Higher-Order Modules
Formal Semantics

April 12, 2010

1 Translation

$$
\begin{array}{rcll}
p & ::= & x \mid p.x & \text{symbolic paths} \\
K & ::= & \Omega \mid \Omega^n \Rightarrow \Omega & \text{kinds} \\
C^s & ::= & \alpha \mid p(\vec{C^s}) & \text{monotypes} \\
C^\lambda & ::= & \lambda\vec{\alpha}.C^s & \text{closed tycons} \\
T & ::= & \mathsf{typ}(C^s) \mid \forall\vec{\alpha}.C^s & \text{closed types} \\
e & ::= & p \mid \lambda x.e \mid e_1 e_2 & \text{terms}
\end{array}
$$

$$
\begin{array}{lrcl}
\text{Core declarations} & d^c & ::= & \mathsf{val}\ x = e \mid \mathsf{type}\ t = C^\lambda \mid \mathsf{datatype}\ \vec{\alpha}\ t \\
 & \mathit{spec} & ::= & \mathsf{structure}\ x : \mathit{sigexp} \mid \mathsf{type}\ \vec{\alpha}\ t \\
 & & \mid & \mathsf{type}\ t = C^\lambda \\
 & & \mid & \mathsf{functor}\ f(x : \mathit{sigexp}_1) : \mathit{sigexp}_2 \\
 & & \mid & \mathsf{val}\ x : T \\
 & \mathit{specs} & ::= & \emptyset_{\mathit{specs}} \mid \mathit{spec}, \mathit{specs} \\
 & \mathit{sigexp} & ::= & x \mid \mathsf{sig}\ \mathit{spec}\ \mathsf{end} \mid \mathit{sigexp}\ \mathsf{where\ type}\ p = C^\lambda \\
 & & \mid & \mathit{sigexp}\ \mathsf{where}\ p_1 = p_2 \\
 & \mathit{strexp} & ::= & p \mid \mathsf{struct}\ d^m\ \mathsf{end} \mid p(\mathit{strexp}) \\
 & & \mid & \mathit{strexp} : \mathit{sigexp} \mid \mathit{strexp} :> \mathit{sigexp} \\
\text{Structure decl} & d^m & ::= & \mathsf{structure}\ x = \mathit{strexp}, d^m \\
 & & \mid & \mathsf{functor}\ f(x : \mathit{sigexp}) = \mathit{strexp}, d^m \mid d^c, d^m
\end{array}
$$

Figure 1: Surface Module Language

$$\Delta ::= \emptyset_{knds} \mid \Delta[\alpha]$$

$$\frac{\alpha \in \Delta}{\Gamma, \Delta \vdash \alpha : \Omega} \tag{1}$$

$$\frac{\Gamma, \Delta \vdash \Gamma(p) : \Omega^n \Rightarrow \Omega \qquad |\vec{C^s}| = n \qquad \Gamma, \Delta \vdash C^s_i : \Omega\ \forall i \in [1, n]}{\Gamma, \Delta \vdash p(\vec{C^s}) : \Omega} \tag{2}$$

$$\frac{\Gamma, \Delta \vdash C^s : \Omega}{\Gamma, \Delta \vdash \mathsf{typ}(C^s) : \Omega} \tag{3}$$

$$\frac{\Gamma, \Delta[\alpha_1] \ldots [\alpha_n] \vdash C^s : \Omega}{\Gamma, \Delta \vdash \forall\vec{\alpha}.C^s : \Omega} \tag{4}$$

$$\frac{\Gamma, \Delta[\alpha_1] \ldots [\alpha_n] \vdash C^s : \Omega}{\Gamma, \Delta \vdash \lambda\vec{\alpha}.C^s : \Omega^n \Rightarrow \Omega} \tag{5}$$

Figure 2: Well-kinding of tycons

$$
\begin{array}{rcll}
\rho & \in & \text{EntityVars} & \text{entity variables} \\
\tau^n & \in & \text{Tycs} & n \text{ is arity} : \Omega^n \Rightarrow \Omega \\
\\
\mathfrak{C}^s & ::= & \alpha \mid \mathfrak{C}^\lambda(\vec{\mathfrak{C}^s}) & \text{semantic monotype} \\
\mathfrak{C}^\lambda & ::= & \lambda\vec{\alpha}.\mathfrak{C}^s \mid \tau^n & \text{semantic tycon} \\
\mathfrak{T} & ::= & \mathsf{typ}(\mathfrak{C}^s) \mid \forall\vec{\alpha}.\mathfrak{C}^s & \text{semantic type expression} \\
\\
\mathbb{C}^s & ::= & \alpha \mid \vec{\rho}(\vec{\mathbb{C}^s}) & \text{relativized monotypes} \\
\mathbb{C}^\lambda & ::= & \lambda\vec{\alpha}.\mathbb{C}^s & \text{relativized tycons} \\
\mathbb{T} & ::= & \mathsf{typ}(\mathbb{C}^s) \mid \forall\vec{\alpha}.\mathbb{C}^s & \text{relativized type expression} \\
\\
\Upsilon & ::= & \emptyset_{ee} \mid \Upsilon[\rho \mapsto \upsilon] & \text{entity environment} \\
R & ::= & \langle \Upsilon^{lcl}, \Upsilon^{clo} \rangle & \text{structure entity} \\
\psi & ::= & \langle \lambda\rho.\varphi; \Upsilon \rangle \mid \langle \lambda\rho.\Sigma; \Upsilon \rangle & \text{functor entity} \\
\upsilon & ::= & \psi \mid R \mid \tau^n & \text{static entity}
\end{array}
$$

Structure entity expression

$$
\begin{array}{rcll}
\varphi & ::= & (\!|\vec{\rho}|\!) & \text{entity path} \\
 & \mid & (\!|\eta|\!) & \text{entity declaration} \\
 & \mid & \theta(\varphi) & \text{functor application} \\
 & \mid & \textbf{let } \eta \textbf{ in } \varphi & \text{local definition} \\
\theta & ::= & \vec{\rho} \mid \lambda\rho.\varphi \mid \lambda\rho.\Sigma & \text{functor entity expression} \\
\eta & ::= & \bullet \mid [\rho = \varphi]\eta \mid [\rho = \theta]\eta \mid [\rho = \mathsf{new}(n)]\eta & \text{entity declarations}
\end{array}
$$

Figure 3: Static entities and entity expressions

$$
\begin{array}{rcll}
s^p & ::= & arity \mid \Sigma \mid \Sigma^f & \text{primary spec} \\
\Sigma & ::= & \emptyset_{sig} \mid [x \mapsto (\rho, s^p)]\Sigma \mid [x \mapsto \mathbb{C}^\lambda]\Sigma \mid [x \mapsto \mathbb{T}]\Sigma & \text{semantic signature} \\
M & ::= & (\Sigma, R) & \text{full signature} \\
\Sigma^f & ::= & \Pi\rho : \Sigma.\Sigma & \text{functor signature} \\
F & ::= & (\Sigma^f, \psi) & \text{full functor signature} \\
\gamma & ::= & \mathfrak{T} \mid \mathfrak{C}^\lambda \mid \Sigma \mid \Sigma^f \mid (\rho, M) \mid (\rho, F) & \text{static binding} \\
\Gamma & ::= & \emptyset_{se} \mid \Gamma[x \mapsto \gamma] & \text{static type environment}
\end{array}
$$

The static bindings for structures and functors include the entity variable to permit direct construction of entity paths during signature extraction, structure path, and functor path elaboration.

Figure 4: Semantic representations

$$\boxed{\Gamma \vdash C^s \Rightarrow_{mt} \mathfrak{C}^s}$$

$$\Gamma \vdash \alpha \Rightarrow_{mt} \alpha \tag{6}$$

$$\frac{\Gamma \vdash \vec{C^s} \Rightarrow_{mt} \mathfrak{C}^s \qquad (\Gamma(p))(\vec{\mathfrak{C}^s}) \Downarrow \mathfrak{C}^s_1}{\Gamma \vdash p(\vec{C^s}) \Rightarrow_{mt} \mathfrak{C}^s_1} \tag{7}$$

$$\boxed{\Gamma \vdash C^\lambda \Rightarrow_{tyc} \mathfrak{C}^\lambda}$$

$$\frac{\Gamma \vdash C^s \Rightarrow_{mt} \mathfrak{C}^s}{\Gamma \vdash \lambda\vec{\alpha}.C^s \Rightarrow_{tyc} \lambda\vec{\alpha}.\mathfrak{C}^s} \tag{8}$$

$$\boxed{\Gamma \vdash T \Rightarrow_{te} \mathfrak{T}}$$

$$\frac{\Gamma \vdash C^s \Rightarrow_{mt} \mathfrak{C}^s}{\Gamma \vdash \mathsf{typ}(C^s) \Rightarrow_{te} \mathsf{typ}(\mathfrak{C}^s)} \tag{9}$$

$$\frac{\Gamma \vdash C^s \Rightarrow_{mt} \mathfrak{C}^s}{\Gamma \vdash \forall\vec{\alpha}.C^s \Rightarrow_{te} \forall\vec{\alpha}.\mathfrak{C}^s} \tag{10}$$

Figure 5: Monotype and Tycon elaboration

$$\boxed{\Gamma, \Upsilon, \Sigma \vdash C^s \Rightarrow^{mt}_{rel} \mathbb{C}^s}$$

$$\Gamma, \Upsilon, \Sigma \vdash \alpha \Rightarrow^{mt}_{rel} \alpha \tag{11}$$

$$\frac{\vec{\rho} = \mathsf{entpath}(\Gamma, \Upsilon, \Sigma, p) \qquad \Gamma, \Upsilon, \Sigma \vdash \vec{C^s} \Rightarrow^{mt}_{rel} \vec{\mathbb{C}^s}}{\Gamma, \Upsilon, \Sigma \vdash p(\vec{C^s}) \Rightarrow^{mt}_{rel} \vec{\rho}(\vec{\mathbb{C}^s})} \tag{12}$$

$$\boxed{\Gamma, \Upsilon, \Sigma \vdash C^\lambda \Rightarrow^{tyc}_{rel} \mathbb{C}^\lambda}$$

$$\frac{\Gamma, \Upsilon, \Sigma \vdash C^s \Rightarrow^{mt}_{rel} \mathbb{C}^s}{\Gamma, \Upsilon, \Sigma \vdash \lambda\vec{\alpha}.C^s \Rightarrow^{tyc}_{rel} \lambda\vec{\alpha}.\mathbb{C}^s} \tag{13}$$

$$\boxed{\Gamma, \Upsilon, \Sigma \vdash T \Rightarrow^{te}_{rel} \mathbb{T}}$$

$$\frac{\Gamma, \Upsilon, \Sigma \vdash C^s \Rightarrow^{mt}_{rel} \mathbb{C}^s}{\Gamma, \Upsilon, \Sigma \vdash \mathsf{typ}(C^s) \Rightarrow^{te}_{rel} \mathsf{typ}(\mathbb{C}^s)} \tag{14}$$

$$\frac{\Gamma, \Upsilon, \Sigma \vdash C^s \Rightarrow^{mt}_{rel} \mathbb{C}^s}{\Gamma, \Upsilon, \Sigma \vdash \forall\vec{\alpha}.C^s \Rightarrow^{te}_{rel} \forall\vec{\alpha}.\mathbb{C}^s} \tag{15}$$

$$\mathsf{entpath}(\Gamma, \Upsilon, \Sigma, p) = \begin{cases} \Sigma_{ep}(p) & \text{if } p \in \Sigma \\ \Upsilon^{-1}(\Gamma(p)) & o.w. \end{cases} \tag{16}$$

Figure 6: Relativization

$\boxed{\Gamma, \Upsilon, \Sigma \vdash \mathit{sigexp} \Rightarrow_{sig} \Sigma'}$

$$\frac{}{\Gamma, \Upsilon, \Sigma \vdash x \Rightarrow_{sig} \Gamma(x)} \tag{17}$$

$$\frac{\begin{array}{c}\Gamma, \Upsilon, \Sigma \vdash \mathit{sigexp} \Rightarrow_{sig} \Sigma' \qquad \Sigma'(p) = (\rho, n) \\ \Gamma, \Upsilon, \Sigma\Sigma' \vdash C^\lambda \Rightarrow_{rel}^{tyc} \mathbb{C}^\lambda \qquad |\mathbb{C}^\lambda| = n\end{array}}{\Gamma, \Upsilon, \Sigma \vdash \mathit{sigexp}\ \textbf{where type}\ p = C^\lambda \Rightarrow_{sig} \mathsf{rebind}(p, \mathbb{C}^\lambda, \Sigma')} \tag{18}$$

$$\frac{\Gamma, \Upsilon, \Sigma \vdash \mathit{specs} \Rightarrow_{specs} \Sigma'}{\Gamma, \Upsilon, \Sigma \vdash \textbf{sig}\ \mathit{specs}\ \textbf{end} \Rightarrow_{sig} \Sigma'} \tag{19}$$

$\mathsf{rebind}(p, \mathbb{C}^\lambda, \Sigma)$ replaces the binding $[x \mapsto (\rho, n)]$ in Σ with $[x \mapsto \mathbb{C}^\lambda]$ where p ends in x.

$\boxed{\Gamma, \Upsilon, \Sigma \vdash \mathit{specs} \Rightarrow_{specs} \Sigma}$

$$\frac{}{\Gamma, \Upsilon, \Sigma \vdash \emptyset_{specs} \Rightarrow_{specs} \emptyset_{sig}} \tag{20}$$

$$\frac{\Gamma, \Upsilon, \Sigma \vdash \mathit{spec} \Rightarrow_{spec} \Sigma' \qquad \Gamma', \Upsilon', \Sigma\Sigma' \vdash \mathit{specs} \Rightarrow_{specs} \Sigma''}{\Gamma, \Upsilon, \Sigma \vdash \mathit{spec}, \mathit{specs} \Rightarrow_{specs} \Sigma'\Sigma''} \tag{21}$$

Figure 7: Signature elaboration

$\boxed{\Gamma, \Upsilon, \Sigma \vdash \mathit{spec} \Rightarrow_{spec} \Sigma'}$

$$\frac{(\rho \text{ fresh in } \Gamma \text{ and } \Upsilon)}{\Gamma, \Upsilon, \Sigma \vdash \textbf{type}\ \vec{\alpha}\ t \Rightarrow_{spec} [t \mapsto (\rho, |\vec{\alpha}|)]} \tag{22}$$

$$\frac{\Gamma, \Upsilon, \Sigma \vdash C^\lambda \Rightarrow_{rel}^{tyc} \mathbb{C}^\lambda}{\Gamma, \Upsilon, \Sigma \vdash \textbf{type}\ t = C^\lambda \Rightarrow_{spec} [t \mapsto \mathbb{C}^\lambda]} \tag{23}$$

$$\frac{\Gamma, \Upsilon, \Sigma \vdash T \Rightarrow_{rel}^{tyc} \mathbb{T}}{\Gamma, \Upsilon, \Sigma \vdash \textbf{val}\ x : T \Rightarrow_{spec} [x \mapsto \mathbb{T}]} \tag{24}$$

$$\frac{\Gamma, \Upsilon, \Sigma \vdash \mathit{sigexp} \Rightarrow_{sig} \Sigma' \qquad (\rho \text{ fresh in } \Gamma \text{ and } \Upsilon)}{\Gamma, \Upsilon, \Sigma \vdash \textbf{structure}\ x : \mathit{sigexp} \Rightarrow_{spec} [x \mapsto (\rho, \Sigma')]} \tag{25}$$

$$\frac{\begin{array}{c}\Gamma, \Upsilon, \Sigma \vdash \mathit{sigexp}_1 \Rightarrow_{sig} \Sigma_1 \qquad (\rho_x \text{ and } \rho \text{ fresh in } \Gamma \text{ and } \Upsilon) \\ \Gamma, \Upsilon, \Sigma[X \mapsto (\rho_x, \Sigma_1)] \vdash \mathit{sigexp}_2 \Rightarrow_{sig} \Sigma_2\end{array}}{\Gamma, \Upsilon, \Sigma \vdash \textbf{functor}\ f(X : \mathit{sigexp}_1) : \mathit{sigexp}_2 \Rightarrow_{spec} [f \mapsto (\rho, \Pi\rho_x : \Sigma_1.\Sigma_2)]} \tag{26}$$

Figure 8: Signature spec elaboration

$\boxed{\Upsilon^{clo}, \Upsilon_1^{lcl} \vdash \Sigma \uparrow \Upsilon_2^{lcl}}$

$$\frac{}{\Upsilon^{clo}, \Upsilon^{lcl} \vdash \emptyset_{sig} \uparrow \emptyset_{ee}} \tag{27}$$

$$\frac{\Upsilon^{clo}, \Upsilon^{lcl} \vdash \Sigma \uparrow \Upsilon'}{\Upsilon^{clo}, \Upsilon^{lcl} \vdash [x \mapsto \mathbb{C}^{\lambda}]\Sigma \uparrow \Upsilon'} \tag{28}$$

$$\frac{\Upsilon^{clo}, \Upsilon^{lcl} \vdash \Sigma \uparrow \Upsilon'}{\Upsilon^{clo}, \Upsilon^{lcl} \vdash [x \mapsto \mathbb{T}]\Sigma \uparrow \Upsilon'} \tag{29}$$

$$\frac{\Upsilon^{clo}, \Upsilon^{lcl}[\rho \mapsto \tau^n] \vdash \Sigma \uparrow \Upsilon' \qquad (\tau \text{ is fresh in } \Upsilon^{clo} \text{ and } \Upsilon^{lcl})}{\Upsilon^{clo}, \Upsilon^{lcl} \vdash [x \mapsto (\rho, n)]\Sigma \uparrow [\rho \mapsto \tau^n]\Upsilon'} \tag{30}$$

$$\frac{\Upsilon^{clo}, \Upsilon^{lcl} \vdash \Sigma' \uparrow \Upsilon' \qquad \Upsilon^{clo}, \Upsilon^{lcl}[\rho \mapsto \langle \Upsilon', \Upsilon^{clo}\Upsilon^{lcl} \rangle] \vdash \Sigma \uparrow \Upsilon''}{\Upsilon^{clo}, \Upsilon^{lcl} \vdash [x \mapsto (\rho, \Sigma')]\Sigma \uparrow [\rho \mapsto \langle \Upsilon', \Upsilon^{clo}\Upsilon^{lcl} \rangle]\Upsilon''} \tag{31}$$

$$\frac{\Upsilon^{clo}, \Upsilon^{lcl} \vdash \Sigma \uparrow \Upsilon'}{\Upsilon^{clo}, \Upsilon^{lcl} \vdash [x \mapsto (\rho, \Pi\rho_x : \Sigma_x.\Sigma_r)]\Sigma \uparrow [\rho \mapsto \langle \lambda\rho_x.\Sigma_r; \Upsilon^{clo}\Upsilon^{lcl} \rangle]\Upsilon'} \tag{32}$$

Figure 9: Signature instantiation

$$\boxed{\Gamma, \Upsilon \vdash d^m \Rightarrow_{decl} (\eta, \Gamma', \Upsilon')}$$

$$\frac{}{\Gamma, \Upsilon \vdash \bullet \Rightarrow_{decl} (\bullet, \emptyset_{se}, \emptyset_{ee})} \tag{33}$$

$$\frac{\Gamma, \Upsilon \vdash e \Rightarrow_{core} \mathfrak{T} \qquad \Gamma[x \mapsto \mathfrak{T}], \Upsilon \vdash d^m \Rightarrow_{decl} (\eta, \Gamma', \Upsilon')}{\Gamma, \Upsilon \vdash \textbf{val } x = e, d^m \Rightarrow_{decl} (\eta, \Gamma', \Upsilon')} \tag{34}$$

$$\frac{\Gamma \vdash C^\lambda \Rightarrow_{tyc} \mathfrak{C}^\lambda \qquad \Gamma[t \mapsto \mathfrak{C}^\lambda], \Upsilon \vdash d^m \Rightarrow_{decl} (\eta, \Gamma', \Upsilon')}{\Gamma, \Upsilon \vdash \textbf{type } t = C^\lambda, d^m \Rightarrow_{decl} (\eta, \Gamma', \Upsilon')} \tag{35}$$

$$\frac{\begin{array}{c} n = |\vec{\alpha}| \qquad \Gamma[t \mapsto \tau^n], \Upsilon[\rho_t \mapsto \tau^n] \vdash d^m \Rightarrow_{decl} (\eta, \Gamma', \Upsilon') \\ (\rho_t \text{ and } \tau \text{ are fresh}) \end{array}}{\Gamma, \Upsilon \vdash \textbf{datatype } \vec{\alpha}\ t, d^m \Rightarrow_{decl} ([\rho_t = \mathsf{new}(n)]\eta, \Gamma', \Upsilon')} \tag{36}$$

$$\frac{\begin{array}{c} \Gamma, \Upsilon \vdash \mathit{strexp} \Rightarrow_{str} (M, \varphi) \qquad M = (\Sigma, R) \qquad (\rho \text{ fresh}) \\ \Gamma[X \mapsto (\rho, M)], \Upsilon[\rho \mapsto R] \vdash d^m \Rightarrow_{decl} (\eta, \Gamma', \Upsilon') \end{array}}{\Gamma, \Upsilon \vdash \textbf{structure } X = \mathit{strexp}, d^m \Rightarrow_{decl} ([\rho = \varphi]\eta, \Gamma', \Upsilon')} \tag{37}$$

$$\frac{\begin{array}{c} \Gamma, \Upsilon, \emptyset_{sig} \vdash \mathit{sigexp} \Rightarrow_{sig} \Sigma_x \qquad \Upsilon, \emptyset_{ee} \vdash \Sigma_x \uparrow \Upsilon_x \qquad R_x = \langle \Upsilon_x, \Upsilon \rangle \\ \Gamma[X \mapsto (\rho, (\Sigma_x, R_x))], \Upsilon[\rho_x \mapsto R_x] \vdash \mathit{strexp} \Rightarrow_{str} ((\Sigma_{res}, \_), \varphi) \\ \theta = \lambda \rho_x . \varphi \qquad \psi = \langle \theta; \Upsilon \rangle \\ \Gamma[f \mapsto (\rho, (\Pi \rho_x : \Sigma_x . \Sigma_{res}, \psi))], \Upsilon[\rho \mapsto \psi] \vdash d^m \Rightarrow_{decl} (\eta, \Gamma'', \Upsilon'') \\ (\rho_x, \rho \text{ fresh}) \end{array}}{\Gamma, \Upsilon \vdash \textbf{functor } f(X : \mathit{sigexp}) = \mathit{strexp}, d^m \Rightarrow_{decl} ([\rho = \theta]\eta, \Gamma'', \Upsilon'')} \tag{38}$$

The resultant Υ must be the local entity environment in order for the structure expression judgment for struct d^m end to properly construct a structure realization.

Figure 10: Module declaration elaboration

$$\boxed{\Gamma, \Upsilon \vdash \mathit{strexp} \Rightarrow_{str} (M, \varphi)}$$

$$\frac{\Gamma(p) = (\vec{\rho}, M)}{\Gamma, \Upsilon \vdash p \Rightarrow_{str} (M, \vec{\rho})} \tag{39}$$

$$\frac{\Gamma, \Upsilon \vdash d^m \Rightarrow_{decl} (\eta, \Gamma', \Upsilon^{lcl}) \qquad \Upsilon \vdash \Gamma' \hookrightarrow \Sigma}{\Gamma, \Upsilon \vdash \mathbf{struct}\ d^m\ \mathbf{end} \Rightarrow_{str} ((\Sigma, \langle \Upsilon^{lcl}, \Upsilon \rangle), (\!|\eta|\!))} \tag{40}$$

$$\frac{\begin{array}{c}\Gamma(p) = (\vec{\rho}, (\Pi X : \Sigma_{par}.\Sigma_{body}, \langle \theta; \Upsilon' \rangle)) \\ \Gamma, \Upsilon \vdash \mathit{strexp} \Rightarrow_{str} (M, \varphi) \\ \Upsilon \vdash (M, \varphi) : \Sigma_{par} \Rightarrow_{match} (M_c, \varphi_c) \\ \varphi_{app} = \vec{\rho}(\varphi_c) \qquad \Upsilon \vdash \varphi_{app} \Downarrow \Upsilon_{app}\end{array}}{\Gamma, \Upsilon \vdash p(\mathit{strexp}) \Rightarrow_{str} ((\Sigma_{body}, \langle \Upsilon_{app}, \Upsilon \rangle), \varphi_{app})} \tag{41}$$

$$\frac{\begin{array}{c}\Gamma, \Upsilon \vdash \mathit{dec} \Rightarrow_{decl} (\eta_{def}, \Gamma_{def}, \Upsilon_{def}) \\ \Gamma_{def}, \Upsilon_{def} \vdash \mathit{strexp} \Rightarrow_{str} (M, \varphi)\end{array}}{\Gamma, \Upsilon \vdash \mathbf{let}\ \mathit{dec}\ \mathbf{in}\ \mathit{strexp} \Rightarrow_{str} (M, \mathbf{let}\ \eta_{def}\ \mathbf{in}\ \varphi)} \tag{42}$$

$$\frac{\begin{array}{c}\Gamma, \Upsilon, \emptyset_{sig} \vdash \mathit{sigexp} \Rightarrow_{sig} \Sigma_{spec} \qquad \Gamma, \Upsilon \vdash \mathit{strexp} \Rightarrow_{str} (M_u, \varphi_u) \\ \Upsilon \vdash (M_u, \varphi_u) : \Sigma_{spec} \Rightarrow_{match} (M_c, \varphi_c)\end{array}}{\Gamma, \Upsilon \vdash \mathit{strexp} : \mathit{sigexp} \Rightarrow_{str} (M_c, \varphi_c)} \tag{43}$$

$$\frac{\begin{array}{c}\Gamma, \Upsilon, \emptyset_{sig} \vdash \mathit{sigexp} \Rightarrow_{sig} \Sigma_{spec} \qquad \Gamma, \Upsilon \vdash \mathit{strexp} \Rightarrow_{str} (M_u, \varphi_u) \\ \Upsilon \vdash (M_u, \varphi_u) : \Sigma_{spec} \Rightarrow_{match} (M_c, \varphi_c) \qquad \Upsilon, \emptyset_{ee} \vdash \Sigma_{spec} \uparrow \Upsilon_{spec}\end{array}}{\Gamma, \Upsilon \vdash \mathit{strexp} :> \mathit{sigexp} \Rightarrow_{str} ((\Sigma_{spec}, \langle \Upsilon_{spec}, \Upsilon \rangle), \varphi_c)} \tag{44}$$

Figure 11: Structure expression elaboration

$$\boxed{\Upsilon \vdash \Gamma \hookrightarrow \Sigma}$$

$$\frac{}{\Upsilon \vdash \emptyset_{se} \hookrightarrow \emptyset_{sig}} \quad (45)$$

$$\frac{\Upsilon \vdash \mathfrak{T} \Rightarrow^{te}_{rel} \mathbb{T} \qquad \Upsilon \vdash \Gamma \hookrightarrow \Sigma_r}{\Upsilon \vdash [x \mapsto \mathfrak{T}]\Gamma \hookrightarrow [x \mapsto \mathbb{T}]\Sigma_r} \quad (46)$$

$$\frac{\Upsilon \vdash \mathfrak{C}^\lambda \Rightarrow^{tyc}_{rel} \mathbb{C}^\lambda \qquad \Upsilon \vdash \Gamma \hookrightarrow \Sigma_r}{\Upsilon \vdash [t \mapsto \mathfrak{C}^\lambda]\Gamma \hookrightarrow [t \mapsto \mathbb{C}^\lambda]\Sigma_r} \quad (47)$$

$$\frac{\Upsilon^{-1}(\tau^n) = \rho \qquad \Upsilon \vdash \Gamma \hookrightarrow \Sigma_r}{\Upsilon \vdash [t \mapsto \tau^n]\Gamma \hookrightarrow [t \mapsto (\rho, n)]\Sigma_r} \quad (48)$$

$$\frac{\Upsilon \vdash \Gamma_r \hookrightarrow \Sigma_r}{\Upsilon \vdash [x \mapsto (\rho, (\Sigma_1, R_1))]\Gamma_r \hookrightarrow [x \mapsto (\rho, \Sigma_1)]\Sigma_r} \quad (49)$$

$$\frac{\Upsilon \vdash \Gamma_r \hookrightarrow \Sigma_r}{\Upsilon \vdash [f \mapsto (\rho, (\Sigma_1^f, \psi))]\Gamma_r \hookrightarrow [f \mapsto (\rho, \Sigma_1^f)]\Sigma_r} \quad (50)$$

Figure 12: Signature extraction

$$\boxed{\Upsilon \vdash (M, \varphi) : \Sigma \Rightarrow_{match} (M_c, \varphi_c)}$$

$$\frac{(\rho_u \text{ is fresh in } \Upsilon) \qquad \Upsilon_a \Upsilon^{clo} \Upsilon, \Sigma_a, \rho_u \vdash \Sigma_s \Rightarrow_{coerce} (\Sigma_c, \Upsilon', \eta)}{\Upsilon \vdash ((\Sigma_a, \langle \Upsilon_a, \Upsilon^{clo} \rangle), \varphi) : \Sigma_s \Rightarrow_{match} ((\Sigma_c, \langle \Upsilon', \Upsilon \rangle), \textbf{let } \rho_u = \varphi \textbf{ in } (\!|\eta|\!))} \tag{51}$$

$$\boxed{\Upsilon, \Sigma_a, \rho_u \vdash \Sigma_s \Rightarrow_{coerce} (\Sigma_c, \Upsilon', \eta)}$$

$$\frac{}{\Upsilon, \Sigma_a, \rho_a \vdash \bullet \Rightarrow_{coerce} (\emptyset_{sig}, \emptyset_{ee}, \bullet)} \tag{52}$$

$$\frac{\Sigma_a(t) = (\rho_a, n) \qquad \Upsilon, \Sigma_a, \rho_a \vdash \Sigma_s \Rightarrow_{coerce} (\Sigma_c, \Upsilon', \eta)}{\Upsilon, \Sigma_a, \rho_u \vdash [t \mapsto (\rho, n)]\Sigma_s \Rightarrow_{coerce} ([t \mapsto \Upsilon(\rho_a)]\Sigma_c, [\rho = \Upsilon(\rho_a)]\Upsilon', [\rho = \rho_u \rho_a]\eta)} \tag{53}$$

$$\frac{\begin{array}{c}\Upsilon \vdash \Sigma_a(x) \equiv \mathbb{T} \\ \Upsilon, \Sigma_a, \rho_a \vdash \Sigma_s \Rightarrow_{coerce} (\Sigma', \Upsilon', \eta)\end{array}}{\Upsilon, \Sigma_a, \rho_a \vdash [x \mapsto \mathbb{T}]\Sigma_s \Rightarrow_{coerce} (\Sigma', \Upsilon', \eta)} \tag{54}$$

$$\frac{\begin{array}{c}\Sigma_a(t) = \mathbb{C}^\lambda \qquad |\mathbb{C}^\lambda| = n \\ \Upsilon, \Sigma_a, \rho_u \vdash \mathsf{replace}(\rho, \mathbb{C}^\lambda, \Sigma_s) \Rightarrow_{coerce} (\Sigma_c, \Upsilon', \eta)\end{array}}{\Upsilon, \Sigma_a, \rho_u \vdash [t \mapsto (\rho, n)]\Sigma_s \Rightarrow_{coerce} ([t \mapsto \mathbb{C}^\lambda]\Sigma_c, \Upsilon', \eta)} \tag{55}$$

$$\frac{\begin{array}{c}\Sigma_a(t) = \mathbb{C}_a^\lambda \qquad \Upsilon \vdash \mathbb{C}_a^\lambda \equiv \mathbb{C}_s^\lambda \\ \Upsilon, \Sigma_a, \rho_u \vdash \Sigma_s \Rightarrow_{coerce} (\Sigma_c, \Upsilon', \eta)\end{array}}{\Upsilon, \Sigma_a, \rho_u \vdash [t \mapsto \mathbb{C}_s^\lambda]\Sigma_s \Rightarrow_{coerce} ([t \mapsto \mathbb{C}_s^\lambda]\Sigma_c, \Upsilon', \eta)} \tag{56}$$

$$\frac{\begin{array}{c}\Sigma_a(x) = (\rho_a, \Sigma_a) \qquad \Upsilon \vdash ((\Sigma_a, \Upsilon(\rho_a)), (\!|\rho_u \rho_a|\!)) : \Sigma_s \Rightarrow_{match} ((\Sigma_c, R_c), \varphi_c) \\ \Upsilon' = \Upsilon[\rho_s \mapsto R_c] \qquad \Upsilon', \Sigma_a, \rho_u \vdash \Sigma_s \Rightarrow_{coerce} (\Sigma', \Upsilon'', \eta')\end{array}}{\Upsilon, \Sigma_a, \rho_u \vdash [x \mapsto (\rho_s, \Sigma_s)]\Sigma_s \Rightarrow_{coerce} ([x \mapsto (\rho_s, \Sigma_c)]\Sigma', \Upsilon'', [\rho_s = \varphi_c]\eta')} \tag{57}$$

$$\frac{\begin{array}{c}\Sigma_a(f) = (\rho_a, \Sigma_a^f) \qquad \Upsilon \vdash ((\Sigma_a^f, \Upsilon(\rho_a)), (\!|\rho_u \rho_a|\!)) : \Sigma_s^f \Rightarrow_{match} (\psi_c, \theta_c) \\ \Upsilon[\rho_s \mapsto \psi_c], \Sigma_a, \rho_u \vdash \Sigma_s \Rightarrow_{coerce} (\Sigma', \Upsilon', \eta)\end{array}}{\Upsilon, \Sigma_a, \rho_u \vdash [f \mapsto (\rho_s, \Sigma_s^f)]\Sigma_s \Rightarrow_{coerce} ([f \mapsto (\rho_s, \Sigma_s^f)]\Sigma', [\rho_s \mapsto \psi_c]\Upsilon', [\rho_s = \theta_c]\eta)} \tag{58}$$

Figure 13: Structure signature matching

$$\boxed{\Upsilon \vdash F \preceq \Sigma^f \rightrightarrows (\psi_c, \theta_c)}$$

$$\frac{\begin{array}{c}\Upsilon \vdash \Sigma_{apar} \uparrow \Upsilon_{apar} \qquad \Upsilon \vdash (\Sigma_{apar}, \langle \Upsilon_{apar} \rangle) \preceq \Sigma_{spar} \rightrightarrows (M_c, \varphi_c) \\ M_c = (\Sigma_c, \Upsilon_c) \qquad \Upsilon[\rho' \mapsto \langle \Upsilon_c \rangle] \vdash \Sigma_{sres} \uparrow \Upsilon_{sres} \\ \Upsilon \vdash (\Sigma_{sres}, \langle \Upsilon_{sres} \rangle) \preceq \Sigma_{ares} \rightrightarrows (M'_c, \varphi'_c) \qquad \theta_c = \lambda \rho'.\varphi'_c\end{array}}{\begin{array}{c}\Upsilon \vdash (\rho_f, \Pi\rho : \Sigma_{apar}.\Sigma_{ares}, \langle \lambda\rho.\varphi; \Upsilon_{cl} \rangle) \preceq \Pi\rho' : \Sigma_{spar}.\Sigma_{sres} \\ \rightrightarrows (\langle \theta_c; \Upsilon \rangle, \theta_c)\end{array}} \tag{59}$$

Figure 14: Functor signature matching

$$\frac{sigexp \Rightarrow K \qquad \vdash strexp \Rightarrow e}{\textsf{functor}\ F(X : sigexp) = strexp \Rightarrow \Lambda X :: K.\lambda X : T.e} \tag{60}$$

$$\frac{\Gamma() \qquad M \Rightarrow \mu}{strexp_0(strexp_1) \Rightarrow @(f[\mu])e} \tag{61}$$

$$\frac{\Gamma \vdash e \rightsquigarrow e'}{\Gamma \vdash \textbf{val}\ x = e \rightsquigarrow \{\rho = e'\}} \tag{62}$$

$$\frac{C^\lambda \Downarrow \tau}{\Gamma \vdash \textbf{type}\ t = C^\lambda \rightsquigarrow \{\rho = \tau\}} \tag{63}$$

$$\frac{\rho = \Upsilon^{-1}(t)}{\Gamma \vdash \textbf{datatype}\ \vec{\alpha}\ t \rightsquigarrow \{\rho = \tau^{|\vec{\alpha}|}\}} \tag{64}$$

$$\frac{\Gamma \vdash strexp \rightsquigarrow e}{\Gamma \vdash \textbf{structure}\ x = strexp \rightsquigarrow \{\rho = e\}} \tag{65}$$

$$\frac{}{\vdash (\Sigma, \langle\Upsilon\rangle) \rightsquigarrow} \tag{66}$$

$$\frac{\vdash strexp \Rightarrow (\tau, e_0)}{\vdash p_0(strexp) \Rightarrow @(p_0[\tau])e_0} \tag{67}$$

$$\frac{\vdash sigexp \Rightarrow \Sigma \qquad \vdash \Sigma \Rightarrow \kappa}{\vdash functor\ F(X : sigexp) = strexp \Rightarrow \Lambda X :: \kappa.\lambda X : \sigma.e} \tag{68}$$

$$\frac{}{\vdash^d type\ t = C^\lambda \Rightarrow \epsilon} \tag{69}$$

$$\frac{\vdash^m strexp \Rightarrow e}{\vdash^d structure\ X = strexp \Rightarrow \rho_x = e} \tag{70}$$

$$\frac{}{\vdash^d functor\ F(X : sigexp) = strexp \Rightarrow \rho_f =} \tag{71}$$

Figure 15: Translation to System F_ω