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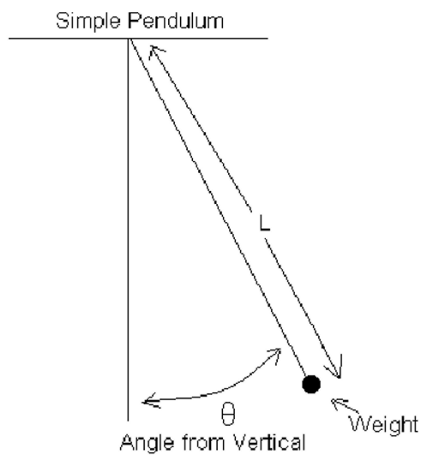
Multicomponent Oscillators

Abstract: In this lab we observed pendulum and mass/spring systems. Simple and double pendulums were set up using an apparatus consisting of a stand, masses, and string. Using the same stand, a mass/spring apparatus was set up. The period of the simple pendulum was measured with a stopwatch and compared to a predicted (calculated) value. The measured and calculated values were close enough to assume the measured value was reasonable. The double pendulum period was measured in both the in-phase and out-of-phase mode. The out-of-phase mode period fell within 10% of the simple pendulum period as predicted. It also matched a predicted (calculated) value. The in-phase and beat-mode periods of the double pendulum were also measured. The spring constant for the mass/spring system was measured by observation of both stretching the spring and watching/timing the period. Stretching the string was more accurately observable, thus likely gave a more accurate answer for the spring constant. Observing the mass/spring system with various masses led to two conclusions: 1) The frequency of the mass/spring movement of the system is always higher than the frequency of the pendulum movement of the system. 2) The more massive the bob, the more likely it is to act like simply a mass/spring system and not like a pendulum.

Procedure:

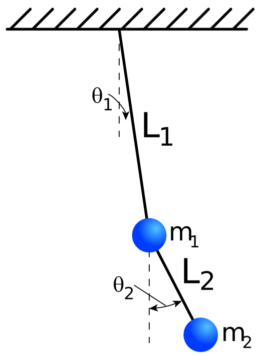
The equipment for this lab consists of: a vertical stand, stopwatch, string, masses, and springs.

Using a vertical stand, we set up a single pendulum with a 0.500 $\pm$ 0.003 meter string (measured from the pivot point to the center of the mass) and a 0.1000 $\pm$ .0001 kilogram mass.



We moved the mass (m) to the left and released making sure that the pendulum oscillations moved in only 2 dimensions. Then we measured the period (T) of several consecutive oscillations so that we could compare our measured results with the expected result from the equation $T = 2\pi\sqrt{\frac{L}{g}}$.

Then, using the same set up, we created a double pendulum. We attached a second mass of 0.0022 $\pm$ 0.0001 kilograms beneath the 0.1000 $\pm$ .0001 kilogram mass via a second string (connected to a hook on the bottom of the first mass) of length 0.496 $\pm$ 0.003 meters from pivot point (hook) to the center of the second mass.

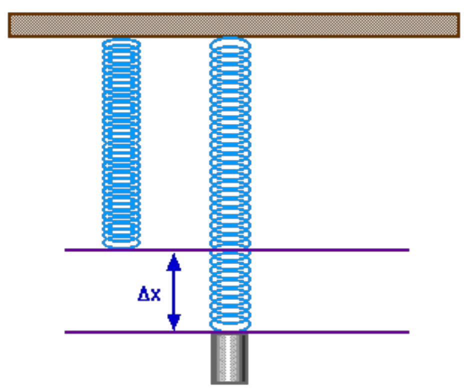


We displaced the top mass about 1cm to the right and the bottom mass about 7 cm to left and released. This made the pendulum oscillate in two dimensions with out of phase behavior, meaning the two masses swung in opposite directions and constant amplitudes. We again measured the period for several consecutive oscillations. We compared this with the period measured for the single pendulum to be sure they were with 5%-10% of each other.

Using the exact same setup, we then displaced the top and bottom masses by 1cm and 7 cm (respectively), but this time in the same direction. We made sure that the amplitudes were constant although each section had a different amplitude (as expected). We again took several consecutive measurements of the period of oscillation.

Again, using the same exact setup, we now displaced only the bottom mass. We saw the two masses transfer energy back and forth. The amplitudes varied as time passed. The time between two successive occurrences of the top mass stopping its oscillation is one beat period.

We removed all pendulum equipment from the stand. Then we attached a spring to the top bar of the apparatus.



We attached several different masses to the spring to determine the spring constant. We attached 0.0500 ± 0.0001, 0.1000 ± 0.0001, and 0.1998 ± 0.0001 kilogram masses. When each individual mass was hung on the spring we measured the change in length of the spring from its initial position with no mass to the new position with a hung mass. This will be compared to the theoretical spring constant. We also measured the frequency of oscillations for the 0.1000 ± 0.0001 kilogram mass.

Finally, the pendulum and spring like behavior of the of three different masses was observed when attached to the spring. We watched and took note of the behavior of the spring/mass setup.
*Images from Wikipedia, my.execpc.com, and ux1.eiu.edu

Results:

The measurements for mass, length, and gravity for this lab are as follows:

String length	Uncert	
0.500	0.003	m
Second Pend		
0.496	0.003	m
100g mass	**Uncert**	
0.100	0.0001	kg
2g mass	**Uncert**	
0.0022	0.0001	kg
Gravity	**Uncert**	
9.818	0.015	m/s2
50g mass	**Uncert**	
0.0500	0.0001	kg
200g mass	**Uncert**	
0.1998	0.0001	kg

Simple Pendulum:

The measurements for the period of the simple pendulum with $L = 0.500 \pm 0.003$ meters and $m = 0.1000 \pm 0.0001$ kilograms are as follows:

10 consecutive oscillations		
Trial	Time for 10 (s)	Time for 1 (s)
1	14.35	1.435
2	14.28	1.428
3	14.22	1.422
4	14.22	1.422

5	14.25	1.425
Avg	14.26	1.426
Uncert	0.09	0.009

The measured period of the single pendulum was 1.426 ± 0.009s

The calculated value for the simple pendulum is:

$$T_{best} = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{.5}{9.818}} \approx 1.418$$

$$T_{+} = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{.5+.003}{9.818+.015}} \approx 1.421$$

$$T_{-} = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{.5-.003}{9.818-.015}} \approx 1.415$$

thus:

$$T_{final} = 1.418 \pm 0.003s$$

These measured and calculated values are close enough to surmise that our measurements are reasonably accurate.

Double Pendulum (Out of Phase):

The measurements for the double pendulum in out-of-phase mode are as follows (note that trial one data was thrown out due to improper measurement):

Trial	Time for 10	Time for 1	5%-10% diff
1	6.84	1.368	4.78%
2	13.25	1.325	7.97%
3	13.22	1.322	8.20%
4	13.09	1.309	9.18%
5	13.09	1.309	9.18%

6	13.25	1.325	7.97%
Avg	13.18	1.318	8.50%
Uncert	0.09	0.009	

The 5%-10% difference column represents the difference between the value found for the single pendulum and the time for one oscillation of the double pendulum. We needed them to be within 10% to be deemed reliable data and they all fell within the expected range.

The measured period was 1.318 ± 0.009s.

We made a frequency prediction then converted it to a period for comparison to this data.

$$\omega_{best} = \sqrt{\frac{g}{L}}\left(1+\frac{1}{2}\sqrt{\frac{m_2}{m_1}}\right) = \sqrt{\frac{9.818}{.5}}\left(1+\frac{1}{2}\sqrt{\frac{.0022}{.100}}\right) \approx 4.760$$

$$\omega_{+} = \sqrt{\frac{g}{L}}\left(1+\frac{1}{2}\sqrt{\frac{m_2}{m_1}}\right) = \sqrt{\frac{9.818+.015}{.5}}\left(1+\frac{1}{2}\sqrt{\frac{.0022+.0001}{.100+.0001}}\right) \approx 4.771$$

$$\omega_{-} = \sqrt{\frac{g}{L}}\left(1+\frac{1}{2}\sqrt{\frac{m_2}{m_1}}\right) = \sqrt{\frac{9.818-.015}{.5}}\left(1+\frac{1}{2}\sqrt{\frac{.0022-.0001}{.100-.0001}}\right) \approx 4.749$$

thus:

$$\omega_{final} = 4.760 \pm .011\,\text{Hz}$$

Period Conversion:

$$T_{best} = \frac{2\pi}{\omega} = \frac{2\pi}{4.760} \approx 1.320$$

$$T_{+} = \frac{2\pi}{\omega} = \frac{2\pi}{4.760+.011} \approx 1.317$$

$$T_{-} = \frac{2\pi}{\omega} = \frac{2\pi}{4.760-.011} \approx 1.323$$

thus:

$$T_{final} = 1.320 \pm .003s$$

This predicted and measured values are very close and within uncertainty, thus our measurements are reasonable.

Double Pendulum (In Phase):

The measurements for the in-phase double pendulum are as follows:

Trial	Time for 10	Time for 1
1	15.31	1.531
2	15.31	1.531
3	15.44	1.544
4	15.25	1.525
5	15.25	1.525
Avg	15.31	1.531
Uncert	0.13	0.013

The final measured period was 1.531 ± 0.013s.

Double Pendulum (Beat Period):

The measurements for the beat period of the double pendulum are as follows:

Trial	Beat Period (s)
	8.88
	9.44
	8.87
	9.91
Avg	9.28
Uncert	0.63

The beat period, time between two successive instances when the top mass ceases to oscillates, was measured as 9.28 ± 0.63s.

Mass and Spring (Spring Constant k):

The measurements for determining the spring constant are as follows (initial un-stretched spring length was 0.108m = xi):

mass (kg)	0.050	0.100	0.150	0.200
stretch (xf) (m)	0.150	0.199	0.248	0.291
xi (m)	0.108	0.108	0.108	0.108
xf-xi	0.042	0.091	0.14	0.183
k=mg/(xf-xi)	11.688	10.789	10.519	10.730
k Avg	10.932			
Uncert	0.756			

The measured k was 10.932 ± 0.756 N/m

The measurements for period and frequency of vertical oscillations is as follows:

Period for 10 (s)	**Period for 1 (s)**	**Frequencies (Hz)**
6.25	0.625	1.60
6.16	0.616	1.62
6.16	0.616	1.62
6.19	0.619	1.62
6.16	0.616	1.62
Avg	0.618	1.62
Uncert	0.007	0.02

thus:

T=0.618 ± 0.007 s

Using $T = 2\pi\sqrt{\frac{m}{k}} \Rightarrow k = \frac{m}{\left(\frac{T}{2\pi}\right)^2}$

$$k_{best} = \frac{.100}{\left(\frac{0.618}{2\pi}\right)^2} \approx 10.336$$

$$k_{+} = \frac{.100}{\left(\frac{0.618 + .007}{2\pi}\right)^2} \approx 10.106$$

$$k_{\_} = \frac{.100}{\left(\frac{0.618 - .007}{2\pi}\right)^2} \approx 10.575$$

thus:

$$k_{final} = 10.336 \pm .239 N/m$$

Although the values for k are fairly close to each other, I believe the first value is more accurate because measuring the change in length of the spring has a much less chance for error than does watching mass move around while connected to the spring.

Mass and Spring (observations):

When we pulled the 50g mass down slightly and released we saw that at first, it did 3 spring-like movements and then went to one pendulum cycle. As time went on, the number of spring-like movements increased and so did the number of pendulum-like movements. Eventually, the system changed totally into a pendulum with no spring bounciness whatsoever.

Length of oscillator as a pendulum = 0.205 +/- .003m

We were able to measure the following:

50g Spr T (s)	50g Pend T (s)	Ratio of f's
2.27	1.06	2.14
2.33	1.03	2.26
2.00	1.03	1.94
100g Spring	100g Pend	Ratio

Cannot measure		
200g Spr	200g Pend	Ratio
Cannot measure		

Frequency as pendulum (50g mass, L = 0.205 +/- .003m):

$$f_{best} = \frac{1}{2\pi}\sqrt{\frac{g}{L}} = 2\pi\sqrt{\frac{9.818}{.205}} \approx 1.101$$

$$f_{+} = \frac{1}{2\pi}\sqrt{\frac{g}{L}} = 2\pi\sqrt{\frac{9.818+.015}{.205+.003}} \approx 1.094$$

$$f_{-} = \frac{1}{2\pi}\sqrt{\frac{g}{L}} = 2\pi\sqrt{\frac{9.818-.015}{.205-.003}} \approx 1.109$$

$$f_{final} = 1.101 \pm .008\,Hz$$

Frequency as mass/spring system (50g mass):

$$f_{best} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{10.932}{.05}} \approx 2.35$$

$$f_{+} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{10.932+0.756}{.05+.0001}} \approx 2.36$$

$$f_{-} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{10.932-0.756}{.05-.0001}} \approx 2.35$$

$$f_{final} = 2.35 \pm .01\,Hz$$

The 50g mass frequencies differ by a factor of approximately 2.13. This seems reasonable as they are predicted to differ by a factor of 2.

Frequency as pendulum (100g mass, L = 0.252 +/- .003m):

$$f_{best} = \frac{1}{2\pi}\sqrt{\frac{g}{L}} = \frac{1}{2\pi}\sqrt{\frac{9.818}{.252}} \approx .993$$

$$f_{+} = \frac{1}{2\pi}\sqrt{\frac{g}{L}} = \frac{1}{2\pi}\sqrt{\frac{9.818+.015}{.252+.003}} \approx .988$$

$$f_{-} = \frac{1}{2\pi}\sqrt{\frac{g}{L}} = \frac{1}{2\pi}\sqrt{\frac{9.818-.015}{.252-.003}} \approx .999$$

$$f_{final} = .993 \pm .005 Hz$$

Frequency as mass/spring system (100g mass):

$$f_{best} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{10.932}{.100}} \approx 1.66$$

$$f_{+} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{10.932+0.756}{.100+.0001}} \approx 1.67$$

$$f_{-} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{10.932-0.756}{.100-.0001}} \approx 1.66$$

$$f_{final} = 1.66 \pm .01 Hz$$

The 100g mass frequencies differ by a factor of approximately 1.67.

Frequency as pendulum (200g mass, L = 0.379 +/- .003m):

$$f_{best} = \frac{1}{2\pi}\sqrt{\frac{g}{L}} = \frac{1}{2\pi}\sqrt{\frac{9.818}{.379}} \approx .810$$

$$f_{+} = \frac{1}{2\pi}\sqrt{\frac{g}{L}} = \frac{1}{2\pi}\sqrt{\frac{9.818+.015}{.379+.003}} \approx .807$$

$$f_{-} = \frac{1}{2\pi}\sqrt{\frac{g}{L}} = \frac{1}{2\pi}\sqrt{\frac{9.818-.015}{.379-.003}} \approx .813$$

$$f_{final} = .810 \pm .003Hz$$

Frequency as mass/spring system (200g mass):

$$f_{best} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{10.932}{.200}} \approx 1.18$$

$$f_{+} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{10.932+0.756}{.200+.0001}} \approx 1.18$$

$$f_{-} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{10.932-0.756}{.200-.0001}} \approx 1.17$$

$$f_{final} = 1.18 \pm .01Hz$$

The 200g mass frequencies differ by a factor of approximately 1.46.

Observations of Mass Spring Behavior (100g and 200g masses):

100g mass:

Doesn't ever completely change to pendulum where the spring doesn't bounce at all. It only becomes slightly pendulum-like.

200g mass:

It's even harder to see the pendulum movement. Even after a very long time, there is even less movement in a pendulum-like manner than there was for the 100g.

Conclusion:

The more massive the bob, the less pendulum like movement there will be. The more massive the bob the more it will act like only a spring-like object moving up and down with no pendulum sway.