ДЗ по матанализу 6 Смирнов Тимофей 236 ПМИ

ДЗ 6.6 Какие из следующих утверждений справедливы при $x \to 0$:

а). $o(x^2) + o(x) = o(x)$

$o(x^2) = o(x) \;\Rightarrow\;$ получаем $o(x^2) + o(x) = o(x) + o(x) = o(x)$ ЧТД

б). $o(x) + x^2 = o(x)$

$x^2 = o(x) \;\Rightarrow\;$ имеем $o(x) + x^2 = o(x) + o(x) = o(x)$ ЧТД

в). $(x + o(x))(2x^2 + o(x^2)) = 2x^3 + o(x^3)$

$x(1+o(1))x^2(2+o(1)) = x^3(1+o(1))(2+o(1)) = x^3(2+3o(1)+o(1)) = x^3(2+o(1)) = 2x^3+o(x^3)$

г). $o(1) - o(1) = 0$

По свойству о-малого $o(1) - o(1) = o(1)$

Но $o(1) \neq 0$ при $x \to x_0 \;\Rightarrow\;$ мы не можем сказать, что $o(1) - o(1) = o(1)$

ДЗ 6.7

а). $\lim\limits_{x\to 0} \dfrac{\arcsin^2(\operatorname{tg} x)}{(\cos(2\sin 2x) - 1)} = \lim\limits_{x\to 0} \dfrac{\arcsin^2(x + o(x))}{\cos(4x + o(x)) - 1} = \lim\limits_{x\to 0} \dfrac{(x + o(x) + o(x + o(x)))^2}{1 - \frac{(4x+o(x))^2}{2} - 1 + o((4x + o(x))^2)} =$
$= \lim\limits_{x\to 0} \dfrac{(x + o(x)))^2}{1 - \frac{16x^2+o(x^2)}{2} - 1 + o(x^2)} = \lim\limits_{x\to 0} \dfrac{x^2 + o(x^2)}{-8x^2 + o(x^2)} = \lim\limits_{x\to 0} \dfrac{1 + o(1)}{-8 + o(1)} = -\dfrac{1}{8}$

б). $\lim\limits_{x\to 1} \dfrac{(1 - \cos(x-1))(4x^2 - 4)}{\ln(x)(x^{\frac{2}{5}} - 1)^2} = \lim\limits_{y\to 0} \dfrac{(1 - \cos(y))(4(y+1)^2 - 4)}{\ln(y+1)((y+1)^{\frac{2}{5}} - 1)^2} =$
$= \lim\limits_{y\to 0} \dfrac{(\frac{y^2}{2} + o(y^2))(4y^2 + 8y)}{(y + o(y))(1 + \frac{2}{5}y + o(y) - 1)^2} = \lim\limits_{y\to 0} \dfrac{2y^4 + 4y^3 + o(y^4) + o(y^3)}{\frac{4}{25}y^3 + o(y^3)} = \lim\limits_{y\to 0} \dfrac{o(y^3) + 4y^3 + o(y^3) + o(y^3)}{\frac{4}{25}y^3 + o(y^3)} =$
$\lim\limits_{y\to 0} \dfrac{4y^3 + o(y^3)}{\frac{4}{25}y^3 + o(y^3)} = \lim\limits_{y\to 0} \dfrac{4 + o(1)}{\frac{4}{25} + o(1)} = 4$

ДЗ 6.8

а). $\lim\limits_{x\to 0} \dfrac{\cos x - \cos 3x}{x^2} = \lim\limits_{x\to 0} \dfrac{1 - \frac{x^2}{2} + o(x^2) - (1 - \frac{9x^2}{2} + o(x^2))}{x^2} = \lim\limits_{x\to 0} \dfrac{4x^2 + o(x^2)}{x^2} = \lim\limits_{x\to 0} \dfrac{4 + o(1)}{1} =$
$\lim\limits_{x\to 0} 4 + o(1) = 4$

б). $\lim\limits_{x\to 0} \dfrac{\cos(a + 2x) - 2\cos(a + x) + \cos a}{x^2} =$
$= \lim\limits_{x\to 0} \dfrac{\cos a \cos 2x - \sin a \sin 2x - 2(\cos a \cos x - \sin a \sin x) + \cos a}{x^2} =$

$$= \lim_{x\to 0} \frac{\cos a(\cos 2x - 2\cos x + 1) + \sin a(2\sin x - \sin 2x)}{x^2} =$$
$$= \lim_{x\to 0} \frac{\cos a(1 - 2x^2 + o(x^2) - 2(1 - \frac{x^2}{2} + o(x^2)) + 1) + \sin a \sin x(2 - 2\cos x)}{x^2}$$
$$= \lim_{x\to 0} \frac{\cos a(-x^2 + o(x^2)) + \sin x(x + o(x))(2 - 2(1 - \frac{x^2}{2} + o(x^2)))}{x^2} =$$
$$= \lim_{x\to 0} \frac{-\cos a(x^2 + o(x^2)) + \sin a(x^3 + o(x^3))}{x^2} = \lim_{x\to 0} \frac{-\cos a(x^2 + o(x^2)) + \sin a(o(x^2))}{x^2} =$$
$$= \lim_{x\to 0} (-\cos a + \sin a o(1)) = -\cos a$$

в). $$\lim_{x\to\infty}(\sqrt[3]{x^3 + 3x^2} - \sqrt{x^2 - 2x}) = \lim_{x\to\infty}\left(x\sqrt[3]{1 + \frac{3}{x}} - x\sqrt{1 + \frac{2}{x}}\right) =$$
$$= \lim_{x\to\infty}\left(x(1 + \frac{1}{x} + o(\frac{1}{x})) - x(1 + \frac{1}{x} + o(\frac{1}{x}))\right) = \lim_{x\to\infty} x \cdot o(\frac{1}{x}) = \lim_{y\to 0} \frac{o(y)}{y} = 0$$

г). $$\lim_{x\to a} \frac{a^x - x^a}{x - a} = \lim_{y\to 0} \frac{a^{y+a} - (y+a)^a}{y} = \lim_{y\to 0} \frac{a^a \cdot a^y - a^a(\frac{y}{a} + 1)^a}{y} = \lim_{y\to 0} \frac{a^a(a^y - 1 - y + o(y))}{y} =$$
$$= \lim_{y\to 0} \frac{a^a(y\ln a - y + o(y))}{y} = \lim_{y\to 0} a^a(\ln a - 1 + o(1)) = a^a \ln a - a^a$$

д). $$\lim_{x\to a} \frac{\ln x - \ln a}{x - a} = \lim_{y\to 0} \frac{\ln(y+a) - \ln a}{y} \lim_{y\to 0} \frac{\ln\left(1 + \frac{y}{a}\right)}{y} = \lim_{y\to 0} \frac{\frac{y}{a} + o(y)}{y} = \lim_{y\to o}\left(\frac{1}{a} + o(1)\right) = \frac{1}{a}$$

е). $$\lim_{x\to 0} \frac{\ln(x^2 + e^x)}{\ln(x^4 + e^{2x})} = \lim_{x\to 0} \frac{\ln(x^2 + 1 + x + o(x))}{\ln(x^4 + 1 + 2x + o(x))} = \lim_{x\to 0} \frac{x^2 + x + o(x) + o(x^2 + x + o(x))}{x^4 + 2x + o(x) + o(x^4 + 2x + o(x))} =$$
$$= \lim_{x\to 0} \frac{x^2 + x + o(x)}{x^4 + 2x + o(x)} = \lim_{x\to 0} \frac{x + o(x)}{2x + o(x)} = \frac{1}{2}$$

ж). $$\lim_{x\to 0}(1 + \mathrm{tg}^2\, x)^{\frac{1}{\ln\cos x}} = \lim_{x\to 0} e^{\frac{\ln(1+\mathrm{tg}^2 x)}{\ln\cos x}} = \lim_{x\to 0} e^{\frac{\ln(1+(x+o(x))^2)}{\ln(1-\frac{x^2}{2}+o(x^2))}} = \lim_{x\to 0} e^{\frac{(x+o(x))^2+o((x+o(x))^2)}{(-\frac{x^2}{2}+o(x^2))+o(-\frac{x^2}{2}+o(x^2))}} =$$
$$= \lim_{x\to 0} e^{\frac{x^2+o(x^2)}{-\frac{x^2}{2}+o(x^2)}} = \lim_{x\to 0} e^{\frac{1+o(1)}{-\frac{1}{2}}+o(1)} = e^{-2}$$

з). $$\lim_{x\to 1}(x^2 + \sin^2(\pi x))^{\frac{1}{\ln x}} = \lim_{y\to 0} e^{\frac{\ln((y+1)^2+\sin^2(\pi y+\pi))}{\ln(y+1)}} = \lim_{y\to 0} e^{\frac{\ln(y^2+2y+1+\sin^2(\pi y))}{y+o(y)}} = \lim_{y\to 0} e^{\frac{\ln(y^2+2y+1+(\pi y+o(y))^2)}{y+o(y)}} =$$
$$= \lim_{y\to 0} e^{\frac{y^2+2y+(\pi y+o(y))^2+o(y^2+2y+(\pi y+o(y))^2)}{y+o(y)}} = \lim_{y\to 0} e^{\frac{y^2+2y+\pi^2y^2+o(y^2)+o(y)}{y+o(y)}} = \lim_{y\to 0} e^{\frac{o(y+2y+o(y)+o(y)+o(y))}{y+o(y)}} = \lim_{y\to 0} e^{\frac{2y+o(y)}{y+o(2)}} =$$
$$\lim_{y\to 0} e^{\frac{2+o(1)}{1+o(1)}} = e^2$$

и). $$\lim_{x\to\pi}\left(\frac{\cos x}{\cos 3x}\right)^{\frac{1}{(\sqrt{\pi x}-\pi)^2}} = \lim_{y\to 0}\left(\frac{\cos(y+\pi)}{\cos(3y+3\pi)}\right)^{\frac{1}{(\sqrt{\pi y+\pi^2}-\pi)^2}} = \lim_{y\to 0}\left(\frac{\cos y}{\cos 3y}\right)^{\frac{1}{\pi(\sqrt{y+\pi}-\sqrt{\pi})^2}} =$$
$$= \lim_{y\to 0}\left(\frac{\cos y}{\cos 3y}\right)^{\frac{1}{\pi^2(1+\frac{y}{2\pi}o(y)-1)^2}} = \lim_{y\to 0} e^{\frac{\ln(\cos y)-\ln(\cos 3y)}{\pi^2(\frac{y}{2\pi}o(y))^2}} = \lim_{y\to 0} e^{\frac{\ln(1-\frac{y^2}{2}+o(y^2))-\ln(1-\frac{9y^2}{2}+o(y^2))}{\pi^2(\frac{y^2}{4\pi^2}+o(y^2))}} =$$
$$= \lim_{y\to 0} e^{\frac{-\frac{y^2}{2}+o(y^2)+o(\frac{y^2}{2}+o(y^2))-(-\frac{9y^2}{2}+o(y^2)+o(\frac{9y^2}{2}+o(y^2)))}{\frac{y^2}{4}+o(y^2)}} = \lim_{y\to 0} e^{\frac{\frac{8y^2}{2}+o(y^2)}{\frac{y^2}{4}+o(y^2)}} = \lim_{y\to 0} e^{\frac{4+o(1)}{\frac{1}{4}+o(1)}} = e^{16}$$

к). $$\lim_{x\to 1} x^{\mathrm{tg}\left(\frac{\pi x}{2}\right)} = \lim_{y\to 0}(y+1)^{\mathrm{tg}\left(\frac{\pi y}{2}+\frac{\pi}{2}\right)} = \lim_{y\to 0}(y+1)^{-ctg\left(\frac{\pi y}{2}\right)} = \lim_{y\to 0} e^{-\frac{\ln(y+1)}{\mathrm{tg}(\frac{\pi y}{2})}} = \lim_{y\to 0} e^{-\frac{y+o(y)}{\frac{\pi y}{2}+o(y)}} = \lim_{y\to 0} e^{\frac{1+o(1)}{\frac{\pi}{2}+o(1)}} =$$
$$e^{-\frac{2}{\pi}}$$