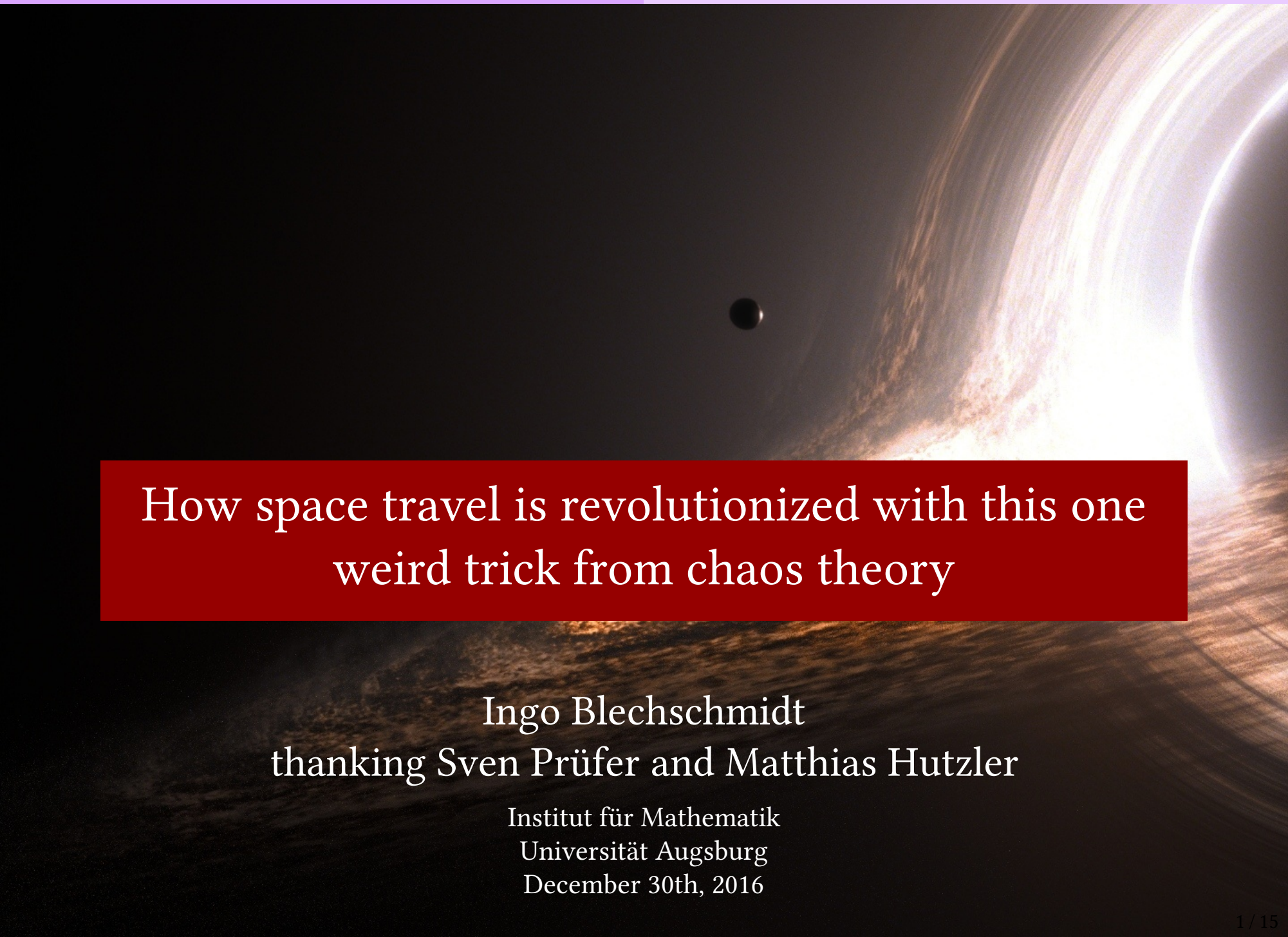

How space travel is revolutionized with this one weird trick from chaos theory

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thanking Sven Prüfer and Matthias Hutzler

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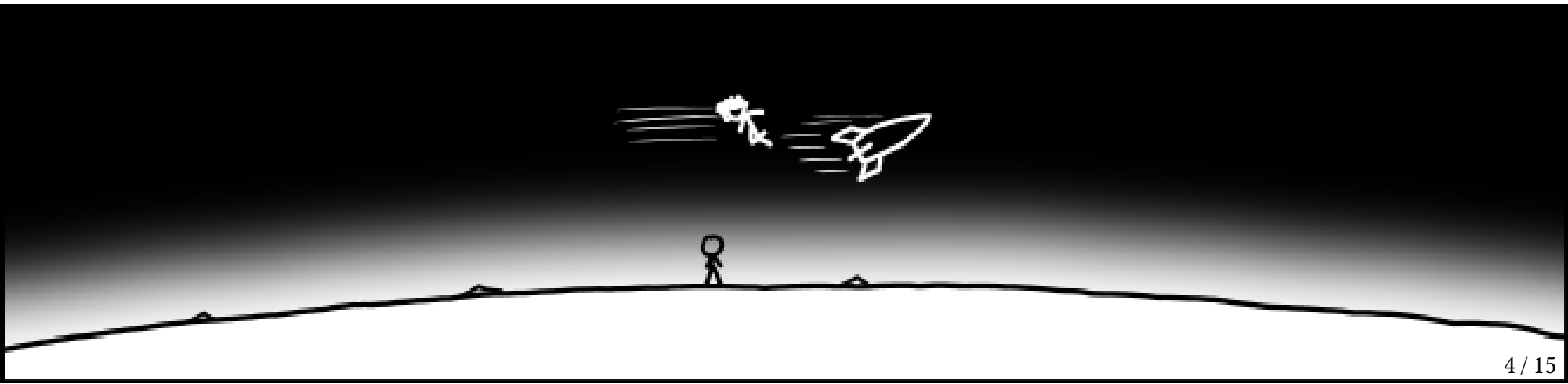
NASA

1. A crash course on orbital mechanics
   - Basic facts
   - Changing orbits
   - The tyranny of the rocket equation

2. One weird trick from chaos theory
   - Lagrangian points
   - Weak stability boundaries
   - The rescue of the Hiten
   - In nature

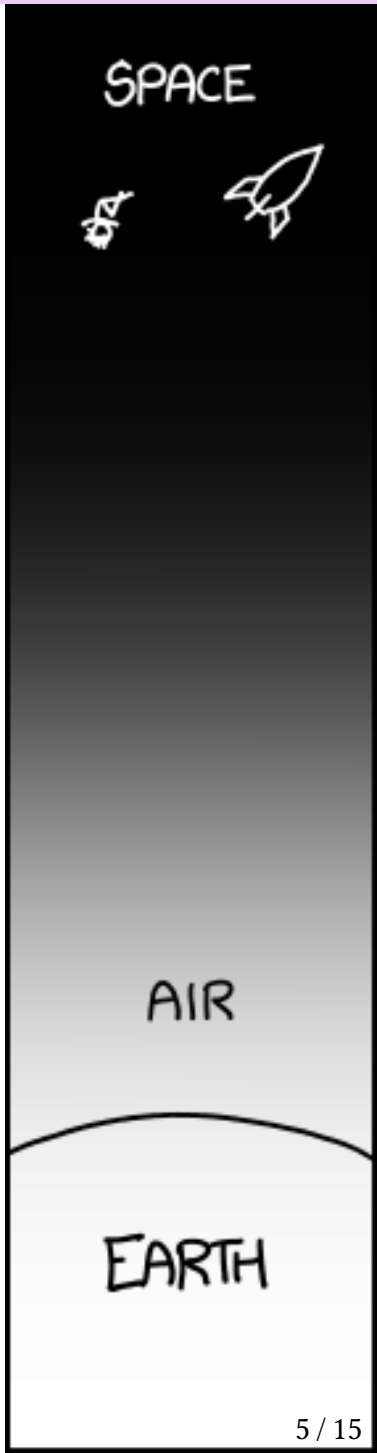
Part I

A crash course on orbital mechanics



Basic facts

- Getting to space is easy.
  The hard part is staying there.
- Gravitational acceleration at
  the height of the ISS is still $\approx 8.7\,\mathrm{m/s^2}$.



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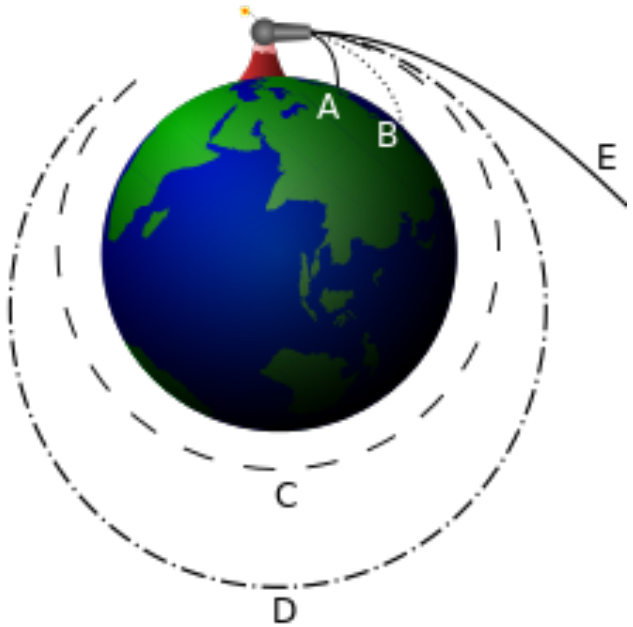

$$F_{\text{centripetal}} = F_{\text{gravitation}} \rightsquigarrow v_1 = \sqrt{GM/r}$$
$$E_{\text{kinetic}} = E_{\text{gravitation}} \rightsquigarrow v_2 = \sqrt{2}\, v_1$$

Basic facts

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body	second escape velocity
Earth	$11.2\,\mathrm{km/s} \approx 40\,000\,\mathrm{km/h}$
Moon	$2.4\,\mathrm{km/s}$
Sun	$618\,\mathrm{km/s}$
Milky Way	$\approx 550\,\mathrm{km/s}$

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Basic facts

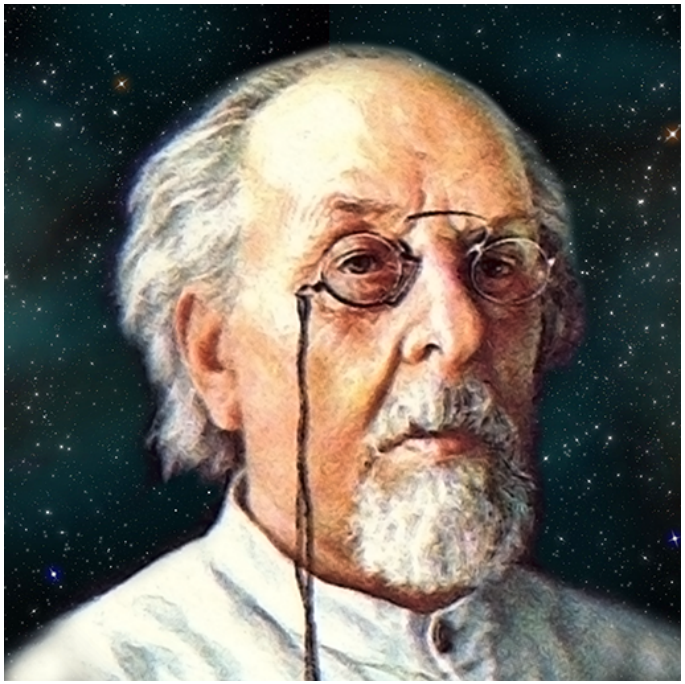
- Getting to space is easy.
  The hard part is staying there.
- Velocity is very important.
- In the **one-body problem**, there are only three kinds of orbits: elliptic, parabolic, and hyperbolic.
- Have your models straight: Earth is …
  1. a perfect ball?
  2. has atmosphere?
  3. rotating?

Changing orbits

"Live demo"

- Changing the phase
- Changing the eccentricity
- Changing the radius
- Changing inclination

The tyranny of the rocket equation



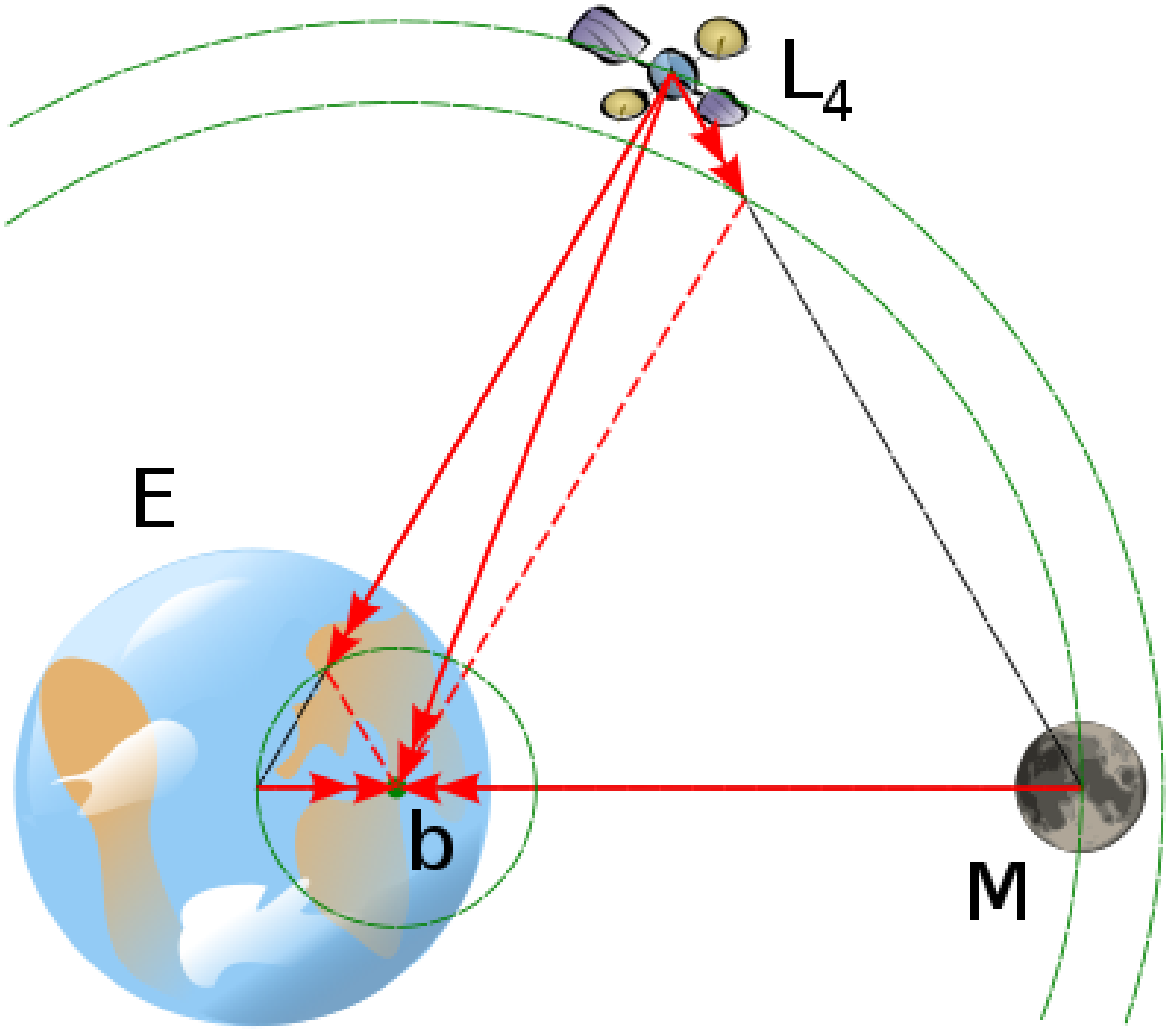
Konstantin Tsiolkovsky (* 1857, † 1935)

$$m_{\text{total}} = m_{\text{payload}} \cdot e^{\Delta v / v_{\text{eff. exhaust}}}$$

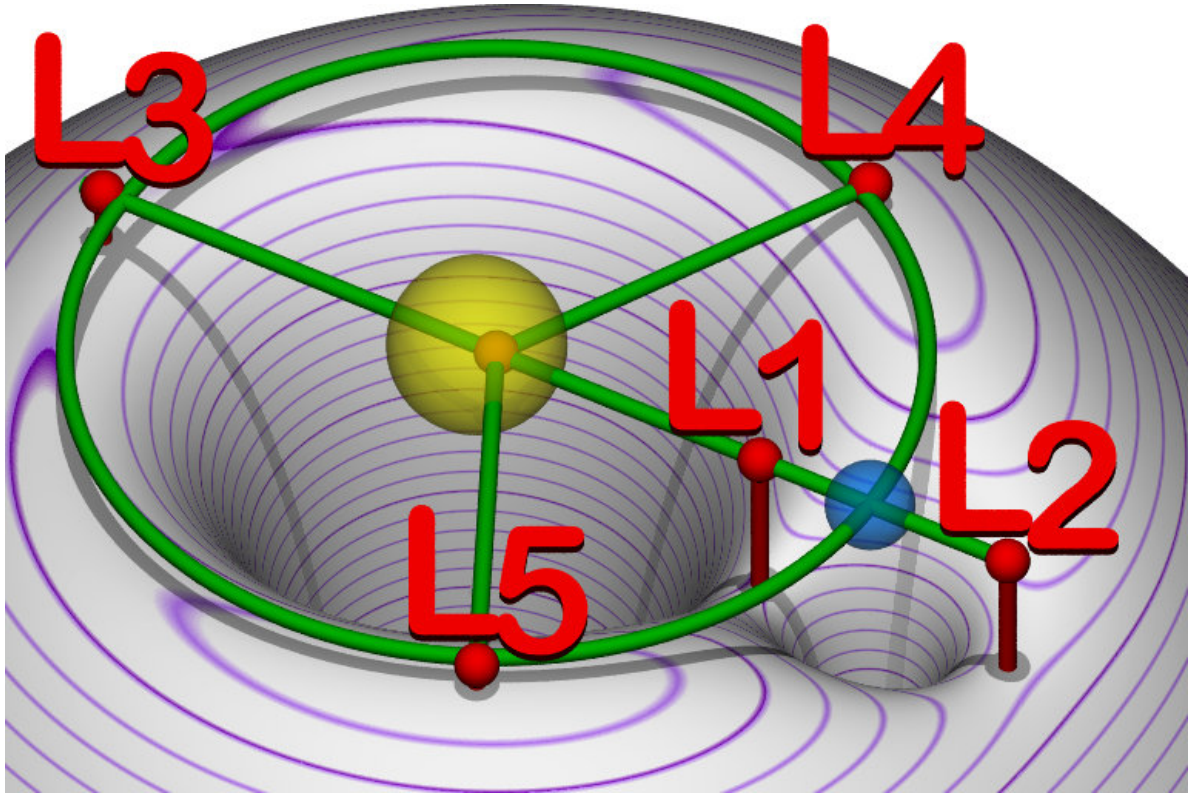
Part II

One weird trick from chaos theory

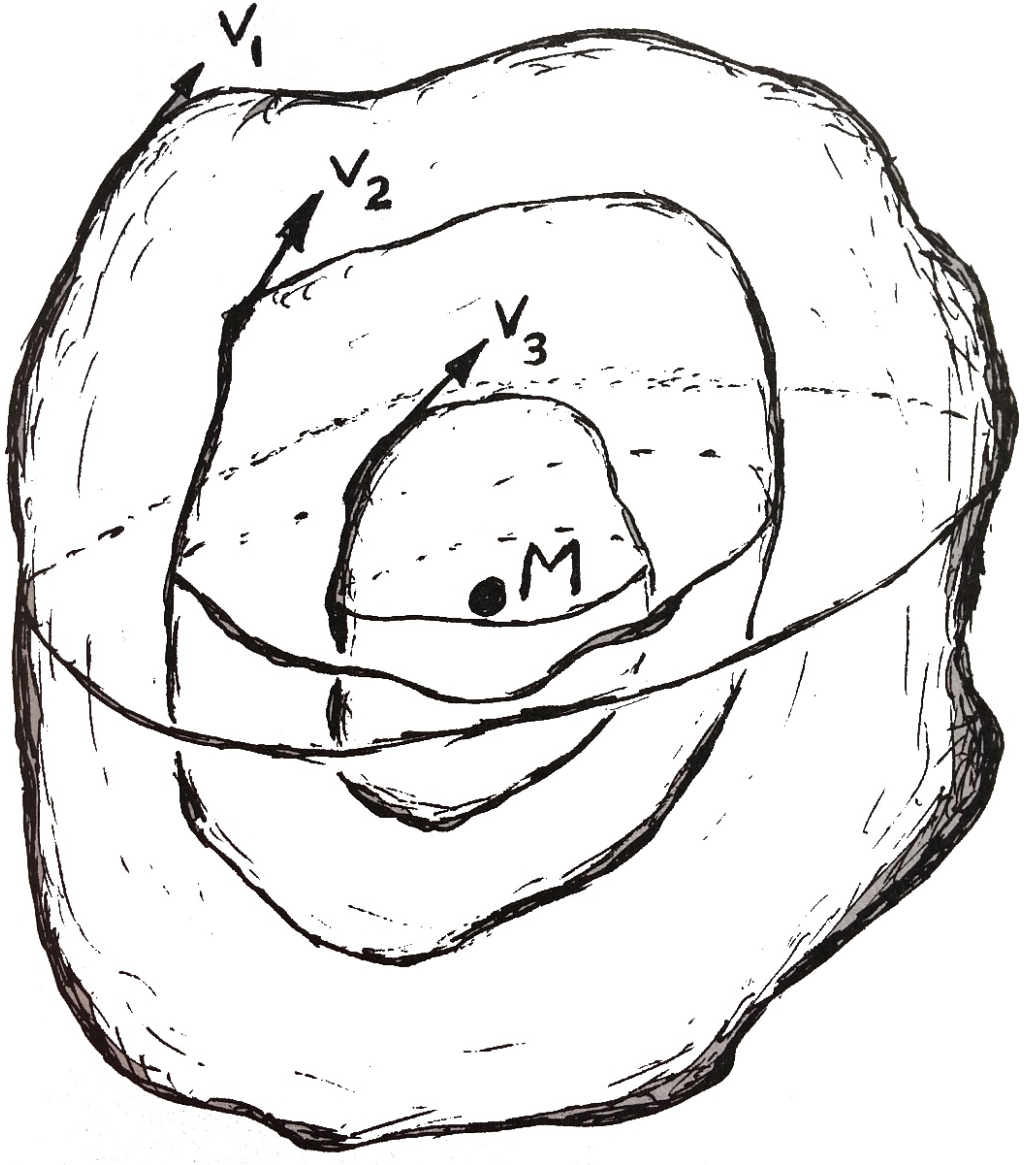
Lagrangian points



Lagrangian points



Weak stability boundaries



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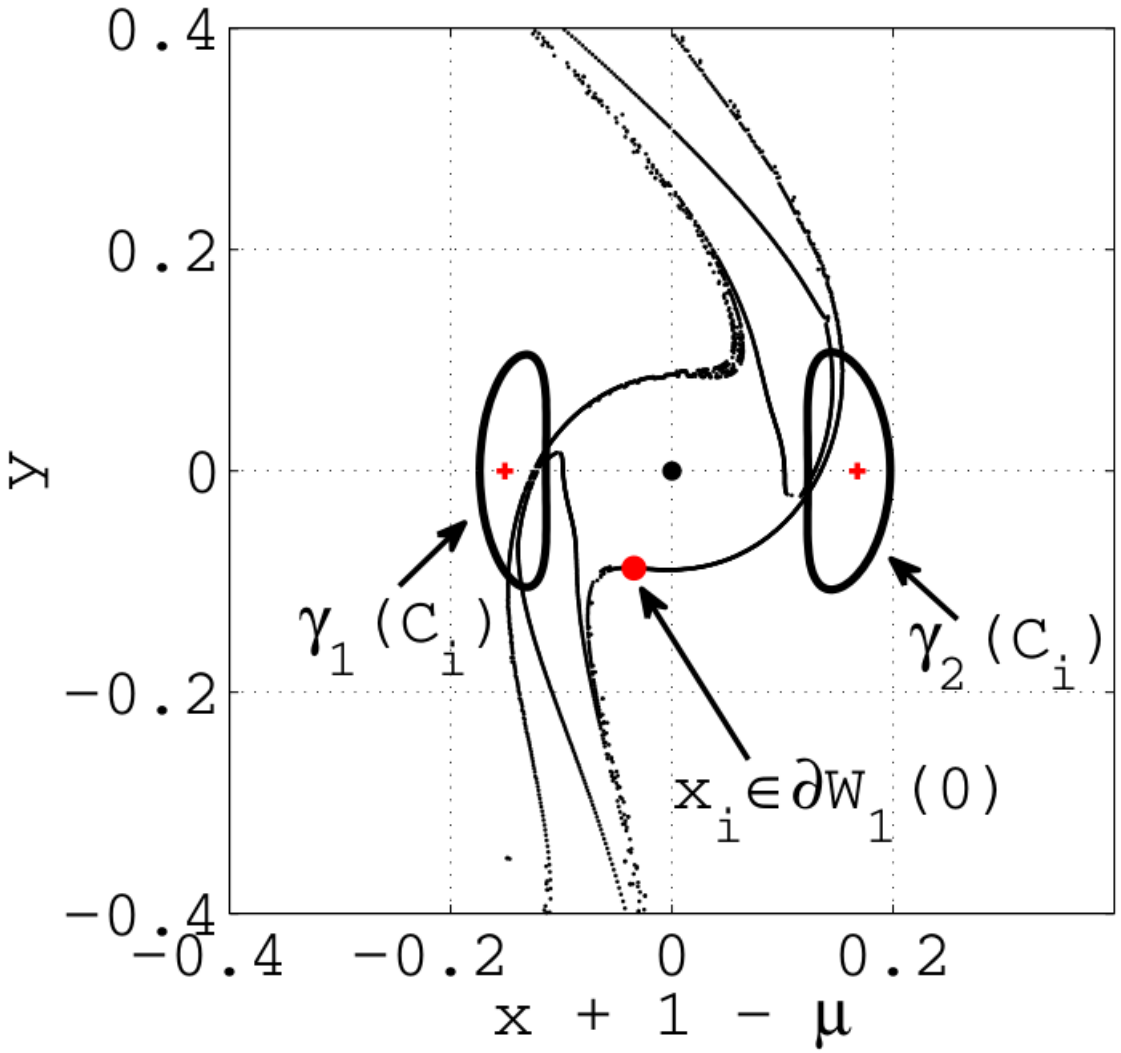
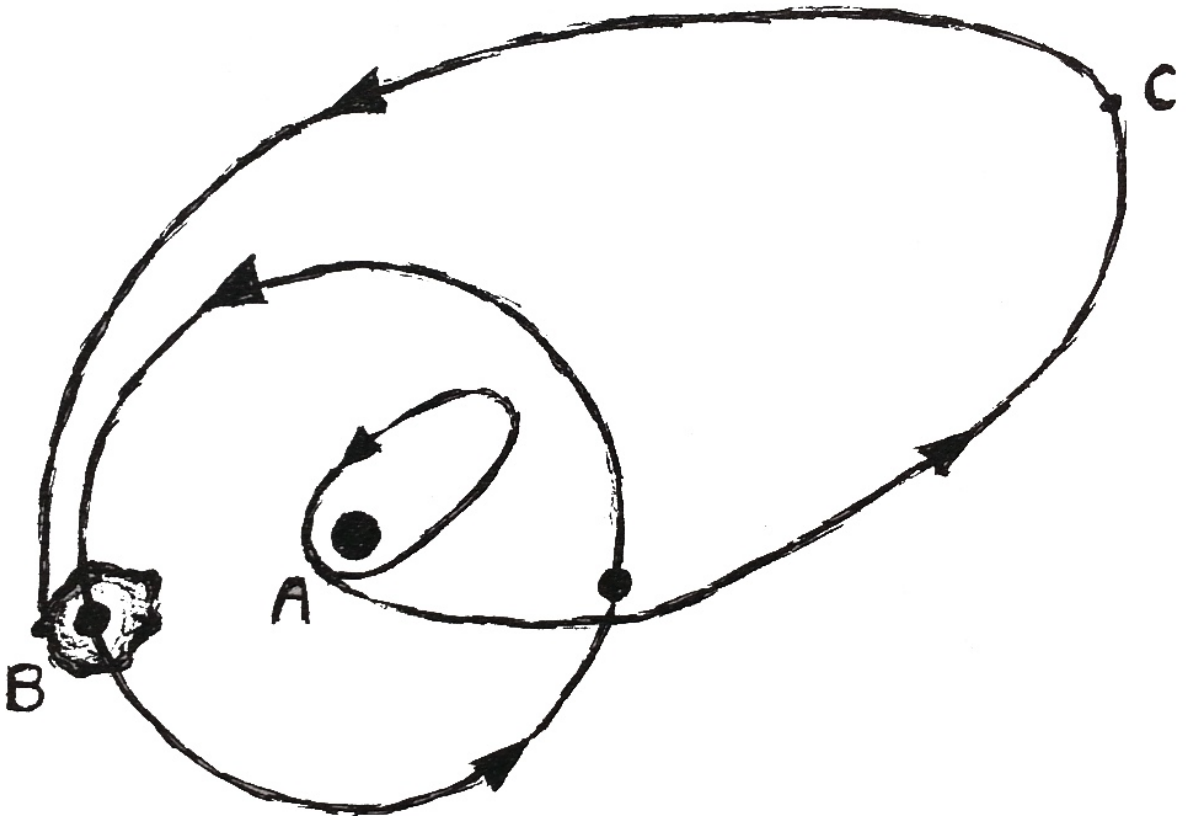


The rescue of the Hiten



In nature

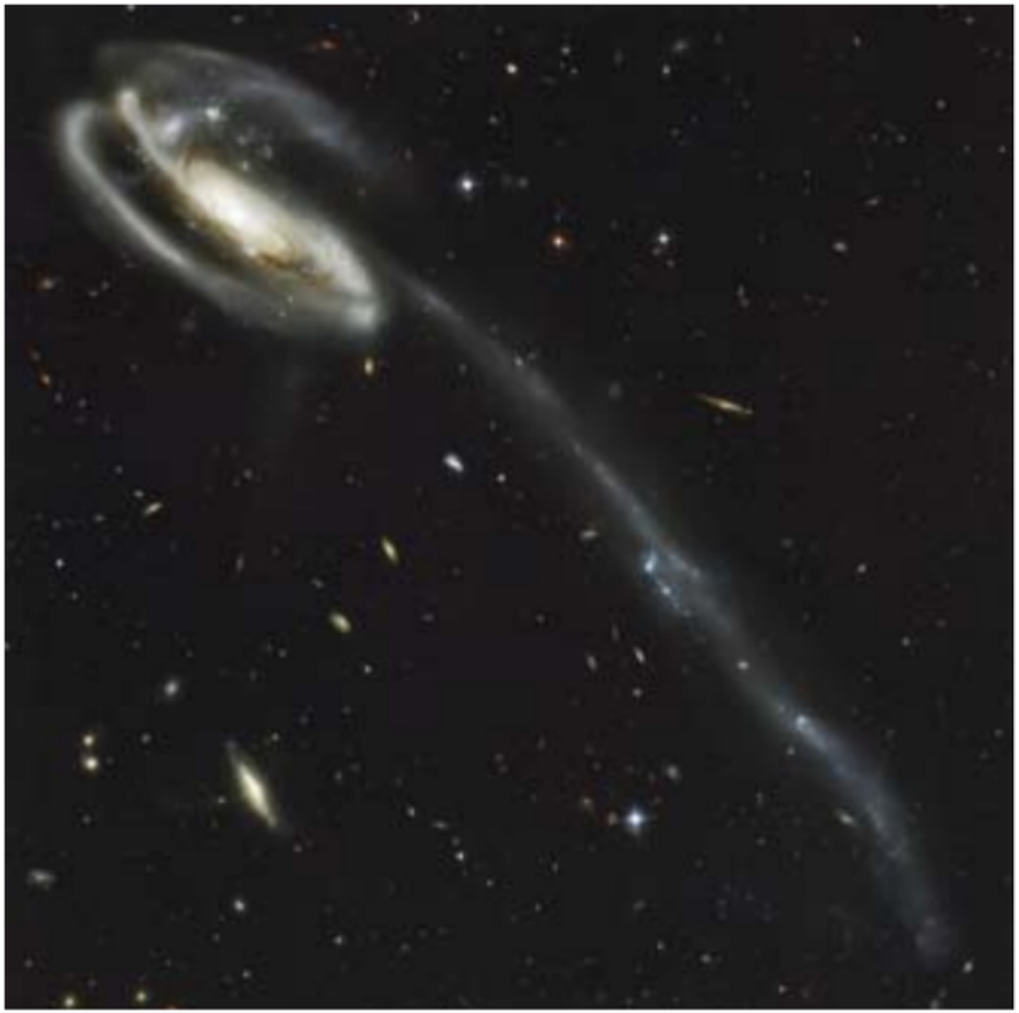


Figure 10. Stars stream outward from the Tadpole Galaxy (Arp 188) along a tubelike channel that stretches for some 280,000 light-years. This conduit (the galactic equivalent of the tubes making up the interplanetary transport network) arose through gravitational interaction with a compact galaxy that can now be seen lurking behind one of the Tadpole's spiral arms. (Courtesy of ACS Science & Engineering Team and NASA.)