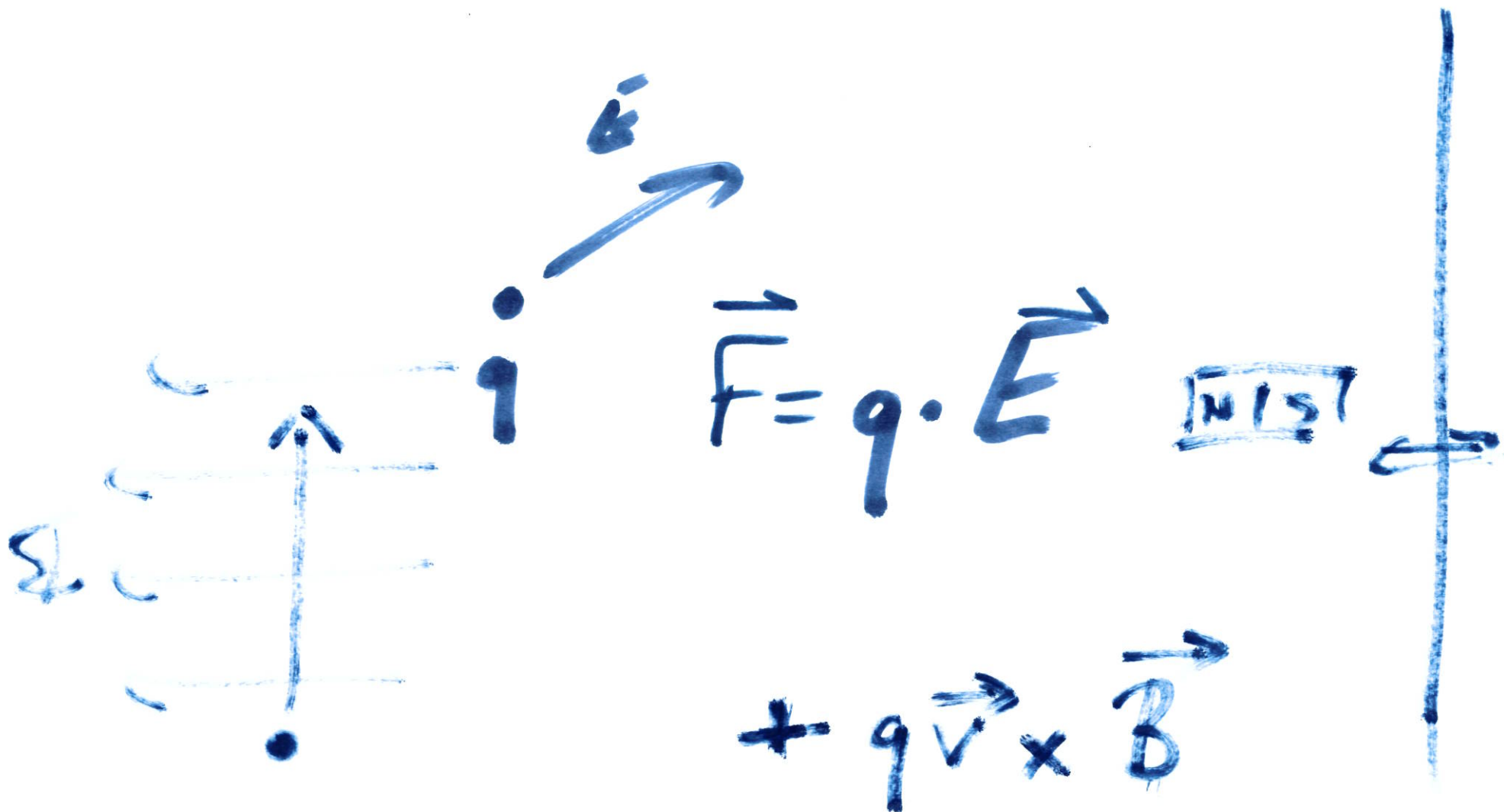


♥ Die Maxwell-Gleichungen ♥

$$\textcircled{1} \vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0} \quad \textcircled{2} \vec{\nabla} \cdot \vec{B} = 0$$

$$\textcircled{3} \vec{\nabla} \times \vec{E} = - \frac{\partial \vec{B}}{\partial t} =: \dot{\vec{B}}$$

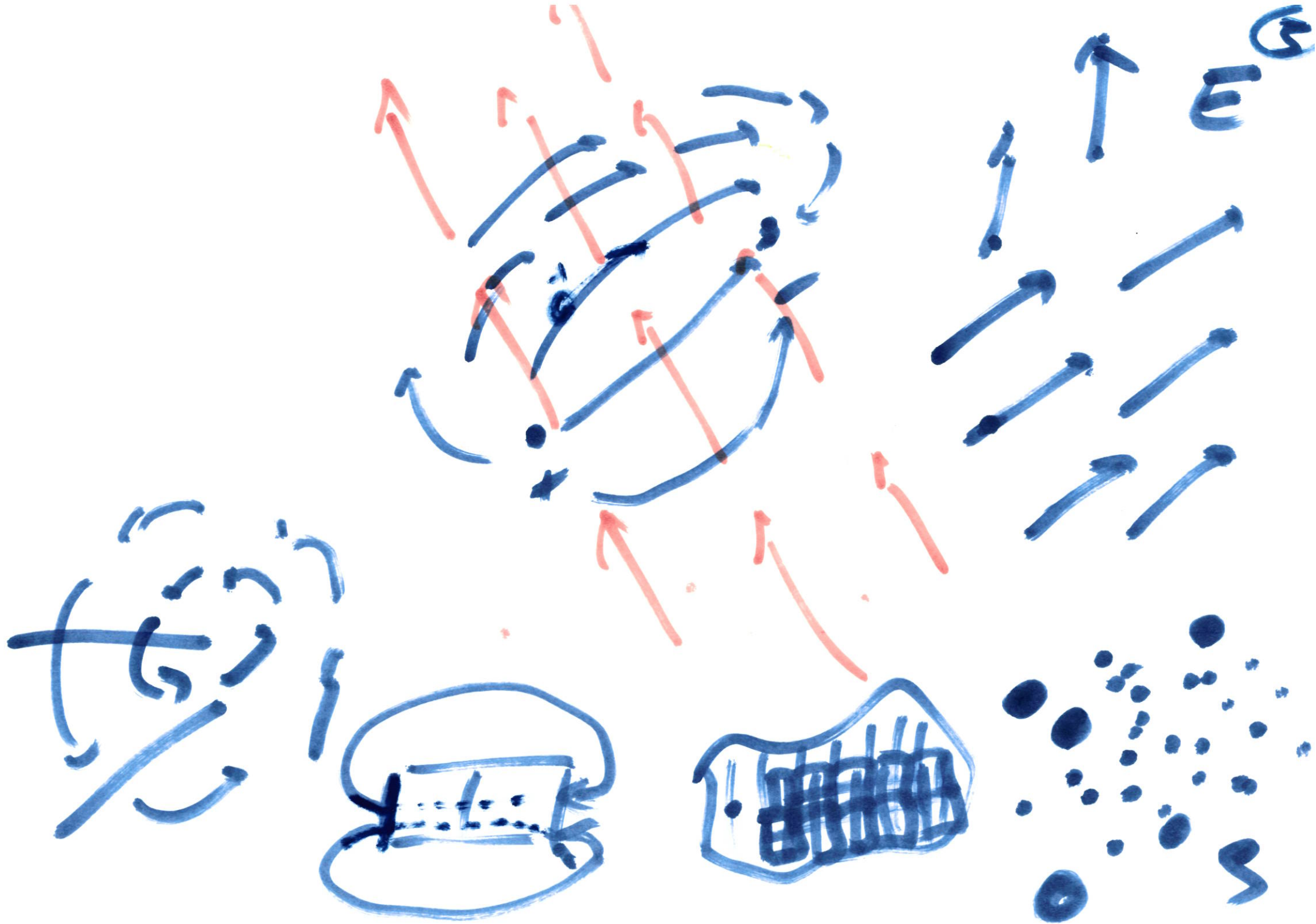
$$\textcircled{4} \vec{\nabla} \times \vec{B} = \mu_0 \vec{j} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$



②

$$\vec{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}, \quad \vec{w} = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}$$

$$\vec{v} \times \vec{w} = \begin{pmatrix} v_2 w_3 - v_3 w_2 \\ v_3 w_1 - v_1 w_3 \\ v_1 w_2 - v_2 w_1 \end{pmatrix}$$



$$\vec{\nabla} = \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix}, \quad \vec{\nabla} \cdot \vec{E} = \frac{\partial E_1}{\partial x} + \frac{\partial E_2}{\partial y} + \frac{\partial E_3}{\partial z}$$

$\vec{E} = \begin{pmatrix} E_1 \\ E_2 \\ E_3 \end{pmatrix}$

④

\parallel
 $\text{div}(\vec{E})$
 \parallel
 de Divergent
 van \vec{E}

$$\frac{\partial}{\partial x} \text{ van } 3x^2 - 9xz \quad \frac{\partial}{\partial y} \quad \frac{\partial}{\partial z}$$

$6x - 9z \quad 0 \quad -9x$

- $\text{div}(\text{Vektorfeld})$ ist ein Skalarfeld ⑤

- $\text{div} \left(\begin{array}{c} \text{Vektorfeld mit Ausstrahlung} \end{array} \right) = \begin{array}{c} \text{Skalarfeld mit Quelle} \end{array}$

- $\text{div} \left(\begin{array}{c} \text{Vektorfeld mit konstanter Divergenz} \end{array} \right) = \begin{array}{c} \text{Skalarfeld mit konstanter Divergenz} \end{array}$

quellen
ist ein
gleiches
Ver

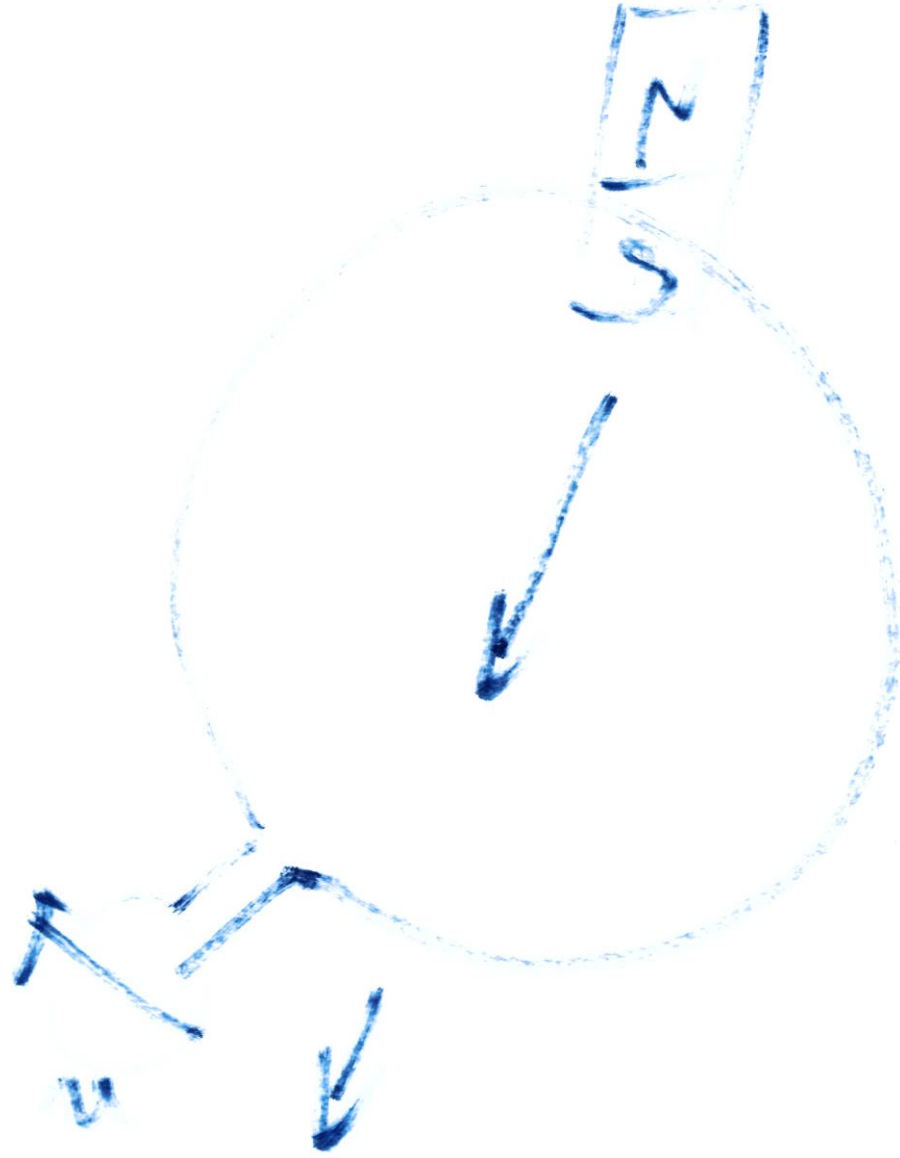
$\text{div} \left(\begin{array}{|c|} \hline \text{Diagram of a vector field with arrows pointing outwards from a central point} \\ \hline \end{array} \right) = \begin{array}{|c|} \hline \begin{array}{cc} \text{+5} & \text{-5} \\ \hline \end{array} \\ \hline \end{array}$

The diagram on the left shows a square region with a vector field represented by red arrows. The arrows originate from a central point and point outwards in all directions, indicating a positive divergence. The entire diagram is enclosed in a blue square frame, which is itself surrounded by a red curved line.

The diagram on the right shows a square region with three red dots. Two dots are located at the top corners, each with a blue label "+5" below it. A third dot is located at the bottom center, with a blue label "-5" below it. This represents a net divergence of zero, as the positive contributions from the top corners are balanced by the negative contribution from the bottom center.

A circled number "6" is written in red in the top right corner.

7



- $\text{rot}(\vec{E}) = \vec{\nabla} \times \vec{E}$

⑧

- $\text{rot}(\text{Vektorfeld})$ ist ein Vektorfeld.

- $\text{rot} \left(\begin{array}{|c|} \hline \text{Vektorfeld} \\ \hline \end{array} \right) = \begin{array}{|c|} \hline \text{Vektor} \\ \hline \end{array}$



9

$$\text{div rot } \vec{V} = 0.$$

$$\Rightarrow \text{div rot } \vec{E} = - \frac{1}{c} \frac{d}{dt} \text{div } \vec{B}$$

$$\Rightarrow (\text{div } \vec{B}) = 0$$

$$\begin{aligned} \textcircled{4} \Rightarrow 0 &= \rho \cdot \text{div } \vec{j} + \rho \cdot \epsilon \frac{\text{div } \vec{E}}{\partial t} \\ &= \rho \cdot (\text{div } \vec{j} + \dot{\epsilon}) \end{aligned}$$

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = -\vec{\nabla} \times \vec{B}$$

(10)

④ $\vec{J} = 0$ \vec{E}

~~$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E})$$~~

~~$$= \vec{\nabla} \cdot (\vec{\nabla} \times \vec{E})$$~~

~~$$= \vec{\nabla} \cdot (\vec{E} \times \vec{B})$$~~

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E})$$

$$= \left(-\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2} \right) \vec{E}$$

$$= -\Delta \vec{E}$$

$$\boxed{-\mu_0 \epsilon_0 \ddot{E} + \Delta E = 0}$$

11

$$\underline{j=0}$$

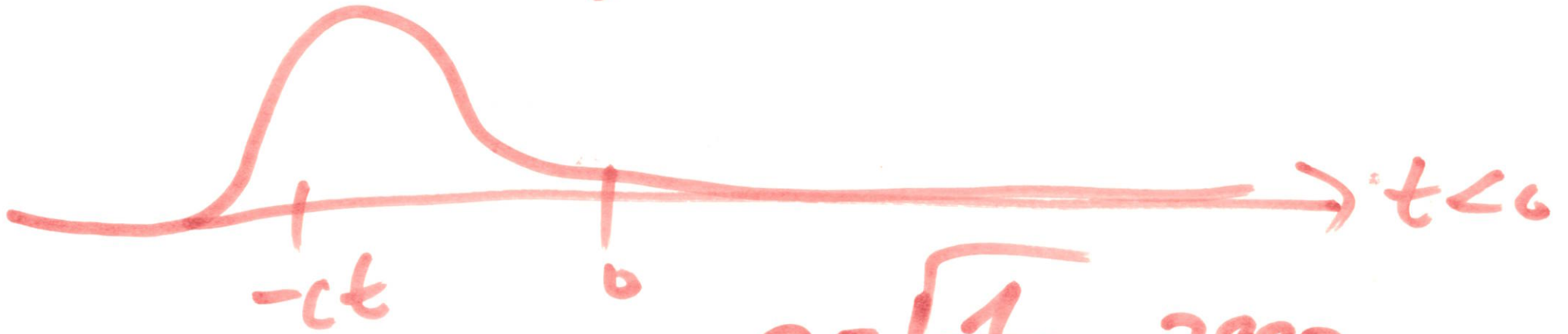
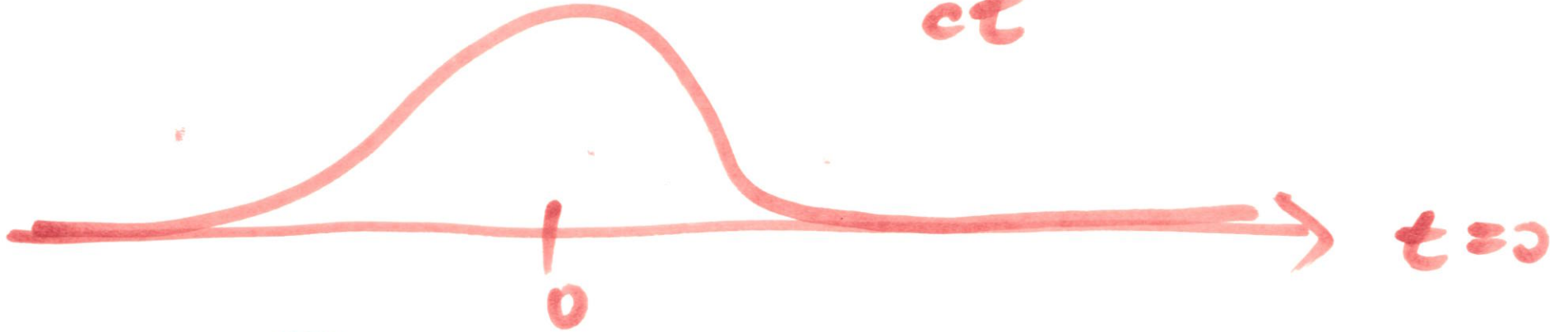
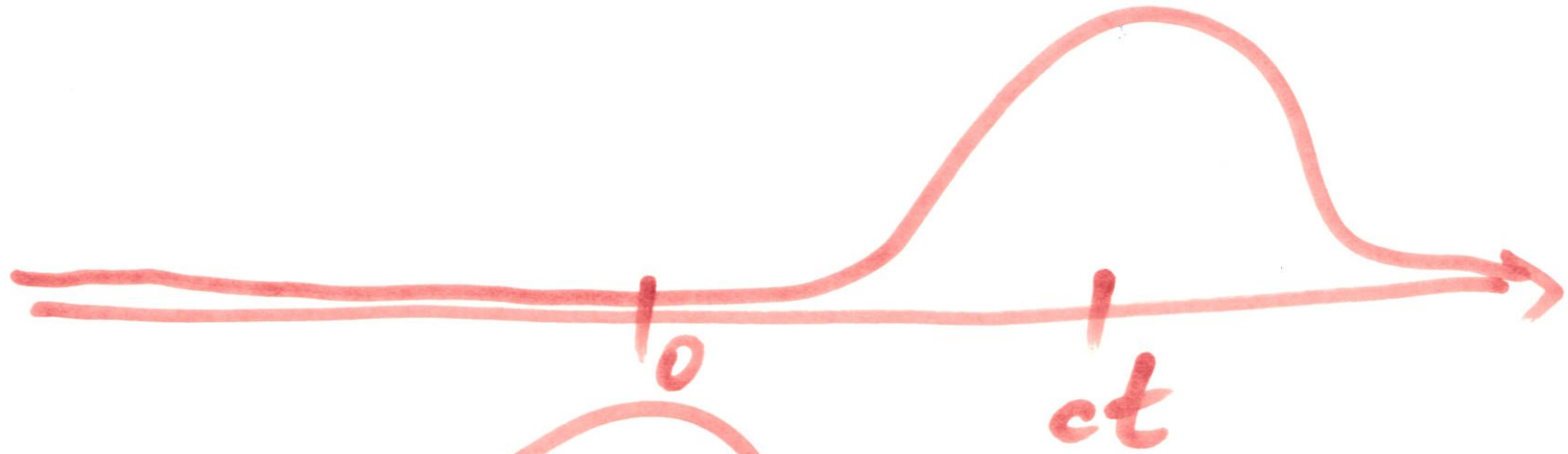
Ausg.: $E_1 = f_1(x - ct)$, $c = \text{const.}$

$$-\mu_0 \epsilon_0 \ddot{E}_1 + E_1'' = 0$$

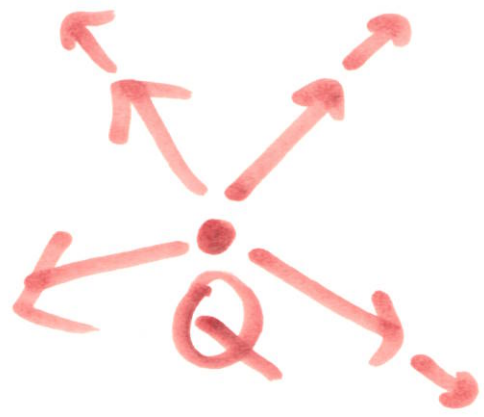
$$-\mu_0 \epsilon_0 (x^2) f_1'' + f_1'' = 0 \Rightarrow$$

$$c = \sqrt{\frac{1}{\mu_0 \epsilon_0}}$$

(12)



$$c = \sqrt{\frac{1}{\mu_0 \epsilon_0}} = 299792458 \frac{\text{m}}{\text{s}}$$



$$E(r) = \frac{1}{4\epsilon_0 \pi} \frac{Q}{r^2}$$

①

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} \frac{\vec{r}}{r}$$

$$\text{div}(\vec{E}) = \frac{\rho}{\epsilon_0}$$

$$r = |\vec{r}|$$

= Länge des Vektors \vec{r}

Wieso impli-

Zieren die
Maxwell-Gln diese

Formel fürs E-Feld einer Punktladung?

Schritt 1:



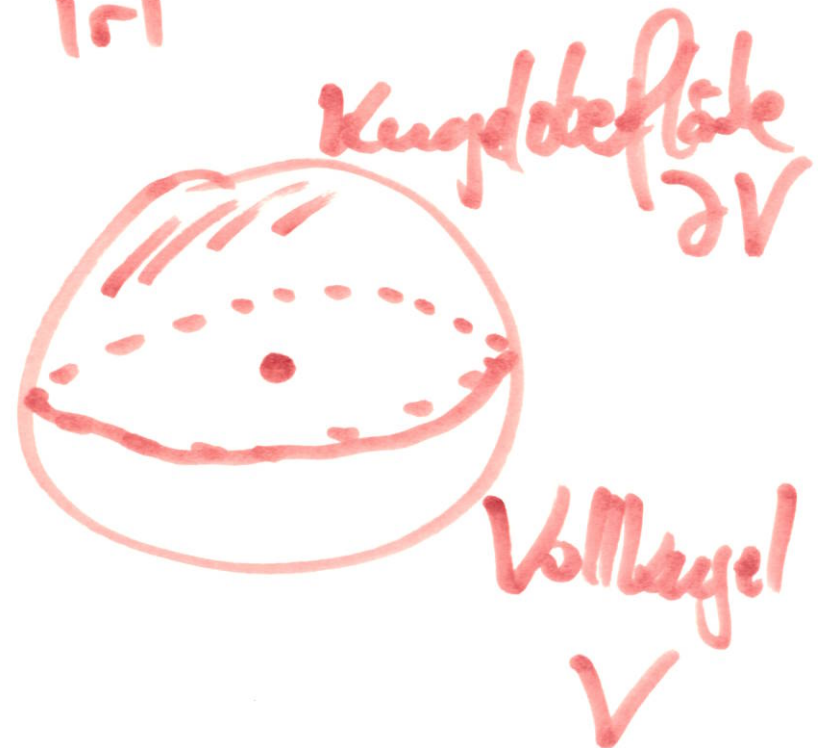
Die Situation ist
Rotationssymmetrisch.

②

Daher muss das \vec{E} -Feld radial sein:

$$\vec{E}(\vec{r}) = f(r) \frac{\vec{r}}{r}$$

Schritt 2: Über integrieren!



③

$$\oint \vec{E} \cdot d\vec{A}$$

∂V

Oberflächenintegral
andere Schreibweise:

$$\oint_{\partial V} \vec{E} \cdot d\vec{A}$$



$$f(r) \cdot 4\pi r^2$$

$$\oint_{\partial V} \vec{E} \cdot d\vec{A} = \int_V \operatorname{div}(\vec{E}) dV \quad (4)$$

\uparrow Stokes \uparrow Volumenintegral,
 andere Schreibweise:

$$\textcircled{1} \int_V \frac{\rho}{\epsilon_0} dV = \frac{1}{\epsilon_0} \int_V \rho dV$$

$$\Rightarrow f(r) = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} \cdot \underbrace{V_j}_{\substack{\text{eingeschlossene} \\ \text{Ladung}}}$$