# ДЗ по линейной алгебре 6 Смирнов Тимофей 236 ПМИ

**№ 1** Найдем матрицу, обратную к $\begin{pmatrix} A & B \\ 0 & C \end{pmatrix}$, A, B, C - невырожденные.

Пусть $A^{-1} = \begin{pmatrix} X_1 & X_2 \\ X_3 & X_4 \end{pmatrix}$, тогда $\begin{pmatrix} A & B \\ 0 & C \end{pmatrix} \cdot \begin{pmatrix} X_1 & X_2 \\ X_3 & X_4 \end{pmatrix} = \begin{pmatrix} E & 0 \\ 0 & E \end{pmatrix}$

$\begin{pmatrix} AX_1 + BX_3 & AX_2 + BX_4 \\ CX_3 & CX_4 \end{pmatrix} = \begin{pmatrix} E & 0 \\ 0 & E \end{pmatrix}$

$CX_3 = 0 \;\Rightarrow\; X_3 = 0 \;\Rightarrow\; BX_3 = 0 \;\Rightarrow\; AX_1 = E \;\Rightarrow\; X_1 = A^{-1}$

$CX_4 = E \;\Rightarrow\; X_4 = C^{-1} \;\Rightarrow\; AX_2 + BX_4 = AX_2 + BC^{-1} = 0 \;\Rightarrow\; X_2 = -BC^{-1}A^{-1}$

**Ответ:** $\begin{pmatrix} A^{-1} & -BC^{-1}A^{-1} \\ 0 & C^{-1} \end{pmatrix}$

**№ 2** $\dfrac{(5+i)(7-6i)}{3+i} = \dfrac{41-23i}{3+i} = (41-23i)(\dfrac{3}{10} - \dfrac{1}{10}i) = 10 - 11i$

**№ 3**

1). $\begin{vmatrix} a+bi & c+di \\ -c+di & a-bi \end{vmatrix} = (a+bi)(a-bi) - (c+di)(-c+di) = (a^2+b^2) - (-c^2-d^2) = a^2+b^2+c^2+d^2$

2). $\begin{vmatrix} \cos\alpha + i\sin\alpha & 1 \\ 1 & \cos\alpha - i\sin\alpha \end{vmatrix} = (\cos\alpha + i\sin\alpha)(\cos\alpha - i\sin\alpha) - 1 =$
$= (1 + (\sin\alpha\cos\alpha - \cos\alpha\sin\alpha)i) - 1 = 0$

3). $\begin{vmatrix} 1 & 0 & 1+i \\ 0 & 1 & i \\ 1-i & -i & 1 \end{vmatrix} = 1 + 0 + 0 - (1+i)(1-i) - 0 - (-i^2) = 1 - 2 - 1 = -2$

**№ 4**

$$\left(\begin{array}{cc|c} 1+i & 1-i & 1+i \\ 1-i & 1+i & 1+3i \end{array}\right) \begin{matrix} +1(2) \\ \phantom{x} \end{matrix} \to \left(\begin{array}{cc|c} 2 & 2 & 2+4i \\ 1-i & 1+i & 1+3i \end{array}\right) \begin{matrix} :2 \\ \phantom{x} \end{matrix}$$

$$\left(\begin{array}{cc|c} 1 & 1 & 1+2i \\ 1-i & 1+i & 1+3i \end{array}\right) \begin{matrix} \phantom{x} \\ -(1-i)(1) \end{matrix} \to \left(\begin{array}{cc|c} 1 & 1 & 1+2i \\ 0 & 2i & 4+4i \end{array}\right) \begin{matrix} \phantom{x} \\ \times\left(-\frac{1}{2}i\right) \end{matrix}$$

$$\left(\begin{array}{cc|c} 1 & 1 & 1+2i \\ 0 & 1 & 2-2i \end{array}\right) \begin{matrix} -1(2) \\ \phantom{x} \end{matrix} \to \left(\begin{array}{cc|c} 1 & 0 & -1 \\ 0 & 1 & 2-2i \end{array}\right)$$

**Ответ:** Получаем ответ: $\begin{pmatrix} -1 \\ 2-2i \end{pmatrix}$

**№ 5** Матрица $A = \begin{pmatrix} 1+i & 1-i \\ 1-i & 1+i \end{pmatrix}$

$\begin{vmatrix} 1+i & 1-i \\ 1-i & 1+i \end{vmatrix} = (1+i)(1+i) - (1-i)(1-i) = 2i + 2i = 4i \neq 0 \ \Rightarrow$ система имеет единственное решение.

Пусть наша искомое решение симеты будет столбцом $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$

1). $x_1 = \dfrac{a_1}{4i}, a_1 = \begin{vmatrix} 1+i & 1-i \\ 1+3i & 1+i \end{vmatrix} = (1+i)^2 - (1-i)(1+3i) = 2 - 4 - 2i = -4 \ \Rightarrow \ x_1 = \dfrac{-4}{4i} = i$

2). $x_2 = \dfrac{a_2}{4i}, a_2 = \begin{vmatrix} 1+i & 1+i \\ 1-i & 1+3i \end{vmatrix} = (1+i)(1+3i) - (1+i)(1-i) = -2 + 4i - 2 = -4 + 4i \ \Rightarrow \ x_2 = (-4+4i)(0 - \dfrac{1}{4}i) = 1 + i$

**Ответ:** $x_1 = i, x_2 = 1 + i$

**№ 6**

1). $-3i = -3(0 + 1i) = -3(\cos\alpha + \sin\alpha i), \alpha = \dfrac{\pi}{2} + 2\pi n, n \in \mathbb{Z}$

2). $1 + i\dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{2}{\sqrt{3}}(\dfrac{\sqrt{3}}{2} + i\dfrac{1}{2}) = \dfrac{2}{\sqrt{3}(\cos\alpha + i\sin\alpha)}, \alpha = \dfrac{\pi}{6} + 2\pi n, n \in \mathbb{Z}$

3). $\dfrac{10 - 6\sqrt{3}i}{2\sqrt{3} - i} = (10 - 6\sqrt{3}i)(\dfrac{2\sqrt{3}}{13} + \dfrac{i}{13}) = 2\sqrt{3} - 2i = 4(\dfrac{\sqrt{3}}{2} - \dfrac{1}{2}i) = 4(\cos\alpha + i\sin\alpha), \alpha = -\dfrac{\pi}{6} + 2\pi n, n \in \mathbb{Z}$

4). $\dfrac{\cos\varphi + i\sin\varphi}{\cos\psi + i\sin\psi} = (\cos\varphi + i\sin\varphi)(\cos\psi - i\sin\psi) = (\cos\varphi \cdot \cos\psi + \sin\varphi \cdot \sin\psi) + i(\sin\varphi \cdot \cos\psi - \cos\varphi \cdot \sin\psi) = \cos(\varphi - \psi) + i\sin(\varphi - \psi)$

**№ 7** $(\sqrt{3} - i)^{32} = (2(\cos -\dfrac{\pi}{6} + i\sin -\dfrac{\pi}{6}))^{32} = 2^{32}(\cos -\dfrac{32\pi}{6} + i\sin -\dfrac{32\pi}{6}) = 2^{32}(\cos -\dfrac{16\pi}{3} + i\sin -\dfrac{16\pi}{3})$

**№ 8**

1). $z^3 = (x+iy)^3 = ((x^2-y^2)+2xyi)(x+yi) = (x^3-3xy^2)+i(3x^2y-y^3) = 1+0i$

Имеем:

$$\begin{cases} x^3-3xy^2 = 1 \\ 3x^2y - y^3 = 0 \end{cases}$$

Отсюда либо $y=0, x=1 \Rightarrow z_1 = 1$, либо $x = -\dfrac{1}{2}, y = (+-)\dfrac{\sqrt{3}}{2} \Rightarrow (-\dfrac{1}{2}(+-)\ i\dfrac{\sqrt{3}}{2})$

2). $z^2 = i = (x+iy)^2 = (x^2-y^2)+2xyi = 0+i$

Имеем:

$$\begin{cases} x^2-y^2 = 0 \\ 2xy = 1 \end{cases}$$

Отсюда $x = y = +-\dfrac{\sqrt{2}}{2} \Rightarrow z_1 = \dfrac{\sqrt{2}}{2}+\dfrac{\sqrt{2}}{2}i, z_2 = -\dfrac{\sqrt{2}}{2}-\dfrac{\sqrt{2}}{2}i$

**№ 9**

1). $\sqrt[3]{2-2i} = \sqrt[3]{\sqrt{8}(\cos-\frac{\pi}{4}+\sin-\frac{\pi}{4}i)} = \sqrt{2}(\cos\frac{-\frac{\pi}{4}+2\pi n}{3}+\sin\frac{-\frac{\pi}{4}+2\pi n}{3}i) =$
$= \sqrt{2}\left(\cos\left(-\frac{\pi}{12}+\frac{2\pi n}{3}\right)+\sin\left(-\frac{\pi}{12}+\frac{2\pi n}{3}\right)i\right)$

$x_1 = \sqrt{2}\left(\cos\left(-\frac{\pi}{12}\right)+\sin\left(-\frac{\pi}{12}\right)i\right)$
$x_2 = \sqrt{2}\left(\cos\left(-\frac{7\pi}{12}\right)+\sin\left(-\frac{7\pi}{12}\right)i\right)$
$x_3 = \sqrt{2}\left(\cos\left(-\frac{5\pi}{4}\right)+\sin\left(-\frac{5\pi}{4}\right)i\right)$

2). $\sqrt[6]{(2-2i)^2} = \sqrt[6]{-8i} = \sqrt{2}\sqrt[6]{-i} = \sqrt{2}\sqrt[6]{-i} = \sqrt{2}\sqrt[6]{\left(\cos(\frac{3\pi}{2}+2\pi n)+i\sin(\frac{3\pi}{2}+2\pi n)\right)} =$
$= \sqrt{2}\left(\cos(\frac{3\pi}{12}+\frac{2\pi n}{6})+i\sin(\frac{3\pi}{12}+\frac{2\pi n}{6})\right)$

1). $x_1 = \sqrt{2}\left(\cos(\frac{3\pi}{12})+i\sin(\frac{3\pi}{12})\right) = 1+i$
2). $x_2 = \sqrt{2}\left(\cos(\frac{7\pi}{12})+i\sin(\frac{7\pi}{12})\right) = \frac{1-\sqrt{3}}{2}+\frac{1+\sqrt{3}}{2}i$
3). $x_3 = \sqrt{2}\left(\cos(\frac{11\pi}{12})+i\sin(\frac{11\pi}{12})\right) = \frac{-1-\sqrt{3}}{2}+\frac{\sqrt{3}-1}{2}i$
4). $x_4 = \sqrt{2}\left(\cos(\frac{15\pi}{12})+i\sin(\frac{15\pi}{12})\right) = -1-i$
5). $x_5 = \sqrt{2}\left(\cos(\frac{19\pi}{12})+i\sin(\frac{19\pi}{12})\right) = \frac{\sqrt{3}-1}{2}+\frac{1-\sqrt{3}}{2}i$

6). $x_6 = \sqrt{2}\left(\cos(\dfrac{23\pi}{12}) + i\sin(\dfrac{23\pi}{12})\right) = \dfrac{1+\sqrt{3}}{2} + \dfrac{1-\sqrt{3}}{2}i$

**№ 10**

$$\sqrt[4]{\frac{-18}{1+i\sqrt{3}}} = \sqrt[4]{-18\cdot\left(\frac{1}{4} - i\frac{\sqrt{3}}{4}\right)} = \sqrt[4]{9\cdot\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)} = |\sqrt{3}|\sqrt[4]{\cos\left(\frac{2\pi}{3} + 2\pi n\right) + i\sin\left(\frac{2\pi}{3} + 2\pi n\right)} =$$

$$= |\sqrt{3}|\left(\cos\left(\frac{\pi}{6} + \frac{\pi n}{2}\right) + i\sin\left(\frac{\pi}{6} + \frac{\pi n}{2}\right)\right)$$

1). $x_1 = |\sqrt{3}|\left(\cos\left(\dfrac{\pi}{6}\right) + i\sin\left(\dfrac{\pi}{6}\right)\right) = \pm\sqrt{3}(\dfrac{\sqrt{3}}{2} + \dfrac{1}{2}i)$

2). $x_2 = |\sqrt{3}|\left(\cos\left(\dfrac{2\pi}{3}\right) + i\sin\left(\dfrac{2\pi}{3}\right)\right) = \pm\sqrt{3}(-\dfrac{1}{2} + \dfrac{\sqrt{3}}{2}i)$

**№ 11**

$(2\sqrt{3} - i)z^4 = 10 - 6\sqrt{3}i$

$z^4 = (10-6\sqrt{3}i)\left(\dfrac{2\sqrt{3}}{13} + \dfrac{i}{13}\right) = 2\sqrt{3}-2i = 4\left(\dfrac{\sqrt{3}}{2} - \dfrac{1}{2}i\right) = 4\left(\cos\left(-\dfrac{\pi}{6} + 2\pi n\right) + i\sin\left(-\dfrac{\pi}{6} + 2\pi n\right)\right)$

$$z = \sqrt{2}\left(\cos\left(-\frac{\pi}{24} + \frac{\pi n}{2}\right) + i\sin\left(-\frac{\pi}{24} + \frac{\pi n}{2}\right)\right)$$

**Ответ:**

1). $z_1 = \sqrt{2}\left(\cos\left(\dfrac{71\pi}{24}\right) + i\sin\left(\dfrac{71\pi}{24}\right)\right)$

**№ 12**

По формуле Муавра $(\cos x + i\sin x)^3 = (\cos 3x + i\sin 3x)$

Так же можено просто перемножетить эти числа 3 раза:

$(\cos x + i\sin x)^3 = (\cos x + i\sin x)^2\cdot(\cos x + i\sin x) = ((\cos^2 x - \sin^2 x) + i(2\sin x\cos x))\cdot(\cos x + i\sin x) = (\cos^3 x - 3\sin^2 x\cos x) + i(3\cos^2 x\sin x - \sin^3 x)$

Два комплексных числа $a + bi$ и $c + di$ равны, если $a = c$ и $b = d$. Мы посчитали $(\cos x + i\sin x)^3$ двумя способами, следовательно мы получилис систему

$$\begin{cases} \cos 3x = \cos^3 x - 3\sin^2 x\cos x \\ \sin 3x = 3\cos^2 x\sin x - \sin^3 x \end{cases}$$