MATH 137 Fall 2020: Practice Assignment 6

Q01. For $f(x) = \dfrac{x+1}{x-1}$, find $f'(x)$ using the limit definition.

Solution. □

Q02. Let $f(x) = \dfrac{ax+b}{ax-b}$ where $a \neq 0$, $b \neq 0$.

(a) Find $f'(x)$ using any method.

Solution. □

(b) Show that for $x \neq \frac{b}{a}$, $abf'(x) < 0$.

Proof. □

Q03. In each case, find $f'(x)$ using any method.

(a) $f(x) = 5^x \sin x + (x^3 + x^2)\cos x$.

Solution. □

(b) $f(x) = \dfrac{x^2 + x - 2}{x^3 + 6}$.

Solution. □

(c) $f(x) = \sqrt{2\tan^2 x + 3}$.

Solution. □

(d) $f(x) = 2^{\sin(\sec x)}$.

Solution. □

Q04. In each case, determine the equation of the tangent to $y = f(x)$ at the point where $x = a$.

(a) $f(x) = x^2$, $a = 3$.

Solution. □

(b) $f(x) = \cos x$, $a = -\dfrac{3\pi}{4}$.

Solution. □

(c) $f(x) = e^x$, $a = \ln \pi$.

Solution. □

(d) $f(x) = 4^x$, $a = -3$.

Solution. $\square$

Q05. Compute $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ and $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$ in each case.

(a) $y = \cos x^2$.

Solution. $\square$

(b) $y = \cos^2 x$.

Solution. $\square$

Q06.

(a) Use the Chain Rule to prove that the derivative of an even function is odd.

Proof. $\square$

(b) Using ONLY the Chain Rule and the Product Rule (and not the Reciprocal/Quotient rules), give an alternative proof of the Quotient Rule. [Hint: $\dfrac{f(x)}{g(x)} = f(x)(g(x))^{-1}$].

Proof. $\square$

Q07. If $y = f(u)$ and $u = g(x)$ where f and g are twice differentiable functions, prove that

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \frac{\mathrm{d}^2y}{\mathrm{d}u^2}\left(\frac{\mathrm{d}u}{\mathrm{d}x}\right)^2 + \frac{\mathrm{d}y}{\mathrm{d}u}\frac{\mathrm{d}^2u}{\mathrm{d}x^2}.$$

Proof. $\square$