ДЗ по математическому анализу 9 Смирнов Тимофей 236 ПМИ

Задача 9.12 Вычислите пределы:

а). **Решение:**

$$\lim_{x\to 0} \frac{e^x - 1 - x}{x^2} = ?$$

- При $x \to 0, e^x = 1 + x + \frac{x^2}{2} + o(x^2)$

$$\lim_{x\to 0} \frac{e^x - 1 - x}{x^2} = \lim_{x\to 0} \frac{1 + x + \frac{x^2}{2} + o(x^2) - 1 - x}{x^2} = \lim_{x\to 0} \frac{\frac{x^2}{2} + o(x^2)}{x^2} = \lim_{x\to 0} \frac{\frac{1}{2} + o(1)}{1} = \frac{1}{2}$$

б). **Решение:**

$$\lim_{x\to 0} \frac{\sqrt{\cos x} - \sqrt[4]{e^{-x^2}}}{x^4} = ?$$

- $\cos x = 1 - \frac{x^2}{2} + \frac{x^4}{24} + o(x^4)$

- $e^{-\frac{x^2}{4}} = 1 - \frac{x^2}{4} + \frac{x^4}{32} + o(x^4)$

- $\sqrt{\cos x} = \left(1 - \frac{x^2}{2} + \frac{x^4}{24} + o(x^4)\right)^{\frac{1}{2}} =$

$$= 1 + \frac{\left(-\frac{x^2}{2} + \frac{x^4}{24} + o(x^4)\right)}{2} - \frac{\left(-\frac{x^2}{2} + \frac{x^4}{24} + o(x^4)\right)^2}{8} + o\left(\left(-\frac{x^2}{2} + \frac{x^4}{24} + o(x^4)\right)^2\right) =$$

$$= 1 - \frac{x^2}{4} + \frac{x^4}{48} + o(x^4) - \frac{x^4}{32} + o(x^4) + o(x^4) = 1 - \frac{x^2}{2} + \frac{x^4}{48} - \frac{x^4}{32} + o(x^4)$$

$$\lim_{x\to 0} \frac{e^x - 1 - x}{x^2} = \lim_{x\to 0} \frac{1 - \frac{x^2}{4} + \frac{x^4}{48} - \frac{x^4}{32} + o(x^4) - \left(1 - \frac{x^2}{4} + \frac{x^4}{32} + o(x^4)\right)}{x^4} =$$

$$= \lim_{x\to 0} \frac{\frac{x^4}{48} - \frac{x^4}{16} + o(x^4)}{x^4} = -\frac{1}{24}$$

в). **Решение:**

$$\lim_{x\to 0} \frac{\cos(\sin x) - \sqrt{1 - x^2 + x^4}}{x^4} = ?$$

- $\sin x = x - \frac{x^3}{6} + \frac{x^5}{120} + o(x^5)$

- $\cos\left(x - \frac{x^3}{6} + \frac{x^5}{120} + o(x^5)\right) = 1 - \frac{\left(x - \frac{x^3}{6} + \frac{x^5}{120} + o(x^5)\right)^2}{2} + \frac{\left(x - \frac{x^3}{6} + \frac{x^5}{120} + o(x^5)\right)^4}{24} + o(x^4) =$

$$= 1 - \frac{x^2 - \frac{x^4}{6} - \frac{x^4}{6} + o(x^4)}{2} + \frac{x^4}{24} + o(x^4) = 1 - \frac{x^2}{2} + \frac{x^4}{6} + \frac{x^4}{24} + o(x^4)$$

- $\sqrt{1-x^2+x4} = (1-x^2+x^4)^{\frac{1}{2}} = 1+\dfrac{(-x^2+x^4)}{2}-\dfrac{(-x^2+x^4)^2}{8}+o(x^4) = 1-\dfrac{x^2}{2}+\dfrac{x^4}{2}-\dfrac{x^4}{8}+o(x^4)$

$$\lim_{x\to 0}\frac{\cos(\sin x)-\sqrt{1-x^2+x^4}}{x^4} = \lim_{x\to 0}\frac{1-\frac{x^2}{2}+\frac{x^4}{6}+\frac{x^4}{24}+o(x^4)-\left(1-\frac{x^2}{2}+\frac{x^4}{2}-\frac{x^4}{8}+o(x^4)\right)}{x^4} =$$
$$= \lim_{x\to 0}\frac{1-\frac{x^2}{2}+\frac{x^4}{6}+\frac{x^4}{24}+o(x^4)-1+\frac{x^2}{2}-\frac{x^4}{2}+\frac{x^4}{8}-o(x^4)}{x^4} = \lim_{x\to 0}\frac{-\frac{x^4}{3}+\frac{x^4}{6}+o(x^4)}{x^4} = -\frac{1}{6}$$

г). **Решение:**

$$\lim_{x\to 0}\frac{\ln\left(\frac{\sin x}{x}\right)+e^{\frac{x^2}{6}}-1}{\ln(\cos x)+\sqrt{1+x^2}-1}$$

- $\sin x = x-\dfrac{x^3}{6}+\dfrac{x^5}{120}+o(x^5)$

- $\cos x = 1-\dfrac{x^2}{2}+\dfrac{x^4}{24}+o(x^4)$

- $e^{\frac{x^2}{6}} = 1+\dfrac{x^2}{6}+\dfrac{x^4}{72}+o(x^4)$

- $\ln\left(\dfrac{\sin x}{x}\right) = \ln\left(1-\dfrac{x^2}{6}+\dfrac{x^4}{120}+o(x^4)\right) = -\dfrac{x^2}{6}+\dfrac{x^4}{120}+o(x^4)-\dfrac{\left(-\frac{x^2}{6}+\frac{x^4}{120}+o(x^4)\right)^2}{2}+o(x^4) =$

$= -\dfrac{x^2}{6}+\dfrac{x^4}{120}-\dfrac{x^4}{72}+o(x^4)$

- $\ln(\cos x) = \ln\left(1-\dfrac{x^2}{2}+\dfrac{x^4}{24}+o(x^4)\right) = -\dfrac{x^2}{2}+\dfrac{x^4}{24}+o(x^4)-\dfrac{\left(-\frac{x^2}{2}+\frac{x^4}{24}+o(x^4)\right)^2}{2}+o(x^4) =$

$= -\dfrac{x^2}{2}+\dfrac{x^4}{24}-\dfrac{x^4}{8}+o(x^4)$

- $\sqrt{1+x^2} = (1+x^2)^{\frac{1}{2}} = 1+\dfrac{x^2}{2}-\dfrac{x^4}{8}+o(x^4)$

$$\lim_{x\to 0}\frac{\ln\left(\frac{\sin x}{x}\right)+e^{\frac{x^2}{6}}-1}{\ln(\cos x)+\sqrt{1+x^2}-1} = \lim_{x\to 0}\frac{-\frac{x^2}{6}+\frac{x^4}{120}-\frac{x^4}{72}+o(x^4)+1+\frac{x^2}{6}+\frac{x^4}{72}+o(x^4)-1}{-\frac{x^2}{2}+\frac{x^4}{24}-\frac{x^4}{8}+o(x^4)+1+\frac{x^2}{2}-\frac{x^4}{8}+o(x^4)-1} =$$
$$= \lim_{x\to 0}\frac{\frac{x^4}{120}+o(x^4)}{\frac{x^4}{24}-\frac{x^4}{8}-\frac{x^4}{8}+o(x^4)} = \lim_{x\to 0}\frac{\frac{x^4}{120}+o(x^4)}{-\frac{5x^4}{24}+o(x^4)} = \frac{\frac{1}{120}}{-\frac{5}{24}} = \frac{\frac{3}{360}}{-\frac{75}{360}} = -\frac{3}{75} = -\frac{1}{25}$$

Задание 9.13 Вычислите с помощью формулы Тейлора число e с точностью до 10^{-7}.

Решение:

Выпишем первые 11 членов ряд Тейлора для экспоненты:

$$f(x) = e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120} + \frac{x^6}{720} + \frac{x^7}{5040} + \frac{x^8}{40320} + \frac{x^9}{362880} + \frac{x^{10}}{3628800}$$

Подставим туда 1 вместо x:

$$f(1) = 1+1+\frac{1}{2}+\frac{1}{6}+\frac{1}{24}+\frac{1}{120}+\frac{1}{720}+\frac{1}{5040}+\frac{1}{40320}+\frac{1}{362880}+\frac{1}{3628800} = \frac{9864101}{3628800} = 2.7182818$$