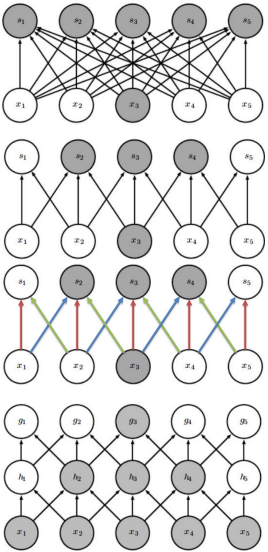


## CNN properties

- Sparse connectivity, neurons act locally
- Shared weights, reduced complexity
- Translational invariance (not rotation)
- Suitable for locally correlated neighbours (patterns)
- Enables some flexibility in input size
- Hierarchical structure of pattern detectors



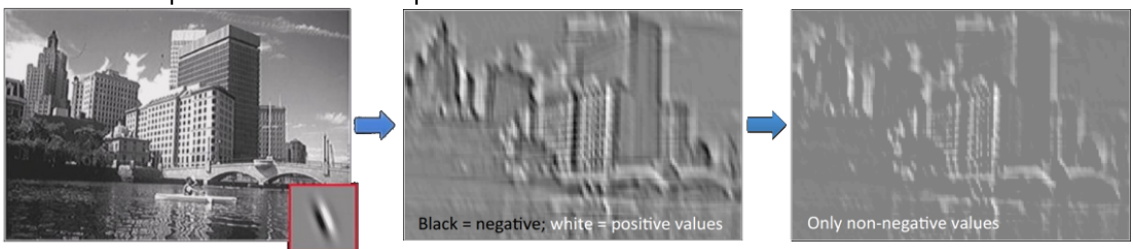



## CNN layer parameters

- Input: a volume of size height x width x depth $H_i \times W_i \times D_i$
- Parameters:
  - Number of filters $K$,
  - Their spatial extent $F$,
  - Stride $S$,
  - zero padding $P$
- Output: a volume of size height x width x depth $H_o \times W_o \times D_o$
  - $W_o = (W_i - F + 2P)/S + 1$
  - $H_o = (H_i - F + 2P)/S + 1$
  - $D_o = K$
- With parameter sharing, there are $F \cdot F \cdot D_i$ weights per filter, i.e. $F \cdot F \cdot D_i \cdot K$ weights and $K$ biases
- Common settings $F \in \{2, \ldots, 15\}$ , $S = 2$, $P = 0$,

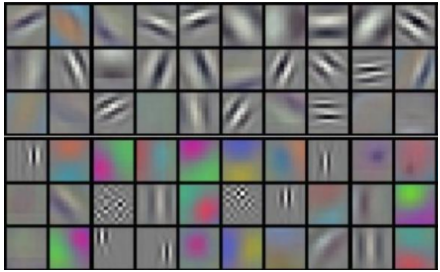
# CNN layer example

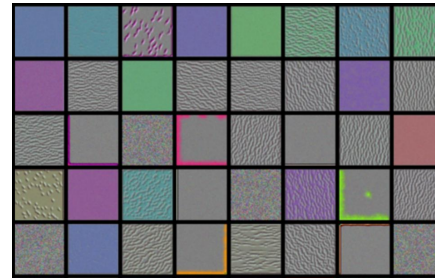
- Filter output and ReLU example



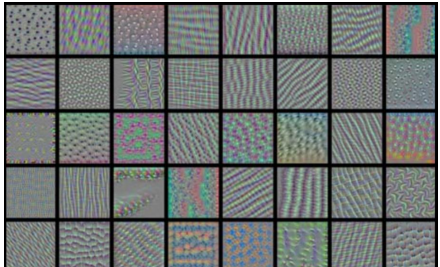
## Convolutional Filters/kernels

Shallow (initial) layers - low level patterns

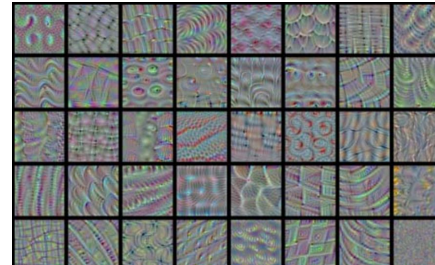




Intermediate layers - more complex patterns

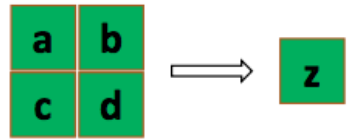


Deep layers - high level patterns



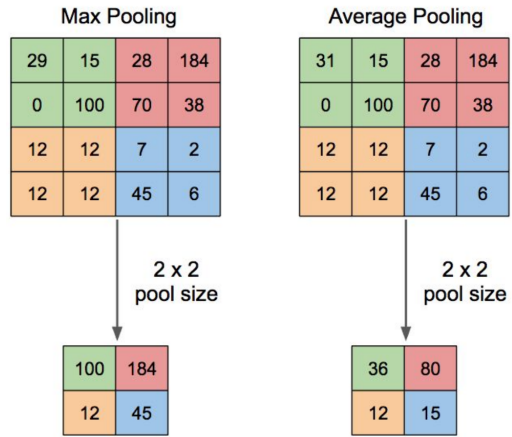
## Pooling layer

- A function that aggregates multiple inputs into a single value
- A summary statistic over regions of the input space identified by a sliding window
  - Non-linear downsampling on activation maps
- Similar to the convolution but the linear transformation is replaced with a pooling operation
- Reduces the size of the layer output (dimensionality)
  - Keeps only the most important information for the next layer
  - Robust against small shifts
  - Faster computations
- Pooling discards information



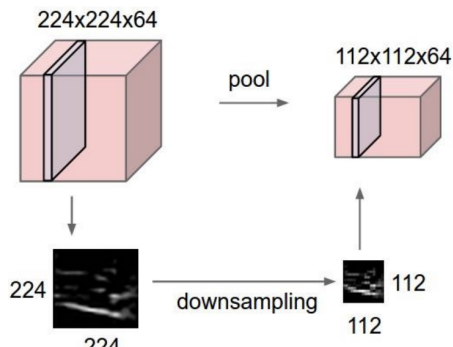
## Pooling methods

- Max pooling
  - Most common in 2D
  - Filters of size 2x2 applied with a stride of 2, takes a max over 4 numbers, discards 75% of the activations.
  - Downsamples at every depth slice (feature map) by 2 both, width & height
  - The depth dimension remains unchanged
  - Has been shown to work well in practice
- Average pooling and L2-norm pooling have also been used
- The gradient is only propagated through the input pixel that contributes to the output value



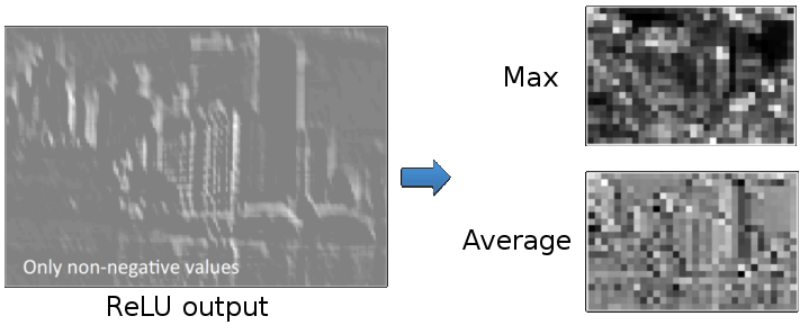
## Pooling parameters

- Input: a volume of size height x width x depth $H_i \times W_i \times D_i$
- Parameters: Spatial extent of the filter $F$, stride $S$, input padding $P$
- Output: a volume of size height x width x depth $H_o \times W_o \times D_o$
  - $W_o = (W_i - F)/S + 1$
  - $H_o = (H_i - F)/S + 1$
  - $D_o = D_i$
- Common settings $F \in \{2, \ldots, 4\}$ , $S = 2$, $P = 0$
- Typically no zero-padding in pooling layers
- Useful for handling inputs of varying size
- No new learnable parameters



# Pooling examples

- Max Pooling
- Average Pooling



# Fully Connected layer

- Basic neural network (MLP)
- Global view on all features
- Used as the final learning phase, which maps extracted features to desired outputs
- Usually adaptive to classification/encoding tasks, with typical sequence

  Convolution → Pooling → Flattening → Full connection → Sigmoid/Softmax/Loss
  - Flattening reshapes feature maps (2D arrays) from a conv. layer to form an input into an FC layer

- Common output is a vector, which is then passed through softmax to represent confidence of classification

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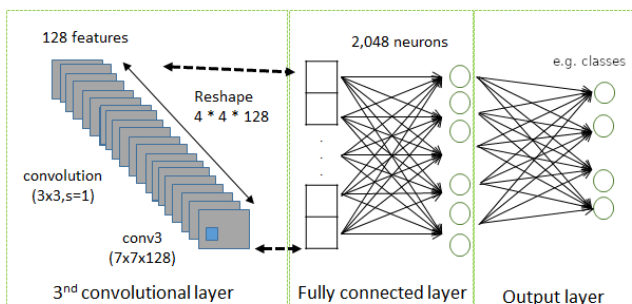
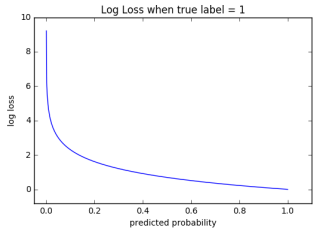


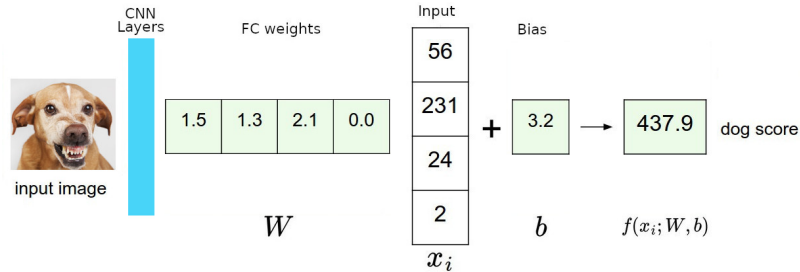
# Loss layer

- Needed for training the network
- Inputs: Network predictions and ground truth (samples from target function)
- Output: Prediction error (scalar)
- Loss function: depends on the task and data
- Classification
  - Softmax (sigmoid) activation function followed by the cross-entropy loss
- Regression
  - L2, L1, Smooth L1

# Loss layer - Binary Classification

- For ground truth label $y$ and network prediction $\hat{y}$
  - with $y \in \{0, 1\}$ $\quad \hat{y} = \frac{e^{W_k x_k + b}}{e^{W_k x_k + b} + 1}$ (sigmoid)
  - Can be interpreted as probability
- Cross-entropy loss function $\ell = -y \log(\hat{y}) + (1 - y) \log(1 - \hat{y})$



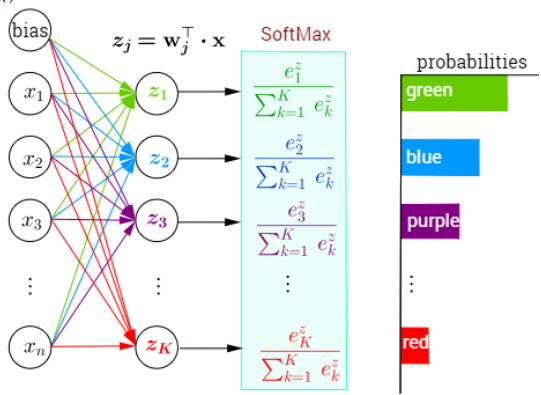

- Loss $\hat{y} = \frac{e^{437.9}}{e^{437.9} + 1} \approx \hat{1}, \quad \ell = -1 \log(\hat{1}) + (1 - 1) \log(1 - \hat{1}) = 0$
  - note that $\hat{y} \in (0, 1)$ is never $0$ or $1$ probability

# Loss layer - Multiclass Classification

- For multi label ground truth $y$ and prediction $\hat{y} \in \mathbb{R}$
- Softmax $\hat{y}_k = \frac{e^{W_k \mathbf{x}_k + \mathbf{b}}}{\sum_i e^{W_i \mathbf{x}_i + \mathbf{b}}}$ and $y_k \in \{0, 1\}$
- Categorical (multiclass) cross entropy error $\ell = -y_k \log(\hat{y}_k)$

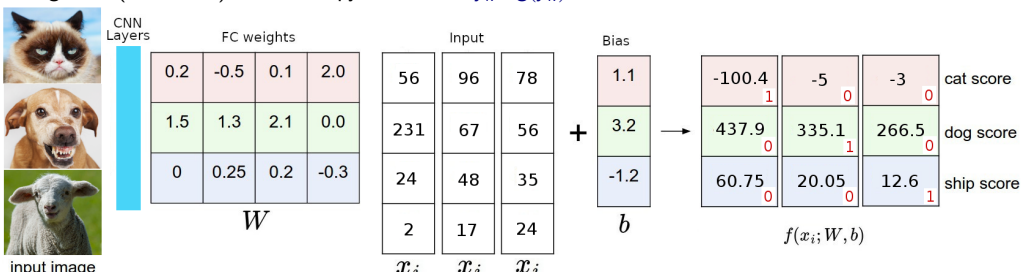
- Better than classification error or MSE
  - Classification Error: ignores confidence
  - Mean Squared Error: too much emphasis on incorrect outputs

$$\mathbf{z} = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ \vdots \\ z_K \end{bmatrix} = \begin{bmatrix} \mathbf{w}_1^\top \\ \mathbf{w}_2^\top \\ \mathbf{w}_3^\top \\ \vdots \\ \mathbf{w}_K^\top \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix}$$



# Loss layer - Multiclass Classification

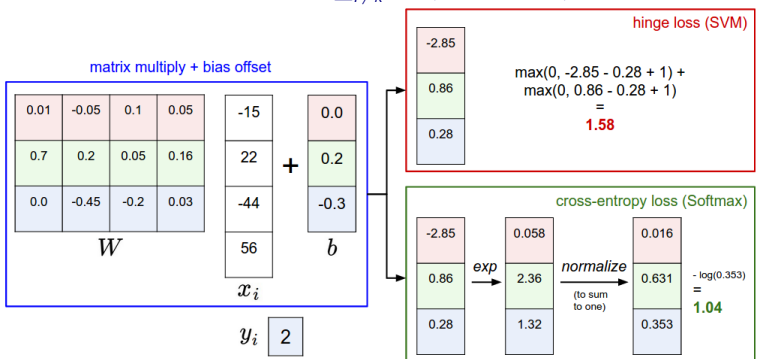
- For multi label ground truth $y$ and prediction $\hat{y} \in \mathbb{R}$
- Softmax $\hat{y}_k = \frac{e^{W_k \mathbf{x}_k + \mathbf{b}}}{\sum_i e^{W_i \mathbf{x}_i + \mathbf{b}}}$
- Categorical (multiclass) cross entropy error $\ell = -y_k \log(\hat{y}_k)$




- cat score $\hat{y}_{1,1} = \frac{e^{-100.4}}{\sum_i e^{-100.4} + e^{437.9} + e^{60.75}}$
- Average Cross Entropy (ACE) loss $\mathcal{L} = \frac{1}{3}[-(1 \log(\hat{y}_{1,1}) + 1 \log(\hat{y}_{2,2}) + 1 \log(\hat{y}_{3,3}))]$

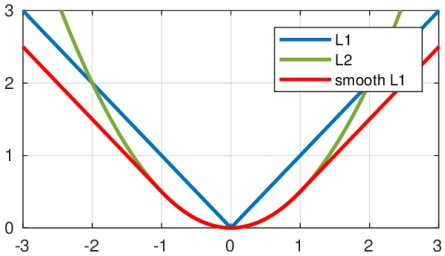
# Loss layer - Multiclass Classification

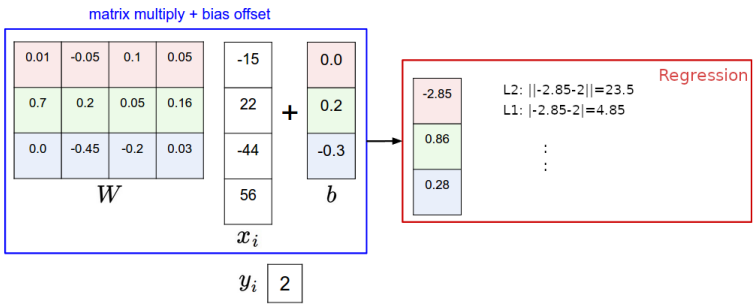
- For multi label ground truth $y \in \mathbb{N}$ and prediction $\hat{y} \in \mathbb{R}$
- Softmax $\hat{y}_k = \frac{e^{W_k \mathbf{x}_k + \mathbf{b}}}{\sum_i e^{W_i \mathbf{x}_i + \mathbf{b}}}$, with cross entropy error $\ell = -y_k \log(\hat{y}_k)$
- Hinge loss for one data sample from class $k$, $\ell = \sum_{i \neq k} \max(0, \hat{y}_i - \hat{y}_k + 1)$



# Loss layer - Regression

- For ground truth value $y \in \mathbb{R}$ and prediction $\hat{y} \in \mathbb{R}$
- L2 Loss: $\ell_{L2} = \|\mathbf{wx} + b - y\| = (\mathbf{wx} + b - y)^2$
- Mean Squared Error: $MSE = \frac{1}{N}\ell_{L2}$
- L1 Loss: $\ell_{L1} = |\mathbf{wx} + b - y|$
- Mean Absolute Error: $MAE = \frac{1}{N}\ell_{L1}$
- Smooth L1 Loss (Huber): $\ell_{L1} - 0.5 \;\; \forall \ell_{L1} \geqslant 0.5$ otherwise $0.5\ell_{L2}$
  - less sensitive to outliers than $L2$



## Practical hints

- Number of filters
  - The number of feature maps directly controls network capacity
  - Depends on the number of available examples and the complexity of the task
  - Computing activations is more expensive than for MLP (convolution operation) even though the neuron is mach smaller than in MLP
  - Initial layers (large inputs) will have less filters then later layers (small but more feature maps),
  - The number of filters can be set to equalise the number of computations and to preserve information at each layer e.g. number of activations.
- Filter shape (size)
  - Depends on the type of data
  - Set to create abstractions at the proper scale
  - Can be larger in first layers eg. 11x11, smaller later 5x5.

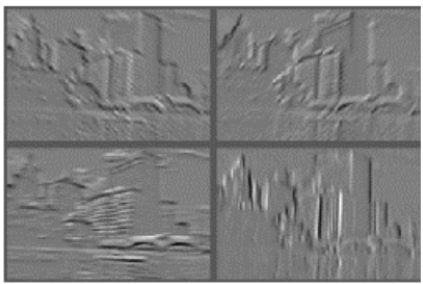
## Practical hints

- Pooling
  - Typical 2x2 or no max-pooling
  - The trend is to use smaller filters, deeper architectures and abandon pooling
  - Large input data may allow 4x4 but this may be discarding too much information.
- Batch normalisation of each feature map
  - e.g. input data in a 2D ConvNet has shape $[N, H, W, C]$, with
    - $N$ is the number of examples in the minibatch
    - $H \times W$ are image height and width
    - $C$ is the number of channels
  - Standard BN would compute $H \times W \times C$ means and stddevs to normalise each feature separately at each spatial location
  - BN in ConvNets instead computes $C$ means and stddevs and normalises jointly for all locations (feature map)
    - to respect the structure (spatial patterns)

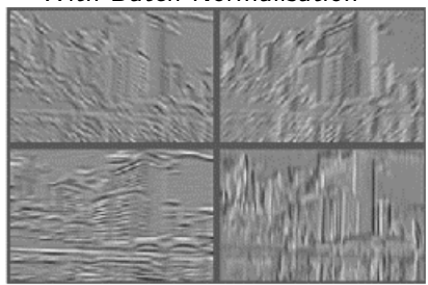
## Practical hints

- Batch normalisation example
  - Per feature map (output of one filter)
  - Zero-mean, one standard deviation
  - Enhanced details with BN

Without Batch Normalisation



With Batch Normalisation



- Loss
  - Classification - Softmax
  - Regression - L2, Smooth L1

# CNN architecture

- Typical sequence of layers in shallow networks
  - $INPUT \rightarrow [CONV \rightarrow Norm \rightarrow RELU \rightarrow POOL]^{(\times 2)} \rightarrow FC \rightarrow RELU \rightarrow FC \rightarrow OUTPUT$
  - $INPUT \rightarrow [CONV \rightarrow RELU \rightarrow CONV \rightarrow RELU \rightarrow POOL]^{(\times 3)} \rightarrow [FC \rightarrow RELU]^{(\times 2)} \rightarrow FC \rightarrow OUTPUT$
- Code example

```
def makeModel(nb_filters):
    model = Sequential()
    model.add(Conv2D(nb_filters, kernel_size, input_shape=(patchsize,patchsize,3), padding = "same"))
    model.add(Activation('relu'))
    model.add(MaxPooling2D(pool_size = pool_size))
    model.add(Conv2D(nb_filters*2, kernel_size, padding = "same"))
    model.add(Activation('relu'))
    model.add(MaxPooling2D(pool_size = pool_size))
    model.add(Conv2D(nb_filters*4, kernel_size, padding = "same"))
    model.add(Activation('relu'))
    model.add(MaxPooling2D(pool_size = pool_size))
    model.add(AveragePooling2D(pool_size = pool_size))
    model.add(Flatten())
    model.add(Dense(128)) # generate a fully connected layer wiht 128 outputs (arbitrary value)
    model.add(Activation('relu'))
    model.add(Dropout(0.5))
    model.add(Dense(3)) # output layer
    model.add(Activation('softmax'))
```

# CNN Summary

- Convolution
- Filters, strides, padding
- Pooling
- FC layer
- Loss layer
- Practicals