Task 3

For task 3 I used FNOs. Since we must predict the next 34 steps and because the timesteps are equally spaced with spacing Δ_t, I decided to learn a translation operator. My model learns to predict the next 35 and not 34 steps since the training data has length 210 and consists of one cycle that is repeated 6 times (210:6=35).
The operator D it learns takes an interval $I_i = [t_i, t_i + 35\Delta_t]$ (in our case $i = 1...5$) and a function $f : \mathbb{R} \to \mathbb{R}$ evaluated only on this interval, i.e. the function $(f\chi_{I_i})(t) = f(t)\chi_{I_i}(t)$ with χ being the characteristic function. Then it maps this to the same function f but evaluated on the next interval shifted by $35\Delta_t$, i.e. $f\chi_{I_{i+1}}$ with $I_{i+1} = [t_i + 35\Delta_t, t_i + 70\Delta_t]$. In conclusion: $D(f, I_i)(t) = f(t)\chi_{I_{i+1}}(t)$.
Concerning the architecture of the model, I chose to use FNOs since they are closely related to CNNs which are translational equivariant. This is a useful property since we have a function that repeats itself in cycles of fixed length. The FNO structure is from the tutorial (`Tutorial 10-FNO.ipynb`). I used 2 FNOs, one for the fluid and one for the solid. The width of one FNO is 512 and it uses 18 modes, such that the `fft` of a tensor with length 35 is not truncated. The training data was prepared with the class `DatasetTask3`. This class overrides the TensorDataset methods `__len__` and `__getitem__`. The initial data for the 210 timesteps is divided into 6 parts of 35 timesteps each, such that the periodicity is enforced. Each of the 5 input-output pairs consists of an input (35 steps) and the consecutive 35 steps as output. The FNO takes as input a tensor of size [35,2] orresponding to $[t, T_{f/s}]$ and returns as output $T_{f/s}$ at the next 35 steps. The loss is then just the MSE between the FNO output and the real output. Since the data has a large range it is normalized for training and then denormalized at the end. The model is trained over 1000 steps with ADAM and a learning rate scheduler.