WAFO Chapter 2

November 26, 2014

1 CHAPTER2 Modelling random loads and stochastic waves

Chapter2 contains the commands used in Chapter 2 of the tutorial and present some tools for analysis of random functions with respect to their correlation, spectral and distributional properties. The presentation is divided into three examples:

Example1 is devoted to estimation of different parameters in the model. Example2 deals with spectral densities and Example3 presents the use of WAFO to simulate samples of a Gaussian process.

Some of the commands are edited for fast computation.

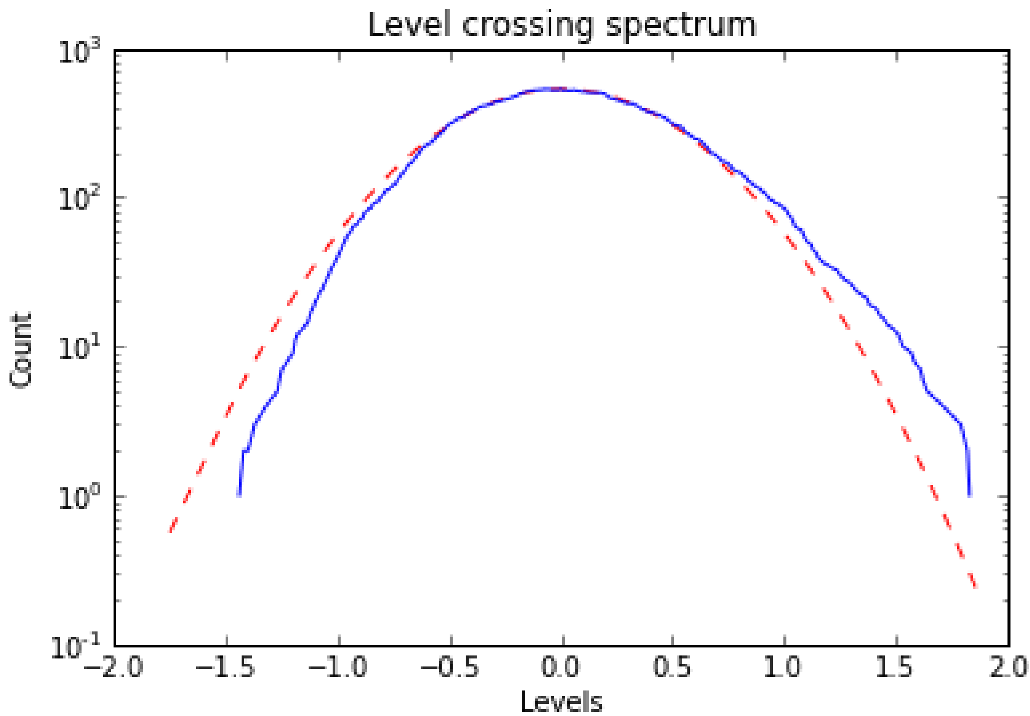
2 Section 2.1 Introduction and preliminary analysis

2.1 Example 1: Sea data

Observed crossings compared to the expected for Gaussian signals

```
In [1]: import wafo
        import wafo.objects as wo
        xx = wafo.data.sea()
        me = xx[:, 1].mean()
        sa = xx[:, 1].std()
        xx[:, 1] -= me
        ts = wo.mat2timeseries(xx)
        tp = ts.turning_points()

        cc = tp.cycle_pairs()
        lc = cc.level_crossings()
        lc.plot()
        show()
```



2.2 Average number of upcrossings per time unit

Next we compute the mean frequency as the average number of upcrossings per time unit of the mean level (= 0); this may require interpolation in the crossing intensity curve, as follows.

```
In [3]: T = xx[:, 0].max() - xx[:, 0].min()
        f0 = np.interp(0, lc.args, lc.data, 0) / T  #! zero up-crossing frequency
        print('f0 = %g' % f0)
```

```
f0 = 0.224071
```

2.3 Turningpoints and irregularity factor

```
In [4]: fm = len(tp.data) / (2 * T)    # frequency of maxima
        alfa = f0 / fm                 # approx Tm24/Tm02

        print('fm = %g, alpha = %g, ' % (fm, alfa))
```

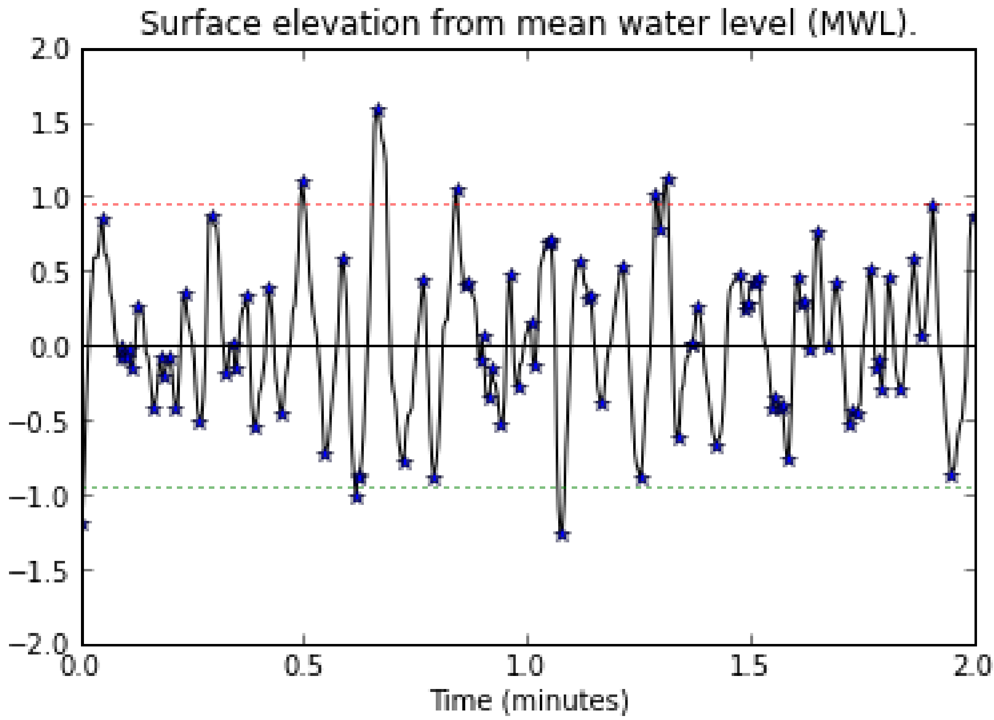
```
fm = 0.456159, alpha = 0.491212,
```

2.4 Visually examine data

We finish this section with some remarks about the quality of the measured data. Especially sea surface measurements can be of poor quality. We shall now check the quality of the dataset {xx}. `It is always good practice to visually examine the data before the analysis to get an impression of the quality, non-linearities and narrow-bandedness of the data.First we shall plot the data and zoom in on a specific region. A part of sea data is visualized with the following commands`

```
In [5]: clf()
        ts.plot_wave('k-', tp, '*', nfig=1, nsub=1)

        axis([0, 2, -2, 2])
        show()
```



2.5 Finding possible spurious points

However, if the amount of data is too large for visual examinations one could use the following criteria to find possible spurious points. One must be careful using the criteria for extremevalue analysis, because it might remove extreme waves that are OK and not spurious.

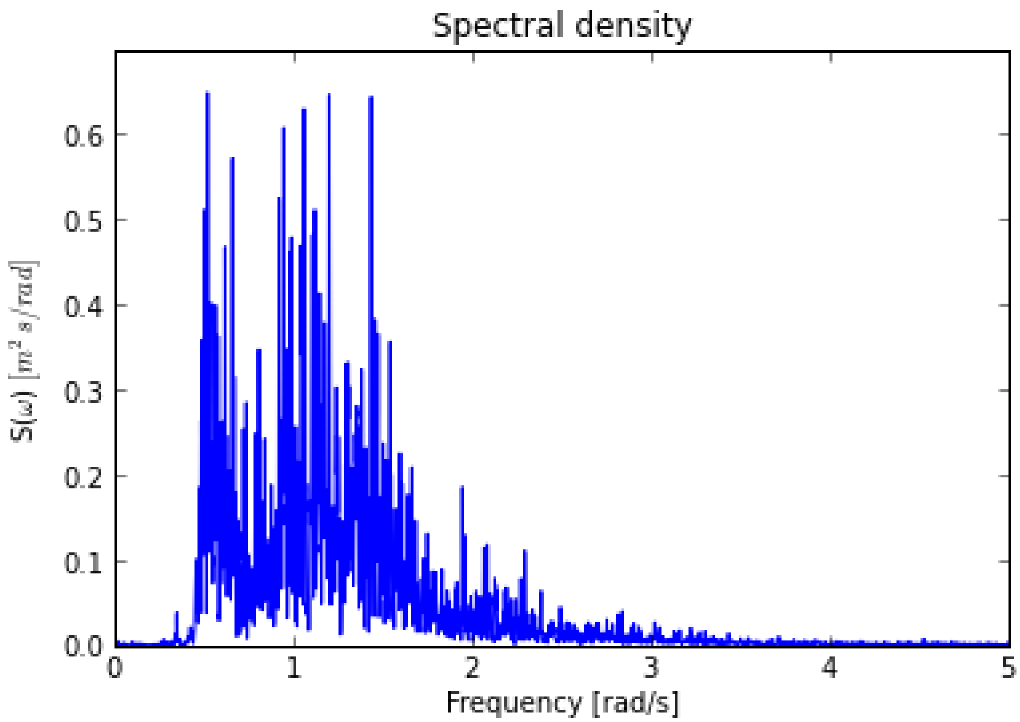
```
In [6]: import wafo.misc as wm
        dt = ts.sampling_period()
        # dt = np.diff(xx[:2,0])
        dcrit = 5 * dt
        ddcrit = 9.81 / 2 * dt * dt
        zcrit = 0
        inds, indg = wm.findoutliers(ts.data, zcrit, dcrit, ddcrit, verbose=True)
```

```
Found 0 spurious positive jumps of Dx
Found 0 spurious negative jumps of Dx
Found 37 spurious positive jumps of D^2x
Found 200 spurious negative jumps of D^2x
Found 244 consecutive equal values
Found the total of 1152 spurious points
```

2.6 Section 2.2 Frequency Modeling of Load Histories

Periodogram: Raw spectrum

```
In [7]: clf()
        Lmax = 9500
        S = ts.tospecdata(L=Lmax)
        S.plot()
        axis([0, 5, 0, 0.7])
        show()
```



2.7 Calculate moments

```
In [8]: mom, text = S.moment(nr=4)
        print('sigma = %g, m0 = %g' % (sa, sqrt(mom[0])))
```

```
sigma = 0.472955, m0 = 0.472955
```

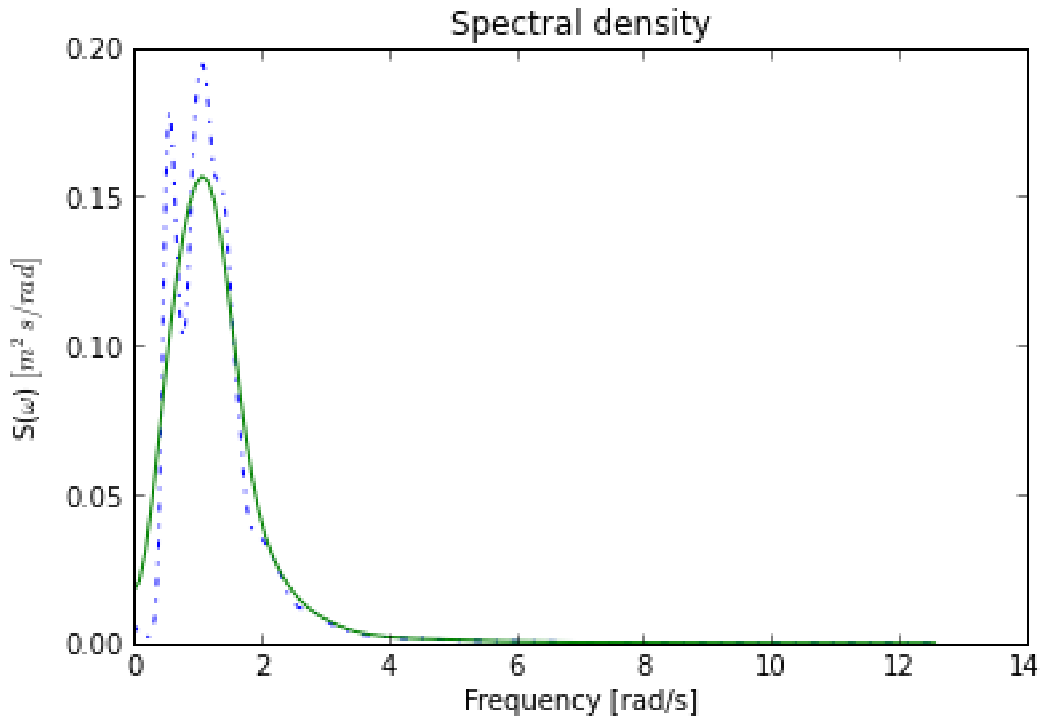
2.8 Section 2.2.1 Random functions in Spectral Domain - Gaussian processes

2.9 Smoothing of spectral estimate

By decreasing Lmax the spectrum estimate becomes smoother.

```
In [9]: clf()
        Lmax0 = 200; Lmax1 = 50
        S1 = ts.tospecdata(L=Lmax0)
        S2 = ts.tospecdata(L=Lmax1)
        S1.plot('-.')
```

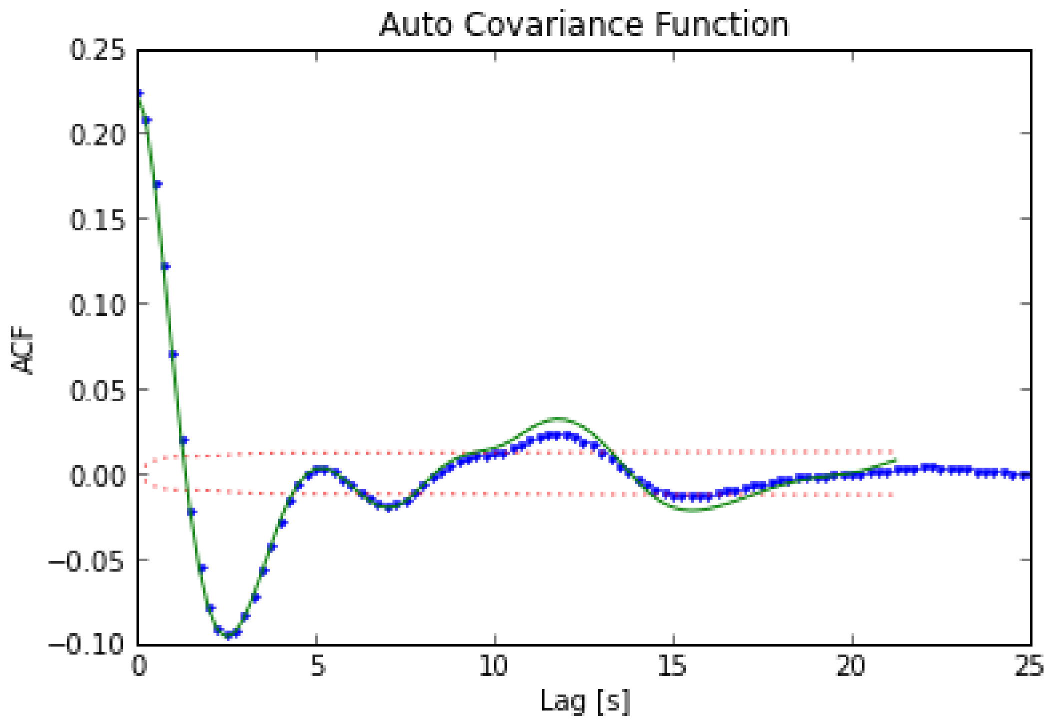
```
S2.plot()
show()
```



2.10 Estimated autocovariance

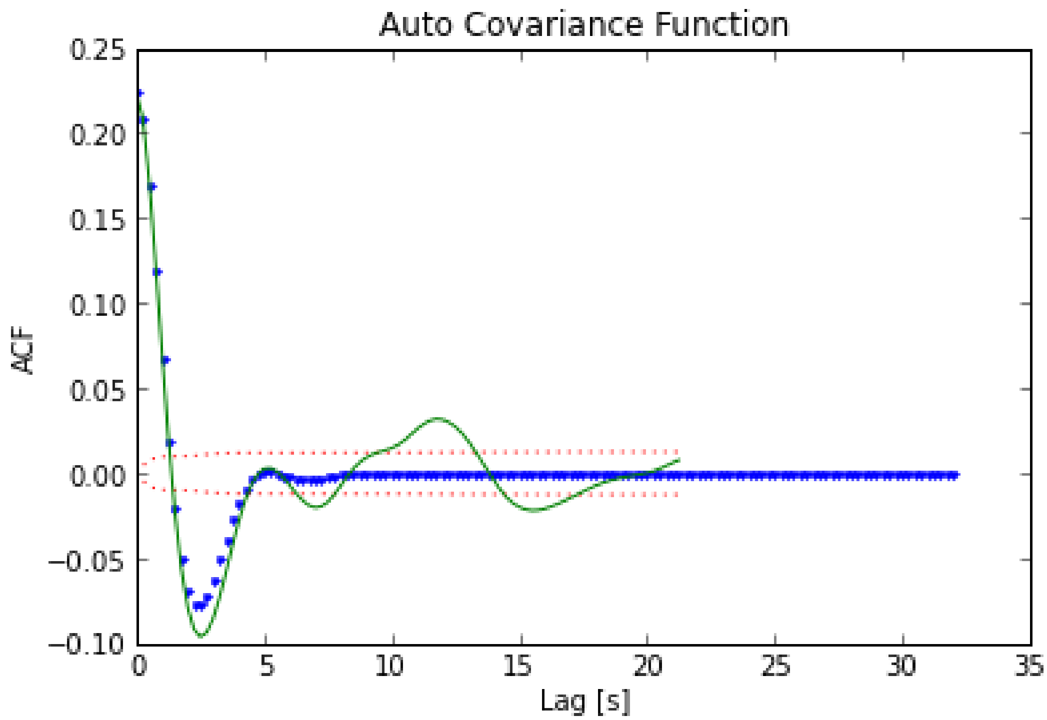
Obviously knowing the spectrum one can compute the covariance function. The following code will compute the covariance for the unimodal spectral density S1 and compare it with estimated covariance of the signal xx.

```
In [10]: clf()
         Lmax = 85
         R1 = S1.tocovdata(nr=1)
         Rest = ts.tocovdata(lag=Lmax)
         R1.plot('.')
         Rest.plot()
         axis([0, 25, -0.1, 0.25])
         show()
```



We can see in Figure below that the covariance function corresponding to the spectral density S2 significantly differs from the one estimated directly from data. It can be seen in Figure above that the covariance corresponding to S1 agrees much better with the estimated covariance function.

```
In [11]: clf()
         R2 = S2.tocovdata(nr=1)
         R2.plot('.')
         Rest.plot()
         show()
```



2.11 Section 2.2.2 Transformed Gaussian models

We begin with computing skewness and kurtosis for the data set xx and compare it with the second order wave approximation proposed by Winterstein:

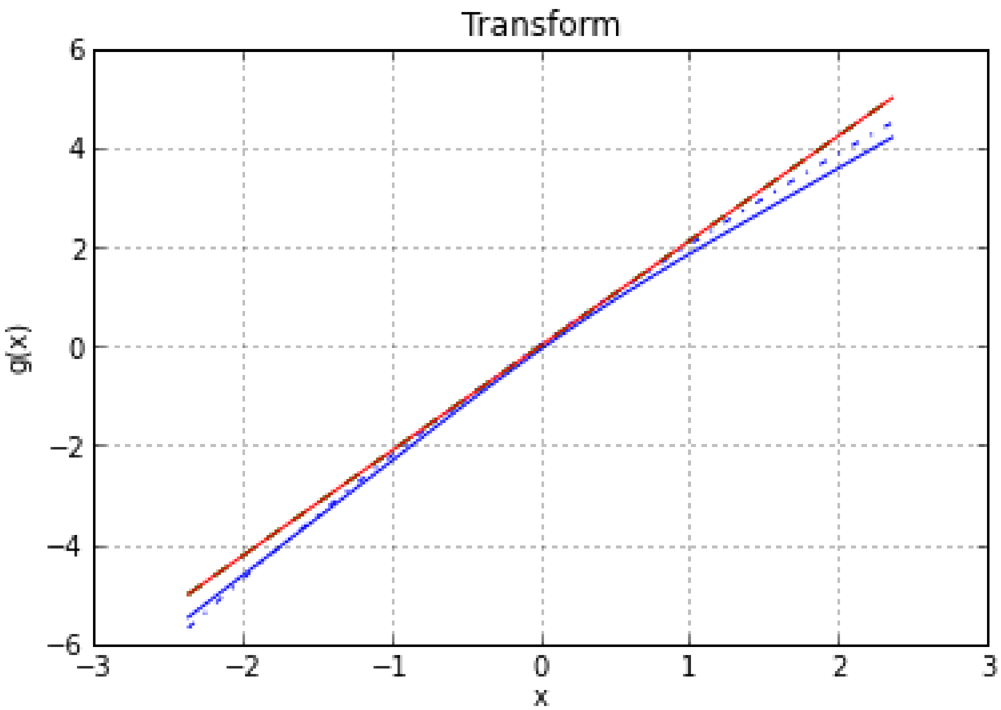
```
In [13]: import wafo.stats as ws
         rho3 = ws.skew(xx[:, 1])
         rho4 = ws.kurtosis(xx[:, 1])

         sk, ku = S1.stats_nl(moments='sk')
```

Comparisons of 3 transformations

```
In [14]: clf()
         import wafo.transform.models as wtm
         gh = wtm.TrHermite(mean=me, sigma=sa, skew=sk, kurt=ku).trdata()
         g = wtm.TrLinear(mean=me, sigma=sa).trdata() # Linear transformation
         glc, gemp = lc.trdata(mean=me, sigma=sa)

         glc.plot('b-') #! Transf. estimated from level-crossings
         gh.plot('b-.') #! Hermite Transf. estimated from moments
         g.plot('r')
         grid('on')
         show()
```



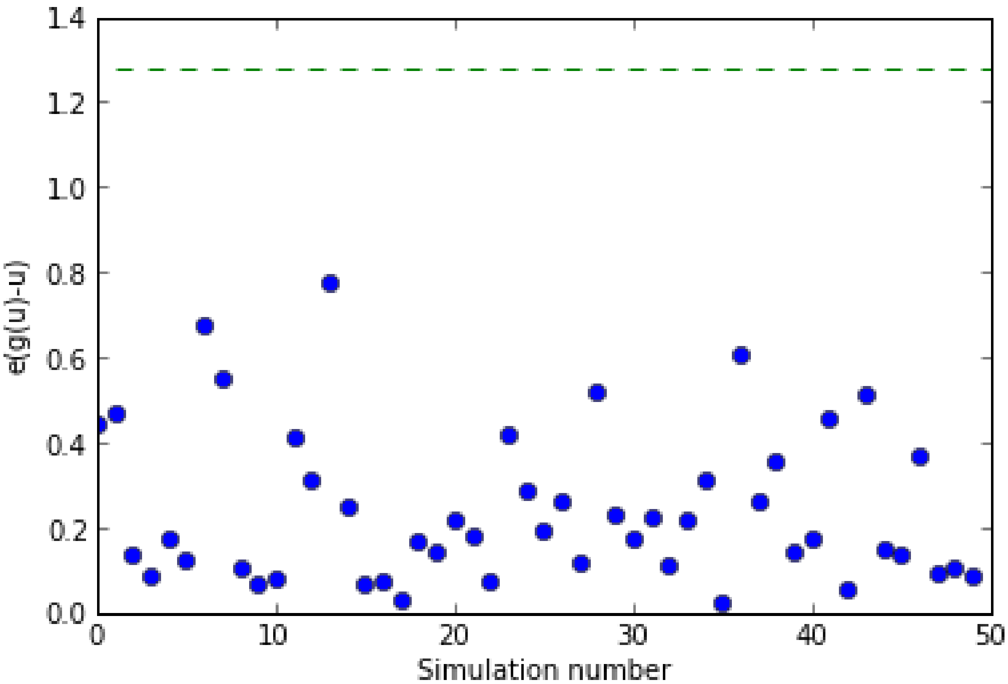
2.12 Test Gaussianity of a stochastic process

TESTGAUSSIAN simulates e(g(u)-u) = int (g(u)-u)^2 du for Gaussian processes given the spectral density, S. The result is plotted if test0 is given. This is useful for testing if the process X(t) is Gaussian. If 95% of TEST1 is less than TEST0 then X(t) is not Gaussian at a 5% level.

As we see from the figure below: none of the simulated values of test1 is above 1.00. Thus the data significantly departs from a Gaussian distribution.

In [15]:
```
clf()
test0 = glc.dist2gauss()
# the following test takes time
N = len(xx)
test1 = S1.testgaussian(ns=N, cases=50, test0=test0)
is_gaussian = sum(test1 > test0) > 5
print(is_gaussian)
show()
```

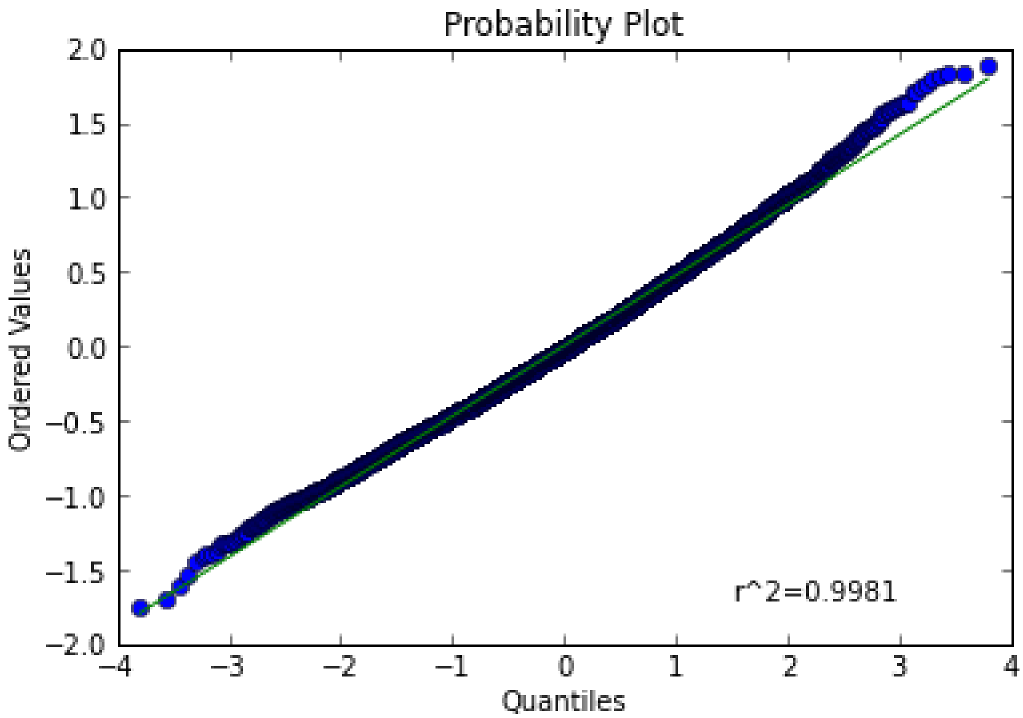
False



2.13 Normalplot of data xx

indicates that the underlying distribution has a ‘‘heavy’’ upper tail and a ‘‘light’’ lower tail.

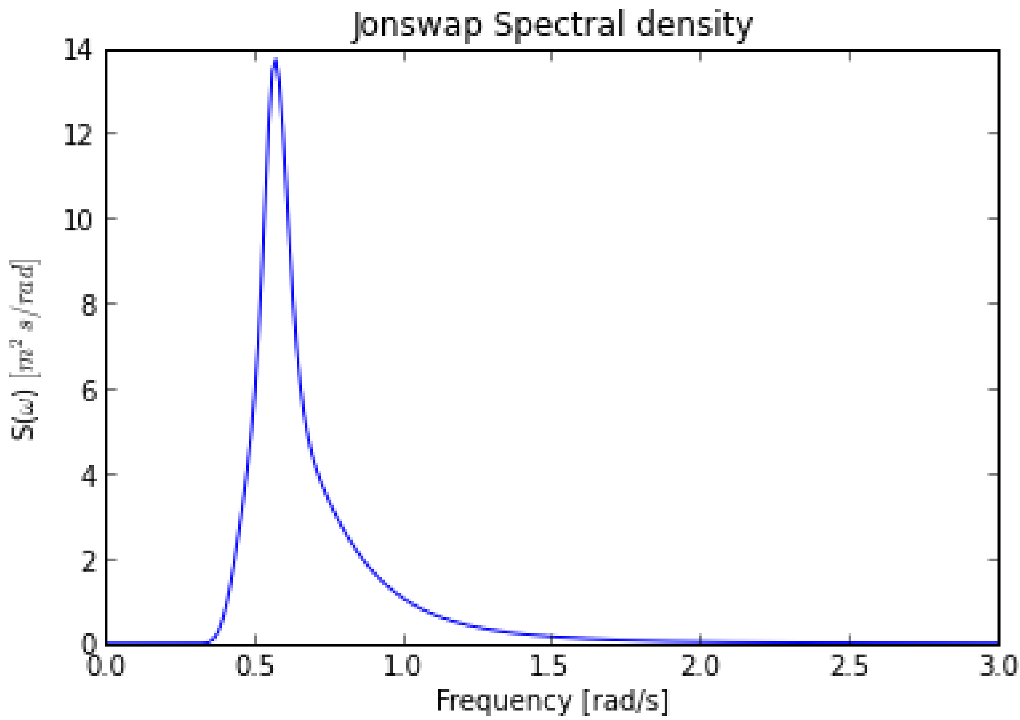
In [16]:
```
clf()
import pylab
ws.probplot(ts.data.ravel(), dist='norm', plot=pylab)
show()
```



2.14 Section 2.2.3 Spectral densities of sea data

Example 2: Different forms of spectra

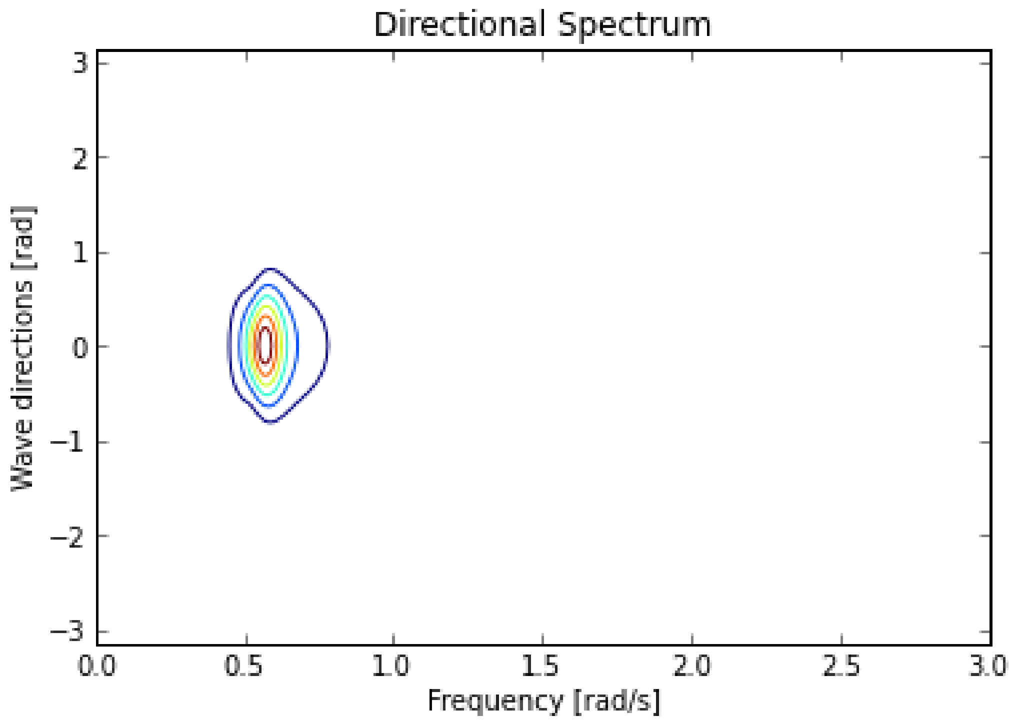
```
In [17]: import wafo.spectrum.models as wsm
         clf()
         Hm0 = 7; Tp = 11;
         spec = wsm.Jonswap(Hm0=Hm0, Tp=Tp).tospecdata()
         spec.plot()
         show()
```



3 Directional spectrum and Encountered directional spectrum

3.1 Directional spectrum

```
In [18]: clf()
         D = wsm.Spreading('cos2s')
         Sd = D.tospecdata2d(spec)
         Sd.plot()
         show()
```



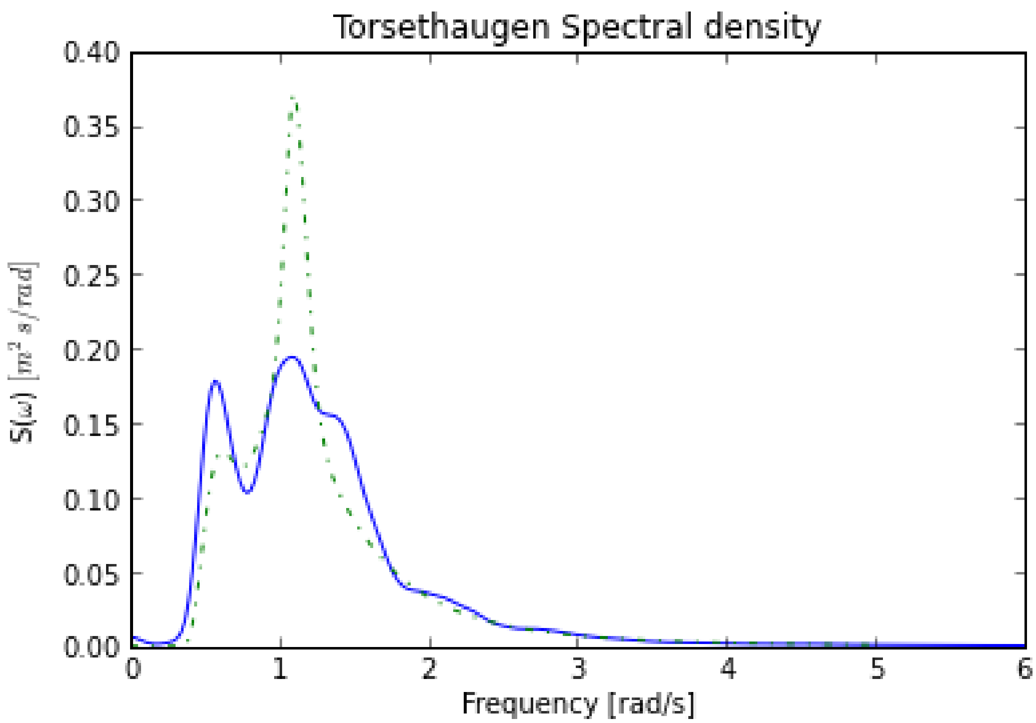
3.2 Encountered directional spectrum

In []:
```
#clf()
#Se = spec2spec(Sd,'encdir',0,10);
#plotspec(Se), hold on
#plotspec(Sd,1,'--'), hold off
##!wafostamp('','(ER)')
#disp('Block = 17'),pause(pstate)
#
##!#! Frequency spectra
#clf
#Sd1 =spec2spec(Sd,'freq');
#Sd2 = spec2spec(Se,'enc');
#plotspec(spec), hold on
#plotspec(Sd1,1,'.'),
#plotspec(Sd2),
##!wafostamp('','(ER)')
#hold off
#disp('Block = 18'),pause(pstate)
#
##!#! Wave number spectrum
#clf
#Sk = spec2spec(spec,'k1d')
#Skd = spec2spec(Sd,'k1d')
#plotspec(Sk), hold on
#plotspec(Skd,1,'--'), hold off
##!wafostamp('','(ER)')
```

```
#disp('Block = 19'),pause(pstate)
#
##!#! Effect of waterdepth on spectrum
#clf
#plotspec(spec,1,'--'), hold on
#S20 = spec;
#S20.S = S20.S.*phi1(S20.w,20);
#S20.h = 20;
#plotspec(S20),  hold off
##!wafostamp('','(ER)')
#disp('Block = 20'),pause(pstate)
#
##!#! Section 2.3 Simulation of transformed Gaussian process
##!#! Example 3: Simulation of random sea
##! The reconstruct function replaces the spurious points of seasurface by
##! simulated data on the basis of the remaining data and a transformed Gaussian
##! process. As noted previously one must be careful using the criteria
##! for finding spurious points when reconstructing a dataset, because
##! these criteria might remove the highest and steepest waves as we can see
##! in this plot where the spurious points is indicated with a '+' sign:
##!
#clf
#[y, grec] = reconstruct(xx,inds);
#waveplot(y,'-',xx(inds,:),'+',1,1)
#axis([0 inf -inf inf])
##!wafostamp('','(ER)')
#disp('Block = 21'),pause(pstate)
#
##! Compare transformation (grec) from reconstructed (y)
##! with original (glc) from (xx)
#clf
#trplot(g), hold on
#plot(gemp(:,1),gemp(:,2))
#plot(glc(:,1),glc(:,2),'-.')
#plot(grec(:,1),grec(:,2)), hold off
#disp('Block = 22'),pause(pstate)
#
##!#!
#clf
#L = 200;
#x = dat2gaus(y,grec);
#Sx = dat2spec(x,L);
#disp('Block = 23'),pause(pstate)
#
##!#!
#clf
#dt = spec2dt(Sx)
#Ny = fix(2*60/dt) #! = 2 minutes
#Sx.tr = grec;
#ysim = spec2sdat(Sx,Ny);
#waveplot(ysim,'-')
##!wafostamp('','(CR)')
#disp('Block = 24'),pause(pstate)
```

3.3 Estimated spectrum compared to Torsethaugen spectrum

In [19]:
```
clf()
fp = 1.1;dw = 0.01
H0 = S1.characteristic('Hm0')[0]
St = wsm.Torsethaugen(Hm0=H0,Tp=2*pi/fp).tospecdata(np.arange(0,5+dw/2,dw))
S1.plot()
St.plot('-.')
axis([0, 6, 0, 0.4])
show()
```

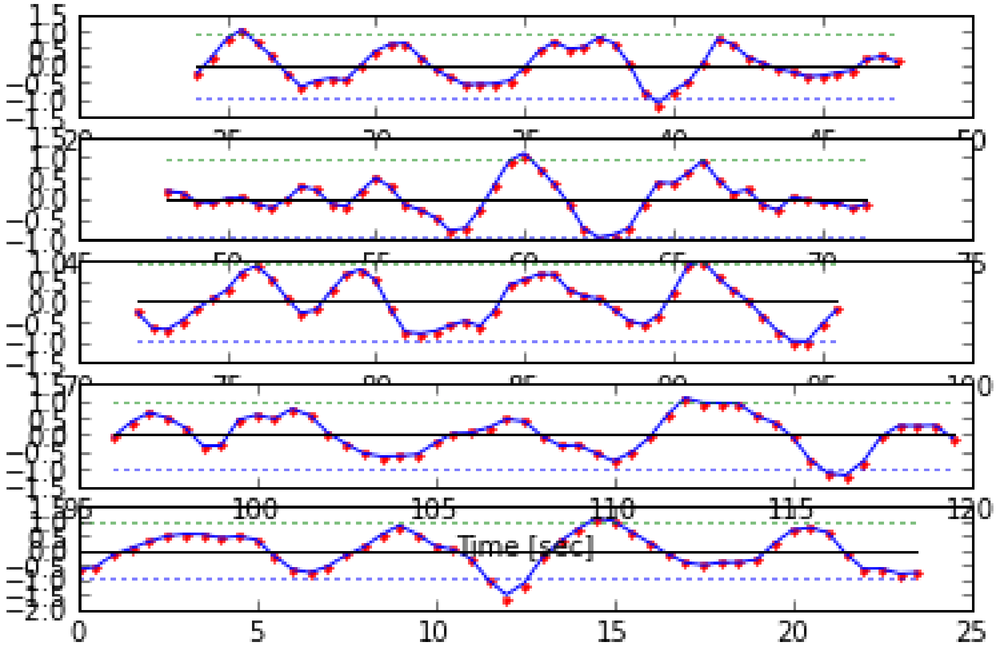


3.4 Transformed Gaussian model compared to Gaussian model

In [20]:
```
dt = St.sampling_period()
va, sk, ku = St.stats_nl(moments='vsk' )
#sa = sqrt(va)
gh = wtm.TrHermite(mean=me, sigma=sa, skew=sk, kurt=ku, ysigma=sa)

ysim_t = St.sim(ns=240, dt=0.5)
xsim_t = ysim_t.copy()
xsim_t[:,1] = gh.gauss2dat(ysim_t[:,1])

ts_y = wo.mat2timeseries(ysim_t)
ts_x = wo.mat2timeseries(xsim_t)
ts_y.plot_wave(sym1='r.', ts=ts_x, sym2='b', sigma=sa, nsub=5, nfig=1)
show()
```



In [ ]: