

## **Report on “The Coupling Time for the Ising Heat-Bath Dynamics & Efficient Optimization for Statistical Inference”- T.L. Hyndman**

This thesis is divided into two parts, the first of which addresses coupling time for Ising heat bath Glauber dynamics. The thesis shows that the distribution of the coupling time for this Markov chain converges to the Gumbel distribution for certain graphs : the 1-d cycle at all temperatures and certain classes of transitive graphs at sufficiently high temperature. The proofs are built on the technique of information percolation introduced by Lubetzsky and Sly in their work showing cutoff for the Ising model.

The second part of the thesis concerns problems in statistical inference for which the author solves optimization problems to find a probability distribution on a discrete support. He improves known bounds in the case of maximum likelihood mixtures and compares them to empirical results.

The thesis is well-written and proofs are clearly explained. While I could not verify and vouch for the correctness of every claim in the, I believe the results to be correct. As such I only have the following minor comments.

### **Minor Comments:**

1. Page 6, para 1, line 9. Change ” at the end of the line to “ and make this change throughout as needed.
2. Page 6, para 3, line 3. Change graph to graphs.
3. Page 7, line 7. You may want to point to the definition of coupling time since it has not yet been defined.
4. Page 9, line 2. Change  $n$  to  $n$ .
5. Page 18, section 2.3.2.3, para1, last line. There are extra commas around the ... Also, it seems like  $t_2$  is missing from the condition.
6. Page 48, last line, sentence starting with “Since our proof...”. It was unclear how you meant a similar approach could extend your results: To the critical temperature? You do address some of this in the conclusions to Part 1, but it might be good to clarify here what you mean.
7. For the second part of the thesis, it might be nice to have a brief summary of your results right before/after explaining Lindsay’s approach in section 6.2. I was left a bit unsure as to what the contribution of the thesis was until I had read through a good deal of Chapter 6. It would be nice to set your results in the context of the introduction is possible. It may also be nice to give a few specific directions for future work in the area in section 6.5.