# Fertility and the Demographic Transition: An Analysis of Swiss Cantonal Fertility Patterns (1888)

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# **Abstract**

In this analysis we study the determinants of fertility across French-speaking Swiss cantons using historical data. Inspired by Gary Becker's study of the Demographic Transition, we analyze how several factors affect fertility rates across a set of cantons, with a justified focus on education and urbanization. Regression analysis reveals evidence of omitted variable bias when education is excluded. Our findings confirm the central role of education in Switzerland's early fertility decline.

# 1. Introduction

In this report, we will look at the determinants of fertility across French-speaking cantons of Switzerland during the demographic transition. This process represents a widespread demographic shift observed across most developed societies, typically marked by declining birth rates and changing family structures. Looking back to the year 1888, we analyze how rurality (determined by the share of the population working in agriculture) and education influenced fertility rates. By using a cross-sectional dataset from the R datasets package, we apply the ordinary least squares (OLS) regression to make an estimate of potential relationships. Several economic theories suggest that rising education levels increase the opportunity cost of having a child, thus provoking a reduction in fertility. As Becker argues in one of his major works on fertility, the decline of fertility could be explained by rational household decisions maximizing utility under constraints, where the cost of raising children increases with modernization. We also study whether excluding education from the model could possibly lead us to Omitted Variable Bias (OVB), specifically in estimating the effect of rurality on fertility.

#### 2. Data

We obtained a dataset from R's datasets package containing the necessary data dated from 1888. The dataset consists of 47 observations on 6 variables. To conduct our analyses, we assumed that the data represents a random sample of the populations of 47 cities through the French-speaking cantons of Switzerland at about 1888. The 6 variables are all quantitative continuous variables and measure the population's participation share as a percentage from 0 to 100. We will be focusing our analysis on the standardized fertility measure and socioeconomic indicators which could have a potential explanation for the decline in fertility during the demographic transition. We gathered the dataset on these 5 variables to explore their potential influence on fertility, (our key variable) adding one after the other. To conduct our analysis, we decided to add another variable called Urbanization (not part of our dataset), which we constructed by using the variable Agriculture. All variables but Fertility give proportions of the population and they were all taken and computed by the Federal Statistical Office.

#### 2.1 Descriptive overview:

- Fertility (Y) is a quantitative variable which represents the fertility level of a given population expressed as a percentage in the dataset (range: 35.00 92.5, mean: 70.14).
- Urbanization  $(x_1)$  is computed as 100 minus the share in agriculture (range: 10.30 98.8, mean: 49.34).

- Education  $(x_2)$  is the percentage of draftees with education beyond primary school (range: 1.00 53.00, mean: 10.98).
- Examination ( $x_3$ ) measures the percentage of draftees in the military receiving the highest mark on army examination (range: 3.00 37.00, mean: 16.49).
- Catholic (x<sub>4</sub>) is the share of the Catholic population (range: 2.15 100.00, mean: 41.14).
- Infant Mortality (x<sub>5</sub>) measures infant deaths per 100 births (range: 10.80 26.60, mean: 19.94).
- **Agriculture** (not used directly) is the percentage of males of the population involved in agriculture as occupation. It is obtained by dividing the number of men employed in agriculture by the total number of active men (range: 1.20 89.70, mean: 50.66).

#### **Distribution of Fertility Levels in 1888:**

Most values lie between 60 and 80, with a slight right skew and a few low outliers.

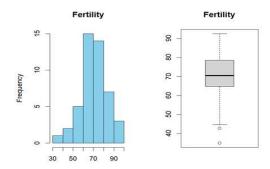


Figure 1: Histogram and Boxplot of Fertility

# 3. Methodology

The OLS estimator models the linear relationship between fertility (Y) and explanatory variables by minimizing the sum of squared residuals. It finds the best-fitting line through the data, estimating unknown population parameters from the sample, using the general equation for simple regression:  $\hat{Y}_i = \hat{\beta}_0 + \hat{\beta}_1 X_i + u_i$ . Thus, we measure the partial effect of a given independent variable X1 on the dependent variable Y, captured by  $\beta_1$ , holding other variables constant:  $\Delta Y = \beta_1 \cdot \Delta X_1$ ; when all other factors are fixed:

$$\hat{\beta}_1 = \frac{\Delta Y}{\Delta X}$$

Our model selection proceeds by gradually expanding the set of regressors and tracking how the estimated coefficients evolve across the addition and subtraction of independent variables.

Special emphasis is placed on the identification of **Omitted Variable Bias (OVB)**, defined by the equation:  $\widehat{\beta}_1 = \beta_1 + \rho X u \frac{\sigma_u}{\sigma_X}$ . OVB is present when an important explanator variable is left out of the regression analysis. It must fulfill two conditions:

- 1. The included regressor (X) must be correlated with the omitted variable (Z):
- 2. the omitted variable Z must itself affect the dependent variable  $Y: \beta_2 \neq 0$ . OVB impacts the OLS estimator, rendering it **biased and inconsistent**

To correct for omitted variable bias, we include the omitted variable as an additional regressor in the linear model:  $Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 Z_i + u_i$ 

# 4. Results

# 4.1 Simple regressions

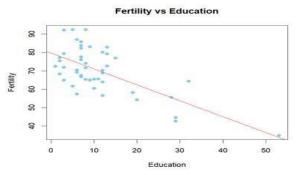


Figure 2: Simple Regression Plot: Fertility vs Education

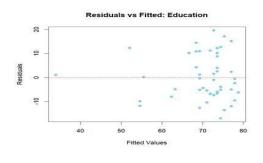


Figure 3: Residuals vs Fitted: Education

$$Fertility_i = 79.61 - 0.862 \cdot Education_i$$

$$Fertility_i = 86.82 - 1.011 \cdot Examination_i$$

$$Fertility_i = 64.43 + 0.139 \cdot Catholic_i$$

$$Fertility_i = 34.52 + 1.787 \cdot InfantMortality_i$$

Model	Coefficient	Significance	R <sup>2</sup>
Fertility ~ Urbanization	-0.194	<i>p</i> = 0.015 *	0.105
Fertility ~ Education	-0.862	<i>p</i> < 0.001 ***	0.428
Fertility ~ Examination	-1.011	<i>p</i> < 0.001 ***	0.404
Fertility ~ Catholic	+0.139	p = 0.001 ***	0.198
Fertility ~ Infant Mort.	+1.787	p = 0.004 **	0.155

With each model iteration, we observe:

- 1. Changes in the **magnitude and sign** of key coefficients (e.g., Urbanization)
- 2. Shifts in **statistical significance** (p-values)
- 3. Improvements in **model fit** (adjusted  $\mathbf{R}^2$ ): we will refer to the adjusted  $\mathbf{R}^2$  as it penalizes the non-parsimonious addition of variables:  $\bar{R}^2 = 1 \left(\frac{n-1}{n-k-1}\right)\left(\frac{SSR}{TSS}\right)$

Regarding OVB, it cannot simply be resolved by increasing the number of observations used to estimate  $\beta_1$ . Particularly, the estimated effect of  $X_1$  on Y may be overstated or understated depending on the direction of the correlation. Moreover, OVB results in the violation of the first OLS assumption  $E(u_i|X_i) = 0$ . In effect, the error term captures all other factors that affect Y but are not included in the model.

#### 4.2 Multiple regressions

Model 1: Urbanization + Education

$$Fertility_i = 77.43 + 0.066 \cdot Urbanization_i - 0.963 \cdot Education_i$$

Adding Education reverses the sign and removes the significance of the Urbanization coefficient, indicating omitted variable bias. Urbanization and Education are positively correlated, while Education negatively affects Fertility. As a result, the negative effect of Education was mistakenly captured by Urbanization in the simple model.

Model 2 (Full Model): Urbanization + Education + Examination + Catholic + Infant Mortality 
$$Fertility = 49.70 + 0.172 \cdot Urbanization - 0.871 \cdot Education - 0.258 \cdot Examiation + 0.104 \cdot Catholic + 1.077 InfantMortality$$

Model	Urbanization	Education	Examinatio n	Catholic	Infant Mortality	Adj. R²
M1: Urban. + Education	+0.066	-0.963			·	0.424
	(p = 0.41)	( <i>p</i> < 0.001 ***)	_			0.424
M2: + (Full Model)	+0.172	-0.871	-0.258	+0.104	+1.077	
	(p = 0.019 *)	(p < 0.001 ***)	(p = 0.315)	(p = 0.005 **)	(p = 0.007 **)	0.671

Additional regression models using three and four variables (e.g., Education + Urbanization + Catholic) are presented in the appendix, along with other specifications exploring alternative variable combinations (fig. 26, 27, 28, 29, 30, 31, 32).

#### **Final model interpretation:**

(Adjusted  $R^2 = 0.671$ ) The final model explains 67.1% of the variation in fertility across Swiss cantons. This is a substantial improvement over the simple regressions, where  $R^2$  ranged between 0.105 and 0.428,

and also over intermediate multivariate models (e.g., Model 1 with adj.  $R^2 = 0.424$ ). This improvement in explanatory power confirms that including all relevant socioeconomic and demographic factors significantly enhances model fit.

- Education has the strongest and most consistent effect: its coefficient is −0.871 and highly significant (p < 0.001). This confirms that higher education levels are strongly linked to lower fertility. According to the final model, a 1 percentage point increase in education is associated with a 0.871-point decrease in the fertility index, holding other variables constant.
- Urbanization, which was negatively associated with fertility in the simple model, becomes **positive** (+0.172, p = 0.019) after controlling for education. This sign reversal reveals **omitted variable** bias: urban areas also tend to be more educated, and the effect of education was wrongly attributed to urbanization in simpler models.
- Catholic population share has a **positive effect** (+0.104, p = 0.005), suggesting that more Catholic regions had higher fertility.
- Infant Mortality also has a positive and significant effect (+1.077, p = 0.007), which may reflect compensatory fertility in high-mortality regions.

#### 4. Conclusion

In conclusion, our analysis highlights that during the demographic transition education had a key role in the reduction of the fertility. At first, urbanization seemed to be an important factor, and we were able to see that after adding other variables, such as education, religion and infant mortality, a presence of omitted variable bias (OVB) appeared. This occurred even more when education is left out of the model.

Our findings support the theory that a higher level of education increases the opportunity cost of having children, leading to lower fertility rates. Moreover, the effect we observed on fertility can partly be explained by other socioeconomics variables, which can be seen in the prolonged models.

However, the analysis has certain limitations, such as the lack of income data. In fact, this is an essential part of the opportunity cost analysis, thus, not having these data limits our ability to fully assess the effects on fertility. Another limitation is the fact that we observed Heteroskedasticity throughout the models, which suggests that the variance of the residuals is not constant across all levels of the independent variable. This may affect the reliability of the estimated standard error.

Finaly, our analysis empathizes the importance of taking key socioeconomic factors into account to avoid biased interpretation and shows fundamentals economics sources on Switzerland's fertility regression.

# 5. Bibliography

Federal Statistical Office. (1888). *Swiss fertility and socioeconomic indicators (1888) data*. [Dataset]. Retrieved from <a href="https://stat.ethz.ch/R-manual/R-devel/library/datasets/html/swiss.html">https://stat.ethz.ch/R-manual/R-devel/library/datasets/html/swiss.html</a>

Bachfischer, M. (2021, March 16). Analysis of Swiss dataset in R. Matthias Bachfischer Blog. Retrieved from <a href="https://bachfischer.me/posts/2021/03/analysis">https://bachfischer.me/posts/2021/03/analysis</a> of swiss dataset in r

Dhar, P. (2017). *Comparative study of clustering techniques on Swiss dataset*. International Journal of Statistical Sciences. Retrieved from <a href="https://www.ripublication.com/ijss17/ijssv12n1">https://www.ripublication.com/ijss17/ijssv12n1</a> 07.pdf

# 6. Appendix

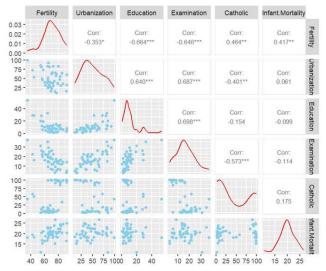


Figure 4: Data exploration

# Dataset Structure and Variable Overview

# **Histograms:**

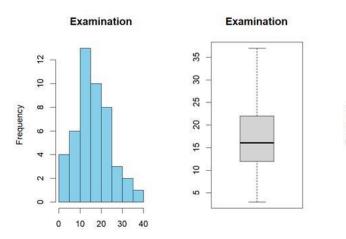


Figure 5: Histogram and boxplot: Examination

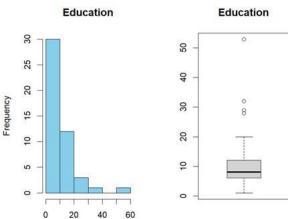
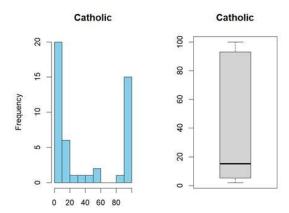


Figure 6: Histogram and boxplot: Education



 $Figure\ 7: Histogram\ and\ boxplot: Catholic$ 

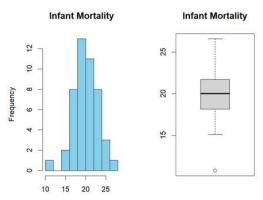


Figure 8: Histogram and boxplot: Infant Mortality

#### SIMPLE REGRESSION OUTPUT

#### Fertility – Urbanization

lm(formula = Fertility ~ Urbanization, data = swiss)

#### Residuals:

Min 1Q Median 3Q Max -25.5374 -7.8685 -0.6362 9.0464 24.4858

#### Coefficients:

Estimate Std. Error t value Pr(>|t|) (Intercept) 79.72455 4.15895 19.169 <2e-16 \*\*\* Urbanization -0.19420 0.07671 -2.532 0.0149 \*

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 11.82 on 45 degrees of freedom Multiple R-squared: 0.1247, Adjusted R-squared: 0.1052 F-statistic: 6.409 on 1 and 45 DF, p-value: 0.01492

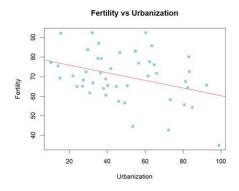


Figure 9 : Scatterplot: Fertility vs Urbanization

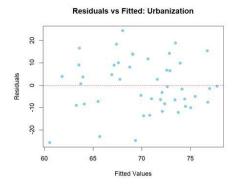


Figure 10: Residuals vs Fitted: Urbanization

# Fertility - Education

```
Call:
lm(formula = Fertility ~ Education, data = swiss)
Residuals:
    Min
             1Q Median
                             3Q
                                    Max
-17.036 -6.711 -1.011
                          9.526
                                19.689
Coefficients:
           Estimate Std. Error t value Pr(>|t|)
                                37.836 < 2e-16 ***
(Intercept)
            79.6101
                         2.1041
Education
             -0.8624
                         0.1448 -5.954 3.66e-07 ***
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 9.446 on 45 degrees of freedom
Multiple R-squared: 0.4406,
                              Adjusted R-squared: 0.4282
F-statistic: 35.45 on 1 and 45 DF, p-value: 3.659e-07
```

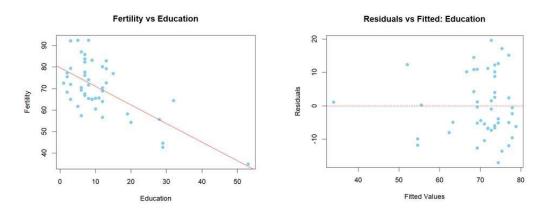


Figure 11 : Scatterplot Fertility vs Education

Figure 12: Residuals vs Fitted: Education

# Fertility - Examination

#### Call: lm(formula = Fertility ~ Examination, data = swiss) Residuals: Median Min **1Q** 3Q Max -25.9375 -6.0044 -0.3393 7.9239 19.7399 Coefficients: Estimate Std. Error t value Pr(>|t|)(Intercept) 86.8185 3.2576 26.651 < 2e-16 \*\*\* Examination -1.0113 0.1782 -5.675 9.45e-07 \*\*\*

--- Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 9.642 on 45 degrees of freedom Multiple R-squared: 0.4172, Adjusted R-squared: 0.4042 F-statistic: 32.21 on 1 and 45 DF, p-value: 9.45e-07

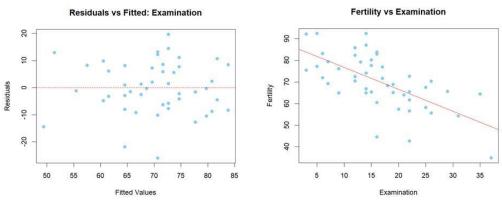


Figure 13: Residuals vs Fitted: Examination

Figure 14: Scatterplot Fertility vs Examination

# Fertility - Catholic

```
Call:
lm(formula = Fertility ~ Catholic, data = swiss)
Residuals:
                 Median
    Min
             10
                             3Q
                                     Max
-35.309 -4.060
                  0.511
                          6.851
                                 16.682
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
                        2.30510 27.950 < 2e-16 ***
(Intercept) 64.42826
Catholic
                                  3.511 0.00103 **
             0.13889
                        0.03956
```

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.05 '.' 0.1 ' ' 1

Multiple R-squared: 0.215, Adjusted R-squared: 0.1976

Residual standard error: 11.19 on 45 degrees of freedom

F-statistic: 12.33 on 1 and 45 DF, p-value: 0.001029

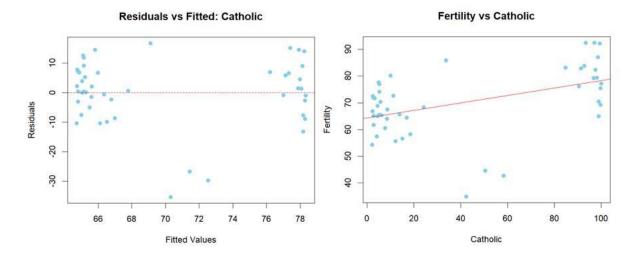


Figure 15 3: Residuals vs Fitted: Catholic

Figure 161 : Scatterplot Fertility vs Catholic

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# Fertility – Infant Mortality

#### Call:

lm(formula = Fertility ~ Infant.Mortality, data = swiss)

#### Residuals:

Min 1Q Median 3Q Max -31.672 -5.687 -0.381 7.239 28.565

#### Coefficients:

Estimate Std. Error t value Pr(>|t|)
(Intercept) 34.5155 11.7113 2.947 0.00507 \*\*
Infant.Mortality 1.7865 0.5812 3.074 0.00359 \*\*

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 11.48 on 45 degrees of freedom Multiple R-squared: 0.1735, Adjusted R-squared: 0.1552

F-statistic: 9.448 on 1 and 45 DF, p-value: 0.003585

#### **Fertility vs Infant Mortality**

# 

Figure 175: Scatterplot Fertility vs Mortality

#### Residuals vs Fitted: Infant Mortality

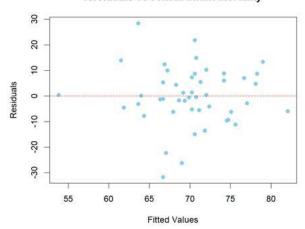


Figure 18: Residuals vs Fitted: Mortality

#### MULTIPLE REGRESSION OUTPUT

#### # Model 1: Urbanization + Education

```
lm(formula = Fertility ~ Urbanization + Education, data = swiss)
Residuals:
    Min
              10
                   Median
                                3Q
                                       Max
-17.3072 -6.6157 -0.9443
                            8.7028 20.5291
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 77.43255
                        3.36644 23.001 < 2e-16 ***
Urbanization 0.06648
                        0.08005
                                  0.830
                                          0.411
            -0.96276
                        0.18906 -5.092 7.1e-06 ***
Education
Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' '1
Residual standard error: 9.479 on 44 degrees of freedom
Multiple R-squared: 0.4492,
                              Adjusted R-squared: 0.4242
F-statistic: 17.95 on 2 and 44 DF, p-value: 2e-06
```

#### Residuals vs Fitted: Urb + Edu

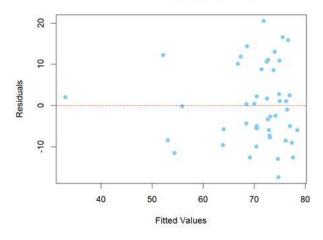


Figure 19: Residuals vs Fitted: Urb + Edu

# # Model 2: Urbanization + Education + Examination

```
lm(formula = Fertility ~ Urbanization + Education + Examination,
    data = swiss)
Residuals:
    Min
            1Q Median
                            3Q
                                   Max
-14.967
        -4.978 -1.045
                         4.906 21.358
Coefficients:
             Estimate Std. Error t value Pr(>|t|)
                        3.33836 24.498 < 2e-16 ***
(Intercept) 81.78413
                                  2.232 0.03084 *
Urbanization 0.18017
                        0.08071
                                 -3.472 0.00119 **
Education
            -0.67242
                        0.19366
Examination -0.79744
                        0.24679 -3.231 0.00237 **
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 8.601 on 43 degrees of freedom
                               Adjusted R-squared: 0.5259
Multiple R-squared: 0.5568,
F-statistic: 18.01 on 3 and 43 DF, p-value: 1.017e-07
```

#### Residuals vs Fitted: + Examination

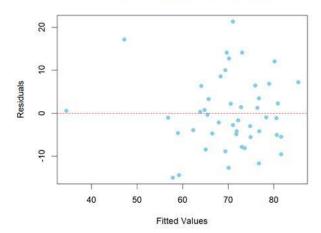


Figure 20: Residuals vs Fitted: + Exa

```
lm(formula = Fertility ~ Urbanization + Education + Examination +
    Catholic, data = swiss)
Residuals:
              1Q
                   Median
                                3Q
-15.7813
         -6.3308
                   0.8113
                            5.7205
                                    15.5569
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
                        4.86832 14.171 < 2e-16 ***
(Intercept)
            68.99087
Urbanization 0.22065
                        0.07360
                                 2.998 0.00455 **
Education
            -0.96161
                        0.19455
                                 -4.943 1.28e-05 ***
                        0.27411
Examination -0.26058
                                 -0.951 0.34722
Catholic
             0.12442
                        0.03727
                                  3.339 0.00177 **
Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
Residual standard error: 7.736 on 42 degrees of freedom
Multiple R-squared: 0.6498, Adjusted R-squared: 0.6164
```

#### Residuals vs Fitted: + Catholic

F-statistic: 19.48 on 4 and 42 DF, p-value: 3.95e-09

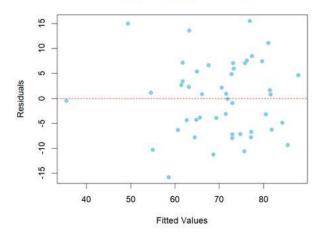


Figure 21: Residuals vs Fitted: + Catholic

# # Model 4: Full Model (Urbanization + Education + Examination + Catholic + Infant Mortality)

# Residuals:

```
Min 1Q Median 3Q Max
-15.2743 -5.2617 0.5032 4.1198 15.3213
```

#### Coefficients:

	Estimate	Std. Error	t value	Pr(> t )	
(Intercept)	49.70378	8.18878	6.070	3.44e-07	***
Urbanization	0.17211	0.07030	2.448	0.01873	*
Education	-0.87094	0.18303	-4.758	2.43e-05	***
Examination	-0.25801	0.25388	-1.016	0.31546	
Catholic	0.10412	0.03526	2.953	0.00519	**
Infant.Mortality	1.07705	0.38172	2.822	0.00734	**

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 7.165 on 41 degrees of freedom Multiple R-squared: 0.7067, Adjusted R-squared: 0.671 F-statistic: 19.76 on 5 and 41 DF, p-value: 5.594e-10

#### Residuals vs Fitted: Full Model

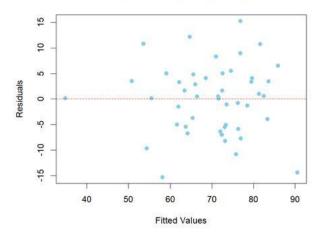


Figure 22: Residuals vs Fitted: + Full Model