STT 4660
Homework #1

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Problem 1

1. The Population is all adults with normal vision.
2. The Parameter is the proportion of these adults who are fooled by the illusion.
3. The Sample is the 100 students in PSYCH 120.
4. The Statistic is the proportion of those 100 students who are fooled by the illusion.

Problem 2

1. We know,
$$Var(X_1) = Var(X_2) = 0.0169$$
2. And,
$$Cor(X_1, X_2) = 0.8$$
3. From the notes, we have the formula,
$$Cor(X_1, X_2) = \frac{Cov(X_1, X_2)}{\sqrt{Var(X_1)Var(X_2)}}$$
4. Rearrange the equation from (3), yielding
$$Cov(X_1, X_2) = Cor(X_1, X_2)\sqrt{Var(X_1)Var(X_2)}$$
5. Substituting values in, we get,
$$Cov(X_1, X_2) = 0.8\sqrt{0.0169 * 0.0169}$$
$$Cov(X_1, X_2) = 0.8\sqrt{(0.0169)^2}$$
$$Cov(X_1, X_2) = 0.8(0.0169)$$
$$Cov(X_1, X_2) = 0.0135$$

6. Now $X_1 - X_2$ has the form $a_1x_1 + a_2x_2$ where $a_1 = 1, a_2 = -1$
7. So, from the notes we have the formula,

$$Var(X_1 - X_2) = (a_1)^2 Var(X_1) + (a_2)^2 Var(X_2) + 2a_1a_2Cov(X_1, X_2)$$

8. Evaluate equation with the known values, yielding,

$$Var(X_1 - X_2) = 1^2(0.0169) + (-1)^2(0.0169) + 2(1)(-1)(0.0135)$$

$$Var(X_1 - X_2) = 0.3380 - 0.0270$$

$$Var(X_1 - X_2) = 0.3310$$

Problem 3

1. List variables: $n = 24$, $\bar{x} = 5.23$, $S = 0.24$, $\alpha = 0.05$
2. We will assume that the popcorn weights follow a normal distribution.
3. Construct the following hypothesis test situation,

$$H_0 : \mu = 5.5$$

$$H_a : \mu \neq 5.5$$

4. Utilize the p-value method and T-test.
5. The equation for T-test is,

$$t_{obs} = \frac{\bar{x} - \mu}{S/\sqrt{n}}$$

6. Substituting in our values, we get,

$$t_{obs} = \frac{5.23 - 5.5}{0.24/\sqrt{24}}$$

$$t_{obs} = -5.511352$$

7. Using R, we can calculate the p-value,

$$p - value = 0.00001324059$$

8. Since $0.00001324059 \leq 0.05$, we reject H_0