Framework for Stochastic Amplitude Estimation

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1 Overview

Aim: estimate θ from samples such that

$$P(X = 1|\theta) = \frac{1}{2}(1 - \cos((4m + 2)\theta)).$$

Cost:

- Number of oracle calls
- Number of shots

Action:

- Stop
- Select m for the next shot

Bellman equation

$$V(s) = \min_{a \in \{m, \text{stop}\}} \mathbb{E}_a \left[V(S) + c(a)\right]$$

The future cost is given by:

$$c(m) = \begin{cases} 2m + 1, & \text{Number of oracle calls} \\ 1, & \text{Number of shots} \end{cases}$$

and for $s = \text{stop}$

$$c(\text{stop}) = \begin{cases} 0, & \|\theta - \tilde{\theta}\| < \varepsilon \\ \infty, & \text{else} \end{cases}$$

Bayesian Framework: Prior $\rightarrow$ posterior: What state could be:

1. Full posterior distribution for θ: $\pi(\theta) \rightarrow \tilde{\pi}(\theta)$

$$\tilde{\pi)}\theta \propto \pi(\theta)P(X = x|\theta).$$

Compact representation: Fourier series or Bessel series

2. Approximation 1: Particle filter - approximate by discrete / point distribution

3. Approximation 2: use moments & moment match to given distribution. Force posterior into Normal or von Mises. Use a von Mise or Normal Prior and get a formula for posterior mean & variance.

Value function: We know how it scales $\varepsilon_1 \to \varepsilon$, linear in the ratio $O(\frac{1}{\varepsilon})$ & quadratic in the ratio $O(\frac{1}{\varepsilon^2})$.

Some frameworks that already touch on this problem:

- Sequential estimation - decision or don't stop
- "Design of experiment" - online decision matrices.