Die Maxwell-Gleichungen

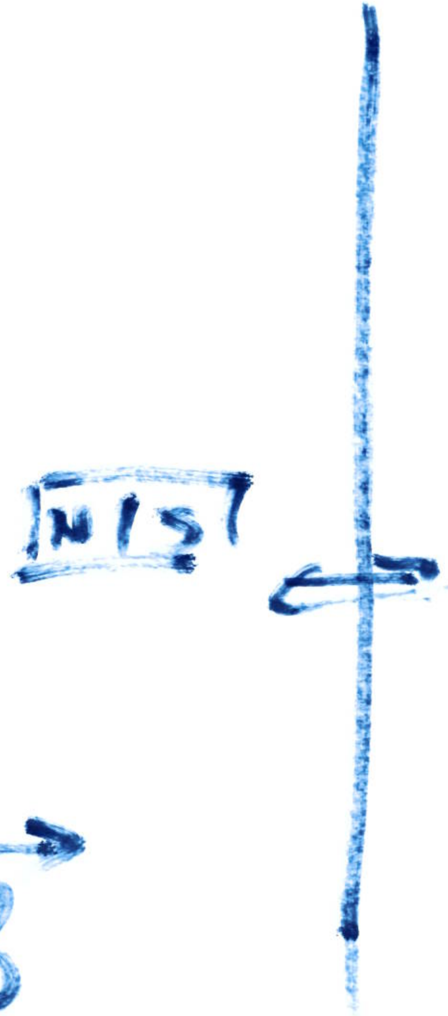
(1) $$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$

(2) $$\vec{\nabla} \cdot \vec{B} = 0$$

(3) $$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} =: \dot{\vec{B}}$$

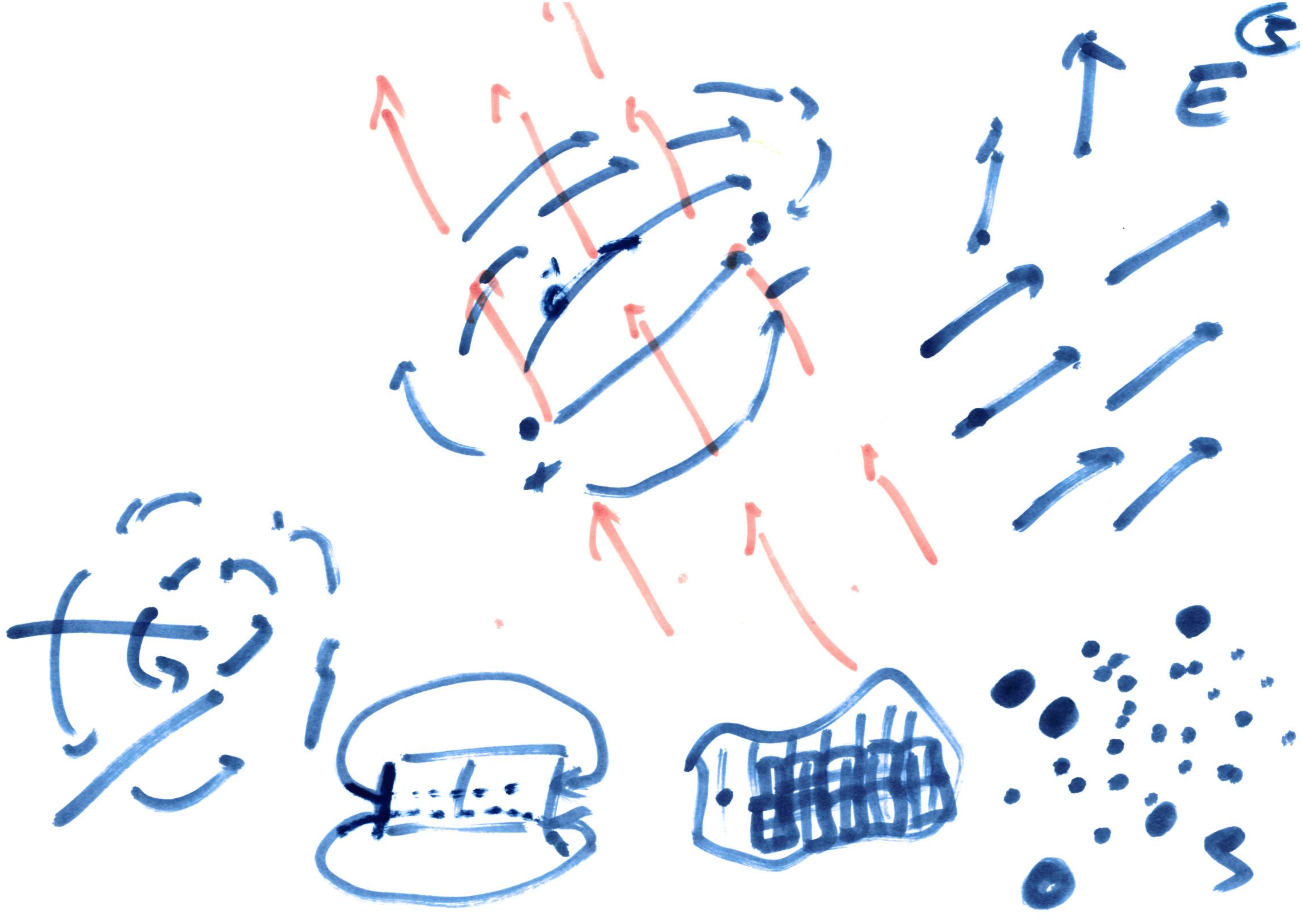
(4) $$\vec{\nabla} \times \vec{B} = \boxed{\mu_0 \vec{j}} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\vec{F} = q\cdot\vec{E} + q\vec{v}\times\vec{B}$$



$$\vec{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}, \quad \vec{w} = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}$$

$$\vec{v} \times \vec{w} = \begin{pmatrix} v_2 w_3 - v_3 w_2 \\ v_3 w_1 - v_1 w_3 \\ v_1 w_2 - v_2 w_1 \end{pmatrix}$$



$$\vec{\nabla} = \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix}, \quad \vec{\nabla} \circ \underbrace{\vec{E}}_{= \begin{pmatrix} E_1 \\ E_2 \\ E_3 \end{pmatrix}} = \frac{\partial E_1}{\partial x} + \frac{\partial E_2}{\partial y} + \frac{\partial E_3}{\partial z}$$

$$= \operatorname{div}(\vec{E})$$

= die Divergenz von $\vec{E}$

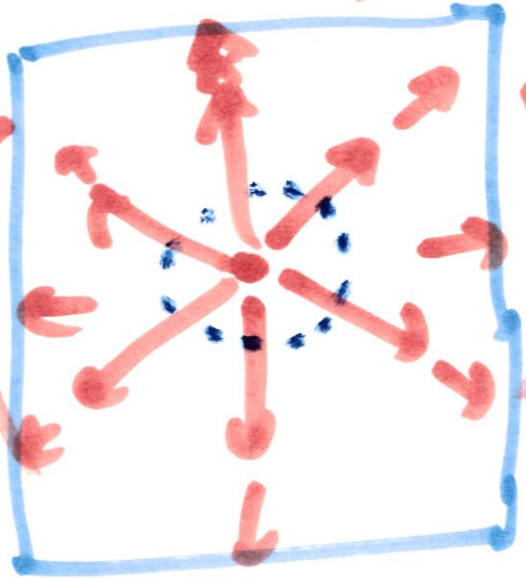
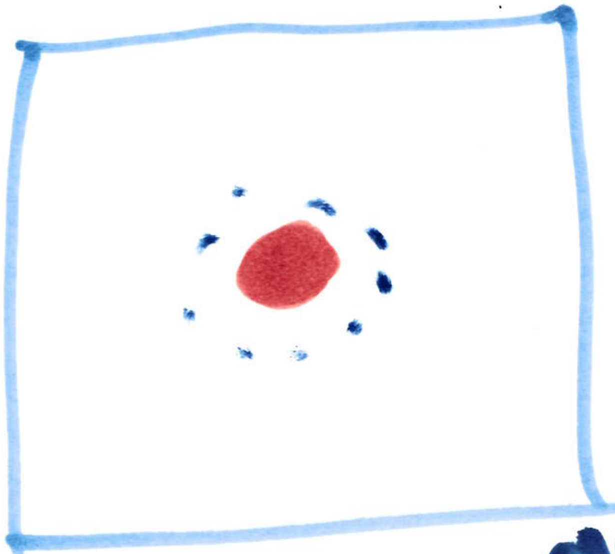
$$3x^2 - 9xz$$

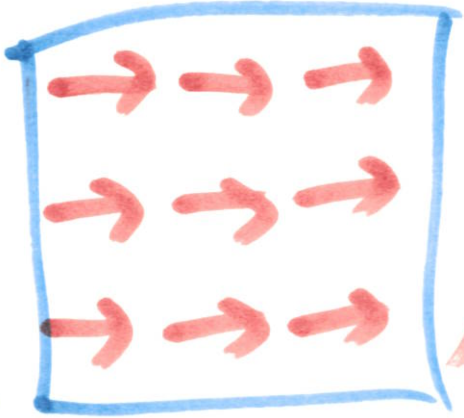
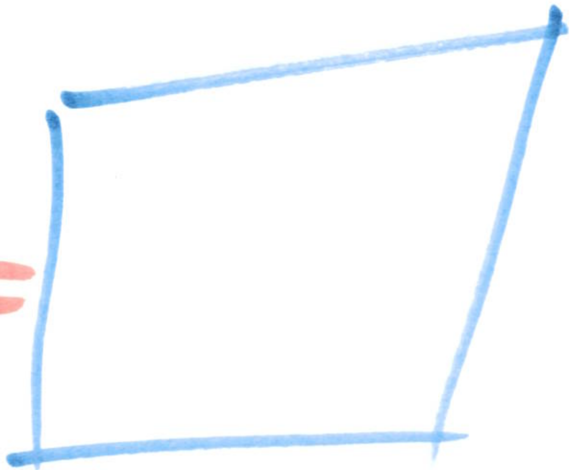
$\frac{\partial}{\partial x}$ nach x abl. $\rightarrow 6x - 9z$

$\frac{\partial}{\partial y} \rightarrow 0$

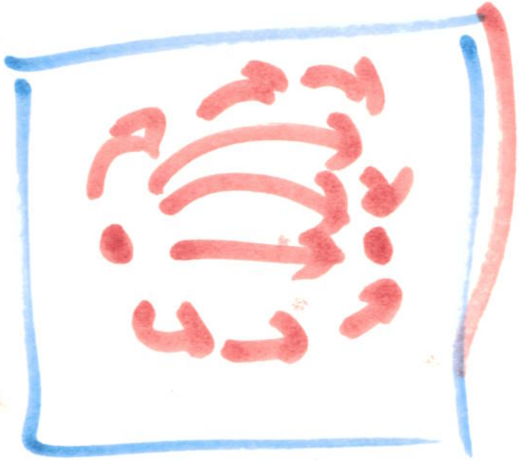
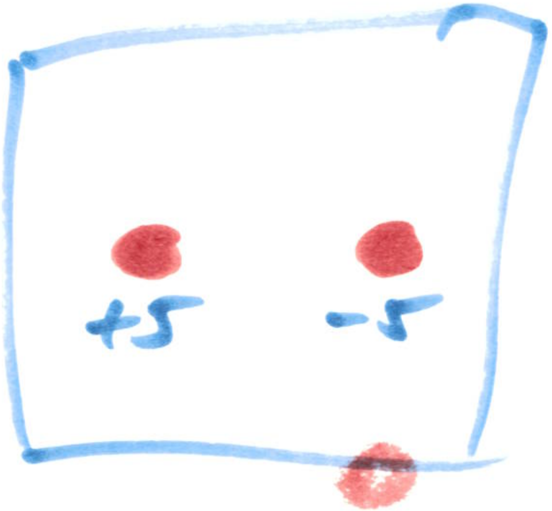
$\frac{\partial}{\partial z} \rightarrow -9x$

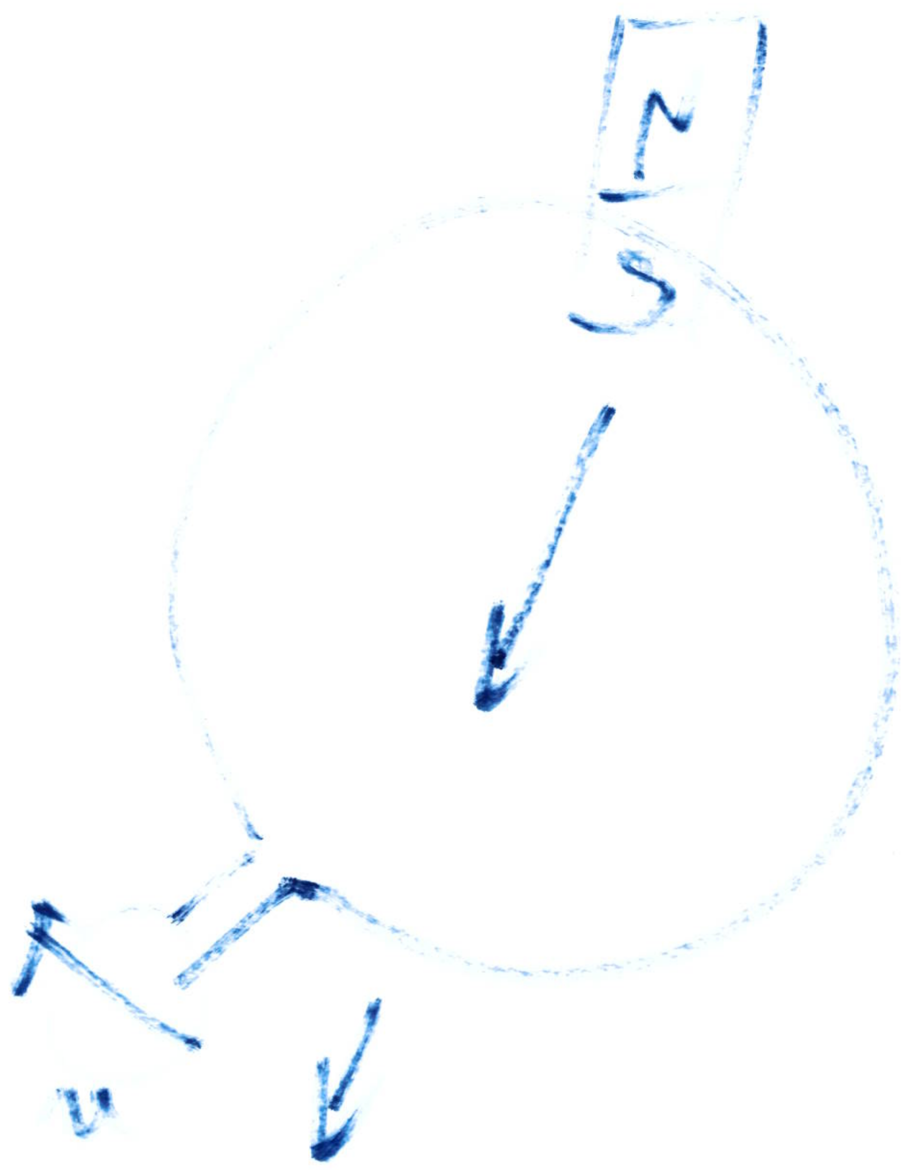
- div(Vektorfeld) ist ein Skalarfeld (S)

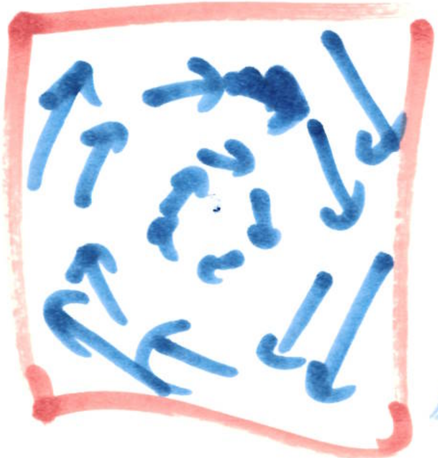
- div () = 

- div () = 

„quellen"
ist ein stark
gebeugtes Ver

· div () = 


N
S

- $\operatorname{rot}(\vec{E}) = \vec{\nabla} \times \vec{E}$
- rot(Vektorfeld) ist ein Vektorfeld.
- rot() = 

$$\operatorname{div} \operatorname{rot} \vec{V} = 0.$$

$$\Rightarrow \operatorname{div} \operatorname{rot} \vec{E} \overset{(3)}{=} - \operatorname{div} \dot{\vec{B}}$$

$$\Rightarrow \left(\operatorname{div} \vec{B}\right)^{\cdot} = 0$$

$$(4) \Rightarrow 0 = \mu_0 \operatorname{div} \vec{j} + \mu_0 \varepsilon_0 \frac{\partial \operatorname{div} \vec{E}}{\partial t}$$
$$\overset{(1)}{=} \mu_0 \left(\operatorname{div} \vec{j} + \dot{\rho} \right)$$

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) \overset{③}{=} -\vec{\nabla} \times \dot{\vec{B}}$$ ⑩

$$\underset{\vec{J} = \vec{0}}{\overset{④}{=}} -\mu_0 \varepsilon_0 \ddot{\vec{E}}$$

$$\cancel{\vec{a} \times (\vec{b} \times \vec{c})}$$
$$= \cancel{\vec{b} \cdot (\vec{a} \cdot \vec{c})}$$
$$\cancel{- \vec{c} \cdot (\vec{a} \cdot \vec{b})}$$

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E})$$
$$= \left(-\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2}\right)\vec{E}$$
$$= -\Delta(\vec{E})$$

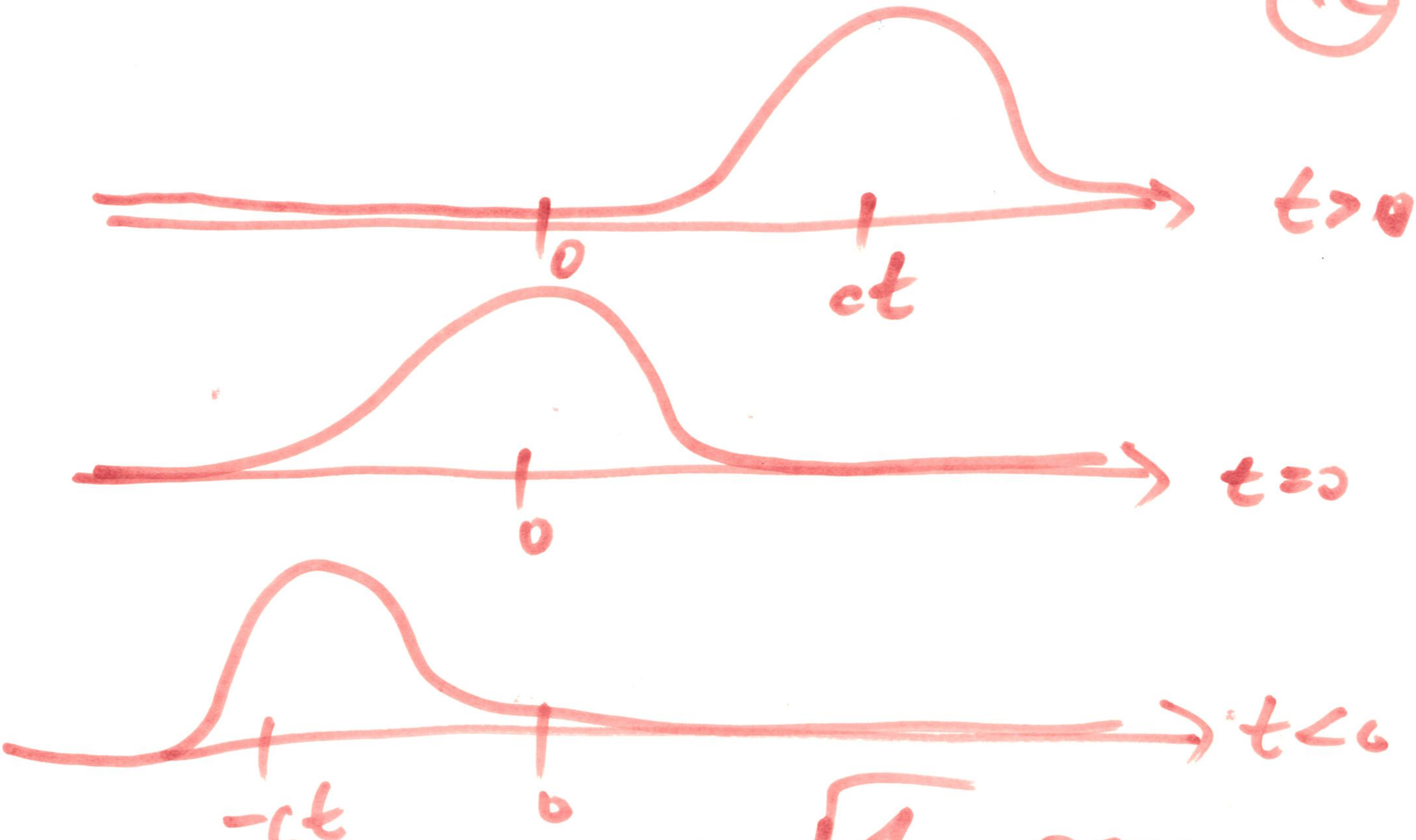
11

$$\boxed{-\mu_0 \varepsilon_0 \ddot{E} + \Delta E = 0} \qquad \underline{j = 0}$$

Ansatz: $E_1 = f_1(x - ct)$, $c = \text{const.}$

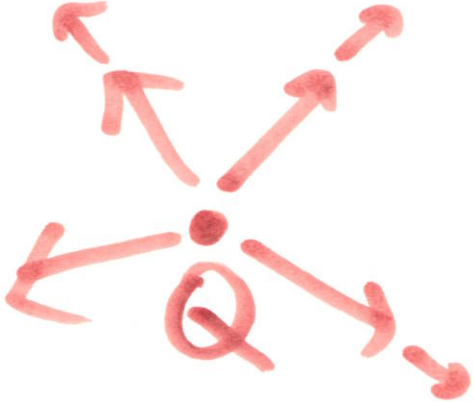
$$-\mu_0 \varepsilon_0 \ddot{E}_1 + E_1'' = 0$$

$$-\mu_0 \varepsilon_0 (+c^2) f_1'' + f_1'' = 0 \Rightarrow c = \sqrt{\frac{1}{\mu_0 \varepsilon_0}}$$



$$c = \sqrt{\frac{1}{\mu_0 \varepsilon_0}} = 299792458 \, \frac{m}{s}$$

1



$$E(r) = \frac{1}{4\varepsilon_0 \pi} \frac{Q}{r^2}$$

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2} \frac{\vec{r}}{\cancel{|\vec{r}|}\, r}$$ 1. $$\mathrm{div}(\vec{E}) = \frac{\rho}{\varepsilon_0}$$

$r = |\vec{r}|$

$=$ Länge des Vektors $\vec{r}$

Wieso implizieren die Maxwell-Gl'n diese Formel fürs E-Feld einer Punktladung?

Schritt 1:

Die Situation ist rotationssymmetrisch.

Daher muss das $\vec{E}$-Feld radialsymmetrisch sein:

$$\vec{E}(\vec{r}) = f(r)\,\frac{\vec{r}}{|r|}$$

Schritt 2: Clever integrieren!

$$\int_{\partial V} \vec{E} \circ d\vec{A} = f(r) \cdot 4\pi r^2$$

Oberflächenintegral

andere Schreibweise:

$$\oiint_{\partial V}$$

$$\int_{\partial V} \vec{E} \circ d\vec{A} = \int_{V} \mathrm{div}(\vec{E})\, dV$$

Stokes

Volumenintegral, andere Schreibweise: $\iiint_V$

$$\overset{(1)}{=} \int_V \frac{\rho}{\varepsilon_0}\, dV = \frac{1}{\varepsilon_0} \int_V \rho\, dV$$

$$= \frac{1}{\varepsilon_0} Q$$

eingeschlossene Ladung

$$\Rightarrow f(r) = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2} \cdot \checkmark$$