Network Measures

In this document I will outline some of the salient network measures and provide a brief plain speaking introduciton to them with straight forward code examples for `R`. Unless otherwise specified all commands will belong to the `igraph` package.

Further for each measure I will identify which kinds of networks it can be applied to. U for undirected graphs, D for directed graphs, B for binary, and V for valued networks.

Density

One of the most basic network measures, it measures the proportion of realized dyads between the maximum number of possible dyads in the network given the number of nodes present.

$U/D/B$

Mean Degree

Another network level measure, returns the mean number of first step connections all nodes in the network has. Mathematically it would be the sum of all node degrees over the network size and this indicates the general level of connectedness in the network.

$U/D/B$

Reciprocity

This measures the degree to which directed ties sent by a node to an alter are returned by alter to the original sending node. This is a network level measure similar to density as it is the proportion of reciprocated edges over all the ties in the network.

```
reciprocity(g) # where g is a graph object
```

D/B

Triad Census

This is not so much a distinct measure which returns one value for a network, but instead counts (performs a *census* really) of all the 16 possible triad isomorphisms in a network. These are isomorphisms, as they are all composed of the same number of nodes, but with different relations. These different relations can be one of three types: Mutual, or reciprocated relations, Asymmetric or non-reciprocated, and Null or no relation (M-A-N is a helpful nemonic). See below for a key which shows the 16 different isomorphisms:

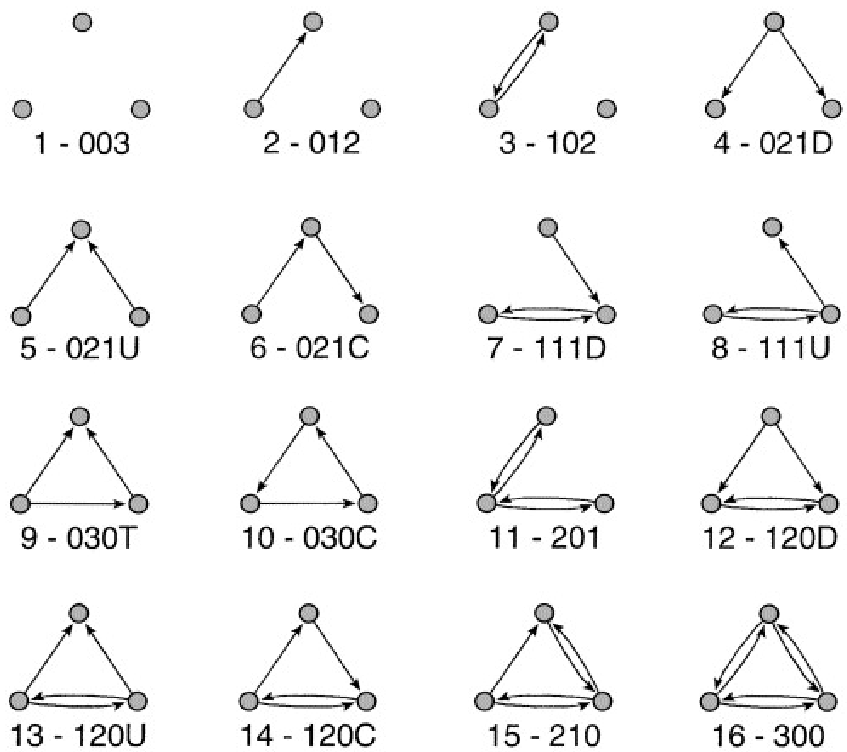

Figure 1: Key with the 16 possible triad isomorphisms in a directed graph.

A triad census with these 16 isomorphisms can only be performed on a directed graph, but there is a version for undirected graphs with fewer isomorphisms.

U/D/B

Size

The number of nodes in a network.

Centrality Measures

These measures are all node level measures, meant to capture how comparatively "important" nodes within the network are.

Degree Centrality

The number of first step connections a node has.

U/D/B/Multigraph

Betweenness Centrality

The number of shortest paths between nodes that the focal node lies on. A node with a comparatively high betweenness centrality in a network is bridging different areas in the network or is a place where information is likely to flow through.

U/D/B

Closeness Centrality

Closeness is the reciprocal of farness (1/farness) where farness is the average distance of a focal node from all other nodes in the graph.

U/D/B

Eigenvector Centrality

This measure of centrality takes into account the degree to which a focal node is connected to other nodes in a network and also the degree of connectedness of the focal nodes neighbors. This is meant to be a more sophisticated centrality measure capturing the relative influence of a node by also capturing the influence of the nodes it is connected to.

There has been some recent criticism of this measure, with the primary critique being that it is highly correlated to degree centrality and most network do not satisfy the theoretical requirements to make this measure meaningful.

U/D/B/V

Bonacich Power

Similar enough to Eigenvector Centrality that I have never been able to distinguish it.

$U/D/B/V$

Community Detection

This is a class of measures which are meant to capture the degree to which a network can be broken up into distinct and meaningful parts. In sociology, we would think of groups and cliques and other meaningful social entities that are embedded in large systems of relations.

The various forms of community detection can be thought of as intermediate measures of network structure, between local and global network measures.

Component Analysis

The most basic form of "network group", and might not be entirely informative for our purposes as our networks are likely to be composed of one component with several isolates because they are so small.

Cliques

Another form of local community structure, and are defined as "fully connected subgrapgs within a network", I suppose that means that groups that have network closure. I imagine cliques are associated with closed triads and that this measure will not be very informative for our own networks

Modularity

This measure "operationalizes our notion of groups above by calculating the proportion of ties that are within groups as opposed to between them." This, I think is the most intuitive means of group detection as a network that has high modularity means that there are distinct "modules" where nodes within them have dense connections with nodes in the shared module and spare connections with those outside the module. It is at bottom all the same really, closure, density, connectedness. Here is an example of how to calculate modularity in `R`:

```
communityMulti <- multilevel.community(g) # where g is a graph object

modularity(communityMulti)
```