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Rotational Dynamics

Abstract: This lab was centered around rotational dynamics. There were 3 parts to the lab. We used a Pasco Rotational Dynamics Platform, a stopwatch, and a mass set to perform three different experiments. In part one we examined varying torque on the apparatus and took several time measurements and then calculated angular acceleration using the equation $\alpha = \frac{2\Delta\theta}{t^2}$ and torque using the equation $\tau = RF$. For the second part we examined varying moment of inertia and compared a predicted angular acceleration to a calculated one using the equations $\alpha = \frac{\tau_{app}}{I}$ and $\alpha = \frac{2\Delta\theta}{t^2}$. In the final part we looked at the conservation of angular momentum by using the equation $\frac{\Delta t_i}{\Delta t_f} = \frac{I_{disk}}{I_{disk} + I_{cylinder}}$ and a disc and hoop setup and stopwatch. The left side of the equation was predicted to match the right. My results for part 1 appear to be reasonable. The results for part 2 match up closely enough to be confident in the data and calculations. For part 3, the results also match up closely enough to be confident in the data and calculations. However, both part 2 and part 3 each have one trial that falls outside of the uncertainty expected.

Results:

Part 1 – Varying Torque:

The raw data for part 1 will be used to calculate angular acceleration for each of the pulley and mass combinations using the equation:

$$\Delta\theta = \frac{1}{2}\alpha t^2$$

We rearrange and solve for angular acceleration, α

$$\alpha = \frac{2\Delta\theta}{t^2}$$

Since each combination went through 4 full revolutions, $\Delta\theta = 8\pi$ radians.

I will assume that there is only random error and that uncertainty in $\Delta\theta$ will be accounted for in the uncertainty of Δt.

I will do a sample calculation using the following data from the raw data sheet:

	Top	Pulley
	Trial	Time (s)
4 full revs	1	17.68
50g	2	17.69
	3	17.72
	avg	17.70
	uncert.	0.02

Sample Calculation for α at top pulley, 4 full revolutions, 50g, with average time 17.70s and uncertainty at ± 0.02:

$$\alpha = \frac{2(8\pi)rad}{(17.70s)^2} \approx .1604 rad/s^2$$ (best)

$$\alpha + = \frac{2(8\pi)rad}{(17.70 + .02s)^2} \approx .1601 rad/s^2$$ (plus uncert)

$$\alpha - = \frac{2(8\pi)rad}{(17.70 - .02s)^2} \approx .1608 rad/s^2$$ (minus uncert)

Final uncertainty = .0004 rad/s^2

$$\alpha\, final = .1604 rad/s^2 \pm .0004 rad/s^2$$

Doing calculations in Excel for all of the angular accelerations yields:

Pulley	Mass(g)	Avg Time (s)	Uncert.(s)	Theta (rads)	α (rad/s^2) Best	Final Uncert. (rad/s^2)	α -	α+
top	50	17.70	0.02	25.1327	0.1604	0.0004	0.1608	0.1601
top	70	14.55	0.014	25.1327	0.2374	0.0005	0.2379	0.2370
top	100	12.07	0.14	25.1327	0.3450	0.0079	0.3532	0.3372
middle	50	14.26	0.02	25.1327	0.2472	0.0007	0.2479	0.2465
middle	70	11.88	0.09	25.1327	0.3562	0.0055	0.3616	0.3508
middle	100	9.95	0.01	25.1327	0.5077	0.0010	0.5087	0.5067
bottom	50	11.52	0.04	25.1327	0.3788	0.0026	0.3814	0.3761
bottom	70	9.60	0.07	25.1327	0.5454	0.0079	0.5535	0.5375
bottom	100	8.03	0.06	25.1327	0.7795	0.0118	0.7913	0.7680

Now I will use the following equation to calculate torque, τ, taking the tension in the string to be mg = $9.818 \pm .015 m/s^2$:

$\tau = RF$ (where R = radius of pulley and F=mg)

I will do a sample calculation for the following raw data from the raw data sheet:

	Top	Pulley
	Trial	Time (s)
4 full revs	1	17.68
.050kg	2	17.69
	3	17.72
	avg	17.70
	uncert.	0.02

Diameter	top	pulley	0.01659 m	r=	0.00830 m

g = 9.818 +/- .015 m/s^2

Sample Calculation:

$$\tau = (.00830m)(.050kg)(9.818m/s^2) \approx 4.074 \times 10^{-3} mN$$ (best)

$$\tau + = (.00830m)(.050kg)(9.818 + .015m/s^2) \approx 4.081 \times 10^{-3} mN$$ (plus uncert)

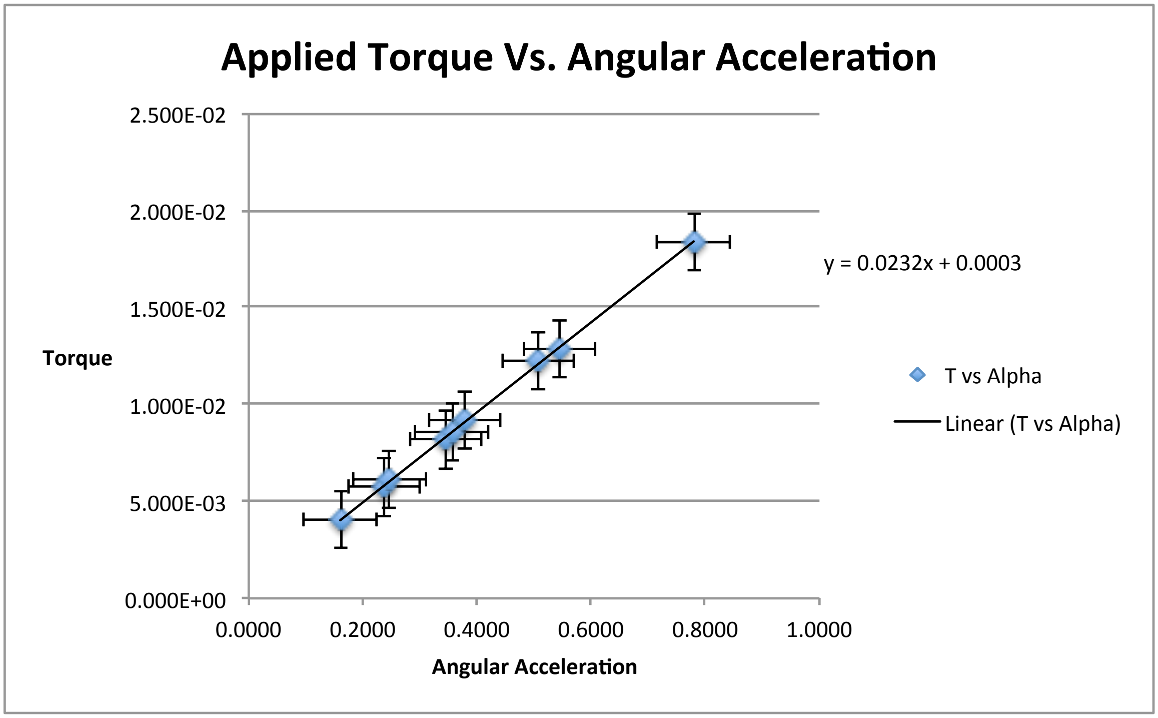
$$\tau - = (.00830m)(.050kg)(9.818 - .015m/s^2) \approx 4.068 \times 10^{-3} mN$$ (minus uncert)

$$\tau\, final = (4.074 \pm 0.006) \times 10^{-3} Nm$$

Doing calculations in Excel for all of the torques yields:

Pulley	Mass kg	Radius m	g m/s^2	g uncert	τ (Nm) Best	Final uncert. (Nm)	τ- (Nm)	τ+ (Nm)
top	0.050	0.00830	9.818	0.015	4.074E-03	6.E-06	4.068E-03	4.081E-03
top	0.070	0.00830	9.818	0.015	5.704E-03	9.E-06	5.696E-03	5.713E-03
top	0.100	0.00830	9.818	0.015	8.149E-03	1.E-05	8.136E-03	8.161E-03
middle	0.050	0.01246	9.818	0.015	6.117E-03	9.E-06	6.107E-03	6.126E-03
middle	0.070	0.01246	9.818	0.015	8.563E-03	1.E-05	8.550E-03	8.576E-03
middle	0.100	0.01246	9.818	0.015	1.223E-02	2.E-05	1.221E-02	1.225E-02
bottom	0.050	0.01872	9.818	0.015	9.190E-03	1.E-05	9.176E-03	9.204E-03
bottom	0.070	0.01872	9.818	0.015	1.287E-02	2.E-05	1.285E-02	1.289E-02
bottom	0.100	0.01872	9.818	0.015	1.838E-02	3.E-05	1.835E-02	1.841E-02

Finally I will graph applied torque versus angular acceleration.



The line likely does not pass through the origin because of the uncertainty in the calculations, although it does look rather close to passing through the origin.

The slope of the line represents a linearly proportional relationship between angular acceleration and torque.

Part 2 – Varying Moment of Inertia:

Since we know the mass and length of the rotating platform we can estimate its moment of inertia if we treat it as a thin rod. We will add to this the contributions from the adjustable masses and treat them like point masses.

I will calculate the torque for each trial from the part 2 data using the equation:

$$\tau = RF$$

I will do a sample calculation using the following raw data from part 2.

	Trial	Time (s)
4 revs, mid pulley	1	9.87
.050 kg	2	9.94
cent. mass .03 m	3	9.96
	avg	9.92
	uncert.	0.05

$$\tau best = (.01246m)(.050kg)(9.818) \approx 6.117 \times 10^{-3} mN$$

$$\tau - = (.01246m)(.050kg)(9.818 - .015m/s^2) \approx 6.107 \times 10^{-3} mN$$

$$\tau + = (.01246m)(.050kg)(9.818 + .015m/s^2) \approx 6.126 \times 10^{-3} mN$$

$$\tau\, final = 6.117 \times 10^{-3} mN \pm .00001 mN$$

Completing these calculations in Excel yields:

radius (m)	g (m/s^2)	g uncert	mass (kg)	τ (Nm) (best)	Final Uncert	τ- (Nm)	τ+ (Nm)
0.01246	9.818	0.015	0.050	6.117E-03	0.00001	0.006107	0.006126

Now I will approximate the angular acceleration using the following equation:

$$\alpha_{pred} = \frac{\tau_{app}}{I}$$

An example of this calculation being:

$$I = \frac{1}{12}mL^2 + 2mr^2 = \frac{1}{12}(.4971)(.507)^2 + 2(.2758)(.03)^2 \approx 1.114 \times 10^{-2}\, kgm^2$$

$$\alpha_{pred}+ = \frac{\tau_{app}}{I} = \frac{(6.117 \times 10^{-3})}{1.114 \times 10^{-2}} \approx .5488 rad/s \qquad \text{(best)}$$

$$\alpha_{pred}+ = \frac{\tau_{app}}{I} = \frac{(6.130 \times 10^{-3})}{1.114 \times 10^{-2}} \approx .5500 rad/s \qquad \text{(plus uncertainty)}$$

$$\alpha_{pred}- = \frac{\tau_{app}}{I} = \frac{(6.107 \times 10^{-3})}{1.114 \times 10^{-2}} \approx .5480 rad/s \qquad \text{(minus uncertainty)}$$

$$\alpha\, final = .5488 \pm .0010 rad/s$$

Using this equation for the predicted values yields:

I (kgm^2)	τ (applied)	α (pred) (rad/s)	L platform (m)	m platform (kg)	m weights (kg)	r (m)	τ+	τ-	α final uncert	α+	α-
1.114E-02	6.117E-03	5.488E-01	0.507	0.4971	0.2758	0.03	6.130E-03	6.107E-03	1.E-03	5.500E-01	5.480E-01
1.418E-02	6.117E-03	4.314E-01	0.507	0.4971	0.2758	0.08	6.130E-03	6.107E-03	9.E-04	4.323E-01	4.307E-01
1.997E-02	6.117E-03	3.063E-01	0.507	0.4971	0.2758	0.13	6.130E-03	6.107E-03	7.E-04	3.070E-01	3.058E-01
2.852E-02	6.117E-03	2.145E-01	0.507	0.4971	0.2758	0.18	6.130E-03	6.107E-03	5.E-04	2.149E-01	2.141E-01
3.983E-02	6.117E-03	1.536E-01	0.507	0.4971	0.2758	0.23	6.130E-03	6.107E-03	3.E-04	1.539E-01	1.533E-01

Using the equation $\alpha = \frac{2\Delta\theta}{t^2}$ yields:

time (s)	theta (rad)	α calculated (rad/s)	time uncert	α+	α-	α Final Uncert
9.92	25.1327	5.108E-01	0.05	5.057E-01	5.160E-01	5.E-03
11.10	25.1327	4.080E-01	0.12	3.993E-01	4.169E-01	9.E-03
13.16	25.1327	2.902E-01	0.07	2.872E-01	2.934E-01	3.E-03
15.66	25.1327	2.050E-01	0.09	2.026E-01	2.073E-01	2.E-03
18.35	25.1327	1.493E-01	0.21	1.459E-01	1.528E-01	3.E-03

A direct comparison yields.

α calculated (rad/s)	α (pred) (rad/s)	uncert
0.5108	0.5488	no overlap
0.4080	0.4314	no overlap
0.2902	0.3063	no overlap
0.2050	0.2145	no overlap
0.1493	0.1536	no overlap

The predicted and calculated angular accelerations are close but are not within uncertainty of each other. Although I think the uncertainty in alpha is too low, so there is likely an error due to platform length.

Part 3 – Conservation of Angular Momentum:

For this section we will use the following data:

Trial	Disk Alone (s)	Disk with cylinder (s)
1	6.62	11.09
4	7.19	11.39
7	6.50	10.55
10	6.65	10.23
11	7.18	11.29

mass of hollow cylinder	1.4396	kg
mass of solid disk	1.4700	kg
circumference of solid disk (r=11.36)	0.714	m
inner diameter of hollow cylinder	0.10731	m
outer diameter of hollow cylinder	0.12736	m
thickness of hollow cylinder	0.00998	m

in combination with the equations:

$$\frac{\Delta t_i}{\Delta t_f} = \frac{I_{disk}}{I_{disk} + I_{cylinder}}$$

$$I_{disk} = .5mr^2 = .5(1.4700)(.1136)^2 \approx 9.485 \times 10^{-3}$$

$$I_{cylinder} = .5m(R_1^{\ 2} + R_2^{\ 2}) = .5(1.4396)(.05366^2 + .06368^2) \approx 4.991 \times 10^{-3}$$

If we solve the above for equation for each trial (letting $\frac{\Delta t_i}{\Delta t_f}$ = A in the chart below and $\frac{I_{disk}}{I_{disk} + I_{cylinder}}$ = B in the chart below) we get:

Trial	A	B
1	0.597	0.655
2	0.631	0.655
3	0.616	0.655
4	0.650	0.655
5	0.636	0.655

We expect these values to be equal. I would predict an uncertainty of +/- .005 So only one result is within my predicted uncertainty.