Performance evaluation: Homework 3

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1 Simulate

Both these questions are answered for $\lambda = 70\ req/s$.

1. The number of requests arrived and number of requests served since the beginning of he simulation is shown on Figure 1. We can see that they arrive at a steady rate and leave the processor at the same rate since the arrival rate λ (70) is low enough for the jobs not to buffer in the processor queue.

2. The number of type i jobs in the queue, for $i = 1, 2$, is shown on Figure 2 and confirms what have been stated in point *1.*. Indeed, the arrival rate of *70 req/s* is low enough so the processes that arrive are almost instantly served.

For the two following questions we also took an arrival rate of $\lambda = 70\ req/s$.

	Type 1 job	Type 2 job
Average response time waiting + service time	17.8 ms	12.3 ms
Average number of jobs served per second	55.9	81.1

We clearly see that the average number of jobs served per second is simply the inverse of the average response time in second.

2 Stationarity

We can see in Figure 3 the plot of number of jobs in the queue for λ from 50 to 250 with a step of 25 for one simulation at each time. We notice that the system become non-stationary for $\lambda \in [100, 115]$. To be more precise on this estimation, we plot in Figure 4 for λ from 105 to 113. These plots confirm that the system become non stationary in this window.
Let's determine analytically for which λ the the system is stationary.
We recall that: $\rho = \lambda *$ average service time, with:

$$\begin{cases} \rho < 1 & \text{system has a stationary regime} \\ \rho > 1 & \text{system has no stationary regime} \end{cases}$$

$$\begin{aligned}
\rho < 1 & \\
&\Leftrightarrow \frac{\lambda}{1000} * \left(E\left[\mathrm{LogN}(\mu, \sigma^2)\right] + E\left[\mathrm{U}(a, b)\right]\right) < 1 \\
&\Leftrightarrow \lambda < \frac{1000}{\left(E\left[\mathrm{LogN}(\mu, \sigma^2)\right] + E\left[\mathrm{U}(a, b)\right]\right)} \\
&\Leftrightarrow \lambda < \frac{1000}{e^{\mu + \frac{\sigma^2}{2}} + \frac{a+b}{2}} \\
&\Leftrightarrow \lambda < \frac{1000}{e^{2 + \frac{0.5^2}{2}} + \frac{0.9+0.7}{2}} \\
&\Leftrightarrow \lambda < 109
\end{aligned} \tag{1}$$

So analytically we found that the system is stationary until $\lambda = 109$, then for higher values, it loses this property.

3 Remove transients

The initial transient period ends when the general tendency of the points changes in curvature (or concavity). To compute the exact time value where the change occurs, and thus remove the initial transient phase, we took the second derivative of the average number jobs in the queue. Then we simply checked where this second derivative changes sign and defined this as the end of the transient phase. Here are the resulting confidence intervals for both cases.

3.1 With initial transient phase

$CI_{median,\lambda=70,type=1} = [1.22, 1.29]$
$CI_{mean,\lambda=70,type=1} = [1.23, 1.30]$
$CI_{median,\lambda=108,type=1} = [23.10, 31.29]$
$CI_{mean,\lambda=108,type=1} = [25.46, 33.71]$
$CI_{median,\lambda=70,type=2} = [1.56, 1.87]$
$CI_{mean,\lambda=70,type=2} = [1.58, 1.90]$
$CI_{median,\lambda=108,type=2} = [24.29, 32.97]$
$CI_{mean,\lambda=108,type=2} = [26.65, 34.90]$

3.2 Without initial transient phase

$CI_{median,\lambda=70,type=1} = [1.42, 1.51]$
$CI_{mean,\lambda=70,type=1} = [1.45, 1.53]$
$CI_{median,\lambda=108,type=1} = [24.00, 33.22]$
$CI_{mean,\lambda=108,type=1} = [27.94, 37.03]$
$CI_{median,\lambda=70,type=2} = [1.17, 1.22]$
$CI_{mean,\lambda=70,type=2} = [1.18, 1.20]$
$CI_{median,\lambda=108,type=2} = [24.16, 33.30]$
$CI_{mean,\lambda=108,type=2} = [28.01, 37.04]$

4 Little's law

Little's Law states that:

> The long-term average number of customers in a stable system L is equal to the long-term average effective arrival rate, λ, multiplied by the average time a customer spends in the system, W; or expressed algebraically: $L = \lambda W$.

Here the arrival rate λ of the type 1 jobs is simply *70* or *108*. On the other hand, for type 2 jobs, the arrival rate λ is defined by the average number of type 1 jobs served per second, which has been computed in part 1. The average response time W for each type of job has also been computed in part 1. Finally, the average number of jobs L of type i in the system, for $i = 1, 2$, has been computed in the previous exercise 3.

Therefore, we have the following results:

4.1 With initial transient phase

- Arrival rate $\lambda = 70$
  - Type 1 jobs
    - $\lambda = 70\ req/sec$
    - $W = 17.8/1000 = 0.0178\ sec$
    - Little's $L = \lambda W = 1.246\ req$
    - Our $L = 1.26\ req$

    Here we can see that Little's law apply and that the average number of jobs in the system.
  - Type 2 jobs
    - $\lambda = 55.9\ req/sec$
    - $W = 12.3/1000 = 0.0123\ sec$
    - Little's $L = \lambda W = 0.68\ req$
    - Our $L = 1.74\ req$

    Here we see that Little's value is close to the computed mean number of jobs in the system, the difference might come from the fact that the $\lambda = 55.9$ is computed out of one simulation.
- Arrival rate $\lambda = 108\ req/s$
  - Type 1 jobs
    - $\lambda = 108\ req/sec$
    - $W = 497/1000 = 0.497\ sec$
    - Little's $L = \lambda W = 53.67\ req$
    - Our $L = 29.56\ req$
  - Type 2 jobs
    - $\lambda = 2\ req/sec$
    - $W = 493/1000 = 0.493\ sec$
    - Little's $L = \lambda W = 0.9868\ req$
    - Our $L = 30.76\ req$

We note that with an arrival rate of $\lambda = 108$ it would require much more time to enter the stationary phase. Hence the Little's law is not even ϵ-satisfied in both cases.

4.2 Without initial transient phase

- $\lambda = 70$
  - Type 1 jobs
    - $\lambda = 70 \ req/sec$
    - $W = 17.8/1000 = 0.0178 \ sec$
    - Little's $L = \lambda W = 1.246 \ req$
    - Our $L = 1.49 \ req$

    Here we see that the Little's law value is really close to our L, we can say that the law applies.
  - Type 2 jobs
    - $\lambda = 55.9 \ req/sec$
    - $W = 12.3/1000 = 0.0123 \ sec$
    - Little's $L = \lambda W = 0.68 \ req$
    - Our $L = 1.19 \ req$

    Here we also see that Little's law is relatively close to our L, the difference might come from the fact that λ come from a single simulation.
- $\lambda = 108$
  - Type 1 jobs
    - $\lambda = 108 \ req/sec$
    - $W = 497/1000 = 0.497 \ sec$
    - Little's $L = \lambda W = 53.67 \ req$
    - Our $L = 32.49 \ req$
  - Type 2 jobs
    - $\lambda = 2 \ req/sec$
    - $W = 493/1000 = 0.493 \ sec$
    - Little's $L = \lambda W = 0.9868 \ req$
    - Our $L = 32.53 \ req$

    We note that with an arrival rate of $\lambda = 108$ it would require much more time to enter the stationary phase. Hence the Little's law is not even ϵ-satisfied in both cases.

5 Parameter estimation and confidence interval

5.1 First experiment

We want a 95% confidence interval for the ϵ parameter.
Assuming we've seen z successes (a success being to see a type 2) over n independent experiment.
In our case z = 0 since we've only seen type 1 jobs, and n = 10 since we picked 10 requests at random. We have to be careful when observing 0 success and in this case the confidence interval for ϵ (probability of seeing a type 2) is $[0, p_0(n)]$ with:

$$\begin{aligned} p_0(n) &= 1 - \left(\frac{1-\gamma}{2}\right)^{\frac{1}{n}} \\ &= 1 - \left(\frac{1-0.95}{2}\right)^{\frac{1}{10}} \\ &\approx 0.31 \end{aligned} \tag{2}$$

So our confidence interval at 95% is: $\epsilon \in [0, 0.31]$
It's a quite large uncertainty for the parameter ϵ knowing that it's a probability and therefore is comprised between 0 and 1. Moreover, our objective is to show that ϵ is nearly 0.
So we can say that 10 samples are not large enough.

Now we can compute the confidence interval at 95% for the λ until which the system is stable.

$$\begin{aligned} &\rho < 1 \\ &\Leftrightarrow \frac{\lambda}{1000} * \left((1-\epsilon)E\left[\mathrm{LogN}(\mu,\sigma^2)\right] + E\left[\mathrm{U}(a,b)\right]\right) < 1 \\ &\Leftrightarrow \lambda < \frac{1000}{((1-\epsilon)E\left[\mathrm{LogN}(\mu,\sigma^2)\right] + E\left[\mathrm{U}(a,b)\right])} \\ &\Leftrightarrow \lambda < \frac{1000}{(1-\epsilon)e^{\mu+\frac{\sigma^2}{2}} + \frac{a+b}{2}} \\ &\Leftrightarrow \lambda < \frac{1000}{(1-\epsilon)e^{2+\frac{0.5^2}{2}} + \frac{0.9+0.7}{2}} \\ &\Leftrightarrow \lambda \in [109, 152] \text{ taking } \epsilon \text{ between 0 and 0.31} \end{aligned} \tag{3}$$

5.2 Second experiment

We want to know the number of samples to be picked so that we can assume: $\epsilon < 95\%$. We keep assuming that we seen only type 1 and so we have z = 0 success, we want to determine n.
Taking our previous formula:

$$\begin{aligned}
0.01 > p_0(n) & \\
& \Leftrightarrow 0.01 > 1 - \left(\frac{1-\gamma}{2}\right)^{\frac{1}{n}} \\
& \Leftrightarrow 0.01 > 1 - \left(\frac{1-0.95}{2}\right)^{\frac{1}{n}} \\
& \Leftrightarrow 0.01 > 1 - (0.25)^{\frac{1}{n}} \\
& \Leftrightarrow -0.99 > -(0.25)^{\frac{1}{n}} \\
& \Leftrightarrow 0.99 < (0.25)^{\frac{1}{n}} \\
& \Leftrightarrow ln(0.99) < \frac{1}{n} ln(0.25) \\
& \Leftrightarrow \frac{ln(0.99)}{ln.0.25} > \frac{1}{n} \\
& \Leftrightarrow \frac{ln(0.25)}{ln.0.99} < n \\
& \Leftrightarrow n > 138
\end{aligned} \tag{4}$$

So we would need 138 samples with only type 1 jobs to assure that $\epsilon \in [0, 0.01]$ with a 95% confidence.

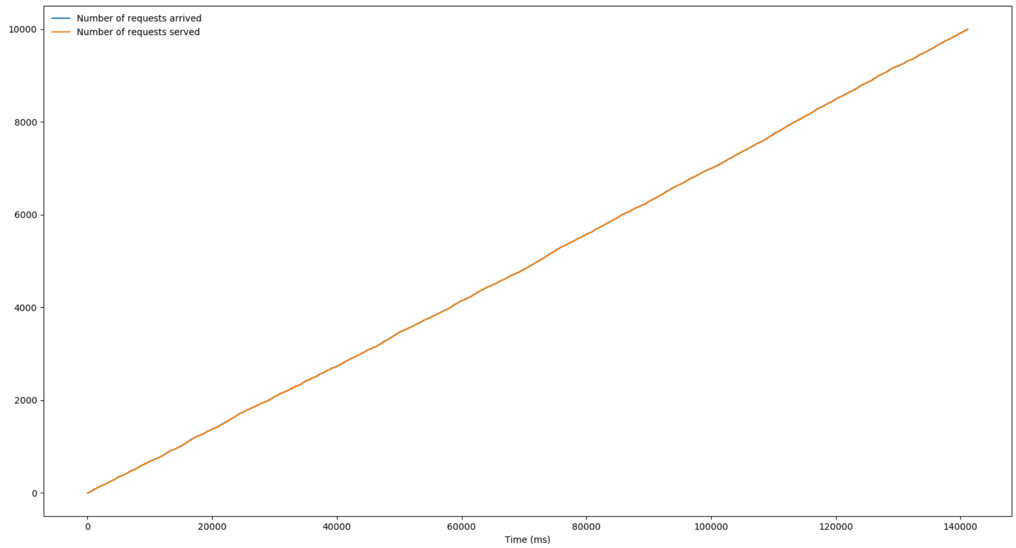

Figure 1: Number of requests arrived and number of requests served since the beginning of the simulation. ($\lambda = 70\ req/s$)

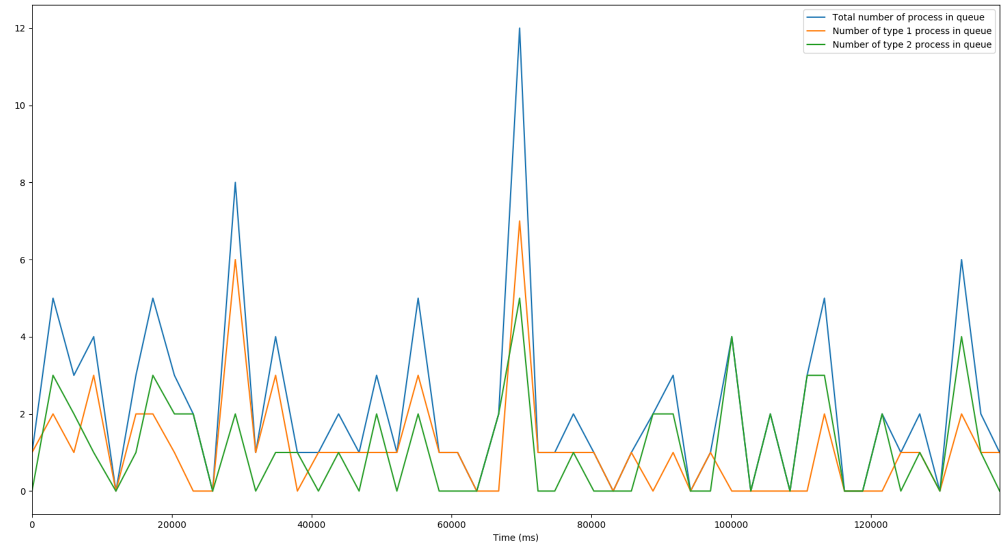

Figure 2: Number of jobs in the processor queue for each type $i = 1, 2$. ($\lambda = 70\ req/s$)

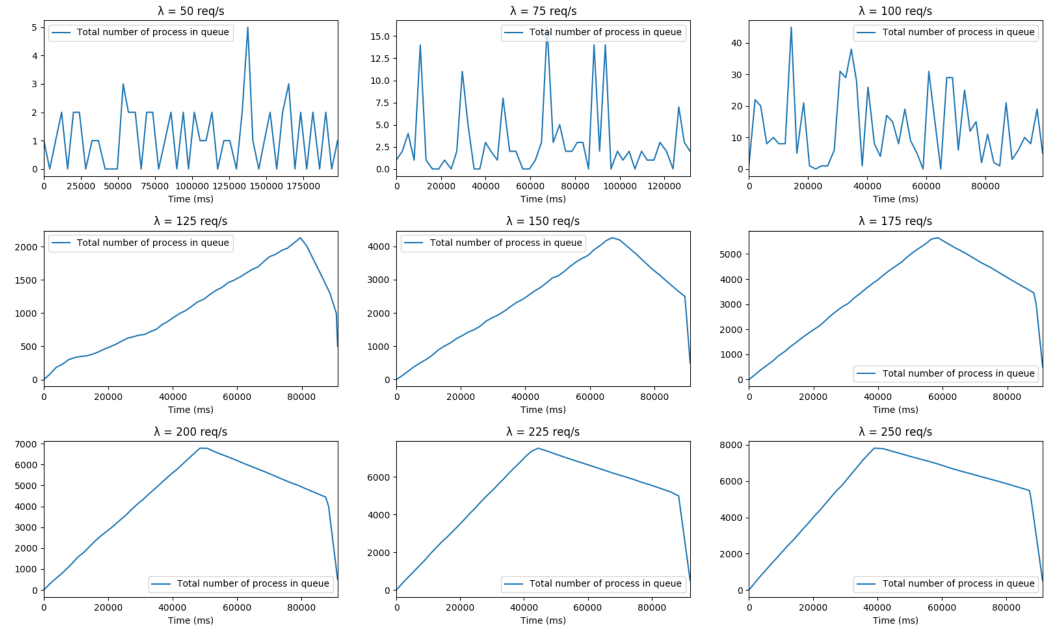

Figure 3: Number of jobs in the processor queue for various values of λ (requests per second).

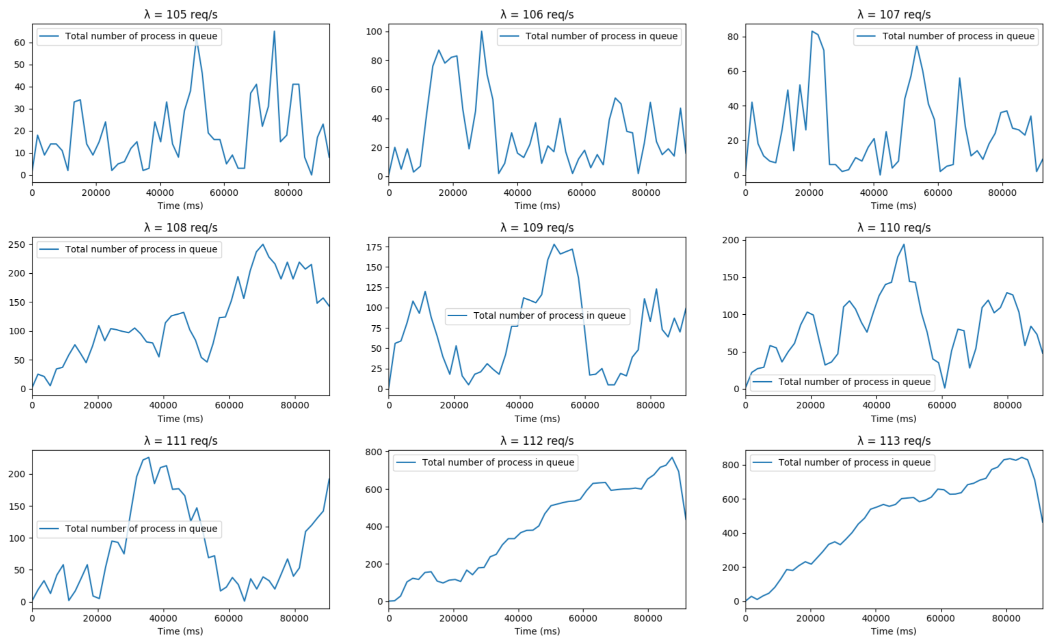

Figure 4: Number of jobs in the processor queue for various values of λ (requests per second).