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Collisions

Abstract:
In this lab we examined the relationship between collisions and momentum. Our goal was to design an experiment and to test the principle of linear momentum conservation in two-dimensional glancing collisions. Using a high-speed digital camera and tracking software we tested three different cases. We used air disks, a weighted air disk, and coins. The software, Tracker, allowed us to analyze motion data and create position as a function of time equations. Using these equations we determined velocity equations. These, along with closely monitoring the video, allowed us to determine momentum using the equation $\vec{p} = m\vec{v}$ immediately before and immediately after the collision. In two cases we were able to verify that momentum is conserved during a collision. In one case we were not able to verify that momentum is conserved during a collision, however, in this case, an external force was likely introduced which would keep momentum from being conserved.

Introduction:

In this lab we study the relationship between collisions and momentum.

Objective:

Design and perform an experiment to test the principle of linear momentum conservation in two-dimensional glancing collisions. We must test under a range of circumstances and draw conclusions based on both our ability to verify momentum conservation and our choice of objects used. The goal is to show that momentum does not change during a collision.

Key Ideas:

Momentum is a vector defined by the equation $\vec{p} = m\vec{v}$.

The rate of change of momentum for a single particle is equal to the net force acting on the particle.

A collision is a brief interaction between two particles. We must determine what we mean by brief as it's related to what other objects in the experiment experience.

The rate of change of total momentum for a system of particles is equal to the net external force on the system. If the net external force on the system is zero, then the total momentum of the system remains constant. In order to accurately test for momentum conservation during collisions, the velocity of each object must be measured just before and just after the collision. In reality, it takes a finite amount of time to measure velocity, whereas we'd like to measure it at an infinitesimal amount of time after the collision. Measuring takes time and during this time unbalanced forces may alter the velocity. Therefore, it is necessary to minimize both external forces and the time allowed to act when performing a collision experiment.

It is necessary to use vector addition to test momentum conservation. The x-components of both objects immediately before the collision should add up to the x-components of both objects immediately after the collision. The y-components of both objects immediately before the collision should add up to the y-components of both objects immediately after the collision.

Therefore:

$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{2f} + \vec{p}_{1f}$$

$$p_{1ix} + p_{2ix} = p_{2fx} + p_{1fx} \quad \text{and} \quad p_{1iy} + p_{2iy} = p_{2fy} + p_{1fy}$$

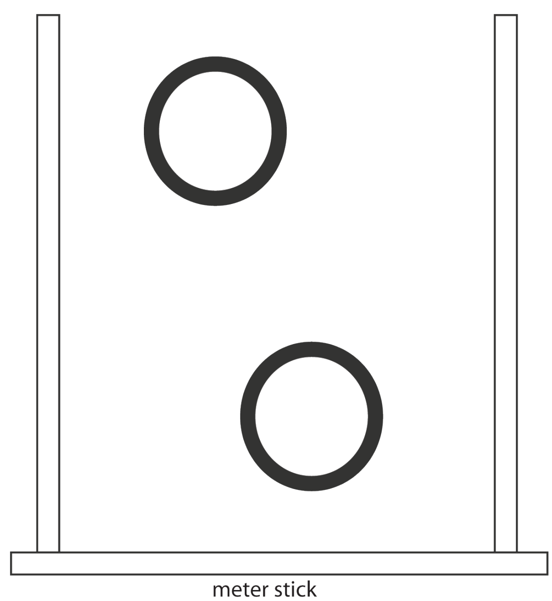
Procedure:

The equipment required for this lab:

A quarter and a nickel, two battery-operated air power soccer disks (they look like miniature hovercraft), a gray metal ring weight, a Casio EX-FH100 or EX-ZR1000 digital video camera with high-speed video capability, a heavy metal stand with brackets, an incandescent light, 2-3 meter sticks, masking tape, and a Macintosh computer with the Tracker video analysis program installed.

Setup:

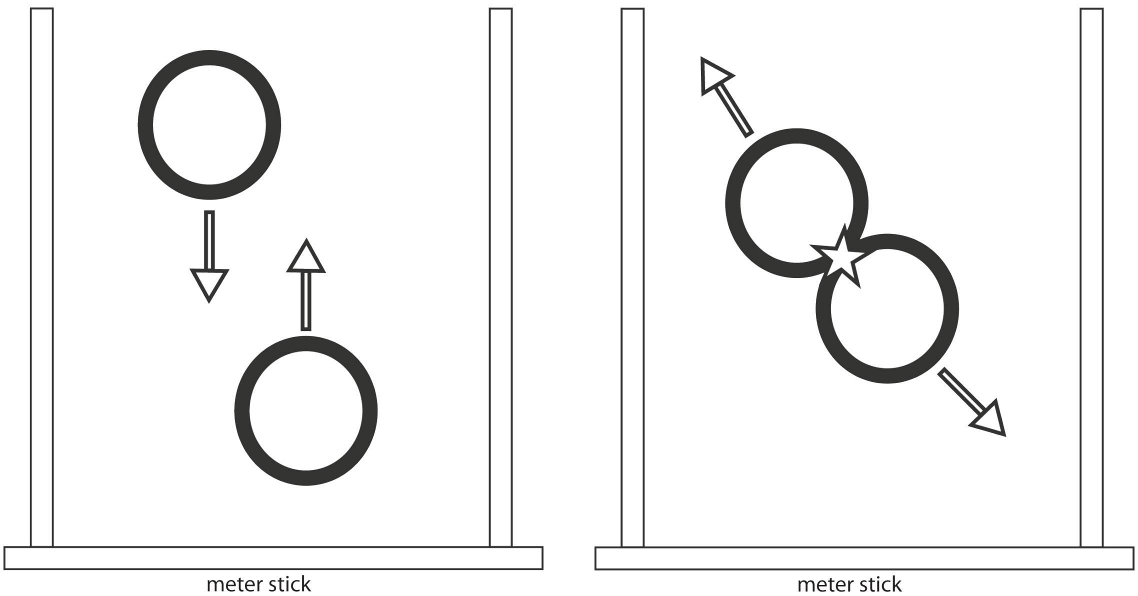
We set up a rectangular grid on the floor using three meter sticks and masking tape. The grid is used both to contain the soccer disks and as a measurement reference in the Tracker software.



Next, we set the camera up on the metal stand using a bracket so that the disks and meter sticks were visible in the viewfinder. We prepared the camera to take high-speed video at 120 fps. We turned the fans of the disks on so they would have minimal friction. They float like hovercraft.

Gathering Data With the Camera:

One person operated the camera while the other person manipulated the disks. The idea was to push the disks towards each other so that they would collide, but the collision needed to be glancing (not head on).



We took several high-speed videos of the disks colliding with each other. We were sure to stay out of the video as much as we could so that we could fully capture the motion of the disks. We saved each video on the memory card of the camera.

Next, we performed the same procedure using the same disks with the gray metal ring weight attached to one of the disks so that we could increase its mass and get data on two objects of the same size with different masses.

Then we set the camera up so that it overlooked a table. On the table we used a rectangular piece of steel to slide coins on and performed the same procedure as above. We used a quarter and a nickel. The frame rate of the camera should be changed to 480 fps for this part of the experiment. We were sure to stay out of the video as much as we could so that we could fully capture the motion of the coin collisions. We saved each video on the memory card of the camera.

Getting the Video into Tracker:

We turned on the Macintosh computer in the lab. We located the Tracker program and opened it. Then using the USB cable in the camera case we transferred our videos to the Macintosh. Once the video was successfully loaded onto the computer, we could transfer any video clips that we wanted to use into the Tracker program.

Results:

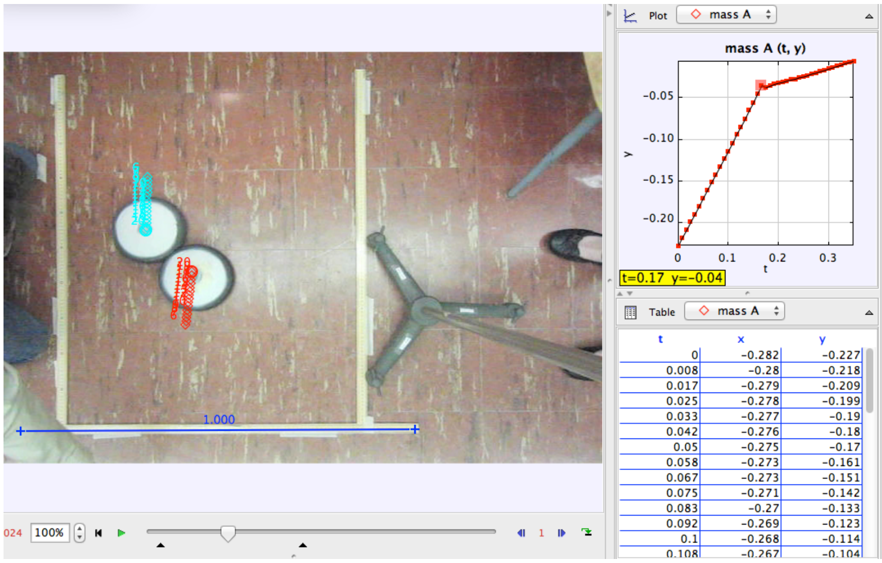
In order to best analyze our video, we used video editing software to trim the clips we wanted to use down into clips containing only the exact footage we wanted to analyze in Tracker. This gave us three trimmed clips: one for the two un-weighted disks trial, one for the coins trial, and one for the weighted disk and un-weighted disk trial. We loaded these three clips into Tracker to analyze them.

I will go over in detail how we analyzed the two un-weighted disks trial using Tracker. I will refer to this trial as UWD from this point forward. The other two trials were analyzed using the same method with a slight variation for the coins (which I will address shortly).

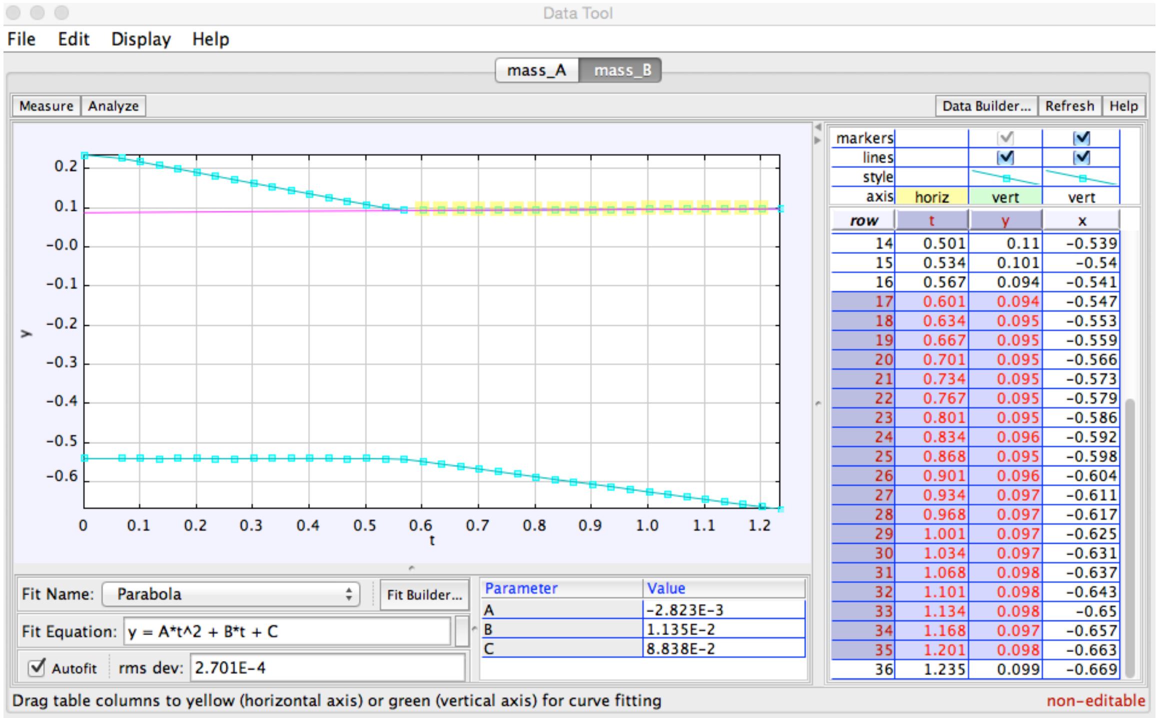
The UWD footage was successfully loaded into Tracker. Tracker has a feature called the calibration tool (the line with 1.000 over it in the photo below) that allows you to set the length scale to a known length in one of your videos. We used this feature and one of the meter sticks in our video to calibrate the scale.

Tracker allows you to analyze the exact piece of footage that you want to study by allowing you to set clip points (the black diamond icons in the photo below). You set these clip points at the start and stop points of the portion of the video you want to analyze.

In order to analyze the footage, we needed to track the disks in the video. Tracker has a nice feature called Autotracker which you can use to automatically track the motion of an object in your video. We allowed Autotracker to track each disk. It produces a time versus position graph of the tracked path for each object. The coin trial was not suitable for use with Autotracker and needed to be tracked manually. This requires clicking on a coin after each frame to manually track the object.



Tracker also has a tool for analyzing and creating an equation for the data produced by tracking. This tool allowed us to create quadratic equations for the position as a function of time for the x and y components of the position for each disk. To determine the equation you must watch the video and decide which part of the path you want the equation of. We needed an equation for each disk for position as a function of time for the time before the collision and the time after the collision. In this case, we needed eight equations: x(t) and y(t) for both disks before the collision and x(t) and y(t) after the collision. In order to create the equations you must scrub through the video to determine what times you want to analyze before and after the equation.



As can be seen in the picture above we made a quadratic equation for y(t) from the times 0.601s – 1.201s. You can see both the times and the number of data points that Tracker uses to make the equation. You can also see a larger version of the graph if you need to visually analyze something (such as the slope) to get immediate information about the data. We chose times before and after the collision based on what we saw in the video. We noted the number of data points. We noted the time immediately before the collision and immediately after the collision, as this is very important for the momentum calculations. We proceeded to use Tracker to get eight equations for each trial.

The equations were put into an Excel spreadsheet. What we were after was a velocity equation for the vector components of each disk before and after the collision. The velocity equation would allow us to determine the components at an instant just before the collision and an instant just after the collision. To get the velocity equations we differentiated the position equations.

A sample calculation is as follows:

$$x(t) = .003154t^2 - .001125t - .4112$$

$$\frac{d}{dt}[.003154t^2 - .001125t - .4112]$$

$$v(t) = .006308t - .001125$$

Going back to the UWD trial, the data is as follows:

Disk Data (No Weight Added)

disk	A	B
mass (kg)	0.3044	0.3044
x(t) before (m)	.003154t^2-.001125t-.4112	.001044t^2-.0006012t-.5391
y(t) before (m)	.03264t^2+.2061t-.1770	-.06249t^2-.2319t+.2399
x(t) after (m)	-.006079t^2+.1989t-.5204	.0003640t^2-.1940t-.4303
y(t) after (m)	.004016t^2-.06038t-.02054	-.002823t^2+.01135t+.08838
vx(t) before (m/s)	.006308t-.001125	.002088t-.0006012
vy(t) before (m/s)	.06528t+.206	-.12498t-.2319
vx(t) after (m/s)	-.012158t+.1989	.000728t-.194
vy(t) after (m/s)	.008032-.06038	-.005646t+.01135
# data points before	15	15
# data points after	19	19
video before times (s)	0-0.501	0-.0501
video after times (s)	0.601-1.201	0.601-1.201

Next we needed to determine uncertainty. We determined that mass and time uncertainty were insignificant compared to velocity uncertainty which varied significantly with the number of data points used to create the position equation. We compared two equations for the same object using a different number of data points. Then we compared two more equations for the same object using a different number of data points. We took the difference between the first terms of those equations in both cases. Then we took the average of that difference as the velocity uncertainty. The results are as follows:

v uncertainty test

A x(t) before (no weight)	A vx(t) before (no weight)	# data points
.003154t^2-.001125t-.4112	.006308t-.001125	14
-.004703t^2+.004356t-.4121	-.009406t+.004356	7
uncert (m/s)	+/- .016	**tests ave. uncert = .012 m/s**

v uncertainty test

A x(t) after (no weight)	A vx(t) after (no weight)	# data points
-.006079t^2+.1989t-.5204	-.012158t+.1989	19
.002796t^2+.1835-.5153	.005592t+.1835	10
uncert (m/s)	+/- .007	**tests ave. uncert = .012 m/s**

From this we plugged the velocity uncertainty into a momentum equation to determine the momentum uncertainty (immediate time after collision = 0.534s).

$$\vec{p}_{xbest} = m\vec{v} = .3044[.006308(0.534) - .001125] \approx 0.000682913 kgm/s$$

$$\vec{p}_{+xbest} = m\vec{v} = .3044[(.006308 + .012)(0.534) - .001125] \approx -0.002633508 kgm/s$$

$$\vec{p}_{-xbest} = m\vec{v} = .3044[(.006308 - .012)(0.534) - .001125] \approx -0.001267682 kgm/s$$

Giving us:

p uncertainty test

A p(x) before (no wt.) best	A p(x) before (no wt.) +	A p(x) before (no wt.) -
0.000682913	0.002633508	-0.001267682
p uncert	+/- 0.002	

Our uncertainty for $\vec{p} = \pm 0.002$

Now that we had the velocity equation and this uncertainty we were able to calculate the components of $\vec{p}$ for UWB:

disk	p x before (kg m/s)	p y before (kg m/s)	p x after (kg m/s)	p y after (kg m/s)	t before coll. (s)	t after coll. (s)
A	0.000682913	0.073317638	5.83E-02	-1.69E-02	0.534	0.601
B	0.000156398	-0.090905809	-5.89E-02	2.42E-03		
Total	0.000839311	-0.017588171	-0.000599494	-0.014488227		
best	0.001	-0.018	-0.001	-0.014		
p uncert +/-	0.002	0.002	0.002	0.002		

As can be seen in the table above the total momentum of the x-components before the collision is within uncertainty of the total momentum of the x-components after the collision and the total momentum of the y-components before the collision is within uncertainty of the total momentum of the y-components after the collision. Therefore in the case for UWB, we've shown that total momentum did not change during the collision.

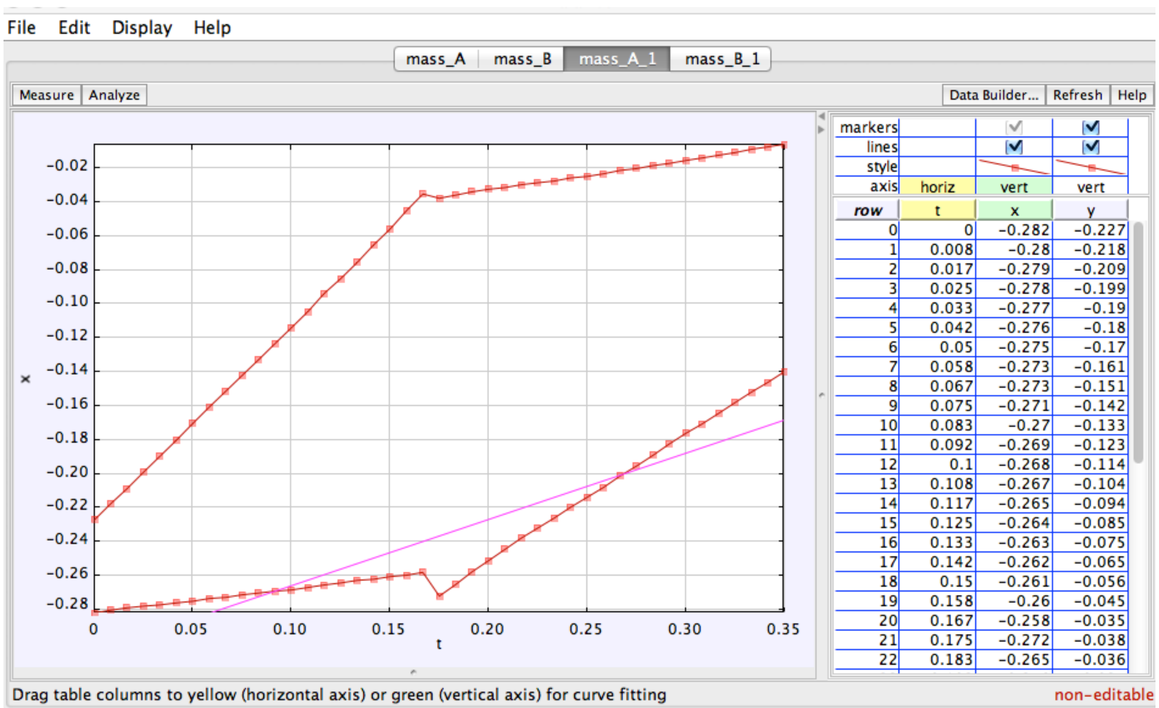
As for the weighted disk and un-weighted disk trial, which I will refer to as WDUD, the data is as follows:

Disk Data (Weight Added)

disk	A	B
mass (kg)	0.4062	0.3044
x(t) before (m)	.05245t^2+.1307t-.2815	.1417t^2-.05612t-.3687
y(t) before (m)	.1633t^2+1.1190t-.2270	-.1322t^2-1.141t+.2615
x(t) after (m)	1.551t^2+.002379t-.3125	- .01384t^2-.8866t-.2256
y(t) after (m)	.3274t^2+.01274t-.04816	.2784t^2-.1319t+.08129
vx(t) before (m/s)	.1049t+.1307	.2834t-.05612
vy(t) before (m/s)	.3266t+1.119	-.2644t-1.141
vx(t) after (m/s)	3.102t+.002379	- .02768t-.8866
vy(t) after (m/s)	.6548t+.01274	.5568t-.1319
# data points before	20	20
# data points after	15	15
video before times (s)	0-0.158	0-0.158
video after times (s)	0.167-0.283	0.167-0.283

disk	p x before (kg m/s)	p y before (kg m/s)	p x after (kg m/s)	p y after (kg m/s)	t before coll. (s)	t after coll. (s)
A	0.060236101	0.476785707	0.213911825	0.050125567	0.1677	0.169
B	-0.002615959	-0.360817459	-0.271304999	-0.011506564		
Total	0.057620142	0.115968248	-0.057393173	0.038619004		
best	0.058	0.116	-0.058	0.039		
p uncert +/-	0.002	0.002	0.002	0.002		

In the WDUD case we were unable to show that momentum was conserved during the collision. However, we did have a glitch in our data for mass A after the collision. It appears that the weight may have been off-centered and blocked the fan during the collision. This could have allowed one of the disks to have made contact with the ground which introduced some friction between the disk and the floor. This would have introduced an outside force into this trial. It's difficult to be positive if this is the cause, but the weight does shake slightly in the video upon impact. The graph of the glitch is shown below.



As for the data with the coin collisions:

Coin Data

Coin	A (Qt)	B (Nk)
mass (kg)	0.0058	0.0051
x(t) before (m)	-.08473t^2+.2970t-.05478	.09937t^2-.2730t+.1442
y(t) before (m)	-.006276t^2-.03711t-.01873	-.01151t^2-.01985t-.02376
x(t) after (m)	.09509t^2-.1786t-.08866	-.1047t^2-.2900t-.02814
y(t) after (m)	.02858t^2-.05238t-.01727	.01424t^2-.04119t-.01853
vx(t) before (m/s)	-.16946t+.297	.19874t-.237
vy(t) before (m/s)	-.012552t-.03711	-.02302t-.01985
vx(t) after (m/s)	.19018t-.1786	-.2094t-.29
vy(t) after (m/s)	.05716t-.05238	.02848t-.04119
# data Points before	11	11
# data points after	21	21
video before times (s)	0-0.334	0-0.334
video after times (s)	0.367-1.034	0.367-1.034

Coin	p x before (kg m/s)	p y before (kg m/s)	p x after (kg m/s)	p y after (kg m/s)	t before coll. (s)	t after coll. (s)
A (Qt)	0.001394322	-0.000239554	-5.95E-04	-1.71E-04	0.334	0.400
B (Nk)	-0.000870166	-0.000140447	-1.91E-03	-1.52E-04		
Total	0.000524156	-0.000380001	-0.002500838	-0.000323163		
best	0.001	0.000	-0.003	0.000		
p uncert +/-	0.002	0.002	0.002	0.002		

We were also able to show, as can be seen in the table above, that the total momentum of the x-components before the collision is within uncertainty of the total momentum of the x-components after the collision and the total momentum of the y-components before the collision is within uncertainty of the total momentum of the y-components after the collision. Therefore in this case for the coins, we've shown that total momentum did not change during the collision.

<u>Conclusion:</u>

We designed an experiment and were able to adequately test the principle of linear momentum conservation in two-dimensional glancing collisions. Using Tracker and a high-speed camera we were able gather data and analyze the collisions of several objects. Tracker allowed us to create quadratic position functions for the objects. Using calculus we were able to determine velocity functions for the objects. We tested the principle with two equally sized air disks of the same mass (UWD), two equally sized air disks with different masses (WDUD), and two coins. In the case of both UWD and the coins we were able to show that total momentum did not change during the collision by measuring the velocity components of both objects immediately before and immediately after the collision and using the equation $\vec{p} = m\vec{v}$. We were not able show that total momentum did not change during the collision for WDUD. It is likely that the weight attached to one of the disks in this case interfered with the fan, causing the disk to touch the ground during the collision. This would have introduced an external force into the system, which would result in total momentum not being conserved during the collision.