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DOES THE EXISTENCE OF "TALENTED OUTLIERS" HELP IMPROVE TEAM PERFORMANCE? MODELING HETEROGENEOUS PERSONALITIES IN TEAMWORK

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Abstract. Personality heterogeneity is an important topic in team management. In many working groups, there exists certain type of people that are talented but under-disciplined, who could occasionally make extraordinary contributions for the team, but often have less satisfactory overall performance. It is interesting to investigate whether the existence of such people in the team does help improve the overall team performance, and if it does so, what are the conditions for their existence to be positive, and through which channel their benefits for the team are manifested. This study proposes an analytical model with a simple structure that sets up an environment to study these questions. It is shown that: (1) feedback learning could be the mechanism through which outliers' benefits to the team are established, and thus could be a prerequisite for outliers' positive existence; (2) different types of teamwork settings have different outlier-positivity conditions: a uniform round-wise punishment for teamwork failures could be the key idea to encourage outliers' existence; for two specific types of teamwork, teamwork that highlights assistance in interactions are more outliers-friendly than teamwork that consists internal competitions. These results well match empirical observations and may have further implications for managerial practice.

1. **Introduction.** The effect of personality heterogeneity on team performance is an important topic in organization studies [18, 16, 17]. Typically, an opposite pair of personality could be identified in a team: on one hand, most people in working teams are disciplined and rule-followers, whose performance displays a great extent of conformity and is highly predictable; on the other hand, in many working groups, a minority of people have great talents but are often accompanied by an unruly personality: they behave as outliers in the team, who can sometimes make extraordinary good performance but display high egos and more often have greater error rates than their teammates [25]. Occasionally, their unforeseen talents could help the team at risk, but their overall performance is less satisfactory than their peers. In most professional teams, the majority of team members are disciplined and only a few are outliers, although individually the extent of ego and the willingness

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to conformity varies person by person [5]. In this paper, we refer to the first type of people as "normal players" and the second type as "talented outliers" [7].

An important empirical observation is that those "talented outliers" frequently appear in sports teams. In fact, one great fun of following sports games is to see some heroic figure suddenly burst out his talents and save the team from an extremely difficult situation. It is even more heart-beating and dramatic if this hero is mediocre or even disappointing at ordinary times, and with a set of unruly and egocentric characters. One good example might be the superstar of the NBA team San Antonio Spurs, Manu Ginobili. He is by any means a crazy outlier who often does not follow team tactics and plays in his own "whimsical" way, while all his teammates are known to be good executives of teamwork in NBA. However, most Spurs fans believe that Manu is indispensable for their team [4]. Once his crazy moves succeed, the whole court goes wild and the whole team are greatly inspired. Although arguably he might have an overall high turnover rate compared to his teammates, Manu could often reverse the situation and save the game when Spurs are at risk. Other NBA examples may be J.R. Smith in Cleveland Cavaliers and Nick Young in Golden State Warriors, and players of such personalities are far from scarce in generally all kinds of sports teams [27]. Although group dynamics in sports team has become a popular research topic [3, 2, 14], studies around personality heterogeneity in the team, especially, around the phenomenon of outliers' existence in teamwork, is far from being saturated. It should also note that Spurs, Warriors and Cavaliers who have outliers in the current team are all considered as among the most competitive teams now in NBA. This might not be a coincidence; outliers' existence in the team may do bring positive effects to teamwork, and having a few creative outliers in regularized teams is believed to be a successful strategy in team management [7].

On the opposite, in other professional teams besides sports teams, especially in industrial organizations, the existence of such "talented outliers" are often considered as negative for teamwork. Although many organizations encourage open conflicts or individualistic thinking, unexpected outcomes and excessive individualism are often not supported at all [11]. How to devise good managerial strategies so as to constrain, instead of facilitate, the conditions in favor of the existence of such unruly "talented outliers" in organizational teams has always been an important issue for administrators [13, 9, 12]. It is then interesting to understand why the outliers' existence arises different attitudes from sports teams and from industrial work groups. Studying the positivity conditions for outliers' existence would also be useful for administrators in industrial organizations to come up with corresponding managerial ideas.

A following question is to identify the channel for outliers to help improve team performance, if they do so. One plausible idea is, their extraordinary talented performance could bring tremendous surprises or extremely creative ideas, which could vastly stimulate the morale of the entire team, and thus the teamwork is improved in subsequent rounds. Specifically, when the team is in difficult situations and team members become less desired to succeed, the outliers' impossible performance and ideas could often bring back hope and greatly inspire the team. This idea described could be attached to a feedback learning mechanism in multiple-round teamworks, where the performance from past rounds has an adjustment effect on people's performance in the current round. Such feedback mechanism essentially embodies the heuristic of "anchor and adjustment", which is a widely known concept and has applied to many fields in engineering and in management [15, 21, 22, 20], and intrinsically similar to the idea of the Kalman Filter [8]. People argue that this mechanism is a natural algorithm adopted by our brain and the neural system [6, 24], and thus becomes one important psychological heuristics that men use when making decisions [23]. It'll be interesting to study whether this feedback learning mechanism is the possible channel in favor of outliers' existence.

Upon the above discussion, in this paper we developed a model to study the conditions for outliers' existence in teamwork. Adopting a simple analytical structure and a minimum set of assumptions, we successfully modeled the positivity condition in favor of the existence of outliers. The feedback learning mechanism is a core component of this model and we applies the idea of "anchor and adjustment" by adding the adjustment term in individual performance. A few important results are derived from the model, through which we tried to answer the questions concerning the existence of conditions for outliers helping the teamwork, the channel for outliers' benefits for the team to be manifested, and the dependence of the positivity conditions for outliers' existence on the nature of the teamwork.

This paper is organized as follows. We set up the base model in Section 2 and derive the first main analytical result in Section 3, where two variants of the model are also studied and corresponding results are shown. In Section 4 we derived the other two main results of this study which led to the major discussion of this paper. In Section 5 we use Monte Carlo simulations to numerically study the model and visualize our analytical results, and the paper is concluded in Section 6.

2. **Model setup.** Suppose there are N members in a team, where N_n of them are "normal players" and $N_o = N - N_n$ are "talented outliers". For convenience, in this paper, we call team members as "players", their work output as "performance" and teamwork periods as "rounds of the game", all of which borrow from the sports game setting. Each player's performance in one single round is a real-valued random variable A. $A > 0$ means the player's performance is successful, and $A < 0$ means the performance is failed, which in real situations may correspond to errors, or "not scoring" of any kind. We assume that, the primary distinction between "normal players" and "talented outliers" lie in the difference in the probability distribution of their performance. Normal players' performance A^N follows a normalized uniform distribution of $[-1, 1]$; outliers' performance is also drawn from a uniform distribution, but with a larger performance range $[-\gamma, \gamma]$ with $\gamma > 1$, which represents their "talents" in terms of a higher upper bound of the performance potential. Note $\gamma < 1$ might be considered as a different type of outliers that have a smaller performance range (e.g. "conservative players"), but is out of the discussion of this study. Next, we assume that outliers' average performance is worse than normal players by a negative shift $m < 0$. This compensates his talents of a larger performance potential: in some cases he can make surprising performance, but at most times, his overall output is less satisfactory than his normal peers. These two assumptions are summarized as the following:

$$A_i^N \sim \text{uniform}[-1, 1] \text{ if player i is a normal player;} \tag{1}$$

$$A_i^O \sim \text{uniform}[-\gamma + m, \gamma + m] \text{ if player i is an outlier.} \tag{2}$$

For meaningful discussion, condition $\gamma + m > 1$ must be satisfied, since if $\gamma + m < 1$, the outlier's "talents" is completely overwhelmed by his higher error rate and his performance is completely inferior to normal players. Also note that we assume a strictly negative shift $m < 0$ for outliers' performance. If $m > 0$, outliers' are both more talented and less error-oriented than normal players; if $m = 0$, the outliers' talents are still not traded-off. In both cases the model will be trivial; if we manage to show that even if this trade-off is established, that outliers' talents are compensated by more errors, the existence of outliers could still be beneficial to the team, the model could then have certain convincing power.

The team participates in a game of multiple rounds. We assume that during each round, every two players in the team interact, and the utility function u_{ij} of the interaction between player i and j depends on each player's performance:

$$u_{ij} = \begin{cases} max(A_i, A_j) & \textbf{if } \ A_i > 0 \ \& \ A_j > 0; \\ -K & \textbf{else}. \end{cases} \tag{3}$$

This means that if two players both produce successful performance ($A_i > 0$ & $A_j > 0$), their interaction will succeed and the better individual performance is counted; if at least one of them has bad performance ($A < 0$), their interaction will fail and a uniform punishment $-K$ will be received ($K > 0$) . After interacting with all other $N - 1$ teammates at this round, the total utility for one player i is the average utility from all his $N - 1$ interactions:

$$U_i = \frac{1}{N-1} \sum_{i \neq j}^{j} u_{ij} \tag{4}$$

U_i evaluates player i's overall interaction performance in one specific round, with a value that is either positive or negative.

In a game of multiple rounds, players' performance at one round is often influenced by the outcomes of past rounds. This feedback learning mechanism is built in our model as an adjustment term of players' performance, which essentially embodies the idea of "anchor and adjustment" [23] and the Kalman Filter [8]. Formally speaking, we assume normal players learn from the utility $U(t)$ of past w rounds with a learning rate $L(t)$, and the adjustment term is essentially the convolution of U and L, which is added to players' performance A:

$$\bar{A}_i^N(t) = A_i^N(t) + \sum_{k=t-1}^{t-w} U_i(k) \cdot L_i(k) \tag{5}$$

where $\bar{A}_i^N(t)$ is the adjusted performance of each player. Here we make our final assumption of outliers' behavior: while normal players' performance is adjusted by learning from past rounds, outliers do not make such adjustment; they are too egocentric to care about past interactions and behave individualistically all the time. Therefore, if player i is an outlier, $L_i(k) = 0$ for all k, and thus $\bar{A}_i^O(t) = A_i^O(t)$, i.e. feedback learning only applies to normal players. This adjustment term $\sum_{k=t-1}^{t-w} U_i(k) \cdot L_i(k)$ complicates the dynamics of the multiple-round teamwork. Note that in deriving analytical results (Section 3 & 4), we assume $w = 1$, i.e. players only learn from the immediate past round.

Finally, at each round of the game, the team performance Q is the sum of all N players' adjusted individual performance $\bar{A}$:

$$Q(t) = \sum_{i=1}^{N} \bar{A}_i(t); Q_g = \sum_t Q(t), \tag{6}$$

where Q_g is the all-round team performance of the entire game. We are interested in studying team performance Q as a function of the number of outliers in the team, i.e. $Q = Q(N_o, N)$. The outliers' existence would be beneficial to the team if the optimal N_o for Q is not zero; and we discuss the favorable condition for outliers under parameters $\{\gamma, m, K, L\}$.

Here summarizes the three assumptions we made on characterizing the behavior of "talented outliers" in teamworks, as is distinct from the behaviors of "normal players" in the team (Figure 1):

(1) Outliers' performance potential is greater than normal players.

(2) However, as a trade-off, outliers' average performance is less satisfactory than normal players.

(3) Unlike normal players who learn from team interactions in past rounds in multiple-round teamworks, outliers do not make such feedback adjustment in their performance.

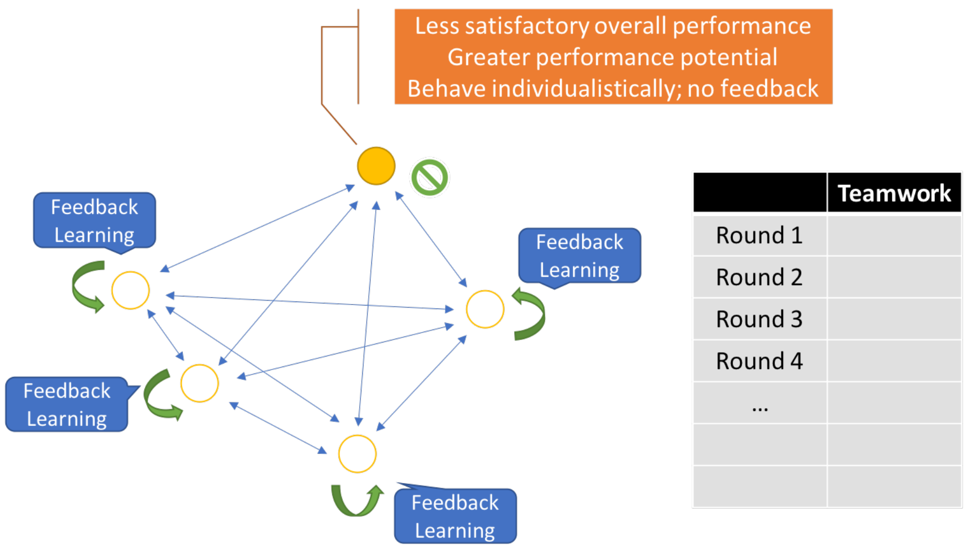

FIGURE 1. *Summary of the model and major assumptions. Teamwork is conducted in a multi-round game setting. In the team, normal players (empty nodes) and "talented" outliers (the solid node) behave differently (orange box). Outliers have worse average performance but a greater performance potential than normal players. Moreover, unlike normal players, outliers do not adjust his performance according to feedbacks from past interactions.*

3. **Positive condition for the existence of outliers.** Based on the above formulation of the model, the first effort is to derive the positive condition for the existence of outliers. Although the model has a stochastic nature and a relatively complex dynamics, after applying a few simplification assumptions, we successfully obtained some analytical results for the outlier-positivity conditions, as specified by the parameter set $\{\gamma, m, K, L\}$. Meanwhile, we studied the role of the feedback learning mechanism in determining the positivity of outliers' existence.

We base our discussion on comparing the team performance Q of the following two scenarios :

Case I. All members in the team are normal players ($N_o = 0, N_n = N$).

Case II. One player is outlier and other team members are normal players ($N_o = 1, N_n = N - 1$).

With all other conditioned fixed, if the team with one outlier (**Case II**) could have a higher performance Q than the team of all normal players (**Case I**) in a specific parameter space, we are then able to conclude that it is possible that the outliers' existence do help the team in certain cases.

Before deriving the conditions, we first look at the feedback learning mechanism. When we shut off the feedback term (i.e, $w = 0$), the expectation of the team performance Q is given by:

$$E(Q) = \sum_{i=1}^{N} E(\bar{A}_i) = \sum_{i=1}^{N} E(A_i) = N_n \cdot 0 + N_o m = N_o m. \tag{7}$$

Therefore, we have the following proposition, as the first main result of this study:

Proposition 1. *The existence of the feedback learning mechanism is a necessary condition in favor of the existence of outliers.*

Proof. Since $m < 0$, (7) implies that when no feedback learning exists, the expectation of team performance declines linearly with the increase of number of outliers in the team, i.e. no-outlier is always the best scenario for the team. □

This result is easily obtained yet has important implications. It suggests that the feedback mechanism in a multiple-round game, that players learn from past interaction performance, may be the exact channel for outliers to bring positive influence to teamwork. This is consistent with the observation in sports game, that a player's crazy move could stimulate the morale of the entire team in subsequent rounds. However, it should note that this result, as well as the identification of the channel, directly comes from our formulation of the model, whose explanatory power should not be overestimated.

In the following discussion we turn on the feedback term and consider feedback learning from past $w = 1$ round. We also assume that all normal players have the same learning rate L, which is a single value since $w = 1$.

When two normal players interact, the expectation of their utility function u_{ij} is:

$$E(u_{ij}) = P(A_i, A_j > 0) \cdot E[max(A_i, A_j)] + P(else) \cdot (-K) = \frac{1}{6} - \frac{3}{4}K. \tag{8}$$

(Note: uniformly distributed $X_i \in [0, 1], X_j \in [0, 1]$, $E[max(X_i, X_j)] = 2/3$.)

Similarly, when a normal player and an outlier interact, the expectation of their utility function u_{io} is:

$$\begin{aligned} E(u_{io}) &= P(A_i, A_o > 0) \cdot E[max(A_i, A_o)] + P(else) \cdot (-K) \\ &= \frac{3(\gamma + m)^2 + 1}{24\gamma} - \frac{3\gamma - m}{4\gamma}K. \end{aligned} \tag{9}$$

Note when $\gamma = 1$, $m = 0$, (9) degenerates into (8), a case where talent outliers are not distinguished from normal players.

The expectation of the team performance Q for **Case I** is:

$$E[Q(N_o = 0)] = \sum_{i=1}^{N} E(\bar{A}_i) = NL \cdot E(u_{ij}) \tag{10}$$

The expectation of the team performance Q for **Case II** is:

$$E[Q(N_o = 1)] = \sum_{i=1}^{N} E(\bar{A}_i) = m + E(u_{io})L + (N-2)E(u_{ij})L \tag{11}$$

Therefore, the positive condition for the existence of one outlier is

$$\begin{aligned} E[Q(N_o = 1)] > E[Q(N_o = 0)] &\Leftrightarrow m + E(u_{io})L + (N-2)E(u_{ij})L > NL \cdot E(u_{ij}) \\ &\Leftrightarrow \frac{m}{L} + \frac{3\gamma + m}{4\gamma}K > \frac{8\gamma - 3(\gamma+m)^2 - 1}{24\gamma} \end{aligned} \tag{12}$$

Define $H = H(\gamma, m, k, L)$ as:

$$H(\gamma, m, K, L) = \frac{m}{L} + \frac{3\gamma + m}{4\gamma}K - \frac{8\gamma - 3(\gamma+m)^2 - 1}{24\gamma} \tag{13}$$

Therefore, $H > 0$ is a necessary condition in the parameter space $\{\gamma, m, K, L\}$ in order that in a game with multiple rounds, it is possible that a team with one outlier could make better performance than the same team with all normal players. Under the constraints $K > 0$, $\gamma > 1$, $L > 0$, $m < 0$ and $\gamma + m > 1$, $H > 0$ could be reached, meaning that the outliers' existence could be beneficial for the teamwork under certain circumstances.

An alternative utility function. Next, keeping the other parts of the model fixed, we investigate a different utility function u'_{ij}:

$$u'_{ij} = \begin{cases} A_i \cdot A_j & \textbf{if } A_i > 0 \ \& \ A_j > 0; \\ -K & \textbf{else}. \end{cases} \tag{14}$$

In this case, when two players both succeed, instead of selecting the better performance of the two players, their performances are both counted and then superposed. Comparing (3) and (14), these two utility functions may correspond to teamwork of two different types and are discussed in Section 4.

Now the expectations of the utility function (equation (8) and (9)) become:

$$E(u'_{ij}) = P(A_i, A_j > 0) \cdot E[A_i \cdot A_j] + P(else) \cdot (-K) = \frac{1}{16} - \frac{3}{4}K. \tag{15}$$

and

$$E(u'_{io}) = P(A_i, A_o > 0) \cdot E[A_i \cdot A_j] + P(else) \cdot (-K) = \frac{(\gamma+m)^2}{16\gamma} - \frac{3\gamma - m}{4\gamma}K. \tag{16}$$

And the positive condition for the existence of one outlier is:

$$\begin{aligned} E[Q(N_o = 1)] > E[Q(N_o = 0)] &\Leftrightarrow m + E(u'_{io})L + (N-2)E(u'_{ij}L > NL \cdot E(u'_{ij}) \\ &\Leftrightarrow \frac{m}{L} + \frac{3\gamma + m}{4\gamma}K > \frac{2\gamma - (\gamma+m)^2}{16\gamma} \end{aligned} \tag{17}$$

Similarly, define

$$H'(\gamma, m, K, L) = \frac{m}{L} + \frac{3\gamma + m}{4\gamma}K - \frac{2\gamma - (\gamma+m)^2}{16\gamma} \tag{18}$$

$H' > 0$ is then the necessary condition for the positive existence of one outlier in the team under utility function (14).

Gaussian distribution of individual performance. In the above discussion we assume players' performance is a random variable A drawn from a uniform distribution: $uniform[-1, 1]$ for normal players and $uniform[-\gamma + m, \gamma + m]$ for outliers. People may argue that the Gaussian distribution might be a more realistic distribution for player's individual performance and here we investigate this variant. We assume normal players' performance $A_i \sim Normal(0, 1)$, and outliers performance $A_o \sim Normal(-m, \gamma)$, keeping the same set of parameters $\{\gamma, m\}$ in specifying the distribution. One problem for adopting the Gaussian distribution is that now it is more difficult to obtain analytical results for outlier-positivity conditions; we could only derive the expectations of interaction utility under equation (14), not equation (3). Under the Gaussian distribution of player's performance, (15) and (16) now become:

$$E(u'_{ij}) = P(A_i, A_j > 0) \cdot E[A_i \cdot A_j] + P(else) \cdot (-K) = \frac{1}{2\pi} - \frac{3}{4}K. \tag{19}$$

and

$$\begin{aligned} E(u'_{io}) &= P(A_i, A_o > 0) \cdot E[A_i \cdot A_j] + P(else) \cdot (-K) \\ &= \frac{\gamma + m}{4\gamma} \cdot \sqrt{\frac{2}{\pi}} \cdot [\gamma^{0.5}\sqrt{\frac{2}{\pi}} exp(\frac{-m^2}{2\gamma}) - m \ \mathrm{erf}(\frac{-m}{\sqrt{2\gamma}})] + \frac{3\gamma - m}{4\gamma}(-K) \end{aligned} \tag{20}$$

Note the above derivation utilized the folded normal distribution. The corresponding positive condition for the existence of outliers, similar to (13) and (18), could be derived accordingly; but in this case the expression is highly complicated due to the existence of the folded normal distribution. We thus have to surrender the analytical effort and continue studying the Gaussian distribution through numerical simulations.

4. **Discussion: Types of teamwork.** Among the five parameters of our model, γ, m, L and w are all behavioral, reflecting individual's characteristics; K is the only teamwork-specific parameter: a large K indicates a teamwork with a high punishment for failed interactions, and a small K corresponds to a mild punishment. Note in this model we only consider $K > 0$, meaning that failed interactions will be penalized. On the opposite, $K < 0$ corresponds to a situation where a small uniform reward will be given if interactions fail. Although this is not completely unrealistic in real teamwork, in this model we constrain our discussion in $K > 0$.

Moreover, looking at (8) and (15), people may argue that there exists an upper bound for K: $K < 2/9$ under utility function (3) and $K < 1/12$ under utility function (14), since $E(u_{ij}) > 0$ has to hold for meaningful discussion such that the expected value of the interactions between two team members is positive. We agree this notion since realistically, for most teamworks the expected utility of team interactions should be positive (otherwise there'll be no need to do teamwork, or form such teams in the first place). However, this is true for a fixed penalty at each round; it is very likely that in certain rounds of a multi-round teamwork there will be a highlighted penalty, e.g. the round before the mid-term sales report, or the final round of a sports game. In these cases, K may go beyond the derived upper bound. Despite this discussion, however, in the current study we do not make any assumption about the dynamics of K in the game.

We have obtained the positivity criteria for outliers' existence $H > 0$ and $H' > 0$ under two forms of utility functions (3) and (14). Comparing the two criteria, we have the following proposition:

Proposition 2. *Under both utility functions (3) and (14), when the team is at risk, meaning that the punishment is high if team interactions fail, the existence of outliers is more beneficial to the team (assuming individual' performance is drawn from a uniform distribution, and team members only learn from the immediate past round).*

Proof. Since

$$\frac{\partial H}{\partial K} = \frac{\partial H'}{\partial K} = \frac{3\gamma + 4m}{4\gamma} > 0, \quad \text{for all } \gamma, m, L \tag{21}$$

this means that monotonously, the larger the punishment for interaction failures is, the more favorable the condition is for outliers' existence in the team. □

This result is consistent with our common sense: the more difficult the situation is for the team, the more likely that outliers' occasional "talented performance" would help the team; on the opposite, when the punishment for the failure of team interactions is reduced, the outliers' performance will become less desired. Essentially, this is because when K is monotonously increased and the other behavioral parameters are kept the same (meaning that the game becomes more difficult for the team yet the member composition of the team is fixed, and so the probability of interaction failures is fixed), then the normal players' mediocre performance gain is no longer enough to compensate the higher penalty, and the outliers' extraordinary performance gain will become valuable to the team in rivaling with the high penalties. Again, this logic is consistent with empirical observations.

Taking Proposition 1 and 2 together, one can see that the uniform punishment K plays a key role in our model. The reason that the existence of outliers could possibly be beneficial to teamwork, instead of being completely harmful to the team due to its less satisfactory overall performance is that, under either utility function (3) or (14), the cost of interaction failures is constrained by the uniform penalty K. Therefore, even if the outlier sometimes gives extremely bad performance at certain rounds, his interactions with his normal peers will only receive a fixed punishment $-K$. The uniform punishment is an implied mechanism in our model that supports outliers' existence. In practice, this is often the case, that multiple-round teamwork is set up to have a uniform failure punishment at each round, for teamwork in sports games, school works/tests, (experimental) academic research, or other teamwork with a regular agenda. For less regularized teamwork, this assumption might not be as appropriated. However, it is still useful to think of this idea in team management: if the administrator would like to encourage outliers' existence in the team, for example, in order to encourage more creativity for the team or to try to elevate the "ceiling" of the team's performance, he may achieve this goal by creating an environment that fixes the round-wise failure punishment of the teamwork, which bottoms the overall performance risk exacerbated by these outliers.

Next, from (13) and (18), we separate the K out of the positivity criterion (K under two schemes (3) and (14) are labelled as K_I and K_{II}) :

$$H > 0 \Leftrightarrow K_I > \frac{8\gamma - 3(\gamma + m)^2 - 1 - \frac{24}{L}\gamma m}{6(3\gamma + m)} \triangleq W_I \tag{22}$$

and

$$H' > 0 \Leftrightarrow K_{II} > \frac{2\gamma - (\gamma+m)^2 - \frac{16}{L}\gamma m}{4(3\gamma+m)} \triangleq W_{II} \tag{23}$$

For convenience, we further label the RHS of (22) and (23) as W_I and W_{II}. We have the following proposition, which is the final main result of this study:

Proposition 3. *When the outliers' performance potential is not extremely higher than that of the normal players (specifically, $\gamma + m < \sqrt{3}$), the type of teamwork under utility function (14) is more favorable to the existence of outliers than the type of teamwork under utility function (3) (assuming individual' performance is drawn from a uniform distribution, and team members only learn from the immediate past round).*

Proof. Since

$$W_I - W_{II} = \frac{10\gamma - 1 - 3(\gamma+m)^2}{12(3\gamma+m)} = \frac{10(\gamma-1) + 3(\sqrt{3}-\gamma-m)(\sqrt{3}+\gamma+m)}{12(3\gamma+m)}. \tag{24}$$

When $\gamma + m < \sqrt{3}$, there is always $W_I > W_{II}$. Recall (22) and (23), this means that in this case the parameter space under the outlier-positive criterion $H > 0$ is a subset of the parameter space under $H' > 0$, i.e., under utility function (14) there is a larger parameter space for the positivity of outliers' existence than that of utility function (3), in the region $\gamma + m < \sqrt{3}$. □

Proposition 3 suggests that, within the space $\{\gamma > 1; m < 0; 1 < \gamma + m < \sqrt{3}; K > 0; L > 0\}$, we could find a parameter set $\{\gamma, m, K, L\}$ such that $H' > 0 > H$, in which case the existence of outliers does not help improve team performance under utility function (3), but they do help the teamwork under utility function (14) (see Section 5, Figure 3).

Besides the teamwork-specific parameter K, the two utility functionals (3) and (14) could also be seen as descriptive of the nature of teamworks; specifically, they represent two different types of teamwork. Equation (14) corresponds to a type of teamwork where one player "assists" the other player's performance and both of their performance outcomes are counted. This is a situation similar to sports games where assistance plays a significant role in scoring. On the other hand, equation (3) correspond to a different situation where two players in an interaction could be seen as forming a "competition", and only the better outcome of their performances gets counted. This is more attached to an organizational setting where, for example, two members in a team both make proposals and the better one get approved. Upon the above distinction of the two types of teamwork, our result (Proposition 3) show that, when the performance potential of outliers do not overrun that of the normal players by a large margin ($<\sim 170\%$ in a quantitative sense; which is often the case), the "assistance-highlighted" type of teamwork is more favorable for the outliers' existence than the "competition-oriented" type of teamwork.

Once more, this result is consistent with the empirical observation, that talented outliers who have high error rates at normal times but can save the team at risky situations, or more generally those "heroic figures" in any team, are more often identified in sports games rather than in industrial organizations. This finding could lead to further implications for managerial practice. If organization administrators expect to avoid the existence of such talented but under-disciplined outliers, one potential strategy could be to introduce in-group competitions into teamwork [1]; on the other hand, if a team supervisor hopes to create the environment that is more favorable for talented outliers, in order to benefit from their sporadic yet extraordinary performance, he could try to adopt the strategy that awards assistance in teamwork, so that the team members will have a greater motivation to come up with unroutined ideas.

5. **Monte Carlo simulations.** Given the stochastic nature of the model, we initiated a series of Monte Carlo (MC) simulations to numerically study the model and to test the analytical results. In all simulation runs, we assume that the team consists of five members ($N = 5$), and the teamwork lasts 100 rounds ($t = 100$). Number of outliers N_o in the team, the parameter set $\{\gamma, m, K, L\}$, as well as the time length of the feedback learning mechanism w, are varied. Simulations were conducted under the two utility types **U1** (equation (3)) and **U2** (equation (14)), and the two distribution types **D1** (uniform) and **D2** (Gaussian). For each set of parameters, > 200 Monte Carlo simulation runs are conducted and the average all-round team performance Q_g is calculated for each run.

Feedback on/off. First we demonstrate equation (7) and test our Proposition 1. We computed the team performance for the $N = 5$ team with N_o ranges from 0 to 5, and compared the results with the feedback mechanism being activated and shut off (Figure 2). The parameter set $\{\gamma, m, K, L\}$ used in Figure 2 satisfies the existence-positivity condition $H' > 0$. As expected, when the feedback learning is turned on and the positivity condition is satisfied, $N_o = 1$ produces better teamwork performance than $N_o = 0$. In Figure 2 the optimal N_o is 1, and more tests show that under different parameters, the optimal N_o ranges from 1 to 5. Proposition 1 is confirmed, that the feedback mechanism could be a prerequisite for the positivity condition of the outliers' existence.

Different utility types and different performance distributions. We studied the two utility types **U1**,**U2**, two distributions of individual performance **D1**, **D2**, and tested our Proposition 3. We compared the overall team performance Q_g for $N_o = 0$ (**Case I**) and $N_o = 1$ (**Case II**) by showing their difference, defined as $\Delta Q_g = Q_g(N_o = 1) - Q_g(N_o = 0)$. $\Delta Q_g > 0$ means that the team performance is better with $N_o = 1$ than with $N_o = 0$, i.e. the existence of outliers is positive for the team, and vice versa. The feedback term is activated. We selected a specific parameter set $\{\gamma, m, K, L\}$ such that $H < 0$ while $H' > 0$, and the simulation results are shown in Figure 3. Proposition 3 is demonstrated, that for the uniform distribution of individual performance **D1**, **U2** is more outlier-friendly than **U1**, as is shown that $\Delta Q_g(\mathbf{D1U2}) > 0 > \Delta Q_g(\mathbf{D1U1})$. We also tested the Gaussian distribution of individual performance **D2**; however, under the same parameter set, both utility functions **U1** and **U2** fail to generate an outlier-friendly environment under the Gaussian distribution of the performance variable ($\Delta Q_g < 0$ for both cases). Other parameter sets are tested and results are similar, which shows that in general the Gaussian distribution of individual performance is more stringent for outliers' positive existence than the uniform distribution.

Feedback learning from multiple rounds. So far we have assumed $w = 1$ in deriving all the results, i.e. normal players only learn from the interaction performance of the immediate past round. In principle, the feedback learning could be drawn from multiple past rounds with a specific learning curve. However, since the choice of the learning curve will inevitably introduce extra unjustified assumptions into the model, we keep $w = 1$ in our main results and only show our investigation of a

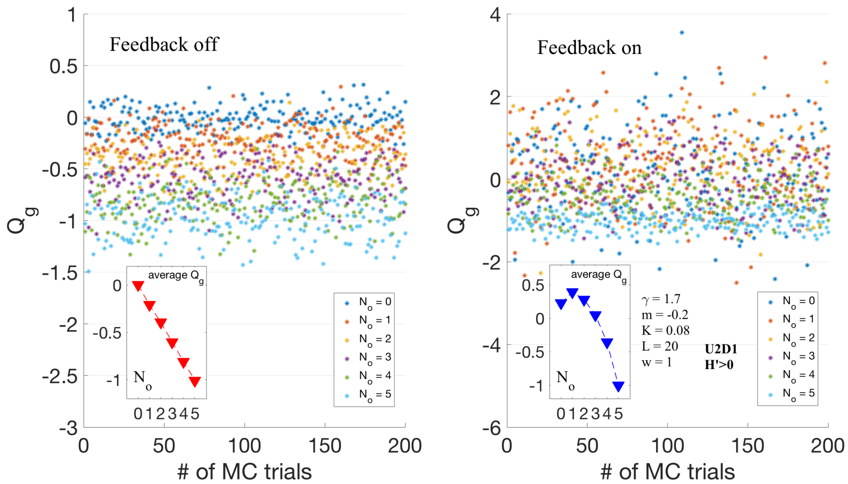

FIGURE 2. *The significance of the feedback learning mechanism as a potential prerequisite for the positivity condition of outliers' existence. Left: no feedback. MC simulation results are consistent with equation (7). Right: the feedback mechanism activated. Inset: average all-round team performance as a function of the number of outliers in the team N_o. Results show that $N_o = 1$ produces the best outcome of teamwork under this parameter set, which satisfies $H' > 0$.*

variant w here. We assume the players learn from past w rounds with a linear learning curve that $L(k) = (L - k + 1)/L$, $k = 1, 2, ...w$, i.e. the past kth round is associated with a learning weight of one-kth of L (see equation (5)). We show ΔQ_g (as in Figure 2) as a function of w, which ranges from 1 to 5. Results from a few individual MC runs are plotted together, for each of the four combinations of U and D (Figure 3). It is interesting that $w = 2$ provides the best ΔQ_g for all four scenarios except **U2D1**, in which case $w = 1$ is the optimal value; yet **U2D1** is only condition that favors outliers' existence ($\Delta Q_g > 0$) under the investigated parameter set. Since the linear learning curve is not a well-justified assumption and we currently don't have much empirical evidence to constrain the value of w, we stop our discussion here and argue that no conclusion could be drawn from this test.

6. **Conclusion.** In this paper, we studied the effect of personality heterogeneity on teamwork performance. Specifically, we modeled the behavior of "talented outliers" in the team, as opposed to "normal players", and studied the positive condition for outliers' existence in teamwork. The model has a simple structure with a stochastic nature, and is built in a multiple-round dynamic setting that highlights the feedback learning mechanism. However, despite its simple structure, a few important analytical results are yielded from the model, and are demonstrated through Monte Carlo simulations. First, we showed that, the feedback learning mechanism could be a prerequisite for the positivity of outliers' existence, i.e. the channel through which the outliers' talented performance get the chance to influence his teammates;

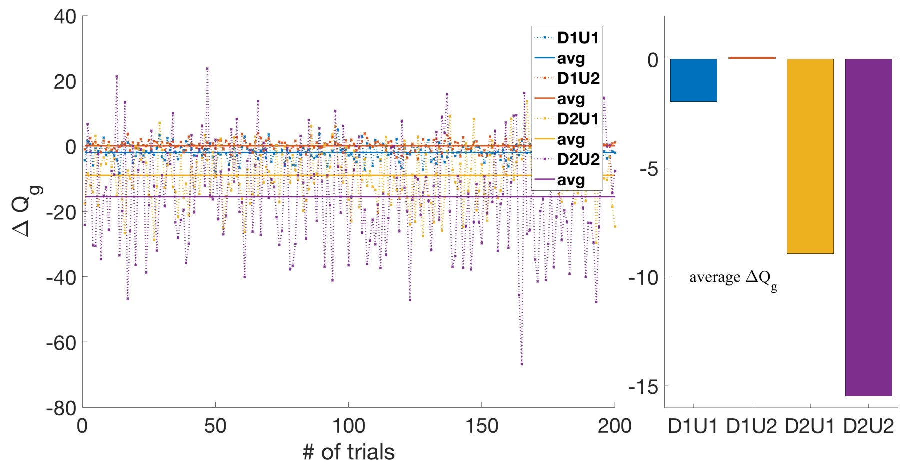

FIGURE 3. *Testing two utility types and two distributions of individual performance. Left: ΔQ_g for individual MC runs; right: average ΔQ_g for all 200 MC runs. Utility function: equation (3):* ***U1****; equation (14):* ***U2****. Performance distribution: uniform:* ***D1****; Gaussian:* ***D2****. $\{\gamma, m, K, L\}$ is chosen such that $H < 0$ (blue;* ***D1U1****) and $H' > 0$ (red;* ***D1U2****). Proposition 3 is demonstrated since $\Delta Q_g(\boldsymbol{D1U2}) > 0 > \Delta Q_g(\boldsymbol{D1U1})$. The Gaussian distribution of individual performance is more stringent for outlier's positive existence than the uniform distribution.*

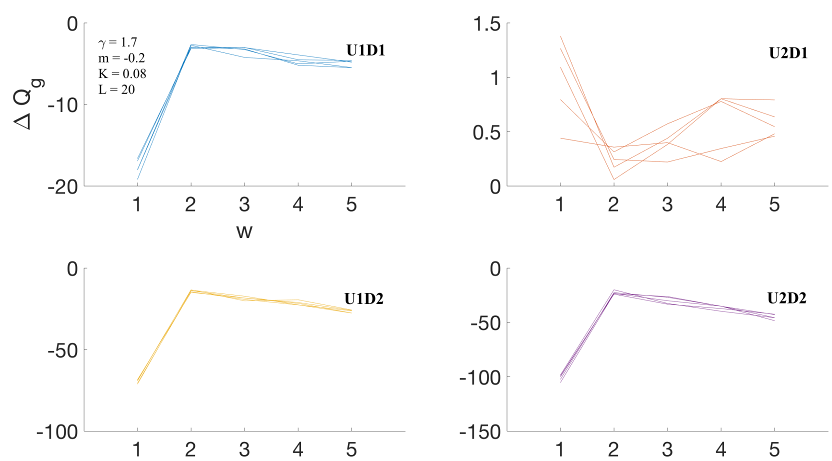

FIGURE 4. *Feedback learning from multiple past rounds. Results show ΔQ_g (as in Figure 2) as a function of w, which ranges from 1 to 5. The results from a few individual runs are plotted together, for all four combinations of U and D. No conclusion could be drawn from this test.*

shutting down this channel, the existence of outliers is no longer beneficial for teamworks. Second, we pointed out that, setting up a uniform round-wise punishment for failures of team interactions could be a key strategy to encourage outliers' emergence in teamworks, and the higher this uniform punishment is, the condition is more favorable for outliers' existence in the team. Last but not least, we showed that, different types of teamwork will leave different room for the positive existence of outliers. Specifically, the type of teamwork that highlights assistance in team interactions is more favorable for the existence of outliers, than the type of teamwork that promotes in-group competitions. All these results do not deviate from empirical observations, and may have some prescriptive power for managerial practice.

In the model, we assume people learn from their past interactions with other teammates and therefore each person's performance will influence all the team members that he interacts with. This could be seen as embodying the idea of "technology spillover" [26] on an individual scale. It is through this effect, together with the feedback learning mechanism, that the outliers' extraordinary performance has a chance to encourage his peers, and thus helps improve the teamwork. Nevertheless, it is worthy to point out that this channel is neutral by nature, since bad peer performance could also propagate and be learned, which is unhealthy for teamwork. Attached to the ideas of feedback learning and technology spillover, it is expected that our model could have further implications for organization studies [19, 10].

Despite its success, this model is nevertheless flawed. One major shortcoming is that it is too difficult to be tested with real data and therefore too abstract to be applied in real situations. A potential application could be in sports data analysis, where this model could be used to help identify heterogeneous personalities in the team by calculating the H identity of each player, given that a set of realistic definitions for $\{\gamma, m, K, L\}$ could potentially be assigned in the sports game setting. For example, in basketball, γ and m could be related to "number of key shots" and "average turnover rate", respectively; K is the uniform scoring points of each round of attack (maybe 2 or 3); and L as the learning rate could be proportional to the inverse of players' age (note that in the simulations, we used arbitrary values of $\{\gamma, m, K, L\}$ to demonstrate our results (Figure 2-4); if these parameters are attached to realistic definitions, their value range should be discussed more carefully). Applications of this sort, if possible, could be useful for team management. Second, in deriving the analytical results we assumed a uniform distribution of individual's performance. Alternatively, the Gaussian distribution is tried but the analytical attempt is not as successful. Moreover, simulations suggest that the Gaussian distribution of individual performance is less favorable for outliers' existence-positivity conditions than the uniform distribution; under the Gaussian distribution, the corresponding propositions may not hold. Since the Gaussian distribution might be viewed as more realistic than the uniform distribution in the setting of this study, then both the explanatory and the prescriptive power of our results might be highly limited. Also, since the current study only investigates two utility functions and two distribution of the individual performance, the results are far from being general. Other utility functionals and distribution types could be tested in this model setting, although it could be even more difficult to yield analytical results. Future efforts could also be made by appointing that a random number of team interactions from random pairs take place at each round of teamwork, for example, through an agent-based modeling approach, as opposed to the current assumption that all possible pairs in the team interact at all rounds. Last but not least, the model is built on two-player interactions in teamwork; in practice, multi-player interactions are nontrivial, which are not captured in the current model, although one could argue that multi-player interactions are essentially made of a sequence of two-player interactions as well.

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