Exqaliber Stage 1 Report

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TBD

1 Introduction

Basic introduction giving definition and contribution

1.1 Amplitude estimation

Details of the amplitude estimation and background on its use, including Quantum Monte Carlo

1.1.1 Statistical amplitude estimation

Introduce idea of statistical learning of parameter to reduce circuit length

1.1.2 Decoherent noise model

Introduce model for decoherent noise and associated effects on statistical problem

1.2 Related work

Review of related work and papers

1.2.1 Quantum phase estimation

I am not sure if we want to make the connection in a public document but it is useful to make in an internal report

1.3 Contribution

This should be written last and include a map for the rest of the paper

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2 Algorithm

2.1 Algorithm description

Give a statement of our algorithm, phrase as an algorithm rather than talking about Bayes updates etc. What is implemented

State is current estimate and variance Rules for updating state given result Rules for selecting next update

Include detail of stopping criteria

2.1.1 Adjustments for noise

Changes needed for decoherence noise

2.2 Bayesian framework

Introduction to Bayesian stats, prior, posterior, and adaptive termination

Introduce idea of using normal distribution

2.2.1 Posterior properties

Posterior mean, variance and expected variance under noiseless and decoherence noise model

2.2.2 Normal approximation of posterior

Details on the use of a normal approximation for posterior distribution and next prior. Include comments about both the circular issue and tails due to indeterminacy.

2.3 Selecting next sample depth

Details of greedy choice (possibly give indication of dynamic programming approach) Noise and noiseless

3 Experimentation

In amplitude estimation, there is an operator $\mathcal{A}$ acting on $n+1$ qubits, such that $\mathcal{A}|\psi\rangle_{n+1} = \sqrt{a}|\psi_1\rangle_n|1\rangle + \sqrt{1-a}|\psi_0\rangle_n|0\rangle$, with $a \in [0,1]$ or, when defining $a = \sin^2\theta_a$ with $\theta_a \in [0,\pi]$,

$$\mathcal{A}_{n+1}|\psi\rangle = \sin\theta_a|\psi_1\rangle_n|1\rangle + \cos\theta_a|\psi_0\rangle_n|0\rangle. \tag{1}$$

The goal is to find θ_a.

One can create an operator $\mathcal{Q} = \mathcal{A}\mathcal{S}_0\mathcal{A}^\dagger\mathcal{S}_{\psi_0}$ with $\mathcal{S}_{\psi_0} = \mathbb{I} - 2|\psi_0\rangle_n\langle\psi_0|_n \otimes |0\rangle\langle 0|$ and $\mathcal{S}_0 = \mathbb{I} - 2|0\rangle_{n+1}\langle 0|_{n+1}$. Now by applying $\mathcal{Q}$ for k times (and thereby applying $\mathcal{A}$ a total of $(2k+1)$ times), the probability of measuring the $|1\rangle$ state follows a Bernoulli distribution with p equal to

$$p = \frac{1}{2}(1 - \cos((2k+1)\theta_a)). \tag{2}$$

Using this equation, one can test different amplitude estimation analytically, by sampling from the Bernoulli distribution directly. We call this direct analytical sampling.

We experimented with our novel amplitude estimation routine. There are two relevant measures for amplitude estimation. These are the time to solution (and its scaling) and the quality of the solution, i.e., the error ϵ. To measure these, we simulate experiments using direct analytical sampling.

As our algorithm uses dynamical updates, depending on the true θ, anomalies can rise for values around integer fractions of π.

3.1 Simulation experiments

Details on setup both with circuits and with direct sampling

3.1.1 Noiseless experiments

3.1.2 Decoherence noise experiments

Focused on direct sampling case

3.2 Real hardware experiments

Include if we decide to carry these out.

4 Discussion

This is for internal discussions not external release Topics to include here include:

- Directions for future including reinforcement learning
- Superconducting and trapped ion thought experiment
- Discussion about general dynamic programming approach for considering error mitigation and correction
- Connection to QPE and applicability of thinking to that setting (maybe include small section on the more complicated statistical challenge of general state)
- Circular distributions and what we know about them (probably subsection of this section or algorithm)

References

[1] gg

A Technical details

Details of technical work if needed